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THE CITY UNIVERSITY

THEORETICAL & EXPERIMENTAL
INVESTIGATION OF CAUSES OF
ELASTIC INSTABILITY OF TALL
STRUCTURES

A Thesis submitted to
THE CITY UNIVERSITY
for the Degree of Doctor of Philosophy

by

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1975

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PREFACE

The author obtained his Diploma in Structural Engineering at Imperial College of Science and Technology, University of London in 1955.

Has designed a wide variety of buildings - educational, residential, swimming pools, warehouses and barracks - but specialised in multi-storey building - offices, banks, hospitals and hotels. Notable examples are Centrepont, London and the new 600 ft. high National Westminster Bank in London. Acted as expert adviser on a number of structural failures including collapse of Ferry Bridge cooling towers and Ronan Point flats. Received I.Struct.E. award for paper on the 'Use and behaviour of large diameter piles in London Clay' 1962. Oscar Faber Bronze Medal awarded by I.Struct.E. for paper on the Drapers Garden Development, 1967.

In 1967 he was registered at The City University to undertake part-time research for the Degree of Master of Philosophy. However, the research was in abeyance for three years due to the Author's commitments in the office.

In 1973 his registration was transferred to the Degree of Doctor of Philosophy.

No part of this thesis has been submitted for the application of any other degree or professional qualification.

SYNOPSIS

This work is concerned with the elastic instability of tall and slender structures. It is now well known that the elastic primary analysis does not always predict accurately the distribution of stresses and deformation in structures generally and this may be a source of serious error in the design. While it is not possible to check the actual stresses in structures, the appearance of cracks and excessive deformation as well as vibration are visual indications of the discrepancy between the predicted and actual response of structures generally and particularly of tall slender frames. In this work, efforts are made to formulate and to include in the matrix other effects, which are often termed as "secondary" and to examine their effect on the response of tall frames and shear walls. Many of these effects result in the non-linear analysis. Particular attention is paid to the matrix formulation of theories in a form suitable for practical application in office conditions. Several experiments are made to check the validity of the assumptions.

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NOTATION

Unless otherwise stated the following symbols are used in this work.

(Other symbols are defined when they first appear):

A	=	Area of cross-section
I	=	Second moment of area of a section
Z	=	Section modulus
τ	=	Stress
ϵ	=	Strain
E	=	Youngs modulus of elasticity
G	=	Shear modulus of elasticity
x	=	Horizontal co-ordinate of a point under consideration
y	=	Vertical co-ordinate of a point under consideration
M, m	=	Bending moment
N	=	Internal axial force
P	=	Applied force
W	=	Lateral load
θ	=	Angle (in radians)
s, c	=	Stability functions
c_{II}, c_{OI}	=	Stability functions in the force-displacement method
f	=	Flexibility coefficient
k	=	Stiffness
V	=	Shear force
Δ, δ	=	Transverse deflection of a member or a joint
λ	=	Axial deformation of a member
h	=	Storey height
L, ℓ	=	Length of beam
ϕ	=	Relative rotation of a flexible connection
μ	=	Poissons ratio
τ	=	Torsional shear stress

1.01 The Object and Scope of the Research

The main object of this research was to develop a practical theory of elastic instability of tall frames and shear walls suitable for office use, to examine the effect of various factors involved, and to check experimentally the validity of the assumptions made and therefore the accuracy of the mathematical formulations.

The following factors are examined:

- (a) the effect of axial forces on the stiffness or flexibility of isolated members and portal frames
- (b) the effect of axial forces on the response of tall frames and shear walls
- (c) the effect of the change of geometry of a system and the action of loads on the response of frames
- (d) the effect of 'out of plumb' construction

Some other possible effects of elastic instability are also examined analytically, but no experimental verification was attempted. These are:

- (e) the effect of elastic deformations of the soil mass, usually referred to as the "soil-structure interaction", and
- (f) the effect of cracks.

This work shows how the various parameters can be included in the matrix in a form suitable for practical application. However, the use of computers is required in most cases to obtain a solution.

The force-displacement method is used in most parts of this work for the following reasons:

- (i) this method gives directly the values of forces and deflections usually required by the designer (the rotations of joints do not enter as indeterminate).
- (ii) the method is based on clear, physical interpretation of the response of the assumed model and this facilitates the setting out of the matrix and helps in avoiding errors
- (iii) the method is very powerful and can be used to formulate some complex structural problems
- (iv) the method is not yet fully explored and efforts were made to use it in the problems of elastic instability of tall frames.

Not dealt with in this work are the following instability problems: the elastic instability arising from vibrations or dynamic loading; instability in the post-elastic range and the visco-elastic response of materials and the ultimate strength of tall frames.

Generally, this research shows that some of the "secondary" effects can represent an important factor in the overall response of tall buildings to gravity and lateral forces - and that by considering these effects, a better correlation can be obtained between calculated and observed response of tall frames.

1.02 Introduction

Instability is generally defined as a mode of elastic or inelastic deformation, buckling or collapse of a structure or a part of it.

For instability to take place, a structure must be initially in motion, although it may be in equilibrium with the forces causing instability and deformations may still be compatible. A collapse is an ultimate bound of instability (generally not dealt with in this work.) The term 'elastic' instability is used when the above action takes place even within the elastic response of materials, although it may not be linearly-elastic.

The above definitions of instability form the basis for analytical formulations in this work.

Elastic instability can arise from various effects such as: excessive slenderness of members, excessive axial forces, eccentricities of loads, crookedness of members or out-of-plumb construction, large lateral deformation, cracked sections, flexible joints, differential settlement of foundations, temperature changes, creep and shrinkage of concrete, etc. Other causes of instability may arise from the dynamic action of wind and earthquakes, and shedding of vortices. The latter are not dealt with in this work.

Instability can occur at high as well as at very low stress level, and can show itself by excessive deformation and by local or overall buckling. Any of the above factors or a combination of them may lead to progressive deformations and even failure. However, elastic instability is limited to excessive deformations when both the requirements of equilibrium and compatibility are still satisfied.

While instability of individual members is well known and is discussed in many textbooks on the subject, analytical formulation of the instability of complex frames and shear walls used in tall buildings is not yet fully resolved, even in the elastic range. Few practical methods of analysis are available suitable for office use which would offer a sufficient degree of accuracy. This research attempts to fill this need.

Most structures deflect more than predicted and show frequent cracks even if the design is based on the accepted level of working stresses.

It is also recognised that the response of structures to loads differs to that predicted by "primary" analyses, and sometimes even reverse response has been observed. It can be expected that one reason for this discrepancy is the effect of elastic instability due to causes listed above.

Theoretical analyses made by various authors confirm that critical loads and the overall stiffness of slender frames can be reduced to an alarming extent by these effects. A stage can be reached when lateral deflections increase much faster than indicated by the primary analysis and buildings may then deflect and vibrate excessively and could become unserviceable. There are recorded cases of this.

The classical energy methods (conservation of energy, virtual work, minimum potential energy and complementary energy) are particularly useful in formulating elastic instability of individual members and a high degree of accuracy is now confirmed. However, direct application of energy methods to full structures is often too complicated and not practical for office use. Other methods are therefore used, based on energy methods and these are mainly the stiffness, flexibility and the force-displacement method.

It has been long known that the stiffness of slender members and their critical loads can reduce rapidly, i.e. the Perry-Robertsons analysis of struts is one example now universally accepted. Similarly, it has been suggested that in certain conditions, tall and slender frames may show a drastic reduction of stiffness even in the elastic range. Many reports are now available, which indicate that even in normal conditions, a considerable reduction of stiffness actually takes place. However, the surprising and not fully resolved fact remains that very few tall buildings actually collapsed in service except those due to the action of earthquake forces, fire or during construction.

It is well known that multi-storey frame structures may buckle laterally under loads significantly less than those predicted by individual column-buckling formulae. In the final stages of collapse a structure behaves as a mechanism and deflections increase without increase of loads, and collapse can take place. More accurate analysis is therefore required, particularly for tall and slender buildings without shear walls or heavy masonry infilled panels. In such frames, stresses and deformations may not be proportional to the magnitude of load, and may depend on the magnitude of the deformations and eccentricities of axial loads. If lateral deformations are sufficiently large, primary moments and deformations can be amplified and instability of members or of the whole structure may occur, even at comparatively small loads.

It is well known that the stiffness and flexibility coefficients of members are a function not only of the span and EI values of members, but also of axial loads, and therefore the critical loads are sometimes defined as loads which correspond to zero stiffness or infinite flexibility matrix.

In principle, therefore, it should be possible to obtain critical loads by setting the stiffness matrix to zero, or the flexibility matrix to infinity. At such loads, deformations and forces become undefined, and buckling modes could be obtained. However, in practice, this approach can supply the solution of some most simple problems only, for idealised frames, without imperfections, and for materials not subjected to yielding. ✓

At a certain stage, such stability may suddenly cease to exist, e.g. a portal frame with column loads may suddenly become unstable, a pre-cast assembly of panels may suddenly collapse. Similarly, a statement that a system is unstable if a small amount of energy supplied from external source causes large deformations of a system is also not strictly true, e.g. a structure can be moved horizontally without any expenditure work at all.

However, if external forces are required to perform positive work to produce any small displacement of a system, then the equilibrium is stable.

Mere existence of stability gives no assurance of safe design and provides no knowledge of the degree of stability. Structures may be in a state of weak stability or in what is termed as "snap-through" stability. These effects do not form a part of this work.

From the practical point of view, a method of analysis is required which could offer even an approximate way of predicting the response of tall buildings to the "secondary" effects listed above. Such a method should take into account any given geometrical configuration of a structure, and its accuracy and the limits of applicability should be verified experimentally.

More accurate analysis is required particularly for tall and slender buildings without rigid shear walls or heavy masonry infilled panels.

Primary static analyses are now well established and are based on the undeformed original configuration of a structure, and assume that the deformations are small in comparison to the sizes of members.

This work aims to investigate various sources of elastic instability, and attempts to formulate some of their effects on structures, particularly on tall buildings, as a repetitive tool in future design. Experimental work aims at verification of the assumptions made.

Generally, the analysis developed confirms that in tall flexible frames, the above mentioned secondary effects can represent an important factor in the response of such frames to gravity and lateral forces.

This thesis takes into account the change of geometry of the loaded system, the effect of gravity and lateral forces on the magnitude of primary stress and deformation as well as the effect of axial deformation and altered stiffness of members. The effect of out-of-plumb construction and imperfections of materials are also considered. It is shown how these parameters can be assembled into a matrix suitable for office work although with the aid of a computer.

1.03 Programme of Experimental Work

The following experiments have been carried out:

- 1.031 Check of the response of a simple cantilever to lateral and axial forces at large deformations, Fig. 1.031.

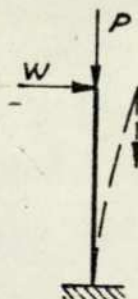


Fig. 1.031 Simple Cantilever.

- 1.032 Check of the response of portal frames to the effect of lateral and axial forces in the non-linear range of stresses, Fig. 1.032 a) and b). Tests were also made using portal frames with different restraints of two columns with hinges at different points and with gravity loads placed unsymmetrically.

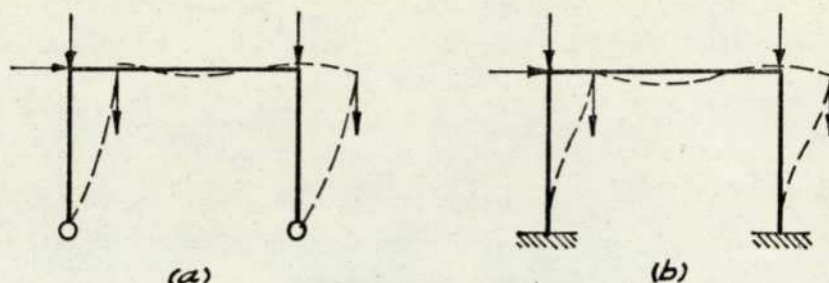


Fig. 1.032 Portal Frames

- 1.033 Check of the response of tall frames to gravity and lateral forces. The former was placed symmetrically and non-symmetrically. Frames shown in Fig. 1.033 a) and b) were made in steel and micro-concrete and that in Fig. 1.033 c) in micro-concrete. Some tests were made on steel frames as shown on Fig. 1.033 a) in which hinges, simulating cracks, were introduced at various places in columns or beams.

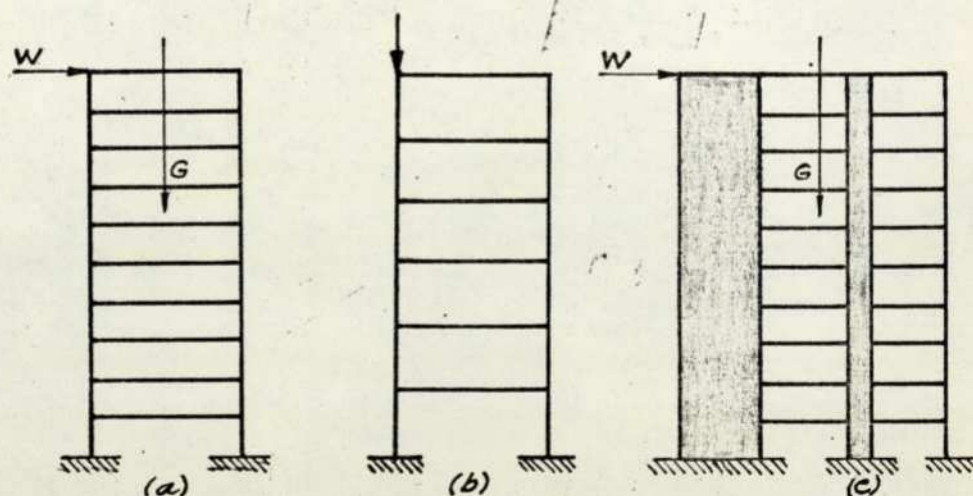


Fig. 1.033 Tall Frames.

- 1.034 Check of the response of out-of-plumb tall frames to gravity and lateral forces. These tests were made on micro-concrete models.

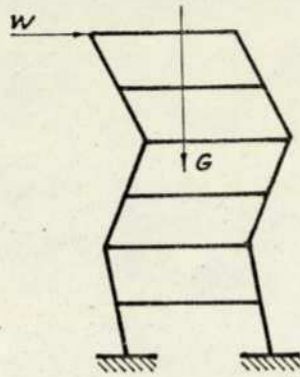


Fig. 1.034 Out-of-plumb Tall Frames.

1.04 General Classification of Instability of Structures

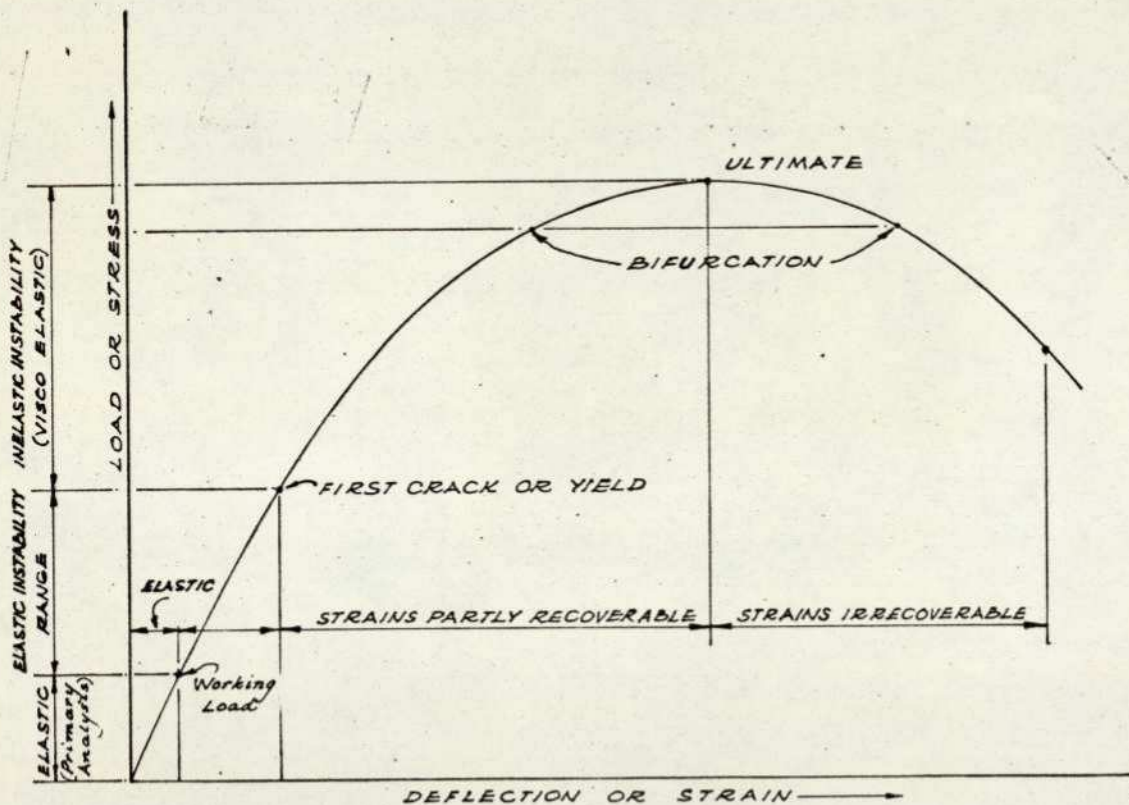


Fig. 1.04 Classification of Instability of Structures.

Although instability need not be a function of the stress level, it is convenient to classify instability in relation to stresses as shown in Fig. 1.04. This is as follows:-

- 1.041 Elastic or primary analysis. This is usually accepted as valid within working loads and is based on the assumptions that
- the relationship between stress and strain is linear
 - that the principle of superposition is valid
 - that deformations are small, and
 - that the action of loads is static

- 1.042 Elastic instability. This is usually considered to take place at higher stresses but below stresses leading to the onset of cracking when:
- a) stresses can still be assumed to be proportional to strains but deformations are beginning to be significant,
 - b) stiffnesses of axially loaded members are reduced, and
 - c) the analysis should more accurately be based on deformed configurations of a structure when the action of loads is affected.
- 1.043 Inelastic instability. This takes place, usually at a high stress level, when stresses are not proportional to strains. The latter are time-dependent and are referred to as visco-elastic. At high stress levels, bifurcation of the configuration can take place and an unstable configuration can exist under decreasing stresses.
- 1.044 Ultimate strength. At this stage, stresses are usually assumed to be fully plastic, the stiffness of a member of a structure reaches zero value (and flexibility is infinite), deflections increase even at decreasing loads and ultimate load is reached. Critical load is a complex function of the properties of materials, loads, geometry and some secondary effects.
- 1.045 The main characteristics which distinguish instability problems from the conventional "elastic analyses" are as follows:-
- a) At certain stages of loading, deflections may become no longer proportional to loads (although the usual assumptions of the linear theory of elasticity may still be used). This is because bending moments become a function of deflections, while the simple bending theory is usually based on the equilibrium conditions being satisfied in the original, undeformed configuration of a system. In other words, in instability analyses the deformations of a structure are not uniquely determined or proportional to loads.
 - b) Buckling is an inherent problem of instability. It is the final stage and a consequence that in instability problems small increments of loads may cause disproportional increase in deflections.
 - c) The existence of critical loads is one of the main features of instability.
 - d) Motion of a structure is an inherent feature of instability, although a structure in motion may, or may not, be in equilibrium with the external forces, and finally,
 - e) the natural frequency of vibrations is a characteristic of instability of a structure.

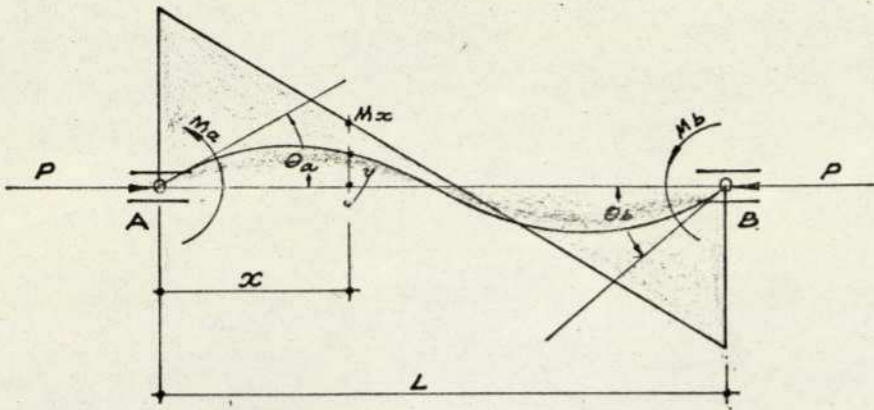


Fig. 1.05 Element Subjected to Axial Load and End Moments

In the analysis of instability problems by stiffness method, stability functions are used, which represent the stiffness and the carry-over factor, as well as their product. Stability functions depend on the dimensions of a member, its material and on the magnitude of axial force applied. These functions are usually calculated from the Fig. 1.05 based on the equation of the deflected line.

Bending moment M_x at any point is:

$$M_x = M_a - (M_a + M_b) \frac{x}{L} + P y = -EI \frac{d^2 y}{dx^2}$$

from which:
$$\frac{d^2 y}{dx^2} + \frac{P}{EI} y = -M_a \frac{L-x}{LEI} + M_b \frac{x}{LEI}$$

Denoting
$$k = \sqrt{\frac{P}{EI}}$$

we obtain:
$$y = A \sin kx + B \cos kx - M_a \frac{L-x}{L-P} + M_b \frac{x}{LP}$$

The constants of integration A and B can be determined from the boundary conditions that:

for $x=0, y=0$, and for $x=L, y=0$

from which:

$$B = \frac{M_a}{P} \text{ and}$$

$$A = -\frac{M_a}{P} \cot kL - \frac{M_b}{P} \operatorname{cosec} kL$$

The slopes at A and B are calculated by taking dy/dx for $x=0$ and $x=L$ respectively:

$$EI \theta_a = \frac{1}{3} M_a L \beta - \frac{1}{6} M_b L \alpha$$

and

$$EI \theta_b = -\frac{1}{6} M_a L \alpha - \frac{1}{3} M_b L \beta$$

in which $\alpha = \frac{6}{k^2 L^2} (kL \operatorname{cosec} kL - 1)$

and $\beta = \frac{3}{k^2 L^2} (1 - kL \cot kL)$

Therefore: $M_a = \left(\frac{EI}{L} \right) k^2 L^2 \left[\frac{\theta_a (1 - kL \cot kL) + \theta_b (kL \operatorname{cosec} kL - 1)}{(1 - kL \cot kL)^2 - (kL \operatorname{cosec} kL - 1)^2} \right]$

or $M_a = \frac{EI}{L} [s \cdot \theta_a + s.c. \cdot \theta_b] \text{ ----- 1.051}$

Similarly: $M_b = \frac{EI}{L} [s.c. \cdot \theta_a + s \cdot \theta_b] \text{ ----- 1.052}$

in which:

$$s = \frac{(1 - kL \cot kL) kL}{2 \tan \frac{kL}{2} - kL} = \frac{12\beta}{4\beta^2 - \alpha^2}$$

$$c = \frac{kL \operatorname{cosec} kL - 1}{1 - kL \cot kL} = \frac{kL - \sin kL}{\sin kL - kL \cos kL} = \frac{\alpha}{2\beta}$$

$$s.c. = \frac{6\alpha}{4\beta^2 - \alpha^2}$$

These values have been tabulated by R.K. Livesley and D.B. Chandler and are also shown in Fig. 1.051.

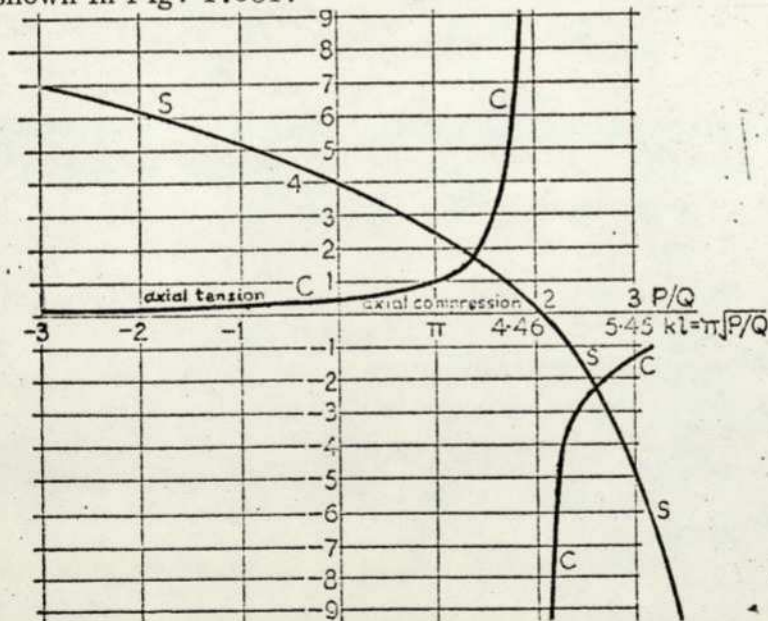


Fig. 1.051 Diagram of stability functions s and c . versus kl (or $\pi\sqrt{P/Q}$ in which P is the axial load and $Q = \pi^2 EI/L^2$)

1.060

Critical Loads Calculated by the Stability Functions

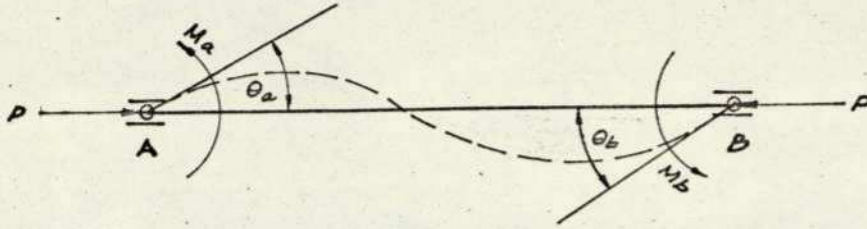


Fig. 1.060 Element Subjected to Axial Load and End Moments.

From Equation 1.051 and 1.052 we have:

$$M_a = \frac{EI}{L} [s \cdot \theta_a + s.c \cdot \theta_b]$$

and

$$M_b = \frac{EI}{L} [s.c \cdot \theta_a + s \cdot \theta_b]$$

At ultimate, M_a and M_b are zero and we have $\theta_a = \theta_b$ so that the stiffness matrix is also zero, i.e.

$$\begin{bmatrix} s & s.c \\ s.c & s \end{bmatrix} = 0$$

or

$$s^2 - s^2 c^2 = 0$$

or

$$(s - s.c)(s + s.c) = 0$$

from which in both cases: $P_{cr} = \frac{\pi^2 EI}{L^2}$ or $P_{cr} = \frac{4\pi^2 EI}{L^2}$

which is Euler's condition for buckling of columns.

The critical load can in theory be calculated by the stiffness or flexibility method. In the first, the determinant of co-efficients in the moment slope-deflection matrix is set to zero, i.e. $\frac{M}{\theta} = 0$

In the second approach the flexibility matrix set out for internal moments tends to infinity, i.e. $\frac{M}{\theta} = \infty$

Fig. 1.061 a) and b) shows diagrammatically the concept of critical load using the stiffness and flexibility approach.

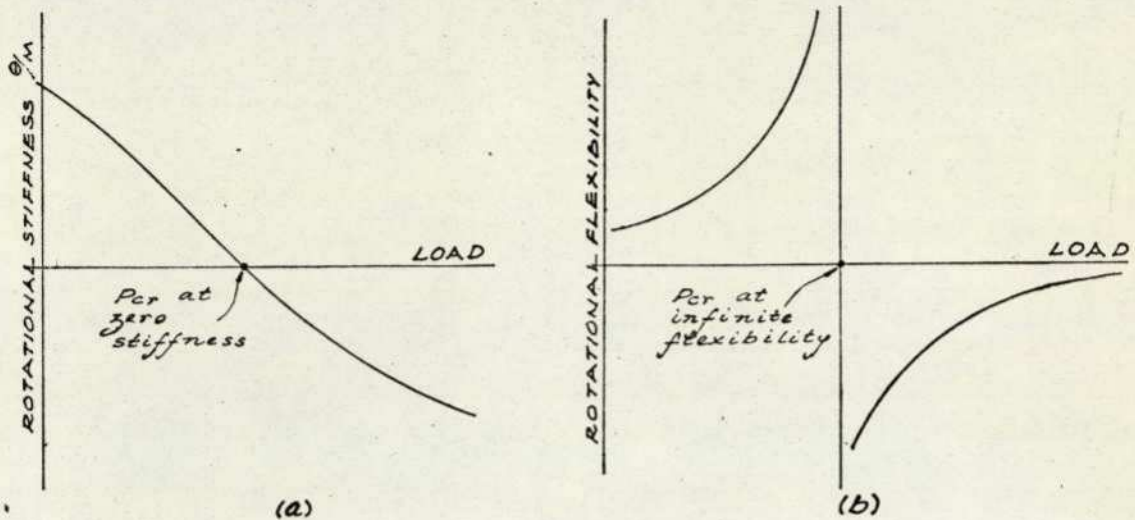


Fig. 1.061 Diagrammatic Concepts of Critical Loads based on Stiffness and Flexibility Methods.

1.070 Flexibility Coefficients for Axially Loaded Columns

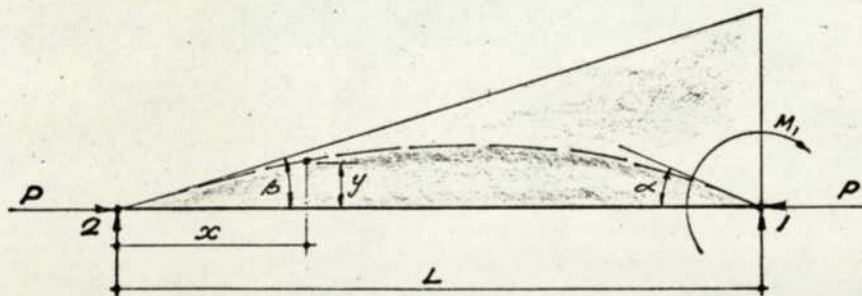


Fig. 1.070 Derivation of Flexibility Coefficients for a Member Subjected to Bending and Axial Force.

A column subjected to end bending moment and an axial force, as shown on Fig. 1.070, is a very useful case in the analysis of elastic stability. It can be shown that the deflection y at a distance x is:

$$y = \frac{M_1}{P} \left(\frac{\sin kx}{\sin kL} - \frac{x}{L} \right)$$

Taking derivatives of this equation we obtain:

$$\alpha = f'' = - \left(\frac{dy}{dx} \right)_{x=L} = - \frac{M_1}{P} \left(\frac{k \cos kL}{\sin kL} - \frac{1}{L} \right)$$

or

$$f'' = M_1 \frac{L}{3EI} \cdot \frac{3}{2U} \left(\frac{1}{2U} - \frac{1}{\tan 2U} \right) = M_1 \frac{L}{3EI} c''$$

Similarly $\beta = f_{21} = \left(\frac{dy}{dx} \right)_{x=0} = \frac{M_1}{P} \left(\frac{k}{\sin kL} - \frac{1}{L} \right) = \frac{M_1 L}{6EI} \left(\frac{1}{\sin 2U} - \frac{1}{2U} \right) \frac{3}{U}$

or $f_{21} = M_1 \frac{L}{6EI} \cdot c_{21}$

In these equations: $k = \sqrt{\frac{P}{EI}}$ and $2U = kL$

The values of coefficients c_{11} and c_{21} are shown in tabular form, Fig. 1.071, and in graphical form, Fig. 1.072.

1.080 Relationship Between the Stability Functions and Flexibility Coefficients

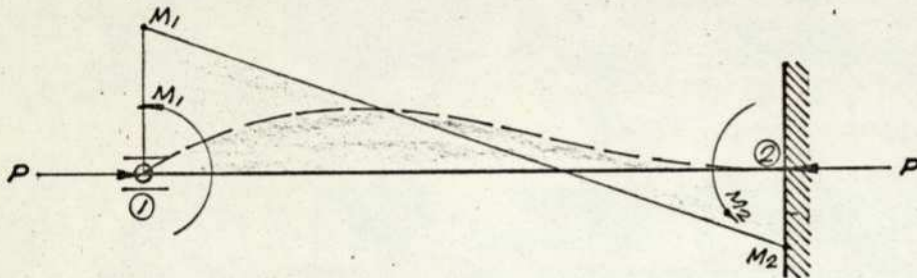


Fig. 1.080 Diagram Used to Derive Relationship Between Stability Functions and Flexibility Coefficients

The stiffness s and the carry-over factor c can be related to the flexibility coefficients f_{11} and f_{21} as follows:-

From Fig. 1.080

$$M_1 f_{11} - M_2 f_{12} = 0,$$

and

$$-M_1 f_{21} + M_2 f_{22} = 0$$

From which

$$\frac{M_1}{\Theta_1} = s = \frac{f_{11}}{f_{11}^2 - f_{21}^2} \quad \text{-----} \quad 1.081$$

and

$$\frac{M_2}{M_1} = c = \frac{f_{21}}{f_{11}} \quad \text{-----} \quad 1.082$$

Or solving for f_{11} and f_{21} we obtain:

$$f_{11} = \frac{1}{s(1 - c^2)} \quad \text{-----} \quad 1.083$$

and

$$f_{21} = \frac{c}{s(1 - c^2)} \quad \text{-----} \quad 1.084$$

$2U=kl$	C_{11}	C_{21}	$2U=kl$	C_{11}	C_{21}	$2U=kl$	C_{11}	C_{21}
0.0	1.0	1.0	2.1	1.5158	1.9493	3.15	-112.9747	-227.1668
0.1	1.0007	1.0008	2.15	1.5616	2.0366	3.20	- 15.7398	- 32.7063
0.2	1.0027	1.0047	2.20	1.6124	2.1336	3.25	- 8.1875	- 17.6312
0.3	1.0061	1.0106	2.25	1.6690	2.2422	3.30	- 5.4154	- 12.0720
0.4	1.0108	1.0189	2.30	1.7325	2.3640	3.40	- 3.0787	- 7.4248
0.5	1.0171	1.0299	2.35	1.8041	2.5022	3.50	- 2.0433	- 5.3769
0.6	1.0249	1.0437	2.40	1.8854	2.6596	3.60	- 1.4572	- 4.2292
0.7	1.0343	1.0603	2.45	1.9786	2.8403	3.70	- 1.0787	- 3.4990
0.8	1.0754	1.0801	2.50	2.0864	3.0502	3.80	- .8128	- 2.9961
0.9	1.0585	1.1033	2.55	2.2124	3.2963	3.90	- .6147	- 2.6314
1.0	1.0737	1.1304	2.60	2.3618	3.5890	4.00	- .4603	- 2.3570
1.1	1.0912	1.1617	2.65	2.5415	3.9421	4.20	- .2317	- 1.9792
1.2	1.1114	1.1979	2.70	2.7619	4.3766	4.40	- .0652	- 1.7429
1.3	1.1345	1.2396	2.75	3.0386	4.9233	4.60	+ .0682	- 1.5962
1.4	1.1610	1.2878	2.80	3.3963	5.6315	4.80	+ .1851	- 1.5152
1.5	1.1915	1.3434	2.85	3.8764	6.5846	5.00	+ .2975	- 1.4914
1.6	1.2266	1.4078	2.90	4.5550	7.9343	6.00	+ 1.8015	- 3.7455
1.7	1.2673	1.4880	2.95	5.5875	9.9915	2π	$\pm \infty$	$\pm \infty$
1.8	1.3147	1.5710	3.00	7.3486	13.5057	6.5	- 2.0242	+ 4.1490
1.9	1.3704	1.6750	3.05	11.0314	20.8629			
2.0	1.4365	1.7993	3.10	23.5659	45.9234			
2.15	1.4742	1.8706						

Fig. 1.071 Flexibility Coefficients for a Beam Subjected to End Bending Moment and Axial Force.

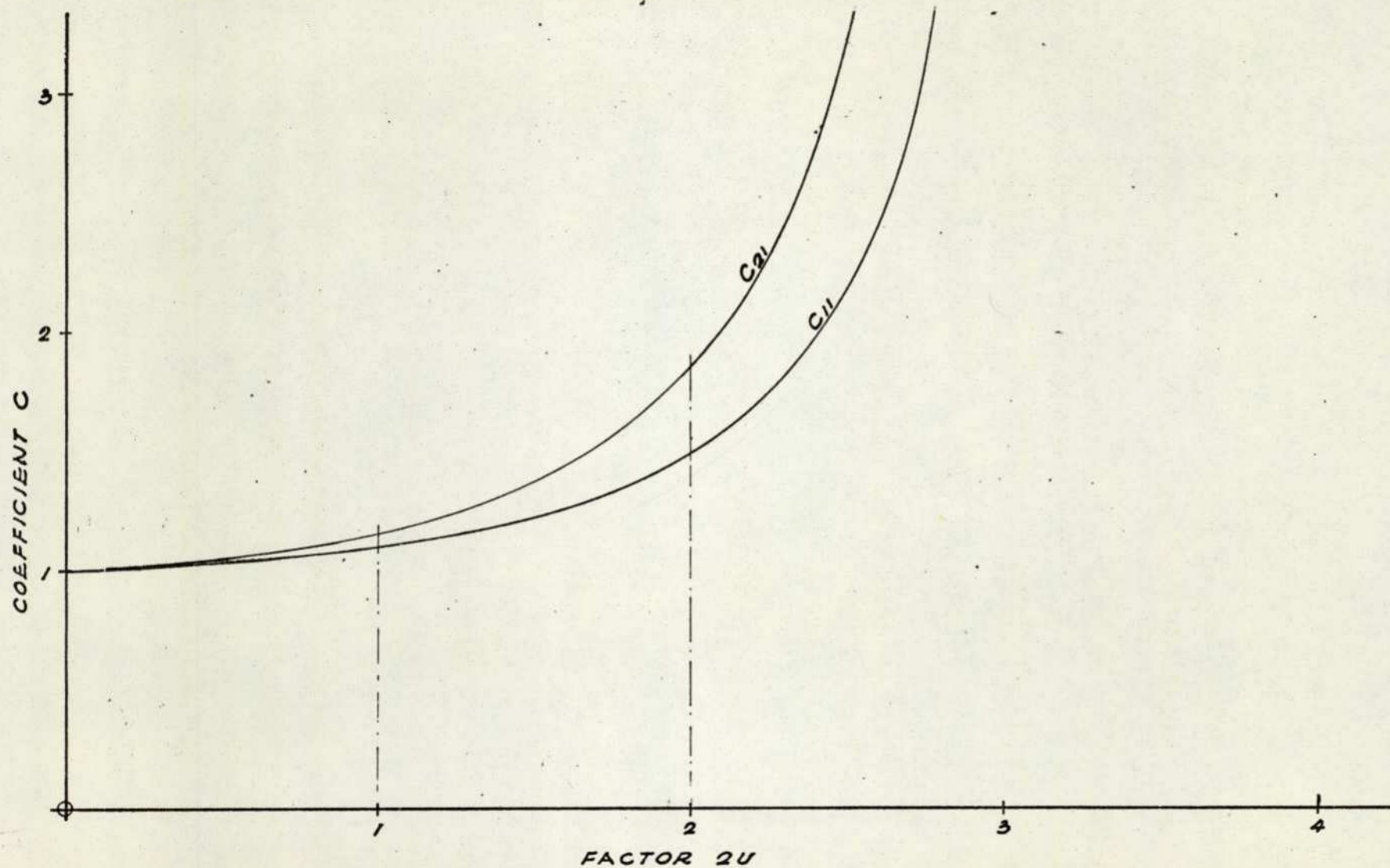


Fig. 1.073 Diagram of Stability Coefficients, C_{II} and C_{2I}

Introducing in Equations 1.081 and 1.082 the values for f_{11} and f_{21} we obtain:

$$s = \frac{kL(\sin.kL - kL\cos.kL)}{2 - 2\cos.kL - kL\sin.kL}$$

$$c = \frac{kL - \sin.kL}{\sin.kL - kL\cos.kL}$$

These coefficients are tabulated by Livesly and Chandler in "Stability Functions for Structural Frameworks".

If the far end of the member is pinned, then the carry-over factor becomes zero, and the stiffness coefficient s can be expressed in terms of S as follows:-

$$M_1 f_{11} = \Theta_1 \text{ and } \frac{M_1}{\Theta_1} = \bar{s} = \frac{1}{f_{11}}$$

Introducing

$$s = \left(\frac{f_{11}}{f_{11}^2 - f_{21}^2} \right)$$

we obtain

$$\bar{s} = s \left[1 - \left(\frac{f_{21}^2}{f_{11}^2} \right) \right]$$

or

$$\bar{s} = s(1 - c^2)$$

For a portal frame subjected to a uniformly distributed load and to column loads P , Fig. 1.081, we have:

$$(M_a f_{aa} + f_{ab} + f_{aa}^{ac}) = \Theta_{ab}$$

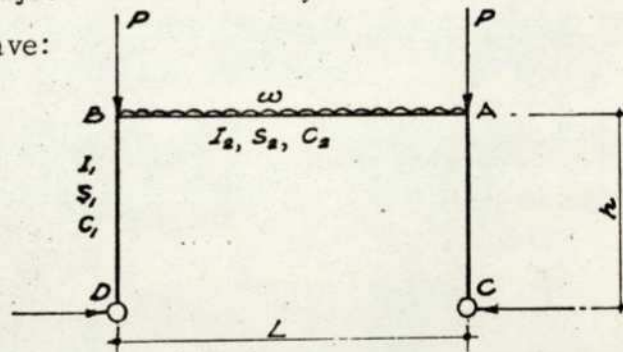


Fig. 1.081 Portal Frame with Uniformly Distributed Load and Column Loads P .

Introducing the values for f from Equation and Θ_{ab} we obtain:

$$H.h \left[\frac{1}{s_2(1 - c_2^2)} - \frac{c_2}{s_2(1 - c_2^2)} + \frac{1}{s_1(1 - c_1^2)} \right] =$$

$$= \frac{\omega L}{P} \left[\frac{\tan k \frac{L}{2}}{k_2 L} - \frac{1}{2} \right]$$

This equation shows the relationship between the horizontal reaction and axial forces in columns, and was first derived by Le-Wu-Lu using a somewhat different approach. The value of P obtained from this equation represents the critical load of the frames.

1.090 Stiffness and Carry-over Factors including Instability Functions

Although the stiffness method is not here used in matrix formulations, it is shown how stiffness and carry-over factors can be calculated including the effect of axial forces by using the force-displacement method. Fig. 1.090 shows a beam fixed at one end but free for angular and axial deformations at the other end. The beam is subjected to a bending moment M and axial force P .

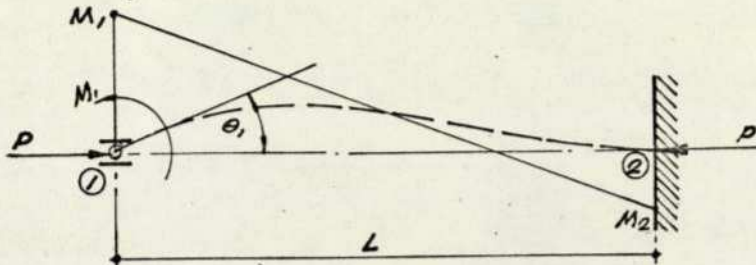


Fig. 1.090 Diagram to Show Relation between Stiffness and Carry-over Factors.

From Fig. 1.090 we can write,

$$\text{At 1} \quad M_1 \frac{c_{11} L}{3EI} - M_2 \frac{c_{12} L}{6EI} = \theta, \quad \text{-----} \quad 1.091$$

$$\text{At 2} \quad -M_1 \frac{c_{21} L}{6EI} + M_2 \frac{c_{22} L}{3EI} = 0 \quad \text{-----} \quad 1.092$$

For a beam with constant second moment of area ($I = \text{constant}$):

$c_{11} = c_{21}$ and $c_{12} = c_{21}$ and from Equations 1.091 and 1.092 we obtain:

$$\text{carry-over factor: } \frac{M_2}{M_1} = \frac{1}{2} \cdot \frac{c_{12}}{c_{11}}$$

$$\text{and stiffness } k = \frac{M_1}{\theta} = \frac{4EI}{L} \cdot \frac{3c_{11}}{(4c_{11}^2 - c_{12}^2)}$$

It is interesting to note that for the stiffness to become zero the value of

$$\frac{3}{2U} \left(\frac{1}{2U} - \frac{1}{\tan 2U} \right) = 0$$

$$\text{or } \tan 2U = 2U$$

$$\text{from which } 2U = \pi$$

$$\text{and } 2U = L \sqrt{\frac{P}{EI}} = \pi$$

i.e. the critical load becomes the Euler's load:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Tabular values of stiffness and carry-over factors are shown in Fig. 1.091 and the data are shown graphically in Fig. 1.092.

$2U$	C_{11}	C_{12}	$\frac{1}{2} \frac{C_{12}}{C_{11}}$	k
0	1.0	1.0	.5000	1.0000
.2	1.0027	1.0047	.5037	.9986
.4	1.0108	1.0189	.5040	.9890
.6	1.0249	1.0437	.5092	.9879
.8	1.0584	1.0801	.5102	.9630
1.0	1.0754	1.1304	.5256	.9580
1.2	1.1114	1.1979	.5389	.9510
1.4	1.1610	1.2878	.5546	.9329
1.6	1.2266	1.4078	.5738	.9117
1.8	1.3197	1.5710	.5975	.8871
2.0	1.4365	1.7993	.6263	.8590
2.2	1.6124	2.1336	.6616	.8193
2.4	1.8854	2.6596	.7053	.7915
2.6	2.3618	3.5890	.7598	.7512
2.8	3.3963	5.6315	.8291	.7063
3.0	7.3486	13.5057	.9189	.6560
π	$\pm \infty$	$\pm \infty$	1.000	

Fig. 1.091 Stiffness and Carry-over Factors.

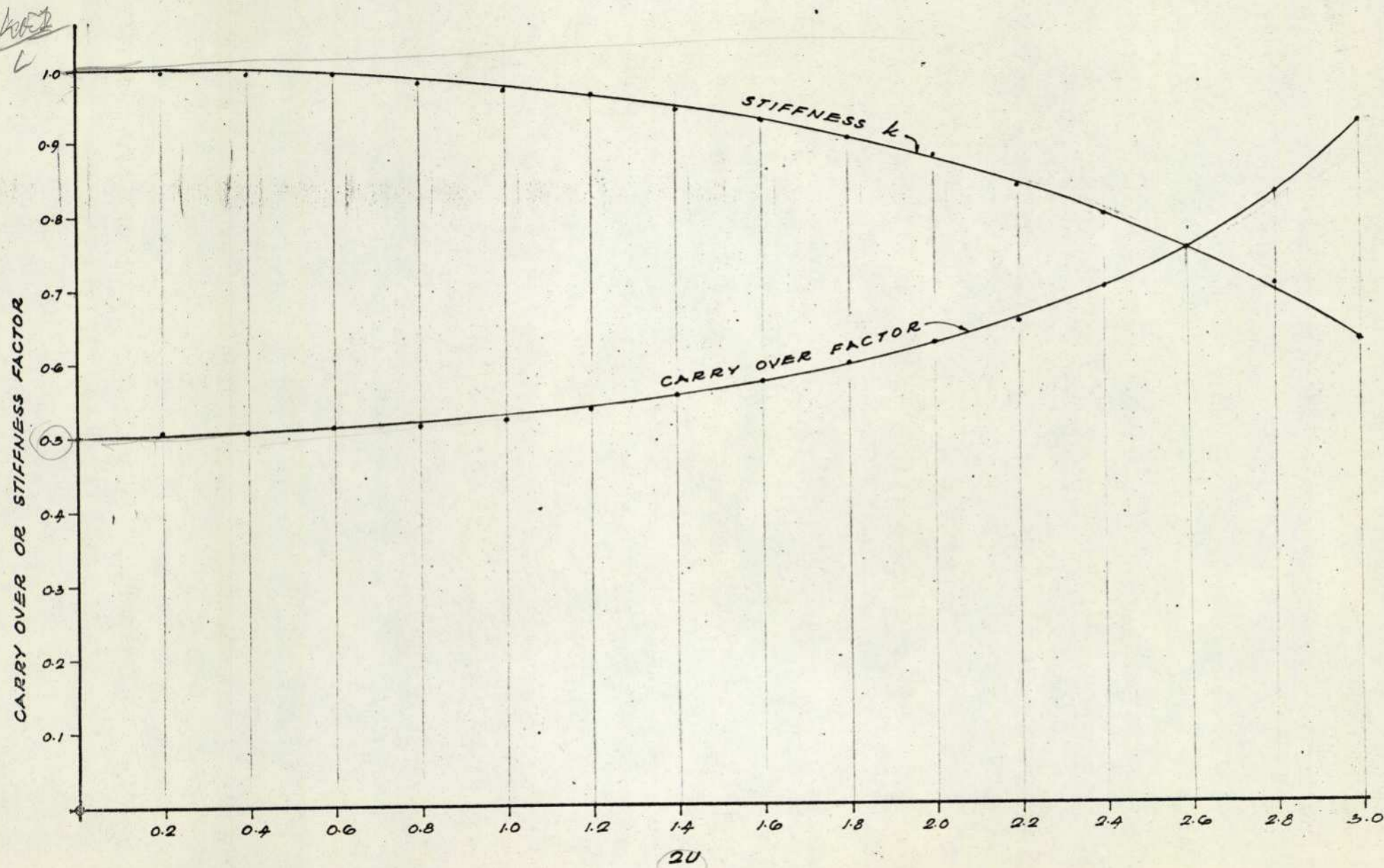


Fig. 1.092 Carry-over and Stiffness Factors as a Function of $2U$ (i.e. the axial load)

1.10 Derivation of the generalised element stiffness matrix including stability functions using the force-displacement method.

Similarly, the element stiffness matrix can be derived including the effect of stability functions s and c , or the coefficients c_{11} and c_{21} . The principle of the force displacement method is used in the derivation.

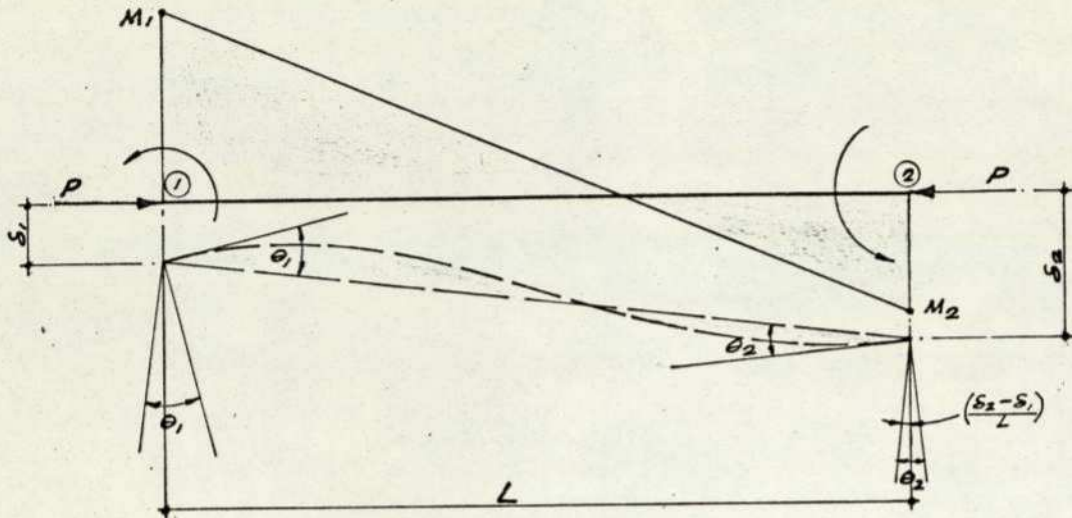


Fig. 1.10 Derivation of the Slope Deflection Equations including Stability Functions.

From Fig. 1.10:

at 1 $M_1 f_{11} - M_2 f_{12} = \theta_1 - \frac{s_2 - s_1}{L}$

at 2 $-M_1 f_{21} + M_2 f_{22} = \theta_2 - \frac{s_2 - s_1}{L}$

substituting: $f_{11} = -\frac{1}{P} \left(\frac{k \cos kL}{\sin kL} - \frac{1}{L} \right)$

$$f_{11} = \frac{L}{3EI} \cdot \frac{3}{2U} \left(\frac{1}{2U} - \frac{1}{\tan 2U} \right)$$

$$f_{11} = c_{11} \cdot \frac{L}{3EI}$$

and

$$f_{12} = f_{21} = \frac{1}{P} \left(\frac{k}{\sin kL} - \frac{1}{L} \right) = \frac{L}{6EI} \cdot \frac{3}{U} \left(\frac{1}{\sin 2U} - \frac{1}{2U} \right)$$

$$f_{12} = c_{21} \cdot \frac{1}{6EI}$$

$M = \frac{2EI}{L} (\theta_A + \theta_B)$

we obtain:

$$M_1 = \frac{2EI}{L} \left[2\theta_1 A_1 + \theta_2 A_2 - A_3 \frac{\delta_2 - \delta_1}{L} \right]$$

$$M_2 = \frac{2EI}{L} \left[\theta_1 A_4 + 2\theta_2 A_5 - A_6 \frac{\delta_2 - \delta_1}{L} \right]$$

in which:

$$A_1 = A_5 = \frac{3C_{22}}{4C_{11}C_{22} - C_{12}C_{21}}$$

$$A_2 = A_4 = \frac{3C_{12}}{4C_{11}C_{22} - C_{12}C_{21}}$$

$$A_3 = A_6 = \frac{3(C_{22} + C_{12})}{4C_{11}C_{22} - C_{12}C_{21}}$$

The shear forces in the beam (excluding the effect of imposed transverse loads) are: $V_1 = -V_2 = \frac{M_1 + M_2}{L} =$

$$= \frac{2EI}{L} \left[\theta_1 (2A_1 + A_4) + \theta_2 (A_2 + 2A_5) - \frac{\delta_2 - \delta_1}{L} (A_3 + A_6) \right]$$

The above equations can be presented in a generalised stiffness form in a matrix form suitable for the analysis of elastic instability problems. This is as follows:

$\frac{4EI}{L} A_1$	$\frac{2EI}{L} A_2$	$\frac{6EI}{L^2} \frac{A_3}{3}$	$-\frac{6EI}{L^2} \frac{A_3}{3}$	$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ V_1 \\ V_2 \end{bmatrix}$
$\frac{2EI}{L} A_4 = A_2$	$\frac{4EI}{L} A_5 = A_1$	$\frac{6EI}{L^2} \frac{A_6}{3}$	$-\frac{6EI}{L^2} \frac{A_6}{3}$	
$\frac{6EI}{L^2} \frac{2A_1 + A_4}{3}$	$\frac{6EI}{L^2} \frac{A_2 + 2A_5}{3}$	$\frac{12EI}{L^3} \frac{A_3 + A_6}{6}$	$\frac{12EI}{L^3} \frac{A_3 + A_6}{6}$	
$-\frac{6EI}{L^2} \frac{2A_1 + A_4}{3}$	$-\frac{6EI}{L^2} \frac{A_2 + 2A_5}{3}$	$-\frac{12EI}{L^3} \frac{A_3 + A_6}{6}$	$-\frac{12EI}{L^3} \frac{A_3 + A_6}{6}$	

Although the above matrix has been developed by the stiffness method, this line of possible research is not pursued further in this work. Preference is given to further formulations by the force-displacement method.

generalised
slope-deflection
eq.

1.11 Review of the Principles of the Force-Displacement Method

Based on the force-displacement method, a system of linear homogeneous equations is set out, expressing the conditions of compatibility of deformations and equilibrium of forces, from which the unknown moments and deflections are calculated simultaneously. The load-deflection characteristics of members are used in terms of angular flexibility coefficients.

The approach is based on the physical fact that a frame is stable for as long as a positive horizontal force is required to produce sideways; the "positive" force being defined as one acting in the direction of the assumed horizontal motion of the frame. This is expressed in the form of a force-displacement matrix.

At first, the procedure is illustrated by a simple portal frame using the classical transcendental functions. Other, more complicated, cases are shown later in the text.

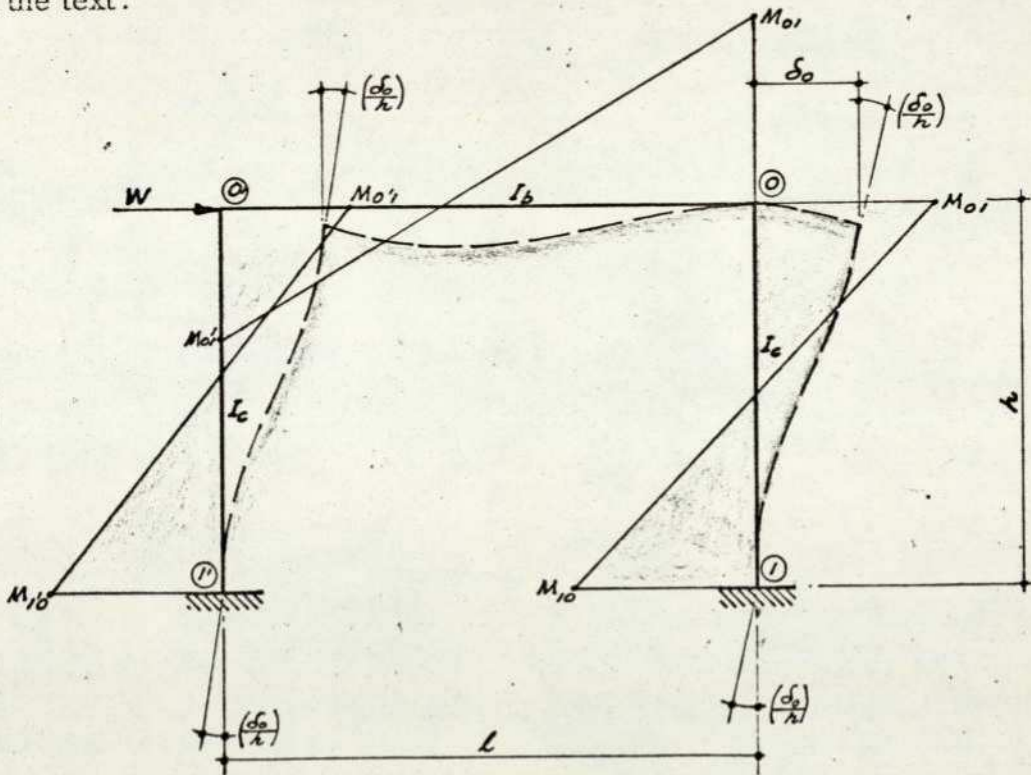


Fig. 1.110 Primary Stresses and Deformations in a Portal Frame

Fig. 1.110 shows a portal frame, subjected to a lateral load W , which is at first analysed for primary effects only. The procedure in setting out of the matrix is as follows: first the structure is assumed to deflect to its unrestrained and final position governed by the deflection δ_0 . In this position, all joints are assumed temporarily relaxed from continuities, which are then simultaneously restored to a correct configuration by the application of unknown forces.

Actual stiffnesses of members are taken into account, and the axial deformations of columns can be included at this stage without difficulties. For compatibility of deformations we can write from Fig. 1.110 that:

$$0. \quad -M_{oi} \frac{l}{6EI_b} + M_{oi} \left(\frac{l}{3EI_b} + \frac{h}{3EI_c} \right) - M_{io} \frac{h}{6EI_c} = \left(\frac{\delta_o}{h} \right)$$

$$1. \quad M_{io} \frac{h}{3EI_c} - M_{oi} \frac{h}{6EI_c} = \frac{\delta_o}{h}$$

For equilibrium of forces: $\left(\frac{M_{oi} + M_{io}}{h} \right) = \left(\frac{W}{2} \right)$

from which $M_{oi} = \frac{3Wnh^2}{2(l+6nh)} \quad \text{-----} \quad 1.110$

$$M_{io} = \frac{Wh(l+3nh)}{2(l+6nh)} \quad \text{-----} \quad 1.111$$

$$\delta_o = \frac{Wh^3}{EI} \cdot \frac{2l+3nh}{12(l+6nh)} \quad \text{-----} \quad 1.112$$

in which $n = \frac{I_b}{I_c}$

If columns are subjected to large axial forces then three changes can take place:

- column stiffnesses (or flexibility) can be altered
- column axial deformations may become significant and may have to be taken into account, and
- deformations may become large and the analysis may have to include the effect of altered geometry of the system.

These are the main factors of the elastic instability of frames systems.

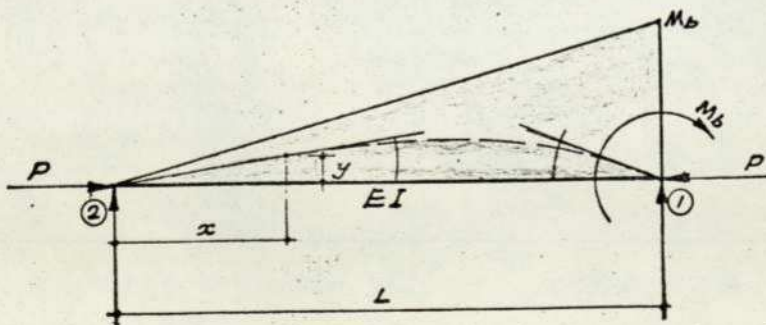


Fig. 1.111 Beam Subjected to Bending Moment and Axial Load

The stiffness of the beam subjected to an axial load and bending moment (Fig. 1.111) has long been solved based on the differential equation of the deflection curve.

$$\frac{d^2 y}{dx^2} = - \frac{M_x}{EI}$$

Timoshenko gives the following solution:

$$y = \frac{M_b}{P} \left(\frac{\sin kx}{\sin kL} - \frac{x}{L} \right) \quad \text{-----} \quad 1.113$$

in which

$$k = \sqrt{\frac{P}{EI}}$$

From Equation 1.113 we obtain:

$$\left(\frac{dy}{dx} \right)_{x=0} = \frac{M}{P} \left(\frac{k}{\sin kL} - \frac{1}{L} \right) = M \frac{L}{6EI} \cdot 6 \left[\frac{1}{2U \sin 2U} - \frac{1}{4U^2} \right]$$

$$\left(\frac{dy}{dx} \right)_{x=L} = \frac{M}{P} \left(\frac{k}{\tan kL} - \frac{1}{L} \right) = M \frac{L}{3EI} \cdot 3 \left[\frac{1}{2U \tan 2U} - \frac{1}{4U^2} \right]$$

in which

$$U = \frac{L}{2} \cdot k = \frac{L}{2} \sqrt{\frac{P}{EI}}$$

The values denoted by:

$$C_{21} = 6 \left[\frac{1}{2U \sin 2U} - \frac{1}{4U^2} \right] = \frac{3}{U} \left(\frac{1}{\sin 2U} - \frac{1}{2U} \right)$$

$$C_{11} = 3 \left[\frac{1}{2U \tan 2U} - \frac{1}{4U^2} \right] = \frac{3}{2U} \left(\frac{1}{2U} - \frac{1}{\tan 2U} \right)$$

represent therefore magnification factors of the flexibility coefficients arising from the presence of axial forces. These coefficients were shown in Fig. 1.071.

Referring to Fig. 1.112 we can now set out the matrix in which account can be taken of the altered geometry of the frame and the stiffnesses of columns (the stiffness of the beam is not appreciably altered because axial forces in the beam are nearly always small with the exception when pre-stressed concrete members are used):

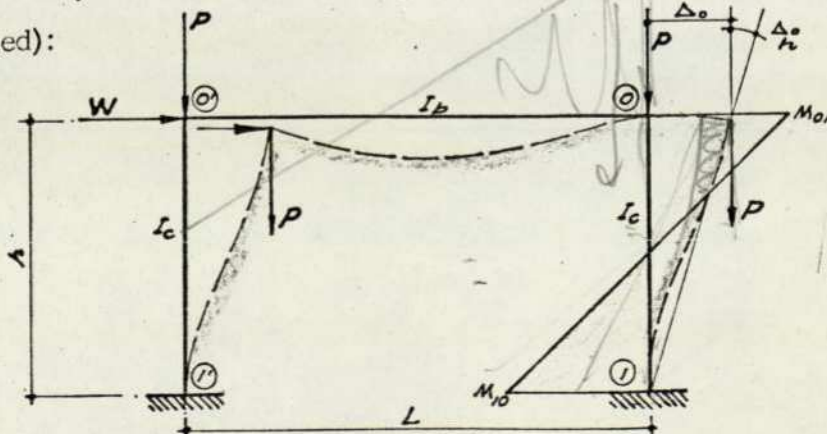


Fig. 1.112 Instability Moments and Deformations in a Portal Frame.

$$0. \quad -M_{o1} \frac{L}{6EI_b} + M_{o1} \left(\frac{L}{3EI_b} + C_{o1} \frac{h}{3EI_c} \right) - M_{io} \frac{h}{6EI_c} C_{o1} = \left(\frac{\Delta_o}{h} \right) \quad \text{-----} \quad 1.114$$

$$1. \quad -M_{o1} \frac{h}{6EI_c} C_{io} + M_{io} \frac{h}{3EI_c} C_{ii} = \left(\frac{\Delta_o}{h} \right) \quad \text{-----} \quad 1.115$$

$$0-0': \quad \frac{M_{o1} + M_{io}}{h} = \frac{1}{2} W + P \frac{\Delta_o}{h} \quad \text{-----} \quad 1.116$$

In equation 1.116 the value of $P \cdot \frac{\Delta_o}{h}$ arises from the horizontal component of the force P, while the axial component affects the flexibility coefficients. This is referred to as the "P.Δ" effect.

The solution of the three equations can be obtained in an explicit form which is convenient for further study of the response of frames.

$$M_{o1} = \frac{3Wh^2}{\left[\frac{2L(3 - C_{ii}Ph^2/EI)}{2C_{ii} + C_{o1}} \right] + 12h} - \frac{Wh}{2} \quad \text{-----} \quad 1.117$$

$$M_{io} = \frac{Wh [L + h(2C_{ii} + C_{io})]}{\left[\frac{2L(3 - C_{ii}Ph^2/EI)}{2C_{ii} + C_{o1}} \right] + 12h} \quad \text{-----} \quad 1.118$$

$$\Delta_o = \frac{Wh^3}{EI} \cdot \left[\frac{\frac{1}{2}L + h(2C_{ii} + C_{io})C_{ii} - \frac{1}{2}hC_{io}}{\left[\frac{2L(3 - C_{ii}Ph^2/EI)}{2C_{ii} + C_{o1}} \right] + 12h} \right] \quad \text{-----} \quad 1.119$$

When $P = 0$, all values of $c = 1$, and the 'instability' moment equations reduce to the 'primary' moment equations, i.e. Equations 1.117, 1.118 and 1.119 revert to Equations 1.110, 1.111 and 1.112 respectively.

Based on the above equations it can be shown, that for small values of axial forces the effect of elastic instability is small. For example, assuming $L = 6m$, $h = 2.7m$, $n = 1$ and $W = 10kN$, we have from the equations 1.117, 1.118 and 1.119:

$$M_{o1} = 0.1824 Wh$$

$$M_{io} = 0.3175 Wh \quad \text{and}$$

$$\Delta_o = 0.07545 Wh^3 / EI$$

Taking $P = 50 \text{ kN}$, we have:

$$C_{ii} = 1.0066, \quad C_{o1} = 1.0115$$

$$M_{o1} = 0.1831 Wh, \quad M_{io} = 0.3203 Wh$$

$$\Delta_o = 0.0766 \frac{Wh^3}{EI}$$

This represents an increase of only 1%.

Based on Equations 1.117, 1.118 and 1.119 the effect of varying axial force can be studied. The following table can be prepared from which Fig. 1.113 is produced.

$P(kN)$	C_{00}	C_{10}	M_{01}/Wh	M_{10}/Wh	$\Delta_0/Wh^3/EI$
0	1	1	.1824	.3175	.0754
50	1.0066	1.0115	.1831	.3203	.0766
100	1.0135	1.024	.1839	.3232	.0768
150	1.0200	1.0340	.1846	.3259	.0789
200	1.0265	1.0466	.1853	.3287	.0801
250	1.0335	1.0575	.1860	.3316	.0814
300	1.0405	1.0710	.1867	.3345	.0827
350	1.0476	1.0838	.1875	.3376	.0840
400	1.0550	1.0970	.1883	.3408	.0854
450	1.0625	1.1240	.1892	.3451	.0868
500	1.0700	1.1300	.1899	.3489	.0887
600	1.0850	1.1507	.1913	.3535	.0912
700	1.1022	1.1814	.1929	.3607	.0945
800	1.1194	1.2124	.1946	.3679	.0979
900	1.1365	1.2435	.1962	.3752	.1015
1000	1.1545	1.2786	.1978	.3831	.1053
1200	1.1958	1.3511	.2013	.4002	.1142
1600	1.2893	1.5240	.2085	.4395	.1359
2000	1.4034	1.7371	.2162	.4872	.1653
2500	1.5518	2.0173	.2250	.5517	.2096
3000	1.7955	2.4860	.2366	.6548	.2938

Fig. 1.113 shows the effect of axial forces on moments and sway in a portal frame. It can be seen that for small axial forces the primary analysis differs comparatively very little from the more accurate analysis. However, when axial forces are large, the effect on moments and deflections can be considerable: the deflection at $P = 3000$ kN is four times larger than that shown by the primary analysis. It follows, therefore, that frames in lower stories of tall buildings may be seriously affected by axial forces and secondary deflections.

It could be concluded that the stage can be reached when the structure will cease to be in equilibrium with the combined effect of wind and instability forces and collapse may occur. Such collapse, however, may also occur from the excessive wind pressure or excessive gravity loads, or from a critical

combination of the two. At present there is no way to study these critical loads in an explicit form because the problem becomes non-linear. The added complication arises from the fact that at many places the structure will enter visco-elastic and even fully plastic distribution of strains.

Fig. 1.114 shows, diagrammatically, the relationship between elastic analyses and elasto-plastic instability analysis. However, the values of the critical loads causing collapse cannot yet be reliably predicted, and must form a separate study.

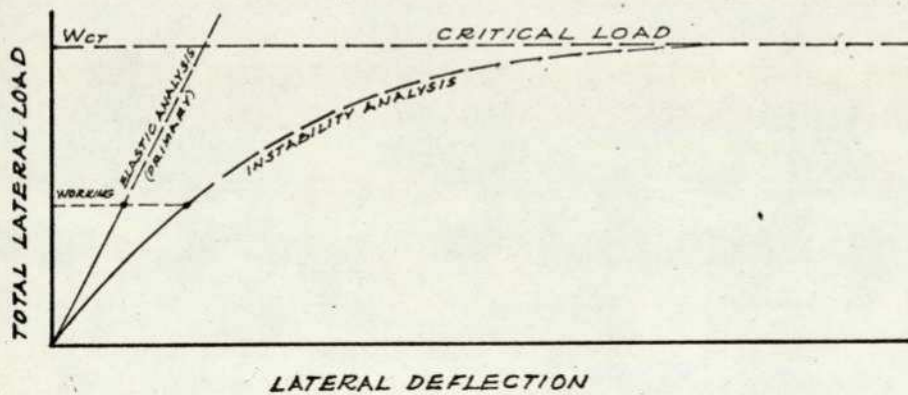


Fig. 1.114 Elastic and Elasto-plastic Deflections.

1.12 Review of the Elastic Instability of Portal Frames

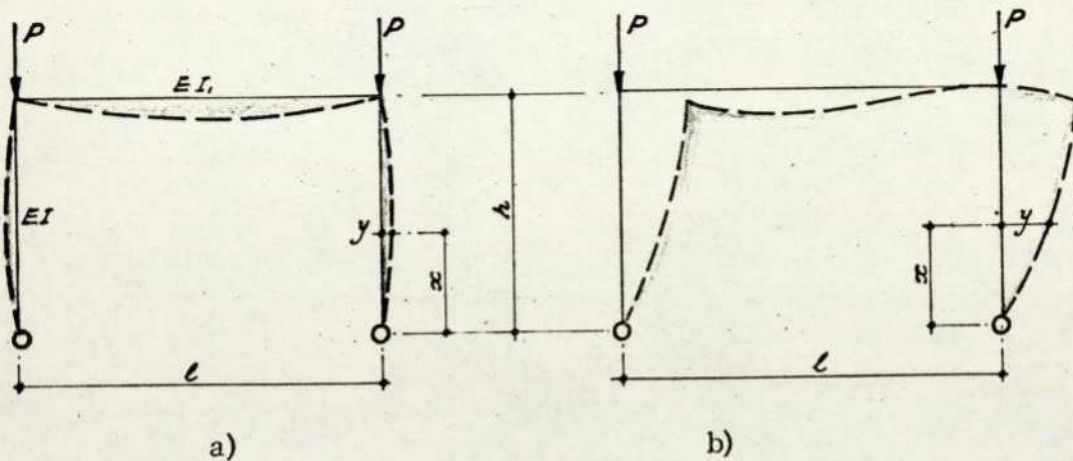


Fig. 1.120 Instability of a Portal Frame.

The theory of stability is well established for portal frames subjected to axial forces only. However, the effect of secondary moments has not been well developed for the stage when frames pass from stable to unstable equilibrium. Calculation of a critical load in a portal frame, Fig. 1.120 (a), without primary moments is usually based on the equation of the deflected line of a structure, i.e.:

$$EI \frac{d^2 y}{dx^2} = -Py$$

To satisfy the condition at the lower edge $y = A \sin kx$ the condition at the upper edge is

$$\left(\frac{dy}{dx} \right)_{x=h} = P \cdot \frac{L}{6EI} \cdot y_{x=h}$$

or

$$k \cos kh = PL \sin kh / 6EI$$

If the beam is infinitely rigid ($EI, = \infty$) then:

$$\cos kh = 0 \quad kh = \frac{\pi}{2}$$

$$P_{cr} = \frac{\pi^2 EI}{4h^2}$$

which is the Euler's critical load for a column fixed at one end and free at the other.

However, under certain loading conditions, another state of equilibrium may exist, called unstable equilibrium, i.e. when bending moments appear. In this state of instability, just after transition from stable equilibrium, the external loads and axial forces no longer form a balanced group of forces, and additional axial shear forces and moments must occur in order to restore equilibrium.

The relations between displacements of joints and the additional forces and moments form basic equations of stability conditions. These equations form a system of homogeneous linear equations.

Since the stability equations are linear and homogeneous, the unknown additional forces and moments are in general equal to zero. This means that no distortions or additional forces occur and the frame is in a state of stable equilibrium. The additional forces appear when the determinant Δ of the system of stability equations vanishes. The criterion $\Delta = 0$, therefore determines values of these forces in unstable states of equilibrium.

Theoretically there is an infinite number of solutions of the equation $\Delta = 0$, with infinite number of different unstable configurations, but the configuration which corresponds to the smallest value of loading determines the critical loading of the frame.

The distortions of the system defined by the condition $\Delta = 0$ are indeterminate, i.e. in this state of unstable equilibrium, the slightest disturbing

moment acting on a member will be sufficient to cause a large distortion of the whole frame, which will not return to its original position when the disturbing force ceases to act.

The Bleich's method of the analysis of stability of plane frames. is based on the system of linear relations between moments and the angles of bar rotations. These two parameters can be used to describe any configuration of the frame in the state of instability. The theory is also based on the fundamental differential equation:

$$EI\tau \frac{d^2y}{dx^2} + M_x = 0 \quad \text{-----} \quad 1.120$$

where y is the deflection at the distance x from the left end of the member, M_x is bending moment at x due to the forces acting on the deflected member in its displaced position.

$\tau = E_t/E$, E_t is the tangent modulus

for $\tau = 1$ the average stress P/A lies in the elastic range, but is a function of P/A if the average stress exceeds the proportional limit.

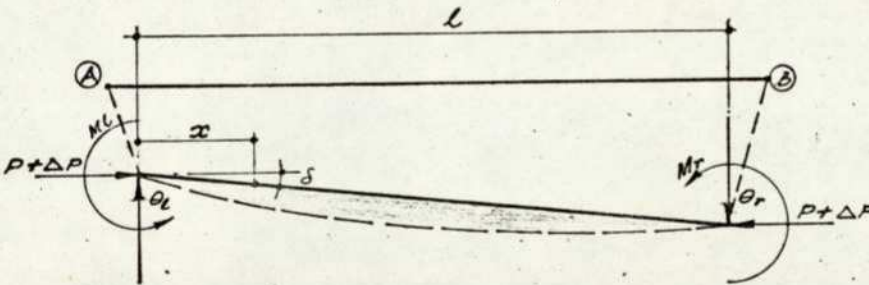


Fig. 1.121 Member Subjected to Moments and Axial Loads.

In Fig. 1.121 member AB in its deflected state is subjected to the following forces: the initial axial forces P due to the external loads ΔP , Q , M_A and M_B are the additional forces and moments. Positive directions of forces, moments and deflections are as shown in Fig.

The bending moment at the distance x from A, after eliminating the shear force Q , can be written as:

$$M_x = M_A \left(1 - \frac{x}{l}\right) + M_B \frac{x}{l} + P y \quad \text{-----} \quad 1.121$$

Equation 1.120 becomes: $EI \frac{d^2y}{dx^2} + P y + M_A \left(1 - \frac{x}{l}\right) + M_B \frac{x}{l} = 0 \quad \text{-----} \quad 1.122$

This equation is satisfied by the expression:

$$y = \frac{M_r}{P} \left[\frac{\sin \phi (1 - x/l)}{\sin \phi} + \frac{x}{L} - 1 \right] + \frac{M_l}{P} \left[\frac{\sin \phi \cdot x/l}{\sin \phi} - \frac{x}{L} \right] \text{----- 1.123}$$

where the stability factor ϕ is:

$$\phi = l \sqrt{P/EI}$$

Equation 1.123 is valid when the term $P.y$ in Eq. 1.122 is positive (when P is a compressive force). If P is tension, $P.y$ becomes negative, and ϕ is imaginary, but the Eq. 1.123 remains valid provided the trigonometric functions are replaced by the corresponding hyperbolic functions.

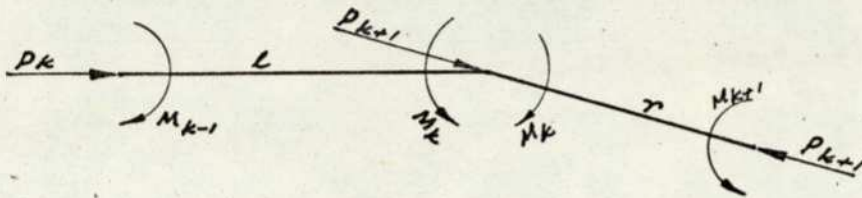


Fig. 1.122 Condition of Continuity at Point of Rigid Joint.

Using the force-displacement method, the stability equations express the condition of continuity at a point where two or more members are rigidly jointed, (Fig. 1.122) using transcendental functions s and c in the form:

$$s = \frac{1}{\phi^2} \left(\frac{\phi}{\sin \phi} - 1 \right) \quad \text{and} \quad c = \frac{1}{\phi^2} (1 - \phi \cot \phi)$$

$$\left(\text{for } \phi = 0, \quad s = \frac{1}{6} \quad \text{and} \quad c = \frac{1}{3} \right)$$

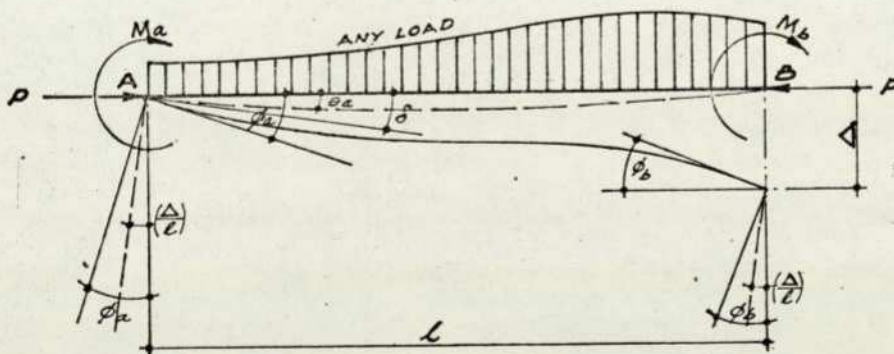


Fig. 1.123 Member Subjected to Variable Loading.

If a beam AB (Fig. 1.123) is subjected to end bending moments M_a and M_b , axial force P , rotations ϕ_a and ϕ_b , and translation Δ of end B with respect to A, then, for equilibrium:

$$M_a \alpha + M_b \beta = \phi_a + \frac{\Delta}{L}$$

$$M_a \beta + M_b \alpha = \phi_b + \frac{\Delta}{L}$$

from which: $M_a = K_s [\phi_a + c\phi_b - (1+c)\delta] + M_{ab}^F$

and $M_b = K_s [\phi_b + c\phi_a - (1+c)\delta] + M_{ba}^F$

when $K = \frac{EI}{l}$ $k = \sqrt{\frac{P}{EI}}$ $U = \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}$

The fixed end moments M_{ab}^F and M_{ba}^F can be calculated from:

$$M_a \alpha + M_b \beta = \Theta_a$$

$$M_b \beta + M_a \alpha = \Theta_b$$

from which, in terms of s and c:

$$M_a = -ks (\Theta_a + c \Theta_b)$$

$$M_b = -ks (\Theta_b + c \Theta_a)$$

Θ_a and Θ_b are end rotations of member AB due to the applied system of loading.

The main difficulty in the stability analysis of frames subjected to axial loads is that the governing equations are of non-linear, transcendental form, and can seldom be solved in an explicit form.

In the stiffness method, the equations of compatibility as well as equilibrium are expressed in terms of displacements. To obtain critical loads and modes of failure, the stiffness determinant would need to be equal to zero. Within working loads however, this is not necessary, and the effect of elastic instability can be obtained directly as described later.

Marchant developed an approximate method for calculating the maximum capacity P_m of an inelastic frame, and related it to the rigid plastic collapse load P_p and the elastic critical load P_e in the form:

$$\frac{1}{P_m} = \frac{1}{P_p} + \frac{1}{P_e}$$

Horne has shown, that for frames bent into a double curvature configuration, the above equation gives close value even when compared with the collapse loads obtained by the more accurate methods based on the elasto-plastic analysis. However, the calculations of 'exact' elastic critical loads requires considerable amount of computing work.

Stevens and Schmidt developed the technique based on the amplification of artificially introduced components of the buckled shape. Their

method utilised the Southwell plot in order to obtain initial estimates of the critical load. Further improvements of the estimated critical load are made by iterative techniques.

Some researchers developed methods for calculating approximate critical load to be specifically used in the Marchant formula. Horne used rigid plastic analysis to derive approximate critical load. Similarly Stevens used energy technique for the same purpose.

Because of considerable difficulties in mathematical formulation of stability problems, simple mechanical models are used to study the problem. Such models consist of rigid bars, which simulate the geometry of the actual system and linearly-elastic springs representing stiffness. In this way, a system with infinite number of degrees of freedom can be represented by an equivalent system with a finite number of degrees of freedom. Furthermore, the response of a system is then described by a set of simultaneous algebraic equations instead of by differential equations. This is an essential improvement, but can be applied to simple configurations only.

A. Chajes uses such models for the analysis of columns and frames, starting with the basic criteria, that, a neutral equilibrium is possible at the transition from stability to instability. For a column to be in equilibrium in a slightly deformed position, the resisting moment at any section $EI\left(\frac{d^2y}{dx^2}\right)$ must be equal to the externally applied moment $P.y$ or:

$$EI \frac{d^2y}{dx^2} + Py = 0$$

from which the column critical load is

$$P_{cr} = \pi^2 \frac{EI}{l^2}$$

Now, using a two bar system (Fig. 1.124) connected by a rotational spring of stiffness $e = M/\theta$

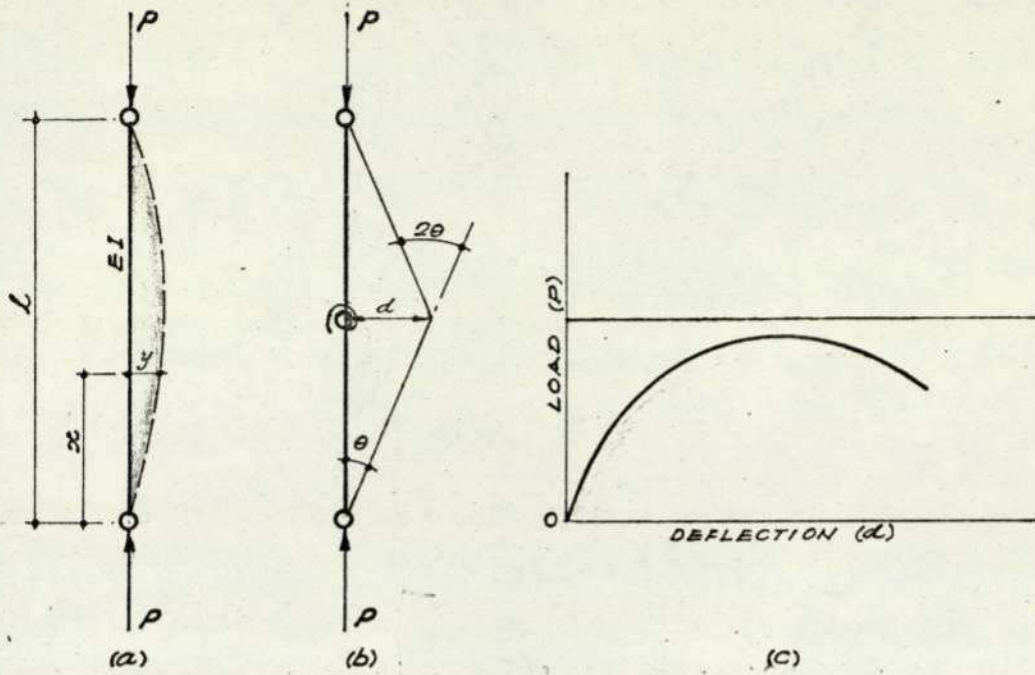


Fig. 1.124 Two-bar System Analogy.

The external moment at mid-height must equal the resisting moment in the spring:

$$P.d = 2\theta.e \quad \text{----- 1.124}$$

in which d is the lateral displacement.

For small deformations $\theta = d/L$ so that

$$d\left(P - \frac{2e}{L}\right) = 0 \quad \text{----- 1.125}$$

which gives a trivial solution 0-1-2 for $d = 0$ and the bifurcation of equilibrium with a critical load.

$$P_c = \frac{2e}{L} \quad \text{----- 1.126}$$

Comparing this critical load with the Euler's load $\pi^2 EI/L^2$, it can be seen that both loads increase with increasing bending stiffness (e or EI/L) and decrease with increasing L . Energy methods can also be used, since the critical load corresponds to the state of neutral equilibrium, i.e. at minimum energy. The total potential energy (existing as the elastic strain energy U and the potential energy of the external forces V) has the following value at equilibrium:

$$\Sigma(U+V) = 0$$

substituting $U = \frac{1}{2}e(2\theta)^2$ and $V = -P\theta^2 L$

We obtain:

$$\theta \left(P - \frac{2C}{L} \right) = 0$$

which is the same as Equation 1.125 obtained by the equilibrium method.

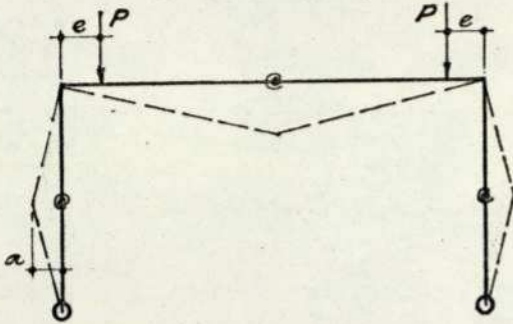


Fig. 1.125 Portal Frame Analogy.

A similar approach applied to a frame such as in Fig. 1.125 leads to a critical load of:

$$P = \frac{3C}{L} \cdot \left[\frac{1}{1 - \left(\frac{e}{2a} \right)} \right] \quad \text{----- 1.127}$$

in which e represents a small distance of the load P from the centre line of a column and a is the deflection at the spring. Equation 1.127 shows that small inaccuracies in primary moments do not affect appreciably the behaviour of frames, particularly symmetrical.

D. Johnson using an energy method has developed a method for analysing the elastic stability of rigid frames subjected to lateral deformations. He assumes the mode of deformations so that compatibility may be satisfied by one standard moment distribution. The results are presented in a graphical form.

Daniels and Lu as well as Armacost use a three step design procedure usually in conjunction with a computer. In the first step tentative sizes are related using the plastic moment balancing method, and critical sway deflections are estimated. Using these results, a second step is made to verify initial sways, sizes and load-carrying capacities. The final step in the design process is to revise the member sizes based on the results of the load-deflection analysis. The Authors suggest that further improvements and simplifications are possible.

Switzky and Ping Chun Wang suggest a method of design for minimum weight based on stability criteria. The results are presented in a series of graphs and tables for two-dimensional steel frames. The Authors

assume that column loads do not cause any bending moments, that the members are ideally straight, and rigidly connected or pin-jointed. The material is assumed to be elastic. Authors have shown that by minimising a stiffness in a fix set as function of weight, a solution of minimum weight can be obtained.

The main difficulty of the above mentioned theories is that they are applicable only to the simplest forces, i.e., columns and portal frames, and rarely can they be used in the complex arrangements often occurring in practice.

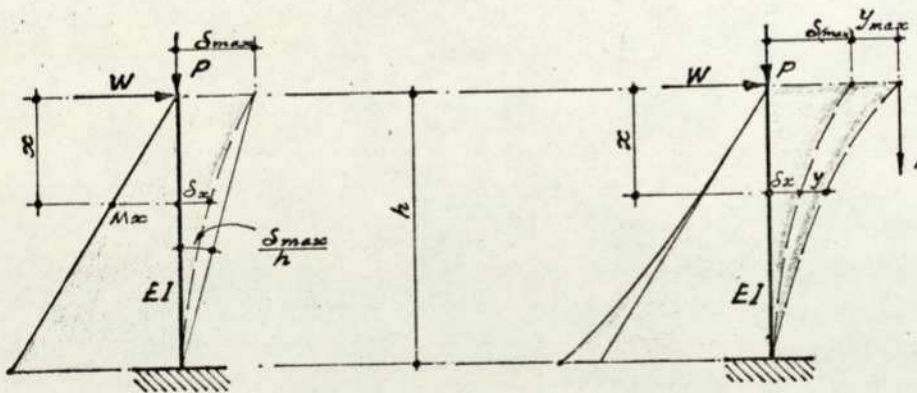
1.13 Elastic Instability of a Cantilever

The instability analysis of a simple cantilever provides a suitable introduction to the analysis of more complex structures.

In this analysis it will be shown that moments and deflections for a simple cantilever subjected to lateral and axial loads can be expressed in terms of moment and deflections obtained under primary effects (lateral loading only). For example:

Additional deflection due to axial load = primary deflection
x magnification coefficient.

Consider the simple cantilever shown in Fig. 1.130.



a) Primary Analysis

b) Instability Effect

Fig. 1.130 Simple Cantilevers used in Analyses.

The primary deflection of a cantilever beam, Fig. 1.130 a), can be calculated from the equation:

$$EI \frac{d^2 \delta}{dx^2} = -M = -Wx$$

from which

$$EI \delta_x = \frac{W}{6} (-x^3 + 3h^2x + 2h^3)$$

For $x = 0$, we obtain $\delta_{max} = \frac{Wh^3}{3EI}$

Alternatively, by the force-displacement method, we obtain:

$$\frac{\delta_{max}}{h} = Wh \cdot \frac{h}{3EI} \quad \text{or} \quad \delta_{max} = \frac{Wh^3}{3EI}$$

Elastic instability analysis gives

$$M_x = P(\delta_{max} - \delta_x)$$

so that

$$EI \frac{d^2y}{dx^2} = -P \left[\frac{Wh^3}{3EI} - \frac{W}{EI} \left(\frac{x^3}{6} + \frac{h^2x}{2} - \frac{h^3}{3} \right) \right]$$

$$EI \frac{d^2y}{dx^2} = -\frac{PW}{6EI} [x^3 - 3h^2x + 4h^3]$$

from which $y = -\frac{PW}{6EI^2} \left[\frac{x^5}{20} - \frac{h^2x^3}{2} + 2h^3x^2 - \frac{11h^4x}{4} + \frac{6h^5}{5} \right]$

and for $x=0$, $y_{max} = -\frac{PWh^5}{5EI} = -\frac{Wh^3}{3EI} \cdot \frac{3Ph^2}{5EI}$

$$y_{max} = \delta_{max} \cdot \frac{3Ph^2}{5EI}$$

in which the value $\frac{3Ph^2}{5EI}$ is a non-dimensional magnification coefficient

Total deflection is therefore:

$$\delta_{max} + y_{max} = \frac{Wh^3}{3EI} \left(1 + \frac{3Ph^2}{5EI} \right)$$

The expression for bending moment including the instability effect becomes:

$$\begin{aligned} M_x &= Wx + P(\delta_{max} + y_{max} - \delta_x - y) \\ M_x &= Wx + P \left[\delta_{max} + \delta_{max} \cdot \frac{3h^2P}{5EI} - \right. \\ &\quad \left. - \frac{W}{6EI} \left(-x^3 + 3h^2x - 2h^3 \right) - \right. \\ &\quad \left. - \frac{PW}{6EI^2} \left(\frac{x^5}{20} - \frac{h^2x^3}{2} + 2h^3x^2 - \frac{11h^4x}{4} + \frac{6h^5}{5} \right) \right] \end{aligned}$$

for $x = h$, $M_{max} = Wh + P \cdot \frac{Wh^3}{3EI} \left(1 + \frac{3h^2P}{5EI} \right) - \frac{PW}{6EI} (-h^3 + 3h^2 - 2h^3) - \frac{P^2W}{6EI^2} \left(\frac{h^5}{20} - \frac{h^5}{2} + 2h^5 - \frac{11h^5}{4} + \frac{6h^5}{5} \right)$

$$M_{max.} = Wh + \frac{PWh^3}{3EI} \left(1 + \frac{3h^2P}{5EI} \right) \quad \text{or}$$

$$M_{max.} = Wh \left[1 + \frac{Ph^2}{3EI} \left(1 + \frac{3h^2P}{5EI} \right) \right]$$

The following example, Fig. 1.131, is used to illustrate the instability effect of a simple cantilever. Assume $h = 300\text{mm}$, $b \times d = 6.35 \times 6.35 \text{ mm}$ with I value of 135.5 mm^4 , $E = 200 \text{ kN/mm}^2$, $P = 100\text{N}$ and $W = 20\text{N}$.

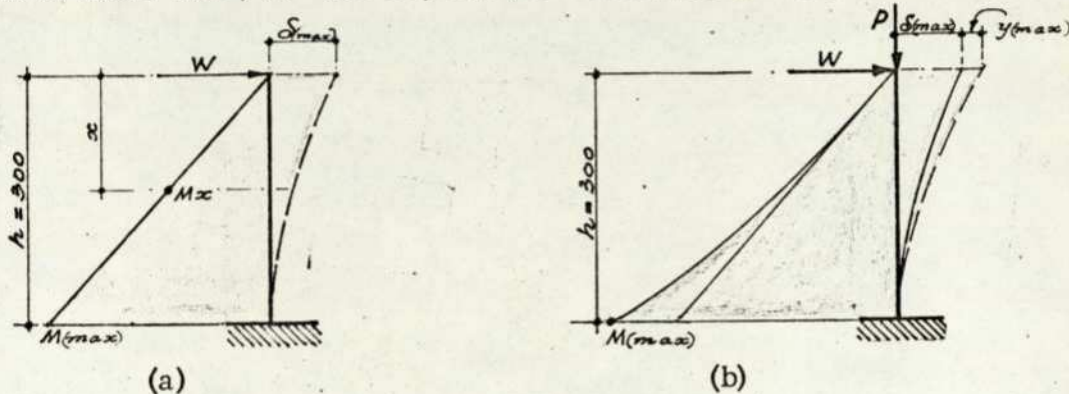


Fig. 1.131 Cantilever Examples

For the primary effect only, Fig. 1.131 (a), we obtain:

$$\delta_{max} = \frac{Wh^3}{3EI} = \frac{20 \times 300^3}{3 \times 200\,000 \times 135.5}$$

$$\delta_{max.} = 6.642 \text{ mm}$$

$$M_{max} = Wh = 20 \times 300 = 6000 \text{ Nmm}$$

Taking into account the instability effect, Fig. 1.131 (b), we obtain:

$$y_{max} = \delta_{max} \cdot \frac{3h^2P}{5EI}$$

$$\text{where } \frac{3h^2P}{5EI} = \frac{3 \times 300^2 \times 100}{5 \times 200\,000 \times 135.5} = 0.1992$$

$$\therefore y_{max} = 0.1992 \delta_{max} = 1.322 \text{ mm}$$

$$M_{max} = Wh + P \frac{Wh^3}{3EI} \left(1 + \frac{3h^2P}{5EI} \right)$$

$$= 600 + 100 \times 6.64(1 + 0.1992) = 6796 \text{ Nmm}$$

Therefore it can be seen that the effect of applying an axial load of 100N to a 300 mm long cantilever already subjected to a lateral load of 20N is (i) to increase the lateral deflection from 6.642 mm to 7.964 mm, i.e. by 20% and (ii) to increase the maximum bending moment by 13%.

2.00 Steel Portal Frames : Theory and Experiment

2.10 Theoretical Analyses of Portal Frames for Primary and Instability Effects

This section is devoted to the development of the theory of the force-displacement method, for a simple portal frame subjected to a lateral point-applied load with and without axial compressive column loads on one or both columns.

The purpose of the analyses was to:

- i) show how the force-displacement method can be applied,
- ii) provide a reference point for further theoretical studies, and
- iii) provide a basis for comparison with experimental data.

To aid comparison, both theoretically and experimentally, a number of structural configurations are analysed in which hinges are introduced into the structure at column foot or column head as shown in Fig. 2.10.

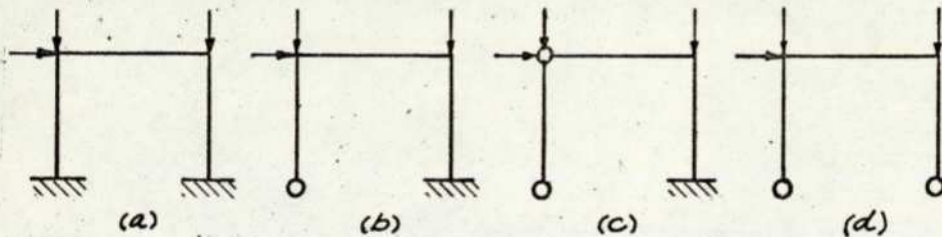


Fig. 2.10 Portal Frames Analysed.

A fixed-ends portal frame, Fig. 2.10 (a), is first analysed in depth giving general solutions for lateral deflection and bending moment at head and foot of column. To enable solution of simultaneous equations manually it is assumed that the frame is symmetrical about a vertical centreline thereby reducing the number of unknown values using the force displacement method to 3.

In a similar way a portal frame with both column feet hinged is analysed in depth. In both cases expressions are derived for deflection and bending moment of the frame in terms of the behaviour of a simple cantilever using reduction factors, e.g. deflection of portal frame = end deflection of cantilever $\left(\frac{Wk^3}{3EI}\right)$ times a reduction factor.

Computer aided analyses based on the force-displacement method are then made for similar frames with an increasing number of hinges introduced.

2.1.1 Theoretical Primary and Secondary Analysis for Fixed-ends Portal Frame

Using the force-displacement method for the primary analysis, compatibility of deformation at O. from Fig. 2.111, gives

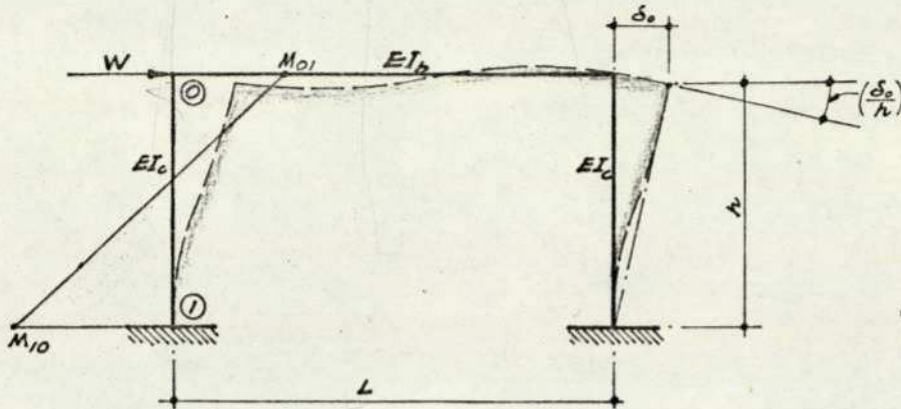


Fig. 2.111 Fixed-ends Portal Frame Analysed for Primary Effects

$$-M_{10} \frac{h}{6EI_c} + M_{01} \frac{h}{3EI_c} + \frac{L}{6EI_b} = \frac{\delta_o}{h} \quad \text{----- 2.1101}$$

$$\text{Similarly at point 1: } -M_{10} \frac{h}{3EI_c} + M_{01} \frac{h}{6EI_c} = -\frac{\delta_o}{h} \quad \text{----- 2.1102}$$

$$\text{Equilibrium condition gives } \frac{M_{01} + M_{10}}{h} = \frac{W}{2} \quad \text{----- 2.1103}$$

Combining equations 2.1101 and 2.1102 gives

$$-M_{10} \frac{h}{2EI_c} + M_{01} \left(\frac{h}{2EI_c} + \frac{L}{6EI_b} \right) = 0$$

Multiplying by EI_c and putting $n = \frac{I_c}{I_b}$

$$-M_{10} \frac{h}{2} + M_{01} \left(\frac{h}{2} + \frac{Ln}{6} \right) = 0 \quad \text{----- 2.1104}$$

$$\text{from 2.1103 } M_{01} + M_{10} = \frac{1}{2} Wh \quad \text{----- 2.1105}$$

$$\text{and from 2.1104, dividing by } h \quad -M_{10} + M_{01} \left(1 + \frac{nL}{3h} \right) = 0$$

$$\text{substituting this } M_{10} \text{ in 2.1105 gives } M_{01} \left(2 + \frac{nL}{3h} \right) = \frac{1}{2} Wh$$

$$\text{therefore } M_{01} = \frac{3Wh^2}{2(6h + nL)} \quad \text{----- 2.1106}$$

$$M_{10} = \frac{1}{2} Wh - M_{01} = \frac{Wh}{2} - \frac{3Wh^2}{2(6h + nL)} = \frac{6Wh^2 + WhnL - 3Wh^2}{2(6h + nL)}$$

$$M_{10} = \frac{Wh(3h + nL)}{2(6h + nL)} \quad \text{----- 2.1107}$$

from 2.1102

$$\delta_o = \frac{h^2}{3EI_c} \cdot M_{10} - \frac{h^2}{6EI_c} \cdot M_{01}$$

$$\delta_o = \frac{h^2}{3EI_c} \left[\frac{Wh(3h+nL)}{2(6+nL)} - \frac{3Wh^2}{4(6h+nL)} \right]$$

$$\delta_o = \frac{Wh^3}{6(6h+nL)EI_c} \left[3h+nL - \frac{3h}{2} \right]$$

$$\delta_o = \frac{Wh^3 (3h/2 + nL)}{6EI_c (6h + nL)} \quad \text{----- 2.1108}$$

A similar procedure is followed now for the elastic instability analysis (Fig. 2.112).

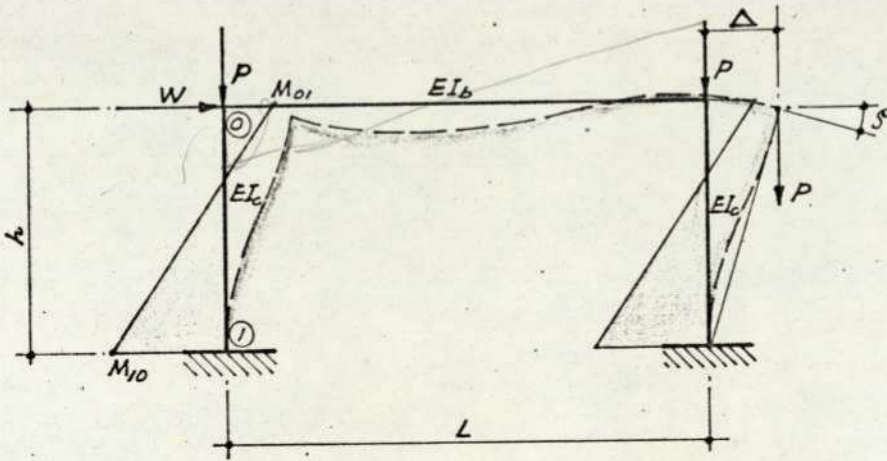


Fig. 2.112 Fixed-ends Portal Frame used for Instability Analysis.

Compatibility of deformation at O:

$$-M_{10} \frac{c_{01}h}{6EI_c} + M_{01} \left(\frac{c_{00}h}{3EI_c} + \frac{L}{6EI_b} \right) = \frac{\Delta}{h} \quad \text{----- 2.1109}$$

Similarly at point 1: $M_{10} \frac{c_{11}h}{3EI_c} - M_{01} \frac{c_{10}h}{6EI_c} = \frac{\Delta}{h} \quad \text{----- 2.1110}$

Equilibrium condition: $\frac{M_{10} + M_{01}}{h} = \left(\frac{W}{2} + P \tan \delta \right) = \frac{W}{2} + \frac{P\Delta}{h} \quad \text{----- 2.1111}$

For members with constant I, $c_{01} = c_{10}$; $c_{11} = c_{00}$

Adding Equation 2.1109 and minus Equation 2.1110

$$-M_{10} \frac{h}{3EI_c} \left(\frac{c_{01} + c_{11}}{2} \right) + M_{01} \left[\frac{h}{3EI_c} \left(c_{00} + \frac{c_{10}}{2} \right) + \frac{L}{6EI_b} \right] = 0 \quad \text{----- 2.1112}$$

from Equation 2.1111 $\frac{M_{10} + M_{01}}{Ph} = \frac{W}{2P} + \frac{\Delta}{h} \quad \text{----- 2.1113}$

adding Equation 2.113 to minus Equation 2.110

$$M_{10} \left(\frac{1}{Ph} - \frac{c_{11}h}{3EI_c} \right) + M_{01} \left(\frac{1}{Ph} + \frac{c_{10}h}{6EI_c} \right) = \frac{W}{2P} \quad \text{----- 2.1114}$$

Multiplying Equation 2.1112 by $6EI_c$ and putting $n = I_c/L$

$$-M_{10}h(c_{10} + 2c_{11}) + M_{01}[h(2c_{11} + c_{10}) + Ln] = 0 \quad \text{----- 2.1115}$$

Multiplying Equation 2.1114 by $6EI_c$

$$M_{10}\left(\frac{6EI_c}{Ph} - 2hc_{11}\right) + M_{01}\left(\frac{6EI_c}{Ph} + c_{10}h\right) = \frac{3EI_c W}{P} \quad \text{----- 2.1116}$$

Dividing Equation 2.1115 by $h(c_{10} + 2c_{11})$

$$-M_{10} + M_{01}\left[1 + \frac{Ln}{h(2c_{11} + c_{10})}\right] = 0 \quad \text{----- 2.1117}$$

Similarly from Equation 2.1116

$$M_{10} + M_{01} \left[\frac{\frac{6EI_c}{Ph} + c_{10}h}{\frac{6EI_c}{Ph} - 2hc_{11}} \right] = \frac{3EI_c W}{\frac{6EI_c}{h} - 2hc_{11}P}$$

$$\begin{aligned} M_{10} + M_{01} \left[\frac{6EI_c + c_{10}h^2P}{6EI_c - 2c_{11}h^2P} \right] &= \frac{3EI_c Wh}{6EI_c - 2c_{11}h^2P} \\ &= \frac{\frac{Wh}{2}}{\left(1 - \frac{2c_{11}h^2P}{6EI_c}\right)} \quad \text{----- 2.1118} \end{aligned}$$

Adding Equation 2.1117 and Equation 2.1118

$$M_{01} \left[1 + \frac{nL}{h(c_{10} + 2c_{11})} + \frac{1 + \frac{c_{10}h^2P}{6EI_c}}{1 - \frac{2c_{11}h^2P}{6EI_c}} \right] = \frac{\frac{Wh}{2}}{1 - \frac{2c_{11}h^2P}{6EI_c}}$$

$$\text{Let } m = \frac{Ph}{6EI_c}$$

$$M_{01} \left[1 + \frac{nL}{h(c_{10} + 2c_{11})} + \frac{1 + mhc_{10}}{1 - 2mhc_{11}} \right] = \frac{Wh}{2} \cdot \frac{1}{1 - 2mhc_{11}}$$

Taking common denominator:

$$\begin{aligned} M_{01} \left[\frac{(1 - 2mhc_{11})h(c_{10} + 2c_{11}) + nL(1 - 2mhc_{11}) + (1 + mhc_{10})h(c_{10} + 2c_{11})}{h(c_{10} + 2c_{11})(1 - 2mhc_{11})} \right] \\ = \frac{Wh}{2} \cdot \frac{1}{1 - 2mhc_{11}} \end{aligned}$$

$$M_{01} = \frac{Wh}{2} \left[\frac{2hc_{11} + hc_{10}}{(1 - 2mhc_{11})(2hc_{11} + hc_{10}) + nL(1 - 2mhc_{11}) + (1 + mhc_{10})(2hc_{11} + hc_{10})} \right]$$

Substituting values values: $h = 256$, $L = 374$, $n = 1$

$$\text{and } m = P \times \frac{256}{6 \times 200000 \times 135.5} = 1.574 \times 10^{-6} P$$

Therefore $M_{01} = 128 \text{ W.A.}$ where A is the square bracket term

The values of A for different values of P have been calculated and set out in tabular form in Fig. 2.113

P (N)	$\frac{2U}{=0.0718/\sqrt{P}}$	C_{11}	C_{01}	A	$\frac{M_{01}}{W}$
0	0	1	1	0.4021	51.468
20	0.3211	1.00709	1.0123	0.4043	51.760
40	0.4541	1.0142	1.0245	0.4089	52.339
60	0.5561	1.0214	1.037	0.4115	52.67
80	0.6422	1.0290	1.0505	0.4145	53.05
100	0.718	1.0360	1.0640	0.4192	53.66
140	0.8495	1.0515	1.0920	0.4264	54.58
180	0.9633	1.0680	1.121	0.4319	55.29
220	1.06496	1.0860	1.151	0.4413	56.49
260	1.1577	1.103	1.182	0.4498	57.58
300	1.2436	1.1215	1.2161	0.4580	58.63
600	1.7587	1.2951	1.534	0.533	68.236

Fig. 2.113 Table of $\frac{M_{01}}{W}$ for values of P.

The curve of M_{01}/W against P is shown in Fig. 2.114. It can be seen that for a given lateral load W, the bending moment increases with increase of axial load P.

Check that for P = 0, the instability value of M_{01} reverts to primary value.

$$\text{For } W = 100 \text{ say, primary } M_{01} = \frac{3 \times 100 \times 256^2}{2(6 \times 256 + 374)} = 5146.8$$

$$\text{and from Fig. 2.113 } M_{01} \text{ (instab)} = 51.468 \times 100 = 5146.8$$

Therefore checks satisfactorily.

It is also of interest to derive the relation between lateral deflection and axial load.

$$\text{From Equation 2.117: } M_{10} = M_{01} \left[1 + \frac{Ln}{2hc_{11} + hc_{10}} \right]$$

Substituting for M_{01} gives:

$$M_{10} = \frac{Wh}{2} \left[1 + \frac{Ln}{2hc_{11} + hc_{10}} \right] \left[\frac{2hc_{11} + hc_{01}}{(1-2mhc_{11})(2hc_{11} + hc_{01} + nL(1-2mhc_{11})) + (1+mhc_{10})(2hc_{11} + hc_{01})} \right]$$

$$M_{10} = \frac{Wh}{2} \left[A + \frac{LnA}{2hc_{11} + hc_{10}} \right]$$

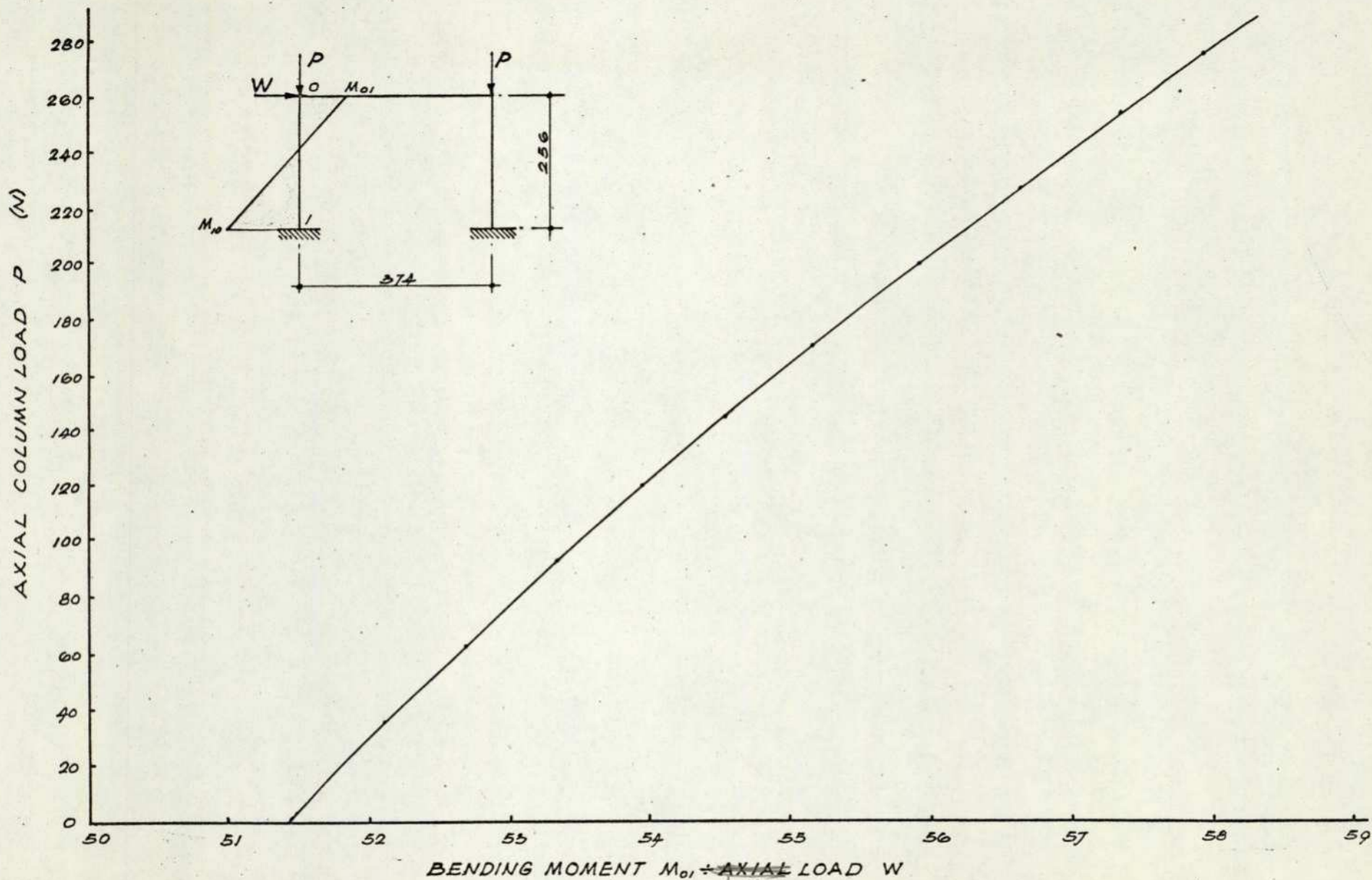


Fig. 2.114 Curve of M_{o1} / W for Various P

And from Equation 2.110

$$\Delta = \left[M_{10} c_{11} - M_{01} \frac{c_{10}}{2} \right] \frac{h^2}{3EI}$$

$$\Delta = \frac{Wh^3}{6EI} \left[A c_{11} + \frac{Ln A c_{11}}{2hc_{11} + hc_{10}} - A \frac{c_{10}}{2} \right]$$

Substituting values:

$$\Delta = 0.1032 WA \left[c_{11} + \frac{374 c_{11}}{512 c_{11} + 256 c_{10}} - \frac{c_{10}}{2} \right]$$

$$\Delta = 0.1032 WAB \quad \text{----- 2.1119}$$

where B = $c_{11} + \frac{374 c_{11}}{512 c_{11} + 256 c_{10}} - \frac{c_{10}}{2}$

To check that for P = 0, the instability value of Δ is equal to the primary value, let W = 100 say

$$\Delta_{\text{primary}} = \frac{100 \times 256^3 \left(\frac{3 \times 256}{2} + 374 \right)}{6 \times 200000 \times 135.5 (6 \times 256 + 374)} = 4.098 \text{ mm}$$

Whereas $\Delta_{\text{instability}} = 0.10318 WAB$

$$\text{where } B = 1 + \frac{374}{512 + 256} - 0.5 = 0.9869$$

$$\text{Therefore } \Delta_{\text{instability}} = 0.10318 \times 100 \times 0.4021 \times 0.9869 = 4.0955$$

Comparing this value with the primary value of 4.0948 shows good agreement.

Using Equation 2.1119, Δ/W can also be tabulated for various P, Fig. 2.115, and is presented graphically in Fig. 2.116.

P (N)	A	B	$\frac{\Delta}{W}$
0	0.4021	0.9869	0.04095
20	0.4043	0.9872	0.04119
40	0.4089	0.9873	0.0416
60	0.4115	0.9874	0.04193
80	0.4145	0.9874	0.042237
100	0.4192	0.9865	0.04267
140	0.4264	0.9862	0.04339
180	0.4319	0.9865	0.0439
220	0.4413	0.9879	0.04499
260	0.4498	0.9876	0.0458
300	0.4580	0.9871	0.04665
600	0.533	0.9871	0.0543

Fig. 2.115 Table of Δ/W for values of P

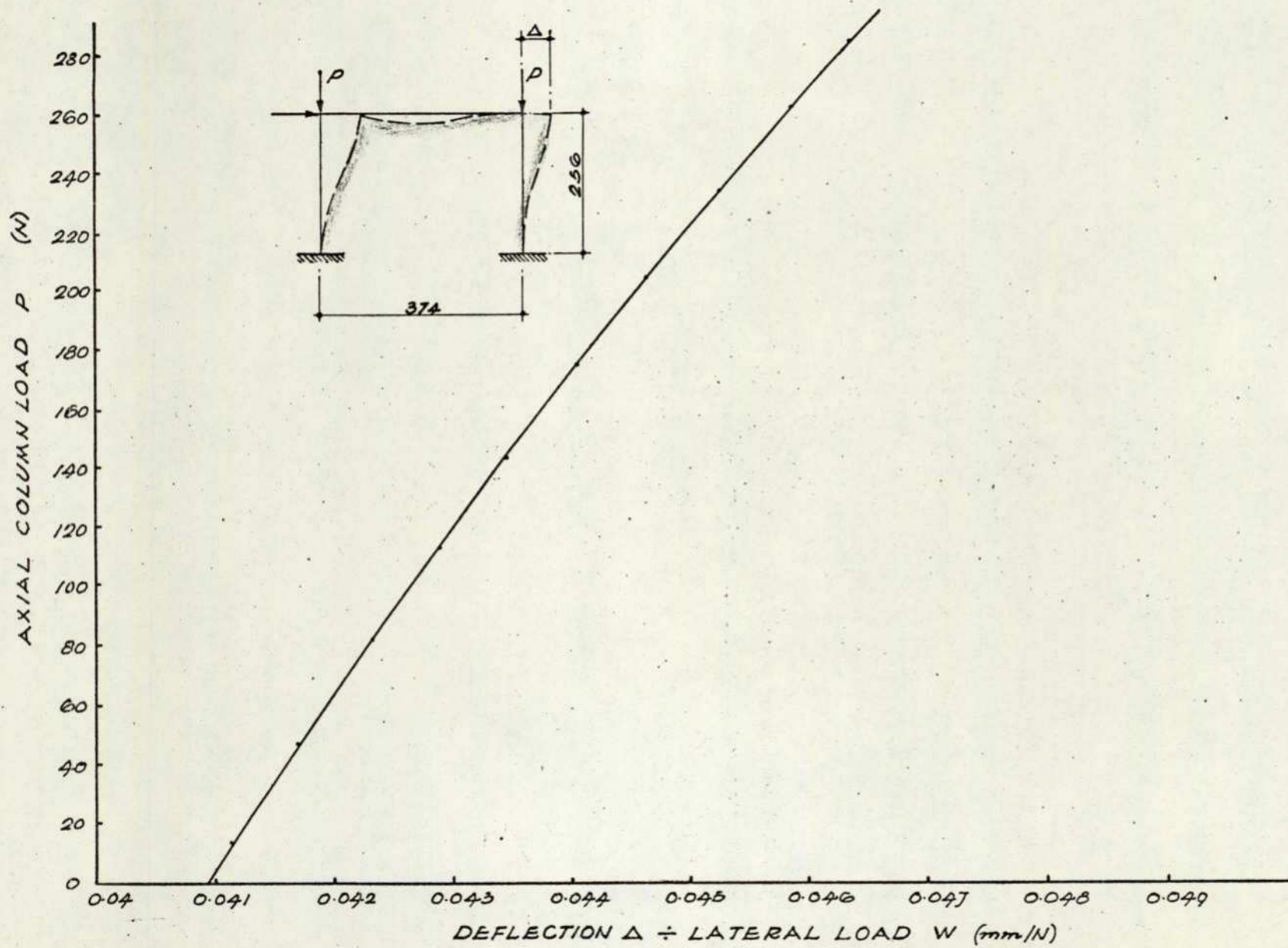


Fig. 2.116 Curve of Δ/W for Various P .

2.1.2 Theoretical Analysis for a Hinged-ends Portal Frame

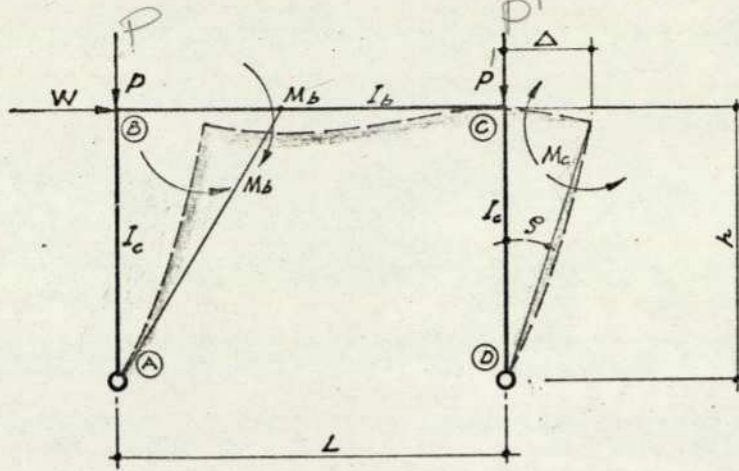


Fig. 2.121 Hinged-ends Portal Frame.

Compatibility of deformation condition at B:

$$M_b \left(\frac{L}{3EI_b} + \frac{c_{bb}h}{3EI_c} \right) - M_c \frac{L}{6EI_b} = \frac{\Delta}{h} \quad \text{----- 2.121}$$

Similarly at C:

$$-M_b \frac{L}{6EI_b} + M_c \left(\frac{L}{3EI_b} + \frac{c_{cc}h}{3EI_c} \right) = \frac{\Delta}{h} \quad \text{----- 2.122}$$

And from the condition of equilibrium of forces:

$$\frac{M_b}{h} + \frac{M_c}{h} = W + P \tan \theta + P \tan \theta \quad \text{----- 2.123}$$

Now if both columns are identical: $I_b = I_c = I$, $M_b = M_c$, $c_{bb} = c_{cc}$

Equation 2.121 becomes:

$$\begin{aligned} \frac{M_b}{3EI} \left(L + c_{bb}h - \frac{L}{2} \right) &= \frac{\Delta}{h} \\ M_b &= \frac{\Delta}{h} \frac{3EI}{\left(\frac{L}{2} + c_{bb}h \right)} \quad \text{----- 2.124} \end{aligned}$$

Equation 2.123 becomes:

$$\begin{aligned} \frac{2M_b}{h} &= W + 2P \frac{\Delta}{h} \quad \text{where } \tan \theta = \frac{\Delta}{h} \\ M_b &= \frac{Wh}{2} + P\Delta \quad \text{----- 2.125} \end{aligned}$$

Equating 2.124 and 2.125 gives:

$$\frac{3EI\Delta}{h(L/2 + c_{bb}h)} = \frac{Wh}{2} + P\Delta$$

Therefore:

$$\Delta \left[\frac{3EI - Ph(L/2 + c_{bb}h)}{h(L/2 + c_{bb}h)} \right] = \frac{Wh}{2}$$

$$\Delta = \frac{Wh^2}{2} \left[\frac{\frac{L}{2} + c_{bb}h}{3EI - Ph\left(\frac{L}{2} + c_{bb}h\right)} \right]$$

$$\Delta = \frac{Wh^3}{3EI} \left[\frac{3EI}{2h} \left(\frac{\frac{L}{2} + c_{bb}h}{3EI - Ph\left(\frac{L}{2} + c_{bb}h\right)} \right) \right] \quad \text{----- 2.126}$$

$$\Delta = \frac{Wh^3}{3EI} \cdot A \quad \text{----- 2.127}$$

In Equation 2.127, $Wh^3/3EI$ is the well known primary deflection of a simple cantilever, and the coefficient A includes a reduction factor for frame action and a magnification coefficient for the instability effect.

Putting $P = 0$ in Equation 2.126 should result in the equation for primary analysis, i.e. when $P = 0$, $c_{bb} = 1$, Δ should revert to the primary deflection which, it can be shown, is equal to

$$\frac{Wh^3}{3EI} \left(1 + \frac{L}{2h} \right) \cdot \frac{1}{2}$$

From equation 2.126,

$$\frac{\left(\frac{L}{2} + c_{bb}h \right) 3EI}{2h \left[3EI - Ph \left(\frac{L}{2} + c_{bb}h \right) \right]} = \frac{\frac{L}{2} + c_{bb}h}{2h \left[1 - \frac{Ph}{3EI} \left(\frac{L}{2} + c_{bb}h \right) \right]}$$

and for $P = 0$ and $c_{bb} = 1$ expression becomes:

$$\frac{\frac{L/2 + h}{2h [1]}} = \frac{\frac{L}{4h} + \frac{1}{2}} = \left(\frac{L}{2h} + 1 \right) \frac{1}{2}$$

This is the same as reduction coefficient for primary effect only, therefore the equations for primary and instability deflections are consistent.

In summary:

$$\Delta = \frac{Wh^3}{3EI} A = \frac{Wh^3}{3EI} \left[\frac{\frac{L}{2} + c_{bb}h}{2h \left[1 - \frac{Ph}{3EI} \left(\frac{L}{2} + c_{bb}h \right) \right]} \right] \quad \text{----- 2.128}$$

For a particular frame configuration of $L = 374\text{mm}$, $h = 238\text{mm}$ and $I = 135.5\text{mm}^4$ (which correspond to the properties of an experimental model) it is possible for different values of P to calculate the coefficient A which can then be compared later with experimental results.

$$2U = L \sqrt{\frac{P}{EI}} = \frac{374}{\sqrt{200\,000 \times 135.5}} \sqrt{P}$$

$$2U = 0.07185 \sqrt{P}$$

Substituting values in Equation 1.128 gives:

$$\frac{\frac{L}{2} + c_{bb}h}{2h \left[1 - \frac{Ph}{3EI} \left(\frac{L}{2} + c_{bb}h \right) \right]} = \frac{187 + 238c_{bb}}{476 - 0.00139P(187 + 238c_{bb})}$$

Values of A for various values of axial load are tabulated in Fig. 2.122.

P (N)	2U = 0.0718 $\sqrt{P}$	c_{bb}	A
0	0	1.00	0.0928
100	.7185	1.0358	1.0428
200	1.0161	1.0776	1.2572
300	1.2445	1.1214	1.5828
400	1.4370	1.1722	2.147
500	1.6066	1.2270	3.3496
600	1.7600	1.2957	7.882
700	1.9098	1.3704	-22.3

Fig. 2.122 Tabulation of A for values of P.

Magnification factor A plotted against P is shown in Fig. 2.123 and in greater detail, for P less than 100N, in Fig. 2.124.

Similarly a magnification factor can be obtained for the bending moment.

From Equation 2.124

$$M_b = \frac{\Delta 3EI}{h\left(\frac{L}{2} + c_{bb}h\right)} = \frac{3EI}{h\left(\frac{L}{2} + c_{bb}h\right)} \cdot \frac{Wh^3}{3EI} \left[\frac{\frac{L}{2} + c_{bb}h}{2h\left[1 - \frac{Ph}{3EI}\left(\frac{L}{2} + c_{bb}h\right)\right]} \right]$$

Simplifying gives

$$M_b = \frac{Wh}{2} \left[\frac{1}{1 - \frac{Ph}{3EI}\left(\frac{L}{2} + c_{bb}h\right)} \right] = \frac{Wh}{2} \cdot B \quad \text{----- 2.129}$$

where B = magnification factor for extra bending moment due to instability effect, i.e.

$$B = \frac{M_b \text{ instability}}{M_b \text{ primary}}$$

For the particular geometrical configuration of the experimental model, Equation 2.129 can be transposed to give:

$$B = \frac{1}{1 - .00139P(187 + 238 c_{bb})}$$

Variation of B with axial load is shown tabulated in Fig. 1.125 and graphically in Figs. 1.123 and 1.124.

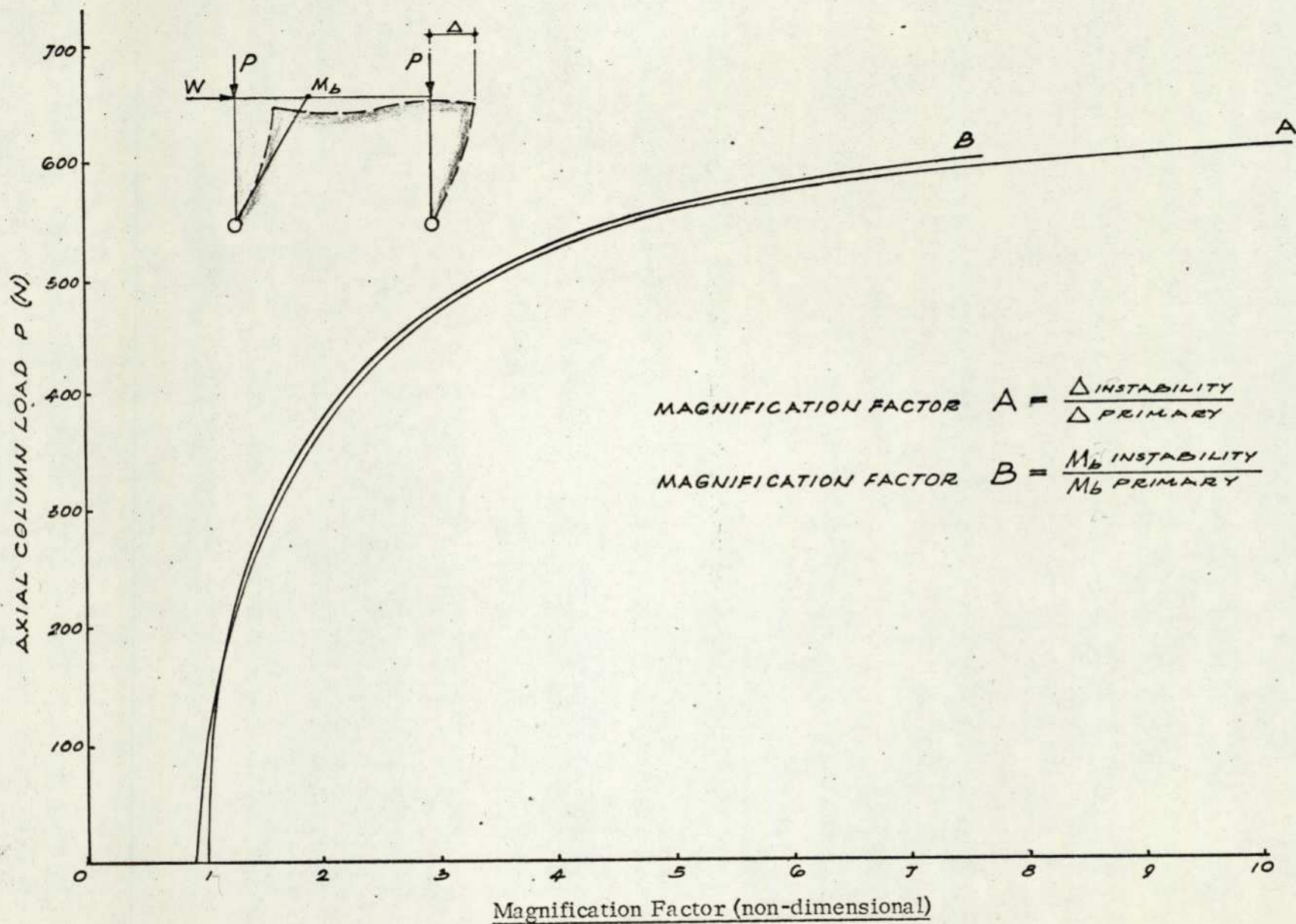


Fig. 2.123 Magnification Factors for Deflection and Bending Moment of a Hinged Portal Frame.

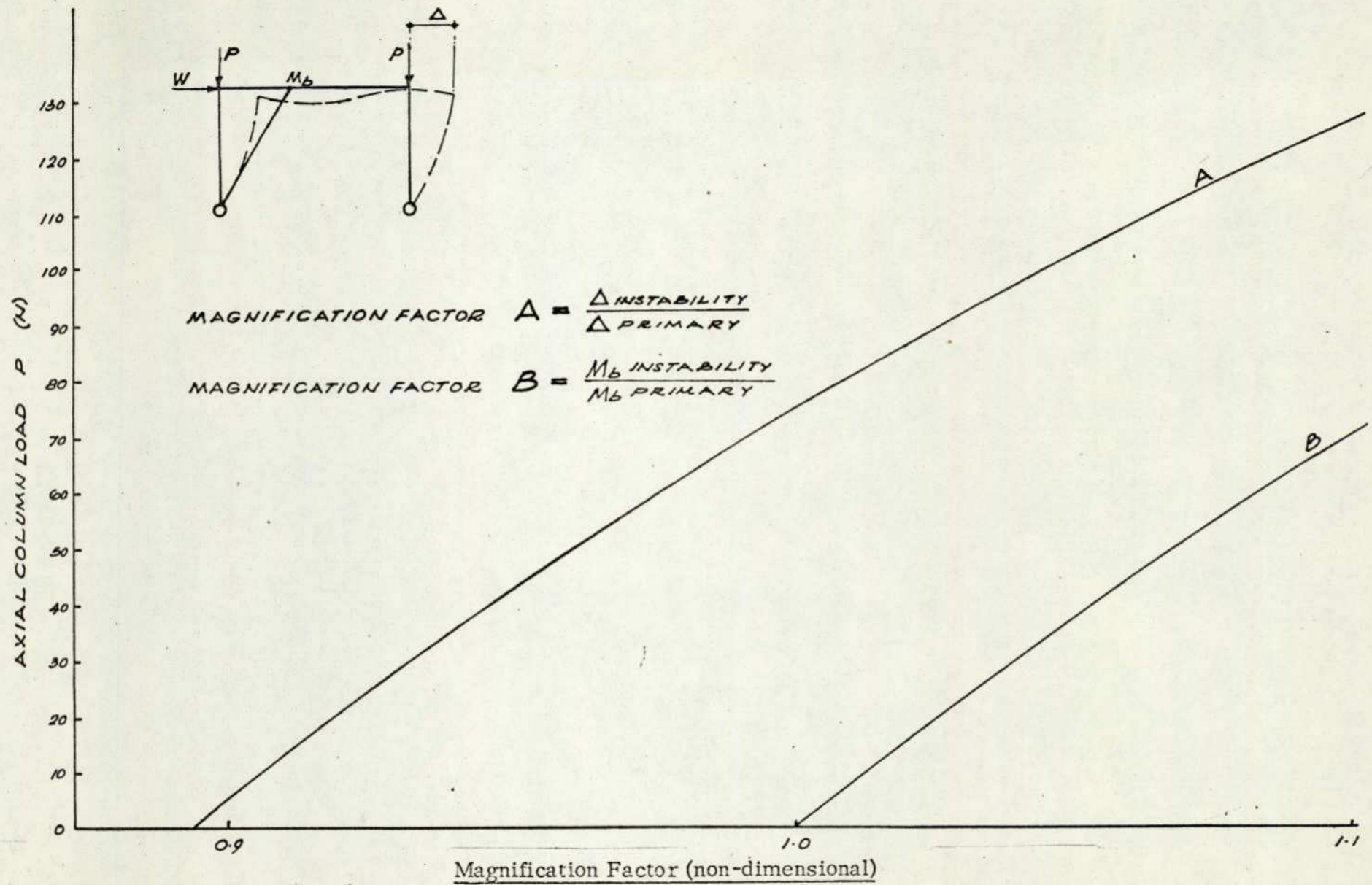


Fig. 2.124 Magnification Factors for a Hinged-Base Portal Frame for Small Axial Column Loads.

P (N)	$2U = .0718\sqrt{P}$	C_{bb}	B
0	0	1	1.0000
20	.3211	1.00709	1.02553
40	.4541	1.10420	1.05263
60	.5561	1.0214	1.08143
80	.6422	1.0290	1.11219
100	.718	1.0364	1.1448
200	1.0161	1.0776	1.3495
300	1.2445	1.1214	1.65837
400	1.4370	1.1722	2.19346
500	1.6066	1.2270	2.3255
600	1.7600	1.2957	7.5642

Fig. 2.125 Variation of Magnification Coefficient B with Axial Load.

In conclusion it has been shown that explicit solutions for deflection and bending moment equations for a hinged-ends portal frame with identical beam and column sections, can be expressed in the following way:

$$\begin{aligned}
 M_b &= \frac{Wh}{2} && \text{----- Primary effect} \\
 M_b &= \frac{Wh}{2} \left[\frac{1}{1 - \frac{Ph}{3EI} \left(\frac{L}{2} + c_{bb}h \right)} \right] && \text{----- Instability effect} \\
 \Delta_b &= \frac{Wh^3}{6EI} \left(1 + \frac{L}{2h} \right) && \text{----- Primary Effect} \\
 \Delta_b &= \frac{Wh^3}{3EI} \left[\frac{\frac{L}{2} + c_{bb}h}{2h \left[1 - \frac{Ph}{3EI} \left(\frac{L}{2} + c_{bb}h \right) \right]} \right] && \text{----- Instability effect.}
 \end{aligned}$$

2.130 Analyses for a Fixed-ends Portal Frame

A single frame with fixed-ends is analysed for three loading conditions: lateral load alone; lateral load with one column axially loaded; lateral load with both columns axially loaded.

2.131 Primary Analysis:

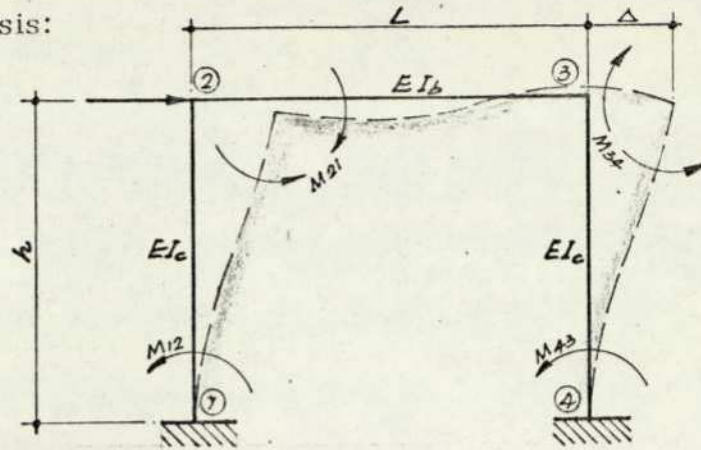


Fig. 2.131 Fixed-ends Portal Frame used in Primary Analysis.

For a symmetrical frame $I_c = I_b = I$, $M_{12} = M_{43}$ and $M_{21} = M_{34}$

Using the force-displacement method for the frame of Fig. 2.131 the compatibility of deformation at the point 1 can be written as:

$$\frac{M_{12}h}{3EI} - \frac{M_{21}h}{6EI} = \frac{\Delta}{h}$$

$$\text{or } M_{12} - \frac{M_{21}}{2} - \Delta \cdot \frac{3EI}{h^2} = 0$$

Similarly, the equation for compatibility of deformation at point 2

becomes:

$$-\frac{M_{12}h}{6EI} + M_{21}\left(\frac{h}{3EI} + \frac{L}{3EI} - \frac{L}{6EI}\right) = \frac{\Delta}{h}$$

$$\text{or } -\frac{M_{12}}{2} + M_{21}\left(1 + \frac{L}{2h}\right) - \Delta \cdot \frac{3EI}{h^2} = 0$$

For equilibrium of forces we can write:

$$\frac{W}{2} = \frac{(M_{12} + M_{21})}{h} \quad \text{or } M_{12} + M_{21} = \frac{Wh}{2}$$

These can be presented in a standard matrix form suitable for computing:

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{3EI}{h^2} \\ -\frac{1}{2} & 1 + \frac{L}{2h} & -\frac{3EI}{h^2} \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} M_{12} \\ M_{21} \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{Wh}{2} \end{bmatrix}$$

In the experimental model the following data were used:

$$I_c = I_b = 135.5 \text{ mm}^4 : L = 374 \text{ mm} : h = 256 \text{ mm} : E = 200,000 \text{ N/mm}^2.$$

Matrix becomes, (for $W = 1N$) :

Condition	Point 1	Point 2		Force Vector
	M_{12}	M_{21}	Δ	
1	1	-0.5	-1240.5	0
2	-0.5	1.7304	-1240.5	0
Shear	1	1	0	128

Computer solution gives:

$$M_{12} = 76.5309 \text{ N.mm}$$

$$M_{21} = 51.469 \text{ N.mm}$$

$$\Delta = 0.04095 \text{ mm}$$

2.132 Instability analysis, one Column Loaded:

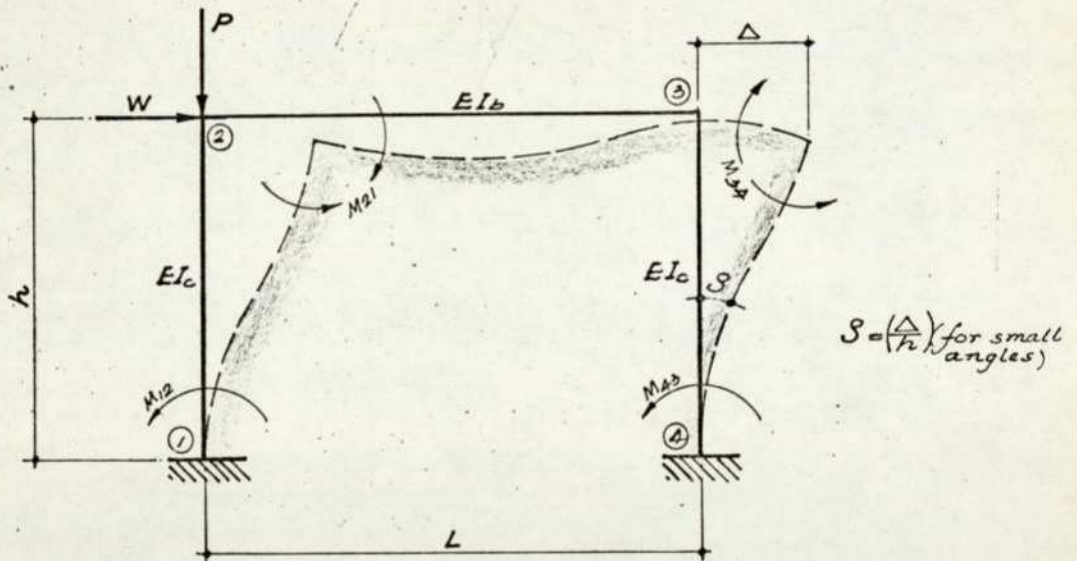


Fig. 2.132 Fixed-ends Portal Frame, one Column Axially Loaded.

Using the force-displacement method (Fig. 2.132) :

Compatibility of deformation at point 1:

$$\frac{M_{12}hC_{11}}{3EI_c} - \frac{M_{21}hC_{21}}{6EI_c} = \frac{\Delta}{h}$$

Multiplying by $6EI_c/h$

$$M_{12}2C_{11} - M_{21}C_{21} - \Delta \frac{6EI_c}{h^2} = 0$$

Similarly at point 2:

$$-\frac{M_{12}hC_{12}}{6EI_c} + M_{21}\left(\frac{hC_{22}}{3EI_c} + \frac{L}{3EI_b}\right) - \frac{M_{34}L}{6EI_b} = \frac{\Delta}{h}$$

$$-M_{12}C_{12} + M_{21}(2C_{22} + 2\eta) - M_{34}\eta - \Delta \frac{6EI_c}{h^2} = 0$$

where $n = \frac{L I_c}{h I_b}$

Similarly at point 3:

$$-\frac{M_{21}L}{6EI_b} + M_{34}\left(\frac{L}{3EI_b} + \frac{h}{3EI_c}\right) - \frac{M_{43}h}{6EI_c} = \frac{\Delta}{h}$$

$$-M_{21}n + M_{34}(2n+2) - M_{43} - \Delta \cdot \frac{6EI_c}{h^2} = 0$$

Again at point 4:

$$\frac{M_{43}h}{3EI_c} - \frac{M_{34}h}{6EI_c} = \frac{\Delta}{h}$$

$$2M_{43} - M_{34} - \Delta \cdot \frac{6EI_c}{h^2} = 0$$

Equilibrium condition:

$$W + P \frac{\Delta}{h} = \frac{M_{12} + M_{21}}{h} + \frac{M_{34} + M_{43}}{h}$$

$$M_{12} + M_{21} + M_{34} + M_{43} - \Delta P = Wh$$

Presented in matrix form, these equations become:

$2C_{11}$	$-C_{21}$	0	0	$-\frac{6EI_c}{h^2}$	M_{12}	=	0
$-C_{12}$	$2(C_{22}+n)$	$-n$	0	$-\frac{6EI_c}{h^2}$	M_{21}		0
0	$-n$	$2(n+1)$	-1	$-\frac{6EI_c}{h^2}$	M_{34}		0
0	0	-1	2	$-\frac{6EI_c}{h^2}$	M_{43}		0
1	1	1	1	$-P$	Δ		Wh

For the model configuration, the following values apply:

$$I_c = I_b = 135.5 \text{ mm}^4 ; h = 256 \text{ mm} ; L = 374 \text{ mm}$$

$$n = \frac{L}{h} = \frac{374}{256} = 1.461 ; E = 200 \text{ kN/mm}^2$$

For $P = 100\text{N}$ (corresponding to a test condition), $2U = L \sqrt{\frac{P}{EI}}$,

$$2U = 0.7185 \text{ for which } C_{11} = 1.03646 \text{ and } C_{12} = 1.06396$$

Matrix becomes (for $W = 1\text{N}$) :

2.07292	-1.0639	0	0	-2481.1	0
-1.06396	4.994	-1.461	0	-2481.1	0
0	-1.461	4.9218	-1	-2481.1	0
0	0	-1	2	-2481.1	0
1	1	1	1	-100	256

Computer solution is:

$$M_{12} = 76.990 \text{ W N.mm} \quad M_{34} = 52.5358 \text{ W N.mm}$$

$$M_{21} = 52.536 \text{ W N.mm} \quad M_{43} = 78.116 \text{ W N.mm}$$

$$\Delta = 0.04179 \text{ W N.mm}$$

2.133 Instability Analysis, Both Columns Loaded:

The portal frame shown in Fig. 2.133 is analysed using the force-displacement method.

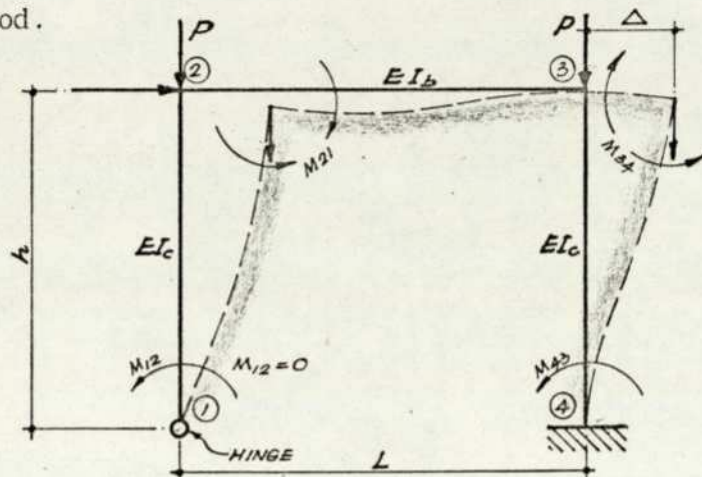


Fig. 2.133 Portal Frame with Fixed-Ends.

Compatibility at deformation at 1:

$$-M_{21} \frac{c_{21}h}{6EI_c} + M_{12} \frac{c_{11}h}{3EI_c} = \frac{\Delta}{h} ; \text{ multiplying by } \left(\frac{6EI_c}{h} \right)$$

$$-M_{21}c_{21} + M_{12}2c_{11} - \Delta \frac{6EI_c}{h^2} = 0 \quad \text{----- (1)}$$

Compatibility of deformation at 2 Let $n = \left(\frac{LI_c}{hI_b} \right)$

$$-M_{12} \frac{c_{12}h}{6EI_c} + M_{21} \frac{c_{22}h}{3EI_c} + \frac{L}{3EI_b} - M_{34} \frac{L}{6EI_b} = \frac{\Delta}{h}$$

$$-M_{12}c_{12} + M_{21}(2c_{22} + 2n) - M_{34}n - \Delta \frac{6EI_c}{h^2} = 0 \quad \text{----- (2)}$$

Compatibility of deformation at 3:

$$-M_{43} \frac{c_{43}h}{6EI_c} + M_{34} \frac{c_{33}h}{3EI_c} + \frac{L}{3EI_b} - M_{21} \frac{L}{6EI_b} = \frac{\Delta}{h}$$

$$-M_{43}c_{43} + M_{34}(2c_{33} + 2n) - M_{21}n - \Delta \frac{6EI_c}{h^2} = 0 \quad \text{----- (3)}$$

Similarly at 4:

$$-M_{34} \frac{c_{43}h}{6EI_c} + M_{43} \frac{c_{44}h}{3EI_c} = \frac{\Delta}{h}$$

$$-M_{34}C_{34} + M_{43}2C_{44} - \Delta \frac{6EI_c}{h^2} = 0 \quad \text{----- (4)}$$

Equilibrium of forces 2-3.

$$\frac{M_{12} + M_{21}}{h} + \frac{M_{34} + M_{43}}{h} = W + \frac{2P\Delta}{h}$$

$$M_{12} + M_{21} + M_{34} + M_{43} - \Delta 2P = Wh \quad \text{----- (5)}$$

General matrix for these conditions is given below:

$2C_{11}$	$-C_{21}$	0	0	$-\frac{6EI_c}{h^2}$	M_{12}	0
$-C_{12}$	$2(C_{22}+n)$	$-n$	0	$-\frac{6EI_c}{h^2}$	M_{21}	0
0	$-n$	$2(C_{33}+n)$	$-C_{43}$	$-\frac{6EI_c}{h^2}$	M_{34}	0
0	0	$-C_{34}$	$2C_{44}$	$-\frac{6EI_c}{h^2}$	M_{43}	0
1	1	1	1	$-2P$	Δ	Wh

For the steel model the following data apply:

$$I_c = I_b = 135.5 \text{ mm}^4 ; L = 374 \text{ mm} ; h = 256 \text{ mm}$$

$$E = 200,000 \text{ N/mm}^2 : n = \frac{374}{256} = 1.461$$

$$\frac{6EI_c}{h^2} = \frac{6 \times 200,000 \times 135.5}{256^2} = 2481.1$$

For P = 100N, corresponding to a load test condition:

$$2U = L \sqrt{\frac{P}{EI}} ; 2U = 0.7185 \text{ for which } C_{11} = 1.03646, \text{ and}$$

$$C_{12} = 1.06396.$$

Matrix becomes (for W = 1N) :

2.07292	-1.06396	0	0	-2481.1	M_{12}	0
-1.06396	4.995	-1.461	0	-2481.1	M_{21}	0
0	-1.461	4.995	-1.06396	-2481.1	M_{34}	0
0	0	-1.06396	2.07292	-2481.1	M_{43}	0
1	1	1	1	-200	Δ	256

Computer solutions are:

$$M_{12} = 78.627 \text{ W (N.mm)}$$

$$M_{43} = 78.627 \text{ W (N.mm)}$$

$$M_{21} = 53.642 \text{ W (N.mm)}$$

$$\Delta = 0.042688 \text{ w (N.mm)}$$

$$M_{34} = 53.642 \text{ W (N.mm)}$$

2.14 Analysis for a Portal Frame with One Column Fixed and the Other Hinged.

A simple portal frame with one foot fixed and the other hinged is analysed for primary effects (lateral load alone) and instability effects (lateral load and axial load on both columns).

2.141 Primary Analysis:

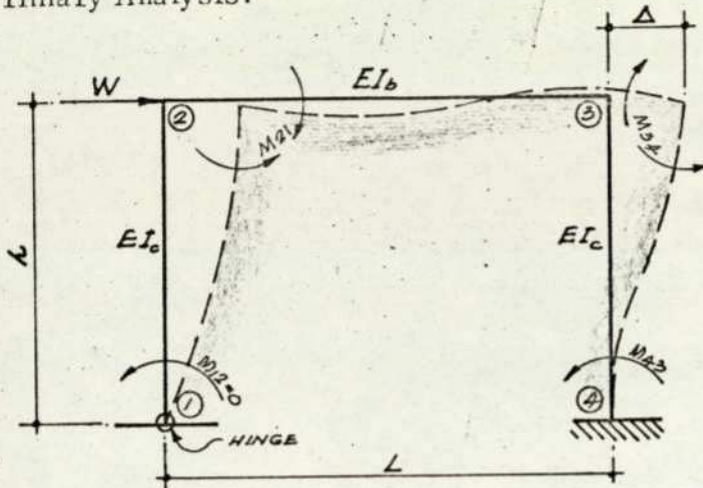


Fig. 2.141 Frame Analysed for Primary Effects.

Using the force-displacement method (Fig. 2.141)

Compatibility of deformation at 2:

$$M_{21} \left(\frac{h}{3EI_c} + \frac{L}{3EI_b} \right) - \frac{M_{34}L}{6EI_b} = \frac{\Delta}{h} \quad ; \quad \text{Let } n = \frac{I_c}{I_b}$$

Multiplying by $\frac{3EI_c}{h}$; $M_{21} \left(1 + \frac{Ln}{h} \right) - M_{34} \frac{Ln}{2h} - \Delta \frac{3EI_c}{h^2} = 0$

Similarly at point 3:

$$-\frac{M_{21}L}{6EI_b} + M_{34} \left(\frac{L}{3EI_b} + \frac{h}{3EI_c} \right) - \frac{M_{43}h}{6EI_c} = \frac{\Delta}{h}$$

or

$$-\frac{M_{21}Ln}{2h} + M_{34} \left(\frac{Ln}{h} + 1 \right) - \frac{M_{43}}{2} - \Delta \frac{3EI_c}{h^2} = 0$$

Similarly at point 4:

$$\frac{M_{43}h}{3EI_c} - \frac{M_{34}h}{6EI_c} = \frac{\Delta}{h}$$

or $M_{43} - 0.5M_{34} - \Delta \frac{3EI_c}{h^2} = 0$

For equilibrium of forces:

$$W = \frac{M_{21}}{h} + \frac{M_{34} + M_{43}}{h}$$

$$Wh = M_{21} + M_{34} + M_{43}$$

Presented in Matrix form:

$$\begin{bmatrix} 1 + \frac{Ln}{h} & -\frac{3EI_c}{h^2} & -\frac{Ln}{2h} & 0 \\ -\frac{Ln}{2h} & -\frac{3EI_c}{h^2} & 1 + \frac{Ln}{h} & -\frac{1}{2} \\ 0 & -\frac{3EI_c}{h^2} & -\frac{1}{2} & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} M_{21} \\ \Delta \\ M_{34} \\ M_{43} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Wh \end{bmatrix}$$

Substituting numerical values for experimental configuration:

$$I_c = I_b = 135.5 \text{ mm}^4 : n = 1 : h = 228.5 \text{ mm} :$$

$$L = 304 \text{ mm} ; E = 200,000 \text{ N/mm}^2$$

Matrix becomes for $W = 1 \text{ N}$:

Condition	Point 2		Point 3	Point 4	Force Vector
	M_{21}	Δ	M_{34}	M_{43}	
2	2.3304	-1557.1	-0.6652	0	0
3	-0.6652	-1557.1	2.3304	-0.50	0
4	0	-1557.1	-0.50	1	0
Shear	1	0	1	1	228.5

Computer solution gives:

$$M_{21} = 51.2469 \text{ W (N.mm)}$$

$$M_{34} = 69.2705 \text{ W (N.mm)}$$

$$M_{43} = 107.9825 \text{ W (N.mm)}$$

$$\Delta = 0.04710 \text{ W (N.mm)}$$

2.142 Instability Analysis:

The simple portal frame (Fig. 2.142), subjected to loads simulating lateral and gravity loads, is analysed for instability effects using the force-displacement method.

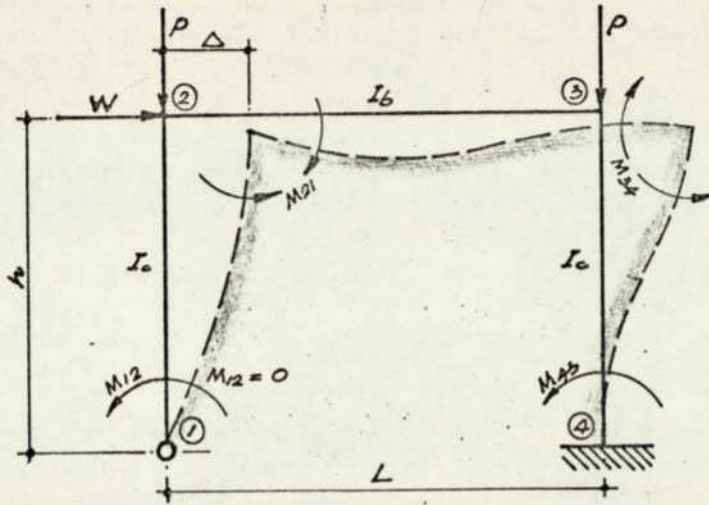


Fig. 2.142 Frame with Fixed and Hinged-Ends,
Analysed for Instability Effects.

Compatibility of deformation at 2:

$$M_{21} \left(\frac{C_{22}h}{3EI_c} + \frac{L}{3EI_b} \right) - M_{34} \frac{L}{6EI_b} = \frac{\Delta}{h}; \text{ Let } n = \frac{L I_c}{h I_b}$$

$$M_{21}(2C_{22} + 2n) - M_{34}n - \Delta \frac{6EI_c}{h^2} = 0 \quad \text{----- (2)}$$

Compatibility of deformation at 3:

$$-M_{21} \frac{L}{6EI_b} + M_{34} \left(\frac{L}{3EI_b} + \frac{C_{33}h}{3EI_c} \right) - M_{43} \frac{h}{6EI_c} = \frac{\Delta}{h}$$

$$-M_{21}n + M_{34}(2n + 2C_{33}) - M_{43} - \Delta \frac{6EI_c}{h^2} = 0 \quad \text{----- (3)}$$

Compatibility of deformation at 4:

$$-M_{34} \frac{C_{34}h}{6EI_c} + M_{43} \frac{C_{44}h}{3EI_c} = \frac{\Delta}{h}$$

$$-M_{34}C_{34} + M_{43}2C_{44} - \Delta \frac{6EI_c}{h^2} = 0 \quad \text{----- (4)}$$

Equilibrium condition 2 - 3:

$$\frac{M_{21} + 0}{h} + \frac{M_{34} + M_{43}}{h} = W + 2P \frac{\Delta}{h}$$

$$M_{21} + M_{34} + M_{43} - \Delta \cdot 2P = Wh \quad \text{----- (5)}$$

Putting $I_c = I_b = I$, the following general matrix is obtained:

$$\begin{bmatrix} 2c_{11} + 2n & -n & 0 & -\frac{6EI_c}{h^2} \\ -n & 2c_{11} + 2n & -1 & -\frac{6EI_c}{h^2} \\ 0 & -c_{10} & 2c_{11} & -\frac{6EI_c}{h^2} \\ 1 & 1 & 1 & -2P \end{bmatrix} \begin{bmatrix} M_{21} \\ M_{34} \\ M_{43} \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Wh \end{bmatrix}$$

For $P = 60N$ (corresponding to a test condition),

$$2U = L \sqrt{\frac{P}{EI}} = 305 \sqrt{\frac{60}{200000 \times 135.5}} = 305 \times 0.001488$$

$$2U = 0.4538 \text{ for which } c_{11} = 1.0148$$

$$\text{and } c_{01} = 1.0248$$

Substituting test data for model:

$$I = 135.5 \text{ mm}^4 ; n = \frac{305 \times 135.5}{228.5 \times 135.5} = 1.3348$$

$$\frac{6EI_c}{h^2} = \frac{6 \times 200000 \times 135.5}{228.5^2} = 3114.2$$

Matrix becomes (for $W = 1$)

$$\begin{bmatrix} 4.6992 & -1.3348 & 0 & -3114.2 \\ -1.3348 & 4.6992 & -1 & -3114.2 \\ 0 & -1.0248 & 2.0296 & -3114.2 \\ 1 & 1 & 1 & -120 \end{bmatrix} \begin{bmatrix} M_{21} \\ M_{34} \\ M_{43} \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 228.5 \end{bmatrix}$$

Computer solution gives

$$M_{21} = 52.564 \text{ W (N.mm)}$$

$$M_{43} = 110.867 \text{ W (N.mm)}$$

$$M_{34} = 70.9377 \text{ W (N.mm)}$$

$$\Delta = 0.048911 \text{ W (mm)}$$

2.15 Analyses for Portal Frame with Double Hinges in One Column

2.151 Primary Analysis:

The portal frame, Fig. 2.151, is assumed to have double hinges in column 12. The frame is analysed for primary effects using the force-displacement method:

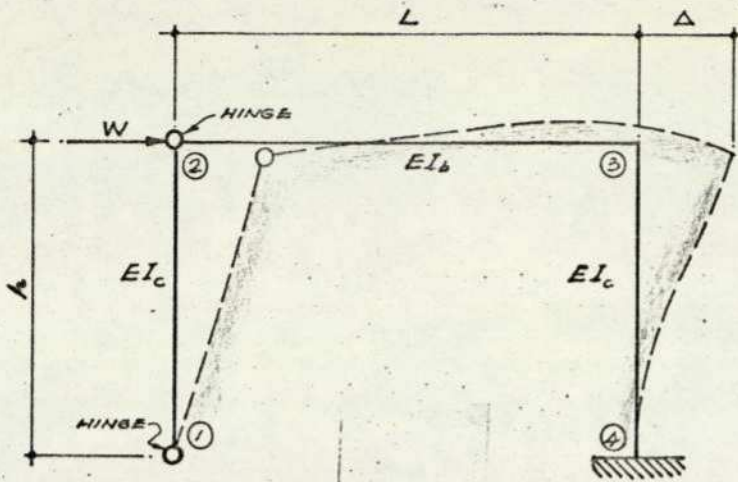


Fig. 2.151 Portal Frame with Double Hinges.

Compatibility of deformation at 3:

$$-M_{43} \frac{h}{6EI_c} + M_{34} \left(\frac{h}{3EI_c} + \frac{L}{3EI_b} \right) = \frac{\Delta}{h}$$

Multiplying by $\frac{6EI_c}{h}$ and putting $n = \frac{LI_c}{hI_b}$

$$-M_{43} + M_{34}(2 + 2n) - \Delta \frac{6EI_c}{h^2} = 0$$

Compatibility of deformation at 4:

$$-M_{34} \frac{h}{6EI_c} + M_{43} \frac{h}{3EI_c} = \frac{\Delta}{h}$$

$$-M_{34} + 2M_{43} - \Delta \frac{6EI_c}{h^2} = 0$$

Equilibrium of forces 2 - 3

$$\frac{M_{34} + M_{43}}{h} = W \quad ; \quad M_{34} + M_{43} = Wh$$

The general matrix for these conditions becomes:

$$\begin{bmatrix} 2+2n & -1 & -\frac{6EI_c}{h^2} \\ -1 & 2 & -\frac{6EI_c}{h^2} \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} M_{34} \\ M_{43} \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Wh \end{bmatrix}$$

In the experimental model the following data apply:

$$I_c = I_b = 135.5 \text{ mm}^4 ; L = 305 \text{ mm} ; h = 228.5 \text{ mm}$$

$$E = 200,000 \text{ N/mm}^2 ; n = \frac{305}{228.5} = 1.3348$$

$$\frac{6EI_c}{h^2} = \frac{6 \times 200,000 \times 135.5}{228.5^2} = 3114.21$$

Numerical matrix becomes (for $W = 1N$)

$$\begin{bmatrix} 4.6696 & -1 & -3114.21 \\ -1 & 2 & -3114.21 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} M_{34} \\ M_{43} \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 228.5 \end{bmatrix}$$

Computer solution gives:

$$M_{34} = 79.0694 \text{ W N.mm}$$

$$M_{43} = 149.430 \text{ W N.mm}$$

$$\Delta = 0.070577W \text{ mm}$$

2.152 Instability Analysis:

The portal frame, Fig. 2.152, subjected to lateral and gravity loads on each column is analysed for instability effects:

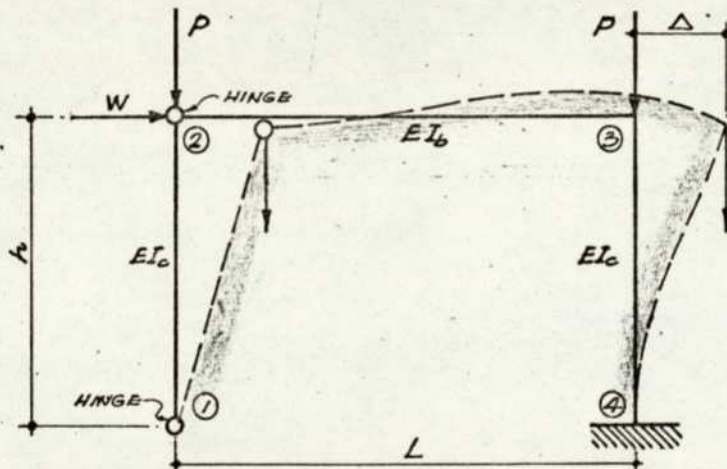


Fig. 2.152 Portal Frame with Double Hinges.

Compatibility of deformation at 4:

$$M_{43} \frac{c_{44}h}{3EI_c} - M_{34} \frac{c_{34}h}{6EI_c} = \frac{\Delta}{h} \quad \text{multiplying by } \frac{6EI_c}{h}$$

$$M_{43} 2c_{44} - M_{34} c_{34} - \Delta \frac{6EI_c}{h^2} = 0$$

Compatibility of deformation at 3:

$$-M_{43} \frac{c_{43}h}{6EI_c} + M_{34} \left(\frac{c_{33}h}{3EI_c} + \frac{L}{3EI_b} \right) = \frac{\Delta}{h} ; \quad n = \frac{LI_c}{hI_b}$$

$$-M_{43} c_{43} + M_{34} (2c_{33} + 2n) - \Delta \frac{6EI_c}{h^2} = 0$$

Equilibrium of forces 2-3

$$\frac{M_{34} + M_{43}}{h} = W + 2P \frac{\Delta}{h} ; \quad M_{34} + M_{43} - 2P\Delta = Wh$$

General matrix for these conditions becomes:

$$\begin{bmatrix} -C_{34} & 2C_{44} & -\frac{6EI_c}{h^2} \\ 2C_{33}+2n & -C_{43} & -\frac{6EI_c}{h^2} \\ 1 & 1 & -2P \end{bmatrix} \begin{bmatrix} M_{34} \\ M_{43} \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Wh \end{bmatrix}$$

For the experimental model the following data apply:

$$I_c = I_b = 135.5 \text{ mm}^4 ; L = 305 \text{ mm} ; h = 228.5 \text{ mm}$$

$$E = 200,000 \text{ N/mm}^2 ; n = \frac{305}{228.5} = 1.3348$$

$$\frac{6EI_c}{h^2} = \frac{6 \times 200000 \times 135.5}{228.5^2} = 3114.21$$

For $P = 60\text{N}$ (corresponding to a load test condition)

$$2U = L \sqrt{\frac{P}{EI}} = 305 \sqrt{\frac{60}{200000 \times 135.5}} = 0.4538$$

Interpolating from the table of flexibility coefficients gives:

$$C_{11} = 1.0148$$

$$C_{01} = 1.0248$$

Numerical matrix (for $W = 1\text{N}$):

$$\begin{bmatrix} -1.0248 & 2.0296 & -3114.21 \\ 4.6992 & -1.0248 & -3114.21 \\ 1 & 1 & -120 \end{bmatrix} \begin{bmatrix} M_{34} \\ M_{43} \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 228.5 \end{bmatrix}$$

Computer solution gives:

$$M_{34} = 82.5821 \text{ W (N.mm)}$$

$$M_{43} = 154.760 \text{ W (N.mm)}$$

$$\Delta = 0.07368 \text{ W (mm)}$$

2.16 Analyses of Portal Frame with Both Columns Hinged

The simple portal frame with both column feet hinged is analysed for primary and instability effects with both columns axially loaded.

2.161 Primary Analysis:

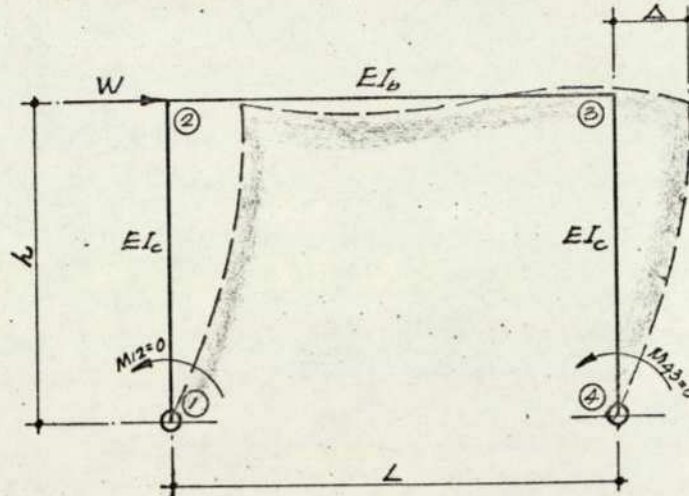


Fig. 2.161 Portal Frame with Hinged Column Feet.

Using the force-displacement method (Fig. 2.161), compatibility of deformation at 2 is:

$$M_{21} \left(\frac{h}{3EI_c} + \frac{L}{3EI_b} \right) - M_{34} \frac{L}{6EI_b} = \frac{\Delta}{h}$$

Multiplying by $6EI_c/h$, letting $n = \frac{LI_c}{hI_b}$ and putting $M_{34} = M_{21}$ as

symmetry exists:

$$M_{21} (2 + 2n) - M_{21} n - \Delta \frac{6EI_c}{h^2} = 0$$

$$M_{21} (2 + n) - \Delta \frac{6EI_c}{h^2} = 0$$

----- 2.1601

Equilibrium 1-2

$$\frac{M_{12} + M_{21}}{h} + \frac{M_{34} + M_{43}}{h} = W$$

As M_{12} and $M_{43} = 0$ and $M_{34} = M_{21}$

$$2M_{21} = Wh$$

$$M_{21} = \frac{Wh}{2}$$

----- 2.1602

Substituting Equation 2.1602 in Equation 2.1601 gives

$$\frac{Wh}{2} (2 + n) \cdot \frac{h^2}{6EI_c} = \Delta$$

----- 2.1603

Substituting values for experimental model: $L = 374\text{mm}$, $h = 238\text{mm}$,

$E = 200,000 \text{ N/mm}^2$, $I_c = I_b = 135.5\text{mm}^4$, gives

$$W \cdot \frac{238}{2} \left(2 + \frac{374}{238} \right) \cdot \frac{238^2}{6 \times 200,000 \times 135.5} = \Delta$$

$$\Delta = 0.1480 W$$

$$M_{2'} = 119 W$$

2.162 Instability Analysis:

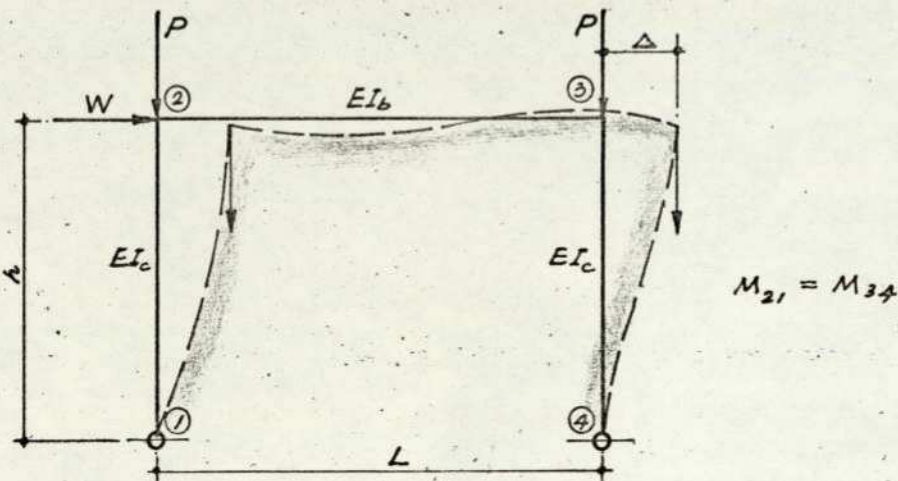


Fig. 2.162 Portal Frame with Hinged Column Feet

Using the force-displacement method, (Fig. 2.162), compatibility of deformation at 2 is:

$$M_{2'} \left(\frac{h c_{22}}{3EI_c} + \frac{L}{3EI_b} \right) - M_{2'} \frac{L}{3EI_b} = \frac{\Delta}{h} \quad \text{Let } n = \frac{LI_c}{hI_b}$$

$$M_{2'}(2c_{22} + n) = \Delta \cdot \frac{6EI_c}{h^2} \quad \text{----- 2.1621}$$

Equilibrium 1 - 2

$$\frac{2M_{2'}}{h} = W + 2P \cdot \frac{\Delta}{h}$$

$$M_{2'} = \frac{Wh}{2} + P\Delta \quad \text{----- 2.1622}$$

Substituting values for the steel model:

$$L = 374\text{mm}, h = 238 \text{ mm}, E = 200,000 \text{ N/mm}^2, I_c = I_b = 135.5 \text{ mm}^4$$

$$\text{For } P = 40\text{N}, 2U = h \sqrt{\frac{P}{EI}} = 238 \sqrt{\frac{40}{200000 \times 135.5}}$$

$2U = 0.2891$ and from table of flexibility coefficients, $C_{22} = 1.006$

$$M_{2'} (2.012 + 1.461) = \Delta \times 2870.5$$

$$M_{2'} = 119 + 50 \Delta \quad \text{for } W = 1$$

$$M_{2'} = 826.28 \Delta$$

$$826.28 \Delta = 119 + 50 \Delta \quad \text{from which}$$

$$\Delta = 0.1533 \text{ and } M_{2'} = 126.66$$

For $W = 1$

$$\Delta = 0.1533 W \text{ and } M_{2'} = 126.66 W.$$

2.17 Comparison of Theoretical Behaviour of Portal Frames Analysed

The lateral deflection and bending moment (M_{23}) data resulting from the computer-aided analyses 2.13 to 2.16 inclusive, have been plotted in Figs. 2.171 and 2.172 respectively. The key diagrams show load arrangement and frame structure with and without hinges.

For a given set of load conditions, the deflections and bending moments would be expected to increase as the number of structural hinges is increased, and this is demonstrated in Fig. 2.171 and Fig. 2.172. It is interesting that the effect upon deflections of introducing a hinge or pin joint in the structure is far greater than the effect of adding axial loads to columns in a given structure. The small instability effect is due to the low level of axial loading in relation to the critical column load - this can be seen from the flexibility coefficients which are near unity. The same pattern emerges for bending moments as can be seen from a comparison of Figs. 2.171 and 2.172.

Some of the curves are used for comparison with the experimental data in 2.20 and 2.30.

2.20 Experiments on Steel Portal Frames

In order to compare the experimental behaviour of portal frames, with and without structural hinges, for instability effects, a number of model portal frames were made from ordinary mild steel.

All members were 6.35 mm ($\frac{1}{4}$ inch) square and were incorporated in three similar frame sizes shown in Fig. 2.201. Dimensions are in millimetres.

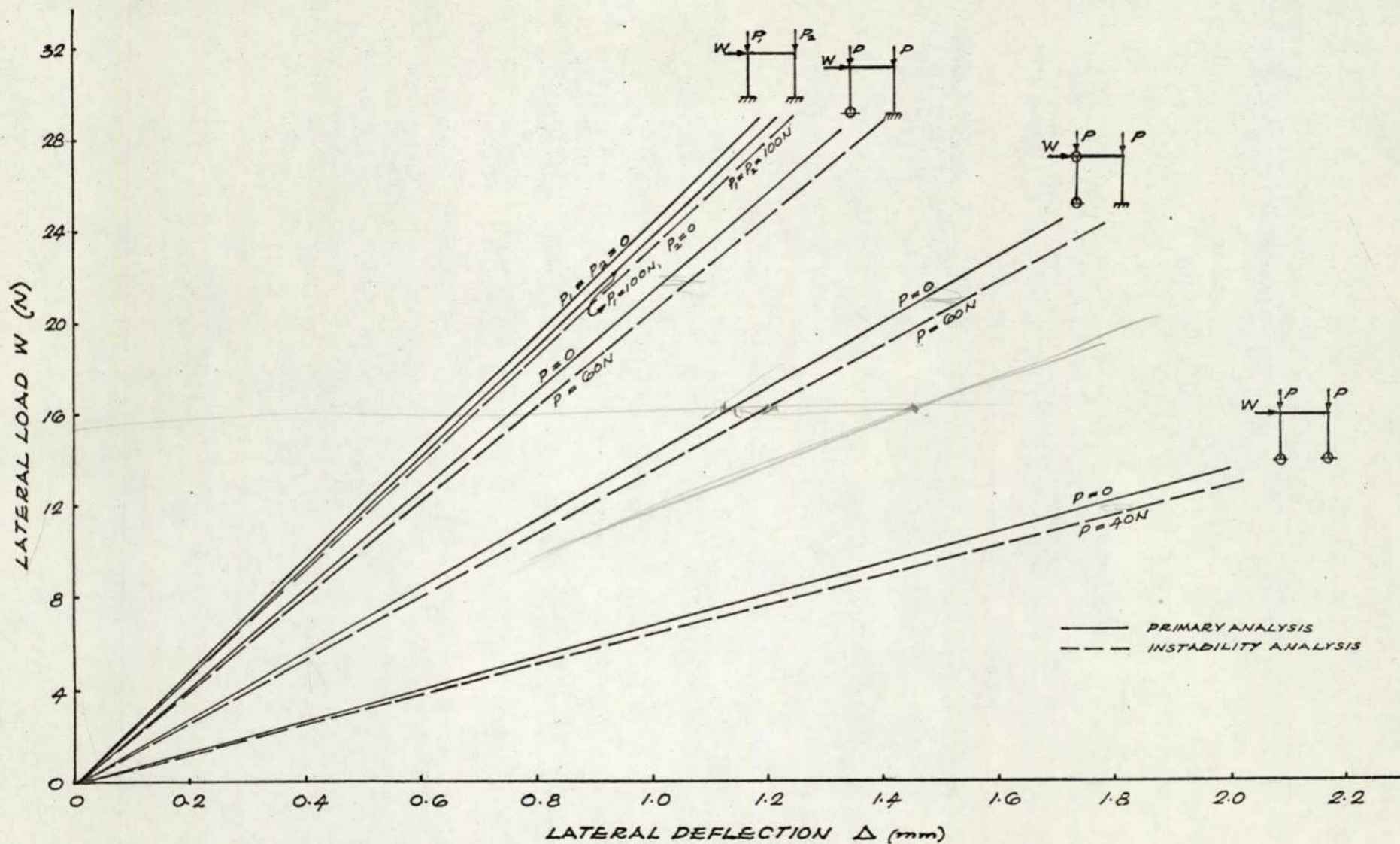


FIG. 2-171 THEORETICAL LOAD/DEFLECTION CURVES FOR PORTAL FRAMES WITH AND WITHOUT HINGES

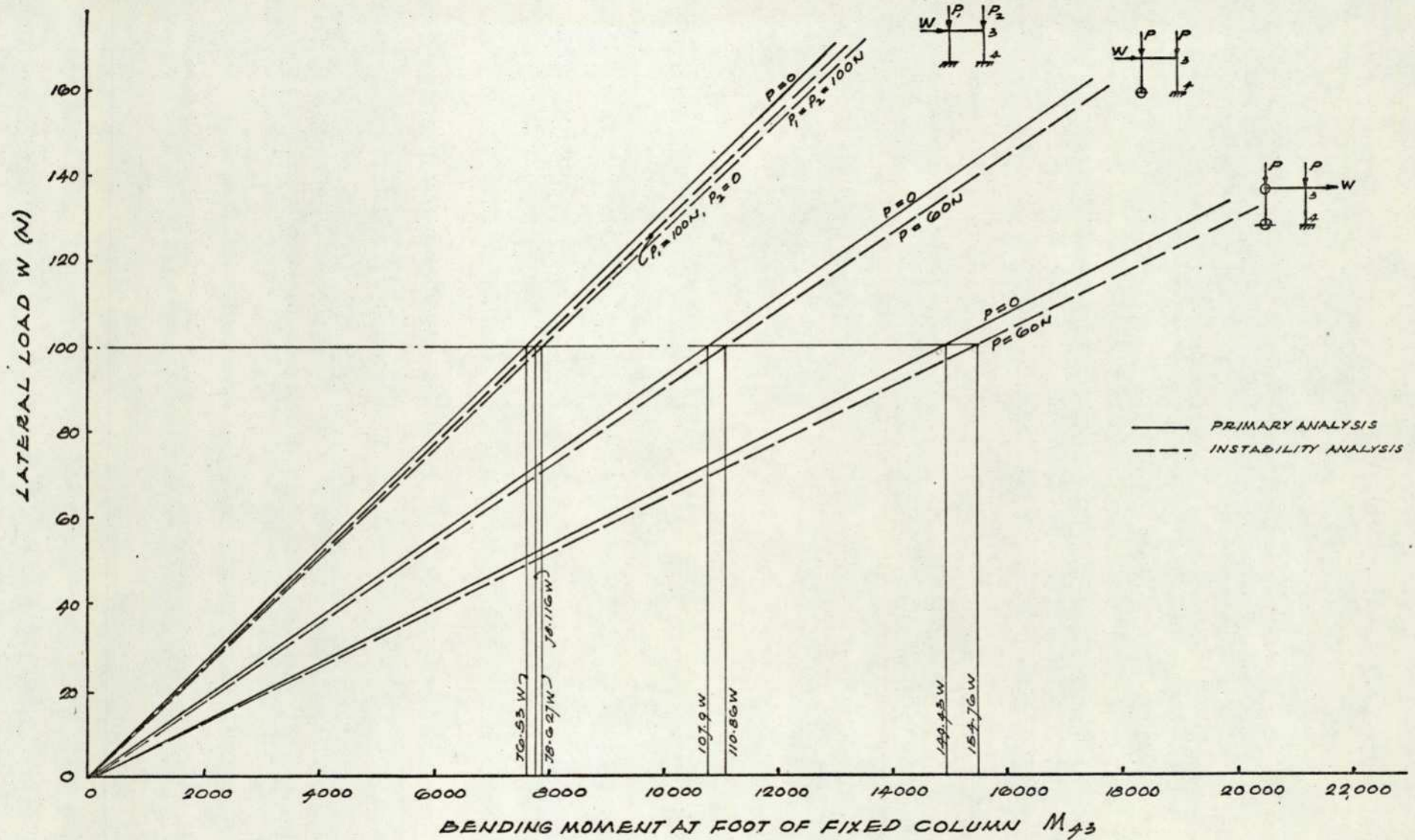


FIG. 2.172 THEORETICAL MOMENT (M_{43}) FOR PORTAL FRAMES WITH AND WITHOUT HINGES

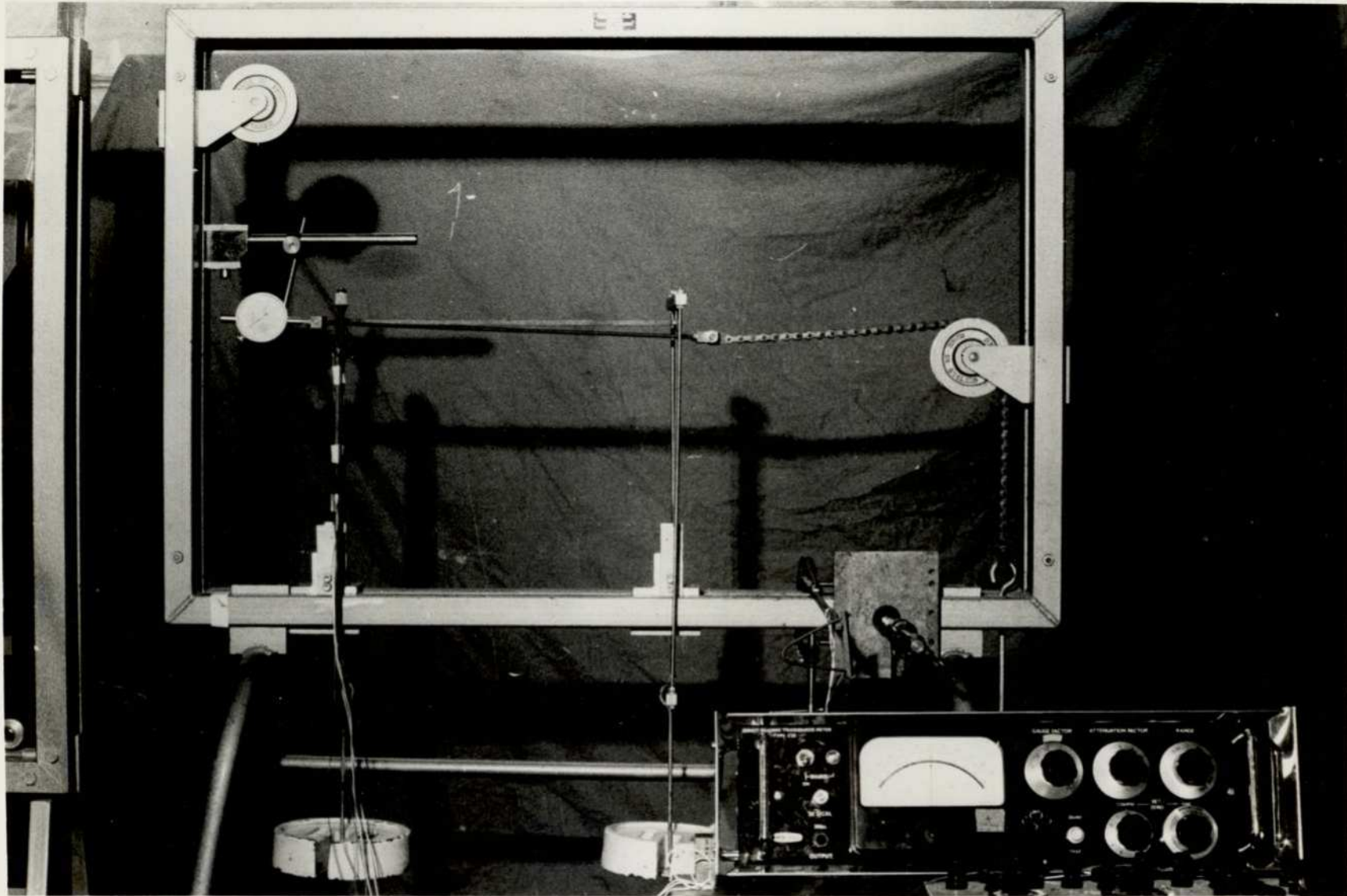


Fig. 2.202 Experimental Apparatus used for Testing Portal Frames

Substituting values gives:

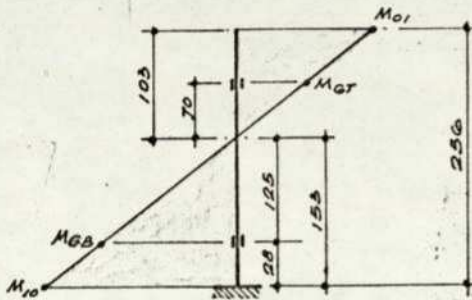
$$\Delta = \frac{W \times 256^3 \left(\frac{3 \times 256}{2} + 374 \right)}{6 \times 200000 \times 135.5 (6 \times 256 + 374)}$$

Theoretical $\Delta = 0.04095 \text{ W mm}$.

From the experimental data, the curve of $W \sim \Delta$ has been plotted in Fig. 2.212. Comparison with the theoretical relation of W & Δ also plotted gives excellent agreement with the experimental performance, i.e.

$$\begin{aligned} \Delta_{\text{theory}} &= 0.04095 \text{ W mm} \\ \Delta_{\text{experiment}} &= 0.0402 \text{ W mm} \end{aligned}$$

The bending moment diagram for one column is shown in Fig. 2.213.



From Equation 2.1106,

$$M_{01} = W \frac{3h^2}{2(6h + L)} = 51.468W$$

$$\text{and } M_{10} = \frac{1}{2}Wh - M_{01}$$

$$M_{10} = 76.532W.$$

Fig. 2.213 Bending Moment Diagram for Column 01.

By similar triangles, point of contraflexure occurs at 153mm from base (assuming linear moment)

Also by similar triangles:

$$M_{10} = M_{GB} \cdot \frac{153}{125} = 1.224 M_{GB}$$

$$M_{01} = M_{Gr} \cdot \frac{103}{70} = 1.4771 M_{Gr}$$

For $W = 36\text{N}$, gauges 1 and 4 read 15 divisions (from Fig. 2.214)

Therefore:

$$\sigma_{Gr} = 1.85 \times 15 = 27.75 \text{ N/mm}^2$$

$$M_{Gr} = \frac{\sigma I}{y} = \frac{27.75 \times 135.5}{3.175} \quad \text{therefore}$$

$$M_{01} = \frac{27.75 \times 135.5}{3.175} \times 1.4771 = 1742 \text{ N.mm}$$

And from theory, for $W = 36\text{N}$, $M_{01} = 51.468 \times 36 = 1852.8$ which, compared with experimental value, shows a difference of 6%.

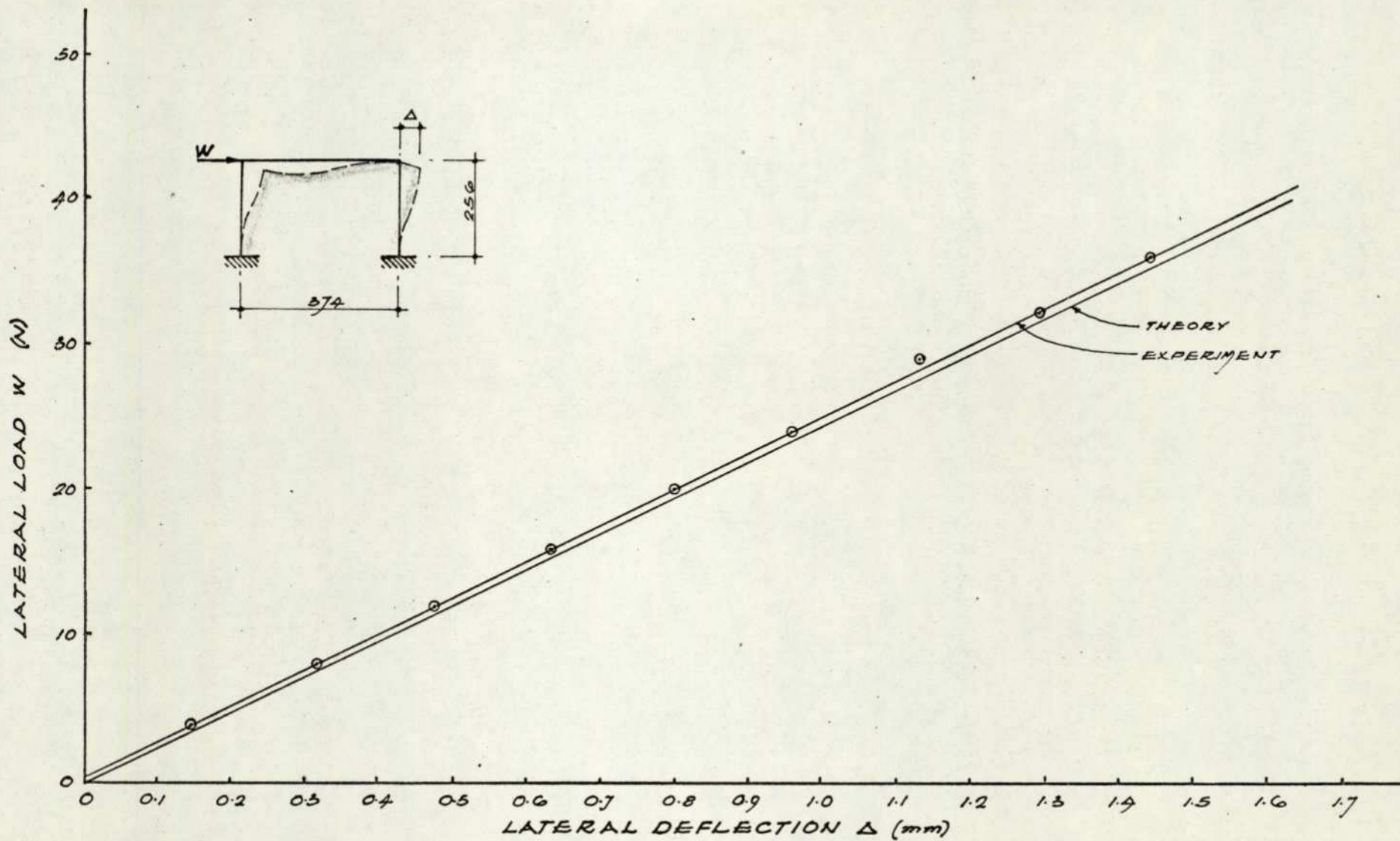


FIG. 2.212 PRIMARY LOAD DEFLECTION CURVE FOR FIXED-ENDS PORTAL FRAME

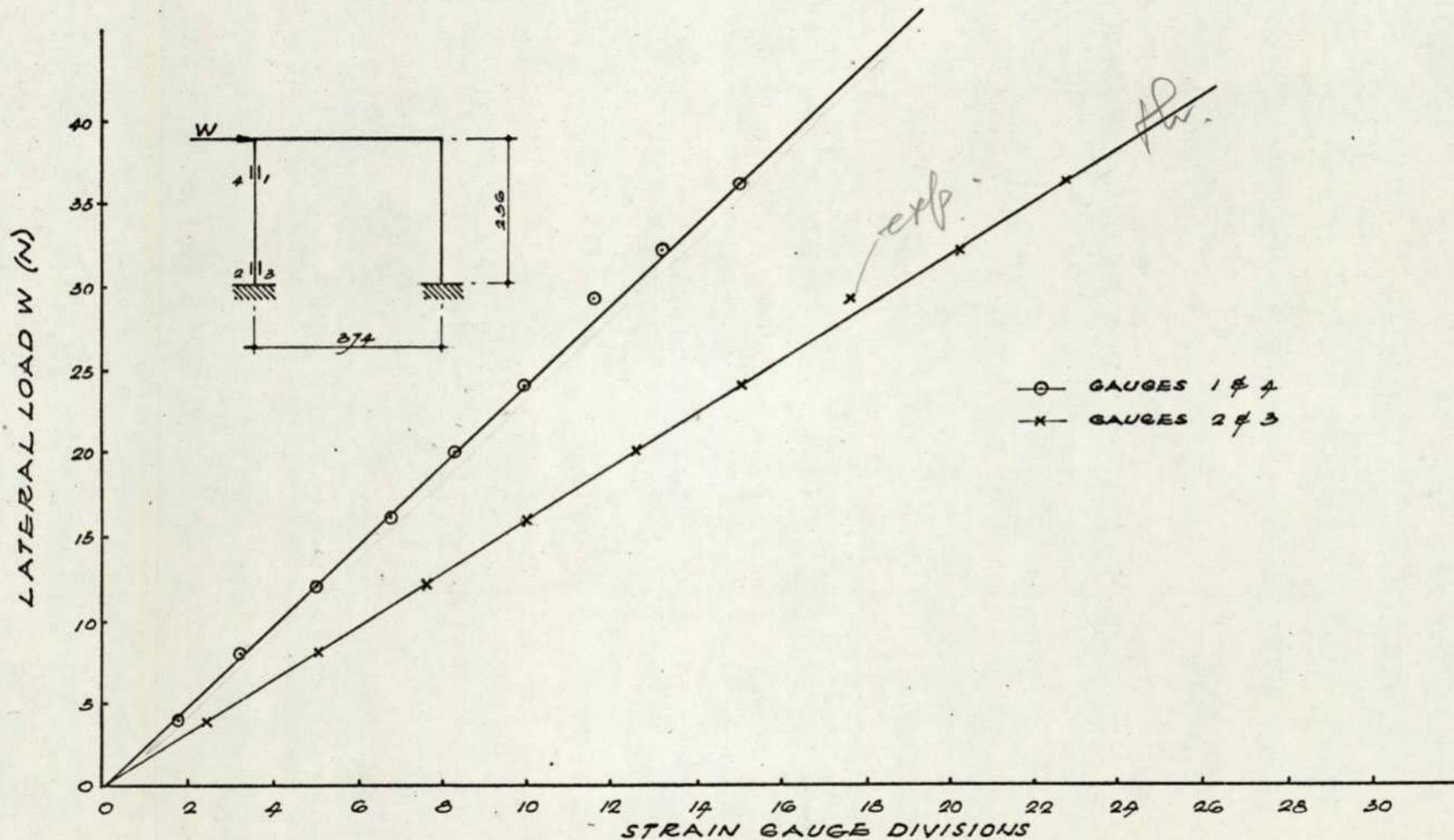


FIG. 2-214 RELATION BETWEEN STRAIN GAUGE READING AND LATERAL LOAD FOR A FIXED-ENDS PORTAL FRAME (PRIMARY EFFECT)

Similarly for $W = 36N$, gauges 2 and 3 read 22.87 divisions (Fig. 2.214)

$$\sigma_{GB} = 1.85 \times 22.87 = 42.318$$

$$M_{GB} = \frac{\sigma I}{y} = \frac{42.318 \times 135.5}{3.175}$$

from which $M_{10} = (M_{GB} \times 1.224) = 2202.4 \text{ N mm.}$

Whereas theoretical $M_{10} = 76.532 \times 36 = 2755.1 \text{ N.mm}$

which represents a difference of 20%

Secondary analysis:

From previous theory (Equation 2.1119) we have:

$\Delta = 0.1032 W.A.B.$ in which the values of A and B were tabulated in Fig. 2.115 for various values of P.

For the experimental conditions $P = 100N$ and $W = 220$, $A = 0.4192$ and $B = 0.9865$.

$$\therefore \Delta_{\text{theory}} = 0.1032 \times 220 \times 0.4192 \times 0.9865$$

$$\Delta_{\text{theory}} = 9.389 \text{ mm which compares well with}$$

$$\Delta_{\text{exp't}} = 9.950 \text{ mm obtained from Fig. 2.215.}$$

Also by comparing the primary deflection with secondary deflection it should be possible, even for small axial column loads, to discern a greater secondary deflection.

2.2.2 Experimental Deflection Data for Steel Portal Frames

The following curves give experimental deflections for frames of increasing flexibility caused by introduction of structural hinges.

Figs. 2.221 and 2.222 relate to primary and instability performance respectively for a portal frame with one column foot fixed and other hinged.

Fig. 2.223 shows the primary deflection performance for a portal frame with a hinge at top and bottom of one column, the other column having fixed-ends. The instability effect is shown in 2.224 for the same frame.

In each case the theoretical performance is plotted for comparison.

Figs. 2.225 to 2.229 are photographs of model portal frames undergoing the full range of tests.

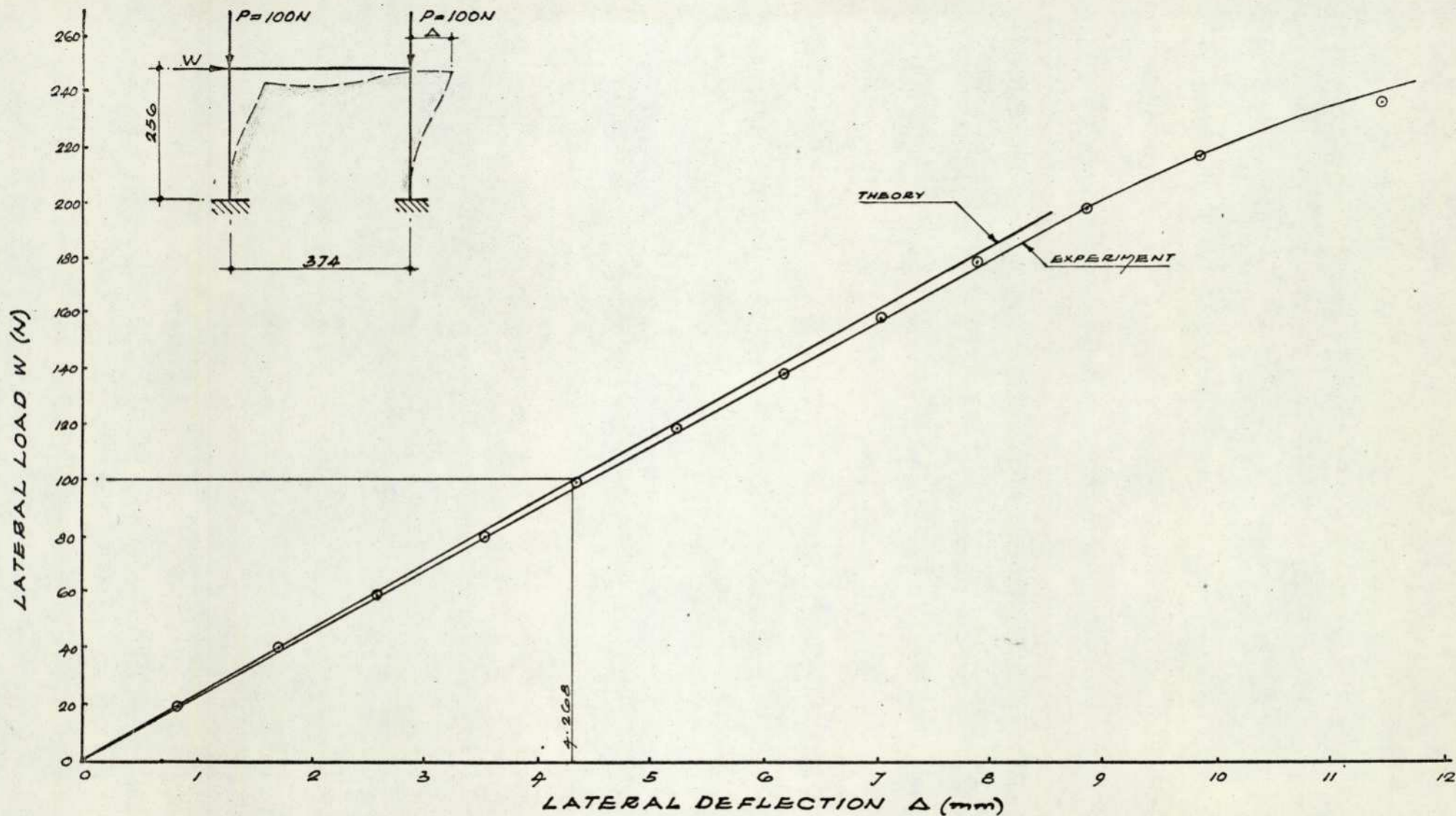


FIG. 2.215 LOAD DEFLECTION CURVES FOR FIXED-ENDS PORTAL FRAME SHOWING INSTABILITY EFFECT

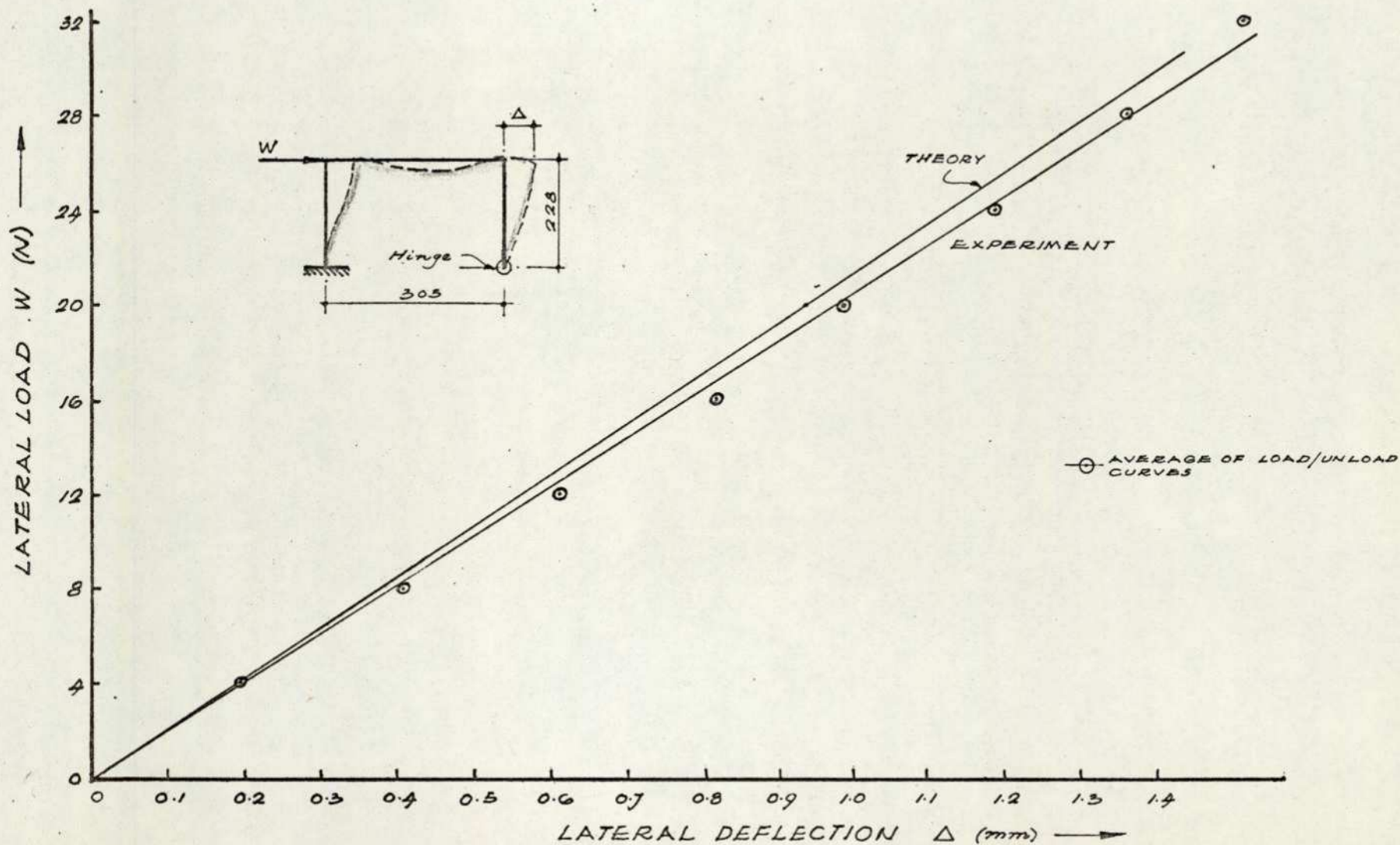


Fig. 2.221 LOAD DEFLECTION CURVES FOR PORTAL FRAME WITH FIXED & HINGED END - PRIMARY EFFECTS.

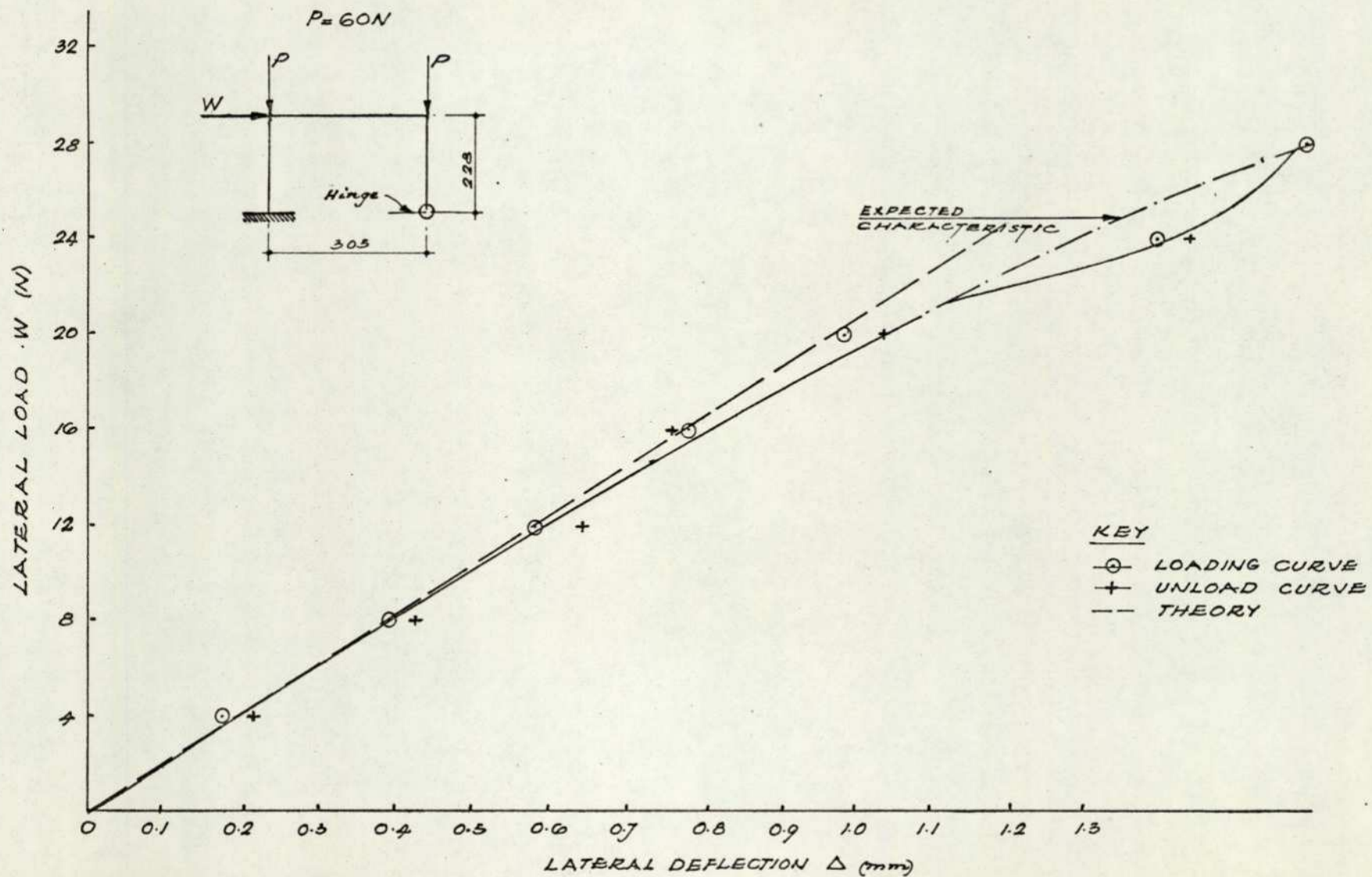


Fig. 2.222

LOAD DEFLECTION CURVE FOR PORTAL FRAME WITH HINGED AND FIXED ENDS-INSTABILITY EFFECT

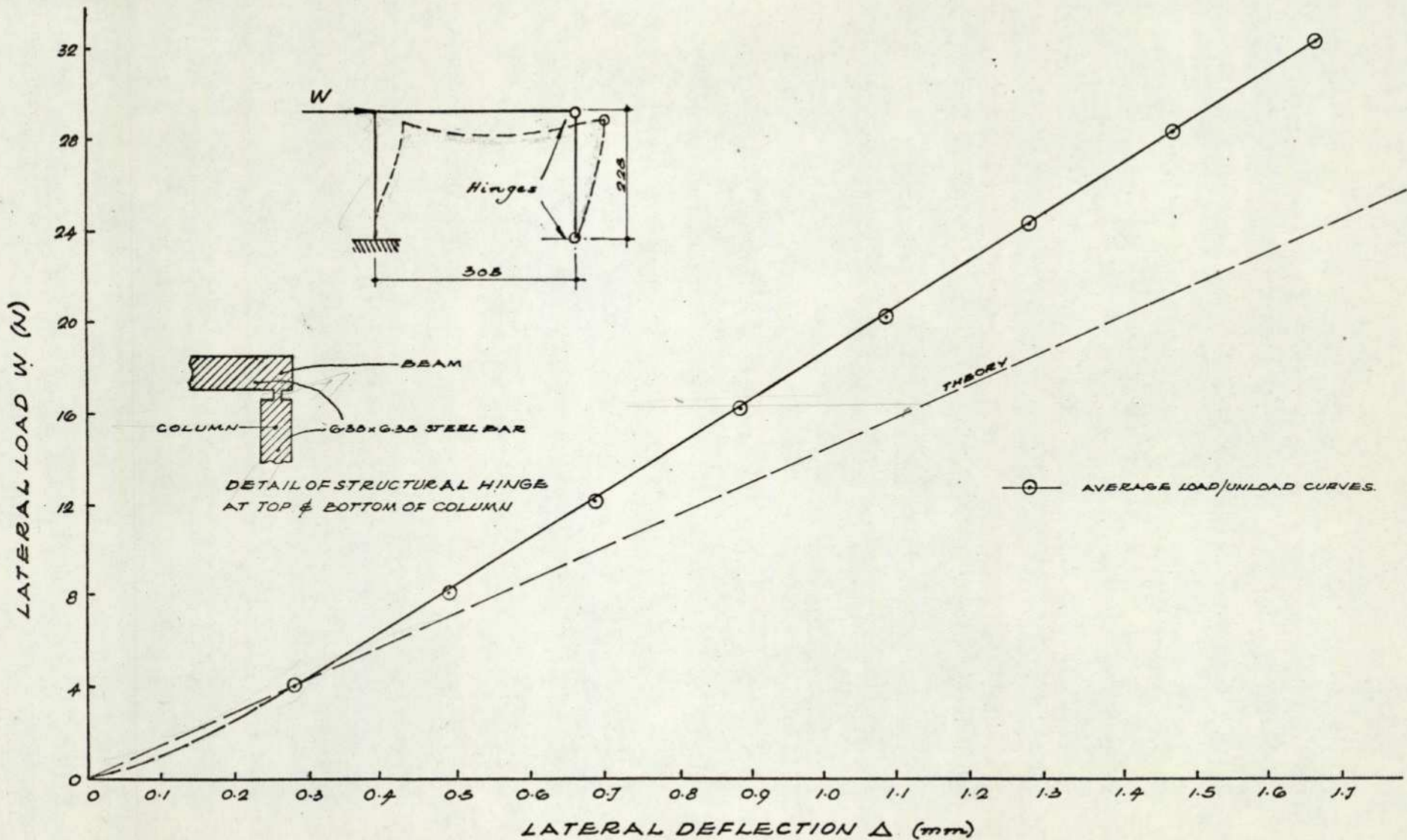


FIG. 2.223 LOAD DEFLECTION CURVE FOR PORTAL FRAME WITH DOUBLE HINGES,
- PRIMARY EFFECTS

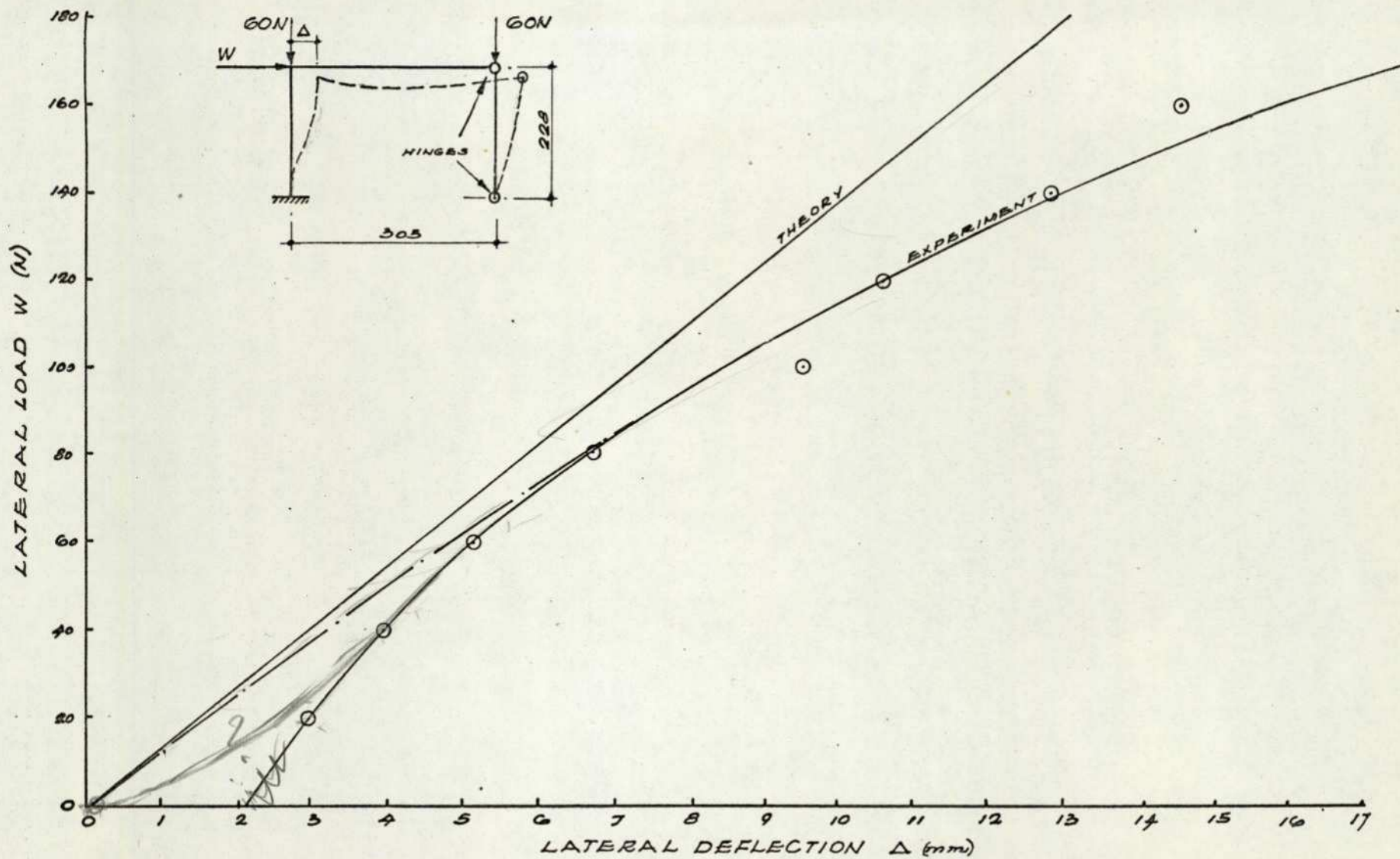


FIG. 2.224 : LOAD/DEFLECTION CURVE FOR PORTAL FRAME WITH DOUBLE HINGES, INSTABILITY EFFECT.

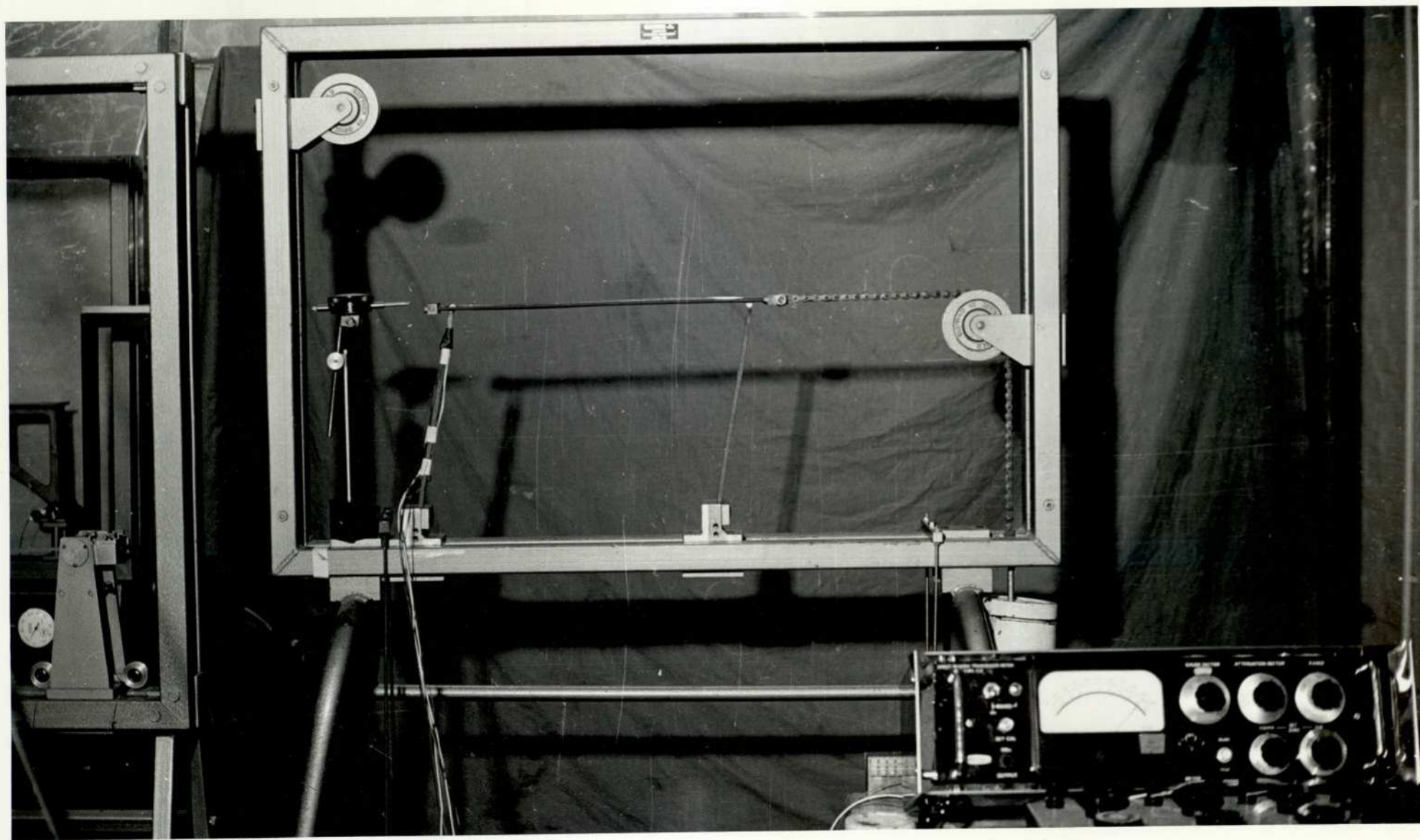


Fig. 2.225 Lateral Load Test on Fixed-ends Steel Portal Frame.

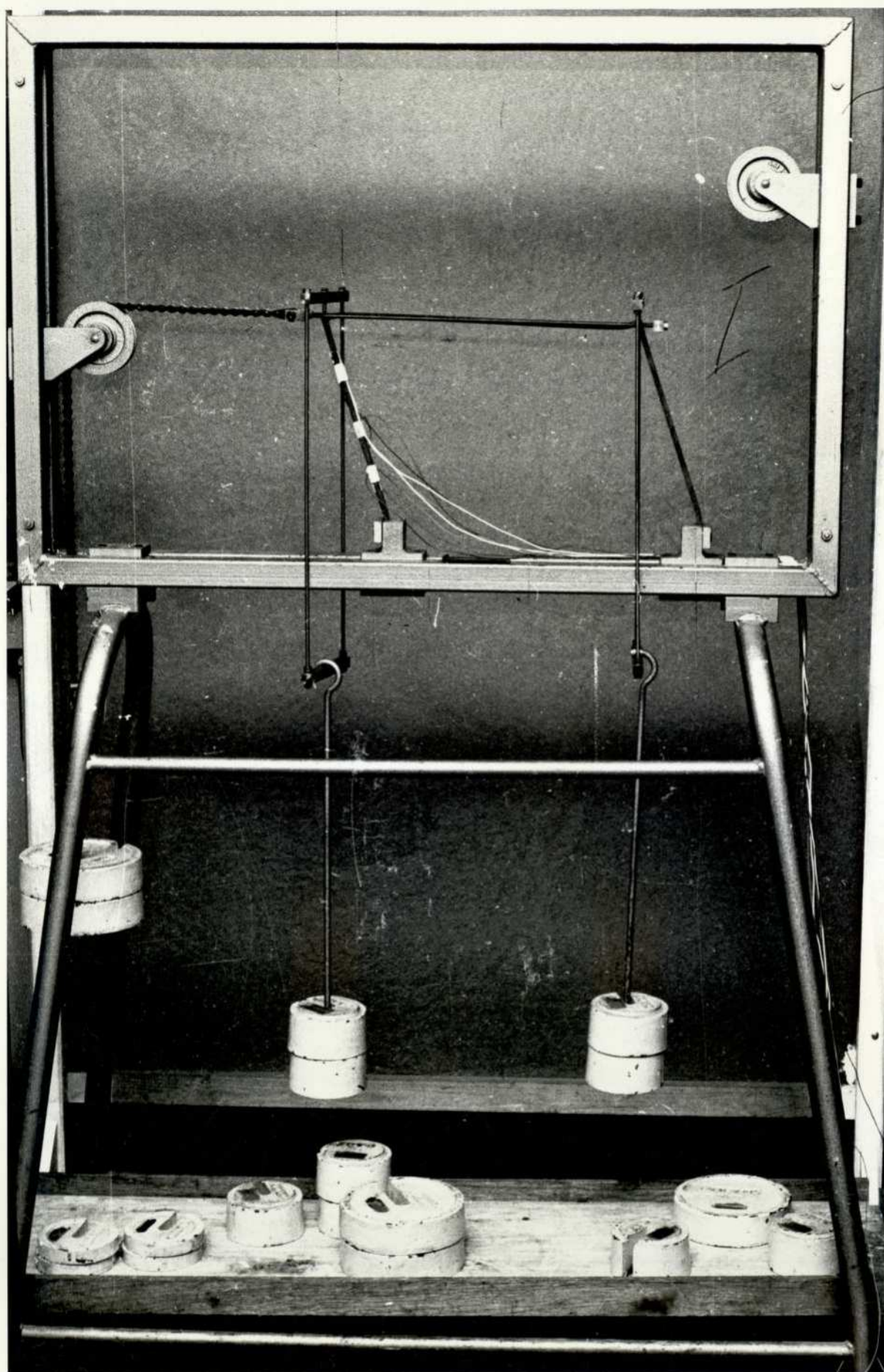


Fig. 2.226 Instability Test on Fixed-ends Portal Frame.

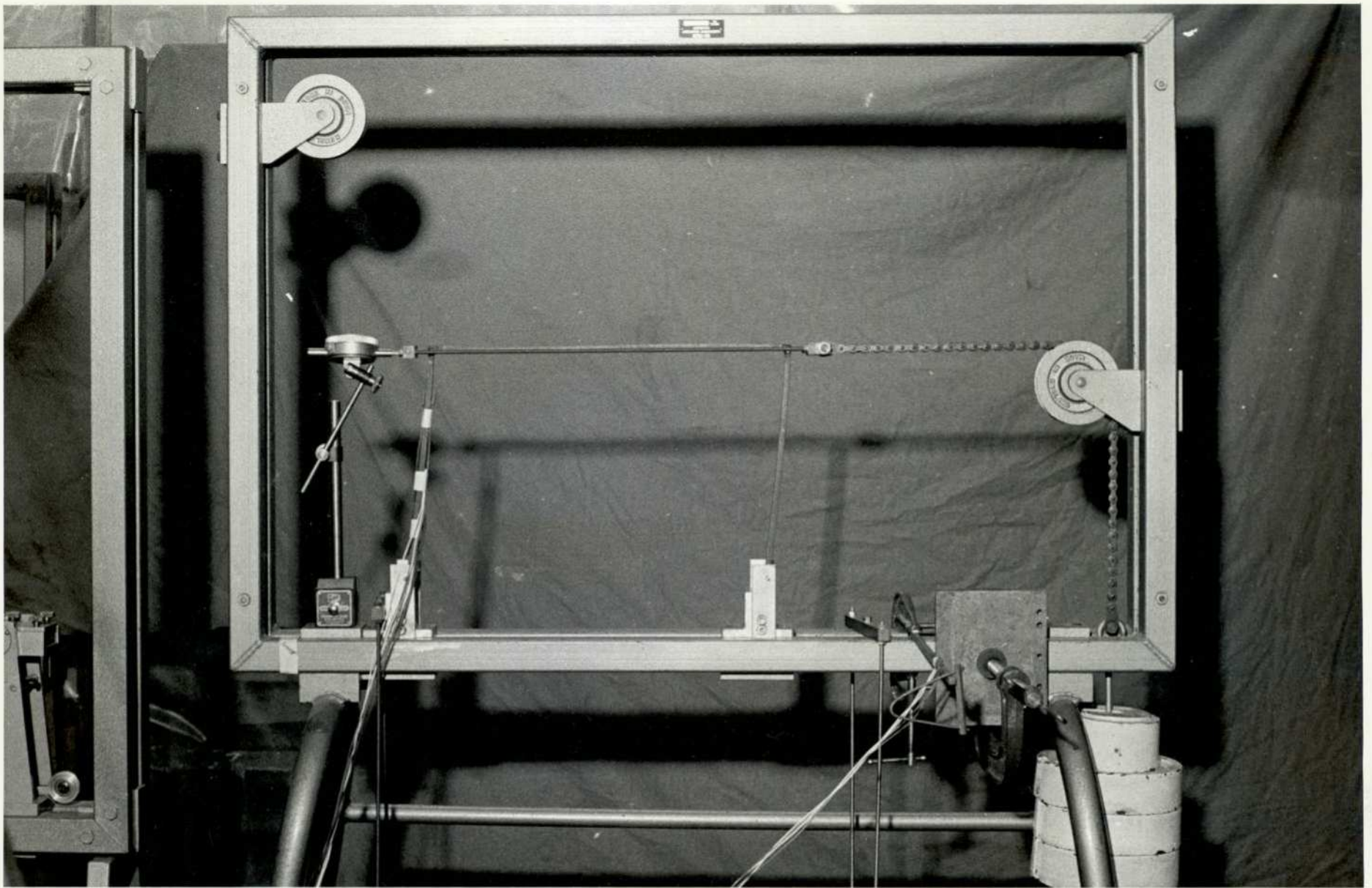


Fig. 2.227 Steel Portal Frame with Hinged-ends Undergoing Lateral Load Test.

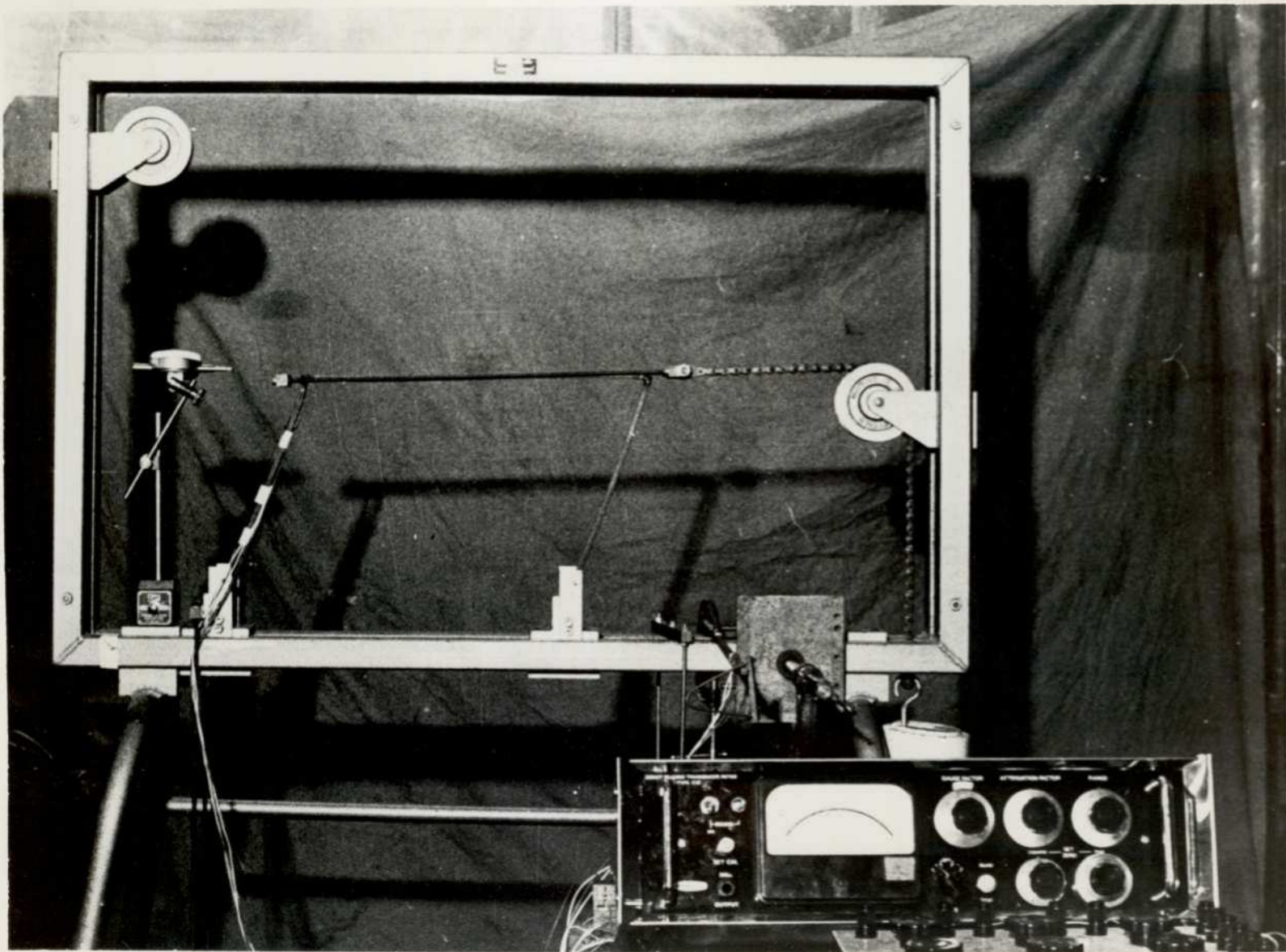


Fig. 2.228 Lateral Load Test on Hinged ends Steel Portal Frame,
Post-yield

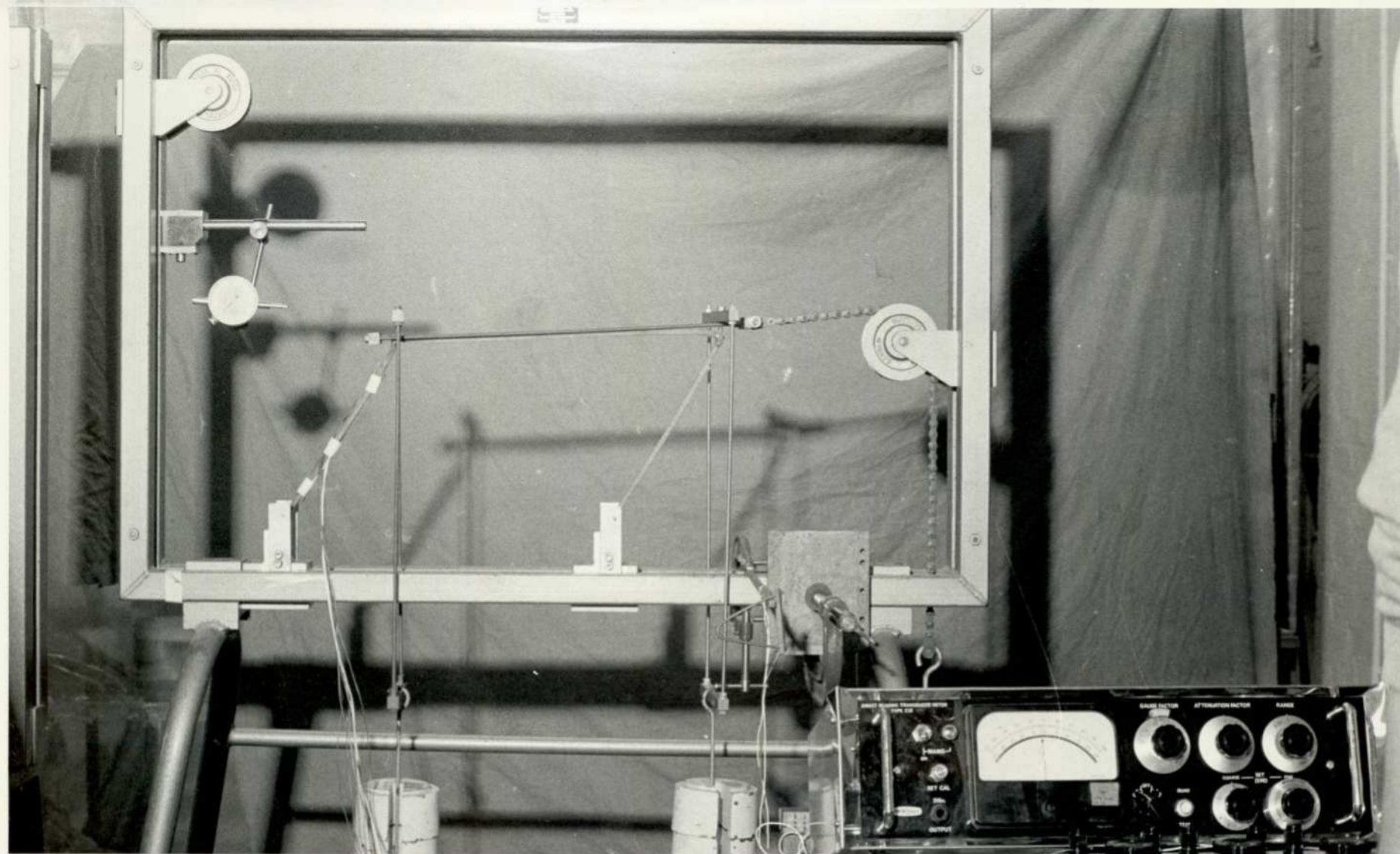


Fig. 2.229 Hinged-ends Steel Portal Frame in Ultimate Stage of Instability Test.

2.23 Experimental Bending Moments for Steel Portal Frames

In this section the load/strain gauge relations for frames of increased flexibility are converted to experimental bending moments at strain gauge positions. These bending moments are then compared with their theoretical counterparts.

2.2.31 Comparison of experimental and theoretical primary bending moments for portal frame with one column fixed and other hinged:

From theoretical analysis 2.141, the bending moment diagram for column 34 is shown in Fig. 2.2311.

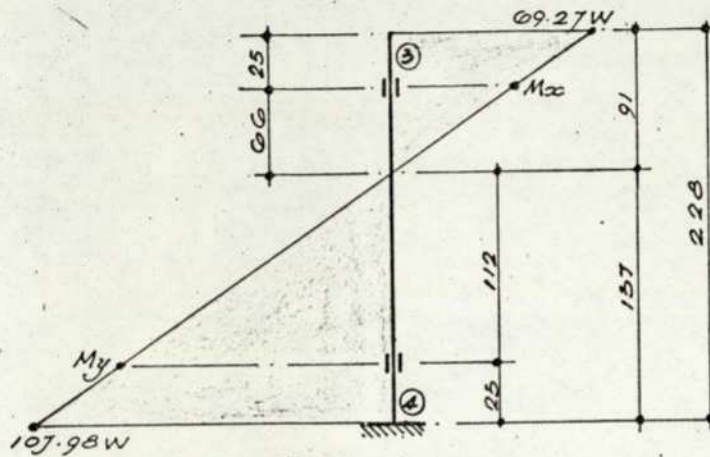


Fig. 2.2311 Bending Moment Diagram for Column 34.

From similar triangles, theoretical moment

$$\text{at } x = M_{xT} = 50.3W \text{ (N.mm), and}$$

$$M_{yT} = 88.2W \text{ (N.mm)}$$

$$\text{for } W = 20N, M_{xT} = 1006 \text{ Nmm}$$

$$M_{yT} = 1764 \text{ Nmm}$$

Experimental values of moments, M_{xE} and M_{yE} , are obtained from relation

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{I\sigma}{y} = \frac{135.5\sigma}{3.175} = 42.67\sigma$$

where σ = No. of strain gauge divisions x calibration factor

where calibration factor = 1.85 n/mm^2 per division. (appendix 1)

$$M = 78.453 \times \text{strain gauge divisions}$$

From the graph of strain gauge divisions/lateral load (Fig. 2.2312), bending moments can be obtained:

$$M_{XE} = 78.953 \times 11 = 868 \text{ Nmm}$$

$$M_{YE} = 78.953 \times 17.8 = 1405 \text{ Nmm.}$$

It can be seen that experimental moments are 13.7% less and 20.3% less than theoretical values at nodes 3 and 4 respectively.

2.232 Comparison of experimental and theoretical instability bending moments for portal frame with one column fixed and other hinged:

From theoretial analysis, the bending moment diagram for column 34 is shown in Fig. 2.2321.

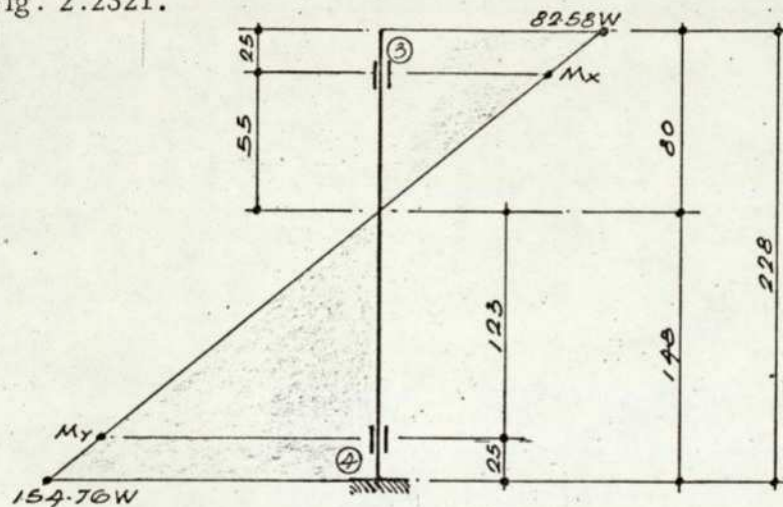


Fig. 2.2321 Theoretical Bending Moment Diagram for Column 34.

From similar triangles, theoretical moments at X = $M_{XT} = 56.77W$ (Nmm)

$$\text{and } M_{YT} = 128.568W \text{ (Nmm)}$$

Using relation $M = 78.953 \times$ strain gauge divisions and data from Fig. 2.2322 relating to gauge Nos. 1 and 2, gives experimental bending moment (for $W = 20\text{N}$):

$$M_{XE} = 78.953 \times \left(\frac{9.5 + 11}{2} \right)$$

$$M_{XE} = 809 \text{ N.mm}$$

Similarly from Fig. 2.2323 for gauges 3 and 4

$$M_{YE} = 78.953 \times \left(\frac{15.6 + 16.8}{2} \right)$$

$$M_{YE} = 1279.0 \text{ N.mm}$$

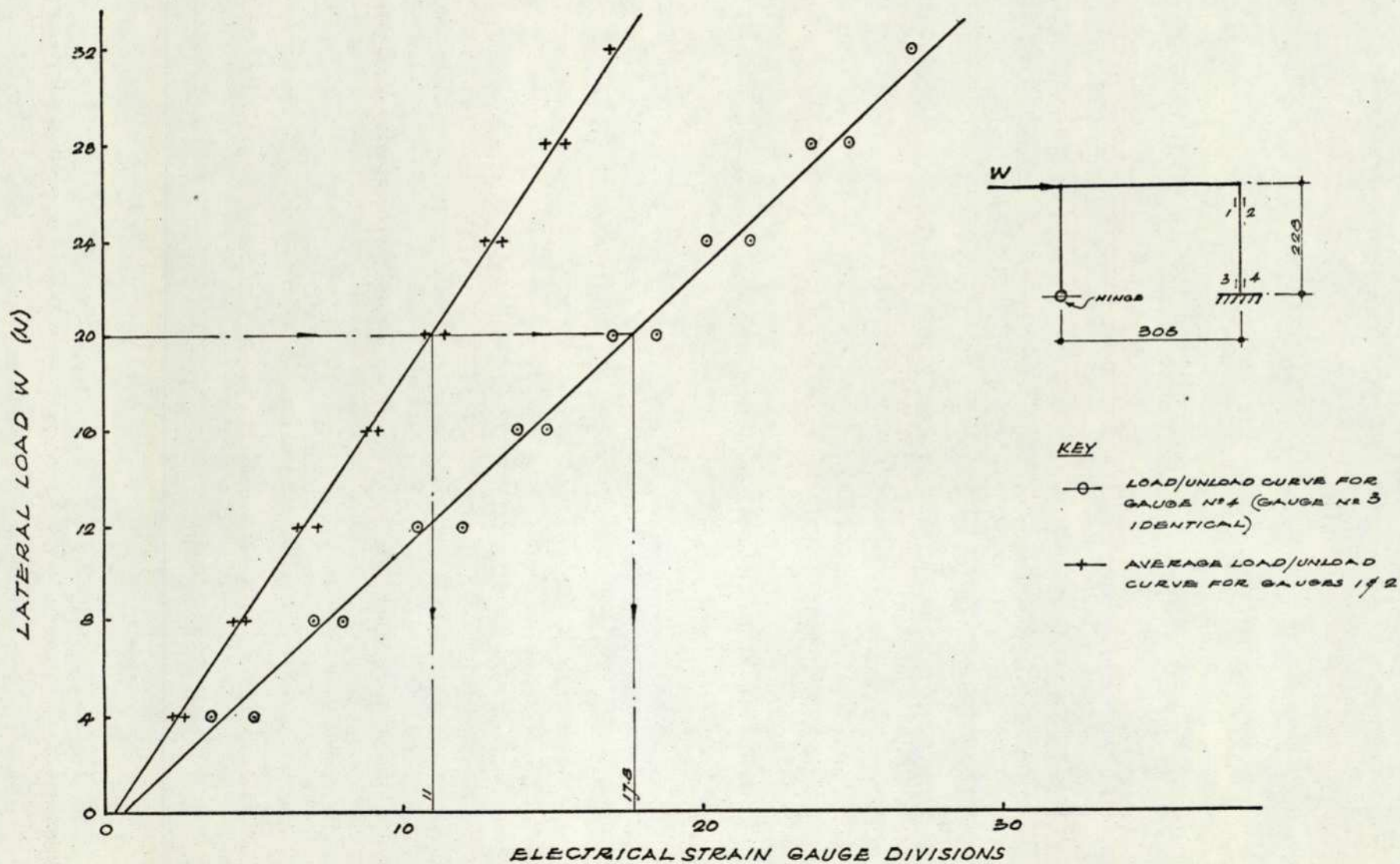


FIG. 2.2312: STRAIN GAUGE READINGS FOR PORTAL FRAME WITH FIXED AND HINGED ENDS - PRIMARY EFFECTS

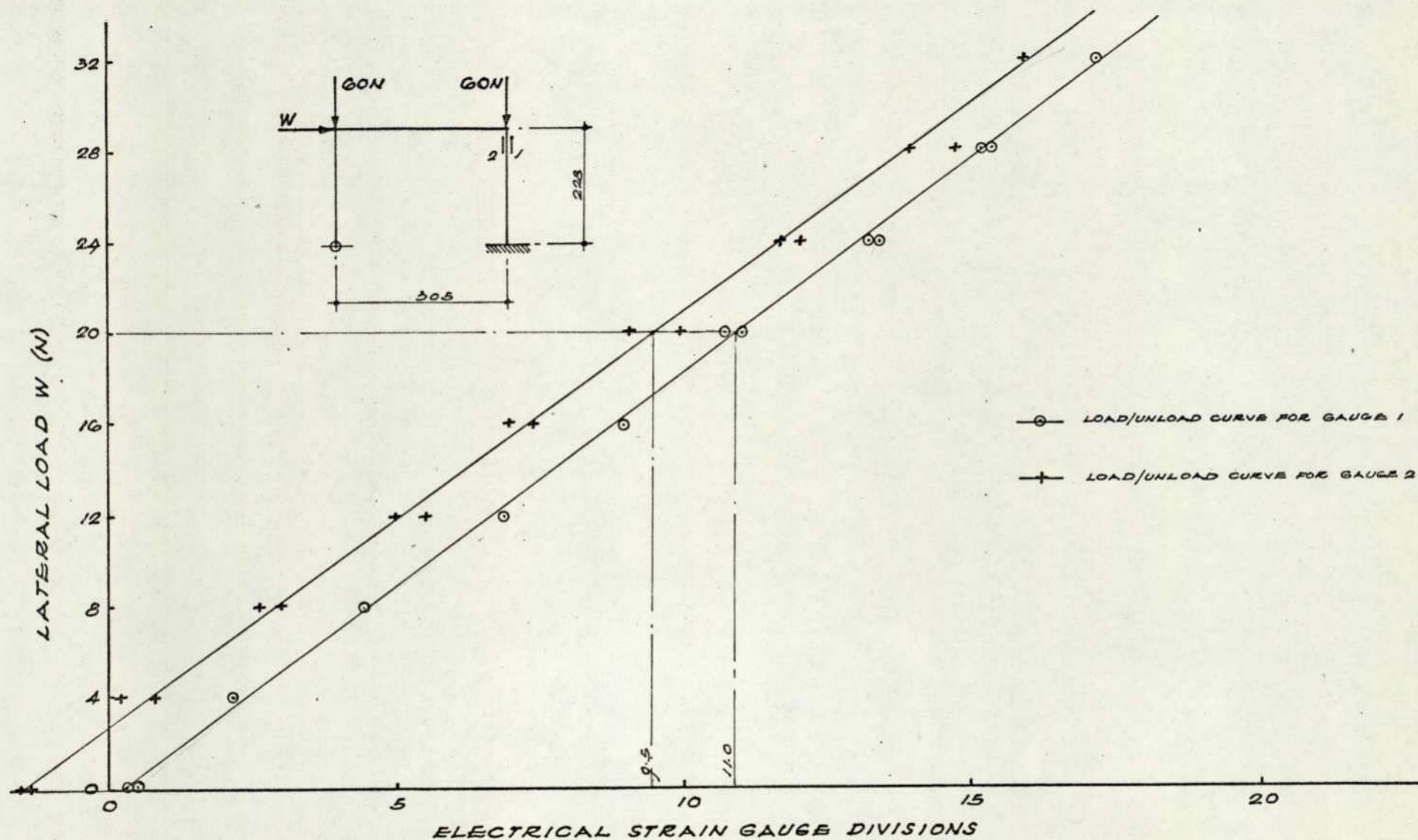


FIG. 2.2322 STRAIN GAUGE READINGS FOR PORTAL FRAME WITH FIXED AND HINGED ENDS
INSTABILITY EFFECT ($P = 60N$)

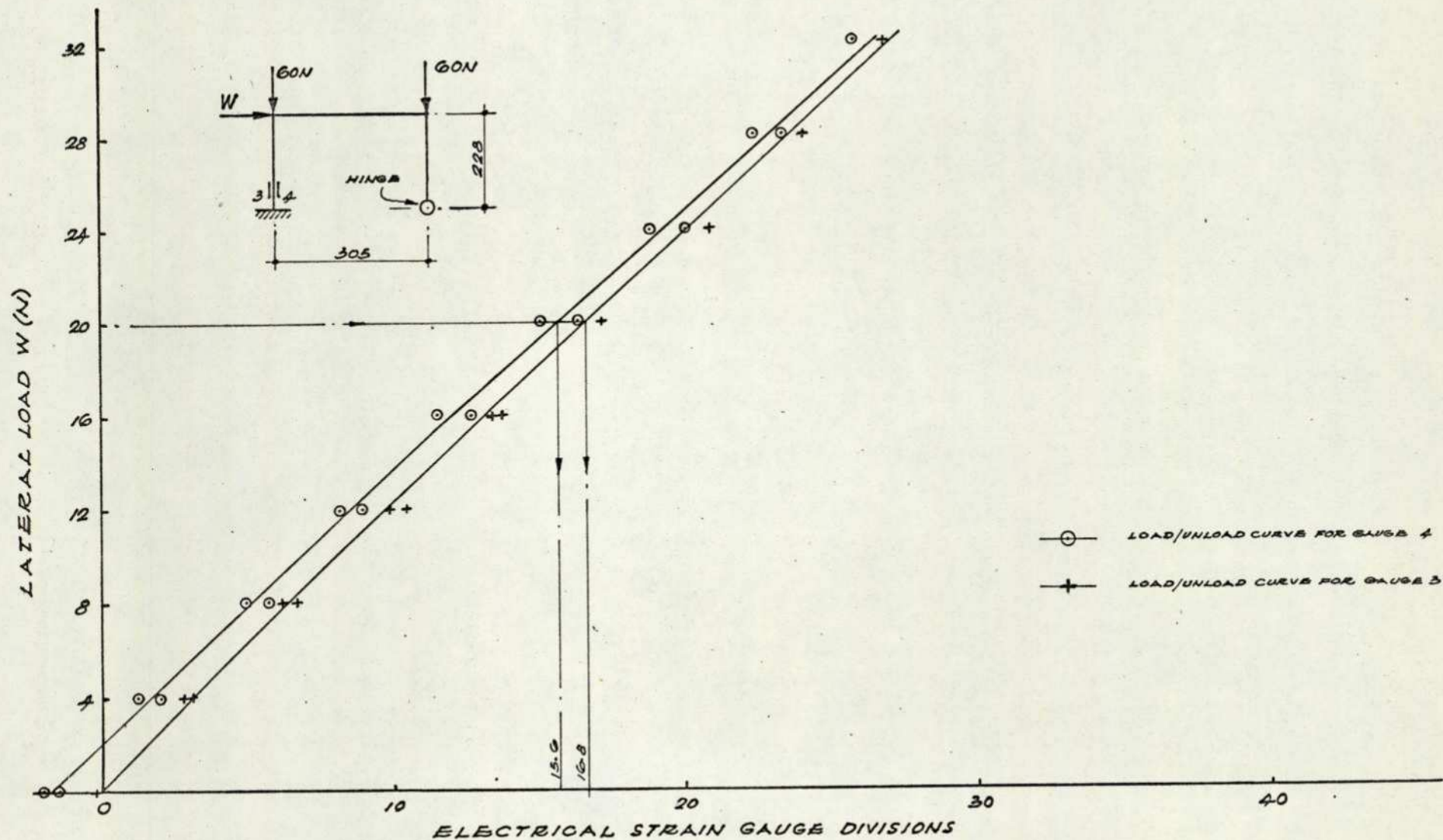


FIG. 2.2323 STRAIN GAUGE READINGS FOR PORTAL FRAME WITH FIXED AND HINGED ENDS
INSTABILITY EFFECTS ($P = 60\text{ N}$)

Comparing these experimental data with the theoretical values of moments ($M_{XT} = 1135$, $M_{YT} = 2572$ N.mm) shows a discrepancy of 28% and 49% for M_{34} and M_{43} respectively.

2.233 Comparison of experimental and theoretical primary bending moments for portal-frame with double hinges in one column:

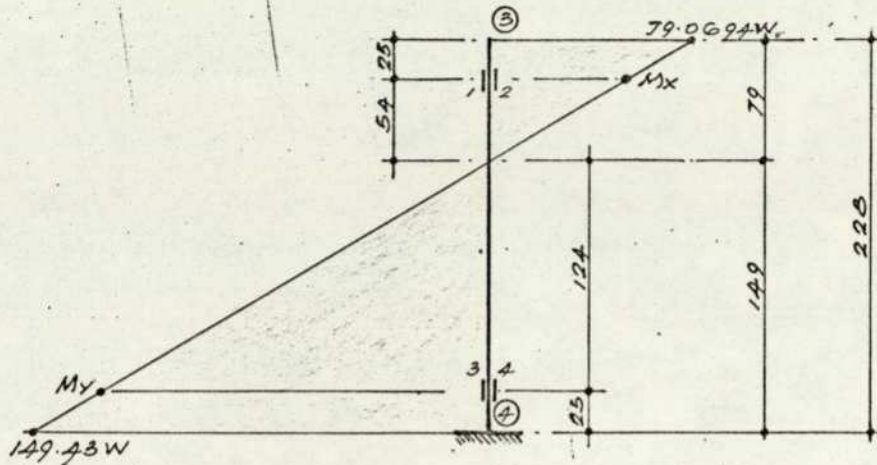


Fig. 2.2331 Theoretical Bending Moment Diagram for Column 34.

From similar triangles, theoretical moments (from 2.142)

$$M_{XT} = 54.043 W \text{ (N.mm)}$$

$$M_{YT} = 124 W \text{ (N.mm)}$$

Using relation $M = 78.953 \times \text{No. of strain gauge divisions}$, where number of strain gauge divisions are obtained from Fig. 2.2332, gives (for $W = 20\text{N}$):

$$M_{XE} = 78.953 \times 10.2 = 805.32 \text{ N.mm}$$

$$M_{YE} = 78.953 \times 17.7 = 1397.46 \text{ N.mm}$$

Whereas theoretical moments for $W = 20\text{N}$, are

$$M_{XT} = 1080.8 \text{ N.mm}$$

$$M_{YT} = 2480 \text{ N.mm}$$

Comparison of theoretical and experimental values shows that experimental moments M_{34} and M_{43} are 25.5% and 43% less than theory predicts.

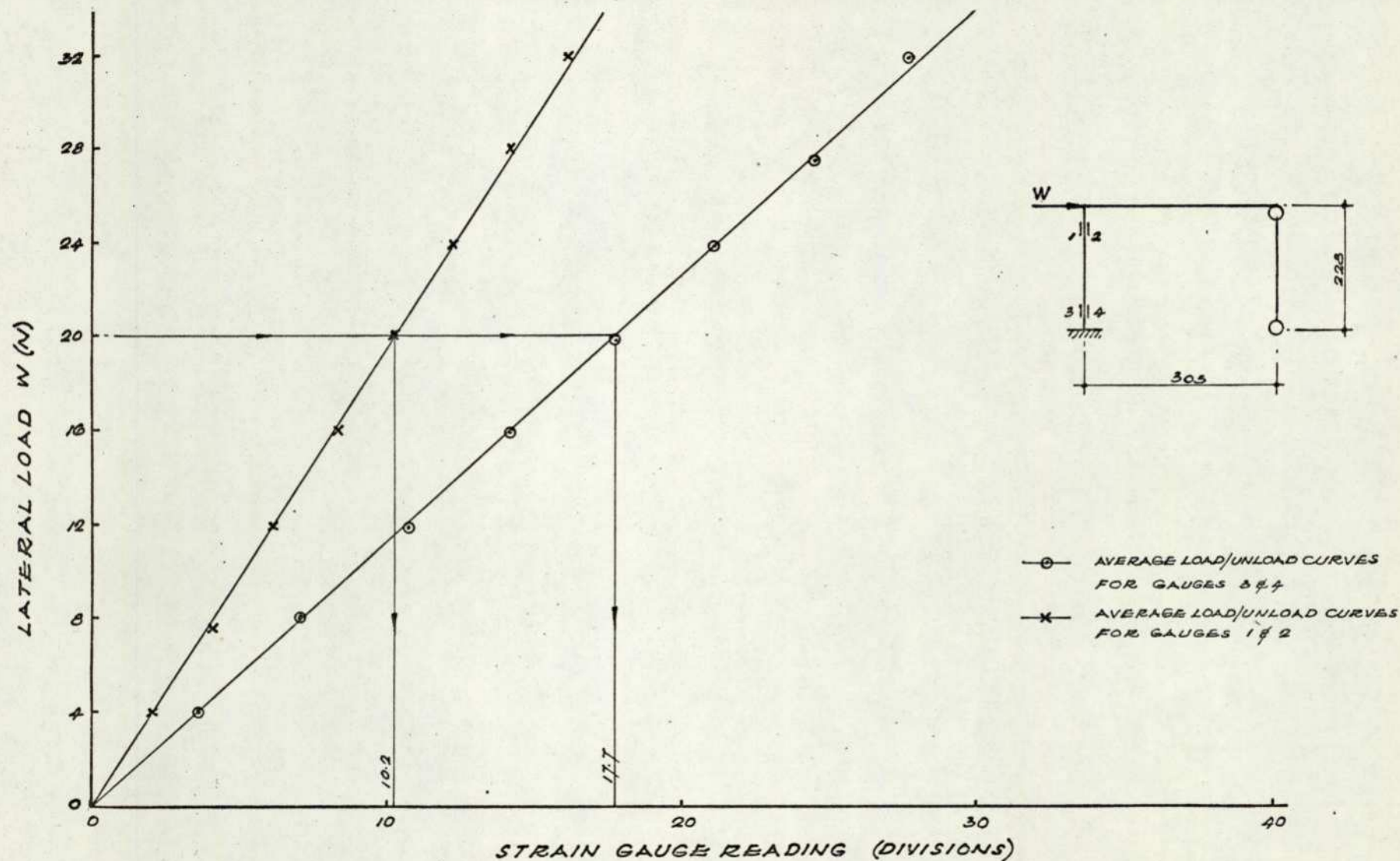


FIG. 2.2332 STRAIN GAUGE READINGS FOR PORTAL FRAME WITH DOUBLE HINGES, PRIMARY EFFECTS.

- 2.234 Comparison of experimental and theoretical instability bending moments for portal frame with double hinges:

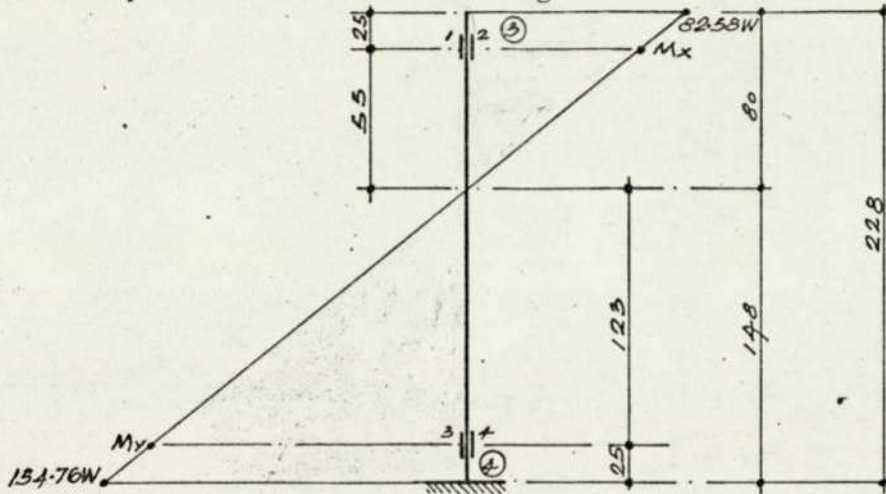


Fig. 2.2341 Theoretical Bending Moment Diagram for Column 34.

From similar triangles, theoretical moments (from 2.152):

$$M_{xT} = 56.77 W \text{ (N.mm)}$$

$$M_{yT} = 128.61 W \text{ (N.mm)}$$

Using relation $M = 78.953 \times \text{number of strain gauge divisions}$ (where number of strain gauge divisions are obtained from Fig. 2.2342) gives, for $W = 100\text{N}$:

$$M_{xE} = 78.953 \times 59 = 4658 \text{ N.mm}$$

$$M_{yE} = 78.953 \times 113 = 8921 \text{ N.mm}$$

Theoretical moments, also for $W = 100\text{N}$, are:

$$M_{xT} = 5677 \text{ N.mm}$$

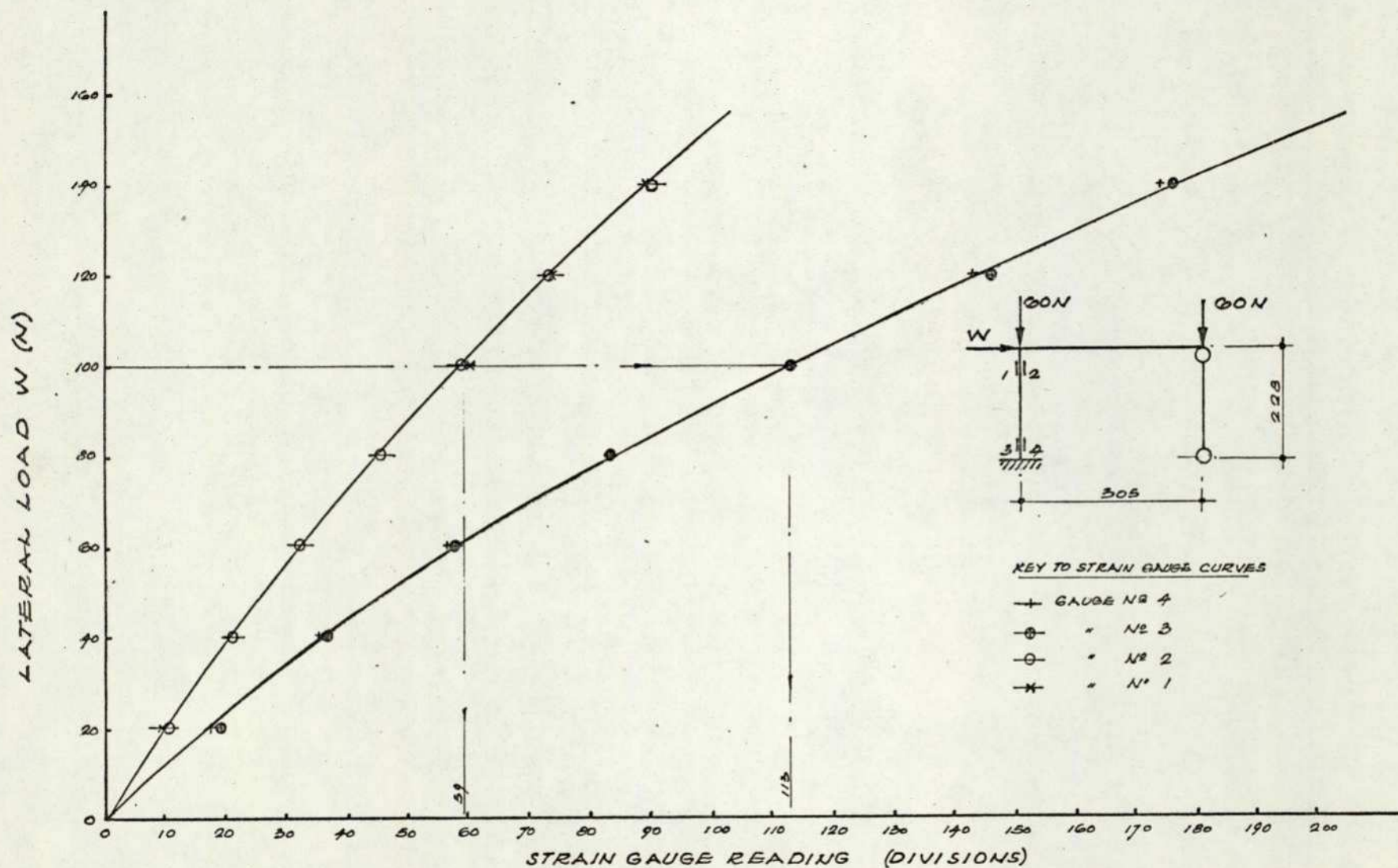
$$M_{yT} = 128.61 \text{ N.mm}$$

Comparison of theoretical and experimental values shows that experimental moments M_{34} and M_{43} are 18% and 30% less than predicted values.

2.30 Discussion of Test Results for Portal Frames

2.31 Deflections:

The correlation of theoretical and experimental deflections was generally very good. This is illustrated by Figs. 2.212, 2.215, 2.221, 2.222 and 2.3101.



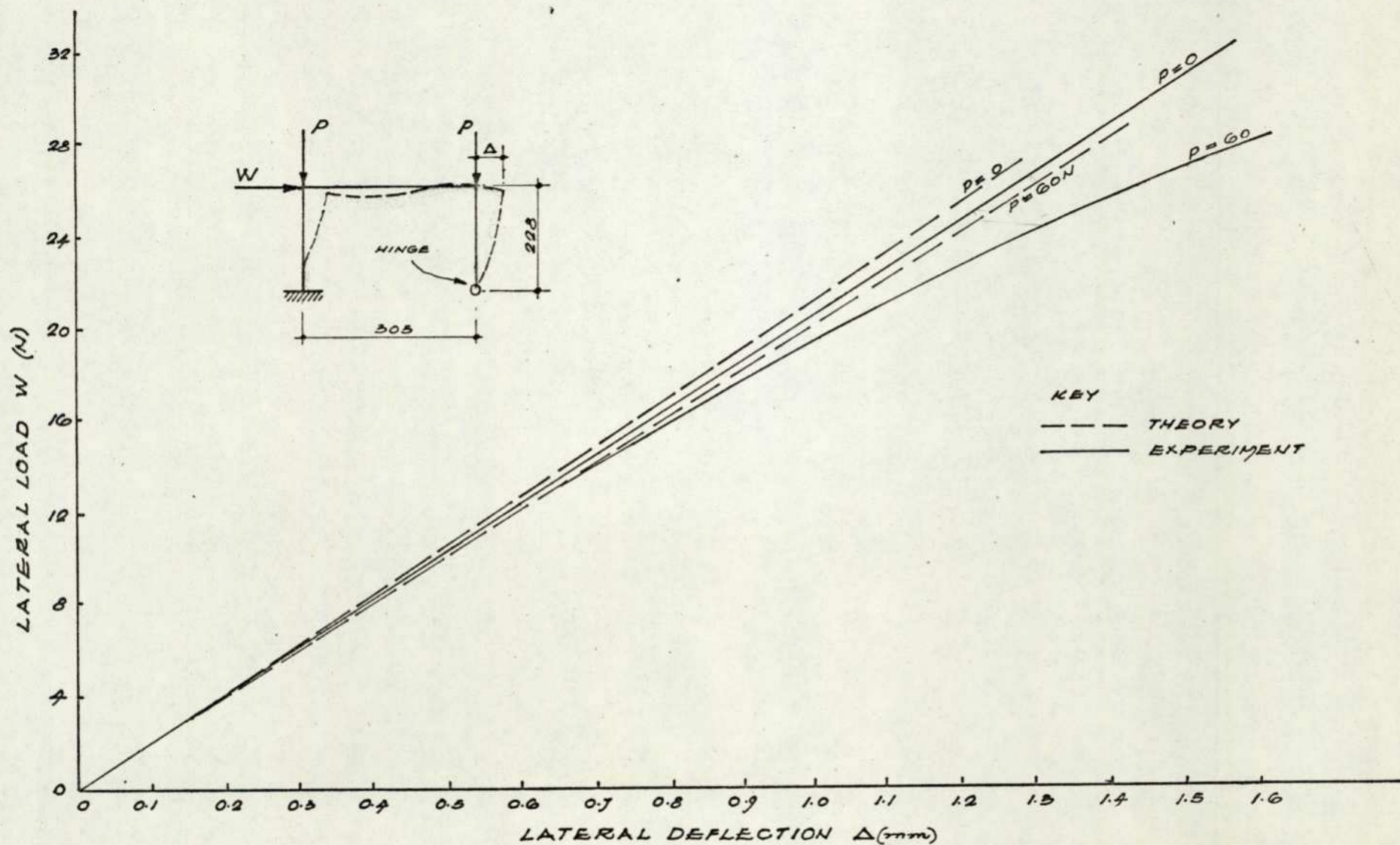


FIG: 2-3101

COMPARISON OF THEORETICAL AND EXPERIMENTAL DEFLECTIONS FOR STEEL PORTAL FRAME WITH FIXED AND HINGED COLUMN FEET.

In the case of the frame with double hinges in one column, Fig. 2.3102, the lesser experimental deflection under primary effects may be attributed to the slight rotational restraint induced by the milled slot arrangement (Fig. 2.223) supposed to represent a hinge. The experimental error of approximately 25% was consistent throughout the load range. For the same frame with combined axial and lateral loading, experimental deflections were more than those predicted from the theory. The excessive deflections under low levels of lateral load, i.e. 0 to 60N, (Fig. 2.224) is more difficult to account for when based on the elastic instability assumption. However it is interesting to note the non-linear shape of the experimental deflection for loads above 100N.

2.32 Bending Moments:

In 2.231 it was shown that experimental moments developed at the top of columns were closer to their theoretical counterparts than for moments developed near the column feet. The fact that in these frames all the measured moments were greater than their theoretical values suggests that the calibration factor (1.85 N/mm^2 per division - see Appendix I) is too high.

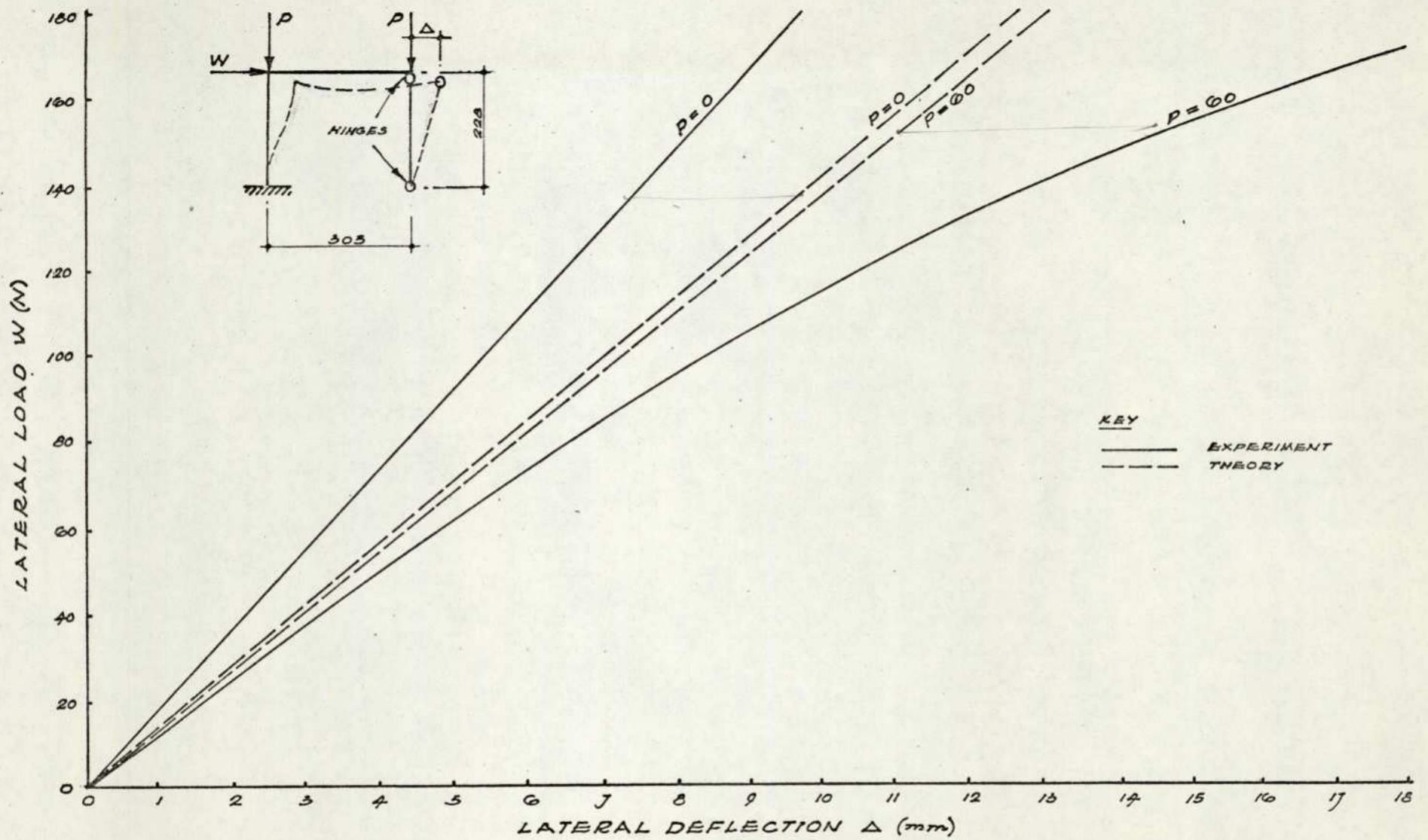


FIG. 2.3102 COMPARISON OF THEORETICAL AND EXPERIMENTAL DEFLECTIONS FOR STEEL PORTAL FRAME WITH DOUBLE HINGES IN ONE COLUMN.

3.00 10-storey Steel Frame.: theory and experiment.

In this section a very slender 10-storey steel model is analysed and tested experimentally for primary and instability effects. For primary effects the model is subjected to a single lateral load applied at the top of the frame. Instability effects are examined with a lateral load applied at the top of the frame and gravity loads applied either at the top or at each nodal point.

The object is to show how the instability analysis is developed for the case where changed flexibility of columns, axial deformations of columns, and large lateral deformations are taken into account. The theoretical performance is then compared with the experimental results.

3.10 Primary Analysis, 10 storey Steel Frame

Fig. 3.11 shows in idealised form the structure to be analysed. This shows assumed shape of moment and deflection diagram. Using the force-displacement method, the typical matrix is set out at the floor level 1-1'. The two compatibility and one equilibrium conditions are as follows:

$$2/1/ \quad (M_{10} + M_{12}) \frac{L}{6EI_b} + M_{12} \frac{h}{3EI_c} - M_{21} \frac{h}{6EI_c} = \frac{\Delta_1 - \Delta_2}{h}$$

$$0/2 \quad -M_{01} \frac{h}{6EI_c} + M_{10} \frac{h}{3EI_c} - M_{12} \frac{h}{3EI_c} + M_{21} \frac{h}{6EI_c} = \frac{\Delta_0 - 2\Delta_1 + \Delta_2}{h}$$

$$EC \ 1-2 \quad \frac{M_{12} + M_{21}}{h} = \frac{1}{2} W_0$$

These equations are now rearranged by multiplying the first two equations by $\frac{6EI_c}{h}$ and the third by h . We obtain:

$$(M_{10} + M_{21})r + 2M_{12} - M_{21} = (\Delta_1 - \Delta_2) \frac{6EI_c}{h^2} \quad \text{----- 3.11}$$

$$-M_{01} + 2M_{10} - 2M_{12} + M_{21} = (\Delta_0 - 2\Delta_1 + \Delta_2) \frac{6EI_c}{h^2} \quad \text{----- 3.12}$$

$$M_{12} + M_{21} = \frac{1}{2} (W_0 h) \quad \text{----- 3.13}$$

in which $r = LI_c/hI_b$ and is referred to as the "frame parameter".

For the test frame this parameter is:

$$r = \frac{376.0}{256.0} = 1.4687$$

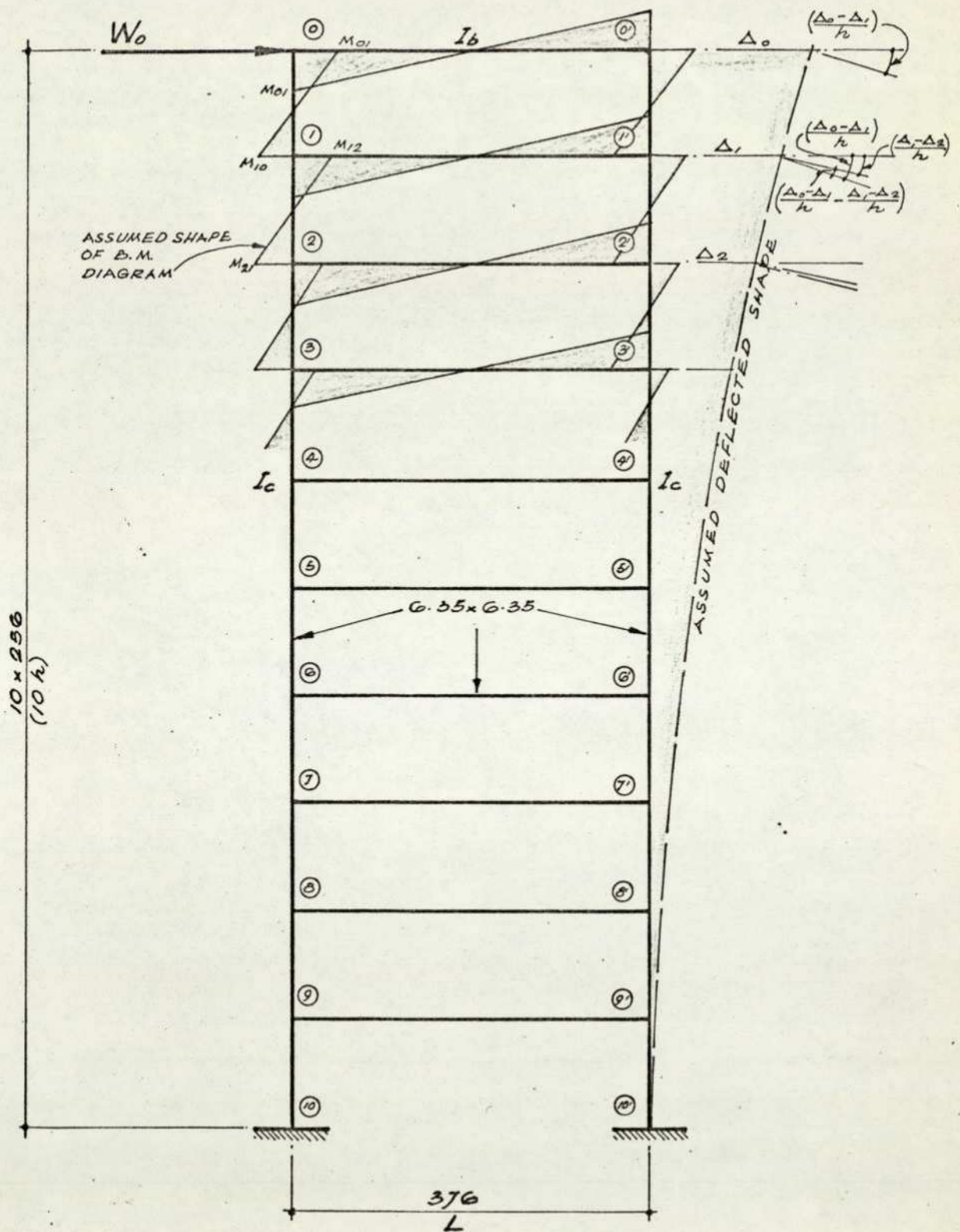


Fig. 3.11 Idealised Frame used in Primary Analysis,
10-storey Steel Frame.

Based on the equations 3.11, 3.12 and 3.13 an element matrix can be set out as well as the overall force-displacement matrix. This is shown in Fig. 3.12.

It should be noted that all bending moments are expressed in terms of $W.h$ and all deflections in terms of $\frac{Wh^3}{6EI_c}$ and also that values obtained from the solution of the matrix are non-dimensional. The solution, therefore represents influence coefficients which are independent of the magnitude of load although it depends on the form of loading used. These coefficients are, theoretically constant within the elastic limit and are convenient to use as a basis for correlation with the experiments. It was expected however that when yield stress is reached at any point in the frame or even when the elastic instability effects take place, then these values will cease to remain constant. The occurrence of this non-linearity was therefore observed with particular interest.

The solution of the general matrix is shown in the bending moment diagram of Fig. 3.13.

3.20 Instability Analysis, 10-storey Steel Frame

This analysis is made to form a basis for comparing the results with those obtained from tests as stated earlier. The following effects are included:

- i) the effect of axial forces on stiffnesses or on flexibility of individual members
- ii) the effect of axial deformations and
- iii) the effect of large deformations (changed geometry of the system and therefore the action of applied loads).

It is assumed that only the columns are subjected to sufficiently high axial forces to affect their elastic properties. In practice these forces arise mainly from the gravity loads, although lateral forces may also add to this effect, particularly for columns in compression.

In setting out the matrix, both the conditions of compatibility of deformations and the equilibrium conditions are affected, similarly as in the portal frames discussed in 2.10.

Referring to the idealised structure, Fig. 3.21, the equations for compatibility of deformation and equilibrium of forces can be set out as follows:-

At point O the compatibility condition becomes:

$$-M_{oi} \frac{L}{6EI_b} + M_{oi} \left(\frac{L}{3EI_b} + \frac{c_{oi} h}{3EI_c} \right) - M_{io} \frac{c_{oi} h}{6EI_c} =$$

EQN NO	COND ^m	0		1			2			3			4			5			6			7			8			9			10	LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₂	M ₄₅	Δ ₄	M ₅₂	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₉	
1	100'	r+2	-1	-1	0	1	(25)																									0
2	0-0'	1	0	1	0	0	(25)																									0.5W/h
3	211'	0	0	r	r+2	-1	-1	0	1	(22)																						0
4	012	-1	-1	2	-2	2	1	0	-1	(22)																						0
5	1-1'	0	0	0	1	0	1	0	0	(22)																						0.5W/h
6	322'			(3)									(19)																			0
7	123			(3)				FE					(19)																			0
8	2-2'			(3)									(19)																			0.5W/h
9	433'						(6)									(16)																0
10	234						(6)				FE					(16)																0
11	3-3'						(6)									(16)																0.5W/h
12	544'							(9)											(13)													0
13	345							(9)				FE							(13)													0
14	4-4'							(9)											(13)													0.5W/h
15	655'									(2)											(10)											0
16	456									(2)						FE					(10)											0
17	5-5'									(2)											(10)											0.5W/h
18	766'												(15)												(7)							0
19	567												(15)						FE						(7)							0
20	6-6'												(15)												(7)							0.5W/h
21	877'																		(18)									(4)				0
22	678																		(18)				FE				(4)					0
23	7-7'																		(18)								(4)					0.5W/h
24	988'																				(21)										(1)	0
25	789																				(21)				FE						(1)	0
26	8-8'																				(21)										(1)	0.5W/h
27	1099'																						(24)									0
28	8910																						(24)									0
29	9-9'																						(24)									0.5W/h
30	109																										(27)	-1	-1	2		0

FIG. 3-12 GENERAL MATRIX FOR PRELIMINARY ANALYSIS — STEEL MODEL

○ DENOTES NUMBER OF ZEROS
r = 1.4687

B.M. DIAGRAM FOR COL.
0'-10' IDENTICAL TO THAT
OF COL. 0-10 BECAUSE
OF FRAME SYMMETRY

LATERAL DEFLECTION
COEFFICIENT (TO BE
MULTIPLIED BY $W h^3 / 6EI$
TO OBTAIN ACTUAL
DEFLECTION

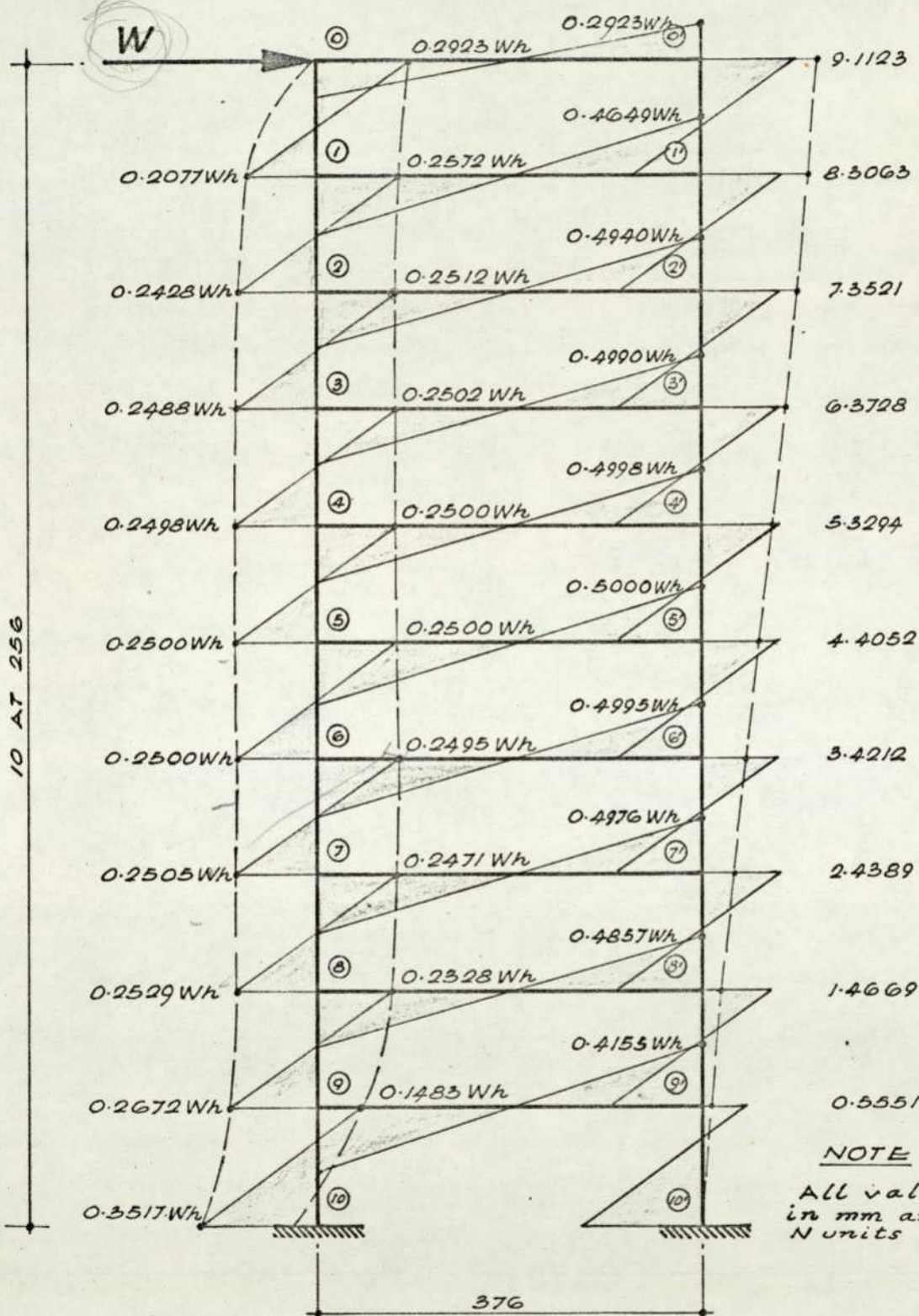


FIG. 3-13 BENDING MOMENT AND DEFLECTION DIAGRAM
- 10 STOREY STEEL MODEL, PRIMARY
ANALYSIS.

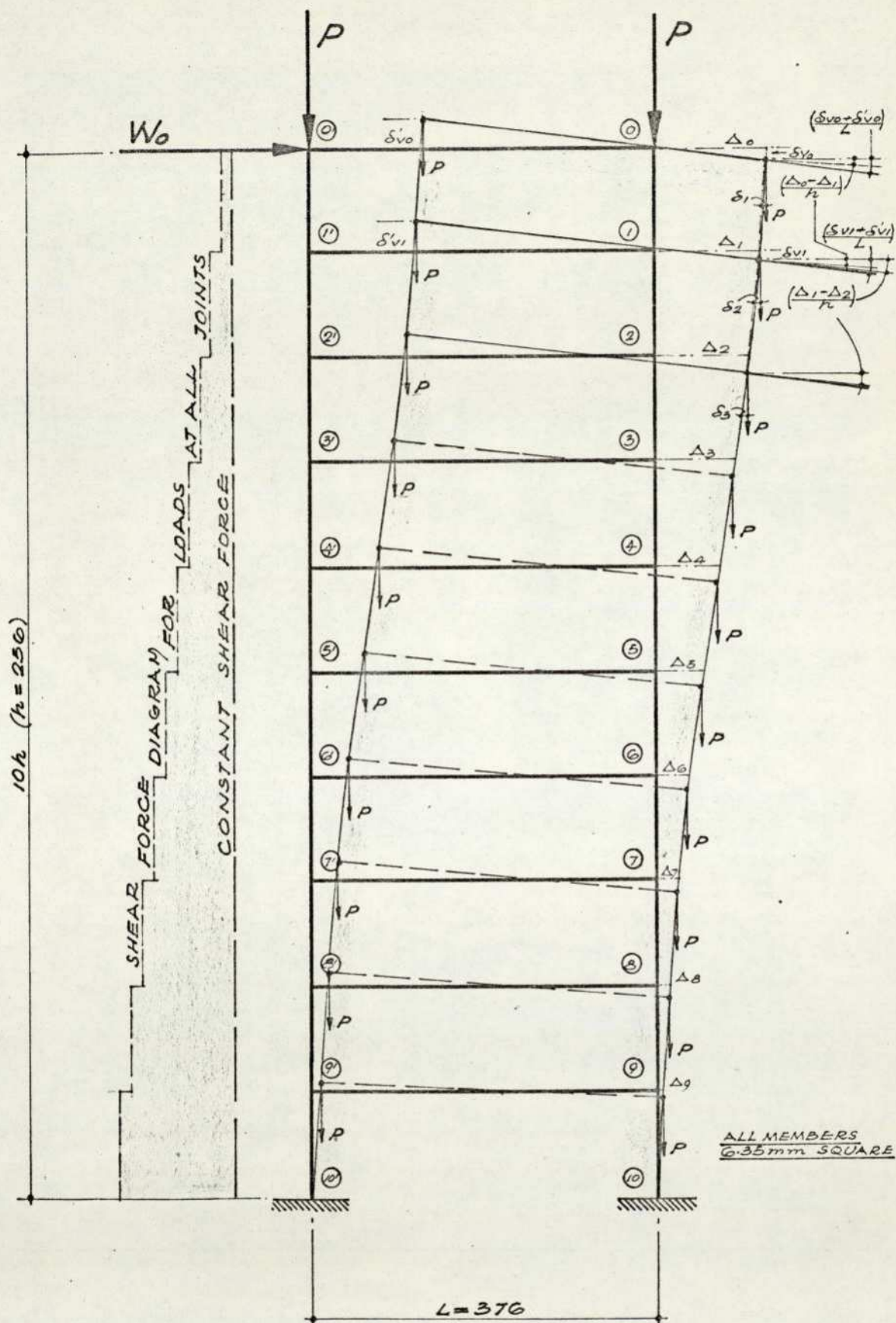


FIG. 3.21 IDEALISED FRAME FOR INSTABILITY ANALYSIS, 10-STOREY STEEL FRAME

$$= \frac{\Delta_o - \Delta_i}{h} - \frac{\delta v_o + \delta v_o'}{L} \quad \text{----- 3.21}$$

and the equilibrium condition becomes:

$$\frac{M_{o1} + M_{10}}{h} = \frac{1}{2} W_o + P \tan \alpha \cdot S_o \quad \text{----- 3.22}$$

The values of c_{oo} and c_{oi} are shown in Fig.1.071 for c and c respectively and:

$$\begin{aligned} \delta v_o &= \frac{P \cdot 10h}{EA} + \frac{2M_{o1}}{L} \cdot \frac{10h}{EA} + \frac{2(M_{10} + M_{12})}{L} \cdot \frac{9h}{EA} + \text{etc.} = \\ &= \frac{2h}{LEA} \left[5PL + 10M_{o1} + 9(M_{10} + M_{12}) + 8(M_{21} + M_{23}) + \text{etc.} \right] \quad \text{---- 3.23} \end{aligned}$$

Introducing equation 3.23 into 3.21 and multiplying both sides of the equation by $\frac{6EI_c}{h}$ we obtain:

$$\begin{aligned} M_{o1} \left(\frac{LI_c}{hI_b} + 2c_{oo} \right) - M_{10} c_{oi} - (\Delta_o - \Delta_i) \frac{6EI_c}{h^2} &= \\ &= - \frac{24I_c}{hL^2A} \left[\text{ditto} \right] \end{aligned}$$

or denoting $r = \frac{LI_c}{hI_b}$ and $p = \frac{24I_c}{hL^2A}$ we have

$$\begin{aligned} M_{o1} (r + 2c_{oo} + 10 \times 2.28 \times 10^{-6}) - 1.312300 \cdot M_{10} + \\ + p (9M_{12} + 8M_{21} + 8M_{23} + 7M_{32} + \text{etc.}) - (\Delta_o - \Delta_i) \frac{6EI_c}{h^2} &= 0 \end{aligned}$$

Taking

$$E_s = 200 \text{ kN/mm}^2$$

$$r = \frac{LI_c}{hI_b} = 1.4682$$

$$p = \frac{24 \times 135.5}{256 \times 376^2 \times 6.35^2} = 2.28 \times 10^{-6}$$

$$2U = L \sqrt{\frac{P}{EI}} = 376 \sqrt{\frac{400}{200 \times 10^3 \times 135.5}} = 1.44$$

$$c_{oo} = 1.1746$$

$$c_{oi} = 1.3123$$

From equation 3.22

$$M_{o1} + M_{10} = \frac{1}{2} W_o h + P (\Delta_o - \Delta_i)$$

or

$$M_{o1} + M_{10} - \frac{Ph^2}{6EI_c} \left[(\Delta_o - \Delta_1) \frac{6EI_c}{h^2} \right] = \frac{1}{2} Wh$$

$$\frac{Ph^2}{6EI_c} = \frac{400 \times 256^2}{6 \times 200 \times 10^5 \times 135.5} = 0.612$$

Similarly for level 1-1', compatibility condition 211' becomes:

$$\begin{aligned} (M_{10} + M_{12}) \frac{L}{6EI_b} + M_{12} \frac{hc_{11}}{3EI_c} - M_{21} \frac{hc_{12}}{6EI_c} = \\ = \frac{\Delta_1 - \Delta_2}{h} - \frac{2\delta v_1}{L} \end{aligned}$$

$$\frac{2\delta v_1}{L} \times \frac{6EI_c}{h^2} = \frac{12I_c}{hL^2A} \left[4.5P + 9M_{o1} + 8M_{10} + 8M_{12} + \text{etc.} \right]$$

$$\begin{aligned} (M_{10} + M_{12})r + M_{12} \cdot 2c_{11} - M_{21}c_{21} = (\Delta_1 - \Delta_2) \frac{6EI_c}{h^2} - \\ - \frac{24EI_c}{hL^2A} \left[4.5P + 9M_{o1} + 8M_{10} + 8M_{12} + \text{etc.} \right] \end{aligned}$$

$$\begin{aligned} + 0.000021 M_{o1} + 1.468,718 M_{10} + 3.968718 M_{12} - \\ - 1.312318 M_{21} + 0.000,016 M_{23} + 0.000016 M_{32} + \\ + 0.000014 M_{34} + 0.000014 M_{43} + 0.000012 M_{45} + \\ + 0.000012 M_{46} + 0.000009 M_{54} + 0.000009 M_{56} + \\ + 0.000007 M_{65} + 0.000007 M_{67} + 0.000005 M_{76} + \\ + 0.000005 M_{78} + 0.000003 M_{87} + 0.000003 M_{89} - \\ (\Delta_1 - \Delta_2) \frac{EI_c}{6h^2} = 0 \end{aligned}$$

2nd condition for compatibility of deformation at 1 is:

$$\begin{aligned} -M_{o1} \frac{c_{o1}h}{6EI_c} + M_{10} \frac{c_{11}h}{3EI_c} - M_{12} \frac{c_{11}h}{3EI_c} + M_{21} \frac{c_{12}h}{6EI_c} = \\ = \frac{\Delta_o - \Delta_1}{h} - \frac{\Delta_1 - \Delta_2}{h} \end{aligned}$$

Multiplying by $6EI_c/h$

$$\begin{aligned} -M_{o1}c_{o1} + 2M_{10}c_{11} - 2M_{12}c_{11} + M_{21}c_{12} = (\Delta_o - 2\Delta_1 + \Delta_2) \frac{6EI_c}{h^2} \\ -1.3123M_{o1} + 2.3492M_{10} - 2.3492M_{12} + 1.3123M_{21} - \\ -(\Delta_o - 2\Delta_1 + \Delta_2) \frac{6EI_c}{h^2} = 0 \end{aligned}$$

Fig. 3.22 shows these conditions presented in numerical matrix form. The solutions given in Fig. 3.23 are plotted in the bending moment and deflection diagram of Fig. 3.24.

SOLUTIONS		M	Δ
0.3724928783E 00	01	01	Δ_0
0.1178813347E 02	02	10	
0.2944493954E 00	00	12	
0.3574863971E 00	00	21	Δ_1
0.1075251142E 02	02	23	
0.3401659515E 00	00	32	Δ_2
0.3541580022E 00	00	34	
0.9526380225E 01	01	43	Δ_3
0.3503109291E 00	00	45	
0.3534133517E 00	00	54	Δ_4
0.8257962537E 01	01	56	
0.3525645267E 00	00	65	Δ_5
0.3532327211E 00	00	67	
0.6980184135E 01	01	76	Δ_6
0.3530706851E 00	00	78	
0.3530933066E 00	00	87	Δ_7
0.5700386329E 01	01	89	
0.3532211202E 00	00	98	Δ_8
0.3524794624E 00	00	910	
0.4420520158E 01	01	109	Δ_9
0.3534736470E 00	00		
0.3488940761E 00	00		
0.3142895409E 01	01		
0.3548279792E 00	00		
0.3277075781E 00	00		
0.1879110947E 01	01		
0.3628020856E 00	00		
0.2024765711E 00	00		
0.6972892115E 00	00		
0.4099264493E 00	00		
ERROR = 0.0000000000E 00			

Fig. 3.23 Computer Solution for Instability Analysis of 10-storey Steel Model.

3.30 Instability Analysis of 10-storey Steel Frame with One Column Axially Loaded.

The analysis is made, using the force-displacement method, for a 10-storey single bay framework with all columns identical (I_c) and all beams identical (I_b). The analysis, developed in general form, takes account of a

EQN. NO	COND ⁿ	0		1			2			3			4			5			6			7			8			9			10	LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₉	
1	100'	3.817423	-1	-1.323	2.210*	1	1.810*	1.810*	0	1.810*	1.810*	0	1.410*	1.410*	0	1.210*	1.210*	0	910*	910*	0	710*	710*	0	410*	410*	0	210*	210*	0	110*	0
2	0-0'	1	0.16/2	1	0	0.16/2		(23)																								0.5WA
3	211'	2110*	0	1.465713	3.465713	-1	-1.32313	1.610*	1	1.810*	1.810*	0	1.410*	1.210*	0	1.210*	910*	0	910*	710*	0	710*	410*	0	410*	210*	0	210*	110*	0	110*	0
4	012	-1.3123	-1	2.3492	-2.3492	2	1.3123	0	-1			(22)																				0
5	1-1'	0	0	0	1	0.16/2	1	0	0.16/2			(22)																				0.5WA
6	322'	2110*	0	1.810*	1.810*	0	1.465713	3.465713	-1	-1.32313	1.610*	1	1.410*	1.210*	0	1.210*	910*	0	910*	710*	0	710*	410*	0	410*	210*	0	210*	110*	0	110*	0
7	123		(3)		-1.3123	-1	2.3492	-2.3492	2	1.3123	0	-1			(19)																	0
8	2-2'		(3)		0	0	0	1	0.16/2	1	0	0.16/2			(19)																	0.5WA
9	433'	2110*	0	1.810*	1.810*	0	1.810*	1.810*	0	1.465713	3.465713	-1	-1.32313	1.410*	1	1.210*	910*	0	910*	710*	0	710*	410*	0	410*	210*	0	210*	110*	0	110*	0
10	234		(6)				-1.3123	-1	2.3492	-2.3492	2	1.3123	0	-1			(16)															0
11	3-3'		(6)				0	0	0	1	0.16/2	1	0	0.16/2			(16)															0.5WA
12	544'	2110*	0	1.810*	1.810*	0	1.810*	1.810*	0	1.410*	1.410*	0	1.465713	3.465713	-1	-1.32313	910*	1	910*	710*	0	710*	410*	0	410*	210*	0	210*	110*	0	110*	0
13	345				(9)					-1.3123	-1	2.3492	-2.3492	2	1.3123	0	-1			(13)												0
14	4-4'				(9)					0	0	0	1	0.16/2	1	0	0.16/2			(13)												0.5WA
15	655'	2110*	0	1.810*	1.810*	0	1.810*	1.810*	0	1.410*	1.410*	0	1.210*	1.210*	0	1.465713	3.465713	-1	-1.32313	710*	1	710*	410*	0	410*	210*	0	210*	110*	0	110*	0
16	456							(12)					-1.3123	-1	2.3492	-2.3492	2	1.3123	0	-1			(10)								0	
17	5-5'							(12)					0	0	0	1	0.16/2	1	0	0.16/2			(10)								0.5WA	
18	766'	2110*	0	1.810*	1.810*	0	1.810*	1.810*	0	1.410*	1.410*	0	1.210*	1.210*	0	1.465713	3.465713	-1	-1.32313	410*	1	410*	210*	0	210*	110*	0	210*	110*	0	110*	0
19	567										(15)					-1.3123	-1	2.3492	-2.3492	2	1.3123	0	-1			(7)					0	
20	6-6'										(15)					0	0	0	1	0.16/2	1	0	0.16/2			(7)					0.5WA	
21	877'	2110*	0	1.810*	1.810*	0	1.810*	1.810*	0	1.410*	1.410*	0	1.210*	1.210*	0	1.465713	3.465713	-1	-1.32313	210*	1	210*	110*	0	110*	110*	0	210*	110*	0	110*	0
22	678											(15)					-1.3123	-1	2.3492	-2.3492	2	1.3123	0	-1			(4)				0	
23	7-7'										(15)					0	0	0	1	0.16/2	1	0	0.16/2			(4)					0.5WA	
24	988'	2110*	0	1.810*	1.810*	0	1.810*	1.810*	0	1.410*	1.410*	0	1.210*	1.210*	0	1.465713	3.465713	-1	-1.32313	210*	1	210*	110*	0	110*	110*	0	210*	110*	0	110*	0
25	789																(21)					-1.3123	-1	2.3492	-2.3492	2	1.3123	0	-1	(1)	0	
26	8-8'																(21)					0	0	0	1	0.16/2	1	0	0.16/2	(1)	0.5WA	
27	1099'	2110*	0	1.810*	1.810*	0	1.810*	1.810*	0	1.410*	1.410*	0	1.210*	1.210*	0	1.465713	3.465713	-1	-1.32313	210*	1	210*	110*	0	110*	110*	0	210*	110*	0	110*	0
28	8910																						-1.3123	-1	2.3492	-2.3492	2	1.3123	0		0	
29	9-9'																						0	0	0	1	0.16/2	1		0.5WA		
30	109																											-1.3123	-1	2.3492	0	

FIG. 3.22 : ANALYSIS OF STEEL MODEL INCLUDING THE EFFECT OF ELASTIC INSTABILITY :

○ DENOTES NUMBER OF ZEROS

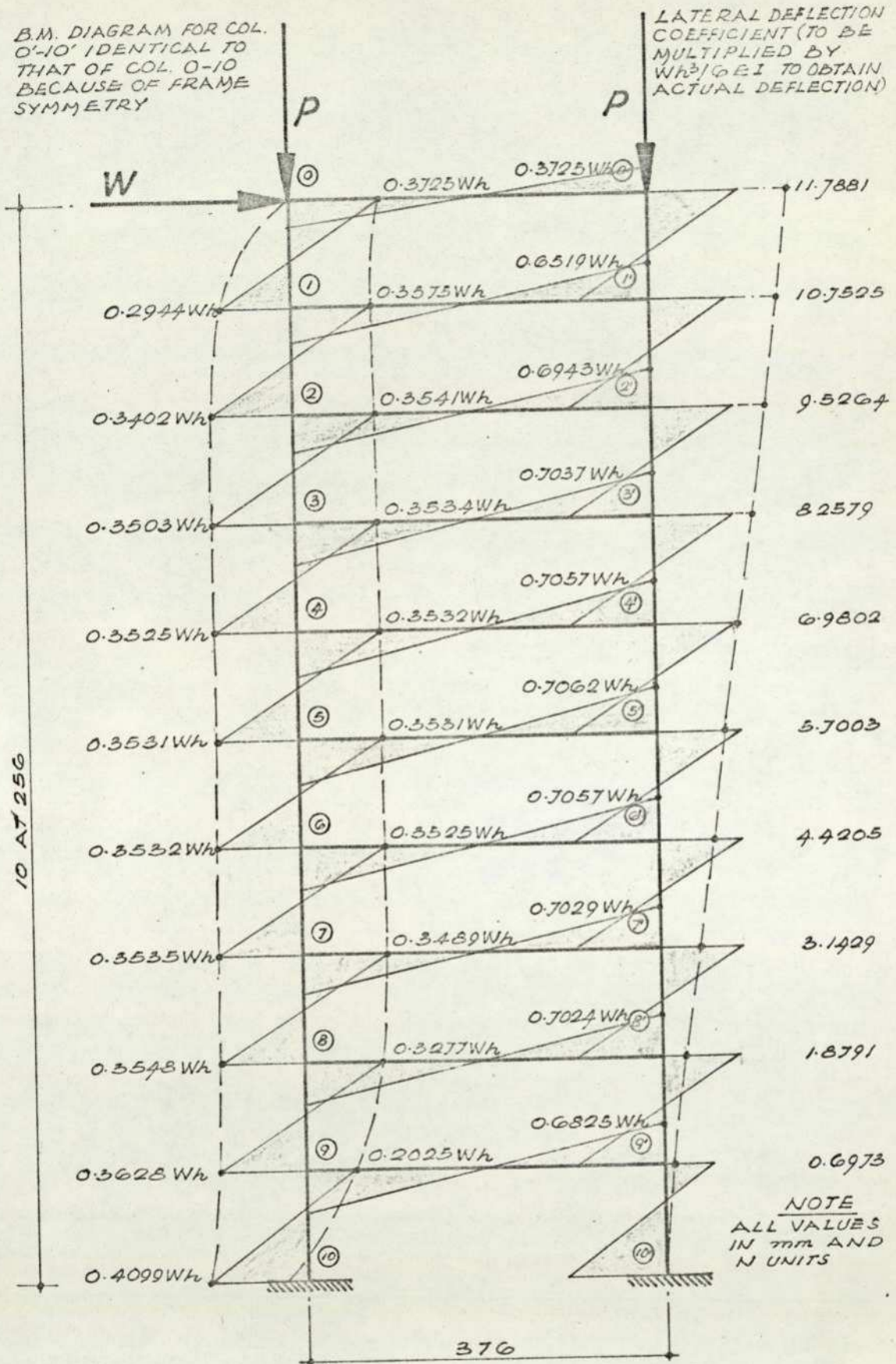


FIG. 3.24 : MOMENT DEFLECTION DIAGRAM FOR 10-STOREY STEEL MODEL. - INSTABILITY EFFECTS.

single lateral load W acting at the top of the frame, and various axial loads acting at each beam/column node over the full height of one side of the 10-storey frame. The idealised structure is shown in Fig. 3.31.

Compatibility of deformation at O

$$100' \quad -M_{10} \frac{h}{6EI_c} + M_{01} \left(\frac{h}{3EI_c} + \frac{L}{3EI_b} \right) - M_{01'} \frac{h}{6EI_b} = \frac{\Delta_o - \Delta_1}{h}$$

Multiplying this and subsequent compatibility of deformation equations by $6EI_c/h$ and equilibrium condition equations by h results in non-dimensional factors of M .

$$-M_{10}r + M_{01}(2+2r) - M_{01'}r - \left(\frac{\Delta_o - \Delta_1}{h} \right) \frac{6EI_c}{h^2} = 0 \quad \text{-----} \quad 3.301$$

Compatibility of deformation at O'

$$00'1' \quad -M_{01} \frac{L}{6EI_b} + M_{01'} \left(\frac{L}{3EI_b} + \frac{hc_{01}}{3EI_c} \right) - M_{10} \frac{hc_{01}}{6EI_c} = \frac{\Delta_o - \Delta_1}{h}$$

$$-M_{01}r + M_{01'}(2r + 2c_{01}) - M_{10}c_{01} - \left(\frac{\Delta_o - \Delta_1}{h} \right) \frac{6EI_c}{h^2} = 0 \quad \text{-----} \quad 3.302$$

Equilibrium condition O-O'

$$\frac{M_{01} + M_{10}}{h} + \frac{M_{01'} + M_{10'}}{h} = W + P_o \left(\frac{\Delta_o - \Delta_1}{h} \right)$$

$$M_{01} + M_{10} + M_{01'} + M_{10'} - P_o \Delta_o + P_o \Delta_1 = Wh \quad \text{-----} \quad 3.303$$

Compatibility of deformation at 1

$$211' \quad -M_{21} \frac{h}{6EI_c} + M_{12} \frac{h}{3EI_c} + (M_{10} + M_{12}) \frac{L}{3EI_b} - (M_{10'} + M_{12'}) \frac{L}{3EI_b} = \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{21} + M_{10} 2r + M_{12}(2+2r) - M_{10'}r - M_{12'}r - \left(\frac{\Delta_1 - \Delta_2}{h} \right) \frac{6EI_c}{h^2} = 0 \quad \text{-----} \quad 3.304$$

$$012 \quad -M_{01} \frac{h}{6EI_c} + M_{10} \frac{h}{3EI_c} - M_{12} \frac{h}{3EI_c} + M_{21} \frac{h}{6EI_c} = \frac{\Delta_o - \Delta_1}{h} - \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{01} + 2M_{10} - 2M_{12} + M_{21} - \left(\Delta_o - 2\Delta_1 + \Delta_2 \right) \frac{6EI_c}{h^2} = 0 \quad \text{-----} \quad 3.305$$

Compatibility of deformation at 1'

$$11'2 \quad -\left(M_{10} + M_{12} \right) \frac{L}{6EI_b} + \left(M_{10'} + M_{12'} \right) \frac{L}{3EI_b} + M_{12} \frac{hc_{11}}{3EI_c} - M_{21} \frac{hc_{21}}{6EI_c} = \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{10}r - M_{12}r + M_{10'} 2r + M_{12'}(2r + 2c_{11}) - M_{21}c_{21} -$$

$$- \left(\Delta_1 - \Delta_2 \right) \frac{6EI_c}{h^2} = 0 \quad \text{-----} \quad 3.306$$

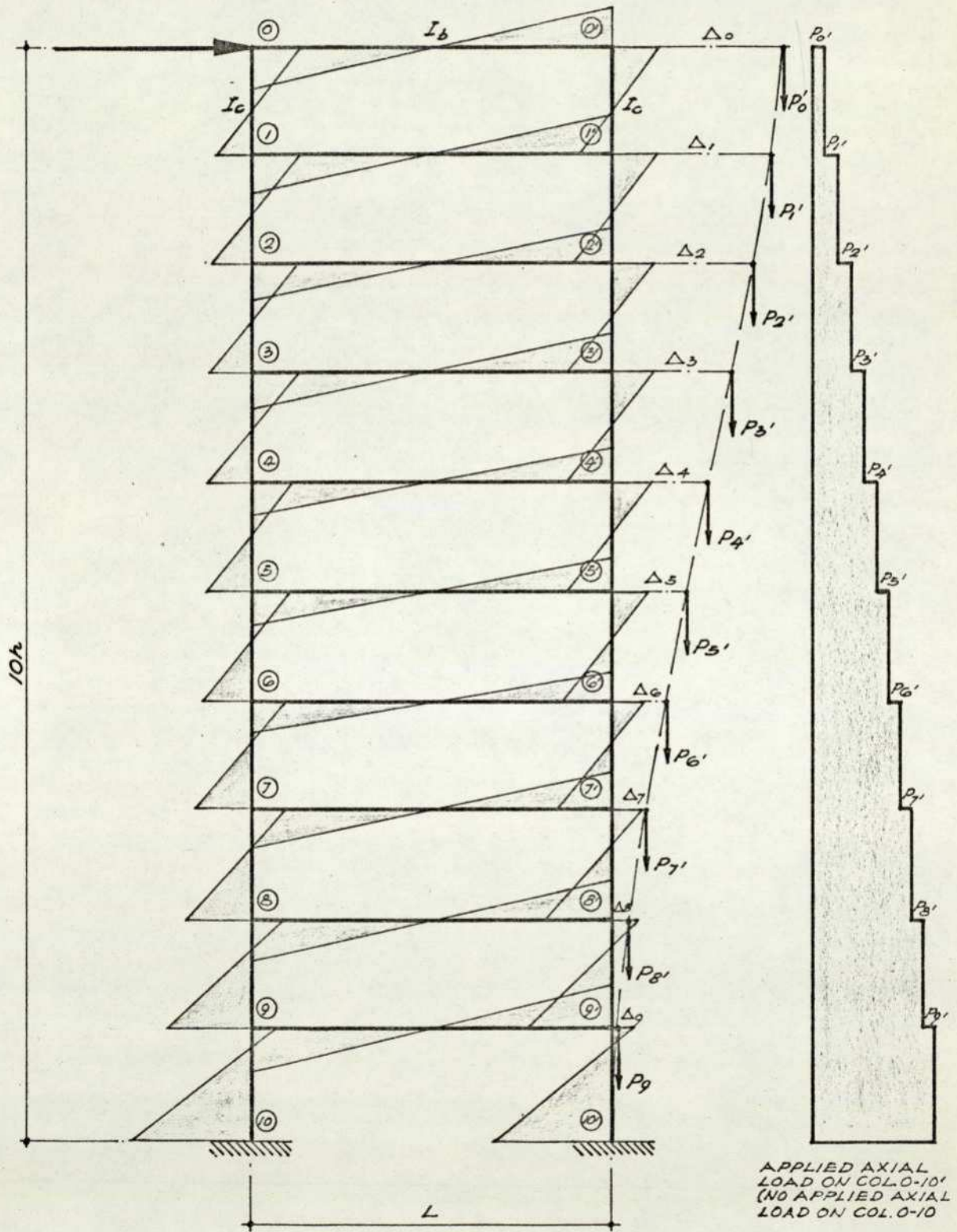


FIG. 3.31 IDEALISED FRAME WITH AXIAL LOADS ON ONE COLUMN, SECONDARY ANALYSIS FOR 10-STOREY STEEL FRAME.

$$0'1'2' -M_{0'1'} \frac{h c_{0'1'}}{6EI_c} + M_{1'0} \frac{h c_{1'0}}{3EI_c} - M_{1'2} \frac{h c_{1'2}}{3EI_c} + M_{2'1} \frac{h c_{2'1}}{6EI_c} =$$

$$= \frac{(\Delta_0 - 2\Delta_1 + \Delta_2)}{h}$$

where $c_{1'0}$ is flexibility coefficient for 1'0 at 1' and

$c_{1'2}$ is flexibility coefficient for 1'2 also at point 1'

$$-M_{0'1'} c_{0'1'} + M_{1'0} 2c_{1'0} - M_{1'2} 2c_{1'2} + M_{2'1} c_{2'1} -$$

$$- (\Delta_0 - 2\Delta_1 + \Delta_2) \frac{6EI_c}{h^2} = 0 \text{ ----- 3.307}$$

Equilibrium condition 1-2

$$\frac{M_{12} + M_{21}}{h} + \frac{M_{1'2} + M_{2'1}}{h} = W + P_0' \frac{(\Delta_0 - \Delta_2)}{2h} + P_1' \frac{(\Delta_1 - \Delta_2)}{h}$$

$$M_{12} + M_{21} + M_{1'2} + M_{2'1} - \Delta_0 \frac{P_0'}{2} - \Delta_2 \left(\frac{P_0'}{2} + P_1' \right) + \Delta_1 P_1' = 0 \text{ -- 3.308}$$

Compatibility of deformation at 2

$$322' -M_{32} \frac{h}{6EI_c} + M_{23} \frac{h}{3EI_c} + (M_{21} + M_{23}) \frac{L}{3EI_b} - (M_{2'1} + M_{2'3}) \frac{L}{3EI_b} =$$

$$= \frac{\Delta_2 - \Delta_3}{h}$$

$$-M_{32} + M_{23} (2 + 2r) + M_{21} 2r - M_{2'1} r - M_{2'3} r -$$

$$-(\Delta_2 - \Delta_3) \frac{6EI_c}{h^2} = 0 \text{ ----- 3.309}$$

$$123 -M_{12} \frac{h}{6EI_c} + M_{21} \frac{h}{3EI_c} - M_{23} \frac{h}{3EI_c} + M_{32} \frac{h}{6EI_c} = \frac{\Delta_1 - \Delta_2}{h} - \frac{\Delta_2 - \Delta_3}{h}$$

$$-M_{12} + 2M_{21} - 2M_{23} + M_{32} - (\Delta_1 - 2\Delta_2 + \Delta_3) \frac{6EI_c}{h^2} = 0 \text{ ----- 3.310}$$

Compatibility of deformation at 2'

$$22'3' - (M_{21} + M_{23}) \frac{L}{6EI_b} + (M_{2'1} + M_{2'3}) \frac{L}{3EI_b} + M_{2'3} \frac{h c_{2'2}}{3EI_c} - M_{3'2} \frac{h c_{2'3}}{6EI_c} = \frac{\Delta_2 - \Delta_3}{h}$$

$$-M_{21} r - M_{23} r + M_{2'1} 2r + M_{2'3} (2r + 2c_{2'2}) - M_{3'2} c_{2'3} - (\Delta_2 - \Delta_3) \frac{6EI_c}{h^2} = 0 \text{ -- 3.311}$$

$$1'2'3' -M_{1'2} \frac{h c_{1'2}}{6EI_c} + M_{2'1} \frac{h c_{2'1}}{3EI_c} - M_{2'3} \frac{h c_{2'3}}{3EI_c} + M_{3'2} \frac{h c_{3'2}}{6EI_c} = \frac{\Delta_1 - \Delta_2}{h} - \frac{\Delta_2 - \Delta_3}{h}$$

$$-M_{1'2} c_{1'2} + M_{2'1} 2c_{2'1} - M_{2'3} 2c_{2'3} + M_{3'2} c_{3'2} -$$

$$-(\Delta_1 - 2\Delta_2 + \Delta_3) \frac{6EI_c}{h^2} = 0 \quad \text{-----} \quad 3.312$$

Equilibrium condition 2-3

$$\frac{M_{23} + M_{32}}{h} + \frac{M_{2'3} + M_{3'2}}{h} = W + \frac{P_0(\Delta_1 - \Delta_3)}{3h} + \frac{P_1(\Delta_1 - \Delta_3)}{2h} + \frac{P_2(\Delta_2 - \Delta_3)}{h}$$

$$M_{23} + M_{32} + M_{2'3} + M_{3'2} = W + \Delta_1 \frac{P_0}{3h} + \Delta_1 \frac{P_1}{2h} + \Delta_2 \frac{P_2}{h} -$$

$$-\Delta_3 \left(\frac{P_0}{3h} + \frac{P_1}{2h} + \frac{P_2}{h} \right) \quad \text{-----} \quad 3.313$$

Remainder of equations can be compiled in matrix form and solved in the usual way.

3.40 Experiments on 10-storey Steel Model

To increase the sensitivity of the frame to high order effects of elastic instability a very slender frame was constructed in which beams and columns were of identical section $\frac{1}{4}$ inch (6.35mm) square, made from ordinary mild steel. The 10-storey single bay model has a span of 376 mm and storey height 256 mm. Fig. 3.41 shows the position of strain and dial gauges as well as the arrangement of lateral forces and one arrangement of gravity loads. Tests were performed for 3 gravity load conditions: axial load P applied only at the top of both columns; axial loads of equal magnitude applied at each node to simulate more nearly the condition in practice where successive floor loads are carried throughout the height of the building; and axial load applied to one column at nodal points.

The four loading conditions are shown in Fig. 3.42. The case a) is used as a control test.

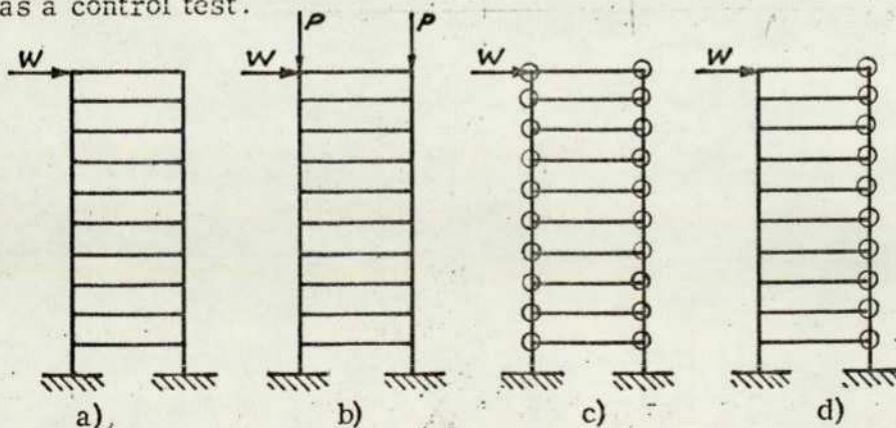


Fig. 3.42 Load Test Conditions.

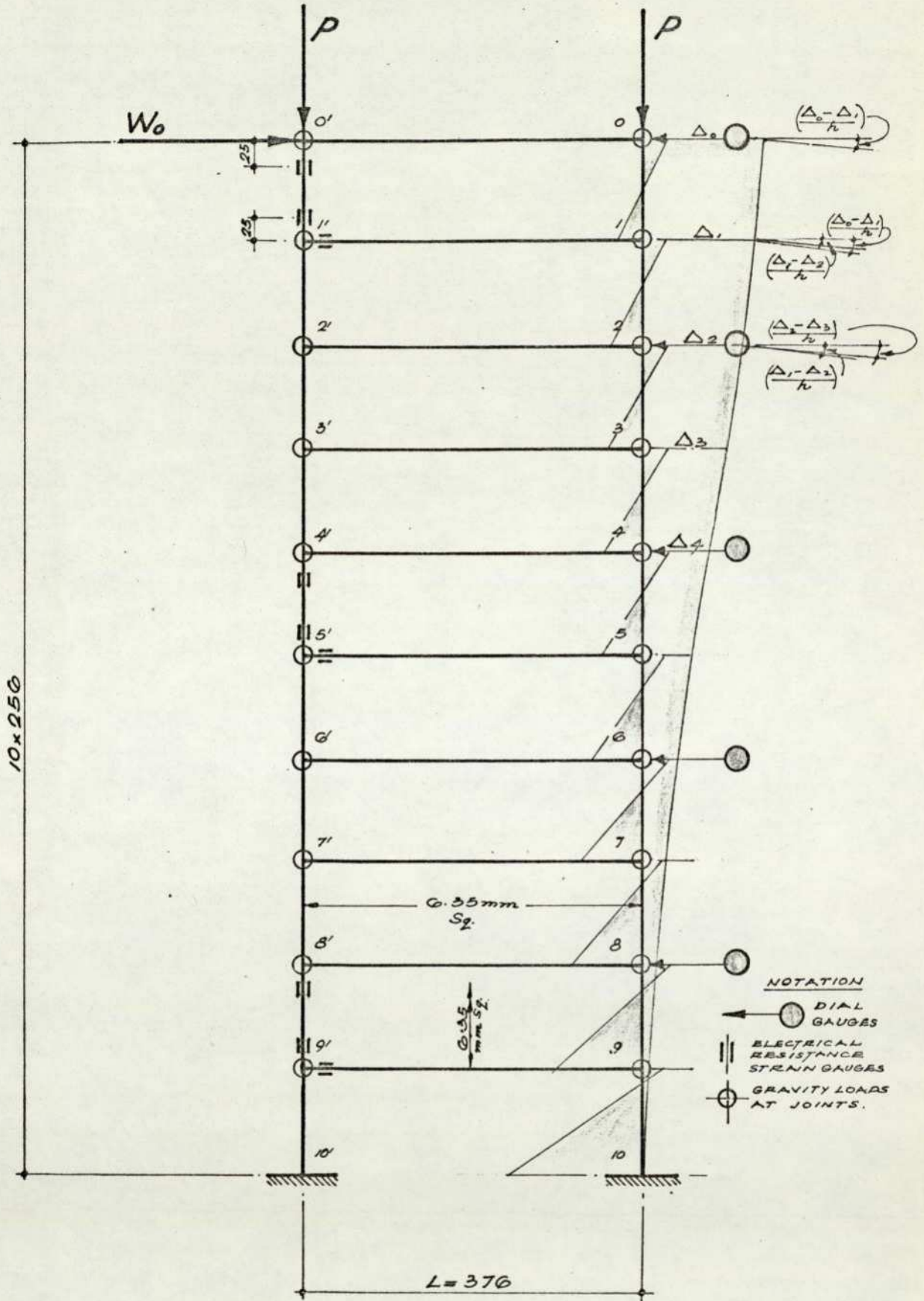


FIG. 3.41 GENERAL ARRANGEMENT OF THE 10-STOREY STEEL MODEL

The experimental apparatus is shown in Fig. 3.43, 3.44 and 3.45.

For the primary load test (Fig. 3.42, case a)) the load deflection performance is shown in Fig. 3.46 for a load limited to 40N. This gave a Δ_o/W ratio of 0.857 mm N⁻¹

In the case of the test taking instability into account (Fig. 3.42, case b)) the non-linear load deflection curve is shown in Fig. 3.47.

3.50 Discussion of Test Results for 10-storey Steel Model

$$\text{Lateral deflection coefficient} \times \frac{Wh^3}{6EI}$$

where lateral deflection coefficient is 9.1123 from primary analysis (Fig. 3.13).

$$\frac{\Delta_o}{W} = \frac{9.1123 \times 256^3}{6 \times 200,000 \times 135.5}$$

$$\frac{\Delta_o}{W} = 0.9402 \text{ mm.N}^{-1} \text{ for primary effects} \quad \text{----- 3.51}$$

Similarly for instability analysis, with axial load P = 400 N,

$$\Delta_o = \text{lateral deflection coefficient} \times \frac{Wh^3}{6EI}$$

$$\frac{\Delta_o}{W} = 11.7881 \times \frac{h^3}{6EI} \quad \text{from Fig. 3.24}$$

$$\frac{\Delta_o}{W} = 1.2163 \text{ mm.N}^{-1} \text{ for instability effects} \quad \text{----- 3.52}$$

Equations 3.51 and 3.52 enable the theoretical load deflection curves to be plotted.

Comparison of data in Figs. 3.13 and 3.24 shows that the instability deflection at the top of frame is 29% more than that for the primary analysis. Similarly, moments are greater. This is shown in Fig. 3.50.

A comparison of theoretical and experimental load deflection curves for primary and instability effects reveals good agreement as shown by Fig. 3.51. It is seen that the analytical curve for instability effect is linear whereas actual deflections give rise to a curvi-linear characteristic resulting in greater deflections.

Moments, both theoretical and experimental, are compared for node 9; the particular moment chosen is M₉₈. The characteristics are shown in Figs. 3.52 to 3.54.

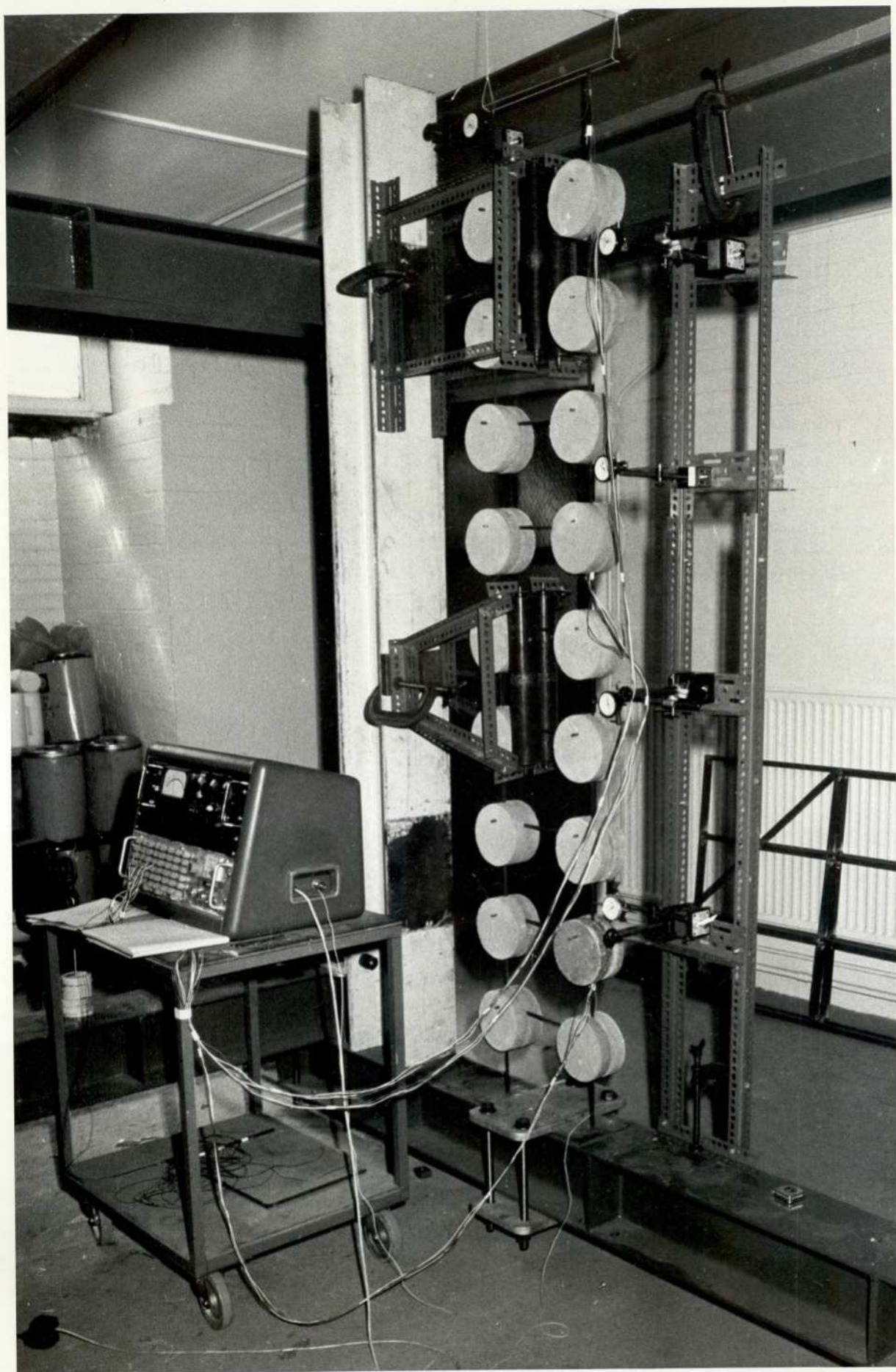


Fig. 3.43 10-storey Steel Model Subjected to Instability Test with Gravity Loads at each Node.

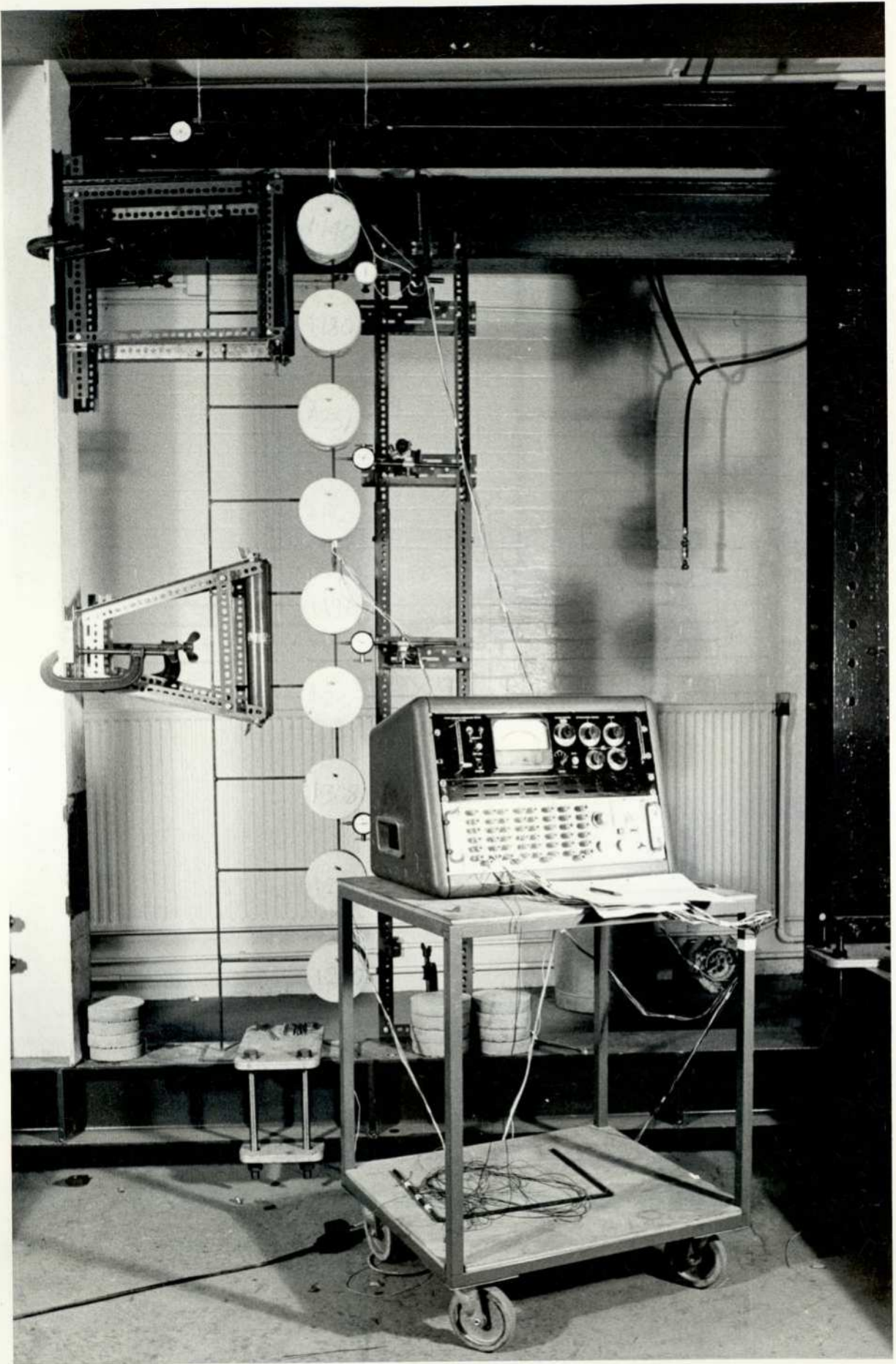


Fig. 3.44 Instability Test on 10-storey Steel Model with Gravity Loads
on One Column.

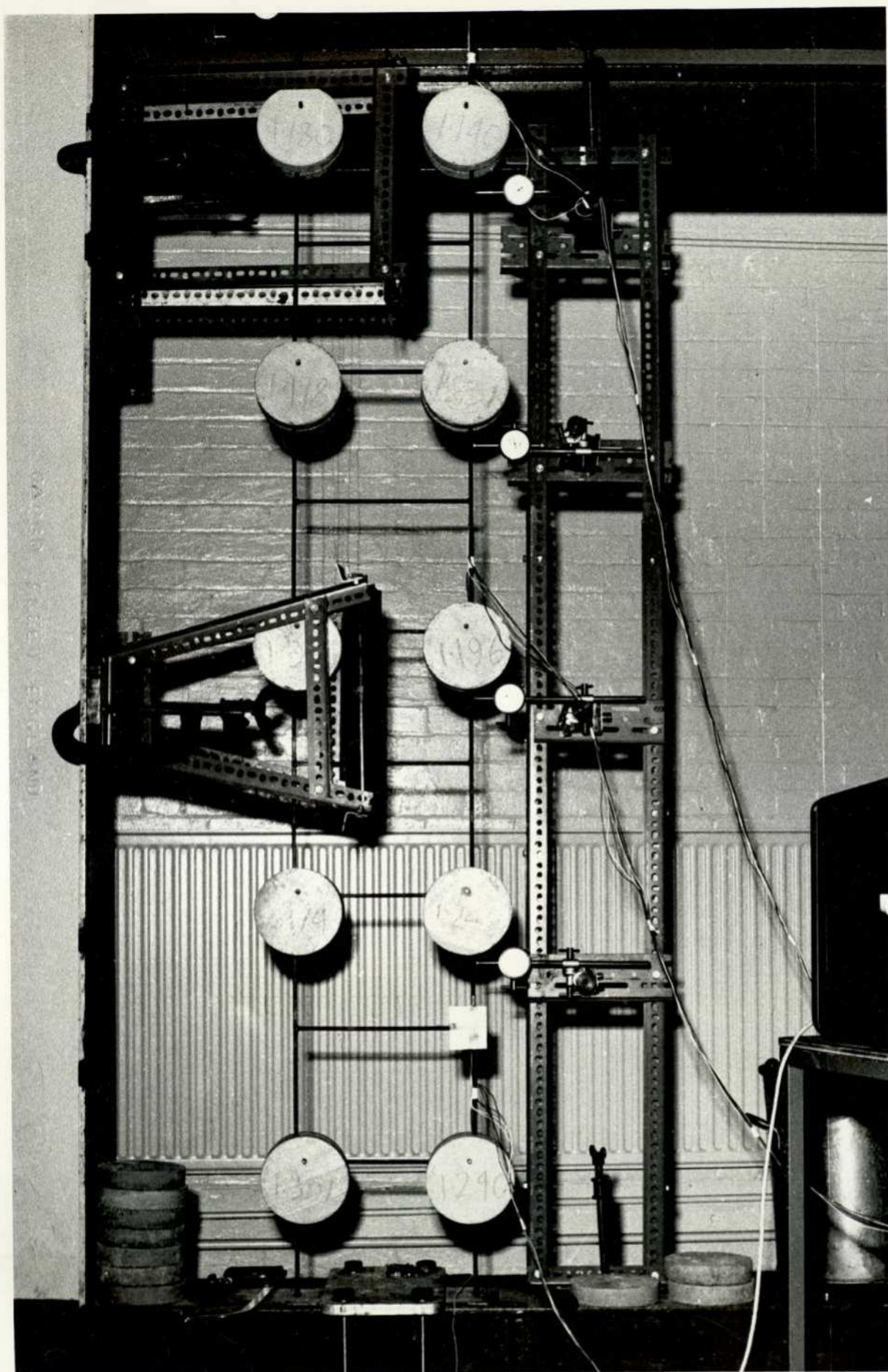


Fig. 3.45 10-storey Steel Model with Gravity Loads at Every Other Node.

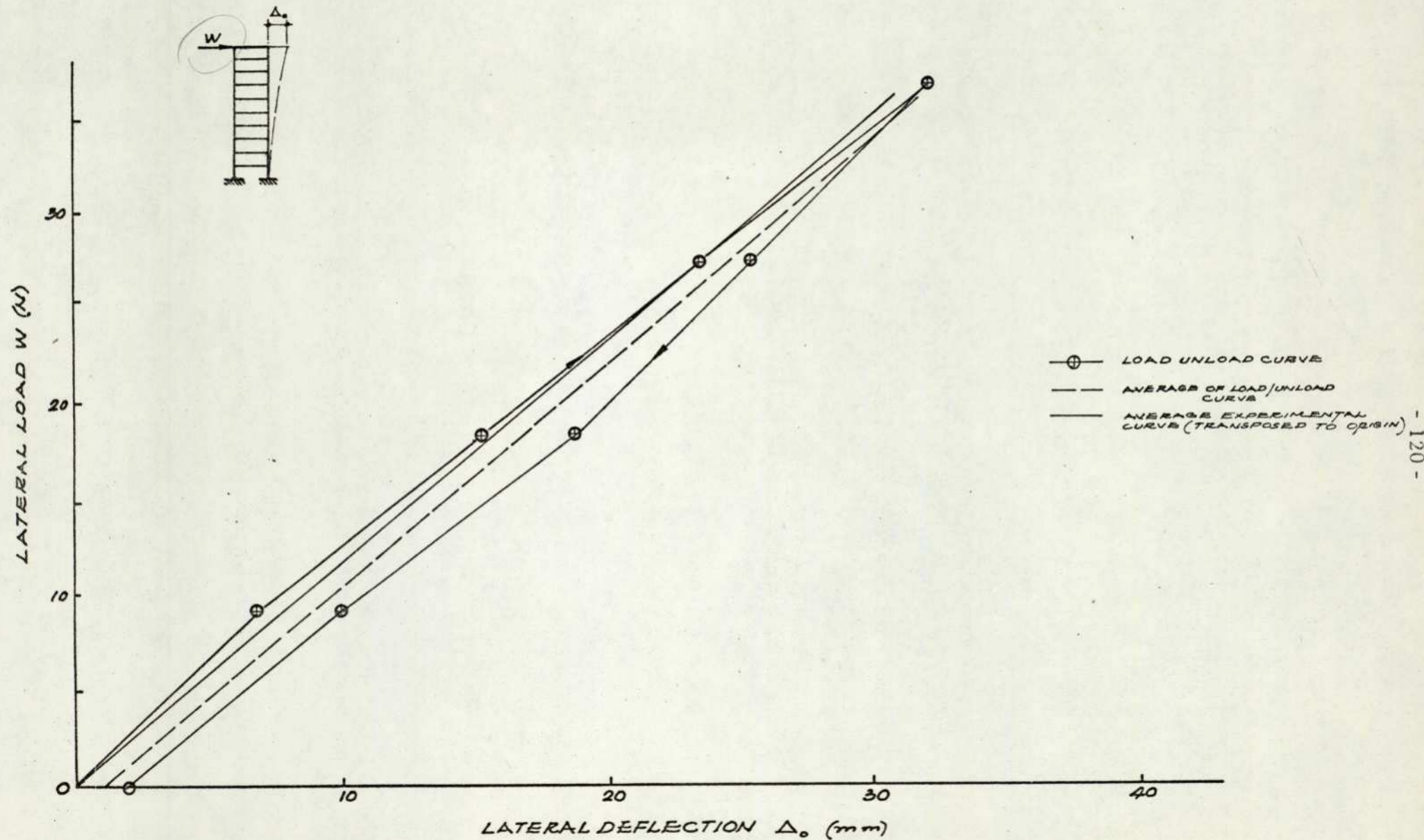


FIG. 3.46. EXPERIMENTAL LOAD/DEFLECTION CURVES, PRIMARY ANALYSIS -
- 10 STOREY STEEL MODEL

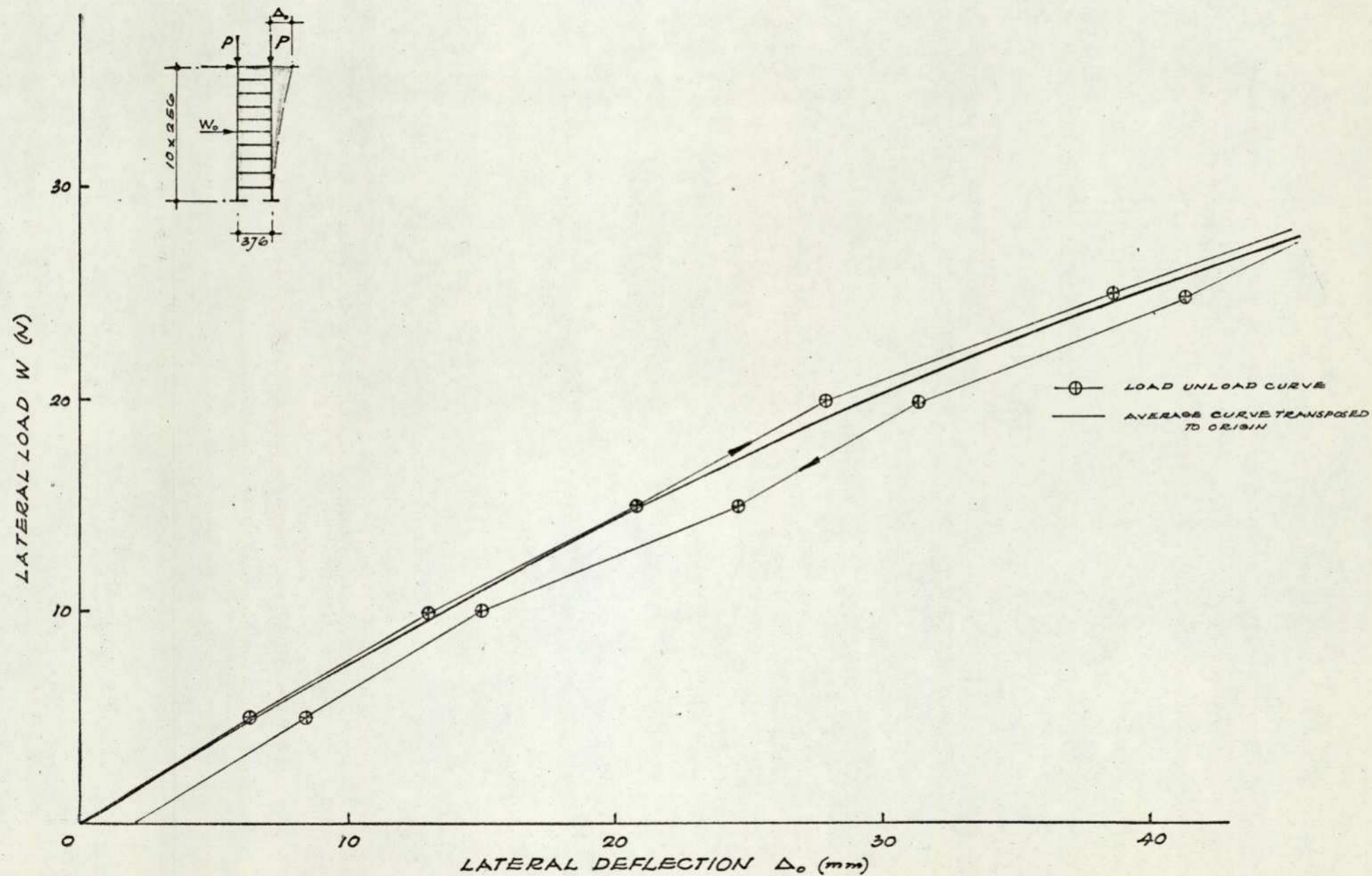


FIG. 3.47. EXPERIMENTAL LOAD/DEFLECTION CURVE, INSTABILITY ANALYSIS
-10 STOREY STEEL MODEL

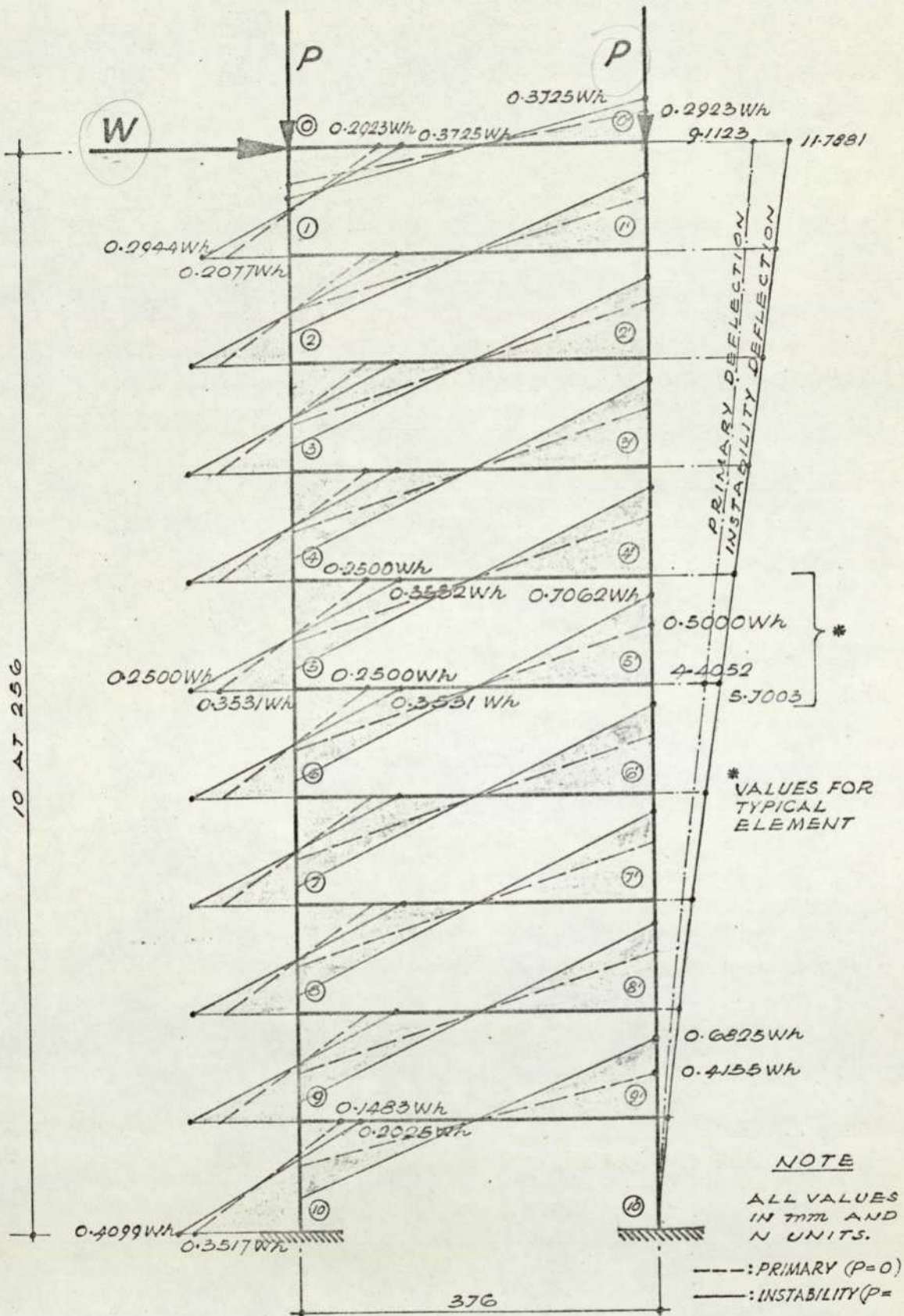


FIG. 3.50 COMPARISON OF MOMENT/DEFLECTION DIAGRAMS FOR PRIMARY AND INSTABILITY ANALYSES OF 10-STOREY STEEL FRAME.

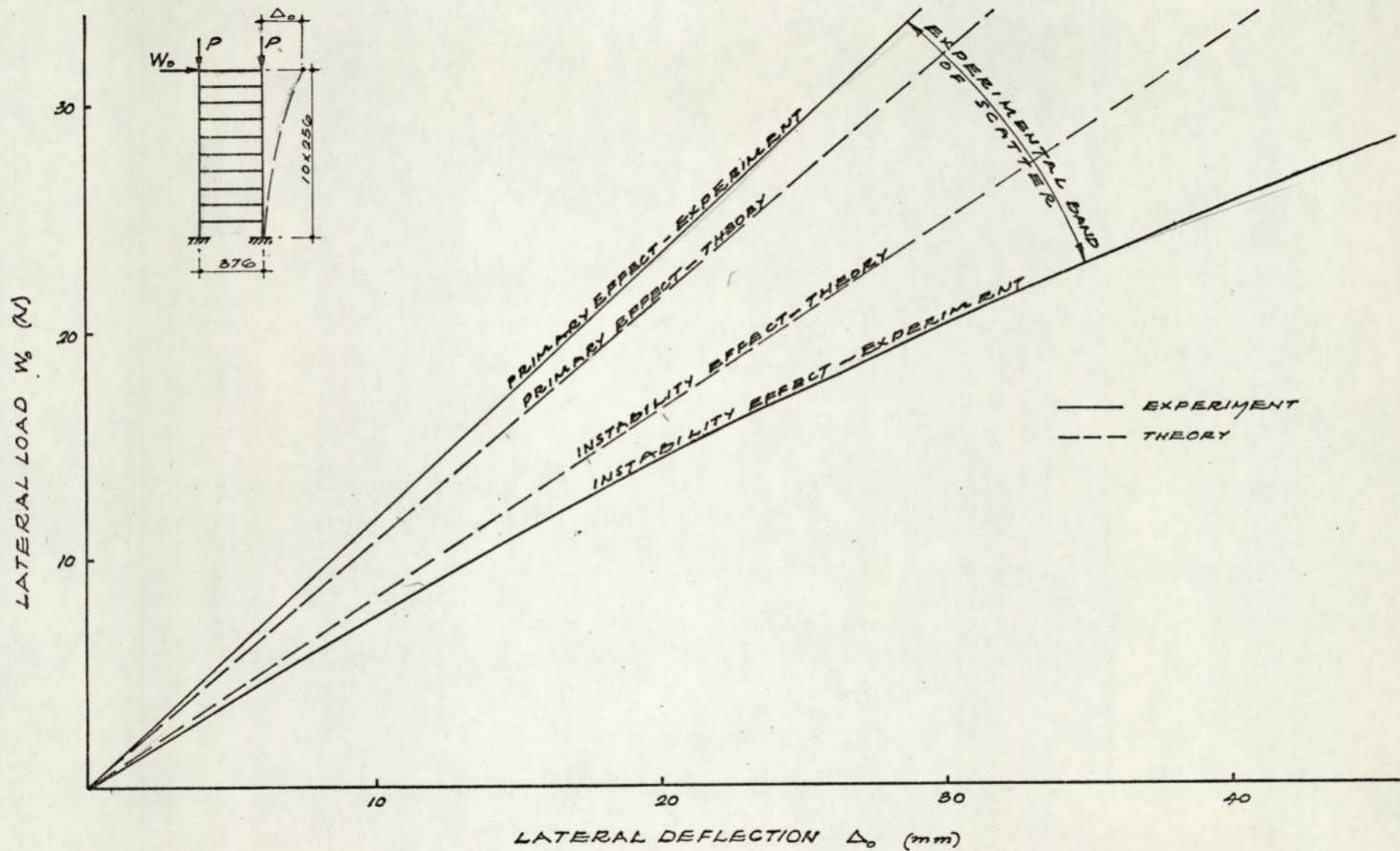


FIG. 3.51 THEORETICAL AND EXPERIMENTAL LOAD DEFLECTION CURVES FOR 10-STOREY STEEL MODEL

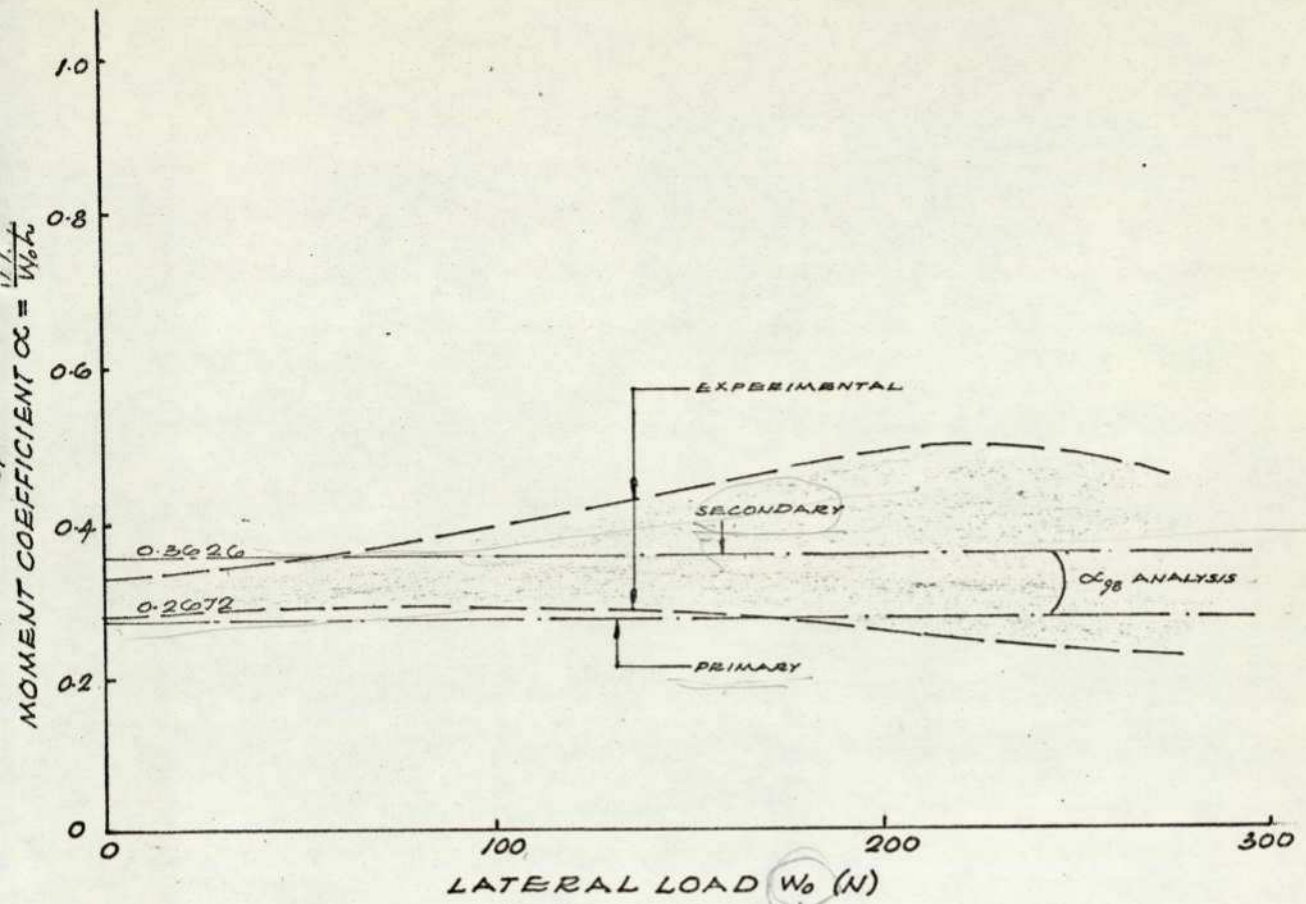


FIG. 3.52 DIAGRAM OF MOMENT COEFFICIENT α_{98} VS LOAD W

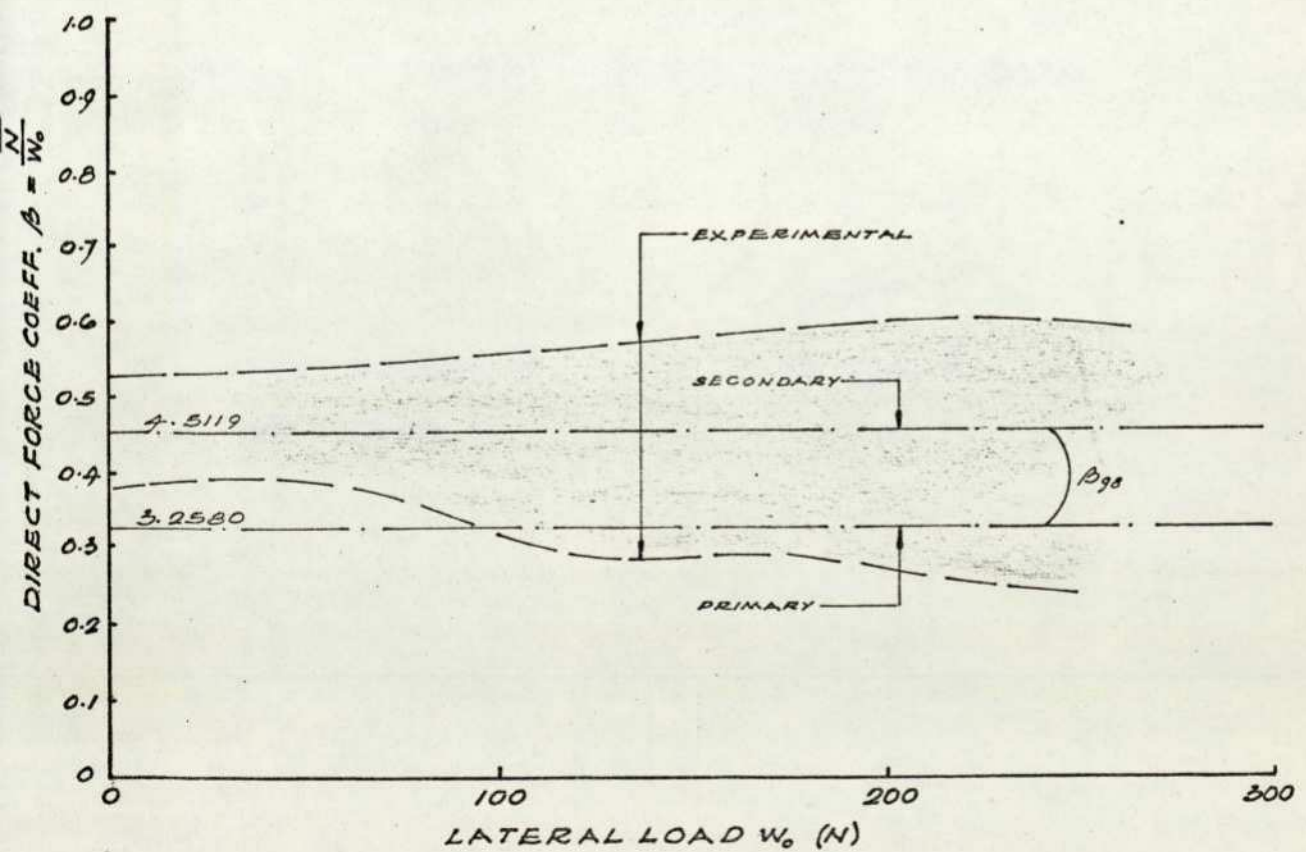


FIG. 3.53 DIAGRAM OF DIRECT FORCE COEFFICIENT β_{98} VS LOAD W

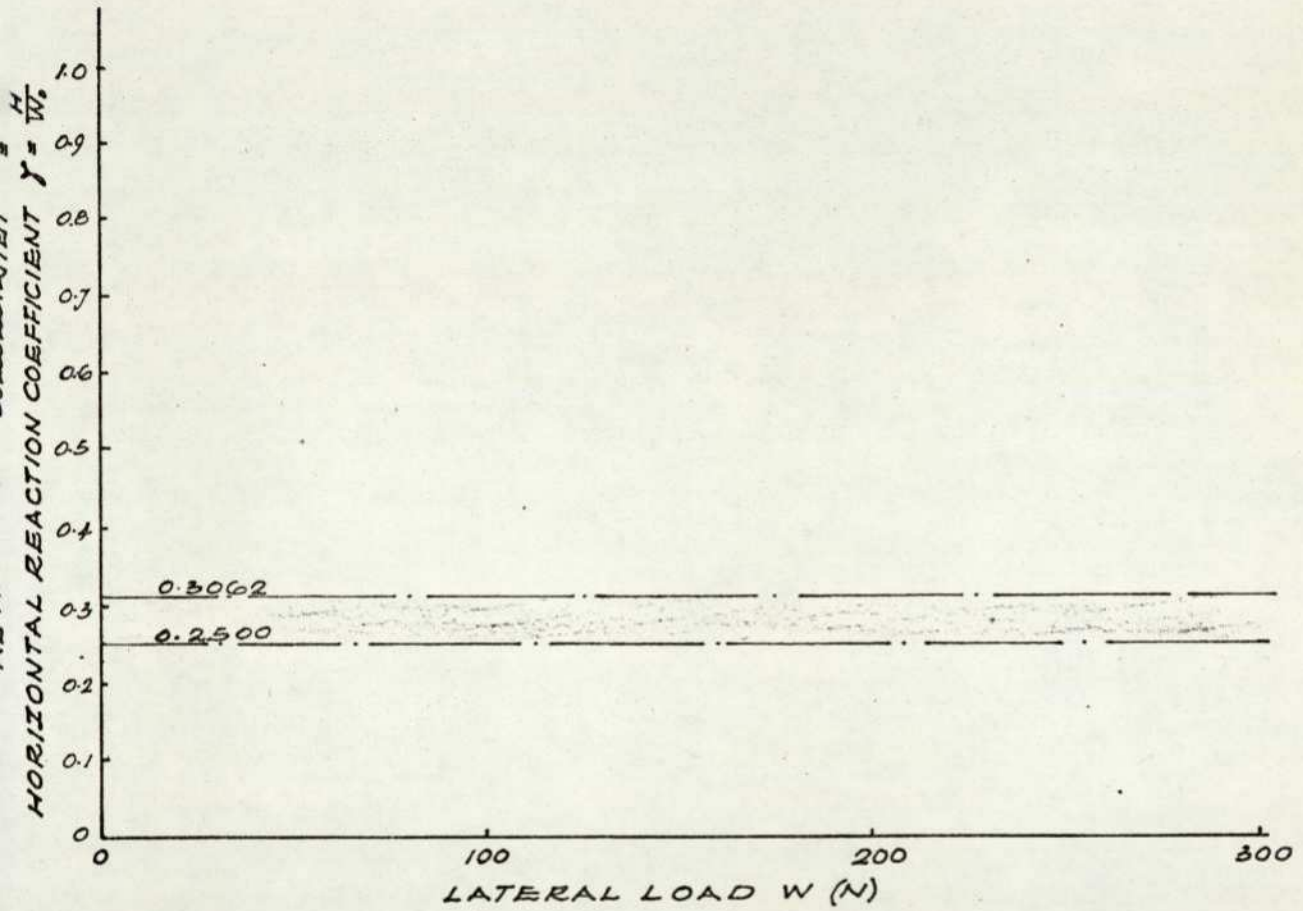


FIG. 3.54 DIAGRAM OF HORIZONTAL REACTION COEFFICIENT γ
 V_s LOAD W

4.00 Concrete Frames with 2 columns : theory and experiment.

Two frames, one 10-storey the other 6-storey were analysed, made and tested for primary and instability effects. For each frame the model is subjected to a single lateral load applied by a hydraulic jack at the top of the frame. Instability effects are examined under the action of varying lateral load at the frame top and a constant axial load applied to each column again at the top of the frame.

The purpose of this work was to develop primary and instability analyses and compare the experimental behaviour with the predicted values of deflections and bending moments.

4.10 Primary Analysis, 10-storey Concrete Frame

Fig. 4.11 shows the idealised form of structure and assumed shapes of deflection and bending moment diagrams.

The primary analysis is carried out using the following data:

$$\text{Columns : } 50 \times 100\text{mm} ; I_c = \frac{1}{12} \times 50 \times 100^3 = 4.16 \times 10^6 \text{ mm}^4$$

$$\text{Beams : } 50 \times 50\text{mm} ; I_b = \frac{1}{12} \times 50^4 = 0.52 \times 10^6 \text{ mm}^4$$

$$\text{Ratio } n = \frac{I_c}{I_b} = 8.0$$

The compatibility condition at the point O is:

$$O \quad M_{o1} \left(\frac{L}{6EI_b} + \frac{h}{3EI_c} \right) - M_{10} \frac{h}{6EI_c} = \frac{\Delta_o - \Delta_1}{h}$$

$$M_{o1} \left(\frac{nL}{6} + \frac{h}{3} \right) - M_{10} \frac{h}{6} + (\Delta_1 - \Delta_o) \frac{EI_c}{h} = 0$$

$$\text{or } 860M_{o1} - 30M_{10} + (\Delta_1 - \Delta_o) \frac{EI_c}{h} = 0 \quad \text{----- 4.11}$$

Equilibrium condition at O becomes:

$$O - O' \quad \frac{M_{o1} + M_{10}}{h} = \frac{1}{2} \cdot W_o$$

$$M_{o1} + M_{10} = 90W_o \quad \text{----- 4.12}$$

Similarly, the element matrix at point one, formed of two compatibility conditions marked as 211' and 012, and one equilibrium condition 1-1', are as follows:-

$$211' \quad (M_{10} + M_{12}) \frac{L}{6EI_b} + M_{12} \frac{h}{3EI_c} - M_{21} \frac{h}{6EI_c} = \frac{\Delta_1 - \Delta_2}{h}$$

Multiplying by EI_c

$$M_{10} \frac{2L}{6} + M_{12} \left(\frac{2L}{6} + \frac{h}{3} \right) - M_{21} \frac{h}{6} + (\Delta_2 - \Delta_1) \frac{EI_c}{h} = 0$$

$$800M_{10} + 860M_{12} - 30M_{21} + (\Delta_2 - \Delta_1) \frac{EI_c}{h} = 0 \quad \text{----- 4.13}$$

$$012 -M_{01} \frac{h}{6EI_c} + M_{10} \frac{h}{3EI_c} - M_{12} \frac{h}{3EI_c} + M_{21} \frac{h}{6EI_c} = \frac{\Delta_0 - \Delta_1}{h} - \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{01} \frac{h}{6} + M_{10} \frac{h}{3} - M_{12} \frac{h}{3} + M_{21} \frac{h}{6} + (-\Delta_0 + 2\Delta_1 - \Delta_2) \frac{EI_c}{h} = 0$$

$$-30M_{01} + 60M_{10} - 60M_{12} + 30M_{21} + (-\Delta_0 + 2\Delta_1 - \Delta_2) \frac{EI_c}{h} = 0 \quad \text{----- 4.14}$$

$$1-1' \quad \frac{M_{12} + M_{21}}{h} - \frac{M_{01} + M_{10}}{h} = \frac{W_1}{2}$$

$$M_{12} + M_{21} - M_{01} - M_{10} = \frac{W_1 h}{2} = 90W_1 \quad \text{----- 4.15}$$

The main matrix can now be set out as shown in Fig. 4.12.

	SOLUTIONS	M	Δ'
0	0.7179720056E 02 0.549329273 E 06	01	Δ_0
1	0.1820279931E 02 0.5962377141E 02 0.4881297645E 06	10 12	Δ_1
2	0.3037522753E 02 0.5074069188E 02 0.4232023685E 06	21 23	Δ_2
3	0.3925930791E 02 0.4314926720E 02 0.3564421707E 06	32 34	Δ_3
4	0.4685073206E 02 0.3514142625E 02 0.2893318765E 06	43 45	Δ_4
5	0.5485857164E 02 0.2491541104E 02 0.2232754197E 06	54 56	Δ_5
6	0.6508458945E 02 0.1017035953E 02 0.1599138474E 06	65 67	Δ_6
7	0.7982964005E 02 -0.1241136012E 02 0.1014945559E 06	76 78	Δ_7
8	0.1024113597E 03 -0.4791063577E 02 0.5137695433E 05	87 89	Δ_8
9	0.1379196354E 03 -0.1043148044E 03 0.1478833237E 05	98 910	Δ_9
10	0.1943148044E 03	109	
ERROR = 0.1854366726E-07			

Fig. 4.13 Computer Solution for Primary Analysis of 10-storey concrete model (for $W_0 = 1$)

EQN. NO	COND ⁿ	0		1			2			3			4			5			6			7			8			9			10	LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₂	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₉	
1	100'	800	-1	-30	0	1								(25)																		0
2	0-0'	1	0	1	0	0								(25)																		90W ₀
3	211'	0	0	800	860	-1	-30	0	1							(22)																0
4	012	-30	-1	60	-60	2	30	0	-1							(22)																0
5	1-1'	-1	0	-1	1	0	1	0	0							(22)																90W ₁
6	322'		(3)														(19)															0
7	123		(3)				FE										(19)															0
8	2-2'		(3)														(19)															90
9	433'			(6)																(16)												0
10	234			(6)						FE										(16)												0
11	3-3'			(6)																(16)												90
12	544'						(9)															(13)										0
13	345						(9)						FE									(13)										0
14	4-4'						(9)															(13)										90
15	655'									(12)																(10)						0
16	456									(12)					FE											(10)						0
17	5-5'									(12)																(10)						90
18	766'												(15)															(7)				0
19	567												(15)				FE											(7)				0
20	6-6'												(15)															(7)				90
21	877'															(18)														(4)		0
22	678															(18)						FE								(4)		0
23	7-7'															(18)														(4)		90
24	988'																		(21)											(1)		0
25	789																		(21)				FE							(1)		0
26	8-8'																		(21)											(1)		90
27	1000'																					(24)										0
28	8910																					(24)										0
29	9-9'																					(24)										90
30	109																									(27)		-30	-1	60		0

FIG. 4-12 MATRIX FOR PRIMARY ANALYSIS, 10-STOREY CONCRETE MODEL

○ - DENOTES NUMBER OF ZEROS

Fig. 4.13 shows the solution of this matrix for $W_o = 1$ and for all other loads equal to zero. The diagram, Fig. 4.14, shows primary moments, forces and deformations which will be used to compare it with the case when secondary effects are included.

Note that actual deflection $\Delta = \text{Computer solution} \times \frac{Wh}{EI_c}$

$$\text{For example, } \Delta_o = \frac{0.54933 \times 10^6 \times 180 \text{ W}}{25,000 \times 4.16 \times 10^6} = 0.0009492W$$

4.20 Instability analysis, 10-storey concrete frame

The instability analysis is carried out on the concrete model shown in Fig. 4.11. An axial load of 35 kN is applied to both columns. The following data were used:

$$L = 600 \text{ mm}; \quad h = 180 \text{ mm}; \quad n = \frac{I_c}{I_b} = 8$$

$$I_c = \frac{1}{12} 50 \times 100^3 = 4.16 \times 10^6 \text{ mm}^4$$

$$\text{For } P = 35 \text{ kN}, \quad \sigma_c = \frac{35000}{50 \times 100} = 7 \text{ N/mm}^2$$

$$\text{From } 2U = L \sqrt{\frac{P}{EI_c}} = 0.37944$$

Therefore from Fig. 1.071, $C_{oo} = 1.0105$ and $C_{oi} = 1.0175$

Setting out the compatibility of deformation conditions, we have for point O:

$$O \quad M_{oi} \left(\frac{L}{6EI_b} + \frac{C_{oo}h}{3EI_c} \right) - M_{io} \frac{C_{oi}h}{6EI_c} = \frac{\Delta_o - \Delta_i}{h} - \frac{2\delta v_o}{L}$$

$$\delta v_o = \frac{2M_{oi}}{10hEA_c} + \frac{2(M_{io} + M_{i2})}{9hEA_c} + \text{etc.}$$

$$= \frac{2}{hEA_c} \left[\frac{M_{oi}}{10} + \frac{(M_{io} + M_{i2})}{9} + \text{etc.} \right]$$

$$\frac{2\delta v_o}{L} = \frac{4}{EA_c L h} \left[\frac{M_{oi}}{10} + \frac{M_{io} + M_{i2}}{9} + \text{etc.} \right]$$

$$O \quad M_{oi} \left(\frac{nL}{6} + \frac{C_{oo}h}{3} \right) - M_{io} \frac{C_{oi}h}{6} + \frac{4}{EALh} \left[\frac{m_{oi}}{10} + \frac{m_{io} + m_{i2}}{9} + \text{etc.} \right] + (\Delta_i - \Delta_o) \frac{EI_c}{h} = 0$$

B.M. DIAGRAM FOR COL. 0-10' IDENTICAL TO THAT OF COL. 0-10' BECAUSE OF FRAME SYMMETRY.

LATERAL DEFLECTION COEFFICIENT (TO BE MULTIPLIED BY WEI_{c10}^{10}/h TO OBTAIN ACTUAL DEFLECTION)

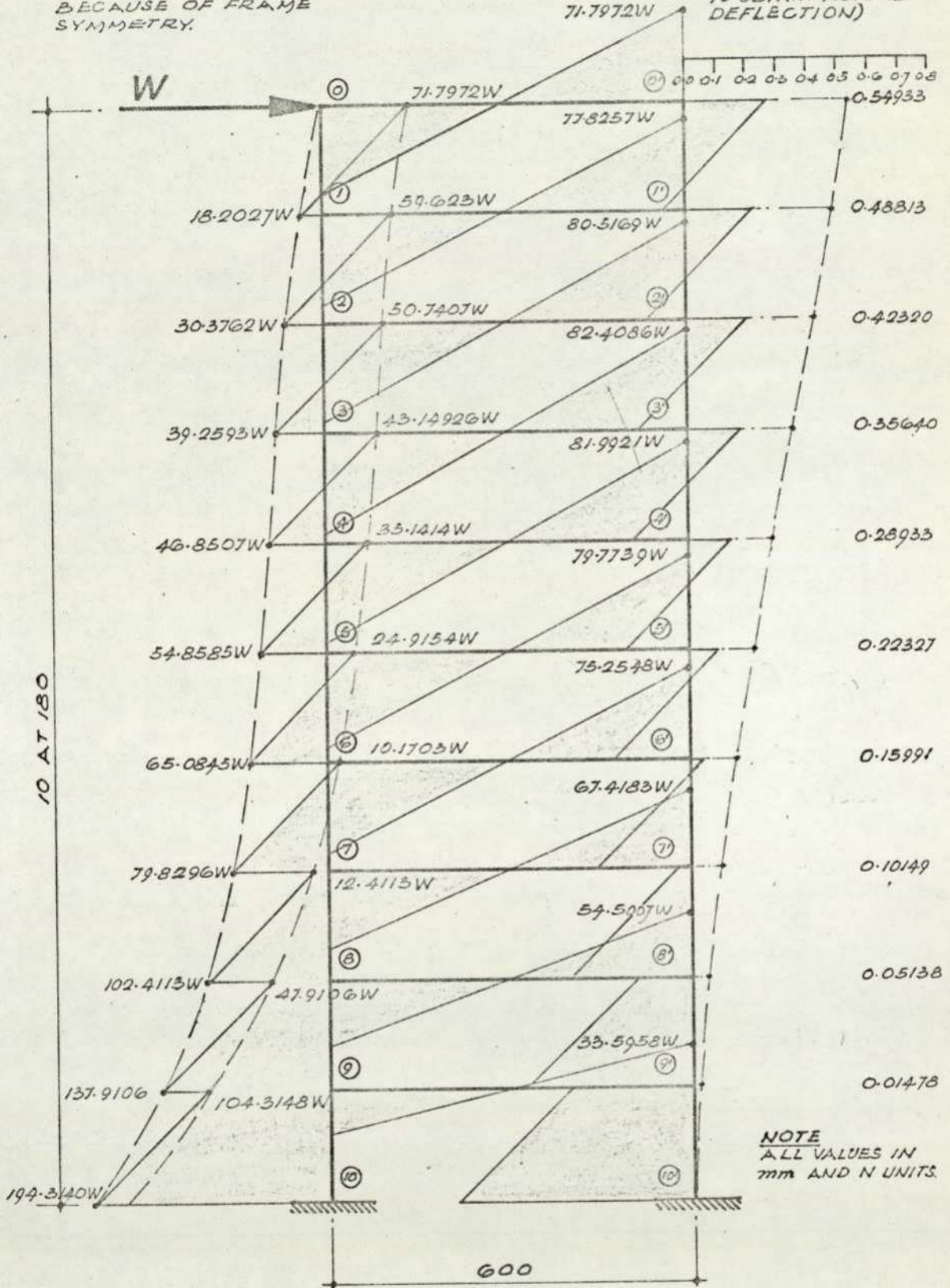


FIG. 4-14 MOMENT/DEFLECTION DIAGRAM FOR 10-STORY CONCRETE MODEL, PRIMARY EFFECTS.

Finally we obtain:

$$\begin{aligned} \underline{0} \quad & 860.0668, 58 M_{01} - 30.52157 M_{10} + 0.003429 M_{12} + \\ & + 0.0038, 58 M_{21} + 0.0038, 58 M_{23} + 0.0044, 091 M_{32} + \\ & + 0.0044, 09 M_{34} + 0.0051, 44 M_{43} + 0.0051, 44 M_{45} + \\ & + 0.0061, 728 M_{54} + 0.0061, 728 M_{56} + 0.0077, 16 M_{65} + \\ & + 0.0077, 16 M_{67} + 0.0102, 28 M_{76} + 0.0102, 88 M_{78} + 0.0154, 32 M_{87} \\ & + 0.0154, 32 M_{89} + 0.0308, 64 M_{98} + 0.0308, 64 M_{910} + (\Delta_1 - \Delta_0) \frac{EI_c}{h} = 0 \end{aligned}$$

Simarly:

$$\underline{211'} \quad (M_{10} + M_{12}) \frac{L}{6EI_b} + M_{12} \frac{c_{11}h}{3EI_c} - M_{21} \frac{c_{12}h}{6EI_c} = \frac{\Delta_1 - \Delta_2}{h} - \frac{2S_{V_1}}{L}$$

$$S_{V_1} = \frac{2M_{01} + 2(M_{10} + M_{12})}{9hEA_c} + \frac{2(M_{21} + M_{23})}{8hEA_c} + etc.$$

$$\frac{2S_{V_1}}{L} = \frac{4}{EA_c L h} \left[\frac{M_{01} + M_{10} + M_{12}}{9} + \frac{M_{21} + M_{23}}{8} + etc. \right]$$

From which:

$$\begin{aligned} \underline{211'} \quad & 0.0034, 2933 M_{01} + 800.0034, 293 M_{10} + 860.0664, 29 M_{12} + \\ & - 30.521142 M_{21} + 0.0038, 58 M_{23} + 0.0044, 0914 M_{32} + \\ & + 0.0044, 0914 M_{34} + 0.0051, 44 M_{43} + 0.0051, 44 M_{45} + \\ & + 0.0061, 728 M_{54} + 0.0061, 728 M_{56} + 0.00716 M_{65} + \\ & + 0.007716 M_{67} + 0.0102, 88 M_{76} + 0.0102, 88 M_{78} + \\ & + 0.0154, 32 M_{87} + 0.0154, 32 M_{89} + 0.0308, 64 M_{98} + \\ & + 0.0308, 64 M_{910} + (\Delta_1 - \Delta_0) \frac{EI_c}{h} = 0 \end{aligned}$$

$$\underline{322'} \quad (M_{21} + M_{23}) \frac{L}{6EI_b} + M_{23} \frac{c_{22}h}{3EI_c} - M_{32} \frac{c_{23}h}{6EI_c} = \frac{\Delta_2 - \Delta_3}{h} - \frac{2S_{V_2}}{h}$$

$$S_{V_2} = \left[\frac{2M_{01} + 2(M_{10} + M_{12}) + 2(M_{21} + M_{23})}{8hEA_c} + \frac{2(M_{32} + M_{34})}{7hEI_c} + etc. \right]$$

$$\frac{2S_{V_2}}{L} = \frac{L}{EA_c L h} \left[\frac{2M_{01}}{8} + \frac{2(M_{01} + M_{12})}{8} + \frac{2(M_{21} + M_{23})}{8} + etc. \right]$$

$$\begin{aligned} & 0.0038, 58 M_{01} + 0.0038, 58 M_{10} + 0.0038, 58 M_{12} + \\ & + 800.0038, 58 M_{21} + 860.0668, 58 M_{23} - 30.5205, 908 M_{32} \end{aligned}$$

$$\begin{aligned}
 &+0.0044, 0914 M_{34} + 0.0051, 44 M_{43} + 0.0051, 44 M_{45} + \\
 &+0.0102, 88 M_{54} + 0.010288 M_{56} + 0.015432 M_{65} + \\
 &+0.015432 M_{67} + 0.0308 M_{78} + \text{etc.}
 \end{aligned}$$

In a similar way the main united force-displacement matrix is set out as shown in Fig. 4.21.

It should be noted that the compatibility conditions between any column and beam now include all moments in the frame, while equilibrium conditions include deformations, i.e., the equilibrium becomes a function of the deformed shape of a structure. In addition, the effect of altered flexibility of compression members, i.e., the effect of instability functions is also included.

Fig. 4.22 shows the distribution of moments, forces and deformations for $W_0 = 1$ and for $P = 35 \text{ kN}$.

The increased deflections and moments due to the instability effect of axial column loads is shown in Fig. 4.23.

4.30 Experiments on 10-storey concrete model

The purpose of tests performed on concrete models was to introduce very high axial forces in columns by means of jacks in order to show instability effects.

At first it was thought to be necessary to make a provision against lateral buckling. This was achieved by joining two frames at a certain distance apart using concrete spacers and bolts as shown in Fig. 4.31. Later this precaution was found to be unnecessary, because no lateral buckling was observed and therefore only single frames were tested as shown in Fig. 4.32. The above figures show the arrangement of the experimental apparatus used.

The details of the 10-storey micro-concrete model used to test for primary and secondary effects is shown in fig. 4.33.

The first test was conducted to assess the correlation between theoretical and experimental deflections and stresses. The model, securely clamped at the base, was loaded with the aid of a single jack at the top.

The load/deflection data are given in Fig. 4.34. The residual deflection, roughly 20% of deflection at the full load of 1200N, suggests that

EQN. NO	COND ⁿ	0		1			2			3			4			5			6			7			8			9			10	LOAD VECTOR	
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₉		
1	100'	800.066 858	-1	-30.323 7067	0.0034 2933	1	800.003 58	860.038 58	0	800.44 0914	800.49 0914	0	0.00544 0	0.00544 0	0	0.0061 728	0.0061 728	0	0.007716 0	0.007716 0	0	0.010288 0	0.010288 0	0	0.015432 0	0.015432 0	0	0.030864 0	0.030864 0	0	0	0	
2	0-0'	1	0.000072	1	0	0.000072						(25)																				90W ₀	
3	211'	0.0034 2933	0	800.003 2933	860.038 2933	-1	-30.323 58	0.0034 58	1	0.0044 0912	0.0044 0914	0	0.00544 0	0.00544 0	0	0.0061 728	0.0061 728	0	0.007716 0	0.007716 0	0	0.010288 0	0.010288 0	0	0.015432 0	0.015432 0	0	0.030864 0	0.030864 0	0	0	0	
4	012	-30.323 58	-1	60.63	-60.63	2	30.323	0	-1			(22)																				0	
5	1-1'	-1	0	-1	1	0.000072	1	0	0.000072			(22)																				0	
6	322'	0.0034 2933	0	800.003 2933	860.038 2933	-1	-30.323 58	0.0034 58	1	0.0044 0914	0.0044 0914	0	0.00544 0	0.00544 0	0	0.0061 728	0.0061 728	0	0.007716 0	0.007716 0	0	0.010288 0	0.010288 0	0	0.015432 0	0.015432 0	0	0.030864 0	0.030864 0	0	0	0	
7	123		(3)		-30.323	-1	60.63	-60.63	2	30.323	0	-1			(19)																	0	
8	2-2'		(3)		-1	0	-1	1	0.000072	1	0	0.000072			(19)																	0	
9	433'	0.0044 0914	0	0.0044 0914	0.0044 0914	0	0.0044 0914	0.0044 0914	0	800.004 40914	860.007 40914	-1	-30.323 58	0.0034 58	1	0.0061 728	0.0061 728	0	0.007716 0	0.007716 0	0	0.010288 0	0.010288 0	0	0.015432 0	0.015432 0	0	0.030864 0	0.030864 0	0	0	0	
10	234				(6)			-30.323	-1	60.63	-60.63	2	30.323	0	-1			(16)														0	
11	3-3'				(6)			-1	0	-1	1	0.000072	1	0	0.000072			(16)														0	
12	544'	0.0034 2933	0	0.0034 2933	0.0034 2933	0	0.0034 2933	0.0034 2933	0	0.0034 2933	0.0034 2933	0	800.003 44	860.007 44	-1	-30.323 58	0.0034 58	1	0.0061 728	0.0061 728	0	0.007716 0	0.007716 0	0	0.010288 0	0.010288 0	0	0.015432 0	0.015432 0	0	0.030864 0	0.030864 0	0
13	345						(9)				-30.323	-1	60.63	-60.63	2	30.323	0	-1				(13)										0	
14	4-4'						(9)				-1	0	-1	1	0.000072	1	0	0.000072				(13)										0	
15	655'	0.0061 728	0	0.0061 728	0.0061 728	0	0.0061 728	0.0061 728	0	0.0061 728	0.0061 728	0	800.006 728	860.007 728	-1	-30.323 58	0.0061 728	1	0.007716 0	0.007716 0	0	0.010288 0	0.010288 0	0	0.015432 0	0.015432 0	0	0.030864 0	0.030864 0	0	0	0	
16	456								(12)					-30.323	-1	60.63	-60.63	2	30.323	0	-1				(10)							0	
17	5-5'								(12)					-1	0	1	1	0.000072	1	0	0.000072				(10)							0	
18	766'	0.007716	0	0.007716	0.007716	0	0.007716	0.007716	0	0.007716	0.007716	0	0.007716	0.007716	0	800.007 16	860.007 16	-1	-30.323 58	0.007716 0	1	0.010288 0	0.010288 0	0	0.015432 0	0.015432 0	0	0.030864 0	0.030864 0	0	0	0	
19	567										(15)					-30.323	-1	60.63	-60.63	2	30.323	0	-1				(7)					0	
20	6-6'										(15)					-1	0	-1	1	0.000072	1	0	0.000072				(7)					0	
21	877'	0.010288	0	0.010288	0.010288	0	0.010288	0.010288	0	0.010288	0.010288	0	0.010288	0.010288	0	800.010 58	860.010 58	-1	-30.323 58	0.010288 0	0	0.015432 0	0.015432 0	0	0.030864 0	0.030864 0	0	0.030864 0	0.030864 0	0	0	0	
22	678											(18)										-30.323	-1	60.63	-60.63	2	30.323	0	-1		(4)		0
23	7-7'											(18)										-1	0	-1	1	0.000072	1	0	0.000072		(4)		0
24	988'	0.015432	0	0.015432	0.015432	0	0.015432	0.015432	0	0.015432	0.015432	0	0.015432	0.015432	0	800.015 52	860.015 52	-1	-30.323 52	0.015432 0	0	0.030864 0	0.030864 0	0	0.030864 0	0.030864 0	0	0.030864 0	0.030864 0	0	0	0	
25	789																					-30.323	-1	60.63	-60.63	2	30.323	0	-1	0	0		
26	8-8'																					-1	0	-1	1	0.000072	1	0	0.000072		0	0	
27	1099'	0.030864	0	0.030864	0.030864	0	0.030864	0.030864	0	0.030864	0.030864	0	0.030864	0.030864	0	800.030 64	860.030 64	-1	-30.323 64	0.030864 0	0	0.030864 0	0.030864 0	0	0.030864 0	0.030864 0	0	0.030864 0	0.030864 0	0	0	0	
28	8910																															0	
29	9-9'																															0	
30	109																															0	

FIG. 4-21 MATRIX FOR INSTABILITY ANALYSIS, 10 STOREY CONCRETE MODEL

○ - DENOTES NUMBER OF ZEROS

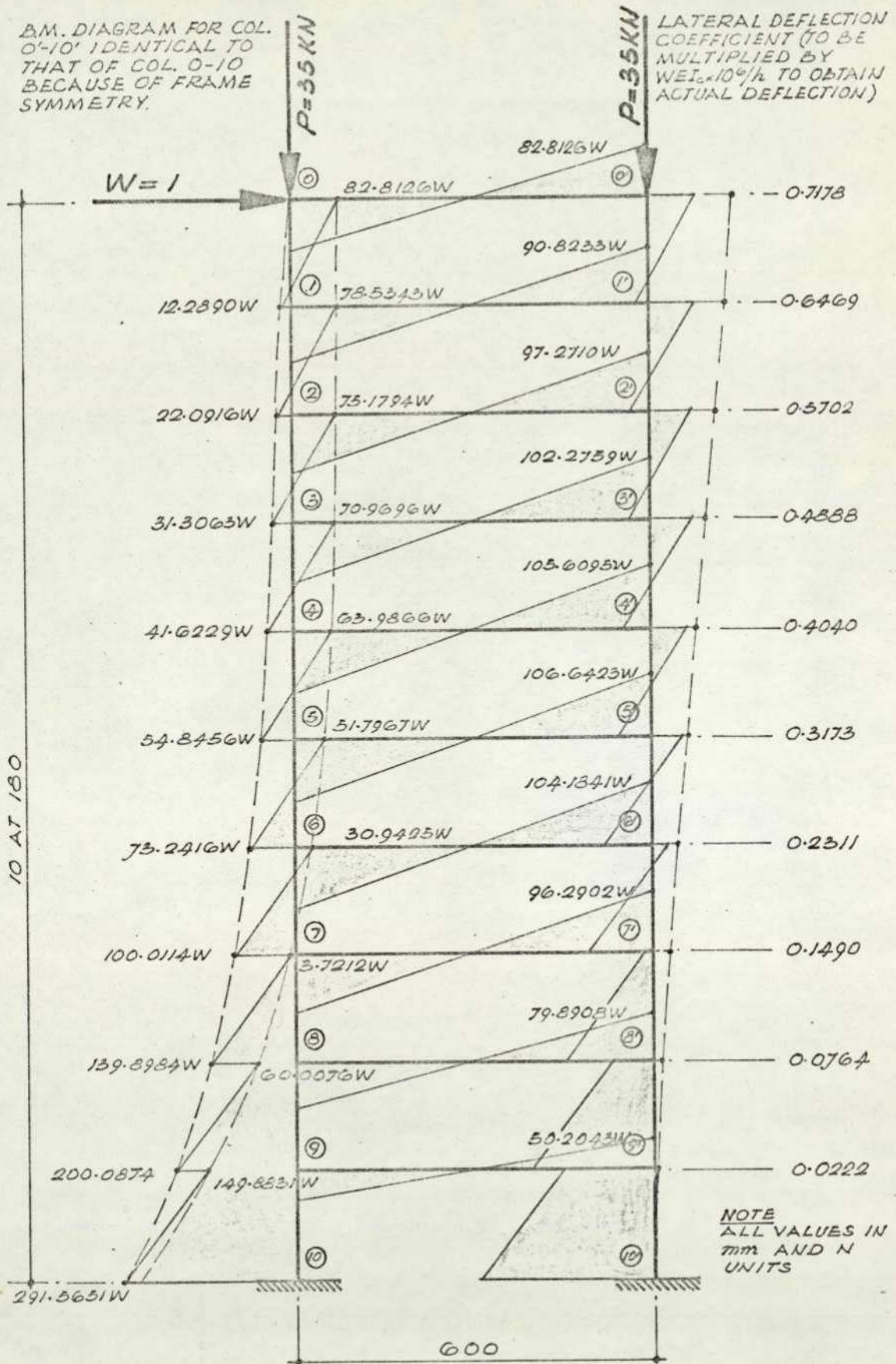
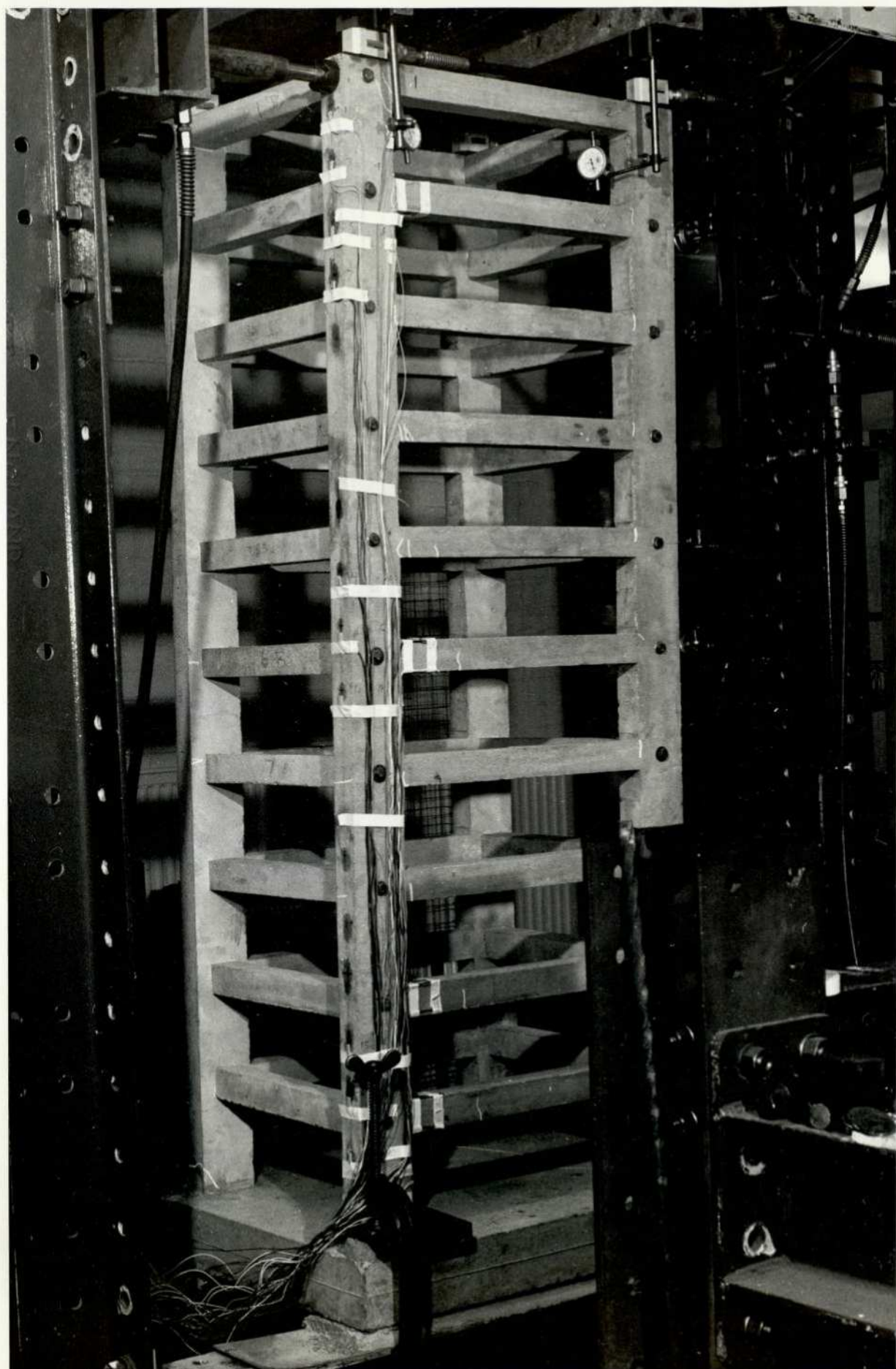


FIG. 4.22 MOMENT/DEFLECTION DIAGRAM FOR 10 STOREY CONCRETE MODEL, INSTABILITY EFFECTS.



4.31 10-storey Concrete Model Tied Together with Bolted Concrete Spacers.

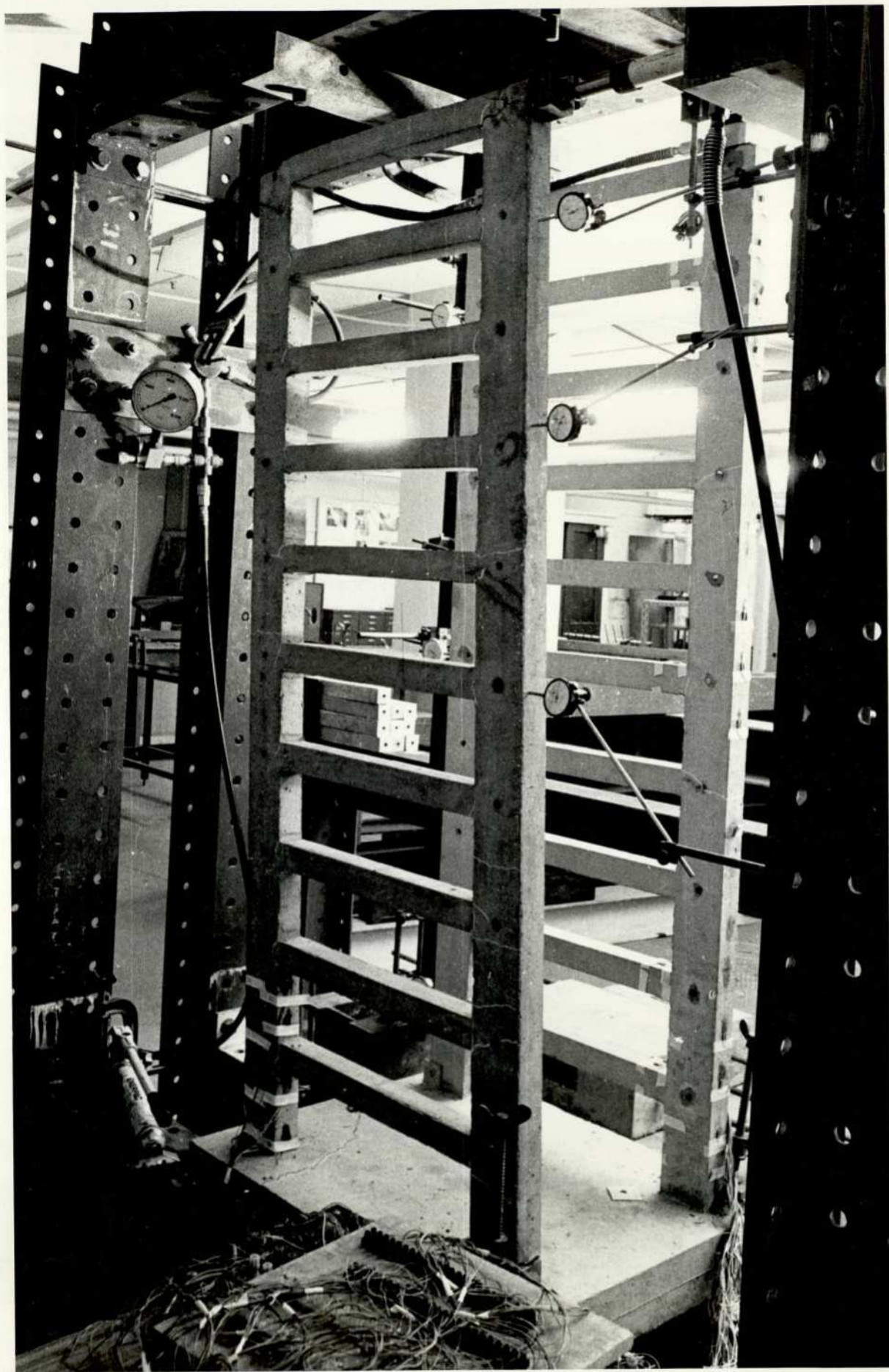


Fig. 4.32 10-storey Concrete Frame Model Tested without Lateral Restraint.

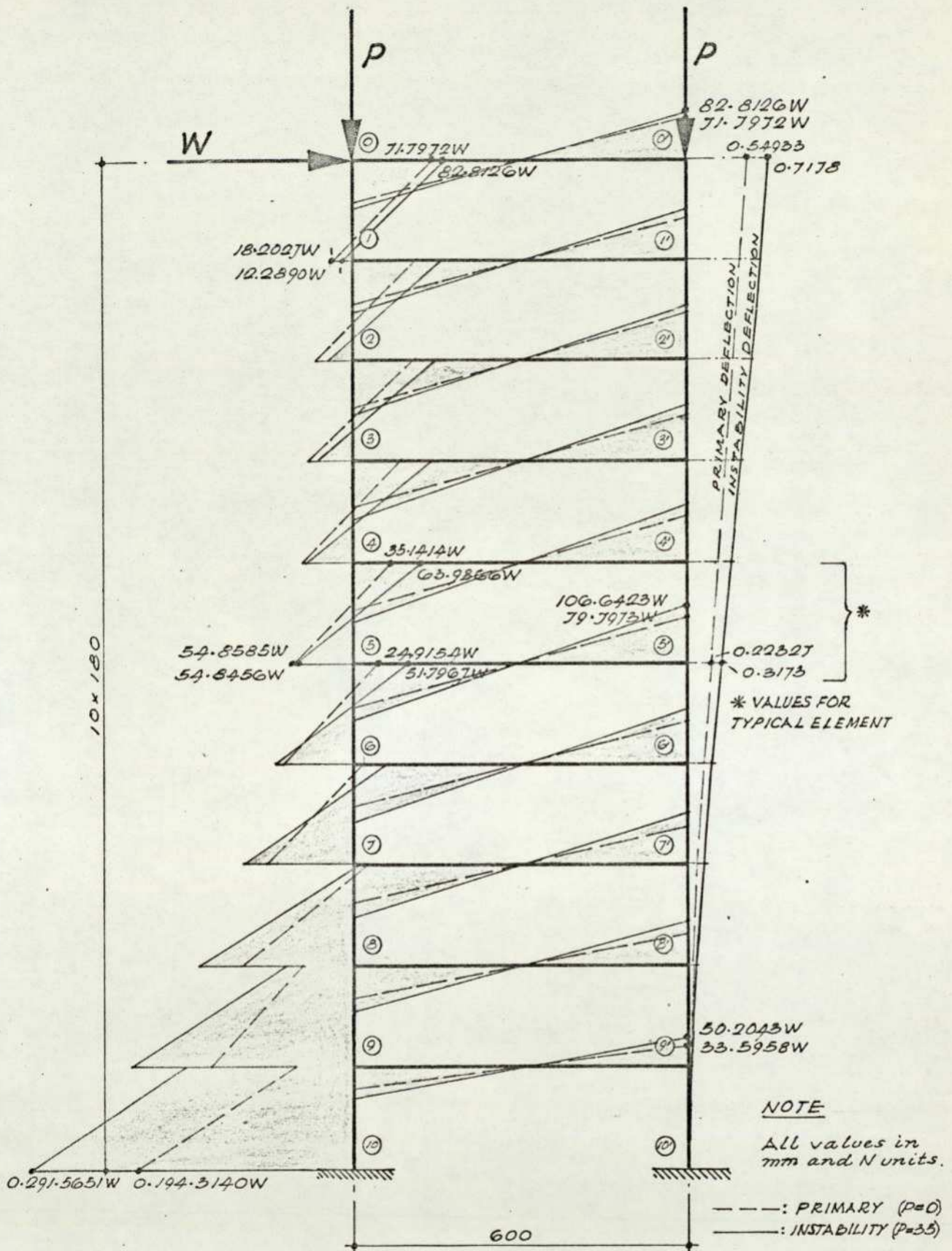


FIG. 4.23 COMPARISON OF ANALYTICAL PRIMARY AND INSTABILITY EFFECTS FOR 10-STOUREY CONCRETE MODEL.

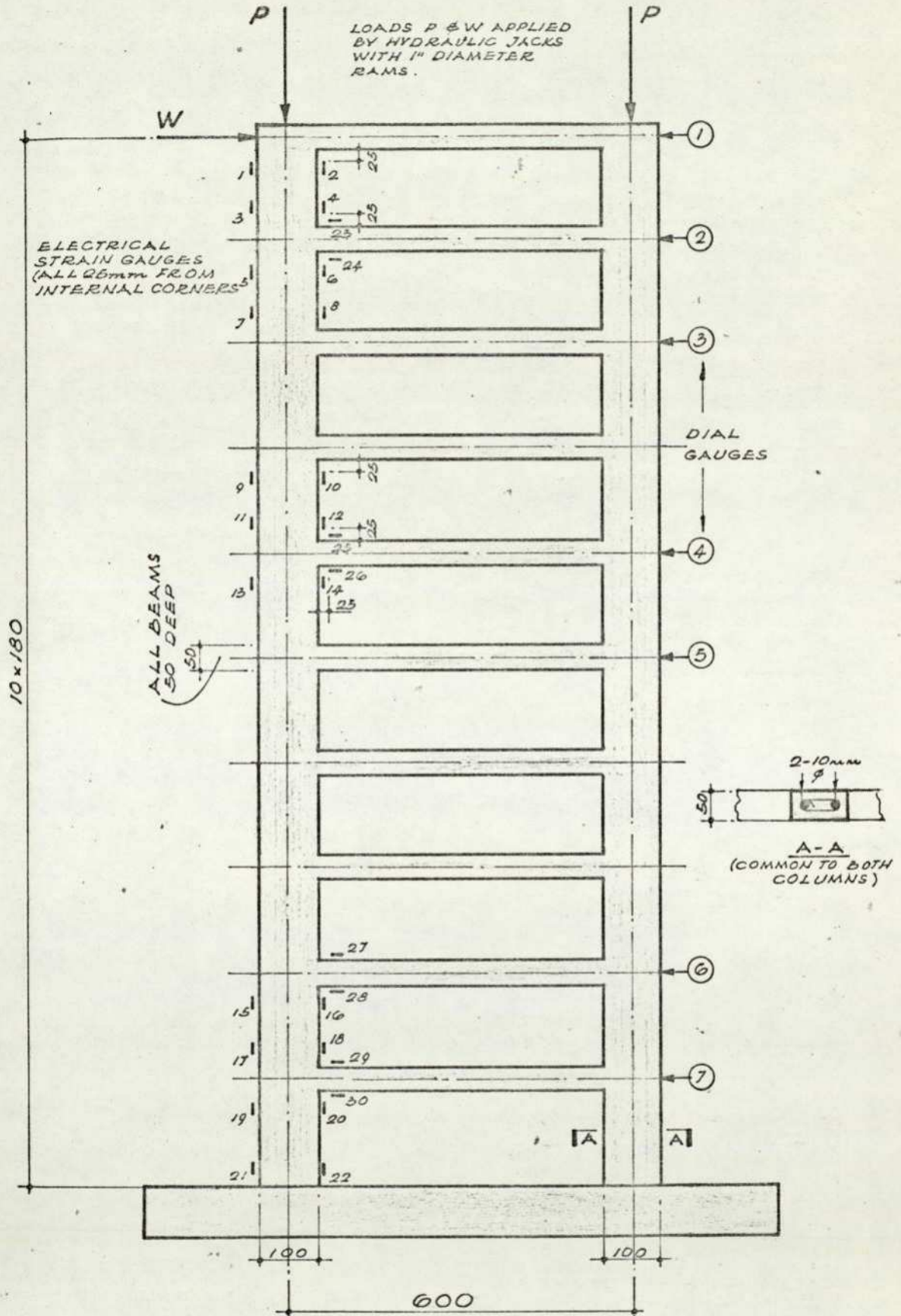


FIG. 4.33 DETAILS OF 10-STOREY CONCRETE TEST MODEL

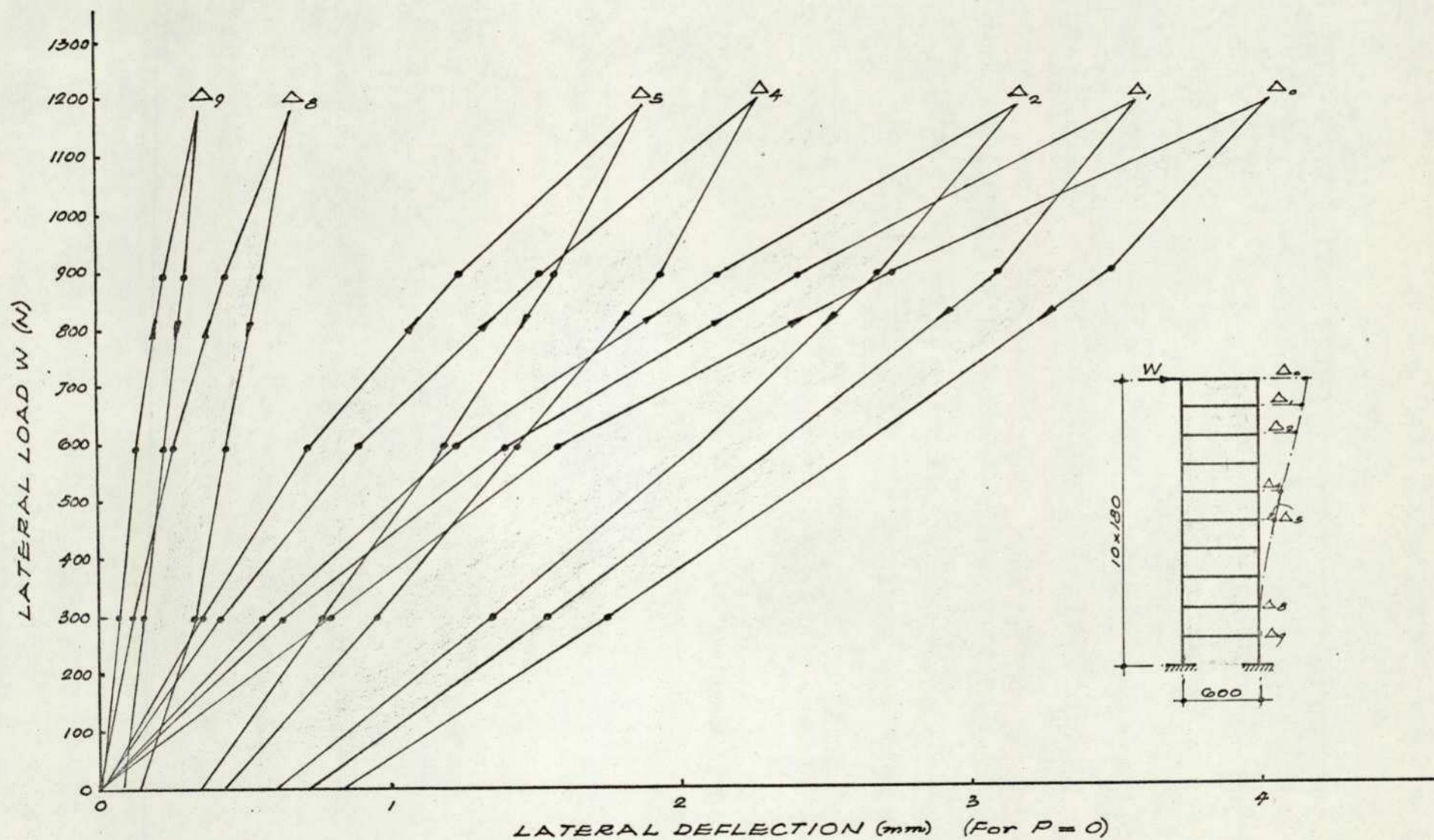


FIG. 4.34 LOAD/DEFLECTION DIAGRAM FOR PRIMARY EFFECTS, 10 STOREY CONCRETE MODEL.

rotation of the whole model may have taken place. It was considered that the magnitude of loading was insufficient to cause cracking of the concrete which could also have resulted in permanent deformation after unloading. In subsequent tests, dial gauges were set up to measure deflections and thus rotation of the model base.

4.40 Discussion of test results for 10-storey concrete model

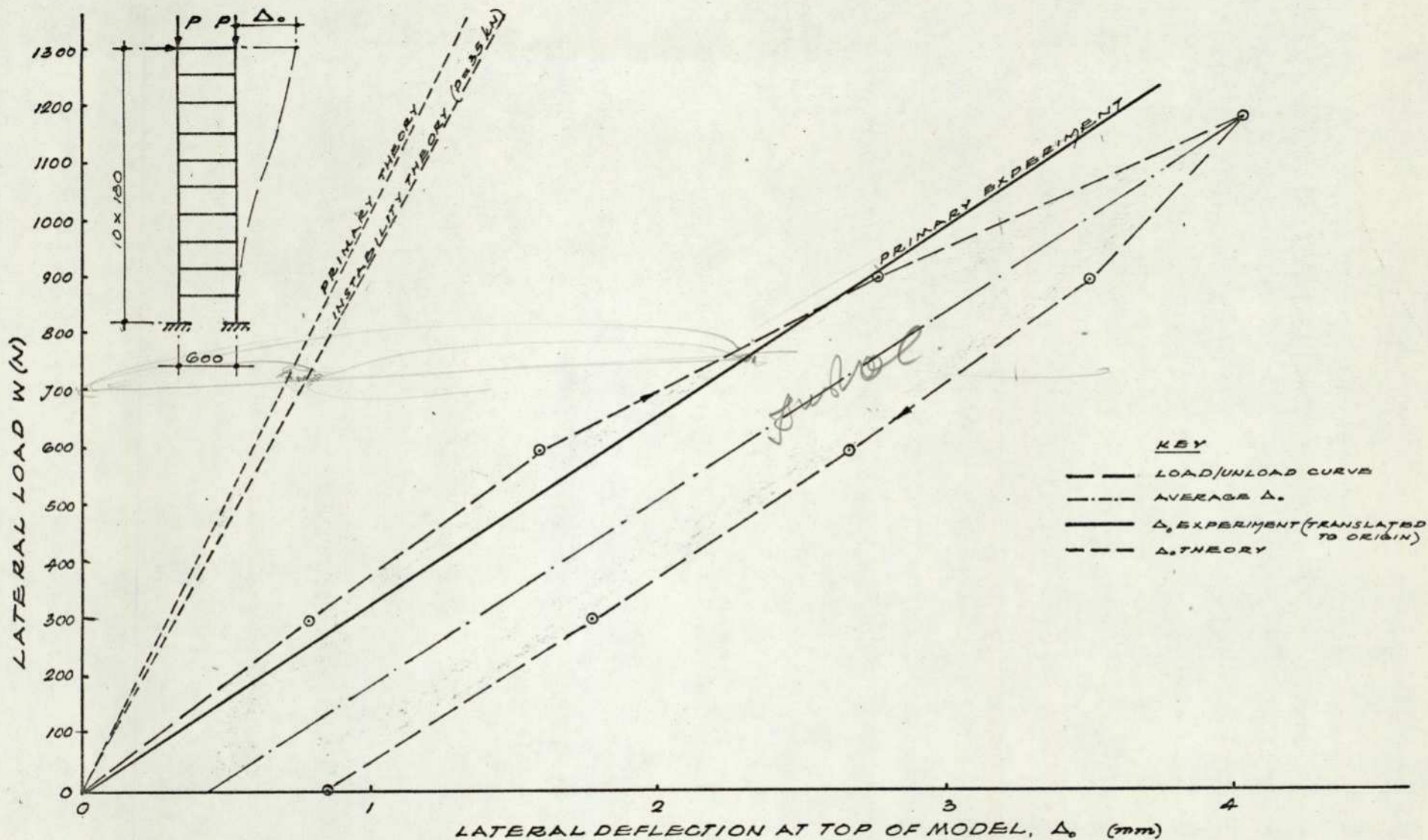
In these tests, the deflections were consistently greater than predicted by theory, being roughly 3 times greater. This poor correlation should be considered in relation to the very small magnitude of deflection, a maximum of 4mm. This poor correlation, Fig. 4.41, is probably due to two effects: rotation of the model base within the test frame under lateral load alone, and secondly micro-cracking of concrete at an early stage of loading, particularly at the joints, when steel was congested to provide for anchorage and laps of bars.

Difficulties were also encountered in the measurement of strain possibly due to the effect of aggregate size on the effective measuring length of the foil strain gauges.

The procedure for this, and subsequent correlations of theoretical and experimental values of bending moment, was as follows:-

- i) From the graph of strain gauge readings for the gauge under consideration, the average gauge reading for a particular load was established.
- ii) Using the calibration data, i.e. that one division of strain gauge reading corresponds to a stress of 0.153N/mm^2 , the stress at the gauge position was calculated.
- iii) From the well known relation $\frac{M}{I} = \frac{\sigma}{y}$ the experimental value of moment at the gauge position was calculated.
- iv) Finally, assuming a linear variation of M with column length between nodes, and using similar triangles, the experimental moment at the particular beam/column node of interest was calculated. This was compared with the theoretical value obtained from the analysis.

Consider moment at position of strain gauges 1 and 2.



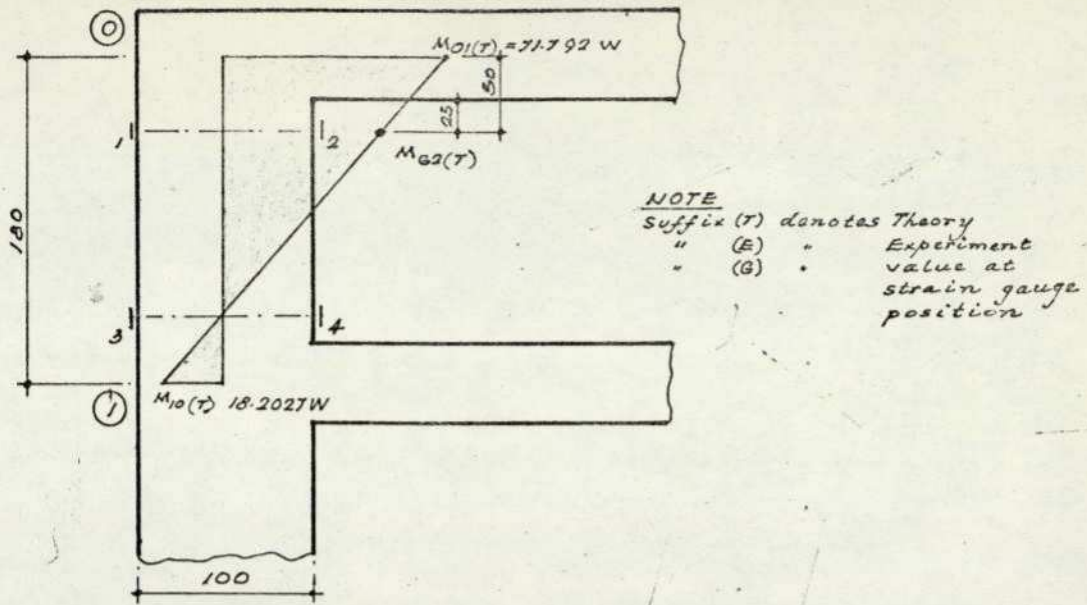


Fig. 4.42 Bending Moment Diagram for Column 01.

From From. 4.43 the number of strain gauge divisions corresponding to $W = 1180\text{N}$ is 4.75.

Therefore $\sigma_{G2(E)} = 4.75 \times 0.153 = 0.7265 \text{ N/mm}^2$

$$M_{G2(E)} = \sigma I / y = \frac{0.7265 \times 4.166 \times 10^6}{50}$$

$$M_{G2(E)} = 60562 \text{ Nmm for } W = 1180\text{N}$$

From Fig. 4.44, the theoretical moment at gauge No. 2 position

$$M_{G2(T)} = \left(90 \times \frac{130}{180} + 18.2027 \right) 1180 = 54280$$

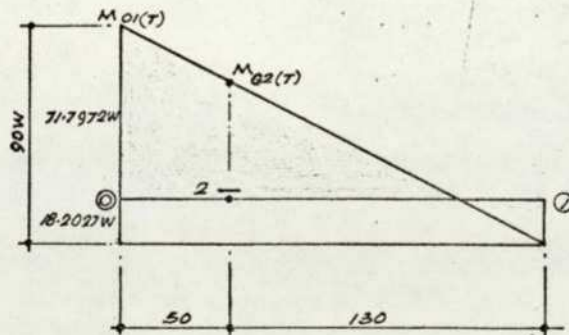


Fig. 4.44 Bending Moment Diagram for Column 01.

Comparison of $M_{G2(E)}$ with $M_{G2(T)}$ shows that the experimental value is 10.4% below the theoretical value.

4.50 Primary Analysis, 6-storey concrete model.

The frame shown in Fig. 4.51 is analysed using the force-displacement method.

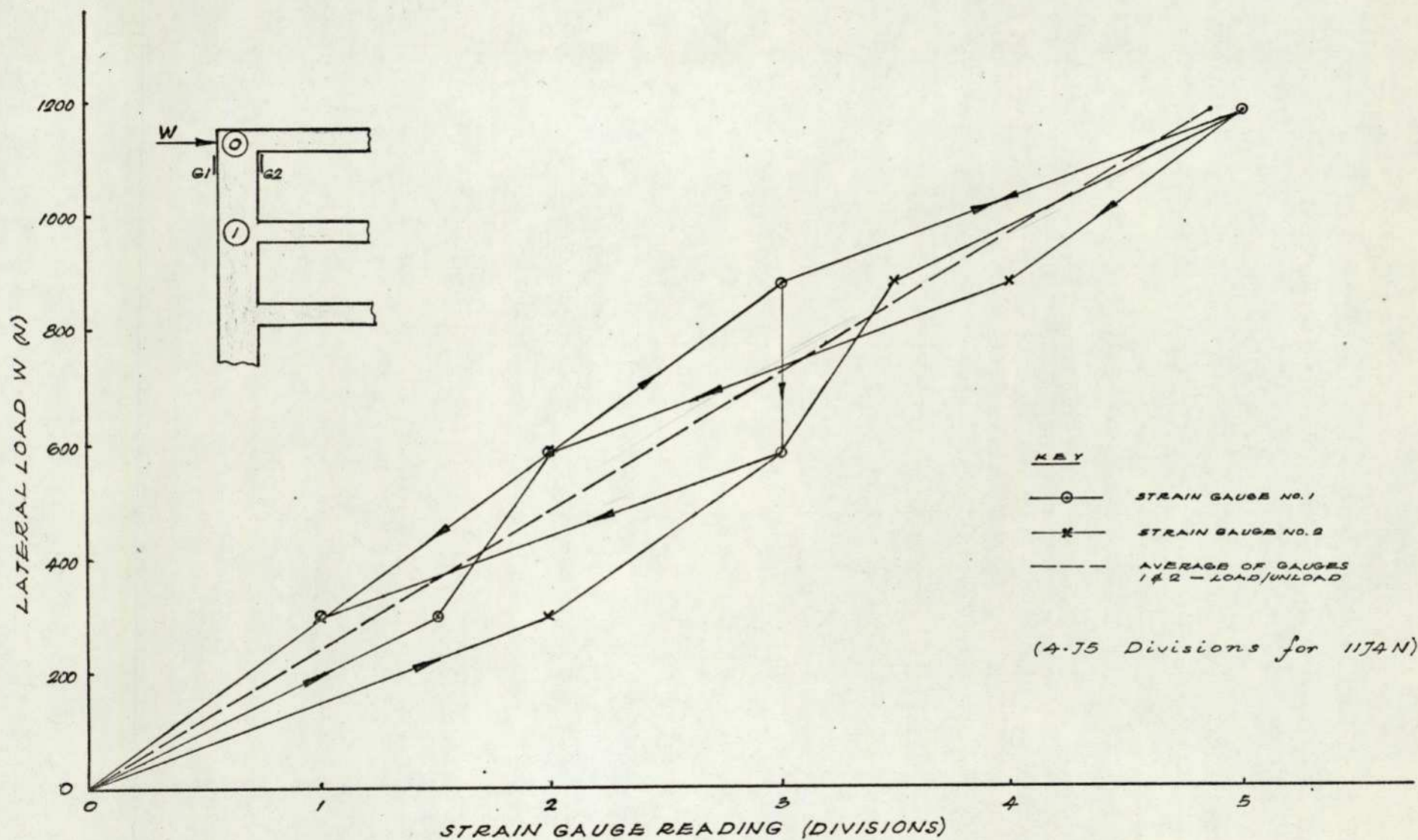


FIG. 4.43 VARIATION OF STRAIN GAUGE READING (GAUGES 1 & 2) WITH LOAD
10-STOREY CONCRETE MODEL (PRIMARY EFFECTS)

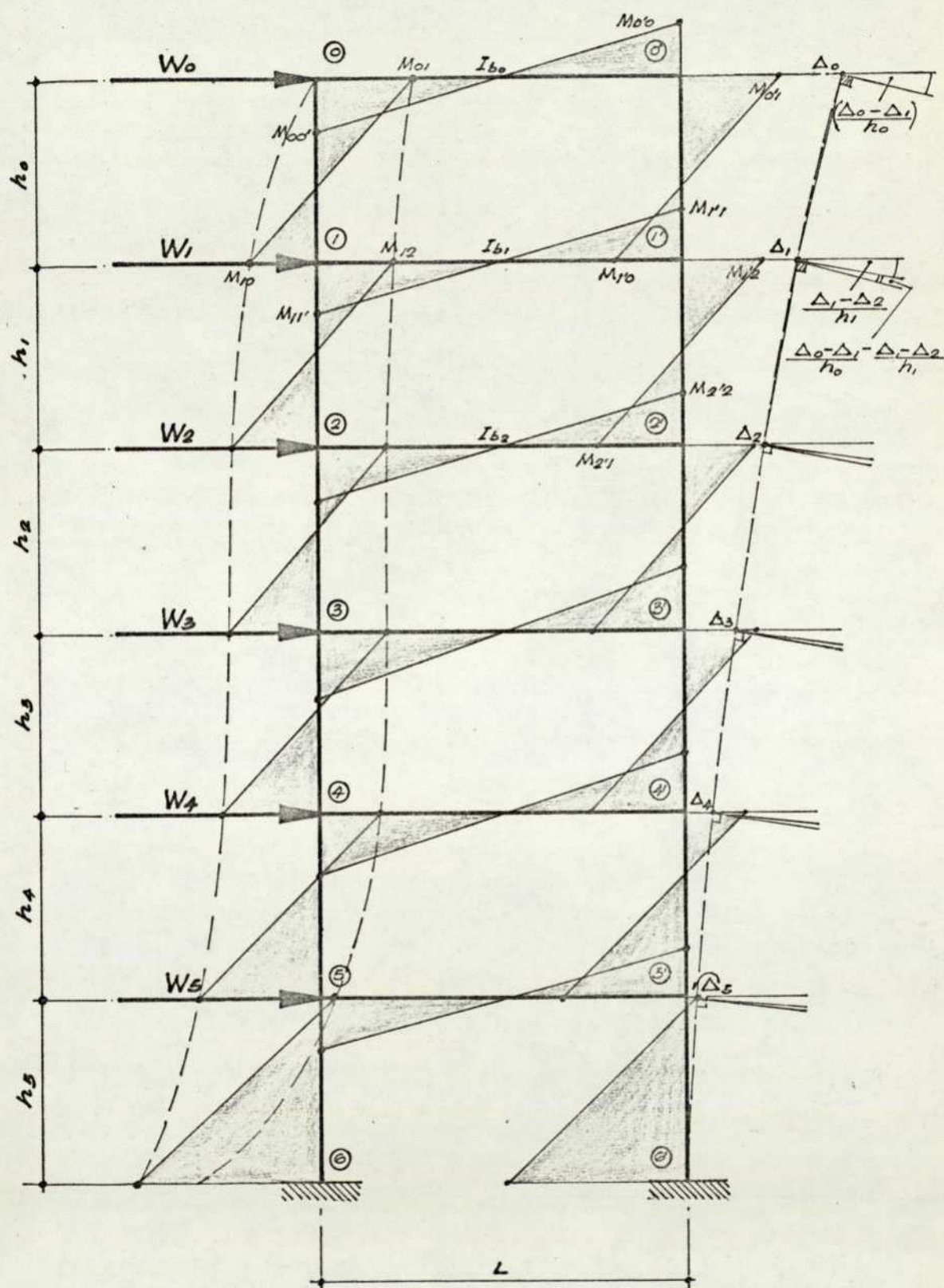


FIG. 4.51 IDEALISED FORM OF 6-STOREY CONCRETE MODEL

Compatibility of deformation at O:

$$\underline{0'0'1} \quad -M_{o1} \frac{L}{6EI} + M_{o1} \frac{L}{3EI} + M_{o1} \frac{h}{3EI} - M_{10} \frac{h}{6EI} = \frac{\Delta_o - \Delta_1}{h}$$

Multiplying this and subsequent compatibility of deformation equations by $6EI/h$ to obtain non-dimensional factors of M , and letting $L/h = r$

$$M_{o1}(r+2) - M_{10}(\Delta_1 - \Delta_o) \frac{6EI}{h^2} = 0$$

Equilibrium at condition O:

$$\frac{2(M_{o1} + M_{10})}{h} = W \quad \therefore M_{o1} + M_{10} = \frac{Wh}{2}$$

Compatibility of deformation at 1:

$$\underline{1'1'2} \quad -(M_{10} + M_{12}) \frac{L}{6EI} + (M_{10} + M_{12}) \frac{L}{3EI} + M_{12} \frac{h}{3EI} - M_{21} \frac{h}{6EI} = \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{10}r - M_{12}r + M_{10}2r + M_{12}2r - M_{21}(\Delta_2 - \Delta_1) \frac{6EI}{h^2} = 0$$

$$M_{10}r + M_{12}(r+2) - M_{21}(\Delta_2 - \Delta_1) \frac{6EI}{h^2} = 0 \dots \dots \dots (3)$$

2nd condition for compatibility of deformation at 1:

$$\underline{0'1'2} \quad -M_{o1} \frac{h}{6EI} + M_{10} \frac{h}{3EI} - M_{12} \frac{h}{3EI} + M_{21} \frac{h}{6EI} = \frac{\Delta_o - \Delta_1}{h} - \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{o1} + 2M_{10} - 2M_{12} + M_{21} - (\Delta_o - 2\Delta_1 + \Delta_2) \frac{6EI}{h^2} = 0 \dots \dots \dots (4)$$

Equilibrium condition at 1:

$$\frac{2(M_{12} + M_{21})}{h} - \frac{2(M_{o1} + M_{10})}{h} = 0$$

$$M_{12} + M_{21} - M_{o1} - M_{10} = 0$$

Similarly at node 5:

6'5'5

$$-(M_{54} + M_{56}) \frac{L}{6EI} + (M_{54} + M_{56}) \frac{L}{3EI} + M_{56} \frac{h}{3EI} - M_{65} \frac{h}{6EI} = \frac{\Delta_s}{h}$$

$$M_{54}r + M_{56}(r+2) - M_{65} - \Delta_s \frac{6EI}{h^2} = 0$$

2nd condition for compatibility of deformation at 5:

4'5'6

$$-M_{45} \frac{h}{6EI} + M_{54} \frac{h}{3EI} - M_{56} \frac{h}{3EI} + M_{65} \frac{h}{6EI} = \frac{\Delta_4 - 2\Delta_s + 0}{h}$$

$$-M_{45} + 2M_{54} - 2M_{56} + M_{65} - (\Delta_4 - 2\Delta_s) \frac{6EI}{h^2} = 0$$

Similarly at base, i.e., node 6:

$$M_{65} \frac{h}{3EI} - M_{56} \frac{h}{6EI} = \frac{\Delta_5}{h}$$

$$2M_{65} - M_{56} - \Delta_5 \frac{6EI}{h^2} = 0$$

Putting $\Delta' = \Delta \frac{6EI}{h^2}$, the equations can be presented in general matrix form as in Fig. 4.52. Substituting numerical values of $h = 300\text{mm}$ and $L = 600\text{mm}$, i.e., $r = 2$, for the model gives the particular matrix in Fig. 4.53.

Computer solution gives the following moment/displacement data (Fig. 4.54):

SOLUTIONS		M	Δ'
0.9063842541E 02	02	01	Δ'_0
0.1930247777E 04	04		
0.5936157458E 02	02	10	
0.7819212708E 02	02	12	Δ'_1
0.1627055650E 04	04		
0.7180787291E 02	02	21	
0.7532221001E 02	02	23	Δ'_2
0.1267371865E 04	04		
0.7467778998E 02	02	32	
0.7341892297E 02	02	34	Δ'_3
0.8971450694E 03	03		
0.7658107703E 02	02	43	
0.6677240483E 02	02	45	Δ'_4
0.5306948746E 03	03		
0.8322759516E 02	02	54	
0.3544310121E 02	02	56	Δ'_5
0.1936706964E 03	03		
0.1145568988E 03	03	65	
ERROR = 0.2359350522E-09			

Fig. 4.54 Computer solution for 6-storey Concrete Frame, Primary Effects.

These values are now converted back into bending moments and deflections, Fig. 4.55. For example, for $W = 530\text{N}$, corresponding to an experimental load on the concrete model, we obtain:

$$M_{01} = 906.38W = 4.83 \text{ kN}$$

$$\Delta_0 = \Delta' \frac{h^2}{6EI} W = \Delta' \times \frac{300^2 \times 533}{6 \times 25 \times 10^3 \times 520833}$$

$$= 1930.2 \times 0.00061$$

$$\Delta_0 = 1.185 \text{ mm}$$

EQN NO	COND ⁿ	0		1			2			3			4			5			6	LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	
1	100'	r+2	-1	-1	0	1		(13)												0
2	0-0'	1	0	1	0	0		(13)												W _A 2
3	2 1 1'	0	0	r	r+2	-1	-1	0	1		(10)									0
4	0 1 2	-1	-1	2	-2	2	1	0	-1		(10)									0
5	1 - 1'	-1	0	-1	1	0	1	0	0		(10)									0
6	3 2 2'	(3)												(7)						0
7	1 2 3	(3)					Fe							(7)						0
8	2 - 2'	(3)												(7)						0
9	4 3 3'					(6)											(4)			0
10	2 3 4					(6)				Fe							(4)			0
11	3 - 3'					(6)											(4)			0
12	5 4 4'								(9)										(1)	0
13	3 4 5								(9)				Fe						(1)	0
14	4 - 4'								(9)										(1)	0
15	6 5 5'										(12)									0
16	4 5 6										(12)				Fe					0
17	5 - 5'										(12)									0
18	6 5													(15)		-1	-1	2		0

FIG. GENERAL MATRIX, 6-STOREY CONCRETE 4.52 MODEL, PRIMARY EFFECTS

○ - DENOTES NUMBER OF ZEROS

1	100'	4	-1	-1	0	1		(13)												0
2	0-0'	1	0	1	0	0		(13)												150
3	2 1 1'	0	0	2	4	-1	-1	0	1		(10)									0
4	0 1 2	-1	-1	2	-2	2	1	0	-1		(10)									0
5	1 - 1'	-1	0	-1	1	0	1	0	0		(10)									0
6	3 2 2'	(3)												(7)						0
7	1 2 3	(3)					Fe							(7)						0
8	2 - 2'	(3)												(7)						0
9	4 3 3'					(6)											(4)			0
10	2 3 4					(6)				Fe							(4)			0
11	3 - 3'					(6)											(4)			0
12	5 4 4'								(9)										(1)	0
13	3 4 5								(9)				Fe						(1)	0
14	4 - 4'								(9)										(1)	0
15	6 5 5'										(12)									0
16	4 5 6										(12)				Fe					0
17	5 - 5'										(12)									0
18	6 5													(15)		-1	-1	2		0

FIG. NUMERIC MATRIX, 6-STOREY CONCRETE 4.53 MODEL, PRIMARY EFFECTS

○ - DENOTES NUMBER OF ZEROS

B.M. DIAGRAM FOR COL.
O'G' IDENTICAL TO
THAT OF COL O-G
BECAUSE OF FRAME
SYMMETRY.

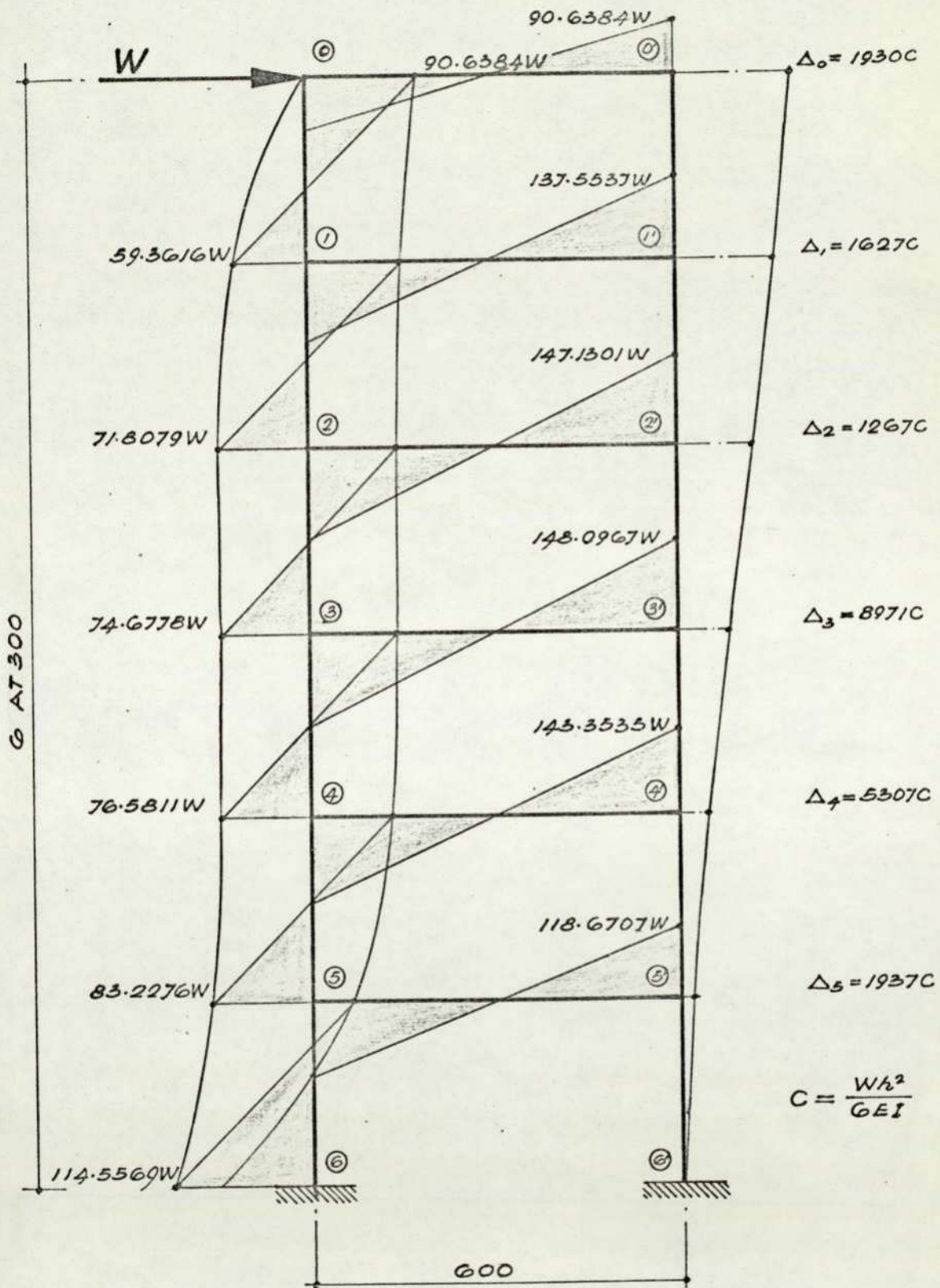


FIG. 4.55 MOMENT/DEFLECTION DIAGRAM 6-STORY
CONCRETE MODEL, PRIMARY EFFECTS

4.60 Experiments on 6-storey concrete model

Two tests were conducted on the microconcrete frame detailed in Fig. 4.61. The first test, using a single lateral load applied at top of frame with a load range 0 to 500N, was conducted to establish primary deflections and moments before the onset of cracking at beam to column joints. The second test was conducted under the combined effect of lateral load up to 900N and axial compressive load on both columns of constant value $P = 2220\text{N}$ acting on each column.

The load deflection curves for the upper 3 storeys of the frame under primary effects is shown in Fig. 4.62. Load deflection curves for the frame subjected to instability effects - varying lateral load with axial compressive load of 2220N on each column - are shown in Fig. 4.63.

Electrical strain gauge readings were made at the positions shown in Fig. 4.61. Considerable variation in their readings presented difficulties in plotting and for this reason the data is omitted.

4.70 Discussion of test results for 6-storey concrete model

From the primary analysis, Section 4.50, the theoretical lateral deflection at the top of the model for a lateral load of 530N was found to be 1.185mm.

Theoretical and experimental values of deflection for primary and instability effects are shown in Fig. 4.71. It can be seen that the experimental primary deflection is greater than the predicted value, i.e. by 56%.

The experimental instability deflection is 14% greater than the primary deflection. This is considered to be a true indication of the instability effect, despite the very small lateral deflections involved, as the onset of cracking - a principal cause of increased flexibility and deflection - is witnessed outside the load range 0 to 530N considered. The increased deflection due to cracking is shown as the discontinuity in the load deflection curve for instability effects, Fig. 4.63, which occurs in the lateral load range 550 to 700N.

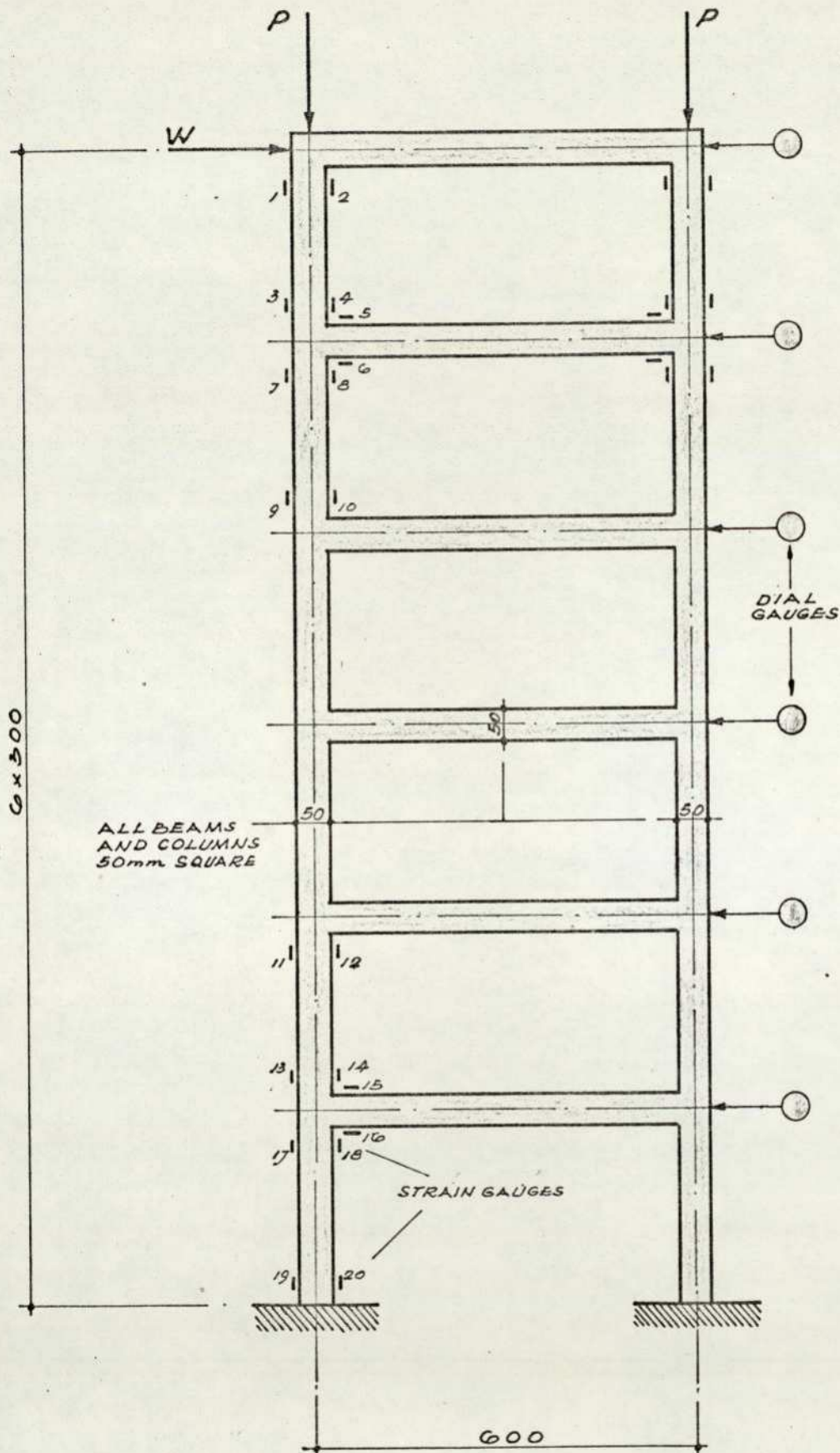


FIG. 4.61 DETAILS OF 6-STORY CONCRETE MODEL

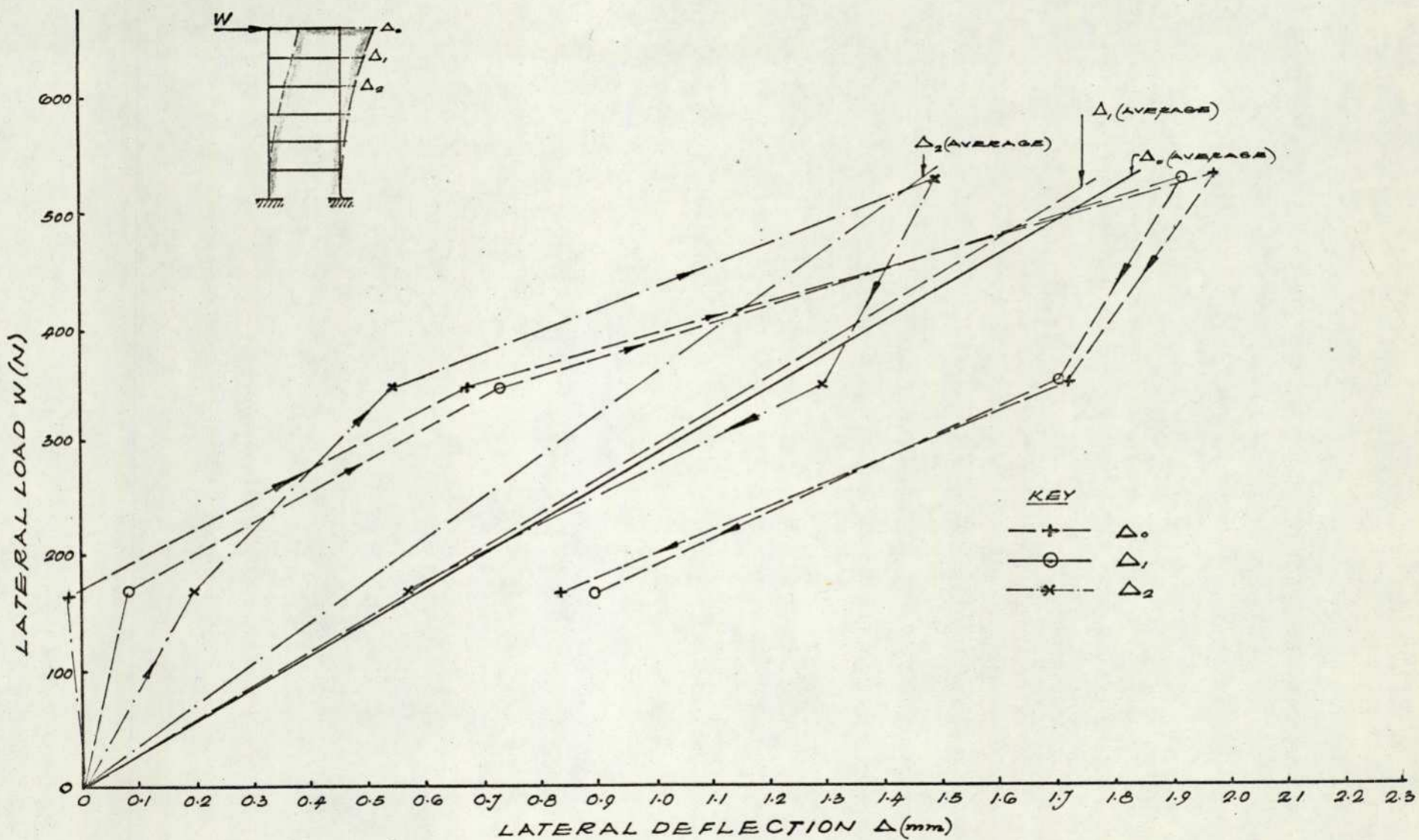


FIG. 4.62 LOAD/DEFLECTION CURVES FOR 6-STOUREY CONCRETE MODEL, PRIMARY EFFECTS

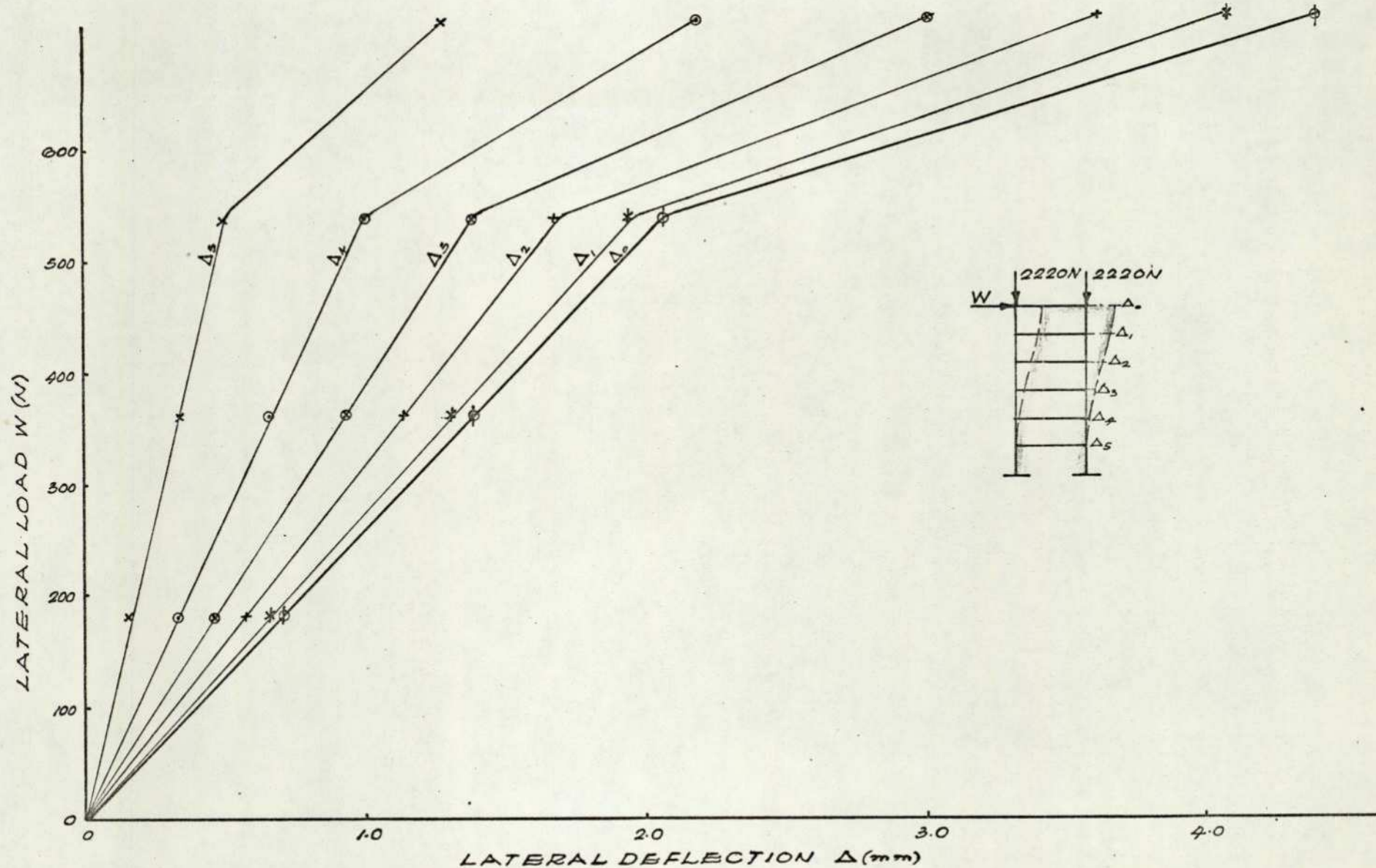


FIG. 4.63 LOAD/DEFLECTION CURVES FOR 6-STORY CONCRETE MODEL
(FOR $P = 2220\text{ N}$)

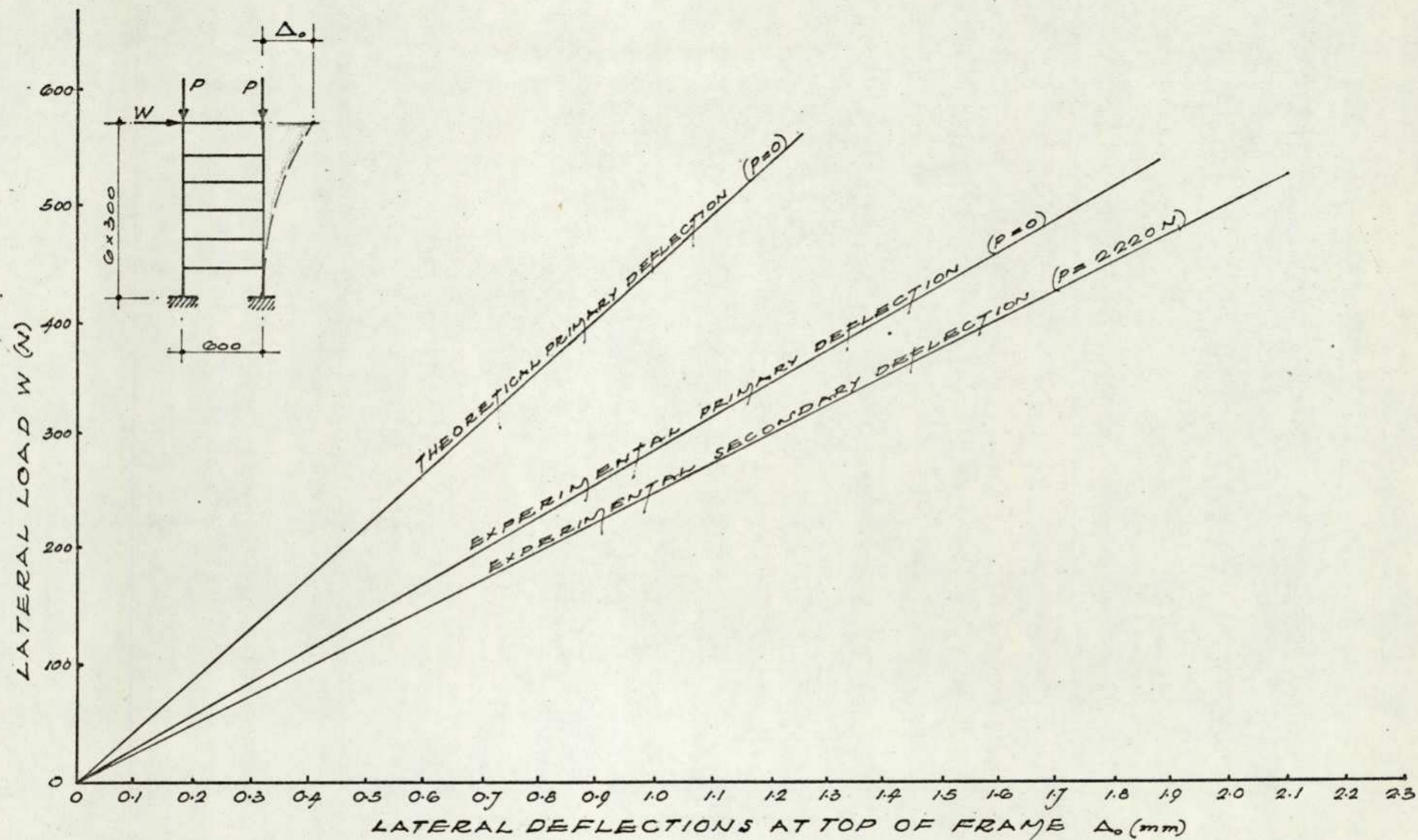


FIG. 4.71 COMPARISON OF THEORETICAL EXPERIMENTAL DEFLECTIONS FOR 6-STOREY CONCRETE MODEL

5.00 Concrete shear wall with 3 columns : theory and experiment

Concrete shear walls with columns of widely differing section properties are a common feature of tall buildings. Here the force-displacement method of analysis is applied to a 6-storey, 3 column shear wall and the theoretical performance compared with the behaviour under test of a micro-concrete model.

5.10 Primary analysis for concrete shear wall

The undeflected and deflected assumed shapes for the frame upon which the analysis is based is shown in Fig. 5.10.

Equations of compatibility of deformation and equilibrium of forces can now be written. These are modified by multiplying the compatibility equations by $6EI_c/h$ and the equations of forces by h . The matrix is thus expressed in terms of non-dimensional coefficients. These are as follows:-

$$100' \quad -M_{10} \frac{h}{6EI_c} + M_{01} \left(\frac{h}{3EI_c} + \frac{L_1}{3EI_b} \right) - (M_{01}' - M_{0'0''}) \frac{L_1}{6EI_b} = \frac{\Delta_o - \Delta_1}{h}$$

$$\text{Let } \frac{L_1 I_1}{I_b h} = n$$

$$-M_{10} + M_{01} (2 + 2n) - M_{01}' n + M_{0'0''} n - (\Delta_o - \Delta_1) \frac{6EI_c}{h^2} = 0 \quad \text{----- 5.101}$$

$$1'0'0'' \quad -M_{1'0} \frac{h}{6EI_2} + M_{01'} \frac{h}{3EI_2} + M_{0'0''} \frac{L_2}{3EI_b} - M_{0''1} \frac{L_2}{6EI_b} = \frac{\Delta_o - \Delta_1}{h}$$

$$\text{Let } \frac{I_1}{I_2} = r ; \quad \frac{L_2 I_1}{I_b h} = m$$

$$-M_{1'0} r + M_{01'} 2r + M_{0'0''} 2m - M_{0''1} m - (\Delta_o - \Delta_1) \frac{6EI_c}{h^2} = 0 \quad \text{----- 5.102}$$

$$00'0'' \quad -M_{01'} \frac{L_1}{6EI_b} + (M_{01}' - M_{0'0''}) \frac{L_1}{3EI_b} - M_{0'0''} \frac{L_2}{3EI_b} + M_{0''1} \frac{L_2}{6EI_b} = 0$$

$$-M_{01} n + M_{01'} 2n - M_{0'0''} (2n + 2m) + M_{0''1} m = 0 \quad \text{----- 5.103}$$

$$0'0''1'' \quad -M_{0'0''} \frac{L_2}{6EI_b} + M_{0''1} \left(\frac{L_2}{3EI_b} + \frac{h}{3EI_2} \right) - M_{1''0} \frac{h}{6EI_2} = \frac{\Delta_o - \Delta_1}{h}$$

$$-M_{0'0''} m + M_{0''1} (2m + 2p) - M_{1''0} p - (\Delta_o - \Delta_1) \frac{6EI_c}{h^2} = 0 \quad \text{----- 5.104}$$

$$\text{Equilibrium at 0: } M_{01} + M_{10} + M_{01'} + M_{1'0} + M_{01''} + M_{1''0} = Wh \quad \text{----- 5.105}$$

$$211' \quad -M_{21} \frac{h}{6EI_1} + M_{12} \frac{h}{3EI_1} + (M_{10} + M_{12}) \frac{L_1}{3EI_b} - (M_{1'0} + M_{1'2} - M_{1''0}) \frac{L_1}{6EI_b} = \left(\frac{\Delta_1 - \Delta_2}{h} \right)$$

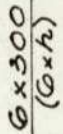


FIG. 5-10 IDEALISED CONCRETE SHEAR WALL USED IN
PRIMARY ANALYSIS

$$-M_{21} + M_{12}(2+2n) + M_{10}2n - M_{1'0}n - M_{1'2}n + M_{1'1''}n -$$

$$-(\Delta_1 - \Delta_2) \frac{6EI_1}{h^2} = 0 \quad \text{-----} 5.106$$

$$012 \quad -M_{01} \frac{h}{6EI_1} + M_{10} \frac{h}{3EI_1} - M_{12} \frac{h}{3EI_1} + M_{21} \frac{h}{6EI_1} = \frac{\Delta_0 - 2\Delta_1 + \Delta_2}{h}$$

$$-M_{01} + 2M_{10} - 2M_{12} + M_{21} - (\Delta_0 - 2\Delta_1 + \Delta_2) \frac{6EI_1}{h^2} = 0 \quad \text{-----} 5.107$$

$$2'1'1'' \quad -M_{2'1} \frac{h}{6EI_2} + M_{1'2} \frac{h}{3EI_2} + M_{1'1''} \frac{L_2}{3EI_2} - (M_{1'0} + M_{1'2}) \frac{L_2}{6EI_2} = \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{2'1}r + M_{1'2}2r + M_{1'1''}2m - M_{1'0}m - M_{1'2}m - (\Delta_1 - \Delta_2) \frac{6EI_1}{h^2} = 0 \quad \text{-----} 5.108$$

$$0'1'2' \quad -M_{0'1} \frac{h}{6EI_2} + M_{1'0} \frac{h}{3EI_2} - M_{1'2} \frac{h}{3EI_2} + M_{2'1} \frac{h}{6EI_2} = \frac{\Delta_0 - 2\Delta_1 + \Delta_2}{h}$$

$$-M_{0'1}r + M_{1'0}2r - M_{1'2}2r + M_{2'1}r - (\Delta_0 - 2\Delta_1 + \Delta_2) \frac{6EI_1}{h^2} = 0 \quad \text{-----} 5.109$$

$$11'1'' \quad - (M_{10} + M_{12}) \frac{L_1}{6EI_b} + (M_{1'0} + M_{1'2} - M_{1'1''}) \frac{L_1}{3EI_b} - M_{1'1''} \frac{L_2}{3EI_b} +$$

$$+ (M_{1'0} + M_{1'2}) \frac{L_2}{6EI_b} = 0$$

$$-M_{10}n - M_{12}n + M_{1'0}2n + M_{1'2}2n - M_{1'1''}(2n + 2m) +$$

$$+ M_{1'0}m + M_{1'2}m = 0 \quad \text{-----} 5.110$$

$$1'1''2'' \quad -M_{1'1''} \frac{L_2}{6EI_b} + (M_{1'0} + M_{1'2}) \frac{L_2}{3EI_b} + M_{1'2} \frac{h}{3EI_3} - M_{2'1} \frac{h}{6EI_3} = \frac{\Delta_1 - \Delta_2}{h}$$

$$M_{1'1''}m + M_{1'0}2m + M_{1'2}(2m + 2p) - M_{2'1}p - (\Delta_1 - \Delta_2) \frac{6EI_1}{h^2} = 0 \quad \text{-----} 5.111$$

$$0'1''2'' \quad -M_{0'1} \frac{h}{6EI_3} + M_{1'0} \frac{h}{3EI_3} - M_{1'2} \frac{h}{3EI_3} + M_{2'1} \frac{h}{6EI_3} = \frac{\Delta_0 - 2\Delta_1 + \Delta_2}{h}$$

$$-M_{0'1}p + M_{1'0}2p - M_{1'2}2p + M_{2'1}p - (\Delta_0 - 2\Delta_1 + \Delta_2) \frac{6EI_1}{h^2} = 0 \quad \text{-----} 5.112$$

Equilibrium at 1

$$M_{12} + M_{21} + M_{1'2} + M_{2'1} + M_{1''2} + M_{2''1} - M_{01} - M_{10} - M_{0'1} - M_{1'0} -$$

$$-M_{0''1} - M_{1''0} = 0 \quad \text{-----} 5.113$$

Similarly at base of frame:

$$6'65 \quad -M_{56} \frac{h}{6EI_1} + M_{65} \frac{h}{3EI_1} = \frac{\Delta_5}{h}$$

$$-M_{56} + 2M_{65} - \Delta_5 \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.114}$$

$$\begin{aligned} 6''6'5' \quad & -M_{56} \frac{h}{6EI_2} + M_{65} \frac{h}{3EI_2} = \frac{\Delta_5}{h} \\ & -M_{56}r + M_{65}2r - \Delta_5 \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.115} \end{aligned}$$

$$\begin{aligned} 6'6''5'' \quad & -M_{56} \frac{h}{6EI_3} + M_{65} \frac{h}{3EI_3} = \frac{\Delta_5}{h} \\ & -M_{56}p + M_{65}2p - \Delta_5 \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.116} \end{aligned}$$

For the dimensions of the shear wall given in Fig. 5.30 the following values apply:

$$I_1 = \frac{1}{12} \times 50 \times 200^3 = 3.33 \times 10^7 \text{ mm}^4$$

$$I_2 = \frac{1}{12} \times 50 \times 100^3 = 4.166 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12} \times 50 \times 50^3 = 5.2083 \times 10^5 \text{ mm}^4$$

Therefore:

$$n = \frac{L_1 I_1}{I_b h} = \frac{450 \times 3.33 \times 10^7}{5.2083 \times 10^5 \times 300} = 96$$

$$r = \frac{I_1}{I_2} = \frac{3.33 \times 10^7}{4.166 \times 10^6} = 8$$

$$m = \frac{L_2 I_1}{I_b h} = \frac{300 \times 3.33 \times 10^7}{5.2083 \times 10^5 \times 300} = 64$$

$$p = \frac{I_1}{I_3} = \frac{3.33 \times 10^7}{5.2083 \times 10^5} = 64$$

$$2(1+n) = 194; \quad 2(n+m) = 320; \quad 2r = 16; \quad 2n = 192$$

The general and numeric matrices can now be set out as shown in Fig. 5.11 and 5.12.

The computer print out given in Fig. 5.13 can be interpreted in terms of moments and deflections as shown in fig. 5.14.

For a lateral load of 8000N (corresponding to a test load), the lateral deflection at top of frame, $\Delta_o = 24902 \text{ C}$ where $C = Wh^2/6EI$.

$$= 24902 \times \frac{8000 \times 300^2}{6 \times 25,000 \times 3.333 \times 10^7}$$

$$\text{i.e. } \Delta_o = 3.5859 \text{ mm for } W = 8000 \text{ N.}$$

This value will be compared with the experimental deflection.

$\bigcirc$ - DENOTES NUMBER OF ZEROS
 $n = \frac{1}{2} \frac{1}{2} \frac{1}{2}$, $m = \frac{1}{2} \frac{1}{2} \frac{1}{2}$, $p = \frac{1}{2}$, $r = \frac{1}{2}$

○ - DENOTES NUMBER OF ZEROS

	M	Δ'
SOLUTIONS		
0.505032633E 02	01	Δ_0
0.249020267E 01	01	
0.1012808450E 03	0'1	
0.5416954503E 02	0'0"	
0.6276008329E 02	0"1	
0.2788766683E 01	10	Δ_1
0.5146753008E 02	12	
0.1962986767E 05		
0.6817380568E 02	1'0	
0.5566545421E 02	1'2	
0.7071309268E 02	1'1"	Δ_2
0.3449329563E 02	1'0	
0.2893876297E 02	1'2	
0.6733684834E 02	21	
-0.1394153343E 02	23	
0.1427718389E 05		Δ_3
0.6419865488E 02	2'1	
0.5836971884E 02	2'3	
0.6766815361E 02	2'2"	
0.3039278949E 02	2'1	
0.2933568973E 02	2'3	Δ_4
0.1282399424E 03	32	
-0.7903891628E 02	34	
0.9163829480E 05		
0.7247212060E 02	3'2	
0.3826593043E 02	3'4	Δ_5
0.6274321737E 02	3'3"	
0.5056496191E 02	3'2	
0.2485941104E 02	3'4	
0.2102910466E 03	43	
-0.1694500933E 03	45	Δ_6
0.4689265373E 04		
0.7771506172E 02	4'3	
0.1522925463E 02	4'5	
0.5268004667E 02	4'4"	
0.2790748643E 02	4'3	Δ_7
0.1791228807E 02	4'5	
0.3200348026E 03	54	
-0.2950110091E 03	56	
0.1374169264E 04		
0.9239404627E 02	5'4	Δ_8
-0.3310481715E 02	5'6'	
0.3352258305E 02	5'5"	
0.2298570184E 02	5'4	
0.5645220682E 01	5'6	
0.5395791276E 03	65	Δ_9
0.6933317044E 02	6'5	
0.1355830772E 02	6'5	
ERRCP = 0.8046627045E-08		

Fig. 5.13 Computer Solutions for Primary Analysis,
Concrete Shear Wall.

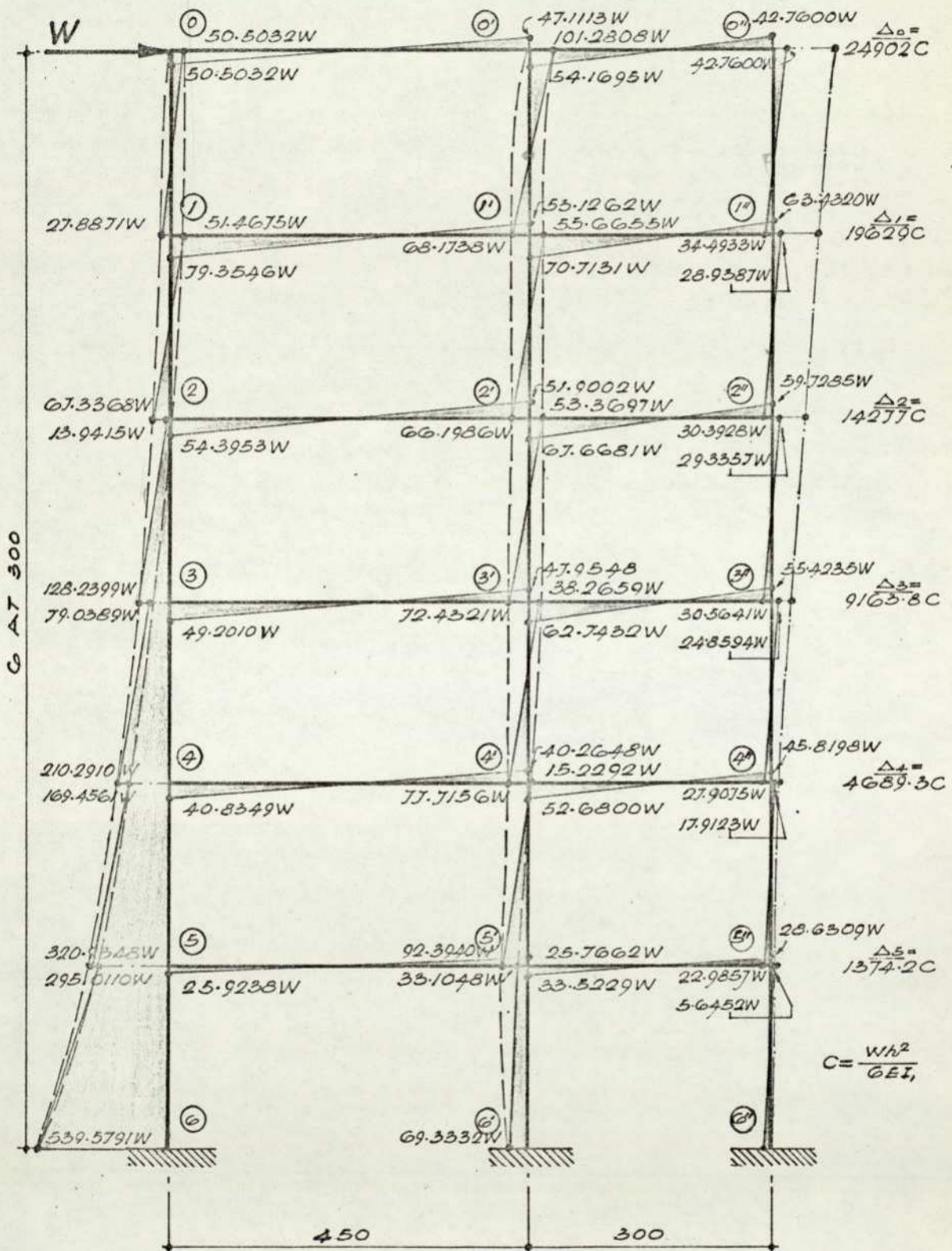


FIG. 5-14. BENDING MOMENT AND DEFLECTION DIAGRAM FOR PRIMARY EFFECTS, 6-STORY CONCRETE SHEAR WALL

5.20

Instability analysis for concrete shear wall

This analysis, for the 3 column shear wall shown in Fig. 5.10, takes account of the increased flexibility of columns due to axial load but ignores axial deformation of columns in the setting out of the equations for compatibility of deformation. Referring to Fig. 5.20 the typical element matrix at level 2 is derived by using the force-displacement method. The compatibility of deformation and equilibrium conditions become:

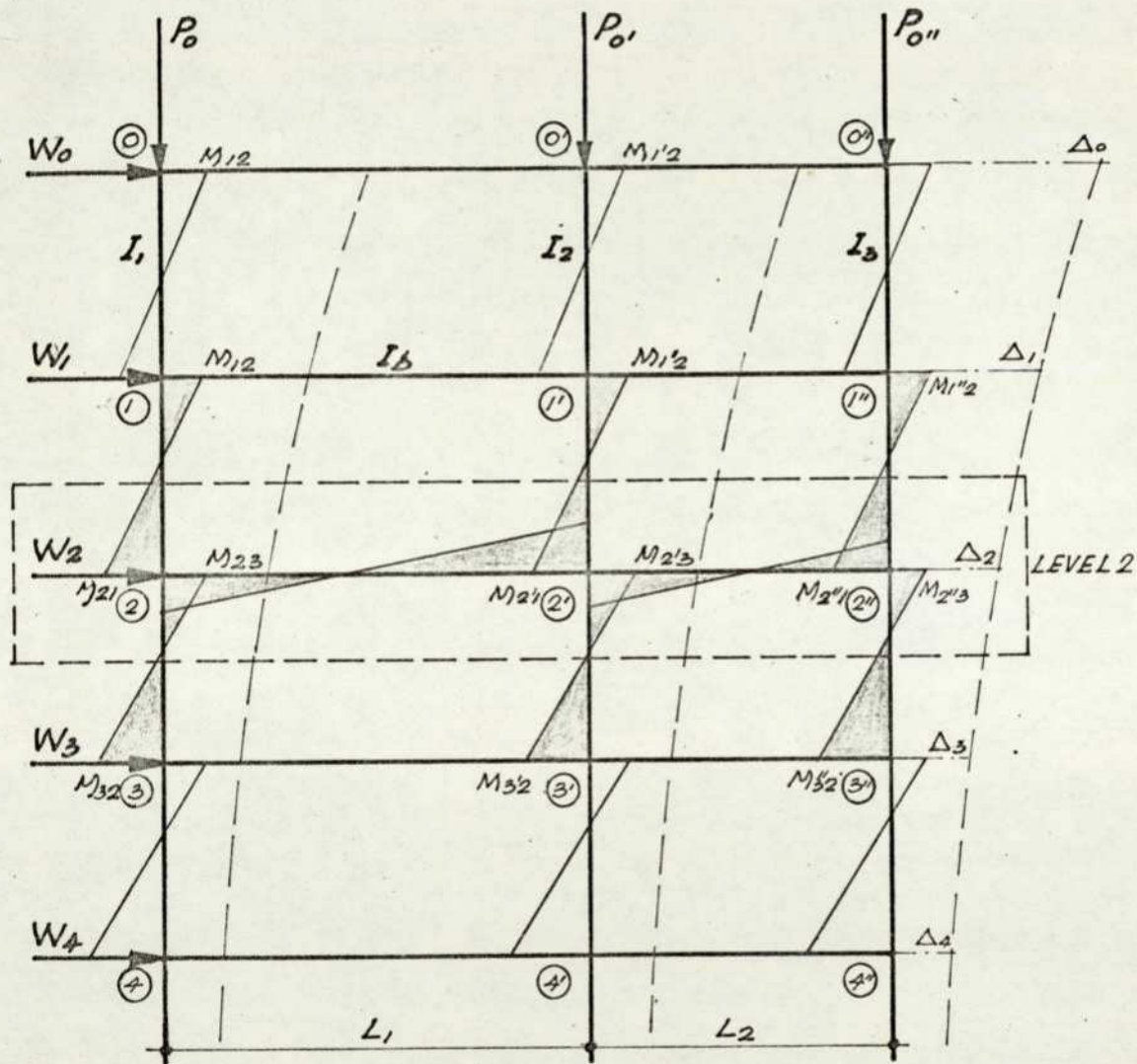


Fig. 5.20 Idealised Structure for Instability Analysis of Concrete Shear Wall.

$$322' \quad -M_{32} \frac{c_{32} h}{6EI_1} + M_{23} \frac{c_{22} h}{3EI_1} + (M_{21} + M_{23}) \frac{L_1}{3EI_b} - \\ - (M_{2'1} + M_{2'3} - M_{2'2''}) \frac{L_1}{6EI_b} = \frac{\Delta_2 - \Delta_3}{h}$$

$$123 \quad -M_{12} \frac{c_{12} h}{6EI_1} + M_{21} \frac{c_{22} h}{3EI_1} - M_{23} \frac{c_{22} h}{3EI_1} + M_{32} \frac{c_{32} h}{6EI_1} = \\ = \frac{\Delta_1 - \Delta_2}{h} - \frac{\Delta_2 - \Delta_3}{h}$$

$$22'2'' \quad - (M_{21} + M_{23}) \frac{L_1}{6EI_b} + (M_{2'1} + M_{2'3} - M_{2'2''}) \frac{L_1}{3EI_1} - M_{2'2''} \frac{L_2}{3EI_b} + \\ + (M_{2''1} + M_{2''3}) \frac{L_2}{6EI_b} = 0$$

$$3'2'2'' \quad -M_{3'2} \frac{c_{32} h}{6EI_2} + M_{2'3} \frac{c_{22} h}{3EI_2} + M_{2'2''} \frac{L_2}{3EI_b} - (M_{2''1} + M_{2''3}) \frac{L_2}{6EI_b} = \\ = \frac{\Delta_2 - \Delta_3}{h}$$

$$1'2'3' \quad -M_{1'2} \frac{c_{12} h}{6EI_2} + M_{2'1} \frac{c_{22} h}{3EI_2} - M_{2'3} \frac{c_{22} h}{3EI_2} + M_{3'2} \frac{c_{32} h}{6EI_2} = \\ = \frac{\Delta_1 - \Delta_2}{h} - \frac{\Delta_2 - \Delta_3}{h}$$

$$2'2'3'' \quad -M_{2'2''} \frac{L_2}{6EI_b} + (M_{2''1} + M_{2''3}) \frac{L_2}{3EI_b} + M_{2'3} \frac{c_{22} h}{3EI_3} - \\ - M_{32''} \frac{c_{32} h}{6EI_2} = \frac{\Delta_2 - \Delta_3}{h}$$

$$1''2''3'' \quad -M_{1''2} \frac{c_{12} h}{6EI_3} + M_{2''1} \frac{c_{22} h}{3EI_3} - M_{2''3} \frac{c_{22} h}{3EI_3} + M_{3''2} \frac{c_{32} h}{6EI_3} = \\ = \frac{\Delta_1 - \Delta_2}{h} - \frac{\Delta_2 - \Delta_3}{h}$$

Equilibrium condition 2-3

$$\frac{M_{23} + M_{32}}{h} + \frac{M_{2'3} + M_{3'2}}{h} + \frac{M_{2''3} + M_{3''2}}{h} = V_{23} + \\ + P_2 \left(\frac{\Delta_2 - \Delta_3}{h} \right) + P_2' \left(\frac{\Delta_2 - \Delta_3}{h} \right) + P_2'' \left(\frac{\Delta_2 - \Delta_3}{h} \right)$$

All compatibility equations are now multiplied by $6EI_1/h$ and equilibrium condition by h , i.e. the matrix is then expressed again in terms of non-dimensional coefficients so that bending moments are in terms of Wh and deflections in terms of $Wh^3/6EI_1$.

$$\text{Denoting} \quad r_1 = \frac{L_1 I_1}{h I_b} \quad r_3 = \frac{I_1}{I_2}$$

$$r_2 = \frac{L_2 I_1}{h I_b} \quad r_4 = \frac{I_1}{I_2}$$

$$322' \quad -M_{32} c_{32} + M_{23} 2c_{22} + (M_{21} + M_{23}) 2 \frac{L_1 I_1}{h I_b} -$$

$$-(M_{21} + M_{23} - M_{2'2''}) \frac{L_1 I_1}{h I_b} + (\Delta_3 - \Delta_2) \frac{6EI_1}{h^2} = 0$$

$$\text{or} \quad 2r_1 M_{21} + M_{23} 2(c_{22} + r_1) - c_{32} M_{32} - r_1 M_{21} - r_1 M_{23} +$$

$$+ M_{2'2''} r_1 + (\Delta_3 - \Delta_2) \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.21}$$

$$123 \quad -c_{12} M_{12} + 2c_{22} M_{21} - 2c_{22} M_{23} + c_{32} M_{32} +$$

$$+ (-\Delta_1 + 2\Delta_2 - \Delta_3) \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.22}$$

$$22'2'' \quad -(M_{21} + M_{23}) \frac{L_1 I_1}{h I_b} + (M_{21} + M_{23} - M_{2'2''}) 2 \frac{L_1 I_1}{h I_b} -$$

$$-M_{2'2''} 2 \frac{L_2 I_1}{h I_b} + (M_{2'1} + M_{2'3}) \frac{L_2 I_1}{h I_b} = 0$$

$$\text{or} \quad -r_1 M_{21} - r_1 M_{23} + 2r_1 M_{21} + 2r_1 M_{23} - 2(r_1 + r_2) M_{2'2''} +$$

$$+ r_2 M_{2'1} + r_2 M_{2'3} = 0 \quad \text{----- 5.23}$$

$$3'2'2'' \quad -M_{32} c_{32} \frac{I_1}{I_2} + M_{23} c_{22} 2 \frac{I_1}{I_2} + M_{2'2''} 2 \frac{L_2 I_1}{h I_b} -$$

$$-(M_{2'1} + M_{2'3}) \frac{L_2 I_1}{h I_b} + (\Delta_3 - \Delta_2) \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.24}$$

$$1'2'3' \quad -M_{12} c_{12} r_3 + M_{21} c_{22} 2r_3 - M_{23} c_{22} 2r_3 + M_{32} c_{32} r_3 -$$

$$-(\Delta_1 - 2\Delta_2 + \Delta_3) \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.25}$$

$$2'2''3'' - M_{2'2''} r_2 + M_{2''2'} 2r_2 + M_{2'3''} (2r_2 + c_{22} r_3) - M_{3''2'} c_{32} r_3 -$$

$$-(\Delta_2 - \Delta_3) \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.26}$$

$$1''2''3'' - M_{1''2''} c_{12} r_4 + M_{2''1''} c_{22} 2r_4 - M_{2''3''} c_{22} 2r_4 + M_{3''2''} c_{32} r_4 -$$

$$-(\Delta_1 - 2\Delta_2 + \Delta_3) \frac{6EI_1}{h^2} = 0 \quad \text{----- 5.27}$$

$$\text{Equilibrium } M_{23} + M_{32} + M_{2'3} + M_{3'2} + M_{2''3} + M_{3''2} - \Delta_2 (P_2 + P_{2'} + P_{2''}) +$$

$$+ \Delta_3 (P_2 + P_{2'} + P_{2''}) = W_0 + W_1 + W_2 \quad \text{----- 5.28}$$

The typical element matrix can then be set out as shown in Fig. 5.21.

5.30 Experiments on 3-column Concrete Shear Wall

A 6-storey model of microconcrete was made to the dimensions given in Fig. 5.30. The reinforcement is shown in Fig. 5.31.

In the test arrangement, 36 electrical strain gauges were bonded to the model in the positions shown in Fig. 5.30. The model was then erected in the test frame (made of large size steel I sections) and 6 dial gauges installed for measuring lateral deflections at each storey level. In addition, dial gauges were set up at the ends of the base, to check for possible rotation of the model base under lateral loading. A further 3 dial gauges were mounted at the top of the model to measure, if possible, axial deformations of each column under the effects of axial loads. The complete test arrangement is shown in Fig. 5.32.

The shear wall was first subjected to axial compressive loads of 4484N acting on each column. Lateral loads were then applied, at the top only, in increments of 300N up to a total of 10,000N. Deflection and strain gauge readings were recorded by eye and the data logger respectively, keeping careful watch for incipient cracking of the concrete. Finally the structure was taken to failure, Fig. 5.33.

EQN NO	COND ^m	1			1'			1''		2			2'			2''			3			3'			3''			LOAD VECTOR
		M ₁₀	M ₁₂	Δ ₁	M _{1'0}	M _{1'2}	M _{1'1''}	M _{1''0}	M _{1''2}	M ₂₁	M ₂₂	Δ ₂	M _{2'1}	M _{2'2}	M _{2'2''}	M _{2''1}	M _{2''2}	M ₃₁	M ₃₂	Δ ₃	M _{3'1}	M _{3'2}	M _{3'3''}	M _{3''1}	M _{3''2}			
1	3 2 2'	0	0	0	0	0	0	0	0	2γ ₁	2C ₂₂ ·2γ ₁	-1	-γ ₁	-γ ₁	γ ₁	0	0	-C ₃₂	0	1	0	0	0	0	0	0		
2	1 2 3	0	-C ₁₂	-1	0	0	0	0	0	2C ₂₂	-2C ₂₂	2	0	0	0	0	0	C ₃₂	0	-1	0	0	0	0	0	0		
3	2 2' 2''	0	0	0	0	0	0	0	0	-γ ₁	-γ ₁	0	2γ ₁	2γ ₁	-2(γ ₁ ·γ ₂)	γ ₂	γ ₂	0	0	0	0	0	0	0	0			
4	3' 2' 2''	0	0	0	0	0	0	0	0	0	0	-1	0	2γ ₂ C ₂₂	2γ ₂	-γ ₂	-γ ₂	0	0	1	-γ ₂ C ₃₂	0	0	0	0			
5	1' 2' 3'	0	0	-1	0	-γ ₂ C ₁₂	0	0	0	0	0	2	2γ ₂ C ₂₂	-2γ ₂ C ₂₂	0	0	0	0	0	-1	γ ₂ C ₃₂	0	0	0	0			
6	2' 2'' 3''	0	0	0	0	0	0	0	0	0	2γ ₂ +C ₂₂ γ ₂	-1	0	0	-γ ₂	2γ ₂	0	0	0	1	0	0	0	-γ ₂ C ₃₂	0			
7	1'' 2'' 3''	0	0	-1	0	0	0	0	-γ ₂ C ₁₂	0	0	2	0	0	0	2γ ₂ C ₂₂	-2γ ₂ C ₂₂	0	0	-1	0	0	0	γ ₂ C ₃₂	0			
8	2 2' 2''										1	-γ ₂ ·γ ₁ ·γ ₂		1			1	1		γ ₂ ·γ ₂ ·γ ₂	1			1	W ₀ +W ₁ +W ₂			

For columns with uniform section throughout height

$$C_{11} = C_{22} = C_{33} = C_{00}$$

$$C_{10} = C_{21} = C_{32} = C_{10}$$

FIG. 5-21 TYPICAL ELEMENT MATRIX INCLUDING INSTABILITY EFFECT FOR CONCRETE SHEAR WALL

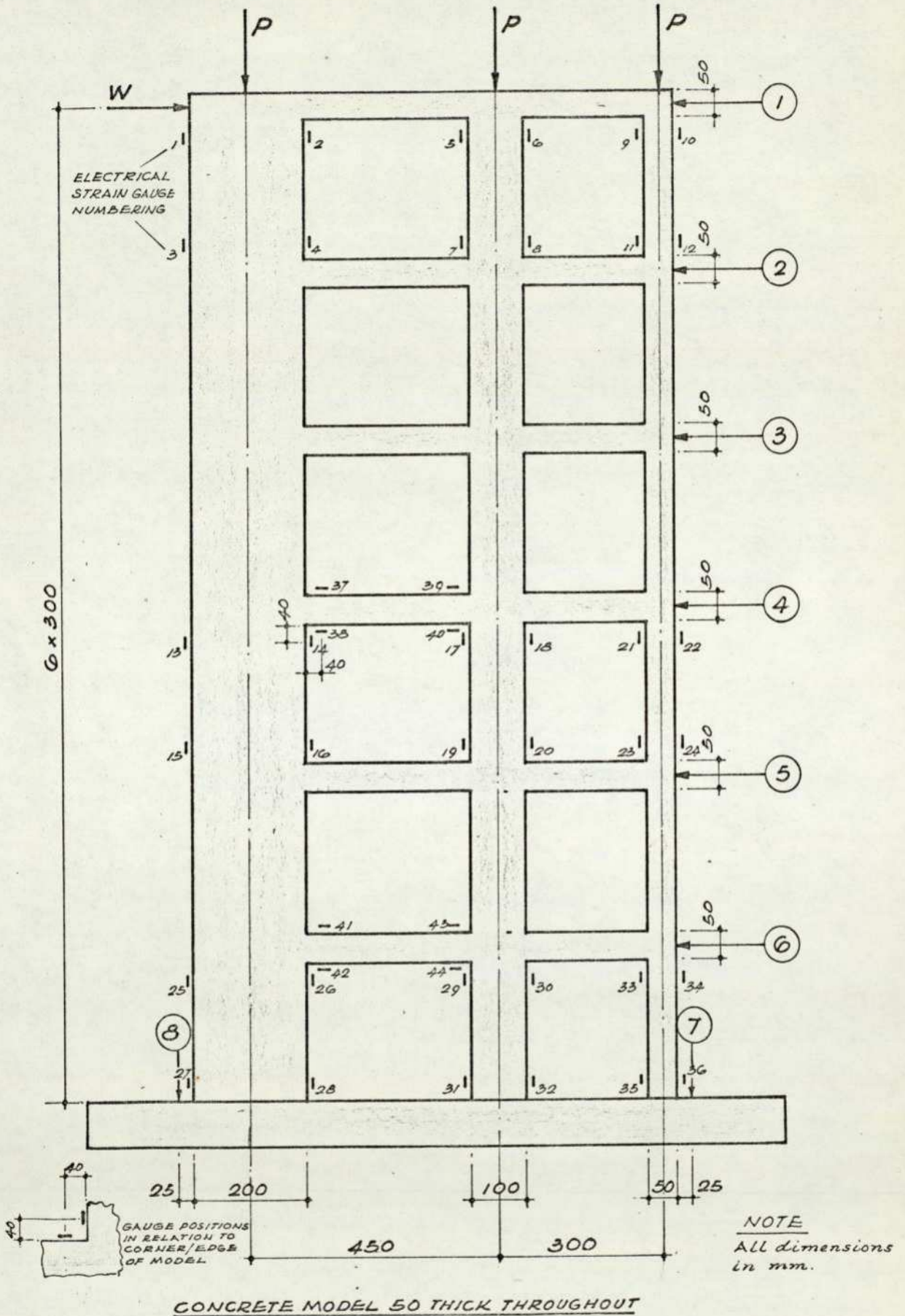
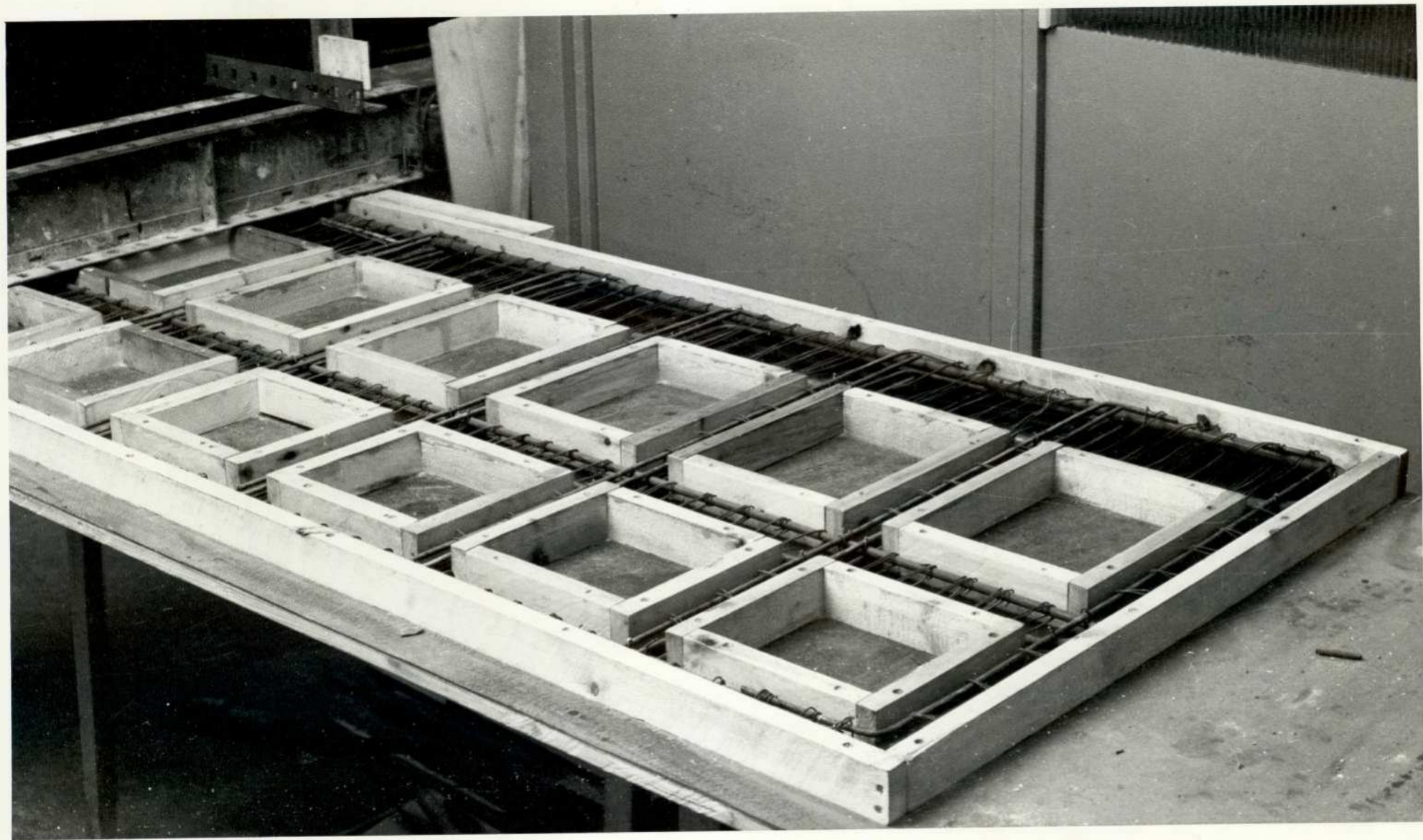


FIG. 5-30 DETAILS OF 6-STORY CONCRETE MODEL SHEAR WALL



5.31 Mould and Reinforcing Steel for 3-column Concrete Shear Wall.



5.32 Experimental Arrangement for 3-column Concrete Shear Wall Model.

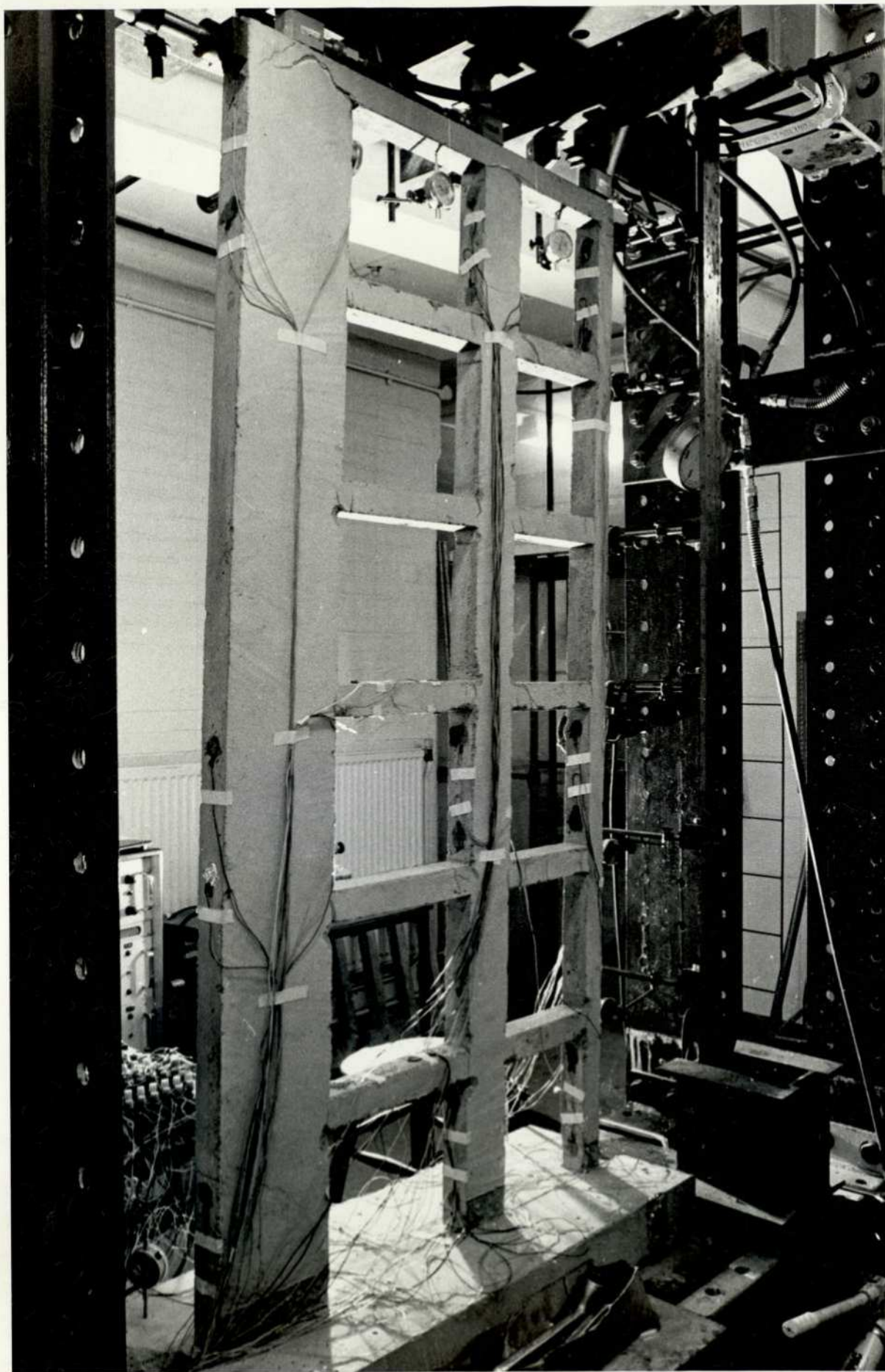


Fig. 5.33 Ultimate Failure of 3-column Concrete Shear Wall Model.

To establish the amount of rotation of the base of the model, readings of gauges 7 and 8 were plotted against lateral load as shown in Fig. 5.34. Taking account of the distance between these gauges and the height of the model, the base rotation was converted to an equivalent lateral deformation at the top of the model and this was deducted from the measured deflection to give the true flexural deflection as shown in Fig. 5.35.

Strain gauge readings were meaningless, being either of very low order or erratically high. For this reason the data were not plotted.

5.40 Discussion of Test Results for Concrete Shear Wall

Comparing the theoretical and experimental deflections, Fig. 5.41, shows that the experimental deflection is 3.5 times greater than the theoretical value. This discrepancy is large but fairly consistent with observed differences of theoretical and experimental deflections from tests on the 10-storey micro-concrete models. Such differences are attributed to extensive micro-cracking of the model caused by shrinkage and the high restraint imposed by the large amount of reinforcing steel used in the model construction.

This phenomena would also account for the erratic readings of the electrical strain gauges making it impossible in this test to correlate theoretical and experimental stresses and moments.

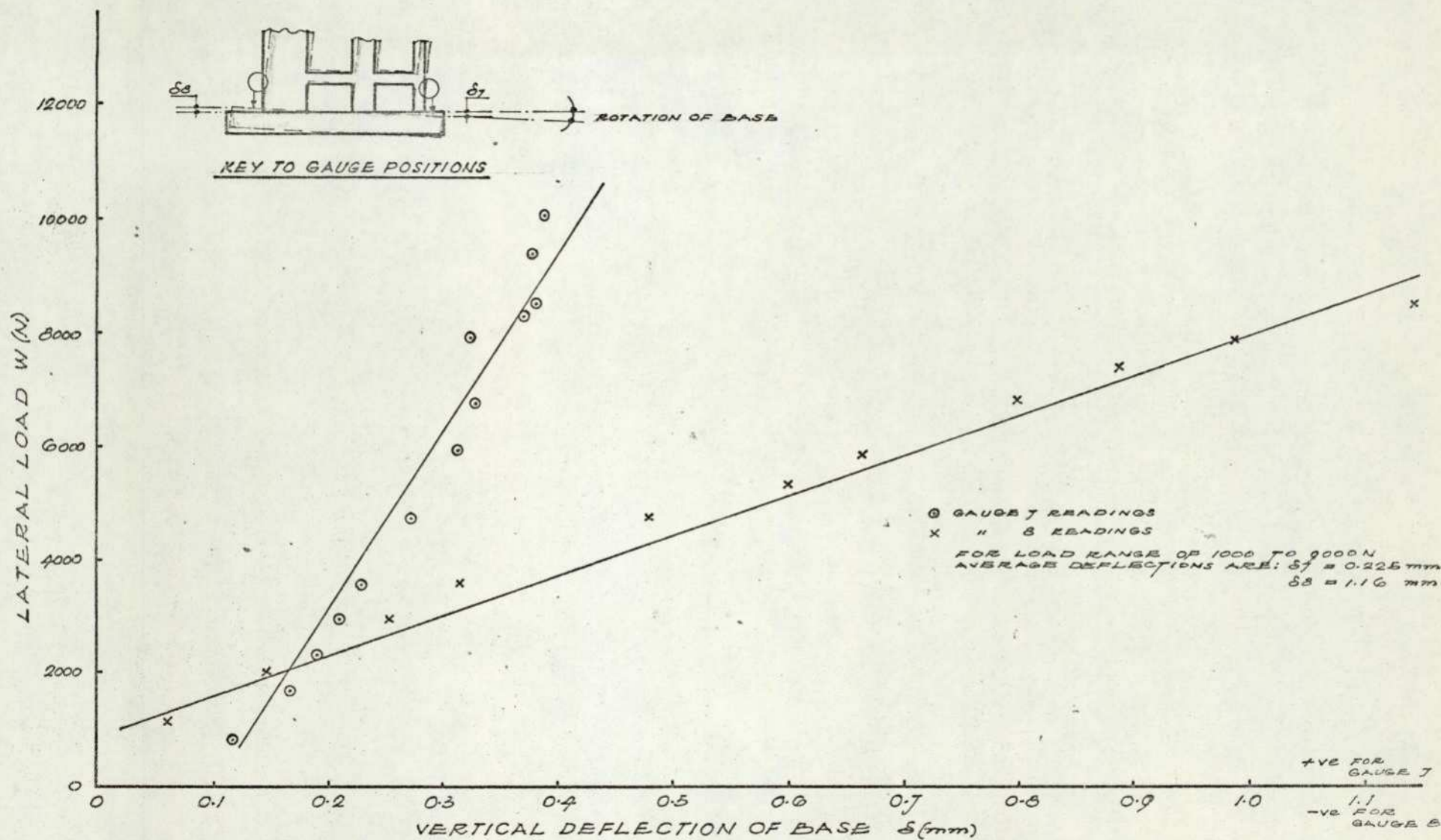


FIG. 5.34. VERTICAL DEFLECTIONS OF BASE OF CONCRETE SHEAR WALL

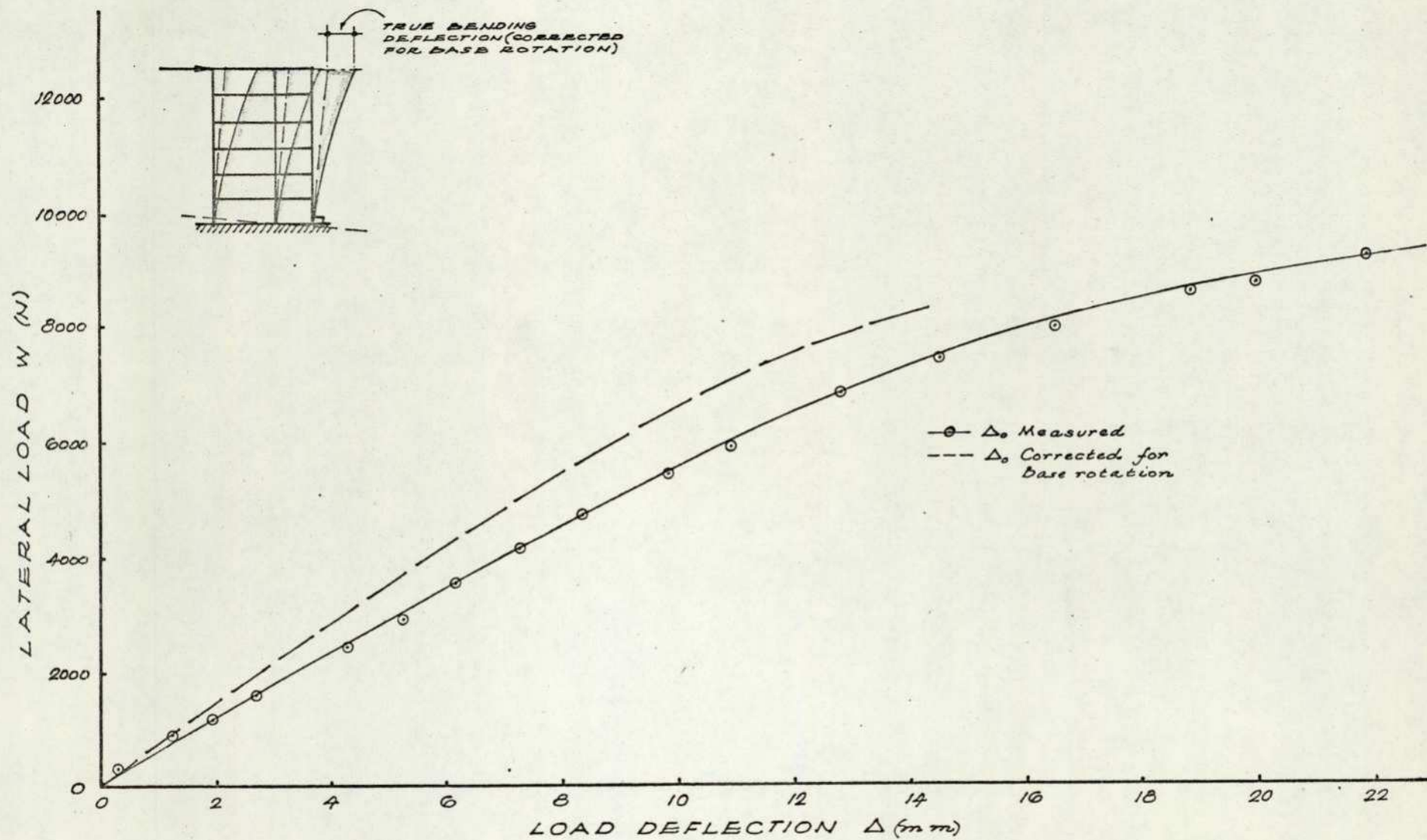


FIG. 5.35 LOAD DEFLECTION CURVES FOR 6-STOREY CONCRETE SHEAR WALL

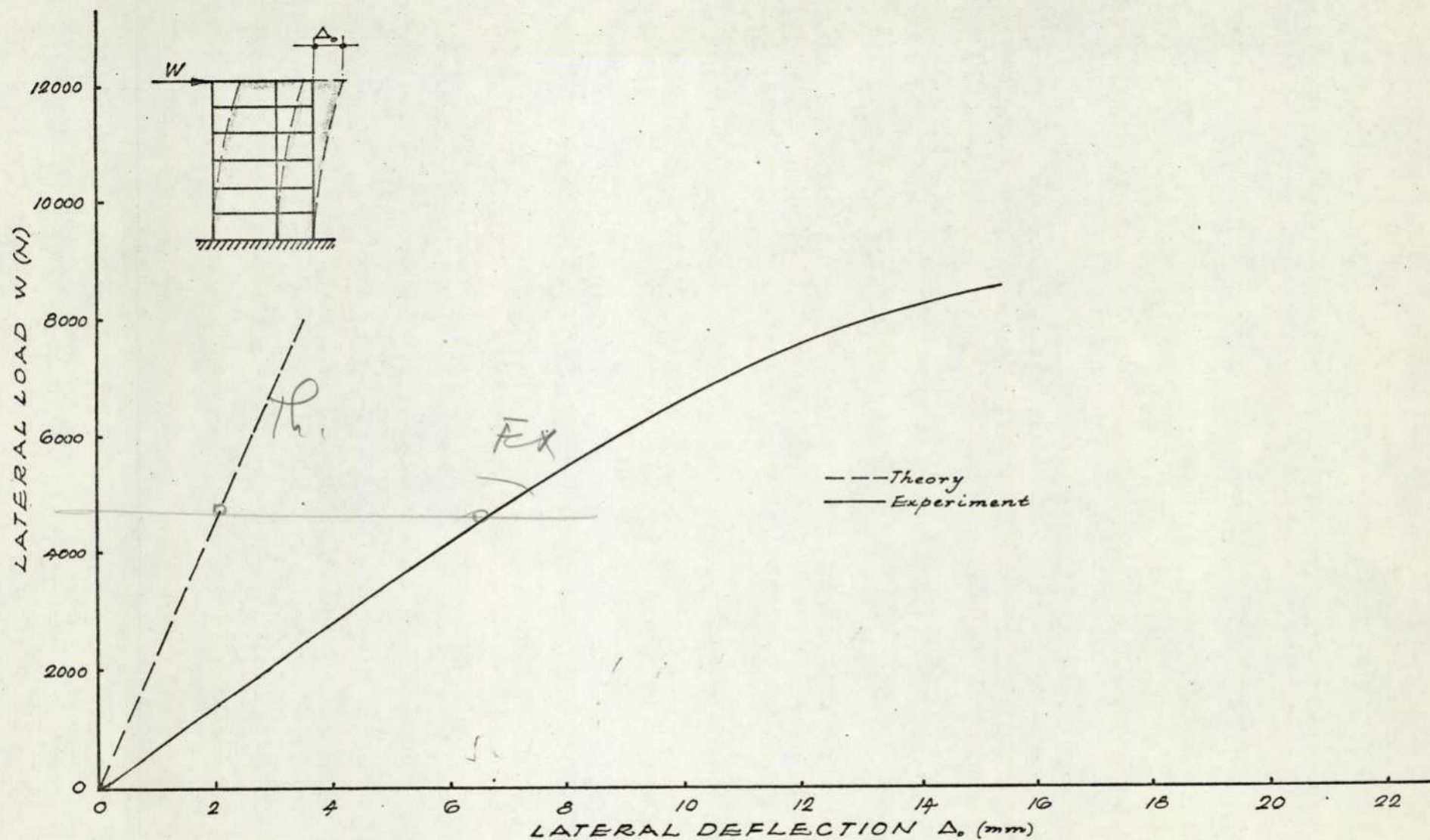


FIG. 5.41 COMPARISON OF THEORETICAL AND EXPERIMENTAL LATERAL DEFLECTION CONCRETE SHEAR WALL

6.00 Elastic Instability of Frames Arising from Out-of-plumb Construction

In practice, all tall buildings are out-of-plumb to a small or large extent and the problem arises: to what extent this is detrimental; what can be considered as an acceptable limit; and how to take this effect into account in the analysis?

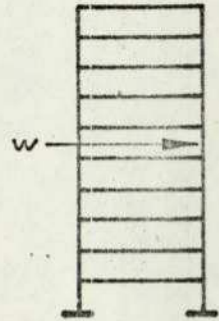
To explore this, several analyses have been made including the effect of out-of-plumb construction subjected to gravity and wind forces, and experiments were made.

A 10-storey frame with practical dimensions, with and without out-of-plumb inaccuracies, is first analysed for primary and instability effects under a range of lateral and vertical loading conditions. Next, using a 6-storey micro-concrete model with large out-of-plumb inaccuracies, the theory is developed and compared with the experimental behaviour of the model.

The analyses are made in two stages. In the first stage only the "primary" analysis is made including the effect of eccentricity of gravity loads on the out-of-plumb frame. In the second stage, some or all of the effects of elastic instability are included, i.e. the effect of axial forces and deformations, the effect of altered stiffnesses (or flexibility) of members, and the effect of altered geometry of deflected structure, i.e. the effect of large deformations.

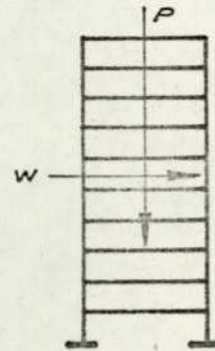
6.10. Analyses of Practical Out-of-plumb Frames

A ten-storey single bay frame of practical dimensions (storey height 3.6m, span 9m) is taken as a basis for the force-displacement analyses for the several cases shown in Fig. 6.100. Six cases are examined. Cases 1 and 2 relate to primary and instability analyses assuming the frame to be straight (free from out-of-plumb inaccuracies) thus providing a basis for comparison of out-of-plumb effects. Cases 3 and 4 consider the out-of-plumb frame under the action of gravity loads with and without lateral loads but ignoring elastic instability. Case 5 includes the instability effect under gravity and lateral loads, while Case 6 examines the effect of lateral (wind) load acting in the opposite direction

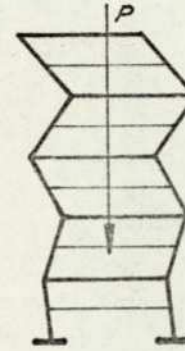


1

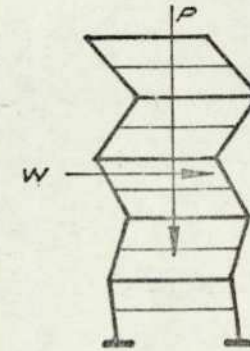
TAKING COLUMN
FLEXIBILITY INTO
ACCOUNT



2

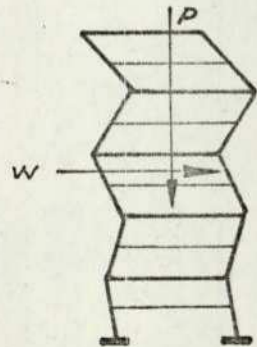


3



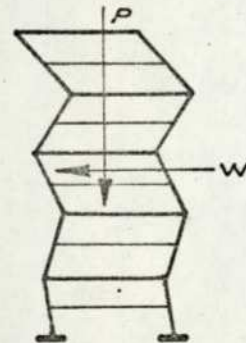
4

TAKING COLUMN
FLEXIBILITY INTO
ACCOUNT



5

OPPOSITE WIND
FORCE



6

NOTES

1. NUMBER OF STOREYS = 10
2. STOREY HEIGHT = 3.6 m
3. SPAN = 9.0 m
4. W = LATERAL LOAD APPLIED AT EACH STOREY LEVEL
5. P = GRAVITY LOAD OF EQUAL MAGNITUDE APPLIED AT EACH BEAM/COLUMN JUNCTION
6. COLUMNS: 0.3m x 0.6m., BEAMS: 0.3m x 0.6m IN SECTION.

Fig: 6.100 ANALYTICAL CASES
CONSIDERED IN OUT-OF-PLUMB
ANALYSES OF A PRACTICAL FRAME

For the first stage the primary analysis is made first in general terms and is then applied to the practical frame and also, later, to the model used in testing.

Based on the idealised out-of-plumb frame of Fig. 6.101, the element force-displacement matrix is set out at the level 2-2' as follows:-

$$123 \quad -M_{12} \frac{h}{6EI_c} + M_{21} \frac{h}{3EI_c} - M_{23} \frac{h}{3EI_c} + M_{32} \frac{h}{6EI_c} = \\ = \frac{\Delta_1 - \Delta_2}{h} - \frac{\Delta_2 - \Delta_3}{h}$$

$$322' \quad -M_{32} \frac{h}{6EI_c} + M_{21} \frac{h}{3EI_c} + (M_{21} + M_{23}) \frac{L}{6EI_b} = \frac{\Delta_2 - \Delta_3}{h}$$

$$\text{Ec. 2} \quad \frac{M_{23} + M_{32}}{h} - \frac{M_{21} + M_{12}}{h} = \frac{W}{2} - P_2 \left(\frac{\Delta_2 + a_2}{h} \right)$$

These equations are now modified by multiplying the first two equations by $6EI_c/h$ and the last by h . The general matrix becomes non-dimensional. This is shown in Fig. 6.102.

In the second stage, the analysis of the effect of out-of-plumb construction includes instability parameters, i.e. the effect of axial forces on the flexibility coefficients and the effect of large deformations. Not included is the effect of axial deformations (the latter has been considered earlier).

The element matrix is based on the following conditions of compatibility of deformations and equilibrium of forces (where $I_c = I_b = I$):

$$100' \quad M_{10} \frac{c_{10} h}{6EI} - M_{01} \left(\frac{c_{00} h}{3EI} + \frac{L}{3EI} \right) - M_{01} \frac{L}{6EI} = \frac{\Delta_0 - \Delta_1}{h}$$

$$\text{or} \quad M_{10} c_{10} - M_{01} \left(2c_{00} + \frac{L}{h} \right) + (\Delta_1 - \Delta_0) \frac{6EI}{h^2} = 0$$

Equilibrium condition $O-O'$:

$$\frac{M_{10} + M_{01}}{h} = \frac{W}{2} - P \left(\frac{a_0 + \Delta_0 - \Delta_1}{h} \right)$$

$$\text{or} \quad M_{01} + M_{10} + \frac{P_0 h^2}{6EI} - \frac{P_0 h^2}{6EI} = \frac{Wh}{2} - P_0 a_0$$

$$211' \quad M_{21} \frac{c_{12} h}{6EI} - M_{12} \left(\frac{c_{11} h}{3EI} + \frac{L}{3EI} \right) - M_{10} \frac{L}{6EI} = \frac{\Delta_2 - \Delta_1}{h}$$

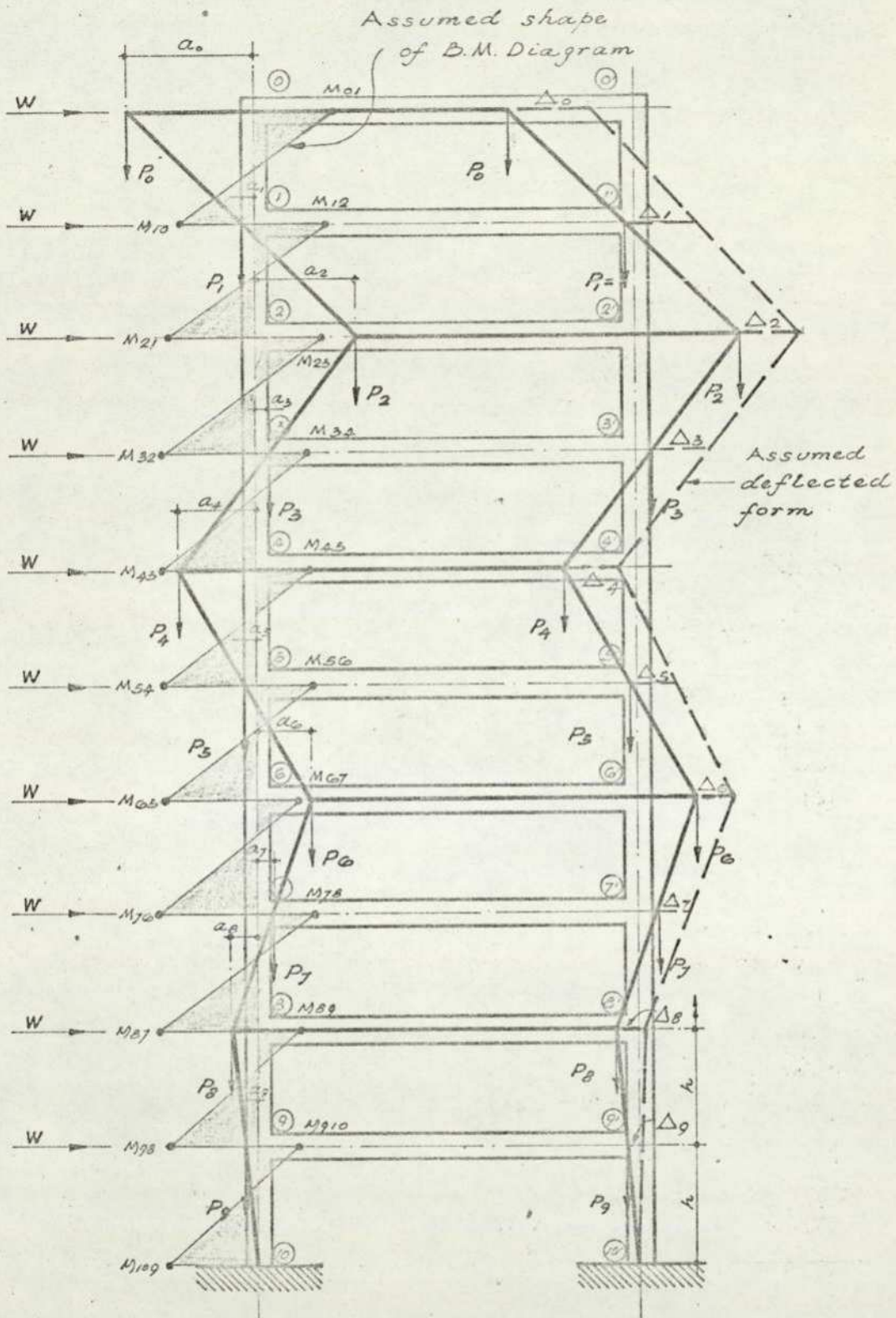


FIG. G.101 OUT-OF-PLUMB PRACTICAL FRAME USED IN ANALYSES

SQU NO	COND ⁿ	1					2			3			4			5			6			7			8			9			10	LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₀	
1	100	-(2+r)	-1	1	0	1																										0
2	0-0	1	0	1	0	P ₂ h																										$\frac{1}{2}Wh - \frac{1}{2}P_2h$
3	211	0	0	-r	-(2+r)	-1	1	0	1																							0
4	012	1	-1	-2	2	2	-1	0	-1																							0
5	1-1	-1	0	-1	1	P ₂ h	1	0	-P ₂ h																							$\frac{1}{2}Wh - \frac{1}{2}P_2h$
6	322				0	0	-r	-(2+r)	-1	1	0	1																				
7	123				1	-1	-2	2	2	-1	0	-1																				
8	2-2				-1	0	-1	1	P ₂ h	1	0	P ₂ h																				
9	233																															
10	234																															
11	3-3																															
12	344																															
13	345																															
14	4-4																															
15	655																															
16	456																															
17	5-5																															
18	766																															
19	567																															
20	6-6																															
21	877																															
22	678																															
23	7-7																															
24	988																															
25	789																															
26	8-8																															
27	1000																															
28	890																															
29	9-9																															
30	109																															

FIG. 6.102 GENERAL MATRIX FOR PRIMARY ANALYSIS OF OUT-OF-PLUMB PRACTICAL FRAME — EFFECT OF GRAVITY AND WIND LOADS

$$r = \left(\frac{L}{h} \right) \left(\frac{I_c}{I_b} \right)$$

$$\text{or} \quad M_{21}c_{12} - M_{12}\left(2c_{11} + \frac{L}{h}\right) - M_{10}\frac{L}{h} + (\Delta_1 - \Delta_2)\frac{6EI}{h^2} = 0$$

$$012 \quad M_{01}\frac{c_{01}h}{6EI} - M_{10}\frac{c_{00}h}{3EI} + M_{12}\frac{c_{11}h}{3EI} - M_{21}\frac{c_{12}h}{6EI} =$$

$$\frac{\Delta_0 - \Delta_1}{h} - \frac{\Delta_1 - \Delta_2}{h}$$

$$\text{or} \quad M_{01}c_{01} - M_{10}2c_{00} + M_{12}2c_{11} - M_{21}c_{12} + (-\Delta_0 + 2\Delta_1 - \Delta_2)\frac{6EI}{h^2} = 0$$

The loading cases for the practical frame shown in Fig. 6.100 are now analysed in sequence. The following data apply to each analysis:
Column and beam sections are identical; depth = 0.6m, thickness = 0.3m.

$$L = 9.0\text{m}, h = 3.6\text{m}, E_c = 28 \text{ kN/mm}^2, I = \frac{1}{12} \times .3 \times .6^3 = 0.0054\text{m}^4,$$

$$EI = 151,200 \text{ kNm}^2, r = L/h = 9/3.6 = 2.5, k = h^2/6EI = 1.428 \times 10^{-5},$$

$$P \text{ applied axially at each node} = 100 \text{ kN i.e. } P_0 = 100 \text{ kN}, P_1 = 200 \text{ kN},$$

$$P_3 = 300 \text{ kN etc.}$$

The frame is first analysed for case 1 (primary analysis for the frame assuming it is straight, i.e. free from out-of-plumb inaccuracies). The diagram of moments and deflection is shown in Fig. 6.103. The diagram of shear forces in beams and axial forces in columns is shown in Fig. 6.104.

The straight frame is now analysed for case 2, Fig. 6.100-instability analysis taking column flexibility into account assuming axially applied loads of 100 kN at all nodes. The flexibility coefficients c for the variable column loadings are given in Fig. 6.105.

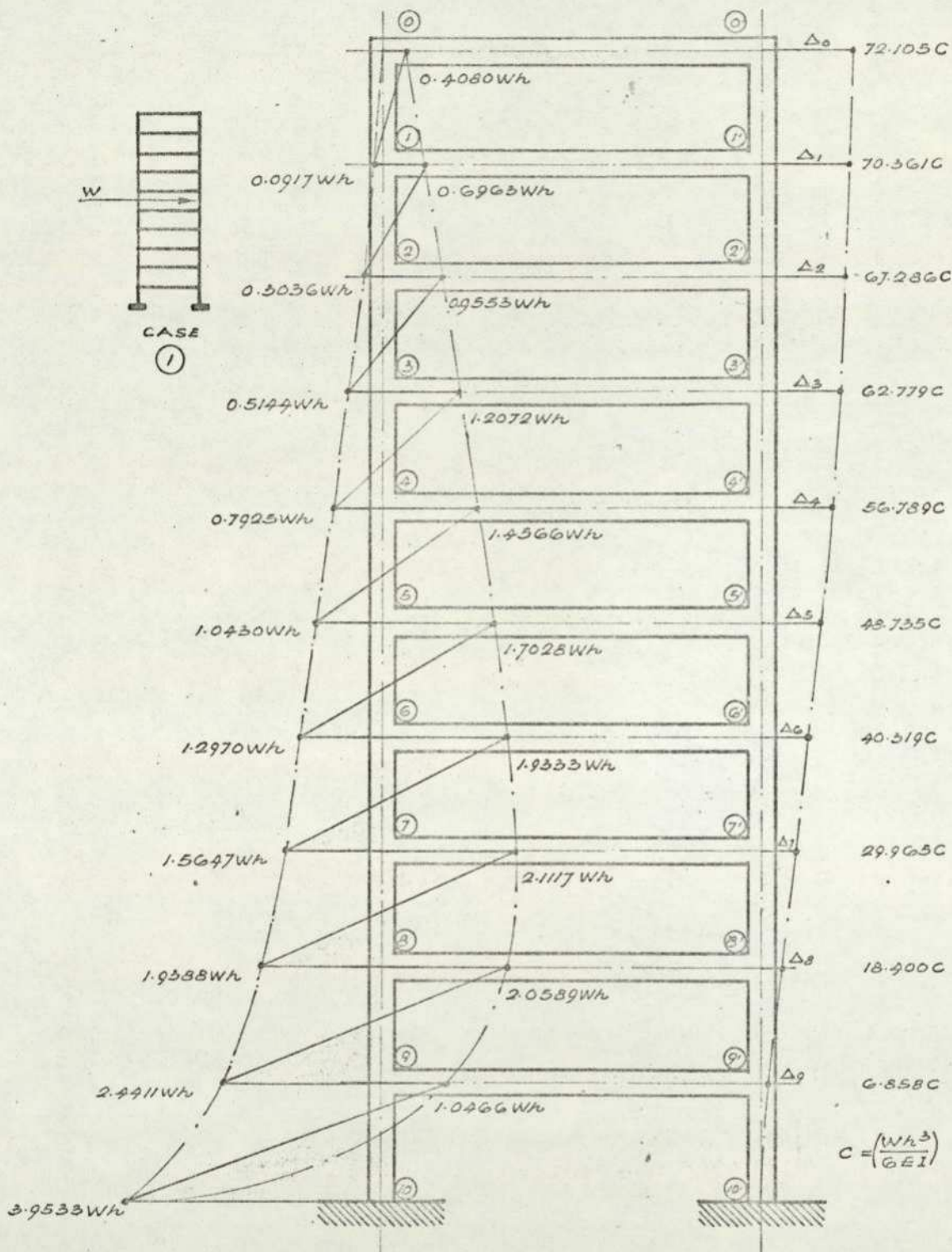


FIG. 6.103 MOMENT/DEFLECTION DIAGRAM FOR 'STRAIGHT' PRACTICAL FRAME, PRIMARY ANALYSIS.

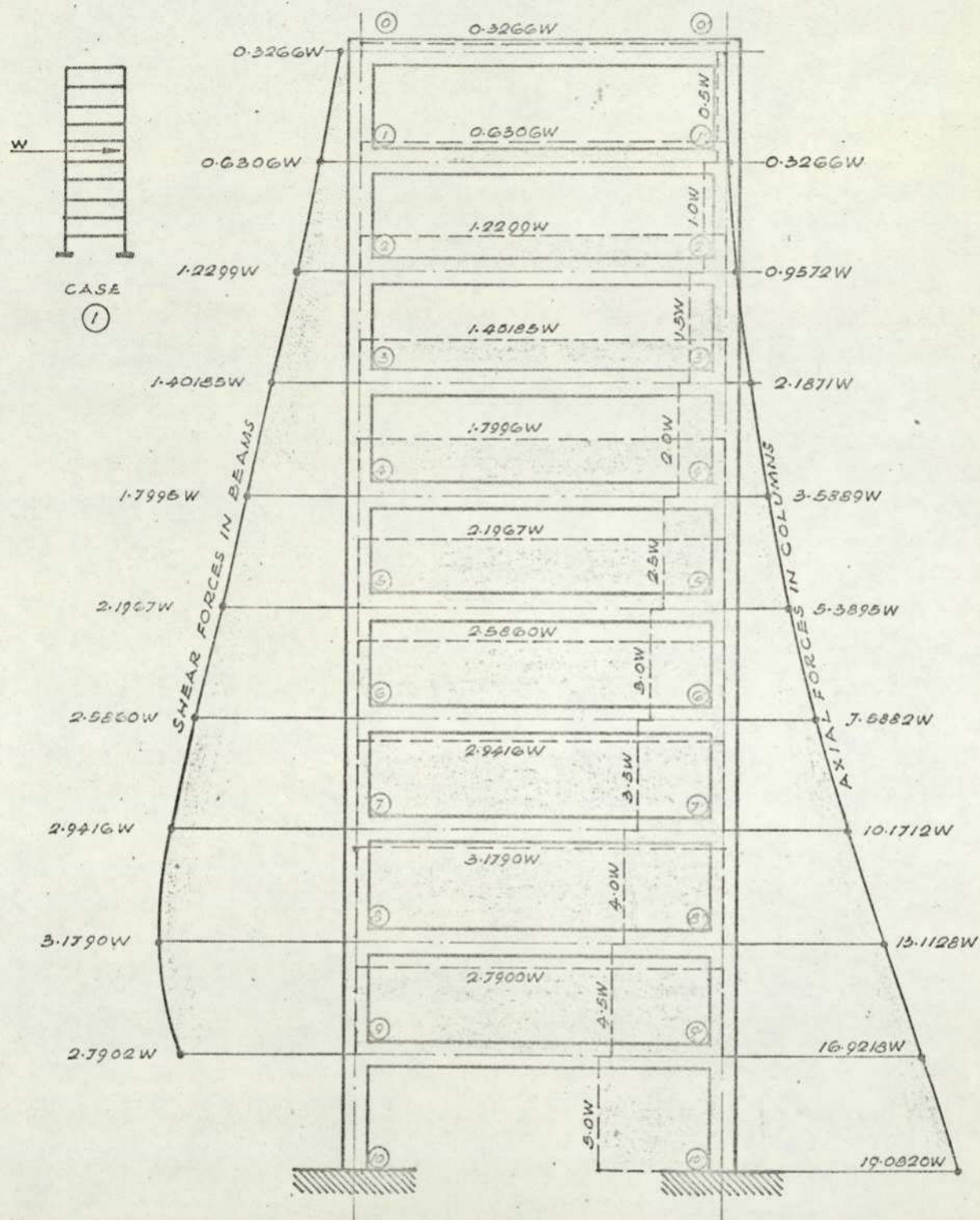


FIG. 6.104 DIAGRAM OF SHEARING AND AXIAL FORCES FOR 'STRAIGHT' PRACTICAL FRAME, PRIMARY ANALYSIS.

Column	2U	Flexibility Coefficients	
01	0.092	$c_{00} = 1.00064$	$c_{01} = 1.0074$
12	0.13	$c_{11} = 1.0013$	$c_{12} = 1.0030$
23	0.159	$c_{22} = 1.0019$	$c_{23} = 1.0031$
34	0.184	$c_{33} = 1.0024$	$c_{34} = 1.0041$
45	0.20	$c_{44} = 1.0027$	$c_{45} = 1.0047$
56	0.225	$c_{55} = 1.0036$	$c_{56} = 1.0062$
67	0.244	$c_{66} = 1.0042$	$c_{67} = 1.0073$
78	0.260	$c_{77} = 1.0046$	$c_{78} = 1.0083$
89	0.276	$c_{88} = 1.0053$	$c_{89} = 1.0091$
910	0.290	$c_{99} = 1.0058$	$c_{9,10} = 1.0100$

Fig. 6.105 Flexibility Coefficients for 10 storey frame with Axial Loads of 100 kN applied at each Node.

The matrix is shown in Fig. 6.106 and the bending moment/deflection diagram in Fig. 6.107.

It is now assumed that certain out-of-plumb inaccuracies exist as shown in Fig. 6.108.

Under gravity loading alone but ignoring the effect of elastic instability, (i.e. case 3 Fig. 6.100) the general matrix is as shown in Fig. 6.109. The resulting bending moment and deflection diagram is given in Fig. 6.110.

The out-of-plumb frame is now analysed for the effect of lateral and gravity forces, elastic instability again being ignored i.e. case 4, Fig. 6.100.

The following data apply:

$$\text{For } \frac{W}{P_0} = 0.1 \quad P_0 = 100 \text{ kN}$$

$$\frac{W}{2} 10W a_0 = Wh \left(\frac{1}{2} - \frac{10a_0}{h} \right) = Wh (0.5 - 0.125)$$

$$Wh (0.5 - 0.125) = 0.375 Wh$$

$$Wh (0.5 - \frac{20a_1}{h}) = 0.25 Wh$$

$$Wh (0.5 + \frac{30a_2}{h}) = 0.792 Wh$$

$$Wh (0.5 + \frac{40a_3}{h}) = 0.890 Wh$$

$$Wh (0.5 - \frac{50a_4}{h}) = 0.152 Wh$$

$$Wh (0.5 - \frac{60a_5}{h}) = 0.082 Wh$$

$$Wh (0.5 + \frac{70a_6}{h}) = 0.792 Wh$$

$$Wh (0.5 + \frac{80a_7}{h}) = 0.832 Wh$$

$$Wh (0.5 - \frac{90a_8}{h}) = 0.375 Wh$$

$$Wh (0.5 - \frac{100a_9}{h}) = 0.361 Wh$$

$$a_0 = a_1 = 45 \text{ mm}$$

$$a_2 = a_3 = 35 \text{ mm}$$

$$a_4 = a_5 = 25 \text{ mm}$$

$$a_6 = a_7 = 15 \text{ mm}$$

$$a_8 = a_9 = 5 \text{ mm}$$

EQN NO	COND	0			1			2			3			4			5			6			7			8			9			10	LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₀		
1	100	2.58126	-1	1.0074	0	1																									0		
2	0-0	1	0.001478	1	0	0.004278																									0.5		
3	211	0	0	2.5	4.5026	-1	1.002	0	1																						0		
4	012	1.0074	-1	2.00178	2.0076	2	1.002	0	-1																						0		
5	1-1	-1	0	-1	1	0.00229	1	0	0.00255																						0.5		
6	322				0	0	2.5	4.5038	-1	1.0031	0	1																			0		
7	123				1.032	-1	2.0025	2.0028	2	1.0031	0	-1																			0		
8	2-2				-1	0	-1	1	0.002578	1	0	0.002578																			0.5		
9	433							0	0	2.5	4.5054	-1	1.0041	0	1																0		
10	234							1.0031	-1	2.0038	2.0048	2	1.0041	0	-1																0		
11	3-3							-1	0	-1	1	0.005712	1	0	0.005712																0.5		
12	544									0	0	2.5	4.5059	-1	1.0047	0	1														0		
13	335									1.0041	-1	2.0048	2.0054	2	1.0047	0	-1														0		
14	2-4									-1	0	-1	1	0.00712	1	0	0.00712														0.5		
15	655												0	0	2.5	4.5084	-1	1.0062	0	1											0		
16	456												1.0047	-1	2.0054	2.0072	2	1.0062	0	-1											0		
17	5-5												-1	0	-1	1	0.005560	1	0	0.005560											0.5		
18	766														0	0	2.5	4.5084	-1	1.0073	0	1									0		
19	567														1.0062	-1	2.0072	2.0084	2	1.0073	0	-1									0		
20	6-6														-1	0	-1	1	0.009996	1	0	0.009996									0.5		
21	877																0	0	2.5	4.5094	-1	1.0082	0	1							0		
22	678																1.0073	-1	2.0084	2.0094	2	1.0082	0	-1							0		
23	7-7																-1	0	-1	1	0.01225	1	0	0.01225							0.5		
24	988																		0	0	2.5	4.5106	-1	1.0091	0	1					0		
25	789																		1.0082	-1	2.0094	2.0106	2	1.0091	0	-1					0		
26	8-8																		-1	0	-1	1	0.01255	1	0	0.01255					0.5		
27	1000																				0	0	2.5	4.5116	-1	1.01					0		
28	900																				1.0091	-1	2.0106	2.0116	2	1.01					0		
29	9-9																				-1	0	-1	1	0.01428	1					0.5		
30	100																												1.01	-1	2.0116	0	

P

W

CASE

(2)

FIG. 6-106 NUMERIC MATRIX FOR INSTABILITY ANALYSIS OF 'STRAIGHT' PRACTICAL FRAME

Defl. are in terms of W_0/GEI
Moments are in terms of W_0

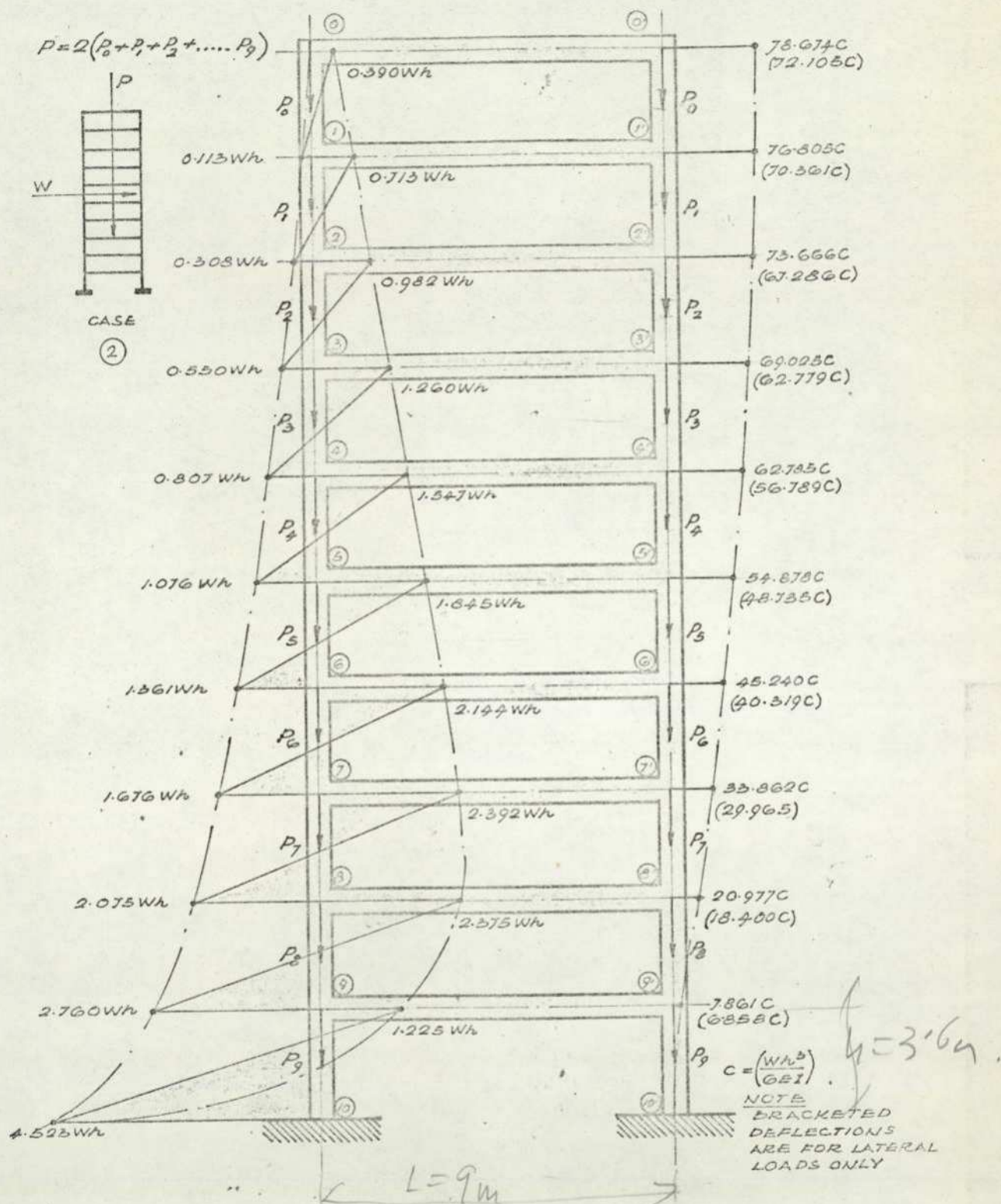


FIG. 6.107 MOMENT/DEFLECTION DIAGRAM FOR 'STRAIGHT' PRACTICAL FRAME, INSTABILITY ANALYSIS.

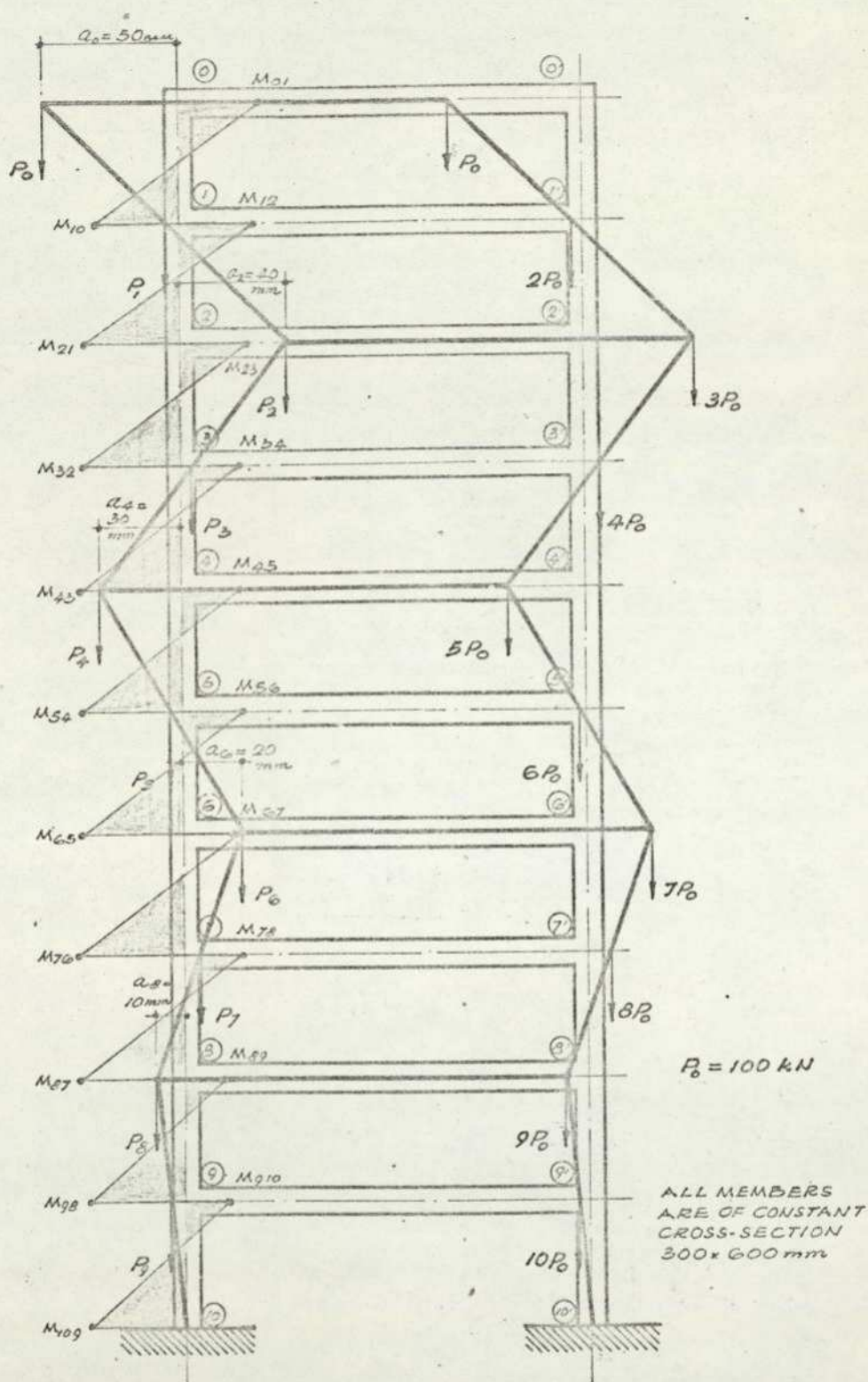
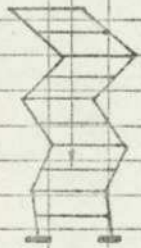


FIG. 6.103 CONFIGURATION OF PRACTICAL OUT-OF-PLUMB FRAME.

EQN NO	COND ⁿ	0		1			2			3			4			5			6			7			8			9			10			LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₀	Δ ₉	M ₁₀₀			
1	100'	(2+r)	-1	1	0	1																									0			
2	0-0'	1	P ₀ k	1	0	-P ₀ k																									-P ₀ a ₀			
3	2/1'	0	0	-r	(2+r)	-1	1	0	1																						0			
4	0/2	1	-1	-2	2	2	-1	0	-1																						0			
5	1-1'	-1	0	-1	1	P ₁ k	1	0	-P ₁ k																						-P ₁ a ₁			
6	3/2'				0	0	-r	(2+r)	-1	1	0	1																			0			
7	1/23				1	-1	-2	2	2	-1	0	-1																			0			
8	2-2'				-1	0	-1	1	P ₂ k	1	0	P ₂ k																			-P ₂ a ₂			
9	4/35						0	0	-r	(2+r)	-1	1	0	1																	0			
10	2/34						1	-1	-2	2	2	-1	0	-1																	0			
11	3-3'						-1	0	-1	1	P ₃ k	1	0	P ₃ k																	-P ₃ a ₃			
12	5/4'								0	0	-r	(2+r)	-1	1	0	1															0			
13	3/45								1	-1	-2	2	2	-1	0	-1															0			
14	4-4'								-1	0	-1	1	P ₄ k	1	0	-P ₄ k															-P ₄ a ₄			
15	6/58										0	0	-r	(2+r)	-1	1	0	1													0			
16	4/56										1	-1	-2	2	2	-1	0	-1													0			
17	5-5'										-1	0	-1	1	-P ₅ k	1	0	P ₅ k													-P ₅ a ₅			
18	7/69												0	0	-r	(2+r)	-1	1	0	1											0			
19	5/67												1	-1	-2	2	2	-1	0	-1											0			
20	6-6'												-1	0	-1	1	-P ₆ k	1	0	P ₆ k											-P ₆ a ₆			
21	8/7'														0	0	-r	(2+r)	-1	1	0	1									0			
22	6/78														1	-1	-2	2	2	-1	0	-1									0			
23	7-7'														-1	0	-1	1	-P ₇ k	1	0	P ₇ k									-P ₇ a ₇			
24	9/88																0	0	-r	(2+r)	-1	1	0	1							0			
25	7/89																1	-1	-2	2	2	-1	0	-1							0			
26	8-8'																-1	0	-1	-1	-P ₈ k	1	0	P ₈ k							-P ₈ a ₈			
27	10/9'																		0	0	-r	(2+r)	-1	1	0	1					0			
28	8/90																		1	-1	-2	2	2	-1	0						0			
29	9-9'																		-1	0	-1	-1	-P ₉ k	1							-P ₉ a ₉			
30	10/9																												1	-1	-2	0		

P



CASE
③

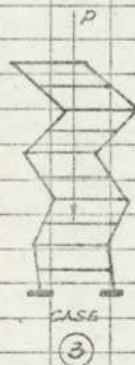


FIG. 6-109 GENERAL MATRIX FOR PRACTICAL OUT-OF-PLUMB FRAME
SUBJECTED TO GRAVITY LOAD ALONE

$$r = \frac{L I_c}{h I_b} \quad k = \frac{h^2}{6EI}$$

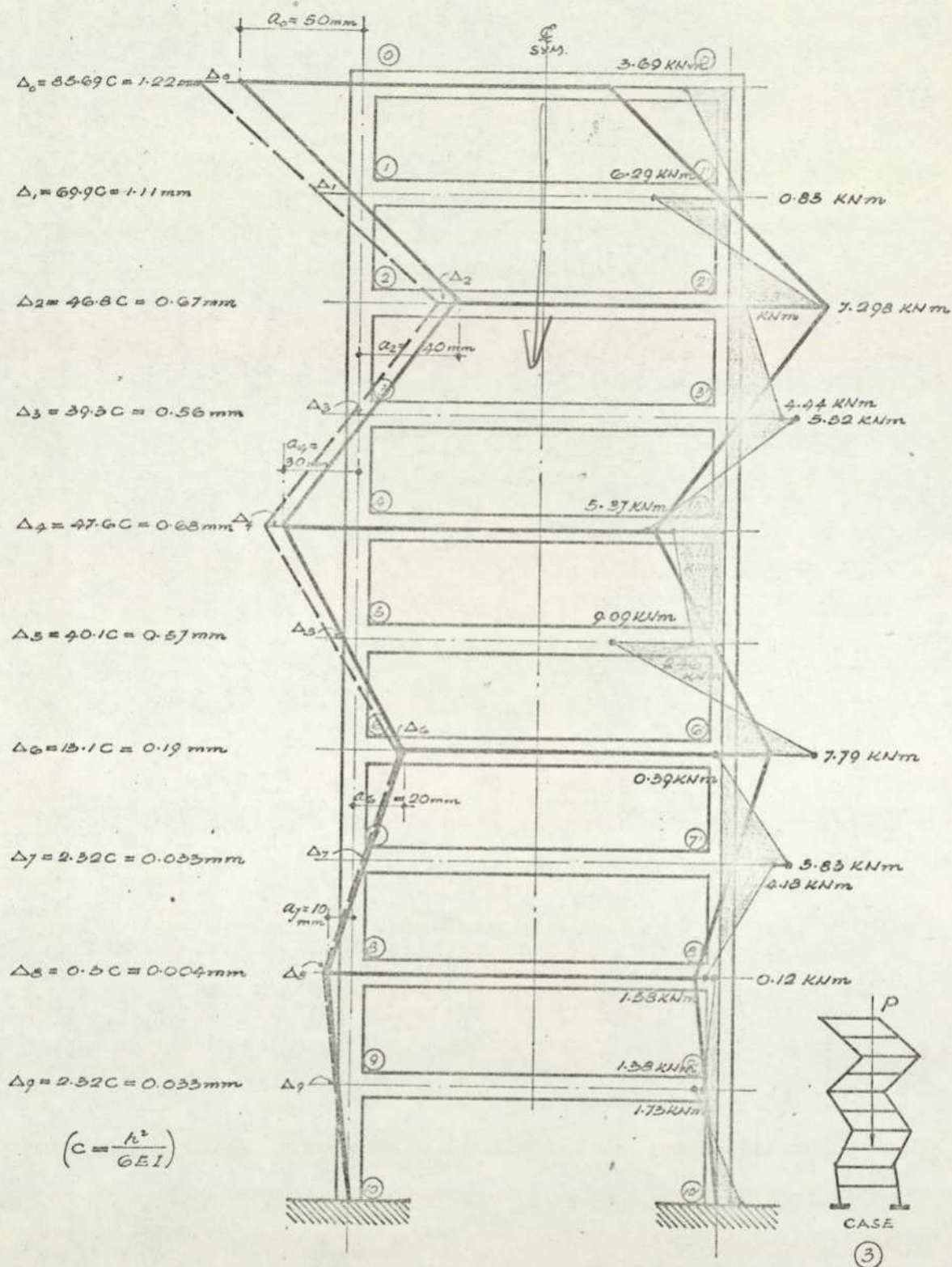


FIG. 6.110 MOMENT/DEFLECTION DIAGRAM FOR PRACTICAL OUT-OF-PLUMB FRAME SUBJECTED TO GRAVITY LOAD ALONE (ELASTIC INSTABILITY IGNORED).

Introducing these values into the general force-displacement matrix, Fig. 6.111, gives the particular matrix of Fig. 6.112. The corresponding diagram of moments and deflections is shown in Fig. 6.113.

Taking elastic instability of the out-of-plumb frame into account under the action of lateral and gravity loads, i.e. case 5 Fig. 6.100, gives the general matrix shown in Fig. 6.114. For $P_o = 100$ kN the particular matrix for $W/P = 0.1$ and $W/P = 0.5$ is given in Fig. 6.115. The appropriate bending moment and deflection diagrams are shown in Fig. 6.116 and 6.117.

The out-of-plumb frame is now analysed for gravity forces and lateral loads of unaltered magnitude but assuming the direction of lateral loads is reversed to represent wind acting in the opposite direction, i.e. case 6 Fig. 6.100.

It is interesting to note that the force-displacement matrix remains the same but the load vector is changed. The load vectors are now as follows:-

LOAD VECTOR FOR $W/P = 0.1$	LOAD VECTOR FOR $W/P = 0.5$
$P_o = 10W$	$P_o = 2W$
$\left(-\frac{Wh}{2} - P_o a_o\right) = Wh\left(-\frac{1}{2} - \frac{10a_o}{h}\right) = Wh(-0.5 - 0.125) = -0.625$	$= -0.525$
$\left(-\frac{Wh}{2} - P_1 a_1\right) = Wh\left(-\frac{1}{2} - \frac{20a_1}{h}\right) = Wh(-0.5 - 0.25) = -0.75$	$= -0.550$
$\left(-\frac{Wh}{2} + P_2 a_2\right) = Wh\left(-\frac{1}{2} + \frac{30a_2}{h}\right) = Wh(-0.5 + 0.292) = -0.208$	$= -0.442$
$\left(-\frac{Wh}{2} + P_3 a_3\right) = Wh\left(-\frac{1}{2} + \frac{40a_3}{h}\right) = Wh(-0.5 + 0.39) = -0.110$	$= -0.420$
$\left(-\frac{Wh}{2} - P_4 a_4\right) = Wh\left(-\frac{1}{2} - \frac{50a_4}{h}\right) = Wh(-0.5 - 0.348) = -0.848$	$= -0.5696$
$\left(-\frac{Wh}{2} - P_5 a_5\right) = Wh\left(-\frac{1}{2} - \frac{60a_5}{h}\right) = Wh(-0.5 - 0.418) = -0.918$	$= -0.5836$
$\left(-\frac{Wh}{2} + P_6 a_6\right) = Wh\left(-\frac{1}{2} + \frac{70a_6}{h}\right) = Wh(-0.5 + 0.292) = -0.208$	$= -0.4416$
$\left(-\frac{Wh}{2} + P_7 a_7\right) = Wh\left(-\frac{1}{2} + \frac{80a_7}{h}\right) = Wh(-0.5 + 0.333) = -0.167$	$= -0.4334$
$\left(-\frac{Wh}{2} - P_8 a_8\right) = Wh\left(-\frac{1}{2} - \frac{90a_8}{h}\right) = Wh(-0.5 - 0.125) = -0.625$	$= -0.525$
$\left(-\frac{Wh}{2} - P_9 a_9\right) = Wh\left(-\frac{1}{2} - \frac{100a_9}{h}\right) = Wh(-0.5 - 0.139) = -0.639$	$= -0.5278$

The bending moment and deflection diagram for $W/P = 0.1$ and 0.5 are shown in Figs. 6.118 and 6.119 respectively.

ROW NO	COND	0		1			2			3			4			5			6			7			8			9			10	LOAD VECTOR			
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₀				
1	100	(2+r)	-1	1	0	1																										0	W _H /2		
2	0-0	1	P ₀ k	1	0	-P ₀ k																										0	-P ₀ /2		
3	211	0	0	-r	-(2+r)	-1	1	0	1																							0	0		
4	012	1	-1	-2	2	2	-1	0	-1																							0	W _H /2		
5	1-1	-1	0	-1	1	P ₀ k	1	0	-P ₀ k																							0	-P ₀ /2		
6	322				0	0	-r	-(2+r)	-1	1	0	1																					0	0	
7	123				1	-1	-2	2	2	-1	0	-1																				0	W _H /2		
8	2-2				-1	0	1	1	-P ₀ k	1	0	P ₀ k																				0	-P ₀ /2		
9	233						0	0	-r	-(2+r)	-1	1	0	1																			0	0	
10	234						1	-1	-2	2	2	-1	0	-1																		0	W _H /2		
11	3-3						-1	0	1	1	-P ₀ k	1	0	P ₀ k																		0	-P ₀ /2		
12	544									0	0	-r	-(2+r)	-1	1	0	1																0	0	
13	345									1	-1	-2	2	2	-1	0	-1																0	W _H /2	
14	4-4									-1	0	-1	1	P ₀ k	1	0	-P ₀ k															0	-P ₀ /2		
15	655												0	0	-r	-(2+r)	-1	1	0	1													0	0	
16	456												1	-1	-2	2	2	-1	0	-1													0	W _H /2	
17	5-5												-1	0	-1	1	P ₀ k	1	0	-P ₀ k												0	-P ₀ /2		
18	766															0	0	-r	-(2+r)	-1	1	0	1										0	0	
19	567															1	-1	-2	2	2	-1	0	-1										0	W _H /2	
20	6-6															-1	0	-1	1	-P ₀ k	1	0	P ₀ k										0	-P ₀ /2	
21	877																		0	0	-r	-(2+r)	-1	1	0	1							0	0	
22	678																		1	-1	-2	2	2	-1	0	-1							0	W _H /2	
23	7-7																		-1	0	-1	1	-P ₀ k	1	0	P ₀ k							0	-P ₀ /2	
24	988																					0	0	-r	-(2+r)	-1	1	0	1				0	0	
25	789																					1	-1	-2	2	2	-1	0	-1				0	W _H /2	
26	8-8																					-1	0	-1	1	-P ₀ k	1	0	-P ₀ k				0	-P ₀ /2	
27	1099																								0	0	-r	-(2+r)	-1	1	0			0	0
28	8910																								1	-1	-2	2	2	1	0			0	W _H /2
29	9-9																								-1	0	-1	1	P ₀ k	1	0			0	-P ₀ /2
30	109																												1	-1	-2	0		0	0

IP

W

CASE 4

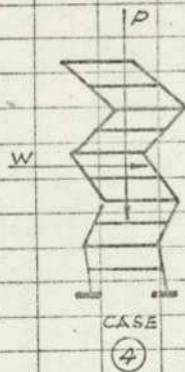


FIG. 6-III GENERAL MATRIX FOR PRACTICAL OUT OF PLUMB FRAME UNDER LATERAL AND GRAVITY LOADING (C OMITTED)

$r = \frac{(I_c)(h)}{I_b(h)}$ $k = \frac{h^2}{6EI}$
Defl. are in terms of $\frac{h^2}{6EI}$

SBN NO	COND	0		1			2			3			4			5			6			7			8			9			10		LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₀		
1	100	-1.5	-1	1	0	1																											
2	0-0	1	0.00198	1	0	0.00098																											
3	211	0	0	-2.5	-4.5	-1	1	0	1																								
4	012	1	-1	-2	2	2	-1	0	-1																								
5	1-1	-1	0	-1	1	0.002556	1	0	0.002556																								
6	322				0	0	-2.5	-4.5	-1	1	0	1																					
7	123				1	-1	-2	2	2	-1	0	-1																					
8	2-2				-1	0	-1	1	0.004224	1	0	0.004224																					
9	433						0	0	-2.5	-4.5	-1	1	0	1																			
10	234						1	-1	-2	2	2	-1	0	-1																			
11	3-3						-1	0	-1	1	0.005712	1	0	0.005712																			
12	544								0	0	-2.5	-4.5	-1	1	0	1																	
13	345								1	-1	-2	2	2	-1	0	-1																	
14	2-4								-1	0	-1	1	0.00714	1	0	0.00714																	
15	655										0	0	-2.5	-4.5	-1	1	0	1															
16	456										1	-1	-2	2	2	-1	0	-1															
17	5-5										-1	0	-1	1	0.008568	1	0	0.008568															
18	766												0	0	-2.5	-4.5	-1	1	0	1													
19	567												1	-1	-2	2	2	-1	0	-1													
20	6-6												-1	0	-1	1	0.009996	1	0	0.009996													
21	877														0	0	-2.5	-4.5	-1	1	0	1											
22	678														1	-1	-2	2	2	-1	0	-1											
23	7-7														-1	0	-1	1	0.011424	1	0	0.011424											
24	988																0	0	-2.5	-4.5	-1	1	0	1									
25	789																1	-1	-2	2	2	-1	0	-1									
26	8-8																-1	0	-1	1	0.012852	1	0	0.012852									
27	1099																		0	0	-2.5	-4.5	-1	1									
28	8110																		1	-1	-2	2	2	-1									
29	9-9																		-1	0	-1	1	0.01428	1									
30	100																																

P

W

CASE
4

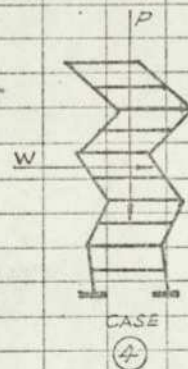


FIG. G-112

NUMERICAL MATRIX FOR PRACTICAL OUT-OF-PLUMB FRAME
UNDER LATERAL AND GRAVITY LOADING (C OMITTED)

Deflections are in terms
of wh^3/GEI .

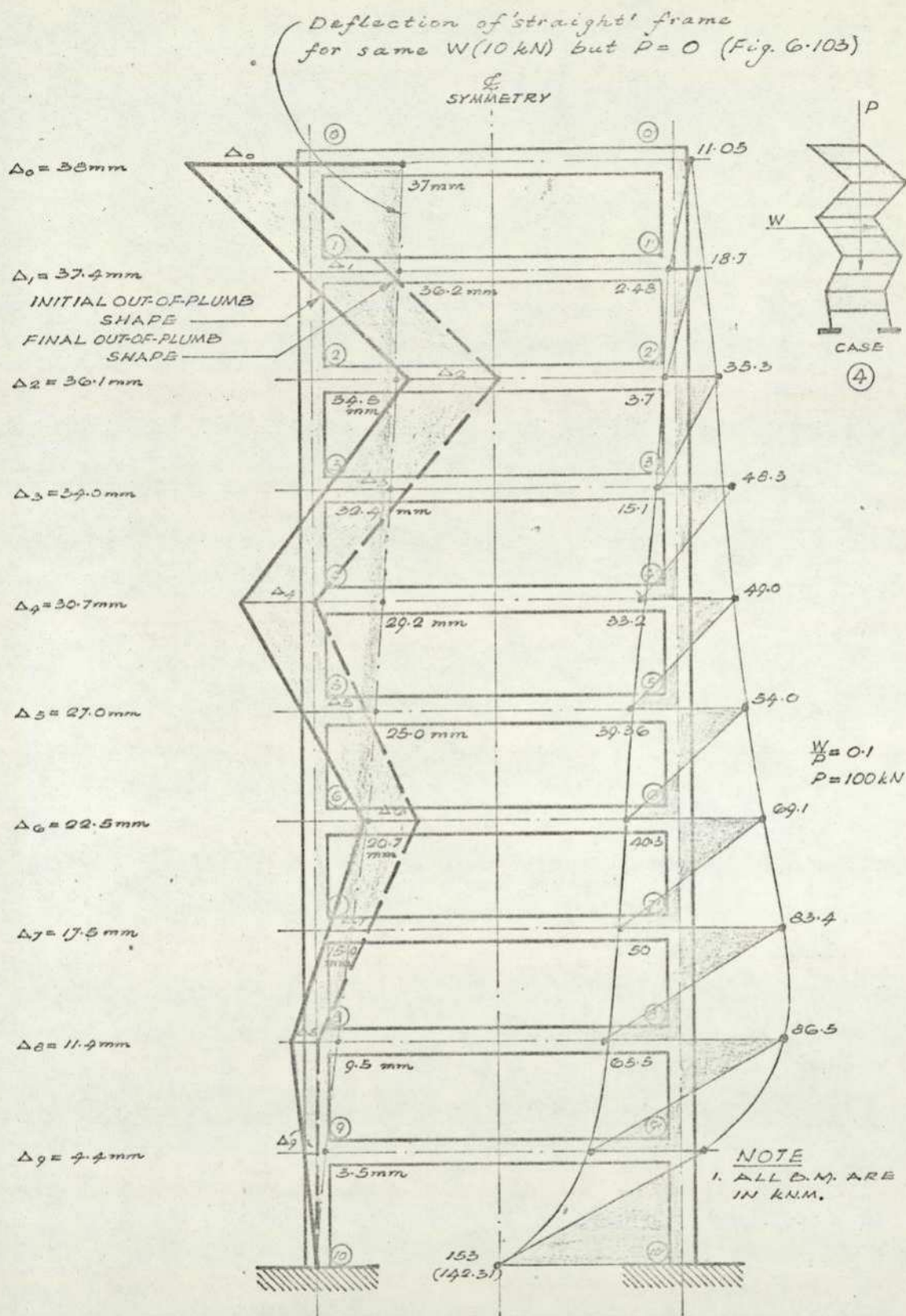


FIG. 6.113 MOMENT/DEFLECTION DIAGRAM FOR PRACTICAL OUT-OF-PLUMB FRAME SUBJECTED TO LATERAL AND GRAVITY LOADS (C OMITTED)

ROW NO	COND	0		1			2			3			4			5			6			7			8			9			10	LOAD VECTOR
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₀	
1	100	(2C ₀ +r)	-1	C ₀	0	1																									0	
2	0-0	1	kP ₀	1	0	-kP ₀																										(W) 33
3	211	0	0	-r	(2C ₁ +r)	-1	C ₁₂	0	1																						0	
4	012	C ₀	-1	-2C ₀	2C ₁	2	-C ₁₂	0	-1																						0	
5	1-1	-1	0	-1	1	kP ₁	1	0	-kP ₁																							(W) 33
6	322																															(W) 33
7	123																															(W) 33
8	2-2																															(W) 33
9	433																															(W) 33
10	234																															(W) 33
11	3-3																															(W) 33
12	544																															(W) 33
13	345																															(W) 33
14	2-4																															(W) 33
15	555																															(W) 33
16	456																															(W) 33
17	5-5																															(W) 33
18	766																															(W) 33
19	567																															(W) 33
20	6-6																															(W) 33
21	377																															(W) 33
22	678																															(W) 33
23	7-7																															(W) 33
24	988																															(W) 33
25	789																															(W) 33
26	8-8																															(W) 33
27	1000																															(W) 33
28	8910																															(W) 33
29	9-9																															(W) 33
30	100																															(W) 33

Diagram of a frame structure with nodes 0 to 10. A vertical load P is applied at node 0. A horizontal load W is applied at node 1. The structure consists of a vertical column and a horizontal beam. The column has nodes 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. The beam has nodes 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. The diagram is labeled CASE 5.

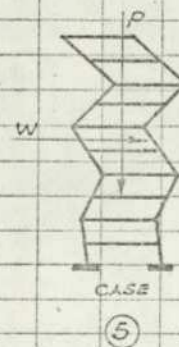


FIG. 6.114 GENERAL MATRIX FOR OUT-OF-PLUMB PRACTICAL FRAME UNDER LATERAL & GRAVITY LOADS, TAKING COL. FLEX. INTO ACCOUNT

$$r = \left(\frac{L}{P} \right), \quad n = \left(\frac{P}{W} \right), \quad k = \left(\frac{1}{GSI} \right)$$

EQN NO	COND	0		1			2			3			4			5			6			7			8			9			10	LOAD VECTORS
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₁₀	Δ ₉	M ₁₀₀	
1	100	4.50128	-1	1.0007	0	1																										0 0
2	0-0	1	0.0014	1	0	0.0014																										0.0014 0.0014
3	211	0	0	-2.5	4.5026	-1	1.0020	0	1																							0 0
4	012	1.0007	-1	0.00128	0.0026	2	1.002	0	-1																							0 0
5	1-1	-1	0	-1	1	0.0026	1	0	0.0026																							0.0026 0.0026
6	322				0	0	-2.5	4.5038	-1	1.0031	0	1																				0 0
7	123				1.002	-1	0.0026	0.0038	2	1.0031	0	-1																				0 0
8	2-2				-1	0	-1	1	0.0038	1	0	0.0037																				0.0037 0.0037
9	033							0	0	-2.5	4.5048	-1	1.0041	0	1																	0 0
10	234							1.0031	-1	0.0038	0.0048	2	1.0047	0	-1																	0 0
11	3-3							-1	0	-1	1	0.0037	1	0	0.0044																	0.0044 0.0044
12	344									0	0	-2.5	4.5054	-1	1.0047	0	1															0 0
13	345									1.0047	-1	0.0048	0.0054	2	1.0047	0	-1															0 0
14	4-4									-1	0	-1	1	0.0054	1	0	0.0055															0.0055 0.0055
15	655											0	0	-2.5	4.5071	-1	1.0062	0	1													0 0
16	456											1.0047	-1	0.0054	0.0072	2	1.0062	0	-1													0 0
17	5-5											-1	0	-1	1	0.0072	1	0	0.0095													0.0095 0.0095
18	766													0	0	-2.5	4.5084	-1	1.0073	0	1											0 0
19	567													1.0062	-1	0.0072	0.0084	2	1.0073	0	-1											0 0
20	6-6													-1	0	-1	1	0.0084	1	0	0.0099											0.0099 0.0099
21	877															0	0	-2.5	4.5094	-1	1.0082	0	1									0 0
22	678															1.0073	-1	0.0084	0.0094	2	1.0082	0	-1									0 0
23	7-7															-1	0	-1	1	0.0094	1	0	0.0114									0.0114 0.0114
24	988																	0	0	-2.5	4.5106	-1	1.0091	0	1							0 0
25	789																	1.0082	-1	0.0094	0.0106	2	1.0091	0	-1							0 0
26	8-8																	-1	0	-1	1	0.0106	1	0	0.0126							0.0126 0.0126
27	1000																			0	0	-2.5	4.5116	-1	1.01	0	1					0 0
28	890																			1.0091	-1	0.0106	0.0116	2	1.01	0	-1					0 0
29	9-9																			-1	0	-1	1	0.0116	1	0.0136						0.0136 0.0136
30	109																															1.01 2.013 2.016 0 0

P

W

CASE 5



FIG. 6-115 NUMERIC MATRIX FOR PRACTICAL OUT-OF-PLUMB FRAME UNDER LATERAL AND GRAVITY LOADS TAKING COL. FLEX. INTO ACCOUNT

Deflection of 'straight' frame
for $W/P = 0.1$ (Fig. G.107)

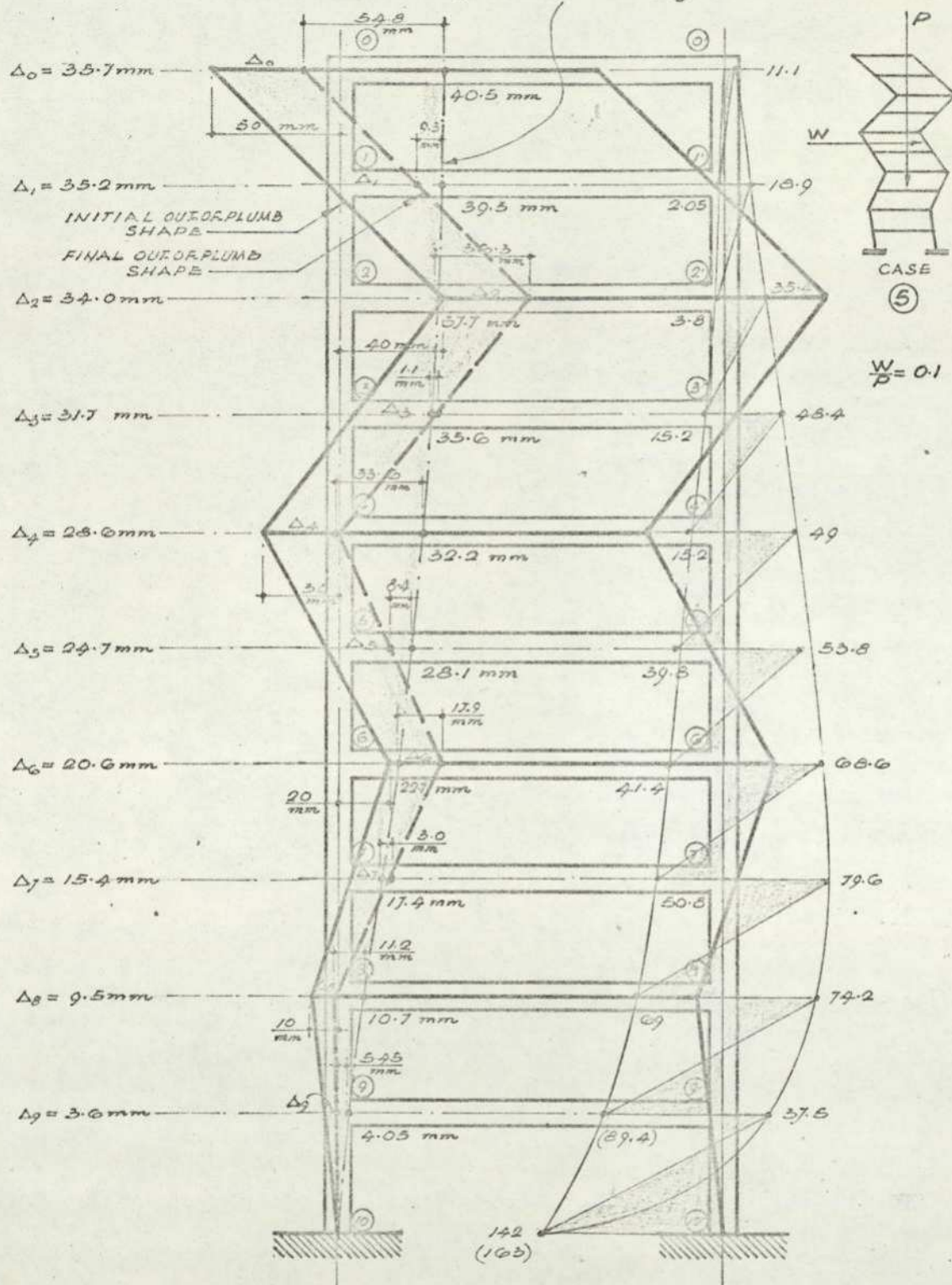


FIG. G.116 MOMENT DEFLECTION DIAGRAM FOR PRACTICAL OUT-OF-PLUMB FRAME SUBJECTED TO LATERAL AND GRAVITY LOADS, INCLUDING THE EFFECT OF ELASTIC INSTABILITY ($W/P_0 = 0.1$)

Deflection of straight frames
for same $W(50 \text{ kN})$ but $P=0$

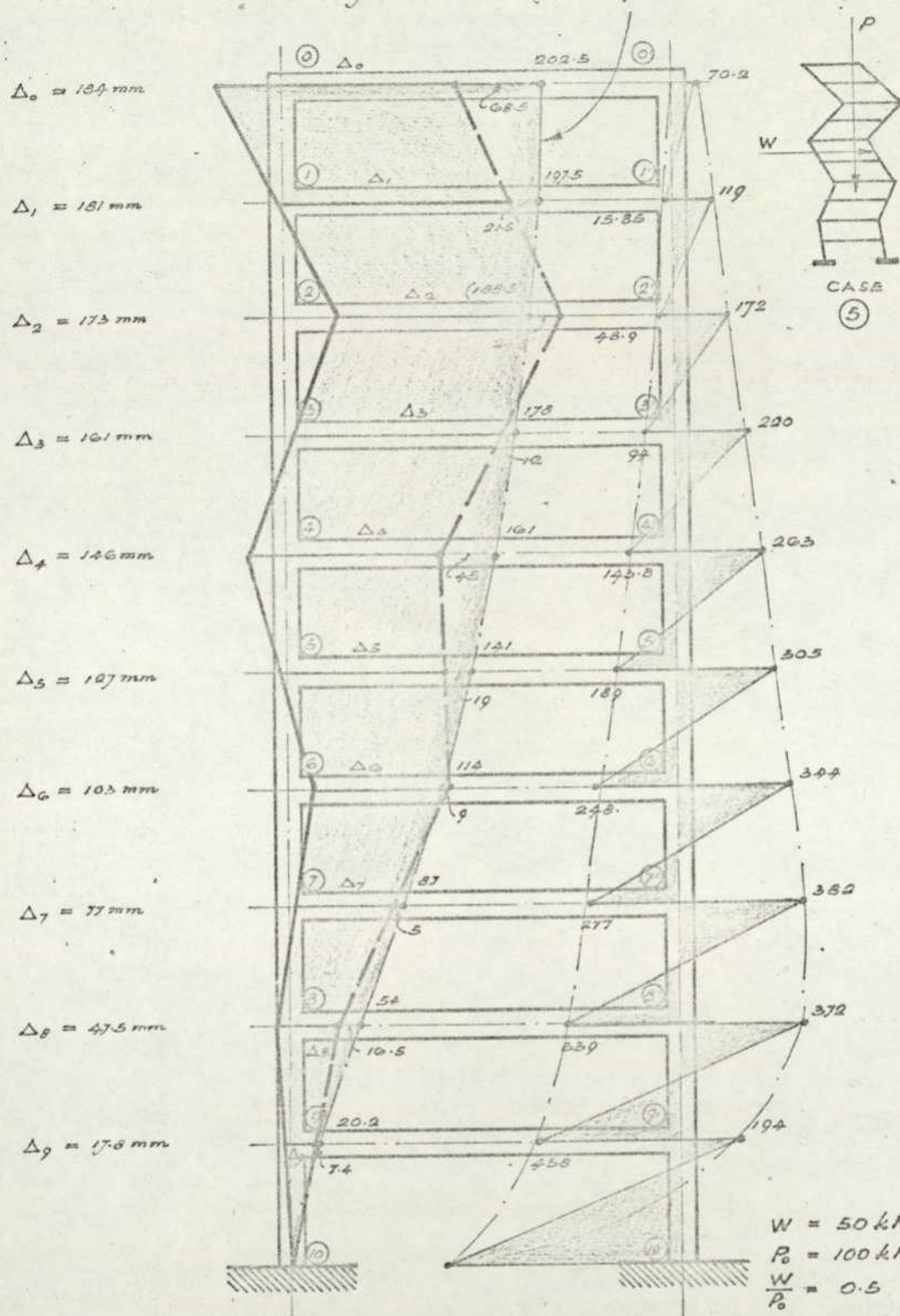


FIG. 6-117 MOMENT DEFLECTION DIAGRAM FOR PRACTICAL OUT-OF-PLUMB FRAME SUBJECTED TO LATERAL AND GRAVITY LOADS INCLUDING THE EFFECT OF ELASTIC INSTABILITY ($W/P_0 = 0.5$)

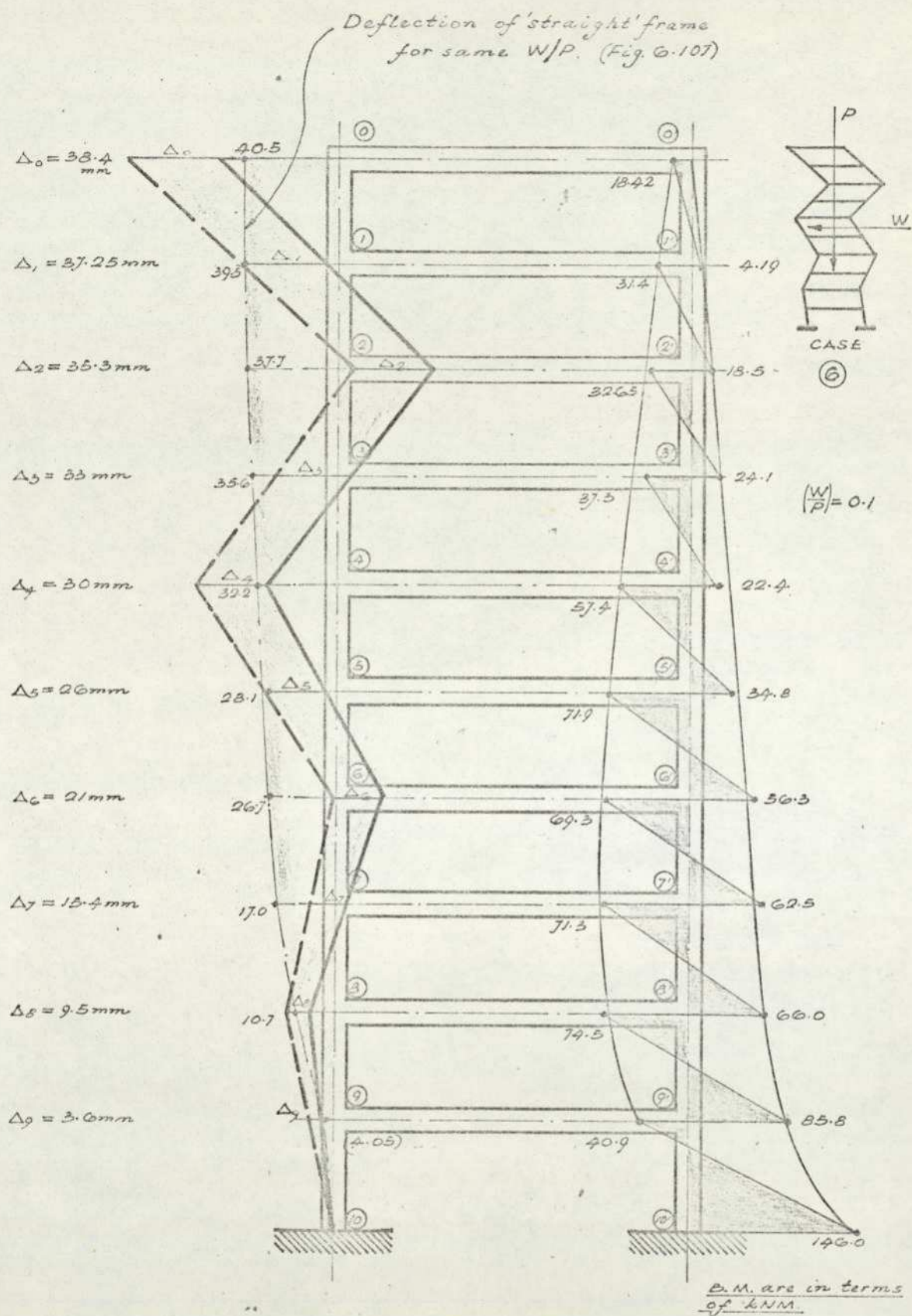


FIG. 6.118 MOMENT DEFLECTION DIAGRAM FOR PRACTICAL OUT-OF-PLUMB FRAME SUBJECTED TO LATERAL LOAD (ACTING IN OPPOSITE DIRECTION) AND GRAVITY LOADS ($W/P = 0.1$).

Deflection of 'straight' frame
for same $W(50\text{ kN})$ but $P=0$

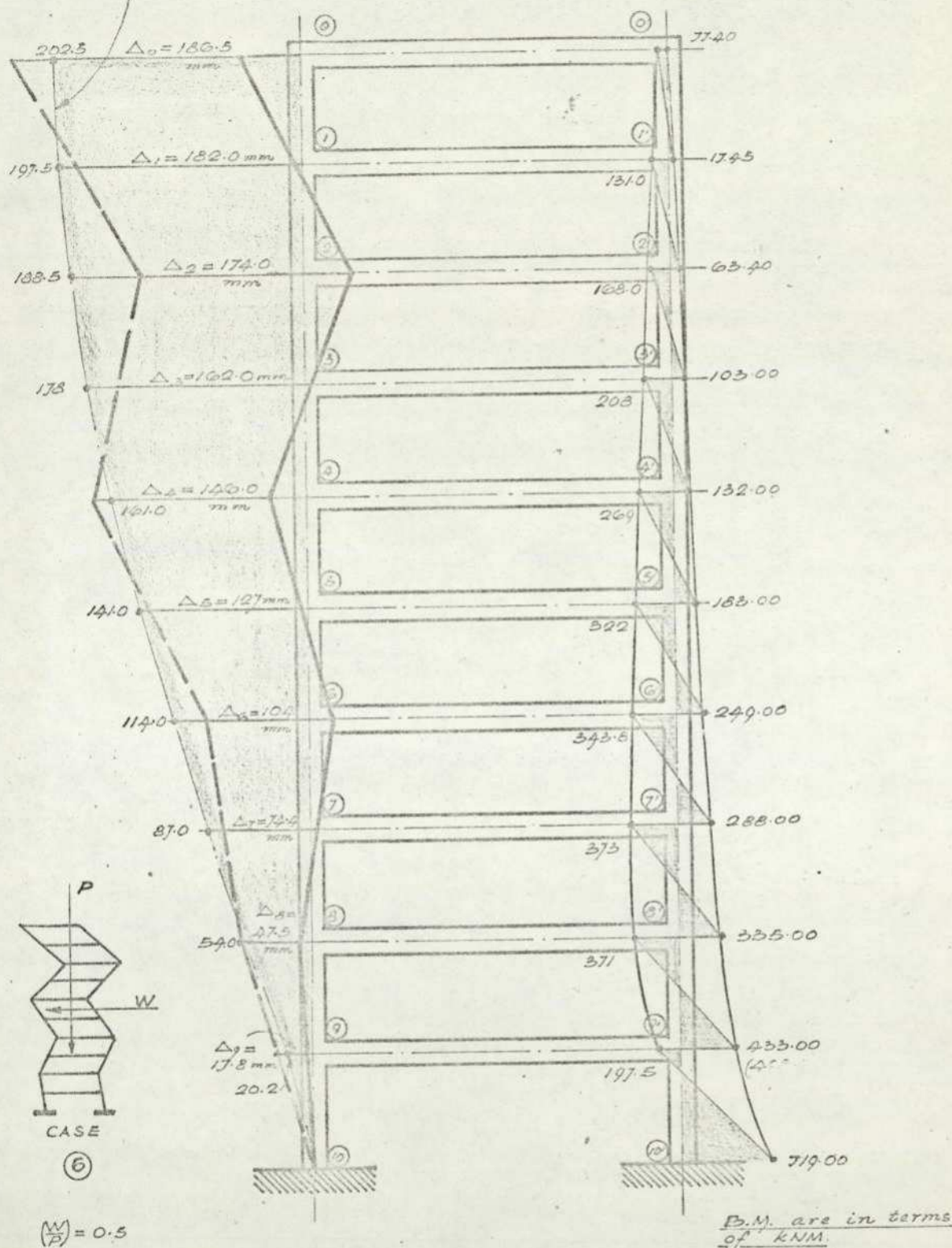


FIG. G.119 MOMENT DEFLECTION DIAGRAM FOR PRACTICAL OUT-OF-PLUMB FRAME SUBJECTED TO LATERAL LOAD (ACTING IN OPPOSITE DIRECTION) AND GRAVITY LOADS ($W/P = 0.5$)

6.20 Analysis of 6-storey Out-of-Plumb Concrete Model

A model concrete frame, shown idealised in Fig. 6.20, is analysed for the gravity load condition in which axial loads P are applied only at the top of model. The theoretical performance will then be compared with the test results.

It is assumed for the purpose of analysis that the frame will sway to the side i.e. the top of each column moves to the left. If, on the other hand, the frame behaves like a concertina, some deflections and moments will be reversed as indicated by negative signs in the computer print-out.

The following frame properties are adopted in the analysis - they apply to the test model discussed later.

$$\begin{aligned}
 a_0 &= 100\text{mm} & a_2 &= 75\text{mm} & a_4 &= 50\text{mm} \\
 h &= 300\text{mm} & L &= 600\text{mm} & E &= 9000\text{N/mm}^2 \\
 & & & & & \text{(See Appendix)} \\
 I_c = I_b &= \frac{1}{12} \times 50^4 = 520833\text{mm}^4 & l_1 &= 312.5\text{mm} \\
 l_2 &= 300.4\text{mm}, l_3 = 301.05\text{mm}, \alpha = 16^\circ 20', \beta = 11^\circ 50', \gamma = 4^\circ 40'
 \end{aligned}$$

Equations for compatibility of deformation are written in the usual way. Instability effects are ignored and it is assumed that beams remain horizontal.

With regard to the equilibrium conditions, the method adopted is to resolve forces at right angles to the column immediately below the node under consideration. Because of the discontinuities in the alignment of columns at nodes 2 and 4 it is necessary to consider the axial and normal components of the vertically applied load P . These are shown in Fig. 6.21.

The equations are formulated as follows:-

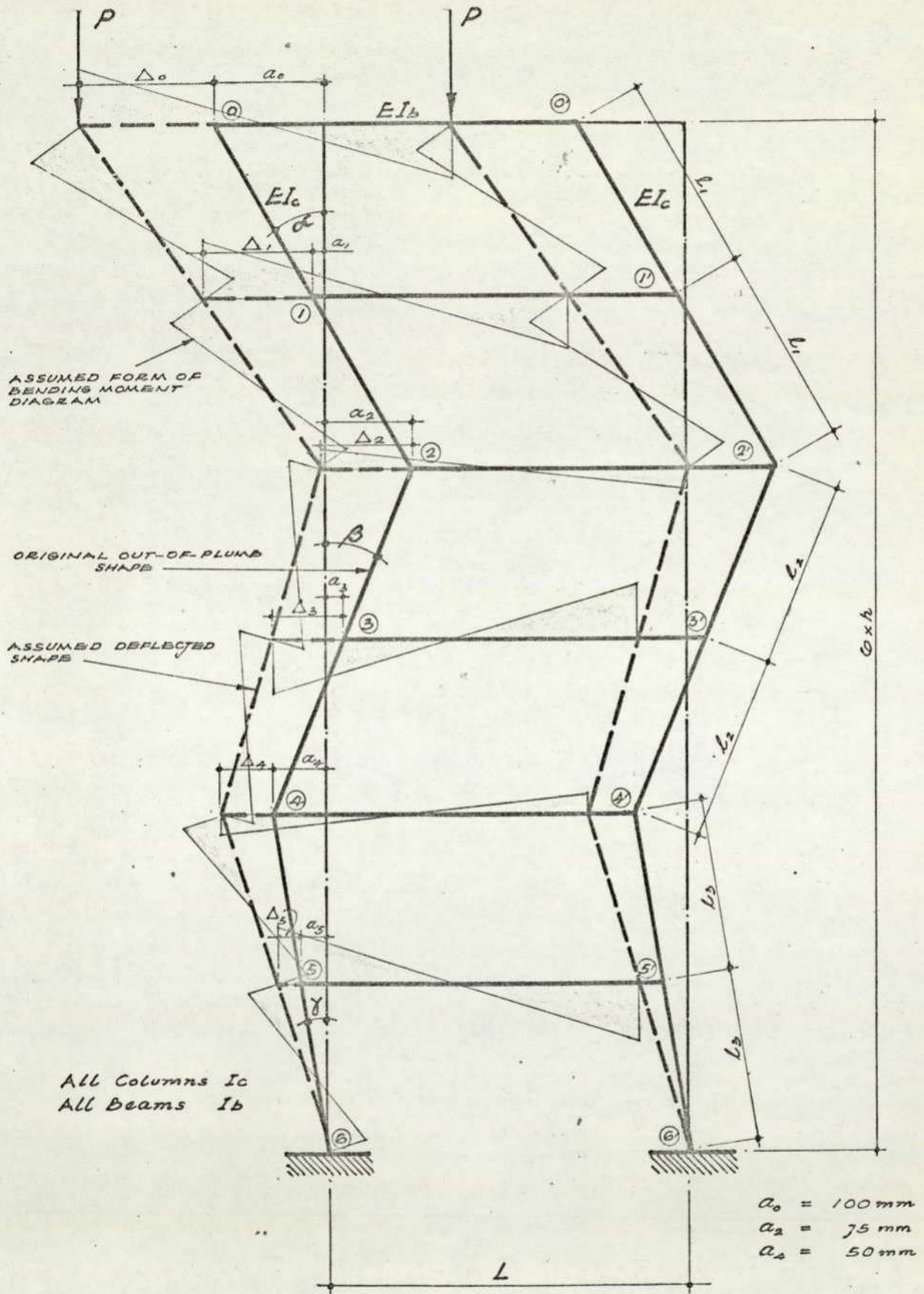
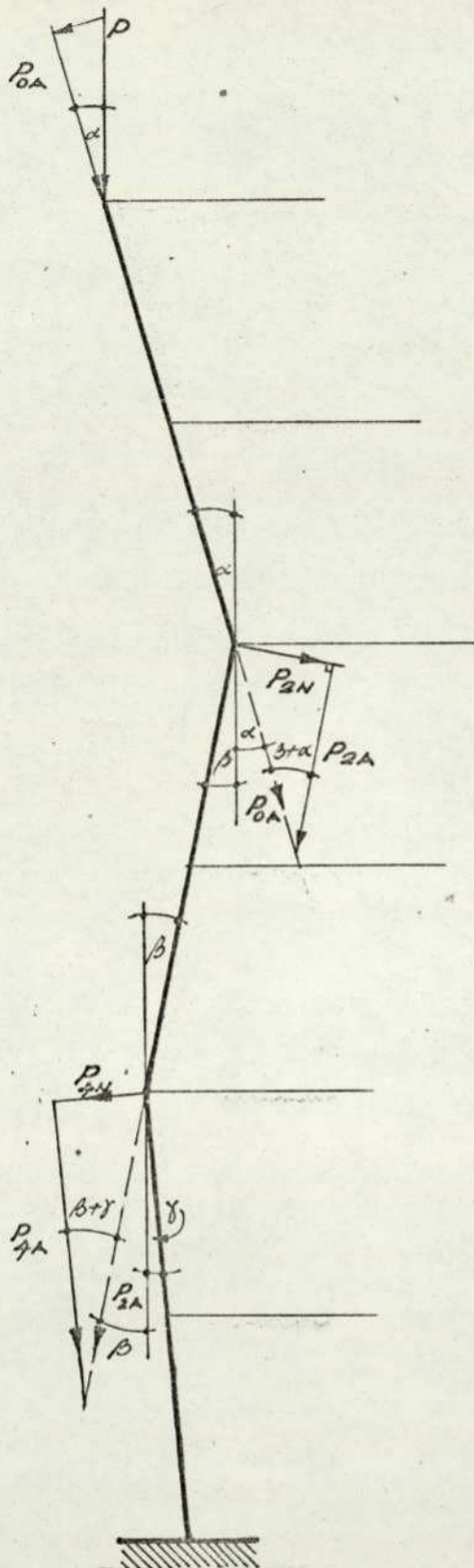


FIG. 6-20 ASSUMED FORM OF BENDING MOMENT AND DEFLECTION DIAGRAM FOR 6-STOREY OUT-OF-PLUMB CONCRETE MODEL



$$\begin{aligned}
 P_{0A} &= P \cos \alpha \\
 P_{2A} &= P_{0A} \cos(\beta + \alpha) \\
 P_{2A} &= P \cos \alpha \cdot \cos(\beta + \alpha) \\
 P_{2N} &= P_{0A} \sin(\alpha + \beta) \\
 P_{2N} &= P \cos \alpha \cdot \sin(\alpha + \beta) \\
 P_{4A} &= P_{2A} \cos(\beta + \gamma) \\
 P_{4A} &= P \cos \alpha \cdot \cos(\alpha + \beta) \cos(\beta + \gamma) \\
 P_{4N} &= P_{2A} \sin(\beta + \gamma) \\
 P_{4N} &= P \cos \alpha \cdot \cos(\alpha + \beta) \cdot \sin(\beta + \gamma)
 \end{aligned}$$

SUBSTITUTING NUMERICAL DATA

$$\begin{aligned}
 P_{2N} &= P \times 0.9596 \times 0.472 \\
 &= 0.4529 P \\
 P_{4N} &= P \times 0.9596 \times 0.881 \times 0.284 \\
 &= 0.24 P
 \end{aligned}$$

FIG. G-21 COMPONENTS OF FORCE USED IN EQUILIBRIUM CONDITIONS.

$$-M_{10} \frac{l_1}{6EI} + M_{01} \left(\frac{l_1}{3EI} + \frac{L}{6EI} \right) = \frac{\Delta_0 \cos \alpha - \Delta_1 \cos \alpha}{l_1}$$

$$-M_{10} + 3.92 M_0 + 2.763 \times 10^5 (\Delta_s - \Delta_o) = 0 \text{ --- } (1)$$

$$EC\ O-O' \quad \frac{M_{O1} + M_{10}}{L_1} = P_{ON} = 0.2812 P$$

$$M_{0,} + M_{1,0} = 87.875P \text{ ————— } (2)$$

$$211' \quad -M_{2,1} \frac{l_1}{6EI} + M_{1,2} \frac{l_1}{3EI} + (M_{1,0} + M_{1,2}) \frac{L}{6EI} = (\Delta_1 - \Delta_2) \frac{\cos \alpha}{l_1}$$

$$-M_{21} + 3.92 M_{12} + 1.92 M_{10} + 2.763 \times 10^5 (\Delta_2 - \Delta_1) = 0 \text{ --- (3)}$$

$$0/2 \quad -M_{01} \frac{l_1}{6EI} + M_{10} \frac{l_1}{3EI} - M_{12} \frac{l_1}{3EI} + M_{21} \frac{l_1}{6EI} =$$

$$= \frac{\Delta_0 \cos \alpha - \Delta_1 \cos \alpha}{l_1} - \frac{\Delta_1 \cos \alpha - \Delta_2 \cos \alpha}{l_1}$$

$$-M_{o1} + 2M_{10} - 2M_{12} + M_{21} - 2.763 \times 10^5 (\Delta_o - 2\Delta_1 + \Delta_2) = 0 \quad \text{--- (4)}$$

$$EC1-1' \quad \frac{M_{01} + M_{10}}{l_1} = \frac{M_{12} + M_{21}}{l_1}$$

$$M_{01} + M_{10} - M_{12} - M_{21} = 0 \text{ ----- } (5)$$

$$322' - M_{32} \frac{l_2}{6EI} + M_{23} \frac{l_2}{3EI} + (M_{21} + M_{23}) \frac{L}{6EI} = \frac{\Delta_2 \cos \beta - \Delta_3 \cos \beta}{l_2}$$

$$- 0.9805 M_{32} + 3.881 M_{23} + 1.92 M_{21} + (\Delta_3 - \Delta_2) \times 2.874 \times 10^5 =$$

$$= 0 \text{ ————— } \textcircled{6}$$

$$\begin{aligned}
 123 \quad & -M_{12} \frac{l_1}{6EI} + M_{21} \frac{l_1}{3EI} - M_{23} \frac{l_2}{3EI} + M_{32} \frac{l_2}{6EI} = \\
 & = \frac{\Delta_1 \cos \alpha - \Delta_2 \cos \alpha}{l_1} - \frac{\Delta_2 \cos \beta - \Delta_3 \cos \beta}{l_2} \\
 & -M_{12} + 2M_{21} - 1.961M_{23} + 0.9805M_{32} - 2.763 \times 10^5 \Delta_1 + 5.637 \times 10^5 \Delta_2 - \\
 & - 2.874 \times 10^5 \Delta_3 = 0 \text{ ----- } (7)
 \end{aligned}$$

$$\begin{aligned}
 EC 2-2' \quad & \frac{M_{12} + M_{21}}{l_1} \cos(\alpha + \beta) - \frac{M_{23} + M_{32}}{l_2} = P_{2N} = 0.453P \\
 & 0.881M_{12} + 0.881M_{21} - 1.019M_{23} - 1.019M_{32} = 141.5P \text{ ----- } (8)
 \end{aligned}$$

$$\begin{aligned}
 433' \quad & -M_{43} \frac{l_2}{6EI} + M_{34} \frac{l_2}{3EI} + (M_{32} + M_{34}) \frac{L}{6EI} = \frac{\Delta_3 \cos \beta - \Delta_4 \cos \beta}{l_2} \\
 & -M_{43} + 3.958M_{34} + 1.958M_{32} + (\Delta_4 - \Delta_3) \times 2.93 \times 10^5 = 0
 \end{aligned}$$

$$\begin{aligned}
 234 \quad & -M_{23} \frac{l_2}{6EI} + M_{32} \frac{l_2}{3EI} - M_{34} \frac{l_2}{3EI} + M_{43} \frac{l_2}{6EI} = \\
 & = \frac{\Delta_2 \cos \beta - \Delta_3 \cos \beta}{l_2} - \frac{\Delta_3 \cos \beta - \Delta_4 \cos \beta}{l_2} \\
 & -M_{23} + 2M_{32} - 2M_{34} + M_{43} - (\Delta_2 - 2\Delta_3 + \Delta_4) \times 2.93 \times 10^5 = 0 \text{ ----- } (10)
 \end{aligned}$$

$$\begin{aligned}
 EC 3-3' \quad & \frac{M_{23} + M_{32}}{l_2} - \frac{M_{34} + M_{43}}{l_2} = 0 \\
 & M_{23} + M_{32} - M_{34} - M_{43} = 0 \text{ ----- } (11)
 \end{aligned}$$

$$544' \quad -M_{54} \frac{l_3}{6EI} + M_{45} \frac{l_3}{3EI} + (M_{43} + M_{45}) \frac{L}{6EI} = \frac{\Delta_4 \cos \gamma - \Delta_5 \cos \gamma}{l_3}$$

$$-M_{54} + 3.993M_{45} + 1.993M_{43} + (\Delta_5 - \Delta_4) \times 3.093 \times 10^5 = 0 \text{ --- (12)}$$

$$\begin{aligned} 345 \quad -M_{34} \frac{l_2}{6EI} + M_{43} \frac{l_2}{3EI} - M_{45} \frac{l_3}{3EI} + M_{54} \frac{l_3}{6EI} = \\ = \frac{\Delta_3 \cos \beta - \Delta_4 \cos \beta}{l_2} - \frac{\Delta_4 \cos \gamma - \Delta_5 \cos \gamma}{l_3} \end{aligned}$$

$$\begin{aligned} -M_{34} + 2M_{43} - 1.964M_{45} + 0.982M_{54} - \Delta_3 \times 2.93 \times 10^5 + \Delta_4 \times 5.97 \times 10^5 - \\ - \Delta_5 \times 3.038 \times 10^5 = 0 \text{ --- (13)} \end{aligned}$$

$$EC 4-4' \quad -\frac{(M_{34} + M_{43})}{l_2} \cos(\beta + \gamma) + \frac{(M_{45} + M_{54})}{l_3} = P_{4N} = 0.2399P$$

$$-0.942M_{34} - 0.942M_{43} + M_{45} + M_{54} = 72.22P \text{ --- (14)}$$

$$655' \quad -M_{65} \frac{l_3}{6EI} + M_{56} \frac{l_3}{3EI} + (M_{54} + M_{56}) \frac{L}{6EI} = \frac{\Delta_5 \cos \gamma}{l_3}$$

$$-M_{65} + 3.993M_{56} + 1.993M_{54} - \Delta_5 \times 3.093 \times 10^5 = 0 \text{ --- (15)}$$

$$\begin{aligned} 456 \quad -M_{45} \frac{l_3}{6EI} + M_{54} \frac{l_3}{3EI} - M_{56} \frac{l_3}{3EI} + M_{65} \frac{l_3}{6EI} = \\ = \frac{\Delta_4 \cos \gamma - \Delta_5 \cos \gamma}{l_3} - \frac{\Delta_5 \cos \gamma - 0}{l_3} \end{aligned}$$

$$-M_{45} + 2M_{54} - 2M_{56} + M_{65} - \Delta_4 \times 3.093 \times 10^5 + \Delta_5 \times 6.186 \times 10^5 = 0 \text{ --- (16)}$$

$$EC 5-5' \quad \frac{M_{45} + M_{54}}{l_3} - \frac{M_{56} + M_{65}}{l_3} = 0$$

$$M_{45} + M_{54} - M_{56} - M_{65} = 0 \text{ --- (17)}$$

$$65 \quad -M_{56} \frac{l_3}{6EI} + M_{65} \frac{l_3}{3EI} = \frac{\Delta_5 \cos \gamma}{l_3}$$

$$-M_{56} + 2M_{65} - \Delta_5 \times 3.093 \times 10^5 = 0 \text{ --- --- --- --- --- } (18)$$

The numerical matrix for these conditions is shown in Fig. 6.22. Computer print-out with solutions is shown in Fig. 6.23. The bending moment and deflection diagram is shown in Fig. 6.24. These values will be compared with experimental values given in the following section.

6.30 Experiments on 6-storey Out-of-Plumb Concrete Model

The micro concrete model shown in Fig. 6.30 has large out-of-plumb inaccuracies built into it in order that axial loads will generate measurable bending stresses before the onset of compression failure.

Electrical strain gauges were adhered to the columns on one side of the frame either side of the beam/column junctions. Dial gauges were used to measure horizontal deflection at each node position and also the vertical deflection at the top of the frame. Vertical load was applied only at the top of each column, using hydraulic jacks separated from the model by roller bearings to allow unrestricted sidesway. Jacking loads were measured with a proving ring. The experimental arrangement is shown in Fig. 6.31.

The load deflection curves are shown in Fig. 6.32. Measured lateral deflections (uncorrected) at the top of the model were corrected for the effect of vertical deflection. Note that vertical deflection alone (no sidesway) would give rise to lateral deflection recorded by the dial gauge because the stem of the gauge registered on the sloping face of the column. From geometry, each unit of vertical deflection at the top resulted in 1/3 unit of horizontal deflection.

From Fig. 6.32, the true lateral deflection of the top of the model, for load P of 2000N, is 2.8 mm. From the theory, the lateral deflection is $0.000465P$ or 0.93 mm for $P = 2000N$. It can be seen that the deflection under test is three times the theoretical deflection.

Strain gauge readings were taken over the complete load range but, because of the detrimental effect of microcracking discussed previously, strain gauge readings have been interpreted only for the small initial load range up to 300N. The data are shown in Fig. 6.32.

EQU. NO.	CONDITION	0		1			2			3			4			5			6	LOAD VECTOR
		M_{01}	Δ_0	M_{10}	M_{12}	Δ_1	M_{21}	M_{23}	Δ_2	M_{32}	M_{34}	Δ_3	M_{43}	M_{45}	Δ_4	M_{54}	M_{56}	Δ_5	M_{65}	
1	1-0-0'	3.92	0.276E6	-1	0	0.276E6				(13)										0
2	EC 0-0'	1	0	1	0	0														87.875
3	2-1-1'	0	0	1.92	3.92	0.276E6	-1	0	0.276E6				(10)							0
4	0-1-2	-1	0.276E6	2	-2	0.553E6	1	0	0.287E6											0
5	EC 1-1'	1	0	1	-1	0	-1	0	0											0
6	3-2-2'		(3)		0	0	1.92	3.881	0.287E6	0.9805	0	0.287E6				(7)				0
7	1-2-3				-1	0.287E6	2	1.901	0.564E6	0.9805	0	0.287E6								0
8	EC 2-2'				0.881	0	0.881	1.019	0	1.019	0	0								121.5
9	4-3-3'					(6)		0	0	1.958	3.958	0.293E6	-1	0	0.293E6			(4)		0
10	2-3-4							-1	0.293E6	2	-2	0.586E6	1	0	0.293E6					0
11	EC 3-3'							1	0	1	-1	0	-1	0	0					0
12	5-4-4'								(9)		0	0	1.993	3.993	0.309E6	-1	0	0.309E6	0	0
13	3-4-5										-1	0.293E6	2	1.964	0.597E6	0.982	0	0.304E6	0	
14	EC 4-4'										0.942	0	0.942	1	0	1	0	0	0	72.22
15	6-5-5'											(12)		0	0	1.993	3.993	0.309E6	-1	0
16	4-5-6													-1	0.309E6	2	-2	0.618E6	1	0
17	EC 5-5'													1	0	1	-1	0	-1	0
18	6-5														(15)		-1	0.309E6	2	0

FIG. 6.22 NUMERICAL MATRIX FOR 6-STORY OUT-OF-PLUMB
CONCRETE MODEL SUBJECTED TO GRAVITY LOADING (P-IN)

$E^6 = 10^6$

○ DENOTES NO OF ZEROS

COEFFICIENT MATRIX (INCLUDING R.H.S. MATRIX):-

0.39200E 01 - 27600E 06 - 10000E 01 0.00000E 00 0.27600E 06 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.10000E 01 0.00000E 00 0.10000E 01 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.87875E 02
0.00000E 00 0.00000E 00 0.19200E 01 0.39200E 01 - 27600E 06 - 10000E 01 0.00000E 00 0.27600E 06 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
- 10000E 01 - 27600E 06 0.20000E 01 - 20000E 01 0.55300E 06 0.10000E 01 0.00000E 00 - 27600E 06 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.10000E 01 0.00000E 00 0.10000E 01 - 10000E 01 0.00000E 00 - 10000E 01 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.28700E 06 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 - 10000E 01 - 27600E 06 0.20000E 01 - 19610E 01 0.56400E 06 0.98650E 00 0.00000E 00
- 28700E 06 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.88100E 00 0.00000E 00 0.88100E 00 - 10190E 01 0.00000E 00 - 10190E 01 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.14150E 03
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
- 29300E 06 - 10000E 01 0.00000E 00 0.29300E 06 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 - 10000E 01 - 29300E 06 0.20000E 01 - 20000E 01
0.58600E 06 0.10000E 01 0.00000E 00 - 29300E 06 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.10000E 01 0.00000E 00 0.10000E 01 - 10000E 01
0.00000E 00 - 10000E 01 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.19930E 01 0.39930E 01 - 30900E 06 - 10000E 01 0.00000E 00 0.30900E 06 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 - 10000E 01
- 29300E 06 0.20000E 01 - 19640E 01 0.59700E 06 0.98200E 00 0.00000E 00 - 30400E 06 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 - 94200E 00
0.00000E 00 - 94200E 00 0.10000E 01 0.00000E 00 0.10000E 01 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.72220E 02
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.19930E 01 0.39930E 01 - 30900E 06 - 10000E 01 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 - 10000E 01 - 30900E 06 0.20000E 01 - 20000E 01 0.61800E 06 0.10000E 01 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.10000E 01 0.00000E 00 0.10000E 01 - 10000E 01 0.00000E 00 - 10000E 01 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00
0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 0.00000E 00 - 10000E 01 - 30900E 06 0.20000E 01 0.00000E 00
SOLUTION NO. 1

01 0.49168724E 02	20 0.46508605E-03	10 0.38706276E 02	12 0.26858466E 02	Δ1 -0.93012213E-04
21 0.61016934E 02	23 -0.48551550E 02	Δ2-0.52266694E-03	32-0.14335717E 02	34 -0.25652545E 02
Δ3-0.32329185E-03	43-0.37234722E 02	45 0.14993909E 02	Δ4-0.80447974E-05	54 -0.20137147E 01
56 0.53739418E 01	Δ5 0.31840012E-04	65 0.76062527E 01		

FIG. 6-23 COMPUTER PRINTOUT OF MATRIX AND SOLUTIONS FOR ANALYSIS OF 6-STORY CONCRETE MODEL

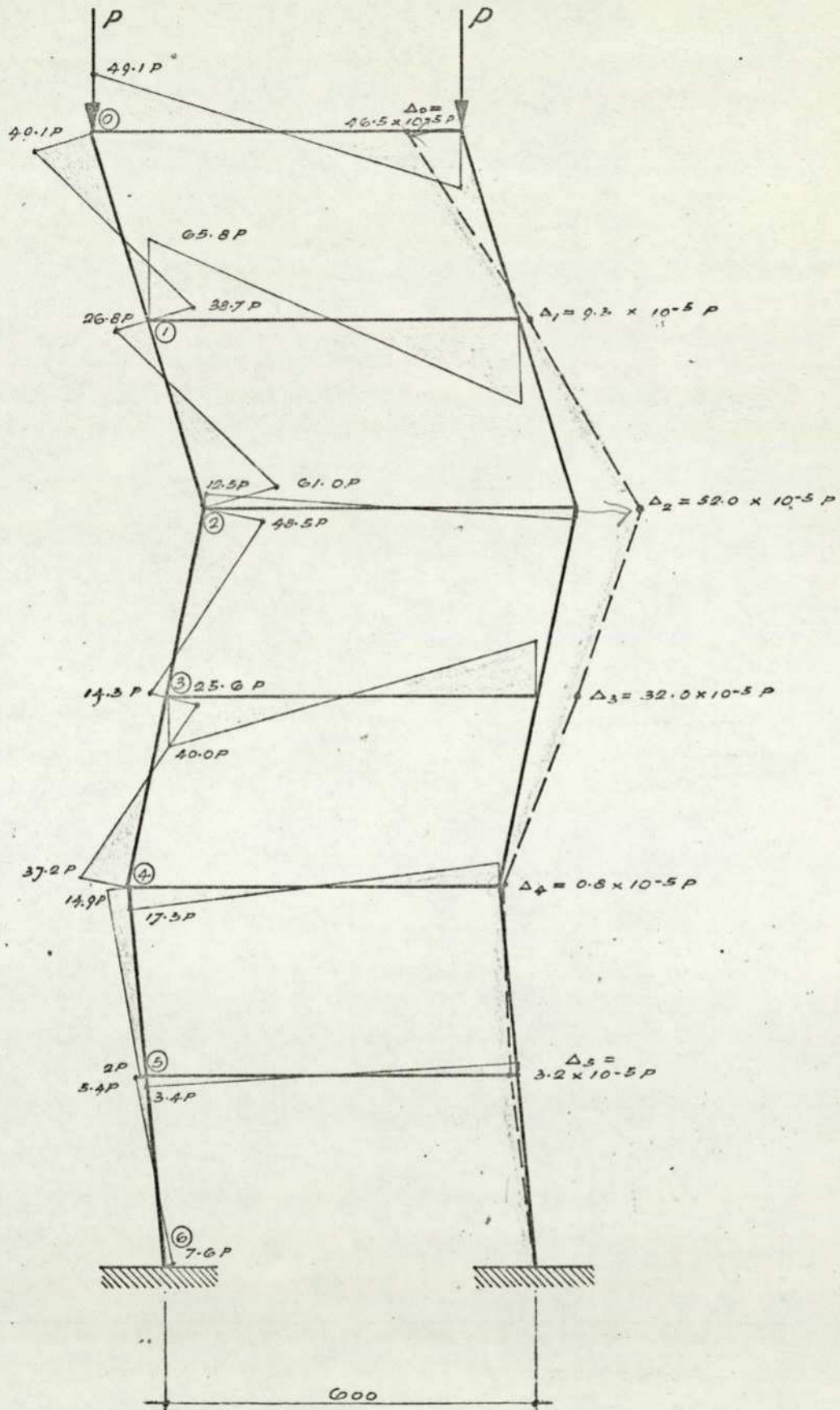


FIG. 6.24 MOMENT DEFLECTION DIAGRAM FOR 6-STORY OUT OF PLUMB CONCRETE MODEL

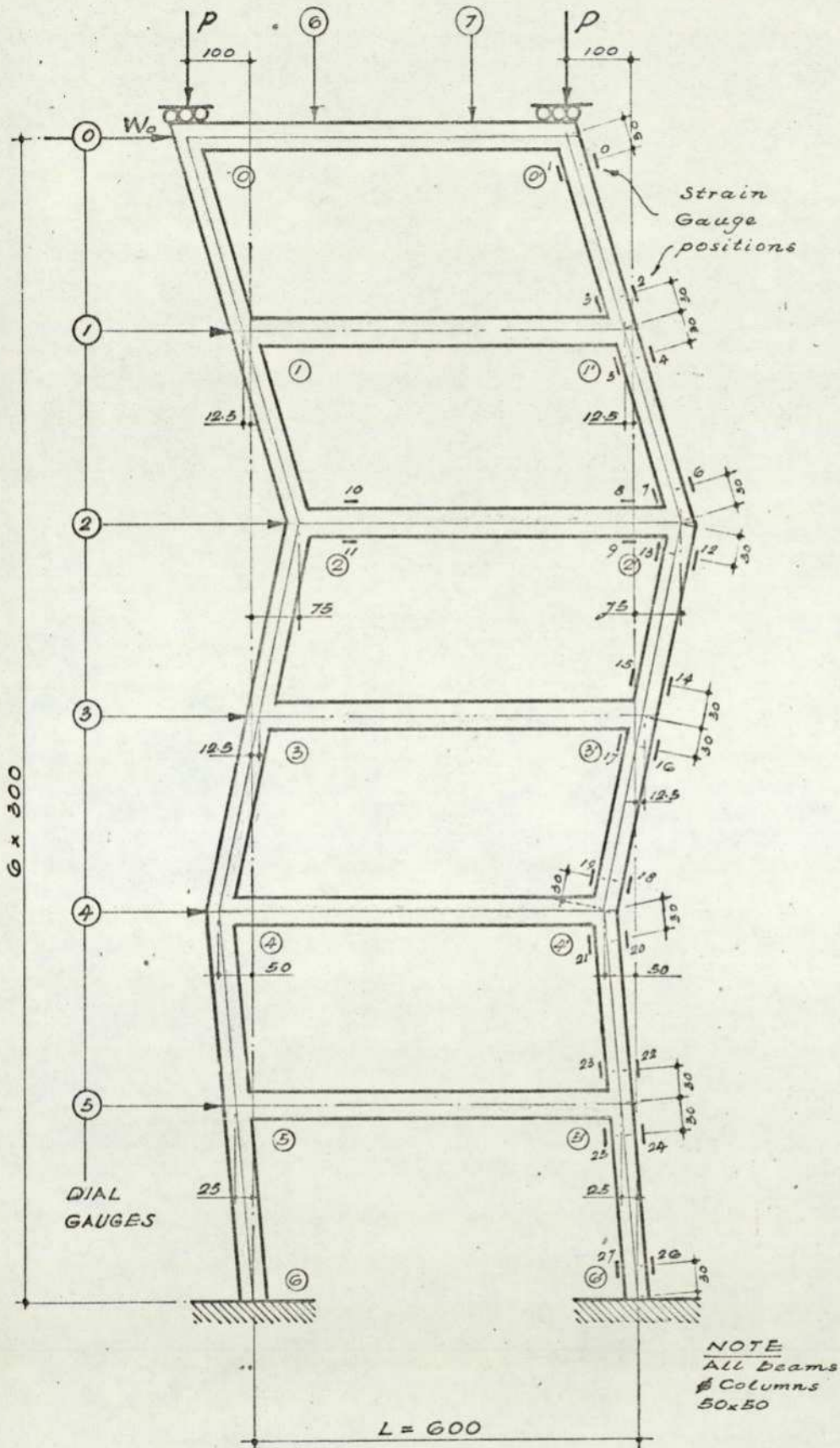


FIG. 6-30 DETAILS OF OUT-OF-PLUMB CONCRETE TEST MODEL



6.31 6-storey Out-of-Plumb Concrete Model with 'Gravity' Loads
Applied at the Top.

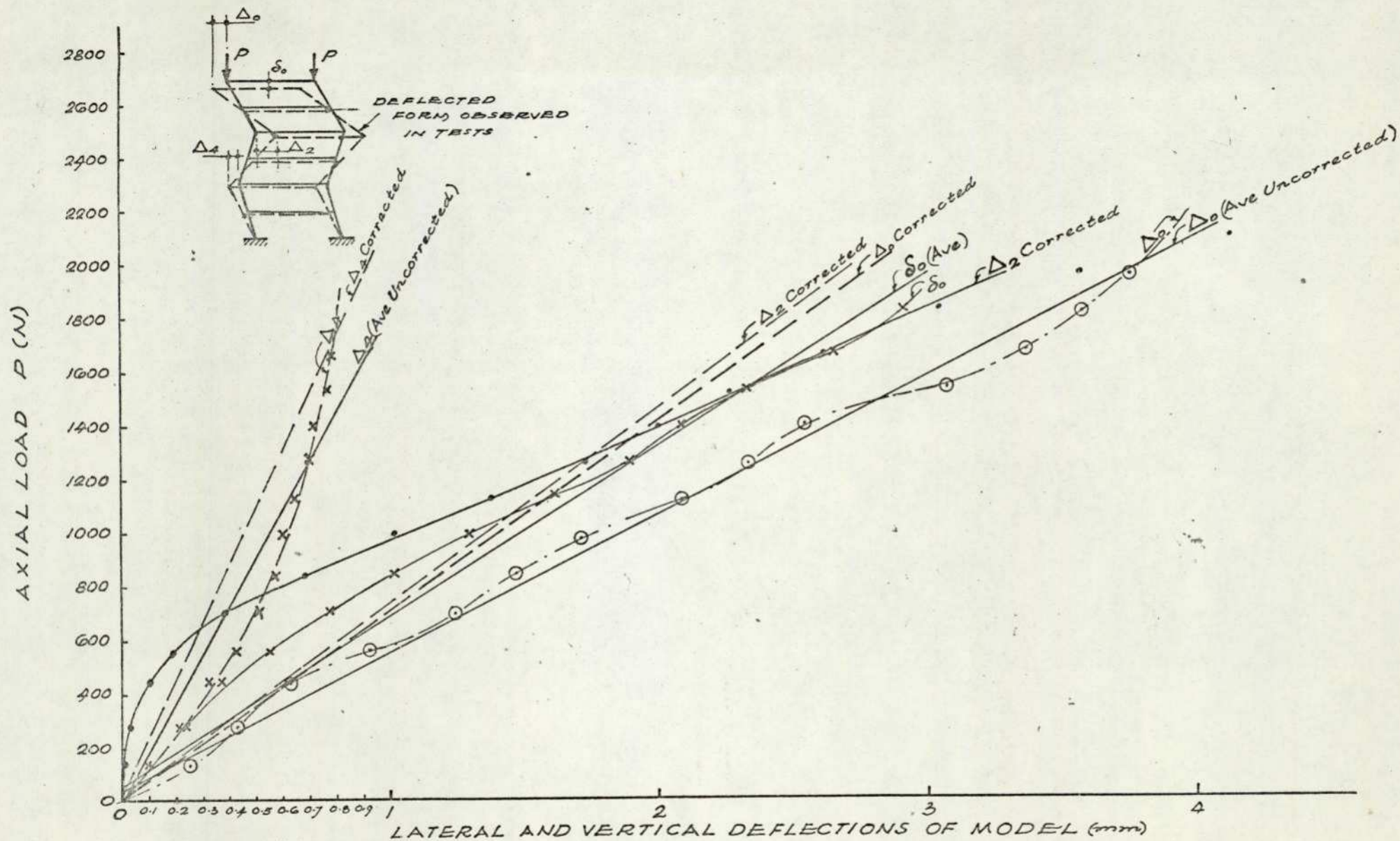


FIG. G.32 LOAD DEFLECTION CURVES FOR 6 STOREY OUT-OF-PLUMB TEST MODEL

LOAD P (N)	STRAIN GAUGE READING (DIVISIONS OF DATA LOGGER)																											
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30.8	0	-2	0	-1	0	+1	+3	0	0	+1	0	-1	0	0	-1	+1	+1	0	-2	0	0	+1	+1	0	0	-184	0	-3
56	0	-2	0	-2	-1	+2	+7	0	0	+1	-1	-1	0	0	-2	+1	+2	-1	-2	+1	-1	+1	+1	0	0	-201	+1	-7
84	0	-2	+1	-3	-1	+4	+13	+2	0	+1	-2	-2	-1	0	-2	+2	+4	-1	-3	+1	-2	+2	+1	0	0	-800	2	-13
112	+1	-2	+1	-3	-1	+5	+18	+2	0	+1	-2	-2	-1	0	-3	+1	+6	0	-4	+1	-3	+2	+2	+1	0	+2	+3	-18
140	+1	-2	+1	-4	-1	+5	+23	+2	0	+1	-3	-2	-1	-1	-4	+2	+9	-1	-4	+2	-3	+2	+2	+1	-1	-202	+3	-24
168	0	-2	+1	-4	-2	+6	+28	+1	-1	+1	-4	-2	-1	-1	-3	+2	+11	-1	-5	+2	-3	+2	+2	0	-1	-800	+3	-29
196	0	-2	+1	-5	-2	+6	+33	+1	0	+1	-4	-3	0	-1	-4	+1	+14	-1	-5	+2	-3	+2	+2	0	-1	-200	+4	-35
224	-1	-2	+1	-5	-2	+6	+38	+1	-1	+1	-4	-3	-1	-2	-4	+1	+16	-2	-5	+2	-4	+1	+2	-1	-1	-201	+4	-41
252	-1	-2	+1	-6	-3	+5	+43	0	0	+1	-5	-3	-1	-2	-4	+1	+19	-2	-5	+2	-4	+1	+2	-1	-1	-200	+5	-47
280	-1	-2	+1	-6	-3	+5	+48	0	-1	+1	-6	-2	0	-2	-4	+1	+21	-2	-5	+3	-4	+1	+2	-1	-1	-787	+5	-52
SIGN OF GAUGE ACCORD TO THEORY	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	-	+	-	+	+	-

* NEGATIVE SIGN INDICATES COMPRESSIVE STRESS

POSITIVE " " TENSILE STRESS

SIGNS ARE FOR EFFECT OF BENDING STRESS ONLY (COMPRESSIVE STRESS DUE TO P TO BE SUPERIMPOSED)

FIG. 6.33 STRAIN GAUGE READINGS FOR 6-STORY OUT-OF-PLUMB CONCRETE
TEST MODEL

Consider the column bending moment M_{10} for load $P = 280\text{N}$. From Fig. 6.33, the bending moment expressed as divisions of data logger output is $\frac{1 - (-6)}{2} = 3.5$. The calibration data in the Appendix (section 11.30) gives 1 division = 675 N.mm. Therefore the experimental bending moment is $3.5 \times 675 = 2,362 \text{ N.mm}$.

The value of the theoretical bending moment using the data in Fig. 6.24, for $P = 280\text{N}$, is $M_{10} = 38.7 \times 280 = 10,836 \text{ N.mm}$. It can be seen that the experimental moment M_{10} is 20% of the theoretical value.

The form of the experimental bending moment diagram is of more interest. Ignoring the readings of faulty strain gauge number 25, the distribution of moments is shown in Fig. 6.34 for the particular value of $P = 280\text{N}$.

6.40 Discussion of Test Results for Out-of-Plumb Model

Corrected lateral deflections at the top of the model are shown to be approximately three times that expected from the theory. This could be attributed to the heavy reinforcement in the model and micro-cracking which modifies the normal structural performance.

While the agreement between experimental and theoretical magnitude of bending moment was poor, there was unmistakable correlation in the form of the bending moment as shown from a comparison of Figs. 6.24 and 6.34.

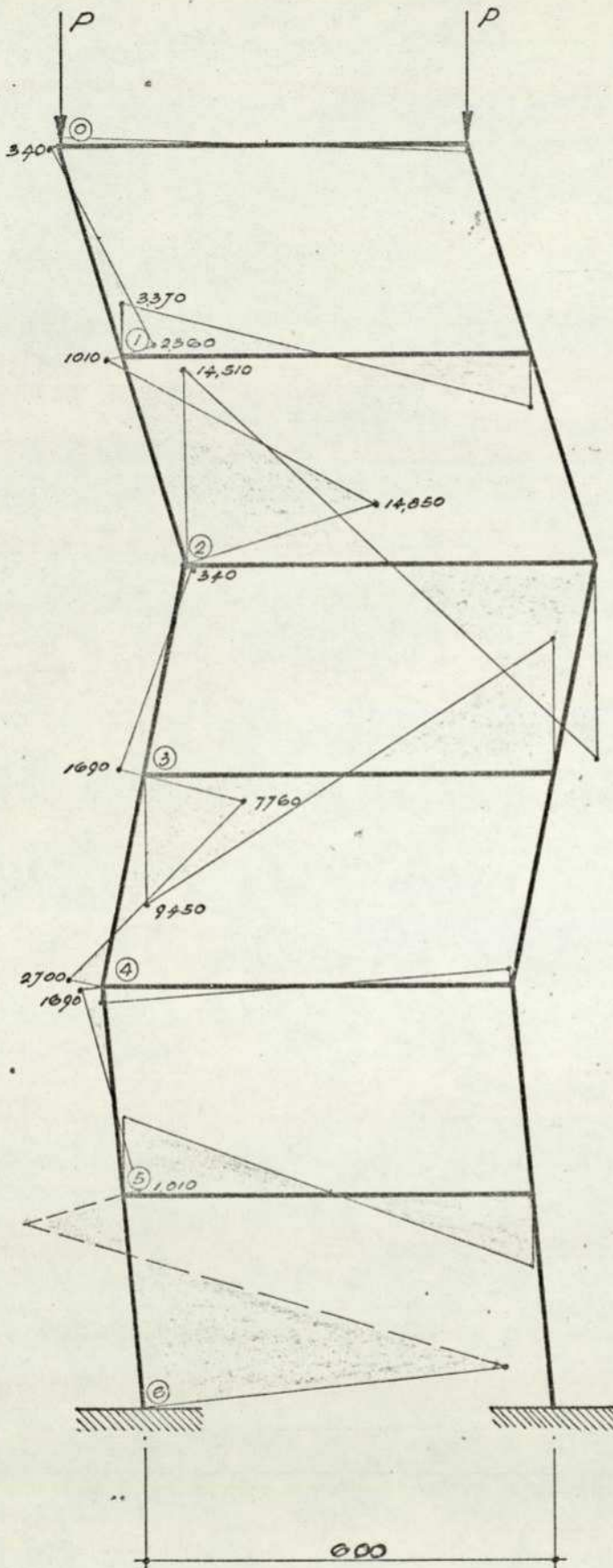


FIG. G-34 FORM OF BENDING BENDING MOMENT OBTAINED FROM TESTS ON G-STOUREY OUT-OF-PLUMB CONCRETE MODEL (FOR $P=280N$)

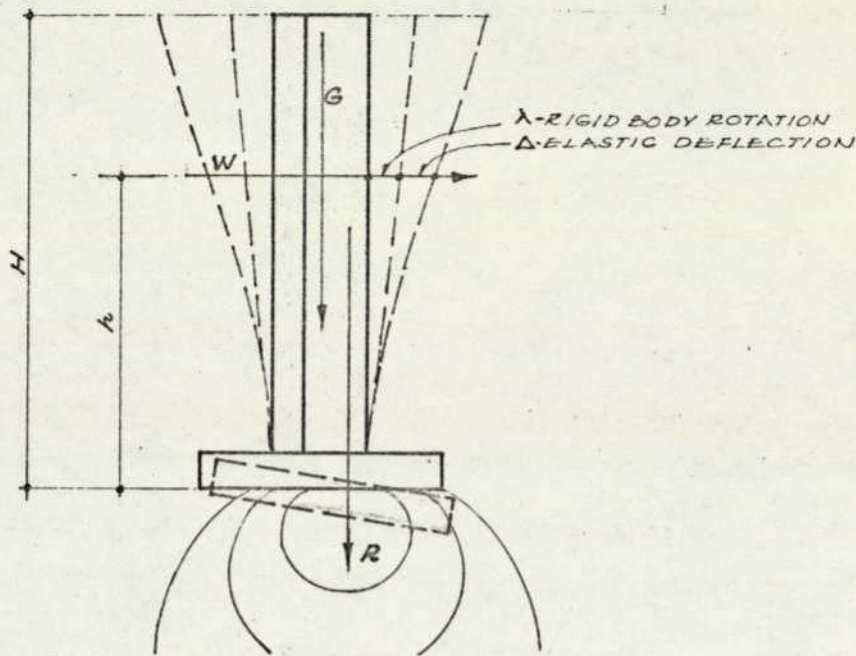


Fig. 7.00 Soil-structure Interaction.

It has long been anticipated that tall and slender buildings may respond significantly to the deformations of the soil subjected to the combined effect of gravity and wind loading. The superstructure may then deform more than predicted by the primary analysis because of the rigid body rotation produced by the deformation of the soil, this is shown in Fig. 7.00. Conversely, stresses and deformations in the soil medium directly under the footing may be affected by the increased eccentricity of the resultant wind and gravity force. This problem, termed as soil-structure interaction, is now well recognised and an added factor in the consideration of elastic instability of tall buildings.

Modern tall buildings can be very slender and although their overall mass tends to be reduced, their foundation area is often small, with high concentration of loads. Such high pressure induced on the soil mass, combined with large eccentricities of the resultant force on the foundation, may produce large deformations of the soil mass. These deformations are both translational as well as rotational and can be plastic or elastic.

While the elastic deformations of the superstructure depend largely on the stiffness or flexibility of the superstructure, the deformations of the soil are a function of the soil properties and eccentricities of loads.

The following analysis is made based on the assumption that:

- a) the soil deformations are elastic, and related to its modulus of elasticity or to the subgrade of soil reaction and that
- b) the action of the forces is static.

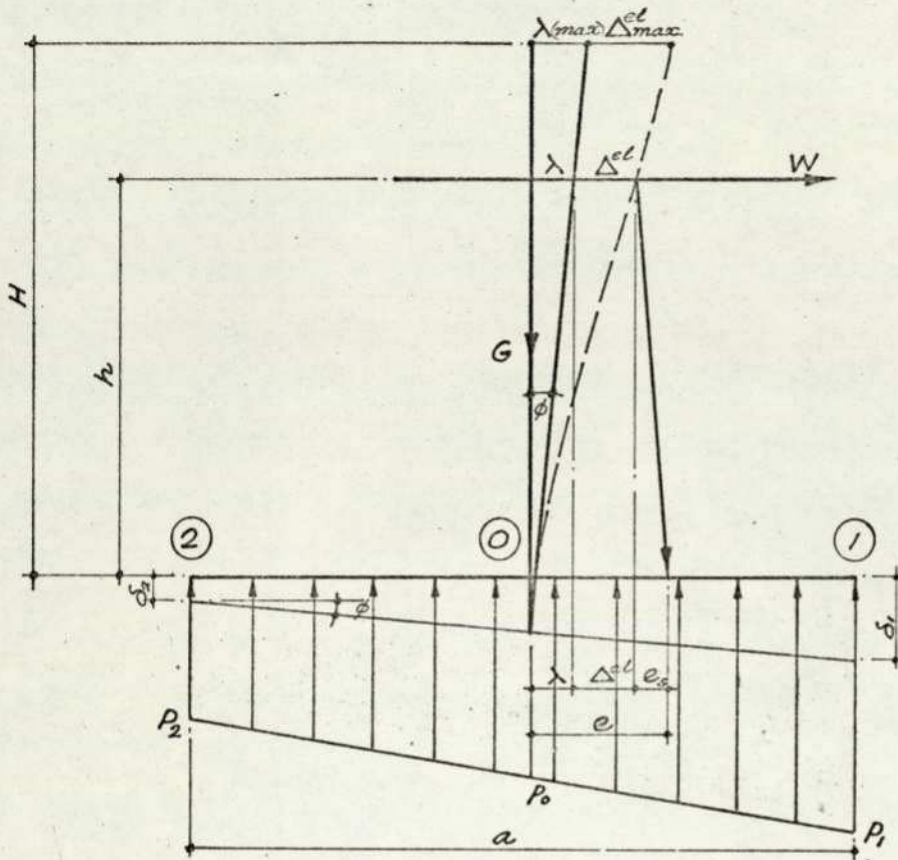


Fig. 7.10 Diagram of Forces and Deflections used in Analysis.

Referring to Fig. 7.10 the following notation is used:-

- | | | |
|---------------|---|--|
| a, b | - | length and width of foundation |
| H | - | total height of building to the underside of foundation |
| h | - | the distance of the resultant wind force from foundation |
| G and W | - | total gravity and wind force respectively, acting on the shear wall considered |
| Δ^{el} | - | elastic deformation of the shear wall |
| ϕ | - | rotation of foundation and superstructure |
| λ | - | deflection of the superstructure due to soil movement alone, and |

e	-	total eccentricity of gravity force
e_s	-	$h \frac{W}{G}$ is the 'static' eccentricity
k	-	is the coefficient of subsoil reaction

From Fig. 7.10

$$\lambda = h \cdot \phi \quad \text{in which} \quad \phi = \frac{\delta_1 - \delta_2}{a}$$

Assuming that $\delta = \frac{p}{k}$, we have $\phi = \frac{p_1 - p_2}{k} \dots\dots\dots 7.10$

For a rigid foundation we can assume that:

$$p_{1,2} = \frac{G}{a \cdot b} \left(1 \pm \frac{Ge}{a} \right) \dots\dots\dots 7.20$$

Substituting Equation 7.20 into Equation 7.10

$$\phi = \frac{G}{kba^2} \left(1 + \frac{Ge}{a} - 1 + \frac{Ge}{a} \right) = \frac{12Ge}{kba^3}$$

or $\phi = \frac{Ge}{kI_f} \dots\dots\dots 7.30$

In which the eccentricity e is:

$$e = \lambda + \Delta^{el} + e_s = \lambda + \Delta^{el} + h \frac{W}{G} \dots\dots\dots 7.40$$

Equation 7.30 is valid for all values of e provided that tension on the soil does not take place, i.e. $e \leq \frac{a}{6}$

Introducing Equation 7.40 into 7.30 we obtain:

$$\lambda = \frac{Geh}{kI_f} = \frac{Gh}{kI_f} \left(\lambda + \Delta^{el} + h \frac{W}{G} \right)$$

or $\lambda = h \left(\frac{W}{G} + \frac{\Delta^{el}}{h} \right) \left(\frac{kI_f}{Gh - 1} \right) \dots\dots\dots 7.50$

From Equation 7.40

$$W = \frac{G}{h} (e - \lambda - \Delta^{el}) = G \frac{e_s}{h} \dots\dots\dots 7.60$$

If $e = \frac{a}{6}$, then from Equation 7.30:

$$\phi = \frac{G \cdot a}{G \cdot k \cdot I_f}$$

and $\lambda = \frac{G \cdot a \cdot h}{G \cdot k \cdot I_f} \dots\dots\dots 7.70$

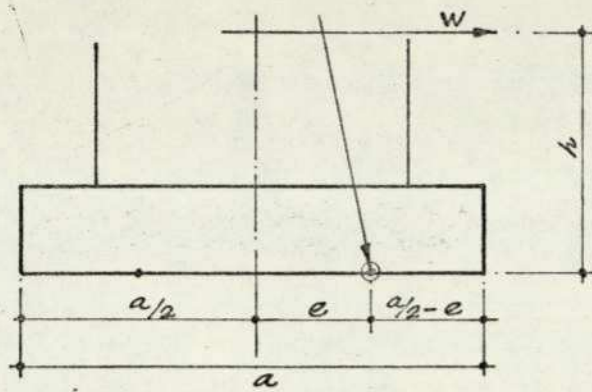


Fig. 7.20 Diagram for Overturning Criteria.

For safety against overturning (Fig. 7.20):

$$G\left(\frac{a}{2} - e\right) \geq N.W.h \quad ('N' \text{ is the safety factor})$$

$$N \leq \frac{G\left(\frac{a}{2} - e\right)}{W\left(\frac{a}{2h} - \frac{e}{h}\right)} \quad \dots\dots\dots 7.80$$

Example:

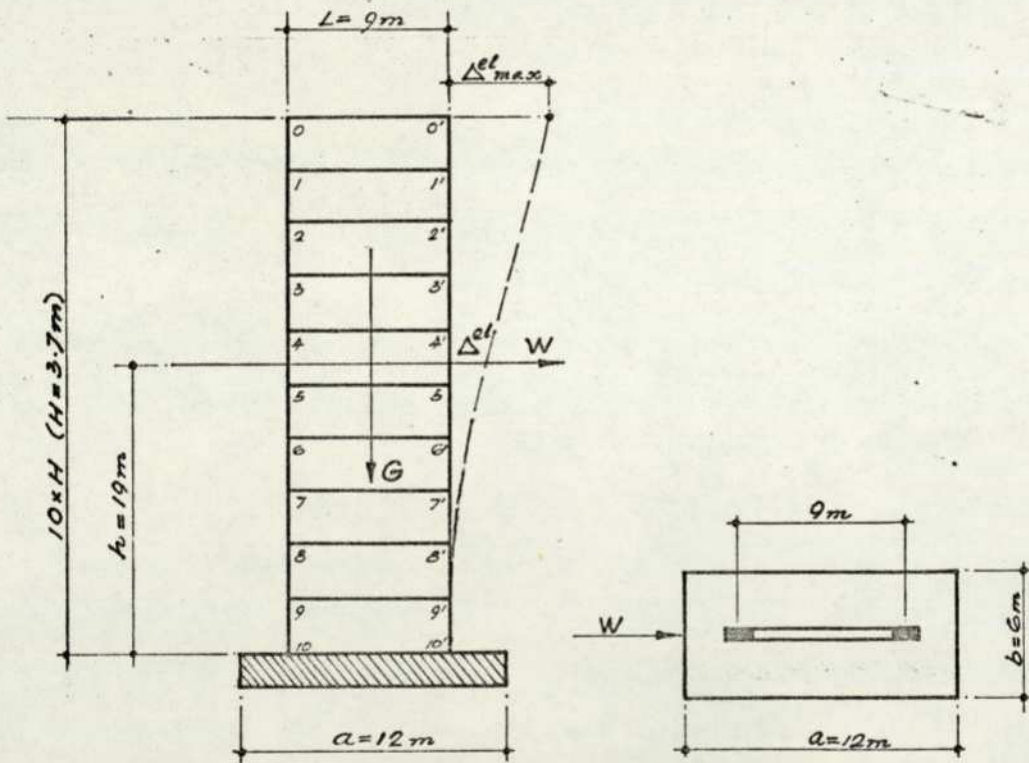


Fig. 7.30 Details of Structure/Foundations in Example.

The following example is made to illustrate the procedure in investigating the effect of soil interaction on movements of the superstructure.

All members in the frame shown in Fig. 7.30 are of constant value of I.

Assumed wind pressure = 21.6 kN per floor

or W total = 216 kN

G = 2,000 kN

k = 1000 t/m³

$I_{\text{beam}} = I_{\text{column}} = 2.22 \times 10^{-4} \text{ m}^4$

$E_s = 2 \times 10^8 \text{ kN/m}^2$

$I_{\text{foundation}} = \frac{1}{12} \times 6 \times 12^3 = 864 \text{ m}^4$

The element force-displacement matrix is set out at the level 1-1' as follows:-

$$211' \quad -M_{21} \frac{h}{6EI_c} + M_{12} \frac{h}{3EI_c} + (M_{10} + M_{12}) \frac{L}{6EI_b} = \frac{(\Delta_1 + \lambda_1) - (\Delta_2 + \lambda_2)}{h}$$

$$\begin{aligned} 012 \quad -M_{01} \frac{h}{6EI_c} + M_{10} \frac{h}{3EI_c} - M_{12} \frac{h}{3EI_c} + M_{21} \frac{h}{6EI_c} = \\ = \frac{(\Delta_0 + \lambda_0) - (\Delta_1 + \lambda_1)}{h} - \frac{(\Delta_1 + \lambda_1) - (\Delta_2 + \lambda_2)}{h} \end{aligned}$$

$$\text{E.C.1-2} \quad \frac{M_{12} + M_{21}}{h} = \frac{1}{2}(2W) + 2P \left[\frac{(\Delta_1 + \lambda_1) - (\Delta_2 + \lambda_2)}{h} \right]$$

Introducing numerical values, the main force-displacement matrix can now be set out. This is shown in the Fig. 7.40. Fig. 7.50 shows the distribution of bending moments and displacement. Values in brackets refer to the primary analysis without the effect of soil interaction. It can be seen that both the bending moments and deflections are appreciably affected.

EQU. NO.	COND.	0		1			2			3			4			5			6			7			8			9			10	LOAD
		M ₀₁	Δ ₀	M ₁₀	M ₁₂	Δ ₁	M ₂₁	M ₂₃	Δ ₂	M ₃₂	M ₃₄	Δ ₃	M ₄₃	M ₄₅	Δ ₄	M ₅₄	M ₅₆	Δ ₅	M ₆₅	M ₆₇	Δ ₆	M ₇₆	M ₇₈	Δ ₇	M ₈₇	M ₈₉	Δ ₈	M ₉₈	M ₉₀	Δ ₉	M ₀₀	WATTS
1	100	2.7	-1.027	-0.6	0	1.027																										0
2	0-1	1	0.0048	1	0	0.0048																										1.8W
3	211	0	0	1.5	2.7	1.0041	-0.6	0	1.0041																							0
4	012	-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041																							3.6W
5	1-2	0	0	0	1	0.0096	1	0	0.0096																							0
6	322				0	0	1.5	2.7	1.0041	-0.6	0	1.0041																				0
7	123				-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041																				0
8	2-3				0	0	0	1	0.0144	1	0	0.0144																				5.4W
9	233						0	0	1.5	2.7	1.0041	-0.6	0	1.0041																		0
10	232						-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041																		0
11	3-4						0	0	0	0.0192	1	0	0	0.0192																		7.2W
12	541								0	0	1.5	2.7	1.0041	-0.6	0	1.0041																0
13	345						-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041																		0
14	2-5						0	0	0	1	0.0241	1	0	0.0241																		9.0W
15	005								0	0	1.5	2.7	1.0041	-0.6	0	1.0041																0
16	456								-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041																0
17	5-6								0	0	0	1	0.0288	1	0	0.0288																10.8W
18	766										0	0	1.5	2.7	1.0041	-0.6	0	1.0041														0
19	567										-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041														0
20	6-7										0	0	0	1	0.0336	1	0	0.0336														12.6W
21	877												0	0	1.5	2.7	1.0041	-0.6	0	1.0041												0
22	678												-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041												0
23	7-8												0	0	0	1	0.0384	1	0	0.0384												14.4W
24	988														0	0	1.5	2.7	1.0041	-0.6	0	1.0041										0
25	789														-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041										0
26	8-9														0	0	0	1	0.0432	1	0	0.0432										16.2W
27	1099																0	0	1.5	2.7	1.0041	-0.6	0	1.0041								0
28	8910																-0.6	1.0041	1.2	-1.2	2.0082	0.6	0	1.0041								0
29	9-10																0	0	0	1	-0.048	1										18.0W
30	100																												-0.6	1.0041	1.2	0

FIG. J.40 : MATRIX FOR 10-STOREY FRAME, USED IN EXAMPLE INCLUDING EFFECT OF SOIL-STRUCTURE INTERACTION

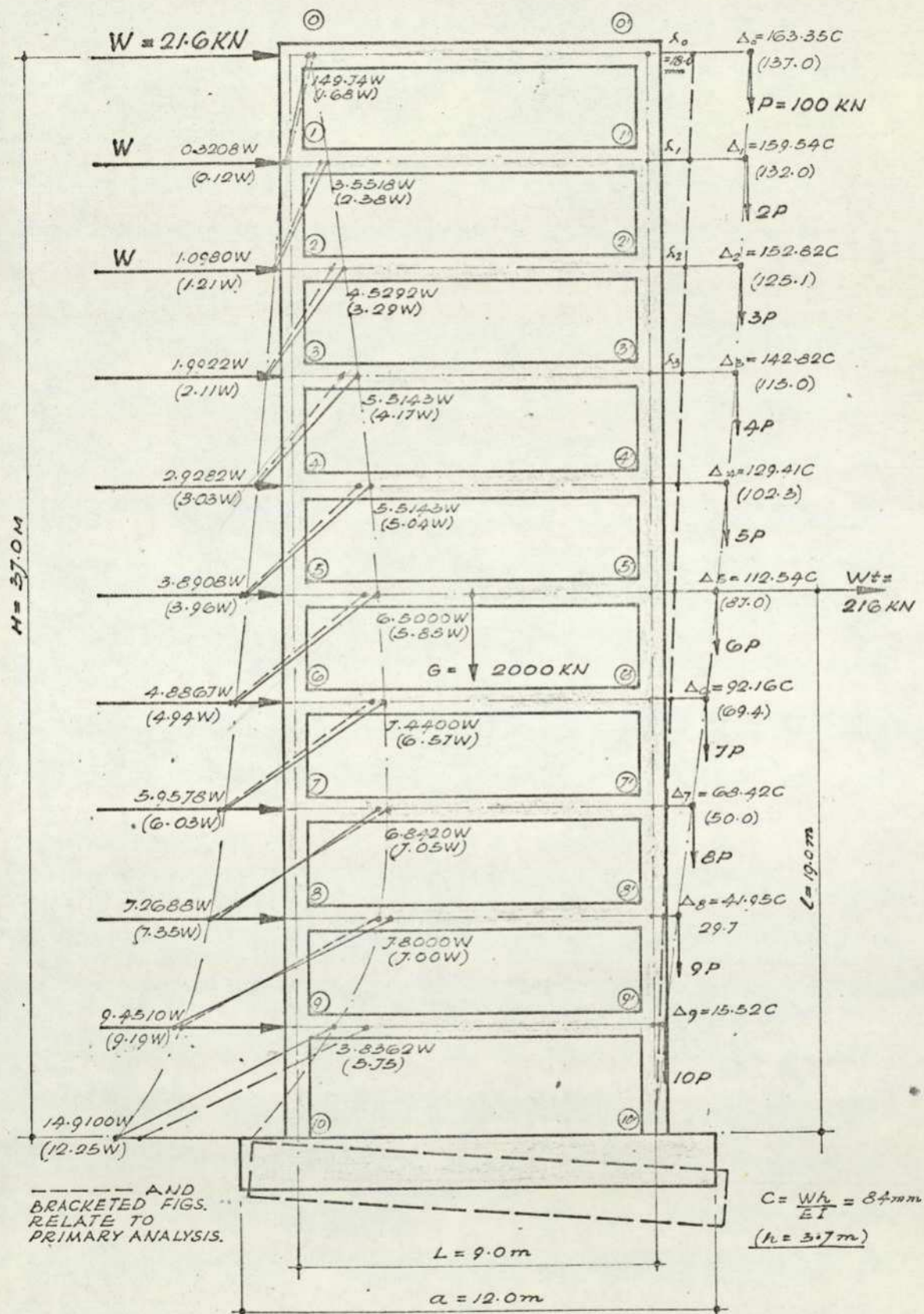


FIG. 7.50 : DIAGRAM OF MOMENTS/DEFLECTIONS FOR 10-STOREY FRAME INCLUDING EFFECT OF SOIL-STRUCTURE INTERACTION

8.00 Effect of Cracks on Elastic Instability of Tall Buildings

Cracks in concrete members develop when tensile strains or stresses reach or exceed the ultimate values for a particular concrete used. Tensile strains and stresses may be from the dead weight of a structure, from the imposed gravity or wind forces, or from other effects such as: shrinkage of concrete, temperature variations, differential settlement of supports, excessive vibrations, etc.

It can be expected that in tall building frames and shear walls, cracks, once formed will have a detrimental effect on the performance, i.e. they may promote redistribution of stresses, may increase deflections and lower the frequency of vibrations.

In the following, an approximate mechanism of crack formation is formulated and then the attempt is made to check its effect on the response of a tall frame.

It is postulated that cracks form when the ultimate tensile strength of concrete is reached. As a result, bond stress must then be reduced to zero at the place of crack, steel is then locally stressed to a higher value than at a place where bond is at its full value at the middle between the cracks. If, at this point, tensile strength in concrete does not reach its ultimate, then further cracks will not appear. If, however, this stress is reached, then further cracks will appear, until in this process an optimal length between the cracks is reached, when tensile stress in concrete at the mid-point between cracks is less than critical.

Fig. 8.01 shows this mechanism related to the stresses in concrete and steel. The following notation is used:

σ_{cc} stress in concrete in compression

$\sigma_{cc'}$ stress in concrete in compression based on ultimate load

C_c resultant compressive force

σ_{ct} stress in concrete in tension

σ_s stress in steel for uncracked section

σ_s	stress in steel for cracked section
T_s	resultant tensile force in steel
β	bond stress in concrete
μ	the width of crack
$n-n$	the position of neutral axis
l'	the distance between cracks
ϵ_s	strain in steel at the place of cracks
E	modulus of elasticity of steel
O	total number of bars.

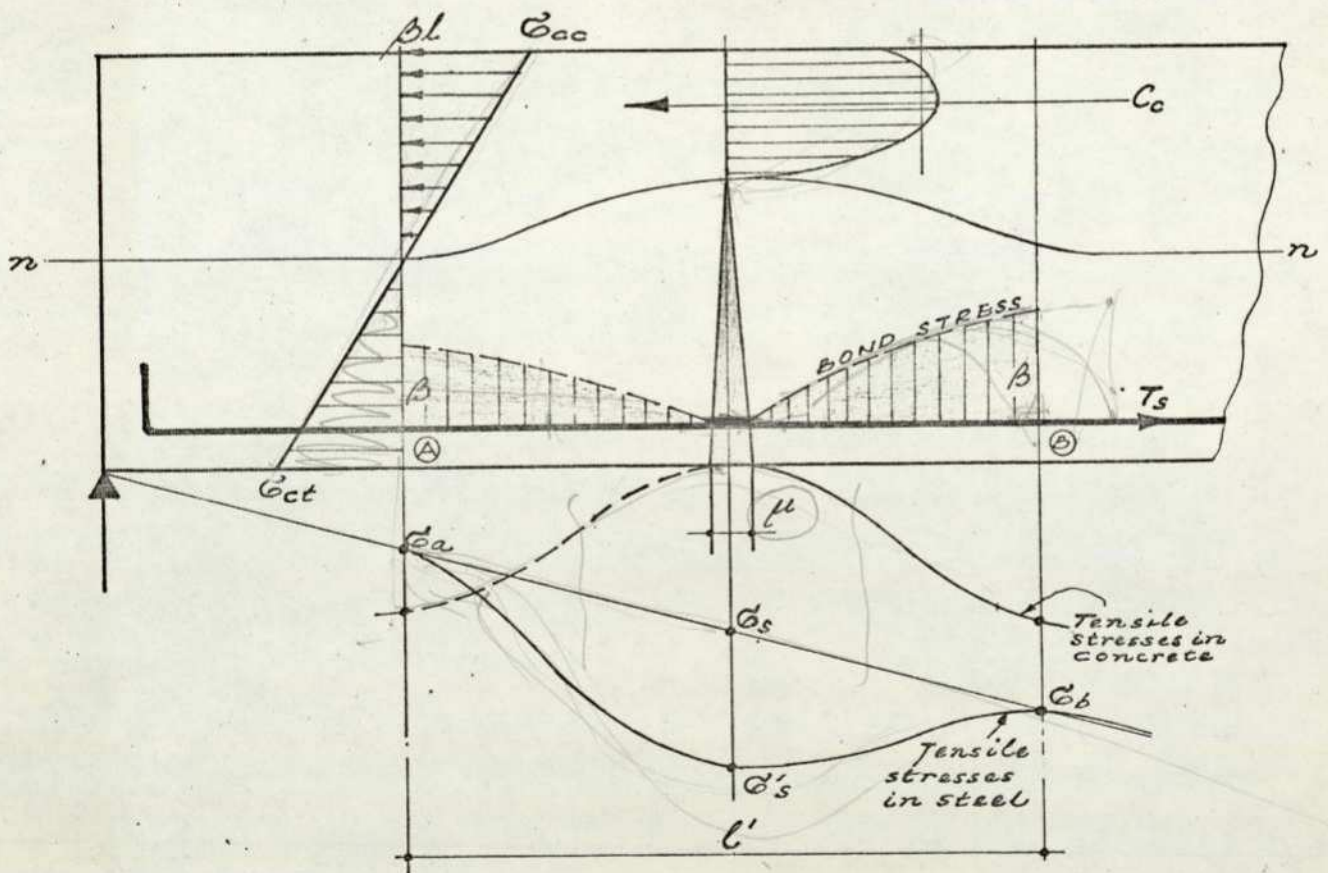


Fig. 8.01 Diagram of the Assumed Mechanism of Crack Formation in Element of Reinforced Concrete.

Based on Fig. 8.01 we can write, that the width of cracks is related to the "excessive" stress in steel ($\sigma'_s - \sigma_s$) when cracks are formed, i.e.

$$\mu = \frac{l'}{2}$$

$$\text{or } \mu = \frac{l'}{2} \left(\frac{\sigma'_s - \sigma_s}{E} \right) \quad \dots\dots\dots 8.10$$

in which the distance l' can be calculated from the condition of equilibrium of internal forces, i.e.

$$T_s = H_s \cdot \sigma_s = O \cdot \frac{l'}{2} \cdot \beta$$

$$\text{or } \frac{l'}{2} = \frac{H_s \cdot \sigma_s}{O \cdot \beta} \quad \dots\dots\dots 8.20$$

When the magnitude of the width of the crack is thus estimated, then its effect on structures can be taken into account as follows:-

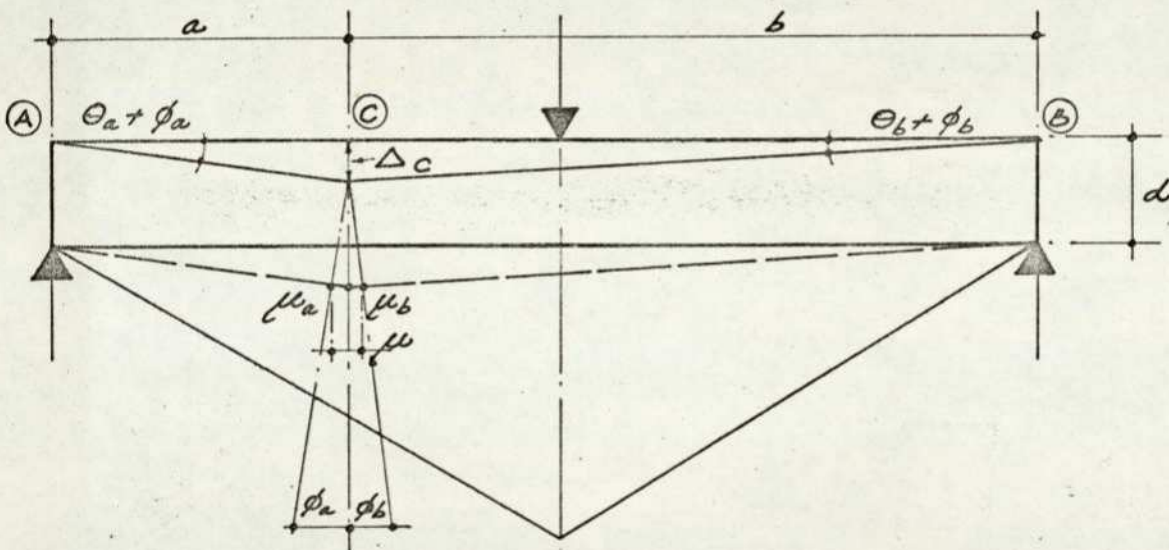


Fig. 8.02 Simply-supported Beam with Crack.

Taking first a simply-supported beam AB as shown in Fig. 8.02 and subjected to any system of loading, the formation of a crack does not affect the distribution of bending moments but affects its deflections. The angular rotations at A and B are now $\theta_a + \phi_a$ and $\theta_b + \phi_b$ in which θ refers to the effect of loading and ϕ to the effect of cracks.

From Fig. 8.02

$$\frac{\mu a}{d} = \frac{\Delta_c}{a}$$

and

$$\frac{\mu - \mu a}{a} = \frac{\Delta_c}{b}$$

from which the additional deflection at C due to the crack only is:

$$\Delta = \mu \frac{a \cdot b}{L \cdot d} \quad \dots\dots\dots 8.30$$

if $a = b = \frac{L}{2}$ then

$$\Delta_c = \mu \frac{L}{4d} \quad \dots\dots\dots 8.40$$

the values of ϕ are:

$$\phi_a = \frac{\Delta}{a} = \mu \frac{b}{L \cdot d}$$

and $\phi_b = \frac{\Delta}{b} = \mu \frac{a}{L \cdot d} \quad \dots\dots\dots 8.50$

if $a = b = L, \mu_a = \mu \frac{b}{L}$ and $\mu_b = \frac{a}{L}$

Example - Taking $L = 10$ m, $a = b = L/2$, $\mu = 1$ mm and $d = 600$ mm we have from

Eqn. 8.50 $\Delta = \mu \frac{L}{4d} = \frac{10,000}{4 \times 600} = 4$ mm

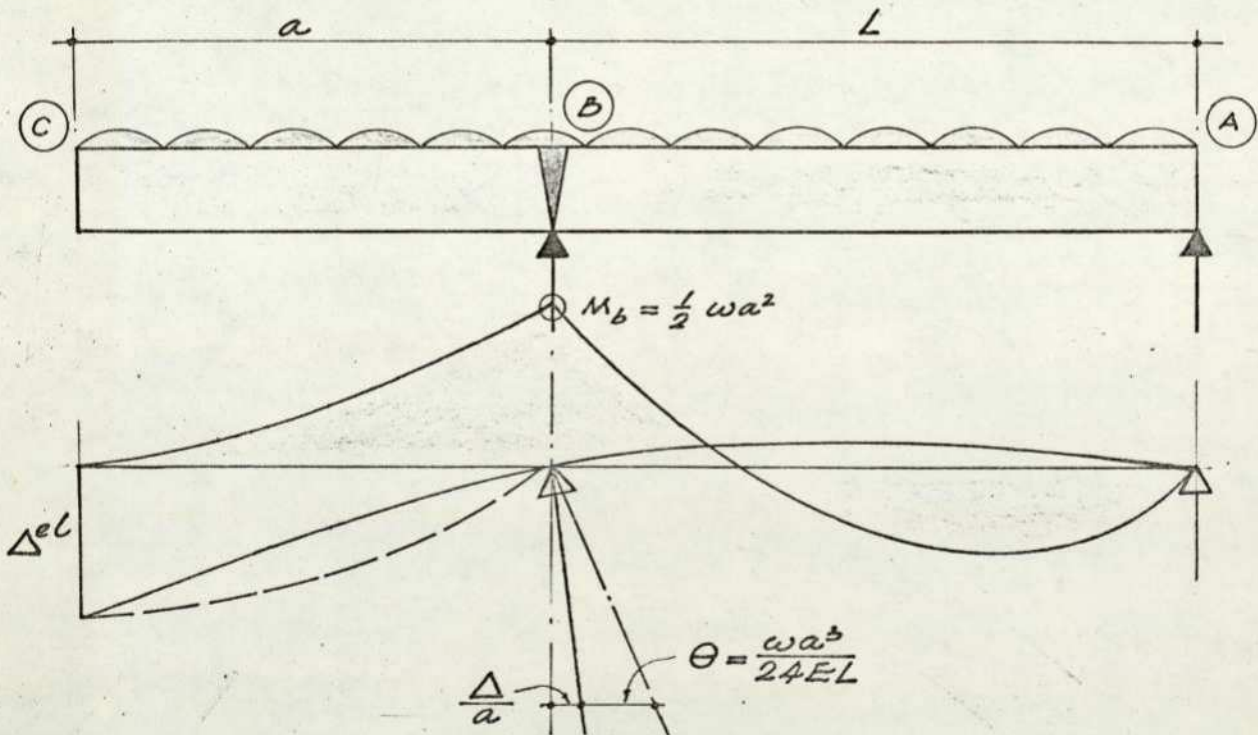


Fig. 8.03 Cantilever Beam with Crack.

To take a step further, a cantilever beam in Fig. 8.03 is used where it is assumed that a crack is formed at B: Using the force-displacement method we can write that at B:

$$M_b \left(\frac{a}{3EI} + \frac{L}{3EI} \right) = \frac{\Delta^{el}}{a} + \frac{W a^3}{24EI}$$

the 'elastic' deflection at C is:

$$\Delta^{el} = \frac{WL^3}{24EI} \left[3 + 4\left(\frac{a}{L}\right)^3 \right]$$

and the deflection of a cracked beam is:

$$\Delta_c = \Delta^{el} + \phi \cdot a = \Delta^{el} + \mu \frac{a}{Ld}$$

Taking e.g. $a = 3000$; $L = 10,000$, $d = 600$, the additional deflection at C for $\mu = 1$ mm is:

$$\phi \cdot a = \mu \frac{a}{d} = 1 \times \frac{3000}{600} = 5 \text{ mm}$$

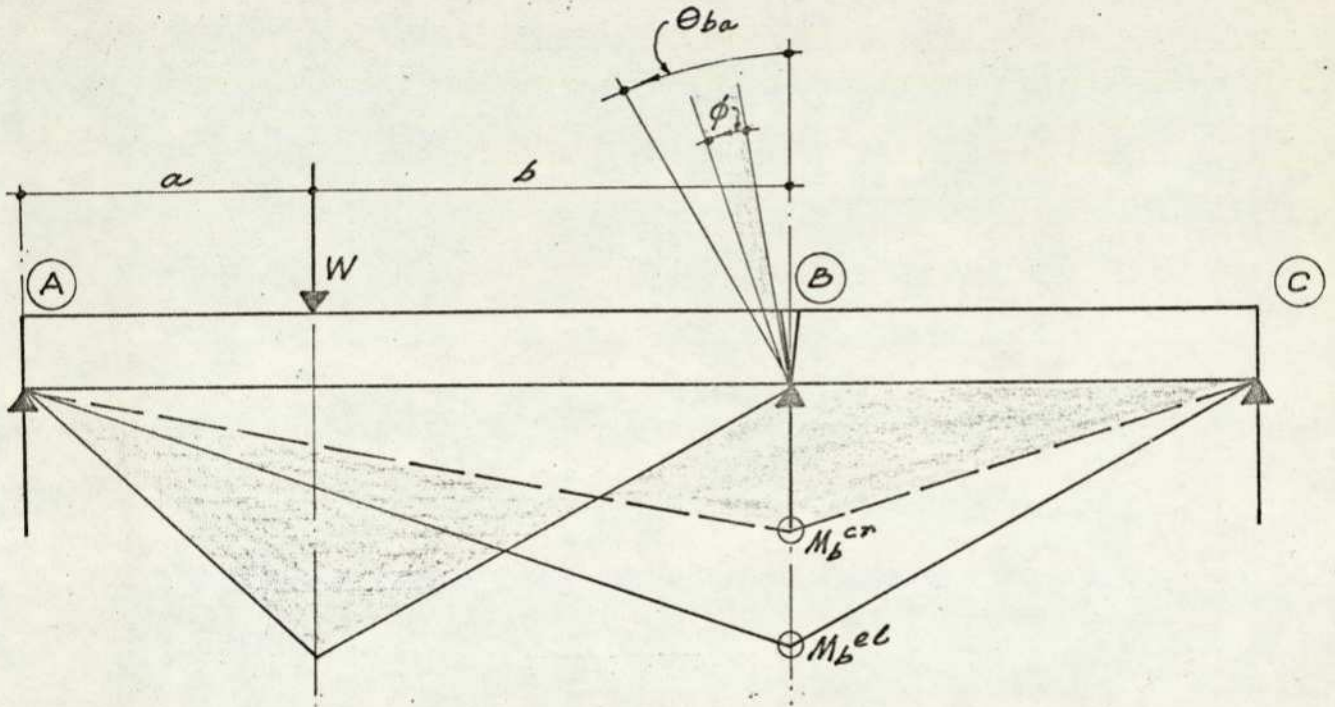


Fig. 8.04 Two Span Beam with Crack at Intermediate Support.

Next, a two-span beam, Fig. 8.04, is used to illustrate the effect of cracks or re-distribution of bending moments as well as deflections. If a crack is formed over the support B, then we can write that

$$M_b \left(\frac{L_1}{3EI_1} + \frac{L_2}{3EI_2} + \phi_b \right) = \Theta_{ba} - \phi$$

in which Θ_{ba} is the rotation BA due to the applied loads and ϕ due to the effect of crack.

Taking $\phi = \frac{\mu}{d}$

we obtain: $M_b = M_o \frac{L_1 + a}{2(L_1 + L_2)} - \frac{3 \cdot \phi \cdot E \cdot I}{L + L_2}$

or
$$M_b^{cr} = M^{al} - \frac{3\phi \cdot EI}{L + L_2}$$

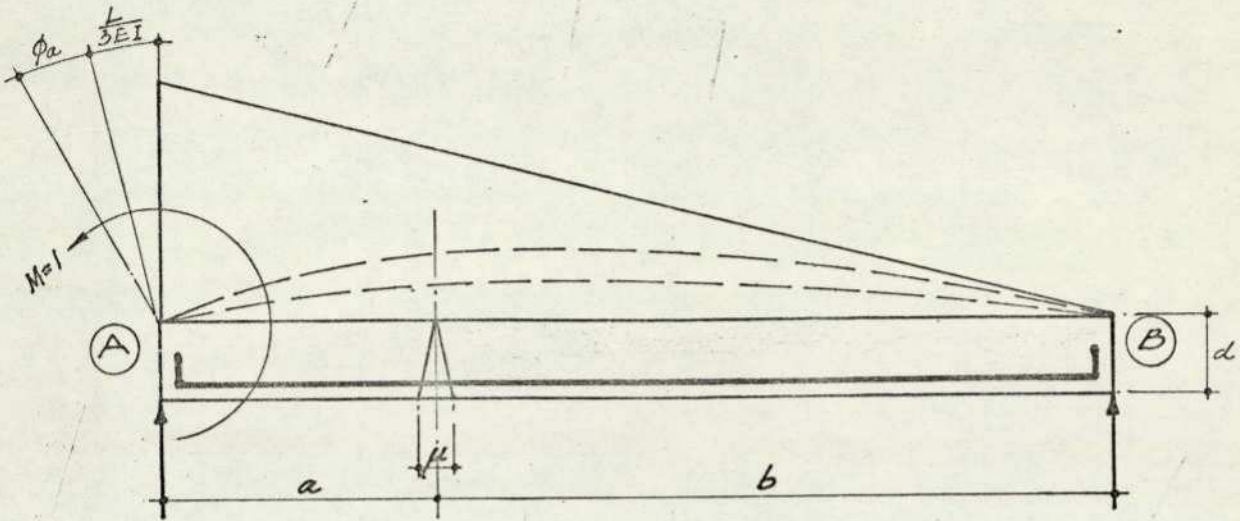


Fig. 8.05 Effect of Cracks on Flexibility Coefficients.

To apply this theory to more complex structures, it is necessary to include the effect of cracks on the flexibility coefficients. Based on Fig. 8.05 these can be written directly as:

$$f_{aa} = \frac{L}{3EI} + \phi_a = \frac{L}{3EI} + \mu \frac{b}{Ld} \quad \dots\dots\dots 8.60$$

and
$$f_{ba} = \frac{L}{6EI} + \mu \frac{a}{Ld} \quad \dots\dots\dots 8.70$$

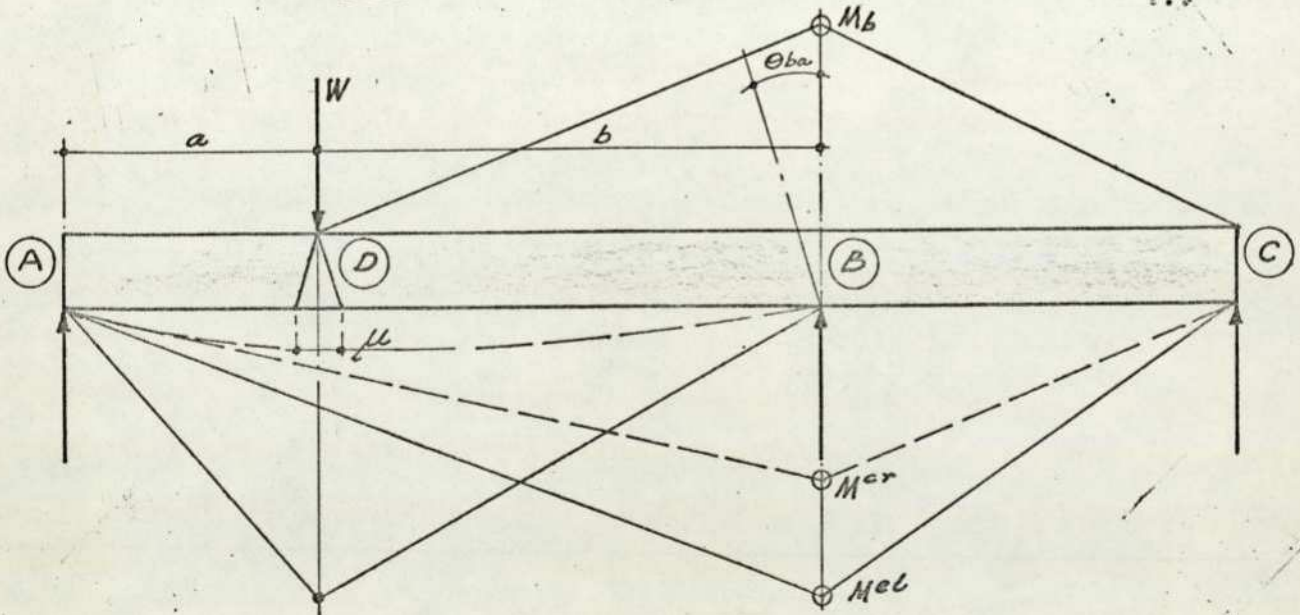


Fig. 8.06 Two-span Beam with Crack at Load Point.

For a two-span beam as shown in Fig. 8.06 in which a crack is formed at D under the load W we can write that:

$$M_b^{cr} \left(\frac{L_1}{3EI_1} + \frac{L_2}{3EI_2} + \mu \frac{a}{L_1 d} \right) = \theta_{ba} + \mu \frac{a}{L_1 d}$$

$$\theta_{ba} = \frac{M_u}{6EI} (L_1 + a)$$

It can be seen that if a crack at D develops, the span bending moments increase and support moment increases. In the limit, for a wide crack, the span moment becomes zero and the support moment is equal to W.b i.e. to the cantilever moment. The beam therefore does not need to collapse, since another plastic hinge is required to form a mechanism.

For a beam to work as a cantilever:

$$M_b \left(\frac{b}{3EI} + \frac{L_2}{3EI} \right) = \frac{\Delta}{b}$$

$$\therefore \Delta = Wb^2 \left(\frac{L_2 + b}{3EI} \right)$$

and since $\Delta = \mu \cdot \frac{a \cdot b}{dL_1}$,

equalising these values we obtain

$$\mu = \frac{W \cdot b \cdot d \cdot L_1 (L_2 + b)}{3a \cdot EI} \dots\dots\dots 8.80$$

i.e. for the width of crack represented by the Equation 8.80 the beam may (theoretically) work as a simple beam BC with a cantilever BD.

A reverse procedure may take place if a further crack develops at B and span bending moment will increase until it is equal to the full value of the static moment $W a \cdot b/L$. Any further increase in cracks results only in increased deflection at D without any change in this moment until stress in steel reaches yield, and fully plastic hinge develops. This is the collapse of the beam.

The mechanics of the effect of cracks can now be applied to tall frames. If we assume that at the place of maximum stress (calculated for homogeneous sections) the crack may appear, then its width can be estimated from the Equation 8.10 and the matrix can be set out to obtain the redistributed bending moments and deflections. The procedure is therefore iterative. To illustrate this, a ten-storey frame is used of dimensions as shown in the previous example, (Fig. 7.30) to facilitate comparison.

The frame is assumed to have a substantial crack in the end of the beam at first floor level such that $M_{\theta\theta} = 0$. The assumed shape of the moment/deflection diagram is shown in Fig. 8.07.

The non-dimensional parameters for this frame are:

$$r = \frac{L I_c}{h I_b} \quad \text{and} \quad q = \frac{E I_c}{h}$$

The equations for compatibility of deformation and for equilibrium are set down as follows:-

$$100' \quad -\frac{h}{6EI_c} M_{10} + \frac{h}{3EI_c} M_{01} + \frac{L}{3EI_b} M_{00'} - \frac{L}{6EI_b} M_{0'0} = \frac{\Delta_0 - \Delta_1}{h}$$

$$-M_{10} + 2M_{01} + 2r(M_{01}) - r(M_{0'1'}) - (\Delta_0 - \Delta_1) \frac{6EI_c}{h^2} = 0$$

$$-M_{10} + (2 + 2r)M_{01} - rM_{0'1'} - q(6\Delta_0) + q(6\Delta_1) = 0$$

$$00'1' \quad -\frac{h}{6EI_c} M_{1'0'} + \frac{h}{3EI_c} M_{0'1'} + \frac{L}{3EI_b} M_{0'0} - \frac{L}{6EI_b} M_{00'} = \frac{\Delta_0 - \Delta_1}{h}$$

$$-M_{1'0'} + 2M_{0'1'} + 2r(M_{0'1'}) - r(M_{10}) - (\Delta_0 - \Delta_1) \frac{6EI_c}{h^2} = 0$$

$$-M_{1'0'} + (2 + 2r)M_{0'1'} - rM_{10} - q(6\Delta_0) + q(6\Delta_1) = 0$$

Equilibrium condition at O-O'

$$\left[\frac{(M_{01} + M_{10})}{h} + \frac{(M_{0'1'} + M_{1'0'})}{h} \right] = 0.5W$$

$$M_{01} + M_{10} + M_{0'1'} + M_{1'0'} = 0.5Wh$$

$$211' \quad -\frac{h}{6EI_c} M_{21} + \frac{h}{3EI_c} M_{12} + \frac{L}{3EI_b} M_{11'} - \frac{L}{6EI_b} M_{1'1} = \frac{\Delta_1 - \Delta_2}{h}$$

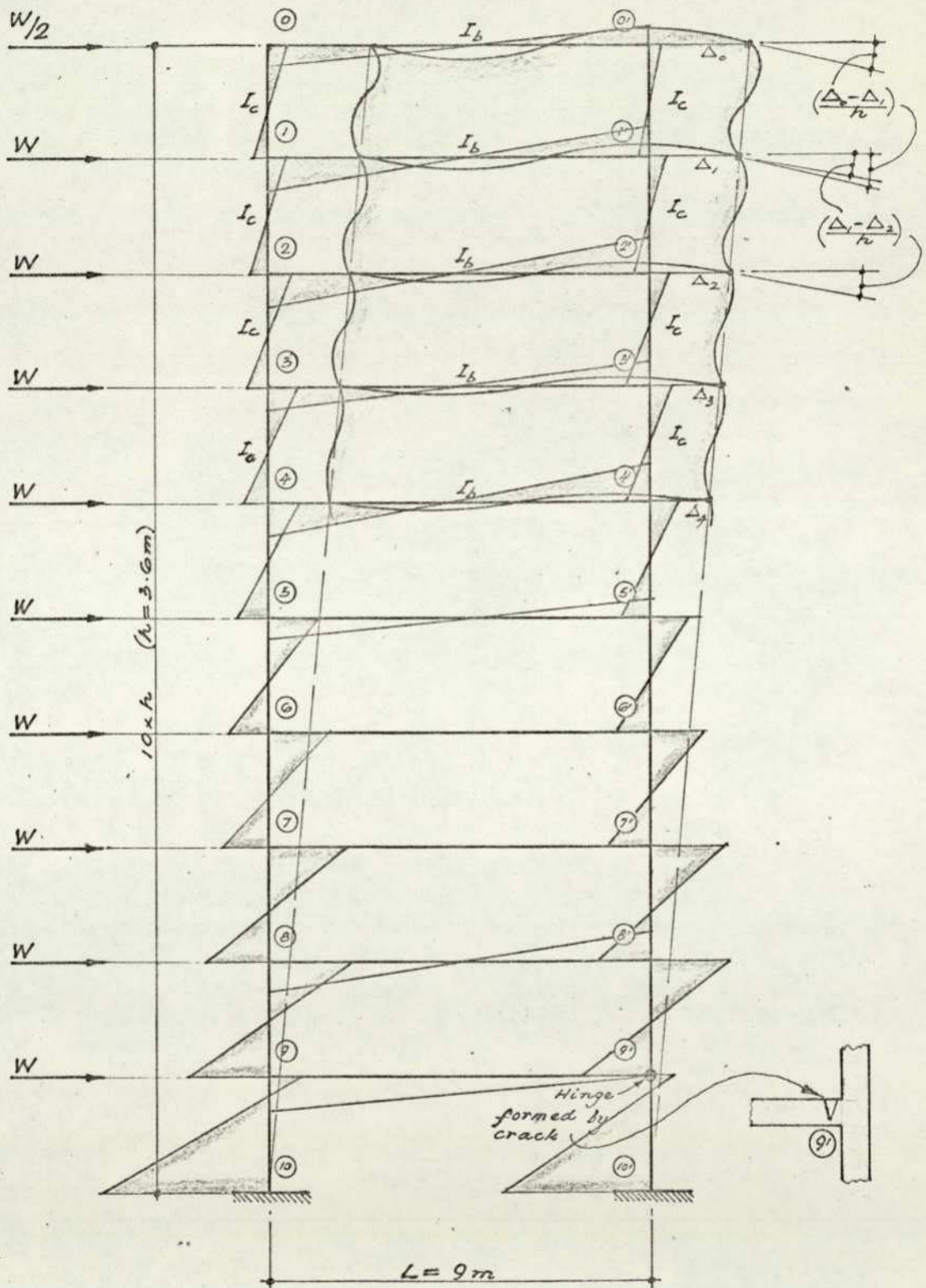


FIG. 8.07 10-STORY FRAME WITH A HINGE AT FIRST FLOOR BEAM END

$$-M_{21} + 2M_{12} + 2r(M_{10} + M_{12}) - r(M_{1'0'} + M_{1'2'}) - (\Delta_1 - \Delta_2) \frac{6EI_c}{h^2} = 0$$

$$-M_{21} + (2+2r)M_{12} + 2rM_{10} - rM_{1'0'} - rM_{1'2'} - 2(\Delta_1) + 2(\Delta_2) = 0$$

$$1'2' \quad -\frac{h}{6EI_c} M_{2'1'} + \frac{h}{3EI_c} M_{1'2'} + \frac{L}{3EI_b} M_{1'1'} - \frac{L}{6EI_b} M_{1'1'} = \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{2'1'} + 2M_{1'2'} + 2r(M_{1'0'} + M_{1'2'}) - r(M_{10} + M_{12}) - (\Delta_1 - \Delta_2) \frac{6EI_c}{h^2} = 0$$

$$-M_{2'1'} + (2+2r)M_{1'2'} + 2rM_{1'0'} - rM_{10} - rM_{12} - 2(\Delta_1) + 2(\Delta_2) = 0$$

$$012 \quad -\frac{h}{6EI_c} M_{01} + \frac{h}{3EI_c} M_{10} - \frac{h}{3EI_c} M_{12} + \frac{h}{6EI_c} M_{21} = \frac{\Delta_0 - \Delta_1}{h} - \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{01} + 2M_{10} - 2M_{12} + M_{21} - 2(\Delta_0) + 2(2\Delta_1) - 2(\Delta_2) = 0$$

$$0'1'2' \quad -\frac{h}{6EI_c} M_{0'1'} + \frac{h}{3EI_c} M_{1'0'} - \frac{h}{3EI_c} M_{1'2'} + \frac{h}{6EI_c} M_{2'1'} = \frac{\Delta_0 - \Delta_1}{h} - \frac{\Delta_1 - \Delta_2}{h}$$

$$-M_{0'1'} + 2M_{1'0'} - 2M_{1'2'} + M_{2'1'} - 2(\Delta_0) + 2(2\Delta_1) - 2(\Delta_2) = 0$$

Equilibrium condition at 1-1

$$-\left[\frac{(M_{01} + M_{10})}{h} + \frac{(M_{0'1'} + M_{1'0'})}{h}\right] + \left[\frac{(M_{12} + M_{21})}{h} + \frac{(M_{1'2'} + M_{2'1'})}{h}\right] = W$$

$$-M_{01} - M_{10} - M_{0'1'} - M_{1'0'} + M_{12} + M_{21} + M_{1'2'} + M_{2'1'} = 1.0Wh$$

$$1099' \quad -\frac{h}{6EI_c} M_{109} + \frac{h}{3EI_c} M_{910} + \frac{L}{3EI_b} (M_{98} + M_{910}) = \frac{\Delta_9}{h}$$

$$-M_{109} + 2M_{910} + 2r(M_{98} + M_{910}) = (\Delta_9) \frac{6EI_c}{h^2}$$

$$-M_{109} + (2 + 2r)M_{910} + 2rM_{98} - 2(6\Delta_9) = 0$$

$$1098 \quad -\frac{h}{6EI_c} M_{89} + \frac{h}{3EI_c} M_{98} - \frac{h}{3EI_c} M_{910} + \frac{h}{6EI_c} M_{109} = \frac{\Delta_8 - \Delta_9}{h} - \frac{\Delta_9}{h}$$

$$-M_{89} + 2M_{98} - 2M_{910} + M_{109} - 2(6\Delta_8) + 2(12\Delta_9) = 0$$

$$10'9'8' \quad -\frac{h}{6EI_c} M_{8'9'} + \frac{h}{3EI_c} M_{9'8'} + \frac{h}{3EI_c} M_{9'10'} + \frac{h}{6EI_c} M_{10'9'} = \frac{\Delta_8 - \Delta_9}{h} - \frac{\Delta_9}{h}$$

$$-M_{8'9'} + 2M_{9'8'} + 2M_{9'10'} + M_{10'9'} - 2(6\Delta_8) + 2(12\Delta_9) = 0$$

Equilibrium condition at 9-9'

$$\begin{aligned} & -\left[\frac{(M_{89} + M_{98})}{h} + \frac{(M_{8'9'} + M_{9'8'})}{h} \right] + \\ & + \left[\frac{(M_{910} + M_{109})}{h} + \frac{(-M_{9'10'} + M_{10'9'})}{h} \right] = W \end{aligned}$$

$$-M_{89} - M_{98} - M_{8'9'} - M_{9'8'} + M_{910} + M_{109} - M_{9'10'} + M_{10'9'} = 1.0 Wh$$

$$10'109 \quad -\frac{h}{6EI_c} M_{910} + \frac{h}{3EI_c} M_{109} = \frac{\Delta_9}{h}$$

$$-M_{910} + 2M_{109} - 2(6\Delta_9) = 0$$

$$10'9'9' \quad -\frac{h}{6EI_c} M_{9'10'} + \frac{h}{3EI_c} M_{10'9'} = \frac{\Delta_9}{h}$$

$$M_{9'10'} + 2M_{10'9'} - 2(6\Delta_9) = 0$$

The numeric matrix for the 50 equations is shown in Fig. 8.08. It should be noted that deflections are shown in terms of Wh^3/EI_c and bending moments in terms of Wh . The distribution of bending moments and deformations is shown in Fig. 8.09.

COMPATIBILITY OR EQUIL. CONDITION	0 0' 1 1' 2 2' 3 3' 4 4' 5 5' 6 6' 7 7' 8 8' 9 9' 10 10'																				LOAD VECTOR
	M 01	M 01	M 01	M 10	M 12	M 12	M 12	M 21	M 23	M 23	M 32	M 34	M 34	M 43	M 45	M 54	M 56	M 56	M 65	M 67	
1-0-0'	7	-6	-35	-1	0	0	0														0
0-0-1'	-25	-6	7	0	0	0	-1														0
EC. 0 - 0'	1	0	1	1	0	0	1														0.5
2-1-1'	0	0	0	5	7	-6	-25	-25	-1	0	0	0									0
1-1-2'	0	0	0	-25	-25	-6	5	7	0	0	0	-1									0
0-1-2	-1	-6	0	2	-2	12	0	0	1	0	-6	0									0
0'-1'-2'	0	-6	-1	0	0	12	2	-2	0	0	-6	1									0
EC. 1 - 1'	-1	0	-1	-1	1	0	-1	1	1	0	0	1									1.0
3-2-2'																					0
2-2'-3'																					0
1-2-3																					0
1' 2' 3'																					0
EC. 2 - 2'																					1.0
4-3-3'																					0
3-3'-4'																					0
2-3-4																					0
2'-3'-4'																					0
EC. 3 - 3'																					1.0
5-4-4'																					0
4-4'-5'																					0
3-4-5																					0
3'-4'-5'																					0
EC. 4 - 4'																					1.0
6-5-5'																					0
5-5'-6'																					0
4-5-6																					0
4'-5'-6'																					0
EC. 5 - 5'																					1.0
7-6-6'																					0
6-6'-7'																					0
5-6-7																					0
5'-6'-7'																					0
EC. 6 - 6'																					1.0
8-7-7'																					0
7-7'-8'																					0
6-7-8																					0
6'-7'-8'																					0
EC. 7 - 7'																					1.0
9-8-8'																					0
8-8'-9'																					0
7-8-9																					0
7'-8'-9'																					0
EC. 8 - 8'																					1.0
10 9 9'																					0
9-9'-10'																					0
8-9-10																					0
8'-9'-10'																					0
EC. 9 - 9'																					1.0
10 - 9																					0
10' - 9'																					0

FIG. 8.03 NUMERIC MATRIX FOR 10-STOREY FRAME WITH HINGE IN BEAM END AT FIRST FLOOR LEVEL

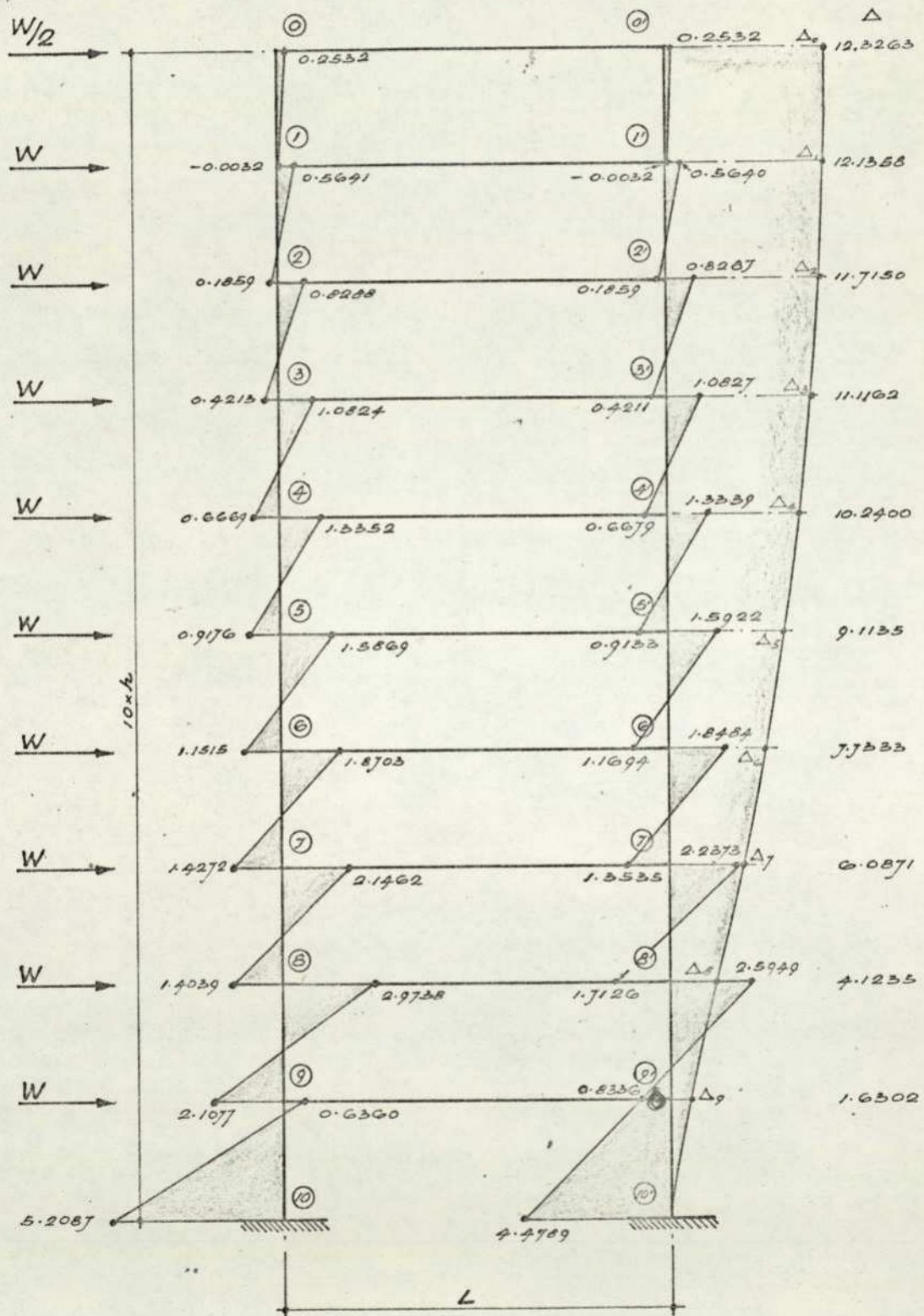


FIG. 8.09 MOMENT DEFLECTION DIAGRAM FOR 10-STORY FRAME WITH CRACK IN BEAM END AT FIRST FLOOR LEVEL

9.00 General Discussion of all Test Results

The correlation of experimental and theoretical bending moments and deflections was generally very good for the models made of steel, i.e. the range of portal frames and the 10-storey frame. The test results, discussed individually under 2.30 and 3.50, show especially close correlation with theory in the case of lateral deflections, both in form and magnitude. The data show, unmistakably, the 'P Δ ' instability effect which gives rise to greater deflections and moments than indicated by a primary analysis. This is shown markedly in the 10-storey frame where the lateral deflections due to the instability effect of P are 30% greater than predicted and observed under primary effects, Fig. 3.50 and 3.51.

The results of the tests in the microconcrete models are discussed in detail under 4.40, 4.70, 5.40 and 6.40. There was in each case reasonable correlation of experimental and theoretical deflections. Generally, however, agreement in the magnitude of theoretical and experimental bending moments was poor, and this could be attributed to the high percentage of steel used in the construction of the models, which could cause micro cracking of the concrete early in the load cycle. Another factor was the use of strain gauges of 7mm gauge length which may have been too short to give average values. In all cases, however, the form of the experimental bending moments agreed with theory.

10.00 General Conclusions

The objects of this research were to examine and to formulate in matrix form the high order effects in the analysis of tall frames suitable for office use and to check experimentally the validity of the assumptions made.

The reason for this effort lies in the fact that the actual response of tall building frames appears to differ considerably from that predicted by the primary analyses, i.e. although no actual failures were recorded so far, there is evidence of excessive deflections, vibrations and cracks which are visible indications of the response being different to that predicted.

It is generally expected that this discrepancy is caused by many factors which are not included in the primary analysis, some of which are difficult to formulate analytically. The main factors are: the effect of axial forces, change of the configuration of the elastic system, inelastic response of joints, out-of-plumb construction, temperature changes, effect of shrinkage and creep, dynamic action of loads, etc.

Some of these effects have been formulated and attempts were made to check the results by carefully altering only one parameter at a time in order to arrive at reliable conclusions. It was obvious from the start that the programme of work was far beyond the scope of one research when e.g. the problem of creep alone would absorb a team of workers full time. Similarly, the effect of shrinkage or temperature changes would warrant a separate work.

However, while the analytical formulation was the main object of the research, some limited amount of experimental work has been done. Analytical formulation was made based on the concept of force-displacement, because the formulation of the matrix involves a basic approach which at all stages has clear physical meaning. Three basic requirements had to be satisfied whatever effect was to be considered i.e. (1) the stress-strain characteristics of the material used or its force-displacements properties

(2) the equilibrium conditions of members and of the whole structure with the imposed forces or effects, i.e. a structure at rest must be in equilibrium until failure occurs, and

(3) the compatibility of deformations must generally apply at all stages with the exception of the ultimate condition. Even at the stage of cracking or inelastic response of joints the compatibility condition, in some modified form, must also hold.

The method of the force-displacements offered the required tool for taking these basic requirements into account. While the primary analysis based on this method is now well established, the high order effects required further work.

It was soon established that some effects lead to the non-linear, although elastic analysis. These were: the effect of axial forces, the loss of stiffness of compression members, the effect of the equilibrium being satisfied in the deformed configuration, the effect of out-of-plumb construction, soil-structure interaction, etc. It was also realised that some other effects could lead to inelastic or even to visco-elastic analysis. The latter were not pursued because the work would require a large team of researchers working over a long period of time.

However, already the first efforts were encouraging when a simple portal frame analysed for the effect of elastic instability showed unmistakable response, different to that predicted by the primary analysis. The precision of measurements was a problem by itself: it was not always easy to eliminate e.g. the effect of movements of the base or the testing frame, temperature changes, or to control exactly the magnitude of axial forces. In spite of these difficulties and within a small allowance of error, the response of some frames, particularly made of steel was unmistakably established, and confirmed that the mathematical formulation can be made.

It was of great concern to the Author, that the formulation of these problems can be made at all, and even of greater concern that this formulation has been at least partly and to a limited extent proved - this opens a possibility of more exact analysis and design to be made in practice.

With regard to the concrete models, it was realised that minute and precise measurements would have to be taken, preferably in humidity and temperature controlled conditions. Even with lack of fully controlled conditions the results proved a definite tendency of the high order effects to show themselves. It appears that the loss of stiffness of compression members is of greater importance than usually accepted. The reason may be because the analysis itself is more complicated being non-linear and has seldom been made so far. In this respect the proposed formulation may encourage its wider use.

In the other group of factors lies the effect of flexible joints. This is a well known problem and the Codes permit the use of this concept. However, the mathematical formulation was always an obstacle in its use. In this respect, the proposed matrix may be useful. Once the inelastic properties of the connection are known (based on laboratory investigation of typical connections) the setting out of the matrix can be made. This part of the research was purely analytical and no experimental verification was envisaged.

The effect of out-of-plumb construction was considered of particular importance. While most structures are out-of-plumb to some degree, there is a lack of information with regard to its effect on the overall performance of a structure in conjunction with gravity or wind loading, and there appears to be no information on how a matrix can be set out taking this effect into account. In practice, the problem is left to a judgement rather than an exact analysis, and arbitrary differences of opinion are expressed with regard to the "max permissible" value of an out-of-plumb construction. It was difficult to precisely answer the question. What is the effect of such constructional errors on stresses and deformations in relation to a similar structure if erected straight? The proposed matrix answers this problem in a quantitative way and offers to account for any form of the out-of-plumb construction. The experiments gave a clear and good correlation with the theory for the form of the bending moment and deflection diagrams. The precise magnitude of these quantities is again a subject for in-depth research perhaps using a steel model so as to avoid the problem of concrete cracking and changes in joint flexibilities.

Much experimental work has been done to check the validity of the mathematical formulation. Although the results not always correlated well, it is felt that there is no reason to suppose that the formulation could be wrong. It is rather the tests which need more careful control and precise measurements than the analysis needs modifications. The latter is based on "large deformations", includes the effects of elastic instability; the compatibility includes the effect of axial forces; while equilibrium includes the so called $P.\Delta$ effect. In this respect the results obtained are viewed as at least encouraging and offering the profession a practical formulation of the problem.

Some efforts were made to formulate the effect of cracks on the response of tall frames. This is a difficult problem, and the way pursued was deliberately different than that used by other Authors in the hope of finding a new way of approach.

A mathematical formulation of the matrix was made and this was applied to simple beams and frames. However, any tests would require a major effort, out of proportion to that envisaged in this work, and the problem was left at its analytical stage only. It is hoped, that some future research can be made along these lines.

Concluding, it is hoped that the main objectives of the effort have been achieved, i.e. analytical formulations of some high order effects has been proposed in a form suitable for practical use in offices.

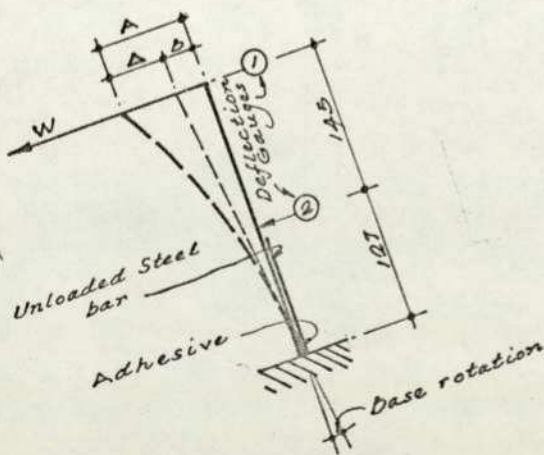
11.00 Appendix: Determination of Elastic Modulus and Strain
Calibration of Model Materials

This appendix gives details of the experimental determination of Young's Modulus of Elasticity for the steels and micro-concretes used in the test models. Also included are the calibration data relating strain to data logger output for the model materials.

11.10 E Value for Steel

Without exception all the steel models used in this work were made from ordinary mild steel bar, $\frac{1}{4}$ inch (6.35 mm) square.

To establish the E value for the particular mild steel used in the manufacture of these models, a flexural test was performed on a steel cantilever and the E value determined from the equation for flexural deflection. To take account of rotation at the base of the cantilever which can occur in the experimental apparatus, a bar of mild steel was adhered to the base of the cantilever as shown in Fig. 11.10. By measuring the deflection of the unloaded bar and assuming that its deflection is unaffected by the curvature at the base of the cantilever, the true flexural deflection can be accurately measured by deducting from the total deflection of the cantilever the deflection due to rotation of the base.



From Fig. 11.10, flexural deflection Δ = total deflection - rotational deflection = A - B. The test data are plotted in Fig. 11.11 from which the flexural deflection for $W = 120N$ is $\Delta = 3mm$. Using the well known relation:

$$\Delta = \frac{Wh^3}{3EI} \quad \text{gives}$$

$$E = \frac{W}{\Delta} \cdot \frac{h^3}{3I}$$

Fig. 11.10 Steel Cantilever

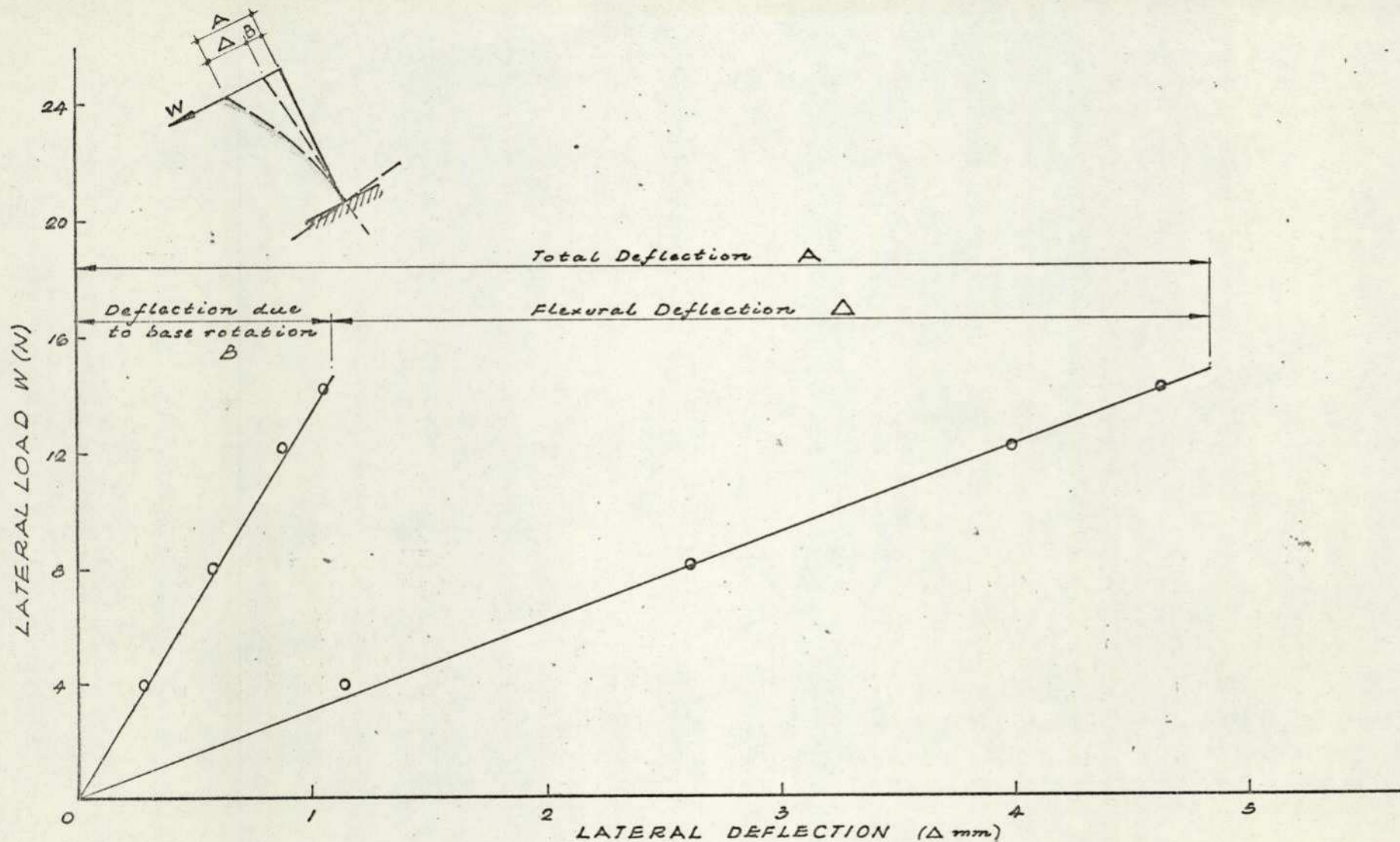


FIG. 11.11 LOAD/DEFLECTION CURVE FOR STEEL CANTILEVER FOR DETERMINING 'E' VALUE

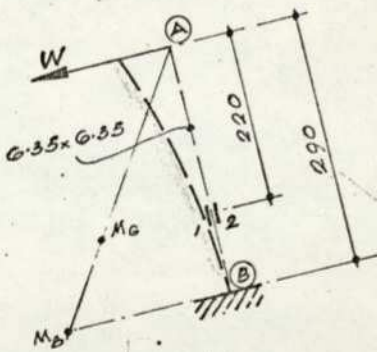
Substituting values gives:

$$E = \frac{12}{3} \times \frac{272^3}{3 \times 135.5} = 198,018 \text{ N/mm}^2$$

This value agrees well with the generally accepted figure of $E = 200 \text{ kN/mm}^2$ for ordinary mild steel. For convenience in all following computations, the above experimental value of $198,018 \text{ N/mm}^2$ has been rounded up to 200 kN/mm^2 .

Calibration factor: For tests on model cantilevers and portal frames the limited number of strain gauge readings meant that a "Boulton and Paul" instrument could be used to measure the micro voltage associated with each gauge. For multi storey models an automatic data logger was employed.

The purpose of the present test was to establish the relation between the instrument reading in divisions and the actual stress. A simple inclined steel cantilever was used equipped with two strain gauges as shown in Fig. 11.12.



The relation between lateral load W and gauge divisions is shown in Fig. 11.13.

$$\text{From } M_g = W \times 220 \text{ (N.mm)} = \frac{\sigma I}{y}$$

the value of σ can be obtained.

Substituting values gives

$$\sigma = \frac{W \times 220 \times 3.175}{135.5} = 0.155 W$$

Fig. 11.12 Steel Cantilever used for Calibration Test.

Therefore for $W = 10\text{N}$, $\sigma = 51.55 \text{ N/mm}^2$, and the number of gauge divisions corresponding to $W = 10\text{N}$ from Figure 11.13 is 27.75.

$$\text{Therefore 1 division} = \frac{51.55}{27.75} = 1.8577 \text{ N/mm}^2$$

This value will be used in all subsequent calculations but rounded down to 1 division = 1.85 N/mm^2 for convenience.

11.20 E values and calibration factors for micro-concrete

To establish Youngs Modulus of Elasticity and Calibration Factors for the particular micro-concrete used in the multi-storey models, four different

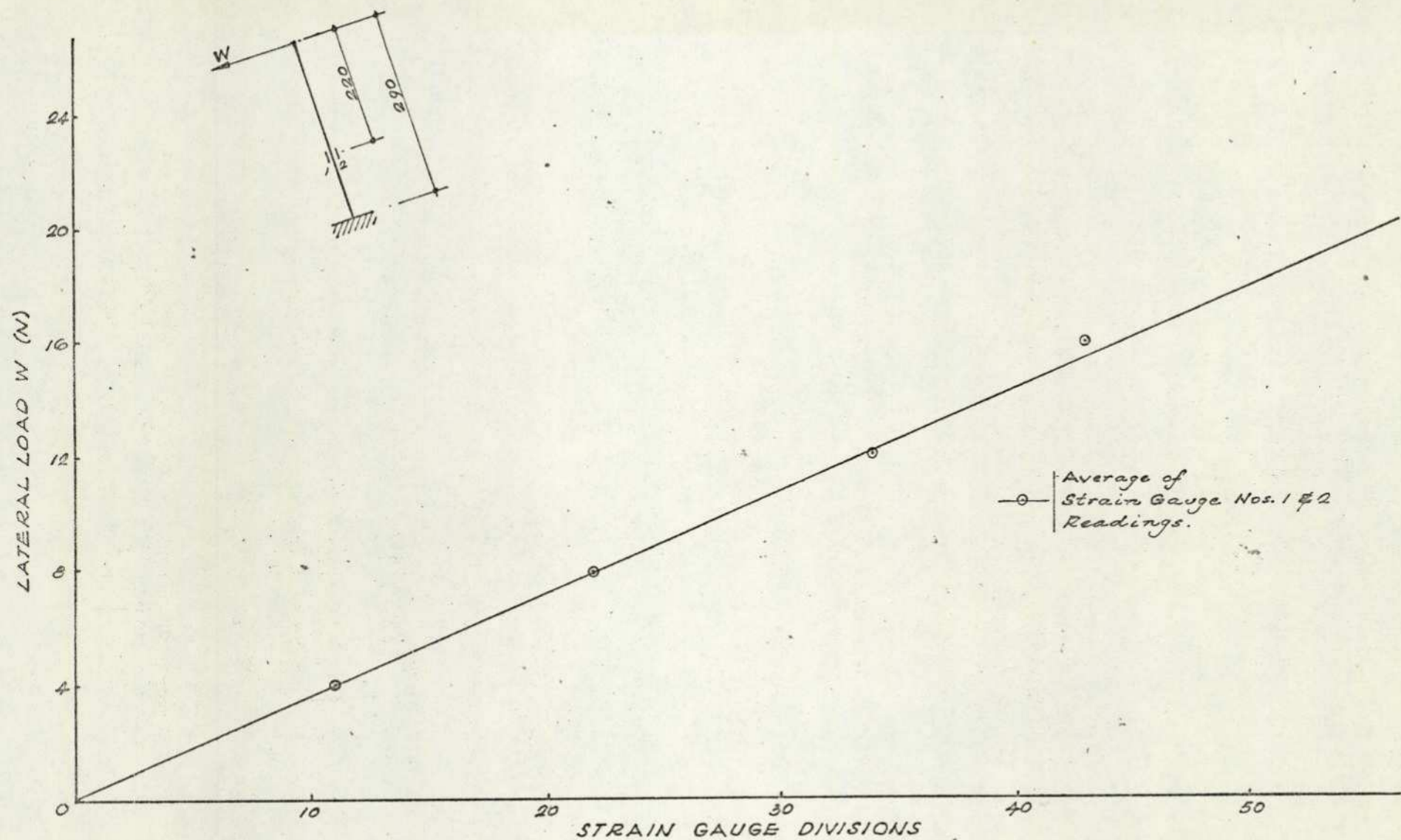


FIG. 11-13 RELATION BETWEEN STRAIN GAUGE DIVISIONS AND LOAD FOR INCLINED STEEL CANTILEVER

tests were conducted on samples of the relevant concrete mix. They were:

1. Uni-axial compressive test on a concrete cylinder.
2. Flexural test on a simply supported beam subjected to four point loading.
3. Flexural test on cantilever representative of concrete used in 'straight' frames
4. Flexural test on cantilever representative of concrete used in out-of-plumb frame.

Control specimens used for calibration tests were cast using a standard mix of ordinary portland cement and natural river washed sand.

Details of the tests and results follow.

Test 1: Uni-axial compressive test on a concrete cylinder

The test sample, 6" diameter and 8" long was mounted in a testing machine with a device for measuring axial deformation at 3 equi-spaced positions around the circumference clamped onto the sample so that deflections over a 5" gauge length could be observed. The load was increased from zero to 15,000 lbs and, as one would expect, dial gauge readings gave linearly increasing readings over this load range. At 10,000 lbs. the average of the 3 gauge readings was 0.00047 ins.

From the relation $E = \frac{\text{STRESS}}{\text{STRAIN}} = \frac{\sigma}{\epsilon} = \frac{WL}{\Delta A}$

where W = axial load = 10,000 x 4.448 N.

ℓ = gauge length = 127 mm

Δ = compressive deflection = 0.012 mm

A = section area = 18,241 mm²

Therefore $E = \frac{10,000 \times 4.448}{0.012} \times \frac{127}{18,241} = 25,807 \text{ N/mm}^2$
 $E = 28.8 \text{ kN/mm}^2$

Test 2. Flexural test on a simply supported concrete beam

The following test was conducted in order to correlate the analogue output of the data logger to the stress in the concrete. It is thus an independent check on the stress per division relation obtained in Test 3 below. The test sample is shown in Fig.11.20.

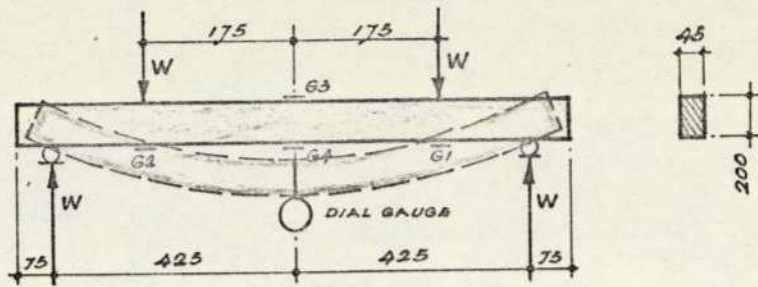
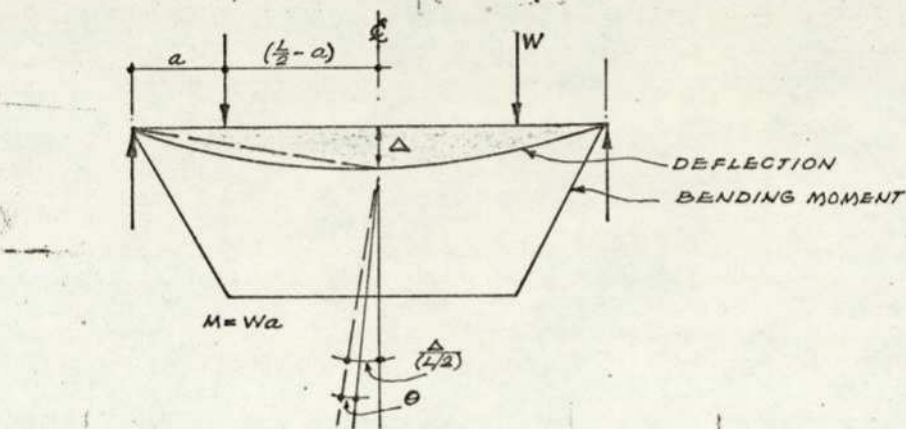


Fig. 11.20 Model Simply Supported Beam used for Test

Using the force displacement method of analysis based on the diagram

below:



$$\theta = \frac{Wa \left(\frac{L}{2} - a \right) \left(\frac{L}{2} + a \right)}{\frac{L}{2} \cdot EI} = \frac{Wa}{12EI} (L^2 - 4a^2)$$

$$M \left(\frac{L}{2} \right) = \left(\frac{\Delta}{\frac{L}{2}} - \theta \right) \quad \text{from which}$$

$$\Delta = \frac{Wa}{24EI} (3L^2 - 4a^2)$$

$$E = \frac{W a}{\Delta 24I} (3L^2 - 4a^2)$$

Substituting values of

$$a = 250 \text{ mm}$$

$$I = \frac{1}{12} \times 45 \times 200^3 = 30 \times 10^6 \text{ mm}^4$$

$$L = 850$$

gives
$$E = \frac{W}{\Delta} \cdot \frac{250}{24 \cdot 30 \cdot 10^6} (2,167,500 - 250,000)$$

$$E = \frac{W}{\Delta} \times 0.6658$$

From Fig. 11.21 the value of W/Δ is equal to 30,075 which gives an E value of 20,024 N/mm².

Again, the relation between gauge reading and stress is derived from $\sigma = My/I$.

$$\begin{aligned} \text{Maximum fibre stress } \sigma &= \frac{W.a.y}{I} \\ \sigma &= \frac{W \times 250 \times 100}{30 \times 10^6} \\ \sigma_{max} &= 0.000833 W \end{aligned}$$

From Fig. 11.22 the number of gauge divisions for gauges 1 and 4 (positioned on extreme fibres subjected to same bending stress) for an applied load $W = 10\text{kN}$ is 12.5. Therefore 12.5 divisions correspond to $0.000833 \times 10,000 \text{ N/mm}^2$, which gives 1 division = 0.6666 N/mm^2 . However, the irregularity in the curves for both these gauges, and the fact that the values are far higher than that obtained in the cantilever test, suggests that cracking may have taken place unnoticed on the tensile face.

Consider therefore gauge No. 3 which is in compression. From Fig. 11.22 number of gauge No. 3 divisions, for $W = 10 \text{ kN}$, is 54.5. Therefore 1 division = 0.1523 N/mm^2 .

Test 3 Flexural test on concrete cantilever representative of concrete used in 'straight' frames

For a simple cantilever, the deflection at the free end is given by the well known relationship:

$$\Delta = \frac{WL^3}{3EI}$$

where W = point load at free end

L = length of cantilever

$$\text{Transposing gives } E = \frac{WL^3}{\Delta 3I}$$

To establish E and also obtain the relationship between gauge reading and stress, a model cantilever of micro concrete was manufactured and strain gauges applied as shown in Fig. 11.23.

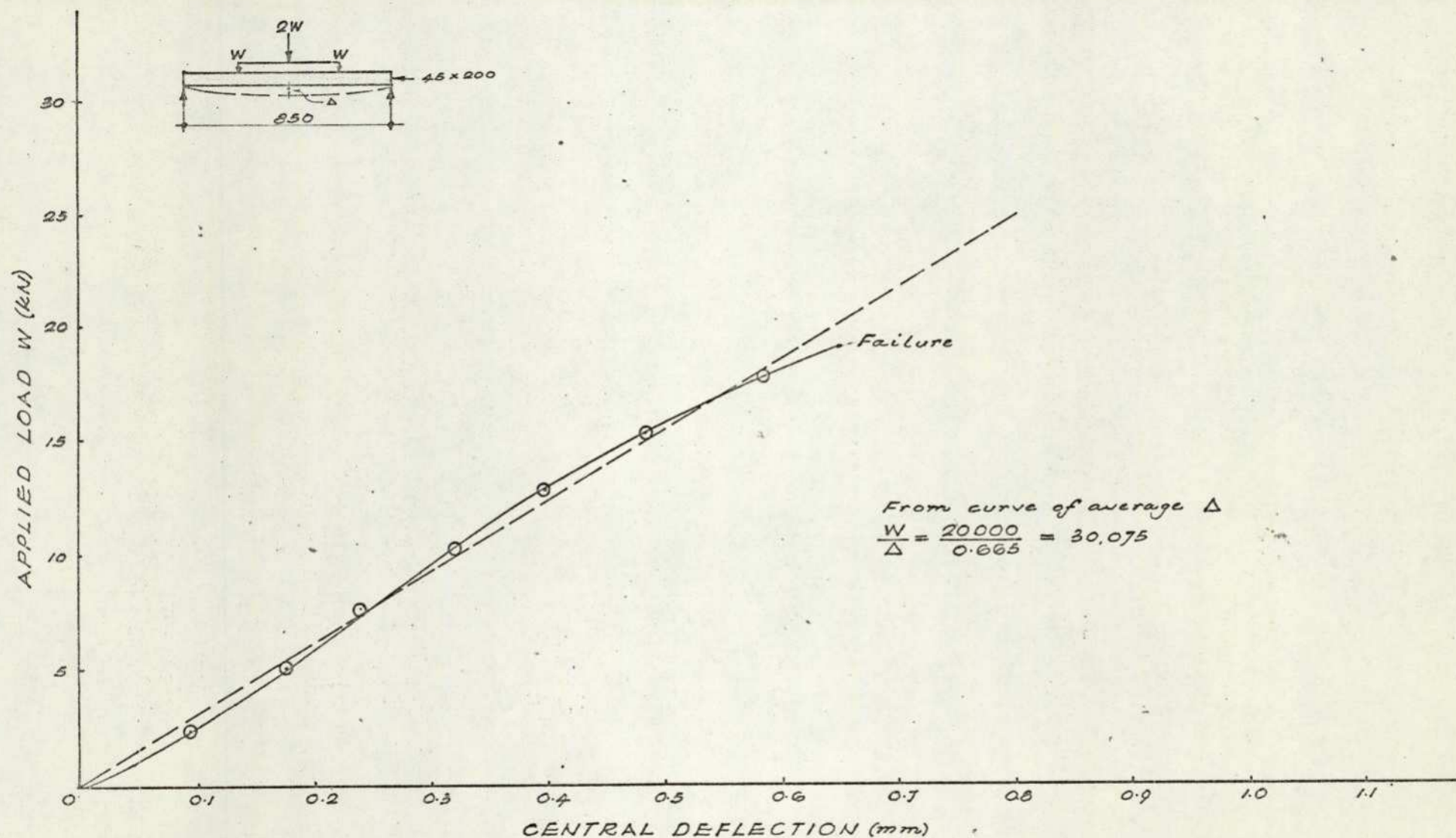


FIG. 11.21 LOAD DEFLECTION CURVE FOR SIMPLY SUPPORTED CONCRETE CALIBRATION BEAM

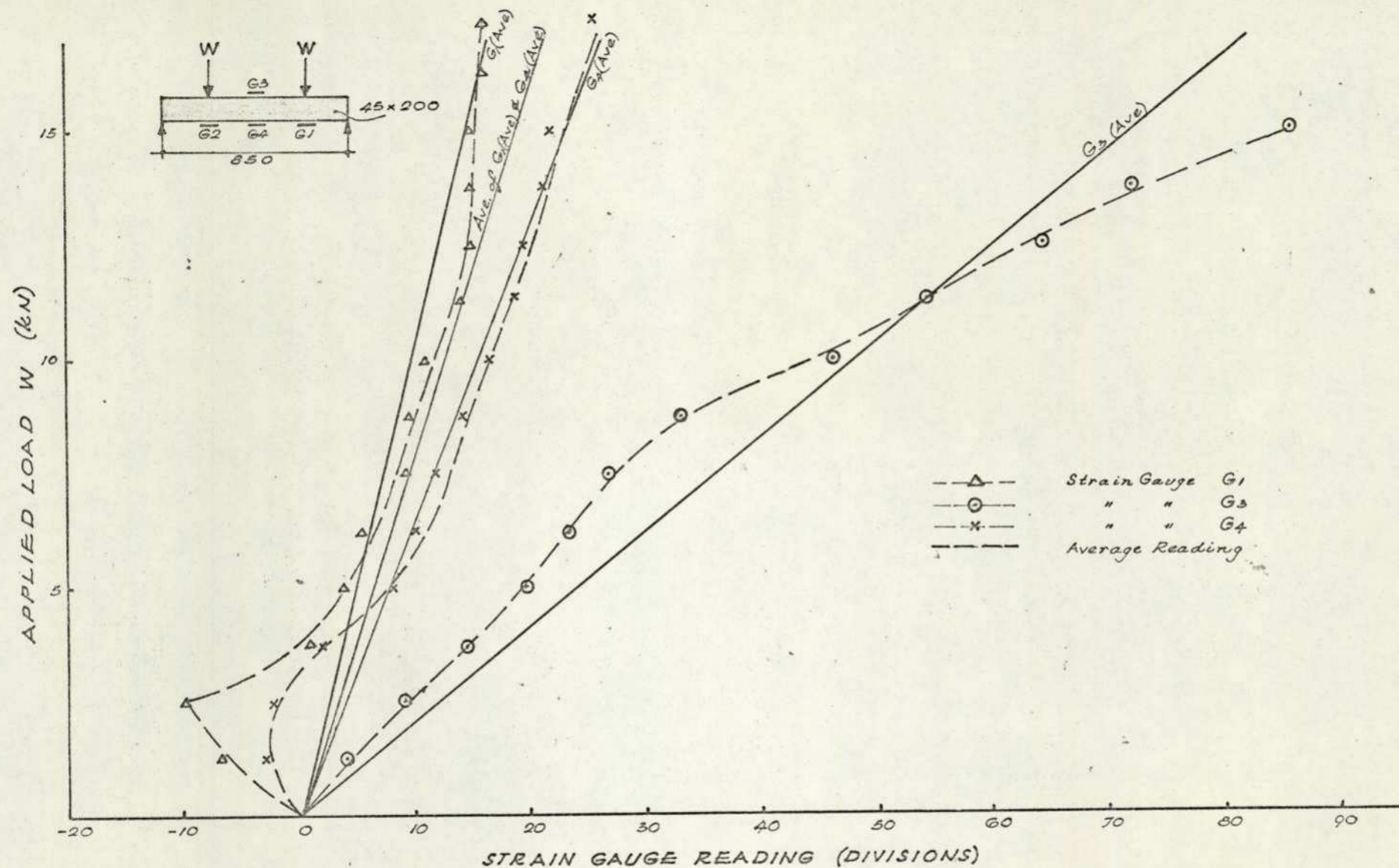


FIG. 11-22 RELATION OF STRAIN GAUGE READING TO APPLIED LOAD FOR SIMPLY SUPPORTED CONCRETE CALIBRATION BEAM

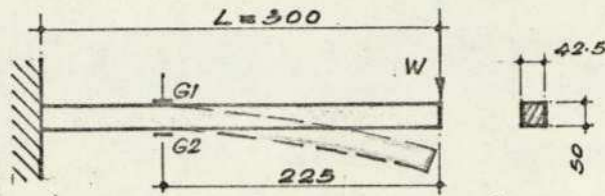


Fig. 11.23 Model Concrete Cantilever used in Test

From $I = \frac{bd^3}{12}$, $I = \frac{1}{12} \times 42.5 \times 50^3 = 443,000 \text{ mm}^4$

$$E = \frac{W}{\Delta} \times \frac{300^3}{3 \times 443 \times 10^3} = \frac{W}{\Delta} \times 20.316$$

where value of W/Δ from Fig. 11.24 = 1250

Therefore $E = 1250 \times 20.316 = 25,395 \text{ N/mm}^2$

Relationship between gauge reading and stress is derived from $\sigma = \frac{My}{I} = \frac{WLy}{I}$

Maximum $\sigma_{(W=120N)} = \frac{120 \times 300 \times 25}{443 \times 10^3} = 2.0316 \text{ N/mm}^2$;

Corresponding strain gauge reading for this load = 13.25 (Fig. 11.25).

Therefore stress/division =

$$= \frac{2.0316}{13.25} = 0.1533 \text{ N/mm}^2$$

Test 4 Flexural test on cantilever representative of concrete used in out-of-plumb model

The test model details are shown in Fig. 11.26.

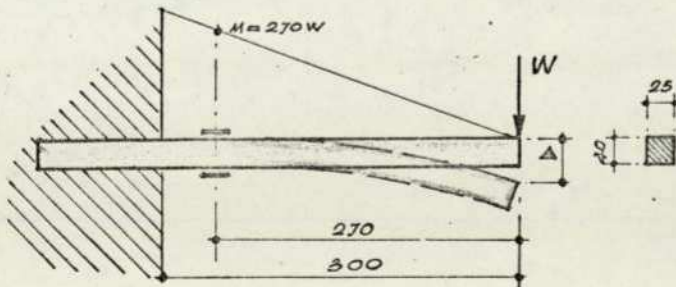


Fig. 11.26 Model Cantilever used in Test

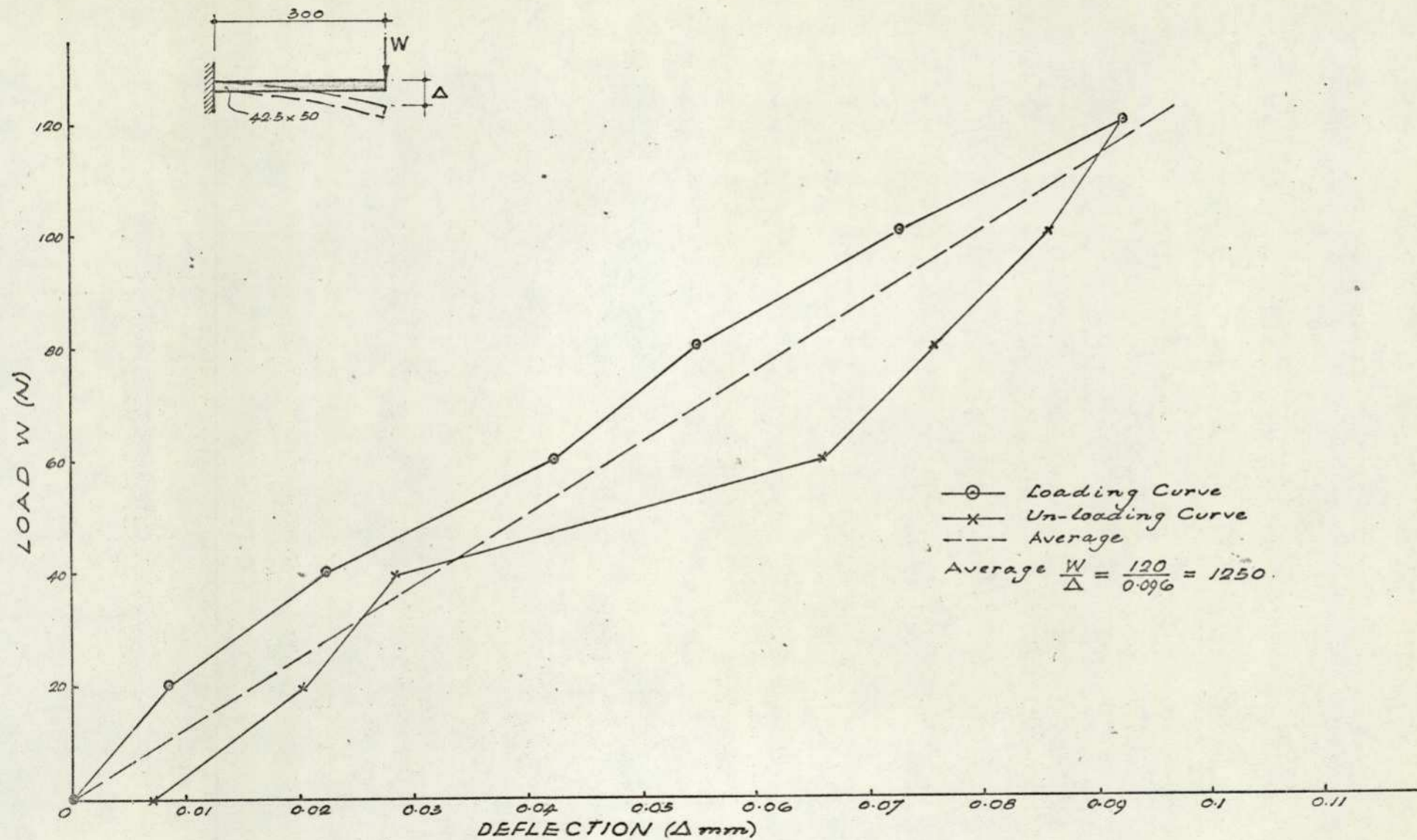


FIG. 11-24 LOAD DEFLECTION CURVE FOR CONCRETE CANTILEVER

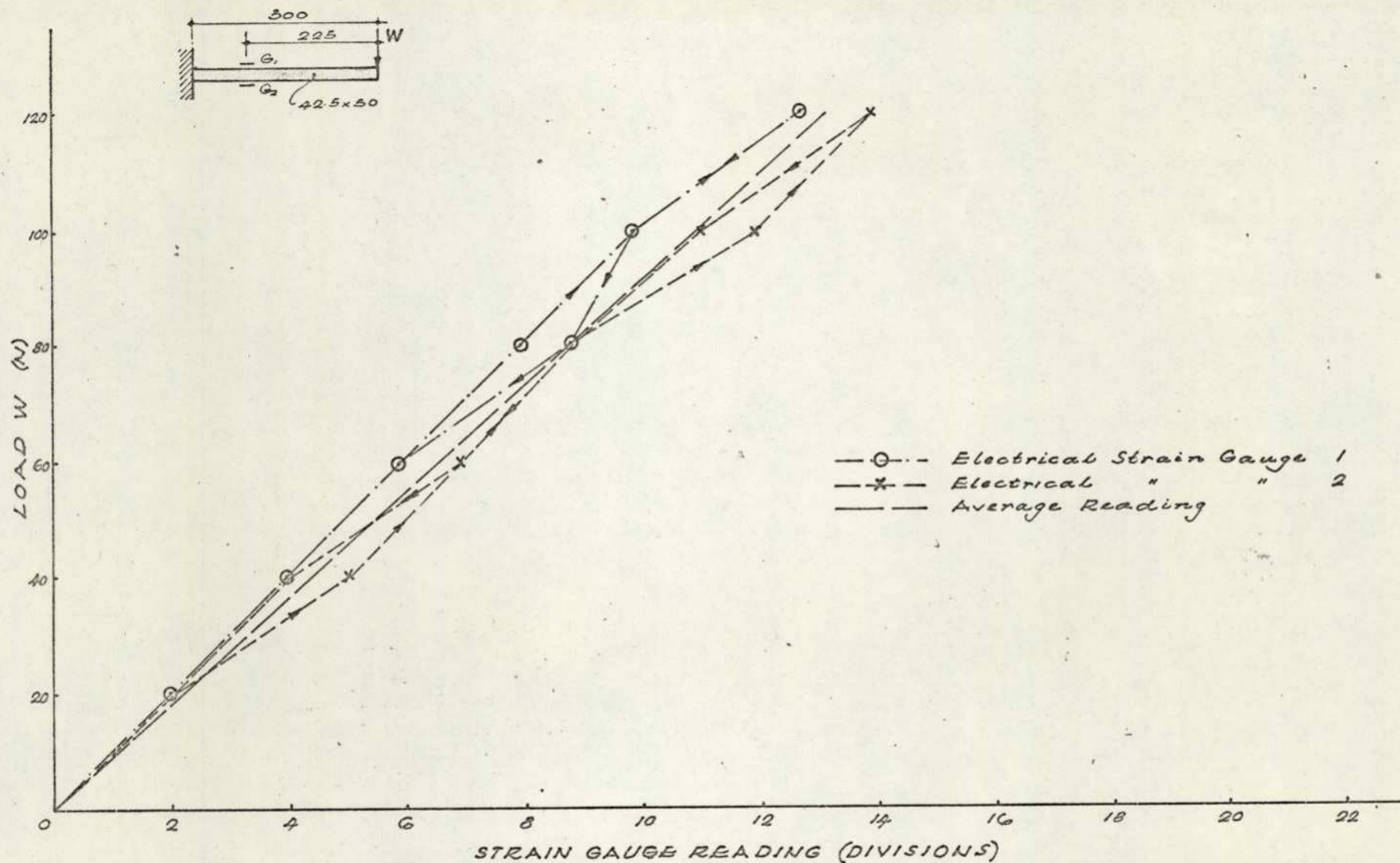


FIG. 11-25 RELATION OF STRAIN GAUGE READING TO APPLIED LOAD FOR CONCRETE CANTILEVER

$$\Delta = \frac{WL^3}{3EI} \quad \text{where } I = \frac{1}{12} \times 25 \times 40^3 = 0.133 \times 10^6 \text{ mm}^4$$

$$E = \frac{WL^3}{\Delta 3I} = \frac{W}{\Delta} \cdot \frac{300^3}{3 \times 0.133 \times 10^6} = 67.5 \frac{W}{\Delta}$$

From Fig. 11.27, average value of $\frac{W}{\Delta} = 133.72 \text{ N/mm}.$

$$\text{Therefore } E = 67.5 \times 133.72 = 9026 \text{ N/mm}^2$$

This is rounded down to 9000 N/mm^2

From the data logger print-out, load increment of $W = 10\text{N}$ resulted in a change of strain gauge reading of 4 divisions, i.e.:

$$4 \text{ divisions} = M = 270 W = 2700$$

$$\text{or } M = 675 \text{ N.mm per div.}$$

11.30 Summary of E value and Calibration Data

- (i) E value for steel = 200 kN/mm^2
- (ii) Stress calibration for steel: 1 division = 1.85 N/mm^2
- (iii) E value for concrete 'straight' frames is average of values in three tests i.e. $\frac{25.8 + 23.395 + 20.024}{3}$ is 23.073 kN/mm^2 or 25 kN/mm^2 rounded up
- (iv) E value for concrete out-of-plumb frame = 9 kN/mm^2
- (v) Stress calibration for concrete 'straight' frames is taken as average of two values $\frac{0.1533 + 0.1523}{2} = 0.1528 \text{ N/mm}^2$ for one division, or $0.153 \text{ N/mm}^2/\text{div.}$ rounded up.
- (vi) Bending moment calibration for concrete out-of-plumb frame is 675 N.mm/div.

These values are used in the analysis of test results in sections 2.30, 3.40, 4.30, 4.60, 5.30 and 6.30.

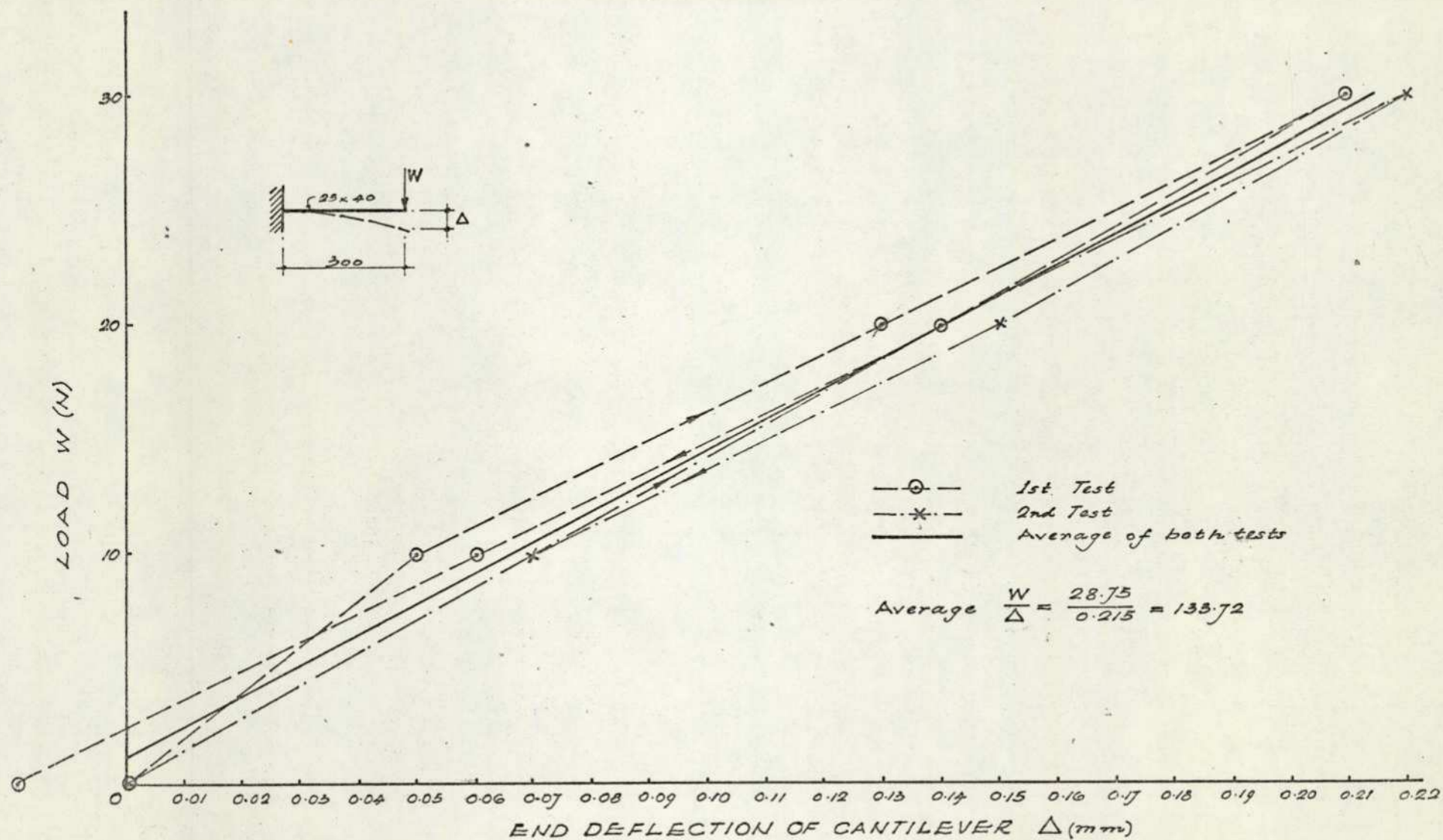


FIG. 11-27 LOAD DEFLECTION CURVE FOR CONCRETE CANTILEVER REPRESENTATIVE OF CONCRETE IN OUT-OF-PLUMB FRAME

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