SEQUENTIAL ADJUSTMENTS OF PHOTOGRAMMETRIC MODELS

by

Mark Richard Shortis

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DECLARATION

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ABSTRACT

The increasing application of close-range photogrammetry as a measurement technique has caused an upsurge in the requirement for digital recording and analysis from single, fully controlled stereopairs. The restitution of such stereopairs is preferably carried out on a stereocomparator using an online minicomputer to process the raw data. This combination has all the advantages of flexibility, high precision, interactive data editing and fast turnaround time. However, the limited processing power and small memories of minicomputers can restrict the sophistication of the restitution possible. The computations are generally broken up into independent interior and exterior orientations, and the control points are considered fixed even though the precision of the photogrammetric measurements may be comparable to the control fixation.

Adjustment of all the photogrammetric observations and the survey measurements for the control is impractical because of the sheer mass of data, but an identical result can be obtained using sequential adjustment. Although the measurements for the control, the interior orientation and the exterior orientation are processed independently, the restitution is improved significantly by using all the available information. Sequential adjustment can be used to strengthen an existing restitution by adding extra observations when and if they are required. Furthermore, the control positions are adjusted on the basis of all the survey and photogrammetric measurements and the precision of the control is improved. The technique can be extended to multiple stereopairs of a single object to obtain a final control set adjusted from many photogrammetric observations.

1. INTRODUCTION

1.1 Analytical Close-Range Photogrammetry

The science of photogrammetry has traditionally been associated with topographic mapping, most recently from aerial photographs, using analogue instruments or approximate methods. Calculations and numerical methods were avoided primarily because of the sheer complexity and size of the computations. The arrival of the digital computer in the nineteen-fifties caused a re-emergence of analytical techniques which had been little used since their original development by Finsterwalder and Pulfrich at the turn of the century. The emphasis was still on aerial mapping, and aerotriangulation advanced rapidly compared with its slow start utilizing approximate or analogue techniques.

It was not until the nineteen-sixties that non-topographic applications of photogrammetry began to be widely exploited. Close-range or terrestrial photogrammetry was used as a non-contact form of measurement in such fields as architectural and archaeological recording, and the techniques of biostereometrics and industrial photogrammetry were born. The majority of this work was limited to analogue instruments with graphical output. Numerical representations of objects in the form of digital sections, strings and terrain models were being developed, but primarily in connection with route planning and design from aerial photography.

During the nineteen-seventies several factors interacted to allow close-range analytical photogrammetry to become a workable and useful method of measurement. For example, the widespread use of computers had made digital analysis and numerical representations familiar to most scientists and engineers, the end users of the products of close-range photogrammetry. The mass storage devices necessary to hold digital representations of objects were available and has proven their reliability. Automated or computer controlled flatbed or drum plotting systems were becoming numerous and extremely attractive in terms of cost-effectiveness. Precisions and tolerances were becoming increasingly demanding in the fields of industrial and structural monitoring and deformation measurement.

The final impetus was provided by the introduction of low-cost, reliable microprocessors and minicomputers. There were immediate advantages in having a dedicated computer online to the stereocomparator, including faster turnaround times, the possibility of data editing during all stages of processing and a general increase in the versatility of the analytical system. Users who had seen the drawbacks of the offline systems were quick to seize the opportunity to commission and develop stereocomparator/digitizer/minicomputer systems. The digitizer interface between the computer and stereocomparator had been well developed by this time for the acquisition of aerotriangulation data and many types were available.

Corresponding systems were developed with stereoplotters, but analogue instruments do not have the precision, versatility or scope for image coordinate refinement of analytical systems based on mono- or stereocomparators. A stereoplotter is severely limited by mechanical restrictions to a close approximation of the normal case of photography, and in the context of ground-based close-range photogrammetry the normal case may not be practical or even desirable. Highly convergent photography is often used to improve precision, and the environment surrounding close-range subjects is sometimes not conducive to the ideal placement of cameras. Industrial and construction sites can cause problems in simply obtaining stereo coverage of the subject because they are so cluttered with both static and dynamic equipment.

More recent advances in the application of computers have seen the introduction of direct-access interfaces and a generation of restitution instruments in the form of analytical plotters. Here the processor controls the operation of the stereocomparator so that the point of interest is always clear of y-parallax. The human operator needs only to position the reference mark and set the depth to obtain an observation. This gives the optimum combination of the flexibility and precision of a stereocomparator with the ease of manipulation of a stereoplotter which has a fully oriented model.

Common to all these systems is the need to re-establish the inner and outer orientations of the cameras at exposure, and the subsequent re-projection to obtain spatial coordinates from measured image positions. The computation of the orientations, known as the restitution of the photographs, is generally carried out in two distinct stages. The interior orientation re-establishes the relationship between the image-forming rays and the camera perspective centre, while the exterior orientation determines the relationship between the camera and the object. The limitations of the early minicomputers restricted the scope of the restitutions to simple direct solutions, but as the processing and memory capacity of such machines increased, the restitutions have become more sophisticated, employing powerful least squares adjustment solutions.

Most of the effort in this area has been directed at the obvious target of fully controlled single stereopairs. Multiple stereopairs and strip and block triangulation are still largely the province of the larger minicomputers or mainframe computers, although microprocessors are often used for the initial data acquisition and editing.

Clearly the complexity of the functional and stochastic models used for the restitution and the sophistication of the least squares adjustment solution will determine the results obtained from the analytical system. The restitution should be designed specifically for the online minicomputer, making the most efficient use of its capacity in memory and processing, and include the maximum amount of information available.

1.2 Metric Cameras and Photographic Imagery

In response to the increasing use of close-range photogrammetry, there has been a steady improvement in the performance and the reliability of close-range metric cameras. Both image geometry and resolution have been subject to the concerted efforts of the manufacturers of both fixed and variable focus cameras. Radial distortions of less than five micrometers and resolutions of fifty line pairs per millimetre are almost standard for wide variety of camera focal lengths and formats.

The photographic medium has also improved over the last two decades with the introduction of polyester based roll films and ultra-flat glass plates. The latter are almost universally employed in close-range work, not only because they are a much more stable base for the emulsion but also due to the lack of efficient film flattening devices in close-range cameras. However there are situations where the use of roll film may be desirable or even essential, such as rapid sequential photography in dynamic situations. Often film is used for a back-up in case of failure of plates.

The re-establishment of the interior orientation of a particular photograph is made possible by the fiducial marks imaged on the photograph and the calibration data supplied with the camera. When the fiducial marks have known positions they can also be used to estimate scale changes and deformations of the image. Even if such changes are very small because of the stability of the emulsion base, four fiducials around the edge of the format do not provide enough information for a conclusive determination. Some cameras have a central fiducial, but only a dense reseau has enough points to obtain good approximations to scale changes and deformations throughout the format area. Some camera calibrations do not provide fiducial marks with known positions, or the positions are known only to a poor precision which makes them of little practical use. In these cases there is no internal control over the scale variations and deformations which may be present in the image.

Solutions for the interior orientations of single stereopairs have largely used only the fiducial information available. The only way to improve the interior orientation without resorting to reseau cameras is to use the control observations, by combining the interior and exterior orientations. Given the stability and resolution of the imaging system and the high precision of a stereocomparator with a resolution of one micrometre, the control observations are quite capable of contributing to the interior orientation. The major advantage of using the control observations is that much more of the format area is utilized to estimate the transformation for the interior orientation.

The normally independent interior and exterior orientations can be brought together into a simultaneous adjustment solution, but this requires the manipulation of a large number of observations. Handling this large mass of data can be avoided if a sequential adjustment is used. Here there is an initial adjustment of the interior orientation which can be checked for a satisfactory result. This adjustment is then phased into the exterior orientation solution to give a result identical to that from an adjustment using all the data simultaneously.

The sequential adjustment technique is particularly applicable to small minicomputers where there are restrictions on memory and processing power.

1.3 Control

The exterior orientation of a stereopair cannot be established without some sort of control in the object which can be identified in the images. Although restitution is possible without control when the camera exterior orientation is known or fixed, the use of control is much more the common case due to greater convenience and superior reliability. Control can take many forms, from individual coordinated points to plumblines and known distances.

In classical aerial photogrammetry the precision of the control was always so high compared with the precision of measuring the photography that the control could be considered fixed and free of error. With the improvements to image quality and the use of precise stereocomparators this is no longer true, and modern analytical aerotriangulation and multistation photography were the first to include allowance for control standard errors in the solutions by least squares adjustment.

The situation is further aggravated in close-range photogrammetry as the control may be hard to configure and errors in the control can often be easily detectable by the photography. Any attempt to restitute a stereopair with incorrect control will naturally lead to a distorted model of the object, so control errors must be allowed for in the solution.

One possible remedy is the simultaneous adjustment of the measurements for the control and the photogrammetric observations, but the amount of data which must be handled precludes the use of small minicomputers. The control is established by measurements relative to an external datum and there may be many redundant observations to provide an overdetermination of the points. The least squares adjustment of these observations alone can fully occupy the minicomputer with regard to both memory and processing.

An identical result to a simultaneous adjustment can be obtained using sequential adjustment, and it is possible to use minicomputers. The control adjustment is carried out first and checked for data errors and an acceptable result. The control observations can then be discarded and only the results of the adjustment, the final coordinates and the corresponding covariances, retained for any subsequent restitution of photographs of that particular object and control set. Where a least squares adjustment is not applicable to the form of control measurement, the precisions of the control can be estimated from experience or any other valid source. The only large masses of data which then need to be manipulated after the initial control adjustment are the actual photogrammetric observations.

1.4 Restitution Using Sequential Adjustment

A restitution by sequential adjustment of the control, the interior and the exterior orientations is the optimum solution because it includes all the available information. Three normally independent adjustments are combined using a minimum of input data. This allows the solution to be carried out on a small minicomputer, retaining all the benefits of a dedicated online processor.

Sequential adjustment can also be used to strengthen an existing adjustment by introducing extra observations if they are required. Analysis of the residuals of a least squares adjustment often indicate that particular additional measurements would be beneficial. The extra data can be incorporated into the solution very efficiently using sequential adjustment. Again the original measurements can be discarded and hence replaced by the new observations. This technique is particularly applicable to online processing of photogrammetric observations, where the review of the initial adjustment, the additional measurements and the subsequent sequential adjustment can be carried out at the operator's convenience and with a minimum amount of computer capacity.

Furthermore, the interior orientation parameters and control point positions are corrected by the sequential adjustment on the basis of the photogrammetric observations for the exterior orientation. The corrected interior orientation parameters can give an insight into the image deformations throughout the format area and adjusted control positions can be beneficial in their own right for the analysis of the photographed object.

The use of all available information must improve the precision of the derived quantities. These derived quantities are used in the re-projection of observed images during data acquisition or digitizing of the subject. It must not be forgotten that this is the ultimate aim of the analytical system and any improvement in precision will contribute toward the quality of the overall result, whatever the purpose of the photography.

Finally, if multiple stereopairs of a single object are taken the technique of sequential adjustment can be extended to use all the photogrammetric observations to improve the control. This improvement in the precision of the control positions can be quite substantial if the photogrammetric intersections have a strong geometry. While this does not pretend to be a rigorous multistation adjustment, it is a useful method of obtaining optimum restitutions for the stereopairs using an online minicomputer.

2. CURRENT RESTITUTION TECHNIQUES

2.1 Interior Orientation

2.1.1 Aim

The aim of an interior orientation is the re-establishment of the relationship between the image-forming rays and the perspective centre of the camera. A suitable transformation between observed comparator coordinates of images and fiducial coordinates of refined image positions must be found to describe the relationship. The parameters of the transformation can be determined by calculation or may be considered known and fixed. Only metric cameras will be dealt with, so the position of the perspective centre relative to the fiducial origin and lens distortions fall into the latter category. Therefore only parameters of a two-dimensional coordinate transformation between the comparator and fiducial systems remain to be determined.

The comparator coordinate system (see Figure 2.1) has an arbitrary origin and orientation, while the camera coordinate system has an origin at the principal point of the photograph and an orientation defined by fiducial marks. The transformation parameters are determined by observing the positions of the fiducial marks in the comparator coordinate system. The known relationship of the fiducial marks to the fiducial coordinate system can then be used to compute a selected number of parameters. The complexity of the transformation and the number of parameters is decided by the number of fiducial marks available and how much information is known about their positions in the fiducial coordinate system.

Fiducial information, focal length value and information about lens distortions are provided by a calibration certificate supplied with the camera by the manufacturer (see Figure 2.2). This information may be supplemented with calibrations made by the user during the working life of the camera (Torlegard 1967, Brown 1971, Bhatti 1973, Scott 1976).

2.1.2 Types of Transformation

The minimum information that can be provided by a camera calibration is the focal length and the principal point position defined by four fiducial marks, usually in the corners of the format or at the

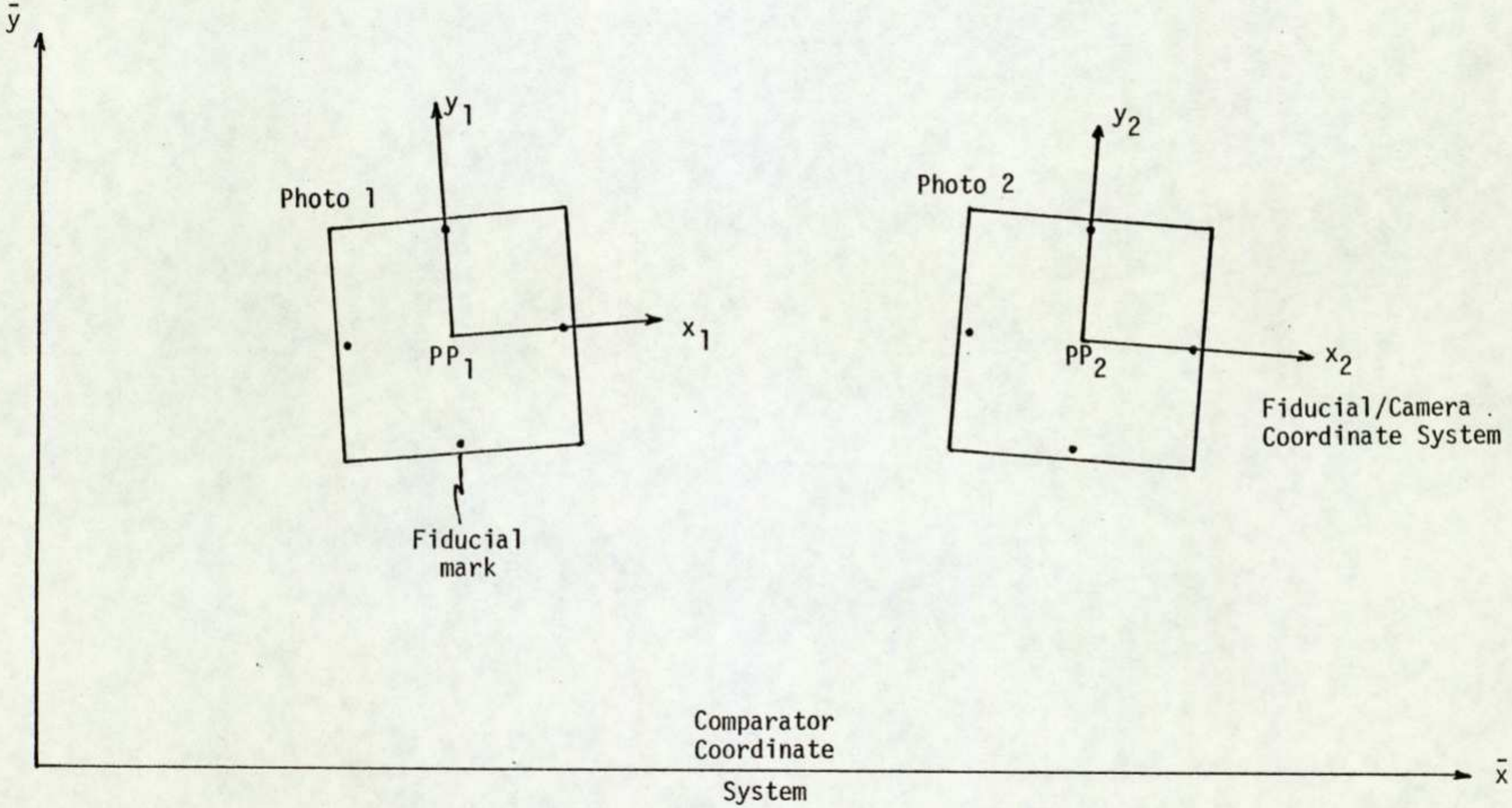


Figure 2.1   Comparator and Fiducial Coordinate Systems

[redacted]

Figure 2.2 Camera Calibration Certificate midpoints of the sides of the frame. This is sometimes the case for close-range cameras where the coordinates of the fiducial marks relative to the principal point are not given by the manufacturer's calibration, or are given to such a low precision that they are of little practical use. The principal point is then merely the intersection of the lines joining opposite fiducial marks. The orientation of the fiducial coordinate system must be arbitrarily defined with respect to one of the fiducial marks.

This transformation is a special case of a rectilinear transformation with only three parameters; two position shifts ($\bar{x}_o$, $\bar{y}_o$) and a rotation (θ):

$$\begin{aligned} x &= \bar{x}_o + \bar{x}\cos\theta - \bar{y}\sin\theta \\ y &= \bar{y}_o + \bar{x}\sin\theta + \bar{y}\sin\theta \end{aligned} \qquad \dots \dots \dots \dots \dots \quad (2.1)$$

where x, y are fiducial or camera coordinates

$\bar{x}, \bar{y}$ are observed comparator coordinates

It is more often the case, however, that fiducial coordinates are known for at least four marks with a precision of the same order as the measuring precision of a comparator. As the four points will yield eight equations, it is possible to apply any of the common four, six and eight parameter transformations (Kupfer 1972, Kratky 1972):

Rectilinear (sometimes called linear conformal or Helmert)

$$\begin{aligned} x &= a_1 + a_3\bar{x} - a_4\bar{y} \\ y &= a_2 + a_4\bar{x} + a_3\bar{y} \end{aligned} \qquad \dots \dots \dots \dots \dots \quad (2.2)$$

Affine

$$\begin{aligned} x &= a_1 + a_3\bar{x} + a_5\bar{y} \\ y &= a_2 + a_4\bar{x} + a_6\bar{y} \end{aligned} \qquad \dots \dots \dots \dots \dots \quad (2.3)$$

Bilinear (psuedo-affine)

$$\begin{aligned} x &= a_1 + a_3\bar{x} + a_5\bar{y} + a_7\bar{x}\bar{y} \\ y &= a_2 + a_4\bar{x} + a_6\bar{y} + a_8\bar{x}\bar{y} \end{aligned} \qquad \dots \dots \dots \dots \dots \quad (2.4)$$

Projective (perspective)

$$x = a_1 + a_3\bar{x} + a_5\bar{y} + a_7\bar{x}\bar{y} - a_8\bar{x}^2$$
$$y = a_2 + a_4\bar{x} + a_6\bar{y} - a_8\bar{x}\bar{y} + a_7\bar{y}^2 \quad \ldots \ldots \ldots \quad (2.5)$$

Deformational

$$x = a_1 + a_3\bar{x} + a_5\bar{y} + a_7\bar{x}\bar{y} + a_8\bar{y}^2$$
$$y = a_2 + a_4\bar{x} + a_6\bar{y} + a_8\bar{x}\bar{y} + a_7\bar{x}^2 \quad \ldots \ldots \ldots \quad (2.6)$$

where $a_1 \ldots a_8$ are the transformation parameters.

A least squares solution by observation equations is generally used to solve for the parameters. Measurements of comparator coordinates are considered to be independent and of equal weight. The number of redundancies in the solution will depend on the number of parameters included.

Higher order transformations are only possible with more than four known fiducial mark positions. Some close-range cameras have a glass back plate which has a fifth fiducial marked near the position of the principal point, and some aerial cameras have eight fiducials marked around the edge of the frame instead of four. The next stage beyond these types is the reseau camera which has a regular grid (of 10 or 20 mm spacing) of points imaged onto the format area.

Large numbers of fiducials and reseaux provide so much information that cubic and quadratic terms can be included in the transformation (Ziemann 1971b, Lampton 1965) and still retain many redundancies in the solution. Unfortunately, high order transformations give rise to the problem that statistically significant and uncorrelated parameters are difficult to select from the many possibilities (Kratky 1972). Alternative treatments which avoid this problem are to use recursive analysis from many photographs taken under similar circumstances (Kupfer 1972), or to employ least squares interpolation between the points of a reseau grid (Kraus 1972).

2.1.3 Applications

In the context of close-range photogrammetry, the choice of a suitable transformation is dictated by the number of fiducial marks available and the emulsion base in common use. Close-range metric cameras rarely have more than five fiducial marks and generally only have four. Glass plates are almost always employed for precise analytical work as they are superior to film in dimensional stability with respect to temperature, humidity and age. Cut or roll film is more sensitive to photographic processing and differential film shrinkage is always a possibility (Jaksic 1972). Furthermore, relatively few close-range cameras have efficient film flattening devices, and pressure plates and vacuum backs can give unpredictable results (Clark 1972). Glass plates are available in a variety of classes of flatness with maximum deviations from a plane ranging between a few tens of microns to a few microns. For these reasons the six parameter affine transformation given by equations (2.3) is generally employed. Image deformations due to lack of flatness or emulsion anomalies are assumed not to be present in significant quantities.

However, some studies with glass plates have shown that significant improvements can be gained using eight parameter transformations (Altman and Ball 1961) and that there can be systematic deformations near the edges of plates (Gollnow and Hagemann 1956). Where film must be used in analytical work, higher order transformations give improved results even where sophisticated film flattening devices are present, but without a reseau, realization of the improvement is unlikely (Brock 1972). Even many fiducial marks around the edge of the frame do not provide enough information to remove deformations throughout the format (Ziemann 1971a).

The only method in current practice which incorporates data supplementary to the camera calibration is the technique of added parameters in aerotriangulation strip and block adjustment (Brown 1974, Ackerman 1980). Here the multitude of observations on object points in a large number of photographs can be used to estimate the systematic image deformations in the camera. This technique has the advantage that it not only removes systematic effects caused by lack of flatness or instability of the emulsion base, but also can remove errors associated with the estimates of atmospheric refraction and lens distortions. Unfortunately this technique is not directly applicable to close-range photogrammetry where the main mode of operation is single stereopairs.

The recursive technique referred to in Section 2.1.2 would not be effective for similar reasons. A block or set of photographs taken with a single camera under similar conditions throughout is necessary for a practical application of the method. The various camera and object configurations and environments encountered in close-range work do not allow a recursive analysis of residual deformations, which in addition, necessitates a reseau camera.

2.2 Exterior Orientation

2.2.1 Aim

The aim of exterior orientation is the re-establishment of the relationship between the camera and the object. A suitable transformation between the refined coordinates of conjugate images and the coordinates of the corresponding object point must be found. The images of the object point on the left and right photographs are defined relative to the perspective centres of the left and right cameras respectively, based on comparator observations and a subsequent interior orientation. The numerical projection of these two images into the object space can only be accomplished if the parameters of the transformation, the positions and orientations of the cameras relative to the object coordinate system, are determined.

The object or spatial coordinate system can be related to some physical aspect of the subject, for example a centre line or datum surface. However, this type of reference is commonly ill-defined (Cooper and Shortis 1978, Cooper 1979) and must be supplemented or superseded by an arbitrary and independent coordinate system. Such a system is defined by control points, which are locations marked on or around the object and measured by some non-photogrammetric means relative to an external datum. Geodetic measurements are commonly employed to fix the positions of control points.

The exterior orientation can be defined with respect to one of the cameras, so that the object coordinate system becomes a function of the fiducial system. Alternately the position and orientation of the cameras can be measured directly relative to some external reference, as is often the case when a photo-theodolite is employed for close-range work. However, both these methods suffer from propagation of errors into the model space. Any slight imprecision in the estimates of the camera parameters will lead to large errors in the object, especially in relation to the base distance between the cameras propagating scale errors into the object space.

By far the most reliable method of establishing the exterior orientation is the use of observed images of points with known spatial coordinates - the control points. If enough control points have coordinates known in both fiducial and object systems, the exterior orientation parameters of the cameras can be computed.

There are six parameters required for each camera; the three coordinates of the position of the camera perspective centre and the three rotations of the fiducial system relative to the object system. In practice more than minimum control is provided for the exterior orientation and the redundant information is processed in a least squares adjustment. The observed image coordinates are considered to be independent and of equal weight.

2.2.2 Types of transformation

2.2.2.1 Collinearity

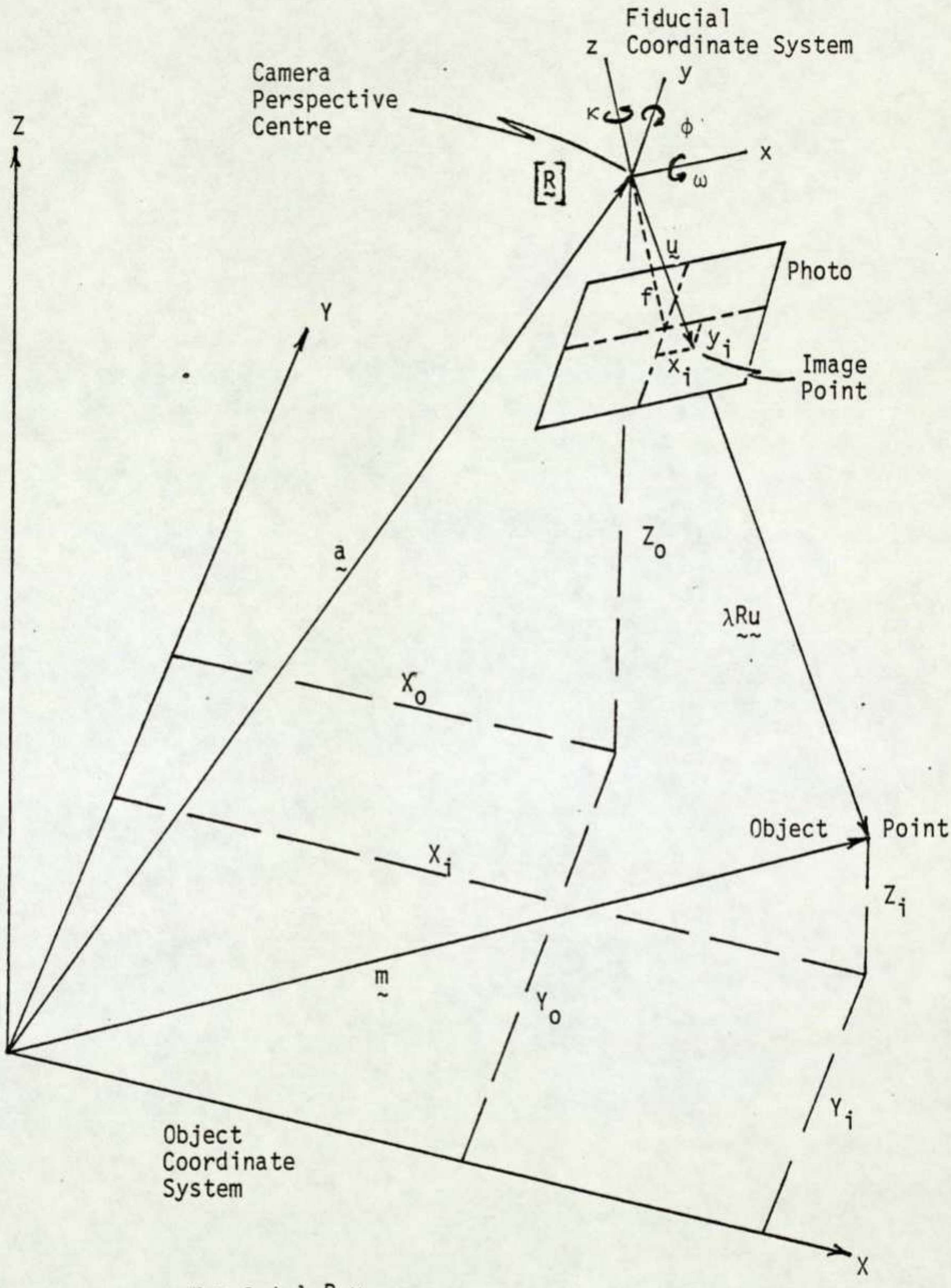
The collinearity condition is the most commonly used exterior orientation solution in analytical photogrammetry. The equations are a development of the projective theory between two planes (Schmid 1954) and state simply that perspective centre of the camera, the photographic image point and the object point must lie on the same line in space (see Figure 2.3). Each object point contributes two equations per camera to the solution (A.S.P. 1966):

$$x_i = -f \frac{r_{11}(X_i - X_o) + r_{21}(Y_i - Y_o) + r_{31}(Z_i - Z_o)}{r_{13}(X_i - X_o) + r_{23}(Y_i - Y_o) + r_{33}(Z_i - Z_o)}$$

$$y_i = -f \frac{r_{12}(X_i - X_o) + r_{22}(Y_i - Y_o) + r_{32}(Z_i - Z_o)}{r_{13}(X_i - X_o) + r_{23}(Y_i - Y_o) + r_{33}(Z_i - Z_o)} \qquad \ldots \ldots \quad (2.7)$$

where x_i, y_i are the fiducial coordinates of the i^{th} image point, corrected for image deformations and lens distortions.

f is the focal length of the camera.



$$\underset{\sim}{m} = \underset{\sim}{a} + \lambda \underset{\sim}{R} \underset{\sim}{u}$$

Figure 2.3 Exterior Orientation - Collinearity Condition

X_i, Y_i, Z_i are the object coordinates of the i^{th} object point.

X_o, Y_o, Z_o are the object coordinates of the camera perspective centre.

$r_{11} \ldots r_{33}$ are the elements of a 3 x 3 orthogonal rotation matrix $\underset{\sim}{R} = (r_{ij})$ where $r_{ij} = f_{ij}(\kappa, \phi, \omega)$

These equations have the advantage that they can be applied to any situation in analytical photogrammetry, from self-calibration of cameras (Kenefick et al 1972) to multistation photography and aerotriangulation (Brown 1958, Schut 1957). For single stereopairs each object point produces four equations, two from each camera. As there is a total of twelve unknown parameters the minimum number of control points is therefore three, with all three spatial coordinates known. Equations (2.7) can be linearized by a Taylor series expansion to first order terms and an iterative least squares adjustment employed for the solution.

The least squares adjustment will minimize the distances between the projected image coordinates and the object locations. Although this is appropriate for the other applications cited above, it is disadvantageous to some extent as far as stereopairs are concerned. The result of the minimization is that the parallax between any conjugate pair of rays to an object point will be selected as the shortest distance between the rays. The shortest distance is a three-dimensional vector, and for single stereopairs it is often more desirable to constrain the parallax to a single component and this is not possible using the collinearity equations. Interpretation of the results of a restitution of a single stereopair is simplified if parallaxes are confined to the single component which corresponds to the y-parallax of classical aerial photogrammetry (i.e. approximately perpendicular to the camera axes and parallel to the y fiducial axes).

Furthermore, parallax discrepancies are not directly obtainable from the collinearity solution which treats the cameras as essentially independent. If the stereopair is observed in a stereocomparator the photographs are at the very least stochastically related and this should be reflected in the covariance matrix of the observations. Single, isolated stereopairs should also be functionally related. The mathematical model of the exterior orientation should connect the two photographs in a way which is not possible in a collinearity solution.

A minor disadvantage of the collinearity equations is that they do not provide information about both discrepancies at control and parallaxes in the object space. The residuals of the adjustment solution give discrepancies in terms of plate coordinates at photograph scale, but the object magnitudes cannot be computed using a simple scale factor in close-range photogrammetry. The photograph and object coordinate systems are not approximately aligned as they can be in aerial work, and there is always the possibility of large depth ranges in close-range subjects. Therefore discrepancies and y-parallaxes at control points and in the object space must be computed by an ancilliary calculation which has no relation to the actual solution.

Such a computation of object space coordinates is yet another disadvantage of the solution, as a least squares adjustment must be used. In multistation photography object points are usually imaged on more than two photographs so that a least squares solution for the coordinates of the point is highly desirable. There may be many equations available to solve for the three unknowns, the three coordinates of the object point. In the case of a single stereopair there are only four equations in the three unknowns and a direct solution would be much more practical. Again a separate calculation is required if the residual parallax is to be determined as a validity check on the observation.

This extended calculation for the coordinates of object points can be extremely disadvantageous for online processing with minicomputers. Such machines can have long cycle times (the interval required to do one operation) resulting in a visible delay between the input of the stereocomparator coordinates and the output of the object space coordinates. This can be critical in the smooth operation of the system during rapid acquisition of profile or string data.

2.2.2.2 Coplanarity

The second common solution to the problem of exterior orientation utilizes the coplanarity condition. This condition can be derived from vector geometry and states that the left and right hand rays to the object point and the base vector between the camera perspective centres must lie in the same plane (Thompson 1968). This can be expressed mathematically as the vector product (A.S.P. 1966):

$$\underset{\sim}{b} \cdot \underset{\sim}{\ell} \times \underset{\sim}{r} = \underset{\sim}{0}$$

or as the determinant:

$$\begin{vmatrix} b_x & b_y & b_z \\ \ell_x & \ell_y & \ell_z \\ r_x & r_y & r_z \end{vmatrix} = 0 \quad \ldots \; \ldots \; \ldots \; \ldots \; \ldots \quad (2.8)$$

where the vectors are defined by Figure 2.4:

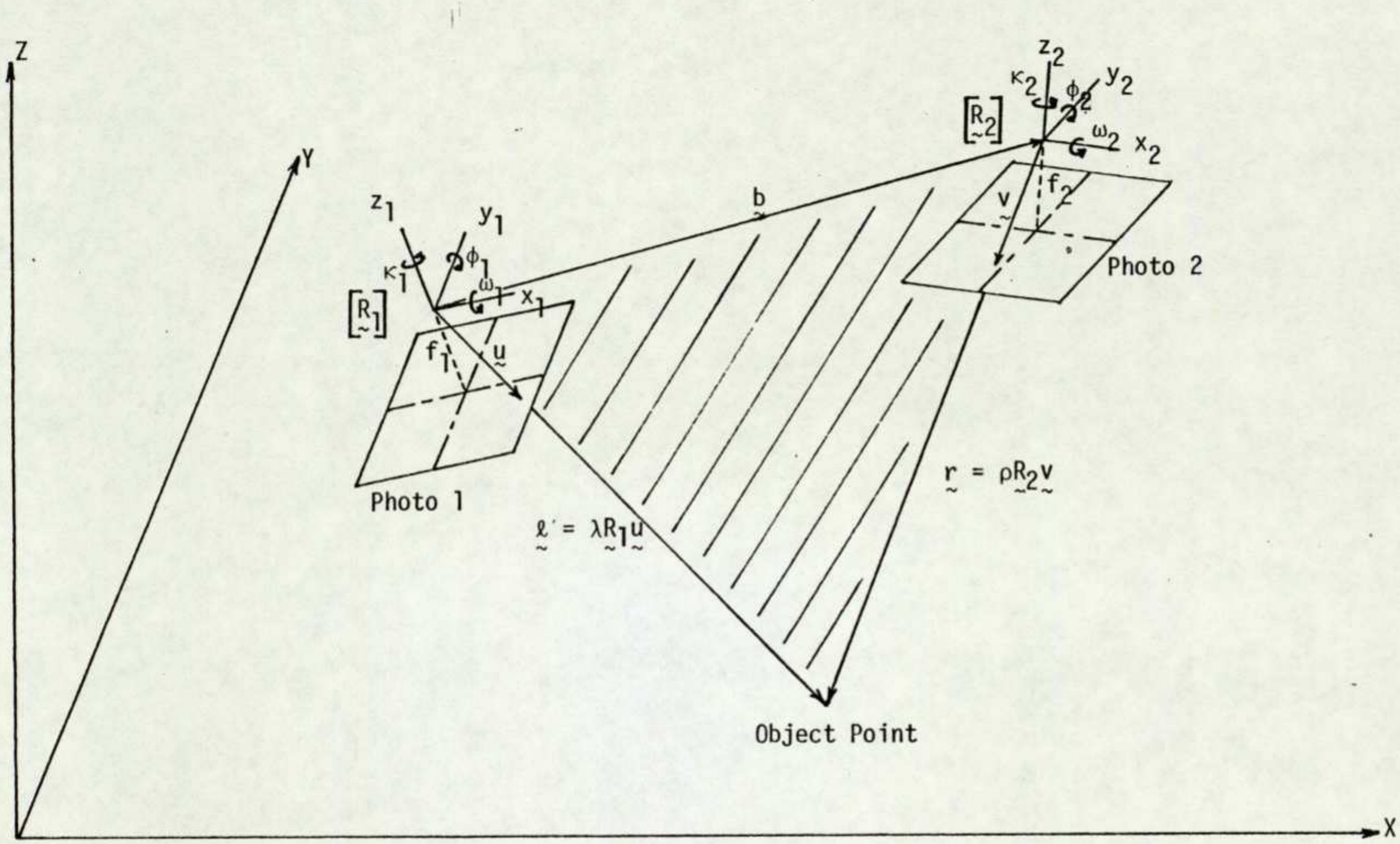
$$\underset{\sim}{b} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \qquad \underset{\sim}{\ell} = \begin{bmatrix} \ell_x \\ \ell_y \\ \ell_z \end{bmatrix} \qquad \underset{\sim}{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

Equations (2.8) can be expanded into the form:

$$b_x(\ell_y r_z - \ell_z r_y) - b_y(\ell_x r_z - \ell_z r_x) + b_z(\ell_x r_y - \ell_y r_x) = 0 \quad \ldots \quad (2.9)$$

The equation can be linearized by a Taylor series expansion to first order terms into a form suitable for solution by an iterative least squares adjustment. Each pair of images of an object point produces only a single equation.

Coplanarity is particularly applicable to single stereopairs because the solution can be used in conjunction with the object coordinate system to minimize a single component of parallax in the object space. As already mentioned in the previous section, the component most readily interpreted is Y-parallax for aerial stereopairs or Z-parallax for terrestrial stereopairs (see Figures 4.1 and 4.4). These components are closely related to the manner in which a photogrammetric operator makes a measurement in the stereomodel (Shortis 1977). It is assumed that the depth or height is set at the level of the object point and then the position of the point is set. Any "error" in the observation is left in the y-parallax (as setting the depth defines the x-parallax).



$$\underset{\sim}{b} - \underset{\sim}{\ell} + \underset{\sim}{r} = \underset{\sim}{0}$$

Figure 2.4 Exterior Orientation - Coplanarity Condition

This concept developed from experience with stereoplotters and is not as valid with respect to stereocomparators where there is direct control over x- and y-parallaxes. However its validity is not diminished as far as interpretation of the observed object points is concerned. For a single pair of photographs it is more convenient to consider an observation in the object space to have only a single component of residual parallax in an appropriate coordinate direction. Residual x- and z-parallaxes (referring to fiducial coordinates) have little meaning when analysing the restitution of an isolated stereopair, so the residual parallax should be confined to the y-component to simplify interpretation. This is particularly applicable to close-range stereopairs where there may be large depth ranges in the object space and the fiducial and object coordinate systems may not correspond. Three-dimensional parallaxes at picture scale become very difficult to interpret in terms of the object and a single component at object scale would be a considerable improvement.

A second advantage of coplanarity in relation to single stereopairs is that it functionally relates the two photographs. While the independent collinearity solution is applicable to multistation photography, if the basic unit of photography is the stereopair then there should be some mathematical relationship between the photographs. This is particularly applicable to close-range photogrammetry where multiple stereopairs are taken to obtain coverage of large subjects, often using stereometric cameras attached to a fixed-length base bar, rather than multiple single photographs. The advantage of this mode of operation is that the photographs can be viewed stereoscopically in the conventional way, thereby adding immeasurably to the interpretation of the object.

The use of the coplanarity solution is confined to relative orientation of photographs because the object coordinates of the observed points do not appear in equation (2.9). It has been used both for the numerical orientation of stereoplotters (Harley 1971) and as a precursor for analytical aerotriangulation by models rather than bundles (Schut 1955).

Hence the drawback of the coplanarity condition is that it cannot be used as a complete exterior orientation, it must be followed by some form of absolute orientation. This is the traditional concept of a relative orientation to obtain a stereomodel free of y-parallax, and then an absolute orientation to fit the stereomodel to the control. A three-dimensional linear conformal transformation (Ghosh 1972) can be employed in conjunction with a least squares adjustment solution. This second series of computations and adjustment will only lead to inefficiency and rounding-off errors in both the restitution and the re-projection, and should be avoided if possible. There is no reason why a single simultaneous solution and adjustment should not be used in analytical photogrammetry.

2.2.2.3 Direct Linear Transformation

The technique of a direct linear transformation or 11-parameter solution was developed from a combination of an affine interior orientation and the collinearity equations (Abdel-Aziz and Karara 1971) for application with non-metric cameras. This approach does not require a conventional interior orientation using fiducial marks as the solution operates directly on observed comparator coordinates. The solution is already in a linear form for a least squares adjustment so that no Taylor expansion, partial differentiation or initial approximations are required:

$$\bar{x}_i + \frac{\ell_1 X_i + \ell_2 Y_i + \ell_3 Z_i + \ell_4}{\ell_9 X_i + \ell_{10} Y_i + \ell_{11} Z_i + 1} = 0$$

$$\bar{y}_i + \frac{\ell_5 X_i + \ell_6 Y_i + \ell_7 Z_i + \ell_8}{\ell_9 X_i + \ell_{10} Y_i + \ell_{11} Z_i + 1} = 0 \quad \ldots \ldots \ldots \quad (2.10)$$

where $\bar{x}_i$, $\bar{y}_i$ are observed comparator coordinates of the i^{th} point

X_i, Y_i, Z_i are the object coordinates of the i^{th} point

$\ell_1 \ldots \ell_{11}$ are the eleven linear coefficients

The linear coefficients can be related to the conventional parameters of interior and exterior orientation if desired. Additional terms can be added to equations (2.10) to account for uncompensated lens distortions (Marzan and Karara 1975) and possibly image deformations. However the solution then has the disadvantage of too many parameters in the functional model, leading to high correlation between parameters and a weakened least squares adjustment.

This solution can be applied to metric photography, but would incur two distinct disadvantages. The first is the relatively large number of parameters. For a single stereopair of photographs there are twenty-two parameters, so that at least six control points are required as a minimum. To obtain a reasonable amount of redundancy in the least squares adjustment and a strong solution, approximately ten well distributed control points would be required. Although in a proportion of close-range photogrammetric projects the provision of a large number of well-placed control points is not a problem, there are cases where the environment or circumstances would make it impractical or even impossible. Inclusion of any additional parameters would increase even further the minimum control requirement.

The second disadvantage is that valuable information provided by the fiducial marks of metric cameras is ignored and the interior orientation is carried out solely on the evidence of control observations. This is the optimum solution for non-metric photography, but any solution for metric photography should always include all the available information. The 11-parameter solution can be used with refined image coordinates from a conventional interior orientation (Altan et al 1978) but here the advantage of computational simplicity has to be weighed against the practical difficulties of providing extra control.

The direct linear transformation also suffers from the drawbacks of the collinearity condition; residual parallaxes must be the shortest distance between conjugate rays, parallaxes and discrepancies at control must be determined by supplementary computations, the two photographs are virtually independent and re-projection is not possible without a least squares adjustment.

2.2.3 Applications and Data Processing

The applications of the collinearity and coplanarity conditions to close-range photogrammetry are far too numerous to mention here and are best left to the excellent bibliographies produced by other authors (Karara 1975 and 1979, Atkinson 1976 and 1980). The direct linear transformation technique has also been applied to close-range photogrammetry using non-metric cameras (Karara 1974, Brandow et al 1976) and metric cameras (Altan 1980), but its use is not as widespread as that of the other two methods.

The instruments used for data acquisition for analytical photogrammetry are many and varied, but high precision monocomparators and stereocomparators predominate (Ghosh 1979). Stereocomparators outnumber monocomparators because the latter require that conjugate image points must be suitably identified or pre-marked by a point transfer device. Stereocomparators do not require preparation of the photographs in this manner and the interpretation of objects is simplified by stereo viewing. On the other hand monocomparators can incorporate a simpler design and can have a slight edge in mechanical reliability and precision of measurement.

Digitizers capable of interfacing comparators to computers have proliferated as a result of advances in electronics and computer hardware. The "hard-wired" modules common in recent years are gradually being replaced by digitizers based on PROM'S (Programmable Read Only Memories), computer software, minicomputer direct access interfaces and timesharing microprocessors (Petrie and Adam 1980).

The use of microprocessors and minicomputers for data processing is also so common that the number of documented systems is too large for even a brief summary to be given here. The general design of such systems is shown in Figure 2.5, where the broken lines represent optional links. The return connection from the computer to the stereocomparator is a feature of so-called analytical plotters. Here the stereocomparator is back driven by the processor to simplify its operation. Once a restitution is carried out the processor can control the movement of the photocarriages (through a feedback loop) to remove any y-parallax at the point of interest. This leaves the operator free to concentrate on setting only the depth and position of the reference mark. However this is a minor distinction as it does not affect the general principles of analytical photogrammetry - restitutions and re-projections are equally applicable to all systems.

The size of the central processor unit (CPU) and the capacity of the accompanying mass storage device are the main factors determining the use of online systems. Small desk-top calculators can be extremely limited in both memory and program steps (Dorrer 1977) so that only simple calculations and data manipulation can be performed. Microprocessors programmable in BASIC and with 8 - 16K words of CPU can have more sophisticated software, especially if a suitable mass

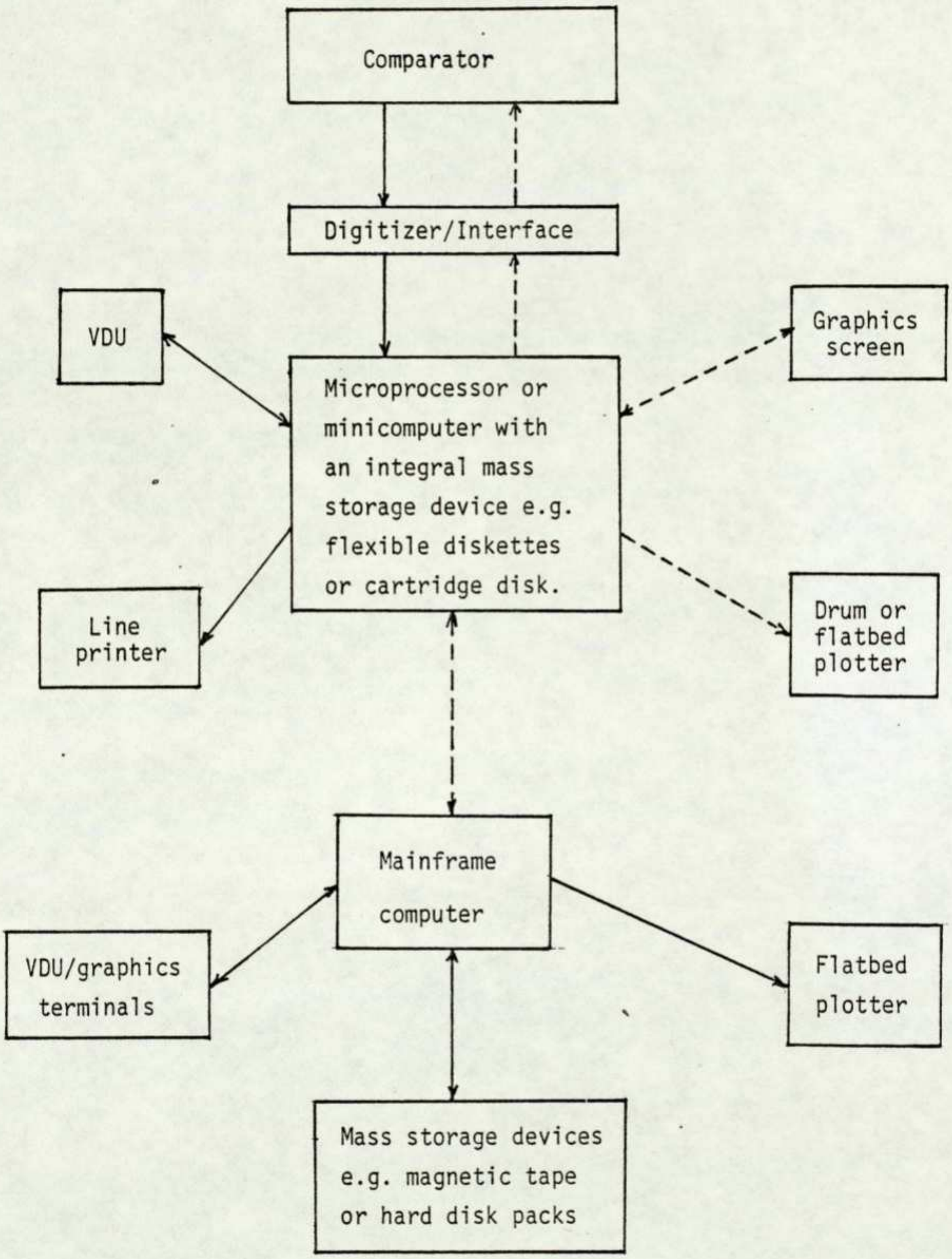


Figure 2.5 Data Acquisition/Processing System for Analytical Photogrammetry

storage device is available, e.g. magnetic cassette or flexible diskette (Grimm and Heimes 1980). Various restitutions are possible but are limited to small numbers of fixed control points or plumblines and distances. Software may have to be modular in nature so that it can be loaded piece by piece into the CPU. This reduces the effectiveness of online processing because the reaction time of the system is increased.

The use of small minicomputers programmable in FORTRAN and with a 16 - 32K word CPU is widespread. These computers occupy the mid-range in both performance and capital outlay and so have become very popular. Software packages can be very sophisticated but are generally restricted to single stereopairs. Interfacing has progressed to the point where some analytical plotters are controlled by this class of processor (Yzerman 1979). If a large mass storage device is available then restitutions can be extended to multistation photography (Haggren et al 1978) but these computations must necessarily be run offline. The majority of minicomputers in this class use small capacity flexible diskettes or cartridge disks, so that the emphasis again is on data acquisition, quantities derived from object coordinates (angles, distances, profiles, volumes) and graphical presentations of data (Seeger 1978).

The third class of minicomputers uses various techniques to access central processor space above 32K. Memory management, virtual arrays and overlays can be used to extend the processor capacity to 256K. These computers have almost unlimited scope for software packages and usually have large capacities for data storage using magnetic tapes and hard disk units. They can be used for analytical aerotriangulation (Jacobsen and Worzyk 1980), data acquisition from many sources (Leatherdale and Keir 1979) and the simultaneous running of many interactive peripherals (Adamec and Ellis 1980).

3. SEQUENTIAL ADJUSTMENT

3.1 Aim

The aim of sequential or phased adjustment is the separate least squares adjustment of a series of groups of observations, culminating in a result which is identical to a simultaneous adjustment of all the observations. The major advantage of sequential adjustment is that after each stage of the adjustment the observations can be discarded. This has particular application to small computers with limited mass storage devices. The processor and storage device can consequently handle much larger adjustments than would be normally possible if large amounts of raw data had to be retained and manipulated.

From each phase of the adjustment the adjusted values of the unknowns and their variance/covariance matrix (hereafter called the covariance matrix) have to be retained to be used in the next phase. Each of these is normally kept for other purposes in any case. The adjusted values are the main result of the adjustment and are used in further calculations. The covariance matrix is usually the subject of some type of error analysis, even if it is only to determine the standard errors of the adjusted values. On the other hand, once the adjustment is complete the initial observations are of no further interest, and it would be very unlikely that they would be used again if the result of the adjustment was satisfactory.

In the context of small computers, the covariance matrix can be retained in a very much reduced storage mode. As the matrix is not interpreted directly but through subsequent calculations, it can be stored in the simplest and most efficient method available. However raw observations must be stored in a manner which makes them easily interpretable and accessible to the operator and interactive editing programs. The observations cannot be stored only as measured values, they must be accompanied by identifiers and other supplementary information, e.g. the type and standard error of the measurement. For both these reasons the storage of measurements in either the processor or the storage device is very inefficient in terms of space and speed of access.

Sequential adjustment can be carried on indefinitely if the results and covariance matrix are retained after each phase. The total number of measurements can be virtually infinite if the measurements are divided up into suitably sized groups for processing in phases.

The number of unknown parameters in the adjustment need not be constant from one phase to the next. Additional unknowns can be included using a slight extension to the algorithm presented in the next section, and examples will be derived in full in later sections. The removal of unknown parameters between phases would require a substantially more complex algorithm which is not given in text as there is no directly related application.

3.2 Theory

Consider two sets of observations or measurements p and q which have been taken to define a set of n unknown parameters. The (p + q) observation equations can be expressed as:

$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{m} + \underset{\sim}{v} \; : \; \underset{\sim}{Q}_m \quad \ldots \;\ldots \;\ldots \;\ldots \;\ldots \qquad (3.1)$$

where $\underset{\sim}{A}$ is a (p + q)x n matrix of linear coefficients

$\underset{\sim}{x}$ is a n x 1 vector matrix of the unknown parameters

$\underset{\sim}{m}$ is a (p + q)x 1 vector matrix of the measurements

$\underset{\sim}{v}$ is a (p + q)x 1 vector matrix of residuals

$\underset{\sim}{Q}_m$ is a (p + q) square symmetric matrix of the weight coefficients associated with the measurements

The principle of least squares adjustment states that:

$$\phi = \underset{\sim}{v}^T \underset{\sim}{Q}_m^{-1} \underset{\sim}{v} \rightarrow \text{minimum} \quad \ldots \;\ldots \;\ldots \;\ldots \qquad (3.2)$$

Differentiating this expression with respect to $\underset{\sim}{x}$ gives the solution:

$$\underset{\sim}{x} = (\underset{\sim}{A}^T \underset{\sim}{Q}_m^{-1} \underset{\sim}{A})^{-1} \underset{\sim}{A}^T \underset{\sim}{Q}_m^{-1} \underset{\sim}{m} \quad \ldots \;\ldots \;\ldots \;\ldots \qquad (3.3)$$

where the superscripts ^{T} and $^{-1}$ refer to the transpose and the inverse of the given matrix, respectively.

The expression (3.1) can be partitioned into its component matrices thus:

$$\begin{bmatrix} A_p \\ A_q \end{bmatrix} x = \begin{bmatrix} m_p \\ m_q \end{bmatrix} + \begin{bmatrix} v_p \\ v_q \end{bmatrix} : \begin{bmatrix} Q_p & 0 \\ 0 & Q_q \end{bmatrix} \qquad (3.4)$$

where A_p and A_q are the matrices of linear coefficients associated with the first and second groups of measurements with dimensions p x n and q x n respectively.

m_p and m_q are the vector matrices of the first and second groups of measurements with dimensions p x 1 and q x 1 respectively.

v_p and v_q are vector matrices of residuals associated with the first and second groups of measurements with dimensions p x 1 and q x 1 respectively.

Q_p and Q_q are square (p x p) and (q x q) matrices of weight coefficients associated with the first and second groups of measurements respectively.

0 is a null matrix of appropriate dimensions.

It can be shown that a block diagonal matrix has an inverse which is simply the inverse of the component submatrices, therefore:

$$Q_m^{-1} = \begin{bmatrix} Q_p & 0 \\ 0 & Q_q \end{bmatrix}^{-1} = \begin{bmatrix} Q_p^{-1} & 0 \\ 0 & Q_q^{-1} \end{bmatrix} \ \dots \ \dots \qquad (3.5)$$

Using expressions (3.3), (3.4) and (3.5) the least squares solution can be obtained for the partitioned matrices:

$$A^T Q_m^{-1} A = \begin{bmatrix} A_p^T & A_q^T \end{bmatrix} \begin{bmatrix} Q_p^{-1} & 0 \\ 0 & Q_q^{-1} \end{bmatrix} \begin{bmatrix} A_p \\ A_q \end{bmatrix} = (A_p^T Q_p^{-1} A_p + A_q^T Q_q^{-1} A_q)$$

$$\underset{\sim}{A}^T \underset{\sim}{Q}_m^{-1} \underset{\sim}{m} = \begin{bmatrix} \underset{\sim}{A}_p^T & \underset{\sim}{A}_q^T \end{bmatrix} \begin{bmatrix} \underset{\sim}{Q}_p^{-1} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_q^{-1} \end{bmatrix} \begin{bmatrix} \underset{\sim}{m}_p \\ \underset{\sim}{m}_q \end{bmatrix} = (\underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{m}_q)$$

Hence $$\underset{\sim}{x} = (\underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{A}_q)^{-1} (\underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{m}_q) \quad \ldots \ldots \ldots \ldots \quad (3.6)$$

In the case of a sequential adjustment, it is assumed that initially only the first group of p measurements are observed and a set of unknown parameters $\underset{\sim}{x}_o$ are estimated from the primary phase of the adjustment:

$$\underset{\sim}{A}_p \underset{\sim}{x}_o = \underset{\sim}{m}_p + \underset{\sim}{v}_o \quad : \quad \underset{\sim}{Q}_p \quad \ldots \ldots \ldots \ldots \quad (3.7)$$

The solution is given by:

$$\underset{\sim}{x}_o = (\underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p)^{-1} (\underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p) \quad \ldots \ldots \ldots \quad (3.8)$$

This can be more conveniently expressed as:

$$\underset{\sim}{x}_o = \underset{\sim}{N}_p^{-1} \underset{\sim}{k}_p$$

And it can be shown that the weight coefficient matrix of the unknowns is equal to the inverse of the normals matrix:

$$\underset{\sim}{Q}_{x_o} = \underset{\sim}{N}_p^{-1} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (3.9a)$$

A rearrangement of this expression gives what are commonly called the normal equations:

$$\underset{\sim}{N}_p x_o = \underset{\sim}{k}_p \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (3.9)$$

Consider now that the second set of q measurements is observed. These observations are adjusted with the first estimates of the unknown parameters regarded as measurements with their weight coefficient matrix taken from the primary phase:

$$\begin{bmatrix} \underset{\sim}{I} \\ \underset{\sim}{A}_q \end{bmatrix} \underset{\sim}{x} = \begin{bmatrix} \underset{\sim}{x}_o \\ \underset{\sim}{m}_q \end{bmatrix} + \begin{bmatrix} \underset{\sim}{v}_1 \\ \underset{\sim}{v}_q \end{bmatrix} : \begin{bmatrix} \underset{\sim}{Q}_{x_o} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_q \end{bmatrix} = \begin{bmatrix} \underset{\sim}{N}_p^{-1} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_q \end{bmatrix} \ldots \quad (3.10)$$

where $\underset{\sim}{I}$ is the identity matrix of appropriate dimensions.

The least squares solution to this set of equations is given by:

$$\underset{\sim}{A}^T \underset{\sim}{Q}^{-1} \underset{\sim}{A} = \begin{bmatrix} \underset{\sim}{I} & \underset{\sim}{A}_q^T \end{bmatrix} \begin{bmatrix} \underset{\sim}{N}_p & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_q^{-1} \end{bmatrix} \begin{bmatrix} \underset{\sim}{I} \\ \underset{\sim}{A}_q \end{bmatrix} = (\underset{\sim}{N}_p + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{A}_q)$$

$$\underset{\sim}{A}^T \underset{\sim}{Q}^{-1} \underset{\sim}{m} = \begin{bmatrix} \underset{\sim}{I} & \underset{\sim}{A}_q^T \end{bmatrix} \begin{bmatrix} \underset{\sim}{N}_p & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_q^{-1} \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_o \\ \underset{\sim}{m}_q \end{bmatrix} = (\underset{\sim}{N}_p \underset{\sim}{x}_o + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{m}_q)$$

Using (Equation (3.9):

$$\underset{\sim}{x} = (\underset{\sim}{N}_p + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{A}_q)^{-1} (\underset{\sim}{k}_p + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{m}_q) \quad \ldots \quad (3.11)$$

An examination of Equations (3.8) and (3.9) will show that this expression is identical to (3.6), the solution for the total set of measurements.

When the sequential adjustment solution is written in the above form it can be seen that for every new set of measurements involving the same set of unknowns, a better least squares estimate of the unknown parameters can be obtained by merely adding the contributions of the new measurements to the normal equations. Only the first estimates of the parameters and their covariance matrix are required from the preceding phase of the adjustment. The adjustment can be extended to many phases by simply partitioning expression (3.1) into the appropriate number of sub-matrices. After each phase the updated estimates and covariance matrix are carried into the next phase to be adjusted with the next group of observations.

3.3 Restrictions

3.3.1 Number of Measurements

The only restriction on the number of measurements per group is on the first group. The initial adjustment must have sufficient observations to define the unknown parameters, i.e. $p > n$. For subsequent phases of the adjustment the size of the group is irrelevant to the theory of the solution, although there may be practical considerations.

However, if additional parameters are introduced for a new phase of the adjustment there must be, in that phase, enough additional measurements to define those new parameters. The parameters from the previous phase of the adjustment are carried in observation equations which are necessary and sufficient for their own definition only.

3.3.2 Correlation

The groups of observations are assumed to be independent, as is implied by the null matrices in the weight coefficient matrix of expression (3.4). Any correlation between groups will invalidate the phased adjustment solution for the unknown parameters given by equation (3.11).

The majority of surveying and photogrammetric measurements can be considered as independent and uncorrelated, so this difficulty would normally not arise. Some measurements generate a group of observation equations, for example image observations for a collinearity condition solution, which can be correlated amongst themselves. However it is unlikely that such a group of measurements would span more than one phase. In any case, the grouping can be ordered to avoid any correlation between phases.

3.3.3 Estimate of the Variance Factor

The estimate of the variance factor for a least squares adjustment described by equations (3.1) and (3.3) is given by:

$$\sigma_o^2 = \underset{\sim}{v}^T \underset{\sim}{Q}^{-1} \underset{\sim}{v}/r = \phi/r \qquad \ldots \ldots \ldots \qquad (3.12)$$

where r is the number of redundancies for the adjustment.

r = m - n if m is the number of measurements and n the number of unknowns of the adjustment.

From equation (3.12) and expression (3.4)

$$\phi_1 = \underset{\sim}{v}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{v}_p + \underset{\sim}{v}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{v}_q \qquad \ldots \ldots \ldots \ldots \qquad (3.13)$$

Similarly, using expression (3.10) it can be shown for the second phase of the sequential adjustment:

$$\phi_2 = \underset{\sim}{v}_1^T \underset{\sim}{N}_p \underset{\sim}{v}_1 + \underset{\sim}{v}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{v}_q \qquad \ldots \ldots \ldots \ldots \qquad (3.14)$$

To be able to obtain the variance factor of the simultaneous adjustment from the sequential adjustment there must be some simple relationship between ϕ_2 and ϕ_1. Evaluating the unknown part of ϕ_2:

$$\underset{\sim}{v}_1^T \underset{\sim}{N}_p \underset{\sim}{v}_1 = (\underset{\sim}{x} - \underset{\sim}{x}_o)^T \underset{\sim}{N}_p(\underset{\sim}{x} - \underset{\sim}{x}_o)$$

$$= (\underset{\sim}{x}^T - \underset{\sim}{x}_o^T)(\underset{\sim}{N}_p \underset{\sim}{x} - \underset{\sim}{N}_p \underset{\sim}{x}_o)$$

$$= \underset{\sim}{x}^T \underset{\sim}{N}_p \underset{\sim}{x} - \underset{\sim}{x}_o^T \underset{\sim}{N}_p \underset{\sim}{x} - \underset{\sim}{x}^T \underset{\sim}{N}_p \underset{\sim}{x}_o + \underset{\sim}{x}_o^T \underset{\sim}{N}_p \underset{\sim}{x}_o \quad \ldots \quad (3.15)$$

From equations (3.8) and (3.9):

$$\underset{\sim}{N}_p = \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p$$

$$\underset{\sim}{k}_p = \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p \quad \therefore \quad \underset{\sim}{k}_p^T = \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \text{ (as } \underset{\sim}{Q}_p^{-1} \text{ is symmetric)}$$

$$\underset{\sim}{x}_o = \underset{\sim}{N}_p^{-1} \underset{\sim}{k}_p \quad \therefore \quad \underset{\sim}{x}_o^T = \underset{\sim}{k}_p^T \underset{\sim}{N}_p^{-1} \quad \text{(as } \underset{\sim}{N}_p^{-1} \text{ is symmetric)}$$

Substituting these into equation (3.15):

$$\underset{\sim}{v}_1^T \underset{\sim}{N}_p \underset{\sim}{v}_1 = \underset{\sim}{x}^T \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \underset{\sim}{x} - \underset{\sim}{k}_p^T \underset{\sim}{N}_p^{-1} \underset{\sim}{N}_p \underset{\sim}{x} - \underset{\sim}{x}^T \underset{\sim}{N}_p \underset{\sim}{N}_p^{-1} \underset{\sim}{k}_p + \underset{\sim}{k}_p^T \underset{\sim}{N}_p^{-1} \underset{\sim}{N}_p \underset{\sim}{x}_o$$

$$= \underset{\sim}{x}^T \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \underset{\sim}{x} - \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \underset{\sim}{x} - \underset{\sim}{x}^T \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p + \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \underset{\sim}{x}_o$$

Using equation (3.7):

$$\underset{\sim}{v}_1^T \underset{\sim}{N}_p \underset{\sim}{v}_1 = \underset{\sim}{x}^T \underset{\sim}{A}^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \underset{\sim}{x} - \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \underset{\sim}{x} - \underset{\sim}{x}^T \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p + \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p + \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{v}_o$$

Factorizing:

$$\underset{\sim}{v}_1^T \underset{\sim}{N}_p \underset{\sim}{v}_1 = (\underset{\sim}{x}^T \underset{\sim}{A}_p^T - \underset{\sim}{m}_p^T)(\underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \underset{\sim}{x} - \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p) + \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{v}_o$$

$$= (\underset{\sim}{A}_p \underset{\sim}{x} - \underset{\sim}{m}_p)^T \underset{\sim}{Q}_p^{-1}(\underset{\sim}{A}_p \underset{\sim}{x} - \underset{\sim}{m}_p) + \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{v}_o$$

$$= \underset{\sim}{v}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{v}_p + \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{v}_o \quad \ldots \ldots \ldots \ldots \ldots \quad (3.16)$$

Thus, from (3.13), (3.14) and (3.16):

$$\phi_1 = \phi_2 - \underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{v}_o \quad \dots \dots \dots \dots \dots \qquad (3.17)$$

The interpretation that can be placed on this final equation is that the variance factor of the simultaneous adjustment cannot be calculated from the results of the sequential adjustment alone, in most practical applications. The only case in which this will be possible is when the initial phase of the sequential adjustment has a necessary and sufficient number of measurements. In this case:

$$n = p$$

$$\therefore \underset{\sim}{v}_o = \underset{\sim}{0} \quad \dots \dots \dots \dots \dots \qquad (3.18)$$

$$\therefore \phi_2 = \phi_1$$

where $\underset{\sim}{v}_o$ is the vector of residuals from the initial phase of the sequential adjustment.

This situation is unlikely to arise as the majority of least squares adjustments have redundant measurements even in the initial stages. Extra observations are added to strengthen the adjustment, usually on the basis of the residuals of the initial phase.

The interpretation of equation (3.17) is supported by the equation for the sequential adjustment (3.10). If the first group of measurements is necessary and sufficient for a solution the simultaneous and sequential adjustments are equivalent and equations (3.18) are satisfied. The initial phase of the adjustment and its contribution to the second phase become identical in the number of observations and hence the number of redundancies. In any case there is no question that the variance factors from the simultaneous and sequential adjustments are unbiased estimators when they are applied to their own adjustments.

The variance factor of the simultaneous adjustment could be computed if the quantity $\underset{\sim}{m}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{v}_o$ was purposely calculated and retained for every phase and if the number of measurements from each phase was known. However this seems impractical and unnecessary, and also tends to defeat the purpose of sequential adjustment. Only the results and the covariance matrix should be required from previous adjustments.

3.3.4 Condition Equations

Up until this stage only observation equations have been dealt with, whereas sequential adjustment is equally applicable to adjustments by condition equations. Condition equations even have the advantage over observation equations that the estimates of the variance factors from simultaneous and sequential adjustments can be equated using only the variance factor from the initial adjustment (Mikhail 1976). Of course there is still the disadvantage that the number of conditions and the factor must be carried from the previous adjustment.

Condition equations are not used here for the sequential adjustment because the equations used for the restitution are not in a form amenable to conditions. The solutions are much better formulated as observation equations and observations are used throughout.

3.4 Applications

3.4.1 Geodetic Survey Observations

The use of sequential adjustment in relation to survey adjustments became necessary due to the comparatively low precision of existing national geodetic control networks in many countries throughout the world with respect to modern instrumentation. The majority of the networks were established or re-surveyed either shortly after the cessation of the Second World War or soon after the first reliable long range EDM (Eletromagnetic Distance Measurement) instuments were introduced. Although the techniques of triangulation and EDM traversing were considered extremely precise at the time subsequent developments in equipment have shown that these survey networks give positions whose precision is much less than that obtainable with the modern instruments.

The introduction of high precision, low cost short- and medium-range EDM has brought millimetre accuracy within the reach of most public and private survey organizations. The majority of control surveys carried out by these organizations are connected to the national datum and many of the photogrammetric control and large construction site surveys will embrace a number of national survey stations. The problem which arises is that the new surveys will be able to detect errors in the relative positions of the national survey stations and conventional adjustments result in large distortions in the measurements between such stations. The surveyor must either accept a substantial drop in the internal precision of the survey by considering the national points fixed, or to come to some sort of compromise between the measurements and the network.

One possible solution is the re-adjustment of part or all of the network incorporating the modern measurements, but this is impractical as the original observations will probably be difficult to locate and interpret. There may be a multitude of measurements which have to be included to adjust only the few points involved in the modern survey. On the other hand, the precision of the network points is sometimes readily accessible or estimable from the data provided by the national organization.

In the light of these factors, sequential adjustment is the best way of reconciling the old and new surveys. Only the national points involved in the modern survey need be included in the adjustment (Cooper and Leahy 1976) and those points are adjusted in position by the new survey and given a higher precision. If enough high precision modern data could be integrated into national networks in this way, the precisions of the national points would eventually become comparable with modern standards.

The application of sequential adjustment to geodetic survey observations is possibly the most practicable and has the highest potential usage of the results of the adjustment. Although this application could be considered as irrelevant to the mainstream of this thesis, it is included as it perhaps demonstrates the benefits of phased adjustment of observations very clearly.

3.4.2 Constrained Variables

The first application of the principle of sequential adjustment to photogrammetric measurement was in the use of constrained variables. Parameters normally considered as unknowns were treated as measurements characterized by estimated standard deviations. This technique was known as the use of weight constraints, but the derived normal equations are very similar to the sequential adjustment expression (3.11). For example, a sequential algorithm for weight constraints in multistation collinearity adjustments was developed (Brown 1958, 1960, 1964):

$$\begin{bmatrix} \dot{\underset{\sim}{N}} + \dot{\underset{\sim}{W}} & \underset{\sim}{N} \\ \underset{\sim}{N}^T & \ddot{\underset{\sim}{N}} + \ddot{\underset{\sim}{W}} \end{bmatrix} \begin{bmatrix} \dot{\underset{\sim}{\delta}} \\ \ddot{\underset{\sim}{\delta}} \end{bmatrix} = \begin{bmatrix} \dot{\underset{\sim}{c}} + \dot{\underset{\sim}{W}}\,\dot{\underset{\sim}{e}} \\ \ddot{\underset{\sim}{c}} + \ddot{\underset{\sim}{W}}\,\ddot{\underset{\sim}{e}} \end{bmatrix} \quad \ldots \quad (3.19)$$

where the single and double dot superscripts refer to camera orientation parameters and object space coordinates respectively.

$\underset{\sim}{N}$ and $\underset{\sim}{c}$ matrices are the contributions from the colleariry equations to the left and right sides respectively of the normal equations.

$\underset{\sim}{\delta}$ vectors are unknown parameters.

$\underset{\sim}{W}$ are a priori weight matrices

$\underset{\sim}{e}$ vectors are differences between the estimated and corrected parameters.

However the above formulation is not intended to be used as a true sequential adjustment. The initial values of the parameters and their covariance matrices are not from apriori adjustment, and in particular the covariance matrices are not full matrices. The parameters are assumed to be independent and off-diagonal terms in the covariance matrices are ignored.

The camera exterior orientation parameters may be provided, say, from an inertial navigation system - one of many auxiliary camera sensors available. The matrix $\dot{\underset{\sim}{W}}$ would then be a diagonal matrix of the estimated weights of the navigation parameters, and the vector $\dot{\underset{\sim}{e}}$ would be composed of the exterior orientation parameters of each camera station.

A similar scheme is used for the object space coordinates. In this case the matrix $\ddot{\underset{\sim}{W}}$ would be either a diagonal matrix in which the weights were estimates, or a block diagonal matrix composed of 3 x 3 submatrices. The object space coordinates may have weights extracted from a previous survey adjustment, but correlation is only allowed between coordinates of the same point. The vector $\ddot{e}$ is then composed of the estimated values of the coordinates of the object points. Normally only the control points have non-zero values for the weights, as the majority of the object space points have completely unknown positions, e.g. passpoints, tiepoints etc.

This diagonality or block-diagonality was stipulated for the matrices in order to obtain large savings in computations. The a priori weight matrices must first be defined as covariance matrices, and if a large block of photographs is involved, the inversion of a full matrix is a substantial calculation. If the matrices are instead diagonal or block diagonal, then only the diagonal terms or submatrices need to be inverted. Furthermore, if the points and photographs in the strip or block are appropriately ordered, a banded normals matrix can be achieved. In this case the technique of recursive partitioning can be used to exploit the banded nature of the matrices and so avoid inversion of the entire matrix (Brown 1968). In any case the effect of the correlation terms on the solution for the camera orientation parameters is assumed to be insignificant.

The algorithm was extended and expanded to be able to include other kinds of constraints of both the camera and the object space coordinates (Case 1961). Chief amongst these are equal elevation constraints for object points on shorelines or water surfaces, heading/statoscope or orbital constraints for the camera exterior orientations and camera interior orientation constraints, which have already been discussed in Chapter 2.

Interior orientation constraints tend to be in a separate category, known as added parameters, because they increase the number of unknowns in the adjustment. Systematic image deformations normally ignored can be modelled by additional parameters which are introduced to the adjustment as new unknowns. The form of the normals matrix in expression (3.19) is altered to a banded and bordered configuration, as the interior orientation unknowns are common to all the photographs, or possibly groups of photographs, in the adjustment. The inversion of the normals matrix can be carried out with the recursive partitioning with only slight modifications.

The aim of all these constraints is of course to improve the multi-station adjustment and to make it more versatile and efficient. The added parameters for the interior orientation and the control point constraints are refinements to the functional model of the adjustment. The camera and object space constraints decrease the reliance on control of the adjustment. Camera constraints actually define elements of the exterior orientations of the cameras, but to such a poor precision that they are of little use by themselves. Object constraints define the relationships between different cameras. In areas of sparse control such auxiliary data can be used to strengthen an otherwise weak adjustment. Alternatively, sensor data and object point constraints can be proposed in advance of the photography to reduce the requirement for expensive ground surveys to provide control.

3.4.3 Online Processing

Sequential adjustment has also been used in the realm of photogrammetric measurement for the phasing of observations into an existing adjustment, with both fixed and variable numbers of unknown parameters (Mikhail and Helmering 1973). However the aim here is not specifically the strengthening of a solution, but improving the efficiency of online data acquisition.

The majority of photogrammetric data acquisitions, whether the acquisition is offline or online, are carried out in what could be called "batch" mode. Large groups of observations are processed in least squares adjustments for fixed numbers of parameters; for example the observations of fiducial marks and control points for the definitions of interior and exterior orientations respectively.

The purpose of sequential adjustment in online processing is to enable the inclusion of single observations in an existing adjustment. That is, once enough observations are taken to define a set of parameters, subsequent observations can be processed individually. These subsequent observations can and probably will introduce new parameters, but each new observation is normally sufficient to define those new parameters. For example, each new point introduced into a collinearity solution of a single stereopair provides four equations but adds three new unknowns (unless the point is a fixed control point).

The main advantage of this type of online data acquisition is that the effect of the addition of the new observation can be checked in terms of changes to the parameters of the adjustment. This is an efficient method of detecting gross errors, because if the parameters are altered beyond a pre-determined limit the observation can be rejected immediately. If a batch mode solution was being employed the error would not be detected until after all the observations were recorded and the operator would have to return to the stereopair to reobserve the offending point. The observation can be rejected very simply by reversing the signs in equation (3.11), the contributions of the observation to the normals are then subtracted rather than added. Similarly, if the operator realizes he has made a mistake in point numbering or identification, the effect of the erroneous point can be quickly and efficiently removed.

The use of equation (3.11) requires that the full normals be inverted for each new observation, and this can be extremely slow in the context of online processing. Instead, extended algorithms have been developed which allow the effect of individual observations to be directly added or removed from the inverted normals. The largest matrix which then needs to be inverted has a dimension equal to the number of equations produced by the observations. Although there are extra matrix manipulations required, it has been shown that it is significantly faster (<u>Helmering</u> 1977).

However, the benefits of this system are largely associated with the collinearity solution where a least squares adjustment (with an inversion of the same order as in the extended algorithm) is mandatory for the calculation of object space coordinates. If a direct computation is used in conjunction with a different solution the technique loses a great deal of its attraction. Error detection can be implemented equally well by testing the magnitude of the residual parallax of the observations, again against a pre-determined limit.

Even if a fast minicomputer is used for the sequential calculation, the return time for the object coordinates can be as much as one to two seconds and this could significantly effect the speed of data acquisition. The majority of small microprocessors have longer cycle times, so this interval could easily double unless the user resorted to machine language programming. Even this may not help, as some minicomputer and microprocessor running systems do not allow combination of routines in low and high level languages and the low level language may not be suited to long numerical calculations. Therefore it may not be possible to use the algorithm without intolerable delays in the data acquisition process.

3.4.4 Control

Allowance for errors in photogrammetric restitutions has been made in various ways for a variety of configurations of photographs. What all the techniques have in common is the combined adjustment of the control and the photogrammetric observations, although the level of sophistication can vary considerably. One technique has already been demonstrated in Section 3.4.2, but further discussion of it and other methods will be given here to fill the catalogue of proposed solutions.

At the highest level of sophistication are true simultaneous adjustments of control and photogrammetric observations. Here the original measurements which define the control are simultaneously adjusted with the photogrammetric measurements (Wong 1975a El Hakim and Faig 1980). Because the number of observations in an overdetermined control survey can be quite large, such simultaneous adjustments must necessarily run offline on large minicomputers or mainframes. The problems caused by many measurements are accentuated when the photogrammetric restitution involves multistation photography, but even if only a single stereopair is involved the manipulation of such a large observation set would probably overtax online processing on microprocessors or minicomputers. If a suitable mass storage device were available the adjustment may be possible using data files, but the performance of the system could deteriorate to the point where the processing could not be called online.

At the other end of the scale, a simple but effective approach may be taken. To allow for the precision of the control in the restitution, the standard errors of the control points can be incorporated into the stochastic models of the photogrammetric observations (Marzan and Karara 1975). The variances of individual control point coordinates are reduced to photograph scale and simply added to those of the observed photo coordinates and according to the principles of error propagation.

This is virutally equivalent to adding a diagonal weight matrix to the left hand side of the normal equations and is the implementation of sequential adjustment at its lowest level of sophistication. No corrections to the control can be computed as the control coordinates are not carried as unknown parameters and the right hand side of the normals is left unchanged. The technique could be extended to use 3 x 3 submatrices to account for correlation between the coordinates of individual control points, but in any case it is only used to weight the control and thereby improve the overall stochastic model of the restitution. The advantages of the technique are that the standard errors of the control can be very simply incorporated and the number of unknown parameters is not increased.

The use of weight constraints is perhaps the next level of sophistication. Here corrections to the control are computed because the coordinates are carried as unknown parameters and the discrepancies are introduced to the right hand side of the normal equations. However the technique still falls short of being a rigorous sequential adjustment because the a priori weight matrix used is not a full matrix. As already discussed in Section 3.4.2, off-diagonal terms are largely ignored and the correlation between control points is not incorporated into the adjustment. This may not significantly effect the restitution or the computed coordinates of object points, but it will effect the corrections to the control. The a posteriori weight coefficient matrix of the control will certainly be quite different from the original weight coefficient matrix supplied from the adjustment of the control measurements.

When used with aerotriangulation strip and block adjustment, the corrections to the control are not exploited as the emphases are generally on the camera orientation parameters and the object space coordinates. In fact the first step of partitioning techniques eliminates $\ddot{\underset{\sim}{\delta}}$ from expression (3.19) to obtain a set of "reduced" normal equations which require less computation:

$$\left[\dot{N} + \dot{W} - N(\ddot{N} + \ddot{W})^{-1} N^T\right]\dot{\delta} = \dot{c} - \dot{W}\dot{e} - N(\ddot{N} + \ddot{W})^{-1}(\ddot{c} - \ddot{W}\ddot{e}) \quad \ldots \quad (3.20)$$

Unknown object space coordinates could then be computed in a piecemeal fashion, rather than calculating $\ddot{\delta}$ as a unit, using the block diagonality of the matrices. The control corrections are not utilized in their own right - perhaps because the control generally had a precision better than the measuring precision of the photogrammetry and was sparsely distributed throughout the photography. Although it is recognized that allowance has to be made for the precision of the control, individual control corrections are not considered to be statistically significant and the adjustment can only give a broad indication of systematic errors in the control.

This argument doesn't apply to close-range photogrammetry where control points can be numerous and the precision of the coordinates may be comparable to or even worse than the measuring precision of the photogrammetry. Rigorous adjustments have been developed and sequential algorithms are often derived directly without reference to the underlying principles (Grun 1978). However the weight matrices used are still diagonal with estimated values, and control corrections are used only as indicators of general systematic deformations. A full covariance matrix from a previous survey adjustment is apparently not used where it is applicable either because the correlations are assumed not to have a significant effect on the results of the adjustment or for the sake of a reduced programming or computational task.

4. METHOD OF RESTITUTION

4.1 Interior Orientation

4.1.1 Transformations Used

The variety of cameras and the possible use of glass plates or roll film in close-range photogrammetry prevent the satisfactory use of a single transformation for all cases of interior orientation. Different solutions must be available to cater for the different combinations possible. The number and diversity of transformations needed is determined by the range of cameras and emulsion bases which are likely to be employed. In practice only three transformations are necessary to cope with almost any situation. If one of the above variables is removed, i.e. if only one type of camera or only glass plates are being used, then only one or two of the following three transformations may be required.

The first transformation required is the affine transformation described by equations (2.3), and could be expected to be the transformation most frequently used. This transformation would be used in conjunction with a camera having at least four known fiducial positions and using glass plates with a high degree of flatness. This combination is widely accepted as producing reliable results in precise analytical close-range photogrammetry.

As the affine transformation has six parameters and a minimum of four known fiducial marks will produce eight equations, there is an excess of information. The two redundancies can be processed conveniently by a least squares adjustment solution by observation equations. Observed comparator coordinates are considered to be independent, whether the observations are carried out on a monocomparator or a stereocomparator.

When a pair of similar photographs is viewed in a stereocomparator the fiducial marks can be observed pseudo-stereoscopically i.e. the fiducial marks on each photograph can be matched together like conjugate images to give an impression of stereo viewing. This technique is designed purely to hasten the measurement process and the final positioning is commonly done monocularly. Therefore the two photographs are presumed to be independent and can be processed in separate adjustments.

The second transformation required is the simple translation and rotation described by equations (2.1). This transformation must be available to accommodate cameras which have the minimum fiducial information, or where the fiducial marks are defined with such a poor precision that they cannot be used as known positions. In this case the principal point of the photograph is defined by the intersection of the lines joining opposite fiducial marks. Although it would be undesirable to use such a camera with roll or cut film for analytical work, the transformation must be used regardless of the emulsion base because of the lack of information.

Equations (2.1) cannot be used directly because no positions are known in the fiducial coordinate system. A least squares solution is not applicable because only the necessary and sufficient information is present to define the unknowns. The three parameters for the transformation must be derived directly from the comparator measurements. It can be shown (see Figure 4.1) that if the four fiducial marks are observed in the comparator coordinate system the principal point position is given by:

$$\bar{x}_o = \{(\bar{x}_2 - \bar{x}_1)(\bar{x}_4\bar{y}_3 - \bar{x}_3\bar{y}_4) - (\bar{x}_4 - \bar{x}_3)(\bar{x}_2\bar{y}_1 - \bar{x}_1\bar{y}_2)\}/S$$

$$\ldots \quad \ldots \qquad (4.1)$$

$$\bar{y}_o = \{(\bar{y}_2 - \bar{y}_1)(\bar{x}_4\bar{y}_3 - \bar{x}_3\bar{y}_4) - (\bar{y}_4 - \bar{y}_3)(\bar{x}_2\bar{y}_1 - \bar{x}_1\bar{y}_2)\}/S$$

where

$\bar{x}_o$, $\bar{y}_o$ are the comparator coordinates of the principal point

$\bar{x}_i$, $\bar{y}_i$ are the comparator corodinates of the i^{th} fiducial mark

$$S = (\bar{x}_4 - \bar{x}_3)(\bar{y}_2 - \bar{y}_1) - (\bar{x}_2 - \bar{x}_1)(\bar{y}_4 - \bar{y}_3)$$

The mean rotation, θ, between the fiducial and comparator coordinate systems is given by:

$$\theta = \frac{1}{2}(\theta_1 + \theta_2) = \frac{1}{2}\{\ \mathrm{atan}(\frac{\bar{x}_2 - \bar{x}_1}{\bar{y}_2 - \bar{y}_1}) + \mathrm{atan}(\frac{\bar{y}_3 - \bar{y}_4}{\bar{x}_2 - \bar{x}_1})\ \} \quad \ldots \quad \ldots \qquad (4.2)$$

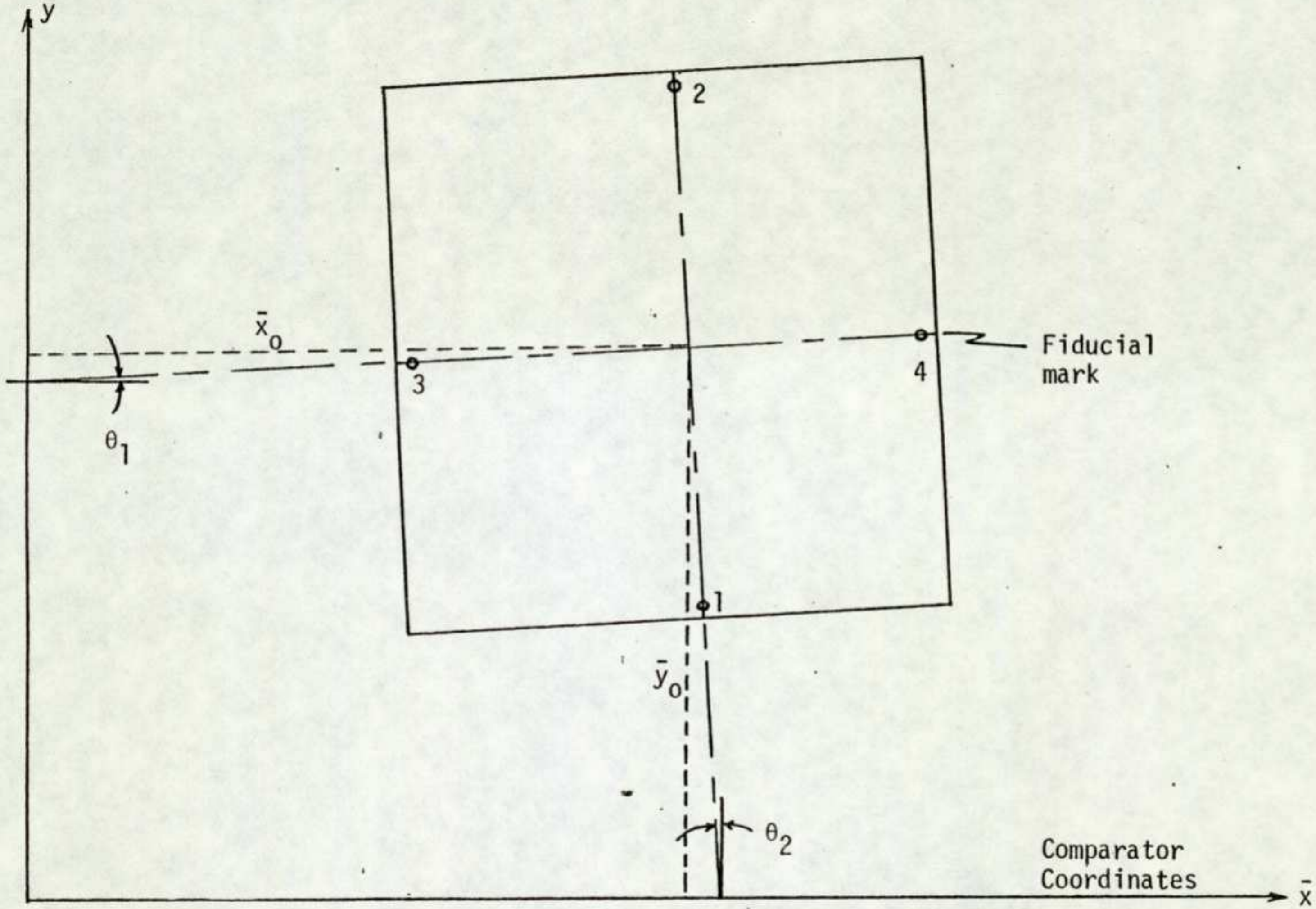


Figure 4.1 Interior Orientation - Simple Translation/Rotation

A mean rotation is used as there is no guarantee that the lines between the fiducial marks will intersect at right angles. One particular line of fiducials could be used as a reference direction, but this would require the line to be uniquely identified by a fixed numbering system. If the format is liable to be used in various orientations on the stereocomparator plate carriers, it is more convenient to employ a numbering system unique with respect to the comparator. The mean rotation is unambiguous and coordinates of fiducial marks from stereo-pairs with differing format orientations can still be compared with ease.

The three parameters can now be substituted into equations (2.1), with the slight alteration that the centre of rotation is shifted to the principal point:

$$x = \cos\theta(\bar{x} - \bar{x}_o) - \sin\theta(\bar{y} - \bar{y}_o)$$

$$y = \sin\theta(\bar{x} - \bar{x}_o) + \cos\theta(\bar{y} - \bar{y}_o)$$

Rearranging:

$$x = (- \bar{x}_o \cos\theta + \bar{y}_o \sin\theta) + \bar{x} \cos\theta - \bar{y} \sin\theta$$

$$\ldots \quad \ldots \qquad (4.3)$$

$$y = (- \bar{x}_o \sin\theta - \bar{y}_o \cos\theta) + \bar{x} \sin\theta + \bar{y} \cos\theta$$

The third and last transformation required is one of the eight parameter transformations described in section 2.1.2. This transformation must be available to deal primarily with the use of cut or roll film in a camera with four or more known fiducial positions. With or without an efficient film flattening device, non-linear image deformations can be present and an affine transformation cannot remove them. An eight parameter transformation may also be used with ordinary glass plates which do not have a certified high quality of flatness. Ultra-flat plates are expensive because the glass is of optical flat quality, so where the highest precision is not required ordinary plates are often used to reduce costs. Eight-parameter transformations can compensate for distortions due to both lack of flatness and non-linear emulsion anomalies.

If a camera with only four known fiducials is used for the photography an exact solution will be obtained, as there are eight unknown parameters and eight equations. If more known fiducial positions are available then there will be redundant information which can be processed by a least squares adjustment. In fact, an adjustment solution is used in all cases for simplicity and convenience.

However, when only four known fiducial marks are present it may be prudent to employ the simpler affine transformation. If an eight-parameter solution is used an exact fit is obtained and there are no residuals from the least squares adjustment. Therefore it is not possible to check for gross errors of measurement, although the magnitudes of the parameters may give a limited indication. The affine transformation can only compensate linear distortions, but the residuals can facilitate error checking.

The effectiveness of the non-linear parameters in removing distortions is the criterion for the selection of a particular eight-parameter transformation. It has been reported that the deformational transformation described by equations (2.6) can remove circular distortions which are often present in all types of films (Kupfer 1972). This report was based on work carried out with aerial cameras and the assumption is made here that film flattening devices, for example vacuum backs and pressure platens, have similar characteristics in close-range cameras. Unfortunately there has been little research in the field of image deformations of close-range cameras and this assumption cannot be verified. In any case the deformational transformation will be superior to the affine transformation when non-linear distortions are present.

4.1.2 Observation Equations

4.1.2.1 Affine Transformation

Repeating the equations (2.3) for an affine transformation:

$$x = a_1 + a_3\bar{x} + a_5\bar{y}$$

$$y = a_2 + a_4\bar{x} + a_6\bar{y}$$

It can be seen that the coefficients are linear and observation equations can be derived by making initial assumptions for the transformation parameters:

$$a_1 = a_1' + \Delta a_1 \qquad a_4 = a_4' + \Delta a_4$$
$$a_2 = a_2' + \Delta a_2 \qquad a_5 = a_5' + \Delta a_5$$
$$a_3 = a_3' + \Delta a_3 \qquad a_6 = a_6' + \Delta a_6 \quad \dots \quad \dots \quad \dots \qquad (4.4)$$

where a_i are the most probably true values of the six parameters

a_i' are the assumed values of the six parameters

Δa_i are small corrections to the six parameters

Substitution into equations (2.3) for the i^{th} fiducial mark will give:

$$x_i - a_1' - a_3' \bar{x}_i - a_5' \bar{y}_i = \Delta a_1 + \Delta a_3 \bar{x}_i + \Delta a_5 \bar{y}_i$$
$$y_i - a_2' - a_4' \bar{x}_i - a_6' \bar{y}_i = \Delta a_2 + \Delta a_4 \bar{x}_i + \Delta a_6 \bar{y}_i \qquad (4.5)$$

These equations can be rearranged into a form suitable for an observation equations least squares adjustment:

$$\Delta a_1 + \bar{x}_i \Delta a_3 + \bar{y}_i \Delta a_5 = \Delta x_i + v_{x_i}$$
$$\Delta a_2 + \bar{x}_i \Delta a_4 + \bar{y}_i \Delta a_6 = \Delta y_i + v_{y_i} \quad \dots \quad \dots \quad \dots \qquad (4.6)$$

where v_{x_i}, v_{y_i} are the residuals of measurement

and $\Delta x_i = x_i - a_1' - a_3' \bar{x}_i - a_5' \bar{y}_i$

$$\Delta y_i = y_i - a_2' - a_4' \bar{x}_i - a_6' \bar{y}_i \quad \dots \quad \dots \quad \dots \qquad (4.7)$$

Equations (4.6) can be expressed in the matrix form $\underset{\sim}{A}\ \underset{\sim}{x} = \underset{\sim}{m} + \underset{\sim}{v}$, and a solution by least squares, obtained according to expression (3.3). For observations on m known fiducial mark positions, the expanded matrix equation is:

$$\begin{bmatrix} 1 & 0 & \bar{x}_1 & 0 & \bar{y}_1 & 0 \\ 0 & 1 & 0 & \bar{x}_1 & 0 & \bar{y}_1 \\ 1 & 0 & \bar{x}_2 & 0 & \bar{y}_2 & 0 \\ 0 & 1 & 0 & \bar{x}_2 & 0 & \bar{y}_2 \\ & \cdot & & & \cdot & \\ & \cdot & & & \cdot & \\ & \cdot & & & \cdot & \\ 1 & 0 & \bar{x}_m & 0 & \bar{y}_m & 0 \\ 0 & 1 & 0 & \bar{x}_m & 0 & \bar{y}_m \end{bmatrix} \begin{bmatrix} \Delta a_1 \\ \Delta a_2 \\ \Delta a_3 \\ \Delta a_4 \\ \Delta a_5 \\ \Delta a_6 \end{bmatrix} = \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta x_2 \\ \Delta y_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_m \\ \Delta y_m \end{bmatrix} + \underset{\sim}{v} \quad \ldots \quad (4.8)$$

It is commonly the case in close-range photogrammetry that the comparator and fiducial coordinates are closely aligned. The matching of alignments is carried out when the photographs are first placed on the comparator, using the mechanical rotation normally provided by the plate carriers. For monocomparators it is not essential, but could be stipulated in the observing procedure. When a stereopair is observed on a stereocomparator it is convenient for the operator to do so. As the majority of stereopairs are taken with zero κ-rotations (see Figure 4.2) conjugate images will be matched throughout the format and the use of the y-parallax control is reduced. If a stereoview can be obtained by manipulation of only the x-parallax the observation of the stereopair is considerably quickened.

As it would be highly unusual for different units to be used between the comparator and fiducial coordinates, the scale factor between the two will be approximately unity. Hence, the following assumptions can be made:

$$a_3 \simeq a_6 \simeq 1$$

$$a_4 \simeq a_5 \simeq 0$$

Using the propagation of variances in equations (4.7), for the observed comparator coordinates of point i:

$$\sigma^2_{\Delta x_i} = (1)^2 \sigma^2_{x_i} + (0)^2 \sigma^2_{y_i} = \sigma^2_{x_i}$$

$$\sigma^2_{\Delta y_i} = (0)^2 \sigma^2_{x_i} + (1)^2 \sigma^2_{y_i} = \sigma^2_{y_i}$$

where $\sigma^2_{x_i}$, $\sigma^2_{y_i}$ are the variances of the observed comparator coordinates of the i[th] fiducial mark. If a series of observations is taken on the fiducials then $\sigma^2_{x_i}$ and $\sigma^2_{y_i}$ should be the variances of the mean positions.

Therefore, as the comparator coordinates are considered to be independent, the weight coefficient matrix is given by:

$$\underset{\sim}{Q} = \begin{bmatrix} \sigma^2_{x_1} & & & & & & \\ & \sigma^2_{y_1} & & & \text{zeros} & & \\ & & \sigma^2_{x_2} & & & & \\ & & & \sigma^2_{y_2} & & & \\ & & & & \ddots & & \\ & \text{zeros} & & & & \sigma^2_{x_m} & \\ & & & & & & \sigma^2_{y_m} \end{bmatrix} \quad \ldots \; \ldots \; \ldots \quad (4.9)$$

A diagonal weight coefficient matrix considerably simplifies the least squares adjustment and the observing procedure on the comparator can be easily regulated to maintain the validity of the above assumptions.

4.1.2.2 Deformational Transformation

The observation equations for the deformational transformation can be derived in a similar fashion, except that there are the two extra parameters as given by equations (2.6):

$$x = a_1 + a_3\bar{x} + a_5\bar{y} + a_7\bar{x}\bar{y} + a_8\bar{y}^2$$
$$y = a_2 + a_4\bar{x} + a_6\bar{y} + a_8\bar{x}\bar{y} + a_7\bar{x}^2$$

Initial values for the eight parameters can be assumed and substituted into the above equations for the i[th] fiducial mark:

$$\Delta a_1 + \bar{x}_i\,\Delta a_3 + \bar{y}_i\,\Delta a_5 + \bar{x}_i\,\bar{y}_i\,\Delta a_7 + \bar{y}_i^2\,\Delta a_8 = \Delta x_i + v_{x_i}$$
$$\Delta a_2 + \bar{x}_i\,\Delta a_4 + \bar{y}_i\,\Delta a_6 + \bar{x}_i^2\,\Delta a_7 + \bar{x}_i\,\bar{y}_i\,\Delta a_8 = \Delta y_i + v_{y_i} \quad \ldots \quad (4.10)$$

where $\Delta x_i = x_i - a_1 - a_3\,\bar{x}_i - a_5\,\bar{y}_i - a_7\,\bar{x}_i\,\bar{y}_i - a_8\,\bar{y}_i^2$

$$\Delta y_i = y_i - a_2 - a_4\,\bar{x}_i - a_6\,\bar{y}_i - a_7\,\bar{x}_i^2 - a_8\,\bar{x}_i\,\bar{y}_i \quad \ldots \quad (4.11)$$

These equations can then be expressed in matrix form suitable for a least squares solution by observation equations:

$$\begin{bmatrix} 1 & 0 & \bar{x}_1 & 0 & \bar{y}_1 & 0 & \bar{x}_1\bar{y}_1 & \bar{y}_1^2 \\ 0 & 1 & 0 & \bar{x}_1 & 0 & \bar{y}_1 & \bar{x}_1^2 & \bar{x}_1\bar{y}_1 \\ 1 & 0 & \bar{x}_2 & 0 & \bar{y}_2 & 0 & \bar{x}_2\bar{y}_2 & \bar{y}_2^2 \\ 0 & 1 & 0 & \bar{x}_2 & 0 & \bar{y}_2 & \bar{x}_2^2 & \bar{x}_2\bar{y}_2 \\ & & \cdot & & & & \cdot & \\ & & \cdot & & & & \cdot & \\ & & \cdot & & & & \cdot & \\ 1 & 0 & \bar{x}_m & 0 & \bar{y}_m & 0 & \bar{x}_m\bar{y}_m & \bar{y}_m^2 \\ 0 & 1 & 0 & \bar{x}_m & 0 & \bar{y}_m & \bar{x}_m^2 & \bar{x}_m\bar{y}_m \end{bmatrix} \begin{bmatrix} \Delta a_1 \\ \Delta a_2 \\ \Delta a_3 \\ \Delta a_4 \\ \Delta a_5 \\ \Delta a_6 \\ \Delta a_7 \\ \Delta a_8 \end{bmatrix} = \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta x_2 \\ \Delta y_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_m \\ \Delta y_m \end{bmatrix} + \underset{\sim}{v} \quad \ldots \quad (4.12)$$

The weight coefficient matrix is exactly the same as that derived for the affine transformation. Observations are again assumed to be independent.

4.1.3 Least Squares Adjustment

The least squares adjustment solution for equations (4.8) and (4.12) is given by equation (3.3):

$$\underset{\sim}{a} = (\underset{\sim}{A}_a^T \underset{\sim}{Q}_a^{-1} \underset{\sim}{A}_a)^{-1}(\underset{\sim}{A}_a^T \underset{\sim}{Q}_a^{-1} \underset{\sim}{m}_a)$$

Because of the simplicity of both the weight coefficient matrix and the equation coefficients, the normal equations can be computed directly with relative ease. The alternative is the formation of the individual matrices and the subsequent matrix multiplications once all the observations have been compiled. This is costly in terms of computer memory and processing compared to determining directly the premultiplied matrices, because there are more matrix elements to be stored and a larger number of operations required.

The solutions are first determined in the normal equation form:

$$(\underset{\sim}{A}_a^T \underset{\sim}{Q}_a^{-1} \underset{\sim}{A}_a)\underset{\sim}{a} = (\underset{\sim}{A}_a^T \underset{\sim}{Q}_a^{-1} \underset{\sim}{m}_a) \quad \ldots \ldots \ldots \ldots \quad (4.13)$$

If the full matrices for the affine transformation are multiplied out the following normal equations are obtained:

$$\begin{bmatrix} \Sigma 1/\sigma_x^2 & 0 & \Sigma x/\sigma_x^2 & 0 & \Sigma y/\sigma_x^2 & 0 \\ & \Sigma 1/\sigma_y^2 & 0 & \Sigma x/\sigma_y^2 & 0 & \Sigma y/\sigma_y^2 \\ & & \Sigma x^2/\sigma_x^2 & 0 & \Sigma xy/\sigma_x^2 & 0 \\ & & & \Sigma x^2/\sigma_y^2 & 0 & \Sigma xy/\sigma_y^2 \\ & \text{symmetric} & & & \Sigma y^2/\sigma_x^2 & 0 \\ & & & & & \Sigma y^2/\sigma_y^2 \end{bmatrix} \underset{\sim}{a} = \begin{bmatrix} \Sigma \Delta x/\sigma_x^2 \\ \Sigma \Delta y/\sigma_y^2 \\ \Sigma x\Delta x/\sigma_x^2 \\ \Sigma x\Delta y/\sigma_y^2 \\ \Sigma y\Delta x/\sigma_x^2 \\ \Sigma y\Delta y/\sigma_y^2 \end{bmatrix} \quad \ldots (4.14)$$

where Σ represents a summation from 1 to m, if m is the number of measurements.

Similarly the normal equations for the deformational transformation are:

$$\begin{bmatrix} \Sigma 1/\sigma_x^2 & 0 & \Sigma x/\sigma_x^2 & 0 & \Sigma y/\sigma_x^2 & 0 & \Sigma xy/\sigma_x^2 & \Sigma y^2/\sigma_x^2 \\ & \Sigma 1/\sigma_y^2 & 0 & \Sigma x/\sigma_y^2 & 0 & \Sigma y/\sigma_y^2 & \Sigma x^2/\sigma_y^2 & \Sigma xy/\sigma_y^2 \\ & & \Sigma x^2/\sigma_x^2 & 0 & \Sigma xy/\sigma_x^2 & 0 & \Sigma x^2y/\sigma_x^2 & \Sigma xy^2/\sigma_x^2 \\ & & & \Sigma x^2/\sigma_y^2 & 0 & \Sigma xy/\sigma_y^2 & \Sigma x^3/\sigma_y^2 & \Sigma x^2y/\sigma_y^2 \\ & \text{symmetric} & & & \Sigma y^2/\sigma_x^2 & 0 & \Sigma xy^2/\sigma_x^2 & \Sigma y^3/\sigma_x^2 \\ & & & & & \Sigma y^2/\sigma_y^2 & \Sigma x^2y/\sigma_y^2 & \Sigma xy^2/\sigma_y^2 \\ & & & & & & \Sigma x^2y^2/\sigma_x^2 + x^4/\sigma_y^2 & \Sigma xy^3/\sigma_x^2 + x^3y/\sigma_y^2 \\ & & & & & & & \Sigma y^4/\sigma_x^2 + x^2y^2/\sigma_y^2 \end{bmatrix} \underset{\sim}{a} = \begin{bmatrix} \Sigma \Delta x/\sigma_x^2 \\ \Sigma \Delta y/\sigma_y^2 \\ \Sigma x\Delta x/\sigma_x^2 \\ \Sigma x\Delta y/\sigma_y^2 \\ \Sigma y\Delta x/\sigma_x^2 \\ \Sigma y\Delta y/\sigma_y^2 \\ \Sigma xy\Delta x/\sigma_x^2 + y^2\Delta y/\sigma_y^2 \\ \Sigma x^2\Delta x/\sigma_x^2 + xy\Delta y/\sigma_y^2 \end{bmatrix} \quad \ldots \ldots (4.15)$$

The elements and the symmetry of the normals coefficient matrices result in a particularly efficient method of obtaining the normal equations. Only one triangular half of the normals coefficient matrix needs to be stored to define it entirely. All of the summation terms in the affine normals appear in the deformational normals, so the simpler transformation can be provided as an option with virtually no extra programming beyond that required for the deformational transformation. The summation form of the elements is particularly appropriate to computer programming, and some elements do not need to be computed as they are always zero. Hence the storage and calculation requirements of the two sets of normal equations are lower than might be expected. Also, the normals coefficient matrices are inverted by a special routine which only requires the upper triangular half of symmetric matrices, resulting in further reductions in storage and programming.

All the terms in the normals coefficient matrix are constant so they do not need to be recomputed for each iteration of the least squares solution. Although the equations are linear and only a single solution should theoretically be required, an iterative solution is used to verify that the corrections to the unknowns are valid and the solution converges to a particular set of parameter values. The corrections from each iteration are added to the current assumed values of the parameters, and the right hand side vector of the normal equations is re-computed for each iteration.

The iterations of the solutions are continued until the corrections become insignificant. Generally three iterations are required to complete a solution. The first computes the major portion of the corrections, the second computes the remainder (caused by poor initial assumptions of the parameters or rounding off errors in the calculations) and the third verifies that the corrections are insignificant. If more iterations are required then the solution will probably not converge. The common cause of solution failures are data errors, point misidentifications or ambiguities and a lack of sufficient data to define the parameters.

4.2 Exterior Orientation

4.2.1 Philosophy of the Method

The discussion of existing techniques of exterior orientation in Chapter 2 can be used as the basis for a list of desirable attributes of an analytical exterior orientation for a close-range stereopair. Such an orientation should have:

(a) a simultaneous solution for relative and absolute orientation, with a single least squares adjustment of all the parameters of the exterior orientation, to increase the efficiency of the processing and avoid propagation of rounding-off errors

(b) a functional relationship between the two photographs if stereopairs are the basic unit of the photography

(c) a suitable stochastic model to describe the observations

(d) the minimum number of parameters to keep control requirements to a minimum and avoid the problems associated with an excessive number of parameters

(e) minimization of discrepancies at control and parallaxes in the object at object scale, to give a clearer picture of errors in stereopairs with large depth ranges

(f) a single component of parallax, in the object coordinate corresponding to the fiducial y-coordinate, to simplify interpretation

(g) direct computation of object coordinates from observed image coordinates without the need for a least squares adjustment.

None of the current techniques of exterior orientation can fulfil all of these requirements without wholesale modifications. Therefore a new solution needs to be developed which has many or all of the above attributes. Such a solution is <u>not</u> proposed as a replacement for existing methods, but merely as an alternative for a specific application. Just as collinearity is associated with multistation photography, coplanarity with relative orientation and DLT with non-metric photography, this solution would be used with single close-range stereopairs.

The exterior orientation developed in the following sections is based on vector equations describing the relationships between the two image points and the corresponding object point. It is assumed that a conventional interior orientation has been carried out to re-establish the relationship between the photograph and the camera perspective centre. It further assumes that the positions of images within the fiducial axis systems are refined coordinates which have had lens and any other distortions removed. A minimum of three fully known control points are required to establish the relationship between the cameras and the object space.

All solutions have their disadvantages, and this particular exterior orientation is no exception. If an object or terrain coordinate system is generally adopted with the Z-axis direction parallel to gravity, there are two camera attitudes which must be considered separately, because the definition of the residual parallax is different.

The first and more common case is the terrestrial stereopair. Here the camera axes are approximately perpendicular to the Z terrain axis (see Figure 4.2) and the residual parallax is confined to a component in the Z terrain direction. The cameras used are generally photo-theodolites or fixed-base stereometric cameras which are tripod mounted. The objects photographed are generally upright, so that the residual parallax is likely to be in the plane of the object. The largest exception to this rule is large scale topographic mapping, but this is unlikely to be carried out using analytical techniques.

The second case is the aerial stereopair. Here the camera axes are approximately parallel to the Z terrain axis (see Figure 4.5) and the residual parallax is confined to a component perpendicular to the Z terrain direction. The cameras used are generally close-range cameras mounted in special brackets (see Figure 6.5) and suspended above or placed below the object. The objects photographed have greater area than height so that this configuration becomes analogous to classical aerial photogrammetry.

The immediate objections which arise are firstly that the division of the solution into two different modes is unnecessary, as one can be mathematically transformed into the other. Unfortunately, such a transformation is not easily accomplished, partly because of the definition of the parallax and also because a consistent coordinate system must be adhered to if a sequential adjustment of the control is contemplated. Both these reasons will be elaborated on during the development of the solution. Furthermore, the inclusion of additional transformations will decrease the efficiency of the online processing system as both the input data and output results must be converted. In the case of sequential adjustment of control, such transformations would become a major delaying factor.

The second objection which arises is that if two distinct modes of solution are employed, how is a stereopair uncharacteristic of either solution processed? Fortunately such borderline cases occur relatively rarely in close-range work with single stereopairs. The camera axes rarely deviate markedly from either parallel or perpendicular to the terrain Z-axis. The case where the camera axis is approximately 45^{o} to the XY plane is much more likely to occur in multistation photography where different aspects of the same object are required, and naturally collinearity would be used in preference to the vector solution.

If such a case did arise in the course of work with single stereopairs then the attitude of the object is the final criterion in deciding which mode of solution to employ. For example, if the camera axes are tilted upwards to photograph a large structure, the terrestrial mode should be employed. The parallaxes will then be in the plane of the structure, and this is advantageous for interpretation of residuals from the solution.

The use of two separate modes of solution for two different aspects of the same problem does not lead to a doubling of the programming task. For example, exactly the same interior orientation package is employed in both programs and similar routines are used to acquire the observations for the exterior orientation, compute the least squares solution and reproject image coordinates. Duplication can be avoided by the use of common subroutines, but too much breaking up of the programs into modules will degrade the online performance of the system. A compromise must be reached between the complexity of the programs and the efficiency of the programming.

4.2.2 Terrestrial Stereopairs

4.2.2.1 Functional Model

The vector equations required for the functional model can be derived by inspection of Figure 4.2. The meaning of the symbols are as follows:

$\underset{\sim}{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$ Position vector of the left hand camera perspective centre in the object coordinate system. "Left" is defined by looking from the photographs toward the object.

$\underset{\sim}{b} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$ Base vector between the left and right camera perspective centres in the object coordinate system.

$\underset{\sim}{c} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}$ Position vector to the control point position in the object coordinate system.

f_1, f_2 Focal lengths of the left and right cameras respectively.

λ, ρ Scale factors associated with the rays between the left and right camera perspective centres and the object point, respectively.

$\underset{\sim}{m} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$ Position vector of a "measurement" in the object space - derived from the mean position of the projections of observed conjugate image points.

$\underset{\sim}{p} \quad \begin{bmatrix} 0 \\ 0 \\ p_z \end{bmatrix}$ Parallax vector between the left and right rays at the measured object point.

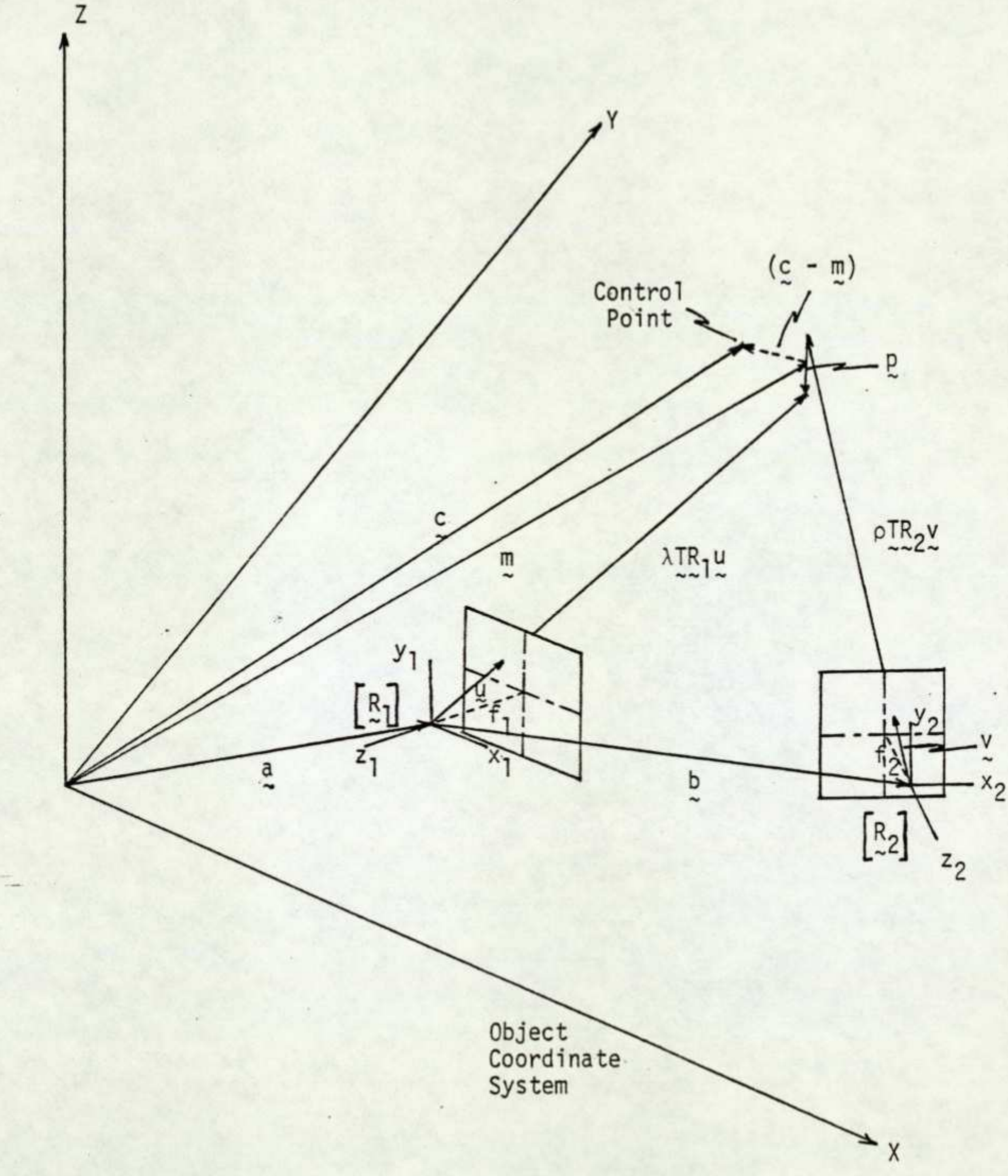


Figure 4.2 Exterior Orientation of Terrestrial Stereopairs

$\underset{\sim}{R}_1, \underset{\sim}{R}_2$ — Orthogonal rotation matrices describing the orientation of the left and right cameras with respect to the object coordinate system, respectively.

$\underset{\sim}{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ — Transformation matrix which converts fiducial coordinates into a system compatible with the object coordinate system.

$\underset{\sim}{u}, \underset{\sim}{v}$ — Fiducial coordinates of the left and right hand images of the object point with respect to the left and right hand camera perspective centres, respectively.

where
$$\underset{\sim}{u} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ -f_1 \end{bmatrix} \quad \underset{\sim}{v} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \\ -f_2 \end{bmatrix} \quad \ldots \quad (4.16)$$

and, ignoring subscripts for the moment:

$$\underset{\sim}{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad \ldots \ldots \ldots \ldots \ldots \quad (4.17)$$

The elements of the rotation matrix are functions of the three axis rotations; phi (ϕ), kappa (κ) and omega (ω). The phi rotation corresponds to the azimuth of the camera axis and is the tertiary rotation:

$$\underset{\sim}{R} = \underset{\sim}{R}\phi \, \underset{\sim}{R}\kappa \, \underset{\sim}{R}\omega \quad \ldots \ldots \ldots \ldots \ldots \quad (4.18)$$

Using (<u>Thompson</u> 1969):

$$\underset{\sim}{R}\phi = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \quad \underset{\sim}{R}\kappa = \begin{bmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underset{\sim}{R}_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix} \quad \ldots \ldots \ldots \quad (4.19)$$

Multiplication of the three matrices yields:

$$
\begin{aligned}
r_{11} &= \cos\phi \cos\kappa \\
r_{12} &= \sin\phi \sin\omega - \cos\phi \sin\kappa \cos\omega \\
r_{13} &= \sin\phi \cos\omega + \cos\phi \sin\kappa \sin\omega \\
r_{21} &= \sin\kappa \\
r_{22} &= \cos\kappa \cos\omega \\
r_{23} &= -\cos\kappa \sin\omega \\
r_{31} &= -\sin\phi \cos\kappa \\
r_{32} &= \cos\phi \sin\omega + \sin\phi \sin\kappa \cos\omega \\
r_{33} &= \cos\phi \cos\omega - \sin\phi \sin\kappa \sin\omega \qquad \ldots \; \ldots \; \ldots \qquad (4.20)
\end{aligned}
$$

Where the left and right rotation matrices can be represented by introducing the subscripts 1 and 2 respectively.

This particular rotation hierarchy is used to be compatible with analogue techniques. Although this may seem to be irrelevant, many close-range projects have photography designed to serve a dual role. It can be used analytically for the capture of numerical data, and used in an analogue instrument for the production of line drawings. In such cases it is useful to be able to transfer the rotations from the numerical restitution to the stereoplotter, to aid the operator in setting up the model. Only if phi is made the tertiary rotation are kappa and omega correct relative to the base for transfer to an analogue instrument.

From Figure 4.2 three vector equations can be derived:

$$
\begin{aligned}
&\underset{\sim}{a} + \lambda \underset{\sim}{T} \underset{\sim}{R}_1 \underset{\sim}{u} = \underset{\sim}{m} + \tfrac{1}{2} \underset{\sim}{p} \\
&\underset{\sim}{a} + \underset{\sim}{b} + \rho \underset{\sim}{T} \underset{\sim}{R}_2 \underset{\sim}{v} = \underset{\sim}{m} - \tfrac{1}{2} \underset{\sim}{p} \\
&\underset{\sim}{b} - \lambda \underset{\sim}{T} \underset{\sim}{R}_1 \underset{\sim}{u} + \rho \underset{\sim}{T} \underset{\sim}{R}_2 \underset{\sim}{v} = - \underset{\sim}{p} \qquad \ldots \; \ldots \; \ldots \qquad (4.21)
\end{aligned}
$$

These three equations are dependent, as any one can be derived from the other two. To reduce these three equations to two independent equations, the first two are meaned to eliminate the half parallaxes and therefore isolate the measurement and parallax vectors:

$$\underset{\sim}{a} + \tfrac{1}{2}\,\underset{\sim}{b} + \tfrac{1}{2}\,\lambda\,\underset{\sim}{T}\,\underset{\sim}{R}_1\,\underset{\sim}{u} + \tfrac{1}{2}\,\rho\,\underset{\sim}{T}\,\underset{\sim}{R}_2\,\underset{\sim}{v} = \underset{\sim}{m}$$

$$\underset{\sim}{b} - \lambda\,\underset{\sim}{T}\,\underset{\sim}{R}_1\,\underset{\sim}{u} + \rho\,\underset{\sim}{T}\,\underset{\sim}{R}_2\,\underset{\sim}{v} = -\,\underset{\sim}{p} \qquad \ldots \quad \ldots \qquad (4.22)$$

This simplifies the formation of the observation equations, as then the vectors ($\underset{\sim}{c}$ - $\underset{\sim}{m}$) and $\underset{\sim}{p}$ can be minimized. At this stage it is convenient to introduce two vectors $\underset{\sim}{\ell}$ and $\underset{\sim}{r}$, such that:

$$\underset{\sim}{\ell} = \begin{bmatrix} \ell_x \\ \ell_y \\ \ell_z \end{bmatrix} = \underset{\sim}{R}_1\,\underset{\sim}{u} \quad \text{and} \quad \underset{\sim}{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \underset{\sim}{R}_2\,\underset{\sim}{v} \qquad \ldots \quad \ldots \qquad (4.23)$$

Equations (4.22) can therefore be expressed:

$$\underset{\sim}{a} + \tfrac{1}{2}\,\underset{\sim}{b} + \tfrac{1}{2}\,\lambda\,\underset{\sim}{T}\,\underset{\sim}{\ell} + \tfrac{1}{2}\,\rho\,\underset{\sim}{T}\,\underset{\sim}{r} = \underset{\sim}{m}$$

$$\underset{\sim}{b} - \lambda\,\underset{\sim}{T}\,\underset{\sim}{\ell} + \rho\,\underset{\sim}{T}\,\underset{\sim}{r} = -\,\underset{\sim}{p} \qquad \ldots \quad \ldots \quad \ldots \qquad (4.24)$$

Expanding these into six scalar equations, using the effect of the transformation matrix, $\underset{\sim}{T}$:

$$\begin{aligned}
a_x + \tfrac{1}{2}\,b_x + \tfrac{1}{2}\,\lambda\,\ell_x + \tfrac{1}{2}\,\rho\,r_x &= m_x \\
a_y + \tfrac{1}{2}\,b_y - \tfrac{1}{2}\,\lambda\,\ell_z - \tfrac{1}{2}\,\rho\,r_z &= m_y \\
a_z + \tfrac{1}{2}\,b_z + \tfrac{1}{2}\,\lambda\,\ell_y + \tfrac{1}{2}\,\rho\,r_y &= m_z \\
b_x - \lambda\,\ell_x + \rho\,r_x &= 0 \\
b_y + \lambda\,\ell_z - \rho\,r_z &= 0 \\
b_z - \lambda\,\ell_y + \rho\,r_y &= -\,p_z \qquad \ldots \quad \ldots \quad \ldots \qquad (4.25)
\end{aligned}$$

As the parallax is confined to one component, the fourth and fifth scalar equations can be used to eliminate the unknown scale factors λ and ρ from the solution. Alternatively, the two scale factors are pre-selected so that two components of the parallax are always zero. Whichever way this step is viewed, the removal of λ and ρ from the solution is beneficial. The scale factors are irrelevant to the solution and if they were carried, two extra unknowns would be added for every new point measured. This of course would contravene the requirements laid down in section 4.2.1, and degrade the performance of online programs.

Using the fourth and fifth scalar expressions as simultaneous equations in λ and ρ:

$$b_x \ell_z - \lambda \ell_x \ell_z + \rho \ell_z r_x = 0$$

$$b_y \ell_x + \lambda \ell_x \ell_z - \rho \ell_x r_z = 0$$

$$\therefore \rho = (b_x \ell_z + b_y \ell_x) / (\ell_x r_z - \ell_z r_x) \quad \ldots \quad \ldots \quad \ldots \quad (4.26)$$

Similarly

$$\lambda = (b_x r_z + b_y r_x) / (\ell_x r_z - \ell_z r_x) \quad \ldots \quad \ldots \quad \ldots \quad (4.27)$$

These values can be substituted into the first three and last scalar expressions in equations (4.25) to obtain four independent equations in only the twelve parameters of the exterior orientation:

$$a_x k + b_x \ell_x r_z + b_y \ell_x r_x = m_x k$$

$$a_y k - b_x \ell_z r_z - b_y \ell_z r_x = m_y k$$

$$a_z k + \tfrac{1}{2} b_x (\ell_z r_y + \ell_y r_z) + \tfrac{1}{2} b_y (\ell_x r_y + \ell_y r_x) + \tfrac{1}{2} b_z k = m_z k$$

$$b_x (\ell_z r_y - \ell_y r_z) + b_y (\ell_x r_y - \ell_y r_x) + b_z k = -p_z k \quad \ldots \quad (4.28)$$

where $k = \ell_x r_z - \ell_z r_x$

These are the final equations of the vector solution and contain the exterior orientation unknowns:

$$a_x, a_y, a_z, b_x, b_y, b_z, \phi_1, \kappa_1, \omega_1, \phi_2, \kappa_2, \omega_2$$

and the pseudo-measurements:

$$m_x, m_y, m_z, p_z$$

which are derived from the projection of observed conjugate image points into the object space.

Referring to equations (4.21), the first two are equivalent to collinearity conditions for the left and right photographs, while the third is an expression of the coplanarity condition which is more recognizable as the last equation of (4.28). Therefore the solution could be said to be a combination of the two conditions. However, because of the form of the equations (4.28), it would be better described as a simultaneous application of a modified coplanarity condition and an absolute orientation of the two photographs. For want of a better name it will simply be referred to as the vector solution.

4.2.2.2 Differential Equations

Equations (4.28) are not in a suitable form for a solution by least squares adjustment of observations. The equations are non-linear and are not related to the control point position. Rearranging the equations:

$$a_x k + b_x \ell_x r_z + b_y \ell_x r_x - m_x k = 0$$

$$a_y k - b_x \ell_z r_z - b_y \ell_z r_x - m_y k = 0$$

$$a_z k + \tfrac{1}{2} b_x (\ell_z r_y + \ell_y r_z) + \tfrac{1}{2} b_y (\ell_x r_y + \ell_y r_x) + \tfrac{1}{2} b_z k - m_z k = 0$$

$$b_x (\ell_z r_y - \ell_y r_z) + b_y (\ell_x r_y - \ell_y r_x) + b_z k + p_z k = 0 \quad \ldots \quad (4.29)$$

They can be now expressed in the functional form:

$$g_1(a_x, b_x, b_y, \phi_1, \kappa_1, \omega_1, \phi_2, \kappa_2, \omega_2, m_x) = 0$$

$$g_2(a_y, b_x, b_y, \phi_1, \kappa_1, \omega_1, \phi_2, \kappa_2, \omega_2, m_y) = 0$$

$$g_3(a_z, b_x, b_y, b_z, \phi_1, \kappa_1, \omega_1, \phi_2, \kappa_2, \omega_2, m_z) = 0$$

$$g_4(b_x, b_y, b_z, \phi_1, \kappa_1, \omega_1, \phi_2, \kappa_2, \omega_2, p_z) = 0 \quad \ldots \quad (4.30)$$

A set of corrections to the exterior orientations can be postulated which will convert the measurement $\underset{\sim}{m}$ into the control position $\underset{\sim}{c}$ and reduce the parallax vector $\underset{\sim}{p}$ to a null vector. In terms of a least squares adjustment, this corresponds to minimizing the vectors $(\underset{\sim}{c} - \underset{\sim}{m})$ and $\underset{\sim}{p}$:

$$g_1(a_x + \Delta a_x, b_x + \Delta b_x, b_y + \Delta b_y, \phi_1 + \Delta\phi_1, \kappa_1 + \Delta\kappa_1, \omega_1 + \Delta\omega_1, \phi_2 + \Delta\phi_2, \kappa_2 + \Delta\kappa_2, \omega_2 + \Delta\omega_2, c_x) = 0$$

$$g_2(a_y + \Delta a_y, b_x + \Delta b_x, b_y + \Delta b_y, \phi_1 + \Delta\phi_1, \kappa_1 + \Delta\kappa_1, \omega_1 + \Delta\omega_1, \phi_2 + \Delta\phi_2, \kappa_2 + \Delta\kappa_2, \omega_2 + \Delta\omega_2, c_y) = 0$$

$$g_3(a_z + \Delta a_z, b_x + \Delta b_x, b_y + \Delta b_y, b_z + \Delta b_z, \phi_1 + \Delta\phi_1, \kappa_1 + \Delta\kappa_1, \omega_1 + \Delta\omega_1, \phi_2 + \Delta\phi_2, \kappa_2 + \Delta\kappa_2, \omega_2 + \Delta\omega_2, c_z) = 0$$

$$g_4(b_x + \Delta b_x, b_y + \Delta b_y, b_z + \Delta b_z, \phi_1 + \Delta\phi_1, \kappa_1 + \Delta\kappa_1, \omega_1 + \Delta\omega_1, \phi_2 + \Delta\phi_2,$$
$$\kappa_2 + \Delta\kappa_2, \omega_2 + \Delta\omega_2, 0) = 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (4.31)$$

These equations can be slighty rearranged:

$$g_1(a_x + \Delta a_x, b_x + \Delta b_x, \quad \ldots \quad , \omega_2 + \Delta\omega_2, m_x + (c_x - m_x)) = 0$$
$$g_2(a_y + \Delta a_y, b_x + \Delta b_x, \quad \ldots \quad , \omega_2 + \Delta\omega_2, m_y + (c_y - m_y)) = 0$$
$$g_3(a_z + \Delta a_z, b_x + \Delta b_x, \quad \ldots \quad , \omega_2 + \Delta\omega_2, m_z + (c_z - m_z)) = 0$$
$$g_4(b_x + \Delta b_x, b_y + \Delta b_y, \quad \ldots \quad , \omega_2 + \Delta\omega_2, p_z + (-p_z)) = 0 \quad \ldots \quad (4.32)$$

and then linearized by a Taylor's series expansion to first order terms only:

$$g_1(a_x, b_x, \ldots, \omega_2, m_x) + \frac{\delta g_1}{\delta a_x}\Delta a_x + \frac{\delta g_1}{\delta b_x}\Delta b_x + \ldots + \frac{\delta g_1}{\delta m_x}(c_x - m_x) = 0$$

$$g_2(a_y, b_x, \ldots, \omega_2, m_y) + \frac{\delta g_2}{\delta a_y}\Delta a_y + \frac{\delta g_2}{\delta b_x}\Delta b_x + \ldots + \frac{\delta g_2}{\delta m_y}(c_y - m_y) = 0$$

$$g_3(a_z, b_x, \ldots, \omega_2, m_z) + \frac{\delta g_3}{\delta a_z}\Delta a_z + \frac{\delta g_3}{\delta b_x}\Delta b_x + \ldots + \frac{\delta g_3}{\delta m_z}(c_z - m_z) = 0$$

$$g_4(b_x, b_y, \ldots, \omega_2, p_z) + \frac{\delta g_4}{\delta b_x}\Delta b_x + \frac{\delta g_4}{\delta b_y}\Delta b_y + \ldots + \frac{\delta g_4}{\delta p_z}(-p_z) = 0 \quad (4.33)$$

Using equations (4.30) and the following partial differentials:

$$\frac{\delta g_1}{\delta m_x} = \frac{\delta g_2}{\delta m_y} = \frac{\delta g_3}{\delta m_z} = -k \quad \text{and} \quad \frac{\delta g_4}{\delta p_z} = +k \quad \ldots \ldots \ldots \quad (4.34)$$

the final form of the linearized observation equations can be derived:

$$\frac{\delta g_1}{\delta a_x}\Delta a_x + \frac{\delta g_1}{\delta b_x}\Delta b_x + \frac{\delta g_1}{\delta b_y}\Delta b_y + \frac{\delta g_1}{\delta \phi_1}\Delta\phi_1 + \ldots + \frac{\delta g_1}{\delta \omega_2}\Delta\omega_2 = k(c_x - m_x)$$

$$\frac{\delta g_2}{\delta a_y}\Delta a_y + \frac{\delta g_2}{\delta b_x}\Delta b_x + \frac{\delta g_2}{\delta b_y}\Delta b_y + \frac{\delta g_2}{\delta \phi_1}\Delta\phi_1 + \ldots + \frac{\delta g_2}{\delta \omega_2}\Delta\omega_2 = k(c_y - m_y)$$

$$\frac{\delta g_3}{\delta a_z}\Delta a_z + \frac{\delta g_3}{\delta b_x}\Delta b_x + \frac{\delta g_3}{\delta b_y}\Delta b_y + \frac{\delta g_3}{\delta b_z}\Delta b_z + \frac{\delta g_3}{\delta \phi_1}\Delta\phi_1 + \ldots + \frac{\delta g_3}{\delta \omega_2}\Delta\omega_2 = k(c_z - m_z)$$

$$\frac{\delta g_4}{\delta b_x}\Delta b_x + \frac{\delta g_4}{\delta b_y}\Delta b_y + \frac{\delta g_4}{\delta b_z}\Delta b_z + \frac{\delta g_4}{\delta \phi_1}\Delta\phi_1 + \ldots + \frac{\delta g_4}{\delta \omega_2}\Delta\omega_2 = kp_z \quad \ldots \quad (4.35)$$

where the values of the partial differentials are given in Appendix A.

Equations (4.35) represent the contribution of one fully known control point to the observation equations of the least squares adjustment. As there are twelve unknown exterior orientation parameters and each fully known control point produces four equations, three such points are required as a minimum solution.

Partially known control points and simple parallax points can be introduced into the solution by including the appropriate equations from the full set of four. If enough partially known points are available a solution may be possible, but there must be enough of each coordinate present to satisfy the geometrical requirements of the solution. The absolute minimum is two points with all three coordinates known, one point with one coordinate known and two points with no coordinates known (parallax points contributing only the fourth equation). This configuration corresponds to the classical aerial relative orientation with five parallax points, followed by an absolute orientation with two fully known control points and one height point.

4.2.2.3 Stochastic Model

The quantities on the right hand sides of the observation equations are not measurements, but derived quantities known as pseudo-measurements. The relationship between the measurements and the pseudo-measurements must be established to determine a suitable stochastic model for use in the least squares adjustment. The weight coefficient matrix used in the adjustment must be appropriate to the measurements or pseudo-measurements on the right hand sides of the observation equations.

If there is a linear relationship between a set of measurements $\underset{\sim}{m}$ and a set of derived quantities $\underset{\sim}{d}$ such that:

$$\underset{\sim}{d} = \underset{\sim}{E}\,\underset{\sim}{m} + \underset{\sim}{f} \qquad \ldots \ldots \ldots \ldots \qquad (4.36)$$

and the weight coefficient matrix associated with $\underset{\sim}{m}$ is $\underset{\sim}{Q}_m$, then the weight coefficient matrix associated with $\underset{\sim}{d}$ can be shown to be:

$$\underset{\sim}{Q}_d = \underset{\sim}{E}\,\underset{\sim}{Q}_m\,\underset{\sim}{E}^T \qquad \ldots \ldots \ldots \ldots \qquad (4.37)$$

by the principles of error propagation.

It has already been shown in Section 4.1.2.1 that if a suitable observation procedure is adhered to, there is a direct relationship between the original comparator observations and the fiducial coordinates. That is, in this case:

$$\underset{\sim}{d} = \underset{\sim}{I}\,\underset{\sim}{m}$$

and therefore

$$\underset{\sim}{Q}_d = \underset{\sim}{I}\,\underset{\sim}{Q}_m\,\underset{\sim}{I}^T = \underset{\sim}{Q}_m \qquad \dots \dots \dots \dots \qquad (4.38)$$

Hence the relationship between the pseudo-measurements and the original comparator observations can be directly determined, bypassing the intermediate stage of fiducial coordinates.

One method of establishing the relationship is to use a Taylor's expansion of differential coefficients, but the form of the vector equations would make this extremely complex. The vector solution has the advantage that the coefficients for the object points are very simple, while having the disadvantage that those for the fiducial coordinates are complicated in the extreme. This is a direct contrast to the collinearity condition where the reverse is true, and the disadvantage of complex control coefficients will be examined further in a later section.

The alternative to this method is to make some assumptions about the physical relationship between the photographs and the object. Simple geometry can then be used to derive the effect of errors in the fiducial coordinates on the pseudo-measurements in the object space. This may be considered to be equivalent to simplifying the differential coefficients, but a much more comprehensible relationship is derived.

Up until this stage, no distinction has been made between mono- and stereocomparators, and the vector solution is equally applicable to both. However, the stochastic models must be different for the two types of machines. Observations on different photographs must be independent when a monocomparator is used as there is no optical correlation of images on the photographs during measurement by the operator. The basic observations are then the independent comparator coordinates for each photograph.

On the other hand, when observations are taken on a stereocomparator there is an optical matching of conjugate images by the operator. Therefore independent comparator coordinates are not measured and there is a basic difference in the quantities observed. Because the operator is viewing two photographs simultaneously and stereoscopically, the mean position on the photographs is observed along with parallaxes between the photographs.

The operator positions the reference mark with respect to both photographs, and observes x-parallax by setting the depth in the stereo image. To retain an independent set of measurements, the y-parallax must be included to define all four photograph coordinates. Although the measurement of y-parallax can be considered to be the relative positioning of the two photographs, if the operator is observing stereoscopically then it does constitute a measurement. The observation set for stereoscopic observations is therefore the mean photo positions and the two parallaxes:

$$\underset{\sim}{m}_s = \begin{bmatrix} x_p \\ y_p \\ p_x \\ p_y \end{bmatrix} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (4.39)$$

While the observation set for monocular observations consists of plate coordinates:

$$\underset{\sim}{m}_m = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{bmatrix}$$

The compatability of the quantities observed on a monocomparator and those that appear in the observation equations lends itself to the use of differential coefficients to derive the relationship between the measurements and pseudo-measurements. However, the emphasis here is on stereocomparators as all the observations carried out in relation to the research were taken on a stereocomparator. As the observed quantities are not those in the observation equations, the relationship for stereocomparators is better derived from an analysis of the physical situation.

4.2.2.4 Weight Coefficients of the Pseudo-measurements

The main assumption which can be made to simplify the physical relationship between the photographs and the object is that the normal case of photogrammetry is present. The camera axes are assumed to be perpendicular to the base and perpendicular to the direction of the object Z-axis. In the majority of cases this assumption will be valid, as single stereopairs are rarely taken on any other configuration.

The only likely exception is the use of convergent photography to improve the precision of the photography in the object space (Abdel - Aziz 1974). However the angle of convergence cannot be large if the photographs are to be observed stereoscopically. Stereofusion of conjugate images and interpretation of the object become increasingly difficult for high convergence angles. The operator's precision of measurement may be affected, nullifying the advantage of the stronger geometry.

The only other exception which is at all likely is the possibility of large tilt angles. However these cases are not common and the magnitude of the tilts must be restricted as again the operator will find stereo observation of such photographs difficult. If very large angles of convergence or tilt are used then it is probable that monocular viewing will be necessary, so that the stochastic model derived here will not be applicable in any case.

For simplicity the normal case of photography is shown in two planes in Figures 4.3 and 4.4. The X - Y plane is shown in Figure 4.3, and the following relationships can be derived using simple geometry:

$$b = \sqrt{(b_x^2 + b_y^2)} \quad , \quad d = \sqrt{(d_x^2 + d_y^2)}$$

$$d_x = m_x - a_x - \tfrac{1}{2}b_x = \tfrac{1}{2}\,\lambda\,\ell_x + \tfrac{1}{2}\,\rho\,r_x$$

$$d_y = m_y - a_y - \tfrac{1}{2}b_y = \tfrac{1}{2}\,\lambda\,\ell_y + \tfrac{1}{2}\,\rho\,r_y$$

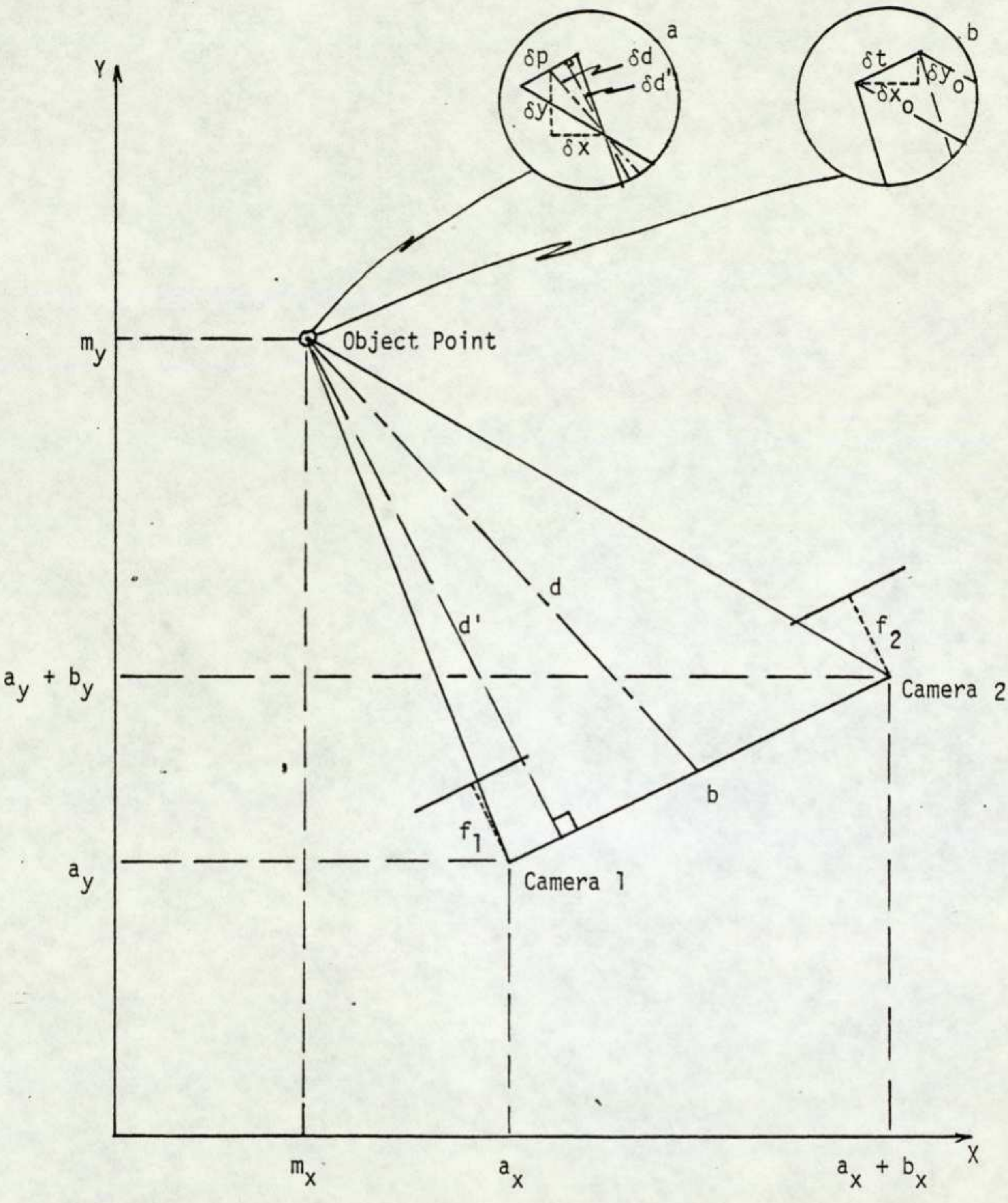


Figure 4.3 Error Propagation in the X - Y Plane
Inset a : Effect of an error in x-parallax
b : Effect of an error in the mean position in the photographs

The error in the object space caused by an error in the mean position on the photographs, δx_p, is first broken up into the effect from each photograph:

$$\delta x_\ell = \delta x_p \; \lambda \; \surd(\ell_x^2 + \ell_z^2) \; / \; \surd(x_1^2 + f_1^2)$$

$$\delta x_r = \delta x_p \; \rho \; \surd(r_x^2 + r_z^2) \; / \; \surd(x_2^2 + f_2^2)$$

and a mean result is taken:

$$\delta t = \tfrac{1}{2} \; (\; \delta x_\ell + \delta x_r \;) = \; s_1 \; \delta x_p$$

where $$s_1 = \tfrac{1}{2} \left\{ \lambda \frac{\surd(\ell_x^2 + \ell_z^2)}{\surd(x_1^2 + f_1^2)} + \rho \frac{\surd(r_x^2 + r_z^2)}{\surd(x_2^2 + f_2^2)} \right\}$$

This can be broken up into coordinate components using similar triangles:

$$\delta x_o = s_1 \; (b_x/b) \; \delta x_p \quad , \quad \delta y_o = s_1 \; (b_y/b) \; \delta x_p$$

The error in the object space caused by an error in x-parallax, δp_x, is given by similar triangles:

$$\delta p = s_1 \; \delta p_x \quad , \quad \delta d/d = \delta p/b$$

$$\therefore \; \delta d = (d/b) \; s_1 \; \delta p_x \qquad \ldots \; \ldots \; \ldots \; \ldots \qquad (4.40)$$

The components are again given by similar triangles:

$$\delta x = (d_x/b) \; s_1 \; \delta p_x \quad , \quad \delta y = (d_y/b) \; s_1 \; \delta p_x$$

Therefore the total errors in the X - Y plane are given by:

$$\delta m_x = \delta x_o + \delta x = (s_1/b) \; (\; b_x \; \delta x_p + d_x \; \delta p_x \;)$$

$$\delta m_y = \delta y_o + \delta y = (s_1/b) \; (\; b_y \; \delta x_p + d_y \; \delta p_x \;) \qquad \ldots \; \ldots \qquad (4.41)$$

Figure 4.4 shows the D'- Z plane, where D' is the direction perpendicular to the base between the cameras. From the diagrams the following relationships can be derived:

$$d_z = m_z - a_z - \tfrac{1}{2} \; b_z = \tfrac{1}{2} \; \lambda \; \ell_z + \tfrac{1}{2} \; \rho \; r_z$$

$$\delta d'/d' = \delta d/d$$

$$\therefore \; \delta d'/d' = \delta p/b \qquad \ldots \; \ldots \; \ldots \; \ldots \qquad (4.42)$$

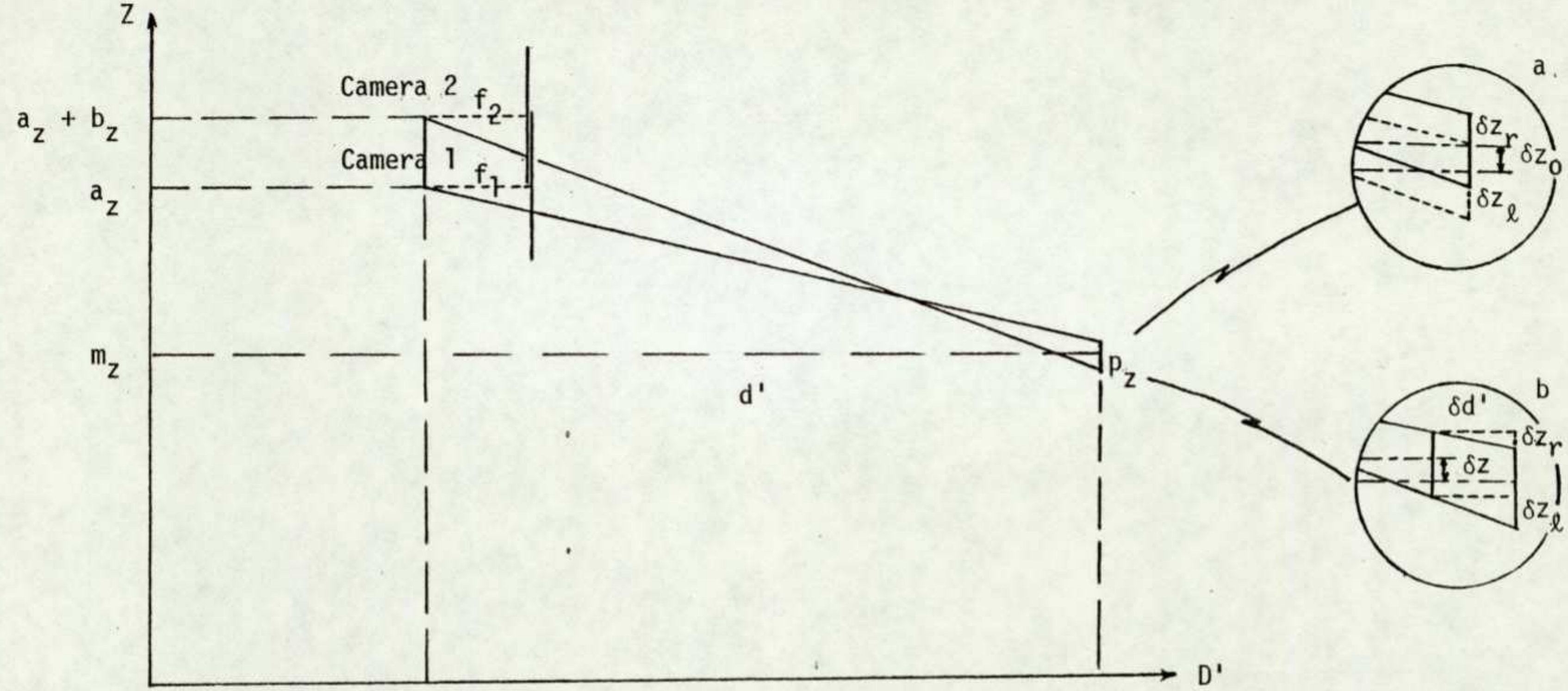


Figure 4.4 Error Propagation in the D' - Z Plane

Inset a : Effect of an error in the mean position in the photographs

b : Effect of an error in x-parallax

The effect of an error in the mean plate position, δy_p, is again broken up into the effect from each photograph:

$$\delta z_\ell = \delta y_p \surd(\lambda^2 \ell_y^2 + d'^2) \;/\; \surd(y_1^2 + f_1^2) = \delta y_p \; s_\ell$$

$$\delta z_r = \delta y_p \surd(\rho^2 r_y^2 + d'^2) \;/\; \surd(y_2^2 + f_2^2) = \delta y_p \; s_r$$

and a mean result is taken:

$$\delta z_o = \tfrac{1}{2} (\delta z_\ell + \delta z_r) = s_2 \; \delta y_p$$

where $s_2 = \tfrac{1}{2} (s_\ell + s_r)$

As b_z is likely to be small for single terrestrial stereopairs and p_z will be small, then $s_\ell \simeq s_r$ and the effect on the Z-parallax of an error in the mean photograph position can be ignored.

The effect of an error in the y-parallax on the photographs on the Z-parallax in the terrain is given by:

$$\delta p_o = s_2 \; \delta p_y$$

An error in the x-parallax in the photographs will affect both the Z-position and Z-parallax in the object space. Referring to Figure 4.4b, the effects are again broken up into the effects on the two photographs:

$$\frac{\delta z_\ell}{\delta d'} = \frac{m_z - a_z}{d'}$$

$$\frac{\delta z_r}{\delta d'} = \frac{m_z - a_z - b_z}{d'}$$

Using equations (4.40), (4.42) and meaning the two effects:

$$\delta z = s_1 \; (d_z/b) \; \delta p_x$$

Subtraction of the two effects will give the error produced in the Z-parallax:

$$\delta p_z = s_1 \; (b_z/b) \; \delta p_x$$

Although b_z is likely to be small, this term may have a significant effect and is included for the sake of completeness. The total errors in the D'- Z plane are:

$$\delta m_z = \delta z_o + \delta z = s_2\ \delta y_p + s_1\ (d_z/b)\ \delta p_x$$

$$\delta p_z = \delta p_o + \delta p_z = s_2\ \delta p_y + s_1\ (b_z/b)\ \delta p_x \qquad \ldots\ \ldots\ \ldots\ (4.43)$$

The errors can be expressed in matrix form:

$$\begin{bmatrix} \delta m_x \\ \delta m_y \\ \delta m_z \\ \delta p_z \end{bmatrix} = s_1 \begin{bmatrix} b_x/b & 0 & d_x/b & 0 \\ b_y/b & 0 & d_y/d & 0 \\ 0 & s_2/s_1 & d_z/b & 0 \\ 0 & 0 & b_z/b & s_2/s_1 \end{bmatrix} \begin{bmatrix} \delta x_p \\ \delta y_p \\ \delta p_x \\ \delta p_y \end{bmatrix} \qquad \ldots\ \ldots\ (4.44)$$

These equations are in the form $\underset{\sim}{s} = \underset{\sim}{E}\ \underset{\sim}{m}$, where $\underset{\sim}{s}$ is the pseudo-measurement vector. If the matrix of weight coefficients associated with the measurements, $\underset{\sim}{m}$, is defined as:

$$\underset{\sim}{Q}_m = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_{px}^2 & 0 \\ 0 & 0 & 0 & \sigma_{py}^2 \end{bmatrix} \qquad \ldots\ \ldots\ \ldots\ (4.44a)$$

Then using expression (4.37):

$$\underset{\sim}{Q}_s = \frac{s_1^2}{b^2} \begin{bmatrix} b_x^2\sigma_x^2 + d_x^2\sigma_{px}^2 & b_x b_y \sigma_x^2 + d_x d_y \sigma_{px}^2 & d_x d_z \sigma_{px}^2 & d_x b_z \sigma_{px}^2 \\ & b_y^2\sigma_x^2 + d_y^2\sigma_{px}^2 & d_y d_z \sigma_{px}^2 & d_y b_z \sigma_{px}^2 \\ & & b^2\frac{s_2^2}{s_1^2}\sigma_y^2 + d_z^2\sigma_{px}^2 & d_z b_z \sigma_{px}^2 \\ \text{symmetric}^{\dagger} & & & b^2\frac{s_2^2}{s_1^2}\sigma_{py}^2 + b_z^2\sigma_{px}^2 \end{bmatrix}$$

$$\ldots\ \ldots\ \ldots\ (4.45)$$

† Weight coefficient and normals coefficient matrices are symmetrical about the leading diagonal, so only the upper triangular halves need to be shown.

This is the weight coefficient matrix for a single observed object point which can be used in the least squares adjustment solution. The matrix is surprisingly uncomplicated compared with that which would be obtained using differential coefficients. Although this weight coefficient matrix makes use of a number of assumptions, it is well known from experience with least squares adjustments that the stochastic model has only a minor influence on the solution. Hence a compromise must be reached between the complexity of the matrix, especially in the light of programming on a minicomputer or microprocessor, and its practical effect. This matrix is perhaps the optimum choice because it will suffice in the majority of cases in close-range photogrammetry, yet it is not overcomplicated.

4.2.3 Aerial Stereopairs

4.2.3.1 Functional Model

The vector equations for the functional model for aerial stereopairs can be derived from Figure 4.5. There are a number of differences from the model for a terrestrial stereopair (Figure 4.2) that are apparent. The first is the different attitude of the cameras with respect to the object coordinate system. The optical axes are approximately parallel to the Z-axis and the transformation matrix $\underset{\sim}{T}$ is no longer necessary.

The second and major difference for the aerial case is that a common kappa rotation of the control must be used as one of the exterior orientation elements instead of the Y-component of the base. The reason for this change concerns the single component of the parallax. The parallax must be confined to the X - Y plane, and unless some action is taken to confine the parallax still further to one of these two coordinate directions, two components will be required. The action of the common kappa rotation is to keep the base direction and the X-axis of the object coordinate system aligned, so that only a single component of the parallax is required (in the Y-axis direction).

If the base Y-component is used and two parallax components are required, the number of unknown parameters cannot be contained to just the twelve exterior orientation elements. Only one scale factor can be eliminated from the vector equations and the other must be carried, adding one extra unknown for every point observed.

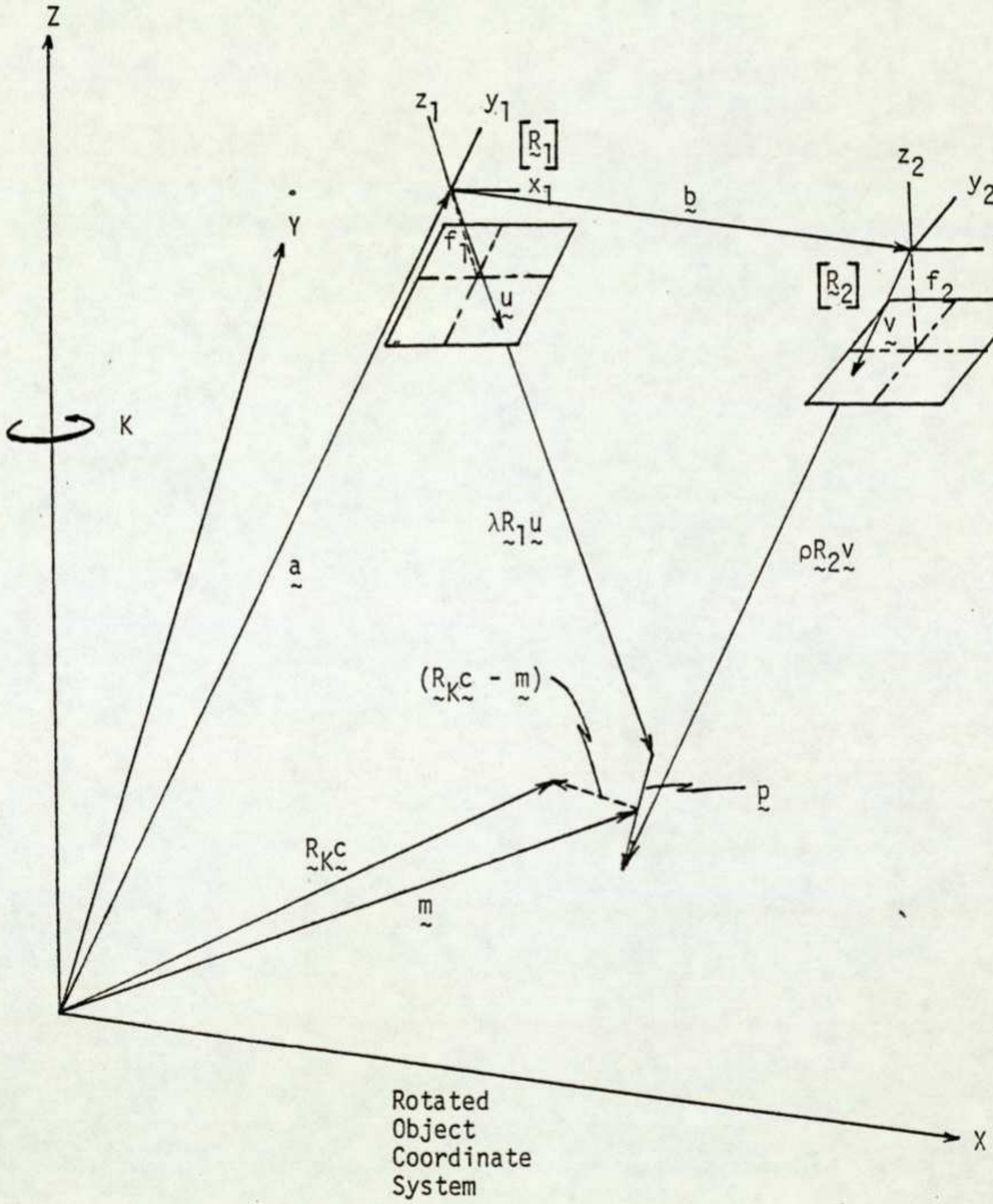


Figure 4.5 Exterior Orientation of Aerial Stereopairs

The disadvantage of the common rotation is that it has the effect of introducing a new object coordinate system related to the original control system through the alignment of the camera base. Therefore a transformation must be applied to all input and output coordinates which would have been better avoided, both because of the extra calculations and the load placed on the online processing. However, the rotation is regarded as an unknown and is determined simultaneously with the other unknown parameters so that the transformation is an integral part of the restitution. In any case this method is preferable to having a variable and increasing number of unknowns.

The only other possibilities are to rotate the camera system onto the control in the solution, or always tailor the control system so that it is approximately aligned with the camera base. The former alternative can be ruled out immediately as any attempt to rotate the camera system, or indeed to functionally constrain the parallax, leads to intolerably complicated equations. The second alternative is more attractive but unfortunately becomes impractical in the long run. The situation will always arise where it is not possible or not desirable to obtain the physical alignment. In the worst case, where the camera base is aligned with the Y-axis, the solution will fail because it will attempt to zero y-parallaxes and minimize x-parallaxes (referring to fiducial coordinates).

Because of the changes in the parallax component and the exterior orientation parameters, some new definitions are required:

$$\underset{\sim}{b} = \begin{bmatrix} b_x \\ 0 \\ b_z \end{bmatrix} \qquad \underset{\sim}{p} = \begin{bmatrix} 0 \\ p_y \\ 0 \end{bmatrix} \qquad \underset{\sim}{R}_K = \begin{bmatrix} \cos K & -\sin K & 0 \\ \sin K & \cos K & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \ldots \quad (4.46)$$

where K is the common kappa rotation.

Another change to the functional model which is not apparent from Figure 4.4 is a change in the rotation hierarchy for the rotation matrices $\underset{\sim}{R}_1$ and $\underset{\sim}{R}_2$. Again this is to make the analytical restitution compatible with analogue restitutions on stereoplotters for the utilization of orientation parameters. The hierarchy is altered to (ignoring the subscripts distinguishing the two cameras):

$$\underset{\sim}{R} = \underset{\sim}{R}_\kappa \, \underset{\sim}{R}_\phi \, \underset{\sim}{R}_\omega \quad \ldots \ldots \ldots \ldots \ldots \quad (4.47)$$

as this is the physical hierarchy of many stereoplotters. Using the matrix expressions (4.19) the elements of the rotation matrix $\underset{\sim}{R}$ can be shown to be:

$$
\begin{aligned}
r_{11} &= \cos\kappa \cos\phi \\
r_{12} &= \cos\kappa \sin\phi \sin\omega - \sin\kappa \cos\omega \\
r_{13} &= \cos\kappa \sin\phi \cos\omega + \sin\kappa \sin\omega \\
r_{21} &= \sin\kappa \cos\phi \\
r_{22} &= \sin\kappa \sin\phi \sin\omega + \cos\kappa \cos\omega \\
r_{23} &= \sin\kappa \sin\phi \cos\omega - \cos\kappa \sin\omega \\
r_{31} &= -\sin\phi \\
r_{32} &= \cos\phi \sin\omega \\
r_{33} &= \cos\phi \cos\omega \qquad \ldots \; \ldots \; \ldots \; \ldots \qquad (4.48)
\end{aligned}
$$

where again the subscripts indicating the left and right fiducial systems are ignored for convenience.

The derivation of the equations for aerial stereopairs resembles closely that for terrestrial stereopairs and a similar set of equations has been developed for the orientation of analogue instruments (Shortis 1977). Therefore the description will be kept relatively brief. The independent vector equations which can be derived from Figure 4.4 are:

$$
\begin{aligned}
\underset{\sim}{a} + \tfrac{1}{2}\, \underset{\sim}{b} + \tfrac{1}{2}\, \lambda\, \underset{\sim}{R}_1\, \underset{\sim}{u} + \tfrac{1}{2}\, \rho\, \underset{\sim}{R}_2\, \underset{\sim}{v} &= \underset{\sim}{m} \\
\underset{\sim}{b} - \lambda\, \underset{\sim}{R}_1\, \underset{\sim}{u} - \rho\, \underset{\sim}{R}_2\, \underset{\sim}{v} &= -\, \underset{\sim}{p} \qquad \ldots \; \ldots \qquad (4.49)
\end{aligned}
$$

Using the zero components of the parallax vector the scale factors can be eliminated:

$$
\begin{aligned}
\lambda &= (b_x\, r_z - b_z\, r_x) / k \\
\rho &= (b_x\, \ell_z - b_z\, \ell_x) / k \qquad \ldots \; \ldots \; \ldots \qquad (4.50)
\end{aligned}
$$

where k has the same value as before.

Substitution into the remaining scalar equations of expression (4.49) gives the four equations in eleven of the twelve orientation parameters and the pseudo-measurements:

$$
\begin{aligned}
&ka_x + b_x\ell_x r_z - b_z\ell_x r_x = km_x \\
&ka_y + \tfrac{1}{2}b_x(\ell_z r_y + \ell_y r_z) - \tfrac{1}{2}b_z(\ell_y r_x + \ell_x r_y) = km_y \\
&ka_z + b_x\ell_z r_z - b_z\ell_z r_x = km_z \\
&\qquad b_x(\ell_z r_y - \ell_y r_z) + b_z(\ell_y r_x - \ell_x r_y) = -kp_y \quad \ldots \qquad (4.51)
\end{aligned}
$$

4.2.3.2 Differential Equations

To derive the differential equations for aerial stereopairs, the vector equations must be first related to the rotated control and the exterior orientation parameters changed in status from assumed or approximate values to most probably true values:

$$
\begin{aligned}
&\dot{k}\dot{a}_x + \dot{b}_x\dot{\ell}_x\dot{r}_z - \dot{b}_z\dot{\ell}_x\dot{r}_x - \dot{k}(c_x\cos\dot{K} - c_y\sin\dot{K}) = 0 \\
&\dot{k}\dot{a}_y + \tfrac{1}{2}\dot{b}_x(\dot{\ell}_z\dot{r}_y + \dot{\ell}_y\dot{r}_z) - \tfrac{1}{2}\dot{b}_z(\dot{\ell}_y\dot{r}_x + \dot{\ell}_x\dot{r}_y) - \dot{k}(c_x\sin\dot{K} + c_y\cos\dot{K}) = 0 \\
&\dot{k}\dot{a}_z + \dot{b}_x\dot{\ell}_z\dot{r}_z - \dot{b}_z\dot{\ell}_z\dot{r}_x - \dot{k}c_z = 0 \\
&\qquad \dot{b}_x(\dot{\ell}_z\dot{r}_y - \dot{\ell}_y\dot{r}_z) + \dot{b}_z(\dot{\ell}_y\dot{r}_x - \dot{\ell}_x\dot{r}_y) = 0 \quad \ldots\ \ldots \qquad (4.52)
\end{aligned}
$$

These equations can be expressed in the functional forms:

$$
\begin{aligned}
&g_5(\dot{a}_x, \dot{b}_x, \dot{b}_z, \dot{\kappa}_1, \dot{\phi}_1, \dot{\omega}_1, \dot{\kappa}_2, \dot{\phi}_2, \dot{\omega}_2, \dot{K}, c_x, c_y) = 0 \\
&g_6(\dot{a}_y, \dot{b}_x, \dot{b}_z, \dot{\kappa}_1, \dot{\phi}_1, \dot{\omega}_1, \dot{\kappa}_2, \dot{\phi}_2, \dot{\omega}_2, \dot{K}, c_x, c_y) = 0 \\
&g_7(\dot{a}_z, \dot{b}_x, \dot{b}_z, \dot{\kappa}_1, \dot{\phi}_1, \dot{\omega}_1, \dot{\kappa}_2, \dot{\phi}_2, \dot{\omega}_2, c_z) = 0 \\
&g_8(\dot{b}_x, \dot{b}_z, \dot{\kappa}_1, \dot{\phi}_1, \dot{\omega}_1, \dot{\kappa}_2, \dot{\phi}_2, \dot{\omega}_2) = 0 \quad \ldots\ \ldots \qquad (4.53)
\end{aligned}
$$

where the dot superscript indicates most probably true value.

The most probably true values can be related to the assumed values used in equations (4.51) and individual corrections:

$$
\begin{aligned}
&\dot{a}_x = a_x + \Delta a_x \\
&\dot{a}_y = a_y + \Delta a_y \\
&\qquad \vdots \\
&\dot{K} = K + \Delta K \quad \ldots\ \ldots\ \ldots\ \ldots \qquad (4.54)
\end{aligned}
$$

and substituted into the functions (4.53):

$$
\begin{aligned}
&g_5(a_x + \Delta a_x, b_x + \Delta b_x, \ \ldots\ , \omega_2 + \Delta\omega_2, K + \Delta K, c_x, c_y) = 0 \\
&g_6(a_y + \Delta a_y, b_x + \Delta b_x, \ \ldots\ , \omega_2 + \Delta\omega_2, K + \Delta K, c_x, c_y) = 0
\end{aligned}
$$

$$g_7(a_z + \Delta a_z, b_x + \Delta b_x, \; \ldots \;, \omega_2 + \Delta\omega_2, c_z) = 0$$

$$g_8(b_x + \Delta b_x, b_z + \Delta b_z, \; \ldots \;, \omega_2 + \Delta\omega_2) = 0 \qquad \ldots \quad \ldots \qquad (4.55)$$

The equations can now be linearized by a Taylor's series expansion, to first order terms only, into a form suitable for a least squares adjustment by observation equations:

$$g_5(a_x, b_x, \ldots, \omega_2, K, c_x, c_y) + \frac{\delta g_5}{\delta a_x}\Delta a_x + \ldots + \frac{\delta g_5}{\delta K}\Delta K = 0$$

$$g_6(a_y, b_x, \ldots, \omega_2, K, c_x, c_y) + \frac{\delta g_6}{\delta a_y}\Delta a_y + \ldots + \frac{\delta g_6}{\delta K}\Delta K = 0$$

$$g_7(a_z, b_x, \ldots, \omega_2, c_z) + \frac{\delta g_7}{\delta a_z}\Delta a_z + \ldots + \frac{\delta g_7}{\delta \omega_2}\Delta\omega_2 = 0$$

$$g_8(b_x, b_z, \ldots, \omega_2) + \frac{\delta g_8}{\delta b_x}\Delta b_x + \ldots + \frac{\delta g_8}{\delta \omega_2}\Delta\omega_2 = 0 \qquad \ldots \quad \ldots \qquad (4.56)$$

Using equations (4.52) with approximate values to resolve the above functional forms, and equations (4.51) to simplify them, the final form of these equations can be arrived at:

$$\frac{\delta g_5}{\delta a_x}\Delta a_x + \ldots + \frac{\delta g_5}{\delta K}\Delta K = k(c_x \cos K - c_y \sin K - m_x)$$

$$\frac{\delta g_6}{\delta a_y}\Delta a_y + \ldots + \frac{\delta g_6}{\delta K}\Delta K = k(c_x \sin K + c_y \cos K - m_y)$$

$$\frac{\delta g_7}{\delta a_z}\Delta a_z + \ldots + \frac{\delta g_7}{\delta \omega_2}\Delta\omega_2 = k(c_z - m_z)$$

$$\frac{\delta g_8}{\delta b_x}\Delta b_x + \ldots + \frac{\delta g_8}{\delta \omega_2}\Delta\omega_2 = kp_z \qquad \ldots \quad \ldots \quad \ldots \qquad (4.57)$$

where the partial differentials are given in Appendix B.

Again three fully controlled points or the equivalent partially controlled points are necessary and sufficient for a solution. Partially known points can be included in the solution by using the appropriate equations from the full set given in (4.57). However only position, height or parallax points can be used, other than points with all three coordinates known. Because the first two equations involve both the X and Y object coordinates they cannot be segregated and used individually as they could be for terrestrial stereopairs. This is not likely to be a disadvantage, as it is very rare to have points with only one of the X or Y position components known.

4.2.3.3 Stochastic Model

The stochastic model for aerial stereopairs can be established in a fashion similar to that for the terrestrial stereopairs. Once more it is assumed that the observation of conjugate image points is carried out on a stereocomparator while viewing a stereo image of the object space. Again a relationship is required between the measurements (x_p, y_p, p_x, p_p)[†] and the pseudo-measurements (m_x, m_y, m_z, p_y) so that equation (4.37) can be implemented to obtain the weight coefficient matrix of the pseudo-measurements.

The normal case of photogrammetry is assumed to simplify the relationship between the photographs and the object space, and is depicted in two separate planes in Figures 4.6 and 4.7. The relationship is further simplified by the fact that the camera base and the X terrain axis are parallel. The effects of errors in the mean x photograph position and the x-parallax in the X - Z plane are shown in Figure 4.6, and the total errors can be shown to be:

$$\delta m_x = s_1 \delta x_p + s_1 (d_x/b_x) \delta p_x$$

$$\delta m_z = s_1 (d_z/b_x) \delta p_x$$

where $$d_x = m_x - a_x - \tfrac{1}{2} b_x = \tfrac{1}{2}(\lambda \ell_x + \rho r_x)$$

$$d_z = m_z - a_z - \tfrac{1}{2} b_z = \tfrac{1}{2}(\lambda \ell_z + \rho r_z)$$

and s_1 has the same value as before.

Similarly the effects of errors in the mean y photograph positon, the x- and y-parallaxes in the Y - Z plane can be derived from Figure 4.7:

$$\delta m_y = s_2 \delta y_p + s_1 (d_y/b_x) \delta p_x$$

$$\delta p_y = s_2 \delta p_p - s_1 (b_z d_y / b_x d_z) \delta p_x$$

where $$d_y = m_y - a_y = \tfrac{1}{2}(\lambda \ell_y + \rho r_y)$$

$$s_2 = \tfrac{1}{2}\{\lambda \sqrt{(\ell_y^2 + \ell_z^2)} / \sqrt{(y_1^2 + f_1^2)} + \rho \sqrt{(r_y^2 + r_z^2)} / \sqrt{(y_2^2 + f_2^2)}\}$$

The errors can be expressed in the matrix form $\underset{\sim}{s} = \underset{\sim}{E}\,\underset{\sim}{m}$:

[†]y-parallax on the photographs is referred to as p_p for aerial stereopairs to avoid confusion with y-parallax in the object space.

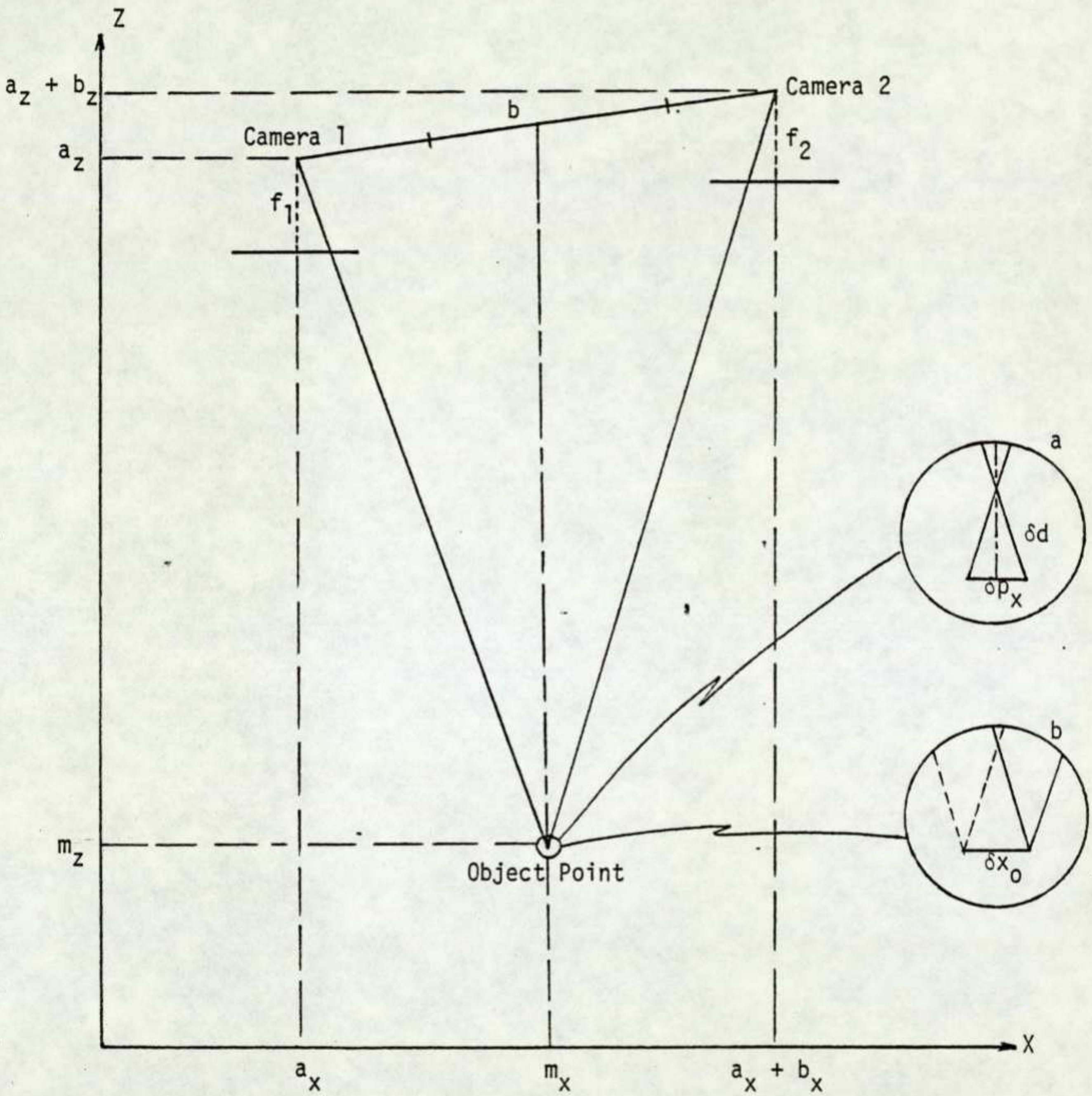


Figure 4.6 Error Propagation in the X - Z Plane
Inset a : Effect of an error in x-parallax
b : Effect of an error in the mean position in the photographs

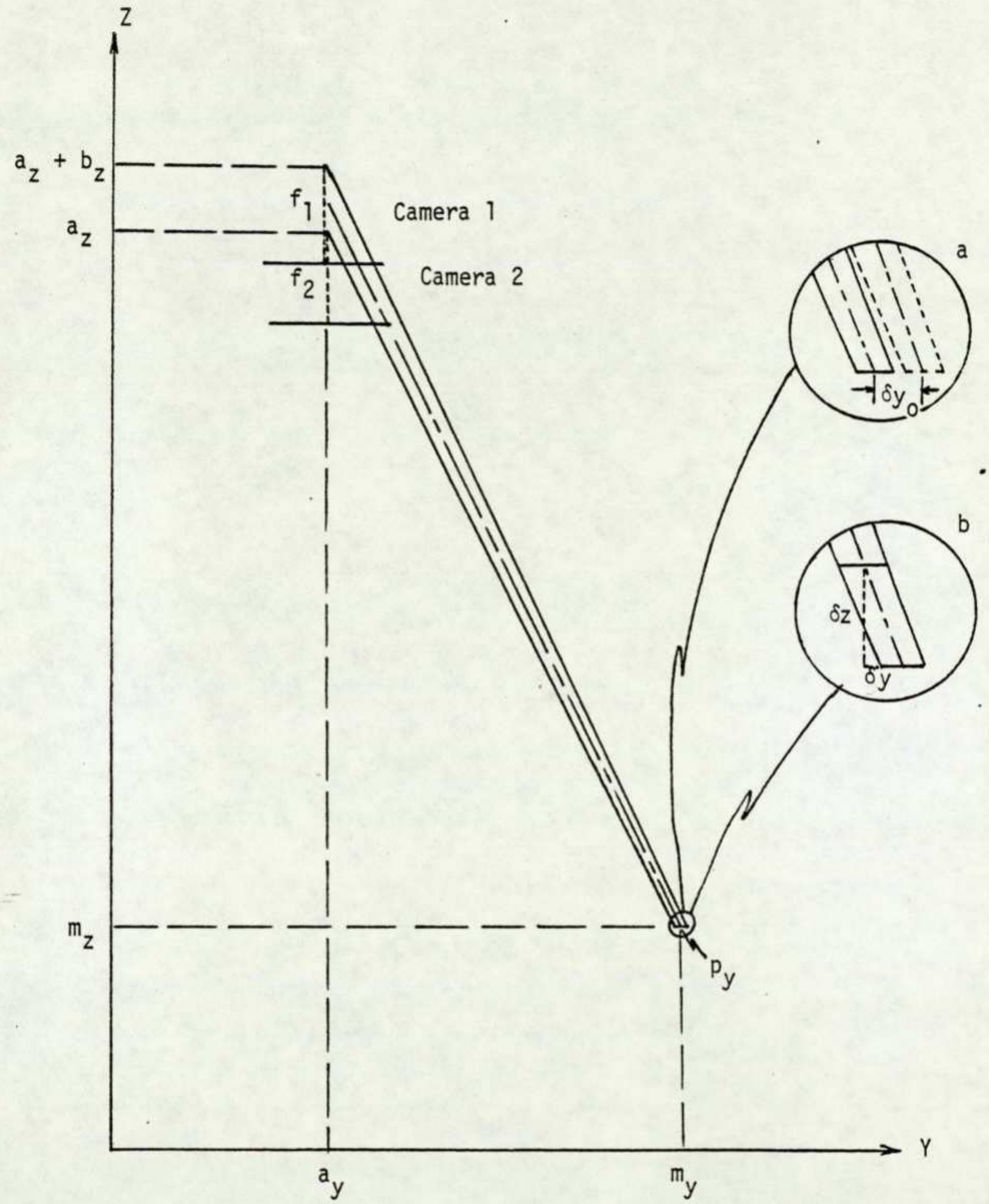


Figure 4.7 Error Propagation in the Y - Z Plane

Inset a : Effect of an error in the mean position in the photographs

b : Effect of an error in x-parallax

$$\begin{bmatrix} \delta m_x \\ \delta m_y \\ \delta m_z \\ \delta p_y \end{bmatrix} = s_1 \begin{bmatrix} 1 & 0 & d_x/b_x & 0 \\ 0 & s_2/s_1 & d_y/b_x & 0 \\ 0 & 0 & d_z/b_x & 0 \\ 0 & 0 & -b_z d_y/b_x d_z & s_2/s_1 \end{bmatrix} \begin{bmatrix} \delta x_p \\ \delta y_p \\ \delta p_x \\ \delta p_p \end{bmatrix}$$

Using expression (4.37) and the same weight coefficient matrix of the measurements given by (4.44a) the weight coefficient matrix of the pseudo-measurements is therefore:

$$\underset{\sim}{Q}_s = \frac{s_1^2}{b_x^2} \begin{bmatrix} b_x^2\sigma_x^2 + d_x^2\sigma_{px}^2 & d_x d_y \sigma_{px}^2 & d_x d_z \sigma_{px}^2 & -(d_x d_y b_z/d_z)\sigma_{px}^2 \\ & b_x^2(s_2^2/s_1^2)\sigma_y^2 + d_y^2\sigma_{px}^2 & d_y d_z \sigma_{px}^2 & -(d_y^2 b_z/d_z)\sigma_{px}^2 \\ & & d_z^2\sigma_{px}^2 & -d_y b_z \sigma_{px}^2 \\ & \text{symmetric} & & b_x^2 \frac{s_2^2}{s_1^2}\sigma_{py}^2 + \frac{d_y^2 b_z^2}{d_z^2}\sigma_{px}^2 \end{bmatrix} \quad (4.59)$$

This weight coefficient matrix is similar in some respects to that derived for terrestrial stereopairs and still remarkably uncomplicated. However it is not exactly the same and could not be made equivalent by a simple interchange of object coordinate axes. Perhaps now it can be seen clearly why a transformation cannot be used to apply a single solution to both terrestrial and aerial stereopairs - both the functional and stochastic models are too different to enable a simple transformation to be a workable proposition.

4.2.4 Least Squares Adjustment

The observation equations for the least squares adjustments of terrestrial and aerial stereopairs can be represented as partitioned matrices, where each set of submatrices refers to a single point. The number of rows in each submatrix depends on the number of equations it contributes to the solution:

$$\underset{m*12}{\begin{bmatrix} \underset{\sim}{A}_1 \\ \underset{\sim}{A}_2 \\ \vdots \\ \underset{\sim}{A}_n \end{bmatrix}} \underset{12*1}{\underset{\sim}{x}} = \underset{m*1}{\begin{bmatrix} \underset{\sim}{s}_1 \\ \underset{\sim}{s}_2 \\ \vdots \\ \underset{\sim}{s}_n \end{bmatrix}} + \underset{m*1}{\underset{\sim}{v}} : \underset{m*m}{\begin{bmatrix} \underset{\sim}{Q}_1 & \underset{\sim}{0} & & \\ \underset{\sim}{0} & \underset{\sim}{Q}_2 & & \\ & & \ddots & \\ & & & \underset{\sim}{Q}_n \end{bmatrix}} \quad \dots \quad \dots \quad \dots \quad (4.60)$$

where n is the number of points observed, $n \geqslant 3$

m is the total number of observation equations, $m \geqslant 12$

$\underset{\sim}{x}$ is the 12 x 1 vector matrix of corrections to the exterior orientation parameters

For terrestrial stereopairs:

$$\underset{\sim}{x}^T = \left[\Delta a_x \; \Delta a_y \; \Delta a_z \; \Delta b_x \; \Delta b_y \; \Delta b_z \; \Delta\phi_1 \; \Delta\kappa_1 \; \Delta\omega_1 \; \Delta\phi_2 \; \Delta\kappa_2 \; \Delta\omega_2 \right]$$

For aerial stereopairs:

$$\underset{\sim}{x}^T = \left[\Delta a_x \; \Delta a_y \; \Delta a_z \; \Delta b_x \; \Delta b_z \; \Delta\kappa_1 \; \Delta\phi_1 \; \Delta\omega_1 \; \Delta\kappa_2 \; \Delta\phi_2 \; \Delta\omega_2 \; \Delta K \right]$$

$\underset{\sim}{v}$ is the m x 1 vector of residuals of measurement

$\underset{\sim}{A}_i$ is the m_i x 12 differential coefficient matrix for the i^{th} point

$\underset{\sim}{s}_i$ is the m_i x 1 vector matrix of pseudo-measurements for the i^{th} point

m_i is the number of equations contributed by the i^{th} point (therefore $m = \sum_{i=1}^{n} m_i$)

$\underset{\sim}{Q}_i$ is the m_i x m_i weight coefficient matrix associated with the i^{th} point

As individual observations are considered to be independent, the weight coefficient matrix for the adjustment is block diagonal with null matrices in all other positions. Advantage can be taken of this sparsely filled matrix to compute directly the normal equations. The benefits of this procedure have already been discussed with respect the adjustment of the interior orientation, and the arguments are reinforced because the matrices involved here have larger dimensions. Computing the normal equations in the form of (4.13):

$$\underset{12*12}{\left[\sum_{i=1}^{n} \underset{\sim}{A}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{A}_i \right]} \underset{12*1}{\underset{\sim}{x}} = \underset{12*1}{\left[\sum_{i=1}^{n} \underset{\sim}{A}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{s}_i \right]} \quad \ldots \quad \ldots \quad (4.61)$$

Equation (4.61) represents a type of step by step processing, not unlike a sequential adjustment, which has been used since the introduction of the first digital computers (<u>Schmid</u> 1956). The observations on each point can be added into the normals as each is taken, or subtracted from them if the point is found to be incorrect or misidentified.

Not only does this avoid the need for storage of the full matrices of (4.60) but it also allows a much smoother flow during online processing. The normal equations are ready for solution immediately after the operator has taken his last observation, so that the resultant corrections can be computed and the iterative solution begun much more rapidly. Although A_i, s_i and Q_i are never permanently stored in the processor between each iteration and must be recomputed, this is not a disadvantage. All three submatrices contain parameters which are updated by each iteration, so they would need to be re-calculated in any case.

The normals matrix is symmetric so that again only the upper triangular half needs to be stored, and the inversion is carried out by the same specialist routine described in Section 4.1.3. An iterative least squares solution must be used because of the assumption that the approximate values are corrected by only first order terms. Approximate values of the exterior orientation parameters must be available to fill the role of the initial assumed values of the parameters for the first iteration. Hence the approximate locations and orientations of the cameras must be known relative to the object coordinate system. This can be a disadvantage when a specific configuration of the cameras is not used for the photography, but the solution will accept inaccurate first assumptions with only an increase in the number of iterations (as long as the camera/object geometry is maintained by the assumptions). The iterations are continued under operator control until a specific convergence criterion is reached - usually that the corrections fall below a significant level in relation to the resolution of the object coordinate system.

5. SEQUENTIAL ADJUSTMENT OF CONTROL AND ORIENTATIONS

5.1 Control Adjustment

5.1.1 Types of Control Measurements

There are several different techniques employed to establish control for close-range stereopairs. The techniques can be distinguished by the type or types of measurement used and whether or not the resultant precision of the control can be determined directly or must be estimated indirectly. The size and shape of the object, the techniques available, the required precision of the control and the required density of control points all contribute to the selection process for an appropriate technique.

Whether or not the object is small and portable enough to be brought into the laboratory is a major consideration in the choice of techniques. If the object is large or immovable and must be photographed and controlled in situ, then geodetic surveying techniques are almost always used. Observation of angles, distances and levels are made with surveying instruments to define a relatively small number of control points. The points may be premarked on or around the surface of the object, or may be at locations defined by physical characteristics of the object.

An advantage of surveying measurements is the wide range of precisions which can be achieved. The type of instruments and the design of the survey network can be selected in advance of the actual survey in order to obtain a specific control precision. The number of redundant measurements taken for each point also plays a large part in determining the resultant precision, and these can also be planned in advance. Redundant measurements are generally included in the survey also to detect gross errors which would otherwise pass unnoticed.

The coordinates of the control points are computed from a least squares adjustment, utilizing any redundant measurements incorporated into the network. The variation of coordinates technique is the basis for the adjustment and the precisions of the coordinates can be estimated from the solution.

The photographic coverage of close-range stereopairs is always small enough in extent to enable the use of plane coordinate systems. If a large topographic or site survey did necessitate a map projection coordinate system it is more convenient to convert the coordinates to a local plane system for each stereopair. The mathematical model for a restitution with non-linear coordinate systems would be extremely complex and rarely used.

Photogrammetric and survey coordinate systems invariably should be right-handed and orthogonal. The Z-axis of a control coordinate system is defined as parallel to gravity by the use of reference level bubbles in the surveying instruments. The origin for the coordinate system may be related to the national datum, but is often arbitrarily defined relative to one of the survey stations if the photogrammetric survey is isolated or self-contained. The orientation of the X - Y plane can be defined relative to true or grid north, or again arbitrarily set if the coordinate system needs no connection to other surveys.

If the subject of the photography can be brought into the laboratory then a wider choice of techniques can be contemplated. The selection of a technique depends in this case not only on those factors already mentioned, but also on whether the control is to be marked on the surface of the object or not. Attaching targets to a small object may obscure too much of the surface and physical marking of the object may be prohibited for various reasons. Photogrammetry is often employed because the method of measurement must be strictly non-contact. Therefore the control must be provided around the object by a suitable frame or background.

The first method for establishing the control is the so-called "micro-geodetic" technique. Observations of angles, distances and levels are employed in a method analogous to geodetic surveying. Theodolites and levels are still the basic instruments used, but distances are measured by a variety of methods ranging from laser interferometry to steel straight-edges. As the size of the object decreases then the positioning and scale variations of distance measuring devices become critical. If the true scale of the object is relatively unimportant (as it is when deformations or departures from design dimensions are required) then a single distance is often measured with suitably high precision and then fixed for further computations. Angular measurements are then used exclusively to establish the positions of the control points in a manner similar to classical triangulation surveys.

This method is equally applicable to control marked on or placed around the object. The equipment required for the observations is readily available and often already in the possession of the surveyor or photogrammetrists. A suitable site can be prepared by making provision for stable theodolite positions in appropriate locations relative to the control network or area set aside for the object. The precision of the control can again be preselected depending on the type of equipment and the number of observations.

The second method is by a three dimensional measuring machine. Such machines vary in sophistication from manual operation and vernier scales to powered operation and digital displays. Positions are measured on the surface of the object by a mechanical probe which is tracked in three orthogonal directions. The origin of the coordinate system is arbitrary but one of the axes is generally related to the direction of gravity by levelling bubbles.

The application of these machines is restricted by the length of travel in the coordinate directions and by their availability. In general the object cannot be larger than 1 - 2 metres in any dimension and the interior of objects can be inaccessible to them. The machines are expensive to hire or buy, are not common and are usually working continuously. The resolution of the scales varies between approximately millimetres and hundredths of millimetres, but the precision is limited by the positioning of the probe by a human operator (although contact with the surface can be semi-automatic). The precision can be improved by repeated measurements and powered machines with automatic recording can produce a large number of control points quickly and efficiently.

The third method is the grid/height bar combination. A grid is drawn or scribed onto a suitable flat surface and the object is placed on the grid for the photography. If the object has a significant depth then length bars can be positioned around the object to control the third dimension (see Figure 8.2). The method is particularly applicable to small objects which cannot have the control marked on the surface, but has the disadvantage that the relationship between the object and the control is destroyed as soon as the object is moved. However if the grid is stable with respect to time then it can be used repeatedly for different objects.

The density of the control can be regulated by the spacing of the grid lines and the number of height bars. The precision of the control depends on the flatness of the surface, the perpendicularity of the bars to the surface and the precisions of the grid and bar lengths. The precision can be improved if particular grid intersections are measured using a coordinatograph and the bars are measured by some type of length gauge.

A similar concept to the grid is the control frame, a portable open structure which is placed around the object to be photographed. Grids have the drawback that they only supply positional information in one plane, whereas a control frame provides three dimensional points all around the object. Both suffer from the difficulty of finding a suitable material and structure that is light and portable yet rigid and stable. At best only a compromise between the two requirements can be reached, and the cost of the materials and construction is often a major consideration.

The frame must be initially calibrated, probably by geodetic techniques, and possibly re-calibrated for each set of photography if it does not have sufficient stability. The frame may need to be able to be dismantled and reassembled if it must be taken to the object, whereas a flat grid is more easily transported. Both are limited to a maximum dimension of 1 - 2 metres by practical considerations and possible applications. Both have the advantage that once configured, the control system is defined relative to the structure and no external reference is required.

5.1.2 Initial Adjustment of Control Measurements

Geodetic measurements can be processed by a least squares adjustment using the technique of variation of coordinates. Measurements of angles, distances and levels can be incorporated into a solution by observation equations. Each measurement contributes one equation which relates the measured quantity to first-order corrections to the coordinates of the appropriate control points (Shortis 1977).

If enough observations are present to define all the points and satisfy the geometrical requirements of the network and coordinate system then a solution is possible.

As the control coordinates are the unknown parameters of the solution, the associated weight coefficient matrix is very easily obtained from the adjustment. According to equations (3.8) and (3.9a) the weight coefficient matrix is computed as part of the solution. This will be a full matrix with non-zero off-diagonal terms describing the correlation between the coordinates. This is the matrix required for input into any subsequent sequential adjustments. It is a symmetric matrix so only one triangular half needs to be retained to define the entire matrix.

An estimate of the variance factor can also be obtained from the initial adjustment of the control measurements according to (3.12). This quantity gives an indication of the accuracy of the a priori estimates of the precisions of the control measurements, based on the residuals of the adjustment. The residuals of the adjustment can be used to detect gross and systematic errors in the initial adjustment before the control coordinates and their weight coefficient matrix are passed on to the sequential adjustment.

In contrast, if a three dimensional measuring machine or a grid/length bar combination is used then there is less scope for checking the control-coordinates and for deriving a reliable weight coefficient matrix. A least squares adjustment is generally not applicable as the values of the control coordinates are obtained independently and often from minimum measurements. Repeated measurements of grid positions, bar lengths and three dimensional measuring machine locations can be taken so that a mean and standard error are available, but this will not provide all the benefits of a least squares adjustment.

Furthermore, there are random and systematic fluctuations which cannot be accounted for by normal practice. The flatness of the grid base or the cyclic errors in 3 - D machine scales can only be checked by elaborate measurement or calibration. The manufacturers' specifications are the only guideline readily available to estimate the significance of any systematic and random errors.

Even if good approximations of the control precisions can be derived from experiment, specifications and experience the weight coefficient matrix constructed for the sequential adjustment will still be incomplete. The matrix will almost certainly be diagonal as it is debatable whether correlations between coordinates can be assessed with any confidence. Any suspected correlation, for example between the coordinates of points on the same grid line, is therefore ignored or assumed to be insignificant.

5.1.3 Weighted Control

The concept of weighted control is derived from observation equations such as (4.35) for terrestrial stereopairs. In the normal course of events the pseudo-measurements (m_x, m_y, m_z, p_z) are considered to be observations with specific standard errors, or in this case a weight coefficient matrix. The control coordinates are fixed and therefore have zero standard errors or a null weight coefficient matrix. If instead the control coordinates are considered to be measurements, the right hand sides of the observation equations take on new meanings. The combined effects of the two measurements in each equation must be taken into account using the laws of propagation of errors.

The weight coefficient matrix of the combined control and psuedo-measurement precisions can be derived using the algorithm for derived quantities (4.37). The weight coefficient matrix of the pseudo-measurements has already been derived and can be defined here with simpler notation:

$$\underset{\sim}{Q}_s = \begin{bmatrix} s_x^2 & s_{xy} & s_{xz} & s_{xp} \\ & s_y^2 & s_{yz} & s_{yp} \\ & & s_z^2 & s_{zp} \\ & & & s_p^2 \end{bmatrix} \quad \ldots \quad \ldots \quad \ldots (5.1)$$

Even if a full coefficient matrix is available for the control, at most only 3 x 3 submatrices on the diagonal are utilized and correlations between the coordinates of different points are ignored for simplicity and convenience. For a control point the weight coefficient matrix can be expressed:

$$\underset{\sim}{Q}_c = \begin{bmatrix} t_x^2 & t_{xy} & t_{xz} \\ & t_y^2 & t_{yz} \\ & & t_z^2 \end{bmatrix} \quad \ldots \; \ldots \; \ldots \; \ldots \; \ldots \quad (5.2)$$

where both matrices are symmetric. The expression (4.37) can be expanded into partitioned form:

$$\underset{\sim}{Q}_d = \begin{bmatrix} \underset{\sim}{E}_c & \underset{\sim}{E}_s \end{bmatrix} \begin{bmatrix} \underset{\sim}{Q}_c & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_s \end{bmatrix} \begin{bmatrix} \underset{\sim}{E}_c^T \\ \underset{\sim}{E}_s^T \end{bmatrix} \quad \ldots \; \ldots \; \ldots \; \ldots \quad (5.3)$$

where from equations (4.34):

$$\underset{\sim}{E}_c = k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \underset{\sim}{E}_s = k \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots \quad (5.4)$$

If expression (5.3) is multiplied out then the following weight coefficient matrix is obtained for terrestrial stereopairs:

$$\underset{\sim}{Q}_{d_t} = k^2 \begin{bmatrix} t_x^2 + s_x^2 & t_{xy} + s_{xy} & t_{xz} + s_{xz} & s_{xp} \\ & t_y^2 + s_y^2 & t_{yz} + s_{yz} & s_{yp} \\ & & t_z^2 + s_z^2 & s_{zp} \\ & & & s_p^2 \end{bmatrix} \quad (5.5)$$

The straightforward additions in this resultant matrix make it a very attractive method of weighting the control. As each control point is observed and the photogrammetric observation equations and weight coefficient matrix are formed, the appropriate control weight coefficient matrix can be searched for and added without interrupting the step by step processing. The contributions to the normal equations can be entered as described in Chapter 4.

The right hand sides of the observation equations for aerial stereopairs are more complex and the matrix $\underset{\sim}{E}_c$ must be altered accordingly:

$$\underset{\sim}{E}_c = k \begin{bmatrix} \cos K & -\sin K & 0 \\ \sin K & \cos K & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \ldots \; \ldots \; \ldots \; \ldots \quad (5.6)$$

If expression (5.3) is again multiplied out, using similar notation:

$$\underset{\sim}{Q}_{d_a} = \begin{bmatrix} r_x + s_x^2 & r_{xy} + s_{xy} & r_{xz} + s_{xz} & s_{xp} \\ & r_y + s_y^2 & r_{yz} + s_{yz} & s_{yp} \\ & & t_z^2 + s_z^2 & s_{zp} \\ & & & s_p^2 \end{bmatrix} \quad \ldots \; \ldots \quad (5.7)$$

where

$$\begin{aligned} r_x &= \cos^2 K\, t_x^2 - \sin 2K\, t_{xy} + \sin^2 K\, t_y^2 \\ r_{xy} &= \tfrac{1}{2} \sin^2 K(t_x^2 - t_y^2) + (\cos^2 K - \sin^2 K) t_{xy} \\ r_y &= \sin^2 K\, t_x^2 + \sin 2K\, t_{xy} + \cos^2 K\, t_y^2 \\ r_{xz} &= \cos K\, t_{xz} - \sin K\, t_{yz} \\ r_{yz} &= \sin K\, t_{xz} + \cos K\, t_{yz} \quad \ldots \; \ldots \; \ldots \; \ldots \quad (5.8) \end{aligned}$$

This weight coefficient matrix is more complicated and as it must be re-computed for each iteration of the solution it does detract from the simplicity of the method. If the weight coefficient matrix of the control coordinates is diagonal then the above expressions are considerably less complex.

5.1.4 Sequential Adjustment of Control

Whereas weighted control is an approximate method, a sequential adjustment of the control is a rigorous solution based on a simultaneous adjustment of the control measurements and photogrammetric observations. Such a simultaneous adjustment can be expressed in the form of the observation equations of expression (3.1), but expanded to show the submatrices:

$$\begin{bmatrix} \underset{\sim}{A}_p & \underset{\sim}{B}_p \\ \underset{\sim}{0} & \underset{\sim}{A}_c \end{bmatrix} \begin{bmatrix} \underset{\sim}{x} \\ \underset{\sim}{c} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{m}_p \\ \underset{\sim}{m}_c \end{bmatrix} + \begin{bmatrix} \underset{\sim}{v}_p \\ \underset{\sim}{v}_c \end{bmatrix} : \begin{bmatrix} \underset{\sim}{Q}_p & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_c \end{bmatrix} \quad \ldots \quad (5.9)$$

where $\underset{\sim}{A}_p$, $\underset{\sim}{x}$, $\underset{\sim}{m}_p$, $\underset{\sim}{v}_p$ and $\underset{\sim}{Q}_p$ have the same meanings as before

$\underset{\sim}{B}_p$ is the matrix of partial differentials of the photogrammetric observation equations with respect to the control coordinates

$\underset{\sim}{A}_c$ is the matrix of coefficients of the equations for the control measurements

$\underset{\sim}{c}$ is the vector matrix of the unknown control coordinates

$\underset{\sim}{m}_c$ is the vector matrix of the control measurements

$\underset{\sim}{v}_c$ is the vector matrix of residuals associated with the control measurements

$\underset{\sim}{Q}_c$ is the matrix of weight coefficients associated with the control measurements.

The above equations assume that the measurement of the control does not include the camera positions. Although it is not unknown for camera stations to be incorporated into geodetic control networks as survey stations, the requirements for the two types of stations are usually different enough to prevent single points taking a dual role. Versatility commonly has a high priority and stations are sited as the need arises. The perspective centres of close-range cameras are generally offset from the axis of rotation, so the camera would have to be considered an "eccentric" station in any case. Therefore it is simpler to consider the control points and camera positions to be independent.

The elements of the coefficient matrix $\underset{\sim}{B}_p$ can be derived by considering the control coordinates as unknowns in the observation equations. The functions of equations (4.32) could be restated:

$$g_1(a_x + \Delta a_x, \ldots , \omega_2 + \Delta\omega_2, m_x + (c_x + \Delta c_x - m_x)) = 0$$
$$g_2(a_y + \Delta a_y, \ldots , \omega_2 + \Delta\omega_2, m_y + (c_y + \Delta c_y - m_y)) = 0$$
$$g_3(a_z + \Delta a_z, \ldots , \omega_2 + \Delta\omega_2, m_z + (c_z + \Delta c_z - m_z)) = 0 \quad \ldots \quad (5.10)$$

The observation equations now become:

$$\frac{\delta g_1}{\delta a_x}\Delta a_x + \ldots + \frac{\delta g_1}{\delta \omega_2}\Delta\omega_2 + \frac{\delta g_1}{\delta m_x}\Delta c_x = k(c_x - m_x)$$
$$\frac{\delta g_2}{\delta a_y}\Delta a_y + \ldots + \frac{\delta g_2}{\delta \omega_2}\Delta\omega_2 + \frac{\delta g_2}{\delta m_y}\Delta c_y = k(c_y - m_y)$$
$$\frac{\delta g_3}{\delta a_z}\Delta a_z + \ldots + \frac{\delta g_3}{\delta \omega_2}\Delta\omega_2 + \frac{\delta g_3}{\delta m_z}\Delta c_z = k(c_z - m_z) \quad \ldots \quad \ldots \quad \ldots \qquad (5.11)$$

The fourth equation is unchanged in each case.

Using the partial differentials (4.34) it can be seen that the coefficient matrix for the i^{th} observation of a terrestrial stereopair:

$$\underset{\sim}{B}_{p_i} = \begin{bmatrix} -k_i & 0 & 0 \\ 0 & -k_i & 0 \\ 0 & 0 & -k_i \\ 0 & 0 & 0 \end{bmatrix} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5.12)$$

Using a similar procedure with the aerial stereopair using equations (4.55) gives a slightly more complicated matrix:

$$\underset{\sim}{B}_{p_i} = \begin{bmatrix} -k_i\cos K & k_i\sin K & 0 \\ -k_i\sin K & -k_i\cos K & 0 \\ 0 & 0 & -k_i \\ 0 & 0 & 0 \end{bmatrix} \quad \ldots \quad \ldots \quad \ldots \qquad (5.13)$$

The matrices show an advantage of the vector solution over the collinearity condition. The equivalent matrix for the collinearity condition does not have zero elements and every term is quite complex. Each object coordinate appears in every equation in a complicated arrangement with the other terms (see equations (2.7)), so the partial differentials are correspondingly complicated. Both the camera and control coordinates are subject to this complexity of coefficients. On the otherhand, the camera and control coordinates in the vector equations are relatively isolated and each only appears once or twice. The partial differentials are therefore much simpler.

The solution to the matrix equation (5.9) is given by (3.3):

$$\begin{bmatrix} \underset{\sim}{x} \\ \underset{\sim}{c} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p & \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{B}_p \\ \underset{\sim}{B}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p & \underset{\sim}{B}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{B}_p + \underset{\sim}{A}_c^T \underset{\sim}{Q}_c^{-1} \underset{\sim}{A}_c \end{bmatrix}^{-1} \begin{bmatrix} \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p \\ \underset{\sim}{B}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p + \underset{\sim}{A}_c^T \underset{\sim}{Q}_c^{-1} \underset{\sim}{m}_c \end{bmatrix} \quad \ldots \quad (5.14)$$

However the adjustment of the measurements establishing the control is normally carried out prior to any photogrammetric measurement or analysis, giving a set of first estimates of the control coordinates $\underset{\sim}{c}_o$. The initial adjustment can be expressed:

$$\underset{\sim}{A}_c \underset{\sim}{c}_o = \underset{\sim}{m}_c + \underset{\sim}{v}_2 \; : \; \underset{\sim}{Q}_c \quad \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \quad (5.15)$$

and the solution is given by:

$$\underset{\sim}{c}_o = (\underset{\sim}{A}_c^T \underset{\sim}{Q}_c^{-1} \underset{\sim}{A}_c)^{-1} \underset{\sim}{A}_c^T \underset{\sim}{Q}_c^{-1} \underset{\sim}{m}_c \quad \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \quad (5.16)$$

which can be expressed in the form of normal equations:

$$\underset{\sim}{c}_o = \underset{\sim}{N}_c^{-1} \underset{\sim}{k}_c \quad \text{or} \quad \underset{\sim}{N}_c \underset{\sim}{c}_o = \underset{\sim}{k}_c \quad \ldots \; \ldots \; \ldots \; \ldots \quad (5.17)$$

The weight coefficient matrix of the first estimates of the control coordinates, according to (3.9a), is given by:

$$\underset{\sim}{Q}_{c_o} = \underset{\sim}{N}_c^{-1} \quad \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots \quad (5.18)$$

A sequential adjustment of the control and photogrammetric measurements can now be carried out using the control coordinates as measurements with precisions provided by the initial adjustment:

$$\begin{bmatrix} \underset{\sim}{A}_p & \underset{\sim}{B}_p \\ \underset{\sim}{0} & \underset{\sim}{I} \end{bmatrix} \begin{bmatrix} \underset{\sim}{x} \\ \underset{\sim}{c} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{m}_p \\ \underset{\sim}{c}_o \end{bmatrix} + \begin{bmatrix} \underset{\sim}{v}_p \\ \underset{\sim}{v}_3 \end{bmatrix} : \begin{bmatrix} \underset{\sim}{Q}_p & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{N}_c^{-1} \end{bmatrix} \ldots \quad (5.19)$$

The solution can be shown to be:

$$\begin{bmatrix} \underset{\sim}{x} \\ \underset{\sim}{c} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p & \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{B}_p \\ \underset{\sim}{B}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p & \underset{\sim}{N}_c + \underset{\sim}{B}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{B}_p \end{bmatrix}^{-1} \begin{bmatrix} \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p \\ \underset{\sim}{N}_c \underset{\sim}{c}_o + \underset{\sim}{B}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p \end{bmatrix} \ldots \quad (5.20)$$

and comparison with the result of the simultaneous adjustment (5.14) will show that these two solutions are identical.

Equations (5.20) represent the sequential adjustment of a set of control coordinates using their full weight coefficient matrix $\underset{\sim}{N}_c$. Even if there are control points which are not photogrammetrically observed, but are part of the network of control measurements, they should be included in the adjustment. Correlations between the control points will cause such points to be adjusted even though they are not photogrammetrically measured. Other observed control points may be correlated through the unobserved points, so to be completely rigorous all points in the control measurement network must be included. Correlations are only likely to be present when geodetic measurements are used to define the control, but as this is the most common method the solution must be general enough to cover the majority of cases.

There is the secondary consideration that it would be impossible to structure the normals given in expression (5.20) if unobserved points were to be removed from the adjustment. This would only be possible if it were known in advance which points were to be observed, and even then a great deal of manipulation of the matrix $\underset{\sim}{N}_c$ would be required. Extra photogrammetric measurements may be included in a sequential adjustment of observations (discussed later in the chapter) subsequent to the first adjustment. If new control points were included then $\underset{\sim}{N}_c$ would require a massive re-arrangement if all points were not retained initially.

The structure of the normals is considerably more complex than the simple control weighting. In this case the photogrammetric observations must be included in the normals in specific locations decided by the locations of the control points in arrays $\underset{\sim}{c}_o$ and $\underset{\sim}{N}_c$. Both the number of known coordinates and the number of fixed coordinates for each control point can vary. That is, some control points may only have the position coordinates known, only the height coordinate known or possibly no coordinates known. In the latter case the point is a parallax point and only contributes to the determination of the exterior orientation elements. In such a case the $\underset{\sim}{B}_{p_i}$ matrix would be a null matrix.

Some control points may be datum points and have some of their coordinates fixed. For example, if the control survey is completely isolated and related to an arbitrary origin, then a particular point may be chosen as the datum. If this point, or any point with any fixed coordinates, is imaged on the photographs and observed, allowance must be made in the normals structure. The numbers of rows and columns in the submatrices for the photogrammetric measurement will vary according to the type of point observed.

However these considerations do not disallow the use of point by point loading of the normals. The expanded structure of the normals is shown in Figure 5.1, where the symbols have the following meanings:

$\underset{\sim}{A}_i$	m_i x 12 submatrix of differential coefficients associated with the exterior orientation parameters for the i^{th} point
m_i	the number of equations contributed by the i^{th} point (equivalent to the number of known coordinates plus one)
$\underset{\sim}{B}_i$	m_i x n_i submatrix of differential coefficients associated with the control coordinates for the i^{th} point
n_i	the number of known and non-fixed coordinates of the i^{th} point
$\underset{\sim}{Q}_i$	m_i x m_i submatrix of weight coefficients associated with pseudo-measurements for the i^{th} point
$\underset{\sim}{m}_i$	m_i x 1 vector submatrix of the pseudo-measurements for the i^{th} point
$\underset{\sim}{c}_i$	n_i x 1 vector submatrix of the corrections to the coordinates of the i^{th} point
m	total number of photogrammetrically observed points
n	total number of control points

As each photogrammetric observation is processed the $\underset{\sim}{A}_i$, $\underset{\sim}{Q}_i$, $\underset{\sim}{B}_i$ and $\underset{\sim}{m}_i$ submatrices are formed and matrix multiplied to produce the following:

$$\underset{\sim}{A}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{A}_i, \quad \underset{\sim}{A}^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{B}_i, \quad \underset{\sim}{B}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{B}_i, \quad \underset{\sim}{A}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{m}_i, \quad \underset{\sim}{B}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{m}_i$$

$$
\left[\begin{array}{c|cccc}
\underset{12*12}{\sum_{i=1}^{m} A_i^T Q_i^{-1} A_i} & \underset{12*n_1}{A_1^T Q_1^{-1} B_1} & \underset{12*n_2}{A_2^T Q_2^{-1} B_2} & \cdots & \underset{12*n_n}{A_n^T Q_n^{-1} B_n} \\
\hline
\underset{n_1*12}{B_1^T Q_1^{-1} A_1} & \underset{n_1*n_1}{B_1^T Q_1^{-1} B_1} & & & \\
\underset{n_2*12}{B_2^T Q_2^{-1} A_2} & & \underset{n_2*n_2}{B_2^T Q_2^{-1} B_2} & & \\
\vdots & & & \underset{n*n}{N_c} \quad \ddots & \\
\underset{n_n*12}{B_n^T Q_n^{-1} A_n} & & & & \underset{n_n*n_n}{B_n^T Q_n^{-1} B_n}
\end{array}\right]
\left[\begin{array}{c}
\underset{12*1}{x} \\
\hline
\underset{n_1*1}{c_1} \\
\underset{n_2*1}{c_2} \\
\vdots \\
\underset{n_n*1}{c_n}
\end{array}\right]
=
\left[\begin{array}{c}
\underset{12*1}{\sum_{i=1}^{m} A_i^T Q_i^{-1} m_i} \\
\hline
\underset{n_1*1}{B_1^T Q_1^{-1} m_1} \\
\underset{n_2*1}{B_2^T Q_2^{-1} m_2} \\
\vdots \\
\underset{n*1}{N_c\, c_o} \\
\vdots \\
\underset{n_n*1}{B_n^T Q_n^{-1} m_n}
\end{array}\right]
$$

Figure 5.1 Normals Structure for Sequential Adjustment of Control

These are then added to the appropriate locations in the normal equations. The full matrices $\underset{\sim}{N}_c$ and $\underset{\sim}{N}_c \underset{\sim}{c}_o$ should already be present and the internal structures resolved so that the contributions of the photogrammetric measurements can be added correctly.

5.1.5 Control Precision

Weight coefficient matrices are used throughout in the derivation of sequential least squares adjustment in preference to covariance matrices. The use of the term variance or covariance matrix implies that an estimate of the variance factor from an adjustment has been obtained and applied to a weight coefficient matrix. The variance factor is given by equation (3.12) and can be used to relate the weight coefficient and covariance matrices of the measurements and unknowns from the adjustment given by (3.3):

$$\underset{\sim}{V}_m = \sigma_o^2 \underset{\sim}{Q}_m$$

$$\underset{\sim}{V}_x = \sigma_o^2 \underset{\sim}{Q}_x = \sigma_o^2 (\underset{\sim}{A}^T \underset{\sim}{Q}_m^{-1} \underset{\sim}{A})^{-1} \quad \ldots \ldots \ldots \ldots \quad (5.21)$$

where $\underset{\sim}{V}_m$ and $\underset{\sim}{V}_x$ are the covariance matrices of the measurements and unknowns respectively.

The use of a covariance matrix instead of a weight coefficient matrix theoretically violates the sequential adjustment algorithms presented in the previous section.

If the variance factor of the initial adjustment is applied to the weight coefficient matrix of the control coordinates, the solutions by simultaneous and sequential adjustments will no longer give identical results. The further the variance factor departs from unity, the larger the discrepancy between the two solutions will become.

On the other hand, if the a priori estimates of the control measurements are not realistic, then the weight coefficient matrix of the control coordinates from the initial adjustment will be unrealistic and the subsequent sequential adjustment is equally invalid on practical grounds. If the control precisions from the initial adjustment are grossly over- or under-estimated then the sequential adjustment will be biassed toward the photogrammetric or control measurements respectively. For example, if the control coordinates have unduly high precisions, the control will tend to be held fixed and the restitution will be distorted.

It could be argued that the variance factor from the initial adjustment should always be included to avoid any possible bias. Such a course could be theoretically justified by simply considering that the simultaneous adjustment uses the covariance rather than the weight coefficient matrix of the control measurements. However, the use of the variance factor cannot be generalized and can only be left to individual cases and an experienced interpreter of least squares adjustments of control measurements. Obviously the two situations of the measurement of one survey not warranting a good variance factor and the measurement of another giving rise to a justifiably poor variance factor must be treated differently.

This problem is more acute if the weight coefficient matrix of the control is not from a least squares adjustment, but is an estimated diagonal matrix. Without the benefits of self-checking provided by an adjustment the values of the precisions may be considerably in error. The only recourse in this situation is to analyse the residuals of the sequential adjustment. The magnitudes of the control corrections and the corresponding precisions can be compared to give some indication of the accuracy of the estimations.

The interpretation of the residuals of a sequential adjustment can be complicated by the interaction of control coordinates and photogrammetric measurements. While the control provided by non-geodetic means can be virtually treated as measurements, control coordinates provided from an initial adjustment of geodetic measurements must be dealt with differently. The weight coefficient matrix will not be diagonal and the influence of the geometry of the survey network will be effected through the correlations. Analysis of the residuals for gross or systematic errors can only be done in the light of that geometry and the relationship of the survey network to the photography.

In the context of close-range stereopairs and online processing, the photogrammetrist is at the least disadvantage as regards error analysis. The control survey and photography is generally included in a single visit to the site, or laboratory, and the processing of the survey and photogrammetric measurements can be carried out with a minimum turnaround time. The shortest delay between the measurement of the control and the interpretation of the residuals of both adjustments should lead to an efficient system of error detection.

5.2 Interior Orientation Adjustment

5.2.1 Estimation of Weight Coefficients

The problem of estimation of the weight coefficient matrix of the unknown parameters also applies to sequential adjustment of the interior orientation. If the interior orientation parameters are obtained from an initial least squares adjustment then the decision must be made whether to include the variance factor or not. If the parameters are not obtained from an initial adjustment then the weight coefficient matrix must be estimated from other sources.

In the cases of affine or deformational transformations the theory of the initial adjustments has been given in Chapter 4. The weight coefficient matrix passed on to the sequential adjustment is again extracted from the solution for the initial adjustment. A variance factor can be computed on the basis of the residuals of the adjustment. Again no hard and fast guidelines can be laid down for the inclusion of the factor and each individual case must be considered on its own merits.

However the physical circumstances of the interior orientation play a much larger part in the decision to use a weight coefficient or a covariance matrix. Large variance factors for interior orientation adjustments may be caused by the presence of systematic errors which have not been adequately described by the transformation. For example, if an affine transformation is used with glass plates of low grade flatness the residuals at the fiducial marks are liable to be large, which will cause a large variance factor. The large residuals are not caused by poor observations, but rather by the inability of an affine transformation to compensate for the non-linear distortions which are present. Inclusion of the variance factor will only result in the underestimation of the precisions of the parameters.

If an inadequate functional model of the interior orientation must be used in the initial adjustment, either because there is insufficient fiducial information or adjustment residuals are required for error checking, the model can be updated in the sequential adjustment. When there is sufficient control the interior orientation parameters can be not only adjusted on the basis of control observations, but the number of parameters can be increased. For example, deformational parameters can be added to an affine transformation to compensate for non-linear distortions which would otherwise not be removed. The inclusion of additional parameters does not demand an increase in the control requirement for the solution, so the list of desirable attributes of the solution is not violated. The additional parameters are not included as unknowns but instead entered in the sequential adjustment as measurements with estimated values and precisions. However, there must be enough control to allow a stable adjustment and provide a reasonable number of redundancies.

In the case of an affine transformation being updated to a deformational transformation, the initial values of the extra parameters must all be zero. The precisions of the extra parameters can only be estimated from experience and experimentation with the particular camera/emulsion combination. It is the random component of the likely fluctuations of the additional parameters which must be estimated for the adjustment. The magnitude will vary depending on the format size of the camera, whether plates or film are used, the method of support and flattening of the emulsion and the processing of the photography.

The systematic components of the fluctuations can only be estimated after a great deal of experience with a particular camera/emulsion combination. The estimates of the systematic components would replace the assumed values of the extra parameters.

The conversion of a simple translation/rotation to an affine or deformational transformation presents a further problem. Not only do any included second order terms need value and precision estimates, but the linear scale factors must also be quantified. It could be expected that the scales in perpendicular directions on the photograph would have both systematic and random fluctuations, primarily depending on the type of emulsion base and the consistency of the photographic processing. Again there is no substitute for experimentation and experience with particular camera/emulsion combinations.

The estimated weight coefficient matrix of a simple translation/rotation is necessarily diagonal, with some terms approximated from the initial interior orientation and some established from experience. The precisions of the principal point coordinates (X_o, Y_o) and the rotation between the comparator and fiducial axes (θ) can be derived from the precision of observation of the fiducial marks and the formulae (4.1) and (4.2). Assuming that the magnitude of the rotation is small, the following expressions can be obtained:

$$\sigma^2_{xo} = \tfrac{1}{4}(\sigma^2_{x1} + \sigma^2_{x2})$$
$$\sigma^2_{yo} = \tfrac{1}{4}(\sigma^2_{y3} + \sigma^2_{y4})$$
$$\sigma^2_{\theta} = 1/16\ \{(\sigma^2_{x1} + \sigma^2_{x2})/Y^2 + (\sigma^2_{y3} + \sigma^2_{y4})/X^2\} \quad \dots \quad \dots \quad (5.22)$$

where σ_{xi}, σ_{yi} standard errors of observation of the $\bar{x}$ and $\bar{y}$ coordinates of the i^{th} fiducial mark (see Figure (4.1) for numbering system)

X, Y half the format dimensions in the $\bar{x}$ and $\bar{y}$ directions respectively

These expressions can then be used in conjunction with equations (4.3) and the expected scale factor fluctuations to estimate the precisions of the six parameters of an affine transformation:

$$\sigma^2_{a_1} = \sigma^2_{xo} \quad , \quad \sigma^2_{a_2} = \sigma^2_{yo}$$
$$\sigma^2_{a_3} = \sigma^2_{\lambda_x} \quad , \quad \sigma^2_{a_4} = \lambda^2_x\sigma^2_{\theta}$$
$$\sigma^2_{a_5} = \lambda^2_y\sigma^2_{\theta} \quad , \quad \sigma^2_{a_6} = \sigma^2_{\lambda_y} \quad \dots \quad \dots \quad \dots \quad \dots \quad (5.23)$$

where σ_{λ_x}, σ_{λ_y} are the standard errors of the expected linear scale fluctuations on the $\bar{x}$ and $\bar{y}$ axes respectively

and a simplified interpretation of the affine transformation is used where λ_x, λ_y are scale factors on the x and y comparator axes and:

$$a_3 = \lambda_x \cos\theta$$
$$a_4 = \lambda_x \sin\theta$$
$$a_5 = -\lambda_y \sin\theta$$
$$a_6 = \lambda_y \cos\theta$$

Without considerable experience with a particular camera/emulsion combination the two scale factors must have estimated values of unity. A camera with minimum fiducial information cannot give any indication of the scale factors. Data on the scale factors can only be given by a direct linear transformation solution, a sequential adjustment of the interior orientation or possibly by analogy with similar cameras which do have known fiducial marks.

If there is enough control information available to give an adequate estimate of the parameters, the potential improvement of the interior orientation is considerable. A full deformational transformation can be derived and utilized compared with a simple translation/rotation with unit scale factors and no second order terms.

5.2.2 Sequential Adjustment of Interior Orientation

There is no analogy to weighted control for the interior orientation and a rigorous sequential adjustment is the only possibility. The sequential algorithm is once more based on the simultaneous solution which can be expressed in expanded matrix form:

$$\begin{bmatrix} \underset{\sim}{A}_a & \underset{\sim}{0} \\ \underset{\sim}{D}_p & \underset{\sim}{A}_p \end{bmatrix} \begin{bmatrix} \underset{\sim}{a} \\ \underset{\sim}{x} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{m}_a \\ \underset{\sim}{m}_p \end{bmatrix} + \begin{bmatrix} \underset{\sim}{v}_a \\ \underset{\sim}{v}_p \end{bmatrix} : \begin{bmatrix} \underset{\sim}{Q}_a & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_p \end{bmatrix} \quad \ldots \ldots \quad (5.24)$$

where $\underset{\sim}{A}_p$, $\underset{\sim}{x}$, $\underset{\sim}{m}_p$, $\underset{\sim}{v}_p$, $\underset{\sim}{Q}_p$ have the same meanings as before.

$\underset{\sim}{D}_p$ is the matrix of partial differentials of the photogrammetric observation equations with respect to the interior orientation parameters

$\underset{\sim}{a}$ is the vector matrix of unknown interior orientation parameters

$\underset{\sim}{m}_a$ is the vector matrix of the fiducial measurements

$\underset{\sim}{v}_a$, $\underset{\sim}{A}_a$, $\underset{\sim}{Q}_a$ are the vector matrix of residuals, and linear coefficient and weight coefficient matrices, respectively, associated with the fiducial measurements

The elements of the coefficient matrix $\underset{\sim}{D}_p$ must be derived by introducing the interior orientation transformation parameters into the equations for the vector solution. Dealing first with terrestrial stereopairs, the fiducial coordinates were deliberately omitted from the function equations (4.30) because at that stage they were considered to be constants. If the interior orientation parameters are to become part of the adjustment then the fiducial coordinates are variables and must be included:

$$g_1(a_x, b_x, \ldots, \omega_2, x_1, y_1, x_2, y_2, m_x) = 0$$
$$g_2(a_y, b_x, \ldots, \omega_2, x_1, y_1, x_2, y_2, m_y) = 0$$
$$g_3(a_z, b_x, \ldots, \omega_2, x_1, y_1, x_2, y_2, m_z) = 0$$
$$g_4(b_x, b_y, \ldots, \omega_2, x_1, y_1, x_2, y_2, p_z) = 0 \quad \ldots \quad \ldots \quad (5.25)$$

The third fiducial coordinates are still omitted because this procedure does not venture into the realms of camera calibration. Metric cameras are assumed throughout with known and stable focal lengths. The principal point position is considered fixed relative to the fiducial marks and adjustment of the translation elements of the interior orientation do not constitute a change in this relationship, i.e. this is not a calibration algorithm.

Further development of the equations (5.25) will assume that a deformational transformation is utilized for the interior orientation. This transformation would be used in the majority of cases as close-range stereopairs generally have enough control to obtain a good solution. A stereopair with near minimum control may demand the use of only an affine transformation, and in such a case it is doubtful whether any great advantage would be gained from a sequential adjustment. Only if the initial interior orientation was a simple translation/rotation would any significant improvement be realized.

Expanding equations (5.25) on the basis of the deformational transformation given by equations (2.6) with additional subscripts to distinguish the left and right photographs:

$$g_1(a_x, b_x, \cdots, \omega_2, a_{11}, \cdots, a_{18}, a_{21}, \cdots, a_{28}, m_x) = 0$$
$$g_2(a_y, b_x, \cdots, \omega_2, a_{11}, \cdots, a_{18}, a_{21}, \cdots, a_{28}, m_y) = 0$$
$$g_3(a_z, b_x, \cdots, \omega_2, a_{11}, \cdots, a_{18}, a_{21}, \cdots, a_{28}, m_z) = 0$$
$$g_4(b_x, b_y, \cdots, \omega_2, a_{11}, \cdots, a_{18}, a_{21}, \cdots, a_{28}, p_z) = 0 \quad \ldots \quad (5.26)$$

Once again corrections to the parameters can be postulated which will convert the measurements (m_x, m_y, m_z, p_z) into the desired control values (c_x, c_y, c_z, 0):

$$g_1(a_x + \Delta a_x, \cdots, \omega_2 + \Delta\omega_2, a_{11} + \Delta a_{11}, \cdots, a_{28} + \Delta a_{28}, c_x) = 0$$
$$g_2(a_y + \Delta a_y, \cdots, \omega_2 + \Delta\omega_2, a_{11} + \Delta a_{11}, \cdots, a_{28} + \Delta a_{28}, c_y) = 0$$
$$g_3(a_z + \Delta a_z, \cdots, \omega_2 + \Delta\omega_2, a_{11} + \Delta a_{11}, \cdots, a_{28} + \Delta a_{28}, c_z) = 0$$
$$g_4(b_x + \Delta b_x, \cdots, \omega_2 + \Delta\omega_2, a_{11} + \Delta a_{11}, \cdots, a_{28} + \Delta a_{28}, 0\ ) = 0 \quad \ldots \quad (5.27)$$

After a modification similar to equations (4.32) the function expressions can be once more expanded by a Taylor's series expansion to first order terms and simplified into the final form of the observation equations for the least squares adjustment:

$$\frac{\delta g_1}{\delta a_x}\Delta a_x + \frac{\delta g_1}{\delta b_x}\Delta b_x + \cdots + \frac{\delta g_1}{\delta \omega_2}\Delta\omega_2 + \frac{\delta g_1}{\delta a_{11}}\Delta a_{11} + \cdots + \frac{\delta g_1}{\delta a_{28}}\Delta a_{28} = k(c_x - m_x)$$
$$\frac{\delta g_2}{\delta a_y}\Delta a_y + \frac{\delta g_2}{\delta b_x}\Delta b_x + \cdots + \frac{\delta g_2}{\delta \omega_2}\Delta\omega_2 + \frac{\delta g_2}{\delta a_{11}}\Delta a_{11} + \cdots + \frac{\delta g_2}{\delta a_{28}}\Delta a_{28} = k(c_y - m_y)$$
$$\frac{\delta g_3}{\delta a_z}\Delta a_z + \frac{\delta g_3}{\delta b_x}\Delta b_x + \cdots + \frac{\delta g_3}{\delta \omega_2}\Delta\omega_2 + \frac{\delta g_3}{\delta a_{11}}\Delta a_{11} + \cdots + \frac{\delta g_3}{\delta a_{28}}\Delta a_{28} = k(c_z - m_z)$$
$$\frac{\delta g_4}{\delta b_x}\Delta b_x + \frac{\delta g_4}{\delta b_y}\Delta b_y + \cdots + \frac{\delta g_4}{\delta \omega_2}\Delta\omega_2 + \frac{\delta g_4}{\delta a_{11}}\Delta a_{11} + \cdots + \frac{\delta g_4}{\delta a_{28}}\Delta a_{28} = kp_z \quad \ldots \quad (5.28)$$

Whereas the partial differentials for the control coordinates were very simple, the differentials for the interior orientation parameters are extremely complicated. Some saving in programming and computation can be made because terms used in the partial differential coefficients for the rotations (see Appendix A) can also be used for these parameters, but in this case the collinearity condition has a clear advantage. Inspection of equations (2.7) shows that the partial differential coefficients for the parameters could hardly be less complicated.

The differential coefficients are given in Appendix C for the terrestrial case only. The development of the extended observation equations for aerial stereopairs need not be included here as it parallels the derivation for terrestrial stereopairs. The difference in the equations due to the use of the common rotation does not affect the developments included in equations (5.25) to (5.28).

The initial adjustment of the interior orientation gives the estimates of the interior orientation parameters $\underset{\sim}{a}_o$ using equations (3.3):

$$\underset{\sim}{A}_a \underset{\sim}{a}_o = \underset{\sim}{m}_a + \underset{\sim}{v}_4 \quad : \quad \underset{\sim}{Q}_a$$

$$\underset{\sim}{a}_o = (\underset{\sim}{A}_a^T \underset{\sim}{Q}_a^{-1} \underset{\sim}{A}_a)^{-1} (\underset{\sim}{A}_a^T \underset{\sim}{Q}_a^{-1} \underset{\sim}{m}_a) = \underset{\sim}{N}_a^{-1} \underset{\sim}{k}_a \quad \dots \quad \dots \quad \dots \qquad (5.29)$$

These equations can be considered as representing the adjustment of the interior orientations of both left and right photographs. In practice the two adjustments would be carried out separately as the same set of programming computations can be used, saving both computer time and memory. The two adjustments can be represented as a single adjustment because they are assumed to be completely independent and it is therefore irrelevant whether they are adjusted together or not. This can be verified by simply partitioning the above matrix equations into the submatrices appropriate to the left and right adjustments:

$$\begin{bmatrix} \underset{\sim}{A}_\ell & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{A}_r \end{bmatrix} \begin{bmatrix} \underset{\sim}{a}_\ell \\ \underset{\sim}{a}_r \end{bmatrix} = \begin{bmatrix} \underset{\sim}{m}_\ell \\ \underset{\sim}{m}_r \end{bmatrix} + \begin{bmatrix} \underset{\sim}{v}_\ell \\ \underset{\sim}{v}_r \end{bmatrix} \quad : \quad \begin{bmatrix} \underset{\sim}{Q}_\ell & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_r \end{bmatrix}$$

$$\begin{bmatrix} \underset{\sim}{a}_\ell \\ \underset{\sim}{a}_r \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}_\ell^T \underset{\sim}{Q}_\ell^{-1} \underset{\sim}{A}_\ell & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{A}_r^T \underset{\sim}{Q}_r^{-1} \underset{\sim}{A}_r \end{bmatrix}^{-1} \begin{bmatrix} \underset{\sim}{A}_\ell^T \underset{\sim}{Q}_\ell^{-1} \underset{\sim}{m}_\ell \\ \underset{\sim}{A}_r^T \underset{\sim}{Q}_r^{-1} \underset{\sim}{m}_r \end{bmatrix} \quad \dots \quad \dots \qquad (5.30)$$

The solution can be derived directly using the fact that block diagonal matrices can be inverted by inverting the component submatrices:

$$\begin{bmatrix} \underset{\sim}{a}_\ell \\ \underset{\sim}{a}_r \end{bmatrix} = \begin{bmatrix} (\underset{\sim}{A}_\ell^T \underset{\sim}{Q}_\ell^{-1} \underset{\sim}{A}_\ell)^{-1} \underset{\sim}{A}_\ell^T \underset{\sim}{Q}_\ell^{-1} \underset{\sim}{m}_\ell \\ (\underset{\sim}{A}_r^T \underset{\sim}{Q}_r^{-1} \underset{\sim}{A}_r)^{-1} \underset{\sim}{A}_r^T \underset{\sim}{Q}_r^{-1} \underset{\sim}{m}_r \end{bmatrix} \quad \dots \quad \dots \quad \dots \quad \dots \qquad (5.31)$$

Therefore the two adjustments can be carried out separately and the block diagonal normals coefficient matrix required for the subsequent sequential adjustment is constructed from the two initial adjustments:

$$\underset{\sim}{N}_a = \begin{bmatrix} \underset{\sim}{N}_\ell & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{N}_r \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}_\ell^T \underset{\sim}{Q}_\ell^{-1} \underset{\sim}{A}_\ell & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{A}_r^T \underset{\sim}{Q}_r^{-1} \underset{\sim}{A}_r \end{bmatrix} \quad \ldots \ldots \ldots \quad (5.32)$$

The sequential adjustment of the interior orientation and photogrammetric observations on the control can be formulated with the interior orientation parameters entered as measurements:

$$\begin{bmatrix} \underset{\sim}{I} & \underset{\sim}{0} \\ \underset{\sim}{D}_p & \underset{\sim}{A}_p \end{bmatrix} \begin{bmatrix} \underset{\sim}{a} \\ \underset{\sim}{x} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{a}_o \\ \underset{\sim}{m}_p \end{bmatrix} + \begin{bmatrix} \underset{\sim}{v}_5 \\ \underset{\sim}{v}_p \end{bmatrix} : \begin{bmatrix} \underset{\sim}{N}_a^{-1} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_p \end{bmatrix} \ldots \ldots \quad (5.33)$$

The solution is again given by expression (3.3):

$$\begin{bmatrix} \underset{\sim}{a} \\ \underset{\sim}{x} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{N}_a + \underset{\sim}{D}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{D}_p & \underset{\sim}{D}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \\ \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{D}_p & \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{A}_p \end{bmatrix}^{-1} \begin{bmatrix} \underset{\sim}{N}_a \underset{\sim}{a}_o + \underset{\sim}{D}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p \\ \underset{\sim}{A}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{m}_p \end{bmatrix} \ldots \quad (5.34)$$

The normals structure for this adjustment is more straightforward than the sequential control adjustment, and once again does not preclude point by point processing:

$$\left[\begin{array}{c|c} \underset{\sim}{N}_a + \sum_{i=1}^{m} \underset{\sim}{D}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{D}_i & \sum_{i=1}^{m} \underset{\sim}{D}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{A}_i \\ 16*16 & 16*12 \\ \hline \sum_{i=1}^{m} \underset{\sim}{A}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{D}_i & \sum_{i=1}^{m} \underset{\sim}{A}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{A}_i \\ 12*16 & 12*12 \end{array}\right] \left[\begin{array}{c} \underset{\sim}{a} \\ 16*1 \\ \hline \underset{\sim}{x} \\ 12*1 \end{array}\right] = \left[\begin{array}{c} \underset{\sim}{N}_a \underset{\sim}{a}_o + \sum_{i=1}^{m} \underset{\sim}{D}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{m}_i \\ 16*1 \\ \hline \sum_{i=1}^{m} \underset{\sim}{A}_i^T \underset{\sim}{Q}_i^{-1} \underset{\sim}{m}_i \\ 12*1 \end{array}\right]$$

$$\ldots \ldots \ldots \quad (5.35)$$

where $\underset{\sim}{D}_i$ is the $m_i \times 16$ submatrix of differential coefficients of the interior orientation parameters

The full matrices $\underset{\sim}{N}_a$ and $\underset{\sim}{N}_a \underset{\sim}{a}_o$ can be computed first and then the submatrices appropriate to each photogrammetric measurement computed and loaded unto the normals as it is observed.

5.3 Adjustment of Additional Observations

5.3.1 Data Storage

The advantages of the sequential adjustment of additional observations are primarily related to online computer processing of photogrammetric measurements. The emphasis is on the observation of control points because extra measurements are commonly taken during this phase of the restitution for a variety of reasons. Although additional control and interior orientation measurements could be included, it is unlikely that such extra measurements would ever be encountered.

The inclusion of additional survey measurements in an online photogrammetric adjustment can be vetoed immediately because programming for variation of coordinates observation equations is tantamount to a simultaneous adjustment. This has already been ruled out due to the limitations on processing power and memory of minicomputers, and should not be required in any case.

Certainly all the measurements taken to define the control would be included in the initial adjustment or configuration of coordinates. The only possibility is therefore the reinstatement of rejected data, and this would happen so infrequently that provision for such a case would be extremely inefficient.

Similarly, all the observations on fiducial marks are normally included in the initial adjustment for the interior orientation. The majority of close-range cameras have only a limited number of fiducial marks to observe and the comparator operator knows from experience how many rounds of observations he must take to achieve a specified precision of measurement. Fiducial marks are clearly distinguishable in the camera format so that gross errors and misidentifications are rare, and extra measurements to resolve such difficulties are once again infrequent and need not be accommodated.

On the other hand, photogrammetric observations on control to define exterior orientations are not as strictly regulated. All control points visible in the stereopair are usually observed, but the number of measurements to each point may vary widely. If the control points are not distributed throughout the object then additional parallax points may be taken to strengthen the exterior orientation. If the interior orientation is included in a sequential adjustment with the exterior orientation then parallax points may be taken to obtain an adequate distribution of points throughout the format. If the control is included in a sequential adjustment then it may be desired to weight points of poor survey precision with many photogrammetric observations.

The consideration of all these requirements will inevitably lead to omitted and insufficient observations for some restitutions. Often the residuals of the adjustment will indicate areas in the object which are lacking observations or where repeated observations on particular control points would be beneficial. Even if all the required observations are taken and the adjustment is otherwise satisfactory, gross errors are bound to occur through ambiguities, indistinct control points and misnumberings. Extra observations may be taken to replace those at fault after they have been deleted from the adjustment.

Additional observations can be incorporated into the adjustment more efficiently using sequential adjustment of observations. If sequential adjustment is not used then a set of dedicated data registers must be retained throughout the computer program for the restitution. These registers hold the current accumulated coordinates of the observations on each control point, as well as some ancillary data. Sequential adjustment only requires the retention of the upper triangular half of the normals matrix, so there is a considerable saving in memory when the number of unknowns is small, for example when only the exterior orientation parameters are involved in the adjustment. The data arrays are no longer dedicated and can be used in other parts of the program.

When the interior orientation parameters and the control coordinates are also included as unknowns in a sequential adjustment the order of the normals matrix can be quite large, nullifying this advantage. But in such a case it becomes mandatory to utilize scratch files on the storage medium of the minicomputer because of its limitations in memory. The weight coefficient matrices from the initial adjustments of the control and interior orientations are written to such a file during the program and read off as required. Sequential adjustment of observations can be employed by over-writing this file after the first adjustment of photogrammetric control observations. If additional observations are required then the file can be read again and manipulated by routines already present.

5.3.2 Sequential Adjustment of Additional Observations

The sequential adjustment of photogrammetric observations and control or interior orientation parameters only have already been derived. A sequential adjustment for a fixed number of parameters has been demonstrated in detail in Section 3.2 and need not be repeated here. The adjustment which is yet to be presented is the sequential adjustment of the control, interior orientation and additional observations. Sequential adjustments excluding the control or interior orientation can be deduced from the following and Sections 5.1 and 5.2 respectively.

The simultaneous adjustment is used as the basis for the sequential adjustment and can be stated:

$$\begin{bmatrix} \underset{\sim}{A}_a & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{D}_p & \underset{\sim}{A}_p & \underset{\sim}{B}_p \\ \underset{\sim}{D}_q & \underset{\sim}{A}_q & \underset{\sim}{B}_q \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{A}_c \end{bmatrix} \begin{bmatrix} \underset{\sim}{a} \\ \underset{\sim}{x} \\ \underset{\sim}{c} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{m}_a \\ \underset{\sim}{m}_p \\ \underset{\sim}{m}_q \\ \underset{\sim}{m}_c \end{bmatrix} + \begin{bmatrix} \underset{\sim}{v}_a \\ \underset{\sim}{v}_p \\ \underset{\sim}{v}_q \\ \underset{\sim}{v}_c \end{bmatrix} : \begin{bmatrix} \underset{\sim}{Q}_a & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{Q}_p & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{Q}_q & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{Q}_c \end{bmatrix} \quad \ldots \quad (5.36)$$

where all the terms have already been defined.

When the above matrices are multiplied out according to (3.3) the least squares solution is obtained:

$$\begin{bmatrix} a \\ x \\ c \end{bmatrix} = \begin{bmatrix} A_a^T Q_a^{-1} A_a + D_p^T Q_p^{-1} D_p + D_q^T Q_q^{-1} D_q & D_p^T Q_p^{-1} A_p + D_q^T Q_q^{-1} A_q & D_p^T Q_p^{-1} B_p + \dots D_q^T Q_q^{-1} B_q \\ A_p^T Q_p^{-1} D_p + A_q^T Q_q^{-1} D_q & A_p^T Q_p^{-1} A_p + A_q^T Q_q^{-1} A_q & A_p^T Q_p^{-1} B_p + A_q^T Q_q^{-1} B_q \\ B_p^T Q_p^{-1} D_p + B_q^T Q_q^{-1} D_q & B_p^T Q_p^{-1} A_p + B_q^T Q_q^{-1} A_q & B_p^T Q_p^{-1} B_p + B_q^T Q_q^{-1} B_q + A_c^T Q_c^{-1} A_c \end{bmatrix}^{-1} \begin{bmatrix} A_a^T Q_a^{-1} m_a + D_p^T Q_p^{-1} m_p + D_q^T Q_q^{-1} m_q \\ A_p^T Q_p^{-1} m_p + A_q^T Q_q^{-1} m_q \\ B_p^T Q_p^{-1} m_p + B_q^T Q_q^{-1} m_q + A_c^T Q_c^{-1} m_c \end{bmatrix} \quad \dots \quad \dots \qquad (5.37)$$

Consider that there are initial adjustments of both the control and interior orientation measurements as described equations (5.15), (5.16) and (5.29), which are to be sequentially adjusted with a first group of p photogrammetric observations on control. The control coordinates and interior orientation parameters are therefore introduced as measurements with precisions defined by the appropriate weight coefficient matrices:

$$\begin{bmatrix} I & 0 & 0 \\ D_p & A_p & B_p \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} a_1 \\ x_o \\ c_1 \end{bmatrix} = \begin{bmatrix} a_o \\ m_p \\ c_o \end{bmatrix} + \begin{bmatrix} v_5 \\ v_p \\ v_3 \end{bmatrix} \; : \; \begin{bmatrix} N_a^{-1} \cdot & 0 & 0 \\ 0 & Q_p & 0 \\ 0 & 0 & N_c^{-1} \end{bmatrix} \qquad (5.38)$$

The control, interior orientation and exterior orientation observations are assumed to be independent so that the weight coefficient matrix is block diagonal. The solution of this set of observation equations produces new estimates of the control coordinates and interior orientation parameters c_1 and a_1 as well as the first estimates of the exterior orientation parameters x_o:

$$\begin{bmatrix} a_1 \\ x_o \\ c_1 \end{bmatrix} = \begin{bmatrix} N_a + D_p^T Q_p^{-1} D_p & D_p^T Q_p^{-1} A_p & D_p^T Q_p^{-1} B_p \\ A_p^T Q_p^{-1} D_p & A_p^T Q_p^{-1} A_p & A_p^T Q_p^{-1} B_p \\ B_p^T Q_p^{-1} D_p & B_p^T Q_p^{-1} A_p & N_c + B_p^T Q_p^{-1} B_p \end{bmatrix}^{-1} * \begin{bmatrix} N_a a_o + D_p^T Q_p^{-1} m_p \\ A_p^T Q_p^{-1} m_p \\ N_c c_o + B_p^T Q_p^{-1} m_p \end{bmatrix} \quad \dots \quad \dots \quad \dots \quad \dots \qquad (5.39)$$

If the matrices $\underset{\sim}{N}_a$ and $\underset{\sim}{N}_c$ are replaced by their expanded forms:

$$\underset{\sim}{N}_a = \underset{\sim}{A}_a^T \underset{\sim}{Q}_a^{-1} \underset{\sim}{A}_a \quad \text{and} \quad \underset{\sim}{N}_c = \underset{\sim}{A}_c^T \underset{\sim}{Q}_c^{-1} \underset{\sim}{A}_c \quad \ldots \quad (5.40)$$

then there is a clear resemblance between equations (5.37) and (5.39), indicating already that the sequential adjustment of the additional q observations will give the correct answer. The structure of the normals in equation (5.39) is again conducive to point by point processing, as can be seen in Figure 5.2. Once more the appropriate submatrices can be formed for each observation and loaded into the structure of the normals.

To enable the next phase of the adjustment to take place, equations (5.39) must be redefined:

$$\begin{bmatrix} \underset{\sim}{a}_1 \\ \underset{\sim}{x}_o \\ \underset{\sim}{c}_1 \end{bmatrix} = \begin{bmatrix} \underset{\sim}{N}_1 & \underset{\sim}{N}_{12} & \underset{\sim}{N}_{13} \\ & \underset{\sim}{N}_2 & \underset{\sim}{N}_{23} \\ & & \underset{\sim}{N}_3 \end{bmatrix}^{-1} \begin{bmatrix} \underset{\sim}{k}_1 \\ \underset{\sim}{k}_2 \\ \underset{\sim}{k}_3 \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Q}_1 & \underset{\sim}{Q}_{12} & \underset{\sim}{Q}_{13} \\ & \underset{\sim}{Q}_2 & \underset{\sim}{Q}_{23} \\ & & \underset{\sim}{Q}_3 \end{bmatrix} \begin{bmatrix} \underset{\sim}{k}_1 \\ \underset{\sim}{k}_2 \\ \underset{\sim}{k}_3 \end{bmatrix} \quad \ldots \quad (5.41)$$

The additional q observations can now be introduced with all the unknown parameters incorporated as measurements with a weight coefficient matrix from the previous phase of the adjustment. This final phase will produce the most probable estimates of the parameters $\underset{\sim}{a}$, $\underset{\sim}{x}$ and $\underset{\sim}{c}$:

$$\begin{bmatrix} \underset{\sim}{I} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{I} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{I} \\ \underset{\sim}{D}_q & \underset{\sim}{A}_q & \underset{\sim}{B}_q \end{bmatrix} \begin{bmatrix} \underset{\sim}{a} \\ \underset{\sim}{x} \\ \underset{\sim}{c} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{a}_1 \\ \underset{\sim}{x}_o \\ \underset{\sim}{c}_1 \\ \underset{\sim}{m}_q \end{bmatrix} + \begin{bmatrix} \underset{\sim}{v}_6 \\ \underset{\sim}{v}_7 \\ \underset{\sim}{v}_8 \\ \underset{\sim}{v}_q \end{bmatrix} : \begin{bmatrix} \underset{\sim}{Q}_1 & \underset{\sim}{Q}_{12} & \underset{\sim}{Q}_{13} & \underset{\sim}{0} \\ & \underset{\sim}{Q}_2 & \underset{\sim}{Q}_{23} & \underset{\sim}{0} \\ & & \underset{\sim}{Q}_3 & \underset{\sim}{0} \\ & & & \underset{\sim}{Q}_q \end{bmatrix} \quad (5.42)$$

The additional measurements are assumed to be independent so that the block diagonality of the weight coefficient matrix can be used to simplify inversion:

$$\begin{bmatrix} \underset{\sim}{Q}_1 & \underset{\sim}{Q}_{12} & \underset{\sim}{Q}_{13} & \underset{\sim}{0} \\ & \underset{\sim}{Q}_2 & \underset{\sim}{Q}_{23} & \underset{\sim}{0} \\ & & \underset{\sim}{Q}_3 & \underset{\sim}{0} \\ & & & \underset{\sim}{Q}_q \end{bmatrix}^{-1} = \begin{bmatrix} \underset{\sim}{N}_1 & \underset{\sim}{N}_{12} & \underset{\sim}{N}_{13} & \underset{\sim}{0} \\ & \underset{\sim}{N}_2 & \underset{\sim}{N}_{23} & \underset{\sim}{0} \\ & & \underset{\sim}{N}_3 & \underset{\sim}{0} \\ & & & \underset{\sim}{Q}_q^{-1} \end{bmatrix} \quad \ldots \ldots \quad (5.43)$$

$$
\left[\begin{array}{cc|c|cccc}
\underset{8*8}{N_\ell} & \underset{8*8}{0} & & & & & \\
\multicolumn{2}{c|}{\underset{16*16}{\sum_{i=1}^{m} D_p^T Q_p^{-1} D_p}} & \underset{16*12}{\sum_{i=1}^{m} D_i^T Q_i^{-1} A_i} & \underset{16*n_1}{D_1^T Q_1^{-1} B_1} & \cdots & & \underset{16*n_n}{D_n^T Q_n^{-1} B_n} \\
\underset{8*8}{0} & \underset{8*8}{N_r} & & & & & \\
\hline
\multicolumn{2}{c|}{\underset{12*16}{\sum_{i=1}^{m} A_i^T Q_i^{-1} D_i}} & \underset{12*12}{\sum_{i=1}^{m} A_i^T Q_i^{-1} A_i} & \underset{12*n_1}{A_1^T Q_1^{-1} B_1} & \cdots & & \underset{12*n_n}{A_n^T Q_n^{-1} B_n} \\
\hline
\multicolumn{2}{c|}{\underset{n_1*16}{B_1^T Q_1^{-1} D_1}} & \underset{n_1*12}{B_1^T Q_1^{-1} A_1} & \underset{n_1*n_1}{B_1^T Q_1^{-1} B_1} & & & \\
\multicolumn{2}{c|}{\vdots} & \vdots & & \ddots \; \underset{n*n}{N_c} \; \ddots & & \\
\multicolumn{2}{c|}{\underset{n_n*16}{B_n^T Q_n^{-1} D_n}} & \underset{n_n*12}{B_n^T Q_n^{-1} A_n} & & & & \underset{n_n*n_n}{B_n^T Q_n^{-1} B_n}
\end{array}\right]
\left[\begin{array}{c}
\underset{8*1}{a_\ell} \\
\\
\underset{8*1}{a_r} \\
\hline
\underset{12*1}{x} \\
\hline
\underset{n_1*1}{c_1} \\
\vdots \\
\underset{n_n*1}{c_n}
\end{array}\right]
=
\left[\begin{array}{c}
N_\ell a_{o\ell} \quad 8*1 \\
\sum_{i=1}^{m} D_i^T Q_i^{-1} m_i \quad 16*1 \\
\underset{8*1}{N_r a_{or}} \\
\hline
\underset{12*1}{\sum_{i=1}^{m} A_i^T Q_i^{-1} m_i} \\
\hline
B_1^T Q_1^{-1} m_1 \quad n_1*1 \\
\vdots \\
N_c c_o \quad n*1 \\
\vdots \\
\underset{n_n*1}{B_n^T Q_n^{-1} m_n}
\end{array}\right].
$$

Figure 5.2 Normals Structure for Sequential Adjustment of Interior Orientation and Control

The solution can now be multiplied out and the right hand side of the normals simplified using (5.41):

$$\begin{bmatrix} \underset{\sim}{a} \\ \underset{\sim}{x} \\ \underset{\sim}{c} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{N}_1 + \underset{\sim}{D}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{D}_q & \underset{\sim}{N}_{12} + \underset{\sim}{D}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{A}_q & \underset{\sim}{N}_{13} + \underset{\sim}{D}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{B}_q \\ \underset{\sim}{N}_{12}^T + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{D}_q & \underset{\sim}{N}_2 + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{A}_q & \underset{\sim}{N}_{23} + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{B}_q \\ \underset{\sim}{N}_{13}^T + \underset{\sim}{B}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{D}_q & \underset{\sim}{N}_{23}^T + \underset{\sim}{B}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{A}_q & \underset{\sim}{N}_3 + \underset{\sim}{B}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{B}_q \end{bmatrix}^{-1} \begin{bmatrix} \underset{\sim}{k}_1 + \underset{\sim}{D}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{m}_q \\ \underset{\sim}{k}_2 + \underset{\sim}{A}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{m}_q \\ \underset{\sim}{k}_3 + \underset{\sim}{B}_q^T \underset{\sim}{Q}_q^{-1} \underset{\sim}{m}_q \end{bmatrix} \quad (5.44)$$

Inspection of this solution in conjunction with equations (5.39) to (5.41) will show that it is identical to the simultaneous solution given by (5.37). The form of the above solution clearly demonstrates the principle of sequential adjustment of observations, as the contributions of the new observations are added to the normal equations just as they were in equation (3.11).

The process of adding new sets of observations can of course be carried on indefinitely, improving the values of the unknown parameters as well as their precisions. Only the current values of the parameters and their weight coefficient matrix need to be retained for use in the next phase of the adjustment.

6. EXPERIMENTAL EQUIPMENT

6.1 Instrumentation

6.1.1 Stereocomparator and Digitizer

The stereocomparator used for the experimental work was a Zeiss Jena Stecometer C (see Figure 6.1). The Stecometer can accept any size photography up to 23 cm by 23 cm and the plate carriers are capable of rotation to align the photographs with the comparator coordinate system. There is a choice of magnifications and measuring marks in the viewing system, and the photographs can be optically rotated to interchange x- and y-parallaxes.

The Stecometer has manual or motor-driven main x and y motions which move the common lower photograph stage and the optics respectively. The differential x and y movements are controlled by small handwheels or footwheels which move the independent right and left upper photograph stages respectively. The movements are sensed by rotary encoders on the lead screws for each carriage motion. The lead screws have a pitch of one millimetre and the encoders are the light source/line grating type with a sensitivity of one thousandth of a rotation, thereby giving the Stecometer a resolution of one micron.

The digitizer used to interpret the signals from the encoders was a Retab NC1000E (see Figure 6.2). The Retab is a hard-wired digitizer which gives a continuous display of the positions in the four axes. The displays are of the nixie tube type showing a sign and six digits and can be zeroed or set to any required value. The coordinates are displayed in millimetres with a least count of one micron.

The Retab has an event counter which is incremented automatically after each observation is taken. This number has four digits, is continually displayed and can also be set to any required value. A unique eight digit point number can be set for any measurement using the thumbwheel selectors on the remote console (on the left in Figure 6.2). A measurement can be registered from either the console or a footswitch.

The original design of the digitizer allowed for two specific output devices, an IBM typewriter and a FACIT 4070 paper tape punch. The typewriter provides a hard copy listing while the tape punch produces a digital record of the data in a medium which is readily acceptable

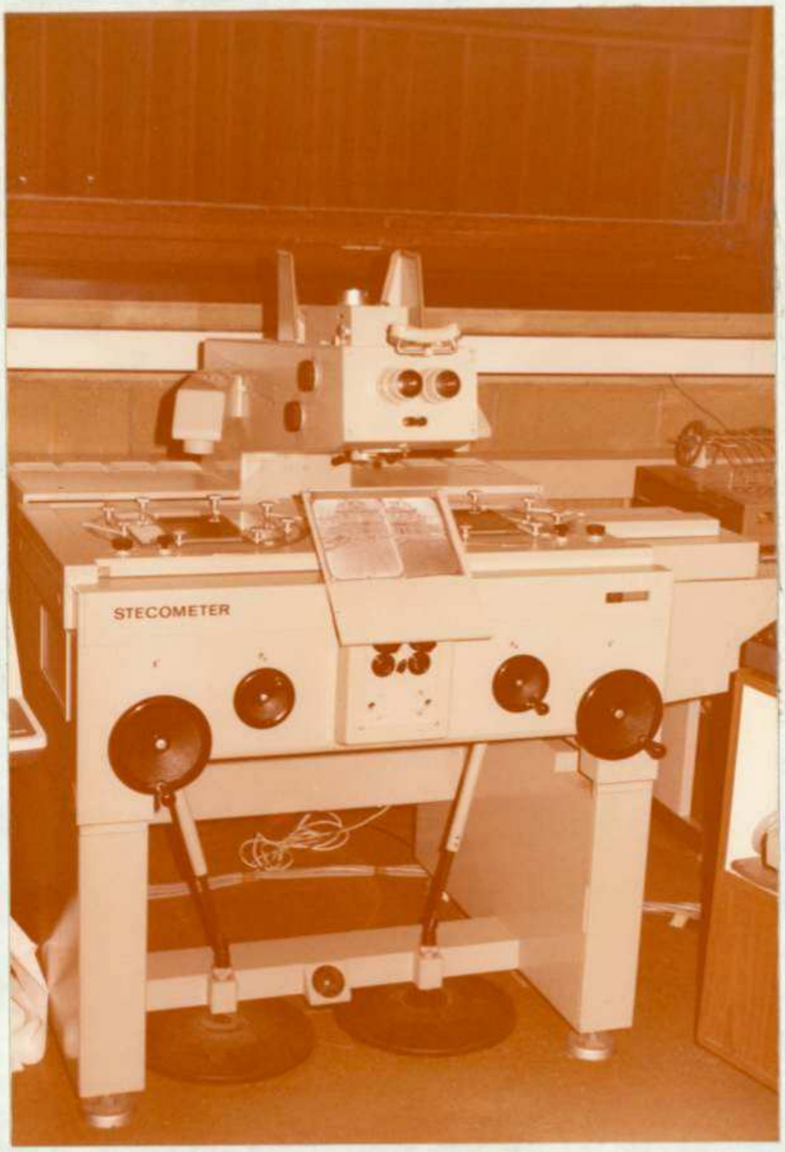


Figure 6.1 Zeiss Jena Stecometer C Stereocomparator

Figure 6.2 Retab NC1000E Digitizer

to most computers. Both devices record the point number, sequence number and the four axis values in the same fixed format.

6.1.2 Photography and Control

The photography for the experimental work was taken with either a Wild Heerbrugg P32 camera or a Zeiss Jena UMK 10/1318 camera (see Figures 6.3 and 6.4). Both are metric cameras with known and stable characteristics and can accept glass plates or roll film. Each is designed specifically for tripod-mounted close-range photogrammetry, but the emphasis is slightly different in each case.

The Wild P32 is a light, portable camera which can be mounted on either a theodolite, a swivel base or a fixed length base bar. The focal length is 64 mm with a format area of 75 mm by 55 mm. The principal point is offset from the centre of the format (see Figure 2.2) and the camera can be rotated about the optical axis to optimise the photographic coverage. The P32 accepts 80 mm by 60 mm glass plates or 120 mm roll film. The emulsion is flattened onto a rear register glass by a pressure platen. The register glass has five engraved fiducial marks which are defined relative to the principal point with a precision of a few microns. The focus of the camera is fixed at 18 m, but at high aperture settings the depth of field extends from approximately 2.5m to infinity. The radial lens distortion does not exceed 5 μm anywhere in the format area.

The Zeiss UMK is a larger, more sophisticated camera which incorporates its own low precision theodolite. The major differences between it and the P32 are that it has variable focus, automatic film winding and less fiducial information. The focus is set in steps from 1·4 m to infinity. The focal length at infinity is 99 mm and steps incrementally with the focus. Whereas the P32 has a manually wound film cassette, the UMK can be fully automatic with a motor drive, which winds the film and then cocks and fires the shutter. Glass plates are accepted and flattened with a pressure platen, while the film cassette uses a vacuum back. Roll film must be cut to the correct width of 130 mm from standard 230 mm aerial film, or 130 mm by 180 mm glass plates can be used.

The effective format area is 160 mm by 110 mm and has only four fiducial marks. Because of the variable focus the positions of the marks relative to the principal point cannot be guaranteed to better than

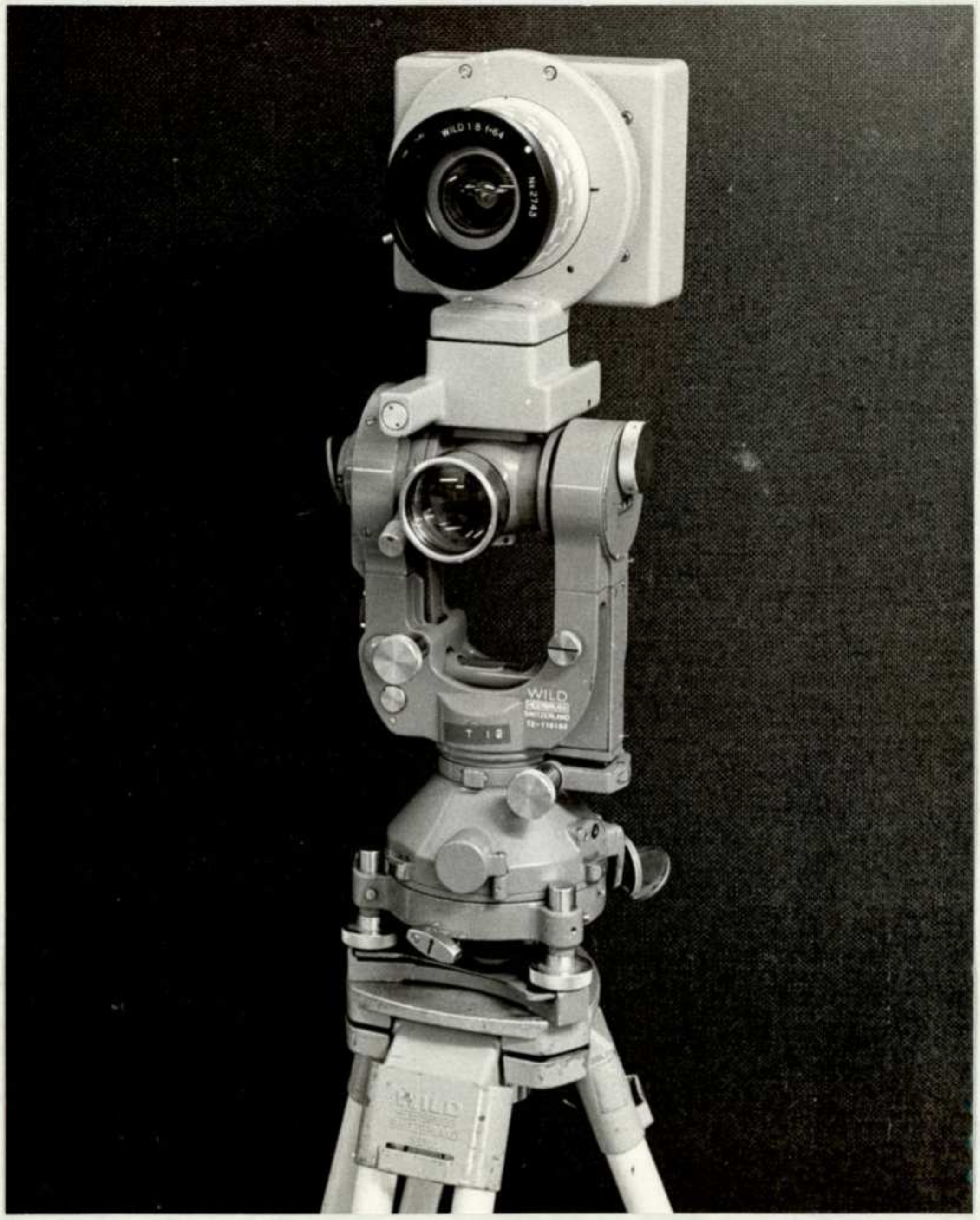


Figure 6.3 Wild Heerbrug P32 Terrestrial Camera and T2 Theodolite

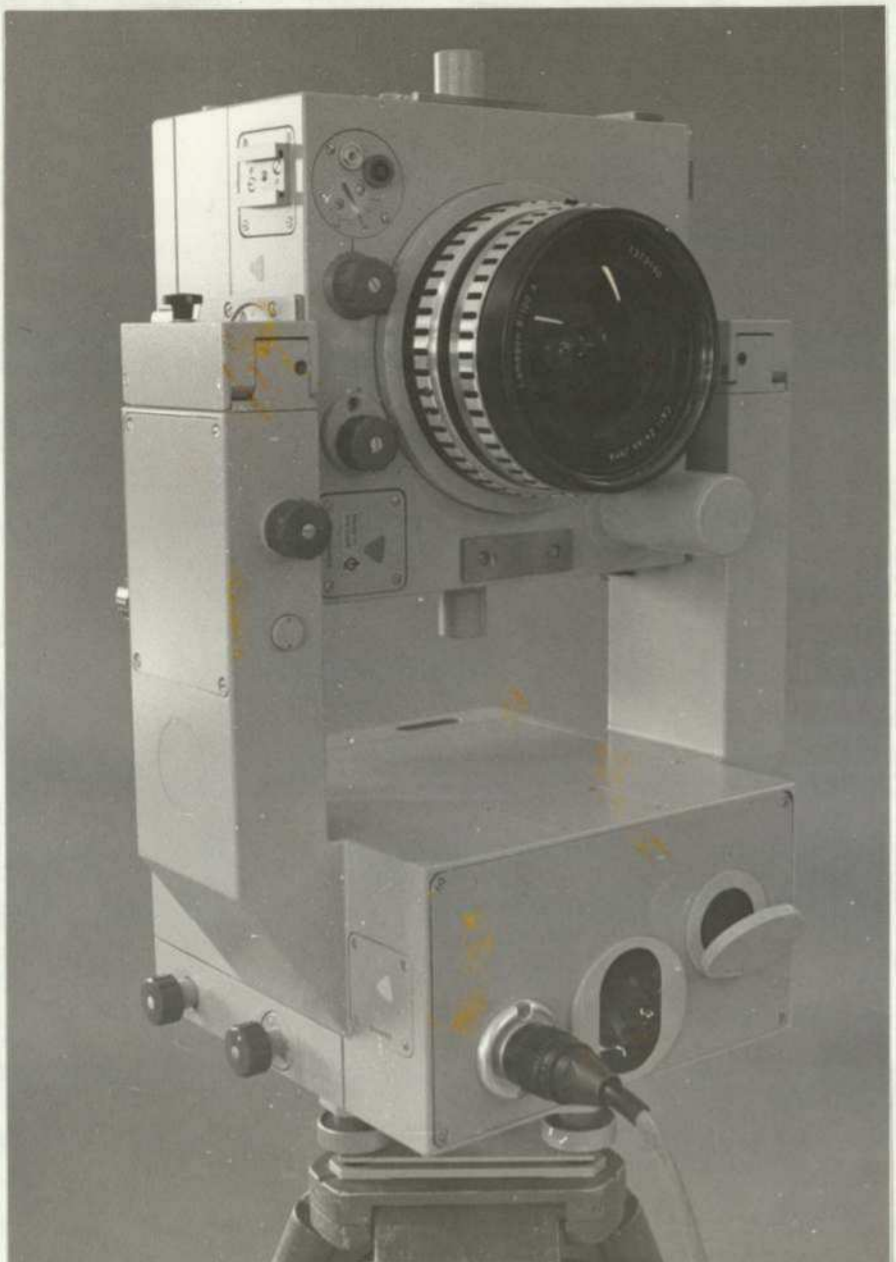


Figure 6.4 Zeiss Jena UMK 10/1318 Terrestrial Camera

20 microns, presumably because the movement of the focussing element in the lens may disturb the position of the principal point. The radial distortion characteristics also change quite considerably with changing focus. At infinity focus the calibrated distortion is less than a few microns while at the closest settings it exceeds 20 microns. The camera can be rotated in its mountings to make best use of the oblong format, but cannot be oriented vertically downwards. A special mount was designed and constructed by Mr.J. Hooker of the Department of Civil Engineering to enable the camera to be used in this attitude (see Figure 6.5).

The P32 is an excellent camera for field and rough country surveys where portability is essential. The UMK is the more versatile camera because of its variable focus and wide angle of view. Also it has an advantage in accuracy because of its longer focal length. Both cameras are preferably used with glass plates. The pressure platen in the P32 is not totally efficient and although the UMK vacuum back is more reliable, the advantage is nullified by the lack of fiducial information. The choice of a camera for a particular project is primarily decided by the camera-to-object distance and the required accuracy. The UMK was used for the majority of the experimental work either because of short ranges or to obtain the maximum accuracy.

The control for the majority of the stereopairs was provided by geodetic techniques. Geodetic measurements were employed both in the field and in the laboratory as they were considered to be by far the most convenient, versatile and reliable method available. For the larger objects photographed there was no other alternative in any case. A wide range of surveying instrumentation was used, including one-second theodolites, short-range EDM, invar bands and various types of levels.

In only two cases was geodetic measurement not used. In the first, an experimental grid/length bar combination which had been developed for controlling small objects in the laboratory was used. In the second, a three-dimensional measuring machine in an industrial laboratory was used for reasons of convenience and time limitations. Each of these methods will be discussed in more detail in respect to the specific projects.

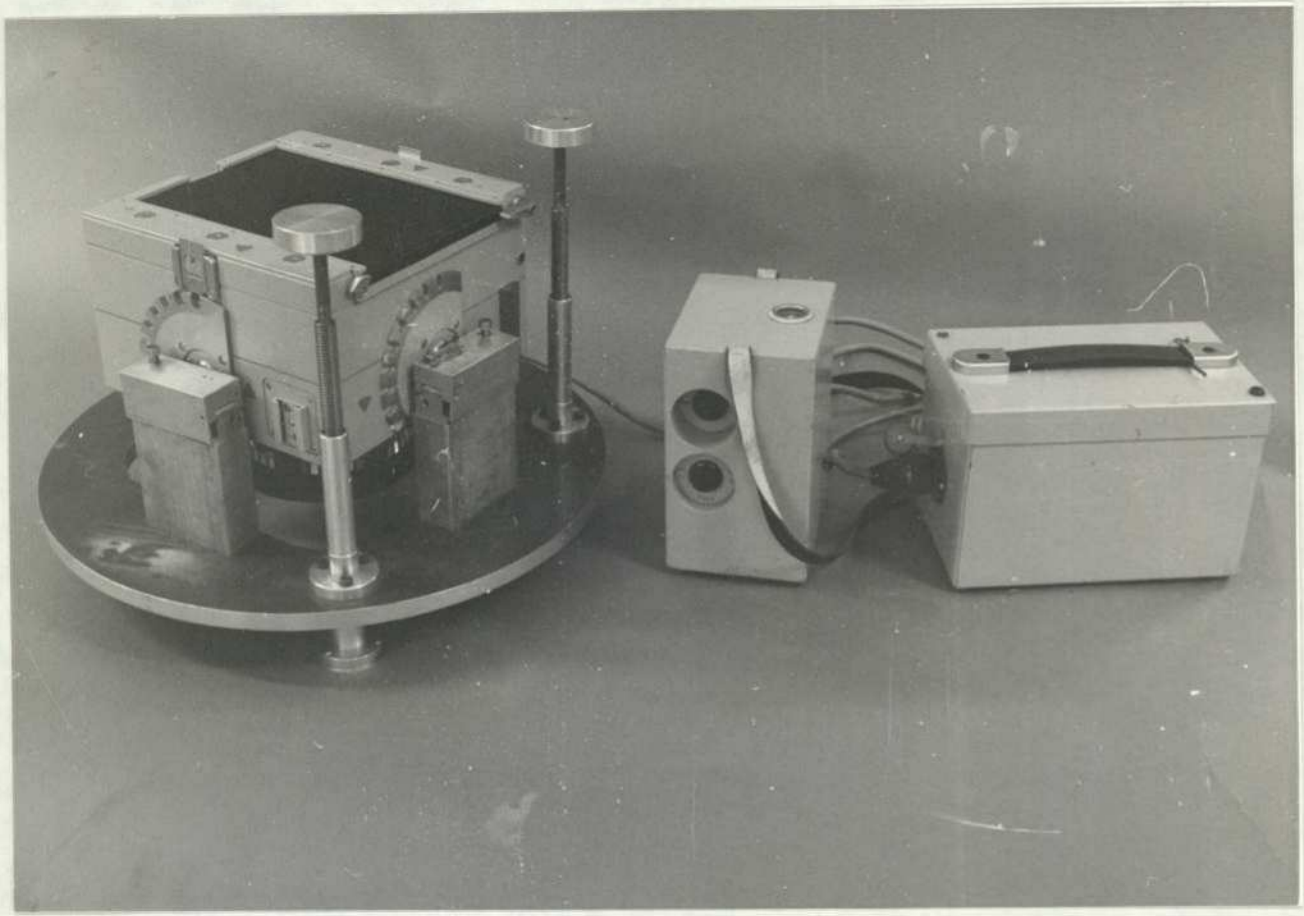


Figure 6.5 UMK in Vertical Mounting

6.1.3 Testing and Calibration

The Stecometer and Retab were periodically tested against a pair of calibrated grid plates placed on the photo carriages of the stereocomparator. A typical calibration consisted of three rounds of pseudo-stereoscopic measurements of ten to twenty grid intersections distributed uniformly throughout the format area. The observations were transformed onto the grid coordinates using the affine transformation and least squares solution described in Chapter 4. The deformational transformation was found to give almost identical results and the extra parameters were barely significant.

The residuals of the adjustment were found to be random in most cases with an even distribution of magnitudes. The root mean square error (RMSE) was generally of the order of 1·5 microns with a maximum error of 3·5 microns. Grid plates have been shown to have precisions of the order of 1 micron, with a maximum error of 2 microns (Szangolies 1968), so therefore the measuring precision of the stereocomparator is of the order of 1 micron.

However, the scale factors on the axes of the comparator showed marginally significant deviations from unity. The factors were 0·99998 and 1·00002 for the x- and y-axes respectively, equivalent to an error of 1·6 microns at the most distant edge of a UMK format. These factors should be applied to any reduction of photogrammetric observations, especially in the case of a camera with minimum fiducial information. The factors should be included in the simple translation/rotation for the interior orientation. If known fiducial positions are available the factors can be ignored as they will be compensated for in the transformation for interior orientation.

The cameras employed for the experimental work were relatively new acquisitions and the manufacturer's original calibrations were considered to be still valid. Different UMK cameras were used, but all were of the same type and of similar age.

The theodolites and levels used in the geodetic control surveys were also periodically tested and adjusted. Invar bands and EDM were calibrated for short-range on a 100 metre base line.

EDM instruments were also calibrated at medium range on a 1200 metre base line and tested for cyclic errors in the laboratory over a double wavelength.

6.2 Data Acquisition and Processing System

6.2.1 Original Offline System

The original system used in conjunction with the Stecometer and Retab was an offline system based on paper tape (see Figure 6.6). The punched tape from the FACIT had to be transported manually from the laboratory to the computer centre and first converted to cards. The paper tape invariably held data errors and point misnumberings which had to be edited from the record. This was conveniently carried out by removing and editing cards from the card deck. Any errors would be noted on the typewriter listing to indicate what changes had to be made.

The processing of the data was carried out on a CDC 6600 mainframe computer at the University of London Computer Centre via a card reader/line printer satellite link. The program used was developed by S. G. Bervoets of the Department of Surveying at the University of Melbourne as an aerotriangulation block adjustment and adapted to close-range photography by M. A. R. Cooper of the Department of Civil Engineering at The City University. The program XYZBLC is composed of six separate modules, of which four must be run in sequence to convert raw comparator measurements into object space coordinates for a single stereopair (Cooper 1979).

Data for single stereopairs was generally held on cards, but data from larger jobs could be catalogued on the CDC computer on disk file. In either case further data editing was rarely avoided. Undetected gross errors or point misidentifications were usually the cause of the problem, with tape or card punching errors occurring almost as frequently. Each module of the program has its own input and output which must be organized correctly, and this was another source of delays in processing.

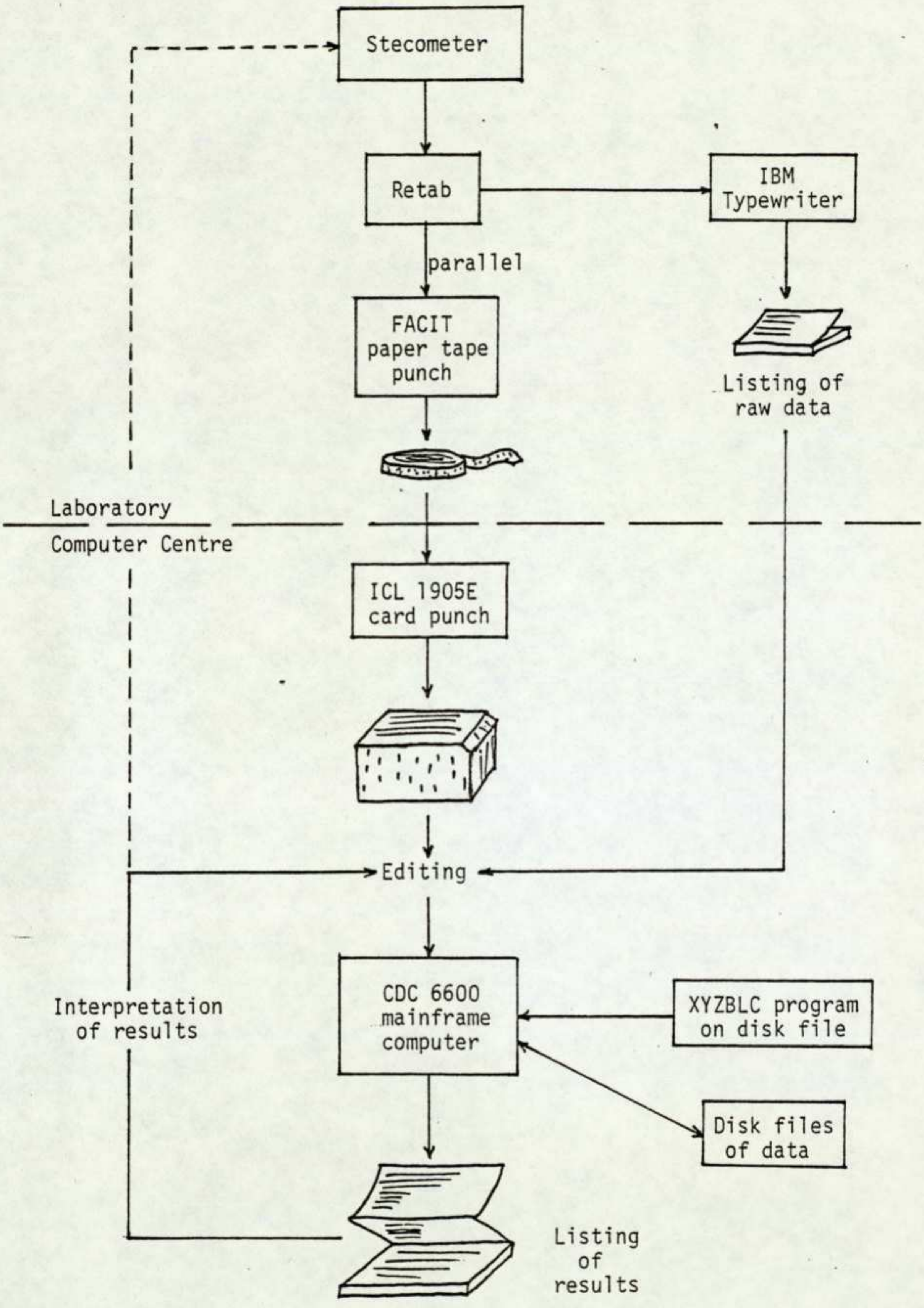


Figure 6.6 Offline Photogrammetric Data Acquisition/Processing System

Once the data had been successfully processed by the system it was occasionally necessary to return to the stereocomparator to reobserve misidentified points or to observe points which had been inadvertently omitted. The turnaround time for this system, that is, the time from the original observations to successful completion with satisfactory results, could range from a few hours as a rare minimum to over a week as a more common maximum. The most common turnaround time was of the order of two days. The observations and the initial editing occupied the first day and at least two full computer runs could require another day or possibly even longer.

The disadvantages and inefficiencies of this and any similar offline system are self-evident. Changes in the medium used for the data are certain to cause difficulties and the use of a program not specifically designed for single close-range stereopairs is very undesirable. However the major disadvantage is the delay between the observations of the stereopair images and the arrival and analysis of the results. By the time the final listing arrives from the mainframe computer the photographs may have been removed from the stereocomparator and the operator will no longer have a detailed knowledge of the photography. This hinders the interpretation of the results and the photographs have to be re-introduced to the comparator if additional observations are required.

6.2.2 Proposed Online System

The online system initially proposed to replace the offline system is shown in Figure 6.7. This system would eliminate all the problems associated with the offline system once a vector solution program was available for the processing of close-range stereopairs. The basic arrangement of the Stecometer and Retab is unchanged, and the dedicated line to the IBM typewriter cannot be altered.

The parallel output normally sent to the tape punch would be instead accepted by a minicomputer, via a parallel input/output interface, to eliminate the physical storage media of paper tape and punched cards. Because this would be a direct link, there is little chance of data errors occurring in the transmission. The IBM typewriter would not be required.

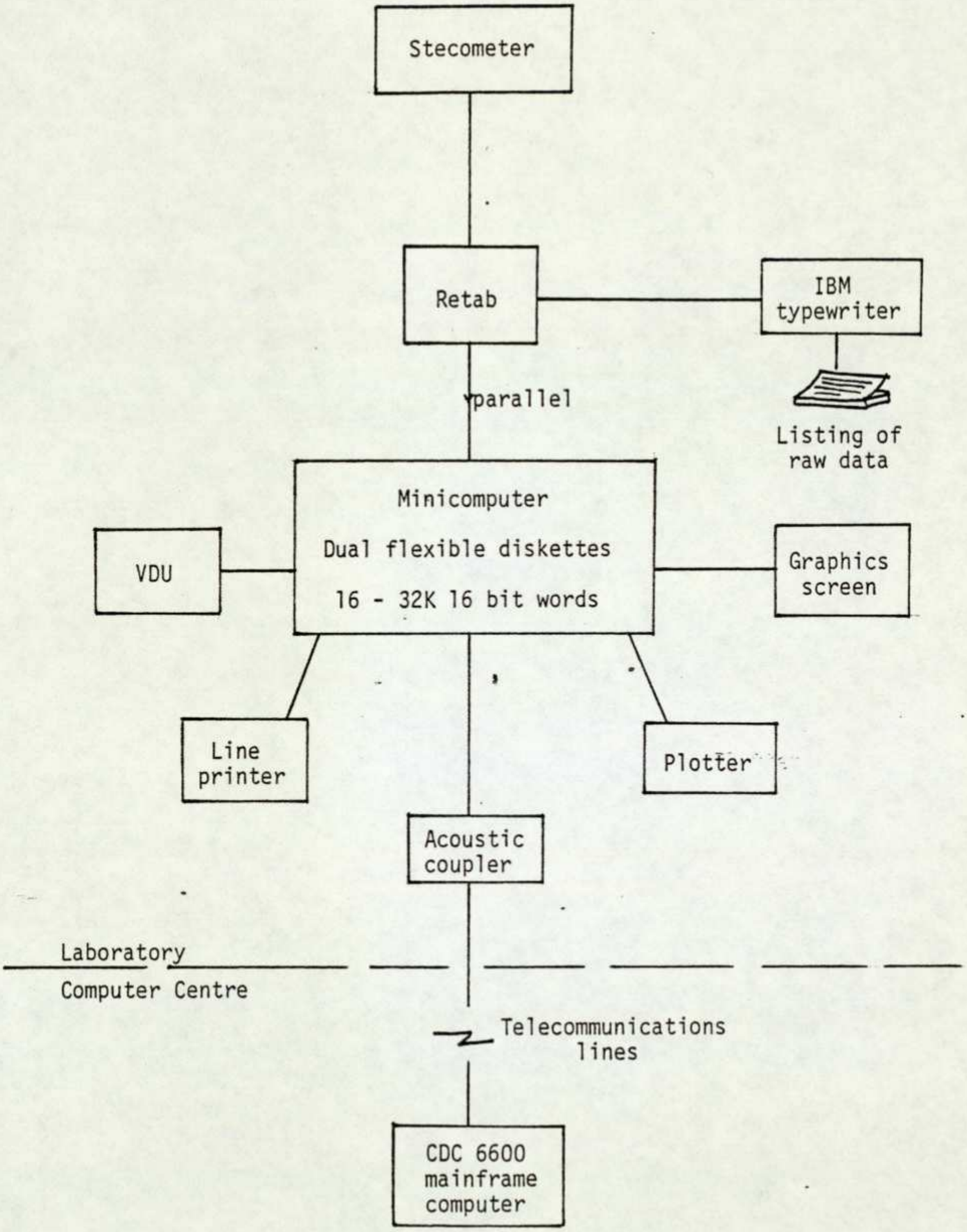


Figure 6.7 Proposed Data Acquisition/Processing System

The minicomputer would have to have at least 16K of 16 bit words for the processing of sophisticated restitutions of single stereopairs, and dual flexible diskettes to store programs and data. Of the high level languages available, Fortran IV is preferable because the majority of surveying and photogrammetric programs are written in this language. Many existing routines on the CDC 6600 would be applicable to the online system and conversions to a new language would be avoided.

The two essential peripherals of the minicomputer would be a visual display unit (VDU) and a line printer. The VDU can display instructions, queries, data and results of analyses to the operator and the line printer can provide hard copies of programs, data and results when required. Three other peripherals must be optional future additions. A graphics screen and some type of plotting table are necessary to make the system completely self-contained. The screen could be used to edit graphical output before it is sent to the table as a final drawing.

The third link would be to an acoustic coupler which in turn would provide a link to a mainframe computer, probably the CDC 6600. This link will enable the transmission of data for aerotriangulation block adjustments which the minicomputer is not capable of processing. The XYZBLC program suite could be transferred in part to the minicomputer to carry out some of the initial computations, reducing the reliance on computer processing outside of the laboratory.

This ideal online computing system would remove all the difficulties associated with offline computing and considerably increase the efficiency of the photogrammetric data acquisition and processing. Data editing can be carried out both before and during processing, reducing the turnaround time to a minimum. Results of a restitution are obtained immediately and any additional observations can be taken if and when they are required. The additional peripherals would enable the system to become almost completely self-reliant, only resorting to assistance from the mainframe when large adjustments of multistation photography were required.

6.2.3 Selection of a Processor

Some criteria for the selection of the minicomputer have already been referred to in the previous section. These will be elaborated on in this section and others will be added, but the main consideration was undoubtedly cost. The ideal system presupposes that enough funds are available to purchase all the items, but in reality there was a definite ceiling on the amount which could be committed both for initial and eventual purchases.

The first effect of this ceiling was the restriction of the processor to the class of machines which would be called "small" minicomputers. Small minicomputers are limited to a central processor unit (CPU) of approximately 32K of 16 bit words of memory by the largest address which can be referred to by one of its 16 bit words ($2^{16}-1 = 177777_8$ = 64K bytes = 32K words). "Large" minicomputers use various techniques to avoid this problem, notably virtual arrays and memory management, which generally double the cost of the CPU alone. Because this class of processor is aimed at a completely different market the total package is generally larger and more versatile, so the gap between small and large minicomputers is considerable in terms of power and cost.

The second effect of the cost ceiling was to limit the number of peripherals to one. A VDU was preferred to a hard copy terminal for a number of reasons. Firstly, although some hard copy is always required, a line printer always gives hard copy whether it is required or not. This is wasteful, especially for the everyday running of the system where the listing is discarded almost as soon as it is printed, and can become costly. On the other hand VDU terminals do not require a continuous supply of paper and are much faster when displaying data or results. Some VDU's were available with an integral screen copying unit and limited graphics, and this was thought to be the best compromise. The screen copier could supply essential listings of programs, data and results, and the graphics could be used to enhance the presentation of displays for the operator.

The second criterion to be considered is the operating system and the programming language. It has already been stated that Fortran IV was absolutely necessary, but this was not a problem as all scientific minicomputers with CPU's larger than 16K supply Fortran as a high level language. However, the operating systems and the interactive capability of the high level languages varied considerably. The operating system should have clear, simple and well-documented instructions to carry out the everyday "housekeeping" of the minicomputer. The system should be capable of allowing simple and versatile communication between the high level language program and the operator at the VDU. Prompts to the operator and data entry by the operator should be made as simple as possible for the efficient execution of online programs.

The third criterion was the size of the processor and the mass storage device(s) associated with it. Between 16K and 32K words were considered to be the acceptable range of CPU memory. Below 16K words restitutions become limited in scope while those above 32K words have already been vetoed by the cost criterion. The mass storage device preferred was dual flexible diskettes (floppy disks). These had proven their reliability and performance and were relatively low in cost. The other options of magnetic tapes or cassettes and fixed or cartridge disks were ruled out by slow access or high cost.

The fourth criterion was the interfacing of peripherals. Even though it was certain to involve an amount of unfamiliar low level language programming, a high speed parallel interface was thought to be the only possibility for the direct connection of the Retab and the minicomputer. The processor would have to have such an interface from the outset, and also have optional serial interfaces for the peripherals which would be added at a later date. Small minicomputers generally have an option on the type and number of interfaces which can be attached over and above the standard interfaces provided. All machines have a single serial interface for the console terminal and many also give a low-speed parallel port.

Other criteria which played a part in the selection process were the documentation of the system and languages, the availability of languages other than Fortran, the portability of the unit, the reputation of the manufacturer and the servicing arrangements. The availability of consultancy services for system programs and hardware was considered to be bonus rather than a criterion.

6.2.4 Interim Online System

The basic system finally purchased for the experimental work is shown in Figure 6.8. A Digital PDP11v03 with 32K words of CPU memory and dual floppy disks was selected in conjunction with VT55 Decscope display (see Figure 6.9). The VT55 has a screen copier and a limited graphics capability. This combination was selected primarily because it fulfilled most of the requirements of the previous section, and the manufacturer had the highest reputation for reliability and documentation.

A parallel interface was included with the system, but its utilization and the consequent machine language programming was avoided at the suggestion of the Chief Electronic Engineer of The City University Computer Unit, [redacted]. The design of the interface is extremely simple in principle. The parallel output from the Retab is converted into serial data and fed into the communication line between the VDU and the minicomputer. The computer reacts as if the signals were coming from the VDU and the VDU reacts as if they were coming from the computer. This means that incoming characters are displayed instantly and can be read by the computer as if they were a keyboard entry from the VDU.

The interface has proved to be very reliable and the typewriter is not used as the verification of raw data is unnecessary. The typewriter is the slowest of any of the devices, so when it is cut out of the system the data transfer is extremely fast. Although the parallel interface would be even faster still, the complicated data retrieval from the interface would slow its entry into the program. The current interface only requires a simple Fortran READ statement.

The running system supplied with the minicomputer as standard was slightly inadequate for the everyday work with the machine in terms of convenience and efficiency. A custom-made system was prepared using a system generation program with a number of alterations incorporated. The operating system routines and monitors occupy approximately 4K words of CPU memory each, leaving 24K words for user programs.

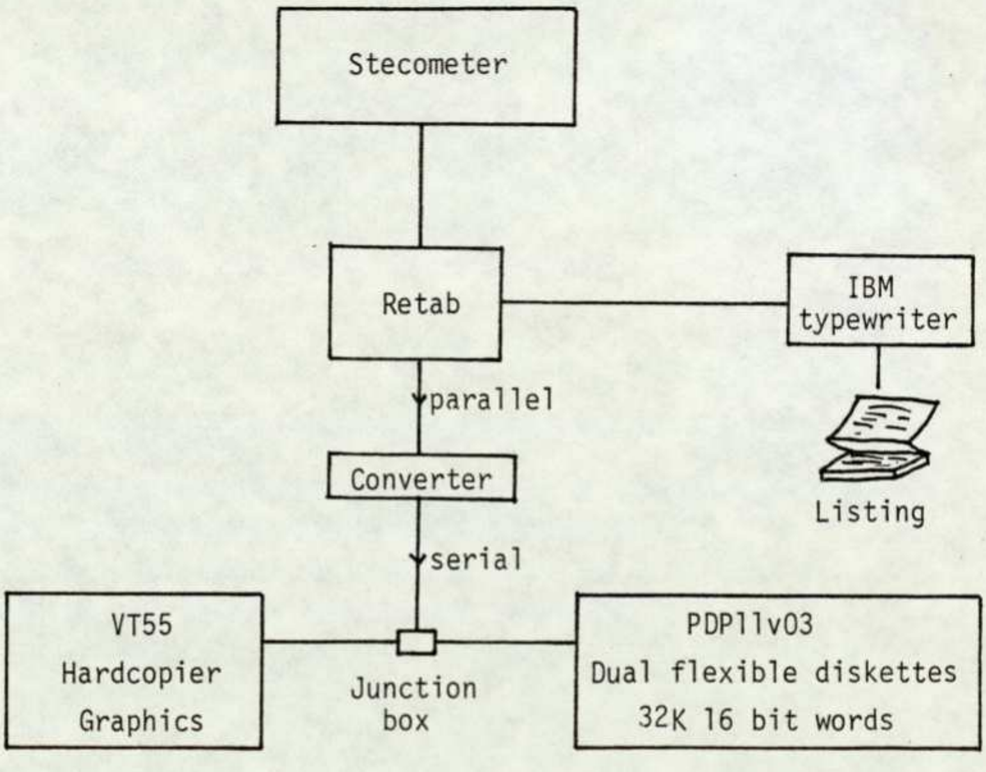


Figure 6.8 Interim Data Acquisition/Processing System

Each floppy disk can record 56K words of programs and/or data, but all the system programs cannot be stored on a single disk. A number of system disks were prepared to fulfil different functions. A simple guide to the use of these disks and the main system commands was prepared for new users of the minicomputer, including an introduction to the file editing techniques available.

This interim system has been updated since the beginning of the research by the addition of another stereocomparator/digitizer combination, a line printer and a replacement digitizer for the Retab. However the same interim system was used for all the experimental work related to the research, to maintain consistency between the observations of different stereopairs.

Figure 6.9 Digital PDP11v03 Minicomputer and VT55 Display Unit

7. INITIAL TESTING

7.1 Preliminary Programs

7.1.1 Fictitious Data Generation

All programs utilizing the vector solution and sequential adjustment were initially tested using fictitious data, both to check the execution of the programs and investigate the reaction of the adjustments to variations in precisions of control and observations. The data was supplied by DATGEN (Data Generation), an interactive program run online on the minicomputer. The program is based on the interior orientation transformations and vector equations already described and is capable of introducing normally distributed random perturbations into both the control positions and the photogrammetric observations. Random variates with a uniform distribution are supplied by an online processor function and converted to normal variates using a short routine based on the central limit theorem.

The main sequence of the program is shown in Figure 7.1, beginning with the input of control and camera specifications and ending with the output of stereocomparator observations. Input and output, the acceptance or rejection of different options and the selection of alternatives are all controlled by prompts to the operator, and this applies to all interactive programs. The prompts are generally in the form of a question to which a yes (Y or 1) or no (N or 0) answer can be given, or in the form of possible choices which require the number of the choice to be selected. In this way the operator chooses which sections of the program are utilized, what computations are performed and most importantly configures the outputs of the program.

The output of the program consists of two or three files of numerical data necessary for the restitution of a stereopair observed on a stereocomparator. The first file contains the coordinates of fiducial marks and control points. The second file is optional and contains the weight coefficient matrix of the control coordinates. The third file consists of the stereocomparator observations of the fiducial marks, the control points and any observed points in the object.

The major features and options of each main section of the program are described briefly below:

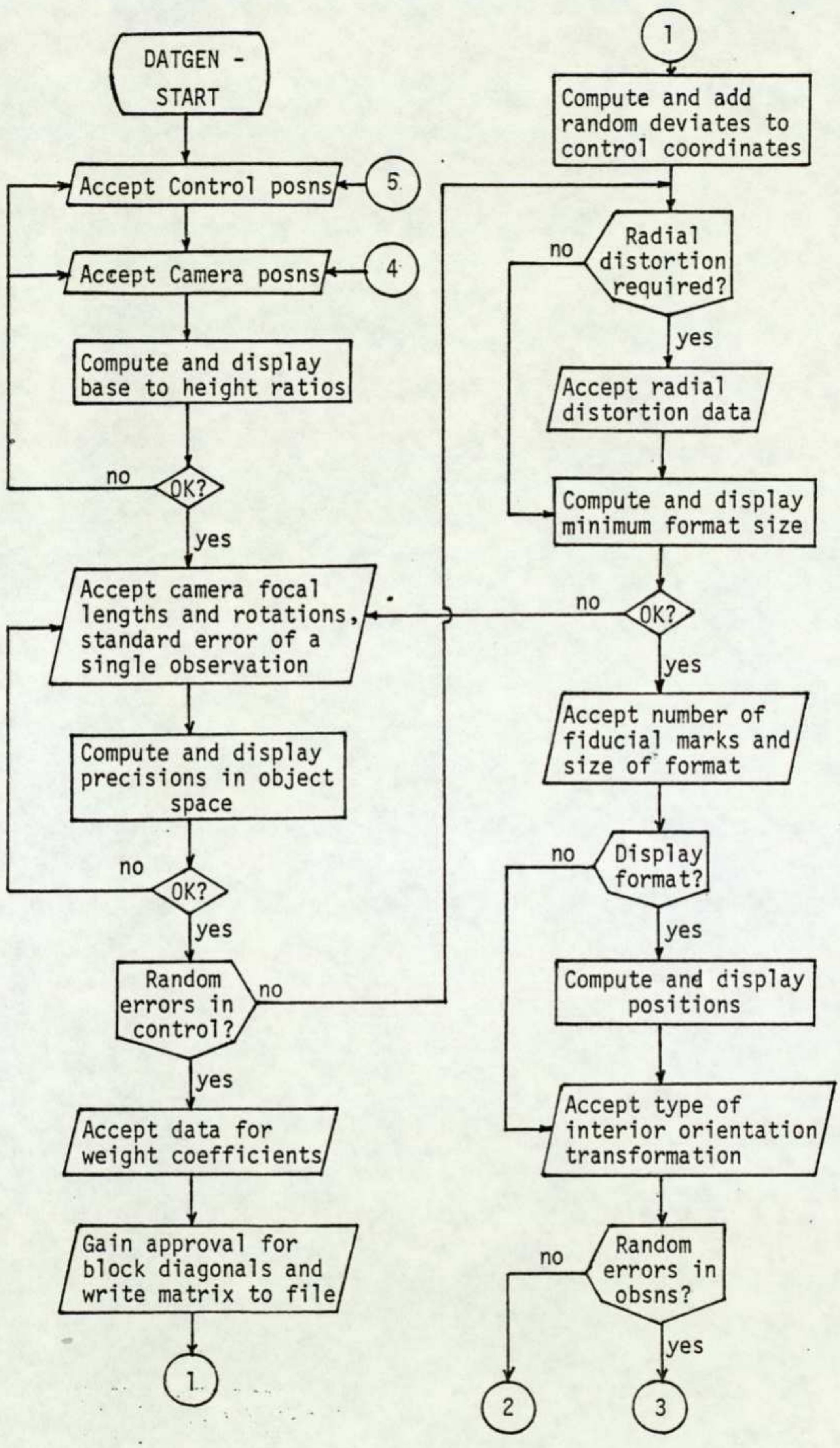


Figure 7.1 Flowchart for Program DATGEN (cont. next page)

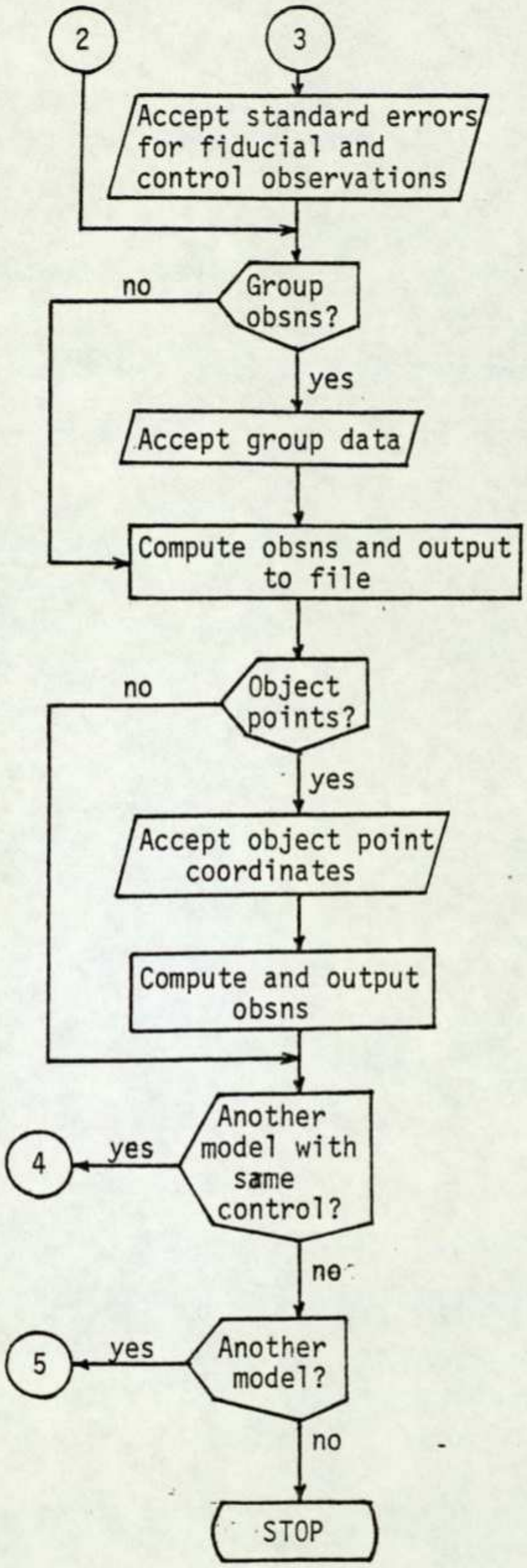


Figure 7.1 Flowchart for Program DATGEN (cont. from 149)

(a) The control coordinates may be typed in at the keyboard, read from a disk file, or selected from a standard set of configurations given the specification of some key dimensions. Fully known, height, position and parallax points are differentiated by control indices indicated in Table 7.1.

(b) Any single pair of camera locations can be specified relative to the control datum. The base distance ratios for the nearest and farthest points are indicated to the operator.

(c) Any focal length and orientation can be specified for the two cameras independently. The expected precisions in the object space are computed for a given standard error of observation on the photographs.

(d) Random errors can be optionally injected into the control coordinates. The weight coefficient matrix is generated by a semi-automatic process as a diagonal, block diagonal or full matrix. The block diagonal component for each control point must be approved by the operator. Each coordinate is perturbed by a random variate with a standard deviation taken from the matrix diagonal.

(e) Radial lens distortion can be optionally specified by a polynomial series expansion to the power twenty, or up to twenty values on a distortion curve with selected spacing and largest radius.

(f) The minimum size of the photograph format to contain all control image positions is indicated to the operator.

(g) Four, five or nine fiducial marks are possible with automatic positioning once the dimensions of the format are specified. The fiducial mark positions can be optionally perturbed by random variates with a specified standard deviation. A display of the fiducial and control image positions for the left photograph is optional.

(h) Either a simple translation/rotation or rectilinear, affine and deformational transformations are available for the interior orientation. The parameters can be entered manually or automatically subject to approval by the operator.

(i) The number of rounds of measurements and their standard errors (optional) can be specified separately for fiducial and control observations. Random variates are injected into the observations if a non-zero standard error is specified. The control observations can be grouped, with any point appearing in any number of groups.

(j) The object coordinates of observed points can be entered via the keyboard or read from a disk file.

(k) The program can be restarted to generate additional stereopairs of the same object control.

Table 7.1 Control Point Indices

Index		Meaning
3	XYZ	coordinates known with statistical error - full control point.
2	XY	coordinates known with statistical error, Z unknown - position point.
1	Z	coordinates known with statistical error, XY unknown - height point.
0	No	coordinates known - parallax point.
-1	Z	coordinate known with statistical error, XY known and fixed - position datum.
-2	XY	coordinate known with statistical error, Z known and fixed - height datum.
-3	XYZ	coordinates known and fixed - datum point.

The various options allow the simulation of real data from any type of camera, object or control distribution. The output can be configured for a simple restitution, a sequential adjustment of control, interior orientation, additional observations or any combination.

7.1.2 Generation and Interpretation of Weight Coefficient Matrices

The maximum number of control points allowed for the restitution programs was determined by the available memory. In the case of a simple vector solution with fixed control and an independent interior orientation, the largest amount of memory was available for data storage and up to thirty control points could be accepted with ease. It is unlikely that any single stereopair would encompass more than thirty control points, primarily because any increase in the number of control points above approximately twenty does not show significant improvements in the restitution, assuming the points are well distributed throughout the object space.

The full weight coefficient matrix for thirty control points could have as many as 8,100 elements if all the points were full control points. The storage of all these elements on disk file can be reduced in two ways. Firstly, all weight coefficient matrices are symmetrical, so only the upper or lower triangular half needs to be retained to define the entire matrix. The routine used to invert weight coefficient matrices requires the upper triangular half stored column by column, so this file structure was adopted.

Such a complex file structure makes direct interpretation or editing extremely difficult, and it is more probable that an entire new matrix would be formed if any changes were required. For these reasons the files were written in binary code rather than ASCII code. Binary code is the most efficient storage method available to the computer, and reduces file sizes by a factor of approximately two. It has the disadvantage that the numbers are not in a form which is meaningful to the operator and many file manipulation routines, but as long as only whole files were dealt with no serious problems were encountered.

Four programs were available to generate and interpret the weight coefficient matrix files, again run interactively on the minicomputer. Two of these programs generated such files and two were for interpretation. The first pair of programs simply generated and interpreted diagonal weight coefficient matrix files as a special case. The generation program required the input of point and index numbers followed by the appropriate number of standard deviations, from the keyboard. The interpretation program displayed the standard deviations point by point once the file name was specified and the file located.

The second pair of programs was more sophisticated. The generation program accepted the details of error ellipsoids rather than simple standard deviations and constructed a block diagonal weight coefficient matrix. A three-dimensional error ellipsoid describes the probability of the position of a point in space and is uniquely defined by the three by three weight coefficient matrix of a full control point (Wong 1975b). A two-dimensional error ellipse can be similarly defined for a point with only the XY position known. The interpretation program could compute such error ellipses from either a block diagonal or full matrix and display them using the limited graphics of the VDU (see Figure 6.9). This program was originally developed in conjunction with a variation of coordinates program for the analysis of survey networks.

The variation of coordinates program was the only other method of generating full weight coefficient matrices apart from DATGEN. The two generation programs mentioned above were utilities designed to produce quickly the matrix required for sequential adjustment of control when non-geodetic measurement techniques were used. The program TDVC (Three-Dimensional Variation of Coordinates) filled the dual role of a convenient method of reducing redundant surveying observations into useful coordinate data and a matrix generator. Originally designed for offline use on a mainframe computer (Shortis 1977), TDVC was converted for interactive use on the minicomputer and updated to accept not only horizontal angles, angles of elevation and slope distances, but also horizontal distances, zenith distances and differences in level. The weight coefficient matrix of the point coordinates is extracted from the least squares adjustment solution and written to a file with optional inclusion of the estimate of the variance factor.

7.2 Restitution Programs Without Control Adjustment

7.2.1 Programs STECT/A

The restitution programs STECT and STECA utilize the aforementioned interior orientation transformations and vector solutions for terrestrial and aerial stereopairs, respectively, to convert observed stereocomparator coordinates of conjugate image points into object space coordinates. The programs are capable of weighting the control and the sequential adjustment of photogrammetric observations. The weights of the control are extracted from either diagonal, block diagonal or full weight coefficient matrices and no scratch files are used to store data or matrices. Up to ten fiducial marks and thirty control points can be included in the independent least squares adjustments for interior and exterior orientations, and the number of observations is unlimited.

The flowchart of STECT/A is shown in Figure 7.2 and the major features and options are described briefly below:

(a) any number of fiducial marks can be specified to a maximum of ten

(b) if fiducial mark positions are unknown, a simple translation/rotation is mandatory

(c) if fiducial mark positions are known then an affine or deformational transformation can be selected. The precisions of the fiducial coordinates can be specified to be included in the adjustment. The least squares solution uses direct computation of the normals

(d) the means and standard errors of the observations are displayed to aid error detection. The interior orientation parameters and residuals of measurement are also displayed

(e) radial distortion can be optionally included in the form of a series expansion or as curve values

(f) if control weighting is required the appropriate submatrices are extracted from a specified weight coefficient matrix file or the diagonal terms only can be entered from the keyboard

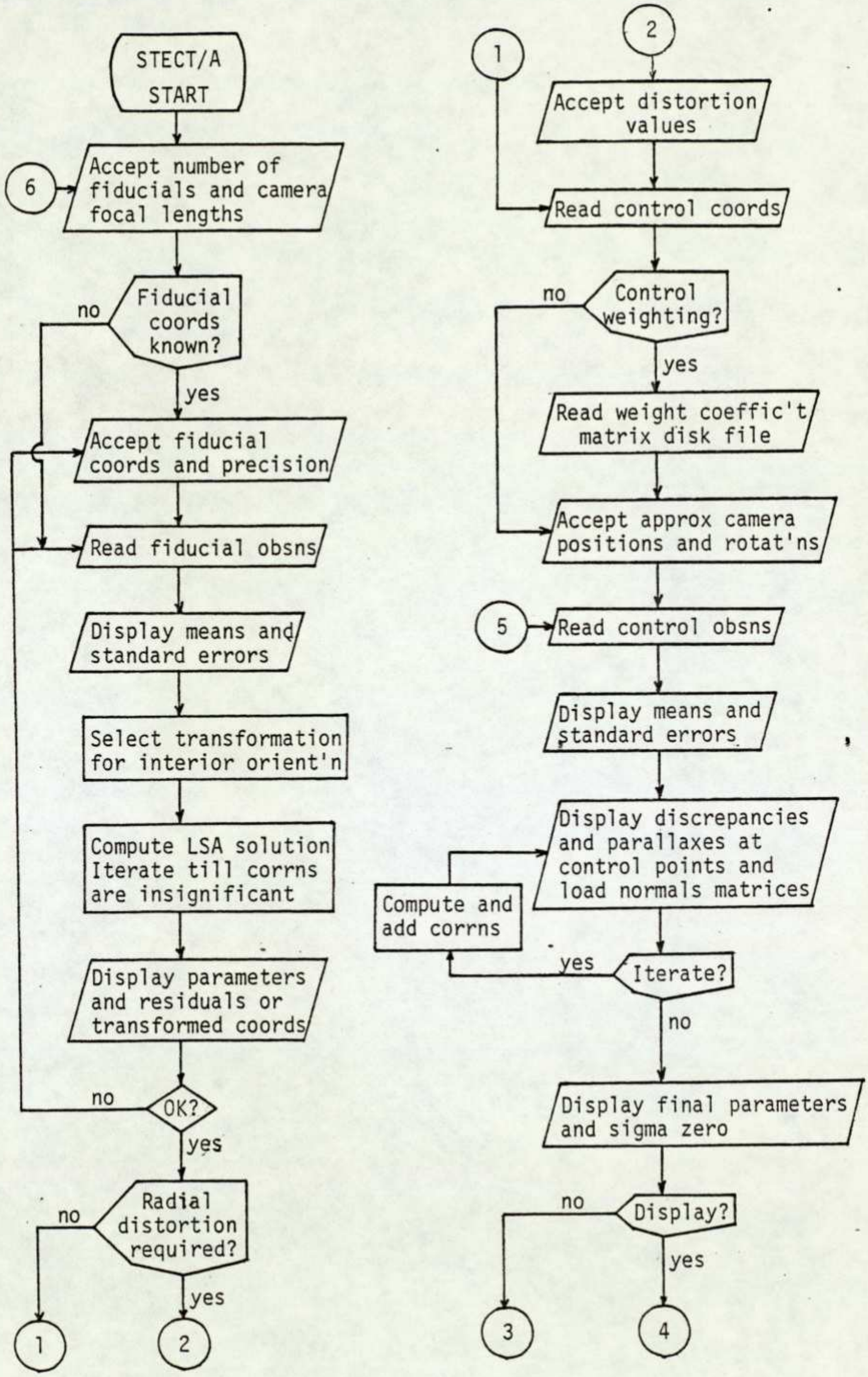


Figure 7.2 Flowchart for Programs STECT/A (cont. next page)

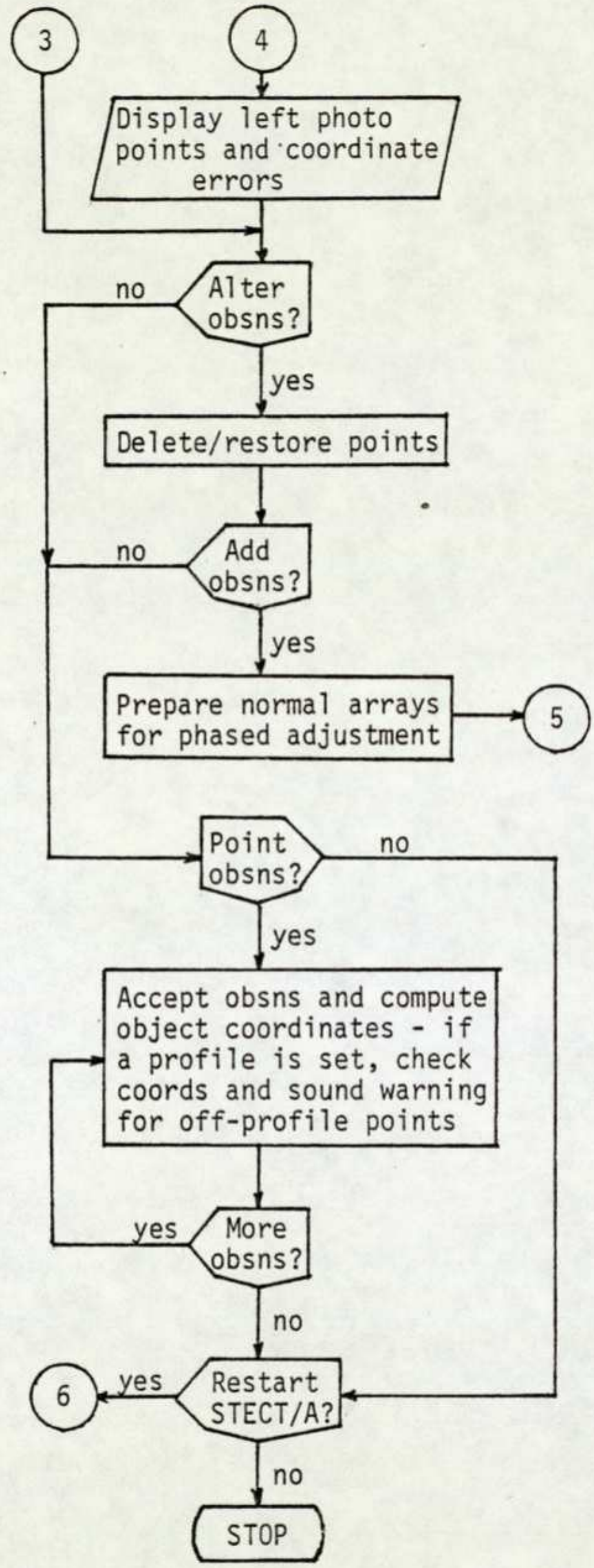


Figure 7.2 Flowchart for Programs STECT/A (cont. from 156)

(g) up to thirty control points can be entered with any index from Table 7.1

(h) the means and standard errors of the control observations are displayed to aid error detection

(i) the computation of the least squares adjustment utilizes point by point processing and direct loading of the normals

(j) the discrepancies and parallaxes at control and the corrections to the parameters are displayed for each iteration (see Figure 7.3). The operator can cease the iterations at any time

(k) the final orientation parameters and an estimate of the variance factor from the solution are displayed after the final iteration (see Figure 7.3)

(l) the standard errors of the exterior orientation parameters are displayed, based on the weight coefficient matrix of the unknowns and the estimate of the variance factor

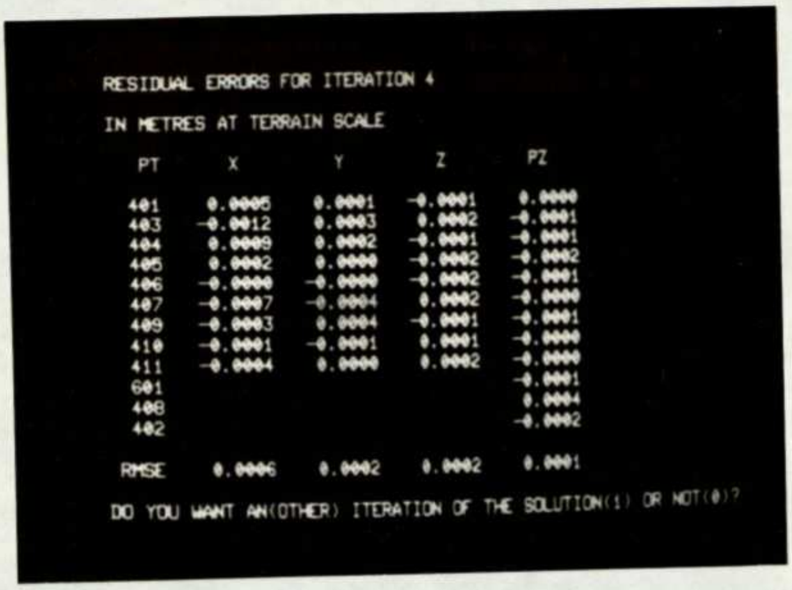
(m) a display of the control point configuration on the left hand photograph is optional. If a discrepancy in any coordinate or the parallax exceeds a set limit the error is indicated by a letter next to the point number (see Figure 7.3)

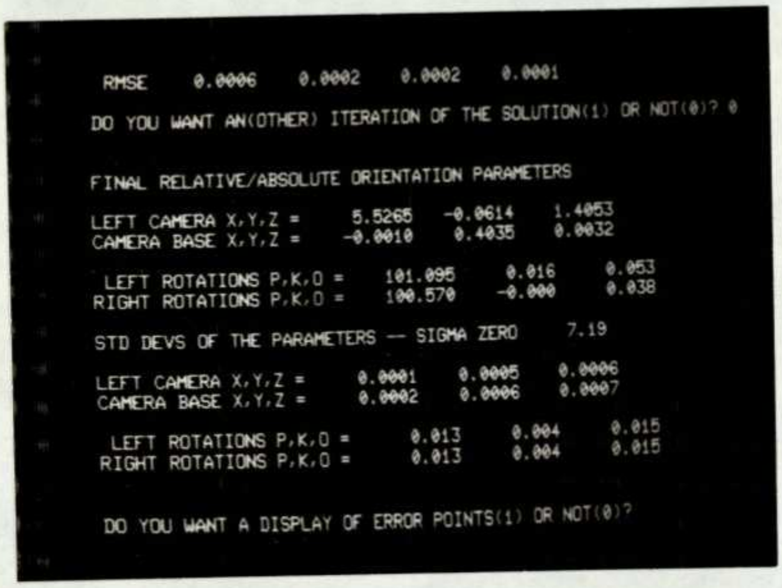
(n) if the restitution is unsatisfactory points can be deleted, previously deleted points can be restored or observations can be added and the least squares solution recomputed

(o) any number of observed points can have either coordinates displayed or written to a disk file

(p) profiles can be specified so that an audible warning is given if the observed point is incorrectly placed.

The program is capable of accepting data from either the digitizer or a disk file. Observations from the stereocomparator can therefore be read online or compiled on a disk file and run offline, but still of course interactively. A common mode of use was to read the fiducial and control observations onto a file and read the observed data points online. This facility was included





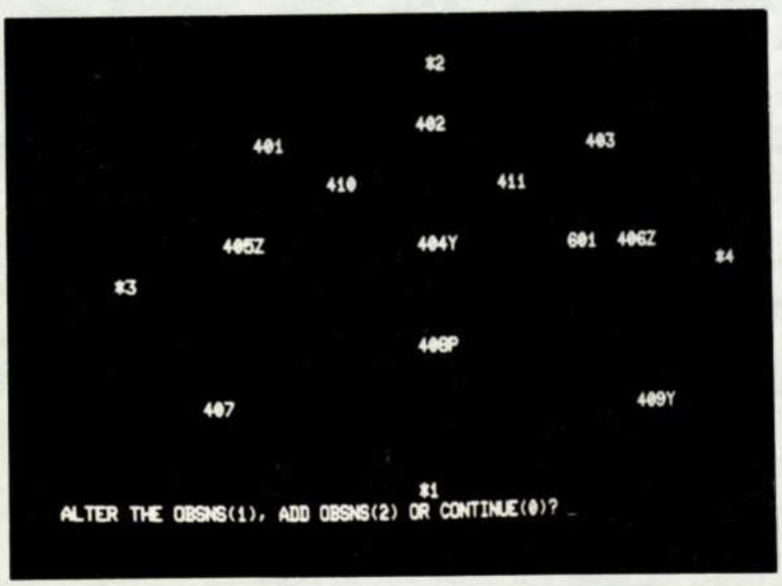


Figure 7.3 STECT/A Displays

soon after the first tests with the program. Although observational errors can be adequately dealt with by the editing techniques provided by the program, data entry on the keyboard cannot be checked before verification by the keyboard monitor. There are certain input data errors which cause an unavoidable crash of the program, and observations run online are immediately lost and must be reobserved. If instead the data for the interior and exterior orientations is on file, the program can be restarted and rerun to the stage where it was lost.

The fiducial and control coordinates can be entered through the keyboard by the operator or read from a disk file. If the optional control weighting is required a full or block diagonal matrix can only be entered from a disk file prepared beforehand by the methods already described in Section 7.1.2. A diagonal matrix can be typed in from the keyboard and all correlations are assumed to be zero. All other data entries are typed in by the operator in response to prompts and queries. The program indicates to the operator when file names or observations are required and performs input/output and computations at the direction of the operator.

Editing control observations is carried out on a point by point basis. The contributions of any point can be removed from or restored to the normals. Control points must be deleted twice to be completely removed; after the first deletion they are still retained as parallax points. Mistakes occur so infrequently in the interior orientation solution that editing is not justified. If, on the basis of the measurement standard errors, parameter values or residual discrepancies, the adjustment is not satisfactory then the observations must be retaken or a new set of fiducial coordinates entered.

These programs are the workhorses of the system and are applicable to most stereopairs encountered. Although corrections to the control points are not computed, the programs will give a satisfactory restitution when there are significant control errors, if control weighting is included. Only when film is used as the emulsion base or there are no known fiducial positions can these programs fail because the interior orientation is ill-defined.

STECT and STECA run in real time, there are no visibly long delays in response to data entry by the operator. The longest computations occur for the least squares adjustments, and neither of the interior and exterior orientations require extensive execution times.

Two or three iterations of the adjustment for the interior orientations of each photograph are usually required and only one or two seconds elapse before the results are displayed. The iterations of the exterior orientation are controlled by the operator and each requires approximately one second per control point. However this execution time is disguised by the display of the discrepancies at control as each is loaded into the normals, so the operator is occupied and unaware of the delay.

The programs were first tested with perfect data using various camera formats, focal lengths and orientations with a number of control configurations. Almost perfect results and zero residuals were obtained in every case after a maximum of three iterations of the solution. Slight rounding-off errors were caused by transmitting the stereocomparator observations from DATGEN only to the nearest micron. Tests were also carried out using photogrammetric observations with random errors, and in every case the estimate of the variance factor was approximately unity. The residual discrepancies and parallaxes at the control points were random with magnitudes increasing with distance from the cameras, an expected result of the stochastic model of the physical situation.

Initial assumptions of the exterior orientation parameters for the least squares solution must be supplied by the operator. This is not a disadvantage in practical cases as the positions of the cameras are always approximately known relative to the object and the rotations are specifically set to obtain a predetermined photographic coverage. However the accuracy of the assumptions will indirectly determine the number of iterations for the solution to reach convergence. Reasonable assumptions will always require only three iterations, but poor assumptions may increase the number to five or six. Impossible assumptions, for example the cameras on the wrong side of the object or pointing in the wrong direction, will cause either an immediate solution failure or a wildly oscillating solution which will not converge.

Finally, tests were carried out with random perturbations injected into the control. The relative precisions of the control and the photogrammetric observations were varied considerably, but the estimate of the variance factor was approximately unity for all stereopairs restituted with control weighting. The departures from unity were slightly more erratic, but this is only to be expected as there are two sources of random error.

Stereopairs run without control weighting when random errors are present in the control coordinates show immediate deterioration in the discrepancies at control and the estimate of the variance factor. The errors in the control distort the restitution and the exterior orientation parameters are forced away from their most probably true values. For example, a theoretical double cube shown in Figure 7.3 was used extensively for testing the accuracy and execution of various restitution programs. The twelve control points were given random errors with a mean standard deviation of 3 mm and appropriate random variates were injected into the coordinates. Four stereopairs were simulated, one from each side of the double cube, using the UMK focal length and format. A standard error of observation of 3 μm was assumed for the stereocomparator observations, giving a mean photogrammetric precision in the object of approximately 5 mm.

The mean results for the root mean square errors of the four stereopairs are shown in Table 7.2. The effect of the control weighting is clearly demonstrated in the reduction of the discrepancies and parallaxes at control, and in the dramatic drop in the variance factor. The last two columns show the errors in the positions and orientations of the cameras, and here the distortion of the restitution by the control errors can be seen.

Table 7.2 Restitutions of Double Cube Stereopairs

Control Weighting	Discrepancies at Control at object Scale (mm)				Variance Factor	Errors in Orientation Elements	
	X	Y	Z	PZ		Position (mm)	Rotation (cc)
No	3·2	3·4	2·9	0·5	9·2	3·0	15·2
Yes	2·8	3·3	2·3	0·3	0·9	1·8	8·4

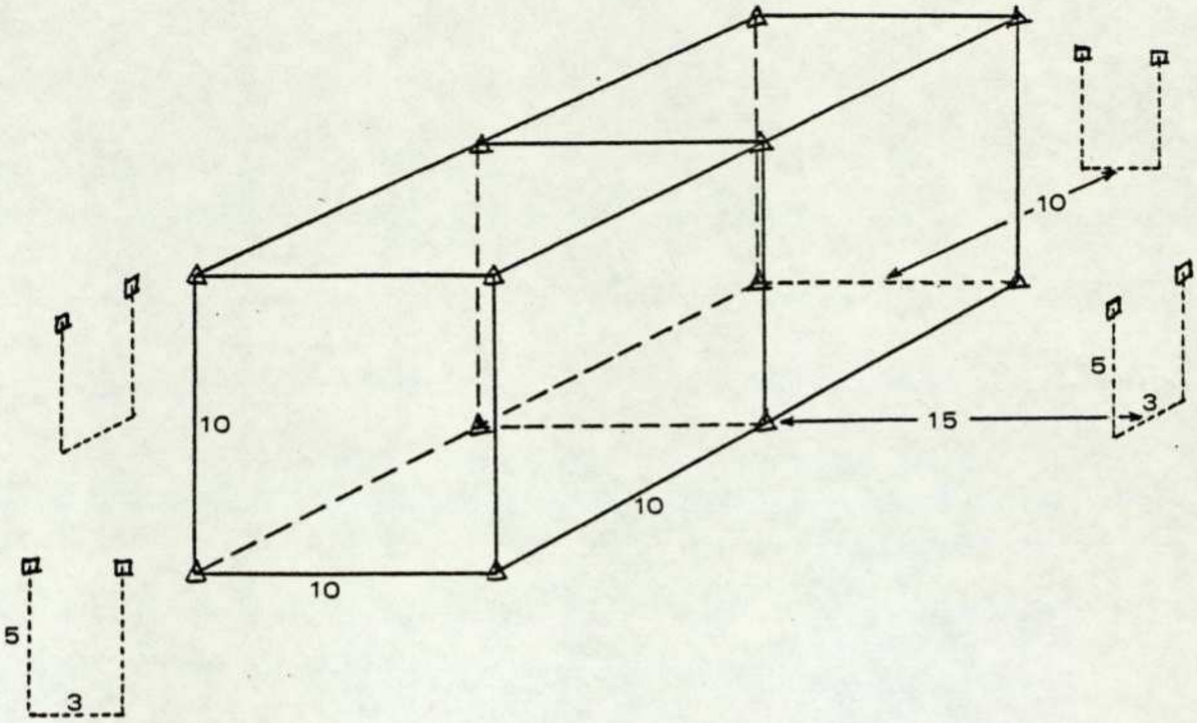


Figure 7.4 Double Cube Test Stereopairs

(Distances are metres, Control : △, Cameras : □)

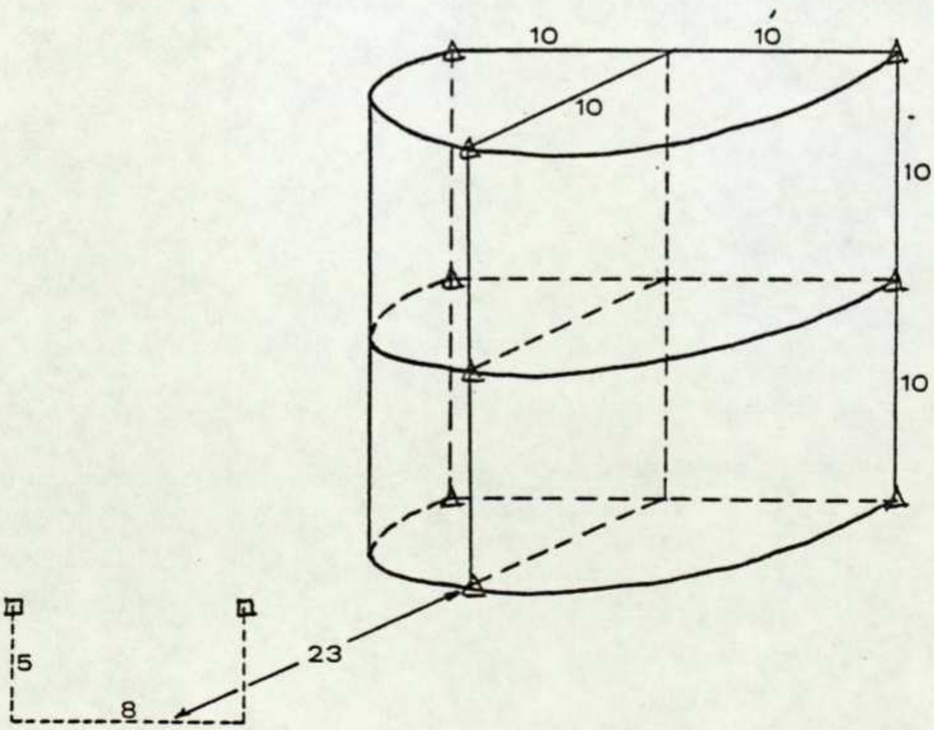


Figure 7.5 Cylinder Test Stereopair

7.2.2 Programs STECI/A

The restitution programs STECI and STECIA have a single major difference form the corresponding programs STECT and STECA. The interior and exterior orientations are no longer independent as the restitution includes a sequential adjustment of the interior orientation parameters. These programs are designed to apply to the cases where the simple restitutions are inadequate, notably when film is used instead of plates, especially in the case where the fiducial marks do not have known positions.

The structure of the programs is very similar to that of STECT/A, except of course for the structure of the normals. A scratch file is used to alleviate the storage problem, as the normals have grown from an order of twelve to twenty-eight. Each separate interior orientation writes the contributions directly to the file ($\underset{\sim}{N}_{\ell}$ and $\underset{\sim}{N}_{r}$ - see section 5.2.2) which is used as a reference for each iteration of the solution. If sequential adjustment of observations is required the file is overwritten with the normals from the first adjustment. Without a scratch file two complete normals would have to be stored in memory (the reference array and the working array) as the reference array must be retained without alteration.

Whereas STECT/A could be considered real time programs, STECI/A have a visible delay during the inversion of the normals and could only be called interactive online programs. From the display of the discrepancies of the last control point to the display of the computed corrections is a delay of approximately seven seconds. There is also a slight but noticeable delay between the display of the discrepancies for each control point. The entire program running time is also increased by the display of corrections and final values of the interior orientation parameters from the sequential adjustment, and the new residuals at fiducial marks.

The display of the corrections to the interior orientation parameters constitutes an additional criterion for the convergence of the least squares solution. The iterations should be pursued until the corrections to both sets of orientation parameters become negligible. A maximum of three to four iterations was generally required if the initial assumptions of the exterior orientation parameters were reasonably accurate. The solution tended to be more sensitive to poor assumptions, sometimes oscillating for five or six iterations.

Sequential adjustment of the interior orientation removes distortions in the restitution caused by random errors in the initial interior orientation estimates of the parameters. If known fiducials are available the initial interior orientations may be disturbed by the propagated effect of random errors in the stereocomparator observations. If known fiducials are not available then a simple translation/rotation must be employed and any scale or deformation errors cannot be accounted for by the initial transformation, and the parameters must be estimated.

No matter what type of interior orientation is initially employed, a deformational transformation is carried by the sequential adjustment. It is theoretically possible to include as many additional interior orientation parameters as are deemed necessary, as they are introduced as measurements and no extra control is required to define them. However, as has already been mentioned in Chapter 2, it is difficult to select statistically significant parameters without a great deal of experimentation with a reseau camera, whereas deformational parameters are known to be significant from experience with initial adjustments for the interior orientation of photographs with known fiducial positions.

Furthermore, the weight coefficients of any new parameters must be estimated for input to the sequential adjustment. Again these cannot be deduced without experience with a reseau camera. In the case of an initial simple translation/rotation, the scale and deformational weight coefficients can be estimated from parallels with other cameras which do have known fiducial positions. Of course any increase in the number of parameters in the sequential adjustment will increase the programming and storage required for the larger normal equations.

To investigate the effectiveness of the restitution many stereopairs were generated with different camera/control configurations and interior orientation transformations. For example, the left hand stereopair of Figure 7.4 was re-generated with scale and deformational variations (but without control errors). The probable magnitude of the variations was estimated as 10 µm per 100 mm from the only practical source available, experience with deformational interior orientation transformations for the P32 camera. Four stereopairs from the same camera positions were generated with scale and deformational variations of zero, one half, two and five times this "standard error." Random errors with a standard deviation of 3 µm were introduced into the fiducial mark positions to represent the uncertainty of the calibration data for the camera. The results of the restitutions are given in Table 7.3.

In the top half of the table four known fiducials were available in positions corresponding to those normally found in a UMK camera. A deformational transformation was carried out for the interior orientation in each case. For each of the four stereopairs the restitution by STECT and STECI are compared in terms of the accuracy of the interior and exterior parameters, the discrepancies at control and the estimate of the variance factor. It is apparent from this table that only slight improvements were gained, no matter what the magnitudes of the variations. This is not unexpected because only a small amount of uncertainty can be imparted to the interior orientation parameters by random errors in the fiducial mark observations and calibrated positions.

The lower half of the table shows the results if known fiducial marks are not available and a simple translation/rotation must be initially employed. Without a sequential adjustment of the interior orientation the restitutions become increasingly distorted as the scale and deformational variations increase. On the other hand, the use of a sequential adjustment consistently reduces the distortions to the level where known fiducials were present. Only at five standard errors does the trend appear to be turning upwards, but this may be because such a level of variation is statistically unlikely and the adjustment will allocate some of the distortion to the exterior orientation parameters and control positions.

During testing, the output of STECI was modified to display the weight coefficient matrix of the unknowns from the adjustment solution in order to investigate the correlations between the interior and exterior orientation parameters. Table 7.4 shows correlation factors between

Stereo-pair	Program	Interior Orientation Parameter RMSE's (μm/100 mm)	Exterior Orientation Parameter RMSE's Position (mm)	Rotation (cc)	Control Discrepancies (mm) X	Y	Z	PZ	Variance Factor
0	STECT	3·4	0·5	1·0	2·3	0·9	0·5	0·3	1·3
	STECI	3·0	0·7	1·2	2·2	0·8	0·4	0·3	1·1
½σ	STECT	2·4	0·5	1·0	2·3	0·9	0·5	0·3	1·3
	STECI	1·9	0·7	1·2	2·2	0·8	0·4	0·3	1·1
2σ	STECT	3·4	0·2	1·5	2·8	1·0	0·5	0·4	1·6
	STECI	2·2	0·5	0·9	2·6	0·9	0·4	0·3	1·1
5σ	STECT	2·9	0·4	1·5	4·3	1·7	1·1	0·5	1·7
	STECI	1·9	0·6	1·6	3·7	1·2	1·0	0·4	1·5
Fiducials									
Mean	STECT	2·9	0·4	1·3	2·9	1·1	0·7	0·4	1·5
	STECI	1·9	0·6	1·2	2·7	0·9	0·6	0·3	1·2
0	STECT	0·9	0·4	1·4	2·9	1·0	0·5	0·3	1·2
	STECI	2·6	0·8	1·5	2·2	0·8	0·4	0·3	1·1
½σ	STECT	3·7	0·7	2·0	3·8	1·6	0·8	0·6	1·9
	STECI	2·2	0·7	1·7	2·2	0·8	0·4	0·3	1·1
2σ	STECT	12·1	2·8	1·6	7·4	2·9	1·4	0·7	2·4
	STECI	2·5	0·3	1·4	2·6	0·9	0·4	0·4	1·5
5σ	STECT	30·8	2·2	8·9	5·6	3·3	3·2	1·2	8·5
	STECI	2·5	0·4	4·0	4·3	1·2	1·1	0·6	3·1
No fiducials									
Mean	STECT	11·9	1·5	3·5	4·9	2·2	1·5	0·7	3·5
	STECI	2·5	0·6	2·2	2·8	0·9	0·6	0·4	1·7

Table 7.3 Results of Sequential Adjustments of Interior Orientation

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_x	-0·04	0·05	1·13	-0·01	-0·02	0·84	-0·04	-0·11
a_y	-0·09	0·02	0·14	0·00	0·00	0·00	0·64	-0·02
a_z	0·00	0·27	0·08	0·02	0·05	0·00	-0·02	0·58
b_x	-0·01	0·00	-0·70	0·00	0·00	-0·59	0·10	0·06
b_y	-0·09	0·03	0·28	-0·01	-0·02	0·08	-0·28	0·01
b_z	0·00	-0·22	-0·07	-0·01	-0·01	0·00	0·01	0·32
ϕ_1	0·50	0·00	0·19	0·01	0·02	-0·01	-0·47	0·02
κ_1	-0·03	0·06	-0·03	-0·71	0·87	0·09	0·05	-0·12
ω_1	0·02	-0·75	-0·09	-0·04	-0·09	-0·06	-0·08	0·07
ϕ_2	0·20	-0·04	-0·02	0·00	0·00	-0·08	-0·32	0·00
κ_2	0·02	-0·06	-0·02	0·02	0·04	0·04	-0·05	0·14
ω_2	0·02	-0·15	-0·02	-0·01	-0·04	-0·02	-0·04	0·35

Table 7.4 Correlation Factors Between Interior and Exterior Orientation Parameters.

the interior orientation parameters of the left hand photograph and exterior orientation parameters typical of the double cube models already discussed in this section. Correlation factors must be used because of the varying magnitudes present, and for two variables a and b the factor is given by:

$$f_{ab} = \frac{\sigma_{ab}}{\sigma_{aa}\sigma_{bb}} \quad \dots \dots \dots \dots \quad (7.1)$$

The strong correlations present can be attributed to the natural consequencies of the physical relationships between the stereo-comparator, fiducial and terrain coordinate systems. For example, the largest values present indicate a strong bond between the X coordinates of the camera stations and the two major scale parameters, a_3 and a_6, in the interior orientation transformation. In this case the X coordinate reflects the distance between the cameras and object, and this is naturally strongly related to the scale of the photographs. Similarly, the rotation factors a_4 and a_5 are strongly linked to the left hand kappa rotation, as both carry out essentially the same task. There are also some strong correlations between the deformational parameters and exterior orientation parameters which give rise to non-linear changes in image positions.

These correlations are the cause of the oscillation of the solution when poor initial assumptions are made. The adjustment over-corrects to compensate for the large discrepancies caused by the poor assumptions, and then must recorrect in the next iteration, resulting in oscillations. Apart from this transitory problem, the solution showed no tendency to be weak or unstable for any of the configurations tested.

Not only ideal control layouts were used; poor and sparse control distributions were also generated and tested. Stereopairs with few control points were found to give almost as large an improvement in the interior orientation, so long as they were well distributed throughout the format. A minimum of one control point in each corner of the format was required to maintain the typical correlations given in Table 7.4. If the control was concentrated in one small area or one quadrant of the format, the correlations became erratic. However the solution showed no instability, the corrections to the interior orientation parameters simply disappeared due to lack of information.

The contributions from the control observations dropped below the level necessary to alter the parameters, given their a priori weight coefficients.

It must be remembered that the above discussion applies to generated data. The presence of systematic and gross errors in image observations will certainly effect the results of the sequential adjustment and any improvements in the interior orientation are subject to their influences.

7.3 Restitution Programs With Control Adjustment

7.3.1 Programs SPAOC/A

The restitution programs SPAOC and SPAOCA (Stereocomparator Phased Adjustment of Observations and Control/Aerial) incorporate a full sequential adjustment of control coordinates according to Section 5.1.4. Sequential adjustment of photogrammetric observations is also possible but the interior orientation is considered to be independent.

These programs are major reconstructions of programs STECT/A. Even with a reduction in the maximum number of control points to twenty, the limitation in CPU memory made it impossible for the programs to be written as a straightforward single unit. STECT/A already utilized a number of common subroutines, but had to be broken up even further so that overlays could be employed to reduce the memory required at any single stage of execution. The overlaying available on the minicomputer is user-generated and loads executable modules from disk file ("large" minicomputers use system generated overlays from additional areas of CPU memory above 32K).

The main difficulty is the order of the normals. If twenty fully known control points were included in the solution the order is seventy-two and the upper triangular half of the matrix requires over 5K of storage alone. The weight coefficient matrix of the control is written to a scratch file which is again read by the exterior orientation adjustment and over-written for a sequential adjustment of observations.

The second cause for large changes was the restructuring of the normals. Whereas STECT/A were structured around the sequence of photogrammetric observations, SPAOC/A had to be designed around the weight coefficient matrix of the control, or the loading of the contributions of observations into the normals would have been impossibly complex (see Section 5.1.4). A full weight coefficient matrix was utilized to include the effects of the correlations between control point coordinates which are invariably produced by the adjustment of geodetic survey measurements.

SPAOC/A have a number of additional outputs, both displayed and written to file. The corrections to the control coordinates are displayed after each iteration of the solution and the final adjusted values are given when the iterations are terminated. The operator therefore again has another criterion for the cessation of the iterations which must be taken into consideration. The convergence of the solution is only complete when the corrections to the control coordinates become negligible.

The adjusted control coordinates can be written to a file, once the solution is complete, if it is desired to compile a new control list based on the sequential adjustment. The new weight coefficient matrix of the control coordinates can also be written to file if desired. This matrix is extracted from the solution of the least squares adjustment, ignoring the weight coefficients of and correlations with the exterior orientation parameters. Whereas the weight coefficient matrix of the control may have further application, once the restitution for a particular stereopair is complete, the exterior orientation parameters and their weight coefficient matrix are of no further interest because a rigorous adjustment of multi-station photography is beyond the scope of the thesis and the processor.

The execution of these programs is considerably slower than the programs previously described. Not only do the normals require a lengthy time lapse for inversion, but the overlaying slows the program throughout. Each executable module must be loaded into a specified area of CPU memory from disk file when it is required, and execution is suspended while this loading takes place. However the waiting

Program	Exterior Orientation Parameter RMSE's		Control Discrepancies (mm)				Variance Factor
	Position (mm)	Rotation (cc)	X	Y	Z	PZ	
STECT	9·8	14·6	6·4	6·3	3·6	1·0	8·3
SPAOC	5·5	10·0	0·1	0·3	0·1	0·4	1·1

Precision	Coordinate (mm)		
	X	Y	Z
Photogrammetric Precision at Control Centroid	0·9'	3·4	1·1
Standard error of control before Adjustment	5·0	7·0	5·0
Standard error of control after adjustment	2·8	4·2	2·9
RMSE of control positions before adjustment	2·6	5·0	8·0
RMSE of control positions after adjustment	2·8	2·4	3·6

Table 7.5 Restitution of Cylinder Stereopair.

times for a response from the minicomputer are never intolerably long, the inversion of the normals for the sequential adjustment of the control requires approximately 20 seconds for a stereopair with 10 fully known control points. Hence the program can still be called online and interactive, but the computation is not in real time.

The effect of a sequential adjustment of control on a restitution is similar to that of control weighting; without the adjustment random deviations of control from their assumed positions distort the restitution and the exterior orientation parameters. The programs were tested with generated data from DATGEN with a variety of camera and object configurations, and the clearest method of demonstrating the effect of the programs is by an example.

One of the standard configurations of control available from DATGEN is shown in Figure 7.5, intended to emulate a realistic photogrammetric survey of a cylindrical storage tank. The photogrammetric observations were assumed to have a precision of 3 μm on the photographs, corresponding to a precision of approximately 4 mm at the centroid of the control. The control was assumed to have been established by geodetic measurements from a base-line, giving a precision of control in the region of one centimetre.

The results of restitutions of this stereopair with STECT and SPAOC are given in Table 7.5. The sequential adjustment of control substantially improves the exterior orientation parameters and the estimate of the variance factor, and the reduction in the discrepancies at the control points indicates clearly the effect of the control errors. Also given in the table are details of the improvement in the control coordinates. The differences between the control coordinates and their most probably true values (as originally defined in DATGEN) are given as root mean square errors before and after the restitution adjustment. The standard errors of the control are given before the adjustment, extracted from the initial weight coefficient matrix of the control coordinates, and after the adjustment, extracted from the weight coefficient matrix output by SPAOC.

The extent of the improvement for any particular stereopair is of course dependent on the relative precisions of the control and the photogrammetric observations. If the control has very high precision compared to the photogrammetric observations then STECT and SPAOC would give essentially identical results. However, if the precision of the control is at all comparable then a better restitution will always be obtained by SPAOC. In any other case similar to the example given, where the two precisions are approximately the same order, the improvement in both the restitution and the control coordinates is quite marked. If the control precision is poorer than the photogrammetric precision by a large factor then the photogrammetric measurements tend to become fixed and the control merely supplies a coordinate datum and the scale of the object.

The effect of correlation between the control coordinates can be demonstrated using the stereopair of the cylinder. The results given in Table 7.4 were obtained with a weight coefficient matrix which had weak correlation between control coordinates. If this correlation between coordinates of points was ignored, i.e. the weight coefficient matrix was assumed to be diagonal, then only a slightly altered restitution resulted. Although individual discrepancies and parallaxes changed, the root mean square discrepancies and parallaxes at control remained constant. The exterior orientation parameters changed with root mean square errors of 0·2 mm and 0·6 cc in position and rotation respectively. These may not be considered significant, but similar alterations of 0·2 mm were made to individual control coordinate correlations and these cannot be ignored if the coordinates are to be of further use.

Perhaps the most significant effect of correlation occurs when the photogrammetric observation of a particular control point is deleted from the solution. If there are no correlations present the coordinates of the point are not adjusted by the solution. Even if only weak correlations are present the point is adjusted, commonly by as much as 0·5 mm, by the influences from other control points in the network.

If large correlations representative of a strong geodetic network are present then the changes to the restitution do become significant when the correlations are ignored. The double cube control set was assumed to have been determined by a tight network of geodetic observations, with the mean weight coefficient matrix for each point given by:

$$\underset{\sim}{Q}_c = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

and also strong correlation between the coordinates of different points (indicated by the off-diagonal submatrix representing the correlation between different points):

$$\underset{\sim}{Q}_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If these correlations are ignored then the exterior orientation parameters do change significantly, in this case with root mean square errors of 3 mm in position and 10 cc in rotation, and the root mean square discrepancies and parallaxes at control show slight differences. The largest changes were to the adjusted control positions, the vector root mean square error was of the order of 3 - 4 mm. Changes of this order of magnitude cannot be tolerated and the full weight coefficient matrix should always be used when the control is supplied by geodetic measurements.

The above discussion refers only to the neglecting of all correlations in the weight coefficient matrix, so that in effect a diagonal matrix is employed in the adjustment. There is an intermediate stage where a block diagonal matrix is employed. Such a matrix ignores only the correlations between coordinates of different points, and this can reduce the effect on the restitution. The correlations between the coordinates of individual points are not neglected, and as these are generally more significant than those between the coordinates of different points, they have a larger effect on any adjustment using the weight coefficient matrix. However, the neglecting of any correlations will have some effect as there is a theoretical violation of the principles of sequential adjustment.

7.3.2 Multiple Stereopairs

The technique of control correction by sequential adjustment of the restitution of stereopairs can be extended to multiple stereopairs. If an object is photographed from more than one standpoint so that some or all of the control points appear in each stereopair, the control coordinates can be adjusted on the basis of all the photogrammetric observations. The technique is not a rigorous adjustment, but it is applicable to interactive programming on a small minicomputer and does produce worthwhile results.

The adjusted coordinates and weight coefficient matrix from the sequential adjustment of each stereopair are merely passed onto the next stereopair as input. The coordinates are then adjusted and the weight coefficient matrix improved by each subsequent restitution. Once all the photogrammetric observations of the control set have been processed then the final coordinates and weight coefficients are the best estimates from the available data.

The only assumptions which need to be made are that estimates of the variance factor are not included in the weight coefficient matrices and that all the control is included initially. Variance factors would invalidate the technique because they would tend to weight differentially the data from each restitution. The coordinates would therefore be adjusted differently depending on the order in which the restitutions were processed, and this was verified by experiment. If variance factors are not used the sequence of data processing is irrelevant. All the control must be present and used throughout for similar reasons. Any new inclusions in the weight coefficient matrix would distort the adjustments, as would the removal of any weight coefficients and their attendant correlations.

This technique must be considered non-rigorous because of neglected correlations. The adjustment of each restitution will introduce correlation between the exterior orientation parameters and the control coordinates, but these are ignored completely. There is no intermediate solution; either the exterior orientation parameters from every camera are included or ignored. If they were included, the solution would be equivalent to a multistation adjustment and the number of unknowns would increase rapidly. For example, four stereopairs and twenty control points would have a normals coefficient matrix with an order of 108 and would require almost half the available CPU memory. To invert such a matrix, either some correlations would have to be ignored and recursive partitioning employed, or a suite of programs developed and run offline to acquire, compile, adjust and display the data.

The value of the method is again best demonstrated by example. The four double cube stereopairs of Figure 7.3 were processed by this method and the results are presented in Table 7.6. The discrepancies between the most probably true values of the control coordinates and the initial values, the adjusted values from each individual adjustment and the adjusted values from the multiple adjustment are given as root mean square errors. As expected, the adjusted coordinates from the multiple technique are the best estimates of the values originally generated by DATGEN. The standard errors from the final weight coefficient matrix are also shown, and are a considerable improvement on the original values shown at the top of the table.

Adjustment		Precision Quantity	Coordinate (mm) X	Y	Z
Initial Control Coordinates		Standard Errors	3	3	3
		RMSE's	2·4	3·0	2·8
Adjusted Coordinates	Front	"	0·9	2·1	1·6
from Individual	Left	"	1·7	1·7	2·0
Adjustments:	Right	"	1·6	1·8	1·8
	Rear	"	0·8	3·0	1·9
	Mean	"	1·3	2·2	1·8
Multiple Stereopairs		Standard Errors	1·4	1·3	1·3
Final Control Coordinates		RMSE's	0·9	1·7	1·6

Table 7.6 Results of Multiple Stereopair Adjustment.

The optimum restitution can be obtained for each stereopair used in the multiple adjustment by running STECT with the final adjusted control coordinates. Theoretically each should be run as the final restitution of the multiple adjustment, but it was found that the simple restitution with the final control set gave almost identical exterior orientation parameters. This implies that the correlations between the orientation parameters and control coordinates do not have a significant effect on the restitutions.

However, once again this is an ideal situation with no systematic errors and an excellent control distribution. In practical cases the assumptions would probably break down and correlations between orientation parameters and particular control points would have an effect. The technique is still a valid application of sequential adjustment in any situation because it will compute the best available estimates of the control coordinates. Restitutions based on these coordinates would have the lowest root mean square discrepancies and parallaxes at control, as the random errors in the coordinates have been minimized.

7.4 Restitution Program With Interior Orientation and Control Adjustment

7.4.1 Program SPIOC

The restitution program SPIOC (Stereocomparator Phased adjustment of Interior orientation, Observations and Control) was the final program developed and commissioned for the research work. This program utilizes the sequential adjustment of the interior orientation, control coordinates and additional observations described by section 5.3.2. The program applies to terrestrial models only, a corresponding aerial program was developed but never commissioned due to time limitations.

SPIOC is yet another major reconstruction of an existing program, SPAOC. The structure of the normals had to be altered considerably, and even greater effort had to be expended in upgrading the efficiency of the computer programming. The maximum order of the normals for this program is 88, and the upper triangular half of the matrix requires almost 8K of CPU memory.

In the light of interactive processing with SPAOC, one measure which could be taken to relieve the pressure on CPU memory was the removal of the online option. Observations on the fiducial marks and control points can only be accepted from a disk file, although observed data points can still be processed online. The removal of this option was not felt to be detrimental to the processing by the program, it would be invariably run offline in any case to avoid the possibility of data loss. Incorrect data entries by the operator were found to be the major cause of program failures and all programs were increasingly run offline. The compilation of observations directly onto a disk file was carried out for each stereopair and then checked for gross errors with the simple restitution programs. The files were edited if required, copied to an archive disk and then used as offline input data to the more sophisticated programs.

The running times for SPIOC exceeded expectations based on experience with SPAOC. The normals required a predictably longer time for inversion, as did the computation of the contributions to the normals for each point. However, the number of displays of parameter corrections and final values which had to be viewed by the operator were a major factor in a large increase in the time required for a stereopair restitution. The number of iterations necessary for the solution was unchanged at three to four depending on the accuracy of the assumed values for the exterior orientation parameters. The total time needed to complete a restitution was never greater than five to ten minutes, depending on the number of control points present, which as a worst case is still extremely favourable when compared to offline processing on a mainframe computer.

Everything which has been discussed with respect to STECI and SPAOC applies to SPIOC in combination. The sequential adjustment of the restitution displays corrections to the interior and exterior orientations and the control coordinates for each iteration, and all the corrections must be considered in relation to convergence of the solution. All adjusted parameters and coordinates are displayed after the final iteration. Restitutions are improved beyond what STECI and SPAOC can achieve as distortions caused by random errors in both the interior orientation parameters and the control coordinates are minimized. These effects have already been adequately demonstrated and will not be repeated here.

7.4.2 Digitizer False Orgins

SPIOC was tested with fictitious data from DATGEN, again with a variety of camera/control configurations, control precisions and interior orientation transformations. In the ordinary course of events no solution instabilities were found and the program gave results which were predictable from experience with STECI and SPAOC, given some interaction between the interior orientation parameters and the control coordinates. Again the variance factors were approximately unity and if a poor control distribution was present then the corrections to the interior orientation parameters decreased due to lack of information.

However, it was during the testing of STECI and SPIOC that a number of solution failures were encountered. All of the data generated by DATGEN had had the stereocomparator origin set at the principal points of the photographs, so that the stereocomparator and fiducial coordinates always had approximately the same magnitudes. It was common practice when acquiring real data to set a false origin at the left hand principal point of (100,100), and this was simulated in several data generation runs in preparation for the processing of real data.

The first problem that these false origin shifts caused was a general increase in the number of iterations in the least squares solutions for the initial interior orientations. This increase can be attributed to rounding-off errors in the larger coordinate values. If a false origin of 100 mm is introduced then six figures are needed to describe the coordinates instead of five, and 16 bit word processors have only six to seven figure significance in real values.

The second problem was the much more serious one, the sequential adjustments of the interior orientations either oscillated wildly before converging slowly, or failed completely. Whether the solution oscillated or failed was found to depend on the camera/control configuration, but the root cause of the problem was the false origin shifts. The large coordinate values falsely weighted the interior orientation parameters in the solution, both in the a priori weight coefficient matrix and in the contributions of the control observations to the normals. This false weighting applied particularly to the deformational parameters, whose linear coefficients were the square of the comparator coordinates. For example, in equations 4.15, the diagonal element of the normals for parameter a_8 would change from 2×10^6 to 2×10^8 for a P32 format, if the origin is falsely shifted from (0, 0) to (100, 100).

This apparent increase in the precision of the parameters is due to the fact that the observations are made relative to the coordinate origin. The precision of measurement is expected to worsen with increased distance from the origin, but as the shift is a false increase, no drop in precision actually occurs. Therefore points observed with an origin shift have a higher relative precision. An analogy can be drawn here with control surveys which may have a high internal precision, but have a poor absolute precision because they must be related to a distant coordinate datum.

The effect of this increase in precision is to enable the interior orientation to dominate the solution by sequential adjustment. The coefficients of the interior orientation parameters involved the large stereocomparator coordinates, while the exterior orientation coefficients involved the smaller fiducial coordinates even though both had the same physical meaning with respect to the photographs. Large discrepancies at control were removed by changes in the interior rather than the exterior orientation parameters, and the solution either oscillated wildly or failed because the geometry of the situation was not satisfied.

Both of these problems were eliminated by simply testing for the origin shift when the original observations were read into the programs. If known fiducials were available the observed coordinates were reduced to the same magnitude by another, equally false, shift. If known fiducials were not available then the observations were reduced to their centroid.

7.4.3 Large Parameter Variations

Another potential difficulty investigated in detail was the magnitudes of the variations in the interior orientation parameters, relative to their estimated weight coefficients. Tests had been carried out with variations between zero and five standard errors, and examples of results are given in Table 7.3 and discussed in Section 7.2.2. The problems described in the previous section had shown that the solution could fail, and there was some doubt about the estimation of the weight coefficients for scale and deformational parameters of cameras without known fiducial positions. If the estimates were severely inaccurate then there was the possibility that the solution could oscillate or even fail.

Accordingly, tests were carried out with different levels of variations in conjunction with under- and over-estimations of the weight coefficients input to the sequential adjustment. If the weight coefficients were grossly over-estimated, i.e. the variations were much smaller than the a priori precisions would indicate, then the variations were compensated and the solution showed no instability, as could be expected.

If the variations were badly under-estimated, i.e. the variations were up to twenty times the a priori precision estimates, then the solution still showed no tendency toward oscillation and not one failure was encountered. Furthermore, the variations were accurately computed by the solution no matter how high the level of significance rose, as long as the control was well-distributed throughout the format. The only indication that the under-estimation was present was an extremely large estimate of the variance factor.

The explanation for this phenomenon is the structure of the normals matrix. If there is sufficient control to provide a good estimation of the large corrections to the interior orientation parameters then the contributions to the normals dominate the a priori weight matrix, i.e. $\underset{\sim}{D}_p^T \underset{\sim}{Q}_p^{-1} \underset{\sim}{D}_p$ dominates $\underset{\sim}{N}_a$ in expression (5.34). Again if the control is poorly distributed the corrections are reduced due to lack of information. The large variance factor is of course caused by the comparison of the large corrections with their a priori weight coefficient estimates.

Hence the solution can detect and correct any level of variation in scale and deformational parameters without special attention or any modification whatsoever. This is particularly advantageous with respect to cameras without known fiducial positions in the format. Badly deformed photographs may not be easily identified by other means and compensation may be equally difficult.

8. EXPERIMENTAL TESTING

8.1 Experimental Data Acquisition/Processing

8.1.1 Stereopairs Observed

The stereopairs used for the experimental testing of the sequential adjustment restitution were selected from projects carried out by the Terrestrial Photogrammetric Unit of the Department of Civil Engineering, with one exception. All photography was black and white with glass plates of medium grade flatness, also with one exception. The stereopairs were chosen to give a wide range of camera/object configurations and the physical characteristics of each are detailed in Table 8.1. The photograph scale was calculated using the distance of the control centroid from the midpoint of the camera base. The depth range for each stereopair was calculated as the difference in the distances to the nearest and farthest control points, given as percentage of the base midpoint to control centroid distance. Where more than one stereopair was involved the values given are means of the individual characteristics.

The stereopairs were also selected to give wide ranges of control distributions and precisions, and details are given in Table 8.2. The type of measurement technique used to provide the control is given and if geodetic measurements were utilized then a variation of coordinates least squares adjustment was always used and the estimate of the variance factor is quoted. The area of the photograph format covered by the control points is given as an approximate percentage of the total format area. The distribution of the points throughout the object space is graded as excellent, good or poor. The mean standard errors of the control are given, either extracted from the initial least squares adjustment of geodetic measurements, or estimated in other cases. For comparison, the precision of photogrammetric measurement was calculated at the control centroid using a standard error of a single stereocomparator observation of 3 μm.

Project Name	Camera	Stereopair Type	Number of Stereopairs	Photograph Scale (mean)	Depth Range (% of Object Distance)	Range of Base to Depth Ratios
British Leyland C40P	UMK/3	Terrestrial	1	1:21	9	1: 8·0 to 1: 8·5
Chrysler Transmission	UMK/3	Aerial	1	1:15	13	1:12·6 to 1:14·4
Chrysler Transmission	UMK/3	Terrestrial	4	1:14	68	1: 1·9 to 1: 3·8
Dorchester	P32/1	Terrestrial	4	1:280	84	1: 4·7 to 1:11·4
Lake Merrimu	P32/2	Terrestrial	1	1:1070	87	1: 5·5 to 1:13·5
Munich Aviary	UMK/4	Aerial	1	1:17	1	1:3·6
S.M.M Propeller	UMK/2	Aerial	1	1:100	10	1: 4·0 to 1: 4·4
St Paul's Cathedral 1978	UMK/1	Terrestrial	1	1:410	17	1: 5·6 to 1: 6·7
St Paul's Cathedral 1980	UMK/4	Terrestrial	1	1:500	23	1: 2·7 to 1: 3·4
York Way Tunnel	UMK/1	Terrestrial	1	1:170	5	1: 2·6 to 1: 2·7

Table 8.1 Physical Characteristics of Observed Stereopairs

Project Name	Measurement Technique/σ_o	Number of Control Points	Format Area Covered (%)	Control Distribution	Control Standard Errors (mm) X	Y	Z	Photo Precision at Centroid (mm) X	Y	Z
British Leyland C40P	3D Machine	9	80	Excellent	0·2	0·2	0·2	0·35	0·09	0·09
Chrysler Transmission/A	Grid/Height Bars	18	80	Posn. - Exc. Depth - Good	0·3	0·3	0·6	0·04	0·04	0·60
Chrysler Transmission/T	Grid/Height Bars	9	30	Good	0·3	0·3	0·6	0·08	0·08	0·09
Dorchester Bath-house	Geodetic/ 2·20	6	40	Excellent	0·8	2·0	1·4	3·3	5·1	2·4
Lake Merrimu	Geodetic/ 0·92	20	80	Excellent	9·8	18·4	4·7	17·9	29·8	3·8
Munich Aviary	Geodetic/ 1·05	11	90	Posn. - Exc. Depth - Poor	0·1	0·1	0·2	0·06	0·06	0·19
S.M.M. Propeller	Geodetic/ 0·54	4	70	Good	0·4	0·4	0·5	0·7	1·1	0·3
St Paul's Cathedral 1978	Geodetic/ 1·26	9	70	Good	1·4	2·8	1·4	1·5	8·1	3·7
St Paul's Cathedral 1980	Geodetic/ 0·96	11	50	Excellent	1·0	2·0	1·0	1·7	4·9	3·0
York Way Tunnel	Geodetic/ 2·48	6	60	Good	0·6	0·5	0·4	0·5	1·3	0·7

Table 8.2 Control Characteristics of Observed Stereopairs

8.1.2 Procedure and Processing

The stereocomparator measurement of the stereopairs was taken with a set procedure wherever possible to retain consistency for the photogrammetric observations. The photographs were always rotated on the plate carriers so that the stereocomparator and fiducial axes were approximately aligned. Three rounds of observations were taken by experienced observers on the fiducial marks and the control points. Parallax points were added at the operator's discretion to fill any gaps in the control distribution or format area. The observations were always taken offline to compile a disk file as a permanent record of the basic data set, as each stereopair was to be computed with different types of sequential adjustment restitutions. The disk file was then edited if required and run with one of the simple restitution programs, STECT or STECA, to detect gross errors in the control or observations. The file could be edited once more and if any further observations were deemed necessary they were again taken offline to provide a permanent record and then added to the original file.

All restitution programs were then run offline with the disk files. Fiducial mark coordinates, control coordinates and weight coefficients were all compiled on disk files prior to the program runs both for convenience and consistency. The positions and orientations of the two cameras were initially estimated from whatever source available for the STECT/A simple restitutions. The results of these restitutions were then used for the subsequently run sophisticated adjustments to keep the numbers of iterations of the solutions to a minimum.

8.1.3 Online Data Acquisition/Processing

Although the data for the comparisons between the simple restitutions and sequential adjustments were compiled onto disk files and processed offline, a great deal of online work was also performed. Most of the models presented in Tables 8.1 and 8.2 were restituted by online data acquisition and processing during some stage of the analysis of the performance of the system. Simple restitutions with control weighting were carried out most often, with occasional sequential adjustments of either the control or the interior orientation.

The greatest benefit of the online processing was the utilization of the available displays and editing techniques to optimize the restitution. As the restitution computations were being performed during the observation of the stereopair the results could be analysed and interpreted by the operator as they were displayed and appropriate action taken. In the majority of cases that action was simply to continue to the next step in the processing, but it was when problems arose that data and result displays proved their worth.

The first display of results occurs immediately after a set of observations are taken for interior or exterior orientation. The mean values of the observations are displayed along with their standard errors. If only one round of observations has been taken the standard errors are assigned an arbitrary value, say 5 μm. If multiple rounds have been taken then the standard errors are the first indicators of blunders or misidentifications. Poorly defined points can be checked to see if they can be measured with an acceptable precision.

The display of results for the interior orientations alters depending on whether the fiducials are considered to have known locations or not. If they are known then the residuals of measurement and the estimate of the variance factor from the adjustment are given, and these are prime indicators of errors in the interior orientation. If fiducial positions are not known then only the transformed positions of the fiducial marks are given. The only check that can be made is the comparison of the transformed positions for the left and right hand photographs for consistency, assuming that the same camera was used.

The display of results for the exterior orientation has three separate indicators of errors in the restitution. The first is the list of discrepancies and parallaxes at control (see Figure 7.3). Gross errors in the photogrammetric observations or control coordinates can be identified by scanning the list for large or uncharacteristic values. The second is the display of the estimate of the variance factor and the standard errors of the exterior orientation parameters from the solution. Gross and systematic errors of any kind will manifest themselves by raising the values of these parameters to unaccountably high levels.

By far the most informative display is the plot of the left hand photograph format. The control points are indicated by their numbers, and if the error in any coordinate or the parallax exceeds a predetermined limit compared to the stochastic model, the error is flagged by an appropriate letter. The limit can be set at any confidence level, and in the majority of cases was five times the standard error. This display shows the distribution of the control and the relationship between it and any errors which may exist. Gaps in the control array and blatantly uncharacteristic errors are shown quite clearly by this display (see Figure 7.3).

The editing options available were used frequently to delete and restore observations from and to the restitution to obtain the optimum solution, but major improvements were more often caused by the addition of observations. The sequential adjustment of photogrammetric measurements was employed to incorporate new observations into the restitution. The display of the photograph format invariably showed up blank areas where parallax points could be observed to strengthen the restitution. Deleted points were often re-observed to establish whether the error was caused by incorrect control coordinates or a poor photogrammetric observation. Isolated or distant control points were commonly observed repeatedly to attempt to strengthen their influence on the restitution and obtain an even emphasis on all control points.

Gross errors were more difficult to detect in the stereopairs restituted with the more sophisticated solutions. The compensating influences of the sequential adjustments tended to disguise blunders. The restitutions were more sensitive to gross errors, the adjustments of the control or the interior orientations changed dramatically in the presence of incorrect photogrammetric observations on control points. Once some experience was gained with the sequential adjustment restitutions such errors could be more easily identified from the corrections to the control or the interior orientation parameters. However, the minimum size of a detectable gross error definitely increases for the sophisticated restitutions.

The major drawback of online processing has already been mentioned in Chapter 7. Because the operator must make many entries into the program from the VDU keyboard many data errors can be made. These errors are first detected by the system's keyboard monitor, and if the error is serious enough (for example: giving alphanumerics in a numerical entry) the program is automatically aborted and all the observational data is lost.

The only way to circumvent this problem is to write each observation to disk file as it is recorded, but if this is taken to its logical extreme the online performance of the system would be seriously degraded. To protect every observation, the disk file would have to be closed and then re-opened every time, and the opening and closing of files require a few seconds each. A compromise must be reached between necessity and practicality, and in this case could take the form of altering the online programs to compile data files of observations for the interior and exterior orientations as they are observed. The files could be closed and re-opened for groups of, say, ten observations or for the entire group for each orientation.

In such a case the online processing could be almost as failsafe as compilation onto disk files. If the program does abort for any reason then the operator only has to restart the program and run it, from the disk file of observations, up to the stage where the abort occurred. Once the restitution is complete the files can be retained as archive records or deleted if they were no longer required.

8.2 Descriptions of Stereopairs

8.2.1 British Leyland C40P

This stereopair is from a project which was to check an as-built model against design dimensions. The model was a truck cabin which was photographed from all four sides. One stereopair was chosen as being representative, as all were essentially similar and were separately controlled. Both graphical and digital output were required so the cameras were set normal to the base with axes horizontal.

The control for the stereopair consisted of ink crosses on small paper stick-on targets which were attached to the surface of the model. The model was mounted on the surface plate of a three-dimensional measuring machine which was used to coordinate the targets. The machine was mechanical in operation and the vernier scales could be read to 0·1 mm. However the precision was estimated to be 0·2 mm from discussion with the operator and some simple experimentation with the metal probe used to point to the targets.

The left hand photograph of the stereopair is shown in Figure 8.1 with control marked by the circles. The results of the restitutions are given in Table 8.3, where all values are in millimetres.

8.2.2 Chrysler Transmission

Five stereopairs were taken of an automotive transmission housing to check the as-built object against its design dimensions. The photography was taken primarily for line drawing production on a stereoplotter, but the stereopairs were first processed by the analytical system to obtain the exterior orientation parameters for the cameras. The parameters were then transferred to the stereoplotter to provide an initial "set-up" for the operator. The cameras of each stereopair were oriented normal to the base and with the optical axes either vertical (aerial stereopair) or horizontal (terrestrial stereopairs).

One stereopair was taken from above the transmission while the other four covered each side from a level slightly higher than the top of the transmission. The base between the two cameras was set incorrectly for the aerial stereopair, so it is segregated from the four essentially similar terrestrial stereopairs in Tables 8.1 and 8.2. The left hand photograph of the aerial stereopair is shown in Figure 8.2 and a typical terrestrial frame is shown in Figure 8.3.

The control for the photography was provided by a grid and height bar arrangement which can be clearly seen in the two figures. This arrangement was designed by [redacted] of the Department of Civil Engineering on the basis of the partial success of an earlier attempt with a grid drawn on polyester draughting foil in combination with calibrated length bars (Cooper and Shortis 1978). The grid is scribed into a sheet of perspex mounted in a wooden support while the bars are made of aluminium. The grid was supplied to a nominal 1 mm accuracy, so selected intersections were coordinated using a Topocart plotting table as a coordinatograph. The aluminium height bars were measured using a length gauge and are located in holes provided in the surface of the perspex. The lower ends of the bars

Figure 8.1 Left Hand Frame, British Leyland C40P Stereopair

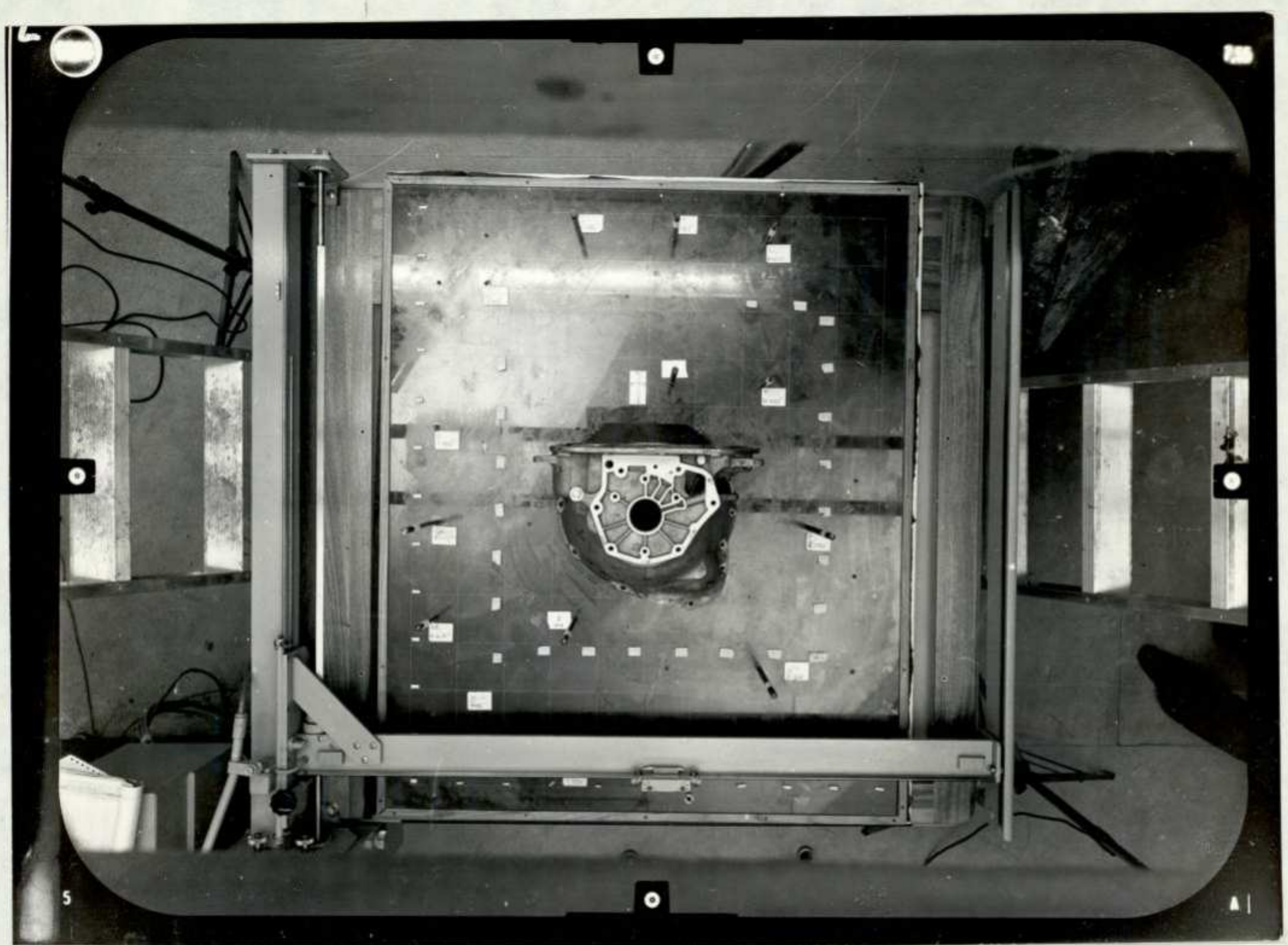


Figure 8.2 Left Hand Frame, Chrysler Transmission Aerial Stereopair

Figure 8.3 Typical Frame, Chrysler Transmission Terrestrial Stereopairs

Program	X	Y	Z	PZ	σ_o
STECT	0·6	0·3	0·2	0·1	7·1
STECT/Weighting	0·6	0·3	0·2	0·1	2·8
SPAOC	0·5	0·2	0·2	0·1	3·0
STECI	0·4	0·2	0·1	0·1	2·4
SPIOC	0·2	0·1	0·0	0·1	2·4

Precision	X	Y	Z
Photogrammetric	0·35	0·09	0·09
Control - a priori	0·20	0·20	0·20
Control - a posteriori	0·17	0·12	0·11
Control - RMSE	0·3	0·1	0·2

Table 8.3 Restitutions of British Leyland C40P Stereopair

Program	X	Y	Z	PZ	σ_o
STECT/A	0·7	0·6	0·5	0·2	11·1
STECT/A - Weighting	0·7	0·6	0·6	0·1	1·9
SPAOC/A	0·2	0·2	0·2	0·1	1·9
STECI/A	0·7	0·6	0·5	0·1	1·8
SPIOC	-	-	-	-	-
STECT/A - Adj. Control	0·4	0·3	0·4	0·2	3·2

Precision	X	Y	Z
Photogrammetric	0·07	0·07	0·19
Control - a priori	0·30	0·30	0·60
Control - a posteriori	0·07	0·07	0·22
Control - RMSE	0·4	0·5	1·0

Table 8.4 Restitutions of Chrysler Transmission Stereopairs

have a shoulder so that they will accurately seat on the surface and stand perpendicular to it. The holes are positioned to a nominal accuracy of 1 - 2 mm, but no convenient and quick way of coordinating them on the Topocart table could be devised.

By placing a steel straight edge against the perspex sheet the departures of the surface from a plane were estimated to be also of the order of 1 - 2 mm. The Topocart table has a precision of 0·1 mm and the length gauge used had a precision of 0·25 mm. Therefore the precisions of the grid points and height bars were estimated as (0·1, 0·1, 0·5) and (0·5, 0·5, 0·7) respectively. Altogether, eighteen control points were provided, but only approximately half of that number appear in each of the terrestrial pairs. Some were behind the object and some height bars were removed from each so as not to obscure the object. The grid was approximately levelled so that the camera orientations would be within the limits set by the stereoplotter rotation ranges.

The results of the restitutions shown in Table 8.4 are generally means for the entire five stereopairs. The disadvantage the aerial pair has in base to height ratio is partially compensated by the higher number of control points, better control distribution and larger format coverage. The results for the program SPIOC are not given. The results for the aerial stereopair could not be included in the means as no equivalent aerial program was developed.

8.2.3 Dorchester Bath-house

The Roman bath-house at Dorchester in Dorset was discovered during some foundation excavations in 1977. A 40 m by 50 m area of the site was uncovered by rescue archaeologists before the land was reclaimed by the owners for construction of the building. The remains of the bath-house were to be preserved by backfilling the trench with sand, but a permanent record of the site was made by four stereopairs photographed with black and white plates and colour film.

One stereopair was taken from each side of the rectangular trench to eliminate as much dead ground as possible and a typical frame is shown in Figure 8.4. Although the photographs were primarily for archive purposes, it was expected that any derived data would be in

Figure 8.4 Typical Frame, Dorchester Bath-house Stereopairs

the form of graphical output from stereoplotters. The cameras were therefore set normal to the base in each case, with a downward tilt of three degrees to obtain as much coverage as possible without exceeding the rotation limits on the stereoplotters.

Eight targets were provided in the trench for control, well distributed throughout the area. The paper targets used could be rotated about their central spike to face any standpoint at the edge of the trench. They were controlled by theodolite intersections from three survey stations established on the outskirts of the site and by conventional levelling from a local datum. An MA100 Tellurometer was used to measure distances between the survey stations, and a few distances between the control points were measured with an invar band to supplement the angle measurements. An average of six targets appeared on each stereopair.

The results of the restitutions given in Table 8.5 are means for the four stereopairs. All four had very similar characteristics.

8.2.4 Lake Merrimu

This stereopair was the only photography not obtained from the Department of Civil Engineering projects. It is in fact taken from earlier research (Shortis 1977) as an extremely well controlled stereopair. The object is an earthen/rockfill dam wall which was selected as a typical subject for terrestrial photography with a high depth range. The cameras were normal to the base and horizontal as the original use of the stereopair was research into stereoplotter orientations.

Twenty control points, the maximum allowed in SPIOC, were chosen from the thirty available in the object. A variety of targets were used; tripod mounted theodolite traversing targets, stick-on paper targets with a drawn cross and wooden staked targets with a painted cross. The coordinates of the points were established by horizontal and vertical theodolite angles from three stations on a baseline. The distances between the baseline stations were measured by an invar band in catenary.

The left hand photograph is shown in Figure 8.5 and the results of the restitutions are given in Table 8.6.

Figure 8.5 Left Hand Frame, Lake Merrimu Stereopair

Program	X	Y	Z	PZ	σ_0
STECT	3·7	2·7	1·7	0·3	7·3
STECT - Weighting	3·2	3·5	1·7	0·4	2·6
SPAOC	2·2	2·4	0·6	0·4	3·8
STECI	3·2	3·3	1·6	0·2	2·4
SPIOC	2·1	2·2	0·6	0·3	3·6
STECT - Adj. Control	2·7	2·1	1·0	0·3	3·7

Precision	X	Y	Z
Photogrammetric	3·3	5·1	2·4
Control - a priori	0·9	2·1	1·4
Control - a posteriori	0·8	2·1	1·0
Control - RMSE	2·6	1·2	1·4

Table 8.5 Restitutions of Dorchester Bath-house Stereopairs

Program	X	Y	Z	PZ	σ_0
STECT	17·2	19·2	4·5	1·7	2·9
STECT - Weighting	18·5	19·2	4·5	1·7	1·1
SPAOC	8·6	10·1	1·1	1·7	1·2
STECI	19·0	19·9	4·4	1·5	1·1
SPIOC	7·5	8.5	1·0	1·5	1·1

Precision	X	Y	Z
Photogrammetric	17·9	29·8	3·8
Control - a priori	9·8	18·4	4·7
Control - a posteriori	8·5	14·5	2·8
Control - RMSE	11·1	14·0	4·1

Table 8.6 Restitutions of Lake Merrimu Stereopair

8.2.5 Munich Aviary

This stereopair was part of an undergraduate student project under the supervision of the Terrestrial Photogrammetric Unit and the Structures Section of the Department of Civil Engineering. A model of a proposed bird aviary for the Munich Zoo was photographed for finite element analysis of the cable network structure. The cameras were mounted above the structure with axes vertical. The left hand photograph is shown in Figure 8.6.

Control was provided by eleven paper targets stuck to the floor around the model. Distances between the points were taken with a two metre steel straight edge in sufficient numbers to provide a few redundancies in the trilateration network. The heights of the points were established by conventional levelling with the same straight edge. Calibrated length bars were placed on the model and levelled, but it was later found that the base had deformed under the weight of the straight edge and the points had to be discarded as control. Unfortunately this left the control almost entirely in a single plane, but it is still a valid test of the restitutions.

The results of the restitutions are given in Table 8.7. There are no entries for the final restitution as this was an aerial stereopair.

8.2.6 S. M. M. Propeller

Stone Manganese Marine Ltd approached the Terrestrial Photogrammetric Unit to carry out a pilot project of photogrammetric measurement of marine propellers. The aim of the project was to represent graphically the amount of material which had to be removed from the propeller to trim the casting down to the design dimensions. The camera was suspended above the propeller and vertical photography taken for stereoplotter sections to be drawn (Cooper 1979).

Four control points were provided around the propeller in the form of cross wire targets laid flat. The positions were controlled by a braced quadrilateral of invar band distances and heights by conventional levelling.

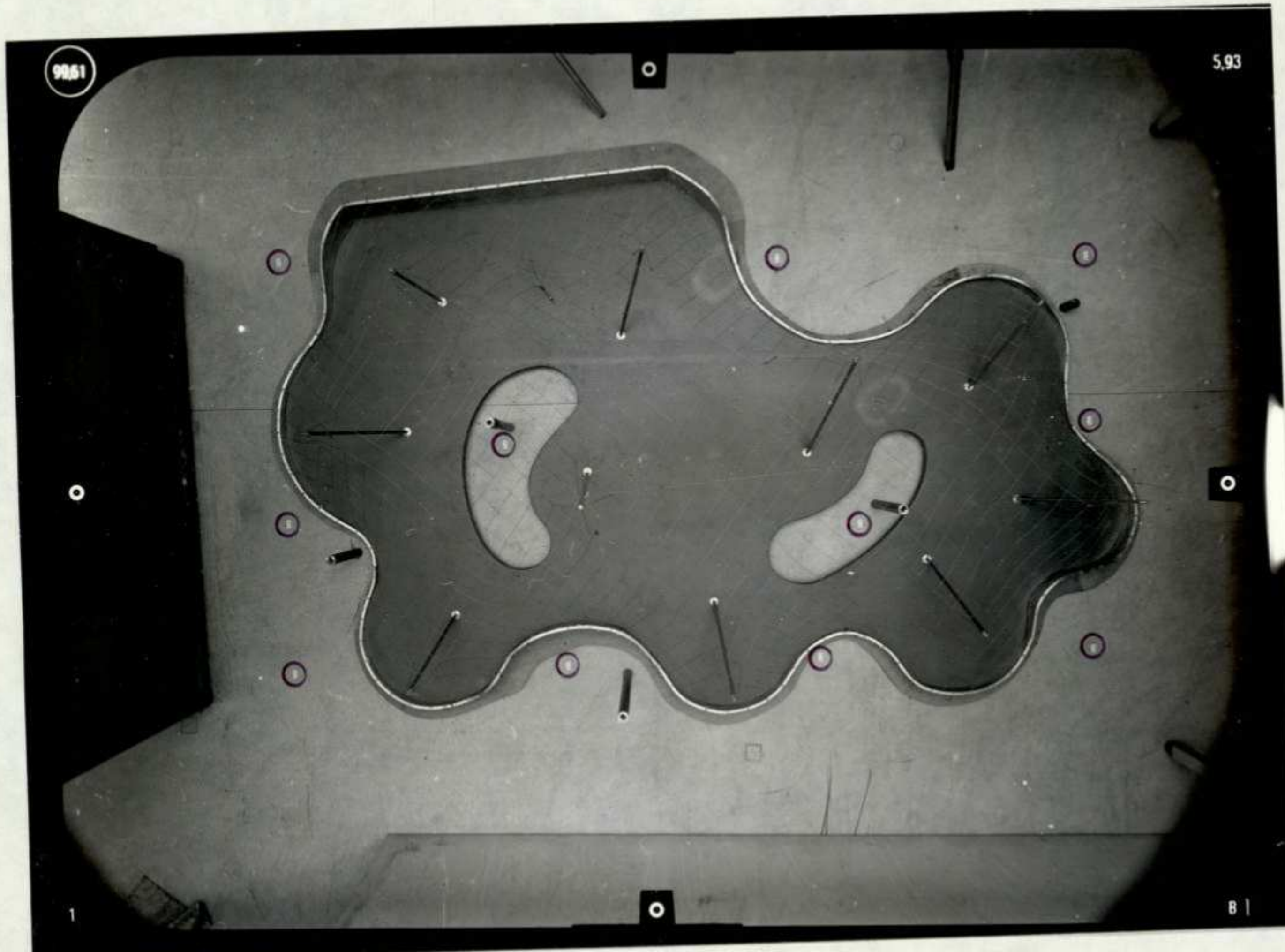


Figure 8.6 Left Hand Frame, Munich Aviary Stereopair

Programs	X	Y	Z	PZ	σ_0
STECA	0·3	0·2	1·0	0·1	3·7
STECA - Weighting	0·2	0·1	1·1	0·1	2·3
SPAOCA	0·2	0·1	0·6	0·1	2·6
STECIA	0·2	0·1	1·1	0·1	2·3
-	-	-	-	-	-
Precision	**X**	**Y**	**Z**		
Photogrammetric	0·06	0·06	0·19		
Control - a priori	0·13	0·12	0·21		
Control - a posteriori	0·11	0·09	0·18		
Control - RMSE	0·4	0·2	0·7		

Table 8.7 Restitutions of Munich Aviary Stereopair

Programs	X	Y	Z	PZ	σ_0
STECA	0·6	1·2	0·4	0·7	9·3
STECA - Weighting	0·3	1·4	0·4	0·6	4·4
SPAOCA	0·2	0·4	0·5	0·6	4·5
STECIA	0·4	0·6	0·1	0·3	2·9
-	-	-	-	-	-
Precision	**X**	**Y**	**Z**		
Photogrammetric	0·7	1·1	0·3		
Control - a priori	0·4	0·4	0·5		
Control - a posteriori	0·2	0·2	0·3		
Control - RMSE	0·6	1·4	0·2		

Table 8.8 Restitutions of Marine Propeller Stereopair

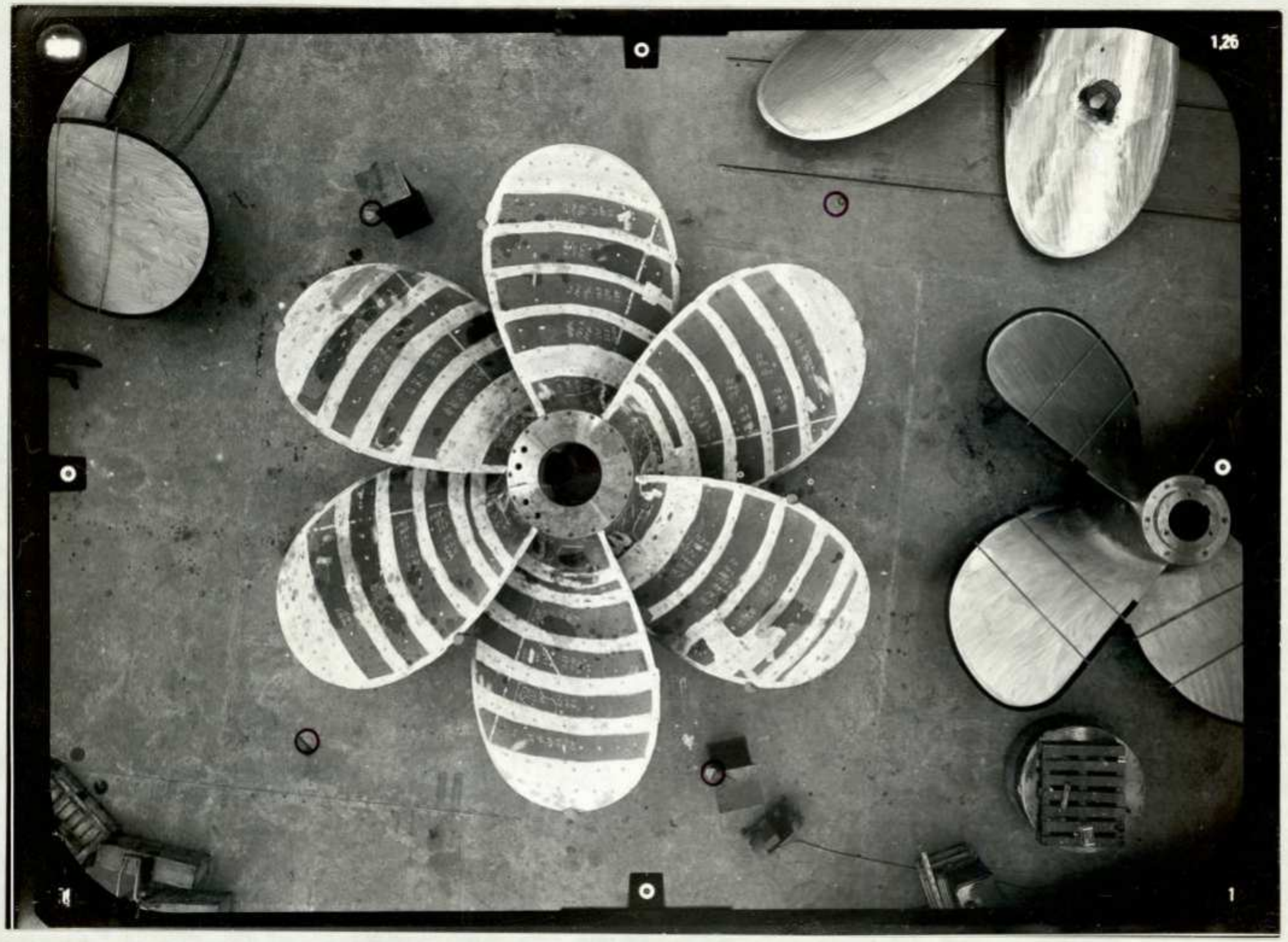


Figure 8.7 Left Hand Frame, S.M.M. Propeller Stereopair

The left hand photograph is shown in Figure 8.7 and the results are given in Table 8.8. Again there are no entries for the final restitution as this was an aerial stereopair.

8.2.7 St. Paul's Cathedral

Monitoring of the facade of the south portico of St. Paul's Cathedral by photogrammetry was another pilot project which has been reported on elsewhere (Cooper and Shortis 1980). Two sets of photography were taken over an interval of two years in an attempt to detect deformations of the structure. Digital profiles were measured on both the stereocomparator and on a stereoplotter, so once again the normal case of photography was employed with the camera axes horizontal.

The stereopairs from both years are included because the camera/object configurations were quite different. In 1978 (see Figure 8.8) the camera was mounted in the basket of a light-cleaning crane and raised ten metres above the ground surface so that the object would fill the format and the minimum camera to object distance could be obtained. In 1980 (see Figure 8.9) the crane was not available and ground stations had to be used at a larger distance from the facade. The base to depth ratio was increased to compensate, but there was a significantly smaller area of the format covered by the object.

The control was provided by theodolite angles to targets on the facade in each case. A three station baseline was permanently established and the distances measured with an MA100 Tellurometer and invar bands in catenary. A level datum was established for the three stations, but the targets were fixed in height by vertical angles. In 1978 nine physical locations (e.g. mortar joints) were selected as control points. In 1980, again to try and compensate for the poorer photography, the lower three were replaced by four paper stick-on targets which increased the depth coverage and were better defined for both the survey and photogrammetric measurements.

These stereopairs were the only photographs observed independently by two experienced operators, and the results given in Table 8.9 and 8.10 are the means for two restitutions in each case. The

Figure 8.8 Left Hand Frame, St. Paul's Cathedral 1978 Stereopair

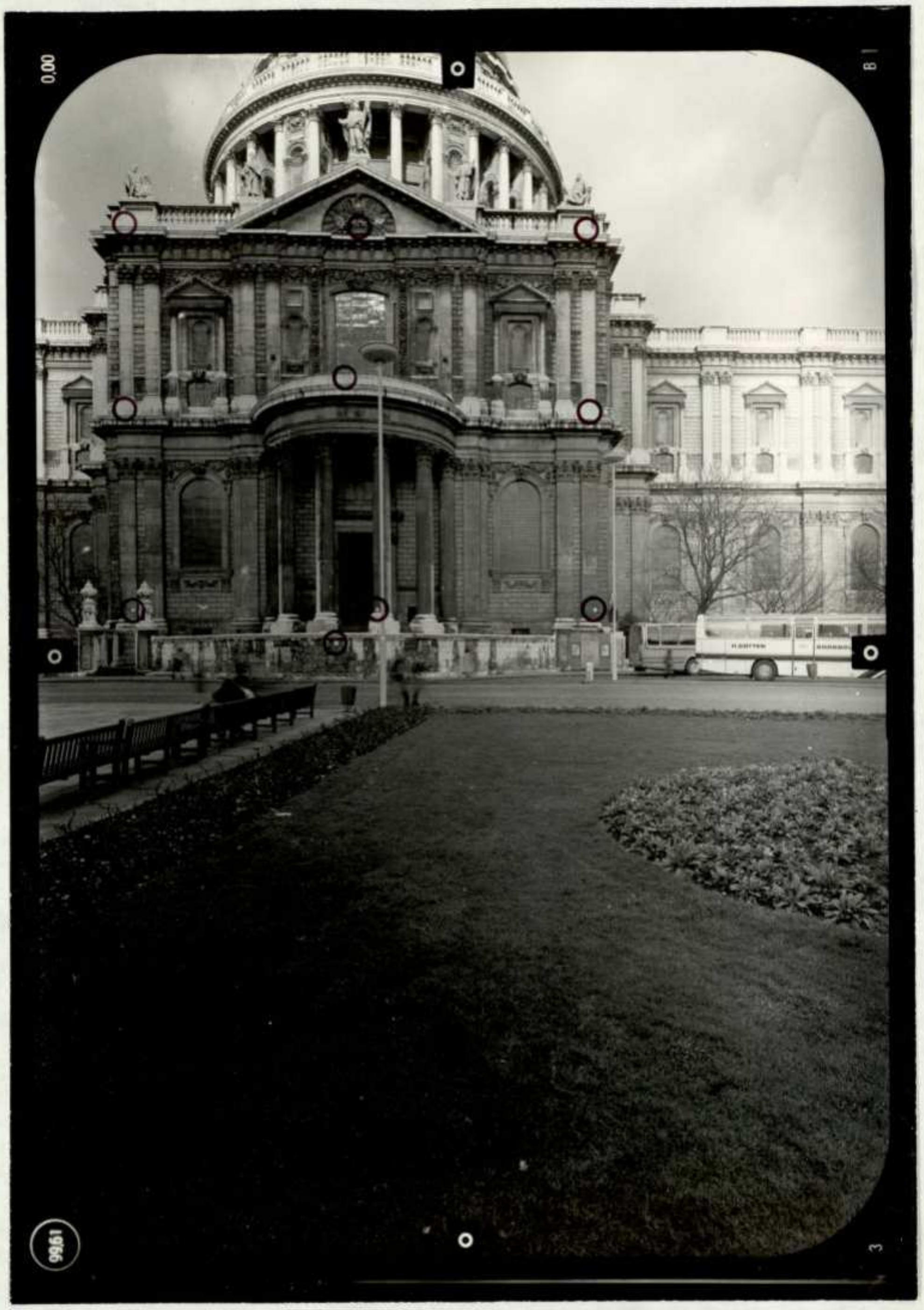


Figure 8.9 Left Hand Frame, St. Paul's Cathedral 1980 Stereopair

Program	X	Y	Z	PZ	σ_0
STECT	4·2	5·7	4·6	3·6	3·8
STECT - Weighting	3·8	7·0	2·8	3·4	2·3
SPAOC	2·1	3·2	2·7	3·6	2·6
STECI	3·2	6·3	2·4	3·3	2·2
SPIOC	1·3	3·0	2·1	3·4	2·2

Precision	X	Y	Z
Photogrammetric	1·5	8·1	3·7
Control - a priori	1·5	3·6	1·4
Control - a posteriori	1·3	2·7	1·2
Control - RMSE	2·0	4·3	1·4

Table 8.9 Restitutions of St. Paul's Cathedral 1978 Stereopair

Program	X	Y	Z	PZ	σ_0
STECT	3·7	4·3	3·9	2·5	4·0
STECT - Weighting	3·8	4·4	3·4	2·3	2·9
SPAOC	2·7	3·5	2·9	2·4	3·4
STECI	3·4	3·8	2·8	2·1	2·5
SPIOC	2·5	2·9	2·5	1·7	2·7

Precision	X	Y	Z
Photogrammetric	1·7	4·9	3·0
Control - a priori	1·0	2·0	1·0
Control - a posteriori	1·0	2·0	1·0
Control - RMSE	0·8	1·7	0·7

Table 8.10 Restitutions of St. Pauls Cathedral 1980 Stereopair

discrepancies and parallaxes at control differed by less than two millimetres at maximum and were generally of the order of one millimetre for the two operators.

8.2.8 York Way Tunnel

This stereopair was part of another deformation survey by analytical photogrammetry carried out as a pilot project in conjunction with the Transport and Road Research Laboratory. Photographs were taken before, during and after the driving of an underground pedestrian tunnel. The aim was to detect any structural deformations of privately owned buildings above the tunnel. Digital profiles were taken on the stereocomparator, but once more there was the possibility of the employment of a stereoplotter so the normal case of photography was used with the camera axes horizontal.

Six physical locations on the building facades (see Figure 8.10) were chosen for control points outside the expected area of deformation, and were used for all stereopairs. The points were coordinated by horizontal and vertical theodolite angles from three stations on a temporary baseline. The distances between the stations were measured with an invar band and conventional levels from a local datum were used to establish the heights.

This was the only case where it was felt that the estimate of the variance factor from the least squares adjustment of the survey measurements should have been included in the weight coefficient matrix of the coordinates. The resultant precision of less than one millimetre was not representative of the survey, primarily because well-defined physical locations could not be found on the facades and stick-on targets could not be employed for legal reasons. Brickwork mortar joints and window frame corners had to be used and could not be defined in position to better than 1 - 2 mm, i.e. the pointing error of the theodolite far out-stripped the definition of the target, and this does not show in the resultant precisions of the coordinates. However, the variance factor was not included so as to retain consistency and to investigate the effect of such systematic errors on the restitutions.

Figure 8.10 Left Hand Frame, York Way Tunnel Stereopair

Programs	X	Y	Z	PZ	σ_o
STECT	5·0	2·5	1·8	0·7	5·2
STECT - Weighting	3·5	2·2	1·7	0·6	3·7
SPAOC	4·9	2·3	1·8	0·7	5·3
STECI	2·8	1·4	1·0	0·6	3·1
SPIOC	2·8	0·9	1·0	0·6	3·4
Precisions	**X**	**Y**	**Z**		
Photogrammetric	0·5	1·3	0·7		
Control - a priori	0·6	0·5	0·4		
Control - a posteriori	0·6	0·5	0·4		
Control - RMSE	0·2	0·3	0·1		

Table 8.11 Restitutions of York Way Tunnel Stereopair.

The results shown in Table 8.11 are from the final stereopair only. All the stereopairs were exactly similar in camera/object and control configuration so inclusion of earlier pairs was not necessary. Mean results were not used as there was a remote possibility that the deformations could have extended beyond the central area, thus distorting the control. Only relative deformations at the centre of the drive area were required, so such small distortions were not considered important with regard to the results of the project.

8.3 Results

8.3.1 Sequential Adjustments of Control

The results of the restitutions of the stereopairs are given in the Tables 8.3 - 8.11. The upper half of each table shows the root mean square discrepancies at the control points and the root mean square parallaxes at control and parallax points in millimetres, for the programs STECT/A and SPAOC/A. The estimates of the variance factors from the adjustment of the exterior orientation or sequential adjustment are also shown. The first entries for STECT/A are from a simple restitution and the second are from a restitution with control weighting. The entries for SPAOC are from a sequential adjustment of the control using the full weight coefficient matrix.

In the two cases of multiple stereopairs the above results are given as means from all the pairs. A third set of entries is shown for STECT/A from simple restitutions of the pairs with adjusted control. The adjusted coordinates were obtained from multiple sequential adjustments of the stereopairs using SPAOC/A, as described in Section 5.1.4.

At the bottom of each table various precisions are shown. Firstly, the photogrammetric precision of measurement at the control centroid and the mean control precision are reiterated for reference. The mean control precisions for each coordinate are simply the means of the standard deviations of that coordinate for all control points, and are labelled a priori to distinguish them as pre-sequential adjustment. The post-sequential adjustment mean control precisions are shown and labelled a posteriori. These standard deviations were extracted from the SPAOC/A sequential adjustment without the inclusion of the estimate of the variance factor. If multiple stereopairs were present then the standard deviations are from the final adjustment in the multiple set.

The last line in each table gives the root mean square corrections made to the control point coordinates on the basis of all available photogrammetric measurements. That is, the differences between the coordinates from the original control or survey measurements and the adjusted coordinates from the sequential adjustment(s) are presented as root mean square errors.

All precision data are given in millimetres, generally to one decimal place, as this is usually the limit imposed by the six to seven figure significance of the processor. However, to clarify the relative precisions of the photogrammetry and pre- and post-adjustment standard errors, two decimal places are shown for some of the short range stereopairs. Rounding-off errors should not be significant as the coordinate values in the control for these stereopairs were only of the order of metres, rather than the tens or hundreds of metres for the longer range stereopairs.

8.3.2 Sequential Adjustments of Interior Orientation

In each of the tables discrepancies, parallaxes and variance factors are given for the restitutions of the stereopairs using programs STECI/A and SPIOC. The entries for STECI/A are from a sequential adjustment of the interior orientation with control weighting. The entries for SPIOC appear only for terrestrial stereopairs, and are from a sequential adjustment of the interior orientation and control, using the full weight coefficient matrix of the control coordinates. The values are again given as root mean square errors in millimetres, and mean results are shown for multiple stereopairs. The mean values for the Chrysler Transmission restitutions by SPIOC would be misleading as the aerial stereopair does not contribute, and are therefore not given.

In addition to the four glass plate stereopairs observed for the Dorchester project, two of the four colour film stereopairs were observed. The other two film pairs were not observed as they showed visible distortions around the edge of the frame due to uneven film transport in the cassette. Mean results for the two film stereopairs are compared to the corresponding glass plate pairs in Table 8.12. As these pairs were only to be used for the investigation of sequential adjustments of interior orientation, only restitutions by STECT and STECI were carried out, both with control weighting.

Program	X	Y	Z	PZ	σ_0
Glass Plates					
STECT	3·0	2·5	1·6	0·4	2·6
STECI	2·9	2·2	1·5	0·2	2·3
Roll Film					
STECT	5·7	8·7	2·5	1·8	6·2
STECI	3·3	6·9	1·9	0·3	3·9

Table 8.12 Comparison of Restitutions of Dorchester Glass Plate and Colour Roll Film Stereopairs

8.4 Analysis and Discussion

8.4.1 Control Weighting and Adjustment

8.4.1.1 Single Stereopairs

The restitutions of all single stereopairs were improved by the more sophisticated solutions. In every case the control precision was significant when compared with the precision of the photogrammetric measurements, and the extent of the improvements is indicated by the reductions in the discrepancies at control and the estimates of the variance factors. The parallaxes are unaffected by the changes in functional and stochastic models and the magnitudes remain constant for the three types of restitution.

The introduction of control weighting into the restitutions reduced the estimates of the variance factors from unaccountably high levels to more acceptable values. The discrepancies at control did not change overall magnitudes, but often were rearranged between the coordinates. The precisions of the control points in the three coordinate directions can differ markedly from one another when the control is established by geodetic measurements, and this differential weighting is reflected in changes in the pattern of discrepancies.

The exterior orientations of the cameras generally changed quite significantly, and where there was a wide variation in the precisions of individual control points the changes were quite substantial. The weighting tended to decrease the influence of poorly defined control points in favour of the well defined points. The a posteriori precisions of the exterior orientation parameters from the solution generally remained constant, as the increase from the larger uncertainties of the control points was balanced by the decrease in the variance factor.

Restitutions of the stereopairs with a sequential adjustment of the control coordinates reduced the discrepancies at control in comparison with the two simpler restitutions. The corrections to the control coordinates were virtually the only source of improvement over the weighted control restitution, as the exterior orientations were altered only slightly by the more sophisticated functional model. The precisions of the exterior orientations were again largely unchanged.

The variance factors for the sequential adjustments invariably increased slightly on the values from the weighted control restitutions. The increase can be attributed to the simpler functional and stochastic models employed by the weighted control restitution. The residuals of measurement are no longer considered to be purely photogrammetric, but are segregated into photogrammetric and control components. Hence the residuals at control will no longer be affected by photogrammetric correlations but will be affected by control correlations. Experiments with fictitious data showed that the increase in the factor was enhanced by strong correlations between the control coordinates.

The York Way stereopair is the only exception to these rules, the discrepancies and the variance factor obtained by the sequential adjustment are not significantly better than those for the simple restitution. Inclusion of the estimate of the variance factor from the initial control adjustment into the weight coefficient matrix gave a slight reduction, but the main cause is undoubtedly systematic errors in the control as discussed in Section 8.2.8.

The magnitudes of the improvements gained by the more complex restitutions were influenced by a number of factors, but by far the most prevalent was the ratio between the precisions of the control and photogrammetric measurements. If the ratio was high and the control coordinates had random errors which were easily detectable by the photogrammetric measurements, then there was a large improvement. The clearest examples of this were the Chrysler stereopairs (see Table 8.4). The photogrammetric precision was better than control precision in all three coordinates, and correspondingly large improvements were gained. The corrections to the control coordinates were also large, compared to their a priori standard deviations.

On the other hand, if the ratio was low then the control coordinates tended to be fixed for the restitutions and there was much less improvement in the discrepancies and variance factors. The most obvious example at this end of the scale was the St. Paul's Cathedral 1980 stereopair, where the control precision was approximately half that of the photogrammetric measurement in all three coordinates. The reductions in the discrepancies and the variance factor were the least of any of the stereopairs. However the corrections to the control were still significant and roughly the order of their a priori precisions.

Every other stereopair had a mixture of ratios for the three coordinate directions, and the general trend of increasing improvement with larger ratios becomes less clear. The trend is confused further by the different camera configurations and control distributions. Some stereopairs have relatively uniform photogrammetric precisions throughout the object space, while others like Dorchester and Lake Merrimu have wide variations. The control precisions can also vary widely throughout the object space when the points are established by geodetic measurements. The interaction of these different precisions will affect the results of the restitution in ways which cannot be deduced from a simple consideration of average or centroidal precisions.

It is not the aim of the analysis of the results to investigate the effects of different camera and control configurations, and it has been discussed thoroughly by other authors (Hottier 1976). The main deduction that can be made from the results is that the restitutions are improved for all types of configurations. The best guide to the magnitude of the improvement is the variance factor, as all influences of configurations, distributions and precisions are incorporated into it by the adjusment.

8.4.1.2 Multiple Stereopairs

All the photogrammetric measurements from the Dorchester and Chrysler projects were utilized to adjust the control coordinates by the technique of multiple sequential adjustment of control. The stereopairs were individually processed by SPAOC/A and in each case the adjusted coordinates and new weight coefficient matrix were used as input to the restitution of the next stereopair. Estimates of the variance factors were not included at any stage so as to avoid any differential weighting of measurements.

The final adjusted coordinates were then checked by repeating the restitution of each stereopair using STECT without control weighting. The mean results for the root mean square errors of all the stereopairs are given in Tables 8.4 and 8.5, indicated by "STECT - Adjusted control". As these final coordinates are the best estimates possible from the available data, it could be expected that the discrepancies, parallaxes and variance factors are the lowest obtainable. The results are certainly a considerable improvement on the initial control set by comparison with the earlier simple restitutions.

The results should theoretically also be an improvement on the mean results of the root mean square errors for the individual sequential adjustments of control, as the maximum amount of random error has been removed from the coordinates. However there is only a slight improvement for the Dorchester stereopairs and a definite deterioration for the Chrysler stereopairs. In the former case the lack of improvement could be attributed to target movement between the photography of stereopairs. Each free-standing target had to be rotated by hand to face the new photographic standpoints, and the possibility of some disturbance to the positions cannot be ruled out. This would distort the final coordinates and reduce the extent of the improvement.

The deterioration for the Chrysler stereopairs can be directly attributed to rounding off errors. Coordinates are only carried to tenths of a millimetre, and a clear picture of the results for these stereopairs could only be obtained if one hundredths of a millimetre were considered. All programs were configured for a maximum of four decimal places of metres as it was never foreseen that five decimal places could be required, and therefore the results cannot be verified.

8.4.2 Interior Orientation Adjustment

8.4.2.1 Stereopairs with Fiducial Control

The initial interior orientations for the stereopairs taken with the P32 cameras utilized the five known fiducial marks and deformational transformations. The coordinates of the fiducials and their estimated 3 μm standard errors were taken directly from the calibration certificates. The estimates of the variance factors from the initial adjustments for the interior orientation parameters were often low, averaging approximately 0·7.

This value was considered to be acceptable, but the factors were not included in the weight coefficient matrices of the parameters passed on to subsequent sequential adjustments. There were variations in the factors, from a minimum of 0·2 to a maximum of 1·3, and differential weighting of sets of the parameters to this extent was felt to be unwarranted. All observations were taken under the same conditions and there was little variation in the definition of the fiducial marks. In any case, the only effect on the sequential adjustment of incorrect initial weight coefficients has been shown to be only a change in the magnitude of the variance factor.

The improvements to the restitutions of the Dorchester and Lake Merrimu glass plate stereopairs due to the sequential adjustment of the interior orientation are relatively minor (see Tables 8.5 and 8.6) both for the control weighted and control adjustment solutions. The vectors of the root mean square discrepancies at control, the root mean square parallaxes and the variance factors showed only small reductions. These results were not unexpected in the light of experience gained with fictitious data tests. If the emulsion base is stable and has good flatness characteristics then the initial interior orientations utilizing known fiducial positions give parameters which are not improved to any significant extent by the addition of control observations.

Analysis of the residuals at the fiducial marks for the eight photographs taken for the Dorchester project showed why the initial interior orientations were giving low variance factors. The mean values of the adjusted residuals from the STECI restitutions of the Dorchester stereopairs are shown in Table 8.13, with their standard errors. These mean values represent the systematic uncertainties in the fiducial mark positions, and have a root mean square of 0·8 μm. These systematic uncertainties were estimated as 3 μm from the calibration certificate, and the over-estimation is the cause of the low variance factors.

Point	x	y
1	0·0003 ± 0·0016	0·0003 ± 0 0018
2	-0·0020 ± 0·0023	-0·0012 ± 0·0020
3	0·0009 ± 0·0010	0·0003 ± 0·0013
4	0·0002 ± 0·0012	0·0001 ± 0·0003
5	0·0003 ± 0·0016	0·0003 ± 0·0018

Table 8.13 Mean Residuals at Fiducial Marks of Wild P32 Camera (mm)

The largest residuals in the table are for point 2, the central point of the format. On the calibration certificate this point is explicitly not given a standard error (see Figure 2.2), presumably because its position is considered to be invariant with respect to the principal point. All points were given equal weight in the interior orientation, as the fixation of a particular point in the array by assigning it infinite weight is an unreasonable restriction which introduces significant distortions. The five fiducial marks are considered to be positioned relative to the principal point with equal precision and the fact that a particular point has zero coordinates is irrelevant to the interior orientation. The lack of correspondence of the central fiducial and the principal point must be determined by a camera calibration and has no relevance to the transformation between the stereocomparator and fiducial coordinate systems.

The average standard error of observation on the fiducial marks from Table 8.13 is 1·4 μm. This value agrees with the value expected from stereoacuity tests of the operator. The standard error of a single observation from the tests was approximately 2 - 3 μm, and as there were three rounds of observations on the fiducials the standard error expected was approximately 1 - 2 μm.

Some of the restitutions were run again using a new value of 1 μm for the fiducial mark uncertainty, and the average value of the variance factors from the initial interior orientations rose to approximately 1·4. This implies that the observation precisions are being under-estimated, and this is much more plausible than the over-estimation indicated earlier by the value of 0·7. The interior orientation parameters and subsequent sequential restitutions were not affected by this alteration to the stochastic model, except by a barely significant increase in the variance factors of the sequential adjustments.

The results for the Dorchester film stereopairs (see Table 8.12) do show substantial improvements to the discrepancies, parallaxes and variance factors. Compared with the values for the corresponding glass plate stereopairs, the film pairs show distinctly poorer results for the simple restitution, but the values for two of the three coordinates and the parallaxes are reduced to the same level by the sequential adjustments. This implies that the film lacks stability and/or flatness as an emulsion base, but the deformational transformation is capable of removing the distortions when the control observations are used to improve the interior orientation.

The less pronounced improvement for the Y-coordinate was caused by a particularly poor result for one of the two stereopairs. The result undoubtedly influences the value of the variance factor for the sequential adjustment, which was higher than could be expected.

8.4.2.2 Stereopairs Without Fiducial Control

The initial interior orientations for the stereopairs taken with the UMK cameras utilized simple translation/rotations. The positions of the four fiducials for these cameras were given with a precision of 20 μm, and this is not sufficient when the observations are taken with a 1 μm stereocomparator. Errors in the positions of the marks of only 5 μm would produce significant scale distortions in the interior orientation and hence affect the subsequent exterior orientation.

Full deformational transformations were carried in the sequential adjustments of the interior orientation, with the weight coefficients of the parameters estimated by the principles described in Section 5.2.1. The random fluctuations in the scale factors and deformational parameters were estimated from grid tests made on glass plates of the type employed for UMK photography.

A glass grid plate normally used to calibrate the stereocomparator was exposed onto the photographic plates, which were then developed in the usual manner. Images of nine grid intersections distributed throughout the format area were then observed monocularly on one of the plate carriers of the stereocomparator. The observed positions were processed by the deformational interior orientation transformation using the known grid positions as control. The resultant scale factors and deformational parameters are shown in Table 8.14, where the $\bar{x}$-axis was aligned with the longer side of the plate format.

The standard errors shown were used as the estimates of the precisions. The means were not used as estimates of the systematic errors in the parameters. The grid test is probably not indicative of plates exposed in the UMK camera, and the standard errors can only be used because they will not effect the restitutions to any great extent.

Plate	$\bar{x}$-scale	$\bar{y}$-scale	$a_7 x 10^7$	$a_8 x 10^7$
1	0·999882	0·999926	2·68	-4·57
2	0·999933	0·999996	-1·01	3·10
3	0·999850	0·999935	-2·44	7·74
4	0·999849	0·999805	-3·50	8·05
5	0·999878	0·999911	-1·20	5·73
Mean	0·999878	0·999915	-1·09	4·01
Standard Error	0·000035	0·000071	2·34	5·19

Table 8.14 Results of UMK Glass Plate Grid Tests

If the values are gross over- or under-estimate then the only effect should be a change in the restitution adjustment variance factors, as predicted by tests with fictitious data.

If the systematic error was introduced then there would be an effect on the restitution because a basic assumption has been altered (this was verified during the fictitious data tests). However these scale factors represent an overall expansion of approximately 20 μm on the long side of the plate. A change of this magnitude cannot be lightly ignored and is definitely worthy of further investigation. The systematic variations for the deformational parameters are less in magnitude, approximately 2 μm at the edges of the plate, but still at a significant level.

The tables of results show that the sequential adjustments of the interior orientations generally gave a reduction in the discrepancies, parallaxes and variance factors for the restitutions of UMK stereopairs. The improvements were certainly more substantial than those gained with the P32 camera with its known fiducials, but again this could be expected from the fictitious data trials. The sequential adjustments of interior orientation show approximately the same improvement whether control weighting or adjustment is used, but this trend is a little uncertain as the aerial pairs for Munich and the Propeller do not have entries for the control adjustment.

The magnitudes of the improvements to the restitutions show no particular trend with respect to the format coverage by the control array. In fact the stereopair with the best distributed control throughout the format area (the Munich Aviary stereopair) is the only one to show no improvement at all. However the control must also be well distributed in the object space, without depth in the control array a change to the image scale can also be implemented by a change in the camera/object distance. Therefore the effect of minor alterations to the interior orientation will be indistinguishable from changes to the exterior orientation and the sequential adjustment will not improve the restitution. The Munich Aviary control array lies in the plane which is nearly perpendicular to the camera optical axes, and this is undoubtedly the reason for the lack of improvement.

Any relationship between improvement of the restitutions and both format coverage and object space distribution of control is blurred by statistical fluctuations in any case. A large sample of different plates for each stereopair would be required before any definite conclusions could be confidently drawn. Again the only real deduction which can be made from the results is that the sequential adjustment of the interior orientation is effective for all types of camera/ object configurations and control distributions, except the case where there is no depth in the control array.

An indication of the random fluctuations in the scale and deformational parameters can be obtained from Table 8.15. The adjusted values of the scale parameters a_3 and a_6, and the deformational parameters a_7 and a_8 are given for the restitutions of the UMK stereopairs with a sequential adjustment of interior orientation and control weighting. The parameters vary from almost no corrections, e.g. Chrysler Terr/1 right hand plate, to very substantial corrections, e.g. Propeller left hand plate.

The standard errors of the scale parameters approximately agree with the values obtained from the grid tests. The agreement is even better if the left hand plate of the Propeller stereopair is eliminated. The values for it are uncharacteristically large and can be rejected on the basis of three standard errors. Either this

Stereopair	Camera	Format	Photograph	a_3	a_6	$a_7 x 10^7$	$a_8 x 10^7$
BL	UMK 3	Horiz.	L	0·999988	0·999996	-1·16	7·84
C40P			R	1·000060	0·999936	0·94	-7·27
Chrysler	UMK 3	Horiz.	L	0·999973	1·000073	-1·60	-2·27
Aerial			R	1·000071	0·999880	1·43	2·36
Chrysler	UMK 3	Horiz.	L	1·000041	0·999941	0·16	0·39
Terr/1			R	0·999996	0·999998	-0·04	-0·21
Chrysler	UMK 3	Horiz.	L	1·000021	0·999915	0·27	1·35
Terr/s			R	1·000023	1·000049	-0·12	-0·49
Chrysler	UMK 3	Horiz.	L	1·000021	0·999960	0·11	-0·38
Terr/3			R	1·000020	1·000003	-0·12	0·52
Chrysler	UMK 3	Horiz.	L	1·000017	0·999968	-0·03	-0·58
Terr/4			R	1·000021	0·999995	0·03	0·74
Munich	UMK 4	Horiz.	L	1·000040	0·999965	0·78	-0·54
Aviary			R	1·000004	0·999999	-0·98	-0·22
S.M.M.	UMK 2	Horiz.	L	0·999823	1·000198	-1·88	1·19
Propeller			R	0·999965	1·000005	-0·71	0·00
St.Paul's	UMK 1	Vert.	L	1·000052	0·999960	-0·61	-0·32
1978			R	1·000037	0·999954	-0·22	0·24
St.Paul's	UMK 4	Vert.	L	1·000039	0·999959	0·54	-0·24
1980			R	1·000069	0·999942	-0·58	1·16
York	UMK 1	Horiz.	L	0·999890	1·000109	-1·18	1·11
Way			R	1·000099	0·999927	0·15	-3·59
Mean				1·000014	0·999991	-0·16	0·04
Std.Err.				0·000052	0·000068	0·83	2·64
Mean	Propeller/L			1·000023	0·999981		
Std.Err.	Deleted			0·000031	0·000051		

Table 8.15 Results of Interior Orientation Sequential Adjustments of UMK Stereopair

plate had outstandingly poor flatness or the UMK pressure platen was not engaged for the exposure (the failsafe shutter lock does not always operate for these cameras). The values for the vertical format St. Paul's Cathedral stereopairs have been transposed in columns to maintain consistency with the long side of the format.

The mean values for the scale factors disagree entirely with the values from the grid tests. Furthermore, there is a discernable tendency for the scale corrections to be approximately equal and opposite, i.e. differential scales are being applied to the two fiducial axes rather than absolute scale changes. This can again be attributed to the lack of depth in the control arrays, as positions in a single plane can only give information on relative scale changes. Absolute scale changes can be repressed by corrections to the exterior orientation parameters. This explanation is verified to some extent by the Chrysler stereopairs. The control array was the best distributed through three dimensions and the stereopairs show the least tendency toward the equal and opposite corrections.

The mean scales do show a surprising correlation with the stereo-comparator scale factors derived from calibration with the grid plates (see Section 6.1.3). In fact these means cancel the stereocomparator scales almost perfectly, straining the bounds of coincidence considerably. It suggests that the most recent calibration may have been influenced by statistical fluctuation and should be repeated with more grid points and a larger number of observations.

The means and standard errors for the deformational parameters bear no resemblance whatsoever to the values gained from the grid tests. This could have been expected as while the photograph plates used in the grid tests were subjected to even pressure by contact with the grid plate, photographic plates in the UMK are only supported at the edge of the frame. What is unexpected is that the magnitudes of the means and standard errors decreased rather than increased.

The values also show the tendency toward equal and opposite corrections, but in this case between left and right photographs rather than perpendicular directions on single photographs. This phenomenon is reduced only when there is a good distribution of control in the object space and a large coverage of the format by control points. None of the stereopairs presented here had this combination, and all show the equal and opposite effect to some extent.

However even the best controlled fictitious data stereopairs still showed a tendency towards equal and opposite corrections to the scale and deformational parameters, and also a reduction in the magnitudes of the corrections expected. This is caused by the interaction of the interior and exterior orientation parameters, and this interaction can never be entirely eliminated. Interior and exterior parameters must be correlated when they perform similar tasks (see Table 7.4), and will tend to compensate for one another. The deformational parameters in particular are strongly correlated to a number of exterior orientation elements, and the results of the sequential adjustment of interior orientations will probably never give absolute values representative of the physical deformations of the emulsion.

The mean positions and standard errors of the fiducial marks from the five Chrysler stereopairs are shown in Table 8.16. All photographs were taken within a short space of time and with the same focus setting, so the standard errors are much larger than could be expected. The numbering system used is the same as Figure 4.1, so the alignments of the fiducial mark axes are relatively stable but the separations of the fiducial marks are extremely variable. This strengthens the case for an initial interior orientation by simple translation/rotation followed by a subsequent sequential adjustment.

Point	x	y
1	-0·002 ± 0·002	-57·448 ± 0·015
2	0·002 ± 0·003	57·545 ± 0·013
3	-80·446 ± 0·007	-0·004 ± 0·007
4	80·721 ± 0·018	0·002 ± 0·009

Table 8.16 Mean Fiducial Mark Positions for Chrysler Stereopairs

The large standard errors may be partially due to the poorer definition of the UMK fiducials. The round spot is too large for accurate pointing and is often over-exposed, making matters worse. However if the variations are primarily due to physical movement of the fiducial marks then the stability of the marks and deformations of the camera body may be significant and worthy of further investigation.

8.4.2.3 Variance Factor

The most sophisticated restitution combined sequential adjustments of the control and the interior orientation, so all possible systematic errors for metric cameras should have been removed. However, the estimates of the variance factors from the adjustments of the observations using SPIOC have an average of approximately 2·5, not unity. This increase in the variance factor above the expected value can only be due to under-estimation of the precisions of the observations, or to systematic errors.

Under-estimation is not unlikely, and the variance factors from the initial interior orientation and control adjustments tend to support this. The average values for these variance factors were 1·4 and 1·3 respectively. This trend would be expected to continue for the control observations for the exterior orientation, and as none of these factors are included in the weight coefficient matrices they would all contribute to the overall variance factor. The estimated weight coefficients of the deformational interior orientation parameters for the UMK photography will not contribute to the final factor, as they are slight over-estimates according to Tables 8.14 and 8.15. Therefore the estimates of the variance factors from the sequential adjustments should have been of the order of 1·5, say.

Hence there must be unmodelled systematic errors, and the obvious sources are in the calibrations of the stereocomparator and the cameras. A more exhaustive test of the Stecometer has already been indicated as desirable by the results of the UMK interior orientations by sequential adjustment. If enough grid intersections were used then the analysis could be extended to the detection of cyclic errors in the lead screws, for example. The cameras could be re-calibrated to provide up-to-date values for the focal lengths, radial distortions and principal point positions. The tangential distortions of the lenses could also be investigated as no data are given in the calibration certificates.

9. CONCLUSIONS

9.1 Effectiveness of the Technique

The combination of the online minicomputer, vector solution and sequential adjustments is an effective method of analytical restitution of close-range stereopairs observed on a stereocomparator. The system has the advantages of speed, versatility and precision while permitting the use of very sophisticated solutions for the restitution of the photographs.

The online minicomputer gives rapid turnaround times for the results of restitutions by carrying out the computations during or immediately after the compilation of the observations. Editing of online or file data can be performed during the observation of the stereopair or as an integral part of the processing of the restitution, eliminating the major disadvantage of offline computations on mainframe computers.

The vector solution of exterior orientation fulfills all the requirements of Section 4.2.1, and is therefore particularly suited to the restitution of single close-range stereopairs. Although the solution has the disadvantage that it must be segregated into two distinct modes of operation, it is the only restitution which has all the desirable attributes for this particular type of photography.

The sequential adjustment of observations was found to be particularly useful during the online processing of data in real time. The option of adding observations to strengthen an initial adjustment can be included with little extra demand on the processing power of the minicomputer. The addition of observations was an integral part of the editing process to obtain the optimum restitution of each stereopair.

The sequential adjustments of both the control and the interior orientation showed marked improvements in the restitutions of all stereopairs tested. Many sets of both fictitious and real data were processed with a wide range of camera/object configurations and control distributions. The testing of real data included stereopairs with control provided by three different techniques and photographs from cameras with and without known fiducial positions in the format.

The sequential adjustment of the control was most effective where the precision of the control coordinates was comparable with the precision of the photogrammetric measurement. This is commonly the case for close-range photography, and if the photogrammetric precision was higher then that of the control fixation then the improvement of the restitution was outstanding. Significant corrections were made to the control coordinates in every case, even when the control precision was higher than the photogrammetric precision. The improvements to the standard errors of the control coordinates caused by the photogrammetric measurements were closely dependent on the ratio of the two precisions. However in the context of close-range photographs the improvement could always be expected to be significant, if not substantial.

Hence the sequential adjustment of control is an excellent method of restitution when the control coordinates cannot be defined with a sufficiently high precision to be considered fixed. Stereopairs which have camera-to-object distances of a few metres or less may have such a high precision of photogrammetric measurement that the provision of fixed control is impossible and a combined adjustment of the photogrammetry and control is the only answer. The corrected control coordinates from the sequential adjustment may be useful in their own right for further computations, or in deformation and monitoring surveys by photogrammetry.

The sequential adjustment of the interior orientation was most effective when the camera did not have fiducial marks which could be considered known. The lack of fiducial information was compensated by estimating the scale variations and deformations in the emulsion using the control observations, resulting in considerable improvements to the restitutions.

The extent of the improvement was determined by the control distribution and pattern. Maximum results were obtained from control arrays which had wide distributions in three dimensions and also covered a large area of the photograph format. The wide distribution of control reduces the correlation between interior and exterior orientation parameters which would otherwise tend to reduce the effectiveness of the solution. The large format coverage gives information on scale variations and deformations over the maximum area of emulsion.

This technique of sequential adjustment of interior orientation enables cameras with little fiducial information to be used for high precision analytical work. The uncertainty of the interior orientation which would normally not allow their use is overcome as long as the control array has the optimum characteristics. The technique also improves the restitutions of photography which does have known fiducials, but the extent of the improvement is naturally much reduced. The fiducial information provided produces an initial interior orientation which is not greatly altered by the inclusion of control observations.

A combined sequential adjustment of control and interior orientation has a potentially wide application in analytical close-range photogrammetry. The maximum precision can be gained from the analytical system, whereas the employment of simple restitutions, fixed control and independent interior orientations would give degraded results. When used in conjunction with online processing and the vector solution, the optimum restitution can be gained for close-range stereopairs with a minimum of time and effort. The data subsequently extracted from the photography will therefore be of the highest precision, and this is the ultimate aim of the system.

9.2 Further Development

There are a number of areas where more research could perhaps emphasize further the effectiveness of sequential adjustment techniques and investigate the precision of close-range photogrammetry. The interior orientation transformations and the vector solution could certainly bear more experimental testing to expand their application and effectiveness.

With particular regard to the experimental testing, a great deal more investigation needs to be carried out with roll film as the emulsion base. The limited amount of work performed on the P32 stereopairs with colour film gave an indication that the improvements gained by a sequential adjustment of the interior orientation could be very substantial. The contribution of the control observations to the definition of the interior orientation throughout the format is enhanced by the lack of stability of the film as an emulsion base. If film were to be used in a camera without known fiducial positions the improvement in the restitution caused by a sequential adjustment of the interior orientation could be remarkable.

The vector solution and sequential adjustments need to be verified with photography which deviates markedly from the normal case. Because all of the stereopairs used in the experimental testing were also destined for orientation and interpretation in stereo-plotters, no convergent or high tilt photography was included. Fictitious data tests indicated that the restitution and sequential adjustments were unaffected, and this could be expected as the mathematical model contains no approximations in this regard. However the stochastic model is derived with the normal case as a basic premise, and deviations from this case may cause errors which are apparent from the analysis of real data. Although radical departures from the normal case are unlikely for single stereopairs, the possibility exists and should be investigated.

The sequential adjustments were found to have an increased sensitivity to gross errors in observations or control coordinates. The presence of such errors was harder to establish because of the compensating effects of changes to the interior orientation or control corrections. To alleviate this problem some type of blunder detection should be incorporated into the restitution solutions. A method which indicates to the operator observations or coordinates which contain gross errors would be extremely useful.

Other types of eight parameter transformations could be tested with the interior orientations of close-range photography. No sufficiently detailed research has been carried out in this field, and the efficiency of emulsion flattening devices in close-range cameras is also very much an unknown factor.

Finally, a logical extension of the sequential adjustment of the interior orientation is the inclusion of those parameters normally considered fixed for metric cameras. The position of the perspective centre relative to the photograph and lens distortions could be considered known for the initial interior orientation, but incorporated into the sequential adjustment as measurements with estimated standard errors. The system would then be self-calibrating and systematic errors in the calibration of the cameras would be eliminated. The technique could also be applied to semi-metric or even non-metric cameras, and there would be no theoretical increase in the minimum control requirement of three fully known points. However, a good control distribution throughout the photograph format and an adequate distribution of points in three dimensions in the object space is always necessary for a practically valid result.

APPENDIX A

Partial differentials for the exterior orientation equations for the terrestrial case (see equations (4.35) in section 4.2.2.2):

Parameter	Function g_1	Function g_2	Function g_3	Function g_4
a_x	k	0	0	0
a_y	0	k	0	0
a_z	0	0	k	0
b_x	$\ell_x r_z$	$-\ell_z r_z$	$\frac{1}{2}(\ell_y r_z + \ell_z r_y)$	$\ell_z r_y - \ell_y r_z$
b_y	$\ell_x r_x$	$-\ell_z r_x$	$\frac{1}{2}(\ell_x r_y + \ell_y r_x)$	$\ell_x r_y - \ell_y r_x$
b_z	0	0	$\frac{1}{2}k$	k
ϕ_1	$AA_1 + BC_1$	$EA_1 + FC_1$	$IA_1 + JB_1 + KC_1$	$OA_1 + PB_1 + QC_1$
κ_1	$AD_1 + BF_1$	$ED_1 + FF_1$	$ID_1 + JE_1 + KF_1$	$OD_1 + PE_1 + QF_1$
ω_1	$AG_1 + BI_1$	$EG_1 + FI_1$	$IG_1 + JH_1 + KI_1$	$OG_1 + PH_1 + QI_1$
ϕ_2	$CA_2 + DC_2$	$GA_2 + HC_2$	$LA_2 + MB_2 + NC_2$	$RA_2 + SB_2 + TC_2$
κ_2	$CD_2 + DF_2$	$GD_2 + HF_2$	$LD_2 + ME_2 + NF_2$	$RD_2 + SE_2 + TF_2$
ω_2	$CG_2 + DI_2$	$GG_2 + HI_2$	$LG_2 + MH_2 + NI_2$	$RG_2 + SH_2 + TI_2$

Where the non-subscripted capital letter coefficients have the following values:

$A = Xr_z + b_x r_z + b_y r_x$ $\quad$ $B = -Xr_x$

$C = -X\ell_z + b_y \ell_x$ $\quad$ $D = X\ell_x + b_x \ell_x$

$E = Yr_z$ $\quad$ $F = -Yr_x - b_x r_z - b_y r_x$

$G = -Y\ell_z - b_y \ell_z$ $\quad$ $H = Y\ell_x - b_x \ell_z$

$I = Zr_z + \frac{1}{2}b_y r_y$ $\quad$ $J = \frac{1}{2}(b_x \ell_z + b_y r_x)$

$K = -Zr_x + \frac{1}{2}b_x r_y$ $\quad$ $L = -Z\ell_z + \frac{1}{2}b_y \ell_y$

$M = \frac{1}{2}b_x \ell_z + b_y \ell_x$ $\quad$ $N = Z\ell_x + \frac{1}{2}b_x \ell_y$

$O = b_y r_y + b_z r_z$ $\quad$ $P = -b_x r_z - b_y r_x$

$Q = b_x r_y - b_z r_x$ $\quad$ $R = -b_y \ell_y - b_z \ell_z$

$S = b_x \ell_z + b_y \ell_x$ $\quad$ $T = b_z \ell_x - b_x \ell_y$

and $X = a_x - c_x$, $Y = a_y - c_y$, $Z = a_z + \frac{1}{2}b_z - c_z$

The subscripted capital letter coefficients have identical forms for the left and right cameras, so ignoring the 1 and 2 subscripts which signify each camera for the moment:

$$A = r_{31}x + r_{32}y + r_{33}z$$

$$B = 0$$

$$C = - r_{11}x - r_{12}y - r_{13}z$$

$$D = - \cos\phi(r_{21}x + r_{22}y + r_{23}z)$$

$$E = \cos\kappa.x - \sin\kappa \cos\omega.y + \sin\kappa \sin\omega.z$$

$$F = \sin\phi(r_{21}x + r_{22}y + r_{23}z)$$

$$G = r_{13}y - r_{12}z$$

$$H = r_{23}y - r_{22}z$$

$$I = r_{33}y - r_{32}z$$

where the r parameters are from the appropriate (left or right) rotation matrix, and (x, y, z) are the appropriate fiducial coordinates. All other parameters are defined in section 4.2.2.1.

APPENDIX B

Partial differentials for exterior orientation equations for the aerial case (see equations (4.56) in section 4.2.3.2):

Parameter	Function g_5	Function g_6	Function g_7	Function g_8
a_x	k	0	0	0
a_y	0	k	0	0
a_z	0	0	k	0
b_x	$\ell_x r_z$	$\frac{1}{2}(\ell_y r_z + \ell_z r_y)$	$\ell_z r_z$	$\ell_z r_y - \ell_z r_y$
b_z	$\ell_x r_x$	$-\frac{1}{2}(\ell_y r_z + \ell_x r_y)$	$-\ell_z r_x$	$\ell_y r_x - \ell_x r_y$
κ_1	$AA_1 + BC_1$	$EA_1 + FB_1 + GC_1$	$UA_1 + LC_1$	$OA_1 + PB_1 + QC_1$
ϕ_1	$AD_1 + BF_1$	$ED_1 + FE_1 + GF_1$	$UD_1 + LF_1$	$OD_1 + PE_1 + QF_1$
ω_1	$AG_1 + BI_1$	$EG_1 + FH_1 + GI_1$	$UG_1 + LI_1$	$OG_1 + PH_1 + QI_1$
κ_2	$CA_2 + DC_2$	$HA_2 + IB_2 + JC_2$	$MA_2 + NC_2$	$RA_1 + SB_2 + TC_2$
ϕ_2	$CD_2 + DF_2$	$HD_2 + IE_2 + JF_2$	$MD_2 + NF_2$	$RD_2 + SE_2 + TF_2$
ω_2	$CG_2 + DI_2$	$HG_2 + IH_2 + JI_2$	$MG_2 + NI_2$	$RG_2 + SH_2 + TI_2$
K	$k(c_x \sin K$ $+ c_y \cos K)$	$-k(c_x \cos K$ $- c_y \sin K)$	0	0

Where the non-subscripted capital letter coefficients have the following values:

$A = Xr_z + b_x r_z - b_z r_x$ $\qquad$ $B = -Xr_x$

$C = -X\ell_z - b_z \ell_x$ $\qquad$ $D = X\ell_x + b_x \ell_x$

$E = Yr_z - \frac{1}{2} b_z r_y$ $\qquad$ $F = \frac{1}{2}(b_x r_z - b_z r_x)$

$G = -Yr_x + \frac{1}{2} b_x r_y$ $\qquad$ $H = -Y\ell_z - \frac{1}{2} b_z \ell_y$

$I = \frac{1}{2}(b_x \ell_z - b_z \ell_x)$ $\qquad$ $J = Y\ell_x + \frac{1}{2} b_x \ell_y$

$U = Zr_z$ $\qquad$ $L = -Zr_x - b_z r_x + b_x r_z$

$M = -Z\ell_z - b_z \ell_z$ $\qquad$ $N = Z\ell_x + b_x \ell_z$

$O = -b_z r_y$ $\qquad$ $P = b_z r_x - b_x r_z$

$Q = b_x r_y$ $\qquad$ $R = b_z \ell_y$

$S = b_x \ell_z - b_z \ell_x$ $\qquad$ $T = -b_x \ell_y$

and $X = a_x - c_x \cos K + c_y \sin K$

$Y = a_y - c_x \sin K - c_y \cos K$

$Z = a_z - c_z$

The subscripted capital letter coefficients have identical forms for the left and right cameras, so ignoring the 1 and 2 subscripts which signify each camera for the moment:

$$A = -r_{21}x - r_{22}y - r_{23}z$$

$$B = r_{11}x + r_{12}y + r_{13}z$$

$$C = 0$$

$$D = \cos\kappa(r_{31}x + r_{32}y + r_{33}z)$$

$$E = \sin\kappa(r_{31}x + r_{32}y + r_{33}z)$$

$$F = -\cos\phi.x - \sin\phi \sin\omega.y - \sin\phi \cos\omega.z$$

$$G = r_{13}y - r_{12}z$$

$$H = r_{23}y + r_{22}z$$

$$I = r_{33}y - r_{32}z$$

Where the r parameters are from the appropriate (left or right) rotation matrix, and (x, y, z) are the appropriate fiducial coordinates. All other parameters are defined in section 4.2.2.1.

APPENDIX C

The partial differentials of the interior orientation parameters in the exterior orientation equations for the terrestrial case (see equations (5.28) in section 5.2.2) have a form very similar to that of the rotations in Appendix A. All the parameters for each photograph have differentials with the same basic form:

Photograph	Function g_1	Function g_2	Function g_3	Function g_4
Left	$AA_1 + BC_1$	$EA_1 + FC_1$	$IA_1 + JB_1 + KC_1$	$OA_1 + PB_1 + QC_1$
Right	$CA_2 + DC_2$	$GA_2 + HC_2$	$LA_2 + MB_2 + NC_2$	$RA_2 + SB_2 + TC_2$

Where the non-subscripted capital letter coefficients have the values given in Appendix A. The subscripted capital letter coefficients have identical forms for the left and right photographs, so ignoring the 1 and 2 subscripts which signify each photograph for the moment, values are given below for the eight interior orientation parameters:

Interior Orientation Parameters	A	B	C
a_1	r_{11}	r_{21}	r_{31}
a_2	r_{12}	r_{22}	r_{32}
a_3	$r_{11}\bar{x}$	$r_{21}\bar{x}$	$r_{31}\bar{x}$
a_4	$r_{12}\bar{x}$	$r_{22}\bar{x}$	$r_{32}\bar{x}$
a_5	$r_{11}\bar{y}$	$r_{21}\bar{y}$	$r_{31}\bar{y}$
a_6	$r_{12}\bar{y}$	$r_{22}\bar{y}$	$r_{32}\bar{y}$
a_7	$r_{11}\bar{y}^2 + r_{12}\bar{x}\bar{y}$	$r_{21}\bar{y}^2 + r_{22}\bar{x}\bar{y}$	$r_{31}\bar{y}^2 + r_{32}\bar{x}\bar{y}$
a_8	$r_{12}\bar{x}^2 + r_{11}\bar{x}\bar{y}$	$r_{22}\bar{x}^2 + r_{21}\bar{x}\bar{y}$	$r_{32}\bar{x}^2 + r_{31}\bar{x}\bar{y}$

Where the r parameters are from the appropriate (left or right) rotation matrix, and $(\bar{x}, \bar{y})$ are the appropriate comparator coordinates.

The partial differentials of the interior orientation parameters for the aerial case can be evaluated in a similar fashion, using the forms for the rotations and the unsubscripted capital letter coefficients from Appendix B.

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