"INTEGRATED SYSTEM OPTIMISATION AND PARAMETER ESTIMATION"

by

ARDAWAN LALUI

Thesis submitted for the
award of the degree of
Doctor of Philosophy

THE CITY UNIVERSITY
LONDON EC1V OHB

DEPARTMENT OF SYSTEMS SCIENCE

February, 1980

CONTENTS

Page

Page

PART 2 DYNAMIC OPTIMISATION

TABLES

FIGURES

Chapter 2:

Chapter 3:

Chapter 4:

Chapter 5:

Chapter 6:

Appendices:

ACKNOWLEGEMENTS

The author wishes to extend his thanks and gratitude to his thesis supervisor Dr. P.D. Roberts for his valuable advice, guidance and interest throughout.

He is also indebted to [redacted] for her excellent typing and spelling.

DECLARATION

The author grants discretionary powers to The City University Librarian to copy the thesis, in whole or in part, without further reference to the author

ABSTRACT

This thesis is concerned with the on-line determination of the optimum operating condition of some systems where the optimum solutions are computed using their simplified mathematical models whose parameters are periodically updated by comparing models and real processes outputs. In each case two problems have to be considered. Firstly, a parameter estimation problem to determine model parameters and, secondly, a system optimisation problem where the model is employed to determine the optimum operating condition to optimise a given performance criterion. In general, the mathematical model will not completely represent the structure of the real system and/or it is deliberately simplified in order to facilitate the computation of the optimal input condition and involve the estimation of fewer parameters. Hence, the two problems interact and several iterations between parameter estimation and system optimisation will be required before the final correct optimum condition is achieved. By treating the two problems of system optimisation and parameter estimation as two interacting subproblems within an integrated scheme and then applying hierarchical system theories of decomposition and subsystem coordination, several alternative techniques which take account of the interaction between the subproblems have been developed. Particular attention is given to a procedure which may be regarded as a modification of the two-step approach, commonly employed in industrial practice, in which the optimisation problem and the parameter estimation problem are treated separately and solved repeatedly until a final converged solution is obtained. The modified technique results in extra terms added to the optimisation objective function to ensure that the correct optimal operating condition is finally achieved on the real process in spite of model inaccuracies.

The technique is shown to perform satisfactorily in simulation studies of the following cases:

(i) Simulation of a chemical plant process where feed flow rate, reaction temperature and recycle ratio controller set points are adjusted to maximise the net rate of return. It has been assumed that the plant achieves its new steady state condition in between each change of set points, so enabling a steady state model to be used for the simulation investigation. In addition, the simulation is employed to investigate the effects of real process measurement errors.

(ii) Development of the above investigation for consideration of how the dynamics of the plant affect the performance of the algorithm.

(iii) Extension of the algorithm to an investigation of using a dynamic mathematical model in order to design a feedback control system where the gains are adjusted to optimise a given performance index.

(iv) Application of the design technique to the optimal feedback control of a synchronous generator system.

CHAPTER 1: INTRODUCTION

Due to increasing shortage of natural resources and energy, and rise in prices of raw materials, it is necessary to operate industrial processes as efficiently as possible. In order to achieve and maintain the most efficient operating condition, on-line digital computer control optimisation can be performed. Employment of such a technique requires a mathematical model whose parameters are estimated by comparing model and corresponding real process outputs. Hence, the parameter estimation problem can be defined as evaluation and adjustment of the unknown parameters within the mathematical model in order that the response of the model is in agreement, as closely as possible, with the response of the system. In other words, the parameter estimation problem is an attempt to determine the unknown parameters in a mathematical model which represents a physical system.

On the other hand, the problem of optimisation is of concern when input conditions to the system are to be determined in order to satisfy a defined performance criterion. The optimisation, or optimal control, is determined from the mathematical model and the results obtained are used for manipulating input feedback controller set points in order to achieve and maintain the process at its optimum steady state operating condition.

Many practical systems require a sophisticated mathematical model. It is well-known that present-day optimal control simulation may require a prohibitive amount of computation for such complex models. It is often advantageous to employ a simplified mathematical model containing fewer parameters to be estimated and which facilitates the computation of the optimal input conditions.

Because of the above deliberate simplification and also due to uncertain structure of the mathematical model, it will often not represent the real system faithfully. Hence, optimal controller set points derived from the model may no longer be optimal as far as the real system is concerned. In this situation the parameters whose values are to be estimated may be dependent on controller set points. These properties cause interaction between the two problems of optimisation and parameter estimation.

In attempting to overcome the interaction between the optimisation and parameter estimation problems, integrated schemes have been developed.

Haimes and Wismer (1972) and Roberts (1977a, b) have described some different approaches to the joint problem, namely the two-step approach, the epsilon constraint approach and the parametric approach.

The two-step approach is the usual technique for tackling the combined problem. In this case the parameter estimation problem and optimisation problem are each treated separately and solved recursively until, hopefully, convergence is obtained to the correct optimum operating condition on the real system. The ε-constraint approach replaces the parameter estimation problem by an inequality constraint related to the accuracy of estimation. The parametric approach formulates the combined problem using a bicriterion function which is then replaced by a parametric formulation.

It is claimed by Haimes (1968) and apparently shown by Olagundoye (1971) that the parametric and ε-constraint approaches are superior to the two-step approach when the model is not a faithful representation of the real system.

In general, the above techniques have failed to take account of the gradient condition, given by Durbeck (1965), and they have only considered model parameters as interconnection constraints. Durbeck (1965) has analysed the two-step approach and has shown that the correct optimal condition at the final converged point will not be obtained unless the derivatives of real process outputs with respect to the manipulable inputs are matched with corresponding model derivatives. Foord (1974) has extended the work of Durbeck (1965) by describing how the model structure may be chosen to make sure that the above gradient condition is met.

Following from the work of Haimes and Wismer (1972), Roberts (1978) has applied hierarchical systems theory to decouple the interaction between the optimisation and parameter estimation problems. This will achieve the required output gradients condition without the use of the appropriate model structure. This technique can be considered as a modified two-step approach which considers model inputs as well as parameters as interconnection constraints. In this approach extra terms are introduced to the optimisation performance index which will overcome the discrepancy existing between model and real process output derivatives. Roberts (1979b) has successfully applied the modified two-step algorithm in a digital simulation investigation of a chemical reactor by using its simplified mathematical model.

It is often difficult to solve an integrated scheme for the entire problem. Hence, it may be desirable to decompose the overall system into a number of smaller subproblems and solve them individually, one at a time. These subproblems are not independent of each other and interactions between subproblems are taken into account by a coordination unit at a higher level where the overall system objectives are specified and achieved. Haimes (1971, 1972) concedes that these further decompositions at intermediate layers may not be worthwhile in small-scale problems.

Roberts (1977a) has given some aspects of multi-level optimisation techniques applied to static and dynamic optimal control, and applied these techniques (Roberts (1977b)) to the ε-constraint and parametric approaches.

Roberts (1979a), in his recent paper, has divided the overall control scheme of a hierarchical structure into, typically, three layers. The lowest layer contains local feedback control loops which maintain the stability of the system. The intermediate layer will provide the set points of the local feedback controllers. The highest layer defines the constraints on the intermediate layer and specifies management decisions.

In this thesis, initially, the performance of the above integrated schemes has been compared with one another and that of the standard two-step approach, as applied in a digital simulation investigation concerned with determining feedflow rate, reaction temperature and recycle ratio controller set points to maximise the net rate of return from a pressurised exothermic chemical plant where a simplified non-linear model is used for system optimisation and parameter estimation. In this investigation, only the modified two-step approach met with success, whereas the other techniques failed to converge, in this particular problem, to the correct optimal solution because they did not take account of the gradient mismatch proposed by Durbeck (1965) and Foord (1974).

The above procedures attempt to determine the optimum steady state operating condition of the plant and ignore dynamic effects. Such steady state optimisation is applicable in the situation where the plant is effectively stabilised by the controllers, and is considered to be valid for slow process disturbances such that the plant operates in a sequence of steady states. The steady state optimisation determines the extremum of a suitable objective function subject to algebraic equality and inequality constraints, and may be required in the operational phase when the task is to determine and maintain the optimum steady state operating condition of the overall system.

In the above steady-state study, the advantages and disadvantages of applying further decomposition using multi-level optimisation techniques have been described. In addition, the effect of measurement noise is investigated.

Only the modified two-step approach will achieve the correct optimum operating condition on the real process in spite of inaccuracies and uncertainties in the structure of the mathematical model. This work has been extended to investigate the effects of real process dynamics on the performance of the modified two-step approach.

Dynamic optimisation is needed when the system states are different from the desired steady state conditions due to dynamic variations in inputs, unwanted disturbances, plant parameter variations, etc. Sometimes it is desirable to change the controller set points in order to bring the states back to their desired levels in an optimal manner. It may also be desirable to move the state of the system optimally from one given steady state condition to a new steady state condition; for instance, during a plant start-up or shut-down. Hence, it is necessary to solve a dynamic optimisation problem of determining the extremum of a defined objective function, which is a functional of the time-dependent state and control signals, subject to the dynamic equations of the model as well as the inequality constraints. Many practical engineering systems require a high number of dynamic equations to represent their mathematical model and consequent requirement of prohibitive amounts of computation using well-established techniques such as the maximum principle and dynamic programming. Hence, for large-scale dynamic optimal control problems there exists considerable motivation for research into alternative techniques, such as a modified two-step approach applied to simplified models in the design of dynamic feedback control systems.

In this thesis the design technique has also been employed to design a simple dynamic feedback control system and has been applied to the optimal feedback control of a synchronous generator system.

It is hoped that the investigations which have been carried out in this thesis will be of importance in the application of practical hierarchical control systems, where a process is under regulatory control at the first level and is supervised at a higher level where the set points of some of the controllers are regulated to optimise a given performance index, using a mathematical model whose structure is uncertain and contains parameters which need to be estimated.

PART 1 STATIC OPTIMISATION

CHAPTER 2: INTEGRATED SCHEMES

This chapter is concerned with determining the optimal operating condition of a system through the use of its steady state mathematical model. This may be specified as solving a static optimisation problem, where the optimisation problem is subject to algebraic equality and inequality constraints. Static optimisation may also be required when the objective is to maintain the optimum steady state operating condition of the system, by manipulating the set points of feedback controllers.

Because of interaction between the problems of optimisation and parameter estimation, some integrated schemes have been considered which attempt to resolve the conflict between these problems. These schemes are based on hierarchical and decomposition control techniques with considerable use of Lagrange theory.

2.1 Steady State Optimisation and Parameter Estimation

The steady state behaviour of a system can be represented by a steady state mathematical model. In general, a steady state mathematical model can be described by a set of algebraic relations, a set of inequality constraints and a set of output relationships given by:

$$\underline{f}(\underline{x}, \underline{u}, \underline{\alpha}) = \underline{0}$$

$$\underline{g}(\underline{x}, \underline{u}, \underline{\alpha}) \leqslant \underline{0}$$

$$\underline{y} = \underline{h}(\underline{x}, \underline{u}, \underline{\alpha})$$

where

$\underline{f}$ = n-vector function of algebraic relationships

$\underline{x}$ = n-vector of model variables

$\underline{\alpha}$ = p-vector of estimated model parameters

$\underline{u}$ = m-vector of manipulated inputs

$\underline{g}$ = r-vector function of algebraic relationships

$\underline{y}$ = q-vector of measurable outputs

$\underline{h}$ = q-vector function

Steady state (static) optimisation can be defined as determining the manipulable inputs, $\underline{u}$, in order to minimise (maximise) a given performance index, $p(\underline{x}, \underline{u}, \underline{\alpha})$, and satisfy the given constraints. Hence, the steady state optimisation can be described as:

$$\begin{aligned} &\min_{\underline{u}} p(\underline{x}, \underline{u}, \underline{\alpha}) \\ \text{s.t. } &\underline{f}(\underline{x}, \underline{u}, \underline{\alpha}) = \underline{0} \\ &\underline{g}(\underline{x}, \underline{u}, \underline{\alpha}) \leq \underline{0} \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.1)$$

The parameter estimation problem can be defined as estimating values of the unknown parameters of the mathematical model in order to achieve the response of the model, as closely as possible, in agreement with that of the real system. This can be achieved by exciting the model and the real system in parallel, applying known inputs $\underline{u}^j$ to both the model and the real system, then, for example, using the weighted least square criterion method over J observations of the real system outputs, $\hat{\underline{y}}^j$. Hence, the unknown parameters, $\underline{\alpha}$, can be determined by minimising a weighted least square criterion, $G(\underline{x}, \underline{u}, \underline{\alpha})$, described as

$$G(\underline{x}, \underline{u}, \underline{\alpha}) = \sum_{j=1}^{J} [\underline{y}^j - \hat{\underline{y}}^j]' W^j [\underline{y}^j - \hat{\underline{y}}^j] \quad \ldots\ldots\ldots\ldots \quad (2.2)$$

with J observations of the real system outputs denoted by

$$\hat{y}_j \ , \quad j = 1, 2, \ldots , J$$

and where W^j is a q x q positive definite weighting matrix.

The parameter estimation problem may then be formulated as:

$$\left.\begin{aligned} &\min_{\underline{\alpha}} \ G(\underline{x}, \underline{u}, \underline{\alpha}) \\ \text{s.t.} \quad &\underline{f}(\underline{x}^j, \hat{\underline{u}}^j, \underline{\alpha}) = \underline{0} \\ &\underline{g}(\underline{x}^j, \hat{\underline{u}}^j, \underline{\alpha}) \leq \underline{0} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.3)$$

It is important to note that the structure of the mathematical model is often uncertain and deliberately simplified in order to facilitate the solution of the optimisation problem. In general, the result of the parameter estimation problem may be dependent on the control inputs, and also the results of the optimisation will be critically dependent upon the validity of the model. These imply that the optimisation and parameter estimation problems interact with each other, the degree of interaction depending upon the accuracy of the structure of the model.

Despite this interaction, the problems of optimisation and parameter estimation are generally treated separately in the literature. This chapter attempts to describe some iterative computational approaches for solving the joint problem. Hence, the combined problem can be described as:

$$\left.\begin{array}{ll} \min\limits_{\underline{u},\underline{\alpha}} \ \{p(\underline{x}, \underline{u}, \underline{\alpha}), G(\underline{x}, \underline{u}, \underline{\alpha})\} & \\ \\ \text{s.t. } \underline{f}(\underline{x}, \underline{u}, \underline{\alpha}) = \underline{0} & \text{optimisation} \\ \quad\ \ \underline{g}(\underline{x}, \underline{u}, \underline{\alpha}) \leq \underline{0} & \text{problem} \\ \\ \quad\ \ \underline{f}(\underline{x}^j, \underline{\hat{u}}^j, \underline{\alpha}) = \underline{0} & \text{parameter estimation} \\ & \text{problem} \\ \quad\ \ \underline{g}(\underline{x}^j, \underline{\hat{u}}^j, \underline{\alpha}) \leq \underline{0} & j = 1, 2, \ldots J \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (2.4)$$

Some integrated schemes which attempt to solve the combined problem, given by equation (2.4), are described in the following sections. These integrated schemes are the two-step approach, the parametric approach, the ε-constraint approach and the modified two-step approach. These approaches, which have been briefly mentioned in chapter 1, are based on hierarchical (multilevel) control theory. Some basic aspects of multilevel optimisation and decomposition will also be discussed.

2.2 Multilevel Optimisation and Decomposition of a Large-Scale System

In a large complex system, it is often complicated, or sometimes impossible, to design a single optimisation technique in order to optimise the performance of the overall system. Hence, it is required to decompose the system into a number of smaller subsystems, each with its own objectives and constraints (i.e. equality and inequality constraints). One of the most common forms which the resulting interconnection of subsystems may take is the hierarchical or multilevel form. In this case each level coordinates the units on the level below it and in turn is coordinated by the units on the level above it. These subsystems can then be solved independently. The solution to these subsystems should be in such a way that, firstly, the interaction between the subsystems is resolved and, secondly, the overall process objectives are attained.

The objective of this section is to describe some different approaches which decompose the overall system into a number of subsystems and solve them independently to give the overall system optimum.

2.2.1 Coupled systems

Consider a system consisting of N coupled subsystems. Each steady state subsystem can be described as:

$$\underline{f}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) = \underline{0}$$

$$\underline{g}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) \leq \underline{0}$$

$$\underline{y}_i = \underline{h}_i(\underline{x}_i, \underline{y}_i)$$

$$\underline{z}_i = \sum_{j=1}^{N} [c_{ij}]\underline{y}_j$$

where

$\underline{x}_i = n_i$ - vector of state variable in subsystem i

$\underline{u}_i = m_i$ - vector of manipulable inputs to subsystem i

$\underline{z}_i = p_i$ - vector of input to subsystem i, from the jth subsystems (j = 1, 2, ... N), which are acting as interconnection variables

$\underline{f}_i = n_i$ - vector function of algebraic relationships

$\underline{g}_i = r_i$ - vector function of algebraic relationships

$\underline{y}_i = q_i$ - vector of outputs from subsystem i

$\underline{h}_i = q_i$ - vector function

$[c_{ij}]$ = constant matrix with 0 or 1 elements

The optimal control problem for each subsystem may be defined as:

$$\left.\begin{aligned} &\min_{\underline{x}_i, \underline{u}_i, \underline{z}_i, \underline{y}_i} \; F_i(\underline{x}_i, \underline{u}_i, \underline{z}_i, \underline{y}_i) \\ &\text{s.t.} \quad \underline{f}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) = \underline{0} \\ &\qquad \underline{g}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) \leq \underline{0} \\ &\qquad \underline{y}_i = \underline{h}_i(\underline{x}_i, \underline{u}_i) \\ &\qquad \underline{z}_i = \sum_{j=1}^{N} [c_{ij}]\underline{y}_i \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.5)$$

The overall objective function of the system $F(\underline{x}, \underline{u}, \underline{z}, \underline{y})$ is assumed to be the sum of the individual subsystems objective functions. Hence, the overall objective function can be described as:

$$F(\underline{x}, \underline{u}, \underline{z}, \underline{y}) = \sum_{i=1}^{N} F_i(\underline{x}_i, \underline{u}_i, \underline{z}_i, \underline{y}_i) \quad \ldots\ldots\ldots\ldots \quad (2.6)$$

and the integrated problem is obtained by minimising the overall objective function given by equation (2.6).

$$\left.\begin{aligned} &\min_{\underline{x},\underline{u},\underline{z},\underline{y}} \quad F(\underline{x}, \underline{u}, \underline{z}, \underline{y}) \\ &\text{s.t.} \quad \underline{f}(\underline{x}, \underline{u}, \underline{z}) = \underline{0} \\ &\qquad \underline{g}(\underline{x}, \underline{u}, \underline{z}) \leq \underline{0} \\ &\qquad \underline{y} = \underline{h}(\underline{x}, \underline{u}) \\ &\qquad \underline{z} = [c]\underline{y} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.7)$$

The integrated steady state optimal control problem defined by equation (2.7) is to be solved, employing multilevel optimisation and decomposition techniques. Emphasis is placed on model-coordination and goal-coordination.

2.2.2 The model coordination method

In this method the optimisation problem is converted into a two-level structure consisting of the supremal and the infimal levels, where the latter is divided into N independent infimal units. This is achieved by fixing the interconnection variables $\underline{z}_i$ and $\underline{y}_i$ in the infimal unit i. Assuming that by fixing $\underline{z}_i$, all the state variables $\underline{x}_i$ will not be fixed, and also considering $\underline{z}_i$ only because $\underline{y}_i = [c]^{-1}\underline{z}_i$, the ith sub-problem at the infimal level becomes

$$\left.\begin{aligned} &p_i(\underline{z}) = \min_{\underline{x}_i,\underline{u}_i} \quad F_i(\underline{x}_i, \underline{u}_i, \underline{z}) \\ &\text{s.t.} \qquad \underline{f}_i(\underline{x}_i, \underline{u}_i, \underline{z}) = \underline{0} \\ &\qquad\qquad \underline{g}_i(\underline{x}_i, \underline{u}_i, \underline{z}) \leq \underline{0} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (2.8)$$

Equation (2.8) represents the resulting equations for the model coordination.

The task of the supremal level is to supply the variables $\underline{z}$ to the infimal level by solving the following problem:

$$\left.\begin{aligned} &\min_{\underline{z}} \ \underline{p}(\underline{z}) \\ &\text{s.t. } \underline{f}_i(\underline{x}_i, \underline{u}_i, \underline{z}) = \underline{0} \\ &\quad\ \ \underline{g}_i(\underline{x}_i, \underline{u}_i, \underline{z}) \leq \underline{0} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ (2.9)$$

where

$$\underline{p}(\underline{z}) = \sum_{i=1}^{N} p_i(\underline{z})$$

The multilevel scheme using the model-coordination method is illustrated in Figure 2.1.

2.2.3 The goal coordination method

In this case the interaction between the sub-problems is removed by "cutting" all links between the subsystems. The independent sub-problems are then solved at the infimal level without satisfying the interconnection constraints, which are considered as additional equality constraints, until the final solution is obtained.

The overall objective function and the additional equality constraints can be combined using Lagrange multipliers. Then the Lagrangian becomes

$$L(\underline{x},\underline{u},\underline{z},\underline{y},\underline{\lambda}) = \sum_{i=1}^{N} \{F_i(\underline{x}_i,\underline{u}_i,\underline{z}_i,\underline{y}_i) + \underline{\lambda}_i' (\underline{z}_i - \sum_{j=1}^{N} [c_{ij}]\underline{y}_i)\} \ (2.10)$$

It is possible to decompose the Lagrangian $L(\underline{x}, \underline{u}, \underline{z}, \underline{y}, \underline{\lambda})$ defined by equation (2.10) into N independent sub-Lagrangians by grouping all terms involving $\underline{x}_i$, $\underline{y}_i$, $\underline{z}_i$ and $\underline{y}_i$. Hence, the i^{th} modified infimal unit problem becomes:

$$\min_{\underline{x}_i,\underline{u}_i,\underline{z}_i,\underline{y}_i} \left\{ L_i = \left[F_i(\underline{x}_i,\underline{u}_i,\underline{z}_i,\underline{y}_i) + \underline{\lambda}_i' \underline{z}_i - \sum_{j=1}^{N} \underline{\lambda}_j' [c_{ji}]\underline{y}_i\right]\right\} \ldots\ (2.11)$$

$$\begin{aligned} \text{s.t. } & f_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) = \underline{0} \\ & \underline{g}_i(\underline{x}_i, \underline{u}_i, \underline{z}_i) \leq \underline{0} \\ & \underline{y}_i = \underline{h}_i(\underline{x}_i, \underline{u}_i) \end{aligned}$$

The task of the supremal unit is to choose Lagrange variables $\underline{\lambda}$ in order to force the interaction balance principle to be satisfied. Interaction balance is achieved when

$$\underline{z} = [c]\underline{y}$$

The multilevel scheme for N interacting subsystems is illustrated in Figure 2.2.

If there is a possibility of eliminating the output variables $\underline{y}$ and interconnection variables $\underline{z}$ in the overall optimisation problem, then the integrated problem can be described by the equations:

$$\left.\begin{array}{l} \min\limits_{\underline{x},\, \underline{u}} \; F(\underline{x}, \underline{u}) \\ \\ \text{s.t. } \underline{f}(\underline{x}, \underline{u}) = \underline{0} \\ \qquad \underline{g}(\underline{x}, \underline{u}) \leq \underline{0} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.12)$$

Formation of the Lagrangian will give

$$L(\underline{x}, \underline{u}, \underline{\lambda}, \underline{\mu}) = F(\underline{x}, \underline{u}) + \underline{\lambda}' \, \underline{f}(\underline{x}, \underline{u}) + \underline{\mu}' \, \underline{g}(\underline{x}, \underline{u}) \; \ldots \quad (2.13)$$

where $\underline{\lambda}$ and $\underline{\mu}$ are Lagrangian multipliers.

The dual function $D(\underline{\lambda}, \underline{\mu})$ can be obtained by minimising the Lagrangian given by equation (2.13)

$$D(\underline{\lambda}, \underline{\mu}) = \min_{\underline{x},\underline{u}} L(\underline{x}, \underline{u}, \underline{\lambda}, \underline{\mu}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.14)$$

The dual problem can be defined as maximising the dual function in order to determine the Lagrange multipliers $\underline{\lambda}$ and $\underline{\mu}$. Hence, the integrated problem can be described as

$$\min_{\underline{x},\underline{u}} F(\underline{x}, \underline{u}) = \max_{\underline{\lambda},\underline{\mu}} \{\min_{\underline{x},\underline{u}} [L(\underline{x}, \underline{u}, \underline{\lambda}, \underline{\mu})]\} \quad \ldots\ldots\ldots\ldots \quad (2.15)$$

The above formulation is referred to as the direct dual formulation and is only valid if there is no duality gap, i.e.

$$\min_{\underline{x},\underline{u}} f(\underline{x}, \underline{u}) - \max_{\underline{\lambda},\underline{\mu}} D(\underline{\lambda}, \underline{\mu}) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.16)$$

The formulation described by equation (2.15) may be decomposed by separating the Lagrangian into N sub-Lagrangians by grouping them as

$$L(\underline{x}, \underline{u}, \underline{\lambda}, \underline{\mu}) = \sum_{i=1}^{N} L_i(\underline{x}_i, \underline{u}_i, \underline{\lambda}, \underline{\mu}) \quad \ldots\ldots\ldots\ldots\ldots \quad (2.17)$$

This can be illustrated as a two-level hierarchical structure, shown in Figure 2.3, where the task of the supremal level is to maximise the Lagrangian in order to supply the Lagrangian multipliers $\underline{\lambda}$ and $\underline{\mu}$ as coordinating variables.

The task of each infimal unit, in turn, is to minimise its sub-Lagrangian $L_i(\underline{x}_i, \underline{u}_i, \underline{\lambda}, \underline{\mu})$, where each sub-Lagrangian can be considered as the sub-system objective function $F_i(\underline{x}_i, \underline{u}_i)$ with its own independent parameters $\underline{x}_i$ and $\underline{u}_i$.

2.2.4 Comment

Some basic aspects of multilevel optimisation which cover goal coordination and model coordination have been briefly represented. It is difficult to suggest or choose a general technique in order to attempt optimising a large-scale system because of the nature of the specific problem. However, it is advantageous to choose a technique with the least number of adjustable variables in both the supremal and the infimal levels.

It is not necessary for the goal coordination approach to satisfy the interconnection constraints until the final solution (i.e. non-feasible coordination). In the model coordination technique, all the variables should be feasible (i.e. feasible coordination). This property of the model coordination method is very useful for on-line optimisation and decomposition, but it would be a disadvantage over the goal coordination if the constraints of the system were over-determined.

It is advantageous to reduce the dimensionality of the sub-system problems in order to make the problems more manageable. This may be achieved by using the sub-system equations to eliminate the need for employing all the variables. For example, it may be possible to eliminate the model variables $\underline{x}$, and output measurements $\underline{y}$ as variables. Hence, the i^{th} sub-problem at the infimal level, in model coordination, will become

$$p_i(\underline{z}) = \min_{\underline{u}_i} F_i(\underline{u}_i, \underline{z})$$

$$\text{s.t.} \qquad \underline{f}_i(\underline{u}_i, \underline{z}) = \underline{0}$$
$$\underline{g}_i(\underline{u}_i, \underline{z}) \leq \underline{0}$$

whereas in the goal coordination the i^{th} problem at the infimal level becomes

$$\min_{\underline{u}_i, \underline{z}_i} L_i(\underline{u}_i, \underline{z}_i)$$

$$\text{s.t. } \underline{f}_i(\underline{u}_i, \underline{z}_i) = \underline{0}$$

$$\underline{g}_i(\underline{u}_i, \underline{z}_i) \leq \underline{0}$$

In this thesis only the goal coordination has been considered further because of the advantages described earlier.

2.2.5 Application of goal coordination to the combined problem of optimisation and parameter estimation

Consider the integrated problem described by equation (2.4). The interconnection between the two problems can be viewed in a schematic form represented by Figure 2.4.

The two problems interact via the unknown parameters $\underline{\alpha}$, and the manipulable inputs $\underline{u}$. In the goal coordination method interactions are removed by cutting the links between the two problems. This is illustrated in Figure 2.5.

The two infimal units are now solved independently at the infimal level without satisfying the interconnection constraints. The task of the supremal unit is to ensure that the interconnection constraints are satisfied when the final solution is obtained, where the interconnection constraints are described by

$$\left.\begin{array}{l} \underline{\sigma} = \underline{\alpha} \\ \underline{v} = \underline{u} \end{array}\right\} \qquad \text{.......................} \; (2.18)$$

In order to clarify the relations between the parameters which have been used in sections 1 and 2 of this chapter, consider the coupled problem defined by equation (2.5). Let us further consider that there are only two sub-problems. Hence, the two coupled subsystems can be illustrated as that of figure 2.6 when they are subjected to the goal coordination method.

The comparison between Figure 2.5 and Figure 2.6 reveals the following points:

(a) Optimisation problem corresponds to sub-system 1

(b) Parameter estimation problem corresponds to sub-system 2

(c) $\underline{\sigma}$ corresponds to $\underline{z}_1$

(d) $\underline{v}$ corresponds to $\underline{z}_2$

(e) $\underline{u}$ corresponds to $\underline{y}_1$

(f) $\underline{\alpha}$ corresponds to $\underline{y}_2$

Hence, from now on the variables $\underline{\sigma}$, $\underline{v}$, $\underline{u}$ and $\underline{\alpha}$ will be used in the analysis of different techniques which attempt to overcome the interaction between the optimisation and parameter estimation problems. $\underline{\sigma}$ and $\underline{u}$ are the unknown parameters and manipulable inputs, respectively, as applied in the optimisation problem. $\underline{\alpha}$ and $\underline{v}$ are the corresponding values of $\underline{\sigma}$ and $\underline{u}$ as applied to the parameter estimation problem.

2.3 The Two-Step Approach

In this case the combined problem described by equation (2.4) is solved as two separate problems recursively, hoping that convergence to the correct optimum condition will occur. As suggested by the title, there are two steps involved to solve the combined problem, starting from an assumed initial estimate u^1 of the inputs.

Step 1: Obtain steady state measurements $\hat{\underline{y}}^1$ by applying the current controller set points values $\underline{u}^1$ to the real system. Then solve the parameter estimation problem

$$\min_{\underline{\alpha}} G(\underline{x}, \underline{u}, \underline{\alpha})$$

s.t. the model equations and constraints

to estimate $\underline{\alpha}^1$ of the model parameters.

Step 2: Solve the optimisation problem

$$\min_{\underline{u}} p(\underline{x}, \underline{u}, \underline{\alpha})$$

s.t. the model equations and constraints

with parameter estimates $\underline{\alpha}^1$ obtained from step 1.
This would give a new set of controller set point values $\underline{u}^2$.

Both minimisations are subject to the given equality and inequality constraints defined by the model. Steps 1 and 2 are repeated until no further improvement is obtained. It is important to note that, in the parameter estimation problem, the number of parameters should not exceed the number of the measurements.

This technique is the one which is most commonly employed in current practice, but makes no attempt to take into account the interaction between the two problems. Durbeck (1965) has analysed the above approach and has shown that the correct optimal condition at the final converged point will be obtained only if the derivatives of real process outputs with respect to manipulable inputs are matched with corresponding model derivatives at the final point. Moreover, it is claimed by Haimes (1972) that the two-step approach is inferior to the integrated approaches to be described below, when the model is not a faithful representation of the real system. Figure 2.7 shows the procedure in schematic form.

2.4 The Parametric Approach

In this case the bi-criterion function given by equation (2.4) is replaced by the parametric formulation described by Haimes (1972):

$$\left.\begin{aligned} &\min_{\underline{u},\underline{\alpha}} \left[\theta.G(\underline{x}, \underline{u}, \underline{\alpha}) - (1-\theta).p(\underline{x}, \underline{u}, \underline{\alpha})\right] \\ &\text{s.t. the model equations and constraints} \\ &\qquad \text{and } 0 < \theta < 1 \end{aligned}\right\} \quad \ldots\ldots \quad (2.19)$$

The above parametric approach converges to the solution of the integrated problems as θ approaches 1 from below without attaining 1 (Haimes (1972)).

A multilevel structure (Roberts (1975)) could be employed to tackle the parametric formulation. This is shown, as the two-level structure, in Figure 2.8 where the task of the infimal level is to determine $\underline{u}$ and $\underline{\alpha}$ to minimise the parametric formulation given by equation (2.18), for any particular value of θ given by the supremal level. The task of the supremal level is to increase the value of θ towards 1 between each complete solution of the infimal level, until no further improvement is obtained.

At lower values of θ, greater emphasis is given to the optimisation problem. As θ increases, more and more emphasis is placed on the parameter estimation problem. The test inputs $\underline{u}$ are applied to the system in order to obtain the required measurements $\hat{\underline{y}}$. These test inputs are applied to the real system at any instant through the controller set points $\hat{\underline{u}}$.

The difference equation given by

$$\theta_n = (1 - \gamma)\theta_{n-1} + \gamma$$

where $0 < \gamma < 1$

can be used to adjust θ, where θ_n is the value of θ at n^{th} iteration, θ_{n-1} is the previous value of θ.

2.4.1 Decomposition of the parametric approach

The bi-criterion formulation given by equation (2.4) contains the optimisation problem of minimising $p(\underline{x}, \underline{u}, \underline{\alpha})$, and parameter estimation problem of minimising $G(\underline{x}, \underline{u}, \underline{\alpha})$, where both problems are subject to the model equations and constraints. Interaction between the two problems occurs through the model parameters $\underline{\alpha}$ and the controllers set points $\underline{u}$; hence, it may be expedient to decompose the two problems by employing the goal coordination technique. This is done by "cutting" all interconnections between the problems. As described in section 2.2.5, this can be achieved by substituting $\underline{\alpha}$ in the optimisation problem and $\underline{u}$ in the parameter estimation problem by some other parameters $\underline{\sigma}$ and $\underline{v}$, respectively. Then the optimisation problem becomes:

$$\min_{\underline{u}} p(\underline{x}, \underline{u}, \underline{\sigma}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.20)$$

and the parameter estimation problem becomes:

$$\min_{\underline{\alpha}} G(\underline{x}, \underline{v}, \underline{\alpha}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.21)$$

subject to the interconnection constraints

$$\underline{v} = \underline{u} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.22)$$

$$\underline{\sigma} = \underline{\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.23)$$

and also the model equations and constraints.

By combining the above two problems with the use of the parametric formulation, the integrated problem becomes

$$\min_{\underline{u},\underline{\alpha}} \{\theta.G(\underline{x}, \underline{v}, \underline{\alpha}) + (1 - \theta).p(\underline{x}, \underline{u}, \underline{\sigma})\} \quad \ldots\ldots\ldots \quad (2.24)$$

subject to the model equations and constraints, and also

$$0 > \theta > 1.$$

By forming a Lagrangian and applying stationarity conditions we obtain:

$$L = \left[\theta.G(\underline{x},\underline{v},\underline{\alpha}) + (1 - \theta).p(\underline{x},\underline{u},\underline{\sigma})\right] + \underline{\mu}'(\underline{\sigma} - \underline{\alpha}) + \underline{\lambda}'(\underline{v} - \underline{u}) \quad (2.25)$$

where $\underline{\lambda}$ and $\underline{\mu}$ are Lagrangian Multipliers and

$$\frac{\partial L}{\partial \underline{u}} = (1 - \theta)\frac{\partial p(\underline{x}, \underline{u}, \underline{\sigma})}{\partial \underline{u}} - \underline{\lambda} = \underline{0} \quad \rightarrow \quad \underline{u} \; \ldots\ldots\ldots \quad (2.26)$$

$$\frac{\partial L}{\partial \underline{\alpha}} = \theta \cdot \frac{\partial G(\underline{x}, \underline{v}, \underline{\alpha})}{\partial \underline{\alpha}} - \underline{\mu} = \underline{0} \quad \rightarrow \quad \underline{\alpha} \; \ldots\ldots\ldots \quad (2.27)$$

$$\frac{\partial L}{\partial \underline{v}} = \theta \cdot \frac{\partial G(\underline{x}, \underline{v}, \underline{\alpha})}{\partial \underline{v}} + \underline{\lambda} = \underline{0} \quad \rightarrow \quad \underline{\lambda} \; \ldots\ldots\ldots \quad (2.28)$$

$$\frac{\partial L}{\partial \underline{\sigma}} = (1 - \theta)\frac{\partial p(\underline{x}, \underline{u}, \underline{\sigma})}{\partial \underline{\sigma}} + \underline{\mu} = \underline{0} \quad \rightarrow \quad \underline{\mu} \; \ldots\ldots\ldots \quad (2.29)$$

$$\frac{\partial L}{\partial \underline{\lambda}} = \underline{v} - \underline{u} = \underline{0} \quad \rightarrow \quad \underline{v} \; \ldots\ldots\ldots \quad (2.30)$$

$$\frac{\partial L}{\partial \underline{\mu}} = \underline{\sigma} - \underline{\alpha} = \underline{0} \quad \rightarrow \quad \underline{\sigma} \; \ldots\ldots\ldots \quad (2.31)$$

Satisfaction of the equation (2.27) gives a modified parameter estimation problem

$$\min_{\underline{\alpha}} \{\theta\, G(\underline{x}, \underline{v}, \underline{\alpha}) - \underline{\mu}'\underline{\alpha}\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.32)$$

and, similarly, equation (2.26) gives the modified optimisation problem

$$\min_{\underline{u}} \{(1 - \theta)\, p(\underline{x}, \underline{u}, \underline{\sigma}) - \underline{\lambda}'\underline{u}\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.33)$$

Both minimisation problems are subject to model equations and constraints. The modified optimisation and modified parameter estimation problems are solved independently at the first level of a hierarchical structure, with fixed values of $\underline{\lambda}$, $\underline{\mu}$ and θ sent down from the second and third levels. The task of the second level is to coordinate the two sub-problems at the first level to achieve interaction balance to satisfy the equality constraints given by equations (2.22) and (2.23). The task of the supremal level is identical to that of the parametric approach without any decomposition. The multilevel structure of this method is shown in Figure 2.9.

2.5 The ε-Constraint Approach

In this approach the parameter estimation problem is replaced by an inequality constraint, such that the combined problem becomes:

$$\left.\begin{aligned} &\min_{\underline{\alpha},\underline{u}} p(\underline{x}, \underline{u}, \underline{\alpha}) \\ &\text{s.t. } G(\underline{x}, \underline{u}, \underline{\alpha}) \leqslant \varepsilon, \\ &\text{model equations and constraints} \\ &\varepsilon > \delta > 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.34)$$

where ε is the minimum tolerated estimation error; by minimising ε the parameter estimation problem is solved. δ is an arbitrary small positive scalar. By using Lagrange-Duality theory the combined problem may be formulated as:

$$D(\mu, \varepsilon) = \min\{p(\underline{x}, \underline{u}, \underline{\alpha}) + \mu\left[G(\underline{x}, \underline{u}, \underline{\alpha}) - \varepsilon\right]\} \quad \ldots\ldots\ldots \quad (2.35)$$

s.t. model equations and constraints,

and

$$\left.\begin{aligned} &\max_{\mu} D(\mu, \varepsilon) \\ &\text{s.t. } \mu \geqslant 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.36)$$

The gradient for the above problem is explicitly given as

$$\nabla_{\mu} D(\mu, \varepsilon) = G(\underline{x}, \underline{u}, \underline{\alpha}) - \varepsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.37)$$

Figure 2.10 illustrates the ε-constraint approach. The infimal level determines the optimal values for $\underline{u}$ and $\underline{\alpha}$ by solving equation (2.35) with fixed values of μ and ε sent down from the second and third level. The task of the second level is to solve the dual problem (2.36) for fixed value of ε sent down from the supremal level. The supremal level gradually decreases the value of ε between each complete solution of the infimal level until there would be no need for further improvement.

Measurements from the real process $\hat{\underline{y}}$ are obtained by applying the current values of $\underline{u}$ to the real system.

2.5.1 Decomposition of the ε-constraint approach

Consider the combined problem described by equation (2.34). In a similar manner to that employed in decomposing the parametric formulation described in section 2.4.1, equation (2.34) may be re-written as:

$$\left.\begin{aligned} &\min_{\underline{u}} \; p(\underline{x}, \underline{u}, \underline{\sigma}) \\ &\text{s.t. } G(\underline{x}, \underline{v}, \underline{\alpha}) - \varepsilon \leqslant 0 \\ &\underline{v} = \underline{u} \\ &\underline{\sigma} = \underline{\alpha} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.38)$$

also s.t. model equations and constraints.

By forming a Lagrangian we have

$$L = p(\underline{x}, \underline{u}, \underline{\sigma}) + \eta\left[G(\underline{x}, \underline{v}, \underline{\alpha}) - \varepsilon\right] + \underline{\lambda}'(\underline{v} - \underline{u}) + \underline{\mu}'(\underline{\sigma} - \underline{\alpha}) \quad (2.39)$$

where η, $\underline{\lambda}$ and $\underline{\mu}$ are Lagrange multipliers. Optimality conditions are:

$$\frac{\partial L}{\partial \underline{u}} = \frac{\partial p(\underline{x}, \underline{u}, \underline{\alpha})}{\partial \underline{u}} - \underline{\lambda} = \underline{0} \quad \rightarrow \underline{u} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.40)$$

$$\frac{\partial L}{\partial \underline{\alpha}} = \eta \frac{\partial G(\underline{x}, \underline{v}, \underline{\alpha})}{\partial \underline{\alpha}} - \underline{\mu} = \underline{0} \quad \rightarrow \alpha_1, \alpha_2, \ldots, \alpha_{p-1}, \eta \quad (2.41)$$

$$\frac{\partial L}{\partial \eta} = G(\underline{x}, \underline{v}, \underline{\alpha}) - \varepsilon = 0 \quad \rightarrow \alpha_p \quad \ldots\ldots\ldots\ldots\ldots \quad (2.42)$$

$$\frac{\partial L}{\partial \underline{v}} = \eta \frac{\partial G(\underline{x}, \underline{v}, \underline{\alpha})}{\partial \underline{v}} + \underline{\lambda} = \underline{0} \quad \rightarrow \underline{\lambda} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.43)$$

$$\frac{\partial L}{\partial \underline{\sigma}} = \frac{\partial p(\underline{x}, \underline{u}, \underline{\sigma})}{\partial \underline{\sigma}} + \underline{\mu} = \underline{0} \quad \rightarrow \underline{\mu} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.44)$$

$$\frac{\partial L}{\partial \underline{\lambda}} = \underline{v} - \underline{u} = \underline{0} \quad \rightarrow \underline{v} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.45)$$

$$\frac{\partial L}{\partial \underline{\mu}} = \underline{\sigma} - \underline{\alpha} = \underline{0} \qquad \rightarrow \underline{\sigma} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.46)$$

Equation (2.40) is satisfied by solving the modified optimisation problem

$$\min_{\underline{u}} \left[p(\underline{x}, \underline{u}, \underline{\sigma}) - \underline{\lambda}'\underline{u}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.47)$$

s.t. given $\underline{\sigma}$ and $\underline{\lambda}$

The modifiers $\underline{\lambda}$ are determined from equations (2.41) to (2.44) to give

$$\underline{\lambda} = \left[\frac{\partial P/\partial \sigma_p}{\partial G/\partial \alpha_p}\right] \frac{\partial G}{\partial \underline{v}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.48)$$

Equation (2.41) is satisfied by solving the modified parameter estimation problem:

$$\min_{\underline{\alpha}} \left[- \underline{\mu}' \underline{\alpha}\right]$$

s.t. $G = \varepsilon$

given $\underline{\mu}$

and model equations and constraints.

$\underline{\mu}$ can be obtained from equation (2.44)

$$\underline{\mu} = -\frac{\partial p(\underline{x}, \underline{u}, \underline{\sigma})}{\partial \underline{\sigma}}$$

Figure 2.11 shows the resulting hierarchical structure and decomposition of the ε-constraint approach.

2.6 Modified Two-Step Approach

Conditions for the two-step approach to determine the correct optimum, in spite of model approximations, have been produced by Durbeck (1965) and analysed by Foord (1974), who show that it is necessary to ensure that the gradients of the real system outputs with respect to the manipulable inputs should finally be matched with the corresponding gradients of the model. Hence, the two-step approach is modified by introducing an extra term to the optimisation performance index to allow for any mismatch between model and system output derivatives. This can be achieved by taking account of model inputs as well as parameters as interconnection constraints.

The optimisation problem may be defined as

$$\left.\begin{array}{l} \min_{\underline{u}} p(\underline{x}, \underline{u}, \underline{\sigma}) \\ \\ \text{s.t. model equations and constraints} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.50)$$

where $\underline{u}$ and $\underline{\sigma}$ are the model inputs and parameters in the optimisation problem.

Let $\underline{v}$ be the input signals employed to disturb the system and the model in order to produce output measurements $\hat{\underline{y}}(\underline{x}, \underline{v})$ which are compared with corresponding model outputs $\underline{y}(\underline{x}, \underline{v}, \underline{\alpha})$.

If there are the same number of measurements as there are model parameters, the parameter estimation problem becomes:

$$\underline{y}(\underline{x}, \underline{v}, \underline{\alpha}) - \underline{y}(\underline{x}, \underline{v}) = \underline{0} \quad \rightarrow \underline{\alpha} \quad \ldots\ldots\ldots\ldots \quad (2.51)$$

The two problems of parameter estimation and optimisation interact through

$$\left.\begin{array}{l} \underline{v} = \underline{u} \\ \underline{\sigma} = \underline{\alpha} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.52)$$

By forming a Lagrangian we obtain

$$L = p(\underline{x},\underline{u},\underline{\sigma}) + \underline{\eta} \left[\underline{y}(\underline{x},\underline{v},\underline{\alpha}) - \hat{y} (\underline{x},\underline{v})\right] + \underline{\lambda}'(\underline{v} - \underline{u}) + \underline{\mu}'(\underline{\sigma} - \underline{\alpha}) \quad \ldots\ldots\ldots\ldots\ldots \quad (2.53)$$

where $\underline{\eta}$, $\underline{\lambda}'$ and $\underline{\mu}$ are Lagrangian multipliers. The optimality conditions may then be written as:

$$\frac{\partial L}{\partial \underline{u}} = \frac{\partial p(\underline{x}, \underline{u}, \underline{\sigma})}{\partial \underline{u}} - \underline{\lambda} = \underline{0} \quad \rightarrow \underline{u} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.54)$$

$$\frac{\partial L}{\partial \underline{\alpha}} = \frac{\partial \underline{y}(\underline{x}, \underline{v}, \underline{\alpha})}{\partial \underline{\alpha}} \underline{\eta} - \underline{\mu} = \underline{0} \quad \rightarrow \underline{\eta} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.55)$$

$$\frac{\partial L}{\partial \underline{v}} = \left\{ \left[\frac{\partial y(\underline{x}, \underline{v}, \underline{\alpha})}{\partial \underline{v}} \right]' - \left[\frac{\partial \hat{y}(\underline{x}, \underline{v})}{\partial \underline{v}} \right]' \right\} \underline{\eta} + \underline{\lambda} = \underline{0} \rightarrow \underline{\lambda} \quad \text{.. (2.56)}$$

$$\frac{\partial L}{\partial \underline{\sigma}} = \frac{\partial p(\underline{x}, \underline{u}, \underline{\sigma})}{\partial \underline{\sigma}} + \underline{\mu} = \underline{0} \qquad \rightarrow \underline{\mu} \quad \text{.............. (2.57)}$$

$$\frac{\partial L}{\partial \underline{\eta}} = \underline{y}(\underline{x}, \underline{v}, \underline{\alpha}) - \hat{\underline{y}}(\underline{x}, \underline{v}) = \underline{0} \qquad \rightarrow \underline{\alpha} \quad \text{.............. (2.58)}$$

$$\frac{\partial L}{\partial \underline{\lambda}} = \underline{v} - \underline{u} = \underline{0} \qquad \rightarrow \underline{v} \quad \text{.............. (2.59)}$$

$$\frac{\partial L}{\partial \underline{\mu}} = \underline{\sigma} - \underline{\alpha} = \underline{0} \qquad \rightarrow \underline{\sigma} \quad \text{.............. (2.60)}$$

Equation (2.54) is satisfied by solving the modified optimisation problem

$$\left.\begin{array}{l} \min\limits_{\underline{u}} \; \{p(\underline{x}, \underline{u}, \underline{\sigma}) - \underline{\lambda}' \, \underline{u}\} \\ \quad \text{s.t. given } \underline{\sigma} \text{ and } \underline{\lambda} \\ \text{and the model equations and constraints} \end{array}\right\} \quad \text{.............. (2.61)}$$

Equation (2.58) defines the parameter estimation problem, which is unchanged. From equations (2.55) - (2.57), $\underline{\lambda}$ is determined to give

$$\underline{\lambda} = \left\{ \left[\frac{\partial \underline{y}}{\partial \underline{v}} \right]' - \left[\frac{\partial \hat{\underline{y}}}{\partial \underline{v}} \right]' \right\} \left\{ \left[\frac{\partial \underline{y}}{\partial \underline{\alpha}} \right]'^{-1} \right\} \frac{\partial p}{\partial \underline{\sigma}} \quad \text{.............. (2.62)}$$

The Lagrange multipliers $\underline{\lambda}$ can be computed from equation (2.62) by perturbation on the model and system. The modified two-step approach can be viewed as the two-level structure shown in Figure 2.12, where the task of the second level is to coordinate the two problems on level 1 by iterating on the coordination variables $\underline{\lambda}$ in order to achieve interaction balance to satisfy the equality constraints, defined by equations (2.22) and (2.23). The modified optimisation problem and the parameter estimation problem are solved independently at the first level of the hierarchical structure for fixed values of the parameters $\underline{\lambda}$ sentdown from level 2. Only $\underline{v}$ is applied to the real system and measurements are taken at the coordination level.

It should be noted that if $\underline{\lambda}$ is set to zero, the scheme is identical to the standard unmodified two-step approach. The above approach is applicable only if the number of measurements from the real system is the same as the number of parameters. A similar algorithm based on the same principle has been developed (see section 5.2) when the number of measurements exceeds the number of parameters.

To control the convergence of the iterative procedure a difference equation of the form

$$\underline{v}_i = \underline{v}_{i-1} + \underline{k}(\underline{u}_i - \underline{v}_{i-1}) \quad \text{........................} \quad (2.63)$$

is used where $\underline{k}$ is a diagonal matrix of gains for regulating the convergence, $\underline{v}_i$ and $\underline{u}_i$ are the values of $\underline{v}$ and $\underline{u}$ at the i^{th} iteration, $\underline{v}_{i-1}$ is the previous value of $\underline{v}$.

2.7 Comments

In the decomposed parametric approach, the modified optimisation and parameter estimation problems are defined by equations (2.32) and (2.33). These are:

modified optimisation problem

$$\min_{\underline{u}} \left[(1 - \theta)p(\underline{x}, \underline{u}, \underline{\alpha}) - \underline{\lambda}'\underline{u}\right]$$

modified parameter estimation problem

$$\min_{\underline{\alpha}} \left[\theta\, G(\underline{x}, \underline{v}, \underline{\alpha}) - \underline{\mu}'\underline{\alpha}\right]$$

subject to model equations and constraints. Ideally, $\underline{\alpha}^*$ are estimated with θ at the highest limit 1; hence, from equation (2.29), $\underline{\mu}$ can be determined to be zero ($\underline{0}$). By substituting $\underline{\mu}$ and θ into modified parameter estimation problem, we will have

$$\min_{\underline{\alpha}} G(\underline{x}, \underline{v}, \underline{\alpha})$$

where $G(\underline{x}, \underline{v}, \underline{\alpha})$ is identical to that given by equation (2.2). If there are the same number of measurements as there are model parameters, the parameter estimation problem may then be considered as

$$y(\underline{x}, \underline{v}, \underline{\alpha}) = \hat{\underline{y}}(\underline{x}, \underline{v}) \quad \rightarrow \quad \underline{\alpha}$$

and with θ at its lower limit, ideally 0, the optimisation problem can be given as

$$\min\{p(\underline{x}, \underline{u}, \underline{\sigma}) - \underline{\lambda}'\underline{u}\}$$

In the decomposed ε-constraint approach, convergence to the true solution occurs with $\varepsilon \rightarrow 0$, hence, with ε equal to zero, the parameter estimation becomes

$$G(\underline{x}, \underline{v}, \underline{\alpha}) = 0$$

Because G is a sum of squares, to be zero all individual squares should be equal to zero,

$$\underline{y}(\underline{x}, \underline{v}, \underline{\alpha}) = \hat{y}(\underline{x}, \underline{v}) \quad \rightarrow \underline{\alpha}$$

Hence, in the limit it can be concluded that the final solution of the decomposed parametric and ε-constraint methods is equivalent to the final solution of the modified two-step approach. However, by using the decomposed problems the computational effort and time may be greatly increased.

2.8 Conclusions

In this section some techniques which attempt to take account of the interaction between the optimisation and parameter estimation problems have been described. The parametric approach and the ε-constraint approach have combined the two problems into a single joint problem, with increased dimensionality. The two-step approach has been modified, by changing the optimisation problem to overcome interaction. All the above techniques are based on multilevel optimisation. Some basic aspects of multilevel optimisation and decomposition covering the goal coordination and the model coordination methods have also been presented. Decomposition is achieved by cutting the links between the optimisation and parameter estimation problems caused by the uncertain model parameters and manipulable inputs. The goal coordination technique has been applied to coordinate the two sub-problems.

Theoretical investigation described in this chapter is to be employed in the application of the techniques to the maximisation of profit from a chemical plant described by a steady-state non-linear mathematical model which has been deliberately simplified in order to reduce computational requirements.

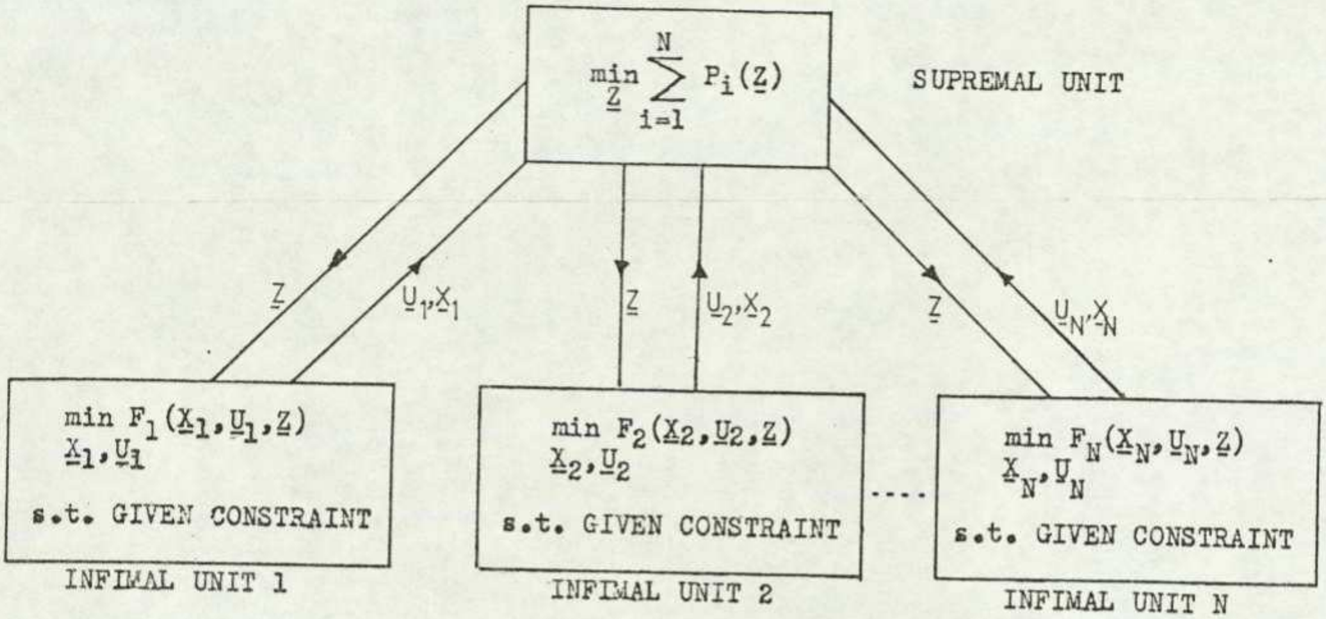


FIG.(2.1) MULTI-LEVEL OPTIMISATION AND DECOMPOSITION USING MODEL COORDINATION.

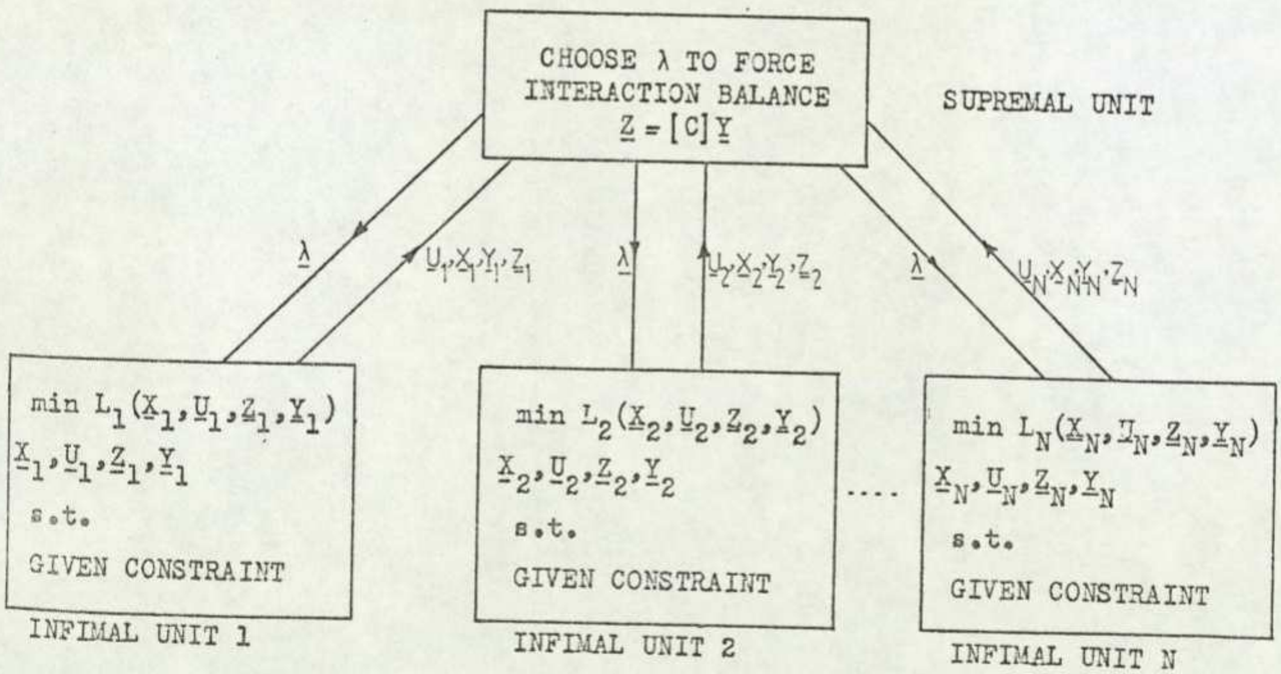


FIG.(2.2) MULTI-LEVEL OPTIMISATION AND DECOMPOSITION EMPLOYING GOAL-COORDINATION.

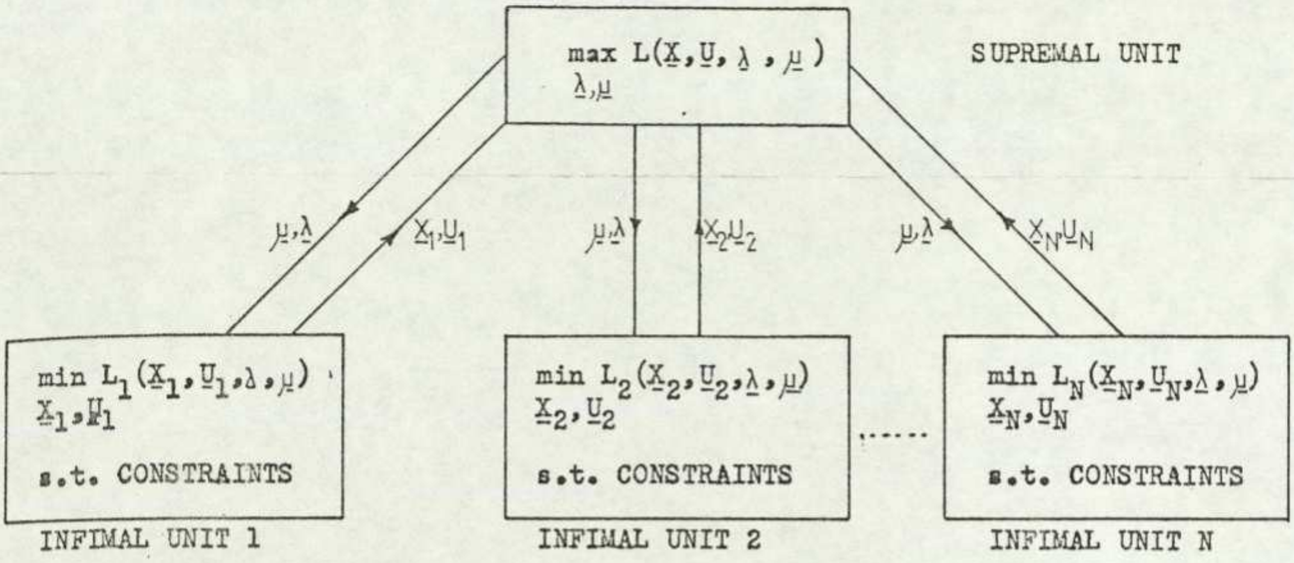


FIG.(2.3) DUAL FORMULATION AND DECOMPOSITION.

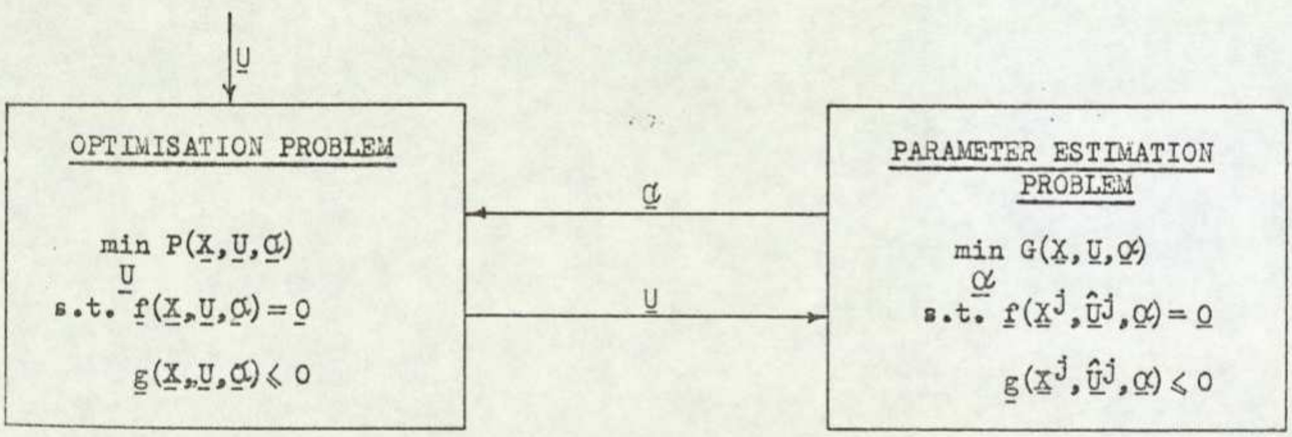


FIG.(2.4) INTEGRATED PROBLEM.

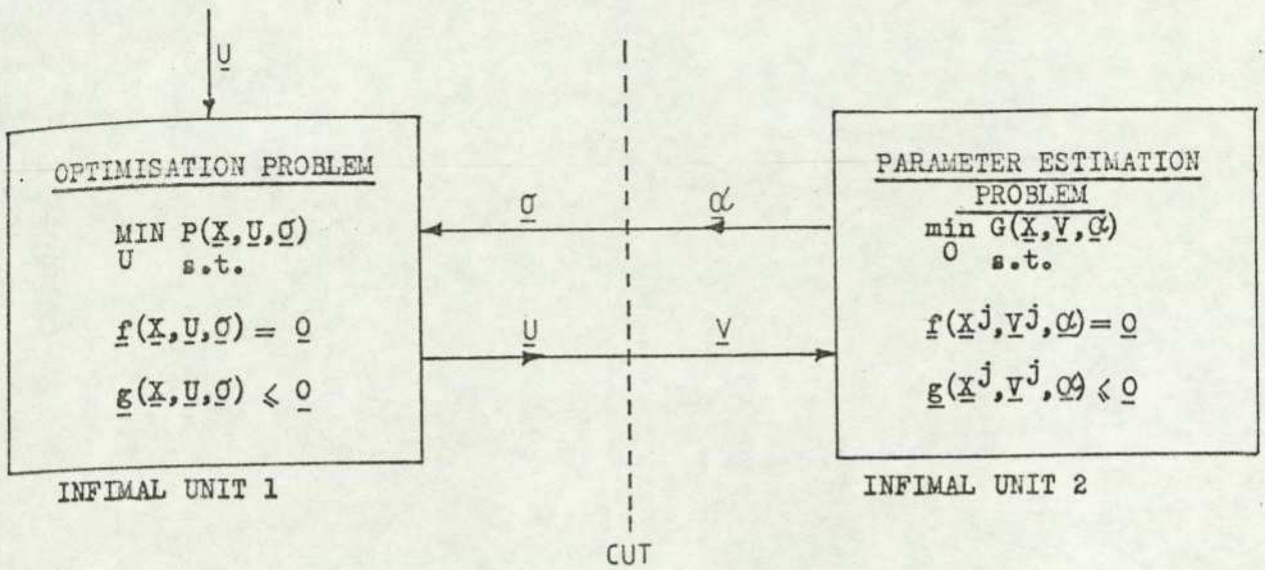


FIG.(2.5) CUTTING LINKS BETWEEN THE PROBLEMS.

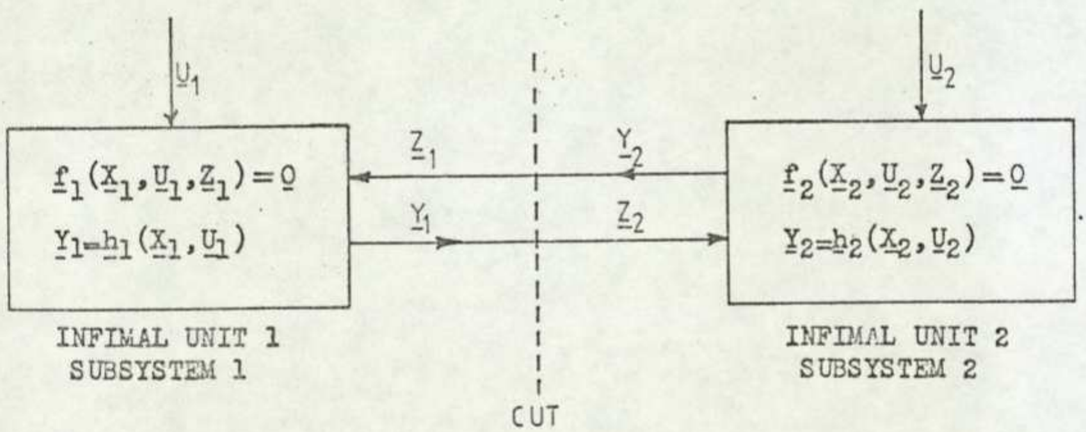


FIG.(2.6) CUTTING BETWEEN SUBSYSTEMS.

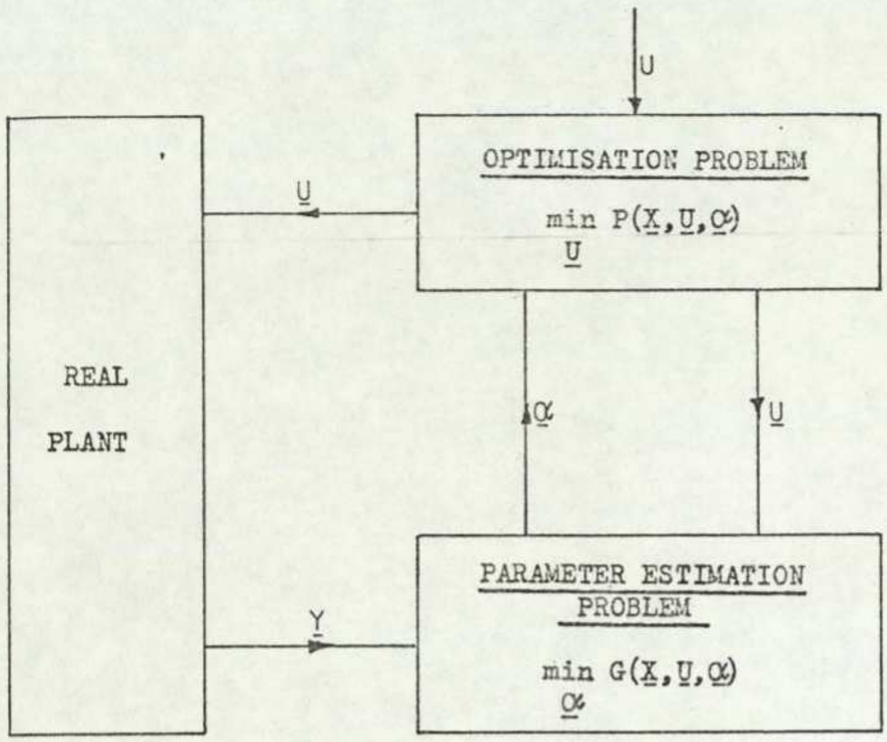
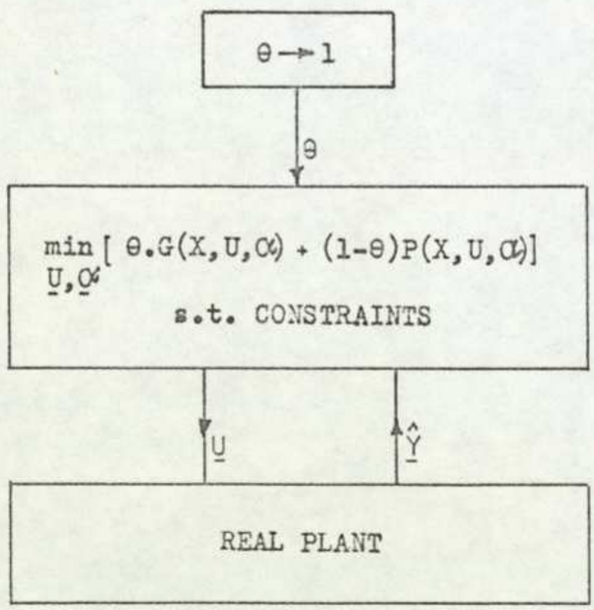
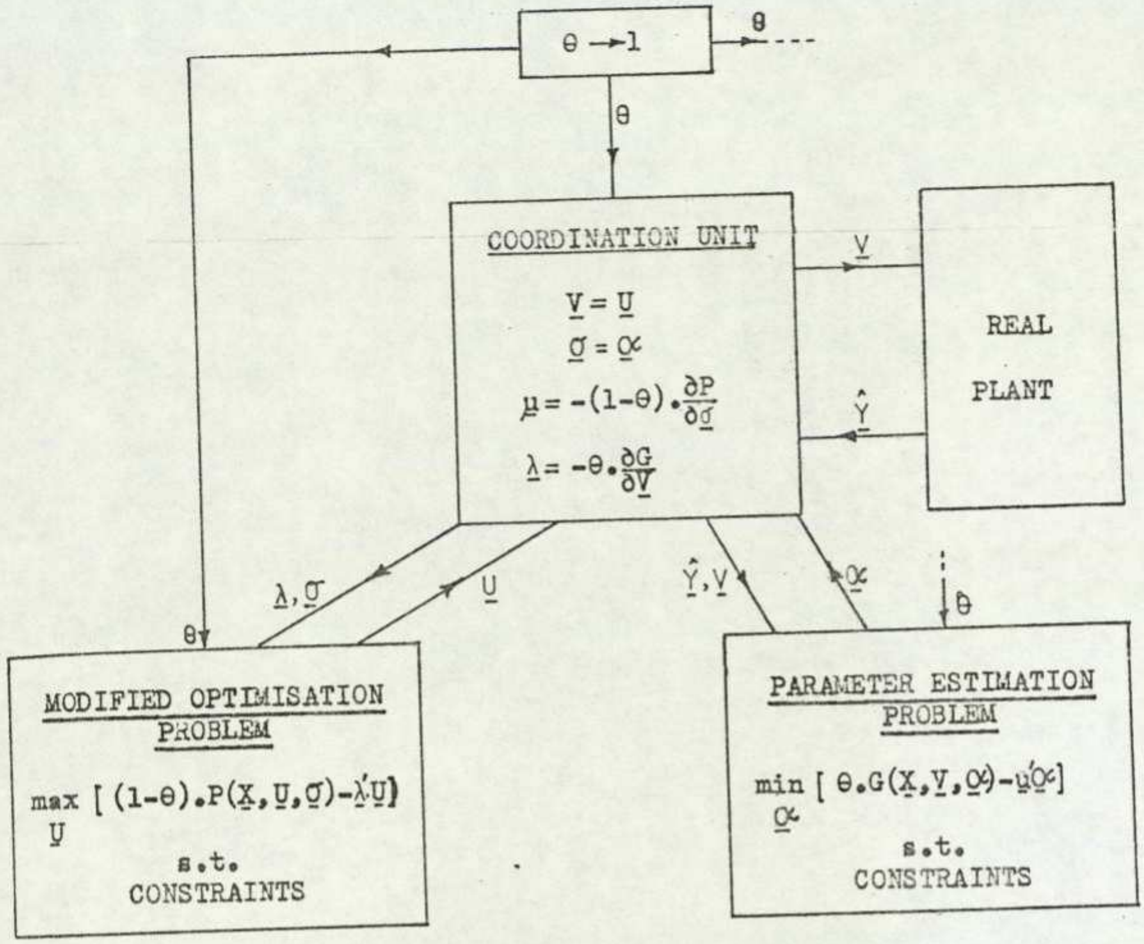


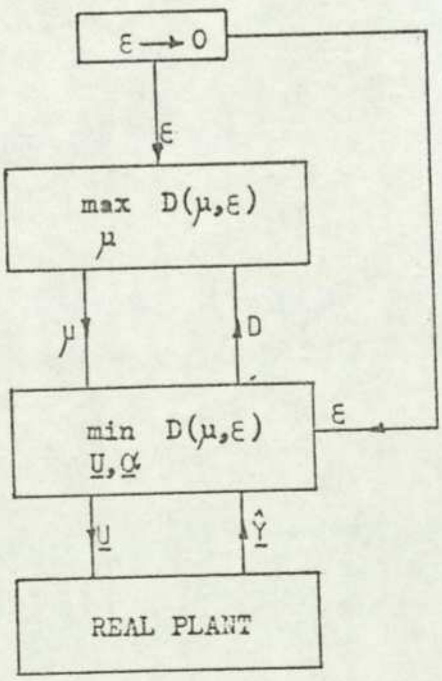
FIG.(2.7) THE TWO-STEP APPROACH.



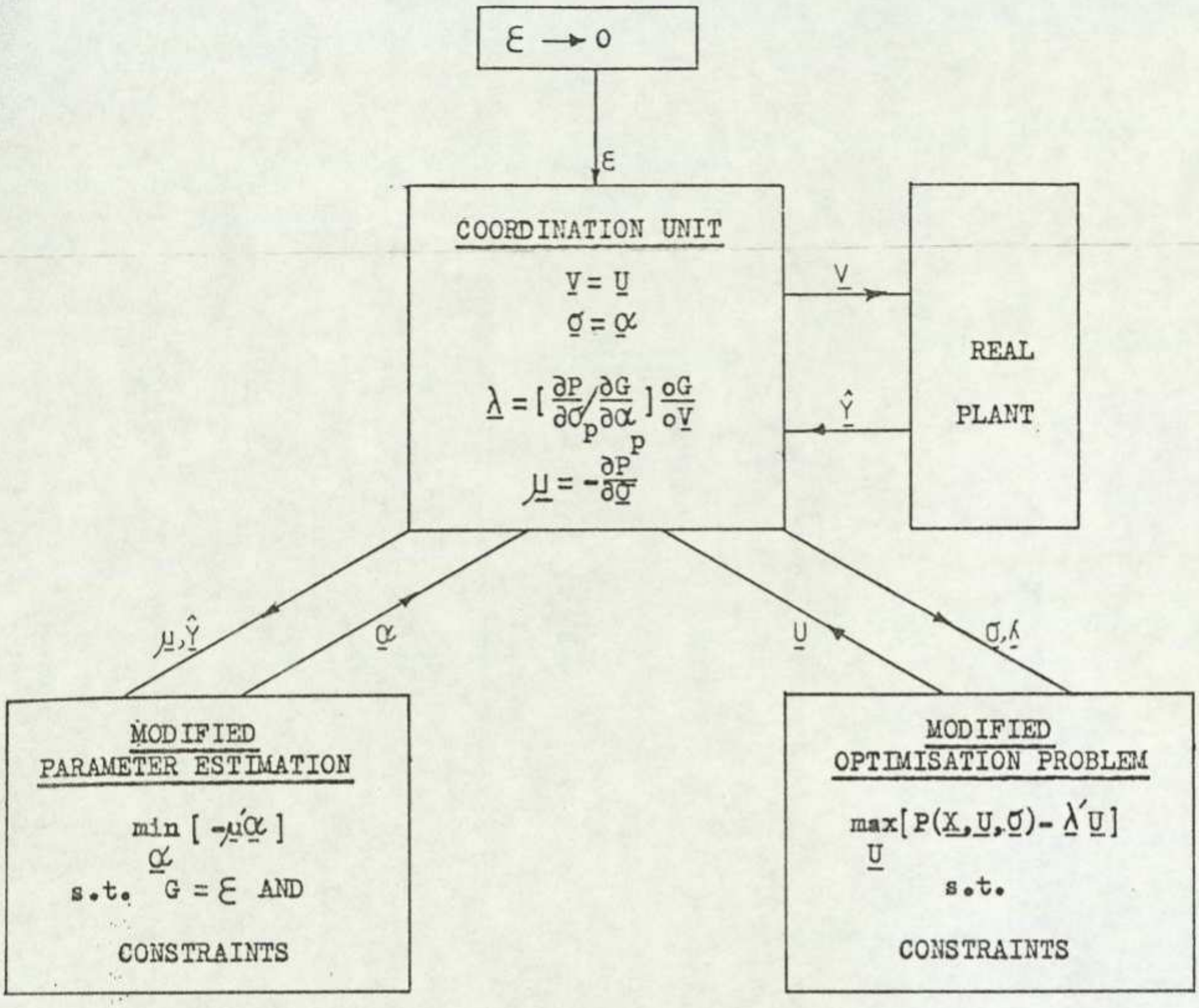
FIG(2.8) THE PARAMETRIC APPROACH WITHOUT DECOMPOSITION.



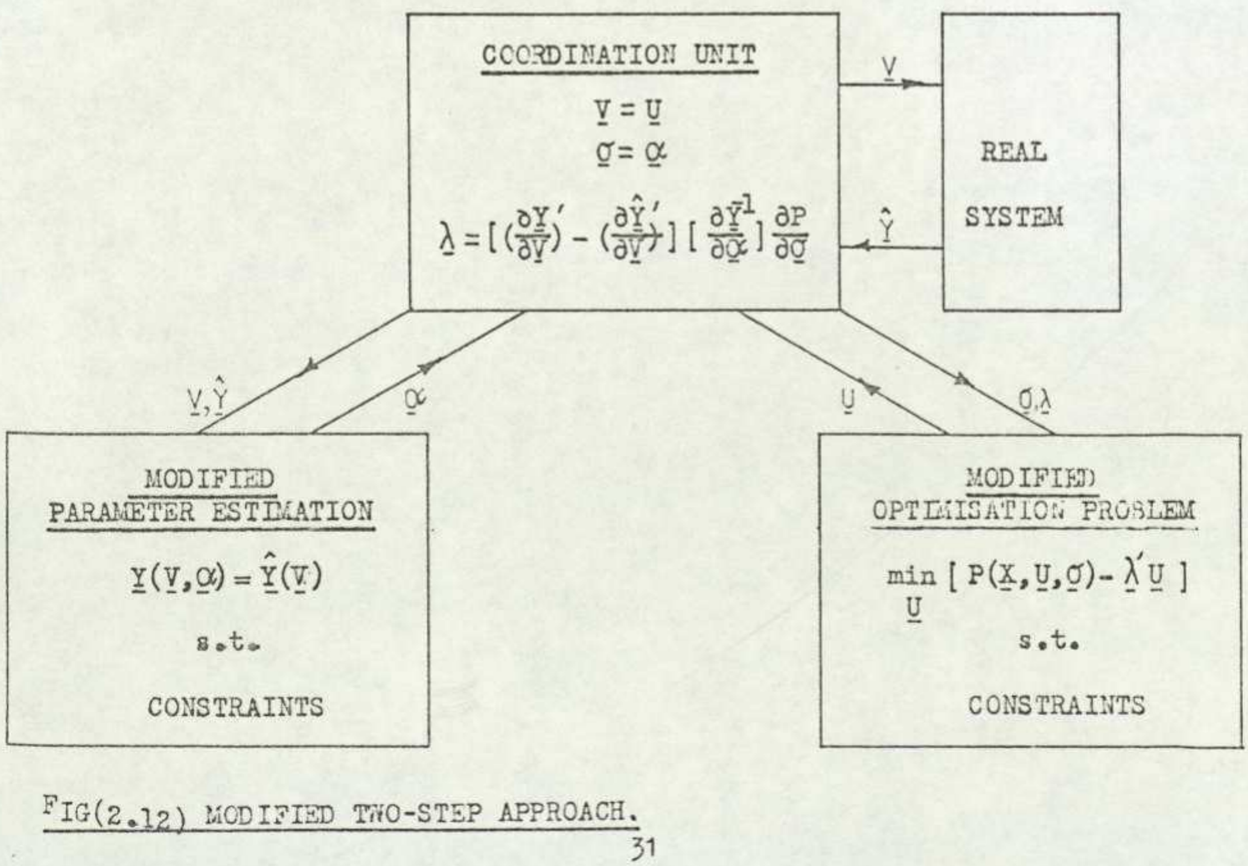
FIG(2.9) DECOMPOSITION OF THE PARAMETRIC APPROACH.



FIG(2.10) ε - CONSTRAINT APPROACH WITHOUT DECOMPOSITION.



FIG(2.11) DECOMPOSITION OF THE ϵ-CONSTRAINT APPROACH.



FIG(2.12) MODIFIED TWO-STEP APPROACH.

CHAPTER 3: APPLICATION OF THE INTEGRATED SCHEMES TO A CHEMICAL PLANT USING ITS STEADY STATE MODEL

In this chapter, the real system and the model of a chemical plant will be described. The integrated schemes which have been described in chapter 2 are now to be implemented using a steady state simplified model of the chemical plant. This is used to determine feedflow rate, reaction temperature and recycle rate controller set points in order to maximise the net rate of return.

The scheme of the chemical plant is taken from the generalised chemical model published by Williams and Otto (1960). For the purpose of this investigation, the above model is described as the "real plant", whereas its simplified mathematical model is termed as the "model". Hence, from now on

"The real plant" = model introduced by Williamsand Otto;
"The model" = the simplified model of the above model.

Normally, in practice, to estimate the unknown parameters of a steady state model, measurements are taken from the real system when it reaches its steady state operating condition. In this chapter, however, the steady state measurements are obtained using the steady state solution of the real plant, and dynamic response considerations are ignored for the present.

3.1 Description of the Plant

Figure 3.1 represents the block diagram of the plant. The plant contains a pressurised continuous stirred tank reactor in which the following chemical reactions take place:

$A + B \rightarrow C$, reaction coefficient, k_1

$C + B \rightarrow P + E$, reaction coefficient, k_2

$P + C \rightarrow G$, reaction coefficient, k_3

Reactant materials A and B are available in pure form. Components C and E are intermediates and/or by-products of the reaction. They have no sale value as chemical products, but can be disposed of as fuels. By-product G is a heavy oily material. The plant is to manufacture a chemical product, P, which has the main sale value.

The heavy oil, waste material G, is removed from the carrier stream in the decanter. In the distillation column, the product P is separated from the effluent stream F_E from the decanter by distillation. F_E contains products of composition A_E, B_E, C_E, E_E and P_E. The discard stream, F_D, is regulated as a certain fraction of the column bottoms flow, F_S. The recycle stream, F_L, then takes all excess and returns it to the reactor. F_A and F_B are the flow rates of the raw materials A and B, respectively. F_R contains products of composition A_R, B_R, C_R, E_R, G_R and P_R. F_L contains products of compositions A_L, B_L, C_L, E_L and P_L. All compositions are considered as mass fractions.

All the reactions which are taking place in the reactor unit are exothermic; hence, a cooling coil is provided and F_W is the mass flow of the cooling water. T_W indicates the outlet temperature of water used as coolant.

The set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$ of the flow rate controller, temperature controller and recycle controller, respectively, are adjusted where the objective is to maximise the net return per hour based on the sales of products F_P and F_D, costs of raw materials A and B, cost of the waste disposal of G, and sales, administration, research, utility and engineering charges.

The dynamic representation of the chemical plant, with the resulting equations of each individual unit, has been given in detail in chapter 4. The steady-state solution can be obtained by equating the time derivatives of the dynamic equations to zero. These equations are represented in the following.

3.1.1 Nomenclature

A = reactant; as subscript, refers to reactant stream

B = reactant; as subscript, refers to reactant stream

C = intermediate component; as subscript, refers to distillation column

C_ρ = heat capacity; subscript gives stream

D = as subscript, refers to discard stream

E = intermediate component; as subscript, refers to decanter material used as distillation column feed

F = flow rate; subscript refers to origin of stream or its identity

G = by-product; as subscript, refers to heavy oil stream

H = heat of reaction

L = as subscript, refers to recycle stream

R = as subscript, refers to reactor

S = as subscript, refers to column reboiler

T = temperature; subscript defines stream and location involved

V = volume; subscript defines location

W = as subscript, refers to water used as coolant

k = reaction rate constant

ρ = specific gravity; subscript defines stream and location involved

t = time

3.1.2 Reactor equations

3.1.2.1 Mass balance of the components

$$F_A + F_L.A_L - F_R.A_R - k_1.A_R.B_R.V_R = 0 \quad \ldots\ldots\ldots\ldots \quad (3.1)$$

$$F_B + F_L.B_L - F_R.B_R - k_1.A_R.B_R.V_R - k_2.B_R.C_R.V_R = 0 \quad \ldots\ldots \quad (3.2)$$

$$F_L.C_L - F_R.C_R + 2.k_1.A_R.B_R.V_R - 2.k_2.B_R.C_R.V_R - k_3.C_R.P_R.V_R = 0 \quad \ldots\ldots\ldots \quad (3.3)$$

$$F_L.E_L - F_R.E_R + 2.k_2.B_R.C_R.V_R = 0 \quad \ldots\ldots\ldots\ldots \quad (3.4)$$

$$F_L.P_L - F_R.P_R + k_2.B_R.C_R.V_R - 0.5\ k_3.C_R.P_R.V_R = 0 \quad \ldots\ldots \quad (3.5)$$

$$1.5.k_3.C_R.P_R.V_R - F_R.G_R = 0 \quad \ldots\ldots\ldots\ldots \quad (3.6)$$

$$F_A + F_B + F_L - F_R = 0 \quad \ldots\ldots\ldots\ldots \quad (3.7)$$

3.1.2.2 Heat balance equations

$$2.k_1.A_R.B_R.V_R.H_1 + 3.k_2.B_R.C_R.V_R.H_2 + 1.5.k_3.C_R.P_R.V_R.H_3 - h_W.A_W.(T_R - T_W) - F_L.C_{\rho L}(T_R - T_L) - F_A.C_{\rho A}(T_R - T_A) - F_B.C_{\rho B}(T_R - T_B) = 0 \quad \ldots\ldots\ldots\ldots \quad (3.8)$$

$$F_W.C_{\rho W}(T_I - T_W) + h_W.A_W(T_R - T_W) = 0 \quad \ldots\ldots\ldots\ldots \quad (3.9)$$

where T_I and A_W are the inlet flow and effective heat transfer area of cooling water.

3.1.3 Decanter equations

$$F_R.A_R - F_E.A_E = 0 \quad \text{.......................... (3.10)}$$

$$F_R.B_R - F_E.B_E = 0 \quad \text{.......................... (3.11)}$$

$$F_R.C_R - F_E.C_E = 0 \quad \text{.......................... (3.12)}$$

$$F_R.E_R - F_E.E_E = 0 \quad \text{.......................... (3.13)}$$

$$F_R.P_R - F_E.P_E = 0 \quad \text{.......................... (3.14)}$$

$$F_G = G_R.F_R \quad \text{.................................. (3.15)}$$

$$F_E + F_G - F_R = 0 \quad \text{............................ (3.16)}$$

3.1.4 Distillation column equations

The relationship between the mass fraction of components P and E has been defined as:

$$P_L = 0.1\, E_L \quad \text{................................ (3.17)}$$

The column equations and reboiler equations can be combined as (see chapter 4):

$$F_E.A_E - F_S.A_S = 0 \quad \text{............................ (3.18)}$$

$$F_E.B_E - F_S.B_S = 0 \quad \text{............................ (3.19)}$$

$$F_E.C_E - F_S.C_S = 0 \quad \text{............................ (3.20)}$$

$$F_E.E_E - F_S.E_S = 0 \quad \text{............................ (3.21)}$$

$$0.1\, F_E.E_E - F_S.P_S = 0 \quad \text{........................ (3.22)}$$

$$F_P = (P_E - 0.1\, E_E)F_E \quad \text{............................ (3.23)}$$

$$F_E - F_P - F_S = 0 \quad \text{.............................. (3.24)}$$

3.1.5 Recycle control equations

$$F_S - F_D - F_L = 0 \quad \text{............................ (3.25)}$$

$$\frac{F_L}{F_S} = \text{constant} = R \quad \text{............................ (3.26)}$$

$$\left.\begin{aligned} \text{also } A_L &= A_S \\ B_L &= B_S \\ C_L &= C_S \\ E_L &= E_S \\ P_L &= P_S \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots (3.27)$$

3.1.6 Systems equations

By substituting equations (3.13) and (3.14) in equation (3.23) we will have:

$$F_P = F_R(P_R - 0.1\, E_R) \qquad \ldots\ldots\ldots\ldots (3.28)$$

Equations (3.15) and (3.16) will give

$$F_E = F_R(1 - G_R) \qquad \ldots\ldots\ldots\ldots (3.29)$$

Substituting for F_E and F_P from the above equations into equation (3.24),

$$F_S = F_R(1 - G_R - P_R + 0.1\, E_R)$$

or

$$F_S = F_R.\beta \qquad \ldots\ldots\ldots\ldots (3.30)$$

where β is given by

$$\beta = 1 - G_R - P_R + 0.1\, E_R \qquad \ldots\ldots\ldots\ldots (3.31)$$

Substituting for F_S in equation (3.26) the result will be

$$F_L = R.\beta.F_R \qquad \ldots\ldots\ldots\ldots (3.32)$$

Combining equations (3.7) and (3.32) will give an expression for F_R to be

$$F_R = -\frac{F_A + F_B}{1 - R.\beta} \qquad \ldots\ldots\ldots\ldots (3.33)$$

By adding equation (3.10) to (3.18) and substituting for F_S and A_S from equations (3.30) and 3.27), respectively, the result will be

$$A_L = A_R/\beta \qquad \ldots\ldots\ldots\ldots (3.34)$$

In a similar manner,

$$B_L = B_R/\beta \qquad \ldots\ldots\ldots\ldots (3.35)$$

$C_L = C_R/\beta$ .... (3.36)

$E_L = E_R/\beta$ .... (3.37)

$P_L = 0.1\ E_R/\beta$ .... (3.38)

By substituting equations (3.34) to (3.38) into equations (3.1) to (3.6) the overall system/plant equations can be summarised as follows:

Summary of the plant equations:

$$\left.\begin{aligned}
&F_A + \frac{F_L.A_R}{\beta} - F_R.A_R - R_1 = 0\\
&F_B + \frac{F_L.B_R}{\beta} - F_R.B_R - R_1 - R_2 = 0\\
&\frac{F_L.C_R}{\beta} - F_R.C_R + 2R_1 - 2R_2 - R_3 = 0\\
&\frac{0.1\ F_L.E_R}{\beta} - F_R.P_R + R_2 - 0.5\ R_3 = 0\\
&\frac{F_L.E_R}{\beta} - F_R.E_R + 2R_2 = 0\\
&1.5\ R_3 - F_R.G_R = 0\\
&F_L = R.\beta.F_R\\
&F_R = -\frac{F_A + F_B}{1 - R.\beta}\\
&\beta = 1 - G_R - P_R + 0.1\ E_R\\
&\text{where}\\
&R_1 = k_1.A_R.B_R.V_R\\
&R_2 = k_2.B_R.C_R.V_R\\
&R_3 = k_3.C_R.P_R.V_R\\
&\text{and}\\
&k_1 = \alpha_1\ e^{-\beta_1/T_R}\\
&k_2 = \alpha_2\ e^{-\beta_2/T_R}\\
&k_3 = \alpha_3\ e^{-\beta_3/T_R}
\end{aligned}\right\} \quad \dots\dots \quad (3.39)$$

The plant is also subject to some inequality constraints due to physical limitations. These are

$$\left.\begin{aligned} 0 \leq R \leq 1.0 \\ 0 \leq F_W \leq F_{Wmax} \\ F_P \leq F_{Pmax} \end{aligned}\right\} \quad \text{.................} \quad (3.40)$$

Flow rates over F_{Pmax} cannot be sold and must be discarded. F_W can be obtained from equation (3.9) to be

$$F_W = h_W.A_W \frac{T_R - T_W}{T_W - T_I} \quad \text{.................} \quad (3.41)$$

where T_W is given by equation (3.8) to be

$$T_W = \frac{1}{h_W.A_W}\Big[2.k_1.A_R.B_R.V_R.H_1 + 3.k_2.B_R.C_R.V_R.H_2 + 1.5.k_3.C_R.P_R.V_R.H_3 - F_L.C_{\rho L}(T_R - T_L) - F_A.C_{\rho A}(T_R - T_A) - F_B.C_{\rho B}(T_R - T_B)\Big] + T_R \quad \text{.................} \quad (3.42)$$

Equations (3.39) and (3.40) constitute the system equations and are solved using a NAG numerical routine (see Appendix A1).

3.2 Model Description

For the purpose of this investigation the model of the reactor is deliberately simplified by neglecting the intermediate component C such that the reaction equations are:

$A + 2B \rightarrow P + E$, reaction rate coefficient k_1

$E + B \rightarrow G$, reaction rate coefficient k_2

with Arrhenius equations

$$k_1 = A_1 EXP(- B_1/T_R) \quad \text{................} \quad (3.43)$$

$$k_2 = A_2 EXP(- B_2/T_R) \quad \text{................} \quad (3.44)$$

where A_1, A_2, B_1 and B_2 are unknown parameters whose values are required to be estimated. The steady state performance of the model can be represented as follows:

3.2.1 Mass balance of the components in the reactor

$$F_A + F_L.A_L - F_R.A_R - R_1.M_A.V_R = 0 \quad \ldots\ldots \quad (3.45)$$

$$F_B + F_L.B_L - F_R.B_R - 2R_1.M_B.V_R - R_2.M_B.V_R = 0 \quad \ldots\ldots \quad (3.46)$$

$$F_L.E_L - F_R.E_R + R_1.M_E.V_R - R_2.M_E.V_R = 0 \quad \ldots\ldots \quad (3.47)$$

$$F_L.P_L - F_R.P_R + R_1.M_P.V_R = 0 \quad \ldots\ldots \quad (3.48)$$

$$R_2.M_G.V_R - G_R.F_R = 0 \quad \ldots\ldots \quad (3.49)$$

where V_R is the effective mass of the reactor contents; M_A, M_B, M_E, M_G and M_P are the molecular weights of components A, B, E, G and P, respectively; and R_1 and R_2 are the reaction rate given by

$$R_1 = \frac{k_1.A_R.B_R^2}{M_A.M_B^2} \quad \ldots\ldots \quad (3.50)$$

$$R_2 = \frac{k_2.E_R.B_R}{M_E.M_B} \quad \ldots\ldots \quad (3.51)$$

From the flow diagram of the model shown in Figure 3.1 the following equations can be derived:

$$F_A + F_B + F_L - F_R = 0 \quad \ldots\ldots \quad (3.52)$$

$$F_R - F_G - F_E = 0 \quad \ldots\ldots \quad (3.53)$$

$$F_E - F_P - F_S = 0 \quad \ldots\ldots \quad (3.54)$$

$$F_S - F_L - F_D = 0 \quad \ldots\ldots \quad (3.55)$$

$$F_G - F_R.G_R = 0 \quad \ldots\ldots \quad (3.56)$$

$$F_P - F_R(P_R - 0.1\,E_R) = 0 \quad \ldots\ldots \quad (3.57)$$

$$F_L - R.F_S = 0 \quad \ldots\ldots \quad (3.58)$$

Equations (3.53), (3.54), (3.56) and (3.57) will give

$$F_S = F_R(1 - G_R - P_R + 0.1\,E_R)$$

or $F_S = F_R.\beta$ $\quad \ldots\ldots \quad (3.59)$

where

$$\beta = (1 + 0.1\,E_R - G_R - P_R) \quad \ldots\ldots \quad (3.60)$$

Hence, from equation (3.58) we will get:

$$F_L = R.\beta.F_R \quad \ldots\ldots \quad (3.61)$$

Substituting equation (3.61) into equation (3.52) will give

$$F_R = \frac{F_A + F_B}{1 - R.\beta} \qquad \ldots\ldots\ldots\ldots\ldots \quad (3.62)$$

In a similar manner to the previous section, for the distillation column and the recycle unit, the following equations can be derived:

$$\left.\begin{aligned} A_L &= \frac{A_R}{\beta} \\ B_L &= \frac{B_R}{\beta} \\ E_L &= \frac{E_R}{\beta} \\ P_L &= \frac{0.1\ E_R}{\beta} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots \quad (3.63)$$

by substituting for F_L, A_L, B_L, E_L and P_L from equations (3.61) and (3.63) into equations (3.45) to (3.49); the steady state performance of the model can be summarised as follows:

<u>Summary of the model equations</u>

$$\left.\begin{aligned} & F_A + A_R(R - 1)F_R - R_1.M_A.V_R = 0 \\ & F_B + B_R(R - 1)F_R - 2R_1.M_B.V_R - R_2.M_B.V_R = 0 \\ & E_R(R - 1)F_R + R_1.M_E.V_R - R_2.M_E.V_R = 0 \\ & F_R(0.1\ E_R.R - P_R) + R_1.M_P.V_R = 0 \\ & R_2.M_G.V_R - G_R.F_R = 0 \\ & A_R + B_R + P_R + E_R + G_R - 1.0 = 0 \\ & R_1 = \frac{k_1.A_R.B_R^2}{M_A.M_B^2} \\ & R_2 = \frac{k_2.E_R.B_R}{M_B.M_E} \\ & k_1 = A_1 \exp(-B_1/T_R) \\ & k_2 = A_2 \exp(-B_2/T_R) \\ & 0 \leq R \leq 1.0 \\ & 0 < F_W \leq F_{Wmax} \\ & \quad F_P \leq F_{Pmax} \end{aligned}\right\} \qquad (3.64)$$

Equations (3.64) are a set of equality and inequality constraints which constitute the steady state mathematical model of the chemical plant.

3.3 Application of the Integrated Schemes to the Chemical Plant

In this chapter it is assumed that actual plant measurements can be taken of the flow rates F_R and F_P when the plant is at its steady state operating condition.

The reaction rate coefficients k_1 and k_2, which have been described by equations (3.43) and (3.44), are functions of A_1, A_2, B_1 and B_2. B_1 and B_2 are chosen to be equal to the mean values of β_1, β_2, and β_3 given by system equations (3.39). Hence, by estimating A_1 and A_2, k_1 and k_2 are determined. Thus, the parameter estimation problem can be defined as determining the unknown parameters A_1 and A_2 in order to have the response of the model agree as closely as possible to that of the real plant. Mathematically, the parameter estimation problem can be defined as

$$\min_{A_1,A_2} G(F_B, T_R, R, A_1, A_2) = W_1(\hat{F}_R - F_R)^2 + W_2(\hat{F}_P - F_P)^2 \qquad (3.65)$$

s.t. equation (3.64)

where $\hat{}$ denotes current measured values. W_1 and W_2 are weighting coefficients.

In the optimisation problem controller set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$ are to be determined in order to maximise the net return per hour which is given (Williams and Otto (1960)) as

$$\max_{F_B,T_R,R} P(F_B, T_R, R, A_1, A_2) = (0.3F_P + 0.0068F_D - 0.02F_A - 0.03F_B - 0.01 F_G) - (2.22 F_R)/8400 - 0.124(0.3 F_P + 0.0068 F_D) \qquad \ldots\ldots\ldots\ldots\ (3.66)$$

s.t. equation (3.64)

where $(0.3 F_P + 0.0068 F_D - 0.02 F_A - 0.03 F_B - 0.01 F_G)$ represents the gross return per hour; $(2.22 F_R)/8400$ represents utility charges; $0.124(0.3 F_P + 0.0068 F_D)$ represents the sales, administration, research and engineering charges.

It should be noted that F_B, T_R and R represent $\underline{u}$, and A_1 and A_2 represent $\underline{\alpha}$, as discussed in the previous chapter.

In general, the optimisation and parameter estimation problems are described by equations (2.1) and (2.3). The model state variables $\underline{x}$ may be eliminated by using the model equality constraints. Then taking into account the interaction between the two problems, the optimisation problem becomes:

$$\min_{\underline{u}} P(\underline{u}, \underline{\sigma})$$

s.t. model equations and constraints

and the parameter estimation problem becomes

$$\min_{\underline{\alpha}} G(\underline{v}, \underline{\alpha})$$

s.t. model equations and constraints

where both problems are subject to interconnection constraints

$$\underline{v} = \underline{u}$$

$$\underline{\alpha} = \underline{\sigma}$$

The above problems are identical to those described by equations (3.65) and (3.66). Hence, the integrated schemes described in chapter 2 may be applied to the chemical plant.

The schematic diagram of the on-line computer control scheme, with three local feedback controllers for regulating F_B, T_R and R, is shown in Figure 3.2.

3.3.1 The two-step approach

To solve the combined problem, start with assumed initial values of the flow $\hat{F}_B$, temperature $\hat{T}_R$, and recycle $\hat{R}$, controller set points.

Step 1: Obtain steady state measurements $\hat{F}_P$ and $\hat{F}_R$ by applying the current controller set points values $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$ to the real plant. Then the parameter estimation problem can be solved to give estimates $\hat{A}_1$ and $\hat{A}_2$.

Step 2: With given parameter estimates $\hat{A}_1$ and $\hat{A}_2$ obtained from the first step, the optimisation problem can then be solved to give a new set of controllers set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$.

Steps 1 and 2 are repeated until no further improvement is obtained. The block diagram of the above procedure can be viewed in Figure 3.3.

3.3.2 The parametric approach

The parametric formulation described by equation (2.19), as applied to the chemical plant, can be written as:

$$\left.\begin{aligned} &\min_{F_B,T_R,R,A_1,A_2} \left[\theta.G(F_B,T_R,R,A_1,A_2) - (1-\theta)P(F_B,T_R,R,A_1,A_2)\right] \\ &\qquad\qquad \text{s.t. equation (3.64)} \\ &\qquad\qquad\quad 0 < \theta < 1 \end{aligned}\right\} \quad (3.67)$$

To solve the above problem, the following steps should be taken:

Step 1: With θ at its lowest limit, assume initial values of the controller set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$.

Step 2: Apply the current controller set point values to the real plant to obtain steady state measurements $\hat{F}_P$ and $\hat{F}_R$.

Step 3: New set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$ and parameter estimates $\hat{A}_1$ and $\hat{A}_2$ can be obtained by minimising the parametric formulation problem given by equation (3.67).

Step 4: Increase value of θ towards 1 (without attaining 1).

Repeat steps 2 to 4 until no further improvement is obtained. It should be noted that, in this technique, it is suggested that measurements from the real process are taken whenever the value of θ is changed. Figure 3.4 represents the block diagram of the scheme.

3.3.3 The ε-constraint approach

Using equations (3.65) and (3.66) as applied to the chemical plant, the ε-constraint approach may be formulated as follows:

$$D(\mu,\varepsilon) = \min_{F_B,T_R,R,A_1,A_2} \{P(F_B,T_R,R,A_1,A_2) + \mu\left[G(F_B,T_R,R,A_1,A_2) - \varepsilon\right]\}$$

$$\text{s.t. equation (3.64)} \quad \ldots\ldots\ldots\ldots\ldots \quad (3.68)$$

$$\left.\begin{array}{l} \max\limits_{\mu} D(\mu, \varepsilon) \\ \quad \text{s.t. equation (3.64)} \\ \qquad\qquad \mu \geqslant 0 \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (3.69)$$

The following steps should be taken to achieve a solution for the combined problem, when the ε-constraint approach is employed:

Step 1: Assume initial values of the controller set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$; and values for μ and ε.

Step 2: Obtain steady state measurements $\hat{F}_R$ and $\hat{F}_P$ by applying the current set points values to the real plant.

Step 3: Determine the optimal values for F_B, T_R, R, A_1 and A_2 by solving equation (3.68) with fixed values of μ and ε.

Step 4: Solve the dual problem given by equation (3.69), with fixed value of ε, for optimal current value of μ.

Step 5: Decrease the value of ε.

Repeat steps 2 to 5 until no further improvement is obtained. The above scheme implies that the real process measurements are taken every time the ε is changed. Figure 3.5 represents the block diagram of the above technique.

3.3.4 The modified two-step approach

The modified two-step approach, as described by equations (2.51), (2.52), (2.61) and (2.62), can be summarised as follows:

Modified optimisation problem

$$\left.\begin{array}{l} \min\limits_{F_B, T_R, R} \{-P(F_B, T_R, R, \sigma_1, \sigma_2) - \lambda_1 F_B - \lambda_2 T_R - \lambda_3 R\} \\ \qquad\qquad \text{s.t. equation (3.64) and} \\ \qquad\qquad \text{given } \sigma_1, \sigma_2, \lambda_1, \lambda_2 \text{ and } \lambda_3 \end{array}\right\} \quad (3.70)$$

Because there are the same number of measurements as there are model parameters, the unknown parameters A_1 and A_2 may be obtained by matching the model outputs. Hence, the parameter estimation problem becomes

$$\left.\begin{array}{l} \underline{y}(\underline{v}, A_1, A_2) = \hat{\underline{y}}(\underline{v}) \rightarrow A_1, A_2 \\ \quad \text{s.t. equation (3.64)} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ (3.71)$$

The above problems are also subject to:

$$\left.\begin{array}{rcl} \sigma_1 & = & A_1 \\ \sigma_2 & = & A_2 \\ F_B & = & v_1 \\ T_R & = & v_2 \\ R & = & v_3 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ (3.72)$$

The modifiers $\underline{\lambda}$ are given by

$$\underline{\lambda} = \left\{ \left[\frac{\partial \underline{y}}{\partial \underline{v}}\right]' - \left[\frac{\partial \hat{\underline{y}}}{\partial \underline{v}}\right]' \right\} \left\{ \left[\frac{\partial \underline{y}'}{\partial \underline{A}}\right]^{-1} \right\} \frac{\partial P}{\partial \underline{\sigma}} \quad \ldots\ldots\ldots\ (3.73)$$

To find a solution to the combined problem using the modified two-step approach, the following steps should be taken:

Step 1: Assume initial values for controller set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$.

Step 2: Let v_1, v_2 and v_3 be equal to the controller set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$, respectively.

Step 3: Obtain steady state measurements $\hat{F}_P$ and $\hat{F}_R$ by applying the current set values v_1, v_2 and v_3 to the real plant.

Step 4: Solve the parameter estimation problem described by equation (3.71) to determine estimates A_1 and A_2.

Step 5: Let σ_1 and σ_2 be equal to A_1 and A_2, respectively. Then find $\underline{\lambda}$ given by equation (3.73).

Step 6: With the given values of $\underline{\sigma}$ and $\underline{\lambda}$, solve the optimisation problem described by equation (3.70) to give a new set of controller set points $\hat{F}_B$, $\hat{T}_R$ and $\hat{R}$.

Repeat steps 2 to 6 until no further improvement is obtained. $\underline{\lambda}$ was obtained by perturbation on both the model and the system. Because in this particular problem, due to non-linearity and also complexity of the problem, it is difficult to evaluate the model derivatives $\frac{\partial y}{\partial \underline{v}}$, $\frac{\partial y}{\partial \underline{A}}$ and $\frac{\partial P}{\partial \underline{\sigma}}$ analytically.

3.4 Results

The simplified mathematical model of the chemical plant has been used to implement, investigate and compare the two-step, the parametric and the ε-constraint approaches as applied to the problem of maximising the net rate of return of the chemical plant. Both the model and the real plant have been simulated on a digital computer, when they were at their steady state operating conditions. Hence, this study is confined to steady state optimisation and measurements.

The effect of measurement noise has also been investigated. All the results obtained are discussed and appear in graphical form.

3.4.1 Inequality constraints

The inequality constraints which have been imposed in this investigation are due to the physical limitations of the chemical plant. In practice, these constraints could be obtained from the a priori physical knowledge which exists on the plant. But, in this case, the digital simulation of the plant had to be performed in order to gain sufficient knowledge of the plant performance. These constraints are used to limit the optimisation search over a finite range of controller set points. The constraints are:

$$0 \leqslant R \leqslant 1.0$$

$$T_{Rmin} \leqslant T_R \leqslant T_{Rmax}$$

$$F_{Bmin} \leqslant F_B \leqslant F_{Bmax}$$

$$0 \leqslant F_W \leqslant F_{Wmax}$$

$$F_P \leqslant F_{Pmax}$$

where subscripts max and min refer to the maximum and minimum values of the corresponding temperature or flow stream, respectively. In practice, process operators often have enough a priori knowledge of the process in order to limit the region of search for the controller set points values.

The above constraints may, or may not, be active. A graph of T_R against F_B with a fixed value of R (Figure 3.7) shows two regions: region A for positive flow of cooling water, and region B for positive rate of return (profit). The common region, C, between A and B, is considered to be the permissible operating region for the plant. The following results can be determined from Figure 3.7:

$$R = 0.55$$
$$F_{Bmin} = 11430.0 \text{ Kg/hr}$$
$$F_{Bmax} = 17140.0 \text{ Kg/hr}$$
$$T_{Rmin} = 65.5 \quad ^{o}C$$
$$T_{Rmax} = 96.0 \quad ^{o}C$$

As R is increased or decreased, region B shifts towards left or right, respectively. With R = 0.4 at the lower limit and R = 0.7 at the higher limit, there is no common region between A and B. Hence, the following limits were obtained for the whole range of R:

$$F_{Bmin} = 9000.0 \text{ Kg/hr}$$
$$F_{Bmax} = 17580.0 \text{ Kg/hr}$$
$$T_{Rmin} = 60 \quad ^{o}C$$
$$T_{Rmax} = 115.5 \quad ^{o}C$$

Figures 3.8 and 3.9 show two more graphs with different values of R.

Flow rate F_P greater than F_{Pmax} cannot be sold and must be discarded.

$$F_P \leqslant F_{Pmax}$$

where $\quad F_{Pmax} = 2160 \text{ Kg/hr}$

Maximum flow of cooling water, F_{Wmax}, which is a physical constraint, is considered to be 13600 Kg/hr.

3.4.2 Digital simulation of the two-step, the parametric, and the ε-constraint approaches without decomposition

Simulation experiments have been performed to investigate the performance of the above approaches as employed to maximise the net rate of return, RRFT, from the chemical plant. The steps which have to be taken in each case have been described in section 3.3 and also represented in block diagrams.

The intermediate values of the control inputs $\underline{u}^n(F_B^{\ n}, T_R^{\ n}, R^n)$ and the parameters $\underline{\alpha}^n(A_1^{\ n}, A_2^{\ n})$ at the n^{th} cycle, are generated by numerical optimisation routine E04CAF (NAG) (see appendix A1), where a cycle can be defined as one complete solution of the optimisation and the parameter estimation problems.

In order to control the convergence of the iterative procedures the difference equation

$$\underline{u}^n = \underline{u}^{n-1} + [k](\underline{u}^{*n-1} - \underline{u}^{n-1})$$

is used where $[k]$ is a diagonal matrix of gain parameters, $\underline{u}^n$ and $\underline{u}^{*n}$ are the values of $\underline{u}$, and optimal $\underline{u}$ at the n^{th} cycle, $\underline{u}^{n-1}$ and $\underline{u}^{*n-1}$ are the previous values of $\underline{u}$ and $\overset{*}{\underline{u}}$.

In these exercises convergence is considered to have occurred when there is no improvement in the iterative procedure as far as the net rate of return, RRFT, is concerned, i.e.

$$|RRFT^{n-1} - RRFT^n| \leq \xi,$$

where ξ is a small positive constant. The procedure will be stopped if after n_{max} cycles the convergence is not obtained where n_{max} is the maximum number of cycles. Figures 3.10, 3.11 and 3.12 illustrate the flow charts of the above integrated schemes.

The steady state behaviour of the real plant which has been presented in section 3.1 has been simulated to obtain the correct controller set points of the real plant in order to maximise the net rate of return on the real process, RRFT. Figure 3.13 shows that variation in RRFT, starting from three non-optimal alternative initial points. The final value of the RRFT was found to be the same in all three cases within 140 iterations. The optimal set points are:

$$\hat{F}_B = 13605 \text{ Kg/hr}$$

$$\hat{T}_R = 77.5 \ ^oC$$

$$\hat{R} = 0.62$$

The simulation experiments were then performed to investigate the performance of the two-step approach, the ε-constraint approach without decomposition and the parametric approach without decomposition when the net rate of return on the model, PRFT, has been maximised to obtain the controller set points. The results of these simulations failed to determine the correct optimum operating condition when applied to the real plant.

Results of these simulations, which have been presented in Figures 3.14, 3.15 and 3.16, show the variation in the net rate of return on the model, PRFT, and the net rate of return on the real plant, RRFT, employing the standard two-step, the ε-constraint and the parametric approaches, respectively. In all three cases the optimisation terminates at the constraint boundary $F_B = F_{Bmin}$ and $T_R = T_{Rmax}$. It should be noted that the final values of RRFT are different in each case, because of non-identical values of R in each approach.

The above results show clearly the unsatisfactory behaviour of these three approaches in this particular application, because they failed to take into account the gradient mismatch which existed between the system and the model outputs with respect to the inputs.

3.4.3 The modified two-step approach

The performance of the modified two-step approach has been investigated by simulation experiments.

The modified two-step algorithm has been simulated in order to investigate the performance of the approach. Results of this simulation have been presented in Figures 3.17, 3.18, 3.19 and 3.20. These figures show the variation in controller settings and net rate of return, starting from four alternative initial points. The results illustrate the rapid convergence of the algorithm for all four initial conditions.

Unlike the previous cases, the modified two-step approach achieves the correct final optimum operating condition because it takes into account the gradient condition described earlier by including the manipulable inputs as interconnection variables as well as the model parameters.

It should be noted that in all four cases the correct final solution on the real process is obtained within 22 cycles.

3.5 Effects of Measurement Noise

In the results discussed above, the real system measurements $\hat{\underline{y}}$ have been assumed to be known precisely. In a real situation, the measurements will be contaminated with noise, i.e.

$$\tilde{\underline{y}}^j = \hat{\underline{y}}^j + \underline{e}^j$$

where J observations of the output are denoted by

$\hat{\underline{y}}^j$ J = 1, 2, ... J and

$\underline{e}^j$ is a random measurement noise

$\tilde{\underline{y}}^j$ is the noisy measurement

Each component of $\underline{e}^j$ is generated by a NAG routine G05AEF (see appendix A1). It has been assumed to be white noise with zero mean and standard deviation σ^2. The percentage noise is defined as:

$$\% \text{ noise} = \frac{\sigma^2}{\tilde{y}_i^{\,j}} \times 100\%$$

Random noise of normal statistical properties with a 10% standard deviation has been added to all measurements taken from the real process.

With initial controller settings at a non-optimal point on the real plant, the modified two-step approach has been applied to the chemical plant under measurement errors. Figures 3.21, 3.22, 3.23 and 3.24 illustrate the performance of the approach when the real system measurements are contaminated with noise. It can be noticed that although measurement errors give rise to a considerable deterioration in the behaviour of the modified algorithm, the controller settings are maintained within a finite region about their optimum values, and on average a greater net rate of return than that of the standard two-step is obtained.

3.6 Conclusions

The integrated techniques which have been described in chapter 2 have been applied to a problem of determining the optimum operating condition of a chemical plant using its steady-state model.

The equations which constitute the plant have been outlined. A mathematical model has been introduced by simplifying the plant equations.

The standard two-step approach, which is the common technique applied in practice, has been employed to the chemical plant. This failed to converge to the correct optimal solution because the gradients of the real plant outputs with respect to the control inputs were not matched with the corresponding gradient of the model.

The ε-constraint and parametric techniques, without any decomposition, have also been applied to the chemical plant using multilevel techniques. Unfortunately, these two integrated techniques did not succeed because they also failed to take account of the above gradient matching. They also showed disadvantages over the two-step approach by having increased dimensionality, which increases the computational burden.

The modified two-step approach, which takes into account the mismatch between the model and the output derivatives, converged to the correct optimal operating condition. The gradient mismatch is taken into account in consequence to employing the manipulable inputs as well as the model parameters as interconnection constraints in the Lagrangian analysis.

It should be noted that if this decoupling condition is also employed in the parametric and ε-constraint approaches, as described in section 2.6, the final results will agree and be optimal. However, further investigation is required to investigate the convergence properties of the above approaches.

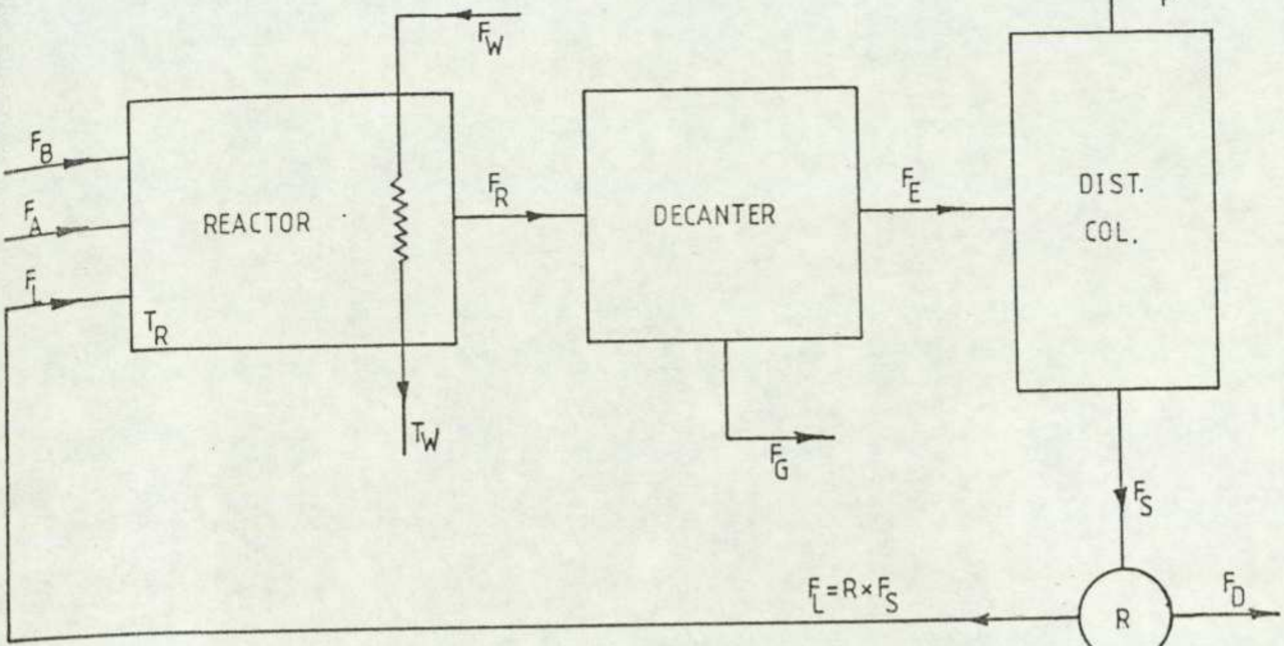


FIG.(3.1) FLOW DIAGRAM OF THE CHEMICAL PLANT.

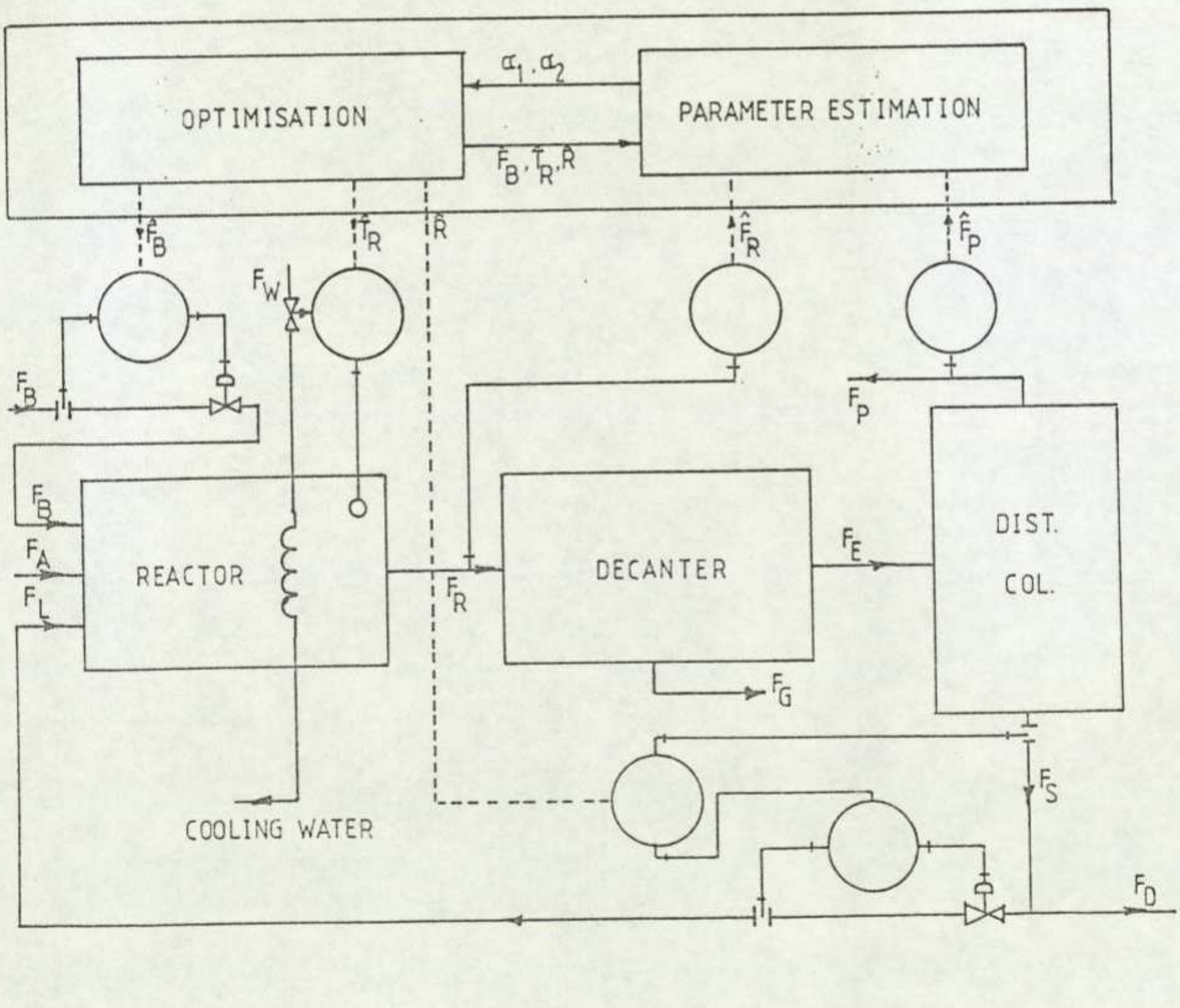
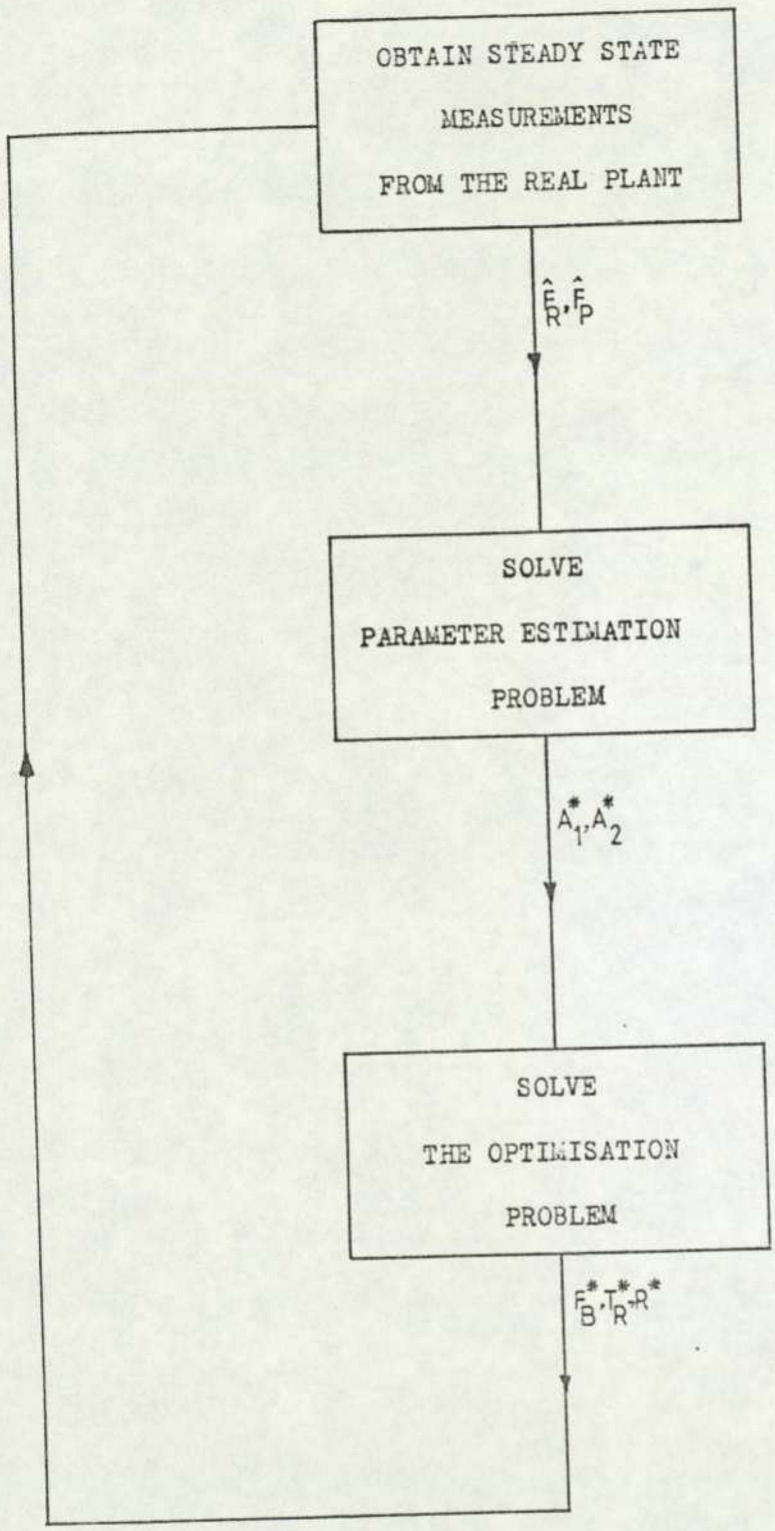


FIG.(3.2) THE CHEMICAL PLANT AND CONTROL SCHEME.



FIG(3.3) BLOCK DIAGRAM OF THE TWO-STEP APPROACH.

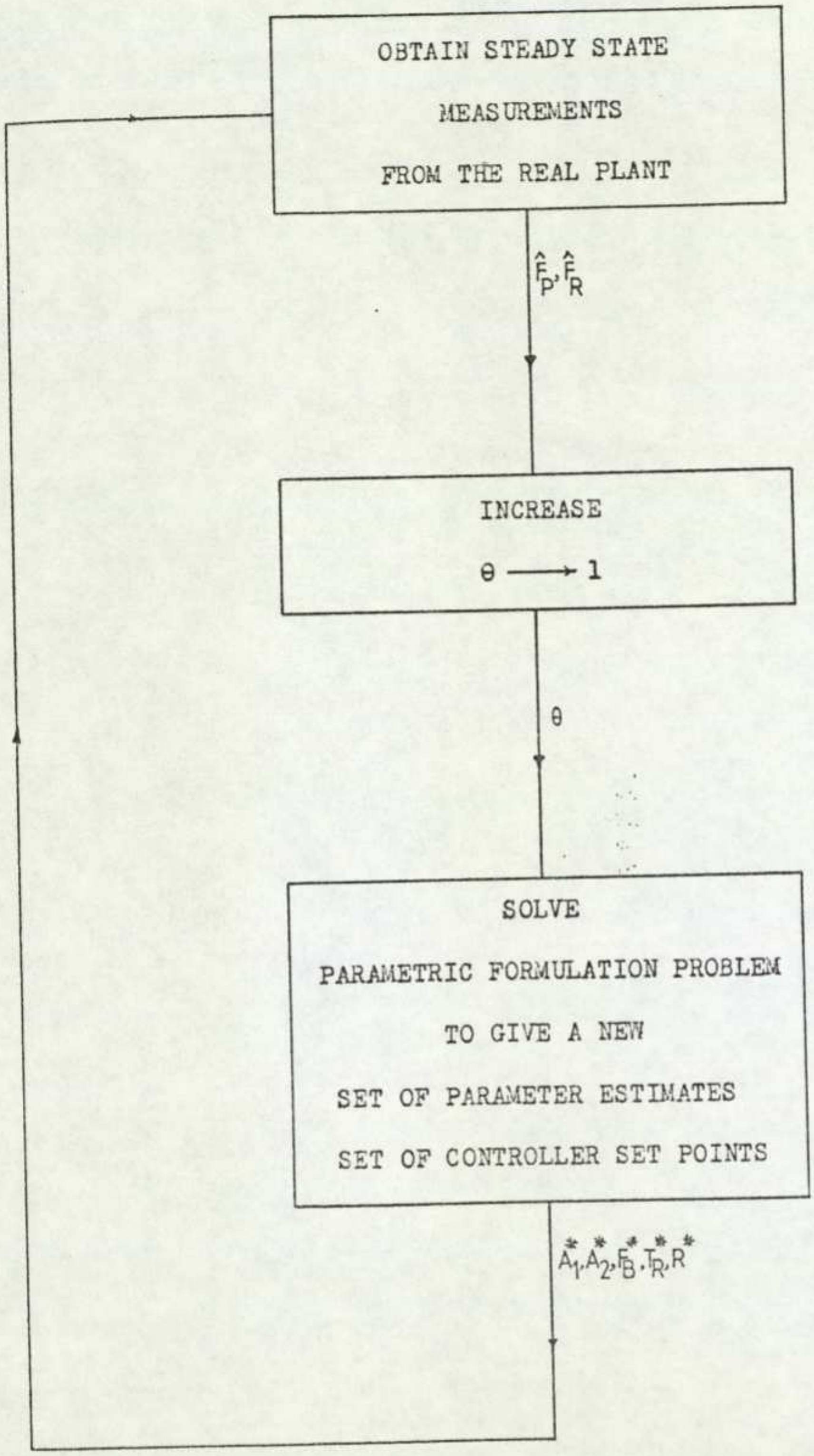


FIG.(3.4) BLOCK DIAGRAM OF THE PARAMETRIC APPROACH.

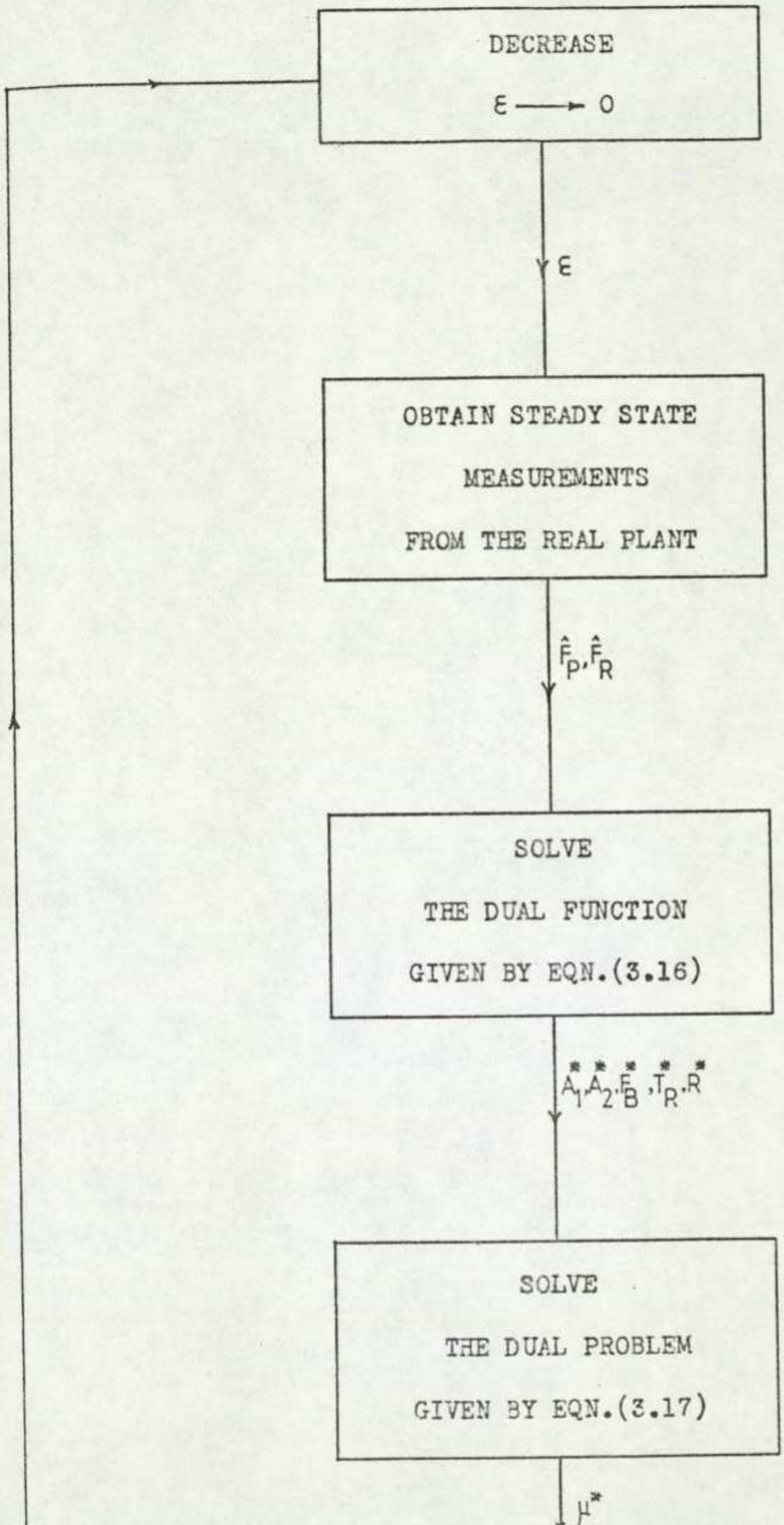
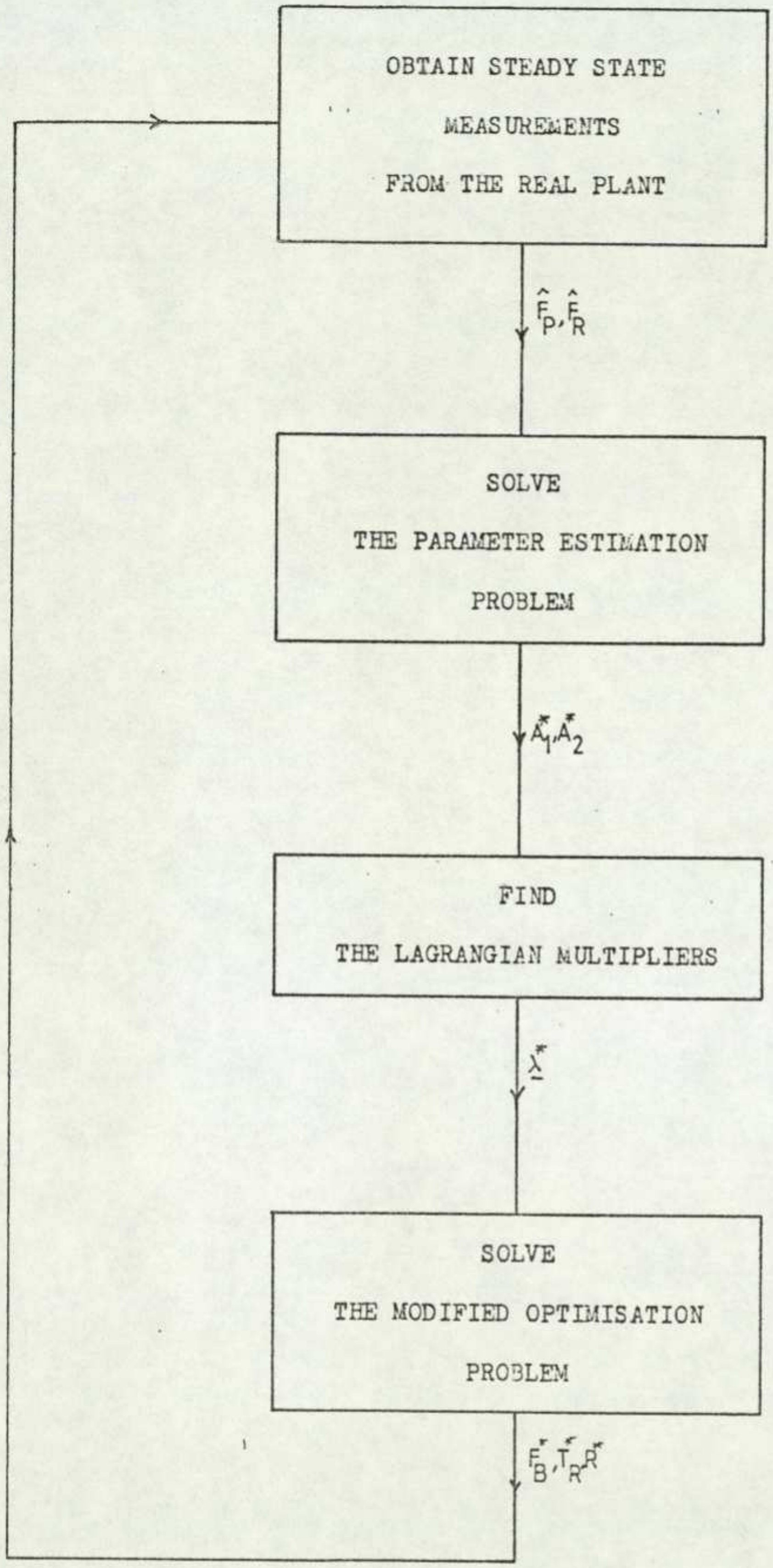


FIG.(3.5) BLOCK DIAGRAM OF THE ε -CONSTRAINT APPROACH.



FIG(3.6) BLOCK DIAGRAM OF THE MODIFIED TWO-STEP APPROACH.

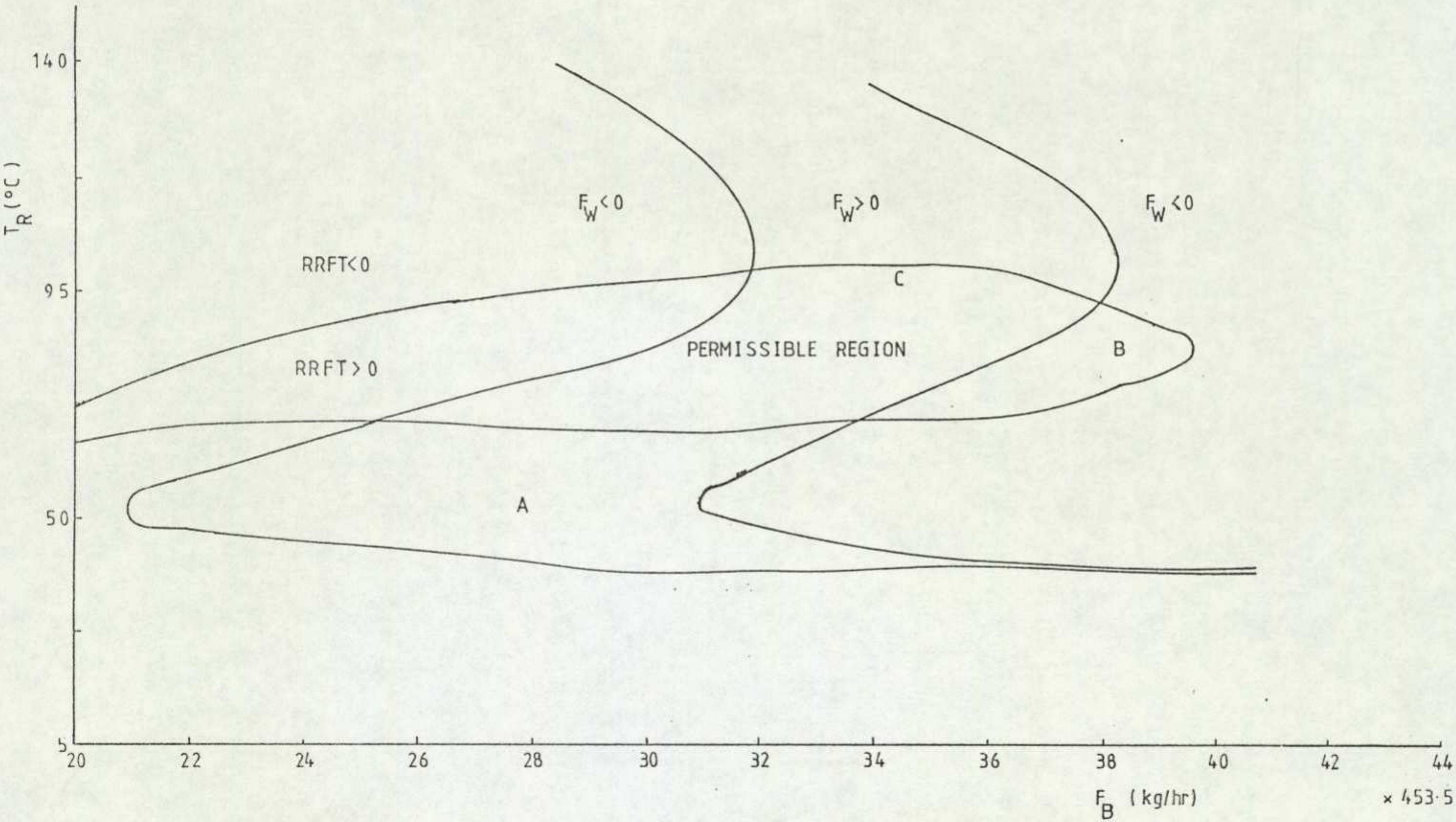


FIG.(3.7) GRAPH OF T_R Vs F_B WITH R = 0.55

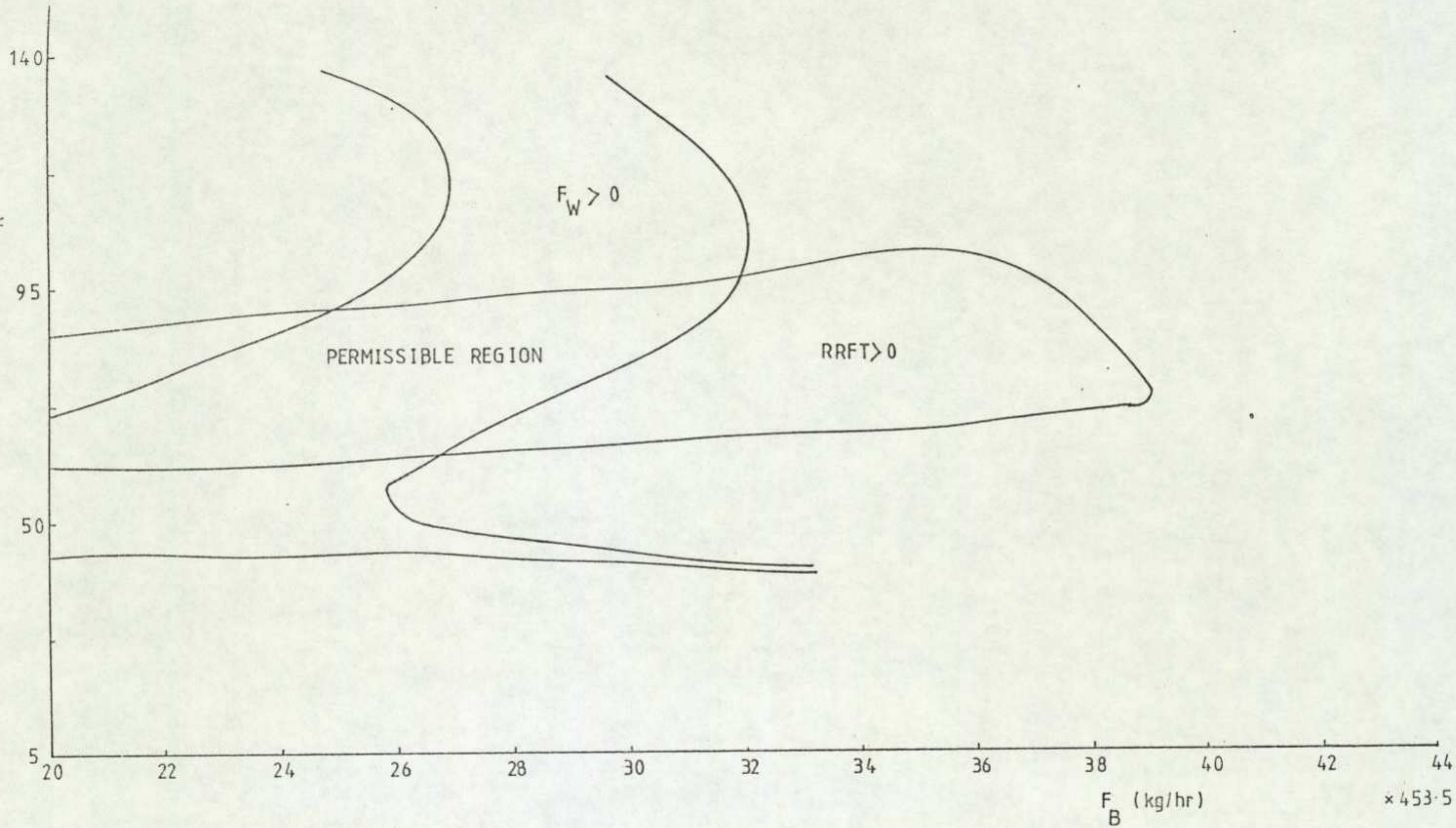


FIG.(3.8) GRAPH OF T_R Vs F_B WITH R= 0.63

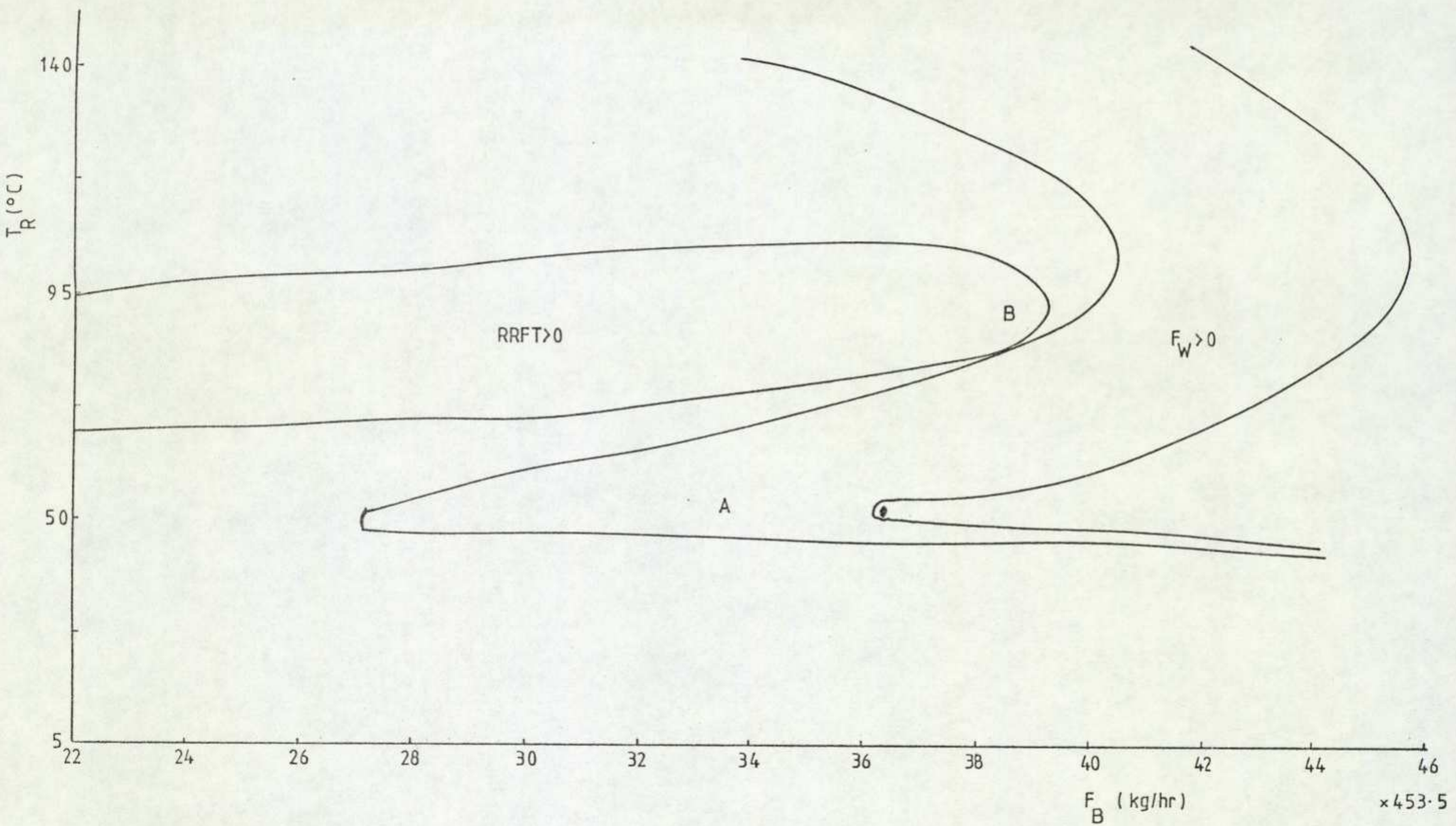
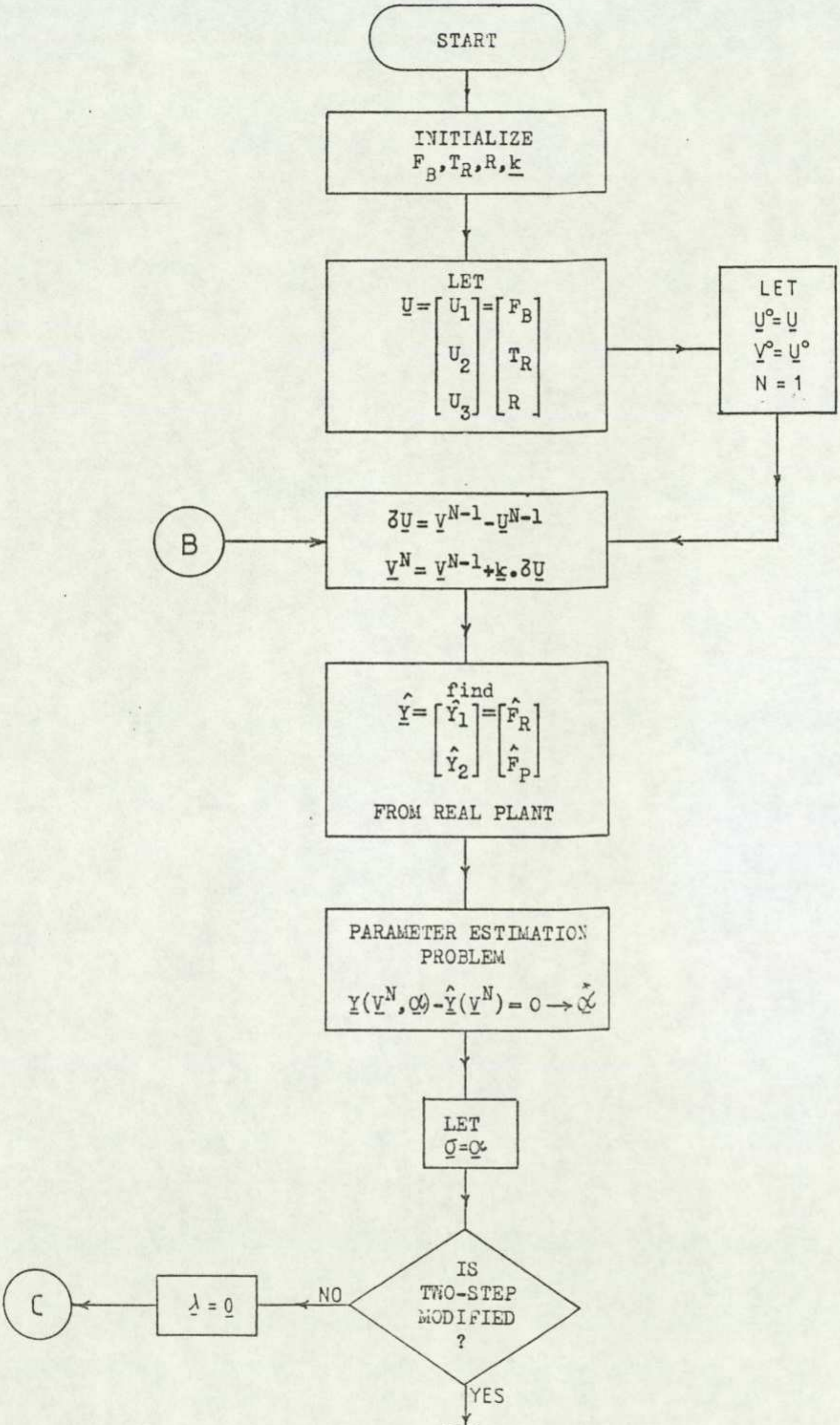
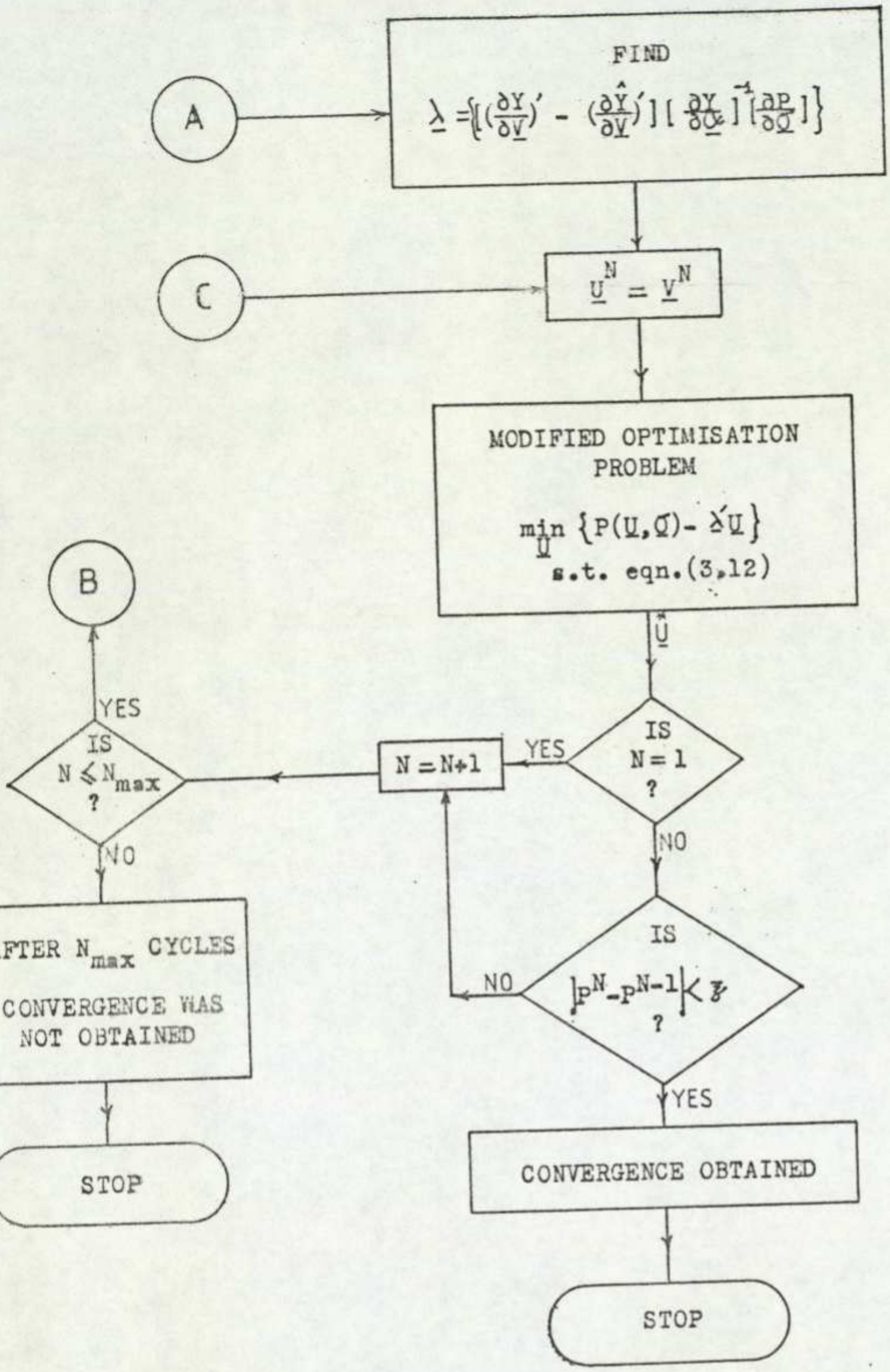
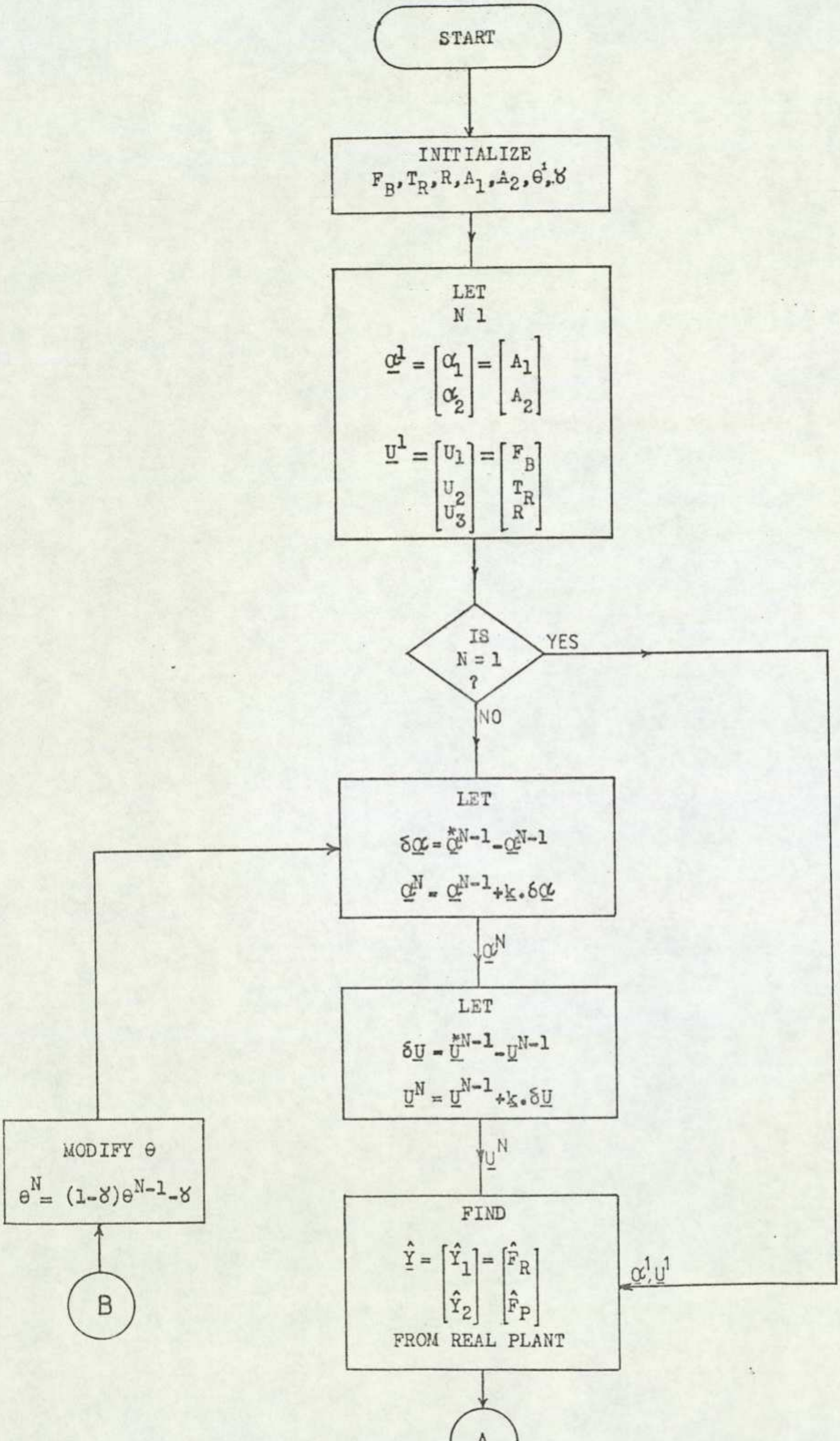


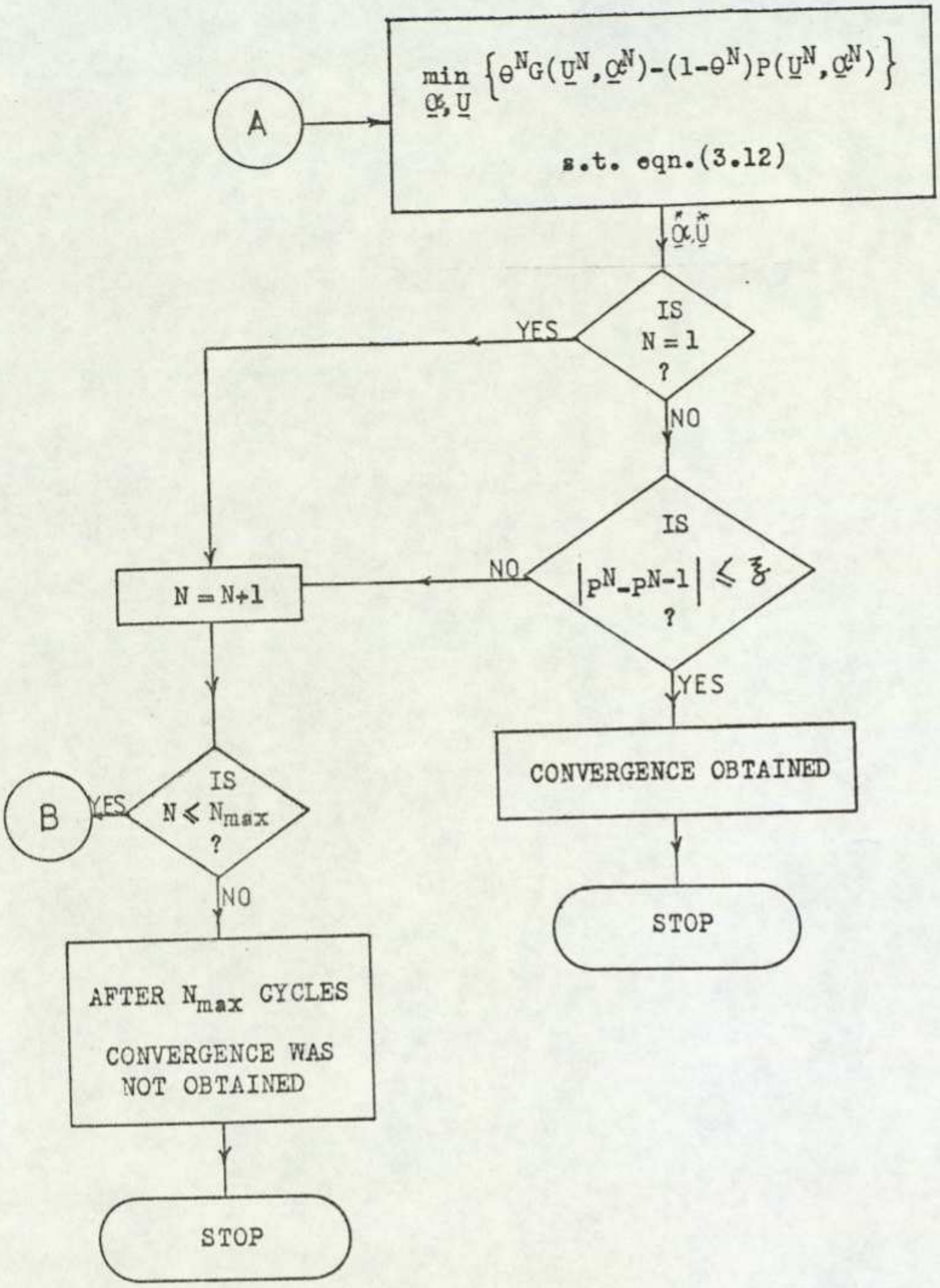
FIG.(3.9) GRAPH OF T_R Vs F_B WITH R = 0.40


START
INITIALIZE
$F_B, T_R, R, \underline{k}$
LET
$\underline{U} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} F_B \\ T_R \\ R \end{bmatrix}$
LET
$\underline{U}^o = \underline{U}$
$\underline{V}^o = \underline{U}^o$
N = 1
B
$\delta\underline{U} = \underline{V}^{N-1} - \underline{U}^{N-1}$
$\underline{V}^N = \underline{V}^{N-1} + \underline{k}.\delta\underline{U}$
find
$\hat{\underline{Y}} = \begin{bmatrix} \hat{Y}_1 \\ \hat{Y}_2 \end{bmatrix} = \begin{bmatrix} \hat{F}_R \\ \hat{F}_P \end{bmatrix}$
FROM REAL PLANT
PARAMETER ESTIMATION
PROBLEM
$\underline{Y}(\underline{V}^N, \underline{\alpha}) - \hat{\underline{Y}}(\underline{V}^N) = 0 \rightarrow \underline{\alpha}^*$
LET
$\underline{\sigma} = \underline{\alpha}$
IS
TWO-STEP
MODIFIED
?
NO
$\underline{\lambda} = \underline{0}$
C
YES
A

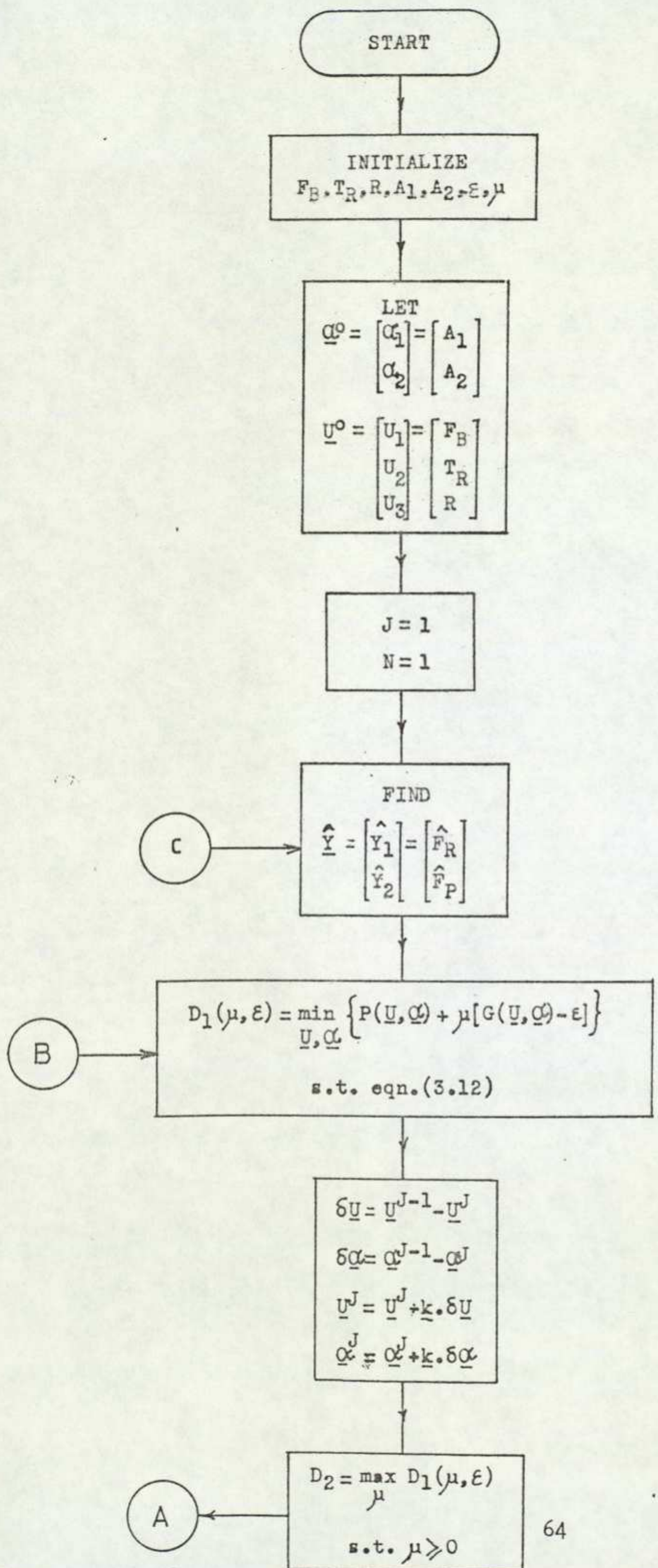


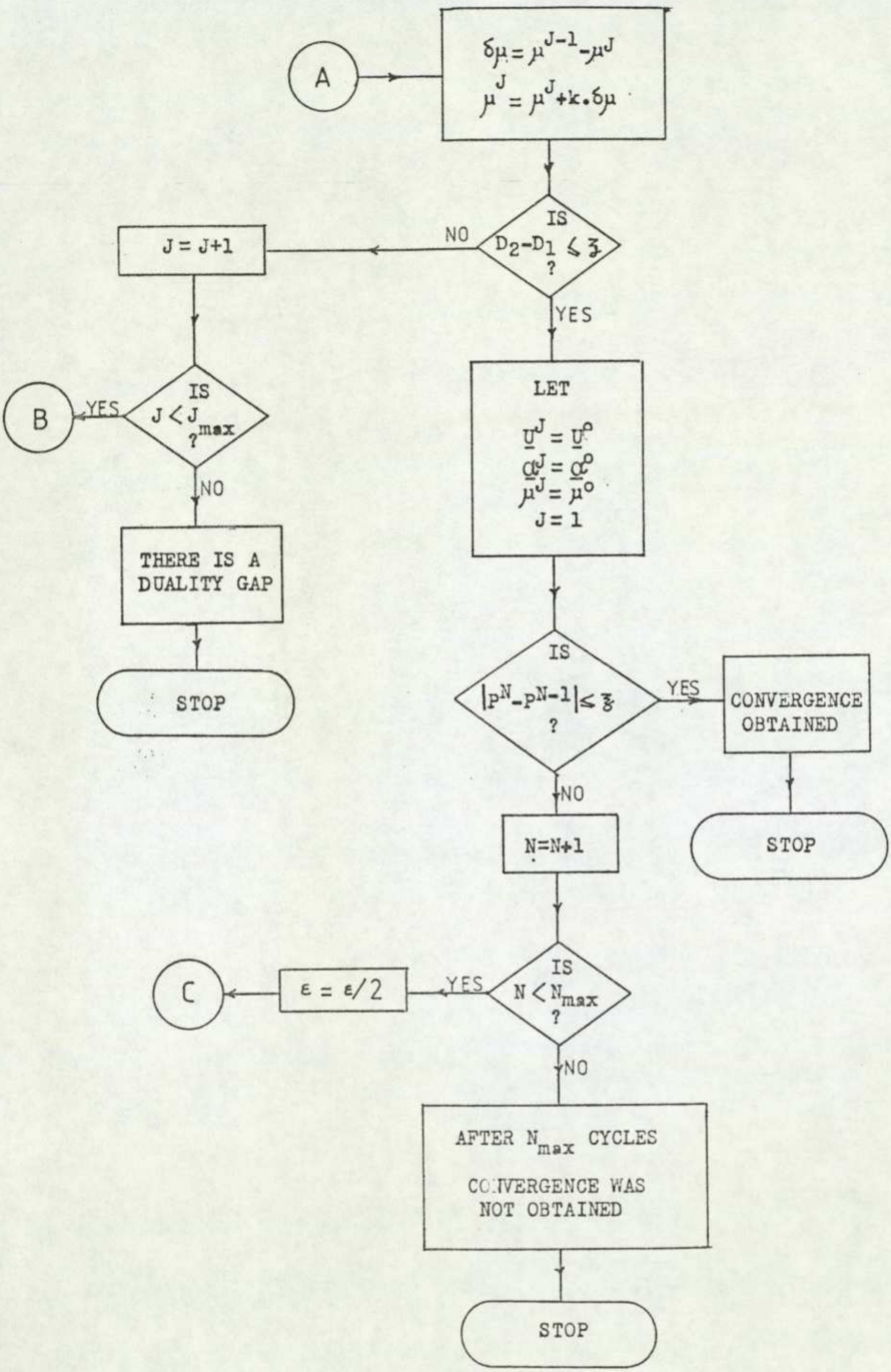
FIG(3.10) FLOW CHART OF THE TWO - STEP APPROACH.


START
INITIALIZE
F_B, T_R, R, A_1, A_2, θ¹, ɣ
LET
N 1
α¹ = [α1 α2] = [A1 A2]
U¹ = [U1 U2 U3] = [F_B T_R R]
IS
N = 1
?
YES
NO
LET
δα = α*N-1 - αN-1
αN = αN-1 + k.δα
αN
LET
δU = U*N-1 - UN-1
UN = UN-1 + k.δU
UN
FIND
Ŷ = [Ŷ1 Ŷ2] = [F̂R F̂P]
FROM REAL PLANT
α¹, U¹
MODIFY θ
θN = (1-ɣ)θN-1 - ɣ
B
A



FIG(3.11) FLOW CHART OF THE PARAMETRIC APPROACH


START
INITIALIZE
$F_B, T_R, R, A_1, A_2, \varepsilon, \mu$
LET
$\underline{\alpha}^o = \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = \begin{bmatrix}A_1\\ A_2\end{bmatrix}$
$\underline{U}^o = \begin{bmatrix}U_1\\ U_2\\ U_3\end{bmatrix} = \begin{bmatrix}F_B\\ T_R\\ R\end{bmatrix}$
J = 1
N = 1
C
FIND
$\hat{\underline{Y}} = \begin{bmatrix}\hat{Y}_1\\ \hat{Y}_2\end{bmatrix} = \begin{bmatrix}\hat{F}_R\\ \hat{F}_P\end{bmatrix}$
B
$D_1(\mu, \varepsilon) = \min_{\underline{U}, \underline{\alpha}} \left\{ P(\underline{U}, \underline{\alpha}) + \mu[G(\underline{U}, \underline{\alpha}) - \varepsilon] \right\}$
s.t. eqn.(3.12)
$\delta\underline{U} = \underline{U}^{J-1} - \underline{U}^J$
$\delta\underline{\alpha} = \underline{\alpha}^{J-1} - \underline{\alpha}^J$
$\underline{U}^J = \underline{U}^J + \underline{k}.\delta\underline{U}$
$\underline{\alpha}^J = \underline{\alpha}^J + \underline{k}.\delta\underline{\alpha}$
$D_2 = \max_{\mu} D_1(\mu, \varepsilon)$
s.t. $\mu \geqslant 0$
A



FIG(3.12) FLOW CHART OF THE ε-CONSTRAINT APPROACH.

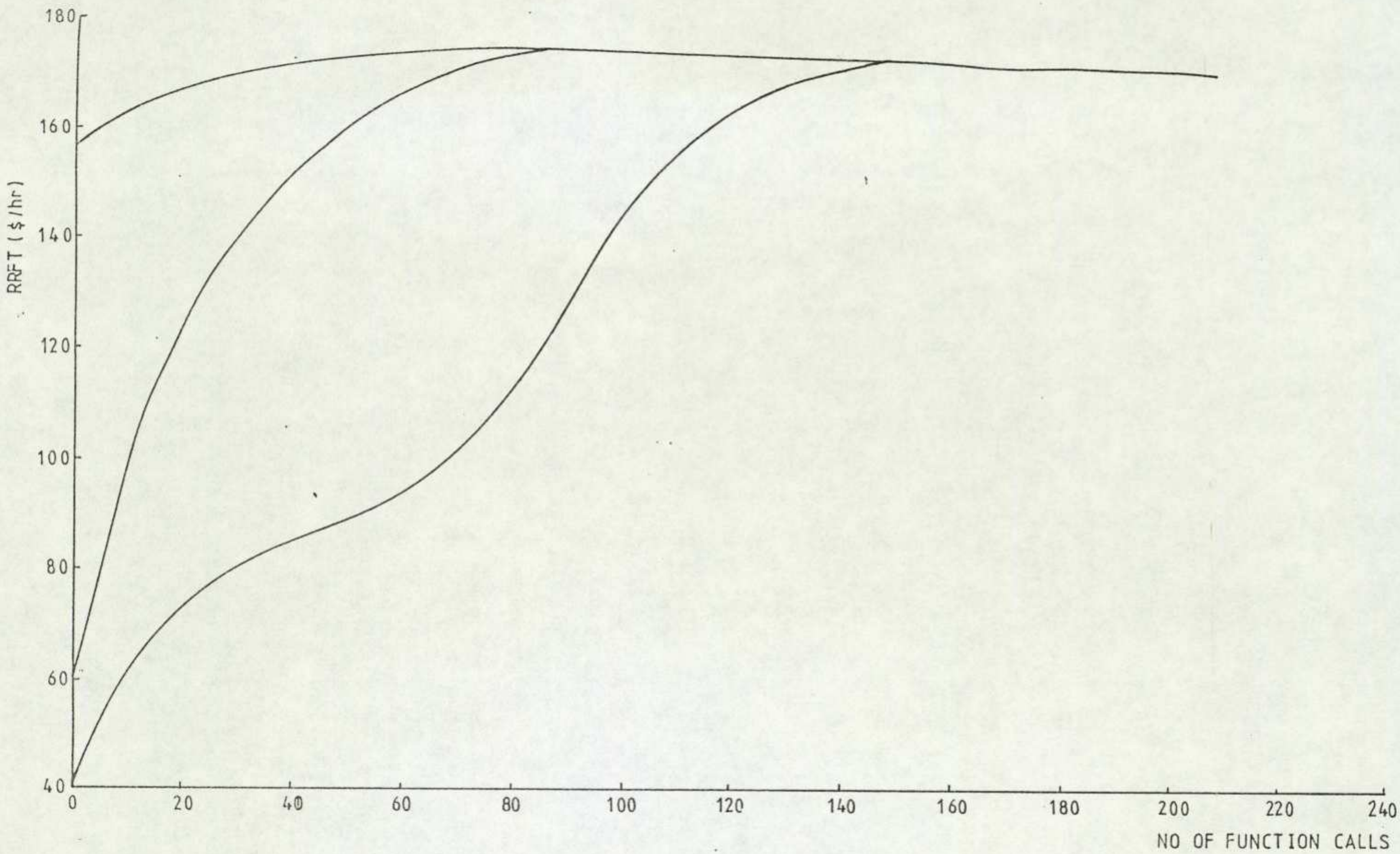


FIG.(3.13) OPTIMISATION OF THE REAL PLANT.

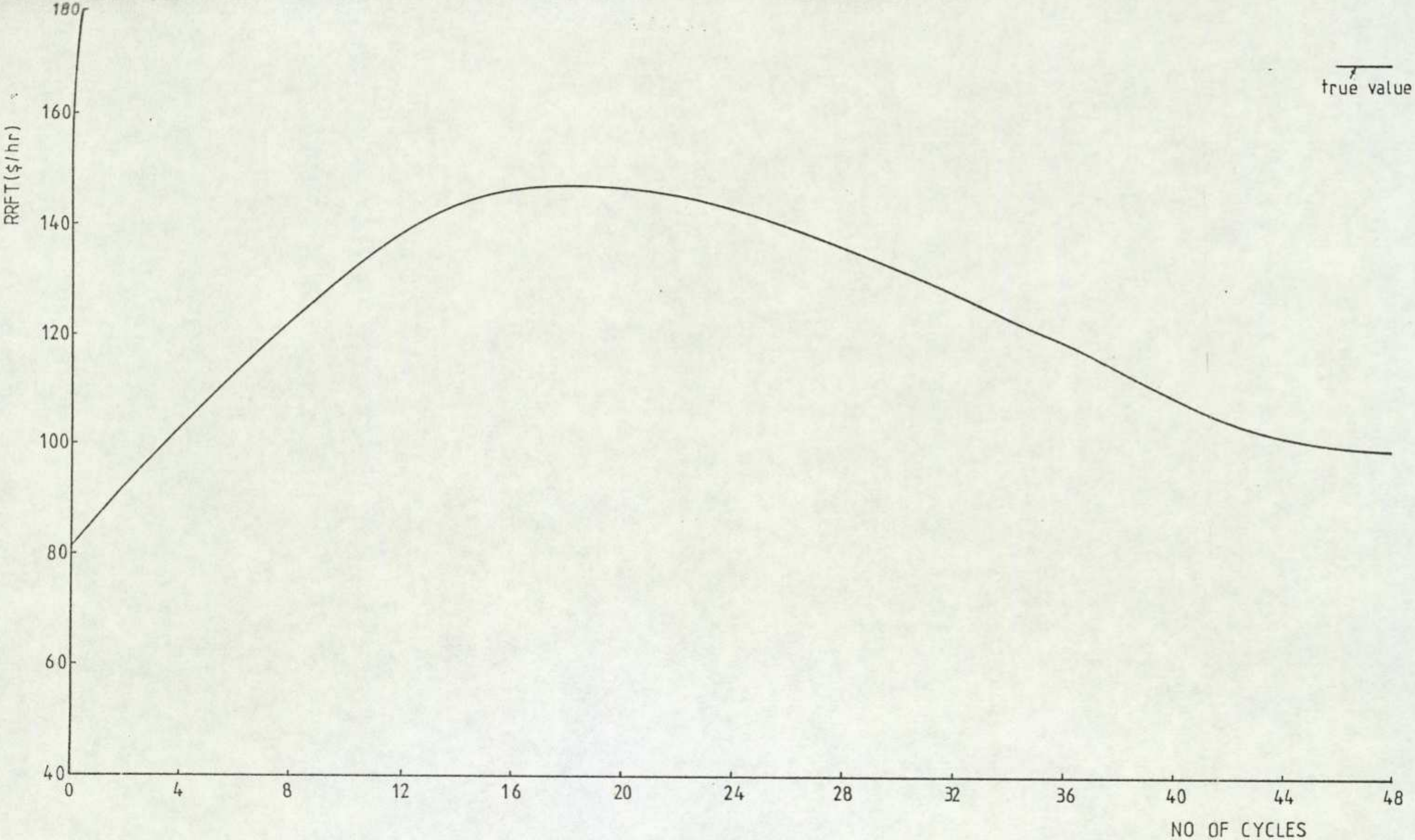


FIG.(3.14) VARIATION OF NET RATE OF RETURN USING THE STANDARD TWO - STEP APPROACH.

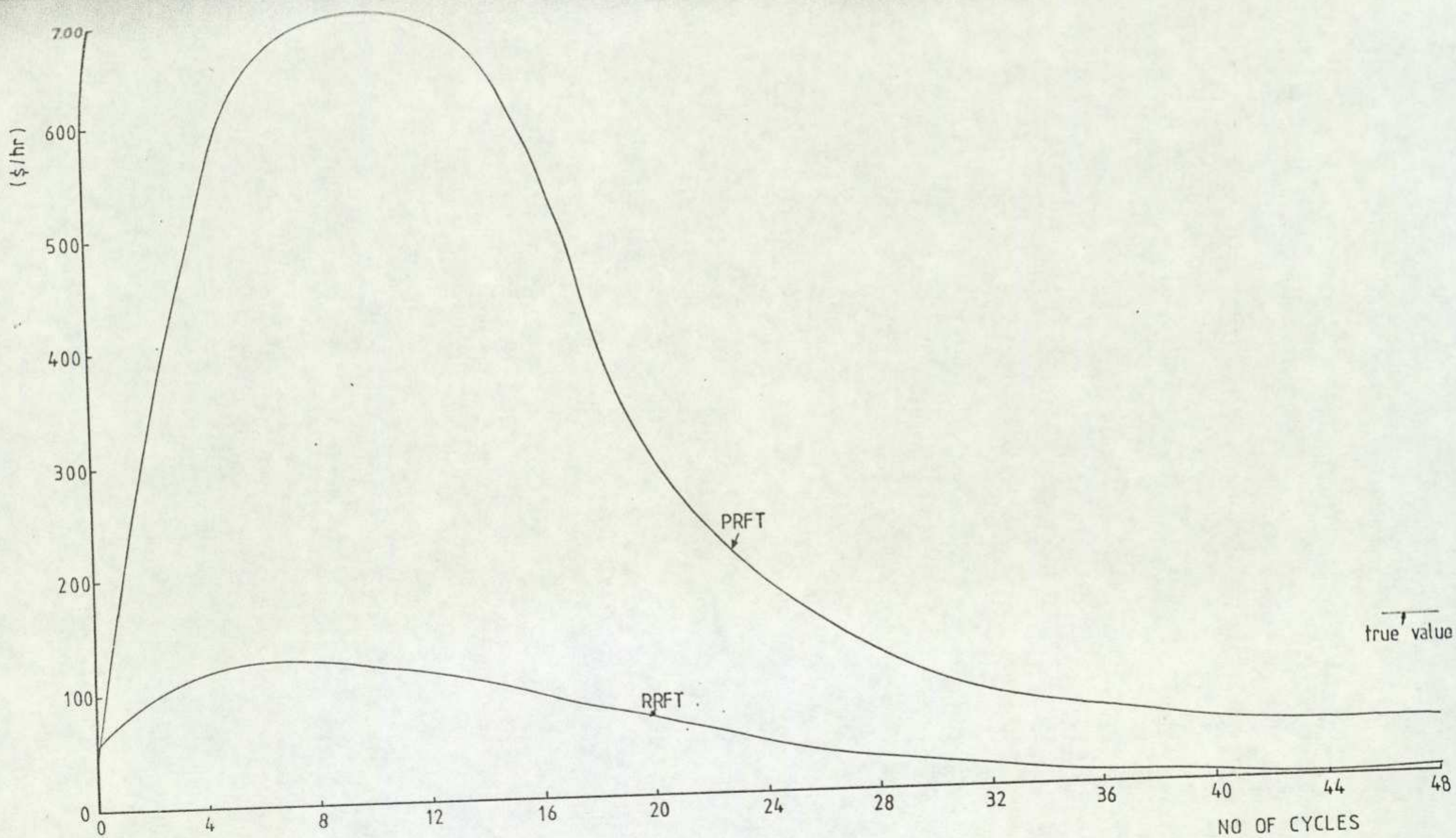


FIG.(3.15) VARIATION OF NET RATE OF RETURN USING THE PARAMETRIC APPROACH.

69

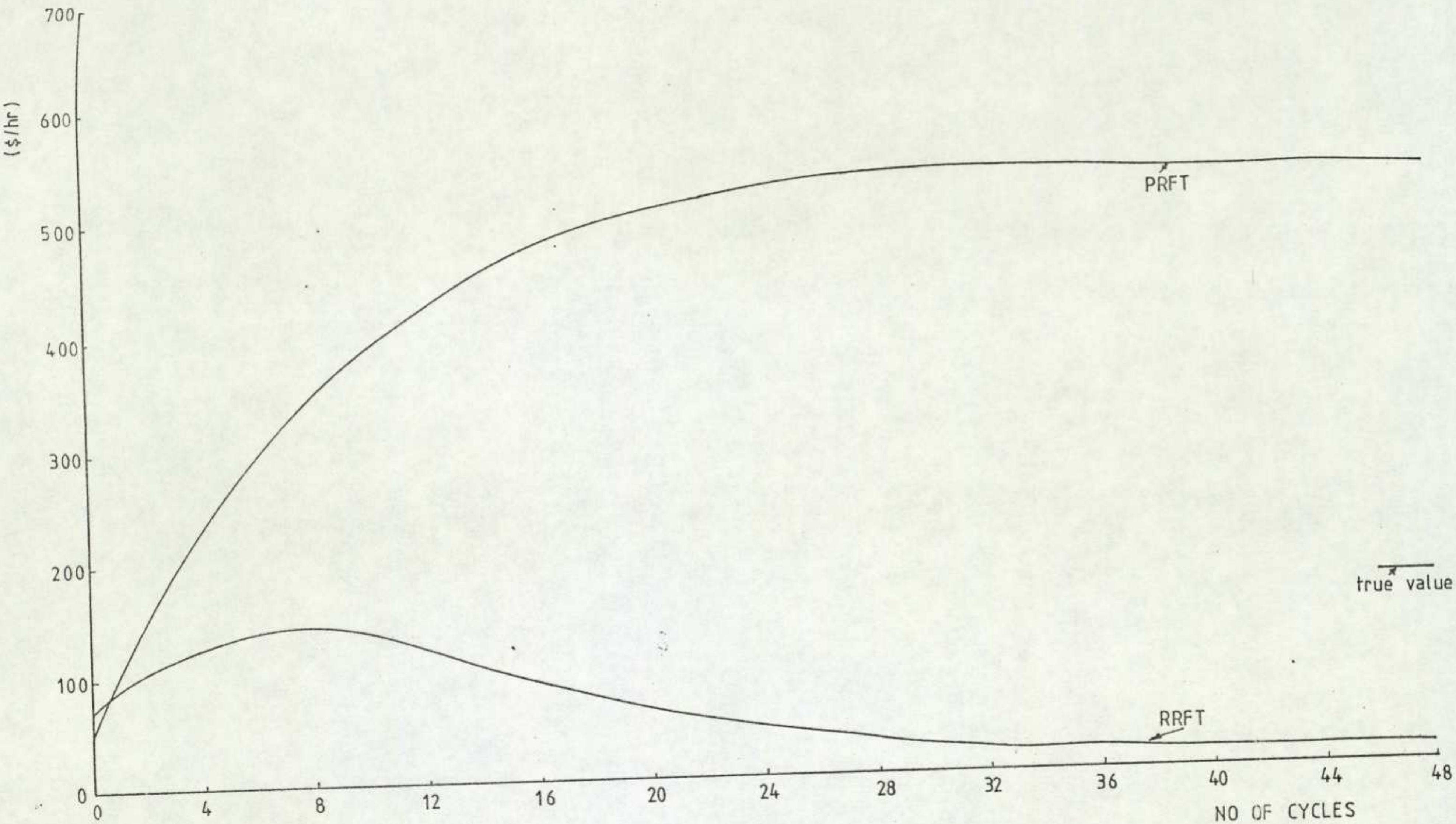


FIG.(3.16) VARIATION OF NET RATE OF RETURN USING THE ε- CONSTRAINT APPROACH.

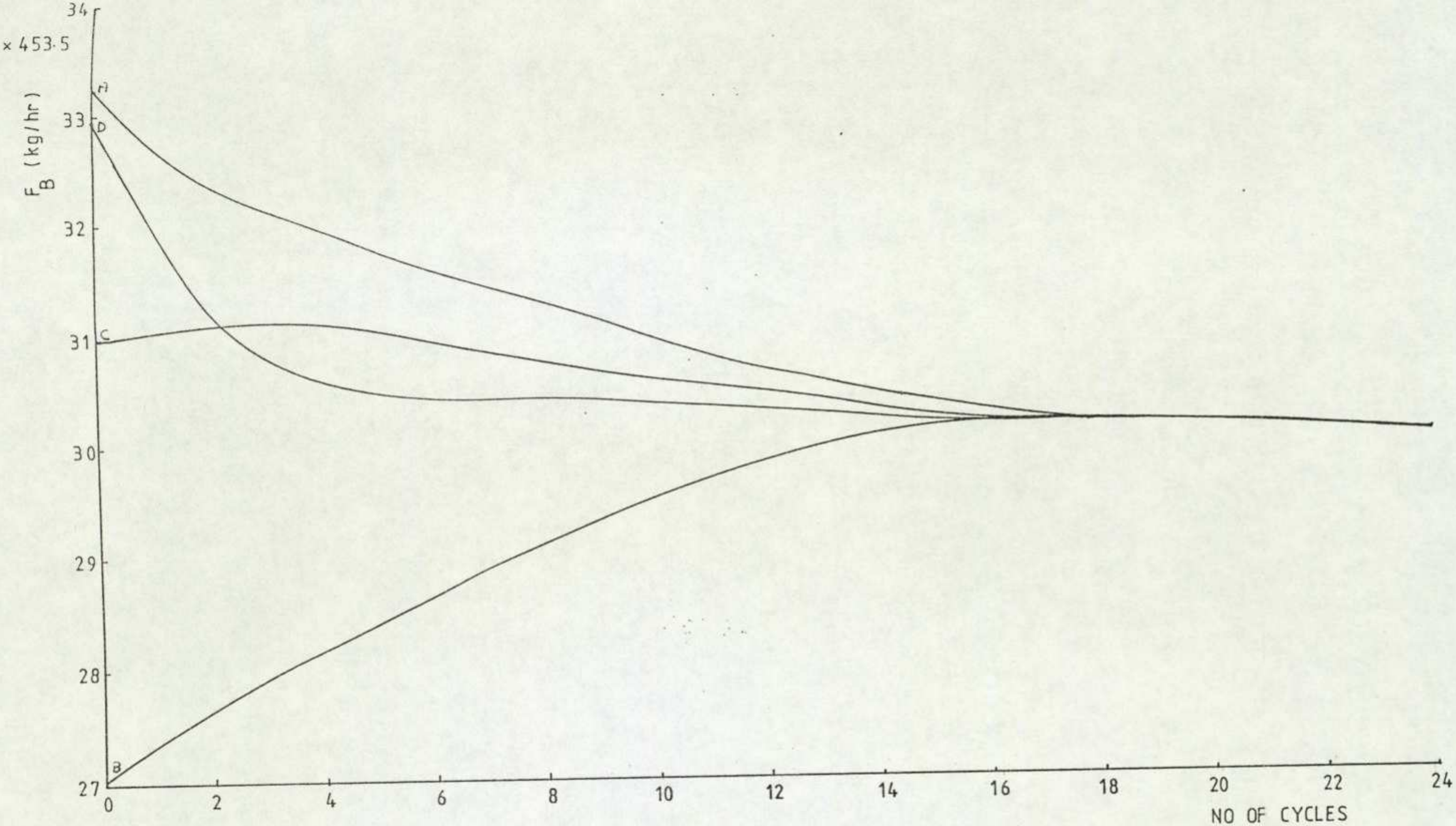


FIG.(3.17) VARIATION OF F_B DURING THE MODIFIED TWO - STEP APPROACH STARTING FROM DIFFERENT POINTS.

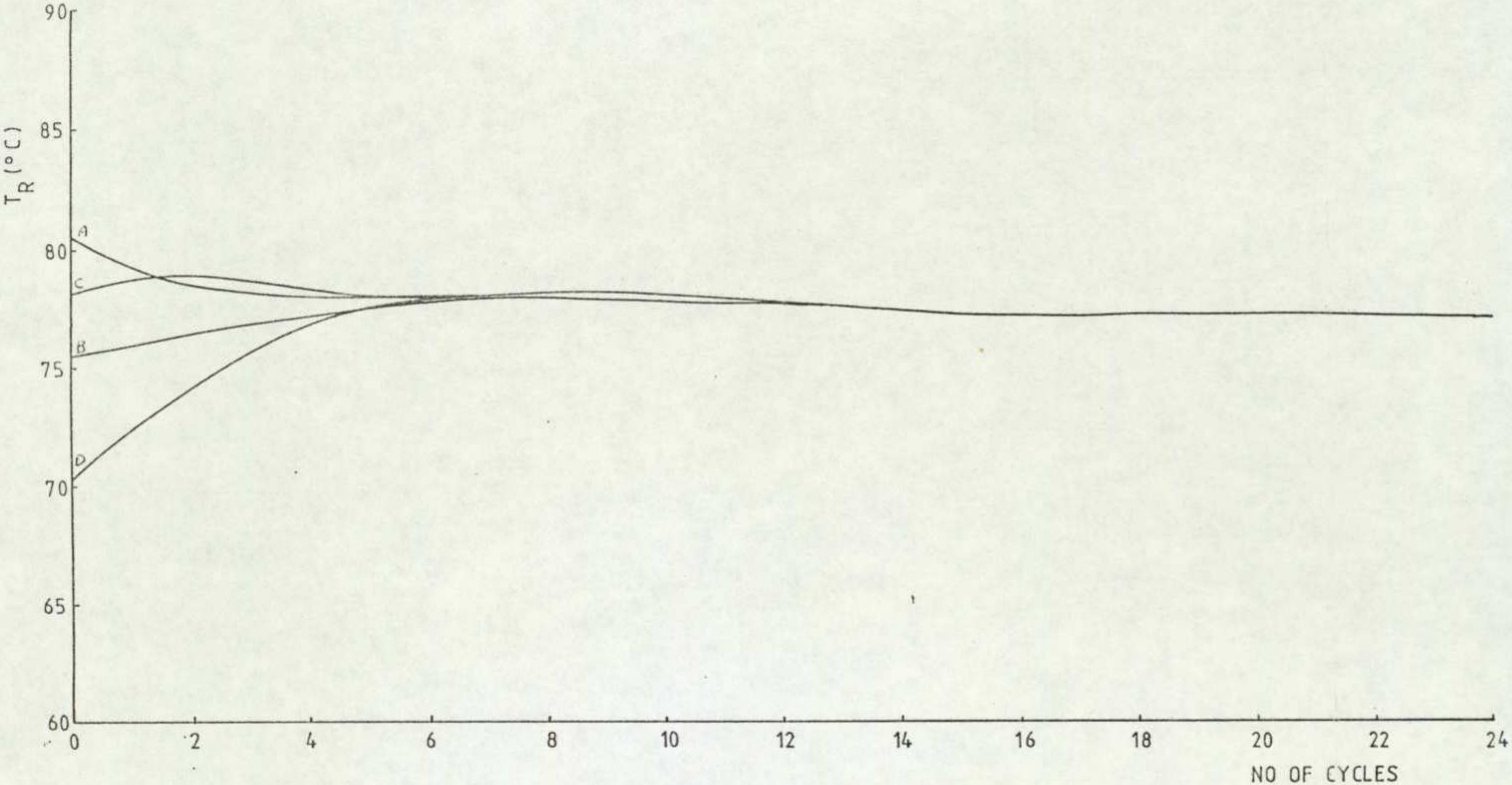


FIG.(3.18) VARIATION OF T_R DURING THE MODIFIED TWO - STEP APPROACH.

72

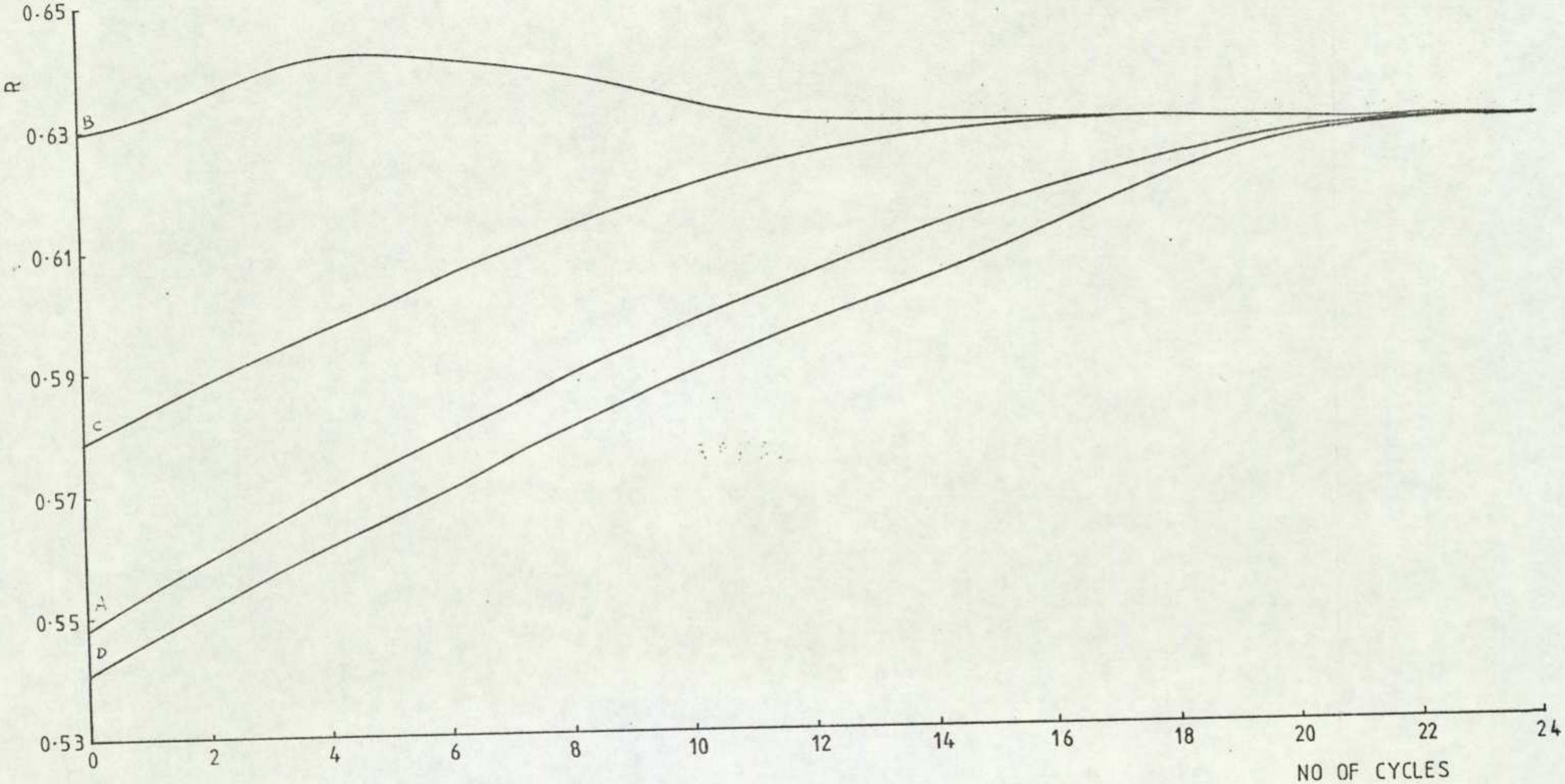


FIG.(3.19) VARIATION OF R DURING THE MODIFIED TWO - STEP APPROACH.

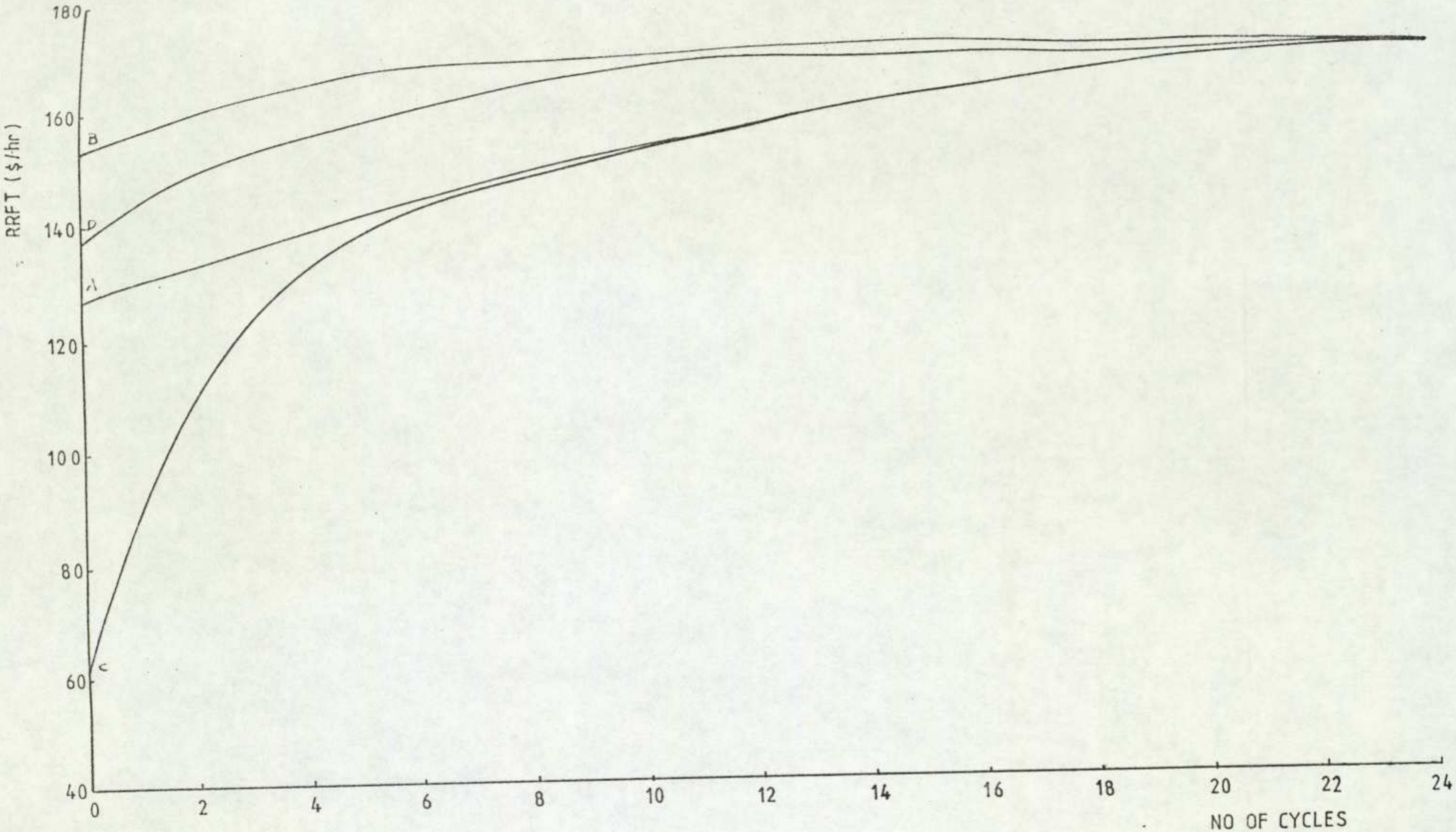


FIG.(3.20) VARIATION OF NET RATE OF RETURN DURING THE MODIFIED TWO - STEP APPROACH.

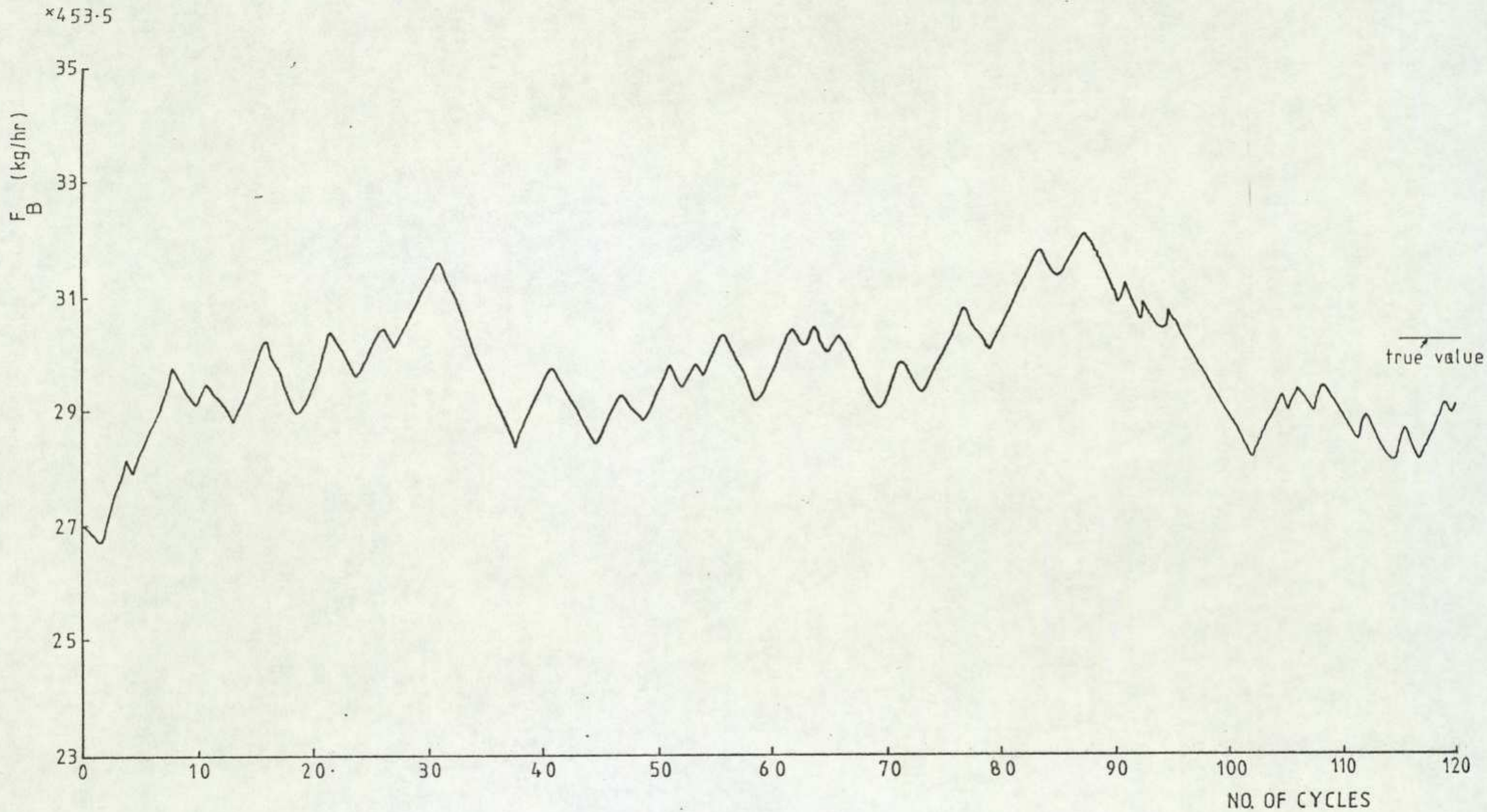


FIG.(3.21) EFFECT OF MEASUREMENT NOISE ON F_B.

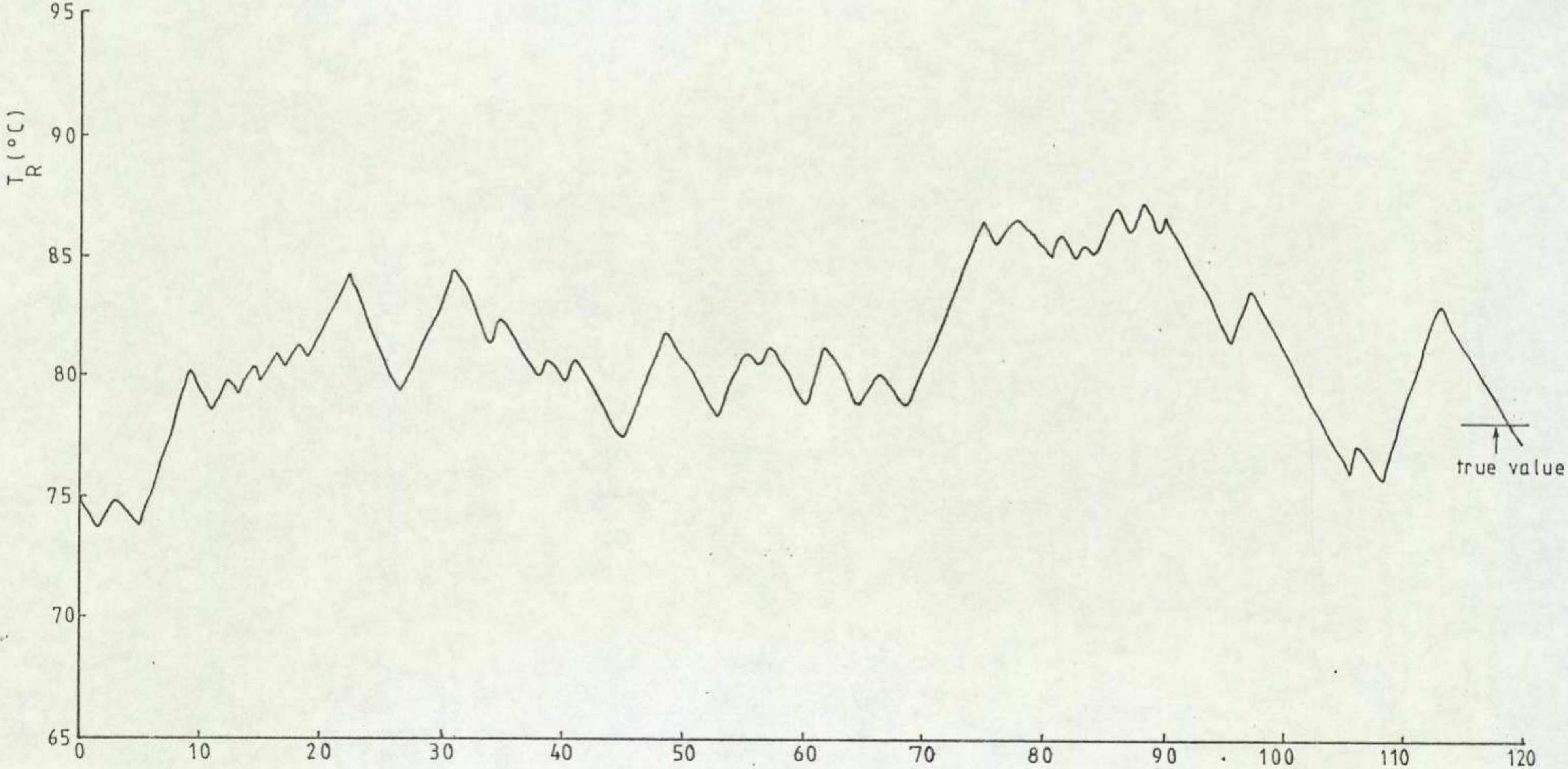


FIG.(3.22) EFFECT OF MEASUREMENT NOISE ON T_R

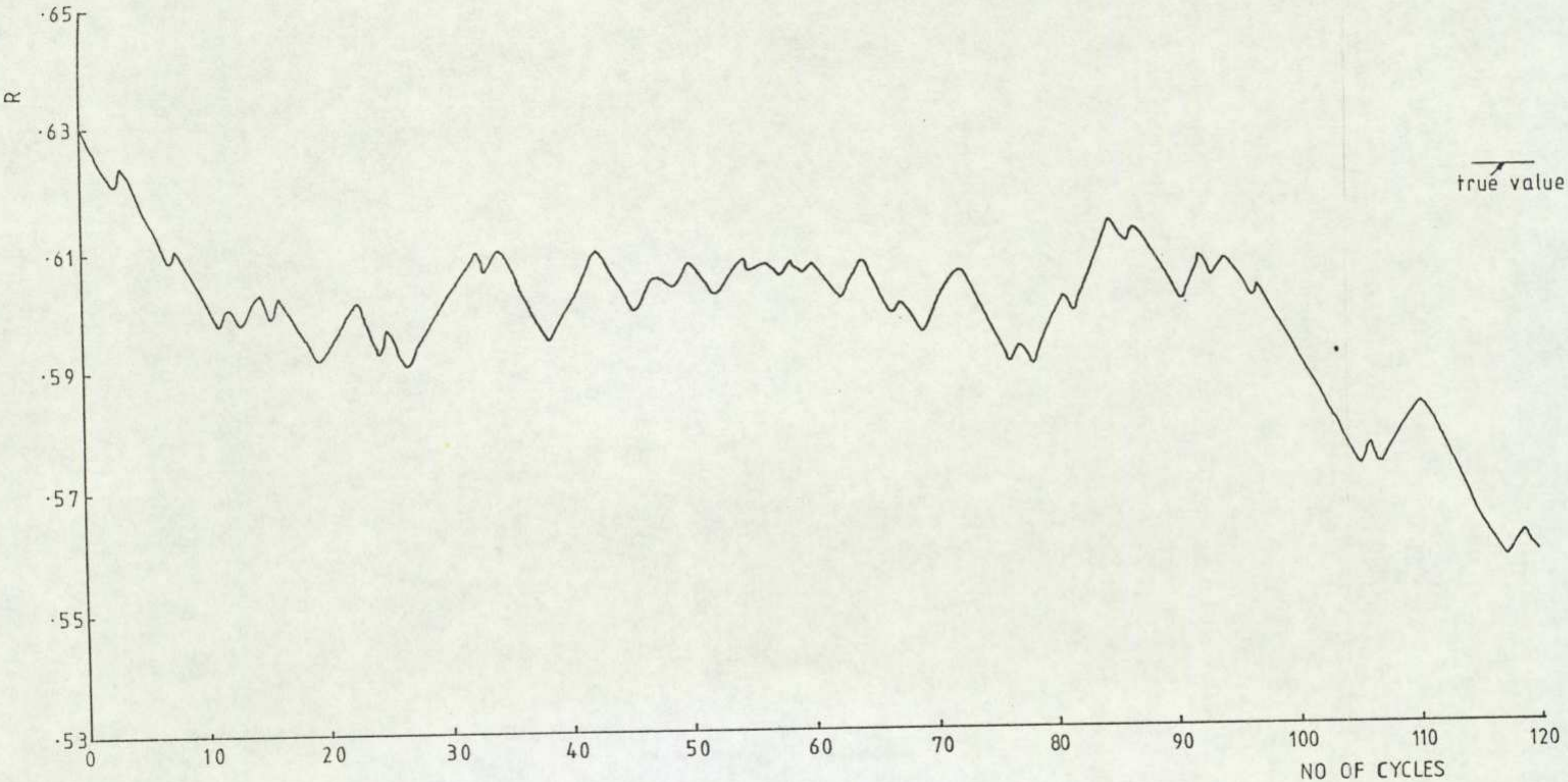


FIG.(3.23) EFFECT OF MEASUREMENT NOISE ON R.

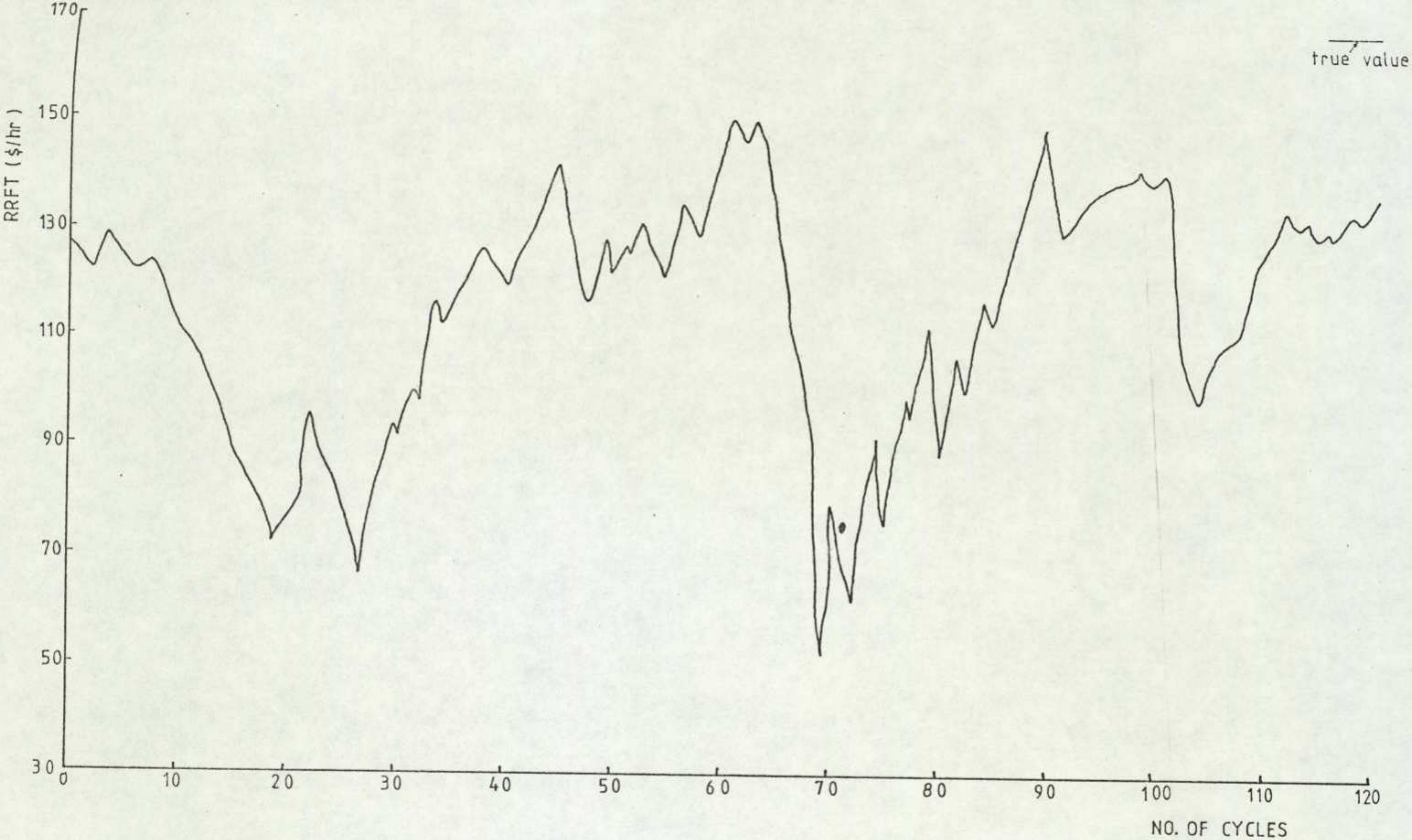


FIG.(3.24) EFFECT OF MEASUREMENT NOISE ON NET RATE OF RETURN.

CHAPTER 4: EFFECTS OF REAL PROCESS DYNAMICS ON THE PERFORMANCE OF THE MODIFIED TWO-STEP APPROACH

In general, the measurements of steady state outputs are obtained from a real dynamic process. Hence, a certain period of time will be required between application of a new set of inputs, $\underline{u}$, and the measurement of outputs, $\hat{\underline{y}}$. In practice, it is considered that this time period should be greater than the dominant time constant of the real system.

If a process is slow, with a long settling time, the optimal operating condition may be found in a relatively short period by predicting the steady state measurements (Bamberger and Iserman (1978)). This can be achieved by simulating the dynamic model during the transient response and estimating its parameters. These estimated parameters can then be updated as the steady state is achieved.

In the previous chapter the real plant measurements were obtained from a steady state mathematical model of the real plant. This procedure ignores the dynamic effects of the real process and determines the optimum steady state operating condition of the plant by manipulating controller set points. In this case the plant operates in a sequence of steady states.

In this chapter the effects of real process dynamics on the performance of the standard and the modified two-step approach have been investigated. The results obtained are compared and the effects of reducing or increasing the time between real plant measurements are investigated.

4.1 Description of the System

The system under investigation is the chemical plant whose steady state model has briefly been introduced in the previous chapter. The plant is taken from the generalised chemical processing model described by Williams and Otto (1960) to those interested in on-line computer control studies. Figure 4.1 illustrates a dynamic block diagram of the plant. The dynamic representation of each unit is described in the following sections.

4.1.1 Reactor

The chemical reactions which take place in the reactor have been given in chapter 3. These are:

$$
\left.\begin{array}{l}
A + B \xrightarrow{k_1} C \\
C + B \xrightarrow{k_2} P + E \\
P + C \xrightarrow{k_3} G
\end{array}\right\} \qquad \ldots\ldots\ldots\ldots\ldots \quad (4.1)
$$

where k_1, k_2 and k_3 are reaction coefficients whose values can be evaluated by using the Arrhenius equation,

$$k_i = A_i \exp(- B_i/T_R) \qquad \ldots\ldots\ldots\ldots\ldots \quad (4.2)$$

where A_i and B_i, i = 1, 2, 3, are known constants given by

$A_1 = 5.9755 \times 10^9$/hr, weight fraction

$B_1 = 6.666 \times 10^3$ ^oK, basis, 0.45359 kg of A or B

$A_2 = 2.5962 \times 10^2$/hr, weight fraction

$B_2 = 8.33 \times 10^3$ ^oK, basis, 0.45359 kg of B

$A_3 = 9.6283 \times 10^{15}$/hr, weight fraction

$B_3 = 1.11 \times 10^4$ ^oK, basis, 0.45359 kg of C

It should be noted that the volume dimension in the reaction coefficient, k, is "kg of mixture" rather than cubic meter. This is satisfactory since specific gravity is considered to be a constant in the reactor.

Heats of reaction of the three reactions are as follows:

Reaction 1: $H_1 = 2.90 \times 10^5$ J/kg of C produced

Reaction 2: $H_2 = 1.16 \times 10^5$ J/kg of E + P produced

Reaction 3: $H_3 = 3.32 \times 10^5$ J/kg of G produced ,

where all reactions are exothermic.

4.1.1.1 Reactors equations

4.1.1.1.1 Mass balance of the components

$$\frac{dA_R}{dt} = \frac{1}{V_R}(F_A + F_L.A_L - F_R.A_R - k_1.A_R.B_R.V_R) \quad \text{..........} \quad (4.3)$$

$$\frac{dB_R}{dt} = \frac{1}{V_R}(F_B + F_L.B_L - F_R.B_R - k_1.A_R.B_R.V_R - k_2.B_R.C_R.V_R) \quad (4.4)$$

$$\frac{dC_R}{dt} = \frac{1}{V_R}(F_L.C_L - F_R.C_R + 2.k_1.A_R.B_R.V_R - 2.k_2.B_R.C_R.V_R - k_3.C_R.P_R.V_R)$$

$$\text{...........} \quad (4.5)$$

$$\frac{dE_R}{dt} = \frac{1}{V_R}(F_L.E_L - F_R.E_R + 2.k_2.B_R.C_R.V_R) \quad \text{...........} \quad (4.6)$$

$$\frac{dP_R}{dt} = \frac{1}{V_R}(F_L.P_L - F_R.P_R + k_2.B_R.C_R.V_R - 0.5.k_3.C_R.P_R.V_R) \quad .. \quad (4.7)$$

$$\frac{dG_R}{dt} = \frac{1}{V_R}(1.5.k_3.C_R.P_R.V_R - F_R.G_R) \quad \text{...........} \quad (4.8)$$

The overall mass balance is

$$F_A + F_B + F_L - F_R = 0 \quad \text{...........} \quad (4.9)$$

where A_R, B_R, C_R, E_R, P_R and G_R are mass fractions of components A, B, C, E, P and G within the reactor product mass flow rate F_R. Similarly, A_L, B_L, C_L, E_L and P_L represent component mass fraction within the recycle mass flow rate, F_L, where it is assumed that component G is completely removed by the decanter. F_A and F_B are the mass flow rates of the raw materials, A and B. V_R is the effective liquid mass in the reactor.

4.1.1.1.2 Heat balance

All reactions are exothermic and the reaction temperature, T_R, is regulated by adjusting the flow rate of the cooling water, F_W.

$$\frac{dT_R}{dt} = \frac{1}{V_R.\rho_R.C_{PR}}\Big[2.k_1.A_R.B_R.V_R.H_1 + 3.k_2.B_R.C_R.V_R.H_2 +$$
$$1.5.k_3.C_R.P_R.V_R.H_3 - h_W.A_W(T_R - T_W) - F_L.C_{PL}(T_R - T_L) -$$
$$F_A.C_{PA}(T_R - T_A) - F_B.C_{PB}(T_R - T_B)\Big] \quad \text{...........} \quad (4.10)$$

$$\frac{dT_W}{dt} = \frac{1}{V_W \cdot \rho_W \cdot C_{PW}} \left[F_W \cdot C_{PW}(T_W - T_I) + h_W A_W (T_R - T_W) \right] \quad \ldots\ldots \quad (4.11)$$

where h_W is a heat transfer parameter; T_A, T_B, T_L, T_W and T_I are the temperature of the raw materials, A and B, recycle flow rate temperature and the outlet and inlet flows of the cooling water; C_{PA}, C_{PB}, C_{PW} and C_{PL} are the specific heats of component A, component B, cooling water and recycle flow rate; ρ_R and ρ_W are specific gravities of reaction and cooling water; A_W and V_W are the effective heat transfer area and volume of cooling water.

4.1.2 Decanter

It is considered that component G is completely removed by the decanter. Hence, the mass balance on component G gives the relationship

$$F_G = G_R \cdot F_R \quad \ldots\ldots\ldots\ldots\ldots \quad (4.12)$$

and the rest of the component mass balances within the decanter are:

$$\frac{dA_E}{dt} = \frac{1}{V_E}(F_R \cdot A_R - F_E \cdot A_E) \quad \ldots\ldots\ldots\ldots\ldots \quad (4.13)$$

$$\frac{dB_E}{dt} = \frac{1}{V_E}(F_R \cdot B_R - F_E \cdot B_E) \quad \ldots\ldots\ldots\ldots\ldots \quad (4.14)$$

$$\frac{dC_E}{dt} = \frac{1}{V_E}(F_R \cdot C_R - F_E \cdot C_E) \quad \ldots\ldots\ldots\ldots\ldots \quad (4.15)$$

$$\frac{dE_E}{dt} = \frac{1}{V_E}(F_R \cdot E_R - F_E \cdot E_E) \quad \ldots\ldots\ldots\ldots\ldots \quad (4.16)$$

$$\frac{dP_E}{dt} = \frac{1}{V_E}(F_R \cdot P_R - F_E \cdot P_E) \quad \ldots\ldots\ldots\ldots\ldots \quad (4.17)$$

Overall mass balance across the decanter will give

$$F_E - F_G - F_R = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (4.18)$$

where A_E, B_E, C_E, E_E and P_E represent mass fraction of components A, B, C, E and P within the decanter output flow rate F_E; F_G is the mass flow rate of the waste material G from the base of the decanter; and V_E is the effective liquid mass in the decanter.

4.1.3 Distillation column

In the following analysis it is assumed that the feed flow, F_E, to the distillation column contains components A, B, C, E and P only. Product P is separated from the feed flow F_E by distillation.

The distillation process has been considered as a column which is a well-mixed tank with a volume equal to total hold up of the column V_C, and a reboiler which likewise may be considered as another well-mixed tank with a volume equal to its given hold up, V_S. It has been shown (by Williams and Otto (1960)) that product P forms a high-boiling azeotrope with the intermediate by-product, E. Thus, some P appears in the column bottom stream, F_S, and therefore in the discard stream, F_D, and the recycle stream, F_L. The following relationship has been defined between the mass fractions of P and E:

$$P_L = 0.1\, E_L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.19)$$

4.1.3.1 Column only

The component mass balances within the column can be defined as

$$\frac{dA_C}{dt} = \frac{1}{V_C}\left[F_E.A_E - (2F_E - F_P)A_C + F_E.A_S\right] \quad \ldots\ldots\ldots\ldots \quad (4.20)$$

$$\frac{dB_C}{dt} = \frac{1}{V_C}\left[F_B.B_E - (2F_E - F_P)B_C + F_E.B_S\right] \quad \ldots\ldots\ldots\ldots \quad (4.21)$$

$$\frac{dC_C}{dt} = \frac{1}{V_C}\left[F_E.C_E - (2F_E - F_P)C_C + F_E.C_S\right] \quad \ldots\ldots\ldots\ldots \quad (4.22)$$

$$\frac{dE_C}{dt} = \frac{1}{V_C}\left[F_E.E_E - (2F_E - F_P)E_C + F_E.E_S\right] \quad \ldots\ldots\ldots\ldots \quad (4.23)$$

$$\frac{dP_C}{dt} = \frac{1}{V_C}\left[0.1\, F_E.P_E - (2F_E - F_P)P_C + F_E.P_S\right] \quad \ldots\ldots\ldots \quad (4.24)$$

Overall mass balance across column will give

$$F_P \;+\; (P_E - 0.1\, E_E)F_E \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.25)$$

where A_C, B_C, C_C, E_C and P_C represent mass fraction of components A, B, C, E and P within the column; A_S, B_S, C_S, E_S and P_S are the mass fraction of components A, B, C, E and P in the reboiler stream, F_S.

4.1.3.2 Reboiler

$$\frac{dA_S}{dt} = \frac{1}{V_S}\left[(2F_E - F_P)A_C - F_S.A_S - F_E.A_S\right] \quad \text{..........} \quad (4.26)$$

$$\frac{dB_S}{dt} = \frac{1}{V_S}\left[(2F_E - F_P)B_C - F_S.B_S - F_E.B_S\right] \quad \text{..........} \quad (4.27)$$

$$\frac{dC_S}{dt} = \frac{1}{V_S}\left[(2F_E - F_P)C_C - F_S.C_S - F_E.C_S\right] \quad \text{..........} \quad (4.28)$$

$$\frac{dE_S}{dt} = \frac{1}{V_S}\left[(2F_E - F_P)E_C - F_S.E_S - F_E.E_S\right] \quad \text{..........} \quad (4.29)$$

$$\frac{dP_S}{dt} = \frac{1}{V_S}\left[(2F_E - F_P)P_C - F_S.P_S - F_E.P_S\right] \quad \text{..........} \quad (4.30)$$

The overall mass balance across the distillation column (column and reboiler) is

$$F_E - F_P - F_S = 0 \quad \text{..........} \quad (4.31)$$

4.1.4 Recycle control

The recycle stream, F_L, is regulated as a certain fraction of the column bottoms flow, F_S, and returns it to the reactor. The discard stream, F_D, then takes all excess. Hence,

$$F_S = F_D + F_L \quad \text{..........} \quad (4.32)$$

$$\frac{F_L}{F_S} = R = \text{constant} \quad \text{..........} \quad (4.33)$$

The mass fraction, A_L, B_L, C_L, E_L and P_L, appearing in the recycle stream, F_L, and also in the column bottom stream, F_S, can be expressed as follows:

$$\left.\begin{aligned} A_L &= A_S \\ B_L &= B_S \\ C_L &= C_S \\ E_L &= E_S \\ P_L &= P_S \end{aligned}\right\} \quad \text{..........} \quad (4.34)$$

Equations (4.1) to (4.34) consist of a set of first-order differential equations together with some algebraic relationships which constitute the dynamic model of the real plant. Some of the equations are non-linear and the dynamic response of the simulated real chemical plant is achieved by using numerical integration routine DO2AJF (see Appendix A1).

The digital simulation of the chemical plant also involves the dynamic equations of the appropriate feedback controllers for regulating the flow rate, F_B; the reaction temperature, T_R, by adjusting the flow rate of cooling water; and the recycle ratio, R. These are simulated as PI controllers without integral wind-up, acting on control valve actuators simulated as single time-constant with velocity limitation. Feedback measurements are obtained from transducers which are also simulated as single time constant with appropriate scaling and limiting (see Appendix A1). Block diagrams of the above process control instrumentations are given in Appendix A2.

4.2 Description of the Model

The model is identical to that which has been described in section 3.2, with the same unknown parameters, parameter estimation problem and optimisation problem.

4.3 The Modified Two-Step Approach

The modified optimisation problem, the parameter estimation problem and the coordination problem, which have been represented in chapter 3 by equations (3.70) to (3.73), can be repeated below as

Modified optimisation problem

$$\left.\begin{array}{l} \min\limits_{F_B,\, T_R, R} \{-P(F_B,\, T_R,\, R, \sigma_1,\, \sigma_2) - \lambda_1 F_B - \lambda_2\, T_R - \lambda_3 R\} \\ \qquad \text{s.t.} \quad \text{model equations and given } \sigma_1,\ \sigma_2 \text{ and } \underline{\lambda}. \end{array}\right\} \quad \ldots\ldots \quad (4.35)$$

Parameter estimation problem

$$\left.\begin{array}{l} \underline{y}(\underline{v},\, A_1,\, A_2) \;=\; \hat{\underline{y}}(\underline{v}) \;\rightarrow\; A_1,\, A_2 \\ \qquad \text{s.t.} \quad \text{model equations} \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (4.36)$$

Coordination problem:

$$\left.\begin{aligned} \sigma_1 &= A_1 \\ \sigma_2 &= A_2 \\ F_B &= v_1 \\ T_R &= v_2 \\ R &= v_3 \\ \underline{\lambda} &= \left\{ \left[\frac{\partial \underline{y}}{\partial \underline{v}}\right]' - \left[\frac{\partial \hat{\underline{y}}}{\partial \underline{v}}\right]' \right\} \left\{ \left[\frac{\partial \underline{y}'}{\partial \underline{A}}\right]^{-1} \right\} \frac{\partial p}{\partial \underline{\sigma}} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots \quad (4.37)$$

The task of each individual unit is identical to that of the previous chapter. But in this case the real process measurements $\hat{\underline{y}}(\underline{v}) = [\hat{F}_R \; \hat{F}_P]'$ are taken from the dynamic model of the system, rather than its steady state model. The required derivatives of $\hat{F}_R$ and $\hat{F}_P$, with respect to the set points, $\underline{v} = [\hat{F}_B \; \hat{T}_R \; \hat{R}]'$, are evaluated by perturbing the system. The modified two-step algorithm assumes that a steady state condition is achieved on the real system before changing to a new set point. This assumption will ignore the dynamic effects of the real process. Hence, due to inaccuracies in process measurements and consequent rise of error in the evaluation of the real process derivatives, it is essential to investigate the effect of the real plant measurements and transient response on the process performance of the above algorithm.

4.4 Results

In order to investigate the effects of real process dynamics on the performance of the modified two-step approach, a dynamic mathematical model of the real plant, together with its associated controllers and instruments, simplified steady state model, parameter estimation problem, optimisation problem and coordination unit described by equations (4.1) to (4.34), (3.64), (4.35), (4.36) and (4.37), respectively, have been simulated on a digital computer.

Initially, the control loops were tuned by selecting appropriate tuning parameters for each loop. This is achieved by performing open loop step response tests on the simulation of the real plant. The measurement time, T_m, was selected to be 60 minutes, where it is assumed that the real process achieves its new steady state condition within that time.

In this investigation it is also assumed that the same inequality constraints which have been obtained in section 3.4.1 are used as a priori knowledge of the real plant in order to limit the area of search for the optimum values of controller set points, $\underline{v} = [\hat{F}_B \ \hat{T}_R \ \hat{R}]'$.

Starting from two alternative initial conditions, A (F_B = 14965 kg/hr, T_R = 71.5°C, R = 0.55) and B (F_B = 15124 kg/hr, T_R = 82.5°C, R = 0.54), Figure 4.2 shows the performance of the modified two-step approach, with measurement time, T_m, equal to 60 minutes. Figures 4.3, 4.4 and 4.5 illustrate the approach of the controllers set point values to their optimal operating point together with a limit cycle type behaviour after 18 iterations, with initial condition starting from A. The controller set points oscillate between the following values:

F_B between 13151.5 kg/hr and 13605 kg/hr

T_R between 79.66°C and 80.11°C

R between .615 and .624

For particular initial condition C (F_B = 15124 kg/hr, T_R = 79.8 °C, R = 0.54), Figure 4.6 shows the performance of the standard two-step approach, as compared with that of the modified one. The results clearly illustrate the unsatisfactory performance of the standard approach.

In the above simulation study, it is assumed that the steady state operating condition is achieved within 60 minutes, before changing the controller set point values to a new point. In order to investigate the effect of real process dynamics on the performance of the modified algorithm, the measurement time, T_m, was reduced to as low as 1 minute. Starting from the initial point C, Figure 4.7 illustrates how the return accumulates over the first 24 hours of operation of the plant. The results show the good performance of the modified two-step algorithm with different measurement times, even when the measurement time, T_m, is reduced to 1 minute. The results also show a considerable deterioration in the behaviour of the modified two-step approach with measurement time, T_m, equal to 5 minutes. This could be due to the fact that the turning points of the temperature response (Figure 4.8) occur approximately every 5 minutes, which could give rise to an error in evaluation of $\underline{\lambda}$. However, further investigation is required to study the cause of this deterioration. Table 4.1 shows the final net rate of return found from the slopes of each curve given in Figure 4.7.

Measurement time T_m (mins)	Final rate of return ($/hr)
60	162
30	162
15	156
10	161
5	159
2	165
1	165

Table 4.1: Net rate of return after 24 hours

Starting from initial condition C, the above simulation experiments illustrate that the greatest net rate of return has been achieved with measurement time of 2 minutes.

Figure 4.8 shows how the flow control loop and recycle control loop follow their set point changes, whereas a considerable slow response of the temperature control loop can be observed. This will give rise to consequent errors in the steady state temperature measurements, which are used to calculate the Lagrangian modifiers, $\underline{\lambda}$. Despite these inaccuracies in calculation of $\underline{\lambda}$, the results illustrate satisfactory overall performance and the robust nature of the algorithm.

In this investigation, with a measurement time, T_m, the modified optimisation problem is solved every 4 x T_m minutes due to computation requirements of the real plant output derivatives during the calculation of the modifiers $\underline{\lambda}$.

4.5 Conclusions

The simulation study of the chemical plant has been used in order to investigate the effect of real system dynamics on the performance of the modified two-step approach. The investigation has shown that the modified algorithm successfully achieves the correct optimal operating condition on the real plant by manipulating the set points of flow rate, $\hat{F}_B$, reaction temperature, $\hat{T}_R$, and the recycle ratio, $\hat{R}$, controllers. This will result in a maximised net rate of return from the plant. Although the measurement time has been reduced to considerably low values, the algorithm is shown to be satisfactory together with its robust nature.

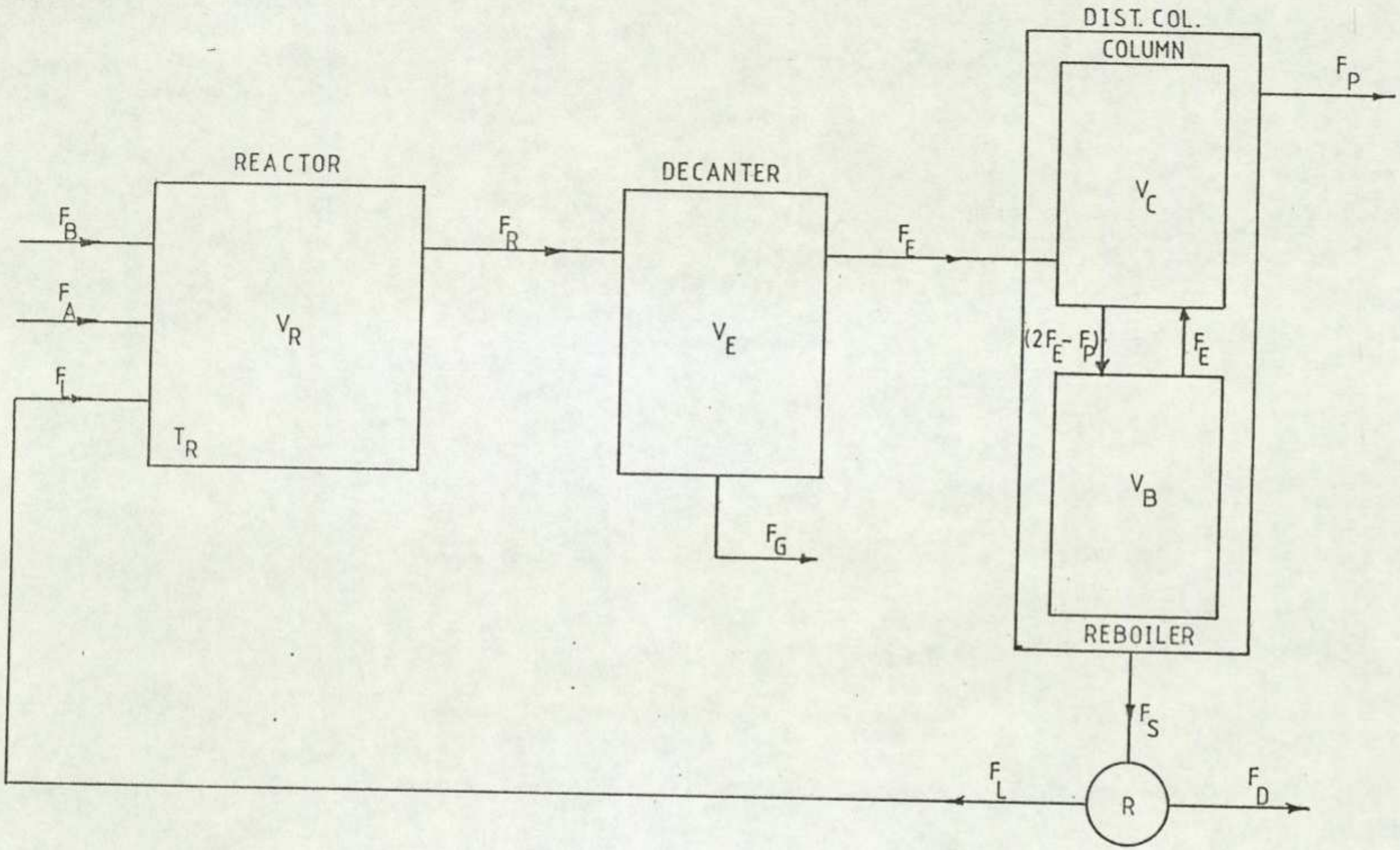


FIG.(4.1) APPROXIMATE DYNAMIC REPRESENTATION OF THE CHEMICAL PLANT.

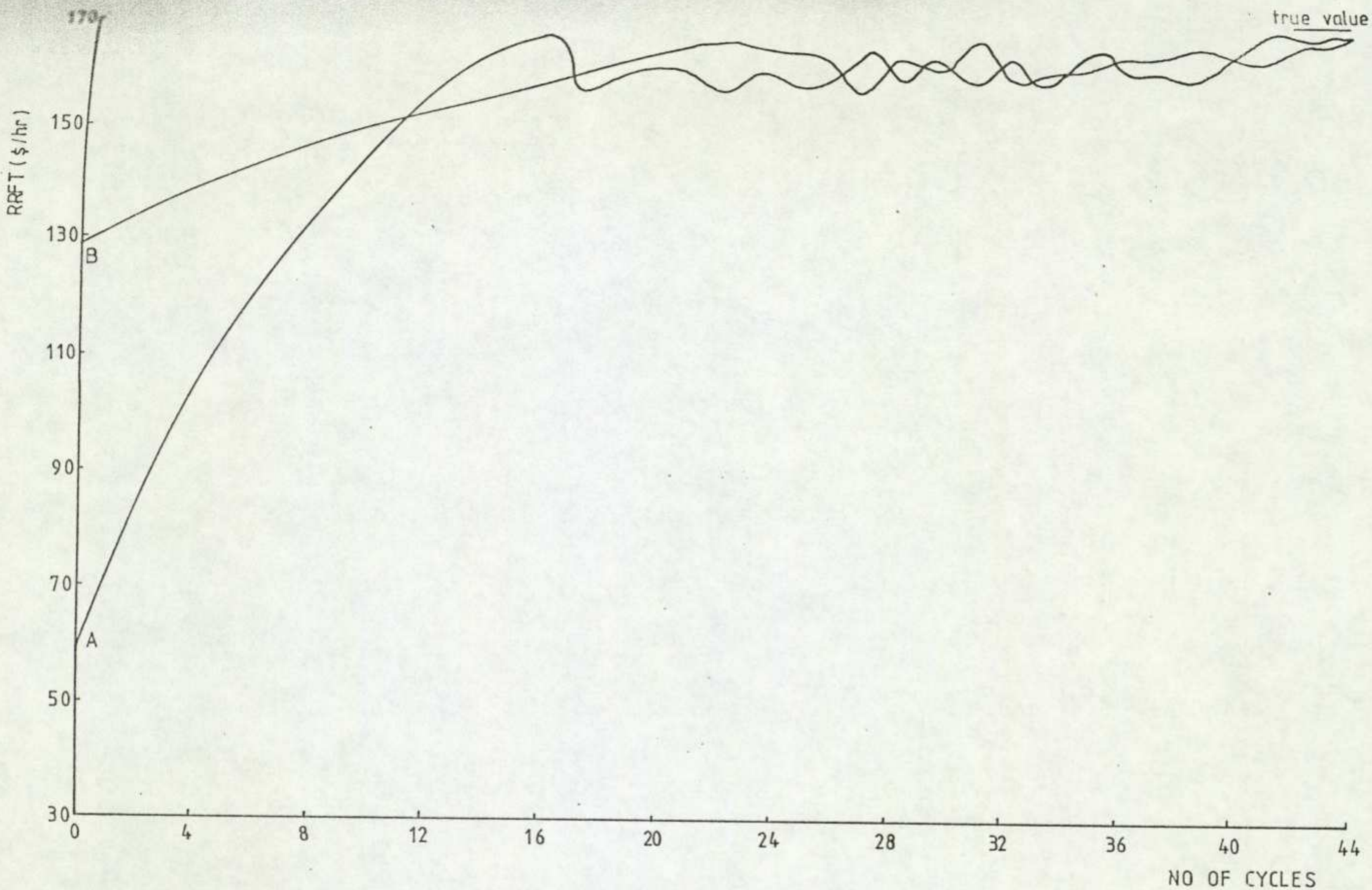


FIG.(4.2) PERFORMANCE OF THE MODIFIED TWO - STEP APPROACH STARTING FROM POINTS A AND B.

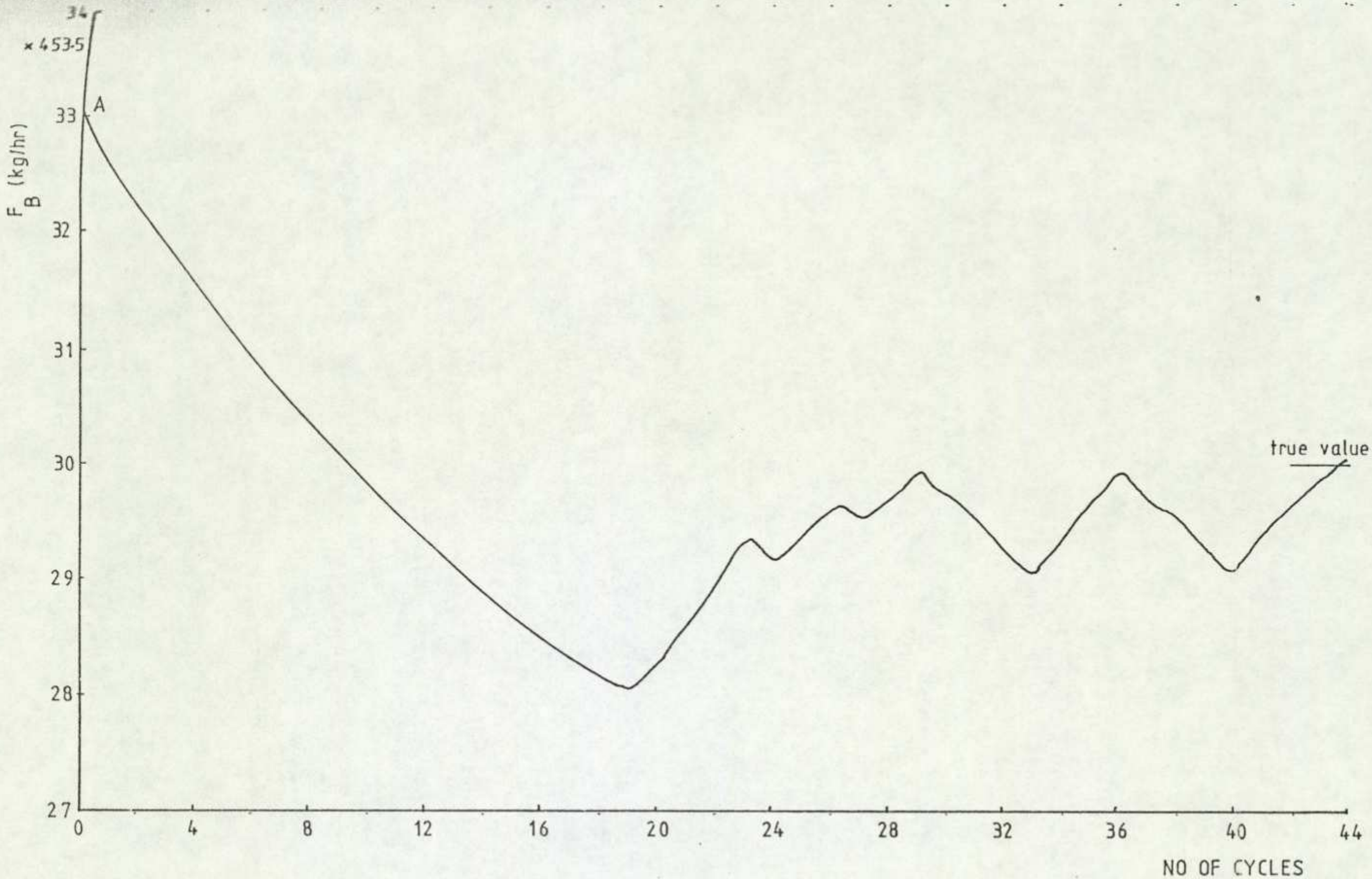


FIG.(4.3) VARIATION OF F_B DURING THE MODIFIED TWO - STEP APPROACH WITH T_m = 60 MINUTES.

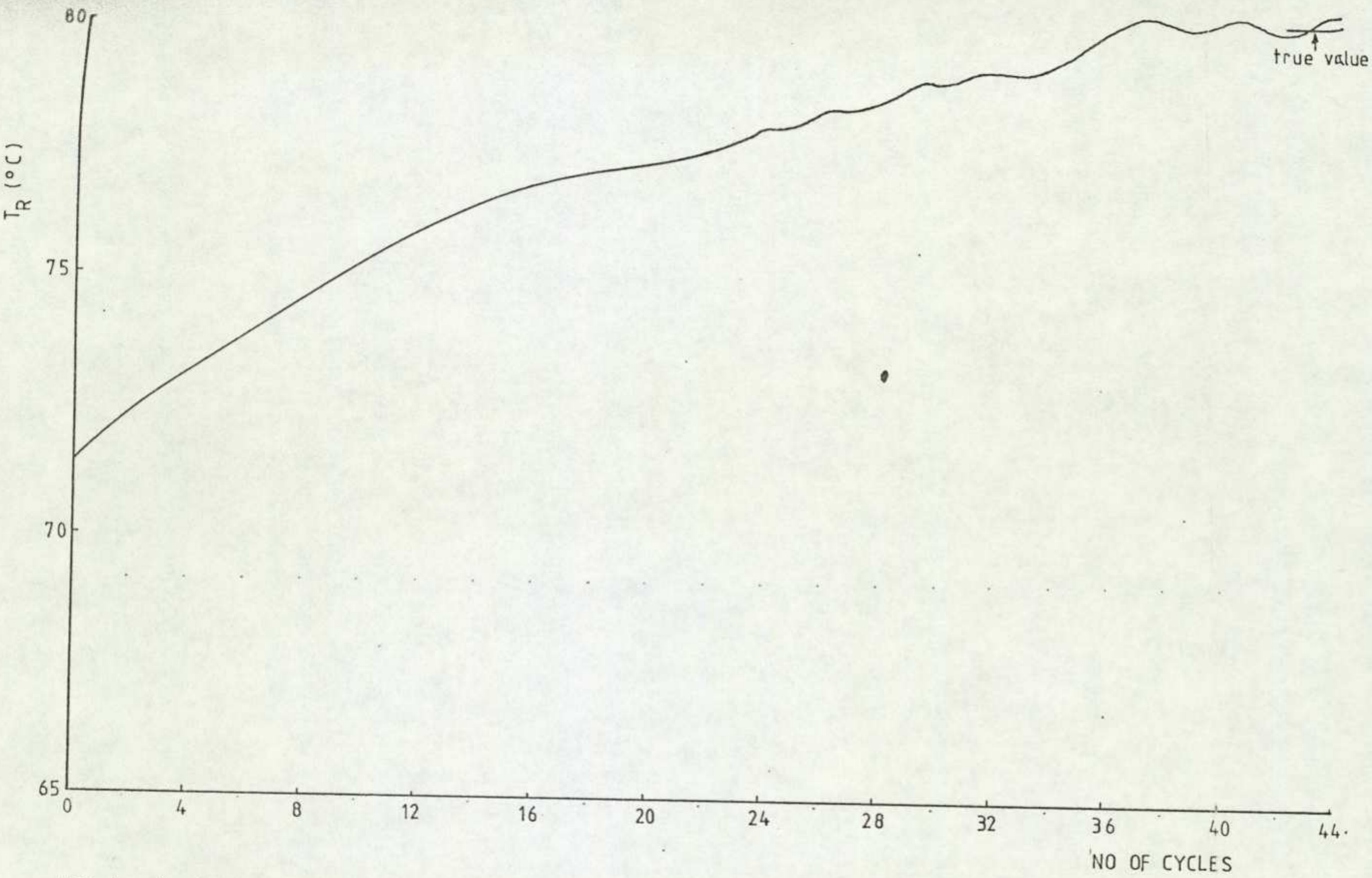


FIG.(4.4) VARIATION OF T_R DURING THE MODIFIED TWO - STEP APPROACH WITH T_m = 60 MINUTES.

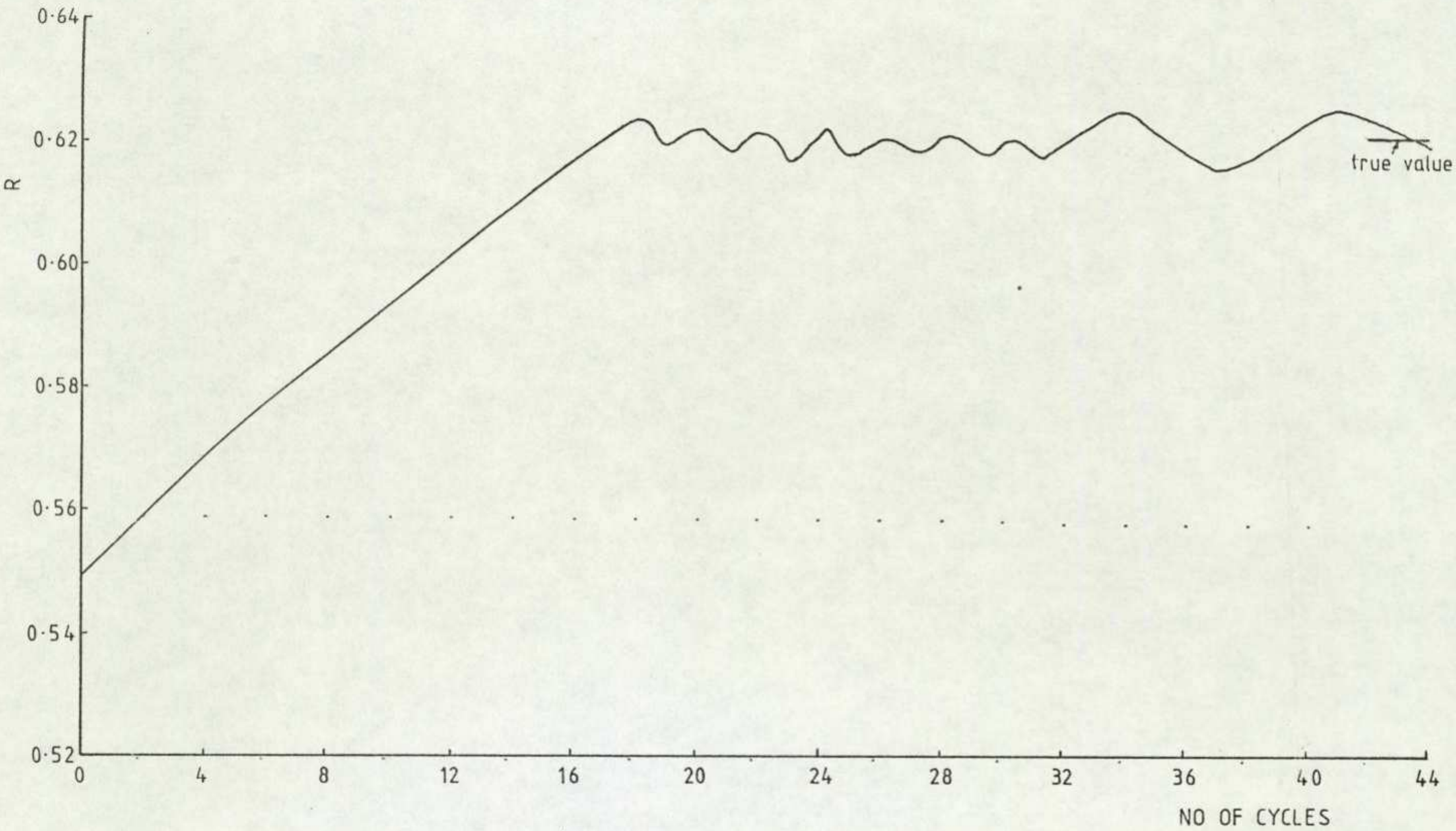


FIG.(4.5) VARIATION OF R DURING THE MODIFIED TWO - STEP APPROACH WITH T_m = 60 MINUTES.

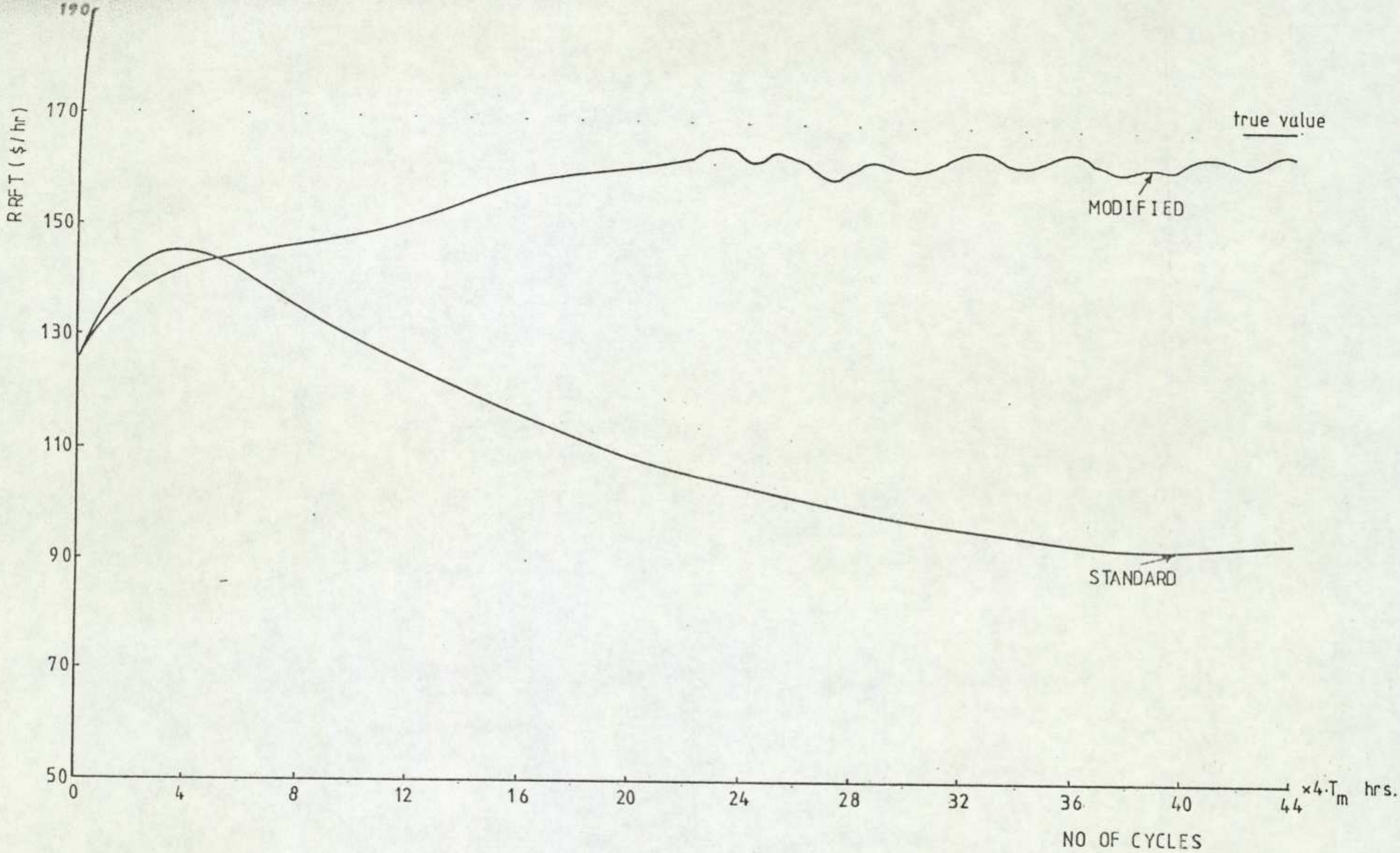


FIG.(4.6) PERFORMANCE OF THE STANDARD AND MODIFIED TWO - STEP APPROACHES WITH MEASUREMENT TIME T_m = 60 MINUTES.

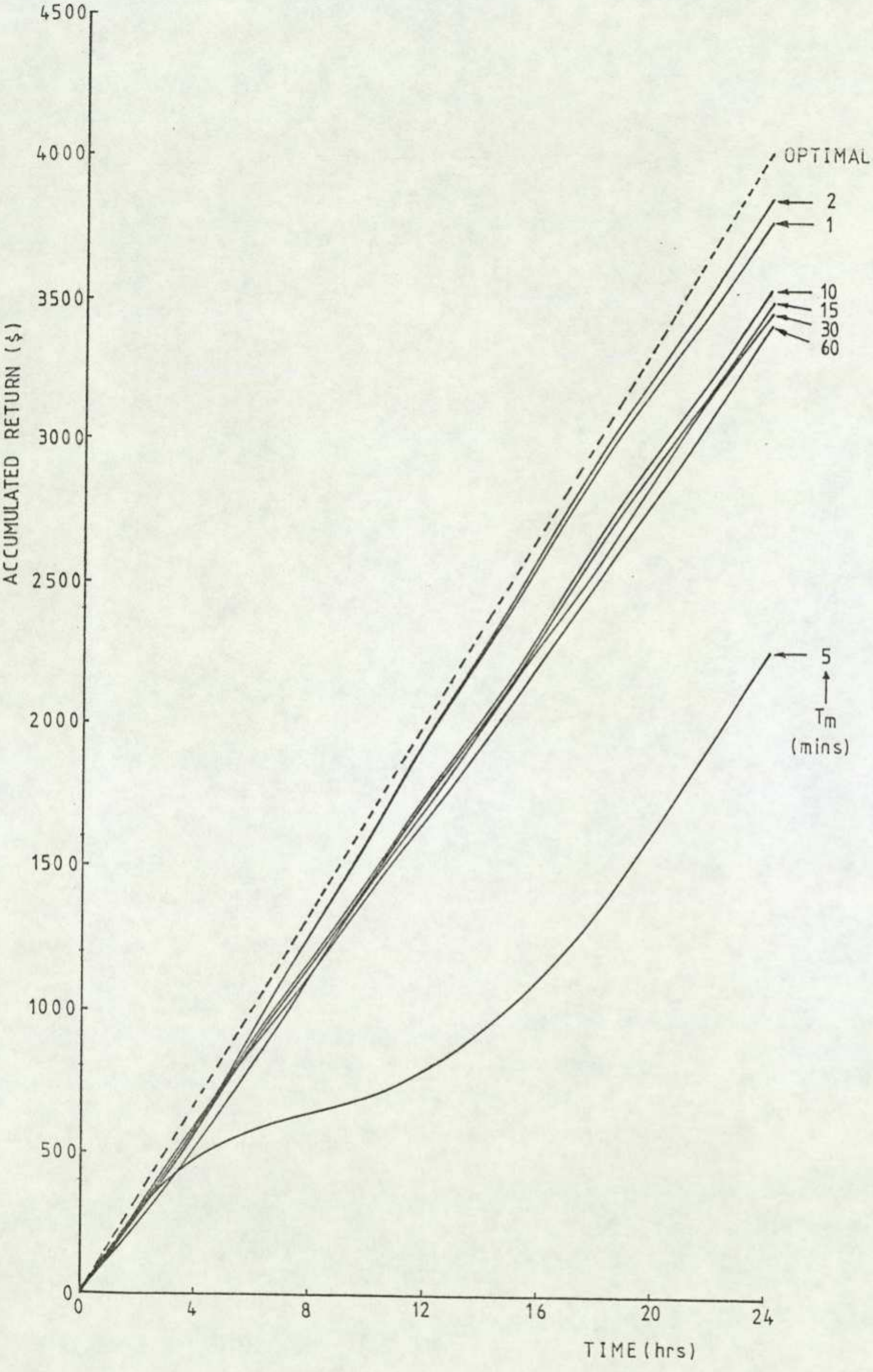


FIG.(4.7) EFFECT OF MEASUREMENT TIME T_m ON ACCUMULATED RETURN.

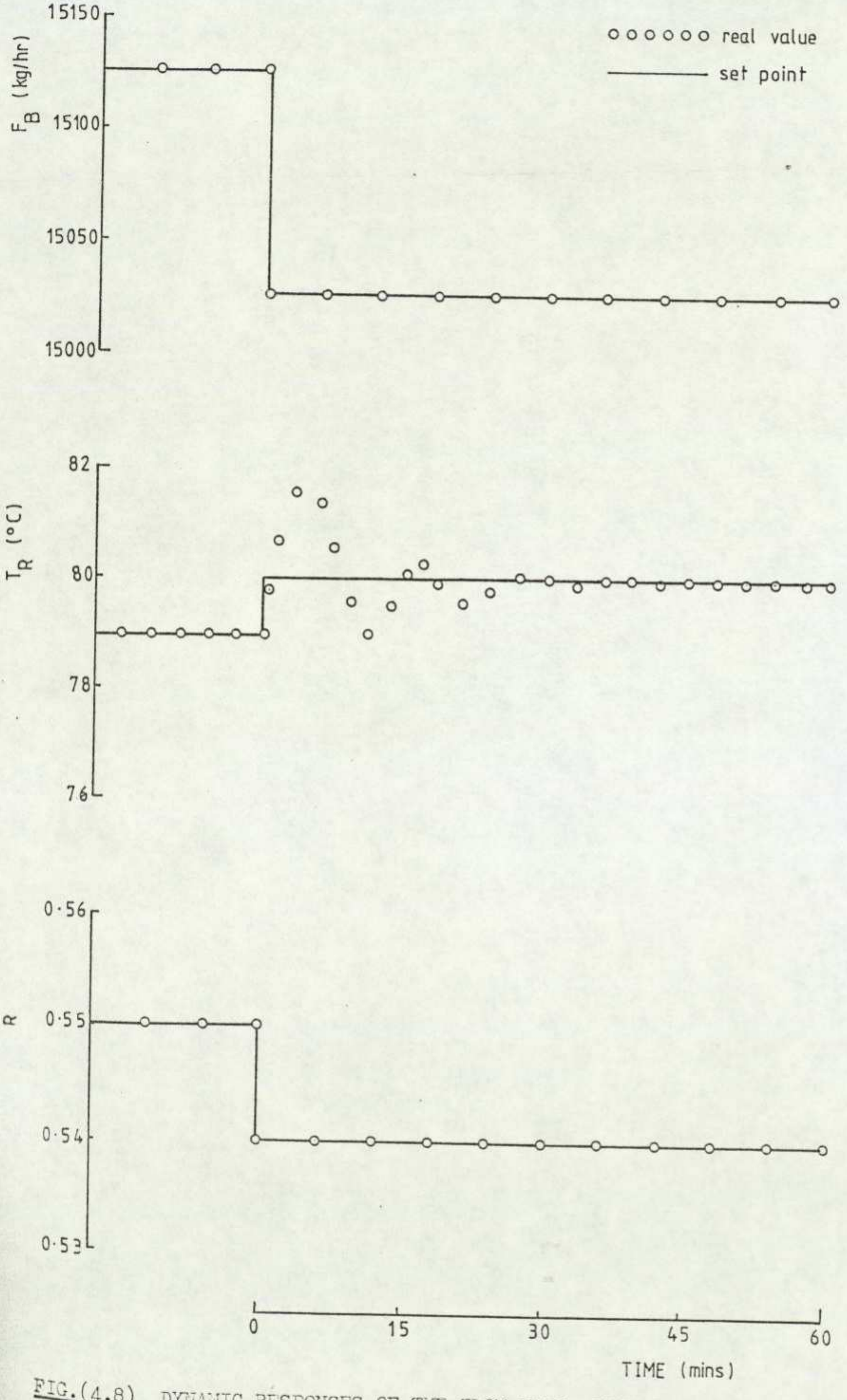


FIG.(4.8) DYNAMIC RESPONSES OF THE FLOW RATE, TEMPERATURE AND RECYCLE RATIO CONTROL LOOPS WITHIN 60 MINUTES.

PART 2 DYNAMIC OPTIMISATION

CHAPTER 5: APPLICATION OF THE MODIFIED TWO-STEP APPROACH IN THE DESIGN OF A DYNAMIC CONTROL SYSTEM

The modified two-step approach which has been described in section 2.6 takes into account the interaction between the two problems of optimisation and parameter estimation. This is achieved by introducing extra terms into the optimisation performance index. So far, the above technique has been confined to steady state optimisation and parameter estimation only. The number of parameters has also been equal to the number of measurements taken from the system.

In this chapter the above technique is to be developed into a more general form, as employed in the design of a dynamic feedback control system.

The design of a non-linear dynamic feedback control system has been given extensive attention in the literature, such as R. Bellman (1961, 19[illegible]), W. H. Fleming and R. W. Rishel (1975), A. A. Fel'dbaum (1965) and K. [illegible] Aström (1970). These authors have used the well-established optimal control synthesis procedures, such as dynamic programming, the maximum principle and variation methods.

It is well-known that the present-day optimal control techniques require a prohibitive amount of computation for a large complex system. Hence, in this chapter the performance of the above algorithm will be tested on a simple feedback dynamic system, using its simplified reduced mathematical model, as a vehicle for the application of the modified two-step approach to more complicated practical systems.

5.1 Dynamic Optimisation and Parameter Estimation

Optimisation techniques generally consist of a mathematical procedure. Therefore it is necessary to represent the behaviour of a system by a set of mathematical relationships which constitute the mathematical model of the system. If the model consists of a set of differential equations, the system is considered to be dynamic, and optimising such a system to achieve a certain performance criterion is termed dynamic optimisation.

Consider a mathematical model which is describing a system by a set of differential equations:

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t), \underline{u}(t), \underline{\alpha}(t))$$

$$\underline{x}(t_o) = \underline{x}_o$$

where

$\underline{f}$ = n-vector function of algebraic relationships

$\underline{x}(t)$ = n-vector of state variables

$\underline{u}(t)$ = m-vector of manipulable inputs

$\underline{\alpha}(t)$ = p-vector of estimated model parameters

t = independent variable (time)

$\underline{x}_o$ = n-vector of initial state at time $t = 0$, (t_o)

The outputs, $\underline{y}(t)$, taken from the model, are defined as:

$$\underline{y}(t) = \underline{h}(\underline{x}(t), \underline{u}(t), \underline{\alpha}(t))$$

where

$\underline{h}$ = r-vector function.

In general, the model is subjected to a set of inequality constraints represented by

$$\underline{g}(\underline{x}(t), \underline{u}(t), \dot{\underline{\alpha}}(t)) \leqslant \underline{0}$$

where

$\underline{g}$ = z-vector function of algebraic relationships.

The dynamic optimisation problem is defined as that of determining the manipulated inputs $\underline{u}(t)$ as functions of time, in order to optimise a given performance index, P, where P is defined by a function whose values depend upon $\underline{x}(t)$, $\underline{u}(t)$ and $\underline{\alpha}(t)$ as time varies in a given interval, $T \geqslant t \geqslant 0$. A general performance function can have the form:

$$P(\underline{x}(t), \underline{u}(t), \underline{\alpha}(t)) = \psi(\underline{x}(T),T) + \int_0^T L(\underline{x}(t), \underline{u}(t), \underline{\alpha}(t))dt \quad .. (5.1)$$

where $\psi(\underline{x}(T))$ is a scalar function of the final values of the system and the integrand L describes the system behaviour during the time interval $0 \leqslant t \leqslant T$.

The dynamic optimisation problem may then be given as:

$$\left.\begin{aligned} &\min_{\underline{u}} \{P(\underline{x}, \underline{u}, \underline{\alpha}) = \psi(\underline{x}(T),T) + \int_0^T L(\underline{x}, \underline{u}, \underline{\alpha})dt\} \\ &\qquad \text{s.t.} \quad \dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, \underline{\alpha}) \\ &\qquad\qquad \underline{g}(\underline{x}, \underline{u}, \underline{\alpha}) \leqslant \underline{0} \\ &\qquad\qquad \underline{y} = \underline{h}(\underline{x}, \underline{u}, \underline{\alpha}) \\ &\qquad\qquad \underline{x}(0) = \underline{x}_0 \\ &\qquad\qquad 0 \leqslant t \leqslant T \end{aligned}\right\} \quad \ldots\ldots (5.2)$$

where $\underline{x}$, $\underline{u}$ and $\underline{\alpha}$ should more correctly be denoted by $\underline{x}(t)$, $\underline{u}(t)$ and $\underline{\alpha}(t)$ because they depend upon time, but functional notation is omitted for convenience.

The parameter estimation problem which estimates the values of the unknown parameters of the model can be obtained by exciting the model and the system in parallel, as shown in Figure 5.1.

Hence, taking a weighted least square fitting criterion as an example, the parameter estimation problem can be described as:

$$\left.\begin{aligned} &\min_{\underline{\alpha}} \{G(\underline{x}, \underline{u}, \underline{\alpha}) = \int_0^T \| \hat{\underline{y}}(t) - \underline{h}(\underline{x}, \underline{u}, \underline{\alpha}) \|_Q^2 \, dt\} \\ &\qquad \text{s.t.} \quad \dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, \underline{\alpha}) \\ &\qquad\qquad \underline{g}(\underline{x}, \underline{u}, \underline{\alpha}) \leqslant \underline{0} \\ &\qquad\qquad \underline{y} = \underline{h}(\underline{x}, \underline{u}, \underline{\alpha}) \\ &\qquad\qquad\quad \hat{y}(t) \end{aligned}\right\} \quad \ldots\ldots (5.3)$$

where $\hat{y}(t)$ denotes measurements from the system.

5.2 Optimal Feedback Control System Design using Integrated System Optimisation and Parameter Estimation

In this section a more general form of the modified two-step approach, based on the same principle as employed in the derivation of the steady state technique, will be developed into an algorithm for the design of a dynamic feedback control system.

Consider the case where a dynamic feedback control system is described by a set of n differential relationships, a set of r output relationships, and a set of r feedback control relationships given by:

$$\underline{\dot{x}}_s = \underline{f}_s(\underline{x}_s, \underline{u}_s)$$

$$\underline{y}_s = g_s(\underline{x}_s, \underline{u}_s)$$

$$\underline{u}_s = \underline{k}_s[\underline{y}_d - \underline{y}_s]$$

where

$\underline{x}_s$ = n-vector of system state variables

$\underline{u}_s$ = r-vector of system control inputs

$\underline{y}_s$ = r-vector of system measurable outputs

$\underline{y}_d$ = r-vector of desired output

$\underline{k}_s$ = r x r - diagonal matrix of system controller gains

$\underline{x}_s$, $\underline{u}_s$ and $\underline{y}_s$ are time-dependent and they should, more correctly, be denoted as $\underline{x}_s(t)$, $\underline{u}_s(t)$ and $\underline{y}_s(t)$.

The simplified mathematical model describing the above system is used to design the optimal feedback control system by computing the optimal feedback gains.

The model can be described by a set of q differential relationships, a set of r output relationships, and a set of r feedback control relationships given by:

$$\underline{\dot{x}}_m = \underline{f}_m(\underline{x}_m, \underline{u}_m, \underline{\alpha})$$

$$\underline{y}_m = g_m(\underline{x}_m, \underline{u}_m, \underline{\alpha})$$

$$\underline{u}_m = \underline{k}_m[\underline{y}_d - \underline{y}_m]$$

where

$\underline{x}_m$ = q-vector of model state variables

$\underline{u}_m$ = r-vector of model control inputs

$\underline{y}_m$ = r-vector of model measurable outputs

$\underline{k}_m$ = r x r - diagonal matrix of model control gains

$\underline{\alpha}$ = p-vector of estimated model parameters.

Subscripts m and s refer to the variables in the model and system, respectively. $\underline{x}_m$, $\underline{u}_m$ and $\underline{y}_m$ are time-dependent.

By employing the above system and model equations into equations (5.2) and (5.3), the following optimisation problem and parameter estimation problem may be formed:

The optimisation problem:

$$\left.\begin{aligned}
&\min_{\underline{k}_m} \{P(\underline{x}_m, \underline{u}_m, \underline{\alpha}) = \psi(x_m(T), T) + \int_o^T L(\underline{x}_m, \underline{u}_m, \underline{\alpha})dt\} \\
&\qquad \text{s.t.} \quad \dot{\underline{x}}_m = \underline{f}_m(\underline{x}_m, \underline{u}_m, \underline{\alpha}) \\
&\qquad\qquad\quad \underline{y}_m = \underline{g}_m(\underline{x}_m, \underline{u}_m, \underline{\alpha}) \\
&\qquad\qquad\quad \underline{u}_m = [\underline{k}_m][\underline{y}_d - \underline{y}_m]
\end{aligned}\right\} \quad (5.4)$$

The parameter estimation problem:

$$\left.\begin{aligned}
&\min_{\underline{\alpha}} \left\{G(\underline{x}_m, \underline{u}_m, \underline{y}_s, \underline{\alpha}) = \int_o^T \left\| \underline{y}_s - \underline{y}_m \right\|_Q^2 dt\right\} \\
&\qquad \text{s.t.} \quad \dot{\underline{x}}_s = \underline{f}_s(\underline{x}_s, \underline{u}_s) \\
&\qquad\qquad\quad \underline{y}_s = \underline{g}_s(\underline{x}_s, \underline{u}_s) \\
&\qquad\qquad\quad \underline{u}_s = [\underline{k}_s][\underline{y}_d - \underline{y}_s] \\
&\qquad\qquad\quad \dot{\underline{x}}_m = \underline{f}_m(\underline{x}_m, \underline{u}_m, \underline{\alpha}) \\
&\qquad\qquad\quad \underline{y}_m = \underline{g}_m(\underline{x}_m, \underline{u}_m, \underline{\alpha}) \\
&\qquad\qquad\quad \underline{u}_m = [\underline{k}_m][\underline{y}_d - \underline{y}_m]
\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (5.5)$$

By using the model equations the integrated reduced problems can be shown as:

$$\left.\begin{aligned}
&\min_{\underline{k}_m} \; P(\underline{k}_m, \underline{\alpha}) \qquad \text{Optimisation problem} \\
&\qquad \text{s.t.} \quad \text{model equations} \\
&\min_{\underline{\alpha}} \; G(\underline{k}_m, \underline{k}_s, \underline{\alpha}) \qquad \text{Parameter estimation problem} \\
&\qquad \text{s.t.} \quad \text{model and system equations}
\end{aligned}\right\} \quad (5.6)$$

Interaction between the two problems occurs through the model parameters $\underline{\alpha}$, and the applied control gains $\underline{k}_m$. The two problems can be decomposed into two independent subproblems by considering the symbols $\underline{\alpha}$ and $\underline{k}_s$ as model parameters and control gains, respectively, in the parameter estimation problem; and $\underline{\sigma}$ and $\underline{k}_m$ as the corresponding symbols in the optimisation problem. The added equality constraints are:

$$\underline{k}_s = \underline{k}_m \qquad \text{.................... (5.7)}$$

$$\underline{\sigma} = \underline{\alpha} \qquad \text{.................... (5.8)}$$

The two problems can then be summarised as follows:

Optimisation problem

$$\min_{\underline{k}_m} P(\underline{k}_m, \underline{\sigma}) \qquad \text{.................... (5.9)}$$

Parameter estimation problem

$$\min_{\underline{\alpha}} G(\underline{k}_s, \underline{\alpha}) \qquad \text{.................... (5.10)}$$

Both problems are subject to model equations, constraints and additional equality constraints described by equations (5.7) and (5.8).

If the optimum parameter estimates occur in the feasible region within the model constraint boundaries, the parameter estimation problem may be considered as determining the unknown parameters $\underline{\alpha}$, to satisfy the estimation gradient condition:

$$\frac{\partial G}{\partial \underline{\alpha}} (\underline{k}_s, \underline{\alpha}) = \underline{0} \quad \rightarrow \quad \underline{\alpha} \qquad \text{.................... (5.11)}$$

Formation of a Lagrangian will give:

$$L = P(\underline{k}_m, \underline{\sigma}) + \underline{\eta}' \frac{\partial G}{\partial \underline{\alpha}} (\underline{k}_s, \underline{\alpha}) + \mathrm{tr}\left\{\underline{\Lambda}' \left[\underline{k}_s - \underline{k}_m\right]\right\} + \underline{\mu}'(\underline{\sigma} - \underline{\alpha}) \qquad (5.12)$$

where $\underline{\eta}$ and $\underline{\mu}$ are vectors of Lagrangian multipliers, and $\underline{\Lambda}$ is a diagonal matrix of Lagrangian multipliers. The conditions for a stationary point are:

$$\frac{\partial L}{\partial \underline{k}_m} = \frac{\partial P}{\partial \underline{k}_m} (\underline{k}_m, \underline{\sigma}) - \underline{\Lambda} = [\underline{0}] \qquad \text{.................. (5.13)}$$

$$\frac{\partial L}{\partial \underline{\alpha}} = \frac{\partial}{\partial \underline{\alpha}} \left[\frac{\partial G'}{\partial \underline{\alpha}} (\underline{k}_s, \underline{\alpha}) \right] \cdot \underline{\eta} - \underline{\mu} = \underline{0} \qquad \text{.................. (5.14)}$$

$$\frac{\partial L}{\partial \underline{k}_s} = \frac{\partial}{\partial \underline{k}_s} \left[\frac{\partial G'}{\partial \underline{\alpha}} (\underline{k}_s, \underline{\alpha}) \right] \cdot \underline{\eta} + \underline{\Lambda} = [\underline{0}] \qquad \text{.............. (5.15)}$$

$$\frac{\partial L}{\partial \underline{\sigma}} = \frac{\partial P}{\partial \underline{\sigma}} (\underline{k}_m, \underline{\sigma}) + \underline{\mu} = \underline{0} \qquad \text{.................. (5.16)}$$

where the elements of the matrices $\left[\frac{\partial}{\partial \underline{\alpha}}(.)\right]$, $\left[\frac{\partial}{\partial \underline{k}_s}(.)\right]$ and the vectors $\frac{\partial G}{\partial \underline{\alpha}}$ and $\frac{\partial P}{\partial \underline{\sigma}}$ are defined as follows:

for $\left[\frac{\partial}{\partial \underline{\alpha}}(.)\right]$ is $\left[\frac{\partial^2 G}{\partial \underline{\alpha}\underline{\alpha}}\right]_{ij} = \frac{\partial^2 G}{\partial \alpha_i \partial \alpha_j}$

for $\left[\frac{\partial}{\partial \underline{k}_s}(.)\right]$ is $\left[\frac{\partial^2 G}{\partial \underline{k}_s \partial \underline{\alpha}}\right]_{ij} = \frac{\partial^2 G}{\partial k_{s_i} \partial \alpha_j}$

for $\left(\frac{\partial G}{\partial \underline{\alpha}}\right)_i$ is $\frac{\partial G}{\partial \alpha_i}$

for $\left(\frac{\partial P}{\partial \sigma}\right)_i$ is $\frac{\partial P}{\partial \sigma_i}$

Equation (5.13) leads to the modified optimisation problem described in the form

$$\min_{\underline{k}_m} \left\{ P(\underline{k}_m, \underline{\sigma}) - tr\left[\underline{\Lambda} . \underline{k}_m\right] \right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (5.17)$$

s.t. model equations and constraints

The solution to the above modified optimisation problem will satisfy equation (5.17), subject to given $\underline{\sigma}$ and $[\underline{\Lambda}]$. The matrix of Lagrange modifiers, $[\underline{\Lambda}]$ can be found from equations (5.14) to (5.16) to be

$$[\underline{\Lambda}] = \left\{ \frac{\partial}{\partial k_s} \left(\frac{\partial G(\underline{k}_s, \underline{\alpha})}{\partial \underline{\alpha}} \right)' \left[G_{\underline{\alpha}\underline{\alpha}} \right]^{-1} \frac{\partial P(\underline{k}_m, \underline{\sigma})}{\partial \underline{\sigma}} \right\} \quad \ldots\ldots\ldots \quad (5.18)$$

The parameter estimation problem which has been described by equation (5.11) is satisfied by solving the parameter estimation problem given by equation (5.10). This problem remains unchanged.

The resulting algorithm may be considered in the hierarchical structured form shown in Figure 5.2, where the task of each unit is identical to that which has been described in chapter 2.

If the model is represented by ordinary algebraic equations rather than dynamic equations and also the number of measurements are the same as the model parameters, then the algorithm will be identical to that of chapter 2, as they are based on the same fundamental principles of Lagrangian analysis and system decomposition.

5.3 Application of the Design Procedure to a Simple Example

In this section it is intended to illustrate the effectiveness of the procedure which has been presented in the previous section. Hence, the above algorithm will be applied to the design of a third order dynamic feedback control system, through the use of its simplified model, where the model is not a true representation of the system.

5.3.1 The system description

Consider a simple dynamic feedback control system, as represented in Figure 5.3. This is a third order dynamic system which may be represented in the following general form:

A set of differential equations

$$\dot{x}_{1s} = x_{2s} \qquad \text{.................} \quad (5.19)$$

$$\dot{x}_{2s} = x_{3s} \qquad \text{.................} \quad (5.20)$$

$$\dot{x}_{3s} = \frac{u_s - 5x_{3s} - x_{2s}}{4.0} \qquad \text{.................} \quad (5.21)$$

an output relationship,

$$y_s = x_{1s} \qquad \text{.................} \quad (5.22)$$

and a control input relationship

$$u_s(t) = k_s.e(t) \qquad \text{.................} \quad (5.23)$$

where e(t) is the error between the system output and the desired output given by:

$$e(t) = y_d - y_s \qquad \text{.................} \quad (5.24)$$

By substituting e(t) from equation (5.24) into equation (5.23), the control input relationship becomes

$$u_s(t) = k_s\left[y_d - y_s\right] \qquad \text{.................} \quad (5.25)$$

where y_d is the desired output of the system and k_s is the controller gain. Hence, the system equations can be summarised as:

$$\left.\begin{aligned} \dot{x}_{1s} &= x_{2s} \\ \dot{x}_{2s} &= x_{3s} \\ \dot{x}_{3s} &= \frac{u_s - 5x_{3s} - x_{2s}}{4.0} \\ y_s &= x_{1s} \\ u_s &= k_s\left[y_d - y_s\right] \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5.26)$$

5.3.2 The model description

The open loop transfer function of the above system is given by

$$T.F_s = \frac{1}{s(1 + s)(1 + 4s)} \quad \ldots\ldots\ldots\ldots \quad (5.27)$$

with its dominant pole at s = 0 and the remaining poles at s = -0.25 and s = -1 (on the s-plane).

To form a simplified model representing the above system, the system transfer function $T.F_s$ is reduced to a second order transfer function given by:

$$T.F_m = \frac{a}{s(1 + bs)} \quad \ldots\ldots\ldots\ldots \quad (5.28)$$

where a and b are the model parameters whose values are to be estimated.

It should be noted that the purpose of this reduction is to have an unfaithful representation of the system. This will examine the effectiveness of the modified two-step approach in the design of a dynamic feedback control system, using its simplified model.

Having defined the model transfer function, the simplified model may be represented as that of Figure 5.4.

The simplified model representing the system can now be formulated in a general form, as follows:

$$\left.\begin{aligned} \dot{x}_{1m} &= x_{2m} \\ \dot{x}_{2m} &= \frac{u_m.a - x_{2m}}{b} \\ y_m &= x_{1m} \\ u_m &= k_m\left[y_d - y_m\right] \end{aligned}\right\}. \quad \ldots\ldots\ldots\ldots \quad (5.29)$$

This model is to be used in order to determine the optimal control gain k_m to minimise the system objective function.

5.3.3 Optimisation and parameter estimation problem

The optimisation problem can be described as that of minimising a given performance index in order to determine the optimal controller gain. In this case the performance index, $F(k_m, a, b)$, is defined by:

$$F(k_m, a, b) = \int_0^\infty (e_m(t)^2 + 0.5\, u_m(t)^2)dt \quad \ldots\ldots\ldots \quad (5.30)$$

s.t. equation (5.29)

Hence, the optimisation problem can be formulated as follows:

$$\min_{k_m} \{F(k_m, a, b) = \int_0^\infty (e_m(t)^2 + 0.5\, u_m^2(t))dt\} \quad \ldots\ldots\ldots \quad (5.31)$$

s.t. equation (5.29)

The parameter estimation problem can be defined as minimising a comparison index in order to find the unknown parameters, a and b. The comparison index, $G(k_s, k_m, a, b)$, is defined by

$$G(k_s, k_m, a, b) = \int_0^\infty (y_s(t) - y_m(t))^2 dt \quad \ldots\ldots\ldots \quad (5.32)$$

The parameter estimation problem may then be formulated as

$$\min_{a,b} \{G(k_s, k_m, a, b) = \int_0^\infty (y_s(t) - y_m(t))^2 dt\} \quad \ldots\ldots\ldots \quad (5.33)$$

s.t. equations (5.26) and (5.29)

Equations (5.31) and (5.33) define the optimisation and the parameter estimation problems respectively. From these two equations it can be seen that the two problems interact via k_m, a, and b. The two problems can be decoupled by employing k_m, a and b in the parameter estimation problem, and k_s, σ_1 and σ_2 as the corresponding gain and parameters in the optimisation problem. This will create some additional equality constraints. Hence, the optimisation problem and parameter estimation problem can be described as follows:

Optimisation problem

$$\min_{k_m} \{F(k_m, \sigma_1, \sigma_2) = \int_0^\infty (e_m^2 + 0.5\, u_m^2)dt\} \quad \ldots\ldots\ldots \quad (5.34)$$

Parameter estimation problem

$$\min_{a,b} \{G(k_s, a, b) = \int_0^\infty (y_s - y_m)^2 dt\} \quad \ldots\ldots\ldots \quad (5.35)$$

The combined problem is subject to the model equations (5.29), the system equations (5.26), and also additional equality constraints given by

$$\left.\begin{aligned} k_s &= k_m \\ \sigma_1 &= a \\ \sigma_2 &= b \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (5.36)$$

5.3.4 The system solution

It is important to find the correct optimal gain k_s^* and optimal value of the performance index F*. This enables comparison of the true solution with that obtained from the model. In order to find the true solution, the optimisation problem described by equation (5.31) is solved subject to the system equations rather than those of the model. Hence, the optimisation problem becomes:

$$\min_{k_s} \{F_s(k_s) = \int_0^\infty (e_s^2(t) + 0.5\, u_s^2(t))dt\} \quad \ldots\ldots\ldots \quad (5.37)$$

s.t. equation (5.26)

Substituting $u_s(t)$ from equation (5.23) into equation (5.37) the above equation becomes

$$\min_{k_s} \{F_s(k_s) = (1 + 0.5\, k_s^2)\int_0^\infty e_s^2(t)dt\} \quad \ldots\ldots\ldots \quad (5.38)$$

s.t. equation (5.26)

The quadratic performance criterion, $\int_0^\infty e_s^2(t)dt$, can be evaluated analytically using the Parseval's Theorem (Jacobs (1974)), provided that the Laplace transform of error, E(S), can be expressed as the ratio of two finite polynomials in S. If the input function to the system is a unit step, E(S) can be evaluated to be:

$$E(S) = \frac{4S^2 + 5S + 1}{4S^3 + 5S^2 + S + k_s} \quad \text{........... (5.39)}$$

Using the table of integrals (Aström (1970)),

$$\int_0^\infty e_s(t)^2 dt = \frac{21k_s + 5}{-8k_s^2 + 10k_s} \quad \text{........... (5.40)}$$

Substituting equation (5.40) into equation (5.38) and minimising F_s, with respect to k_s $\left(\text{i.e. } \frac{\partial F_s}{\partial k_s} = 0 \rightarrow k_s^*\right)$, the resulting k_s^* is found to be

$$k_s^* = 0.314$$

with optim al value of function

$$F_s^* = 5.17,$$

where * refers to the optimal values.

5.3.5 Application of the algorithm to the simple problem

The scheme which has been described in section 5.2 is to be applied to the simple example. Hence, the modified optimisation problem, the parameter estimation problem and the coordination unit problem can be summarised as follows:

Modified optimisation problem

$$\min_{k_m} \left\{\left[F_m(k_m, \underline{\alpha}) - \Lambda.k_m\right] = \int_0^\infty (e_m^2(t) + 0.5\, u_m^2(t))dt - \Lambda.k_m\right\} \quad \text{......... (5.41)}$$

s.t. equation (5.29)

Parameter estimation problem

$$\min_{a,b} \{G(k_s, a, b) = \int_0^\infty (y_s(t) - y_m(t))^2 dt\} \quad \ldots\ldots\ldots \quad (5.42)$$

s.t. equations (5.26) and (5.29)

Coordination unit problem

$$\left.\begin{aligned} k_s &= k_m \\ \sigma_1 &= a \\ \sigma_2 &= b \\ \Lambda &= \frac{\partial}{\partial k_s} \begin{bmatrix} \frac{\partial G}{\partial a} & \frac{\partial G}{\partial b} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 G}{\partial a^2} & \frac{\partial^2 G}{\partial a \partial b} \\ \frac{\partial^2 G}{\partial b \partial a} & \frac{\partial^2 G}{\partial b^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F_m}{\partial \sigma_1} \\ \frac{\partial F_m}{\partial \sigma_2} \end{bmatrix} \end{aligned}\right\} \quad \ldots\ldots \quad (5.43)$$

Solution of the parameter estimation problem given by equation (5.42) will update the model parameters, $a = \sigma_1$ and $b = \sigma_2$. Then the coordination unit computes a new value of the modifier Λ using equation (5.43). The new values of Λ, σ_1 and σ_2 are then transmitted to the modified optimisation problem, equation (5.41), which updates the value of $k_m = k_s$, and the procedure is repeated until no further improvement is observed.

If the model is not a faithful representation of the system, several iterations between the two problems will be needed before the final converged solution is obtained. In order to control the stability of these iterative loops, the relationship $k_s = k_m$ in the coordination unit is implemented by a difference equation of the form

$$k_s^{\,i} = k_s^{\,i-1} + k\{k_m^{\,i} - k_s^{\,i-1}\} \quad \ldots\ldots \quad (5.44)$$

where $k_s^{\,i}$ and $k_m^{\,i}$ are the values of k_s and k_m at i^{th} iteration, $k_s^{\,i-1}$ is the previous value of k_s. The loop gain, $0 < k \leqslant 1$, determines the change made to the system controller gain, k_s, from one iteration to the next. This ensures that an excessive alteration is not made.

In the above iterative procedure, the parameter estimation comparison index, $G(k_s, a, b)$; the optimisation performance index, $F_m(k_m, \sigma_1, \sigma_2)$; and the Lagrangian modifier, Λ, have been evaluated analytically using Parseval's Theorem and numerically using the NAG library of routines (see appendix A1).

5.3.6 Results

In order to determine the effectiveness of the procedure as applied to the design of a dynamic control system, the system has been designed analytically using Parseval's Theorem. The optimal controller gain, k_s^*, is found to be equal to 0.314 with the performance index, F_s^*, at its minimum value of 5.17.

The standard unmodified two-step approach can be conducted by setting the Lagrangian multipliers, Λ, to zero. Figure 5.5 shows variation of performance index, F_s, of the system starting from two alternative initial points, as well as the corresponding variation in gain, k_s, respectively. These results clearly show the unsatisfactory behaviour of the standard two-step approach when applied to this particular problem. In this case the controller gain, k_s, is found to be 0.445 with $F_s = 5.5$. The results imply that the output gradient of the system with respect to the controller gain, $\partial y_s/\partial k_s$, is not matched with that of the model $\left(\text{i.e. } \frac{\partial y_s}{\partial k_s} \neq \frac{\partial y_m}{\partial k_m}\right)$. Hence, an algorithm is required which takes into account the above gradient condition to the solution of the overall problem.

Figure 5.6 shows variation in performance index, F_s, by applying the modified algorithm starting from the same two alternative initial points and employed using the standard two-step approach. It was observed that although the correct final optimum performance index, F_s, on the system is obtained within 0.2%, there is an error of about 7% in the controller gain. Values of k_s and F_s in this case are found to be

$$k_s = 0.336$$

$$F_s = 5.18$$

The error could be due partly to non-zero values of the gradients, $\frac{\partial G}{\partial a}$ and $\frac{\partial G}{\partial b}$, and partly to a flat minimum. In addition to showing the variation in F_s, Figure 5.6 illustrates the corresponding variation in the gain k_s and the behaviour of the modifier, Λ, observed during optimisation.

In this exercise convergence is considered to have occurred when further improvement of the performance index, F_s, is less than some positive constant ε. Typical times to converge to the optimal solution are shown in Table 5.1.

The overall solution to the combined problem of the optimisation and parameter estimation has been conducted in two ways:

(i) G, F and Λ were found numerically and then the iterative procedure (i.e. the modified two-step) was employed;

(ii) G, F and Λ were found analytically (using Parseval's Theorem) before using the iterative procedure.

Table 5.2 gives a summary of the results obtained with

$$\varepsilon = 1.0\,E - 3$$
$$k = 0.1$$

This compares the final system performance index, F_s, and the controller gain, k_s, produced by the standard two-step approach and the modified two-step approach (including (i) and (ii)) with that of the true solution.

5.4 Conclusions

In this chapter a more general algorithm, which attempts to take account of the inherent interaction between system optimisation and parameter estimation, is employed to design a dynamic control system using its simplified mathematical model. This algorithm is based on the same fundamental principles of Lagrangian analysis and system decomposition as employed in the derivation of the steady state algorithm, described in chapter 3.

The technique has been successfully applied in a design of a dynamic feedback control system by adjusting the controller gain to minimise a given performance index. The model used has been deliberately simplified in order to demonstrate the effectiveness of the technique. The results obtained show that, without the required modification, the two-step approach fails to determine the correct optimum condition on the system. However, the modified two-step algorithm successfully minimises the given performance index.

Loop gain k	Number of cycles
0.1	39
0.2	18
0.5	5
0.7	8

Table 5.1: Convergence time for $\varepsilon = 10^{-3}$

	True Solution	Standard two-step	Modified two-step	
			Analytical	Numerical
K_S	0.314	0.445	0.336	0.337
a	-	1.428	1.250	1.250
b	-	7.161	6.285	6.305
F_S	5.174	5.501	5.184	5.185

Table 5.2: Summary of the final values

The requirement of the modified two-step algorithm, to have $\frac{\partial G}{\partial \underline{\alpha}}$ equal to zero in the Lagrangian analysis and also to measure second derivatives of real process outputs, imposes an important practical limitation to the technique. However, in this particular simulation study, although the value of the optimal controller gain, k_s, is not as accurate as its theoretical value, the technique still minimises the given performance index to the correct optimal value within acceptable errors (about 0.2%).

The results also show that the convergence of the iterative technique can be regulated by appropriate choice of gain parameters within the iterative loop.

In the next chapter the algorithm will be applied to a more complicated and practical dynamic system using its simplified mathematical model.

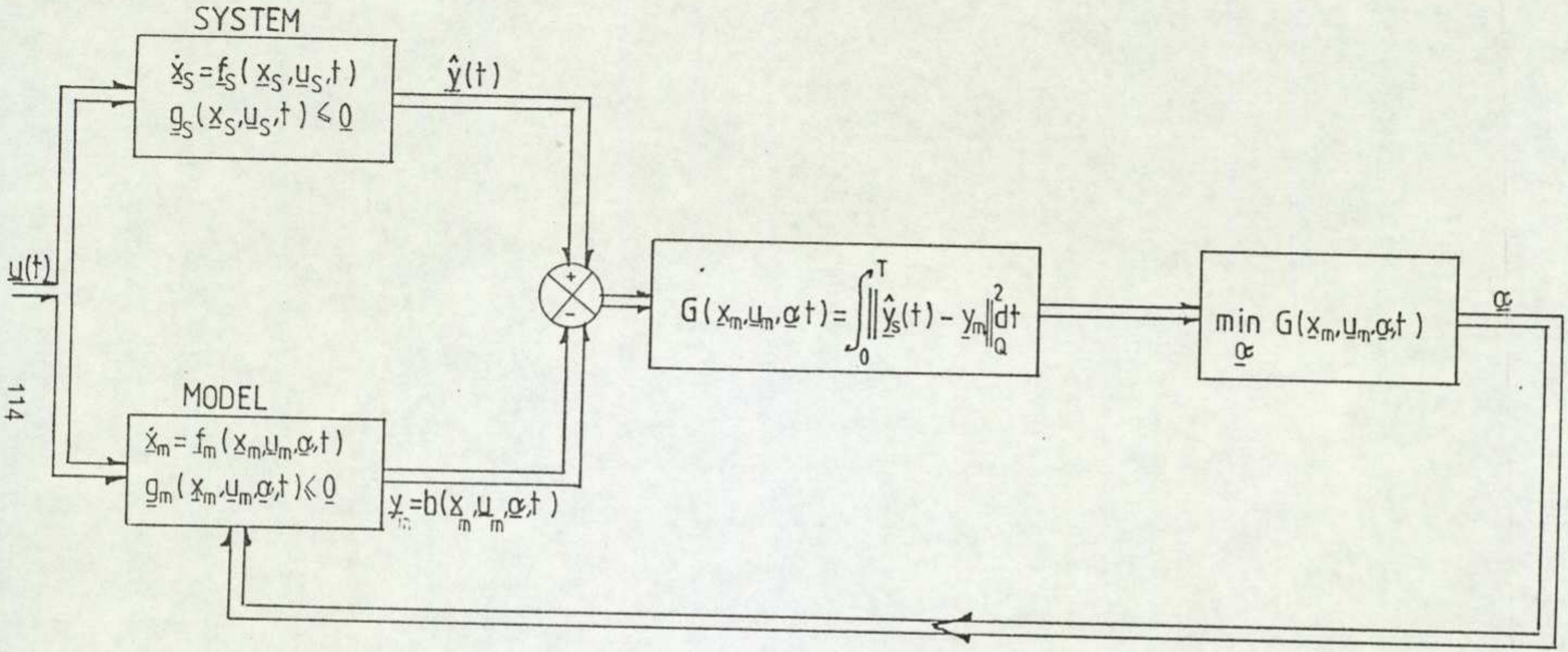


FIG (5.1) PARAMETER ESTIMATION PROBLEM

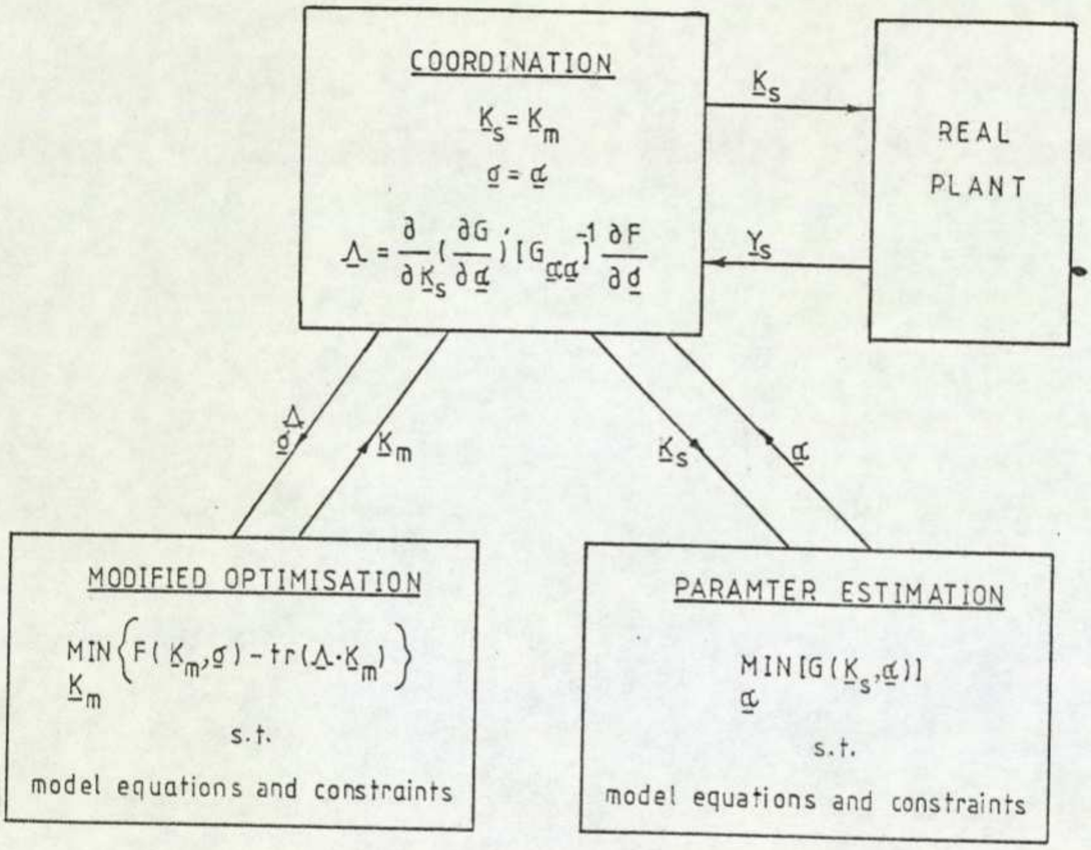


FIG.(5.2) HIERARCHICAL STRUCTURE OF THE DESIGN PROCEDURE.

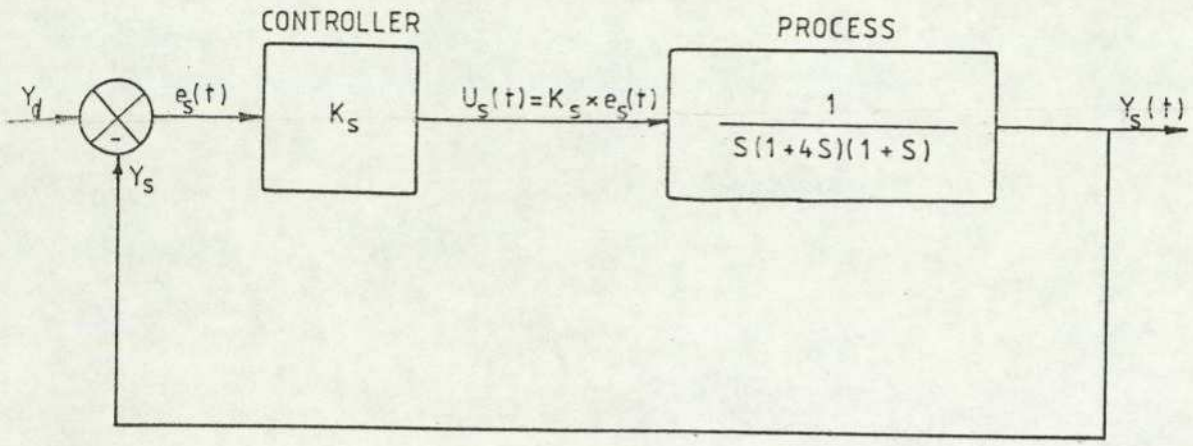


FIG.(5.3) A UNITY FEEDBACK CONTROL SYSTEM.

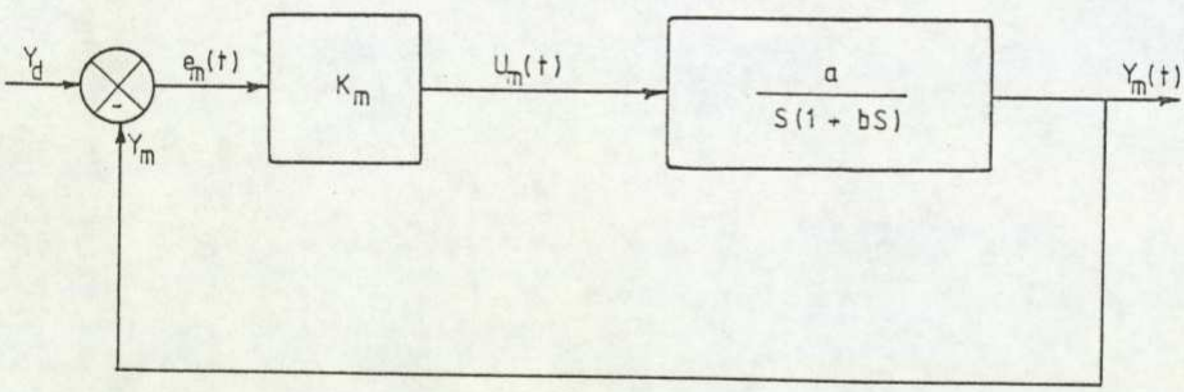


FIG.(5.4) THE BLOCK DIAGRAM OF THE SIMPLIFIED MODEL.

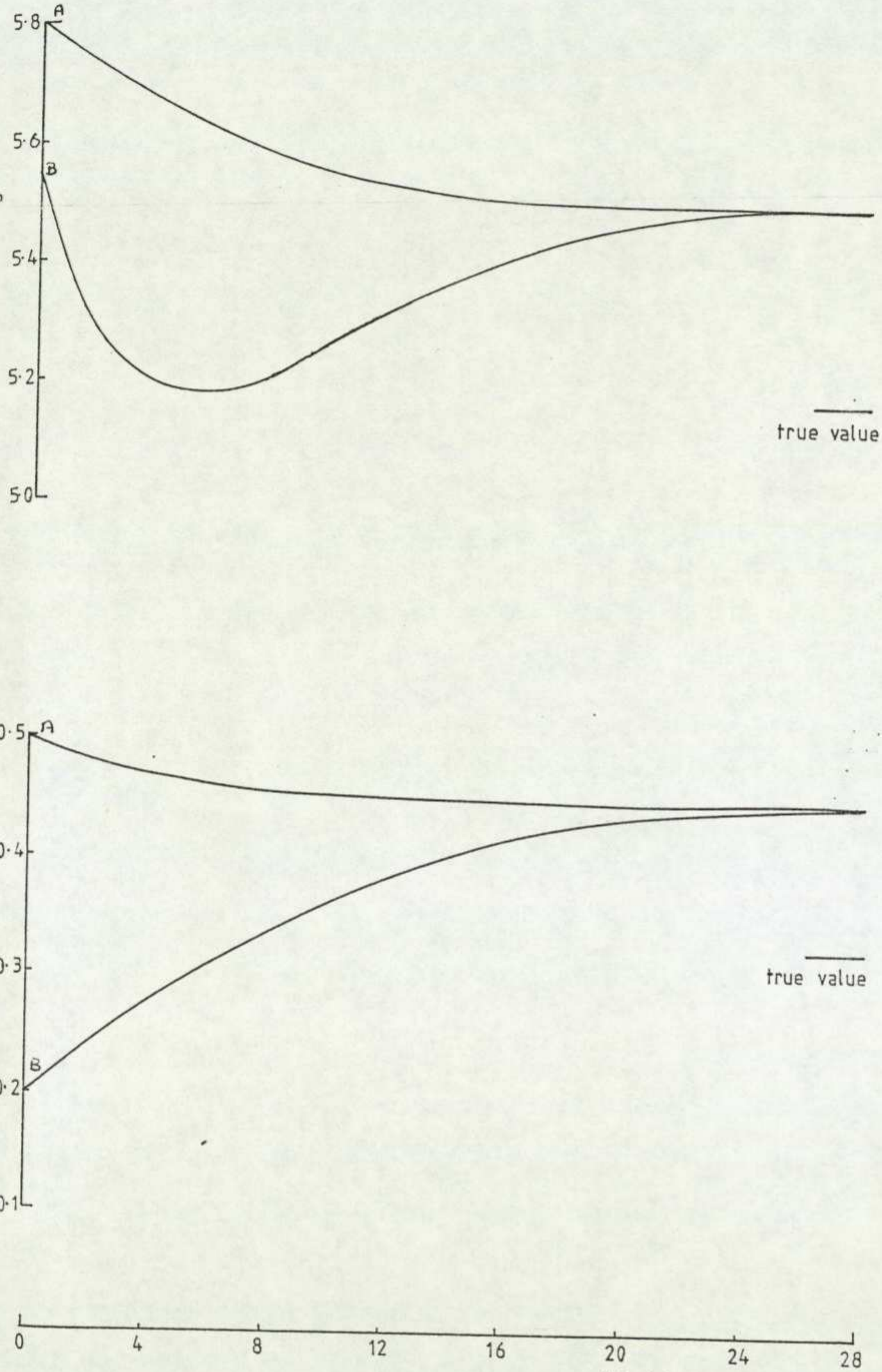


FIG.(5.5) VARIATION OF THE PERFORMANCE INDEX F_s AND GAIN CONTROLLER SETTING K, DURING THE STANDARD TWO-STEP APPROACH.

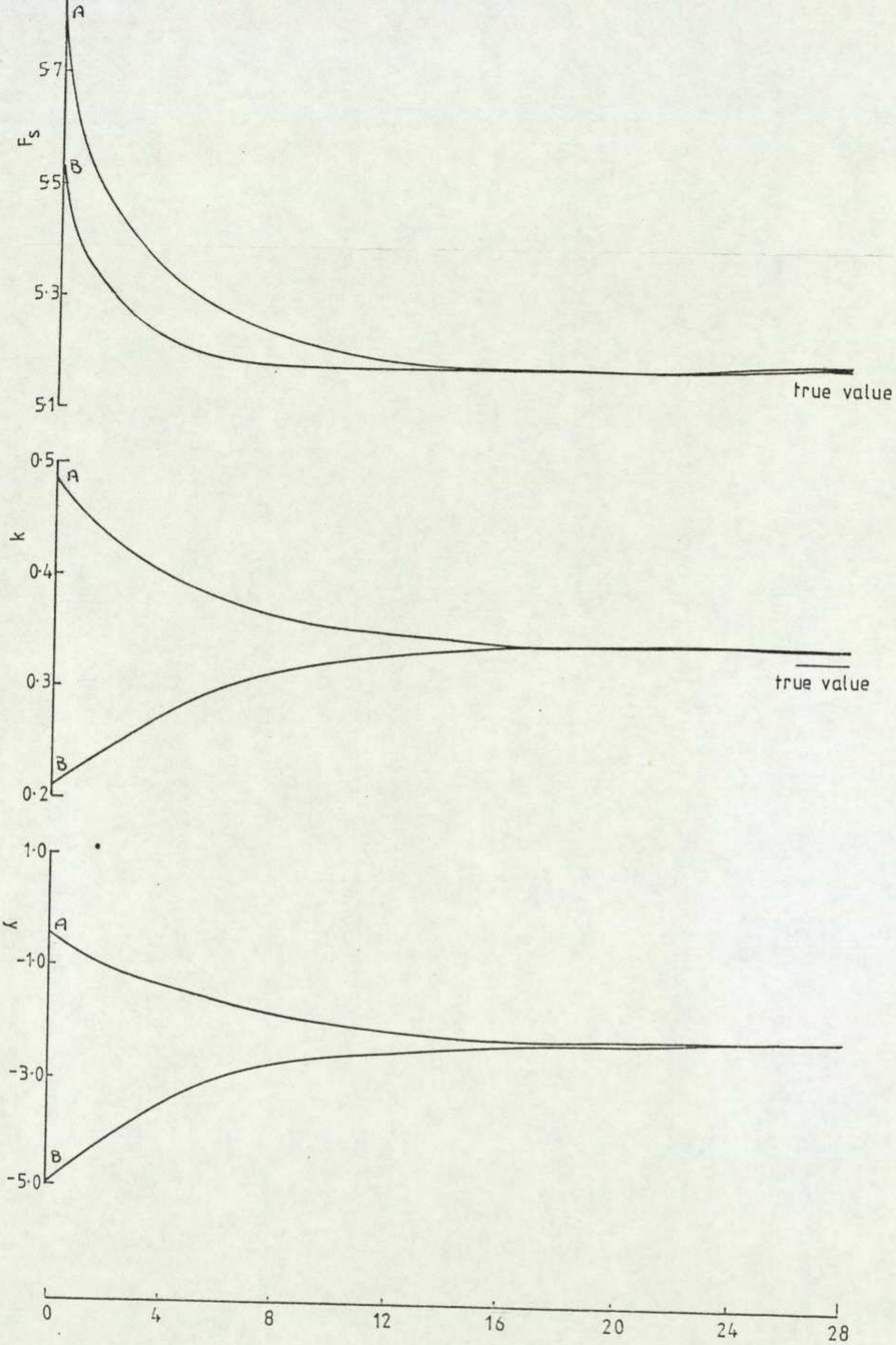


FIG.(5.6) VARIATION OF PERFORMANCE INDEX J_s, GAIN CONTROLLER K AND OPTIMISATION MODIFIER λ DURING THE MODIFIED TWO-STEP APPROACH.

CHAPTER 6 - APPLICATION OF THE MODIFIED TWO-STEP APPROACH TO THE DESIGN OF OPTIMAL FEEDBACK CONTROL OF A SYNCHRONOUS GENERATOR

In this chapter the modified two-step approach will be employed in order to design an optimal feedback control synchronous generator. This is done by designing a controller gain matrix in order to minimise a given performance index, using the generator's simplified mathematical model. A comparison to the standard two-step approach, as far as the convergence properties and accuracy at which the real system optimum operating condition are concerned, will be made.

The system which will be considered in this chapter consists of a synchronous machine together with its associated controllers. Some small time constants, small time delays and other small parameters are neglected so that a high-order time delay system can be approximated by a lower-order ordinary model (considered as the system).

The above system exhibits both fast and slow dynamic properties simultaneously. Hence, the mathematical model is formed by neglecting the dynamics of those equations whose steady state is reached much faster.

Later, the model is further simplified by linearising any non-linear differential equations which exist in the non-linear model.

6.1 System Description

The power system under investigation consists of a synchronous machine unit connected to an infinite bus, Figure 6.1. It has both a voltage regulator and speed governor controls. This type of system has been considered in the literature (Reddy and Sannuti (1976); Yu, Vongsuriya and Wedman (1970); Anderson (1971); Miles (1961); Demello and Concordia (1969); and Prabhashankar and Janischewsyj (1968)). The basic machine equations were originally developed by Park (1929).

6.1.1 List of symbols

d = direct

q = quadrature

g = gate opening

g_f = governor control-loop signal

h = water head

i_d = d-axis armature current

i_q = q-axis armature current

k_i = proportionality constant i = 1, 2, 3

M = inertia constant

P_i = mechanical power input

P_e = electrical power output

p_o = initial value of mechanical power input

u_1 = exciter control input

u_2 = governor control input

v_o = constant bus voltage

v_d = d-axis bus voltage

v_q = q-axis bus voltage

v_f = field voltage

v_s = exciter control voltage

x_d = d-axis synchronous reactance

x'_d = d-axis transient reactance

x_q = q-axis synchronous reactance

x'_q = q-axis transient reactance

x''_q = q-axis subtransient reactance

τ_o = open-circuit field time constant

τ_e = exciter time constant

τ_g = gate time constant

τ_s = time constant of the control voltage amplifier

τ_d = governor actuator time constant

τ_w = water time constant

μ_e = exciter gain

μ_s = gain of voltage amplifier

μ_a = governor actuator gain

σ = total droop (permanent plus transient)

ψ_d = d-axis flux linkage

ψ_q = q-axis flux linkage

ψ_f = field flux linkage

δ = torque angle (radians)

ω_o = synchronous speed

ω = speed (rad/sec)

θ_1 = time delay in hydraulic head

θ_2 = time delay associated with gate opening

6.1.2 Synchronous machine

Park's model describing the dynamic characteristics of a synchronous machine in per-unit form is given by:

$$v_d = \frac{d\psi_d}{dt} - r\, i_d - \psi_q.\omega \quad \text{........... (6.1)}$$

$$v_q = \frac{d\psi_q}{dt} - r\, i_q + \psi_d.\omega \quad \text{........... (6.2)}$$

$$v_f = \frac{d\psi_f}{dt} + x_d i_d + \omega\psi_d \quad \text{,.......... (6.3)}$$

where

$$\omega\psi_d = G(s).v_f - x_d(s)i_d \quad \text{........... (6.4)}$$

$$\omega\psi_q = -x_q(s)i_q \quad \text{........... (6.5)}$$

where the operators $G(s)$, $x_d(s)$ and $x_q(s)$ are:

$$G(s) \Rightarrow \begin{cases} G(0) = 1 \\ G(\infty) = 0 \end{cases} \quad \text{i.e. } G(s) = \frac{1}{1 + \tau_o S} \quad \text{........... (6.6)}$$

$$x_d(s) \Rightarrow \begin{cases} x_d(0) = x_d \\ x_d(\infty) = x'_d \end{cases} \quad \text{i.e. } x_d(s) = \frac{x_d + \tau_o x'_d S}{1 + \tau_o S} \quad \text{........... (6.7)}$$

$$x_q(s) \Rightarrow \begin{cases} x_q(0) = x_q \\ x_q(\infty) = x_q - x_q'' \end{cases} \quad \text{with } x_q'' = 0 \quad x_q(s) = x_q \quad \ldots \quad (6.8)$$

By neglecting the armature resistance, r, time derivatives of the direct and quadrature axes flux linkages, $\frac{d\psi_d}{dt}$ and $\frac{d\psi_q}{dt}$, the Park's equations become:

$$v_d = -\psi_q \omega \quad \ldots\ldots\ldots\ldots\ldots \quad (6.9)$$

$$v_q = \psi_d \omega \quad \ldots\ldots\ldots\ldots\ldots \quad (6.10)$$

by substituting the above operators given by equations (6.6) to (6.8) in equations (6.4) and (6.5)

$$\omega\psi_q = -x_q \, i_q \quad \ldots\ldots\ldots\ldots\ldots \quad (6.11)$$

$$\omega\psi_d = \frac{v_f}{1 + \tau_o S} - \frac{1 + \tau_d S}{1 + \tau_o S} x_d \, i_d \quad \ldots\ldots\ldots\ldots\ldots \quad (6.12)$$

where $\tau_d = \tau_o \frac{x'_d}{x_d}$.

Equation (6.12) can be written as

$$\omega\psi_d = v_{fx} - x_d \, i_d \quad \ldots\ldots\ldots\ldots\ldots \quad (6.13)$$

where

$$v_{fx} = \frac{v_f + \tau_o S(x_d - x'_d) i_d}{1 + \tau_o S} \quad \ldots\ldots\ldots\ldots\ldots \quad (6.14)$$

Equations (6.3) and (6.13) will give

$$\frac{d\psi_f}{dt} = v_f - v_{fx} \quad \ldots\ldots\ldots\ldots\ldots \quad (6.15)$$

Substituting for v_{fx} from equation (6.14) in equation (6.15), the results will give

$$\psi_f = \tau_o \{v_f - \frac{d\psi_f}{dt} - x_d \, i_d + x'_d \, i_d\} \quad \ldots\ldots\ldots\ldots \quad (6.16)$$

Equations(6.16) and (6.3) will then give

$$\psi_f = \tau_o [\psi_d \omega + x'_d \, i_d] \quad \ldots\ldots\ldots\ldots \quad (6.17)$$

The machine terminal voltage v_o is related to v_d, v_q and the torque angle δ by

$$v_d = v_o \sin\delta \qquad \text{.............. (6.18)}$$

$$v_q = v_o \cos\delta \qquad \text{.............. (6.19)}$$

Solving for i_d from equation (6.17) and substituting in equation (6.19) for v_q

$$i_d = \frac{\psi_f}{\tau_o x'_d} - \frac{v_o \cos\delta}{x'_d} \qquad \text{.............. (6.20)}$$

From equations (6.10) and (6.18),

$$i_q = \frac{v_o \sin\delta}{x_q} \qquad \text{.............. (6.21)}$$

The power of the electromechanical energy conversion is

$$P_e = i_q v_q + i_d v_d - P_o \qquad \text{.............. (6.22)}$$

Initial power input P_o is assumed to be zero, hence,

$$P_e = i_q v_q + i_d v_d \qquad \text{.............. (6.23)}$$

Substituting for i_q, i_d, v_q and v_d from equations (6.18) to (6.21), in equation (6.23)

$$P_e = \frac{v_o v_f \sin\delta}{\tau_o x'_d} + \frac{(x'_d - x_d)}{2x_d x'_d} v_o^2 \sin2\delta \qquad \text{....... (6.24)}$$

The equation of motion for the rotating masses of the generator set is

$$\frac{d\omega}{dt} = \frac{1}{M}(P_i - P_e) \qquad \text{.............. (6.25)}$$

The last equation is obtained from

$$P_i = D.\omega - M\frac{d\omega}{dt} + P_e \qquad \text{.............. (6.26)}$$

where the damping coefficient, D, is neglected (Yu, Vongsuriya and Wadman (1970)).

Since the performance of a hydraulic turbine depends on turbine gate opening, g, net head, h, and the speed of rotation ω, the mechanical power input P_i is given by (Reddy and Sannuti (1976)).

$$P_i = k_1 h(t - \theta_1) + k_2 g(t - \theta_2) + k_3\omega \quad \ldots\ldots\ldots\ldots \quad (6.27)$$

where Θ_1 and Θ_2 are pure time delays.

Substituting equations (6.24) and (6.27) in equation (6.25) yields:

$$\frac{d\omega}{dt} = \frac{1}{M}\left[k_1 h(t - \theta_1) + k_2 g(t - \theta_2) + k_3\omega - \frac{v_o\, v_f \sin\delta}{\tau_o\, x'_d} - \frac{(x'_d - x_d)}{2x'_d x_q} v_o^2 \sin 2\delta \right] \ldots \quad (6.28)$$

Equations (6.14), (6.15) and (6.20) will give

$$\frac{d\psi_f}{dt} = v_f - \frac{x_d \psi_f}{\tau_d x'_d} + \frac{(x_d - x'_d)}{x'_d} v_o \cos\delta \quad \ldots\ldots\ldots\ldots \quad (6.29)$$

The torque angle δ is related to speed by

$$\frac{d\delta}{dt} = (\omega - 1)\omega_o \quad \ldots\ldots\ldots\ldots \quad (6.30)$$

Equations (6.28) to (6.30) constitute the synchronous machine equations.

6.1.3 Exciter control-voltage

A control signal u_1 is fed into the summing junction of the exciter-voltage regulator system through a transfer function G_s,

$$G_s = \frac{\mu_s}{1 + \tau_s S} \quad \ldots\ldots\ldots\ldots \quad (6.31)$$

The exciter of the machine has a transfer function G_e,

$$G_e = \frac{\mu_e}{1 + \tau_e S} \quad \ldots\ldots\ldots\ldots \quad (6.32)$$

shown in the forward branch of Figure 6.2.

The exciter system equations may be written as

$$\frac{dv_f}{dt} = -\frac{v_f}{\tau_e} + \frac{\mu_e}{\tau_e}(v_o - v_s) \quad \ldots\ldots\ldots\ldots \quad (6.33)$$

$$\frac{dv_s}{dt} = -\frac{v_s}{\tau_s} + \frac{\mu_s}{\tau_s}(u_1) \quad \ldots\ldots\ldots\ldots \quad (6.34)$$

Equations (6.33) and (6.34) constitute the exciter control equations.

6.1.4 Governor control system

A control signal u_2 is fed into a summing junction of the governor system through a servo motor (Figure 6.3). The motor has a transfer function of the form

$$G_m = \frac{\mu_a}{1 + \tau_a S} \qquad \text{............. (6.35)}$$

The governor system transfer function G_g and the hydraulic operator G_ω are shown in the forward branch of Figure 6.3, where

$$G_g = \frac{1}{\sigma + \tau_g S} \qquad \text{............. (6.36)}$$

$$G_{\dot{\omega}} = \frac{-t_\omega S}{1 + 0.5\ \tau_\omega S} \qquad \text{............. (6.37)}$$

The governor system equations may be summarised as follows:

$$\frac{dg}{dt} = -\frac{\sigma g}{\tau_g} + \frac{1}{\tau_g}\left(-\frac{\omega}{\omega_o} - g_f\right) \qquad \text{............. (6.38)}$$

$$\frac{dg_f}{dt} = -\frac{g_f}{\tau_a} + \frac{\mu_a}{\tau_a} u_2 \qquad \text{............. (6.39)}$$

$$\frac{dh}{dt} = -2\frac{dg}{dt} - \frac{2}{\tau_\omega} h \qquad \text{............. (6.40)}$$

6.1.5 Complete system equations

The system equations consist of some known numerical data, taken from Reddy and Sannuti (1976), given below:

$x_d = 1.0$	$x'_d = 0.5$	$x_q = 0.6$
$\tau_o = 10.0$	$M = 5.0$	$\omega_o = 1.0$
$\mu_e = 2.5$	$\tau_e = 0.25$	$\tau_s = 0.2$
$\mu_s = 2.0$	$\sigma = 0.45$	$\tau_g = 0.1$
$\mu_a = 1.0$	$\tau_a = 0.1$	$\tau_\omega = 0.5$
$k_1 = 1.67$	$k_2 = -1.52$	$k_3 = 0.217$
$v_o = 1.05$	$\theta_1 = 0.025$	$\theta_2 = 0.025$

The time delays, $\theta_1 = \theta_2 = 0.025$, are not significant and can be neglected in equation (6.28) to save computer run time. However, the effect of the time delays in governor-gate opening and hydraulic head, with higher values of θ_1 and θ_2, has been given by Reddy and Sannuti (1976). Substituting the above numerical values in the equations of the machine, the exciter and the governor system, the complete system equations together with the control inputs can be summarised as follows:

$$\frac{d\delta}{dt} = \omega - 1 \qquad \text{............} \quad (6.41)$$

$$\frac{d\omega}{dt} = 0.0433\omega - 0.0422\ \psi_f \sin\delta + 0.0367 \sin 2\delta - 0.304g + 0.334h \qquad \text{............} \quad (6.42)$$

$$\frac{d\psi_f}{dt} = v_f - 0.2\ \psi_f + 1.05 \cos\delta \qquad \text{............} \quad (6.43)$$

$$\frac{dv_f}{dt} = -4\ v_f - 10\ v_s + 10.5 \qquad \text{............} \quad (6.44)$$

$$\frac{dv_s}{dt} = -5\ v_s + 10\ u_1 \qquad \text{............} \quad (6.45)$$

$$\frac{dg}{dt} = -4.5g - 10\omega - 10\ g_f \qquad \text{............} \quad (6.46)$$

$$\frac{dg_f}{dt} = -10\ g_f + 10\ u_2 \qquad \text{............} \quad (6.47)$$

$$\frac{dh}{dt} = -4h + 9g + 20\omega + 20\ g_f \qquad \text{............} \quad (6.48)$$

Equations (6.41) to (6.48) consist of a complete set of system equations. δ, ω, ψ_f, v_f, v_s, g, g_f and h are the state variables. The system is non-linear because of the non-linear terms of equations (6.42) and (6.43). u_1 and u_2 are the controls given by:

$$u_1 = k_{11}(\delta - \delta_d) + k_{12}(\omega - \omega_d) + u_{1d} \qquad \text{............} \quad (6.49)$$

$$u_2 = k_{21}(\delta - \delta_d) + k_{22}(\omega - \omega_d) + u_{2d} \qquad \text{............} \quad (6.50)$$

where k_{11}, k_{12}, k_{21} and k_{22} are the elements of a simple proportional multivariable controller matrix $\underline{k}$, given by

$$\underline{k} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

The controller $\underline{k}$ is to be designed so that the power level goes from a given initial value (assumed to be zero) to a desired final value. The final desired real power P_d is assumed to be 0.735 and the reactive power Q_d is 0.034 (Reddy and Sannuti (1976)). In the system equations (6.41) to (6.50), the power can be controlled by controlling the power angle, δ. The final desired values of δ_d, ω_d and ψ_{fd} are calculated from the final desired power level.

The performance index, J_S, has been defined as

$$J_S = 50(\delta(5) - \delta_d)^2 + \int_0^5 \Big[10(\delta - \delta_d) + 10(\omega - \omega_d)^2 + 0.1(\psi_f - \psi_{fd})^2 +$$

$$100(u_1 - u_{1d})^2 + 100\ (u_2 - u_{2d})^2\Big] dt \qquad \text{.......... (6.51)}$$

where the initial conditions are:

$\delta(0) = 0 \qquad \omega(0) = 1.0 \qquad \psi_f(0) = 10.5$

$\psi_f(0) = 0 \qquad h(0) = 0 \qquad g(0) = 0$

The final conditions are:

$\delta_d = 0.375 \qquad \omega_d = 1.0 \qquad \psi_{fd} = 11.2$

$u_{1d} = 0.272 \qquad u_{2d} = -0.845$

where the values u_{1d} and u_{2d} are found from the steady state solution by equating the derivatives to zero.

6.1.6 Summary of the system equations

The dynamic representation of the synchronous machine as well as its associated control systems are summarised as follows:

$$\dot{x}_{1s} = x_{2s} - 1.0 \qquad \text{.......... (6.52)}$$

$$\dot{x}_{2s} = 0.0433.x_{2s} - 0.0422\ x_{3s} \sin x_{1s} + 0.0367 \sin 2x_{1s} - 0.304\ x_{6s} + 0.334\ x_{8s} \qquad \text{.......... (6.53)}$$

$$\dot{x}_{3s} = x_{4s} - 0.2\ x_{3s} + 1.05 \cos x_{1s} \qquad \text{.......... (6.54)}$$

$$\dot{x}_{4s} = -4\ x_{4s} - 10.x_{5s} + 10.5 \qquad \text{.......... (6.55)}$$

$$\dot{x}_{5s} = -5\ x_{5s} + 10\ u_{1s} \qquad \text{.......... (6.56)}$$

$$\dot{x}_{6s} = -4.5\ x_{6s} - 10\ x_{2s} - 10\ x_{7s} \qquad \text{.......... (6.57)}$$

$$\dot{x}_{7s} = -10\ x_{7s} + 10\ u_{2s} \qquad \text{.......... (6.58)}$$

$$\dot{x}_{8s} = -4.x_{8s} + 9\,x_{6s} + 20\,x_{2s} + 20\,x_{7s} \quad \ldots\ldots\ldots \quad (6.59)$$

$$u_{1s} = k_{11s}\left[x_{1d} - x_{1s}\right] + k_{12s}\left[x_{2d} - x_{2s}\right] + u_{1ds} \quad \ldots\ldots \quad (6.60)$$

$$u_{2s} = k_{21s}\left[x_{1d} - x_{1s}\right] + k_{22s}\left[x_{2d} - x_{2s}\right] + u_{2ds} \quad \ldots\ldots \quad (6.61)$$

$$J_s = 50(x_{1s}(5) - x_{1d})^2 + \int_0^5 \{10(x_{1s} - x_{1d})^2 + 10(x_{2s} - x_{2d})^2 + 0.1(x_{3s} - x_{3d})^2 + 100(u_{1s} - u_{1sd})^2 + 100(u_{2s} - u_{2ds})^2\}dt \quad \ldots\ldots\ldots \quad (6.62)$$

where $\delta = x_1$, $\omega = x_2$, $\psi_f = x_3$, $v_f = x_4$, $v_s = x_5$, $g = x_6$, $g_f = x_7$, $h = x_8$, $\delta_d = x_{1d}$, $\omega_d = x_{2d}$, $\psi_{fd} = x_{3d}$; the dots denote $\frac{d}{dt}$, and subscript s refers to the variables in the system equations.

6.2 Model Description

6.2.1 Non-linear model

A simplified third order mathematical model can be formed by neglecting the dynamic equations of v_f, v_s, g, g_f and h, assuming that they reach their steady state much faster. In addition, the exciter control voltage and governor time constant may be neglected by taking the exciter gain μ_e and the total droop σ as the unknown parameters whose values are to be estimated in order that the model represents the system as closely as possible. Hence, from the original system equations given by equations (6.28) to (6.30) the model equations can be given as:

$$\frac{d\delta_m}{dt} = \omega_m - 1 \quad \ldots\ldots\ldots\ldots\ldots \quad (6.63)$$

$$\frac{d\omega_m}{dt} = \frac{1}{M}\left[k_1 h_m + k_2 g_m + k_3 \omega_m - \frac{v_o\, v_{fm} \sin\delta_m}{\tau_o\, x'_d} - \frac{(x'_d - x_d)}{2x'_d\, x_q} \cdot v_o^2 \sin 2\delta_m\right] \quad \ldots\ldots\ldots\ldots\ldots \quad (6.64)$$

$$\frac{d\psi_m}{dt} = v_{fm} - \frac{x_d\,\psi_{fm}}{x'_d\,\tau_d} + \frac{(x_d - x'_d)}{x'_d} v_o \cos\delta_m \quad \ldots\ldots\ldots\ldots \quad (6.65)$$

$$v_{fm} = \alpha_1(v_o - \mu_s u_{1m}) \quad \text{.............. (6.66)}$$

$$g_m = -\alpha_2 \left(\frac{\omega_m}{\omega_o} - \mu_a u_{2m}\right) \quad \text{.............. (6.67)}$$

$$u_{1m} = k_{11m}(\delta_d - \delta_m) + k_{12m}(\omega_d - \omega_m) + u_{1dm} \quad \text{......... (6.68)}$$

$$u_{2m} = k_{21m}(\delta_d - \delta_m) + k_{22m}(\omega_d - \omega_m) + u_{2dm} \quad \text{......... (6.69)}$$

By substituting the numerical values in the above equations the complete model equations become:

$$\dot{x}_{1m} = x_{2m} - 1 \quad \text{.............. (6.70)}$$

$$\dot{x}_{2m} = (0.0433 + 0.304\,\alpha_2)x_{2m} - 0.0422\, x_{3m}.\sin x_{1m} + 0.0367 \sin 2x_{1m} + .304\,\alpha_2\, u_{2m} \quad \text{.............. (6.71)}$$

$$\dot{x}_{3m} = 0.2\, x_{3m} + 1.05 \cos x_{1m} - 4.0\,\alpha_1(1.05 - 2u_{1m}) \quad \text{.............. (6.72)}$$

$$u_{1m} = k_{11m}(x_{1d} - x_{1m}) + k_{12m}(x_{2d} - x_{2m}) + u_{1dm} \quad \text{...... (6.73)}$$

$$u_{2m} = k_{21m}(x_{1d} - x_{1m}) + k_{22m}(x_{2d} - x_{2m}) + u_{2dm} \quad \text{...... (6.74)}$$

where the dot denotes $\frac{d}{dt}$ and subscript m denotes the variables associated with the model equations. x_{1m}, x_{2m} and x_{3m} are the model state variables. The model is also non-linear because of the non-linear terms of equations (6.71) and (6.72).

6.2.2 Linear model

By neglecting the dynamic equations of v_f, v_s, g, g_f and h, assuming that they reach their steady state much faster, the system equations (6.41) to (6.48) can be written as

$$\dot{x}_{1m} = x_{2m} - 1.0 \quad \text{.......... (6.75)}$$

$$\dot{x}_{2m} = 0.0433\, x_{2m} - 0.0422\, x_{3m} \sin x_{1m} + 0.0367 \sin 2x_{1m} - 0.304\, x_{6m} + 0.334\, x_{8m} \quad \text{.......... (6.76)}$$

$$\dot{x}_{3m} = x_{4m} - 0.2\ x_{3m} + 1.05 \cos x_{1m} \qquad \text{..........} \quad (6.77)$$

$$-4\ x_{4m} - 10\ x_{5m} + 10.5 = 0 \qquad \text{..........} \quad (6.78)$$

$$-5\ x_{5m} + 10\ u_{1m} = 0 \qquad \text{..........} \quad (6.79)$$

$$-4.5\ x_{6m} - 10\ x_{2m} - 10\ x_{7m} = 0 \qquad \text{..........} \quad (6.80)$$

$$-10\ x_{7m} + 10\ u_{2m} = 0 \qquad \text{..........} \quad (6.81)$$

$$-4\ x_{8m} + 9\ x_{6m} + 20\ x_{2m} + 20\ x_{7m} = 0 \qquad \text{..........} \quad (6.82)$$

From equations (6.82) and (6.80),

$$x_{8m} = 0 \qquad \text{..........} \quad (6.83)$$

Equations (6.78) and (6.79) will give

$$x_{4m} = -5\ u_{1m} + 2.625 \qquad \text{..........} \quad (6.84)$$

Combination of equations (6.80) and (6.81) yields

$$x_{6m} = (-10\ x_{2m} - 10\ u_{2m})/4.5 \qquad \text{..........} \quad (6.85)$$

Substituting for x_{4m}, x_{6m} and x_{8m} in equations (6.76) and (6.77), the model equations may then be summarised as:

$$\dot{x}_{1m} = x_{2m} - 1.0 \qquad \text{..........} \quad (6.86)$$

$$\dot{x}_{2m} = 0.68\ x_{2m} - 0.0422\ x_{3m} \sin x_{1m} + 0.0367 \sin 2x_{1m} + 0.675\ u_{2m} \qquad \text{..........} \quad (6.87)$$

$$\dot{x}_{3m} = -0.2\ x_{3m} + 1.05 \cos x_{1m} - 5\ u_{1m} + 2.625 \qquad \text{......} \quad (6.88)$$

The Jacobian matrix of $\dot{\underline{x}}_m$ with respect to the states $\underline{x}_m$ evaluated at the point of linearisation will be:

$$J = \begin{bmatrix} 0 & 1 & 0 \\ J_{21} & 0.68 & J_{23} \\ J_{31} & 0 & -0.2 \end{bmatrix}$$

where
$$\left.\begin{aligned} J_{21} &= 0.0422\ \alpha_2 \cos \alpha_1 + 2 \times 0.0367 \cos 2\alpha_1 \\ J_{23} &= -0.0422 \sin \alpha_1 \\ J_{31} &= -1.05 \sin \alpha_1 \end{aligned}\right\} \text{.....} \quad (6.89)$$

where α_1 and α_2 are the unknown parameters, whose values should be estimated. Hence, the linearised simplified third order model may be written as

$$\dot{x}_{1m} = x_{2m} - 1 \quad \text{.............} \quad (6.90)$$

$$\dot{x}_{2m} = J_{21} x_{1m} + 0.68 x_{2m} + J_{23} x_{3m} + 0.675 u_{2m} \quad \text{......} \quad (6.91)$$

$$\dot{x}_{3m} = J_{31} x_{1m} - 0.2 x_{3m} - 5 u_{1m} + 2.625 \quad \text{.............} \quad (6.92)$$

$$u_{1m} = k_{11m}(x_{1d} - x_{1m}) + k_{12m}(x_{2d} - x_{2m}) + u_{1dm} \quad \text{......} \quad (6.93)$$

$$u_{2m} = k_{21m}(x_{1d} - x_{1m}) + k_{22m}(x_{2d} - x_{2m}) + u_{2dm} \quad \text{......} \quad (6.94)$$

Equations (6.90) to (6.94) constitute the linearised model of the synchronous generator.

6.3 Formulation of the Parameter Estimation Problem and the Optimisation Problem

6.3.1 Parameter estimation problem

The parameter estimation problem can be defined by

$$\min_{\underline{\alpha}} \{G(\underline{k}_s, \underline{\alpha}) = \int_0^5 \left[\omega_1(\delta_s - \delta_m)^2 + \omega_2(\omega_3 - \omega_m)^2 + \omega_3(\psi_{fs} - \psi_{fm})^2\right] dt\}$$

s.t. model and system equations .... (6.95)

where $\underline{\omega}_i$ (i = 1, 2, 3) are the weighting coefficients. In order to solve the above problem, equation (6.95) can be replaced by:

$$\min_{\underline{\alpha}} \{G(\underline{k}_s, \underline{\alpha}) = \sum_{n=0}^{N_s} \left[\omega_1^n(\delta_s^n - \delta_m^n)^2 + \omega_2^n(\omega_s^n - \omega_m^n)^2 + \omega_3^n(\psi_{fs}^n - \psi_{fm}^n)^2\right]\}$$

s.t. model and system equations .... (6.96)

where N_s is the number of samples taken in a fixed interval. This will give n samples per measurement where

$$n = N_s + 1$$

6.3.2 Optimisation problem

The optimisation problem can be defined as

$$\min_{\underline{k}_m} \{J_m(\underline{k}_m, \underline{\sigma}) = 50(\delta_m(5) - \delta_d)^2 + \int_o^5 \Big[10(\delta_m - \delta_d)^2 + 10(\omega_m - \omega_d)^2 +$$

$$0.1(\psi_{fm} - \psi_{fd})^2 + 100(u_{1m} - u_{1d})^2 + 100(u_{2m} - u_{2d})^2\Big] dt\}$$

s.t. the model equations .... (6.97)

6.3.3 Coordination components

In the coordination unit we will have:

$$k_{11m} = k_{11s}$$
$$k_{12m} = k_{12s}$$
$$k_{21m} = k_{21s}$$
$$k_{22m} = k_{22s}$$
$$\sigma_1 = \alpha_1$$
$$\sigma_2 = \alpha_2$$

6.4 Discussion of Results

The synchronous generator system, its simplified dynamic model, parameter estimation, optimisation and coordination unit, described in previous sections, have been simulated on a digital computer and several investigations have been conducted of the performance on the system and the overall scheme.

6.4.1 Performance of the real system

In order to investigate the effectiveness of the overall scheme, the real system was simulated and an optimisation routine was employed to determine the optimal value of the controller gain matrix $[\underline{k}_s]$. This minimises the performance index J_s, which has been defined by equation (6.62). Table 6.1 illustrates the result of the optimisation of the real system starting from three different initial conditions. As can be observed from the results, the performance index, J_s, has converged to the same optimal point. Figure 6.4 shows the variation of J_s as k_{ij} is varied, keeping the other elements of $[k]$ at their optimal values. The results clearly show the "flatness" of the minimum point.

INITIAL CONDITIONS					OPTIMAL SOLUTION				
[k]				Performance index J_S	[k]				Performance index J_S^*
k_{11}	k_{12}	k_{21}	k_{22}		k_{11}	k_{12}	k_{21}	k_{22}	
0	0	0	0	5841	-0.099	0.046	-0.238	1.8869	2.2
1.0	1.0	1.0	1.0	39.08	〃	〃	〃	〃	〃
0.5	0.5	0.5	0.5	36.02	〃	〃	〃	〃	〃

Table 6.1: Convergence of the real system to the optimal solution

6.4.2 Performance of the integrated schemes

The simplified mathematical model was formed by reducing the order of the system from eighth to third. The simplified model contains a set of non-linear differential equations. The reduced model was further simplified by linearising the non-linear model. Hence, there exist two models upon which the performance of the overall scheme has been conducted.

6.4.2.1 Non-linear model

In this case the non-linear model was simulated and the performances of the following schemes were investigated.

6.4.2.1.1 Performance of the standard two-step approach

Figures 6.5, 6.6 and 6.7 show the performance of the standard unmodified two-step approach, starting from three different initial conditions. Figure 6.8 shows the corresponding variation of the controller gain settings. Although the result obtained for J_s, employing the two-step approach, does not converge to the true minimum value, its performance cannot be considered as unsatisfactory. As can be seen, the results are in agreement. These results imply that the output gradients with respect to controller gain settings are nearly matched between the real process and the simplified model at the final converged point.

6.4.2.1.2 Performance of the modified two-step approach

Figures 6.5, 6.6 and 6.7 also show the performance of the modified two-step algorithm, starting from the same three alternative initial points as are employed using the standard two-step approach. The results illustrate the rapid convergence from the initial points A and C (B started near to the optimum point) and then a zigzag type behaviour close to the optimum point. Figure 6.9 shows the corresponding variation of the controller gain. Figure 6.10 illustrates the behaviour of the Lagrangian modifiers, starting from point C, which illustrates the oscillation of the modifiers around zero. This signifies the matching of the output gradients of the model and the system without any additional terms in the modified optimisation problem (i.e. $\underline{\lambda} = 0$ for standard two-step approach).

6.4.2.2 Linear model

The non-linear model which has been described in section 6.2.1 was linearised. The performance of the standard and modified two-step approach was investigated.

6.4.2.2.1 Performance of the standard two-step approach

Figures 6.11 and 6.12 show the performance of the standard approach, starting from two alternative initial conditions. In both cases the results illustrate the convergence of the unmodified algorithm close to the true minimum. These results also imply a fairly well matched gradient between the real system and the simplified linear mathematical model. It should be noticed that they are not as good as the previous results as the model is a less perfect representation of the real system. Figure 6.13 shows the corresponding variation of the controller gain.

6.4.2.2.2 Performance of the modified two-step approach

Figures 6.11 and 6.12 also show the variation of the performance index, J_s, employing the modified two-step approach, starting from the same initial conditions as used above. The results illustrate a very rapid convergence, initially, and then behaving in an oscillatory manner as it gets closer to the minimum value.

This oscillation is expected as it is a difficult task to obtain zero values of the gradients $\left[\frac{\partial G}{\partial \underline{\alpha}}\right]$ as the theory of the modified two-step algorithm requires.

Figure 6.14 shows the variation of the controller gain, starting from initial conditions C and D.

Figure 6.15 shows the behaviour of the modifiers, $\underline{\Lambda}$, during the optimisation, starting from initial condition C. It can be observed that the modifiers also oscillate around zero.

It should be noted that the controller gain matrix elements, k_{11}, k_{12}, k_{21} and k_{22} do not converge to the true optimal values. This is partly due to non-zero values of $\frac{\partial G}{\partial \underline{\alpha}}$ and partly due to flat curves shown in Figure 6.4 at their minimum values.

6.5 Conclusions

The algorithm which has been presented in chapter 5 attempts to take account of the inherent interaction between parameter estimation and optimisation when an imperfect mathematical model is employed to determine the optimal control gain of a dynamic control system. In this chapter the above algorithm has been employed to the optimal feedback control of a synchronous generator system, in order to minimise a defined performance index.

The mathematical model which has been employed in this study has been deliberately simplified in order to show the effectiveness of the algorithm and also facilitate the solution of the model. Two mathematical models have been proposed:

(i) Simplified model with non-linear reduced number of differential equations;

(ii) As in (i) when the differential equations have been linearised.

The modified two-step algorithm was successfully applied in the design of the feedback control of a synchronous generator by adjusting the controller gain to minimise the given performance index, J_s. As has been explained in a previous chapter, the requirement of the modified two-step approach to have $\frac{\partial G}{\partial \underline{\alpha}}$ equal to zero in the Lagrangian analysis will give rise to a slight error if the above zero gradient condition is not met.

Unlike the previous cases, the standard two-step algorithm also showed success in the design of the above system. In case (i) it even illustrated advantage over the modified two-step approach as far as the simplicity of the algorithm, and also more accurate results, were concerned. This implies that the output gradients with respect to controller gains are matched between the real system and the simplified non-linear model at the final converged point.

In case(ii), because the model is a less accurate representation of the system and also the above gradient condition is not met exactly, the modified two-step approach showed slight advantage over the standard algorithm.

The rapid dynamic convergence of the modified two-step technique can be observed in this particular problem. Further research is required to investigate the dynamic convergence, together with the final convergence of the modified approach.

It can be concluded that if the structure is uncertain of a mathematical model whose parameters are to be estimated, and if the gradients between model and plant are not closely matched at the optimum point, then the application of the modified two-step algorithm is advantageous.

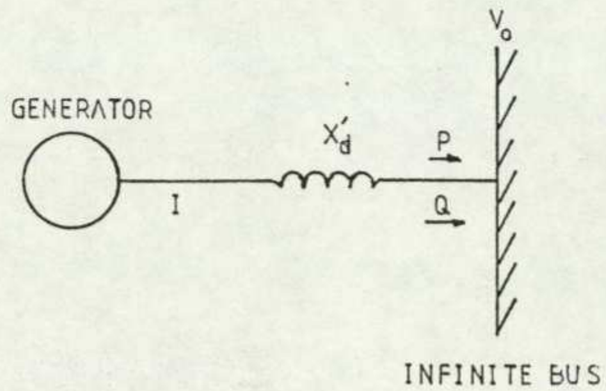


FIG.(6.1) SYNCHRONOUS GENERATOR SYSTEM.

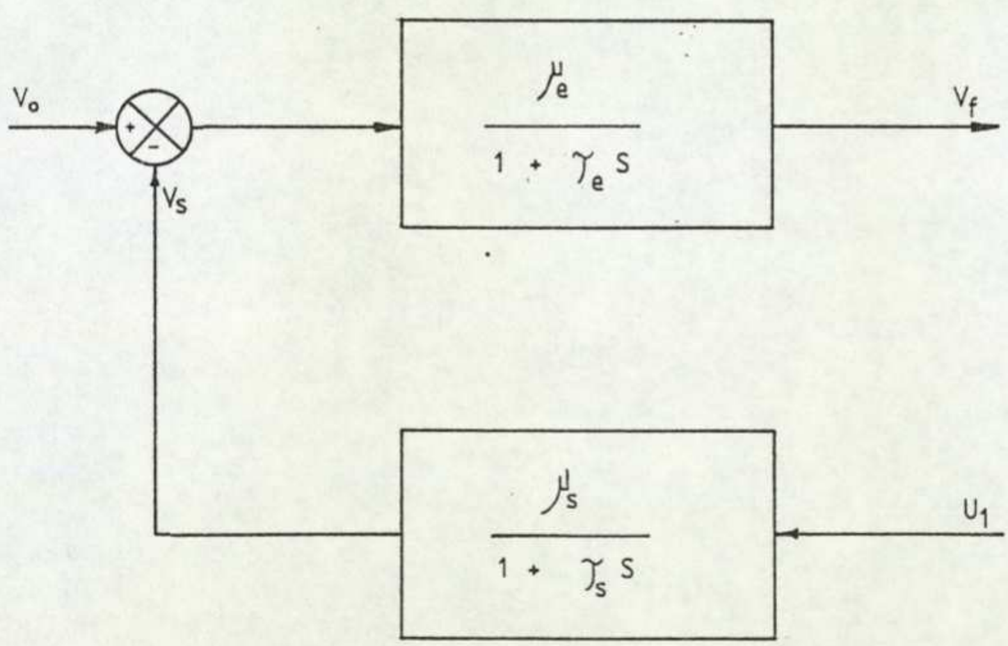


FIG.(6.2) EXCITER VOLTAGE CONTROL SYSTEM.

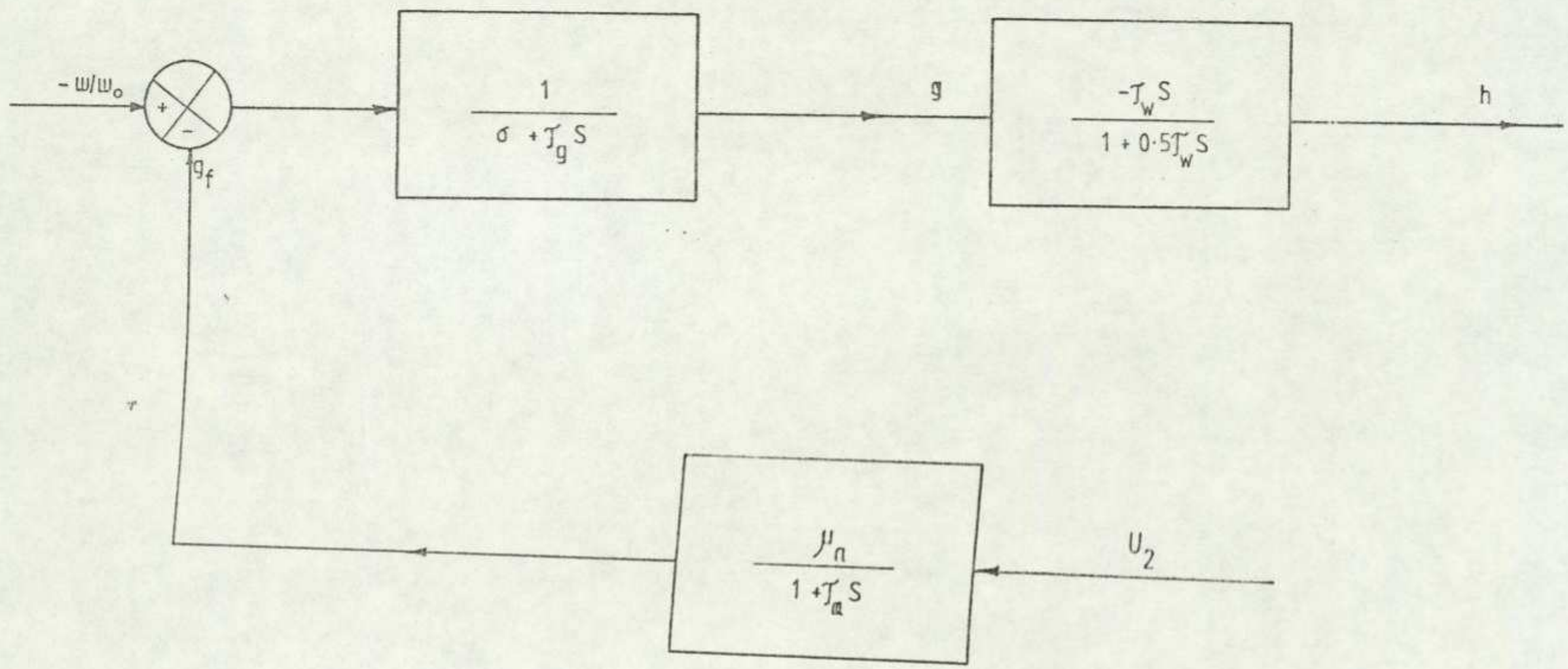
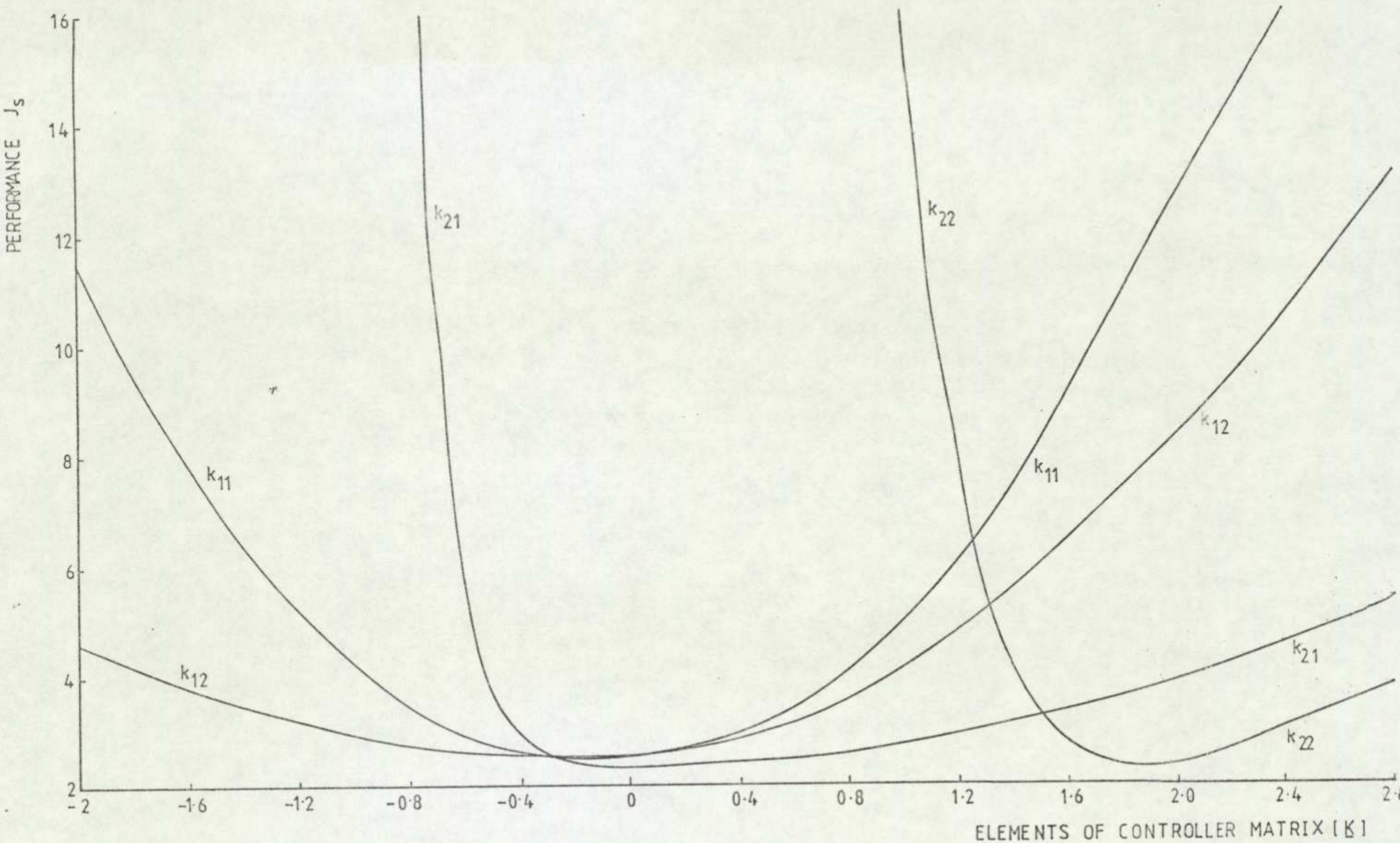


FIG.(6.3) HYDRAULIC GOVERNOR CONTROL SYSTEM.



FIG(6.4) VARIATION OF THE PERFORMANCE INDEX WITH CONTROLLER GAIN.

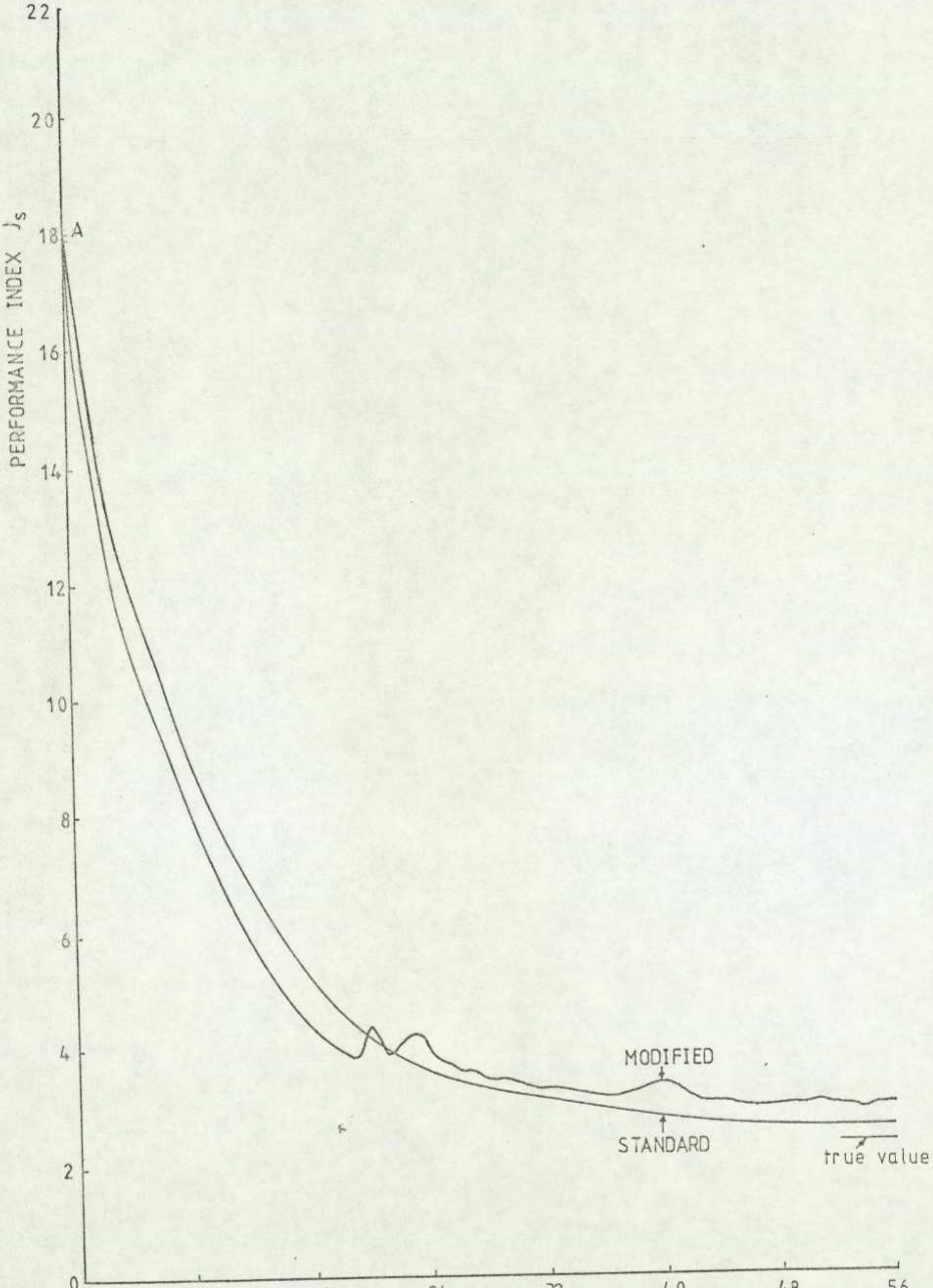


FIG.(6.5) VARIATION OF THE PERFORMANCE INDEX J_s, USING THE NON-LINEAR MODEL, STARTING FROM INITIAL CONDITION A.

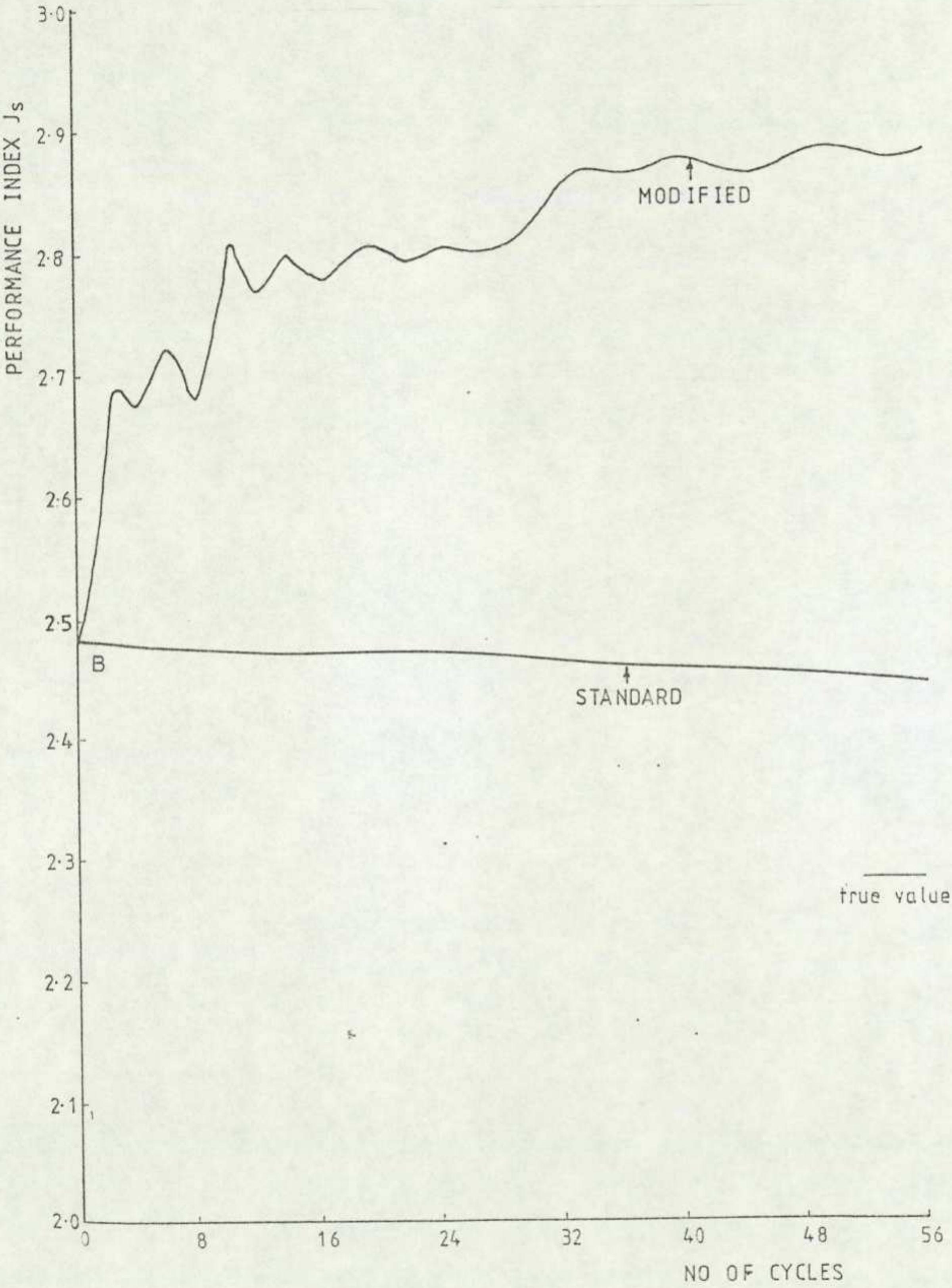


FIG.(6.6) VARIATION OF THE PERFORMANCE INDEX J_s, STARTING FROM INITIAL CONDITION B.

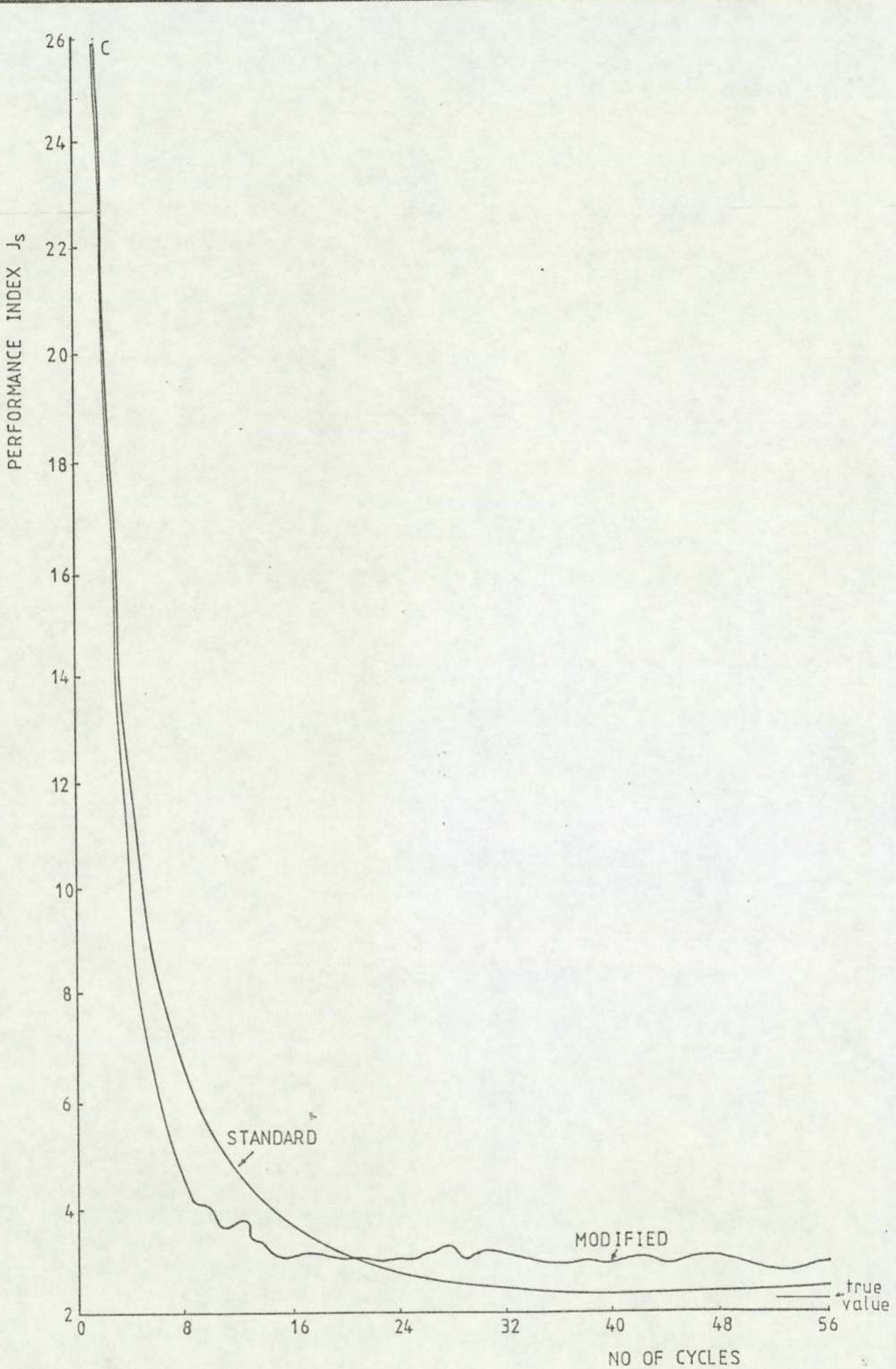
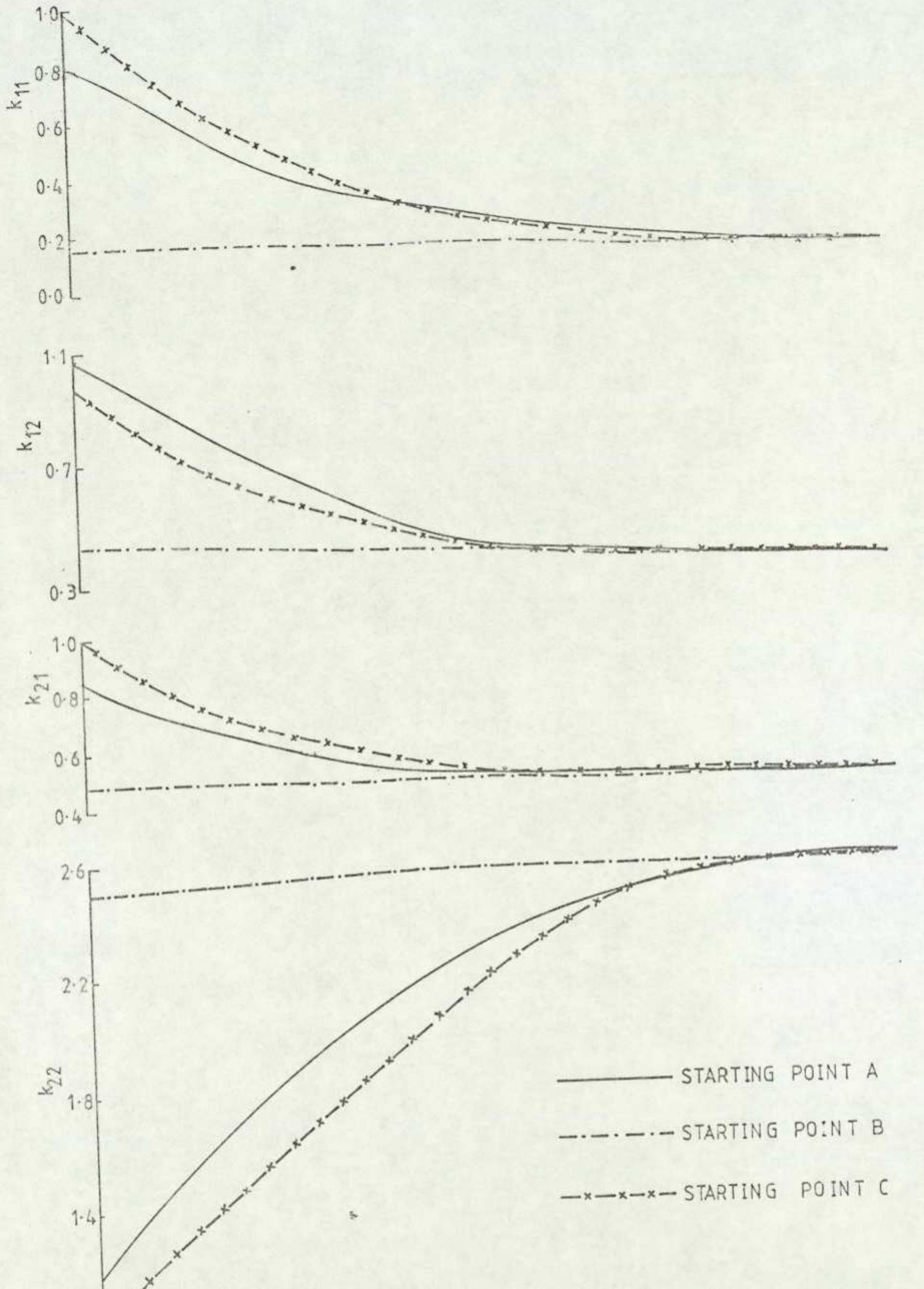


FIG.(6.7) VARIATION OF THE PERFORMANCE INDEX J_s, USING THE NON-LINEAR MODEL, STARTING FROM INITIAL CONDITION C.



FIG(6.8) VARIATION OF THE CONTROLLER GAIN USING NON-LINEAR MODEL DURING THE STANDARD TWO-STEP APPROACH.

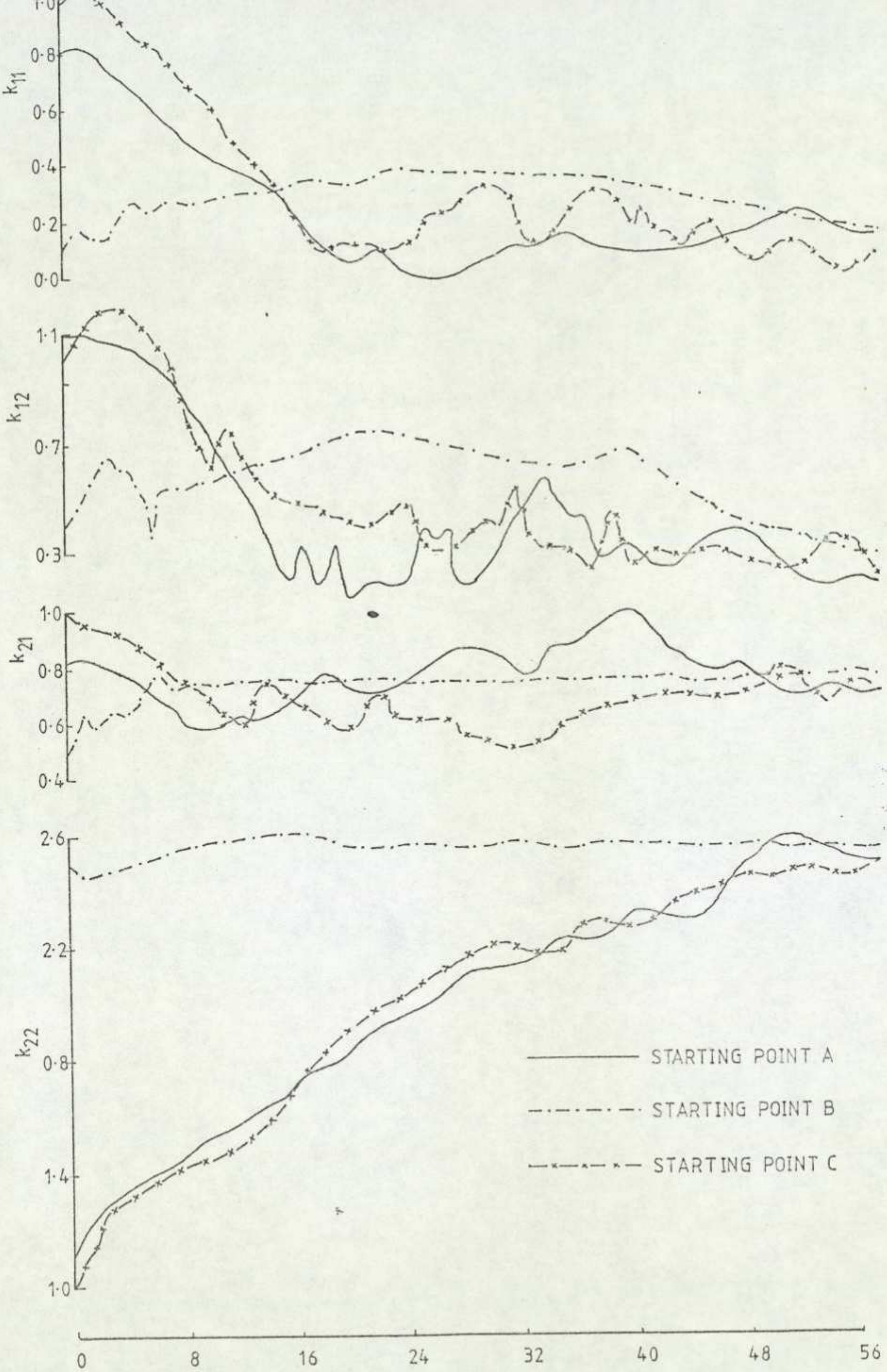


FIG.(6.9) VARIATION OF THE CONTROLLER GAIN USING NON-LINEAR MODEL DURING THE MODIFIED TWO-STEP APPROACH.

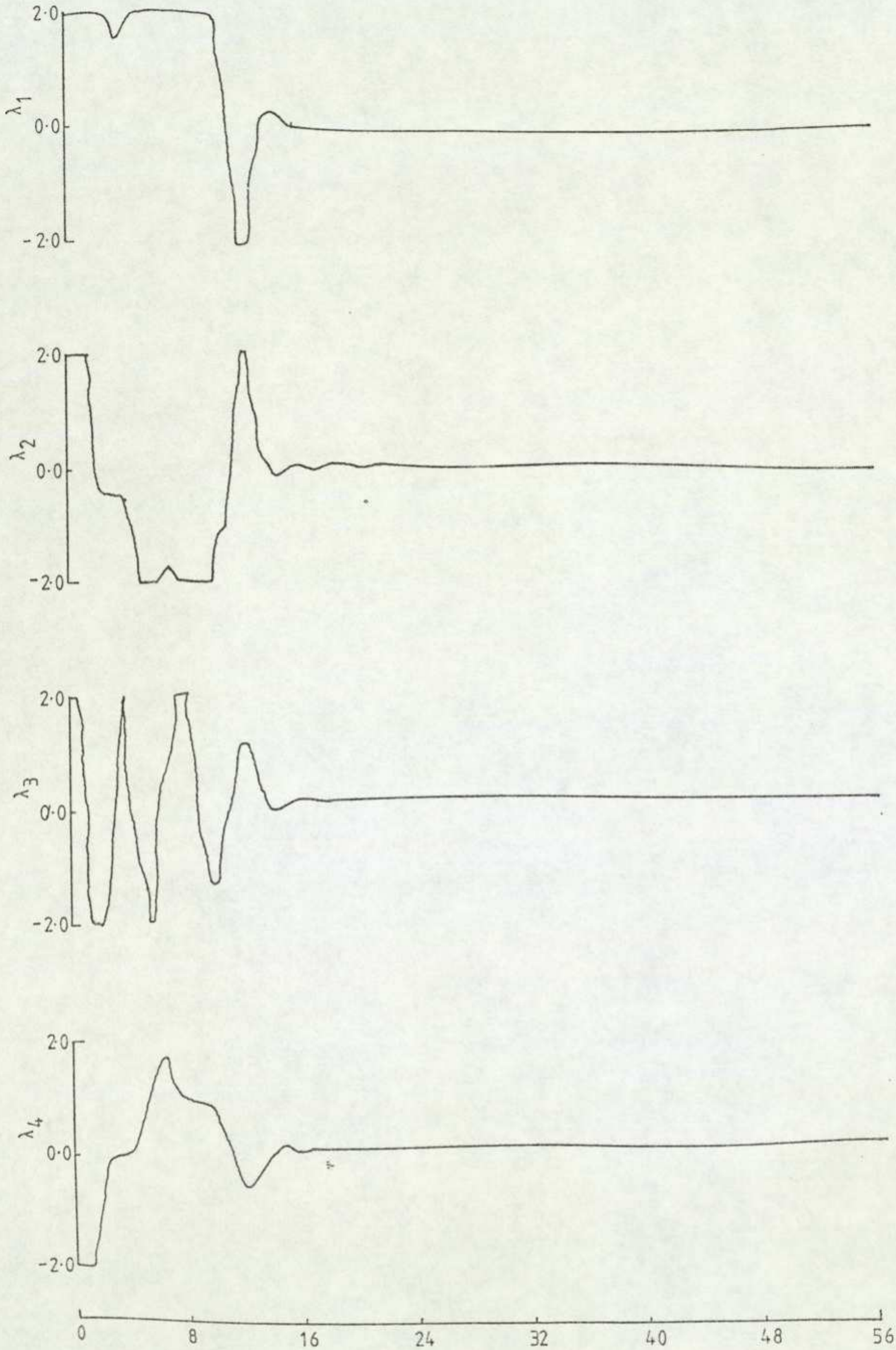


FIG.(6.10) VARIATION OF OPTIMISATION MODIFIERS USING NON-LINEAR MODEL DURING MODIFIED TWO-STEP APPROACH STARTING FROM INITIAL CONDITION C.

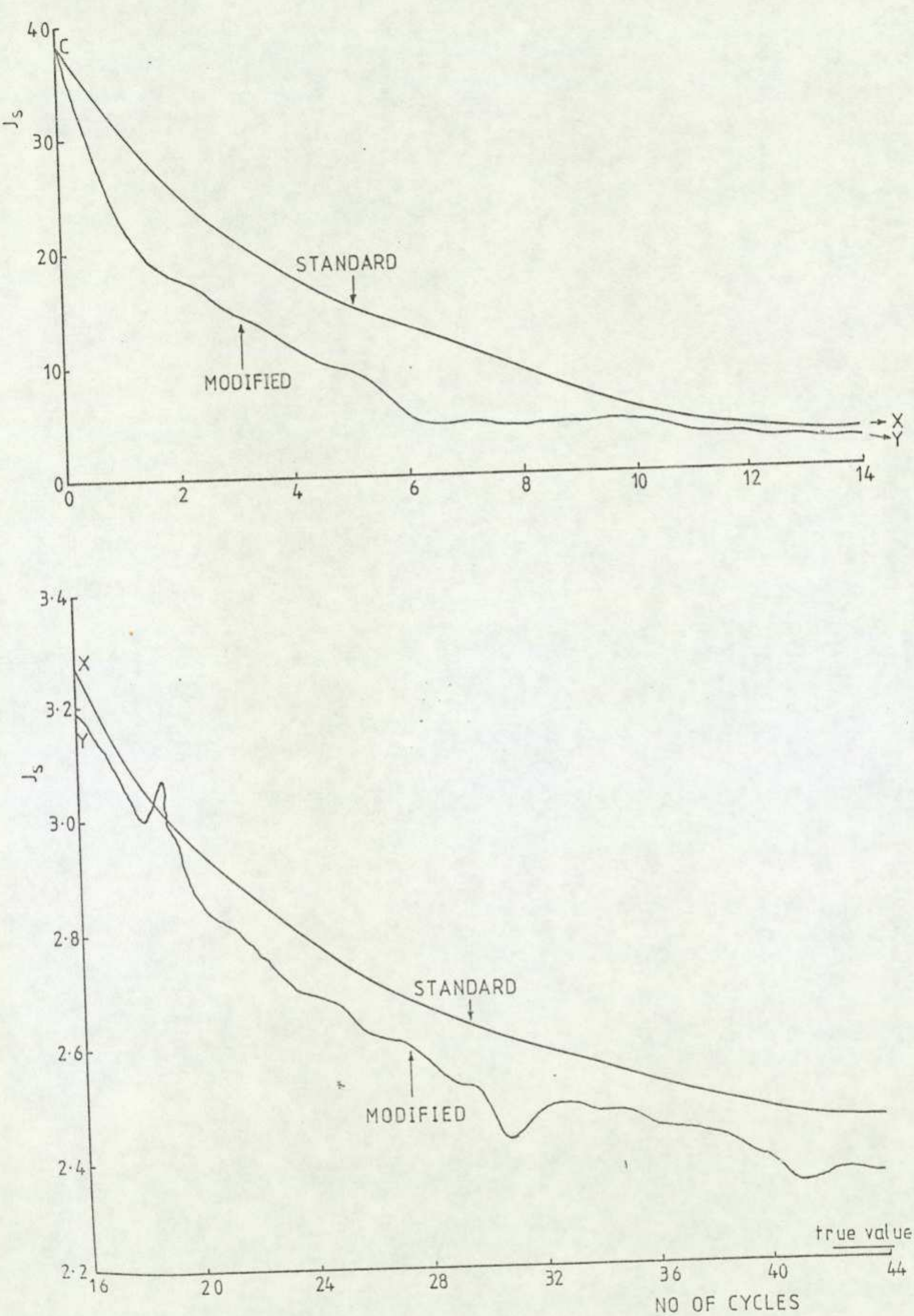


FIG.(6.11) VARIATION OF THE PERFORMANCE INDEX J_s, USING THE LINEAR MODEL, STARTING FROM INITIAL CONDITION C.

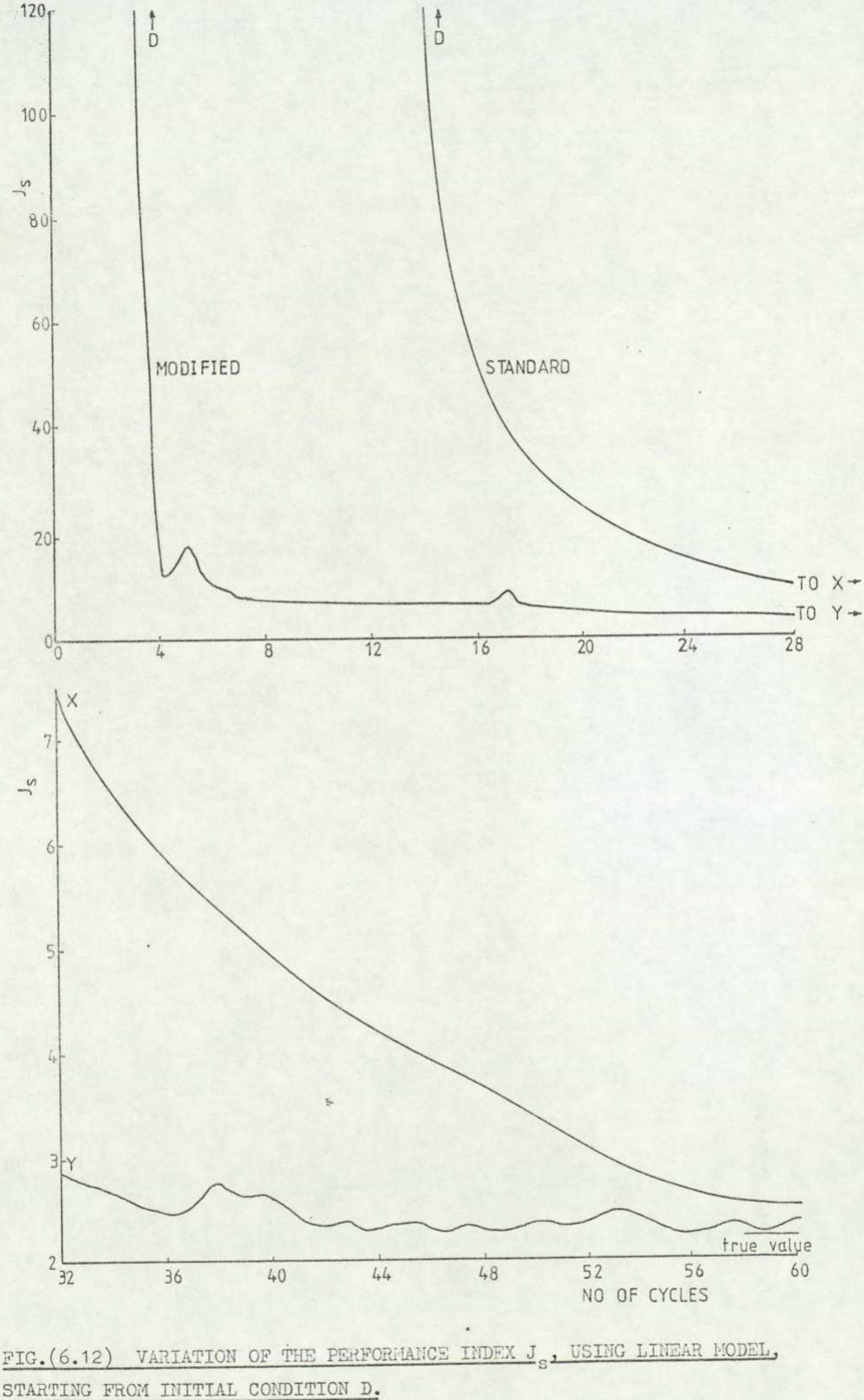


FIG.(6.12) VARIATION OF THE PERFORMANCE INDEX J_s, USING LINEAR MODEL, STARTING FROM INITIAL CONDITION D.

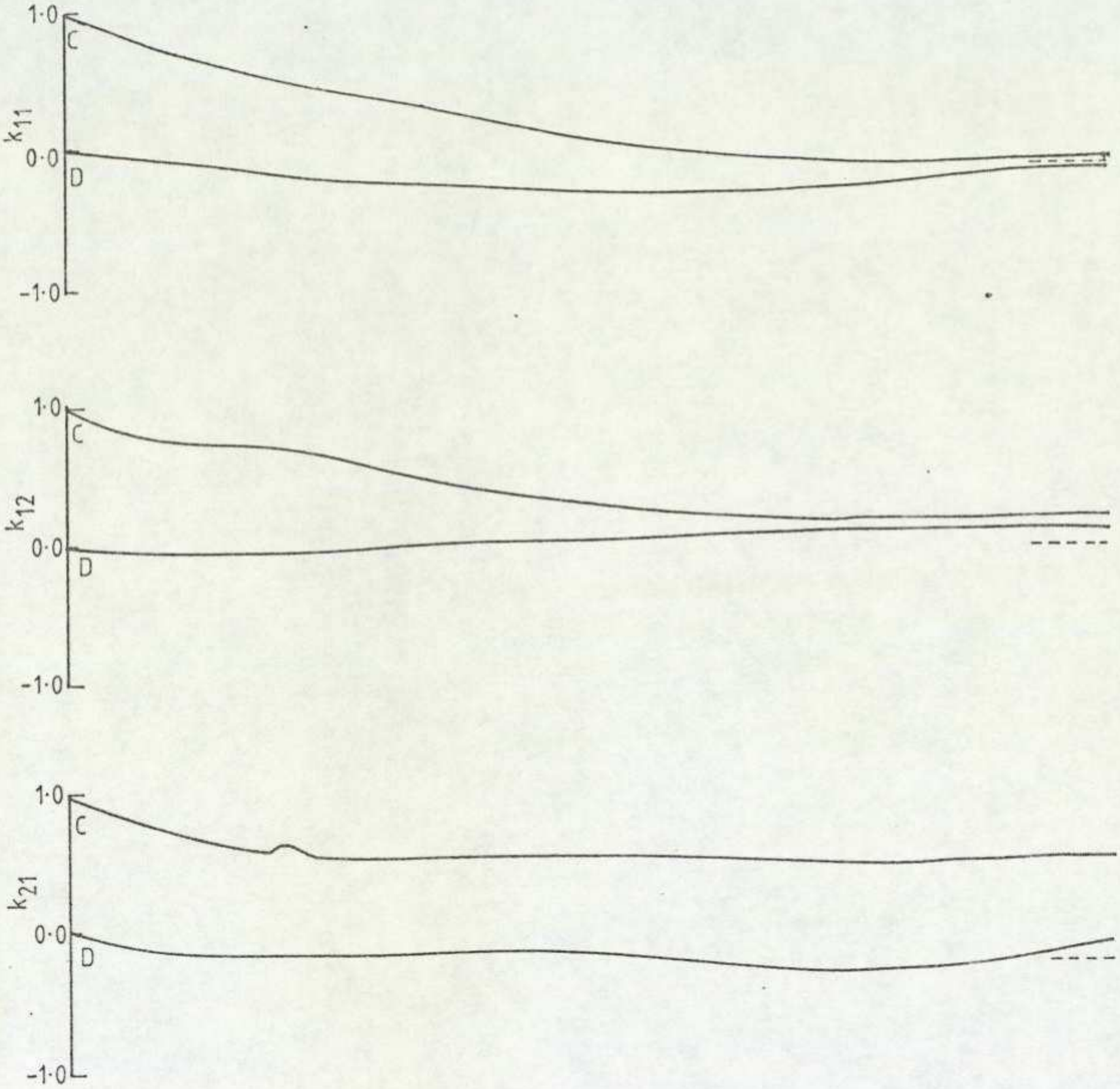


FIG.(6.13) VARIATION OF THE CONTROLLER GAIN, USING THE LINEAR MODEL, DURING THE STANDARD TWO-STEP APPROACH.

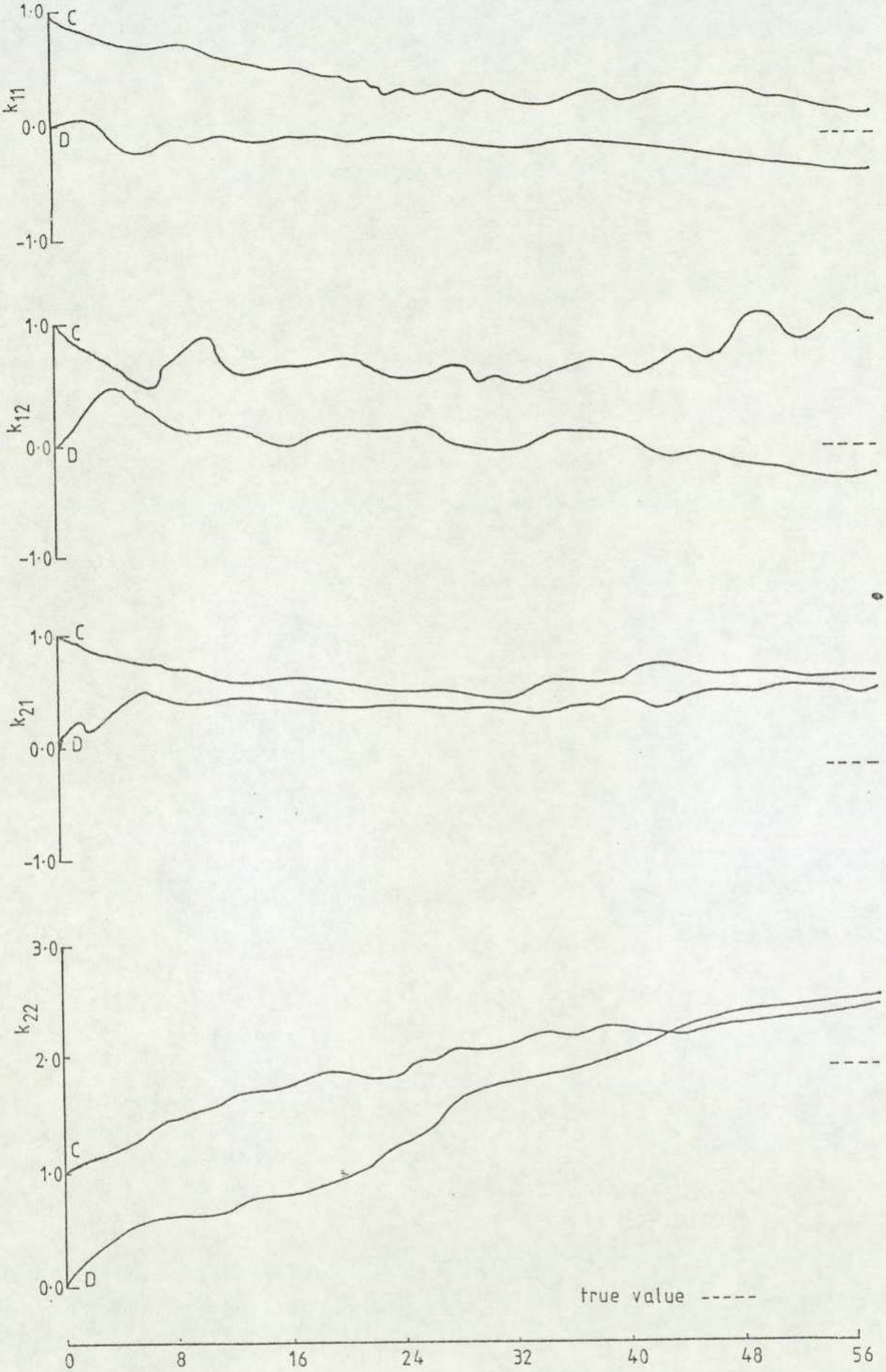


FIG.(6.14) VARIATION OF THE CONTROLLER GAIN, USING THE LINEAR MODEL, DURING THE MODIFIED TWO-STEP APPROACH.

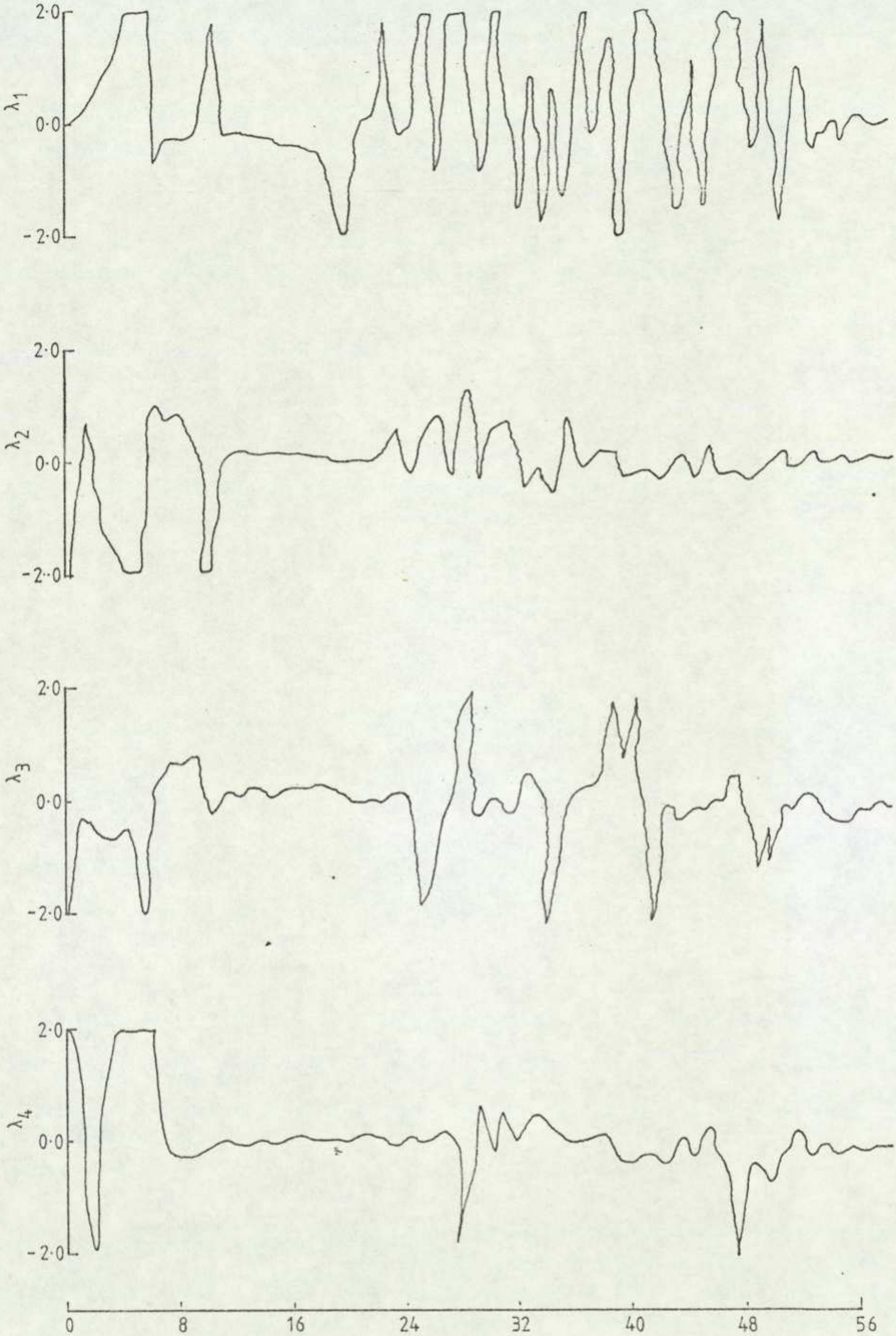


FIG.(6.15) VARIATION OF THE OPTIMISATION MODIFIERS USING LINEAR MODEL DURING THE MODIFIED TWO-STEP APPROACH, STARTING FROM INITIAL CONDITION C.

CHAPTER 7: CONCLUSIONS

Some techniques which attempt to take account of the interaction between parameter estimation and system optimisation have been reviewed. These techniques have been applied to a problem determining the steady state optimum operating condition of a chemical plant through the use of an adaptive steady-state model whose parameters are estimated by comparing model outputs with those of the real plant. The objective is to maximise the net rate of return obtained from the chemical plant by manipulating the set points of flow, temperature and recycle ratio controllers.

The standard two-step approach, which is the common technique applied in practice, has been applied to the chemical plant. This failed to converge to the correct optimal solution because the gradients of the real plant outputs with respect to the control inputs were not matched with the corresponding gradients of the model.

Both the ε-constraint approach and the parametric approach, which attempt to take account of the interaction between the two problems, combine the two problems into a single joint problem. These techniques, without any decomposition, have been applied to the chemical plant using multilevel techniques. Unfortunately, these two approaches did not show any success because they also failed to take account of the above gradient matching. They also showed disadvantages over the two-step approach by having increased dimensionality which increases the computational burden.

The two-step approach is modified by introducing extra terms to the optimisation performance index to allow for any mismatch between the model and the real system output derivatives. This is achieved by taking account of model inputs as well as the model parameters as interconnection constraints.

The ε-constraint and the parametric approaches were decomposed by cutting the linkage between the two problems caused by uncertain model parameters and the control inputs, then coordinating the two modified subproblems at a higher level. From the theory given, it can be seen that in the limit, the final solution of these methods is identical with the final solution of the modified two-step approach.

Using the above decomposed problems will greatly increase the computational time; hence, in this work they have not been investigated further. However, additional study is required to investigate the convergence properties of the above decomposed problems.

Later, the effect of real process dynamics on the performance of the modified two-step approach, which successfully obtained the correct optimum in spite of error in the structure of the mathematical model, was investigated using the simulation study of the above plant. The results obtained have shown the robust nature of the algorithm in that it performs well even when the measurement time between change of set point and record of plant measurement value is considerably less than the time constant of the plant.

The modified two-step approach, which has taken account of the inherent interaction between parameter estimation and system optimisation, has also been used for determining the optimal feedback control of non-linear dynamic systems. This uses the same fundamental principles of Lagrangian analysis and systems decomposition as that of determining the optimal steady-state operating condition of the chemical plant. The procedure has been successfully applied to a simple dynamic optimal control problem and then applied to the control of a synchronous generator. The requirement of the modified two-step approach to satisfy the zero estimation gradient condition,which occurs when the number of unknown parameters is not the same as the number of measurements, and also to evaluate second partial derivatives of the estimation comparison index, imposes important practical limitations on the algorithm. Hence, it is disadvantageous to use the modified two-step approach when the gradients between model and real process are closely matched at the optimum point, even when the model structure is in error.

In both modified algorithms the parameter estimation problem remains unchanged and the optimisation problem is modified by including an extra term $\underline{\lambda}'\underline{u}$. Hence, if $\underline{\lambda}$ is set to zero, the standard two-step approach is obtained. This shows that both algorithms are simple modifications of the standard two-step algorithm.

On the basis of the investigation and simulation studies carried out in this work, it can be concluded that the modified two-step algorithm has application to a wide range of adaptive control systems involving mathematical models whose structure is uncertain and contains parameters to be estimated. However, further research is required to investigate the convergence properties of the technique as applied to dynamic control systems.

APPENDICES

APPENDIX A1: Numerical routines

The Numerical Algorithm Group, Ltd. (N.A.G.) has produced a library of routines for solving a wide range of numerical problems. The following N.A.G. routines have been employed in this thesis in order to solve different types of numerical problems.

CO5NAF: Finds a solution to an n non-linear equation which is based on the FORTRAN subroutine given by Powell (1968).

DO2AEF: Integrates a system of first-order ordinary differential equations over a range, using Gear's method for stiff systems of equations. The method is based on ideas given by Gear (1971), and is intended to solve "stiff" systems of equations.

DO2AJF: Integrates a system of first-order ordinary differential equations over a range, using a divided-difference formulation of Gear's method described in Craigie (1975) and a variant of the method described in Krogh (1973). It is primarily intended for "stiff" systems of equations.

EO4CAF: Minimises a general function of n independent variables using the conjugate direction method based on Powell (1964). Derivatives of the function need not be supplied.

EO4FAF: Finds an approximation to a minimum of the sum of squares of n non-linear residuals, based on Peckham's optimisation algorithm (Peckham (1970)).

EO4JAF: Is a quasi-Newton algorithm for finding a minimum of a given function, subject to fixed upper and lower bounds on variables of the function, using function values only (Gill and Murray (1976, 1972)).

FO1AAF: Calculates the approximate inverse of a real matrix by Crout's method (Wilkinson and Peinsch (1971)).

FO1CKF: Returns with the results of the multiplication of two matrices, B and C, in the matrix A.

FO1CJF: Forms the transpose of a matrix.

G05AEF: Returns a pseudo-random number from the normal distribution, subject to given mean value and standard deviation.

EB365DYSP: Is called from the user supplied master program on USER-LIBRARY 2 (The City University) and has the following subprograms:

TRAN: Simulates a transducer which has single time constant together with appropriate scaling.

VACT: Simulates an actuator for a control valve. It is simulated as a single time constant together with velocity limitation. The coulomb friction is simulated as a dead band. The facility provided for allowing different time constant, stroke time and friction values during opening the valve than used for closing the valve. The valve position is limited and normalised to lie in the range 0 to 1.

CPID: Simulates a general continuous PI or PID controller without integral wind-up.

A1.1 References

Box, G.E.P., and Muller, M. E., 1958,
"A note on the generation of random normal deviates", Ann. Math. Stats., vol. 29, pp. 610-611

Craigie, J. A. I., 1975,
"A variable order multistep method for stiff systems of ordinary differential equations", University of Manchester, Numerical Analysis Report 11

Gear, C. W., 1971,
"Numerical initial value problems in ordinary differential equations", Prentice-Hall, ch. 9

Gill, P. E., and Murray, W., 1972,
"Quasi-Newton methods for uncertained optimisation", Journal of the Institute of Mathematics and its Applications, vol. 9, pp. 91-108

Gill, P. E., and Murray, W., 1976,
"Minimisation subject to bounds on the variables", National Physical Laboratory Report NAC 72

Krogh, F. T., 1973,
"Algorithm for changing the step-size used by a multistep method", SIAM. J. Num. Anal. 10, pp. 949-956

Peckham, G., 1970,
"A new method for minimising a sum of squares without calculating gradients", Computer J., vol. 13, pp. 418-420

Powell, M. J. D., 1964,
"An efficient method for finding the minimum of a function of several variables without calculating derivatives", Computer J., vol. 7, pp. 155-162

Powell, M. J. D., 1968,
"A FORTRAN subroutine for solving systems of non-linear algebraic equations", Harwell Report AERE-R5947, H.M.S.O.

Wilkinson, J. H., and Reinsch, C., 1971,
"Handbook for Automatic Computation", vol. II, Linear Algebra, Springer-Verlag

N.A.G. Library Manual, Nottingham, Algorithm Group. Document Nos:

NAGFLIB: 133/384: Mk5: Jan 76 - C05NAF
NAGFLIB: 793/504: Mk5: Nov 74 - D02AEF
NAGFLIB: 1386/0: Mk5: Feb 76 - D02AJF
NAGFLIB: 1267/148: Mk5: Oct 75 - E04CAF
NAGFLIB: 1310/428: Mk5: Nov 75 - E04FAF
NAGFLIB: 1435/0: Mk6: May 77 - E04JAF
NAGFLIB: 1069/622: Mk5: Sep 75 - F01AAF
NAGFLIB: 925/764: Mk5: Jul 75 - F01CKF
NAGFLIB: 897/285: Mk5: Jul 75 - F01CJF
NAGFLIB: 883/448: Mk5: Nov 74 - G05AEF

APPENDIX A2

Models of process control instrumentation

A2.1 Transducer

Figure A1 represents the block diagram of a single time constant transducer together with the required change, where:

x = Input signal value (input)

x_o = Minimum value of the input signal (data)

x_m = Maximum value of the input signal (data)

y_o = Minimum value of the output signal (data)

y_m = Maximum value of the output signal (data)

L_L = Lower saturation limit (data)

U_L = Upper saturation limit (data)

T = Time constant (data)

y = Output signal

A2.2 Proportional and Integral Controller without Integral Wind-up

Figure A2 represents the block diagram of (PI) controller, where:

S_p = Controller set point (input)

M_v = Measured value (input)

K = Controller gain (data)

T_i = Integral action time (data)

$e(t)$ = Controller error

C = Controller output

L_L = Lower saturation limit (data)

U_L = Upper saturation limit (data)

A2.3 Control Valve Actuator

Figure A3 represents a block diagram of a control valve actuator, where:

P = Input pressure (input)

P_o = Input pressure which completely opens the valve (data)

P_c = Input pressure which completely closes the valve (data)

F_o = Coulomb friction during valve opening (data)

F_c = Coulomb friction during valve closing (data)

T_{co} = Opening exponential time constant (data)

T_{cc} = Closing exponential time constant (data)

T_{so} = Opening strcke time (data)

V_p = Valve stem position (normalised) (output)

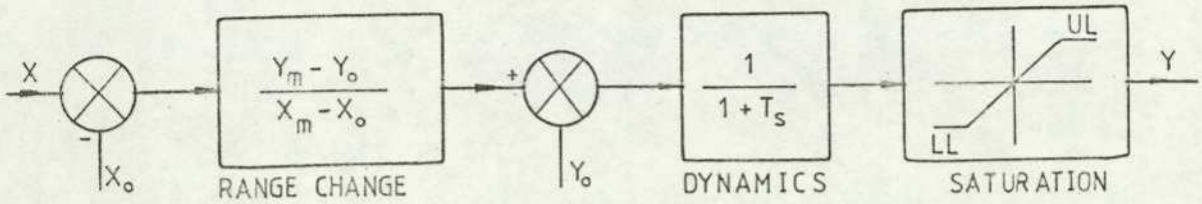


FIG.(A1) BLOCK DIAGRAM OF A TRANSDUCER.

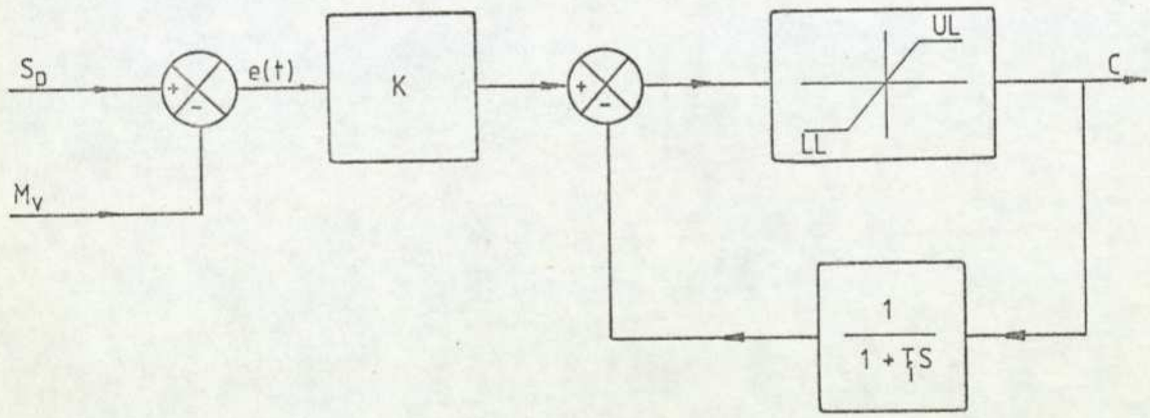


FIG.(A2) BLOCK DIAGRAM REPRESENTATION OF A (PI) CONTROLLER.

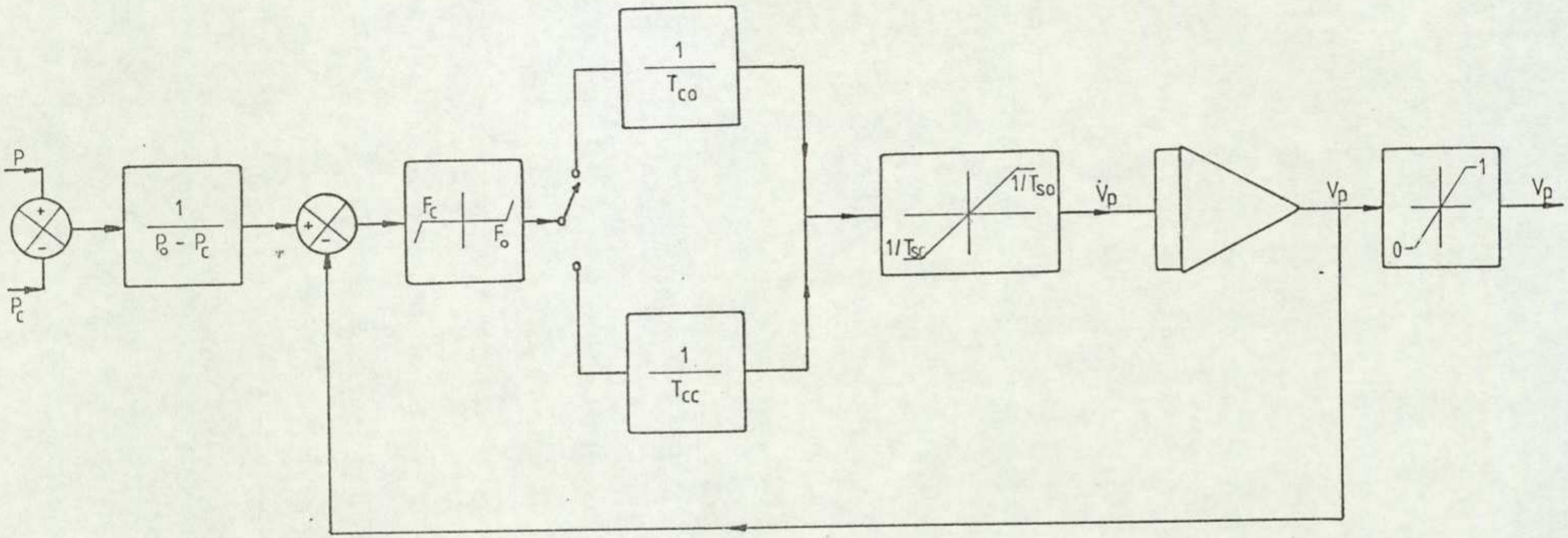


FIG. (A3) BLOCK DIAGRAM OF A CONTROL VALVE ACTUATOR.

REFERENCES AND BIBLIOGRAPHY

Anderson, J. H., 1971,
"The control of a synchronous machine using optimal control theory", Proc. IEEE, vol. 59, No. 1, pp. 25-35

Aström, K. J., 1970,
"Introduction to stochastic control theory", Academic Press, New York

Bamberger, W., and R. Isermann, 1978,
"Adaptive on-line steady state optimisation of flow dynamic processes", Automatica, 14, pp. 223-230

Bellman, R., 1961,
"Adaptive control processes: A Guided Tour", Princeton University Press

Bellman, R., 1967,
"Introduction to the mathematical theory of control processes", Academic Press

Concordia, C., 1944,
"Steady-state stability of synchronous machine as affected by voltage regulator characteristics", AIEE Trans., vol. 63, pp. 215-220

Demello, F. P., and C. Concordia, 1969,
"Concepts of synchronous machine stability as affected by excitation control", IEEE Trans., vol. PAS-88, No. 4, pp. 316-327

Durbeck, R. C., 1965,
"Principles for simplification of optimising control models", Ph.D. Thesis, Case Western Reserve University

Fel'dbaum, A. A., 1965,
"Optimal control systems", Academic Press

Fleming, W. H. and R. W. Rishel, 1975,
"Deterministic and stochastic optimal control", Springer-Verlag

Foord, A. G., 1974,
"On-line optimisation of a petrochemical complex", Ph.D. Thesis, University of Cambridge

Haimes, Y. Y., R. L. Perring and D. A. Wismer, 1968,
"Identification of aquifer parameters by decomposition and multi-level optimisation", Israel J. Tech. 6, pp. 322-329

Haimes, Y. Y., and D. A. Wismer, 1971,
"Integrated system modeling and optimisation via quasilinearisation", J. Opt. Theory, Applic. 8, pp. 100-109

Haimes, Y. Y., and D. A. Wismer, 1972,
"A computational approach to the combined problem of optimisation and parameter estimation", Automatica, vol. 8, pp. 337-347

Jacobs, O. L. R., 1974,
"Introduction to control theory", Clarendon Press, Oxford

Lalui, A., 1976,
"Integrated identification and optimisation of a chemical reactor", M.Sc. Thesis, The City University (London)

Lalui, A., and P. D. Roberts, 1979,
"Investigation into some techniques for integrated system optimisation and parameter estimation", Research Memo. DSS/AL-PDR/179, The City University (London)

Lasdon, L. S., 1970,
"Optimisation theory for large-scale systems", Macmillan

Leal, D. J., 1974,
"Integrated identification and optimisation", M.Sc. Thesis, The City University (London)

Olagundoye, O. B., 1971,
"Efficiency and the ε-constraint approach for multicriterion systems", M.Sc. Thesis, Case Western Reserve University

Park, R. H., 1929,
"Two-reaction theory of synchronous machines, Part 1: Generalized method of analysis", AIEE Trans., vol. 48, pp. 716-730

Reddy, P. B., and P. Sannuti, 1976,
"Asymptotic approximation method of optimal control applied to a power-system problem", Proc. IEE, Vol. 123, No. 4, pp. 371-376

Roberts, P. D., 1977a,
"Multilevel approach to the combined problem of system optimisation and parameter identification", Int. J. Systems Sci., vol. 8, No. 3, pp. 273-299

Roberts, P. D., 1977b,
"Integrated system optimisation and parameter estimation of a chemical reactor using a multilevel approach", 5th IFAC/IFIP Conference on digital computer applications to process control, pp. 551-557

Roberts, P. D., 1978,
"Algorithm for integrated system optimisation and parameter estimation", Electronics Letters, 16th March, vol. 14, No. 6, pp. 196-197

Roberts, P. D., 1979a,
"Hierarchical control and decomposition of a chemical plant", Int. J. Systems Sci., vol. 10, No. 2, pp. 207-223

Roberts, P. D., 1979b,
"An algorithm for steady-state optimisation and parameter estimation", Int. J. Systems Sci., Vol. 10, no. 7, pp. 719-734

Roberts, P. D., and T. W. C. Williams, 1979c,
"On an algorithm for combined system optimisation and parameter estimation", 5th IFAC Symposium on Identification and Parameter Estimation, Darmstadt, F.R. Germany, September

Roberts, P. D., 1979d,
"Process optimisation using uncertain models", 3rd Instrumentation Seminar/1st Intrinsic Safety Symposium, Salvador Brazil, 4-9 Nov.

Savas, E. S., 1965,
"Computer control of industrial processes", McGraw-Hill

Singh, M. G., and A. Titli, 1979,
"Hierarchical feedback control for large dynamical systems", Int. J. Systems Sci., vol. 8, No. 1, pp. 31-47

Singh, M. G., and A. Titli, 1978,
"Systems: Decomposition, Optimisation and Control", Pergamon, Oxford

Williams, T. J., and Otto, R. E., 1960,
"A generalised chemical processing model for the investigation of computer control", AIEE Trans. Commun. Elect., vol. 79, pp. 458-473

Wismer, D. A. (Ed.), 1971,
"Optimisation methods for large-scale systems", McGraw-Hill

Yu, Y. N., K. Vongsuriya and L. N. Wedman, 1970,
"Application of an optimal control theory to a power system", IEEE Trans., vol. PAs-89, No. 1, pp. 55-62