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CONTROLLED DENSITY METHODS IN
PHOTOELASTICITY

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ABSTRACT

A photographic method is presented that provides full-field fractional fringe orders without any auxiliary equipment on lens or diffused-light polariscopes. The boundary lines between dark and bright bands on a high contrast film and the equidensity lines obtained by pseudo-solarization or an Agfacontour film represent lines of constant fractional fringe order in an isochromatic pattern. The position of these lines in the field of the model and hence their fractional order can be changed by variation of exposure time and/or load when high contrast or Agfacontour film is exposed directly in the camera, or by changing the copying exposure time when an original negative on low-contrast film is copied through yellow light onto Agfacontour film. The boundary or equidensity lines can be calibrated in terms of fractional order or principal stress difference by means of a separate calibration model such as a beam in pure bending or in terms of fractional order only by means of a self-calibration procedure when a separate calibration model cannot be used. Fringe orders can be measured to an accuracy of 1% or better and the sensitivity for evaluation is considerably increased (50 times or more).

The undesirable effects of a central intense spot normally obtained at the image plane of the camera on a lens polariscope can be reduced or eliminated by using a small camera aperture or a semi-diffused light polariscope.

Special problems investigated include: study of optical creep and residual stresses, determination of stress concentration factor when the image on the film is very small, determination of full-field fractional fringe orders in slices of three-dimensional stress-frozen models.

Full-field fractional isochromatic fringes without isoclinics can be obtained by photographic double exposure on a plane polariscope so that quarter-wave plates are not needed for accurate evaluation.

(ii)

A number of photographic methods are proposed to obtain pure and sharpened isoclinics in sensitive model materials. The equal intensity concept allows accurate point-by-point construction of isoclinics over the whole field when the isoclinic is known at one point and the isochromatic pattern is available. With a single orientation of the crossed polaroids, three isoclinic parameters can be obtained simultaneously over an extended region.

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Note: References are not numbered but given in alphabetical order of the first named author. In the text, references are indicated suitably by either 'Coker (1911)' or '(Coker, 1911)'.

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CHAPTER 1
INTRODUCTION AND LITERATURE SURVEY

Photoelasticity is an optical means of obtaining the lines of constant principal stress difference (isochromatics) and lines of constant principal stress directions (isoclinics) over the full field of a loaded model.

Isochromatic information from photoelasticity is used (i) to evaluate free boundary stresses, (ii) to determine interior maximum shear stress, and (iii) to provide basic data for the separation of individual principal stresses. Since in some situations only a few integral and half-order isochromatics can be seen, a number of compensation methods have been developed to provide fractional isochromatic fringe order at those points through which no integral or half order isochromatic passes. The overall accuracy of photoelastic analysis largely depends on the accuracy with which fractional fringe orders can be measured. Point methods and full field methods of fractional fringe order measurement may be distinguished.

1.1 POINT METHODS OF FRACTIONAL FRINGE ORDER DETERMINATION

1.1.1 Compensators

A compensator is an additional device placed on the polariscope either in front or at the back of the model in a predetermined orientation (parallel to either principal stress direction). It produces relative retardations or fringe orders which can be varied in a known manner by known amounts. The resultant observed retardation is the algebraic sum of the retardation in model and compensator and is usually made equal to zero. The additional retardation produced by the compensator to achieve the condition of zero retardation at a point in an isochromatics field gives a measure of the retardation or of the fractional fringe order in the model at the point under investigation.

A compensator may be a tension or compression bar made of a photoelastic material, variable retardation being obtained by changing the load on the bar. This is known as the Coker (1911)*

* For arrangement of references see Note at the end of the Table of Contents.

compensator and is based on an idea originated by Wertheim (1854).

The second type of compensator makes use of permanently doubly refracting material such as quartz, mica or cellophane to introduce additional retardation. A classical description of a number of such compensators namely those of Babinet, Soleil, Rayleigh, De Forest, Palmer, Brace, Szivessy, Sénarmont, Richartz, *et al* has been given by Jerrard (1948). Out of these, only two, i.e. those by Babinet and Soleil (also known as Babinet - Soleil) are usually described in the text books on photoelasticity. Jerrard limits his discussion to the measurement of phase difference whereas Richartz and Hsu (1949) have described a technique for the complete determination of the state of elliptical polarisation in each compensator.

Compensators of the third type are the known "permanent compensators" with frozen-stress isochromatic patterns in models of either a tapered bar in tension, (Frocht, 1948a) or a beam in pure bending (Jellyman and Milne, 1948). These permanent compensators can also be used for determining integral orders of fringes when counting is not possible. They are of value in situations where no integral fringe is visible.

The advantage of compensators is that they show which principal stress is the larger and by how much it is larger than the other principal stress. Their disadvantages are:- (i) the method is a point method, (ii) it requires additional equipment, (iii) prior determination of the principal stress directions is necessary, and (iv) visual judgement of extinction is required.

1.1.2 Goniometric Methods

Goniometric methods of fractional fringe order measurement are the well known Sénarmont (or Friedel) and Tardy methods of determining fractional fringe orders by the exclusive use of elements of the polariscope itself.

The Sénarmont method uses only the quarter-wave plate adjacent to the analyser whereas the Tardy method uses both quarter-wave plates. After the isoclinic parameter determination at the point, these plates are arranged in such a manner that the magnitude

of the angle by which the analyser is rotated to bring a neighbouring integral fringe to the point is a measure of fractional fringe order. The methods are simple in operation, require no additional equipment and can be highly accurate. Disadvantages are:- (i) Determination at a point only, (ii) laborious and time consuming work if a large number of readings is required, (iii) need for accurate determination of isoclinics, (iv) extinctions to be judged visually, (v) need for monochromatic light and accurate quarter-wave plates, and (vi) unsuitability for model materials which show high optical creep.

The Sénarmont and Tardy methods are both subject to retardation errors in quarter-wave plates and to inaccuracies in isoclinic parameter determination, and their relative merits have been widely discussed. From consideration of the retardation error alone, Jessop (1953) has recommended the Sénarmont method, whilst consideration of the errors in isoclinic angle only favours the Tardy method as shown by Chakrabarti and Machin (1969). According to Sathikh and Bigg (1972), the simultaneous consideration of retardation and isoclinic error indicates no unique criterion favouring the one or the other.

Practical ranges of values of retardation errors have been reported by Brown and Hickson (1950) and Jessop (1953). Definite values of isoclinic setting errors are not available (Jessop, 1953), (Chakrabarti and Machin, 1969), (Frocht, Pih and Landsberg, 1954). Flynn (1970) has described two theorems and one corollary in order to explain the use of suitably oriented quarter-wave plates and plane polarisers in the various variations of the Senarmont method. Fishburn and Froust (1972) have described a goniometric compensation technique where the polariser alone is rotated in order to avoid the disturbance of optical elements between the model and viewing screen.

The accuracy of all the pointwise compensation methods depends to a great extent on the accuracy with which the positions for extinction or minimum intensity are determined. To obtain accurate visual determination of extinction, a half-shade device

has been used by Goranson and Adams (1933) for the Sénarmont method and by Hariharan (1960) for the Babinet compensator. This half-shade device works on the principle of a photometric match between two parts of a divided field.

Brown and Hickson (1950) have proposed the use of a photometer to detect the condition of minimum intensity in the Tardy method, and have measured isochromatics to within ± 0.002 fringe. Frocht, Pih and Landsberg (1954) have also used a photometric device and have measured isoclinics to within $\pm 0.3^\circ$ and isochromatics by a Babinet - Soleil compensator to within ± 0.002 fringe. Photometers have here been used to eliminate the personal judgement of the observer in determining the setting which produces maximum extinction.

1.1.3 Photometric Methods

Robertson (1965) has measured by a photometer the magnitude of intensity transmitted by the analyser as a function of isochromatic and isoclinic fringes at the observed point. Insertion and adjustment of quarter wave plates are thus eliminated. The method is influenced by the presence of flare due to internally reflected light and by the inaccuracy in quarter-wave plates. An isochromatic error as large as $\frac{1}{30}$ th of a fringe and an isoclinic error of $2\frac{1}{2}^\circ$ has been reported.

Cloud (1968) has described an improvement which reduces the effects of zero slope of the intensity curve and of noise in the photometer when measuring minimum intensities. If the angle is determined on either side of the minimum where the amplitude reaches some arbitrarily assigned value, then for a signal to noise ratio of 100, the indeterminacy in minimum intensity angle, usually amounting to $\pm 5.73^\circ$, is reduced to $\pm 1.15^\circ$ by measurements made where intensity is one-half the maximum. Barishpol'skii and Radchenko (1972) have obtained isoclinic and isochromatic parameter simultaneously by measuring intensity at a point twice with different orientations of the polariscope elements.

The photometric method has the advantage of a high degree of accuracy in measurements. Its disadvantages are that (i) it is a point method, (ii) it requires additional equipment, (iii) a

stable light source is necessary, (iv) it is sensitive to non-uniform illumination of the field and to noise in the photometer and (v) it gives inaccurate readings in regions of high stress gradients because of the size of measuring aperture.

1.2 FULL FIELD METHODS OF FRACTIONAL FRINGE ORDER DETERMINATION

1.2.1 Fringe Multiplication Techniques

Full-field methods have lately gained interest. Post (1955) fringe multiplication technique provides a number of fringes between two integral or half-order fringes by placing of a slightly inclined partial mirror on both sides of the model and by isolation for observation and recording purposes of a ray which has passed through the model a certain number of times. The technique allows elimination of residual fringe effects that cause errors in the point methods, and it increases the sensitivity of full-field isochromatic measurements.

However, it requires a lens type polariscope with critical focussing, longer exposures than usual, a brighter light source, and special photographic treatment. After five passes through the model the light at the screen has only two per cent of the intensity after one passage. Since slightly inclined mirrors are used, the reflected rays pass through slightly different points on each passage through the model and the photoelastic effect is averaged over an area increasing with distance and included angle between the mirrors.

Fringe multiplication techniques have been used by Wells and Post (1958) for dynamic analysis, by Taylor and Schweiker (1959) and Post (1966, 1970a) for three-dimensional problems and by Mylonas and Drucker (1961) for the study of torsion. Kenny and Hugill (1967) and McNicholas (1970) have suggested modifications in the design of the polariscope to make it suitable for fringe multiplication in three-dimensional slices.

Using a diffused light polariscope, Dally and Ahimaz (1962) have critically exposed high-contrast film separately to dark and to bright background fields and have superimposed the two negatives with proper alignment so as to obtain on yet another negative a mixed field pattern containing quarter-order fringes. Lack of

success with a simple double exposure of one film has been attributed to non-linearities in the characteristic curve of the film. However, the intensity equations of dark field and bright field circular polariscopes indicate that no isochromatic fringe pattern will appear in double exposure if equal exposure times are used for dark and bright field and that the fringe numbers are not increased even for different dark and bright field exposures (Meyer and Mehrotra, 1974).

Cernosek (1972) has outlined theoretically a modified Tardy method usable as a full-field compensation method. A third set of polariser and quarter-wave plate, a beam splitter and a mirror are added to the normal equipment and Tardy compensation is applied repeatedly. Fringes are wider than usual and errors exceed those of the normal Tardy point method.

Chiang (1971) has proposed a full-field method of interpolation with light of different wavelengths. The required different light sources add to costs and for accurate evaluation may require different quarter-wave plates. Since the stress-optic coefficient depends on wavelength, the evaluation is not linear. Results appear inconclusive in certain regions between zero and half-order and between half-order and first-order fringes.

1.2.2 Controlled Density Techniques in Literature

Chiang and Khetan (1974) have proposed an equidensity method of full-field interpolation. The fringe order is a continuous function of intensity which can be converted into density of a photographic negative and traced out by a microdensitometer or isodensitracer, either continuously or discretely at several hundred data points between integral and half-order fringes. Since the method requires recording of the fringe pattern on the linear range of the film characteristic, it fails at points around zero light intensity and may also lead to errors at very high intensity (Meyer and Mehrotra, 1975). Uneven background illumination may affect the results.

Evaluation depends on the $\sin^2$ relationship between density and fringe order, and the reading device must be calibrated

with a standard density wedge. Resolution of the density measuring device and noise level of the film determine the smallest readable fractional order and a long development time is required to produce a required gamma value of unity.

Schwieger and Haberland (1956, 1956a) were first in applying to photoelasticity the photographic process of pseudo-solarization or Sabattier effect to obtain equidensity lines. Using a plane polariscope, they obtained in a pointwise analysis, the fractional fringe orders in situations where no full fringes were seen because all isochromatic orders were below one. Schwieger and Spuida (1965) applied the equidensity technique to study the birefringence pattern obtained in the case of peripheral impact of a spherical weight on a circular glass ring.

Thadani (1972) has obtained by pseudo-solarization equidensity lines in a model having a large number of isochromatic fringes. These lines have been erroneously called Mackie lines (Meyer, Mehrotra and Pabani, 1975). Two equidensity lines representing constant fractional fringe orders are obtained between any two neighboring integer or half order fringes. If a point on the equidensity line is matched to a point on the model, the magnitude of the fractional fringe order at this point can be determined by a compensation technique. With this information, the fractional fringe orders of various equidensity lines can be established. Any error in the compensation technique is transmitted to the calculated fringe order and further errors will result if the points on the photograph and the model are not identical.

Agfa-Gaevert have recently marketed a special equidensity film "Agfacontour" invented by Ranz (1970). In a direct procedure, Friedrich (1974) has obtained on Agfacontour equidensity lines representing constant fractional fringe order for an isochromatic fringe pattern. For the calibration of the fringe orders Friedrich suggests an approximate graphical procedure (described in Chapter 5 below), which however, has not yet been applied to a practical problem and which does not take account of the effects of a non-uniform field on the equidensities.

In the work reported here, a full-field photographic method of fractional fringe order measurement is developed (Meyer and Mehrotra, 1974). This method does not require any additional equipment and gives permanently recorded information - without much effort or cost. A high contrast film is used in a lens polariscope to obtain isochromatic fringe photographs containing dark and bright bands with no intermediate grey tones. The sharply defined boundary lines between dark and bright bands represent fractional fringe orders.

The fringe order represented by these boundary lines is determined by a calibration model such as a beam in pure bending. The position of the boundary lines and their fractional orders can be changed by varying the exposure time or the load so that a sufficient number of fractional fringes can be obtained. The calibration result for one given exposure time can be transferred to any other exposure time.

A phenomenon similar to boundary lines is obtained in the form of equidensity lines when either an exposed film is pseudo-solarized or a special Agfacontour film is exposed in the camera. The equidensities are also obtained by copying an original photograph, preferably on a low-contrast film, onto Agfacontour film. The equidensity lines can be calibrated in terms of fractional fringe order in a manner similar to that used for boundary lines.

The boundary line method and equidensity methods can be called controlled density methods because they depend on accurate control of densities in the photographic film for the purpose of fractional fringe order measurements.

1.2.3 Controlled Density Techniques described in this Thesis.

Chapter 2 presents the photoelastic and photographic theories. In Chapter 3, the boundary line method is described in detail. Section 3.3 outlines various procedures for self-calibration which are used in situations where a separate calibration model is not available. The boundary line method has been applied to the problem of a disk under diametral compression and a good agreement with theory has been obtained. Various optical and photographic factors which can influence the density are studied in Section 3.7.

The controlled density methods are sensitive to no-load non-uniformity of light intensity at the camera image plane. The problem of uniformity of illumination is discussed in Chapter 4 where an attempt has been made to outline the possible causes of this non-uniformity. A number of techniques used to solve the problem of non-uniform illumination are also described. The most important observation of this chapter is that the use of a small camera aperture improves the non-uniformity to a great extent whilst leading to a negligible loss of light intensity.

Chapter 5 outlines the pseudo-solarization method and the use of Agfacontour film to obtain equidensity lines representing constant fractional fringe orders. This method supplements the boundary line method and the two methods are compared in this Chapter. In fact, equidensity and boundary lines both represent lines of constant intensity in the camera image plane. It has been found that one single photograph of a fringe pattern suitably exposed on low-contrast film can be used to obtain equidensities over the whole field of the model by copying on Agfacontour with different copying exposure times.

Chapter 6 is concerned with the solution of some specialised problems with the help of controlled density methods. Section 6.1 presents the boundary line method used with the diffused light polariscope. Unlike Post's fringe multiplication technique, this method is applicable to diffused light polariscopes. Section 6.2 describes a study of creep properties of CR-39 by the boundary line method. Residual stresses are investigated in Section 6.4. Section 6.3 indicates the applicability of the method in situations where the maximum fringe order is less than $\frac{1}{2}$. The effect of pre-exposure is considered in Section 6.5. Some additional model studies are described in Section 6.6. Section 6.7 gives an analysis of three-dimensional slices using one of the self-calibration procedures. The method is shown to be applicable to full field isochromatic determination in three-dimensional slices even when no separate calibration model is available.

Chapter 7 deals with a double-exposure procedure for photographing pure isochromatics in a plane polariscope. The use of quarter-wave plates is shown to be unnecessary because the proposed controlled density methods provide full field fractional isochromatic orders in a plane polariscope.

1.3 IMPROVEMENTS IN THE DETERMINATION OF ISOCLINICS.

The positions of isoclinic fringes depend upon the orientation of crossed polariser and analyser with respect to a fixed reference. Different values of the isoclinic parameter (isoclinic angle) are obtained by simultaneous rotation of crossed polariser and analyser in a plane polariscope in discrete steps. The isoclinics do not appear independently of isochromatics except in models with extremely low sensitivity.

The definition and accuracy of isoclinics are severely restricted because of interaction between isoclinics and isochromatics and also because of difficulties in obtaining sharp isoclinics. Isoclinic angles are usually determined visually by finding as best as possible the centre lines of the dark bands. This inspection is difficult where the isoclinics are broad bands, i.e. where (i) principal stress directions change very slowly, (ii) isoclinic interact with zero-order isochromatics and (iii) isoclinics interact with residual stresses at low loads. Isoclinics can also be photographed with complete elimination of isochromatics. Isoclinics are used for plotting of stress trajectories. Usually the stress trajectories are drawn graphically from curves of isoclinics traced at intervals of 5 or 10° . Photographic methods of obtaining stress trajectories by multiple exposures of different isoclinic parameter from 0 to 90° through a grid have been described by Vasarhelyi (1951), Morris (1956) and Havard (1967). Havard has used multiple exposures in the camera whereas Vasarhelyi and Morris have used multiple exposures of photographs of different isoclinic parameters on yet another film.

Isoclinics are also used in separation of normal stresses in two and three-dimensional photoelasticity. Although prediction of failure of ductile material requires information about the

maximum shear stress (isochromatics) only, separation of the principal stresses within the boundary is necessary for brittle materials where a knowledge of maximum principal stress is essential. A high accuracy in isoclinic measurement is necessary for the sub-slice technique of obtaining solutions for general three dimensional stress distributions in a single model (Frocht, Pih and Landsberg, 1954).

Photometric methods have been used by Brown and Hickson (1950), Frocht, Pih and Landsberg (1954) and Robertson (1965) in order to locate the positions of isoclinics accurately to within $\pm \frac{1}{2}^\circ$ in point-by-point exploration. Accurate photometric work requires a uniformly illuminated field, and accuracy of measurement of minimum intensity is restricted by the noise level of the instrument unless remedial measures are taken as explained by Cloud (1968).

In order to increase the accuracy of isoclinic measurement, Dunn (1945) has suggested a photographic method in which the load on the model is varied so as to blur out the isochromatic pattern obtained in monochromatic light and to leave the isoclinics superimposed on a uniform grey background. No experimental results are given.

Circularly polarised light has been used by Srinath and Sarma (1971) and Barishpol'skii and Radchenko (1972) to obtain isoclinic parameters in pointwise analysis employing intensity measurements. Srinath and Sarma have described a method which requires the coupling of the second quarter-wave plate and analyser and which leaves undisturbed the elements of the polariscope between light source and model. The method uses quarter-wave plates and the errors in isoclinics due to inaccurate quarter-wave plates are not considered. Barishpol'skii and Radchenko obtained simultaneously isochromatics and isoclinic parameters by absolute measurement of intensity at a point by two sets of orientations of the polariscope elements.

Becker and Colao (1969) and Cernosek and Perla (1970) have developed methods of determining the isoclinic parameters from measurements of isochromatic fringe orders.

Becker and Colao require three separate measurements, (a) of the fringe order n_0 of a stress frozen tensile fringe pattern of known tensile direction θ in the photoelastic field, (b) of the order n_1 in the loaded model, (c) of the order of n of stress frozen and model fringe pattern superimposed. A nomogram derived from stress transformation equations then gives for measured ratios n_1/n_0 and n/n_0 the angle $\phi + \theta$ where ϕ is the isoclinic angle at the point of measurement. Measurements require either two models, one with the frozen pattern and one without, or one model and a frozen tension strip inserted in the field in a known direction. If the model is symmetrical, a tension strip can be bonded to one half of the model. With the usual compensation methods of partial fringe order determination the method has the advantage of accuracy but the disadvantages of a pointwise and time consuming method.

Cernosek and Perla use a "linearly" polarised field (polariser and first quarter-wave plate parallel, second quarter-wave plate and analyser also parallel and at 90 degrees to the first set). If polariser and analyser are rotated through $+5^\circ$ and -5° respectively, a set of "secondary isochromatic fringes" appears in the field of the loaded model. A second set is obtained on reversing the rotations. If the two sets are superposed, the intersections of corresponding lines lie on the isoclinic line for the parameter given by the direction of the original "linear" field. Hence two photographs are generally required for each isoclinic line which must be constructed from the superposed picture. Even if the secondary isochromatics are sharpened as described in this thesis, the procedure is lengthy and the resultant accuracy depends on finding the lines of minimum intensity.

Parmeter and Rahn (1971) have used a method of simultaneously recording several isoclinic patterns in dynamic photoelasticity or photoviscoelasticity where isoclinic parameters must be measured quickly. The rotation of the plane of polarisation of light is achieved by using two rotationally active substances (Levulose and Karo syrup), which are complementary to each other, one between the polariser and the model and another between the model and analyser.

These substances rotate dispersively the plane of polarisation of light i.e. different wavelengths are rotated by different amounts. Thus a multi-wavelength beam of light polarised in a given plane by the polariser is dispersed in its direction of polarisation by the substance and therefore each wavelength responds to different isoclinic patterns as it passes through the model. The observation of the image through different narrow-band interference filters reveals a family of isoclinic patterns, each filter giving one particular isoclinic parameter. The recording system is complex and requires a high-speed camera at each image location, and all the cameras must be synchronised.

The second part of the thesis (Chapter 9) describes a number of photographic methods of obtaining pure isoclinics (i.e., independent of isochromatics) for optically sensitive models such as those made of CR-39 or Araldite. It is shown that a considerable sharpening of the isoclinics is possible. In addition the boundary line concept has been used in restricted situations to obtain more than one isoclinic parameter for only one setting of crossed analyser and polariser. In this part of the work, the author had the help of a M.Sc. project undertaken by Mr. Kremezis under the author's supervision.

1.4 APPENDICES

Appendix I to VI present those parts of the work for the Ph.D. degree that has already been published or submitted for publication.

Appendix I, a paper to be published in July 1975 issue of the "Journal of Strain Analysis", describes the pseudo-solarisation method of producing equidensity lines, which are lines of constant fractional fringe order. Various methods had been developed before the advent of Agfacontour film, and the paper clarifies the photographic terms and techniques and proposes a number of simplifications and improvements in the pseudo-solarisation technique.

Appendix II, a paper submitted for publication to "Experimental Mechanics", deals with the use of Agfacontour film for the generation of equidensity lines by direct photography on the polariscope or by contact copying from a low-contrast negative of the isochromatic fringe

pattern. This Appendix should be read in conjunction with Chapter 5 of this Thesis. In fact, Chapter 5 has been presented in a relatively short manner because Appendix II already presents the main lines of the new technique of equidensities. It gives the first comprehensive treatment of this powerful new technique.

Appendix III, a summary of a paper read at the Fourth Canadian Congress of Applied Mechanics held at Montreal in May 1973, introduces briefly the boundary line method of Chapter 3 and the technique of calibration with a separate calibration test piece.

Appendix IV, presented at the fifth International Conference on Experimental Stress Analysis at Udine (Italy) in May 1974, contains a detailed investigation into the boundary line method of fractional fringe order measurement using a high-contrast film. The calibration method by a separate test piece (beam in bending) is presented in great detail. This appendix should be read in conjunction with Chapter 3 of this thesis.

Appendix V reproduces a discussion contribution by F.P. Chiang on the paper given in Appendix IV and the author's reply. Both will be printed in the Proceedings of the Fifth International Conference on Experimental Stress Analysis and mainly concern the advantages of pre-exposure in overcoming difficulties of fringe order measurements near the intensity minima (see also Chapter 6.5 of Thesis).

In Appendix VI, published in the May 1975 issue of Experimental Mechanics, a technique by Chiang and Khetan of densitometric measurement of fractional fringe orders is compared with the boundary line method described in this Thesis. Each method has its advantages and disadvantages. The discussions of Appendix V and VI refer to the same authors and the same publications.

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CHAPTER 2

ANALYSIS OF POLARISCOPE AND WORKING METHODS

2.1 POLARISCOPES

In its simplest form the polariscope consists of a light source, a loaded model placed between two polaroids (polariser near light source, analyser near viewing point), and a recording or viewing arrangement of camera or screen. If the planes of polarisation are parallel, a plane polarised field with bright background is obtained, if they are crossed, the background is dark. Plane fields show a combined pattern of isoclinics and isochromatics. If two quarter-wave plates with crossed planes of retardation at 45° to the polarisation planes are inserted on either side of the model, the polarisation is circular and isoclinics do not appear.

In the diffused-light polariscope (Fig. 2.1.1a) the light source may be large and consist of a bank of incandescent or fluorescent lamps with a plate of flashed opal glass to provide a uniform diffusion of light. The only lens used is the projection or the camera lens.

In a lens-type polariscope (Fig. 2.1.1b) the light source is small or focussed to a point so that a fairly parallel beam of light through the model can be obtained by means of a field lens.

2.1.1 Light Source and Filters

Polariscopes usually have a monochromatic source of light in addition to white light.

In a diffused polariscope, fluorescent tubes are frequently employed. Durelli and Lake (1951) have described the difficulties in obtaining a monochromatic light of narrow waveband by this source even with a Kodak Wratten filter No. 77. An extended light source may require the use of a filter near the camera when large size filters are not available. This may not be desirable because monochromatic quarter-wave plates are used and also because of the non linear spectral transmission characteristics of different elements of the polariscope (Pindera

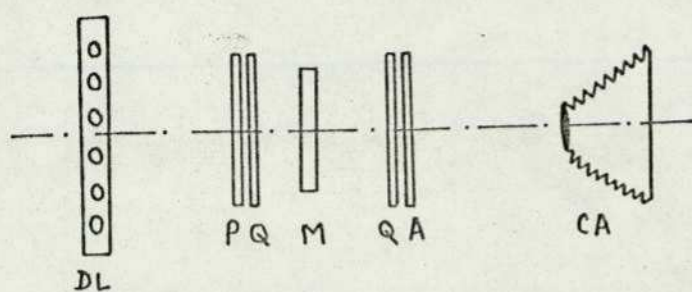


Fig. 2.1.1a Diffused light polariscope.

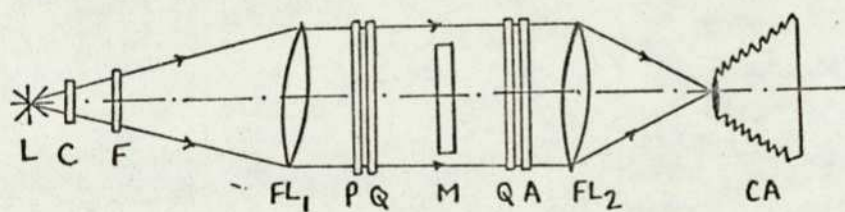


Fig. 2.1.1b Lens-type polariscope.

- | | |
|------------------------------------|-------------------------------------|
| A = Analyser | FL ₂ = Second field lens |
| C = Condenser housing | L = Point light source |
| CA = Camera | M = Model |
| DL = Diffused light source | P = Polariser |
| FL ₁ = First field lens | Q = Quarter-wave plate |

and Sinha, 1972). However, Navaratna (1962) has described a simple method of making large inexpensive filters to isolate the mercury green line.

The monochromatic light source for a lens-polariscope is either a sodium vapour lamp or a high-pressure mercury arc lamp. Sodium vapour lamps have a low intensity of illumination but require no filters because 99% of light is transmitted at $5890 \text{ \AA}$ and $5896 \text{ \AA}$. A suitable filter (usually Kodak Wratten No. 77) is used to isolate the $5461 \text{ \AA}$ green line from a mercury discharge lamp which emits intensely brilliant concentrated light having characteristics as described by Bourne (1943). True monochromatic light is difficult to obtain and laser light is the nearest approximation to it. Its application to photoelasticity has been studied by Taylor, Bowman, North and Swinson (1966). Projection lamps are commonly used to obtain concentrated white light.

2.1.2 Polarisers

Early polarisers were made from nicol prisms. Modern polarisers are invariably made from H-type polaroid sheets and are often bonded between two glass plates. The performance characteristics of polaroid polarisers have been described by Shurcliff (1962).

2.1.3 Quarter-wave Plates

These are mica sheets producing uniform relative retardations equal to one quarter of a wavelength of the monochromatic light used. Therefore, a quarter-wave plate must be matched to the particular wavelength of light.

Recently two or more different wavelengths of light have been used (i) for counting integral fringe orders (Pant, 1963; Srinath and Sarma, 1973), (ii) in automatic photoelasticity (Young, 1969; Redner, 1974) and (iii) in fractional fringe order measurement (Chiang, 1971). If the wavelength of light differs from that of the quarter-wave plates, a correction must be applied which introduces a non-linearity into the formula for Tardy or Sénarmont compensation.

As an example, let a quarter-wave plate matched for $6000 \text{ \AA}$ be used in light of $4000 \text{ \AA}$. The relative retardation of $1500 \text{ \AA}$ is then not a quarter-wave of the monochromatic $4000 \text{ \AA}$ light, and this

light is not changed from plane to circular polarisation.

The application of light of multiple wavelength therefore requires achromatic quarter-wave plates. Cernosek (1973) has shown that a nearly achromatic quarter-wave plate can be assembled from a set of commercially available quarter-wave plates. West and Makas (1949) and Panchratnam (1955) have also described methods of obtaining nearly achromatic quarter-wave plates. Panchratnam uses three sheets of mica oriented in a particular manner whilst West and Makas use either a single sheet of suitable composition (cellulose nitrate) or combine two or more sheets.

Even for a single monochromatic wavelength, it is difficult to produce true quarter-wave plates because of limitations in obtaining an accurate thickness of mica sheet. However, Mindlin (1937a) has shown that small errors in quarter-wave plate have a negligible effect on the precision of photoelastic analysis. The effect of inaccurate quarter-wave plates on goniometric compensation methods have been described by Mindlin (1937), Hickson (1952), Jessop (1953), Robertson (1965), Young (1969), Cernosek and Perla (1970) and Sathikh and Bigg (1972).

2.1.4 Field Lenses and Camera

Single element lenses can be used for monochromatic light but must be of good optical quality to eliminate the effects of monochromatic aberrations. The first field lens is placed so that its focal plane lies at the light source or at the source aperture.

A camera is used to photograph the fringe pattern with the camera lens placed at the focal plane of the second field lens. For visual observations a screen or a microscope can be used instead of camera.

2.1.5 Automatic Polariscopes

An automatic recording and displaying polariscope system has been described by Redner (1974). Automatic collection of photoelastic data is desirable where a substantial number of point readings are required (shear - difference method) or where a digital or remote signal is required (photoelastic - coating technique). Redner also summarises work by other authors on automatic polariscopes, notably Young (1969) and Allison and Nurse (1971).

2.2 OPTICAL ANALYSIS

Optical analyses of lens polariscopes have been given by Mylonas (1948), Jessop (1948) and Post (1970). Mylonas has shown that a compact and intense source is needed to get a sharp image in a strongly illuminated field. A condenser lens C (Fig. 2.1.1b) near the light source has been shown by Jessop and by Mylonas to be advantageous. If a diaphragm is placed on the axis of the bench in the focal plane where the condenser forms a real image of the light source, the aperture of the diaphragm can be reduced to concentrate the light beam. Thus the light is transmitted to the field lens from the central area of the image, and therefore ultimately from the corresponding central area of the lamp (Jessop, 1948).

Light from any point of the source aperture spreads out in spherical waves. The first field lens FL_1 collimates the light and converts it into plane waves. The second field lens FL_2 re-focusses these waves to a unique point (neglecting diffraction effects) in its focal plane. The camera lens is located in this plane. Thus light from a unique point in the source aperture illuminates the entire model, but passes through a unique point in the camera lens.

2.2.1 Finite Light Source and Parallel Beam of Light

If a true point source of light is placed in the focal plane of the field lens FL_1 , a simple divergent beam which changes to a parallel beam after passing through the field lens (broken lines in Fig. 2.2.1) is obtained. However, a finite source (or source aperture) of diameter d_s gives a divergent beam which on passage through FL_1 is converted into a beam consisting of elemental cones with parallel axes. The apex angle of these elemental cones w is given by

$$w \approx \frac{d_s}{f_1} \quad (2.2.1)$$

where f_1 is the focal length of the field lens FL_1 .

Hence each point in the mid-plane of the model receives a ray not from a unique point in the field lens but from a circle of diameter d_1 given by

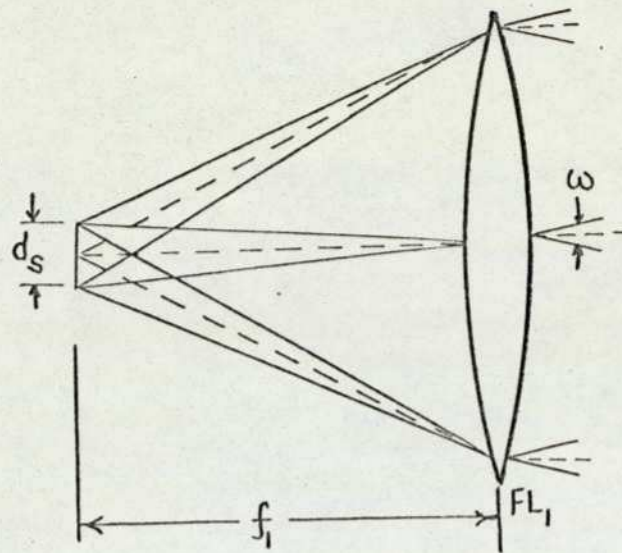


Fig. 2.2.1 Effect of finite light source on parallelism of light.

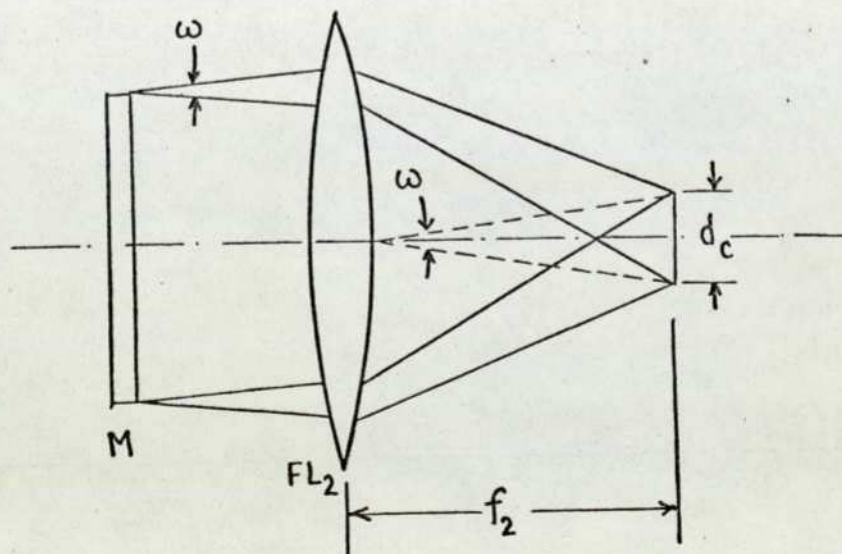


Fig. 2.2.2 Effect of angle of cone on the light focussed by field lens.

$$d_1 \approx w l_1 \quad (2.2.2)$$

where l_1 is the distance between the field lens FL_1 and the model. Similarly the cone of light from one point of the model will intersect the second field lens FL_2 on a circle of diameter $d_2 \approx w l_2$ where l_2 is the distance between the model and the second field lens.

In addition, the size of the useful field is slightly less than the size of the lenses and depends on w, l_1 and l_2 . Hence the field lenses FL_1 and FL_2 are best arranged in close vicinity of the model. Post (1970) therefore places them between the model and the polaroids. However, this arrangement requires stress free lenses.

Behind the second field lens the beam converges (Fig. 2.2.2) so that in the focal plane it has a circle of diameter

$$d_c = w f_2 \approx d_s \frac{f_2}{f_1} \quad (2.2.3)$$

where f_2 is the focal length of the second field lens FL_2 .

A source aperture of diameter d_s thus gives an image of diameter d_c in the plane of the camera lens and d_c can be reduced if d_s and/or the ratio f_2/f_1 is reduced. The size of d_c also limits the size of the camera aperture which must exceed d_c if some of the outer rays are not to be cut off and the field near the boundary is not to be darker than the rest.

2.2.2 Effect of Cone Angle on Stress-Gradients

The isochromatic fringes accurately represent the stress difference in the model if either the cone angle w is small or the stress gradient in the model is low. For severe stress gradients, the smallest possible value of w is desired. If w is large, errors arise because different rays of any cone pass through a region of the model over which stresses change rapidly.

2.2.3 Positions of Field and Camera Lenses

The field lenses FL_1 and FL_2 (Fig. 2.1.1b) are usually placed in front of the polariser and behind the analyser so that internal stresses in the glass of the lenses do not affect the fringe pattern (see however Post, 1970).

For optimum performance, the camera lens must be in the focal plane of the second field lens FL_2 . This condition precludes movement of the camera lens so that for changes in magnification the camera lens must be changed for one of different focal length.

2.2.4 Location of Object Plane

Post (1970) has shown that the camera lens must be focussed onto the plane of the model. If the camera is focussed onto a different plane, contrast is reduced and for large differences the position and shape of the isochromatics will be affected.

2.2.5 Magnification of Image

When the model is small, an accurate determination of fringe order requires magnification of its image in the film plane. This can be achieved by a camera lens of long focal length or by a reduction in the distance between the second field lens and the model, for instance by placing a stress free second field lens between model and analyser, as suggested by Post (1970). A combination of both can give very large magnifications indeed.

2.2.6 Internal Reflection

The lens-type polariscope incorporates many glass surfaces each of which reflects a small fraction of the incident light. A part of the reflected ray is lost but a small fraction is re-reflected and transmitted towards the camera. This causes non-uniformity of the field as discussed in Chapter 4. Murray (1949) has analysed the effect of reflected light in an optical system with many coated or uncoated reflecting surfaces.

2.2.7 Optics of the Diffused-Light Polariscope

Durelli and Lake (1951) have investigated the effects of ray obliquity in diffused-light polariscopes. Using the 'wrapped wavefront approach', Post (1970) has studied the resolution and contrast of fringes in regions of high stress gradients and the optimum location of the object plane.

2.3 SPECIFICATIONS OF THE BENCH USED

In the present investigation, a 6" (150 mm) lens-type polariscope made by Jessop-Leach has been used (Fig. 2.3.1). It has a 250 w high-pressure mercury lamp for monochromatic light and a 100 w projection lamp for white light. The electric supply to the mercury lamp contains a choke to minimise voltage fluctuations.

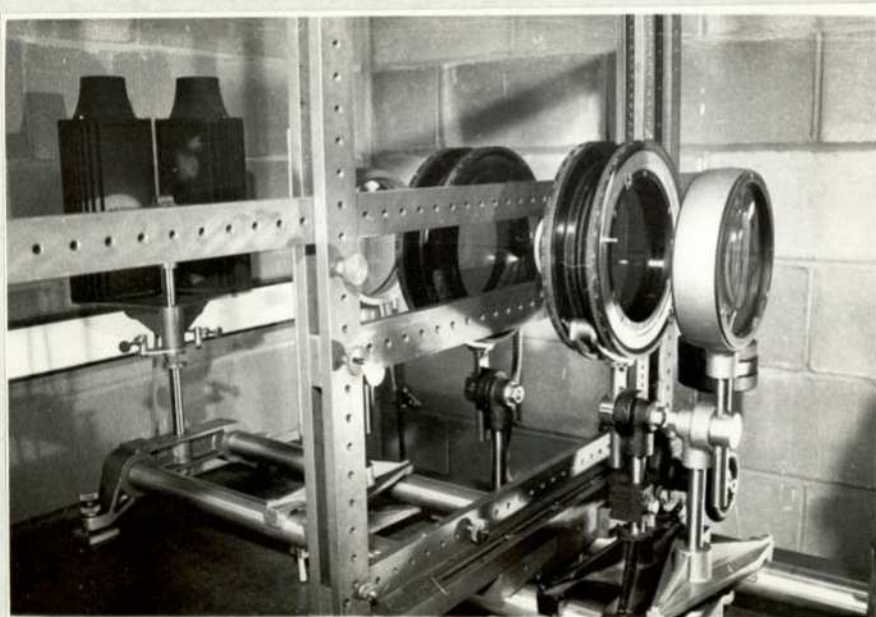
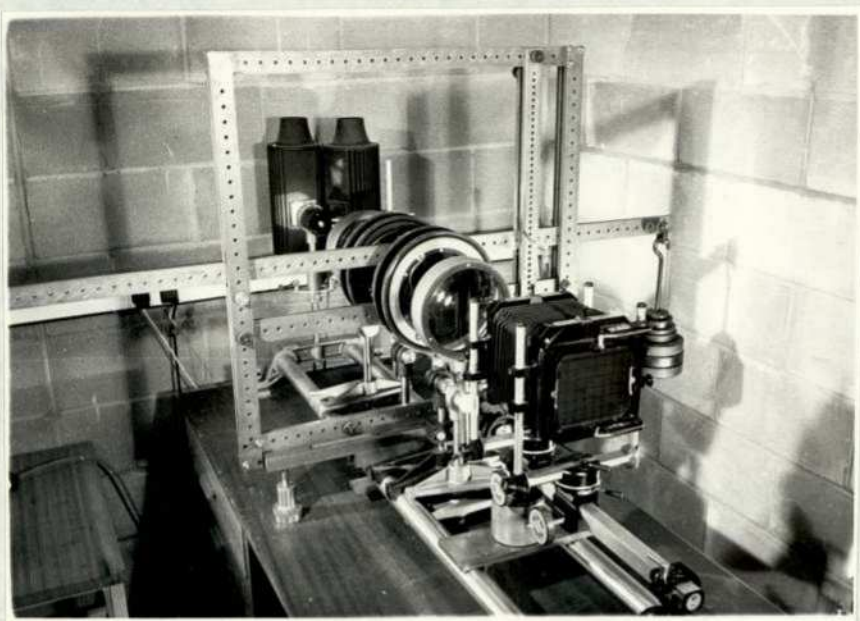


Fig. 2.3.1

6 in. Jessop-Leach Bench

An Ilford filter no. 807 was used in the earlier experiments and was later combined with a Kodak 77A filter. The polarisers, quarter-wave plates and field lenses are 6" (150 mm) in diameter.

The first field lens is placed at a distance equal to its focal length $f_1 = 18"$ (457 mm) from the aperture of the condenser housing of the lamp. The camera lens is placed in the focal plane of the second field lens with $f_2 = 8"$ (203 mm).

A universal loading frame for tensile, compressive and flexural loading by dead weights and levers is provided.

2.3.1 The Monochromatic Light Source

A compact 250 W Osram-G.E.C. high-pressure mercury lamp of type ME/D has been used in the bench. The characteristics of high pressure mercury vapour lamps have been described by Elenbaas (1965) and their photographic properties by Bourne (1943). According to the manufacturer's catalogue (Osram - G.E.C., 1969) the bulb operates at a pressure of 20 atm (2MN/m^2) and the small arc length and width of 3.75 and 2.5 mm provide the nearest practical approach to a point source. The 200-250 V A.C. mains supply contains a series choke and, if required, a voltage stabilizer can be inserted between mains and choke.

The relative spectral energy curve of the lamp used is shown in Fig. 2.3.2. The visible region contains predominant spectral lines at $4047/8 \text{ \AA}$ (Purple), $4358 \text{ \AA}$ (Blue), $5461 \text{ \AA}$ (Green) and $5770/5791 \text{ \AA}$ (Yellow), all with a half bandwidth of less than 1% of the central wavelength. These lines are superimposed on a continuous spectrum of relatively small energy. Elenbaas has shown that the spectral lines are broadened under high mercury vapour pressure.

2.3.2 Filters

The transmission characteristics of the filters used are given in Table 2.3.1. and their values in the neighbourhood of the green line are plotted in Fig. 2.3.3. It is seen that the combination of filters no. 807 and 77A reduces the band of emergent light to about $300 \text{ \AA}$ ($5300 - 5600 \text{ \AA}$). At the wavelength of $5461 \text{ \AA}$ the transmission is 10.3% for 807, 64% for 77A, and 6.6% for 807 and 77A combined.

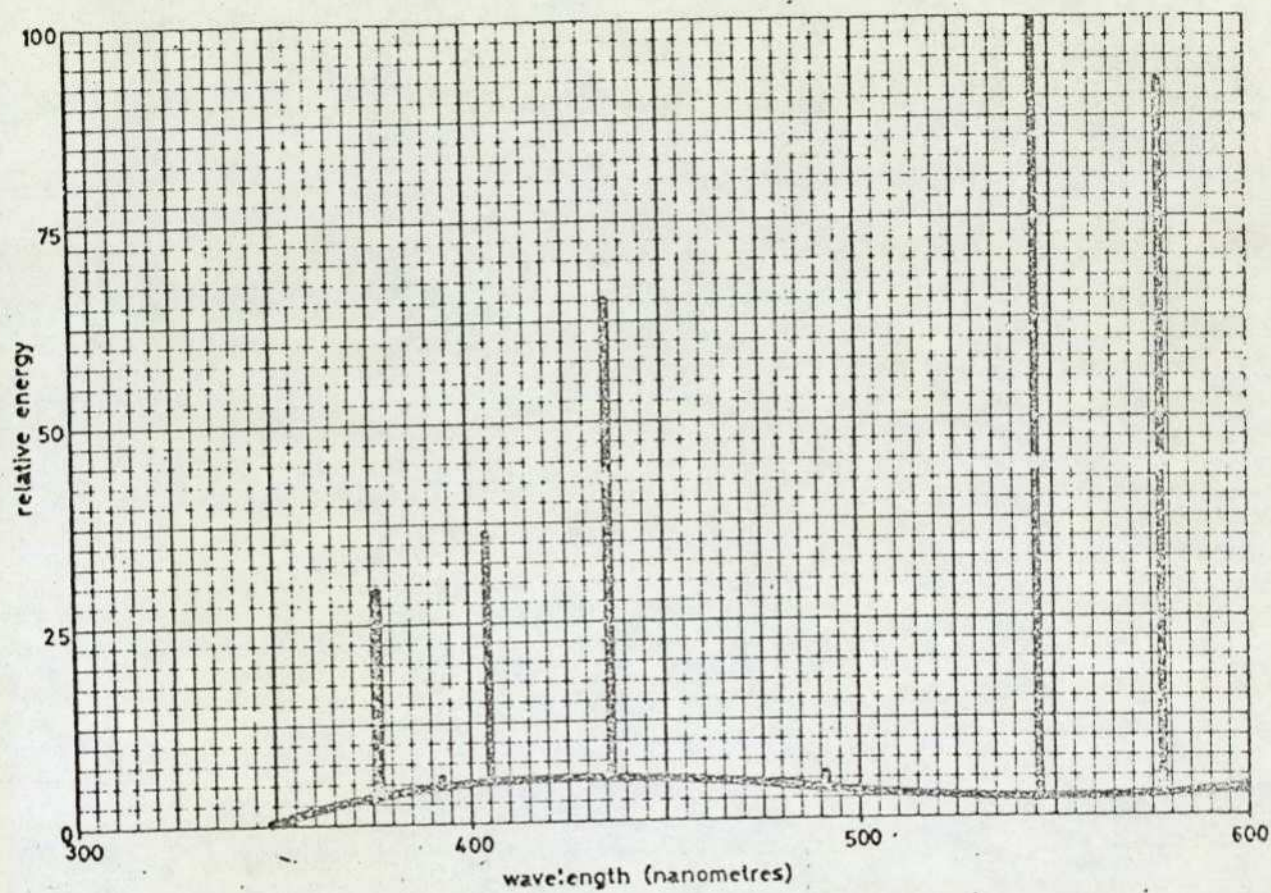


Fig. 2.3.2 Typical spectral energy curve for ME/D lamp.

Table 2.3.1Percent Transmission of Filters at Different Wavelengths

Wavelength Å	5100	5200	5300	5400	5500	5600	5700	6000
Ilford 807	1.8	15	22	16	6.9	2.2	0.5	-
Kodak 77A	0.1	5.35	1.90	35.0	71.8	63.1	-	1.6
807 + 77A	.0018	0.8	0.42	5.6	4.95	1.39	-	-

Wavelength Å	6100	6200	6300	6400	6500	6600	6700	6800
Ilford 807	-	-	-	-	-	-	-	-
Kodak 77A	32.1	78.0	79.5	86.5	89.2	89.0	79.5	62.5
807 + 77A	-	-	-	-	-	-	-	-

Wavelength Å	6900	7000	7100	7200
Ilford 807	-	0.4	6.3	25
Kodak 77A	62.4	83.0	-	-
807 + 77A		0.33	-	-

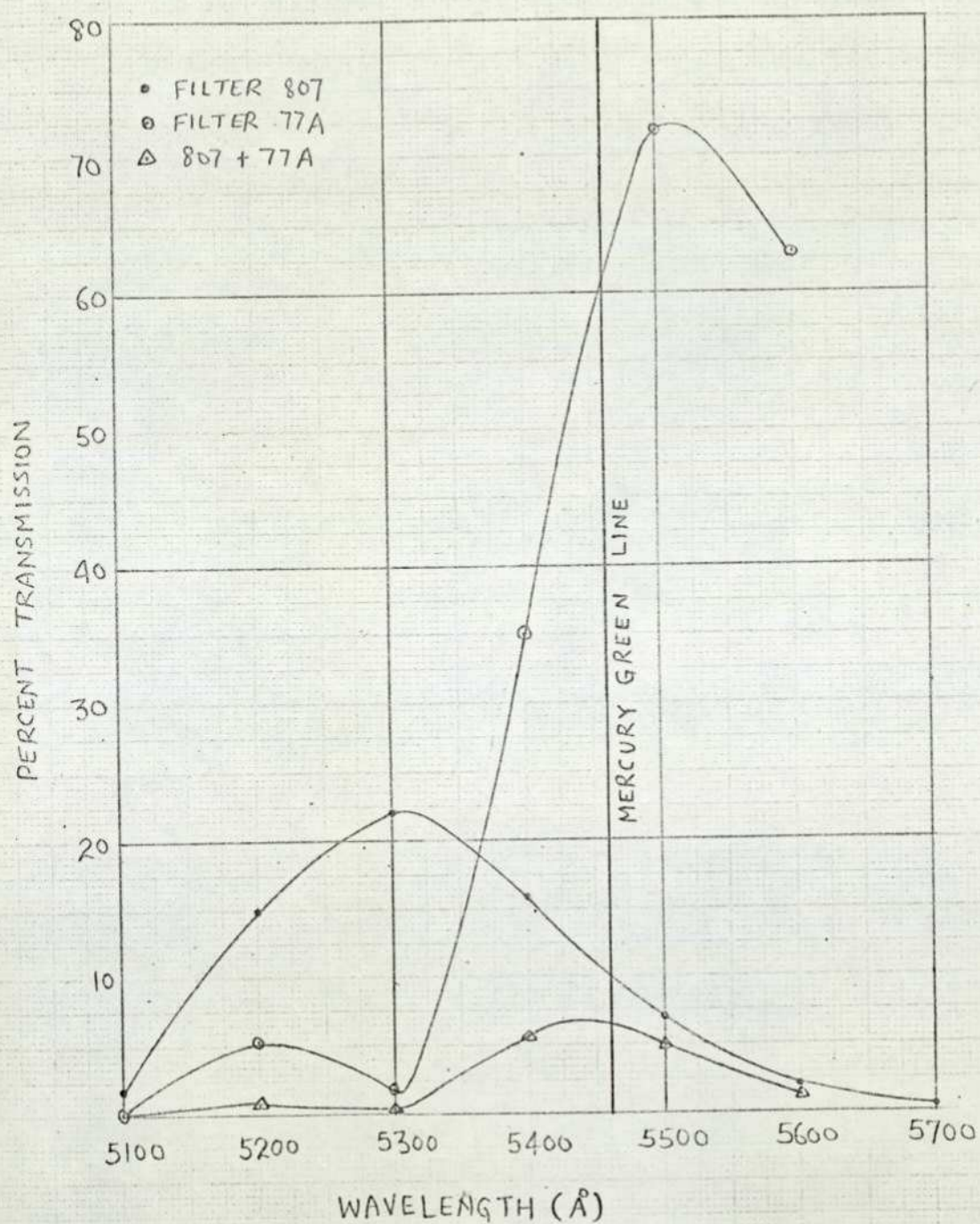


Fig. 2.3.3 Percent transmission of filters.

The yellow lines of the mercury lamp are suppressed almost completely.

Assuming that the relative spectral energy emitted by the lamp at 5461 Å is 100, the relative energy transmitted through the filters at other wavelengths can be computed in accordance with Figs. 2.3.2 and 2.3.3 and is given in Table 2.3.2. The improvement in the light spectrum obtained by the combination of 807 and 77A filters is evident.

Table 2.3.2.

Relative spectral energy of mercury vapour lamp with different filter.

Wavelength Å	5461	5100	5200	5300	5400	5500	5600	5700
Filter 807	100	0.44	3.64	5.34	3.88	1.67	0.53	0.12
Filter 77A	100	0.004	0.21	0.74	1.37	2.80	2.46	-
807 + 77A	100	-	0.30	0.16	2.12	1.87	0.53	-

A reasonably monochromatic light of narrow band width has been obtained.

2.3.3 Camera

The 125 x 100 mm De Vere Devon technical camera is mounted on a monorail. To allow multi-directional adjustments, both the lens and film ends are supported on vertical and horizontal rails. The two ends of the camera are connected by standard bellows so that the distance between lens and film can be varied. A rotating springback supports a 125 x 100 mm film or plate holder. A ground glass permits to view and focus the image by moving the film end with respect to the lens end.

A f 5.6/150 mm Symmar lens is used in the camera. The opening of the diaphragm (or aperture) can be varied from f 5.6 to f45 to allow control of the light reaching the film. The shutter mechanism provides exposures from 1 sec. to 1/400 sec.

2.4 INTENSITY EQUATIONS OF THE CIRCULAR POLARISCOPE

The intensity distribution of the light emerging from the analyser of a polariscope depends on the orientation of its various elements, on the stress field in the model and on the maximum intensity transmitted through the polariscope. Mindlin (1937) and Jerrard (1948) have given the most general equation of the polariscope assuming arbitrary orientation for polariser, analyser and wave plates and general values of retardations produced by wave plates and model. The equations for particular cases can be derived from the general equation which also helps in the formulation of an error analysis.

In a dark field circular polariscope, intensity distribution I_d of light incident on the camera film in a region covered by the model is given by

$$I_d = I_o \sin^2 \frac{\Delta_d}{2} = I_o \sin^2 \frac{\pi r_d}{\lambda} = I_o \sin^2 \pi N_d \quad (2.4.1)$$

where

- I_o = maximum intensity of light incident upon the film (image plane of the model) in the region covered by the model,
- Δ_d = phase difference,
- r_d = retardation or linear phase difference,
- N_d = relative retardation or fringe order,
- λ = wavelength of light used.

In literature, Δ has also been called relative phase difference, relative phase retardation, angular phase difference or phase retardation.

For a bright field circular polariscope the equation is

$$I_b = I_o \cos^2 \frac{\Delta_b}{2} = I_o \cos^2 \frac{\pi r_b}{\lambda} = I_o \cos^2 \pi N_b \quad (2.4.2)$$

The value of I_o in dark and bright field set-up is expected to be equal because the only difference is a rotation by 90° of the analyser.

The equations (2.4.1) and (2.4.2) assume

- (i) complete extinction of light by crossed polariser and analyser,
- (ii) no reflection by the various elements of polariscope,
- (iii) no differential absorption over the field.

If these assumptions are not satisfied, the equation (2.4.1) can be modified to

$$I = I_1 + I_0 \sin^2 \pi N_d \quad (2.4.3)$$

where,

I_1 is the background intensity of the crossed polariscope, i.e. the uniform intensity obtained in the dark set-up in the region of the unloaded model.

It is likely that I_1 is not constant over the field but varies over the area because of differential absorption, internal reflection or selective incomplete extinction. A constant value of I_1 can easily be taken into account in calibration but a varying value of I_1 is detrimental to all photoelastic work. Its effects and remedies will be discussed in Chapter 4.

Errors may arise if (i) the quarter-wave plates do not give retardations equal to one-fourth of the wavelength of light used, (ii) the monochromatic light has a wavelength different from that for which the quarter-wave plates have been matched, and (iii) white light is used.

If the two quarter-wave plates are identical but not accurate for the monochromatic light used, the intensity is given by (Baud, 1931)

$$I = I_0 (1 - \sin^2 2\theta \cos^2 \Delta_0) \sin^2 \frac{\Delta_d}{2} \quad (2.4.4)$$

where,

Δ_0 is the phase retardation of the quarter-wave plates and θ is the isoclinic parameter.

Mindlin (1937a) has studied the distortion in the isochromatic fringe pattern when the two quarter-wave plates are not identical.

2.5 MATERIAL AND MODEL FRINGE CONSTANTS

The constants relating the optical effect (fringe order) to the stresses (principal stress difference) in the model are determined by calibration.

Let

- Δ = relative phase difference introduced by the model
- N = relative retardation or fringe order expressed in fraction or multiples of the wavelength λ of the light
- $\sigma_1 - \sigma_2$ = principal stress difference at a point in the model
- n_1, n_2 = indices of refraction of the two principal planes at a point of the model
- r = linear phase difference = $N\lambda$
- C = relative stress-optic co-efficient
- t = thickness of the model
- F = model fringe constant based on the principal stress difference, psi/fringe
- f = material fringe constant based on the principal stress difference, psi-in/fringe

The photoelastic equations then are

$$\Delta = 2\pi N = \frac{2\pi t}{\lambda} (n_1 - n_2) \quad (2.5.1)$$

$$C = \frac{n_1 - n_2}{\sigma_1 - \sigma_2} = \frac{N\lambda}{t(\sigma_1 - \sigma_2)} \quad (2.5.2)$$

$$F = \frac{\sigma_1 - \sigma_2}{N} = \frac{\lambda}{Ct} \quad (2.5.3)$$

$$f = F \cdot t = \frac{\lambda}{C} = \frac{(\sigma_1 - \sigma_2)t}{N} \quad (2.5.4)$$

Sometimes F and f are defined on the basis of maximum shear stress; their values will then be one half of those obtained on the basis of principal stress difference.

2.5 PROPERTIES OF CR-39

Columbia resin, CR-39, is the model material used in the present investigation. It is an allyl type of thermosetting plastics material. Its properties have been investigated by Coolidge (1948) and Clark (1956) and are summarised in the following:

(i) CR-39 has a colourless, highly transparent, glass-like appearance. The surfaces of sheets supplied are highly polished and need not be machined.

(ii) High initial (residual) stresses exist in the plane of the sheet, i.e. through the thickness of the plate, and cannot be eliminated by annealing. They do not affect two-dimensional work if the plate is kept normal to the incident light but exclude oblique incidence observations.

(iii) CR-39 is almost free from time edge effects and does not require an oven for storage.

(iv) CR-39 is brittle and requires considerable care in machining to avoid chipping at the edges. However, it has been found that edges can be machined without chipping if the model is sandwiched between scrap CR-39 or Araldite sheets covering the edge to be machined.

(v) CR-39 creeps slightly under load but the creep is linear so that fringe orders are proportional to stress at any given time after loading (Durelli, Lake and Phillips, 1952).

Properties of CR-39 reported in literature are:

Refractive index	=	1.505
f	=	83.5 - 99 psi-in/fringe
E	=	$(2.0 - 3.76) \times 10^5$ psi
ν	=	0.42 - 0.44
Tensile strength	=	6000 - 7000 psi

2.7 PHOTOGRAPHY IN PHOTOELASTICITY

2.7.1 Photographic Fundamentals

The discussion in this section will be confined to black and white photography because colour photography is no longer necessary for photoelastic applications. The purpose of photography is to obtain a permanent record of intensity (or brightness) variations caused by the loaded model. A camera is required to form the image of the subject, and a light sensitive film or plate records the image.

The amount of light energy reaching the film is controlled by (i) the diaphragm or aperture opening which controls the amount of light and (ii) the shutter speed which controls the time for access of light to the film. The aperture is usually indicated by

the f number which is the ratio of the focal length of the camera lens to the diameter of lens opening determined by the aperture. For a focal length of 150 mm, a f number of 8 indicates a lens opening of $150/8 = 18.75$ mm in diameter. The shutter speed is usually given in fractions of a second so that a shutter speed of 100 indicates an exposure of $1/100$ second.

The product of light intensity and exposure time is a measure of the light energy incident on the film and is called "exposure". If intensity varies from point to point in an image, the exposure varies accordingly because each point is exposed for the same time. A latent image is then formed on the film and made visible by development.

The developed film contains a distribution of fine silver particles and their accu^{mu}lation is known as density. Because of their minute size, these silver particles appear black. The density formed at a particular point on the film is a non-linear function of the exposure received by the point and this function is defined by a characteristic curve which depends on the type of film and on the development. The density D is usually expressed as the logarithm of the opacity (O) which is the ratio of the intensity of incident light (I_i) to that of transmitted light (I_t) and the reciprocal of the transmission T :

$$(O) = \frac{I_i}{I_t} = \frac{1}{T} \quad (2.7.1)$$

$$D = \log (O) = -\log T \quad (2.7.2)$$

A typical characteristic curve of a film or plate which relates density to the logarithm of exposure for given or assumed conditions of development is shown in Fig. 2.7.1. This curve is also known as $D/\log E$ curve or $H \& D$ (Hurtford and Druffield) curve. It has a toe AB, a straight line portion BC and a shoulder CD.

At very small exposures below the exposure inertia or threshold exposure E_0 , the density, known as the fog density D_0 , is not affected by variations in exposure. If no well defined exposure inertia point exists, the point of intersection of the straight lines AB and BC (Fig. 2.7.1) may serve as a suitable

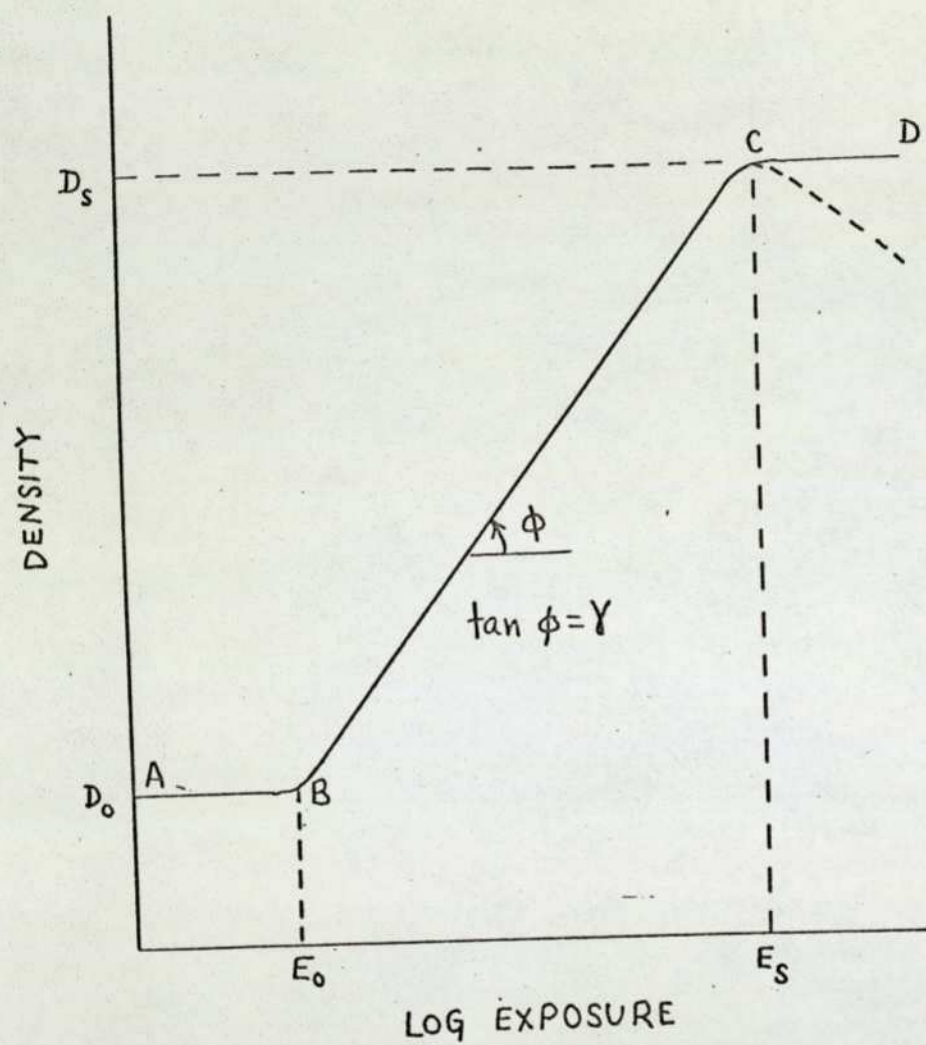


Fig. 2.7.1 Characteristic curve of films.

definition for E_0 . At exposures greater than E_0 , the density varies linearly with variations in log exposure.

The rate of change of density or the slope of the straight line is called the contrast index or γ number of the film. A film with a high γ is known as a high-contrast film and one with low γ is called a low-contrast film. The γ value of a film is not constant but varies with the development time used.

The maximum or saturation density D_s is obtained for an exposure E_s which may be called saturation exposure. The density once again remains constant at D_s or changes only by small amounts with increase in exposure beyond E_s .

In some films, CD may actually have a negative slope as shown by dotted lines in Fig. 2.7.1. so that density decreases with increasing exposure. This characteristic has been used to obtain a direct positive image in reversal films.

The interval $(E_s - E_0)$ is usually called the range of exposure of the film. At an arbitrary point in this range, the exposure E is related to the corresponding density D by

$$\gamma = \frac{D - D_0}{\log(E / E_0)}$$

or

$$D = D_0 + \gamma \log (E / E_0) \quad E_0 \leq E \leq E_s \quad (2.7.3)$$

Outside the range we have

$$D = D_0 \quad E \leq E_0 \quad (2.7.4)$$

$$D \simeq D_s \quad E \geq E_s \quad (2.7.5)$$

2.7.2 Films

The films or plates used in photography may be classified on the basis of their colour response into (a) Orthochromatic films sensitive to ultra-violet, blue and green, and (b) panchromatic films sensitive to all visible wavelengths. Less common films exist that are sensitive to blue colour only.

In addition, films are characterised by their contrast index. A panchromatic as well as an orthochromatic film may have a moderate, medium, high or extremely high contrast (or γ -value). Sometimes, films are simply classified into low and high-contrast films.

A third basis for classification of films is speed of the film expressed in the ASA or DIN number of the film. A high number indicates a high speed film requiring for the same light intensity a smaller exposure time than a low speed film. The characteristic curve of a high speed film is situated to the left of that of a low speed film and has a lower value of E_0 .

On a negative the bright areas of the object appear dark. To get the correct brightness distribution of the object, the negative must be printed on yet another sensitive material. Alternatively, a reversal film can be used to obtain a positive image directly on the film. A combined positive and negative image with intermediate transparent lines (equidensity lines) can be obtained in one step by normal development in a special type of film known as Agfacontour and described in Chapter 5.

2.7.3. Films with $\gamma = 1$

Films with $\gamma = 1$ are required for some specialised applications (Chiang and Khetan, 1974 ; Talukder, 1974). In practice it is rather difficult to obtain $\gamma = 1$ correctly, because even with a standard development process, variations in gamma value may occur in different films for the same pack.

The following films have been suggested as a result of an enquiry for film having the following three characteristics:-

(i) $\gamma = 1$, (ii) very low fog density and (iii) sensitive to green light.

Kodak: Kodak separation negative film 4131, type 1. Developed in DK 50 solution (1 + 2) for 6 minutes with continuous agitation. Fog density = 0.03.

Ilford: Commercial ortho film as used here and described in Section 2.7.4.

Agfa Gavaert: Agfapan 25 Professional developed in Studional for 6 minutes.

Chiang and Khetan (1974) have used Kodak Plus-X Pan Professional film developed in Kodak D-76 developer (1 + 1) for 22 minutes. This film (see Section FM-36 of Kodak Data Book on Applied Photography) has basically $\gamma = 0.5$ and an excessively long development time has been used by Chiang and Khetan to obtain $\gamma = 1$.

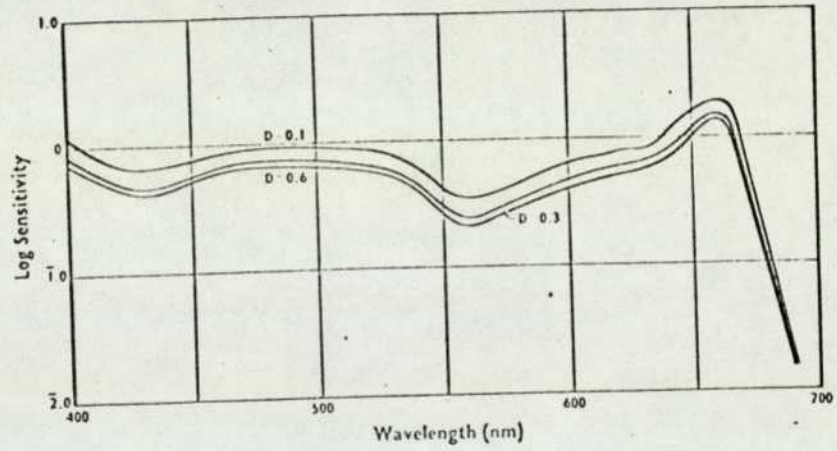
2.7.4 Characteristics of Films Used

In addition to Agfacontour (described in Chapter 5), the following types of films have been used:

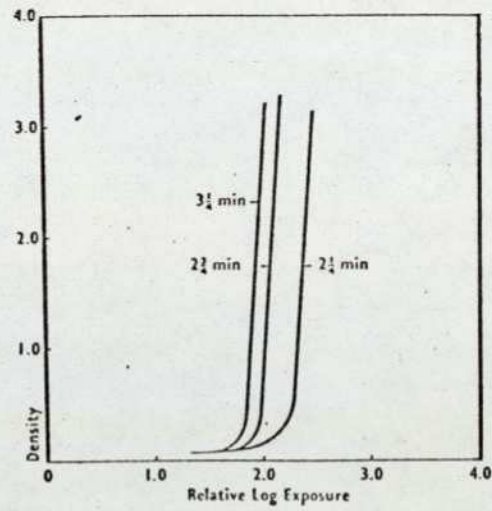
1. Kodalith Pan Film 2568: This is a fast panchromatic film of extremely high contrast. Its 'Estar' base provides a very high degree of dimensional stability. The film must be developed in Kodalith Super developer in order to obtain the extreme contrast. Since the film is panchromatic, development must be carried out in total darkness. Its spectral sensitivity and characteristic curve are shown in Fig. 2.7.2.
2. Kodalith Ortho Film 2556, Type 3: This is similar to Kodalith pan film except that it is orthochromatic and slightly slower. Its development may be carried out under a deep red filter. Spectral sensitivity and characteristic curve are shown in Fig. 2.7.3.
3. Ilford Commercial Ortho Film: This is a very fast orthochromatic film of medium contrast and can be developed to $\gamma = 1$. For this purpose, Ilford recommend dish development in Ilford Phenisol liquid developer solution of 1 + 9 and 20°C with out of phase agitation for 2½ minutes. This produces a low fog level of 0.08. A drop of 30 seconds in development time reduces the gamma value to 0.92 and a rise of 30 seconds increases it to 1.13. Fig. 2.7.4 shows the spectral sensitivity & characteristic curve.

2.7.5 Processing

The processing of an exposed film involves development, rinsing, fixing, washing and drying. A chemical developer transforms the latent image of the exposed film into a visible image with a given distribution of densities. If the silver forming the densities is derived from the emulsion, the process is known as

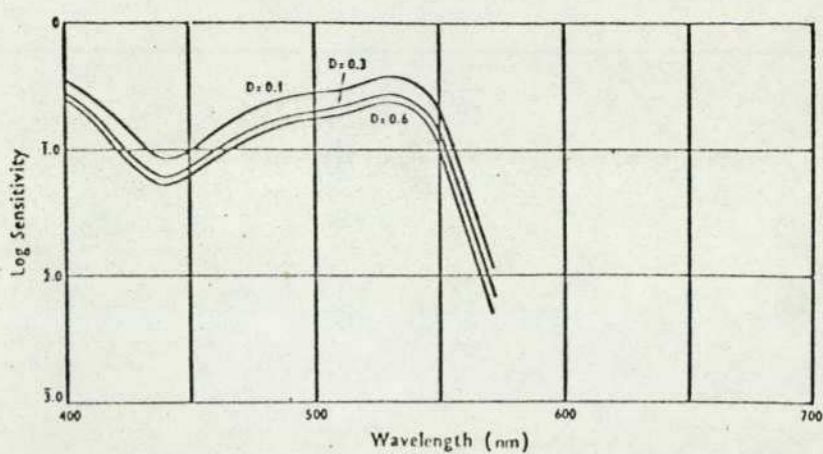


(a) Spectral sensitivity curve.

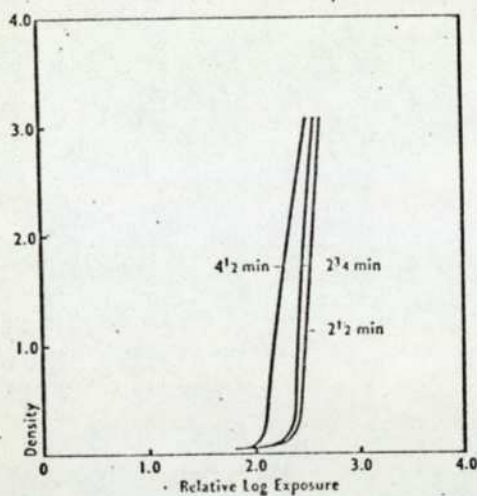


(b) Characteristic curve (super developer at 20°C).

Fig. 2.7.2 Kodalith pan film 2568.



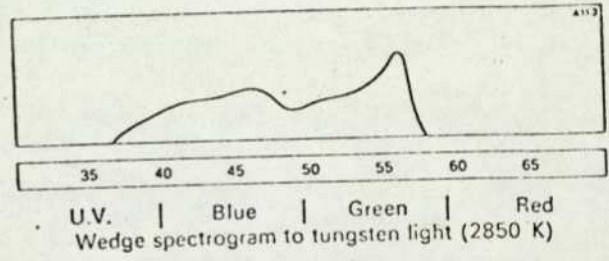
(a) Spectral sensitivity curve.



(b) Characteristic curve (super developer at 20°C).

Fig. 2.7.3 Kodalith ortho film 2556, type 3.

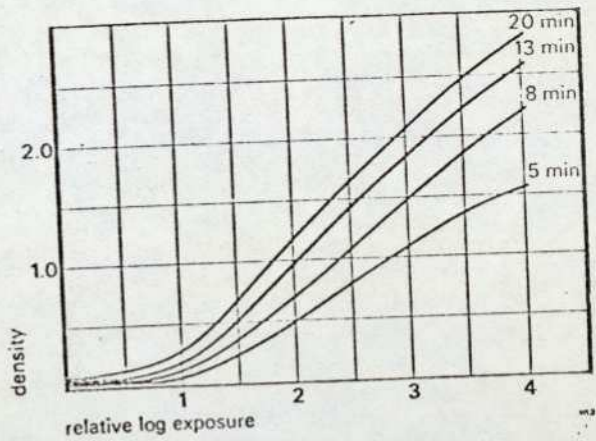
Wedge Spectrogram



(a) Spectral sensitivity curve.

Sensitometric Curves

Developed in ID-2 (1+2)
at 20°C (68°F) with
continuous agitation.



(b) Characteristic curve.

Fig. 2.7.4 Ilford commercial ortho film.

chemical development, and if it is provided by the developer, the process is a physical development.

The film is placed in a dish containing a certain volume of appropriate developer at a constant temperature of 20°C and is kept in the developer for a certain development time during which the developer is agitated in a particular manner. The development of the silver image is checked by rinsing the film in either water or diluted acetic acid.

The film is then fixed in a chemical solution known as fixer which removes the unused sensitive material of the film in order to make the image stable to light. The next process of washing removes unwanted soluble chemicals from the film. Lastly the film is dried either naturally or in hot air.

The densities formed in the image depend not only on the exposure, as shown in Fig. 2.7.1, but also on processing and particularly development. Two films exposed identically will give different density distributions if different development parameters are used. The main parameters are:-

1. Development time
2. Temperature of the developer
3. Mode of Agitation.

The effect of the development time is usually indicated by the manufacturer and a normal development time is recommended for a given film and developer. If the development time is exceeded, the film is overdeveloped and more densities than normal are formed. Similarly underdevelopment gives less density than normally expected. The time of development also influences the gamma value of the developed film and curves showing variations of gamma with development time are given by the manufacturer.

A development temperature of 20°C is recommended. With a different temperature the developed density changes even if the recommended development time is used. A small change in temperature, say $\pm 1^{\circ}\text{C}$, has a negligible effect on the final result.

A film or developer should be agitated to obtain a uniform development of the film. The mode of agitation, whether intermittent,

continuous or none, also affects the final densities formed. For consistent and accurate work, the various parameters of manual processing have been standardised and are described in ASA no. PH 4.1 - 1962 entitled "Manual Processing of Black-and-White Photographic Films and Plates".

Further information on photography is given by Encyclopedia (1969), Engel (1969), Ilford (1958), Langford (1972).

2.7.6 Precautions in Photographic Work

Keen (1973) has suggested that the following precautions be taken to obtain reproducible results in photographic work as used here:

1. Fresh developer is to be used for each development.
2. Only the developer recommended by the manufacturer is to be used at controlled temperature and time.
3. Development is to be carried out immediately after exposure but a constant interval of time between exposure and processing is desirable.

Keen also describes a number of difficulties in obtaining a distortion free print from a negative and has suggested that exposure inertia is influenced by a number of factors and may not be the same for two batches of the same film.

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CHAPTER 3

THE BOUNDARY LINE METHOD OF FULL FIELD

FRACTIONAL FRINGE ORDER MEASUREMENT

3.1 THEORY

The intensity of monochromatic light transmitted through any point of a model in a circular polariscope is given by

$$I_d = I_o \sin^2 \frac{\Delta_d}{2} = I_o \sin^2 \pi N_d \quad (2.4.1)$$

$$I_b = I_o \cos^2 \frac{\Delta_d}{2} = I_o \cos^2 \pi N_b \quad (2.4.2)$$

Photography converts the intensity distribution of the isochromatic pattern into a density distribution on the photographic film in accordance with density-exposure relationship given by the H-D curve, Fig. 2.7.1.

The relative phase difference produced in the loaded model varies the light intensity in the image plane from point to point so that with a fixed exposure time t different points on the film receive different exposures E . No visible density variation is developed in regions where the exposure is less than exposure inertia, and these regions appear uniformly bright on the developed negative, dark on the final print, so that it is usually difficult to pinpoint on a negative or print the exact loci of absolute zero intensity.

Regions where the exposure E (product of intensity and exposure time) exceeds E_o appear increasingly dark on the developed negative and increasingly bright on the print. If either the exposure time is so long or the exposure latitude ($E_s - E_o$) so small that saturation density is reached, then it may be difficult to pinpoint on the film the loci of absolute maximum intensity. Moreover, according to the sine square law of Equations (2.4.1) and (2.4.2), small changes in relative retardation N_b or N_d are difficult to detect in regions of maximum or minimum intensity.

These aspects of photoelastic photography are shown diagrammatically in Fig. 3.1.1 where the curve (a) represents Equation (2.4.1) relating intensity I_d to fringe order N in a dark field circular polariscope. For a given exposure time t , curve (a) also represents exposure to a certain scale and for a particular film let E_0 be the exposure inertia and E_s the saturation exposure as shown. As a general rule, if the exposure time t is increased, the ordinates of E_0 and E_s in Fig. 3.1.1 will be reduced, and if t is reduced, the ordinates of E_0 and E_s will be increased.

For the fringe orders falling within the intervals Q_5Q_4 , Q_1OP_1 and P_4P_5 , the exposure is less than E_0 and the corresponding regions will appear bright in the developed negative whilst for the fringe orders between Q_1 and Q_4 or between P_1 and P_4 the exposure exceeds E_0 and the corresponding regions will be darkened to a varying extent on the developed negative. In the intervals between Q_3 and Q_2 and between P_2 and P_3 , the exposure exceeds E_s and the corresponding regions will appear uniformly dark on the developed negative.

The resultant density distribution D is shown by the density curve (b) of Fig. 3.1.1. Thus density variations are obtained only over small regions, namely those corresponding to the intervals Q_4Q_3 , Q_2Q_1 , P_1P_2 and P_3P_4 , and the density remains constant and equal to D_0 or D_s over fairly large areas. If the film and exposure time are such that E_s is beyond the maximum given exposure, density variations are obtained over large areas because the intervals Q_3Q_2 and P_2P_3 will then not show a flattening of the density curve (b).

Normally it is not easy to eliminate the flat parts of the density curve in the intervals where it has the value D_0 . However these intervals can be reduced by an increased exposure time and in some cases eliminated altogether by an increased exposure time accompanied by a small amount of background intensity so that the exposure retains a small finite value even at the minimum where the fringe order is N or $N + 1$, or $N - 1$, etc. From the

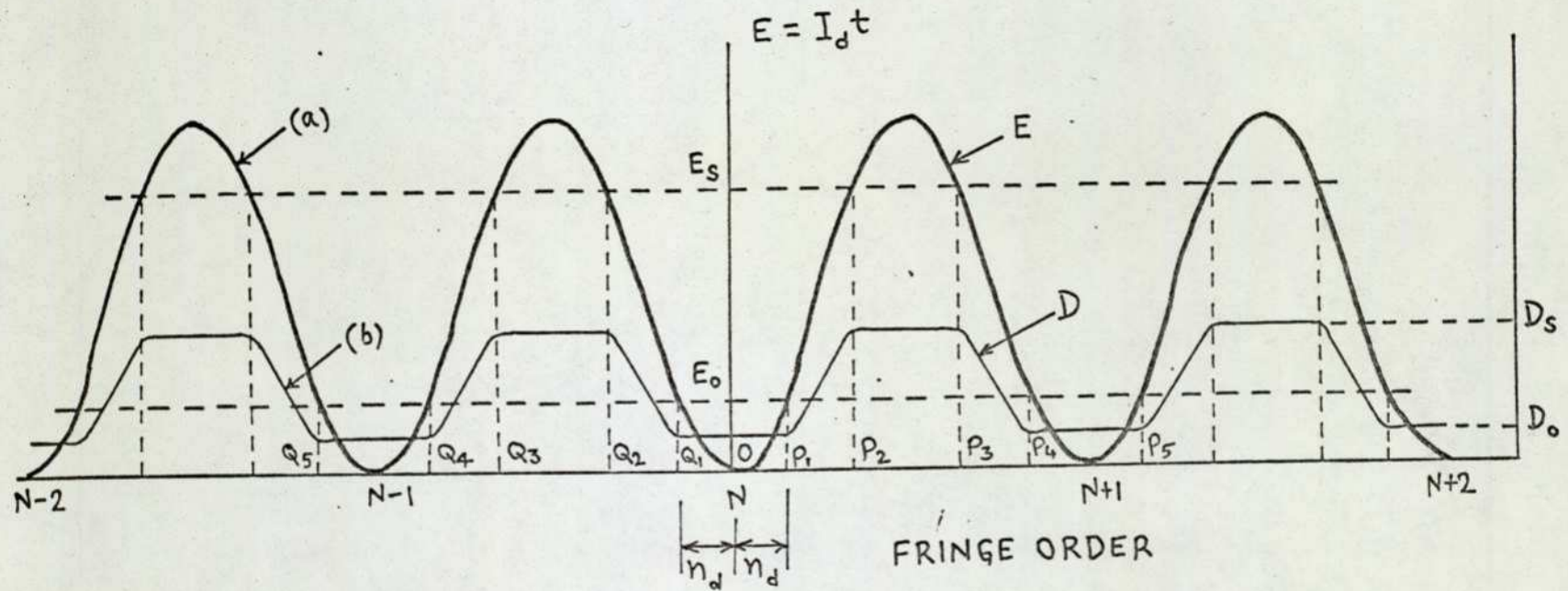


Fig. 3.1.1 Intensity, exposure and density variations with fringe order.

I_d = Intensity, E = Exposure, t = Exposure time,
 D = Density, N = Fringe order, n_d = Fractional
fringe order in the dark field.

density curve it is obvious that small changes in fringe order will be difficult to detect in the regions of maximum or minimum intensity.

Photographs of isochromatic fringe patterns thus normally contain dark bands, bright bands, and intermediate grey tones depending on the value of the contrast ratio and on the development process. The intermediate grey tones can be eliminated almost completely if a high-contrast film (very high γ) is processed with a suitable developer. The developed densities then show an almost abrupt transition from the fog density D_0 to the maximum density D_s where the exposure just exceeds E_0 , and the intervals Q_4Q_3 , Q_2Q_1 , P_1P_2 and P_3P_4 then are very small. The photographs of fringe patterns on such films thus show dark and bright bands with clearly defined boundary lines along which

$$E_0 = I_d t_d = I_0 t_d \sin^2 \pi N_d \quad (3.1.1)$$

$$E_0 = I_b t_b = I_0 t_b \cos^2 \pi N_b \quad (3.1.2)$$

for dark and bright field set-up respectively.

Since E_0 , I_0 , t_b & t_d have constant values for any photograph taken, developed and printed under controlled conditions, N_d and N_b must each have a constant value for all points of any given boundary line. Each boundary line is thus a locus of constant (fractional) fringe order of a value determined by the abscissae $Q_4 (\approx Q_3)$, $Q_1 (\approx Q_2)$, $P_1 (\approx P_2)$ and $P_4 (\approx P_3)$ in Fig. 3.1.1. Since these values are controlled by the photographic process, the application of this technique may be called a 'controlled density method'.

If the same exposure time $t_b = t_d$ is chosen for bright field and dark field photographs, the values of N_b and N_d are related by

$$\sin \pi N_d = \cos \pi N_b$$

$$\text{or } (N_b \pm N_d) = \pm m - \frac{1}{2} \quad (m = 1, 3, 5, \dots) \quad (3.1.3)$$

Equation (3.1.3) will be strictly true only if I_0 is the same for dark field and bright field setting of the polariscope.

In particular let the boundary line fringe orders in bright and dark field be counted as proper fractions n_b, n_d in the same sense from the same integer or half-order fringe N so that $N_b = N \pm n_b$ and $N_d = N \pm n_d$. Since $0 < n_b < \frac{1}{2}$ and $0 < n_d < \frac{1}{2}$, Equation (3.1.3) then simplifies to

$$/ n_b + n_d / = \frac{1}{2} \quad (3.1.4)$$

In classical photoelasticity, only the centre lines of the dark and bright bands are evaluated in terms of integral or half-order fringes. However, since under controlled photographic conditions each boundary line represent a constant fractional fringe order which can be determined by calibration, the photographed isochromatic pattern can also be evaluated from such boundary lines.

The positions of the boundary lines and their fractional orders can be altered by variation of any of the following three controlling parameters of the photographic process:

- (i) the camera exposure time,
- (ii) the maximum uniform light intensity in the image plane of the camera, and
- (iii) the processing parameters such as development time, development temperature and mode of agitation.

The product of parameters (i) and (ii) make up the exposure which can be widely varied, whilst changes in processing parameters are limited by the need to have a consistent and convenient processing technique. Hence it is advantageous to vary either the camera exposure time or, by means of given filters, the maximum light intensity and to keep the remaining two parameters constant. In the present work, the camera exposure time is varied because this is easy, repeatable and convenient. An unlimited variation in exposure time is not possible but usually a range of exposure times is available for any film and variations within this range will produce shifts in the boundary lines.

Two boundary lines always appear between centres of any two neighbouring dark (or bright) fringe bands, for instance at the

points P_1 and P_4 between the fringe orders N and $(N + 1)$ in Fig.

3.1.1. Alternatively, two boundary lines at P_1 and Q_1 are obtained symmetrically on either side of an integral order N and separated from N by fractional order differences $\pm n_d$ of a magnitude $|n_d|$ depending on the camera exposure. If m exposure times are used in repeated photographs, $2m$ different lines of fractional fringe order are thus obtained. This number is again doubled if a bright field is used in addition to the dark field.

A change in loading of the model alters the positions of all isochromatic lines, except those of zero order, and thus also the positions of the boundary lines. However, for constant photographic parameters the fractional orders of the boundary lines are not thereby changed. Such a change therefore allows plotting of a single curve or pattern of fringe order per unit load. If it is used in combination with the described changes in exposure time, experimental points or boundary lines of fractional fringe orders can be obtained well distributed over the entire range of fringe orders appearing in the photographs except in the immediate vicinity of any stationary zero-order fringe, where other methods also find difficulties. The limitations inherent in the intensity law of Equations (2.4.1) and (2.4.2) and in the exposure inertia and saturation exposure of the film can thus be eliminated almost entirely. This procedure of load variation will not normally be possible with slices of three-dimensional model.

It has been assumed here that I_0 can be kept constant over the whole field of the model. This is true only if the model is free from residual stresses, mottle and edge effects and if the polariscope gives a uniform illumination. However, ordinary photoelastic work requiring numerical evaluation and other methods of fractional fringe order measurement are also influenced by failure to ensure these conditions. The resultant defects can usually be identified by the irregularities they produce in the pattern. Nevertheless, selection of a model material as free as possible from defects is advisable.

On the other hand, the proposed method lends itself to the detection of such defects by photography under controlled conditions when the model is loaded or lightly loaded. If necessary, spurious fringe patterns may be subtracted from those obtained under the test loading and the effects thus eliminated from evaluation.

3.2 CALIBRATION ON A SEPARATE TEST PIECE

Theoretically, if the exposure inertia E_0 of the high-contrast film, the exposure time t , and the maximum light intensity I_0 at the image plane are known, the corresponding fractional fringe orders can be calculated from Equation (3.1.1) or (3.1.2). However, E_0 and I_0 are usually unknown and their experimental determination can be tedious. The characteristic curve given by the manufacturer is only a guide so that for accurate work E_0 must be determined under actual working conditions. A sensitive photometer is required for accurate determination of I_0 . Experimental calibration is therefore preferred especially as for evaluation in terms of stresses, the stress-optic constant must always be found by experimental calibration.

Any arrangement for which the principal stress difference is known from theory may serve as calibrating device. Photographs of the calibration fringe pattern are taken with the exposure time and under the photographic conditions to be used with the test model. The calibration model must be made of the same material and thickness as the test model so that the value of I_0 is the same for both cases. For a given exposure time, the boundary line passes through points of known principal stress difference in the calibration model so that their (fractional) fringe orders can be calibrated directly in terms of stress difference. For the same exposure times, the boundary lines in the fringe patterns of the test model and calibration model will represent the same fractional fringe orders.

A beam in pure bending with a linear stress distribution offers a particularly simple procedure for calibration. As shown in Fig. 3.2.1, let a_1 and a_2 be the widths of the dark and bright bands as seen on the negative of a dark-field fringe pattern, b_1

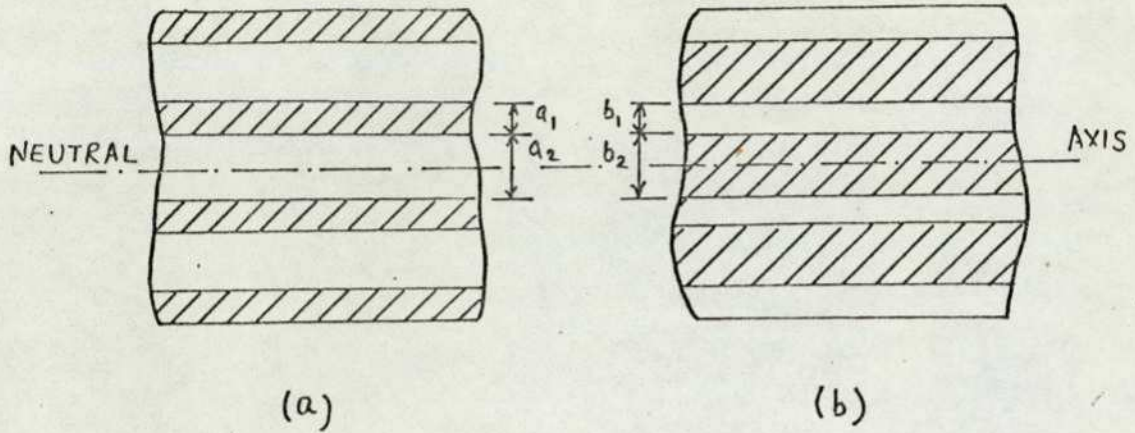


Fig. 3.2.1 Schematic representation of fringe pattern in a beam under pure bending as it appears on a photographic negative with (a) dark field and (b) bright field.

and b_2 the widths of the bright and dark bands in a corresponding negative of a bright-field pattern. Then the distances $(a_1 + a_2)$ and $(b_1 + b_2)$ correspond to one full fringe order and the partial fringe order can be derived from

$$n_d = \frac{a_2/2}{a_1 + a_2}, \quad n_b = \frac{b_2/2}{b_1 + b_2} \quad (3.2.1)$$

where n_d and n_b are counted from the common centre line of bands of width a_2 and b_2 respectively. If this centre line corresponds to a fringe of integral order N , as in Figure 3.2.1 where $N = 0$, then

$$N_b = N + n_b, \quad N_d = N + n_d \quad (3.2.2)$$

the sign being inferred from the general fringe pattern. Further, in a bright-field pattern, the centre lines of the dark bands on the negative or of the bright bands on the print can always be interpreted as integral fringe orders.

For the same exposure time $t_d = t_b$ in dark and bright fields, $b_1 = a_2$ and $b_2 = a_1$, so that $n_b + n_d = \frac{1}{2}$ as required by Equation (3.1.4).

Alternatively, n_b or n_d may be taken as the ratio of half the distance between two boundary lines surrounding the neutral axis to the distance between any two lines separated by one intermediate line. For greater accuracy, corresponding distances between any suitably chosen lines may be considered.

The described procedure requires separate calibration for each exposure time employed with the test model. However, calibration at a single known exposure time t_1 in a bright or dark field may also be used for any other exposure time t_2 (Meyer, Mehrotra and Talukder, 1973). Let the fractional fringe orders (with $N = 0$) for the exposure times of t_1 and t_2 be n_1 and n_2 respectively. Equation (3.1.1) or (3.1.2) leads to

$$t_1 \sin^2 \pi n_1 = t_2 \sin^2 \pi n_2 \quad (3.2.3)$$

$$\text{and} \quad t_1 \cos^2 \pi n_1 = t_2 \cos^2 \pi n_2 \quad (3.2.4)$$

in the dark or bright field respectively. Hence, if n_1 and t_1 are known, n_2 can be calculated for any other exposure time t_2 . In addition, the simple Equation (3.1.4) permits to derive for a given exposure time t from a known dark field fractional order the bright field fractional order, or vice versa.

Thus one single calibration photograph taken at a known exposure time under controlled conditions allows the determination of (i) material or model fringe value in the usual manner and (ii) the proper fractional order of the boundary lines by a simple measurement of two distances in the photograph. The fractional orders for other exposure times and for different background can

then be determined for similarly controlled optical and photographic conditions. The calibrated fractional fringe order must be determined very accurately if this fringe order is to be used to calculate fringe orders at different exposure times.

3.3 SELF CALIBRATION TECHNIQUES

In two-dimensional as well as three-dimensional photoelasticity, a special type of test model may be encountered or it may be possible to modify the model shape so that a part of the model itself can be used for calibration. This technique has been called self-calibration (Heywood, 1969 and Frocht, Guernsey and Landsberg, 1953).

Self-calibration involves here the determination of fractional fringe orders solely on the basis of a study of the boundary lines obtained in the test model. This will be useful in situations where only fringe orders are required (for stress-concentrations) or where the material fringe constant for the model under investigation is known beforehand and its separate determination is not contemplated. It can also be used for 3-dimensional slices where normally the material fringe constant is determined from a separate small model such as a $\frac{1}{4}$ " diameter circular piece. Self-calibration is also often useful in dynamic photoelasticity because of difficulties in arranging for a separate calibration model.

Self-calibration can be based on a knowledge of E_0 and I_0 (see Chapter 3.2) but practical difficulties oppose application of this particular method. Several other self-calibration techniques are described in the following sections:

3.3.1 Graphical Methods for a Given Photograph

3.3.1.1 Friedrich's Method

In connection with equidensity work in photoelasticity which is similar to the boundary line method, Friedrich (1974) has suggested the following technique which can be directly applied to the self-calibration of boundary lines: A cross-sectional reference line through the fringe pattern is considered. The points of intersection of the equidensities or boundary lines with this line are the data points for the abscissa in a plot. The ordinates in this plot are the integral fringe orders (for dark field set-up)

nearest to the boundary lines at the given points of intersection. The resultant set of data points is used to obtain two curves - one joining the points at which the actual fringe orders of the boundary lines are above the integral order and the other at which they are below the integral order. The difference in ordinate values between these two curves is equal to twice the required fractional fringe order n_d .

This procedure ignores the effect of high fringe gradients on the fringe spacings. If the calibrating line passes through areas of appreciable stress gradient, the extrapolated vertical distances may differ markedly from $2n_d$ in some places and the final average vertical distance may still show an appreciable error. Such errors will also appear in the averaging method by integration of the areas between the two curves, as recommended by Friedrich. The accuracy of the procedure also depends on the availability of a large number of data points which may not be available in some applications.

The difficulties associated with Friedrich's method are reduced in the two following alternative graphical procedures.

3.3.1.2 Consecutive Estimates

A calibration line through the photoelastic pattern is considered and the intersections of the boundary lines with the calibration line are plotted as abscissae of a curve the ordinates of which will represent fringe orders. From an estimate of the proximity of the boundary line to the integral fringe, a fractional fringe value n is assumed for the boundary line as a first approximation. Then the assumed fringe orders for various boundary lines are $N \pm n$ where N is an integer, and two separate curves, one for $N + n$ (upper curve) and another for $N - n$ (lower curve), can be plotted over the calibrating line as the abscissa. If the two curves are coincident, the value of the fractional fringe order n must implicitly have been assumed correctly because the fringe order then has a single and unique value at each point of the calibrating line.

In practice, the assumed value n will probably not be correct and hence the upper and lower curves will not at first coincide. The ordinate difference separating them is a measure of the difference between the assumed and the correct value of n . If the difference is large, a new value of n must be assumed. A guide for the second assumption for n is obtained by finding out which of the two curves of first approximation has the lower ordinate values. If the $(N - n)$ curve is below the $(N + n)$ curve, too large a value of n has been assumed in the first approximation. If the $(N + n)$ curve is below the $(N - n)$ curve, the first approximation has been too small.

Instead of assuming a second approximation, the first estimated value of n can be improved by measuring the small residual area between the two curves of the first approximation and dividing it by the length over which the area has been measured. One-half of the average residual ordinate thus obtained may be added to or subtracted from the first assumed value following the rules already described.

This procedure overcomes the difficulties associated with the Friedrich's method and lends itself to self-calibration even in regions of high stress gradients. With the concept of residual area, just one assumption for n may be sufficient for most cases.

For an application of this method, there must exist at least two integral fringes along the calibration line. Since the procedure is independent of fringe gradients, the calibration line can be deliberately chosen to pass through regions of high spatial frequency of fringes so that many data points are obtained for the first curves. It is then possible to apply the technique to any model and even to slices of three-dimensional models. Difficulties arise only if over the whole field of the model the maximum fringe order is less than two.

3.3.1.3 Approximation by Average

A calibration line taken within the plane of the model is so chosen that it embraces the maximum number of fringes. The positions of integral and half order fringes (i.e. the centre lines of bright and dark bands on the negative of a dark field pattern)

are marked on the calibration line. Their orders are plotted as a continuous curve of fringe order against position on the calibration line. Then the fringe order at each intersection of a boundary line with the calibration line is estimated. The difference between this order and the neighbouring integral order gives the fractional order n for the particular set of boundary lines considered. The magnitude of n will vary in general along the calibration line and have an average value n_a which may be assumed to be the final value of n . The accuracy of fractional fringe order estimation thus depends on the number of fringes covered by the calibration line. The greater this number, the higher will be the accuracy.

3.3.2 Extension of Graphical Methods to several Exposures

The procedures described provide calibration for one exposure time only. Calibration values at other exposures can be obtained by adopting the procedure outlined by Meyer, Mehrotra and Talukder (1973). However, the use of this procedure implies that any error in the basic calibrated value (which is quite likely to occur in this case) will be transmitted to all other calibration results. Alternatively the following method may be used:

Either the results for each exposure time can be calibrated by the method outlined, or if combined boundary lines for different exposure times are available, these can be calibrated on a single calibration line provided each boundary line has first been identified with its particular exposure time. In fact, one value of fractional fringe order n is associated with each exposure time used so that all the boundary lines can be uniquely identified with a given fringe order.

To increase the accuracy of the methods, more than one calibration line through the model may be considered.

3.3.2.1 Application to Frozen Stress Slices

Situations where the maximum fringe order over the whole field is less than two are uncommon in two-dimensional photoelasticity but may be rather frequent in three-dimensional slices. If such a slice has just one or two fringes only, then perhaps another slice with a larger number of fringes may be found in the same three-dimensional model or the model may be so arranged that it contains a slice with many fringes. This slice can then be analysed and

the results applied to the slice with a small number of fringes, provided (i) the thickness of the two slices are the same, (ii) similar optics and polariscope conditions are used for both and (iii) similar film and photographic parameters are used for both.

This fact may be used in three-dimensional analysis where a large number of slices are involved. Out of many slices, the one with the largest number of fringes can be selected for calibration and the results applied to the fringe patterns of all the other slices.

3.3.3 Compensation Method

Two-dimensional models as well as three-dimensional slices can be self-calibrated by a procedure suggested by Thadani (1972). On the photograph of the fringe pattern, the position of a point on a given boundary line is established with respect to a set of co-ordinate axes and the fractional fringe order at the corresponding point in the field of the model on the photoelastic bench is determined by any compensation method.

The Tardy or Sénarmont method can be used for this purpose and not much time is lost because the fractional fringe order needs to be determined at one point only. The fractional fringe orders of all the other boundary lines can be found by inspection of the distribution of integral order fringes over the field.

This method offers the problem of exact coincidence of the chosen point on the photograph and in the field of the model on the polariscope. Further, any error in the compensation method will be transmitted to the calibration result. However, accuracy can be increased if the method is applied to more than one point and average result is taken.

The method has the advantage that it can be used even if the maximum fringe order in the model is less than unity (provided a boundary line is formed on the photograph).

When several exposure times are used to shift the locations of the boundary lines, a calibration line may be again considered on the photographs of fringe patterns and its intersections with boundary lines for different exposure times be established. The fractional fringe orders for various exposure times are then

determined by considering a corresponding line in the field of the model on the polariscope and obtaining the fringe orders at corresponding points by means of pointwise compensation.

3.3.4 Density Method

The calibration procedure suggested by Chiang and Khetan (1974) requires a densitometer and is based on the strict condition that all the exposed points fall on the straight line portion of the characteristic curve of the negative film. This condition is virtually impossible to satisfy for high contrast films. Therefore, this method cannot be applied to self-calibration for the boundary line method.

Low-contrast films are used for fractional fringe order measurements in the equidensity method described in Chapter 5. However, even with low-contrast films, the condition that all the exposed points fall on the straight line portion of the characteristic curve, can be satisfied only if the light passing through a crossed polariser and analyser is not fully extinguished. Further, the method requires accurate determination of the γ value of the film used, in addition to measurement of density at the location of a boundary line and of maximum and minimum densities in the surrounding region. The relationship between fringe order and densities in a dark field polariscope is given by

$$N = \frac{1}{\pi} \sin^{-1} \left[\frac{10^{(\frac{D}{\gamma})} - 1}{10^{(\frac{D_{\max}}{\gamma})} - 1} \right]^{\frac{1}{2}} \quad (3.3.1)$$

with $\gamma = 1$, the above equation reduces to

$$N = \frac{1}{\pi} \sin^{-1} \left[\frac{10^D - 1}{10^{D_{\max}} - 1} \right]^{\frac{1}{2}} \quad (3.3.2)$$

The application of the method is thus restricted considerably and offers no special advantage over Thadani's approach except that the polariscope is required once only and all the subsequent evaluation proceeds from one low-contrast photograph of the isochromatic pattern.

3.3.5 Example

The method of Section 3.3.1.3 has been applied in a study of the equidensity results for a model of a disk under diametral compression. The equidensity pattern for a given exposure time is shown in Fig. 3.3.1, and the fractional fringe orders are $n_b = \pm 0.278$ as determined by means of a beam calibration model (see Equation 3.2.1).

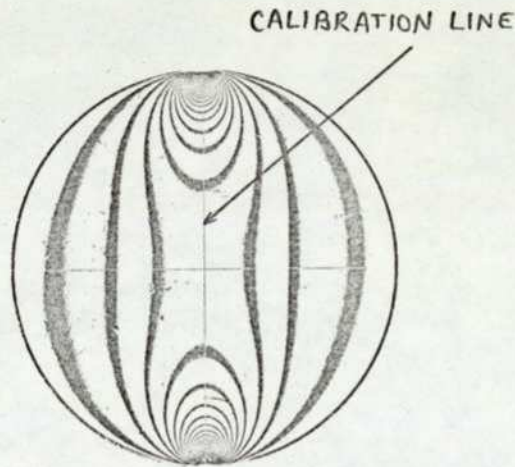


Fig. 3.3.1

Camera exposure time (Ilford) = 1/10 s
Contact exposure time (Agfacontour) = 75 s

A calibration line connecting the two loading points in Fig. 3.3.1 will give the maximum number of data points. It has not been found necessary that the calibration line extends to the boundary of the model at its ends; it can start and end inside the model if that seems convenient. It is desirable, however, that the calibration

line is straight rather than curved because distances on this line must be measured. Further, care must be taken that the calibration line does not run parallel to an equidensity or boundary line.

The positions of intersections of equidensity lines and calibration line have been determined on a X - Y recorder. To a given relative scale, the readings of position are 144 and 202 for the equidensity lines neighbouring on the second order fringe, 235 and 258 for those near the third order fringe, 269 and 280 for the fourth order, 286 and 292 for the fifth order. The positions for the integral orders 2, 3, 4 and 5 thus are 173, 246.5, 274.5 and 289. These are plotted in Fig. 3.3.2 giving fringe order as a function of measured position.

From the curve, the fractional fringe orders n_b of the two equidensities adjacent to the third order are -0.21 and +0.33, those adjacent to the fourth order are -0.25 and +0.30 with an average of ± 0.272 . The fractional fringe orders for the positions 202 and 286 have not been used in averaging because the curve cannot be extrapolated accurately to fringe orders below 2.0 or above 5.0. The value of ± 0.272 obtained by self-calibration is very close to that of ± 0.278 determined from a beam calibration model.

Normally, such a high accuracy will not be obtained from a single curve. In general, to increase the accuracy, a number of calibration lines may be considered and their intersections with the equidensities or boundary lines be determined as accurately as possible. It has been found that large variations in calibration values will occur if even only a single point of intersection is determined inaccurately.

3.4 APPLICATION TO DIAMETRICALLY COMPRESSED DISK

In order to investigate the applicability of the method, models of a beam and of a disk were cut from the same sheet of CR-39. The beam subjected to pure bending moment serves as a calibration model and the disk loaded in diametral compression serves as a test model with a known theoretical solution.

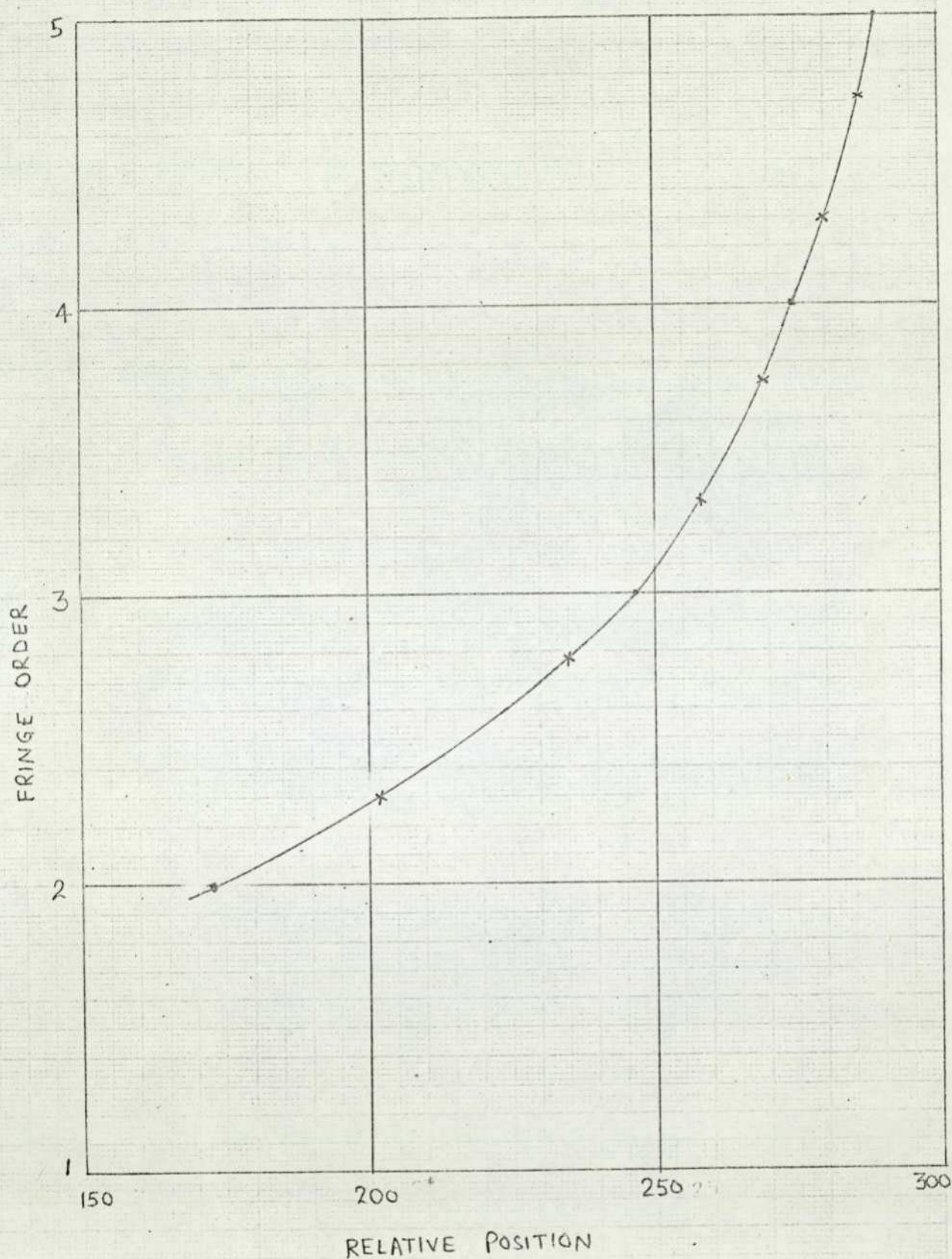


FIG. 3.3.2.

FRINGE ORDER ALONG THE CALIBRATION LINE
FOR A DISK UNDER DIAMETRAL COMPRESSION

The fringe pattern was photographed on Kodalith Ortho Film 2556, Type 3, which was developed in Kodalith Super developer for 3 minutes at 20°C under continuous agitation. For consistency of procedure, the developer was used once only and discarded after use. This did not entail much loss of the developer since dish development was used and two films were always developed simultaneously.

As the fringe patterns were photographed on 5" x 4" (127 x 102 mm) films, with an image magnification factor of about 0.6, the widths of the fringes in the calibration beam and positions of the boundary lines in the test model could be measured by conventional means. However, for greater precision, an X - Y reader (supplied by PCD Ltd., Farnborough, Hants.) was used. This eliminated the need and disadvantages of printing the negatives, because the positions of the boundary lines can be determined on the negatives. On prints the boundary lines are not necessarily obtained at the same relative positions.

The CR-39 beam 10" x 1.53" x 0.247" (254 x 39 x 6.3 mm) was loaded in pure bending through four holes on the neutral axis and observed on a lens-type polariscope with a high pressure mercury arc source and green filter (Ilford no. 807) for $\lambda = 546.1$ nm. At this wavelength, the material fringe value based on principal stress difference was 102 lbf/in fringe (18 kN/m fringe).

Dark background photographs of the fringe pattern under a constant bending moment were taken in quick succession with exposure times of $\frac{1}{2}$, 1, 2, 3, 5 and 10 seconds. The experiment was finished within 5 minutes from the instant of load application so that creep of CR-39 could not noticeably influence the final results. (This may be compared with the long times required by Tardy or Sénarmont compensation). The widths of the dark and bright bands on the negative were measured at leisure on the X - Y reader to within 0.002" (0.05 mm) and the fractional fringe orders n_d and n_b were calculated from Equation (3.2.1).

The results in Table 3.4.1 show that $t \cdot \sin^2 \frac{\pi}{2} n$ is sufficiently constant for all the exposure times, Equation 3.2.3, so that calibration with a single known exposure time in fact suffices

to determine the fringe order for any other exposure time.

Table 3.4.1

Relationship of fractional fringe orders for different exposure times

Exposure time t (seconds)	Fractional Fringe Order n_d	$\theta = 180 n_d$ (degrees)	$\sin^2 \pi n_d$	$t \cdot \sin^2 \pi n_d$
$\frac{1}{2}$	0.253	45.54	0.5094	0.2547
1	0.172	30.96	0.2647	0.2647
2	0.117	21.06	0.1291	0.2582
3	0.095	16.98	0.0853	0.2559
5	0.074	13.32	0.0532	0.2660
10	0.051	9.18	0.0255	0.2550

Calibration with bright background confirmed Equation (3.1.4), a time of 2 second producing for instance a fractional fringe order of 0.383.

For the same bending moment, Figs. 3.4.1 and 3.4.2 show representative prints of the fringe patterns taken with dark background and exposure times of 1 and 2 seconds, Figs. 3.4.3 and 3.4.4 those for bright background and exposure times of $\frac{1}{2}$ and 2 seconds. The shift in the boundary line between dark and bright bands due to change in exposure time is very evident for dark as well as bright background. Fig. 3.4.4 shows some aspects of non-uniformity at the centre of the field, and these will be described in detail in Chapter 4.

The test model is a circular disk of diameter d and thickness t subjected to a diametrically compressive load P as shown in Fig. 3.4.5. With co-ordinate axes (x, y) passing through the centre of the disk, the dimensionless co-ordinate values are

$$k_1 = \frac{x}{d/2}, \quad k_2 = \frac{y}{d/2} \quad (3.4.1)$$

The isochromatic fringe order at the centre of the disk is given by Frocht (1948a),

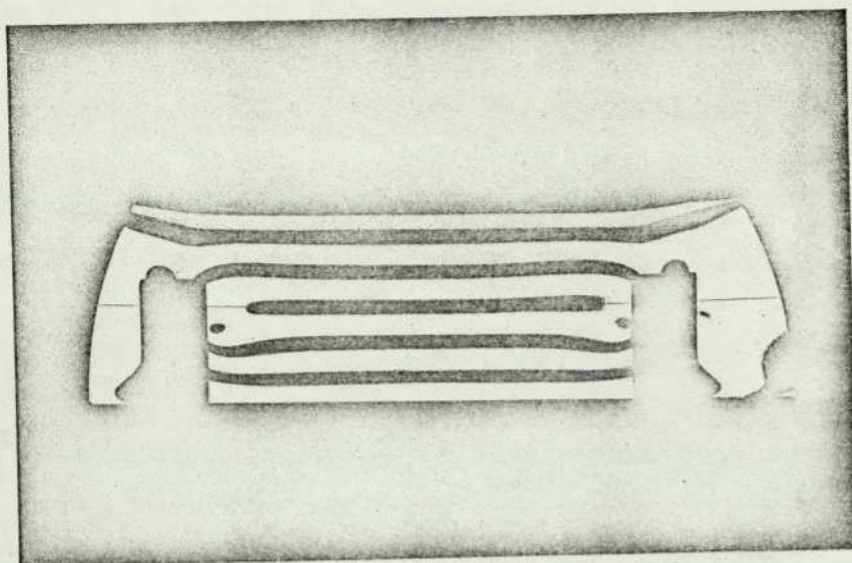


Fig. 3.4.1

Exposure time= 1 sec
Camera aperture = f5.6

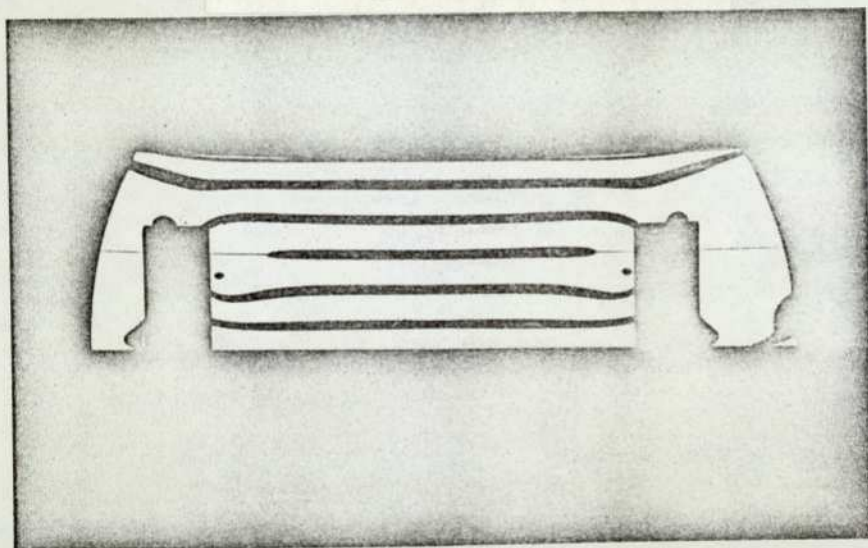


Fig. 3.4.2

2 s, f5.6

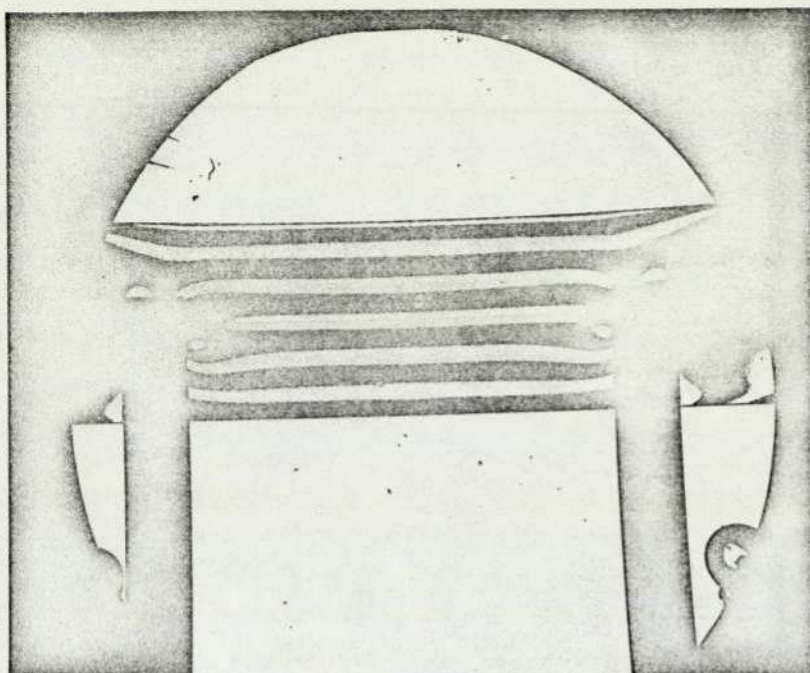


Fig. 3.4.3

1/2 s, f5.6

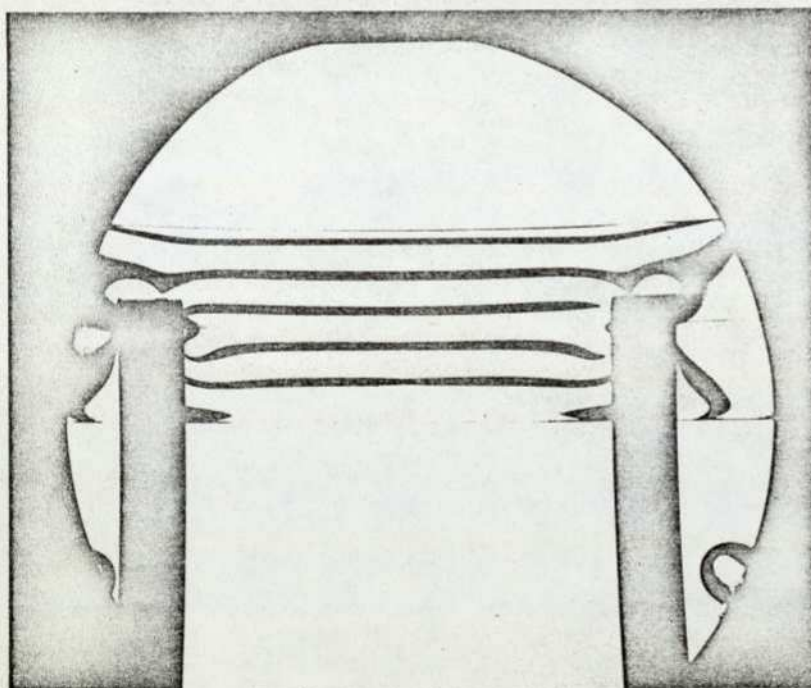


Fig. 3.4.4

2 s, f5.6

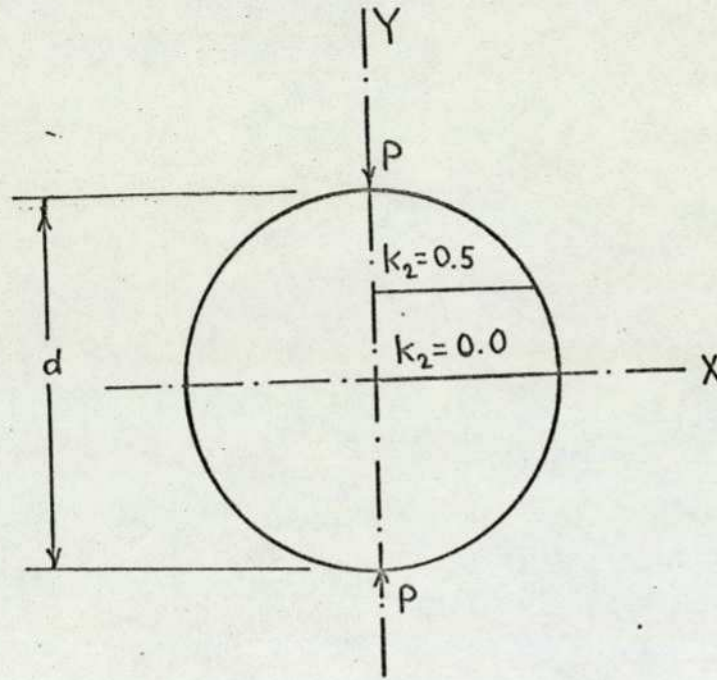


Fig. 3.4.5 Circular disk under diametral compression ($k_2 = 2y/d$).

$$N_o = \frac{8P}{\pi df} \quad (3.4.2)$$

where f is the material fringe value defined as the difference in principal stress producing one fringe per unit thickness of the model. The ratio of fringe order N_d at any point in the disk to N_o at its centre, if written in terms of the dimensionless co-ordinates, is

$$\frac{N_d}{N_o} = \frac{1 - k_1^2 - k_2^2}{(1 + k_1^2 + k_2^2)^2 - 4k_2^2} \quad (3.4.3)$$

as can be derived from Frocht (1948a). The value of N_d for any given load P can thus be calculated from Equations (3.4.2) and (3.4.3).

Finally, the isoclinic parameter θ at any point in the disk is given by

$$\tan \theta = \frac{2k_1 k_2}{1 - k_2^2 + k_1^2} \quad (3.4.4)$$

Equations (3.4.3) and (3.4.4) assume that the load acts at a point though in practice it acts over a finite area. However, Kuno (1932) has found that, provided the angle subtended by the loaded arc at the centre is small, the stress distribution under a load distributed over a small area is the same as that under a point load.

In the experiments, a CR-39 disk of 3" (76mm) diameter was used. Weights of 25, 35, 40 lb (11.35, 15.9, 18.2 kg) applied with a lever magnification of about 5.3 gave $N_0 = 1.1, 1.54, 1.76$. With the 25 lb weight, 12 photographs of isochromatics were taken at exposure times of $\frac{1}{2}, 1, 2, 3, 5, 10$ seconds with dark and bright backgrounds. For each of the two higher loads, 8 photographs were obtained at exposure times of $\frac{1}{2}, 1, 2, 3$ seconds with dark and bright backgrounds. The reported calibration results gave the fringe orders for the various boundary lines in each case.

Figs. 3.4.6 to 3.4.9 are representative dark background prints of fringe patterns for a load of 40 lb at an exposure of 1 and 2 seconds, for 35 lb and 2 seconds, and for 25 lb and 2 seconds. Hence, Figs. 3.4.6 and 3.4.7 show the shift in the positions of the boundary lines due to changes in exposure time only, whilst Figs. 3.4.7, 3.4.8, and 3.4.9 show the shifts in the positions of the boundary lines due to variation in load only.

The theoretical fringe orders for the three loads, determined along the horizontal diameter $k_2 = 0.0$ and along a horizontal line $k_2 = 0.5$ midway between the centre of the disk and the loading point, are plotted as continuous curves in Figs. 3.4.10 and 3.4.11. where the experimental points are also entered. The good agreement is quite obvious. Points of half and integer fringe order have not been used in this analysis.

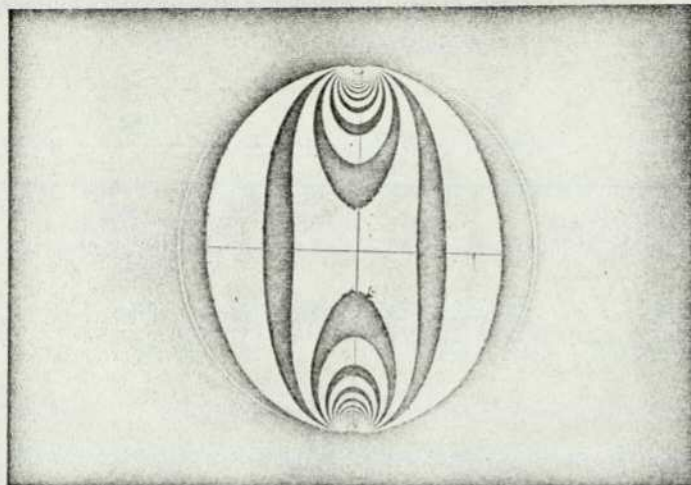


Fig. 3.4.6

1 s, f5.6

Load = 212 lbf

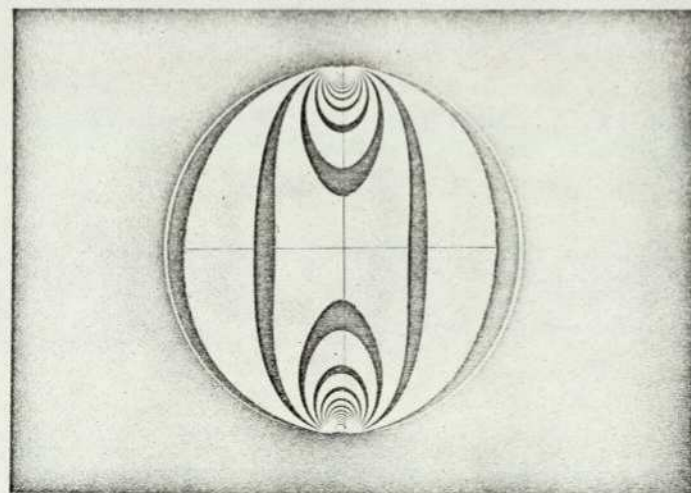


Fig. 3.4.7

2 s, f5.6

Load = 212 lbf

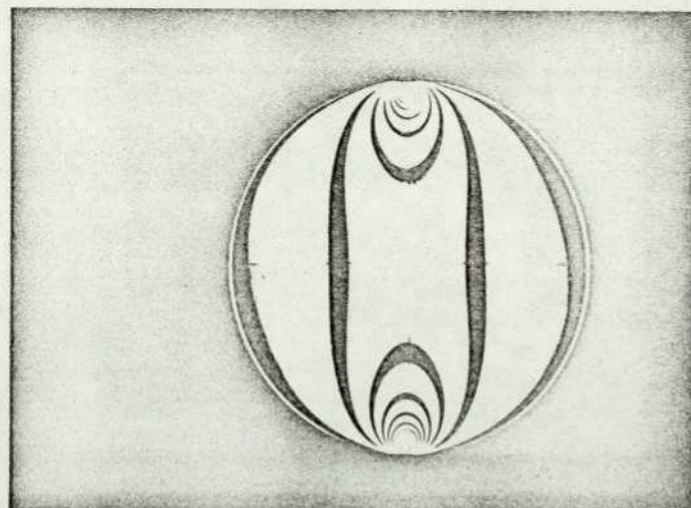


Fig. 3.4.8

2 s, f5.6

Load = 186 lbf

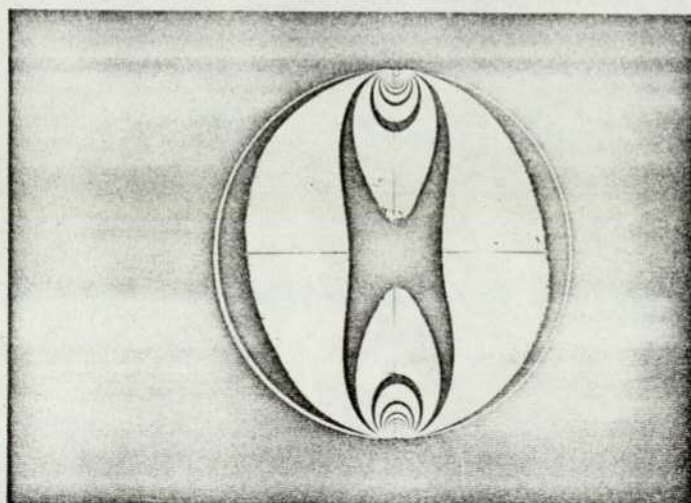


Fig. 3.4.9

2 s, f5.6

Load = 133 lbf

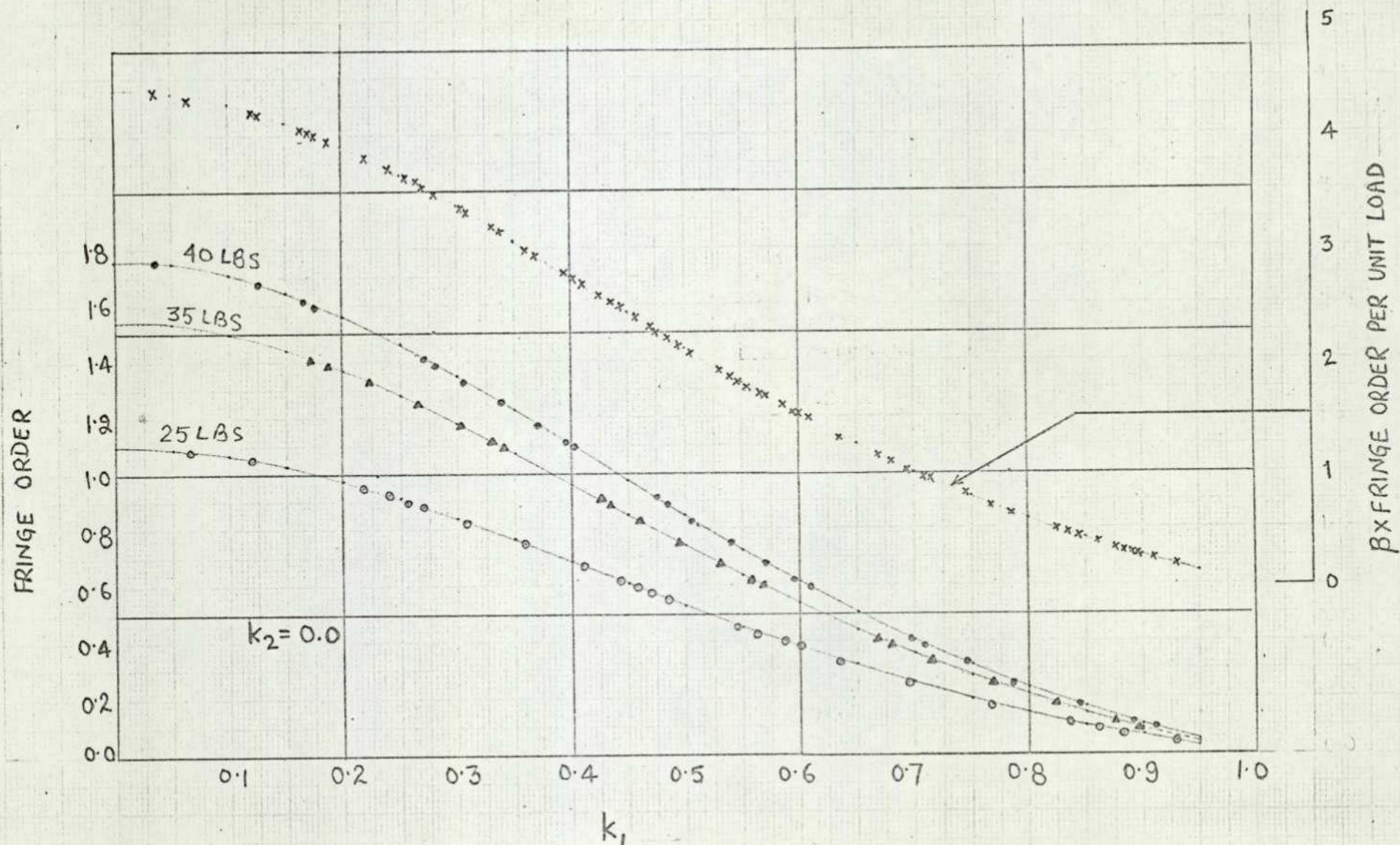


FIG. 3.4.10
 GRAPH OF FRINGE ORDER AGAINST k_1 ALONG $k_2=0.0$ OF A DIAMETRICALLY COMPRESSED DISK

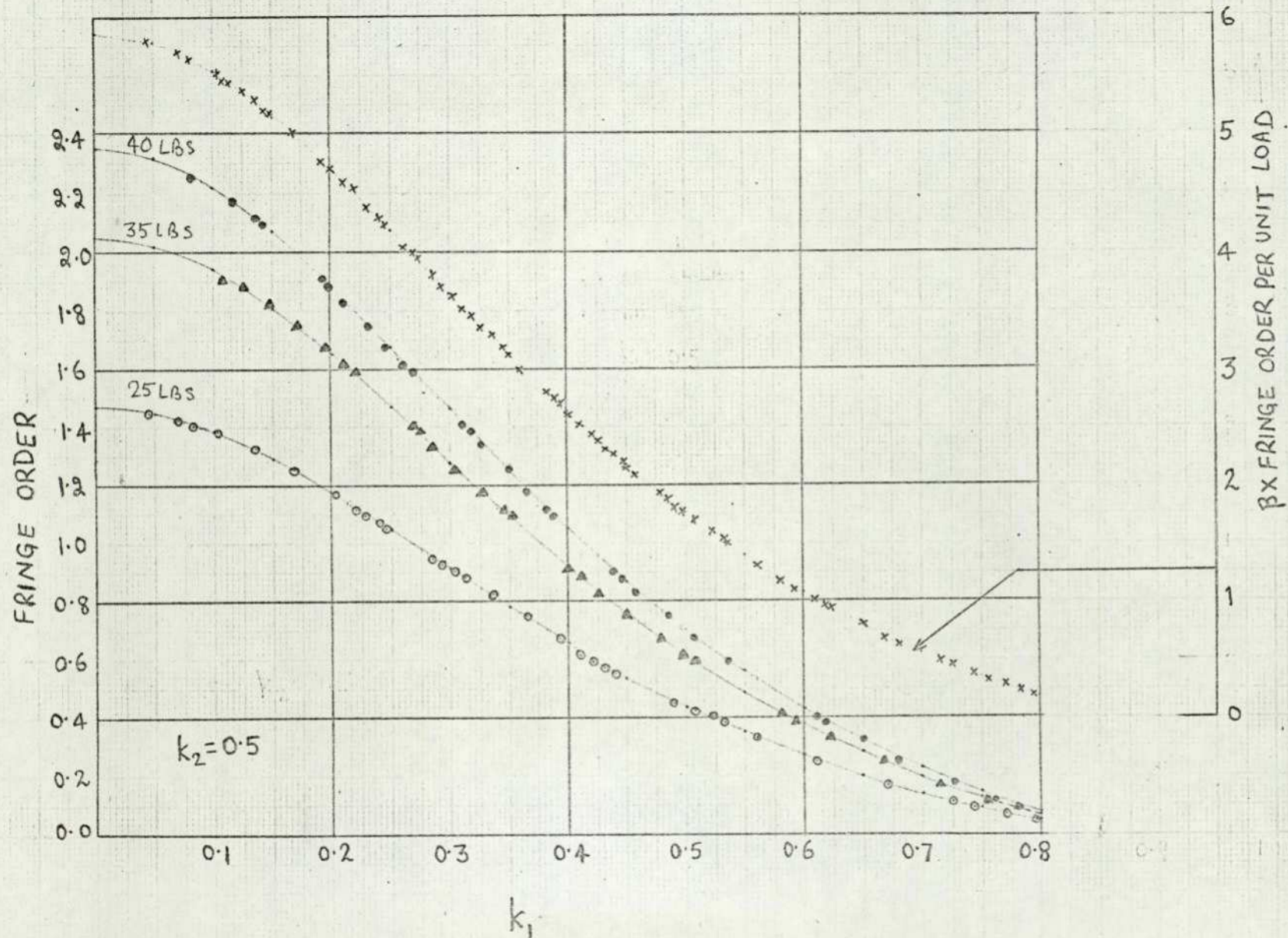


FIG. 3.4.11

GRAPH OF FRINGE ORDER AGAINST k_1 ALONG $k_2 = 0.5$ OF A CIRCULAR DISK UNDER DIAMETRAL COMPRESSION

The results for the three different loads can be combined in one curve of fringe order per unit load as also shown in Figs. 3.4.10 and 3.4.11. where the scale factor β for the combined curves refers their fringe order value to a fictitious applied weight of 100 lb. The combined variations in load level and exposure time are thus seen to give a great number of well distributed experimental points. Although no attempt was made to obtain the optimum distribution of experimental points, the sensitivity of the method is here already $\frac{1}{50}$ th of a fringe or better.

Since the method depends on the accurate production of densities, it requires due care in photographic work. The most important criterion is not in obtaining the absolute values of a particular factor, say developer temperature, but in maintaining identical conditions for test and calibration model for each exposure time. If a single calibration result is used for all exposure times, then due care must be taken to reproduce identical optical and photographic conditions for all the test model photographs taken at different exposure times. Obviously these requirements on the maintenance of identical optical and photographic conditions can be relaxed if no calibration model is used and the fringe orders are determined by one of the self-calibration techniques (Section 3.3).

3.5 SENSITIVITY AND ACCURACY OF THE METHOD

The proposed method is highly sensitive for isochromatic fringe order measurement. If load and exposure time are varied, an increase in sensitivity equivalent to a fringe multiplication factor of 50 or more is quite easily achieved without any reduction in light intensity as in Post's fringe multiplication technique. Even where the load cannot be varied, as in three-dimensional slices, a high sensitivity is possible if the exposure time can be varied at will. The use of pre-exposure, Section 6.5, helps in increasing the latitude of exposure time when discrete exposure times only are available in the camera.

For a particular situation, there is always a range of exposure times in which sharply defined fringes can be obtained.

In the absence of any possibility of load variations, the sensitivity of the method depends on the number of different exposure times that can be used within that range. A large variation in light intensity or a small change in film development time also helps in increasing the sensitivity.

The problem of lack of latitude in exposure associated with high contrast film can be overcome by photographing the fringe pattern on a low-contrast film and then copying it onto Agfacontour film as described in Chapter 5. This method gives a considerably higher sensitivity and, by proper design of experiment, permits to achieve any desired sensitivity.

Taking 33 boundary lines with fringe orders between 0 and $1\frac{1}{2}$ intersecting the horizontal centre line of the disk of Fig. 3.4.5., a maximum deviation of only 0.01 fringe from the theoretical curve has been obtained. The overall accuracy will depend on

- (i) the extent to which similar optical and photographic conditions are reproduced for the calibration beam and the test model,
- (ii) the quality of the reading equipment,
- (iii) the aptitude of the observer.

The last two factors also influence accuracy in conventional compensation. The first factor will lead to large errors if the photographic and optical conditions for the calibration beam and test model are not identical. One of the self-calibration procedures (section 3.3) may then be used.

If the optical and photographic conditions are kept uniform and constant (see Section 3.7), the accuracy of the method is determined by the precision with which the position of boundary lines can be located. With an X - Y reader capable of 0.05 mm resolution and a distance of 10 mm per full fringe on the calibration beam, the fractional order of a line can be determined to within 0.01 or better. In reading on the test models the error then depends on the fringe gradient. The total error in the measurement of higher order fringes will be more than in the measurement of lower order fringes. This fact can be used in keeping the error within 1 to 2%.

3.6 DIFFICULTIES NEAR THE INTENSITY EXTREMES

The proposed method does not always provide fringe orders in the immediate vicinity of intensity maxima and minima. If only one background, black or white, is used, then fractional fringe orders near the intensity maxima may be unobtainable because the shortest usable exposure time is dictated by discrete steps in shutter speed of the camera. A given speed may give maximum exposure just below the exposure inertia of the film, which must be exceeded before any visible densities are developed. The next possible speed may already give the exposure which is too large to give boundary lines near the maxima of intensity.

The problem is not as severe near intensity minima because large exposure times can give boundary lines very close to the intensity minima. This fact can be used to overcome the difficulties at intensity maxima by changing the background so that a region of intensity maxima changes into a region of intensity minima.

A slight change in load generally displaces the integer or half order fringes, i.e. the regions of intensity maxima and minima, except for the zero fringe. With the concept of unit load this method can also be used for the fringe orders in the immediate vicinity of intensity minima or maxima.

An imperfect polariscope, i.e. a polariscope with polaroids that do not completely extinguish the light in the crossed position, also helps in obtaining fringe orders near the intensity minima. Such imperfections do not alter the position of intensity minima but for the same exposure time the imperfect polariscope gives boundary lines much closer to the intensity minima than can be obtained with a perfect polariscope.

3.7 FACTORS AFFECTING DENSITY DISTRIBUTION IN FRINGE PATTERN

The following optical and photographic parameters influence the accuracy and sensitivity of the method, and their constancy is particularly important when a separate calibration model is used to establish the fringe orders represented by the boundary lines. Many of the conditions discussed in the following can be relaxed if a self-calibration procedure is used.

3.7.1 Symmetry of Light Source

The light source as well as the camera aperture opening must be symmetrical and aligned on the axis of the polariscope. A set of photographs taken to study the effect of asymmetry of light source or aperture have shown that a small asymmetry of the light source (i.e. a few degrees off from the symmetrical position) does not influence the density distribution but a large asymmetry affects uniformity and also reduces the maximum intensity of light.

Fig. 3.7.1 shows the fringe pattern obtained with a symmetrical light source, Fig. 3.7.2 that for a light image asymmetrically focussed with respect to the camera aperture opening. All other optical and photographic parameters are the same for both. Thus the effect of asymmetrical placing of the focussed light image at the camera aperture is quite evident.

3.7.2 Uniformity of Illumination

Even when the light source and its focussed image are symmetrical with respect to the polariscope axis, the illumination at the image plane of the camera may not be uniform throughout the field and especially in certain regions near the edges and at the centre.

As a result of such a non-uniform illumination, spurious densities are formed in regions where the actual intensity is higher than the uniform intensity and a loss of density may occur where the intensity is less than the uniform intensity. The possible causes for non-uniform illumination and various methods to reduce the non-uniformity are considered separately in Chapter 4.

3.7.3 Misfocus

If a separate calibration beam is used, the beam and the test model are best placed in the same position on the polariscope, i.e. at equal distance from the second field lens. If this condition is not satisfied, the focussed image of the test model and calibration piece may not be obtained for the same position of the film plane (Post, 1970).

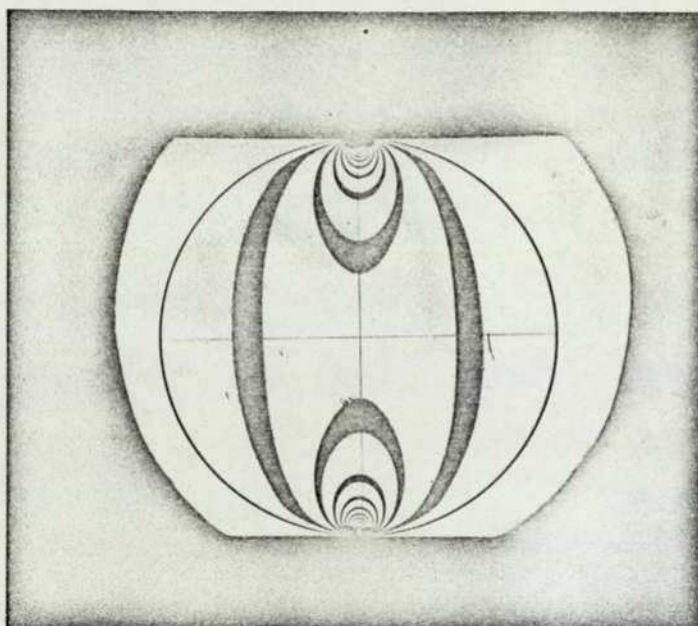


Fig. 3.7.1

10 s, f45

Symmetrical light source

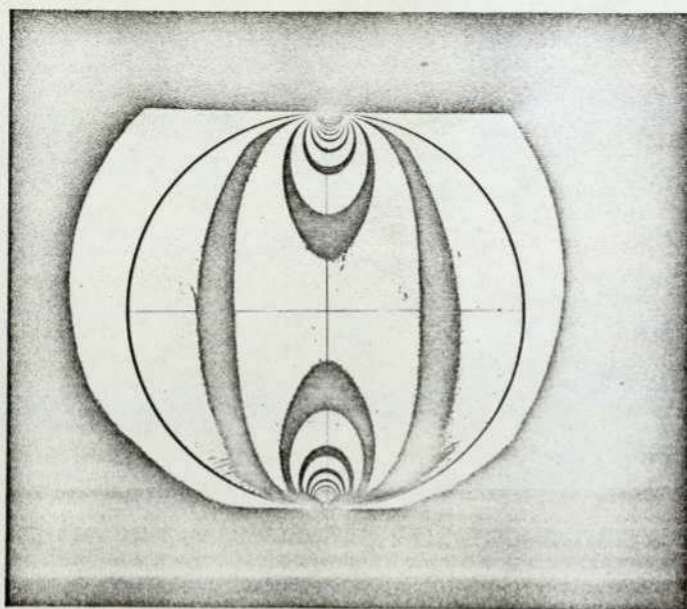


Fig. 3.7.2

10 s, f45

Unsymmetrical light source

For identical optical conditions the film plane also must not be displaced. A detailed study of the effect of misfocus was made by changing the position of the film plane with respect to the camera lens. This study was also intended to verify the claim by Post (1970) that misfocus results in distortion and displacement of the fringe pattern.

Figs. 3.7.3 and 3.7.4 show the fringe patterns obtained when the film plane is at its focussed distance of 18.0 and at a distance of 21.0 cm from the camera lens, all other parameters being the same. It is obvious that misfocus does not produce any non-uniformity in the fringe pattern.

Measurements on the location of the fringes indicate that there is neither distortion nor shift in position of the fringes. The centre line of the half-order fringe in both photographs is at $k_1 = 0.652$, $k_2 = 0.0$. On the horizontal diameter ($k_2 = 0.0$), the boundary lines are obtained at k_1 values of 0.560 and 0.744 in Fig. 3.7.3 and at 0.576 and 0.728 in Fig. 3.7.4.

Hence, although the same exposure time is used, identical fractional fringe orders are not obtained in focussed and misfocussed fringe patterns. It is found that with an increase in distance between camera lens and film plane, the maximum available light at the film plane and hence the total exposure decreases.

A slight misfocus thus is not detrimental to this work provided the relationship between the model and the film plane is maintained for both the test model and the calibration beam.

As a result of misfocus the magnification of the image is also changed. Bringing the film plane nearer to the camera lens decreases the magnification whereas taking it further away increases the magnification.

3.7.4 Consistency of Exposure Time

The exposure time must be identical for the calibration beam and the test model. The camera shutter can reproduce identical exposure times without difficulty. However, when a stop watch is used for a large exposure time, care must be taken to eliminate as far as possible any human error in reading the stop watch and opening and closing the shutter. Alternatively, a timer circuit and solenoid may be used to produce consistent long exposures.

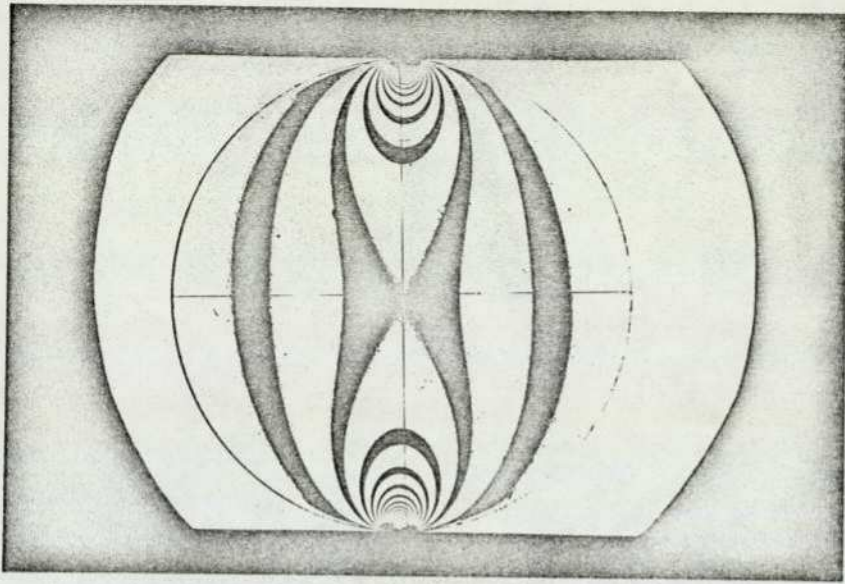


Fig. 3.7.3

6 s, f22

Distance between film plane
and camera lens = 18 cm

In focus

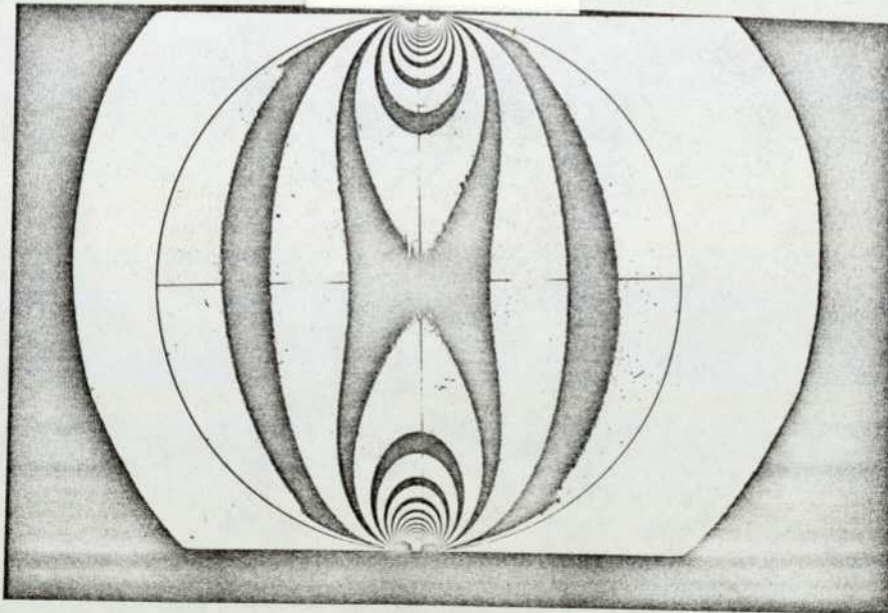


Fig. 3.7.4

6 s, f22

Distance between film plane
and camera lens = 21 cm

Out of focus

When a single calibration is used to obtain results of various exposure times, care must be taken to see that all exposure times are actually in the intended proportions to the calibrating exposure time. The camera shutter or a stop watch are reliable for this purpose but difficulties were encountered with an enlarger timer as shown in Table 3.7.1.

Table 3.7.1

Comparison of Indicated and Actual Time given by an enlarger timer.

Indicated Time, sec	2.0	3.0	4.0	5.0	6.0	8.0	10.0
Actual Time, sec	2.5	3.7	4.8	5.8	6.9	9.0	11.2

Although the maximum difference between indicated and actual time is small, it may be large enough to affect results calculated from a single experimentally determined calibration value.

3.7.5 Batch Variations of the Film

For consistency of density development, the same batch of film and preferably the same pack of film is to be used for both the calibration beam and the test model so that calibration results can be used without involving substantial errors. The photographs Figs. 3.7.5 and 3.7.6 taken at 3 sec exposure time under identical optical and photographic conditions except for variations in batch of Kodalith film, show the large variations that can be obtained.

3.7.6 Developer Temperature

Manufacturers recommend a developer temperature of 20°C. As a rule, a higher temperature results in larger densities and a lower temperature in smaller densities. The overall effect can be very pronounced.

Kodalith Super developer is particularly sensitive to lower temperatures. A detailed controlled study of the effect of variations in developer temperature on the densities and consequently on the fractional fringe orders has been made with a model of a beam in pure bending photographed under otherwise identical optical and photographic conditions. The results are summarised in Table 3.7.2.

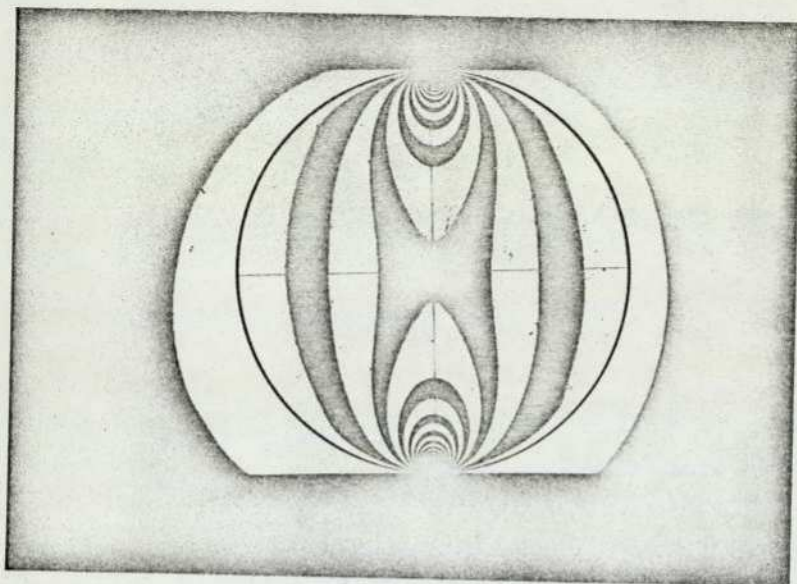


Fig. 3.7.5

3 s, f45

Film batch 581

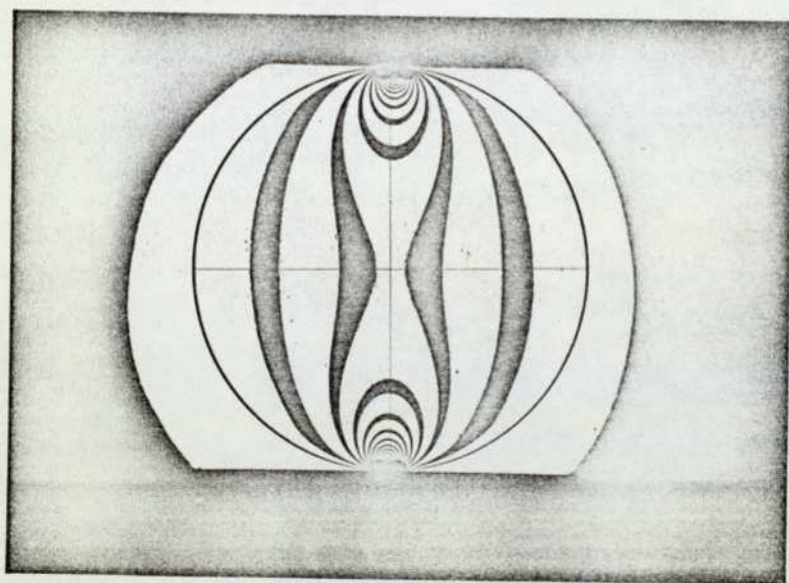


Fig. 3.7.6

3 s, f45

Film batch 551

Table 3.7.2Variations in fractional fringe order with developer temperature.

Developer Temperature, °C	18.0	19.5	20.5	22.0	24.0
Fractional Fringe Order	0.2197	0.1869	0.1857	0.1741	0.1684

A variation in temperature of 0.5°C on either side of 20°C has no appreciable effects on the fringe orders but a decrease in temperature of 2°C gives about 18% change in fractional fringe order and an increase of 2°C a change of 9.6%.

Figs. 3.7.7 and 3.7.8 show representative fringe patterns obtained with developer temperatures of 18°C and 22°C.

3.7.7 Mode of Agitation

Identical modes of agitation have been used for developing the photographs for the calibration beam and test model.

For normal processing, the mode may be

- (a) intermittent agitation,
- (b) continuous agitation, or
- (c) no agitation,

depending on whether the developing tray is rocked a few times every 15 seconds, throughout the development time, or left undisturbed throughout development.

The effect of each of these three modes is shown in turn in the photographs 3.7.9 to 3.7.11 of fringe patterns taken of a diametrically compressed disk under the same load and with the same exposure time of 3 sec.

It is found that continuous agitation gives the least density, no agitation the maximum density, and intermittent agitation intermediate value of density. These appear as brightness differences in Figs. 3.7.9 to 3.7.11. Continuous and intermittent agitation produce only a small difference, but no agitation definitely shows large density variations from the other two modes.

It is rather surprising that no agitation produces the maximum density. This is contrary to what happens with most other developers. In Kodalith Super developer used here, the products

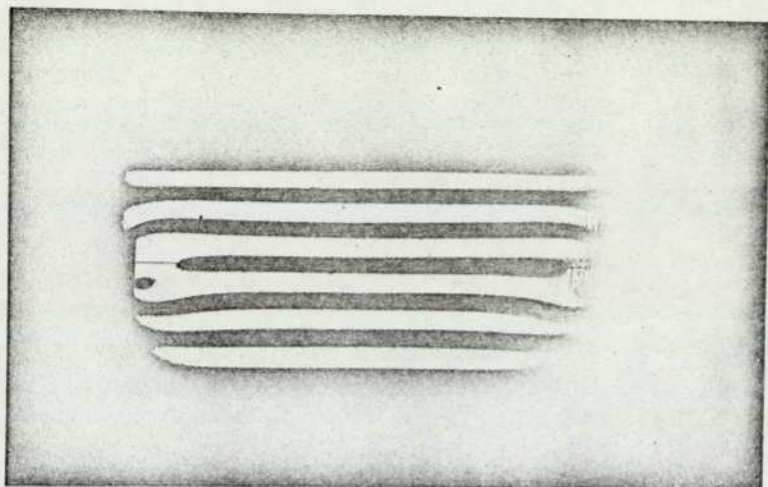


Fig. 3.7.7

3 s, f45

Developer temperature = 18°C

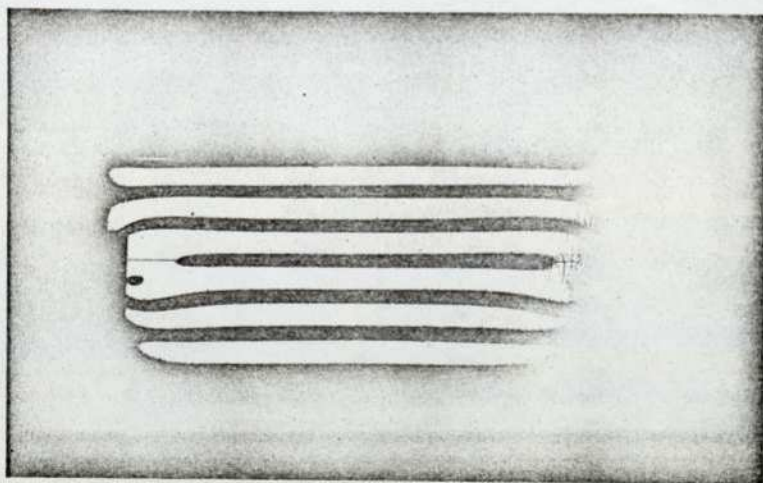


Fig. 3.7.8

3 s, f45

Developer temperature = 22°C

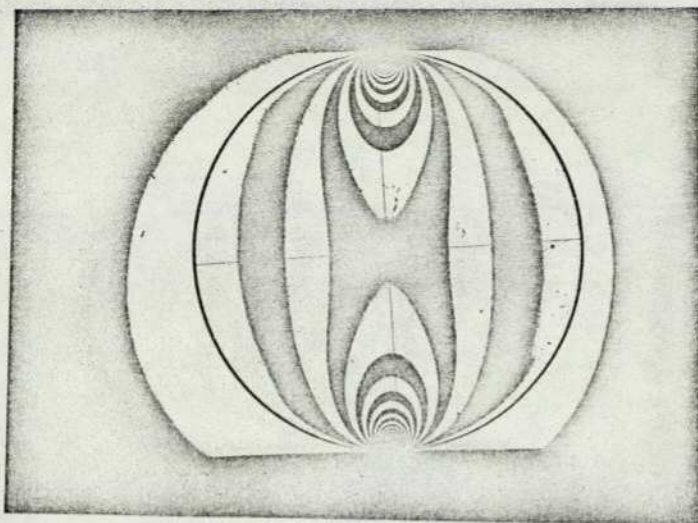


Fig. 3.7.9

3 s, f45

Intermittent agitation

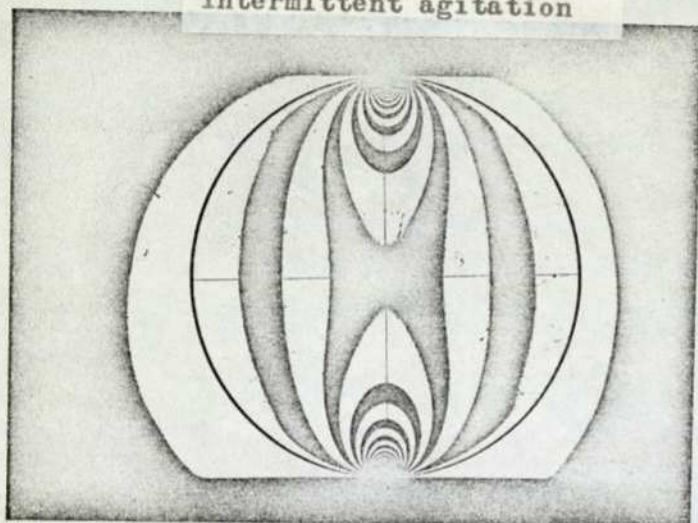


Fig. 3.7.10

3 s, f45

Continuous agitation

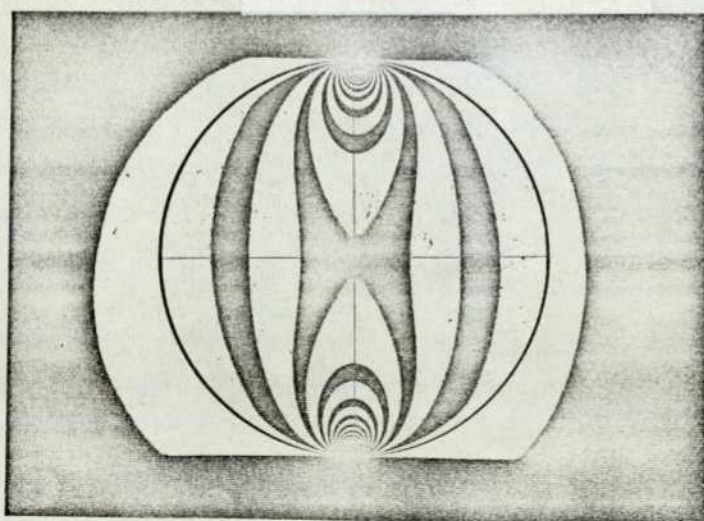


Fig. 3.7.11

3 s, f45

No agitation

of the development reactions apparently exert by themselves a strong developing action. They are moved away by continuous or intermittent agitation but considerably affect the development of densities when there is no agitation.

In the present work, intermittent agitation was mostly used. For accurate work, an agitation procedure as recommended in ASA no. PH4.1-1962 may be used.

3.7.8 Condition of Developer

It is always best to use fresh developer and to discard it after use. Large variations in densities can occur if the developer is oxidised (left in a partially filled bottle for a long time) or exhausted (used).

Figs. 3.7.12 and 3.7.13 show results obtained with fresh and with oxidised developer without any agitation. Figs. 3.7.14 and 3.7.15 compare results obtained with fresh and with used developer under intermittent agitation.

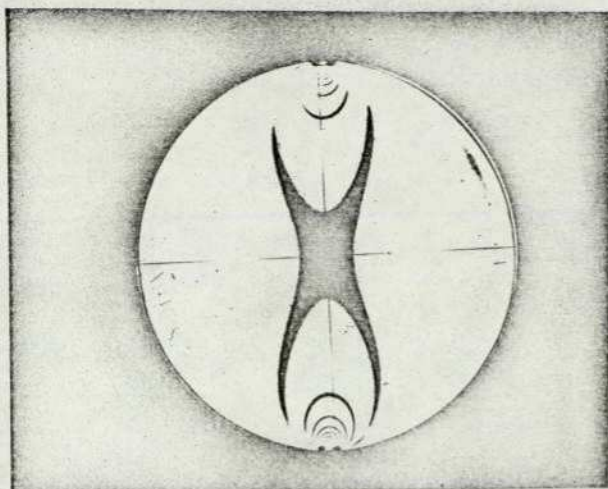


Fig. 3.7.12

5 s, f45
Fresh developer
No agitation

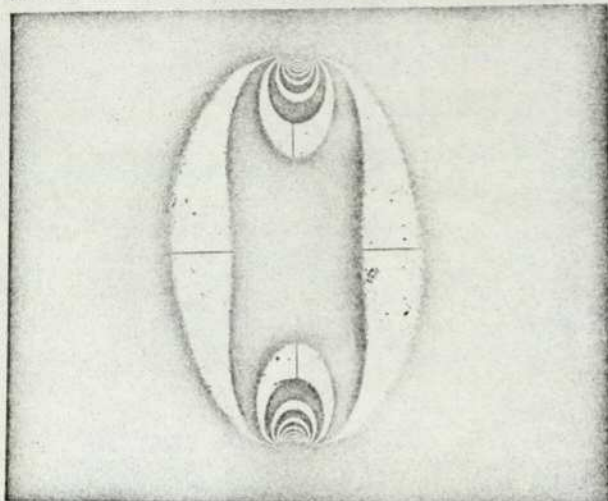


Fig. 3.7.13

5 s, f45
Oxidised developer
No agitation

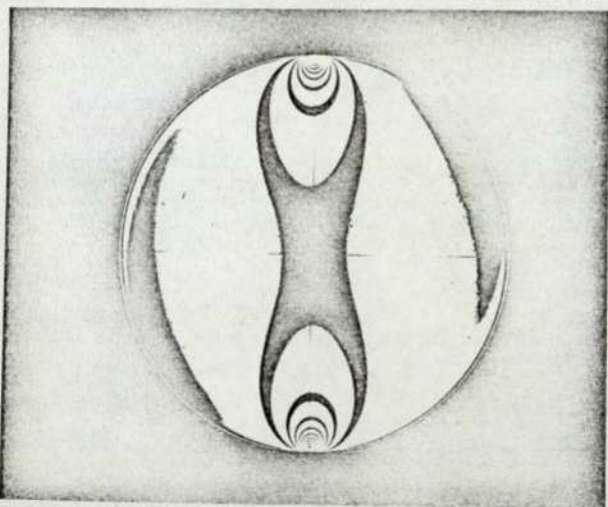


Fig. 3.7.14

2 s, f45
Fresh developer
Intermittent agitation

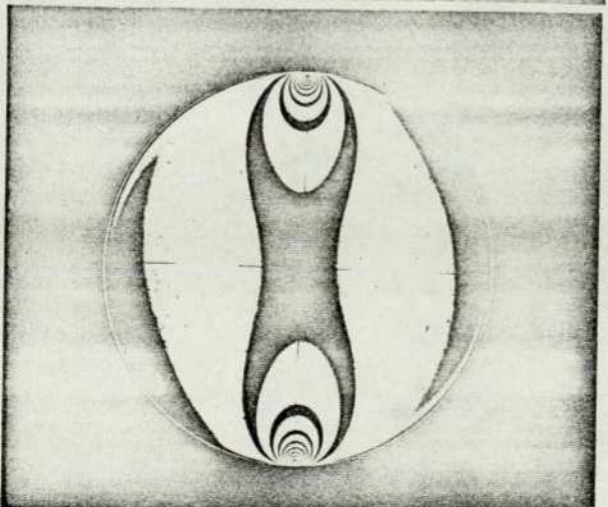


Fig. 3.7.15

2 s, f45
Used developer
Intermittent agitation

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CHAPTER 4

UNIFORMITY OF ILLUMINATION

4.1 DEFINITION OF THE PROBLEM

The model divides the whole field into (i) the part of the field covered by the model and (ii) the remaining part not covered by the model. The intensity of light in these two parts may differ, but the intensity equation of the polariscope (Equation 3.1.1) assumes a uniform constant intensity distribution at least in the field covered by the model. Since the model may be placed anywhere in the field, a uniform illumination of the whole field is obviously desirable.

It has been found that particularly in lens polariscopes the no load intensity is not entirely uniform. A field is usually obtained which has an intense spot at the centre, is uniform throughout most of the remaining field, but has a reduced intensity near the periphery, as shown schematically in Fig. 4.1.1.

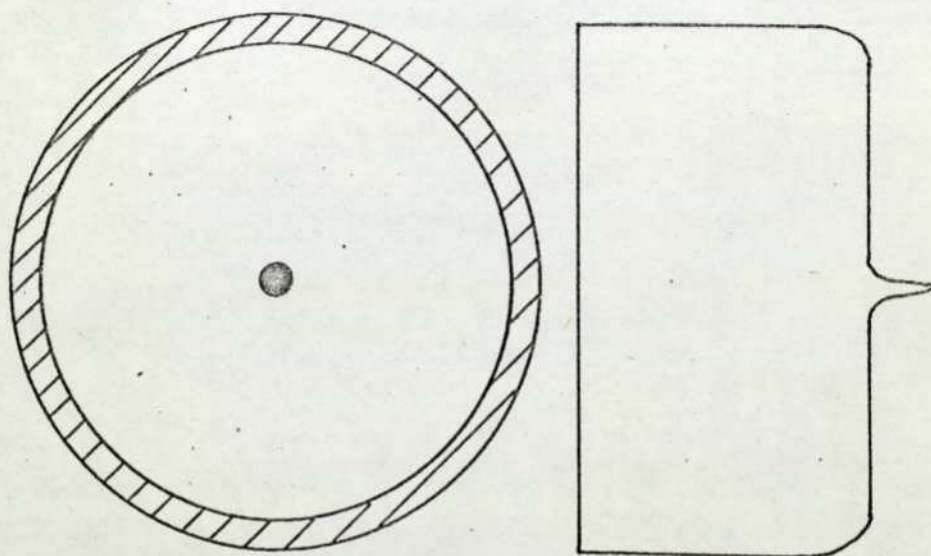


Fig. 4.1.1 Non-uniformity of illumination at the camera image plane.

The non-uniform illumination along the periphery is due to diffraction from the edges of the optical elements of the bench. It is not detrimental because its net effect is only to reduce the size of the field slightly. Irrespective of its shape, the model can always be easily placed in the field so as not to include any portion of the periphery.

The intense spot at the centre of the field, although it covers only a small area, is not desirable. In most situations it will not be possible to place the model in the field without including the central region, unless the model size is very small compared to the field. The central non-uniformity is found to be more severe in bright background than in the dark field set-up of the polariscope.

If the model includes the central region, the intense spot will disturb the fringes in its neighbourhood, and therefore regions of higher order fringes are preferably kept away from the centre. In the classical evaluation of the centre lines of dark and bright band as isochromatic fringes of half or integer order and in conventional compensation methods, the effects of the central spot are not very severe for lower order fringes.

However, non-uniformity at the centre seriously affects all the intensity methods (Photometric methods) and the density methods of Schwieger and Haberland (1956), Thadani (1972), Chiang and Khetan (1974), Friedrich (1974) and Meyer and Mehrotra (1974). It distorts equidensities obtained by whichever technique. The resultant intensity (or density) is the sum of the intensity due to the isochromatic pattern in a uniform field and that due to the additional intense central illumination.

Any non-uniformity in the field of observation can easily be demonstrated and checked if the fringe pattern of a beam in pure bending is photographed with different exposure times on Agfacontour or high-contrast film. In uniform light, all the photographed patterns show bands of uniform width over the relevant span of the beam. A non-uniform band width indicates a non-uniform distribution of light and the indicated effect increases with increasing exposure.

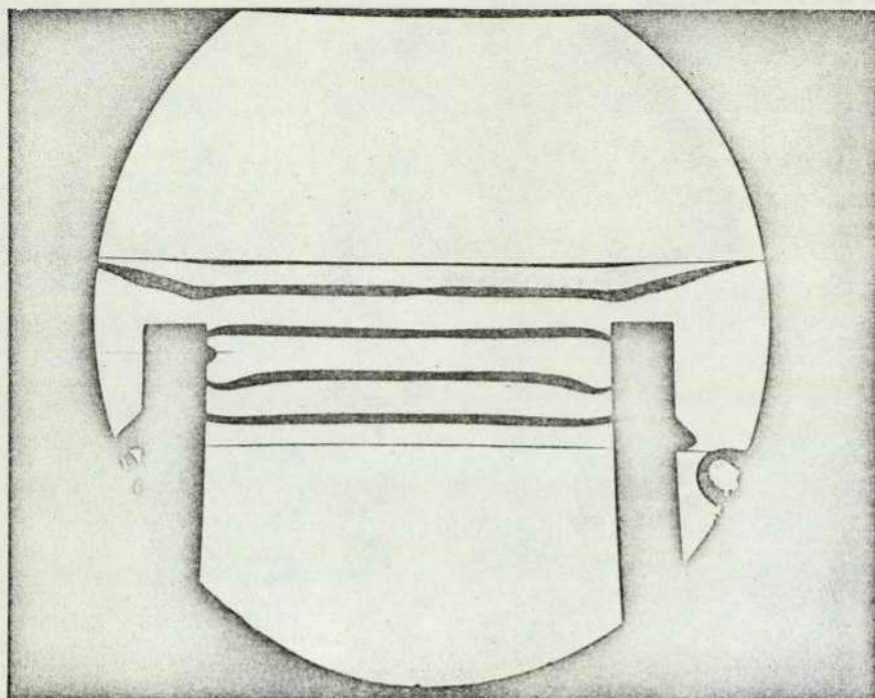


Fig. 4.1.2

3 s, f5.6

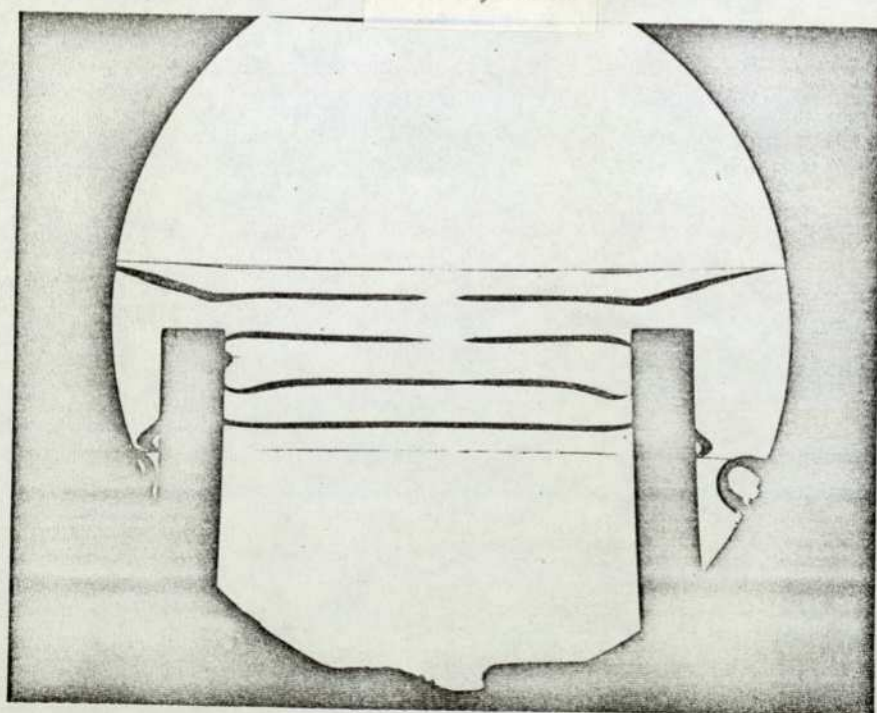


Fig. 4.1.3

5 s, f5.6

The photographs Figs. 4.1.2 and 4.1.3, taken with 3 and 5 second exposure, show the effect of the non-uniform field and its reinforcement with increased exposure in bright background.

4.2 CAUSES OF NON-UNIFORMITY

Some of the probable factors causing non-uniform illumination are:

- (a) Optical disturbances in the model,
- (b) Imperfect polaroids,
- (c) Diffraction from the edges of elements of the polariscope and camera shutter,
- (d) Internal reflection in the lens system.

Selective non-uniformity of the field will result if the optical properties of the model vary from point to point, e.g. if the transmissibility of the model is different at different points.

Normally imperfect polaroids, i.e. polaroids producing incomplete extinction of light when crossed, give a uniform background intensity of an amount depending on the wavelength of light (Shurcliff, 1962; Grabau, 1937) for a particular type of sheet used. A non-uniform background intensity will result if for some reason the transmittance of the crossed polaroids varies from one point to another in the field.

Diffraction of light from the edges of the elements of the polariscope and from the camera shutter results in a decrease in the intensity of light towards the periphery of the field. A slight reduction in the working diameter will exclude this non-uniformity.

Internal reflections (double reflection) of light from the field lenses and covering glasses of the polarisers and quarter-wave plates seem to be the most important cause of the bright spot at the centre of the field.

When light passes from air to glass or glass to air, a small percentage is reflected back. Fig. 4.2.1, where M denotes the model, FL₂ the second field lens, CL the camera lens and CA the camera, shows how the rays A incident on FL₂ are partly reflected as rays B and C.

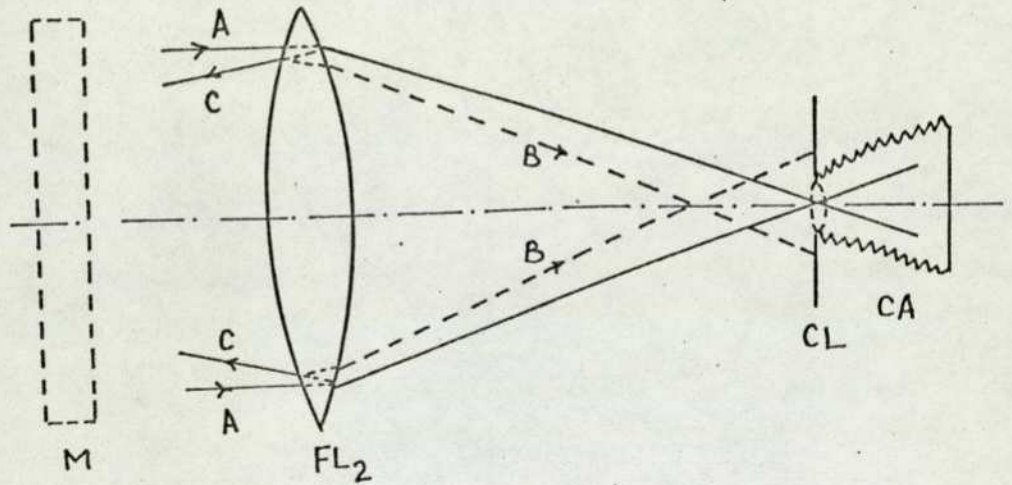


Fig. 4.2.1 Internal reflections from the field lens.

A major part of the rays C is lost to the image but a small part, B in Fig. 4.2.1., is re-transmitted, and can be shown to cause a ghost image or flare spot on the film or screen plane.

According to Encyclopedia (1969), most lenses produce ghost images if a brilliant light source is included in the field of view. These images form a particular kind of flare spot in the form of distorted images of the light source. Similarly, Ilford (1958) observes that not all the re-transmitted light is spread uniformly over the film. Some of it may form an image of the light source which is more or less out of focus. The author has in fact found on the photoelastic bench that the central spot is an image of the light source and that different images are obtained for mercury vapour and tungsten lamps.

Due to internal reflections in the first field lens on a photoelastic bench, the rays between the two lenses are not parallel. A small fraction forms oblique, i.e. converging/diverging, rays (Birch, 1975). The author has verified this fact by placing a screen behind the first field lens and slowly increasing its distance from the lens. In addition to the uniformly illuminated field, an intense spot is then gradually focussed and again taken out of focus at the centre of the field as the screen is moved away from the lens. Therefore, various planes beyond the first field lens show different effects of internal reflection depending upon whether the spot is focussed or diffused. It is thus advantageous not to place the model in a plane where the central spot is focussed.

If one looks obliquely from the polariscope field into the camera, then the total light in the plane of the camera lens manifests itself in:

- (i) a central intense light spot indicating the main converging rays giving the desired picture,
- (ii) a number of focussed light spots near the centre of the lens indicating images caused by those rays of internal reflection from the polariscope elements which are converging at the camera lens plane,
- (iii) a diffused illumination covering an extended area in the plane of the camera lens and indicating those converging rays caused by internal reflections that do not form an image of the light source at the plane of the camera lens,
- (iv) a diffused illumination covering an extended area in the plane of the camera lens and indicating divergent rays (B in Fig. 4.2.1) also caused by internal reflections.

All these four types of rays are shown schematically in Fig. 4.2.2. Looking into the camera lens obliquely, the unaided eye will see images due to the rays (i) and (ii) when the camera lens is covered by a paper or lens cap. On the other hand, illuminations due to rays (iii) and (iv) are difficult to detect by naked eye because of their extremely low intensity.

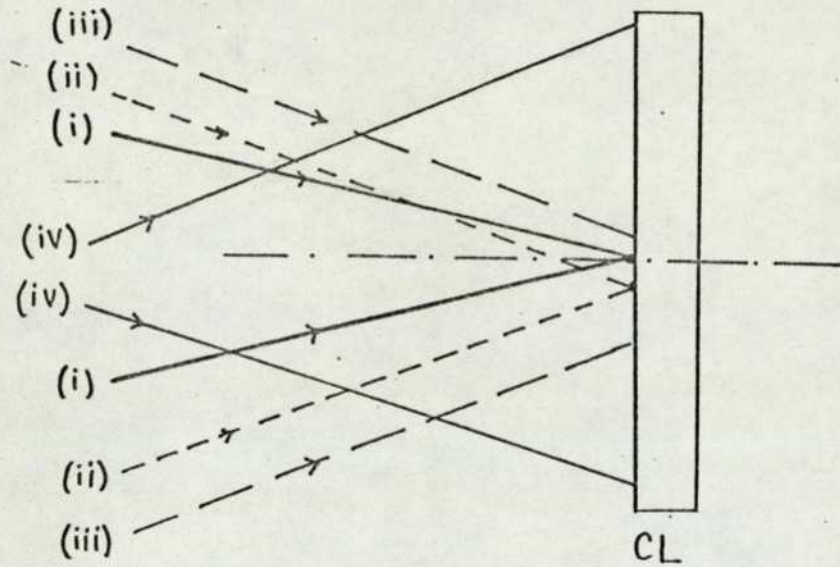


Fig. 4.2.2 Four types of rays incident at the camera lens.

The following summarises the effect which the rays (i) to (iv) have at the film plane of the camera:

- (a) The rays (i) form the main image of the model,
- (b) The rays (ii) produce stray light in the form of uniform background illumination which is superimposed on that due to imperfect polaroids (Robertson, 1965),
- (c) The rays (iii) form an image of the source just behind the camera lens and thereafter propagate as diverging rays towards the film plane where they give a uniform flare similar to that of the rays (ii),
- (d) The rays (iv) converge towards the film plane and give a number of diffused or focussed central spots depending on the position of the film plane (Birch, 1975).

Thus the central intense spot is produced by the type (iv) rays. This can be proved by the following experiment: At the camera lens plane, the main centrally focussed light spot (i), as well as the light spots (ii) and a part of the rays (iii) are blocked out by a small piece of black paper cut to correct size to cover all these images at the camera lens plane. The screen of the camera then still shows a central light spot which proves beyond any reasonable doubt that type (iv) rays exist at the camera lens plane and cause the central bright spot at the film plane.

A reduction in aperture of the camera lens thus can improve the uniformity at the centre of the field by cutting off at least some of the rays of type (iv). This has also been verified experimentally (see Section 4.3.3).

The effects of internal reflection depend on the number of air-glass surfaces in the bench. They are therefore more severe in lens type than in diffused light polariscopes and also in circular than in plane polariscopes. The influence of the number of surfaces on the reflected light and ghost images has been described by Murray (1949).

4.3 METHODS TO OBTAIN UNIFORM ILLUMINATION

As the reasons for the non-uniformity in illumination are fairly well established, certain remedial measures can be recommended: (i) models should be free from stresses and mottle, (ii) the polaroids should have maximum extinction when crossed and the value of extinction, if not absolutely zero, should be constant throughout the field, (iii) the outer field of the polariscope should not be used, (iv) lenses and other glass surfaces should be provided with antireflection coating (blooming). However, faced with a given set of polaroids and lenses without antireflection coatings, the author has used the following methods to improve the uniformity of illumination at the centre of the field.

4.3.1 Diaphragm in Front of the Camera Lens

In order to reduce the unwanted focussed and diverging reflected rays of light of the system, i.e. to improve the uniformity

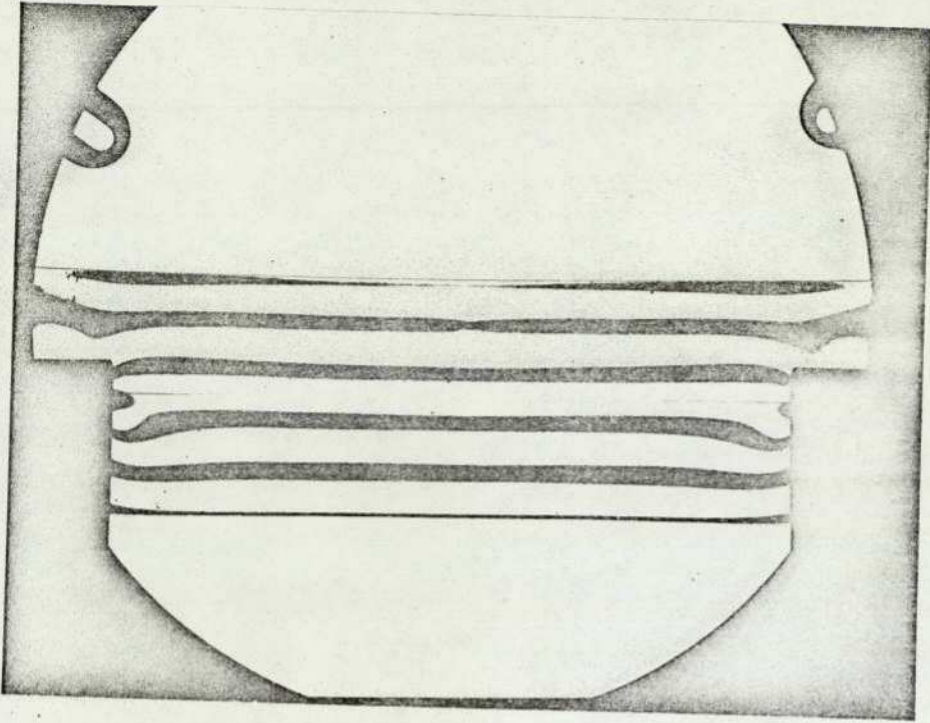


Fig. 4.3.1

Exposure time = 3 s
Without diaphragm

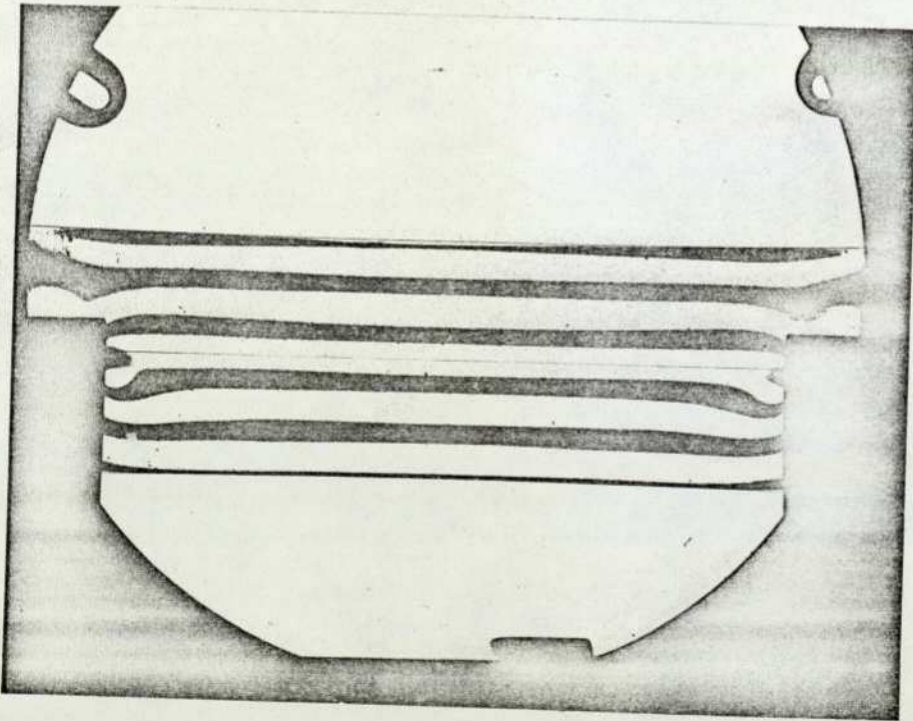


Fig. 4.3.2

Exposure time = 3 s
With diaphragm

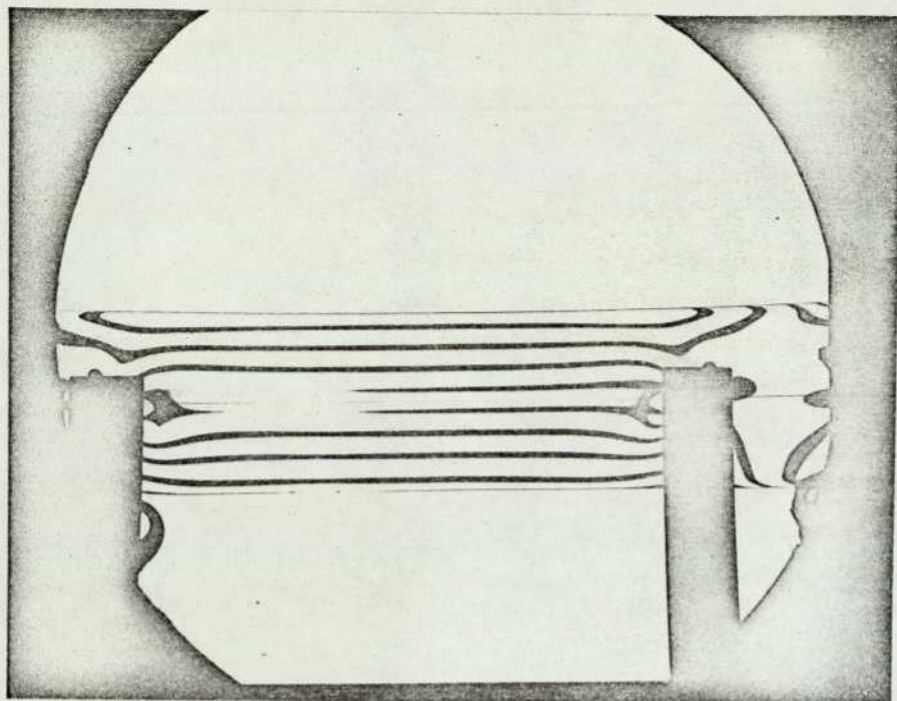


Fig. 4.3.3

Exposure time = 2 s
With diaphragm

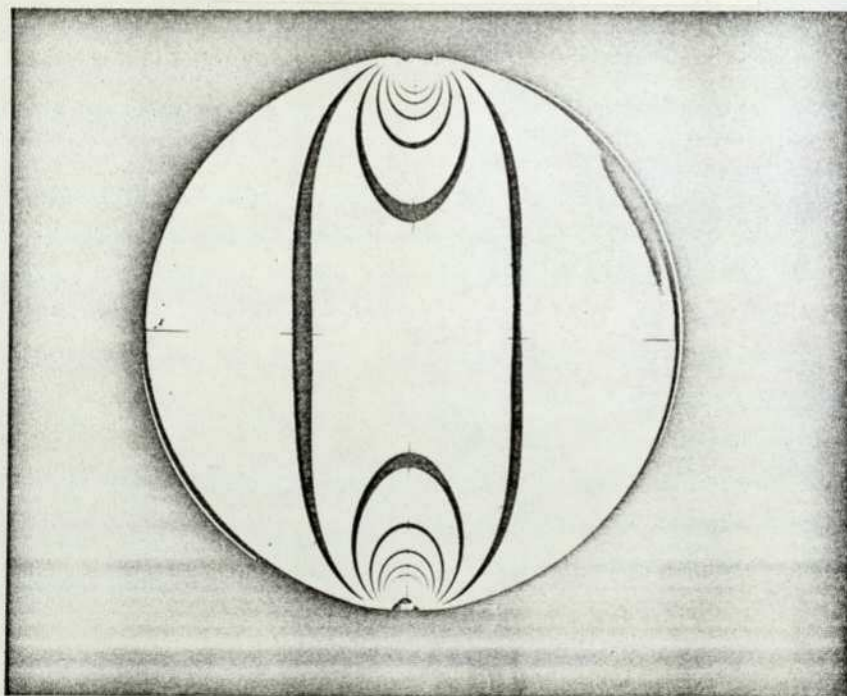


Fig. 4.3.4

Exposure time = 3 s
With diaphragm

at the centre of the field, a diaphragm was mounted in front of the camera lens as near to it as possible. The diaphragm was opened to the smallest extent that still permitted passage of the central focussed beam of light and yet reduced as much as possible the re-transmission of the reflected light to the camera film plane. The camera aperture itself was used at its maximum opening f5.6.

The improvement at the centre of the field is shown by a comparison of Fig. 4.3.1 taken without and Fig. 4.3.2 taken with the diaphragm. However, after a large number of results had been obtained, the method was finally abandoned because the general uniformity of illumination depended very critically on the position and opening of the diaphragm. Hence, although uniformity at the centre was improved, additional non-uniformities, particularly in regions outside the centre of the field, as shown in Figs. 4.3.3. and 4.3.4. could be introduced by incorrect positioning and opening of the diaphragm.

4.3.2 Talukder's Procedure

Superposition of a negative, as suggested by Talukder (1974) to improve the uniformity of illumination, was also tried. An Ilford Commercial Ortho film exposed to the field alone without the model was developed to $\gamma = 1$ and used as a filter in photographing the fringe pattern on Kodalith film. The exposure time required was about 4 to 5 times higher than in the normal experiments.

A representative result thus obtained (Fig. 4.3.5) shows

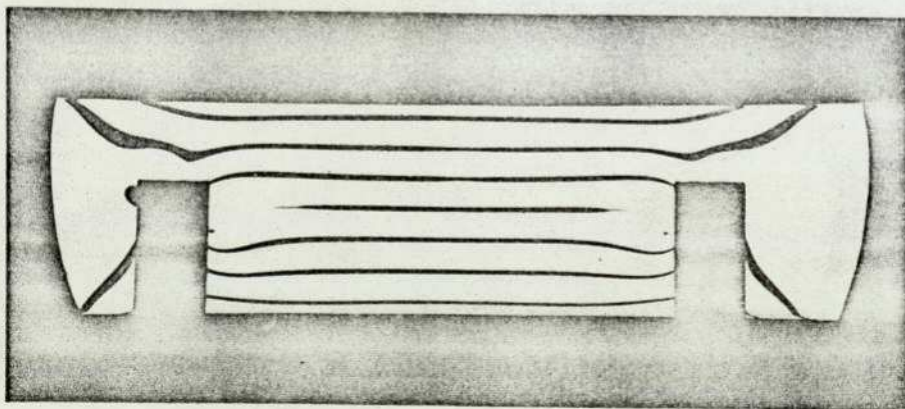


Fig. 4.3.5

30 s, f5.6

that the central uniformity is not completely removed. This may be due to (i) inaccuracy in assuming that a gamma value of unity has been obtained on the filter film and (ii) difficulties in aligning the low contrast film used as a filter with the field to be photographed on the high contrast film.

4.3.3 Small Camera Aperture

In the earlier part of the investigation, the camera aperture had been used fully open (f5.6) in order to collect the maximum available light. It had been assumed from published photographic theory that reduction in the size of the aperture by one stop would decrease the light intensity by one-half and therefore require twice the exposure time. This theoretical deduction appeared to preclude the use of a small camera aperture.

After the failure of attempts to use a diaphragm in front of the camera lens, it was decided to investigate the appearance of the uniformity at the centre of the field when smaller camera apertures were used.

This investigation has shown conclusively a definite improvement in the uniformity at the centre of the field with small camera apertures, i.e. large f numbers. In addition it has proved that the use of a small camera aperture does not lead to the loss in intensity suggested by the photographic formula.

It is shown in Appendix II that for the same exposure time, the fringe pattern of a beam in pure bending taken on Agfacontour film at an aperture of f5.6 shows a fractional fringe order of $n_d = +0.332$ whilst a reduced aperture of f45 leads to $n_d = +0.327$. This represents a loss of

$$\frac{I_1 - I_2}{I_1} = \frac{\sin^2 \pi N_1 - \sin^2 \pi N_2}{\sin^2 \pi N_1} = \frac{\sin^2 0.332 \pi - \sin^2 0.327 \pi}{\sin^2 0.332 \pi} = 2\frac{1}{2}\%$$

in the light intensity when the aperture changes from f5.6 to f45. According to photographic relationship, the intensity at f45 is supposed to be reduced to 1.5% of that at f5.6, representing a net loss of 98.5%.

Thus the relationship between exposure time and aperture as given in photographic books cannot be applied to the lens-

polariscope. The insignificant loss of light with reduced aperture is due to the fact that the size of the focussed image on the camera lens is smaller than the smallest aperture available in the camera.

The improvement with reduced aperture in the uniformity of illumination at the centre of the field can be explained by considering Fig. 4.2.2. As the aperture is reduced, some of type (iii) and an increasing portion of the diverging reflected rays of type (iv), the main cause of the non-uniformity at the centre, are blocked from reaching the film plane (see Fig. 4.2.2) and this results in an increasingly uniform illumination at the centre.

When the light is focussed on the plane of the camera lens, a reduced aperture results in a reduction of the total available field because the rays near the periphery of the field are blocked by the camera shutter. This is not undesirable if the field available at the smallest camera aperture is sufficient to cover the whole model. The author has found that it is not necessary to use the smallest possible camera aperture to obtain a uniform central field. In fact, an intermediate value of aperture and hence a much larger field can be used.

Therefore, if the loss in the size of the field with reduced aperture is such that the available field does not cover the model, the next higher aperture can be used. This will increase the size of the field while maintaining at a low level the detrimental effect of the central bright spot.

It is also found that the field is not reduced in size if the light is focussed on the camera shutter instead of the camera lens. However, this remedy is similar to that of putting a diaphragm in front of the camera lens and may result in non-uniformities in regions other than the centre of the field. Therefore, light from the second field lens should always be focussed on the camera lens and not on the camera shutter.

Figs. 4.3.6 to 4.3.9 show fringe patterns in a disk under diametral compression, photographed with an exposure time of 4 sec. and with camera apertures of f5.6, f8, f11 and f22. The load is

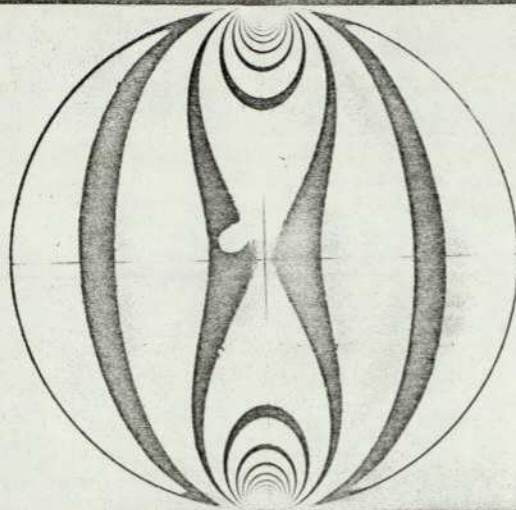


Fig. 4.3.6

4 s, f5.6

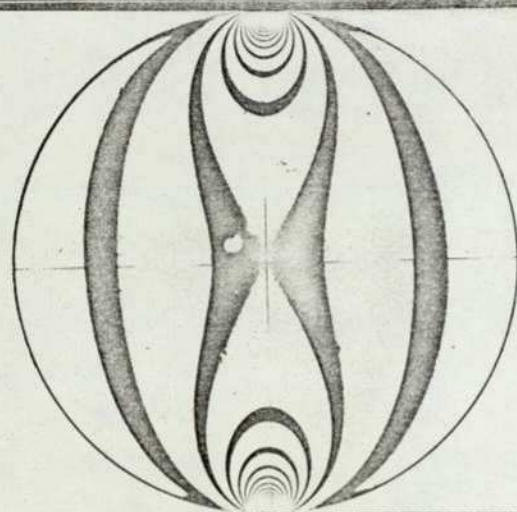


Fig. 4.3.7

4 s, f8

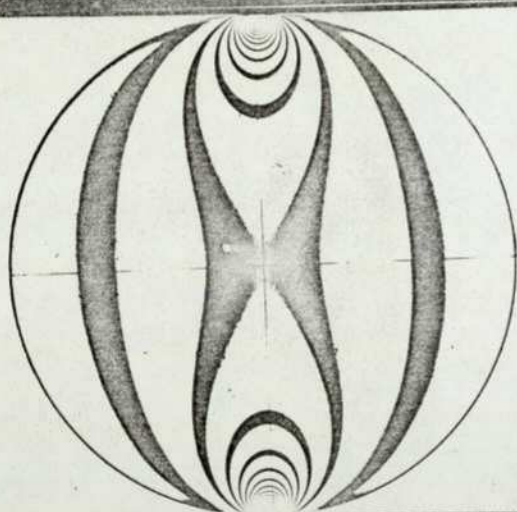


Fig. 4.3.8

4 s, f11

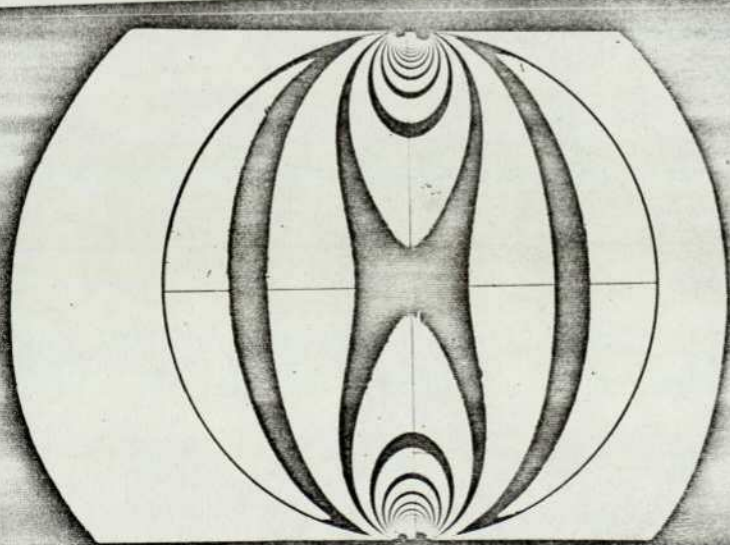


Fig. 4.3.9

4 s, f22

chosen so that the centre of the disk is in a bright region on the negative, or in a dark region on the print. It is observed that at full opening of $f5.6$ a bright spot appears near the centre on the print and that its size decreases as the aperture is reduced. It disappears altogether at a relatively high aperture opening of $f22$. Two more observations arise from these results: (i) there is a negligible loss of intensity when the aperture decreases from $f5.6$ to $f22$ and (ii) the reduction in the size of the field is very small when the aperture decreases from $f5.6$ to $f22$ because light has been focussed on the camera lens.

Figs. 4.3.10 to 4.3.13 show the fringe pattern of a beam in pure bending photographed with an exposure time of 5 sec and with camera apertures of $f5.6$, $f11$, $f22$ and $f45$. The considerable non-uniformity at the centre of the field is quite evident in Fig. 4.3.10 whilst a reasonably uniform field has been obtained at an aperture of $f22$. The model is placed slightly off the centre of the field and the progressive loss in size of the field is obvious. The diameter of the field is reduced by 5, 22 and 36% at apertures of $f11$, $f22$ and $f45$ compared with $f5.6$.

All the demonstrated results here show a bright background. It has been observed that the non-uniformity is more severe in the bright field than with dark background and that with a dark field, a fair uniformity at the centre of the field is obtained at a much larger camera aperture than with a bright background.

4.3.4 Semi-Diffused Light Polariscopes

A fairly uniform illumination at the image plane is obtained with a semi-diffused light polariscope, which is simply a lens polariscope with a diffusing screen placed between the light source and the first field lens. In the present investigation, this screen has been tracing paper. With this tracing paper, the available light intensity is reduced to about one-fiftieth of the original value. Nevertheless, the exposure time requirements are less than those for a diffused light polariscope. The optical adjustment of the various elements of a semi-diffused light polariscope is not as critical as that for a lens polariscope, and it prevents the out-of-focus image of the light source itself from being superimposed on the image of the model.

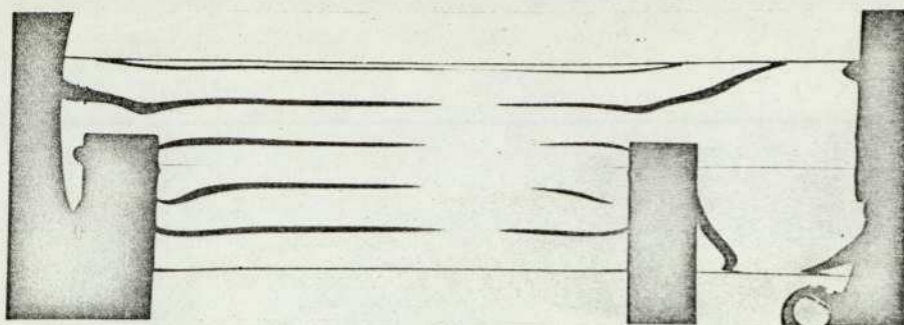


Fig. 4.3.10

5 s, f5.6

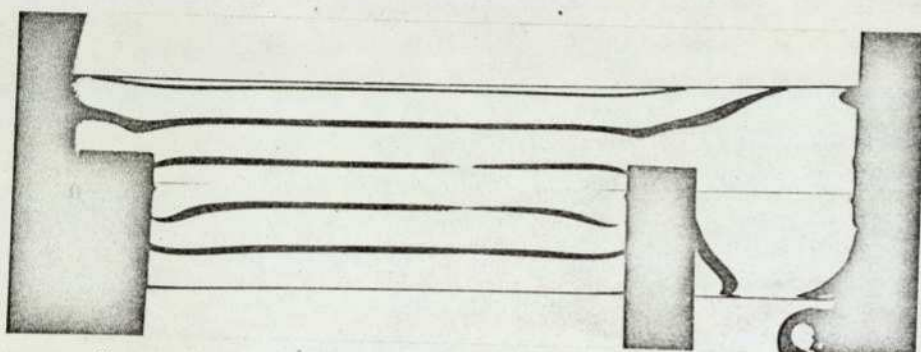


Fig. 4.3.11

5 s, f 11

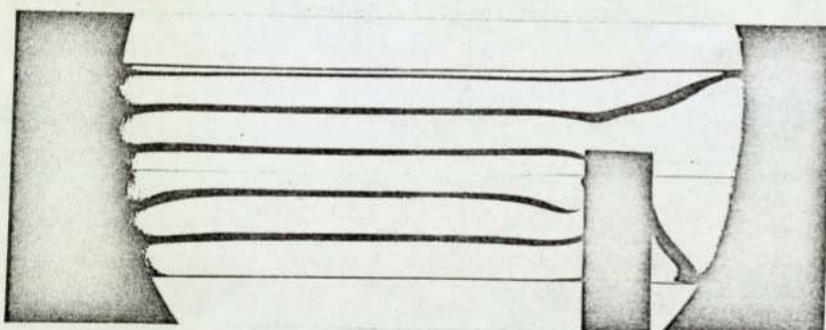


Fig. 4.3.12

5 s, f22

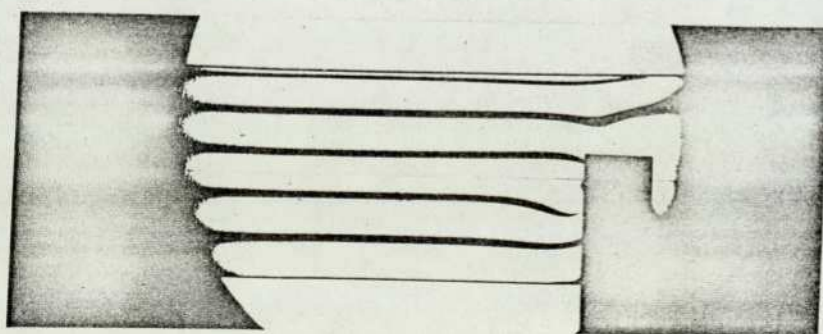


Fig. 4.3.13

5 s, f45

The semi-diffused light polariscope thus combines effectively the characteristics of a lens and a diffused-light polariscope and, as seen here, combines most of the advantages of both while cancelling out the characteristic disadvantages.

Fig. 4.3.14 shows the fringe pattern of a beam in pure bending photographed on a semi-diffused light polariscope with bright background, 360 sec exposure time, and a camera aperture of f5.6. A uniform fringe pattern is evidently obtained. This result compares favourably with Fig. 4.3.15 photographed on a lens polariscope with bright background, 5 sec exposure time, and a camera aperture of f5.6, which shows some non-uniformity near the centre of the field below the neutral axis of the beam. This proves conclusively that a semi-diffused light polariscope gives a uniform field at the camera image plane.

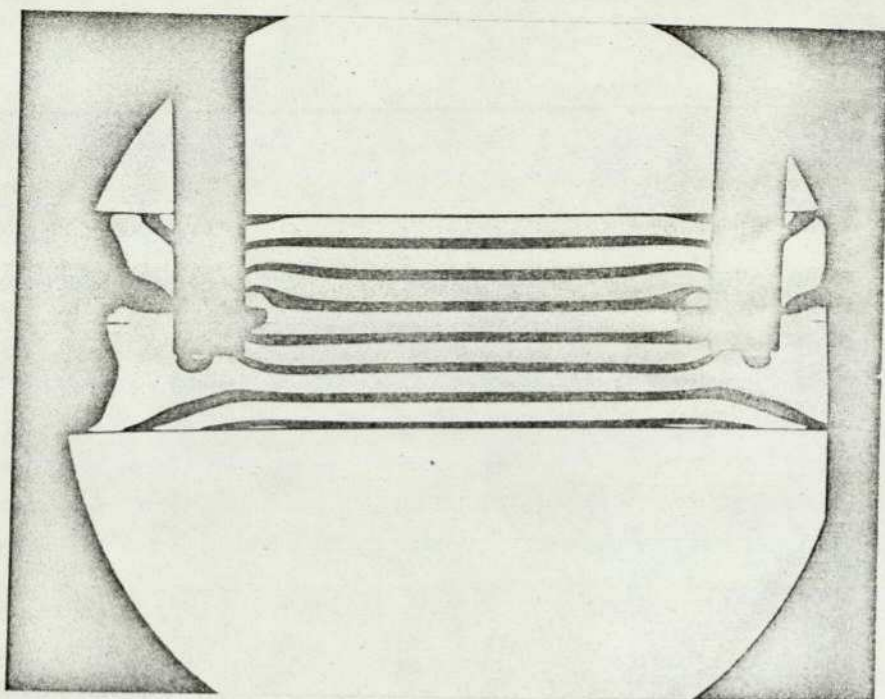


Fig. 4.3.14

360 s, f5.6

Semi-diffused polariscope

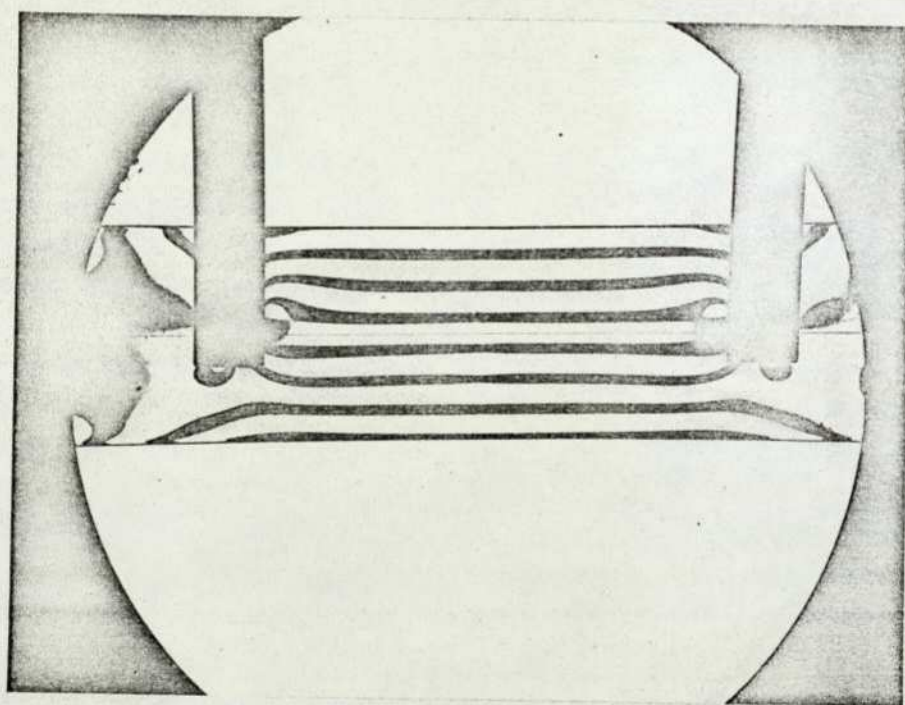


Fig. 4.3.15

5 s, f5.6

Lens polariscope

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CHAPTER 5

EQUIDENSITOMETRY IN PHOTOELASTICITY.

5.1 INTRODUCTION

Equidensitometry comprises all methods of measurement based on equidensities, i.e. points, lines or curves of equal density on a photograph. The procedure for obtaining specific curves of equal density from a photographic print or negative is known as "equidensitometering". Lau and Krug (1968) have described in detail a number of methods of producing curves of equal density, namely point by point photometering, two-layer method, Sabattier effect or pseudo-solarisation, solarisation, Clayden effect, scattered light, density-relief, etc.

The equidensity lines obtained in a photograph of an isochromatic fringe pattern in photoelasticity have functions similar to those of boundary lines between dark and bright bands described in Chapter 3. Both represent lines of constant intensity in the camera image plane and hence both are lines of constant fractional fringe order on a circular polariscope. On the density scale, a boundary line on a high contrast film always represents a density equal to the fog density of the film, whereas an equidensity line may represent any value of density between fog and saturation density.

Various applications of equidensity lines in photoelasticity have been described in Section 1.2.2. They have been produced by two different methods: (i) by pseudo-solarisation (Section 5.2) and (ii) on Agfacontour film (Section 5.3).

5.2 PSEUDO-SOLARISATION METHOD

Until very recently, the photographic procedure of pseudo-solarisation or Sabattier effect has been the most popular method of obtaining equidensity lines. Pseudo-solarisation and its application to full-field fractional fringe order measurements have been described in Appendix I which consists of a paper accepted for publication in the Journal of Strain Analysis. The experimental work on pseudo-solarisation has been carried out under the author's supervision for the purpose of an undergraduate project (Pabani, 1975).

Appendix I defines and differentiates between various terms such as solarisation, pseudo-solarisation or Sabattier effect, and Mackie lines, which have been used erratically in literature. Conventional photographic procedures for pseudo-solarisation are described and some novel improvements outlined. A number of important points associated with the process of pseudo-solarisation are also described.

Pseudo-solarisation has been applied to photoelasticity in order to obtain sharp equidensity lines in photographs of isochromatic fringe patterns. These lines represent full field fractional fringe orders of a magnitude determined by calibration on a calibration model, e.g. a beam in pure bending. The positions of the equidensity lines in pseudo-solarised photographs can be altered by using different exposure times in photographing the original.

For a given isochromatic pattern in a photoelastic model, the lines obtained by pseudo-solarisation with different exposure times and calibrated for each exposure time can be superimposed photographically on one film so that a combined pattern results which contains many pseudo-solarised lines representing as many different fractional fringe orders in the whole field.

The sensitivity of isochromatic measurement is 0.02 of a fringe or better. The results obtained are influenced by the non-uniformity of illumination in the image plane of the camera. The causes and remedies for these non-uniformities have been described in Chapter 4.

5.3 AGFACONTOUR FILM

Agfacontour, a new film developed by Ranz (1970), can be used to obtain equidensities in photographs of isochromatic fringe patterns either directly on the polariscope or by copying from an ordinary negative. The method has been described in Appendix II which consists of a paper submitted for publication in Experimental Mechanics.

Appendix II describes fully the Agfacontour film and compares its characteristics with those of a conventional film. With single

exposure and normal development, Agfacontour gives a combined negative and positive image with a transparent line interposed between the two images. Such transparent lines are first order equidensity lines similar to those obtained by the more complicated process of pseudo-solarisation (Section 5.2).

Available literature suggests that Agfacontour film is primarily used for copying from conventional negatives. However, the author has obtained equidensity lines by exposing Agfacontour film directly in the camera of a lens type polariscope with mercury lamp and green filter. Agfacontour gives a larger range of exposure times than Kodalith films, and variations in this range produce equidensities in different locations. The constant fractional fringe orders represented by these equidensities are obtained by identically exposing on Agfacontour the isochromatic fringe pattern of a separate calibration model of a beam in pure bending. Equidensities obtained with different exposure times can be combined in a single photograph following the procedure outlined in Section 5.2.

Equidensities are also obtained if a conventional black and white negative of an isochromatic pattern is copied onto Agfacontour film. The original may be a high-contrast or a low-contrast negative.

Original negatives on high-contrast film (e.g. Kodalith), in which the exposure and hence the density variations are limited to narrow regions, always give sharp first-order equidensity lines regardless of whether yellow filters are used in copying or not. Because of the small latitude in exposure of the original film variations in exposure time for copying such films do not always result in equidensities spread over the whole field.

This problem is overcome by the use of low-contrast negatives (e.g. Ilford Commercial Ortho film). The best low-contrast film for this purpose is one with a low exposure inertia and a large exposure range. It is then possible to convert all the intensity variations in the image plane of the polariscope into corresponding density variations on the low-contrast film and to obtain a wide spread of equidensities when the developed negative is copied onto Agfacontour. The disadvantage of using low-contrast film for copying

is that first order equidensities are less sharp than those obtained with high-contrast film. If no yellow filters are used, i.e. in white copying light, broad equidensities are obtained for a certain critical range of copying exposure time and equidensities fail to appear if the copying exposure time is beyond this range. On the other hand, equidensities are sharpened if yellow filters are used in copying. With properly selected exposure times equidensities can then be obtained at any desired location. Thus a single original negative on low-contrast film may yield equidensities and hence fractional fringe orders over the whole field of a photoelastic model including regions adjacent to lines of minimum intensity.

Equidensities, whether derived from high-contrast or low-contrast originals, can be calibrated very simply if a calibration beam in pure bending made from the same sheet of material as the test model is photographed under identical conditions and with the same exposure times. If corresponding photographs of beam and test model are also copied under identical conditions, the resultant equidensities represent the same fractional fringe order in both copies and their numerical values are determinable from the fringe pattern of the beam.

Agfacontour film has also been used for sharpening the isoclinics (Section 9.8) and can be used for obtaining more than one isoclinic parameter simultaneously (Section 9.10).

5.4 PRACTICAL CONSIDERATIONS

The method of equidensity lines is closely analogous to that of boundary lines. Therefore, the problems arising in both these methods are also similar. The various parameters which may affect the equidensity results are the same as those described in Section 3.7 in connection with the boundary line method. The light source can easily be made symmetrical (Section 3.7.1) and non-uniformity of illumination (Section 3.7.2) can be improved by one of the methods outlined in Section 4.3.

If a separate calibration model is used, the optical and photographic parameters (Section 3.7.3 to 3.7.8) must be maintained identical for test and calibration model. The conditions for these parameters (Section 3.7.3 to 3.7.8) can be relaxed if no calibration model is used and fractional fringe orders of equidensity

lines are established by self-calibration (Section 3.3).

The accuracy of the equidensity method depends mainly on the accuracy of locating the positions of equidensity lines in test model and calibration beam. A P.C.D. digital data reader has been used for this purpose. Since the Agfacontour negative is completely black except for the equidensity zones or lines, their positions are difficult to read on a negative. Copies on high-contrast film or paper provide well defined black lines on bright background and thus increase the accuracy of reading and reduce its physical strain.

Figs. 5.4.1 and 5.4.2 show the appearance of equidensity lines in the models of calibration beam and diametrically compressed disk.

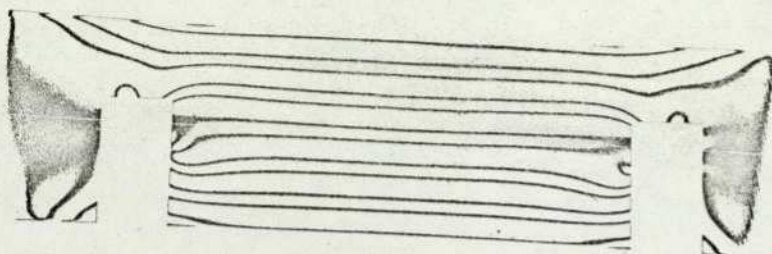


Fig. 5.4.1

Camera exposure time = $1/10$ s

Contact exposure time = 30 s

Copying exposure time = 30 s

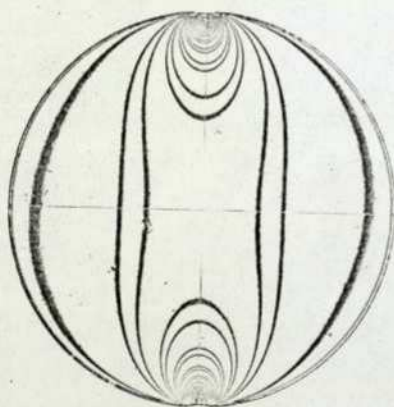


Fig. 5.4.2

Camera exposure time = $1/10$ s

Contact exposure time = 30 s

Copying exposure time = 30 s

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CHAPTER 6
SOME PROBLEMS SOLVED BY CONTROLLED
DENSITY TECHNIQUES

6.1 FRACTIONAL FRINGE ORDERS IN A DIFFUSED LIGHT POLARISCOPE

The fringe multiplication technique of Post (1955) cannot be used on a diffused light polariscope, but the described photographic methods of fractional fringe order measurement can be successfully used on it. Variations in exposure time shift the positions of the boundary or equidensity lines as in the case of a lens-type polariscope and the lines can be calibrated either by a separate calibration beam or by a self-calibration procedure.

The lens polariscope (Fig. 2.1.1b) used in the investigations was converted into a diffused light polariscope by removing the first field lens and attaching a sheet of ordinary tracing paper to the polariser. In addition to diffusing the light the tracing paper transmitted more light than a diffusing screen. Other elements of the bench such as point source of mercury light, green filter, quarter-wave plates and second field lens were undisturbed.

Normally in a diffused light polariscope the second field lens is also removed. In the present investigation the results are obtained with and without the second field lens in the bench. The second field lens directs the light towards the camera lens in a guided path so that the exposure time is considerably reduced and model material showing creep, such as CR-39, can be used.

For the present investigation, the diffused light polariscope offers an advantageously large latitude in exposure time and a reduced non-uniformity of illumination, but the fringe order at any point of the film is the average of the order over a small area surrounding the corresponding point in the model.

6.1.1 The Diffused Light Polariscope without a Second Field Lens

Models of a beam and a disk were cut from the same sheet of Araldite for an investigation of their fringe patterns in a diffused light polariscope without a second field lens. Araldite was chosen instead of CR-39 because of the large exposure times required.

A bending moment of 103.3 lbf-in (11.76 Nm) was applied to the beam. Exposure times of 2, 3, 5 and 10 minutes in a dark-field monochromatic set-up resulted in boundary lines of fractional fringe orders of 0.226, 0.217, 0.144 and 0.107. Figs. 6.1.1 and 6.1.2 show fringe patterns for 3 and 10 minutes exposure. The corresponding fringe patterns of a disk under 200 lbf(896N) compression taken with 3 and 10 min. therefore also show fractional fringe orders of 0.217 and 0.107 (Figs. 6.1.3 and 6.1.4).

The shift in the position of the boundary lines with exposure time is evident. A loss in discrimination with increased exposure time is clearly seen in Fig. 6.1.4 where the fringes are completely lost near the loading point, whilst for the shorter exposure of Fig. 6.1.3 these fringes are very distinct. On the other hand, Figs. 6.1.1 and 6.1.2 show that the illumination tends to fall off towards the outside.

6.1.2 The Diffused Light Polariscope with a Second Field Lens

With the shorter exposure times on this polariscope, the fringe patterns in CR-39 beam and disk models can be studied. Exposure times here range from $\frac{1}{2}$ to 4 minutes. Hence if small differences in transmission of light through CR-39 and Araldite are ignored, about 3 to 4 times more light is available with a second field lens than without it. As the polariscope also affords a large latitude in variations of exposure time and since the loads can also be varied, a dark background is used exclusively.

The thickness of the model is 0.1998" (0.507 cm), the width of the beam 1.305" (3.31 cm) and the diameter of the disk 3.712" (9.43 cm). The distance between model and camera lens is 18.5" (47 cm) and that between model and film plane is 24.8" (63 cm). The focal length of the camera lens is 5.9" (15 cm).

The beam is subjected to a bending moment of 88.6 lbf-in (10.1 Nm) and, in order to eliminate creep effects, photographs of fringe pattern are always taken 20 minutes after the instant of loading. The material fringe value at this instant is 93.6 lbf/in-fringe and remains pretty well constant for the four consecutive photographs taken with the dark background and 1, 2, 3, 4 minutes exposure. This indicates that creep effects are negligible during

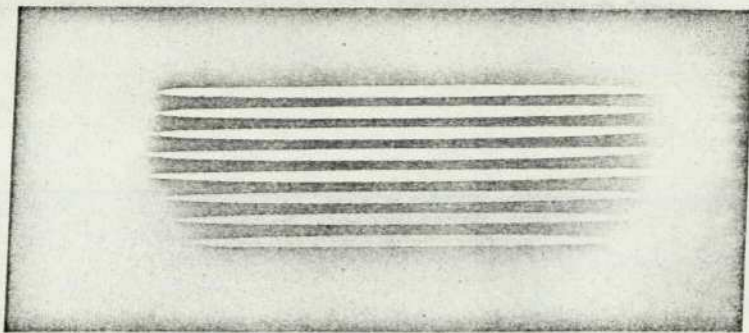


Fig. 6.1.1

Exposure time = 3 min

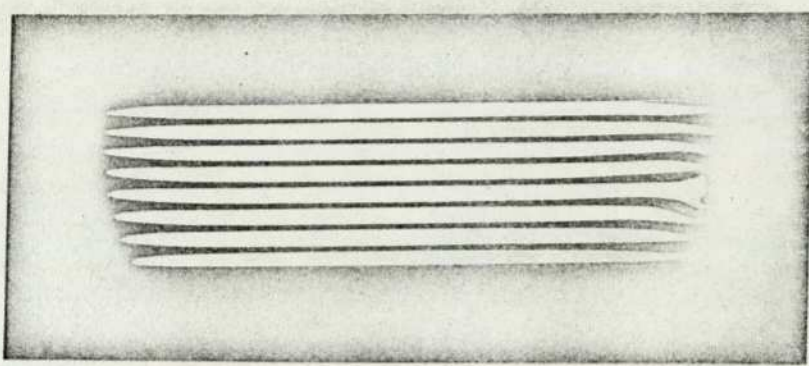


Fig. 6.1.2

Exposure time = 10 min

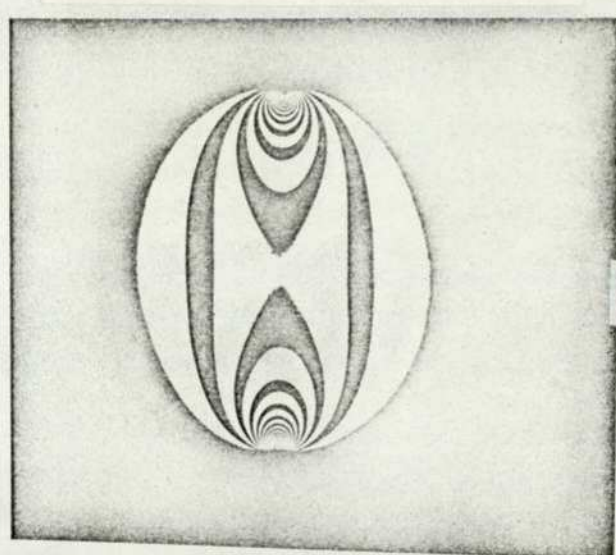


Fig. 6.1.3

Exposure time = 3 min

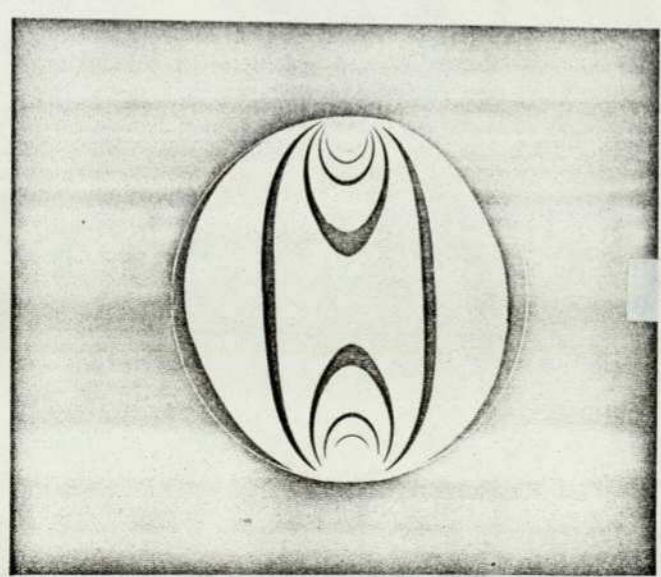


Fig. 6.1.4

Exposure time = 10 min

the 10 minutes of the experiment provided 20 minutes have elapsed before the first photograph is taken. The fractional fringe orders in the four photographs are found to be 0.225, 0.144, 0.125, 0.099 and Figs. 6.1.5, 6.1.6 show representative fringe patterns obtained for 2 and 4 minutes exposure.

The disk is diametrically compressed by loads of 40, 45, 50 and 55 lbf (178, 200, 222 and 245 N) applied at a leverage of 5.0. With an assumed material fringe value of 93.6 lbf/in, the theoretical fringe order N_0 at the disk centre then are 1.47, 1.65, 1.83 and 2.02.

Fringe pattern have been photographed again 20 minutes after load application and Figs. 6.1.7., 6.1.8. show representative fringe patterns obtained for a compressive force of 200 lbf (890 N), dark background and exposure times of 2 and 4 minutes. Hence the fractional fringe orders of the boundary lines are 0.144 for Fig. 6.1.7. and 0.099 for Fig. 6.1.8. If the positions of the intersections of boundary lines with the horizontal diameter ($k_2 = 0.0$) are determined on an X - Y reader, the experimental and theoretical fringe orders at these intersections are in good agreement as shown in Table 6.1.1.

Table 6.1.1

Comparison of theoretical and experimental fringe order along $k_2 = 0.0$ for a disk under diametral compression.

k_1	0.296	0.320	0.408	0.432
Experimental Fringe Order	1.144	1.099	0.901	0.856
Theoretical Fringe Order	1.133	1.086	0.906	0.849

Thus the proposed method of full field fractional fringe order measurements by boundary lines can be applied on a diffused light polariscope. No creep correction is required, even for photographs taken 20 minutes after loading, if the material fringe value is also obtained for the same time after loading.

6.2. OPTICAL CREEP OF CR-39 MODEL MATERIAL

Since CR-39 has been used extensively, the effect of its creep on the fringe pattern has here been measured. If results from

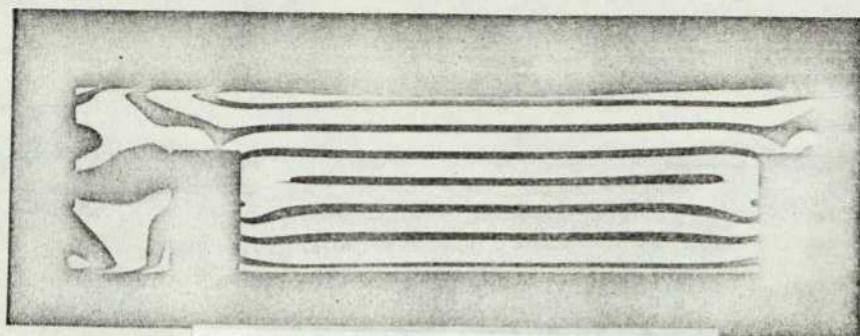


Fig. 6.1.5

Exposure time = 2 min

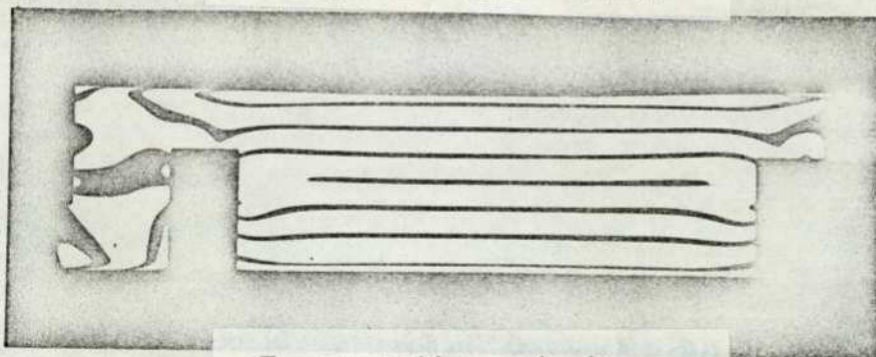


Fig. 6.1.6

Exposure time = 4 min

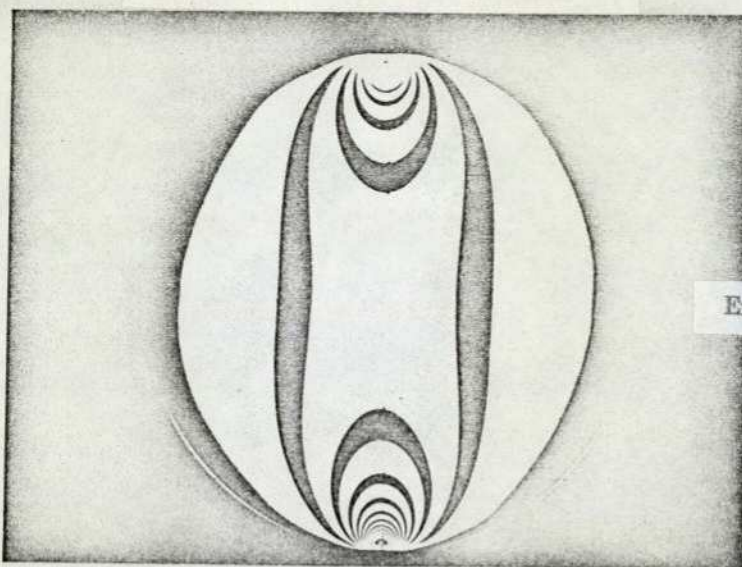


Fig. 6.1.7

Exposure time = 2 min

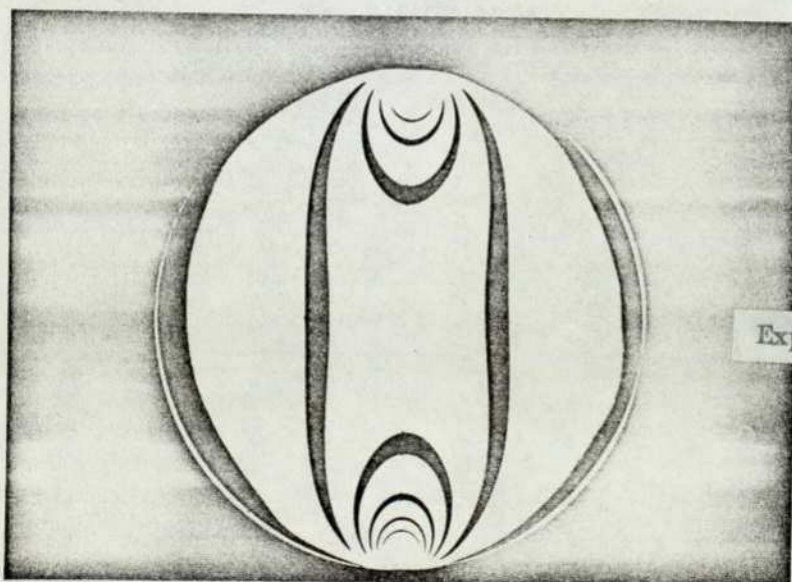


Fig. 6.1.8

Exposure time = 4 min

a CR-39 calibration beam model are to be applied to a disk model, the exposures for the two models should preferably be made at identical times after the instant of loading and the maximum fringe orders (except perhaps at the point of loading) should be of the same order.

In creep studies the percent increase in fringe order at various times is usually determined with respect to the fringe order at zero time. Kuno (1935) has shown that fringe orders cannot be observed at zero-time because:- (i) it is not easy to apply load suddenly without starting vibration and to observe the optical effect at the same time, and (ii) creep may be remarkably rapid immediately after loading so that a slight lapse of time in measuring may give a considerable error in the optical effect observed.

Point methods of measuring fringe orders are unsuitable for creep studies because of the time involved in taking measurements, but exposure of a film recording the fringe pattern photographically can be completed within 5 seconds of the instant of fairly rapid loading. This gives a better approximation to zero time than any other available method of fringe order observation.

A CR-39 beam in pure bending has thus been photographed at different times after gradual but rapid application of a constant load, and the maximum fringe orders have been measured with an X - Y reader. The results are given in Table 6.2.1. and plotted in Fig. 6.2.1.

Table 6.2.1.

Change in maximum fringe order at various instant after loading.										
Time after loading, mins.	0	1	2	5	10	20	30	45	60	90
Maximum fringe order	2.644	2.677	2.705	2.728	2.793	2.834	2.838	2.872	2.886	2.910
Percent change	-	1.2	2.3	3.2	5.6	7.2	7.3	8.6	9.1	10.1

It is seen that the rate of change of fringe order with time is high up to about 20 minutes after loading and then slows down

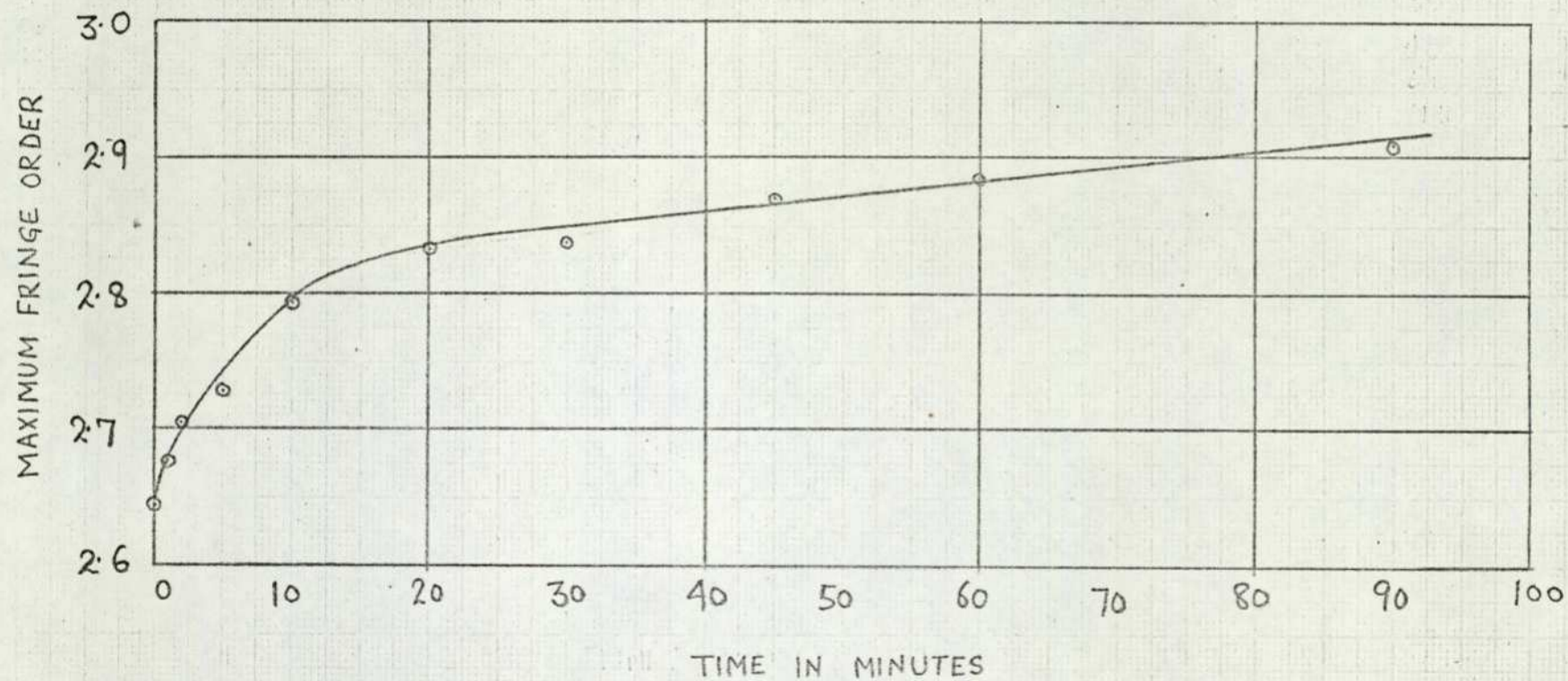


FIG. 6.2.1.
CREEP OF CR-39

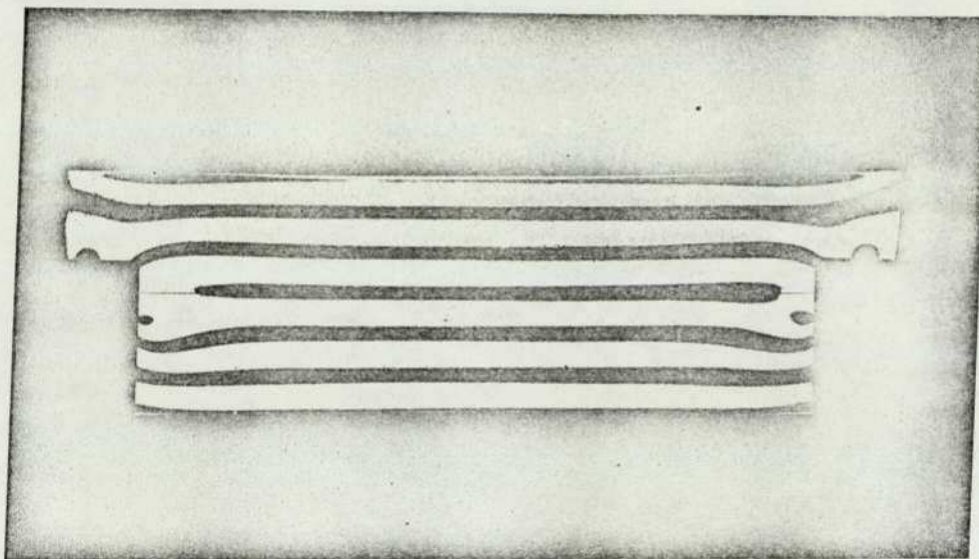


Fig. 6.2.2

Exposure time = 1 s

Photograph taken 90 min
after load application

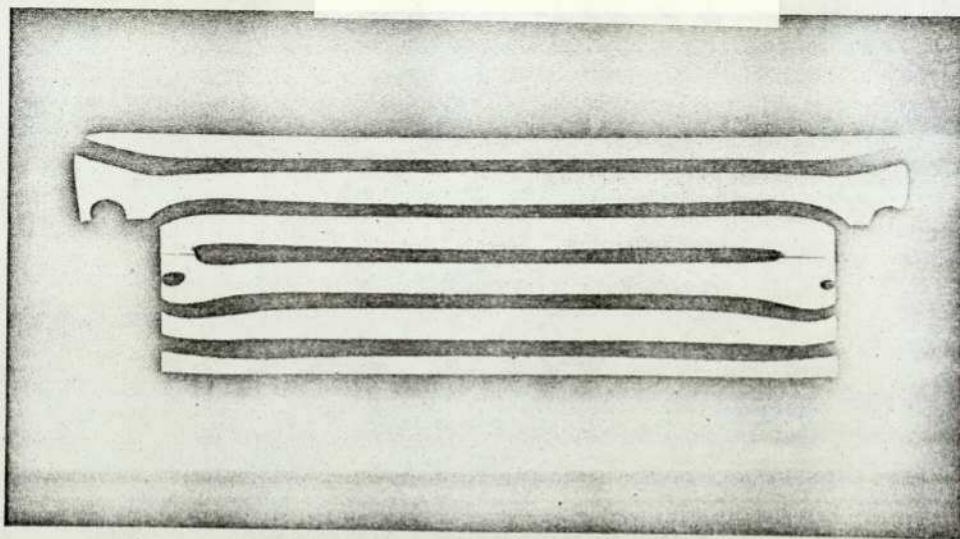


Fig. 6.2.3

Exposure time = 1 s

Photograph taken at the
instant of load application

considerably. Between 0 and 10 minutes, the rate of change of fringe order is 0.0149 per minute, between 10 and 20 minutes it is 0.0041 per minute, and between 20 and 60 minutes it is 0.0013 per minute. These values are obtained for a maximum fringe order of 2.644 at zero time.

Photographs taken at 0 and 90 minutes (Figs. 6.2.3 and 6.2.2) show clearly the shift in the positions of the fringes.

6.3 FRINGE ORDERS UNDER LOW LOADS

The methods of fractional fringe order presented here give results even where the maximum fringe order over the whole field is less than $\frac{1}{2}$ so that no discrete fringes are seen in dark or bright background. As an example, a beam under pure bending has been subjected to a small load giving a maximum fringe order less than $\frac{1}{2}$.

Photographs were taken in dark field with exposures of 1, 2, and 5 sec. (Figs. 6.3.1, 6.3.2 and 6.3.3). Obviously the resultant fringe patterns cannot be used for calibration but for the exposure times used, the previously calibrated fringe order of the boundary lines are known to be 0.172, 0.117 and 0.074.

Thus by variations of exposure time, such low fringe orders over the whole field can easily be determined. Any asymmetry seen in the fringe patterns of Figs. 6.3.1 to 6.3.3 is due to the presence of initial stresses (see next Section).

6.4 DETERMINATION OF INITIAL STRESSES IN MODELS

If an unloaded model is observed in a polariscope, the fringe order throughout the field is not zero. Due to combined effects of the manufacturing process, machining stresses and time edge effects, practically all models have some initial stresses along the edges. In most situations it is rather difficult to eliminate these stresses completely. Hence, for accurate analysis and particularly where boundary stresses are required, these initial stresses must be determined, preferably before loading of the model because after unloading the initial stresses may have been modified by interaction with stress due to relaxation of the model.

Since the methods presented here can be used to obtain fringe orders with a maximum of less than $\frac{1}{2}$, they have been applied

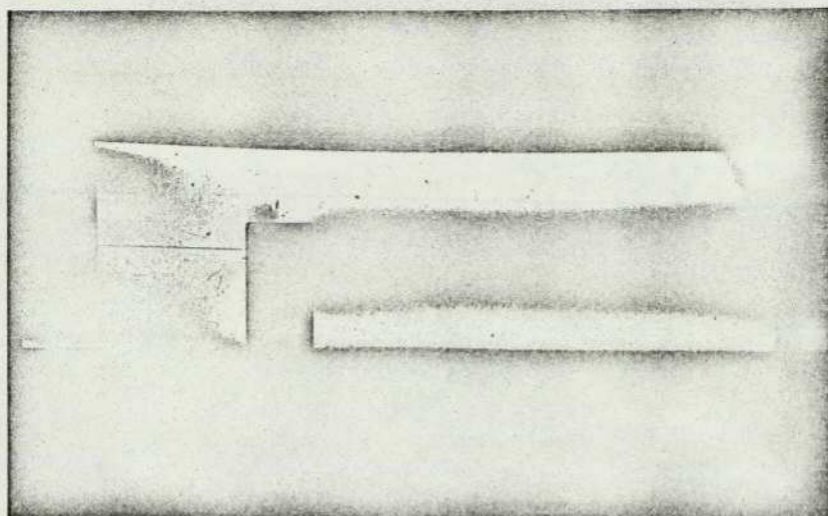


Fig. 6.3.1

1 s, f5.6

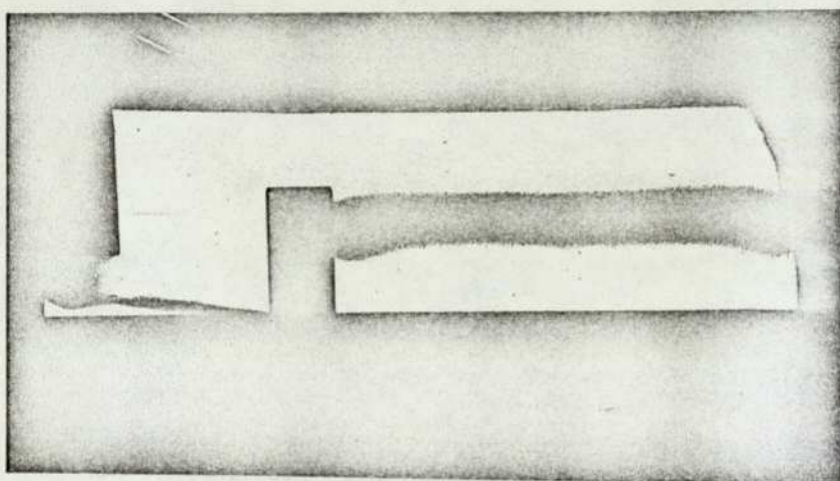


Fig. 6.3.2

2 s, f5.6

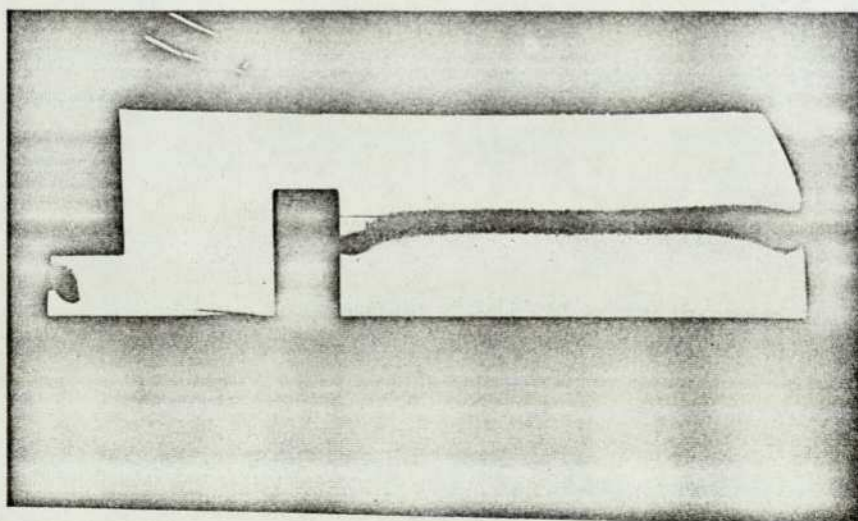


Fig. 6.3.3

5 s, f5.6

to full-field initial stress measurements by photographing the unloaded model on dark background at different exposure times.

The negative usually shows a boundary line between the dark band near the edges and a central bright area. This boundary line represents a certain fractional fringe order of a magnitude that depends on the initial stresses and on the exposure time and can be obtained by calibration.

As an example an unloaded CR-39 beam was photographed in dark background with exposure times of 1 and 2 sec. (Figs. 6.4.1 and 6.4.2). The fractional fringe orders corresponding to 1 and 2 sec. exposures are 0.172 and 0.117. Larger exposure times will shift the boundary line further towards the neutral axis. The initial stresses seen here are compressive (Frocht, 1948) and their magnitudes are observed to be unequal on the two horizontal edges of the beam.

Assuming a linear stress distribution inside the edge of the model, the maximum initial stress at the very edge of the beam can be obtained by linear extrapolation from values obtained at two or more internal points. Thus the fringe orders at the two edges of the beam of Figs. 6.4.1 , 6.4.2 are found to be 0.208 (bottom edge) and 0.22 (top edge). Alternatively, inspection of the fringe patterns obtained for a range of exposure times can give an estimate of the fringe order at the edges.

In pure bending of the beam, these initial stresses add to the loading stresses on the compression side and subtract on the tension side. As a result, the fringe order on the tension side is lower and the fringes themselves are broader than on the compression side. This effect can be observed near the edges of a beam in bending where the central portion should be symmetrical and free from this effect. (see Fig. 6.4.3). The shift in the position of neutral axis due to initial stresses has been found to be negligible.

In the solution of a general problem, it is desirable therefore to determine the distribution of the initial stresses along the boundaries of the model. The initial fringe order can then be added to or subtracted from the final fringe order

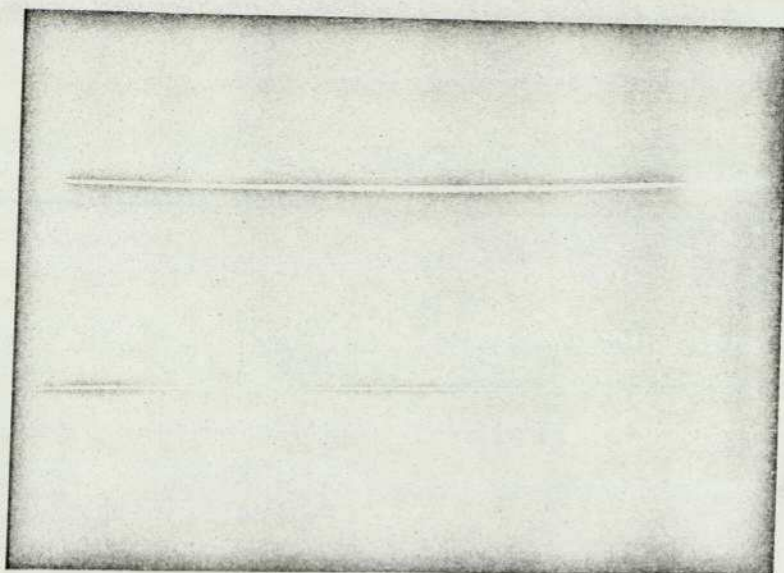


Fig. 6.4.1

1 s, f5.6
Bending moment = 0

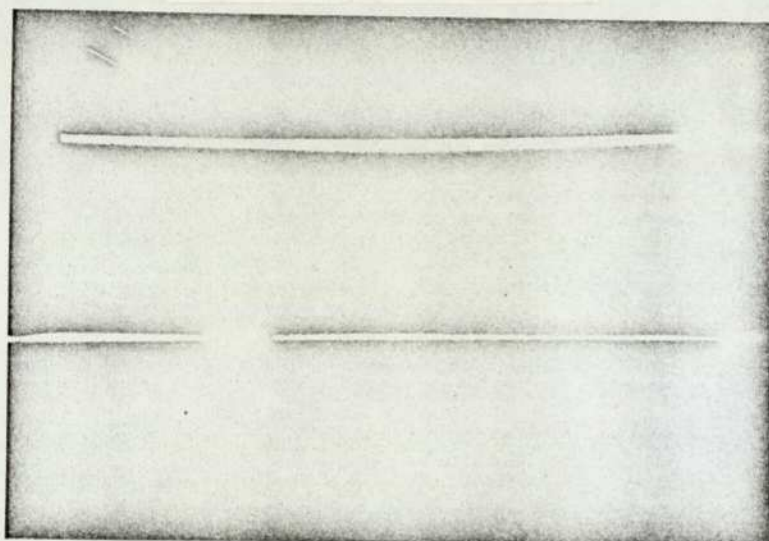


Fig. 6.4.2

2 s, f5.6
Bending moment = 0

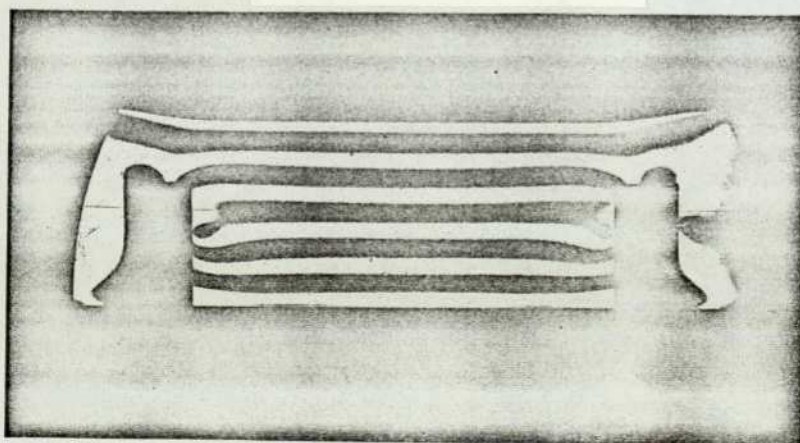


Fig. 6.4.3

1/2 s, f5.6
Bending moment = 73.8 lbf-in

observed so that the desired fringe order due to loading alone can be obtained.

6.5 EFFECT OF PRE-EXPOSURE

In pre-exposure an extended area of the film is exposed to uniform light before it is exposed to the fringe pattern. The light source may be the polariscope bright field without the model. This pre-exposure offers a number of advantages in the work presented here and in similar work.

Jones (1955) has shown that pre-exposure considerably reduces the minimum photographically detectable light energy, and a specific amount of pre-exposure (to a level at which the gradient of the characteristic curve is 0.3 times the maximum gradient) also minimises the noise level of the film.

Chiang (1974a) in his densitometric technique of fractional fringe order measurement has used pre-exposure to overcome the difficulty encountered near the intensity minima. The problems introduced by the use of pre-exposure in Chiang's approach have been described by Meyer and Mehrotra (1974a).

In the present work, pre-exposure has been used as an additional means of obtaining different fractional fringe orders, and it can be used when a cheap camera does not give the needed range of exposure times. For the available range of exposures in the camera, the use of pre-exposure helps in obtaining those fractional orders which would not be obtained otherwise.

In general photoelastic work, pre-exposure can be used successfully for fringe sharpening and this procedure will be more simple and convenient than Post's fringe sharpening method involving partial mirrors. Further, fringe sharpening by pre-exposure can be applied to both diffused light and lens-type polariscope. Although this method of fringe sharpening is fundamentally similar to that obtained by using large exposure time, it has been found that for the same maximum exposure, sharper fringes are obtained by the method of pre-exposure than by direct exposure.

Fig. 6.5.1 shows a plot of exposure against location across a photoelastic field. With E_0 as the exposure in $\circ$ ertia and E_m as the direct maximum exposure, the fringe width normally

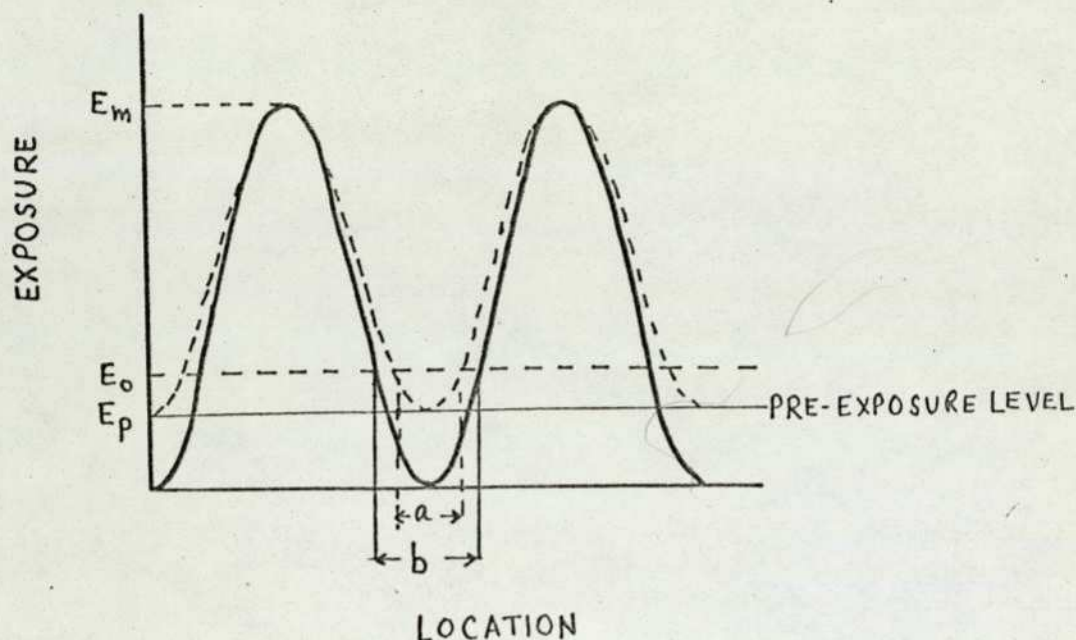


Fig. 6.5.1 Effect of pre-exposure on fringe sharpening.

obtained on high contrast film is given by the distance b . If a uniform pre-exposure $E_p < E_o$ is given to the film and the main exposure adjusted so that the maximum exposure remains E_m , sharper fringes of width $a < b$ are obtained. Their sharpness will increase as E_p approaches E_o . For $E_p > E_o$ the definition of the fringes will be lost completely.

Figs. 6.5.2, 6.5.3 and 6.5.4 show the sharpening of fringes obtained for a main exposure of $\frac{1}{2}$ sec and pre-exposures of $\frac{1}{25}$, $\frac{1}{5}$ and $(\frac{1}{5} + \frac{1}{10})$ sec. They also show that for one main exposure time a shift in the positions of the boundary lines can be obtained by varying the amount of pre-exposure.

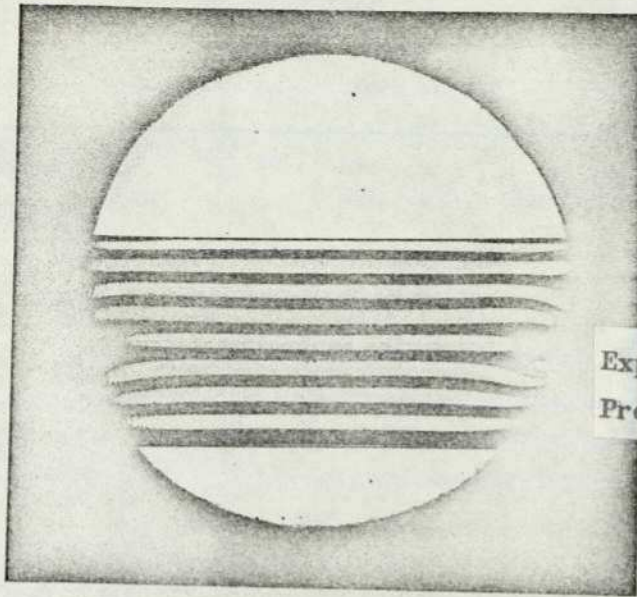


Fig. 6.5.2

Exposure time = $1/2$ s
 Pre-exposure time = $1/25$ s

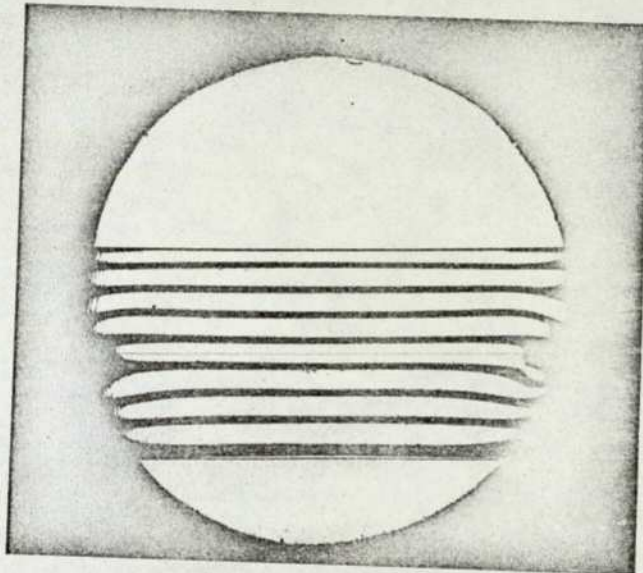


Fig. 6.5.3

Exposure time = $1/2$ s
 Pre-exposure time = $1/5$ s

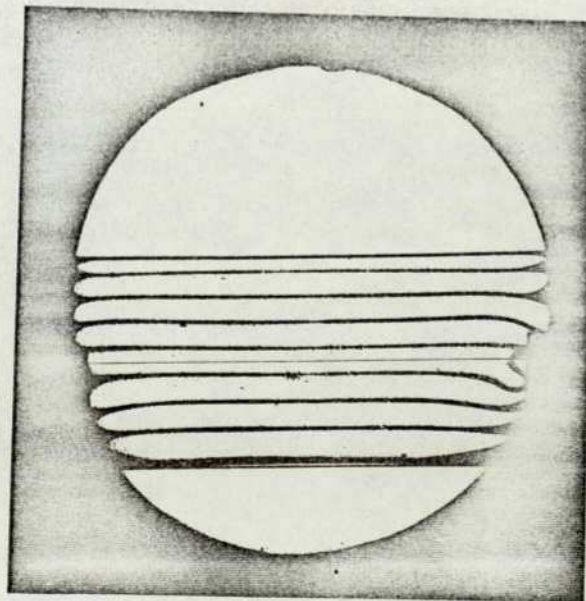


Fig. 6.5.4

Exposure time = $1/2$ s
 Pre-exposure time =
 $(1/5 + 1/10)$ s

In the equidensity work with low contrast originals, pre-exposure of the original can be used effectively to bring all the exposed points onto the straight-line portion of its characteristic curve (see Chapter 5).

6.6 STRESSES IN VARIOUS MODELS

A number of different models, all cut from the same sheet of CR-39, have been investigated by the boundary line method, namely a beam in pure bending, a solid disk under diametral compression, a diametrically compressed disk with a central hole, and a tension strip with a central hole.

The boundary lines have been calibrated in terms of fractional fringe order in the beam in pure bending for one exposure time only. The fractional fringe orders corresponding to other exposure times have been obtained from Equation (3.2.3).

The beam is 1.305" (3.31cm) high and 0.200" (0.508 cm) thick. The fringe pattern shown in Fig. 6.6.1 has been obtained 10 minutes after loading on dark background with 5 sec. exposure and a camera aperture of f16. Except for the exposure time and background, these parameters are maintained constant for all models.

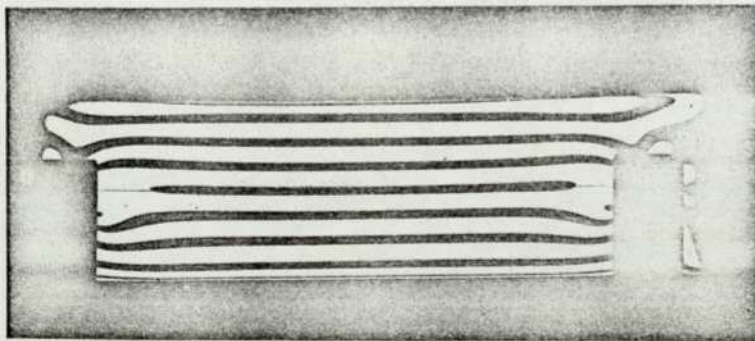


Fig. 6.6.1

5 s, f16

A maximum fringe order of 3.63 has been obtained for an applied bending moment of 103.3 lbf-in (11.67 Nm). This results in a material fringe value of 100.3 lbf/in - fringe based on the principal stress difference and on a delay of 10 minutes after loading.

The fractional fringe order at 5 sec. exposure has been found to be 0.170. Other fractional fringe orders have been obtained as follows:

If the fractional fringe orders at exposure times t_1 and t_2 are n_1 and n_2 , then

$$t_1 \sin^2 \pi n_1 = t_2 \sin^2 \pi n_2 \quad (3.2.3)$$

Hence for an exposure t_1 of 5 sec. and $n_1 = 0.170$,

$$t_1 \sin^2 \pi n_1 = 5 \times \sin^2 (\pi \times 0.170) = 1.296$$

and for any fractional fringe order n at any other exposure time t ,

$$t \sin^2 \pi n = 1.296 \quad (6.6.1)$$

Equations (6.6.1.) and (3.1.4.) have been used to compile Table 6.6.1.

Table 6.6.1

Fractional fringe orders n at various exposure times t calculated from a fractional fringe order of 0.170 at 5 sec. exposure time in dark background. n_d, n_b refer to dark and bright background respectively.

Exposure Time t_1 sec.	$\sin \Pi n_d =$ $\sqrt{1.296/t}$	Πn_d degrees	n_d	$n_b = \frac{1}{2} - n_d$
3	0.6574	41.1	0.228	0.262
4	0.5692	34.7	0.193	0.307
5	-	-	0.170	0.330
8	0.4025	23.6	0.131	0.369
10	0.3600	21.1	0.117	0.383
15	0.2939	17.1	0.0949	0.405
20	0.2546	14.6	0.0814	0.419

6.6.1. Diametrically Compressed Disk with Central Hole

The diameter of the disk is 3.992" (10.14 cm), its thickness 0.200" (0.508 cm) and the hole diameter 0.5" (1.27 cm). A compressive load of 160 lbf (712 N) has been applied and the fringe photographs have been taken 10 minutes after load application with 3, 5, 10 and 20 sec. exposure for dark background and with 3, 4, 8 and 15 sec. exposure for bright background. The fractional fringe orders under these conditions are given in Table 6.6.1.

The positions of the boundary lines have been determined on an X - Y reader along $k_2 = y/r = 0.0$ (see Section 3.4) and their fractional fringe orders are as given in Table 6.6.1., the integer orders being deduced by inspection. A plot of fringe order against $k_1 = x/r$ is shown in Fig. 6.6.2. Similar plots can be drawn for any other line on the model. The resultant increase in the sensitivity of fringe order measurements is obvious and has been obtained for one given load only. Sensitivity can be further increased if a number of different loads are used.

Figs. 6.6.3 and 6.6.4. show representative fringe patterns taken with 4 and 15 sec exposure against a bright background.

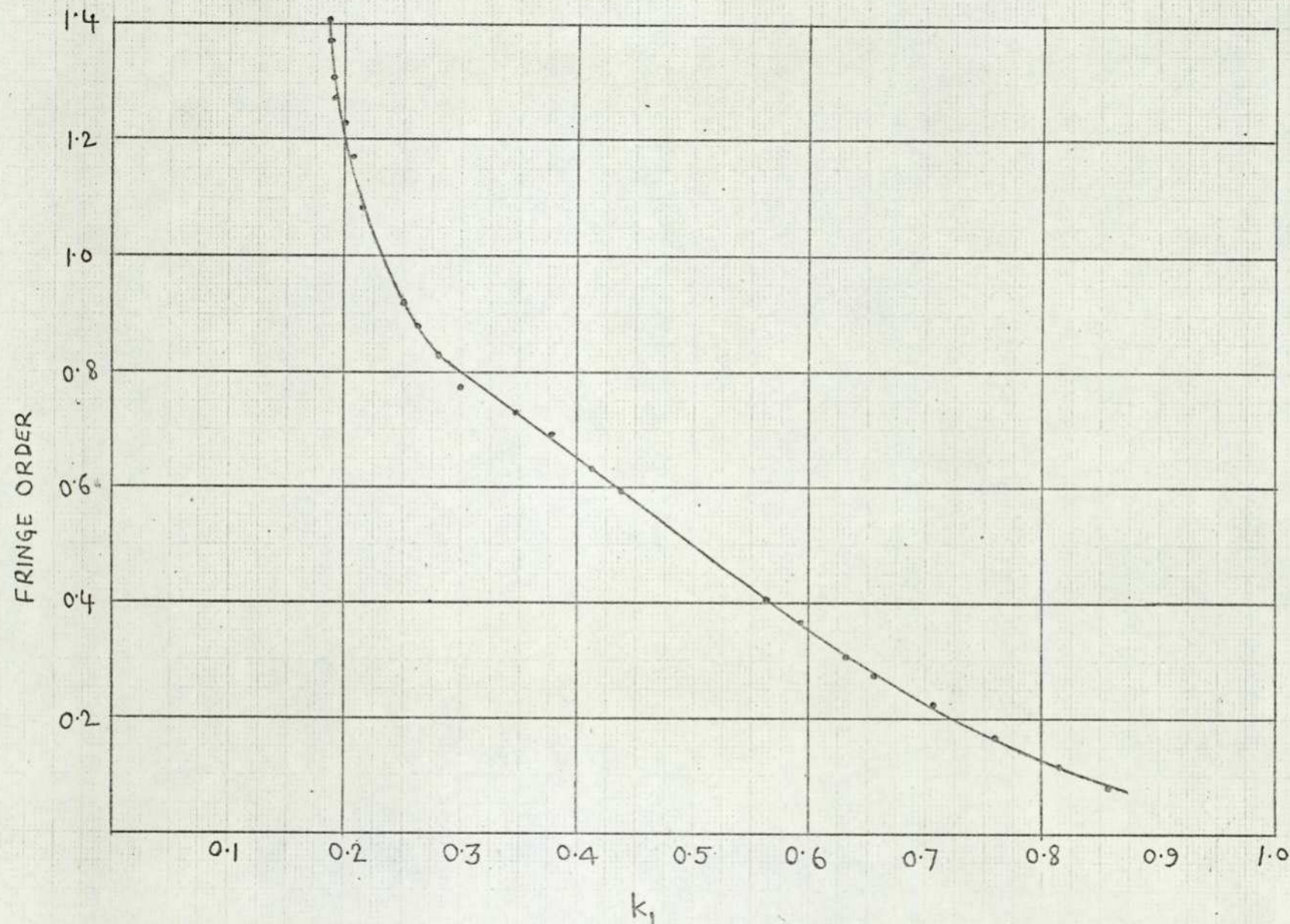


FIG. 6.6.2.

FRINGE ORDER ALONG $k_2=0.0$ FOR A DIAMETRICALLY COMPRESSED DISK WITH CENTRAL HOLE

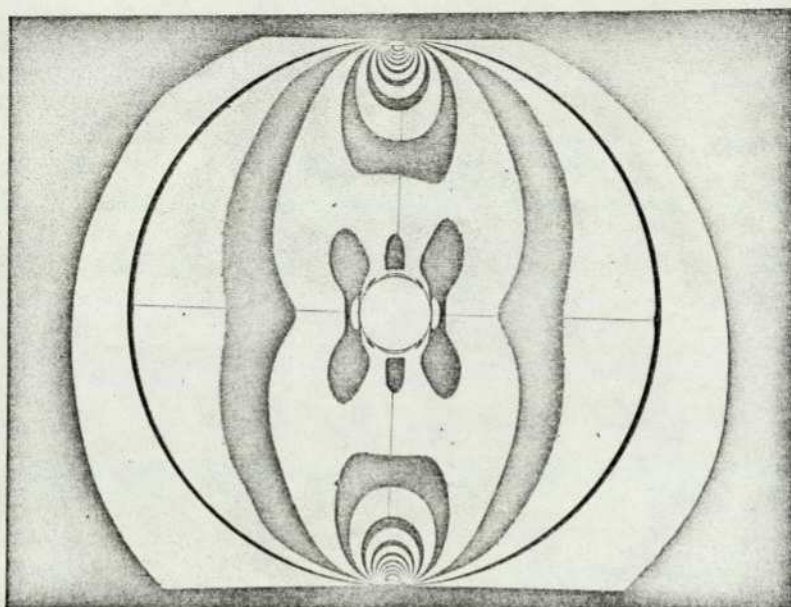


Fig. 6.6.3

4 s, f16

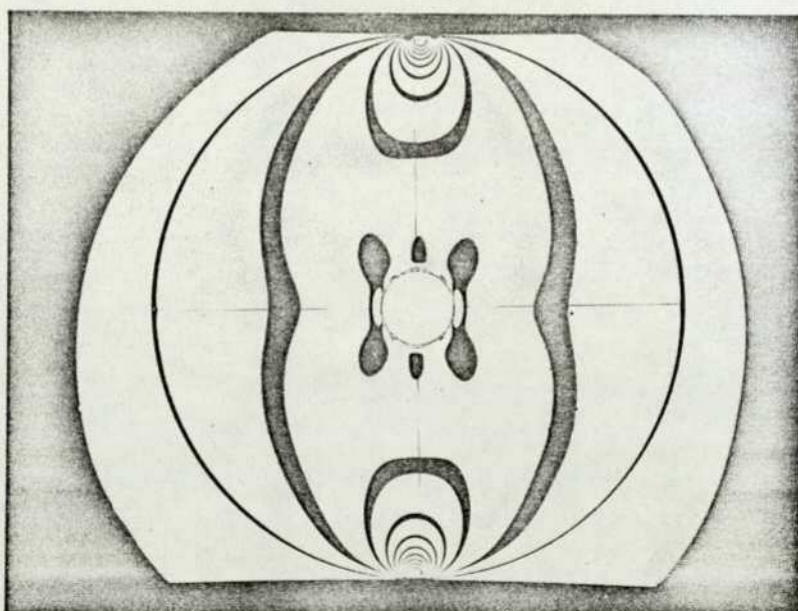


Fig. 6.6.4

15 s, f16

6.6.2 Tension Strip with Central Circular Hole

A tension strip cut from the same sheet of CR-39 as the previous models is 0.86" (21.8 mm) wide and has a central circular hole of diameter 0.50" (12.7 mm). The ratio of the hole diameter to width is therefore 0.58.

The model has dimensions that are rather small for photographic recording. A similar problem is encountered in slices of three-dimensional model (Section 6.7), but it is aggravated here because the boundaries of the model must be accurately defined for an estimate of the stress concentration factor. If the photographic image of the model on the film is small, the position of the fringe boundary lines and of the model outlines cannot be established accurately unless a highly sensitive recording instrument is used.

The problem can be solved in one of the following ways:

(i) Suitable optics inserted in the polariscope produce a large magnification of the image of the model in the polariscope camera.

(ii) The photograph of the fringe pattern is enlarged in the process of printing. In this case the negative of a fringe pattern in a beam in pure bending cannot normally serve for calibration and self-calibration must be used if the fringe patterns of tension strip and beam taken with identical exposure times are not available.

(iii) If a calibration fringe pattern has been taken with the same exposure time, the following procedure can be used:

(a) The photographs of the fringe patterns in calibrating beam and tension strip are enlarged consecutively by the same amount in the enlarger and processed together so that the fractional fringe orders on both enlarged prints are comparable.

(b) The enlargement described in (a) is made directly on Agfacontour film. This procedure has been used with three-dimensional slices (Section 6.7).

As fringe photographs of calibration model and tension strip taken on dark background with 4 sec. exposure were available, both negatives were mounted together in the negative carrier of the enlarger and printed on a single photographic paper instead of being printed separately. This was possible only because of the

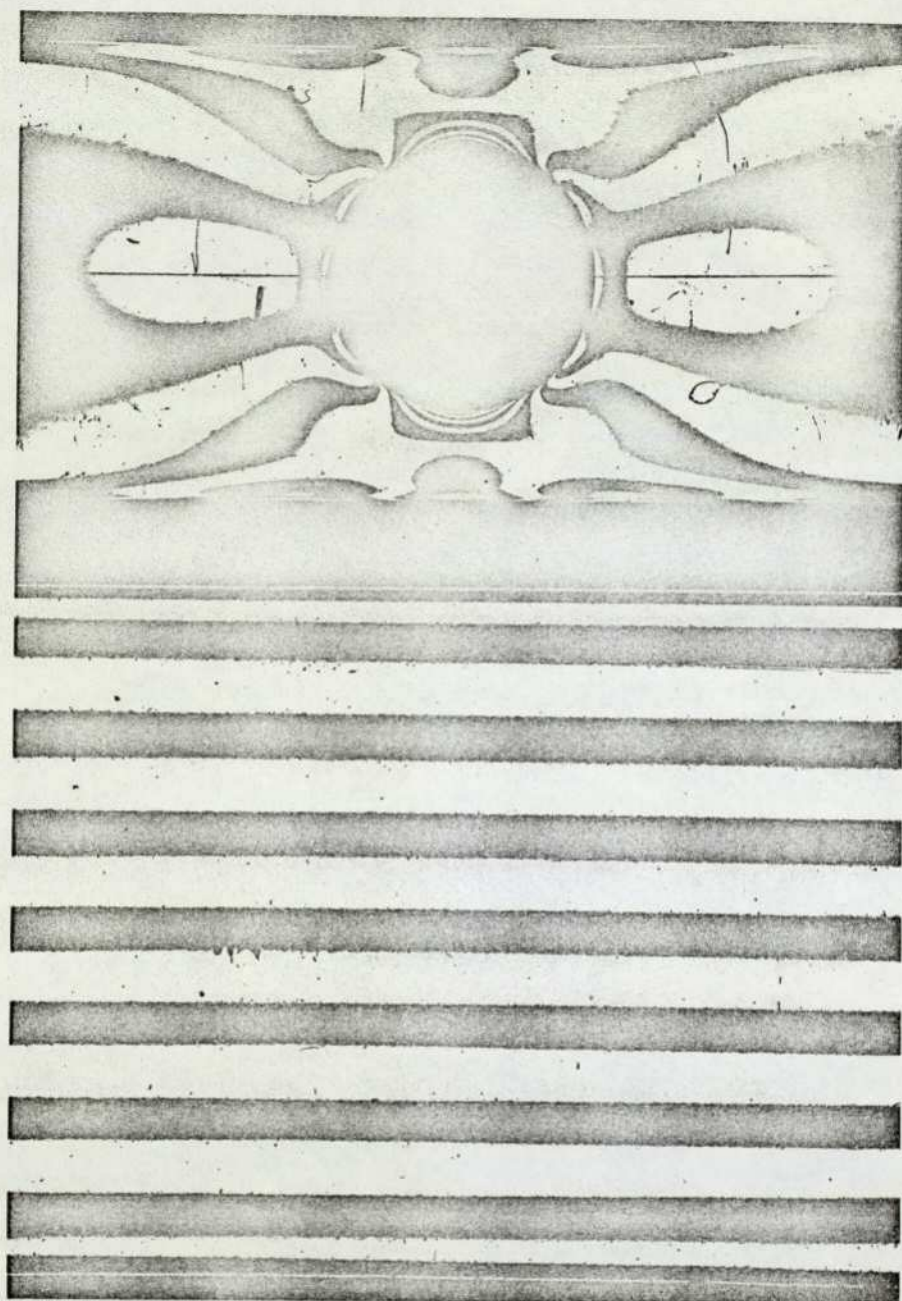


Fig. 6.6.5

4 s, f16

Enlarged 4X

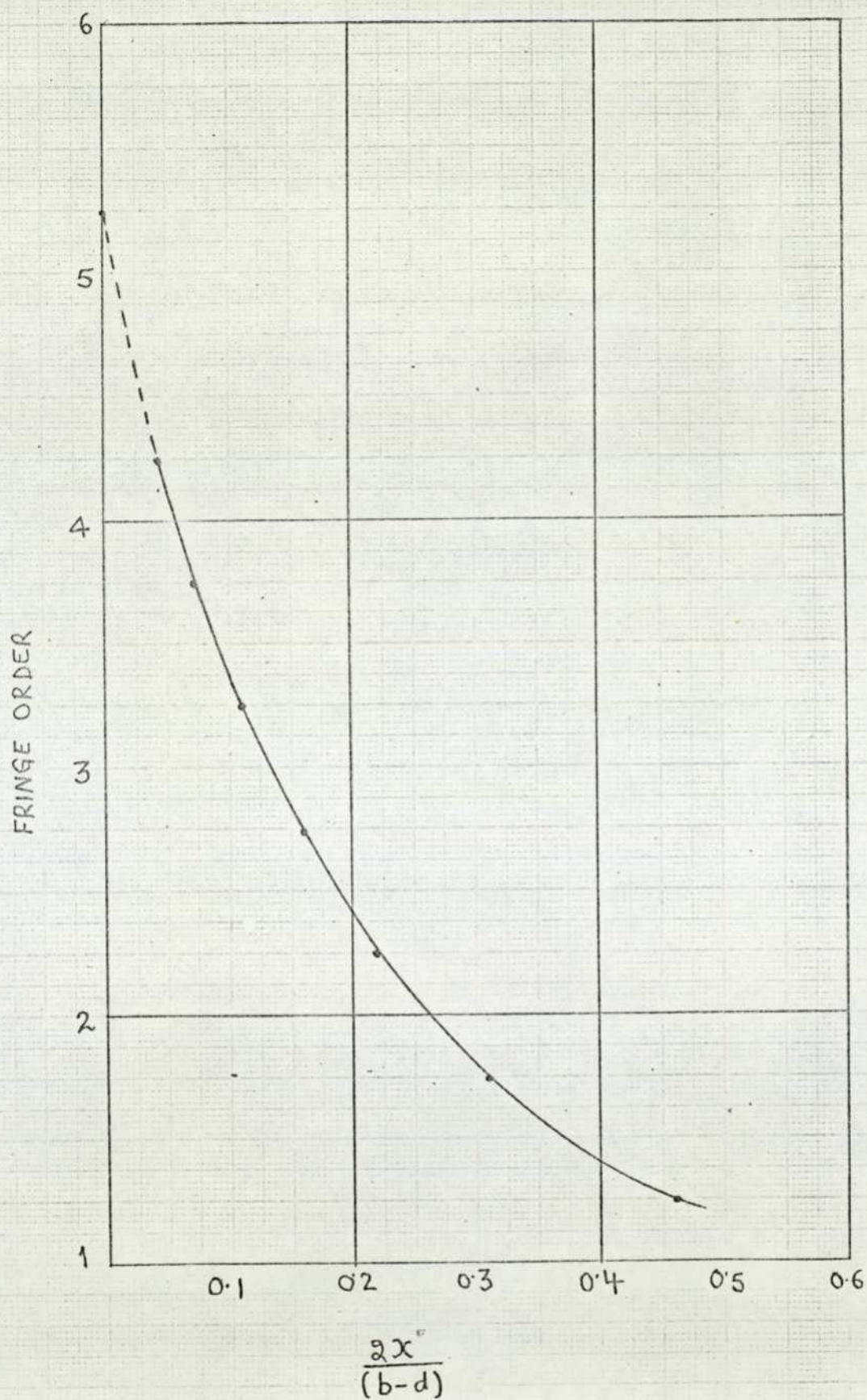


FIG. 6.6.6.

GRAPH OF FRINGE ORDER AGAINST $\frac{2x}{(b-d)}$ FOR
A TENSION STRIP WITH CENTRAL HOLE

dark background and similar efforts with bright background did not succeed.

Fig. 6.6.5 shows a print of the combined fringe patterns of beam and tension strip four times enlarged from the image on the negative. The common fractional fringe order of the boundary lines as determined from the beam pattern is 0.234.

The positions of the boundary lines across the net section of the tension strip as determined on an X - Y reader are given in Table 6.6.2. The fringe orders are plotted in Fig. 6.6.6 against a dimensionless abscissa scale $2x/(b - d)$ where b is width of the plate, d the diameter of the hole, and x the distance from the edge of the hole (Fig. 6.6.7).

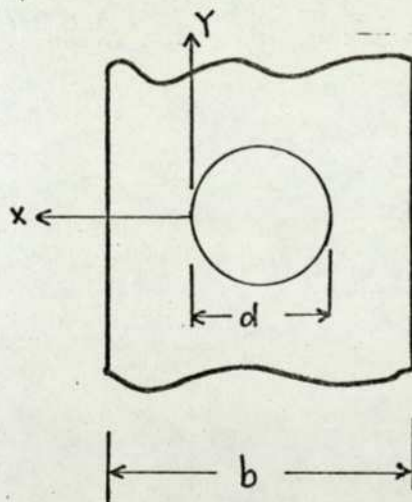


Fig. 6.6.7 Tension plate with central hole.

Table 6.6.2

Fringe orders across the net section of a tension strip.

$2x/(b - d)$	0.04	0.07	0.11	0.16	0.22	0.31	0.46
Fringe order	4.234	3.766	3.234	2.766	2.234	1.766	1.234

The values in Table 6.6.2 have been extrapolated to $x = 0$ at the edge of the holes by Newton's formula (Section 10.4.2)

$$f(x) = y_0 + (x - x_0) \delta(x_0, x_1) + (x - x_0)(x - x_1) \delta(x_0, x_1, x_2) + \dots$$

$$\dots + (x - x_0)(x - x_1) \dots (x - x_{n-1}) \delta(x_0, x_1, \dots, x_n)$$

(6.6.2)

where

y_0 is the first value at x_0

$$\delta(x_0, x_1) = \frac{y_0 - y_1}{x_0 - x_1}, \quad \delta(x_0, x_1, x_2) = \frac{\delta(x_0, x_1) - \delta(x_1, x_2)}{x_0 - x_2}$$

$$\delta(x_0, x_1, x_2, x_3) = \frac{\delta(x_0, x_1, x_2) - \delta(x_1, x_2, x_3)}{x_0 - x_3} \text{ and so on.}$$

From Table 6.6.2

$$\delta(x_0, x_1) = -16.7$$

$$\delta(x_0, x_1, x_2) = +59.5$$

$$\delta(x_0, x_1, x_2, x_3) = -264$$

$$\delta(x_0, x_1, x_2, x_3, x_4) = +1000$$

$$\delta(x_0, x_1, x_2, x_3, x_4, x_5) = -2152$$

$$\delta(x_0, x_1, x_2, x_3, x_4, x_5, x_6) = +1736$$

Substitution in Equation (6.6.2) gives

$$\begin{aligned} f(x) &= 4.234 + (-.04)(-16.7) + (-.04)(-.07)(+59.8) \\ &\quad + (-.04)(-.07)(-.11)(-264) + (-.04)(-.07)(-.11)(-.16)(1000) \\ &\quad + (-.04)(-.07)(-.11)(-.16)(-.22)(-2152) \\ &\quad + (-.04)(-.07)(-.11)(-.16)(-.22)(-.31)(1736) \\ &= 4.234 + 0.67 + 0.167 + 0.0813 + 0.0493 + 0.0233 + 0.0058 \\ &= 5.23 \end{aligned}$$

This good convergence of the series is obtained from one photograph only. A number of photographs will further increase the accuracy of the result but only at the expense of involved calculations.

The polynomial extrapolation suggested by Allison (1965) has not been used here because it would have involved, even in the present example, a solution of seven simultaneous equation.

The maximum fringe order of 5.23 is obtained for a load of 110 lbf (493 N). A load of 95 lbf (426 N) produces just one fringe in a plain section remote from the hole. Hence, the nominal fringe order in a plain strip under 110 lbf load is $110/95 = 1.16$ and, for a ratio of hole diameter to width of 0.58, the stress concentration factor based on the net section is

$$\frac{5.23}{1.16} \times \left(\frac{b-d}{b} \right) = \frac{5.23}{1.16} \times (1 - 0.58) = 1.89$$

This result has been calculated only as an example and is not claimed to be specially accurate. It shows that the technique, if used carefully, may increase the accuracy in the determination of stress concentration factors even where clearly defined model boundaries are not obtained or where the higher order fringes are poorly defined.

6.6.3 Fringe Patterns near the Loading Point of a Solid Disk under Diametral Compression.

The basic theory for contact stress distributions is due to Hertz (1881) and Thomas and Hoersch (1930). However, these theories apply to long cylinders or toroids in contact and not to flat disks where the maximum fringe order and shape of the fringes are also influenced by the type of contact area at the loading points. The problem has been investigated by Kuno (1932) and Frocht (1948a).

Frocht has shown that the maximum fringe order differs for load application through a pin or a flat bar. Kuno has proved that for a small contact area on a disk of radius R the maximum stress occurs below the central loading point at a distance $0.965 R$ from the centre of the disk. The present investigation has been made to demonstrate that the boundary line and equidensity

methods can be used to determine the maximum fringe order near the loading point when load is applied through a flat bar.

In order to study the fringe pattern near the loading point accurately, it is necessary to enlarge the fringe pattern. The difficulty of enlargement has been discussed in Section 6.6.2. For the present investigation, a calibrated pattern of fractional order $n = 0.18$ was available on Agfacontour film and its enlargement on photographic paper (Fig. 6.6.8) did not distort the positions of the equidensities.

The clarity of the equidensities near the loading point and the excellent definition of the model outlines are remarkable. The maximum fringe order occurs at a distance of about $0.035 R$ below the loading point and its magnitude is accurately 9.18 because an equidensity just emerges at the point of maximum fringe order.

If an equidensity does not emerge just at the point of maximum fringe order, a different exposure time can be used or the maximum fringe order may be estimated or extrapolated from a knowledge of neighbouring fringe orders. Between the point of maximum fringe order and the loading point the fringe orders are clearly seen to decrease but this region is extremely difficult to investigate.

6.7 FULL-FIELD FRACTIONAL ISOCHROMATICS IN SLICES OF THREE-DIMENSIONAL MODELS

If the isochromatic fringe pattern of the slices is photographed on a high contrast film, the resultant boundary lines between dark and bright bands represent fractional fringe orders. The positions of the boundary lines are shifted and their fractional fringe orders changed by varying the exposure time.

If a photograph of the fringe pattern is taken on a low contrast film, Ilford Commercial Ortho, with an appropriate exposure time, equidensity lines can be obtained by contact copying on Agfacontour film (see Chapter 5). A single original negative, if correctly exposed, is found to give equidensities over the whole field if the contact copying exposure time is varied.

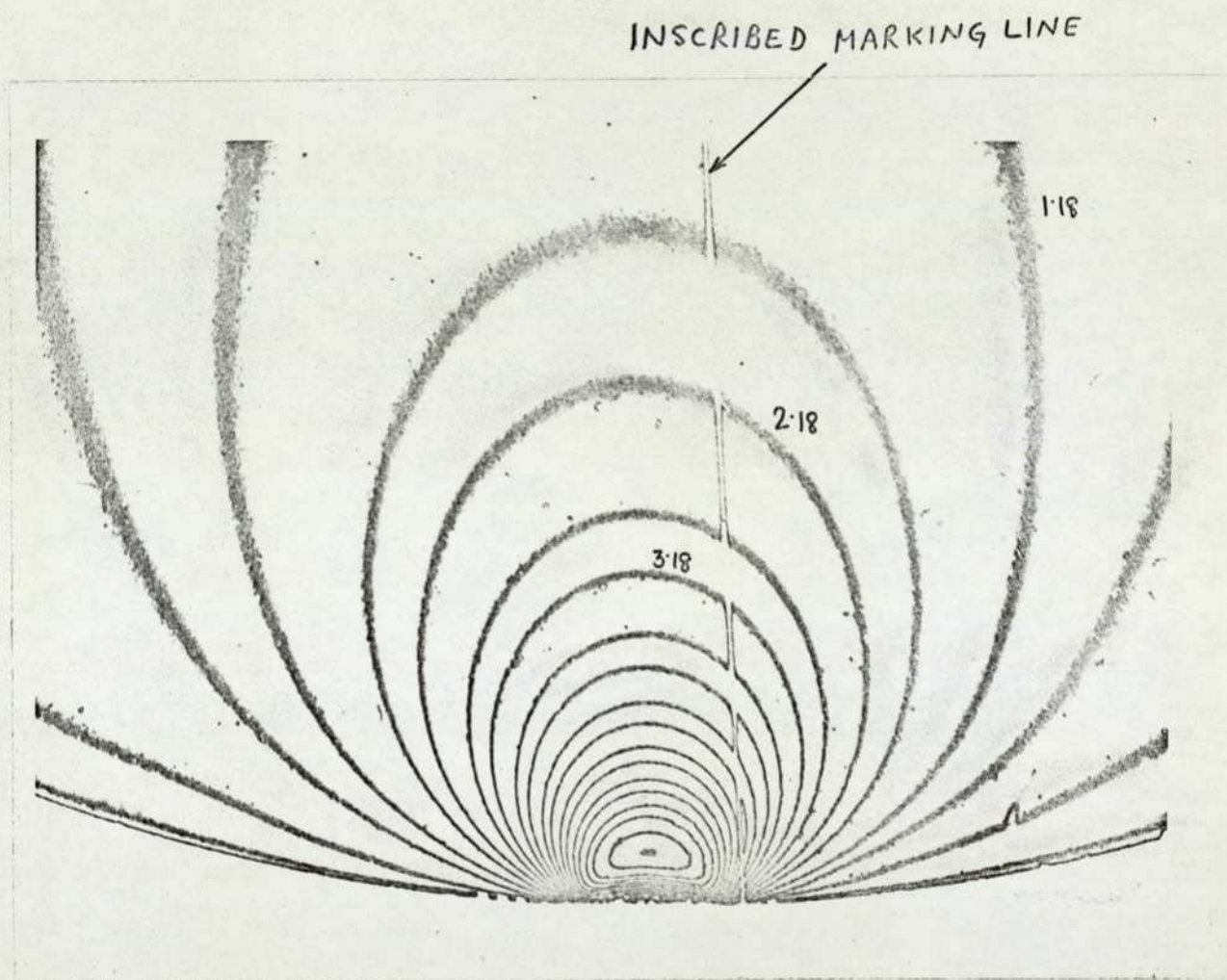


Fig. 6.6.8

Camera exposure time (Ilford) = $1/5$ s
Enlarger exposure time (Agfacontour) = 3 min

Enlarged about 8X

A part of the test model or a part of the main casting may serve as a separate two- or three-dimensional calibration model with a known stress distribution such as a sphere or disk under diametral compression, a tapered bar in tension, or a beam in pure bending, etc. Its dimensions (width or diameter) should not be too small, say $\frac{1}{4}$ " (6.35 mm), because photography of the specimen is involved and for very small specimens the polariscope must have optical elements providing a large magnification.

The calibration piece is subjected to the frozen stress cycle simultaneously with the test model and storage and analysis conditions are also maintained identical for the test model and the calibration piece which must finally be machined to the same thickness as the model slices, even if a two-dimensional calibration piece is used, so that the transmissibility of the calibration piece and the slices is the same. In fact, in three-dimensional photoelasticity the calibration model used to determine the material fringe value can be modified slightly so as to provide also the calibration of the boundary or equidensity lines directly in terms of the principal stress difference.

In situations where a separate calibration model is either not available or can only be made so small that meaningful photography of the fringe pattern is rather difficult, the boundary lines or equidensities of the slices can be calibrated by one of the methods of self-calibration described in Section 3.3. These self-calibration methods provide calibration in terms of fringe orders only, so that the material fringe value must be established by other means.

In the present investigation, no separate calibration model has been used because a complete solution of three-dimensional problems is not intended. A number of slices obtained from Dr. Fessler of Nottingham University and from the Photoelastic Laboratory of The City University have been photographed and two of these have been analysed as examples by the compensation method of self-calibration (Section 3.3.3).

6.7.1 Example 1

Figs. 6.7.1 and 6.7.2 show isochromatic fringe patterns taken on Kodalith Ortho film with 45 and 90 sec exposure against a

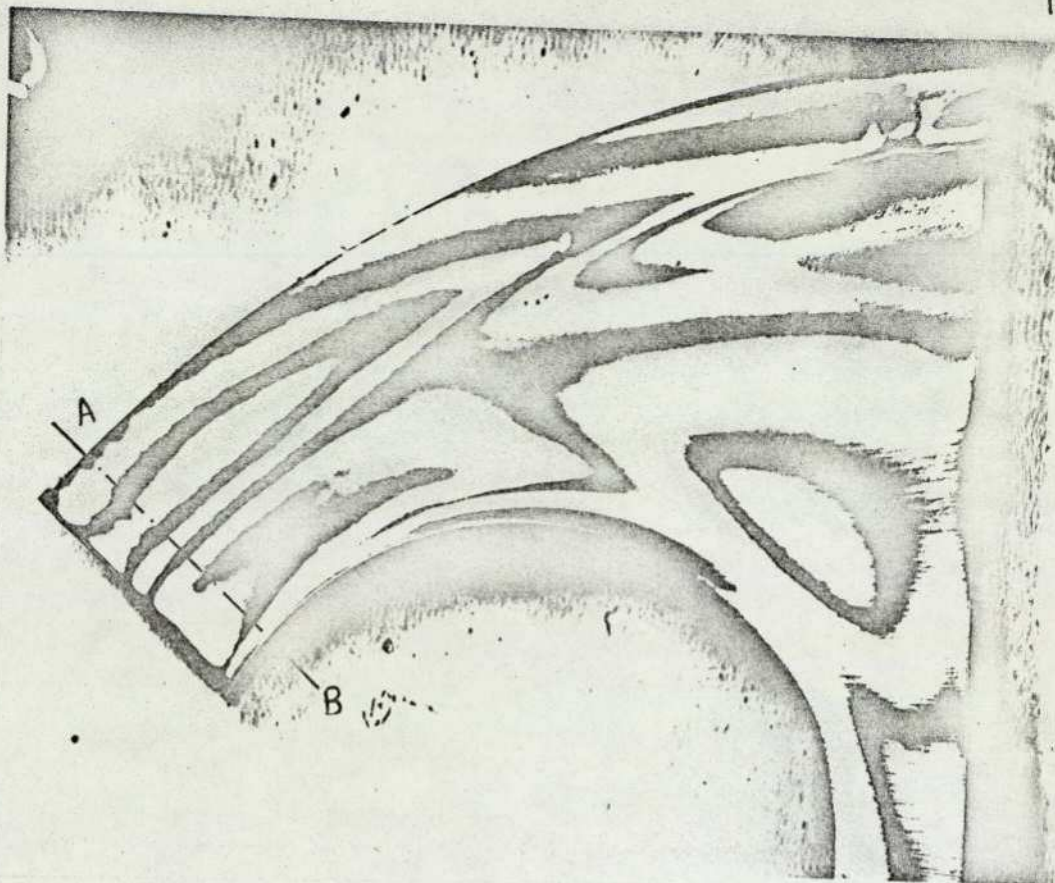


Fig. 6.7.1

45 s, f5.6

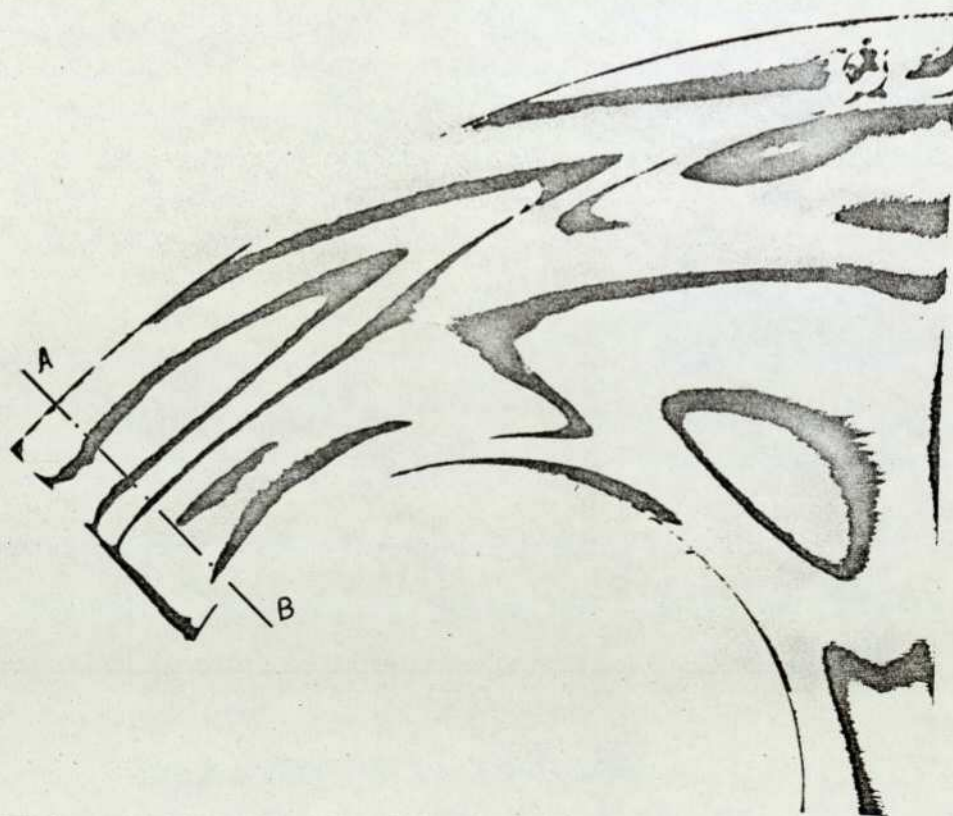


Fig. 6.7.2

90 s, f5.6

dark background. The slice has been obtained from Dr. Fessler and represents a horizontal section through one of the exhaust valve ports in a compound cylinder head. A relatively long exposure is required because of the poor transparency of the slice.

For the section AB on both photographs the positions of the boundary lines have been determined on the X - Y reader and the isochromatic orders at the corresponding points of the model have been obtained directly on the polariscope by Tardy's method. They are $N+0.24$ in Fig. 6.7.1 and $N+0.18$ in Fig. 6.7.2, where N is an integer. Thus fractional fringes of known order are obtained over the whole field and further fractional orders can be obtained simply by changing the camera exposure time. In this self-calibration procedure, no restriction need be imposed on the photographic process or on the optics of the polariscope.

6.7.2 Example 2

The equidensity method has been used for the analysis of a slice taken from the end closure of a cylindrical pressure vessel.

A single photograph of the isochromatic fringe pattern has been taken on a low contrast film, Ilford Commercial Ortho, with the relatively long exposure time of 1 sec so that all the intensity variations in the image plane can be converted into density variations on the film. Fig. 6.7.3 shows a print of the fringe pattern.

The negative has been copied with yellow filters onto Agfacontour film. Variations of contact exposure time give equidensities in different locations so that from a single photograph on low contrast film equidensities can be obtained over the whole field.

Contact printing is undesirable if the slices are small or their image in the polariscope camera cannot be magnified sufficiently. Hence the small image on the Ilford film has here been magnified and printed onto Agfacontour film in an enlarger. The magnification is limited by the size of Agfacontour film available.

Representative results obtained from the fringe pattern of Fig. 6.7.3 with a magnification of two and enlarger exposure time of 1.5, 5 and 15 minutes are shown in Figs. 6.7.4 to 6.7.6.

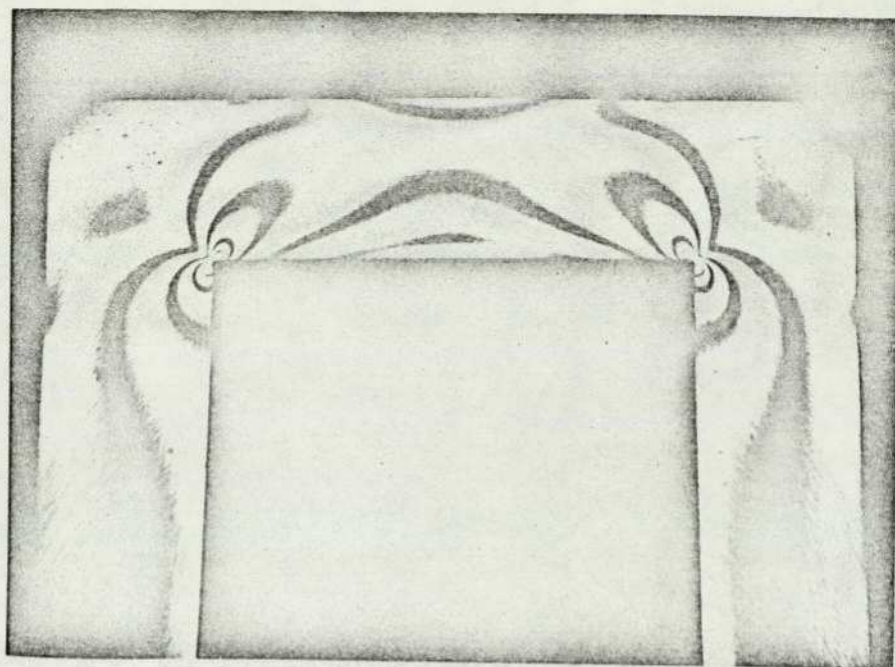


Fig. 6.7.3

Ilford Comm. Ortho Film
Exposure time = 1 s
Camera aperture = f5.6

Enlarged print

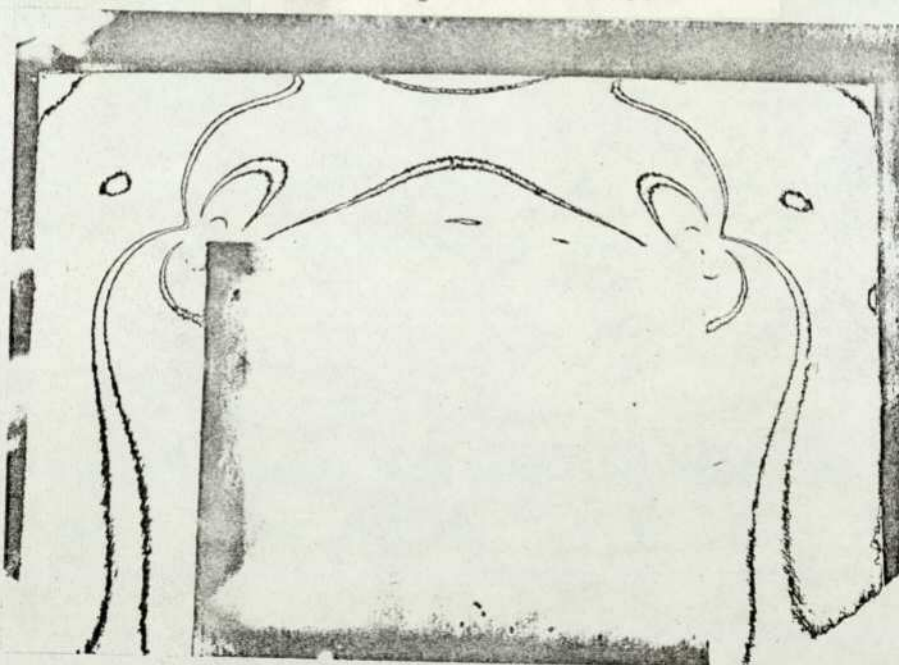


Fig. 6.7.4

Enlarger exp. time = 1.5 min

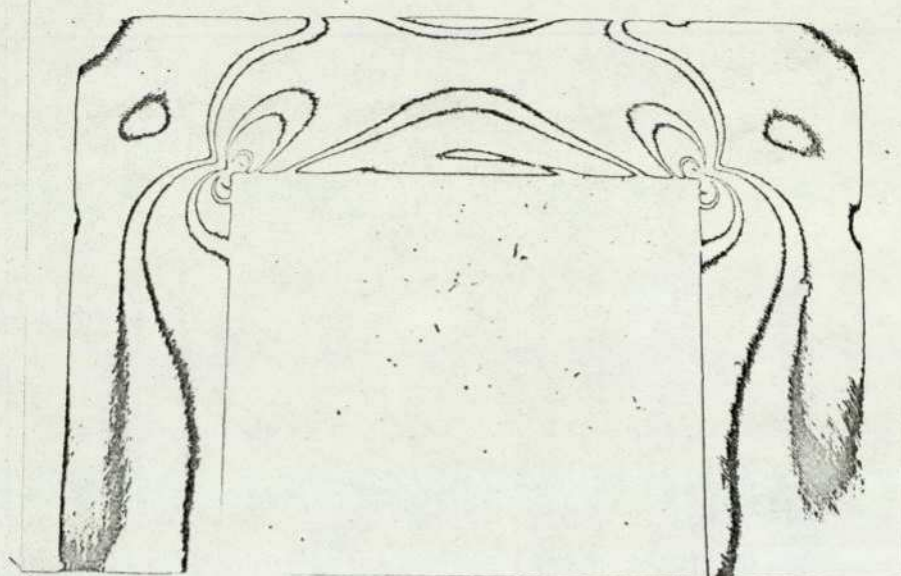


Fig. 6.7.5

Enlarger exp. time = 5 min

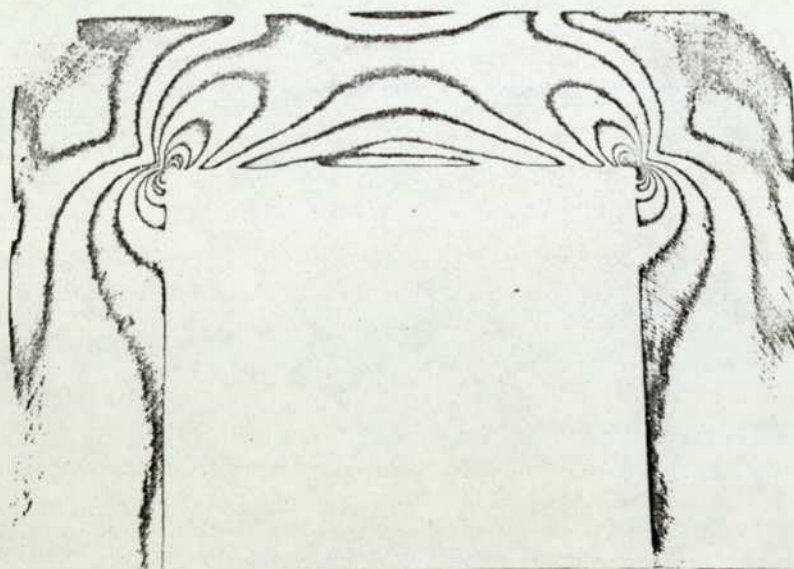


Fig. 6.7.6

Enlarger exp. time = 15 min

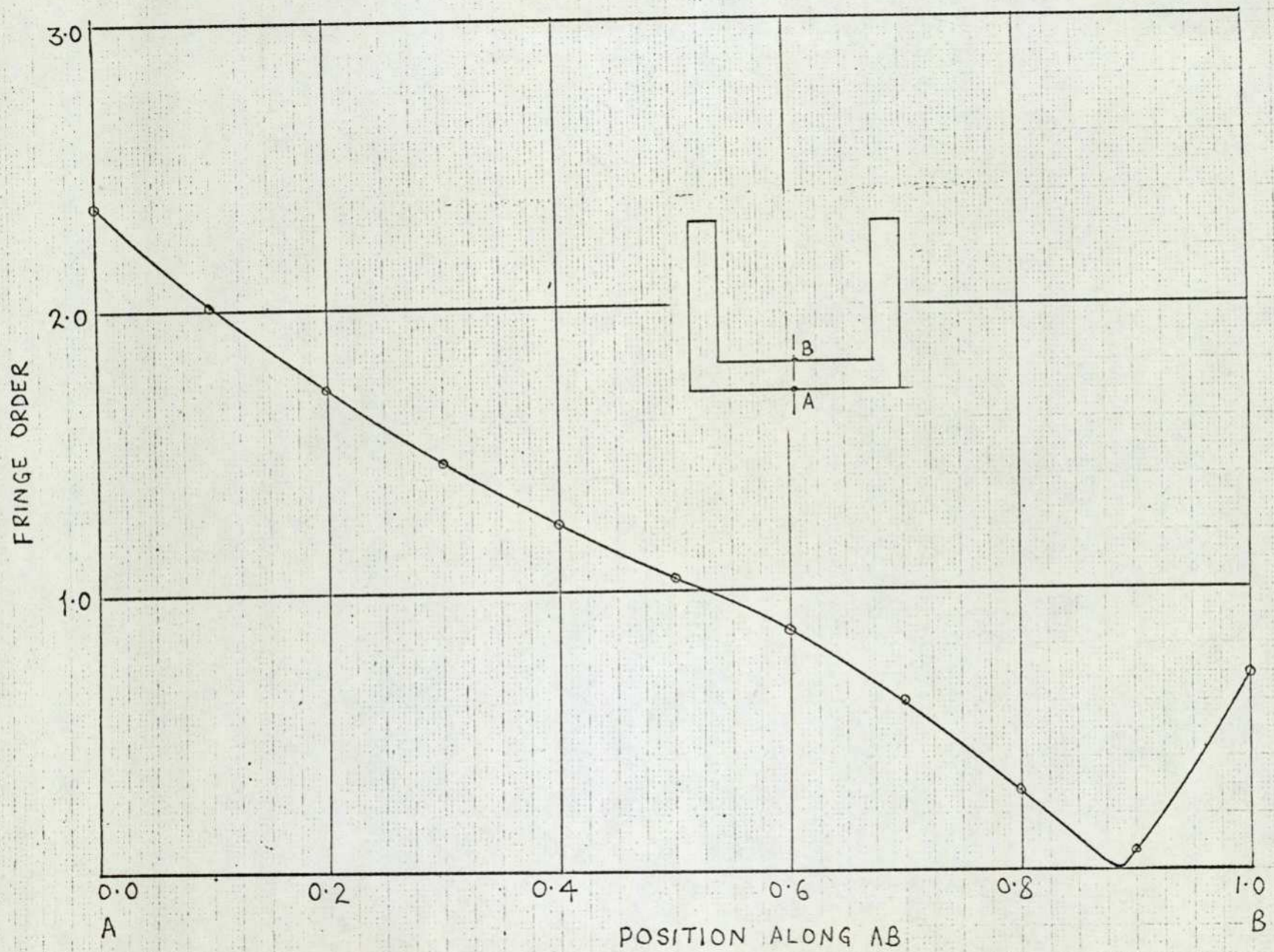


FIG. 6.7.7.

Very long exposures are required because the Agfacontour film is slow and the image is enlarged. A slightly faster Agfacontour film or a more intense light source in the enlarger would reduce the exposure time. The photographs have been enlarged onto Agfacontour with a set of yellow filters and as a result sharp first-order equidensities have been obtained.

The equidensity lines can be self-calibrated in terms of fractional fringe orders if the fringe orders along a line AB are obtained by Tardy's method on the bench and plotted as shown in Fig. 6.7.7. The positions of the equidensity lines in Figs. 6.7.4 , 6.7.5, 6.7.6 can then be determined on a X - Y reader and the fringe orders for these positions read off Fig. 6.7.7 giving $N^{+0.04}$, $N^{+0.11}$ and $N^{+0.24}$ for Figs. 6.7.4., 6.7.5, and 6.7.6.

In this representative example, a single photograph has given fractional fringe orders from $^{+0.04}$ to $^{+0.24}$. Higher values will require still longer exposures time. Equidensities over the whole field can be obtained from one film only if this original film has a low exposure inertia and a large exposure range.

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CHAPTER 7

PURE ISOCHROMATICS OBTAINED IN A PLANE POLARISCOPE

7.1 INTRODUCTION

In classical photoelasticity, quarter-wave plates and circularly polarised light have always been used to eliminate isoclinics, i.e. to obtain pure isochromatics in the photoelastic fringe pattern. The circular polariscope is also used to obtain fractional fringe orders by Tardy or Sénarmont compensation.

The use of quarter-wave plates involves several difficulties (see Sections 2.1.3 and 2.4). The plates are suitable for only one wavelength of light and it is difficult to obtain plates that give a retardation exactly equal to one quarter wavelength of the monochromatic light used on the bench.

It may, therefore, be advantageous if a plane polariscope can be used to obtain pure isochromatic fringes of full and fractional orders free of any contribution by isoclinics. The photoelastic bench will then be simpler, cheaper, and probably more accurate.

Koch and Szigo (1954), Schwieger and Haberland (1956) and Kuske and Robertson (1974) have suggested double exposure of a film with crossed polaroids once oriented at θ^0 and the other time at $(\theta + 45)^0$. This process of double exposure gives only integral orders of pure isochromatic fringes.

The author has combined this technique of double exposure with the techniques already described in this thesis in order to obtain with a plane dark-field polariscope pure isochromatics of fractional order on high contrast film (Kodalith) or on Agfacontour film. Although double exposure with parallel polaroids on a polariscope can be used for half-order fringes, there are certain difficulties in extending the boundary line method to double exposure on a plane bright-field polariscope (see Section 7.5).

7.2 INTENSITY EQUATIONS OF THE PLANE POLARISCOPE

Let the analyser A be inclined at an angle ϕ to the polariser P (Fig. 7.2.1) and let the principal stress direction X_1 make an angle α with the polariser P.

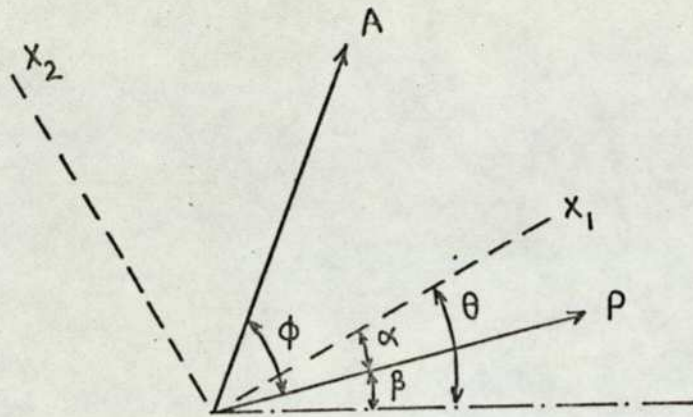


Fig. 7.2.1 Plane polariscope with polariser at an angle β to the horizontal and ϕ to the analyser. X_1 and X_2 are principal stress directions at the point.

The intensity of light emerging at any point then is (Jerrard, 1948; Schwieger and Haberland, 1956)

$$I = I_0 \left[\cos^2 \phi + \sin 2\alpha \sin 2(\phi - \alpha) \sin^2 \frac{\Delta}{2} \right] \quad (7.2.1)$$

where $\Delta = 2\pi N$. This equation leads to the intensity equations for various specialised cases:

(i) Crossed Plane Polariscope

For the plane polariscope with crossed polaroids and hence dark background ($\phi = 90^\circ$),

$$I_d = I_0 \sin^2 2\alpha \sin^2 \pi N \quad (7.2.2)$$

(ii) Parallel Plane Polariscope

If the polaroids are parallel, the background is bright ($\phi = 0^\circ$) and

$$I_b = I_0 \left[1 - \sin^2 2\alpha \sin^2 \pi N \right] \quad (7.2.3)$$

Equations (7.2.2) and (7.2.3) show that the intensity at an observed point depends (i) on the isochromatic order N and (ii) on the angle α between principal stress direction and direction of polarisation. Conventionally the angle between the principal stress direction and the horizontal or vertical is called the isoclinic parameter θ so that $\alpha = \theta - \beta$ if β is the angle between the polarisation plane and the horizontal or vertical (Fig. 7.2.1).

If under a constant load ($N = \text{constant}$) the coupled polariser and analyser are rotated, then ^{at} any observed point P the intensity changes between a minimum and maximum according to

$$I_d/I_o = k \sin^2 2(\theta - \beta), \text{ i.e. } 0 \leq I_d/I_o \leq k \quad (7.2.4)$$

for crossed polaroids and

$$I_b/I_o = 1 - k \sin^2 2(\theta - \beta), \text{ i.e. } (1 - k) \leq I_b/I_o \leq 1 \quad (7.2.5)$$

for parallel polaroids, $k = \sin^2 \frac{\Delta}{2}$ being constant for the point under consideration.

7.3 DOUBLE EXPOSURE ON A PLANE POLARISCOPE

If on a plane polariscope a single film is exposed twice with equal exposure times $t_1 = t_2 = t$ and if between the two exposures the coupled polaroids are both rotated through 45° anticlockwise or clockwise, then for a constant load on the model (constant N) the Equation (7.2.1) together with $\Delta = 2\pi N$ and $E = I_1 t_1 + I_2 t_2$ produces at any point of the film the combined exposure

$$\begin{aligned} E &= I_o t \left[\cos^2 \phi + \sin 2\alpha \sin 2(\phi - \alpha) \sin^2 \pi N + \cos^2 \phi \right. \\ &\quad \left. + \sin 2(\alpha \pm 45^\circ) \sin 2(\phi - \alpha \mp 45^\circ) \sin^2 \pi N \right] \\ &= I_o t \left[2\cos^2 \phi + \sin^2 \pi N \left\{ \sin 2\alpha \sin 2(\phi - \alpha) - \cos 2\alpha \cos 2(\phi - \alpha) \right\} \right] \\ &= I_o t (2\cos^2 \phi - \cos 2\phi \sin^2 \pi N) \end{aligned} \quad (7.3.1)$$

It is seen that the isoclinic angle $\alpha = \theta - \beta$ vanishes and that the combined exposure depends only on the coupling angle ϕ between polariser and analyser and on the isochromatic fringe order N . No isoclinics appear on the developed film which, for

uniform I_0 , shows a uniform background density produced by an exposure $2I_0 t \cos^2 \phi$ and a superimposed pure isochromatic pattern. The contrast of the isochromatic fringes depends on the ratio of maximum to background density, and maximum contrast is obtained for minimum background density which corresponds to $\phi = 90^\circ$.

For crossed polaroids (dark field), $\phi = 90^\circ$ and

$$E = E_d = I_0 t \sin^2 \pi N \quad \text{i.e. } 0 \leq E_d \leq I_0 t \quad (7.3.2)$$

For parallel polaroids (bright field), $\phi = 0^\circ$ and

$$E = E_b = I_0 t (2 - \sin^2 \pi N) \quad \text{i.e. } I_0 t \leq E_b \leq 2I_0 t \quad (7.3.3)$$

Finally for $\phi = 45^\circ$ not only the isoclinics but also the isochromatic pattern disappears since

$$E = E_{45} = I_0 t \quad (7.3.4)$$

Polarisers with correctly crossed or parallel polaroids are discussed in Section 7.4 to 7.6

Equation (7.3.1) is plotted in Fig. 7.3.1 which shows

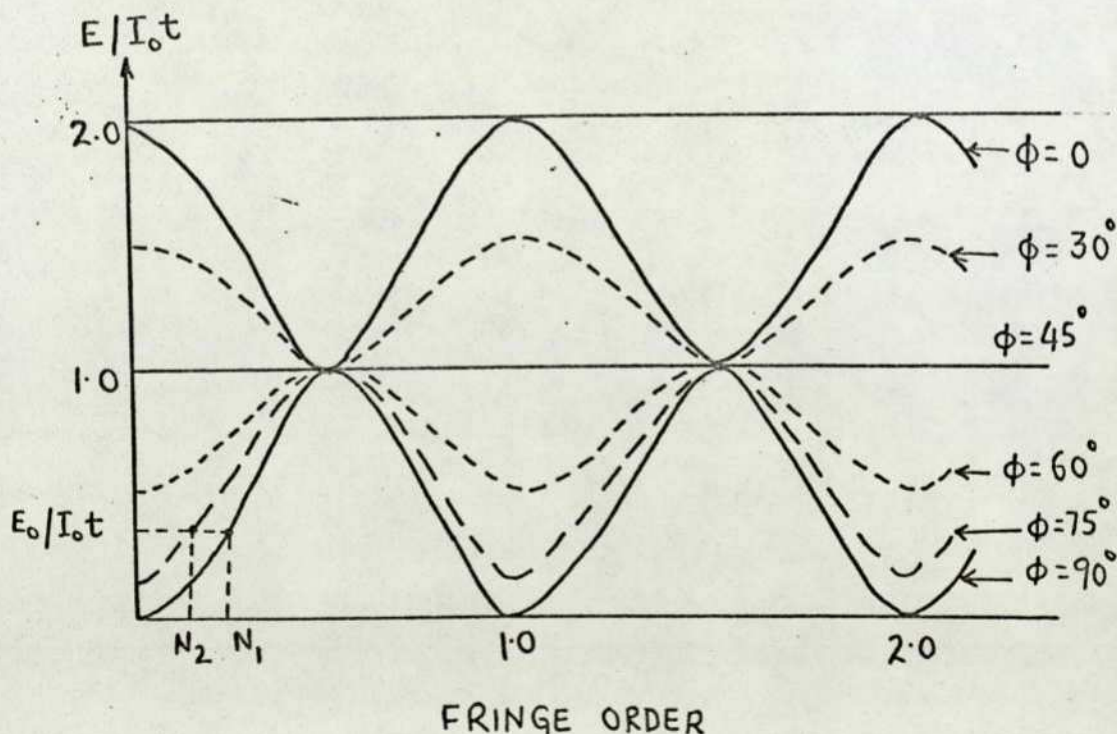


Fig. 7.3.1 Exposure variation in double exposure on a plane polariscope with different angle ϕ between the polariser and analyser.

curves of exposure vs fringe order for various values of ϕ from 0 to 90° . This shows that for the same exposure time, the exposure variation differs for different values of ϕ . Therefore, if E_0 is the exposure inertia of the film, a variation in the value of ϕ , say from 90° to 75° , will result in a shift in the position of the boundary or equidensity line from N_1 to N_2 (Fig. 7.3.1) even when the exposure time is kept constant (see Chapter 3).

This shift is obtained as long as exposure producing background density, i.e. $2I_0 t \cos^2 \phi$ is less than the exposure inertia of the film. Hence if large variations in the positions of boundary lines are required, the use of a slow film with a high E_0 is desirable. With a suitable variation in ϕ and in the exposure time t , the boundary lines representing fractional isochromatic orders can then be obtained anywhere in the field. It is even possible to choose ϕ so as to raise the minimum combined exposure to a level just below the exposure inertia E_0 of the film and to obtain in this way boundary or equidensity lines of very nearly or even exactly integral order.

Double exposure with $\phi = 45^\circ$ (Equation 7.3.4) or double exposure with plane parallel and plane crossed polariscope (Equations 7.2.2 and 7.2.3) can be adopted for pre-exposing the film through the correctly loaded model. If I_0 is not uniform, then either double exposure will give a negative which after development may be used as a filter reducing or eliminating the non-uniformities (see Section 4.3.2). The method has the advantage that the intensity distribution reaches the film through the final set-up which then remains undisturbed except for the re-orientation of the polaroids. Obviously the difficulty remains of correctly inserting the developed negative in front of the undeveloped film in the camera.

The technique of generating pure isochromatics and eliminating all traces of isoclinics by double exposure thus depends for its accuracy on three conditions:

- (i) Both exposures must have the same time $t_1 = t_2 = t$.

(ii) The film and the model must not move between the first and the second exposure so that the two exposures are accurately superimposed.

(iii) The coupled polaroids must be rotated through exactly 45° between exposures.

These conditions are not difficult to fulfil within satisfactory limits (see Chapter 7.7). In addition, the usual care, as already described, must be taken that the calibration model and test model are always photographed under identical conditions.

7.4 CROSSED POLARIDS AND HIGH CONTRAST FILM

The analyser and polariser of the polariscope are crossed accurately to $\phi = 90^\circ$ and the quarter-wave plates are removed. Although white light can be used, for obvious reasons monochromatic light is usually preferred.

7.4.1 Monochromatic Light Source

Filters 77A and 807 were used with mercury vapour light source. A camera aperture of f45 was chosen in order to reduce the non-uniformity of illumination at the centre of the field. The beam and disk models of CR-39 as described in Section 3.4 were once more investigated. The beam had a thickness of 0.247" (6.3 mm) and a height of 1.53" (39 mm), the disk a diameter of 3" (76 mm) and the same thickness.

The isochromatic fringe pattern of the beam in pure bending is obtained by doubly exposing the film at 15° and 60° orientation of the crossed polariser and analyser. It provides calibration not only for the material constant but also for the boundary lines in terms of fractional fringe orders. The positions of the boundary lines are varied as usual by changes in load or in the common exposure time $t_1 = t_2 = t$.

The disk under diametral compression has been used as a test model with the calibration results obtained from the beam. The fringe orders in the disk thus obtained experimentally are compared with theory.

Fringe photographs of the beam in bending with double exposure times of 1 + 1, 2 + 2, 3 + 3, 4 + 4 and 5 + 5 seconds produce boundary lines with fractional fringe orders of ± 0.273 ,

± 0.195 , ± 0.161 , ± 0.130 and ± 0.103 respectively.

Thus a variation from 0.273 to 0.103 is obtained by just 5 variations in exposure time. Figs. 7.4.1 and 7.4.2 are representative photographs taken with (2 + 2) and (5 + 5) sec exposure for a bending moment of 157.5 lbf-in. (17.8 Nm) which produces at the extreme fibre a maximum fringe order of 3.96 as estimated from the positions of the boundary lines. This information together with the dimensions of the beam give a material fringe value of 101.9 lbf/in. fringe (18 kN/m-fringe) based on principal stress difference, the same value as in Section 3.4.

The complete elimination of isoclinics cannot be appreciated fully from the fringe pattern of a beam in pure bending but is evident in the photographs of the disk under diametral compression.

Fringe photographs of the disk under diametral compression have been taken at double exposure times of 1 + 1, 2 + 2, 3 + 3 and 5 + 5 seconds under 175 lbf load giving a fringe order of 1.46 at the centre of the disk and at 1 + 1, 2 + 2, 3 + 3 and 4 + 4 seconds under 250 lbf load giving 2.08 fringes at the centre. The fringe orders of the boundary lines are found from the calibration results of the beam for the corresponding exposure times. The positions of the boundary lines on the horizontal diameter normal to the load (i.e. $k_2 = y/r = 0.0$) are measured on the X - Y reader. Experimental and theoretical fringe orders for the loads of 175 and 250 lbf are compared in Table 7.4.1.

Table 7.4.1

Experimental and theoretical fringe orders along $k_2 = y/r = 0.0$ in a disk under diametral compression.

(a) 250 lbf (1112 N) load

$k_1 = x/r$	0.184	0.208	0.224	0.408	0.44	0.448	0.46
Exp. Fringe Order	1.870	1.839	1.805	1.273	1.195	1.161	1.13
Theo. Fringe Order	1.876	1.828	1.793	1.275	1.177	1.152	1.117

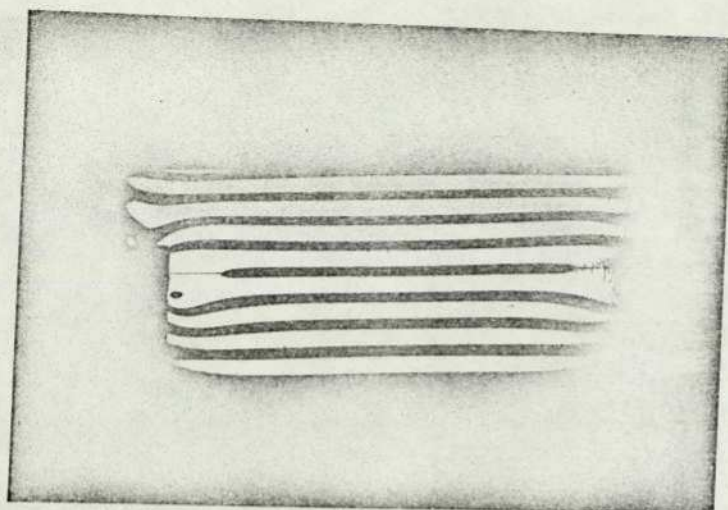


Fig. 7.4.1

(2+2) s, f45

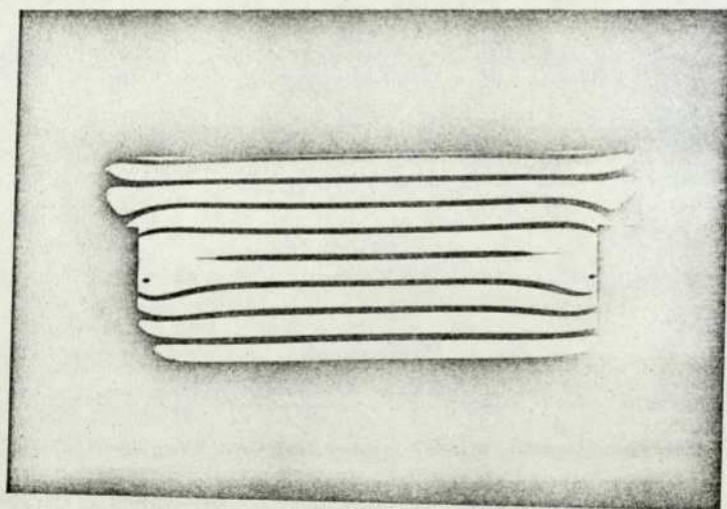


Fig. 7.4.2

(5+5) s, f45

(b) 175 lbf (778 N) load

$k_1 = x/\gamma$	0.208	0.264	0.288	0.308	0.420	0.44	0.448	0.50
Exp. Fringe Order	1.273	1.195	1.161	1.103	0.897	0.839	0.805	0.727
Theo. Fringe Order	1.280	1.184	1.146	1.099	0.892	0.824	0.806	0.716

The experimental accuracy depends mainly on the accurate measurement of position which has been kept within 0.002" (0.05 mm). The average difference between theoretical and experimental fringe order is less than 0.01 fringe and in most individual results it is less than 0.02 fringe, which can be regarded as a good agreement. The ratios of the fringe orders measured under 250 and 175 lbf load at $k_1 = 0.208, 0.44$ and 0.448 are 1.44, 1.42 and 1.44 whilst the theoretical fringe ratio equals the load ratio of 1.43.

Figs. 7.4.3 and 7.4.4 are representative fringe patterns obtained at double exposure times of (2 + 2) and (5 + 5) sec. in the disk under 175 lbf diametral compression. Similarly Figs. 7.4.5 and 7.4.6 have been obtained at exposure times of (2 + 2) and (4 + 4) sec under 250 lbf compression. The complete elimination of isoclinics is evident.

7.4.2 Use of White Light

As no quarter-wave plates are involved, white light can be used to photograph isochromatics by double exposure in a plane crossed polariscope. The result will normally differ from that for monochromatic light because with white light the net effect depends on the wavelengths and their relative energies. On panchromatic film, the isochromatic patterns for monochromatic and white light have been found to be quite different because the film is receptive to various colours of the white light.

On the other hand, blue-sensitive or orthochromatic film gives very similar results in monochromatic and white light because the film itself acts as a colour filter. Hence white light and a blue-sensitive film can replace the monochromatic light source and thus further reduce the cost of polariscopes for accurate numerical work.

A blue-sensitive film has not been used here but the results obtained on Kodalith orthochromatic film 2556 and on Kodalith panchromatic film 2568 have been compared.

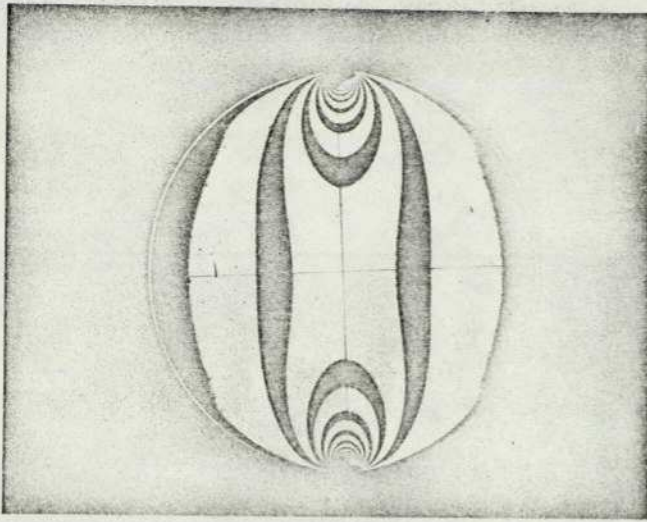


Fig. 7.4.3

(2+2) s, f45
Load = 175 lbf

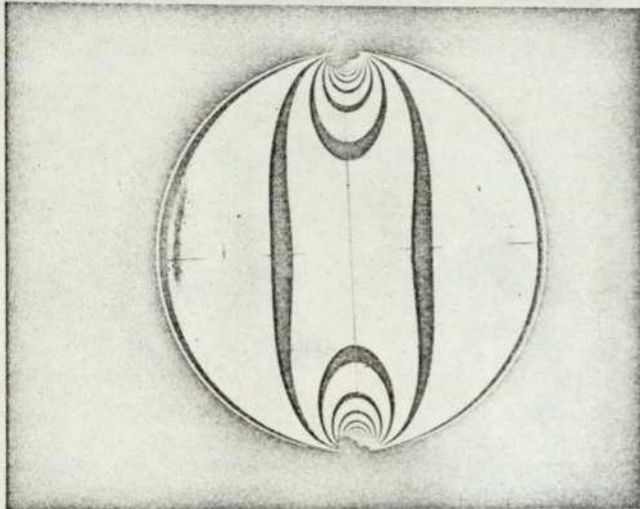


Fig. 7.4.4

(5+5) s, f45
Load = 175 lbf

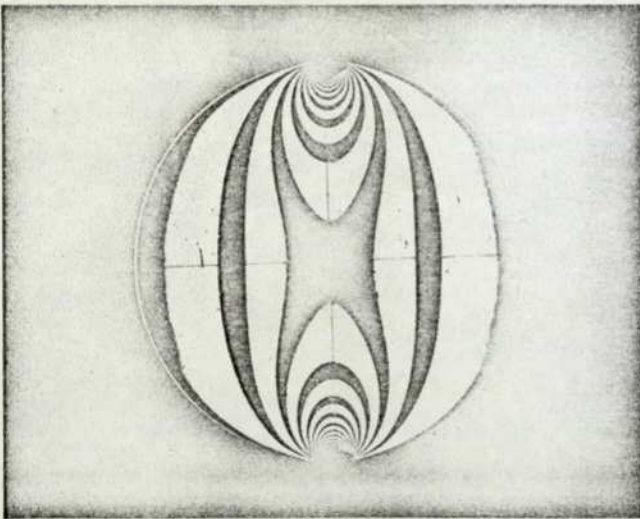


Fig. 7.4.5

(2+2) s, f45
Load = 250 lbf

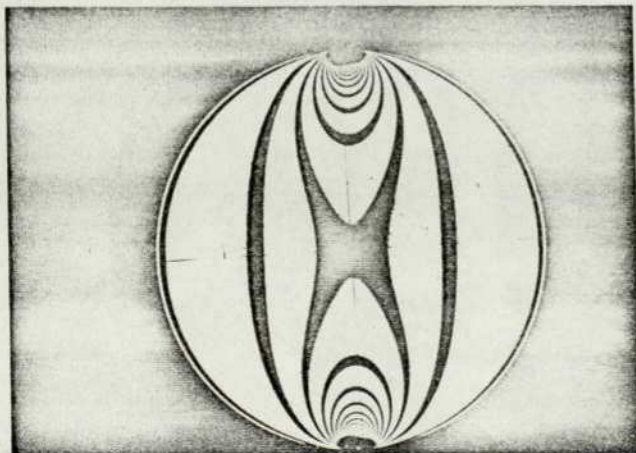


Fig. 7.4.6

(4+4) s, f45
Load = 250 lbf

Loss of fringes near the loading point is due to use of small camera aperture of f45.

Figs. 7.4.7 and 7.4.8 show the isochromatic fringe patterns of two stress-frozen disks photographed in monochromatic and in white light by double exposure on Kodalith panchromatic film 2568 in a plane crossed polariscope. The differences in the patterns of the larger disk are obvious. In the pattern of the smaller disk the horizontal diameter is intersected by three fringes in white light (Fig. 7.4.8) and by four fringes in monochromatic light (Fig. 7.4.7).

On orthochromatic film Kodalith 2556 in monochromatic light (Fig. 7.4.9) and in white light (Fig. 7.4.10) the fringe patterns are similar. A small difference is expected because the monochromatic pattern has been taken in green light whereas in white light the film is sensitive to blue as well as to green. This also explains why in white light the higher order fringes tend to get washed out.

7.5 PARALLEL POLAROIDS AND HIGH CONTRAST FILM

Equation 7.3.3 shows that the resultant doubly exposed fringe pattern is again independent of the isoclinic parameter θ and hence exhibits pure isochromatics and possibilities of fractional fringe order interpretation. However, the exposure varies here between $E_b = I_0 t$ for half-order isochromatics and $E_b = 2I_0 t$ for integer orders so that the minimum exposure is one-half of the maximum. This reduces the contrast of the fringe patterns even if high-contrast films are used.

Although contrast can be improved by as small an exposure time as possible, it is not advisable to use the boundary line method of fractional fringe order measurements on a plane parallel polariscope. It is likely that variations in Φ between 90° and 45° coupled with the use of slow film (high E_0 value) will serve to supply all the required information (see Fig. 7.3.1).

Figs. 7.5.1 and 7.5.2 show pure isochromatics obtained in the models of beam and disk by double exposure of 1 + 1 second on a plane polariscope with parallel polaroids. The negative of Fig. 7.5.2 has been underdeveloped (2 minutes instead of 3 minutes). The centres of the black fringes in the shown print represent half orders. Comparison of Fig. 7.5.1 (1 + 1 second) with 7.4.2

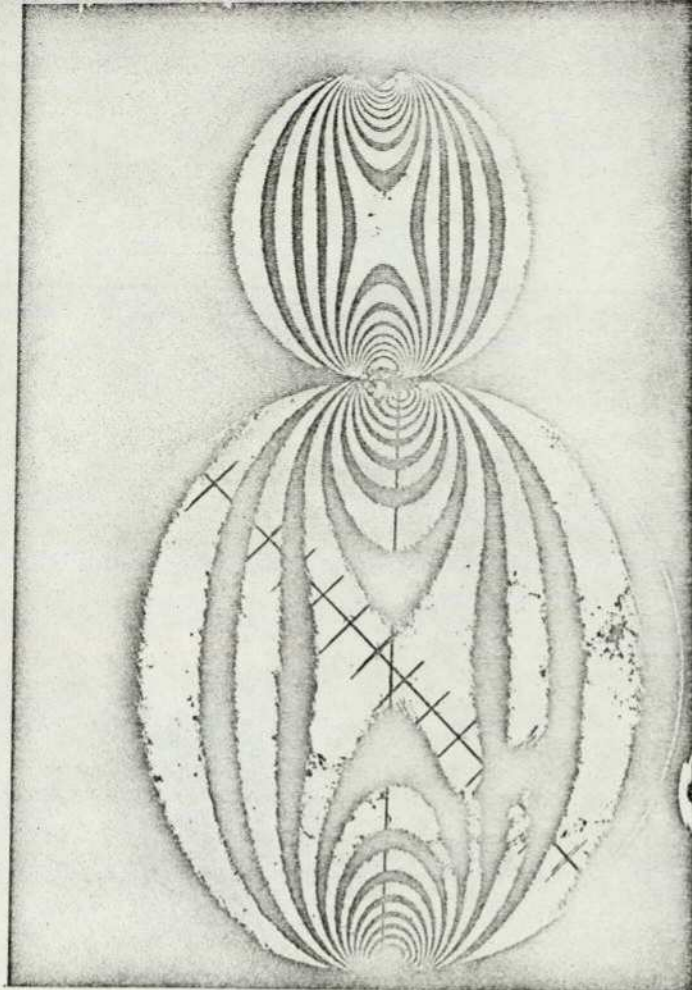


Fig. 7.4.7

Kodalith pan film
Monochromatic light
Exposure time = $(1/5+1/5)$ s

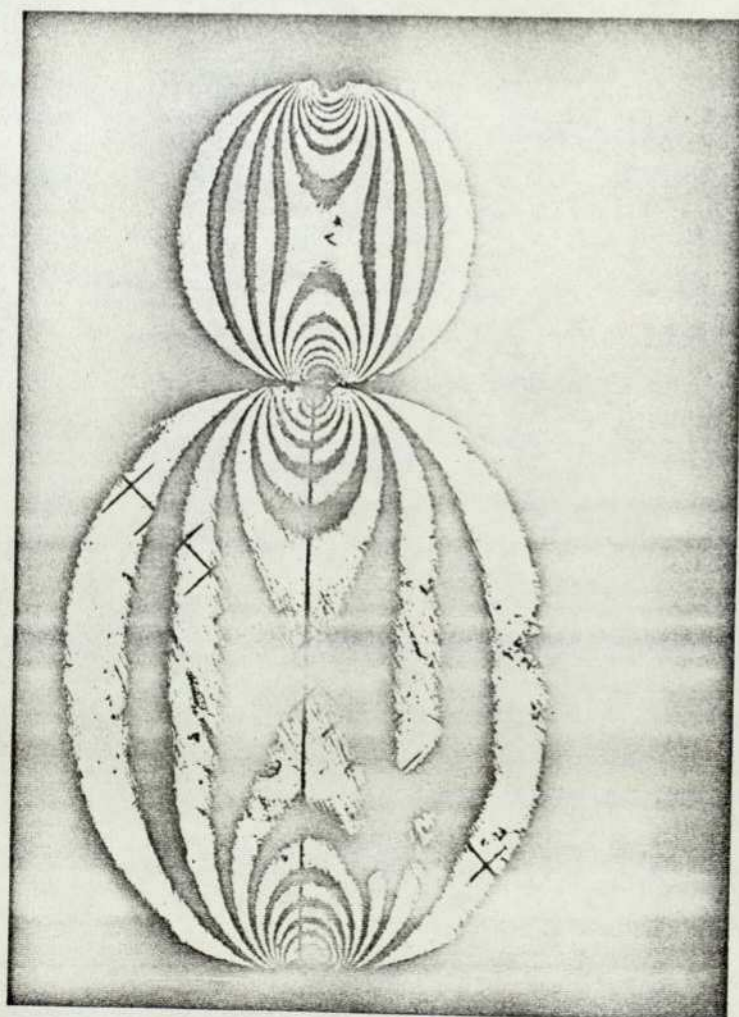


Fig. 7.4.8

Kodalith pan film
White light
Exposure time =
 $(1/100+1/100)$ s

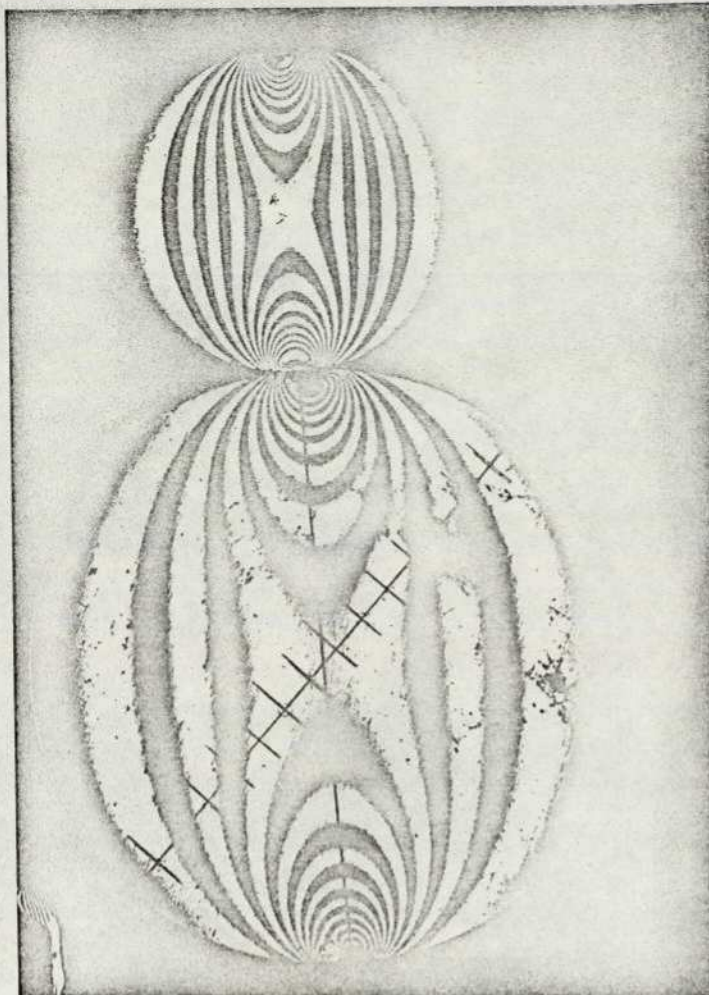


Fig. 7.4.9

Kodalith ortho film
Monochromatic light
Exposure time = $(1+1)$ s

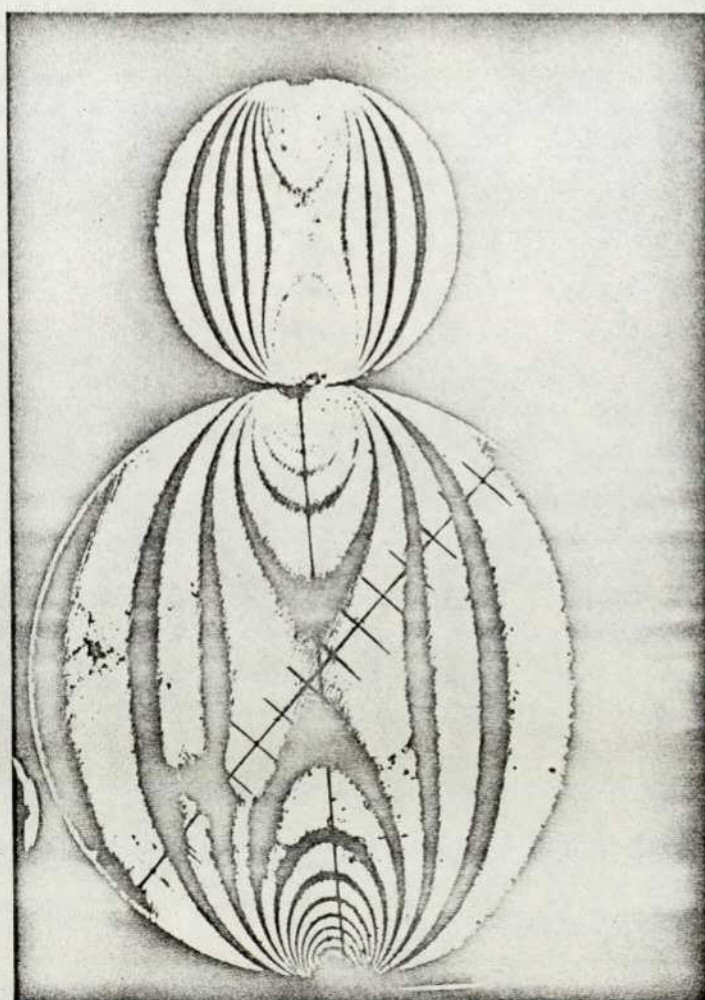


Fig. 7.4.10

Kodalith ortho film
White light
Exp. time = $(1/2+1/2)$ s

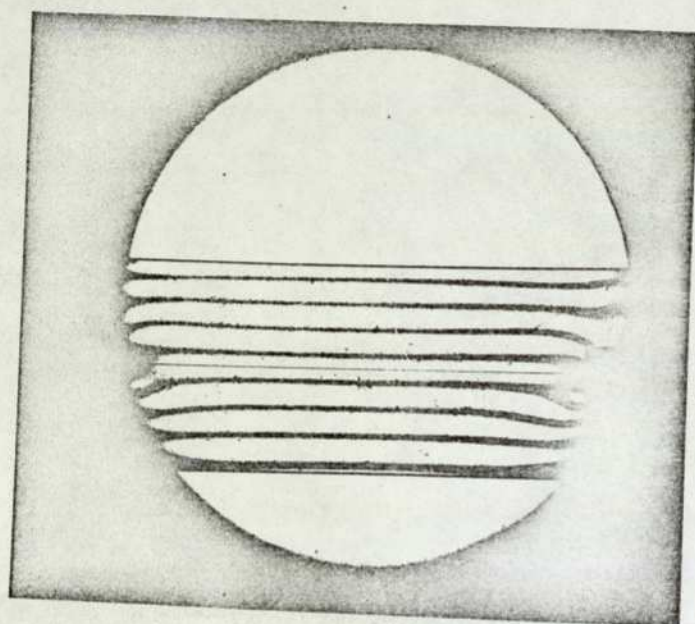


Fig. 7.5.1

Exposure time = $(1+1)$ s
parallel polaroids

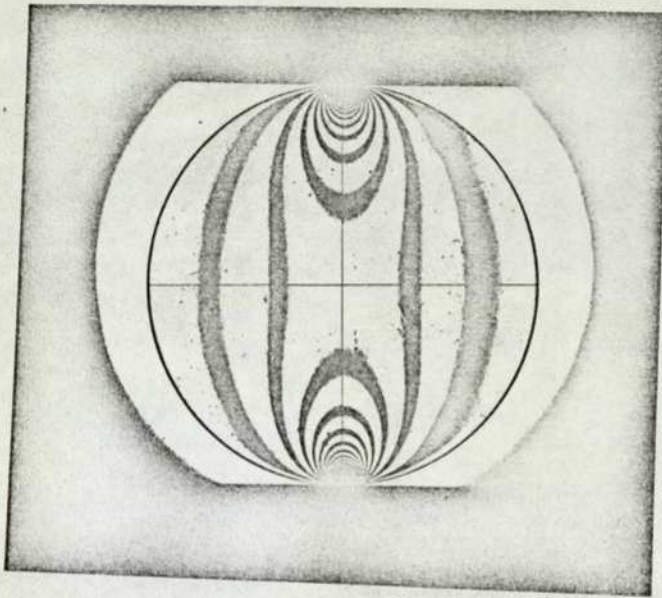


Fig. 7.5.2

Exposure time = (1+1) s
parallel polaroids

(5 + 5) second shows the reduction of exposure time required for photographs taken on the plane parallel polariscope. As an alternative the exposure time for printing or contact copying the original negative may be increased.

7.6 USE OF AGFACONTOUR FILM

Equidensity lines on Agfacontour film may replace the boundary lines on high contrast film in an evaluation of pure isochromatics obtained by double exposure on a plane polariscope. In direct photography on the polariscope, monochromatic light or a set of yellow filters is required. If copies on Agfacontour are taken from a low-contrast panchromatic or orthochromatic original, white light may be used for the original.

For a CR-39 beam under a pure bending moment of 137.8 lbf-in (15.6 Nm), Figs. 7.6.1 and 7.6.2 show equidensity lines obtained on a plane polariscope with crossed polaroids by direct photography and double exposures of 45 + 45 and 60 + 60 seconds in mercury green light with filters 77A and 807 and with a camera aperture of f45. The fractional fringe orders are ± 0.234 and ± 0.185 . Figs. 7.6.3 and 7.6.4 show equidensity lines obtained under the same conditions for a disk under 250 lbf diametral compression. Obviously the technique lends itself to full-field fractional fringe order determination.

For a plane polariscope with parallel polaroids it may be best to photograph the fringe pattern by double exposure on a low-contrast film and then to contact copy the original onto Agfacontour film with varying exposure times. However, double exposure with ϕ varying from 90° to 45° is likely to give all the necessary information so that this additional technique may not be required.

7.7 ERROR ANALYSIS FOR THE PLANE POLARISCOPE

The model is assumed to be correctly placed and loaded. Possible calibration errors have been treated elsewhere (Section 3.5) as have non-uniformities in I_0 (Section 4.1). According to Equation (7.2.1) the remaining errors peculiar to the plane polariscope are those in ϕ and α . Combined isoclinic and isochromatic patterns obtained by single exposure are still important in some applications and the errors in ϕ and α may influence the



Fig. 7.6.1

Agfacontour film
(45+45) s, f45



Fig. 7.6.2

Agfacontour film
(60+60) s, f45

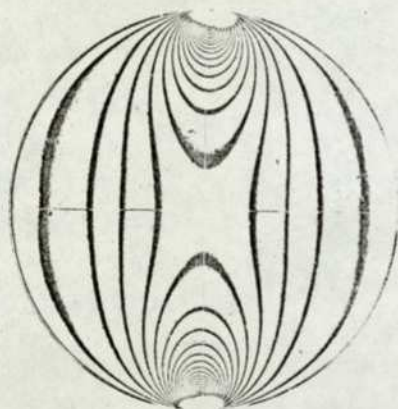


Fig. 7.6.3

Agfacontour film
(45+45) s, f45

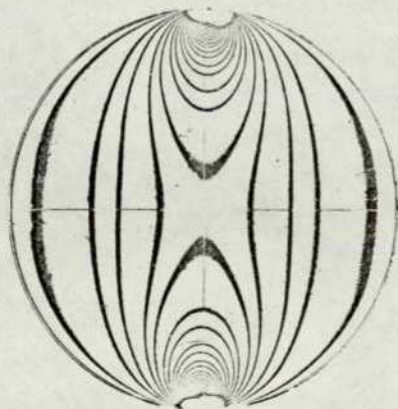


Fig. 7.6.4

Agfacontour film
(60+60) s, f45

accuracy of isoclinic determination (Section 9.9). In double exposure, errors also arise from faulty rotation between exposures.

7.7.1 Single Exposure with Crossed Polaroids

For $\phi = 90^\circ \pm \epsilon_1$ and correct α , Equation (7.2.1) yields with $\Delta = 2\pi N$

$$I_d = I_o \left[\sin^2 \epsilon_1 + \sin 2\alpha \sin 2(\alpha \mp \epsilon_1) \sin^2 \pi N \right] \quad (7.7.1)$$

The minimum intensity on the isoclinic centre line ($\alpha = 0$) and on isochromatics of integral fringe order N is not zero but

$$I_{d \min} = I_o \sin^2 \epsilon_1 \quad (7.7.2)$$

A possible error of $\epsilon_1 = \frac{1}{2}$ degree gives $I_{d \min} = 0.00008 I_o$ and even an impossibly large error of $\epsilon_1 = 2$ degrees gives only $0.0012 I_o$.

For $\phi = 90^\circ$ and $\alpha \pm \epsilon_2$ instead of α , Equation (7.2.1) leads to

$$I_d = I_o \left[\sin^2 2(\alpha \pm \epsilon_2) \sin^2 \pi N \right] \quad (7.7.3)$$

With a minimum intensity on the isoclinic centre line ($\alpha = 0$) of

$$I_{d1} = I_o \sin^2 2\epsilon_2 \sin^2 \pi N \quad (7.7.4)$$

and with a zero intensity on isochromatic lines of integral fringe order. The maximum intensity $I_d = I_o$ occurs where half order isochromatics intersect isoclinics $\alpha \pm \epsilon_2 = 45^\circ$.

If both errors occur in combination,

$$I_d = I_o \left[\sin^2 \epsilon_1 + \sin 2(\alpha \pm \epsilon_2) \sin 2(\alpha \mp \epsilon_1 \pm \epsilon_2) \sin^2 \pi N \right] \quad (7.7.5)$$

7.7.2 Single Exposure with Parallel Polaroids

For $\phi = \pm \epsilon_1$ and correct $\alpha \gamma / \epsilon_1$, Equation (7.2.1) gives

$$I_b = I_o \left[\cos^2 \epsilon_1 - \sin 2\alpha \sin 2(\alpha \mp \epsilon_1) \sin^2 \pi N \right] \quad (7.7.6)$$

The maximum intensity, now indicating isoclinic lines ($\alpha = 0$) and isochromatics of integral order, is reduced by a negligible amount to

$$I_{b \max} = I_o \cos^2 \epsilon_1 \quad (7.7.7)$$

The minimum intensity occurs at intersections of half-order isochromatics with lines of $\alpha = 45^\circ$ and is

$$I_{b \min} = I_o \sin^2 \epsilon_1 \quad (7.7.8)$$

which is identical with Equation (7.7.2) for the dark-field set up.

For $\phi = 0$ and $\alpha = \pm \epsilon_2$, Equation (7.2.1) supplies

$$I_b = I_o \left[1 - \sin^2 2(\alpha \pm \epsilon_2) \sin^2 \Pi N \right] \quad (7.7.9)$$

The maximum intensity $I_b = I_o$ indicates integral - order isochromatics and lines of $\alpha = \pm \epsilon_2$. Hence the isoclinics are in error by ϵ_2 . The minimum intensity $I_b = 0$ occurs where half order isochromatics meet isoclinics of $\alpha \pm \epsilon_2 = 45^\circ$.

Finally for both errors, ϵ_1 and ϵ_2 , combined

$$I_b = I_o \left[\cos^2 \epsilon_1 - \sin 2(\alpha \pm \epsilon_2) \sin 2(\alpha \mp \epsilon_1 \pm \epsilon_2) \sin^2 \Pi N \right] \quad (7.7.10)$$

7.7.3 Double Exposure

For incorrectly crossed polaroids with $\phi = 90 \pm \epsilon_1$ and correct α , the total combined exposure at any point of the field of the model as given by Equation (7.3.1) is

$$E = E_{di} = I_o t (2 \sin^2 \epsilon_1 + \cos 2 \epsilon_1 \sin^2 \Pi N) \quad (7.3.11)$$

Since the small error ϵ_1 is constant over the whole field, its effect is simply to increase slightly the minimum combined exposure and to reduce correspondingly the maximum contrast. The maximum exposure $E_{i \max} = I_o t$ remains unaffected as does the fundamental type of dark-field type of pure isochromatic pattern that emerges.

For the parallel field with $\phi = \pm \epsilon_1$ and correct $\alpha > \epsilon_1$, Equation (7.3.1) results in

$$E = E_{bi} = I_o t (2 \cos^2 \epsilon_1 - \cos 2 \epsilon_1 \sin^2 \Pi N) \quad (7.7.12)$$

The maximum combined exposure for integral order isochromatics is reduced by a negligible amount as is the maximum contrast.

The minimum exposure $E_{i \min} = I_o t$ remains unaffected as does the fundamental type of bright-field pure isochromatic pattern.

Thus the isochromatic fringe pattern remains unaffected by an error in ϕ . As shown in Fig. 7.3.1, a change in ϕ can be used advantageously in shifting the position of boundary lines. As long as calibration and test model are photographed with the same setting $\phi \pm \epsilon_1$, no error arises in this technique.

If the value of ϕ is correctly known and if an incorrect $\alpha \pm \epsilon_2$ is used instead of α , then Equation (7.3.1) shows that this error makes no difference whatever to the result because the isoclinic values are self-cancelling. Similarly if both errors ϵ_1 and ϵ_2 are involved, Equation (7.3.1) leads to the conclusion that ϵ_2 cancels out and that ϵ_1 can be used to advantage as already described.

Finally, if for correct ϕ and α the coupled polaroids are rotated through an angle $45^\circ \pm \epsilon$ instead of 45° , we obtain instead of Equation (7.3.1)

$$\begin{aligned} E &= I_0 t \left[2 \cos^2 \phi + \sin^2 \pi N \left\{ \sin 2\alpha \sin 2(\phi - \alpha) + \sin 2(\alpha + 45^\circ \pm \epsilon_1) \right. \right. \\ &\quad \left. \left. \sin 2(\phi - \alpha - 45^\circ \mp \epsilon_1) \right\} \right] \\ &= I_0 t \left[2 \cos^2 \phi + \sin^2 \pi N \left\{ \sin 2\alpha \sin 2(\phi - \alpha) - \cos 2(\alpha \pm \epsilon_1) \right. \right. \\ &\quad \left. \left. \cos 2(\phi - \alpha \mp \epsilon_1) \right\} \right] \end{aligned}$$

Hence for a dark field ($\phi = 90^\circ$)

$$\begin{aligned} E &= I_0 t \sin^2 \pi N \left[\sin^2 2\alpha + \cos^2 2(\alpha \pm \epsilon) \right] \\ &= I_0 t \sin^2 \pi N \left[\sin^2 2\alpha + (\cos 2\alpha \cos 2\epsilon \mp \sin 2\alpha \sin 2\epsilon)^2 \right] \\ &\simeq I_0 t \sin^2 \pi N \left[\sin^2 2\alpha + \cos^2 2\alpha \mp (2 \cos 2\alpha \sin 2\alpha) 2\epsilon \right. \\ &\quad \left. + (\sin^2 2\alpha) 4\epsilon^2 \right] \\ &\simeq I_0 t \sin^2 \pi N (1 \mp 2\epsilon \sin 4\alpha) \end{aligned} \tag{7.7.13}$$

and for a bright field

$$\begin{aligned} E &= I_0 t \left[2 - \sin^2 \pi N \left\{ \sin^2 2\alpha + \cos^2 2(\alpha \pm \epsilon) \right\} \right] \\ &\simeq I_0 t \left[2 - \sin^2 \pi N (1 \mp 2\epsilon \sin 4\alpha) \right] \end{aligned} \tag{7.7.14}$$

Consequently pure isochromatics are not obtained for a faulty rotation of the coupled polariser and analyser. The basic isochromatic pattern is overlaid by small variations indicating the isoclinic pattern with maximum (or minimum) exposure for $\alpha = 22\frac{1}{2}^\circ, 112\frac{1}{2}^\circ$, etc. For a practically possible error of $\epsilon = \pm \frac{1}{2}$ degree $\simeq \frac{1}{100}$ radian, maximum overlaid variation due

to isoclinic influences is ± 2 per cent. Although this may be permissible in work with integral and half order fringes, it does slightly affect the accuracy of boundary line or equidensity work.

7.8 POTENTIALITIES OF THE METHOD

Double exposure in a plane polariscope has the following advantages over the use of a circular polariscope for the evaluation of isochromatic fringe orders:

- (i) Quarter-wave plates are not required.
- (ii) Errors due to inaccurate quarter-wave plates are avoided.
- (iii) Accurately monochromatic light is not needed.
- (iv) The same polariscope elements are used for isoclinics and isochromatics.
- (v) The non-uniformity of illumination at the centre of the field is slightly reduced because of the absence of two elements, namely the quarter-wave plates.
- (vi) A simple and cheap bench can be used even for accurate evaluation.

Boundary line (high-contrast film) and equidensity line (Agfacontour film) techniques can both be used for fractional fringe order determination if the polaroids are crossed. For parallel polaroids it is best to use the combination of low-contrast and Agfacontour film described in Section 7.6. However, it is likely that the correctly and incorrectly crossed orientations of the polaroids will supply all the information required (Section 7.3) so that the parallel orientation need not be chosen.

The main disadvantage of this method is that the pure isochromatics uninfluenced by isoclinics cannot be viewed directly over the whole field. However, they can normally be seen in different regions if in monochromatic light the isoclinics are moved away by rotation of the polaroids or if white light is used. A restriction of the method is that the coupled polaroids must be rotated through exactly 45° between the two exposures.

In view of the considerable advantages of this technique, it is quite possible that future research using polariscopes will adopt it and dispense entirely with quarter-wave plates.

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CHAPTER 8

PHOTOGRAPHY OF ISOCLINICS

8.1 INTRODUCTION

The boundary line and equidensity methods can also be applied in the photography of isoclinics. The exposure E of the film in a plane polariscope with crossed polariser and analyser is given by

$$E = It = I_0 t \sin^2 2\alpha \sin^2 \pi N \quad (8.1.1)$$

where I = intensity, I_0 = maximum intensity transmitted by the model, t = exposure time, $\alpha = \theta - \beta$ = angle between the isoclinic parameter θ of the principal stress direction X_1 and the plane of polarisation (β) of the analyser, N = fringe order. Hence, if N has a given constant value everywhere, the boundary or equidensity lines of the photoelastic pattern will represent two isoclinic lines denoting $\alpha = \pm \Delta$ or $\theta = \beta \pm \Delta$.

Therefore with the plane of polarisation of the analyser at an angle β to the reference direction (crossed polariser and analyser making an angle β with the horizontal and vertical reference directions, see e.g. Fig. 8.9.1) and with N constant, the two boundary lines will represent isoclinics of parameters $\beta + \Delta$ and $\beta - \Delta$ and the centre line between the two lines will approximate to the isoclinic line of parameter $\theta = \beta$.

The main difficulty is the need to keep N constant throughout the field. No isoclinic is seen when the model is unloaded, and under any load, even a small load applied to a relatively insensitive model material, N is likely to change at least over some of the field. Even where such a change is not directly visible to the naked eye, it will affect the positions of the boundary or equidensity lines so that errors are introduced into an accurate determination of isoclinic parameters by these methods. More conventional methods of recording isoclinics and evaluating their centre lines are less affected by these errors.

Since the use of an optically poorly sensitive material is not suitable for fringe order determination, its use for the determination of isoclinic parameters requires that two separate models are made for a single problem. It appears therefore more advantageous to investigate methods of eliminating the isochromatics from patterns of highly sensitive materials. This is the purpose of this part of the investigation which deals with improvements in the conventional methods of recording isoclinics.

In addition, a number of methods are described that will sharpen the isoclinics at the same time so that they can be located accurately. Each of these methods was tested by the author with the assistance of Mr. Kremezis who also carried out additional experiments

under the author's supervision for incorporation in his M.Sc. project thesis (Kremezis, 1975).

9.2 VARIATION OF LOAD DURING EXPOSURE OF FILM

When the load is varied, the isochromatics are displaced (except for zero order) but the isoclinics remain undisturbed. Density will therefore be developed on the (negative) film in the regions of isochromatics, but since the intensity along the isoclinic lines remains zero irrespective of any load variations, no densities are formed along these lines. White light is used and satisfactory results have been obtained on panchromatic and on orthochromatic films. With monochromatic light, large variations in loads are required before the effects of the isochromatics can be eliminated from the resultant photograph.

This procedure cannot be used if the exposure time is so small (e.g. for high speed films) that the load cannot be varied sufficiently during this time. Further, this method is not suitable for three-dimensional slices which are not usually loaded.

Figures 9.2.1 and 9.2.2 show the 5^0 isoclinic line obtained with white light in a CR-39 model of a disk under diametral compression. Fig. 9.2.1 has been taken on a Kodalith ortho film 2556 with 20 sec exposure, Fig. 9.2.2 on a Kodalith panchromatic film 2568 with 2 sec exposure. The complete elimination of the isochromatics and the sharpening of isoclinics is very evident.

Comparison of Figures 9.2.1 with 9.2.2 indicates that in white light a better result is obtained on panchromatic than on orthochromatic films. The traces of isochromatics in Fig. 9.2.1 may be due either to the use of orthochromatic film with white light or to improper load variations. The loss of the isoclinic line near the loading point in Fig. 9.2.2 is primarily caused by over exposure used to achieve a considerable sharpening of the isoclinics. Overexposure is also responsible for the irregular outlines of the model in Fig. 9.2.2. Both these problems can be overcome by a shorter exposure time but only at the expense of much broader isoclinics as shown in Fig. 9.2.3 taken on panchromatic film with an exposure of only 1 sec instead of 2 sec. Further it seems that the interruptions of the isoclinic lines in Figures 9.2.1 and 9.2.2 are caused by non-uniformity of illumination at the image plane of the camera.

A result obtained with monochromatic light is shown in Fig. 9.2.4 of a 5^0 isoclinic in a model of a disk under diametral compression. The film has been exposed on Kodalith ortho film for 10 sec during which the load on the model has been varied. Comparison with the results obtained in white light shows that the isochromatics are not eliminated completely in monochromatic light.

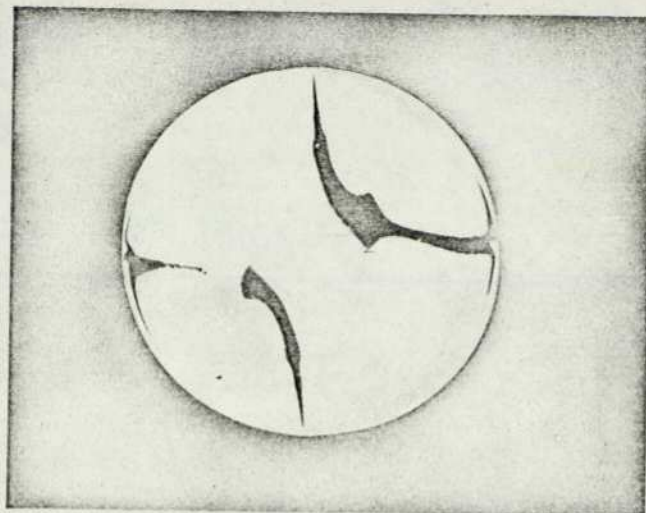


Fig. 8.2.1
Kodalith ortho film
White light, 20s

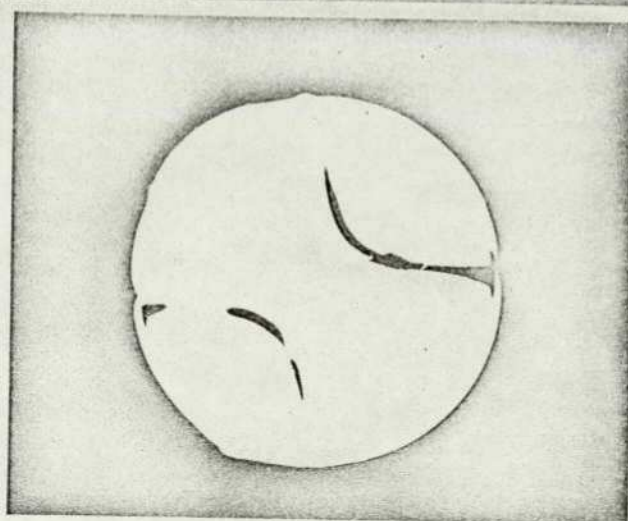


Fig. 8.2.2
Kodalith pan film
White light, 2 s

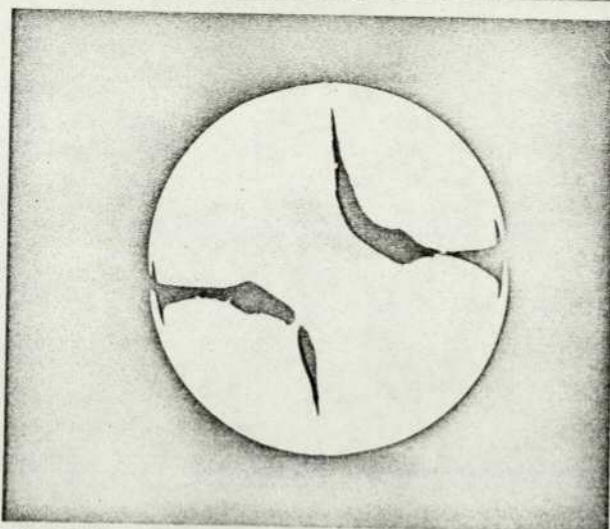


Fig. 8.2.3
Kodalith pan film
White light, 1s

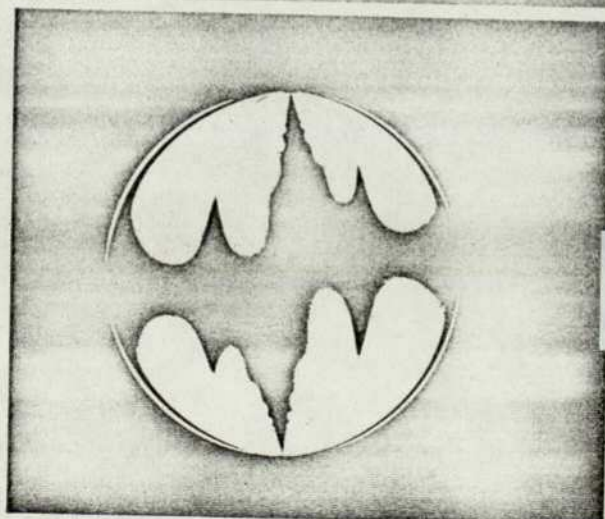


Fig. 9.2.4

Fig. 8.2.4
Kodalith ortho film
Monochromatic light, 10 s

9.3 VARIATION OF LOAD WITH MULTIPLE EXPOSURE OF ONE FILM

This method is similar to that of Section 9.2 but a multiple exposure is used and a different load for each exposure. The procedure permits recording of pure isoclinics with small exposure times that can then be divided into two or more smaller sub-exposure times (provided these are available in the camera). The same film is exposed each time, and the load on the model is varied suitably between exposures. White light is again preferred and panchromatic or orthochromatic film can be used.

Fig. 9.3.1 shows an excellent result of 5° isoclinics obtained on panchromatic film 2568 in white light for a CR-39 model of a disk under diametral compression. Three exposures of $\frac{1}{5}$ sec each have been made for compressive loads of 50, 100 and 150 lbf. Obviously this method is preferable to that of Section 9.2 where loads are varied during exposure.

9.4 APPLICATION OF LARGE LOADS

This technique (Frocht, 1948) involves the use of white light, a heavily loaded model, and panchromatic film. The isochromatic colours then blur in the resultant brightness throughout most of the field and only the isoclinic lines remain dark. However, excessive loads are undesirable because of large deformations, the possibility of creep, and the danger of failure of the model. Additional difficulties arise where the isoclinic line intersects low order isochromatics.

Fig. 9.4.1 shows the 5° isoclinics obtained in white light under a large diametrically compressive load of 290 lbf applied to a CR-39 model of a disk with a hole. This photograph on orthochromatic film has been taken from the report by Kremezis (1975).

9.5 OVER-EXPOSURE OF FILMS

A moderate load is applied to the model and the film is over-exposed in white light. Densities are then formed over all areas of the film except where the intensity is a minimum, i.e. along the isoclinics and zero isochromatics. The overexposure cannot be taken too far, as otherwise the definition of the isoclinics may be impaired. Obviously this method can also be used for slices of three-dimensional models. Panchromatic film is preferable to orthochromatic film in this method.

Fig. 9.5.1 shows 0° isoclinics in a stress-frozen model of Araldite photographed on orthochromatic film. Their saw-tooth appearance indicates the areas of relatively small exposures where isochromatics emerge from the isoclinics. A panchromatic film will reduce this effect. Because of overexposure the outlines of the model are not sharp.

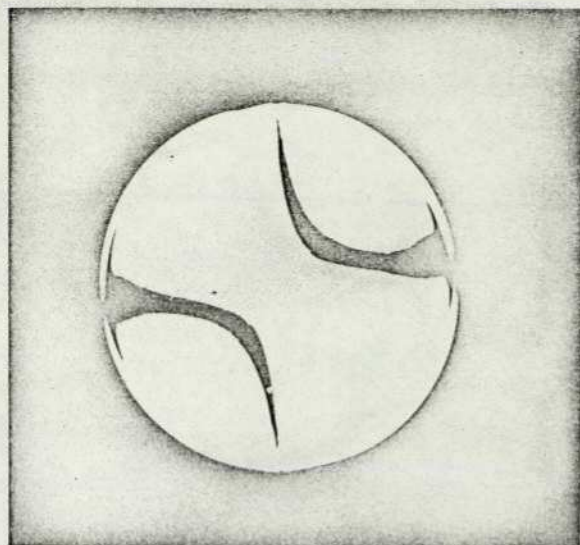


Fig. 8.3.1
Kodalith pan film
White light, $1/5$ s each
for loads 50, 100 & 150 lbf

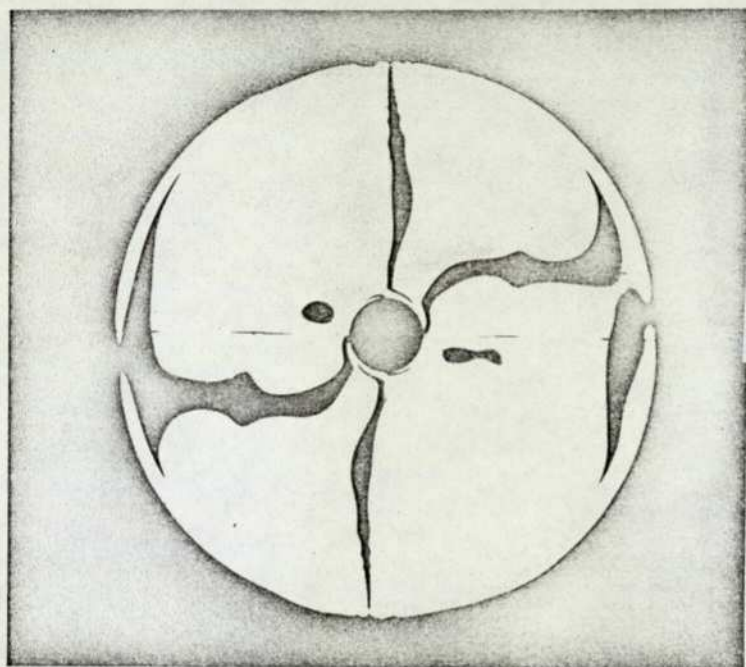


Fig. 8.4.1
Kodalith ortho film
White light, 1 s

Load = 290 lbf

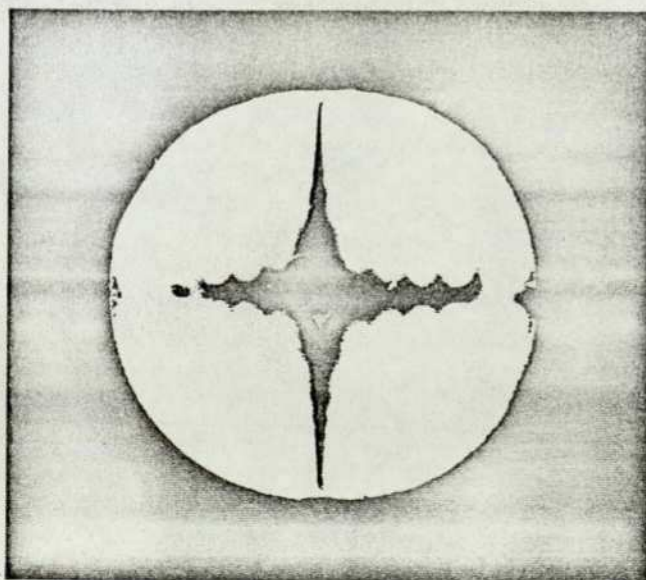


Fig. 8.5.1
Kodalith ortho film
White light, 10 s

Frozen stress slice

9.6 APPLICATION OF VERY SMALL LOADS

If small loads are applied, a major portion of the model will be free from isochromatic lines and, since the fringe order is not entirely zero, the isoclinics will be visible and can be recorded photographically. Monochromatic light and orthochromatic film may be used because no interference with isochromatics is obtained over a large part of the field.

The resultant isoclinics are usually broad but can be sharpened by either overexposure or overdevelopment of the film. The isoclinics will be disturbed in the small regions where isochromatics are present but there will normally be no difficulty in extrapolating the isoclinics over these small regions.

Fig. 9.6.1 shows the 5° isoclinics for a lightly loaded Araldite model of a tension bar with a hole. White light and Kodalith panchromatic film 2568 have been used. Fig. 9.6.2 shows the 5° isoclinics of a disk under diametral compression photographed in white light on Kodalith orthochromatic film 2556. The difficulty of obtaining the isoclinic line near its junction with a zero degree isochromatic is very obvious. The machining stresses in the model hide the zero isochromatic at the very edge and the isoclinic is clearly seen to indicate the 5° point of the boundary.

9.7 SUPERPOSITION OF A NEGATIVE

The effect of non-uniformities of light intensities on the recording of isoclinics may be reduced by superposition of a negative as suggested by Talukder (1974). The combined isoclinic and isochromatic pattern is recorded on a low contrast film with $\gamma = 1$ (Ilford Commercial ortho film developed in Phenisol (1 + 9) for $2\frac{1}{2}$ minutes) and with over-exposure. The film is developed and after drying is placed in the camera cassette in front of an unexposed high-contrast film. The developed negative then acts as filter which reduces considerably the passage of light through the darkened regions whilst light is transmitted preferentially through a narrow region surrounding the isoclinic.

Exposure through such a filter sharpens the isoclinics and reduces the non-uniformity of the field. Due care must be taken to see that the positions of the model and of the film plane remain undisturbed over the two exposures. The detailed mechanics of the process have been described by Kremezis (1975).

9.8 USE OF AGFA CONTOUR FILM

Agfa contour film used instead of Kodalith in the preceding methods for pure isoclinic recording gives excellent results. Usually two equidensity lines are produced and the centre line between them is

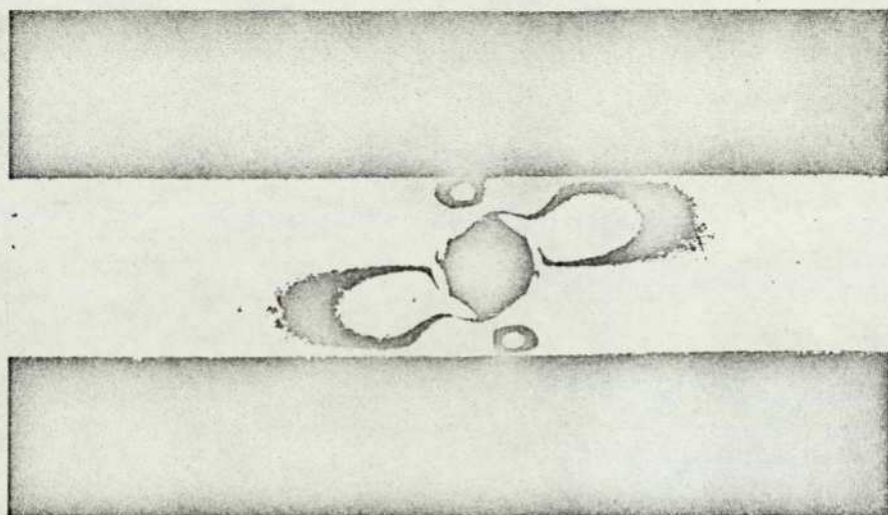


Fig. 8.6.1
Kodalith pan film
White light, 5 s

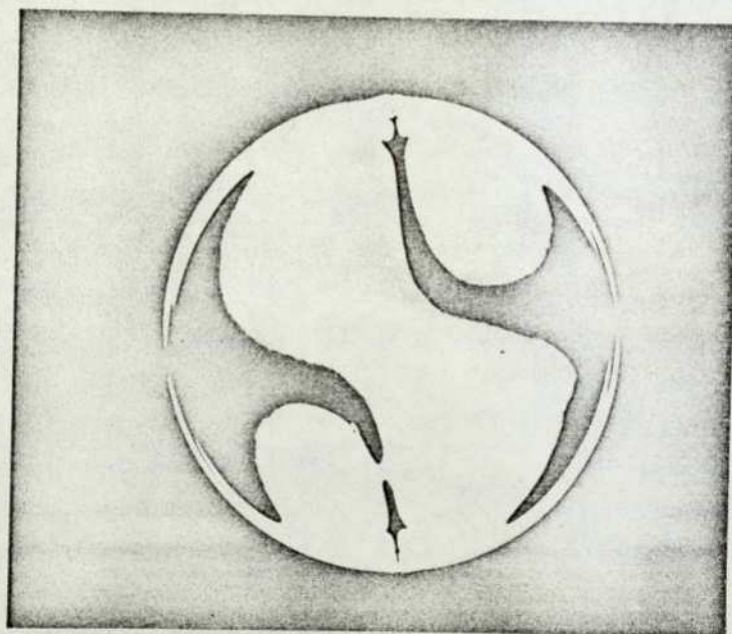


Fig. 8.6.2
Kodalith ortho film
White light, 15 s

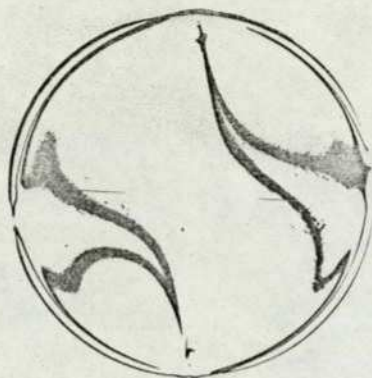


Fig. 8.8.1
 Agfacontour film, 480 s, 25 μ f
 White light, yellow filter

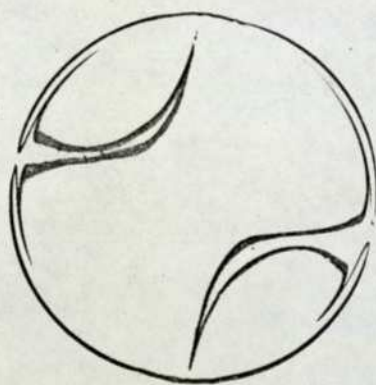


Fig. 8.8.2
 Agfacontour film
 White light, yellow filters

75, 100, 125, 150 μ f
 60 s at each

the isoclinic. By proper variation of exposure time, the distance between the two lines can be reduced and the isoclinic centre line thus 'sharpened'.

Fig. 9.8.1 is the result of directly exposing Agfacontour film in white light with yellow filters directly to the optical pattern in the camera on the bench. It shows the 10^0 isoclinics in a disk of CR-39 under a low diametrically compressive load of 25 lbf only. In Fig. 9.8.2 the same isoclinics in the same model have been obtained by multiple exposure of 60 sec. at each of four different loads, namely 75, 100, 125 and 150 lbf. The improvement is drastic. Both photographs show spikes due to the zero isochromatic just inside the edge which has been affected by machining stresses. However, in Fig. 9.8.2 the isoclinic centre line can easily be interpolated over the affected zone.

9.9 METHOD OF EQUAL INTENSITIES

The equation for the light intensity I emerging at any given point from the analyser in a plane polariscope is (see Eqn. 8.1.1)

$$I = I_0 \sin^2 \pi N \sin^2 2\alpha \quad (9.9.1)$$

where I_0 is the maximum intensity transmitted through the analyser, N the isochromatic fringe order, and α the angle between the principal stress direction (isoclinic parameter θ) and the plane of polarisation of the polariser, which is at 90° to the analyser and at an angle β to a given reference directions (see Fig. 9.9.1).

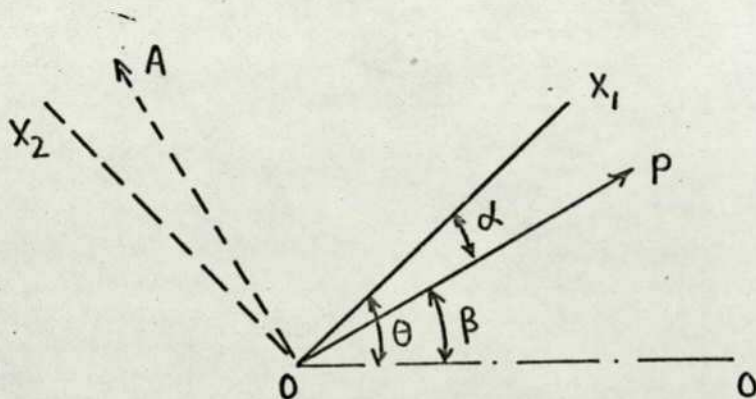


Fig. 9.9.1 Isoclinic parameter θ and positions of analyser and polariser.

Equation (8.9.1) can be written

$$I = I_0 \sin^2 \pi N \sin^2 2(\theta - \beta) \quad (8.9.2)$$

or
$$I = I_0 \sin^2 \pi N \sin^2 2(\beta - \theta) \quad (8.9.3)$$

Thus for a particular orientation β of the crossed polariser and analyser, the intensity at a point of the field will vary with the isoclinic parameter θ , if the isochromatic fringe order is constant over the field, or with θ and N if both isochromatic and isoclinic values vary from point to point. Two points of the model with isochromatic values N_1 , N_2 and θ_1 , θ_2 will transmit the same intensity I if

$$\sin^2 \pi N_1 \sin^2 2(\theta_1 - \beta) = \sin^2 \pi N_2 \sin^2 2(\theta_2 - \beta)$$

or
$$\sin^2 2(\theta_2 - \beta) = \frac{\sin^2 \pi N_1}{\sin^2 \pi N_2} \cdot \sin^2 2(\theta_1 - \beta) \quad (8.9.4)$$

Hence if photographs are taken under controlled conditions and with a known orientation β of the crossed analyser and polariser, then θ_2 can be calculated once N_1 , N_2 and θ_1 are known. As the isochromatic values N can usually be determined accurately, the isoclinics can be found point by point over the whole field if the isoclinic parameter θ_1 at one point is known. This method does not require the difficult separation of isoclinics from isochromatics.

The lines of equal intensity are also lines of equal density which can easily be obtained by the boundary line method on high contrast film or by the equidensity method on Agfacontour film if the combined pattern of isoclinics and isochromatics is photographed as described in Chapters 3 and 5. The positions of the lines in the combined pattern can be changed photographically by varying one or more of the following three parameters: (i) exposure time; (ii) orientation of crossed analyser and polariser (angle β), and (iii) load on the model.

Fig. 8.9.2 shows clearly defined boundary lines between dark and bright regions and all these lines define a constant value of intensity so that equation (8.9.4) can be applied to any two points anywhere on these lines. For instance, in Fig. 8.9.2, taken on Kodalith Orthochromatic film 2556 with $\beta = 45^\circ$, an exposure of 5 sec, and a compressive load of 175 lbf (778.4 N), the equidensity line intersects the horizontal diameter at the point A ($k_1 = x/r = 0.406$, $k_2 = y/r = 0$) where the isoclinic parameter is known to be zero. An earlier analysis has shown that under a load of 175 lbf the fringe orders at A and at B ($k_1 = 0.406$, $k_2 = 0.5$) are 0.9 and 0.85 respectively. Hence the isoclinic parameter θ_B at B is given by

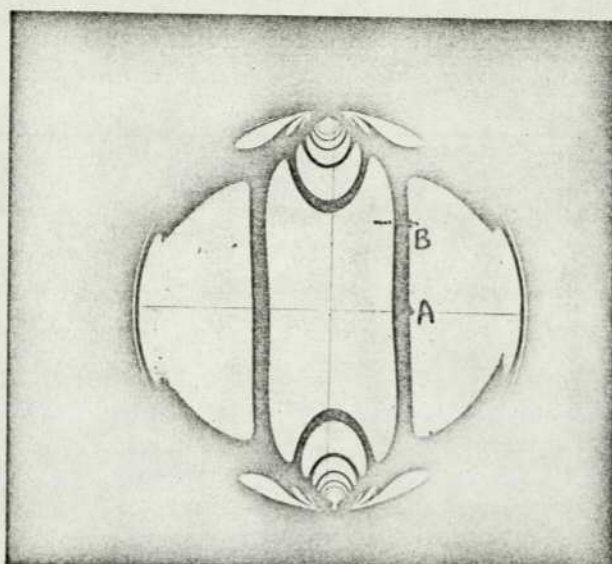


Fig. 8.9.2
Kodalith ortho film, 5 s
 $\beta = 45^\circ$, Load = 175 lbf

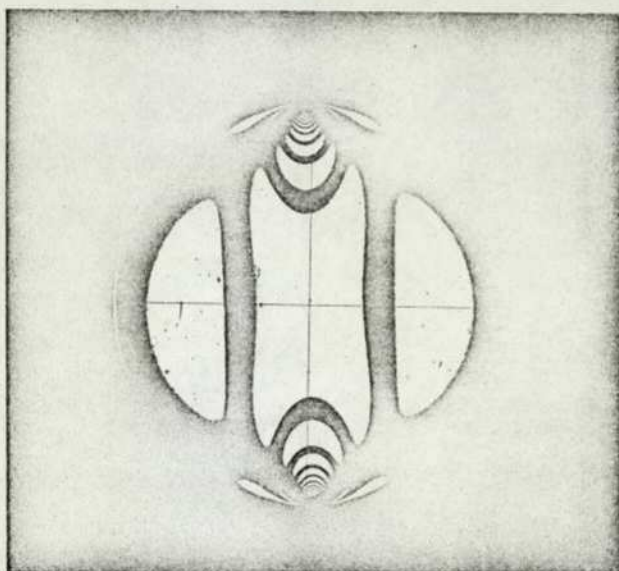


Fig. 8.9.3
Kodalith ortho film, 2 s
 $\beta = 45^\circ$, Load = 175 lbf

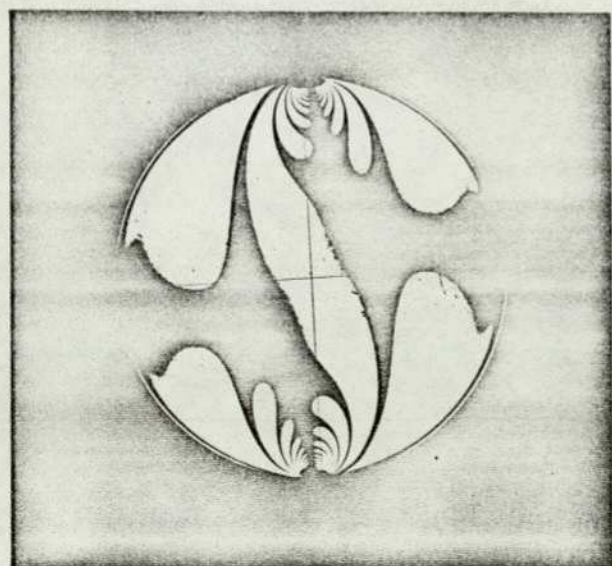


Fig. 8.9.4
Kodalith ortho film, 5 s
 $\beta = 10^\circ$, Load = 175 lbf

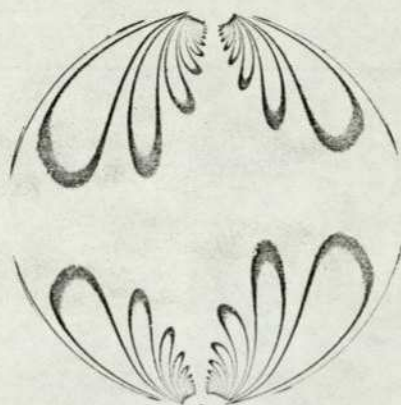


Fig. 8.9.5
Agfacontour film, 120 s
 $\beta = 5^\circ$, Load = 225 lbf



Fig. 8.9.6
Agfacontour film, 135 s
 $\beta = 15^\circ$, Load = 225 lbf



Fig. 8.9.7
Agfacontour film, 180 s
 $\beta = 45^\circ$, Load = 225 lbf

$$\sin^2 2(45^\circ - \theta_B) = \frac{\sin^2(\pi \times 0.9)}{\sin^2(\pi \times 0.85)} \times \sin^2 2(45^\circ - 0) = 0.463$$

or $\theta_B = 23.55^\circ$

This is in good agreement with theoretical value $\theta_B = 23.9^\circ$.

This point method, although lengthy, gives a good accuracy in isoclinic measurements if the isochromatics are separately determined accurately by one of the methods outlined in this thesis. Correct photography and a sensitive X-Y recorder is all that is required.

By comparison with Fig. 8.9.2, Fig. 8.9.3 taken with 175 lbf load, $\beta = 45^\circ$ and 2 sec exposure shows that a change in exposure time shifts the positions of the boundary lines. Fig. 8.9.4 taken with 175 lbf load, $\beta = 10^\circ$ and 5 sec exposure shows by comparison with Fig. 8.9.2 that the boundary lines of the combined pattern change their positions when β changes.

Similar results are seen in Figures 8.9.5, 8.9.6 and 8.9.7 taken directly on Agfacontour film with 225 lbf load, with $\beta = 5^\circ$, 15° and 45° , and with 120, 135 and 180 sec exposure. All the black lines on any one of these photographs have a constant value of intensity. If the isoclinic angle is known at any one point of any black line, the isoclinics at all other points of the black lines on the same photograph can be determined from the isochromatic information alone.

8.10 SIMULTANEOUS RECORDING OF SEVERAL ISOCLINICS

If the model is lightly loaded, the isochromatic order will either be constant or vary only slightly over extended regions of the model. The equation of the plane polariscope with crossed polariser and analyser at an angle β to a given reference direction can then be written as

$$I = k \sin^2 2\alpha = k \sin^2 2(\theta - \beta) \quad (8.10.1)$$

where k is constant or approximately constant.

Thus for a given orientation β of the crossed analyser and polariser, the intensity depends on the isoclinic parameter θ at any point. The boundary lines or equidensities obtained on high-contrast or Agfacontour film therefore represent lines of $\alpha = \pm\Delta$, i.e. of constant isoclinic parameters given by $\theta = \beta \pm \Delta$, where Δ is the small difference between the known central isoclinic value β and the values appropriate to the two neighbouring lines and can be determined by calibration.

The positions of the isoclinic boundary or equidensity lines and hence their values of Δ vary with exposure time. Thus in a single photograph, isoclinics are obtained for $(\beta + \Delta)$, $(\beta - \Delta)$ and, by interpolation, also for β .

The advantage of this method is that more than one isoclinic parameter is recorded simultaneously and that the isoclinic lines are sharp lines and not broad bands. A large number of isoclinics of different parameters can be obtained if one photograph is taken on a low-contrast film and later copied with various exposure times onto Agfacontour film to give equidensities representing isoclinics.

Difficulties associated with this method are: (i) regions of constant or slowly varying N may be difficult to obtain, (ii) three-dimensional slices having a large number of fringes cannot be investigated, and (iii) calibration is required for the differential isoclinic angle Δ .

This calibration may be made by any of the following methods:

(i) use of a separate calibration model in which the isoclinic distribution is known, (ii) use of known properties of the isoclinics (e.g. symmetry) allowing to assign an isoclinic parameter to any of the lines obtained on a photograph, and (iii) determination on the polariscope or from photographs (see Section 8.9) of the isoclinic parameter at a point through which a given boundary or equidensity line passes.

Calibration methods are described fully in a report by Kremezis (1975). Consideration of the problem of a disk under diametral compression has shown that the two boundary or equidensity lines are in fact approximately true isoclinics in a lightly loaded model.

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CHAPTER 9

CONCLUSIONS

9.1 BOUNDARY LINE TECHNIQUE (Chapter 3)

9.1.1 Significance of Boundary Lines

Instead of the centre lines of dark and bright bands representing fringes of integer or half order, the boundary lines between such bands can be chosen as fractional fringes for the evaluation of the isochromatic pattern. On high-contrast film (e.g. Kodalith), these boundary lines are sharply defined, a particularly valuable feature where the bands themselves are broad and irregular.

The method always gives a view of the whole field and is applicable to lens-type and diffused-light polariscopes without any additional equipment apart from the usual photographic facilities and, for accuracy, an X-Y reader to determine the positions of the boundary lines within the plane of the model. It thus offers considerable advantages over the more common full-field or point methods of fractional fringe order measurement.

9.1.2 Methods for Obtaining Different Fractional Orders

Changes in photographic exposure time within the permissible exposure range provide a great number of different full-field fractional fringe orders. Changes in load magnitudes further extend the sensitivity of the method through the construction of curves or patterns of fringe order per unit load. The photographic information about all the full-field fractional fringe orders is available on film negatives in permanent form and can be analysed without prints at any convenient time.

The difficulty in obtaining a boundary line near the intensity minima and maxima can be overcome by change in load magnitude or exposure times (Section 3.6) or pre-exposure of the film (Section 6.5) or double exposure on a plane polariscope (Section 7.4), if dark and bright backgrounds are used suitably.

With changes in exposure time and load level, fractional orders to within less than 0.02 fringe can easily be obtained because one of the boundary lines can always be made to pass through practically any point in the model. In Fig. 3.4.10., about 90 well distributed fringes have been obtained between orders of 0 and 1.75.

A considerable freedom of choice exists for the required number of different exposure times and load levels so that additional efforts can always be restricted to what at any stage is found necessary. At the same time, an overall view of the experimental achievement is available at any stage of investigation.

Actual photoelastic observations by photography can be completed within a very few minutes, usually less than 5 minutes for one set of photographs at different exposure times. This compares favourably with the time required in point by point compensation techniques.

If a single photograph of the photoelastic pattern is first taken with appropriate exposure on a low-contrast film (e.g. Ilford commercial Ortho), the developed negative can be copied on high-contrast film with various copying exposure times giving boundary lines of different fractional orders. Photoelastic observation on the polariscope is then limited to a single exposure and the method can be applied in dynamic photoelasticity or photoviscoelasticity, where no other existing method gives fractional fringe orders. One single photograph provides a permanent record of all the fractional fringe order information likely to be needed.

9.1.3 Calibration of Boundary Lines

A simple calibration procedure involving photography of the fringes in a model of a beam in pure bending determines, without any significant effort, the stress-optical coefficient of the model as well as the proper fraction of the isochromatic order represented by the boundary lines in the photograph of the test model, taken and processed under identical conditions. The integral orders must be determined by inspection of the boundary line pattern or by conventional methods.

Normally a separate calibration is required for each exposure time used but a single calibration at a known exposure time also permits calculation of the fractional orders of the boundary lines for other known exposure times and for transition from dark to bright background fields or vice versa (Section 3.2 and Section 6.6). Strict adherence to photographic and optical conditions is required when this technique is used.

The calibration method can be applied in direct photography on high-contrast film on the polariscope and in photography on low-contrast film with copying on high-contrast film.

Where a separate calibration model cannot be employed or where identical photographic and optical parameters cannot be maintained, self-calibration in terms of fractional fringe orders can be adopted. This also requires knowledge of the integral orders by inspection or by conventional methods.

Graphical methods of self-calibration (Sections 3.3.1 and 3.3.2) can be used where isochromatics of at least a few integral or half orders are present.

The compensation method of self-calibration (Section 3.3.4) is suitable even when no integral or half order isochromatics are present. A point on the photograph with boundary lines is matched with a point on the model in the bench, for which the fractional order is determined by conventional compensation.

The density method of self-calibration (Section 3.3.4) requires recording of the fringe pattern on the linear portion of a film with $\gamma = 1$ in addition to density measurements on the negative by means of a densitometer equipment.

The graphical and density methods require only a single exposure on the photographic bench and can therefore be applied in dynamic photoelasticity.

Self-calibration techniques can be applied to frozen-stress slices of three-dimensional models when a separate calibration model is not available or when slices are of different thickness (Section 6.7).

9.1.4 Precision of Boundary Line Measurements

The precision of the method depends mainly on the accuracy of locating the boundary lines, on maintaining controlled photographic conditions and, where a separate calibration model is used, on transmission of the same maximum light intensity through both calibration and test model. No other external factors are involved.

Fringe orders can easily be measured to an overall resolution better than 0.01 fringe in regions of relatively low fringe gradients and on a calibration beam in bending. Resolution of high order fringes may be slightly worse because measurement for boundary line locations offers difficulties of resolution over the small distances normally separating high order fringes.

Experience has shown that except in regions with steep gradients the overall error in fringe order measurement and location is normally less than 1%. A major part of this is due to manual or instrumental errors in establishing the positions of the boundary lines.

Resolution and precision can be further improved by photographic enlargement of the boundary line negative or by enlargement in copying from a low-contrast negative.

9.2 EQUIDENSITY LINE TECHNIQUE (Chapter 5)

9.2.1 Photographic Generation of Equidensities

Equidensities generated from photographs of the photoelastic fringe pattern are similar to boundary lines between dark and bright bands and represent lines of constant fractional fringe order. They are obtained either by the photographic process of pseudo-solarisation or on a special film known as Agfacontour. The use of Agfacontour film shows marked advantages over pseudo-solarisation.

Agfacontour can be directly exposed in the polariscope camera in monochromatic green or yellow light. Alternatively, photographs of an isochromatic pattern on low-contrast film can be contact copied on Agfacontour if yellow filters are used in copying. Without filters Agfacontour copies may fail to give equidensities.

Normally, first-order equidensity lines obtained by direct exposure or contact copy are sufficiently sharp for accurate measurement. Where necessary contact copy of a photograph with first-order equidensities will yield two second-order equidensity lines of much improved sharpness for each first-order equidensity copied.

The second paragraph of Section 9.1.1 applies equally to this technique.

9.2.2 Methods for Obtaining Different Fractional Orders

Since equidensity lines are analogous to boundary lines, the methods summarised in Section 9.1.2 can all be applied in equidensity techniques with Agfacontour film replacing high-contrast film. The effort involved is about the same for both methods and so are their advantages.

Agfacontour permits a larger latitude for variation of exposure time than does Kodalith film.

9.2.3 Calibration of Equidensity Lines

All the calibration methods for boundary lines (Section 9.1.3) can be applied to equidensity lines.

9.2.4 Precision of Equidensity Line Measurements

The resolution and accuracy obtainable with equidensity lines are normally the same as with boundary lines (see Section 9.1.4).

In special cases, a better resolution may be obtained by copying enlargements (Fig. 6.6.8) and/or use of second-order equidensities.

9.3 OPTICAL AND PHOTOGRAPHIC REQUIREMENTS (Chapters 3, 4)

9.3.1 Optical Parameters

Optical parameters for boundary lines or density formation are: symmetry of light source, misfocus, consistency of exposure time, uniformity of illumination.

The light source and camera aperture should be symmetrical with respect to the axis of the polariscope.

The test model and the calibration beam should be placed in the same position along the bench.

On a lens polariscope the image should be focussed on the camera lens.

If the required exposure time is not available in the camera, a timer circuit will give repeatable though less accurate exposure times; a stop watch can be used for long exposure times.

Boundary and equidensity line methods are sensitive to variations in the uniformity of maximum light transmission at the image plane. Such variations occur owing to (i) diffraction from the edges of the bench elements, (ii) internal reflection in the lens and other optical elements, (iii) optical disturbances in the model and in imperfect polaroids.

A region of reduced light transmission obtained at the periphery of the field must be eliminated by a decrease in the usable diameter of the field. The undesirable non-uniformity arising from (ii) at the centre of the field can be reduced by one or several of the following measures: (a) bloomed lenses and coated glass surfaces, (b) a smaller camera aperture, (c) a translucent screen placed between light source and first field lens.

Diffuse-light polariscopes require very good translucent screens for greatest uniformity of maximum light transmission and a great distance between model and camera lens so that the obliquity of light rays does not falsify the fractional fringe line readings.

9.3.2 Photographic Parameters

Photographic parameters for density development are: specific properties of batch or pack of film, mode of agitation, developer temperature, condition of developer. Thorough understanding of these parameters is essential.

There are no restrictions on the values of the photographic parameters when a self-calibration procedure is used.

Where a separate calibration model is used and calibration results are obtained separately for different exposure times, photographic parameters must be the same for test and calibration model. The same batch of film and preferably the same pack should generally be used and films carrying the calibration and test fringe patterns should be processed under identical conditions regarding mode of agitation and developer temperature. In addition, fresh developer should be used each time when a new film is processed.

Where a separate calibration model is used and calibration results at one exposure time are applied to derive the results for various other exposure times, the absolute value of the photographic parameters must be maintained strictly constant for all photographs calibrated from the same model.

9.4 APPLICATIONS (Chapter 6)

The boundary and equidensity line techniques have been applied on a diffused-light polariscope without a first field lens and with or without a second field lens to obtain fractional orders in a calibration beam in pure bending and in a disk under diametral compression (Section 6.1).

Difficulties arising from creep of model materials are minimised by the short time taken up on the polariscope and by photographing calibration beam and test model with the same exposure time at identical intervals after load application, when the creep rate has decreased sufficiently. If the copying method is used, creep can be investigated with great precision in single photographs taken at various intervals after loading (Section 6.2).

Fractional fringes have been obtained in models showing less than 0.5 fringe overall (Section 6.3).

The effects of initial stresses due to machining have been observed and their magnitudes determined as fractional orders. (Section 6.4).

Pre-exposure of film has been used as an additional means of varying the position and fringe order of boundary lines (Section 6.5).

Calibration results for one exposure time have been applied to photographs obtained with other exposure times of a diametrically compressed circular disk with a central hole. (Section 6.6).

Enlargement on photographic paper of the fringe patterns on negatives of calibration and test model permits accurate location of fractional fringes when the image of the test model on the film is very small. This technique in combination with Newton's extrapolation formula has given a very good experimental value for the maximum stress concentration in a tensile strip with a central circular hole (Section 6.6).

The maximum fringe order near the contact area of a disk under diametral compression has been determined by photographic enlargement of the equidensity lines obtained on Agfacontour (Fig. 6.6.8).

Boundary line and equidensity methods have been applied to frozen-stress slices of three-dimensional models with fractional fringe orders determined by the compensation method of self-calibration (Section 6.7).

9.5 DOUBLE EXPOSURE ON A PLANE POLARISCOPE (Chapter 7)

Pure isochromatics without isoclinics are obtained on a plane polariscope without quarter-wave plates if the photoelastic pattern is photographed by double exposure of equal time and if the coupled polariser and analyser are rotated through 45° between the exposures. The angle of orientation ϕ between polariser and analyser does not affect this principle but only the contrast of the resultant isochromatic fringes (Fig. 7.3.1).

The boundary and equidensity line techniques allow this method to be applied to sensitive numerical evaluation so that quarter-wave plates need never be used.

Variations in the angle ϕ give an additional means of varying the fractional orders of boundary or equidensity lines, but an error analysis shows that angles between 20° and 70° should not normally be used.

Monochromatic light is normally preferred and is a necessity for photography on Agfacontour film.

White light can be used for the boundary line method but fringe patterns obtained on panchromatic film then differ significantly from those obtained on orthochromatic film or in monochromatic light (Figs. 7.4.7 to 7.4.10). White light can give a fringe pattern similar to one obtained in monochromatic light provided a blue-sensitive film is used.

Use of a parallel plane polariscope offers no advantage over a crossed plane polariscope except for generally shorter exposure times. It has the disadvantage of lower sensitivity due to reduced fringe contrast.

9.6 DETERMINATION OF ISOCLINICS (Chapter 8)

Boundary and equidensity line techniques can be used with additional models of low isochromatic sensitivity to obtain relatively sharp isoclinic fringes by over-exposure, free from interference by isochromatics.

If the same model is used for isochromatic and isoclinic photography, various methods applied simply or in combination reduce or eliminate the effects of isochromatics and at the same time sharpen the isoclinics:

- (a) Variation of loading during exposure,
- (b) Multiple exposure under various loads,
- (c) Application of large loads in white light with photography on panchromatic film,
- (d) Application of small loads so that isochromatics remain below the first fringe order,
- (e) Over-exposure of the film,
- (f) Exposure of a high-contrast film through an already developed low-contrast negative placed as a filter immediately in front of the high-contrast film.

From a combined boundary or equidensity line pattern of isoclinics and isochromatics, the isoclinic parameter at any required point on any of the lines in the whole field can be calculated if the isoclinic parameter at one point and the fringe orders at that point and at the required point are known. This point by point method is lengthy but accurate and does not require separation of isoclinics.

The described techniques can be applied to two-dimensional models and to three-dimensional frozen-stress slices.

In regions of a lightly loaded model, where the fringe order is approximately constant, the boundary or equidensity lines represent isoclinics of parameters $\beta \pm \Delta$ where β is the parameter indicated by the polaroid orientations. The relatively small angle Δ can be varied by variation of exposure time and calibrated by a separate model, e.g. a disk under diametral compression. Thus two isoclinics are combined on one photograph and a third, the isoclinic β , can usually be obtained by interpolation.

9.7 OUTLOOK

The techniques described can now be regarded as established in principle and in practice. Further refinements will arise from practical applications for industrial purposes and in research. Possible fields are:

- (i) Replacement of conventional compensation techniques in industrial problems,
- (ii) Comparison with results from automatic polariscopes (Allison and Nurse, 1971) for regions of very steep strain gradients,
- (iii) Applications in combination with the photoelastic layer technique,
- (iv) Optical creep and relaxation behaviour of photoelastic materials,
- (v) Photoplasticity,
- (vi) Dynamic and quasi-static photoelasticity,
- (vii) Evaluation of isopachic and moiré fringes,
- (viii) Evaluation of fringes in holography or in any other optical interference phenomena.

Combinations of several of the described techniques or their combinations with other techniques will lead to methods of particular advantage in special cases. Examples are:

- (a) Since boundary lines can be considered as "zero-order" equidensities which by copying on Agfacontour can be converted into first-order equidensities, so the boundaries of first-order equidensities can be considered as boundary lines from which second-order equidensities may be generated. The boundary line technique can then be applied to first-order equidensity photographs to give lines of increased sharpness

without any need for further photographic copying. This method will be particularly advantageous for enlargements of photographs or enlarged copies.

- (b) The boundary or equidensity line technique can be applied to fringe photographs obtained by the fringe multiplication technique of Post (1970a). In special cases this combination may increase the sensitivity by an order of magnitude.
- (c) The calibration technique by separate model may be combined with the technique of densitometry by Chiang and Khetan (1974). Difficulties of finding films with $\gamma = 1$ and of interpreting the densitometric trace then disappear and the evaluation of isochromatics, starting from a single low-contrast negative of the test model and one of the calibration model, can be made entirely automatic. Graphical self-calibration techniques combined with densitometry may also lead to an automatic process.
- (d) The point-by-point method of evaluation for the isoclinics also lends itself to automation in connection with automation of evaluation for isochromatics, provided the isoclinic value at one point of the equidensity concerned is known.

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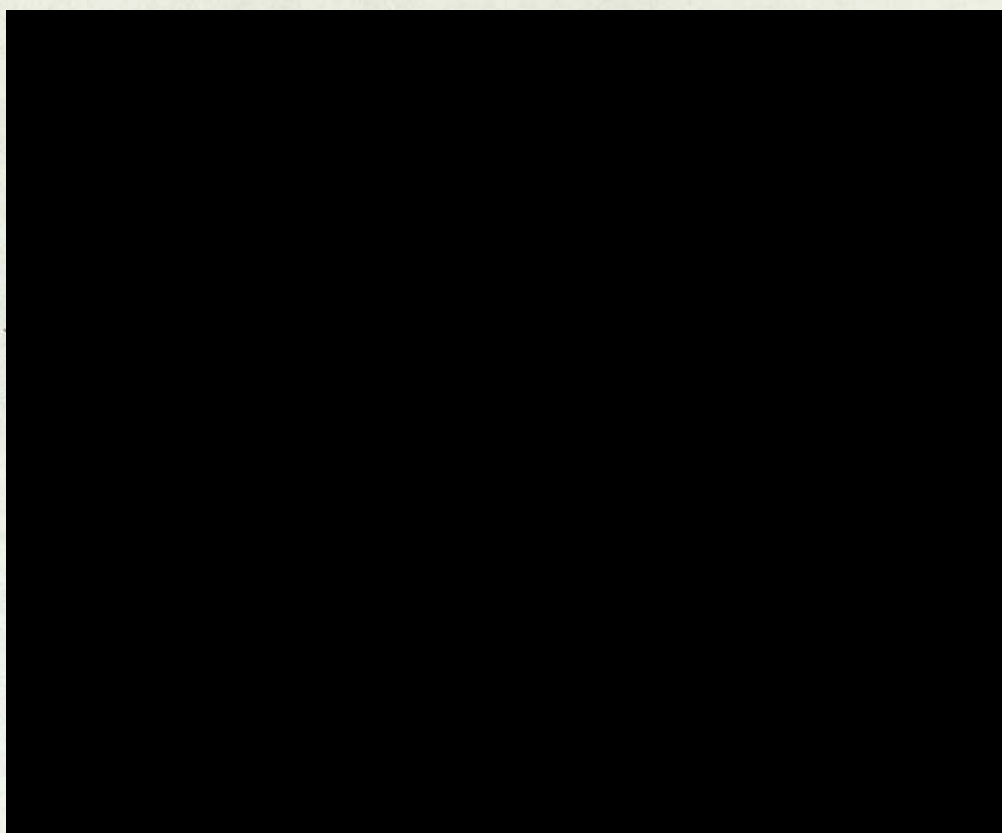
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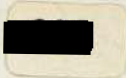
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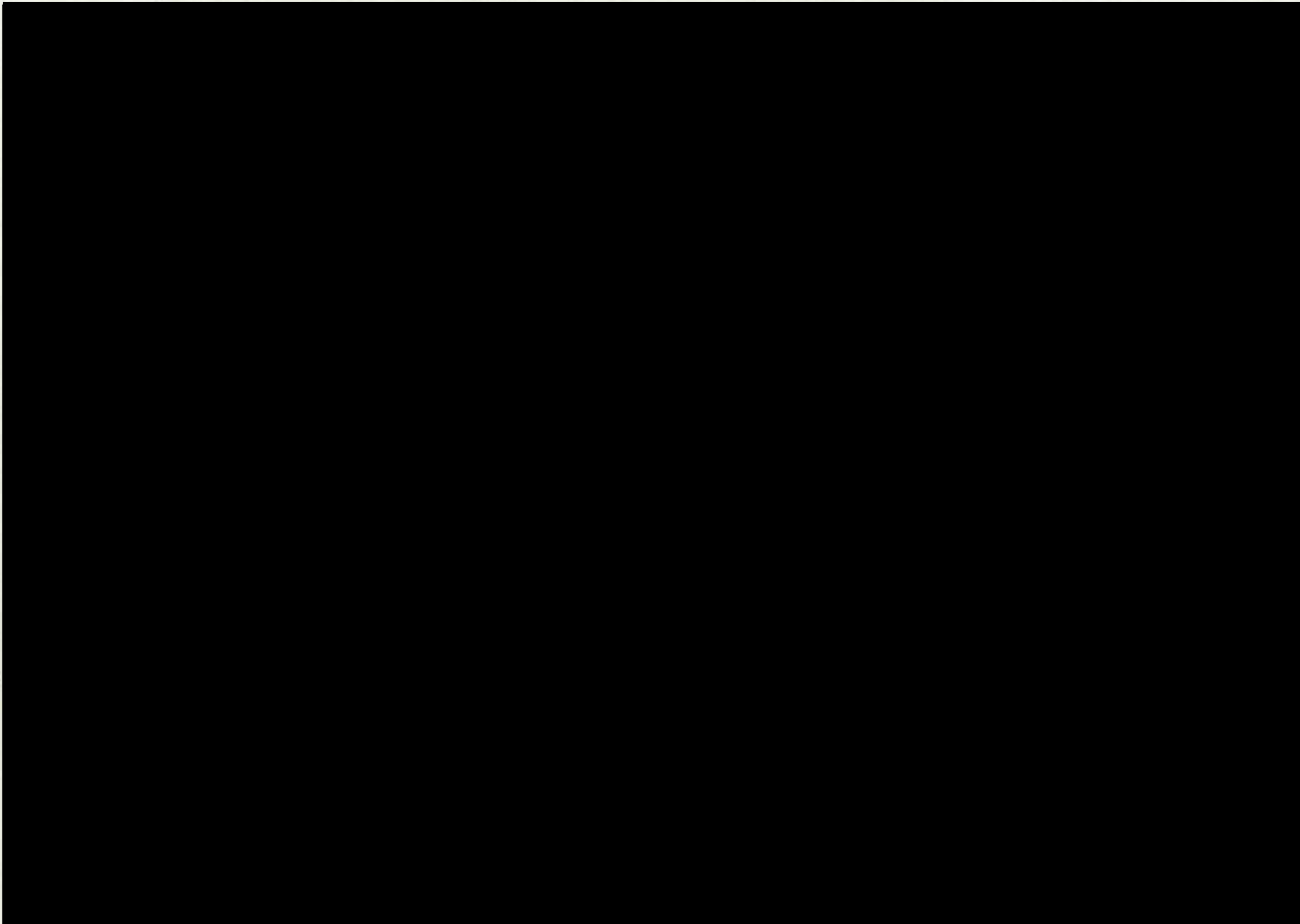
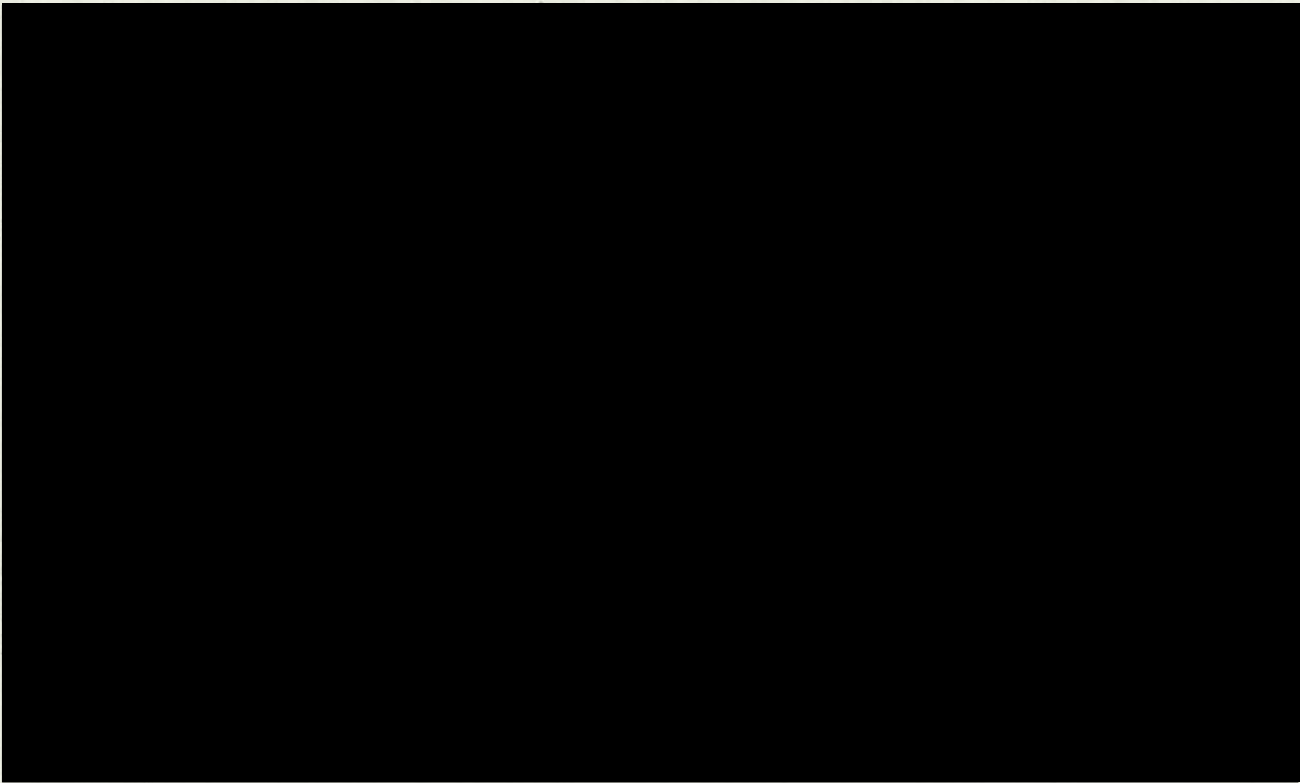
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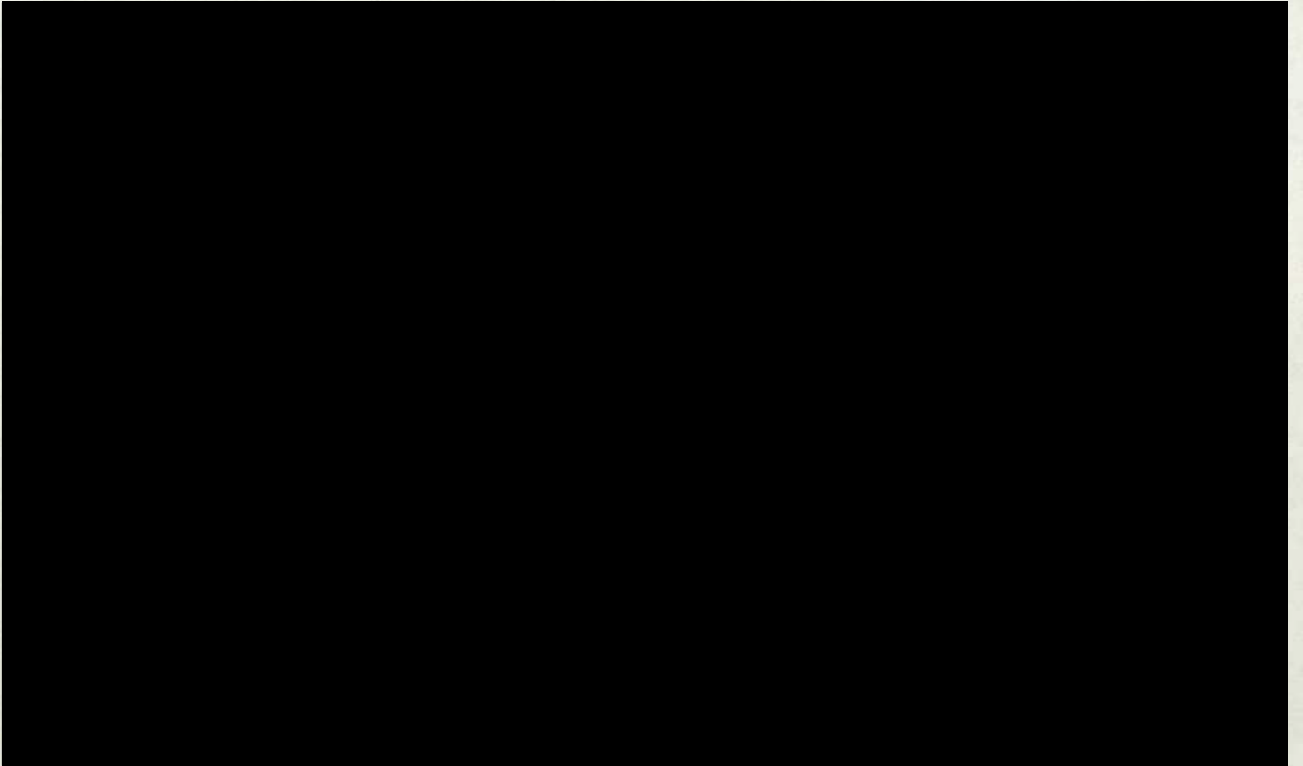
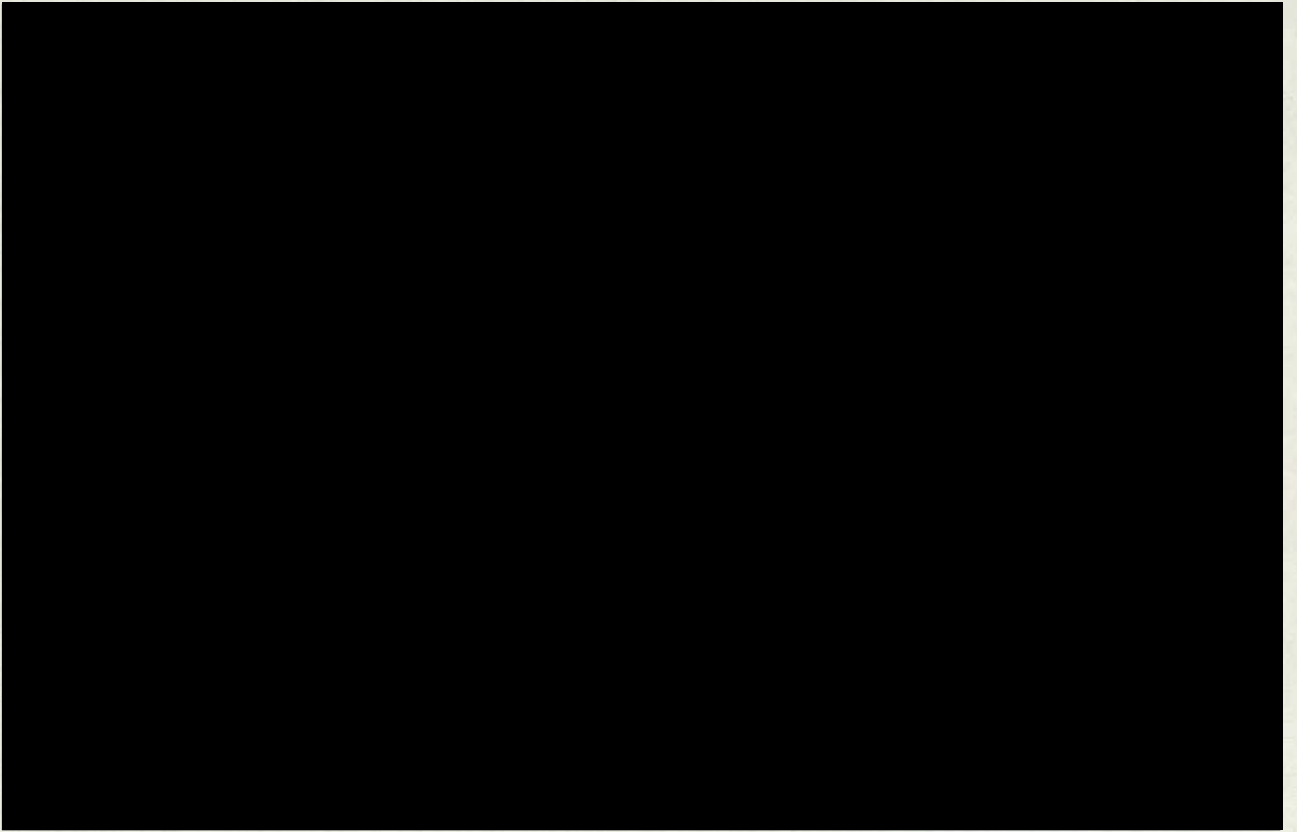
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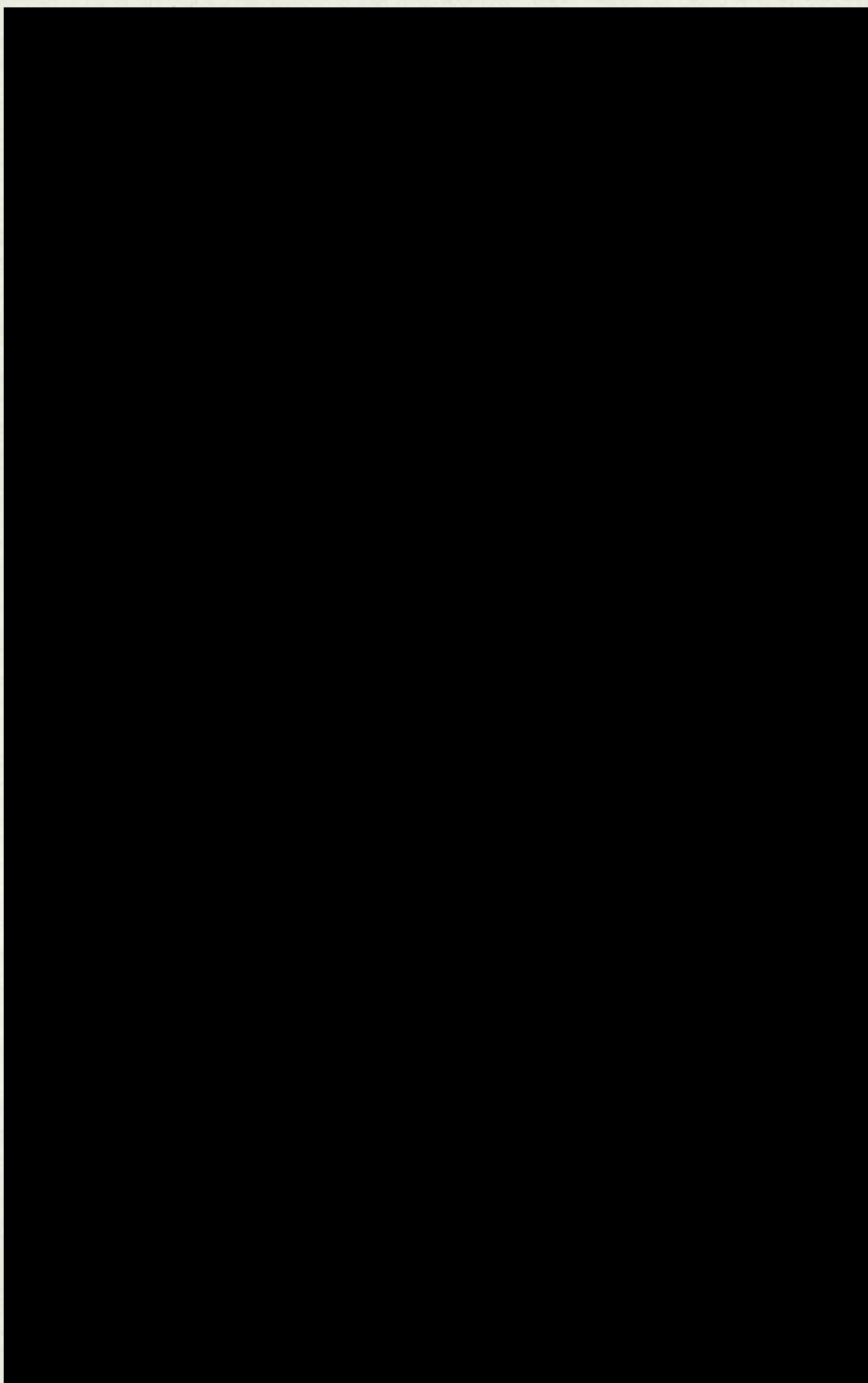
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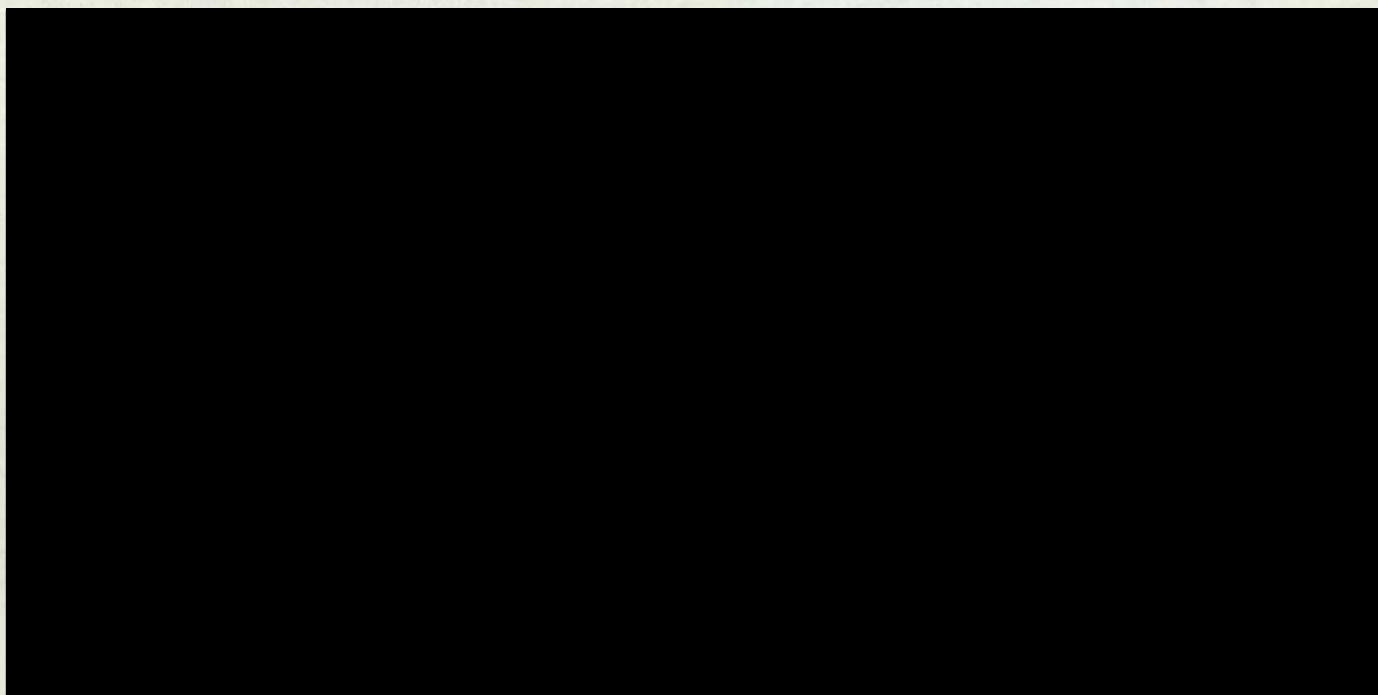
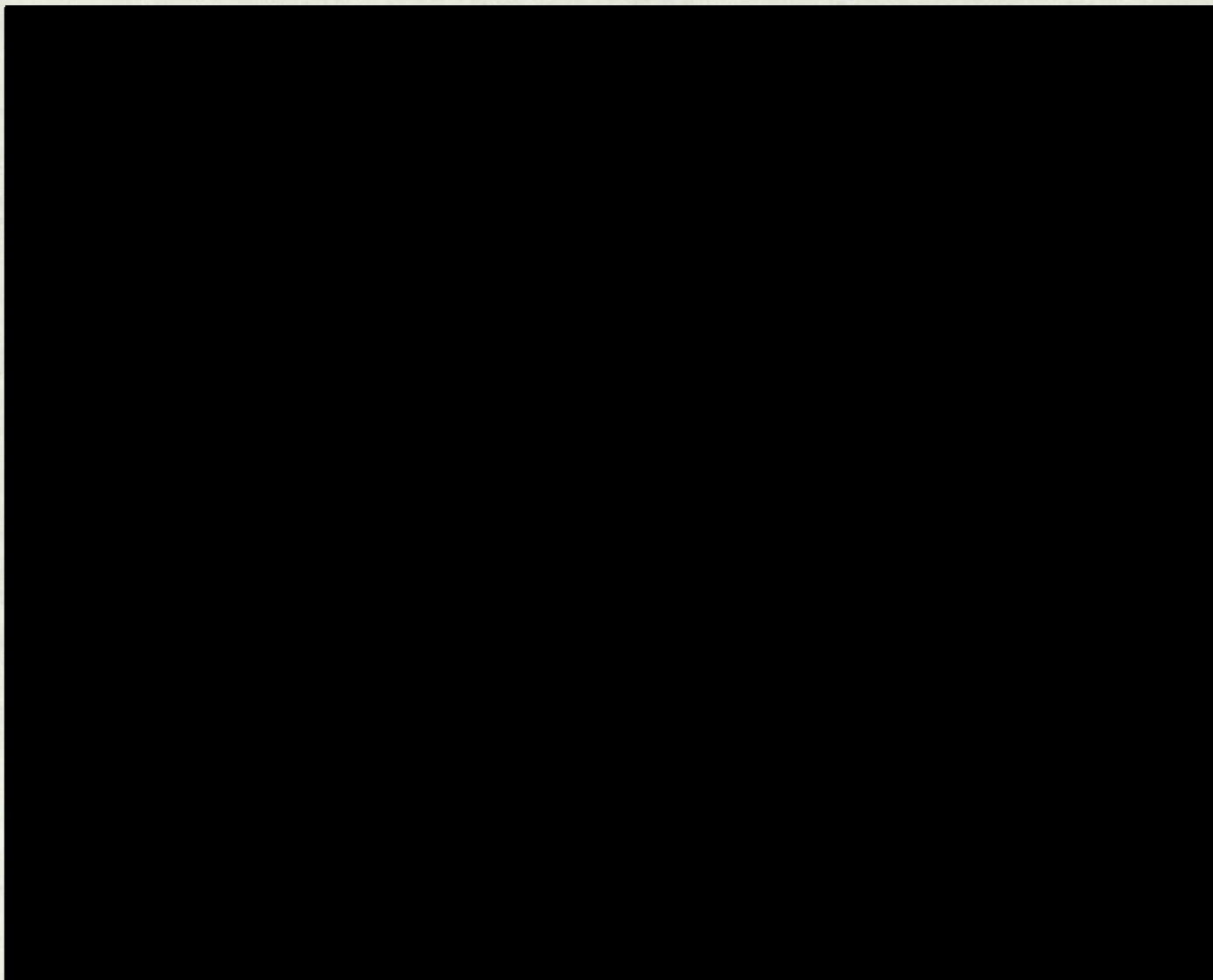
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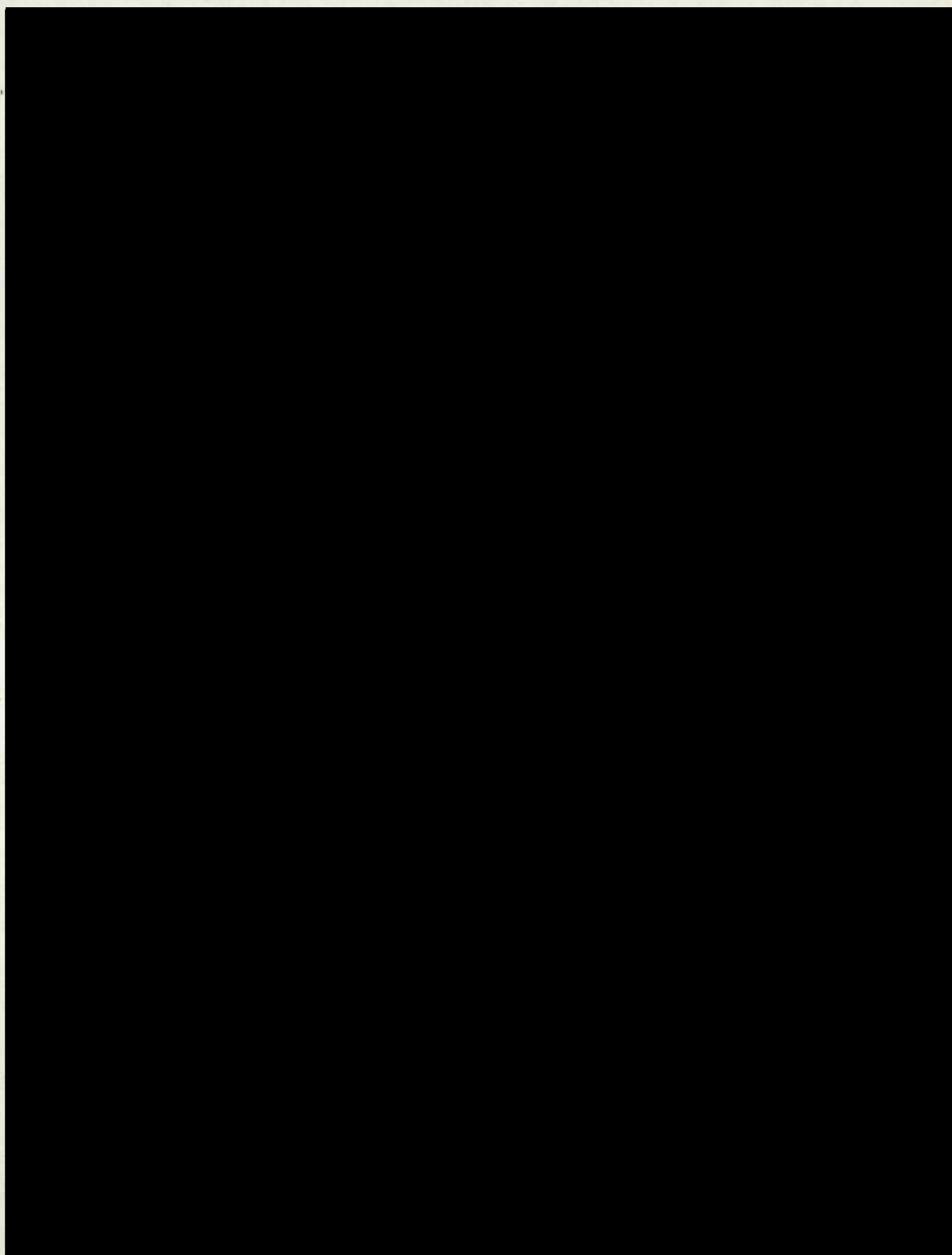
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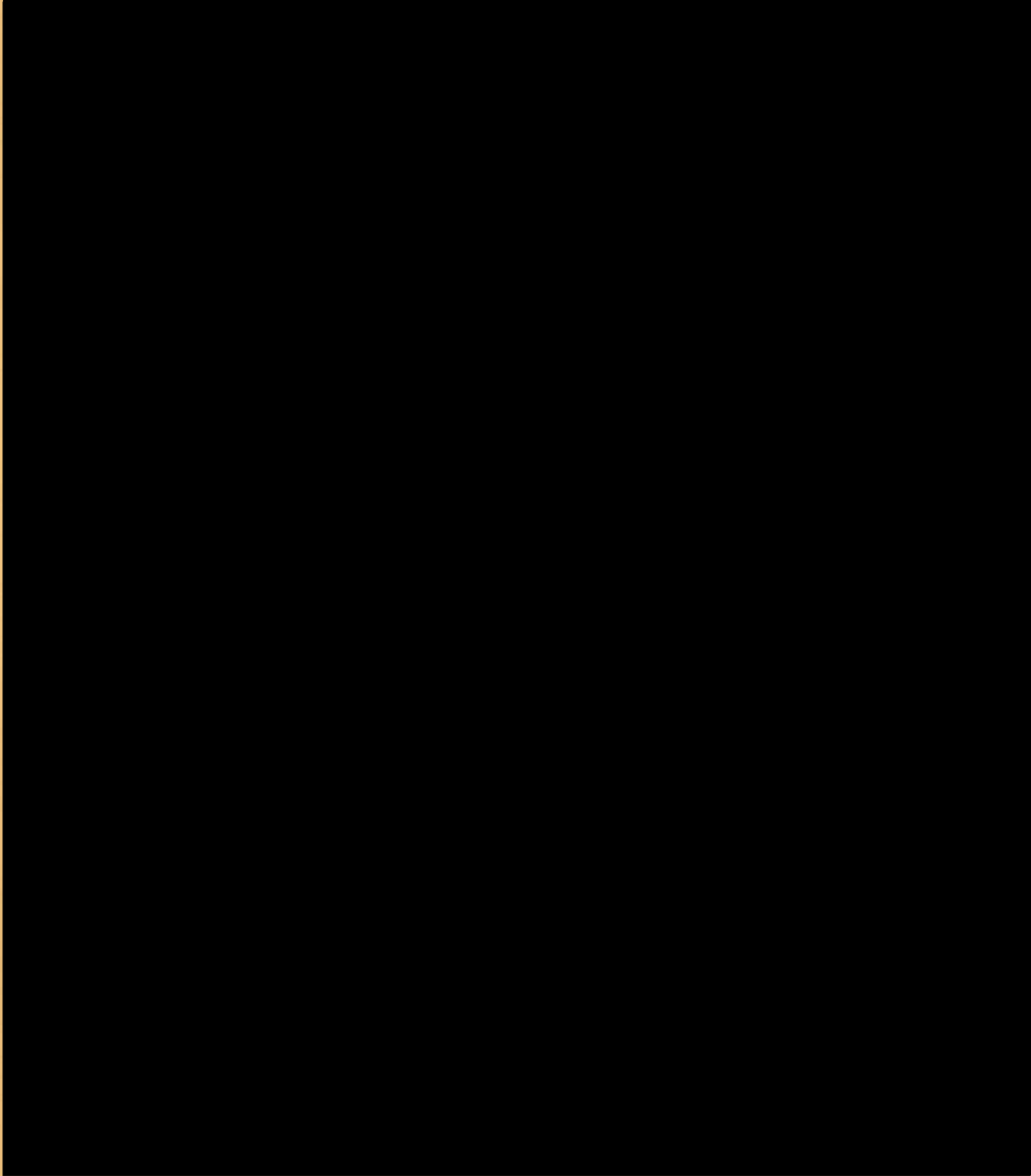


A PHOTOGRAPHIC METHOD FOR DETERMINING FRACTIONAL FRINGE ORDERS

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The City University,
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St. John Street,
London, EC1V 4PB,
U.K.

Dr. N.K. Das Talukder
Reader in Mechanical Engg.,
Banaras Hindu University,
Varanasi-5,
India



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CISM UDINE

REPRINT FROM THE

PROCEEDINGS OF THE FIFTH
INTERNATIONAL CONFERENCE
ON EXPERIMENTAL STRESS ANALYSIS

UDINE, ITALY, 1974

PAPER 42

PHOTOGRAPHIC METHOD FULL-FIELD FRACTIONAL FRINGE ORDER DETERMINATION

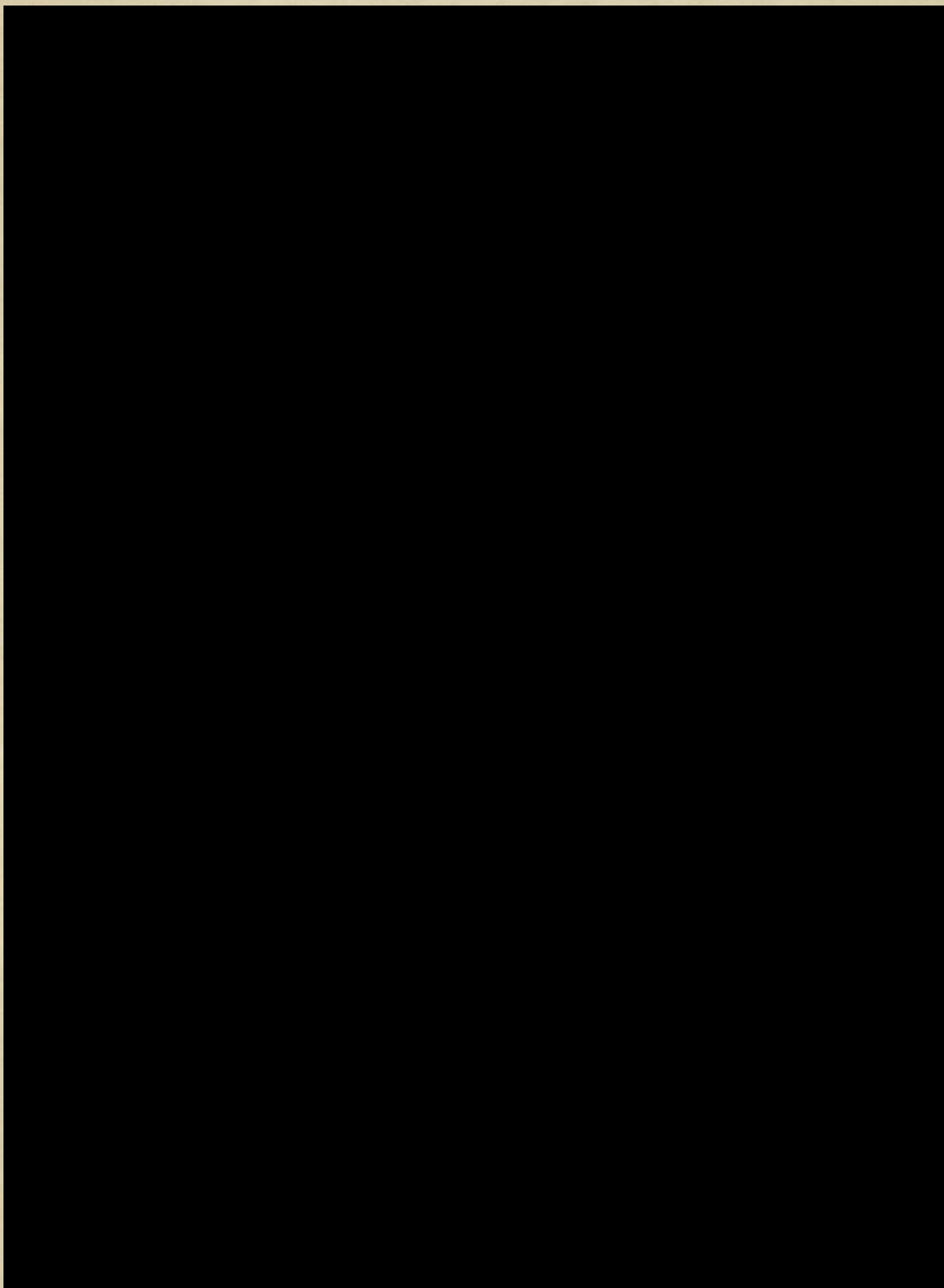
DÉTERMINATION GLOBALE DES ORDRES ISOCHROMATIQUES
FRACTIONNAIRES PAR MÉTHODE PHOTOGRAPHIQUE

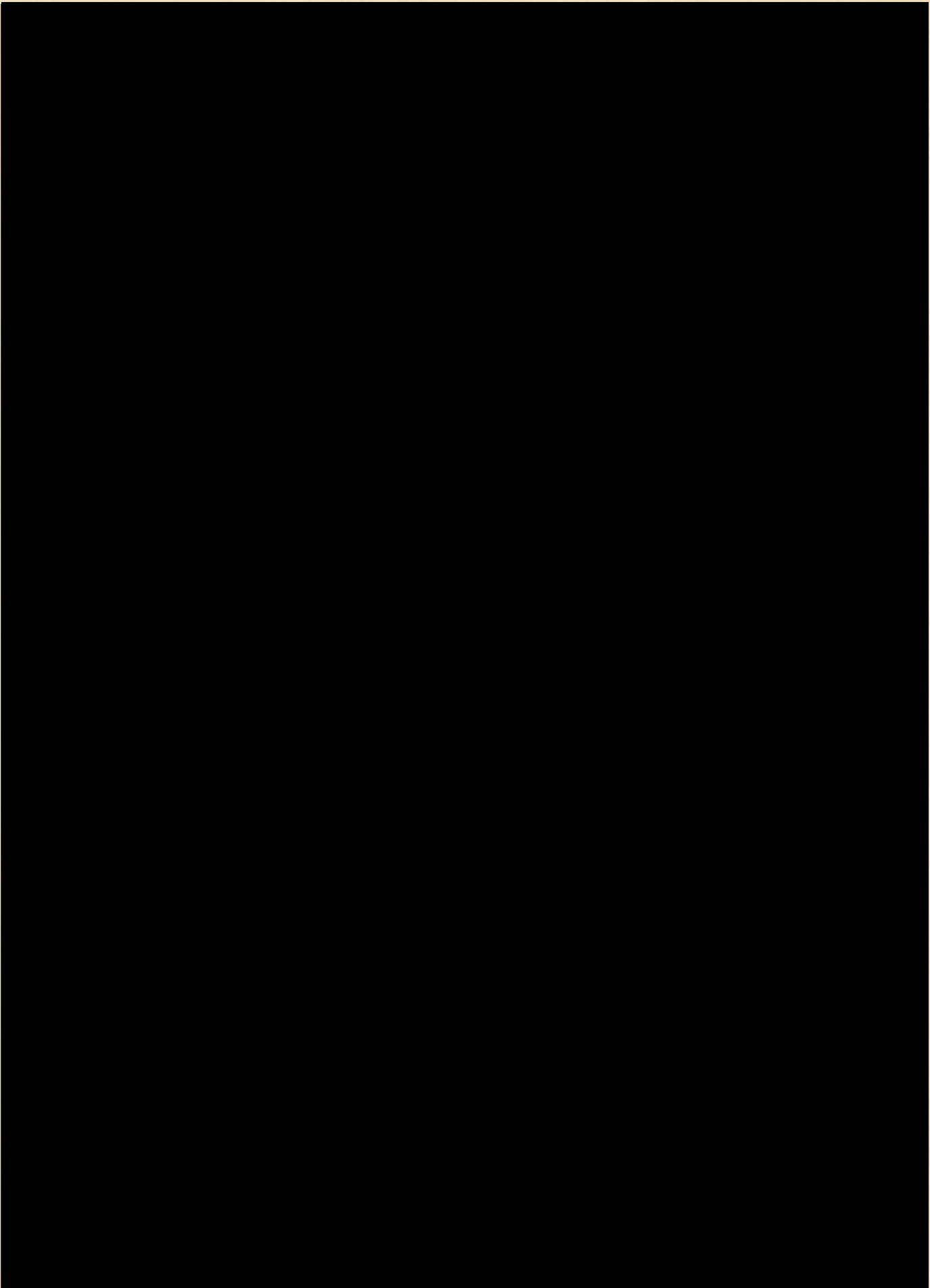
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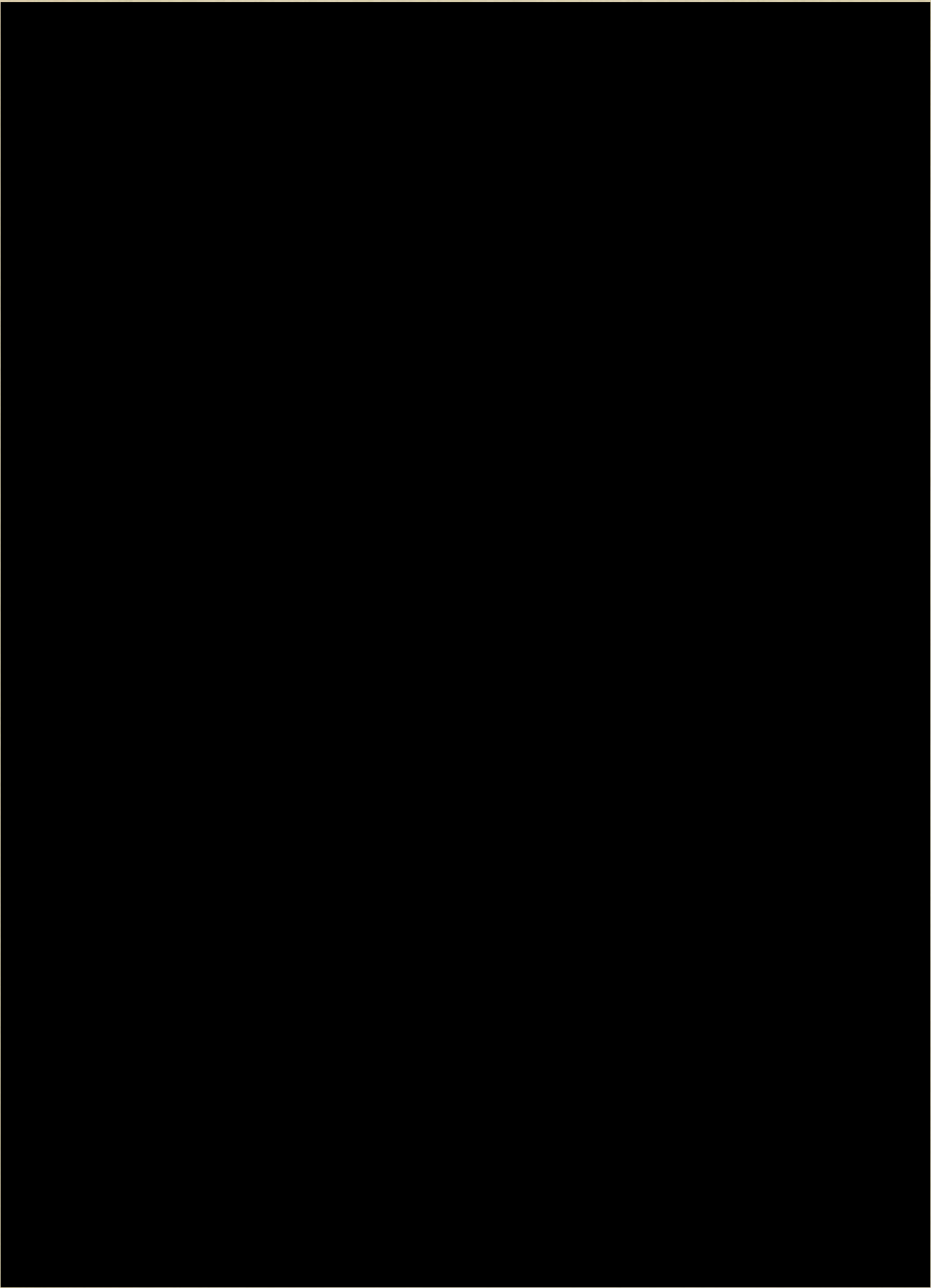
M. L. Meyer,* C. L. Mehrotra*

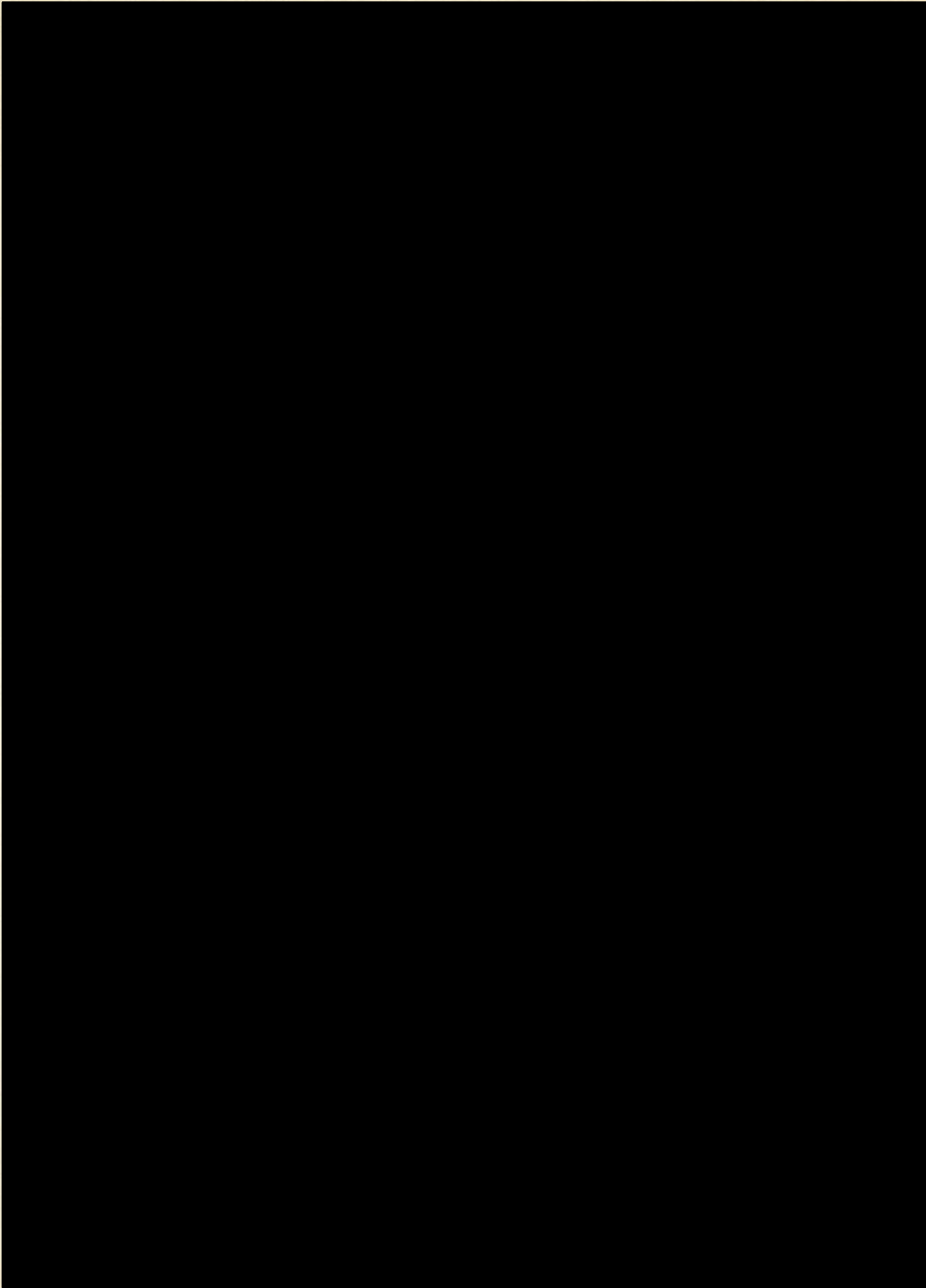
In photoelasticity, very high contrast film with suitable developer has been used to obtain photographs of isochromatic patterns of dark and bright bands with well-defined boundary lines. For a given set of photographic parameters, these boundary lines represent constant fractional fringe orders, numerically determinable by a calibration procedure. For any particular loading condition, the positions and hence the numerical values of the boundary lines can be altered widely by variations in exposure time. Thus the fractional fringe order distribution over the whole field can be obtained on both lens and diffuse types of polariscope without any auxiliary equipment.

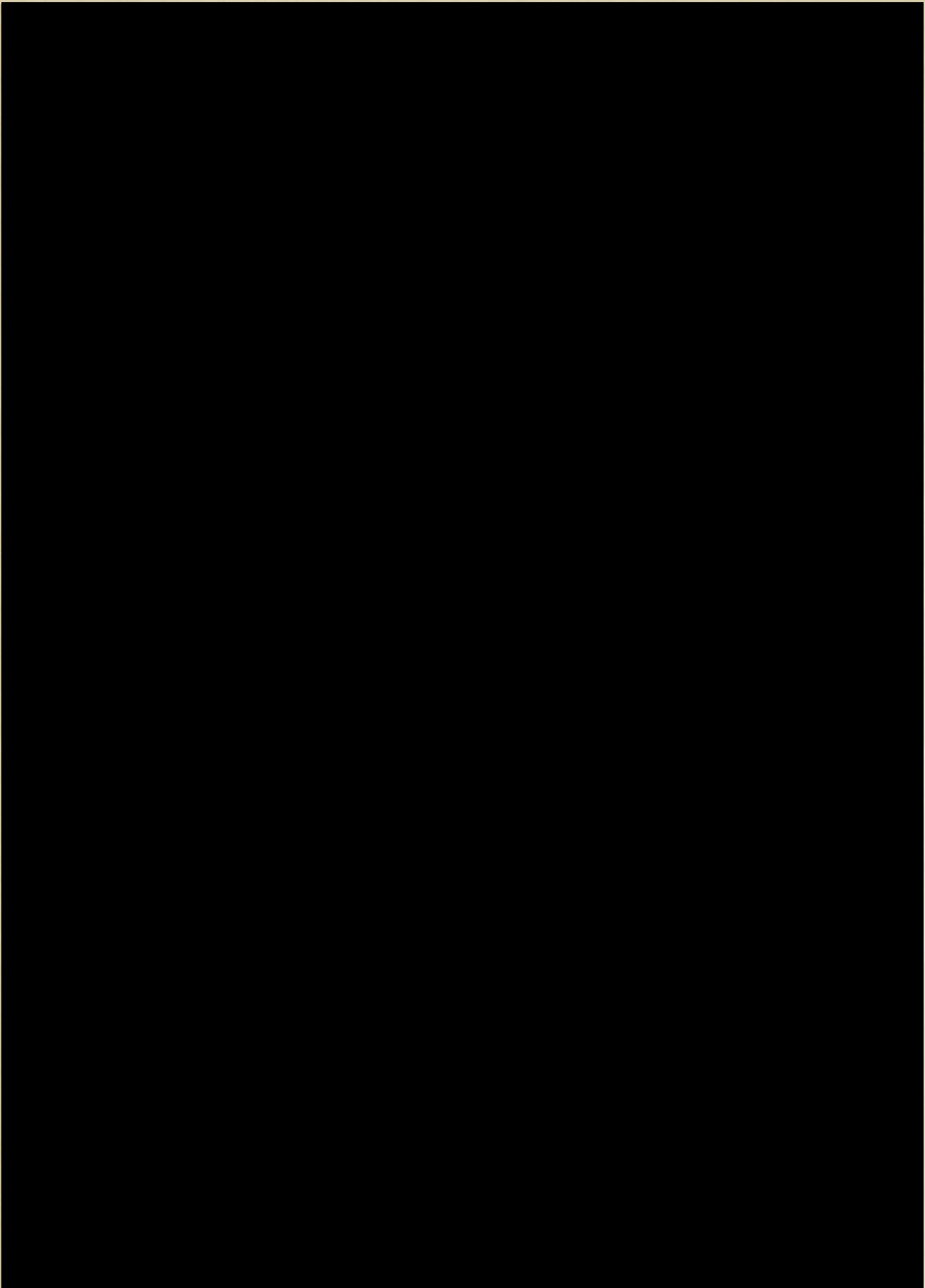
* The City University, St. John Street, London, EC1V 4PB,
U. K.

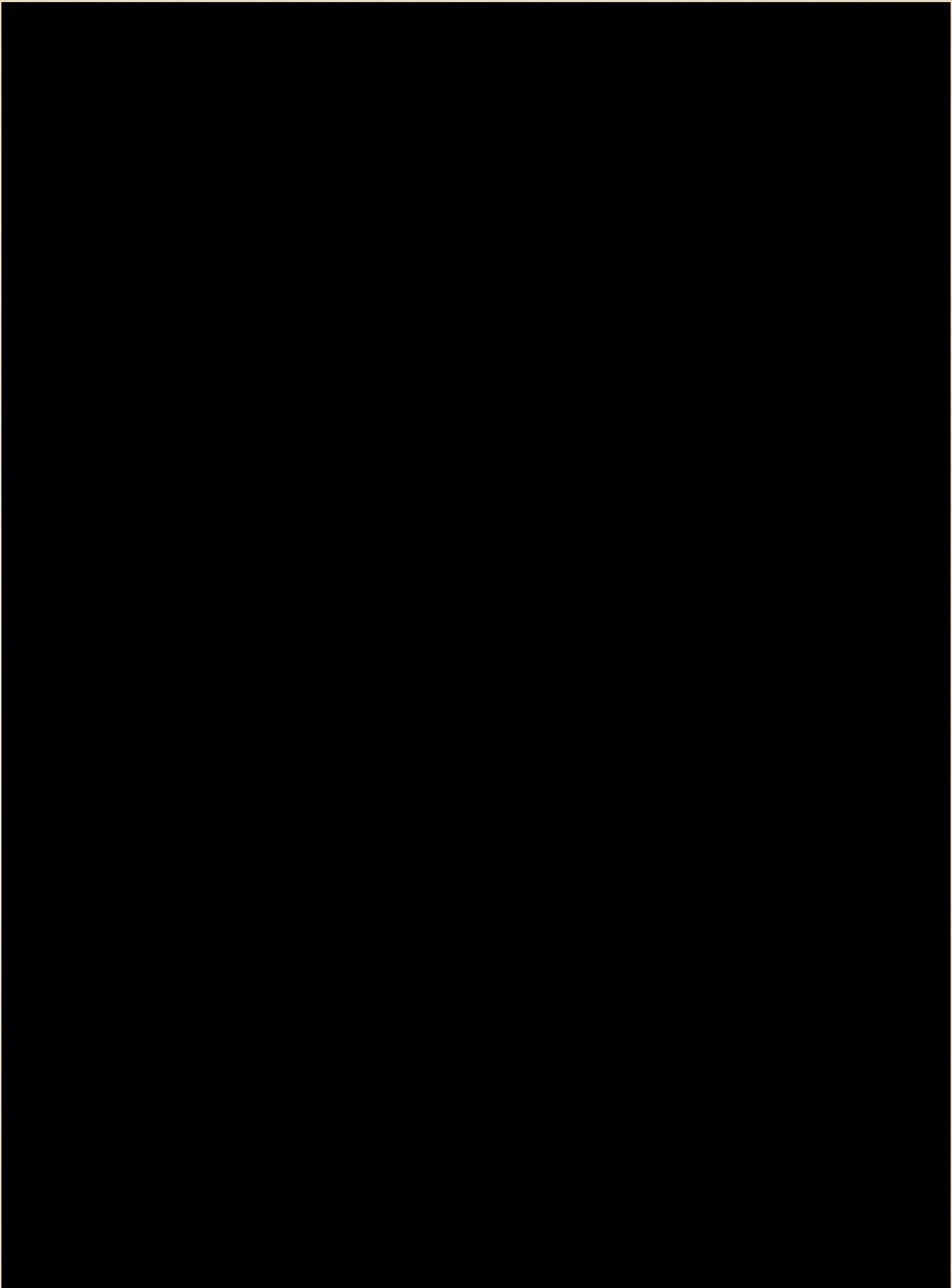


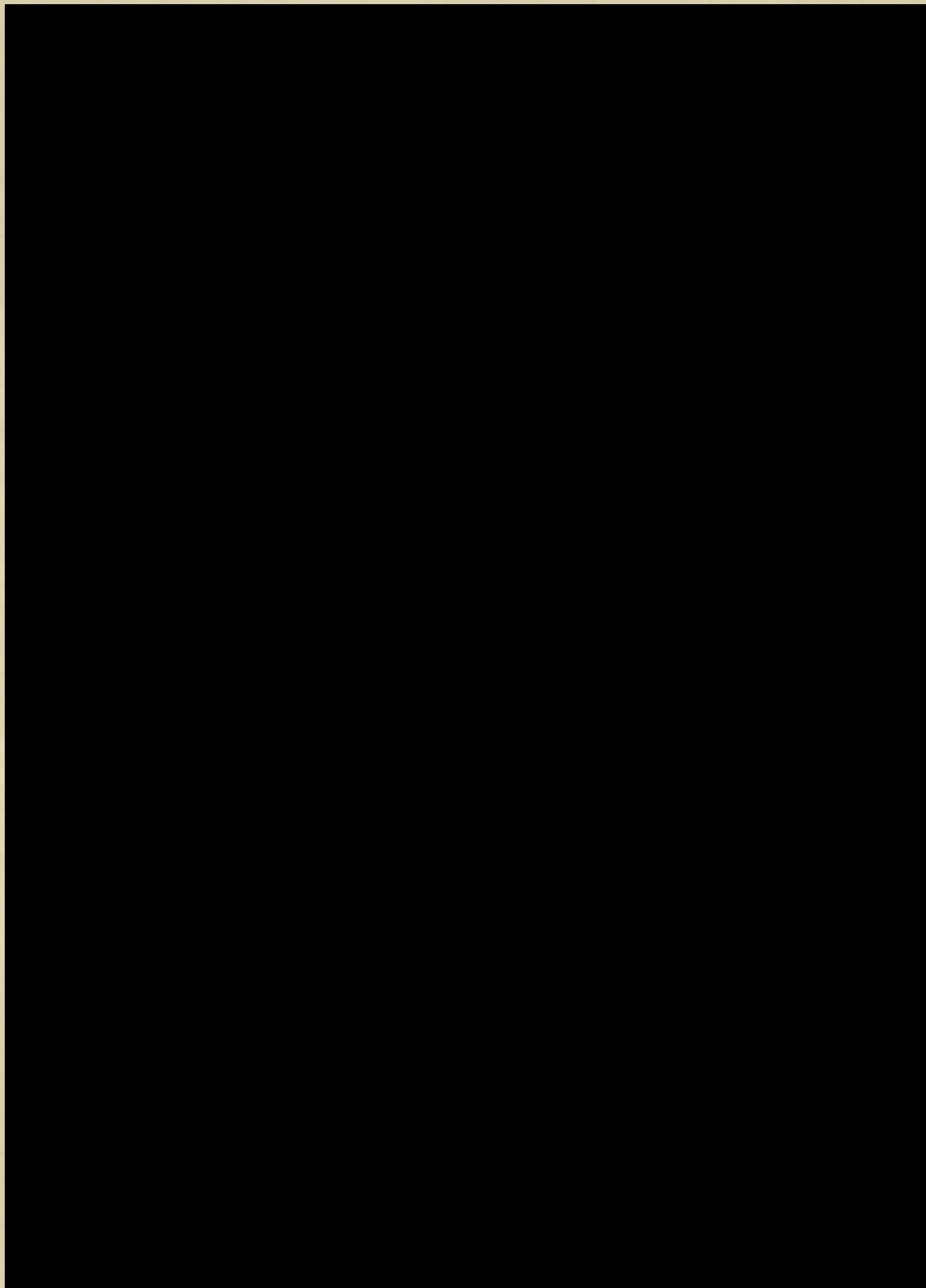


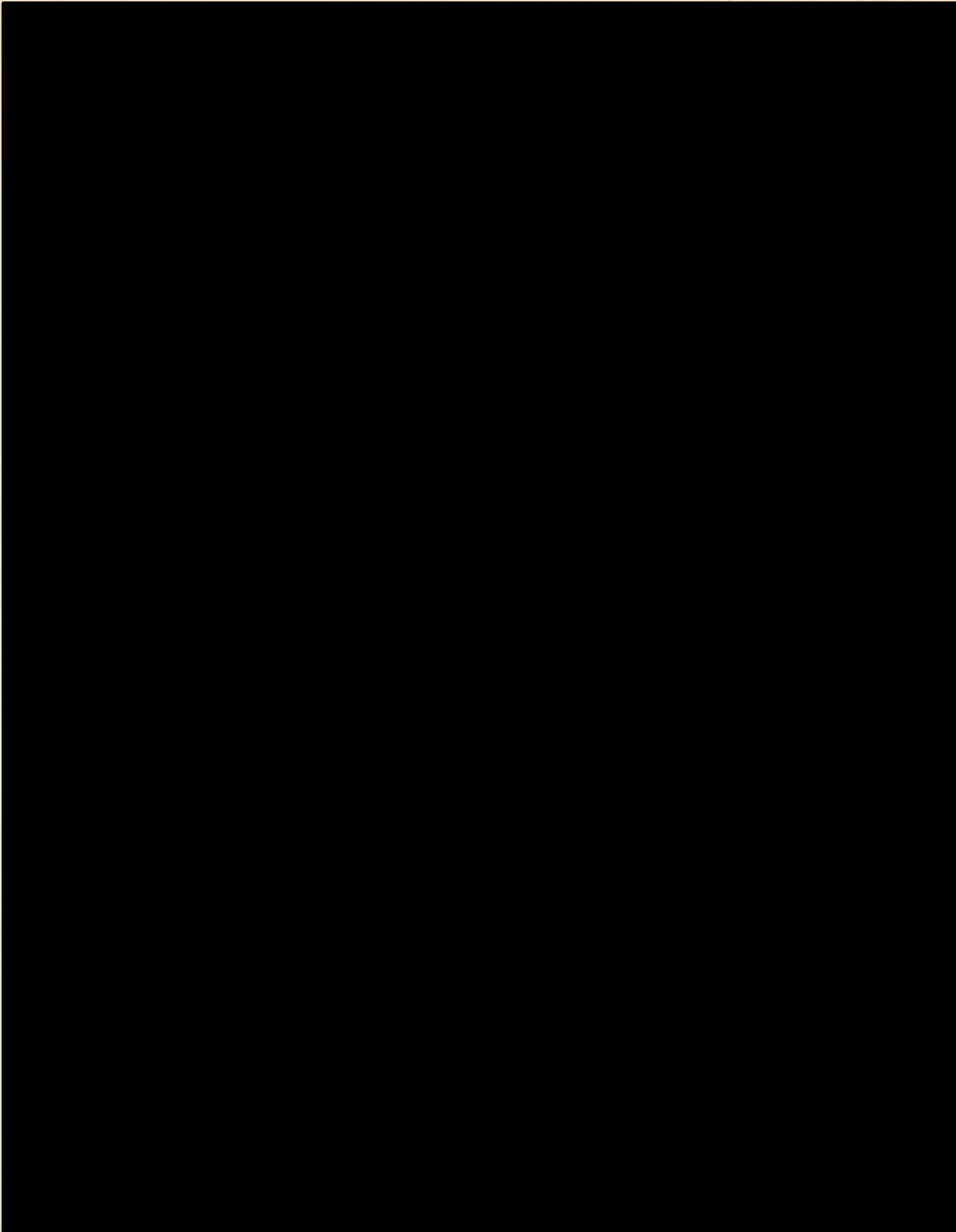












FIFTH INTERNATIONAL CONFERENCE ON EXPERIMENTAL STRESS ANALYSIS

UDINE - ITALY

27th - 31st MAY 1974

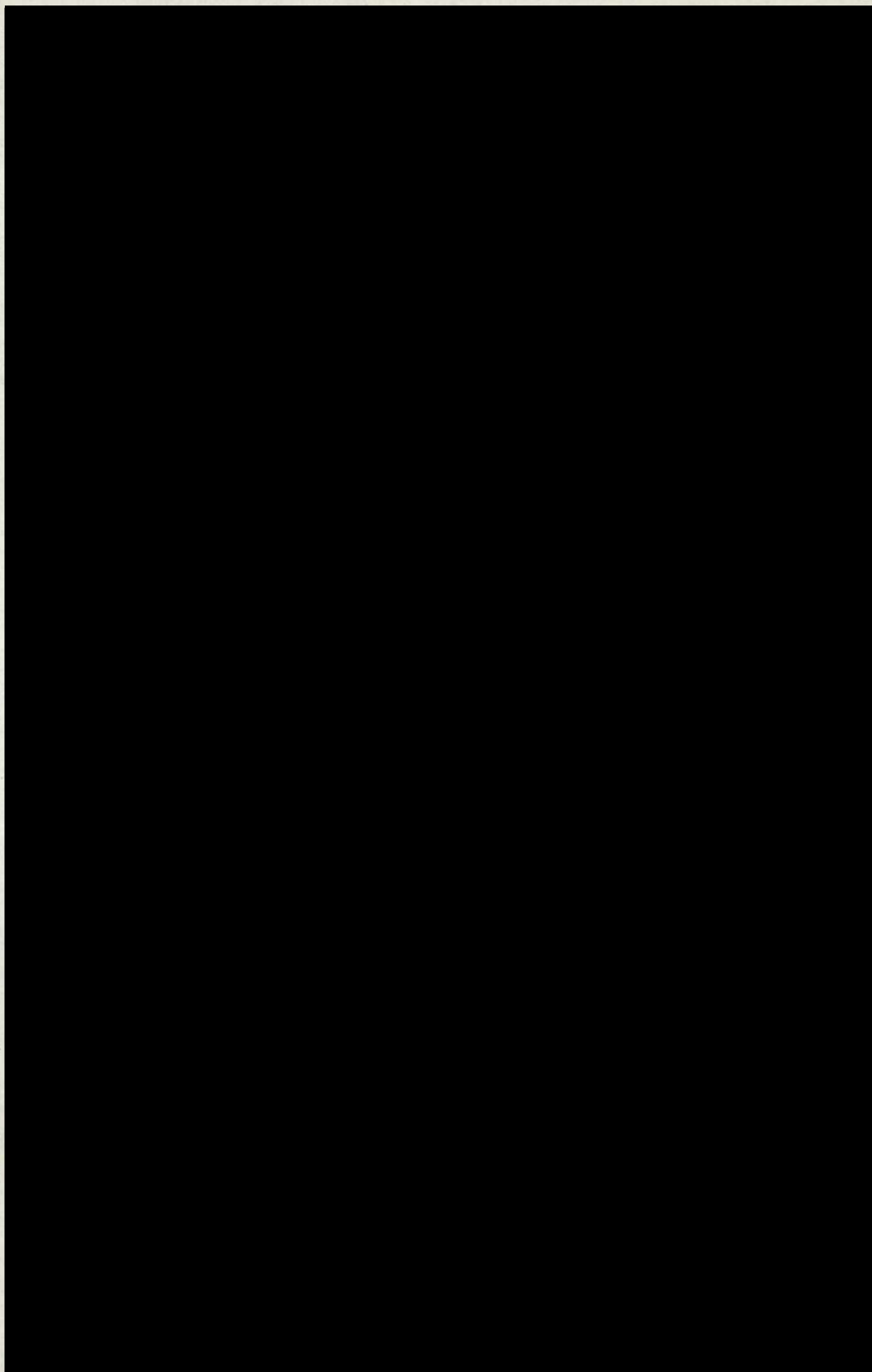
DISCUSSION FORM

Written Contributions to the Discussion and Scripts of verbal remarks must be as short as possible, not to exceed 600 words and two illustrations, and be in English, French, German or Italian. Please write clearly. Before the publication, the Editor reserves the right to contact the contributors to suggest any necessary modification.

If more than one sheet is used, please write your name at the head of each sheet.

Please hand in slides before discussions begin.





- 2 -



A New Method to Increase the Sensitivity of Photoelasticity

Paper by F. P. Chiang and R. P. Khetan, was published in EXPERIMENTAL MECHANICS, January 1974, Pages 29-32

Discussion

by M. L. Meyer and C. L. Mehrotra

M. L. Meyer and C. L. Mehrotra are associated with the City University, London.

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Indexed by Chronological Order,
Author and Topic

Covering Volume I (1943) through Volume XXX (1973), this *single* publication constitutes the *only* source of reference for *every paper* published by the SESA during its first *thirty years* of existence.

For more details, write to:

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Westport, CT 06880