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DEVELOPMENT OF THE DIFFRACTION TYPE ELECTRO - OPTIC
MODULATORS AND DEFLECTORS

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Thesis presented for the Degree of Doctor of Philosophy in the
faculty of Engineering of the City University

1975

ABSTRACT

This thesis deals with the modulation and deflection of an optical beam, utilising the electrically induced refractive index variations in electro-optic crystals. The modulator and deflector configurations considered utilise phase gratings produced by surface deposited interdigital electrodes on a crystal face. When the electrodes are arranged symmetrically, a Raman-Nath type of diffraction pattern is obtained in the far field. However an asymmetrically electroded arrangement gives rise to preferential deflection of the zero order light beam to a higher order diffraction position. A typical unoptimised device, which has been developed during the course of this work, and having asymmetrical electrodes deflected about 81 % of the incident light having a wavelength of $0.63 \mu\text{m}$ to the first order at an applied d.c. voltage of 157 Volts. 53 % of the incident energy was deflected to the second order with this device at an applied d.c. voltage of 255 Volts.

Acoustic resonances were observed in the devices developed when operated under a.c. conditions. In the symmetrical electrode arrangement both bulk and surface wave resonances were observed. However in the asymmetrical arrangement only bulk waves were observed. The bulk resonances were almost completely suppressed by placing an absorber on the face opposite to the electrode face.

An extension of the principle of the internal reflection modulator device to a double reflection modulator produced about 50 % reduction in the zero order extinction voltage

CONTENTS

Chapter 1 .	<u>Introduction</u>	
1.1	General introduction	1
1.2	The arrangement of the thesis	4
Chapter 2 .	<u>Electro-optic phase gratings</u>	
	Introduction	6
2.1	Diffraction grating effects produced by microwave oscillations in electro-optic crystals	7
2.2	Diffraction gratings in electro-optic crystals produced by interdigital electrodes	10
2.3	Diffraction gratings in electro-optic materials produced by other methods	19
Chapter 3	<u>Electro-optic effects in crystals</u>	
	Introduction	25
3.1	The index ellipsoid	26
3.2	Electro-optic effects in crystals	30
3.3	Electro-optic effects induced by interdigital electrodes on a crystal	33
3.4	Principal of the theoretical derivation of the total change in the index of refraction	43
Chapter 4 .	<u>The theoretical evaluation of the field induced in the crystal by different electrode configurations</u>	
	Introduction	45
4.1	Potential distribution along the crystal face	46
4.2	Potential distribution inside the crystal	48
4.3	Electric field distribution inside the crystal	57
Chapter 5 .	<u>Initial investigation of the performance of the internal reflection modulator</u>	
	Introduction	71

5.1	Phase change produced by reflection from an interface	72
5.1.1	Reflection at a dielectric-dielectric interface	72
5.1.2	Reflection at a dielectric-conductor interface	73
5.2	Preliminary experiments on reflection at interface	75
5.3	Investigation of phase change produced by reflection of light from a LiNbO_3 crystal-air or metal interface	76
5.3.1	Subdiffraction fringes produced by copper electrodes on LiNbO_3	80
	* <u>Publication</u> : Noise generated in the total internal reflection electro-optic diffraction deflector / modulator	84
5.3.2	Variation of the zero order intensity with angle of incidence and with zero applied voltage	93
5.3.3	Variation of the light intensity in the zero order with applied d.c. voltage	98
5.4	Elimination of the subdiffraction pattern from the far field	100
5.4.1	Theoretical consideration	100
5.4.2	Experimental observations with a crystal having a MgF_2 film on the reflecting face	104
5.4.3	Variation of the light intensity in the zero order with applied d.c. voltage to the improved device	107
5.4.4	Variation of the zero order extinction voltage with angle of incidence	110
<u>Chapter 6 . Deflection of light using diffraction gratings</u>		
	Introduction	112
6.1	Theoretical considerations of light beam deflection techniques	113

6.1.1	Conventional blazed gratings	113
6.1.2	Bragg diffraction	115
6.2	The development of a new electro-optic deflector	123
6.3	Experimental performance of the novel diffraction deflector	130
	* <u>Publication</u> : A novel electro-optic diffraction deflector	132
6.3.1	Initial observations of the performance of the diffraction deflector	135
6.3.2	Modulation of the zero order energy by diffraction deflector	137
6.3.3	Performance of the device as an optical beam deflector	140
6.3.3.1	Variation of the first order intensity with applied voltage	140
	* <u>Publication</u> : An assessment of the blazed grating electro-optic deflector	143
6.3.3.2	Effect of the ratio of the electrode spacing on coupling efficiencies	147
6.3.4	Variation of the second order intensity with applied voltage	148
6.4	Variation of the zero order extinction voltage with angle of incidence	151
6.5	Variation of the first order maximum voltage with angle of incidence	153
6.6	Variation of the first order maximum intensity with angle of incidence	153
<u>Chapter 7 . Acoustic waves in surface electroded devices</u>		
	Introduction	157
7.1	Primary and secondary electro-optic effects in crystals	158

7.1.1	Acoustic waves in LiNbO_3	160
7.2	Acoustic resonances observed in the symmetrical and asymmetrical electrode device arrangements	162
7.2.1	Initial observations of resonance in the device	162
7.2.2	Observations of resonance in the device with an absorber	168
7.2.3	Summary of observations	163
7.3.1	Acoustic waves in a Y cut LiNbO_3 crystal	177
7.3.2	Summary of the observations for the Y cut LiNbO_3 crystal	183
7.3.3	Qualitative analysis of the observations with the Y cut LiNbO_3 crystal	185
7.3.4	Effect of MgF_2 on the generation of surface waves	185
7.4	Optical effect due to acoustic resonance	188
7.5	Performance of the device at a.c. and d.c. applied voltages	190
<u>Chapter 8 . Multireflection electro-optic modulator</u>		
	Introduction	194
8.1	Theoretical consideration of the multireflection modulator	195
8.2	Variation of the zero order extinction voltage with angle of incidence	197
8.3	Comparison of the variation of the zero order intensity with applied voltage for the single and double internal reflection arrangements	199
	* <u>Publication</u> : An improved diffraction type electro-optic modulator / deflector	202
<u>Chapter 9 . Preparation of the experimental device</u>		
	Introduction	206
9.1	Production of the experimental sample	207
9.1.1	Device requirements	207
9.1.2	Crystal cutting	208

9.1.3	Crystal polishing	209
9.2	Preparation of the sample for deposition	211
9.3	Preparation of the photo-mask	212
9.4	Deposition of MgF_2 film and the electrode metal	212
9.4.1	Deposition of MgF_2 film	212
9.4.2	Deposition of electrode metal	213
9.5	Etching of the electrodes	214
<u>Chapter 10 . Conclusion, suggestions for further work</u>		
	<u>and device applications</u>	
10.1	Conclusion	218
10.2	Suggestions for further work	220
10.2.1	Linearisation of the field	221
10.2.2	Optimisation of the electrode structure	221
10.2.3	Optimisation of the crystal orientation	221
10.2.4	Use with unpolarised light	221
10.2.5	Low operating voltage modulator / deflector	222
10.3	Device applications	222
<u>Appendix</u>		
A	The experimental arrangement	224
B	Propagation characteristics of electromagnetic waves	229
B.1	Fresnel's formulae	229
B.2	Reflection at a dielectric-conductor boundary	234
C	Estimation of the intensity of the diffracted order beams produced by a phase grating	237
D	Computer programme	243
	<u>References</u>	253
	<u>Acknowledgements</u>	257

CHAPTER 1

INTRODUCTION

1.1 General introduction

The advance of laser technology has lead to the potential application of the laser in such fields as optical communications, optical data processing, radar and ranging, optical logics and memory systems. These applications often require means of impressing information on to the laser beam. This may be achieved by modulating the amplitude, phase or frequency of the beam.

A controlled removal of energy from the incident beam of light results in the modulation of the beam. If this energy is directed into a particular and definable direction, then the modulating device also fulfills the function of a deflector. The different principles of operation that can be adopted to give both modulation and deflection of the beam may be classified as

- (i) Modulation and deflection by mechanical means.
- (ii) Modulation and deflection resulting from acousto-optic interactions.
- (iii) Modulation and deflection resulting from electro-optic interactions.

- (i) The mechanical modulator: The mechanical modulator (a rotating mirror, prism, mechanical shutter) may be arranged to produce a variation of the intensity of the beam (amplitude or intensity modulation) (1). This amplitude modulation may be accompanied by the deflection of the laser beam.

(ii) The acousto-optic modulator: The interaction of sound waves and electromagnetic waves at optical frequencies can be used to modify the incident beam by deflecting it in space and/or altering its fundamental frequency. It has been pointed out (2) that the frequency of the scattered light will be Doppler shifted by an amount equal to the acoustic frequency. However, this effect is noticeable only when the acoustic frequency is appreciable compared to the optical frequency. If the acoustic wave inside the medium is assumed to produce a sinusoidally periodic refractive index variation, and if the optical and acoustic wavefronts are perpendicular to each other, a symmetrical diffraction pattern is obtained in the far field. This is known as the Raman-Nath type of scattering. Alternatively, if the two wavefronts are at oblique incidence and the angle of incidence satisfies the Bragg reflection law, the diffracted light will appear in a single spot and the light beam is said to have undergone Bragg type scattering. These interactions are further considered in Chapter 6. The principles, design considerations and the experimental observations of the acousto-optic modulators have been given in such references as (3-10).

(iii) The Electro-optic modulator: The application of an electric field to an electro-optic crystal cause changes in refractive indices of the medium. A detailed analysis of this effect is given in Chapter 3. An appropriately polarised beam of light will experience a corresponding change in phase which can be utilised to give intensity modulation. The conventional

types of electro-optic modulators are the longitudinal and transverse modulators. In the longitudinal modulator the electric field and the direction of light propagation are parallel while in the transverse modulator, they are mutually perpendicular to each other (11-13). In the transverse modulator, if a refractive index gradient exists in the direction perpendicular to the direction of propagation of the optical beam, then the trajectory of the incident beam will be bent in the direction of the gradient (1).

Thin film versions of both acousto-optic and electro-optic deflectors have been produced and are reported in such references as (14-17).

This thesis has been concerned with the development of a novel electro-optic modulator/deflector. The electro-optic modulator/deflector offers the advantage that the possible speeds of operation are appreciably greater than that obtained by other methods. The particular type of electro-optic modulator/deflector considered in this thesis has been of the diffraction grating type. The diffraction grating inside an electro-optic crystal may be conveniently produced by having surface deposited interdigital electrodes on properly chosen faces of a crystal (18). The depth of penetration of the field inside the crystal is normally small and, therefore, in order to utilise the intense field region Akira Mishiaki (19) and Seikhor-Rylski (20) have proposed that the optical beam is reflected from the crystal electrode interface.

The initial work in this thesis was an investigation into the operation of the internal reflection modulator proposed by these workers. A theoretical and experimental assessment was made of

the phenomena occurring when light is reflected from the crystal-electrode interface. It was found that the earlier internal reflection modulators proposed by Akira Nishiwaki and Scibor-Rylski generated an undesirable background scattered light. As a result of initial work undertaken it has been possible to produce a noise free internal reflection modulator.

These earlier diffraction grating type modulators diffracted the light into the higher orders in the form of a Raman-Nath pattern. These type of deflectors are, therefore, not particularly efficient for deflecting a large percentage of energy from one order to another.

As a result of the work carried out in this thesis an electro-optic deflector has been produced which will deflect over 80% of the incident energy and it is to be expected that using the technique developed better performance will be obtained. These results give encouragement for further work to be undertaken to produce an integrated electro-optic deflector which may be utilised in many applications; one important application is a trunking switch in optical communications. The final section of the work in the thesis has been to assess the high frequency performance of the device.

1.2. The arrangement of this thesis

The initial chapters of the thesis consider the theoretical background necessary to establish the principles of operation of the electro-optic diffraction type modulators and deflectors. These principles are covered by Chapter 2 which considers the different techniques which are available to generate phase gratings in an electro-optic crystal. Chapter 3 discusses the electro-optic effects in crystals.

The penetration of the field into the crystal due to the interdigital electrodes is one of the important factors determining the operation of the devices considered in this thesis. This is covered in Chapter 4.

Chapter 5 discusses the initial work undertaken to assess the performance of the modulator/deflector of the type proposed by Akira Nishiwaki, and Scibor-Ryliski (19, 20), which operates on the principle of internal reflection of the beam off the crystal electrode interface and considers the modifications necessary to eliminate the appreciable amount of scattered light which was found to be generated in this type of modulator/deflectors.

Chapter 6 gives an exhaustive theoretical analysis of deflection by diffraction gratings. From this work, the possibility of an asymmetrically electroded electro-optic deflector is discussed and the experimental results obtained from such a device is given.

The acoustic resonances resulting from alternating voltages applied to the devices at high frequency operation are included in Chapter 7.

The concept of a multireflection modulator to produce a low operating voltage device is discussed in Chapter 8.

The technology involved in the production of the above mentioned devices are important and, therefore, the procedures adopted for device production are summarised in Chapter 9.

Conclusions of the theoretical and experimental work undertaken during the course of the work of this thesis are given in Chapter 10. The potential application of the devices developed and suggestions for further research work are included in this chapter.

CHAPTER 2

ELECTRO-OPTIC PHASE GRATINGS

INTRODUCTION

An optical beam will be diffracted into different orders when it is transmitted through a diffraction grating. A controlled removal of the energy from the zero-order of the beam is possible by varying the grating effect. This results in the modulation of the zero-order beam. The grating effect required to diffract the incident light may be produced inside a crystal by different methods. The different methods available for the production of the grating effect using the electro-optic properties of the crystal are considered in this chapter. Also the performance of some of the different types of electro-optic grating modulators are summarised.

The different methods that could be adopted to produce the grating effect in the electro-optic crystal will be classified as follows:

- (1) Methods using microwaves
- (2) Methods using interdigital electrodes
- (3) Other methods

These different classes are considered below in some detail.

2(i) Diffraction grating effects produced by microwave oscillations in electro-optic crystals

With the advance of laser technology, light modulation by microwaves in an electro-optic media has received considerable study. The passage of an electromagnetic wave, having a frequency in the microwave region, inside an electro-optic crystal produces a periodic variation in the index of refraction (21). Thus a beam of light passing through the perturbed medium in a correctly chosen direction, experiences a phase grating. The interaction of the microwave and the optical wave cause the optical beam to be diffracted into many orders. This results in a decrease of the energy in the main or the zero order beam. This decrease in energy is proportional to the applied microwave power (22). If the deflected beam is resolvable from the zero order beam, the decrease in intensity of the zero order beam could be measured and this order is said to be intensity modulated. The deflected beam will be resolvable only when the deflected angle is greater than twice the angular spread of the incident beam. The deflection angle is greater if the microwave phase velocity in the medium is low. Maxwells equations show that the phase velocity decreases with an increase of the dielectric constant of the medium. Therefore, in a practical phase grating modulator, it is desirable that the medium has a high dielectric

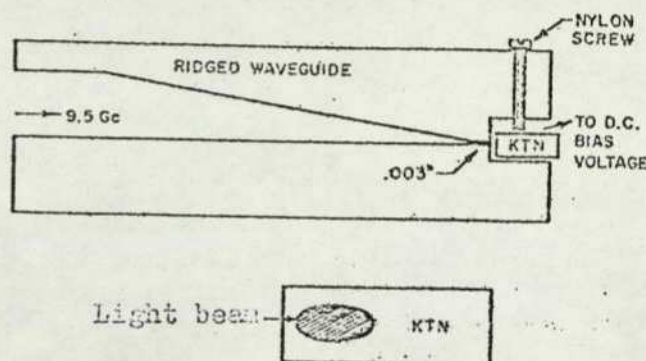
constant at the microwave frequencies.

In certain classes of materials, ferroelectrics for example, the dielectric constants and the electro-optic coefficients at microwave frequencies are large (23). ABO_3 perovskites are a class of materials that possess very large microwave dielectric constants (10^4). Examples of this class of materials are $BaTiO_3$, $KTaO_3$, $KTa_{1-x}Nb_xO_3$ (KTN). In particular, when x equals 0.65, the Curie temperature of KTN is 283°K . The measured dielectric constant of KTN, a quadratic electro-optic material, at room temperature (295°K) is of the order of 10^4 (23).

When measuring the efficiency of KTN as a microwave light beam modulator it is usual to consider the microwave power required to produce a phase retardation of π in the optical beam. Considerable reduction in this power may be obtained by applying a DC bias voltage to the sample so that it is biased with a phase retardation of $n\pi$ where n is a positive integer (24). This DC bias then produces in effect a linear transverse electro-optic effect; it has been reported (23) that at a wave length of $0.5\ \mu\text{m}$, a power reduction of at least one order has been achieved using KTN as the modulating material. An increase in the magnitude of the electro-optic coefficients occurs near the Curie point. Hence, the complexities involved in having long interaction lengths and critical velocity matching is reduced working near this temperature. However, the problem of matching microwaves into high dielectric constant materials become more significant.

In reference (21) it was reported that the microwave frequency electro-magnetic waves were launched into the high dielectric constant material KTN using a single ridged waveguide as shown in

figure 2.1. The ridge height was tapered over a distance of several microwave wavelengths from 0.497 in. to 0.003 in. Using microwave pulses of width 2 μ sec. and peak power of a few hundred watts at 9.5 GHz, the authors have reported that about 9.5% of the energy in the zero order was transferred to the higher orders.



It was further reported that calculations have shown that in an optimised system, complete extinction of the zero order beam could be achieved with about 5 watts of microwave power.

Fig. 2.1 Ridged waveguide transducer

The variation of the refractive index produced by the application of a field to the ABO_3 perovskites in the para-electric phase has been extensively studied (25). It is reported that for these electro-optic materials, the change in the index of refraction produced by an applied electric field varied by less than 15% between each material when this change was measured under identical conditions. Other materials in which grating effects are produced with the passage of electromagnetic waves at microwave frequencies are XDP, CuCl, ZrS. Such effects have been observed in KDP by Kaminow (26). This author adjusted the dimensions of his arrangement so that the microwave phase velocity in the medium approximates to the light velocity when the cavity is excited in a predetermined mode. It was reported that the half wave voltage required was of the order of 7KV. These measurements were made at a wavelength of 0.8 μ m. Comparison of the changes in the electro-optic coefficients (r_{33} - r_{13})

produced by the microwaves in ABO_3 perovskites show that the change is about two orders of magnitude greater than in KDP (25). Experiments have shown that the reactive drive power and the power dissipation in ABO_3 type of modulators are about one quarter of that required in KDP type modulators. It has been reported that CuCl as a modulating material (27) offers a power saving of between one and two orders of magnitude over the KDP modulators, but the material is not available in large crystals of consistent optical quality. Observations of microwave modulation in ZnS by this method have also been reported by Domenico and Anderson (28).

2.2 Diffraction gratings in Electro-optic crystals produced by interdigital electrodes

The diffraction grating produced by the passage of an electromagnetic wave at microwave frequencies has been considered in section 2.1. A similar grating could be produced by the application of potentials to an interdigital electrode structure which has been deposited on to the polished face of an electro-optic crystal. Alternatively the electrode system could be brought into close contact with the crystal face. The different ways of arranging the electrodes to produce the diffraction grating which have been considered by previous workers (references 18,29,30,31) are shown in figure 2.2.

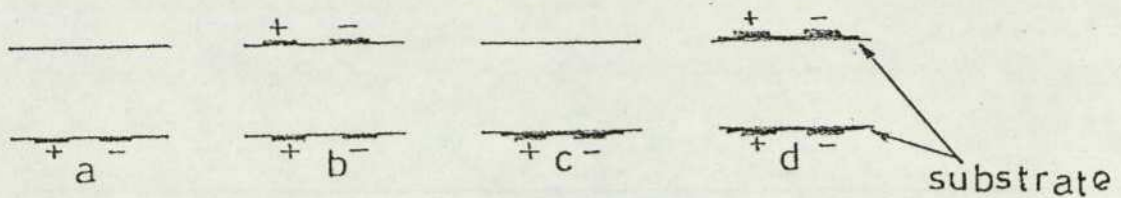


Fig 2.2 Modulator arrangements

- a) See reference 19, 20, 30
- b) See reference 18, 29, 30, 31
- c) See reference 30
- d) See reference 30

The application of a potential to the interdigital electrode system causes a fringing field to penetrate into the crystal. This spatially periodic electric field changes the index of refraction of the crystal. Hence, a polarised beam of light, which passes through a suitably oriented crystal, experiences a diffraction phase grating. The interaction of the electro-optically induced phase grating effect and the optical beam, which is similar to the interaction of the phase grating due to an acoustic beam and an optical beam, produces a diffraction pattern in the far field. If a sinusoidal periodic change in refractive index is assumed, and if the two wavefronts are perpendicular to each other then the far field pattern obtained will be of the Raman-Nath type. The angular deflection of the different orders of the diffracted beam is a function of the wavelength of the incident light and the periodicity of the induced electro-optic phase grating. The amount of energy coupled to the higher orders is a function of the phase difference impressed on the optical beam by the grating, which is a function of the change in the refractive index. The change in the index of refraction is proportional to the electric field in linear electro-optic crystals eg. LiNbO_3 , XDP (the Pockels effect) and proportional to the square of the electric field in the quadratic electro-optic crystals eg KTN (the Kerr effect).

In the practical applications of these interdigital electro-optic grating deflectors, the different aspects that have to be taken into consideration are

- i) The linearity of modulation of the zero-order energy
- ii) The complete extinction of the zero-order energy
- iii) The total number of resolvable spots produced by the phase grating
- iv) The angle between the different diffracted orders

The variation of the intensity of the zero order (or the energy coupled into the higher orders) depends on the variation of the index of refraction of the optical modulating medium. When the light energy is deflected into the higher orders by the application of a voltage to the modulator, the zero order beam is said to be intensity modulated. The diffracted orders are separated by an angle equal to the ratio of the wavelength of the incident light to the periodicity of the induced phase grating. In order to resolve the diffracted orders completely, the width of the incident beam must be greater than twice the electrode pitch (13).

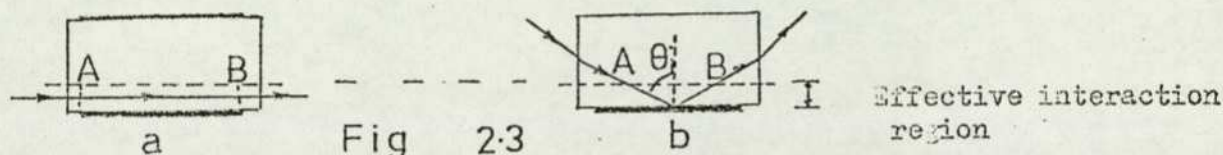
The depth of penetration of the fringing field produced by the interdigital electrodes depends on the electrode pitch (the detailed analysis of this effect is given in chapter 4). Reducing the electrode pitch increases the angular separation of the higher orders but reduces the depth of penetration of the field. Thus the difficulty arises of ensuring that the light beam approaches close to the electrodes in order to take advantage of the high field intensity.

We may consider two approaches to this problem.

- (1) The light beam travelling parallel to the base of the crystal which carries the electrodes.

- (2) The light beam reflecting off the base of the crystal which carries the electrodes.

These two arrangements are illustrated in figure 2.3



- a) Parallel transmission
b) Internal reflection transmission

In case 'a' the practical difficulty arises, in a simple bulk modulator, of ensuring that the light beam passes sufficiently close to the electrodes to take advantage of the high field region. In case 'b' this problem is overcome by reflecting the light from the base of the crystal, but of course the effective interaction length may be small.

In the design of a modulator of the above type, important factors to consider are the choice of the crystal, the relative orientation, and the electrode structure. The fringing field inside an electro-optic crystal causes:

- a) Rotation of the index ellipsoid
b) Variation of the indices of refraction

The detailed study of the properties of the crystal groups (refer chapter 3) shows that classes 6mm and 3mm are suitable crystals for this type of diffraction modulator. These classes of crystals have reasonably large electro-optic coefficients.

Optical modulators, in which the beam of light travels parallel to the electrodes (Fig.2.3a), using the electro-optic material LiNbO_3 as the modulating media have been reported (18,29,30). In the arrangement considered in these references, the electrodes were deposited on the YZ plane of the crystal. The length of the electrodes were along the Y direction. The optic axis of the crystal was along the Z axis (Fig.2.4).

The incident beam of light was polarised along the Z axis and was transmitted along the Y direction. It was reported that under optimum conditions, 10%

modulation of the zero order was obtained with a 30V drive signal (18). The electrode pitch was 300 μm and the length of the

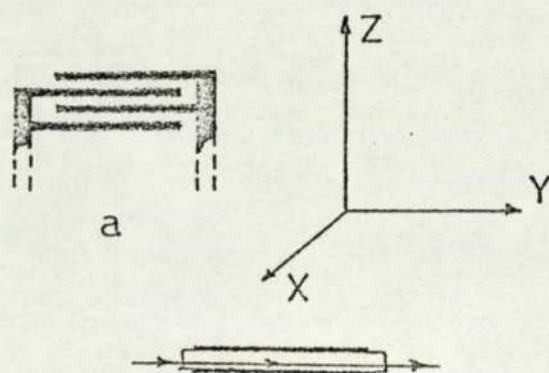


Fig . 2.4

Modulator arrangement

a) electrode configuration

electrodes 6mm. Complete extinction of the zero order at 24 V has been reported by deBarros and Wilson (30). Although not stated by the authors it might be assumed that this reduction in the operating voltage may have been due to the beam being brought closer to the crystal-electrode interface. The electrode pitch was again 300 μm but the length of the electrodes were 7 mm. Grating effects produced in a 45° cut ADP crystal by interdigital electrodes in contact with the crystal base has been reported by Scibor-Rylski (32).

Preliminary reports of a sandwich type structure (Fig.2.2d) has been given by deBarros and Wilson (30). These authors have reported

that a higher extinction voltage was required with the glass substrate than without when operated at high speeds. However, owing to the large difference in the resistivities of glass and LiNbO_3 , the arrangement failed to operate with direct current. In all these earlier references, the wavelength of the incident light was $0.6328 \mu\text{m}$.

Diffraction effects produced by an interdigital electrode structure deposited on sputtered thin films of LiTaO_3 have been reported (16). Thin films of LiTaO_3 were deposited on glass substrates by radio-frequency sputtering. The light beam was coupled into this thin film by two rutile prisms. Incident coherent light of wavelength $0.6328 \mu\text{m}$ was focused on the input prism base and a TEO mode was launched and propagated under the gold interdigital electrodes at the first order Bragg angle. A similar Bragg diffraction modulator using a periodic electrode structure has been reported by Hammers and Phillips (17). In this report, the authors diffused metallic niobium into LiTaO_3 substrates producing a high index layer of $\text{LiNb}_x\text{Ta}_{1-x}\text{O}_3$. Since the index of refraction of this layer is large compared to that of the surrounding, this could be used as an optical wave guide. The effective thickness of the layer and the index of refraction can be controlled to produce single mode wave guides. The incident light was guided into this thin film by SrTiO_3 prisms. The experiment has been performed at wavelengths of 0.4976 , 0.5592 and $0.6328 \mu\text{m}$. It is reported that over 80% of the guided light was deflected at 3.5V , 4.5V and 6.5V at the above wavelengths respectively.

The electrode pitch and the length of the electrodes were $7.6 \mu\text{m}$ and 3 mm respectively.

Diffraction gratings have been produced in liquids using interdigital electrodes and Nitrobenzene as the active wave guide material (33,34). The modulating field was applied with an integrated electrode structure deposited on to a substrate. The substrate, a microscope slide, acts as the base of the channel containing the electro-optic material. The experimental arrangement used by Polky and Harris (34) is shown in

figure 2.5. It was reported that with electrodes of length 5 mm and electrode pitch $24\text{ }\mu\text{m}$, 50% modulation of the incident optical energy was obtained with an applied voltage of 200V. The wavelength of the incident light was $0.6328\text{ }\mu\text{m}$.

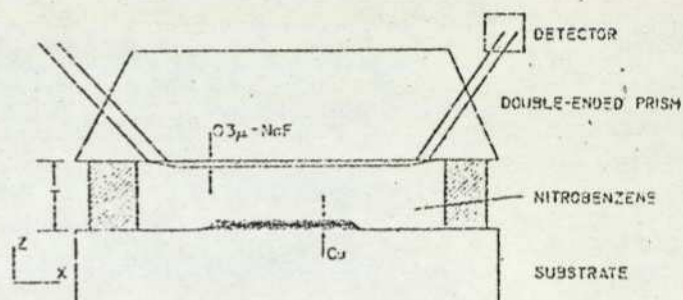


Fig. 2-5

Liquid electro-optic modulator

These modulators which have been considered are arranged with the light beam travelling parallel to the electrodes. The deflectors and modulators whose operating principle is based on the internal reflection of the optical beam off the base carrying the electrodes will now be considered.

The experimental arrangement considered by Akira Nishiwaki (19) is shown in figure 2.6. The electro-optic crystal used was LiTaO_3 .

The interdigital electrodes were deposited on the YZ surface of the crystal. The incident light was polarised with the E vector parallel to the Z axis and the

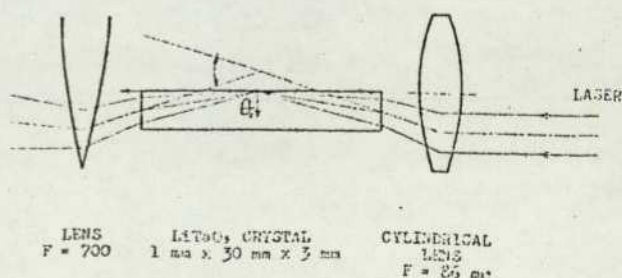
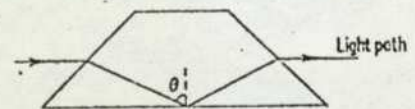


Fig. 2-6

Internal reflection modulator (19)

beam of light was focussed on to the base, the YZ plane of the crystal, by the cylindrical lens. The width and the spacing of the electrodes were arranged to be equal to $20\text{ }\mu\text{m}$. As seen in Fig. 2.6 the interaction length varies with the angle of incidence θ_i . The zero order extinction voltage is now a function of the angle of incidence θ_i . It is reported by this author (19) that for an angle of incidence of about 0.02 radians, the minimum intensity of the zero order was obtained at an applied voltage of about 25 volts. The incident light was of wavelength $0.6328\text{ }\mu\text{m}$.

A similar type of modulator using LiNbO_3 as the modulating media has been reported by Scibor-Rylski (20,32,35). The performance of the device with the electrodes deposited on the XY and YZ planes are presented in these references. The optic axis was along the Z axis, and the incident beam of light was polarised with the E vector in the direction of the optic axis. This author reports in these references (20,32,35) that at an angle of incidence of 2° complete extinction of the zero order is obtained at 40V. The experimental arrangement used by him is as shown in figure 2.7. The spacing between the electrodes and the electrode width was made equal to 0.1 mm. The end faces of the crystal were angled to refract the incident beam of light on to the base of the crystal carrying the electrodes. The wavelength of the incident beam of light was $0.6328\text{ }\mu\text{m}$.



Electrode structure

Fig. 2-7

T.I.R. Modulator (20)

A bulk diffraction modulator also operating on the principles of internal reflection of the beam inside the medium has been reported by Kirkby (36). The modulator material was a prism of Lead Lanthanum Zirconate Titanate ceramic (PLZT), which exhibits quadratic electro-optic effects. The ceramic is reported to have had a composition of $\text{Pb}_{0.865} \text{La}_{0.09} \text{Zr}_{0.65} \text{Ti}_{0.35} \text{O}_3 \cdot \text{An}$ interdigital electrode pattern of chromium-gold was deposited on the broad face of the right angled isosceles prism (Fig.2.8). The incident light refracted through one of the short faces of the prism is reflected at the base carrying the electrodes and is refracted out at the other short face. It is reported that when the device was operated at a wavelength of $0.633 \mu\text{m}$ with an electrode period of 0.3 mm , and θ equalling 20° , complete extinction of the zero order was obtained at about 100 volts.

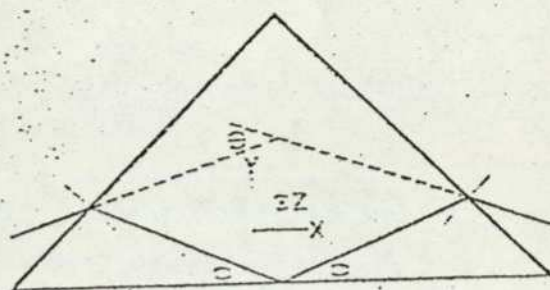


Fig. 2.8

T.I.R. Modulator (36)

2.3 Diffraction gratings in electro-optic materials produced by other methods

The modulators discussed in section 2.2 have all been arranged with the electrodes uniformly spaced and with electrode spacing equal to the electrode width. This section considers other arrangements that have been used to impress a periodic phase profile on the transmitted beam of light.

In the arrangement considered by Hammer (37) the voltage controlled phase grating was produced by columns of electrodes on one face of an electro-optic crystal slab and a grounded plane on the opposite face. The arrangement is

shown in Figure 2.9. The crystal used was a Z cut LiNbO_3 . The elements of a single column were connected by a common bus. The dimensions 'd' and 'a' are different for each column. When a voltage was applied across any particular column and the ground plane, a

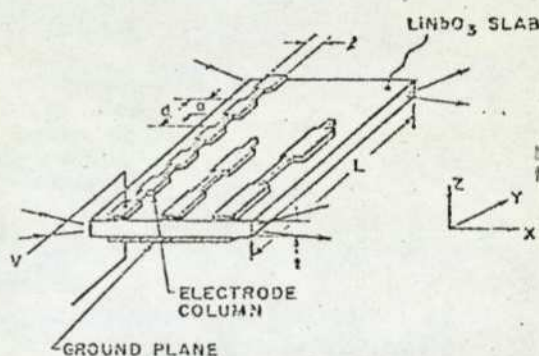


Fig. 2.9

Digital electro-optic modulator

grating effect is produced which was characteristic of that column. The Y periodic electric field causes a periodic perturbation in the refractive index. The incident light, polarised in the Z direction, hence encounters a periodic phase change and is thus diffracted into the different orders. When the electrode spacing 'd' is equal to the width 'a' of the electrode the author assumed that the variation of the refractive index in the Y direction will be that of a square wave near the upper surface. Near the ground plane that index will

be uniform and in the mid plane the author assumed that the variation approximated to a sinusoidal function. By applying a potential difference between any one column of electrodes and the ground plane the author, Hammer, was able to diffract an incident beam of light. As the interorder angle of diffraction is a function of the geometry of the electrode column, the beam could be diffracted with independent angles of diffraction. It has been reported that for an incident beam having a wavelength of $0.6328 \mu\text{m}$ and with the electrode pitch equalling 0.3 mm , 60% and 74% modulation of the zero order was obtained at 600V and 800V respectively

An optical modulator which impresses a square phase profile on the emergent beam has been proposed by Toshio Motoki (33). The arrangement used by this author is as

shown in figure 2.10. In this arrangement a periodic crystal structure was placed in a uniform electric field. As shown in figure 2.10, an array of identical LiNbO_3 rods were bound to form a slab. The Z axis of the neighbouring crystals were oriented in

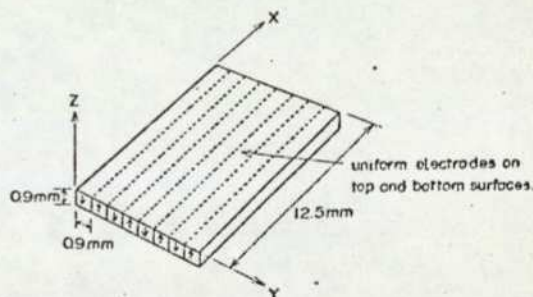


Fig. 2.10

Array type modulator

opposite direction to each other. Electrodes on both XY planes form a plane parallel capacitor and in effect produce an approximately uniform field within the capacitor. Since the optic-axes of the adjacent crystals are in opposite directions, the field inside the capacitor periodically changes the index of refraction of the composite slab of LiNbO_3 . The phase differences induced have a uniform rectangular distribution. A beam of light directed along the X axis then experiences a periodic rectangular phase change, and

the energy from the zero order is distributed into the higher orders. It is reported that 100% modulation of the zero order beam was achieved with an applied voltage of 68 V. The author has also indicated from his theoretical study that by fabricating four $0.3 \times 3 \times 12.5$ mm LiNbO_3 crystals, the array modulator would give 100% modulation with an applied voltage of 23V. The incident beam of light had a wavelength $0.6328 \mu\text{m}$.

A new form of grating arrangement to produce beam deflection has been reported by Wright and Wilson (39). As discussed earlier in this section, when the electrode spacings are uniform and equal to the electrode width it may be assumed that an approximate sinusoidal perturbation of the refractive index will occur. When alternate polarities are applied to the adjacent electrodes the transmitted beam is symmetrically diffracted into higher orders and the far field pattern is of the Raman Nath type. However, in this diffraction pattern no one order beam intensity is dominant. By appropriate shaping of the impressed phase grating, it is possible to deflect a large percentage of the light energy into a single higher diffraction order.

Considering the theoretical sawtooth phase profile as shown in figure 2.11, if the phase delay

θ is adjusted so that $\theta = m\pi$

where m is an integer then

maximum amount of energy will be transferred from the zero order

beam to the m th order beam i.e.

the m th order diffraction band

will be dominant. Since the phase

profile is periodic the wave could be represented by its Fourier

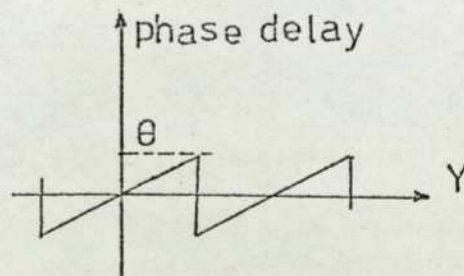


Fig. 2.11

Ideal sawtooth phase profile

components. In the experimental arrangement of Wright and Wilson (39) (figure 2.12) the first three Fourier components of the sawtooth phase profile was impressed on the incident wave front by having three different sets of symmetrical electrode structure on the crystal faces. The electrode lengths, spacings, and voltages applied to the electrodes were adjusted to correctly give the first three Fourier components of the desired sawtooth phase profile.

The electro-optic crystal was LiNbO_3 . The optic axis of the crystal was along the Z axis. The electrodes were deposited on both ZY planes of the crystal. The pitch of the electrodes producing

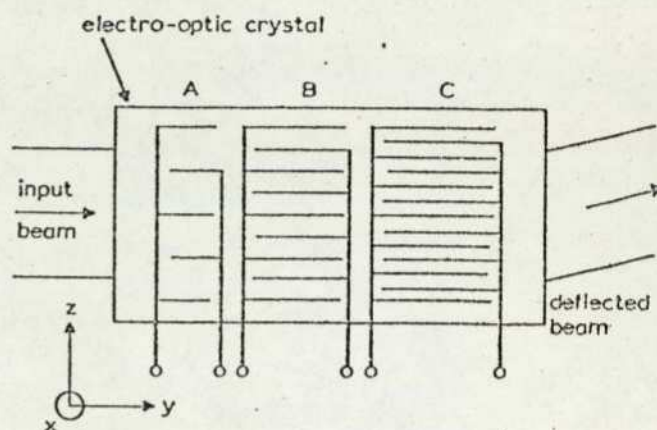


Fig. 2.12

Cascaded electrode arrangement

the fundamental component of the phase delay was $300 \mu\text{m}$. The length of the electrodes producing the fundamental and the two harmonic components were 0.94, 2.36 and 3.68 mm respectively. The individual voltages applied to the electrodes were adjusted so that maximum light energy in the first order was obtained.

The voltages required for the fundamental, first and the second harmonics were 185 V, 40 V and 19 V respectively. It is reported that with these voltages applied to the electrodes, it was possible to deflect 58% of the incident optical power into either the +1 or the -1 order depending on the voltage polarities.

The deflection of an optical beam by this type of sawtooth phase profile or blazed grating has also been reported by Yuichi Ninomiya

(40). The electro-optic material was a slab of LiNbO_3 . The faces perpendicular to the Z and Y axes were polished and the optic axis was parallel to the Z axis. The electrodes shown in figure 2.13 was deposited on both faces of the crystal. Similar shaped electrodes were exactly opposite to each other. Opposite polarities were applied to the adjacent electrodes on one face of the crystal, and the polarities were reversed on the corresponding opposite electrodes on the opposite face of the crystal. The change in the index

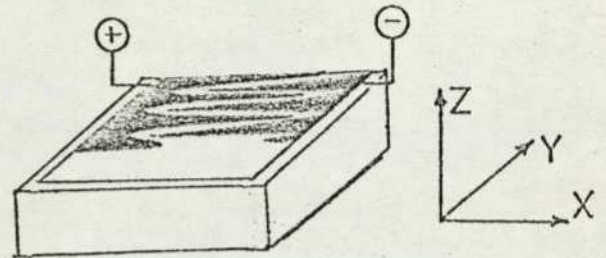


Fig 2.13

of refraction by the applied electric field is shown in Figure 2.14. The incident beam of light from a He-Ne laser was focussed to travel along the X

Deflector arrangement (40)

direction. The beam then experiences the sawtooth phase profile, and will be deflected to a higher

order. It is reported that with an applied voltage of about 50 Volts, the beam is deflected to the first order. It is to be

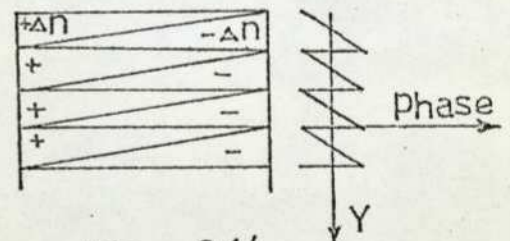


Fig. 2.14

noted that as this device is a voltage controlled phase grating, the voltage may be adjusted to

Phase change impressed on light

deflect the beam to any higher order. It is reported that when the voltage was increased to 597 Volts, the beam was deflected to the 12 th order. In an idealised arrangement, the author has stated that it should be possible to obtain 630 resolvable spots, the voltage range required being from 0 to 800 Volts.

A modified version of the above type of deflector has been reported in a later publication by this author, Yuichi Minomiya (41). The construction of this later deflector is shown in figure 2.15. The modulator was constructed from a periodic array of prisms where the electro-optic material was LiNbO_3 . The optic axes of the adjoining crystals were inverted; the electrodes were of a parallelogram shape and were deposited exactly opposite to each other on the two faces perpendicular to the Z axis. The voltages applied to the adjacent electrodes on the same face were alternately inverted (ref. Fig. 2.15) and the polarities were reversed between corresponding electrodes on the opposite face of the crystal. In this arrangement, since the optic-axis of the adjoining

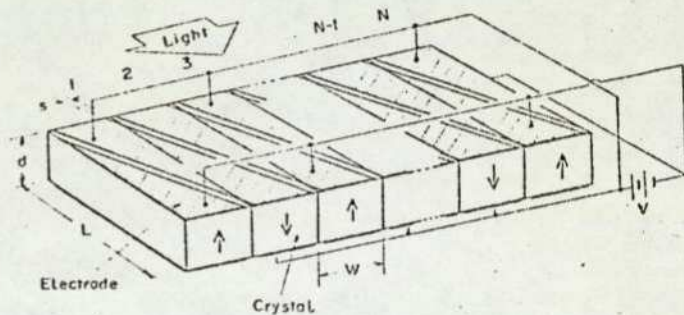


Fig. 2.15

Prism array deflector

crystals were alternately changed, the author considered that the impressed sawtooth phase profile in this arrangement more closely approximates the ideal sawtooth wave than in his earlier arrangement. He has reported that with nine elements each of dimension $3.0 \times 0.35 \times 18.5$ mm it was possible to get 180 Rayleigh points with $\pm 580\text{V}$ of driving voltage. His theoretical calculations showed that with fifty four LiNbO_3 crystals of dimension $0.2 \times 0.8 \times 21$ mm it is possible to have 1030 Rayleigh points for $\pm 64\text{V}$ drive. The extinction ratio of this deflector was over 32 db.

CHAPTER 3

ELECTRO-OPTIC EFFECTS IN CRYSTALS

INTRODUCTION

The optical properties of a crystal can be altered by the application of an external electric field. The electrically induced changes of the crystal properties may, in some cases, be utilised to modulate an optical beam. In this chapter we shall consider the nature of propagation of electromagnetic waves inside an optically anisotropic crystal and the variation of the optical properties with an applied electric field. In particular, the induced refractive index changes occurring inside a LiNbO_3 crystal, caused by the application of a voltage to an interdigital electrode system deposited on to a crystal face are analysed in some detail.

3.1 The Index ellipsoid

Some properties of a crystal vary with the direction of measurement. The crystal is then said to be anisotropic for these properties which are generally represented by a second-rank tensor (eg permittivity, electrical conductivity, dielectric susceptibility) or a third-rank tensor (eg piezo-electric moduli) or a higher rank tensor (fourth-rank tensor to describe the stiffness constant of a crystal) (42).

Consider an electromagnetic wave passing through an anisotropic crystal. The propagation characteristics will depend on the direction of propagation of this wave. Let $\underline{E}$ and $\underline{D}$ be the electric and displacement vectors respectively. These vectors are then related as

$$D_k = \epsilon_{kl} E_l \quad 3.1$$

For an isotropic crystal ϵ_{kl} is a scalar denoting the permittivity and, therefore, $\underline{D}$ and $\underline{E}$ will be in the same direction. However, in an anisotropic crystal ϵ_{kl} is a second-rank permittivity tensor. The subscripts refer to a Cartesian co-ordinate system where both k and l have the values of x , y and z . These reference axes are fixed with respect to the crystal axes.

The stored electrical energy density ξ is given by

$$\xi = \frac{1}{2} \cdot \underline{E} \cdot \underline{D} \quad 3.2$$

Substituting equation 3.1 in 3.2 the expression becomes

$$\xi = \frac{1}{2} E_k \epsilon_{kl} E_l \quad 3.3$$

The second rank tensor ϵ_{kl} of an anisotropic material consists of nine elements, but by considering the rate of change of stored energy it can be shown (43) that

$$\epsilon_{kl} = \epsilon_{lk} \quad 3.4$$

Therefore, the permittivity tensor will be symmetric and have only six independent elements. This symmetrical tensor can be diagonalised by appropriately choosing the reference co-ordinate axes.

The new sets of axes are called the principal dielectric axes and equation 3.1 may be written with reference to these new axes as

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad 3.5$$

where X, Y and Z correspond to the principal dielectric axes (44). by substituting equation 3.5 in equation 3.2 the stored energy is obtained as

$$\mathcal{E} = \frac{1}{2} \left[\frac{D_x^2}{\epsilon_x} + \frac{D_y^2}{\epsilon_y} + \frac{D_z^2}{\epsilon_z} \right] \quad 3.6$$

The dielectric tensor K_{kl} is related to the permittivity tensor ϵ_{kl} as (42)

$$K_{kl} \epsilon_0 = \epsilon_{kl} \quad 3.7$$

and referred to the principal axes X, Y, Z of equation 3.5

$$K_x = \frac{\epsilon_x}{\epsilon_0} \quad K_y = \frac{\epsilon_y}{\epsilon_0} \quad \text{and} \quad K_z = \frac{\epsilon_z}{\epsilon_0}$$

K_x , K_y , and K_z are known as the principal dielectric constants and are equal to the square of the index of refraction in the X , Y , and Z directions respectively.

Let n_x , n_y , and n_z be the refractive indices in the direction of the principal axes X , Y , Z respectively.

Equation 3.6 now reduces to

$$1 = \frac{1}{2\epsilon_0\epsilon} \left[\frac{D_x^2}{n_x^2} + \frac{D_y^2}{n_y^2} + \frac{D_z^2}{n_z^2} \right] \quad 3.8$$

This equation shows that the constant energy density surface in the D space defined by D_x , D_y , and D_z is an ellipsoid. In order to find the indices of refraction it is convenient to analyse the equation of the index ellipsoid. This equation is obtained by replacing

$$\frac{D_x}{\sqrt{2\epsilon_0\epsilon}}, \frac{D_y}{\sqrt{2\epsilon_0\epsilon}} \quad \text{and} \quad \frac{D_z}{\sqrt{2\epsilon_0\epsilon}} \quad \text{by } X, Y \text{ and } Z \text{ respectively}$$

The equation 3.8 is then written as

$$\frac{X^2}{n_x^2} + \frac{Y^2}{n_y^2} + \frac{Z^2}{n_z^2} = 1 \quad 3.9$$

This equation now corresponds to an ellipsoid. The ellipsoid is known as the index ellipsoid or the optical indicatrix.

An optical beam travelling inside a crystal in any arbitrary direction is resolved into two components. The polarisation of the two components will be mutually perpendicular to each other. The index ellipsoid is generally used to determine the two D vectors and the two refractive indices associated with the two components of the wave. Geometrically these parameters are found by considering the plane of intersection of the index ellipsoid and the plane normal to the incident beam which passes through the origin of the ellipsoid. The intersection curve is elliptical and half the values of the major and minor axes correspond to the two refractive indices. The directions of the D vectors are parallel to these major and minor axes and the phase velocities of the two components are determined by the respective refractive indices.

In a crystal, the symmetry elements determine the relative magnitudes of the elements of the dielectric tensor. When all the refractive indices are equal the ellipsoid reduces to a sphere and the crystal is said to be optically isotropic. When two refractive indices are equal then the ellipsoid reduces to a spheroid and the crystal is said to be uniaxial. If all the refractive indices are different, the optical indicatrix is a general ellipsoid and the crystal is said to be biaxial. The biaxial crystals have two circular planes passing through the origin, the uniaxial crystal, one circular plane and in the isotropic crystals there are an infinite number of circular planes passing through the origin. When an optical beam is directed along the normal to these circular planes, the two mutually perpendicularly polarised beams will experience equal values of refractive indices.

The direction of the normal to the circular plane is called the optic axis of the crystal and is in general defined as the Z axis (44). Hence, for the uniaxial crystals, the index ellipsoid reduces to

$$\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} = 1 \quad 3.10$$

where $n_x = n_y = n_o$ and $n_z = n_e$.

n_o and n_e are called the refractive indices of the ordinary and extraordinary rays. In the index ellipsoid if n_o is greater than n_e , the crystal is called a negative uniaxial crystal (eg LiNbO_3) and if n_o is less than n_e then the crystal is called a positive uniaxial crystal (eg LiTaO_3)

3.2 Electro-optic effects in crystals

The application of an external electric field changes the elements of the permittivity tensor. As seen earlier from equation 3.7 the refractive index is a function of the permittivity. Therefore, the propagation characteristics of an electromagnetic wave will be altered when an electric field is applied to the crystal. For simplicity, assume that the displacement vector and the electric field are in the same direction. In general, these vectors are related as

$$D = k^o E + \alpha E^2 + \beta E^3 + \dots \quad 3.11$$

where $k^0, \alpha, \beta, \dots$ are constants (42).

The permittivity tensor ϵ_{kl} is defined as $\partial D_k / \partial E_l$ (42)

Differentiating equation 3.11 gives

$$\epsilon = k^0 + 2\alpha E + 3\beta E^2 + \dots \quad 3.12$$

From equation 3.7 the dielectric constant is then given as

$$K = \frac{1}{\epsilon_0} [k^0 + 2\alpha E + 3\beta E^2 + \dots] \quad 3.13$$

The dielectric constant is equal to the square of the refractive index and, therefore, by representing equation 3.13 in terms of the refractive index, we deduce that

$$n = n^0 + aE + bE^2 + \dots \quad 3.14$$

n^0 is the refractive index at zero applied field and $a, b, \dots$ are constants. In the above equation, the linear term (aE) contributes to the linear electro-optic effect (Pockels Effect) and the quadratic term (bE^2) contributes to the quadratic electro-optic effect (Kerr Effect).

In crystals which possess inversion symmetry, the refractive index is unaltered by a reversal of the applied field. This occurs only when the constants in equation 3.14 with odd powers of E are zero. Therefore, in these crystals the linear electro-optic effect cannot exist. However, quadratic effects can occur in crystals with inversion symmetry.

It is to be remembered that the equation 3.14 was derived under the assumption that D and E are parallel. When this does not occur and D and E are not parallel, an external electric field applied to the

crystal may again change the dimensions of the axes of the ellipsoid (equation 3.9) but the ellipsoid may rotate. The deformed index ellipsoid may then be represented in general terms as

$$a_{11}X^2 + a_{22}Y^2 + a_{33}Z^2 + (a_{23} + a_{32})YZ + (a_{31} + a_{13})ZX + (a_{12} + a_{21})XY = 1 \quad 3.15$$

and since the permittivity tensor is symmetrical (Equation 3.4)

$$a_{23} = a_{32} \quad a_{31} = a_{13} \quad \text{and} \quad a_{12} = a_{21}$$

Following the Voigt notation (43) this equation may be written as

$$a_1X^2 + a_2Y^2 + a_3Z^2 + 2a_4YZ + 2a_5ZX + 2a_6XY = 1 \quad 3.16$$

The crystal considered in this thesis is LiNbO_3 which exhibits the linear electro-optic effect. Therefore, we shall restrict ourselves to the linear electro-optic variation only.

For crystals which exhibits linear electro-optic effects, the changes in the index of refraction may be represented in terms of the linear electro-optic coefficients and in the matrix notation these may be written as (43)

$$\Delta \left[\frac{1}{n^2} \right]_i = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad 3.17$$

where $E_1 = E_x$, $E_2 = E_y$ and $E_3 = E_z$

(The elasto-optic effect is neglected in this equation. Refer

Chapter 7 for the complete expression for the change in refractive index)

The coefficients $a_1, \dots, a_6$ of equation 3.16 may be conveniently represented in terms of the refractive indices as

$$a_i = \frac{1}{n_i^2} + \sum_{k=1}^3 r_{ik} E_k \quad 3.18$$

where $i=1,2, \dots, 6$ r_{ik} is a third rank electro-optic tensor.

When $i = 4, 5$ or 6 , from equation 3.9, $1/n_i$ is zero as no terms of the form YZ , ZX or XY appears in that equation. These are the terms in the equation 3.16 which gives rise to the rotation of the index ellipsoid.

It was stated earlier in this section that the linear electro-optic effect can occur only in crystals lacking an inversion symmetry.

This requirement of no inversion symmetry is necessary for linear piezoelectric effects to occur in crystals and a (3×6) tensor defined as d_{ji} describes this property. The form of the r_{ij} and d_{ji} tensors are uniquely determined by the point group symmetry of the crystals. The form of the electro-optic tensor is obtained by replacing d_{ji} by r_{ij} and is defined by a (6×3) tensor (43).

The total number of non vanishing electro-optic coefficients can be determined by the crystal symmetry.

3.3 Electro-optic effect induced by interdigital electrodes on crystal

In the previous section it was shown that the index ellipsoid can be deformed, resulting in a change in the phase velocities of the optical wave by the application of an electric field. This dependence of the phase velocities on the applied electric field can be utilised to modulate and deflect an optical beam.

This thesis considers the effects of a periodic corrugation of the refractive index inside the electro-optic medium. An appropriately polarised incident beam which experiences this perturbation will be diffracted into the higher orders. The periodic variation of the refractive index is achieved by applying a potential to a periodic electrode structure deposited on an electro-optic crystal. The performance of modulators and deflectors are generally characterised by the voltages required for extinction of the zero order and the voltages required for deflecting the incident energy to the higher orders.

In electro-optic crystals, the application of an external electric field causes:

- i) a change in the lengths of the ellipsoid axes (refractive indices)
- ii) a rotation of the index ellipsoid giving rise to a rotation of the planes of polarisation of the beam.

This second factor is not desirable in the present application as the voltages required for the diffraction effect are dependent on the plane of polarisation of the incident beam and, therefore, suitable crystals must be selected which will give a large change in the length of the ellipsoid axes but with zero or negligible contribution to the rotation of the ellipsoid.

A close study of the available electro-optic crystals with large electro-optic coefficients suggests that (45, 46, 47, 48) the triclinic, monoclinic and orthorhombic classes of crystal are not suitable for this purpose (a few of the last two classes of the crystals have reasonably large electro-optic coefficients but they are difficult to produce as large crystals with consistent optical

quality). Therefore, the following discussions will be broadly based on the remaining crystal classes.

When an electric field is applied to the crystal the deformed index ellipsoid equation may be written using equations 3.17 and 3.18 as

$$\begin{aligned} & \left(\frac{1}{n_x^2} + r_{11} E_x + r_{12} E_y + r_{13} E_z \right) X^2 + \left(\frac{1}{n_y^2} + r_{21} E_x + r_{22} E_y + r_{23} E_z \right) Y^2 + \\ & \left(\frac{1}{n_z^2} + r_{31} E_x + r_{32} E_y + r_{33} E_z \right) Z^2 + 2(r_{41} E_x + r_{42} E_y + r_{43} E_z) YZ + \\ & 2(r_{51} E_x + r_{52} E_y + r_{53} E_z) ZX + 2(r_{61} E_x + r_{62} E_y + r_{63} E_z) XY = 1 \end{aligned} \quad \text{--- 3.19}$$

Consider an interdigital electrode system deposited on to a polished face of a crystal. The reference coordinate axes are taken to be along the principal axes of the crystal. For simplicity, assume that the incident light is travelling along the direction of the axes (Fig 3.1). The electrodes are in the XY plane and are

periodically spaced along the Y direction. The electrode lengths are along the X axis. When alternate polarities are applied to the adjacent electrodes, a fringing field penetrates into the crystal.

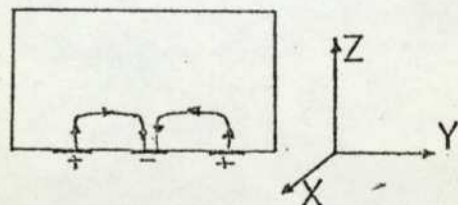


Fig 3.1

Crystal-electrode configuration

The expressions for the different components of the fields are derived in chapter 4. In order to ensure that the incident beam experiences a periodically varying refractive index, the optical beam should be directed along the length of the electrodes (X axis). In the arrangement considered in figure 3.1 the electric field inside the crystal will have components E_z and E_y along the Z and Y directions respectively. The optical indicatrix will be deformed

by these two components and the modified index ellipsoid may be written from equation 3.19 as

$$\begin{aligned} & \left(\frac{1}{n_x^2} + r_{12} E_y + r_{13} E_z \right) X^2 + \left(\frac{1}{n_y^2} + r_{22} E_y + r_{23} E_z \right) Y^2 + \\ & \left(\frac{1}{n_z^2} + r_{32} E_y + r_{33} E_z \right) Z^2 + 2(r_{42} E_y + r_{43} E_z) YZ + \\ & 2(r_{52} E_y + r_{53} E_z) ZX + 2(r_{62} E_y + r_{63} E_z) XY = 1 \end{aligned} \quad 3.20$$

When the incident beam is along X direction, the ellipse which determines the two refractive indices is

$$\left(\frac{1}{n_y^2} + r_{22} E_y + r_{23} E_z \right) Y^2 + \left(\frac{1}{n_z^2} + r_{32} E_y + r_{33} E_z \right) Z^2 + 2(r_{42} E_y + r_{43} E_z) YZ = 1 \quad \text{--- 3.21}$$

The first two terms determine the lengths of the major and minor axes of the ellipse and the third term corresponds to the rotation of the ellipse. An identical equation can be obtained for the case when the electrodes are deposited on the ZX plane with electrode lengths along the X axis.

For the cases where the electrodes are deposited on the XY or YZ planes and the electrodes are parallel to the Y axis, the equation of the ellipse reduces to

$$\left(\frac{1}{n_x^2} + r_{11} E_x + r_{13} E_z \right) X^2 + \left(\frac{1}{n_z^2} + r_{31} E_x + r_{33} E_z \right) Z^2 + 2(r_{51} E_x + r_{53} E_z) ZX = 1 \quad \text{--- 3.22}$$

A similar equation can be obtained for electrodes deposited on the ZX or ZY planes with the incident beam traversing in the Z direction.

It can be seen from these equations that the electro-optic coefficients r_{41} , r_{52} and r_{63} do not contribute to either the change in length of the axes of the ellipsoid or to the rotation of the ellipsoid. If the geometries are as chosen, then crystal classes

having these three non-vanishing r_{ij} coefficients, but with all other coefficients equal to zero, are not suitable for this application. These are the 422, 42m tetragonal 43m cubic and 622 hexagonal classes of crystals (42). However these crystals (eg KDP) may be used when the crystal is not cut along the principal axes.

In the remaining classes of crystals, materials with relatively large linear electro-optic coefficients are LiTaO_3 , BaTiO_3 , $\text{Ba}_{1-x}\text{Sr}_x\text{TaO}_3$, and LiNbO_3 . LiNbO_3 was the crystal chosen for the work in this thesis. This was because these crystals are relatively cheap and readily available with consistent optical qualities. LiNbO_3 is a ferro-electric (Curie Temperature 1483°k), uniaxial crystal. The point group symmetry is 3m, and by convention (42) the reference Z axis was chosen parallel to the optic axes. In this convention, the electro-optic matrix for LiNbO_3 may be written as

$$r_{ij} = \begin{bmatrix} 0 & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & 0 \\ r_{61} & 0 & 0 \end{bmatrix} \quad 3.23$$

It can be seen that the coefficients of the electro-optic matrix satisfy the earlier condition that other elements beside r_{41} , r_{52} and r_{63} should be present if the principal axes are chosen to define the crystal faces.

Substituting the above electro-optic tensor coefficients in equation 3.19 the index ellipsoid reduces to

$$\left(\frac{1}{n_x^2} + r_{12}E_y + r_{13}E_z\right)X^2 + \left(\frac{1}{n_y^2} + r_{22}E_y + r_{23}E_z\right)Y^2 + \left(\frac{1}{n_z^2} + r_{33}E_z\right)Z^2 + 2(r_{42}E_y)YZ + 2(r_{51}E_x)X + 2(r_{61}E_x)XY = 1 \quad 3.24$$

In the electro-optic tensor defined in equation 3.23 only four of the coefficients are independent for the crystal LiNbO_3 and for this crystal (78,79,80)

$$\begin{aligned} r_{22} &= -r_{12} = -r_{61} = 3.4 \times 10^{-12} \text{ m/V} \\ r_{51} &= r_{42} = 28.0 \times 10^{-12} \text{ m/V} \\ r_{13} &= r_{23} = 8.6 \times 10^{-12} \text{ m/V} \\ r_{33} &= 30.8 \times 10^{-12} \text{ m/V} \end{aligned}$$

In designing an efficient modulator/deflector the direction of propagation of light must be selected so that the coefficient r_{33} is retained in the final ellipse obtained. The modulators described in the thesis were arranged with the crystal faces defined by the principal axes. However, it is expected that improved performance would be obtained by choosing an optimum axis configuration. This is indicated in Chapter 10 of this thesis.

The different electrode planes that could be considered using a crystal cut along the principal axes are as shown in figure 3.2. The case of electrodes parallel to the Z axis have been neglected, since the component of the field along this axis reduces to zero, consequently the change in the refractive index is small.

- i) Electrodes in the XY plane
and electrodes parallel to
X axis
- ii) Electrodes in the XY plane
and electrodes parallel to
Y axis
- iii) Electrodes in the YZ plane
and electrodes parallel to
Y axis
- iv) Electrodes in the XZ plane
and electrodes parallel to
X axis

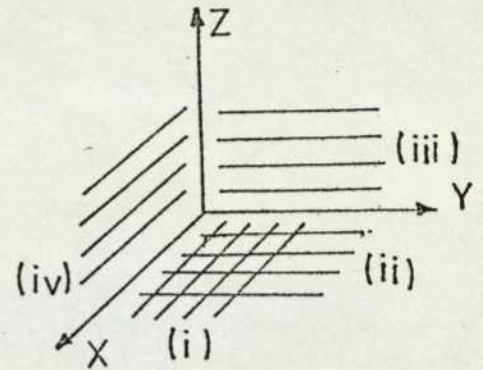


Fig 3-2

Electrode planes

As discussed in Chapter 2 in order to experience the intense field region inside the crystal, the incident beam of light is reflected off the face of the crystal carrying the electrodes. Consider the first arrangement where the electrodes are deposited on the XY plane and the electrode fingers are parallel to the X axis (Fig. 3.3)

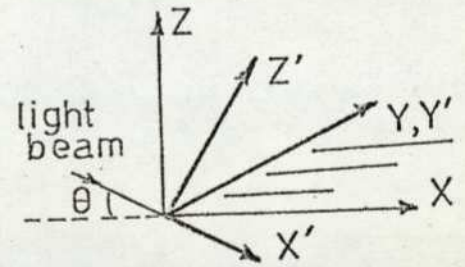


Fig 3-3

Coordinate transformation

The modified index ellipsoid is given by

$$\left(\frac{1}{2} + r_{12}E_y + r_{13}E_z\right)X^2 + \left(\frac{1}{2} + r_{22}E_y + r_{23}E_z\right)Y^2 + \left(\frac{1}{2} + r_{33}E_z\right)Z^2 + 2(r_{42}E_y)YZ = 1 \quad 3.25$$

where $n_o = n_x = n_y$, $n_e = n_z$. This equation is obtained by equating E_x to zero in the equation 3.24. Let the incident beam be in the XZ

plane and in a direction X' . This direction X' is at an angle θ with the X axis. The two refractive indices experienced by the two components of the beam are given by the magnitude of the semi-major and semi-minor axes of the ellipse formed by the intersection of the ellipsoid given by equation 3.25 and $X' = 0$ plane. In order to obtain the expression for the ellipse, first transfer the reference coordinate system to the new axes $X' Y' Z'$ (Fig. 3.3) and then substitute $X' = 0$. In the $X' Y' Z'$ coordinate system, the ellipse of intersection is

$$\left[\frac{\sin^2 \theta}{n_o^2} + \frac{\cos^2 \theta}{n_e^2} + (r_{12} E_y \sin^2 \theta + r_{13} E_z \sin^2 \theta + r_{33} E_z \cos^2 \theta) \right] Z'^2 + \left[\frac{1}{n_o^2} + (r_{22} E_y + r_{23} E_z) \right] Y'^2 + 2(r_{42} E_y \cos \theta) Y'Z' = 1 \quad \text{--- 3.26}$$

In the above equation, the quantities in the brackets shaped as () are introduced by the applied field.

This equation may be written as

$$\frac{Y'^2}{n_{y'}^2} + \frac{Z'^2}{n_{z'}^2} + 2 \frac{1}{n_{y'z'}} Y'Z' = 1 \quad \text{3.27}$$

Where $n_{y'}^2$, $n_{z'}^2$, $n_{y'z'}^2$ correspond to the reciprocal of the coefficients of Y'^2 , Z'^2 and $Y'Z'$ of equation 3.26. It is seen that the cross term $Y'Z'$ which causes the rotation of the ellipse appears in this equation. Let the angle of rotation of the principal axes of the ellipse be α . This angle is given by the expression

$$\tan 2\alpha = (2/n_{y'z'}^2) / (1/n_{y'}^2 - 1/n_{z'}^2) \quad \text{3.28}$$

Substituting the magnitudes of the r_{ij} coefficients for LiNbO_3 which are given on page 33 into this expression, the numerator is found to be very small compared to the denominator and, therefore, the angle of rotation will be negligible. The changes in the indices of refraction can be obtained from equation 3.26 by expanding the coefficients of Y'^2 , Z'^2 binomially. For the change in the index of refraction along the Y' axis we have

$$n_{y'}^2 = \frac{1}{\frac{1}{n_0^2} \left[1 + \frac{r_{22} E_y + r_{23} E_z}{1/n_0^2} \right]} \quad 3.29$$

Taking the square root and expanding binomially then

$$n_{y'} = n_0 \left[1 - \frac{1}{2} n_0^2 (r_{22} E_y + r_{23} E_z) \right] \quad 3.30$$

and, therefore, the change in the index of refraction is

$$\Delta n_{y'} = -\frac{1}{2} n_0^3 (r_{22} E_y + r_{23} E_z) \quad 3.31$$

Similarly for the change in the Z' direction

$$\Delta n_{z'} = -\frac{1}{2} \frac{(r_{13} \sin^2 \theta - r_{33} \cos^2 \theta) E_z - r_{22} \sin^2 \theta E_y}{\left[\frac{\sin^2 \theta}{n_0^2} + \frac{\cos^2 \theta}{n_e^2} \right]^{3/2}} \quad 3.32$$

In this equation r_{12} has been replaced by $-r_{22}$

Similar expressions could be obtained for the other three cases and the results are summarised in Table 3.1. In all these cases the rotation of the ellipse is negligible.

TABLE 3.1

Electrode system	Change in refractive index
case(i)	$\Delta n_{y'} = -\frac{1}{2} n_o^3 (r_{22} E_y + r_{13} E_z)$ $* \Delta n_{z'} = -\frac{1}{2} \frac{(r_{13} \sin^2 \theta + r_{33} \cos^2 \theta) E_z - r_{22} E_y \sin^2 \theta}{\left[\frac{\sin^2 \theta}{n_o^2} + \frac{\cos^2 \theta}{n_e^2} \right]^{\frac{3}{2}}}$
case(ii)	$\Delta n_{x'} = -\frac{1}{2} n_o^2 r_{13} E_z$ $* \Delta n_{z'} = -\frac{1}{2} (r_{13} E_z \sin^2 \theta + r_{33} E_z \cos^2 \theta) / \left(\frac{\sin^2 \theta}{n_o^2} + \frac{\cos^2 \theta}{n_e^2} \right)^{\frac{3}{2}}$
case(iii)	$* \Delta n_{z'} = -\frac{1}{2} n_e^3 r_{33} E_z$ $\Delta n_{x'} = -\frac{1}{2} n_o^3 (r_{13} E_z - 2r_{22} E_x \cos \theta \sin \theta)$
case(iv)	$* \Delta n_{z'} = -\frac{1}{2} n_e^3 r_{33} E_z$ $\Delta n_{y'} = -\frac{1}{2} n_o^3 [-r_{22} E_y (\sin^2 \theta - \cos^2 \theta) + r_{13} E_z]$

The changes in the index of refraction in the Z' direction which are shown starred in Table 3.1 are larger than those of the other corresponding changes. This is because the change in the refractive index along the Z' direction contains the r_{33} coefficient (30.8×10^{-12} m/V) which is very large compared with the other coefficients (r_{13} , r_{22}). It is apparent from this table that, in the first two cases, the periodic variation of the component of the field perpendicular to the electrode surface determines the variation of the refractive index. However, in the last two cases the component of the field which is parallel to the electrode surface determines the variation of the index of refraction. For this reason and as will be seen in the experimental description of the devices constructed during the course of this work, for a symmetrical phase grating (Chapter 5) any one of the four cases

can be selected. However, in the asymmetrical phase grating (Chapter 6) only the first two cases can be used.

Using these equations, the total change in the refractive index and, hence, the total phase change impressed on the transmitted beam can be calculated. Consider

the experimental arrangement shown in figure 3.4. When the incident beam reflects off the electrode face, it experiences an electric field changing in intensity,

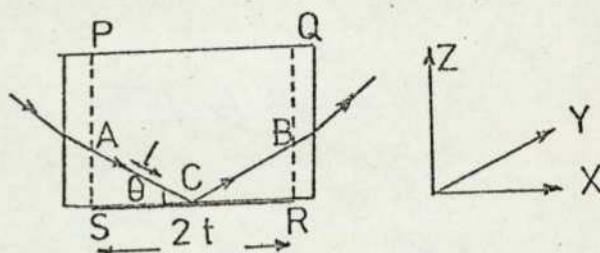


Fig. 3.4

E_z and E_y , as these are functions of Z and Y .

Internal reflection arrangement

In the following section the expressions for the total change in the refractive index and phase change impressed on the transmitted beam are derived.

3.4 Principle of the theoretical derivation of the total change in the index of refraction

The method of deriving the changes occurring in phase is given below. In the derivation it is assumed that the electric field is confined to the region PQRS of the crystal (Fig 3.4).

Let the total length of the electrodes be $2t$. The light is reflected at the mid length of the electrodes (C).

The total phase change impressed on the transmitted beam when the light travels through ACB is $\Delta \phi$ and is given by

$$\Delta \phi = \frac{4\pi}{\lambda} \int_{t \tan \theta}^0 \Delta n_{z'} \frac{dz}{\sin \theta} \quad 3.33$$

where λ is the incident wavelength. The relevant expressions for the components of the electric field are derived in the next chapter.

CHAPTER 1

INTRODUCTION

The theoretical evaluation of the field induced in the
crystal by different electrode configurations

The propagation characteristics of electromagnetic radiation inside electro-optic crystals change when an electric field is applied to the crystal. When an interdigital electrode structure is deposited on to the face of an electro-optic crystal and a voltage is applied, a fringing field penetrates into the crystal. This fringing field periodically changes the index of refraction of the crystal, thus causing a phase grating to appear inside the medium. When a beam of light, polarised in the proper direction, experiences this phase grating the incident light energy is diffracted into the higher orders. In order to estimate the energy coupled into the higher orders, it is necessary to estimate the phase change impressed on the transmitted beam of light when it traverses through the perturbed medium. The phase change impressed by the medium is a function of the applied field. In this chapter, the expressions for the different components of the electric field intensities produced inside the electro-optic crystal when a voltage is applied to different electrode structures deposited on to the face of the crystal are derived.

4.1. Potential distribution along the crystal face

Consider the electrode arrangement of figure 4.1 deposited on to the polished face of a crystal where the voltages on alternate electrodes are $+V_0$ and $-V_0$. The analytic

solution of the field inside the crystal is only possible if the boundary voltages on the crystal face is defined. An exhaustive solution for this voltage may be found by numerically solving the Laplaces equation. The exact

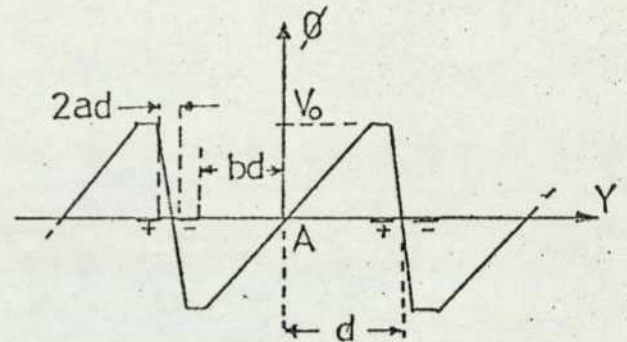


Fig. 4.1

voltage distribution between the electrodes is dependent, among other factors, upon the cross section of the electrodes and the regularity of these electrodes.

It was considered that the detailed solution for the boundary voltage was not warranted as only an indication of the field distribution inside the crystal is required, and thus the assumption that the voltage is uniformly distributed between the electrodes was made. The electrodes are assumed to be perfectly conductive so that the potential remains a constant ($+V_0$ or $-V_0$) along the electrodes. Figure 4.1 shows the assumed voltage distribution along the boundary face of the crystal.

Let the periodic pitch of the profile be $2d$ and the spacing between the neighbouring electrodes, $2ad$ and $2bd$.

The potential distribution (ϕ_e) along the crystal surface may be represented by the Fourier series as

$$\phi_e = A_0 + \sum_n A_n \cos \frac{n\pi}{d} y + \sum_n B_n \sin \frac{n\pi}{d} y \quad 4.1$$

where $n = 1, 2, 3, \dots$ and A_0, A_n, B_n are constants of the Fourier series.

These coefficients A_0, A_n , and B_n may be obtained by Fourier analysis. Choosing the reference axes through A as in figure 4.1, it is apparent that the wave profile exhibits sine symmetry and, therefore, A_0 and A_n reduces to zero.

B_n is given as the solution of the equation describing the Fourier coefficients, thus

$$B_n = \frac{2}{\lambda} \int_0^{\lambda} \phi_e \sin \frac{n\pi}{d} y \, dy \quad 4.2$$

where

$n = 1, 2, 3, \dots$ and λ is the wavelength
For the voltage profile shown in figure 4.1

$$B_n = 2V_0 \left[\frac{\sin(bn\pi)}{n\pi \cdot b n\pi} - \frac{\cos(n\pi) \cdot \sin(an\pi)}{n\pi \cdot an\pi} \right] \quad 4.3$$

where

$n = 1, 2, 3, \dots$

by substituting in equation 4.1

$$\phi_e = \sum_n 2V_0 \left[\frac{\sin(bn\pi)}{n\pi \cdot b n\pi} - \frac{\cos(n\pi) \sin(an\pi)}{n\pi \cdot an\pi} \right] \sin \frac{n\pi}{d} y \quad 4.4$$

where

$n = 1, 2, 3, \dots$

In particular for the case where the spacing between the adjacent electrodes is the same ie $2ad=2bd$, ϕ_e reduces to

$$\phi_e = \sum_n 4V_0 \frac{\sin(an\pi)}{n\pi \cdot an\pi} \sin \frac{n\pi}{d} y \quad 4.5$$

where

$n = 1, 3, 5, \dots$

4.2 Potential distribution inside the crystal

The Laplace equation for the scalar potential ϕ in a rectangular co-ordinate system is given by

$$\epsilon_1 \frac{\partial^2 \phi}{\partial x^2} + \epsilon_2 \frac{\partial^2 \phi}{\partial y^2} + \epsilon_3 \frac{\partial^2 \phi}{\partial z^2} = 0 \quad 4.6$$

ϵ_1, ϵ_2 and ϵ_3 are the dielectric constants in the directions X, Y and Z respectively. The dielectric media is assumed to be charge free. In figure 4.2 the Z axis is perpendicular to the plane of the electrodes and the length of the electrodes are parallel to the X axis.

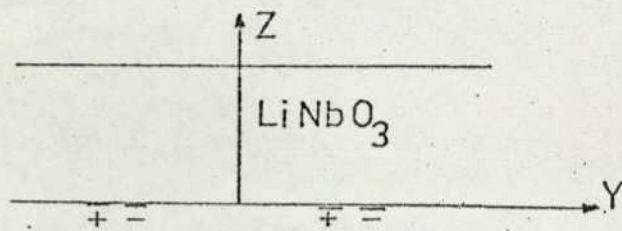


Fig. 4.2

Crystal-electrode orientation

The solution of equation 4.6 may be obtained by the method of substitution. Assume a solution to the above equation is of the form

$$\phi = f(x)g(y)h(z) \quad 4.7$$

where $f(x)$ is a function of x only and non zero
 $g(y)$ is a function of y only and non zero
 $h(z)$ is a function of z only and non zero

Substituting this general solution of ϕ in the Laplace equation, i.e. equation 4.6 and dividing by $f(x)g(y)h(z)$ then

$$\epsilon_1 \frac{1}{f(x)} \frac{\partial^2 f(x)}{\partial x^2} + \epsilon_2 \frac{1}{g(y)} \frac{\partial^2 g(y)}{\partial y^2} + \epsilon_3 \frac{1}{h(z)} \frac{\partial^2 h(z)}{\partial z^2} = 0 \quad \text{---} 4.8$$

Since each term of equation is a function of one variable only, each of these terms must individually reduce to a constant for the equation to hold for any arbitrary value of X, Y, and Z. The restriction on the values of these constants is that the sum of the constants must reduce to zero to satisfy the right hand side of the

equation.

In the arrangement of figure 4.2 the electrodes were parallel to the X axis. Hence, for a given value of Y and Z, the potential remains constant for all values of X. Therefore ϕ is a function of Y and Z only. Hence it follows that

$$\epsilon_2 \frac{1}{g(y)} \frac{\partial^2 g(y)}{\partial y^2} + \epsilon_3 \frac{1}{h(z)} \frac{\partial^2 h(z)}{\partial z^2} = 0 \quad 4.9$$

Dividing by ϵ_2 this equation becomes

$$\frac{1}{g(y)} \frac{\partial^2 g(y)}{\partial y^2} + \frac{\epsilon_3}{\epsilon_2} \frac{1}{h(z)} \frac{\partial^2 h(z)}{\partial z^2} = 0 \quad 4.10$$

Each of these terms, being functions of one variable only, must reduce to a constant and thus

$$\frac{1}{g(y)} \frac{d^2 g(y)}{dy^2} + k_y^2 = 0 \quad \text{or} \quad \frac{d^2 g(y)}{dy^2} + k_y^2 g(y) = 0 \quad 4.11$$

$$\frac{\epsilon_3}{\epsilon_2} \frac{1}{h(z)} \frac{d^2 h(z)}{dz^2} + k_z^2 = 0 \quad \text{or} \quad \frac{d^2 h(z)}{dz^2} + k_z^2 \frac{\epsilon_2}{\epsilon_3} h(z) = 0 \quad 4.12$$

where k_y and k_z are arbitrary constants and the only restriction on these constants is that

$$k_y^2 + k_z^2 = 0 \quad 4.13$$

From equation 4.11, if the general solution of $g(y)$ is assumed to be of the form

$$g(y) = A_1 \sin(k_y y) + A_2 \cos(k_y y) \quad 4.14$$

where A_1 and A_2 are arbitrary constants

then as $k_z^2 = -k_y^2$ (from equation 4.13) the general solution of $h(z)$ is of the form

$$h(z) = B_1 \exp \left[\left(k_y \sqrt{\frac{\epsilon_2}{\epsilon_3}} \right) z \right] + B_2 \exp \left[- \left(k_y \sqrt{\frac{\epsilon_2}{\epsilon_3}} \right) z \right] \quad 4.15$$

where B_1 and B_2 are arbitrary constants.

The general solution of the Laplace equation would then be of the form

$$\phi = \left[A_1 \sin(k_y y) + A_2 \cos(k_y y) \right] \left[B_1 e^{\frac{k_y \sqrt{\epsilon_2}}{\epsilon_3} z} + B_2 e^{-\frac{k_y \sqrt{\epsilon_2}}{\epsilon_3} z} \right] \quad 4.16$$

The arbitrary constants A_1 , A_2 , B_1 , B_2 and k_y of the above solution may be evaluated by considering the known boundary conditions.

- 1) When $Z = 0$, the value of the potential changes sign with a change in the sign of Y

$$\text{ie.} \quad \begin{array}{cc} \phi & = -\phi \\ \frac{z=0}{y=y} & \frac{z=0}{y=-y} \end{array}$$

ie. ϕ exhibits sine symmetry with respect to Y . Therefore, the potential distribution at the surface of crystal electrode interface must exhibit sine symmetry.

$$\text{Therefore} \quad A_2 = 0$$

- 2) Along the Y axis, ϕ is equal to zero when Y is zero or an integer multiple of d . Thus k_y will be equal to $m\pi/d$ where m is zero or an integer. From equation 4.16 it can be seen that the case of m equal to zero is trivial, since this is the

case of the potential equal to zero for all values of Y .

Therefore, in general

$$\begin{aligned}\phi &= \sum_m (B_{1m} e^{\sqrt{\frac{\epsilon_2}{\epsilon_3}} \frac{m\pi}{d} z} - B_{2m} e^{-\sqrt{\frac{\epsilon_2}{\epsilon_3}} \frac{m\pi}{d} z}) A_{1m} \sin \frac{m\pi}{d} y \\ &= \sum_m (C_{1m} e^{\sqrt{\frac{\epsilon_2}{\epsilon_3}} \frac{m\pi}{d} z} - C_{2m} e^{-\sqrt{\frac{\epsilon_2}{\epsilon_3}} \frac{m\pi}{d} z}) \sin \frac{m\pi}{d} y\end{aligned}\quad 4.17$$

where

$$m = 1, 2, 3, \dots$$

- 3) As Z tends to infinity, the potential ϕ tends to zero, and this condition is satisfied in the above equation by placing C_{1m} equal to zero and thus

$$\begin{aligned}\phi &= \sum_m C_{2m} e^{-\sqrt{\frac{\epsilon_2}{\epsilon_3}} \frac{m\pi}{d} z} \sin \frac{m\pi}{d} y \\ \text{where } m &= 1, 2, 3, \dots\end{aligned}\quad 4.18$$

- 4) When $Z=0$ the potential distribution ϕ is given by the potential distribution along the electrode - crystal boundary

$$\begin{aligned}\phi_e &= 2V_0 \sum_n \left[\frac{\sin(bn\pi)}{n\pi \cdot bn\pi} - \frac{\cos(n\pi) \sin(an\pi)}{n\pi \cdot an\pi} \right] \sin \frac{n\pi}{d} y \\ \text{where } n &= 1, 2, 3, \dots\end{aligned}$$

C_{2m} may now be determined by equating these two equations

(4.4 and 4.18) at $Z=0$, and, hence, it follows that the general expression for the potential distribution inside the crystal is

$$\begin{aligned}\phi &= 2V_0 \sum_n \left[\frac{\sin(bn\pi)}{n\pi \cdot bn\pi} - \frac{\cos(n\pi) \sin(an\pi)}{n\pi \cdot an\pi} \right] e^{-\sqrt{\frac{\epsilon_2}{\epsilon_3}} \frac{n\pi}{d} z} \sin \frac{n\pi}{d} y \\ \text{where } n &= 1, 2, 3, \dots\end{aligned}\quad 4.19$$

In particular if we consider an electrode arrangement such that the electrode spacings are equal i.e. $2ad=2bd$ then the potential distribution inside the crystal is given by

$$\phi = 4V_0 \sum_n \frac{\sin(n\pi a)}{n\pi \cdot n\pi a} e^{-\sqrt{\frac{\epsilon_2}{\epsilon_3}} \frac{n\pi}{d} z} \sin \frac{n\pi}{d} y \quad 4.20$$

where

$$n=1,3,5,\dots$$

In determining the potential distribution inside the crystal it is necessary to distinguish between the zero frequency case where the equivalent resistive network determines the potential distribution and the high frequency case when the equivalent capacitive network determines the potential distribution. In the case where the resistive network determines the potential distribution inside the crystal, it has been assumed that the resistances in all directions are the same, and under these conditions, the distribution of ϕ is determined by the Laplace equation of the form

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad 4.21$$

This assumption has been made as further information on the samples used in the experiment was not available. The manufacturers figure for the electrical resistance of LiNbO_3 is $5 \times 10^{10} \Omega \text{m}$ (2). However, no advice was given as to whether the resistances varied in the different directions for the samples obtained, but it was of interest to establish the possible differences of the diffraction effect that might be expected between the two equivalent networks considered for the crystal. That is the high frequency case given by equation 4.19 and the zero frequency case given by the solution of equation 4.21. Later experimental work (ref. Chapter 7) has indicated that the

assumption that the resistances in different directions are the same appears to be reasonable

The solution of equation 4.21 will be similar to that of 4.20 with ϵ_2 and ϵ_3 omitted from the solution. That is

$$\phi = 4V_0 \sum_n \frac{\sin(n\pi a)}{n\pi a n\pi} e^{-\frac{n\pi}{d}z} \sin \frac{n\pi}{d}y \quad 4.22$$

where

$$n = 1, 3, 5, \dots$$

From the solution given by the expression 4.19 it can be seen that the potential distribution inside the crystal is a function of the electrode pitch (d) and the electrode spacing (2ad, 2bd). For simplicity if it is assumed that the spacing between adjacent electrodes are equal then the potential ϕ is a function of 'a' and 'd'.

In order to study the nature of the potential distribution inside the crystal, a particular electrode arrangement was considered. In this arrangement, the magnitude of 'a' was taken as 0.25 . This gives a symmetrical electrode configuration where the electrode width is equal to the electrode spacing. The value of 'd' , the electrode pitch, is assumed to be 0.1 mm, as this is of the order of the value chosen for the experimental arrangement.

In determining the potential distribution inside the crystal, two different cases should be considered.

- i) Applied voltage at zero frequency
- ii) Applied voltage at a high frequency

In the first case when the applied voltage is at zero frequency, the potential distribution inside the crystal is given by the equation 4.22 . It is seen from this equation that the potential distribution inside the crystal is the same when the electrodes are in the plane of the optic axis

or perpendicular to it. However, the potential distribution in the high frequency case changes with the electrode plane. Therefore two electrode configurations can be considered for the high frequency case. One, the electrode plane along the optic axis, and the second, the electrode plane perpendicular to the optic axis. The dielectric constants along and perpendicular to the optic axis were assumed to be 28.6 and 84.6 respectively. These values were taken as they correspond to the dielectric constants of LiNbO_3 , the material used in the work described in the thesis. The potential distributions for the zero frequency and the high frequency case with electrodes in the plane perpendicular to the optic axis are given in figures 4.3.1 a and 4.3.2 a respectively. Figures 4.3.1 b and 4.3.2 b shows the penetration of the potential inside the crystal. Similar plots can be obtained for the high frequency case when the electrodes are in the plane parallel to the optic axis.

Comparision of figures 4.3.1 b and 4.3.2 b show that, in the arrangement considered, the penetration of the potential is larger in the anisotropic case than in the isotropic case.

The depth of penetration of the potential inside the crystal is an important consequence for an arrangement when the light travels parallel to the electrodes as described in Chapter 2. However, for the internal reflection modulator, this penetration depth while important may not be the most important determining factor. In this internal reflection arrangement, the sensitivity is determined by the integral of the product of the field intensity and the incremental path length.

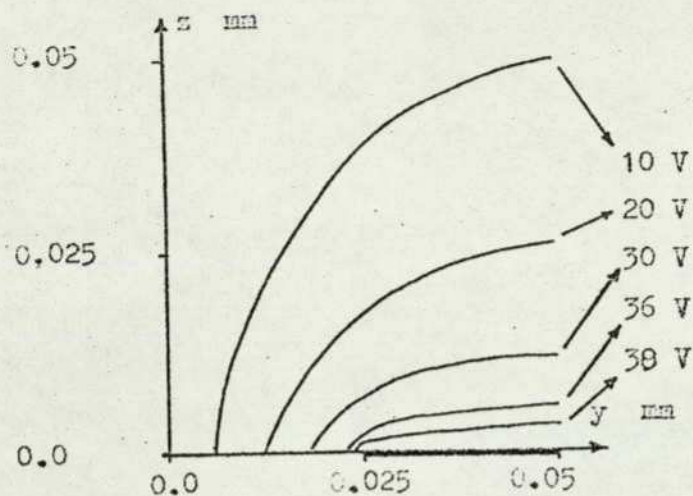


Fig. 4.3.1 a Potential distribution inside the crystal (Isotropic case)
 $a = 0.25$ $d = 0.1$ mm

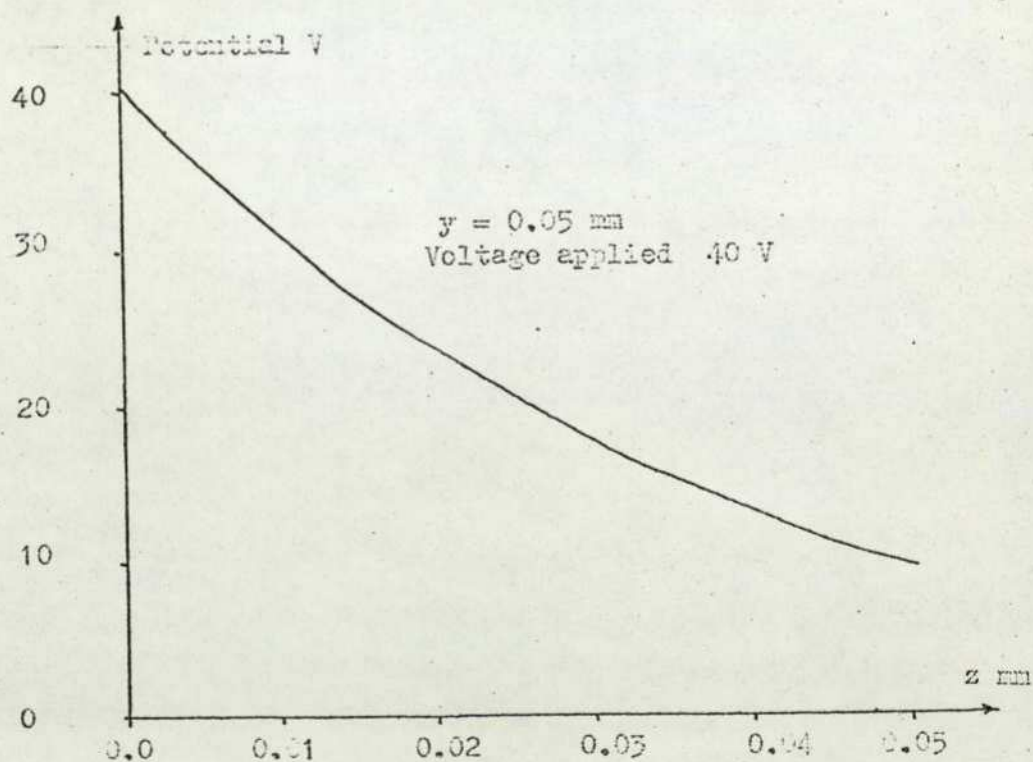


Fig. 4.3.1 b Potential penetration inside the crystal (Isotropic) case
 $a = 0.25$ $d = 0.1$ mm

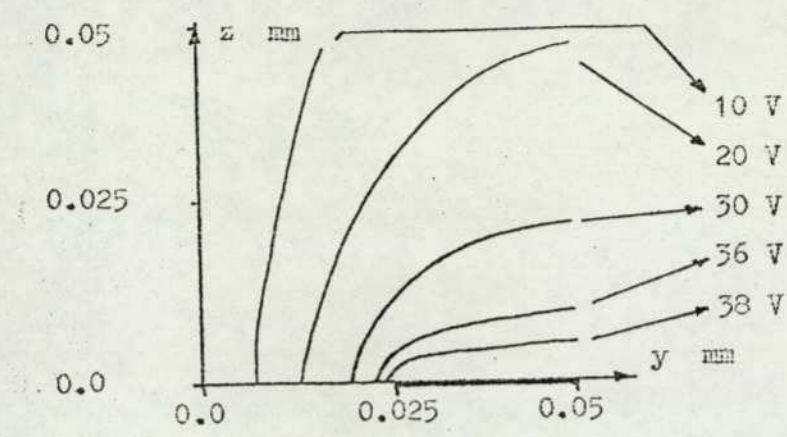


Fig. 4.3.2 a Potential distribution inside the crystal (Anisotropic) case
 $a = 0.25$ $d = 0.1$ mm

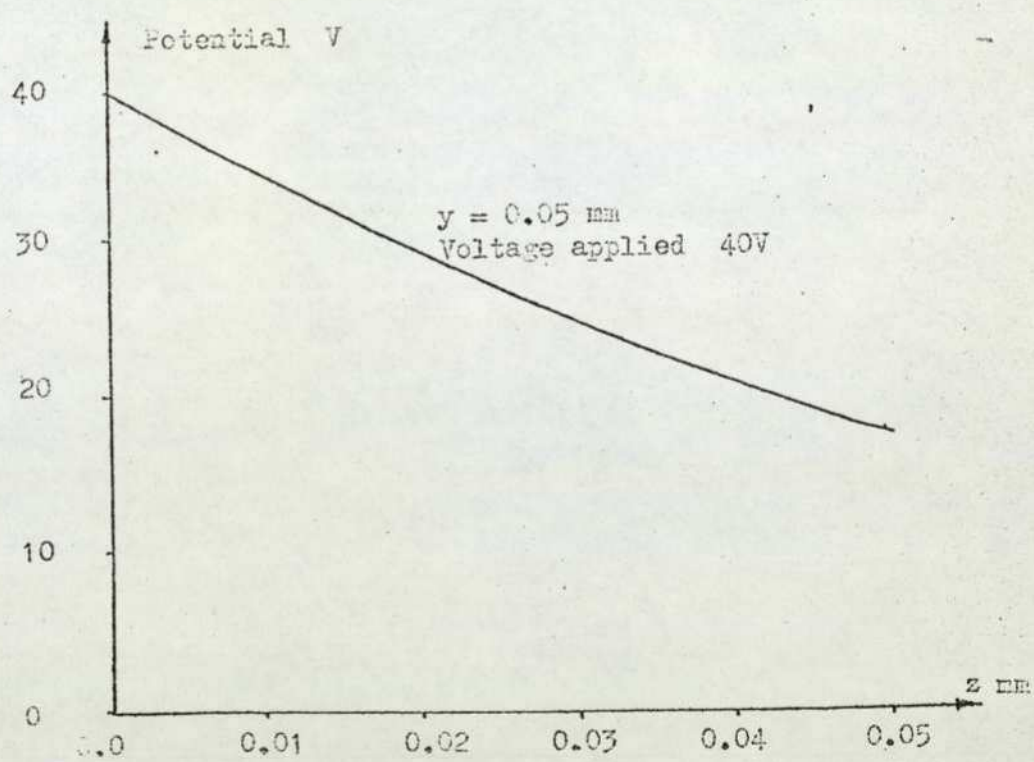


Fig. 4.3.2 b Potential penetration inside the crystal (Anisotropic) case
 $a = 0.25$ $d = 0.1$ mm

4.3 Electric Field distribution inside the crystal

In this section expressions are derived for the electric field intensities inside the crystal. The electric field intensities (or the component of the field) is calculated from the derivatives of the potential

$$\begin{aligned} \underline{E} &= - \text{grad}(\phi) \\ &= - \left(\underline{i} \frac{\partial}{\partial x} + \underline{j} \frac{\partial}{\partial y} + \underline{k} \frac{\partial}{\partial z} \right) \phi \end{aligned} \quad 4.23$$

where $\underline{i}$, $\underline{j}$, $\underline{k}$ are the unit vectors along the X, Y, Z directions. As discussed earlier, as the electrodes are arranged parallel to the X axis, there is no variation in potential along the X direction. Therefore, the component of the field along the X direction is zero.

$$\underline{E}_x = 0 \quad \underline{E}_y = -\underline{j} \frac{\partial \phi}{\partial y} \quad \underline{E}_z = -\underline{k} \frac{\partial \phi}{\partial z} \quad 4.24$$

For the general case where the electrode spacings $2ad$ and $2bd$ are not equal, the potential ϕ is given by the equation 4.19, and from this equation the derivatives E_y and E_z are obtained as

$$\begin{aligned} E_x &= 0 \\ E_y &= -2V_0 \sum_n \left[\frac{\sin(bn\pi)}{n\pi b n\pi} - \frac{\cos(n\pi) \sin(n\pi a)}{n\pi a n\pi} \right] e^{-\frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_3}} \frac{n\pi}{d} z} \frac{n\pi}{d} \cos \frac{n\pi}{d} y \end{aligned} \quad 4.25$$

$$E_z = 2V_0 \sum_n \left[\frac{\sin(bn\pi)}{n\pi b n\pi} - \frac{\cos(n\pi) \sin(n\pi a)}{n\pi a n\pi} \right] e^{-\frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_3}} \frac{n\pi}{d} z} \left(\frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_3}} \frac{n\pi}{d} \right) \sin \frac{n\pi}{d} y \quad 4.26$$

where

$$n = 1, 2, 3, \dots$$

For the case when $a=b$, the components of the field are given by

$$E_x = 0$$

$$E_y = -4V_0 \sum_n \frac{\sin(an\pi)}{n\pi \cdot an\pi} e^{-\frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_3}} \frac{n\pi}{d} z} \frac{n\pi}{d} \cos \frac{n\pi}{d} y \quad 4.27$$

$$E_z = 4V_0 \sum_n \frac{\sin(an\pi)}{n\pi \cdot an\pi} e^{-\frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_3}} \frac{n\pi}{d} z} \left(\frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_3}} \frac{n\pi}{d} \right) \sin \frac{n\pi}{d} y \quad 4.28$$

where
 $n=1, 3, 5, \dots$

From the above equations (4.25) and (4.26) it can be seen that the components of the field intensities are functions of the periodicity of the electrodes and the electrode spacings. Hence, a detailed study of the variation of these components of the fields with the periodicity and spacing of the electrodes will be advantageous in choosing the proper electrode structure for any desired application. For simplicity, an electrode structure with equal spacings between the electrodes deposited on to a polished face of a crystal is assumed. Six different arrangements of electrodes to analyse the manner in which the electric field penetrates into the crystal are considered. Three of these assume constant 'd' and varying 'a' while the other three have constant 'a' and varying 'd'. The different values of 'a' and 'd' chosen are summarised in Table 4.1.

TABLE 4.1

Constant d			Constant a	
1)	$a = 0.125$	$d = 0.10 \text{ mm}$	4)	$a = 0.25$ $d = 0.075 \text{ mm}$
2)	$a = 0.25$	$d = 0.10 \text{ mm}$	5)	$a = 0.25$ $d = 0.10 \text{ mm}$
3)	$a = 0.375$	$d = 0.10 \text{ mm}$	6)	$a = 0.25$ $d = 0.125 \text{ mm}$

In estimating the variations of the components of the field with the above parameters, it is assumed that the electrodes are parallel to the X axis of the co-ordinate system and, therefore E_x reduces to zero. As discussed earlier, for the isotropic case where the resistances are assumed equal in all directions, the potential distribution inside the crystal remains unaltered when the plane of the electrodes is parallel or perpendicular to the optic axis.

However, in the high frequency anisotropic case ($\epsilon_2 \neq \epsilon_3$) the field distribution inside the crystal will vary with the plane on which the electrodes are deposited. The different possibilities considered for the arrangement of the electrodes and the corresponding components of field intensities that are of interest are summarised in Table 4.2.

TABLE 4.2

	Anisotropic case	Isotropic case
Electrodes $\perp$ to the optic axis	1) Component of $E_{\perp}$ to the electrode structure	3) Component of $E_{\perp}$ to the electrode structure
	2) Component of $E_{\parallel}$ to the electrode structure	4) Component of $E_{\parallel}$ to the electrode structure
Electrodes $\parallel$ to the optic axis	5) Component of $E_{\perp}$ to the electrode structure	7) Component of $E_{\perp}$ to the electrode structure
	6) Component of $E_{\parallel}$ to the electrode structure	8) Component of $E_{\parallel}$ to the electrode structure

Of course for the isotropic case from Table 4.2, case 3 is identical to 7, and case 4 to 8. In order to study the variation of these six different cases in some detail, we shall first see the variation of the perpendicular components of the field across the electrode width.

Figure 4.4.1 shows the variations of the perpendicular component of the induced electric field with distance across the electrode

widths for case 2 ($a=0.25$ $d=0.1$ mm) of Table 4.1. These variations were computed for the zero frequency case where ϵ_2 and ϵ_3 are omitted from equation 4.28. The E_z component of the field was calculated for three different values of Z , the perpendicular distance from the electrodes. The three values of Z being 0.025, 0.05 and 0.075 mm respectively. It can be seen from these plots (Fig.4.4.1) that when Z is equal to 0.025mm and 'a' is 0.25, the theoretical maximum occurs at a distance of about 0.02 mm from the centre of the electrodes. This may have been due to the electric field concentration at the edges of the electrodes. However, when the value of Z was increased, this effect was not present and the maxima occurred at the centre of the electrodes. Therefore, as an indication of the variation of the perpendicular component of the electric field, with distance, for the different electrode-crystal configurations, the variations of this component along the centre line of the electrodes was studied.

A similar plot for the variation of the parallel component of the electric field (E_y) is shown in figure 4.4.2. This plot shows that the maximum of this component occurs at the centre line of the spacing. Therefore, the variation of the parallel component with distance from the electrodes was studied along the perpendicular line passing through the centre of the electrode spacing.

Figure 4.5.1-3 shows the variation of the perpendicular component of the field with distance from the electrode face for the isotropic and the two anisotropic cases respectively. The numbers of the different curves on each figure corresponds to the case numbers of Table 4.1.

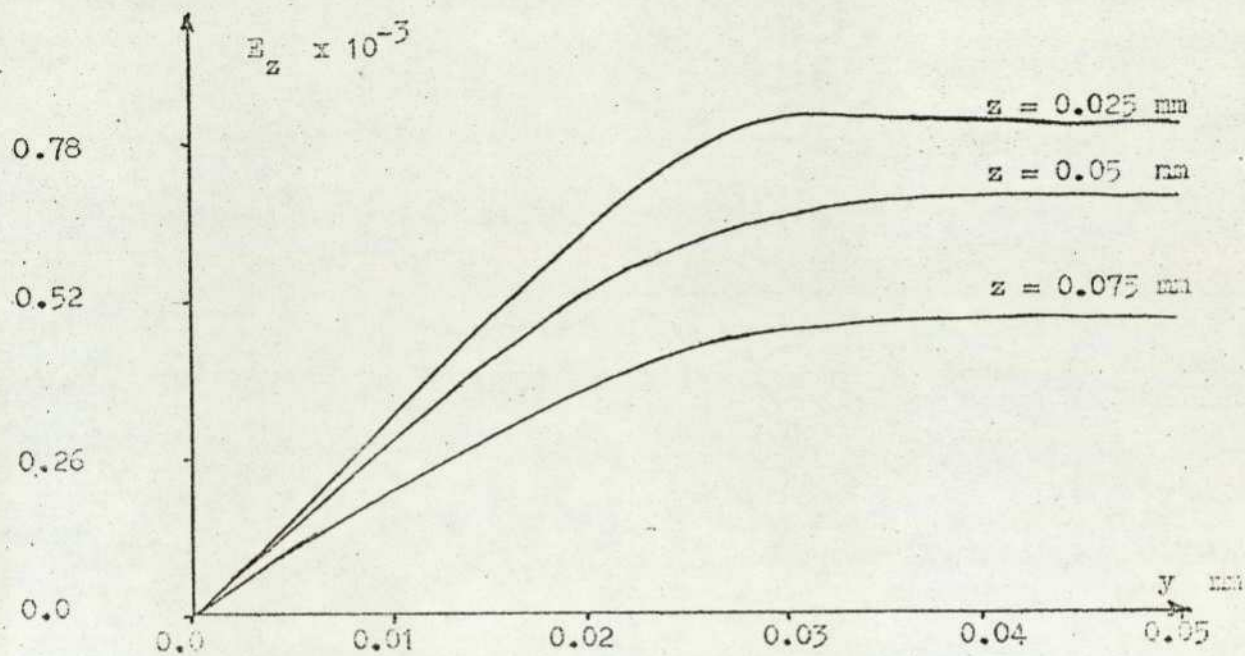


Fig 4.4.1 Variation of E_z with y

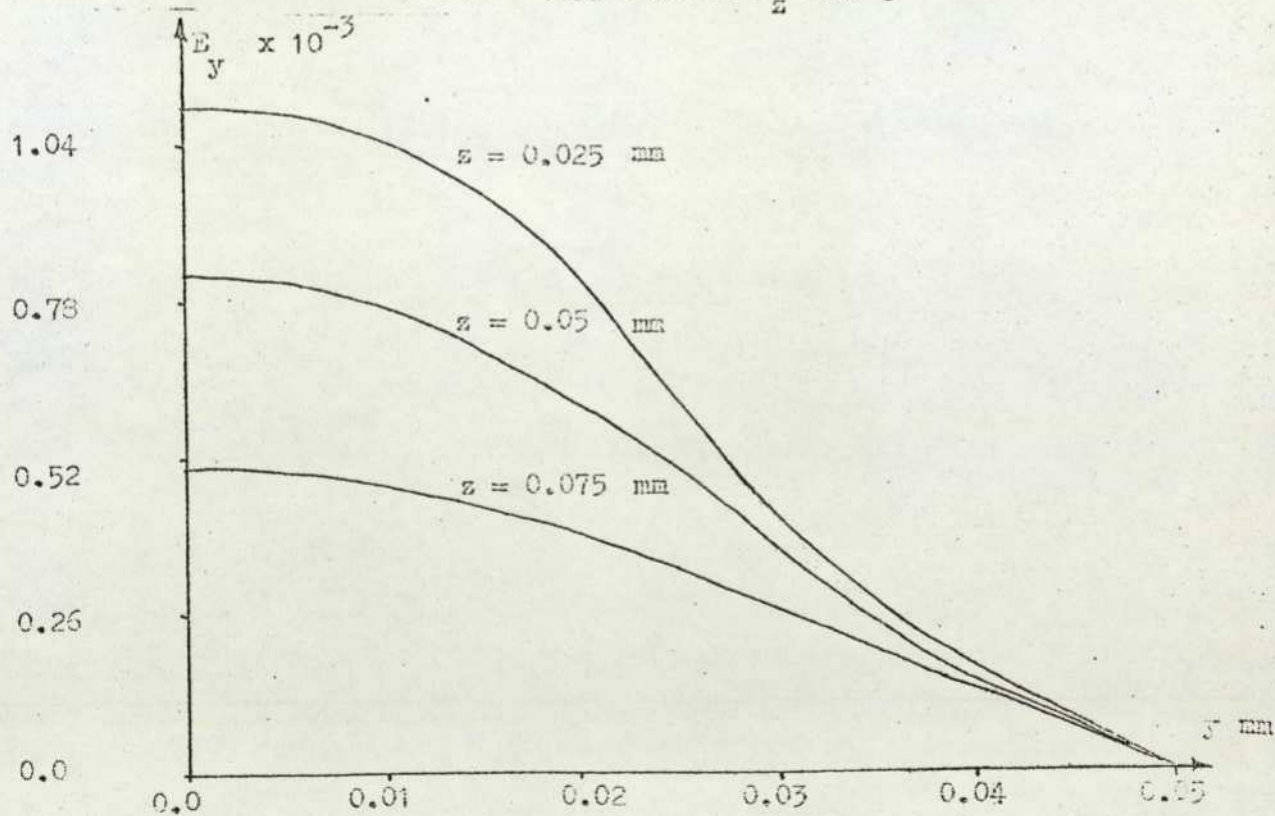


Fig 4.4.2 Variation of E_y with y

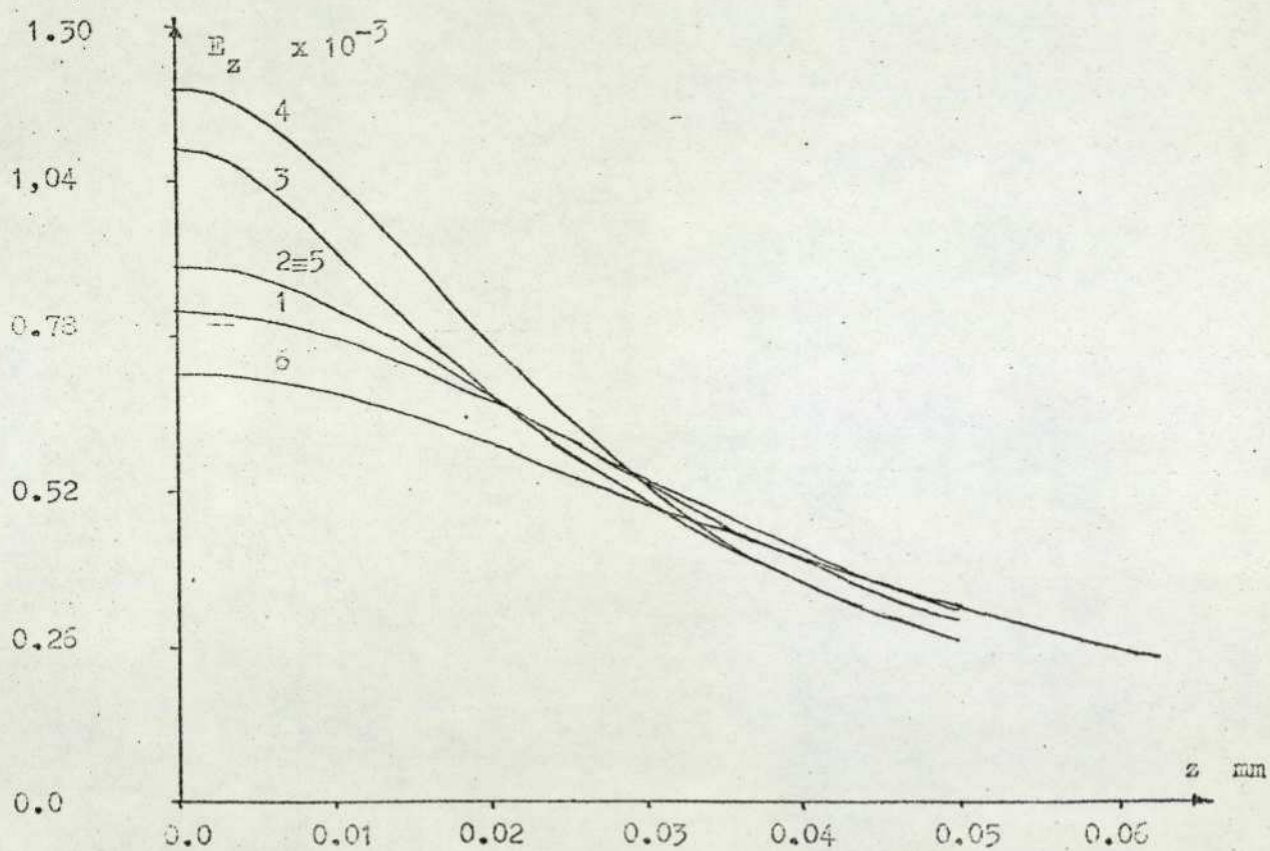


Fig. 4.5.1 Variation of E_z with z
 Isotropic case
 Electrodes in the XY plane

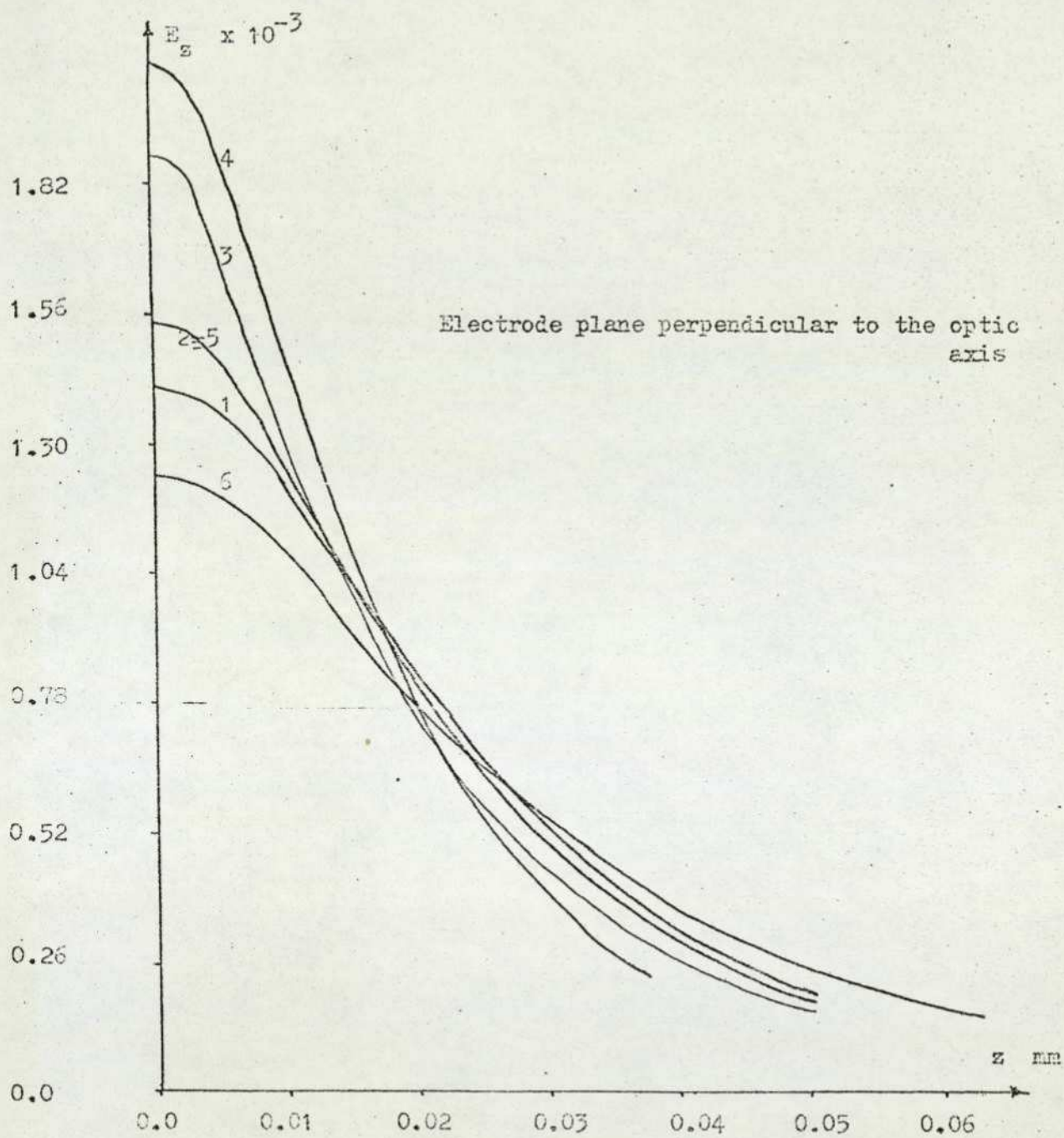


Fig 4.5.2 Variation of E_z with z
 Anisotropic case $\epsilon_1 = 84.6$ $\epsilon_3 = 28.6$
 Electrodes in the XY plane

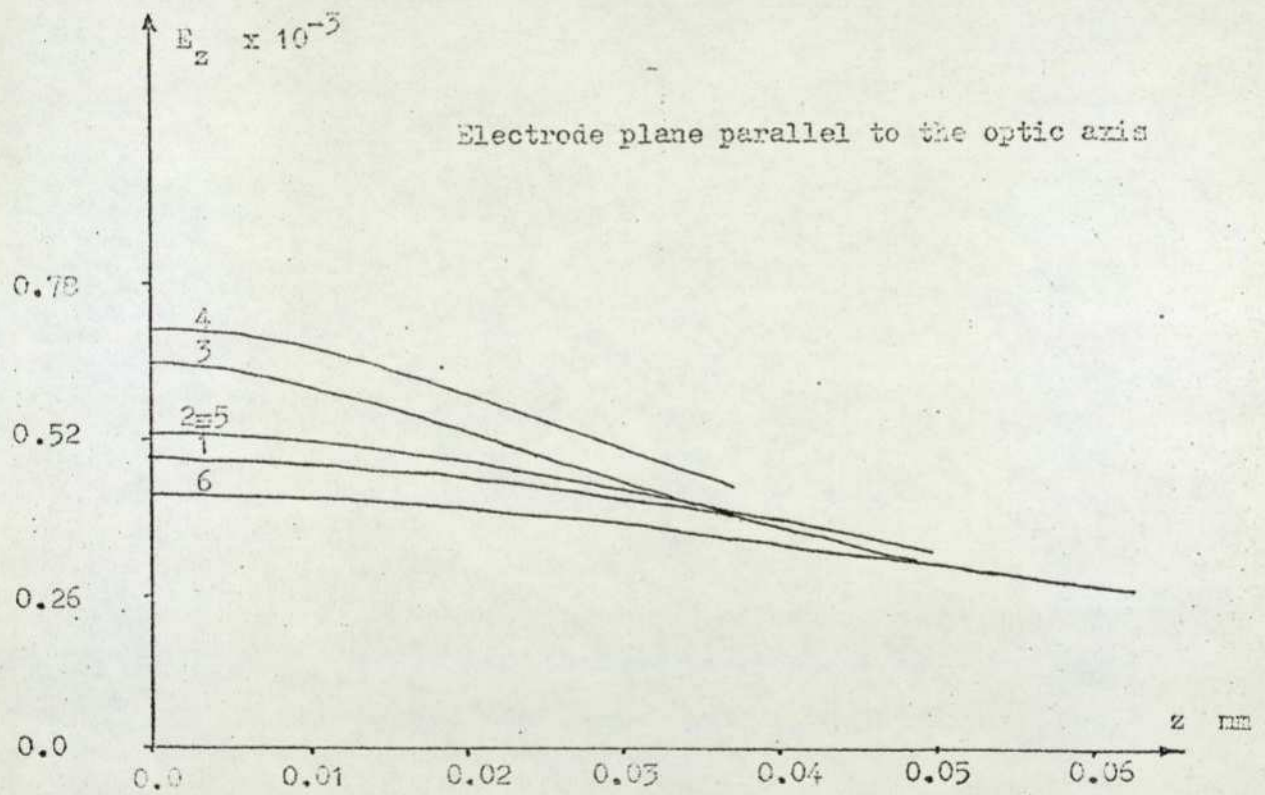


Fig. 4.5.3 Variation of E_z with z
 Anisotropic case $\epsilon_1 = 28.6$ $\epsilon_3 = 84.6$
 Electrodes in the XY plane

Let E_z denote the perpendicular component of the field measured along the line taken through the centre of an electrode, and Z denote the perpendicular distance from the centre of the electrode. Graphs 1, 2 and 3 of figure 4.5.1-3 show the variation of E_z with Z when the electrode pitch remains unaltered and the electrode spacings varied. It can be seen from these plots that the magnitude of E_z at the electrode-crystal interface increases with an increase of the electrode spacings. However, this component decreased more rapidly with an increase of Z when the electrode spacings are increased. The plots 4, 2 and 5 (corresponding to cases 4, 2 and 5 of Table 4.1) show that the magnitude of E_z at the electrode-crystal interface increases with decrease of electrode pitch. However, a more rapid decrease of E_z with an increase of Z occurs as the electrode pitch is decreased.

Summarised are the important deductions obtained from these plots.

- 1) At Z equals zero, E_z increases with an increase of the electrode spacing when 'd' is kept constant.
- 2) At Z equal to zero, E_z increases with a decrease in the electrode pitch when 'a' is kept constant.

The anisotropic arrangement where the optic axis is perpendicular to the electrode surface (figure 4.5.2) and the isotropic case (figure 4.5.1) clearly indicates that for LiNbO_3 :

- 3) With the electrodes on the plane perpendicular to the optic axis, at a distance less than $d/4$, E_z is greater than that obtained in the isotropic case.
- 4) At a distance very much greater than $d/4$, E_z is less in the first arrangement than in the second arrangement.

(Note: The arrangement where the optic axis is parallel to the electrode surface is not of great practical importance when the perpendicular component of the field is considered. The detailed study of this effect is given in Chapter 3.)

- 5) At Z equal to zero, the value of E_z for the isotropic case is less than that for the anisotropic case, but the decrease of the magnitude of E_z with increase in Z is much sharper for the anisotropic case than that in the isotropic case.
- 6) In the isotropic arrangement, the magnitude of E_z decreases to approximately half of its value at $Z=0$ when the distance from the electrode surface is approximately $d/3$. For the anisotropic case, a similar decrease in E_z occurred when Z is considered at a distance of about $d/4$ from the electrode surface.

The variation of the electric field component parallel to the electrode surface with distance from the electrode face is now considered. The earlier graph (figure 4.4.2) shows that the maximum value of the parallel component of the electric field occurs at the centre of the electrode spacing. Let E_y denote the parallel component of the field measured along the centre of an electrode spacing, and Z is again assumed to denote the perpendicular distance from the centre of the electrode spacing. Equation 4.27 indicates that at the interface i.e. $Z=0$, the magnitude of E_y is the same for the isotropic and for the two anisotropic cases when the electrodes are either perpendicular or parallel to the optic axis.

The variation of E_y with Z for these three different arrangements are shown in figures 4.6.1-3.

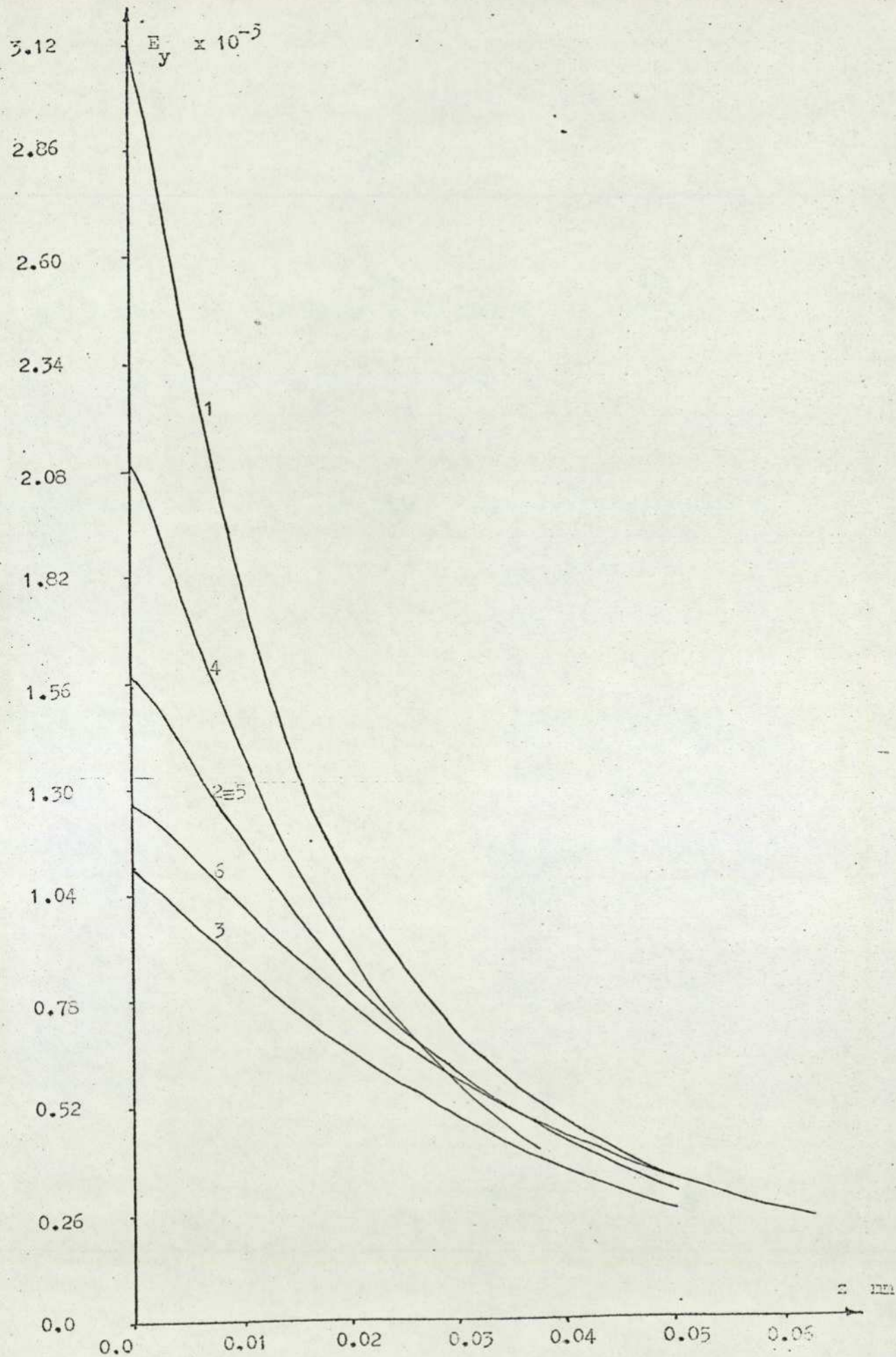


Fig 4.6.1 Variation of E_y with z
 Isotropic case
 Electrodes in the XY plane

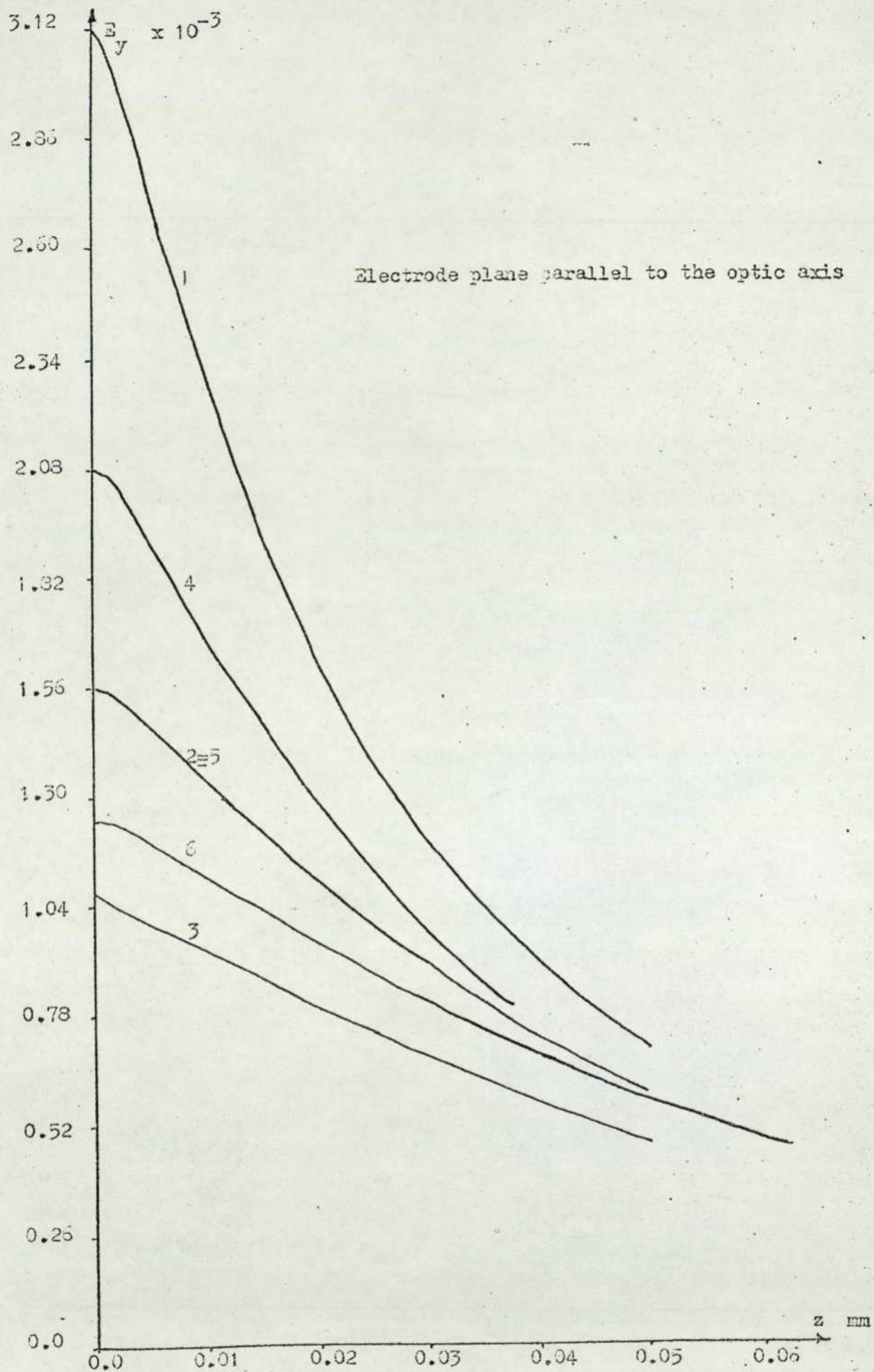


FIG. 4.6.2 Variation of E_y with z
 Anisotropic case $\epsilon_3 = 84.6$ $\epsilon_1 = 23.6$
 Electrodes in the XY plane

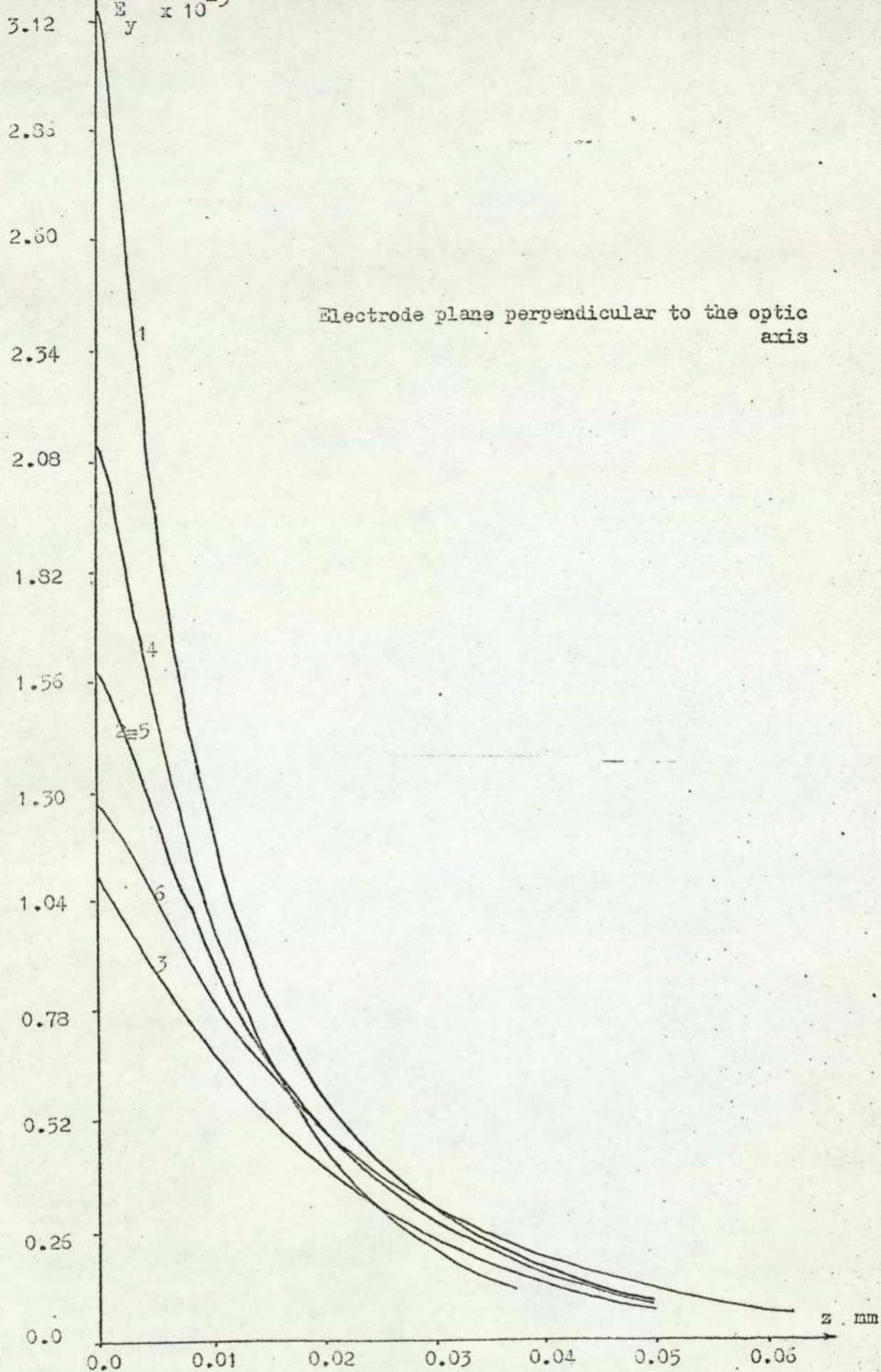


Fig 4.6.3 Variation of E_y with z
 Anisotropic case $\epsilon_3 = 29.6$ $\epsilon_1 = 84.6$
 Electrodes in the XY plane

The variation of E_y with z when the electrode plane is perpendicular to the optic axis is not important for the applications considered in this thesis. Detailed analysis of this case is given in Chapter 3. From the three figures it is apparent that if constant electrode pitch but varying electrode spacing is considered then, at Z equal to zero, E_y increases with a decrease in the spacing ($2ad$). In the plots 4, 2, and 5 where the electrode width is equal to the spacing of the electrodes but the electrode pitch (d) varied, it is seen that at Z equal to zero E_y increases with a decrease in the electrode pitch (d).

Comparing the plots for the isotropic and the anisotropic case where the electrode plane is parallel to the optic axis, even though the magnitudes of E_y at the interface are equal, the drop of E_y with increasing Z is sharper for the isotropic case than for the anisotropic case. However, it is interesting to note that for the electrode configurations studied, in the region of Z less than $d/4$, the magnitude of E_y in the anisotropic arrangement always remains greater than for the isotropic arrangement.

The plots of the different parallel and perpendicular components of the field intensities show that the magnitudes of the components decrease considerably when the distance from the electrode surface is increased. Hence, in order to ensure that the incident beam of light encounters the intense field region, two approaches may be adopted viz. passing the light beam very close to the electrode crystal interface or reflecting the incident beam off the electrode-crystal interface.

CHAPTER 5

INITIAL INVESTIGATION OF THE PERFORMANCE OF THE INTERNAL

REFLECTION MODULATOR

INTRODUCTION

The application of a voltage to an interdigital electrode structure deposited on to the polished face of an electro-optic crystal causes a fringing field to penetrate into the crystal. The detailed investigation of the variation of the magnitude of this field with the distance from the electrodes shows (ref. Chapter 4) that the field intensity drops sharply with an increase in distance from the electrodes. To take advantage of the high field intensity, two different approaches may be adopted. First we allow the incident optical beam to pass in the close vicinity of the electrodes. Alternatively the beam may be allowed to reflect off the crystal electrode interface. The performance of devices operating on both approaches have been reported by earlier workers (ref. Chapter 2). An internal reflection modulator was constructed at the start of this work and its operation tested. After careful alignment and setting up of the device it was apparent that a subdiffraction pattern was present in the far field even with zero applied voltage. This pattern had not been reported in the earlier literature and had apparently been overlooked by these earlier workers. In this chapter we shall identify the causes of the appearance of this subdiffraction pattern and the means of removing this pattern from the far field without degrading the performance of the device.

5.1 Phase changes produced by reflection from an interface

Electromagnetic waves are divided into the reflected and transmitted components when the waves encounter a boundary.

Let us assume that the first medium is a pure dielectric one (ie conductivity equal zero). The second medium will be considered to be either a pure dielectric or a conductor. The characteristics of the reflected and the transmitted wave are a function of the properties of both media.

5.1.1 Reflection at a dielectric-dielectric interface

Consider the case when reflection of electromagnetic waves occurs at a dielectric-dielectric interface, and assume that the refractive indices of the two media are n_1 and n_2 respectively as shown in figure 5.1.

If n_1 is assumed to be greater than n_2 , then by Snells Law, it is possible to have total internal reflection (T.I.R.) of an electromagnetic wave at the interface when the wave

travels from the medium 1 towards medium 2. When T.I.R.

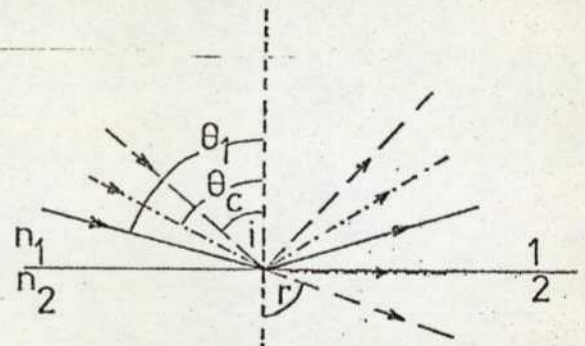


Fig 5-1

Reflection of e.m. waves at
a boundary

takes place at a boundary, the amplitude of the reflected beam is equal to that of the incident beam (ref. Appendix B).

However, the phase of the reflected wave is changed. When T.I.R. occurs, an evanescent wave appears in the second medium, the amplitude of which decays exponentially and no energy is carried away from the interface by these evanescent waves.

Let θ_c be the critical angle for the two media of refractive indices n_1 and n_2 .

If a beam of light is transmitted through the media 1 towards the media 2 at an angle of incidence θ_1 (figure 5.1), and if θ_1 is greater than θ_c , then T.I.R. occurs at the boundary between 1 and 2. If the incident beam is assumed to be polarised with the E vector in the plane of incidence, the phase change produced on reflection may be obtained from the Fresnel's equation as

$$\tan \frac{\delta_1}{2} = \frac{n_1^2}{n_2^2} \frac{\sqrt{n_1^2 \sin^2 \theta_1 - n_2^2}}{n_1 \cos \theta_1} \quad 5.1$$

where $\theta_1 > \theta_c$. Detailed derivation of this equation is given in Appendix B. A similar equation could be derived for the case when the incident optical beam is polarised with the E vector perpendicular to the plane of incidence.

5.1.2 Reflection at a dielectric-conductor interface

Consider the case when medium 1 is again a pure dielectric but medium 2 is now conductive. Maxwell's theory shows that the index of refraction of a conductor may be represented as a complex quantity. Let the complex refractive index of the conductor be n_2 , then

$$n_2 = n + ik \quad 5.2$$

where n and k are the refractive index and absorption coefficient of the metal respectively. Again if the incident wave is assumed to be polarised with the E vector in the plane of incidence, then by substituting this complex index of refraction in the Fresnel's

equation, the phase change produced by reflection at the interface can be calculated (ref. Appendix B). Let δ_2 be the phase change produced on reflection, then

$$\tan \delta_2 = \frac{XY - UV}{XV + UY} \quad 5.3$$

$$X = (n^2 - k^2) \cos \theta_1 \quad Y = 2nk \cos \theta_1 + n_1 B$$

$$U = 2nk \cos \theta_1 - n_1 B \quad V = (n^2 - k^2) \cos \theta_1 + n_1 A$$

$$A^2 - B^2 = n^2 - k^2 - n_1^2 \sin^2 \theta_1 \quad AB = nk$$

and θ_1 is the angle of incidence

A similar equation could be derived for the case when the incident optical beam is polarised with the E vector perpendicular to the plane of incidence.

Equations 5.1 and 5.3 show that the phase changes δ_1 and δ_2 are functions of the angle of incidence. This initial theory indicates that the phase change from a dielectric-dielectric interface differs from that of a predominantly dielectric-conductive interface. If the conductor is arranged in a symmetrical series of strips on the reflecting surface, then a periodic phase profile will be impressed on the reflected beam. This periodic phase front will cause the beam to be diffracted into different orders and in the far field a diffraction pattern will be obtained. Thus it would be expected that in the T.I.R. modulator constructed with a series of conducting electrode strips on the reflecting surface of the crystal, a subdiffraction pattern will be generated in the far field at zero applied voltage.

5.2 Preliminary experiments on reflection at interfaces.

Initial experiments were performed with glass as the dielectric medium to support the conclusions deduced theoretically in section 5.1.2.

A right angled isosceles glass prism with optically flat faces (flatness to $\lambda/4$ where $\lambda = 0.6\mu\text{m}$) was chosen as the optical medium.

Copper interdigital electrodes with spacing equal to the width of the electrodes were deposited on the longest face of the prism

ABC by using photolithographic

techniques. The electrode

fingers were parallel to AB. The

incident beam, polarised with the E vector in the plane of the incidence, was allowed to pass through one short face AC so that the angle of incidence on face AB exceeds the critical angle of the glass-air interface. As shown in figure 5.2, after reflection the beam emerges through the second short face BC. The reflected beam was displayed on a distant screen and was found to consist of diffraction bands. The inter-order angle was equal to $\sin^{-1}(\lambda/d)$ where λ is the incident wavelength ($0.6328\mu\text{m}$) and d is the electrode pitch.

When the electrodes were removed from the face AB and when the light was again reflected off the prism base AB the far field pattern observed was free from any diffraction fringes.

It was noted that when the electrodes were deposited on a substrate and the electrodes brought into close contact with the face AB, no diffraction pattern appeared in the far field when

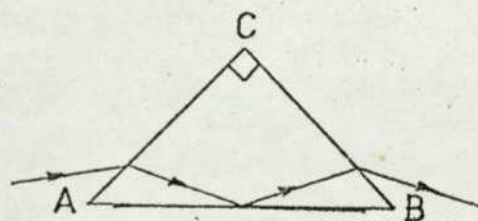


Fig 5.2

Optical path inside a glass prism

the light was reflected off the face AB. The absence of the diffracted orders in this experiment could be attributed to the existence of a small air gap between the electrodes and the prism face so that the incident beam sees only a uniform glass-air interface.

Findings of this preliminary experiment clearly indicate that when the light is incident at an angle greater than the critical angle as calculated for the two dielectric media (glass-air) a periodic phase corrugation is impressed on the reflected beam due to the presence of the interdigital electrodes on the reflecting face, thus causing a diffraction pattern to appear in the far field.

5.3 Investigation of Phase Change produced by reflection of light from a LiNbO_3 crystal-air or LiNbO_3 crystal-metal interface.

The electro-optic medium considered in this thesis is LiNbO_3 , a uniaxial electro-optic material of refractive indices 2.28 and 2.20 for the ordinary and extraordinary rays respectively. If the second medium is assumed to be air, then the critical angle θ_c can be calculated using Snells Law. Although the Snells Law cannot be applied directly to find the critical angle for the extraordinary ray, Snells Law may be used to determine the direction of the (49) wave normal. A negligible error only will be introduced by assuming this angle for the critical angle.

For the ordinary ray $\theta_c \approx 26^\circ$

For the extraordinary ray $\theta_c \approx 27^\circ$

Hence, if the angle of incidence is greater than the corresponding critical angles, the incident beam will be totally internally reflected. However, if interdigital electrodes are deposited on the polished reflecting face, then the incoming beam will now see two

different interfaces namely the crystal-air interface and the crystal-metal interface. Thus a periodic phase corrugation will be impressed on the reflected beam and a diffraction pattern obtained at the far field.

The relative orientations of the crystal and the electrodes structures considered (Chapter 3) are:

- (i) The plane of the electrodes is perpendicular to the optic axis and the incident optical beam polarised with the E vector in the plane of incidence.
- (ii) The plane of the electrodes is parallel to the optic axis and the incident light polarised with the E vector perpendicular to the plane of incidence.

A theoretical analysis of a modulator arranged as (i) above was made. The metal electrodes were assumed to be either copper or chromium. The refractive indices and the absorption coefficients of copper and chromium at $0.6 \mu\text{m}$ are taken to be 0.186, 2.98 and 3.19, 2.26 (50) respectively. The refractive index of LiNbO_3 is taken as 2.2. As shown in equations 5.1 and 5.3 the phase change produced on reflection at an interface is a function of the angle of incidence. The phase change (δ_1 and δ_2) impressed on the reflected wave was computed from these two equations and the variation of δ_1 and δ_2 with angle of incidence θ is plotted in figure 5.3.

It is apparent from this graph that the diffraction effect produced by the interdigital electrodes will only be eliminated when the angle of incidence is either close to the critical angle or near 90° , the grazing incidence. As a measure of the efficiency of an electro-optic diffraction modulator it is usual to measure the voltage required to produce complete extinction of the zero order beam. The theoretical

and practical results show (Section 5.4.4) that the zero order extinction voltage increases with a decrease in the angle of incidence. Hence, in the practical modulator, working with small angles of incidence is not desirable.

Considering the second possibility of working at grazing incidence, this case now reduces to the arrangement where the incident beam of light is directed to travel parallel to the electrodes. This raises the problem of ensuring that the optical beam travels in the active field region of a bulk modulator. Hence, the approach of selectively choosing the angle of incidence to overcome the diffraction effect due to the two periodic interfaces is not suitable in a practical internal reflection modulator. Since the zero order extinction voltage decreases with an increase in the angle of incidence, lower operating voltages will be achieved only when the angle of incidence is large. When the angle of incidence is large but less than 90° , the reflected phase front will be periodically altered thereby causing a diffraction pattern to appear in the far field. Having established theoretically the existence of the phase corrugation impressed by reflection off the face of the crystal carrying the electrodes, we shall now consider the practical significance of this effect.

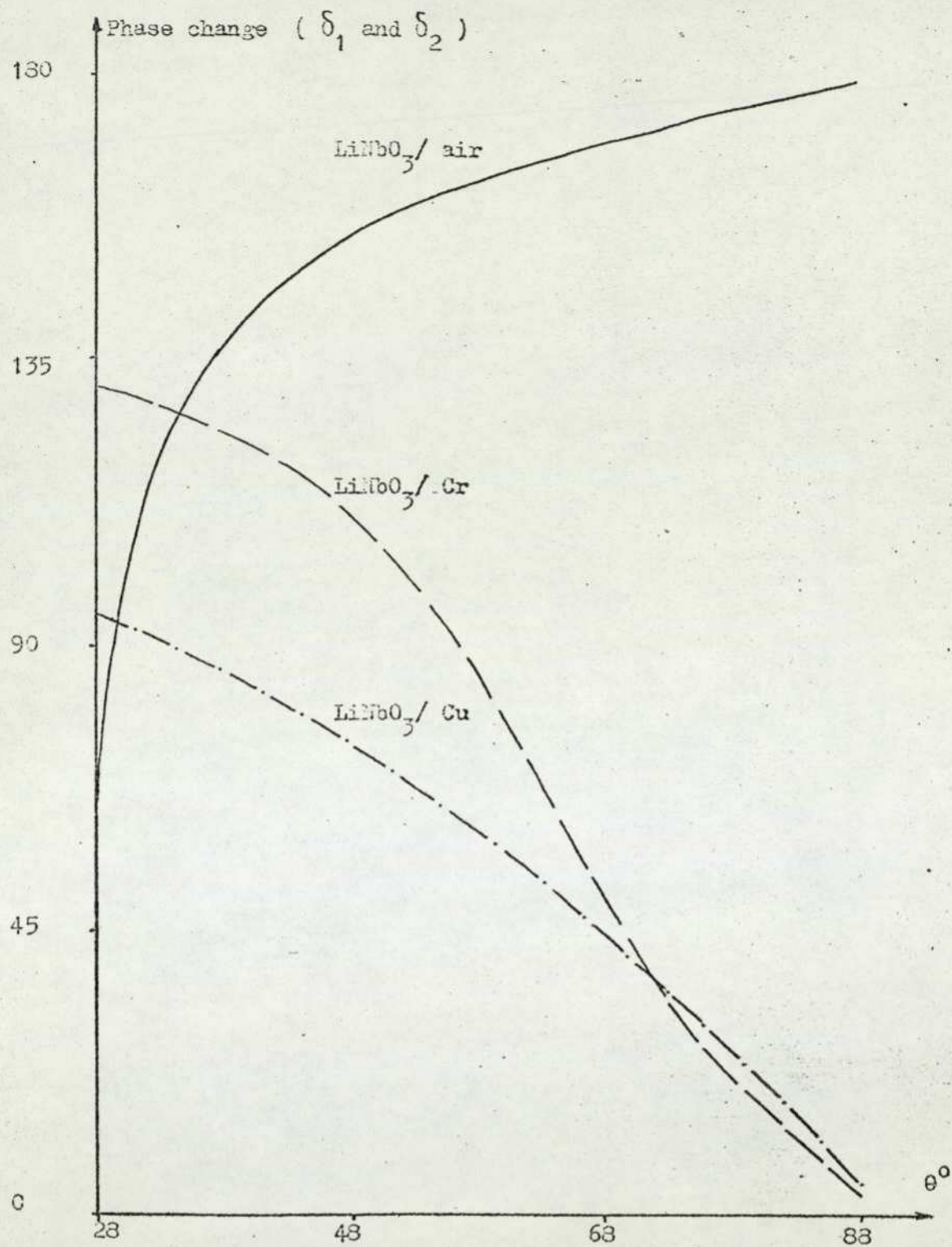


Fig 5.3 Variation of phase change with angle of incidence

5.3.1. Sub diffraction fringes produced by Copper electrodes on LiNbO_3

An electro-optic diffraction modulator was constructed from a Z cut LiNbO_3 crystal. Copper interdigital electrodes of width equal to the electrode spacing were deposited on the polished face (XY) of the crystal by conventional photo lithographic techniques (ref Chapter 9 for detailed sample preparation). The dimensions of the crystal were 4 mmx 3.5 mm x 20.0mm. The electrode fingers were along the X axis which had the longest dimension. The two faces used for light transmission perpendicular to the X axis were also polished. The orientation of the crystal and the electrode structure are represented in figure 5.4.1.

The incident beam of light polarised with the E vector along the plane of incidence was allowed to reflect off the base of the crystal carrying the electrodes. The experimental arrangement is schematically represented in figure 5.4.2. It is apparent from the electrode structure of figure 5.4.1, that when an optical beam of angle of incidence greater than the critical angle is reflected off the face carrying the electrodes, a Y periodic rectangular phase profile is impressed on the reflected beam, and thus the beam is diffracted into different orders. The diffraction pattern obtained in the far field is symmetrical. The energy coupled into the higher order is a function of the relative variations of δ_1 and δ_2 which are functions of the angle of incidence. The angle of incidence was adjusted to be 84° ($\gg \theta_c$) and the far field pattern produced by the reflected beam was carefully studied. A photograph of the far field pattern obtained is shown in figure 5.5.1. The intensity distribution was measured by a movable photomultiplier carrying a straight slit.

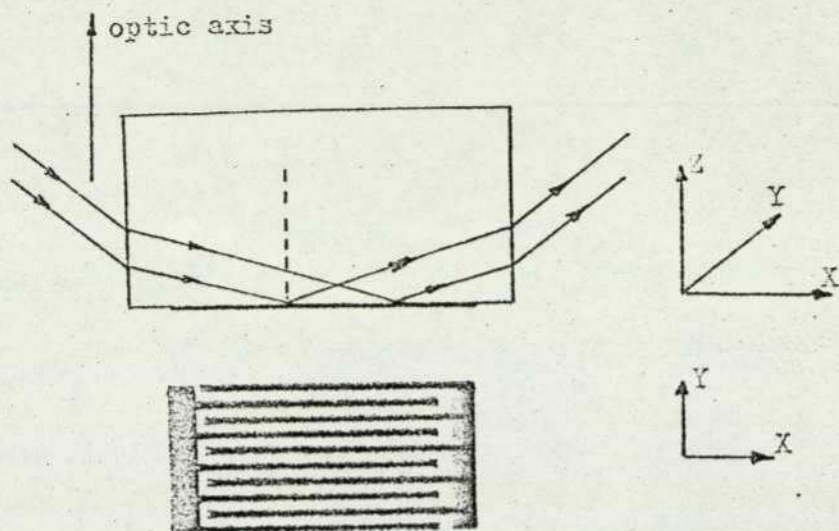


Fig 5.4.1 Modulator configuration

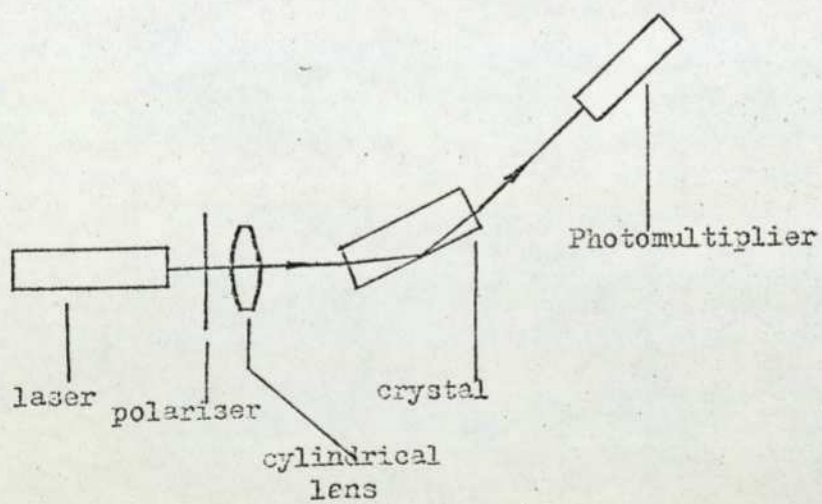


Fig 5.4.2 Optical arrangement

Details of the experimental system is given in Appendix A. The width of the slit was along the Y direction. The photomultiplier was moved along the Y direction and the corresponding intensities (I) were recorded. For the same angle of incidence the incident beam of light was allowed to reflect off a section of the crystal base which was not covered by the electrodes. In this case the far field pattern was free from any diffraction pattern. The photomultiplier was positioned to record the peak intensity (I_0) of the light through the straight slit. Figure 5.5.2 shows the variation of the ratio (I/I_0) with distance along the Y direction. The angular separation of these diffracted orders was equal to $\sin^{-1}(\lambda/d)$ where λ is the incident wave length ($0.6328 \mu\text{m}$) and d is the electrode pitch (0.218 mm).

(The enclosed publication page 84 gives the performance of a similar experimental modulator).

Fig 5.5.1 Far field pattern

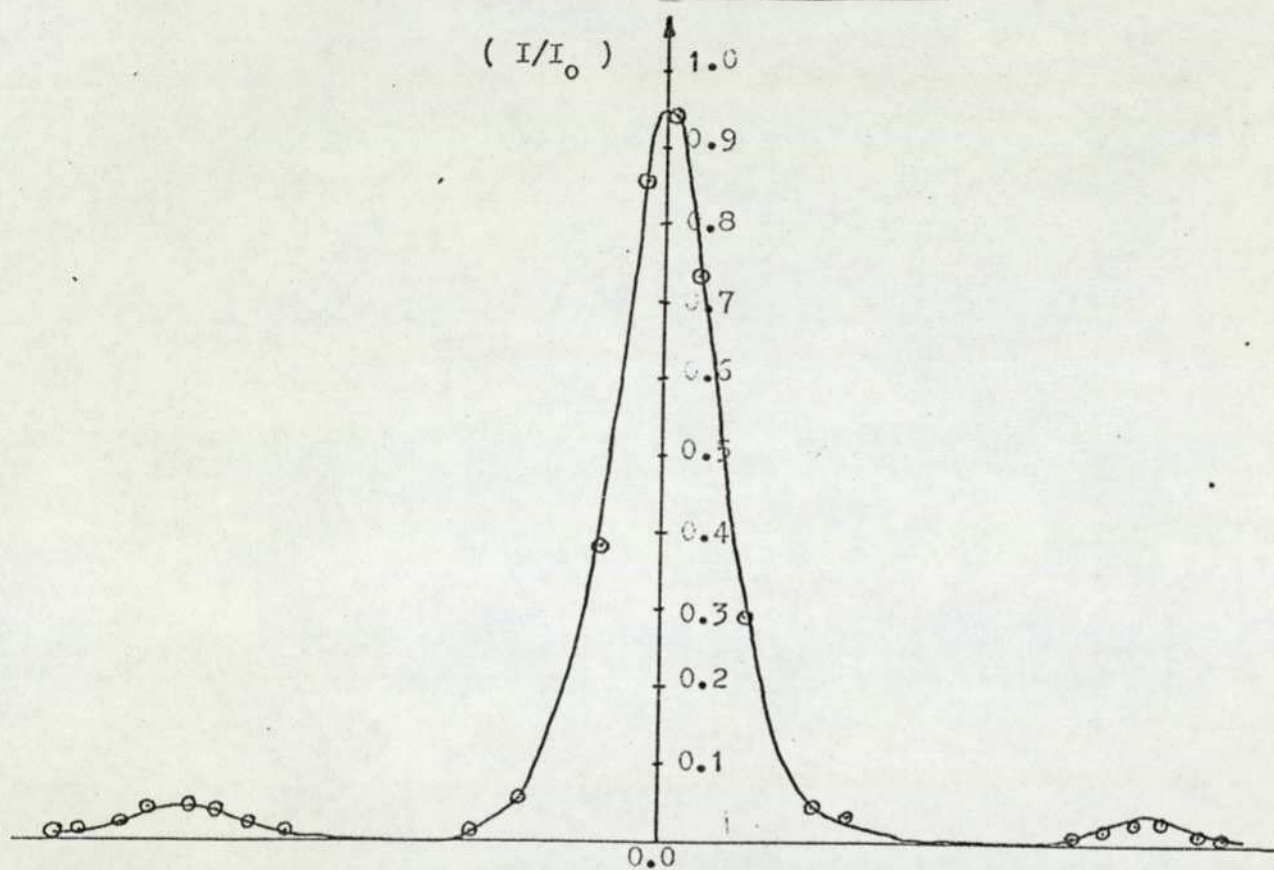
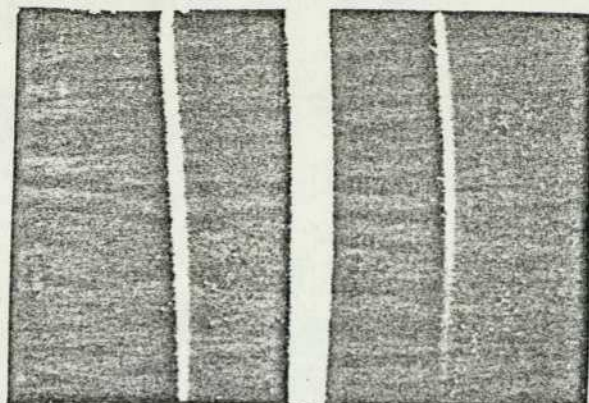


Fig 5.5.2 Far field intensity distribution

Noise generated in the total internal reflection electro-optic diffraction deflector/modulator

V Ramachandran and A W Palmer

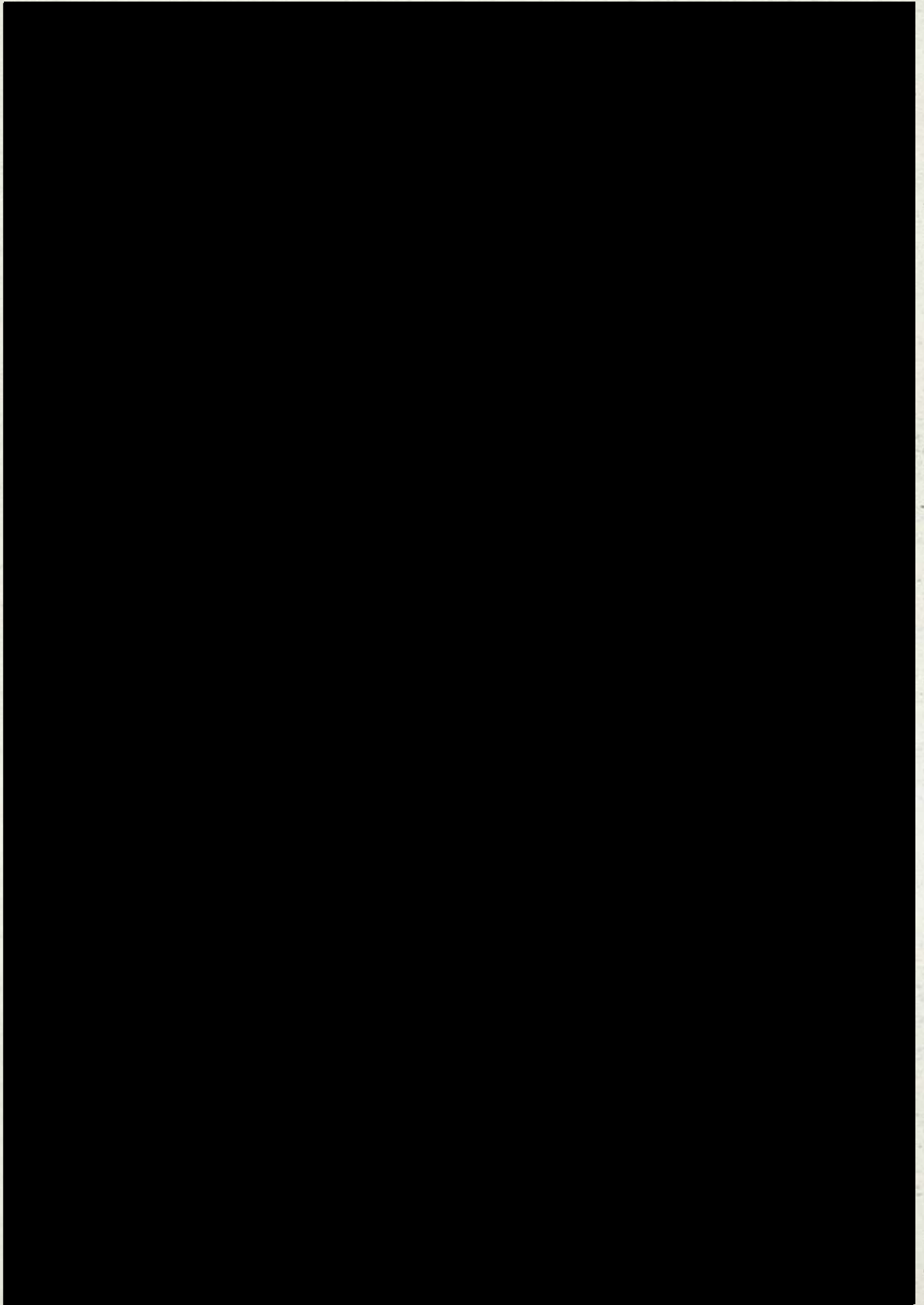
Department of Electrical and Electronic Engineering, City University, St John Street, London, EC1V 4PB.

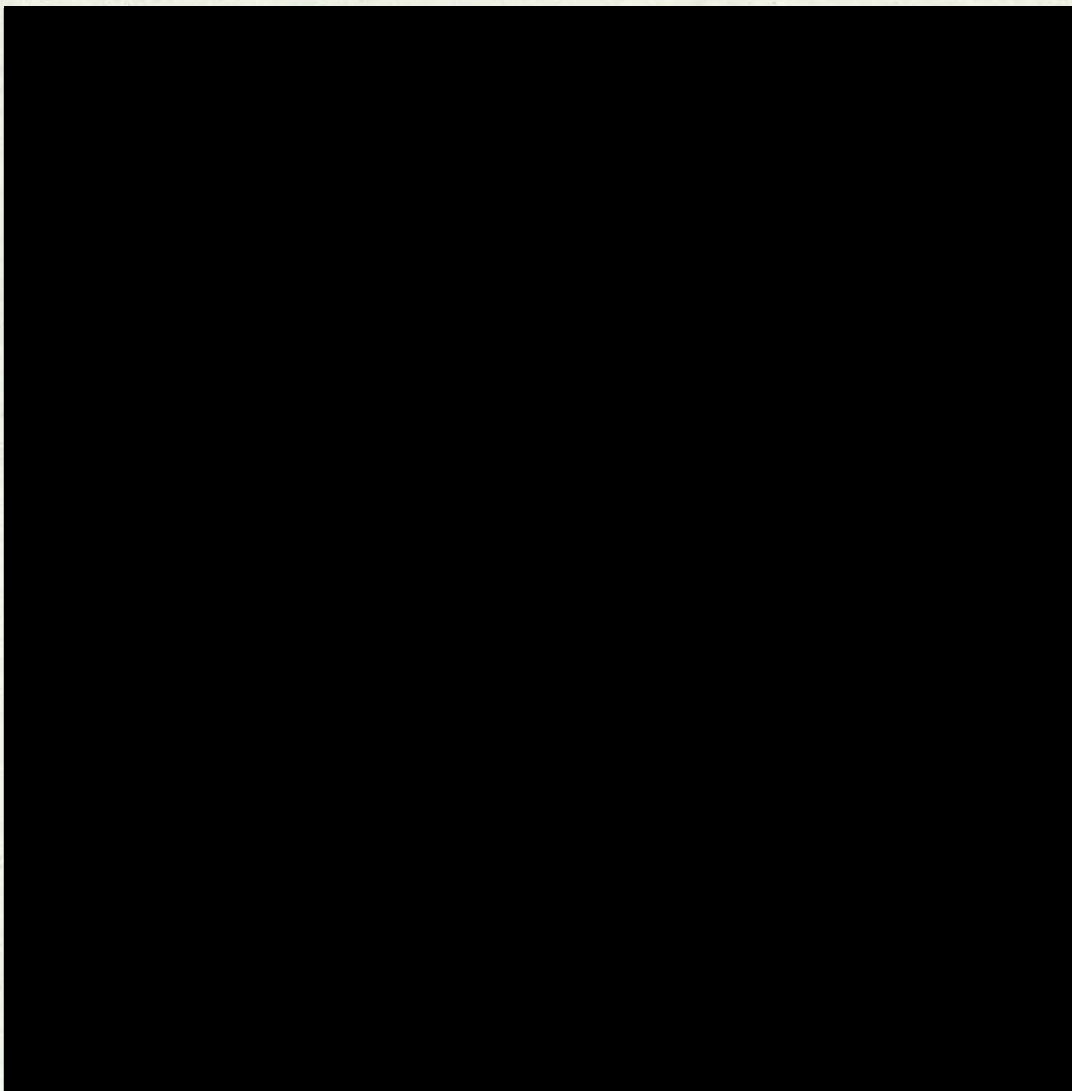
Received 26 July 1974, in final form 18 September 1974

Abstract. The total internal reflection electro-optic deflector/modulator has so far been limited by the subdiffraction patterns that appear in the far field when light is transmitted through the device with zero applied voltage. This paper isolates the causes of these subdiffraction patterns and describes a simple means of removing them from the far field without affecting the performance of the device. The results of measurements taken at zero applied voltage are included and give an indication of the order of this effect.

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The figure 5.5.1 and 5.5.2 clearly indicates the presence of a symmetrical diffraction pattern in the far field at zero applied voltage.

When a voltage is applied to the electrodes a fringing field penetrates into the crystal. The variation of the components of the field along Y axis has already been shown in figures 4.5.1 and 4.5.2 of Chapter 4. In the arrangement so far considered in this section the electrode plane

was perpendicular to the optic axis and the incident light was polarised with the E vector in the plane of the optic axis (ZX plane). The phase change impressed on the reflected beam by the induced field is given by

$$\Delta\phi = -\frac{4\pi}{\lambda} \int_0^t \frac{1}{\tan\theta} \frac{(r_{13} \sin^2\theta + r_{33} \cos^2\theta) E_z - r_{22} E_y \sin^2\theta}{\left[\frac{\sin^2\theta}{n_o^2} + \frac{\cos^2\theta}{n_e^2} \right]^{3/2}} dz \quad \text{--- 5.4}$$

(Ref. Chapter 3 for the detailed derivation of this equation).

Where E_z and E_y are the components of the field along the Z and Y direction respectively and r_{ij} are the electro-optic coefficients and θ the glancing angle. The expressions for E_z and E_y are given by equations 4.25 and 4.26 (Chapter 4) respectively. The change in phase produced by an applied DC voltage of 24 volts was computed and the variation of the phase change with Y is given in figure 5.6. Comparing figure 5.6 and figure 4.4.1 (Chapter 4) it can be seen that the variation of the change in phase follows the mirror image of the contour of the variation of E_z about the Y axis. In the computation of phase change from equation 5.4 it was assumed that the electrodes are uniform along the X direction and that the voltage between the electrodes on the crystal boundary is uniformly distributed. The length of the electrodes is 5.4 mm. The phase

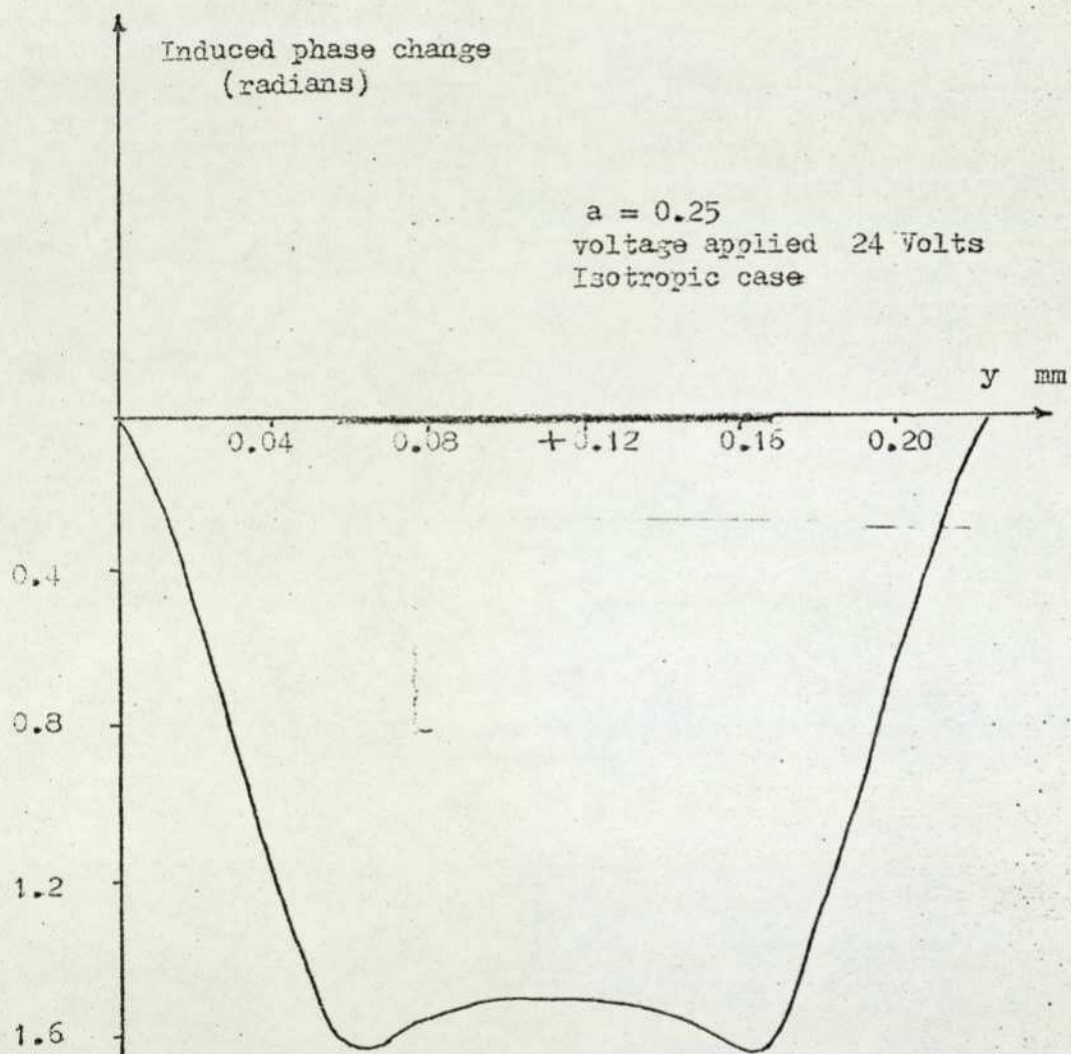


Fig 5.6 Induced phase grating profile

changes impressed on the optical beam by the induced electro-optic effect (A) and by the reflection off the crystal-electrode interface (B) are given in figure 5.7. It is apparent from this figure that the rectangular profile (B) caused by reflection from the non-uniform interface is of half the periodicity of that of graph A.

The diffraction theory indicates that the angular separation of the diffracted order varies inversely with the pitch of the grating. Therefore, the first order band produced by reflection off the crystal-electrode interface should coincide with the second order band produced by the electro-optic effect.

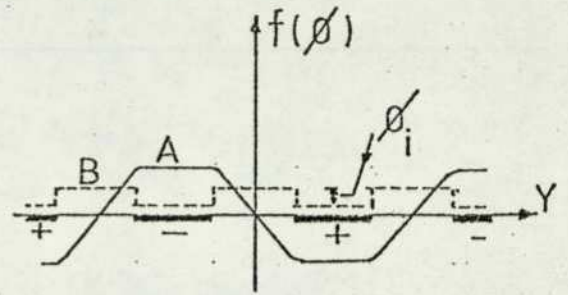


Fig. 5.7
Phase change profiles inside
the crystal

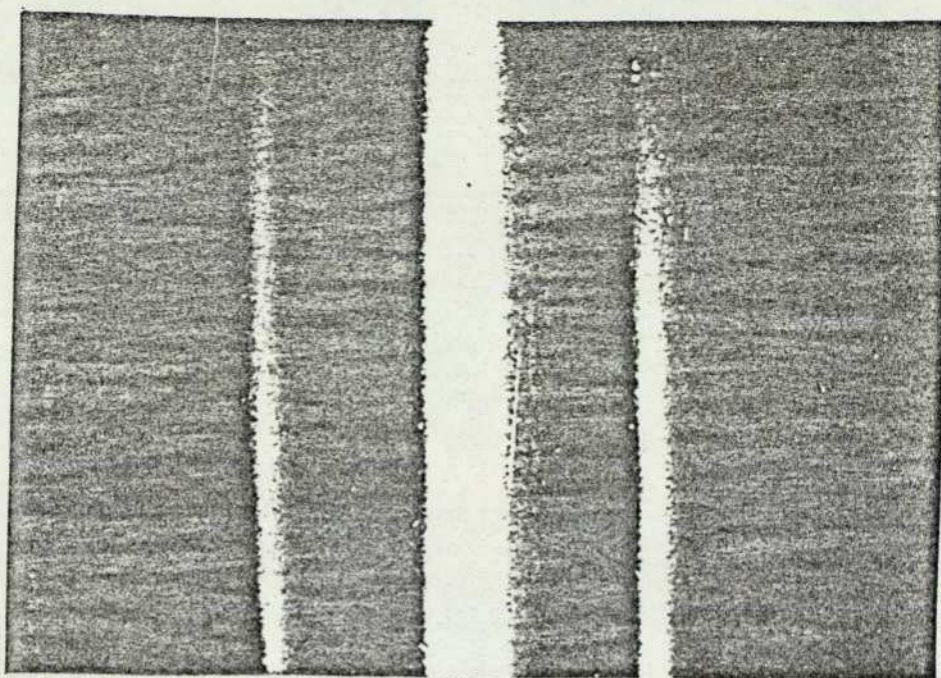


Fig 5.8 a Far field pattern at zero applied voltage

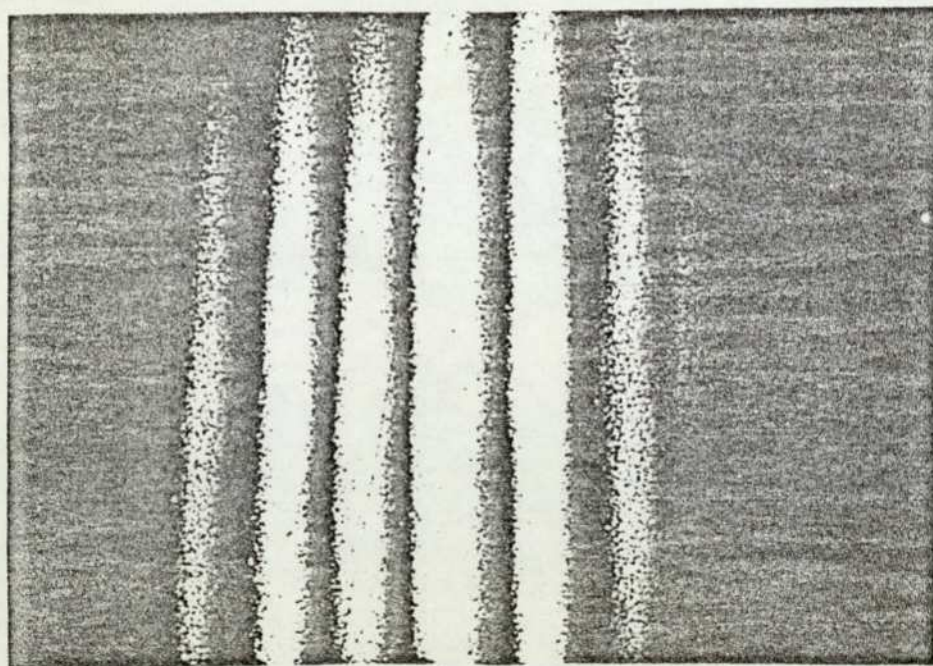


Fig 5.8 b Far field pattern at 24 V applied voltage

Figure 5.8 compares the photographs of the far field pattern obtained before and after applying a voltage (24 volts) to the electrodes. The angle of incidence was again 84° and shows this prediction to be valid.

5.3.4. Variation of the zero order intensity with angle of incidence and with zero applied voltage

From equations 5.1 and 5.3 it is apparent that the phase change impressed on the reflected wave due to the electrodes is a function of the angle of incidence. Consequently the energy coupled to the higher orders also should be a function of the incident angle. At any given angle of incidence, the phase corrugation impressed by the two different interfaces, crystal-electrode and crystal-air, can be represented by a rectangular wave. The magnitudes of the corresponding phase changes can be obtained from figure 5.3. The phase change impressed on the reflected wave on reflection off the crystal base is represented by curve B of figure 5.7. The electrodes are assumed to be uniform and rectangular in cross section. This periodic phase change can be represented mathematically as a Fourier series.

$$f(\phi) = U_0 + \sum_n U_n \cos \frac{2\pi n}{d} y \quad 5.5$$

where

$$n = 1, 2, 3, \dots$$

$f(\phi)$ is the function representing the phase change (Fig. 5.7) and if this function is known, the Fourier coefficients of the series may be calculated.

Let $U_1, U_2, U_3, \dots, U_i, \dots$ be the Fourier coefficients of the square wave profile impressed on the reflected wave. If the reflected wave is diffracted into different orders then the amplitude of the n th order diffracted beam is given by

$$\phi_n = \phi_0 \sum_{k_2, k_3, \dots = -\infty}^{\infty} J_{n-2k_2-3k_3-\dots}(U_1) \times J_{k_2}(U_2) \times J_{k_3}(U_3) \times \dots$$

— 5.6

where ϕ_0 is the amplitude of the undiffracted beam and ϕ_n the amplitude of the n th order diffracted beam produced by a phase grating of any periodic shape. $J_{k_i}(U_i)$ denotes the Bessel coefficient of order k_i and argument U_i . The detailed derivation of this equation is given in Appendix C. This equation is reasonably amenable to computer solution as the higher order Bessel functions are quickly convergent. The first four Fourier components were taken as representing the square wave and the approximation to the square wave that these first four components give is shown in figure 5.9.

These four Fourier coefficients were calculated for angles of incidence between 80° and 89° and by the substitution of these coefficients, the percentage energy lost from the zero

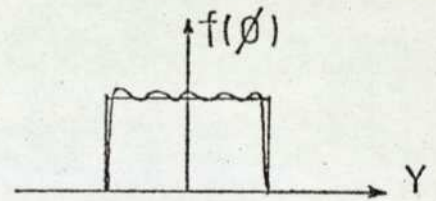


Fig. 5.9

Phase profile synthesised from four Fourier components

order for each angle of incidence was calculated. Curve B of figure 5.10 shows the predicted percentage loss of energy from the zero order.

The intensity distribution was measured by a photomultiplier. The laser light (wavelength = $0.6328 \mu\text{m}$) was directed on to the base of the crystal as described in section 5.3.1. Initially the light was directed on to a section of the base not covered by the electrodes. A straight slit was placed in front of the photomultiplier such that the slit was parallel to the fringe pattern obtained in the far field. The photomultiplier was then adjusted to give a maximum recording

across the section of the beam covered by the slit. This maximum output was assumed to be proportional to the intensity of the undiffracted zero order beam and denoted by I_0 . At the same angle of incidence and with the light beam now directed at the crystal-electrode interface, the photomultiplier was positioned to give the maximum recording of the section of the zero order beam covered by the slit. The maximum energy in the second case is denoted by I . The distance of the photomultiplier from the crystal being kept constant in both cases.

The percentage loss of energy from the zero order, at any angle of incidence was defined as

$$\% \text{ loss} = \left[\frac{I_0 - I}{I_0} \right] \times 100$$

The above measurements were repeated for different angles of incidence. The experimental variation of the percentage loss with the angle of incidence is given in figure 5.10.A. It is apparent from these graphs (figure 5.10A and B) that the predicted variation of the percentage loss of the energy from the zero order was less than that obtained experimentally. This difference between the theoretical and the experimental values may in part be attributed to

- i) The scattering of the incident light by any irregularities on the crystal surface
- ii) The scattering of light by the presence of any foreign particles (photoresist, etching agent, dust)
- iii) The presence of an amplitude grating due to the complex index of refraction of the electrode structure.

The detailed analysis of the contributions by these different factors

is beyond the scope of this thesis and, therefore, we shall restrict ourselves to considering the different means of eliminating the subdiffraction pattern from the far field without degrading the performance of the device.

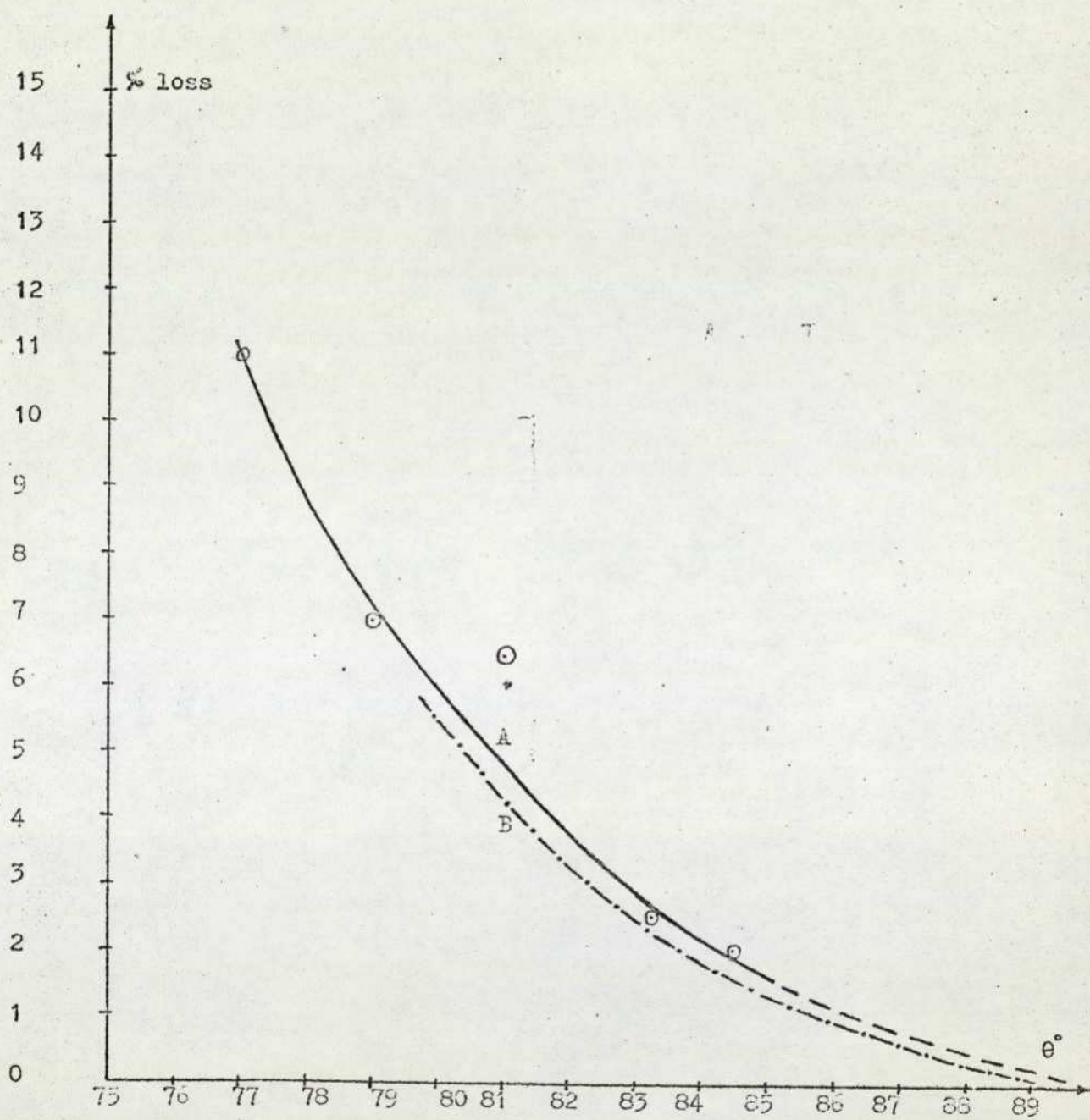


Fig 5.10 Variation of the percentage loss of energy from the zero order with angle of incidence

5.3.5. Variation of the light intensity in the zero order with applied DC voltage

The electro-optically induced phase change which occurs in the interdigital electrode device is proportional to the applied voltage (equation 5.4). Therefore, for a fixed angle of incidence, . . . the energy removed from the zero order will also be a function of the applied voltage. As a measure of the efficiency of this device, the variation of the zero order energy with applied voltage was studied. The intensity was again measured by a photomultiplier arranged as described in the previous section. The experimental arrangement is as shown in Fig. 5.4.2. The voltage V applied to the electrodes was gradually increased and the intensity of the zero order beam was measured by recording the photomultiplier output (I_V). The angle of incidence was 87° . This angle was chosen in order to achieve a compromise between the operating voltage (zero order extinction) and the reasonable ease of aligning the device. The position of the crystal was changed and the incident beam was allowed to reflect off the base of the crystal which was not covered by the electrodes. At the same angle of incidence and with the photomultiplier slit in the same position relative to the zero order the output was recorded as I_0 . Figure 5.11 shows the variation of the ratio (I_V/I_0) is less than unity when the applied voltage is zero. This graph shows that minimum energy in the zero order was obtained with an applied voltage of 83 volts. At this voltage the ratio (I_V/I_0) was about 0.013, that is about 1.3% of the incident light was remaining in the zero order at 83 volts.

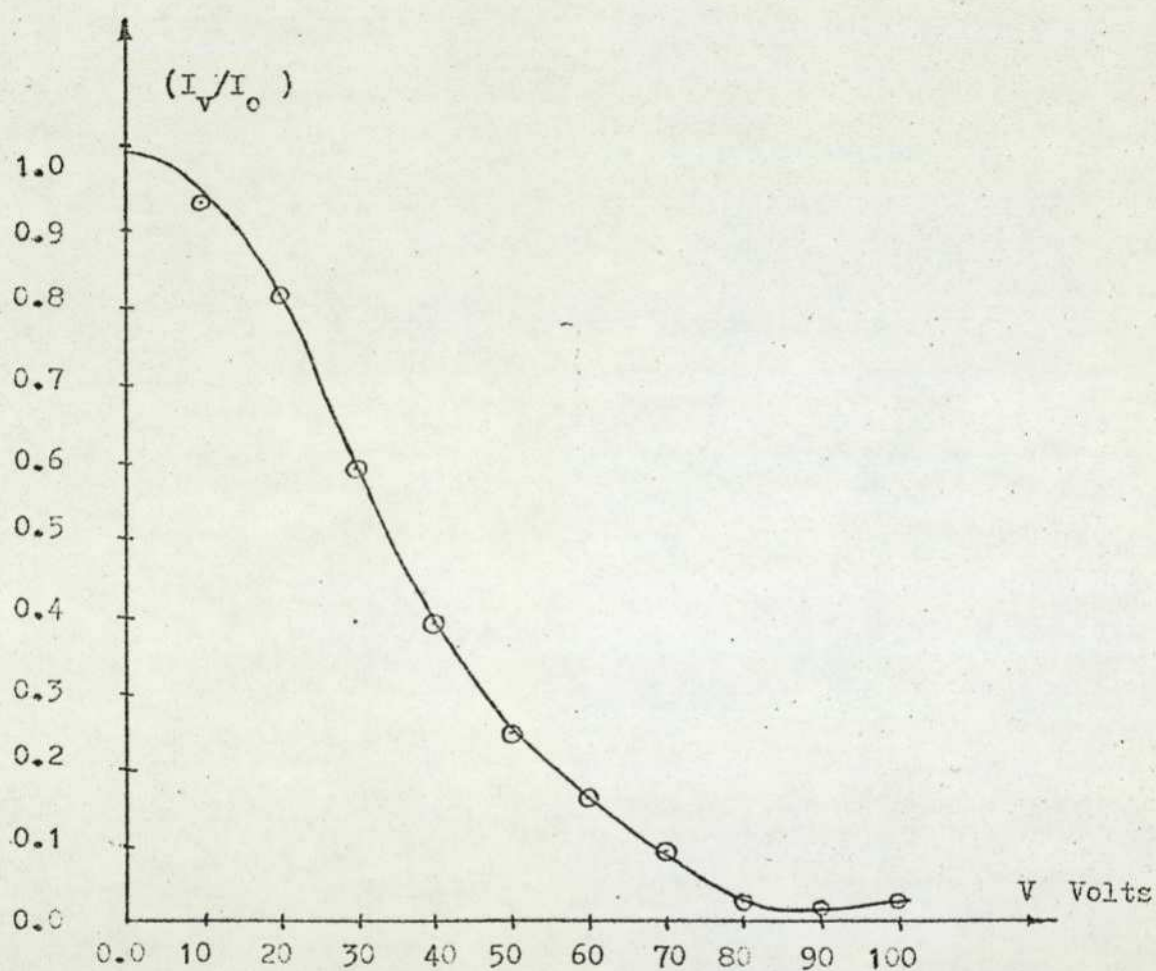


Fig 5.11 Variation of the zero order intensity with voltage

5.4 Elimination of the Sub-Diffraction pattern from the far field

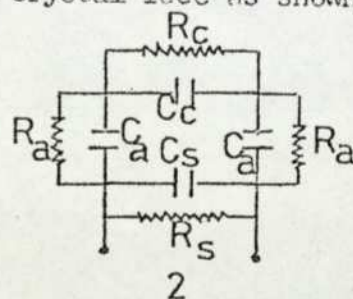
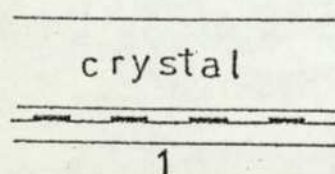
5.4.1. Theoretical considerations

The findings of the experiments described in the earlier section shows that when the beam of light is reflected off the base of the crystal carrying the electrodes, a part of the incident energy is diffracted into the higher orders. The grating effect by the electrodes will be eliminated only when the incident beam of light sees a uniform reflecting interface. Hence, any attempts to remove the subdiffraction pattern from the far field must ensure that the incident beam experiences a uniform interface on reflection.

As seen from the earlier experiment with the glass prism, one method of removing the subdiffraction pattern from the far field is by having the electrodes deposited on to a substrate and bringing the electrodes into close contact with the crystal face as shown in figure 5.12.1.

Fig 5.12

- 1) Experimental arrangement
- 2) Equivalent network



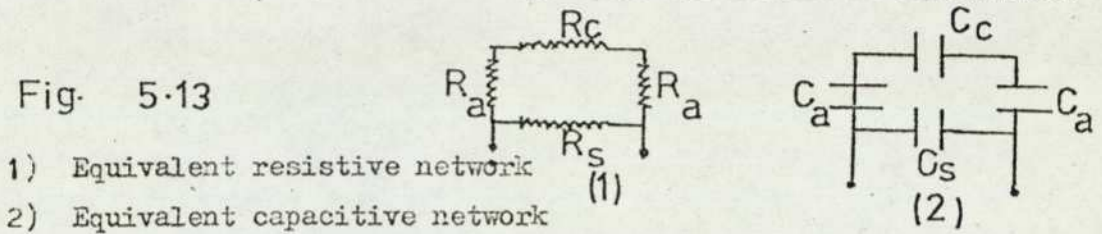
The potential distribution inside the crystal and air gap can be analysed by assuming the equivalent network shown in figure 5.12.2. where C_a , C_s , and C_c are the contribution to the total capacitance between the electrodes. C_a is the capacitance in the air gap between the electrodes, C_s is the capacitance due to the direct air gap and C_c is the capacitance of the crystal between the electrodes respectively. R_a , R_s and R_c are the resistances in parallel with the respective capacitances C_a , C_s and C_c .

In this arrangement the induced electro-optic effect and the the diffraction produced by this arrangement can be conveniently analysed by considering

- i) The zero frequency condition
- ii) The high frequency condition.

When a zero frequency (DC) voltage is applied to the electrodes the resistive network shown in figure 5.13.1 should be considered.

Fig. 5.13



- 1) Equivalent resistive network
- 2) Equivalent capacitive network

Let the voltage applied to the crystal be V_0 .

The potential difference V_c across R_c representing the resistive path in the crystal is

$$V_c = \frac{V_0}{2R_a + R_c} R_c \quad 5.7$$

Similarly, the potential difference V_a across R_a the resistive component representing the air gap is

$$V_a = \frac{V_0}{2R_a + R_c} R_a \quad 5.8$$

Owing to the large differences in the resistivities of air and the electro-optic medium LiNbO_3 , the potential drops drastically in the air gap.

Consequently the electric field induced in the crystal is considerably reduced. In the high frequency case the equivalent capacitive network shown in figure 5.13.2 determines the potential distribution.

If Z_a , Z_c , and Z_s are taken as the impedances of C_a , C_c , and C_s then the potential difference V_c across C_c is

$$V_c = \frac{V_0}{2Z_a + Z_c} Z_c = \frac{V_0}{2C_c + C_a} C_a \quad 5.9$$

and the potential difference V_a across C_a is

$$V_a = \frac{V_0}{2Z_a + Z_c} Z_a = \frac{V_0}{2C_c + C_a} C_c \quad 5.10$$

If the air gap has a thickness of $0.5 \mu\text{m}$, a reasonable minimum distance that could be achieved, then a device having electrode dimensions of the order considered in figure 5.4.1 would have the air gap capacitance C_a appreciably greater than C_c . Therefore, at high frequencies the potential drop in the air gap would be negligible. Hence, this device could be operative at high frequencies and the subdiffraction pattern eliminated.

However, practical difficulties arise in ensuring that the electrode surface is optically flat and maintains a uniform spacing with the crystal face and the spacing is not distorted by foreign particles. If the electrode surface is not uniformly spaced with the crystal face, the potential impressed on the crystal face will vary from point to point and consequently the field distribution inside the crystal will arbitrarily vary. Therefore, if this approach is adopted, it is apparent that the subdiffraction pattern produced by the electrodes is eliminated at the expense of the performance of the device.

Another approach to eliminate the subdiffraction pattern is by depositing a thin film of a different material on the reflecting face of the crystal and then depositing the electrodes on the exposed face of this film. The thickness of this film should be determined by the depth of penetration of the evanescent wave in the second medium (film) i.e. the thickness required to maintain

a uniform phase change on reflection irrespective of the third medium. The third medium referred to being either the electrodes or air. The thin film material can either be a metal or a dielectric. First consider a thin metallic film being deposited on the reflecting face. In metal films, if it is assumed that only a single reflection takes place at the interface, then it can be shown that when the thickness of the film is approximately half the incident wavelength, the phase change produced by the film is equal to that produced by a bulk metal (51,52). On the exposed surface of this film, electrodes of the same metal can be deposited. The wave length of the incident light is $0.6328\mu\text{m}$. Therefore, in order to suffice the required condition the thickness of the metal film should be of the order of $0.3\mu\text{m}$. Even though very thin films of metals possess large resistance, a metallic film of $0.3\mu\text{m}$ thickness will act as a pure conductor giving rise to a short circuit between the electrodes. Hence, this approach is not adoptable in practice.

Alternatively a thin dielectric material can be evaporated on the surface of the crystal and the electrodes could then be deposited onto the film. To obtain T.I.R. off the crystal-film interface, the refractive index of the medium of the film should be less than that of the crystal (Refractive index of LiNbO_3 is 2.2). Another requirement of this medium is that it should be hard and resistant to the etching agents.

The thickness of the film should be of the order of the incident wave length ($0.6\mu\text{m}$) to produce T.I.R. and the incident beam will then see a uniform interface. Known technology with thin film techniques suggests that glass and magnesium flouride (MgF_2) are

well suited for this purpose. The refractive indices of glass and MgF_2 film are 1.5 and 1.39 respectively (50). As MgF_2 thin film deposition does not involve a great deal of complexity, it was chosen as the material to provide the thin film on the crystal base.

5.4.2 Experimental observations with a crystal having a MgF_2 film on the reflecting face

The crystal which was used in the earlier experiments (section 5.3) was again used as the modulator material. The Cu electrodes on the polished XY face of crystal were removed and a thin film of MgF_2 of thickness of the order of $0.6 \mu\text{m}$ was deposited on the reflecting XY face. (Ref. Chapter 9 page 206 for the detailed deposition process of MgF_2). The crystal was then baked in an oven at 80°C for about 12 hours. A thin layer of Cu was deposited on the exposed face of the MgF_2 film. Interdigital electrodes of dimensions equal to that described in section 5.3 was etched on this Cu layer by conventional photolithographic methods (See chapter 9 page 206). The experimental arrangement was again similar to that shown in figure 5.4.2 and the angle of incidence was adjusted to be 84° . This angle of incidence was chosen for comparison purposes. The pattern obtained on a distance screen is as shown in figure 5.14.1. The intensity distribution was measured by a photomultiplier in the manner described in the section 5.3.1 and the variation of (I/I_0) as defined in section 5.3.1 along the Y direction is shown in figure 5.14.2. It is apparent from these figures that the far field pattern obtained is free from any diffraction pattern. Therefore, it is concluded that a thin film of MgF_2 of thickness of the order of the incident wavelength acts as a bulk material when the

angle of incidence is greater than the critical angle θ_c and the incident beam sees a uniform interface. It should be remembered that the angle θ_c is now determined by the two materials LiNbO_3 and MgF_2 rather than previously when θ_c was determined by LiNbO_3 and air. The indices of refraction of MgF_2 and LiNbO_3 are taken as 1.39 and 2.2, and θ_c can now be calculated from Snells Law.

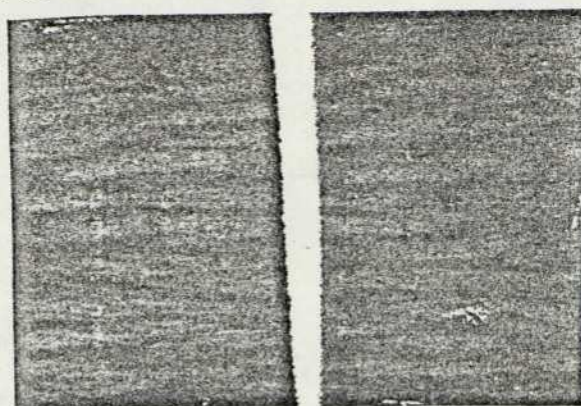
$$\sin \theta_c = \frac{1.39}{2.2}$$

$$\theta_c \approx 38^\circ$$

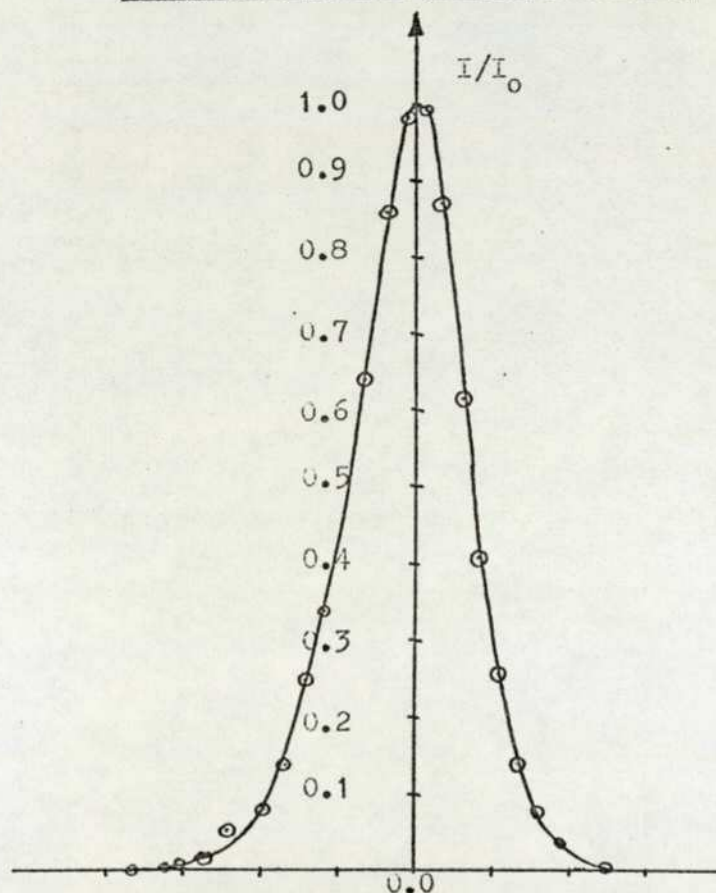
5.11

The performance of this improved device is given in the following two sections.

Fig 5.14 a Far field.pattern (Applied.voltage=0)



(i)



(ii)

Fig 5.14 Far field intensity distribution

5.4.3. Variation of the light intensity in the zero order with d.c. voltage applied to the modified device

As discussed in section 5.3.2 the variation of the intensity in the zero order is a function of the angle of incidence and the applied voltage. First the variation of the zero order intensity with applied voltage at a constant angle of incidence will be studied in some detail. The experimental arrangement is similar to that shown in figure 5.4.2 and the angle of incident was chosen as 87° as in the previous experiment so that the performance of the device without MgF_2 film and the improved device (with MgF_2) could be compared. The photomultiplier with the slit was positioned at the centre of the zero order as described in section 5.3.3. The voltage V applied to the electrodes was gradually increased and the variation of the zero order intensity (I_v) recorded. The crystal was positioned so that the incident beam reflects off the section of the crystal face which was not covered by the electrodes and the intensity I_0 was recorded. The variation of (I_v/I_0) with voltage V of this improved device is shown in figure 5.15 (a). The graph (b) shows the performance of the earlier device without MgF_2 and graph c shows the theoretically predicted variation of (I_v/I_0) with voltage. This has been computed from equations 5.4 and 5.6. In this computation it was again assumed that the total contribution by the first four Fourier components of the phase profile was equal to that of the impressed phase change.

Examination of these graphs show that

- (i) With the device without MgF_2 the ratio (I_v/I_0) at zero applied voltage was less than unity.

- (ii) With the improved device the ratio (I_v/I_o) is equal to unity since no energy is lost to the higher orders at zero applied voltage.
- (iii) The minimum zero order intensity was obtained at an applied voltage of 83 volts with the earlier arrangement without MgF_2 .
- (iv) Complete extinction of the zero order was obtained with the improved device at a voltage of 77 volts.

(It was estimated that the light detector used in the above experiments was capable of recording a minimum ratio of I_v/I_o of at least 0.96%)
- (v) The theoretically predicted zero order extinction voltage was 75 volts.

These observations show that the deposition of the MgF_2 film does not degrade the performance of the device at DC voltages, in fact a small decrease in the zero order extinction voltage was noted. Although it would be reasonable to expect that the insertion of MgF_2 between the crystal and the electrodes would degrade the performance of the device due to the voltage drop across the film in fact an improvement has occurred. A possible explanation is that the presence of MgF_2 on the crystal base provides a uniform resistive layer across the crystal face. Therefore, the variation of the potential across the face is improved giving a more uniform field distribution along the crystal face and an improvement in the shape of the diffraction grating. Of course the validity of the theoretically derived zero order extinction voltage must be questioned as this derivation assumes a uniformly distributed voltage between the electrodes. Without a resistive layer a high concentration of the

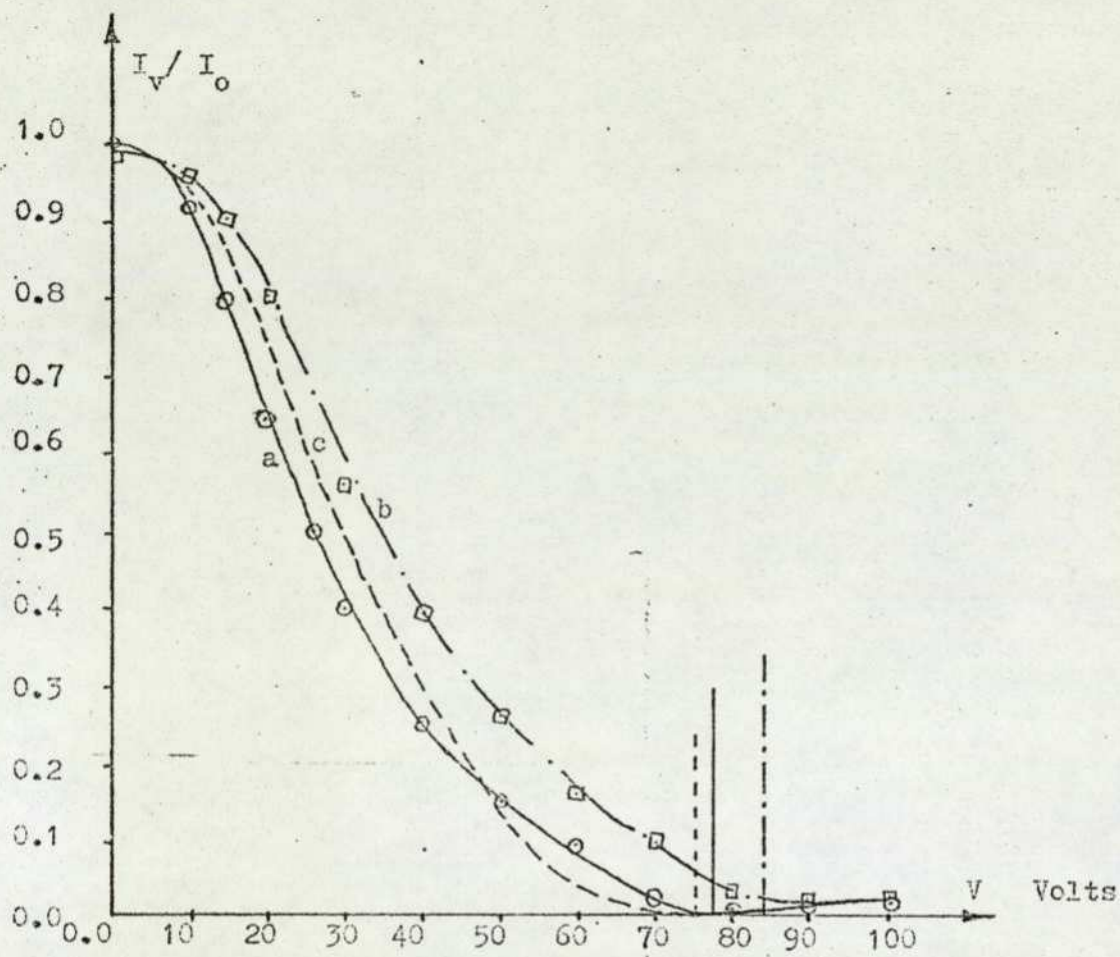


Fig 5.15 Variation of the zero order intensity with applied voltage

- a) Device with MgF_2
- b) Device without MgF_2
- c) Predicted variation

field will occur at the electrode edges and a non linear voltage distribution will be expected to occur across the space between the electrodes.

5.4.4. Variation of the zero order extinction voltage with angle of incidence

From equation 5.4 it is seen that the change in refractive index inside the crystal varies with angle of incidence θ and the applied voltage. Therefore, for each angle of incidence, the zero order extinction voltage will vary. This variation was systematically studied for this improved device. The experimental arrangement was again similar to that shown in figure 5.4.2. The photomultiplier was positioned, as described in the previous sections. The angle of incidence was chosen to be large (in the range of 85 to 89°) and the voltage applied to the crystal to produce complete zero order extinction was measured.

Figure 5.16 shows the variation of the zero order extinction voltage with angle of incidence. This graph shows that the extinction voltage decreased with an increase in the angle of incidence.

Therefore, it is concluded that a low operating voltage modulator is attainable when the angle of incidence is large. However, at large angles of incidence the difficulties in aligning the crystal to ensure that the beam of light reflects off the crystal-electrode interface is increased.

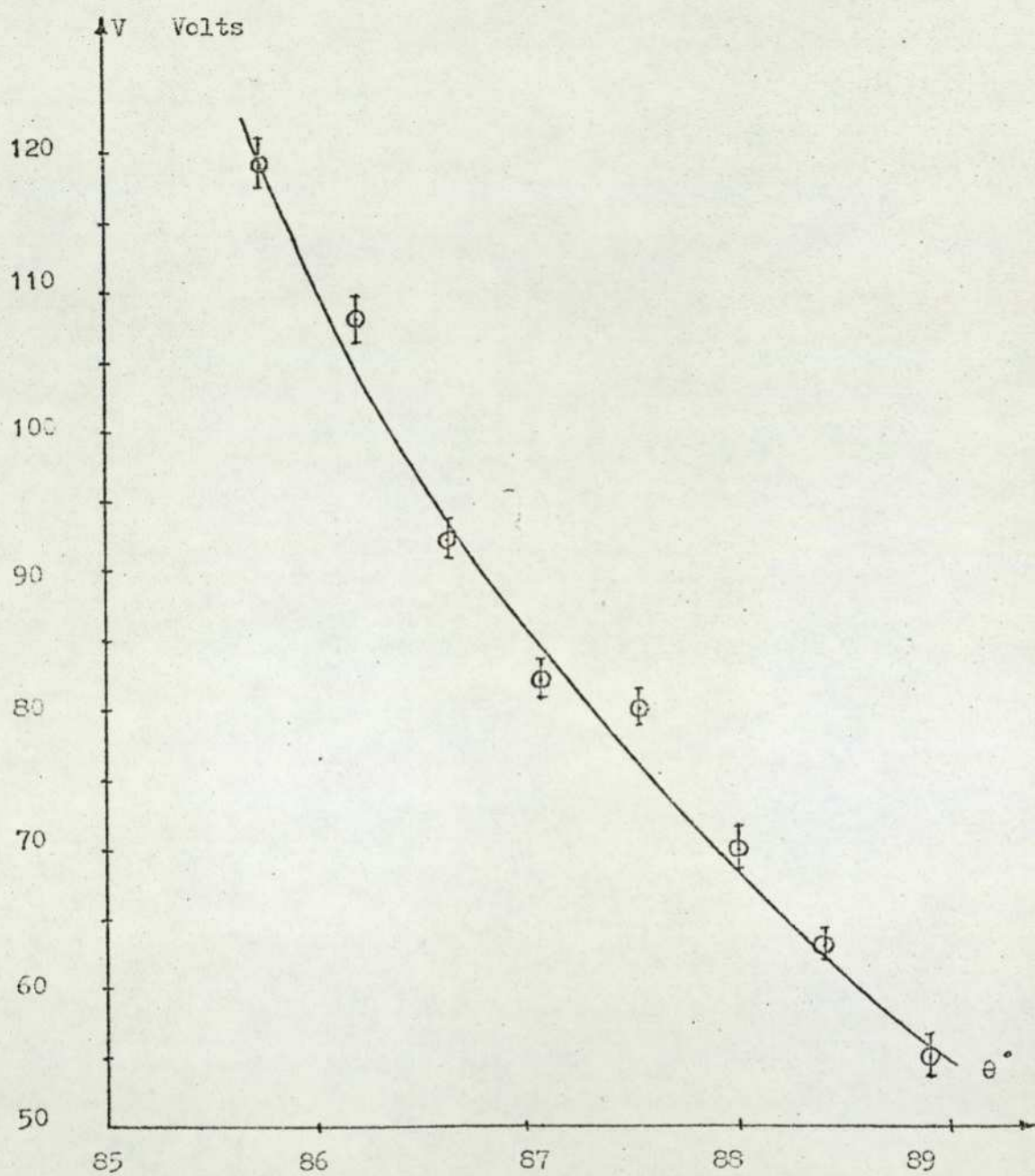


Fig 5.16 Variation of the zero order extinction voltage with angle of incidence

CHAPTER 6

DEFLECTION OF LIGHT USING DIFFRACTION GRATINGS

INTRODUCTION

An optical beam which is incident on a diffraction grating (amplitude, phase or both) will be diffracted into different orders. If the grating element consists of only a sinusoidal phase variation and the incident light is normal to the axis of the phase variation, then the far field pattern produced by this grating is of the Raman-Nath type and the diffraction pattern is symmetrical about the zero order. However, if the diffraction grating is blazed or the incident light is at oblique incidence to the axis of phase variation, then the diffracted energy is not symmetrically distributed about the zero order. In this chapter, the different beam deflection techniques using diffraction gratings are analysed in some detail.

A novel form of voltage controlled blazed grating using the electro-optic crystal LiNbO_3 is discussed and the performance of this deflector under d.c. conditions is included in this chapter.

6.1.Theoretical considerations of light beam deflection techniques

An optical beam normally incident to a sinusoidal phase perturbation will undergo Raman-Nath type of diffraction and the diffracted orders will be symmetrical about the zero order. The total number of resolvable spots produced on diffraction is a function of the angular spread of the optical wave vector and the spread of the diffracting perturbation(3).

The different diffracted beam positions provide different points for addressing (Fig.6.1). There-

fore, this type of grating could be used as a multiport switch.

However, as the diffracted beam is symmetrical about the zero

order, this device suffers dupli-

cation of the far field information. If this device is operated in the Bragg mode, that is if the optical beam is incident to the perturbation at the appropriate angle (the Bragg angle) a large percentage of the incident energy is deflected to a single higher order position. Alternatively the light may be deflected to a higher order position by appropriately blazing the grating.

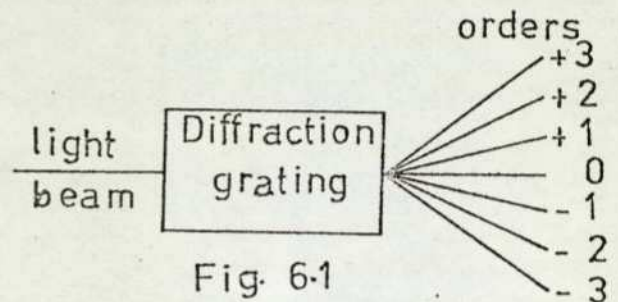
In the following sections we shall consider the different methods for beam deflection by utilising a diffraction grating.

6.1.1. Conventional Blazed gratings

Diffraction gratings may be classified into two categories

- i) The amplitude gratings
- ii) The phase gratings

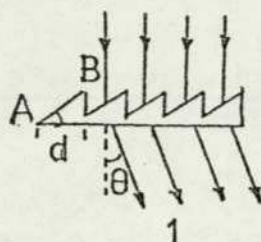
There are two general subtypes of each of these two categories; transmission gratings and reflection gratings. Reflection gratings are almost always phase gratings. These gratings can be blazed and the two types of gratings are illustrated in figures 6.2.1 and



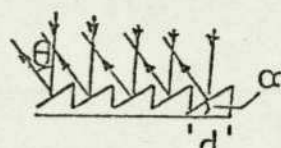
Multiport switch

6.2.2 respectively.

Fig. 6.2



Transmission grating



Reflection grating

These diffraction gratings are conventionally produced by ruling a medium (transparent medium for the transmission gratings and opaque and reflecting medium for the reflection gratings) with a special edge (eg. the natural edge of carborundum crystal or a diamond). The cut faces are optically flat and are at an angle

α to the flat face of the media as shown in the above figures.

First consider the transmission blazed gratings. Assume that the refractive index of the medium is μ . In this arrangement the blazing angle α is chosen so that the deviation produced when the optical beam travels through the grating is equal to the angle of diffraction of one of the orders.

If the angles α and θ are assumed to be small, then

$$\sin \alpha \approx \alpha$$

$$\sin \theta \approx \theta$$

and applying Snells law

$$\theta = (\mu - 1)\alpha \quad 6.1$$

$$\text{If } \theta \text{ satisfies the condition } d \sin \theta = n\lambda \quad 6.2$$

where d is the grating spacing, λ the incident wavelength and n is the diffracted order number, then the beam will be deflected to the n th order (53).

In order to determine the far field intensity pattern, it is convenient to analyse the blazed grating using the Fourier Transform.

The transmission grating can be imagined as a large rectangular

aperture with a periodic phase change across it. The diffraction pattern obtained is similar to that of a rectangular slit but spatially shifted to a diffracted order position (53). Figure 6.3

shows the shape of this diffracted pattern drawn for the case of $n=1$. It can be seen from equation 6.1 that θ is a function of the refractive index of the medium and this index is a func-

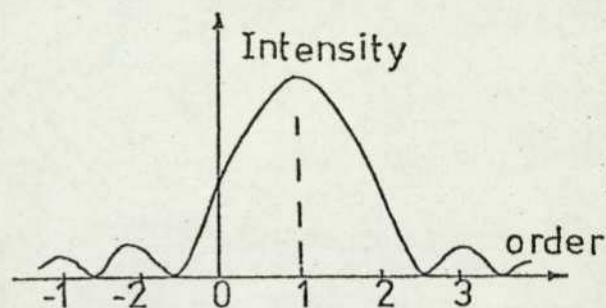


Fig. 6-3 Intensity distribution

tion of the incident wave length. Hence, this type of transmission grating can only be used to deflect energy for a particular incident wavelength.

Considering the reflection grating, in order to deflect the energy from the zero order to the n th order, θ should satisfy the equation

$$n\lambda = d \sin \theta \quad 6.3$$

$$\text{and } \theta = 2\alpha \quad 6.4$$

Since the angle of reflection is not a function of the incident wavelength, the reflection blazed gratings are more versatile and a spectrum of wavelengths may be deflected to one of the higher order positions.

It can be seen from the above four equations that when α is determined, it is possible to deflect the energy only to one of the higher order positions.

6.1.2. Bragg diffraction

An optical beam will undergo Raman-Nath type diffraction when the diffraction element consists of a sinusoidal phase variation and the incident optical beam is perpendicular to the axis of phase variation (2). However, the incident beam undergoes Bragg diffraction

when the angle between the incident wavefront and the axis of phase variation satisfies the Bragg reflection law. In this case, constructive interference of the beam occurs, and in the far field a particular order intensity will be dominant. When the incident beam is not perpendicular to the axis of the phase variation, and not at the Bragg angle, there can be other types of diffraction which cannot be strictly called Bragg type diffraction (54)

Consider the constructive and destructive interference of optical rays passing through a sinusoidally perturbed phase grating. The phase grating is assumed to be transparent for the incident wavelength concerned.

In figure 6.4 let the horizontal lines represent the plane of equal phase points inside the medium.

The separation (or the periodicity) of these lines is 'd' and the width of the perturbation is taken as 'w'. $A_1 D_1$ and $B_3 B_3''$ are the incident and the reflected wavefronts respectively. The incident rays make the glancing angle ϕ

with the lines representing the plane of equal phase points (phase fronts). Let the reflected rays travel at an angle θ to the phase fronts. The conditions under which constructive interference occurs in the θ direction may now be examined in four special cases

i) Considering the scattering points B_2 and A_2 ,

if

$$\theta = \phi$$

6-5

then the path lengths $A_1 A_2 A_3$ and $B_1 B_2 B_3$ are equal. Therefore,

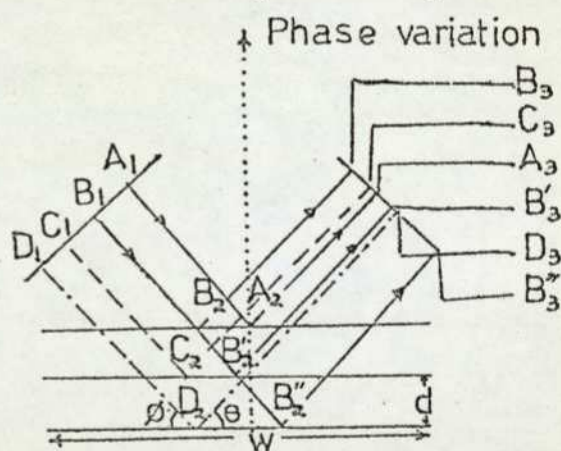


Fig. 6.4

Diffraction/Deflection of light
by a sinusoidal phase variation

there will be reinforcement of all scattered rays from this plane. This is of course similar to the mirror reflection (partially reflecting mirror).

ii) Consider the scattering of a single wave $B_1 B_2$. The different scattering points are considered as B_2, B'_2, B''_2 etc. Constructive interference will occur in this case if path lengths $B_1 B_2 B_3, B_1 B'_2 B'_3, B_1 B''_2 B''_3$ are equal, or the difference between them is $n\lambda$ where n is an integer order number and λ is the incident wavelength. From trigonometric construction it can be shown that the above condition is satisfied when

$$n\lambda = \frac{[1 - \cos(\phi + \theta)]}{\sin \phi} \quad 6.6$$

and thus constructive reinforcement will occur in the θ direction. The more phase fronts the light beam passes in the transparent medium, the greater will be the reinforcement and, hence, the intensity increases. This will occur when 'd' is small and 'w' is large.

iii) Let us assume that the above two conditions (equation 6.5 and 6.6) are satisfied simultaneously. Substituting equation 6.5 in 6.6 gives

$$\frac{n\lambda}{d} = 2 \sin \phi \quad \text{Bragg condition} \quad \text{---6.7}$$

Under these conditions, the intensity of the n th order is very much greater than the intensity obtained in the first two cases. Again it should be remembered that the intensity will be large when the optical beam passes through a large number of phase fronts. In particular, if the incident beam passes through less than two phase fronts, the second type (ii) of diffraction cannot exist and, hence no Bragg diffraction can exist.

iv) In this last case, consider that the width of the perturbation 'w' is less than $C_2 B_2^1$. Then a light ray, for example $B_1 B_2$ or $C_1 C_2$ will only cross a small portion of the perturbation and the scattering will occur only at such points as B_2, C_2, D_2, B_2^1 etc. If the path lengths $B_1 B_2 B_3, C_1 C_2 C_3, D_1 D_2 D_3$ differs by $n\lambda$ reinforcement of the waves takes place, and in the far field, intensity domination occurs in the θ direction. The condition then satisfied is

$$\frac{n\lambda}{d} = (\sin \phi + \sin \theta) \quad 6.8$$

Again in this case the energy on reinforcement will be less than that in (iii). Therefore, it may be concluded that the intensity of the deflected beam when the Bragg condition is satisfied is very much greater than that obtained in the other three cases.

An optical beam may be deflected to the different orders when the phase change impressed by a grating is blazed. In the two arrangements discussed in the two sections, the required phase change was produced by

- a) Physically blazing the grating (6.1.1)
- b) By directing the optical beam at an oblique incidence to the phase fronts (6.1.2).

In the first arrangement in order to deflect the light from an order to another order, the blazing angle should be changed. In the second arrangement, the deflection of the beam is obtained by varying the angle of incidence. In an arrangement where acoustic wave (or microwave) inside the medium produces the sinusoidally perturbed diffraction element, the frequency of the acoustic wave (or microwave) can be altered to produce deflection of the beam to higher order

positions. Again the Bragg condition should be satisfied for efficient beam deflection.

The use of an electro optic crystal to produce a blazed grating has, therefore, been considered as an alternative technique. The deflector of this type appears to offer the only possible method of conveniently switching a high proportion of the incident energy to a large number of different positions.

Previous workers (39, 40) have attempted to obtain an approximation to the phase profile (blazed phase change) of figure 6.5 by applying voltages to suitably arranged electrodes deposited on the electro-optic crystal. The techniques used by these earlier workers has been described in chapter 2. One of these deflectors, proposed by Yuichi Ninomiya (40) employed an array of electro optic prisms. It can be seen in this reference that the construction of the diffraction element (the array of prisms) will be difficult and expensive to manufacture. Another method described in chapter 2 is due to Bright and Wilson (39). These workers synthesised a saw tooth phase change by having a cascaded electrode structure. We shall consider the potential of

this technique further. Consider the blazed phase profile of figure 6.5. This may be expressed in a Fourier series as

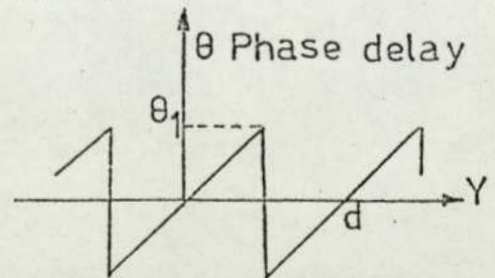


Fig. 6.5

Ideal sawtooth

$$\theta(y) = \sum_{m=1}^{\infty} U_m \sin \frac{2\pi m}{d} y \quad 6.9$$

where d is the periodicity of the phase profile and U_m is the amplitude of the m th harmonic component

The diffraction pattern corresponding to $\theta(y)$ is then given by the Fourier Transform as

$$a(u) = \int_{-\infty}^{\infty} \exp(i\theta(y)) \exp(-iuy) dy \quad 6.10$$

where $U = k \sin \alpha$, $k = 2\pi/d$ and α is the angle of diffraction (53).

When the phase delay θ_1 is equal to $n\pi$ ($n = 1, 2, 3, \dots$) then the diffracted pattern contains a maxima at the n th order i.e. the n th order intensity is dominant (55, 56). However, in a practical electro-optic deflector, the phase change impressed by the electrically induced phase grating only approximates to the ideal saw tooth phase change.

Consider an optical beam of amplitude ϕ_0 diffracted by a diffraction grating of an arbitrary shape. The amplitude of the n th order diffracted beam is given by

$$\phi_n = \phi_0 \sum_{k_2, k_3, \dots = -\infty}^{\infty} J_{n-2k_2-3k_3-\dots}^{(U_1)} \times J_{k_2}^{(U_2)} \times J_{k_3}^{(U_3)} \times \dots \quad \text{--- 6.11}$$

where $U_1, U_2, U_3, \dots$ are the Fourier coefficients of the phase profile impressed on the transmitted beam and $J_{k_i}^{(U_i)}$ denotes the Bessel coefficient of argument U_i and order k_i (detailed derivation of equation 6.11 is given in Appendix C).

If the phase delay θ_1 of figure 6.5 is equal to π then the amplitudes of the harmonics are determined by equation 6.8 and have the values

$$\begin{aligned} U_1 &= 2.0 & U_2 &= -1.0 \\ U_3 &= 0.667 & U_4 &= -0.5 \end{aligned}$$

Consider that the first (i.e. fundamental) component of the ideal saw tooth profile is impressed on the transmitted beam. Then the first order amplitude $(\phi_1)_1$ is given from equation 6.11 as

$$(\phi_1)_1 = \phi_0 J_1^{(U_1)} \quad 6.12$$

This is equivalent to impressing a sinusoidal phase change.

Let the intensity of the undiffracted and the first order beam be

I_0 and $(I_1)_1$ respectively

then

$$\frac{(I_1)_1}{I_0} = [J_1(U_1)]^2 \quad 6-13$$

The intensity is assumed to be proportional to the square of the amplitude. If it is assumed that the contributions by the fundamental and the first harmonic components are impressed on the transmitted beam, then the intensity of the first order beam $(I_1)_2$ is given by

$$(I_1)_2 = I_0 \left[\sum_{k_2=-\infty}^{\infty} J_{1-2k_2}(U_1) \times J_{k_2}(U_2) \right]^2 \quad 6-14$$

Similar expressions can be obtained for the inclusion of an increasing number of Fourier components of the ideal sawtooth. These equations are reasonably amenable

to the computer and Table 6.1

summarises the percentage of energy coupled into the first order when the impressed phase change constitutes an increasing number of the Fourier components

of the ideal sawtooth. These results are obtained by computer compilation. The results were calculated by using the programme given in Appendix D.

TABLE 6.1

No. of components	% intensity in the 1 st order
1	33.2
2	55.4
3	65.8
4	72.7

The approximation to the saw tooth which will be obtained by the inclusion of these Fourier components is given in Figure 6.6.

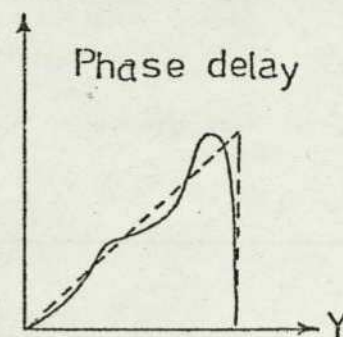


Fig. 6-6

Approximation of the sawtooth profile

Wright and Wilson have reported

that in their device (39), the first three components of the ideal sawtooth were obtained by applying voltages to three independent electrode structures deposited on the electro-optic medium LiNbO_3 . As discussed in chapter 2, the electrode lengths, spacings, and the individual voltages applied to the electrodes were adjusted to give maximum intensity of the first order beam. It is reported that this maximum intensity was 58% of the incident energy.

The advantage of the electro-optic deflectors over the conventional blazed gratings or the Bragg deflectors is that the phase change impressed on the optical beam can be varied by varying the applied voltages. Hence, the beam could be deflected to any order by applying the correct voltages to the electrodes.

From Table 6.1 it can be seen that an increasing percentage of the incident energy can be coupled to the first order when an increasing number of Fourier components contribute to the phase change. This could be achieved by having a large number of individual electrode sets to produce the required contribution to the different Fourier components. However, practical difficulties arise in fabricating a large number of electrodes on a crystal face. Another limiting factor is the diffraction effect occurring inside the crystal due to the optical beam arriving at the later electrode stages

after having already suffered diffraction by the electrodes at the earlier stages. For these reasons, an entirely new approach to the problem was considered during the work undertaken in this thesis and is reported in the following section.

6.2. The development of a new electro-optic deflector

The diffraction effects produced by the application of a voltage to a symmetrical interdigital electrode structure were discussed in the previous chapter. In this section we shall discuss the effect of asymmetrically arranging the electrodes on an electro-optic crystal to produce the blazed grating effect.

Consider the electrode arrangement of figure 4.1 (Chapter 4) which is deposited on the polished XY face of LiNbO_3 crystal (figure 6.7). From equation 4.25 and 4.26, the expressions for E_z and E_y components of the field are given by

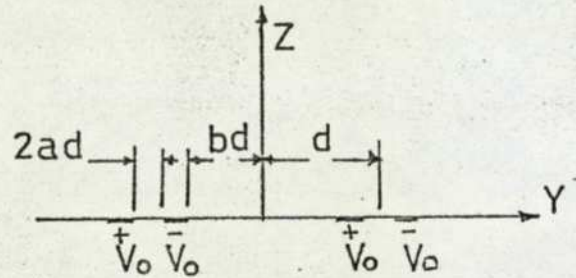


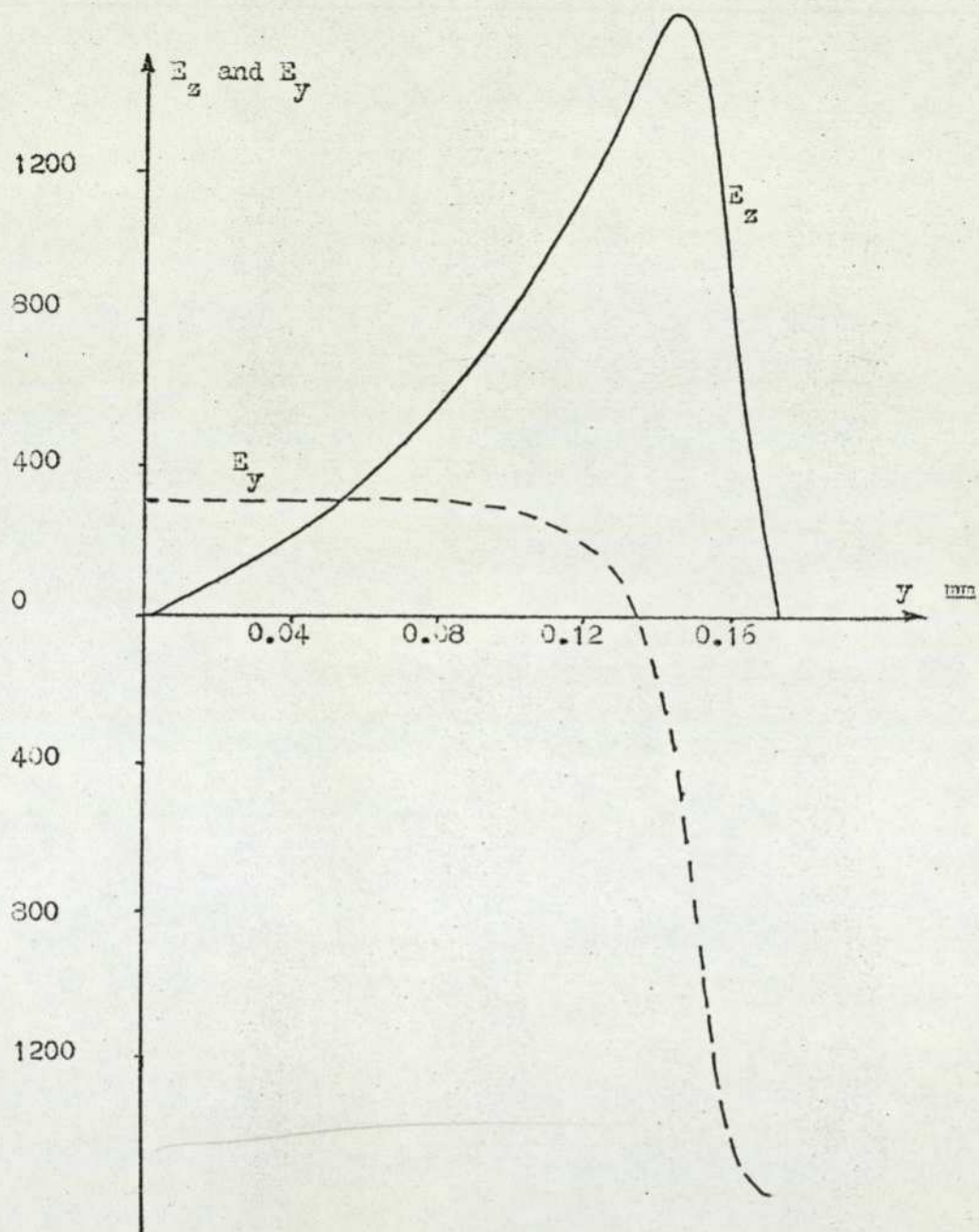
Fig. 6.7

Asymmetrical electrode arrangement

$$E_z = 2V_0 \sum_n \left[\frac{\sin(bn\pi)}{n\pi} - \frac{\cos(n\pi)\sin(an\pi)}{n\pi} \right] e^{-\frac{n\pi}{d}z} \frac{n\pi}{d} \sin \frac{n\pi}{d} y \quad \text{---6.15}$$

$$E_y = -2V_0 \sum_n \left[\frac{\sin(bn\pi)}{n\pi} - \frac{\cos(n\pi)\sin(an\pi)}{n\pi} \right] e^{-\frac{n\pi}{d}z} \frac{n\pi}{d} \cos \frac{n\pi}{d} y$$

where
 $n = 1, 2, 3, \dots$ ---6.16



$$2ad = 0.045$$

$$2bd = 0.275 \text{ mm}$$

$$2d = 0.344 \text{ mm}$$

$$z = 0.005 \text{ mm}$$

Fig 6.8 Variation of E_z and E_y with y

These equations are applicable in the zero frequency conditions (ref. Chapter 4). The variation of E_z and E_y along the Y axis can now be computed from these equations. Taking 2ad, 2bd and 2d as equal to $45 \mu\text{m}$, $275 \mu\text{m}$ and $344 \mu\text{m}$ respectively, the variation of the components of the electric field were computed and are represented in figure 6.8. These dimensions were chosen so as to coincide with those of the experimental arrangement. The applied d.c. voltage was taken as 100 volts. These components were calculated at a distance of $10 \mu\text{m}$ from the interface ($z = 10 \mu\text{m}$).

From these plots it can be seen that the variation of the E_z component of the electric field follows an approximate sawtooth shape which is not the case with the E_y component. Therefore, a modulator was constructed so that the variation of the refractive index inside the crystal is governed by the E_z component of the field which will impress a sawtooth phase corrugation on the transmitted beam.

In the case of LiNbO_3 , when the optic axis is parallel to the Z axis and a voltage is applied to the electrodes, the total change in the path length (ΔL_z) is given by

$$\Delta L_z = - \frac{1}{\sin \alpha} \int_0^{\frac{1}{\tan \alpha}} \frac{(r_{13} \sin^2 \alpha + r_{33} \cos^2 \alpha) E_z - r_{22} \sin^2 \alpha E_y}{\left[\frac{\sin^2 \alpha}{n_o^2} + \frac{\cos^2 \alpha}{n_e^2} \right]^{3/2}} dz \quad \text{---6.17}$$

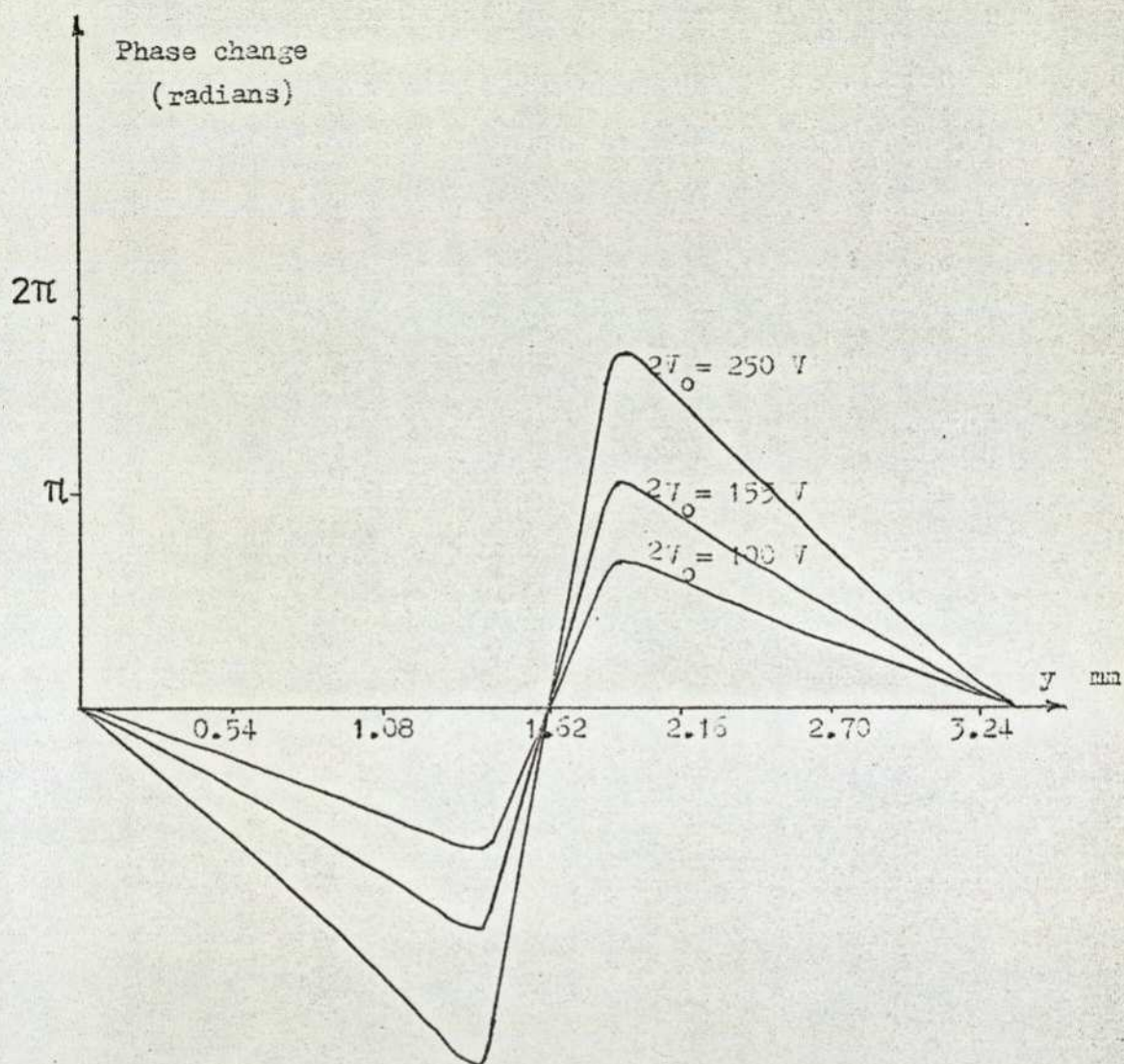


Fig 6.9 Variation of phase change with y (varying V_0)

where r_{ij} are the electro optic coefficients, α is the glancing angle (Fig.6.11.3) and $2l$ is the length of the electrode.

From this expression it can be seen that since r_{13} , r_{22} are small relative to r_{33} , the variation of ΔL_z along the Y axis approximately follows the contour of the mirror image of the shape of E_z about the Y axis.

The phase change impressed on the transmitted beam is proportional to the change in path length. The phase change $\Delta\phi$ impressed on the beam is given by

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta L_z \quad 6.18$$

Figure 6.9 shows the computed variation of the phase change $\Delta\phi$ along the Y axis obtained from equation 6.18. In this computation (see 6.17) the length of the electrodes l and the glancing angle α were taken as 3.5 mm and 4.75° . From equation 6.17 it is seen that the phase change is a function of the applied voltage. Therefore, to study the variation of $\Delta\phi$ with voltage, the computations were performed for different applied voltages. Graphs i, ii, and iii of figure 6.9 correspond to voltages of 100, 155 and 250 volts respectively. It can be seen from these plots that the shape of the phase change profile closely approximates to the sawtooth shape. Therefore, a modulator built with a LiNbO_3 crystal having the optic axis along the Z direction and electrodes deposited asymmetrically on the XY plane of the crystal gives the possibility of producing a blazed grating.

From figure 6.5 which shows the ideal sawtooth, when the phase delay ϕ_1 is equal to $n\pi$ where n is an integer, the maximum amount of energy will be deflected from the incident beam to the n th

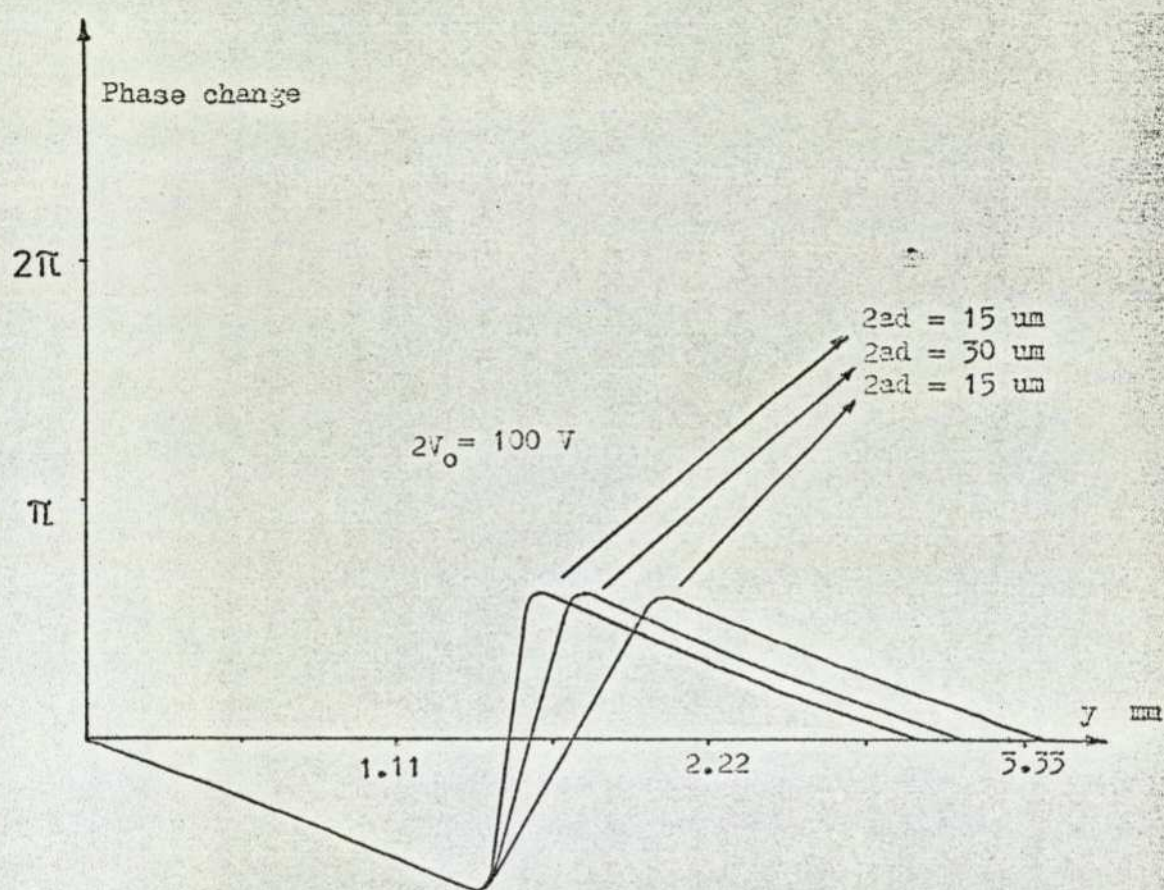


Fig 6.10 Variation of phase change with y (varying $2ad$)

order. The plots in figure 6.9 show that when the applied voltages are 155 and 250 volts, the phase delay θ_1 is approximately equal to π and 2π respectively. Hence, with applied voltages approximately equal to these values, it would be expected that the maximum energy should be deflected to the first and the second orders respectively. The variation of the phase change impressed by electrode structures with varying electrode spacing ($2ad$) was studied theoretically and figure 6.10 shows this variation along the Y axis. The graphs a, b and c show the variation of the phase change for $2ad$ equalling 60, 30 and 15 μm respectively. The magnitude of $2bd$ was kept unaltered (275 μm). These graphs indicate that a better approximation of the sawtooth shape is obtained when the spacing $2ad$ is small. Therefore, in order to achieve a reasonable approximation of the sawtooth shape of figure 6.5 the electrode spacing $2ad$ should be decreased. The minimum separation of the electrodes that could be achieved is determined by

- i) The breakdown of the dielectric and/or the breakdown across the surface between the electrodes.
- ii) The feasibility of etching electrodes with narrow spacings.
- iii) The depth of penetration of the electric field into the crystal.

i) Electrical breakdown

A limitation that will prevent the reduction of the electrode spacings with a consequential increase in electric stress, will be the ultimate breakdown between the electrodes. This breakdown may be due to the surface breakdown between the electrodes or to the breakdown in the crystal or to the breakdown of the media separating the electrodes.

In the device developed, in which the electrodes were deposited on the face of the crystal and separated in the air, breakdown would most probably occur either in the air or across the crystal surface. In the present form of this device, it would be expected that the breakdown strength may be greatly increased by coating the electrodes and the crystal base with an insulating layer.

ii) Etching of electrodes with narrow spacing

The electrode structures considered in the experimental work were produced by conventional photolithographic techniques. The processes used in the preparation of the electrode structures are discussed in detail in chapter 9. It is expected that such preparation in a highly controlled manner will ultimately result in a resolution of the order of a few microns. The minimum practical separation obtainable with the available facilities at this University is possibly of the order of ten micrometres. (Experimental value $45\text{ }\mu\text{m}$). However, with modern techniques such as electron beam scanning (57) it is possible to obtain submicrometre resolution.

iii) Penetration of the electric field

A detailed analysis of the penetration of the electric field inside the electro-optic crystal (LiNbO_3) is given in Chapter 4.

6.3 Experimental performance of the novel diffraction deflector

In order to study the performance of the deflector developed on the principle described in the previous section, a device was constructed using the electro-optic crystal LiNbO_3 . The optic axis of the crystal

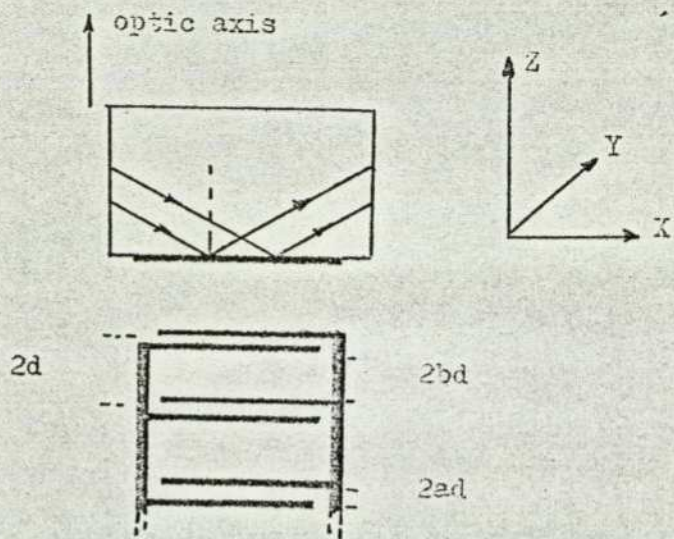


Fig 6.11 a Deflector configuration

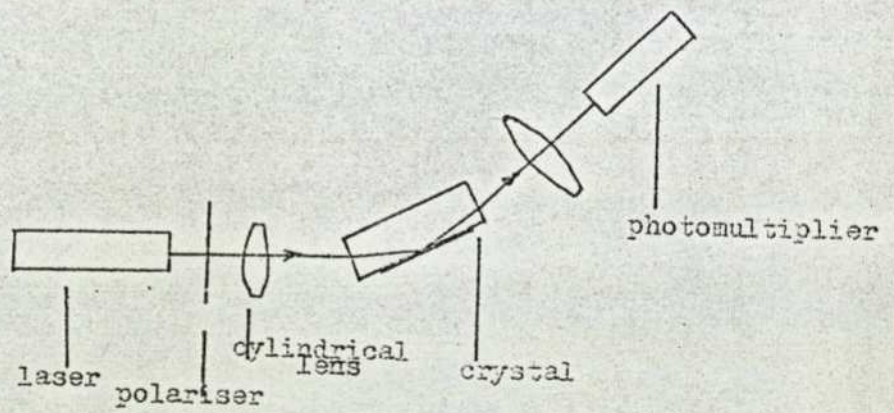


Fig 6.11 b Optical arrangement

LETTER TO THE EDITOR

A novel electro-optic diffraction deflector

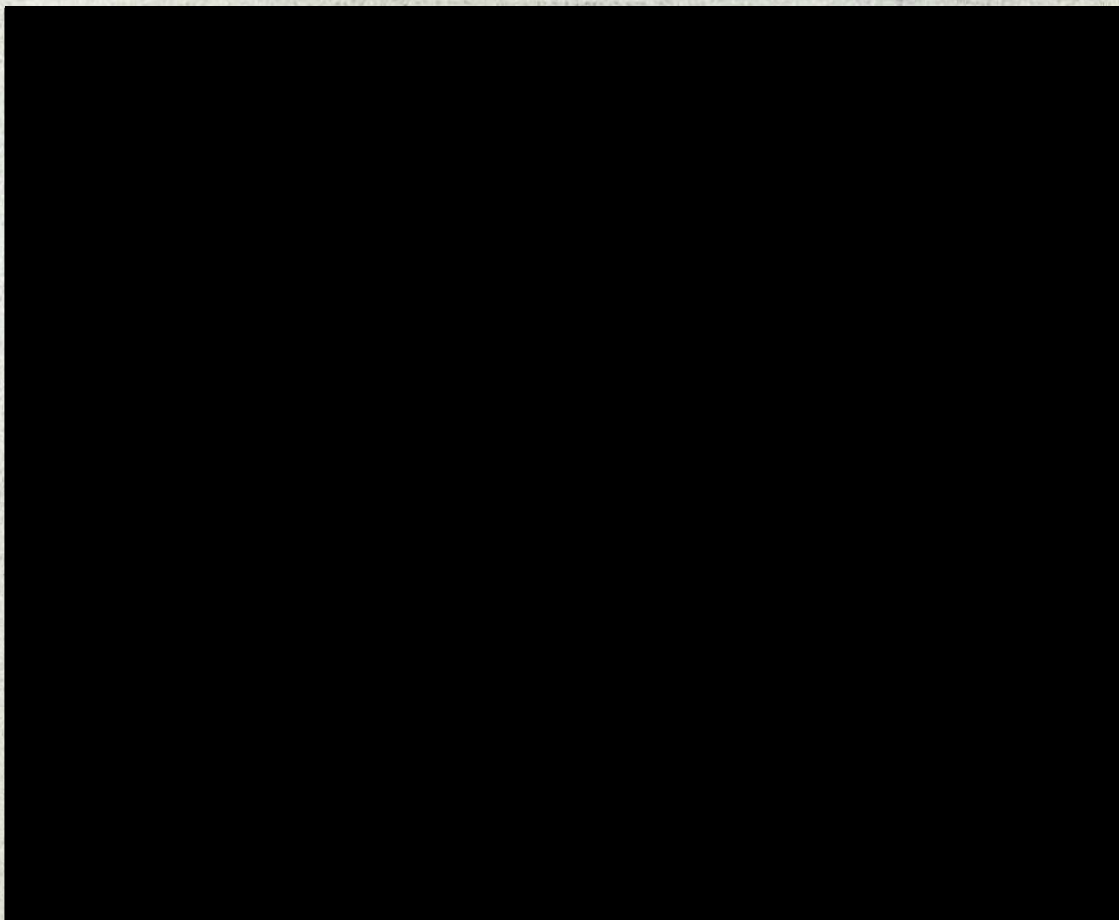
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Received 18 March 1975

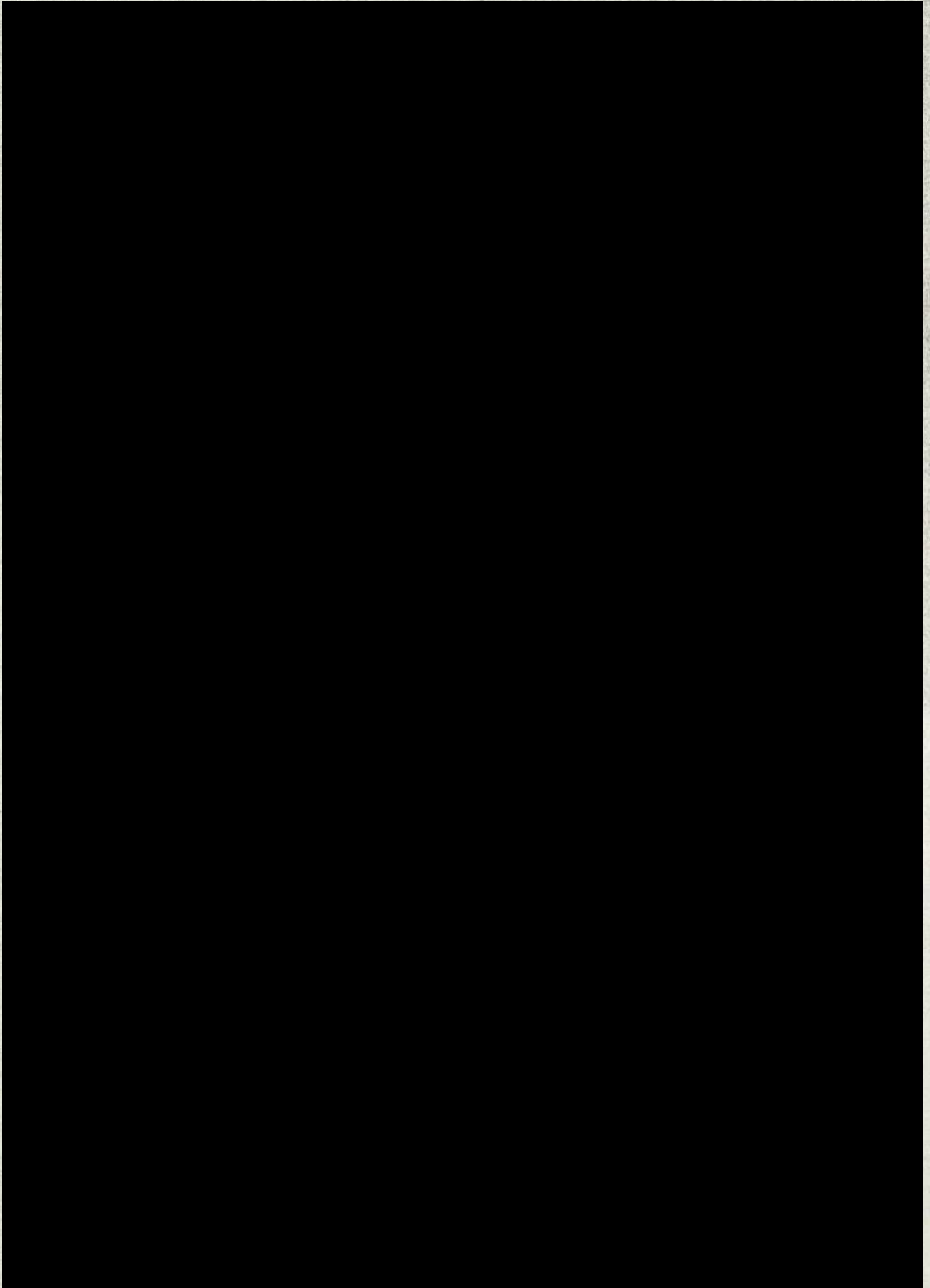
Abstract. New developments of the electro-optic light deflector using total internal reflection of the light are given. By arranging the electrodes in the manner described it has been possible to deflect over 80% of the transmitted light from the zero to either of the first orders. The arrangement is ideally suited for further miniaturization and it is anticipated that switching between the orders will eventually be possible with an applied voltage of only a few volts. Suitable applications include beam steering, multi-stage deflectors and information processing.

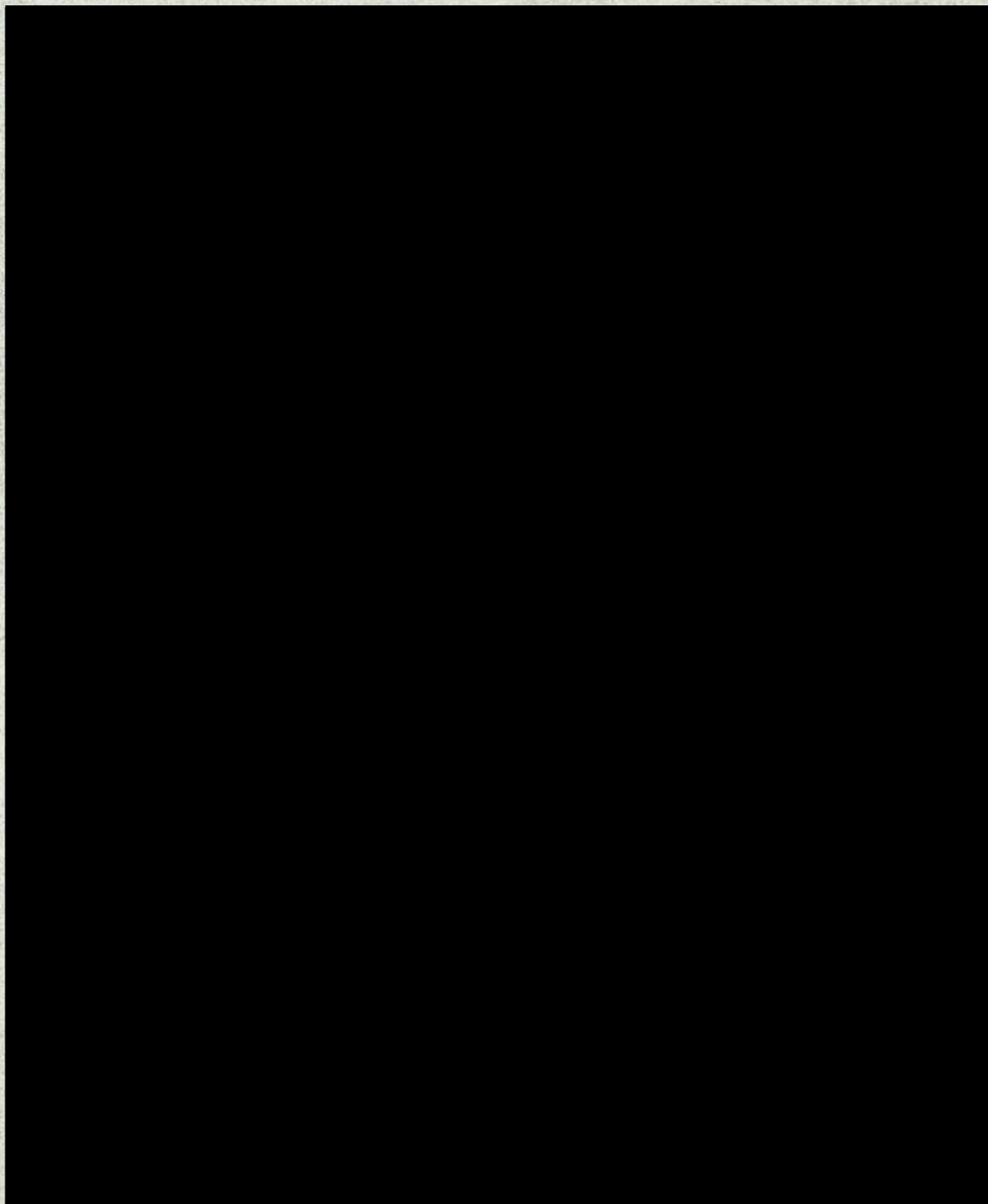


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Letter to the Editor





was along the Z axis (Fig. 6.11a). A thin film of MgF_2 (thickness $0.6 \mu\text{m}$) was deposited on the polished XY face of the crystal to prevent the generation of the subdiffraction pattern (ref. chapter 5). The two faces perpendicular to the X axis were also polished for light transmission. The crystal was baked in an oven for 12 hours at a temperature of 80°C . This produced a hard MgF_2 film. The electrode structure shown in figure 6.11.b was then deposited on the exposed surface of the MgF_2 film.

6.3.1 Initial observations of the performance of the diffraction deflector

The experimental arrangement used is shown in figure 6.11.b. The incident beam of light was polarised with the E vector along the plane of the optic axis. With this arrangement the E_z component of the field dominates the phase change impressed on the beam. This incident beam was directed towards the base of the crystal to ensure that it passes through the active field region. The angle of incidence was greater than the critical angle for the LiNbO_3 and MgF_2 interface.

The photomultiplier carrying the straight slit arranged in the manner described in chapter 5 (page 71) was used to measure the intensity of the reflected beam. The reflected beam was focussed by a second cylindrical lens so that the fringes appearing in the far field were shorter in length than the straight slit of the photomultiplier.

At zero applied voltage, the photomultiplier was scanned across the far field fringe pattern and the intensity distribution recorded on an XY recorder is shown in figure 6.12.A. The total area under this curve is proportional to the total intensity of the undiffracted (zero order) beam.

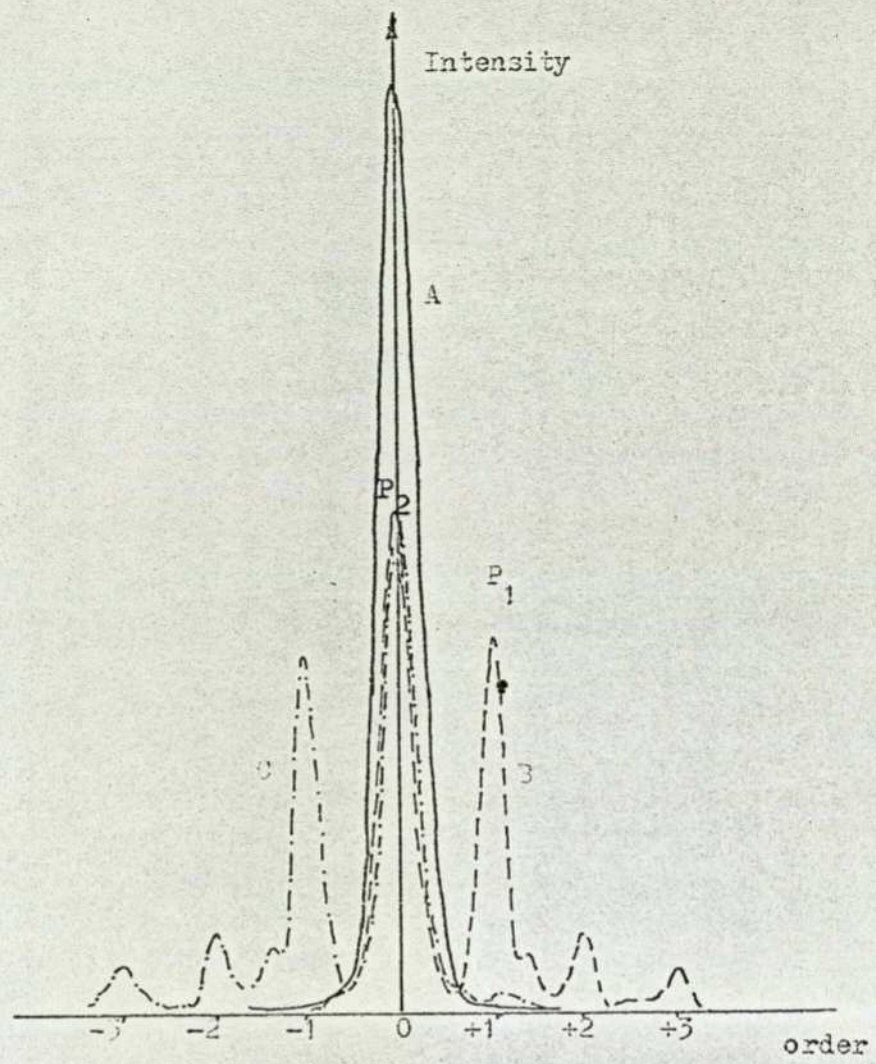


Fig 6.12 Far field intensity distribution

The voltage applied to the electrodes was increased and a diffraction pattern obtained on a distant screen. The intensity distribution in the far field was again measured by scanning the photomultiplier across the diffraction fringes. The intensity distribution at an applied voltage of 100 volts d.c. is shown in figure 6.12.B. The angular separation of the peaks P_1 and P_2 corresponds to the first order diffracted angle ($\sin^{-1} \lambda / 2d$) where λ is the incident wave length of $0.6328 \mu\text{m}$ and $2d$ is the periodicity of the phase profile as defined in figure 6.5 ($2d = 344 \mu\text{m}$). It can be seen that the +1 order intensity is very much greater than the -1 order intensity.

The voltage polarities were interchanged and the far field pattern was studied with an applied voltage of 100 volts d.c. The intensity distribution obtained is shown in figure 6.12.c. In this case the -1 order intensity is very much greater than that of the +1 order. This is to be expected since with the reversal of the polarities, the phase change is reversed.

A comparison of the curves B and C show that the intensity in the +1 order is equal to that in the -1 order when the polarities are changed.

The observations of this initial experiment indicate that a bulk modulator built with the electrode structure arranged asymmetrically will act like a blazed grating, deflecting the incident light energy to a higher order.

6.3.2 Modulation of the zero order energy by the diffraction deflector

The initial experiment with the new device shows that when a voltage is applied to the electrode structure, the incident beam is diffracted to higher orders and the diffraction pattern is not symmetrical about the zero order. When the light energy is deflected to the higher

orders, a proportionate decrease in energy will occur in the zero order. This decrease in energy varies with the applied voltage. Therefore, the deflector can be operated as a voltage controlled intensity modulator. In the following section, the performance of the deflector when used in the intensity modulation mode is studied. The device was arranged as in figure 6.11.b. The glancing angle α was taken as 4.75° . It was seen from the performance of the symmetrical electrode structure modulator (chapter 5) that the operating voltages and the zero order extinction voltages decreases with a decrease in the glancing angle α . Hence, this angle of 4.75° was chosen as a compromise between the low operating voltage and the difficulties arising when aligning at very shallow angles of incidence. The intensity of the reflected beam was measured by the photomultiplier arranged in the manner described in the previous section. At zero applied voltage, the photomultiplier was positioned to give maximum output for the section covered by the straight slit. This reading was assumed to be proportional to the energy of the undiffracted zero order beam (I_0). As the voltage applied to the electrodes was gradually increased, the intensity remaining in the zero order gradually decreased and the photomultiplier output was recorded (I_V). The voltage was increased until a minimum energy was obtained in the zero order.

Figure 6.13 shows the variation of the ratio (I_V/I_0) with the applied voltage V . It can be seen from this figure that the decrease in the intensity of the zero order is approximately linear in the range of 40 to 100 volts. Therefore, it is apparent that operating in this range, the device could be used as a linear intensity modulator.

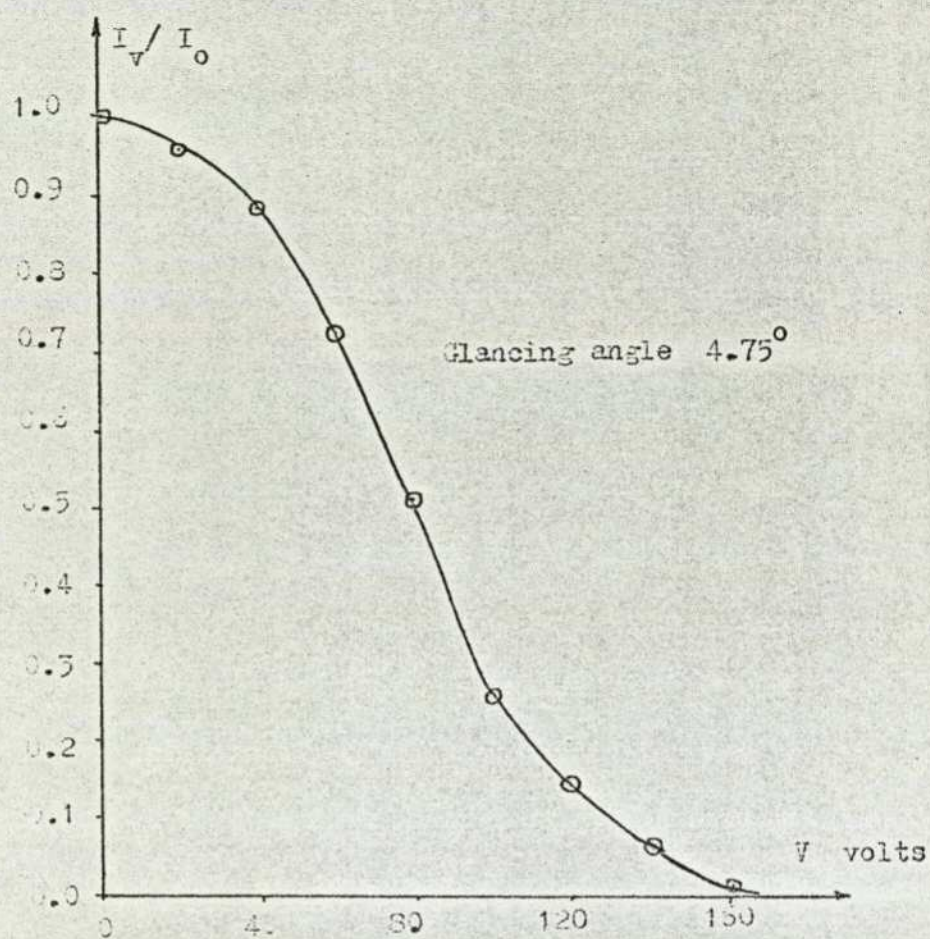


Fig 6.13 Variation of the zero order intensity with applied voltage

6.3.3 Performance of the device as an optical beam deflector

The potential applied to the asymmetrical electrode structure causes an effective blazed grating to appear inside the electro-optic medium. The blazing of the grating is a function of the applied voltage and, therefore, the energy coupled to the higher orders should also vary with the applied voltage. In this section the variation of the first order intensity with applied voltage is experimentally studied in some detail.

6.3.3.1 Variation of the first order intensity with applied voltage

The experimental arrangement was again similar to that shown in figure 6.11.c. The voltage applied to the electrodes was gradually increased and the far field pattern was displayed on a distant screen. The photomultiplier arranged in the manner described in section 6.3.1 was scanned across the first order fringe pattern at constant speed. The output of the photomultiplier was recorded on an XY plotter where the intensity was given by the Y ordinate and the X ordinate was set at a constant time scale. The area under this curve was taken to be proportional to the first order diffracted energy $(I_1)_V$ at any given applied voltage V . The incident energy I_0 was measured in the manner described in section 6.3.1. The angle of incidence was again taken as 85.25° ($90^\circ - 4.75^\circ$). The experiment was repeated for different values of applied voltages and the corresponding $(I_1)_V$ was measured. Figure 6.14 shows the variation of the ratio $(I_1)_V / I_0$ with applied voltage V .

In this graph, the ratio $(I_1)_V / I_0$ peaks at a voltage of 157 volts. That is maximum amount of energy is transferred from the zero order to the first order at this voltage. The far field intensity distribution at 157 volts d.c. is shown in figure 6.15. At this applied

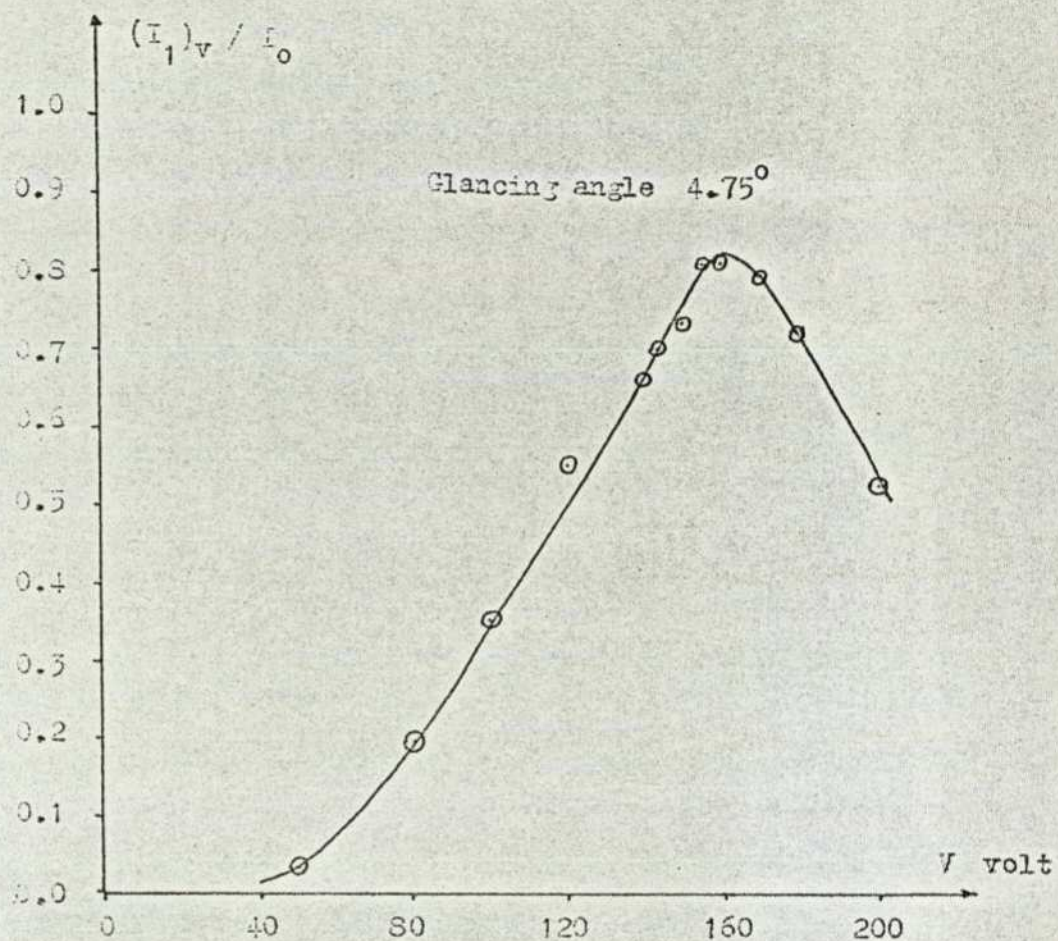


Fig 6.14 Variation of the first order intensity with applied voltage

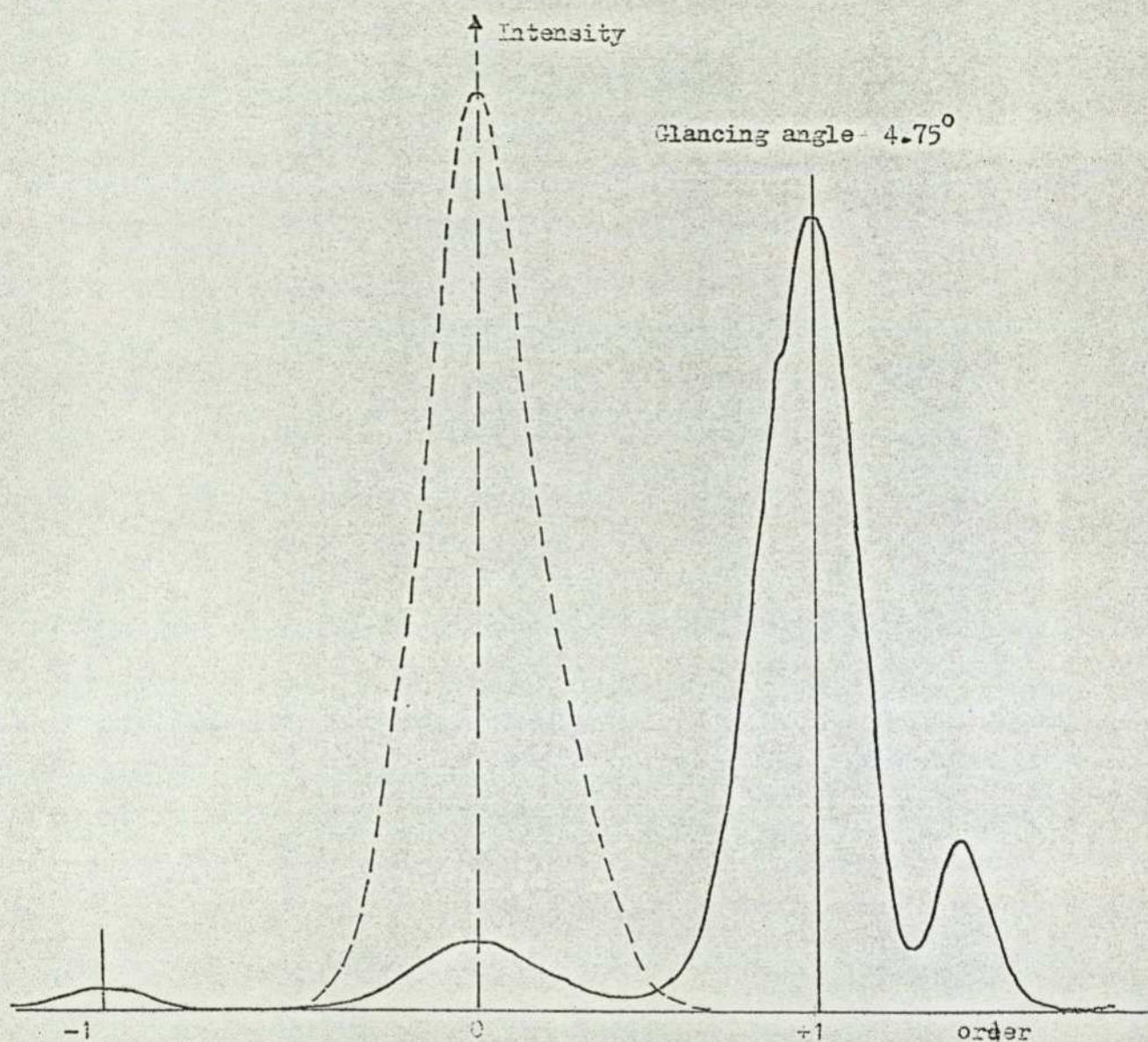


Fig 6.15 Far field intensity distribution (First order maximum intensity)

Applied voltage = 15V

An assessment of the blazed grating electro-optic deflector

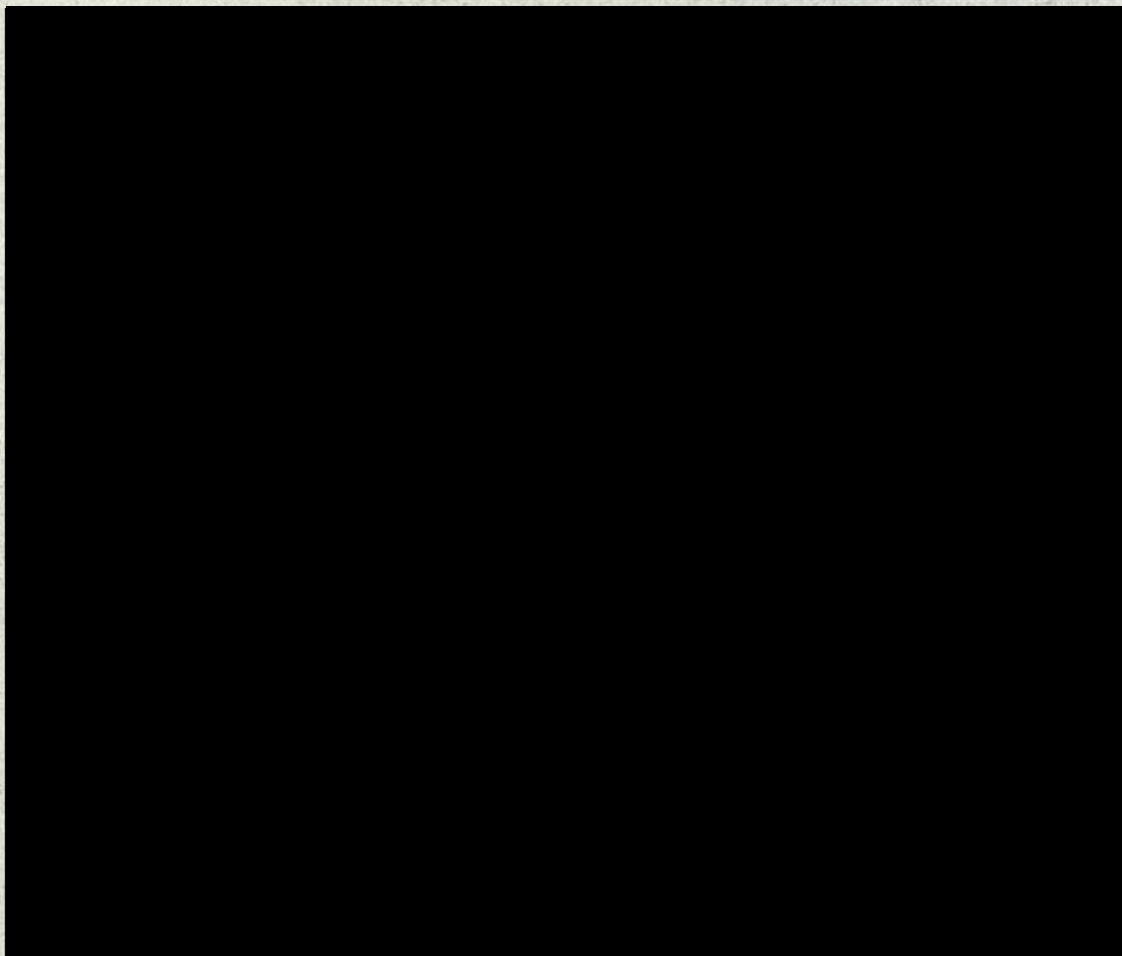
V Ramachandran,[†] A W Palmer[†] and M T V Scibor-Rylski[‡]

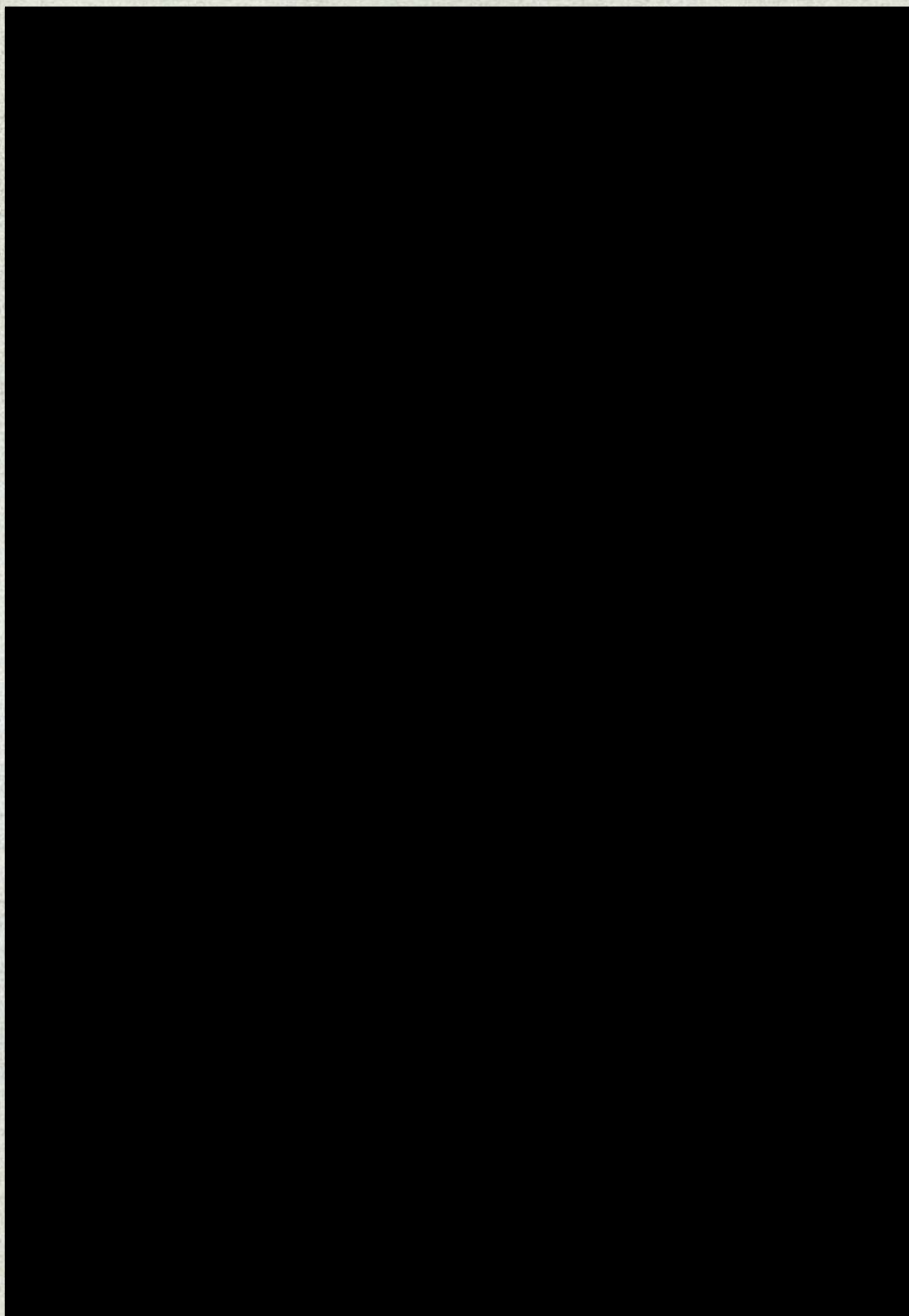
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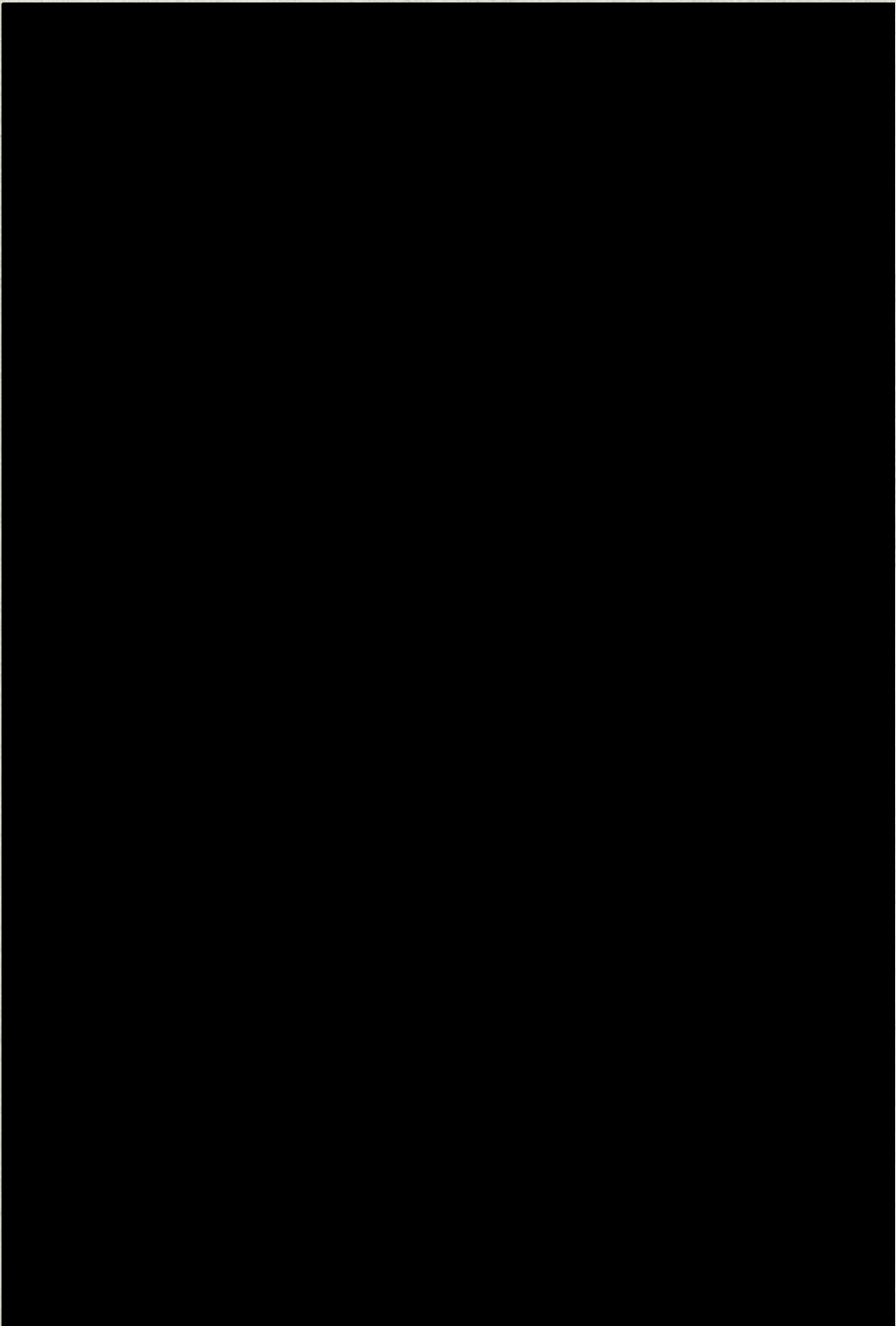
[‡] Xerox-Research UK, Church Road, Welwyn AL6 9LU

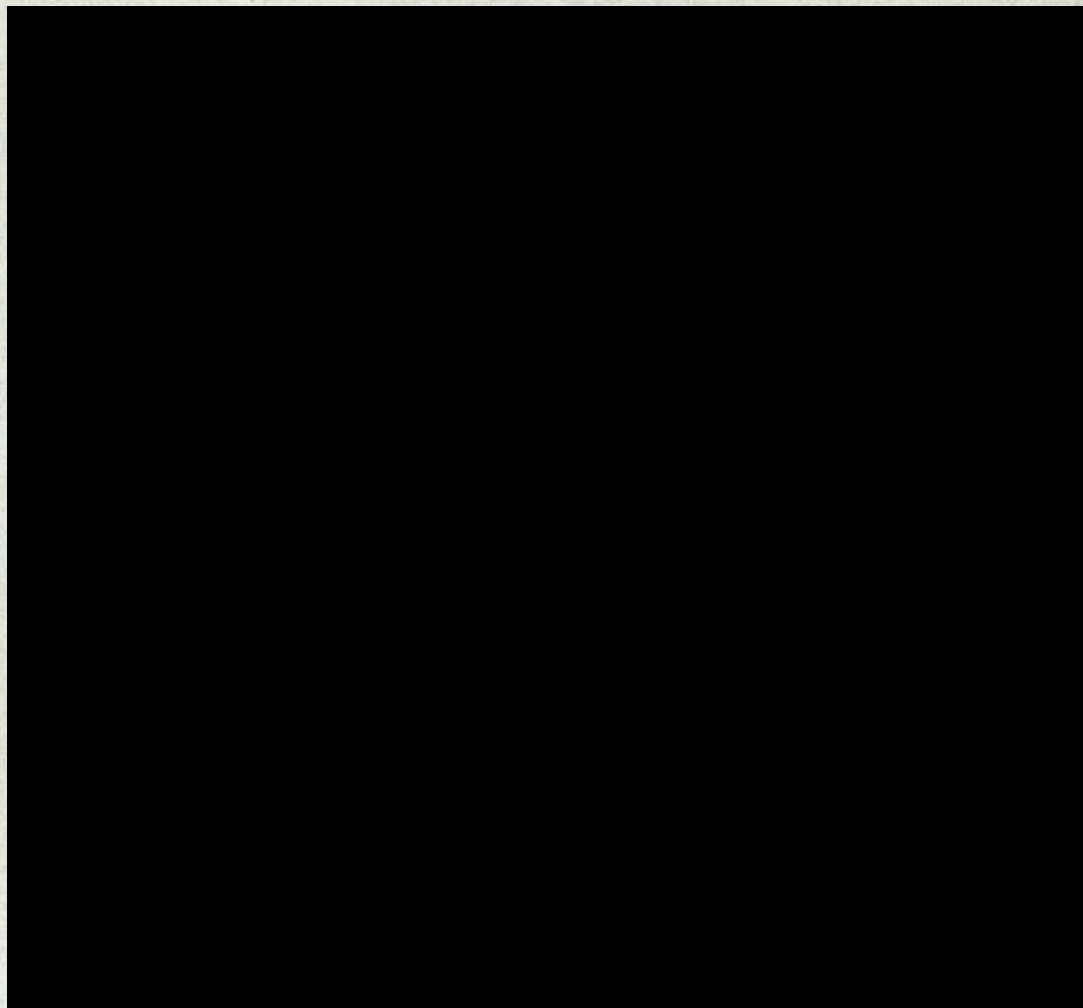
Received 18 June 1975

Abstract. Electro-optic blazed grating deflectors have been used to switch light selectively into a particular higher order. This paper assesses the theoretical efficiency of blazed phase gratings which contain an increasing number of Fourier components of the ideal blazed grating, and gives an electroding modification which will impress more than four Fourier components of the periodic phase change on the optical beam.









voltage, the percentage energy coupled into the first order is 81%. From table 6.1 it can be seen that when the first four Fourier components of the ideal sawtooth phase change are impressed on the transmitted beam, a maximum of 72.7% of the incident energy is transferred to the first order. Even though it is premature to conclude the number of Fourier components contributing to the phase change, it would be expected that the new arrangement impresses more than four Fourier components of the phase change on the transmitted beam.

The experimental results closely agree with the theoretical prediction from graphs 6.8(ii), that when the applied voltage is 155 volts, the phase delay θ_1 is approximately equal to π and, therefore, it should be expected that the beam is deflected to the first order, the condition for maximum energy to be transferred to the first order is that the phase delay θ_1 is equal to π

6.3.3.2 Effect of the ratio of the electrode spacing on the coupling efficiencies

A second deflector was prepared in the manner described in section 6.3. In this new deflector, the electrode spacing $2ad$ was again equal to $45 \mu\text{m}$ but the spacing $2bd$ (figure 6.7) was reduced from $344 \mu\text{m}$ to $172 \mu\text{m}$. This effectively decreased the ratio of 'b' to 'a' from about 8 to 4. The electrode widths ($12 \mu\text{m}$) were the same in both cases.

Preliminary observations with this experimental device showed that a maximum of 56% of the incident energy was coupled into the first order. The angle of incidence was again taken as 85.25° . This reduction in the amount of energy coupled to the first order is to be expected as a better approximation of the ideal sawtooth phase profile will only

be achieved when the electrode spacing $2bd$ is large. Typically when the dimensions of $2ad$ and $2bd$ (figure 6.7) are equal, the electrode configuration reduces to the symmetrical electrode arrangement and, therefore, a symmetrical diffraction pattern is obtained in the far field.

6.3.4 Variation of the 2nd order intensity with applied voltages

The variation of the first order intensity with voltage was discussed in the previous section. A similar variation was observed for the second order intensity and is given in the following section.

The experimental arrangement used was again as shown in figure 6.11.c. The voltage applied to the electrodes was gradually increased and the photomultiplier was scanned across the second order fringe as described in section 6.3.3. The area under this curve was taken to be proportional to the intensity $(I_2)_V$ of the second order beam at any applied voltage V . The variation of the ratio $(I_2)_V/I_0$ with applied voltage V is shown in figure 6.16. The angle of incidence was again adjusted to 85.25° . It is seen from this figure that maximum energy from the incident beam is deflected to the second order when the applied voltage is 255 volts. This maximum percentage was 53%. This relatively small coupling efficiency to the second order by the device will be improved by reducing the pitch of the phase grating relative to the incident beam width. From the electrode structure in figure 6.11.B it can be seen that the periodicity of the phase profile is 0.344 mm. The laser beam width measured was approximately 2.5 mm. The graph 6.8 (iii), shows that when the voltage applied to the electrodes is 250 volts the phase delay θ_1 is approximately equal to 2π . Therefore, it would be expected that maximum energy will be deflected to the second order at a voltage

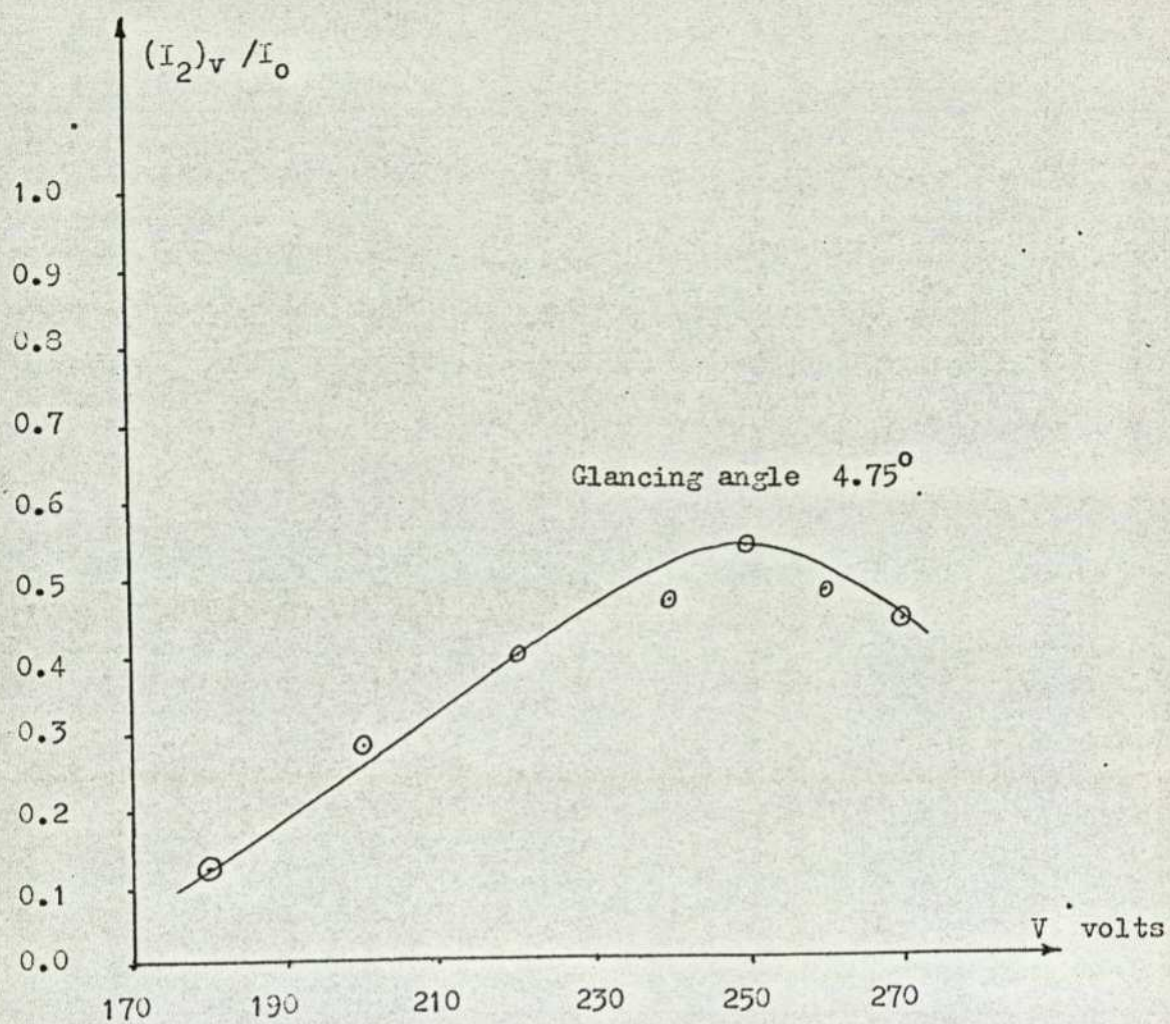


Fig 6.16 Variation of the second order intensity with applied voltage

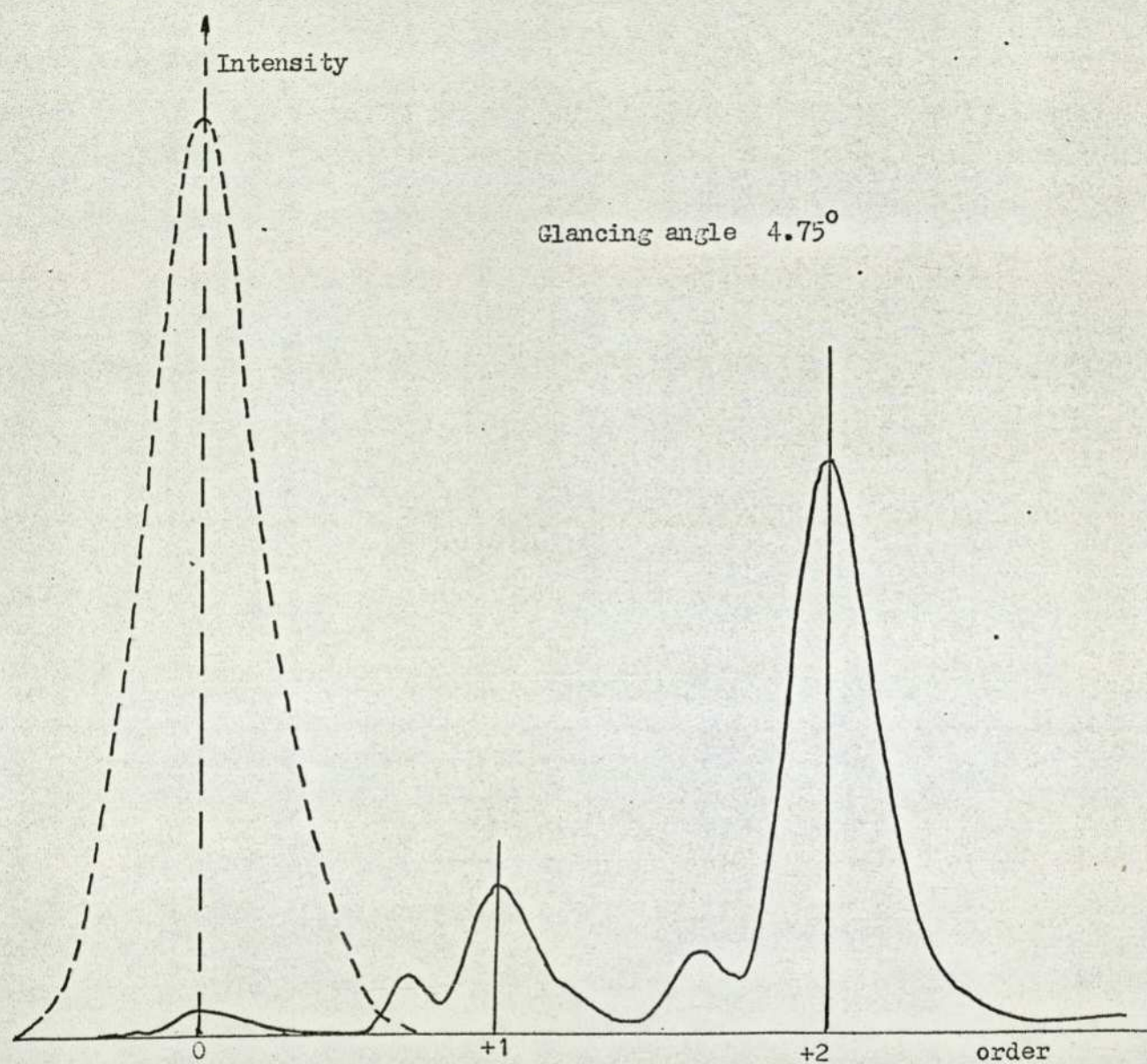


Fig 6.17 Far field intensity distribution (Second order maximum intensity)
Applied voltage = 255 V

approximately equal to this value. As seen, the experimental value is in reasonable agreement with this prediction.

6.4 Variation of the zero order extinction voltage with angle of incidence

The equation 6.18 shows that the phase change impressed on the transmitted beam is a function of the applied voltage and the angle of incidence. The experiments with the modulator having the symmetrical electrode structure (Chapter 5) show that the zero order extinction voltage decreases with an increase in the angle of incidence. These observations may be theoretically reasoned thus as follows. As the angle of incidence decreases, the interaction region for the periodic refractive index perturbation and the optical wave is decreased, and, therefore, in order to impress the equivalent phase variation on the transmitted beam, the voltage applied to the electrodes should be increased. A similar variation occurs in the deflector with the asymmetrical electrodes, and in this section the variation of the zero order extinction voltage with angle of incidence is studied experimentally.

The experimental arrangement was similar to that shown in figure 6.11.c. The photomultiplier arranged in the manner described in section 6.3.2 was used to measure the intensity of the zero order beam. The photomultiplier was positioned to give the maximum output of the section covered by the slit. The voltage applied to the electrodes was gradually increased so that minimum intensity of the zero order was obtained i.e. the photomultiplier output was a minimum (zero). This was repeated for varying angles of incidence and the corresponding zero order extinction voltage measured. Figure 6.18 shows the variation of this voltage with glancing angle. The graph shows,

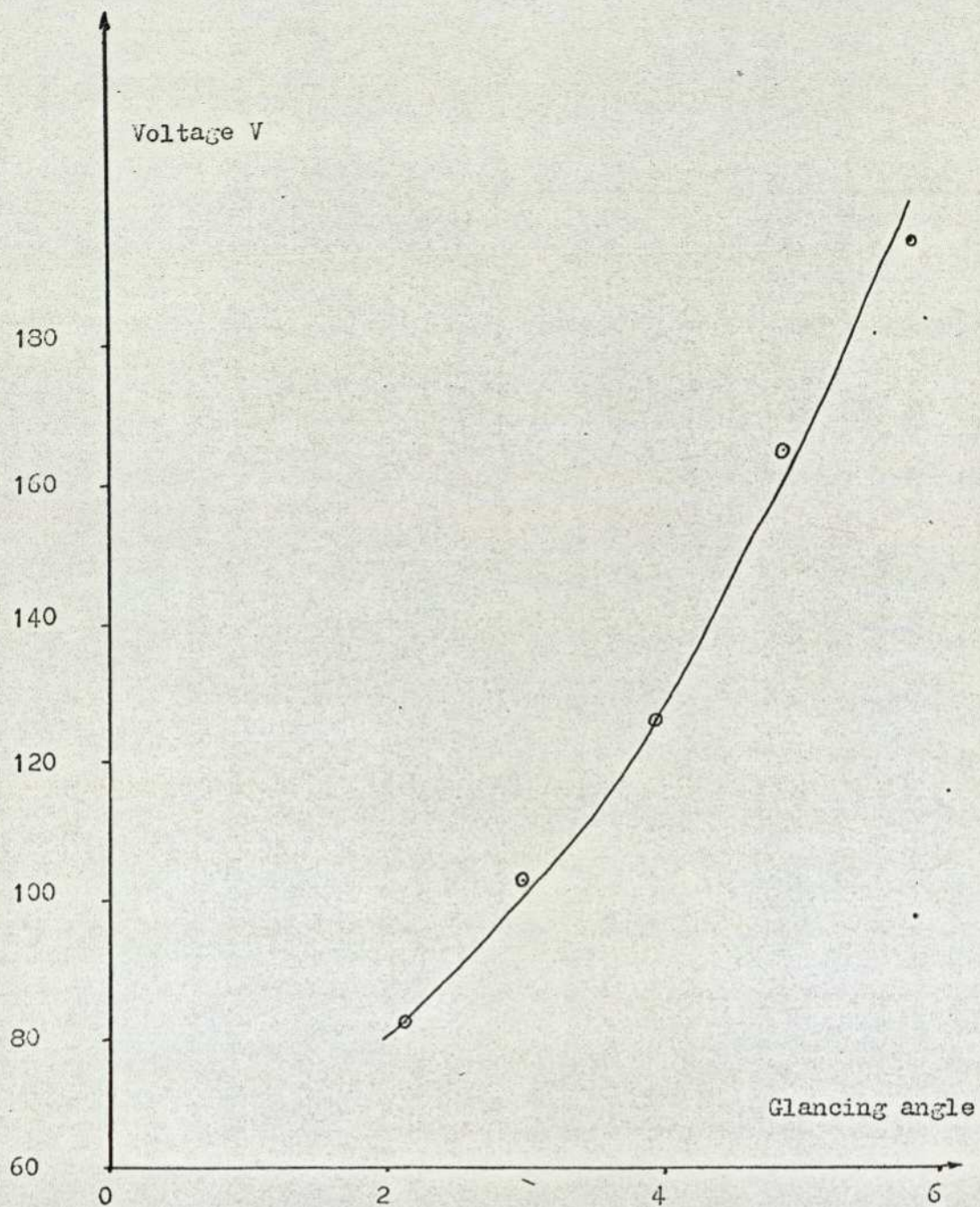


Fig 6.18 Variation of the zero order extinction voltage
with glancing angle
(glancing angle = $90 - \text{angle of incidence}$)

as expected, that the zero order extinction voltage decreased with an increase in the angle of incidence.

6.5 Variation of the first order maximum voltage with angle of incidence

The experimental observations of section 6.4 clearly indicates that as the angle of incidence decreases the required phase variation is impressed on the transmitted beam only by increasing the applied voltage. A similar effect is expected for the variation of the voltage required for the maximum energy in the first order, and this was experimentally verified.

The experimental arrangement was again similar to that shown in figure 6.11.b. The photomultiplier slit was positioned at the first order maximum position. The voltage applied to the crystal was gradually increased until the output of the photomultiplier reached a maximum. This was repeated for different angles of incidence and the variation of the first order maximum voltage with the glancing angle is shown in figure 6.19. The graph shows that the voltage required for first order maximum decreases with an increase in the angle of incidence.

6.6 Variation of the first order maximum intensity with angle of incidence

Experimental observations in the previous section show that the voltage required to give maximum energy in the first order increases with a decrease in the angle of incidence. In this section, the maximum percentage of energy coupled into the first order at different angles of incidence is studied in some detail.

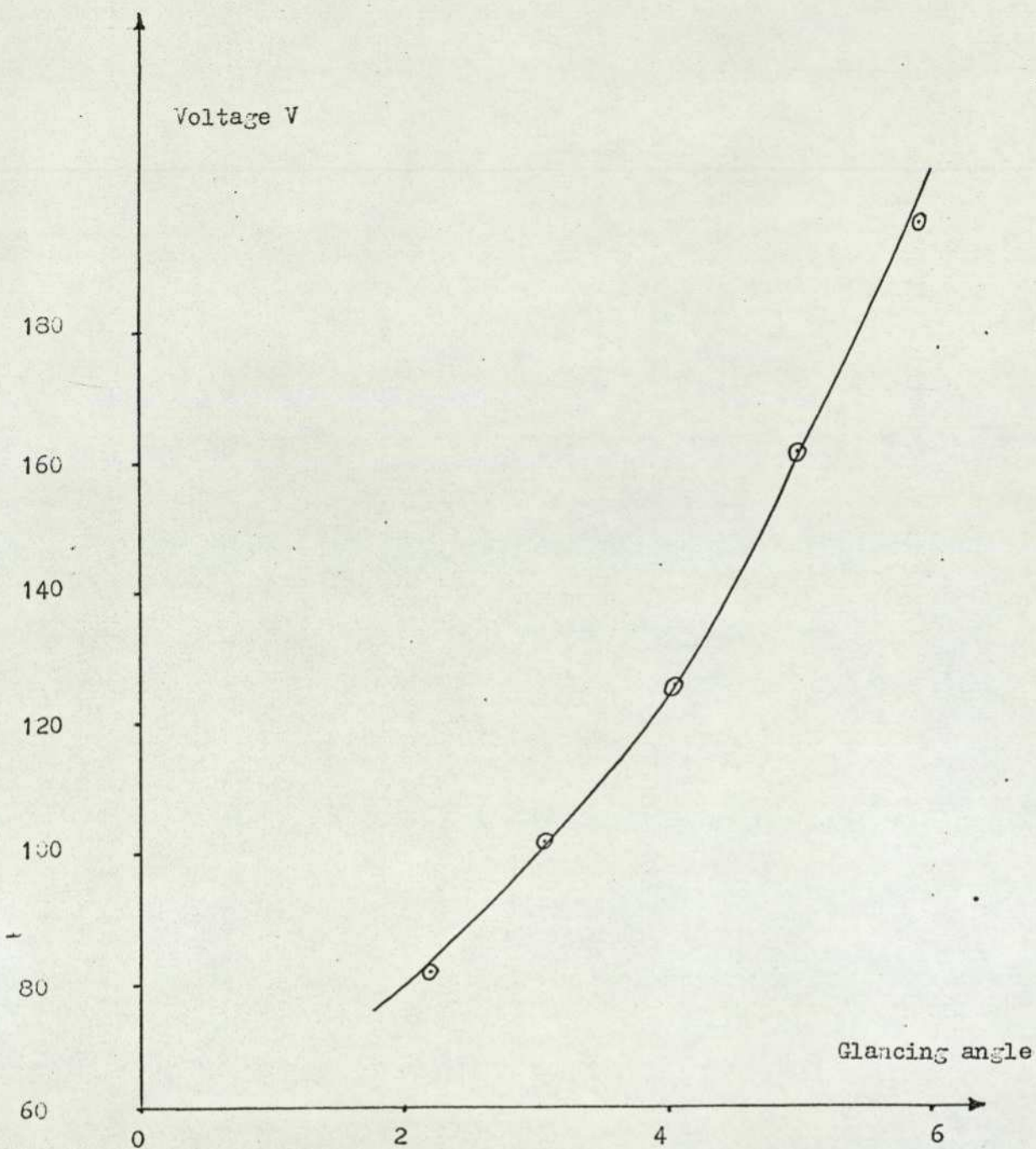


Fig 6.19 Variation of the first order maxima voltage with
glancing angle
(glancing angle = 90° - angle of incidence)

The experimental arrangement was similar to that described in section 6.3.3. The angle of incidence of the light beam was adjusted to be 86.5° . At this angle of incidence, the voltage required for the maximum intensity in the first order was 104 Volts. The incident undiffracted energy and the maximum first order energy were measured in the manner described in sections 6.3.1 and 6.3.3.1 respectively. It was found that 80.7% of the incident energy was coupled to the first order at an applied voltage of 104 volts. This compares with 81% measured for an angle of incidence of 85.25° , in section 6.3.3.1. Figure 6.20 shows the calculated variation of the phase change along the Y axis at an angle of incidence of 86.5° and at an applied voltage of 104 volts. Comparison of figures 6.20 and 6.9(ii) indicates that the magnitude of the phase profile is not appreciably altered at the two angles of incidence (86.5° and 85.25°). Therefore, it is reasonable to expect, and this is confirmed experimentally in this section, that the percentage of energy coupled to the first order will be unaffected by varying the angle of incidence over the range considered.

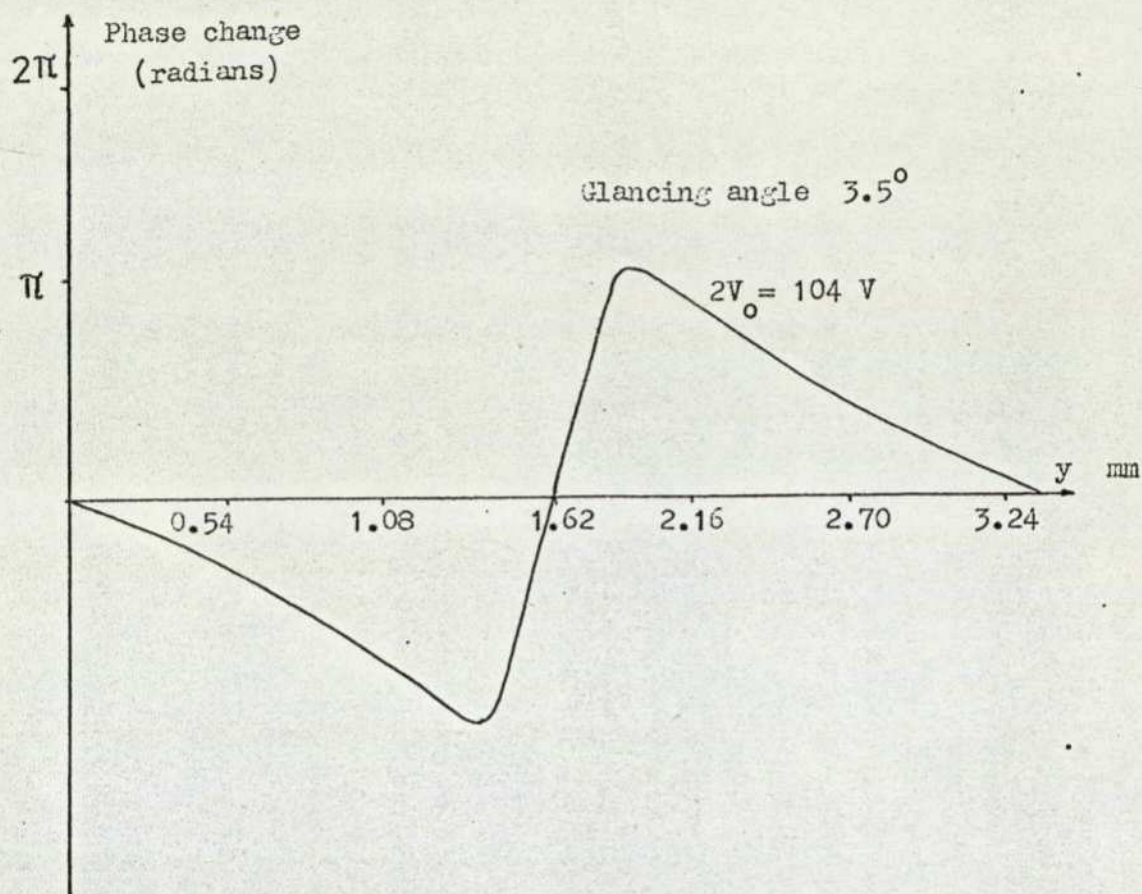


Fig 6.20 Variation of phase change with y

CHAPTER 7

ACOUSTIC WAVES IN SURFACE ELECTRODED DEVICES

INTRODUCTION

The changes in the refractive indices caused by acoustic waves inside a piezoelectric crystal may be utilised to modulate an incident optical beam. However, the performance of a device, whose operation is based on the electro-optic variations of the refractive index, may be affected by the piezoelectric resonances when operated with AC signals.

In this chapter the resonances observed in the experimental devices used in the earlier chapters are isolated, and possible means of eliminating these effects are investigated.

7.1 Primary and Secondary electro-optic effects in crystals

In Chapter 3 the index ellipsoid and the electro-optic deformation of the ellipsoid caused by an applied electric field was considered in some detail. In general when an electric field is applied to a piezoelectric crystal the shape of the crystal is altered. This effect is known as the converse piezoelectric effect (42), and in tensor notation this effect is represented as

$$e_{jk} = d_{ijk} E_i + S_{jkmn} T_{mn} \quad 7.1$$

where E_i represents the component of the electric field intensity inside the crystal, d_{ijk} is the piezoelectric tensor, S_{jkmn} is the adiabatic compliance tensor, T_{mn} is the stress tensor and e_{jk} denotes the strain tensor which describes the change of the shape of the crystal (43).

When the crystal is deformed the index of refraction of the crystal will be changed. The strain in the crystal is related to the change in the refractive index by the elasto-optic coefficients of the crystal, and the total change of the index of refraction is conveniently represented as

$$\left[\frac{1}{n^2} \right]_i = \sum_{k=1}^6 r_{ik} E_k + \sum_{j=1}^6 p_{ij} e_j \quad 7.2$$

where $i = 1, 2, \dots, 6$ and r, p, e , are the electro-optic, elasto-optic and strain tensor coefficients respectively. E is the component of the field intensity and n is the refractive index (Cf equation 3.17 Chapter 3).

In the above equation, the first and the second term of the right hand side of the equation represents the contributions by the electro-optic and piezo-electric effects respectively. The electro-optic effect that would be obtained if the crystals were not strained is called the primary (or true) effect and that due to the piezoelectricity and photo elasticity is called the secondary (or false) effect (42). In the above equation (7.2) the two terms do not necessarily have the same sign.

If the crystal is free from external forces the observed effect is the sum of the primary and the secondary effects and since the algebraic signs of the two contributions are not necessarily equal, the combined effect may be greater or smaller than the primary effect (45). If the applied electric field is at a frequency corresponding to an acoustic resonant frequency of the device, then the magnitude of the secondary effect may be considerably larger than that of the primary effect. The resonant frequencies of the device are determined by the dimensions of the crystal and for surface waves, the geometry of the electrodes.

The application of an alternating electric field to a crystal which for example is in the shape of a rectangular parallelepiped causes, in general, nine acoustic waves to travel inside the bulk of the crystal. These are one quasi-longitudinal wave and two quasi-shear (62) waves along the normals to the faces of the crystal (62). However, the symmetry of the crystal causes a reduction in the number of waves capable of being generated inside the medium by an electric field.

7.1.1 Acoustic waves in LiNbO_3

The electro-optic medium considered in this thesis is LiNbO_3 . Some of the properties of this crystal are

- i) Low ultrasonic loss
- ii) Large piezoelectric coefficients
- iii) Large elasto-optic coupling efficiencies

In strongly piezoelectric crystals, in order to launch acoustic waves inside the crystal it is not necessary to use an external transducer. The crystal will act as its own transducer. Therefore, in the devices considered in the earlier chapters, it is expected that the secondary effect will be appreciable at the resonant frequencies.

If the LiNbO_3 sample is assumed to be an infinite plate and the electric field is applied by infinite electrodes deposited on the flat faces of the plate (60) then the different modes of propagation of elastic waves inside the medium can be summarised as in Table 7.1.

TABLE 7.1

Direction of field	Modes of propagation of elastic waves
X	Two quasi-shear waves.
Y	One quasi-shear wave and a quasi-longitudinal wave.
Z	Pure-longitudinal wave propagating along the Z axis.

In the arrangements described in Chapters 5 and 6, the electric field inside the crystal was produced by the application of a voltage to the electrode structures deposited on the crystal face. These transducing electrodes on the face of the crystal provide a periodic array source for the generation of the waves inside the bulk of the crystal and

direct coupling to surface waves. For simplicity if the symmetrical arrangement of Chapter 5 is consi-

dered the electric field inside the crystal may be approximately represented as shown in figure 7.1.

In order to analyse the nature of propagation of the acoustic waves inside the crystal, the contribution to these waves by the two components (E_z and E_y) of the field should be studied, in finite arrangement, in contrast to the infinite plate arrangement considered in the results of Table 7.1.

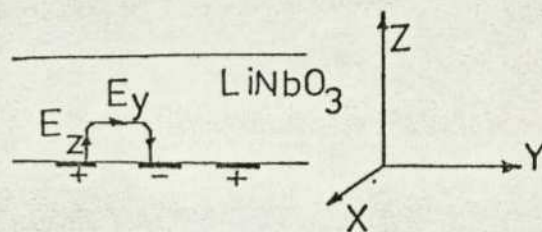


Fig. 7.1

Symmetrical electrode arrangement

Theoretical estimates of the contributions made by these two periodically varying components of the field is very difficult. It was not possible, in the time devoted towards this aspect of the work to undertake the detailed theoretical analysis of this problem. However, attempts were made to experimentally analyse the different resonances observed during the course of this work.

Applying alternating polarities to the adjacent electrodes of the arrangement shown in figure 7.1 produces (58-67)

- i) Acoustic waves inside the crystal
- ii) Acoustic waves on the surface of the crystal (Rayleigh Waves)

For Rayleigh waves, the energy of the wave is confined within a thin layer of the crystal close to the surface. The amplitudes of these waves decreases exponentially when the distance from the surface is increased. The surface wave velocity is given with a maximum error of 0.5% by the approximate formula.

$$V_s = V_t \frac{(0.87 + 1.12\sqrt{S})}{(1 + \sqrt{S})} \quad 7.3$$

where V_s and V_t are the velocities of the surface and the bulk transverse waves which are in the same direction of propagation as the surface waves respectively, $\sqrt{S}$ is the Poissons ratio (59)

7.2 Acoustic resonances observed in the symmetrical and asymmetrical electrode device arrangements

At the resonance frequencies the impedance of the device will abruptly change (60)

The devices considered in Chapters 5 and 6 were constructed using a Z cut LiNbO_3 crystal. Symmetrical (Chapter 5) and asymmetrical (Chapter 6) electrodes were deposited on the polished XY face of the crystal. The electrode fingers were along the X axis. A thin layer of MgF_2 was deposited on the crystal face prior to the deposition of the electrodes.

The impedance of the device was measured by the Vector Impedance Meter available at the City University.

The procedure for measuring the impedance is described in the following section.

7.2.1 Initial observations of resonances in the device

The crystal was mounted inside the holder as shown in figure 7.2 (The photographs of the holders used for different electrode configurations

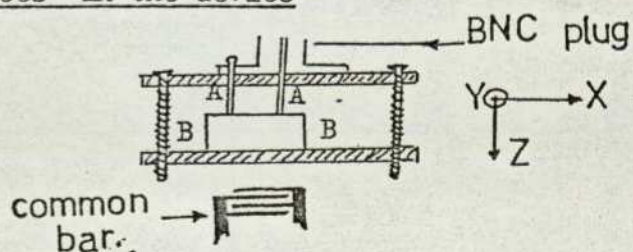


Fig. 7-2

are shown in Appendix A (page 223). The separation of the connecting leads AA was adjusted so that the leads made contact with the centre of the common bar (fig.7.2) connecting the electrodes. The contacts between the leads A, A and the electrodes were made by applying a conducting paint (silver paint) between the leads and these bars. In the measurement of the impedance it is necessary to ensure that a good contact is made between the bars and the leads, but care should be taken to ensure that no stress is exerted on the crystal face as any stress may alter the resonant frequencies.

The lengths of the connecting leads AA should be made as small as possible to reduce the contribution of the 'stray' impedance by the leads. However, to facilitate easy connection, a compromise length of 7 mm was chosen. The composite structure was then connected directly to the Vector Impedance Meter via the BNC plug. The electrical resonant frequency of the arrangement was found to be beyond the range of measurements.

The frequency range of the impedance meter is from 0.5MHz to 108MHz. The frequency at which the impedance was measured was manually altered and the impedances were directly recorded on an XY plotter. The X and Y axes represented the frequencies and the impedance respectively. The experimental plots are given in figure 7.3A and 7.4A respectively. It can be seen that the impedance gradually decreases as the frequency of measurement is increased. This is due to the decrease in the impedance by the bulk capacitance of the device. However, at discrete frequencies the impedance changes abruptly (or fluctuates). The phase of the device was also measured and the variation was similar to that of the impedance. The phase

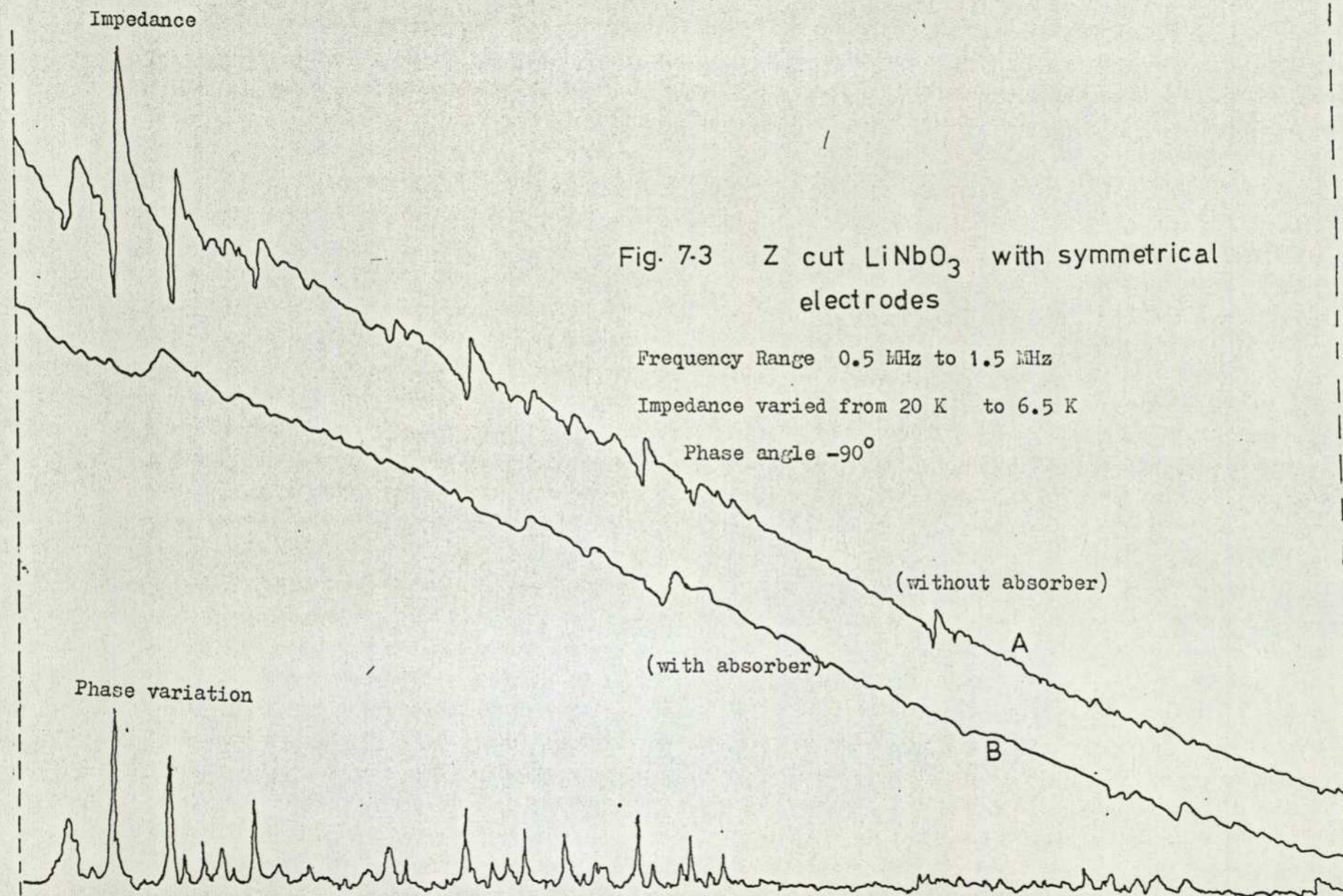


Fig 7.3

- 1) Frequency range 35 MHz to 108 MHz
Impedance varied from 0.3 K to 0.09 K
- 2) Frequency range 1.5 MHz to 4.5 MHz
Impedance varied from 6.5 K to 2.15 K

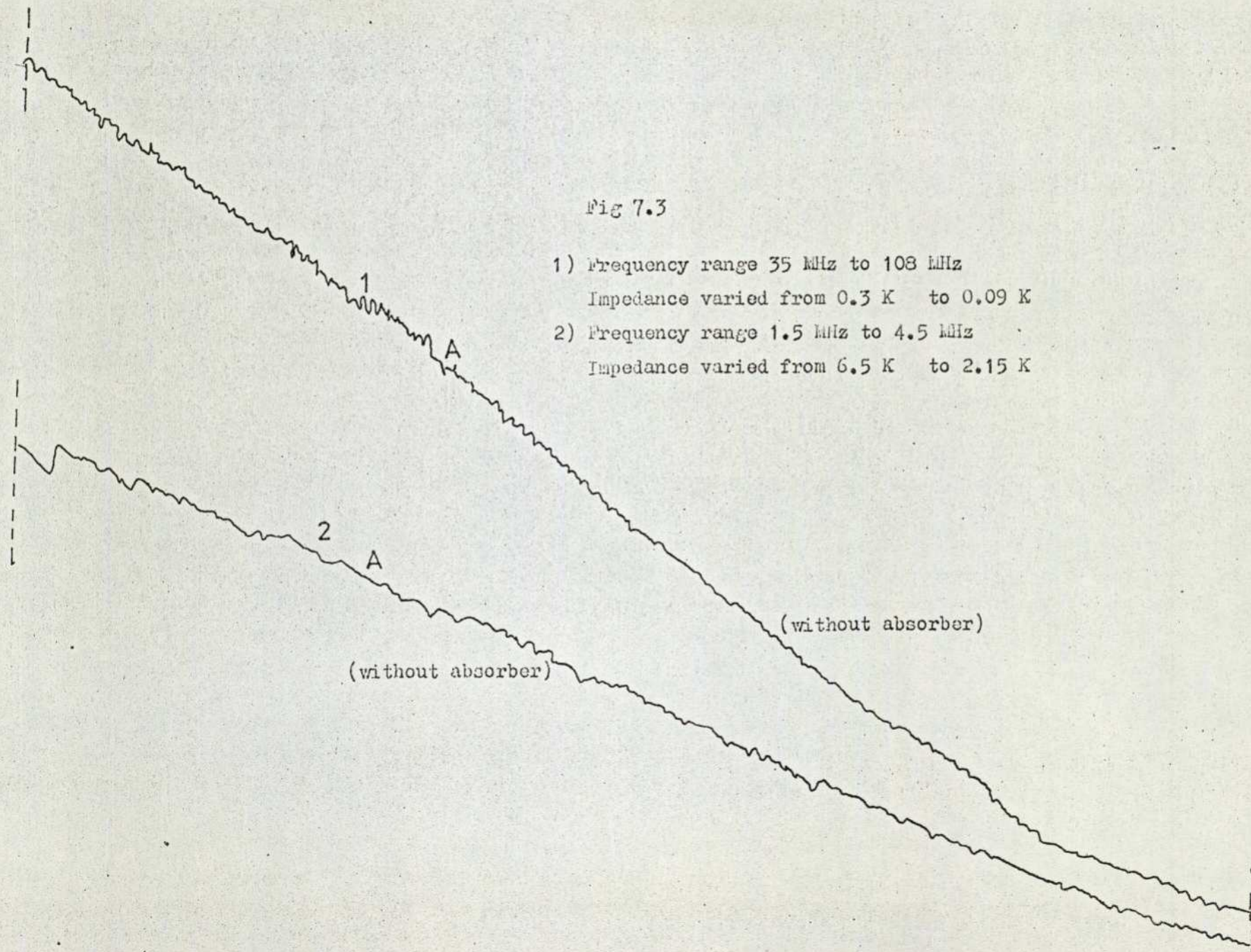


Fig 7.3

Frequency range 4.5 MHz to 14.0 MHz

Impedance varied from 2.15 K to 0.68 K

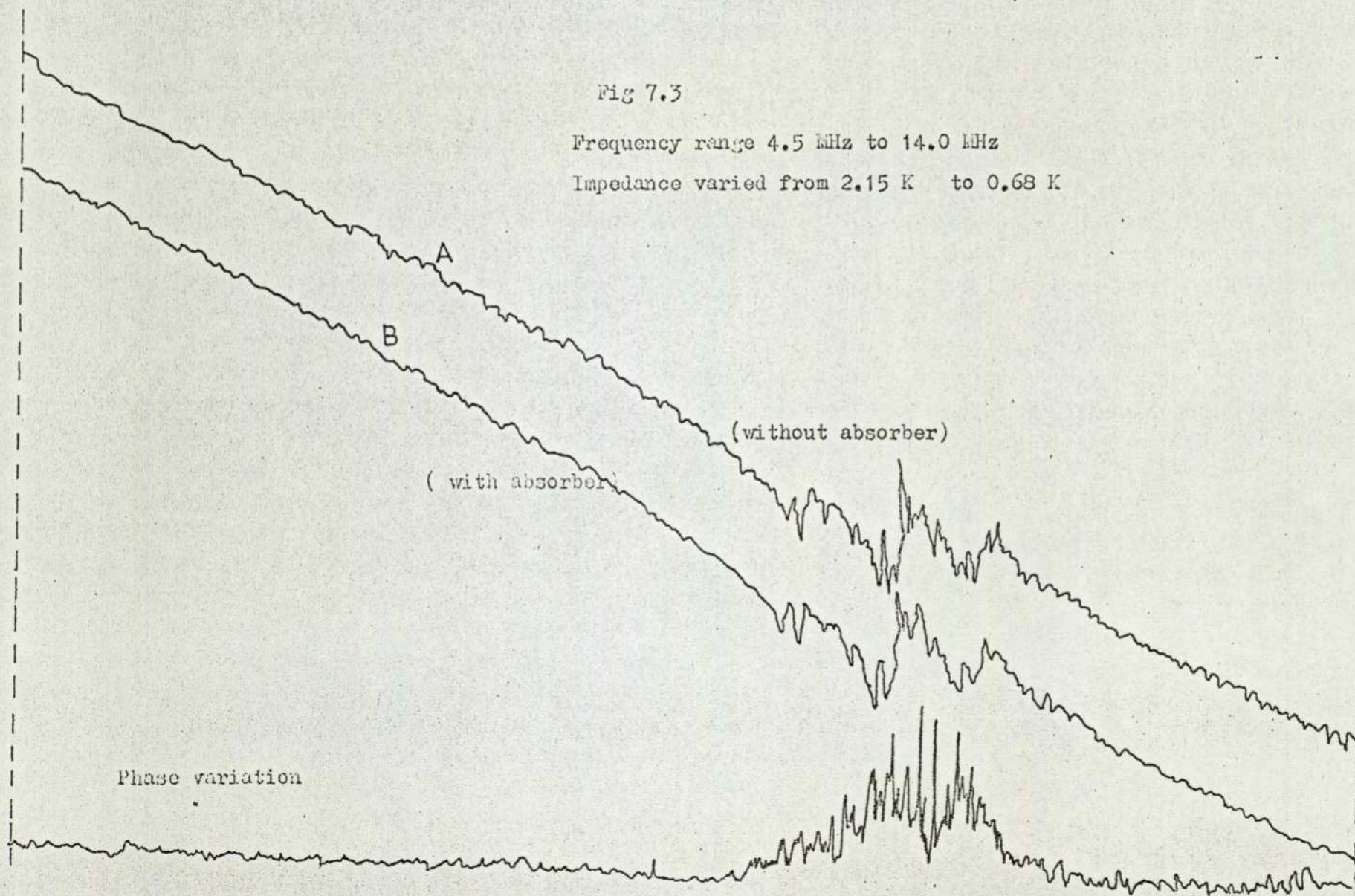
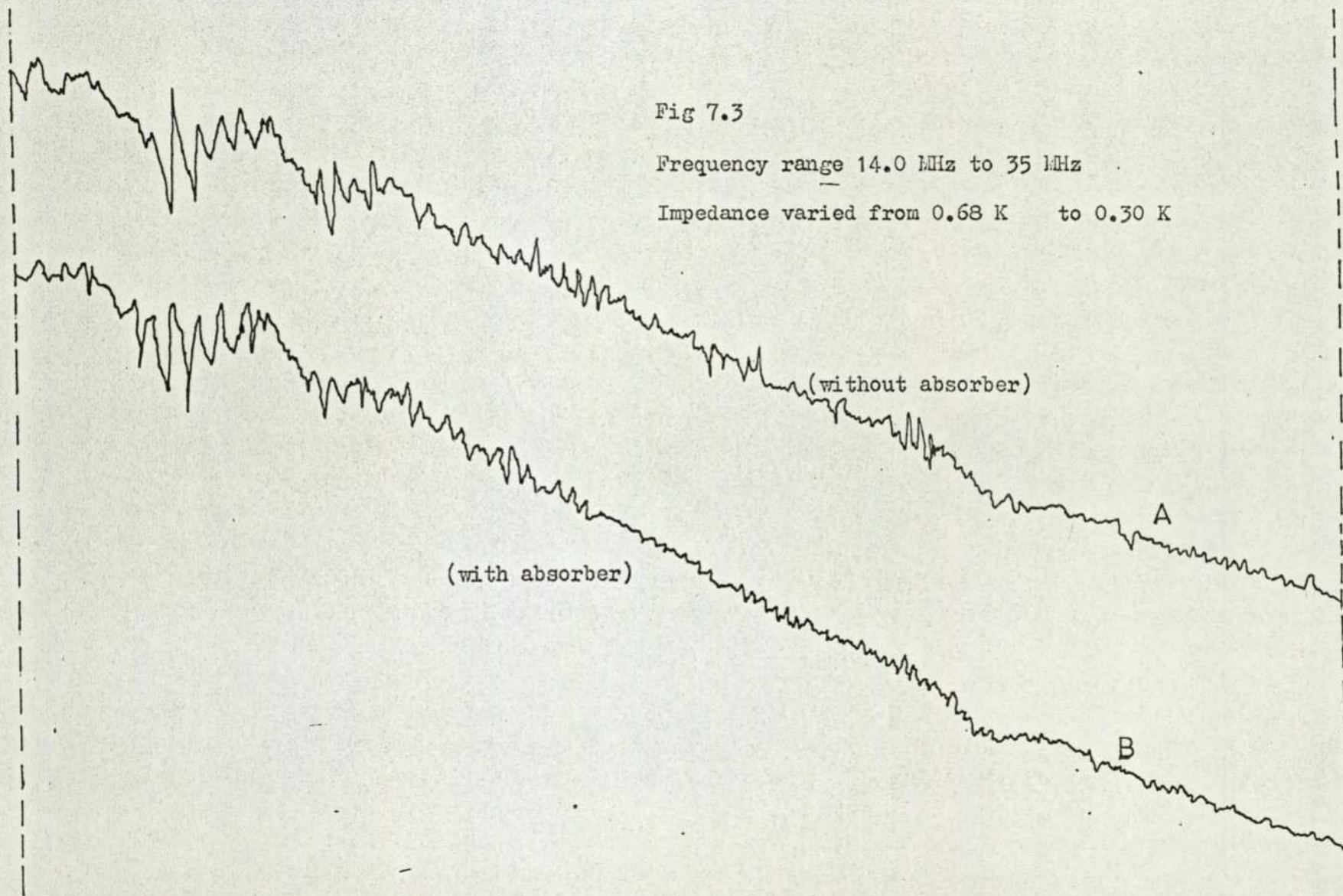


Fig 7.3

Frequency range 14.0 MHz to 35 MHz

Impedance varied from 0.68 K to 0.30 K



angle was -90° and changed abruptly at frequencies where the impedance abruptly changes.

The following procedure was adopted to isolate the elastic waves causing the resonances.

7.2.2 Observation of resonance in the device with an absorber

The acoustic waves produced inside the piezoelectric crystal may be completely or partially absorbed by having a proper absorber attached to the faces of the crystal. The absorber may be a soft material (rubber or plasticine) or a liquid. Alternatively the waves may be coupled out when the crystal is efficiently adhered to a second solid medium. In order to isolate the causes of the different resonances observed in the earlier section, attempts were made to absorb the acoustic waves produced inside the crystal.

A thick layer of plasticine was placed between the crystal base BB and the holder. The impedance was again measured in the manner described in the earlier section and the experimental plots are given in figures 7.3B and 7.4B.

7.2.3 Summary of the observations

When acoustic resonance occurs inside the medium, the electrical impedance of the crystal will abruptly change. The nature of the variation (either an increase or a decrease) of the impedance can only be determined by knowing the exact equivalent circuit of the arrangement. The equivalent circuits of the above devices are rather complicated. The frequency range over which the change from the minimum impedance to the maximum impedance occurs is of the order of 0.05 MHz. Therefore, although it is difficult to define exactly the resonant frequency because of the complexity of the equivalent

circuit, very little error will be introduced by defining the resonant frequency as the frequency at which maximum impedance occurs.

i) Symmetrical electrode arrangement

For the arrangement without the absorber at the base BB, a large change of the impedance was observed at 0.54 MHz. A considerably smaller variation was also observed at discrete frequencies in the range of 0.5 MHz to 1.5 MHz. However, when the absorber was placed on the base BB of the crystal these resonances were almost completely suppressed. Therefore, it may be assumed that the resonances observed in this range were due to elastic waves travelling along the Z direction of the crystal. On the graph covering the range of 4.5 MHz to 14.0 MHz an appreciable regular fluctuation of impedance was observed from 7.8 MHz to 9.5 MHz. The maximum variation was at 8.7 MHz. These fluctuations were not appreciably altered by the presence of the absorber. Similar fluctuations were observed on the plot covering the frequency range of 14 MHz to 35 MHz. These variations had about one hundredth of impedance change that was observed at 0.54 MHz. The impedance was found to decrease gradually with very little fluctuations in the range of 35 MHz to 108 MHz.

ii) Qualitative analysis of the observations

The previous observations clearly indicate that the resonance observed at 0.54 MHz was due to the waves travelling along the Z direction. If this resonance is assumed to be due to a fundamental frequency longitudinal wave travelling along the Z axis, then the wavelength of this resonance wave is approximately equal to twice the dimension of the crystal in the Z direction ie 14 mm (ref.Chapter 5).

The velocity of the longitudinal wave along the Z axis is 7330.6 m/s (47). From this data the resonant frequency of the longitudinal wave in the fundamental mode may be calculated and is equal to 0.53 MHz. This corresponds closely to the measured resonant frequency of 0.54 MHz and it would thus appear that the observed resonance may be attributed to a longitudinal wave along the Z direction. The deviation in the calculated and measured frequencies is within the range of uncertainty in deducing the exact frequency at which resonance occurs. The other small variations of impedances observed in the range of 0.5 to 1.5 MHz may be due to surface irregularities of the second unpolished face perpendicular to the Z axis.

The bulk waves inside the crystal can also produce surface waves which travel on and near the crystal face. In addition, the interdigital electrodes deposited on a strongly piezoelectric crystal can couple surface waves to travel on the crystal face (47). When alternate polarities are applied to the adjacent electrodes, efficient coupling of the surface waves will occur only when the electrode pitch is equal to half the wavelength of the surface waves (or a multiple for the harmonics). The electrode pitch used in the symmetrical arrangement is equal to 0.217 mm. The surface wave velocity along the Y direction in the XY face of the crystal is 3900 m/s (60). Therefore, the fundamental frequency required for surface wave generation is 8.27 MHz. This is in close agreement with the resonance observed which was not affected by the presence of the absorber on the base (7.8 MHz to 9.5 MHz). The deviation between the calculated and the peak resonance frequency observed is about 5%.

Impedance

Fig. 7.4 Z cut LiNbO_3 with
asymmetrical electrodes

Frequency range 0.5 MHz to 1.5 MHz

Impedance varied from 23.5 K to 13.3 K

A

(without absorber)

Fig 7.4

Frequency range 0.5 MHz to 1.5 MHz

Impedance varied from 23.5 K to 13.3 K

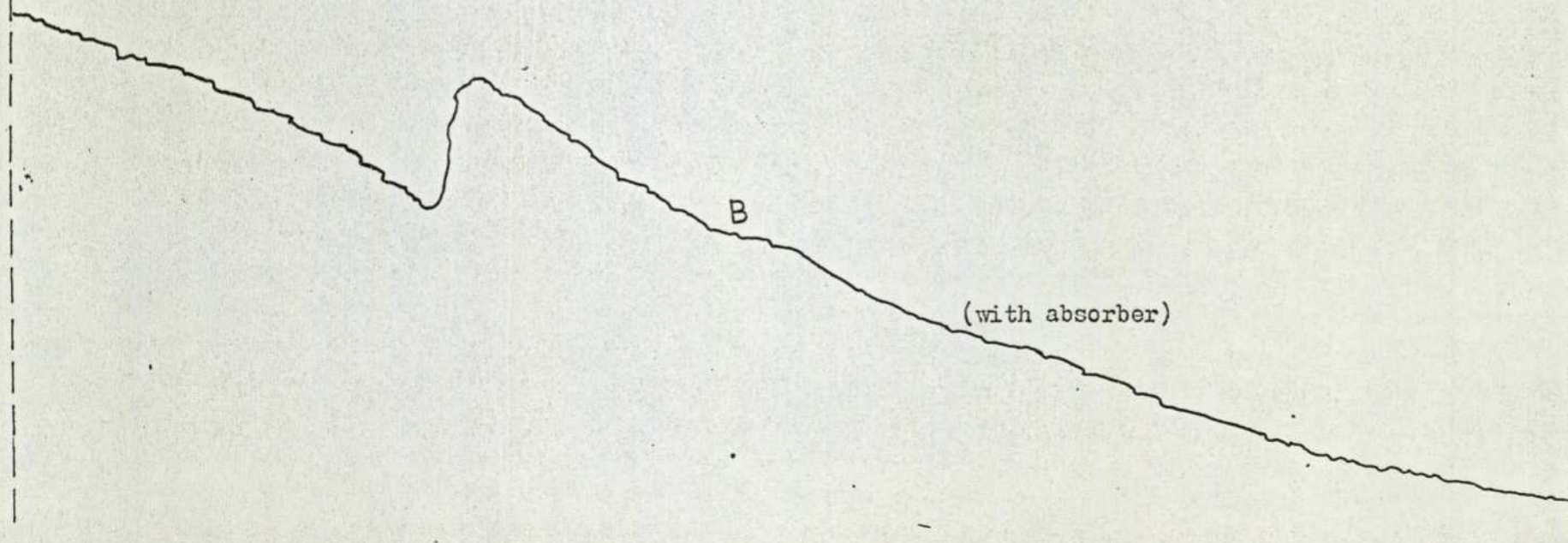


Fig 7.4

Frequency range 1.5 MHz to 4.5 MHz

Impedance varied from 13.3 K to 4.5 K

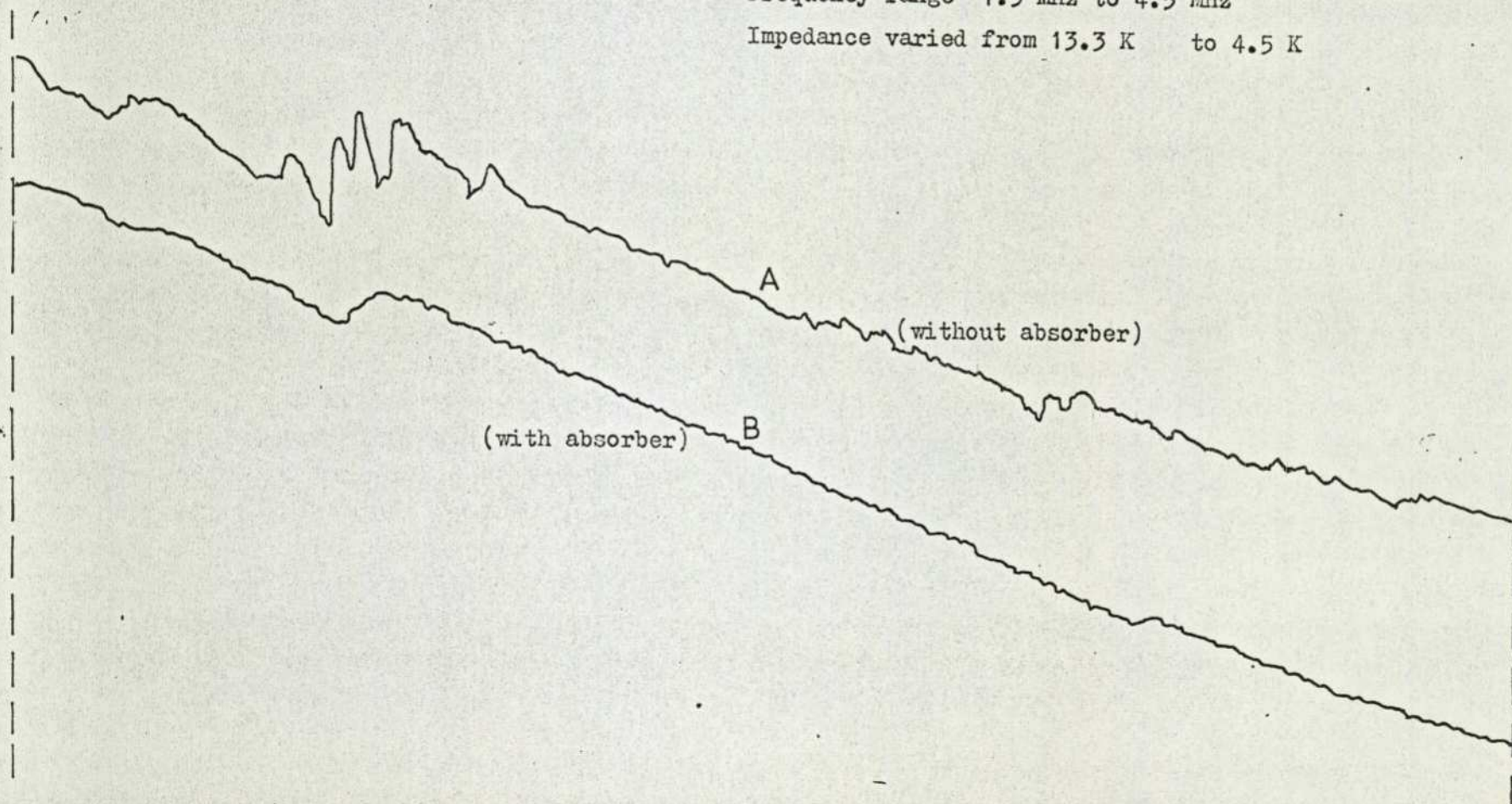


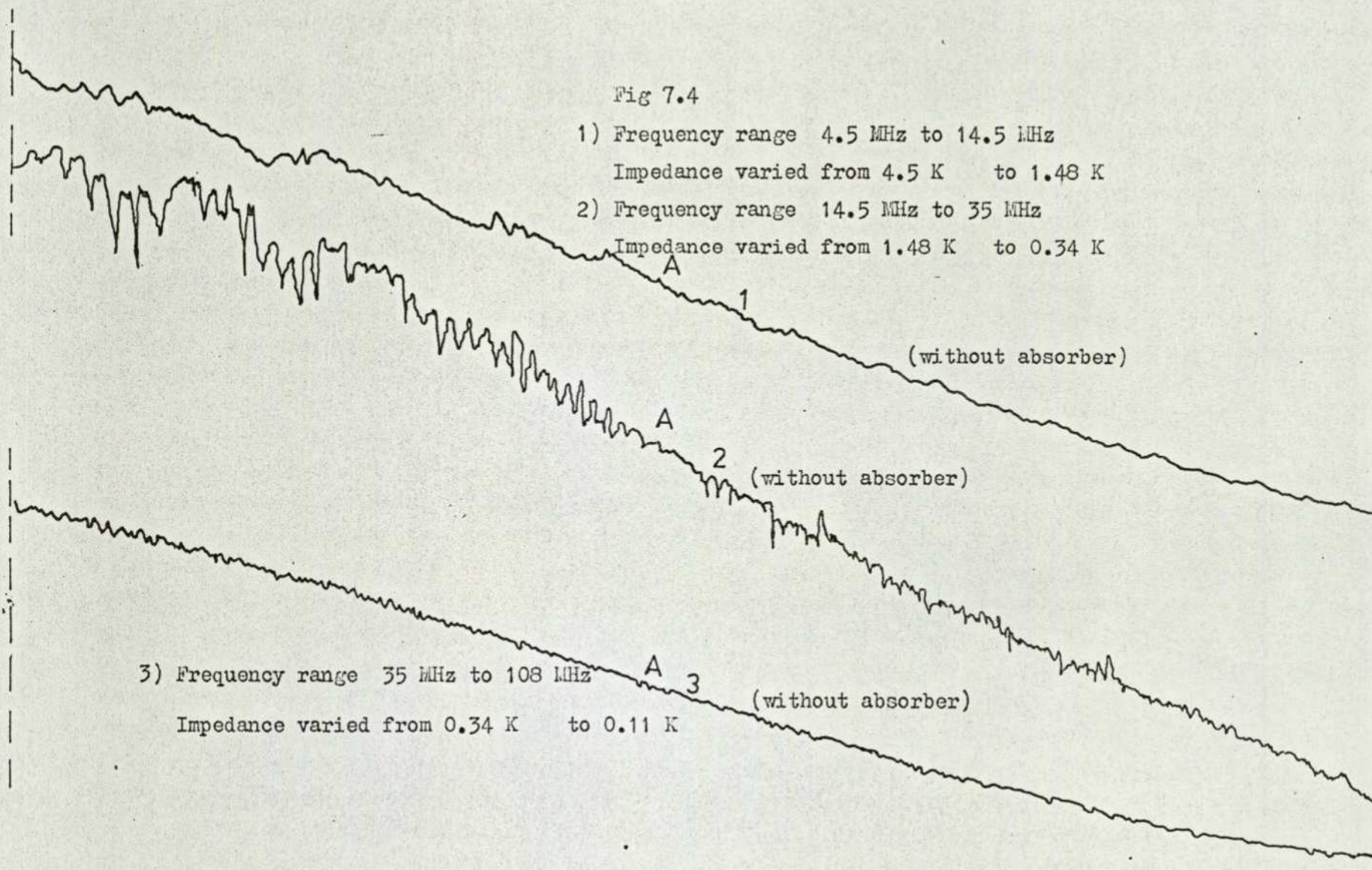
Fig 7.4

1) Frequency range 4.5 MHz to 14.5 MHz

Impedance varied from 4.5 K to 1.48 K

2) Frequency range 14.5 MHz to 35 MHz

Impedance varied from 1.48 K to 0.34 K



3) Frequency range 35 MHz to 108 MHz

Impedance varied from 0.34 K to 0.11 K

This deviation may be attributed to the uncertainty in the determination of the frequency at which resonance is occurring and also due to the device irregularities.

These observations clearly indicate that in the symmetrically electroded arrangement, two types of waves are produced by the application of an alternating voltage to the electrodes.

iii) Asymmetrical electrode arrangement

For the arrangement without the absorber at the base of the crystal, a large change in the impedance was observed at 0.64 MHz. Considerably smaller variations were also observed at about 1.77 MHz. All these variations were almost completely suppressed by the presence of the absorber at the base of the crystal indicating that these resonances were due to waves travelling along the Z direction. A random fluctuation of the impedance was observed in the range of 14.0 to 25 MHz. The magnitudes of these fluctuations were negligible when compared with the variation obtained at 0.64 MHz. In the other ranges of frequencies upto 108 MHz the impedance decreased gradually with an increase in the frequency.

iv) Qualitative analysis of the observations of the asymmetrical arrangement

If the resonance observed in the range 0.5 MHz to 1.5 MHz is due to a longitudinal wave in the Z direction then the wave length of the resonant wave is 6 mm (dimension of the crystal in the Z direction is 3 mm ref. chapter 6). Therefore, the resonant frequency should be 1.22 MHz as the velocity of this wave is 7330.6 m/s (47). However, if the resonance is assumed to be due to a shear wave along the Z direction, the resonant frequency can be calculated using the shear

wave velocity along the Z direction. This velocity is 3714 m/s (47). The resonant frequency of this wave is then 0.62 MHz. Experimentally a large increase in the impedance was observed at 0.64 MHz. Assuming this resonance is due to the shear wave, the deviation of the calculated and measured frequencies is about 5%. This may again be due to the uncertainty in the assumption of the frequency measurement. The other small variations of the impedances in the range of 0.5 MHz to 1.5 MHz may be due to the surface irregularities of the unpolished XY face of the crystal. The resonance observed at about 1.77 MHz may have been due to the third harmonic of the shear wave.

In the asymmetrically electroded arrangement, if the electrode spacing between the pairs of the electrodes ($2d = 344 \mu\text{m}$ ref. figure 6.11 Chapter 6) were chosen to be equal to an even multiple of the pitch of the closest pair electrodes ($67 \mu\text{m}$) then it is possible to launch surface waves along the face of the crystal. The wavelength of these waves is twice the pitch of the closest pair of electrodes. In the experimental arrangement this pitch is equal to $57 \mu\text{m}$. Therefore, the frequency required for surface wave generation is about 29 MHz. However, no resonance was observed experimentally at this frequency. This may have been due to the distance $2d$ not being an even multiple of the closest electrode pitch, as this relationship would be required to sustain the surface waves after it had been launched from the closely spaced electrodes.

7.3.1 Acoustic waves in a Y cut LiNbO_3 crystal

Detailed experimental analysis of the different types of waves propagating inside the crystal can be made only when all the faces of the crystal sample are well polished. This facilitates a uniform reflection of the waves at the crystal faces. The availability of the LiNbO_3 samples have restricted choice to the selection of a Y cut LiNbO_3 crystal and in the following sections the acoustic resonances observed in this configuration are discussed. The dimensions of the crystal are 1.35 cmx 0.3 cmx 0.72 cm along the X, Y and Z directions respectively (see fig.7.5). One of the XZ faces was polished (flatness is of the order of $\lambda/4$ where $\lambda = 0.53 \mu\text{m}$) and the other faces of the crystal were polished to give a reasonably smooth finish to the surface. In

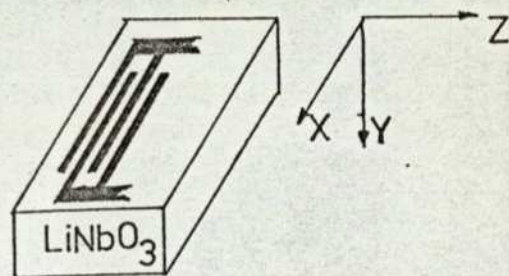


Fig. 7.5
Y cut crystal arrangement

order to study the effect of the MgF_2 film on the propagation of surface waves, a symmetrical electrode structure as described in section 7.2 was chosen.

A thin layer of MgF_2 was first deposited on the polished ZX plane of the crystal. The symmetrical electrode structure was then deposited on the exposed MgF_2 surface, with its fingers along the X axis. The crystal was mounted in the holder in a manner similar to that described in section 7.2 and the impedance was measured in the range of 0.5 MHz to 108 MHz without any external absorber attached to the crystal.

Impedance

Fig. 7-6 Y cut LiNbO_3 with
symmetrical electrodes

Frequency range 0.5 MHz to 1.5 MHz

Impedance varied from 16.2 K to 5.62 K

(without absorber)

Fig 7.6

Frequency range 0.5 MHz to 1.5 MHz

Impedance varied from 16.2 K to 5.62 K

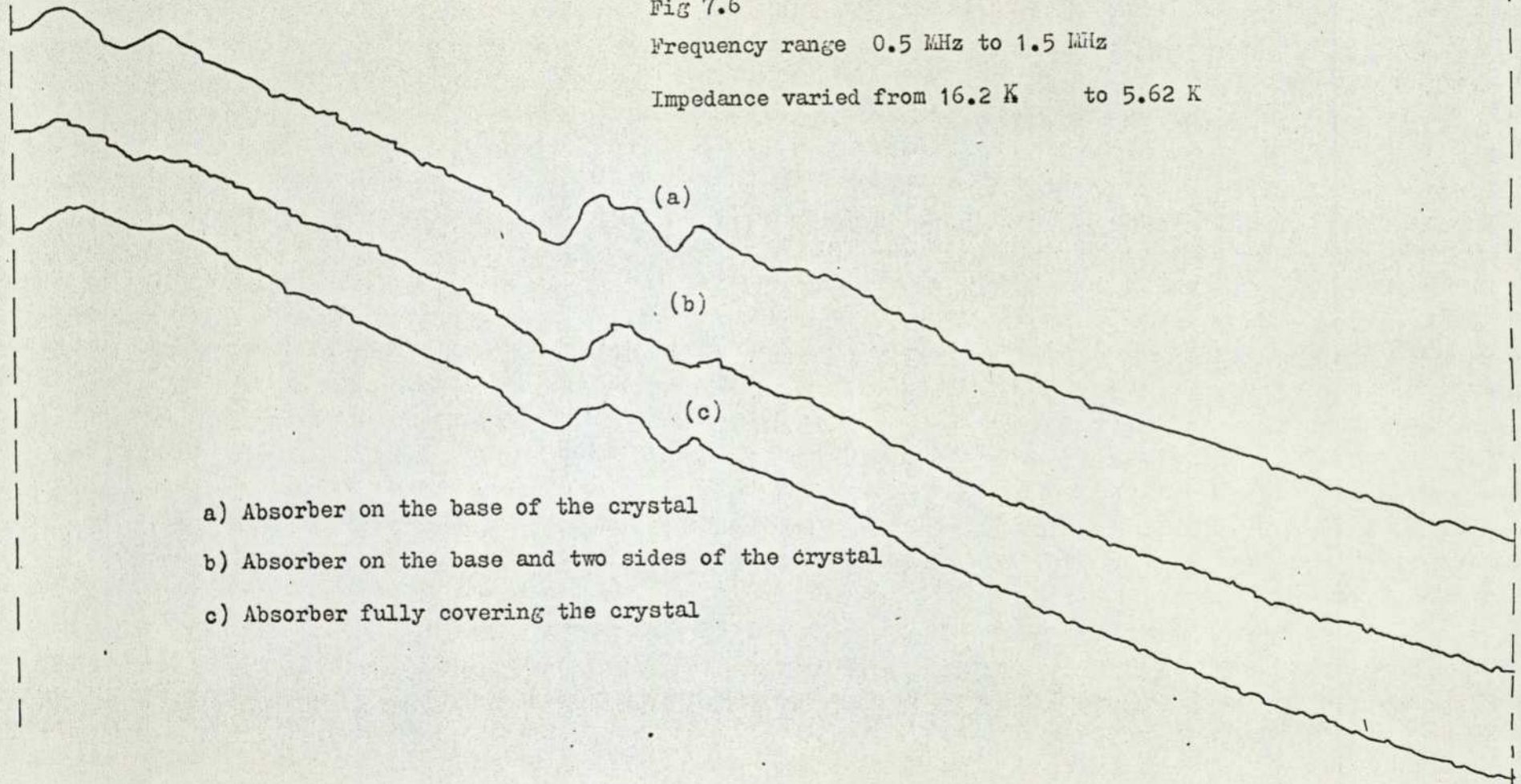


Fig 7.6.d

1) Frequency range 1.5 MHz to 4.5 MHz

Impedance varied from 5.62 K to 1.75 K

2) Frequency range 35 MHz to 108 MHz

Impedance varied from 0.26 K to 0.1 K

(without absorber)

(without absorber)

Fig 7.6 e

Frequency range 4.5 MHz to 14.0 MHz

Impedance varied from 1.75 K to 0.54 K

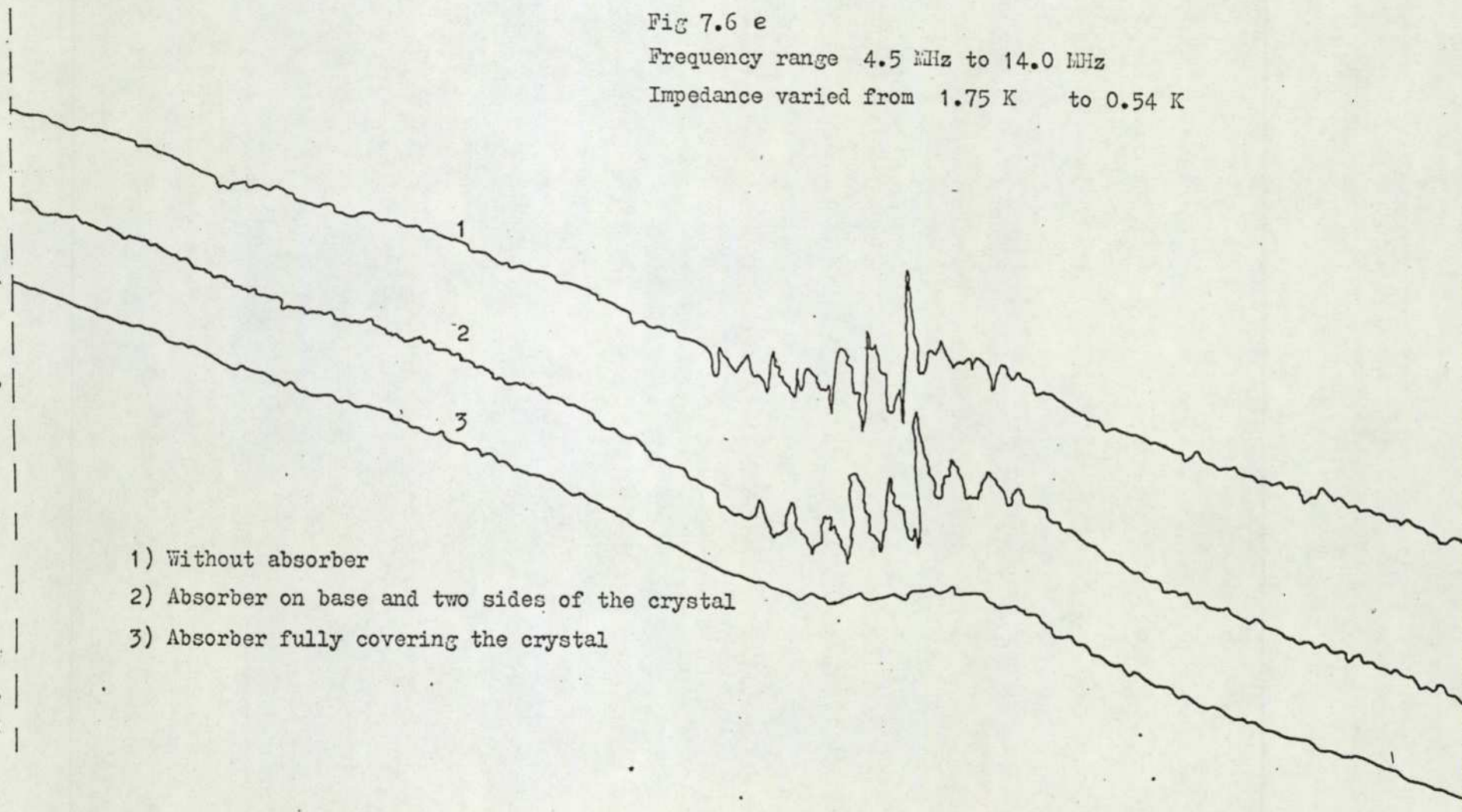
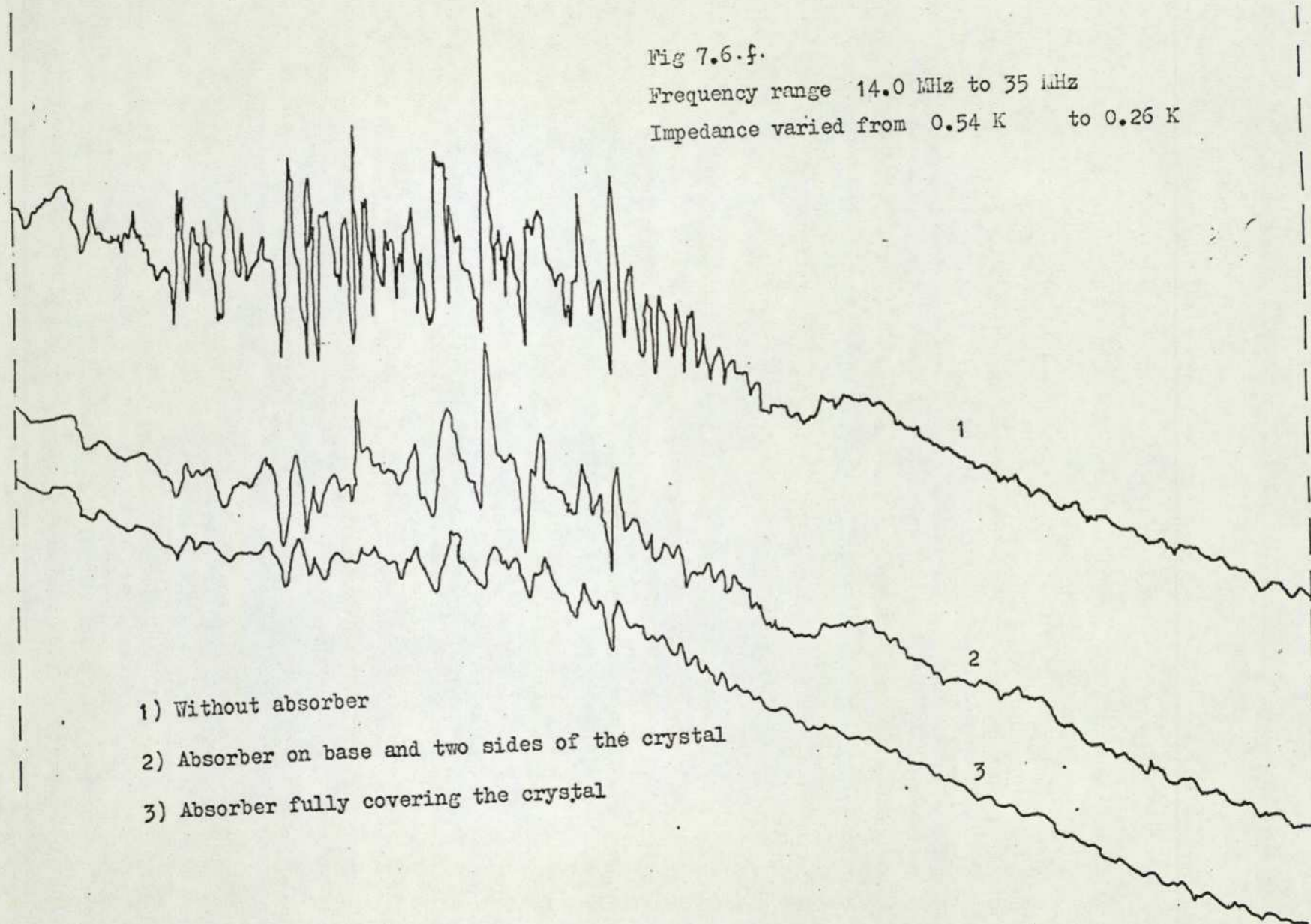


Fig 7.6.f.

Frequency range 14.0 MHz to 35 MHz

Impedance varied from 0.54 K to 0.26 K



In the frequency range of 1.5 to 4.5 MHz and 35.0 to 108 MHz the impedance varied uniformly. The experimental plots are given in figure 7.6d. Rapid variations of the impedance were observed in the other frequency regions and the following tests were carried out to isolate the resonances observed.

- i) With plasticine on the base of the crystal
- ii) With plasticine on the base and the two sides perpendicular to the Z axis
- iii) With plasticine completely covering all the six faces of the crystal.

In the frequency range 0.5 MHz to 1.5 MHz, the above three tests were carried out and the variation of the impedance is shown in figures 7.6 a, b and c. In the other two regions 4.5 MHz to 14.0MHz and 14.0MHz to 35.0MHz, tests with the arrangements described in (ii) and (iii) were carried out and the impedance variations are given in figures 7.6 e and 7.6f respectively.

The second arrangement will isolate any reflection of the waves taking place on the two faces perpendicular to the Z axis. In this arrangement the absorber was purposely put over the edge of the ZX face so that the reflection of the surface waves at the abrupt edge of the face may be nullified. By putting the absorber over the electrode surface (arrangement iii) the surface waves can be absorbed.

7.3.2 Summary of the observations for the Y cut LiNbO_3 crystal

Case (i) Frequency range 0.5 MHz to 1.5 MHz

A large variation of the impedance was observed at 0.76 MHz when the crystal was mounted inside the holder without any absorber. It can be seen from the plot (figure 7.6a) that small variations of the impedance

occur around 0.52 and 0.68 MHz. A small fluctuation of the impedance occurs at about 1.1 MHz. These variations were almost completely suppressed when the absorber was placed on the base (Test i) of the crystal. Therefore, these variations of the impedance must have been produced by waves travelling along the Y direction. In the arrangements described in tests (ii) and (iii), no appreciable variation of the impedance from that of test (i) was noted.

Case (ii) Frequency range 4.5 MHz to 14.0 MHz

In the arrangement with no absorber attached to the crystal, a fluctuation of the impedance was observed in the frequency range of 7.2 MHz to 8.8 MHz. The maximum variation of the impedance was obtained at 8.18 MHz. The fluctuation was not appreciably changed when measurements were made with the arrangements of tests (i) and (ii). However, with the third arrangement, where the absorber was placed on the electrode face the resonance observed was completely suppressed. Therefore, the resonance observed in this range may be attributed to surface waves on the crystal face.

Case (iii) Frequency range 14.0 MHz to 35.0 MHz

The impedance in the range of 14.0 MHz to 25.0 MHz fluctuated when measurements were made without any absorber. The magnitudes of the fluctuations were about one hundredth of that observed at 0.75 MHz. No appreciable change in the fluctuations were observed in tests (i) and (ii). However, when the crystal was completely covered by the absorber, the fluctuations were appreciably reduced.

7.3.3 Qualitative analysis of the observations with the Y cut LiNbO_3 crystal

If the resonance observed at the frequency of 0.76 MHz is assumed to be due to a longitudinal (quasi-longitudinal) wave along the Y direction, the wavelength of this wave is approximately equal to 0.6 cm. The velocity of a quasi-longitudinal wave along the Y axis is 6837.9 m/s (47). Therefore, the resonant frequency is 1.14 MHz. This is not in agreement with the experimental observation. However, if the wave is assumed to be a quasi-shear wave along the Y direction, the velocity of it being 4466.7 m/s (47), the calculated resonance frequency is 0.74 MHz. The deviation in the frequencies measured and calculated is about 2%. Therefore, the resonance occurring at 0.76 MHz may be attributed to the quasi-shear wave travelling along the Y direction of the crystal.

The resonance observed at 8.18 MHz was not affected by the presence of the absorber on the base and the two faces perpendicular to the Z axis. However, the resonance was completely suppressed by the presence of the absorber on the electrode face. The electrode pitch used in the experiment is 0.217 mm (ref. section 7.2.4). The surface wave velocity along the Z direction is 3490.0 m/s (60). Therefore, the frequency of the surface wave is 8.23 MHz. This is in close agreement with the experimental observations.

7.3.4 Effect of MgF_2 on the generation of surface waves

In the arrangements so far considered, a thin film of MgF_2 was deposited on the crystal face prior to the deposition of the electrodes. In this section the effect of the MgF_2 film on the generation of the surface waves is considered. The crystal was washed in concentrated

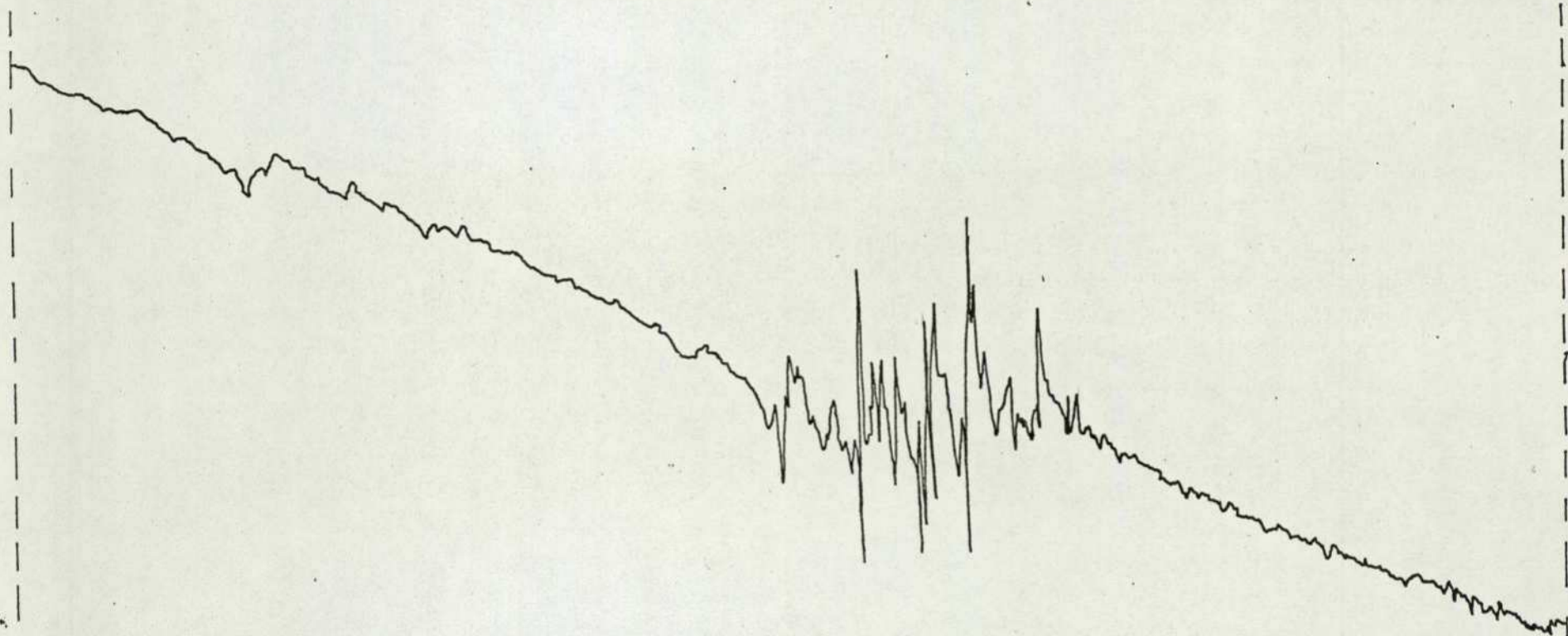


Fig. 7.7 Y cut LiNbO_3 with symmetrical electrodes

This arrangement is without the MgF_2 film

Frequency range 4.5 MHz to 14.0 MHz

Impedance varied from 1.8 K to 0.56 K

Fe_2Cl_3 solution to remove the MgF_2 layer. The interdigital electrodes used in section 7.3.1 were deposited directly on the crystal face. The crystal was mounted inside the crystal holder and the impedance was measured in the frequency range of 4.5 MHz to 14.0 MHz. The variation of the impedance in this frequency range is given in figure 7.7. It can be seen that the resonances (or the fluctuations) observed in the range of 7.0 MHz to 8.8 MHz are very much larger than that observed in the crystal with the MgF_2 layer. The maximum variation of the impedance was again at a frequency approximately 8.18 MHz.

The experimental observations of the devices so far considered suggests that:

i) In the Z cut crystal, with the symmetrical electrodes in the XY plane and with electrode fingers parallel to the X axis, two types of waves are produced by the application of an alternating voltage to the electrodes.

a) Bulk waves which can be absorbed almost completely by a suitable absorber at the face parallel to the electrode face.

b) Surface waves on the crystal face.

ii) In the Z cut crystal, with assymmetrical electrodes on the XY plane of the crystal and with electrode fingers parallel to the X axis, only one type of wave is produced, namely Bulk waves inside the crystal. These waves may be absorbed by putting an absorbing medium on the face parallel to the electrode face.

iii) In the Y cut crystal with symmetrical electrodes on the ZX plane two types of waves are produced:

- a) Bulk waves which can be absorbed by a suitable absorber placed on the face parallel to the electrode face.
- b) Surface waves on the electrode face. The surface waves were found to be completely absorbed by an absorber on the electrode face.

iv) Comparison of the surface wave resonance observed in the Y cut crystal, with and without the MgF_2 film, indicates that the thin film of MgF_2 on the ZX plane of the crystal reduces the surface wave resonance. Reasons for this may be postulated but no attempts were made to analyse this effect in detail.

7.4 Optical effect due to acoustic resonance

The Y cut LiNbO_3 crystal as described in section 7.3.1 was arranged in the manner described in Chapter 5 (figure 5.4.2). The incident optical beam was polarised with the E vector along the plane of the optic axis. The glancing angle of the incident beam with the X direction was adjusted to be equal to 2.5° . The performance of the device was examined by a net-work analyser. The experimental arrangement is schematically represented in figure A.2 of Appendix A. The photomultiplier was positioned at the centre of the zero order beam as described in Chapter 5. The net-work analyser compares the electrical input signal to the device and the output from the photomultiplier. The net-work analyser was set at 0.8 MHz symmetrical sweep with a band width of 0.1 MHz. The output of the analyser was recorded on an XY plotter. The X and the Y ordinates corresponds

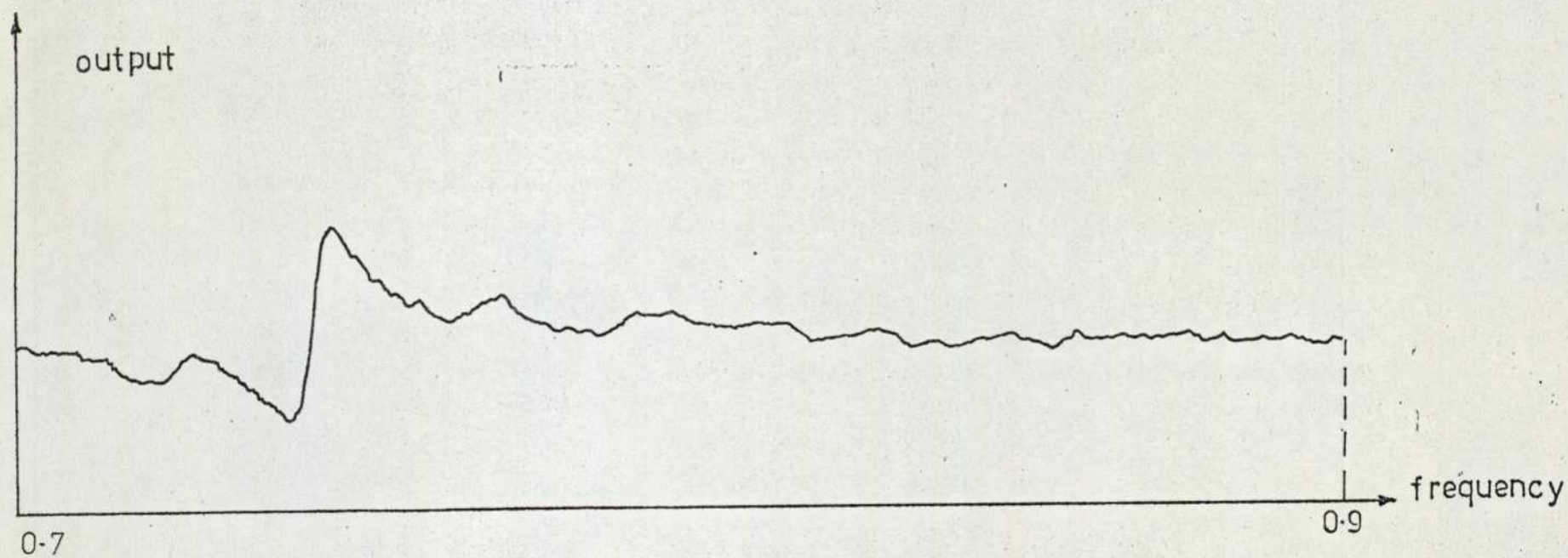


Fig. 7.8

Net-work analyser output

to the frequency and the output of the analyser respectively. The experimental plot is shown in figure 7.8. It is apparent from this plot that a resonance is present at about 0.78 MHz. With this arrangement a resonance of the electrical impedance was observed at about 0.76 MHz. Therefore, it could be concluded that the optically observed resonance was due to the acoustic resonance produced inside the crystal.

7.5 Performance of the device at a.c. and d.c. applied voltages

In deriving the expressions for the field components (Chapter 4) in the high frequency case, it was assumed that the potential distribution inside the crystal is governed by the equivalent capacitive network. Accordingly, the relative dielectric constants along and perpendicular to the optic axis were taken as 28.6 and 84.6 respectively. These were the values supplied by the manufacturers. However, due to the lack of information of the resistivities, in the d.c. case, it was assumed that the resistivities are equal in all directions. Hence, it is important to verify the validity of this assumption.

A simple experiment was undertaken to compare the performance of the device at a.c. and d.c. applied voltage. The experimental arrangement was again similar to that shown in Chapter 5 (figure 5.4.2). The angle of incidence was adjusted to 88° . The photomultiplier arranged in the manner described in Chapter 5, was used to detect the intensity of the zero order beam.

A 30 Volts 0.5 MHz signal was applied to the electrodes, and the output of the photomultiplier was measured by an oscilloscope. The alternating voltage supply was isolated and a d.c. voltage supply was

connected to the electrodes. The voltage applied to the electrodes was gradually increased until the decrease in the intensity of the zero order beam was equal to that produced in the a.c. condition. The voltage required was 24 volts.

The energy removed from the zero order is a function of the applied voltage (ref. Chapter 5). The application of a voltage to the interdigital electrodes impresses a periodic phase change on the transmitted optical beam, and a diffraction pattern is obtained in the far field. If the decrease in the energy from the zero order is the same when the applied voltage is 30 volts (peak) a.c. or 24 volts d.c., then the phase change impressed in both these arrangements should be the same at the two voltages. Figure 7.9 shows the variation of the phase changes along the Y axis for the above two cases. The phase changes were calculated from equation 3.33. In the a.c. condition, equations 4.27 and 4.28 were assumed to represent the E_y and E_z component of the field. Justification that at this frequency the whole of the equivalent network may be described in terms of the dielectric constants as assumed in equations 4.27 and 4.28 is given by the phase reading at this frequency of -90° , indicating a capacitive network. In the d.c. condition equations 4.27 and 4.28 (ϵ_2 and ϵ_3 omitted) were assumed to be the E_y and E_z component of the field inside the crystal. It is apparent from the figure 7.9 that the changes in phase in the a.c. and d.c. conditions are approximately equal. Therefore, the diffraction effect produced in the two cases must be the same.

In the derivation of the equation 4.22, it was assumed that the resistivities of the crystal are the same in all

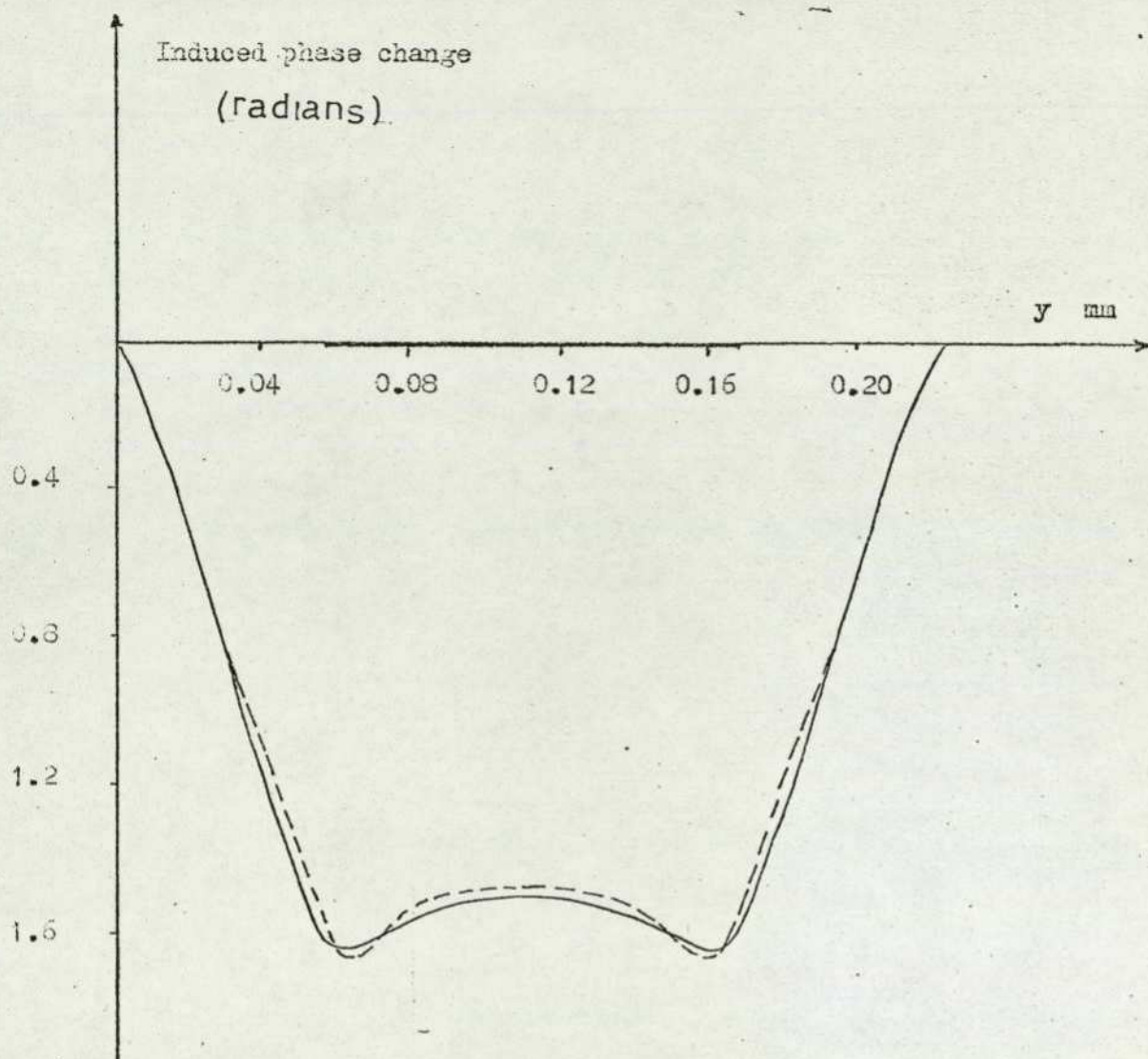


Fig 7.9 Induced phase grating profile

- Isotropic case (Applied voltage = 24 V)
 - - - - Anisotropic case (Applied voltage = 30 V)

directions. Comparison of the performance of the device in a.c. and d.c. conditions indicates that this assumption is reasonable.

MULTIREFLECTION ELECTRO-OPTIC MODULATOR

INTRODUCTION

Experimental observations with internal reflection modulators, described in Chapters 5 and 6 show that the zero order extinction voltage decreases with an increase in the angle of incidence. However, when the angle of incidence is large, practical limitations are encountered in aligning the modulator. An extension of the principle of single internal reflection modulator to a multireflection modulator to give a low operating voltage at a comparatively small angle of incidence is described in this chapter. Initial observations with a double reflection modulator gives about 50% reduction in the zero order extinction voltage.

8.1 Theoretical consideration of the multi-internal reflection modulator

Theoretical derivations and the experimental observations discussed in Chapters 4,5 and 6 show that a periodic variation of the refractive index inside an electro-optic medium may be produced by the application of a potential difference to the interdigital electrodes deposited on to the crystal face. If the electrode structure is assumed to be symmetrical, as considered in Chapter 5, then the components of the fringing field are given by

$$E_x = 0$$

$$E_y = -4V \sum_n \frac{\sin(an\pi)}{n\pi a n\pi} e^{-\frac{n\pi}{d}z} \frac{n\pi}{d} \cos \frac{n\pi}{d}y \quad 8.1$$

$$E_z = 4V \sum_n \frac{\sin(an\pi)}{n\pi a n\pi} e^{-\frac{n\pi}{d}z} \frac{n\pi}{d} \sin \frac{n\pi}{d}y \quad 8.2$$

where

$$n = 1, 3, 5, \dots$$

These expressions are with respect to the coordinate system shown in figure 3.1 where the electrodes are deposited on the polished XY face of the electro-optic crystal, LiNbO_3 , with the electrode fingers in the X direction. $2V$ is the applied d.c. voltage, d is the electrode pitch and $2ad$ is the spacing between the electrodes.

From these equations it can be

seen that the E_z and the E_y components of the field decreases exponentially with an increase of z . The computer plots given in Chapter 4 (figures 4.5 and 4.6) indicates that, typically, when z is equal to $d/2$, the magnitudes

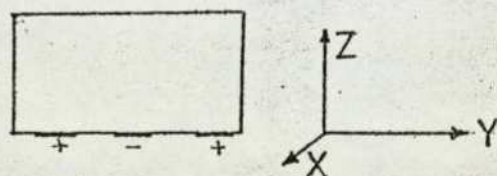


Fig. 8.1

Modulator configuration

of these components are approximately equal to one quarter of that at the crystal electrode interface.

The fringing field inside the crystal produces a periodic variation of the refractive index inside a properly oriented crystal. When a suitably polarised beam of light encounters this periodic variation of the refractive index, a periodic phase change will be impressed on the transmitted beam and thus will be diffracted into different orders. From the diffraction theory it can be shown that the inter order angle will be increased by decreasing the electrode pitch. However a decrease in the electrode pitch will decrease the penetration of the electric field inside the crystal. Therefore in order to take advantage of the high field intensity region, the incident optical beam has to be focused to travel very close to the base of the crystal. Modulators operating on this principle have been reported by St. Ledger and Ash (18). Although the arrangement lends itself to integrated circuit applications, as a bulk modulator the device is possibly limited due to practical alignment difficulties.

Alternatively, in order to ensure that the incident optical beam experiences the intense field region, it may be reflected off the crystal electrode interface. An account of the investigations with an internal reflection modulator is given in Chapter 5. The operating characteristics of a diffraction based modulator are generally characterised by the zero order extinction voltage and/or the voltage required to deflect the incident energy to the higher orders. It can be seen from section 5.4.4.(Chapter 5) of this thesis that the zero order extinction voltage decreases with an increase in the angle of incidence.

Consider the internal reflection modulator arrangement of figure 8.2.

AB denotes the effective field region. When the angle of incidence θ_i is decreased, the effective interaction length of the optical beam is decreased.

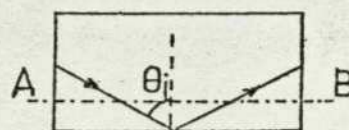


Fig- 8-2

Single reflection arrangement

Therefore in order to impress an equivalent amount of phase change on the optical beam, the applied voltage should be increased. However if the optical beam is allowed to undergo two reflections, as shown in figure 8.3 the total

interaction region is increased and therefore the total phase change impressed on the optical beam is increased.



Fig- 8-3

Double reflection arrangement

Consequently the voltage required for the zero order extinction is decreased. The total number of reflections that the optical beam can undergo will be determined by the diffraction effect inside the crystal.

In order to experimentally verify this effect, a modulator was constructed with a thin LiNbO_3 crystal and the initial investigations with this device are given in the following sections.

8.2 Variation of the zero order extinction voltage with angle of incidence

A thin LiNbO_3 crystal dimensions $28.5\text{mm} \times 4\text{mm} \times 0.72\text{mm}$ was taken as the modulating material. These dimensions were along the X, Y and Z directions respectively (figure 1 page 202). The two faces

perpendicular to the Z axis from which reflection occurs were polished. (Ref. Chapter 9 for the detailed description of the polishing procedure). The two faces perpendicular to the X axis were also polished for the light transmission. To prevent the generation of the subdiffraction pattern by the presence of the two different reflection boundaries (crystal-electrode and crystal-air; Ref. Chapter 5), a thin layer of MgF_2 (thickness $\approx 0.6\mu\text{m}$) was deposited on the faces perpendicular to the Z axis. Symmetrical interdigital electrodes of electrode pitch 0.2 mm and electrode spacing of 0.1mm were deposited on one of the magnesium fluoride films. The lengths of the electrodes were 22mm and were in the X direction (figure 1b, page 202).

The variation of the zero order extinction voltage with angle of incidence was studied. The experimental arrangement used for this measurement is similar to that described in section 5.4.4 of Chapter 5 and is shown in figure 8.4.

The incident light was polarised with the E vector along the plane of incidence and the intensity of the zero order beam was measured by a photomultiplier carrying a straight slit. The photomultiplier was positioned such that the length of the slit was along the centre of the zero order beam. The voltage applied to the crystal was gradually increased until the photomultiplier reading was minimum (zero). The procedure was repeated for a varying angles of incidence and the variation of the zero order extinction voltage with the glancing angle (θ) is given in figure 2 (page 202). It is seen from this

graph that the zero order extinction voltage decreased with an increase in the angle of incidence. This observation is in agreement with the theoretical prediction.

8.3. Comparison of the variation of the zero order intensity with applied voltage for the single and double internal reflection arrangements

In order to study the effect of the double reflection, the incident beam was initially allowed to undergo one internal reflection inside the crystal and the modulation of the zero order beam energy with applied voltage was measured.

The experimental arrangement was similar to that shown in figure 8.4. The angle of incidence was adjusted to be equal to 79.5° . This angle was chosen so that it would be possible to also obtain two reflections off the crystal electrode interface at the same angle of incidence by properly positioning the crystal. The intensity of the zero order beam at zero applied voltage was measured and taken as I_0 .

The voltage applied to the electrodes was gradually increased and the corresponding intensities, I_v , were measured. Graph b of figure 3 in the following paper (pages 202) shows the variation of the ratio I_v/I_0 with voltage when the optical beam undergoes one internal reflection. This paper gives a summary of the multi internal reflection modulator.

The crystal was then positioned so that the incident beam suffers two internal reflections for the same angle of incidence (79.5°). The variation of the zero order intensity with voltage was again measured. Graph c of figure 3 (page 202) shows the variation of

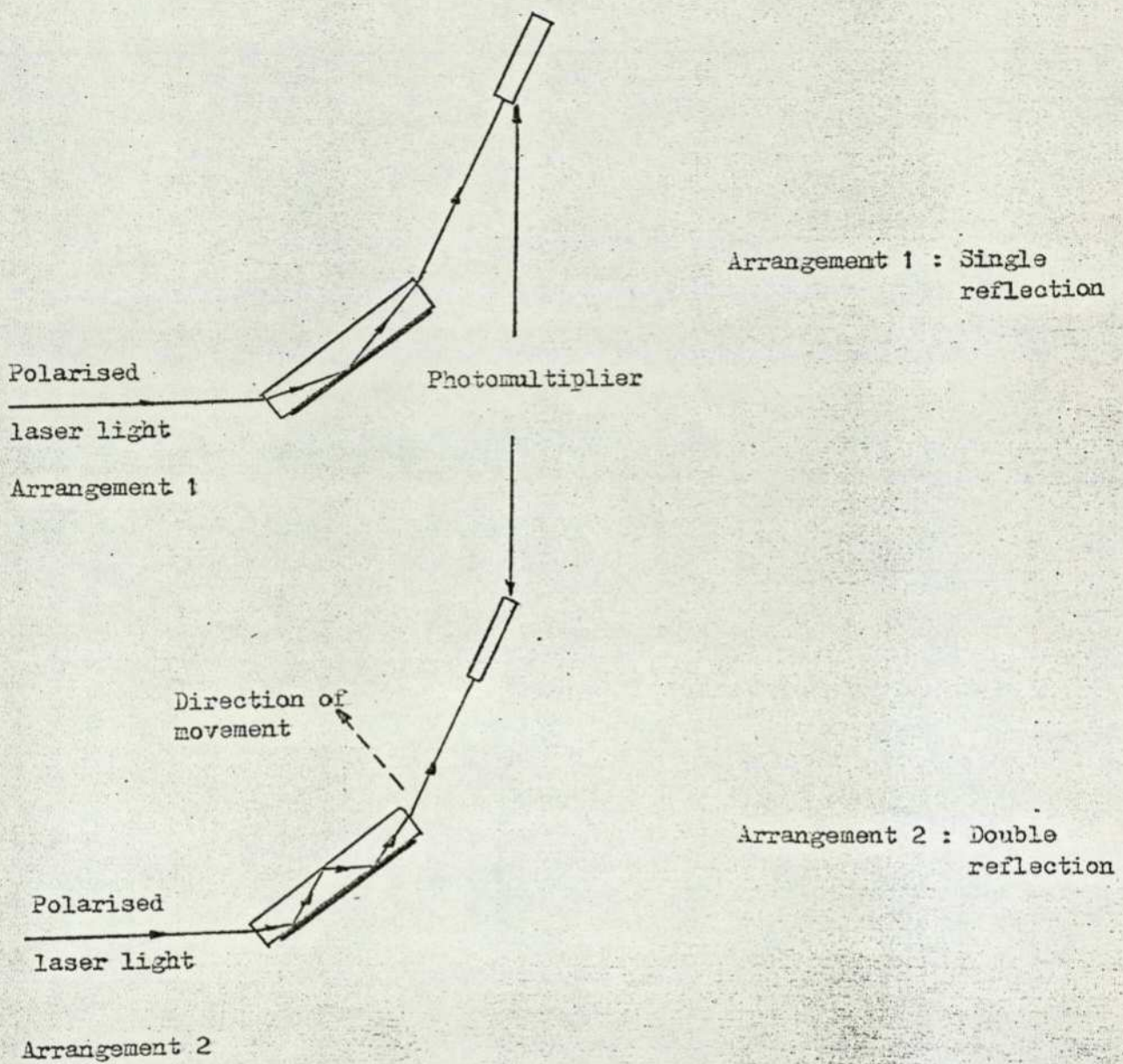


Fig 8.4 Optical arrangement for the single and double reflection configurations

I_v/I_o with voltage when the optical beam undergoes two internal reflections. Comparison of these two graphs shows that the voltage required to produce minimum intensity in the zero order is approximately halved when two internal reflections take place on the crystal electrode interface.

The final section of the paper summarising the performance of the multireflection device (pages 205) gives an account of the advantages of this device.

LETTER TO THE EDITOR

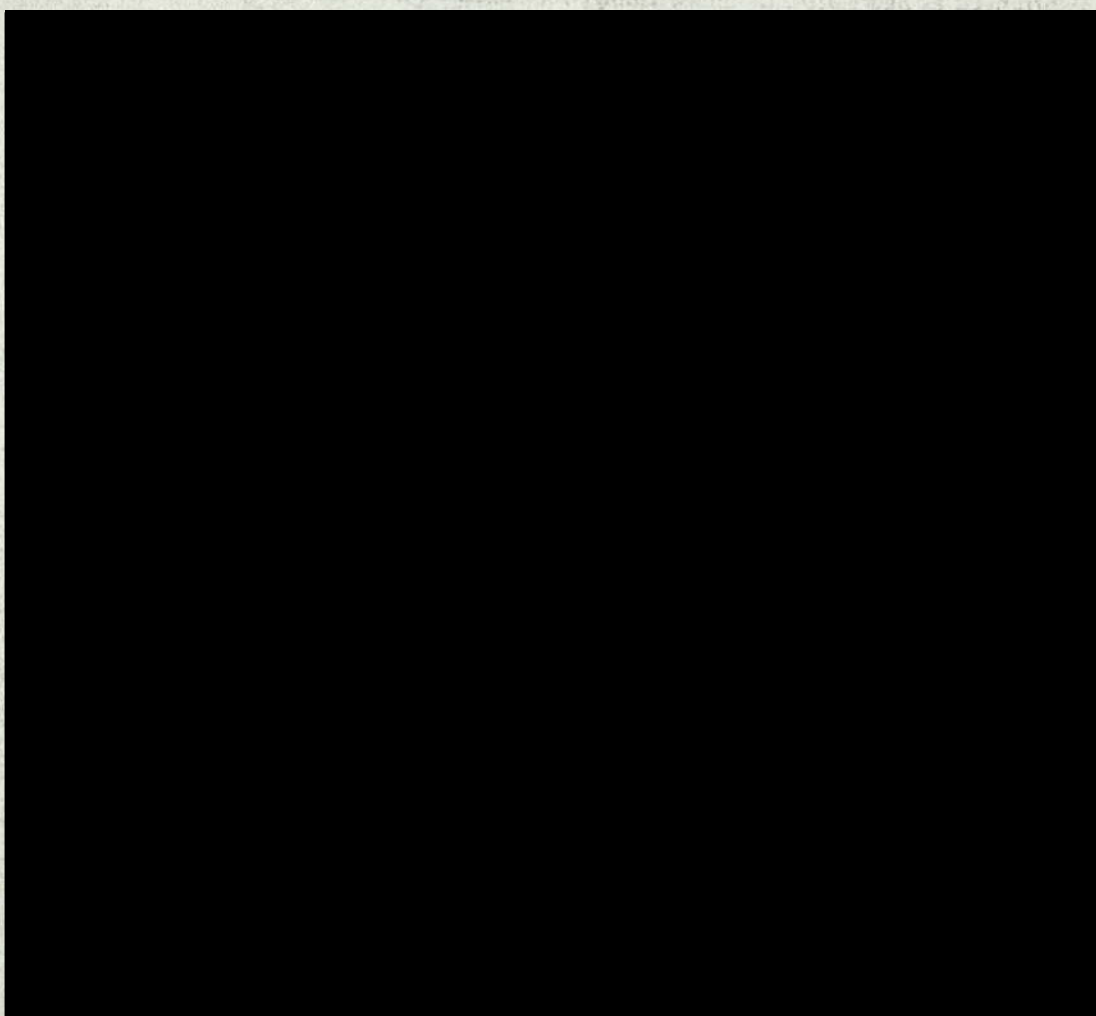
An improved diffraction-type electro-optic modulator/deflector

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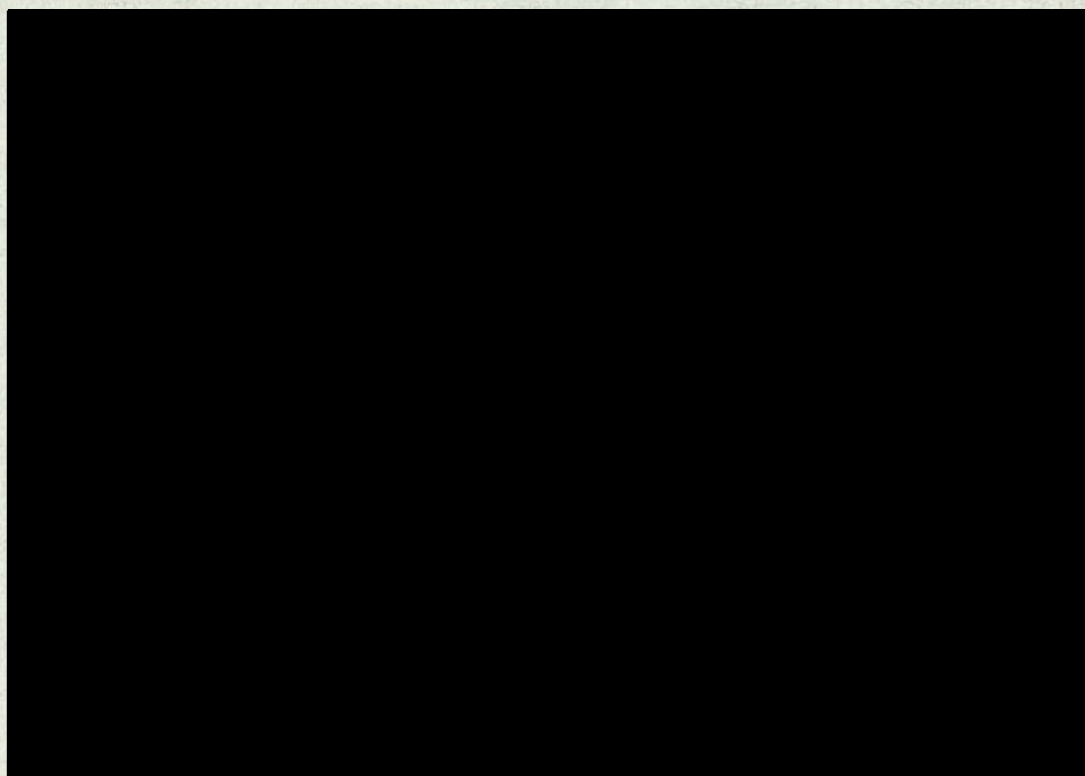
Received 9 September 1975

Abstract. A low-voltage diffraction-type modulator using multireflection of the optical beam off the crystal-electrode interface is discussed. Experimental observations with a double-reflection modulator gives 50% reduction of the zero-order extinction voltage. This approach could be extended to give a further reduction in the operating voltage.









PREPARATION OF THE EXPERIMENTAL DEVICE

INTRODUCTION

The devices discussed in the earlier chapter give promise to a variety of applications such as beam steering, Q switching and Mode-locking of a laser cavity (Ref. Chapter 10). A device which is utilised in an application should be easy to manufacture. Hence it is appropriate that a considerable amount of emphasis is given to the production of these devices. The devices considered in this thesis were produced at The City University. In this chapter the preparation of the experimental samples considered in the earlier chapters are discussed in some detail. The problems of practical limitations encountered in developing these devices and suggestions for the elimination of these limitations are discussed in detail.

9.1 Production of the experimental samples

The electro-optic crystal considered in this thesis is LiNbO_3 , a uniaxial crystal. The physical properties of this crystal indicates that it is relatively hard (hardness 6 moh), non hygroscopic, and resistant to most chemicals. The material is also highly temperature resistant. The Curie temperature is 1483°K . Devices used inside a laser cavity (eg Q switching or Mode locking) are required to be accurately polished and the preparation is highly critical, to minimise the losses of the device. However when the device is used external to the cavity (eg. modulator or deflector) the losses may not be so critical and therefore the tolerances on the device accuracy may be relaxed. The devices described in the earlier chapters were used externally of the laser cavity. In the following sections the preparation of these devices are discussed.

9.1.1. Device requirements

The two arrangements considered in the earlier chapters were

- i) Symmetrically electroded internal reflection light modulators
- ii) Asymmetrically electroded internal reflection light deflectors

In the first arrangement, the electrodes can be deposited, either parallel or perpendicular to the optic axis of the crystal (the Z axis Ref. Chapter 3). Both these arrangements will give rise to a symmetrical periodic perturbations of the refractive index in the crystal. However in the second arrangement when the electrodes are asymmetrically situated on the crystal face the perpendicular component of the electric field follows an approximate sawtooth variation.

Therefore the electrodes should be deposited on the XY surface of the crystal so that the r_{33} coefficient dominates the change in the refractive index. The first stage of producing the device is the production of a suitably oriented crystal. In the following section the production of a crystal with the Z axis perpendicular to the plane of the electrodes is considered. A similar procedure could be adopted for crystal samples of different orientations.

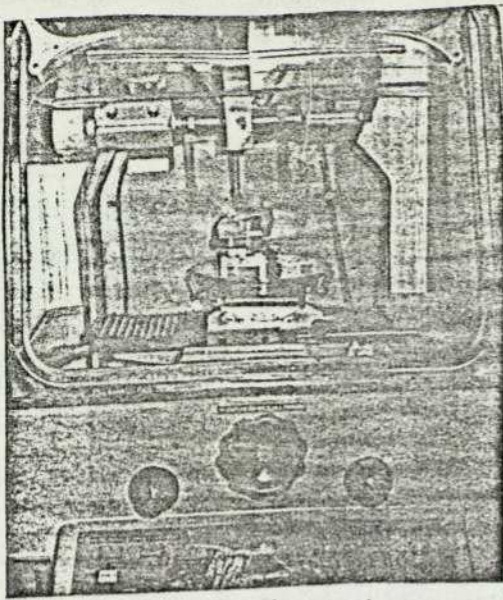
9.1.2. Crystal cutting

LiNbO_3 crystals are generally grown by Czochralski method with the Z axis of the crystal along the axis of growth. The crystal boule is cylindrical in shape. In the samples obtained, it was seen that two protrusions were present along the length of the crystal boule. The lines joining the centre of a circular section to the positions of the protrusions made an angle of 120° with each other and the radius vectors approximately corresponded to either of the Y axis of the crystal. The crystal was mounted rigidly on the horizontal platform of the crystal cutting machine (Meyer and Burger Ltd.,) (See Fig. 9.1). The plane of the platform was adjusted so that the Z axis of the crystal was perpendicular to the plane of the saw. The cutting saw of the machine was a circular blade whose edge was impregnated with diamond powder. The rate of rise of the horizontal platform was adjusted so that chipping of the sample is prevented while cutting. During the cutting process, a lubricant (Castrol Ilogrind 483) was sprayed along the cutting edge. The lubricant further absorbed the undue heat evolved during the cutting.

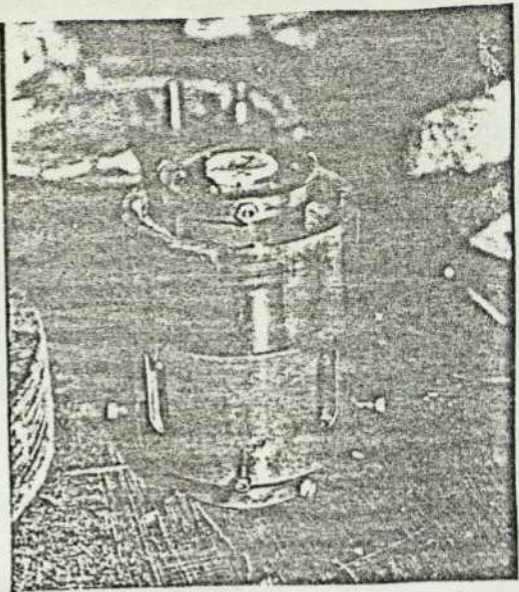
The exact direction of the axes of the crystal slice was determined by conventional X-ray diffraction techniques. The slice was then cut into a rectangular parallelepiped with faces perpendicular to the X and Y axis.

9.1.3. Crystal polishing

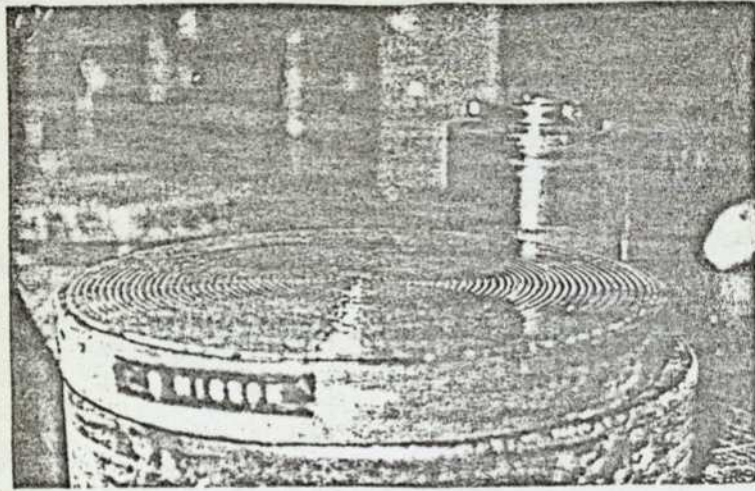
The crystal was polished by the polishing machine available in this department. The base of the crystal (XY face) was first polished. The crystal was held in position inside a rectangular slot in a brass block by paraffin wax with the XY face exposed. The brass block was then attached to the chuck of the polishing machine (Fig.9.1). The polishing plate was a lead lap and hyprez lapping compound (Engis Ltd.,) was used as the polishing/lubricating agent. First the lubricant with larger diamond powder ($14\text{ }\mu\text{m}$) was sprayed on the polishing lap. The pressure exerted on the crystal and the speed of revolution of the lap were adjusted to give reasonably smooth crystal surface. The polishing lap was then changed and a spray containing $1\text{ }\mu\text{m}$ diamond powder was used to give the final polish of the crystal surface. Alternatively Ceriroughe powder (Thorium Ltd) with water could be used as the lubricant. No scratches on the polished surface could be detected by the eye and the flatness of the surface was checked using an interferometer. The interference pattern contained only one half of a fringe across the whole crystal face ($25\text{mm} \times 5\text{mm}$).



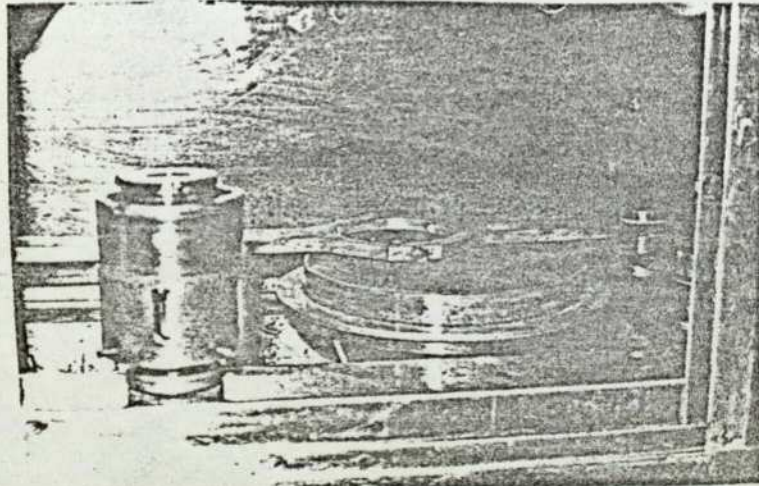
Crystal cutting machine



Polishing machine chuck



Lead polishing lap



Crystal polishing machine

Fig. 9.1

This indicated that the flatness of the surface is $\lambda/4$ across the face of the crystal. The incident wavelength was $\approx 0.53 \mu\text{m}$

A similar procedure was adopted to polish the two faces perpendicular to the X axis.

The crystal polishing was attempted using a pelon pad instead of a lead lap. This quickly produced a polished surface but when viewed under the interferometer, the face appeared to be curved and about eight circular fringes were present in the interference pattern.

9.2 Preparation of the sample for deposition

The polished crystal was first washed in acetone for four hours.

This removed any organic contamination on the crystal. The crystal

was then washed in running tap water for 30 minutes. The crystal

was dried and put in a bath of potassium dichromate solution with concentrated H_2SO_4 and was left in it for 12 hours. This removes

any inorganic contamination (salts) from the crystal. The sample

was taken out from the solution and the cleaning agent was allowed

to drip off from the crystal. Glass slides were used to hold the

crystal since the solution attacks metals. This solution is highly

corrosive and any contact to the skin or to cloth should be avoided.

It is important that the crystal is almost free from the solution,

otherwise when put in water, due to the evolution of heat ($\text{H}_2\text{SO}_4 + \text{H}_2\text{O}$)

the crystal will shatter. The crystal was washed in running deionised

water for two hours and then dried in a stream of flowing air. The

crystal was then baked in an oven for an hour at a temperature of

100°C . Care should be taken to avoid any form of contact with the

polished surfaces of the crystal, especially the surface on which the

electrodes are to be deposited.

9.3 Preparation of the Photo-mask

The electrode structure was etched on the crystal face by conventional photolithographic techniques (Ref. section 9.5). The photo-mask required for this was produced by photo-reduction of an enlarged version of the required electrode pattern. The master pattern was prepared by cutting on a Kimoto Strip film, procured from speciality Products. The dimension of the 'master' was about 200 times larger than that of the required size. The original pattern was then photo-reduced, in stages, to the required dimension. If the reduction is done in two stages, then the reduced pattern is a replica of the original, but if the reduction is done in three stages then the pattern obtained will be the negative of the original. The masks used in the production of the devices considered in this thesis were produced in two reductions. When the master is reduced, any flaws in the original will be greatly reduced and can usually be neglected. The masks required for the symmetrical electrode structures were made on an Ilford N4E50 film. For the asymmetrical arrangement a Kodak High Resolution plate was used.

9.4 Deposition of MgF_2 film and the electrode metals

9.4.1 Deposition of MgF_2 film : A molybdenum boat was used to evaporate MgF_2 pellets (BDH chemicals). The pellets used varied in size (4-40 mesh). The crystal was clamped inside the vacuum chamber (Edwards High Vacuum Ltd) by metallic clips at about 20cm from the boat. The oscillating crystal of the thin film thickness monitor was

clamped adjacent to the crystal and at the same distance from the boat. The chamber was first evacuated by a rotary pump. This gives a pressure of approximately 0.005 Torr. The chamber was then connected to an ion pump and the pressure was reduced to about 10^{-6} Torr.

The MgF_2 was first covered with a shutter. The current passing through the boat was gradually increased until the pellets melted completely. When the MgF_2 pellets melt, it is possible for air pockets to be trapped under the molten material. Therefore the current was slightly increased until all the air which may be trapped inside escapes. If this is not carried out, then when the deposition of MgF_2 commences, the MgF_2 will 'spit' on the crystal. The current was reduced and the shutter covering the MgF_2 removed. The current was again increased and the thickness of the MgF_2 deposited on the thickness monitor crystal was measured directly from the frequency of oscillation of the crystal. The thickness of the film deposited on the LiNbO_3 crystal was assumed to be equal to that on the oscillating crystal. The LiNbO_3 crystal was then baked in an oven for 12 hours at a temperature of 30°C . This produced a hard MgF_2 film. This film is reasonably resistant to water and to a dilute solution of ferric chloride (copper etching solution). However this film was affected by the chromium etching solution (ref. section 9.5).

9.4.2. Deposition of electrode metal

The electrode metal can be either chromium, a combination of chromium and copper, or copper. It is seen from the previous section that the MgF_2 film is affected by the Cr etching solution. Therefore only a thin

film of Cu (thickness $1\ \mu\text{m}$) was deposited on the MgF_2 film. Thicker Cu films increase the difficulties of etching to give a high resolution. Cu can be evaporated using the boat and the procedure for deposition is similar to that described for MgF_2 .

9.5 Etching of the electrodes

The photo-mask used was positive and therefore the Cu metallic surface was covered with positive photoresist (AZ 1350 positive working photoresist-Shipley Europe Ltd.). The crystal was clamped on a spinning table and rotated at a rate of approximately 2000 rpm. This produced a reasonably uniform layer of photoresist of approximate thickness $1\ \mu\text{m}$ on the Cu surface. The rate of spinning should be large to prevent 'Bead-ing' of the photoresist at the edge of the crystal. This is necessary to ensure that the photo-mask is in good contact with the photoresist film. The crystal was then dried in an oven for 30 minutes at a temperature of 60°C . The ultra violet light source used for exposure was an ordinary sun ray lamp (Pifco). The emergent beam was not focused. The photo-mask was placed over the surface covered with photoresist. If the photo-mask is not in good contact with the photoresist surface, then due to the diffraction of UV light at the edges of the mask pattern, the reproduction on the photoresist will deteriorate. The time of exposure was 7 minutes. The photoresist has the property of being affected by the UV light, and the section of the photoresist layer which is exposed to the UV light can be dissolved. AZ developer (Shipley Europe Ltd) was used to dissolve the exposed photoresist. The crystal was then washed in distilled water and dried in an oven for one hour at 60°C .

The crystal was then put in the Cu etching agent. The time for etching the Cu varied with the concentration of the etching solution. The concentration was chosen such that the time required for etching the pattern is not less than one minute. The etched crystal was washed and dried in an oven for 30 minutes.

The remaining photoresist was then removed using the photoresist remover (AZ remover - Shipley Europe Ltd). The crystal was then again washed with distilled water.

The two faces previously polished for light transmission were cleaned with acetone by a lens tissue.

The compositions of the developer and etching solutions used were

AZ developer:- 1 part of AZ developer: 2 parts distilled water

Cu etching solution:- 1 part of FeCl_3 (Hopkin and Williams):

50 parts of distilled water

1 part of FeCl_3 :7 parts of distilled water

(if Cr and Cu are used).

Cr etching solution:-

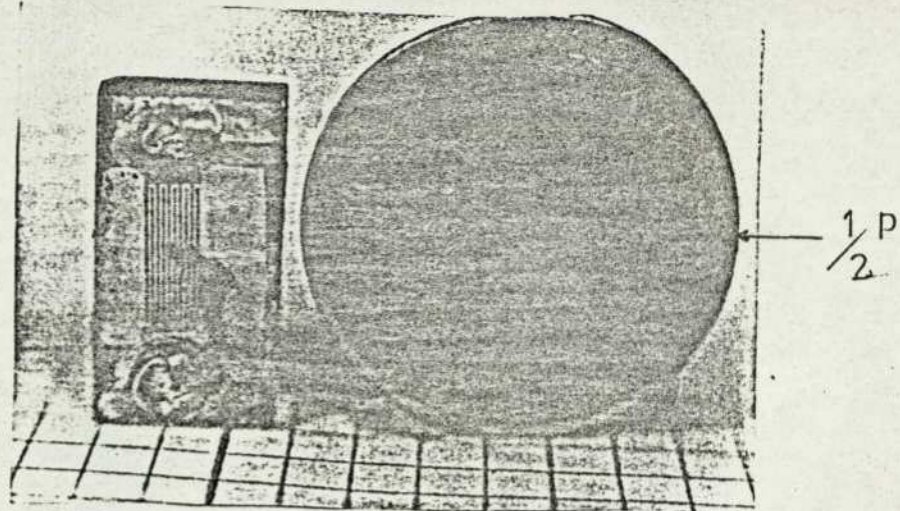
a) a solution was made with

20 gms of $\text{K}_3\text{Fe}(\text{CN})_6$: 10 gms NaOH:60ml distilled water

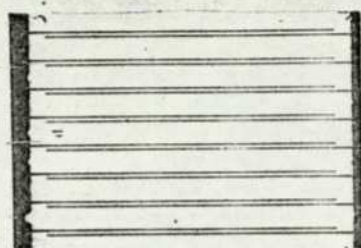
this solution was diluted as

1 parts (Volume) of solution 'a': 3 parts (Volume)distilled
water

The etched electrode structures on the crystal base is shown in
photographs 9.2



Symmetrical electrode arrangement



Asymmetrical electrode arrangement

Fig 9.2 Symmetrical and asymmetrical electrode arrangements

CHAPTER 10

CONCLUSION, SUGGESTIONS FOR FURTHER WORK AND DEVICE APPLICATIONS

10.1 Conclusion

It is apparent from the brief survey of electro-optic modulators given in Chapter 2 of this thesis that the grating effect produced inside an electro-optic crystal can be used to modulate and/or to deflect an incident optical beam. A convenient method of producing the diffraction grating effect inside the crystal is by using surface deposited electrodes. A close study of the electro-optic effects in crystals have shown that among the readily available crystals, crystals belonging to classes 3m and 6m are suitable for using in the principal cut modes. The experimental devices were constructed with LiNbO_3 crystals, a 3m class material. Theoretical analysis of the different types of electrode configurations suggest that by the proper choice of electrode arrangement and crystal orientation, the device may be used to produce a symmetrical or an asymmetrical diffraction pattern. The expressions for the components of the field intensities show that these magnitudes decrease exponentially with an increase of the distance from the electrodes. Two different approaches have been considered to utilise the intense field region in the close neighbourhood of the electrodes.

- i) Passing the light beam parallel and in the close vicinity of the electrodes.
- ii) Reflecting the incident beam off the crystal-electrode interface.

In the internal reflection arrangement constructed with symmetrical electrodes on the polished XY face of a Z cut LiNbO_3 crystal, it was found that a subdiffraction pattern appeared in the far field at zero applied voltage. Detailed examination of this pattern indicates that this pattern was due to the presence of the interdigital electrodes on the reflecting face of the crystal.

A thin layer of MgF_2 deposited on the crystal face prior to the deposition of the electrodes removed this degrading effect. The modulation efficiency (zero order extinction voltage) of the improved device was not degraded by the presence of the thin substrate layer. Infact the zero order extinction voltage was reduced and it closely agreed with the theoretically predicted value. At 87° angle of incidence, the d.c. extinction voltages for the device without the MgF_2 and with the MgF_2 film are 84 & 77 volts respectively. The theoretically predicted value was 75 volts. This improvement may be due to the high resistance of the MgF_2 , linearising the potential between the electrodes along the crystal face, thus giving an improved diffraction profile. No attempt in the thesis has been made to optimise the thickness of this film. At certain thickness of this film the sensitivity the device will be expected to decrease due to loss of field in the MgF_2 film.

The deflection of light by blazed diffraction gratings is dicussed. Theoretical estimations show that when the first four Fourier components of an ideal sawtooth phase profile of peak phase delay (θ_1 - figure 6.5) equal to π is impressed on the transmitted beam, a maximum of 72.7 % of the incident energy is deflected to the first order. Synthesising individual electrode sets to produce these Fourier components is laborious and therefore a novel approach of asymmetrically electroding the device has been adopted to impress the blazed sawtooth phase corrugation. Theoretical analysis shows that only the perpendicular component of the field can be used for this application, and this is achieved with a Z cut LiNbO_3 sample. With the device constructed, a maximum of 81% of the incident energy was deflected to the first order with an applied voltage of 157 volts. When the voltage was increased to 255 volts, 53% of the incident energy was deflected to the second order. This lower percentage of the energy may be attributed to the relative coarseness of the electrode structure. The reversal of the voltage to the electrodes caused corresponding deflections to the negative orders. The angle of incidence was 85.25° . Theoretical computations showed that in the arrangement considered the phase delay θ_1

is equal to π and 2π when the applied voltages are 155 and 250 volts respectively. The zero order extinction voltage was 150 volts. Since the zero order extinction voltage (or the voltage required to produce a higher order maxima) is a function of the angle of incidence, a low operating voltage can be achieved by choosing a large angle of incidence. Typically when the angle of incidence is 2° , the zero order extinction voltage and the first order maximum voltages are 79 and 80 volt respectively

Acoustic resonances may effect the performance of the device when operated with a.c. voltages. In the devices constructed the acoustic resonances observed were due to bulk waves and Rayleigh waves ^{The later was observed in} (symmetrical electrode arrangement). The bulk waves were almost completely absorbed when an absorber was placed on the face opposite to the electrode face. The surface waves were completely absorbed by the absorber on the electrode face. The thin film of MgF_2 deposited prior to the electrode deposition reduced the magnitude of the surface wave resonance of the device.

A multireflection modulator offers the possibility of a low operating voltage bulk device at a relatively small angle of incidence. The double reflection arrangement considered in this thesis gives about 50% reduction in the zero order extinction voltage.

10.2 Suggestions for further work

The asymmetrically electroded light deflector offers the possibility of preferentially deflecting the light to the higher orders. The alternate approaches are to use the Bragg deflector, which is unsatisfactory as for each order and for constant electrode pitch (in the symmetrically electroded arrangement), a new angle of incidence of the light must be selected. The acoustic deflector potentially does not offer the operating speed of an electro-optic device and inherently requires greater drive power. It would therefore seem that further development of the device

initially suggested in the thesis is well warranted.

The questions that have been raised in this work are summarised as;

10.2.1) Linearisation of the field

This is discussed in Chapter 4, and briefly it would appear that it may be possible to optimise the shape of the electro-optic grating by utilising the resistive and dielectric properties of films which might be deposited between the electrodes. The material which may be used for this purpose is SnO_2 . If the conducting layer material is not affected by the etching agent, this layer may be deposited on the MgF_2 film prior to the deposition of the electrode metal. If the conducting layer is affected, then it may be deposited on the crystal face after etching the electrodes. Care should be taken to remove any foreign particles (etching agent, photo-resist) from the electrode spacing prior to the deposition of the conducting layer

10.2.2) Optimisation of the electrode structure

The initial experiments (section 6.3.3.2 Chapter 6) show that in the asymmetrical electrode arrangement the intensity of the first order varies for varying electrode pair spacing. This indicates that an optimum ratio of the electrode pair spacing to the closest electrode spacing is possible for efficient light deflection.

10.2.3) Optimisation of the crystal orientation

The experimental work described in the thesis deals with a device operating in the principal cut mode. It is expected that an efficient utilisation of the electro-optic coefficients can be achieved by optimising the crystal orientation.

10.2.4) Use with unpolarised light

In the arrangements considered the incident light was polarised by an external polariser. If the crystal faces are oriented so that two reflections are possible in the crystal and if the faces are adjusted so that Brewster's reflection law is satisfied at the first reflection then

the light will be polarised. This polarised beam may then reflect off at another face of the crystal where the electrodes are deposited.

10.2.5) Low operating voltage modulator/deflector

- i) Multiple reflection : The arrangements described in the thesis could be extended to have multiple reflections off the crystal-electrode interface(s). The additional capacitance contributed by having long electrode lengths may be overcome by having independent electrode structures at the predetermined reflecting positions
- ii) Thin film version : A drastic reduction in the operating voltages is possible when the incident beam is passed parallel to the electrode face but in the close vicinity of the electrodes. This may be achieved by coupling the light to an electro-optic waveguide. A deflector of this type may be envisaged as operating at logic voltage levels and as such would find applications in systems utilising optical logics.

10.3 Device applications

- i) The modulator arrangement with symmetrical electrodes produces a symmetrical diffraction pattern in the far field. Hence this device could be used as a multiport switch.
- ii) The arrangement with asymmetrical electrodes deposited on the XY plane of a Z cut crystal gives rise to an asymmetrical intensity distribution in the far field. Hence this device could be used as a beam deflector. However when the electrodes are deposited in a plane parallel to the optic axis, the far field pattern will be symmetrical about the zero order.
- iii) Modulation characteristics of the incident beam by these devices show that complete extinction of the zero order beam is possible by applying an appropriate voltage to the electrodes. Therefore it is expected that these devices could be used inside a laser cavity to switch or mode lock the laser. The insertion losses by the device should be absolutely minimal.

A P P E N D I X

- A. The experimental arrangement
- B. The propagation characteristics of the reflected and transmitted beam produced when an incident optical beam experiences a boundary.
- C. The derivation of the intensity of the diffracted order beam when an incident optical beam experiences a phase grating of an arbitrary shape.
- D. The computer programme which calculates the intensity of the nth order beam when the phase grating is produced by the electro-optic effect in LiNbO_3 .

APPENDIX A

THE EXPERIMENTAL ARRANGEMENT

The optical arrangement used in the earlier experiments described in chapters 5 - 8 is schematically represented in figure A. 1.

(1) The optical source : The optical source used was a HeNe laser.

The CW output of this laser at a wavelength of $0.6328 \mu\text{m}$ is $\sim 0.5 \text{ mW}$.

(2) The polariser : The polariser was used to select the required polarisation of the incident beam.

(3) The condensing lens : A cylindrical lens was used to condense the incident light on to the electrode structure. This lens was necessary, since at very shallow angles of incidence, the whole cross section of the incident beam will not get reflected off from the electrode structure.

(4) The crystal mounting :

i) The crystal holder : The crystal holders used in the experimental arrangements with the symmetrical and the asymmetrical electrode structures are shown in figures A .2 a and A .2 b respectively

ii) The crystal holder mounting table : The crystal holder was fitted on to a platform whose plane can be tilted with respect to a reference plane by three adjustable screws. The reference plane of this arrangement was fitted on to the mounting table which had provisions of rotation in the horizontal and vertical planes.

(5) The light detector : The detector used in the experiments was a photomultiplier (Mullard 56 TYP) carrying a straight slit. In the arrangement considered in Chapter 6, the detector was attached to a servometer which provided a uniform movement of the detector in a particular direction.

(6) The chart recorder : The photomultiplier output was recorded on an XT recorder. The record chart was driven at a uniform speed, the X ordinate

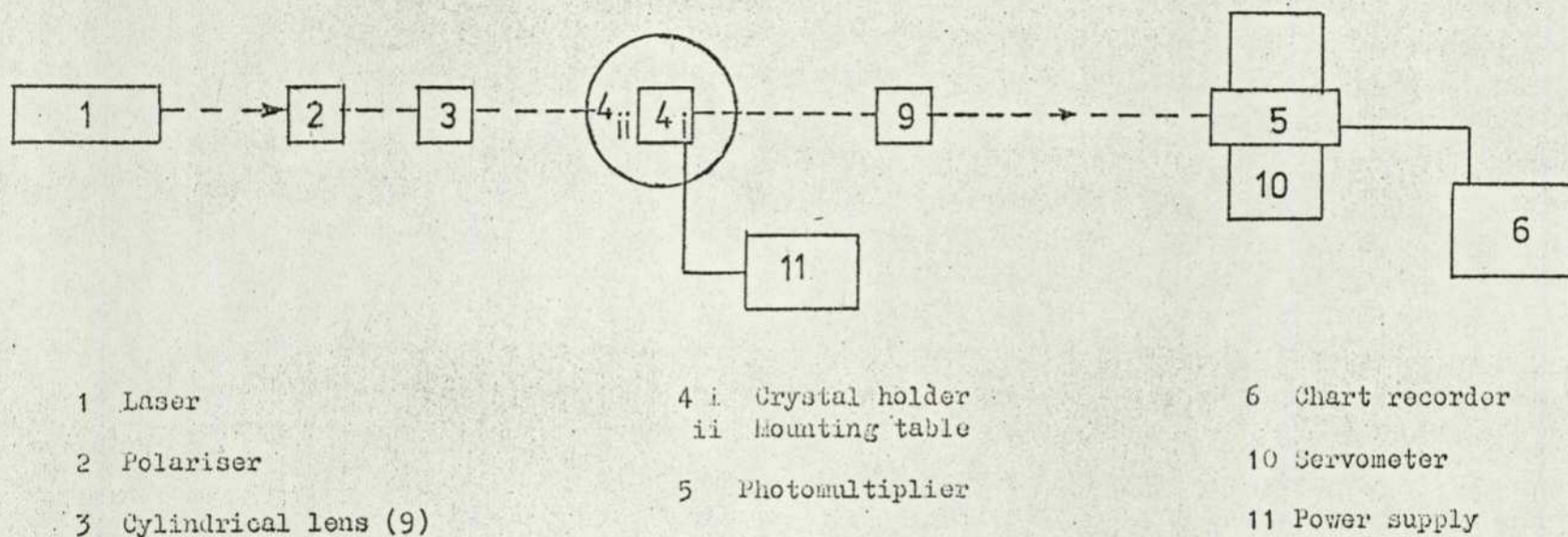
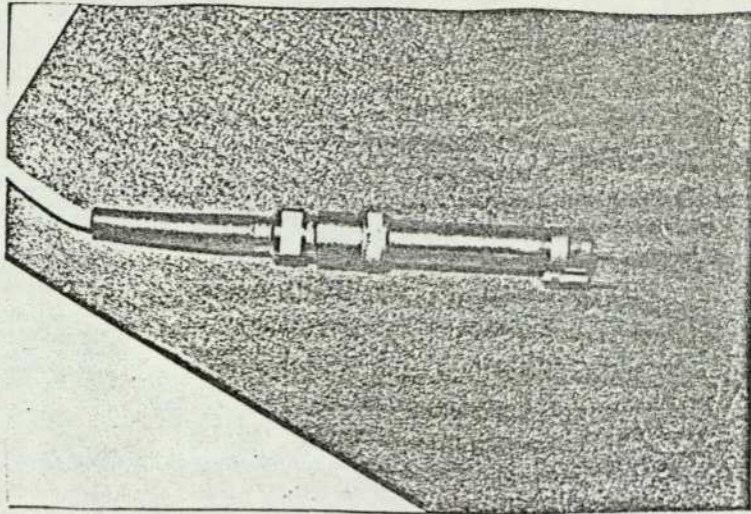
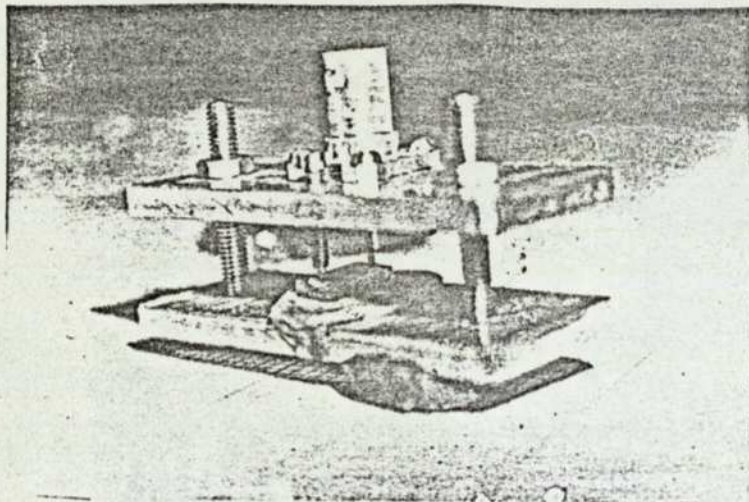


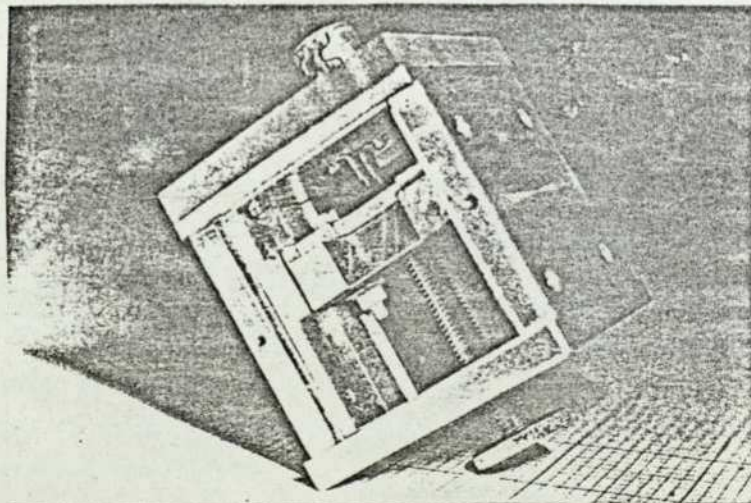
Fig. A.1. The optical arrangement



Connction lead of the Impedance meter

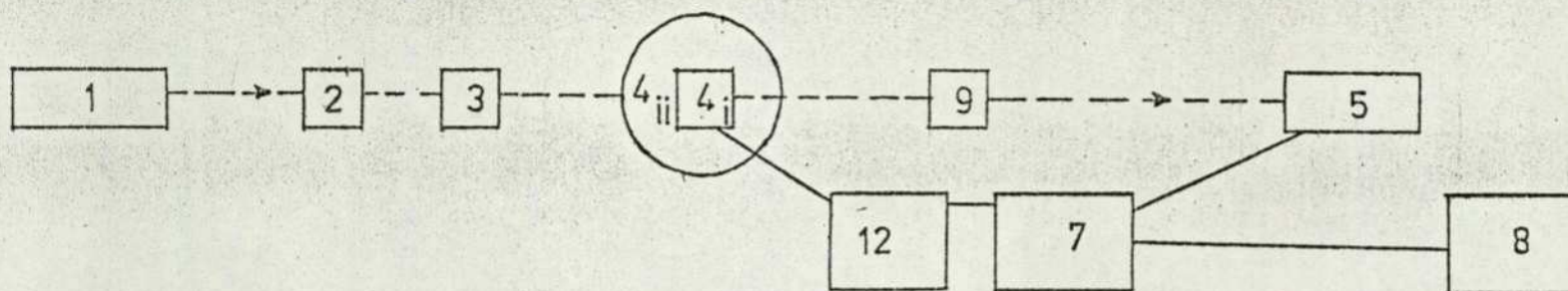


Crystal holder for the symmetrical arrangement



Crystal holder for the asymmetrical arrangement

Fig. A.2



Components labeled

1, 2, 3, 4i, 4ii, 5, and 9 corresponds
to those shown in figure A.1

7 Net-work analyser

8 XY recorder

12 Oscillator

Fig. A.3 Arrangement of the net-work analyser

represents the photomultiplier output at any instant of time

The experimental arrangement used for analysing the high frequency performance of the device is schematically represented in figure A.3

(7) The net-work analyser : A net-work analyser was used to analyse the output of the photomultiplier

(8) The XY recorder : An XY recorder was used to record the output of the network analyser. The X axis corresponds to the frequency of measurement and the Y axis corresponds to the output of the analyser.

(In the measurement of the vector impedance, Chapter 7, the X and the Y axes correspond to the frequency and the Impedance of the device respectively)

APPENDIX B

Propagation characteristics of electromagnetic waves

The propagation characteristics of the reflected and transmitted electromagnetic waves produced when the incident beam experiences a boundary is conveniently analysed by considering the Fresnel's Formulae. In the following section these formulae are derived starting from first principle.

B.1 Fresnel's formulae

When an electromagnetic wave falls on a boundary between two homogeneous media of different optical properties, it is split into two waves,

- i) A wave in the second medium - Transmitted wave
- ii) A wave travelling back in the first medium - Reflected wave

Let $Z=0$ plane be the surface of separation of the two media of refractive indices n_1 and n_2 and $Y=0$ is the plane of incidence of the e.m wave. The electric vector of the electromagnetic wave can be resolved into two components namely E_n and E_p where

E_n is the component which is normal to the plane of incidence and E_p the component parallel to the plane of incidence. The normal component is taken along the positive Y direction. Similar notations have been used for the reflected and transmitted waves (Fig.B.1)

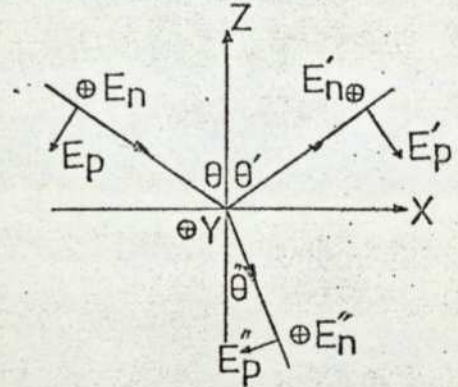


Fig. B.1

The components of the incident, reflected and transmitted wave along the X,Y,Z directions are $(E_x, E_y, E_z), (E'_x, E'_y, E'_z)$ and (E''_x, E''_y, E''_z) respectively and are given -

For the incident wave

$$\begin{aligned} E_x &= -E_p \cos \theta \exp i\omega \left[t - (n_1/c) (x \sin \theta - z \cos \theta) \right] \\ E_y &= E_n \exp i\omega \left[t - (n_1/c) (x \sin \theta - z \cos \theta) \right] \\ E_z &= -E_p \sin \theta \exp i\omega \left[t - (n_1/c) (x \sin \theta - z \cos \theta) \right] \end{aligned} \quad \text{--- B.1}$$

For the reflected wave

$$\begin{aligned} E'_x &= E'_p \cos \theta' \exp i\omega' \left[t - (n_1/c) (x \sin \theta' + z \cos \theta') \right] \\ E'_y &= E'_n \exp i\omega' \left[t - (n_1/c) (x \sin \theta' + z \cos \theta') \right] \\ E'_z &= -E'_p \sin \theta' \exp i\omega' \left[t - (n_1/c) (x \sin \theta' + z \cos \theta') \right] \end{aligned} \quad \text{--- B.2}$$

and for the transmitted wave

$$\begin{aligned} E''_x &= -E''_p \cos \theta'' \exp i\omega'' \left[t - (n_2/c) (x \sin \theta'' - z \cos \theta'') \right] \\ E''_y &= E''_n \exp i\omega'' \left[t - (n_2/c) (x \sin \theta'' - z \cos \theta'') \right] \\ E''_z &= -E''_p \sin \theta'' \exp i\omega'' \left[t - (n_2/c) (x \sin \theta'' - z \cos \theta'') \right] \end{aligned} \quad \text{--- B.3}$$

where c is the velocity of light in a vacuum, θ, θ' and θ'' are defined in figure B.1 and ω, ω' and ω'' are the frequencies of the incident, reflected and transmitted waves respectively.

When the dielectric constant and the magnetic permeability are changed abruptly, the electromagnetic wave satisfies the following four boundary conditions -

- 1) The normal component of the magnetic induction is continuous
- 2) - do - electric - do -
- 3) The tangential component of the magnetic field is continuous
- 4) - do - electric - do -

At the boundary, the conditions 3 and 4 hold at all times and at all values of x and y only if ω, ω' and ω'' are equal to each other (68)

All the boundary conditions are satisfied at the boundary plane ($Z=0$) when the arguments of the exponentials for the incident, reflected and transmitted waves are all identical.

Then

$$\theta = \theta'$$

$$\text{and } n_1 \sin \theta = n_2 \sin \theta'' \text{ (Snells Law)}$$

If the intrinsic impedances of the two media are Z_1 and Z_2 , then for the boundary condition 4

$$- E_p \cos \theta + E'_p \cos \theta = - E''_p \cos \theta'' \quad \text{B.4}$$

$$\text{and} \quad E_n + E'_n = E''_n \quad \text{B.5}$$

and for the boundary condition 3

$$-\frac{E_n \cos \theta}{Z_1} + \frac{E'_n \cos \theta}{Z_1} = -\frac{E''_n \cos \theta''}{Z_2} \quad \text{B.6}$$

and

$$\frac{E_p}{Z_1} + \frac{E'_p}{Z_1} = \frac{E''_p}{Z_2} \quad \text{B.7}$$

Using the equation B.4 - B.7 the Fresnels relations can be obtained. Let R_p and R_n be the reflection coefficients of the components parallel and normal to the plane of incidence. T_p and T_n are the corresponding transmission coefficients, then

$$\frac{E'_p}{E_p} = R_p = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2} \quad \text{B.8}$$

$$\frac{E'_n}{E_n} = R_n = \frac{n_2 \cos \theta_2 - n_1 \cos \theta_1}{n_2 \cos \theta_2 + n_1 \cos \theta_1} \quad \text{B.9}$$

$$\frac{E''_p}{E_p} = T_p = \frac{2 n_1}{n_2 \cos \theta_1 + n_1 \cos \theta_2} \quad \text{B.10}$$

and
$$\frac{E_n'}{E_n} = T_n = \frac{2 n_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \quad \text{B.11}$$

Where θ and θ'' for the previous relationship are now replaced by θ_1 and θ_2 respectively. In the above equations since $n_1, n_2, \theta_1, \theta_2$ are real quantities R_p, R_n, T_p and T_n are real. If n_1 is greater than n_2 then by Snells Law a limiting case is obtained when

$$n_1 \sin \theta_1 = n_2 \quad \text{B.12}$$

and the angle of incidence is called the critical angle. When the angle of incidence is greater than the critical angle, the angle of refraction is given by

$$\cos \theta_2 = \left[1 - \left(\frac{n_1}{n_2} \sin \theta_1 \right)^2 \right]^{\frac{1}{2}} \quad \text{B.13}$$

But since $(n_1/n_2) \sin \theta_1$ is greater than 1, $\cos \theta_2$ will be imaginary and written as (44)

$$\cos \theta_2 = -i \sqrt{\left(\frac{n_1}{n_2} \sin \theta_1 \right)^2 - 1} \quad \text{B.14}$$

Considering equation B.8, since $\cos \theta_2$ is a complex quantity the reflection coefficient R_p will also be a complex quantity and may be represented as $r_1 \exp(i\delta_1)$ where δ_1 is the phase change introduced on reflection.

Substituting for $\cos \theta_2$ in B.8 the equation becomes

$$r_1 e^{i\delta_1} = \frac{n_2^2 a + i n_1^2 b}{n_2^2 a - i n_1^2 b} \quad \text{B.15}$$

where $a = n_1 \cos \theta_1$ and $b = \sqrt{n_1^2 \sin^2 \theta_1 - n_2^2}$

In the above equation, the modulus is unity and the incident wave is said to have undergone total internal reflection. The phase change, impressed on the reflected wave under these conditions is given as

$$\tan \frac{\delta_1}{2} = \frac{n_1^2}{n_2^2} \frac{\sqrt{n_1^2 \sin^2 \theta_1 - n_2^2}}{n_1 \cos \theta_1} \quad \text{B.16}$$

Similar calculations could be done for the normal component of the electric field. When total internal reflection occurs an evanescent wave appears in the second medium which rapidly attenuates on the far side of the boundary. Since all the energy is reflected back in the first medium it can be concluded that no energy is carried away by the evanescent wave.

B.2 Reflection at a dielectric - conductor boundary

Consider the medium 2 as a conductive medium (metal). The refractive index of a metal may be represented as a complex quantity and given as

$$n_2^* = n - ik \quad \text{B.17}$$

where n and k are the refractive index and absorption coefficient of the metal. The angle of refraction in the second medium is (51) defined as a complex quantity θ_2^* . Substituting for n_2 and θ_2 in equation B.8 gives

$$R_p^* = r_2 e^{i\delta_2} = \frac{n_2^* \cos \theta_1 - n_1 \cos \theta_2^*}{n_2 \cos \theta_1 + n_1 \cos \theta_2^*} \quad \text{B.18}$$

where δ_2 is the phase impressed on the reflected wave.

From Snells Law

$$\sin \theta_2^* = (n_1 / n_2^*) \sin \theta_1 \quad \text{B.19}$$

$\cos \theta_2^*$ may be determined from equation B.19 and substituted into B.18

$$\text{This gives} \quad r_2 e^{i\delta_2} = \frac{(p - i q) \cos \theta_1 - n_1 (p' - i q)^{\frac{1}{2}}}{(p - i q) \cos \theta_1 + n_1 (p' - i q)^{\frac{1}{2}}}$$

where

— B.20

$$p' = n^2 - k^2 - n_1^2 \sin^2 \theta_1$$

$$p = n^2 - k^2$$

$$q = 2 n k$$

By expanding equation B.20 and separating the real and imaginary parts we get

$$\tan \delta_2 = \frac{X Y - U V}{X V + U Y} \quad \text{B.21}$$

where

$$X = p \cos \theta_1 - n_1 A$$

$$Y = q \cos \theta_1 + n_1 B$$

$$U = q \cos \theta_1 - n_1 B$$

$$V = p \cos \theta_1 + n_1 A$$

$$A^2 - B^2 = p'$$

$$\text{and } 2AB = q$$

It can be shown from equation B.20 that the modulus of the equation is not equal to unity.

In an arrangement where interdigital electrodes are deposited on the base of a crystal and an optical beam is reflected off from the base, two effects will give rise to a diffraction pattern in the far field.

These are

- i) The phase grating effect due to the difference in δ_1 and δ_2
- ii) An amplitude grating due to the absorption of the light energy by the metallic electrodes.

APPENDIX C

Estimation of the intensity of the diffracted order beams produced by a phase grating

The interaction of a periodic phase grating and an optical beam gives rise to a diffraction pattern in the far field. One of the methods of producing this type of grating is by the passage of acoustic waves inside the medium. An exhaustive study of acousto-optic interaction is given in such references as 69-77

In this appendix the basic theoretical considerations of this interaction phenomena developed by Klein, Cook and Mayer (74) and Zankel and Hiedemann (75) are summarised, and these are extended to the electro-optically induced diffraction phase gratings. In the following discussions the electro-optic medium is assumed to be transparent to the incident beam of light and the phase variation impressed on the optical beam is taken to have an arbitrary periodic shape. The phase change impressed on the transmitted optical beam is produced by the electro-optically induced change of refractive index. This change of refractive index will be small.

Consider plane monochromatic waves incident on the electro-optic medium. The electric vector of the incident beam may be represented as

$$E = E_0 \exp [i (\omega_0 t - \mu_0 \underline{k}_0 \cdot \underline{r})] \quad C.1$$

where ω_0 is the angular velocity and $\underline{k}_0$ is the wave vector of the optical beam in free space respectively. $\underline{r}$ denotes the position vector and μ_0 is the index of refraction of the unperturbed medium.

Let the refractive index of the perturbed medium be μ , then

$$\mu = \mu_0 + \Delta\mu \quad C.2$$

where $\Delta\mu$ is the small variation in the refractive index along the Y direction (Fig. C.1). When the optical beam passes through this perturbed media, the electric vector may be written as

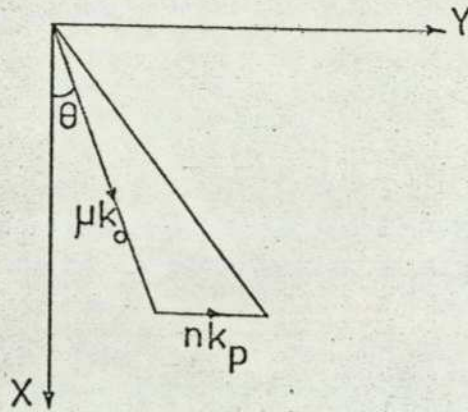


Fig C-1

$$E = E_o \exp [i (\omega_o t - \mu k_o \cdot \underline{r})] \quad C.3$$

This equation may be re-written as

$$E = E_1 \exp [i (\omega_o t - \mu k_o \cdot \underline{r})] \quad C.4$$

where E_1 varies periodically along the y axis. E_1 can therefore be expanded in a Fourier Series and is given as

$$E_1 = \sum_{n=-\infty}^{\infty} \phi_n \exp (-ink_p y) \quad C.5$$

where ϕ_n are the Fourier coefficients and k_p is the wave vector of the perturbation, whose direction is shown in fig. C.1.

Substituting for E_1 in equation C.4

$$E = \sum_{n=-\infty}^{\infty} \phi_n \exp [i(\omega_o t - (nk_p y + \mu k_o \cdot \underline{r}))] \quad C.6$$

The above equation may be re-written as

$$E = \psi \exp (i\omega_o t) \quad C.7$$

where
$$\psi = \sum_{n=-\infty}^{\infty} \phi_n \exp [-i (nk_p y + \mu_{00} k_o \cdot r)] \quad C.8$$

Since $\mu_{00} k_o \cdot r$ is independent of n , it can be taken outside the summation quantity. If the medium is assumed to be non-magnetic, then the wave equation of the optical beam inside this medium is

$$\Delta^2 E = \left[\frac{\mu}{c} \right]^2 \delta^2 E / \delta t^2 \quad C.9$$

Substituting equation C.7 in C.9,

$$\frac{\delta^2 \psi}{\delta x^2} + \frac{\delta^2 \psi}{\delta y^2} + \frac{\delta^2 \psi}{\delta z^2} = - \mu^2 k_o^2 \psi \quad C.10$$

In the L.H.S. of equation C.10, without loss of generality it can be assumed the ψ is independent of z and now substituting equation C.8 in the L.H.S. of equation C.10,

$$\begin{aligned} \text{L.H.S (C.10)} &= \sum_{n=-\infty}^{\infty} \left[- \left\{ (\mu_o k_o \sin \theta + nk_p)^2 + (\mu_o k_o \cos \theta)^2 \right\} \phi_n \right. \\ &\quad \left. - 2 i \mu_o k_o \cos \theta \frac{\delta \phi}{\delta z} n \right] \exp [-i (nk_p y + \mu_o k_o \cdot r)] \quad C.11 \end{aligned}$$

$\frac{\delta^2 \phi}{\delta z^2}$ is assumed to be negligible as ϕ is a slowly varying function in the z direction. Using equation C.2, and assuming $(\Delta \mu)^2$ to be negligible, the R.H.S. of equation C.10 reduces to

$$\text{R.H.S. (C.10)} = - k_o^2 \mu_o^2 \psi - 2 k_o^2 \mu_o \Delta \mu \psi \quad C.12$$

As stated earlier, $\Delta \mu$ is a periodically varying quantity along the y direction and can therefore be expanded in a Fourier series as

$$\Delta \mu = \sum_{k=-\infty}^{\infty} \frac{V_k}{2i} \left[\exp (-ik k_p y) + \exp (ik k_p y) \right] \quad C.13$$

where V_k are the amplitude of the Fourier components of the refractive index.

Substituting equation C.13 and C.8 in C.12

$$\begin{aligned} \text{R.H.S. (C.10)} = & -k_o^2 \mu_o^2 \psi - 2k_o^2 \mu_o \left[\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \phi_n \exp \left[-i (nk_p y + \mu_o k_o \cdot r) \right] \right. \\ & \left. \frac{V_k}{2i} \exp \left[(-i k k_p y) + \exp i k k_p y \right) \right] \right] \end{aligned} \quad \text{C.14}$$

Equating equations C.11 and C.14, then

$$\begin{aligned} & \sum_{n=-\infty}^{\infty} \left[- \left\{ (\mu_o k_o \sin \theta + nk_p)^2 + (\mu_o k_o \cos \theta)^2 \right\} \phi_n - 2i \mu_o k_o \cos \theta \frac{\delta \phi}{\delta z} n \right] \exp(-ink_p y) \\ = & - 2k_o^2 \mu_o \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \phi_n \exp(-ink_p y) V_k \left[\exp(-ikk_p y) + (\exp i k k_p y) \right] \\ & - k_o^2 \mu_o^2 \sum_{n=-\infty}^{\infty} \phi_n \exp(-ink_p y) \end{aligned} \quad \text{C.15}$$

Equating the coefficients of the individual terms with $\exp(-ink_p y)$ and dividing by $-i \mu_o k_o \cos \theta$, it can be deduced that

$$\begin{aligned} 2 \frac{\delta \phi_n}{\delta z} + \frac{(\mu_o k_o \sin \theta + nk_p)^2 + (\mu_o k_o \cos \theta)^2}{i \mu_o k_o \cos \theta} \phi_n = \\ - \frac{k_o}{\cos \theta} \sum_{k=-\infty}^{\infty} V_k (\phi_{n-k} - \phi_{n+k}) + \frac{k_o \mu_o}{i \cos \theta} \phi_n \end{aligned} \quad \text{C.16}$$

This equation now reduces to

$$2 \frac{\delta \phi_n}{\delta z} + i \left[nk_p \tan \theta + \frac{(nk_p)^2}{i \mu_o k_o \cos \theta} \right] \phi_n = - \sum_{k=-\infty}^{\infty} \frac{k_o}{\cos \theta} V_k (\phi_{n-k} - \phi_{n+k}) \quad \text{C.17}$$

When θ is small, $\tan \theta$ is also small, and when k_p is very much smaller than k_o , equation C.17 reduces to

$$\frac{2 \delta \phi_n}{\delta z} + \sum_{k=-\infty}^{\infty} \frac{k_o}{\cos \theta} V_k (\phi_{n-k} - \phi_{n+k}) = 0 \quad \text{C.18}$$

Representing equation C.18 in terms of the phase change impressed on the beam, the solution of the above equation (76) is obtained as

$$\phi_n = \phi_0 \sum_{k_2, k_3, \dots, k_i, \dots = -\infty}^{\infty} J_{n-2k_2-3k_3 \dots -ik_i} (U_1) \times J_{k_2} (U_2) \times J_{k_3} (U_3) \times J_{k_i} (U_i) \times \dots \quad C.19$$

where ϕ_0 and ϕ_n are the amplitudes of the incident and the diffracted beams respectively. U_i is the amplitude of the i^{th} harmonic component of the phase change. $J_{k_i} (U_i)$ denotes the Bessel coefficient of order k_i and argument U_i .

If the variation in the refractive index is time dependent, then in equation C.13, $\Delta\mu$ is replaced by a time dependent quantity. This condition will then give U_i as time dependent, and thus ϕ_n will be a time varying quantity.

The electro-optically induced phase change impressed on the optical beam is given by equation 333 chapter 3, and the Fourier components can be calculated from this equation.

In particular, if the phase variation is assumed to be sinusoidal then $U_2, U_3, \dots$ will all reduce to zero and the amplitude of the diffracted beam is given by

$$\phi_n = \phi_0 J_n (U_1) \quad C.20$$

As the intensity is proportional to the square of the amplitude, then

$$\frac{I_n}{I_0} = [J_n (U_1)]^2 \quad C.21$$

where I_n and I_0 are the intensities of the n^{th} and zero order respectively.

Equation C.19 is reasonably amenable to the computer calculation and Appendix D gives the computer program which calculates the intensity of the diffracted order beams when the diffraction element is assumed to be produced by interdigital electrodes on a LiNbO_3 crystal.

In this computer programme the electro-optically induced phase change may be represented by four Fourier components.

APPENDIX D

COMPUTER PROGRAMME

The computer programme calculates the intensity of the diffracted order beams when the incident beam experiences a phase grating.

The grating effect is produced inside an electro-optic crystal by applying a d.c. voltage to the interdigital electrodes deposited on the base of the crystal. The crystal considered in this thesis is LiNbO_3 and the relevant electro-optic coefficients are used in this programme.

The electrodes are taken to be in the plane perpendicular to the Z axis with the electrode lengths along the X direction. The electrode arrangement is assumed to be asymmetrical. However, the programme can be modified for the symmetrical electrode structure by substituting equal values for A and BA (lines 21 and 22). In the d.c. condition the dielectric constant E_1 does not equal E_2 .

The programme uses a sub-routine available at The City University computer library which calculates the zero order Bessel coefficients

The calculations included in the programme are:

- i) The path length
- ii) The corresponding phase change
- iii) The Fourier coefficients of the phase profile
- iv) The zero order Bessel coefficients
- v) The higher order Bessel coefficients

- vi) The ratio of the amplitude of the diffracted beam to the undiffracted beam
- vii) The ratio of the intensity of the diffracted order beam to the intensity of the undiffracted beam

The calculation (vi) considers the contribution of the first four harmonics of the phase profile. It is expected that a better approximation of the ratio of the amplitudes will be obtained by modifying the programme to incorporate the contribution by more harmonics.


```

PROGRAM INTE(INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT)
DIMENSION T(71),TN(70),U(21),UN(20),V(21),VN(20),W(21),WN(20),Z(5
1,C(71),CN(70)
INTEGER P
REAL NO,NE,INTS

```

C ELECTRODE CHARECTERISTICS

```

DIMENSION T(71),TN(71),U(71),UN(71),V(71),VN(71),W(71),WN(71)
1,Z(10),C(71),CN(71)
DO 999 III=1,71
  T(III)=0.0
  TN(III)=0.0
  U(III)=0.0
  UN(III)=0.0
  V(III)=0.0
  VN(III)=0.0
  W(III)=0.0
  WN(III)=0.0
  Z(III)=0.0
  C(III)=0.0
  CN(III)=0.0
999 CONTINUE
A=0.0445/0.338
BA=0.275/0.338
D=0.169
TL=1.745

```

C CRYSTAL PROPERTIES

```

R13=8.6*10.0**(-9)
R33=30.8*10.0**(-9)
R22=3.4*10.0**(-9)
E1=84.6
E3=28.6
NO=2.28
NE=2.20
WAVEL=0.0006328
PI=4.0*ATAN(1.0)
E1=E2

```

C INCIDENT ANGLE

```

Q1=4.75
22 Q=(Q1*PI)/180.0

```

C INTEGRATED PATH CHANGE

```

A1=R13*(SIN(Q))*(SIN(Q))
A2=R33*(COS(Q))*(COS(Q))
A3=A1+A2
V0=7.0
DO 780 II=1,12
  P=1
57 DIVT=100.0
  DIV=100.0
52 HATS=0.0
  HATC=0.0

```



```

      Q2=360.0/(2.0*DIV)
      Y=(2.0*D)/(2.0*DIV)
112  Q3=(P*Q2*PI)/180.0
      G=0.0
      M=0
      M1=10
20  M=M+1
      N=M
      B11=(2.0*V0*SIN(N*PI*BA))/(N*PI*N*PI*BA)
      B12=((2.0*V0*SIN(N*PI*A))*COS(N*PI))/(N*PI*N*PI*A)
      B1=B11-B12
      B2=SIN((N*PI*Y)/D)
      B3=SQRT(E1/E3)
      B4=1.0-EXP((-B3*N*PI*TL*TAN(Q))/D)
      B5=-B1*B2*B4*A3
      C3=COS((N*PI*Y)/D)
      C4=(SIN(Q))*(SIN(Q))
      C7=(-R22*B1*C3*C4*B4)/(B3)
      D1=B5+C7
      D2=((SIN(Q))*(SIN(Q)))/(NO*NO)
      D3=((COS(Q))*(COS(Q)))/(NE*NE)
      D4=D2+D3
      D5=D4*D4*D4
      D6=SQRT(D5)
      G1=D1/D6
      G2=G1/SIN(Q)

```

C

INTEGRATED PHASE CHANGE

```

      RI=(G2*2.0*PI)/WAVEL
      G=G+RI
      IF(M-M1)20,25,30
25  CONT=G
30  IF(M-M1)20,60,60
60  CT=G-CONT
      CTA=ABS(CT)
      GA=ABS(G)
      CA=(CTA/GA)
      IF(CA-0.005)65,65,70
70  M1=M1+5
      GO TO 20

```

C

FOURIER TERMS

```

65  HAS=G*SIN(Q3)
      HAC=G*COS(Q3)
      HATS=HATS+HAS
      HATC=HATC+HAC
      DY=(2.0*D)/DIV
      DQ2=360.0/DIV
      Y=Y+DY
      Q2=Q2+DQ2
      IF(Y-0.345)110,135,135
135 TERMS=(2.0*HATS)/DIV
      TERMC=(2.0*HATC)/DIV
      IF(DIV-DIV1)190,80,190
80  TS=TERMS

```



```

TC=TERMC
192 CONTS=TERMS-TS
ACS=ABS(CONTS)
ACTS=ABS(TERMS)
ACSI=ACS/ACTS
IF(ACSI-0.01)100,100,95
95 DIV=DIV+50.0
GO TO 50

```

C TERMS ARE SIN FOURIER TERMS ORDER P

```

100 WRITE(2,35)P,TERMS,TERMC,DIV
Z(P)=TERMS
WRITE(2,35)N,DIVT,ACS,ACTS
P=P+1
IF(P-5)57,145,145
145 WRITE(2,55)A,D,V0,01
55 FORMAT(4F20.5)

```

C ZERO ORDER BESSEL COEFFICIENTS FUNDAMENTAL AND 3HARMONICS

```

T(1)=S17AAF(Z(1))
U(1)=S17AAF(Z(2))
V(1)=S17AAF(Z(3))
W(1)=S17AAF(Z(4))
WRITE(2,91)T(1),U(1),V(1),W(1)
91 FORMAT(4F10.4)
DO 150 I=1,4
N=1
A1=1.0
A2=(Z(I)/2.0)
X=A2*A2

```

C HIGHER ORDER BESSEL COEFFICIENTS

```

11 B1=(A2)**N
A1=A1*N
B2=(B1/A1)
B3=X/(1*(N+1))
B4=(B3*X)/(2*(N+2))
B5=(B4*X)/(3*(N+3))
B6=(B5*X)/(4*(N+4))
B7=(B6*X)/(5*(N+5))
B8=(B7*X)/(6*(N+6))
B9=(B8*X)/(7*(N+7))
B10=(B9*X)/(8*(N+8))
B11=(B10*X)/(9*(N+9))
B12=(B11*X)/(10*(N+10))
B13=(B12*X)/(11*(N+11))
B14=(B13*X)/(12*(N+12))
B=B2*(1-B3+B4-B5+B6-B7+B8-B9+B10-B11+B12-B13+B14)
AB=ABS(B)
IF(AB.LT.0.00001) GO TO 150
IF(I.EQ.1) GO TO 7
IF(I.EQ.2) GO TO 8
IF(I.EQ.3) GO TO 9
IF(I.EQ.4) GO TO 10

```



```

7  NK1=N+1
   T(NK1)=B
   TN(NK1)=(-1)**N*T(NK1)
   WRITE(2,35)NK1,Z(I),T(NK1),TN(NK1)
35  FORMAT(10X,I6,3(E18.4,5X))
   GO TO 37
8  NK2=N+1
   U(NK2)=B
   UN(NK2)=(-1)**N*U(NK2)
   WRITE(2,35)NK2,Z(I),U(NK2),UN(NK2)
   GO TO 37
9  NK3=N+1
   V(NK3)=B
   VN(NK3)=(-1)**N*V(NK3)
   WRITE(2,35)NK3,Z(I),V(NK3),VN(NK3)
   GO TO 37
10 NK4=N+1
   W(NK4)=B
   WN(NK4)=(-1)**N*W(NK4)
   WRITE(2,35)NK4,Z(I),W(NK4),WN(NK4)
37  N=N+1
   IF(N-20)11,150,150
150 CONTINUE
   MNK2=NK2
   MNK3=NK3
   MNK4=NK4
   WRITE(2,600)MNK2,MNK3,MNK4
600 FORMAT(10X,3I10)

```

C INTENSITY ESTIMATION

```

J=2
S11=0.0
S22=0.0
S33=0.0
S44=0.0
S55=0.0
S66=0.0
S77=0.0
S88=0.0

```

C M2 M3 M4 NON ZERO

```

DO 501 NK2=2,MNK2
DO 501 NK3=2,MNK3
DO 501 NK4=2,MNK4
M2=NK2-1
M3=NK3-1
M4=NK4-1
CALL DATA(NK1,NK11,NK12,NK13,NK14,NK15,NK16,NK17,NK18,M2,M3,M4,J)
NK1=NK11
CALL POS(ANK1,NK1,T,TN,C,CN)
S1=C(NK1)*U(NK2)*V(NK3)*W(NK4)
NK1=NK12
CALL POS(ANK1,NK1,T,TN,C,CN)
S2=C(NK1)*UN(NK2)*V(NK3)*W(NK4)
NK1=NK13

```



```

CALL POS(ANK1,NK1,T,TN,C,CN)
S3=C(NK1)*U(NK2)*VN(NK3)*W(NK4)
NK1=NK14
CALL POS(ANK1,NK1,T,TN,C,CN)
S4=C(NK1)*U(NK2)*V(NK3)*WN(NK4)
NK1=NK15
CALL POS(ANK1,NK1,T,TN,C,CN)
S5=C(NK1)*UN(NK2)*VN(NK3)*W(NK4)
NK1=NK16
CALL POS(ANK1,NK1,T,TN,C,CN)
S6=C(NK1)*U(NK2)*VN(NK3)*WN(NK4)
NK1=NK17
CALL POS(ANK1,NK1,T,TN,C,CN)
S7=C(NK1)*UN(NK2)*V(NK3)*WN(NK4)
NK1=NK18
CALL POS(ANK1,NK1,T,TN,C,CN)
S8=C(NK1)*UN(NK2)*VN(NK3)*WN(NK4)
S11=S11+S1+S2+S3+S4+S5+S6+S7+S8
501 CONTINUE
WRITE(2,666)S11

```

C M2 EQUALS ZERO M3 M4 NON ZERO

```

DO 502 NK3=2,MNK3
DO 502 NK4=2,MNK4
M3=NK3-1
M4=NK4-1
M2=0
CALL DATA(NK1,NK11,NK12,NK13,NK14,NK15,NK16,NK17,NK18,M2,M3,M4,J)
NK1=NK11
CALL POS(ANK1,NK1,T,TN,C,CN)
S1=C(NK1)*U(1)*V(NK3)*W(NK4)
NK1=NK13
CALL POS(ANK1,NK1,T,TN,C,CN)
S3=C(NK1)*U(1)*VN(NK3)*W(NK4)
NK1=NK14
CALL POS(ANK1,NK1,T,TN,C,CN)
S4=C(NK1)*U(1)*V(NK3)*WN(NK4)
NK1=NK16
CALL POS(ANK1,NK1,T,TN,C,CN)
S6=C(NK1)*U(1)*VN(NK3)*WN(NK4)
S22=S22+S1+S3+S4+S6
502 CONTINUE
WRITE(2,666)S22

```

C M3 EQUALS ZERO M2 M4 NON ZERO

```

DO 503 NK2=2,MNK2
DO 503 NK4=2,MNK4
M2=NK2-1
M4=NK4-1
M3=0
CALL DATA(NK1,NK11,NK12,NK13,NK14,NK15,NK16,NK17,NK18,M2,M3,M4,J)
NK1=NK11
CALL POS(ANK1,NK1,T,TN,C,CN)
S1=C(NK1)*U(NK2)*V(1)*W(NK4)
NK1=NK12

```



```
CALL POS(ANK1,NK1,T,TN,C,CN)
S2=C(NK1)*UN(NK2)*V(1)*W(NK4)
NK1=NK14
CALL POS(ANK1,NK1,T,TN,C,CN)
S4=C(NK1)*U(NK2)*V(1)*WN(NK4)
NK1=NK17
CALL POS(ANK1,NK1,T,TN,C,CN)
S7=C(NK1)*UN(NK2)*V(1)*WN(NK4)
S33=S33+S1+S2+S4+S7
```

503 CONTINUE

WRITE(2,666)S33

C M4 EQUALS ZERO M2 M3 NON ZERO

```
DO 504 NK2=2,MNK2
DO 504 NK3=2,MNK3
M2=NK2-1
M3=NK3-1
M4=0
CALL DATA(NK1,NK11,NK12,NK13,NK14,NK15,NK16,NK17,NK18,M2,M3,M4,J)
NK1=NK11
CALL POS(ANK1,NK1,T,TN,C,CN)
S1=C(NK1)*U(NK2)*V(NK3)*W(1)
NK1=NK12
CALL POS(ANK1,NK1,T,TN,C,CN)
S2=C(NK1)*UN(NK2)*V(NK3)*W(1)
NK1=NK13
CALL POS(ANK1,NK1,T,TN,C,CN)
S3=C(NK1)*U(NK2)*VN(NK3)*W(1)
NK1=NK15
CALL POS(ANK1,NK1,T,TN,C,CN)
S5=C(NK1)*UN(NK2)*VN(NK3)*W(1)
S44=S44+S1+S2+S3+S5
```

504 CONTINUE

WRITE(2,666)S44

C M2 M3 EQUALS ZERO M4 NON ZERO

```
DO 505 NK4=2,MNK4
M4=NK4-1
M3=0
M2=0
CALL DATA(NK1,NK11,NK12,NK13,NK14,NK15,NK16,NK17,NK18,M2,M3,M4,J)
NK1=NK11
CALL POS(ANK1,NK1,T,TN,C,CN)
S1=C(NK1)*U(1)*V(1)*W(NK4)
NK1=NK14
CALL POS(ANK1,NK1,T,TN,C,CN)
S4=C(NK1)*U(1)*V(1)*WN(NK4)
S55=S55+S1+S4
```

505 CONTINUE

WRITE(2,666)S55

C M3 M4 EQUAL ZERO M2 NON ZERO

```
DO 506 NK2=2,MNK2
M2=NK2-1
```



```
M3=0
M4=0
CALL DATA(NK1,NK11,NK12,NK13,NK14,NK15,NK16,NK17,NK18,M2,M3,M4,J)
NK1=NK11
CALL POS(ANK1,NK1,T,TN,C,CN)
S1=C(NK1)*U(NK2)*V(1)*W(1)
NK1=NK12
CALL POS(ANK1,NK1,T,TN,C,CN)
S2=C(NK1)*U(NK2)*V(1)*W(1)
S66=S66+S1+S2
506 CONTINUE
WRITE(2,666)S66
```

C M4 M2 EQUAL ZERO M3 NON ZERO

```
DO 507 NK3=2,MNK3
M3=NK3-1
M4=0
M2=0
CALL DATA(NK1,NK11,NK12,NK13,NK14,NK15,NK16,NK17,NK18,M2,M3,M4,J)
NK1=NK11
CALL POS(ANK1,NK1,T,TN,C,CN)
S1=C(NK1)*U(1)*V(NK3)*W(1)
NK1=NK13
CALL POS(ANK1,NK1,T,TN,C,CN)
S3=C(NK1)*U(1)*V(NK3)*W(1)
S77=S77+S1+S3
507 CONTINUE
WRITE(2,666)S77
```

C M2 M3 M4 ALL EQUAL ZERO

```
IF(J.EQ.0) GO TO 700
IF(J.EQ.1) GO TO 701
IF(J.EQ.2) GO TO 702
IF(J.EQ.3) GO TO 703
700 S88=TN(3)*U(1)*V(1)*W(1)
GO TO 704
701 S88=TN(2)*U(1)*V(1)*W(1)
GO TO 704
702 S88=T(1)*U(1)*V(1)*W(1)
GO TO 704
703 S88=T(2)*U(1)*V(1)*W(1)
704 WRITE(2,666)S88
AMP=S11+S22+S33+S44+S55+S66+S77+S88
WRITE(2,666)AMP
INTS=AMP*AMP
WRITE(2,666)INTS
666 FORMAT(1F10.5)
V0=V0+7.0
780 CONTINUE
STOP
END
```


ROUTINE POS

76/76 OPT=2 TRACE

UMRCC FTN 4.2.178

```
SUBROUTINE POS(ANK1,NK1,T,TN,C,CN)
DIMENSION T(71),TN(70),C(71),CN(70)
IF(NK1.GE.0) GO TO 53
ANK1=IABS(NK1)
NK1=ANK1+1
C(NK1)=TN(NK1)
GO TO 43
53 NK1=NK1+1
C(NK1)=T(NK1)
43 CONTINUE
RETURN
END
```

ROUTINE DATA

```
SUBROUTINE DATA(NK1,NK11,NK12,NK13,NK14,NK15,NK16,NK17,NK18,
1M2,M3,M4,J)
NK11=J-2*M2-3*M3-4*M4-2
NK12=J+2*M2-3*M3-4*M4-2
NK13=J-2*M2+3*M3-4*M4-2
NK14=J-2*M2-3*M3+4*M4-2
NK15=J+2*M2+3*M3-4*M4-2
NK16=J-2*M2+3*M3+4*M4-2
NK17=J+2*M2-3*M3+4*M4-2
NK18=J+2*M2+3*M3+4*M4-2
RETURN
END
```


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ACKNOWLEDGEMENTS

The author wishes to thank Dr.A.W. Palmer for his continuous encouragement, advice and assistance during this work. Acknowledgements are due to Dr.F.C. [REDACTED] Dr.D.C. [REDACTED], Dr.M. [REDACTED] and Mr.R. [REDACTED] for the useful discussions rendered during the course of the work.

Many thanks are due to Mr.A.T. [REDACTED] for help and guidance with the crystal cutting and polishing and to Mr. W. [REDACTED] for help with the photography.

The author wishes to express his gratitude to Xerox Research (UK) limited and Croydon Precision Instruments for the financial assistance provided during the course of this work.

Finally many thanks are extended to Miss. L. [REDACTED] and to Mr. A.D. [REDACTED] for their patient typing of this thesis.