



City Research Online

City St George's, University of London

Citation: Catchesides, J. E. (1975). The illusion of apparent oscillation.
(Unpublished Doctoral thesis, The City University)

This is the accepted version of the paper.

This version of the publication may differ from the final published version. To cite this item please consult the publisher's version.

Permanent repository link: <https://openaccess.city.ac.uk/id/eprint/38092/>

Copyright and Reuse: Copyright and Moral Rights remain with the author(s) and/or copyright holders. Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge, unless otherwise indicated, provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way. For full details of reuse please refer to [City Research Online policy](#).

CITY UNIVERSITY LIBRARY



3 8008 02041 1038

THE ILLUSION OF
APPARENT OSCILLATION

JANE ELIZABETH
CATCHESIDES

A THESIS SUBMITTED
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
THE CITY UNIVERSITY
LONDON

AUGUST, 1975.

ABSTRACT

The illusion of apparent oscillation produced by a rotating trapezoidal object (generally known as Ames' window illusion) was investigated. Previous explanations of this phenomenon were reviewed, and found to be inadequate to relate all the different parameters of the illusion. The trapezoidal stimulus was analysed in terms of the three main types of visual information that it provides: the speed of rotation; the surface pattern; and the continuous perspective transformation, projected onto the frontal plane. This transformation was considered in some detail, and it was found that it could be summarised by considering the elliptical loci produced by projection of the corner points of the trapezoid as it rotated. A computer program was produced to plot these loci for any particular trapezoid.

Hypotheses were proposed concerning the relative importance of these three main types of visual information. These were tested in a series of six experiments.

The results obtained in this study, and the published results of similar studies by others, were considered in detail. The data was analysed in terms of the different types of information investigated.

In general, the results supported the proposed hypotheses. Speed of rotation was found to have a significant effect on the type of motion (rotation or oscillation) seen; there was found to be a fundamental change in the nature of the illusion with speeds greater than 30 rpm. Surface pattern, however, was found to be relatively unimportant.

But the most significant finding was that consideration of the projected transformation of the shapes used enabled the type of motion seen to be accurately predicted, by following a simple set of rules. This was found not only for the experiments reported here, but also for the published results of similar studies by others.

ACKNOWLEDGMENTS

The experimental work was supervised by Professor R. Fletcher (Visual Science) and Dr. S.H. Miller (Psychology) of the City University; I am most grateful for their help and guidance.

Particular thanks are due also to [REDACTED] for their skill in constructing the apparatus, to [REDACTED] for the photography, and to [REDACTED] for her patience and excellent typing.

To all those who have offered their assistance, advice, and encouragement, my thanks.

This work was carried out in the Department of Ophthalmic Optics and Visual Science, of the City University and was supported by a research grant from the University from 1969 - 1972.

J.E.C.
August, 1975.

CONTENTS

Abstract	(i)
Acknowledgements	(ii)
1. <u>INTRODUCTION</u>	1
2. <u>VISUAL MECHANISMS</u>	5
3. <u>THE ROTATING TRAPEZOID ILLUSION</u>	13
Description of Phenomenon	13
Previous Explanations	17
Summary	23
4. <u>ANALYSIS OF THE TRAPEZOIDAL ILLUSION</u>	25
STORED INFORMATION	26
Mathematical Analysis	26
Projection of a Rotating Rectangle	27
Projection of a Rotating Trapezoid	34
Projection of an Oscillating Trapezoid	35
Equation to the locus of a corner point	36
Computer Programs	40
Discussion	43
Analysis of Differential Angular Velocity Cue	44
Other types of Stored Information	49
CURRENT INFORMATION	51
Projective Transformations	51
Surface Pattern	60
Speed of Rotation	61
Summary	62

5.	<u>EXPERIMENTAL INVESTIGATION - GENERAL DESIGN</u>	63
	General Methodology	64
	The Apparatus	66
	Motor Systems	69
	Trapezoids	76
	Film Presentation	76
	Recording Results	79
	Subjects	80
6.	<u>EXPERIMENTAL INVESTIGATION - DETAILS AND RESULTS</u>	85
	E1 EFFECT OF SHAPE AND SURFACE PATTERN	
	Introduction	86
	Experimental Design	88
	Results	93
	Multiple Comparisons	94
	Discussion	101
	E2 EFFECT OF SPEED OF ROTATION	
	Introduction	104
	Experimental Design	105
	Results	107
	Discussion	111
	E3 HEIGHT RATIO AND AXIS OF ROTATION POSITION	
	Introduction	114
	Experimental Design	115
	Results	119
	Discussion	125
	E4 PROJECTIONS OF A RECTANGLE	
	Introduction	138
	Experimental Design	138
	Results	141
	Discussion	144

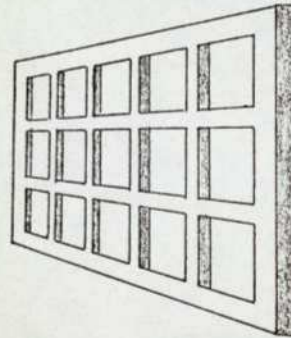
E5	WITHIN - S AND BETWEEN - S VARIABILITY	
	Introduction	149
	Experimental Design	149
	Results	151
	Discussion	154
E6	EFFECT OF FIXATION POSITION	
	Introduction	155
	Experimental Design	155
	Results	158
	Discussion	160
	SUMMARY OF EXPERIMENTAL RESULTS	161
7.	<u>EXPERIMENTAL INVESTIGATION - DISCUSSION</u>	163
	Speed of Rotation	164
	Surface Pattern	167
	Shape	
	Outline Shape (1)	178
	Outline Shape (2)	179
	'Partial' Shapes	181
	Judgment of Slant	185
	Outline Size and Viewing Distance	188
	Viewing Distance	188
	Outline Size	190
	Parallel Projections	193
	Monocular/Binocular View	199
	Subjective Variables	201
	Summary	207
8.	<u>CONCLUSIONS</u>	210
	REFERENCES	213
	APPENDIX	222

SECTION 1

INTRODUCTION

The illusion investigated in this study is generally known as Ames' rotating trapezoidal window (Ames, 1951). In its most common form, it consists of a trapezoid of sheet metal, with holes cut-out to resemble window panes and shadows painted to give the appearance of thickness (see Fig.1.1).

Fig. 1.1
Trapezoidal Window
(After Ames, 1951)



The trapezoid is mounted upon a central vertical axis and slowly rotated at a constant speed. But it is not seen to rotate; instead, it appears to oscillate with the longer vertical side always apparently in front of the shorter side.

The illusion is a particularly strong and compelling one, and produces the same effect with practically all observers. Even those who are fully aware of the true shape of the window, and of the nature of its movement, continue to perceive apparent oscillation instead of veridical rotation.

Despite the almost cursory treatment of visual illusions in many texts on the psychology of vision, and their dismissal as 'errors of judgment' or 'mistaken interpretations', research has continued almost unabated since the great upsurge of interest in "geometrisch-optische Täuschung" (Oppel, 1855) towards the end of the nineteenth century.

The fact that illusions can occur in practically every aspect of visual activity has long been recognised, and is reflected in the variety of theories that have been proposed to explain them. Most of these theories have concentrated on the geometrical-optical illusions, and many have offered explanations for only one particular figure. For example, Boring (1942) summarises twelve different theories which appeared between 1889 and 1902 to explain the well-known 'arrow' illusion (Müller-Lyer, 1889), many of which add little to a mere description of the illusion.

Some explanations have stressed physiological factors at either retinal or cortical level; others have attributed illusions to errors in information sampling, while a third group has emphasised aspects of perceptual learning and perspective, such as phenomenal depth, in accounting for illusory perception.

Useful summaries of the main categories of explanations of visual illusions are to be found in Boring (1942), Woodworth and Schlosberg (1954) and Over (1968). A much more thorough treatment is provided by Robinson (1972).

Many of these theories have tried to explain all illusions by the same mechanism. This is unlikely to be possible, since illusions have been found to occur in so many different areas of perception. But just because of this generality of occurrence, illusions can provide very valuable research tools for investigating many aspects of perception. We cannot begin to understand the visual system until we can understand its limitations, and why it breaks down in certain circumstances. But what is that visual system?

It may be considered as an information - processing, decision - making mechanism which relates incoming information provided by the current visual scene (or stimulus) to stored information derived from previous stimuli.

The general nature of the perceptual system as an information - processor is considered in Section 2, to provide a basis for the analysis and discussion of the illusion considered in this study.

Gregory (1968) has summarised succinctly the essential problem in perception: how do the momentary patterns of light reaching the retina of the eye convey knowledge of external objects ? It has been suggested that the brain can most efficiently do this by means of stored models (Gregory, 1968, 1970¹), against which current information may be assessed and future trends predicted.

Although such a 'look-up' system has many advantages over a purely 'stimulus-response' system, it also has several disadvantages. For example, if the incoming data is ambiguous, or contains very little information, an incorrect perceptual model or mechanism may be called up, and an illusory situation may result.

Attempts at classification of illusions are usually unsuccessful - mainly because many of them fall into more than one category, and many offer very subtle differences in effect. But one characteristic which many of the geometric - optical illusions have in common is that they provide the visual system with very little information, since most of them consist of just a few simple lines.

It might appear, superficially, that the less information contained in a display, the more certain should be the interpretation, in that there is little to cause confusion. But the evidence provided by these simple line figures indicates that the opposite is in fact true, for with a minimal amount of information, a variety of different hypotheses may fit the incoming data, and the wrong one may well be chosen. As the amount of information increases, the number of possible hypotheses decreases, until only one interpretation is usually possible. Our normal visual world is so rich in constantly - changing information that illusions rarely occur.

The Ames trapezoidal window illusion is different from many other visual illusions in one important respect. It provides a constantly - changing stimulus that is rich in information, and as such is more like a 'normal' visual scene.

Why, then, is the perceptual system persistently fooled by it ? Several different explanations have been proposed in the past, and these are considered in Section 3. It is found that none of these theories are satisfactory to explain the many different parameters involved in this illusion.

The various different sources of information contained in the trapezoidal window are analysed in Section 4, and are compared with possible sources of stored information (internal models, to use Gregory's term).

It is suggested that the most potent source of information is the continuous perspective transformation of the window as it rotates. This information is considered in some detail, and it is shown that the transformation can be summarised by considering the loci of the corner points of the trapezoid as it rotates. A computer program TROT is described, which will plot these loci for any (specified) trapezoidal shape as viewed from any (specified) distance. Consideration of these graphs enables the type of motion seen (rotation or oscillation) to be predicted, by following a simple set of rules.

The effect of speed of rotation and surface pattern are also considered in Section 4, and three simple hypotheses are proposed concerning the relative significance of these three main sources of information.

A series of six separate, but inter-related, experiments was designed to test these hypotheses. The general design and methodology are described in Section 5. Details of each experiment, and the results obtained are given in the following Section.

The results of this study are discussed in detail, and compared with the published data obtained by other workers, in Section 7. Various factors that could not be investigated in this study are also considered.

Predictions of the type of motion seen (from TROT) were made for all the shapes used both in this study and in all the similar studies by others. It is found that these predictions agree well with the results obtained.

General conclusions from this study of Ames rotating trapezoidal window illusion are drawn in Section 8, and the status of the proposed hypotheses is evaluated.

SECTION 2

VISUAL MECHANISMS

A brief survey of some current concepts of the way in which incoming information is processed by the visual system provides the necessary basis for this study. The advantages to be derived from analysing the perceptual system in information - processing terms has been eloquently expressed by Haber (1969, particularly pp.2-5) and Broadbent (1965), among others.

The essential problem in perception, as Gregory (1968) has neatly summarised it, is how the fleeting patterns of light reaching the retina can convey knowledge of, or provide information about, external objects. It has been suggested that this can most efficiently be done by means of stored 'schemata' (Bartlett, 1932), 'templates' (Welford, 1970) or 'internal models' (Craik, 1943; Gregory, 1968, 1970), against which current information may be assessed and future trends predicted.

This is an attractive idea, and one that has been developed in some detail (particularly by Gregory - see, for example: 1970, 1971, 1974). Such a 'look-up' system has several advantages over a purely 'stimulus-response' system:

- (1) it is essentially predictive, and can have zero reaction-time delay in suitable circumstances.
- (2) it can achieve high performance with a limited rate of information transmission
- (3) it can continue to function through data gaps
- (4) it can infer to non-sensed features of object situations
- (5) it can 'respond' to absence of features or objects
- (6) it can continue to function when the input changes in kind
- (7) it can extract signals from 'noise'; if the internal models are highly redundant, they can be called up with minimal sensory information.

- (8) Provided a particular situation is similar to the situation for which a 'model' was developed, behaviour will generally be appropriate.

But a 'look-up' system also has some disadvantages.

When the current situation is sufficiently similar to past situations which have been selected and combined to give an internal model, but the current situation differs in crucial respects, then the system will be systematically misled by its model; an illusory situation may then result.

These perceptual models, derived from past experience, must presumably integrate information from all the senses about typical characteristics of classes of objects. For example, stored data about objects categorised as "tables" must relate information from the visual sense (tables are typically brown, with wood-grain texture, and rectangular) with information from the touch sense (relating also to rectangularity, and to hardness) and from the other senses (tables usually do not squeak, or make any other sounds - nor do they smell).

No suggestions have yet been made as to how the perceptual system is able to integrate these various types of information, and store and re-call them for future use.

We do not know what sort of object characteristics are utilised to re-call a stored model, nor have we any idea of the complexity of such models. For instance, is there one model that stores 'rectangular' information, another for 'brown, wood-grain' information, and a third that integrates these two to give 'table' - which must be 'hard' ?

But the concept of perceptual models, although in themselves complex, tends in some ways to over simplify the situation. This can perhaps best be illustrated by considering the perception of a simple, static, visual scene - an apple lying on a table.

The incoming sensory information is provided by the retinal image - a distorted, aberrated pattern of coloured light on the retina, which provides a mosaic of more or less stimulated points.

But far more data is provided by that retinal mosaic than can be handled

by the visual system and can be perceived, so that key features must be selected, integrated and organised into the percept of two objects - an apple and a table.

Some of the ways in which such features may be selected have been indicated by recent work that has demonstrated the existence of physiological coding mechanisms at both retinal and cortical levels (eg. Hubel, 1963; Weisstein, 1969; Kuffler, 1973).

The selection of information must also be a function of attention, for typically more attention is paid to some data, whilst the rest are relegated to a 'background' which is largely ignored, and commonly cannot be reported in much detail (Welford, 1970; Broadbent, 1958). The table in our example, then, may be recognised as 'a table', but details of its colour, surface-texture, etc may not be noticed.

But the apple, the main object of attention, is not interpreted only as a cold, impersonal green sphere, suspended in time. Rather it is seen "in the light of past experience" of apples, and in relation to the present state of being. It is seen as something which has texture, taste, smell, which we may or may not like to eat; and that is dependent upon whether we like apples, whether this one looks inviting, and whether we are hungry. These non-optical aspects will be interpreted differently by different observers, depending upon their particular past experience and present state; but it is these non-optical aspects of visual perception which are in many ways the most significant.

The incoming sensory data - the momentary, two-dimensional aberrated patterns of light imaged on the retina - is in itself trivial. It only achieves significance when it can be interpreted in terms of real, three-dimensional objects. For it is objects that are biologically important - we spend our lives identifying them, classifying them, using them, reacting to them.

So visual perception must, then, involve even more than interpretation, for having interpreted the stimulus, some response must be made (even if it is the negative response of 'doing nothing'). For that response to be appropriate, the perceptual system must be able to predict events in the immediate future, not only anticipating the simultaneous change in sensory input, but also allowing for the finite time taken to process and react to the original sensory information. An appropriate strategy

for action may then be determined.

These essential stages in the process known as visual perception may be represented diagrammatically by a flow-chart such as Fig.2.1. From such a simplified model it is possible to summarise visual perception as: the prediction of events in the immediate future, based on the interpretation of retinal images from the immediate past, enabling the strategy of our actions to be determined.

This model is deliberately naïve in its over-simplification; but it can be expanded, complicated and made more precise by analysing each stage of the process in turn. The detailed models produced by other workers on specific parts of the system can be incorporated; for example, the model derived by Sperling (1967) on immediate memory, or the information-flow diagram of Broadbent (1958) derived from studies of hearing, could be included.

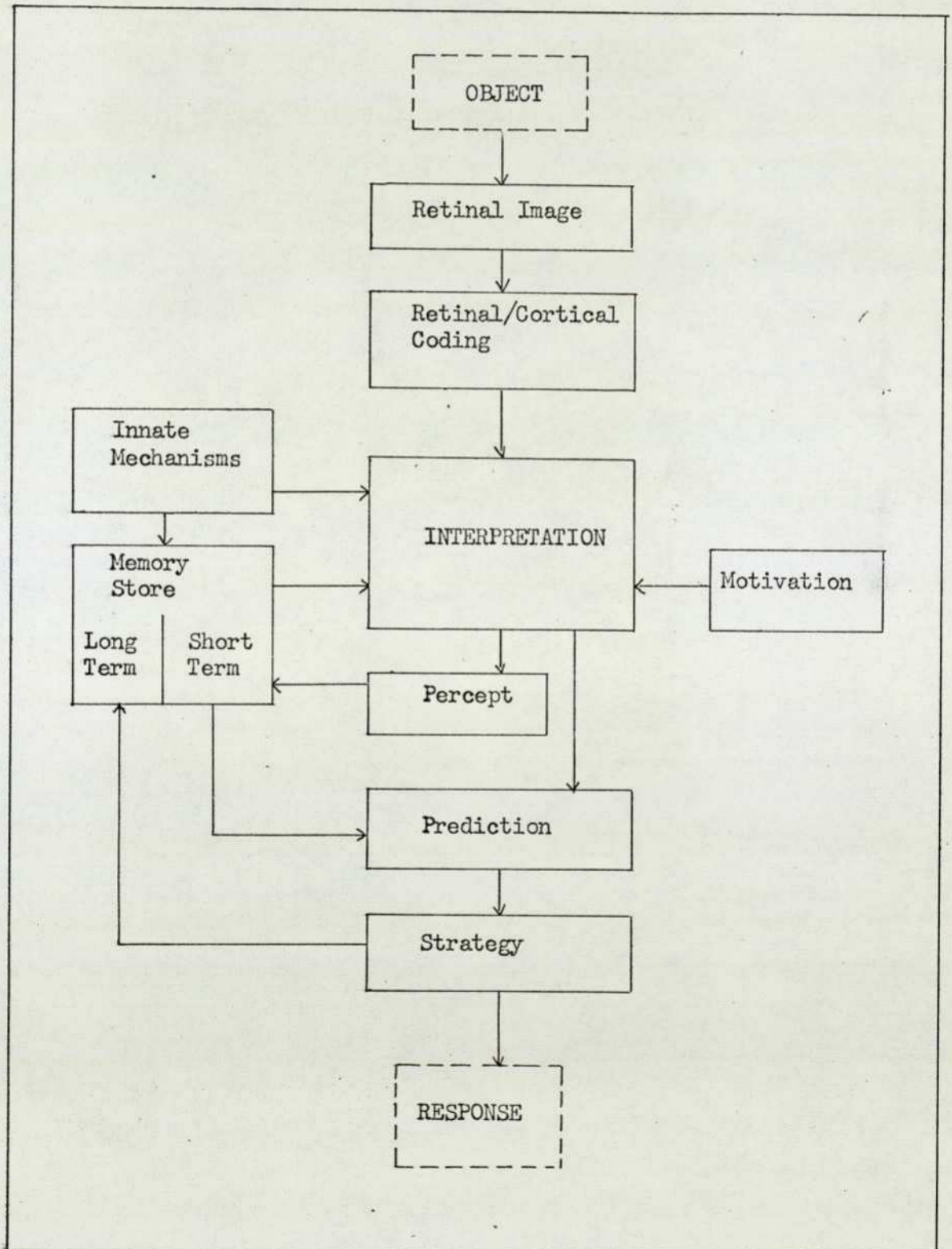
So far we have been concerned only with the interpretation of a static scene. But most visual scenes are not static, but continuously changing. Indeed if the scene becomes static (by stabilising the retinal image) the percept quickly fades (Pritchard, 1958).

The changing relationship between successive 'static' scenes (which may be considered as the intervals between saccads (Sperling, 1963)) provides further vital information for the interpretation of incoming sensory data.

Since the Ames trapezoid illusion considered in this study is a moving one, we will look at movement perception in more detail.

It has been suggested by Gibson (1957) and by Gibson & Gibson (1957) that the relationship between successive visual scenes provides one of the most fundamental sources of visual information available to us. As an object moves relative to an observer (ie. either object stationary and observer moving or vice versa) the retinal image of that object is transformed in a particular precise way for a certain kind of relative movement. The type of transformation is determined solely by the geometry of the scene, and is regular, systematic and repeatable for any particular type of relative movement.

Fig. 2.1



Gibson summarises succinctly the geometry of projective transformations, and the distinction between polar and parallel projections:

"Continuous perspective transformations: In geometry, any form or pattern on one plane which is a projection of a form or pattern lying on some other plane is called a perspective transformation. These forms are static. When the point of projection (the focus of the sheaf of lines which connect the pair of forms, point for point) is near the two planes, we speak of a central or polar projection; when this point of projection is at a very great distance from the planes, we speak of a parallel projection..... When the two planes are parallel the difference between the projected form and the given form is one of size only; it is called a similarity transformation. When the two planes are not parallel, the difference between the forms is that to which common meaning applies the term "perspective," or sometimes the term "foreshortening." In geometry, both the difference of size and that of shape are classed as perspective transformations.

We can now speak of motion. When the angle or the distance of the first plane relative to the second plane is altered, the projected form is correspondingly altered. The fact is that the relation between any earlier and any later projected form is also a perspective transformation. The relation holds between any two of its stages in times. Hence, the motion in question may be described as a continuous series of perspective transformation. It is a relation between a temporal series or family of static forms, not merely between two forms. Any such moving transformation can be analyzed by six parameters corresponding to the six components of the possible movements of the first plane - that is, three of translation and three of rotation."

(from Gibson, 1957, p289,290)

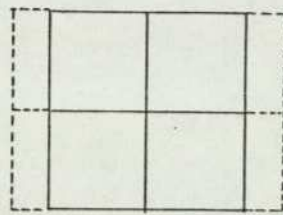
Hay (1966) has extended Gibson's analysis, and has defined eight parameters (rather than Gibson's six) which would define any rigid motion in depth. The eight parameters can be divided into four types, each applying to both horizontal and vertical dimensions of the image. These are image translation, stretching, shearing and foreshortening; they are illustrated in Fig.2.2.

Hay used matrix algebra to determine these transformations in general terms, for any moving object of constant size and shape.

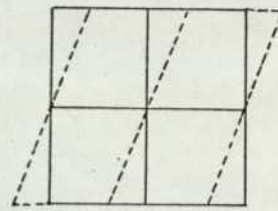
Fig. 2.2

(After Hay, 1966)

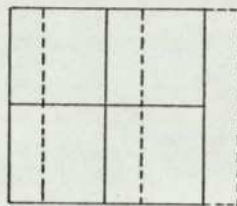
Horizontal Parameters of Projective Transformation



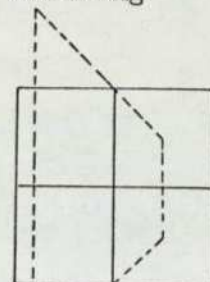
Stretching



Shearing



Translation



Foreshortening

The concept of continuous perspective transformations as a source of information for movement perception has obtained fairly wide acceptance (see, for example, Haber 1973).

It must be closely related to the phenomena of shape and size constancy, which have long been recognised.

In fact, projective transformations may be considered as an extension of them, for if these constancies provide us with stable percepts of static objects in space, then projective transformations provide us with stable percepts of moving objects in space - ie. they provide information both about the object, and about its movement.

In studying brain function we can analyse a particular scene in many different ways, and so find different types of information that may be utilised by the visual system.

Any visual scene contains a wide variety of sources of information; the perceptual system may use any or all of this information in viewing the scene; we can only postulate which types of information might be used. It is unlikely that "bits" of information (in the information - theory sense) are utilised. But internal models may be of many kinds : they may store data about horizontal, vertical or angular transformations,

or about colour, size or shape changes - or they may integrate these different sources into a continuous perspective transformation that in turn provides information about relative motion in depth.

In order to study brain function, then, we can consider these different types of information in turn, vary them systematically, and note the effect on the resulting percept. We might then be able to suggest which types of information are actually utilised by the perceptual system.

SECTION 3

THE ROTATING TRAPEZOID ILLUSION

The illusion investigated in this study is named after its originator, Adelbert Ames Jr., who devised it about 30 years ago as one of a series of demonstrations designed to illustrate the transactionalist approach to visual perception.

The illusion of apparent oscillation produced by a rotating trapezoid is a particularly compelling one, and produces consistent effects with a wide variety of observers. It is unlike many other visual illusions in that it provides a complex, constantly - changing stimulus which is rich in visual information - and yet the perceptual system is persistently fooled by it.

This study was designed to investigate the reasons for this.

DESCRIPTION OF PHENOMENON:

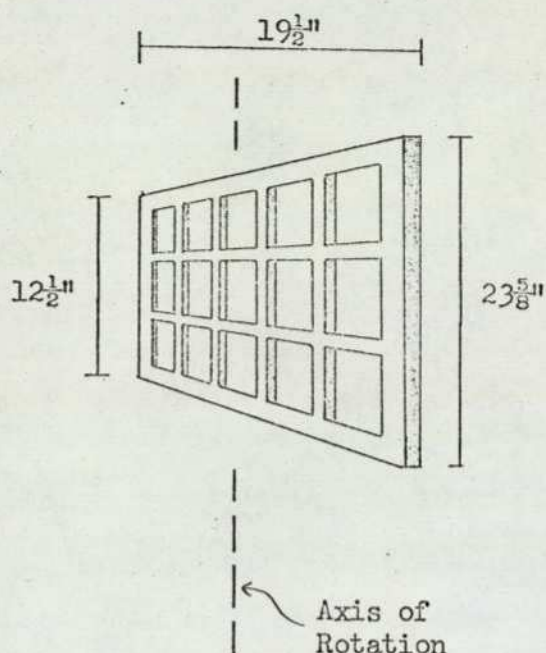
The construction and appearance of the illusion have been adequately reported by Ittelson (1952), but it is necessary here to describe briefly the appearance of the 'window' as it rotates, for the benefit of those unfamiliar with the phenomenon.

A trapezoid of sheet metal, with holes cut out to resemble window - panes, and shadows painted to give the appearance of thickness (see Fig.3.1) is mounted on a vertical shaft and driven by an electric motor so that it slowly rotates at a constant speed. As the trapezoid rotates, it appears to oscillate and change in size with the longer vertical side always apparently in front of the shorter side. When various objects are attached to the window they may appear to become detached or may be strangely contorted. For example, when a tube is suspended through one of the openings, part of the time the tube and the apparently rectangular window are perceived as moving in opposite directions. When they 'meet', several different effects may be seen.

Fig. 3.1
AMES' TRAPEZOIDAL WINDOW

(This is a view of the
trapezoid perpendicular
to the line of sight)

(After Ames, 1951)



The most commonly reported are that the tube either "bends around" or else "cuts through" the apparent rectangle. When a cube is attached to the shorter end of the trapezoid, it is seen as travelling in a circular path about the apparently oscillating rectangle, so that during part of its movement the cube "breaks away" and seems to float freely in space.

Ames (1951) constructed various shapes and sizes of window, with differing numbers of 'window panes', and concluded that the "most effective in producing alteration of the observer's perception" was the shape illustrated in Fig.3.1, with painted shadows indicating thickness, although no quantitative evidence was offered to support his conclusion (Ames, 1951, p28).

The trapezoid described by Ittelson (1952, and 1968), however, was somewhat different from this (Fig.3.2), and since this was available in kit form many subsequent investigators of the phenomenon have used this latter trapezoid as the basis for their experiments.

For the data obtained in this investigation to be comparable with the

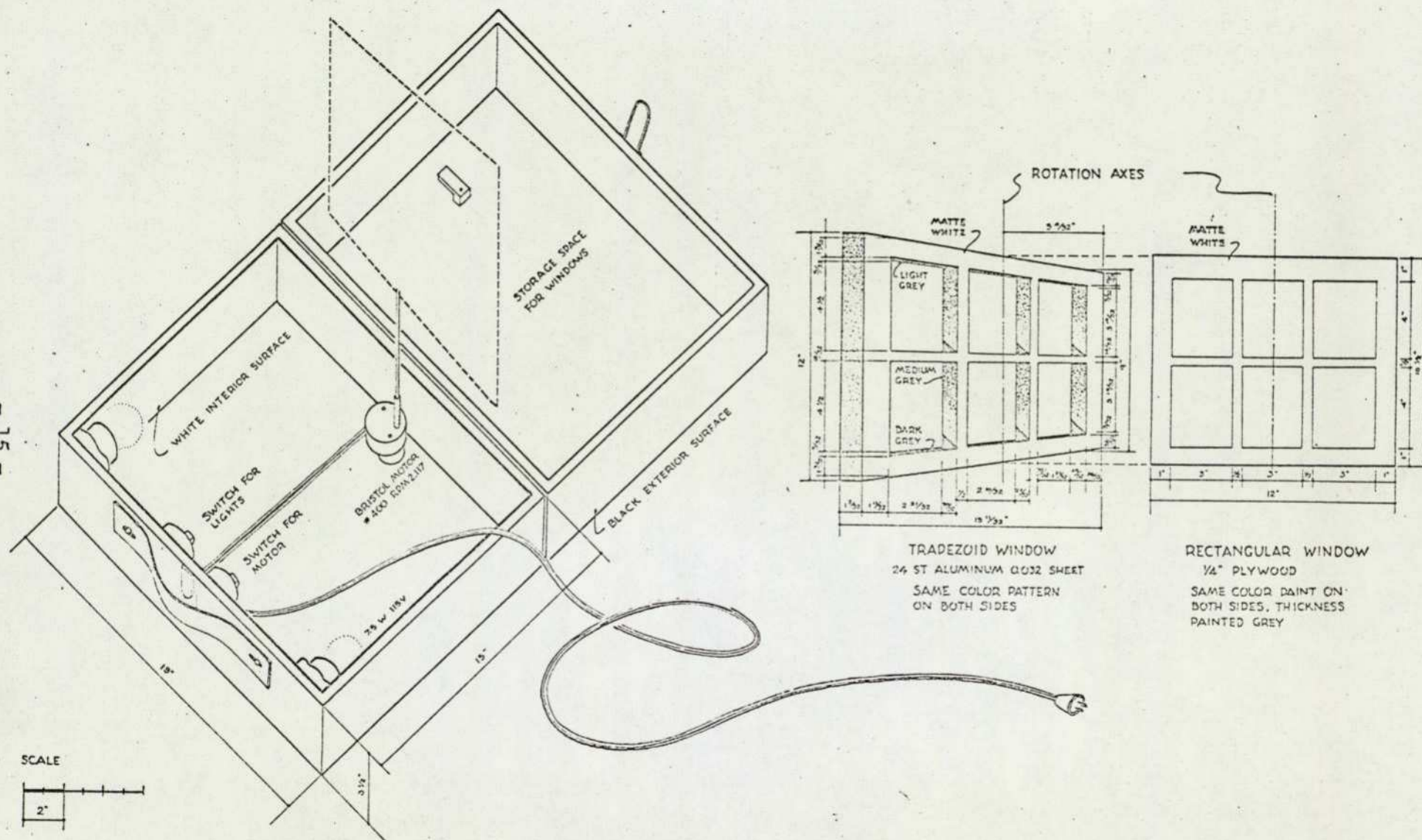


Fig.3.2

Apparatus drawing for the rotating trapezoid demonstration.

(N. H. Loom, 1952)

data obtained by others, it would seem reasonable that the stimulus shapes should be comparable. The basic stimulus shape used was therefore chosen to be the same as that shown in Fig.3.2 (but see p 88 for further details).

CHOICE OF DIMENSIONS:

The dimensions of Ames' original trapezoid and the viewing distances used were given in inches and feet. Again, since most investigators have used these dimensions, and since they are represented conveniently by whole numbers, this will be continued here for ease of comparison. The metric equivalents are given when this is helpful.

The rest of this Section will review briefly the various explanations and theories that have been proposed to explain the Ames rotating trapezoid illusion, and will then summarise the main factors thought to contribute to the illusion, that have been investigated by others.

PREVIOUS EXPLANATIONS

The first detailed description of the trapezoidal window illusion was by Ames (1951), although it is a typical example of a much larger class of illusions in which rotary motion is misperceived as oscillation.

Possibly the first example of such an illusion was reported in 1759 by Porterfield (Boring, 1942) as the windmill illusion, later reiterated by Miles (1929). A similar effect with the rotating blades of an electric fan was described by Kenyon (1898), and more recent work has investigated the effect with circles, ellipses, rectangles and irregular shapes (e.g. Pastore, 1952; Mulholland, 1963¹; Cross & Cross, 1969). The explanations reviewed here will concentrate on the rotating trapezoid illusion, since the most detailed work has been concerned with this phenomenon.

Ames (1951) attempted to account for the appearance of the rotating trapezoidal window by comparing its image on a ground glass screen representing the retina, with the image of a rotating rectangle. He concluded that:

".... there is no possibility of accounting for these appearances in causal terms of interactional effects between the objective phenomena and subjective appearances..... the most reasonable answer lay in bringing into consideration the past experience of the observer...."

(Ames, 1951, p.13, 14)

The explanation put forward by Ames was based on the fact that the retinal image of a rectangular object (such as a door, window, etc) is almost always trapezoidal. But through experience one learns to interpret these trapezoidal images as representing rectangles inclined to the line of sight. Any such trapezoidal form will therefore be interpreted as rectangular, with its apparently longer edge always apparently closer to the observer, so that the rotating trapezoid

appears to be an oscillating rectangle.

Ames' explanation is based, not on geometrical considerations, but on a detailed description comparing the actual and apparent positions of both a rotating rectangle and a rotating trapezoid.

Pastore (1952) demonstrated, and Canestrari (1956) and Mulholland (1956, 1958) confirmed, that apparent oscillation could be produced with a number of small cut out figures such as a circle, ellipse, triangle and irregular shapes, besides a trapezoid. The fact that apparent oscillation could be obtained with these other shapes formed the basis of Pastore's criticism of Ames' explanation; his misunderstanding of Ames' explanation has been critically analysed by Kilpatrick (1953).

But Pastore did propose an alternative explanation, which he did not develop in detail, but which is very similar to the ideas of several later writers. He suggested that the illusion was due to two factors: the (implied) ambiguity of the retinal image ("the movement of points across the retina"), and the misperception of tilt such that oscillation occurred when perceived tilt differed from the actual tilt. These concepts will be considered in more detail shortly.

The apparent oscillation of a rotating trapezoidal window has been analysed by Graham (1963) in terms of (1) the differential angular velocities existing between selected points on the surface of the window, and (2) linear perspective. He attempted to show that the differential angular velocity cue was ambiguous - that is, it was independent of whether the window was moving towards or away from the observer. This source of information could not then, he suggested, be used to determine the actual direction of motion so that the observer was forced to rely solely on linear perspective, choosing to see the shorter size as always further away.

But Graham's analysis of the differential angular velocity cue as ambiguous depends upon several assumptions, which seriously limit its applicability. In a footnote (1963, p.1022) he quotes a formula relating the angular velocity to the position of the window, the viewing distance and the points on the window being observed. This formula will be derived later (p.44), and will be critically analysed then. Suffice to say here that the analysis (and the formula) holds only for an infinite viewing distance (ie. for parallel projection) and not for

polar projections.

Graham emphasises that his explanation, like that of Pastore, states that the oscillation depends upon stimulus conditions, and is not to be accounted for in terms of past experience with trapezoids. He states that the observer

".....resolves the ambiguity of movement parallax cues by depending on perspective cues. The short end of the window is always seen as farther from the subject than the long end; and the ambiguous parallax cues are interpreted as in harmony with perspective cues."

(Graham, 1963 p.1022)

But he offers no reason at all why the shorter side of the window should always be seen as further away. Such a perspective interpretation is surely dependent upon past experience, which is a very similar concept to that of Ames.

Criticism of Graham's analysis of the differential angular velocity cue has been made by Macrae and Power (1969). They state, quite correctly that the cue is not ambiguous unless the object is at infinity.

But their own analysis goes to the other extreme and considers the situation where the viewing distance is relatively short compared to the size of the window; the illusion of apparent oscillation is not usually found in this situation. Their analysis is, however, correct for short distances; this will be shown later (p 37).

A "general theory" of apparent reversal of rotary motion in depth was advanced by Day and Power (1965), based upon the premise that there is an identity of horizontal retinal motion projections for points on a rotating object for both clockwise and anticlockwise rotation; if this is so, then apparent reversals will occur spontaneously, provided cues to true orientation or movement are absent, in the same way that spontaneous fluctuations of apparent orientation may be observed with two dimensional figures such as the Necker cube and Schroeder staircase.

Day and Power emphasise that these spontaneous reversals are due to the ambiguity of 'retinal motion', and depend entirely on chance. They suggest that if cues to motion direction were available, then apparent reversals would either be reduced in frequency or eliminated, for example, one such cue is that provided by retinal disparity. Apparent reversals are not reported with binocular viewing unless the viewing distance is greater than that within which retinal disparity is normally operative.

Their basic premise, which is very similar to that of Graham (1963), and leans heavily on his analysis, is subject to the same criticism as Graham's. It may be valid for parallel projections (ie. an infinite viewing distance), but it is not valid for polar projections (unless the relevant motion perspective cues are beneath their psychophysical thresholds). The shapes and viewing distance used in their published experiments (Day and Power 1963, 1965) certainly did not provide parallel projections; this will be discussed further later.

Power (1965) reiterated the same principle - he summarised his theoretical position as follows:

"....the apparent reversal of rotating shapes is a function of the equivalence of the retinal projections for the rotation and oscillation of the object. That is, the horizontal retinal projections for the two directions of motion (clockwise and counter-clockwise) are identical and thus perceptually ambiguous. In the absence of information for depth, therefore, perception of motion direction is largely a matter of chance; an observer may perceive the object as rotating sometimes clockwise and sometimes counter-clockwise. The retinal equivalence of the two directions of rotary motion has been quantitatively analysed by Graham (1963)"

After Power (1965, p.346)

But Power's later criticism of Graham (Macrae and Power, 1969) must equally apply to his own analysis. The projections are only identical (and therefore ambiguous) for parallel projections - not for a finite viewing distance.

Similar criticism of Day and Power's theory has been made by Hershberger (1967); he analysed (qualitatively) the transformation of the retinal

image of a rigid object rotated in depth in terms of the relative displacements of individual points of the image along two co-ordinates, one vertical and one horizontal. Herschberger proceeded to show experimentally that even a one-dimensional display of the horizontal polar projection of a rotating stimulus object resulted in veridical perception. Thus Day and Powers "general theory" of apparent reversal of rotary motion in depth is not as general as they supposed. It applies only where motion perspective cues as to the direction of rotation are sub-threshold.

The statistical significance of Herschberger's experimental results has been challenged by Pitz (1968), in a theoretical note illustrating the Bayesian method of hypothesis testing. But whether his experimental results are valid or not, Herschberger's fundamental criticism of Day and Power's theory still stands.

The explanation advanced by Cook, Mefferd and Wieland (1967) suggested that apparent motion reversals are a consequence of an automatic, abrupt, apparent reversal of orientation in depth, analagous to a perspective reversal. They found experimentally that these reversals (1) are systematic, non-random events and (2) occur quite readily in the absence of misleading perspective cues or in the presence of valid depth cues.

Their findings supported the earlier view (Ames, 1951; Pastore, 1952; Graham, 1963) that apparent orientation determines apparent direction of rotation; but they were directly counter to the suggestion of Day and Power (1965), that the reversals were random, spontaneous events.

The hypothesis that monocular "pictorial" depth cues such as aerial perspective, interposition and texture gradient, determine the apparent oscillation of a rotating stimulus object has been proposed by Canestrari and Farne (1969), although accurate quantification of their observations awaits publication.

The use of interposition and shadow as important cues were also suggested by Cross and Cross (1969); since the shadow patterns painted on the surface of the trapezoidal window do not reverse during the rotation, they provide additional cues to oscillation (see Fig.4.19 p.60). But these can only be secondary (reinforcing) cues, since the illusion can be shown to be just as strong with planar unpatterned stimulus

objects (p. 103).

A detailed study by Zegers and Murray (1962) was designed to test the hypothesis that:

".... the perception of illusion represents a failure of the visual apparatus to pick up and convey to consciousness sufficient information whereby the viewing subject sees external reality correctly."

(Zegers and Murray, 1962, p.116)

Various parameters were selected for investigation which on an a priori basis were thought to be susceptible to misinterpretation, but the overall conclusion of their research was that the perception of the illusion was due to the failure not of a single cue, but of many. Wide individual differences, and the restricted number of observers used, limited the usefulness of their results, but their basic hypothesis certainly comes closest to that tested in this investigation.

SUMMARY

Ames' (1951) explanation that because of previous familiarity with trapezoidal projections of rectangular shapes, a rectangular model must be applied to all such shapes, has often been criticised as being too specific, in that it is applicable to this particular situation only, and not applicable to the other demonstrable cases of misinterpreted rotary motion.

This criticism has led to various hypotheses attempting to relate the illusion to quantifiable stimulus characteristics, but such criticism is really unjustifiable. Ames' explanation was only specific since he was referring to a specific stimulus shape; the principle that past experience may cause an inappropriate model to be assigned to the stimulus may be applied far more generally.

Several explanations have depended upon the identity or ambiguity of cues provided by the approaching or receding object. If such cues were truly ambiguous, then oscillation would be found 50% of the time. It will be seen later (p.193) that this is found with symmetrical objects at distances where motion perspective cues are subthreshold. But with objects such as Ames trapezoid, oscillation is a regular, systematic occurrence - certainly not random.

Various explanations have depended essentially on the confusing depth cues provided by the shapes used. These are really only a logical extension of Ames' explanation, for if the trapezoidal shape is interpreted as a rectangle seen in perspective, then the apparently longer vertical edge must always be closer to the observer - and this is what causes the shape to be seen in the wrong orientation in depth.

The explanations of the rotating trapezoid illusion reviewed here are inadequate to relate all the different parameters of the illusion. The factors that have been investigated in previous studies of this phenomenon have included shape, size, and surface pattern, judgment of slant and various subjective variables. These studies will be discussed in detail in Section 7, when the published data will be compared with the results obtained in this investigation. They are mentioned here to indicate the variety of possible approaches to the subject.

SECTION 4

ANALYSIS OF THE TRAPEZOIDAL ILLUSION

The problem posed by the rotating trapezoid is obviously not a simple one; the studies discussed above have indicated that the illusion is due to the interaction of several conflicting sources of information. So the basic problem still remains - what are the essential cues necessary to produce apparent oscillation instead of veridical rotation of an object?

In terms of the model of visual perception discussed earlier (p. 8) the interpretation of the direction of motion of a rotating object must relate the information contained in the present stimulus with stored models of the apparent transformations undergone by different shapes as they move in different ways relative to the observer.

The information contained in the present stimulus can be analysed in a variety of ways; it may be considered in the information theory sense of 'bits' of information, or it may be considered as a weighted function of its component parts.

Analysis in terms of bits of information becomes very laborious for a complex situation such as this, and is probably less useful, when comparison with other similar situations is attempted, than the second possible approach.

The various sources of information contained in the rotating trapezoidal window may be summarised as follows:

- outline shape
- outline size
- surface pattern
- axis of rotation
- speed of rotation
- viewing distance
- monocular/binocular view

The interpretation of these factors (ie. the apparent direction of rotation seen) will be considered in detail later, but it may be considered as a probability function incorporating the information content of the present stimulus, and stored information or past experience, (relating to previous stimuli).

STORED INFORMATION

The manner in which information derived from past experience with rotating or oscillating objects may be stored and analysed can only be considered hypothetically, since there is no experimental evidence to suggest how this might be done. A mathematical analysis of the basic parameters involved might indicate a possible approach.

MATHEMATICAL ANALYSIS

Now every static light pattern entering an eye and forming a retinal image could represent an infinite number of objects, since every such

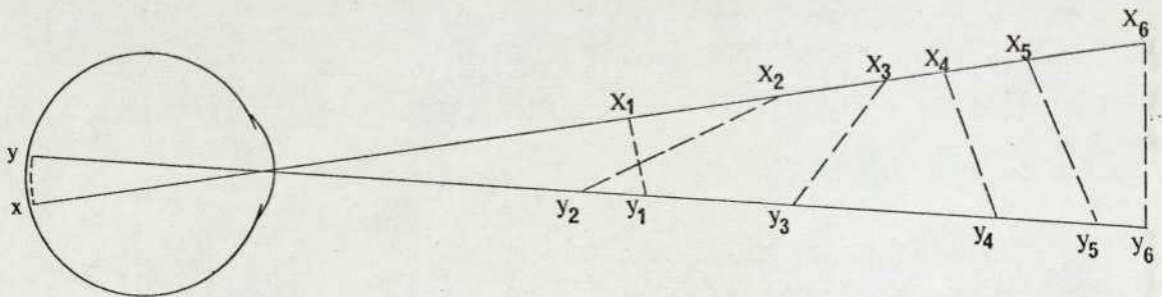


Fig. 4.1

Various line objects
producing the same
retinal image, XY

static pattern could represent the projection of an object from an infinite number of points (see Fig. 4.1).

But as soon as that object moves relative to the observer, the number of possible interpretations of the image is considerably reduced, because the projection of the object can vary only in particular very limited ways for certain kinds of relative movement. For a known type of relative movement (eg. either observer stationary and object moving or vice versa) the variation in the retinal image is completely determined for an object of constant size and shape.

PROJECTION OF A ROTATING RECTANGLE:

Consider now the projection of a rectangle as it is rotated about a central vertical axis, with its centre on the same horizontal plane as the observer's line of sight, the observer remaining stationary. (Fig. 4.2)

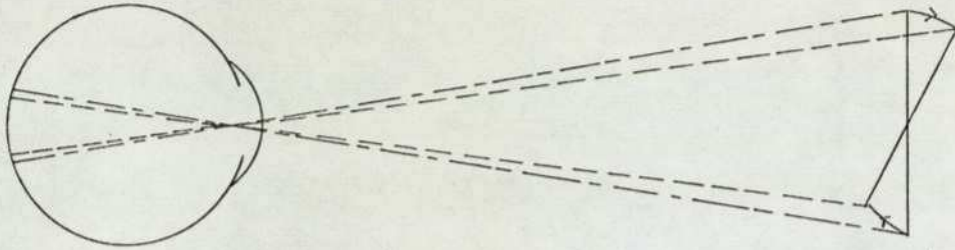
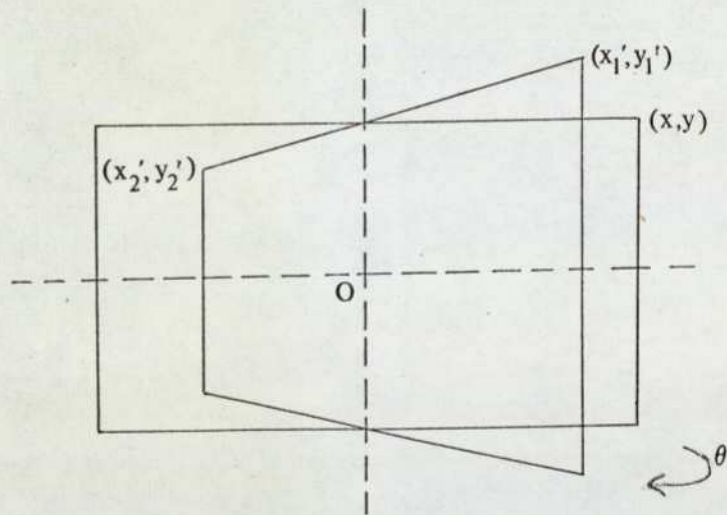


Fig. 4.2

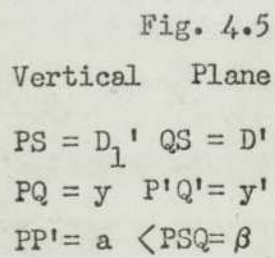
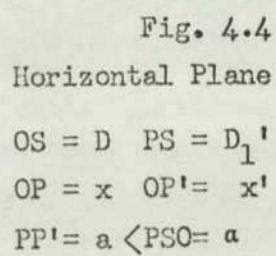
Fig. 4.3



Consider the rectangle to have dimensions $2x \times 2y$, with centre at the origin O .

Let the rectangle rotate clockwise through an angle θ such that the dimensions of the trapezoid corresponding to the projection of the new position of the rectangle back to the frontal plane are $x_1' + x_2'$ horizontally and $2y_1'$ & $2y_2'$ vertically, as shown in Fig.4.3.

Let the subject S be situated a distance D from O , and on the same horizontal plane as O .



From Fig.4.4

$$x' = D \cdot \tan \alpha$$

But $\tan \alpha = \frac{x \cdot \cos \theta}{(D - x \cdot \sin \theta)} \quad (\Delta PNS)$

Thus $x' = \frac{D \cdot x \cdot \cos \theta}{D - x \cdot \sin \theta} \quad - (1)$

From Fig.4.5

$$y' = (D_1' + a) \cdot \tan \beta$$

But $\tan \beta = \frac{y}{D_1'}$

Thus $y' = \frac{y(D_1' + a)}{D_1'} \quad - (2)$

From ΔOPS (horizontal plane)

$$(D_1')^2 = D^2 + x^2 - 2D \cdot x \cdot \cos (90 - \theta)$$

$$(D_1')^2 = D^2 + x^2 - 2D \cdot x \cdot \sin \theta$$

From $\Delta OPP'$ (horizontal plane)

$$a^2 = x^2 + x'^2 - 2x \cdot x' \cdot \cos \theta$$

But $x' = \frac{D \cdot x \cdot \cos \theta}{D - x \cdot \sin \theta}$

Thus $a^2 = x^2 + \frac{D^2 x^2 \cos^2 \theta}{(D - x \cdot \sin \theta)^2} - \frac{2x^2 D \cdot \cos^2 \theta}{(D - x \cdot \sin \theta)}$

$$a^2 (D - x \cdot \sin \theta)^2 = x^2 (D^2 - 2D \cdot x \cdot \sin \theta + x^2 \sin^2 \theta) + 2x^3 D \sin \theta \cdot \cos^2 \theta - x^2 D^2 \cos^2 \theta$$

$$a^2 (D - x \cdot \sin \theta)^2 = x^4 \sin^2 \theta - 2x^3 D \sin^3 \theta + x^2 D^2 \sin^2 \theta$$

$$= x^2 \sin^2 \theta (x^2 - 2x D \sin \theta + D^2)$$

Thus $a = \frac{x \cdot \sin \theta (x^2 - 2x D \sin \theta + D^2)^{\frac{1}{2}}}{(D - x \cdot \sin \theta)}$

$$\text{Thus} \quad x' = \frac{D \cdot x \cdot \cos \theta}{D - x \cdot \sin \theta} \quad y' = \frac{D \cdot y}{D - x \cdot \sin \theta}$$

The values of x' and y' for a rectangle 15" x 10" ($x = 7.5"$, $y = 5"$) and a viewing distance of 15 feet were calculated at 20° intervals, and are tabulated below (Table 4.1). Fig. 4.7 shows the projection of the rectangle for $\theta = 0^\circ, 20^\circ, 40^\circ, 60^\circ, 80^\circ, 90^\circ$.

Table 4.1

PROJECTION OF A RECTANGLE FOR A DISTANCE OF 15 FEET

Angle	$2y_1'$	$2y_2'$	x_1'	x_2'
0°	10.00"	10.00	7.50	- 7.50
20	10.14	9.85	7.15	- 6.95
40	10.27	9.74	5.90	- 5.59
60	10.37	9.65	3.89	- 3.62
80	10.42	9.61	1.36	- 1.25
90	10.43	9.60	0.00	- 0.00

The most significant feature of Fig.4.7 is that while x' varies from $- 7.5" \rightarrow + 7.5"$, y' varies very little relative to y - in fact a maximum percentage difference in the height of the two sides of about 4% compared to about 40% for Ames' trapezoid.

It is interesting to note that there is no actual rectangle for which Ames' trapezoid could be a possible projection onto the frontal plane, since substitution of Ames' dimensions in the formulae calculated above produces an inequality. The dimensions of Ames' trapezoid, however, were calculated by projection onto a plane oblique to the line of regard. This is illustrated in Fig.4.8 (after Ames, 1951, Fig.2).

The reasons for this complicated derivation are not clear.

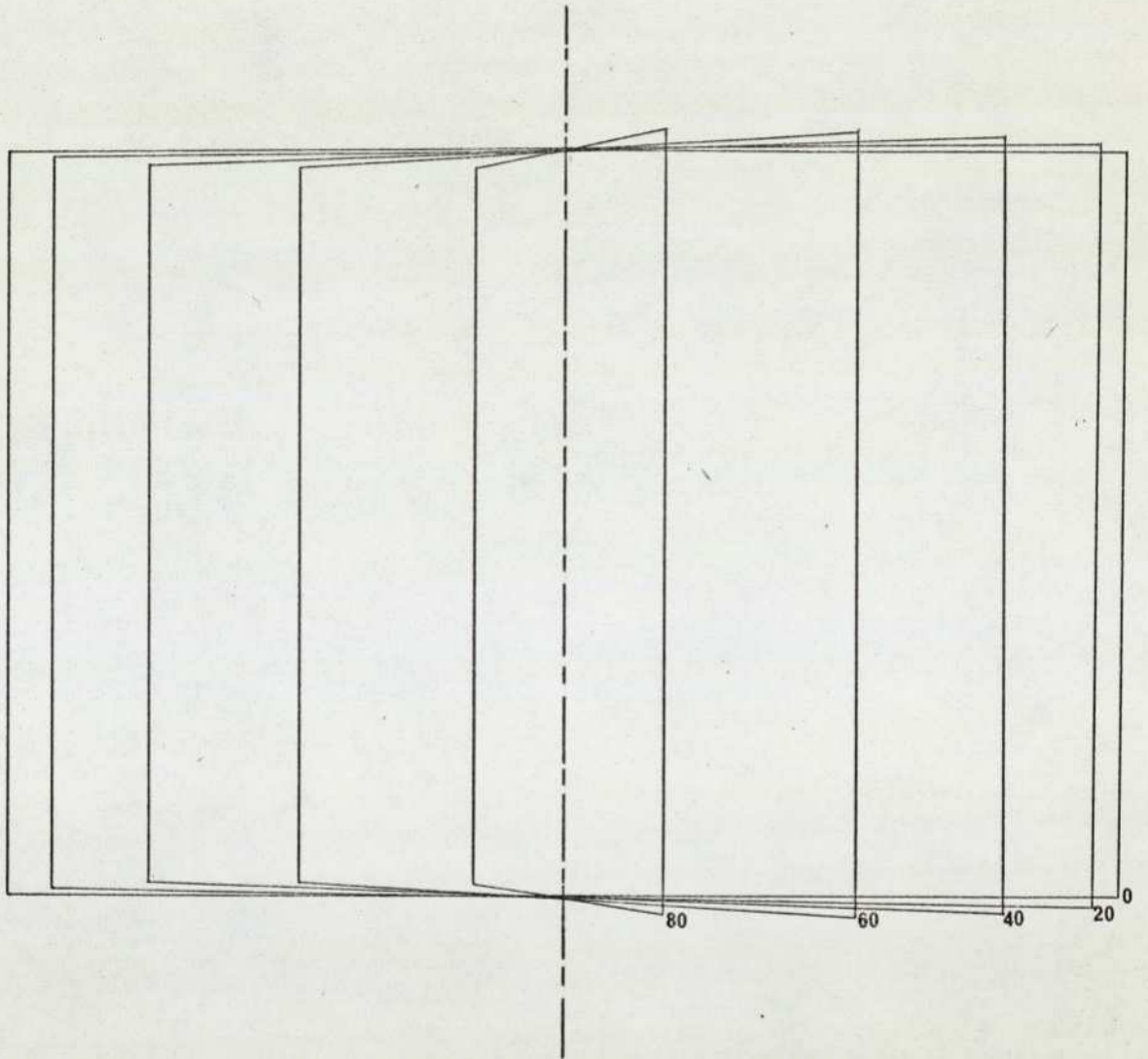
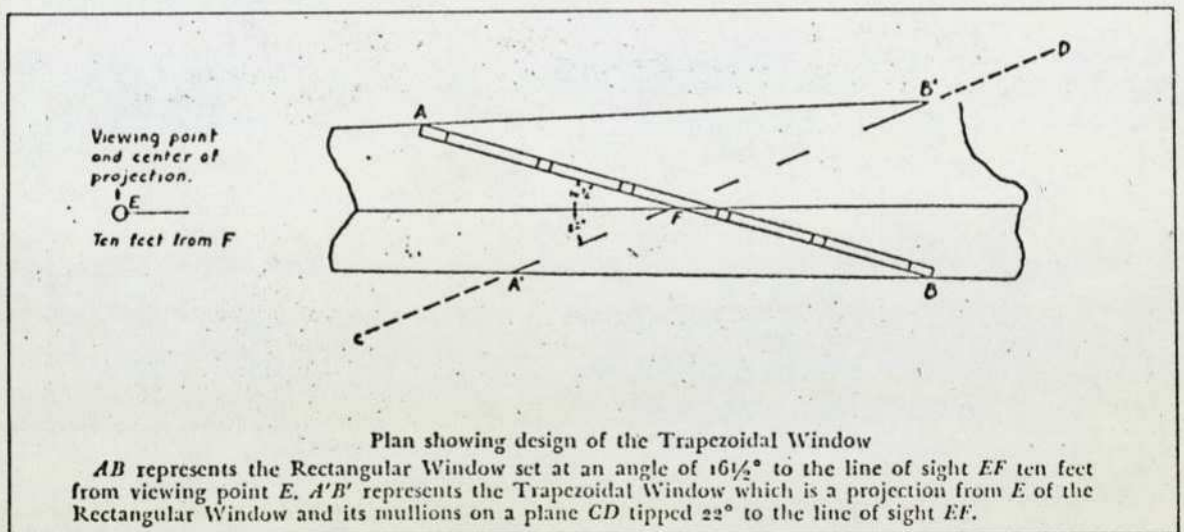


Fig. 4.7

Projections of a
Rectangle
Rotated through 80° .

Fig. 4.8

(From Ames, 1951)



PROJECTION OF A ROTATING TRAPEZOID:

Modification of the formulae derived to calculate the projection of a rotating rectangle can be made to enable the projection of a rotating trapezoid to be calculated.

Consider the isosceles trapezoid shown in Fig.4.9, symmetrical about the x-axis and rotating about the y-axis. Then the trapezoid is defined by the corner points (x_1, y_1) and (x_2, y_2) , where $y_1 > y_2$. (When $y_1 = y_2$ the trapezoid becomes a rectangle.)

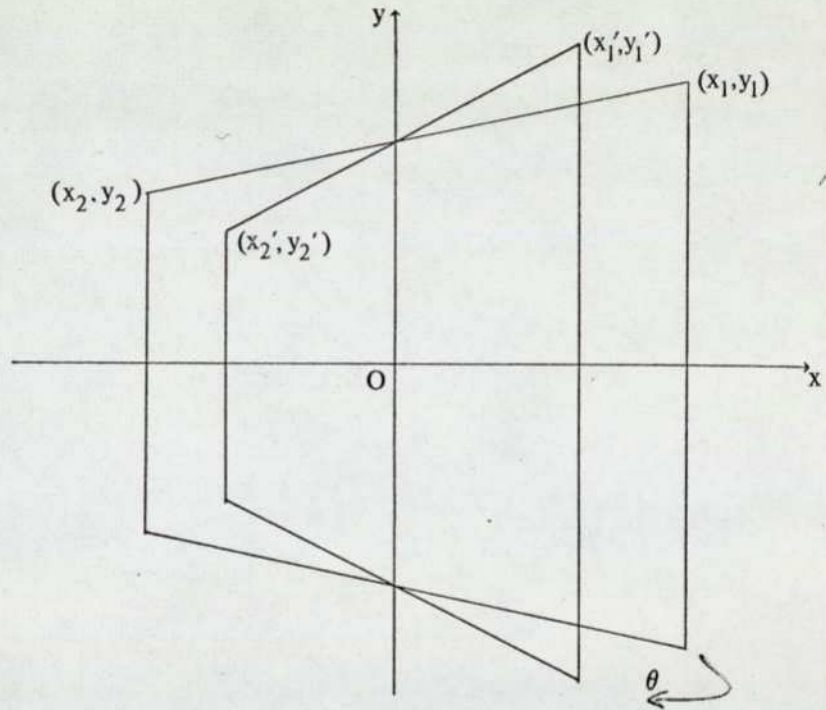


Fig. 4.9

Consider the trapezoid to be rotated clockwise through an angle θ , and viewed from a distance D on the same horizontal level as the origin.

Then if (x_1', y_1') is the projection of (x_1, y_1)

and (x_2', y_2') is the projection of (x_2, y_2) , it may be shown that:

$$x_1' = \frac{D \cdot x_1 \cdot \cos \theta}{D - x_1 \cdot \sin \theta}$$

$$y_1' = \frac{D \cdot y_1}{D - x_1 \cdot \sin \theta}$$

$$x_2' = \frac{D \cdot x_2 \cdot \cos \theta}{D + x_2 \cdot \sin \theta}$$

$$y_2' = \frac{D \cdot y_2}{D + x_2 \cdot \sin \theta}$$

— [T.P.]

PROJECTION OF AN OSCILLATING TRAPEZOID:

Simple manipulation of the data obtained from calculating the projection of a rotating trapezoid may be made to show the transformations produced by an oscillating trapezoid, with either the longer end or the shorter end closer to the observer. For the former, values of θ from 0° to 180° only would be considered, and for the latter case values of θ from 180° to 360° only would be considered.

This analysis has concentrated on the transformations undergone by the corner points of the rotating or oscillating trapezoid.

But every point (x,y) on the surface of the trapezoid must, obviously, undergo the same transformation.

However, since the corner points provide the greatest source of information (in terms of Attneave's analysis (1954)), and since the co-ordinates of the 4 corner points determine the apparent outline shape of the trapezoid at any moment in time it seems profitable to concentrate on their analysis.

It may be useful, therefore, to consider the loci of the corner points as they rotate or oscillate.

EQUATION TO THE LOCUS OF THE POINT (xy)

It will be shown that the projection of the locus of any point (x,y) on the rotating trapezoid is an ellipse; the characteristics of that ellipse will be examined.

Let the projection of the point (x,y) after rotating through an angle θ be (x',y').

Then from equations (T.P.), (page 34) we had:

$$x' = \frac{Dx \cos \theta}{D - x \sin \theta} \quad - \textcircled{1}$$

$$y' = \frac{Dy}{D - x \sin \theta} \quad - \textcircled{2}$$

$$\textcircled{2} \rightarrow \sin \theta = \frac{D(y' - y)}{x y'}$$

$$\textcircled{1}/\textcircled{2} \rightarrow \cos \theta = \frac{x'}{x} \frac{y}{y'}$$

Squaring and adding gives

$$D^2 (y' - y)^2 + x'^2 y'^2 - x^2 y'^2 = 0$$

$$D^2 y'^2 - 2D^2 y \cdot y' + D^2 y^2 + x'^2 y'^2 - x^2 y'^2 = 0$$

$$x'^2 y'^2 + y'^2 (D^2 - x^2) - 2D^2 y \cdot y' + D^2 y^2 = 0$$

$$\begin{aligned} x'^2 y'^2 + (D^2 - x^2) \left[y' - \frac{D^2 y}{(D^2 - x^2)} \right]^2 &= \frac{D^4 y^2}{(D^2 - x^2)} - D^2 y^2 \\ &= \frac{D^2 y^2 x^2}{(D^2 - x^2)} \end{aligned}$$

Put $X = x'$

$$Y = y' - \frac{D^2 y}{(D^2 - x^2)}$$

Then:
$$\left[\frac{(D^2 - x^2)}{D^2 x^2} \right] X^2 + \left[\frac{(D^2 - x^2)^2}{D^2 x^2 y^2} \right] Y^2 = 1$$

Thus the locus of any point (x, y) on the rotating trapezoid is an ellipse, and the locus of the particular corner point (x_1, y_1) will be given by:

$$\left[\frac{(D^2 - x_1^2)}{D^2 x_1^2} \right] X_1^2 + \left[\frac{(D^2 - x_1^2)^2}{D^2 x_1^2 y_1^2} \right] Y_1^2 = 1$$

Where $X_1 = x_1'$ $Y_1 = y_1' - \frac{D^2 y_1}{(D^2 - x_1^2)}$

Characteristics of the ellipse:

1. The ellipse is not symmetrical about the line

$$y = y_1$$

Rather it is symmetrical about the line

$$Y_1 = 0$$

i.e. $y_1' = \frac{D^2 y_1}{(D^2 - x_1^2)}$

2. The range of values of y_1' is :

$$y_1'_{\max} - y_1'_{\min} = \frac{2D x_1 y_1}{(D^2 - x_1^2)}$$

3. The maximum value of x_1' is not x_1 , but rather

$$x_1'_{\max} = \frac{\pm D x_1}{(D^2 - x_1^2)^{\frac{1}{2}}}$$

4. When x_1 is small relative to D,

$$y_1' \rightarrow y_1$$

and $x_1' \max \rightarrow \pm x_1$

and the locus of the corner point tends to the straight line $y = y_1$.

These points are illustrated by Figs. 4.10 and 4.11, which show the loci of the projections of a corner point of the same trapezoid for viewing distances of 3.5 ft and 15.5 ft respectively.

The projection of the advancing corner of the trapezoid is obviously different from the projection of the receding corner. The advancing trapezoid produces the upper half of the ellipses, and the receding trapezoid gives the lower half.

These two halves are only identical when the ellipse becomes a straight line (i.e. when $D = \infty$, parallel projection).

But the two halves will become psychophysically equal when the vertical diameter of the ellipse is sub-threshold (such as shown in Fig.4.11).

Fig. 4.10

$$\begin{aligned}x_1 &= 7.82 & y_1 &= 6.50 \\x_2 &= -5.43 & y_2 &= 4.50 \\D &= 3.50 \text{ ft.}\end{aligned}$$

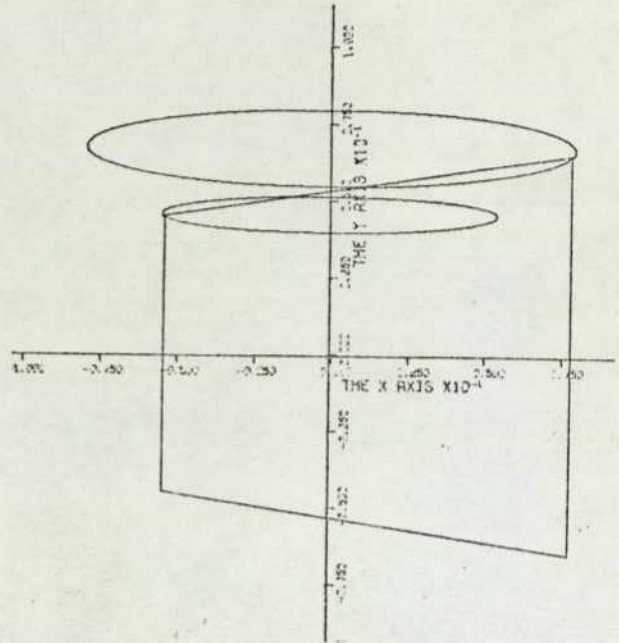
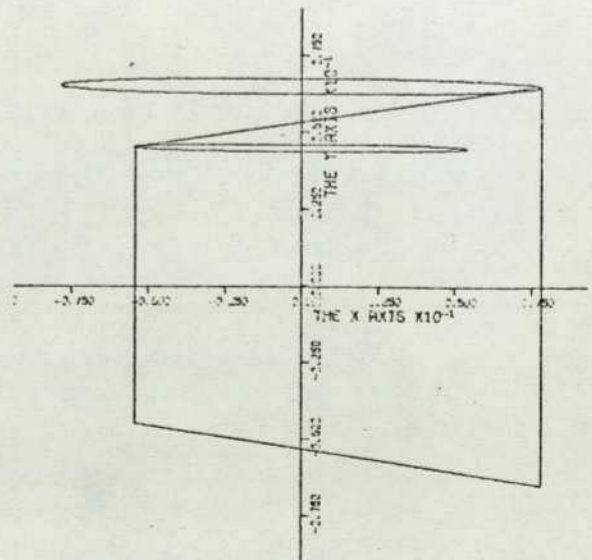


Fig. 4.11

$$\begin{aligned}x_1 &= 7.82 & y_1 &= 6.50 \\x_2 &= -5.43 & y_2 &= 4.50 \\D &= 15.50 \text{ ft.}\end{aligned}$$



COMPUTER PROGRAMS.

A computer program TRAP was written to calculate the projection of each of the stimulus shapes used in the following experiments, at 10° intervals as it was rotated about the vertical axis, for viewing distances of 5, 10, 15, 20 and 25 feet.

TRAP was modified to calculate the projection at 10° intervals for each of the stimulus shapes used not only in the present series of experiments, but also those used by previous investigators, for specified viewing distances. This new program, TROT, then plotted the locus of the points (x_1', y_1') and (x_2', y_2') on the graph plotter, together with the outline of the stimulus shape when perpendicular to the observer (ie. in the 0° position).

Table 4.2 illustrates a typical example of the print-out from TROT.

The viewing distance and the size of the trapezoid (determined by (x_1, y_1) and (x_2, y_2)) are given at the top of the table, and the values of (x_1', y_1') and (x_2', y_2') are tabulated for values of θ from 0° to 360° . The example shown is for Ames (1951) trapezoid.

The loci of the points (x_1, y_1) and (x_2, y_2) are illustrated by the graph, Fig. 4.12. These are the smooth curves drawn through the points (x_1', y_1') and (x_2', y_2') respectively.

The program TROT is printed in Appendix 1, and will be discussed further shortly.

TRAPEZOID PROJECTION
FOR A DISTANCE OF 20.0 FEET

	X1= 12.30	Y1= 11.81	X2= -7.20	Y2= 6.25
ANGLE	X1DASH	Y1DASH	X2DASH	Y2DASH
0.	12.3000	11.8100	-7.2000	6.2500
10.	12.2219	11.9160	-7.0539	6.2176
20.	11.7644	12.0207	-6.6971	6.1865
30.	10.9323	12.1206	-6.1432	6.1576
40.	9.7433	12.2123	-5.4112	6.1318
50.	8.2294	12.2926	-4.5241	6.1096
60.	6.4356	12.3585	-3.5088	6.0917
70.	4.4197	12.4075	-2.3950	6.0786
80.	2.2494	12.4378	-1.2144	6.0706
90.	-0.0000	12.4480	0.0000	6.0680
100.	-2.2494	12.4378	1.2144	6.0706
110.	-4.4197	12.4075	2.3950	6.0786
120.	-6.4356	12.3585	3.5088	6.0917
130.	-8.2294	12.2926	4.5241	6.1096
140.	-9.7433	12.2123	5.4112	6.1318
150.	-10.9323	12.1206	6.1432	6.1576
160.	-11.7644	12.0207	6.6971	6.1865
170.	-12.2219	11.9160	7.0539	6.2176
180.	-12.3000	11.8100	7.2000	6.2500
190.	-12.0063	11.7058	7.1277	6.2827
200.	-11.3591	11.6066	6.8359	6.3148
210.	-10.3860	11.5149	6.3303	6.3452
220.	-9.1218	11.4334	5.6240	6.3729
230.	-7.6076	11.3639	4.7369	6.3970
240.	-5.8886	11.3081	3.6960	6.4167
250.	-4.0136	11.2674	2.5340	6.4313
260.	-2.0333	11.2426	1.2883	6.4403
270.	0.0000	11.2342	-0.0000	6.4433
280.	2.0333	11.2426	-1.2883	6.4403
290.	4.0136	11.2674	-2.5340	6.4313
300.	5.8886	11.3081	-3.6960	6.4167
310.	7.6076	11.3639	-4.7369	6.3970
320.	9.1218	11.4334	-5.6240	6.3729
330.	10.3860	11.5149	-6.3303	6.3452
340.	11.3591	11.6066	-6.8359	6.3148
350.	12.0063	11.7058	-7.1277	6.2827
360.	12.3000	11.8100	-7.2000	6.2500

Table 4.2

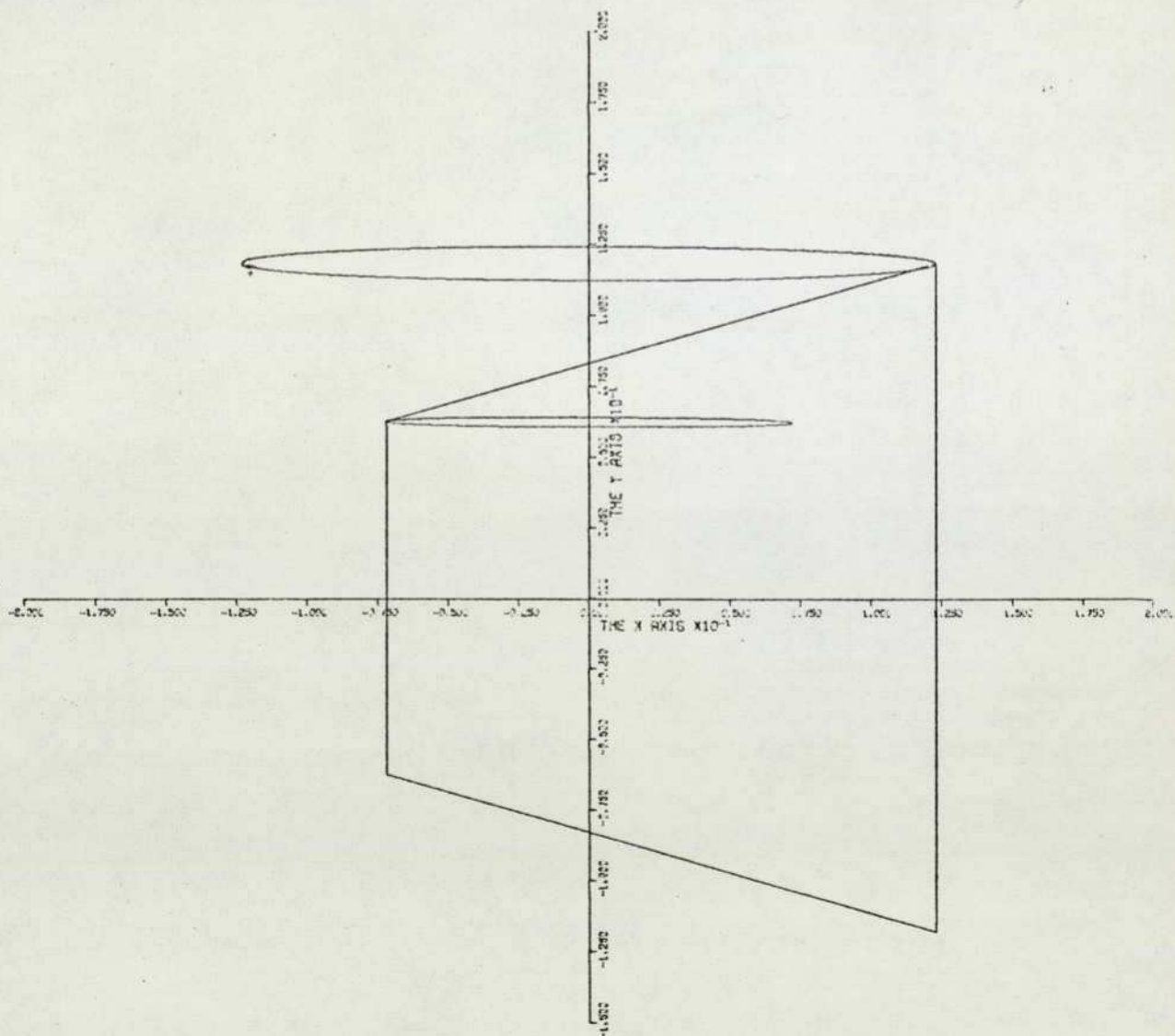
PROJECTION OF AMES' (1951)
TRAPEZOID,
CALCULATED BY TROT.

Fig. 4.12

PROJECTION OF AMES' (1951)
 TRAPEZOID,
 PLOTTED BY TROT.

TRAPEZOID PROJECTION

X1= 12.30 Y1= 11.81
 X2= -7.20 Y2= 6.25
 D= 20.00 FEET



DISCUSSION

This analysis of the mathematical transformation undergone by points on a rotating (or oscillating) stimulus object provides one example of what Gibson (1957) has termed "continuous perspective transformation". The utilisation of such transformations as cues to motion perception was discussed earlier (p 10).

Since just isolated points have been considered, the analysis can apply to points on any rotating stimulus object - be it rectangle, trapezoid, circle, ellipse, or any other shape. To find the apparent change in the shape of any rotating object, the projection^{of} key points on the periphery may be considered.

To simplify the analysis, only two dimensional shapes have been considered here (i.e. negligible thickness). But the transformation of three-dimensional shapes could easily be computed, by considering each plane in turn.

No suggestion has been made that the brain does actually store information in this way, but this kind of simplified mathematical analysis provides a useful basis for further discussion.

The only other study that has considered mathematically factors affecting the rotating trapezoid illusion was that of Graham (1963); this was referred to earlier (p 18). It may be useful here to consider in more detail the comparison between Graham's analysis of the situation, and that developed here.

Graham (1963) referred to the "differential angular velocity" existing between selected points on the surface of a moving object, and attempted to show that this was ambiguous.

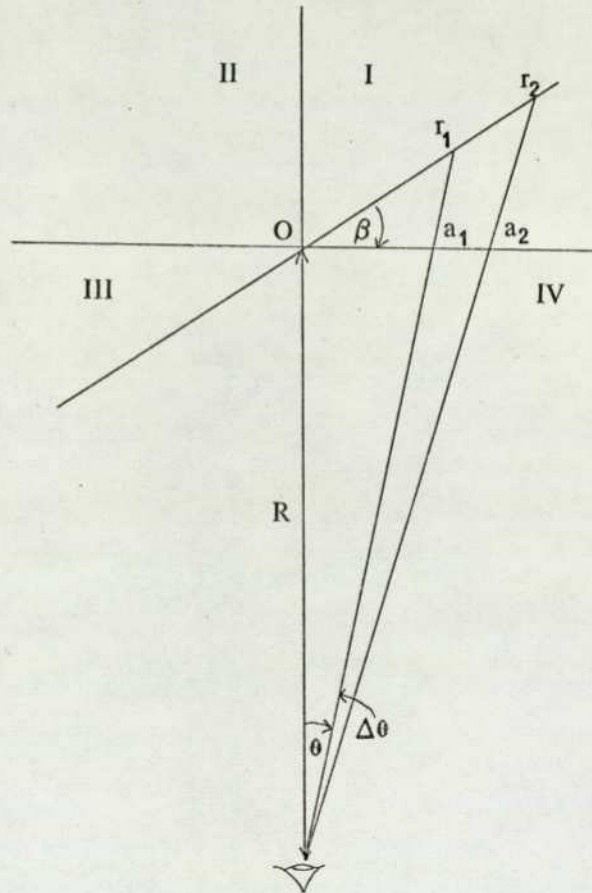


Fig. 4.13

(After Graham, 1963)

Showing a Receding
Window in Quadrant I

Graham considers 2 points r_1 and r_2 lying on one side of the vertical axis in the same horizontal plane (see Fig.4.13). These points are defined, presumably, such that the length $Or_1 = r_1$ and $Or_2 = r_2$ (where $r_2 = r_1 + \Delta r$). It is supposed that the eye, distant R from the axis, fixates these 2 points such that θ varies with the position of the window, β . The points a_1 and a_2 are the projections of r_1 and r_2 respectively in the frontal plane.

It is stated that (for R much greater than r_1 and Δr) the differential angular velocity varies in terms of stimulus conditions approximately in the following way:

$$\frac{d \Delta \theta}{dt} = \left[-R \Delta r \right] \left[(R \sin \beta + r_1 + r_1 \cos^2 \beta) (R + r_1 \sin \beta)^{-3} \right] \frac{d\beta}{dt}$$

This relationship was merely stated by Graham, so that before discussing its limitations it is necessary to rederive the formula and assess its assumptions.

It was shown above (Fig.4.9, equations (T.P.)) that;

$$x' = \frac{Dx \cos \theta}{D - x \sin \theta} \quad y' = \frac{Dy}{D - x \sin \theta}$$

Transposing this to Graham's notation, we have:

$$\begin{aligned} x &\equiv r \\ x' &\equiv a \\ D &\equiv R \\ \theta &\equiv 360 - \beta \end{aligned}$$

Note: The equations (T.P.) were derived by considering the observer to be fixating the centre of the rotating trapezoid. Graham's derivation considers the observer to be fixating the points r_1, r_2 . But the projection in the frontal plane is independent of the point of fixation.

Therefore;

$$a_1 = r_1' = \frac{R \cdot r_1 \cdot \cos \beta}{R + r_1 \sin \beta} \quad (1)$$

and

$$\begin{aligned} a_2 = r_2' &= \frac{R \cdot r_2 \cdot \cos \beta}{R + r_2 \sin \beta} \\ &= \frac{R (r_1 + \Delta r) \cos \beta}{R + (r_1 + \Delta r) \sin \beta} \quad (2) \end{aligned}$$

$$\begin{aligned} \text{But } a_1 &= R \tan \theta \\ a_2 &= R \tan (\theta + \Delta \theta) \end{aligned}$$

$$\text{Thus } (1) \rightarrow R \tan \theta = \frac{R \cdot r_1 \cdot \cos \beta}{R + r_1 \sin \beta} \quad (3)$$

$$(2) \rightarrow R \tan (\theta + \Delta \theta) = \frac{R (r_1 + \Delta r) \cos \beta}{R + (r_1 + \Delta r) \sin \beta} \quad (4)$$

If θ is small, such that $\tan \theta = \theta$,
 subs. (3) in (4):

$$\frac{R \cdot r_1 \cos \beta}{R + r_1 \sin \beta} + R \cdot \Delta \theta = \frac{R (r_1 + \Delta r) \cos \beta}{R + (r_1 + \Delta r) \sin \beta}$$

$$\text{Thus } \Delta \theta = \frac{(r_1 + \Delta r) \cos \beta}{R + (r_1 + \Delta r) \sin \beta} - \frac{r_1 \cos \beta}{R + r_1 \sin \beta}$$

$$\Delta \theta = \frac{(r_1 \cos \beta + \Delta r \cdot \cos \beta) (R + r_1 \sin \beta) - (r_1 \cos \beta) (R + r_1 \sin \beta + \Delta r \sin \beta)}{(R + r_1 \sin \beta + \Delta r \sin \beta) (R + r_1 \sin \beta)}$$

$$\Delta \theta = \frac{R \cdot \Delta r \cdot \cos \beta}{R^2 + 2Rr_1 \sin \beta + R \Delta r \sin \beta + r_1^2 \sin^2 \beta + r_1 \Delta r \sin^2 \beta}$$

$$\Delta \theta = \frac{R \cdot \Delta r \cdot \cos \beta}{R^2 + 2R \sin \beta (r_1 - \Delta r/2) + r_1^2 \sin^2 \beta (1 + \Delta r/r_1)}$$

If Δr is very small, $\Delta r/2 \approx 0$ $\Delta r/r_1 \approx 0$

$$\text{Thus } \Delta \theta = \frac{R \cdot \Delta r \cdot \cos \beta}{R^2 + 2R \cdot r_1 \cdot \sin \beta + r_1^2 \sin^2 \beta}$$

$$\Delta \theta = \frac{R \cdot \Delta r \cdot \cos \beta}{(R + r_1 \sin \beta)^2}$$

$$\text{Thus } \frac{d\Delta \theta}{dt} = \left[\frac{(-R \cdot \Delta r \cdot \sin \beta) (R + r_1 \sin \beta)^2 - 2(R \Delta r \cos \beta) (R + r_1 \sin \beta) (r_1 \cos \beta)}{(R + r_1 \sin \beta)^4} \right] \frac{d\beta}{dt}$$

$$\frac{d\Delta \theta}{dt} = \left[\frac{[-R \cdot \Delta r] [(R + r_1 \sin \beta) (\sin \beta) + 2 \cos \beta (r_1 \cos \beta)]}{(R + r_1 \sin \beta)^3} \right] \frac{d\beta}{dt}$$

$$\frac{d\Delta \theta}{dt} = [-R \cdot \Delta r] [R \sin \beta + r_1 + r_1 \cos^2 \beta] [R + r_1 \sin \beta]^{-3} \cdot \frac{d\beta}{dt}$$

Thus Graham's formula is correct, assuming that his assumptions are valid. But are they ? What are the limitations caused by these assumptions ?

The most important assumptions are:

$$\begin{aligned} \theta &= \tan \theta \\ \text{[not that } \Delta\theta &= \tan \Delta\theta] \\ \text{and } R &\gg r_1 \end{aligned}$$

Graham is therefore considering only very long viewing distances - i.e. practically parallel projection, not the more typical polar projections. The essential differences between parallel and polar projections have already been considered.

It is also assumed that Δr is very small - ie. he is considering two points very close together on the surface of the window. And r_1, r_2 are in the same horizontal plane, so that only the horizontal differential angular velocities are being considered.

What does Graham's formula indicate ? As it stands, it is not particularly meaningful. Graham states (1963, p.1022) that the differential angular velocity cue is ambiguous because a receding window in quadrant I (Fig.4.13) provides the same information as an advancing window in quadrant IV (Fig.4.14).

But does it ? Consider the same window in the two positions shown in Figs. 4.13 and 4.14, such that $\alpha = 360 - \beta$.

Then applying Graham's formula, we have for position 1 (Fig.4.13).

$$\frac{d.\Delta\theta}{dt} = f(R, r_1, \beta) \quad - (5)$$

and for position 2 (Fig.4.14)

$$\begin{aligned} \frac{d.\Delta\theta}{dt} &= f(R, r_1, \alpha) \\ &= f(R, r_1, (360-\beta)) \quad - (6) \end{aligned}$$

But (5) $\neq$ (6)

since $\sin(360-\beta) = -\sin\beta$

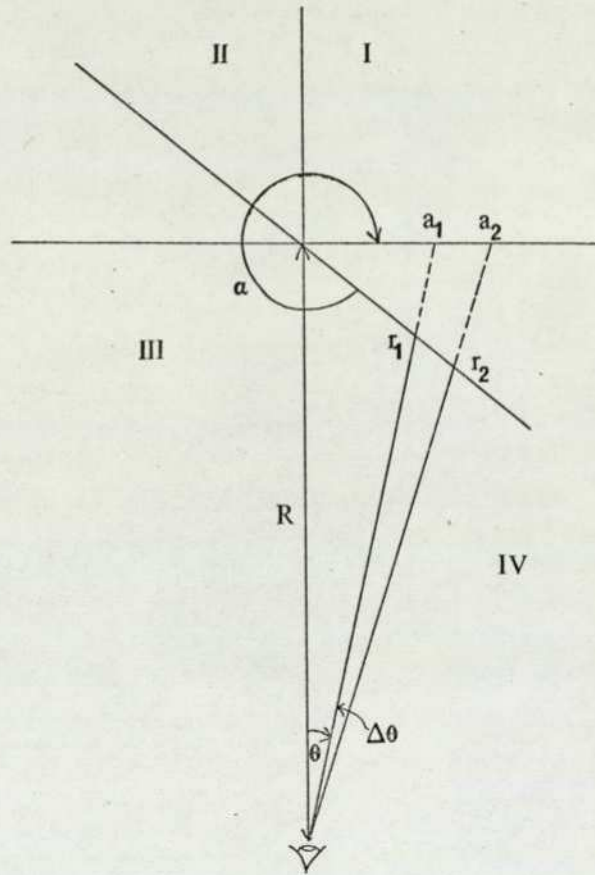


Fig. 4.14

Showing an Advancing
Window in Quadrant IV
 $\alpha = 360 - \beta$

It is possible that there may be some other angle, δ , in quadrant IV such that

$$\frac{d\Delta\theta}{dt} = f(R, r_1, \delta) = f(R, r_1, \beta)$$

but this is unlikely, since $\sin\beta$ will always be negative and $\sin\delta$ always positive.

Thus the differential angular velocities in the two positions are not equal. There must, therefore, be some other reason for the ambiguity.

Other types of Stored Information

Only one possible source of information relevant to the interpretation of the direction of rotation has been considered in detail - that is the continuous perspective transformation undergone by the rotating trapezoidal shape.

What are the other types of stored information with which incoming information may be compared ?

There is probably some indication of the frequency with which different types of motion have been encountered in the past. It would seem intuitively that rotation is a fairly fundamental type of movement, which is met frequently in everyday experience. But oscillation is probably encountered almost as frequently - not oscillation in the strict mathematical sense of simple harmonic motion through 180° , but oscillation in the sense of doors swinging on their hinges, or tree branches blown by the wind.

Shape constancy must also be relevant here. Rectangles are encountered frequently, and their retinal images are almost always trapezoidal; but trapezoids themselves are rarely met. The existence of shape constancy has long been recognised, and its importance in providing a stable visual world has been demonstrated frequently.

Normal observers rarely report any change in the apparent shape of a rotating trapezoid which appears to oscillate, although logically the shape should appear to change. However Ames (1951) and Cook et al (1967) have remarked that changes in shape were reported by experienced (and critical) observers, who were making careful and analytical descriptions of their percepts.

Depth perception and the judgment of slant may also be considered. But it is important to differentiate between static slant perception, and the slant of a moving object. This will be discussed further later (p 185).

These, then, are some of the relevant types of information which may be stored by the perceptual system. The incoming information will be compared with these stored models, and a judgment made of the type and direction of motion being seen.

CURRENT INFORMATION

The sources of information contained in the rotating trapezoidal window were summarised above as :

- outline shape
- outline size
- surface pattern
- axis of rotation
- speed of rotation
- viewing distance
- monocular/binocular view

The interpretation of all these factors (i.e. the direction of rotation seen) must relate this incoming information to the information that has been obtained from past experience and which will have been stored by the perceptual system in various ways - possibly in some of the ways that have been suggested.

PROJECTIVE TRANSFORMATIONS

Probably one of the most powerful sources of stored information is the projective transformation undergone by any object as it moves in any way relative to an observer. Hay's analysis (1966) has shown that this transformation can be considered in terms of 4 basic parameters, each of which could operate either horizontally or vertically. (i.e. image translation, stretching, shearing and foreshortening).

The particular apparent transformations undergone by a particular class of shapes (trapezoids) has been demonstrated mathematically for two particular types of movement (rotation and oscillation) relative to a stationary observer.

If the difference between the transformation due to rotation and the transformation due to oscillation were sub-threshold, then an ambiguous situation would result and errors in the resulting percept would be expected.

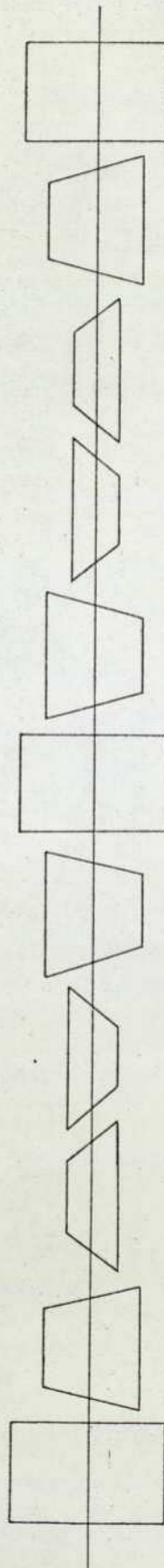
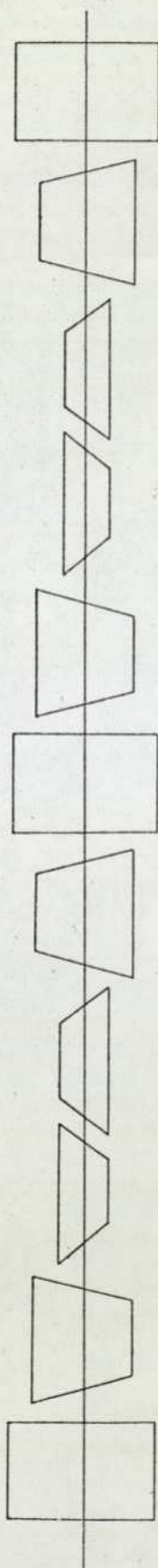
But if the situation were completely ambiguous, the resulting percept should be 50% correct and 50% incorrect. This is not the result

Fig. 4.15.

Transformation of a Rectangle during:

(1) Clockwise Rotation

(2) Oscillation



obtained with the rotating trapezoid illusion, for typically apparent oscillation is seen more than 90% of the time with stimuli such as Ames' window (see p.103).

Why, then, should there be such a strong bias towards perceived oscillation rather than rotation ?

Consider the essential difference between the transformation due to rotation and the transformation due to oscillation for a particular class of trapezoids - rectangles. This is represented in Fig.4.15 with the vertical dimensions exaggerated for clarity.

Note that immediately after the 0° position, the apparently longer vertical edge of the rectangle (i.e. the actually nearer edge) is always on the right for clockwise rotation, whereas it alternates between right and left for oscillation.

This is true for all values of x , y and D (using the same notation as above), and can be simply demonstrated mathematically:

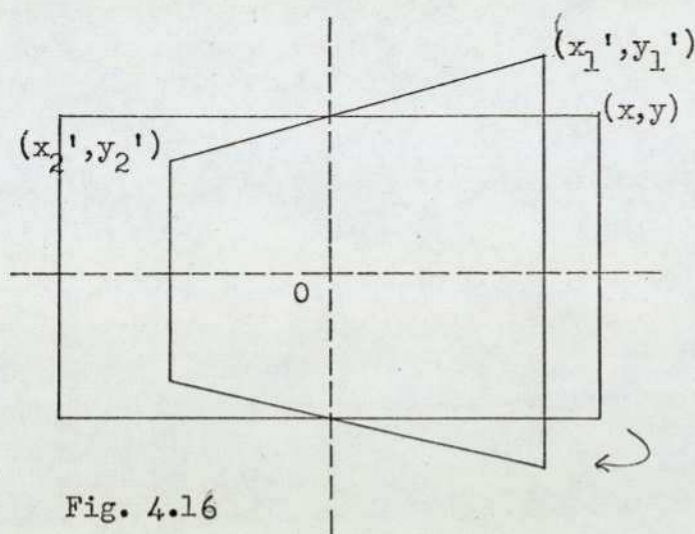


Fig. 4.16

Consider the rectangle $2x \times 2y$, viewed from a distance D and rotated through an angle θ . Then if the projected shape transformation is defined by the points (x_1, y_1) (x_2, y_2) it was shown above (p. 34) that

$$x_1' = \frac{Dx \cos \theta}{D - x \sin \theta} \quad y_1' = \frac{Dy}{D - x \sin \theta}$$

$$x_2' = \frac{Dx \cos \theta}{D + x \sin \theta} \quad y_2' = \frac{Dy}{D + x \sin \theta}$$

$$x_1' = \frac{Dx \cos \theta}{D - x \sin \theta} \quad y_1' = \frac{Dy}{D - x \sin \theta}$$

$$x_2' = \frac{Dx \cos \theta}{D + x \sin \theta} \quad y_2' = \frac{Dy}{D + x \sin \theta}$$

The necessary condition is that

$$y_1' - y_2' > 0$$

But $y_1' - y_2' = Dy \left[\frac{1}{D - x \sin \theta} - \frac{1}{D + x \sin \theta} \right]$

$$y_1' - y_2' = \frac{2 D x y \sin \theta}{D^2 - x^2 \sin^2 \theta} \quad \text{--- (R)}$$

Thus $y_1' - y_2' > 0$

if $D^2 \geq x^2 \sin^2 \theta$

But max value $x^2 \sin^2 \theta = x^2$

$\therefore D^2 \geq x^2$ or $D \geq x$

(D must be positive)

But D must be greater than x (or else the observer would be hit on the nose each time the rectangle rotated).

Thus $y_1' - y_2' > 0$

Thus if a rectangle is rotated about a vertical axis through its centre, the actually nearer vertical edge will always be the apparently longer vertical edge (this will always be the edge on the right for clockwise rotation).

Similarly, it can be shown that if the rectangle is oscillated about the same axis, the actually nearer vertical edge will again always be the apparently longer edge, but this will now alternate between right and left.

But equation (R) also shows that

$$\begin{aligned} &\text{as } D \rightarrow \infty \\ &\text{then } y_1' - y_2' \rightarrow 0 \end{aligned}$$

The apparent difference in height between the 2 vertical sides will therefore be small for most values of D (since D is large w.r.t. x, y). Fig 4.17 illustrates the projection of the corner points of a rectangle 10" x 15" for viewing distances of 5, 10, 15, 20 and 25 feet.

The maximum difference in the apparent lengths of the two vertical sides is indicated by the vertical diameters of the elliptical loci of the corner points (i.e. when $\theta = 90^\circ$). Fig.4.17 indicates that this is small (possibly subthreshold) for values of $D > 15$ feet.

If we cannot distinguish these differences in height, then rotation and/or oscillation of the rectangle will be ambiguous - and this is what is found experimentally (eg. Braunstein and Payne, 1968; Pastore, 1952; Cross and Cross, 1969).

But if we can distinguish the difference in height - i.e. if the difference in the apparent lengths of the 2 vertical sides is supra-threshold - then this provides a strong and unambiguous cue to the direction of rotation, and the apparently longer side will be the actually nearer side.

Consider now the projection of a rotating trapezoid. Although the actually longer edge is further away from the observer during half of the rotation, it is still apparently longer, so that this longer edge alternates between right and left in the same way as for a rectangle oscillating through somewhat less than 180° . This is so provided that the difference in height between the two vertical edges of the trapezoid is such that the elliptical loci of the end points are separated by some amount which is supra-threshold.

This, then, provides us with the basis for an hypothesis to explain the apparent oscillation seen with the rotating trapezoidal window: If the elliptical loci of the corner points are narrow (sub-threshold) and well - separated (supra-threshold), then rotation will always be seen as oscillation with the apparently longer side

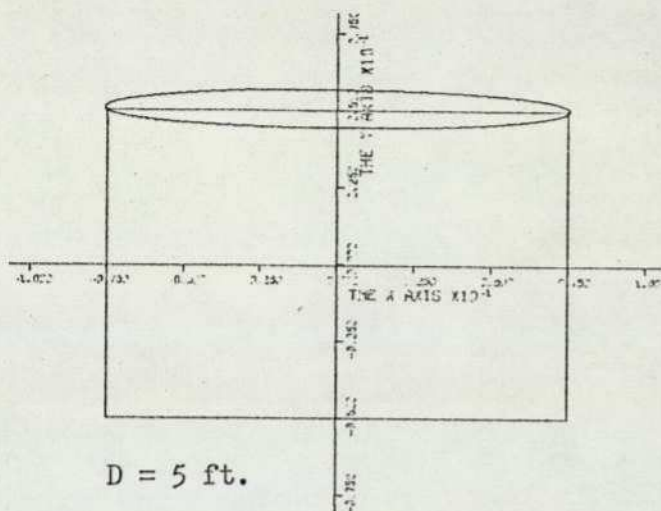
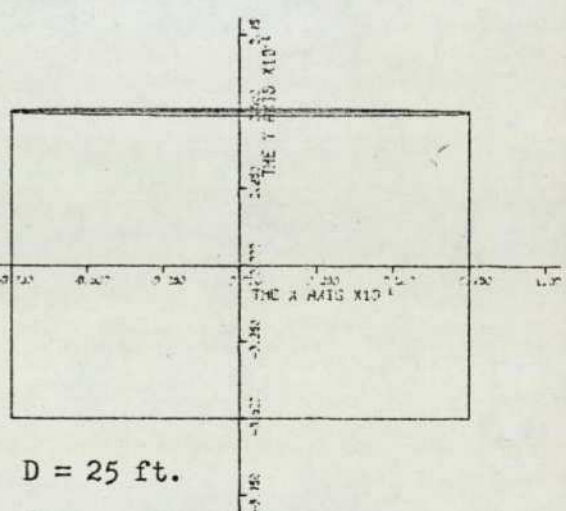
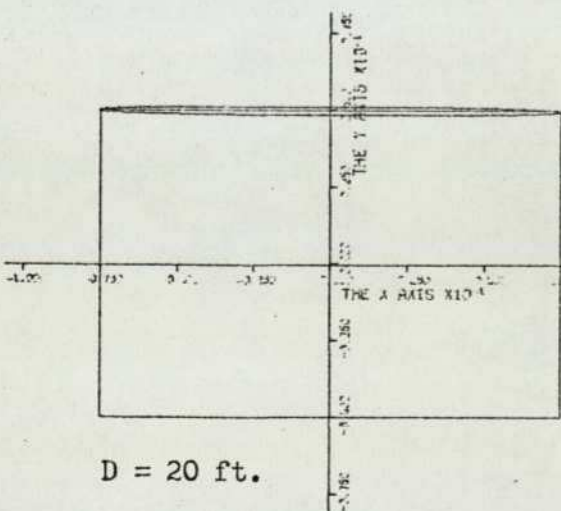
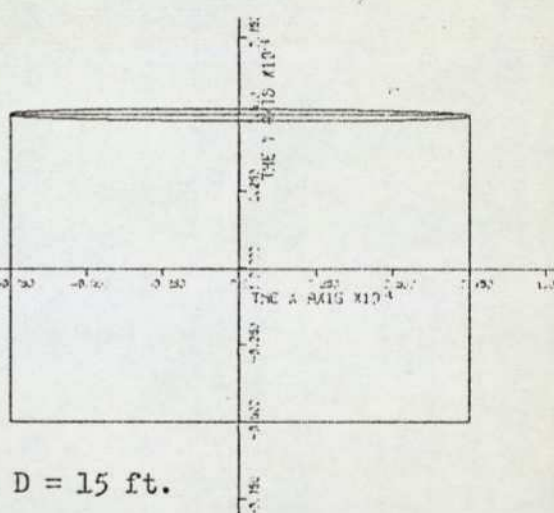
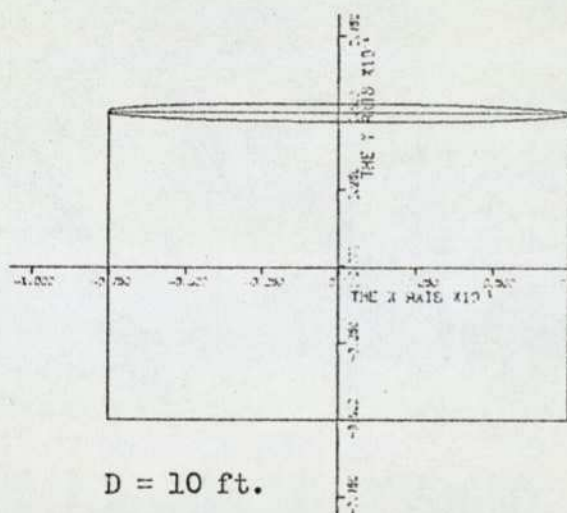


Fig. 4.17

PROJECTIONS OF A
10" x 15" RECTANGLE
FOR VARIOUS VIEWING
DISTANCES.



seen as apparently nearer (see Fig.4.18 (1)).

If the elliptical loci are closer together (nearly touching, or overlapping) then there will be some confusion, and sometimes rotation will be seen, sometimes oscillation (see Fig.4.18 (2)). The strong tendency for shape constancy may reinforce the oscillation cues, and add to the confusion.

If the ellipses are wide (supra-threshold) (whether they overlap or not) then the difference in the two positions can be easily distinguished, and rotation will be seen (Fig.4.18 (3)).

This may be summarised as follows:

- | | | |
|---------------------|---|--|
| Rectangle - D large | - | ambiguous cues
(ellipses narrow and overlapping, cannot be distinguished) hence sometimes see rotation, sometimes oscillation. |
| Rectangle - D small | - | unambiguous cues
(ellipses wide, can distinguish differences in height and therefore see true direction) apparently longer side = actually nearer side. |
| Trapezoid - D large | - | unambiguous cues
(ellipses narrow - small change in apparent length between front and back) apparently longer side seen as apparently nearer side therefore rotation seen as oscillation. |
| Trapezoid - D large | - | ambiguous cues
(ellipses closer together - maybe overlapping - confusion) sometimes see rotation , sometimes oscillation |
| Trapezoid - D small | - | unambiguous cues
(ellipses wide - large change in apparent length, so can distinguish between front and back) rotation seen as rotation, oscillation seen as oscillation. |

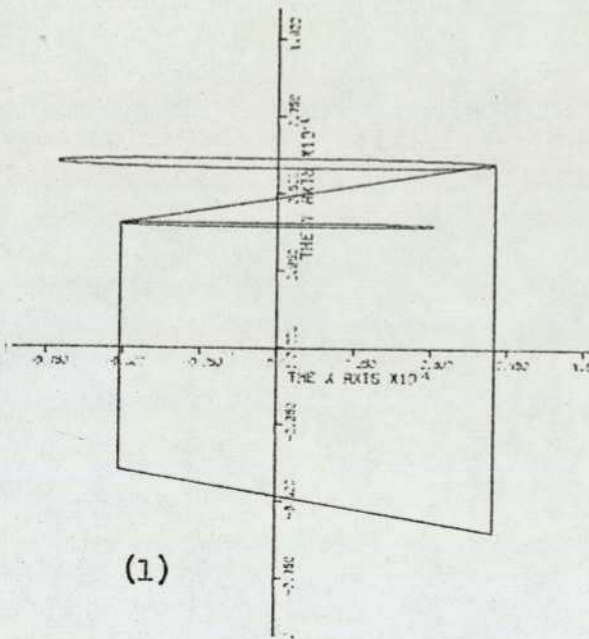
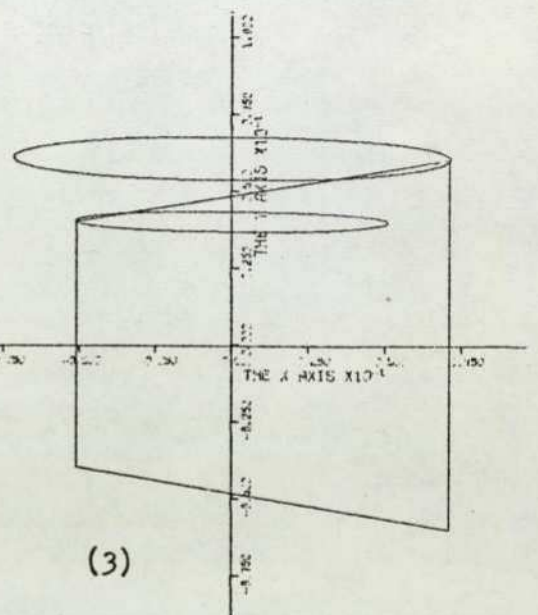
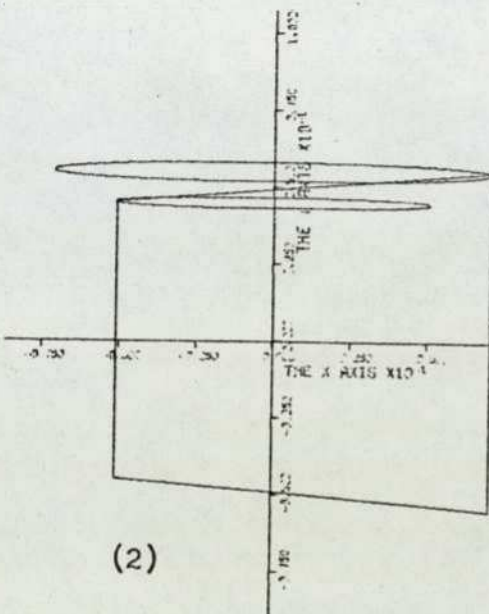


Fig. 4.18



It has been suggested, then, that the most important source of information when viewing a rotating trapezoid object is the projective transformation that it undergoes. This transformation may be summarised by considering the elliptical loci of the projections of the corner points of the trapezoid.

This transformation depends upon several of the factors listed above as contributing towards the interpretation of the direction of motion. The outline shape and size, axis of rotation position, and viewing distance are inter-related in the formulae derived above. The actual values of each of these dimensions determines absolutely the projection of that particular shape as it is either rotated or oscillated, and thus will determine whether the differences in those 2 projections will be either sub - or supra - threshold.

These factors together then probably provide the most important source information.

The computer program TROT (p.40) can be used to plot the elliptical loci of the corner points for any particular trapezoid, viewed at any particular distance.

Consideration of the graphs obtained enables the type of motion seen (rotation or oscillation) to be predicted, by following the simple set of rules summarised above.

The other main sources of information contained in the rotating trapezoid window illusion will now be considered briefly.

SURFACE PATTERN

Surface pattern provides a rather different source of information; the pattern may be chosen to either conflict with, or to reinforce, the outline shape of the stimulus, and may be either two-dimensional, or apparently three-dimensional.

If the surface pattern produces a three - dimensional effect (i.e. with shadows) it may add to the oscillation cues. Fig. 4.19 shows eight successive views of an artificially shadowed two-dimensional trapezoid in rotation (top), and the same views of a genuine three-dimensional trapezoid (below).

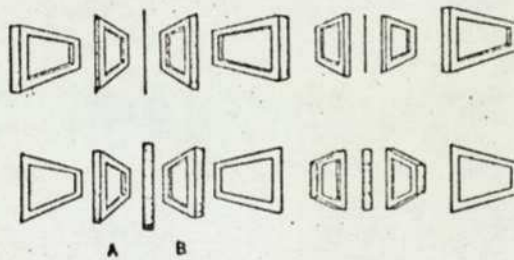


Fig. 4.19

(From Cross & Cross, 1969)

Since the two sides of the two dimensional trapezoid are painted alike, the shadow patterns do not reverse during the second half of the rotation as they do for the solid rotating trapezoid, and hence may indicate oscillation rather than rotation.

If the surface pattern conflicts with the outline shape, or if the patterns is different on the 2 sides of the object, this will provide additional cues to the direction of motion.

But it is suggested that the surface pattern is very much a secondary source of information. The projective transformation undergone by any rotating rectangular object is quite independent of the pattern of that object and provides a far more fundamental cue to the direction of rotation.

SPEED OF ROTATION

The speed of rotation obviously affects the rate of change of the projected pattern on the retina, and this in turn may affect the rate of interpretation of that changing pattern.

A very slow moving pattern may be essentially boring so that when two alternative interpretations are possible the percept switches more rapidly from one to the other.

A very fast changing pattern may not allow the perceptual system time to differentiate adequately between possible hypotheses, so frequently causing the wrong interpretation to be made.

It is suggested that there is a middle range over which variations in the speed of rotation have little effect on the resulting percept. Over this range, there is adequate time for incoming sensory information to be processed, and a decision made as to the type of motion being observed.

SUMMARY

It is suggested that the misinterpretation of the direction of motion of a rotating trapezoidal object is due to the misapplication of stored hypotheses based on the continuous perspective transformations undergone by rectangular objects. The illusion therefore depends upon past experience with rectangular objects, but does not depend upon the frequency with which oscillation and/or rotation have been seen in the past.

The illusion depends primarily upon the strong and unambiguous cue provided by rectangular objects viewed at short distances, that the apparently longer side is the actually nearer side.

How does this hypothesis compare with the explanations proposed by others? Ames (1951) stated that the illusion was due to past experience with rectangles, but he did not attempt to clarify which aspects of that past experience were the cause.

Graham's (1963) analysis of the differential angular velocity cue as ambiguous has been shown to be inadequate to explain the illusion at shorter viewing distances; the same can be said of Day and Power's (1965) analysis of 'retinal motion' cues.

Pastore (1952) and Mefferd et al (1966) suggested that it was the misperception of slant which caused the illusion. Whilst this may hold for stationary objects, it is difficult to relate to moving ones - and it does not suggest which particular stimulus characteristics are necessary for slant to be misperceived.

The explanation proposed here has attempted to analyse the illusion in terms of the various types of information it contains, and has suggested that the most important source of information is the continuous perspective transformation of the rotating object. This transformation is dependent upon both the outline shape and size, the axis of rotation position, and viewing distance. Secondary sources of information, such as surface pattern and speed of rotation, have also been considered. A series of six separate, but inter-related experiments was designed to investigate the relative importance of these various factors.

SECTION 5

EXPERIMENTAL INVESTIGATION:

GENERAL DESIGN

It was suggested in Section 2 that if the information content of an illusory stimulus was varied systematically, and related to the resulting percept, the types of information utilised by the brain for veridical perception could be investigated.

The Ames trapezoidal illusion provides a suitable stimulus for investigation, since the resulting percept is readily quantifiable. It is obviously a complex phenomenon, involving the interaction of several different types of visual cues. The stimulus parameters selected for investigation in the present study were:

- outline shape
- surface pattern
- speed of rotation
- $y_1:y_2$ ratio
- $x_1:x_2$ ratio (See Fig.4.9, p.34)
- fixation position.

Perspective effects (including monocular/binocular viewing) were not investigated specifically because of limitations due to the space available for experimentation. Previous work on the effects of viewing distance (eg. Braunstein & Payne, 1968) produced results consistent with the theoretical mathematical analysis discussed above.

Another possible variable was the contrast level between the stimulus shapes and the background. Preliminary experiments (not reported in detail here) showed this to be insignificant, and it was thought that the primary factors affecting the illusion would be of a higher order than that of contrast.

Of the many possible subject variables that could be investigated, only the following were chosen:

- within-subject variability
- between-subject variability
- sex.

Sex was included since it was relatively simple to incorporate this factor as an element of the experimental design, but restrictions on the number and type of subjects available meant that factors such as age, intelligence and mental state could not be included. It was considered that this did not present a major criticism of the experimental investigation, since these factors were relatively unimportant so far as the hypothesis under test was concerned.

Zegers & Murray (1962) showed that age was not an important variable in the perception of the illusion, and cross-cultural studies such as those of Allport & Pettigrew (1957) indicated that the illusion still occurred (although to a somewhat reduced extent) in more 'primitive' cultures than our own. The occurrence of the illusion in various abnormal mental conditions was investigated by Mulholland (1956, 1958), but there are several major criticisms of his work which will be discussed further later.

These, then were the factors investigated in six separate, but inter-related, experiments.

GENERAL METHODOLOGY:

The same basic method was used for every experiment. The stimulus shapes were cut from sheet aluminium and spray-painted flat white; they were evenly illuminated from above, creating no shadows, and were viewed against a matt black background.

Each shape was in turn mounted upon a vertical shaft attached to a variable - speed motor, and rotated in a horizontal plane about a vertical axis. They were filmed as they rotated, and the subjects viewed a two-minute film presentation following a one-minute 'trial run'.

The measure used was the frequency of an apparent change in direction, calculated as a percentage of the maximum possible, assuming two possible apparent oscillations for every complete revolution. Since the viewing time was kept constant, this maximum possible obviously depended upon the speed of rotation.

Precise details for each experiment will be given in the description of that experiment.

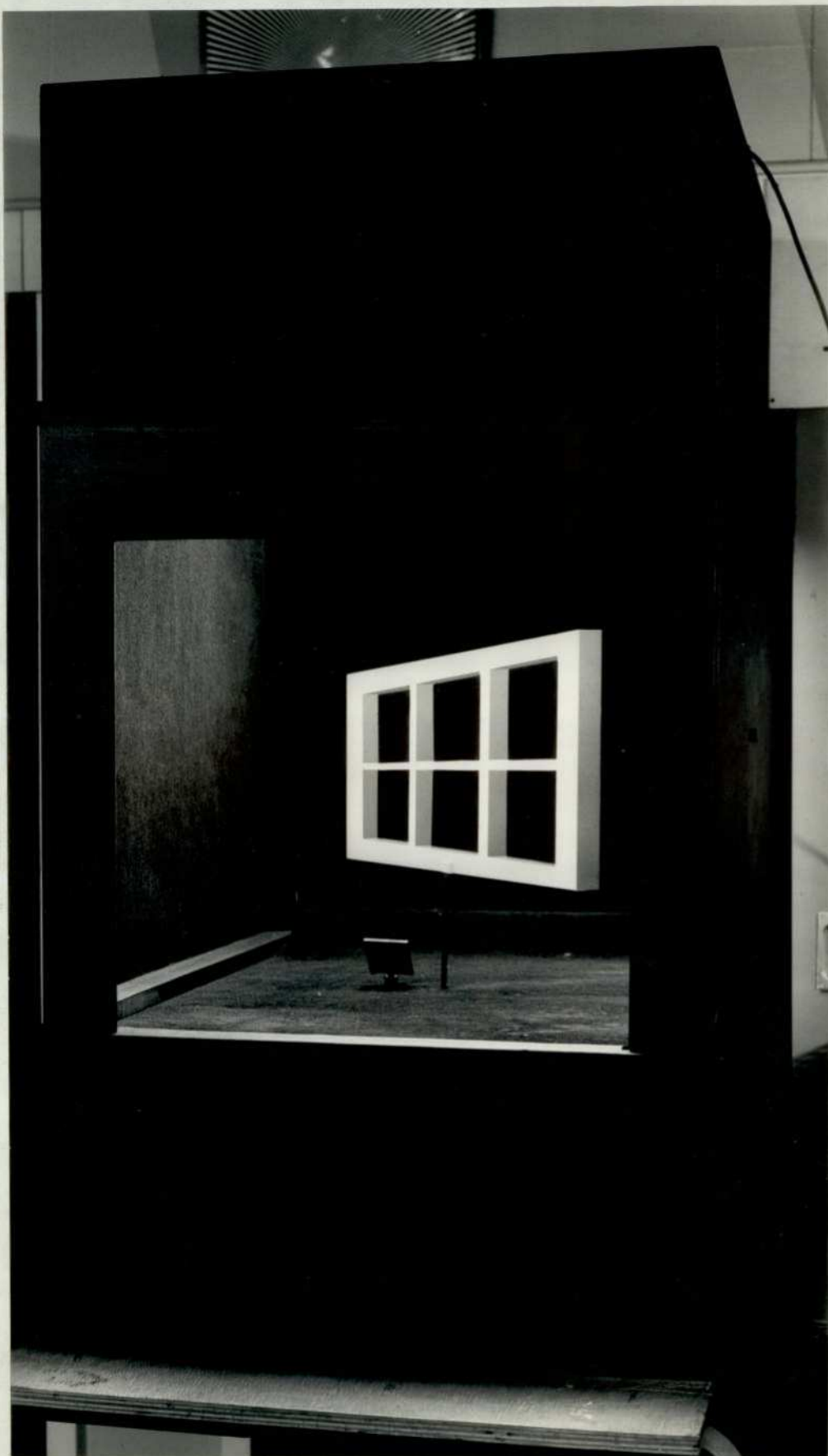


Fig. 5.1
The Apparatus.

THE APPARATUS:

The apparatus was originally designed to be viewed directly by the subject (S), and so was constructed in such a way that all the mechanics of the system were hidden from view. However, for reasons detailed below film presentation was eventually used, and the apparatus was modified accordingly.

The apparatus consisted basically of three boxes, mounted one upon the other, as shown in the photographs - Figs.5.1 & 5.3; Fig.5.2 shows the three sections diagrammatically, together with their dimensions.

The bottom section housed the motor systems, which were accessible through a drop-flap opening in the side (shown open in Fig.5.3). The vertical shaft upon which the stimulus shapes were mounted, passed through a hole in the top of this box. A second hole was made for the position-indicator system described below.

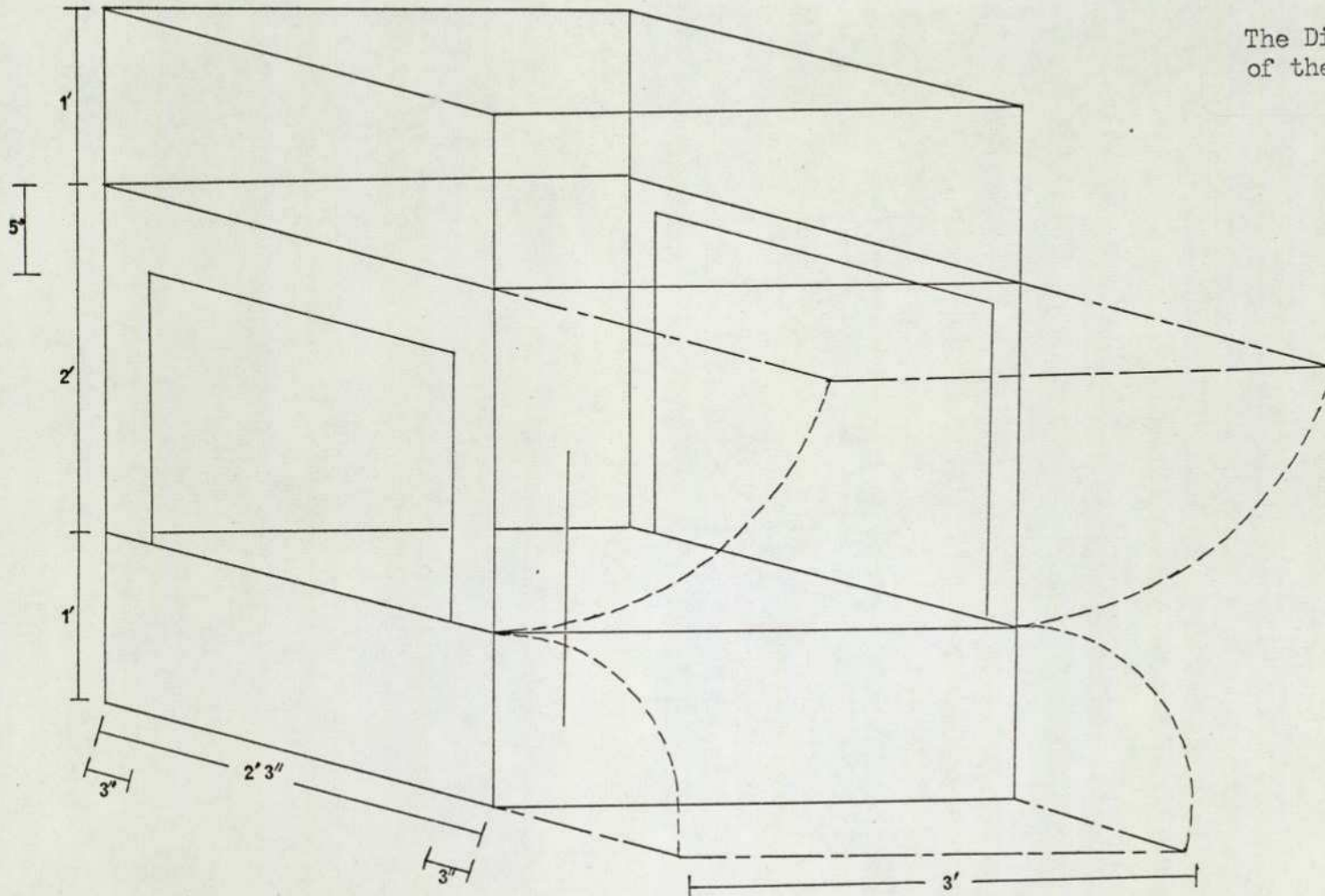
The sides, back, and base of the middle section were completely lined with black flock paper to give as dark a surround as possible. Trial films, however, showed that this background was not 'black' enough when filmed, so the back section was cut out and the apparatus was positioned in the doorway of a small dark-room whilst being filmed. One side of this middle section could be raised so that the stimulus shapes could easily be changed.

The lighting system was housed in the top section; this consisted of four 20watt 'daylight' fluorescent tubes, each 2 ft long, mounted lengthwise at the top with an $\frac{1}{2}$ inch thick 030 opal diffusing screen 8 inches below them, giving even illumination over the screen.

The 'proscenium arch' at the front of the middle section screened the opal diffuser from direct view. The inside of both the top and bottom sections were painted flat white, and the whole outside painted matt black.

Fig.5.2

The Dimensions
of the Apparatus.



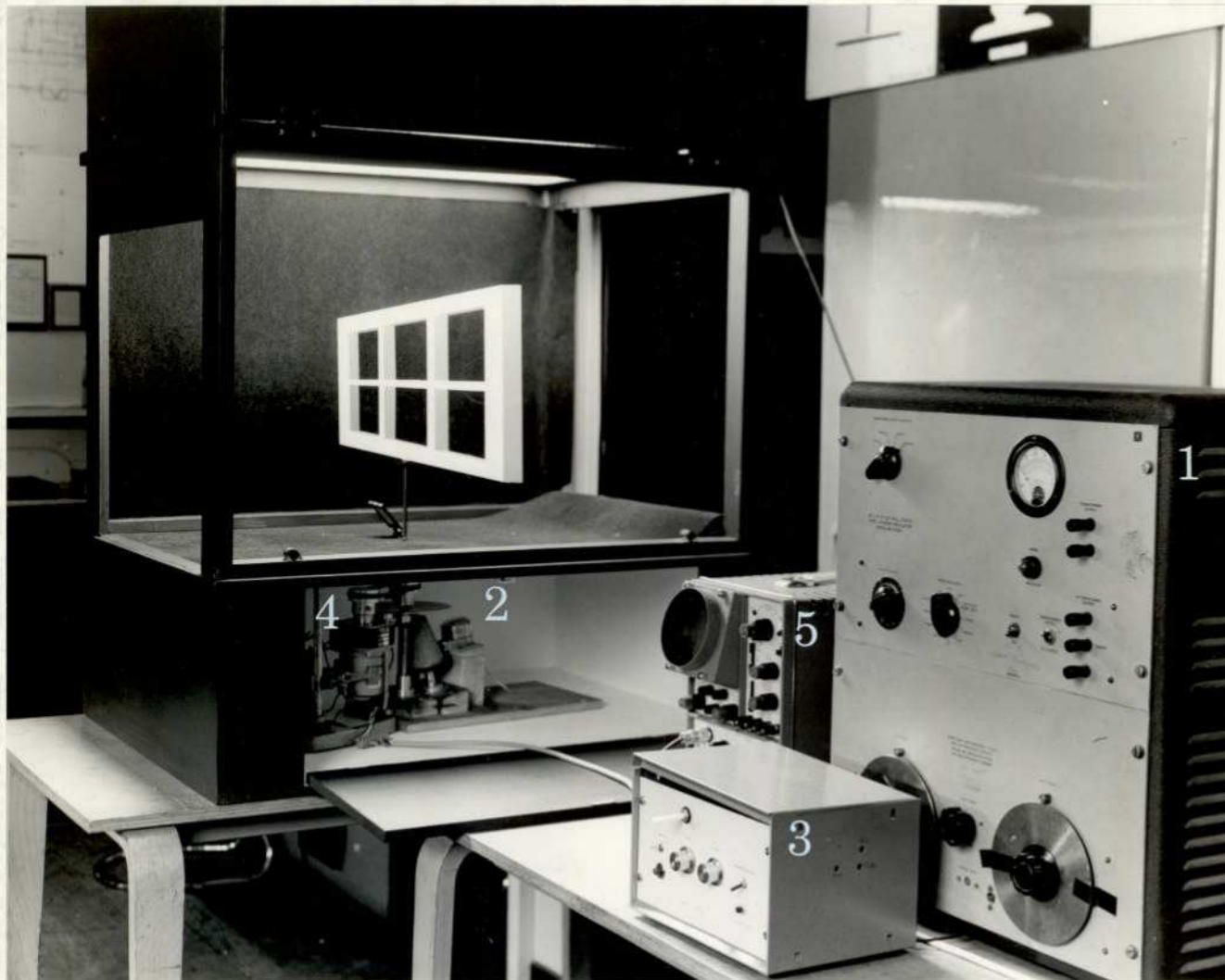


Fig. 5.3

Side View of the Apparatus showing:

1. Frequency Generator to alter speed of
2. Synchronous Motor
3. Control Box for
4. Stepping Motor
5. Oscilloscope to check speed of rotation.

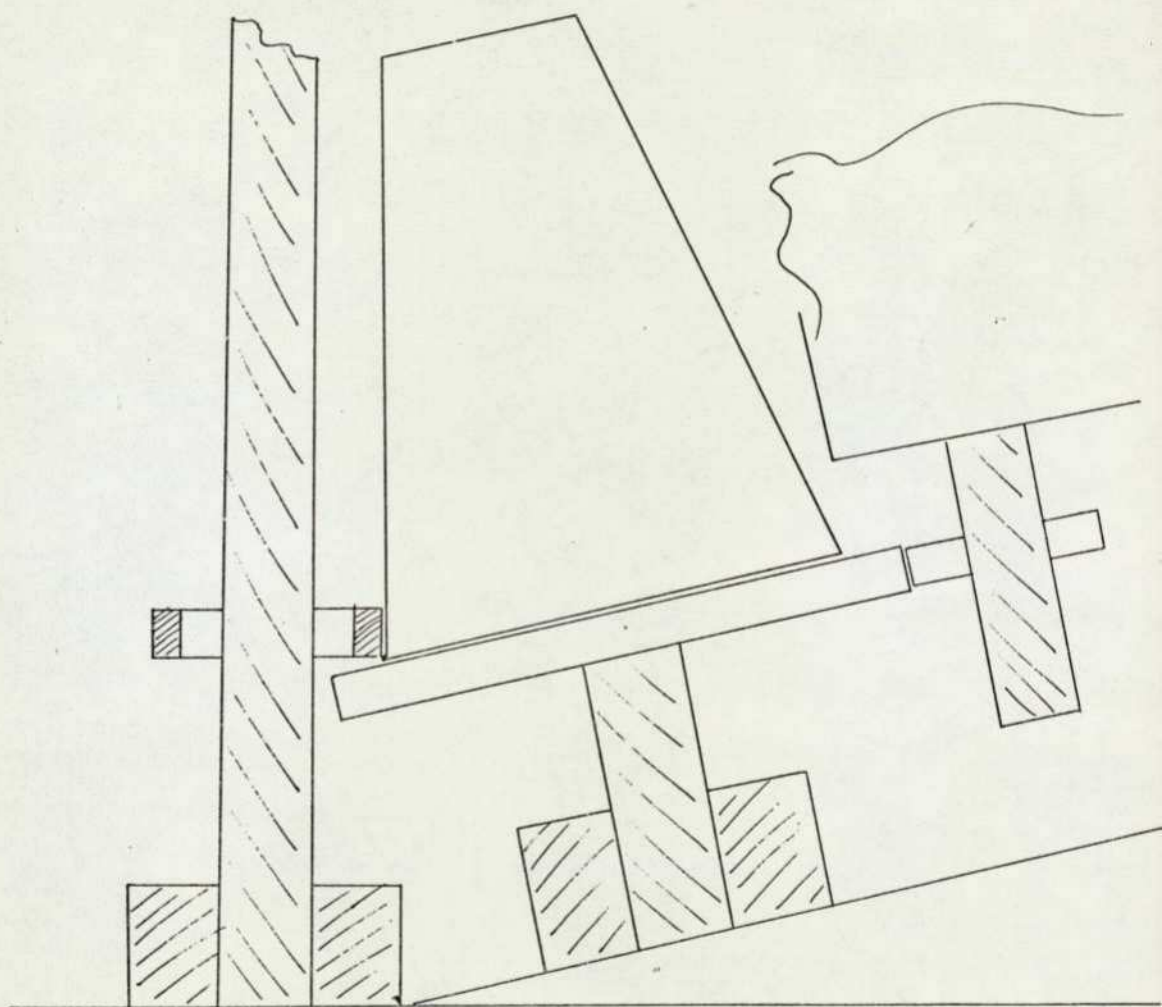


Fig. 5.4

Cone Gear for
Brush Motor.

MOTOR SYSTEMS:

The experimental design required a quiet variable - speed reversible motor which would give stable and repeatable results over a minimum range of 2 to 32rpm. Several methods were tried in an attempt to satisfy these requirements.

The first motor tried was a Parvalux series wound brush motor internally geared to give a nominal final speed of 10rpm with an input of 230/250volts A.C. Attempts were made to vary this speed with a voltage transformer, but this was found to be reasonably stable over a range of 8 - 16rpm only.

Auxiliary gearing systems were tried in an attempt to increase this range, the one finally chosen being shown in Fig.5.4. This consisted of a 3:1 reduction gear, upon which was mounted a wooden cone giving a continuously variable gear with a maximum range of 1:2. A variable-position rubber wheel drove the vertical shaft upon which the shapes were mounted, when it was in contact with this cone.

This combination of variable cone height and voltage variation gave the required range of speeds, but even with a stabilised power supply their repeatability from day to day was unacceptable.

Another feature of this motor was the noise it produced, which was considered to be unacceptable for use with direct viewing by S's.

The brush motor was then replaced by a silent Crouzet synchronous motor giving 60rpm at 50cps; this is shown most clearly in Fig.5.5, where it is in the position previously occupied by the brush motor. The speed was varied by altering the frequency of the input signal to the motor with the frequency generator shown in Fig.5.3, and it was found possible to obtain the required range by this means.

Unfortunately the frequency generator became more and more unstable with time, and gave increasingly large day to day variations which meant that recalibration was necessary almost daily. This was especially so over the higher speeds required, until eventually the generator broke down completely and a third method had to be devised. (The frequency generator could not be replaced since similar high-power types were no longer available). The most successful motor used was an Impex 4 phase stepping motor (PD 18) (see Fig.5.5). The vertical shaft turning the various stimulus shapes

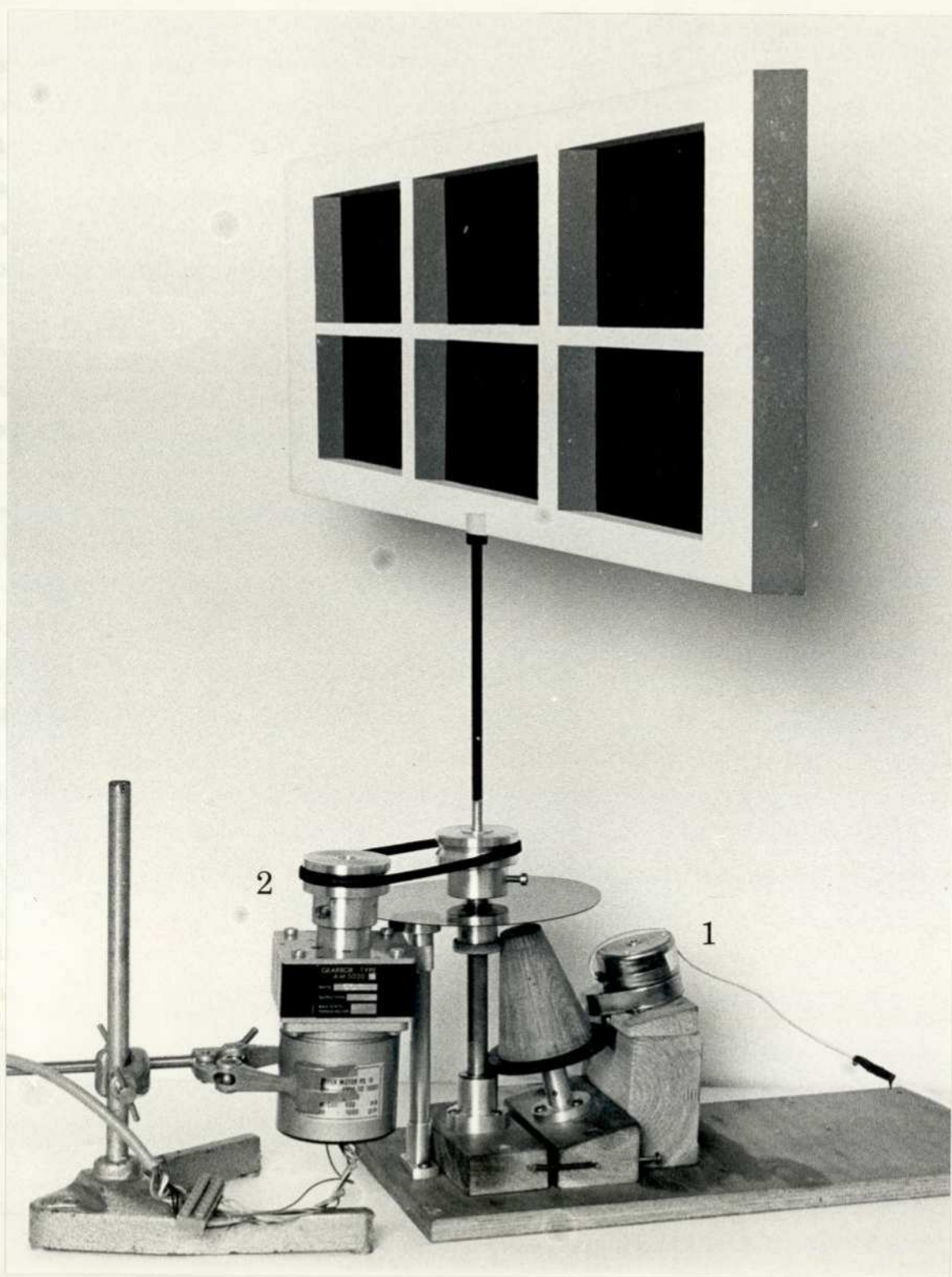


Fig. 5.5

Two of the Motor Systems:

1. Synchronous Motor
2. Stepping Motor

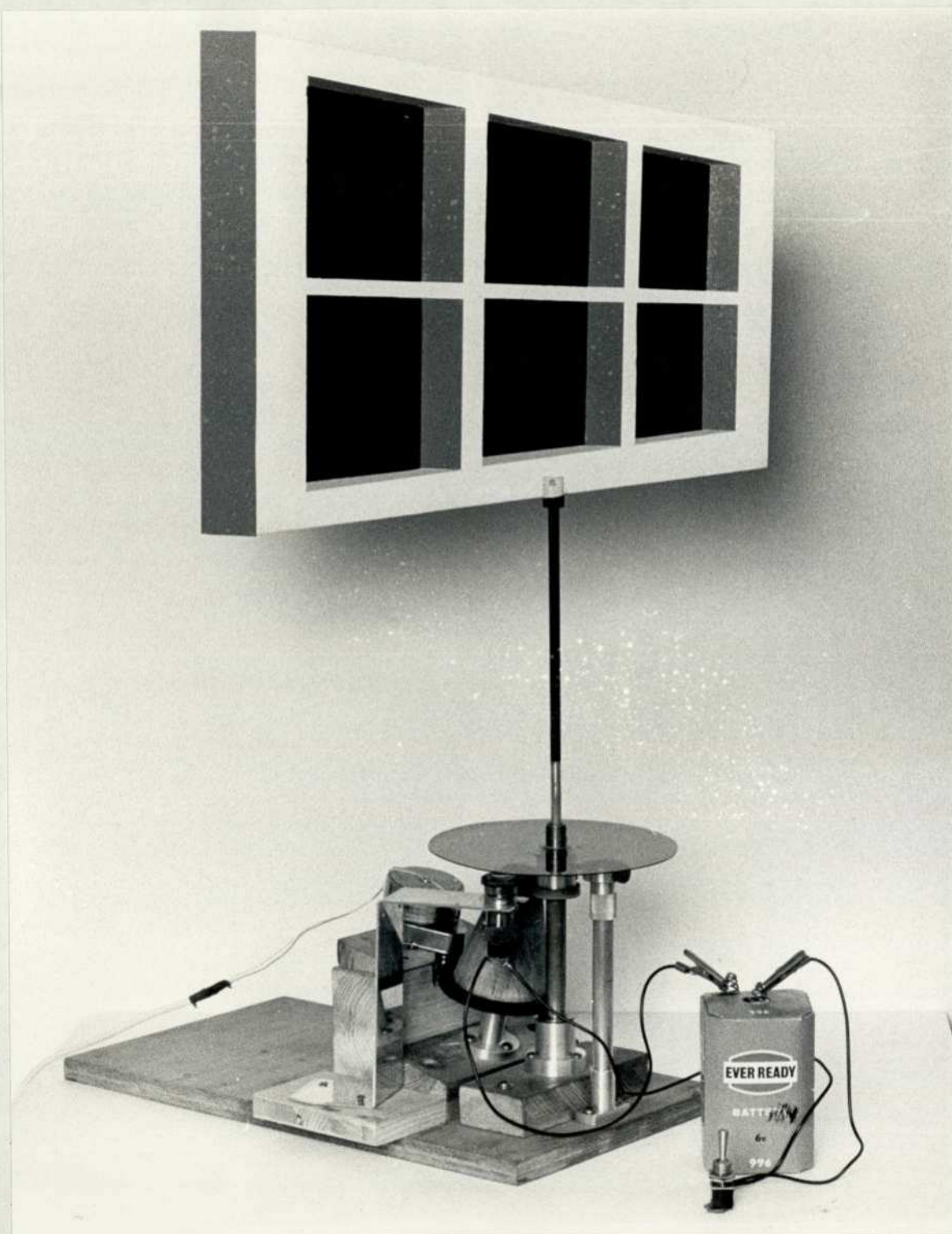


Fig. 5.6

Polaroid Transducer
for checking motor speeds.

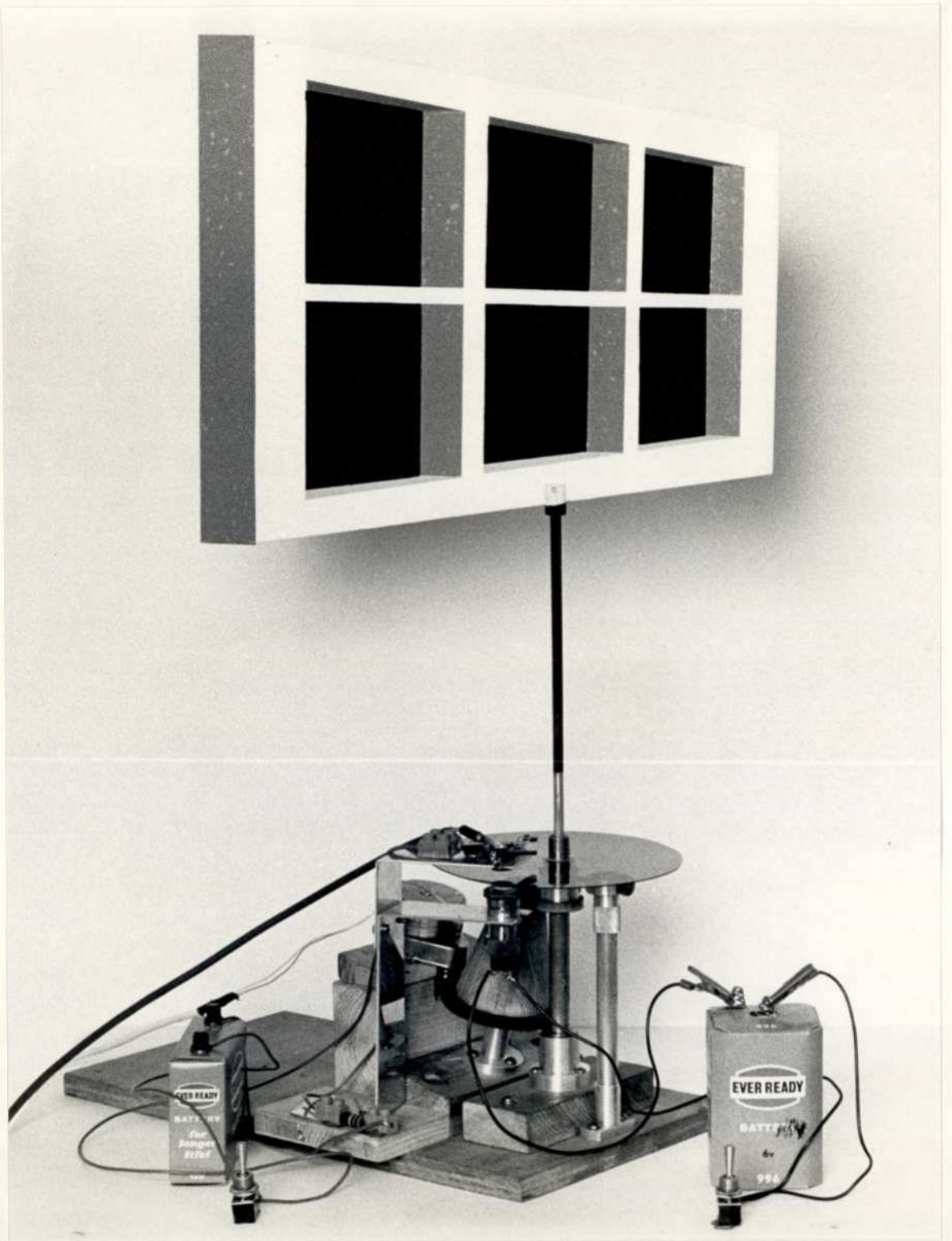
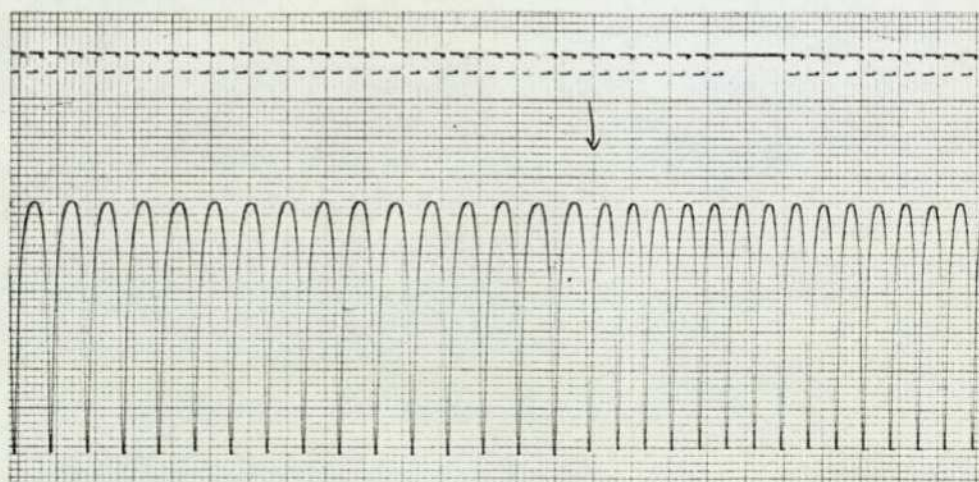
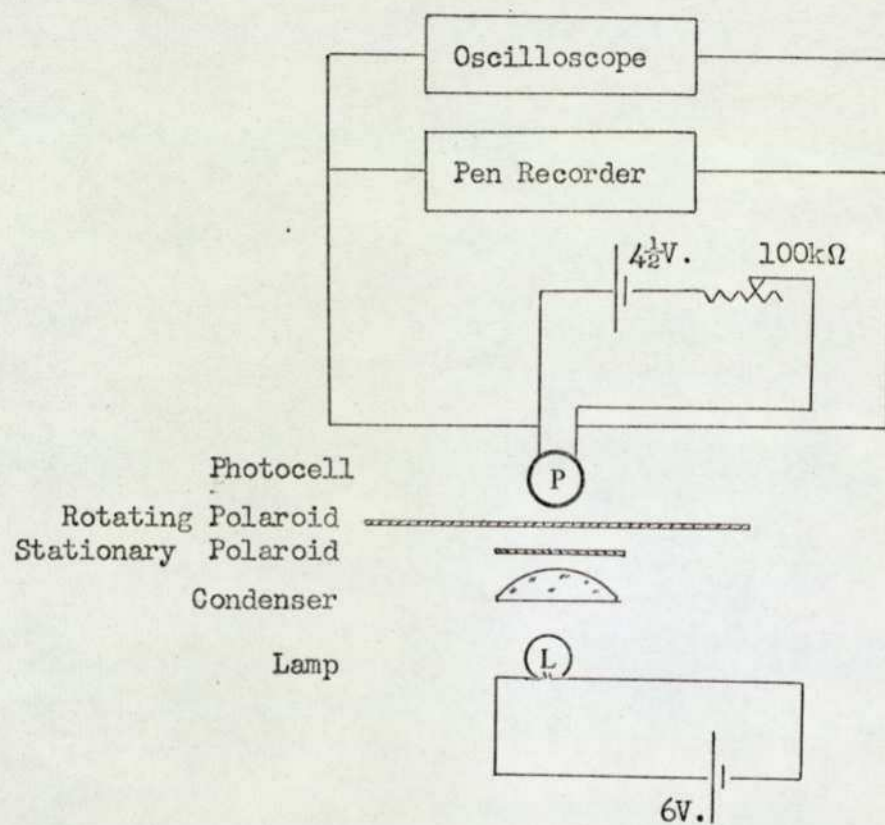


Fig. 5.7
Polaroid Transducer
with photo-cell.

Typical Output from Pen Recorder.

Fig. 5.8

Circuit Diagram for
Polaroid Transducer



Typical Output from Pen Recorder.

TRAPEZOIDS:

All the different shapes used were cut from 20gauge sheet Dural, which was both strong and rigid, and could be cut with great accuracy (± 0.5 mm.) The precise dimensions are given in the experimental details below. Each was bored with a small screw hole about $\frac{1}{4}$ inch from the lower edge, indicating the axis of rotation position; they were slotted into the vertical shaft and screwed tight so that they were held rigidly. Three different vertical shafts were constructed - one for each of the trapezoid shapes used in experiments 1 & 2, with the slot angled precisely to hold the trapezoid in the correct position, and the third with a horizontal slot. All the other shapes were cut with a short horizontal section to fit squarely into this holder.

All the shapes were spray-painted first with anti-rust metal primer, and then with several coats of U-Spray matt white. The shadow effects for experiment 1 were produced with Letrafilm 175M (dark grey), 173M (mid-grey) and 171M (light grey). The cut-out effects were achieved by sticking on black flock paper. This gave a very realistic three-dimensional effect, and was much more accurate than actually cutting out the holes. Such a 'window' is shown clearly in Fig.5.7.

FILM PRESENTATION:

Several factors contributed to the choice of film presentation as the most convenient and suitable form, some of which have been indicated already. The complete control over the ambient conditions and the standardisation that this permits was the most important single factor.

The experimental design employed meant that each S viewed at least two different shapes, with a short trial run followed by 2 minutes observation. With film presentation this was easy to achieve - the timing was precise, each S saw exactly the same piece of film, and the film could be put together in the required order. But with direct observation the motor had to be started, and adjusted to give the required speed when necessary, and then stopped and the stimulus shape replaced by another; the motor

was re-started, re-adjusted, and so on. This was a tedious process for both S and the experimenter and was obviously much less accurate and more time-consuming.

The type of film used was Kodak 16mm 4-X negative, type 7224, shot in 40 second takes at 18 frames/sec. The projection speed was also at 18 frames/sec. The only source of illumination used was the light-box above the rotating shapes, all the extra room lights being switched off. The camera height was level with the centre of the shapes, and the distance from the camera to the axis of rotation was 12.5 feet; this distance was limited by the dimensions of the room available for filming, with a dark-room that could be used to provide the necessary black background.

Since it was only possible to film short takes of each shape used, several prints of each take were obtained and edited together to give the required length of film. These were then re-edited in random order (see below) to give:

trial run Shape 1 : 1 minute
blank black : 10 seconds approx.
observation Shape 1 : 2 minutes
blank black : 10 seconds approx.
trial run Shape 2 : 1 minute
etc...

The actual order in which the film was edited together is indicated by table 5.1 (p. 81).

It was considered important that no information should be lost to the subject through using film presentation, and after careful consideration it was thought that this condition had been met. The film was essentially a monocular view of the rotating stimulus objects, viewed at a distance of 12.5 feet. This method of presentation had the additional advantage of effectively eliminating depth information provided by accommodation, convergence or binocular cues.

The Bauer F6 Automatic cine projector was 13.5 feet from the screen, and the subject (using monocular vision) was 12.5 feet from the projection screen, as shown in Fig.5.9.

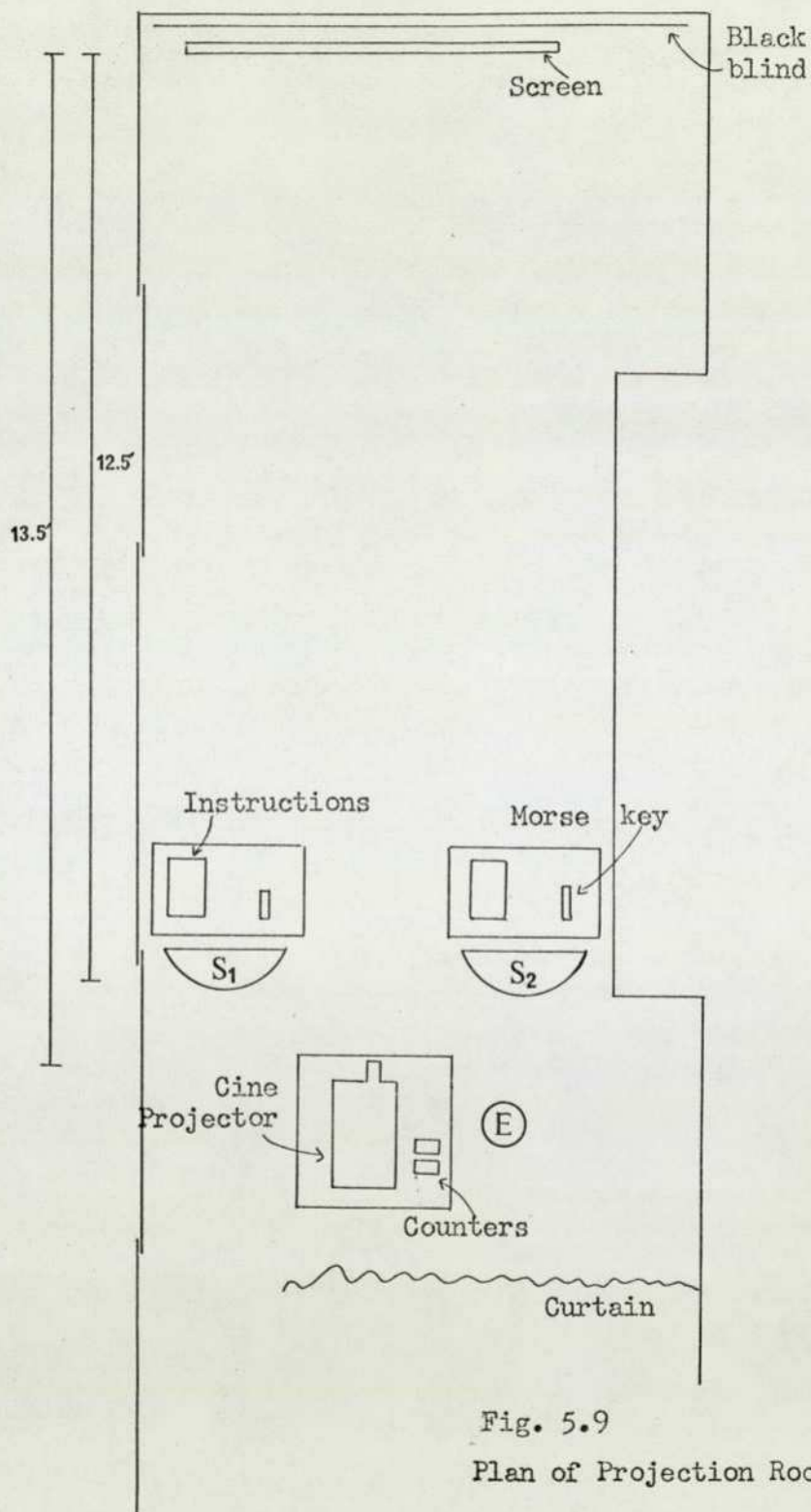


Fig. 5.9
Plan of Projection Room.

However, although the shapes were filmed, projected and viewed at practically the same distance, differences in the lens systems of the camera and cine projector resulted in a magnification of the shapes on the screen of approximately X2 (precise values are given with the details of each experiment).

This magnification resulted in the effective size of the shapes used being much larger than those used in many other studies of apparent oscillation. The limitations that this imposes on comparisons with the results of other studies is discussed critically in Section 7.

The projection screen used was of rigidly mounted hardboard, 6ft tall x 5ft wide, and painted matt white; a rigid screen was chosen to eliminate any possible distortion due to movement of the screen itself. The room was dark, although not completely black, and the illumination on the screen was:

White window 40 lux maximum

'Black' background 0.75 lux.

RECORDING RESULTS:

The measure of the illusion used for these experiments was the frequency of apparent oscillation during a two minute period, calculated as a percentage of the maximum possible.

Subjects were asked to signal an apparent change in direction of motion by pressing a micro-switch roller, which activated an electro-mechanical counter; since these counters were non-resettable, their values were recorded both before and after a period of observation.

An alternative method of recording results was used during preliminary experiments, and was adapted for use with film presentation, although the method was eventually abandoned.

The polaroid transducer was arranged in such a way that the peak on the pen-recorder trace occurred when the stimulus shape was exactly perpendicular to the line of sight, indicating the 'zero', and S's micro-switch triggered an event-marker on the pen-recorder. A trace was then obtained which indicated exactly where, during a 360° revolution, the apparent oscillations occurred. For the purposes of these preliminary experiments

subjects were asked to indicate when they saw the shapes stop, and where they appeared to start moving again. Occasionally these stop-start positions were almost simultaneous, but often there was a considerable interval between them.

This method was modified for film presentation by removing the photocell (as in Fig.5.6) and allowing the light to pass through a small hole in the top of the base box onto a mirror inclined at 45° to the top of the box, so that the light was reflected towards the camera.

As the shapes rotated, this small light patch varied in intensity sinusoidally and could be monitored when the film was projected by mounting a photocell in the appropriate place on the projection screen.

This method of recording results obviously provided far more information than the simple frequency count, but the analysis proved to be far too lengthy when a large number of observations were required, and it was therefore abandoned for the main series of experiments.

SUBJECTS:

Experiments 1 to 4 were run simultaneously, the subjects used being selected from the students and staff of the Department. The only condition imposed was that each S should have 6/6 V.A. (or better) in the eye used for making observations, with spectacle correction if necessary.

Six S's (3male & 3 female) observed each condition in each experiment. Each S made a maximum of 3 observations, one from each of three groups:

Group 1 - experiment 1 - 21 conditions

Group 2 - experiments 2&4 - 19 conditions

Group 3 - experiment 3 - 24 conditions

Each group was considered to consist of 24 conditions (some of which might be "no observation" in groups 1 & 2), and a number (1 - 24) was assigned to each treatment condition. These numbers were then arranged in random order for each group, as tabulated below (Table 5.1)

Then S1 viewed Group 1 condition 5; Group 2 condition 3; Group 3 condition 13;
S2 viewed Group 2 condition 3; Group 3 condition 13; Group 1 condition 12;
and so on.

Table 5.1: RANDOM ORDERS FOR PRESENTATION
OF 24 CONDITIONS IN EACH OF
THREE EXPERIMENTAL GROUPS.

Group 1	Group 2	Group 3
5	3	13
12	18	11
24	16	2
13	22	16
10	1	4
6	23	18
17	4	23
18	8	8
16	21	20
19	7	22
14	13	9
7	24	17
15	2	24
23	15	1
21	11	5
3	10	15
8	19	14
22	12	6
20	14	7
9	17	10
2	20	19
11	6	3
4	5	21
1	9	12

The original random orders were modified slightly to eliminate identity (or close similarity) between the conditions observed by a single subject from the three different groups, and no subject had more than one "no observation" condition.

A total of 144 subjects were required, 72 males and 72 females, and two subjects (1 male & 1 female) were tested simultaneously.

The following instructions were typed out and stuck on the table for S to read:

"Watch the film on the screen carefully. You will be shown a series of different moving shapes, which may be turning sometimes in one direction, sometimes in another; sometimes they will change direction & sometimes they won't change at all.

All you have to do is to press the lever in front of you each time you see a change in direction. Your observations may be different from your neighbour's - don't worry if they are. There is no right or wrong answer - I am interested only in what you observe.

Use just one eye; there is an occluder on the table for you to cover the other eye. There will be a shorter practice run before each set of readings.

Any questions?

Do no discuss the experiment with anyone outside this room."

Instructions to S's,
Experiments 1 - 4.

S's were asked if they understood the instructions; if in any doubt they were invited to ask questions after the first trial run, when the film could be stopped. They were also asked to fixate the centre of the screen.

The noise of the cine projector successfully masked the faint clicking of the micro-switches, and examination of the results showed that there was no interaction between pairs of subjects.

Experiments 5 & 6 required only four S's each, all of whom were very familiar with the Ames trapezoidal illusion to minimise possible learning effects with repeated observations. They were run individually, the only modification in the instructions being in the second paragraph, for which the following was substituted:

"Please press the lever in front of you each time you see a change in the direction in which the shape is moving."

This summarises the main features of the general design of the six experiments carried out in this study. Details of the apparatus and method used in a particular experiment will be given in the description of that experiment, in the next section.

SECTION 6

EXPERIMENTAL INVESTIGATION DETAILS AND RESULTS

The main features of the general design of the six experiments carried out in this study were summarised in Section 5.

Details of the dimensions of the trapezoids used, and the detailed experimental design will be given with the description of each particular experiment.

The factors investigated in the six experiments were:

- E1 Effect of Shape and Surface Pattern
- E2 Effect of Speed of Rotation
- E3 Height Ratio and Axis of Rotation Position
- E4 Projections of a Rectangle
- E5 Within - S and Between - S Variability
- E6 Effect of Fixation Position

The results obtained in these experiments will be compared with the published data obtained in similar studies by others, in Section 7.

EXPERIMENT 1

EFFECT OF SHAPE & SURFACE PATTERN

INTRODUCTION

It was suggested in Section 4 that the most important single factor contributing to the illusion of apparent oscillation produced by a rotating trapezoidal object is the outline shape of the object, and that surface pattern provides a secondary source of information.

In general, this has been confirmed by other studies; but many of these have used irregular or conflicting surface patterns (eg. Day and Power, 1963; Zegers and Murray, 1962; Cross and Cross, 1969).

Ames' emphasis on 'window' patterns relates to his transactional explanation of the phenomenon, but since his choice of the "most effective" shape and pattern for producing apparent oscillation (Ames, 1951) was not supported by any quantitative data, it was decided that these factors should be investigated first.

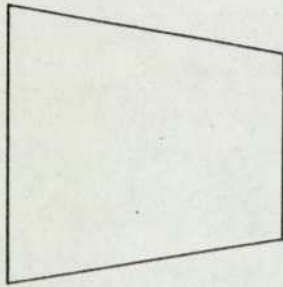
Attneave (1954) has shown that visual information is concentrated along contours (i.e. regions where colour changes abruptly) and is further concentrated at those points on a contour at which direction changes most rapidly (i.e. at angles or peaks of curvature).

The sequence illustrated in Fig. 6.1 thus provides a graded series of increasing information, although it is patently not an equal-interval scale. This sequence may be sub-divided into three sub-series - numbers 3, 5, 7 and numbers 1, 4, 6, investigating the effects of numbers of 'windows' with and without 'shadows' respectively; and numbers 1, 2, 3 investigating the effect of altering the outline contour.

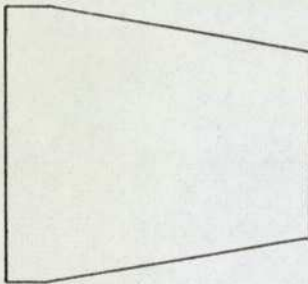
The sequence, then, provides three different aspects of pattern information that may be analysed separately.

Fig. 6.1.1

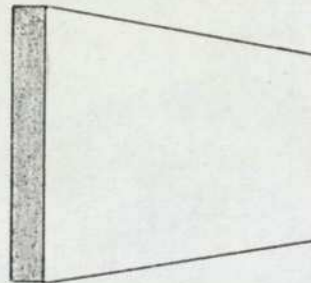
INCREASING PATTERN
INFORMATION



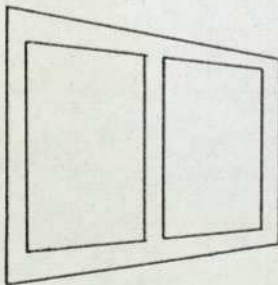
1. Solid
Matt White
No shadows
No windows



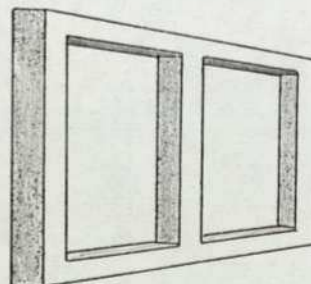
2. Solid white
'Window'
shape



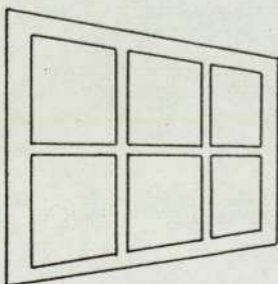
3. White
1 shadow
'Window'
shape



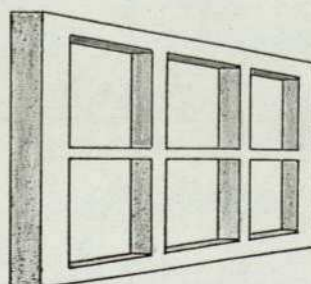
4. White frame
2 windows
No shadows



5. White frame
2 windows
Grey shadows



6. White frame
6 windows
No shadows



7. White frame
6 windows
Grey shadows

EXPERIMENTAL DESIGN

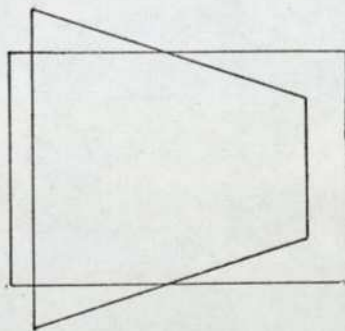
The effect of increasing the amount of information provided by the surface pattern was investigated at 7 different levels for each of three different outline shapes. Shape 2 (labelled T2) was equivalent to Ittelson's version of Ames' trapezoid (Ittelson, 1952); Shape 1 (labelled T1) was a more exaggerated trapezoid; and Shape 3 (labelled R) was a rectangle.

The outline dimensions of the shapes used are shown in Fig.6.1.2.

The dimensions of the rectangle were chosen such that trapezoids could be possible 'perspective' representations of the rectangle rotated about a central vertical axis (judged visually, rather than mathematically). This is illustrated in Fig.6.1.3, which shows the shapes superimposed.

Fig. 6.1.3

Shapes
T1 and R



Shapes
T2 and R

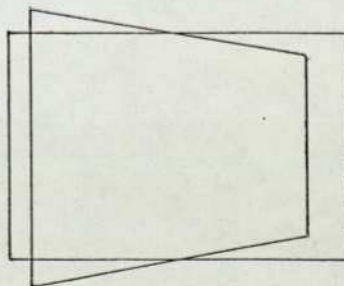


Fig 6.1.2

PATTERN	SHAPE		
P1	T1	T2	R
P2			
P3			
P4			
P5			
P6			
P7			

The basic experimental design is shown in Table 6.1.1; the table also indicates the number allocated to each treatment condition for the random allocation of subjects (see page 80, section 5).

Table 6.1.1

Pattern Information*	Shape		
	T1	T2	R
P1	1	2	3
P2	4	5	6
P3	7	8	9
P4	10	11	12
P5	13	14	15
P6	16	17	18
P7	19	20	21
* "no observ- ation"	22	23	24

* see Fig. 6.1.2

* see p. 81, section 5

Three male and three female subjects viewed each treatment condition. There were therefore three factors being investigated - shape (at 3 levels), surface pattern (7 levels) and sex (2 levels).

All the shapes were rotated at a constant speed of 5rpm, and film presentation was used as described in Section 5.

Thus over a 2 minute observation period a possible total of 20 apparent changes in direction could be observed.

The magnification produced by the projection system was X2, so that the outline dimensions of the shapes on the screen were actually twice those shown in Fig. 6.1.2.

The randomised group design for the 3 factor analysis of variance used may be summarised as follows:-

Factors

Shape	= A	- 3 levels
Pattern	= P	- 7 levels
Sex	= C	- 2 levels

Thus there are $(3)(7)(2) = 42$ treatment combinations.

$n = 3$ for all treatments

Thus total $(3)(42) = 126$ observations.

Then the total sum of squares with $(126-1) = 125$ d.f. can be partitioned into the following components with degrees of freedom as shown at the right:-

Main effects:	A	2
	P	6
	C	1
2 factor interactions:	A x P	12
	A x C	2
	P x C	6
3 factor interactions:	A x P x C	12
Within treatment:	error	$125-41 = 84$ d.f.

Then A1 = T1	P1	C1 = Male
A2 = T2	P2	C2 = Female
A3 = R	P3	} see Fig. 6.1.2
	P4	
	P5	
	P6	
	P7	

RESULTS

All the observed changes in direction of rotation were recorded as a percentage of the possible maximum.

The spread of the data was found to approximate more closely to a normal distribution after using a square-root transformation (see Edwards, 1969; Bartlett, 1936; Boneau, 1960).

This procedure was followed for all the results from experiments 1 to 4.

$$\text{Thus } X' = \sqrt{X + 1}$$

Where X' = "normalised" or transformed data

X = number of oscillations observed, expressed as % of maximum possible.

(Here $X = 100x/20$, where

x = number of oscillations observed).

Thus 2 minutes observation at 5 rpm should provide a theoretical maximum of

$$x = 20 \text{ oscillations}$$

$$X = 100\%$$

$$X' = \sqrt{100 + 1} = 10.05$$

and a minimum of

$$x = 0 \text{ oscillations}$$

$$X = 0\%$$

$$X' = \sqrt{1} = 1$$

The range of values of X' was thus 1→10.05.

The results obtained in this experiment are summarised in table 6.1.2.

Table 6.1.2

SUMMARY OF RESULTS OF EXPERIMENT 1

Each cell entry is the sum of $n = 3$ observations
(transformed data)

Sex	Pattern Information	Shape			Σ
		T1	T2	R	
C _M	P1	29.13	29.64	3	61.77
	P2	26.48	15.12	6.77	48.37
	P3	28.22	21.77	5.32	55.31
	P4	29.90	29.90	3	62.80
	P5	29.90	24.39	12.97	67.26
	P6	28.59	23.91	3	55.50
	P7	30.15	30.15	6.77	67.07
C _F	P1	30.15	30.15	3	63.30
	P2	26.50	17.94	3	47.44
	P3	29.65	30.15	3	62.80
	P4	29.64	28.82	3	61.46
	P5	30.15	26.74	17.18	74.07
	P6	28.05	22.65	3	53.70
	P7	30.15	30.15	4.45	64.75
Σ		406.66	361.48	77.45	845.59

A summary of the 3 factor analysis of variance is given in table 6.1.3.

Table 6.1.3.
SUMMARY OF ANALYSIS OF VARIANCE FOR EXPERIMENT 1

Source of Variation	Sum of Squares	d.f.	Mean Square	F	α
Shape - A	1516.5487	2	758.274	348.8	0.001
Pattern - P	74.4155	6	12.403	5.7	0.001
Sex - C	0.7083	1	0.708		
A x P	101.2506	12	8.437	3.9	0.001
P x C	5.7340	6	0.955		
A x C	3.0641	2	1.532		
A x P x C	1.8715	12	0.156		
Within treatments	182.6327	84	2.174		
Total	1886.2254	125			

This shows that the outline shape had the most significant effect upon the amount of apparent oscillation produced ($\alpha = 0.001$). The surface pattern (P) and the A x P interaction were also significant ($\alpha = 0.001$). The sex of the subjects (C) had absolutely no effect on the amount of oscillation seen. (Because of this, none of the other interaction effects reached significance).

Scheffé's test for multiple comparisons (see Edwards, 1969 page 150) was used to test for significant differences between the various shapes and surface patterns used. (Scheffé, 1953)

This test can be used for making any and all comparisons of interest on a set of R means including those comparisons that may be suggested by the values of the means themselves. Thus, unlike most other statistical comparisons, it is not necessary to plan the particular comparisons in advance.

The only condition necessary before applying Scheffé's test to the results of any experiment is that the treatment mean square must be significantly larger than the within treatment mean square. If the

treatment mean square is not significant, then no comparison will be judged significant by Scheffé's test. This test is more conservative than other procedures for testing planned orthogonal comparisons - that is, larger differences will be required for significance. A lower standard might therefore be considered (eg. $\alpha = 0.10$ rather than $\alpha = 0.05$).

An example of the application of Scheffé's test is given in table 6.1.4. which is a multiple comparison between the $k (=3)$ different shapes used in this experiment.

The treatment sums to be compared are the sums of all the different observations on each shape in turn (given in the bottom row, table 6.1.2). Thus each sum is based on $(7) \cdot (2) \cdot (3) = 42$ observations.

Table 6.1.4.
POSSIBLE COMPARISONS BETWEEN $k = 3$
SHAPES, USING SCHEFFE'S TEST

(1) Comparison	(2) $\sum T_1$ 407	(3) $\sum T_2$ 361	(4) $\sum R$ 77	(5) $\sum a_i^2$	(6) D_i	(7) D_i^2	(8) $MS_{D_i} = \frac{D_i^2}{n \sum a_i^2}$
T1 vs T2	1	-1	0	2	46	2116	25.2
T1 vs R	1	0	-1	2	330	108900	1296.4
T2 vs R	0	1	-1	2	284	80656	960.2
T1 vs T2+R	2	-1	-1	6	376	141376	561
T2 vs T1+R	-1	2	-1	6	238	56644	224
R vs T1+T2	1	1	-2	6	614	376996	1496

It is not necessary that all of the comparisons shown in the table be made, but we can make any or all of the comparisons that are of interest with probability equal to or greater than $(1-\alpha)$ that all the statements concerning the significance of the comparisons are true.

Let any given comparison of the treatment sums be represented by D_i . The values of D_i , given in column (6) of table 6.1.4. are obtained by multiplying each of the treatment sums, given at the top of the table, by the corresponding coefficients in each row. For a given D_i to be a comparison, we require only that $\sum a_i = 0$ for the comparison; that is, for the sum of the coefficients for the comparison to be equal to zero. It is not necessary that the various comparisons be orthogonal. Then, with each sum based upon the same number of observations, we have

$$MS_{D_i} = \frac{D_i^2}{n \sum a_i^2}$$

as the mean square for the comparison of interest (column (8) of table 6.1.4).

Then, as a test for significance, we may find

$$F' = \frac{MS_{D_i}}{s^2}$$

where s^2 is the error mean square of the analysis of variance and

$$F' = (k - 1) F$$

F is the tabled value of F for $(k - 1)$ and $k(n - 1)$ d.f.

Then, for significance, we have

$$MS_{D_i} \geq (F') (s^2)$$

Multiple Comparison for Shape (A)

$$\begin{aligned}k &= 3 \text{ levels} \\n &= 42 \text{ observations} \\s^2 &= 2.17 \text{ (see table 6.1.3)}\end{aligned}$$

For significance,

$$\begin{aligned}MS_{D_i} &\geq (F') (s^2) \\&\geq (k - 1) (F) (s^2)\end{aligned}$$

Tabled values of F for $(k - 1) = 2$ and $k (n - 1) = 123$ d.f.

are $F = 7.32$ for $\alpha = 0.001$

$F = 5.54$ for $\alpha = 0.005$

$$\begin{aligned}\text{Thus } MS_{D_i} &\geq 31.77 \text{ for } \alpha = 0.001 \\&\geq 24.04 \text{ for } \alpha = 0.005\end{aligned}$$

Thus from table 6.1.4, the comparison between T1 and T2 is significant with $P = 0.5\%$, and all the other comparisons are significant with $P = 0.1\%$.

The results of Scheffé's test thus showed that there was a significant difference ($P = 0.5\%$) in the amount of apparent oscillation produced by the two trapezoidal shapes, with T1 giving more oscillation than T2 (Ames shape). It was also shown that the two trapezoidal shapes produced very many more oscillations than the rectangle ($P = 0.1\%$).

Multiple Comparisons for Pattern (P)

The sequence of surface patterns used in this experiment provided a graded series of increasing information (see Fig.6.1). This sequence may be sub-divided into 3 sub-series - P3, P5, P7 and P1, P4, P6, investigating the effects of numbers of 'windows', with and without 'shadows' respectively; and P1, P2 and P3 investigating the effect of altering the outline contour.

Significant differences between the amounts of apparent oscillation produced by these various treatment conditions can be investigated in several ways - for example, by considering a 3 factor analysis of

Comparison	ΣP_1	ΣP_2	ΣP_3	ΣP_4	ΣP_5	ΣP_6	ΣP_7	Σa_i^2	D_i^2	D_i	$\frac{MS_{D_i}}{D_i^2} / n \Sigma a_i^2$
	125	96	118	124	141	109	132				
P_1 vs P_2	1	-1	0	0	0	0	0	2	29	841	23.4
P_2 vs P_3	0	1	-1	0	0	0	0	2	22	484	13.4
P_1 vs P_3	1	0	-1	0	0	0	0	2	7	49	1.4
$P_1 + P_4 + P_6$ vs $P_3 + P_5 + P_7$	-1	0	1	-1	1	-1	1	6	33	1089	10.1
P_7 vs P_1	-1	0	0	0	0	0	1	2	7	49	1.4
P_7 vs P_6						-1	1	2	23	529	14.7
P_5 vs P_4				-1	1			2	17	289	8.0
P_5 vs P_2		-1			1			2	45	2025	<u>56.3</u>
$P_5 + P_7$ vs P_3			-2		1		1	6	37	1369	12.7
$P_5 + P_7$ vs P_1	-2				1		1	6	23	529	4.9
$P_4 + P_6$ vs P_1	-2			1		1		6	17	289	2.7

TABLE 6.1.5
COMPARISONS BETWEEN SURFACE PATTERNS

variance with shape, windows and shadows as the 3 factors, and a 2 factor analysis of variance with shape and outline contour as the factors.

A simpler approach is to use Scheffé's test to look for significant differences between the treatment sums. The various comparisons are summarised in table 6.1.5.

Then $k = 7$ levels
 $n = 18$ observations
 $S^2 = 2.17$

For significance, $MS_{Di} \geq (F') (S^2)$
 $\geq (k - 1) (F) (S^2)$

Tabled values of F for $(k - 1) = 6$ and $k (n - 1) = 119$ d.f.

are $F = 4.04$ for $\alpha = 0.001$
 $F = 1.87$ for $\alpha = 0.10$

Thus $MS_{Di} \geq 52.6$ for $\alpha = 0.001$
 ≥ 24.3 for $\alpha = 0.10$

Thus from table 6.1.5, the comparison between P5 and P2 is significant with $\alpha = 0.001$, but none of the other comparisons reach significance.

Thus the pattern producing the highest number of apparent oscillations (P5) produced significantly more than the pattern causing the lowest number of apparent oscillations, but increasing the numbers of windows, or adding shadows, had no significant effect.

Since there was such a significant difference between the amount of oscillation produced by the two trapezoidal shapes (T1 and T2), and that produced by the rectangle (R), it was considered that the effect of surface pattern on the 2 trapezoidal shapes may have been somewhat different from the overall effect. A comparison for pattern was therefore carried out with the treatments summed over T1 and T2 only, and both levels of C (instead of T1, T2 and R and both levels of C). These comparisons are shown in table 6.1.6.

Comparison	$\Sigma P1$	$\Sigma P2$	$\Sigma P3$	$\Sigma P4$	$\Sigma P5$	$\Sigma P6$	$\Sigma P7$	Σa_i^2	D_i	D_i^2	$MS_{D_i} = \frac{D_i^2}{n} \Sigma a_i^2$
	119	86	110	118	111	103	120				
P1 vs P2	1	-1	0	0	0	0	0	2	33	1089	45.4
P2 vs P3		-1	1					2	24	576	24
P1 vs P3	1		-1					2	9	81	3.4
P7 vs P6						1	-1	2	17	289	12.0
P3 + P5 + P7 vs P1 + P4 + P6	-1	0	1	-1	1	-1	1	6	1	1	0.01
P7 vs P2		1					-1	2	34	1156	48.2
P5 vs P2		1			-1			2	25	625	26

TABLE 6.1.6.
COMPARISONS BETWEEN SURFACE
PATTERNS FOR TRAPEZOID SHAPES ONLY

Then $k = 7$ levels
 $n = 12$ observations
 $s^2 = 2.17$

For significance,

$$\begin{aligned} MS_{Di} &\geq (k - 1) (F) (s^2) \\ &\geq 13.02 (F) \end{aligned}$$

Tabled values of F for $(k - 1) = 6$ and $k(n - 1) = 77$ d.f.

$$\begin{aligned} \text{are } F &= 3.40 && \text{for } \alpha = 0.005 \\ F &= 1.86 && \text{for } \alpha = 0.10 \end{aligned}$$

$$\begin{aligned} \text{Thus } MS_{Di} &\geq 44.3 && \text{for } \alpha = 0.005 \\ &\geq 24.2 && \text{for } \alpha = 0.1 \end{aligned}$$

Thus from table 6.1.6 it can be seen that again there was a significant difference ($\alpha = 0.005$) between those patterns producing the highest amounts of oscillation (P1 & P7), and that producing the lowest amount (P2); but the other comparisons were not significant.

It was predicted that the pattern P2 would produce fewer oscillations than any of the other patterns, since this was the distorted outline shape, without any shadow to give meaning to the outline. This irregularity appears to have been adequate to reduce the amount of oscillation for these trapezoidal shapes.

These comparisons, then, have shown that there was not a gradual increase in the amount of oscillation with increasing pattern information. Neither was there any significant difference as the numbers of 'windows' or the amount of 'shadow' was increased. In fact, it was found that P1 and P7 produced practically identical frequencies of apparent oscillation for these trapezoidal shapes.

The fact that the A x P interaction (table 6.1.3) was significant, indicated that the effect of surface pattern had been different with different outline shapes.

But consideration of the effect of surface pattern on the two trapezoidal shapes has indicated that this was similar to the overall effect. The surface pattern must therefore have had a different effect on the amounts of oscillation produced by the rectangle.

This is shown most clearly by the Graph 6.1.1 (page 102), which shows the amount of oscillation produced (expressed as a percentage of the maximum possible) by each of the different surface patterns for the three shapes used. (The results shown are the means for each pattern, summed over both levels of C.)

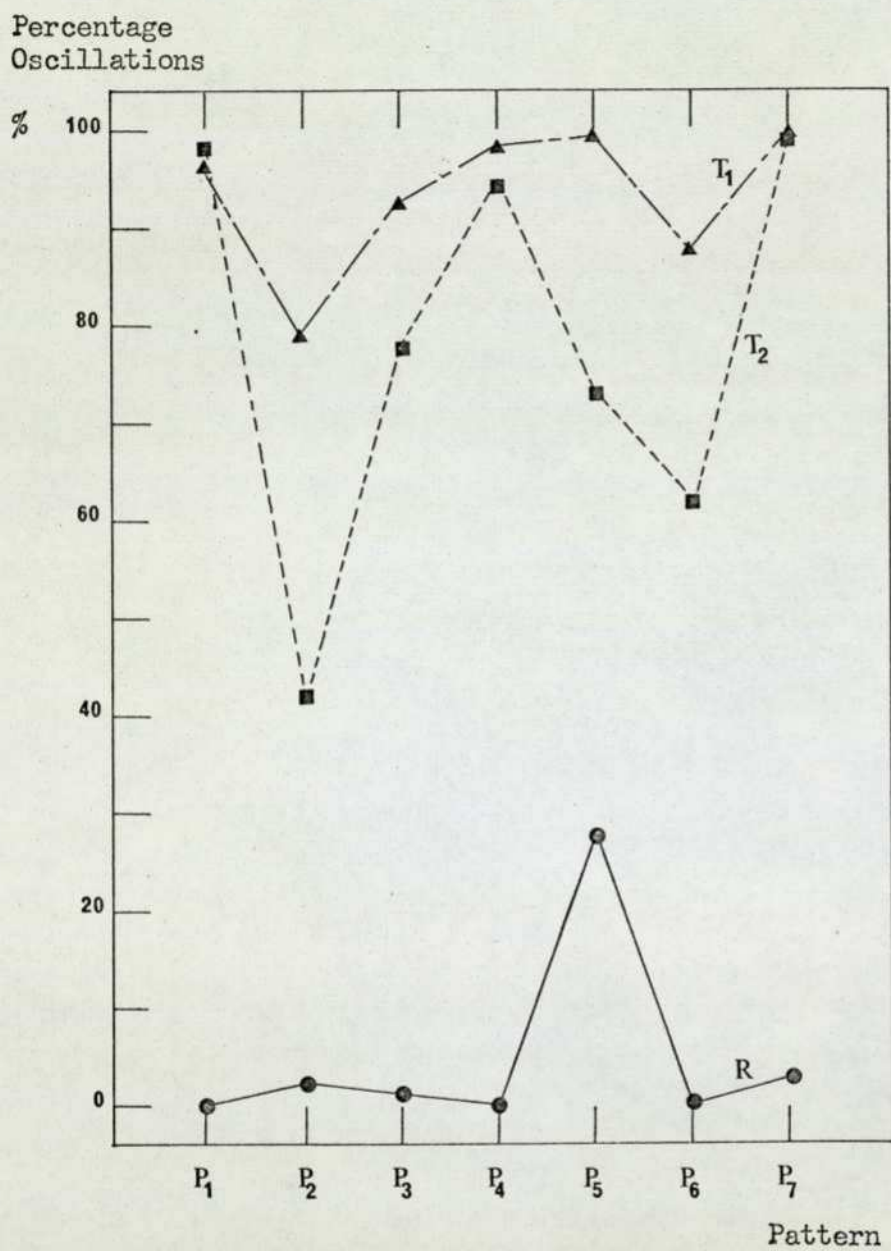
Clearly the pattern P5 (2 windows, with shadows) produced far more oscillations with the rectangle than any other pattern. Why this particular pattern should be so effective is unclear, particularly since only three of the six subjects who observed it recorded any oscillations. This may, therefore, be just a chance result, but the effect should be investigated further.

DISCUSSION

Three factors thought to contribute to the illusion of apparent oscillation with a rotating trapezoidal object were investigated in this experiment; these were the outline shape, surface pattern, and the sex of the subject.

It was found that the sex of the subjects used in the experiment had practically no effect on the results obtained (see table 6.1.3).

Because of this, it was possible to combine the observations of both male and female subjects when considering the detailed effects of outline shape and surface pattern. There were then 6 observations per treatment condition, instead of three.



Graph 6.1.1

Graph showing percentage of oscillations produced by each surface pattern for 3 different shapes.
(See Fig.6.1.2. for details)

The results obtained supported the hypothesis that the most important single factor in producing apparent oscillation is the outline shape of the rotating stimulus object (section 4, p 62).

But Ames' (1951) claim to have found the "most effective" shape for producing apparent oscillation was not verified by these results, since the shape T1 produced significantly more ($\alpha = 0.005$) apparent oscillations than did the Ames' shape T2. Presumably the "most effective" or optimum shape would produce apparent oscillation continuously; in this experiment Ames' trapezoid produced 77% of the maximum possible number of oscillations, whilst the more exaggerated trapezoid T1 produced 93% oscillations. With the rectangle, however, veridical rotation was seen 95% of the time.

The effect of altering the outline shape will be considered in more detail in Experiment 3, when the amount of oscillation obtained with these (and other) shapes will be compared with predictions based on consideration of the elliptical projections of the corner points as they rotate.

Although the analysis of variance showed that surface pattern had a significant effect on the amount of apparent oscillation produced ($\alpha = 0.001$), the detailed comparisons of the various treatment conditions did not indicate the nature of that effect.

Although there was a significant difference between the patterns producing most oscillations (P5 for the rectangle, but P7 for T1 and T2) and the patterns producing least oscillations (P2 for T1 & T2), there was certainly not a gradual increase in the amount of oscillation with increasing pattern information. Neither was there a significant difference as the numbers of 'windows' or the amount of 'shadow' was increased. In fact, when the two trapezoidal shapes alone were considered, it was found that P1 and P7 produced practically identical frequencies of apparent oscillation.

Surface pattern, then, was a secondary source of information in this experiment. (It is possible, however, that in a situation where the outline shape was more ambiguous, the addition of surface pattern may have had a more significant effect).

EXPERIMENT 2

EFFECT OF SPEED OF ROTATION

INTRODUCTION

Ames (1951) suggested rotation speeds of 3 - 6 rpm as optimum for demonstrating the apparent oscillation of a rotating trapezoid. Subsequent examinations of this phenomenon have used speeds ranging from 5 rpm (Haber, 1965), through 8 rpm (Day and Power, 1963) to over 30 rpm (Freeman and Pasnak, 1968; Mulholland, 1963; Zenhausern, 1969).

Despite these large variations, investigators have compared their results without attempting to account for the fundamental discrepancies in their experimental designs.

The speed of rotation obviously affects the rate of change of the projected pattern on the retina, and so affects the rate of change of incoming information ; this may in turn affect the rate of interpretation of that changing pattern.

It was suggested above (Section 4, p. 61) that a very slow moving pattern may be essentially boring so that when two alternative interpretations are possible the percept may switch from one to the other, whereas a very fast changing pattern may not allow the brain time to differentiate adequately between possible perceptual hypotheses, so frequently causing the wrong interpretation to be made. It was also suggested that there may be a middle range over which variations in the speed of rotation have little effect on the resulting percept. Over this range, there is adequate time for incoming sensory information to be processed, and a decision made as to the type of motion being observed.

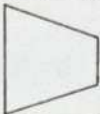
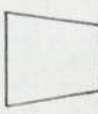
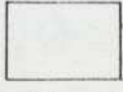
This hypothesis was tested by investigating the effect of altering the speed of rotation over a range of between 2 rpm and 32 rpm. (This covered the range of speeds that had been used by most other investigators of apparent oscillation).

If the hypothesis was verified, then the results from experiments incorporating rotation speeds from this middle range could safely be compared; but for faster or slower speeds it would be necessary to say that a different (although related) phenomenon was being investigated.

EXPERIMENTAL DESIGN

The effect of altering the angular velocity was investigated at five different levels for each of the three planar matt white shapes used in Experiment 1, pattern 1. The experimental design is summarised in table 6.2.1, which also indicates the number allocated to each treatment condition for the random allocation of subjects (see p.80, section 5).

Table 6.2.1.

	Speed (rpm)		Shape		
	Nominal	Actual	 A1 = T1	 A2 = T2	 A3 = R
S ₁	2	1.8	1	2	3
S ₂	4	4	4	5	6
S ₃	8	9.5	7	8	9
S ₄	16	16	10	11	12
S ₅	32	35	13	14	15

Three male and three female S's viewed each treatment condition. There were therefore three factors being investigated - shape (at 3 levels), speed (5 levels) and sex (2 levels).

Differences between the nominal and actual speeds of rotation were due to limitation of the stepper motor being used (p.72.).

The magnification caused by the projection system was again X2, so that the sizes of the shapes on the screen were the same as in Experiment 1.

The randomised group design for the 3 factor analysis of variance used may be summarised as follows:

Factors:

Shape	= A	- 3 levels
Speed	= S	- 5 levels
Sex	= C	- 2 levels

Thus there are $(3) (5) (2) = 30$ treatment combinations.

$n = 3$ for all treatments

Thus total $(3) (30) = 90$ observations

Then the total sum of squares with $(90-1) = 89$ d.f. can be partitioned into the following components with degrees of freedom as shown at the right:

Main effects	A	2
	S	4
	C	1
2 - factor interactions	A X S	8
	A X C	2
	S X C	4
3 - factor interaction	A X S X C	8
Within treatment	Error	$89-29 = 60$ d.f.

Then A1 = T1	S1 = 2 rpm	C1 = M
A2 = T2	S2 = 4 rpm	C2 = F
A3 = R	S3 = 8 rpm	
	S4 = 16 rpm	
	S5 = 32 rpm	

RESULTS

The results obtained are summarised in table 6.2.2. (see p.92 for details of the manipulation of the raw data).

Table 6.2.2
SUMMARY OF RESULTS OF EXPERIMENT 2

Each cell entry is sum of $n = 3$ observations

Sex	Speed of Rotation	T ₁	Shape T ₂	R	Σ
C _M	S ₁	29.19	29.51	14.24	72.94
	S ₂	29.52	22.0	10.54	62.06
	S ₃	20.44	24.18	3.0	47.62
	S ₄	30.07	22.62	7.77	60.46
	S ₅	14.43	14.09	5.43	33.95
C _F	S ₁	26.92	27.81	18.81	73.54
	S ₂	30.15	27.49	9.14	66.78
	S ₃	24.34	25.76	4.69	54.79
	S ₄	29.11	19.67	6.44	55.22
	S ₅	21.44	19.46	4.63	45.54
Σ		255.62	232.58	84.70	572.90

A summary of the 3 factor analysis of variance is given in table 6.2.3. This shows that both the outline shape and the speed of rotation had very significant effects on the amount of apparent oscillation produced ($\alpha \geq 0.001$). The sex of the subjects had no effect on the amount of oscillation seen, and none of the interaction effects were significant at $\alpha \geq 0.25$.

Source of Variation	Sum of Squares	d.f.	Mean Square	F	α
Shape - A	573.4137	2	286.71	59.38	> 0.001
Speed - S	144.5778	4	36.14	7.48	> 0.001
Sex - C	3.9426	1	3.94		
A x S	49.8032	8	6.22	1.29	< 0.25
A x C	0.6345	2	0.32		
S x C	9.1575	4	2.29		
A x S x C	14.9253	8	5.62	1.16	< 0.25
Within treatments	289.7259	60	4.83		
Total	1086.1805	89			

TABLE 6.2.3.
SUMMARY OF ANALYSIS OF VARIANCE FOR EXPERIMENT 2

Scheffé's test for multiple comparisons (see p. 94) was used to test for significant differences between the shapes and speeds investigated. The results of these comparisons are shown in tables 6.2.4 and 6.2.5., and are discussed below.

Table 6.2.4.
COMPARISONS BETWEEN SHAPES

Comparison	$\sum_{256}^{T_1}$	$\sum_{233}^{T_2}$	$\sum_{85}^R$	$\sum a_i^2$	Di	Di^2	MS_{Di}
T_1 vs T_2	1	-1	0	2	23	529	8.8
T_1 vs R	1	0	-1	2	171	29241	487
T_2 vs R	0	1	-1	2	148	21904	365
$T_1 + T_2$ vs R	1	1	-2	6	319	101761	565

Table 6.2.5.
COMPARISONS BETWEEN SPEEDS

Comparison	$\sum_{146}^{S_1}$	$\sum_{129}^{S_2}$	$\sum_{102}^{S_3}$	$\sum_{116}^{S_4}$	$\sum_{79}^{S_5}$	$\sum a_i^2$	Di	Di^2	MS_{Di}
S_1 vs S_2	1	-1	0	0	0	2	17	289	8
S_4 vs S_5	0	0	0	1	-1	2	37	1369	38
S_1 vs $S_2 + S_3 + S_4 + S_5$	4	-1	-1	-1	-1	20	158	24964	69
S_5 vs $S_1 + S_2 + S_3 + S_4$	-1	-1	-1	-1	4	20	177	31329	87
$S_1 + S_2$ vs $S_3 + S_4 + S_5$	1.5	1.5	-1	-1	-1	7.5	115.5	13340	99

Multiple Comparisons for Shape (A)

k = 3 levels

n = 30 observations

S^2 = 4.83 (see table 6.2.3)

For significance,

$$MS_{D_i} \geq (k - 1) (F) S$$

Tabled values of F for $(k - 1) = 2$ and $k (n - 1) = 87$ d.f.
are $F = 7.54$ for $\alpha = 0.001$
 $F = 1.41$ for $\alpha = 0.25$

Thus for significance,

$$MS_{D_i} \geq \begin{array}{ll} 72.84 & \text{for } \alpha = 0.001 \\ 13.62 & \text{for } \alpha = 0.25 \end{array}$$

Thus from table 6.2.4, the comparison between the two trapezoidal shapes showed that there was no significant difference in the frequency of apparent oscillation. But the comparisons between the trapezoidal shapes and the rectangle showed very significant differences ($P > 0.1\%$).

Multiple Comparisons for Speed (S)

$k = 5$ levels
 $n = 18$ observations
 $S^2 = 4.83$

Tabled values of F for $(k - 1)$ and $k (n - 1) = 85$ d.f.
are $F = 5.12$ for $\alpha = 0.001$
 $F = 4.04$ for $\alpha = 0.005$
 $F = 3.58$ for $\alpha = 0.01$
 $F = 2.01$ for $\alpha = 0.1$

Thus for significance,

$$MS_{D_i} \geq \begin{array}{ll} 98.9 & \text{for } \alpha = 0.001 \\ 78.0 & \alpha = 0.005 \\ 69.2 & \alpha = 0.01 \\ 38.8 & \alpha = 0.1 \end{array}$$

Then from table 6.2.5, it can be seen that the slowest speed of rotation ($S_1 = 2$ rpm) produced significantly more apparent oscillations than any of the other speeds ($P = 99\%$), and the fastest speed ($S_5 = 32$ rpm) produced fewer oscillations than any of the others ($P = 99.5\%$). The difference between S_4 and S_5 just failed to reach significance at the 10% level.

DISCUSSION

Three factors thought to contribute to the illusion of apparent oscillation were investigated in this experiment; these were outline shape, speed of rotation, and sex of the subject.

As in the first experiment., (E1), the outline shape was found to have the most significant effect on the number of apparent oscillations produced. The shapes used were the same as in E1 (pattern1), so that the results obtained are directly comparable.

Scheffé's test showed that the 2 trapezoidal shapes both produced many more oscillations than the rectangle ($\alpha < 0.001$); this was so for the complete range of speeds used, as is shown clearly by the Graph 6.2.1. In this experiment there was no significant difference between the amount of oscillation produced by Ames' trapezoid (T_2) and the more exaggerated trapezoid (T_1).

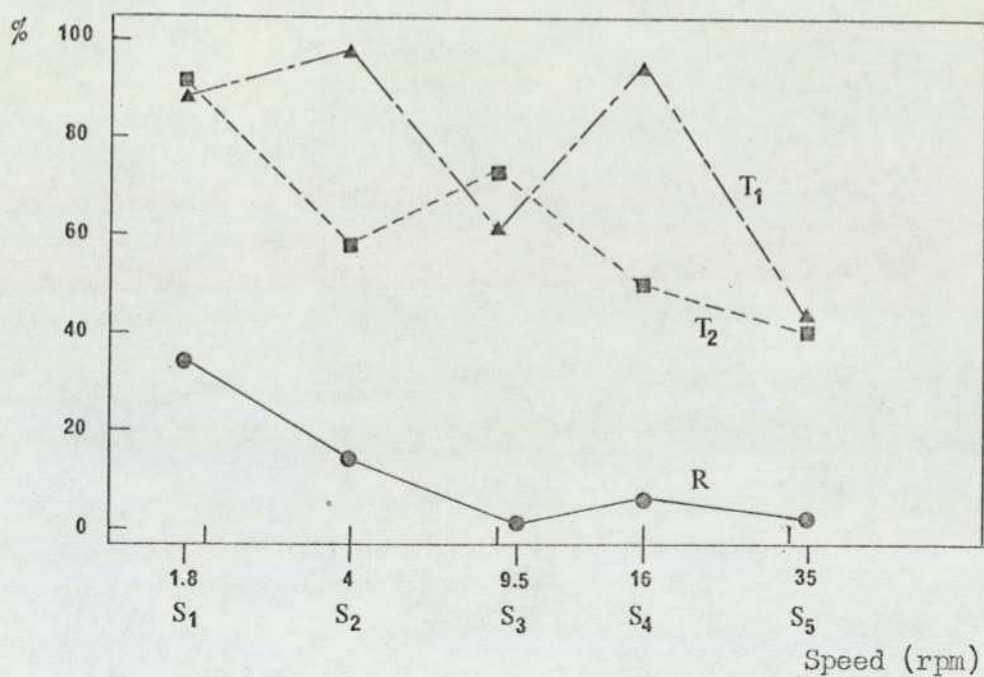
The sex of the subjects used had no significant effect on the results obtained; this verified the result obtained in E1.

The speed of rotation was found to have a very significant effect ($\alpha < 0.001$) upon the amount of apparent oscillation produced.

Over the middle range of speeds used (4→16 rpm) there was little variation in the illusory effects. For T_1 and T_2 there was almost continuous apparent oscillation, with just occasional complete rotations seen. But for the rectangle R there was almost continuous rotation, with very few oscillations.

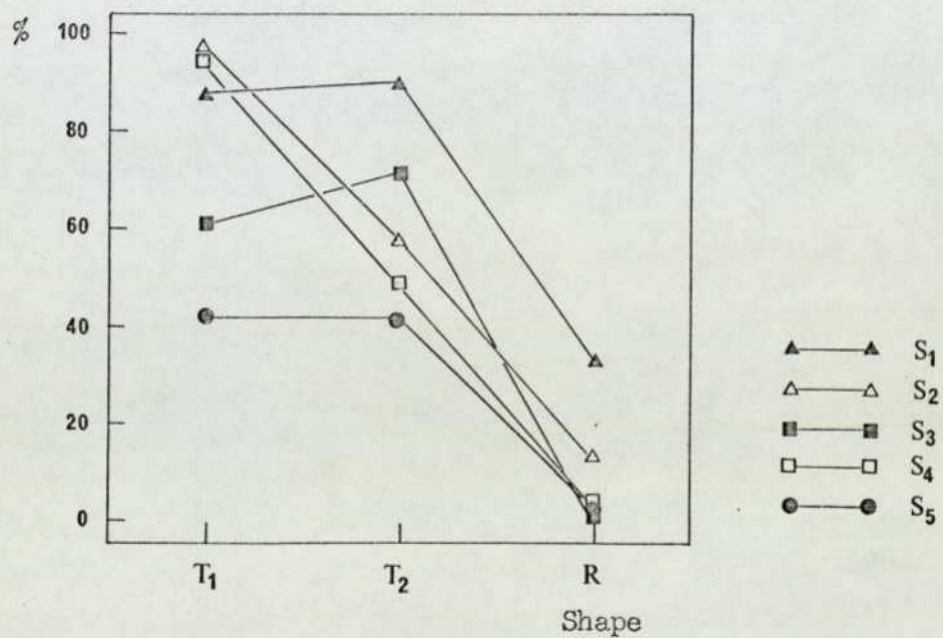
The slowest speed (2 rpm) gave significantly ($\alpha < 0.01$) more oscillations than any of the other speeds -- particularly for the rectangle (see

Percentage
Oscillations



Graph 6.2.1

Percentage
Oscillations



Graph 6.2.2

Graph 6.2.2). This supports the hypothesis, which suggested that for slow speeds 'boredom', or loss of visual interest, could result in alternative interpretations of the same visual scene being perceived in turn. (Since the rectangle provided more ambiguous information than the trapezoids (see page 57) this shape would be more susceptible to these spontaneous fluctuations in interpretation).

The fastest speed used (35 rpm) gave significantly fewer apparent oscillations than any other speed ($\alpha = 0.005$). The pattern of results obtained at this speed was very different from all the others. The observations tended to be grouped in spells of either complete rotation or continuous oscillation, these spells lasting for 5 - 10 secs. each. The standard deviation of the results obtained was higher for this speed than for any other, and several S's actually called out "Slow down! " (or some such) during the trial runs, indicating the difficulty they experienced in interpreting this rapidly moving stimulus. This difficulty has not been commented on in any other study, but surely it cannot be unique to the S's used in this experiment.

The results of this experiment, then, verified the hypothesis under test - i.e. that over a modest range of angular velocities (approx 4 $\rightarrow$ 16 rpm) the illusion of apparent oscillation produced by a rotating trapezoidal object is a stable and repeatable effect.

At slower speeds visual 'boredom' can confound the illusion, whereas at higher speeds inadequate 'decision time' produces confusing results.

EXPERIMENT 3

HEIGHT RATIO & AXIS POSITION

INTRODUCTION

The results of the previous 2 experiments indicated that the outline shape was the most significant single factor in the production of the illusion of apparent oscillation.

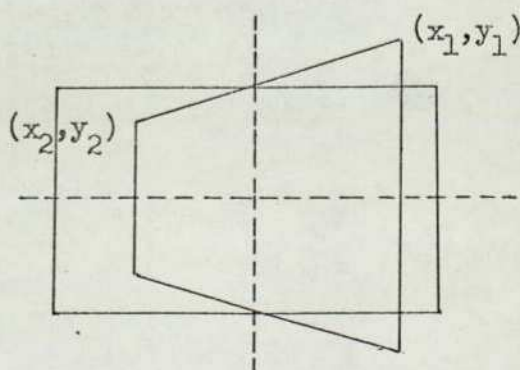
The hypothesis detailed in Section 4 (p59), suggested that the most important source of information when viewing a rotating trapezoidal object is the projective transformation that it undergoes. The shapes used so far have been either exaggerated trapezoids (which have produced almost continuous apparent oscillation), or a rectangle (which has produced consistent rotation). Consideration of the projective transformation undergone by these particular shapes shows that these results are entirely consistent with that hypothesis.

But there must be some change over point from apparent oscillation to veridical rotation as the height ratio decreases, and the elliptical loci of the corner points of the trapezoid come closer together (or maybe overlap). It was suggested that there may be a range of shapes that cause some confusion, with sometimes rotation being seen, and sometimes oscillation.

It was shown above, that for a constant viewing distance D , the relative positions of the elliptical loci of the corner points of the rotating trapezoid depended on both the ratio of the height of the vertical sides ($y_1 : y_2$), and upon the axis of rotation position ($x_1 : x_2$) (See p 37).

Fig. 6.3.1

$$x_1 > x_2$$



A secondary source of information is also provided by the axis of rotation position. If $x_1 > x_2$, it provides reinforcing "perspective" cues, in that the trapezoidal shape (when perpendicular to the observer) looks like the typical graphical representation of a rectangle rotated about a central vertical axis (See Fig. 6.3.1). If $x_1 < x_2$ this conflicts with the 'perspective' information.

EXPERIMENTAL DESIGN

The effect of altering the height ratio of the vertical sides of a trapezoid was investigated at six different levels for each of four different axes of rotation positions. The viewing distance was maintained constant throughout at 12.5 feet, so that only those parts of the hypothesis relating to large viewing distance ($D \gg x$) were investigated.

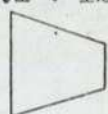
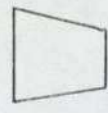
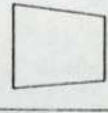
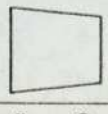
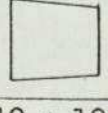
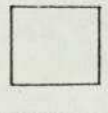
All the trapezoids used were planar matt white shapes similar to those used in E2 and in E1 (Pattern 1), and the angular velocity was maintained constant throughout at 5 rpm. The experimental design is summarised in table 6.3.1., which also indicates the number allocated to each treatment condition for the random allocation of subjects. Three male and three female S's viewed each of the 24 treatment conditions.

The actual sizes of the shapes constructed for this experiment are shown in the table. These shapes were filmed at a different time to those used in E1 and E2, and the slightly different camera settings used resulted in a magnification of 2.15 x on the projection screen. The projected sizes are also shown in table 6.3.1.

The conditions 3 and 7, then, did not correspond exactly to the shapes T_1 and T_2 used in experiments 1 and 2, as originally intended. However, the angular sizes were close enough for useful comparisons to be made.

There were therefore three factors being investigated in this experiment - height ratio (at 6 levels), axis of rotation position (4 levels) and sex (2 levels).

Table 6.3.1

AXIS OF ROTATION POSITION ($x_1 : x_2$)				
Actual (Projected)	5.1 : 7.1 (10.96 : 15.26)	6.1 : 6.1 (13.11 : 13.11)	7.1 : 5.1 (15.26 : 10.96)	8.1 : 4.1 (17.41 : 8.81)
HEIGHT RATIO ($2y_1 : 2y_2$)				
Actual (Projected)	 B1	 B2	 B3	 B4
14 : 6 (30.1 : 12.9)  A1	1	2	3	4
12 : 8 (25.8 : 17.2)  A2	5	6	7	8
11.5 : 8.5 (24.7 : 18.3)  A3	9	10	11	12
11 : 9 (23.6 : 19.3)  A4	13	14	15	16
10.5 : 9.5 (22.6 : 20.5)  A5	17	18	19	20
10 : 10 (21.5 : 21.5)  A6	21	22	23	24

The randomised group design for the 3 factor analysis of variance used may be summarised as follows:

Factors

'Shape' -	$y_1 : y_2$	=	A	6 levels
Rotation position	$x_1 : x_2$	=	B	4 levels
Sex		=	C	2 levels

Thus there are $(6)(4)(2) = 48$ treatment combinations.

$n = 3$ for all treatments

Thus total $(48)(3) = 144$ observations

Then the total sum of squares with $(144-1) = 143$ d.f. can be partitioned into the following components, with degrees of freedom as shown at the right:

Main effects:	A	5
	B	3
	C	1
2 factor interactions:	A x B	15
	B x C	3
	A x C	5
3 factor interactions:	A x B x C	15

Within treatment error $(143 - 47) = 96$ d.f.

Then	$A_1 = 14 : 6$	$B_1 = 5.1 : 7.1$	$C_1 = M$
	$A_2 = 12 : 8$	$B_2 = 6.1 : 6.1$	$C_2 = F$
	$A_3 = 11.5 : 8.5$	$B_3 = 7.1 : 5.1$	
	$A_4 = 11 : 9$	$B_4 = 8.1 : 4.1$	
	$A_5 = 10.5 : 9.5$		
	$A_6 = 10 : 10$		

Sex	Rotation Position ($x_1 : x_2$)	"Shape" ($y_1 : y_2$)						Σ
		A_1	A_2	A_3	A_4	A_5	A_6	
C_M	B_1	30.15	26.10	23.98	26.53	23.31	3.0	133.08
	B_2	29.90	28.53	26.58	23.91	16.65	6.0	131.56
	B_3	30.15	29.90	29.10	26.53	20.44	4.45	140.57
	B_4	26.65	25.42	25.42	25.06	18.82	3.0	124.35
C_F	B_1	28.27	29.90	29.90	21.55	12.88	3.0	125.50
	B_2	29.90	28.32	29.64	25.67	21.05	6.58	141.16
	B_3	30.15	30.15	30.15	25.98	18.85	3.0	138.28
	B_4	25.66	27.75	27.49	25.20	13.35	3.0	122.44
Σ		230.83	226.06	222.25	200.43	145.35	32.03	1056.95

TABLE 6.3.2.
SUMMARY OF RESULTS OF EXPERIMENT 3

Each cell entry is sum of $n = 3$ observations

RESULTS

The results obtained are summarised in table 6.3.2. (see p 92 for details of the manipulation of the raw data).

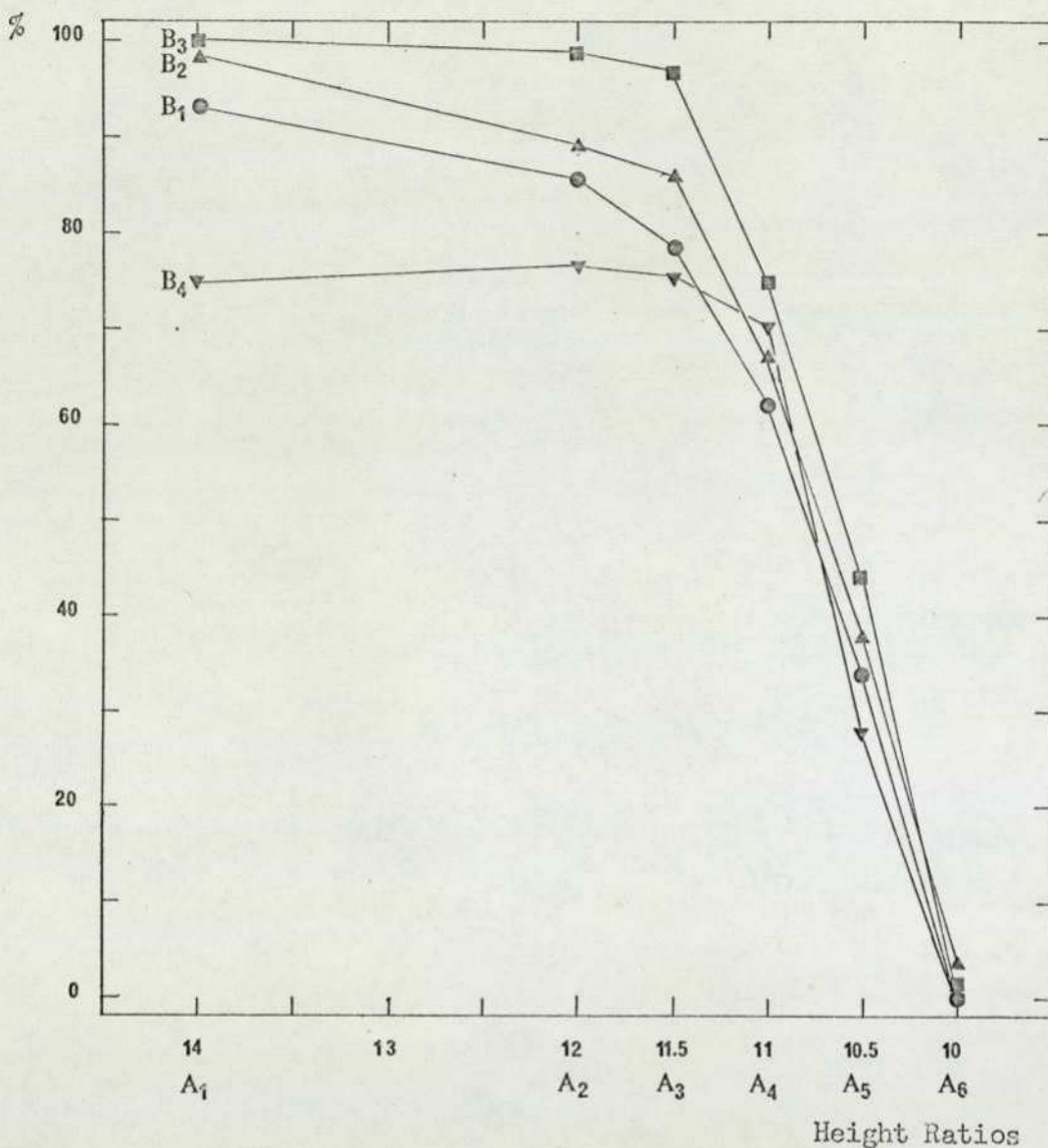
A summary of the 3 factor analysis of variance is given in table 6.3.3. This shows that the height ratio ($y_1 : y_2$) had the most significant effect upon the amount of apparent oscillation produced ($\alpha < 0.001$). The axis of rotation position ($x_1 : x_2$) was also significant, but only at the 10% level ($\alpha = 0.10$). The sex of the subject had no effect on the amount of oscillation seen, and none of the interactions produced significant results.

Table 6.3.3.
SUMMARY OF ANALYSIS OF VARIANCE FOR EXPERIMENT 3

Source of Variation		Sum of Squares	d.f.	Mean Square	F	α
$y_1 : y_2$	A	1246.44	5	249.30	111.64	0.001
$x_1 : x_2$	B	17.27	3	5.76	2.68	0.10
Sex	C	0.03	1	0.03		
A x B		8.77	15	0.58		
B x C		4.37	3	1.46		
A x C		15.72	5	3.14	1.41	
A x B x C		24.12	15	1.61		
Within treatments		214.38	96	2.23		
Total		1531.10	143			

The results are shown most clearly by the graphs 6.3.1 and 6.3.2. Scheffé's test for multiple comparisons was used to test for significant differences between the various height ratios and axis of rotation positions used. The results of these comparisons are shown in tables 6.3.4. and 6.3.5., and are discussed below.

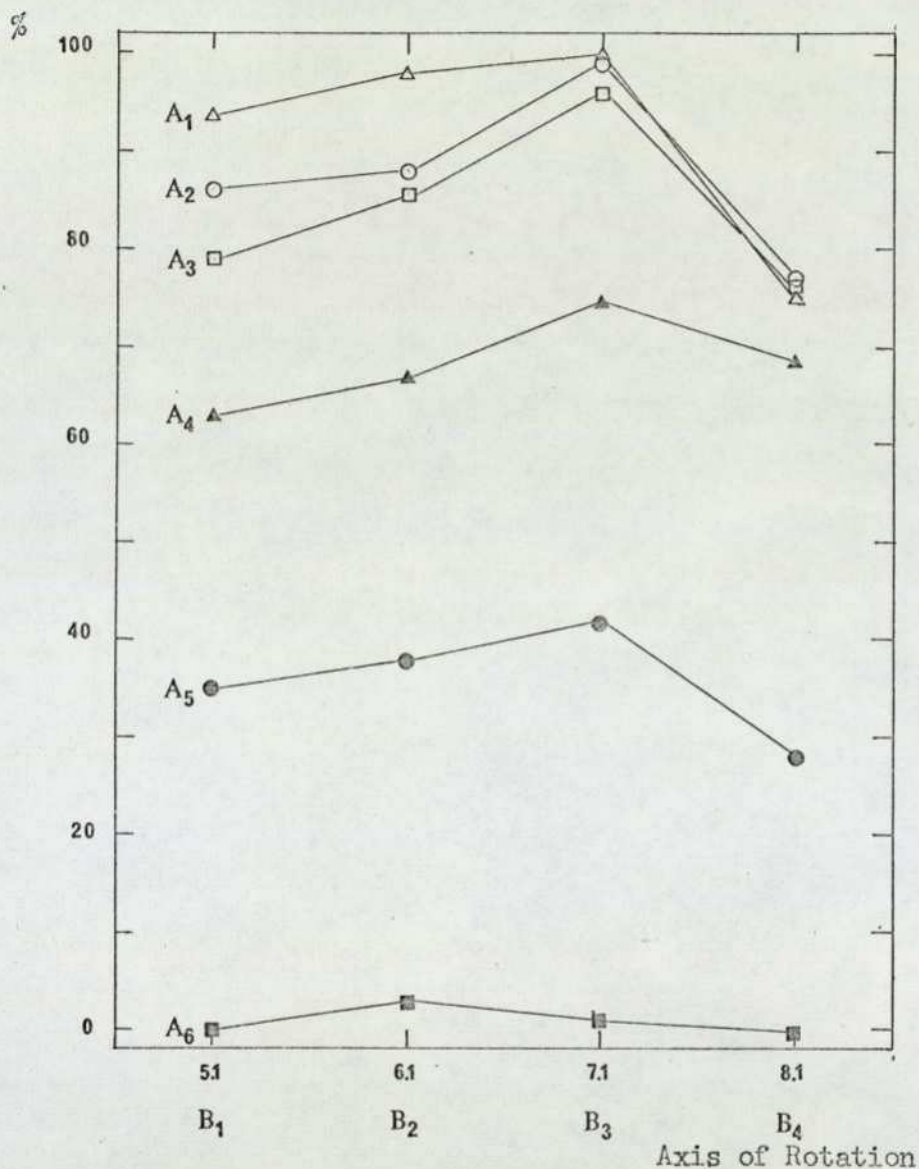
Percentage
Oscillations



Graph 6.3.1

Graph showing percentage of oscillations produced by each height ratio, for 4 different axis of rotation positions.
(See Table 6.3.1 for dimensions)

Percentage
Oscillations



Graph 6.3.2

Graph showing percentage of oscillations produced by each axis of rotation position, for 6 different height ratios.
(See Table 6.3.1 for dimensions)

Comparison	$\sum A_1$ 231	$\sum A_2$ 226	$\sum A_3$ 222	$\sum A_4$ 200	$\sum A_5$ 145	$\sum A_6$ 32	$\sum a_i^2$	D_i	D_i^2	$MS = \frac{D_i^2}{n \sum a_i^2}$
A_1 vs A_2	1	-1	0	0	0	0	2	5	25	0.5
A_2 vs A_3	0	1	-1	0	0	0	2	4	16	0.3
A_3 vs A_4	0	0	1	-1	0	0	2	22	484	10.1
A_4 vs A_5	0	0	0	1	-1	0	2	55	3025	63
A_5 vs A_6	0	0	0	0	1	-1	2	113	12769	266
$A_1 + A_2 + A_3$ vs $A_4 + A_5 + A_6$	1	1	1	-1	-1	-1	6	302	91204	633.4
$A_1 + A_2 + A_3 + A_4$ vs $A_5 + A_6$	1	1	1	1	-2	-2	12	525	275625	957

TABLE 6.3.4.
COMPARISONS BETWEEN HEIGHT RATIOS

Comparison	$\sum_{259}^{B_1}$	$\sum_{273}^{B_2}$	$\sum_{279}^{B_3}$	$\sum_{247}^{B_4}$	$\sum_{ai^2}$	D_i	D_i^2	MS_{D_i}
B_1 vs B_2	1	-1	0	0	2	14	196	2.7
B_2 vs B_3	0	1	-1	0	2	6	36	0.5
B_3 vs B_4	0	0	1	-1	2	32	1024	14.2
$B_2 + B_3$ vs $B_1 + B_4$	-1	1	1	-1	4	46	2116	14.7
B_3 vs $B_1 + B_2 + B_4$	-1	-1	3	-1	12	58	3364	7.8

TABLE 6.3.5.
COMPARISONS BETWEEN AXIS OF ROTATION POSITIONS

Multiple Comparisons for $y_1:y_2$ Ratio

$$\begin{aligned}k &= 6 \text{ levels} \\n &= 24 \text{ observations} \\S^2 &= 2.23 \text{ (see table 6.3.3.)}\end{aligned}$$

Tabled values of F for $(k - 1) = 5$ and $k(n - 1) = 138$ d.f.
are $F = 4.41$ for $\alpha = 0.001$

Then for significance

$$\begin{aligned}MS_{Di} &\geq (k - 1) (F) (S^2) \\&\geq 49.17 \text{ for } \alpha = 0.001\end{aligned}$$

Thus from table 6.3.4., there was no significant difference in the amount of oscillation produced by the shapes A_1 to A_4 , but there were marked differences between these shapes and A_5 and A_6 .

Multiple Comparisons for $x_1:x_2$ Ratio

$$\begin{aligned}k &= 6 \text{ levels} \\n &= 36 \text{ observations} \\S^2 &= 2.23\end{aligned}$$

Tabled values of F for $(k - 1) = 3$ and $k(n - 1) = 140$ d.f.
are $F = 2.12$ for $\alpha = 0.10$
 $F = 1.38$ for $\alpha = 0.25$

Then for significance

$$\begin{aligned}MS_{Di} &\geq (k - 1) (F) (S^2) \\&\geq 14.18 \text{ for } \alpha = 0.10 \\&\quad 9.23 \text{ for } \alpha = 0.25\end{aligned}$$

From table 6.3.5., the amount of oscillation produced by the axis of rotation positions B_2 and B_3 was significantly higher ($\alpha < 0.10$) than that produced by the other 2 positions.

DISCUSSION

Three factors thought to contribute to the illusion of apparent oscillation with a rotating trapezoidal object were investigated in this experiment. These were the height ratio of the vertical sides, the axis of rotation position, and the sex of the subject.

The sex of the subjects used had no significant effect on the amount of apparent oscillation seen; this verified the results obtained in the previous 2 experiments.

The ratio of the lengths of the vertical sides of the rotating trapezoids had a very significant effect upon the number of apparent oscillations perceived ($\alpha < 0.001$). This is shown most clearly by the graphs 6.3.1 and 6.3.2.

The number of apparent oscillations decreased as the height ratio decreased, being almost 100% with the steepest trapezoid used, and less than 2% for the rectangle.

There was a steady decrease in the amount of apparent oscillation produced by the shapes A_1 to A_4 , for all four axis of rotation positions, although the decrease was not statistically significant. However, the drop between A_4 and A_5 (75% to 40% oscillation) and between A_5 and A_6 (40% to 2% oscillation) were both significant at $p = 0.1\%$.

The graphs 6.3.3 to 6.3.6 show the loci of the end points of the shapes A_1 to A_4 respectively, for the same viewing distance ($D = 12.5\text{ft}$) and axis of rotation position ($x_1:x_2=7.1:5.1$). (Graphs produced by computer program TROT).

These loci are well separated for A_1 , A_2 and A_3 , and are therefore quite unambiguous. Although the ellipses are fairly wide (particularly for A_1) they are well separated and the actually longer edge is always apparently longer, even when it is behind the shorter edge. Since the apparently longer edge is seen as the apparently nearer edge, the trapezoid will be seen to oscillate. This agrees with the results obtained.

The loci for A_4 are becoming much more ambiguous; the ellipses are much closer together - in fact almost adjacent when $\theta = 270^\circ$ so that the lengths of the vertical sides are apparently almost equal. But

Graph 6.3.3

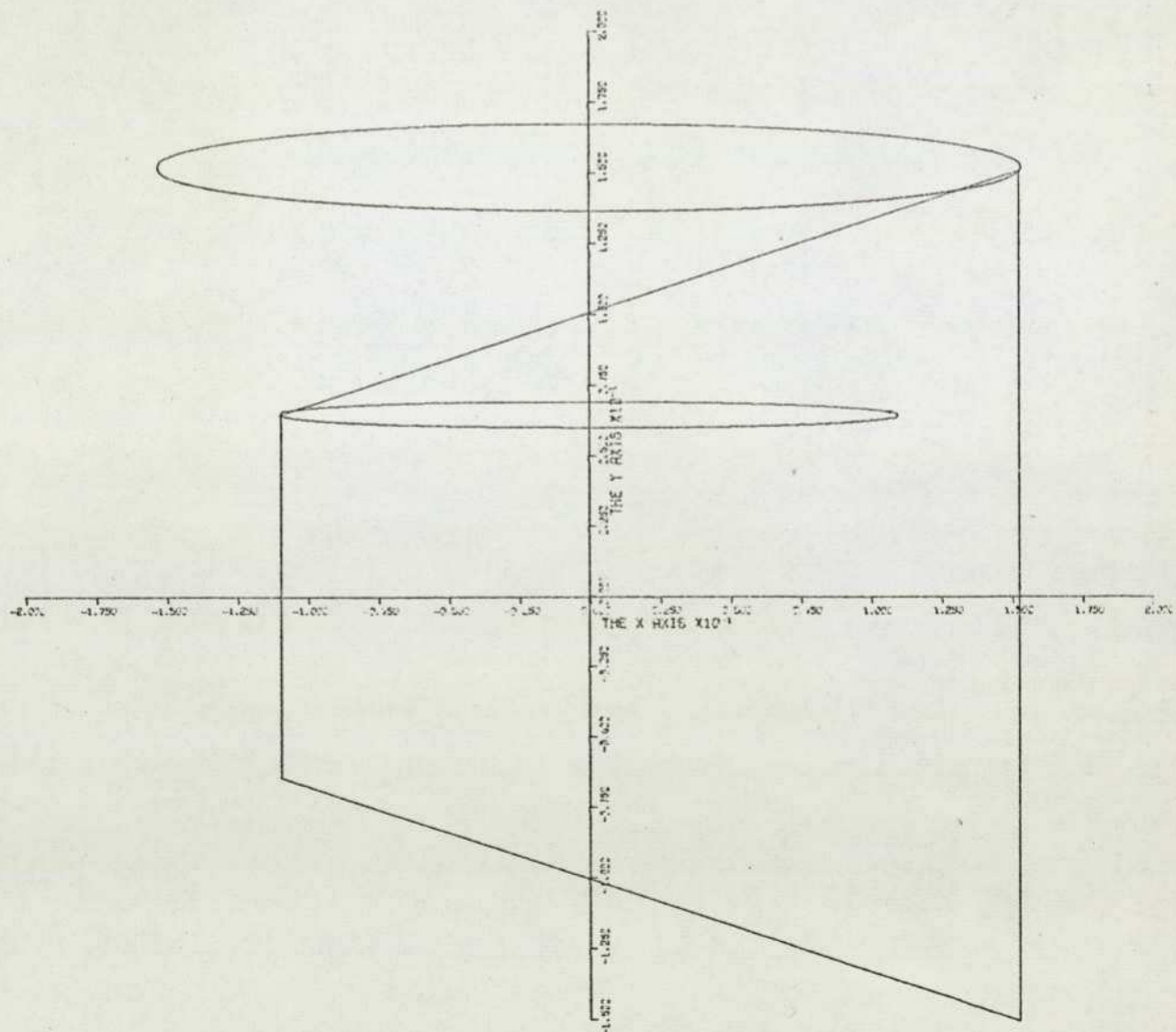
(A1, B3)

TRAPEZOID PROJECTION

X1= 15.26 Y1= 15.05

X2= -10.96 Y2= 6.45

D= 12.50 FEET



Graph 6.3.4

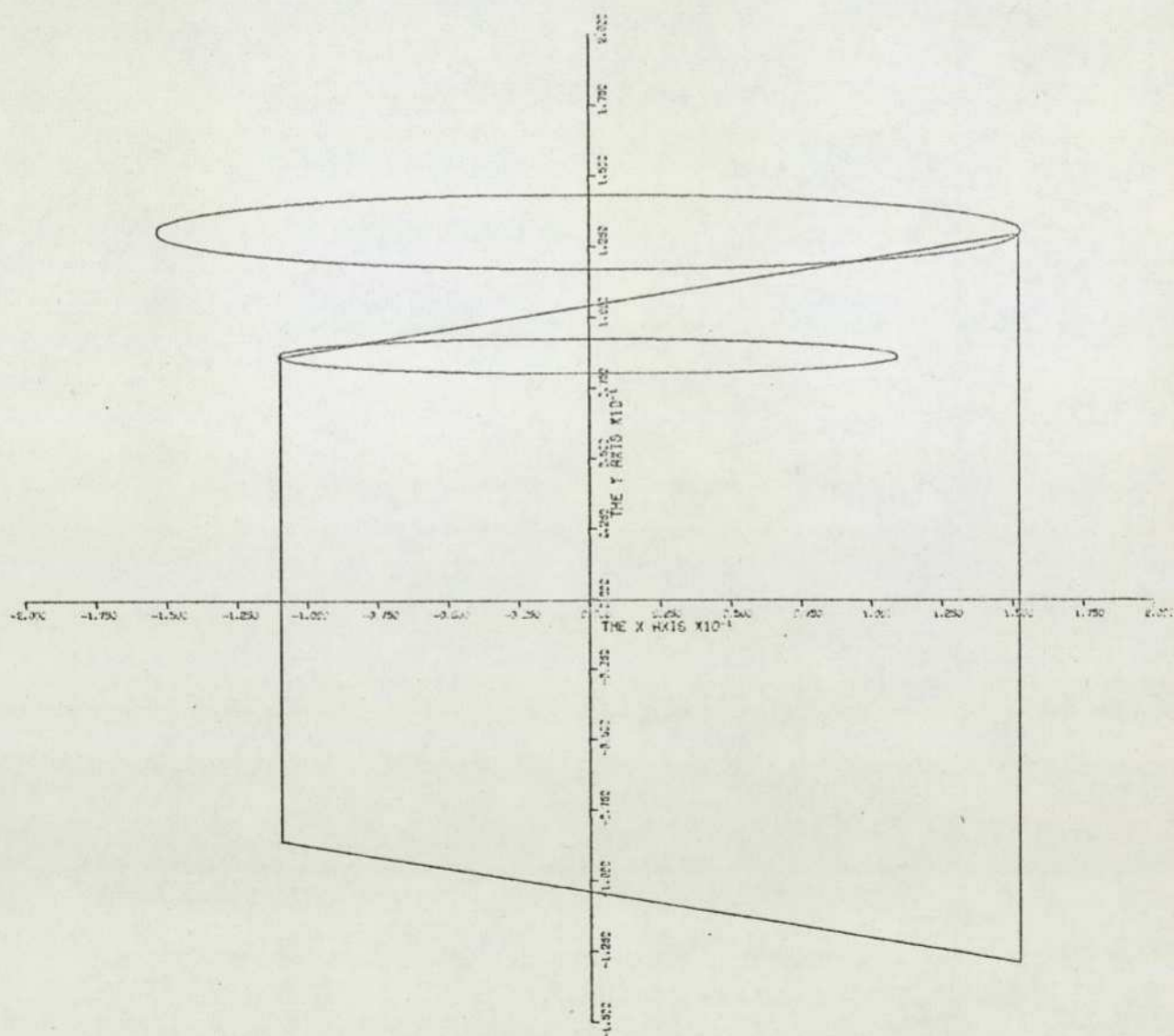
(A2, B3)

TRAPEZOID PROJECTION

X1= 15.26 Y1= 12.90

X2= -10.96 Y2= 8.60

D= 12.50 FEET



Graph 6.3.5

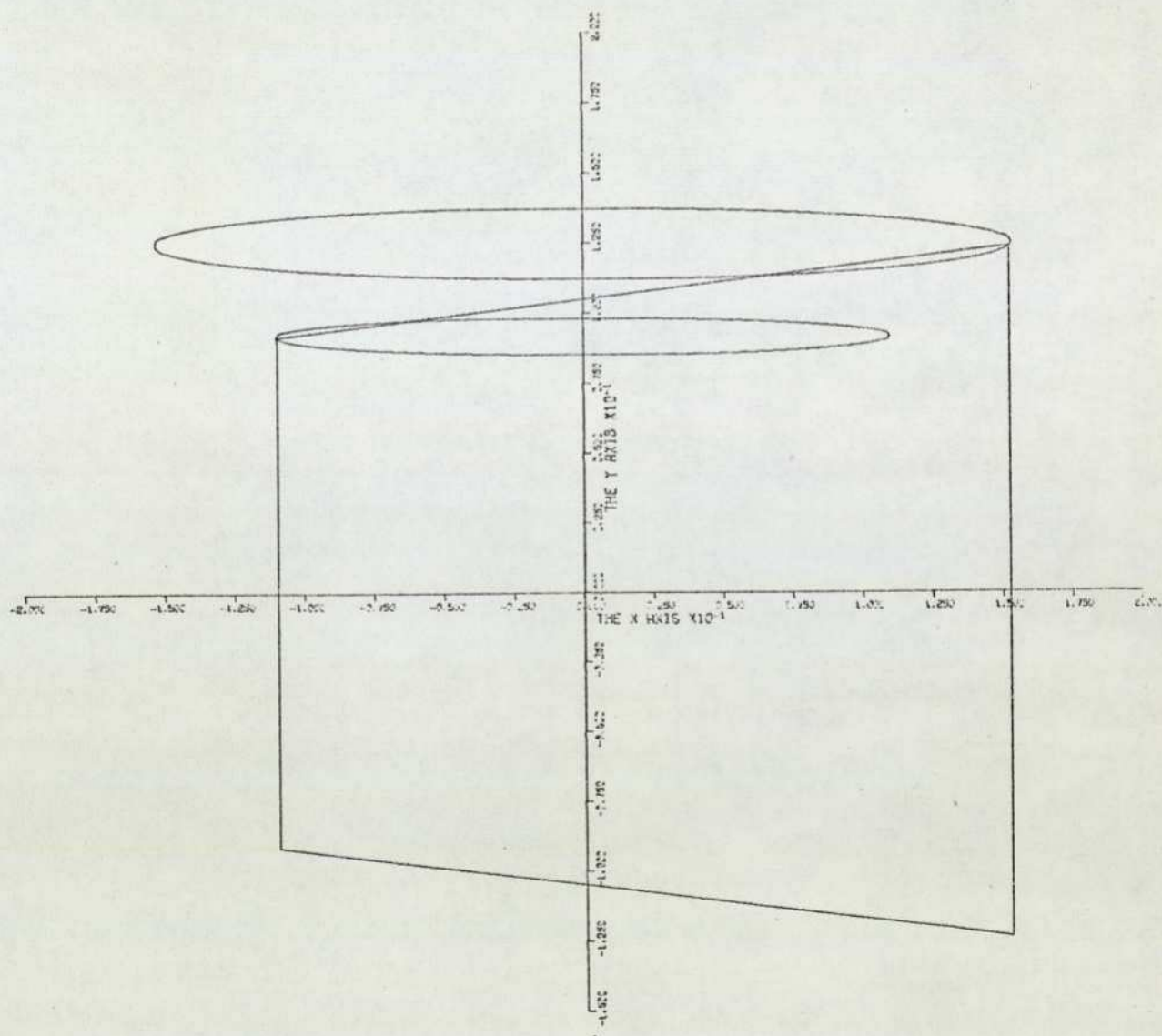
(A3, B3)

TRAPEZOID PROJECTION

$X1 = 15.26$ $Y1 = 12.36$

$X2 = -10.96$ $Y2 = 9.14$

$D = 12.50$ FEET



Graph 6.3.6

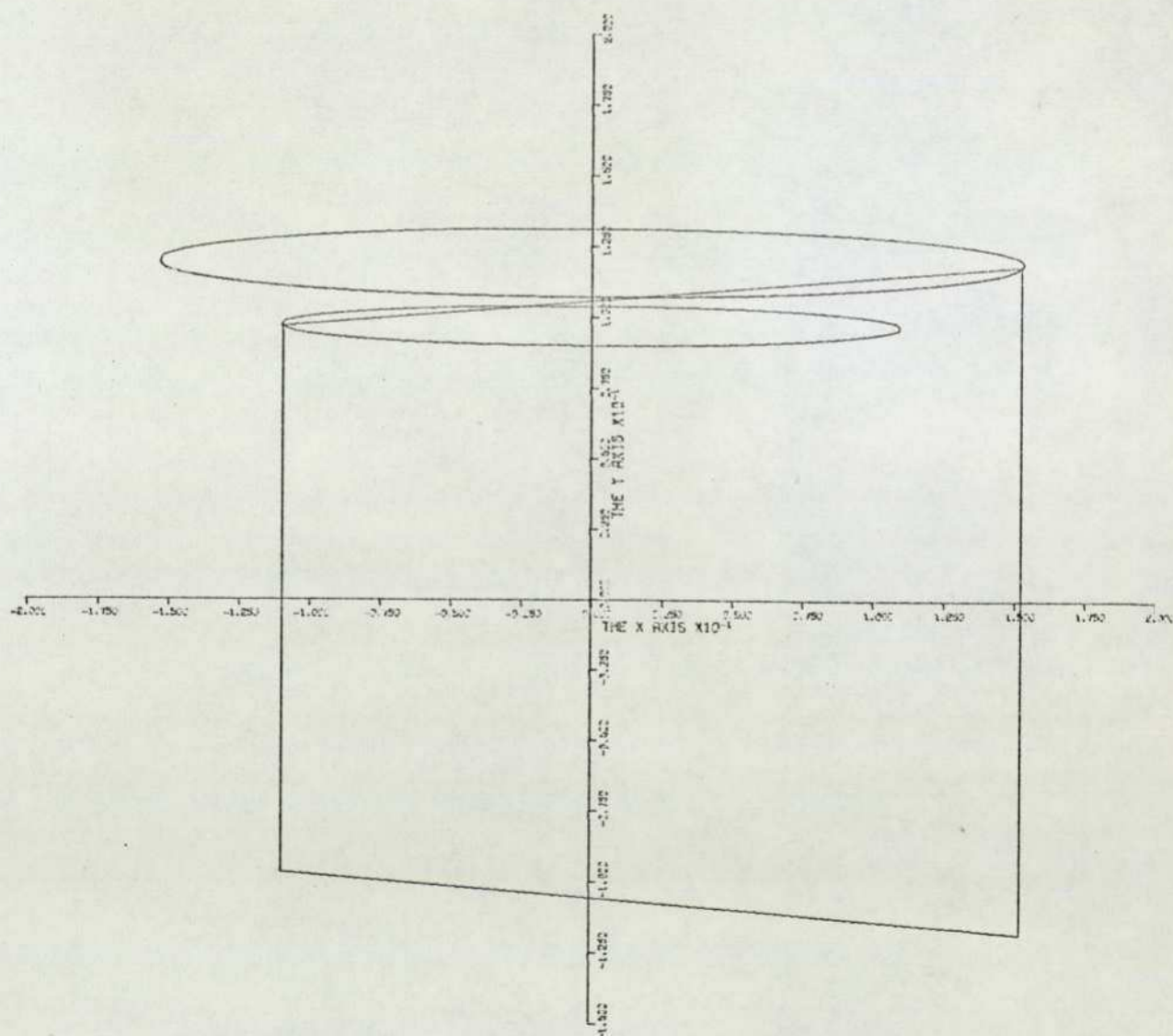
(A4, B3)

TRAPEZOID PROJECTION

X1= 15.26 Y1= 11.82

X2= -10.96 Y2= 9.67

D= 12.50 FEET



Graph 6.3.7.

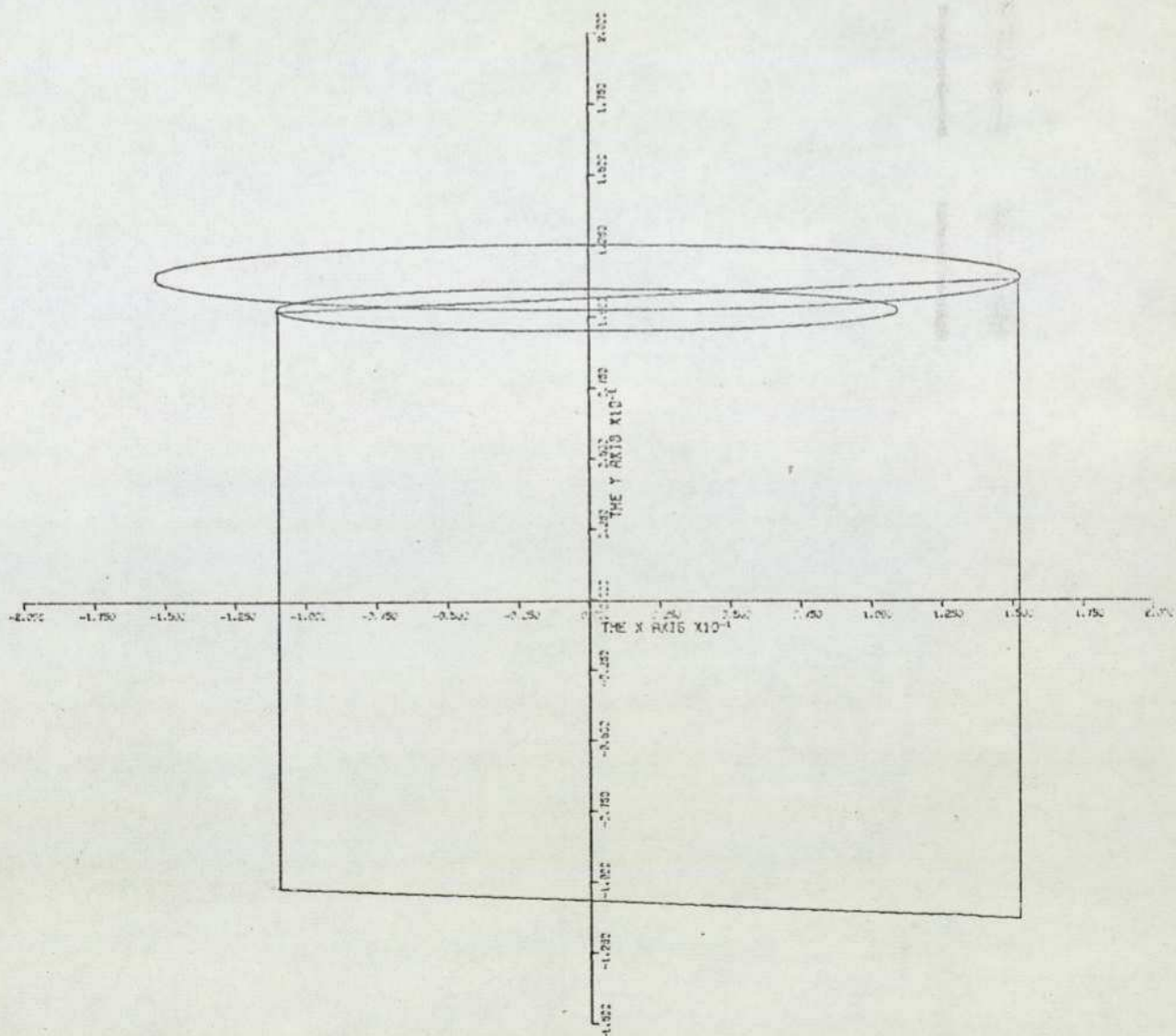
(A5, B3)

TRAPEZOID PROJECTION

$X1 = 15.26$ $Y1 = 11.29$

$X2 = -10.96$ $Y2 = 10.21$

$D = 12.50$ FEET



Graph 6.3.8

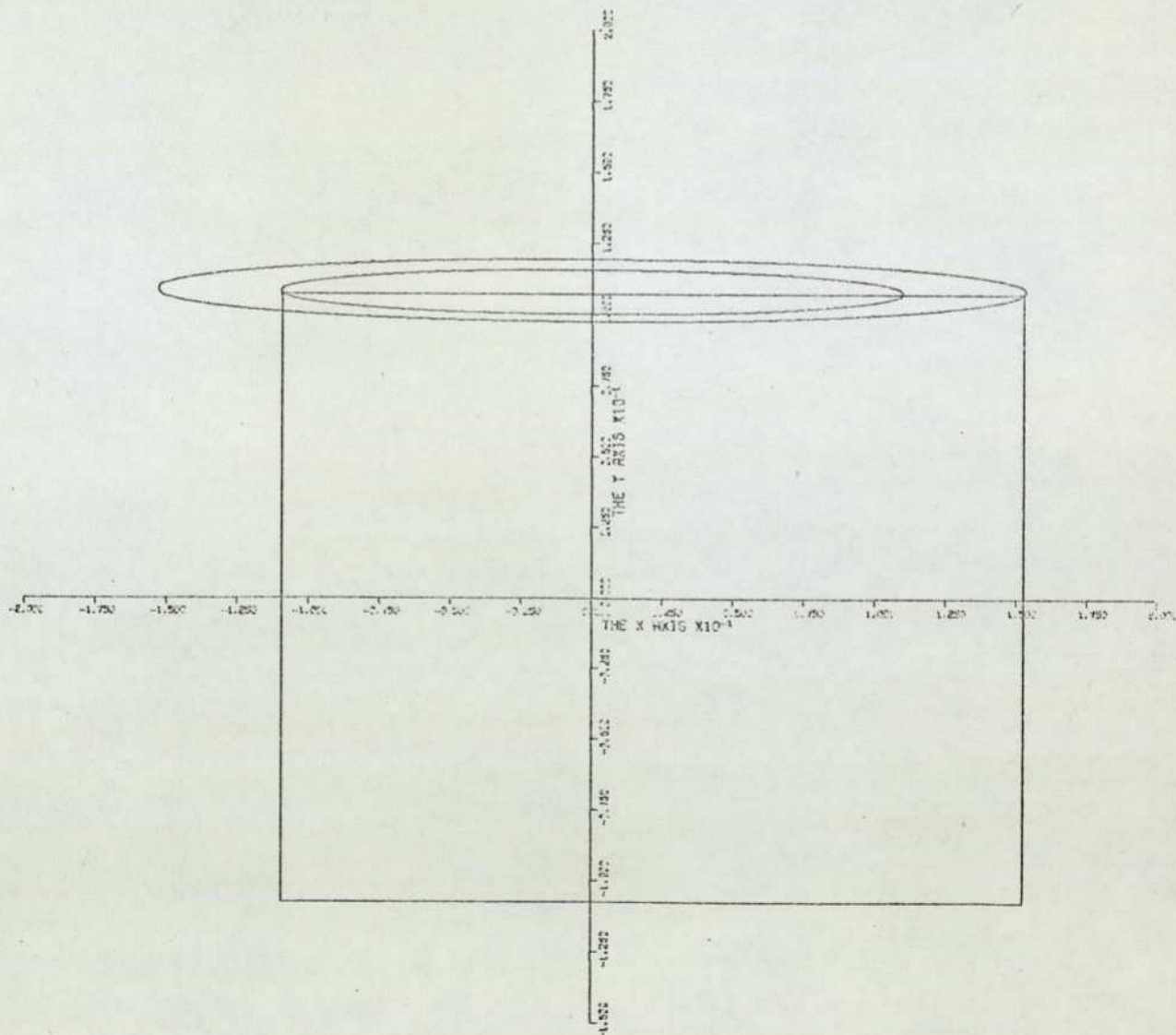
(A6, B3)

TRAPEZOID PROJECTION

$X1 = 15.26$ $Y1 = 10.75$

$X2 = -10.96$ $Y2 = 10.75$

$D = 12.50$ FEET



Graph 6.3.9

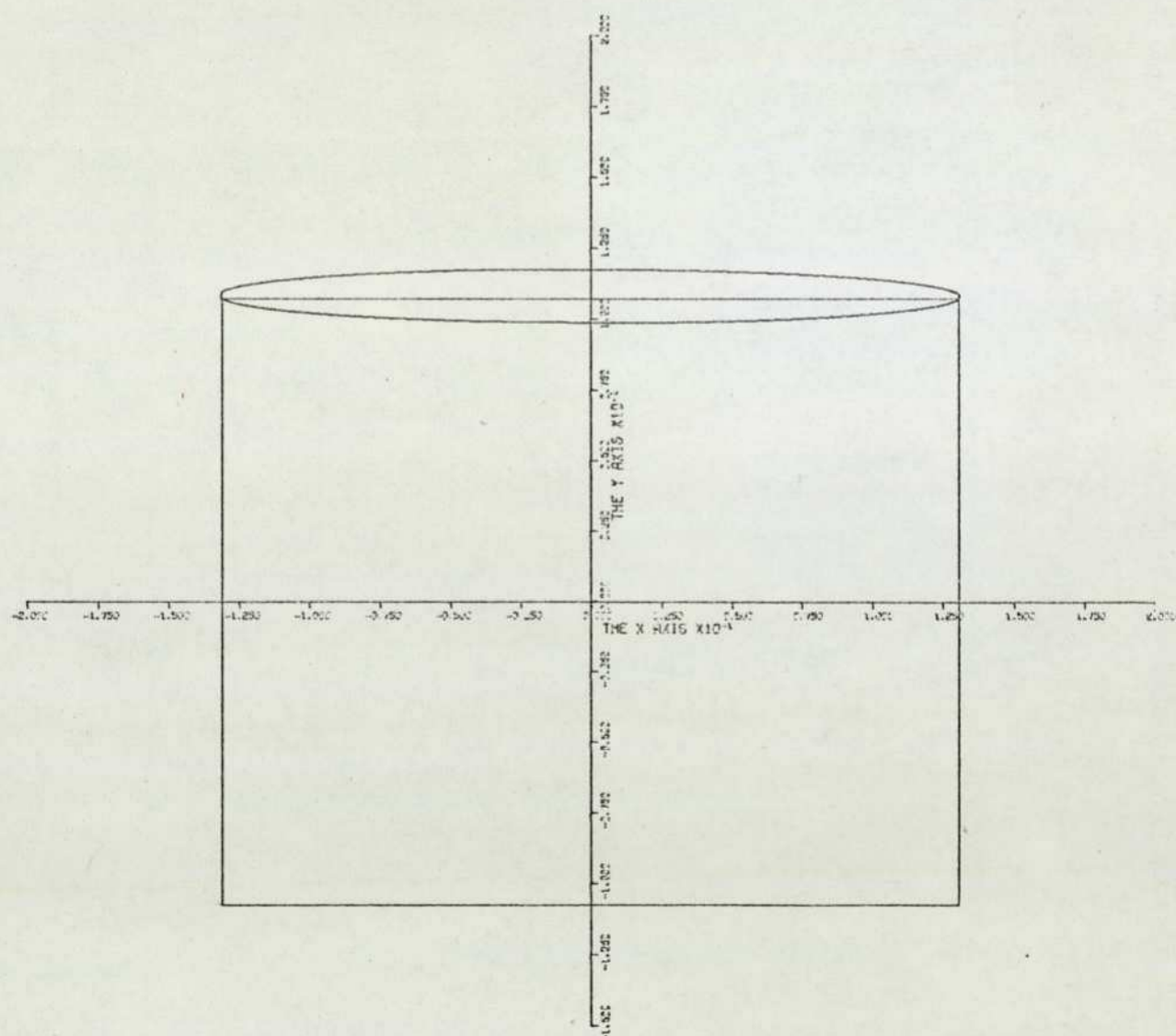
(A6, B2)

TRAPEZOID PROJECTION

X1= 13.11 Y1= 10.75

X2= -13.11 Y2= 10.75

D= 12.50 FEET



there is remarkably strong "shape constancy" when viewing these trapezoids - none of the subjects used commented on an apparent change in the shape of the moving object. (Although Ames (1951) referred to it with practiced subjects). The ambiguity of the projection of this shape is reflected in the decrease in the amount of apparent oscillation produced. The graph 6.3.7. plots the loci for the shape A_5 , the ellipses here partially overlap so that the actually longer edge is not the apparently longer edge for a complete revolution. This degree of ambiguity would suggest that about 50% rotation and 50% oscillation would be perceived - and this was in fact found.

The elliptical loci of the corner points of the rectangle of course overlapped and so provided ambiguous information (since the vertical diameters of the ellipses were small relative to D). (See Graph 6.3.8). One would therefore expect some spontaneous oscillations to occur. But the results indicate that the "off centre" axis of rotation positions provided enough additional information for the hypothesis of rotation to be chosen in preference to that of oscillation; practically no oscillation was obtained with these conditions.

However, when the axis of rotation was central (See Graph 6.3.9) the elliptical loci of the corner points of the rectangle were completely superimposed, and so quite ambiguous. In this condition some spontaneous apparent oscillations did occur (See Graph 6.3.2).

Variations in the axis of rotation position were also found to have a significant effect on the amount of apparent oscillation produced, although to a much smaller extent than the height ratios. ($p = 10\%$). The graphs 6.3.10 show the variations in the elliptical loci of a particular trapezoid ($y_1:y_2 = 11.5 : 8.5$) with different axis of rotation positions (viewing distance held constant). The effect is obviously far less marked than that produced by altering the height ratio.

In condition B_3 ($x_1:x_2 = 7.1 : 5.1$) the position of the axis of rotation was displaced towards the shorter end of the trapezoidal shapes, thus corresponding approximately to the 'perspective' view of a rectangle rotated about a central axis (Fig 6.3).

Graph 6.3.10

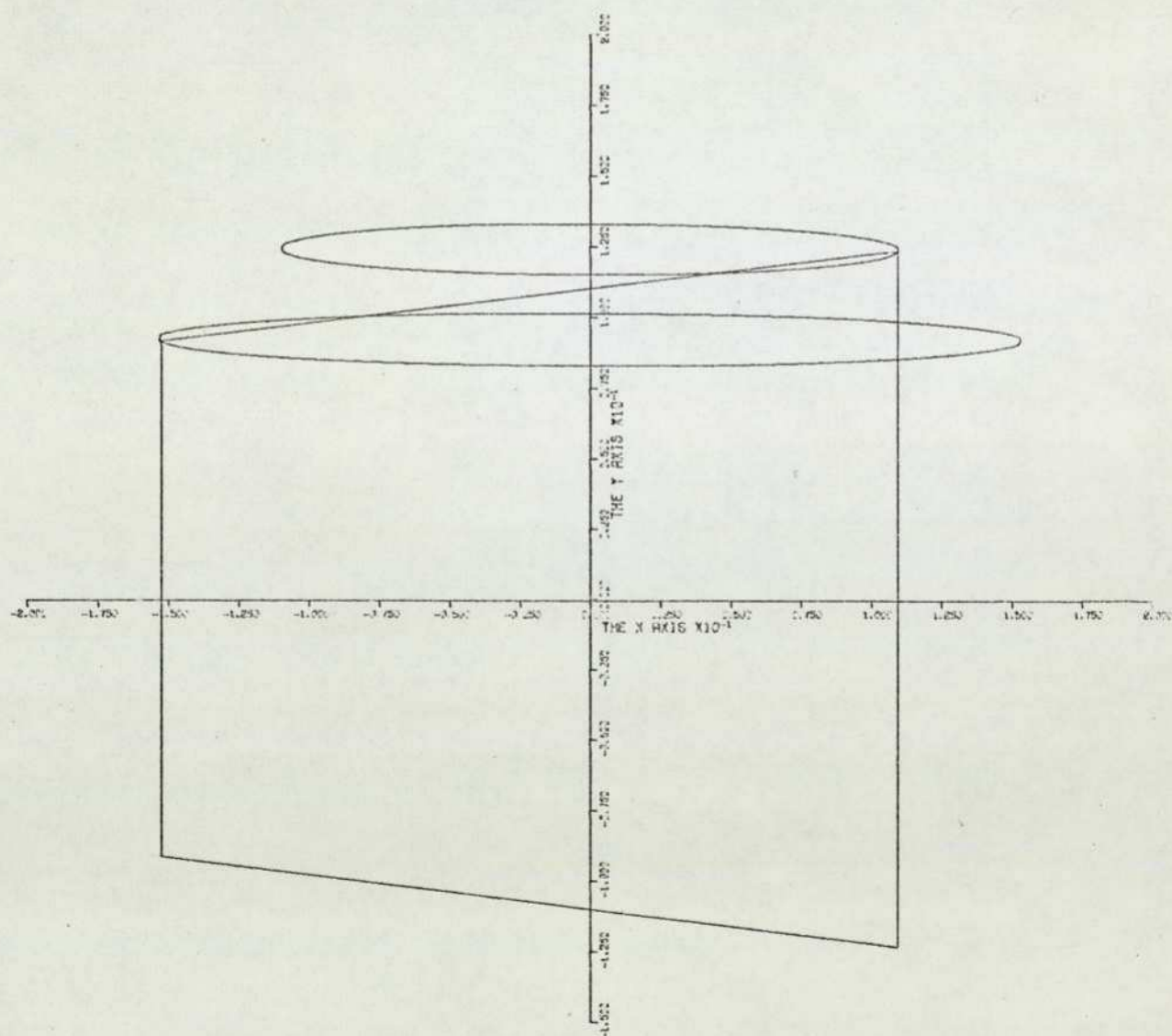
(A3, B1)

TRAPEZOID PROJECTION

X1= 10.96 Y1= 12.36

X2= -15.26 Y2= 9.14

D= 12.50 FEET



Graph 6.3.10

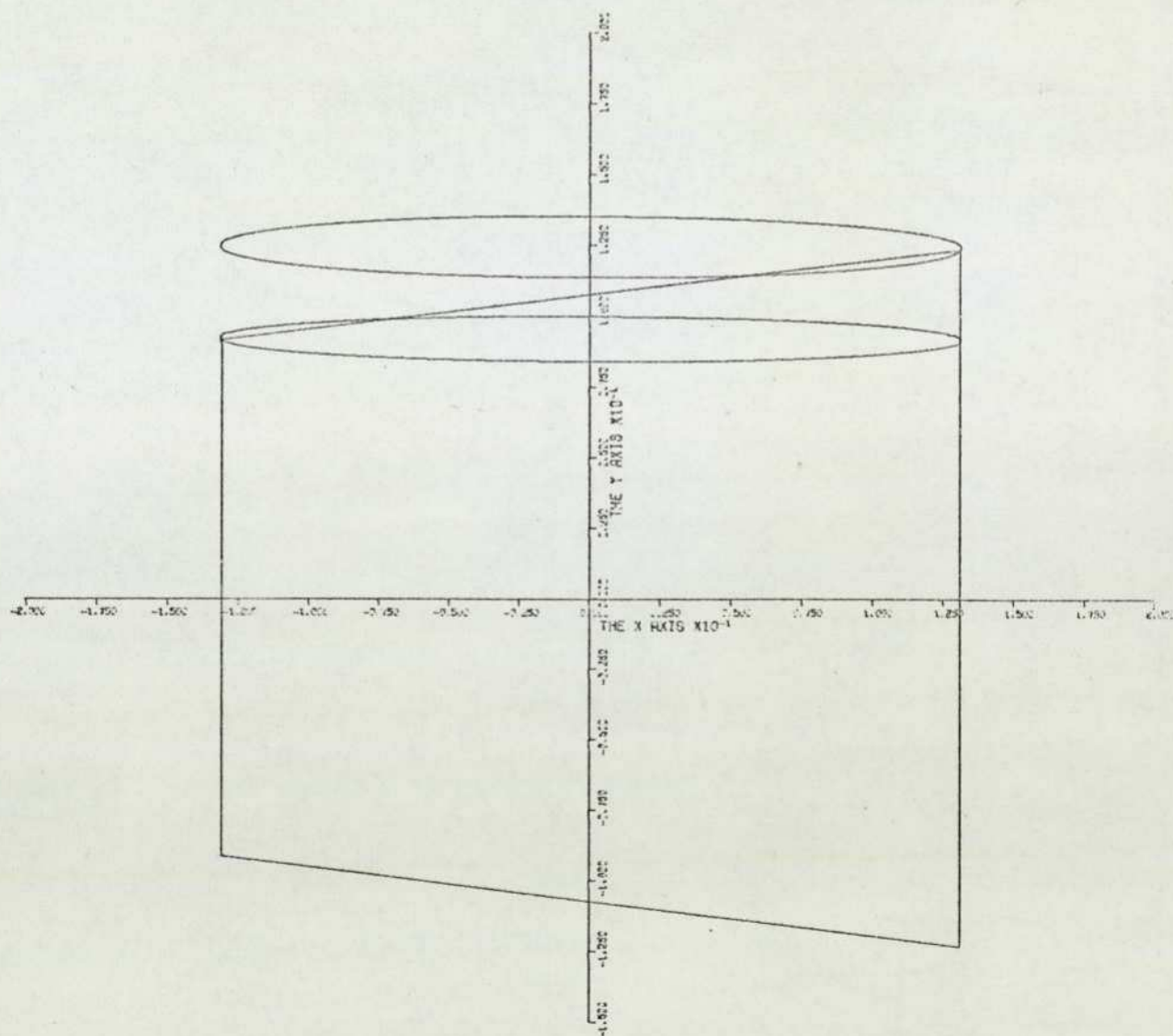
(A3, B2)

TRAPEZOID PROJECTION

X1= 13.11 Y1= 12.36

X2= -13.11 Y2= 9.14

D= 12.50 FEET



Graph. 6.3.10

(A3, B4)

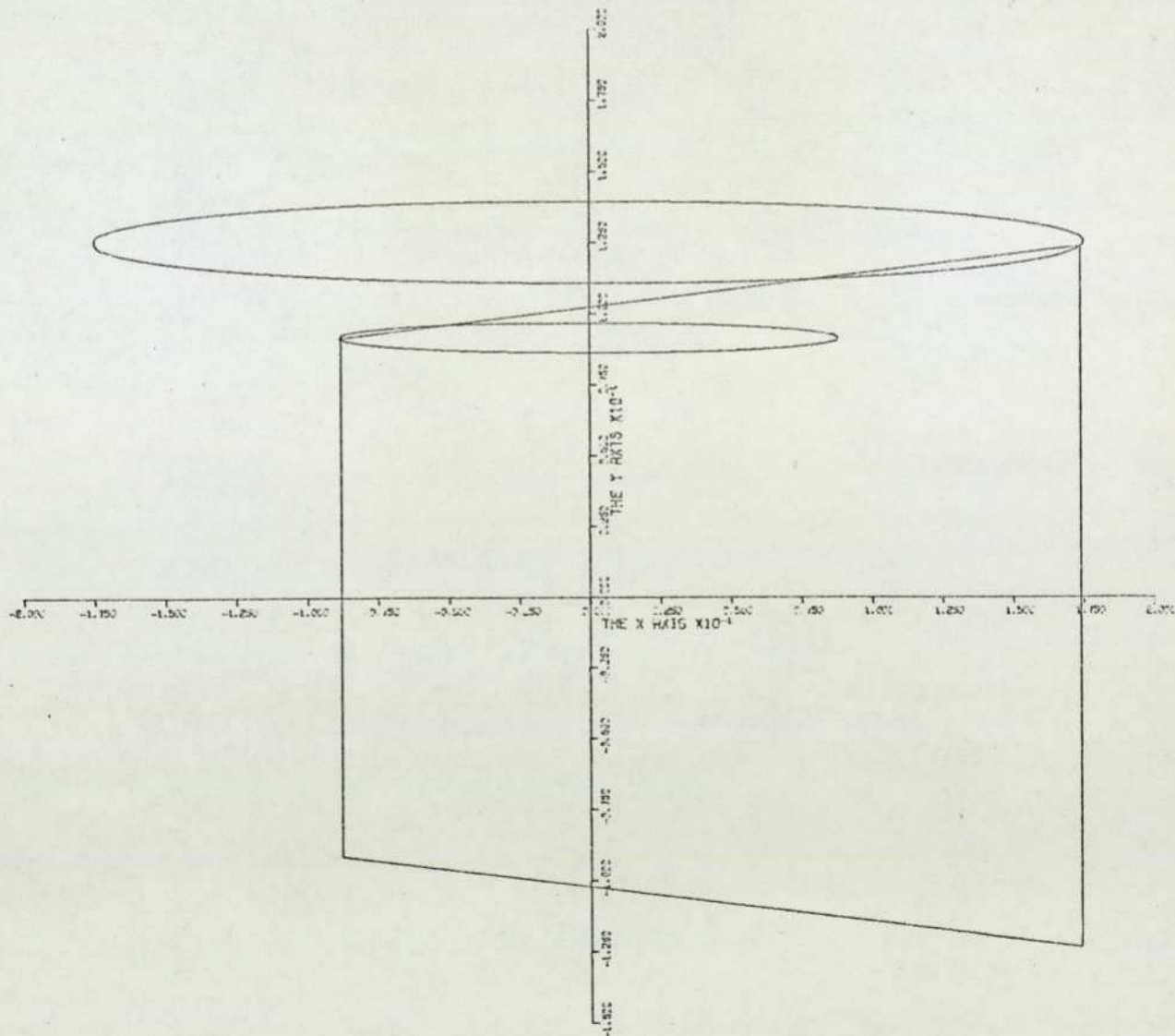
(See also Graph 6.3.5, (A3, B3)).

TRAPEZOID PROJECTION

X1= 17.41 Y1= 12.36

X2= -8.81 Y2= 9.14

D= 12.50 FEET



This position produced more apparent oscillations than the other 3 positions for all of the trapezoidal shapes, although the increase was not statistically significant (See Graphs 6.3.1 and 6.3.2).

The second most effective position for producing apparent oscillation for all the trapezoidal shapes was with the axis of rotation central ($x_1:x_2 = 6.1 : 6.1$). As mentioned above, this position produced more oscillations with the rectangle than any other.

These two 'central' axis positions together produced significantly more apparent oscillations than the other 2 positions ($p = 10\%$).

The results of this experiment, then, verified the hypothesis under test. Both the height ratio and the axis of rotation position were found to contribute to the illusion, with the height ratio having by far the most significant effect. The results obtained compared well with predictions made by considering the elliptical loci of the projections of the corner points of the rotating trapezoids.

EXPERIMENT 4

PROJECTIONS OF A RECTANGLE

INTRODUCTION

The mathematical analysis above (Section 4, p 31) showed that the projection of an actual rectangle when rotated about a central axis and viewed at a distance of about 15 feet produced a much less exaggerated trapezoidal shape than that used by Ames. It was decided, therefore, to investigate the percept resulting from viewing the rotation of such a projection of a rectangle.

Four shapes were constructed, equivalent to the actual projections of the 10" x 15" planar matt white rectangle used in E1 pattern1, at angles of 0° , 20° , 40° and 60° respectively from the plane perpendicular to the line of regard.

There was no particular theoretical concept under test in this experiment - it was included purely for interest. It was, however, possible to predict the outcome by considering the relative positions of the elliptical loci of the corner points of the shapes used, as calculated and plotted by TROT.

EXPERIMENTAL DESIGN

Four planar matt white shapes were constructed, equivalent to the actual projections of a 10" x 15" rectangle at angles of 0° , 20° , 40° and 60° respectively from the plane perpendicular to the line of regard. These four shapes are shown diagrammatically in Fig.6.4.

The actual dimensions of these shapes are given in table 6.4.1. The camera settings used when these shapes were filmed produced a magnification of 2.5 x on the projection screen. The projected sizes of the four shapes are also shown in table 6.4.1.

The speed of rotation used was 3 rpm throughout. Three male and three female subjects viewed each of the four shapes. There were thus only two factors being investigated in this experiment - projected shape of a rectangle (4 levels) and sex (2 levels).

Fig. 6.4

The 4 shapes used
in Experiment 4.

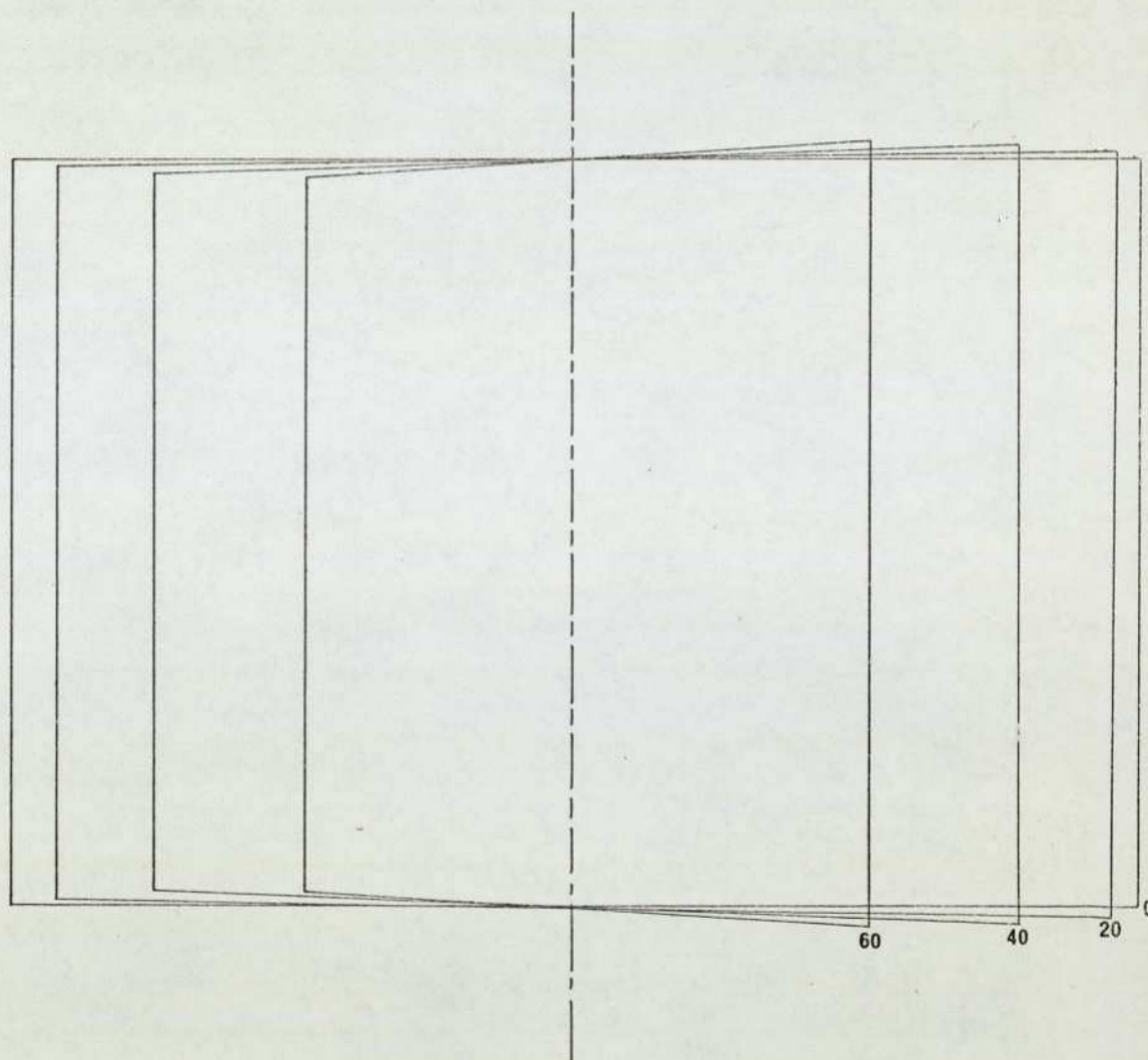


Table 6.4.1

PROJECTIONS OF A 10" x 15" RECTANGLE VIEWED AT 15FT

Angle of Rotation	Actual Dimensions			
	x_1	y_1	x_2	y_2
0°	7.5	5.0	-7.5	5.0
20°	7.15	5.07	-6.95	4.93
40°	5.90	5.14	-5.60	4.87
60°	3.89	5.19	-3.62	4.83
Dimensions on Projection Screen				
	x_1	y_1	x_2	y_2
0°	18.75	12.5	-18.75	12.5
20°	17.87	12.67	-17.37	12.32
40°	14.75	12.85	-14.00	12.17
60°	9.72	12.97	- 9.05	12.07

The design for the two factor analysis of variance may be summarised as follows:

Factors

Shape : A - 4 levels
Sex : C - 2 levels

Thus there are (4) (2) = 8 treatment combinations.

The number of observations, $n = 3$ for all treatments.

Thus total (3) (8) = 24 observations.

Then the total sum of squares with $(24 - 1) = 23$ d.f. can be partitioned into the following components with degrees of freedom as shown at the right:

Main effects	A	3
	C	1
2 factor interaction	A x C	3
Within treatment	error	$23 - 7 = 16$ d.f.

Then	$A_1 = 0^\circ$	$C_1 = M$
	$A_2 = 20^\circ$	$C_2 = F$
	$A_3 = 40^\circ$	
	$A_4 = 60^\circ$	

RESULTS

The results obtained are summarised in table 6.4.2. (see p 92 for details of the manipulation of the raw data).

Table 6.4.2
SUMMARY OF RESULTS OF EXPERIMENT 4

Each cell entry is sum of $n = 3$ observations

	A_1	A_2	A_3	A_4	Σ
C_1	3	4.45	3	26.37	36.82
C_2	3	3	3	26.37	35.37
Σ	6	7.45	6	52.74	72.19

A summary of the 2 factor analysis of variance is given in table 6.4.3. This shows that both of the factors investigated, as well as the A x C interaction, had a significant effect upon the amount of apparent oscillation produced. ($\alpha < 0.001$).

Scheffé's test for multiple comparisons (table 6.4.4) showed that shape A_4 (the 60° projection) produced significantly more apparent oscillations than any other shape ($\alpha = 0.001$). (In fact only one other condition (A_2C_1) produced any oscillations at all). The results are summarised graphically in graph 6.4.1.

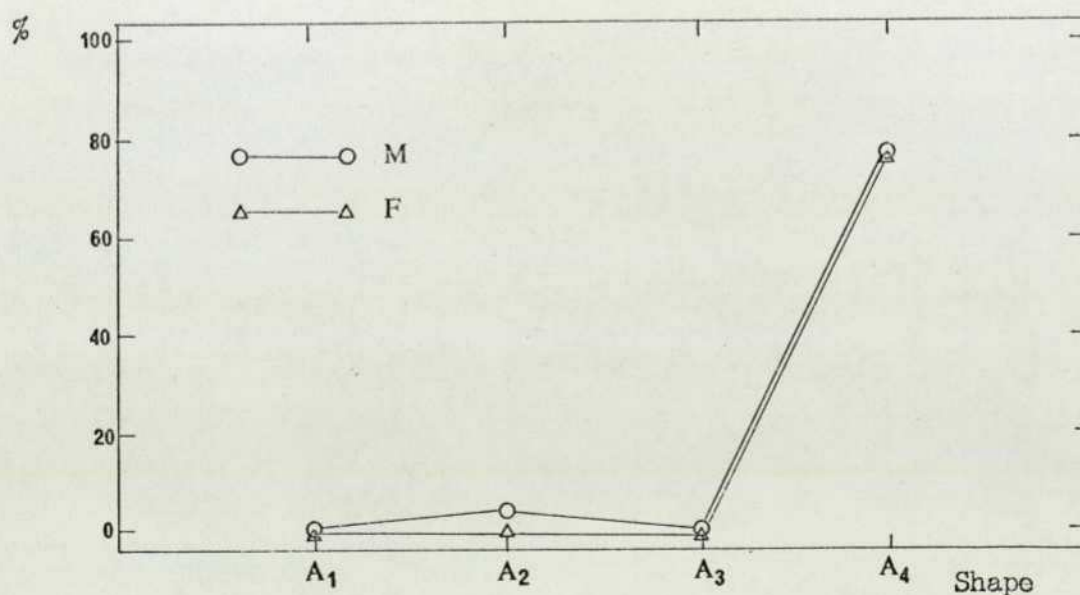
Table 6.4.3.

SUMMARY OF ANALYSIS OF VARIANCE FOR EXPERIMENT 4

Source of Variation	Sum of Squares	d.f.	Mean Square	F	α
Shape - A	267.77	3	89.26	233.96	0.001
Sex - C	12.21	1	12.21	32.01	0.001
A x C	11.86	3	3.95	10.36	0.001
Within treatments	6.10	16	0.38		
Total	274.22	23			

Graph 6.4.1

Percentage
Oscillations



Comparison	$\sum_{A_1}^6$	$\sum_{A_2}^{7.5}$	$\sum_{A_3}^6$	$\sum_{A_4}^{52.7}$	$\sum a_i^2$	D_i	D_i^2	$MS_{D_i} = \frac{D_i^2}{n \sum a_i^2}$
A_4 vs $A_1 + A_2 + A_3$	-1	-1	-1	3	12	138.6	19210	533.6
A_4 vs A_2	0	-1	0	1	2	45.2	2043	170.25

Table 6.4.4.
COMPARISONS BETWEEN SHAPES

Multiple Comparisons for Shape (A)

$k = 4$ levels

$n = 6$ observations

$S^2 = 0.38$ (see table 6.4.3)

Tabled value of F for $(k - 1) = 3$ and $k(n - 1) = 20$ d.f.

is $F = 8.10$ for $\alpha = 0.001$

Then for significance, $MS_{D_i} \geq (k - 1)(F)(S^2)$

≥ 9.23 for $\alpha = 0.001$

DISCUSSION

This was the only experiment in which sex had a significant effect upon the results - i.e. there was a difference between the results obtained with male and female observers.

There are two possible reasons for this. Firstly the numbers of observers used was much smaller than in the previous three experiments; and secondly, the error Mean Square obtained was very small. This may, therefore, be a chance result. One could expect at least one anomalous result in a series of experiments such as this.

Practically no apparent oscillations were obtained with the shapes A_1 to A_3 (the projections of the rectangle at angles 0° , 20° , and 40° respectively). These shapes are all practically rectangular and the graphs 6.4.2, 3 and 4 indicate the ambiguities of the loci of their corner points.

Graph 6.4.5, however, shows that the 60° projection has become almost recognisably trapezoidal, and the elliptical loci overlap by an amount that makes apparent oscillation more likely (cf Graph 6.3.7) Oscillation was, in fact, found with this shape.

Although there was no particular theoretical basis for this experiment, the results obtained are consistent with predictions based on the hypothesis tested in experiment 3.

Graph 6.4.2

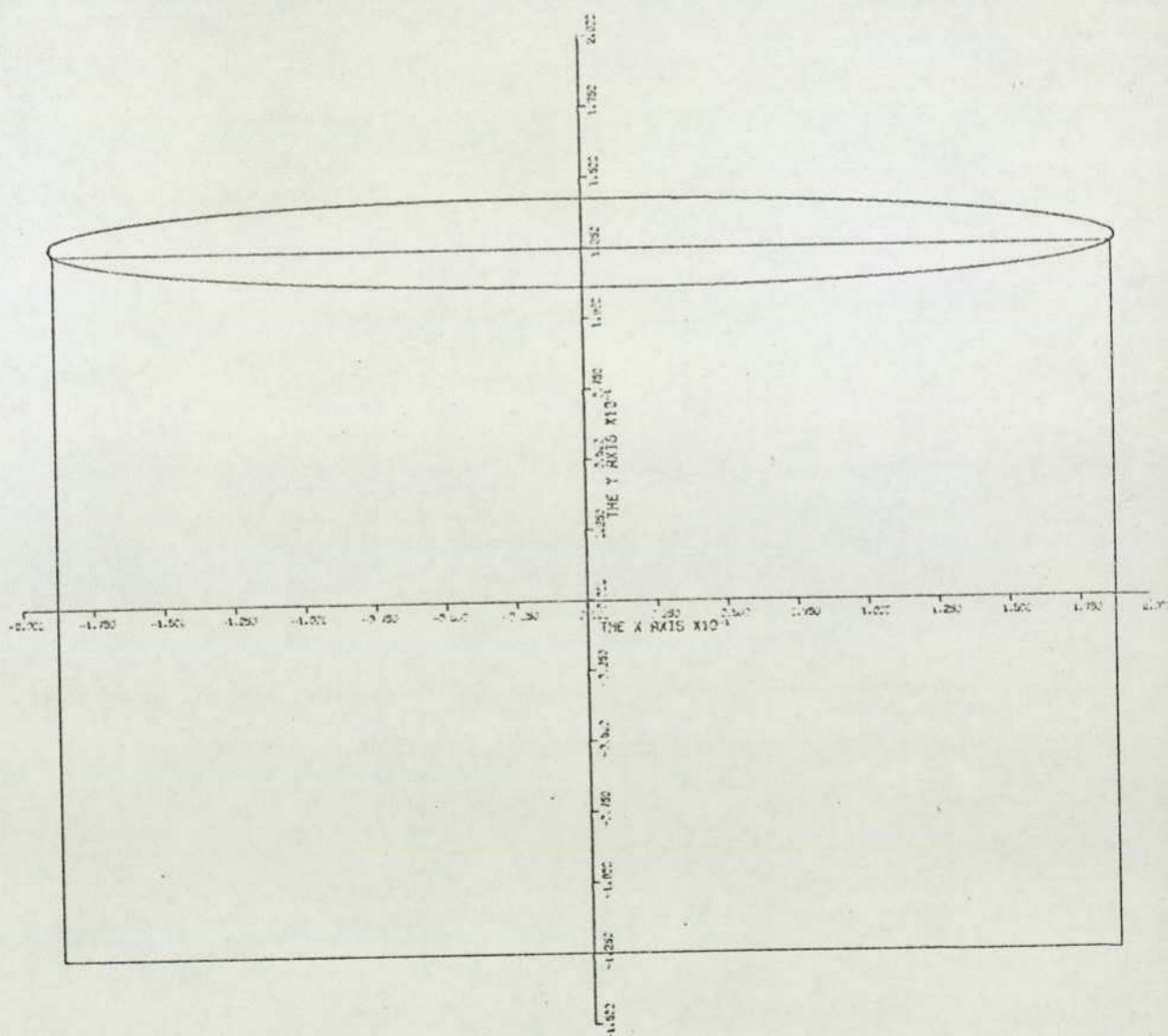
(A_1)

TRAPEZOID PROJECTION

$X1 = 18.75$ $Y1 = 12.50$

$X2 = -18.75$ $Y2 = 12.50$

$D = 12.50$ FEET



Graph 6.4.3

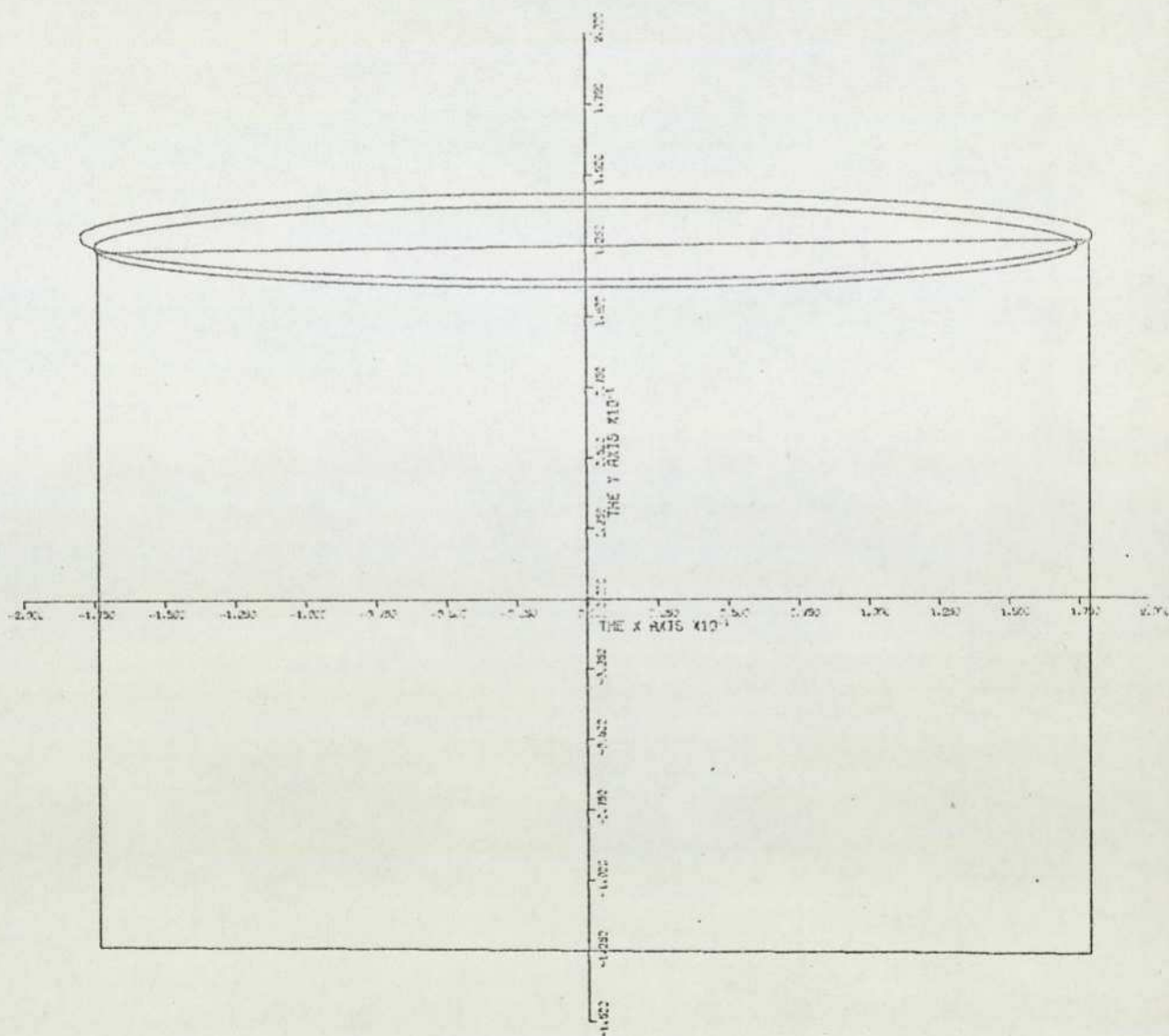
(A_2)

TRAPEZOID PROJECTION

$X1= 17.87$ $Y1= 12.67$

$X2=-17.37$ $Y2= 12.32$

$D= 12.50$ FEET



Graph 6.4.4

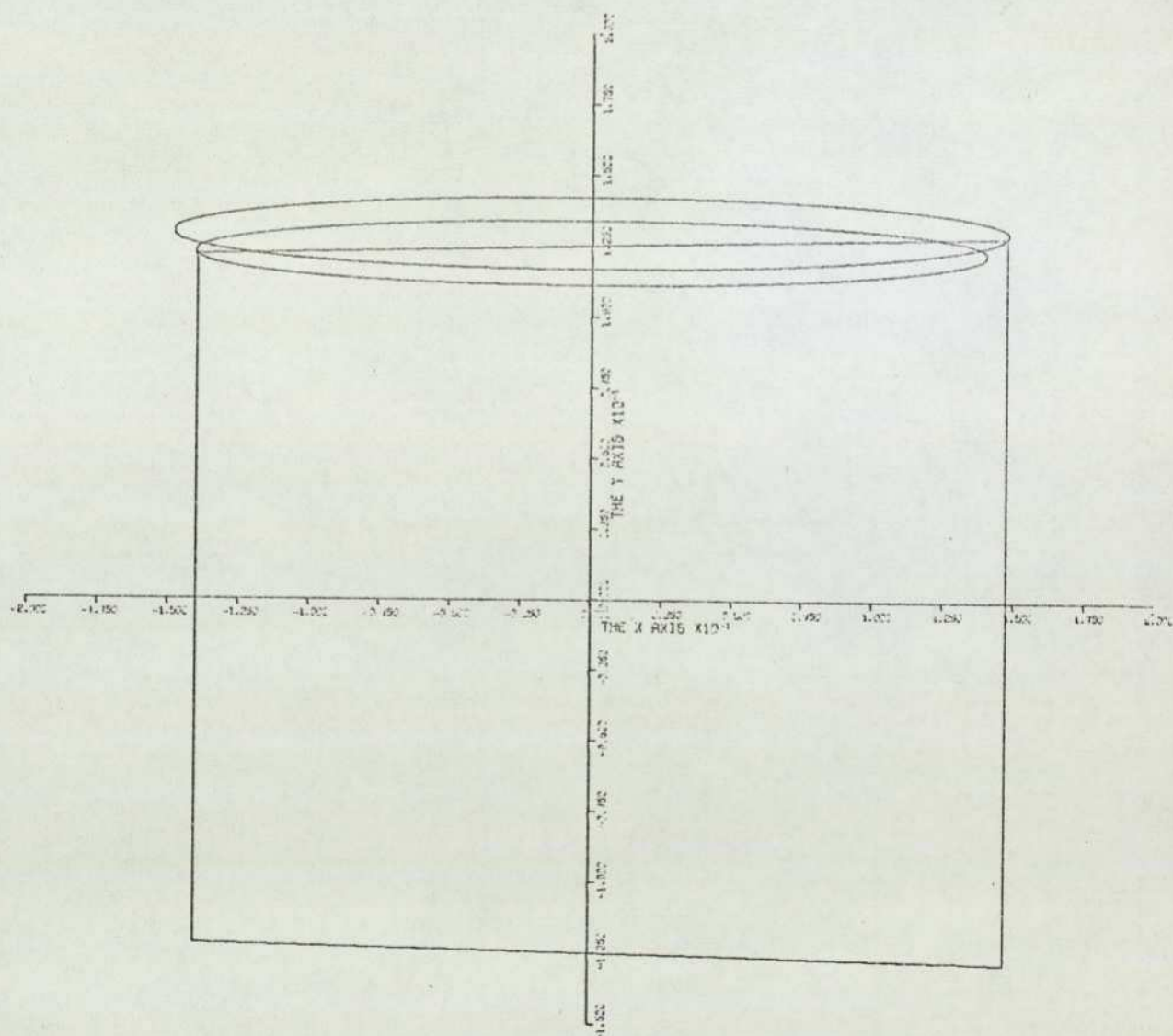
(A_3)

TRAPEZOID PROJECTION

$X1= 14.75$ $Y1= 12.85$

$X2=-14.00$ $Y2= 12.17$

$D= 12.50$ FEET



Graph 6.4.5

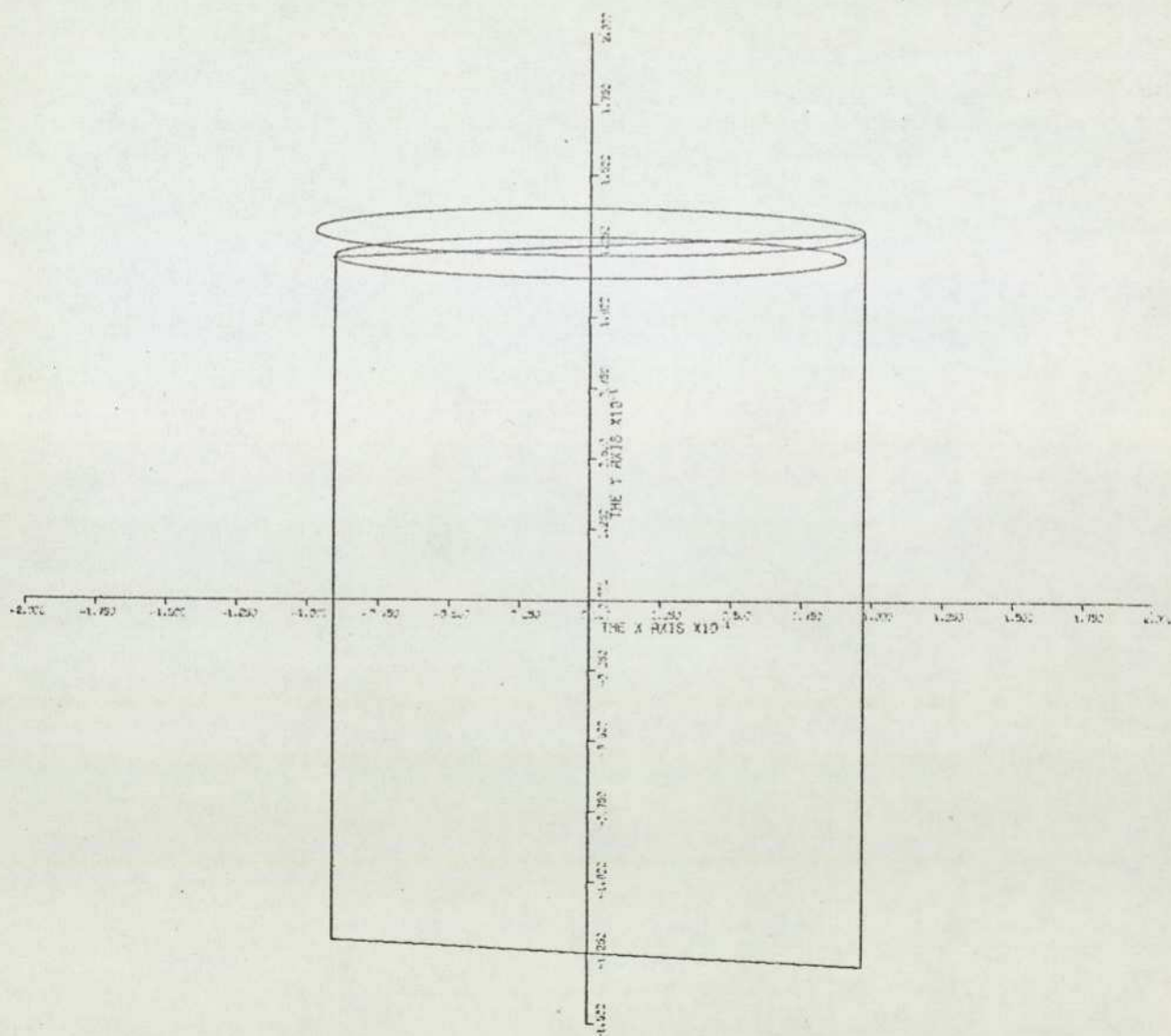
(A_4)

TRAPEZOID PROJECTION

X1= 9.72 Y1= 12.97

X2= -9.05 Y2= 12.07

D= 12.50 FEET



EXPERIMENT 5

WITHIN-S & BETWEEN-S VARIABILITY

INTRODUCTION

The hypothesis detailed in Section 4 (p 57) suggested that those shapes that were almost rectangular, and not obviously trapezoidal, provided ambiguous cues as to the direction of rotation because of the overlap of the elliptical loci of their corner points. When viewing these ambiguous shapes, then, sometimes rotation and sometimes oscillation would be seen.

This would result in both the between - S and within - S variability being greater for these shapes than for any others.

The results from E3 gave a larger standard deviation for shapes in this category, but since only one observation was made by each S, the within - S variability could not be tested.

It was decided, therefore, to run a separate experiment for this purpose.

EXPERIMENTAL DESIGN

Four subjects, each of whom was very familiar with the Ames illusion and with the experimental set up (to avoid any possible learning effects) were tested on each of 4 shapes on four consecutive days.

The 4 lengths of film were taken from E3, and were of planar matt white shapes rotating at 5 rpm. The dimensions of the shapes used are shown in Table 6.5.1 (usual notation).

Table 6.5.1

DIMENSIONS OF SHAPES USED IN E5

Shape	Actual Dimensions	
	$y_1:y_2$	$x_1:x_2$
A ₁	12 : 8	7.1 : 5.1
A ₂	11 : 9	7.1 : 5.1
A ₃	10.5 : 9.5	7.1 : 5.1
A ₄	10 : 10	7.1 : 5.1
Dimensions on Projection Screen		
	$y_1:y_2$	$x_1:x_2$
A ₁	25.8 : 17.2	15.26 : 10.96
A ₂	23.65 : 19.35	15.26 : 10.96
A ₃	22.57 : 20.42	15.26 : 10.96
A ₄	21.5 : 21.5	15.26 : 10.96

The subjects thus saw a rectangle (A₄), an obviously trapezoidal shape (A₁) and two intermediate shapes (A₂ and A₃).

The number of apparent changes in direction during a 2 minute trial was taken as a measure of the illusion.

A randomised block design was employed (Edwards, 1969), with each S being considered as a block.

Then the factors investigated may be summarised as:

Shape (A)	4 levels	3 d.f.
Days (D)	4 levels	3 d.f.
A x D		9 d.f.
Blocks (S's)	4 levels	3 d.f.

RESULTS

The results obtained are shown in table 6.5.2.

Table 6.5.2.

RESULTS OF EXPERIMENT 5

Each cell entry is 1 observation

Days	Subjects	A ₁ 12:8	A ₂ 11:9	A ₃ 105:95	A ₄ 10:10	Σ
D ₁	S ₁	19	20	20	2	61
	S ₂	20	20	8	0	48
	S ₃	20	19	19	4	62
	S ₄	20	10	1	1	32
	Σ	79	69	48	7	
D ₂	S ₁	20	20	10	2	52
	S ₂	19	20	10	0	49
	S ₃	20	20	20	4	64
	S ₄	20	8	5	0	33
	Σ	79	68	45	6	
D ₃	S ₁	20	20	20	0	60
	S ₂	20	19	10	0	49
	S ₃	20	20	20	3	65
	S ₄	20	10	19	2	51
	Σ	80	69	69	5	
D ₄	S ₁	20	18	2	0	40
	S ₂	20	20	8	0	48
	S ₃	20	20	20	2	62
	S ₄	20	20	18	0	58
	Σ	80	78	48	2	
Σ		318	284	210	20	832

The analysis of variance is summarised in table 6.5.3.

Table 6.5.3.

SUMMARY OF ANALYSIS OF VARIANCE FOR EXPERIMENT 5

Source of Variation	Sum of Squares	d.f.	Mean Square	F	α
Shape - A	3326.5	3	1108.83	67.67	0.001
Days - D	21.875	3	7.29		
A x D	90.625	9	10.07		
Blocks (S's)	201.625	3	67.21	4.10	0.025
Error (= Blocks x treatments)	737.375	45	16.39		
Total	4378	63			

This shows that the outline shape again had the most significant effect ($\alpha = 0.001$) on the amount of apparent oscillation seen.

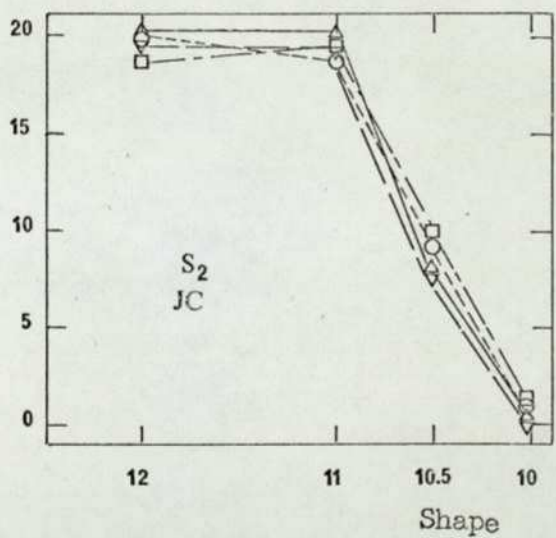
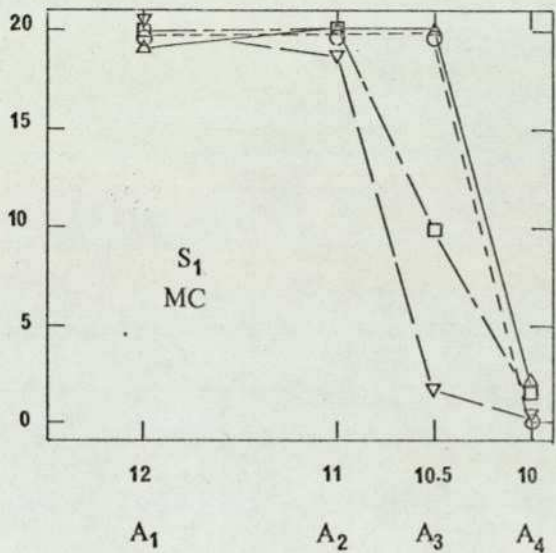
The day to day variation (i.e. the within - S variability) did not produce any significant variation in the amount of oscillation seen. The results obtained, then, showed that the observations made by each individual subject were remarkably consistent from one day to the next. This is illustrated in the graph 6.5.1., which shows the number of oscillations produced by each shape for each of the 4 days, for the 4 subjects separately.

The analysis of variance also showed that the between - subject variability (Blocks) was significant at the 2.5% level. This can also be seen from the graphs. The results for all 4 subjects are remarkably consistent for both the trapezoid (A_1) and the rectangle (A_4), but there is considerable variation between subjects for the 2 intermediate shapes ($A_2 + A_3$)

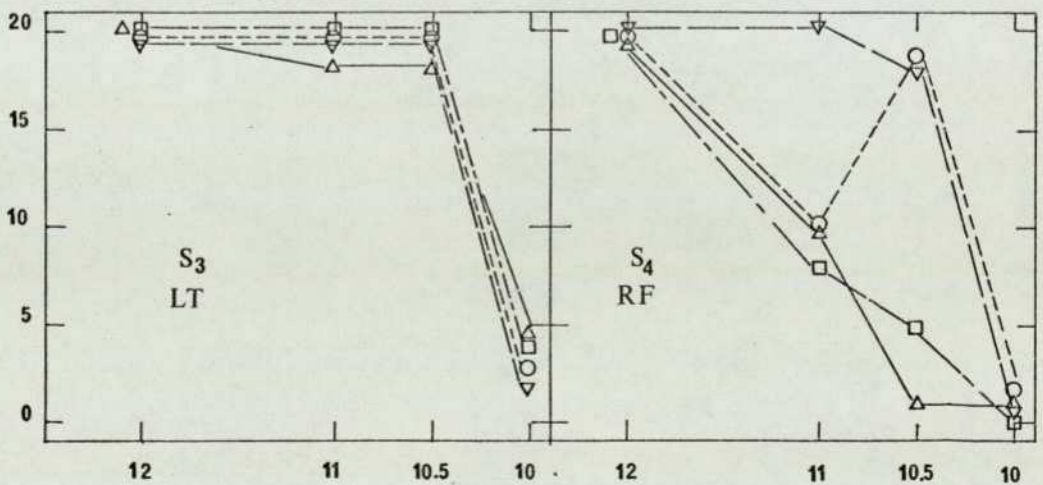
Graphs 6.5.1

Number of
Oscillations

(See Table 6.5.1 for
details of dimensions)



△—△ D₁
□—□ D₂
○—○ D₃
▽—▽ D₄



DISCUSSION

The hypothesis detailed in Section 4 (p.57) anticipated that the between-S and within-S variability would be greater for those intermediate shapes that produced ambiguous cues to the direction of rotation. This ambiguity can be defined in terms of the elliptical loci of the projections of the corner points as the shapes rotate. If those loci are close together, or partially overlapping, then the apparent lengths of both the actually longer side and the actually shorter side may be very similar for part of a revolution, and confusion of the resulting percept will be inevitable.

The elliptical loci of the corner points of the 4 shapes used in this experiment are shown graphically in E3 - see Graph 6.3.4, 6.3.6, 6.3.7 and 6.3.8.

The results of this experiment indicated that the perceptual interpretations made by each individual subject were remarkably stable (i.e. the within-S variability was statistically insignificant).

But the results obtained with different subjects did show a significant variation ($\alpha = 0.025$). However, this variation only occurred with those intermediate shapes ($A_2 + A_3$) that were of special interest in this experiment (see Graph 6.5.1). The results for all 4 subjects were quite remarkably consistent for the obviously trapezoidal shape (A_1) and for the rectangle (A_4). The information provided by the projections of these shapes, then, was unambiguous and resulted in a stable percept - either continuous oscillation with the trapezoid or continuous rotation with the rectangle. (This was predictable from the graphs 6.3.4 and 6.3.8 - see comments in E3).

The results of this experiment, then, supported the hypothesis that the between subject variability would be greater for those shapes providing ambiguous cues to the direction of rotation. The perceptual interpretations made by each individual subject, however, were found to be more stable than was anticipated.

EXPERIMENT 6

EFFECT OF FIXATION POSITION

INTRODUCTION

Preliminary experiments had indicated that the number of apparent oscillations observed may vary with the point fixated on the rotating shape. This was supported by several spontaneous remarks by S's in later experiments, although all the S's were instructed to fixate the centre of the shapes during the observation runs.

It was decided, therefore, to investigate this effect in more detail.

EXPERIMENTAL DESIGN

The effect of varying the point of fixation was investigated by using the three different fixation positions indicated in Fig.6.6.1.

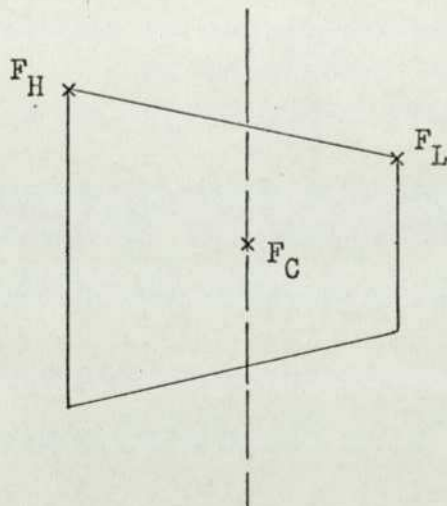


Fig. 6.6.1

F_H = Fixate top of longer side
 F_L = Fixate top of shorter side
 F_C = Fixate centre

Four different shapes were used, the dimensions of which are shown in table 6.6.1. (usual notation). The 4 lengths of film were again taken from E3, and were of planar matt white shapes rotating at 5 rpm.

Table 6.6.1

DIMENSIONS OF SHAPES USED IN E6

Shape	Actual Dimensions	
	$y_1 : y_2$	$x_1 : x_2$
A ₁	12 : 8	7.1 : 5.1
A ₂	11 : 9	7.1 : 5.1
A ₃	10 : 10	7.1 : 5.1
A ₄	10 : 10	6.1 : 6.1
Dimensions on Projection Screen		
	$y_1 : y_2$	$x_1 : x_2$
A ₁	25.8 : 17.2	15.26 : 10.96
A ₂	23.65 : 19.35	15.26 : 10.96
A ₃	21.5 : 21.5	15.26 : 10.96
A ₄	21.5 : 21.5	13.11 : 13.11

Four subjects were chosen, each of whom was very familiar with the Ames illusion and with the experimental set up being used, to minimise any possible learning effects.

The experiment thus had a 3 x 4 factorial design, the factors being:

Shape	(A)	4 levels	3 d.f.
Fixation	(F)	3 levels	2 d.f.

A randomised block design was again employed, each S being considered as one block. All 4 S's viewed each of the 12 different treatment combinations, presented in random order in 3 sessions per subject. (E.5 had shown that the day to day variation within subjects was not statistically significant).

The number allotted to each different treatment condition is indicated in table 6.6.2, and the random orders for the presentation of each condition to each subject are shown in table 6.6.3.

Table 6.6.2.

3 x 4 FACTORIAL DESIGN FOR E6*

Shapes	Fixation Positions		
	F _H	F _C	F _L
A ₁	1	2	3
A ₂	4	5	6
A ₃	7	8	9
A ₄	10	11	12

* Note: Condition 10 = Condition 12

Table 6.6.3.

RANDOM ORDERS FOR PRESENTATION OF
CONDITIONS TO EACH SUBJECT

S ₁	S ₂	S ₃	S ₄
11	7	2	3
6	6	10	4
2	1	4	9
1	8	3	1
9	4	11	7
4	9	6	6
10	11	12	5
5	2	1	8
8	5	8	12
12	3	7	11
3	10	9	10
7	2	5	2

The experimental set up was the same as that used in experiments 1 - 5. A small light "pointer" was used by the experimenter to indicate the fixation position required during each trial run. The instructions given were to "follow this corner" (for F_H and F_L) or to "fixate the centre, here" (for F_C), and to depress the morse key whenever the rotating shapes appeared to change direction.

The number of apparent changes in direction during a 2 minute run was taken as a measure of the illusion.

RESULTS

The results obtained are shown in table 6.6.4. and the analysis of variance is summarised in table 6.6.5.

Table 6.6.5.

SUMMARY OF ANALYSIS OF VARIANCE FOR E6

Source of Variation	Sum of Squares	d.f.	Mean Square	F	α
Shape - A	3354.06	3	118.02	86.20	0.001
Fixation position - F	40.87	2	20.44	1.57	0.25
A x F	11.62	6	1.94		
Blocks - S's	17.73	3	5.91		
Error (=Blocks x treatments)	428.02	33	12.97		
Total	3852.31	47			

This showed that the outline shape of rotating stimulus objects again had a highly significant effect on the amount of apparent oscillation observed ($\alpha = 0.001$). But the fixation position used was significant only at the 25% level, and the differences between subjects (blocks) was statistically insignificant.

Table 6.6.4.

RESULTS OF EXPERIMENT 6

Each cell entry is 1 observation

Fixation Positions	Subjects	Shapes				Σ
		A ₁	A ₂	A ₃	A ₄	
F _H	S ₁	5	10	4	2	21
	S ₂	20	17	0	0	37
	S ₃	20	20	0	0	40
	S ₄	20	20	0	0	40
	Σ	65	67	4	2	138
F _C	S ₁	19	19	4	6	48
	S ₂	20	20	4	2	46
	S ₃	20	20	0	0	40
	S ₄	20	20	0	0	40
	Σ	79	79	8	8	174
F _L	S ₁	8	17	6	4	35
	S ₂	20	20	0	0	40
	S ₃	20	18	0	0	38
	S ₄	20	20	0	0	40
	Σ	68	75	6	4	153
Σ		212	221	18	14	465

DISCUSSION

It was found that varying the fixation position did not have the anticipated effect of altering the amount of apparent oscillation perceived, since this factor was statistically significant at the 25% level only, and so many may well have occurred by chance. This result is fortunate, since it indicates that possible slight variations in the fixation positions used by all the subjects in experiments 1 - 5 are unlikely to have influenced the results unduly. All the subjects in these earlier experiments had been instructed to fixate the centre of the rotating objects, but no definite indicator (eg. a black cross on the screen) was provided.

[The mathematical analysis of the projection the rotating trapezoidal shapes was based on an arbitrary fixation position at the centre of each shape. The actual fixation position does not affect the projection in the frontal plane, but it does affect the projection of the image onto the retina. This was the main reason for central fixation being chosen for these experiments, so that the results could be directly compared with the computed projections of each shape.]

SUMMARY OF RESULTS

By far the most significant factor found to influence the amount of apparent oscillation seen when viewing a rotating stimulus object was the outline shape of that object. Every experiment carried out showed this factor to be statistically significant at <0.001 level.

More detailed investigation of the effect of outline shape (E3) showed that for those shapes that were obviously trapezoidal (i.e. the elliptical loci of their corner points were narrow, and well separated), the actually longer vertical edge was always seen as the apparently longer edge, and was therefore seen as the apparently nearer edge. These shapes were therefore seen to oscillate.

But for rectangular shapes, where the elliptical loci overlapped completely, the shapes were seen as continually rotating.

For the intermediate shapes, where the elliptical loci were either very close together or partially overlapping, an ambiguous situation resulted so that sometimes rotation and sometimes oscillation was seen. This was found in all the experiments in which such shapes were used (E3, E4, E5 and E6). It was also found that there was more variation from subject to subject with these shapes than with any others (E5).

Of the other sources of information contained in the rotating stimulus objects, the speed of rotation seems to be the most important (E2). It was found that over a moderate range of speeds (say 4 to 16 rpm), the speed of rotation had little effect on the amount of oscillation perceived. But for slower speeds 'boredom', or lack of visual interest, could cause spontaneous fluctuations in the direction of rotation seen (cf Necker's cube and other reversible objects). And for faster speeds the perception - interpretation - decision time was inadequate; the observations tended to be grouped in spells of 5 to 10 seconds, of either complete oscillation or complete rotation.

It was found that surface pattern was very much a secondary source of information in the interpretation of these rotating figures (E1). The addition of 'window' or 'shadow' patterns had little effect on the amount of oscillation seen.

However, it should be pointed out that only 'extreme' shapes were used in that experiment. Maybe with the more ambiguous outline shapes used in E3, surface pattern may have had a more significant effect.

The point of fixation (E6) was found to have an insignificant effect on the amount of oscillation produced.

The results in general, then, support the hypotheses detailed in Section 4, which considered the relative importance of the various sources of information available in the rotating stimulus shapes. The main source of information that was not investigated here was the viewing distance. Limitations on the space available for experimentation precluded this.

However, this factor will be considered in the next Section, where the results obtained in this study will be compared with the published results of other investigations.

SECTION 7

EXPERIMENTAL INVESTIGATION DISCUSSION

In this section, the results obtained in the experiments just described will be discussed in more detail, and will be compared with the results obtained in similar studies by others.

The various parameters that may contribute to the illusion are considered separately, including some factors not specifically investigated in the present series of experiments. The status of the hypotheses proposed above will then be evaluated.

SPEED OF ROTATION

One of the most significant parameters found in the present series of experiments was the effect of altering the speed of rotation on the amount of apparent oscillation produced.

It was found that over a moderate range of speeds (about 4 - 16 rpm) the illusion of apparent oscillation produced by a rotating trapezoidal object is a stable and repeatable effect.

At slower speeds visual 'boredom' can confound the illusion; if the stimulus is even slightly ambiguous, (eg. with a rectangle) the percept will alternate between possible interpretations, providing a more interesting scene than the single interpretation.

The experiment also indicated that observation became difficult with speeds of about 35 rpm. The range of speeds between 16 and 35 rpm was not tested so the actual changeover point from consistent to variable responses cannot be estimated. But with speeds as high as 35 rpm the number of possible reversals were more than one per second - i.e. the interpretation - decision - reaction time was less than one second per reversal. Although such an interval may be adequate for a single decision, it is postulated that this cannot be maintained for more than a few seconds. This should be compared with the maximum decision rate of about 0.5 sec per decision possible for the human neural system (Gregory, 1970¹).

Subjects used in E2 reported spontaneously that they were soon completely confused by objects rotating at 35 rpm, and their observations tended to be grouped in spells of either complete rotation or oscillation, these spells lasting 5 - 10 secs each. Several subjects actually called out "Slow down!" (or some such), and reported afterwards that they felt they just did not have time to decide what it was they were seeing.

Ames suggested speeds of 3 - 6 rpm as optimum for viewing the illusion, but he provides no quantitative evidence for this. Only two other studies have systematically varied the rotation speed.

Zegers and Murray (1962) carried out a detailed series of experiments, investigating several parameters of the illusion. In the first two of their experiments they used rotation speed of 4 (or 5) rpm, 30, 60 and 90 rpm. For their subsequent experiments they used 30 rpm only. Zegers and Murray make no comment about the effects of increasing the speed of rotation, but from their tabulated results it is clear that 100% oscillation was found with their obviously trapezoidal shapes at all speeds; and rotation was seen with the rectangle at the slowest speed; but for the intermediate shapes, the number of 'reversals' increased as the speed increased until sometimes 100% reversals were obtained with the rectangle at 90 rpm. It should be noted that:

"The non-quantitative comments of the subjects indicated that, at the higher speeds, the predominant illusion with the rectangle was an in - and - out pulsating, accordian effect"

(Zegers and Murray, 1962 p19)

Similar 2-dimensional "pulsating" effects were found by Mulholland (1956) with various shapes rotated at 78 rpm. In the same paper, Mulholland describes a brief investigation of the effect of speed of rotation on a trapezoidal window and a rectangular window, carried out using small (3.5" diameter) shapes viewed at 9 feet. He found that between 12 - 15 rpm, the perceived motions were predominantly those reported by Ames. When the speed was increased to 27 - 30 rpm, the rectangle was seen as rotating and the trapezoid as oscillating, but these motions were not as frequent nor as persistent as they were at a slower speed. Some of the time, the rectangle was seen to oscillate and the trapezoid to rotate. Expansion - contraction of both figures in the fronto-parallel plane was also perceived. At 90 rpm, the regular motions described by Ames were seldom perceived. Apparent oscillation of both stimulus objects was very frequent and persistent, as was expansion - contraction.

It is difficult to understand why so many investigators have used rotation speeds of 30 rpm or more, particularly when they claim to be investigating the 'Ames illusion', and Ames recommended much slower speeds.

But the results obtained in this investigation (E2) appear to be confirmed by the qualitative comments of others - that at high speeds (>20 rpm, say) the nature of the illusion changes. It is not such a stable and repeatable effect, and other illusory types of movement are seen besides regular apparent oscillation - eg. in and out pulsation, irregular oscillation etc.

This phenomenon is almost certainly a function of perceptual - processing - decision - time.

Investigators using these higher speeds must, therefore, be said to be examining a related, but different, illusion.

SURFACE PATTERN

Ames (1951) described the mullioned window of Fig.4.1 as the "most effective" in producing apparent oscillation rather than veridical rotation. He offers no quantitative evidence to support this conclusion, but in an appendix to his monograph, he illustrated some of the patterns that were tested (Fig.7.1) and described their effectiveness:

"It was found that white plane surfaces (see Fig. 7.1, row a), although they "produced" alteration of the observer's perception, were least effective, irrespective of the degree of trapezoidal variation; that more effect was "produced" by the frame-shaped surfaces (see Fig. 7.1, row b); still more when mullions were added (see Fig. 7.1, row c); still more when shadows were painted on the surface to represent a three-dimensional window frame (see Fig. 7.1, row d); and still more when there were more mullions or panes (see Fig. 7.1, row e). It was also found that larger trapezoids produced more unequivocal effects than smaller ones."

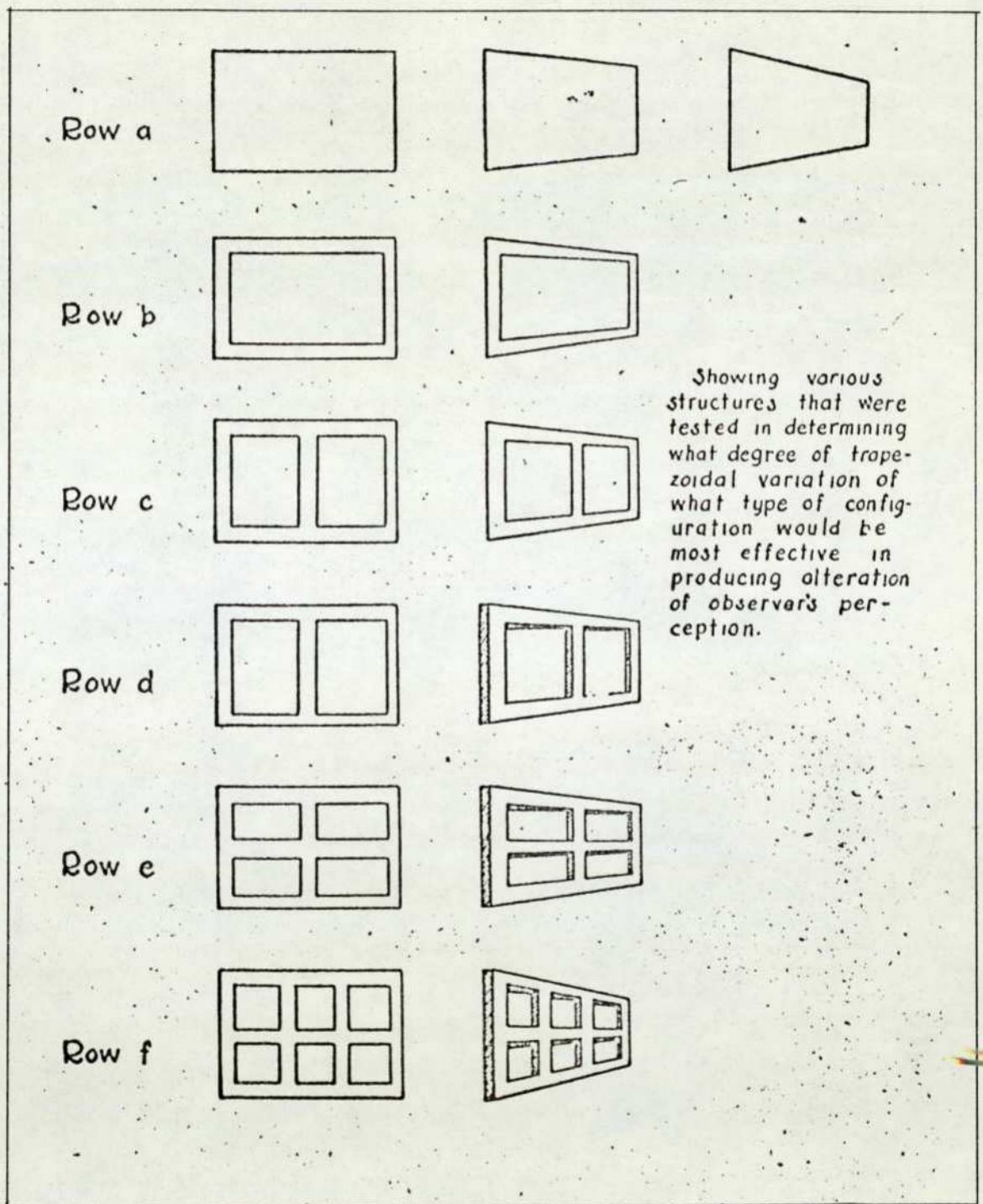
(Ames, 1951, p.29)

The experiment reported here (E1) indicated that surface pattern did have a significant effect upon the amount of oscillation produced, but the detailed comparisons of the various treatment conditions did not indicate the nature of that effect. Although there was a significant difference between the pattern producing most oscillations, and that producing least, there was certainly not a gradual increase in the amount of oscillation with increasing pattern information. In fact the planar white unpatterned shapes (P1) produced just as many oscillations as the shapes with full window and shadow patterns (P7) when the trapezoids alone were considered (see Fig. 6.1.1). With the rectangle, however, P5 produced far more oscillations than any other pattern; there is no obvious reason for this - it may just be a chance result.

Gradually increasing the amount of information provided by the surface pattern, then, did not produce the systematic increase in the amount of oscillation suggested by Ames. In this experiment, outline shape was shown to be a far more important source of information.

Fig. 7.1

(From Ames, 1951)



The 'shadow' effects used in E1 were similar to those illustrated by Ittelson (1952) (see Fig. 3.2), but with the pattern inverted so that the darker shadow was at the top of each 'window pane'. This made the pattern consistent with typical 'sunlight' shadow - i.e. with light incident from above, rather than below. Other investigators have copied Ittelson's pattern (eg. Graham, 1963, 1965; Allport and Pettigrew, 1957; Spitz, 1964).

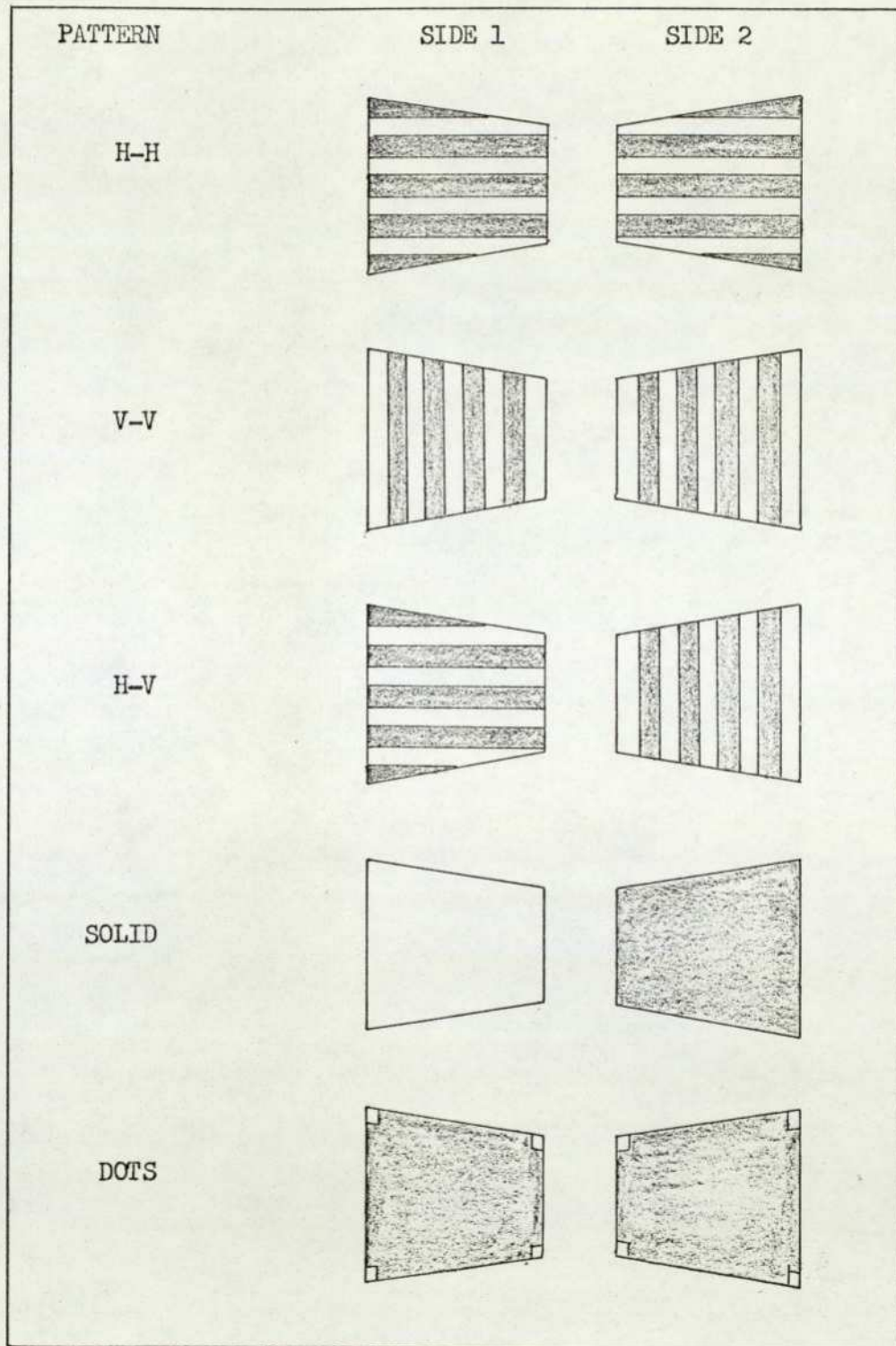
Surface pattern was one of several parameters of the illusion to be investigated in the series of experiments by Zegers and Murray (1962). In their second experiment they found that the addition of a 'six window' pattern did not significantly affect the illusion. In a further experiment (experiment 3) they investigated the effect of various different types of surface pattern, including stripes and dots (see Fig. 7.2).

The patterns were luminous and viewed against a black background. They found that there was no significant difference in the amount of oscillation produced, even by the obviously 'unbalanced' pattern with horizontal stripes on one side and vertical stripes on the other. There were, however, wide individual variations and certain aspects of their experimental procedure may cast doubt on the usefulness of their results - they used very long viewing sessions (about one hour each), every subject viewed every condition, and they used a speed of rotation of 30 rpm throughout. They admitted there were 'numerous occurrences' of types of motion other than oscillations or rotations. This was probably due to the high speed of rotation used.

Yet another type of surface pattern was investigated in Zegers and Murray's fourth experiment.

Fig. 7.2

PATTERNED TRAPEZOIDS USED BY
ZEGERS AND MURRAY, Experiment 3



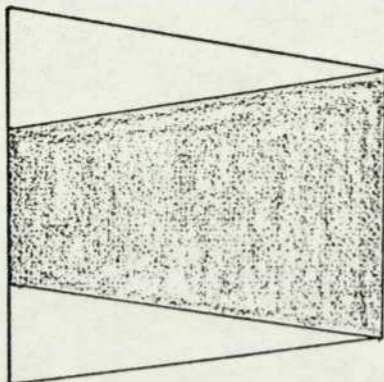


Fig. 7.3

This was a grey trapezoid shape painted onto the white trapezoid, whose angle of slant was opposite to that of the outer trapezoid (Fig. 7.3). They anticipated that the opposing directions of slant of the inner and outer trapezoids would negate the effect of perspective in the production of the illusion as the trapezoid was rotated. But again there was no significant effect, from which they concluded :

"...that subjects viewed the trapezoid as a whole, and that patterning was reduced to a subordinate position"

(Zegers and Murray 1964 p.10)

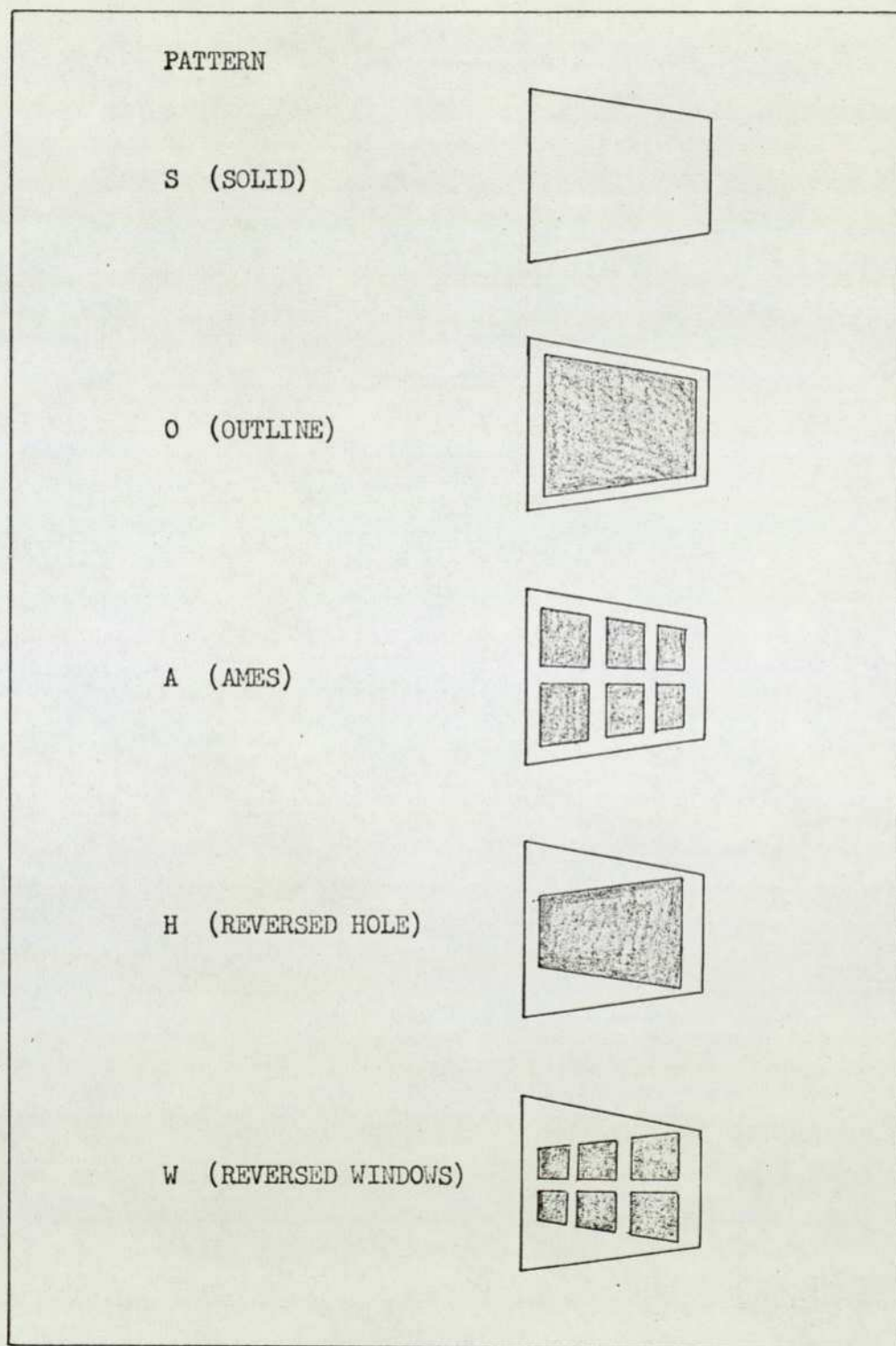
Zegers and Murray seemed determined to find some type of surface pattern that affected the illusion, in accordance with Ames' description. In a fifth experiment, they used the patterns shown in Fig.7.4, and they found that there was a significant effect. The Ames pattern always produced the highest number of reversals, and the 'reversed' patterns produced fewer reversals than the others.

But again every subject viewed every condition of the experiments (see comments on p 204), and the same high speed of rotation (30 rpm) was used.

The effect of surface pattern, was also investigated by Day and Power (1963, 1965). They used a mullioned window similar to, but

Fig. 7.4

Patterned Trapezoids used by
Zegers & Murray, Experiment 5.



about half the size of, that used by Ames; a pattern of vertical bars; and a plain white surface (see Fig. 7.5(1)).

Day and Power found that these surface patterns had no significant effect which, they suggested, is contrary to expectancy from the transactionalist hypothesis. This is not, of course, strictly true since Ames suggested that it was familiarity with rectangles per se (which may be of any colour or surface pattern) rather than familiarity with a particular window pattern that was important. However, Ames did emphasise that increasing the amount of 'windowness' increased the strength of the illusion.

Further experiments by Power and Day (1973) investigated the effect of surface pattern with elliptical shapes. Their revised theory (based on Day, 1972) suggested that mis-interpretation of cues to orientation results in mis-perception of the direction of movement; they tested this theory by investigating surface pattern as a cue to orientation direction.

The patterns used are shown in Fig.7.5(2). They predicted increasing amounts of oscillation as surface pattern became more misleading; thus they expected an increase in the frequency of reversal with patterns 1, 3, 2 and 4 respectively. This prediction was verified by the results obtained.

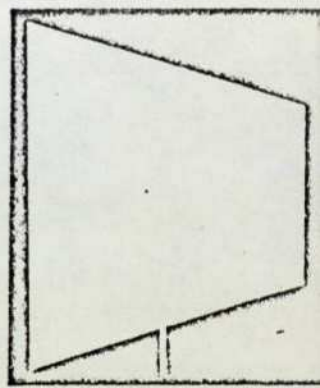
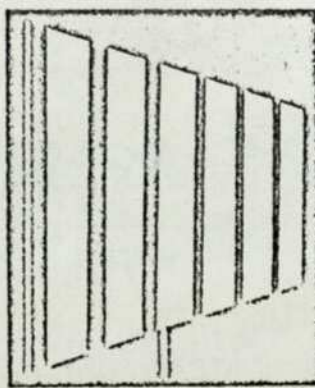
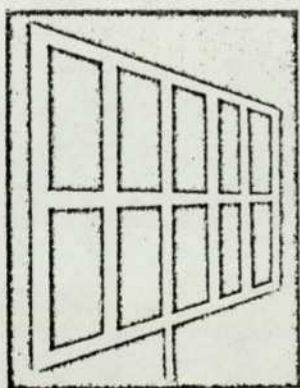


Fig. 7.5.(1)

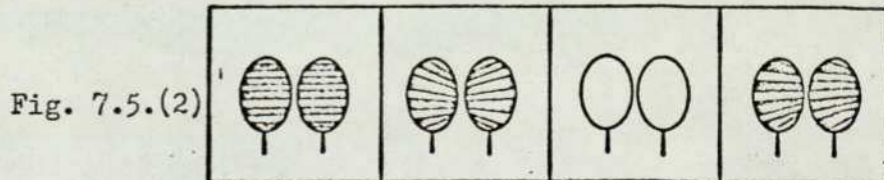


Fig. 7.5.(2)

Power and Day stated that "the outline stimulus was eliminated by using elliptical objects" (p.218), but how this is so remains unexplained. However, this does suggest that when outline shape provides a less compelling cue to direction of movement, surface pattern becomes more important.

Wallace and Cramer (1975) extended the experiments of Power and Day (1973), investigating the effects of the same surface patterns (Fig.7.5 (2)) with circular targets as well as ellipses; they obtained very similar results, but the circles always produced fewer reversals than the elliptical targets.

A variety of cutout shapes with and without artificial shadows were used by Cross and Cross (1969) (see Fig.7.6).

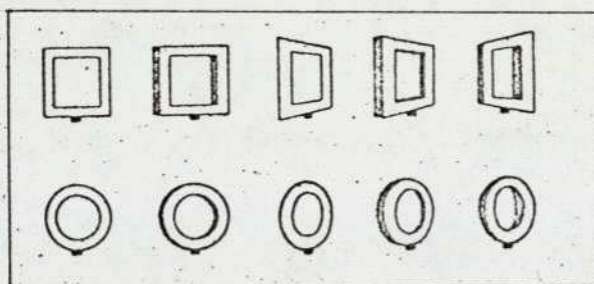


Fig. 7.6

(From Cross & Cross, 1969)

They found that 'shadowed' and 'perspectived' (sic) objects produced more illusory reversals than those without. But they used small objects (2" diameter) at a reasonably long viewing distance (10 feet) providing practically parallel projection, so that a high percentage of spontaneous reversals could be expected with all the shapes used. The percentage of reversals obtained by Cross and Cross for the various different shapes covered a narrower range than that obtained in most other studies, and although their results were significant at $p < 0.01$, most workers have achieved significance levels if $p < 0.001$ or better.

The validity of their results may also be challenged because of the high speed of rotation used ($33\frac{1}{3}$ rpm). Their results probably reflect the difficulty experienced by the subjects in making their judgments.

The effect of a somewhat different type of surface pattern was investigated by Zenhausern (1969). He had previously shown (Zenhausern, 1968) that an illusion of rotation could be obtained with a trapezoid oscillating through an angle of 120° , with its shorter end in the frontal plane (the method of achieving this oscillation, and whether or not it was simple harmonic is not clear).

The two illusions (i.e. illusory rotation and illusory oscillation) were compared with respect to 'perspective' and distance; the perspective was provided by the cutout patterns of 'windows' in the two targets, the external dimensions of which were identical. The patterns used are shown in Fig.7.7. It was thought that since the internal configuration of the 'C' trapezoid enhanced the external shape (i.e. the window decreased in width and height, as did the internal mullions from large end to small end) it provided more cues to perspective than target 'A'.

He found that with the condition of veridical rotation, the C target produced significantly more reversals than target A, but with veridical oscillation target A produced more illusory experiences (rotations) than target C.

Zenhausern concluded from this that "increases in perspective result in increases in the perception of oscillating targets and decreases in perspective result in increases in the perception of rotation with oscillating targets" (his emphasis).

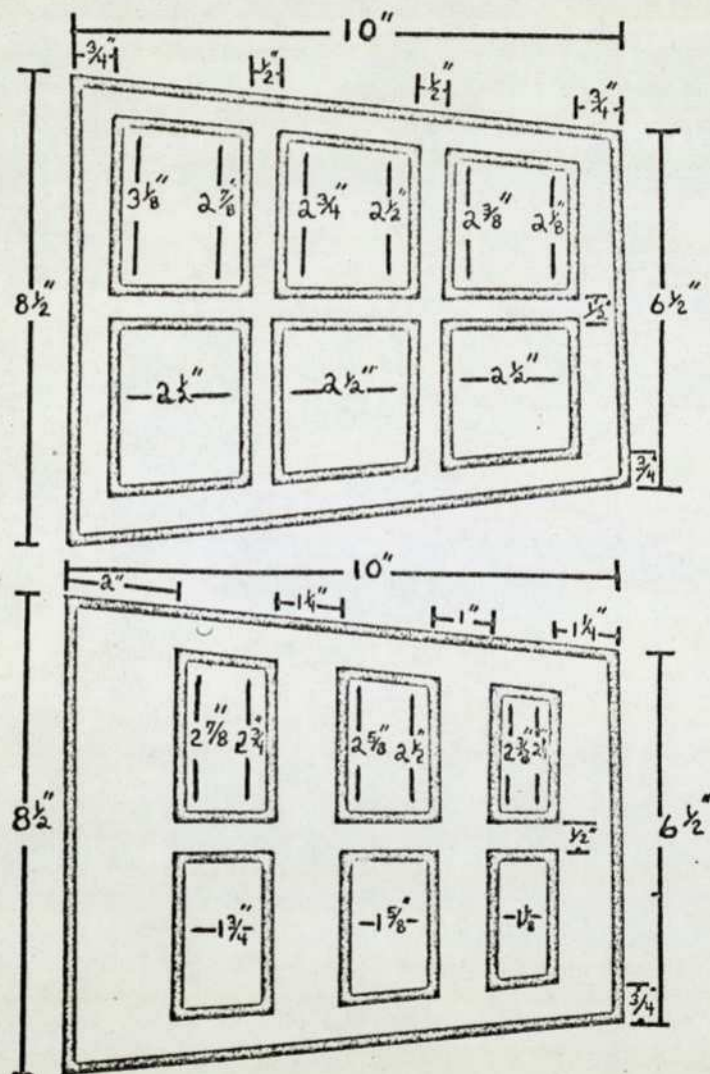
These somewhat ambiguous findings are subject to the same criticism as those of Cross and Cross (1969) in that the targets were rotated at 31 rpm (or oscillated at 62 opm). The number of illusory experiences, either actual or expressed as a percentage, were not given by Zenhausern, so that comparison with the results of similar studies is not possible.

The general conclusion from these various studies, then, is that surface pattern per se does not effect the illusion of apparent oscillation produced by a rotating trapezoidal target.

But it should be noted that practically all the shapes used have been either exaggerated trapezoids, or rectangles, such that the

Fig. 7.7

(From Zenhausern, 1969)



Dimensions of the A trapezoid (top) and C trapezoid (bottom)

projective transformation undergone by the corner points provided unambiguous cues as to the direction of motion. Had the shapes used been more ambiguous (eg. height ratios of 11:9 or 10.5:9.5) then the addition of surface pattern may have had a more significant effect.

It was suggested earlier (p 60) that painting symmetrical shadow patterns on both sides of the trapezoid may add to the oscillation cues; but it has been shown that removing these shadow patterns, or even replacing them with conflicting patterns on the two sides of the trapezoid (Zegers and Murray, 1962) does not reduce the amount of oscillation.

Surface pattern, then, is of minor importance in the perception of the illusion.

SHAPE

Outline Shape (1)

The experiments reported here indicated that the most important single factor in producing the illusion of apparent oscillation was the outline shape of the object.

Ames' (1951) claim to have found the 'most effective' shape for producing apparent oscillation was not verified since the more exaggerated trapezoidal shape used (T1) consistently produced more apparent oscillations than did the Ames shape (T2).

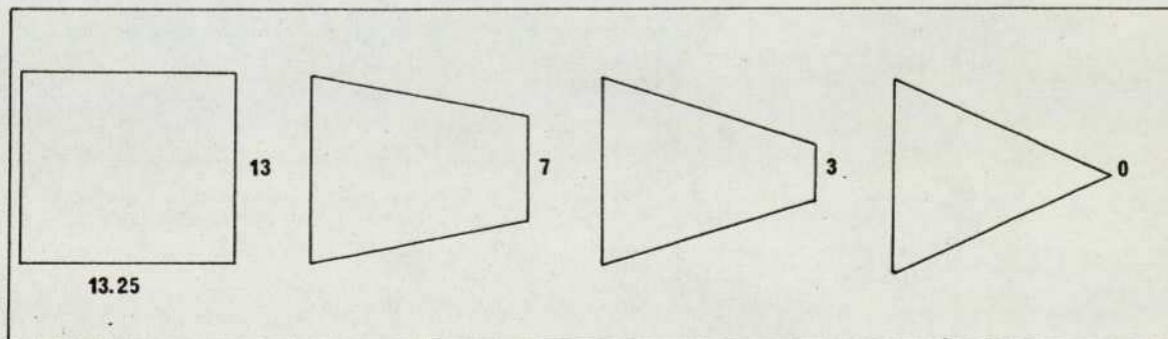
The detailed investigation of the effect of altering the outline shape (E3) indicated that for those shapes that were obviously trapezoidal (i.e. the elliptical loci of their corner points were narrow, and well separated), the actually longer vertical edge was always the apparently longer edge, and was therefore seen as the apparently nearer edge. These shapes were therefore seen to oscillate.

But for rectangular shapes, where the elliptical loci overlapped completely, continuous rotation was seen.

For the intermediate shapes, where the elliptical loci were either very close together or partially overlapping, an ambiguous situation resulted so that sometimes rotation and sometimes oscillation was seen.

The only other study to vary the trapezoidal shape systematically was that of Zegers and Murray (1962).

In their first experiment, Zegers and Murray used the four shapes shown in Fig. 7. 8, and viewed at 15 feet.



For the slower speeds used, they found that the three trapezoidal shapes produced systematic oscillations, whilst the rectangle was nearly always seen to rotate.

The effect of trapezoidal slant was investigated in more detail in their 2nd experiment, in which the length of the shorter side was reduced more gradually, being respectively 12, 11, 10, 9 and 8 in. It was found that the percentage of reversals increased as the ratio of the vertical edges increased, with continuous oscillation being observed with ratios of 13 : 11 or more.

This result could be predicted from consideration of the elliptical loci of the corner points of the shapes used. Fig.7.9 shows plots from TROT of the shapes with height ratios 13 : 11 and 13 : 12 used by Zegers and Murray. With the first shape, the loci are separated and unambiguous, but with the second shape they are practically touching and so providing ambiguous information. In this case sometimes oscillation and sometimes rotation would be seen - which was found by Zegers and Murray.

Outline Shape (2)

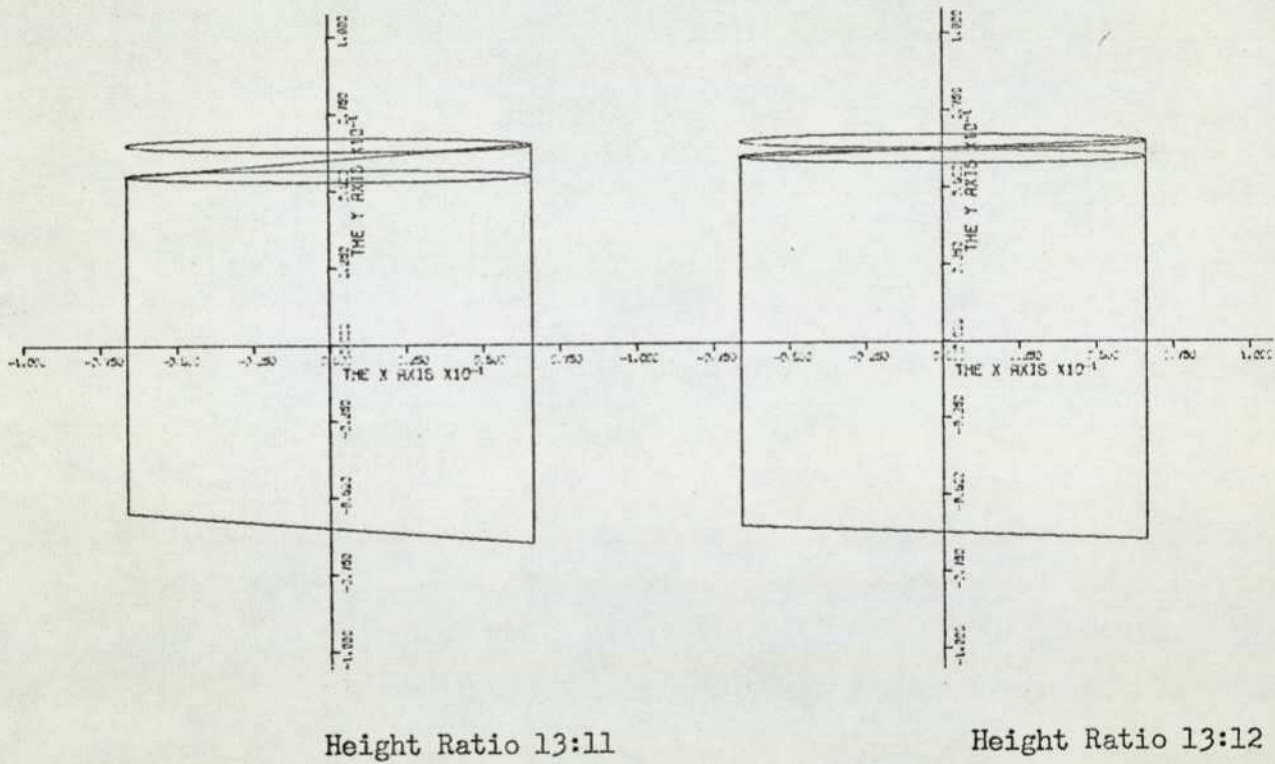
The existence of an illusion of apparent oscillation with shapes other than a trapezoid has been demonstrated by several studies, the aim of most of which has been to disprove the transactionalist theories of Ames et al.

Pastore (1952) reported that oscillations occurred with small circles, ellipses, triangles, and several other shapes, but he gave no indication of the frequency of reversals. Mulholland (1956, 1958, 1963¹) found both regular and irregular oscillations, as well as expansion - contraction and figure ground reversal with similar small shapes rotated at 78 rpm. Both these results were almost certainly due to the small objects producing near-parallel projections (and therefore ambiguous information) at the long viewing distances used.

Day and Power (1963, 1965), Braunstein and Payne (1968), Cross and Cross (1969) and Wallace and Cramer (1975) have all compared the amount of oscillation produced by circles and ellipses, and all have

Fig. 7.9

Projections of Shapes used
by Zegers & Murray (Experiment 2)



found that ellipses reverse more frequently than circles (but less than trapezoids). The reasons for this are unclear, and none of these studies have offered an explanation.

'Partial' Shapes

Several studies have been concerned to find factors contributing to the illusion by analysing it in terms of lines or angles rotating about a vertical axis.

Power (1967) considered that there were two basic cues giving information as to the true direction of rotation provided by rectangular shapes. These were: (1) the changing relationship between the vertical ends of the object; (2) the changing relationship between the horizontal edges. There were also two secondary cues: (3) the changing relationship between the edges parallel to the axis of rotation; and (4) the changing relationship between the edges perpendicular to the axis of rotation. The six shapes shown in Fig.7.10 were produced to investigate the relative importance of these various cues, and from the results obtained Power concluded that the cue to true rotation direction for a rectangular shape is the 'seesaw' motion of the edges perpendicular to the axis, and that the relative size change of the edges parallel to the axis is

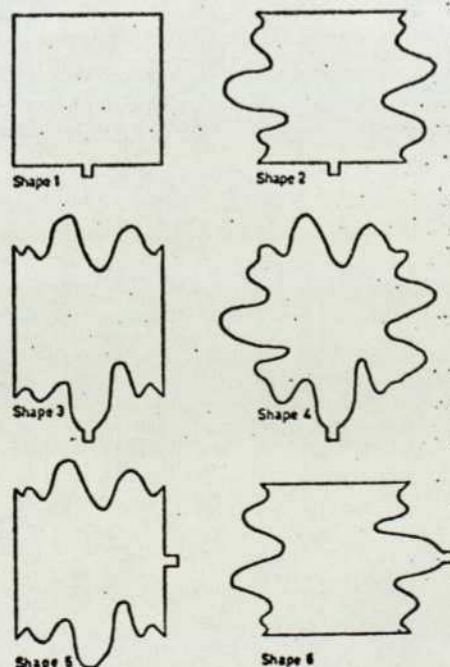


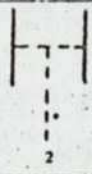
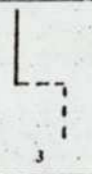
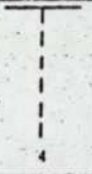
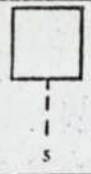
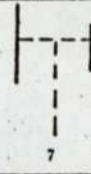
Fig. 7.10
(From Power, 1967)

ineffective as a cue to direction of rotation.

Murch (1970) considered that, whilst the irregular sides employed by Power (1967) may have reduced the effectiveness of each cue, it may not have eliminated then. He suggested that Power's experiment could be re-interpreted to mean that the orderly transformations in the rotating figures provide perspective cues.

Murch produced the series of figures shown in Fig.7.11, which was designed to test further the effectiveness of the size and 'see-saw' cues. From his results (also shown in Fig.7.11) Murch concluded that the cues provided by orderly changes in perspective (provided by the transformation of the line elements within the target) may be the most important indicators of true rotary motion.

Fig. 7.11 (From Murch, 1970)

Measurement	Figure object						
							
M	8.93	11.32	17.21	15.04	6.61	21.82	17.18
SD	8.00	9.23	10.28	11.51	5.53	8.56	11.02

"As a known object rotates in space, certain orderly transformations in the relationship between elements of the figure, and the figure and background, can be noted. These changes would include perspective changes and relative size changes of the elements of the target. From these changes the observer develops a perception of rotational direction. The frequency with which observers fail to properly perceive the motion of the object would depend upon the relative strengths and clarity of the transformation of the elements of the objects. If the shapes are unknown, irregular, or circular, then these transformations are necessarily ambiguous."

(Murch, 1970, p.85)

Murch does not develop this idea further, but obviously his concept is very similar to that developed in detail in this study.

The information available for determining the rotation direction was analysed in a different way by Braunstein (1971). The three sources of information that he studied were the angles which the vertical contours form with the horizontal contours, the relative extents of the vertical contours and the position of the axis of rotation relative to the vertical contours. The forms used are shown in Fig.7.12.

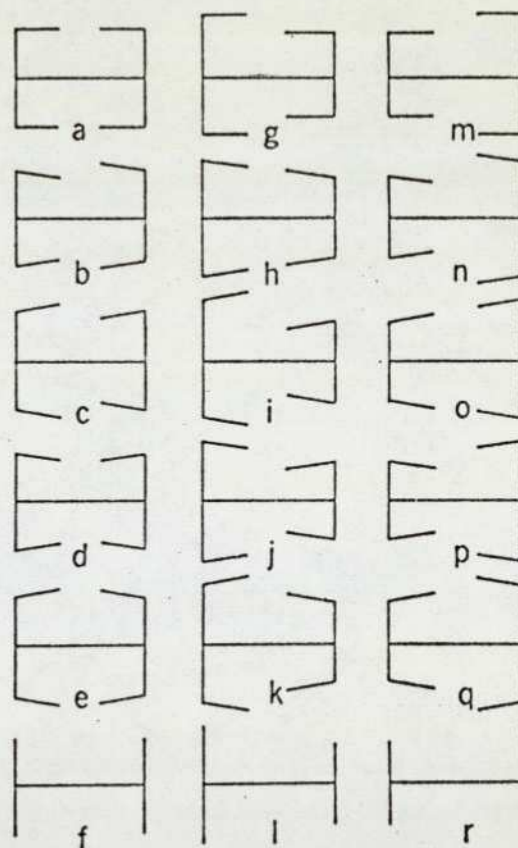


Fig. 7.12

(From Braunstein, 1971)

The 18 forms. (The Ss viewed white forms against a black background. The axis of rotation was either midway between the vertical lines or displaced to the right.)

Braunstein found that accuracy of direction judgments was highest for the form most similar to a rectangle, and lowest for the forms most similar to a trapezoid (Forms h and o, Fig.7.12)

It was also found that accuracy was greatest for forms containing right angles, even when the vertical contours were unequal, and lowest for forms in which one vertical contour was enclosed in acute angles and the other in obtuse angles, even when the vertical contours were equal. Within angle relationships, accuracy was

generally greater when the vertical contours were equal.

But as Zegers and Murray (1962, p.9) pointed out, it is the perception of the whole rotating shape that is important, not just isolated parts of it. It is unlikely that the visual system analyses the stimulus into its horizontal, vertical or angular components in this sort of way. It is the continuous transformation of the whole shape that is important, and as was shown earlier, the projective transformation of every point on the object depends upon its three-dimensional relationship to the viewing point (ie. x, y, and D locate the point in 3 - dimensions). It is the subtle differences in the continuous perspective transformations between different points on the object that provide the cues to motion direction.

JUDGEMENT OF SLANT

Several writers have considered the illusion of apparent oscillation produced by a rotating stimulus object to be either due to, or to result in, a misjudgement of the angle of slant relative to the fronto-parallel plane. Consequently several measurements have been made of the ability and accuracy to judge slant.

Judgement of slant has been considered in general terms in studies such as those of Freeman (1965) and Flock (1964, 1965).

The apparent orientation of a static trapezoidal window and five other shapes, was judged in an experiment by Day and Power (1965). They found that when the true orientation was at either 60° or 120° , the apparent orientation of an ellipse, rectangle and three irregular shapes was approximately correct. But the trapezoid window was always judged such that its longer edge was seen as closer.

In a further experiment (Day and Power, 1965) judgement of the apparent orientation of objects in the frontoparallel plane was followed by assessments of the initial apparent direction of movement, immediately after the various shapes started to rotate. It was found that the relationship between judged orientation and judgements of initial direction of motion achieved significance only for the trapezoidal shape, and not for any other shape giving similar frequencies of apparent oscillation (eg. ellipse or irregular shape).

Day and Power concluded from this that although misperception of orientation resulted in misperception of initial motion direction for the trapezoid window (as Pastore, 1952, had suggested), it was not a necessary condition for the misperception of motion direction, since it did not occur with all shapes.

A similar experiment by Cook, Mefferd and Wieland (1967), compared judgements of orientation of static figures in 6 different positions (120° , 135° , 150° , 210° , 225° , 240°) with judgements of the initial direction of rotation.

They found that 90% of these judgements with all the figures were in agreement (ie. both veridical or both non-veridical).

Cook et al suggested that the discrepancy between their results and those of Day and Power was due to the fact that the latter's shapes

were initially situated in the frontoparallel plane. Mefferd et al (1966) had previously shown that spontaneous apparent reversals in depth occurred only when there was an actual difference in depth between points on the object - not in the frontal plane. This was verified to some extent by the results of Cook et al's second experiment, in which they found that apparent reversals never occurred in the frontal plane.

However, an experiment by Catchesides (1969) had measured the apparent stop and re-start positions for both a rotating trapezoid and an oscillating rectangle. It was found that the mean of these (for both shapes) consistently occurred in the frontal plane.

Similar results were obtained in preliminary experiments for ^{the}present study when the polaroid transducer and pen-recorder mentioned in Section 5 (p.79) were used to measure the apparent stop and start positions. Again the mean of these occurred in the frontal plane. If Cook et al had measured the stop position as well as the re-start position, they would probably have obtained similar results.

The 'planes of apparent reversal' were used as a measure of the illusion in a study by Freeman and Pasnak (1968), in which various parameters were investigated. They used small targets rotated at 30 rpm, but since they give no indication of the frequency of apparent reversal, it is difficult to compare their data with that obtained by others.

The plane of apparent reversal was also measured by Olson (1974). He used a variety of trapezoidal shapes, computed as geometrical projections of rectangles at specific inclinations to the line of sight. Olson measured the plane of apparent reversal for the rotating trapezoids, and the apparent slant of the same trapezoids when stationary; he seemed surprised to find a close correlation between the two measures. But surely this result was predictable. Apparent reversal would occur when the trapezoid reached its maximum width horizontally (i.e. perpendicular to the line of sight, for most viewing distances). This is the same position as that used for judging static slant, so that we could expect the apparent position of the trapezoid to be the same in both cases.

A more significant finding was the close correspondence between perceived, and perspective - implied, slant for all the shapes used. But these relationships do not indicate why the trapezoids should appear to oscillate. Judgements of apparent slant with targets a few degrees either side of the frontoparallel plane may provide some interesting data.

Investigations that have compared judgements of object slant with judgements of initial direction of rotary motion have failed to distinguish the essential difference between judgement of a static display and judgement of a continuously moving one.

The judgement of a static display can be infinitely ambiguous, since any static pattern of light reaching the retina can represent an infinite number of objects.

But as soon as that object moves, the number of possible interpretations is drastically reduced, since the projection of the object can vary only in particular very limited ways for certain kinds of relative movement. The sources of information utilised by the perceptual system, then, in interpreting the two situations, are probably very different.

OUTLINE SIZE AND VIEWING DISTANCE

These two factors will be considered together, since they determine not only the angular size of the rotating stimulus objects, but also the projective transformations that those objects undergo.

The various trapezoidal shapes used in other studies have all been based (more or less) on Ames original shape, but with significant variations in the overall size and viewing distance.

The computer program TROT was used to plot the elliptical loci of the projections of the corner points for each of the shapes used in these various studies. From the graphs, the type of motion seen could be predicted; these predictions were then compared with the actual results obtained. Frequent reference will be made to these graphs; in general, it can be said that there was wide agreement between the predictions and the results.

Viewing Distance

The viewing distance used in this series of experiments was kept constant through-out at 12.5 feet. This was shorter than originally intended, but was limited by the size of the room available for filming and viewing the various trapezoidal shapes.

Most studies have used viewing distances of 10 feet or more, although some have used considerably less than this (eg. Day and Power (1963 et seq) used 52 inches).

Several investigators have compared the effects at two different viewing distances - usually 10 feet and 20 feet (eg. Allport and Pettigrew, 1957).

The main studies investigating the effect of viewing distance on the rotating trapezoid illusion have been by Zegers and Murray (1962), and by Braunstein and Payne (1968).

Braunstein and Payne introduced the concept of 'perspective' as a means of expressing the relationship between viewing distance and the size of the object.

'Perspective' was defined as:

"the ratio of the projection of a unit distance on the nearest part of the figure, to the projection of the same distance on the most distant part of the figure"

(Braunstein and Payne, 1968, p.400)

It may be useful to compare this with the equations to the projection of a rotating trapezoid derived earlier (Equation T.P., p. 34).

We had;

$$x_1' = \frac{Dx_1 \cos \theta}{D - x_1 \sin \theta} \quad y_1' = \frac{Dy_1}{D - x_1 \sin \theta}$$

$$x_2' = \frac{Dx_2 \cos \theta}{D + x_2 \sin \theta} \quad y_2' = \frac{Dy_2}{D + x_2 \sin \theta}$$

$$\text{Then } \frac{y_1'}{y_2'} = \frac{y_1}{y_2} \frac{D + x_2 \sin \theta}{D - x_1 \sin \theta}$$

$$\text{Then 'Perspective' } = \frac{D + x_2 \sin \theta}{D - x_1 \sin \theta}$$

when $\theta = 90^\circ$

Thus the ratio of the apparent heights of the two sides when edge-on to the observer is equal to the ratio of the actual heights of the two sides, multiplied by the 'perspective'.

The effect of perspective upon the perception of a rotating rectangle trapezoid, circle and ellipse was investigated by Braunstein and Payne, using computer - generated 16 mm motion-picture sequences. Perspectives of 1.00, 1.25, 1.50, 1.75 and 2.00 were used, at a speed of rotation of 8 rpm.

They found that the Ames effect - perceived rotation of the rectangle and oscillation of the trapezoid - occurred at the 1.25 perspective, but with parallel projection a high proportion of oscillations and incorrect rotations were obtained with all shapes. At the greatest perspectives (1.75 and 2.00) almost all responses for both figures

were correct.

Figure 7.13 illustrates the projective transformations of the corner points of the trapezoid used by Braunstein and Payne, with 'perspectives' translated to equivalent viewing distances.

The ellipses produced at 1.25 perspective (Fig. 7.13 (b)) are quite narrow, and therefore unambiguous, so that constant oscillation would be predicted; but for higher perspectives the width of the ellipses provides adequate information for the true direction of rotation to be determined. (Similar graphs were obtained for the rectangle used).

Zegers and Murray (1962, experiment 3) studied the effect of viewing distance by viewing a 13 inch trapezoid at ten different distances, ranging from 3.5 feet to 17 feet. (The speed of rotation used was again 30 rpm).

They found that the amount of oscillation increased as the distance increased, up to 11 feet, and then remained fairly constant.

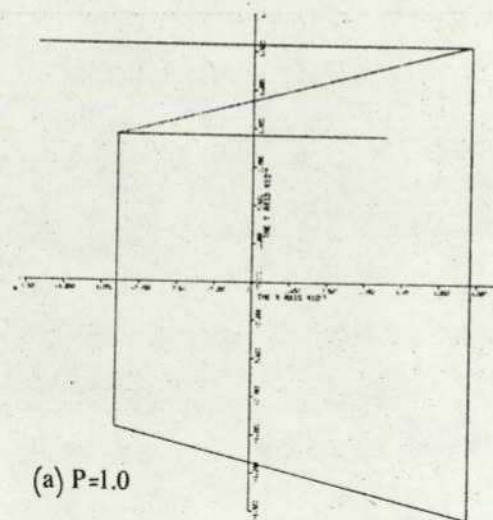
Again, this result could be predicted from Fig. 7.14 (from TROT), by considering the vertical diameters of the elliptical loci of the corner points.

It has been shown, then, that viewing distance has a significant effect upon the perception of the illusion. Several writers have interpreted this finding as an indication that the angular subtense of the rotating stimulus object is a significant factor in the illusion. We will return to this idea shortly.

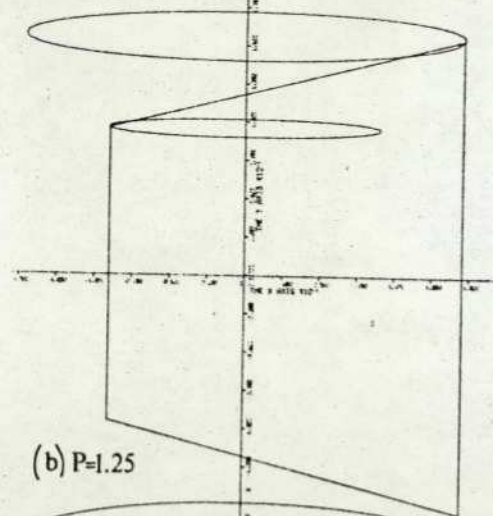
Outline Size

There have been many variations in the size of the objects used for experiments on apparent oscillation, ranging from about 2 inches to over 30 inches diameter.

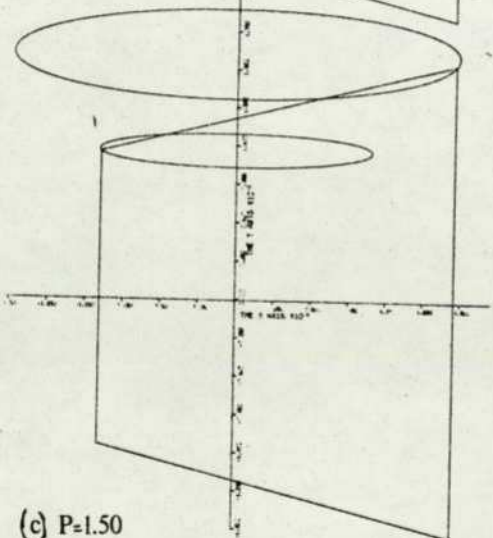
With the smallest shapes, near-parallel projection is likely so that some apparent oscillations are probable with any shape. But it has been seen that for larger shapes, it is the relationship between size and distance that is important in determining the continuous perspective transformation of the rotating object.



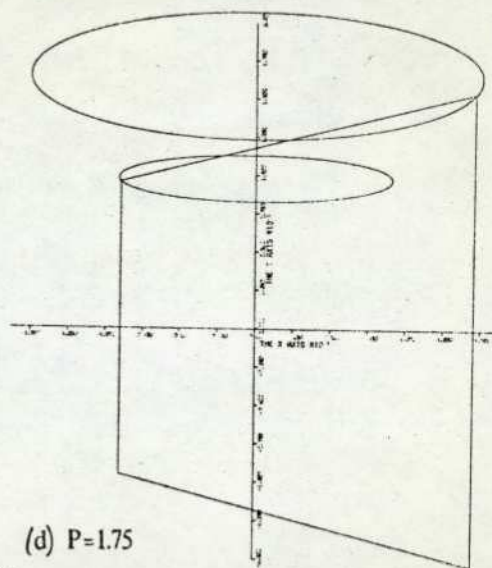
(a) $P=1.0$



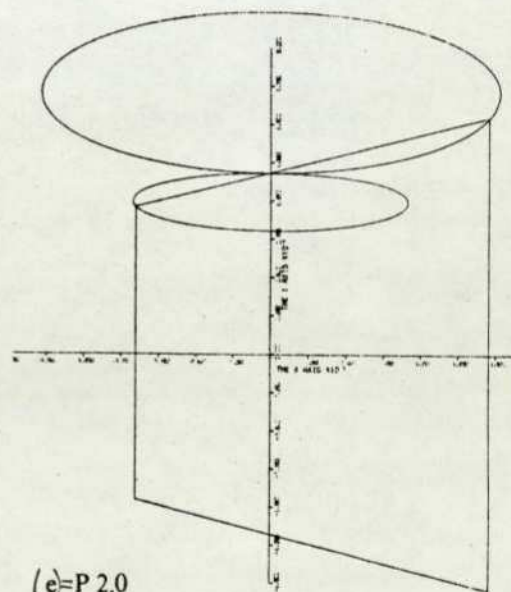
(b) $P=1.25$



(c) $P=1.50$



(d) $P=1.75$



(e) $P=2.0$

Fig. 713

TRAPEZOID PROJECTION

$X1= 14.57$ $Y1= 15.45$
 $X2= -9.01$ $Y2= 9.56$

PROJECTIONS OF TRAPEZOID USED
 BY BRAUNSTEIN & PAYNE (1968),
 SHOWN $\frac{1}{2}$ SIZE.

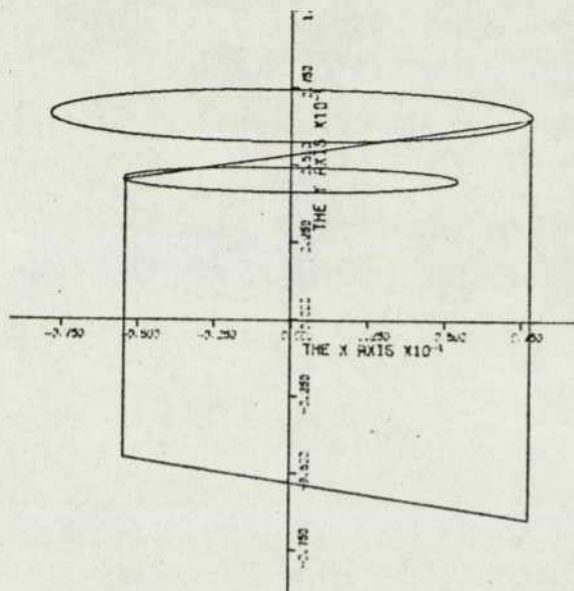


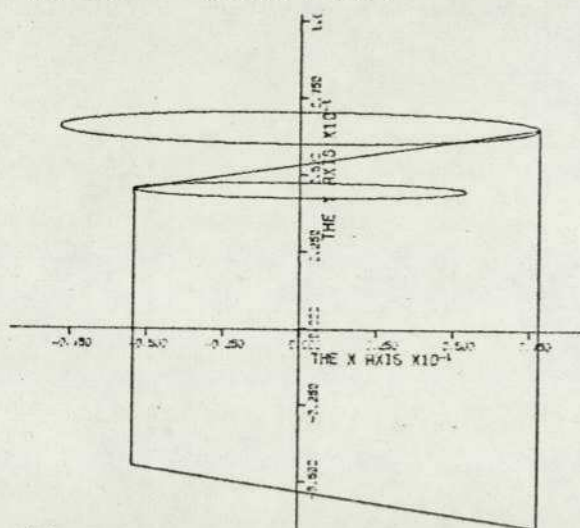
Fig.7.14
Projections of Trapezoid
Used by Zegers & Murray
(1962, E3), at 5 of
their Viewing Distances.

TRAPEZOID PROJECTION

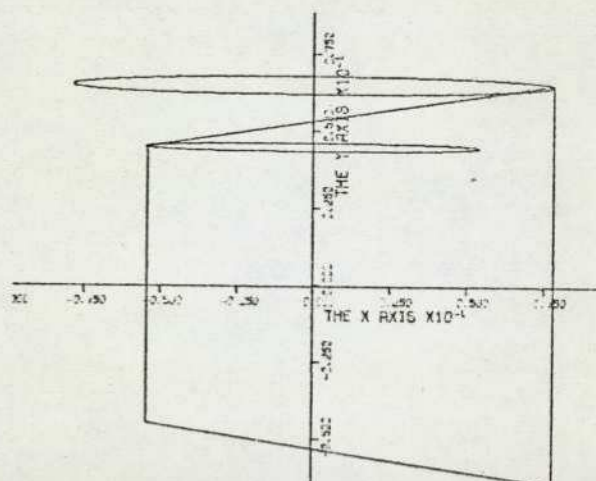
X1= 7.82 Y1= 6.50

X2= -5.43 Y2= 4.50

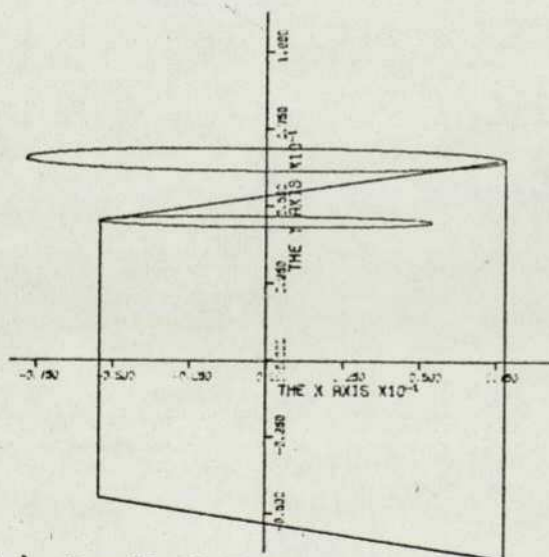
(a): D= 5.00 FEET



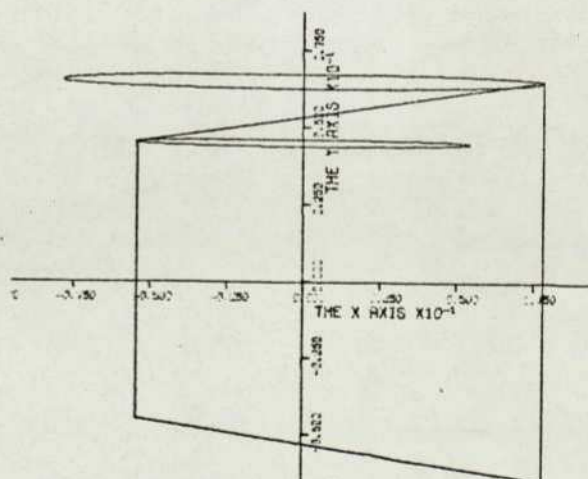
(b): D = 8.00 FEET



(d): D = 14.00 FEET



(c): D = 11.00 FEET



(e): D = 17.00 FEET

A further experiment by Zegers and Murray (experiment 3, part 2) investigated the effect of altering the outline size with a constant viewing distance of 11 feet. They constructed a set of five different trapezoids, each of which subtended the same visual angle (at 11 feet) as the trapezoid in part 1, viewed at five different distances.

They found that the amount of oscillation increased as the size of the trapezoids decreased for the 2 largest shapes, and then remained fairly constant.

Again, this result is predictable from consideration of the loci of the corner points of the trapezoids (Fig.7.15).

Zegers and Murray combined the results from parts 1 and 2 of their experiment, and concluded that the important factor affecting the amount of oscillation was the visual angle subtended by the trapezoids.

But it is not visual angle per se that is important. Comparison of Figs.7.14 (a) and 7.15 (a) (Zegers and Murray trapezoids subtending equal visual angles) indicates that the nature of the ellipses is the same in both cases. (Fig. 7.15 (a) is a magnification of Fig. 7.14 (a)). Thus consideration of visual angle alone masks the important parameter - i.e. the projective transformation - which is dependent upon both size and distance, and is thus dependent only indirectly on visual angle. To illustrate this point, the angular sizes of various trapezoids used in other studies are tabulated below, (Table 7.1), together with an indication of the type of motion observed. It can readily be seen that there is no consistent relationship between the two factors.

Parallel Projections

It was seen earlier that when the viewing distance becomes infinitely great, then conditions of parallel projection obtain and the cues to the direction of motion will be infinitely ambiguous regardless of the size of the object (the elliptical loci of corner points become straight horizontal lines in this case).

Projections of 5 different trapezoids used by Zegers & Murray (1962, E3, part 2), subtending same visual angles as those shown in Fig. 7.14

(a): $X_1 = 17.19$ $Y_1 = 14.30$
 $X_2 = -11.96$ $Y_2 = 9.90$
 $D = 11.00$ FEET

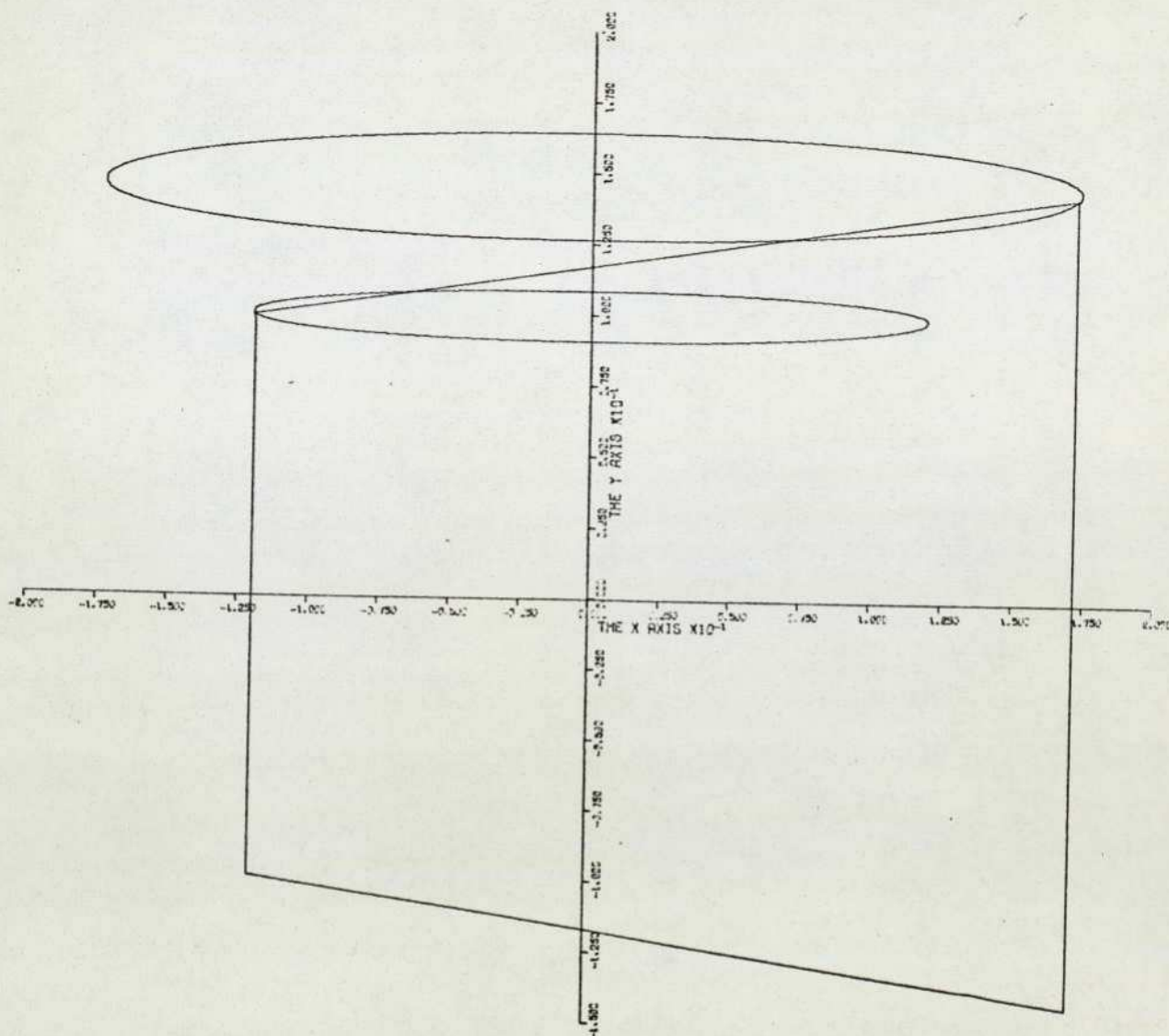
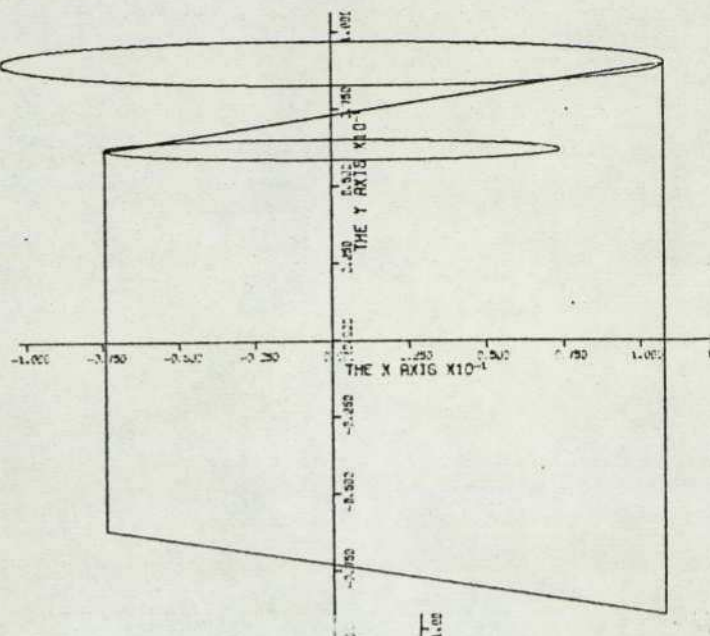


Fig. 7.15 (cont.)

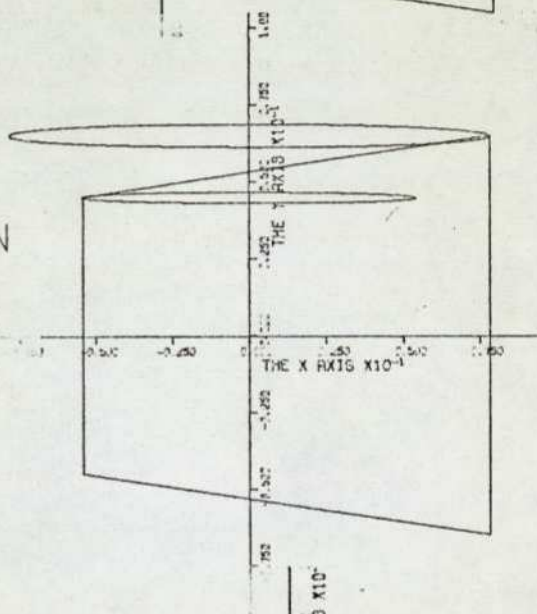
TRAPEZOID PROJECTION

(b): $X_1 = 10.80$ $Y_1 = 8.94$
 $X_2 = -7.40$ $Y_2 = 6.18$
 $D = 11.00$ FEET



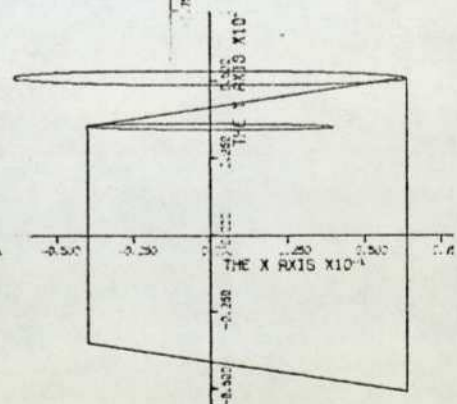
TRAPEZOID PROJECTION

(c): $X_1 = 7.82$ $Y_1 = 6.50$
 $X_2 = -5.43$ $Y_2 = 4.50$
 $D = 11.00$ FEET



TRAPEZOID PROJECTION

(d): $X_1 = 6.40$ $Y_1 = 5.10$
 $X_2 = -4.00$ $Y_2 = 3.53$
 $D = 11.00$ FEET



TRAPEZOID PROJECTION

(e): $X_1 = 5.15$ $Y_1 = 4.20$
 $X_2 = -3.40$ $Y_2 = 2.91$
 $D = 11.00$ FEET

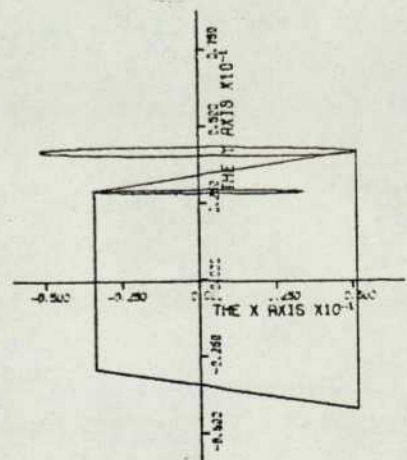


Table 7.1

ANGULAR SIZE AND MOTION SEEN FOR PARTICULAR TRAPEZOIDS

AUTHOR (DATE)	ANGULAR SIZE $2y_1 : 2y_2 : (x_1 + x_2)$	MOTION SEEN	ILLUSTRATION OF SHAPE (FIGURE NUMBER)
Ames (1951)	$10^\circ \quad 6^\circ \quad 9^\circ$	oscillation	4.1
Zegers & Murray (1962)	$5\frac{1}{2}^\circ \quad 4^\circ \quad 6^\circ$	oscillation	7.14 (c) and 7.15 (c)
"	$12^\circ \quad 8^\circ \quad 12^\circ$	mainly rotation	7.14 (a) and 7.15 (a)
Day & Power (1963)	$13^\circ \quad 7^\circ \quad 10^\circ$	only 50% oscillation	7.18
Braunstein & Payne (1968)	$23\frac{1}{2}^\circ \quad 15^\circ \quad 18\frac{1}{2}^\circ$	oscillation	7.13 (b)
	$23\frac{1}{2}^\circ \quad 15^\circ \quad 18\frac{1}{2}^\circ$	rotation	7.13 (d)
Catchesides (1975)	$9^\circ \quad 7^\circ \quad 10^\circ$	oscillation	6.3.5

This is equivalent to the situation with Lissajou's figures, and some spontaneous reversals of the direction of apparent movement would be expected.

It is also equivalent to the situation used by Gregory (1970) in back-projecting rotating wire objects or silhouettes which appear to change their direction of movement spontaneously.

Parallel projection was effectively the viewing condition used by Pastore (1952), Cross and Cross (1969) and Mulholland (1956 and 1958), when small targets were viewed at large distances. The ambiguity of the projections obtained resulted in spontaneous oscillations being observed with all the targets used - including rectangles. Other types of motion were also seen - for example, two dimensional pulsation and irregular oscillations. Figure 7.16 is typical of the graphs obtained for these shapes.

As mentioned above the computer-generated display produced by Braunstein and Payne (1968) resulted in at least 50% oscillation being observed with all shapes when parallel projection was used.

A study by Graham and Gillam (1970) was designed to test Graham's (1963) hypothesis that reversals of the trapezoidal window are due to the resolution of ambiguous differential angular velocity cues by utilising linear perspective cues.

Criticism of Graham's earlier paper was recognised by emphasising that the differential angular velocity cue was only ambiguous for parallel projection; consequently they attempted to use parallel projection for the investigation, by using back projection onto an opal glass screen.

But in fact they were forced to use a divergent beam of light (rather than a parallel one) because of spherical aberration produced by the simple optical system used. [This is a common difficulty in experiments using back projection; many authors (studying, for example, kinetic depth perception) assume that any back-projection system is equivalent to parallel projection, which of course is not so. The only simple way of achieving a large aberration free field of view is to use a large aperture parabolic mirror, such as that used by Gregory (1970³)].

Fig. 7.16

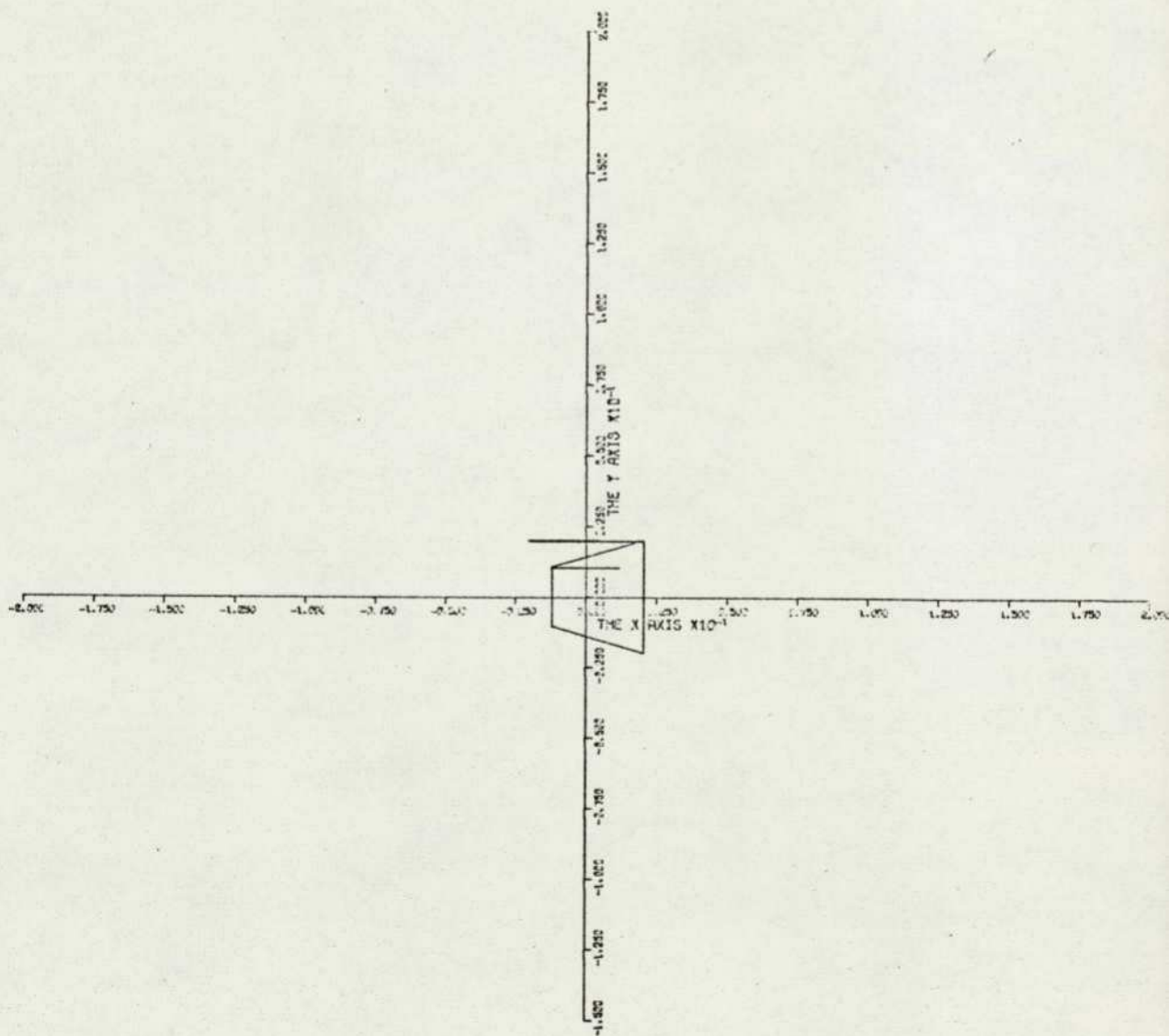
(Mulholland, 1956)

TRAPEZOID PROJECTION

X1= 2.05 Y1= 2.00

X2= -1.20 Y2= 1.04

D= 9.00 FEET



Graham and Gillam found, as predicted, that oscillations occurred regularly and systematically twice per revolution in the frontal plane; they also assessed the position of the shorter side during two consecutive revolutions and found that it was always apparently further from the subject. This data provided evidence to support Graham's (1963) theory, and contradicted Day and Powers (1965) hypothesis that the "spontaneous" nature of reversals preclude a dependence on cue resolution by linear perspective.

Monocular/Binocular View

Practically all the studies carried out at viewing distances of 15 feet or less have used monocular vision, whilst those above 15 feet have used binocular vision. In general, no reasons have been given for this choice, beyond some occasional references to the role of retinal disparity as a depth cue (see, for example, Gibson, 1950; Graham, 1965). Only one experiment has compared monocular/binocular viewing at the same (15 feet) distance, (Zegers and Murray, 1962, experiment 1). No significant effect was found

Qualitative comments from various studies suggest that the differences between monocular and binocular viewing may be summarised as in Fig.7.17.

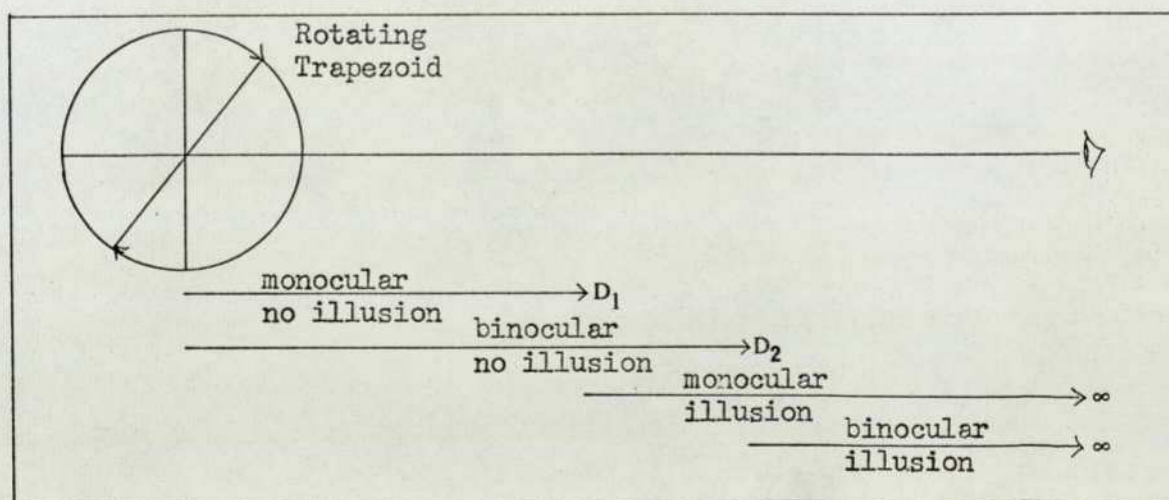


Fig. 7.17

Effect of monocular/binocular viewing on the illusion of Apparent Oscillation

Thus the illusion breaks down at a longer viewing distance with binocular vision than it does with monocular vision; but the actual values of D_1 and D_2 will depend also on the size of the rotating object.

Retinal disparity obviously provides an additional source of depth information when viewing rotating targets. As such it provides a physiological cue which (together with accommodation and convergence) has been effectively eliminated from most studies of the psychological factors involved in perceiving the illusion of apparent oscillation.

SUBJECTIVE VARIABLES

Studies of the effect of subjective variables on the Ames illusion have met with varied, and not always consistent results.

Several studies have reported wide individual variations in the results they obtained with different subjects; but this may well have been largely a function of the experimental design used. For example, Zegers and Murray (1962) frequently commented on individual variations; but they often used 4 or 5 viewing sessions of an hour or more, which must surely have resulted in some fatigue (and possibly boredom). They also used a speed of rotation of 30 rpm; the difficulties this may have caused were discussed earlier (p.164).

Other studies, in contrast, have reported on the surprising consistency of the results obtained, and the repeatability of the data (eg. Catchesides, 1969).

An experiment reported here (E5, p. 149) investigated between-subject and within-subject variability for four S's who were all very familiar with both the rotating trapezoid illusion, and the particular experimental set up being used, so that learning effects during the experiment should have been minimal. Only one of the S's (JC) was aware of the hypothesis under test.

This experiment indicated that the within-S variability was insignificant (i.e. day-to-day variation), but the between-S variability was significant (at $p=2.5\%$) for the shapes used, which were the critical 'change-over' shapes from E3.

Two of the subjects used in E5 and E6 (MC. and JES.) reported that they occasionally saw apparent reversal of the rectangle in the sagittal plane. These reversals were ignored when calculating the results of E5 and E6, but their occurrence supports the reports of Mefferd et al (1966), that with experienced subjects more critical judgements can result in a variety of perceptual effects.

The more naïve S's used in the other experiments did not observe these effects, and their observations are probably more typical of the 'normal' perception of this illusion.

The investigation by Day and Power (1963) into the effects of shape and surface pattern (see p. 171) showed a significant ($p = 5\%$) and consistent practice effect, with the second trial in both experiments showing a higher frequency of oscillation; this effect was found with both practiced and naive subjects.

Their finding may be supported by several anecdotal reports that the effect of apparent reversal takes time to build up.

Similar findings were not, however, obtained in the present series of experiments. This may have been because the 'trial run' used before each period of observation was sufficient to stabilise the percept. No explanation of this practice effect has yet been offered.

There have been several anecdotal reports that the observer's knowledge of the true shape of the trapezoidal window has little effect on his perceptual experience (eg. Kilpatrick, 1953). Even observers who are extremely familiar with the phenomenon are still subject to the illusion.

Haber (1965) attempted to determine whether the perceptual experience of the trapezoidal illusion could be modified by controlled experience, and by "disconfirmatory knowledge of any prior assumptions made by the perceiver".

The trapezoid used was similar in shape and size to that of Ames (1951); it was viewed binocularly at 20 feet, and was rotated at 5 rpm. There were 7 trials in each of 5 experimental conditions, and the results obtained are summarised in Table 7.2. Haber's findings have not been repeated in any other investigation.

In a series of experiments using various designs and various instructional sets, Mulholland (1963²) found that S's reported a given perceived motion more often when instructed to try to perceive that motion, indicating clearly the necessity of controlling instructions and suggestions when studying these phenomena.

Similar results were obtained by Power (1965); he used two experimental groups, one of which was instructed to report oscillations and the other rotation. The mean frequency of apparent reversal was always higher for the oscillation group than for the rotation group.

Table 7.2

SUMMARY OF HABER'S (1965) RESULTS

EXPERIMENTAL CONDITION	n	% OF SUBJECTS SHOWING	
		INITIAL OSCILLATION	FINAL OSCILLATION
1. Control: No additional information Trial 1-7	10	80%	80%
2. Gradual Information: S given more information after each trial up to discussion and handling	28	89%	36%
3. Early Information: discussion and handling before Trial 1	10	40%	40% ? (not stated)
4. Late Information: discussion after Trial 5, handling after Trial 6	18	88%	Trial 6 - 28% Trial 7 - 11%
5. Distance Variation: discussion before Trial 1; S varied distance to try to maintain rotation	7	Median distance 8 feet for illusion - not affected by information	

It was almost certainly the effect of instructional set (possibly inferential, if not directly given) which produced Haber's results. Observer's who are well-practiced in viewing illusory oscillation often report that they can choose to see either rotation or oscillation at will, but with as passive a viewing set as possible apparent oscillation nearly always returns.

Zenhausen et al (1970) showed significant order effects in the perception of the trapezoidal illusion, when S's were exposed to all the experimental conditions. The results are complex, and the presentation of the raw data might facilitate interpretation; but the finding of such order effects provides evidence against the use of a "within-S's design" (such as that used by Zegers and Murray 1962) in which a single S is exposed to all conditions.

The various subjective variables considered so far are all concerned primarily with aspects of experimental design. Other, more general, subjective factors have been investigated. These include age, culture, and mental state.

Zegers and Murray (1962) tested 32 children aged between 4 years and 5 years 11 months with both a trapezoidal and a rectangular target, rotating at 18 rpm and viewed at 20 feet. Almost all the children saw the trapezoid oscillating more than 50% of the time, but the rectangle was nearly always seen as rotating.

They concluded from this that if learning and past experience are essential factors for a subject to perceive the illusion, then these are acquired before the age of 4 years. (Attempts to test 3 year old children were abandoned because reliable quantification proved impossible).

A study of the effect of cultural influences upon the perception of apparent oscillation was carried out by Allport and Pettigrew (1957); they compared the illusion with rural Zulus (who lived in a 'circular' rather than rectangular environment), urban Zulus and Europeans. They used a trapezoid very similar to that of Ittleson (1952) (see

Fig.3.2), and they found that under optimum viewing conditions (i.e. 20 feet monocularly) all the subject-groups saw practically continuous oscillation. But under more marginal conditions (10 feet binocularly), the group of rural Zulus perceived the illusion significantly less frequently than the other groups.

Allport and Pettigrew also found that nearly one-third of the S's reported oscillation at some time with a rectangular window; this may be an artefact produced by the slow (< 2 rpm) speed of rotation used (see p.164).

They concluded that their results obtained under optimal conditions could provide evidence for either a nativistic or an empiricistic interpretation, but that

"object connotation (meaning) based on closely relevant cultural experience helps to determine the nature of the perceived movement under marginal conditions"

(Allport and Pettigrew, 1957, p.113
their emphasis)

This experiment was later re-evaluated by Slack (1959), who concluded that the results were perfectly consistent with a strong empiricistic interpretation, which would predict no difference between the special groups studied under optimal conditions, but which would predict a large difference under marginal conditions. This conclusion was based upon consideration of the relationship between the basic differential of the illusion (rectangle vs. trapezoid) and the basic culture-differential (not circularity vs. rectangularity, but less rectangularity vs. more rectangularity) as they interact in the presence of 'give away' cues. Slack's interpretation is clear, concise and convincing.

A comparison between mental retardates and normals of approximately equal chronological age showed that the retardates were just as susceptible to the rotating trapezoid illusion as were the normals (Spitz, 1964); there was no significant difference in the results obtained with the 2 groups. This was in contrast to earlier investigations with other types of illusion (eg. Spitz and Blackman, 1959) in

which the illusions took longer to induce, were smaller in degree and persisted for a longer period of time in retardates than in normals. Spitz concluded from this that the trapezoid illusion was of a fundamentally different type from the others he had investigated, being induced by reducing and distorting distance cues, and causing the observer to make assumptions based on past experience.

These various studies, then, have indicated that the illusion is surprisingly universal, and produces consistent effects with a wide variety of subjects.

SUMMARY

The data obtained in the present series of experiments has been compared with the published results of other investigations.

One of the main factors found to affect the illusion was the speed of rotation used. The experiment reported in Section 6 (E2) provides the most detailed examination of this, and the results obtained are consistent with the qualitative comments of others. In general it may be said that at high speeds (> 20 rpm, say) the nature of the illusion changes: it is not such a stable and repeatable effect, and other illusory types of movement are seen besides regular apparent oscillation - eg. in-and-out pulsation, irregular oscillation, etc.

Studies using rotation speeds of 30 rpm or more have generally obtained much less than 100% oscillation with trapezoidal shapes that would have produced continuous oscillation at slower speeds. These studies have also obtained significant amounts of oscillation with rectangles. Investigators using these higher speeds must, therefore, be said to be examining a related, but different, illusion. These results support the hypothesis detailed in Section 4 (p. 61).

Surface pattern was found to have a relatively minor effect upon the illusion of apparent oscillation produced by a rotating trapezoidal object. Many different types of surface pattern have been investigated, but it has been found that even painting conflicting patterns on the 2 sides of the trapezoid does not reduce the illusion.

The most important result obtained from the comparisons of these different studies is that consideration of the projective transformations undergone by the various shapes used enables practically all the results obtained by others to be predicted.

The only exception to this is the experiments of Day and Power (1963, 1965), who obtained approximately 50% oscillations with trapezoidal shapes for which nearly 100% oscillations would be predicted (Fig. 7.18). This is the only study for which such a low rate of oscillation was obtained, which suggests that maybe some other aspect of their experimental design affected the results. For example, possibly

Fig. 7.18

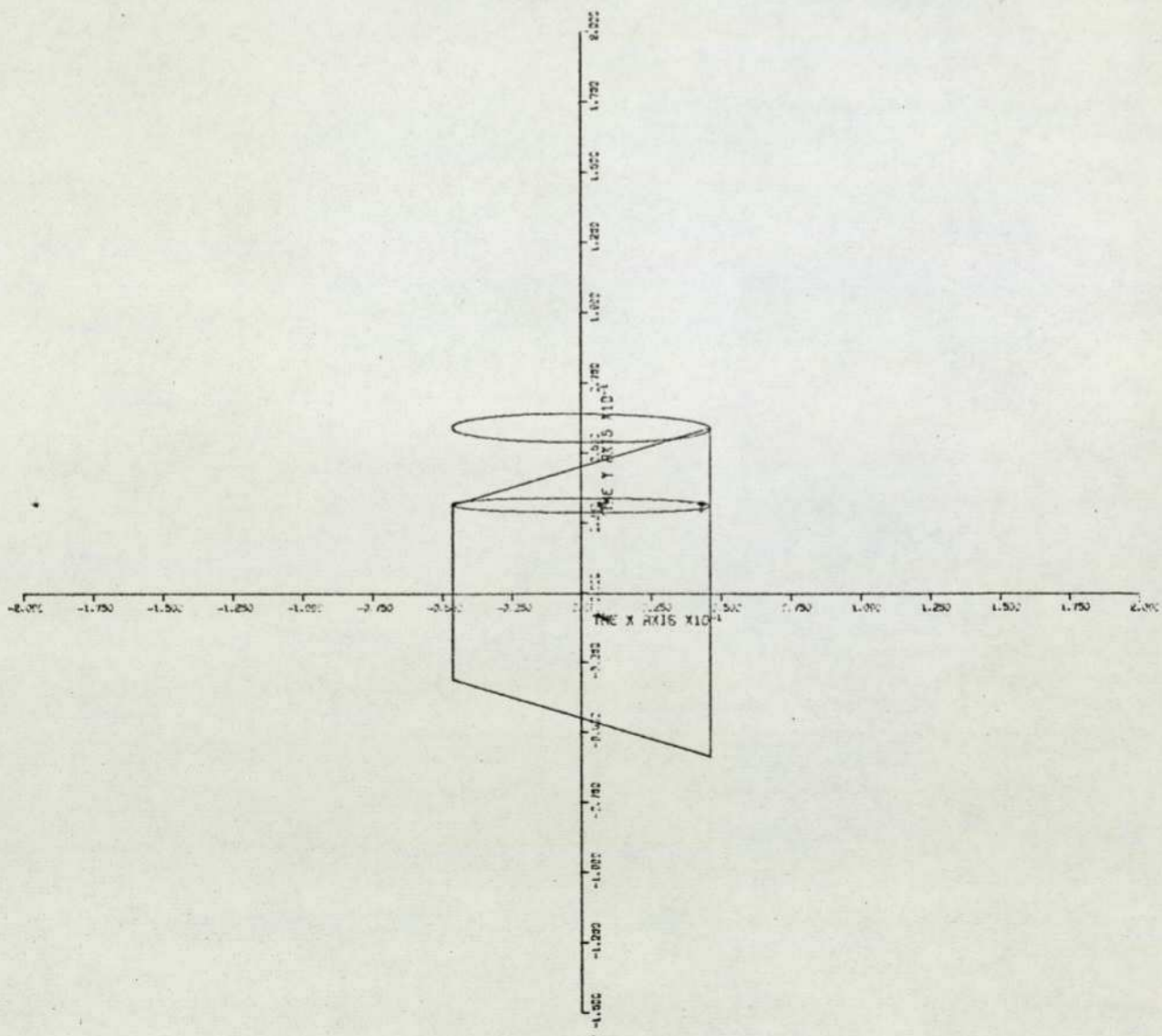
(Day & Power, 1963)

TRAPEZOID PROJECTION

X1= 4.62 Y1= 5.87

X2= -4.62 Y2= 3.13

D= 4.33 FEET



uneven lighting on the rotating shapes, or the very short viewing distance used (4.33 feet), may have affected the results.

Analyses of the illusion in terms of horizontal, vertical or angular components, as well as size, distance or shape components, were seen to be inadequate to predict the type of motion seen. The continuous perspective transformation undergone by the rotating shapes inter-relates all these other components. These results then, were consistent with the hypothesis detailed in Section 4.

It should not be surprising that the brain is able to synthesise all these different factors and interpret the changing incoming information in terms of a rigid object moving in depth.

Studies of some of the various subjective variables that may effect the illusion have indicated that although it may be affected to some extent by the instructions to the observers the illusion is a surprisingly universal one, and produces consistent results with a wide variety of subjects.

SECTION 8

CONCLUSIONS

The most useful explanations of visual illusions analyse them in terms of the types of information that they contain; to be satisfactory, they must also relate them to known aspects of brain function. Previous theories of the Ames rotating trapezoidal window illusion have relied on either vague and unspecific concepts (such as 'past experience') or specific detailed hypotheses (such as 'differential angular velocity') to explain the illusion.

Since the illusion contains several different types of information, the percept seen is likely to inter-relate this information. Any explanation, then, must also inter-relate the information available, and consider the relative importance of the different types of information. This is what has been attempted in this study.

It is not realistic to analyse visual scenes into minute components - 'bits' of information, or even horizontal or vertical components. These are useful concepts for mathematical analysis, and help us to formulate what is happening, but it seems most unlikely that the brain handles information in this way.

It is more likely that the perceptual system utilises changes in a visual scene; one of the most fundamental changes is the rate of change of one object relative to another.

Probably the most fundamental concept in this respect is change in size - a larger object means a nearer object, so that if two objects are changing in relative size, but one is always bigger than the other, then that one is always nearer. (This relationship is so fundamental that it may well be innate - if not, it must be learnt very quickly). If the change in size is large relative to the size of the object, then this is likely to be interpreted as a change in depth of the object (relative to a stationary observer).

This study has been concerned with a particular class of shapes - trapezoids. These shapes are unfamiliar in our normal visual world, although rectangular objects often provide us with trapezoidal retinal images.

A particular type of movement relative to a stationary observer has been considered - rotation around a 'central' vertical axis - and it has been seen that the apparent change in shape of the trapezoid as it rotates follows a simple elliptical locus. This continuous perspective transformation of the shape is quite fundamental. By considering the loci of the two corner points we can summarise the whole change in the apparent shape.

It has been seen that by considering this projective transformation, the type of motion that will be seen can be predicted, by following a few simple rules:

- (1) If the elliptical loci are narrow (vertically) and separated (vertically), then the actually longer edge will be seen as the apparently longer edge, and rotation will be seen as oscillation.
- (2) If the elliptical loci are narrow, but touching or overlapping, then there will be some confusion (ambiguous information) and sometimes oscillation, sometimes rotation will be seen.
- (3) If the elliptical loci are wide, whether they are touching or not, (which occurs with short viewing distances) then the change in apparent length will be interpreted as a change in depth, and the true direction of rotation will be seen.

The projective transformation provides the most fundamental and important source of information when viewing a rotating trapezoidal object.

The formulae derived earlier indicated that the transformation is due to the size, shape, viewing distance and the axis of rotation position, and so integrates many of the types of information that have been considered separately by others.

These other studies, as well as those that have analysed the illusion in terms of its horizontal, vertical or angular components, have been unable to predict the type of motion that would be seen with other shapes.

But consideration of the projective transformation of the whole shape is able to do this.

But although the continuous perspective transformation of the rotating shapes integrates many of the type of information contained therein,

it does not provide the only source of information.

Surface pattern has been found to affect the movement seen, although to a much smaller extent. It has been found that conflicting patterns (i.e. different patterns on the two sides of the trapezoid) do not reduce the illusion, so surface pattern must be an unimportant source of information with obviously trapezoidal shapes.

But if the transformation provides ambiguous information (i.e. sometimes oscillation, sometimes rotation, was seen) then surface pattern may be utilised to resolve the ambiguity. This has not yet been investigated.

A more important source of information is the speed of rotation. This obviously determines the rate of change of the pattern on the retina, and hence the rate of change of incoming information. It seems that if the rate of change is too fast (probably >20 rpm, certainly >30 rpm) the perceptual system does not have time to interpret the incoming information, and the resulting percept consists of 'blocks' of either rotation or oscillation for a few seconds each. This occurs for rectangular as well as obviously trapezoidal shapes, and so supersedes even the information provided by the projective transformation. The effect is so fundamental that studies that have used speeds >30 rpm must be said to have investigated a different, although related, illusion.

The results of this study, then, provide some data about the types of information used by the perceptual system in interpreting rotary motion in depth. The interpretation is considered to be a function of the speed of rotation, the continuous perspective transformation, and the surface pattern of the rotating trapezoidal shapes.

REFERENCES

- ALLPORT, G.W. and PETTIGREW, T.F. (1957) Cultural influence on the perception of movement; the trapezoidal illusion among Zulus. *J. Abnormal Social Psychol.*, 55, 104-113.
- AMES, A. Jr. (1951) Visual Perception and the Rotating Trapezoidal Window. *Psychological Monograph*, 65, (7): whole no.324.
- ATTNEAVE, F. (1954) Some Informational Aspects of Visual Perception. *Psychol. Rev.*, 61, 183-193.
- BARTLETT, F.C. (1932) Remembering. Cambridge University Press, Cambridge, England.
- BARTLETT, M.S. (1936) Square-root transformation in analysis of variance. *J. Royal Statistical Society Supplement*, 3, 68-78.
- BONEAU, C.A. (1960) The effects of violations of assumptions underlying the 't' test. *Psychological Bulletin* 1960, 57, 49-64.
- BORING, E.G. (1942) Sensation and Perception in the history of Experimental Psychology. D. Appleton-Century, New York.
- BRAUNSTEIN, M.L. (1971) Perception of rotation in figures with rectangular and trapezoidal features. *J. exp. Psychol.*, 91, (1), 25-29.
- BRAUNSTEIN, M.L. and PAYNE, J.W. (1968) Perspective and the Rotating Trapezoid. *J. opt. Soc. Am.*, 58, (3), 399-403.

- BROADBENT, D.E. (1958) Perception and Communication.
Pergamon Press, London.
- BROADBENT, D.E. (1965) Information Processing in the Nervous System.
Science, 150, (3695), 457-462.
- CANESTRARI, R. (1956) Osservazioni sul fenomeno del trapezio
ruotante.
Rivista di Psicologia, 50, 1-20.
- CANESTRARI, R. and FARNE, M. (1969) Depth cues and apparent
oscillatory motion.
Percept. Mot. Skills, 29, 508-510.
- CATCHESIDES, J.E. (1969) Production and Discussion of Illusion
Models - A Study of Visual Illusions.
Unpublished Undergraduate Project Report,
The City University.
- COOK, T.H., MEFFERD, R.B. and WIELAND, B.A. (1967) Apparent Reversals
of orientation (Perspective Reversals) in
depth as determinants of apparent reversals
of rotary motion.
Percept. Mot. Skills, 24, 691-702.
- CRAIK, K.J.W. (1943) The Nature of Explanation.
London, Cambridge University Press.
- CROSS, J. and CROSS, J. (1969) The misperception of rotary motion.
Percept. Psychophys., 5, 94-96.
- DAY, R.H. (1972) Visual Spatial Illusions: A general
explanation.
Science, 175, 1335-1340
- DAY, R.H. and POWER, R.P. (1963) Frequency of apparent reversal of
rotary motion in depth as a function of
shape and pattern.
Australian j. Psychology, 15, 162-174.

- DAY, R.H. and POWER, R.P. (1965) Apparent Reversal (Oscillation) of Rotary Motion in Depth: An Investigation and a General Theory.
Psychological Review, 72, (2), 177-127.
- EDWARDS, A.L. (1969) Experimental Design in Psychological Research.
London, Holt, Rinehart and Winston, 3rd Ed.
- FLOCK, H.R. (1964) Some conditions sufficient for accurate monocular perceptions of moving surface slants.
J. exp. Psychol. 67, 560-572.
- FLOCK, H.R. (1965) Optical texture and linear perspective as stimuli for slant perception.
Psych. Rev., 7, (6), 505-514.
- FREEMAN, R.B., Jr. (1965) Ecological Optics and Visual Slant.
Psych. Rev., 7, (6) 501-504.
- FREEMAN, R.B., Jr. and PASNAK, R. (1968) Perspective determinants of the rotating trapezoid illusion.
J. exp. Psychol., 76, 94-101.
- GIBSON, J.J. (1950) The Perception of the Visual World.
Allen and Unwin/Houghton Mifflin.
- GIBSON, J.J. (1957) Optical Motions and Transformations as Stimuli for Visual Perception.
Psychol. Rev., 64, (5), 288-295.
- GIBSON, J.J. and GIBSON, E.J. (1957) Continuous Perspective transformations and the perception of rigid motion.
J. exp. Psychol., 54, 129-138.
- GRAHAM, C.H. (1963) On some aspects of real and apparent visual movement.
J. opt. Soc. Am., 53, 1019-1025.

- GRAHAM, C.H. et al (1965) Vision and visual perception.
John Wiley and Sons, New York, N.Y.
- GRAHAM, C.H. and GILLAM, B.J. (1970) Occurance of Theoretically Correct Responses during Rotation of the Ames Window.
Percept. & psychophysics, 8, (5A), 257-260.
- GREGORY, R.L. (1968) Visual Illusions.
Scientific American, 219, (5), 66-76.
- GREGORY, R.L. (1970 ¹) On how little information controls so much behaviour.
IN: A.T. Welford and L. Houssiadas (Eds.)
Contemporary Problems in Perception.
Taylor & Francis Ltd., London.
- GREGORY, R.L. (1970 ²) The intelligent eye.
Weidenfeld and Nicolson, London.
- GREGORY, R.L. (1971) Seeing in the Light of Experience.
Lighting Research Technology, 3, (4), 247-250.
- GREGORY, R.L. (1974) Concepts and Mechanisms of Perception.
London, Duckworth.
- HABER, R.N. (1965) Limited Modification of the Trapezoidal Illusion with Experience.
Am . J. Psychol., 78, 651-655.
- HABER, R.N. (Ed.) (1969) Information-Processing Approaches to Visual Perception.
New York, Holt, Rinehart and Winston, Inc.
- HABER, R.N. (1973) Psychology of Visual Perception.
New York, Holt, Rinehart and Winston, Inc.

- HAY, J.C. (1966) Optical Motions and Space Perception. An extension of Gibson's Analysis. Psychol. Rev., 73, (6), 550-565.
- HERSHBERGER, W.A. (1967) Comment on "Apparent reversal (oscillation) of rotary motion in depth". Psychol. Rev., 74, 235-238.
- HUBEL, D.H. (1963) Integrative Processes in Central Visual Pathways of the Cat. J. Opt. Soc. Am., 53, (1), 58-65.
- ITTELSON, W.H. (1952) The Ames Demonstrations in Perception. Princeton University Press.
- ITTELSON, W.H. (1968) The Ames Demonstrations in Perception - together with an interpretative manual, by Adelbert Ames Jr. with a new introduction by W.H. Ittleson. Hafner Publishing Co., New York.
- KENYON, F.C. (1898) A curious optical illusion connected with an electric fan. Science, 8, 371-372.
- KILPATRICK, F.P. (1953) The Ames oscillatory effect: A reply to Pastore. Psychological Review, 60, 76-79
- KUFFLER, S.W. (1973) The single-cell approach in the visual system and the study of receptive fields. Invest. Ophthalm., 12, 794-810.
- MEFFERD, R.B., WIELAND, B.A., COOK, T.H., SADLER, T.G., BENTON, R.G. and REDDING G.M. (1966) Analysis of Perspective Reversals and associated apparent motions using a perspective - bound movement illusion. Percept. Mot. Skills, 22, 835-858.

- MILES, W. (1929) Figure for the windmill illusion.
J. gen Psychol., 2, 143-145.
- MULHOLLAND, T. (1956) Motion perceived while viewing rotating stimulus objects.
Am. J. Psychol., 69, 96-99.
- MULHOLLAND, T. (1958) The 'swinging disc' illusion.
Am. J. Psychol., 71, 375-382.
- MULHOLLAND, T. (1963 ¹) Notes concerning "nonmotion" responses while viewing rotating stimulus objects.
J. Psychol., 56, 227-232.
- MULHOLLAND, T. (1963 ²) Instructional sets and motion perceived while viewing rotating stimulus objects.
J. Psychol., 56, 233-237.
- MULLER-LYER, F.C. (1889) Optische Urteilstäuschungen.
Archiv für Physiologie, Supplement-band, 263-270.
- MURCH, G.M. (1970) The Perception of Rotary Motion.
J. exp. Psychol., 86, (1), 83-85.
- MACRAE, K.D. and POWER, R.P. (1969) Critical comment on One Aspect of Graham's "On Some Aspects of Real and Apparent Visual Movement".
J. opt. Soc. Am., 59, (8), 1002.
- NECKER, L.A. (1832) Observations on some remarkable phenomenon seen in Switzerland; and an optical phenomenon which occurs on viewing of a crystal or geometrical solid.
Phil. Mag., 1, 329-337.

- OLSON, R.K. (1974) Slant judgments from static and rotating trapezoids correspond to rules of perspective geometry.
Percept. Psychophys., 15, (3), 509-516.
- OPPEL, J. (1855) Uber geometrisch optische Täuschungen.
Jahresbericht des physikal. Vereins zu Frankfurt a. M. p.37.
- OVER, R. (1968) Explanations of Geometrical Illusions.
Psychol. Bull., 70, (6), 545-562.
- PASTORE, N. (1952) Some remarks on the Ames oscillatory effect.
Psychol. Rev., 59, 319-323.
- PITZ, G.F. (1968) An example of Bayesian Hypothesis Testing: The Perception of Rotary Motion in Depth.
Psychol. Bull., 70, (4), 252-255.
- POWER, R.P. (1965) The effect of instructions on the Apparent Reversal of Rotary Motion in depth.
Quart. J. exp. Psychol., 17, 346-350.
- POWER, R.P. (1967) Stimulus Properties which reduce apparent reversals of rotating rectangular shapes.
J. exp. Psychol., 73, 595-599.
- POWER, R.P. and DAY, R.H. (1973) Constancy and illusion of apparent direction of rotary motion in depth: Tests of a theory.
Percept. Psychophys., 13, (2), 217-223.
- PRITCHARD, R.M. (1958) Visual Illusions viewed as Stabilised Retinal Images.
Quart. J. exp. Psychol., 10, 77.

- ROBINSON, J.O. (1972) The Psychology of Visual Illusion.
London, Hutchinson University Library.
- SCHEFFE, H. (1953) A method for judging all contrasts in the
analysis of variance.
Biometrika, 40, 87-104.
- SLACK, C.W. (1959) Critique on the interpretation of cultural
differences in the Ames trapezoid.
Am. J. Psychol., 72, 127-131.
- SPERLING, G. (1963) A model for visual memory tasks.
Hum. Factors, 5, 19-31.
- SPERLING, G. (1967) Successive approximations to a Model
for short-term memory.
Acta Psychologica, 27, 285-292.
- SPITZ, H.H. (1964) Comparison of Mental Retardates and Normals
on the rotating trapezoidal window illusion.
J. abnorm. soc. Psychol., 68, 574-578.
- SPITZ, H.H. and BLACKMAN, L.S. (1959) A comparison of mental retardates
and normals on visual figural after-effects
and reversible figures.
J. abnorm. soc. Psychol., 58, 105-110.
- WALLACE, B. and CRAMER, D.E. (1975) Apparent Reversals for Circular
and Elliptical Stimuli.
Percept. Mot. Skills, 40, 215-219.
- WEISSTEIN, N. (1969) What the Frog's Eye tells the Human Brain:
Single Cell Analysers in the human visual
system.
Psychol. Bull., 72, (3), 157-176.

- WELFORD, A.T. (1970) Perceptual selection and integration.
IN: A.T. Welford and L. Houssiadass (Eds.)
Contemporary Problems in Perception.
Taylor & Francis, London.
- WOODWORTH, R.S. and SCHLOSBERG, H. (1954) Experimental Psychology.
Holt, New York.
- ZEGERS, R.T. and MURRAY, P. (1962) Perception of distortion: an
experimental and clinical study.
Tech. Rep. NAVTRADEVCEEN, 506-1.
- ZENHAUSERN, R. (1968) The Perception of Rotation with an
Oscillating Trapezoid.
Psychonomic Science, 13, 79-80.
- ZENHAUSERN, R. (1969) Effect of Perspective on two Trapezoid
Illusions.
Percept. Mot. Skills, 28, 1003-1009.
- ZENHAUSERN, R., HAUSEN, K. and SOBRIEN, S. (1970) Order effects in
perception of two trapezoidal illusions.
Percept. Mot. Skills, 31, 335-366.

APPENDIX

PROGRAM TROT

```

014 MASTER TROT
015 DIMENSION XD1(37),YD1(37),XD2(37),YD2(37),YD3(37),YD4(37)
016 PI=3.1415927
017 K=-30
018 CALL HGPSTART(35HTRAPEZOID PROJECTION, J CATCHESIDES,35)
019 200 READ(1,201) X1,Y1,X2,Y2,D
020 201 FORMAT(5F6.2)
021 IF(Y1=50.) 202,99,99
022 202 WRITE(2,100)
023 100 FORMAT(1H1,20X,20HTRAPEZOID PROJECTION)
024 WRITE(2,104) D
025 104 FORMAT(1H ,20X,18HFOR A DISTANCE OF ,F4.1,5H FEET)
026 WRITE(2,101) X1,Y1,X2,Y2
027 101 0FORMAT(1H0,20X,3HX1=,F6.2,4X,3HY1=,F6.2,4X,3HX2=,F6.2,
028 101 14X,3HY2=,F6.2)
029 WRITE(2,103)
030 103 0FORMAT(1H0,10X,5HANGLE,5X,6HX1DASH,7X,6HY1DASH,7X,
031 103 16HX2DASH,7X,6HY2DASH)
032 K=K+30
033 D=D*12
034 DO 20 I=1,37
035 A=10.0*(I-1)
036 R=A*PI/180
037 XD1(I)=D*X1*COS(R)/(D-(X1*SIN(R)))
038 YD1(I)=D*Y1/(D-(X1*SIN(R)))
039 XD2(I)=D*X2*COS(R)/(D-(X2*SIN(R)))
040 YD2(I)=D*Y2/(D-(X2*SIN(R)))
041 WRITE(2,105) A,XD1(I),YD1(I),XD2(I),YD2(I)
042 105 FORMAT(1H ,10X,F4.0,4(5X,F8.4))
043 XD1(I)=(XD1(I)/2.5)+13.0+K
044 YD4(I)=(-YD1(I)/2.5)+9.0
045 YD1(I)=(YD1(I)/2.5)+9.0
046 XD2(I)=(XD2(I)/2.5)+13.0+K
047 YD3(I)=(-YD2(I)/2.5)+9.0
048 YD2(I)=(YD2(I)/2.5)+9.0
049 20 CONTINUE
050 D=D/12
051 50 0CALL HGPAXIS(5.0+K,9.0,30H THE X AXIS,
052 50 1-30,16.0,0.0,-20.0,2.5)
053 60 0CALL HGPAXIS(13.0+K,3.0,30H THE Y AXIS,
054 60 1-30,14.0,90.0,-15.0,2.5)
055 CALL HGPDASHIN(XD1(1),YD1(1),XD2(1),YD2(1),0.0)
056 CALL HGPlot(XD2(1),YD3(1),2,0)
057 CALL HGPlot(XD1(1),YD4(1),2,0)
058 CALL HGPlot(XD1(1),YD1(1),2,0)
059 CALL HGPSURVE(XD1,YD1,37,0,0.0,0.0)
060 CALL HGPSURVE(XD2,YD2,37,0,0.0,0.0)
061 CALL HGPSYMBL(15.0+K,21.5,0.35,20HTRAPEZOID PROJECTION,0.0,20
062 CALL HGPSYMBL(15.0+K,20.4,0.35,13HX1= Y1=,0.0,13)
063 CALL HGPSYMBL(15.0+K,19.7,0.35,13HX2= Y2=,0.0,13)
064 CALL HGPSYMBL(15.0+K,19.0,0.35,14H D= FEET,0.0,14)
065 CALL HGPNUMBER(16.0+K,20.4,0.35,X1,0.0,0,2,2)
066 CALL HGPNUMBER(19.0+K,20.4,0.35,Y1,0.0,0,2,2)
067 CALL HGPNUMBER(16.0+K,19.7,0.35,X2,0.0,0,2,2)
068 CALL HGPNUMBER(19.0+K,19.7,0.35,Y2,0.0,0,2,2)
069 CALL HGPNUMBER(16.0+K,19.0,0.35,D,0.0,0,2,2)
070 GO TO 200
071 99 CALL HGPFINISH
072 STOP
073 END

```