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COUPLED MICROSTRIP LINES
AND RELATED STRUCTURES WITH
REFERENCE TO DIRECTIONAL COUPLERS.

by

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A Thesis presented for the
Degree of Doctor of Philosophy

to

The City University

Department of Electrical & Electronic Engineering

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ABSTRACT.

The properties of coupled microstrip lines are investigated with the aim of improving the performance of microstrip proximity directional couplers. A closely related transmission line structure termed, for convenience, a microstrip/slot line combination, is also investigated. This supports two dominant modes, easily made degenerate by the addition of a second lowloss high dielectric material. The investigation is followed by the construction of an experimental 3 dB coupler, based upon the microstrip/slot line structure.

Ring resonators are used to measure the normalised guide wavelengths of the modes of the transmission line structures. For coupled microstrip lines the dispersions of the even- and odd-modes are shown to be approximately linear between 1 and 12 GHz, and well characterised by recent theoretical analyses. The normalised guide wavelengths of the dominant modes of the microstrip/slot line combination are also measured using ring resonators.

A new method of measuring the characteristic impedance of transmission lines is presented, and has been used to measure the frequency dependence of the characteristic impedance of microstrip lines. The characteristic impedances of the dominant modes of the microstrip/slot line combination are also determined.

The theoretical analysis of the microstrip transmission line is also studied with attention on the ability and ease at which available methods could analyse the more complex microstrip/slot line combination. The recent Least Square technique applied to mode matching solutions is tested on L-shaped waveguides and then applied to the microstrip problem in a way more general than was previously carried out. TE, in addition to TM solutions are given for the L-shaped waveguide.

INTRODUCTION.

One of the major advances in microwave engineering in the last two decades has been in the technology of microwave integrated circuits. Since its introduction, the earlier forms - strip line, triplate and microstrip - have been complemented by slot line and coplanar waveguide. Along with improved analytical and measurement techniques, there has been much sophistication in the fabrication of the circuits and in the use of materials. Microstrip has gained in importance over the other forms of planar structures, since the high dielectric substrates allow easy construction of hybrid structures that give good electrical performance.

The microwave integrated circuit, despite its obvious advantage of size, has not met with such widespread acceptance as might have been expected. This can be attributed to problems associated with mass production, since the master drawings, photographic reduction, etching and materials, have to be held to high tolerances for correct electrical performance.

The subject of this thesis is not concerned directly with these difficulties, although with any new technique, it is important to consider the practical problems involved. Directional couplers are perhaps among the most widely used passive components in microwave integrated circuits, yet when designed for microstrip circuits certain properties of the dielectric inhomogeneity lead to a deterioration in their performance. The dielectric inhomogeneity itself makes theoretical studies of microstrip complicated, and as an academic problem most interesting. Engineers have, however, achieved a lot without accurate knowledge of the microstrip parameters. A large number of measurements have been taken, and are presented here, to obtain the frequency-dependence of the even- and odd-mode normalised guide wavelengths of a pair of coupled microstrip lines. The results show that the dispersion of each mode, up to a frequency of 12 GHz, is approximately linear. The guide wavelengths of the

two dominant modes of a microstrip/slot line combination are measured in a similar way over a wide frequency range, and it is found that with suitable modifications, the modes are degenerate, that is for a given frequency the modes have identical phase velocities. It is explained that the condition of degeneracy is a condition for optimum directivity and isolation for proximity directional couplers.

The characteristic impedance of transmission lines is discussed in some detail with particular reference to microstrip lines. The definition of characteristic impedance lacks uniqueness, and hence is a difficult quantity to measure. A new method of measurement is proposed, which relates to the usual definition used in theoretical approaches, and an attempt is made at measuring the frequency-dependence of a single microstrip line. The measurements have enabled a recently introduced form of directional coupler to be more fully understood, and a section of the thesis is devoted to outlining the parameters of its transmission line structure to enable a preliminary design of this type of coupler to be made, and to provide practical measurements upon which future development of the theoretical analysis may be based. Once this has been achieved, full design parameters can be calculated, covering wider possibilities than the limited practical measurements.

A theoretical approach for the solution of the microstrip transmission line is given in this thesis. The method, based on a previously used microstrip line analysis, has been extended to form a more general method, suitable for solving not only structures such as the L-shaped waveguide, but also the microstrip line and the more complex microstrip/slot line combination. The theoretical work presented here is aimed at solving this last structure, and thus is possibly not the best approach for the solution of the simpler structures.

CHAPTER 1MICROSTRIP PROXIMITY DIRECTIONAL COUPLERS1.1 Introduction

The problems encountered in practical microstrip proximity directional couplers have already been briefly outlined in the introduction. Before the problems can be properly understood and improvements suggested, it is necessary to understand the way in which directional couplers operate. To this end, some of the theoretical techniques, used to analyse directional couplers, will be discussed in this Chapter. Each theoretical approach gives a slightly different physical understanding of the operation of a directional coupler, and together they give a background on which modifications or alternative solutions, that effect an improved performance, can be examined. Microstrip directional couplers can be subdivided into two categories, namely branch line and proximity couplers. Both types have received extensive investigation since their introduction in the mid-1950's, the history of their initial development being well documented by Mathaei, Young and Jones (1) and Levy (2).

Branch line couplers consist of two parallel transmission lines, which are joined by two or more transmission lines termed branch lines. At each junction of a branch line to a main line, the signal partially splits into the two transmission lines and is partially reflected. The lengths of the branch lines, their impedance, and their spacing, can be arranged so that the individual waves reinforce in one direction of propagation, and interfere destructively in the other. This type of coupler can give good directivity and isolation, though for weak coupling, due to the requirements of high impedance branch lines, the coupler is difficult to fabricate.

The natural coupling between a pair of microstrip lines can be utilised to make a directional coupler, called a proximity coupler. When the two transmission lines are sufficiently close to each other, waves in each line interact in such a manner that

for certain line dimensions good directivity, isolation and input match can be achieved. It is the improvement of this form of coupler that is of prime consideration in this thesis.

Microstrip proximity directional couplers can be made to give a wide range of coupling ratios without the fabrication difficulties of branch line couplers. However, the directivity and isolation is generally worse for the more weakly coupled lines.

1.2 Coupled-Mode Theory.

Coupled-mode theory is concerned with the coupling of a wave in one transmission system to a wave in another transmission system. It can be applied to a wide variety of problems, but in the context of microwave directional couplers it is generally used to describe the action of certain waveguide couplers. For microstrip proximity couplers, it provides a more rigorous theoretical treatment at higher microwave frequencies.

The coupling of two modes is a relatively simple case, and has been well treated by Johnson (3) and Louiselle (4). Adopting Johnson's notation, the transmission line equations for each mode are:

$$j \frac{\partial a_1}{\partial z} - H_{11} a_1 - H_{12} a_2 = 0 \quad \dots 1.1$$

$$j \frac{\partial a_2}{\partial z} - H_{22} a_2 - H_{21} a_1 = 0 \quad \dots 1.2$$

where a_1 is mode 1, and a_2 is mode 2. H_{11} and H_{22} are the uncoupled propagation constants of modes 1 and 2 respectively. H_{12} is the coupling coefficient from mode 1 to mode 2, and H_{21} is the coupling coefficient from mode 2 to mode 1. If one assumes a_1 and a_2 to have form

$$a_1(z) = A_1' e^{-j\beta_1 z} + A_1'' e^{-j\beta_2 z} \quad \dots 1.3$$

$$a_2(z) = A_2' e^{-j\beta_1 z} + A_2'' e^{-j\beta_2 z} \quad \dots 1.4$$

then equations 1.1 and 1.2 become

$$\beta a_1 = H_{11} a_1 + H_{12} a_2 \quad \dots 1.5$$

$$\beta a_2 = H_{21} a_1 + H_{22} a_2 \quad \dots 1.6$$

and hence

$$\beta = \frac{H_{11} + H_{22}}{2} \pm \sqrt{\left(\frac{H_{11} - H_{22}}{2}\right)^2 + H_{12} H_{21}} \quad \dots 1.7$$

where both values for β must satisfy equations 1.5 and 1.6.

This equation is, perhaps, the key to the coupling problem. The two solutions correspond to the phase constants of each mode influenced by the other mode. Both phase constants are real when H_{12} and H_{21} are of the same sign, that is, modes 1 and 2 convey the power in the same direction. Under these conditions 'passive' coupling is said to take place. The waves in each transmission line interact by a periodic exchange of power. If the power in the two modes propagates in opposite directions, that is when H_{12} and H_{21} are opposed in sign, the two phase constants become complex, as shown by the following relation:

$$|H_{12} H_{21}| > \left(\frac{H_{11} - H_{22}}{2}\right)^2 \quad \dots 1.8$$

Coupling under this condition is termed 'active'. The interaction between the two modes no longer produces a periodic exchange of power, but either growing or decaying waves are formed.

The process of active coupling is often associated with the design of amplifiers and oscillators, and is not considered in this thesis. Passive coupling, however, must be considered in more detail, since the normal modes of a microstrip proximity coupler fulfil the conditions for this type of coupling. Certain results will now be stated without reiterating their derivation to be found in reference 3. If the power in modes a_1 and a_2 are P_1 and P_2 respectively, and no input is applied to the mode a_2 at

$z=0$, then the modal powers P_1 and P_2 at a position z are given by:

$$P_1 = + |a_1(0)|^2 \cos^2 H_{12} z \quad \dots 1.9$$

$$P_2 = + |a_1(0)|^2 \sin^2 H_{12} z \quad \dots 1.10$$

where $a_1(0)$ is the initial condition of the mode a_1 at $z=0$. Being a passive structure, the total power does not vary, though complete energy transfer is achieved when

$$H_{12}L = \pi/2 \quad \dots 1.11$$

and the modes are exactly synchronised, or degenerate, with each other. L is the physical length of the coupled section.

This theory of coupling will be found to be a secondary form of coupling in the microstrip configurations investigated. For coupled microstrip lines, the two normal modes, namely the even- and odd-mode, are generally assumed to be decoupled, H_{12} and H_{21} being zero. It will be shown in Chapter 2 that this is not true for a microstrip/slot combination, although coupling is weak enough to ignore in a practical design.

1.3 Coupled-Mode Analysis using a Quasi-TEM Approximation

Coupled transmission lines can be analysed approximately by considering a quasi-TEM mode to propagate in each line. Krage and Haddad (5) analysed coupled microstrip lines in this manner. The quasi-TEM mode approximation allows the electric and magnetic fields to be considered separately. Four differential equations related the voltage and current with the self- and mutual-inductance and capacitance in each line. These equations were then converted into a normal mode form by substituting four variables, called mode amplitudes, which were related to the voltage, current and characteristic impedance, in such a way that the power flowing was equal to the square of the mode amplitudes. With some rearrangement, the coupled-mode equations, which related the

mode amplitudes and phase constants of the coupled lines, were formed. Inductive and capacitive coupling coefficients replaced the identities formed from the self- and mutual-inductance and capacitance of each transmission line. The coupled-mode equations were then solved assuming the mode amplitudes to change with the position z , in the following manner:

$$a(z) = a_0 e^{j\gamma z} \quad \dots 1.12$$

where a_0 is the peak value of the mode amplitude $a(z)$, and γ the propagation constant. The solution to the coupled-mode equations was as follows:

$$\gamma = \pm j\beta_0 \sqrt{1 \pm \delta} \quad \dots 1.13$$

where

$$\beta_0 = \sqrt{\frac{\beta_1^2 + \beta_2^2}{2} - \beta_1 \beta_2 k_L k_C} \quad \dots 1.14$$

and

$$\delta = \sqrt{1 - \frac{\beta_1^2 \beta_2^2}{\beta_0^4} (1 - k_L^2)(1 - k_C^2)} \quad \dots 1.15$$

where β_1 and β_2 are the phase constants of each line, k_L is the inductive coupling coefficient, and k_C is the capacitive coupling coefficient. When the coupled lines are identical the propagation constant had four roots, except for $k_L = \pm k_C$, when it has two. The roots are such that when $k_L = \pm k_C$ two modes are formed with different phase velocities from the superposition of the four mode amplitudes. The phase velocities are equal when the modes are degenerate, that is when $k_L = \pm k_C$.

Krage and Haddad proceeded to use the linear combination of the four mode amplitudes to form the two propagating modes of the system. The performance of a coupler was then investigated theoretically for various combinations of k_L , k_C and phase constant. The method, although only approximate, does give some physical insight into the coupling mechanism. For example, the forward

and backward waves on a line can be seen to be decoupled when the line's impedance is invariant with length, and the coupling between the waves on each line can be seen to be directional when $k_L = k_c$.

1.4 Microstrip Directional Coupler Analysis

The analysis of a pair of identical coupled microstrip lines, uniformly spaced over an electrical length l , is usually carried out in terms of the even- and odd-modes of a four port network. This technique, a derivation of Bartlett's bisection theory for symmetrical two-port networks (6), is well known and can be applied to any symmetrical four-port network allowing it to be considered as the superposition of two two-port networks.

Line of symmetry

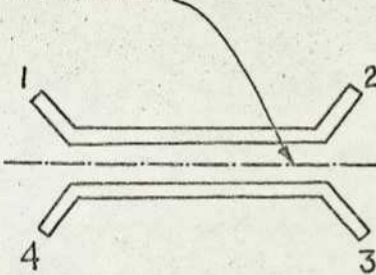


Figure 1.1

Even-mode excitation is achieved by applying equal amplitude and in-phase signals to ports 1 and 4 of Figure 1.1. A maximum of potential occurs along the line of symmetry, allowing a magnetic wall to be placed along it without affecting the field distribution. Odd-mode

excitation occurs when signals with equal amplitude, but with 180 degree phase difference, are applied to ports 1 and 4. A potential zero occurs along the line of symmetry allowing an electric wall to be substituted for the magnetic wall of the previous case. For each mode the problem is reduced to a two-port network, which allows much easier analysis. Superposition of the two field distributions corresponds to the field distribution which would occur when only Port 1 is excited, this latter condition being the requirement of the analysis. Besides being able to calculate the field distributions, the reflection and transmission coefficients for each mode can be found when the line is terminated at each end

by a known impedance, Z_0 . This can conveniently be achieved by using the transfer matrix for each excitation, these being:

$$\begin{bmatrix} V_{1e} \\ I_{1e} \end{bmatrix} = \begin{bmatrix} \cos \theta_e & jZ_{oe} \sin \theta_e \\ \frac{j}{Z_{oe}} \sin \theta_e & \cos \theta_e \end{bmatrix} \begin{bmatrix} V_{2e} \\ I_{2e} \end{bmatrix} \quad \dots 1.16$$

$$\begin{bmatrix} V_{1o} \\ I_{1o} \end{bmatrix} = \begin{bmatrix} \cos \theta_o & jZ_{oo} \sin \theta_o \\ \frac{j}{Z_{oo}} \sin \theta_o & \cos \theta_o \end{bmatrix} \begin{bmatrix} V_{2o} \\ I_{2o} \end{bmatrix} \quad \dots 1.17$$

where Z_{oe} and Z_{oo} are the impedances of the coupled line section under even- and odd-mode excitation respectively. If the transfer matrix can be assumed to be of the form:

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix} \quad \dots 1.18$$

then the reflection coefficient, Γ , is,

$$\Gamma = \frac{A + B/Z_0 - C.Z_0 - D}{A + B/Z_0 + C.Z_0 + D} \quad \dots 1.19$$

the transmission coefficient, τ , is,

$$\tau = \frac{2}{A + B/Z_0 + C.Z_0 + D} \quad \dots 1.20$$

and the input impedance, Z_1 , is

$$Z_1 = \frac{A + B/Z_0}{C + D/Z_0} \quad \dots 1.21$$

The vector amplitudes of the signals at each port are found by superposition, these being:

$$a_1 = \frac{1}{2} (\Gamma_{oe} + \Gamma_{oo}) \quad \dots 1.22$$

$$a_2 = \frac{1}{2} (\tau_{oe} - \tau_{oo}) \quad \dots 1.23$$

$$a_3 = \frac{1}{2} (\tau_{oe} + \tau_{oo}) \quad \dots 1.24$$

$$a_4 = \frac{1}{2} (\Gamma_{oe} - \Gamma_{oo}) \quad \dots 1.25$$

where the suffixes to the vector amplitudes refer to the number

of the port, and the suffixes oe and oo refer to the even- and odd-mode excitation respectively.

When the electrical lengths of the line under even- and odd-mode excitation are equal, it can be shown (1) that the input to the coupler will be matched when

$$Z_o = \sqrt{Z_{oo} \cdot Z_{oe}} \quad \dots 1.26$$

If this applies, then

$$a_1 = 0 \quad \dots 1.27$$

$$a_2 = \frac{jc \sin \theta}{\sqrt{1-c^2} \cos \theta + j \sin \theta} \quad \dots 1.28$$

$$a_3 = 0 \quad \dots 1.29$$

$$a_4 = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2} \cos \theta + j \sin \theta} \quad \dots 1.30$$

where the coupling factor, c , is given by:

$$c = \frac{Z_{oe}/Z_{oo} - 1}{Z_{oe}/Z_{oo} + 1} \quad \dots 1.31$$

and θ is the electrical length of the coupled line. Equation 1.31 can be arranged to give the following design relationships:

$$Z_{oe} = Z_o \sqrt{\frac{1+c}{1-c}} \quad \dots 1.32$$

$$Z_{oo} = Z_o \sqrt{\frac{1-c}{1+c}} \quad \dots 1.33$$

Generally the electrical length is different for each of the modes and equation 1.26 is only approximately true for weakly coupled lines. By substituting θ_e for the electrical length under even-mode excitation, and θ_o for the electrical length under odd-mode excitation, it can be shown (2) that the condition for perfect match and isolation occurs when:

$$\frac{Z_{oo} \cdot Z_{oe}}{Z_o^2} = \frac{Z_{oe} \sin \theta_o + Z_{oo} \sin \theta_e}{Z_{oe} \sin \theta_e + Z_{oo} \sin \theta_o} \quad \dots 1.34$$

This will mean that the match and isolation will be frequency dependent. Further theoretical investigation reveals that a 5% difference in electrical lengths reduces the isolation to about 28 dB. This isolation is independent of coupling for weakly coupled lines, causing the directivity to be worse for loosely coupled lines than for tightly coupled lines.

1.5 Equalisation of the Phase Velocities in Coupled Microstrip Lines.

Recently there has been some attention paid to the problem of equalisation of the phase velocities of the even- and odd-modes in a pair of coupled microstrip lines. The previous section showed how a small difference in electrical lengths of each mode, within the coupler, causes poor isolation and bad directivity. Several methods to make the two modes degenerate have been investigated by others. Initial work considered the coupled line as propagating quasi-TEM modes. Inductive and capacitive coupling coefficients were calculated for the structure, the solution to the equalisation being to equate these two coupling coefficients. It was found that the capacitive coupling coefficient was less than the inductive coupling coefficient, and practical modifications to the structure were designed to increase the former. This could be achieved by several different methods. An insulating metallic overlay situated a small distance directly above the lines, or an overlay of insulating material placed over the lines, have very similar effect. Also both methods can be combined to the same end. An alternative solution is to have an air, or a substrate with a low dielectric constant, between the dielectric and ground plane. With all methods of this nature not only the phase velocity, but also the characteristic impedance of the mode will be altered. The calculation of these parameters for a single microstrip line is discussed in Chapter 4. It is sufficient to say here that it is a difficult theoretical problem because of the dielectric inhomogeneity.

The presence of a dielectric inhomogeneity constrains the theoretical methods to approximations, such as assuming the

propagating mode to be quasi-TEM, or to use a hybrid mode analysis with its associated convergence difficulties. Okugawa (7) analysed embedded coupled lines (a dielectric overlay on conventional coupled microstrip lines) using the quasi-TEM approximation and obtained values for the effective dielectric constant for both even- and odd-mode excitation. He calculated the characteristic impedance using the relationships which are only valid for TEM lines, and showed that for certain thicknesses the two modes were degenerate, but it was not substantiated with any practical evidence. The characteristic impedance of microstrip lines is discussed in Chapter 3, and from the measurements presented some doubt is cast onto the accuracy of Okugawa's impedance calculations.

Krage and Haddad have published two analyses of various structures which equalise the even- and odd-mode phase velocities in microstrip proximity couplers. The first of these used the quasi-TEM approximation in a variational method (8). The modified structures analysed were a dielectric overlay, a dielectric overlay with a metallic conductor, and an air gap between the dielectric and the ground plane. All structures showed, that with suitable dimensions, equalisation of the inductive and capacitive coupling coefficients could be achieved. This implied that good directivity and isolation could be achieved in a coupler. The characteristic impedance of each mode was not known, but could be calculated from the effective dielectric constant using similar relationships to those used by Okugawa. Their second method (9), which is more recent, determines the frequency dependent characteristics for some of the configurations discussed previously. This was achieved using the Fourier matching technique with a hybrid mode formulation (see Chapter 4). Convergency problems related to this method were not discussed, though the way in which the matching technique was formulated may have diminished the problem. The variation of the effective dielectric constant and of the impedance, with normalised frequency, were calculated for the dielectric overlay modification, and for a substrate made out of two different dielectric materials. The dielectric overlay modification considered, showed that mode degeneracy would occur

at a frequency where excitation of higher order modes would make it impracticable. The other structure analysed was a double layer structure where a thin layer of teflon (dielectric constant of 2.2) was placed between the Alumina substrate (dielectric constant of 9.7) and the ground plane of a conventional microstrip proximity coupler. This was shown to exhibit mode degeneracy at a low normalised frequency. Possible coupler directivities were estimated from the results. The frequency-dependency of the characteristic impedance of each mode for these modifications was not given. It was suggested that to obtain mode degeneracy with this type of modification, thinner substrates would be required at higher frequencies.

Kowalski and Pregla (10) have also theoretically analysed the dielectric overlay structure, but with different dielectrics and dimensions than those used by Krage and Haddad. A variational expression was used which had previously given good results for coupled microstrip lines. Their results covered a wider frequency range than those of Krage and Haddad, showing two points of mode degeneracy. Details of the characteristic impedance of both modes were given, showing little variation over the wide frequency range considered. Higher order modes were not discussed, though surface waves were considered in some detail for the single layer structure.

1.6 An Alternative Form of Proximity Directional Coupler.

The problems of equalisation of the phase velocities of the even- and odd-modes may be approached in an alternative way to those expressed in the previous section. It can be seen that the performance of a practical proximity coupler is well modelled by the analysis of section 1.4. This fact allows one to conclude that the even- and odd-modes are decoupled with respect to each other, indeed they are often treated separately in theoretical analyses. The proximity directional coupler can therefore be viewed as a transmission line which supports two decoupled modes, being excited at each end by two microstrip transmission lines. In principle the two modes and the transmission line, in which they propagate, are quite arbitrary, the only

requirements being that at each end of the transmission line they couple suitably into two microstrip lines, and the two decoupled modes are degenerate. It was perhaps using this principle that de Ronde (11) investigated the possible applications of a microstrip line over a ground plane slot. Various proposals were examined by him, these being directional couplers, a magic tee, and a wideband lumped proximity coupler. The directional couplers operated using the principle of coupled modes, outlined in section 1.2, since when a microstrip line and a ground plane slot are not parallel the two dominant modes which exist in such a structure are not decoupled. Transfer of energy from one mode to the other, and directionality of coupling, will occur depending upon the coupled length and the coupling coefficient. The magic tee is of little importance to this subject, though it is an interesting application of the combination of microstrip lines and ground plane slots. The lumped proximity coupler, also described, gave 3 dB coupling and a directivity greater than 20 dB over a 2 to 8 GHz bandwidth. The design was derived from a standard microstrip proximity coupler, for if the gap between the two microstrip lines could be regarded as a slot, then it could be substituted for a slot in the ground plane. By making the length of the coupled region very much less than a wavelength, and placing an overlay capacitor over the slot, the coupler was almost of lumped form, giving a near invariant frequency response.

More recently Garcia (12) published details of a 3 dB directional coupler which had a similar basic structure to that of de Ronde. There was no overlay in the coupled region, though two special openings in the ground plane, parallel to the slot, had been introduced for tuning purposes. The outputs from the direct port and coupled port corresponded to that which one would expect from a standard quarter-wavelength proximity coupler. Over the frequency band, 2.5 to 10.5 GHz, the power at the isolated port was at its worst case 18 dB, indicating a worst case directivity of 15 dB. The input VSWR was between 1.05 and 1.25. The coupler had been developed empirically, the final design being the best of a number of trial combinations of top conductor, slot length and width.

CHAPTER 2

MEASUREMENT OF THE NORMALISED GUIDE WAVELENGTH OF COUPLED MICROSTRIP LINES.

2.1 Introduction

The resonance of a length of transmission line is only possible at a number of discrete frequencies. At the line's resonant frequencies it supports a standing wave in which the energy is alternately stored in the electric and magnetic fields. The nature of the terminations placed at each end of the line determines how the physical length of the line is related to the wavelength of the standing wave. It is from the use of this phenomenon that the wavelength of a particular propagating mode can be established at a particular frequency. The resultant information being more usefully expressed, either by normalisation with respect to the free space wavelength at the same frequency, or in terms of the phase velocity, v_p , of the mode in the line which is related to wavelength, λ_g , and frequency, f , by

$$v_p = \lambda_g \cdot f \quad \dots 2.1$$

A resonant section of any transmission line can be made by imposing open- or short-circuits at the ends of the line. The standing wave is excited through a loosely coupled probe in such a way as to minimise the effect it might have on the resonator. It has been shown by Troughton (13), that the end effect in the open- or short-circuit causes an inaccuracy in measurements on microstrip resonators that should be accounted for in any subsequent calculations. For this reason, an alternative is often chosen whereby the transmission line is formed in such a way that the two free ends are joined (14, 15). Open- or short-circuit end effects are not present, allowing the wavelength to be related to the physical dimensions without recourse to assumptions as to the nature of the end effect.

Measurement of the variation of phase velocity with

frequency of single microstrip lines using ring resonators has been well documented (14, 15, 18). The results of these, as well as other measurements by Zysman and Varon (17), generally agree at zero frequency with the theoretical predictions of Bryant and Weiss (19), and their variation with increasing frequency agrees with the recent frequency dependent theoretical approaches (9, 15, 17, 18, 20-23).

The phase velocities of the two modes present in a coupled microstrip line configuration have been measured and compared with a theoretical calculation by Denlinger (15). His method involved the use of a hoop resonator - a cylinder with the two lines on the outer surface, the ground plane on the inner surface. Although close agreement with a theoretical approach was achieved, the results were limited to a magnesium titanate substrate, which had a relative permittivity of 16.24 and a thickness of 0.1041 cm., the coupled lines being such that $W/H = 0.375$ and $S/H = 0.25$.

Napoli and Hughes (24) have also measured the phase velocities of the two modes. A linear open-circuited resonator was used, the loose coupling to it being at each end. The results of their experiment showed that they had measured the difference in the two guide wavelengths to be between 15% and 17%, which compared to a 7% difference calculated from results of Bryant and Weiss, and 12% from results of Johnson (25). The difference between the guide wavelengths of the two modes is an important factor in the design of a directional coupler, as has been shown in Chapter 1.

The ring resonator has been analysed by several people (27, 28) to account for the effect of curvature. The analysis techniques used a radial waveguide model, and showed that if the ratio of line width W to mean radius R , is such that $W/R < 0.1$, higher order radial modes are generally not excited. Other theoretical work (29, 30) has been to investigate the behaviour of rings which have some asymmetry or are asymmetrically excited. The ring resonator is shown to support two degenerate modes which are generally uncoupled with respect to each other. When asymmetry

of some kind is introduced, they couple together allowing two values of phase constant for any one frequency. This results in either double peaks to the resonance condition, or with less coupling, a broadening of the response about resonance. For nominally symmetrical ring resonators this effect, produced by small errors in the construction of the resonator, will lower the measured Q factor without the implied frequency change.

2.2 Measurement of the Even- and Odd-Mode Guide Wavelengths of Coupled Microstrip Lines.

It has been shown that the directivity obtained from conventional microstrip proximity couplers is dependent upon the difference between the even- and odd-mode guide wavelengths. Apart from the low frequency theoretical solutions of Bryant and Weiss, and the measurements of Napoli and Hughes, little has been done to enumerate these parameters. The purpose of this experiment was to provide more information about the propagation of the even- and odd-mode in coupled microstrip lines. It was hoped to measure the frequency dependence of the guide wavelengths, and to provide a check on the results of Napoli and Hughes' experiment (24), which had shown discrepancies with the theoretical solution by Bryant and Weiss.

Concentric rings were chosen for the resonator in order to eliminate the difficulties of the open- or short-circuited ends associated with linear resonators. The external connections to the resonator were made on two separate 50 ohm lines tangential to this outer ring, and diametrically opposed. This method of coupling to the resonator is an original aspect of the experimental procedure. Its purpose was to match the input and output of the resonator to the measuring equipment, and to provide coupling into, and out of, the resonator. Apart from complying with these conditions, it also allowed easy identification of the two modes, since they give different transfer characteristics between the input and output circuits. Figure 2.1 is a photograph of a ring resonator, discussed in Section 2.3, and a typical transfer plot

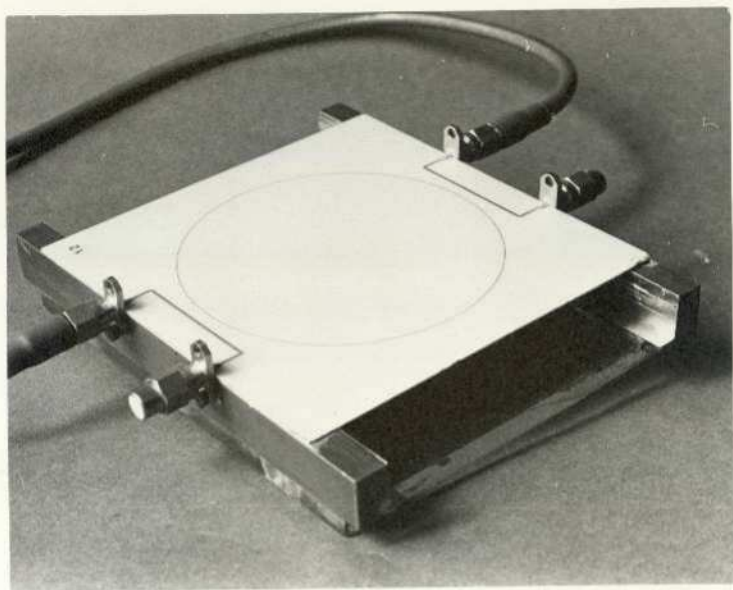


Figure 2.1 A Microstrip/Slot Line Ring Resonator.

is shown in Figure 2.2.

A possible explanation for this behaviour is now given. The inner ring will have a higher Q factor than the outer ring, because it is loaded less by the input and output circuits. Power is preferentially coupled into the even-mode excitation, and, because of the inner ring's higher Q factor, the peak voltage on the inner ring will be greater than that on the outer ring. An odd-mode is thus excited which is orientated so as to oppose the even-mode field in the outer ring. The phase of each mode shifts through 180 degrees at the output point of the resonator for each successive resonance, even- and odd-modes of similar orders combining to form a transfer characteristic as in Figure 2.2a. The odd-mode outer ring resonance is at a lower frequency than that of the inner ring, and its associated opposition to the even-mode field causes a dip in the transfer characteristic. The resonance of the odd-mode in the inner ring is in sympathy with the even-mode field, causing a peak in the transfer characteristic. If the two modes have different dispersive effects, it is possible for modes of different orders of resonance to be superimposed. Because of the 180 degree phase shift mentioned above, modes in which the order of resonance differ by an odd number, superimpose in such a way that the shape of the odd-mode resonance is reversed, as in Figure 2.2b.

The input and output circuits were weakly coupled to the resonator by virtue of their distance at the nearest points of proximity. The degree of coupling achieved was not as easy to adjust as for the case of probe coupling that Troughton used (26), giving in this experiment a Q factor between 100 and 250. If one can regard the ring at resonance as a parallel resonant circuit, the relationship between resonant frequency f , with a finite Q, and that frequency f_0 , with an infinitely large Q is:

$$\frac{f}{f_0} = \left(1 - \frac{1}{Q^2}\right)^{\frac{1}{2}} \quad \dots 2.2$$

from which a Q of 100 will give a 0.005% lowering of the actual resonant frequency. (0.0013% for a Q of 200).

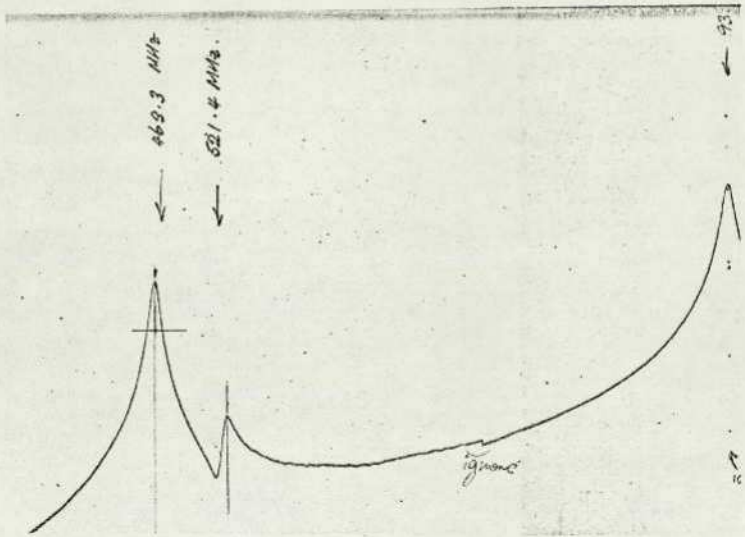


Figure 2.2a

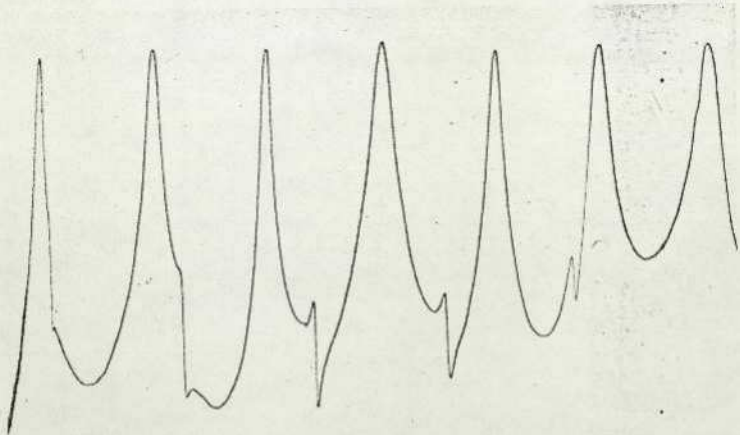


Figure 2.2b Transfer Characteristics of a Coupled Microstrip Line Ring Resonator.

Three coupled ring structures, of different dimensions, were chosen. Their choice was to give close approximation to some of the dimensions that Napoli and Hughes measured, and to provide details of the modes present in both weakly and strongly coupled lines. Each structure in turn was fabricated on Coors ADS 995 Alumina, of thickness 0.635 mm. (equivalent to 0.025 of an inch). This was necessary since these substrates are not cut and polished, but are in the 'as fired state'. There is, thus, a spread in dielectric constant, flatness, and tolerances between substrates from the same batch. As a check upon repeatability and accuracy, the measurements were carried out on a total of three substrates, nominally identical. The thickness of the copper conductors was approximately 0.013 mm. in order that the thickness would be several skin depths at the lowest resonant frequency.

The resonant frequencies of the even- and odd-modes were measured using a Hewlett Packard 537A wavemeter which has a maximum error of 0.17%. For the even-mode, the two frequencies associated with the fundamental resonances of the inner and outer rings, designated f_i and f_o respectively, were measured using loosely coupled probes above the substrate. This technique also established that these resonances were due to the different circumferences of the inner and outer rings, and not due to coupling of modes that travel in opposite directions around the resonator, as had been shown to occur in slightly asymmetrical rings by Wolff (30). The apparent resonant frequency f_s was measured using the coupling arrangement on the substrate. Letting v_{pe} be the phase velocity for the even-mode, then:

$$v_{pe} = f_i C_i = f_o C_o \quad \dots 2.3$$

or

$$\frac{f_i}{f_o} = \frac{C_o}{C_i} \quad \dots 2.4$$

where C_i and C_o are the effective circumferences of the even-mode for the inner and outer rings respectively. They are assumed to lie symmetrically about the mean circumference, the centre line

of the coupled line structure, from a consideration of the low frequency charge distribution by Bryant and Weiss (31). Denoting the radii of the circles with circumferences C_i and C_o as r_i and r_o respectively:

$$\frac{r_o}{r_i} = \frac{f_i}{f_o} \quad \dots 2.5$$

further

$$r_i + r_o = 2r_m \quad \dots 2.6$$

where r_m is the mean radius, obtained from measurements using a precision travelling microscope. Consequently, the circumference of the structure C_s , which corresponds to the apparent resonant frequency f_s can be related to either of the two separate resonances:

$$C_s = \frac{2\pi r_i f_i}{f_s} = \frac{2\pi r_o f_o}{f_s} \quad \dots 2.7$$

and from solving simultaneously 2.5 and 2.6,

$$r_i = \frac{2r_m}{(1 - f_i/f_o)} \quad \dots 2.8$$

and

$$r_o = \frac{2r_m}{(1 - f_o/f_i)} \quad \dots 2.9$$

thus

$$\lambda_g = \frac{C_s}{N} \quad \dots 2.10$$

where N is the resonance number, and

$$\lambda_o = \frac{2\pi f_s}{c} \quad \dots 2.11$$

where c is the velocity of light.

The circumference used for the odd-mode was derived in a similar manner. The measured resonant frequency f_s being

chosen for convenience as the lower frequency resonance, remembering that this will be either a maxima or minima. Where tightly coupled rings did not allow f_i and f_o to be identified separately, C_s for each mode was found by arithmetically scaling its value, for loosely coupled rings, to a new one which was similarly related to the geometry of the tightly coupled rings by the following identity:

$$C_s = 2\pi\tau_m - 2\pi S \left(\frac{r'_m - \frac{C'_s}{2\pi}}{S'} \right) \quad \dots 2.12$$

where the primed variables correspond to the previous results, and S is the width of the gap between the lines.

The results of this experiment, compared with those from Napoli and Hughes' experiment and Bryant and Weiss' theoretical solution are given in Table 2.1. The theoretical solutions were computed on a CDC 6600 computer, using programs CP34 and CP63, from the IEE Computer Library. The dielectric constant for the experiment was taken as 10.3 rather than the value of 9.7 which was used in the paper published on these measurements (see Appendix 2). The reason for making this change is given in the discussion of this experiment in Chapter 6. The frequency dependence of the two modes is displayed in Figure 2.3.

2.3 Measurement of the Guide Wavelengths of the Dominant Modes of a Microstrip/Slot Combination.

A direct development from the experiment described in the previous section was to include a narrow slot in the ground plane directly under the coupled microstrip lines. The aim was to reduce the difference between the even-mode and odd-mode phase velocities, by reducing the proportion of the power flowing in the dielectric for the former, and increasing it for the latter. A slot, with a width to height ratio, D/H , of unity, under coupled lines of $W/H = 0.8$ and $S/H = 0.6$, reduced the ratio of even- to odd-mode guide wavelength from 1.1325 to 1.1010, at a frequency of 2 GHz. With this arrangement, however, a third dominant mode is also excited which has a phase velocity very

		This Experiment $\epsilon_r = 10.3$ (1GHz)			Napoli & Hughes (24) $\epsilon_r = 10.4$ (8GHz)		
S/H		1.112	0.69	0.19	1.00	0.528	0.12
W/H		0.86	0.80	0.30	0.96	0.863	0.363
A	λ_{ge}/λ_0	0.369	0.371	0.391	0.355	0.356	0.372
	λ_{go}/λ_0	0.411	0.419	0.435	0.403	0.414	0.424
	$\lambda_{go}/\lambda_{ge}$	1.114	1.129	1.112	1.135	1.16	1.14
B	λ_{ge}/λ_0	0.369	0.369	0.386*	0.364	0.367	0.383*
	λ_{go}/λ_0	0.405	0.410	0.420*	0.402	0.409	0.418*
	$\lambda_{go}/\lambda_{ge}$	1.098	1.111	1.088*	1.104	1.114	1.090*

* Inaccuracy expected because S/H is outside the program limits (see Program description CP34, reference 19)

A denotes experimental work, and B denotes theoretical work.

Table 2.1.

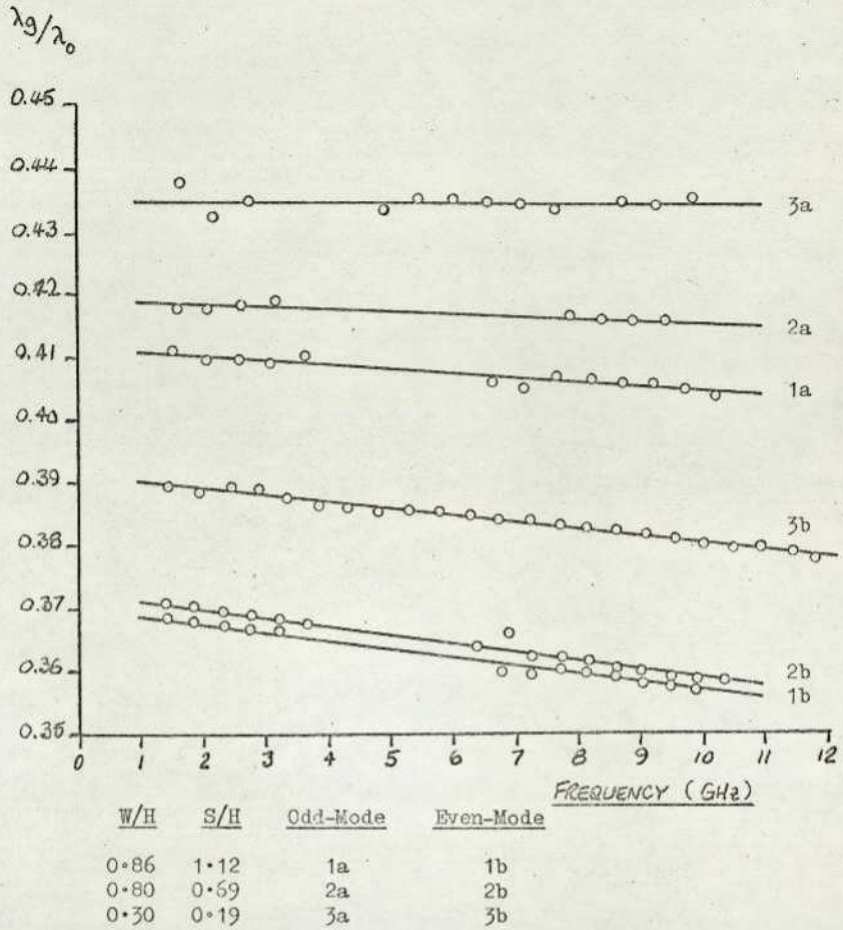


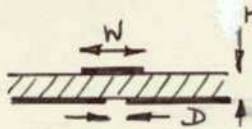
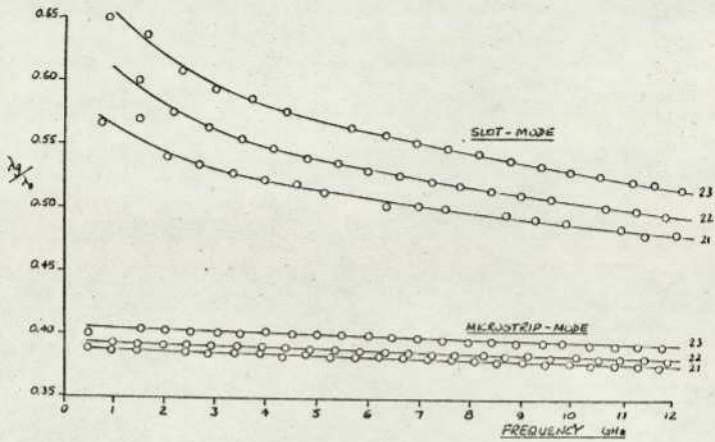
Figure 2.3 Frequency Dependence of the Even- and Odd-Modes of Coupled Microstrip Lines.

much higher than either of the other two dominant modes.

The introduction of a third mode serves only to complicate the already difficult problem of the proximity directional coupler. Elimination of the odd-mode, by joining the pair of lines on the upper surface of the substrate, causes the transmission line to have two dominant modes - a microstrip-mode and a slot-mode. It will be seen that the microstrip-mode is similar to the even-mode of coupled microstrip lines. The slot-mode, however, has no similarity with the odd-mode of coupled microstrip lines, showing close similarity with the slot-mode described by Cohn (32).

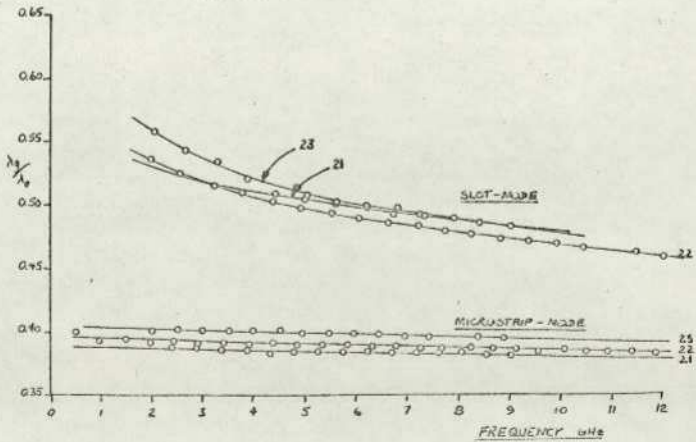
Ring resonators were used to measure the guide wavelengths of these modes. The input and output circuits were more weakly coupled to the rings than they were for the coupled microstrip line resonators. The reason for this being that the Q factor of the coupled microstrip line resonator may have been limited by loading due to the input and output circuits. Three different widths of line and three different widths of slot were chosen allowing nine possible combinations of line and slot. Each structure in turn was fabricated on the same tile. Problems of registration of the slot under the line were eliminated, somewhat crudely, by using film rather than glass plates as photographic positives. The films for the top and bottom surfaces were then formed into an envelope into which the substrate could be placed for exposure under the ultra-violet lamp. In order to obtain quickly an insight into the line/slot performance, only five out of the nine possible combinations were tested. Of these, three had a similar top line, this being the intermediate choice, combined with different width slots. The results for the variation of guide wavelength with frequency is given in Figure 2.4. In determining these results, the circumference of the ring resonator was calculated from the mean geometrical diameter, since the electrical circumference for each mode was closely approximated by the mean geometrical circumference. This fact was proved by comparing a resonance taken using the input/output circuitry, to the same resonance measured with an external probe directly over the slot or line.

Figure 2.4 Variation, with frequency, of the Microstrip- and Slot-Modes of a Microstrip/Slot Line Combination.



Number	W/H	D/H
21	1.0	0.1
22	1.0	0.5
23	1.0	1.0

Figure 2.5 Variation, with frequency, of the Microstrip- and Slot-Modes of a Microstrip/Slot Line Combination when loaded with 0.025 inch thick Alumina.



It was found that coupling between the two modes was weak, since the phase velocity of the slot-mode could be modified with little effect on the microstrip-mode. The desired modification to the slot-mode was to reduce the phase velocity, so that, ideally, the mode would become degenerate with the microstrip-mode. The reduction in phase velocity could be achieved in one of two ways. Firstly by positioning an insulated conducting strip over the slot, the fields associated with the mode in the slot are deformed, with the net effect that more power flows in the dielectric material, reducing the phase velocity of the mode. Secondly, the phase velocity of the slot mode could be reduced by placing a piece of dielectric material directly beneath the slot. The former method was employed by de Ronde (11) for his directional coupler. The gap between the overlay conductor and the ground plane was possibly critical, and the structure needed to be carefully designed to avoid exciting resonances associated with the extra conductor. It was the second alternative that was adopted for the reduction of the slot-mode's phase velocity. The decision was made taking account of the simplicity with which dielectrical loading of the slot could be achieved, and the fact that the technique was novel.

Three different forms of dielectric loading were tested. They were a 0.025 inch thick alumina substrate, a 0.025 inch and a 0.040 inch thick zirconate substrate. The results are shown in Figures 2.5 to 2.7 for the same parameters as for the measurements on un-loaded lines shown in Figure 2.4. These results are for constant line width and varying slot width combinations. Results shown in Figures 2.8 to 2.11 are for constant slot width and varying line width combinations. This required two further ring combinations to be measured, bringing the total number of ring combinations measured to five. It can be seen that the two modes become degenerate for both thicknesses of the zirconate material, and also that the loading material had little effect on the guide wavelength of the microstrip-mode.

The widest slot yielded results which made it more easy to see that the two modes are indeed weakly coupled.

Figure 2.6 Variation, with Frequency, of the Microstrip- and Slot-Modes of a Microstrip/Slot Line Combination when loaded with 0.025 inch thick Zirconate.

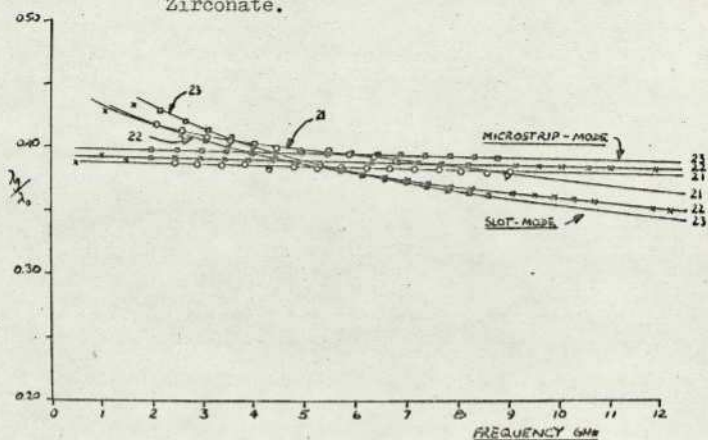


Figure 2.7 Variation, with Frequency, of the Microstrip- and Slot-Modes of a Microstrip/Slot Line Combination when loaded with 0.040 inch thick Zirconate.

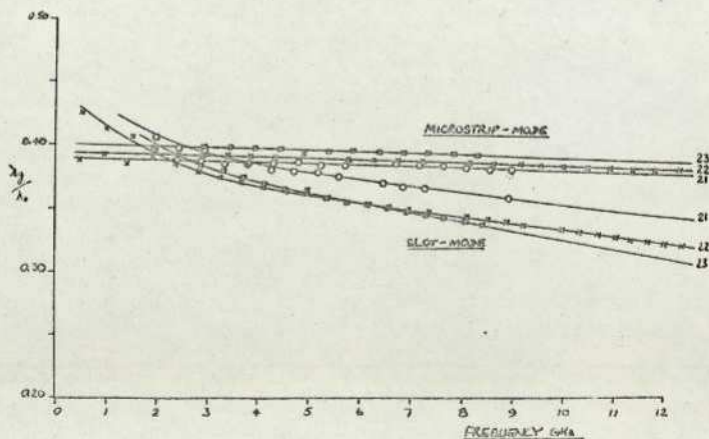
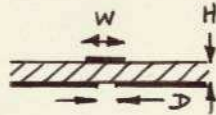
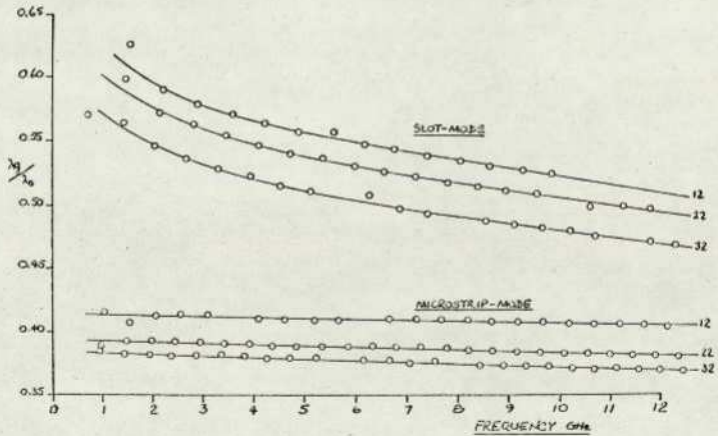


Figure 2.8 Variation, with Frequency, of the Microstrip- and Slot-Modes of a Microstrip/Slot Line Combination.



Number	W/H	D/H
12	0.2	0.5
22	1.0	0.5
32	2.0	0.5

Figure 2.9 Variation, with Frequency, of the Microstrip- and Slot-Modes of a Microstrip/Slot Line Combination when loaded with 0.025 inch thick Alumina.

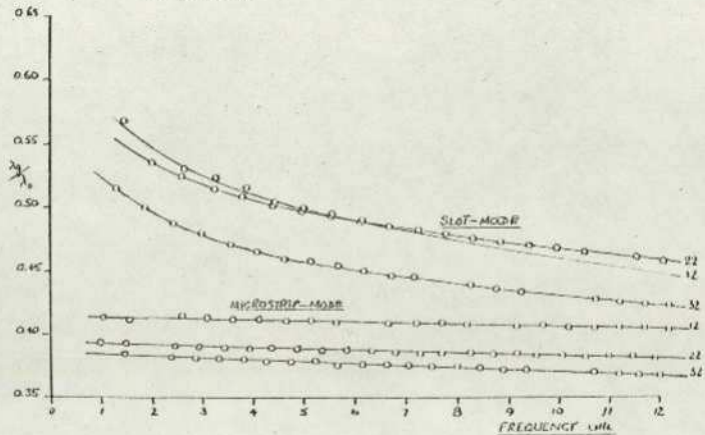


Figure 2.10 Variation, with Frequency, of the Microstrip- and Slot-Modes of a Microstrip/Slot Line Combination when loaded with 0.025 inch thick Zirconate.

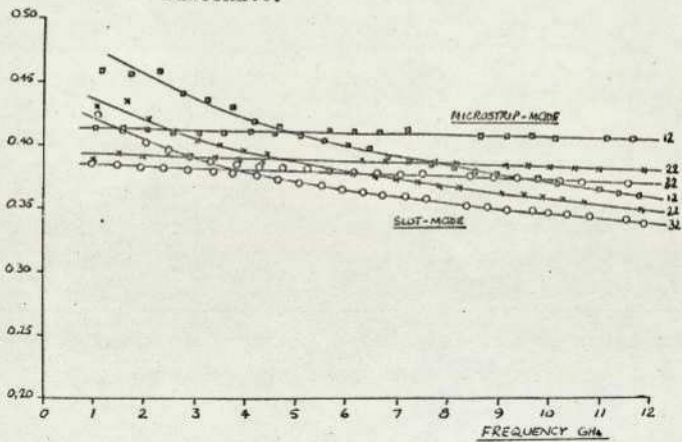


Figure 2.11 Variation, with Frequency, of the Microstrip- and Slot-Modes of a Microstrip/Slot Line Combination when loaded with 0.040 inch thick Zirconate.

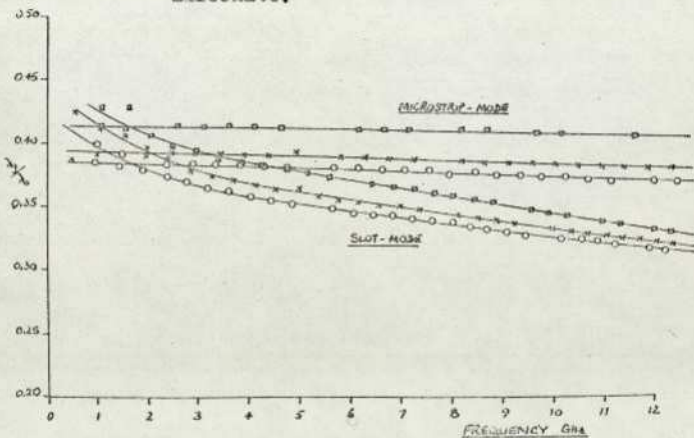


Figure 2.12 is a more detailed plot of the guide wavelengths of the two modes around the point of degeneracy for this case. It is easily seen that the diagram is characteristic of two modes passively coupled (3). It has been stated that in this case the mode coupling is weak. This is evident from Figure 2.12, for it can be calculated that the required coupling length for complete power exchange would be 87.5 cms. The reason for the mode coupling is thought to be partially due to the curvature, and partially due to small perturbations in the ring's fabrication which would cause mode coupling. The results from these various graphs can be expressed in a different, but more useful, form. It was decided, on the basis of the results, to use a 0.025 inch zirconate substrate as the loading material. The following results are limited to that case.

It can be calculated that the average slope of the microstrip-mode's normalised guide wavelength is $0.0014 \frac{\lambda_g/\lambda_0}{\text{GHz}}$. By extrapolating each curve to zero frequency, the relation of the microstrip-mode's normalised guide wavelength to W/H can be determined for various slot widths. This result is shown in Figure 2.13, where the slot is loaded with 0.025 inch zirconate material. It is seen, as might be expected, that the normalised guide wavelength is increased as the slot width is increased, and is lowered slightly for larger slot widths when the slot is loaded with 0.025 inch zirconate material. The information in Figure 2.13 may be extrapolated to other frequencies, up to approximately 12 GHz by reducing the value of the normalised guide wavelength by 0.0014 for each gigahertz.

The dispersion of the slot-mode, in the presence of the microstrip line is not linear, and thus cannot be treated in such a simple manner as the microstrip-mode above. However, for the limited practical cases measured, Figure 2.14 shows the variation of the slot-mode's normalised guide wavelength with D/H for a fixed W/H , and Figure 2.15 shows the variation of the slot-mode's normalised guide wavelength with W/H for a fixed D/H . The measured variation of normalised guide wavelength with slot width was unexpected, since it was thought originally that

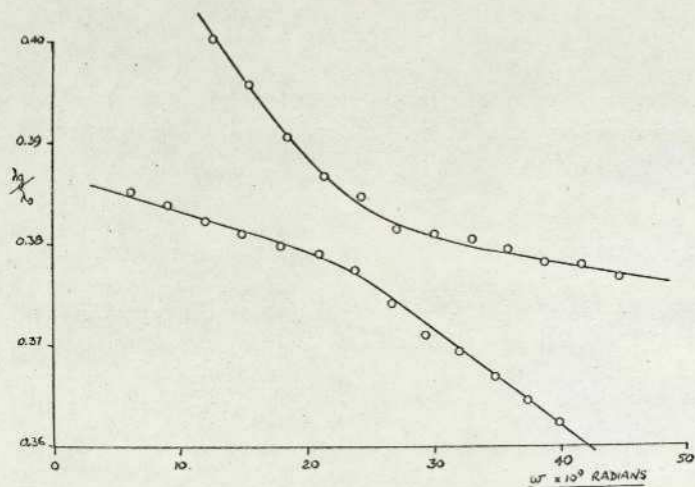


Figure 2.12 Detail of the Point of Degeneracy of the Microstrip- and Slot-Modes.

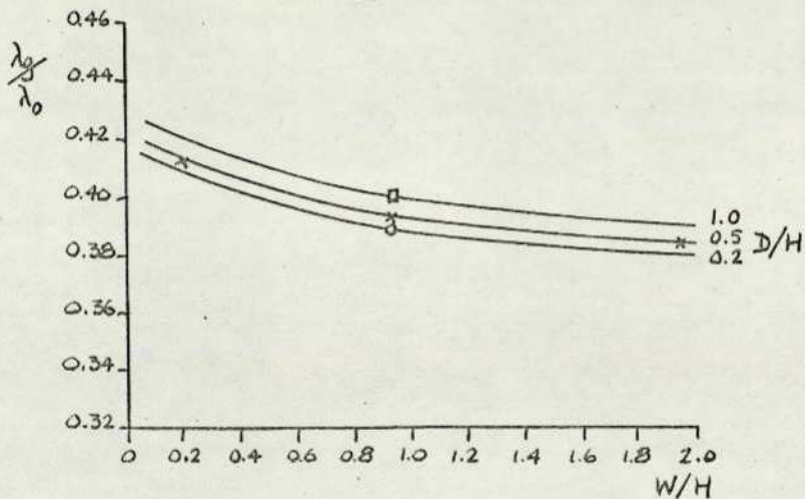


Figure 2.13 Variation of the Normalised Guide Wavelength of the Microstrip-Mode with W/H , loaded with 0.025 inch thick Zirconate.

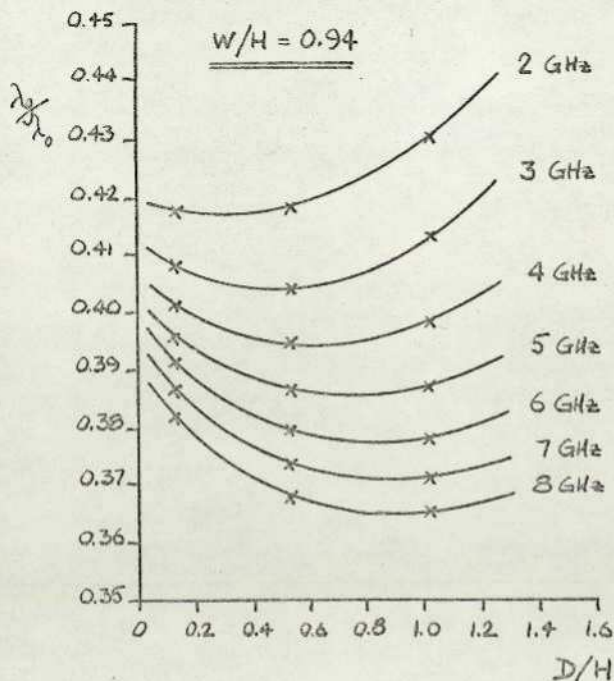


Figure 2.14 Variation of the Normalised Guide Wavelength of the Slot-Mode with D/H for a fixed W/H loaded with 0.025 inch thick Zirconate.

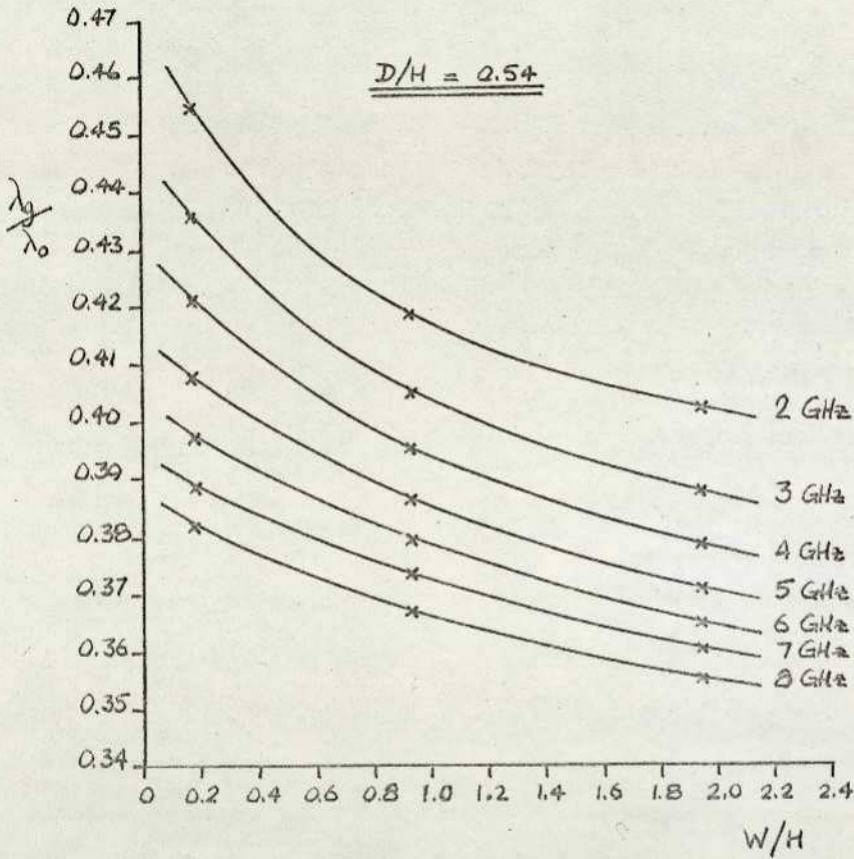


Figure 2.15 Variation of the Normalised Guide Wavelength of the Slot-Mode with W/H for a fixed D/H loaded with 0.025 inch thick Zirconate.

dielectric loading of the slot would merely lower the normalised guide wavelengths of modes of different slot widths by a proportional amount, and not in the very non-linear manner of Figure 2.14. The variation with W/H , however, was as expected.

Before this section is completed an interesting phenomenon, discussed in the introduction to this Chapter, was noticed. In some of the experiments it was found that double peaks occurred on the slot-mode resonances, which could be eliminated by shifting the dielectric loading material slightly. This double peak phenomenon was attributed to asymmetry in the positioning of the dielectric loading material, since, by Wolff (30), two degenerate modes present in the ring structure would couple together if any asymmetry existed. It was found also that if a small flaw in the top strip existed, a double peak in the resonances of the microstrip-mode would result.

CHAPTER 3MEASUREMENT OF THE IMPEDANCE OF
MICROSTRIP LINES AT MICROWAVE FREQUENCIES.3.1 Introduction and Definition of Impedance.

The concepts of characteristic impedance and phase constant, allow a microwave transmission line to be considered as a simple network, enabling its electrical performance to be predicted using network theory used in other areas of Electrical Engineering. The propagation of energy in a transmission line depends upon the relation of the free space wavelength of the wave propagation to the physical shape and construction of the line. With an arbitrarily shaped line, or waveguide, it is generally possible for the energy to propagate down it in a variety of different ways. These modes of propagation, as they are termed, can be grouped into a number of distinct classifications, namely:

- a) Transverse electric and magnetic mode (TEM)
- b) Transverse electric mode (TE, or H mode)
- c) Transverse magnetic mode (TM, or E mode)
- d) Longitudinal section electric mode (LSE)
- e) Longitudinal section magnetic mode (LSM)
- f) Hybrid mode (EH)

Although energy may propagate in a transmission line in more than one mode at the same frequency, systems are often designed to avoid this, making use of one particular mode. The choice is usually straightforward, being the dominant mode which propagates at a frequency below the lowest frequency at which the first higher order mode can propagate. Occasionally it may be desired for a higher order mode to be used, in which case the other modes have to be suppressed.

To use simple network theory for any particular mode, the characteristic impedance and phase constant for that mode in

the transmission line must be known. Chapter 2 has dealt with measurement of the guide wavelength, λ_g , of coupled microstrip lines which is equivalent to measuring the phase constant, β , since the two parameters are related simply by:

$$\beta = \frac{2\pi}{\lambda_g} \quad \dots 3.1$$

Because the concept of characteristic impedance is used to quantify in a certain manner the behaviour of a number of totally different propagating modes, it is not surprising that its definition is not exact, but varies depending upon the mode. These definitions can be placed in four categories.

Firstly, one might intuitively choose the definition that the characteristic impedance is equal to the maximum transverse voltage divided by the total current flowing in the direction of propagation. For simple transmission lines at low frequencies, supporting TEM propagation, the following relationships can be shown to be true:

$$Z_0 = \sqrt{L/C} = L \cdot v_p = 1/C \cdot v_p \quad \dots 3.2$$

where Z_0 is the characteristic impedance, L the inductance per unit length of the line, C the capacitance per unit length of the line, and v_p the phase velocity of the mode. The voltage may be found from taking a line integral of the electric field between two points of maximum potential difference, and the current from a closed line integral of magnetic intensity around one of the conductors.

Secondly, where the voltage is difficult to define an alternative definition can be derived from the relation of the complex time-average power flow, P , to the voltage, V , and current, I , given by:

$$P = \frac{1}{2} V I^* = \frac{1}{2} Z_0 I I^* \quad \dots 3.3$$

the asterisk denoting the complex conjugate. The time-average complex power flow across a surface S is given by the integral of the complex Poynting vector over the surface (33),

$$P = \frac{1}{2} \iint_S (\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{a} \quad \dots 3.4$$

the real part represents the real power flow and the imaginary part, the stored energy in the electric and magnetic fields. Thus, the power flowing along the line is

$$\begin{aligned} P &= \frac{1}{2} \operatorname{Re} \iint_S (\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{a}_z \, dx \, dy \\ &= \frac{1}{2} Z_0 I I^* \end{aligned} \quad \dots 3.5$$

hence

$$Z_0 = \frac{\operatorname{Re} \iint_S (\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{a}_z \, dx \, dy}{I \cdot I^*} \quad \dots 3.6$$

The magnitude of the complex Poynting vector in the direction of propagation, $(\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{a}_z$ is given by:

$$(\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{a}_z = E_x H_y^* - E_y H_x^* \quad \dots 3.7$$

The third alternative definition is somewhat related to the previous one, since the time-average power flow given in terms of current and impedance in equation 3.3, can also be given in terms of voltage and impedance, thus:

$$P = \frac{1}{2} V I^* = \frac{1}{2} \frac{V V^*}{Z_0} \quad \dots 3.8$$

and, similar to above

$$Z_0 = \frac{V V^*}{\operatorname{Re} \iint_S (\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{a}_z \, dx \, dy} \quad \dots 3.9$$

Finally, there is the impedance defined by the direct ratios of the electric and magnetic fields. This is usually

termed the wave impedance. It has special significance in providing a relative impedance in waveguides, since the voltage and current in a waveguide can be defined in such a way that their ratio equals the wave impedance, their product equals the power flow, and they are proportional to the transverse electric and magnetic fields respectively. This concept allows simple network theory to be easily applied without the requirement of a concept of an exact impedance (34).

Individually the definitions, where applicable, will yield different impedances for the same transmission line. It is important, in order to preserve the validity of the use of network theory, that throughout a system containing transmission lines and other components the impedance definition is consistent.

The measurement of characteristic impedance of a transmission line is often required when it is not readily calculated. The lack of unique definition and the requirement that it be related to some norm, often means that the measurement technique is one of comparison with a standard line or known mismatch. The test line is incorporated into a simple system, and a response is measured, which allows the relation of the test line's unknown characteristic impedance and the standard to be calculated. The connection of the line under test to the standard line, which may be part of the measuring instrument, can give rise to a significant junction discontinuity. When there is an abrupt cross-sectional geometrical change in these two lines, the junction discontinuity may be severe enough to greatly affect the accuracy of the measurement.

The methods traditionally used for measuring the characteristic impedance of high frequency transmission lines usually use a slotted line or waveguide. The slotted transmission line is of a standard known characteristic impedance, or in the case of waveguide, it is regarded as having the reference impedance to which everything is normalised. A short length of the unknown test line is connected to the end of the slotted line. The input impedance of the line is measured with the slotted line

when the test line is both open- and short-circuited. Assuming the junction discontinuity is negligible, and the open- and short-circuits to be at the same reference plane, then the characteristic impedance is given by (35):

$$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}} \quad \dots 3.10$$

where Z_{oc} is the input impedance of the test line open-circuited and Z_{sc} the input impedance of the same line short-circuited. Apart from ignoring the problem of the junction discontinuity, the method requires accurate open- and short-circuit terminations to the test line. For microstrip and slot line this is difficult to achieve in practice, so an alternative form of the same basic method could, perhaps, be used. In this method the test line is terminated in a known transition, Z_t , and then connected to the slotted line. The input VSWR of the line is then measured over a range of frequencies such that the line's electrical length changes by more than half a wavelength. It will be seen that the VSWR passes through a maximum and minimum value, and the characteristic impedance given by:

$$Z_0 = Z \sqrt{VSWR_{max} \cdot VSWR_{min}} \quad \dots 3.11$$

where Z is the impedance of the standard line.

A method of determining the characteristic impedance of transmission lines was put forward by Seckelmann (36), in which nodal shifts of the standing waves in a slotted line are measured. Although he derived the method for lossless, dispersionless lines without a junction discontinuity, good agreement was achieved with other methods when measuring microstrip lines. A voltage node is located in a coaxial slotted line, to which the open- or short-circuited line is connected. The node is shifted by varying the electrical length between the open- or short-circuit and the node, by changing the frequency of the signal applied to the line. The shift in nodal position is recorded at a number of points over a wide frequency range, from which a graph can be drawn. The characteristic impedance of the test

line can then be found, either from the extreme value of nodal shift, or from the loci of the extreme values of nodal shift.

Recently several new instruments have enabled different techniques to be established, and, apart from improving accuracy, they have greatly reduced the time taken for the measurements to be performed. The two main instruments referred to are the Time Domain Reflectometer (T.D.R.), and the Network Analyser.

The T.D.R. is an instrument which applies a step waveform to a transmission line, and displays the return signal caused by small mismatches along the line which partially reflect the incident waveform. The input step waveform, and the return signals, travel at a finite velocity in the transmission line, giving a time delay between reflections arriving back at the T.D.R. from different mismatches. Thus, it can be seen that the information related to the reflection of the junction discontinuity and the line's characteristic impedance is separated. The instrument is limited by the finite rise-time of the step waveform, which imposes an upper frequency limit to the frequency over which the measurement is valid, and a limit to the resolution of closely spaced mismatches.

The Network Analyser is an instrument capable of displaying the transmission and reflection coefficients, or input impedance. When associated with a swept frequency source it is a very powerful tool, with good engineering accuracy. Junction discontinuities at the interface between the device and the analyser have recently been given considerable attention. With computer correction not only the effects of junction discontinuities, but also some of the errors in the analyser, can be eliminated. Computer controlled and corrected Network Analysers are still expensive, but where there is access to a small computer, on-line correction is relatively simple. Shurner has published a number of papers (37, 38) dealing with this topic, and more recently Da Silva and McPhun (39) showed that correction could be achieved without the requirement of known resistive terminations. Because of the existence of a number of factors not accounted for by these methods, a letter included in Appendix 1 shows that similar

overall accuracy can be achieved without accurate frequency measurement, a prerequisite of other correction methods.

3.2 A New Method for Measuring the Characteristic Impedance of Transmission Lines.

3.2.1 Introduction

A new method of measuring the characteristic impedance of transmission lines, which is described in this section, has, in part, been the subject of a paper to be found in Appendix 1. The new method bears some similarity with that of the nodal shift method mentioned in the previous section, though instead of measuring the actual shift of the nodes, the change of phase at a datum plane is observed. It will be shown that the phase angle of the reflection coefficient, at the transition of the unknown line to a standard line, is dependent partially upon the unknown line's characteristic impedance. For simplification of the mathematics involved, the unknown line can be terminated in either an open- or short-circuit. In the following discussion an open-circuited line will be considered, and further, it will be lossless, and with a junction discontinuity. This will insure that the basic principles be easily understood. These complexities will be considered in detail in the remaining sub-sections of this section.

The input impedance of a length of unknown lossless transmission line is given by:

$$Z_{\text{input}} = Z_1 \left(\frac{Z_L \cos \beta l + j Z_1 \sin \beta l}{Z_1 \cos \beta l + j Z_L \sin \beta l} \right) \quad \dots 3.12$$

where Z_1 is the characteristic impedance of the unknown line, and Z_L the impedance of the load terminating the line at its far end. When the line is open-circuited, its impedance becomes

$$Z_{\text{input}} = -j Z_1 \cot \beta l \quad \dots 3.13$$

The input impedance of the line is seen to be completely reactive. It could be regarded as a 'lumped' impedance which terminates a standard line of characteristic impedance Z_0 . The reflection coefficient, Γ , of such an arrangement is:

$$\Gamma = \rho / \phi = \frac{Z_{\text{input}} - Z_0}{Z_{\text{input}} + Z_0} \quad \dots 3.14$$

Substituting for Z_{input} , the phase angle of the reflection coefficient, ϕ , is:

$$\phi = \arctan \left(\frac{2Z \cot \beta l}{1 - Z^2 \cot^2 \beta l} \right) \quad \dots 3.15$$

where $Z = Z_1/Z_0$, the normalised characteristic impedance of the unknown line. This function is plotted for various values in Figure 3.1. It can be seen that, when $Z = 1$

$$\phi = n\pi - 2\beta l \quad \dots 3.16$$

where $n = 0$ for an open-circuited line, and 1 for a short-circuited line.

When Z is different from unity, the phase angle of the reflection coefficient deviates either positively or negatively from its value when $Z = 1$. The deviation of the phase angle of the reflection coefficient, $\delta\phi$, is given by:

$$\delta\phi = \arctan \left(\frac{2Z \cot \beta l}{1 - Z^2 \cot^2 \beta l} \right) + 2\beta l \quad \dots 3.17$$

This function is plotted for various values of Z in Figure 3.2. It is seen that all maxima and all minima are of equal amplitude, dependent only upon the value of Z . To find the maximum value of equation 3.17, it is differentiated with respect to βl , thus

$$\frac{d(\delta\phi)}{d(\beta l)} = 2 - \frac{2Z}{\sin^2 \beta l (1 + Z^2 \cot^2 \beta l)} \quad \dots 3.18$$

at a maxima it can be shown that

$$\beta l = n\pi + \text{Arctan}(\sqrt{Z}) \quad \dots 3.19$$

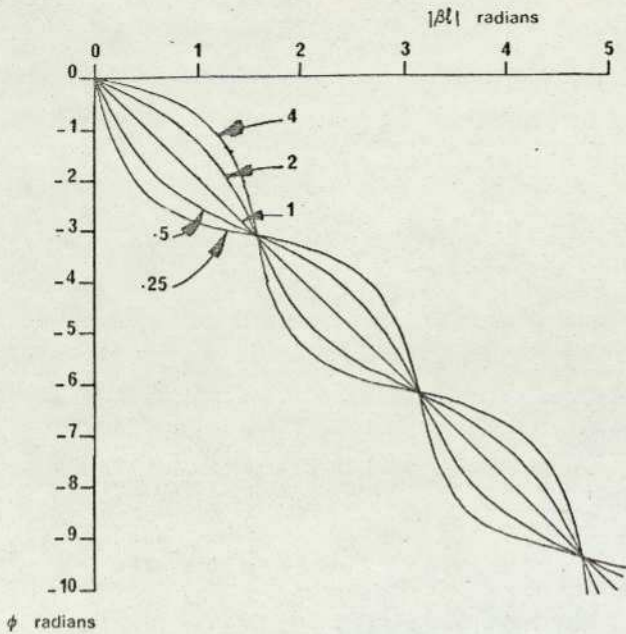


Figure 3.1 Variation of the Phase Angle of the Reflection Coefficient for Various Line Impedances.

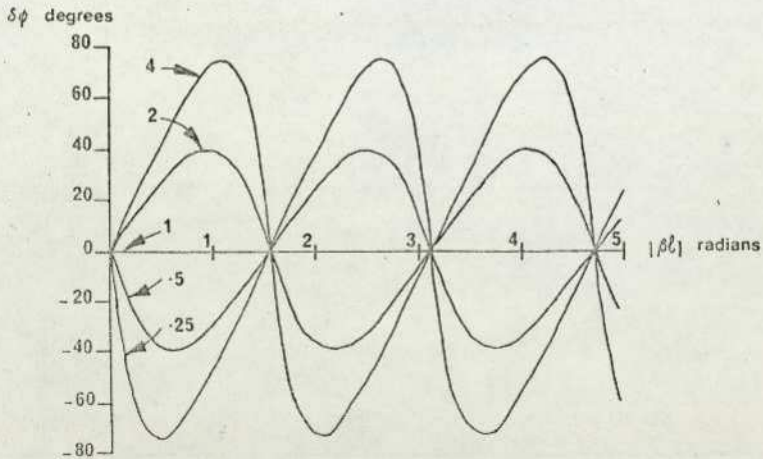


Figure 3.2 Deviation of the Phase Angle of the Reflection Coefficient for Various Line Impedances.

where Arctan is taken as its principle value. Substituting this into equation 3.18, after some rearrangement, the relation between Z and βl is found to be given by:

$$Z = \tan^2 \left(\frac{\delta\phi - 3n\pi}{4} \right) \quad \dots 3.20$$

This equation has an infinite number of solutions, one of which lies between the following limits,

$$-\pi \leq \delta\phi \leq \pi \quad \text{for} \quad 0 \leq Z \leq \infty \quad \dots 3.21$$

and is plotted in Figure 3.3. It will be seen that the modulus of $\delta\phi$ is plotted, ignoring its sign. The characteristic impedance Z_1 can be obtained from Figure 3.3 by measuring the extreme values of phase deviation.

Practical measurements can be made using the Hewlett Packard Network Analyser. This particular instrument is chosen for two reasons. Firstly, because it is able to measure the phase angle of reflection coefficient at a junction of the unknown open- or short-circuited line, and secondly, it incorporates a mechanically variable length of line, which, when correctly adjusted, enables the analyser to display the required information concerning the phase angle of the reflection coefficient. The unknown open- or short-circuited line is connected to the 'unknown input' in the reflection test unit of the analyser. The input frequency to the analyser is adjusted so that the line's electrical length changes by at least one wavelength, and the reference plane extension is adjusted until two peak values of phase angle have the same amplitude. The peak phase amplitude is then recorded and the characteristic impedance determined. In practice it will be seen later that this procedure is made necessarily simple because of the initial assumptions. When a more complex transmission line model is considered, the measurement procedure has to be modified slightly.

3.2.2 Lossy Transmission Line.

A lossless transmission line was considered in the

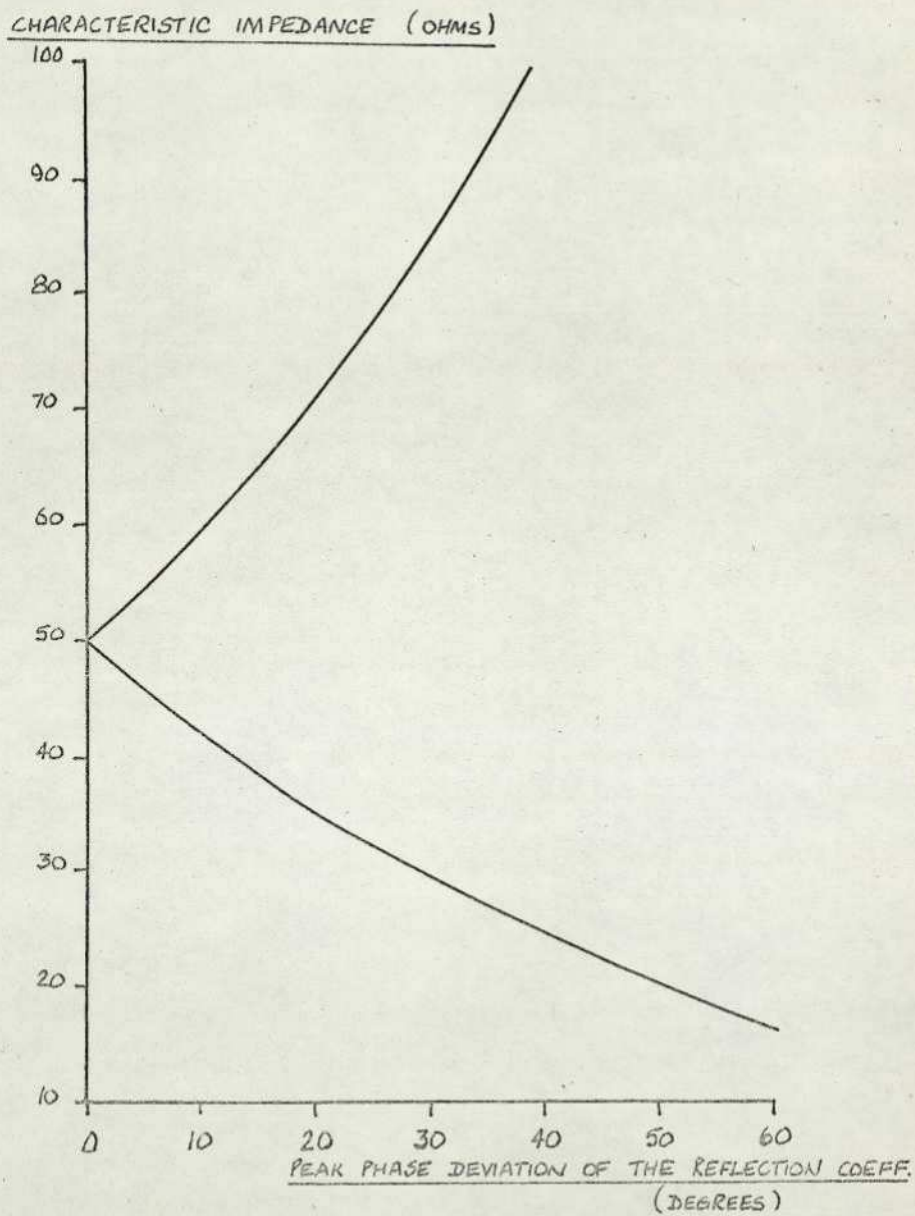


Figure 3.3 Variation of Characteristic Impedance with Peak Phase Angle Deviation.

previous sub-section for introducing the basic theory of the method. In this sub-section the line will be assumed to be lossy in order that its effect upon the measurement of the line's characteristic impedance can be found. The loss mechanisms can be due to either dielectric, conductor or radiation losses, the result being that the propagation constant, γ , reverts to its most general form,

$$\gamma = \alpha + j\beta \quad \dots 3.22$$

where β is the phase constant, and α the attenuation constant.

To show that the method is equally suitable for short-circuited lines, the theory to follow will be derived for lines terminated with a short-circuit. The input impedance of a lossy line, of characteristic impedance Z_1 , terminated with a load Z_L is:

$$Z_{\text{input}} = Z_1 \left(\frac{Z_L \cosh \gamma l + Z_1 \sinh \gamma l}{Z_1 \cosh \gamma l + Z_L \sinh \gamma l} \right) \quad \dots 3.23$$

which for $Z_L = 0$, reduces to

$$Z_{\text{input}} = Z_1 \tanh \gamma l \quad \dots 3.24$$

substituting for γ then:

$$Z_{\text{input}} = Z_1 \left(\frac{\tanh \alpha l + j \tanh \beta l}{1 + j \tanh \alpha l \cdot \tanh \beta l} \right) \quad \dots 3.25$$

Following a similar procedure to that in the previous sub-section, it can be found that the magnitude, ρ , and phase angle, ϕ , of the reflection coefficient at the junction to a standard line is given by:

$$\rho = \sqrt{(x^2 + y^2)/u^2} \quad \dots 3.26$$

and

$$\phi = \arctan(y/x) \quad \dots 3.27$$

when $Z = Z_1/Z_0$, then x, y and u are

$$x = \tanh^2 \alpha l (Z^2 - \tan^2 \beta l) + (Z^2 \tan^2 \beta l - 1) \quad \dots 3.28$$

$$y = 2Z \tan \beta l (1 - \tanh^2 \alpha l) \quad \dots 3.29$$

$$u = (Z \tanh \alpha l + 1)^2 + \tan^2 \beta l (Z + \tanh^2 \alpha l)^2 \quad \dots 3.30$$

When the line is lossless, equation 3.27 becomes:

$$\phi = \arctan \left(\frac{2Z \tan \beta l}{Z^2 \tan^2 \beta l - 1} \right) \quad \dots 3.31$$

It can be seen that this equation is similar to equation 3.15, for the open-circuit lossless line, if Z is replaced by its reciprocal.

Realistic values of α , the attenuation constant, were chosen from calculations published by Horton, Easter and Gopinath (40). They calculated a value of 0.577 nepers/metre for a microstrip line, $W/H = 1.0$, on a 0.635 mm. thick alumina substrate, at a frequency where the skin depth is of the same order as the conductor thickness. With a value of attenuation constant four times that of the practical case, it was found that the peak values, of phase deviation, only increased by between 0.3% and 0.4% of those obtained with a lossless line. The peak value of the deviation of the phase of the reflection coefficient, given by equation 3.27 was not found by differentiation, but by using a function maximisation routine on the computer.

It is interesting to note that being a passive structure, the magnitude of the reflection coefficient is always equal to, or less than, unity. When loss is present it is always less than unity, varying cyclically as shown in Figure 3.4. A practical measurement is shown in Figure 3.5, taken of a microstrip line on a 0.040 inch thick zirconate substrate. Attenuation in the transmission line can be seen to be increasing with frequency, due to increasing dielectric loss, since the zirconate material is being operated above the recommended limit (41) of 5 GHz. It may perhaps be required in some cases to evaluate the characteristic

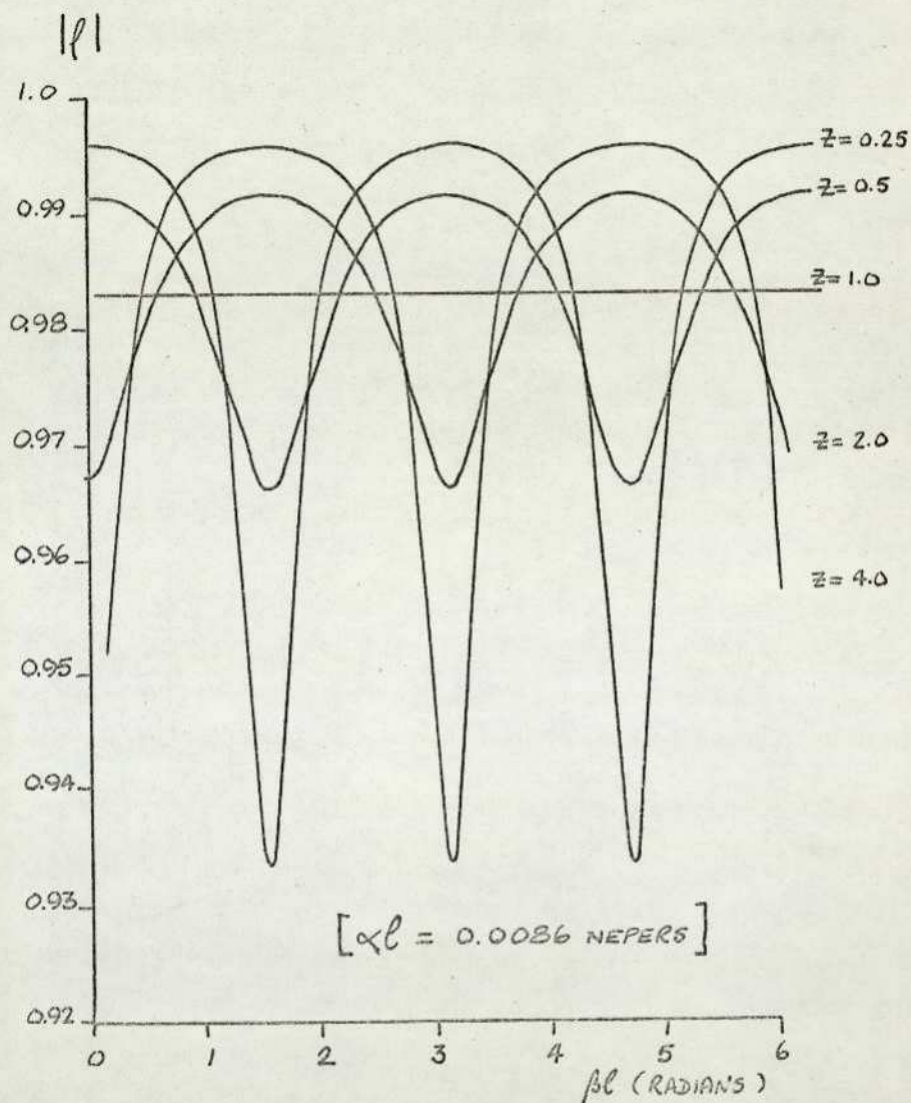


Figure 3.4 Variation of the Magnitude of the Reflection Coefficient for a Lossy Short-Circuited Line.

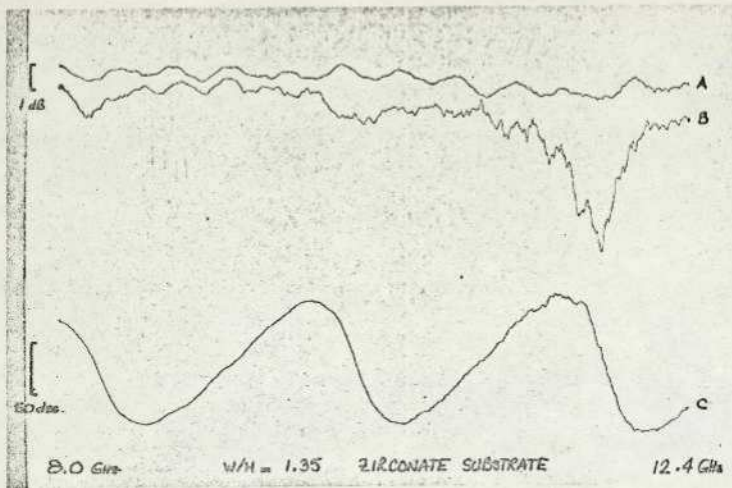


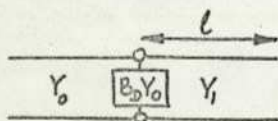
Figure 3.5 A Practical Measurement of the Reflection Coefficient.

impedance when the line is lossy. Figure 3.6 can be used to assess the loss in the line, by approximate knowledge of its characteristic impedance, and measurement of the minimum value of the magnitude of the reflection coefficient.

3.2.3 Transmission Line with a Capacitive Junction Discontinuity.

It has previously been stated that one of the complications in measuring the characteristic impedance of microstrip lines, is the existence of a junction discontinuity at the junction of the line to a standard coaxial line. Caulton, Hughes and Sobel (42) reported that this discontinuity for a coaxial to microstrip transition is mainly represented by a shunt capacitance, so the following discussion will be limited to this case, although the technique is adaptive to different discontinuities.

The simplest way of representing a shunt capacitive discontinuity is shown diagrammatically in Figure 3.7 for an open-circuited transmission line. The discontinuity is shown by $B_D Y_0$, where B_D is the normalised junction susceptance. The characteristic admittance of the microstrip line is Y_1 , and



that of the standard line, Y_0 . Formulating the reflection coefficient in the same way as before, the phase angle, ϕ , is given by:

Figure 3.7

$$\phi = \arctan\left(\frac{-2(Y \tan \beta l + B_D)}{1 - Y^2 \tan^2 \beta l - 2Y B_D \tan \beta l - B_D^2}\right) \quad \dots 3.32$$

where $Y = Y_1/Y_0$, and B_D the normalised junction susceptance.

This simple model, of the microstrip line and junction discontinuity, was not found to be sufficiently close to the practical case. The frequencies at which the peaks of the phase deviation occurred practically, were not related to the frequencies found theoretically. It was decided to investigate this problem with a more flexible model, shown in Figure 3.8.

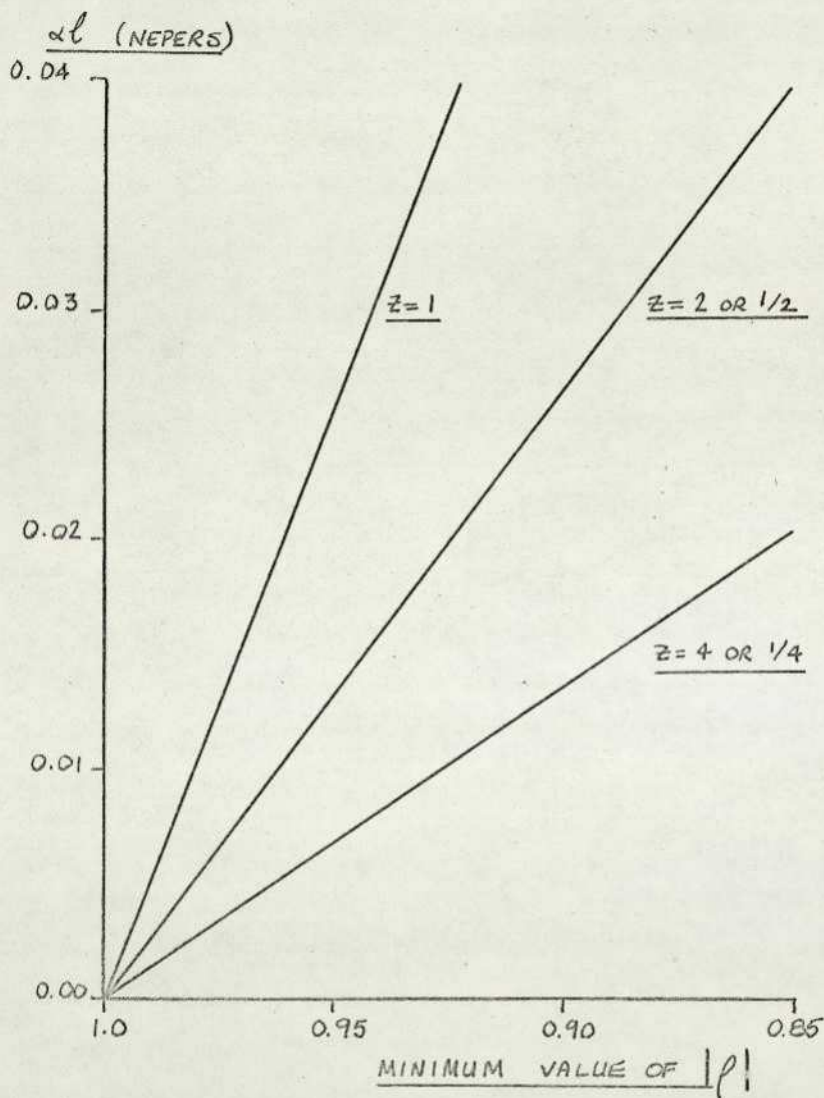


Figure 3.6 Variation of Line Loss with the Minimum Value of the Magnitude of the Reflection Coefficient, for various Line Impedances.

The junction susceptance is shown to be located a small length,

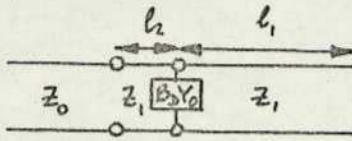


Figure 3.8

l_2 , along the microstrip line. The new model is nearer to the practical case since fringing fields at the discontinuity, and the short overlay of the connector

on the microstrip line would tend to position the centre of their combined effect a short distance along the microstrip line.

With this configuration the phase angle of the reflection coefficient, ϕ , by similar reasoning as given before, can be shown to be given by:

$$\phi = \arctan \left(\frac{2xy}{x^2 - y^2} \right) \quad \dots 3.33$$

where

$$x = \sin \beta l_2 (Z \tan \beta l_1 + Z^2 B_0) - Z \cos \beta l_2 \quad \dots 3.34$$

$$y = \cos \beta l_2 (\tan \beta l_1 + Z B_0) + \sin \beta l_2 \quad \dots 3.35$$

in which $Z = Z_1/Z_0$. The phase deviation, $\delta\phi$, is given by

$$\delta\phi = \arctan \left(\frac{2xy}{x^2 - y^2} \right) + 2\beta l \quad \dots 3.36$$

where $l = l_1 + l_2$. When the junction discontinuity is completely capacitive, it will be found that successive peaks of equation 3.36 decrease non-linearly with increasing βl , the minimas decreasing more rapidly than the maximas. The peaks of equation 3.35 are computed using a golden section algorithm, and the average slope of three successive maxima and three successive minima is found in order to cancel the slope. This operation produces maxima and minima which lie symmetrically about a horizontal reference line. It bears similarity with the practical realisation on the Network Analyser, where the reference plane extension, in

the reflection test unit, is adjusted until neighbouring maxima and minima lie symmetrically about a horizontal reference line.

Figures 3.9 to 3.11 show the results of a computer program, described in Appendix 2, the aim of which was to characterise the theoretical behaviour of a practical microstrip line on 0.635 mm. thick alumina. Experimental details of the measurements and results are given in Section 3.3. The computed results are for a line of similar dimensions, and phase constant. The line was assumed lossless since the magnitude of the reflection coefficient was close to unity in the practical measurements, and the junction discontinuity susceptance, in Figures 3.9 to 3.11, are for B_D equal to $0.0125^{(A)}$, $0.0250^{(B)}$ and $0.0500^{(C)}$ at a frequency of 1 GHz, rising linearly with increasing frequency. These values of junction susceptance can be related to the voltage standing wave ratio, S , that they would cause in an otherwise matched transmission line system by:

$$\beta_D = \frac{S-1}{\sqrt{S}} \quad \dots 3.37$$

or in terms of the junction susceptance, B_D , by

$$S = 1 + \frac{B_D^2}{2} \pm \frac{B_D}{2} \sqrt{4 + B_D^2} \quad \dots 3.38$$

3.2.4 Transmission Line with a Capacitive Junction Discontinuity and Radiation from its Open-Circuited End.

This particular problem was considered because the results of the previous sub-section showed close agreement with practical measurements given in the next section. The microstrip lines that were measured, used open-circuits for the termination as they are easier to fabricate. The model, shown in Figure 3.12, was investigated to see if the effect of radiation

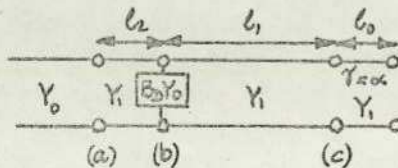


Figure 3.12

from the open-circuited end, significantly affected the phase characteristic of the reflection coefficient. The capacitive discontinuity, $B_D Y_0$, is situated a distance

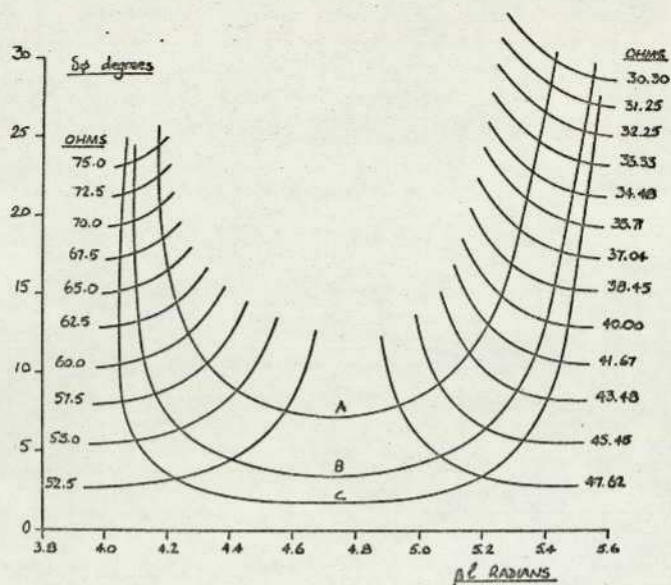


Figure 3.9a $l_2 = 0.0$ mm

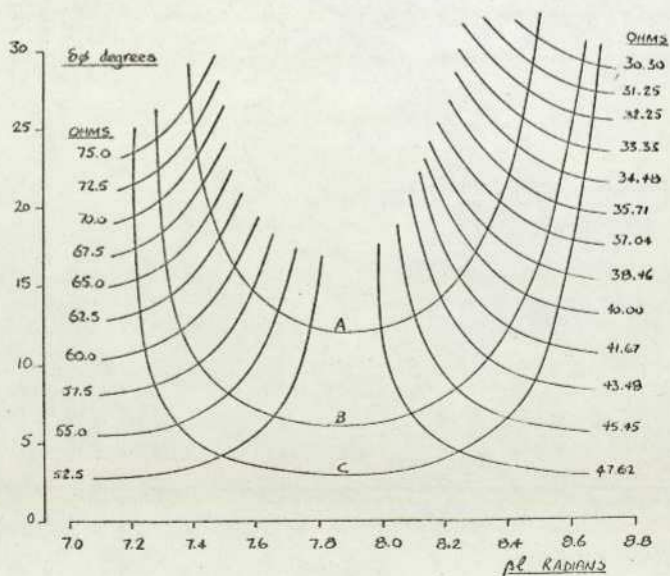


Figure 3.9b $l_2 = 0.0$ mm

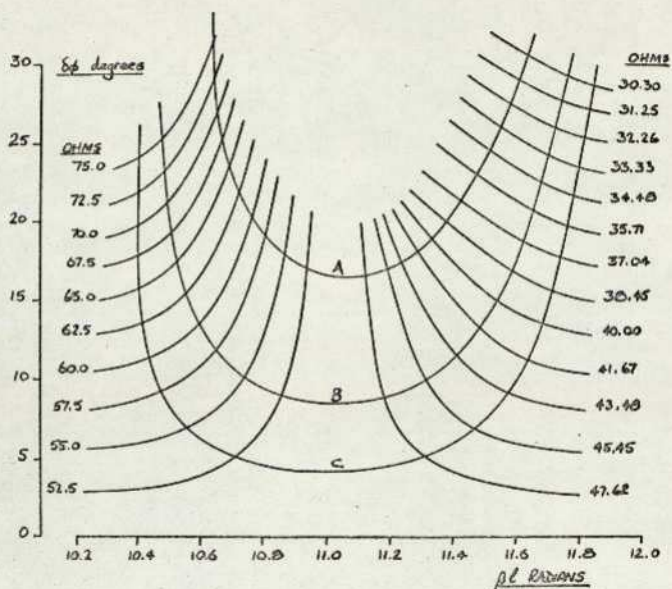


Figure 3.9c $l_2 = 0.0$ mm

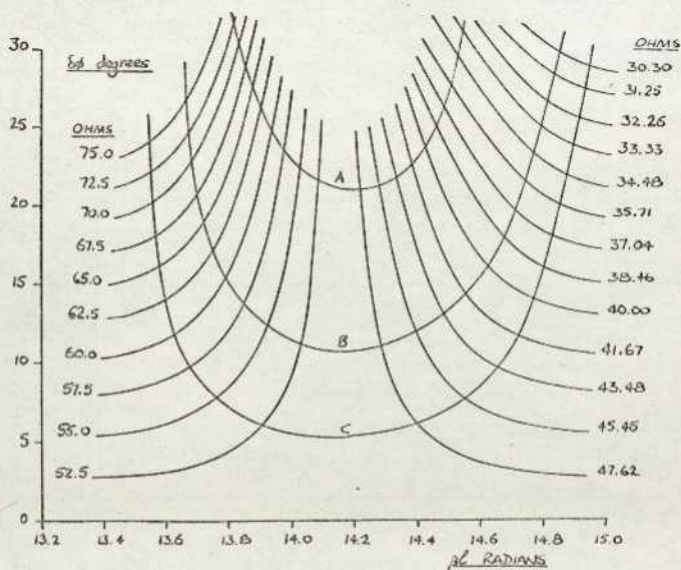
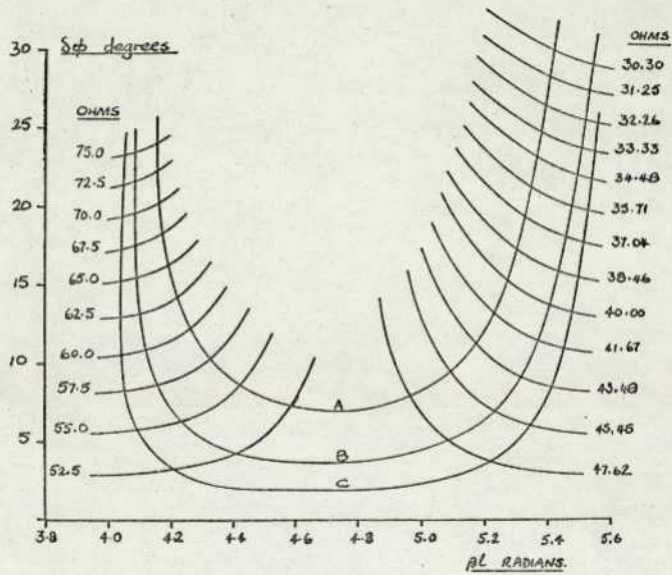
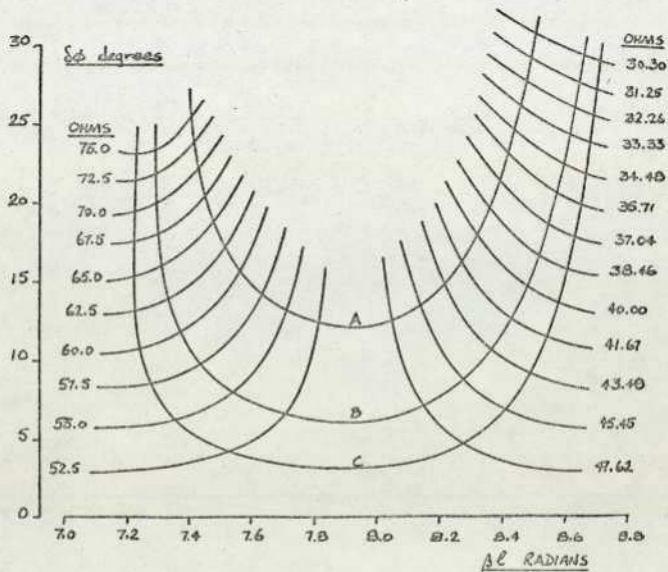


Figure 3.9d $l_2 = 0.0$ mm

Figure 3.10a $l_2 = 0.2$ mmFigure 3.10b $l_2 = 0.2$ mm

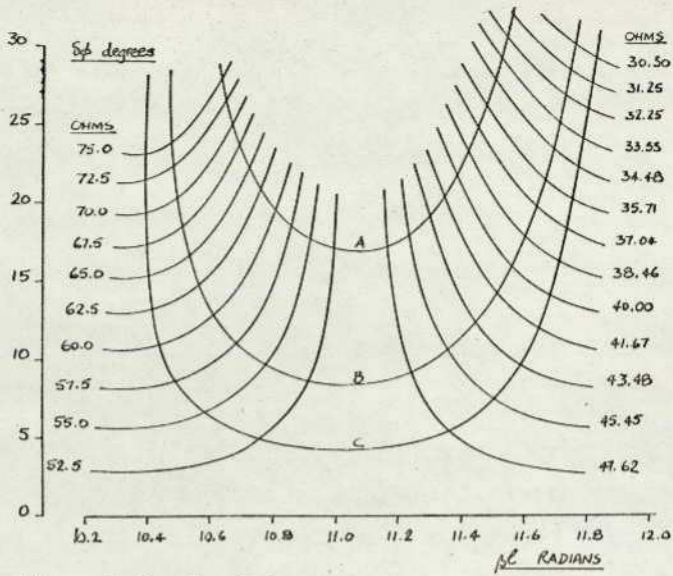


Figure 3.10c $l_2 = 0.2$ mm

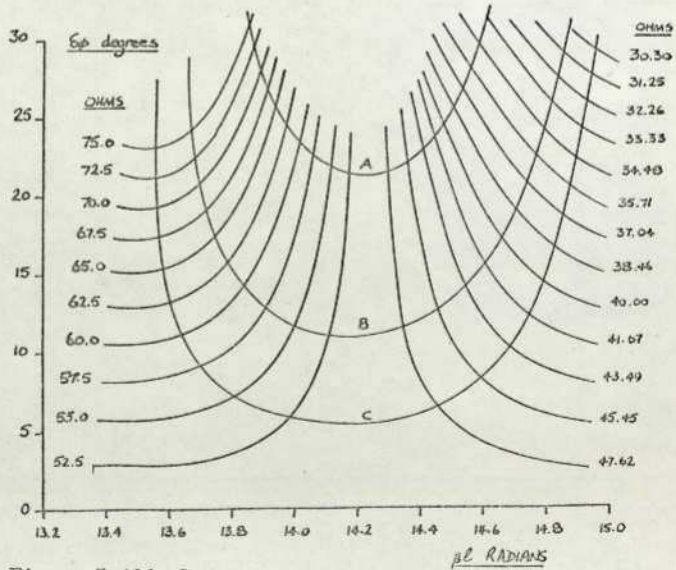


Figure 3.10d $l_2 = 0.2$ mm

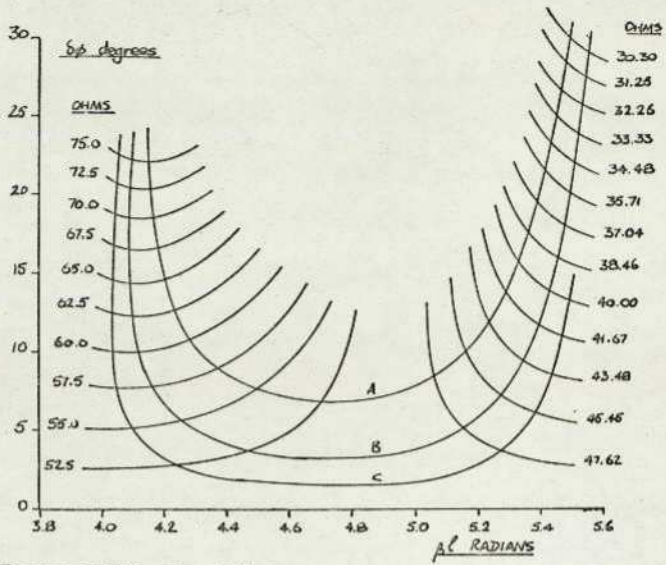


Figure 3.11a $l_2 = 1.0$ mm

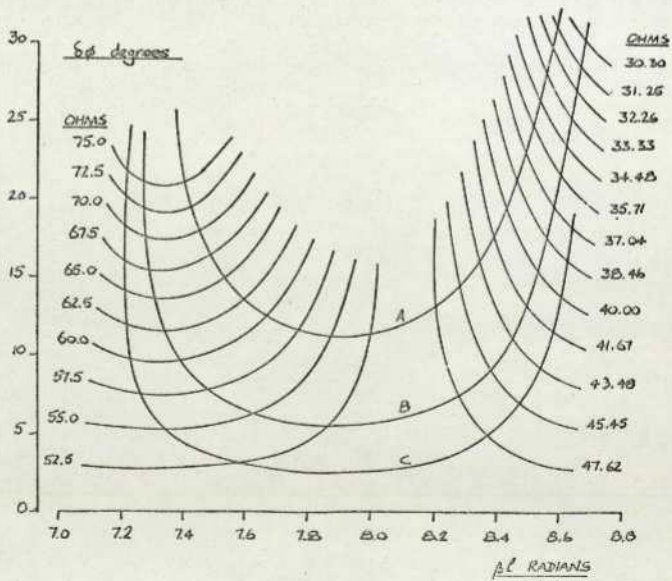
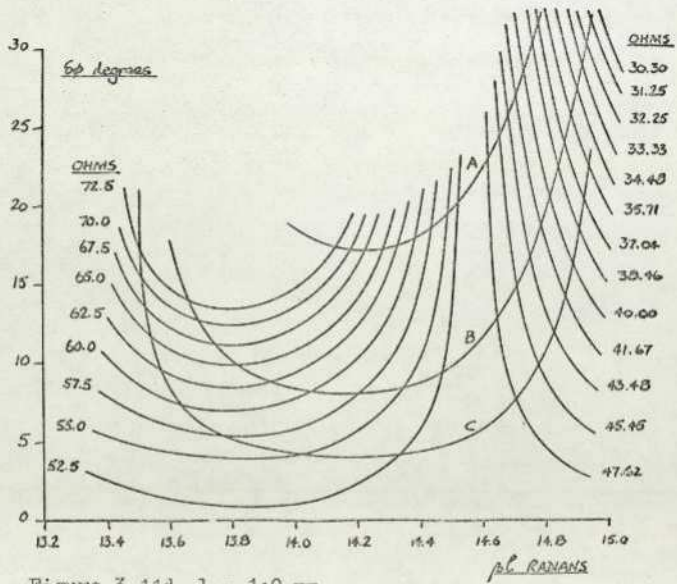
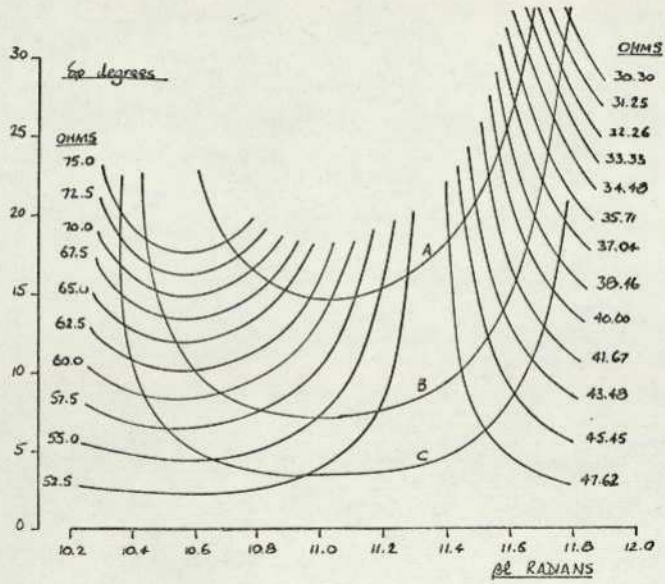


Figure 3.11b $l_2 = 1.0$ mm



l_2 from the junction of the microstrip line. The open-circuited end is at the end of an infinitely short length of lossy line, which has the same characteristic admittance, Y_1 , as the rest of the line. The phase deviation, $\delta\phi$, is calculated in a similar way as previously, and is found to be given by:

$$\delta\phi = \arctan \left(\frac{np - qm}{mp + qn} \right) + 2\beta l \quad \dots 3.39$$

in which

$$m = r \tanh \mathcal{R}l_0 - (\cos \beta l_2 (\tan \beta l_1 + \mathcal{Z}B_D) + \sin \beta l_2) \quad \dots 3.40$$

$$n = s \tanh \mathcal{R}l_0 + (\sin \beta l_2 (\mathcal{Z} \tan \beta l_1 + \mathcal{Z}^2 B_D) - \mathcal{Z} \cos \beta l_2) \quad \dots 3.41$$

$$p = r \tanh \mathcal{R}l_0 + (\cos \beta l_2 (\tan \beta l_1 + \mathcal{Z}B_D) + \sin \beta l_2) \quad \dots 3.42$$

$$q = (\sin \beta l_2 (\mathcal{Z} \tan \beta l_1 + \mathcal{Z}^2 B_D) - \mathcal{Z} \cos \beta l_2) - s \tanh \mathcal{R}l_0 \quad \dots 3.43$$

and

$$r = \mathcal{Z}(\tan \beta l_1 \cos \beta l_2 + \sin \beta l_2 - \mathcal{Z}B_D \tan \beta l_1 \sin \beta l_2) \quad \dots 3.44$$

$$s = \cos \beta l_2 - \mathcal{Z}B_D \tan \beta l_1 \cos \beta l_2 - \tan \beta l_1 \sin \beta l_2 \quad \dots 3.45$$

When $\mathcal{R}l_0$ is zero, there is no loss at the open-circuit, and the equation for the phase deviation, $\delta\phi$, reverts to equation 3.36, in the previous sub-section. When $\mathcal{R}l_0$ is real and non-zero, the lossy transmission line, length l_0 , causes attenuation without changing the phase of the signal, and thus simulates radiation from the open-circuit.

Detailed results for this model are not included here, though they can easily be obtained using the same computer program as that used in the previous sub-section. As $\mathcal{R}l_0$ is increased, but kept real, the peak phase deviation is increased slightly for the same characteristic impedance and junction susceptance. From practical considerations, however, this increase is small compared to the magnitude of the measurement errors. The effect of radiation from the open-circuit can thus be ignored until the

measurement techniques are improved.

3.3 Measurement of the Characteristic Impedance of Microstrip Lines.

The transverse inhomogeneity of microstrip geometry raises some difficulty in defining its characteristic impedance. When the propagation of energy is approximated by assuming a quasi-TEM mode, its characteristic impedance can be related to the inductance and capacitance per unit length. Thus, as the frequency increases, the capacitance per unit length also increases since the effective dielectric constant is increased. This results in the characteristic impedance falling. Recently, this result was concluded by Denlinger (15), and Deutsch and Jung (43), and found to be true practically by Napoli and Hughes (44). Napoli and Hughes measured the characteristic impedance relative to a TEM line, concluding that its impedance decreased more rapidly than the increase in the effective dielectric constant alone would cause. Doubt in some of the assumptions used by Napoli and Hughes, caused Krage and Haddad (9) to question their conclusions, since, using a hybrid mode formulation and a power/longitudinal current flow definition for characteristic impedance, they found that the characteristic impedance of microstrip lines initially decreased slightly before steadily increasing with frequency. Kowalski and Pregla (45), and Schmitt and Sarges (23), both calculated the characteristic impedance of microstrip in two ways, and obtained similar results for similar methods. The two methods used were, firstly, the definition of the voltage in the centre of the strip divided by the longitudinal current, and, secondly, by the power flow divided by the longitudinal current squared. In both cases the characteristic impedance increased from zero frequency, the rate of increase being greater for narrower strips.

3.3.1 A Practical Measurement.

The characteristic impedance has been measured using the method described in the previous section. A Hewlett Packard

8710A Network Analyser, in conjunction with its associated Reflection Test Unit, Harmonic Frequency Analyser and a Swept Frequency source, was used to measure the phase of the Reflection Coefficient. An important consideration, before measurements could be made, was that the total electrical length of the microstrip test line and connectors was less than the available Reference Plane Extension. The microstrip lines were constructed on 0.635 mm. thick Alumina substrates, and made approximately 3.5 cms. long, in order to comply with the length restriction. The termination at the far end of the lines was, in all cases, an open-circuit, the open-circuit being merely an abrupt end of the strip with the ground plane continued a further 1.58 cm. to the end of the substrate.

The microstrip line was connected to the unknown input of the Reflection Test Unit by means of a modified OSM 954/CC 3mm. coaxial/strip line connector and an OSM/APC converter. The connector was modified in a way similar to the modification suggested by Caulton, Hughes and Sobel (42). Instead of cutting the centre conductor off flush with the flange and adding a small tab, the tab was formed by filing down the centre conductor. The sweep generator was adjusted so that it covered a frequency band such that the electrical length of the microstrip line changed by more than half a wavelength. The Reference Plane Extension was then adjusted until successive peak phase deviations of the Reflection Coefficient were symmetrically placed in a horizontal plane. Because the microstrip has a capacitive junction susceptance, and the transmission line is dispersive, the peak phase deviations increase with frequency and the locus of their mean value is a curve rather than a straight line. These complications cause the adjustment of the Reference Plane Extension to be somewhat arbitrary; the error incurred for microstrip lines was sufficiently large, in this case, to make calculation of the phase constant, as suggested in the paper included in Appendix 1, impossible. The peak values of phase deviation, and their associated frequencies, were measured. The length of the microstrip line was also measured and corrected for its open-circuit end effect, $\Delta\ell$, by adding

$$\Delta\ell = 0.2 \times H$$

... 3.46

where H is the height or thickness of the substrate. The equivalent length of TEM supporting line, ℓ_o , was then found from

$$\ell_o = \ell_e / (\lambda_g / \lambda_o) \quad \dots 3.47$$

where λ_g / λ_o is the normalised guide wavelength for the line, and ℓ_e is its corrected physical length. The electrical length, $\beta_o \ell_o$ is then found from the relationship,

$$\beta_o \ell_o = \frac{2\pi f \ell_o}{c} \quad \dots 3.48$$

where c is the velocity of light, and f is the frequency. The normalised guide wavelength was determined from Bryant and Weiss' results using knowledge of the width of the line to substrate height ratio, W/H . It was corrected for its variation with frequency by using the following empirical formula.

$$\frac{\lambda_g}{\lambda_o} = \left. \frac{\lambda_g}{\lambda_o} \right|_{f=0} + mf \quad \dots 3.49$$

where f is the frequency in GHz and m is the rate of change of λ_g / λ_o with frequency. For this measurement, m was taken as -0.0014 , derived from ring resonator measurements similar to those in the previous Chapter. The characteristic impedances of the microstrip lines were found, using the graphs in Figures 3.9 to 3.11, from the maximum values of phase deviation and their associated electrical lengths, obtained from the frequencies of peak phase deviation using equation 3.48.

The importance of the position of the junction susceptance can be seen from the results shown in Figure 3.13. This shows the same set of practical results, evaluated using the theory established in the previous section, for various positions of junctions susceptance in the line. As the frequency is increased, the loci of the various susceptance positions diverge. A similar presentation is used in Figure 3.14, the difference being that here, the practical data was obtained where the connector had an overlap of 0.4 mm. Comparison of the two sets of results reveals that a similar characteristic impedance is obtained for each case, if the

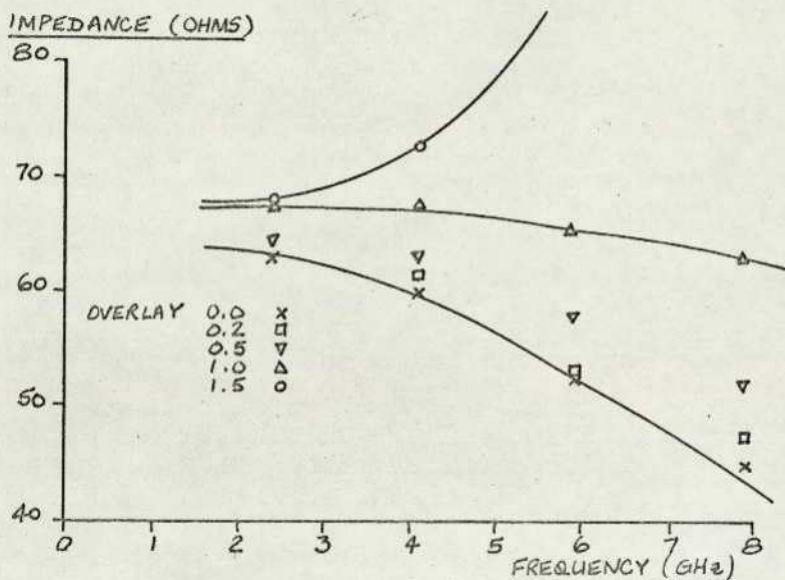


Figure 3.13 Evaluation of Line Impedance with a Practical Connector overlay of 2.2mm, for various Positions of Junction Susceptance.

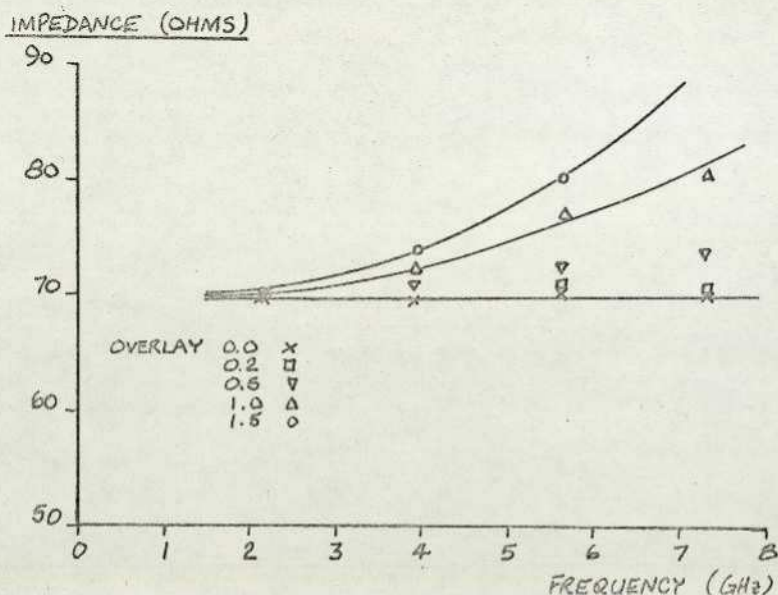


Figure 3.14 Evaluation of Line Impedance with a Practical Connector overlay of 0.4mm, for various Positions of Junction Susceptance.

junction susceptance is positioned half the distance of the overlap along the line. A small change in its positioning has a large effect on the behaviour of the calculated impedance at high frequencies. Thus, a small overlap of the connector on the microstrip line will not only reduce the junction discontinuity, but will also aid the accuracy of the measurement by making the position of the junction susceptance more certain. Further observations show that at low frequencies, the characteristic impedance is almost independent of the junction susceptance's position, and significantly lower than the various theoretical predictions and T.D.R. measurements show. A comparison of these results with the theoretical predictions is given in Figure 3.15.

3.4 Measurement of the Characteristic Impedance of the Dominant Modes of a Microstrip/Slot Line Combination.

The configuration, the cross-section of which is shown diagrammatically in Figure 3.16, supports two modes, a microstrip-mode and a slot-mode.

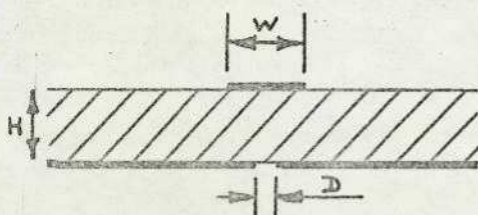


Figure 3.16

Their phase velocities have been measured using ring resonators (see Chapter 2) showing that for a normal slot the phase velocity of the microstrip-mode is only slightly altered from that of a true microstrip line.

Direct measurement of the slot-mode impedance has not been attempted. The problem of making a wideband connection to a standard TEM line without large junction discontinuities has not been overcome. The characteristic impedance has been determined somewhat crudely by constructing a microstrip/slot line coupler, and relating its performance to a theoretical model with various slot line impedances as trial values. The details of this experiment are given in Chapter 5.

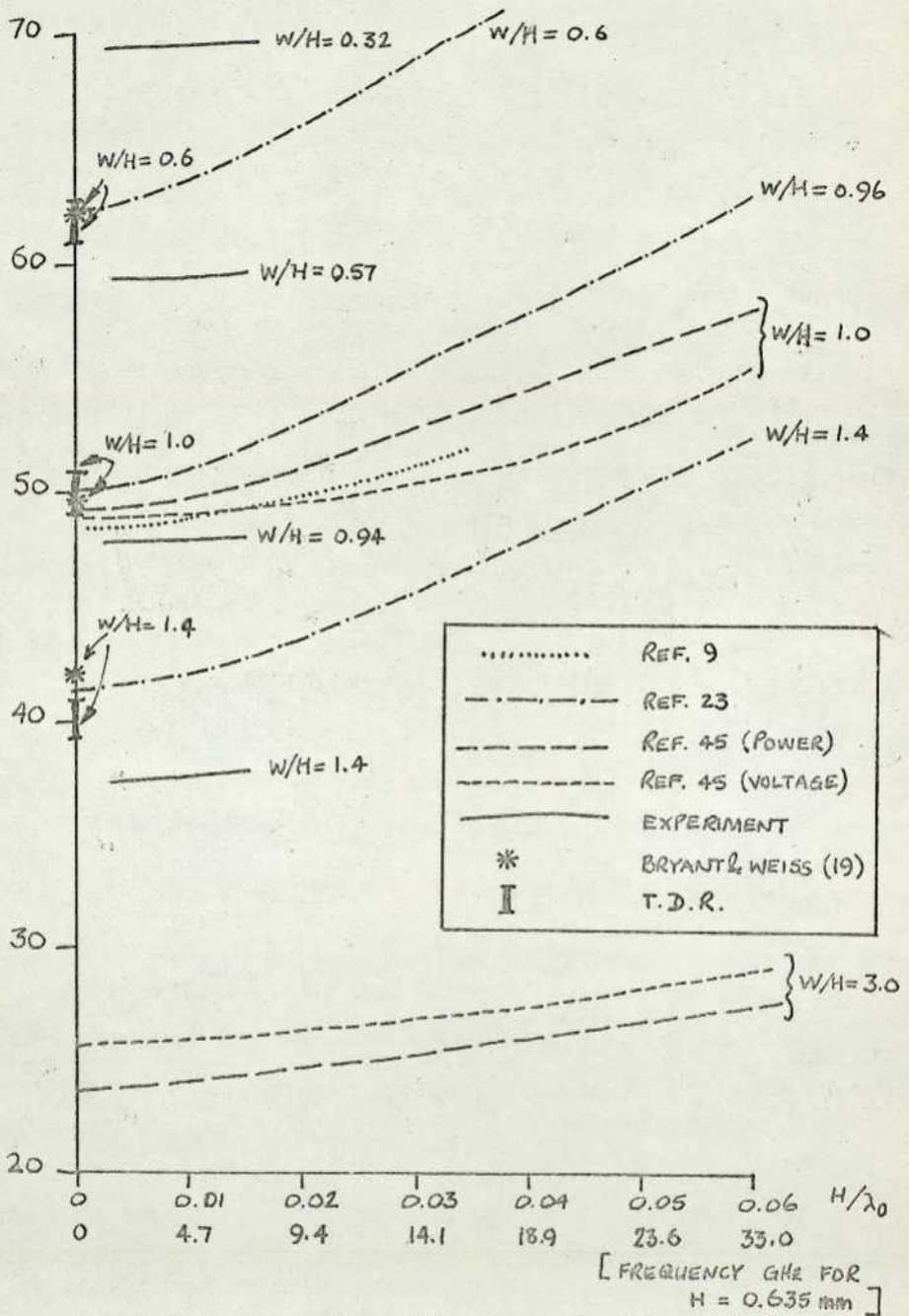


Figure 3.15 A Comparison of the Practical and Theoretical Characteristic Impedance of Microstrip.

The microstrip-mode impedance, however, has been directly measured. Because of the similarity in cross-section between the two transmission lines, the characteristic impedance of the microstrip-mode in the microstrip/slot line combination was measured with respect to a microstrip line. This avoided the technical problem of joining the microstrip/slot line to a coaxial line. A microstrip line was constructed on an Alumina substrate, 0.635 mm. thick, and a slot 2 cms long etched, under the line, in the ground plane, such that on either side of the microstrip/slot line there was a section of microstrip line. From T.D.R. measurements it was found that the slot caused the characteristic impedance to be higher than for the same line dimensions without a slot in the ground plane. Three different sized slots were used, the results being shown graphically in Figure 3.17.

% INCREASE OF MICROSTRIP-MODE
IMPEDANCE OVER IMPEDANCE OF
A SIMILAR MICROSTRIP LINE

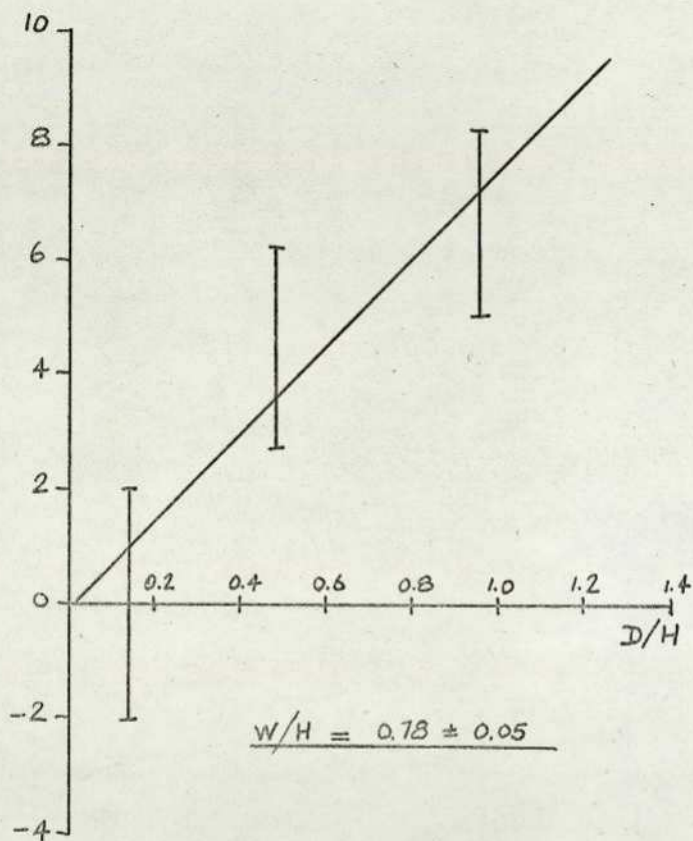


Figure 3.17 Variation of the Microstrip-Mode Impedance with D/H when loaded with 0.025 inch thick Zirconate.

CHAPTER 4A THEORETICAL APPROACH FOR THE SOLUTION OF FIELD PROBLEMS USING
A LEAST SQUARE CRITERION4.1 Introduction

In the preceding Chapters, attention has been centred upon the measurement of various parameters in order to design a microwave directional coupler. The design has been based entirely upon these measurements, which if a widely differing performance were required, or different materials used, would necessitate the performance of a completely new set of measurements. The theoretical solutions of microstrip transmission lines and related structures are complicated mainly by the presence of the transverse inhomogeneity. Since no analytical approach has been found which yields a closed form solution, a range of numerical methods have been used to find an approximate solution. Early methods for solving microstrip were based upon a quasi-static approach which assumed a quasi-TEM mode to propagate (31, 51-54). More recently, attention has been concentrated on determining the dispersive nature of microstrip by using either methods based on a hybrid mode analysis (9, 17, 22, 23, 45, 55-67), or frequency dependent models (18, 69).

The directional coupler described in this thesis supports two different dominant modes which are uncoupled and dispersive. A theoretical method, based upon a hybrid mode analysis, that would be applicable to general structures more complex than microstrip, is required. It was thought that the method of error minimisation using a Least Square criterion published by LaRiviere (67) offered the best possibility of extension to characterise structures more complex than, but similar to, microstrip. Finite Element (63) or Finite Difference (60) approaches require large amounts of computer storage, and the use of non-linear discretisation grids to maintain accuracy near regions of rapidly varying fields. The approach using singular integral equations by Mittra and Itch (65) showed that

for microstrip, results of similar accuracy could be achieved with a 2×2 matrix compared to a 50×50 matrix using Finite Difference techniques. The singular integral technique, however, is not easily applied to more complicated structures due to the increased complexity of the algebraic manipulations involved.

Hybrid mode analysis techniques using Fourier Matching and Variational expressions have also been developed by LaRiviere and others (51, 59, 64). The main disadvantages with these approaches for microstrip are that the relative convergence of a pair of semi-infinite integrals is not known explicitly, and that large orders of determinant have to be solved. Because the field singularities at the edge of the strip are often ignored in the hybrid mode formulation, the rate of convergence is slow. With added complexity, however, this latter problem may be eliminated by application of the 'edge condition' (70) as an extra boundary condition in the hybrid mode formulation. The relative convergence problem has been solved explicitly using this condition for a bifurcated waveguide by Mittra (71). The problem for microstrip remains unsolved, with methods having to resort to the use of trial ratios. Applying these techniques to structures similar to, but more complex than, microstrip would, in general, slow the rate of convergence, and make the relative convergence problem more intractable.

The Least Square error minimisation technique is well known as a method of optimising the fit of two sets of functions. The technique can be generally applied in one of two ways. Firstly as an interior method where the field, which is completely enclosed by the boundaries, conforms to a given equation, the kernel of which is given by an approximate solution which conforms to the boundary conditions. The error is squared and integrated over the entire enclosed surface, the best approximate solution being that which is found when the integral is at its minimum. The alternative approach is to find an exact function which satisfies the linear operator equation within the region, and then to integrate the square of this function when it is subjected to the boundary condition, along the boundaries of the region. This approach is termed a boundary method.

An application of the boundary method, for the solution of scattering problems, has been published by Davis (72). Boundary conditions at the interface of sub-regions or at a conducting surface where the basis functions of the region do not satisfy the boundary condition, lead to a boundary residual which can be minimised by application of the Least Square criterion. He proved the 'boundary residual' method to be rigorously convergent, and showed that the convergence is guaranteed for regions where singularity in the fields may occur, as long as the 'boundary residuals' are evaluated along contours which end at the point of singularity, though the rate of convergence may be slower.

An alternative approach to that adopted by LaRiviere is given in this Chapter for a solution to microstrip. The method is perhaps more general, and can be extended to solve the propagation problem of the two modes supported in the directional coupler described in Chapter 5. The essential difference of approach is that basis functions for the fields in a transverse section of the enclosed microstrip line, satisfy all the boundary conditions imposed by the metallic conductors, excluding the 'edge condition' at the edge of the strip. Thus non-zero boundary residuals only exist within the enclosed region, allowing it to be considered as an interior method, although the formulation of the Least Square criterion is not integrated over the enclosed surface, but along contours within the region. In section 4.2, the L-shaped waveguide is used as a test for the method, though formulated in a different manner to that of LaRiviere's work. The transverse section of the guide was subdivided in a similar manner to the way in which the transverse section of the microstrip line of section 4.3 would be subdivided. In addition to comparing TM solutions with those obtained by LaRiviere, TE solutions were also computed. The dominant TE solution is important since it is upon this mode that the field singularity at the edge of the strip has its greatest effect.

4.2 Theory applied to an L-Shaped Waveguide.

4.2.1 TM Modes in an L-Shaped Waveguide.

Consider the L-shaped waveguide in Figure 4.1.

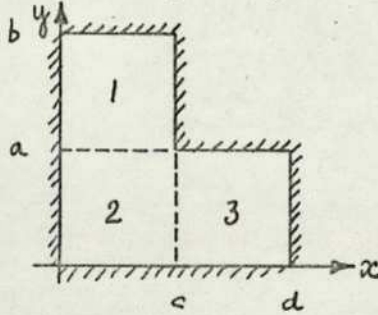


Figure 4.1

The fields within these sub-regions can be expressed by equations which must satisfy, (a) the boundary between sub-regions, and (b) Maxwell's equations within each sub-region. Further, as it is required to solve an eigenvalue problem, then these field expansions must satisfy the Helmholtz Equation:

$$\nabla^2 \chi + k^2 \chi = 0 \quad \dots 4.1$$

where χ is a scalar

$$\nabla \text{ is the vector operator } \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

k^2 is the wave number, or eigenvalue of the partial differential equation.

For TM modes there is no longitudinal magnetic field, i.e.

$$H_z = 0 \quad \dots 4.2$$

thus, if Q is the magnetic scalar potential, defined such that

$$H_z = Q(x, y) \cdot e^{-j\beta z} \quad \dots 4.3$$

where β is the phase constant, then

$$Q = 0 \quad \dots 4.4$$

Thus, within each region only the electric field has to conform to Helmholtz equation:

$$\nabla^2 \psi + k^2 \psi = 0 \quad \dots 4.5$$

where the scalar electric potential ψ is related to the longitudinal electric field, by the definition:

$$E_z = \psi(x, y) \cdot e^{-j\beta z} \quad \dots 4.6$$

Solutions of the partial differential equation 4.6, which conform to the boundary conditions, $\psi=0$, are given in terms of unknown coefficients, namely $a_n, \bar{c}_n, \tilde{c}_n$ and e_n :

$$\psi_1 = \sum_{n=1}^N (a_n \sin \alpha_n^{(1)} x \cdot \sin \beta_n^{(1)} (b-y)) \quad \dots 4.7$$

$$\psi_2 = \sum_{n=1}^N (\bar{c}_n \sin \alpha_n^{(1)} x \cdot \sin \beta_n^{(1)} y + \tilde{c}_n \sin \beta_n^{(2)} x \cdot \sin \alpha_n^{(2)} y) \quad \dots 4.8$$

$$\psi_3 = \sum_{n=1}^N (e_n \sin \beta_n^{(2)} (d-x) \cdot \sin \alpha_n^{(2)} y) \quad \dots 4.9$$

where the separation coefficients of the partial differential equation are given by:

$$\alpha_n^{(1)} = \frac{n\pi}{c} \quad ; \quad \beta_n^{(1)} = \sqrt{k_A^2 - \alpha_n^{(1)2}} \quad \dots 4.10$$

$$\alpha_n^{(2)} = \frac{n\pi}{a} \quad ; \quad \beta_n^{(2)} = \sqrt{k_A^2 - \alpha_n^{(2)2}} \quad \dots 4.11$$

and k_A^2 is the wave number, related to the angular velocity, ω , the velocity of light, c , and the phase velocity, β , by:

$$k_A^2 = \frac{\omega^2}{c^2} - \beta^2 \quad \dots 4.12$$

Applying Maxwell's curl relationships within each sub-region, the components of the fields, which are tangential and normal to the interfaces of the sub-regions, are given as follows:

$$E_x = \frac{1}{\epsilon_1 k_0^2 - \beta^2} \left(-j\omega\mu \frac{\partial Q}{\partial y} - j\beta \frac{\partial \psi}{\partial x} \right) \quad \dots 4.13$$

$$E_y = \frac{1}{\epsilon_r k_0^2 - \beta^2} \left(j\omega\mu \frac{\partial Q}{\partial x} - j\beta \frac{\partial V}{\partial y} \right) \quad \dots 4.14$$

$$H_x = \frac{1}{\epsilon_r k_0^2 - \beta^2} \left(-j\beta \frac{\partial Q}{\partial x} + j\omega\epsilon \frac{\partial V}{\partial y} \right) \quad \dots 4.15$$

$$H_y = \frac{1}{\epsilon_r k_0^2 - \beta^2} \left(-j\beta \frac{\partial Q}{\partial y} - j\omega\epsilon \frac{\partial V}{\partial x} \right) \quad \dots 4.16$$

From the knowledge that the tangential electric field at any boundary is continuous, the tangential electric fields can be matched between regions 1 and 2, and regions 3 and 2, allowing the unknown coefficients a_n and e_n to be expressed in terms of $\bar{c}_n$ and $\tilde{c}_n$. The component E_z in each region is given by equation 4.6, and E_x and E_y by equations 4.13 and 4.14 respectively, remembering that Q is zero. It is found that the following relationships result:

$$a_n = \bar{c}_n \quad \dots 4.17$$

$$e_n = \tilde{c}_n \quad \dots 4.18$$

Across an interface between two non-conducting regions, the total surface current density per unit width is zero, and the tangential components of the magnetic field are continuous. It is upon this relationship that the Least Square error minimisation criterion is defined. The boundary residual R , along a contour is given by:

$$R = \int_C \left\{ (H_{z_1} - H_{z_2})(H_{z_1} - H_{z_2})^* dt + (H_{t_1} - H_{t_2})(H_{t_1} - H_{t_2})^* dt \right\} \quad \dots 4.19$$

where H_t indicates the transverse direction parallel to the interface between the two regions, suffixed 1 and 2, and the asterisks indicate the complex conjugate.

It is at this stage that the normal mathematic notation can be best replaced by computer programming notation, especially for the names of certain variables. This aids the checking of the computer program against the mathematics, and thus helps to

eliminate errors.

In the L-shaped waveguide under consideration, the two boundary residuals become:

$$R_{1-2} = \int_0^c \sum_{n=1}^N \left\{ \bar{c}_n \text{HXIA}(N) \cdot \sin \alpha_n^{(1)} x + \tilde{c}_n \text{HXIB}(N) \cdot \sin \beta_n^{(2)} x \right\} \\ \sum_{m=1}^M \left\{ \bar{c}_m \text{HXIA}(M) \sin \alpha_m^{(1)} x + \tilde{c}_m \text{HXIB}(M) \sin \beta_m^{(2)} x \right\}^* dx \quad \dots 4.20$$

and

$$R_{2-3} = \int_0^a \sum_{n=1}^N \left\{ \bar{c}_n \text{HYIA}(N) \cdot \sin \beta_n^{(1)} y + \tilde{c}_n \text{HYIB}(N) \cdot \sin \alpha_n^{(2)} y \right\} \\ \sum_{m=1}^M \left\{ \bar{c}_m \text{HYIA}(M) \sin \beta_m^{(1)} y + \tilde{c}_m \text{HYIB}(M) \sin \alpha_m^{(2)} y \right\}^* dy \quad \dots 4.21$$

where the one dimensional H arrays are:

$$\text{HXIA}(N) = -j \frac{2\omega\epsilon_0 \beta_n^{(1)}}{k_A^2} \cos \beta_n^{(1)} a \quad \dots 4.22$$

$$\text{HXIB}(N) = -j \frac{(-1)^n \alpha_n^{(2)} \omega\epsilon_0}{k_A^2} \quad \dots 4.23$$

$$\text{HYIA}(N) = j \frac{(-1)^n \alpha_n^{(1)} \omega\epsilon_0}{k_A^2} \quad \dots 4.24$$

$$\text{HYIB}(N) = j \frac{2\omega\epsilon_0 \beta_n^{(2)}}{k_A^2} \cos \beta_n^{(2)} c \quad \dots 4.25$$

Expanding equations 4.20 and 4.21, it will be seen that each boundary residual is expressed in terms of a pair of the unknown coefficients, $\bar{c}_n$ and $\tilde{c}_n$, with the product of two of the one dimensional H arrays, and an integral. Thus, equations 4.20 and 4.21 become:

$$R_{12} = \sum_{n=1}^N \sum_{m=1}^M \left\{ \bar{c}_n \bar{c}_m \text{AA12} + \bar{c}_n \tilde{c}_m \text{AB12} + \tilde{c}_n \bar{c}_m \text{BA12} + \tilde{c}_n \tilde{c}_m \text{BB12} \right\} \quad \dots 4.26$$

and

$$R_{23} = \sum_{n=1}^N \sum_{m=1}^M \left\{ \bar{c}_n \bar{c}_m \text{AA23} + \bar{c}_n \tilde{c}_m \text{AB23} + \tilde{c}_n \bar{c}_m \text{BA23} + \tilde{c}_n \tilde{c}_m \text{BB23} \right\} \quad \dots 4.27$$

where the variables AA12, AB12, AA23, AB23, etc, are computed

for each n and m , being given by the following set of equations, written in Fortran, the computer programming language. Here, only, the asterisk denotes multiplication.

```

AA12=HX1A(N)*C01NJG(HX1A(M))*CO1(N,M)
AB12=HX1A(N)*C02NJG(HX1B(M))*CO2(N,M)
BA12=HX1B(N)*C03NJG(HX1A(M))*CO3(N,M)
BB12=HX1B(N)*C04NJG(HX1B(M))*CO4(N,M)
AA23=HY1A(N)*C05NJG(HY1A(M))*CO5(N,M)
AB23=HY1A(N)*C06NJG(HY1B(M))*CO6(N,M)
BA23=HY1B(N)*C07NJG(HY1A(M))*CO7(N,M)
BB23=HY1B(N)*C08NJG(HY1B(M))*CO8(N,M)

```

... 4.28

where the two dimension C arrays are given by:

$$\begin{aligned}
 C01(N,M) &= \int_0^c \sin \alpha_n^{(1)} x \cdot (\sin \alpha_m^{(1)} x)^* dx \\
 C02(N,M) &= \int_0^c \sin \alpha_n^{(1)} x \cdot (\sin \beta_m^{(2)} x)^* dx \\
 C03(N,M) &= \int_0^c \sin \beta_n^{(2)} x \cdot (\sin \alpha_m^{(1)} x)^* dx \\
 C04(N,M) &= \int_0^c \sin \beta_n^{(2)} x \cdot (\sin \beta_m^{(2)} x)^* dx \\
 C05(N,M) &= \int_0^a \sin \beta_n^{(1)} y \cdot (\sin \beta_m^{(1)} y)^* dy \\
 C06(N,M) &= \int_0^a \sin \beta_n^{(1)} y \cdot (\sin \alpha_m^{(2)} y)^* dy \\
 C07(N,M) &= \int_0^a \sin \alpha_n^{(2)} y \cdot (\sin \beta_m^{(1)} y)^* dy \\
 C08(N,M) &= \int_0^a \sin \alpha_n^{(2)} y \cdot (\sin \alpha_m^{(2)} y)^* dy
 \end{aligned}$$

... 4.29

Adding equations 4.26 and 4.27 together, and writing the result using matrix notation, then

$$x^T A x = R$$

... 4.30

where R is the total boundary residual, the vector $x = (\bar{c}_1, \bar{c}_2, \dots, \bar{c}_N, \tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_N)$, x^T is the transpose of x being

$x^T = (\bar{c}_1, \bar{c}_2, \dots, \bar{c}_M, \tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_M)$, and A is the matrix formed by the coefficients AA_{12} , AB_{12} , etcetera. It can be seen that this equation has a quadratic form, but also that because the vector x is not implicit in the problem, since the magnitude of the scalar fields are not fixed, the expression must be normalised in order that the form be minimised. Dividing by the inner product, equation 4.30 becomes:

$$\frac{x^T A x}{x^T x} = R \quad \dots 4.31$$

This form is well known as being the Raleigh quotient (73, 74), which has the property that for $x^T x = 1$, its value equals the smallest eigenvalue of the matrix A when the vector x equals the eigenvector associated with that eigenvalue. Hence, the problem reduces to one of solving the eigenvalue problem:

$$(A - I\lambda)\bar{x} = 0 \quad \dots 4.32$$

To find the minimum boundary residual it is thus only required to vary the frequency with all other parameters fixed, and to find the minimum value of λ_1 , the smallest eigenvalue from equation 4.32. The vector x is equal to the eigenvector associated with λ_1 .

Since the problem is formulated from basis functions which are without a true scalar reference, it is instructive to imagine that they do have one and that it is fixed by the relation $x^T x = 1$. Thus, the smallest eigenvalue of A is $x^T A x$, and the problem is one of solving the eigenvalue equation 4.32. Each region is not necessarily bound to a common numerical scale, and intentional scaling or effects of the higher harmonics of the basis functions will change the magnitudes of the components of the basis functions. The inner product in this case will change and will no longer be equal to unity. It can easily be shown that an eigenvalue of A is given by

$$\lambda = \frac{\bar{x}^T A \bar{x}}{\bar{x}^T \bar{x}} \quad \dots 4.33$$

Thus, since the inner product is no longer unity, its effect is

to impose different numerical shifts to all the eigenvalues of A , so that generally λ_1 , will no longer be its smallest eigenvalue though it still will be representative of the value of the quadratic form in question.

Where the physical problem is simple, and the scalar fields similar in each sub-region, singularities and intentional scaling absent, the eigenvalues of A , which correspond to the value of the quadratic form, will perhaps be given by the smallest eigenvalue of A . This is true for this case, but when microstrip is considered in section 4.3, it is certainly not true. Moreover, the variation due to larger matrix orders re-scales the eigenvalue problem in such a way that the eigenvalues of larger order matrices are numerically greater than those of smaller matrices. It should be remembered, however, that the frequency at which the minimum of the eigenvalue occurs, and the ratios of the components of the associated eigenvector, are seemingly unaltered by this phenomenon.

Matrix A is formed when the elements AA_{12} , AB_{12} etc. in equations 4.26 and 4.27 are arranged as in the following diagram, Figure 4.2. For this problem it is seen numerically that the matrix is real symmetric, though more generally it

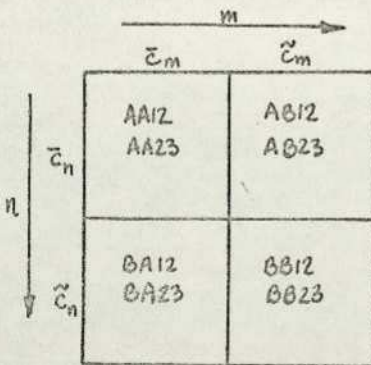


Figure 4.2

is hermitian, as is, for example, the case of the TE solution for L-shaped waveguide considered in the next sub-section. The decision to multiply rows and columns by $-j$ and j respectively, was not necessary since the matrix was already real. It was decided, however, that this would not be done for a general case in order to gain simplification at

at the expense of the increased computer storage requirements of a complex matrix. The algorithms used for obtaining the eigenvalues and eigenvectors were taken from a book by J.H. Wilkinson and C. Reinsch (75), which contains a collection of algorithms for solving linear algebraic systems and the algebraic eigenvalue problem. One possibility would be to form the augmented matrix.

$$\begin{bmatrix} A & -B \\ B & A \end{bmatrix} \quad \dots 4.34$$

from the complex matrix $A + jB$, and use methods applicable to real symmetric matrices. One of the most efficient of these, the Householder Tridiagonalization (76), has a slight disadvantage that where the elements of a matrix have widely varying orders of magnitude, they must be organised such that the smallest are in the top left hand corner, and the largest in the bottom right hand corner. LaRiviere used this routine in conjunction with the methods of Bisection (77) to find the eigenvalues. This latter procedure is an extremely flexible method of finding some or all of the eigenvalues, and eigenvectors, of a tridiagonalised matrix to any degree of accuracy. LaRiviere found matrix scaling necessary to order the magnitudes of its elements as required for the Householder algorithm, the optimum scaling factor giving the fastest convergence of the problem. Because of the fact that the augmented matrix produces pairs of eigenvalues, and eigenvectors, which would require pairing after computation, and also because of the scaling, it was decided to use algorithms which would deal directly with complex matrices and thus avoid these difficulties.

The procedure recommended in Wilkinson and Reinsch's book for a general complex matrix is to use a complex form of the LR algorithm called 'Comlr 2' (78). This computes all the eigenvalues and eigenvectors of a complex Hessenberg Matrix. The general complex matrix can first be transformed to an upper-Hessenberg form, using the routine 'Comhes' (79). These two routines are particularly computable, since the multipliers used in the similarity transformations to the upper-Hessenberg form are stored

below the sub-diagonal in the lower half triangle of the Hessenberg matrix. 'Comlr 2' uses this information to form the eigenvectors of the original matrix A rather than the Hessenberg matrix. The eigenvalues are unaffected by the similarity transformations. Where the elements in the matrix are of widely different orders of magnitude, the norm, $\|A\|_1$, of the matrix can be reduced using a complex version of the routine 'Balance' (80). This has been implemented in all the problems so far solved, but without showing much difference in the value of the computed eigenvalue.

An outline of the computer program necessary to compute the eigenvalues and eigenvectors, is given in Appendix 2. The program was executed on a CDC7600 computer at the University of London Computing Centre. The required solution for each of the TM modes was each mode's associated wave number, k_A^2 . With β and the waveguide dimensions set to unity, i.e. the waveguide cross-section made up of three unit squares, k_A^2 was varied noting the value of a particular eigenvalue. The eigenvalue associated with each mode reached a minimum value for a particular value of k_A^2 . The minima of these eigenvalues were initially obtained graphically, but later improved accuracy was achieved by using a golden section algorithm to locate each minima, the eigenvalue of interest being recognised numerically by means of its associated eigenvector which only changed slowly with k_A^2 . The results obtained are given in Table 4.1, all figures accurate to the last significant figure.

This test of the Least Square error minimisation technique, applied to solving the eigenvalue problem of an L-shaped waveguide, can be seen to be successful when comparison is made between the results in Table 4.1 and those obtained by LaRiviere, where the fields were matched along a diagonal between the apex of the guide and its external corner, and Fox et. al. (81) who solved the similar eigenvalue problem of an L-shaped membrane. The comparison is given in Table 4.2, where LaRiviere's results have been multiplied by π^2 since his unit square had sides of length π .

Eigenvalue 1 at $k_A^2 = 9.64$ was obtained since the separation constants, $\beta_n^{(1)}$ and $\beta_n^{(2)}$ given in equations 4.10 and 4.11, passed through zero at π^2 (9.869). This made the matrix so

Eigenvalue No.	Wave Number, k^2 for given Matrix Order				
	2x2	4x4	6x6	8x8	10x10
1	-	-	-	-	-
2	15.7098	15.3642	15.2844	15.2519	15.2352
3	-	-	-	-	-
4	29.7197	29.5203	29.5159	29.5167	29.5174
5	33.3636	32.7136	32.5659	32.4893	32.4376
6	-	41.6990	41.6090	41.5656	41.5419
7	-	45.6995	45.2993	45.1627	45.0957
8,9	-	-	-	-	-
10	-	-	56.6342	56.6234	56.6208

Table 4.1.

Eigenvalue No.	Fox et al.	LaRiviere		Computed	
		% diff.		% diff.	
1	9.6397238	-	-	*	-
2	15.19725	15.1972	-0.001	15.2352	+0.25
3	19.73920	19.7392	-	*	-
4	29.52148	29.5230	+0.005	29.5172	-0.014
5	31.91263	32.8372	+2.90	32.4376	+1.65
6	41.4745	42.4620	+2.38	41.5419	+0.187
7	44.948	44.7616	-0.42	45.0957	+0.328
8,9	49.3480	49.3480	-	*	-
10	56.7096	57.6217	-1.608	56.6208	-0.157

Table 4.2.

* See Text.

ill-conditioned around this value that the eigenvalue could not be seen to pass through a minimum. Eigenvalues 3, 8 and 9 have eigenfunctions which are multiple squares, and thus with the contours chosen to evaluate the Least Square criterion, the residuals are small, and are satisfied exactly. The non-appearance of the expected minima for these eigenvalues may be due to this fact, causing the minima to be very sharp, and thus easily missed in a discrete search. The fact that eigenvalue 3 is equal to $2\pi^2$, and eigenvalues 8 and 9 are equal to $5\pi^2$ can be easily seen from equations 4.10 and 4.11. A typical plot of the change of an eigenvalue with k^2 is given in Figure 4.3.

4.2.2 TE Modes in an L-Shaped Waveguide.

The previous sub-section showed that good results could be achieved with moderate sized matrices when considering the TM modes in an L-shaped waveguide, where the scalar electric fields exactly satisfied the Dirichlet boundary condition, $\psi = 0$, around the waveguide, and the scalar magnetic field was zero by definition. A more complex guided wave structure, such as a single microstrip line or two coupled microstrip lines, would involve both scalar electric as well as scalar magnetic fields, since the dominant mode contains both longitudinal electric and magnetic fields. By solving the same waveguide for its TE modes, the scalar electric field will be zero since there is no longitudinal electric field component, and the scalar magnetic field, Q , will conform to the Helmholtz equation,

$$\nabla^2 Q + k^2 Q = 0 \quad \dots 4.35$$

with Neumann boundary conditions at the waveguide walls:

$$\frac{\partial Q}{\partial n} = 0 \quad \dots 4.36$$

n being the direction normal to any surface of the waveguide.

A solution of 4.35 with Neumann conditions, which is analogous to that of equation 4.5 in the previous sub-section,

SMALLEST EIGENVALUE $\times 10^{-6}$

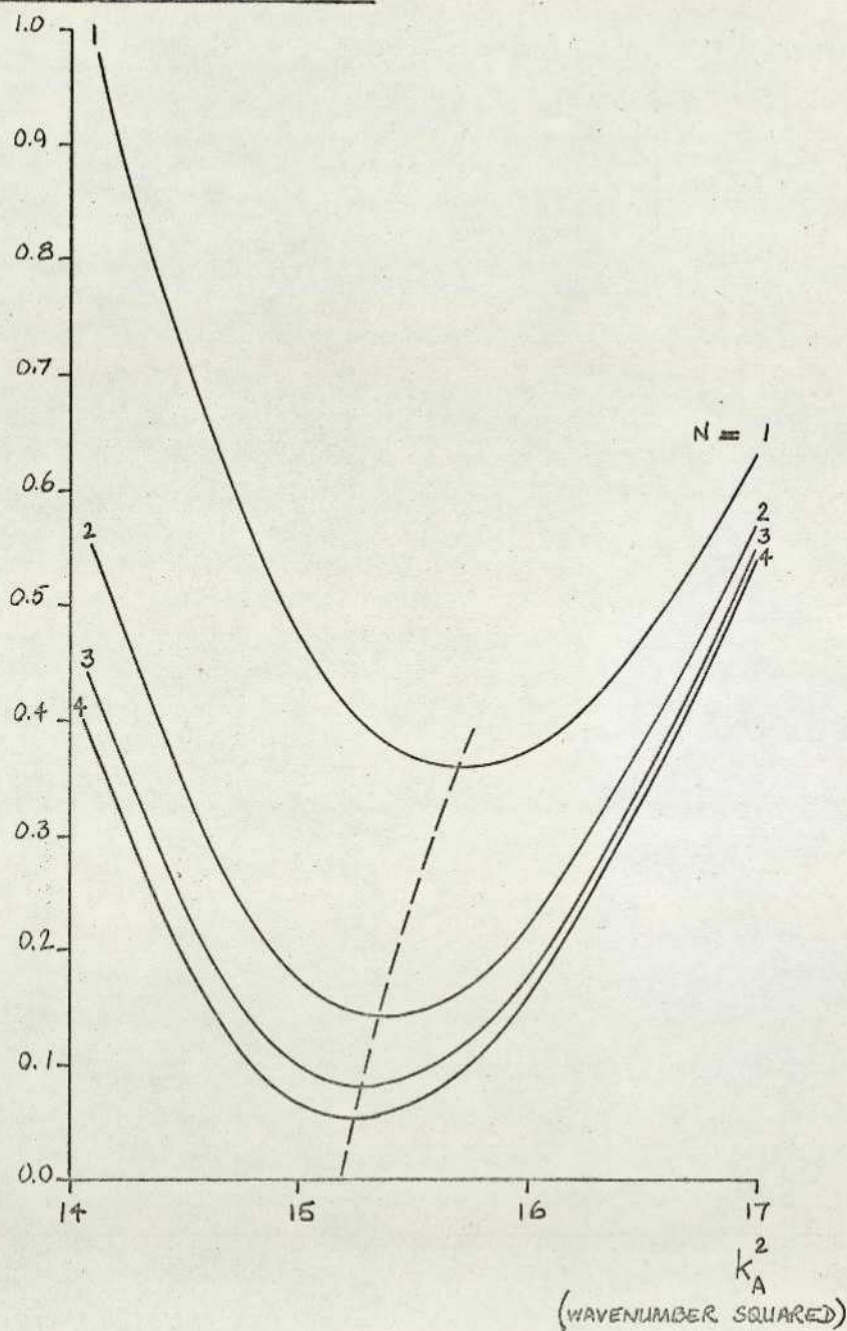


Figure 4.3 Variation of the Value of the Eigenvalue about its Minimum Value of k_A^2 , for Various Matrix Orders.

is as follows:

$$Q_1 = \sum_{n=0}^N (b_n \cos \alpha_n^{(1)} x \cdot \cos \beta_n^{(1)} (b-y)) \quad \dots 4.37$$

$$Q_2 = \sum_{n=0}^N (\bar{d}_n \cos \alpha_n^{(1)} x \cos \beta_n^{(1)} y + \tilde{d}_n \cos \beta_n^{(2)} x \cos \alpha_n^{(2)} y) \quad \dots 4.38$$

$$Q_3 = \sum_{n=0}^N (f_n \cos \beta_n^{(2)} (d-x) \cos \alpha_n^{(2)} y) \quad \dots 4.39$$

However, it is seen that at the internal 270° corner, the Neumann condition is not satisfied by these equations. The separation constants are previously given in equations 4.10 and 4.11.

Matching the tangential electric fields along the boundaries between sub-regions simplifies the above equations by establishing the following identities:

$$b_n = -\bar{d}_n \quad \dots 4.40$$

and

$$f_n = -\tilde{d}_n \quad \dots 4.41$$

Applying the Least Square error minimisation criterion of equation 4.19, yields in this case two sets of equations for each boundary, namely one for the H_z field match, and one for either the H_x or H_y depending upon the boundary. Thus from the above equations, the boundary residual along the contours between sub-regions 1 and 2, and sub-regions 2 and 3 become

$$R_{1-2} = R_{y_{1-2}} + R_{x_{1-2}} \quad \dots 4.42$$

and

$$R_{2-3} = R_{y_{2-3}} + R_{y_{2-3}} \quad \dots 4.43$$

where

$$R_{y_{1-2}} = \sum_{n=0}^c \left\{ \bar{d}_n HZ1A(N) \cos \alpha_n^{(1)} x + \tilde{d}_n HZ1B(N) \cos \beta_n^{(2)} x \right\} \\ \sum_{m=0}^M \left\{ \bar{d}_m HZ1A(M) \cos \alpha_m^{(1)} x + \tilde{d}_m HZ1B(M) \cos \beta_m^{(2)} x \right\}^* dx \quad \dots 4.44$$

$$R_{x_{1-2}} = \sum_{n=1}^N \left\{ \bar{d}_n HXIA(N) \sin \alpha_n^{(1)} x + \tilde{d}_n HXIB(N) \sin \beta_n^{(2)} x \right\} \\ \sum_{m=1}^M \left\{ \bar{d}_m HXIA(M) \sin \alpha_m^{(1)} x + \tilde{d}_m HXIB(M) \sin \beta_m^{(2)} x \right\}^* dx. \quad \dots 4.45$$

$$R_{y_{2-3}} = \sum_{n=1}^N \left\{ \bar{d}_n HZ2A(N) \cos \beta_n^{(1)} y + \tilde{d}_n HZ2B(N) \cos \alpha_n^{(2)} y \right\} \\ \sum_{m=1}^M \left\{ \bar{d}_m HZ2A(M) \cos \beta_m^{(1)} y + \tilde{d}_m HZ2B(M) \cos \alpha_m^{(2)} y \right\}^* dy \quad \dots 4.46$$

$$R_{y_{2-3}} = \sum_{n=1}^N \left\{ \bar{d}_n HY2A(N) \sin \beta_n^{(1)} y + \tilde{d}_n HY2B(N) \sin \alpha_n^{(2)} y \right\} \\ \sum_{m=1}^M \left\{ \bar{d}_m HY2A(M) \sin \beta_m^{(1)} y + \tilde{d}_m HY2B(M) \sin \alpha_m^{(2)} y \right\}^* dy \dots 4.47$$

where

$$HZIA(N) = -2 \cos \beta_n^{(1)} a \quad \dots 4.48$$

$$HZIB(N) = -(-1)^N \quad \dots 4.49$$

$$HXIA(N) = -j \frac{2\beta \cdot \alpha_n^{(1)}}{k_A^2} \cos \beta_n^{(1)} a \quad \dots 4.50$$

$$HXIB(N) = -j \frac{\beta \cdot \beta_n^{(2)} (-1)^N}{k_A^2} \quad \dots 4.51$$

$$HZ2A(N) = -(-1)^N \quad \dots 4.52$$

$$HZ2B(N) = -2 \cos \beta_n^{(2)} c \quad \dots 4.53$$

$$HY2A(N) = -j \frac{\beta \cdot \beta_n^{(1)} (-1)^N}{k_A^2} \quad \dots 4.54$$

$$HY2B(N) = -j \frac{2 \alpha_n^{(2)} \beta}{k_A^2} \cos \beta_n^{(2)} c \quad \dots 4.55$$

In a similar manner to the case in sub-section 4.2.1, the total boundary residual for each contour can be written:

$$R_{12} = \sum_{n=1}^N \sum_{m=1}^M \left\{ \bar{d}_n \bar{d}_m AA12 + \bar{d}_n \tilde{d}_m AB12 + \tilde{d}_n \bar{d}_m BA12 + \tilde{d}_n \tilde{d}_m BB12 \right\} \dots 4.56$$

$$R_{23} = \sum_{n=1}^N \sum_{m=1}^M \left\{ \bar{d}_n \bar{d}_m AA23 + \bar{d}_n \tilde{d}_m AB23 + \tilde{d}_n \bar{d}_m BA23 + \tilde{d}_n \tilde{d}_m BB23 \right\} \dots 4.57$$

where the matrix coefficients are given by:

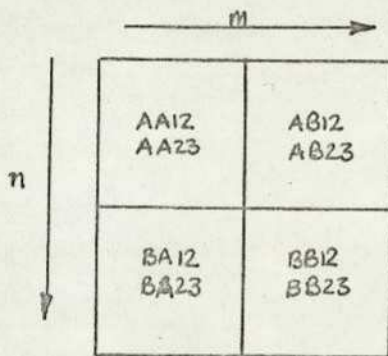
$$\begin{aligned}
AA12 &= HX1A(N) * C\phi NJG(HX1A(M)) * CO1(N,M) \\
&\quad + HZ1A(N) * C\phi NJG(HZ1A(M)) * CO5(N,M) \\
AB12 &= HX1A(N) * C\phi NJG(HX1B(M)) * CO2(N,M) \\
&\quad + HZ1A(N) * C\phi NJG(HZ1B(M)) * CO6(N,M) \\
BA12 &= HX1B(N) * C\phi NJG(HX1A(M)) * CO3(N,M) \\
&\quad + HZ1B(N) * C\phi NJG(HZ1A(M)) * CO7(N,M) \\
BB12 &= HX1B(N) * C\phi NJG(HX1B(M)) * CO4(N,M) \\
&\quad + HZ1B(N) * C\phi NJG(HZ1B(M)) * CO8(N,M) \\
AA23 &= HY2A(N) * C\phi NJG(HY2A(M)) * CO9(N,M) \\
&\quad + HZ2A(N) * C\phi NJG(HZ2A(M)) * C13(N,M) \\
AB23 &= HY2A(N) * C\phi NJG(HY2B(M)) * C10(N,M) \\
&\quad + HZ2A(N) * C\phi NJG(HZ2B(M)) * C14(N,M) \\
BA23 &= HY2B(N) * C\phi NJG(HY2A(M)) * C11(N,M) \\
&\quad + HZ2B(N) * C\phi NJG(HZ2A(M)) * C15(N,M) \\
BB23 &= HY2B(N) * C\phi NJG(HY2B(M)) * C12(N,M) \\
&\quad + HZ2B(N) * C\phi NJG(HZ2B(M)) * C16(N,M)
\end{aligned}
\quad \dots 4.58$$

The integrals being:

$$\begin{aligned}
CO1(N,M) &= \int_0^c \sin \alpha_n^{(1)} x (\sin \alpha_m^{(1)} x)^* dx \\
CO2(N,M) &= \int_0^c \sin \alpha_n^{(1)} x (\sin \beta_m^{(2)} x)^* dx \\
CO3(N,M) &= \int_0^c \sin \beta_n^{(2)} x (\sin \alpha_m^{(1)} x)^* dx \\
CO4(N,M) &= \int_0^c \sin \beta_n^{(2)} x (\sin \beta_m^{(2)} x)^* dx \\
CO5(N,M) &= \int_0^c \cos \alpha_n^{(1)} x (\cos \alpha_m^{(1)} x)^* dx \\
CO6(N,M) &= \int_0^c \cos \alpha_n^{(1)} x (\cos \beta_m^{(2)} x)^* dx \\
CO7(N,M) &= \int_0^c \cos \beta_n^{(2)} x (\cos \alpha_m^{(1)} x)^* dx \\
CO8(N,M) &= \int_0^c \cos \beta_n^{(2)} x (\cos \beta_m^{(2)} x)^* dx \\
CO9(N,M) &= \int_0^a \sin \beta_n^{(2)} y (\sin \beta_m^{(1)} y)^* dy \\
C10(N,M) &= \int_0^a \sin \beta_n^{(1)} y (\sin \kappa_m^{(2)} y)^* dy
\end{aligned}
\quad \dots 4.59$$

$$\begin{aligned}
 C11(N,M) &= \int_0^a \sin \alpha_n^{(2)} y (\sin \beta_m^{(1)} y)^* dy \\
 C12(N,M) &= \int_0^a \sin \alpha_n^{(2)} y (\sin \alpha_m^{(2)} y)^* dy \\
 C13(N,M) &= \int_0^a \cos \beta_n^{(1)} y (\cos \beta_m^{(1)} y)^* dy \\
 C14(N,M) &= \int_0^a \cos \beta_n^{(1)} y (\cos \alpha_m^{(2)} y)^* dy \\
 C15(N,M) &= \int_0^a \cos \alpha_n^{(2)} y (\cos \beta_m^{(1)} y)^* dy \\
 C16(N,M) &= \int_0^a \cos \alpha_n^{(2)} y (\cos \alpha_m^{(2)} y)^* dy
 \end{aligned}$$

Forming the matrix A, from the boundary residual equations 4.56 and 4.57, the elements, given by equation 4.58, are arranged as in the diagram - Figure 4.4. The eigenvectors of A are found in the same way as before, though in this case the matrix is



hermitian rather than real symmetric. All the eigenvalues are real indicating that the matrix is positive definite. The results obtained for the first two eigenvalues are given below in Table 4.3

Figure 4.4

Eigenvalue	Wave Number, k_a^2 for given Matrix Order				
No.	2 x 2	4 x 4	6 x 6	8 x 8	10 x 10
1	1.3660	1.4810	1.4899	1.4914	1.4915
2	3.3317	3.5418	3.5399	3.5381	3.5369

Table 4.3

In order to check the rate of convergence and overall accuracy, it was hoped to be able to compare these results with

other results published for L-shaped waveguide solutions with Neumann boundary conditions. However, no published results could be found. A ridge waveguide of the basic dimensions, as shown

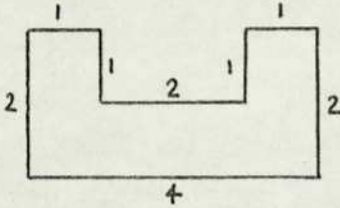


Figure 4.5

in Figure 4.5, has been analysed for both TE and TM modes. For TE modes, where Neumann boundary conditions are used, the symmetry of the ridge waveguide would make each eigenfunction of the L-shaped waveguide

equivalent to successive even eigenfunctions of the ridge waveguide. The relationships between the eigenvalue numbers being given in Table 4.4.

Comparison of the results with those of Bulley and Davis (82) and Beaubien and Wexler (83) for ridge waveguide, is given in Table 4.5. The results of Table 4.3 are for k_A^2 , or where 'a' is the size of the unit square, as $(k_A a)^2$. Bulley and Davis quote their results for k_0^2 , the value of 'a' being 0.5. To allow comparison with $(k_A a)^2$ here 'a' is unity, their results have to be divided by a factor of four. Beaubien and Wexler published results for k_A , 'a' being unity, thus for comparison their results are squared. Eigenvalues 3 and 4 are ill-conditioned, since for $k_A^2 = \pi^2$ the separation constants $\beta_n^{(1)}$ and $\beta_n^{(2)}$ pass through zero.

4.3 Theory Applied to an Enclosed Microstrip Line.

The previous section has shown a hybrid mode analysis incorporating a Least Square criterion to minimise the matching of fields between sub-regions, for which the type of field solution is known. The L-shaped waveguide was chosen to test the technique, since the form of the sub-regions would be similar to those employed in an enclosed microstrip solution considered in this section. The dominant mode for microstrip is symmetrical either

Eigenvalue No.		Eigenfunctions for L-shaped Waveguide
Ridge Waveguide	L-shaped Waveguide	
1	-	-
2	1	symmetric
3	-	-
4	2	anti-symmetric
5	-	-
6,7	3,4	symmetric square

Table 4.4

Eigenvalue No.	Bulley and Davis	Beaubien and Wexler	Computed 10x10 matrix
1	1.5160	1.4677	1.4915
2	3.539	3.5329	3.5369
3,4		-	-

Table 4.5

side of the strip, allowing some initial simplification. The transverse section is divided into sub-regions as shown in

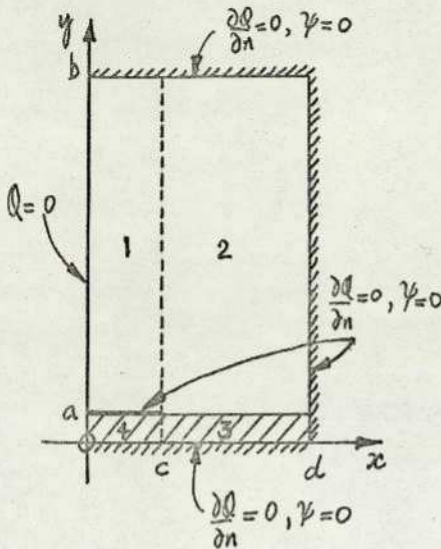


Figure 4.6

Figure 4.6. No assumptions have been made regarding E_z and H_z , hence the Helmholtz equation has to be satisfied for both electric and magnetic scalar fields,

$$\nabla^2 \psi + k^2 \psi = 0 \dots 4.60$$

$$\nabla^2 Q + k^2 Q = 0 \dots 4.61$$

subject to the boundary conditions shown in Figure 4.6. It will be noticed that the line of symmetry is a magnetic wall, which allows transverse

even symmetry in both the electric and magnetic fields.

The following equations are the solutions to the Helmholtz equations above, giving the scalar fields in each sub-region.

$$\psi_1 = \sum_{n=1}^N \left(a_n \frac{\cos \beta_n^{(1)} x}{\cos \beta_n^{(1)} c} \sin \alpha_n^{(1)} (b-y) \right) \dots 4.62$$

$$Q_1 = \sum_{n=1}^N \left(b_n \frac{\sin \beta_n^{(1)} x}{\sin \beta_n^{(1)} c} \cos \alpha_n^{(1)} (b-y) \right) \dots 4.63$$

$$\begin{aligned} \psi_2 = \sum_{n=1}^N & \left(\bar{c}_n \frac{\sin \beta_n^{(1)} (d-x)}{\sin \beta_n^{(1)} (d-c)} \sin \alpha_n^{(1)} (b-y) \right. \\ & \left. + \tilde{c}_n \sin \alpha_n^{(2)} (d-x) \frac{\sin \beta_n^{(2)} (b-y)}{\sin \beta_n^{(2)} (b-a)} \right) \dots 4.64 \end{aligned}$$

$$\begin{aligned} Q_2 = \sum_{n=1}^N & \left(\bar{d}_n \frac{\cos \beta_n^{(1)} (d-x)}{\cos \beta_n^{(1)} (d-c)} \cos \alpha_n^{(1)} (b-y) \right. \\ & \left. + \tilde{d}_n \cos \alpha_n^{(2)} (d-x) \frac{\cos \beta_n^{(2)} (b-y)}{\cos \beta_n^{(2)} (b-a)} \right) \dots 4.65 \end{aligned}$$

$$\psi_3 = \sum_{n=1}^N \left(\tilde{e}_n \frac{\sin \beta_n^{(3)}(d-x)}{\sin \beta_n^{(3)}(d-c)} \sin \alpha_n^{(3)} y + \tilde{e}_n \sin \alpha_n^{(2)}(d-x) \frac{\sin \beta_n^{(2)} y}{\sin \beta_n^{(2)} a} \right) \quad \dots 4.66$$

$$Q_3 = \sum_{n=1}^N \left(\tilde{f}_n \frac{\cos \beta_n^{(3)}(d-x)}{\cos \beta_n^{(3)}(d-c)} \cos \alpha_n^{(3)} y + \tilde{f}_n \cos \alpha_n^{(2)}(d-x) \frac{\cos \beta_n^{(2)} y}{\cos \beta_n^{(2)} a} \right) \quad \dots 4.67$$

$$\psi_4 = \sum_{n=1}^N \left(g_n \frac{\cos \beta_n^{(3)} c}{\cos \beta_n^{(3)} c} \sin \alpha_n^{(3)} y \right) \quad \dots 4.68$$

$$Q_4 = \sum_{n=1}^N \left(h_n \frac{\sin \beta_n^{(3)} c}{\sin \beta_n^{(3)} c} \cos \alpha_n^{(3)} y \right) \quad \dots 4.69$$

the separation constants being:

$$\alpha_n^{(1)} = \frac{n\pi}{(b-a)} \quad ; \quad \beta_n^{(1)} = \sqrt{k_A^2 - \alpha_n^{(1)2}} \quad \dots 4.70$$

$$\alpha_n^{(2)} = \frac{n\pi}{(d-c)} \quad ; \quad \beta_n^{(2)} = \sqrt{k_A^2 - \alpha_n^{(2)2}} \quad ; \quad \beta_n^{(2)} = \sqrt{k_D^2 - \alpha_n^{(2)2}} \quad \dots 4.71$$

$$\alpha_n^{(3)} = \frac{n\pi}{a} \quad ; \quad \beta_n^{(3)} = \sqrt{k_D^2 - \alpha_n^{(3)2}} \quad \dots 4.72$$

where

$$k_A^2 = k_0^2 - \beta^2 \quad \dots 4.73$$

and

$$k_D^2 = \epsilon_r k_0^2 - \beta^2 \quad \dots 4.74$$

It is seen that these scalar field equations bear a similar resemblance to those for the L-shaped waveguide considered in the last section. The singularity in the scalar magnetic field around the edge of the strip is present, and will be seen to have a more severe influence on the convergence of the problem as the frequency increases. Some normalisation of the scalar fields has been effected by dividing functions such as $\cos \beta_n^{(1)} x$ in equation 4.62 by its maximum value in its particular sub-region, namely $\cos \beta_n^{(1)} c$ in this example. This normalisation has

the effect of reducing the numerical range of the elements in the final matrix, causing it to be better conditioned and reducing the effects of rounding errors. This type of normalisation is absolutely justified since the scalar fields are arbitrary.

Matching the tangential electric fields as before simplifies the problem by reducing the number of unknown coefficients from twelve to five, since it is found that:

$$a_n = \bar{c}_n \quad \dots 4.75$$

$$b_n = \bar{d}_n \tan \beta_n^{(1)}(d-c) \cdot \tan \beta_n^{(1)}c \quad \dots 4.76$$

$$g_n = \bar{e}_n \quad \dots 4.77$$

$$h_n = \bar{f}_n \tan \beta_n^{(3)}(d-c) \tan \beta_n^{(3)}c \quad \dots 4.78$$

$$\tilde{e}_n = \tilde{c}_n \quad \dots 4.79$$

$$\tilde{f}_n = \tilde{c}_n \frac{\beta_n \alpha_n^{(2)}}{\omega \mu \beta_n^{(2)} \tan \beta_n^{(2)} a} \left(\frac{k_D^2}{k_A^2} - 1 \right) - \tilde{d}_n \frac{k_D^2 \beta_n^{(2)} \tan \beta_n^{(2)}(b-a)}{k_A^2 \beta_n^{(2)} \tan \beta_n^{(2)} a} \quad \dots 4.80$$

This last relationship shows that coupling between the electric and magnetic scalar fields is taking place across the dielectric interface.

Matching the fields along each boundary, the Least Square criterion takes on an exactly similar form as before, though of course each boundary residual is more complex. Each boundary residual is now considered in turn.

a) Boundary residual between sub-regions 1 and 2

Taking the longitudinal magnetic field first, the component of this boundary residual is given by

$$R_{z1-2} = \int_{-a}^b \left\{ \sum_{n=1}^N \bar{d}_n H_z I A(n) \cos \alpha_n^{(1)}(b-y) + \tilde{d}_n H_z I B(n) \cos \beta_n^{(2)}(b-y) \right\} \cdot \left\{ \sum_{m=1}^M \bar{d}_m H_z I A(m) \cos \alpha_m^{(1)}(b-y) + \tilde{d}_m H_z I B(m) \cos \beta_m^{(2)}(b-y) \right\}^* dy \quad \dots 4.81$$

where

$$HZIA(N) = \tan \beta_n^{(1)}(d-c) \tan \beta_n^{(1)}c - 1 \quad \dots 4.82$$

$$HZIB(N) = - \frac{(-1)^N}{\cos \beta_n^{(2)}(b-a)} \quad \dots 4.83$$

The transverse magnetic field which is parallel to the boundary in question is the H_y field, the residual being:

$$\begin{aligned} R_{y1-2} = & \int_a^b \left\{ \sum_{n=1}^N \bar{d}_n HYIA(N) \sin \alpha_n^{(1)}(b-y) + \tilde{d}_n HYIB(N) \sin \beta_n^{(2)}(b-y) \right. \\ & + \bar{c}_n HYIC(N) \sin \alpha_n^{(1)}(b-y) + \tilde{c}_n HYID(N) \sin \beta_n^{(2)}(b-y) \Big\} \cdot \\ & \left\{ \sum_{m=1}^N \bar{d}_m HYIA(m) \sin \alpha_m^{(1)}(b-y) + \tilde{d}_m HYIB(m) \sin \beta_m^{(2)}(b-y) \right. \\ & + \bar{c}_m HYIC(m) \sin \alpha_m^{(1)}(b-y) + \tilde{c}_m HYID(m) \sin \beta_m^{(2)}(b-y) \Big\}^* dy \quad \dots 4.84 \end{aligned}$$

where

$$HYIA(N) = \frac{j\omega\epsilon_0\beta_n^{(1)}}{k_A^2} \left(\tan \beta_n^{(1)}c - \frac{1}{\tan \beta_n^{(1)}(d-c)} \right) \quad \dots 4.85$$

$$HYIB(N) = - \frac{j\omega\epsilon_0\alpha_n^{(2)}(-1)^N}{k_A^2 \sin \beta_n^{(2)}(b-a)} \quad \dots 4.86$$

$$HYIC(N) = - \frac{j\omega\epsilon_0\alpha_n^{(1)}}{k_A^2} (\tan \beta_n^{(1)}(d-c) \tan \beta_n^{(1)}c - 1) \quad \dots 4.87$$

$$HYID(N) = \frac{j\omega\epsilon_0\beta_n^{(2)}(-1)^N}{k_A^2 \cos \beta_n^{(2)}(b-a)} \quad \dots 4.88$$

b) Boundary residual between sub-regions 3 and 4.

Considering this in a similar manner yields the following equation for the boundary residual from the H_z fields:

$$\begin{aligned} R_{z3-4} = & \int_0^a \left\{ \sum_{n=1}^N \bar{f}_n HZ3A(N) \cos \alpha_n^{(3)}y + \tilde{f}_n HZ3B(N) \cos \beta_n^{(2)}y \right. \\ & + \bar{d}_n HZ3C(N) \cos \beta_n^{(2)}y \Big\} \cdot \left\{ \sum_{m=1}^M \bar{f}_m HZ3A(m) \cos \alpha_m^{(3)}y \right. \\ & + \tilde{f}_m HZ3B(m) \cos \beta_m^{(2)}y + \bar{d}_m HZ3C(m) \cos \beta_m^{(2)}y \Big\}^* dy \quad \dots 4.89 \end{aligned}$$

where

$$H\bar{Z}3A(N) = \tan \bar{\beta}_n^{(3)}(d-c) \tan \bar{\beta}_n^{(3)}c - 1 \quad \dots 4.90$$

$$H\bar{Z}3B(N) = \frac{k_D^2 \beta_n^{(2)} (-1)^n \tan \beta_n^{(2)}(b-a)}{k_A^2 \bar{\beta}_n^{(2)} \sin \bar{\beta}_n^{(2)} a} \quad \dots 4.91$$

$$H\bar{Z}3C(N) = -\frac{\beta_n \alpha_n^{(2)} (-1)^n}{\omega \mu \bar{\beta}_n^{(2)} \sin \bar{\beta}_n^{(2)} a} \left(\frac{k_D^2}{k_A^2} - 1 \right) \quad \dots 4.92$$

Taking the H_y fields, the boundary residual is given by:

$$\begin{aligned} R_{y3-4} = & \int_0^a \left\{ \sum_{n=1}^N \tilde{c}_n H\bar{Y}3A(N) \sin \bar{\beta}_n^{(2)} y + \tilde{d}_n H\bar{Y}3B(N) \sin \bar{\beta}_n^{(2)} y \right. \\ & + \bar{e}_n H\bar{Y}3C(N) \sin \alpha_n^{(3)} y + \bar{f}_n H\bar{Y}3D(N) \sin \alpha_n^{(3)} y \left. \right\} \cdot \left\{ \sum_{m=1}^M \tilde{c}_m H\bar{Y}3A(M) \sin \bar{\beta}_m^{(2)} y \right. \\ & + \tilde{d}_m H\bar{Y}3B(M) \sin \bar{\beta}_m^{(2)} y + \bar{e}_m H\bar{Y}3C(M) \sin \alpha_m^{(3)} y + \bar{f}_m H\bar{Y}3D(M) \sin \alpha_m^{(3)} y \left. \right\}^* dy. \quad 4.93 \end{aligned}$$

where

$$H\bar{Y}3A(N) = \frac{-j\alpha_n^{(2)} (-1)^n \left(\frac{\beta^2}{k_D^2 \sin \bar{\beta}_n^{(2)} a} \left(\frac{\beta^2}{\omega \mu} \left(\frac{k_D^2}{k_A^2} - 1 \right) + \omega \epsilon_0 \epsilon_r \right) \right)}{k_D^2 \sin \bar{\beta}_n^{(2)} a} \quad \dots 4.94$$

$$H\bar{Y}3B(N) = j\frac{\beta (-1)^n \beta_n^{(2)} \tan \beta_n^{(2)}(b-a)}{k_A^2 \sin \bar{\beta}_n^{(2)} a} \quad \dots 4.95$$

$$H\bar{Y}3C(N) = j\omega \epsilon_0 \epsilon_r \bar{\beta}_n^{(3)} \left(\tan \bar{\beta}_n^{(3)}c - \frac{1}{\tan \bar{\beta}_n^{(3)}(d-c)} \right) \quad \dots 4.96$$

$$H\bar{Y}3D(N) = j\frac{\beta \alpha_n^{(3)} \left(\tan \bar{\beta}_n^{(3)}(d-c) \tan \bar{\beta}_n^{(3)}c - 1 \right)}{k_D^2} \quad \dots 4.97$$

c) Boundary residual between sub-regions 2 and 3.

The boundary residual for the H_z fields is given as follows:

$$\begin{aligned} R_{z2-3} = & \int_c^d \left\{ \sum_{n=1}^N \bar{a}_n H\bar{Z}2A(N) \cos \beta_n^{(1)}(d-x) + \tilde{a}_n H\bar{Z}2B(N) \cos \alpha_n^{(2)}(d-x) \right. \\ & + \bar{f}_n H\bar{Z}2C(N) \cos \bar{\beta}_n^{(3)}(d-x) + \bar{c}_n H\bar{Z}2D(N) \cos \alpha_n^{(2)}(d-x) \left. \right\} \cdot \\ & \left\{ \sum_{m=1}^M \bar{a}_m H\bar{Z}2A(M) \cos \beta_m^{(1)}(d-x) + \tilde{a}_m H\bar{Z}2B(M) \cos \alpha_m^{(2)}(d-x) \right. \\ & + \bar{f}_m H\bar{Z}2C(M) \cos \bar{\beta}_m^{(3)}(d-x) + \bar{c}_m H\bar{Z}2D(M) \cos \alpha_m^{(2)}(d-x) \left. \right\}^* dx \dots 4.98 \end{aligned}$$

where

$$HX2A(N) = \frac{(-1)^N}{\cos \beta_n^{(1)}(d-c)} \quad \dots 4.99$$

$$HX2B(N) = 1 + \frac{k_D^2 \beta_n^{(2)} \tan \beta_n^{(2)}(b-a)}{k_A^2 \beta_n^{(2)} \tan \beta_n^{(2)} a} \quad \dots 4.100$$

$$HX2C(N) = - \frac{(-1)^N}{\cos \beta_n^{(1)}(d-c)} \quad \dots 4.101$$

$$HX2D(N) = - \frac{\beta_n^{(2)}}{\omega \nu \beta_n^{(2)} \tan \beta_n^{(2)} a} \left(\frac{k_D^2}{k_A^2} - 1 \right) \quad \dots 4.102$$

Finally the transverse fields parallel to the boundary, the H_x fields, give the following boundary residual:

$$\begin{aligned} R_{x2-3} = & \int_c^d \left\{ \sum_{n=1}^N \bar{c}_n HX2A(N) \sin \beta_n^{(1)}(d-x) + \tilde{c}_n HX2B(N) \sin \alpha_n^{(2)}(d-x) \right. \\ & + \bar{d}_n HX2C(N) \sin \beta_n^{(1)}(d-x) + \tilde{d}_n HX2D(N) \sin \alpha_n^{(2)}(d-x) \\ & + \bar{e}_n HX2E(N) \sin \beta_n^{(3)}(d-x) + \bar{f}_n HX2F(N) \sin \beta_n^{(3)}(d-x) \Big\} \\ & + \sum_{m=1}^M \left\{ \bar{c}_m HX2A(M) \sin \beta_m^{(1)}(d-x) + \tilde{c}_m HX2B(M) \sin \alpha_m^{(2)}(d-x) \right. \\ & + \bar{d}_m HX2C(M) \sin \beta_m^{(1)}(d-x) + \tilde{d}_m HX2D(M) \sin \alpha_m^{(2)}(d-x) \\ & + \bar{e}_m HX2E(M) \sin \beta_m^{(3)}(d-x) + \bar{f}_m HX2F(M) \sin \beta_m^{(3)}(d-x) \Big\} \text{doc.} \quad 4.103 \end{aligned}$$

where

$$HX2A(N) = - \frac{j \omega \epsilon_0 \alpha_n^{(1)} (-1)^N}{k_A^2 \sin \beta_n^{(1)}(d-c)} \quad \dots 4.104$$

$$\begin{aligned} HX2B(N) = & j \left(\frac{\beta_n^{(2)} \alpha_n^{(2)2} (k_D^2/k_A^2 - 1)}{k_D^2 \omega \nu \beta_n^{(2)} \tan \beta_n^{(2)} a} \right. \\ & \left. - \omega \epsilon_0 \left(\frac{\beta_n^{(2)}}{k_A^2 \tan \beta_n^{(2)}(b-a)} + \frac{\epsilon_r \beta_n^{(2)}}{k_D^2 \tan \beta_n^{(2)} a} \right) \right) \dots 4.105 \end{aligned}$$

$$HX2C(N) = - \frac{j \beta_n^{(1)} (-1)^N}{k_A^2 \cos \beta_n^{(1)}(d-c)} \quad \dots 4.106$$

$$HX2D(N) = - j \frac{\beta_n^{(2)} \alpha_n^{(2)}}{k_A^2} \left(1 + \frac{\beta_n^{(2)} \tan \beta_n^{(2)}(b-a)}{\beta_n^{(2)} \tan \beta_n^{(2)} a} \right) \quad \dots 4.107$$

$$HX2E(N) = - \frac{j \omega \epsilon_0 \epsilon_r \alpha_n^{(3)} (-1)^N}{k_D^2 \sin \beta_n^{(3)}(d-c)} \quad \dots 4.108$$

$$HX2F(N) = \frac{j\beta_n \overline{\beta_n^{(3)}} (-1)^n}{k_D^2 \cos \beta_n^{(3)} (d-c)} \quad \dots 4.109$$

The equations that give the total boundary residuals in terms of the matrix coefficients are not included in this section, as they have been in section 4.2, since, because of their increased number, their inclusion would tend to distract from the main purpose of this section. For completeness these equations are given in Appendix 3.

Although one of the main attributes of the Least Square error minimisation technique is guaranteed convergence, it has been shown by Davis (72) and LaRiviere(67) that the use of scaling factors can speed up the rate of convergence. Davis used a scaling factor which provided greater weighting to the electric or magnetic walls in the scattering problem, whereas LaRiviere employed scaling factors to provide weighting between the various longitudinal and transverse components of each boundary residual.

It was not felt that a detailed study of the effects of scaling factors on this particular problem would, in the time available, add much to the extension of the method to more complex structures, such as coupled microstrip or the microstrip/slot combination. Thus, in order to give some comparison with previous work by others, a preliminary choice of scaling factors was made such that the various components of each boundary residual were similar in magnitude for small matrix orders. The selection of scaling factors using small matrices was used by LaRiviere, though he also optimised them for maximum rate of convergence. Similar dimensions and permittivities to those previously employed by others (17, 65, 67) were used, though accurate comparison was only possible with LaRiviere's results for Fourier Matching, the other results being displayed graphically.

The results obtained using the same routines as for the L-shaped waveguide are compared with LaRiviere's results

for Fourier Matching in Table 4.6. The differences between results presented here and his results are expressed as a percentage in an adjacent column, and shown graphically in Figure 4.7.

The results show that the convergence is not continuous, but appears to oscillate. It would appear, particularly from Figure 4.6, that certain numbers of expansion terms satisfy the problem better than others. For example, when $n = 3$, the solution is less stable with frequency than it is for $n = 2$ or 4. Another observation of relevance is that more expansion terms are necessary to obtain solutions of similar accuracy as the frequency increases. The question of the absolute numerical value of the eigenvalue was discussed in section 4.2. The eigenvalues of interest with this problem show that as more expansion terms are taken, further scaling of the matrix is taking place. This seems to be the case since the minimum value of the eigenvalues, for an increasing number of expansion terms, does not steadily decrease, but appear to be unrelated to each other. The eigenvectors are extremely useful, because of this fact, in enabling the loci of the eigenvalues to be plotted with frequency. Without them, interpretation of the eigenvalues would be almost impossible, particularly for the solutions less stable with frequency.

Tables 4.7 and 4.8 give the eigenvectors for the solutions for two different matrix sizes. The complete set of unknown coefficients in equations 4.62 to 4.69 can be found using the relationships given in equations 4.75 to 4.80. This has been done for one set of values to show the eigenfunction. A computer program, which is not described here, used these coefficients to compute the electric and magnetic scalar potentials over a number of points within the microstrip cross-section. After normalisation they were plotted, as shown in Figure 4.8, using a routine called SYMVU, which gives a perspective view of a surface. The program was executed on the University of London Computing Centre's CDC7600 computer, which has the microfilm plotting routine SYMVU on permanent file. In Figure 4.8, the ground plane is along the left hand surface; the rear and right hand edges signify the walls of the box, and the front surface represents

β	LaRiviere's results	This Method					
	100x100 Matrix	12x12 Matrix		18x18 Matrix		24x24 Matrix	
	(GHz)	GHz	%	GHz	%	GHz	%
100	1.943	1.89	-2.70	1.94	-0.16	1.93	-0.67
150	2.898	2.83	-2.34	2.94	+1.45	2.88	-0.41
200	3.839	3.73	-2.89	3.95	+2.90	3.83	-0.234
250	4.766	4.62	-3.07	5.02	+5.35	4.76	-0.126
300	5.680	5.52	-2.82	6.07	+6.68	5.69	+0.568
350	6.584	6.38	-3.10	7.10	+7.84	6.60	+0.24
400	7.479	7.25	-3.05	-	-	7.52	+0.79
450	8.364	8.10	-3.16	-	-	8.43	+0.79
500	9.242	8.92	-3.50	-	-	9.70	+4.96
550	10.112	-	-	-	-	-	-
600	10.976	10.62	-3.54	-	-	-	-

Table 4.6

The dimensions in Figure 4.6 for the above results are:

$$a = 1.27 \text{ mm.}$$

$$b = 12.7 \text{ mm.}$$

$$c = 0.635 \text{ mm.}$$

$$d = 6.35 \text{ mm.}$$

and the relative permittivity, $\epsilon_r = 8.875$.

% DIFFERENCE WITH 100x100 MATRIX
FOR FOURIER MATCHING (67)

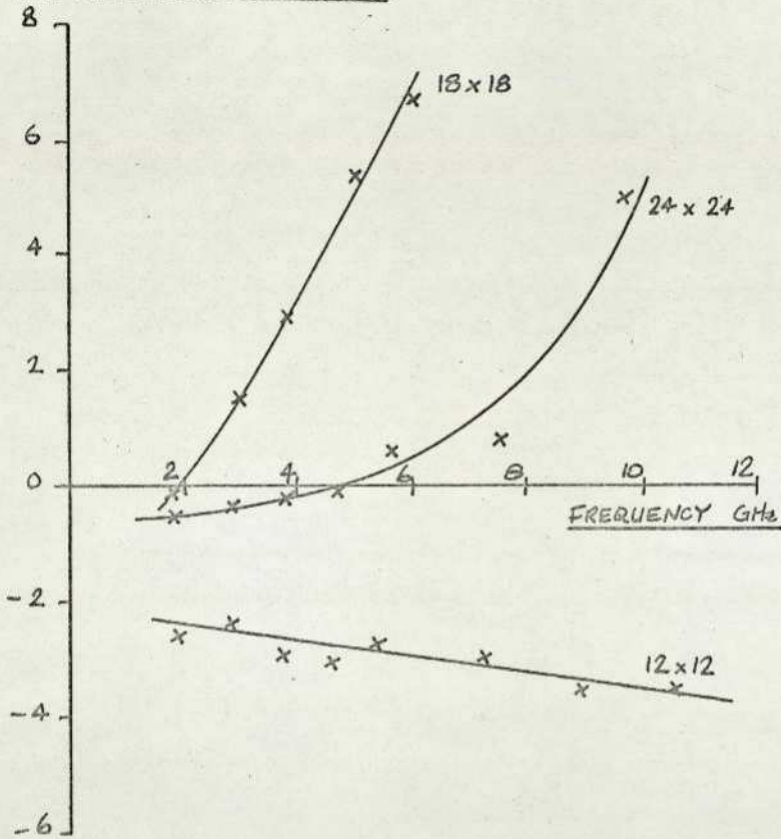


Figure 4.7 Convergence of the Microstrip Solution.

Beta	$\bar{c}$	$\tilde{c}$	$\bar{d}$	$\tilde{d}$	$\bar{e}$	$\bar{f}$
150	1.5 -05	6.7 -03	-1.9 -01	1.0 00	-1.2 -03	-3.6 -02
	-1.8 -05	8.9 -03	9.9 -02	3.6 -01	1.7 -03	7.8 -03
200	4.3 -05	6.1 -03	-1.9 -01	1.0 00	-1.1 -03	-3.9 -02
	-4.1 -05	9.4 -03	9.9 -02	3.6 -01	1.3 -03	7.8 -03
250	5.8 -05	6.8 -03	-1.9 -01	1.0 00	-9.8 -04	-4.3 -02
	-4.0 -05	1.0 -02	9.9 -02	3.7 -01	1.1 -03	8.3 -03
300	5.9 -05	8.5 -03	-1.9 -01	1.0 00	-9.2 -04	-5.2 -02
	-2.4 -05	1.1 -02	9.7 -02	3.8 -01	9.2 -04	8.7 -03
350	4.7 -05	1.1 -02	-1.8 -01	1.0 00	-8.6 -04	-6.0 -02
	1.7 -06	1.3 -02	9.2 -02	3.9 -01	7.6 -04	9.0 -03
400	1.1 -04	6.2 -03	-2.2 -01	1.0 00	-8.2 -04	-5.8 -02
	-4.8 -05	9.3 -03	9.7 -02	3.9 -01	8.5 -04	1.0 -02
450	7.2 -05	1.0 -02	-1.9 -01	1.0 00	-7.7 -04	-6.7 -02
	-9.1 -06	1.2 -02	8.8 -02	4.1 -01	6.8 -04	9.9 -03
500	1.0 -04	7.3 -03	-2.1 -01	1.0 00	-7.6 -04	-6.7 -02
	-3.9 -05	9.9 -03	9.1 -02	4.1 -01	7.4 -04	1.1 -02

Table 4.7

Matrix size: 12 x 12

Beta	$\bar{c}$	$\bar{c}'$	$\bar{d}$	$\bar{d}'$	$\bar{e}$	$\bar{f}$
150	-5.2 -04	2.1 -02	-1.3 -01	1.0 00	-2.2 -03	-7.4 -02
	3.0 -04	2.4 -02	1.1 -01	-3.0 -01	3.1 -03	2.0 -02
	-2.6 -04	4.2 -02	-6.3 -02	-8.4 -01	-3.6 -03	-9.0 -03
	2.6 -04	4.7 -02	3.9 -02	-2.5 -01	-8.7 -04	-2.4 -03
200	3.5 -04	-1.3 -02	9.6 -02	-8.4 -01	1.3 -03	6.1 -02
	-1.2 -04	-2.3 -02	-1.1 -01	5.5 -01	-1.8 -03	-1.7 -02
	8.7 -05	-3.9 -02	6.4 -02	1.0 00	1.5 -03	7.1 -03
	-1.0 -04	-3.4 -02	-4.0 -02	2.8 -01	3.6 -03	2.1 -03
250	-3.3 -04	2.3 -02	-5.7 -02	1.0 00	-1.1 -03	-8.9 -02
	2.2 -04	2.8 -02	8.6 -02	-2.0 -01	1.5 -03	2.6 -02
	-2.1 -04	4.7 -02	-5.2 -02	-8.8 -01	-2.7 -04	-8.6 -03
	2.2 -04	5.5 -02	-3.2 -02	-2.8 -01	-7.0 -03	-3.0 -03
300	2.6 -04	-2.1 -02	4.0 -02	-9.5 -01	8.6 -04	8.6 -02
	-1.6 -04	-2.8 -02	-8.3 -02	3.1 -01	-1.1 -03	-2.6 -02
	1.6 -04	-4.7 -02	5.1 -02	1.0 00	2.8 -05	7.9 -03
	-1.8 -04	-5.2 -02	-3.2 -02	3.2 -01	6.1 -03	2.9 -03
350	2.0 -04	-1.8 -02	1.7 -02	-8.5 -01	6.3 -04	8.0 -02
	-1.3 -04	-2.6 -02	-6.8 -02	3.2 -01	-8.3 -04	-2.5 -02
	1.3 -04	-4.4 -02	4.4 -02	1.0 00	-1.2 -04	6.7 -03
	-1.5 -04	-4.7 -02	-2.8 -02	3.2 -01	4.9 -03	2.7 -03
400	1.7 -04	-1.7 -02	-1.3 -02	-7.8 -01	4.9 -04	7.7 -02
	-1.2 -04	-2.5 -02	-4.9 -02	2.8 -01	-6.4 -04	-2.4 -02
	1.3 -04	-4.2 -02	3.3 -02	1.0 00	-2.2 -04	5.5 -03
	-1.4 -04	-4.6 -02	-2.2 -02	3.3 -01	4.2 -03	2.5 -03
450	1.6 -04	-1.7 -02	-5.8 -02	-7.1 -01	3.8 -04	7.0 -02
	-1.3 -04	-2.4 -02	-2.3 -02	2.1 -01	-4.8 -04	-2.2 -02
	1.4 -04	-4.1 -02	1.8 -02	1.0 00	-2.9 -04	3.8 -03
	-1.5 -04	-4.6 -02	-1.3 -02	3.5 -01	3.5 -03	2.1 -03
500	3.5 -04	-2.8 -02	-3.9 -01	-4.5 -01	3.3 -05	-4.1 -02
	-3.2 -04	-3.5 -02	1.4 -01	-5.3 -01	-5.4 -05	4.8 -03
	3.2 -04	-5.2 -02	-7.6 -02	1.0 00	-2.3 -04	-2.0 -03
	-3.0 -04	-8.2 -02	4.5 -02	5.2 -01	1.7 -03	-2.8 -04

Table 4.8

Matrix size: 24 x 24

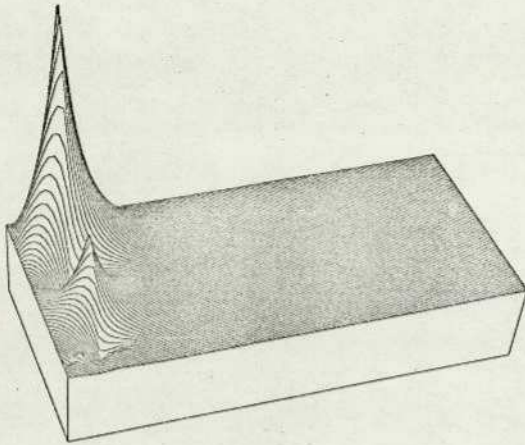


Figure 4.8a

SCALAR ELECTRIC POTENTIAL FOR A MICROSTRIP LINE ER=8.875 N=4 BETA=150
 *AZIMUTH = 108 *ALTITUDE = 45
 *WIDTH = 3.00 *HEIGHT = 5.00
 * BEFORE FORESHORTENING 10/04/74

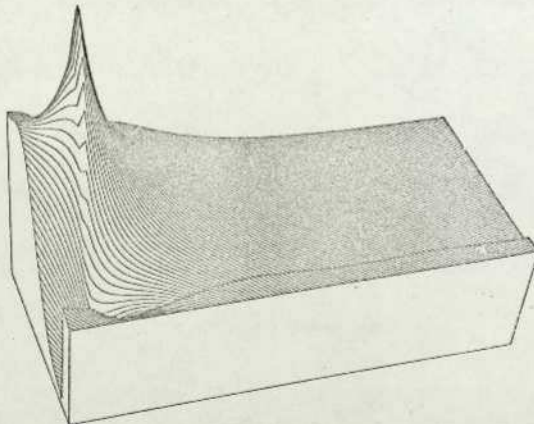


Figure 4.8b

SCALAR MAGNETIC POTENTIAL FOR A MICROSTRIP LINE ER=8.875 N=4 BETA=150
 *AZIMUTH = 108 *ALTITUDE = 45
 *WIDTH = 3.00 *HEIGHT = 5.00
 * BEFORE FORESHORTENING 10/04/74

the magnetic wall along the line of symmetry; the position of the strip is parallel to the ground plane and is approximately 1 cm from the front left hand corner, extending $\frac{1}{2}$ cm inwards.

CHAPTER 5APPLICATION OF MICROSTRIP/SLOT LINE PARAMETERS TO A PRACTICAL
DIRECTIONAL COUPLER DESIGN5.1 Introduction

At this point in the thesis, the various results obtained will be unified to allow a fuller understanding of the operation of a particular form of proximity directional coupler. One alternative to a coupled microstrip line proximity directional coupler is that which uses a microstrip/slot line combination, first published by de Ronde(11). No references to the characteristics for the dominant modes of this structure were found, therefore the results contained within this thesis were used for the practical design of a coupler. This alternative was chosen since although it had been shown that the even- and odd-modes of coupled microstrip lines could be made to be degenerate, this was technologically difficult. The results in Chapter 2 showed degeneracy was practically very easy to achieve for the microstrip/slot line combination.

Chapter 2 is devoted to the measurement of normalised guide wavelength, which has been carried out for both modes of the microstrip/slot line combination with several forms of dielectric loading of the slot. The results for the microstrip-mode were as expected, being close to those for a single microstrip line when the slot width is small compared to the width of the top line. The results for the slot-modes were of a similar overall shape to those obtained theoretically by Cohn (32) for the more conventional slot-mode, but had a much lower overall normalised guide wavelength. The very non-linear variation of normalised guide wavelength for various slot widths was rather unexpected, when loaded with 0.025 inch thick zirconate material and with the top line width kept constant.

Chapter 3 introduces a new method of measurement of characteristic impedance, and supports, with practical measurements,

recent claims that the impedance of single microstrip lines generally increases with frequency, if only slightly at the lower microwave frequencies. This result, which is of interest in all applications of microstrip, can allow one to design wideband structures with more certainty of their practical performance. Because of the similarity between the dominant mode of a single microstrip line and the microstrip/slot line combination, it is reasonable to assume that the frequency dependence of their characteristic impedances will be similar. The characteristic impedance of the microstrip-mode has been measured in comparison to the impedance of a single microstrip line by Time Domain Reflectometry. The slot line impedance was not measured by any of the methods discussed in Chapter 3, since the structure supports a dominant mode which is TE, and cannot be connected directly to a transmission line supporting a TEM mode without a mode converter of some type. For most of the measurement techniques, as was explained in Chapter 3, impedance is normally measured by relating the unknown line to a known line. Measurement with a mode converter would still not give a unique result since it would depend on parameters of the converter which are in turn dependent upon the definition of impedance. The method used to characterise the slot-mode impedance will be discussed later.

5.2 Directional Coupler Design Fundamentals.

In the preceeding Chapters emphasis has been placed upon the properties of the transmission line which supports the two dominant modes. For the microstrip/slot combination, it has been shown that one of the basic requirements for a good directional coupler, namely mode degeneracy, can be achieved with a 0.025 inch thick zirconate substrate placed over the slot line. In addition to knowing the normalised guide wavelength of each mode, the characteristic impedance of the microstrip-mode is also known. This mode has even symmetry about the transmission line. The only unknown parameter of the transmission line structure is the slot-mode characteristic impedance. This mode has odd symmetry about the transmission line, and its characteristic impedance

is equal to twice the odd-mode impedance, Z_{oo} , of the transmission line, when the normal mode analysis of section 1.3 is considered. With knowledge of this mode impedance, the directional coupler can be modelled for the particular characteristic desired.

The practical realisation of a directional coupler, so that it performs in a similar manner to the ideal coupler of the theoretical analysis, is of major importance. It is easy in a theoretical analysis of a proximity directional coupler to imagine four ports completely isolated from each other. Each of these ports is connected, via a matched line which is not coupled to any other part of the circuit, to a point on the transmission line structure which causes the coupling to take place. Consider the standard microstrip proximity directional coupler. Figure 5.1 shows a typical coupler where the adjacent ports are at 90 degrees

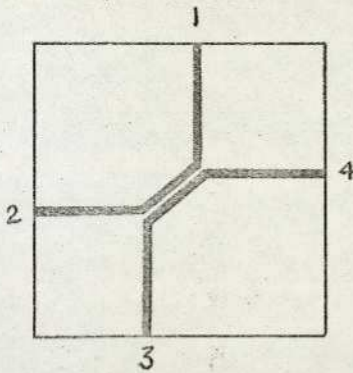


Figure 5.1

to each other. The field pattern set up in the coupled transmission line region can be regarded as the superposition of an even- and odd-mode. Thus, for an incident wave at a port, its energy will be split between the two modes depending upon their apparent characteristic impedances. If two in-phase waves are applied to adjacent ports, 1 and 4, it is easily seen that the

even-mode will be excited, and conversely the odd-mode will be excited with a 180 degree phase difference at two adjacent ports. In all practical microstrip proximity directional couplers it is difficult to see how weak coupling can be avoided between the input lines to the directional coupler. The angle subtended by adjacent

input lines will not only influence the degree of this coupling, but will also affect the energy split between the two modes in the coupler. The reason for this being that an abrupt change of field distribution takes place at the point where the input lines meet the coupler. The way in which the input lines join the coupler seems to be a matter of preference, since couplers are made with many different input line configurations. It is possible that the previous lack of accurate knowledge of the transmission line parameters, in the coupled section, has meant that too many unknowns existed for the effects of the input and output lines to be analysed.

By viewing the microstrip proximity directional coupler in a rather different manner, the similarity with the microstrip/slot line coupler will be much easier to see. Imagine a wave on one of the input lines of the coupler. When the wave reaches the coupled section its energy will be redistributed, since there is a number of ways in which it can propagate. These are simply, the two modes of the coupled regions, a return wave on the input line, a wave propagating away from the coupled region in the output line adjacent to the input line, and possibly some radiation. The energy in the even- and odd-modes propagates along the coupled section until it reaches the end where the two lines from the remaining ports join the coupled lines. The energy in each mode can be radiated, reflected back along the coupled section, or if good symmetry exists, be divided equally between the two output lines. Studying this last possibility in more detail it can be seen that the waves in the output lines excited by the even-mode will be in-phase with each other, whereas the waves excited by the odd-modes will bear a 180 degree phase difference. Thus, the resultant waves in each of the output ports will be different depending upon how the separate components interfere. By suitable design, the unwanted reflections and radiations can be reduced, and the even- and odd-mode impedances chosen to provide the required split of energy between the three output ports.

The directional coupler made from the microstrip/slot line combination, achieves its desired effect in a very similar

manner to that just described for the microstrip proximity coupler. The basic physical structure is shown in Figure 5.2. The input wave propagates along microstrip line 1, towards the microstrip/slot line combination, where its energy is divided. The energy in

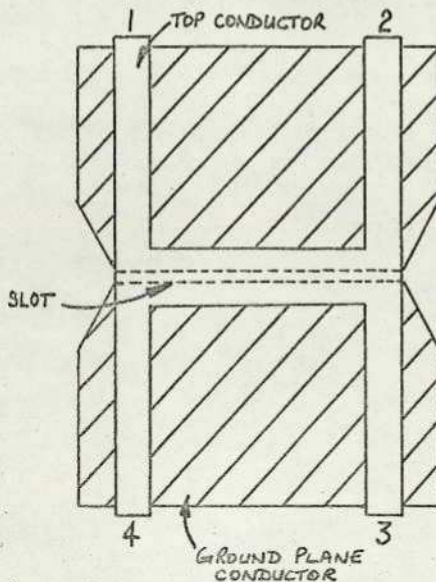


Figure 5.2

the input wave is shared by the two dominant modes supported in the microstrip/slot line combination, a reflected wave directed outward from the coupler in line 4 and a radiated wave from the open-circuited end of the ground plane slot. The energy propagating along the microstrip/slot line combination is further divided between a reflected wave which travels

back along the microstrip/slot line combination, waves which travel away from the coupler in lines 2 and 3, and some radiation from the open-circuited ends of the ground plane slot. As before, interference occurs between the waves formed in both lines 2 and 3. Energy from the microstrip-mode is split equally into lines 2 and 3, the waves being in-phase with each other due to the mode's even symmetry about the line. The energy from the slot-mode also splits equally between lines 2 and 3, but since the mode has odd symmetry about the slot, there will be a 180 degree phase difference between the waves in each line. This means that complete cancellation of the waves can occur in one line giving rise to the device's directional properties.

In this simplified view of the coupler a lot of detail has been ignored, but together with the more rigorous approach, which is given in a subsequent section of this Chapter, a better

understanding of the coupler should be possible. With this type of coupler the way in which the microstrip lines join the microstrip/slot line combination, and the effectiveness of the open-circuit at each end of the ground plane slot, will affect the performance of the coupler. Both published couplers of this form (11, 12) have had adjacent input lines at approximately 90 degrees to one another. However, the configuration of Figure 5.2 was used in most practical experiments, since it was felt that a good open-circuit in the ground plane slot at the junction of the two input lines would be achieved with this arrangement.

5.3 Theoretical Model of the Microstrip/Slot Line Coupler.

The modelling of a directional coupler using transfer matrices has already been covered in Chapter 1, section 1.4. In this section the technique will be applied to the microstrip/slot line coupler where the input lines radiation from the open-circuit, and the discontinuity at the junction of the coaxial to microstrip connector, will also be considered. Figure 5.3 is a photograph of an actual microstrip/slot line coupler. It can be seen that two lines of symmetry are present, though for the purposes of the even- and odd-mode analysis, the line of symmetry considered will be that which is parallel to the microstrip/slot line. Applying even-mode excitation, the structure reduces to a two port network given by the equivalent circuit of Figure 5.4. Each section of the equivalent circuit

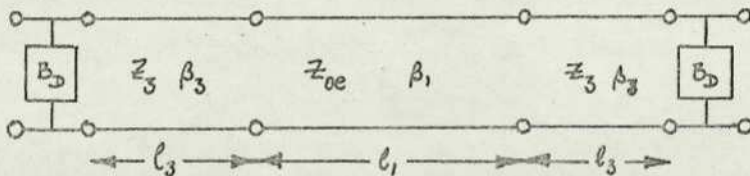


Figure 5.4

has a physical significance. At the left of Figure 5.4 there is a shunt susceptance B_D which represents the junction discontinuity

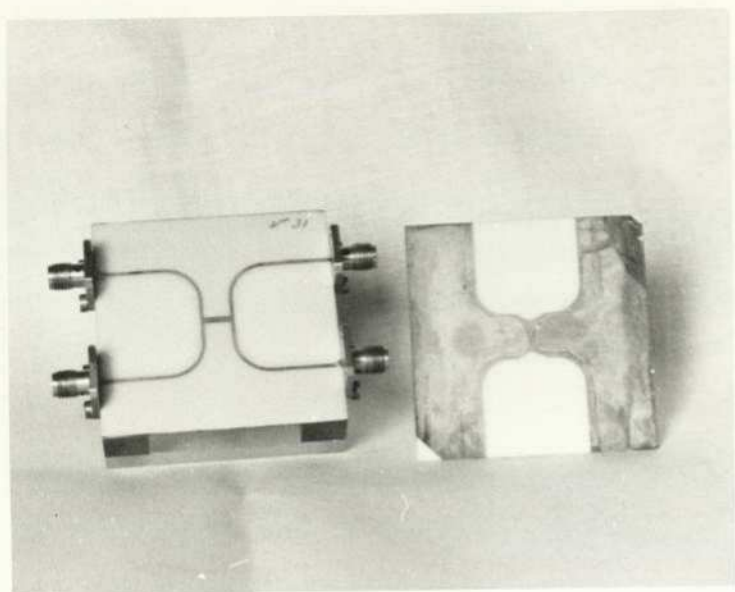


Figure 5.3 A Microstrip/Slot Line Directional Coupler.

at the coaxial to microstrip interface. This is followed by a line of length l_3 , phase constant β_3 and characteristic impedance Z_3 , which represents the microstrip line that connects to the microstrip/slot line combination. Under even-mode excitation the microstrip/slot line combination is represented by a line of length l_1 which has a phase constant β_1 and a characteristic impedance, Z_{oe} , which is the even-mode impedance of the microstrip/slot line combination. Finally, there is another length of microstrip line and a junction discontinuity similar to those described previously. The transfer matrix of the complete equivalent circuit is the product of the individual transfer matrices for each section. The transfer matrix for the junction discontinuity is $[B]$ given by

$$[B] = \begin{bmatrix} 1 & 0 \\ j\beta_3 f & 1 \end{bmatrix} \quad \dots 5.1$$

The microstrip line is given by $[C]$:

$$[C] = \begin{bmatrix} \cos \beta_3 l_3 & j \sin \beta_3 l_3 \\ j \frac{1}{Z_3} \sin \beta_3 l_3 & \cos \beta_3 l_3 \end{bmatrix} \quad \dots 5.2$$

and the microstrip/slot line combination, under even-mode excitation by $[D]$:

$$[D] = \begin{bmatrix} \cos \beta_1 l_1 & j Z_{oe} \sin \beta_1 l_1 \\ j \frac{1}{Z_{oe}} \sin \beta_1 l_1 & \cos \beta_1 l_1 \end{bmatrix} \quad \dots 5.3$$

Thus the transfer matrix for the complete coupler under even-mode excitation $[A]_e$ is:

$$[A]_e = [B][C][D][C][B] \quad \dots 5.4$$

The same coupler will now be considered under odd-mode excitation. Figure 5.5 shows the equivalent circuit. It can be immediately noticed that there are two extra transmission line sections which

have not been given a dimension of length. These zero length,

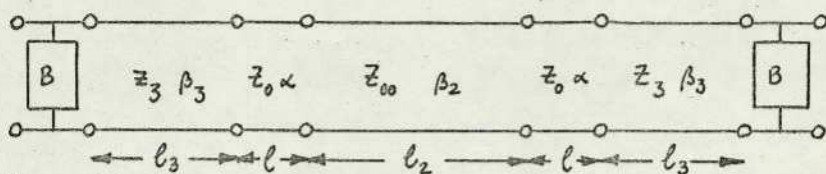


Figure 5.5

zero phase constant sections with characteristic impedance Z_0 , only contribute attenuation in their transfer function, which is given by [E]:

$$[E] = \begin{bmatrix} \cosh \alpha l & \sinh \alpha l \\ \sinh \alpha l & \cosh \alpha l \end{bmatrix} \quad \dots 5.5$$

where the product αl is a number in nepers which represents the attenuation (see Chapter 3, section 3.2.3). These sections of attenuation merely serve to simulate the energy which is radiated from the open-circuit at each end of the ground plane slot. Since the field in the slot is only excited under odd-mode excitation the lossy section was not included in the even-mode equivalent circuit. The other difference from the even-mode equivalent circuit is in the representation of the microstrip/slot line. As expected its characteristic impedance is different, being given by the odd-mode impedance, Z_{00} . The length, l_2 , and phase constant, β_2 are also different to account for all possibilities in the coupler's performance. Thus, the transfer function for this section is given by [F]:

$$[F] = \begin{bmatrix} \cos \beta_2 l_2 & j Z_{00} \sin \beta_2 l_2 \\ j \frac{1}{Z_{00}} \sin \beta_2 l_2 & \cos \beta_2 l_2 \end{bmatrix} \quad \dots 5.6$$

and the transfer matrix for the complete coupler under odd-mode excitation, $[A]_o$ is

$$[A]_o = [B][C][E][F][E][C][B] \quad \dots 5.7$$

Equations 5.4 and 5.7 can be used directly to form the reflection and transmission coefficients for each excitation, from which the signal at each port can be found. This procedure has already been discussed in Chapter 1, section 1.4.

5.4 Determination of the Slot-Mode Impedance.

This section has been included as part of this Chapter, rather than in Chapter 3, since the slot-mode impedance is determined from the performance of the coupler in which it is used. It is not in the strict sense a measurement of the characteristic impedance of the slot-mode of a microstrip/slot line combination since the configuration of the coupler will have some effect.

The analysis developed in the previous section was used to model the performance of a number of couplers similar to that shown in Figure 5.3. The couplers were made with microstrip/slot line combinations close to those measured in Chapter 2. In this way the phase constants β_1 and β_2 in equations 5.3 and 5.6 will be known as a function of frequency. In the computer program, which calculates the performance of the coupler, (see Appendix 2, program G046) the two phase constants are calculated in different ways. β_1 , the even-mode phase constant, is computed from the zero frequency normalised guide wavelength, by assuming that it changes linearly with frequency at a rate of -0.0014 per GHz. The normalised guide wavelength for the slot-mode is very non-linear, so first the guide wavelength is found by interpolating between successive values taken at one gigahertz intervals. From this, β_2 , the phase constant for the odd-mode, is calculated.

The input lines to the coupler were made to be approximately 50 ohms. The 3 mm. coaxial microstrip converters were assumed to exhibit a capacitive discontinuity. This assumption, by virtue of its use in the measurements of Chapter 3, has already been shown to be a good approximation at a junction between a 50 ohm 3 mm. coaxial and a 50 ohm microstrip line. The value of the normalised susceptance, B_D , is given in terms of only the VSWR

for the junction, by equation 3.38 in Chapter 3. For this computation the VSWR, due to the discontinuity, was 1.0125 at 1 GHz. Thus matrix $[B]$ of equation 5.1 can be computed, where f is the frequency in gigahertz.

The length and width of the lines leading from each port to the coupler were measured. The ratio W/H was used to determine their characteristic impedance and phase constant, from the results of Bryant and Weiss (19). These items of information are sufficient to allow matrix C of equation 5.2 to be computed, though assumptions are made that all of the four lines have similar length and impedance. Other assumptions are that the impedance of each line is constant over its entire length and that its length is the same for both excitations of the coupler.

Matrices $[D]$ and $[F]$ of equations 5.3 and 5.6, can be formed from the knowledge of the microstrip- and slot-mode parameters gained in previous Chapters. It has already been discussed how β_1 and β_2 are derived. The impedance of the microstrip-mode is taken from the curves shown in Figure 3.17 of Chapter 3. The even-mode impedance, Z_{oe} used in the calculations, is equal to twice the impedance of the microstrip-mode. The slot-mode impedance, which is unknown, is equal to twice the odd-mode impedance, Z_{oo} . Various trial values of Z_{oo} are used in the computation, the value which gives the best approximation to the coupler's measured characteristics being deemed the correct value for Z_{oo} . The lengths l_1 and l_2 are derived from measurements, however, they are somewhat arbitrary since an estimation has to be made where each mode effectively terminates.

The value of αl in the matrix for the lossy section, equation 5.5, was determined empirically. Measurements of the signals from each port, and the reflected signal of the input port, were summed. Any power not accounted for was assumed to be radiated. Trial values of αl were used to find that value which gave the same percentage of power loss in the two cases.

The results for the even- and odd-mode impedances are

shown in Figure 5.6. It should be stressed that these results are only approximate, but are sufficiently accurate to allow the initial design of couplers. The results for the even-mode impedance were taken from the microstrip-mode impedances given previously (Figure 3.17, Chapter 3). The odd-mode impedances were the impedance values which gave the best agreement between practical measurements and theoretical predictions, for five test couplers. The photograph, Figure 5.3, of two of the couplers, shows the top of one coupler, and the ground plane of another. The photograph shows them rotated through 90 degrees with respect to each other. The ground plane of the input microstrip lines was cut away on one side, so that it extended 1.27 mm (approximately two line widths) from the edge of the top line. This was so that a large area of the ground plane could be cut away, to make a good wideband open-circuit at the end of the slot-line. Tests were carried out on the Time Domain Reflectometer which indicated that the ground plane modifications had little effect on the characteristic impedance of the microstrip line. The end of the slot was directly underneath the junction of the two microstrip input lines and the microstrip line of the coupler. To achieve this, it can be seen that it was necessary to cut the ground plane away in the form of a 'v' at each end of the slot line. The five couplers used to determine the odd-mode impedance of the coupler, were all identical apart from the widths of the microstrip/slot line combination.

Tests were carried out on the coupler with the microstrip/slot line absent. They showed that the power coupled to the isolated port and direct port was -35 dB and -40 dB respectively. This amount of coupling would limit the power from the isolated port to -35 dB. In the practical cases measured, it was seen that the power from the isolated port was generally much larger than -35 dB, the stray coupling between the input lines having little effect.

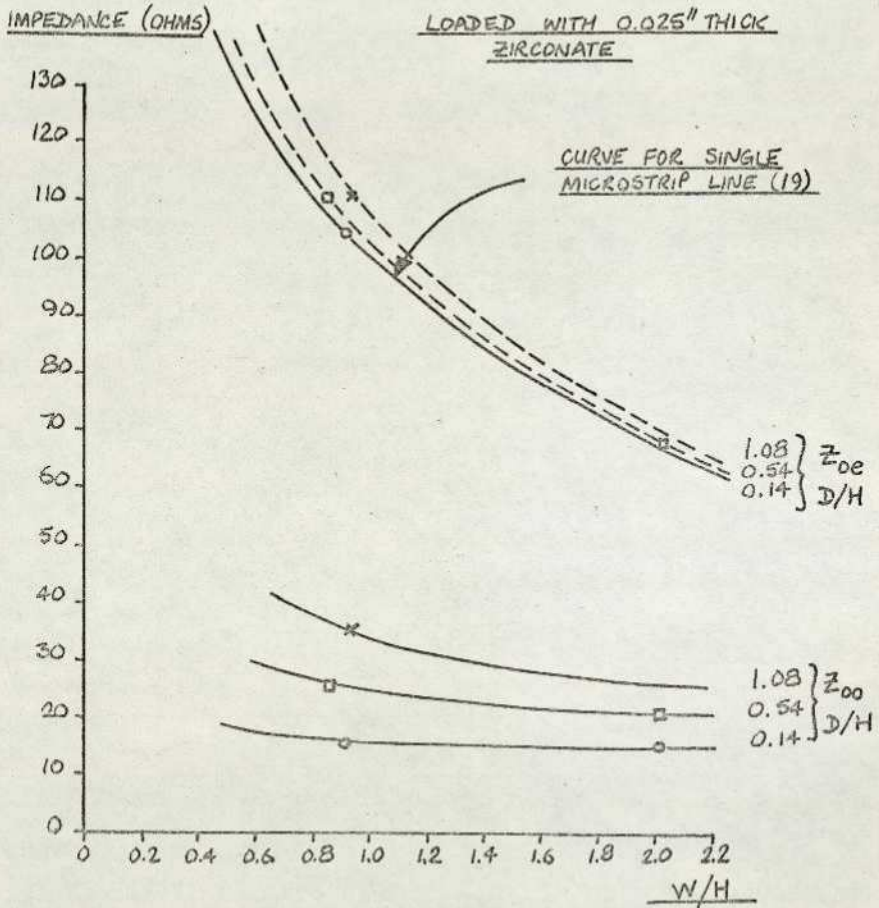


Figure 5.6 Values obtained Practically for the Even- and Odd-Mode Impedances of the Microstrip/Slot Line Combination, between 3 and 6 GHz.

5.5 A Design for a 3 dB Proximity Coupler.

With the last of the parameters of the microstrip/slot line combination known, a design for a proximity coupler can be made. To test the validity of the empirical curves for the even- and odd-mode impedances given in Figure 5.6, a 3 dB coupler was designed. For this particular choice of coupling, the input power is split equally between the coupled and direct ports. Equations 1.32 and 1.33 relating the even- and odd-mode impedances to the coupling coefficient can be expressed in terms of the power at the coupled port, in decibels:

$$Z_{oe} = 50 \left\{ \left(1 + 10^{\text{dB}/20} \right) \left(1 - 10^{\text{dB}/20} \right) \right\}^{1/2} \quad \dots 5.8$$

$$Z_{oo} = 50 \left\{ \left(1 - 10^{\text{dB}/20} \right) \left(1 + 10^{\text{dB}/20} \right) \right\}^{1/2} \quad \dots 5.9$$

Where dB represents the coupling required in decibels, in this case dB = -3.0. For a 3 dB coupler, it is found that $Z_{oe} = 123$ ohms and $Z_{oo} = 20.7$ ohms. From the characteristics given in Figure 5.6, the unique values for W/H and D/H are found to be approximately 0.66 and 0.2 respectively. Choosing a similar coupler design to those previously tested, a coupler was fabricated with measured W/H = 0.655 and D/H = 0.191. Its performance, given in Figure 5.7, shows the outputs of the coupled and direct ports to be approximately equal over the frequency range 4.0 to 5.0 GHz, with a minimum directivity of 17 dB. The input VSWR, over nearly an octave bandwidth about this centre frequency, being better than 1.5.

The computer program that calculates the response of the model coupler, was used as a theoretical check of this coupler. In the setting up of the program, the following parameters were used:

Length of input lines	= 3.0 cm.
Length of microstrip line	= 6.0 mm.
Length of slot line	= 7.3 mm.
Z_3	= 53 ohms.
Z_{oo}	= 20 ohms.
Z_{oe}	= 120 ohms.
Junction VSWR	= 1.0125
α_1	= 0.2 nepers

The performance obtained with these parameters is shown in Figure 5.8. The power in the direct and coupled ports are very accurately predicted, that in the isolated port being somewhat worse, since the rippling is not in-phase in the two cases. In the evaluation of the slot line impedance it was necessary to match a theoretical solution to the practical measurements. In most cases, a good match could be achieved, most deviation between the two results occurring in the matching of the two rippling functions in the isolated port. It could be seen from the use of the computer program for the model, that this rippling was sensitive to changes in the input lines and junction discontinuity in addition to changes in the microstrip/slot line combination.

In defining the model, an approximation was made in which complete symmetry between the four ports was assumed to exist. It seems that in the practical case bad alignment of the mask in the photolithographic process would cause the input line length to be dissimilar, and etching tolerances would cause them to have different and non-uniform impedances. It was decided for these reasons to accept the differences between the responses of the isolated ports in the two cases, when they were closely approximated, but rippled slightly out of phase with each other.

Removing the junction discontinuity from the theoretical model, by setting the junction VSWR to unity, affected the response of the isolated port by lowering its power by about one decibel. The other ports were affected by approximately two orders of magnitude less than the isolated port. Removing the mismatched input lines by changing their input impedances from 53 ohms to 50 ohms, imposes a much greater change on the response of the isolated port, the other ports again being hardly affected. The response of the model under these conditions is shown in Figure 5.9. It is seen that the rippling in the isolated port is completely removed, the directivity now being greater than 20 dB. Removal of the rippling also reveals certain design faults, which will be discussed in the next Chapter. It shows

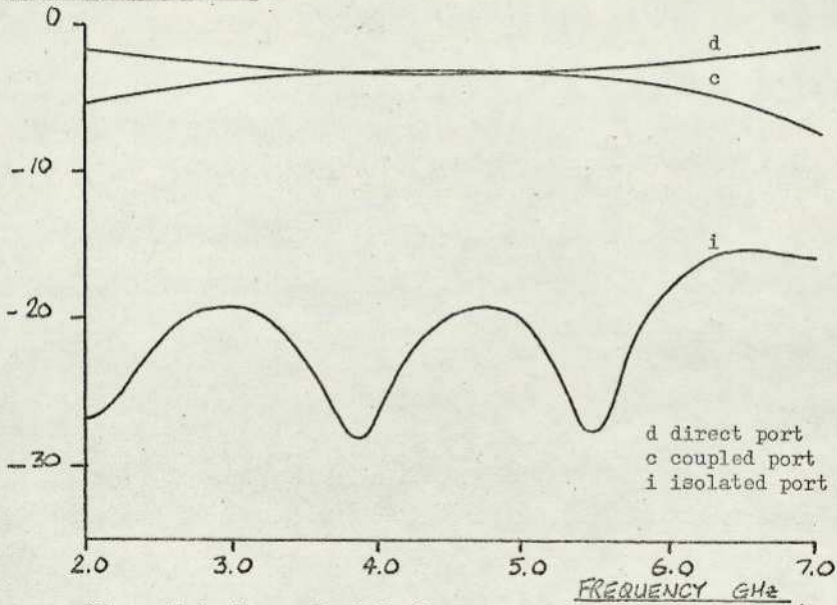
COUPLED POWER dB.

Figure 5.8 Theoretical Performance of the 3 dB Microstrip/Slot line Coupler.

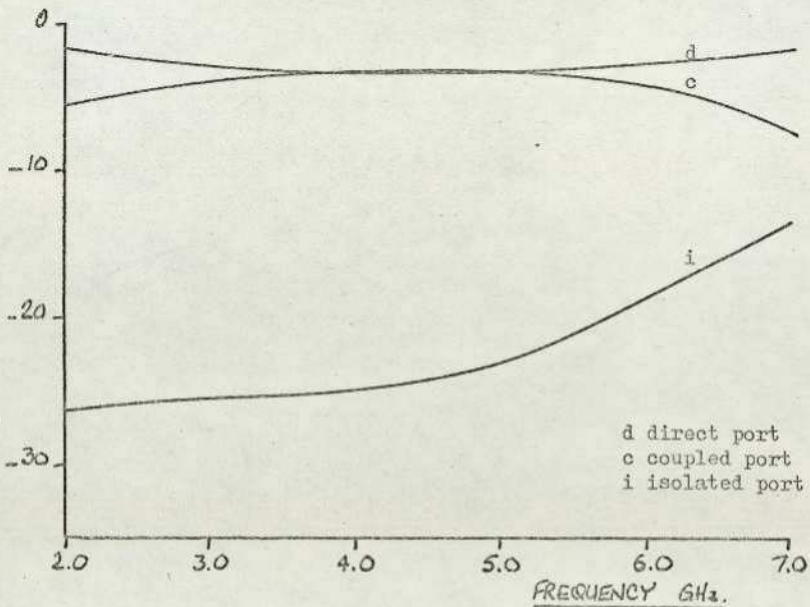
COUPLED POWER dB

Figure 5.9 Theoretical Performance of the 3 dB Microstrip/Slot Line Coupler with input/output Line Mismatches removed.

that the minimum response of the isolated port does not match the centre frequency of the direct and coupled ports.

The remaining aspect of this experimental coupler, not so far described, is the loss due to radiation. From the matching of the theoretical models to the practical couplers responses it has been found that the loss is approximately 4% for couplers with slots having width to height ratios of around 0.2, to 25% for width to height ratios of 1.0.

CHAPTER 6.DISCUSSION6.1 Introduction.

The procedures and results contained in the experimental and theoretical sections of this thesis will now be discussed in some detail. In general, conclusions will not be made in order that they may be more clearly expressed in the next Chapter. In both the introduction to the thesis, and in Chapter 1, it has been emphasised that the design and performance of a microstrip proximity directional coupler can only be predicted from the knowledge of certain parameters of microstrip and related transmission line structures.

Each section of this Chapter will be devoted to considering one of the main topics where either experimental or theoretical work, or both, has been carried out. The topics will be discussed in the order in which they appear in the thesis, with the exception that the theoretical study of microstrip will be considered last.

6.2 Phase Velocity Measurements.

The fact that the phase velocity and normalised guide wavelength of a mode are simply related, has allowed their measurement to be made from the knowledge of the physical length and resonant frequencies of a resonator which supports the mode in question. This technique has been investigated in Chapter 2, resulting in the measurement of the even- and odd-mode normalised guide wavelengths of coupled microstrip lines, and the normalised guide wavelengths of the two dominant modes of a microstrip/slot line combination. The resonator configuration adopted in each case was that of a ring, thus eliminating the problems associated with open- or short-circuit end effects, encountered with other forms of resonators. The ring resonator, however, does have some

disadvantages for this application in that for lower order resonances, around the ring, the curvature is sufficient to cause a perturbation of resonant frequency. It has been shown by Horton and Easter (40) that this effect is small when the ring's circumference is greater than three times the wavelength.

Over the frequency range 1 to 12 GHz, the results for the even- and odd-mode normalised guide wavelength showed the dispersion to be approximately linear. The normalised guide wavelengths of the even- and odd-mode changed by approximately -0.0013 per GHz and -0.0004 per GHz respectively, for medium coupled lines. The fact that the odd-mode was less dispersive and had a larger phase velocity than the even-mode is easy to understand if one considers the static field configuration for each mode. The energy for the odd-mode is concentrated less in the dielectric than for the even-mode, allowing the phase velocity to be nearer its free space value, and less dependent upon the effects of the dielectric and the dielectric inhomogeneity.

Apart from Denlinger's measurements of the even- and odd-mode guide wavelengths of coupled microstrip lines on magnesium titanate substrates (15), and Napoli and Hughes' measurements (24), there were no published results which could be used as a comparison for the experimental results. Denlinger's results are unsuitable for use for comparison purposes because of the large difference of relative permittivity of the substrate material to the one used in this experiment. One of the original intentions of the experiment in Chapter 2 was to check the experimental results of Napoli and Hughes. This was because they had found a much larger difference between the normalised guide wavelengths of the two modes, than was postulated by Bryant and Weiss' theoretical analysis. Napoli and Hughes found the difference to be between 13% for weakly coupled lines, and 17% for strongly coupled lines. In their experiments, a substrate with a relative permittivity of 10.4 was used. From Chapter 2, using a substrate with a relative permittivity of 10.3, the measured differences were between 11.2% and 12.9% for the same cases. The theoretical differences, calculated using Bryant and Weiss' computer program (19), for both substrates

lay between 8.8% and 11.1%. One reason, pointed out in the letter published on this experiment (Appendix 1), is that Napoli and Hughes' results were obtained at 8 GHz, whereas Bryant and Weiss' theoretical analysis is a quasi-static approach, only valid at low frequencies.

Publication of the results of the even- and odd-mode normalised guide wavelengths of coupled microstrip lines, shown in Chapter 2, has resulted in them being used for comparison for two theoretical approaches. Getsinger (68) used the data to show that his Microstrip Dispersion Model was valid for coupled microstrip lines. This was true if the total mode impedances were employed in the evaluation of certain parameters needed to characterise the model. Kowalski and Pregla (10) also used the experimental results as a comparison for their theoretical method based on a variational expression.

Getsinger's LSE-mode model for microstrip predicted the following dispersion relationships:

$$\epsilon_c = \epsilon_s - \frac{\epsilon_s - \epsilon_{e0}}{1 + G(f^2/f_p^2)} \quad \dots 6.1$$

where

$$f_p = \frac{Z_0}{2\sqrt{\mu H}}$$

$$G \triangleq 0.6 + 0.009Z_0 \quad \dots 6.2$$

and ϵ_c the frequency-sensitive effective dielectric constant

ϵ_{e0} the effective dielectric constant at zero frequency

ϵ_s the substrate dielectric constant

Z_0 the characteristic impedance at zero frequency

μ 31.92 nH/inch

G an empirical parameter

H the substrate thickness

The empirical parameter G can be found either from equation 6.2 or from the 'inflection-point method' described in Getsinger's earlier paper which was concerned with a Microstrip Model (18).

For the coupled line case, good agreement was found to occur between the measurements and predictions of equation 6.1. This is perhaps not particularly surprising since ϵ_{eo} is derived from the experimental data, ensuring a good curve fit near zero frequency. The empirical parameter G , which can be derived from the actual data, or from equation 6.2, sets the position of an inflection point in the frequency variation of the effective dielectric constant. In general, for the microstrip lines considered, this will occur above 12 GHz. It has been shown that the model has not been used to verify the practical results, but that the results have been shown to be consistent with the operation of a simple model. By knowledge of the approximate impedance and zero frequency effective dielectric constant for each mode, the frequency dependence of the effective dielectric constant is found. The even-mode does appear to be characterised better than the odd-mode, which is most likely because of the mode's closer similarity with the dominant mode of a microstrip line, which the model represents.

The theoretical analysis of Kowalski and Pregla is more useful as a verification of the practical measurements than Getsinger's model. Although the odd-mode is closely characterised, the measured normalised guide wavelength exhibits a consistently higher effective dielectric constant than predicted over the frequency range considered. Since Kowalski and Pregla's theory agrees with Bryant and Weiss' results at zero frequency, it appears that the even-mode normalised guide wavelength that was measured, was in error. The reason for this was at first unclear, though one explanation would be that the relative permittivity of the substrate was higher than 9.7 as had been found to be true for an alumina material, from a different manufacturer, by Napoli and Hughes (84). This possibility was investigated and surprisingly it was found that in November 1971 the manufacturers had changed their data sheet relating to this material (85), to the effect that the dielectric constant of ADS 995 Alumina had been increased from 9.8 to 10.3. This higher figure for the dielectric constant has been used for the

calculations given in Chapter 2, but the change was not realised until after the paper on this experiment had been published and the results used by Kowalski and Pregla, and by Getsinger. The accuracy of Getsinger's model is unaffected since the parameters are obtained from the data, but the theoretical prediction of Kowalski and Pregla was computed assuming a dielectric constant of 9.7. Thus, their results for the effective dielectric constant for each mode were lower than were calculated from the measurements. If their theoretical predictions were re-calculated for a dielectric constant of 10.3, the even-mode effective dielectric constant would be nearer to the measured values, and the odd-mode slightly greater. The latter effect could be the result of a finite conductor thickness of approximately $13\mu m$, since the theoretical analysis assumes infinitely thin lossless conductors.

The almost linear frequency dependence of the normalised guide wavelength of the even- and odd-modes of coupled microstrip lines was established by the measurements described in Chapter 2. Publication of the results (see Appendix 1) promoted a reply from Napoli and Hughes (86), since in previous publications, (44, 84) they had shown empirically that the frequency dependence of the normalised guide wavelengths of the two modes is approximately square law. In their correspondence, they expressed the propagation constant, γ , as a function of s in the complex frequency domain:

$$\gamma = a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + \dots \quad \dots 6.3$$

such that for real frequencies, that is when $s = j\omega$

$$\gamma = ja_1\omega - a_2\omega^2 - ja_3\omega^3 + a_4\omega^4 \quad \dots 6.4$$

since

$$\gamma = \alpha + j\beta \quad \dots 6.5$$

then

$$\beta = \alpha_1 \omega \left(1 - \frac{\alpha_3}{\alpha_1} \omega^2 + \dots \right) \quad \dots 6.6$$

showing the deviation to be square law. They suggested that losses or curvature within the resonators had affected the results, giving the linear frequency dependence. It was felt that the above argument required certain assumptions about the nature of the microstrip line to be valid. Tackling the problem in a different manner, from first principles, it can be seen that

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad \dots 6.7$$

where R is the resistance, L the inductance, G the conductance, and C the capacitance, all per unit length, of a transmission line. If $R \ll \omega L$ and $G \ll \omega C$, then from equation 6.7 it can easily be shown that

$$\beta \approx \omega \sqrt{LC} \left(1 + \frac{1}{8\omega^2} \left(\frac{R}{L} - \frac{G}{C} \right)^2 \right) \quad \dots 6.8$$

showing the deviation to be square law as before. It is unlikely in the case of microstrip that the line parameters will be constant with frequency. The most obvious of these, in the complex frequency plane, is given by:

$$\begin{aligned} C &= f(s) \\ &= C_0 + C_1 s + C_2 s^2 + \dots \end{aligned} \quad \dots 6.9$$

for real frequencies $s = j\omega$, then

$$C(j\omega) = C_0 - j\omega C_1 - \omega^2 C_2 + \dots \quad \dots 6.10$$

assuming C to be real for all frequencies, C_1 , C_3 etcetera must be zero, or more simply

$$C = f(s^2) \quad \dots 6.11$$

The propagation constant can still be expressed by equation 6.7, the transmission line equation, or by the power series of equation 6.3 with modified coefficients. The deviation of the phase constant which results is again square law. Generalising

further, C may be a function of σ and ω separately, that is

$$C = f(\sigma, \omega) \quad \dots 6.12$$

Under this condition, equation 6.3 is no longer valid, and the propagation constant must be expressed by two series, one with C dependent on σ and one with it dependent on ω . The resulting expression for the phase constant, β , contains all powers of ω thus giving rise to a first order linear deviation of the phase constant.

It can be seen from the foregoing discussion that a square law deviation of the phase constant with frequency results when the transmission line parameters are invariant with frequency. It has been shown that by imposing certain conditions on the capacitance, the frequency dependence of the phase constant is more general, containing all powers of ω . Since it is likely that the inductance per unit length will also be a function of σ and ω , the almost linear frequency dependence of the normalised guide wavelength that was measured, has some justification theoretically.

The preliminary investigation of a microstrip/slot line configuration has shown that mode degeneracy of its two dominant modes exists under certain forms of dielectric loading of the ground plane slot. The experimental results had to be limited from practical considerations to a few specific microstrip/slot line combinations. They are intended to be used as data to which theoretical predictions can be compared, since the theoretical analysis can give complete characterisation of the configuration without the measurement and constructional errors of the practical case. The errors in measurement are similar to those of the coupled microstrip lines, the ring and coupling arrangements being of similar dimensions. It has been explained in Chapter 2, that the circumference of the resonator was taken to be the mean geometrical circumference, further the importance of the positioning of the dielectric loading material was noted. The results for the normalised guide wavelength of the microstrip-mode were close to expectations, being very similar to those for

the even-mode of a pair of coupled microstrip lines. It has been seen that the normalised guide wavelength of the slot-mode, of the microstrip/slot line combination, varies in a non-linear manner quite unlike that of the odd-mode of a pair of coupled microstrip lines. It is this difference, between the modes, that enables simple modifications to effect mode degeneracy in the microstrip/slot line configuration. The change of the normalised guide wavelength of the slot-mode with D/H for a fixed W/H (see Figure 2.14) has shown, for a particular frequency, that in general two values of slot width have the same normalised guide wavelength. It is seen that the normalised guide wavelength at a particular frequency is relatively invariant over a small range of slot widths. To comment further on this fact would require further measurements in order that this function be plotted for other values of W/H .

6.3 Impedance Measurements.

A measurement of the characteristic impedance of some microstrip lines has been presented in Chapter 3, in which, by using a novel method, the characteristic impedance has been determined over the frequency range 1 - 8 GHz. The results show the measured characteristic impedance to be lower than that calculated by Bryant and Weiss at zero frequency, and the frequency dependent solutions of Kowalski and Pregla (45), and Schmitt and Sarges (23).

The reason for the measured characteristic impedances being lower than those theoretically predicted is not fully understood, partially because the inherent inaccuracies did not allow precise enough measurements to be made. A starting point in discussing this result is to reconsider the theoretical model and its relation to the practical case. The use of a shunt capacitance to represent the junction discontinuity, caused by the coaxial to microstrip line transition, was based upon observations by Caulton, Hughes, and Sobel (42). A more recent paper by Wight, Jain, Chudobiak and Makios (87) describes the

transition in terms of a shunt capacitance and series inductance. The values at 10 GHz, for a 0.025 inch thick Alumina substrate, using an OSM 244-4A coaxial to microstrip transition, were $\omega L = 9$ ohms and $\omega C = 1.1$ mmho. This more complex model had been considered prior to the publication of Wight et al., but it was thought it would add complications to a method, where existing errors in the measurement technique had not been reduced. The remainder of the model is possibly adequate considering the accuracy of the practical measurements. The fact that the discrepancy between the theoretical and measured impedances are worse when the test and standard line are of different impedance does tend to point towards the presence of an impedance discontinuity parasitic, which may be in the form of a series inductance (87).

The results that have been obtained do show an increase in value with increasing frequency, a trend more exaggerated with higher impedance lines. This observation backs the recent theoretical conclusions of Krage and Haddad (9), Kowalski and Pregla (45) and Schmitt and Sarges (23). It may be argued that if the reason for the apparent low values is due to bad modelling of the transition, or due to not accounting for an impedance discontinuity parasitic, then this may shift, as well as change the values of the peaks of the computed phase deviations, such that the overall impedance characteristic is valid. This, however, is mere conjecture; one fact which may be relevant is that an accidental lifting of the top strip, over a very short length of line, adjacent to the transition, caused the peak phase deviations to increase by approximately 50% in value, with only a very small change in their peak frequency. If a very short length of line, lifted from the substrate, can be represented in an equivalent circuit as a series inductance, then it is very likely that the presence of a small series inductance at the transition, in the transmission line model, would effect an increase in the measured characteristic impedance.

The Time Domain Reflectometer (T.D.R.) was used on the same microstrip lines to measure the characteristic impedance.

Higher impedance values resulted, but they were in between those obtained using the phase shift method and the theoretical predictions. The problems associated with the use of the T.D.R. have already been enunciated, it only being necessary to state that the main error that existed was due to the physical size of the microstrip line, its transition and open-circuited end being too small for the rise time of the step waveform of the T.D.R.

Partly because of the uncertainties related above, the phase shift method of impedance measurement was not used for measuring the impedance of the microstrip-mode in the microstrip/slot line combination. Instead, the T.D.R. was used to obtain quickly some value for the impedance relative to a microstrip line of the same dimensions. The results are as expected, with very small slots having little effect upon the impedance. The effects become noticeable when the slot is greater than approximately half the top strip width.

Measuring the characteristic impedance of the slot-mode of the microstrip/slot line combination, proved more difficult than the measuring of the microstrip-mode because it is a TE mode. It has been shown that impedance measurement generally involves comparison with a coaxial line which supports a TEM mode. When the two are connected, mode conversion occurs, affecting the measurement to an unknown extent. In Chapter 5, the characteristic impedance of this mode has been determined whilst under its required operating conditions as a directional coupler. The method, although lacking elegance, has been shown to yield reasonably consistent results, and is sufficiently accurate to allow directional couplers to be designed to give particular performances.

6.4 Directional Coupler Design.

In Chapter 5, it has been explained how a directional coupler can be made from a microstrip/slot line combination.

With this structure the phase velocity measurements of Chapter 2 have shown that degeneracy of the two dominant modes is easily achieved in the frequency band 2 to 8 GHz, with possibilities of degeneracy outside this band. This is a condition that cannot be easily achieved in a conventional microstrip proximity coupler.

For the optimisation of the design of a coupler, and for characterisation of the slot-mode impedance, a theoretical model of the complete coupler was derived. Using a digital computer, the performance of the coupler's model could be calculated. In constructing the model, various assumptions were made in order to gain simplification, making the analysis more straightforward and reducing the number of input parameters to the problem. The coupler was assumed to be completely symmetrical, with all of the transmission lines having uniform impedances and propagation constants along their lengths. At the coaxial to microstrip line transition, the junction discontinuity was assumed to be completely described by a shunt capacitance, as in the measurements in Chapter 3, and radiation from the ground plane slot was assumed to come only from the open-circuited ends.

The results of the theoretical model predict that, in the absence of mismatched input lines to the coupler, the minimum power from the isolated port will not occur at the centre frequency of the coupler. This is a result of the fact that the coupler was made similar to the couplers used in the determination of the slot-mode impedance, excepting the widths of the microstrip and slot lines in the microstrip/slot line combination. The length of this section was made equal to a quarter wavelength at 4.5 GHz, this being approximately the centre of the range over which mode degeneracy was found to occur with 0.025 inch thick zirconate substrate as the loading dielectric. With the microstrip/slot line combination necessary to produce the desired mode impedances, mode degeneracy would occur between 5 and 6 GHz for an ideal coupler. Because, however, the effective length over which the slot-mode propagated was longer than that of the

microstrip-mode, the minimum of the power from the isolated port occurred at a lower frequency.

6.5 A Theoretical Solution for Microstrip.

A theoretical solution for a microstrip transmission line has been presented in Chapter 4. A Least Square criterion was applied to a hybrid mode formulation in a way that is directly applicable to the more complex configuration found in the microstrip/slot line combination. Throughout Chapter 4, some discussion of the reasons for the way in which the problem was tackled, have already been given. At the risk of repetition the more important points relating to the theoretical analysis will now be discussed.

The TE and TM solutions to the L-shaped waveguide problem provided a good test for the theoretical method since the internal 270 degree corner produced sub-regions that were similar to those encountered in the microstrip analysis. The computer program was tested by two cases, and it showed none of the numerical problems that LaRiviere (67) had noted. In his solution for the TM modes of the L-shaped waveguide, a real matrix was formed. If any elements were imaginary, they were made real by multiplying the rows and columns by $-j$ and j respectively. The Householder Tridiagonalisation (76) and the method of Bisection (77) were then used by him to compute the eigenvalues. One of the disadvantages of this procedure is that ordering of the matrix coefficients is required, which means that the scalar fields have to be multiplied by a certain scaling factor. The scaling factor is chosen as the one which gives the fastest convergence of the solution. The formulation given in Chapter 4 has purposely set out to be more general, and thus a complex matrix has been formed, and the algorithms 'Comhes' (79) and 'Comlr 2' were used to compute all of the eigenvalues and eigenvectors. These algorithms seem numerically very stable for solving the type of matrices produced and if problems are encountered a complex version of 'Balance'

(80) can be used to reduce the norm of the matrix.

Unlike LaRiviere's work, scaling of the matrix by dividing the scalar field by a factor such as $ne^{2\pi n}$, where n is the harmonic number, made little, if any, change to the convergence of the computed eigenvalues. This shows that the algorithms used here were relatively insensitive to the numerical ordering of the matrix, and further that the eigenvalue system of the problem is unaffected by scaling of the complete scalar fields by a common factor.

The results for the first three TE modes of the L-shaped waveguide were surprisingly accurate when it is remembered that the boundary condition on the internal 270 degree corner is not rigorously enforced. It was considered important that both TE and TM modes of the L-shaped waveguide were found, since the dominant mode of the shielded microstrip line is formed from the coupling of the dominant LSE and LSM modes of a similarly constructed guide, but with the strip absent.

A major difference in the formulation of the microstrip solution was that both scalar electric and scalar magnetic fields were present. They are often related to each other in this type of analysis by the wave impedance of free space, but it was decided that little reason existed for adopting this relationship in this case, so for complete generality it was decided to keep the two scalars decoupled.

A number of different scaling factors were tried in the solution of the microstrip problem to attempt to find the most sensitive boundary residuals. The boundary residual between regions 1 and 2 was the most stable of these and changed little in value with frequency. The boundary residual which was the most sensitive was that between regions 2 and 3. When the magnetic scalar potentials are relatively large in comparison to the electric scalar potentials, the H_z component of this boundary residual passes through a minimum near to the required solution. Generally, this relationship between the electric

and magnetic scalar fields does not exist and the H_z component of the boundary residual merely changes with frequency and matrix size in a continuous manner. The H_x component of this boundary residual changes at a much greater rate due mainly to the contribution by FF23, defined in Appendix 3. This component of the boundary residual becomes increasingly large with increasing matrix order, and thus masks the contributions from other coefficients. For this reason, the H_x component of the boundary residual was reduced by a large amount by the action of the scaling factors till its value was similar to the other boundary residuals. The boundary residual between regions 3 and 4 changed less quickly with frequency and matrix order, and was equated to the other boundary residuals by scaling factors close to unity.

A contour representation of the electric and magnetic scalar fields has been shown in Figure 4.8. It shows that between regions 2 and 3, along the air/dielectric interface, violent oscillations occur in the scalar potentials. Because these oscillations are related to the number of expansion terms of the scalar fields, they may have little physical significance. The coefficient $\tilde{d}_n$ in equation 4.65 partially controls the scalar magnetic field of region 2, which is in turn a component of the eigenvector. From Tables 4.7 & 4.8, the component of the eigenvector associated with $\tilde{d}_n$ can be seen to be greater for larger values of n . Ideally, one would imagine a scalar magnetic field quickly decaying away from the edge of the strip, in which case, $\tilde{d}_n$ would decrease with increasing n .

The solutions, one of which is shown in Figure 4.7, show that for the various matrix sizes the convergence is unique, and is close to LaRiviere's solutions obtained from Fourier Matching. The convergence problem was not investigated fully, partly because of the uncertainty over the reason for the oscillations of the scalar potentials along the air/dielectric interface, and partly because the intention of this study was to develop an existing method to a point where it is easily applied to more complex structures, such as the microstrip/slot

line combination. It has recently been stated by Hofmann (88) that there is not a relative convergence problem in mode matching solutions for microstrip, since he has shown convergence to be unique and the edge condition on the strip to be satisfied. In LaRiviere's approach, there was a relative convergence problem using Fourier Matching for the microstrip problem. These two seemingly conflicting results, may point to a difference which results from using a different choice of sub-regions, since it is thought that Hofmann used four sub-regions (89) similar to the cross-section used in Chapter 4, and LaRiviere used only two sub-regions. If this is so it may mean that in the latter case the edge condition on the strip was not satisfied in the solution. With the Least Square approach to mode matching, it has already been said that there is no relative convergence problem as unique convergence is guaranteed if the contours along which the residuals are evaluated, start or end at any field singularity present, so that every singularity lies at one end of a boundary residual.

CHAPTER 7.CONCLUSIONS7.1 Properties of Single and Coupled Microstrip Lines and Related Structures.

This thesis is concerned with the properties of single and coupled microstrip lines and the microstrip/slot line combination, in relation to their use in microstrip directional couplers. The single microstrip line is perhaps the most well known of these transmission lines. The zero frequency solutions for single microstrip lines of Bryant and Weiss (19) are supplemented by the recent frequency dependent solutions of Mittra and Itoh (65), Kowalski and Pregla (45), Krage and Haddad (9) and Schmitt and Sarges (23). All of these solutions, except that of Mittra and Itoh, have given both the variation of phase constant, in some form, and impedance with frequency. They have postulated, contrary to previous opinion, that the characteristic impedance of a microstrip line increases with frequency. In order that the directional coupler can be made compatible with other microstrip structures, the characteristic impedance of a single line is an important parameter which must be known over the operational frequency band. A measurement of impedance variation with frequency, of microstrip lines on 0.025 inch thick Alumina, has been presented in Chapter 3 using a new measurement technique. Although the impedance values are lower than expected, the variation has been shown practically, for the first time, to be constant or to increase slightly over the frequency range 1 to 8 GHz.

A zero frequency solution by Bryant and Weiss (19) and frequency dependent solutions by Kowalski and Pregla (45) and Krage and Haddad (9) have also been published for a pair of coupled microstrip lines. Little practical verification of these solutions was available, indeed an experiment by Napoli and Hughes (44) had found some disagreement between their results and theoretical predictions. The even- and odd-mode guide

wavelengths have been measured for a number of different coupled line configurations on 0.025 inch thick Alumina, and the results published (see Appendix 1). Agreement between these results and the theoretical predictions has been improved recently, due to knowledge that the dielectric constant of the Alumina used was higher than that originally stated by the manufacturer. The variation of the normalised guide wavelength with frequency has been found to be approximately linear between 1 and 12 GHz, the even-mode changing by approximately -0.0013 per GHz and the odd-mode by -0.0004 per GHz. Thus, the ratio of the odd- to even-mode guide wavelength increases with frequency, typically by $+0.003$ per GHz. In Napoli and Hughes' experiment, this ratio was found to be much larger than the theoretical predictions showed, however, it has been shown here that inaccuracies of interpolating Bryant and Weiss' results and comparison of these with practical measurements taken at 8 GHz, are sufficient to account for the differences.

The normalised guide wavelengths, of the dominant modes of a structure made from a microstrip line with parallel ground plane slot, have been measured between 1 and 12 GHz. The structure, which for simplicity is called a microstrip/slot line combination, has been shown to support two dominant modes, one mode being very similar to the even-mode of a coupled microstrip line, and the other mode being very similar to the dominant mode of a slot line. Degeneracy of the two modes has been achieved by loading the ground plane slot with a zirconate substrate 0.025 inches thick. It can be seen that other materials and thicknesses can be used as the loading material, and, thus, in principle, it is possible to make the dominant modes, of a large number of configurations, degenerate. The characteristic impedance of the microstrip-mode of this combination has been measured. For small slot widths, it has been shown to be close to the characteristic impedance of a single microstrip line. The characteristic impedance of the slot-mode has been deduced from the performance of a directional coupler, in which the microstrip/slot line combination was used. These parameters have been found to be measured with sufficient

accuracy to allow a directional coupler to be designed.

7.2 A Proximity Directional Coupler using the Microstrip/Slot Line Combination.

This type of coupler, first used by de Ronde, has been investigated in some detail. A theoretical model has been made and used both to predict the coupler's performance and to provide details of the characteristic impedance of the slot-mode of the microstrip/slot line combination.

A 3 dB coupler was designed using the measured parameters of the microstrip- and slot-modes given in previous Chapters. The resulting characteristics showed that 3 dB coupling was achieved, but that the directivity was impaired by rippling of the power in the isolated port. Decreasing the power in the isolated port directly increases the directivity. It can be seen that the mismatched output lines causes power to be reflected back into the coupler. Reflected power from the direct port will thus split, in a 3 dB coupler, equally into the input port and the isolated port. In practice, the VSWR in the output lines is not only due to mismatched lines, but also to the presence of the junction discontinuities at each end. It has been demonstrated that the major cause of the rippling in the 3 dB coupler, was the 53 ohm impedance of the output lines. The coupler and the loads were all matched to 50 ohms.

Several design faults have been discussed in the previous Chapter, but one problem that remains to be dealt with is that of direct coupling across the length of the coupler. In Chapter 5, it can be seen that for the length used in the experimental determination of the slot-mode impedance, and in the 3 dB coupler, the effect is that the extra power coupled to the isolated and direct ports is -35 and -40 dB respectively. Thus, if rippling due to the mismatched input lines was reduced, causing the power in the isolated port to decrease, the direct coupling may become

noticeable. With couplers with higher centre frequencies, where the length of the microstrip/slot line is shorter, the level at which direct coupling becomes noticeable will be proportionally higher.

The limitation on the directivity is not only the amount of power in the isolated port, but also the power level in the coupled port, since the directivity of a coupler is the ratio of the coupled to isolated power. Thus, the difficulty in achieving low powers in the isolated port restricts the attainable directivity. This means that good directivity together with weak coupling can only be achieved with careful design and close tolerancing in the coupler and its associated input and output lines and terminating impedances.

It has been stated in Chapter 5 that appreciable power is radiated from the coupler for the larger slot widths. This problem, which is more serious for those couplers which have smaller amounts of coupled power, is thought to be due to radiation from the open-circuited ends of the ground plane slot. This aspect of the coupler will require further investigation before the radiation can be minimised, and will be discussed further in Chapter 8.

7.3 Theoretical Consideration of Microstrip and Related Structures.

In Chapter 4, a theoretical approach, first used by LaRiviere (67), has been applied to solving the microstrip problem in a more general way. Being more general, the method produces a hermitian matrix, rather than as in LaRiviere's study, a real symmetric matrix. Different algorithms were required to find the eigenvalues and eigenvectors of the matrix. These algorithms were found to be much less dependent upon the numerical ordering of the matrix than those LaRiviere used. This eliminated the need for one of the scaling operations that had previously been found to be necessary in order to obtain rapid convergence.

Tests on the method were carried out by solving the first few modes of both the TE and TM configurations in an L-shaped waveguide. By using smaller sub-regions in defining the problem, and hence increasing the lengths along which matching is performed, results of similar accuracy to those obtained by LaRiviere could be achieved for similar sizes of matrix. This means that fewer harmonics of the basis functions are used to secure a solution of similar accuracy, and, therefore, the basis functions are nearer to their form for the complete problem. The tests on the L-shaped waveguide were important since sub-regions of similar shape to those used in the test were also used in the microstrip solution. The tests also showed that the equations chosen to represent the magnetic scalars, even though they did not fully define the internal 270 degree corner, gave solutions of good accuracy.

Solutions were achieved for the microstrip problem which converged uniquely, and for similar size matrices to those used by LaRiviere, similar accuracy was achieved with little optimisation of the scaling coefficients. In Chapter 6, the problem of the rippling of the scalar fields along the dielectric interface has been discussed. It seems unlikely that this rippling is correct or is a result of the Least Square method. An error in the formulation of the boundary residual or in the matching of the tangential electric fields along the dielectric interface, would appear to be the most likely reason for the rippling.

The eigenvectors have been shown to be of prime importance in the determination of the eigenvalue minimum in solving the matrix. They have also been used to compute the electric and magnetic scalar fields, from which the vector fields can be found and a characteristic impedance defined.

Despite the problem of the rippling of the scalar potentials, the method has been relatively easy to apply to the microstrip problem. Merely by increasing the number of

sub-regions and matrix size, the more complex problem of the microstrip/slot line combination can be solved. The theoretical approach thus shows possibilities of relatively easy extension to enable the solution of the more complicated transmission line problems.

CHAPTER 8.SUGGESTIONS FOR FURTHER WORK.

There are several parts of this study where further work could lead to a better understanding of the problems, which may then result in an improved design for a proximity directional coupler. The areas where further work would be beneficial will now be considered.

The measurement of the characteristic impedance of microstrip using the 'phase shift method' has not been brought to its final conclusion. Various reasons for this have been discussed in Chapter 6. Improvements in the measurement technique to reduce errors in the setting of the reference plane extension, and elimination of small phase errors due to the instrument, would justify the adoption of a more complex theoretical model for the line and its transition. Throughout this study, for convenience only, open-circuited lines were used in the measurements. Some investigation of the effects of the open-circuit have been carried out theoretically, but for completeness, practical measurements should be made on short-circuited lines. Further supplementary measurements of the characteristic impedances of the microstrip- and slot-modes in the microstrip/slot line combination, would allow greater flexibility in coupler design, and a greater understanding of the relationship of the characteristic impedance and the physical structure of the transmission line.

The normalised guide wavelengths of the microstrip- and slot-modes in the microstrip/slot line combination have been measured for five of the original nine possible structures, and the dielectric loading has been confined to 0.025 inch thick Alumina and Zirconate, and 0.040 inch thick Zirconate substrates. With further measurements, in addition to increasing the knowledge of these parameters, it is possible that mode degeneracy may be achieved over a wider frequency range than has been so far discovered.

The proposal that further practical characterisation,

of the dominant modes of the microstrip/slot line combination, should be carried out, emphasises the need for a theoretical solution. Before the Least Square technique is applied to this problem, there is the question of the oscillations of the scalar potentials along the dielectric interface, in the microstrip problem, to be investigated. It may also be possible that certain boundaries can be eliminated from the Least Square minimisation procedure without seriously impairing the convergence of the solution. This simplification may, for example, be made between sub-regions 1 and 2 in the microstrip problem considered. If it can be shown that this is a valid simplification for the microstrip problem, then it should help in reducing the complexity of the problem for the microstrip/slot line combination.

A further reduction in computational time and storage requirements can be made by virtue of the fact that for the microstrip solution, a certain normalisation of the basis functions was made which quite unintentionally reduced the matrix from a hermitian to a real symmetric form. The normalisation can be seen in equations 4.62 to 4.69 of Chapter 4. Thus, the matrix can be stored in half the area that the complex matrix occupied. The algorithms that LaRiviere (67) used did show problems with the numerical ordering of the matrix coefficients, so an alternative treatment for real matrices suggested by Peters and Wilkinson (78) is to be preferred in this case. The procedure to be followed for a general real matrix is to first balance the matrix, using 'Balance' (80), then reduce it to a real Hessenburg form by procedures 'Elmhes', 'Dirhes', or 'Orthes' (79). The transformation matrix is then derived using 'Elmtrans', 'Dirtrans' or 'Ortrans' (78) and the eigenvalues and eigenvectors of the balanced matrix found using 'Hqr 2' (78). The eigenvectors of the original matrix are recovered using 'Balbak' (80). These algorithms are essentially the versions for real matrices of the routines that have been found successful in dealing with the complex matrices.

In Chapter 5, it has been shown that by using the measured characteristics of the dominant modes of the microstrip/slot line

combination, a 3 dB directional coupler can be made. Some of the problems that affect the performance of a coupler have been discussed, but not investigated in any depth. In the context of this type of coupler, one is concerned with the way in which the four input/output lines join the microstrip/slot line combination. The configuration used in this study was not optimised, and is likely to be rather unsatisfactory at high frequencies. Experimental investigation should reveal how the performance is affected by the angle subtended between two adjacent input/output lines, and the area and shape of the ground plane removed at the end of the slot line.

APPENDIX 1.PUBLISHED PAPERS.

The publications included in this appendix have been made during the course of this research. The full references for these are as follows:

Gould, J.W. & Talboys, E.C., 'Even- and Odd-Mode Guide Wavelengths of Coupled Lines in Microstrip', Electron. Lett., 8, 9th March 1972, pp 121-122.

Gould, J.W., 'Transmission Line Impedance Measurement at Microwave Frequencies', Electronics and Power, 6th September, 1973, pp 363-365.

Gould, J.W. & Rhodes, G.M., 'Computer Correction of a Microwave Network Analyser without accurate Frequency Measurement', Electron. Lett., 9, 18th October 1973, pp 494-495.

The following articles have been removed for copyright reasons

Gould, J.W. & Talboys, B.C., 'Even- and Odd-Mode Guide Wavelengths of Coupled Lines in Microstrip', Electronics Letters, 8, 9th March 1972, pp 121-122

Gould, J.W., 'Transmission Line Impedance Measurement at Microwave Frequencies', Electronics and Power, 6th September, 1973, pp 363-365

Gould, J.W. & Rhodes, G.M., 'Computer Correction of a Microwave Network Analyser without accurate Frequency Measurement', Electronics Letters, 9, 18th October 1973, pp 494-495.

APPENDIX 2.DESCRIPTIONS OF COMPUTER PROGRAMS REFERENCED IN THE TEXT.

Program Name: G039.

Description.

Program G039 is used to evaluate the maximum and minimum deviation from the linear slope of the phase of the reflection coefficient of a transmission line model. The program can be used to evaluate several different models, and has computed the data presented in Chapter 3.

Referring to Chapter 3, a function for $\delta\phi$ is taken and the peak values, together with their values of βl , are computed. The transmission line models, which contain a junction susceptance, have peak values which are more negative as βl increases. The program evaluates the average slope of these in order to add an equal but opposite slope, making the peak values symmetrical about zero. Since the slope is non-linear, this process is carried out using linear interpolation. The program is written in FORTRAN and is executed on a CDC7600 computer. The following Subroutines and Functions are used in the program.

- GOLDEN : Golden Section algorithm used for evaluating the maxima or minima of a function given by FUNCTION DYDX.
- FIT : Finds, using a Least Square criterion, the best line through a number of points.
- DYDX : This FUNCTION routine provides the values of the function which is to be maximised or minimised. It is used in SUBROUTINE GOLDEN. The routine is quite complex since the equations for $\delta\phi$ generally require an arctangent to be evaluated. The computer only provides solutions between $\pm \pi/2$ for an

arctangent, so an integer number of $-\pi$ have to be added to obtain a smoothly changing function.

The required function for $\delta\phi$ must be put into FUNCTION DYDX in the form of ϕ and is called C in the program. The parameter AL is set equal to 9.0, since this is the approximate reference plane extension in centimetres. The total length of microstrip line is equal to 3.5 cm., given in the program by X1 + X2. Input to the main program is in the form of a data card giving BD, the normalised junction susceptance, and a flag called ISTOP, which is unity when the last value of BD has been read. The flow chart and program are given at the end of this Appendix.

Program Name: G043.

Description.

This program computes the matrix and its eigenvalues and eigenvectors for the problems that have been analysed using the Least Square technique of Chapter 4. The program, which is on microfilm at the end of this Appendix, is purposely written so that different problems can be solved with minimal changes to the basic program. From Chapter 4, it is seen that each element of the matrix is formed from the summation of a number of coefficients, each of which is formed from the product of two other coefficients and an integral. The integrals are of two possible types, being either a sin product, or a cos product, evaluated by FUNCTION SS or CC respectively, the results being stored in a two dimensional 'C' array. The coefficients, with which the integrals are multiplied, are each stored in a one dimensional 'H' array.

For a particular problem, the integrals are evaluated in SUBROUTINE SINT, the 'H' arrays in SUBROUTINE HARRY and the matrix, called A6, formed in SUBROUTINE ARR1 and ARR2, depending upon the complexity of the problem. The algorithms, mentioned

in Chapter 4, which are used to compute the eigenvalues and eigenvectors, were supplied by Oxford University Computing Laboratory, since they had already transliterated these algorithms into FORTRAN for use in the NAG Library. Their SUBROUTINE names correspond in the following manner to those already mentioned:

FO1AVF	}	Complex Balancing
FO1AVZ		
FO1AMF		'Comhes'
FO2ARF		'Comlr 2'
AO2AAF	}	Further Subroutines called by the above Subroutines.
AO2ABF		
AO2ACF		
FO1AAF		

The NAG Library was intended for implementation on the ICL1900 series of computers; small changes were made in a few cases to optimise their execution on the CDC7600 computer.

For maximum execution speed it is desirable for large arrays stored in a COMMON BLOCK in a SUBROUTINE, to be set to local variables within the subroutine. For the microstrip problem, this was found to produce a large number of successive calls to the COMMON BLOCK which the compiler had difficulty in optimising. For this reason the formation of the matrix was split into two subroutines, namely ARR1 and ARR2.

The computing of the separation constants is carried out in the main program, in addition to the setting of certain variables such as the structural parameters and electrical constants. The main program also provides the statements which control the printing of the results. The eigenvectors, obtained from FO2ARF ('Comlr 2'), are normalised arbitrarily, and, thus, before being printed, they are first normalised using SUBROUTINE VNORM. A typical time for producing an 18 x 18, 12 x 12, and 6 x 6 matrix and evaluating their eigenvalues and eigenvectors is approximately 1 second on the CDC7600 computer.

Program Name: G046.

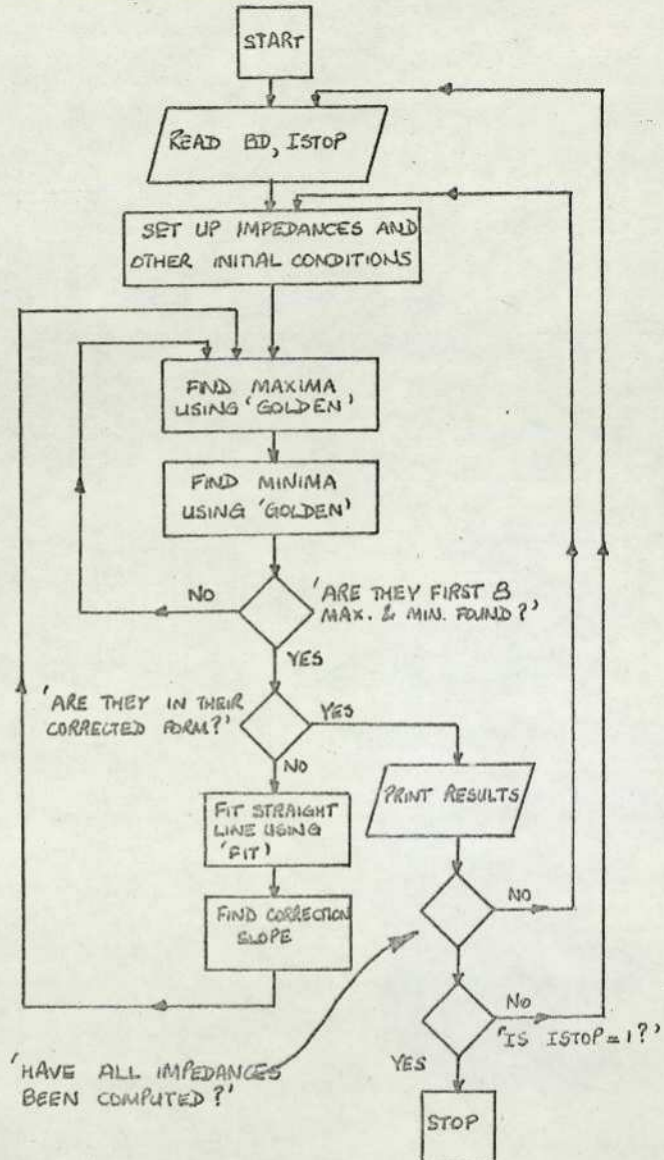
Description.

This program computes the performance of the proximity directional coupler described in Chapter 5. It accepts a number of parameters which correspond to the physical coupler that it is modelling, these are:

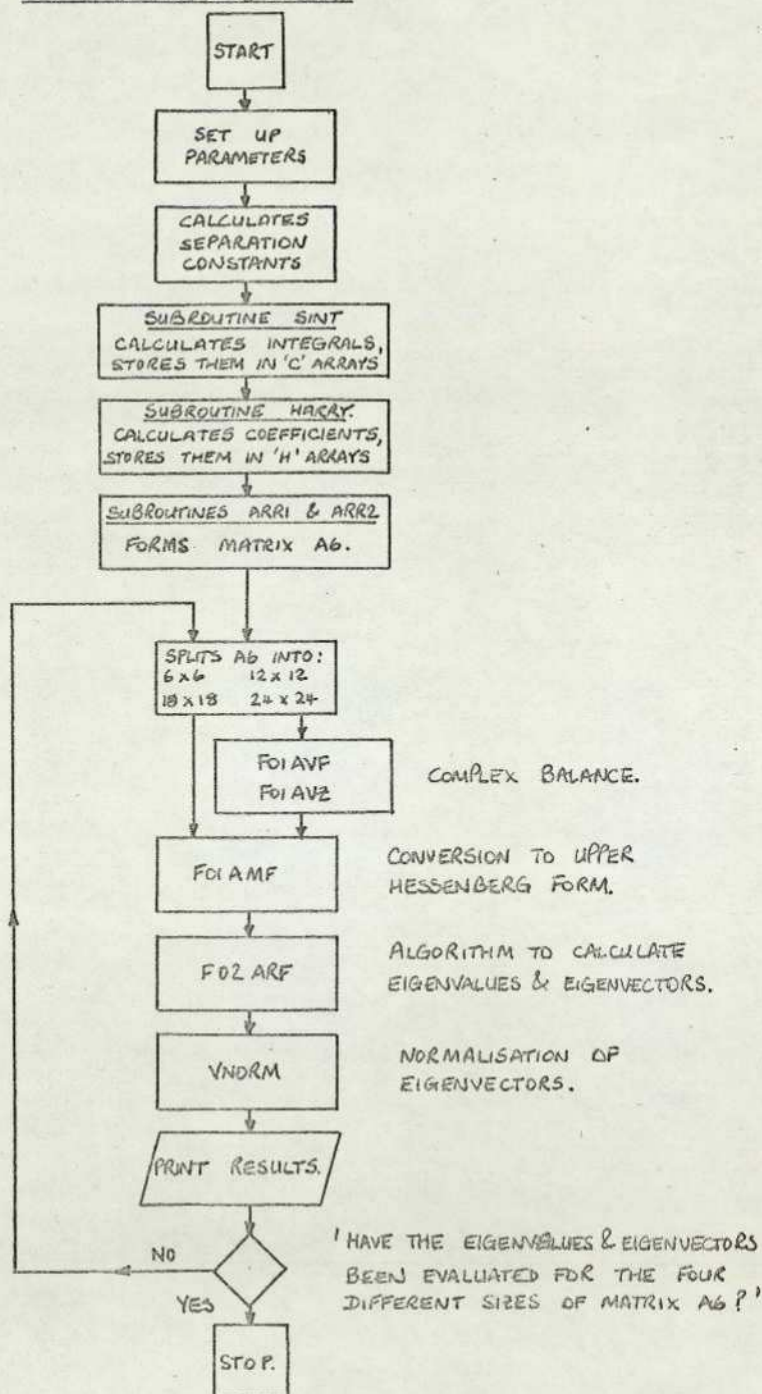
- Z₁ Odd-mode characteristic impedance
- Z₂ Even-mode characteristic impedance
- Z₃ Input/output line characteristic impedance
- L₁ Length of coupler with odd-mode excitation
- L₂ Length of coupler with even-mode excitation
- L₃ Length of each input/output line
- B₁ Phase constant of odd-mode
- B₂ Phase constant of even-mode
- B₃ Phase constant of the input/output lines
- AL Constant which simulates radiated losses
- S VSWR that the junction discontinuity causes in a matched system

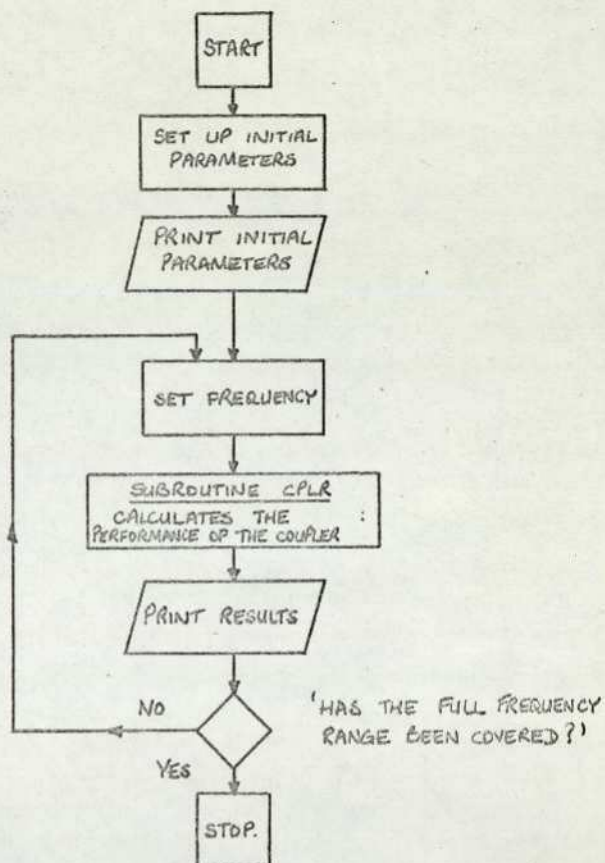
The transfer matrices, given in Chapter 5, are formed in SUBROUTINE CPLR. They are multiplied together, using SUBROUTINE MULT, to form the total transfer matrix for the coupler. The power from each of the four ports is then computed, and is expressed as an amplitude and phase angle. Since the structure of the computer program is very similar to that previously used by the author for Branch Line Couplers (90), the flow chart is not included in this appendix. The program is included as a microfilm.

Flow Chart for Program G039



Flow Chart for Program G043



Flow Chart for Program G046

APPENDIX 3BOUNDARY RESIDUAL EQUATIONS FOR THE MICROSTRIP SOLUTION OF CHAPTER 4

The Boundary Residuals for the microstrip solution of Chapter 4 were not included in the text for reasons of their length. They are given below for each of the boundaries in turn. Further variables are defined such as ZAN, TAN etc. which are merely to speed the execution of the program. The equations are written in the computer programming language FORTRAN, where the asterisk denotes multiplication.

a) Boundary between Regions 1 and 2.

Defining,

$$ZAN=HZ1A(N)$$

$$ZAM=C/NJG(HZ1A(M))$$

$$ZBN=HZ1B(N)$$

$$ZEM=C/NJG(HZ1B(M))$$

$$TAN=HY1A(N)$$

$$TAM=C/NJG(HY1A(M))$$

$$TBN=HY1B(N)$$

$$TEM=C/NJG(HY1B(M))$$

$$TCN=HY1C(N)$$

$$TCM=C/NJG(HY1C(M))$$

$$TDN=HY1D(N)$$

$$TDM=C/NJG(HY1D(M))$$

then,

$$AA12=TAN*TAM*CO5(N,M)$$

$$AB12=TAN*TEM*CO6(N,M)$$

$$AC12=TAN*TCM*CO5(N,M)$$

$$AD12=TAN*TDM*CO6(N,M)$$

$$BA12=TEB*TAM*CO7(N,M)$$

$$BB12=TEB*TEM*CO8(N,M)$$

$$BC12=TEB*TCM*CO7(N,M)$$

$$BD12=TEB*TDM*CO8(N,M)$$

$$CA12=TCN*TAM*CO5(N,M)$$

$$CB12=TCN*TEM*CO6(N,M)$$

$$CC12=TCN*TCM*CO5(N,M)+ZAN*ZAM*CO1(N,M)$$

$$CD12=TCN*TDM*CO6(N,M)+ZAN*ZEM*CO2(N,M)$$

$$DA12=TDN*TAM*CO7(N,M)$$

$$DB12=TDN*TEM*CO8(N,M)$$

$$DC12=TDN*TCM*CO7(N,M)+ZBN*ZAM*CO3(N,M)$$

$$DD12=TDN*TDM*CO8(N,M)+ZBN*ZEM*CO4(N,M)$$

and the boundary residual BR12 is given by:

$$BR12=AA12+AB12+AC12+AD12+BA12+BB12+BC12+BD12+CA12+CB12+CC12+CD12+DA12+DB12+DC12+DD12$$

b) Boundary between Regions 2 and 3.

Defining,

ZAN=HZ2A(N)
 ZAM=C/NJG(HZ2A(M))
 ZBN=HZ2B(N)
 ZBM=C/NJG(HZ2B(M))
 ZCN=HZ2C(N)
 ZCM=C/NJG(HZ2C(M))
 ZDN=HZ2D(N)
 ZDM=C/NJG(HZ2D(M))
 TAN=HX2A(N)
 TAM=C/NJG(HX2A(M))
 TBN=HX2B(N)
 TBM=C/NJG(HX2B(M))
 TCN=HX2C(N)
 TCM=C/NJG(HX2C(M))
 TDN=HX2D(N)
 TDM=C/NJG(HX2D(M))
 TEN=HX2E(N)
 TEM=C/NJG(HX2E(M))
 TPN=HX2F(N)
 TFM=C/NJG(HX2F(M))

then,

AA23=TAN*TAM*C26(N,M)
 AB23=TAN*TEM*C27(N,M)
 AC23=TAN*TCM*C26(N,M)
 AD23=TAN*TIM*C27(N,M)
 AE23=TAN*TEM*C32(N,M)
 AF23=TAN*TFM*C32(N,M)
 BA23=TBN*TAM*C28(N,M)
 BB23=TBN*TEM*C29(N,M)+ZDN*ZDM*C20(N,M)
 BC23=TBN*TCM*C28(N,M)+ZDN*ZAM*C19(N,M)
 BD23=TBN*TIM*C29(N,M)+ZDN*ZBM*C20(N,M)
 BE23=TBN*TEM*C34(N,M)
 BF23=TBN*TFM*C34(N,M)+ZDN*ZCM*C25(N,M)
 CA23=TCN*TAM*C26(N,M)
 CB23=TCN*TEM*C27(N,M)+ZAN*ZDM*C18(N,M)
 CC23=TCN*TCM*C26(N,M)+ZAN*ZAM*C17(N,M)
 CD23=TCN*TIM*C27(N,M)+ZAN*ZBM*C18(N,M)
 CE23=TCN*TEM*C32(N,M)
 CF23=TCN*TFM*C32(N,M)+ZAN*ZCM*C23(N,M)
 DA23=TDN*TAM*C28(N,M)
 DB23=TDN*TEM*C29(N,M)+ZBN*ZDM*C20(N,M)
 DC23=TDN*TCM*C28(N,M)+ZBN*ZAM*C19(N,M)
 DD23=TDN*TIM*C29(N,M)+ZBN*ZBM*C20(N,M)
 DE23=TDN*TEM*C34(N,M)
 DF23=TDN*TFM*C34(N,M)+ZBN*ZCM*C25(N,M)
 EA23=TEN*TAM*C31(N,M)
 EB23=TEN*TEM*C33(N,M)
 EC23=TEN*TCM*C31(N,M)
 ED23=TEN*TIM*C33(N,M)
 EE23=TEN*TEM*C30(N,M)
 EF23=TEN*TFM*C30(N,M)

$$\begin{aligned}
FA23 &= TFN * TAM * C31(N, M) \\
FB23 &= TFN * TFM * C33(N, M) + ZCN * ZDM * C24(N, M) \\
FC23 &= TFN * TCM * C31(N, M) + ZCN * ZAM * C22(N, M) \\
FD23 &= TFN * TFM * C33(N, M) + ZCN * ZEM * C24(N, M) \\
FE23 &= TFN * TFM * C30(N, M) \\
FF23 &= TFN * TFM * C30(N, M) + ZCN * ZCM * C21(N, M)
\end{aligned}$$

and the boundary residual BR23 is given by:

$$\begin{aligned}
BR23 &= AA23 + AB23 + AC23 + AD23 + AE23 + AF23 + BA23 + BB23 + BC23 + BD23 + BE23 + BF23 + \\
&\quad CA23 + CB23 + CC23 + CD23 + CE23 + CF23 + DA23 + DB23 + DC23 + DD23 + DE23 + DF23 + \\
&\quad EA23 + EB23 + EC23 + ED23 + EE23 + EF23 + FA23 + FB23 + FC23 + FD23 + FE23 + FF23
\end{aligned}$$

c) Boundary between Regions 3 and 4.

Defining,

$$\begin{aligned}
ZAN &= HZ3A(N) \\
ZAM &= C/NJG(HZ3A(M)) \\
ZBN &= HZ3B(N) \\
ZEM &= C/NJG(HZ3B(M)) \\
ZCN &= HZ3C(N) \\
ZCM &= C/NJG(HZ3C(M)) \\
TAN &= HY3A(N) \\
TAM &= C/NJG(HY3A(M)) \\
TBN &= HY3B(N) \\
TBM &= C/NJG(HY3B(M)) \\
TCN &= HY3C(N) \\
TCM &= C/NJG(HY3C(M)) \\
TDN &= HY3D(N) \\
TDM &= C/NJG(HY3D(M))
\end{aligned}$$

then,

$$\begin{aligned}
BB34 &= TAN * TAM * C13(N, M) + ZCN * ZCM * C12(N, M) \\
BD34 &= TAN * TBM * C13(N, M) + ZCN * ZEM * C12(N, M) \\
BE34 &= TAN * TCM * C14(N, M) \\
BF34 &= TAN * TDM * C14(N, M) + ZCN * ZAM * C11(N, M) \\
DB34 &= TBN * TAM * C13(N, M) + ZBN * ZCM * C12(N, M) \\
DD34 &= TBN * TBM * C13(N, M) + ZBN * ZEM * C12(N, M) \\
DE34 &= TBN * TCM * C14(N, M) \\
DF34 &= TBN * TDM * C14(N, M) + ZBN * ZAM * C11(N, M) \\
EB34 &= TCN * TAM * C15(N, M) \\
ED34 &= TCN * TBM * C15(N, M) \\
EE34 &= TCN * TCM * C16(N, M) \\
EF34 &= TCN * TDM * C16(N, M) \\
FB34 &= TDN * TAM * C15(N, M) + ZAN * ZCM * C10(N, M) \\
FD34 &= TDN * TBM * C15(N, M) + ZAN * ZEM * C10(N, M) \\
FE34 &= TDN * TCM * C16(N, M) \\
FF34 &= TDN * TDM * C16(N, M) + ZAN * ZAM * C09(N, M)
\end{aligned}$$

and the boundary residual BR34 is given by:

$$\begin{aligned}
BR34 &= BB34 + BD34 + BE34 + BF34 + DB34 + DD34 + DE34 + DF34 + EB34 + ED34 + EE34 + EF34 + \\
&\quad FB34 + FD34 + FE34 + FF34
\end{aligned}$$

The C arrays are defined by the following set of equations:

$$C01(N,M) = \int_a^b \cos \alpha_n^{(1)}(b-y) (\cos \alpha_m^{(1)}(b-y))^* dy$$

$$C02(N,M) = \int_a^b \cos \alpha_n^{(1)}(b-y) (\cos \beta_m^{(2)}(b-y))^* dy$$

$$C03(N,M) = \int_a^b \cos \beta_n^{(2)}(b-y) (\cos \alpha_m^{(1)}(b-y))^* dy$$

$$C04(N,M) = \int_a^b \cos \beta_n^{(2)}(b-y) (\cos \beta_m^{(2)}(b-y))^* dy$$

$$C05(N,M) = \int_a^b \sin \alpha_n^{(1)}(b-y) (\sin \alpha_m^{(1)}(b-y))^* dy$$

$$C06(N,M) = \int_a^b \sin \alpha_n^{(1)}(b-y) (\sin \beta_m^{(2)}(b-y))^* dy$$

$$C07(N,M) = \int_a^b \sin \beta_n^{(2)}(b-y) (\sin \alpha_m^{(1)}(b-y))^* dy$$

$$C08(N,M) = \int_a^b \sin \beta_n^{(2)}(b-y) (\sin \beta_m^{(2)}(b-y))^* dy$$

$$C09(N,M) = \int_0^a \cos \alpha_n^{(3)} y (\cos \alpha_m^{(3)} y)^* dy$$

$$C10(N,M) = \int_0^a \cos \alpha_n^{(3)} y (\cos \beta_m^{(2)} y)^* dy$$

$$C11(N,M) = \int_0^a \cos \beta_n^{(2)} y (\cos \alpha_m^{(3)} y)^* dy$$

$$C12(N,M) = \int_0^a \cos \beta_n^{(2)} y (\cos \beta_m^{(2)} y)^* dy$$

$$C13(N,M) = \int_0^a \sin \beta_n^{(2)} y (\sin \beta_m^{(2)} y)^* dy$$

$$C14(N,M) = \int_0^a \sin \beta_n^{(2)} y (\sin \alpha_m^{(3)} y)^* dy$$

$$C15(N,M) = \int_0^a \sin \alpha_n^{(3)} y (\sin \beta_m^{(2)} y)^* dy$$

$$C16(N,M) = \int_0^a \sin \alpha_n^{(3)} y (\sin \alpha_m^{(3)} y)^* dy$$

$$C17(N,M) = \int_c^d \cos \beta_n^{(1)}(d-x) (\cos \beta_m^{(1)}(d-x))^* dx$$

$$C18(N,M) = \int_c^d \cos \beta_n^{(1)}(d-x) (\cos \alpha_m^{(2)}(d-x))^* dx$$

$$C19(N,M) = \int_c^d \cos \alpha_n^{(2)}(d-x) (\cos \beta_m^{(1)}(d-x))^* dx$$

$$C20(N,M) = \int_c^d \cos \alpha_n^{(2)}(d-x) (\cos \alpha_m^{(2)}(d-x))^* dx$$

$$C21(N,M) = \int_c^d \cos \beta_n^{(3)}(d-x) (\cos \beta_m^{(3)}(d-x))^* dx$$

$$C22(N,M) = \int_c^d \cos \beta_n^{(3)}(d-x) (\cos \beta_m^{(1)}(d-x))^* dx$$

$$C23(N,M) = \int_c^d \cos \beta_n^{(1)}(d-x) (\cos \beta_m^{(3)}(d-x))^* dx$$

$$C24(N,M) = \int_c^d \cos \beta_n^{(3)}(d-x) (\cos \alpha_m^{(2)}(d-x))^* dx$$

$$C25(N,M) = \int_c^d \cos \alpha_n^{(2)}(d-x) (\cos \beta_m^{(3)}(d-x))^* dx$$

$$C26(N,M) = \int_c^d \sin \beta_n^{(1)}(d-x) (\sin \beta_m^{(1)}(d-x))^* dx$$

$$C27(N,M) = \int_c^d \sin \beta_n^{(1)}(d-x) (\sin \alpha_m^{(2)}(d-x))^* dx$$

$$C28(N,M) = \int_c^d \sin \alpha_n^{(2)}(d-x) (\sin \beta_m^{(1)}(d-x))^* dx$$

$$C29(N,M) = \int_c^d \sin \alpha_n^{(2)}(d-x) (\sin \alpha_m^{(2)}(d-x))^* dx$$

$$C30(N,M) = \int_c^d \sin \beta_n^{(3)}(d-x) (\sin \beta_m^{(3)}(d-x))^* dx$$

$$C31(N,M) = \int_c^d \sin \beta_n^{(3)}(d-x) (\sin \beta_m^{(1)}(d-x))^* dx$$

$$C32(N,M) = \int_c^d \sin \beta_n^{(1)}(d-x) (\sin \beta_m^{(3)}(d-x))^* dx$$

$$C33(N,M) = \int_c^d \sin \beta_n^{(3)}(d-x) (\sin \alpha_m^{(2)}(d-x))^* dx$$

$$C34(N,M) = \int_c^d \sin \alpha_n^{(2)}(d-x) (\sin \beta_m^{(3)}(d-x))^* dx$$

APPENDIX 4.FABRICATION OF MICROSTRIP STRUCTURES ON ALUMINA.

This Appendix is concerned with the metalization and photoetching of the Alumina substrates, and, to a lesser extent, with the production of the photographic masks for the photolithographic process.

Throughout this study, Coors ADS 995 Alumina was used as the substrate. This ceramic is 99.5% pure aluminium oxide, Al_2O_3 , with traces of calcium oxide and magnesium silicate. Recently, its structure has been altered by the manufacturers, with the result that its density has increased slightly, the main outward effect, as far as its microwave use is concerned, being an increase of its dielectric constant from 9.8 to 10.3.

In order to obtain good adhesion of the desired metal, in this case copper, a 'seeding' layer of chromium was first deposited onto the Alumina. The chromium forms strong molecular links with the aluminium oxide. Subsequent metals, such as copper or gold, are able to form stronger bonds with the chromium than they would have directly with the Alumina. For good adhesion of the metal to the Alumina, the surface of the Alumina must be very clean. There are several processes by which the substrate can be degreased, and solids and organic impurities, on the surface, be removed. These processes will not be discussed here since little difference between subsequent adhesive properties of any metal films deposited was found for a number of these intricate methods that were tried. A very straightforward method, which was finally adopted was as follows:

- 1) Clean with abrasive powder (if necessary)
- 2) Rinse in tap water
- 3) Rub with liquid detergent
- 4) Rinse in tap water
- 5) Immerse for about 1 minute in 'Neutraclean' (Shipley Chemicals Ltd.)
- 6) Rinse in tap water

- 7) Dry in oven at 100°C , and bake for 1 hour
- 8) Cool in a desiccator

It is likely that this method of cleaning was as effective as the more intricate methods, because of the fact that the evaporator, which was used to deposit the metals, was not used under the cleanest conditions, and contamination of the substrate surface probably occurred during the outgassing of the electron gun, used to heat the chromium. After about $400\text{ \AA}$ of chromium had been deposited, copper, heated in a molybdenum boat, was evaporated together with the chromium for about ten seconds, after which the chromium evaporation was terminated. Copper evaporation was terminated after several thousand angstrom units had been deposited.

The thickness of the copper on each side of the substrate was plated up to a thickness of approximately 0.0005 inch, by an electrolytic process using the following solution:

- 1) 10 litres of copper plating solution, containing:
 - a) Sulphuric acid 56 g/l
 - b) Copper sulphate 200 g/l
 - c) Potash alum 12 g/l
- 2) Thiourea 0.1 g
- 3) Sucrose 2 g
- 4) Teepol 2 g

The substrate to be plated was placed between two copper anodes, and the plating current was adjusted so that the current density was 0.175 amps per square inch of the substrate, for plating on both surfaces. With this rate of deposition, approximately 0.0005 inch of copper is deposited in 35 minutes. A plate/de-plate cycle of 6:1 was used to improve the surface finish of the electroplated copper.

After the plating was finished, the substrate was thoroughly washed in de-ionised water to remove all remnants of the plating solution, as this may cause bad adhesion of the

photosensitive resist, to be applied later. The substrate is then dried and stored in air at room temperature, if possible, for about three weeks to improve the adhesion of the metal film to the alumina (91).

Photosensitive resist AZ1350 (Shipley Chemical Ltd.) was used for the photolithographic process. Before application, the surface was cleaned using 'Neutraclean' (also from Shipley Chemicals Ltd.), washed and then dried in an oven at 40° C. The substrate was cooled in a desiccator. The photoresist, applied with a syringe, was spun onto the substrate at approximately 550 rev/min for two minutes. At this speed, a sufficiently thick layer was achieved avoiding 'pin-holing' due to the surface roughness of the electroplated copper. The photoresist was dried in a re-circulating oven at 40° C for 20 minutes. The photoresist was exposed to ultra-violet light through the required photographic mask for between 45 seconds and 10 minutes. The lamp used was a Philips Ultra-violet lamp, the exposure times, for a lamp to substrate distance of two feet, had a wide range due to ageing of the lamp, the latter figure being reached after a few hundreds of hours use. The photoresist image was produced after 30 seconds of development in a 1:2 dilution of developer with water. After washing it was dried at room temperature. It was found that the final preparation of the surface, application of the photoresist and assembly of the substrate and photographic master, should be carried out in a clean area within the dark room. To this end the walls, floors and ceilings were sealed, and a polythene curtain was erected to seal off half of the dark room. A fan blew air into this half, via a filter, so as to form a positive pressure, the filter removed all but sub-micron sized particles.

The copper was etched using a 60% solution of ferric chloride. The chromium was etched using a solution made from one part of solution A and three parts of solution B, these solutions being as follows:

Solution A	Distilled water	500 ml.
	Sodium hydroxide	250 g.
Solution B	Distilled water	1500 ml.
	Potassium ferricyanide	500 g.

These etchants are very suitable since they do not etch the other metal, however, the photoresist should be removed before the chromium is etched, since it is dissolved by the chromium etchant, reducing the alkalinity of the solution. The substrate should also be well washed after use in the copper etchant, otherwise a reaction between the two etchants occurs which can result in the substrate being stained.

The photographic masters were produced on Ilford N4E-50 line film. N4E-50 is a 0.004 inch thick polyester film giving good dimensional stability, acceptable resolution for microstrip applications and is relatively inexpensive. Being a blue sensitive film, a blue transmission light source was used to illuminate through the master drawing. The master drawing was made using either red 'cut-and-peel' material or black crepe paper on translucent film. Using a Polaroid MP-3 land camera, mounted horizontally, the correct exposure was found to be 3 seconds at an aperture of f/8. Phenisol (Ilford Ltd.), diluted one part of Phenisol to four parts of water, completely developed the image to its ultimate density in about 4 minutes. After rinsing, the image was fixed in IF-23 (Ilford Ltd.) for a time equal to twice the time taken for the film to clear. After washing for about 10 minutes, the film can be dried.

REFERENCES.

1. Matthaei, G.L., Young, L., & Jones, E.M.T., 'Microwave Filters, Impedance-Matching Networks and Coupling Structures', McGraw Hill Book Co., 1964, Chapter 13.
2. Young, L. (Editor), 'Advances in Microwaves Volume 1', Academic Press, 1966, pp115-209.
3. Johnson, C.C., 'Field and Wave Electrodynamics', McGraw Hill Book Co., 1965, Chapter 9.
4. Louisell, W.H., 'Coupled mode and parametric electronics', J. Wiley & Sons, Inc., 1960, Chapter 1.
5. Krage, M.K., and Haddad, G.I., 'Characteristics of Coupled Microstrip Transmission Lines - I: Coupled-Mode Formulation of Inhomogeneous Lines', IEEE Transactions, MTT-18, April, 1970, pp 217-222.
6. Scott, R.E., 'Linear Circuits', Addison-Wesley Publishing Co. Inc., 1960, pp 176-180.
7. Okugawa, S., 'Calculation of characteristics of embedded coupled microstrip lines with finite thickness', Electron. Lett. 8, 5th Oct., 1972, pp 494-495.
8. Krage, M.K., and Haddad, G.I., 'Characteristics of Coupled Microstrip Transmission Lines - II: Evaluation of Coupled-Line Parameters', IEEE Transactions, MTT-18, 4, April, 1970, pp 222-228.
9. Krage, M.K., and Haddad, G.I., 'Frequency-Dependent Characteristics of Microstrip Transmission Lines', IEEE Transactions, MTT-20, 10, October, 1972, pp 678-695.

10. Kowalski, G., and Pregla, R., 'Dispersion Characteristics of Single and Coupled Microstrips with Double-Layer Substrates', Band 27, March, 1973, pp 125-130. A.E.Ü.
11. de Ronde, F.C., 'A new class of microstrip directional couplers', IEEE Transactions, G-MTT Int. Symp. Dig., May, 1970, pp 184-186.
12. Garcia, J.A., 'A Wide-Band Quadrature Hybrid Coupler', IEEE Transactions, MTT-19, July, 1971, pp 660-661.
13. Troughton, P. 'Design of Complex Microstrip Circuits by Measurement and Computer Modelling', Proc. IEE, 188, 3/4, 1971, paper 6371E.
14. Troughton, P., 'Measurement Techniques in Microstrip', Electron. Lett., 5, pp 25-26.
15. Denlinger, E.J. 'Frequency Dependence of a Coupled Pair of Microstrip Lines', IEEE Transactions, MTT-18, 1970, pp 731-732.
16. Arnold, S. 'Dispersive effects in microstrip on alumina substrates', Electron. Lett., 5, 1969, pp 673-674.
17. Zysman, G.I., and Varon, B., 'Wave propagation in microstrip transmission lines', G-MTT Int. Symp. Dig., (Dallas, Texas), 5-7th May, 1969.
18. Getsinger, W.J., 'Microstrip Dispersion Model', IEEE Transactions, MTT-21, Jan., 1973, pp 34-39.
19. Bryant, T.G., and Weiss, J.A., 'Parameters of Microstrip', Electron. Lett., 5, 1969, pp 517-519.
20. Carlin, H.J., 'A simplified model for Microstrip', IEEE Transactions, MTT-21, 1973, pp 589-591.

21. Mittra, R., and Itoh, T., 'A New Technique for the Analysis of the Dispersion Characteristics of Microstrip Lines', IEEE Transactions, MTT-19, Jan., 1971, pp 47-56.
22. Kowalski, G., and Pregla, R., 'Dispersion Characteristics of Shielded Microstrips with Finite Thickness', Arch. Elektr. & Übertragungstechnik, Band 25, Heft 4, 1971, pp 193-196.
23. Schmitt, H.J., and Sarges, K.H., 'Wave Propagation in Microstrip', Nachrichtentechnische Z., Heft 5, 1971, pp 260-264.
24. Napoli, L.S., and Hughes, J.J., 'Characteristics of Coupled Microstrip Lines', R.C.A. Rev., September, 1970.
25. Johnson, K.M., 'A High Performance Integrated Microwave Circuit Frequency Quadrupler', IEEE Transactions, SC-3, June, 1968, pp 82-86.
26. Troughton, P., 'High Q Factor Resonators in Microstrip', Electron. Lett., 4, 1968, pp 520-522.
27. Wolff, I., and Knoppik, N., 'Microstrip Ring Resonators and Dispersion Measurements on Microstrip Lines', Electron. Lett., 7, 1971, pp 779-781.
28. Wu, Y.S., and Rosenbaum, F.J., 'Mode Chart for Microstrip Ring Resonators', IEEE Transactions, MTT-21, 1973, pp 487-489.
29. Wested, J., and Andersen, E., 'Resonance Splitting in Non-Uniform Ring Resonators', Electron. Lett., 8, 1972, pp 301-302.
30. Wolff, I., 'Microstrip Bandpass Filter using Degenerate Modes of a Microstrip Ring Resonator', Electron. Lett., 8, 1972, pp 302-303.

31. Bryant, T.G., and Weiss, J.A., 'Parameters of Microstrip Transmission Lines and Coupled Pairs of Microstrip Lines', IEEE Transactions, MTT-16, 1968, pp 1021-1027.
32. Cohn, S.B., 'Slot Line on a Dielectric Substrate', IEEE Transactions, MTT-17, December, 1969, pp 768-778.
33. Collin, R.E., 'Field Theory of Guided Waves', McGraw Hill, 1960, p 10, 121-125.
34. Ramo, S., Whinnery, J.R., and Van Duzer, T., 'Fields and Waves in Communication Electronics', J. Wiley and Sons, Inc., 1965, p 585.
35. King, R.W.P., 'Transmission Line Theory', Dover Publications, Inc., pp 282-283.
36. Seckelmann, R., 'Nodal Shift Measurements to determine Transmission Line Properties', Microwave J., 1968, 11, pp 69-72.
37. Shurmer, H.V., 'Swept-Frequency Microwave Measurements with On-Line Computer Correction', Annual Rept. for 1970, The Univ. of Warwick.
38. Shurmer, H.V., 'Low-Level Programming for the on-line correction of Microwave Measurements', Radio & Electron. Eng., 1972, 41, pp 357-364.
39. Da Silva, E.F., and McPhun, M.K., 'Calibration of a Microwave Network Analyser for Computer-Corrected S Parameter Measurements', Electron. Lett., 9, pp 126-128.
40. Horton, R., Easter, B., and Copinath, A., 'Variation of Microstrip Losses with Thickness of Strip', Electron. Lett., 7, 26th August, 1971, pp 490-491.

41. 'Zirconate Substrate Ceramic Z2 - Specification', G.E.C. Ltd., Hirst Res. Centre, Wembley, Middlesex.
42. Caulton, M., Hughes, J.J., and Sobel, H., 'Measurement of the Properties of Microstrip Transmission Lines for Microwave Integrated Circuits', IEEE Transactions, MTT-19, January, 1971, pp 30-39.
43. Deutsch, J., and Jung, H.J., 'Measurement of the Effective Dielectric Constant of Microstrip Lines in the Frequency Range from 2 GHz to 12 GHz', Nachrichtentechnische Z., December, 1970, pp 620-624.
44. Napoli, L.S., and Hughes, J.J., 'High Frequency Behaviour of Microstrip Transmission Lines', R.C.A. Rev., 30, June, 1969, pp 268-270.
45. Kowalski, G., and Pregla, R., 'Dispersion Characteristics of Single and Coupled Microstrips', Arch. Elektr. & Übertragungstechnik, 26, June, 1972, pp 276-280.
46. Wheeler, H.A., 'Transmission-Line Properties of Parallel Strip Separated by a Dielectric Sheet', IEEE Transactions, MTT-13, March, 1965, pp 172-185.
47. Green, H.F., 'The Numerical Solution of some important Transmission Line Problems', IEEE Transactions, MTT-13, September, 1965, pp 676-692.
48. Silvester, P., 'TEM Wave Properties of Microstrip Transmission Lines', Proc. IEE, 115, January, 1968, pp 43-48.
49. Yamashita, E., and Mittra, R., 'Variational Method for the Analysis of Microstrip Lines', IEEE Transactions, MTT-16, April, 1968, pp 251-256.

50. Stinehelfer, H.E., 'An Accurate Calculation of Uniform Microstrip Transmission Lines', IEEE Transactions, MTT-16, July, 1968, pp 439-444.
51. Yamashita, E., 'Variational Method for the Analysis of Microstrip-Like Transmission Lines' IEEE Transactions, MTT-16, August, 1968, pp 529-535.
52. Judd, S.V., Whiteley, I., Clowes, R.J., and Rickard, D.C., 'An Analytical Method for Calculating Microstrip Transmission Line Parameters', IEEE Transactions, MTT-18, February, 1970, pp 78-87.
53. Mittra, R., and Itoh, T., 'Charge and Potential Distributions in Shielded Striplines', IEEE Transactions, MTT-18, March, 1970, pp 149-156.
54. Eaves, R.E., and Bolle, D.M., 'Guided Waves in Limit Cases of Microstrip', IEEE Transactions, MTT-18, April, 1970, pp 231-232.
55. Farrar, A., and Adams, A.T., 'A Potential Theory Method for Covered Microstrip', IEEE Transactions, MTT-21, July, 1973, pp 494-495.
56. Deschamps, G.A., 'Theoretical Aspects of Microstrip Waveguides', Trans. IRE, MTT-2, January, 1954, pp 100-103.
57. Black, K.G., and Higgins, T., 'Rigorous Determination of the Parameters of Microstrip Lines', Trans. IRE, MTT-3, March, 1955, pp 93-113.
58. Wu, T.T., 'Theory of the Microstrip', J. Appl. Phys., 28, March, 1957, pp 299-302.
59. Hornsby, J.S., and Gopinath, A., 'Fourier Analysis of a Dielectric-Loaded Waveguide with a Microstrip Line', Electron. Lett., 5, 12th June, pp 265-267. 1969

60. Hornsby, J.S., and Copinath, A., 'Numerical Analysis of a Dielectric-Loaded Waveguide with a Microstrip Line-Finite Difference Methods', IEEE Transactions, MTT-17, September, 1969, pp 684-690.
61. Gelder, D., 'Numerical Determination of Microstrip Properties using the Transverse Field Components', Proc. IEE, 117, April, 1970, pp 699-703.
62. Davis, J.B., and Corr, D.G., 'Computer Analysis of Fundamental and Higher-Order Modes in Single and Coupled Microstrip', Electron. Lett., 6, December, 1970, pp 806-808.
63. Daly, P., 'Hybrid-Mode Analysis of Microstrip by Finite-Element Methods', IEEE Transactions, MTT-19, January, 1971, pp 19-25.
64. Denlinger, E.J., 'A Frequency Dependent Solution for Microstrip Transmission Lines', IEEE Transactions, MTT-19, January, 1971, pp 30-39.
65. Mittra, R., and Itoh, T., 'A New Technique for the Analysis of the Dispersion Characteristics of Microstrip Lines', IEEE Transactions, MTT-19, January, 1971, pp 47-56.
66. Corr, D.G., and Davis, J.B., 'Computer Analysis of the Fundamental and Higher-Order Modes in Single and Coupled Microstrip', IEEE Transactions, MTT-20, October, 1972, pp 669-678.
67. LaRiviere, H.J.A., 'Methods for Solving Microstrip and Related Transmission Line Problems', Ph.D. Thesis, Department of Electronic and Electrical Engineering, University College London, June, 1973.
68. Getsinger, W.J., 'Dispersion of Parallel-Coupled Microstrip', IEEE Transactions, MTT-21, March, 1973, pp 144-145.

69. Carlin, H.J., 'A Simplified Circuit Model for Microstrip' IEEE Transactions, MTT-21, September, 1973, pp 589-591.
70. Mittra, R., and Lee, S.W., 'Analytical Techniques in the Theory of Guided Waves', Macmillan, 1971, pp 4-11.
71. Mittra, R., 'Relative Convergence of the Solution of a Doubly Infinite Set of Equations', J. Res. of NBS - Dept. Rad. Prop., Vol. 67D, No. 2, March-April, 1963.
72. Davis, J.B., 'A Least-Squares Boundary Residual Method for the Numerical Solution of Scattering Problems', IEEE Transactions, MTT-21, February, 1973, pp 99-104.
73. Wilkinson, J.H., 'The Algebraic Eigenvalue Problem', Clarendon Press, 1965, pp 172-174.
74. Fox, L., 'An Introduction to Numerical Linear Algebra', Clarendon Press, 1964, pp 46-48.
75. Wilkinson, J.H., and Reinsch, C., 'Handbook for Automatic Computation, Volume II: Linear Algebra', Springer-Verlag, 1971.
76. *ibid.*, pp 212-226.
77. *ibid.*, pp 249-256.
78. *ibid.*, pp 372-395.
79. *ibid.*, pp 339-358.
80. *ibid.*, pp 315-326.
81. Fox, L., Henrici, P., and Moler, C., 'Approximations and Bounds for eigenvalues of elliptic operators, Siam J. Numer. Anal, 4, no. 1., 1967.

82. Bulley, R.M., and Davis, J.B., 'Computation of Approximate Polynomial Solutions to TE Modes in arbitrarily shaped Waveguides', IEEE Transactions, MTT-17, August, 1969, pp 440-446.
83. Beaubien, M.J., and Wexler, A., 'Unequal-Arm Finite-Difference Operator in the Positive-Definite Successive Overrelaxation (PDSOR) Algorithm', IEEE Transactions, MTT-18, December, 1970, pp 1132-1149.
84. Napoli, L.S., and Hughes, J.J., 'Foreshortening of Microstrip Open Circuits on Alumina Substrates', IEEE Transactions, MTT-19, January, 1971, pp 559-561.
85. Coors Data Sheet, Bulletin 1400, November, 1971, Coors Porcelain Co., 600 Ninth St., Golden, Colorado, U.S.A.
86. Napoli, L.S., and Hughes, J.J., Private Correspondence, 21st December, 1971.
87. Wight, J.S., Jain, O.P., Chudobiak, W.J., and Makios, V., 'Equivalent Circuits of Microstrip Impedance Discontinuities and Launchers', IEEE Transactions, MTT-22, January, 1974, pp 48-52.
88. Hofmann, H., 'Relative Convergence in mode-matching solutions of microstrip problems', Electron. Lett., 10, 18th April, 1974, pp 276-277.
89. Hofmann, H., 'Dispersion of ferrite-filled microstrip-line' Arch. Elektronik & Übertragungstechnik, 28, 1974 (to be published).
90. Gould, J.W., 'Asymmetric Branch Line Directional Couplers in Microstrip', B.Sc. Special Project, January, 1970, The City University.

91. Maissel, L.I., and Glang, R., 'Handbook of Thin Film Technology', McGraw Hill Book Co., 1970.


```

PROGRAM G039 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
DIMENSION YMAX(8),XMAX(8),YMIN(8),XMIN(8),YP1(8),XP1(8),YP2(8),
1XP2(8),YNMAX(8),YNMIN(8),YFIT(6),XFIT(6)
DIMENSION PDX(4),II(8),JJ1(4),XPDX1(8),XPDX2(8)
5 COMMON /COM1/IX,DX,YLAST,XLAST,BD,Z,YNEXT
COMMON /COM2/GL
II(1)=1
II(2)=1
II(3)=2
10 II(4)=2
II(5)=3
II(6)=3
II(7)=4
II(8)=4
15 DO 19 L=1,8
YNMIN(L)=0.0
XMIN(L)=0.0
YNMAX(L)=0.0
XMAX(L)=0.0
20 19 CONTINUE
1 READ(5,2) GL,ISTOP
2 FORMAT(1X,F6.4,2X,I1)
WRITE(6,2) GL,ISTOP
BD=0.025
25 DO 26 L=1,2
DO 4 IZ=1,21
Z=0.95+0.05*FLOAT(IZ)
IF (L.EQ.2) Z=1.0/Z
K=0
30 DX=0.0
PDX(1)=0.0
PDX(2)=0.0
PDX(3)=0.0
PDX(4)=0.0
35 5 IX=0
X1=0.0
X2=3.14159
J=1
40 6 M=1
I=II(J)
DX=PDX(I)
CALL GOLDEN(X1,X2,X,Y,M)
YNMAX(J)=YNEXT
YMAX(J)=Y
45 XMAX(J)=X
XPDX1(J)=X+XPDX
IF (J.GT.1) GO TO 7
IX=0
GO TO 8
50 7 YLAST=YNMIN(J-1)-3.5
XLAST=XMIN(J-1)
X1=XMIN(J-1)+2.5
X2=XMIN(J-1)+3.5
8 M=0
55 CALL GOLDEN(X1,X2,X,Y,M)

```

G039 000
G039 005
G039 006
G039 007
G039 008
G039 009
G039 010
G039 011
G039 012
G039 013
G039 014
G039 015
G039 016
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G039 035
G039 040
G039 041
G039 042
G039 043
G039 044
G039 045
G039 050
G039 055
G039 060
G039 065
G039 070
G039 086
G039 088
G039 075
G039 077
G039 080
G039 085
G039 089
G039 090
G039 095
G039 100
G039 105
G039 110
G039 112
G039 113
G039 115
G039 120

YNMIN(J)=YNEXT

G039 123

YMIN(J)=Y

G039 125

XMIN(J)=X

G039 130

XPD_X2(J)=X+X_KDX

G039 133

J=J+1

G039 135

X1=XMAX(J-1)+2.5

G039 140

X2=XMAX(J-1)+3.7

G039 145

IF(L.EQ.2) X2=XMAX(J-1)+3.5

G039 147

YLAST=YNMAX(J-1)-4.0

G039 150

IF(L.EQ.2) YLAST=YNMAX(J-1)-2.8

G039 153

XLAST=XMAX(J-1)

G039 155

IF(J.LT.9) GO TO 6

G039 160

IF(K.EQ.1) GO TO 11

G039 210

DO 10 J=1,8

G039 170

YP1(J)=YMAX(J)

G039 175

XP1(J)=XMAX(J)

G039 180

YP2(J)=YMIN(J)

G039 185

XP2(J)=XMIN(J)

G039 190

10 CONTINUE

G039 195

JJ1(1)=1

G039 196

JJ1(2)=3

G039 197

JJ1(3)=5

G039 198

JJ1(4)=6

G039 199

DO 21 I=1,4

G039 200

J1=JJ1(I)

G039 201

J2=J1+2

G039 202

DO 24 K=1,3

G039 203

K1=J1+K-1

G039 204

YFIT(K)=YMAX(K1)

G039 205

YFIT(K+3)=YMIN(K1)

G039 206

XFIT(K)=XMAX(K1)

G039 207

XFIT(K+3)=XMIN(K1)

G039 208

24 CONTINUE

G039 209

CALL FIT(YFIT,XFIT,SLOPE1,CONST1,1,3)

G039 210

CALL FIT(YFIT,XFIT,SLOPE2,CONST2,4,6)

G039 212

AMINX=XFIT(1)

G039 213

AMAXX=0.0

G039 214

DO 25 K=1,6

G039 215

IF(XFIT(K).LT.AMINX) AMINX=XFIT(K)

G039 216

IF(XFIT(K).GT.AMAXX) AMAXX=XFIT(K)

G039 217

25 CONTINUE

G039 218

XFIT(1)=(AMINX+AMAXX)/2.0

G039 219

YFIT(1)=SLOPE1*XFIT(1)+CONST1

G039 220

YFIT(2)=SLOPE2*XFIT(1)+CONST2

G039 221

AVGY=(YFIT(1)+YFIT(2))/2.0

G039 222

PDX(I)=-AVGY/XFIT(1)

G039 223

21 CONTINUE

G039 224

K=1

G039 225

GO TO 5

G039 226

11 IF(IZ.GT.1) GO TO 12

G039 230

WRITE(6,13) BD

G039 295

13 FORMAT(/1X,33HNORMALISED JUNCTION SUSCEPTANCE =,F6.4)

G039 240

WRITE(6,14)

G039 245

14 FORMAT(/7X,6HNORM Z,3X,14HMAXIMAS X ,3X,14HMINIMAS X ,

G039 250

14X,14HMAXIMAS X ,3X,14HMINIMAS X)

G039 255

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100

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110

	12	DO 18 J=1,8	G039 261
		YMAX(J)=YMAX(J)*360.0/(2.0*3.14159)	G039 262
		YMIN(J)=YMIN(J)*360.0/(2.0*3.14159)	G039 263
		YP1(J)=YP1(J)*360.0/(2.0*3.14159)	G039 264
115		YP2(J)=YP2(J)*360.0/(2.0*3.14159)	G039 265
	18	CONTINUE	G039 266
		K=1	G039 268
		WRITE(6,15) Z,YP1(K),XP1(K),YP2(K),XP2(K),YMAX(K),XMAX(K),YMIN(K),	G039 269
		1XMAX(K)	G039 270
120	15	FORMAT(/6X,1H ,F6.4,2(2X,F8.3,1X,F7.4,1X,F8.3,1X,F7.4))	G039 271
		DO 16 J=2,8	G039 272
		WRITE(6,17) YP1(J),XP1(J),YP2(J),XP2(J),YMAX(J),XMAX(J),YMIN(J),	G039 290
		1XMIN(J)	G039 292
	17	FORMAT(1H ,12X,2(2X,F8.3,1X,F7.4,1X,F8.3,1X,F7.4))	G039 297
125	16	CONTINUE	G039 310
	4	CONTINUE	G039 315
	26	CONTINUE	G039 320
		IF(ISTOP.NE.1) GO TO 1	G039 325
		STOP	G039 335
130		END	G039 340

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
4102 G039

VARIABLES	SN	TYPE	RELOCATION
4776 AMAXX		REAL	4775 AMINX REAL
4777 AVGY		REAL	4 BD REAL COM1
4772 CONST1		REAL	4774 CONST2 REAL
1 DX		REAL	0 GL REAL COM2
4763 I		INTEGER	5140 II INTEGER ARRAY
4754 ISTOP		INTEGER	0 IX INTEGER COM1
4755 IZ		INTEGER	4761 J INTEGER
5150 JJ1		INTEGER	4766 J1 INTEGER
4767 J2		INTEGER	4756 K INTEGER
4770 K1		INTEGER	4753 L INTEGER
4762 M		INTEGER	5134 PDX REAL ARRAY
4771 SLOPE1		REAL	4773 SLOPE2 REAL
4764 X		REAL	5126 XFIT REAL ARRAY
3 XLAST		REAL	5010 XMAX REAL ARRAY
5030 XMIN		REAL	5154 XPDX1 REAL ARRAY
5164 XPDX2		REAL	5050 XP1 REAL ARRAY
5070 XP2		REAL	4757 X1 REAL
4760 X2		REAL	4765 Y REAL
5120 YFIT		REAL	2 YLAST REAL COM1
5000 YMAX		REAL	5020 YMIN REAL ARRAY
6 YNEXT		REAL	5100 YNMAX REAL ARRAY
5110 YNMIN		REAL	5040 YP1 REAL ARRAY
5060 YP2		REAL	5 Z REAL COM1

FILE NAMES	MODE	2036	OUTPUT	0	TAPE5	FMT	2036	TAPE6	FMT
0 INPUT									

EXTERNALS	TYPE	ARGS	2036	OUTPUT	0	TAPE5	FMT	2036	TAPE6	FMT
0 FIT		6								
0 GOLDEN		5								

STATEMENT LABELS	2036	OUTPUT	0	TAPE5	FMT	2036	TAPE6	FMT
4137 1	4626	2				0	4	
4173 5	4200	6				4225	7	
4241 8	0	10				4451	11	
4457 12	4642	13				4653	14	FMT
4701 15	0	16				4722	17	FMT
0 18	0	19				0	21	
0 24	0	25				0	26	

COMMON BLOCKS	LENGTH
COM1	7
COM2	1

STATISTICS	PROGRAM LENGTH	576
PROGRAM LENGTH	11008	576
BUFFER LENGTH	40748	2108
COMMON LENGTH	108	8


```

SUBROUTINE GOLDEN (AA,BB,XX,Y,M)
DIMENSION A(21),B(21),X1(21),X2(21),YX1(21),YX2(21)

GOLDEN SECTION ALGORITHM FOR MAXIMISING OR MINIMISING THE
FUNCTION Y=F(X)

```

```

      AA  INITIAL STARTING POINT FOR X
      BB  DITTO
      Y    PEAK FUNCTION VALUE
      M    1 FOR MAX, 0 FOR MIN

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```

      X=XX
      N=1
      TAU=0.618
      A(1)=AA
      B(1)=BB
      X1(1)=B(1)-TAU*(B(1)-A(1))
      X2(1)=A(1)+TAU*(B(1)-A(1))
4     X=X1(N)
      YX1(N)=DYDX(X)
      X=X2(N)
      YX2(N)=DYDX(X)
      N=N+1
      IF (N.GT.20) GO TO 2
      IF (M.5,5,6
5     IF (YX1(N-1).LT.YX2(N-1)) GO TO 3
      GO TO 7
6     IF (YX1(N-1).GT.YX2(N-1)) GO TO 3
      A(N)=X1(N-1)
      B(N)=B(N-1)
      X1(N)=X2(N-1)
      X2(N)=B(N-1)-(1.0-TAU)*(B(N-1)-A(N))
      GO TO 4
3     B(N)=X2(N-1)
      A(N)=A(N-1)
      X2(N)=X1(N-1)
      X1(N)=A(N-1)+(1.0-TAU)*(B(N)-A(N-1))
      GO TO 4
2     IF (YX2(N-1).GT.YX1(N-1)) X=X2(N-1)
      IF (YX2(N-1).LT.YX1(N-1)) X=X1(N-1)
      D=ABS((YX2(N-1)-YX1(N-1))/YX2(N-1))
      IF (D.LT.10.0E-15) X=(X1(N-1)+X2(N-1))/2.0
      Y=DYDX(X)
      XX=X
      RETURN
      END

```

```

GOLD 000
GOLD 005
GOLD 010
GOLD 015
GOLD 020
GOLD 025
GOLD 030
GOLD 035
GOLD 040
GOLD 045
GOLD 050
GOLD 054
GOLD 055
GOLD 060
GOLD 065
GOLD 070
GOLD 075
GOLD 080
GOLD 085
GOLD 090
GOLD 095
GOLD 100
GOLD 105
GOLD 110
GOLD 115
GOLD 120
GOLD 125
GOLD 130
GOLD 135
GOLD 140
GOLD 145
GOLD 150
GOLD 155
GOLD 160
GOLD 165
GOLD 170
GOLD 175
GOLD 180
GOLD 185
GOLD 190
GOLD 192
GOLD 195
GOLD 200
GOLD 203
GOLD 205
GOLD 210

```


SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
2 GOLDEN

VARIABLES	SN	TYPE	RELOCATION
215 A		REAL	0 AA REAL F.P.
242 B		REAL	0 BB REAL F.P.
214 D		REAL	0 M INTEGER F.P.
212 N		INTEGER	213 TAU REAL
211 X		REAL	0 XX REAL F.P.
267 X1		REAL	314 X2 REAL ARRAY
0 Y		REAL	341 YX1 REAL ARRAY
366 YX2		REAL	

EXTERNALS	TYPE	ARGS	
ABS	REAL	1	DYDX REAL 1

STATEMENT LABELS

130 2		106 3	31 4
0 5	INACTIVE	57 6	64 7

STATISTICS

PROGRAM LENGTH	4138	267
----------------	------	-----

SUBROUTINE FIT(Y,X,SLOPE,CONST,N1,N2)

FIT 000

DIMENSION Y(10),X(10)

FIT 005

SUMX=0.0

FIT 010

SUMY=0.0

FIT 015

SUMXX=0.0

FIT 020

SUMXY=0.0

FIT 025

DO 1 J=N1,N2

FIT 030

SUMX=SUMX+X(J)

FIT 035

SUMY=SUMY+Y(J)

FIT 040

SUMXX=SUMXX+X(J)*X(J)

FIT 045

SUMXY=SUMXY+X(J)*Y(J)

FIT 050

1 CONTINUE

FIT 055

AN=FLOAT(N2+1-N1)

FIT 053

$$SLOPE = (SUMXY/SUMX - SUMY/AN) / (SUMXX/SUMX - SUMX/AN)$$

FIT 060

CONST=SUMY/AN-SLOPE*SUMX/AN

FIT 065

RETURN

FIT 070

END

FIT 075

5

10

15

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
2 FIT

VARIABLES	SN	TYPE	RELOCATION
101 AN		REAL	0 CONST REAL F.P.
100 J		INTEGER	0 N1 INTEGER F.P.
0 N2		INTEGER	0 SLOPE REAL F.P.
74 SUMX		REAL	76 SUMXX REAL
77 SUMXY		REAL	75 SUMY REAL
0 X		REAL	0 Y REAL ARRAY F.P.

EXTERNALS	TYPE	ARGS
0 FLOAT	REAL	1

STATEMENT LABELS
0 1STATISTICS
PROGRAM LENGTH 122B 82


```

FUNCTION DYDX(XX)
REAL M,NV
COMMON /COM1/IX,DX,YLAST,XLAST,BD,Z,YNEXT
COMMON /COM2/GL
5  X=XX
   AL=9.0
   FREQ=30.0*X/(2.0*3.14159*AL)
   CAP=BD/0.314159
   BD1=3.14159*FREQ*CAP/10.0
10  XPDX=X+DX*X/2.0
   X1=34.7*X/35.0
   X2=0.3*X/35.0
   T1=TAN(X1)
   S2=SIN(X2)
15  C2=COS(X2)
   XV=Z*(T1*C2+S2-Z*BD1*T1*S2)
   YV=C2-Z*BD1*T1*C2-T1*S2
   M=XV*TANH(GL)-(C2*(T1+Z*BD1)+S2)
   NV=YV*TANH(GL)+(S2*(Z*T1+Z*Z*BD1)-Z*C2)
20  P=XV*TANH(GL)+(C2*(T1+Z*BD1)+S2)
   Q=S2*(Z*T1+Z*Z*BD1)-Z*C2-YV*TANH(GL)
   C=(NV*P-Q*M)/(M*P+Q*N)
   Y=ATAN(C)
   IF(IX.NE.0)GO TO 1
25  IX=1
   YLAST=0.0
   XLAST=0.0
   1 IF(X.NE.0.0)GO TO 2
   Y=0.0
30  YNEXT=0.0
   XPDX=0.0
   2 XNEXT=X
   IF(Y)30,31,32
35  N=-1
   GO TO 33
   31 IF(X)32,34,32
   34 YNEXT=0.0
   GO TO 35
   32 N=0
40  33 N=N+1
   YNEXT=-(FLOAT(N)*3.14159-Y)
   D=YNEXT-YLAST
   IF(X.GT.XLAST)GO TO 36
   IF(ABS(D).LE.3.14159)GO TO 37
45  GO TO 33
   37 IF(D)38,36,36
   38 YNEXT=YNEXT+3.14159
   36 IF(D)35,35,33
   35 YLAST=YNEXT
50  DYDX=YNEXT+2.0*XPDX
   XLAST=X
   RETURN
   END

```

```

DYDX 000
G039DYDX
DYDX 005
G039DYDX
DYDX 006
DYDX 007
DYDX 010
DYDX 015
DYDX 020
DYDX 025
DYDX 031
DYDX 031
G039DYDX
G039DYDX
G039DYDX
G039DYDX
G039DYDX
G039DYDX
G039DYDX
G039DYDX
G039DYDX
G039DYDX
G039DYDX
DYDX 045
DYDX 050
DYDX 055
DYDX 060
DYDX 065
DYDX 070
DYDX 075
DYDX 080
DYDX 085
DYDX 090
DYDX 095
DYDX 100
DYDX 105
DYDX 110
DYDX 115
DYDX 120
DYDX 125
DYDX 130
DYDX 135
DYDX 140
DYDX 145
DYDX 146
DYDX 147
DYDX 148
DYDX 149
DYDX 150
DYDX 155
DYDX 160
DYDX 165
DYDX 170
DYDX 175

```


SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
2 DYDX

VARIABLES	SN	TYPE	RELOCATION				
264 AL		REAL		4	BD	REAL	COM1
267 BD1		REAL		302	C	REAL	
266 CAP		REAL		275	C2	REAL	
306 D		REAL		1	DX	REAL	COM1
260 DYDX		REAL		265	FREQ	REAL	
0 GL		REAL	COM2	0	IX	INTEGER	COM1
261 M		REAL		305	N	INTEGER	
262 NV		REAL		300	P	REAL	
301 Q		REAL		274	S2	REAL	
273 T1		REAL		263	X	REAL	
3 XLAST		REAL	COM1	304	XNEXT	REAL	
270 XPDx		REAL		276	XV	REAL	
0 XX		REAL	F.P.	271	X1	REAL	
272 X2		REAL		303	Y	REAL	
2 YLAST		REAL	COM1	6	YNEXT	REAL	COM1
277 YV		REAL		5	Z	REAL	COM1

EXTERNALS	TYPE	ARGS				
ABS	REAL	1		ATAN	REAL	1
COS	REAL	1		FLOAT	REAL	1
SIN	REAL	1		TAN	REAL	1
TANH	REAL	1				

STATEMENT LABELS

153	1		160	2		0	30	INACTIVE
166	31		171	32		172	33	
0	34	INACTIVE	217	35		215	36	
211	37		0	38	INACTIVE			

COMMON BLOCKS	LENGTH
COM1	7
COM2	1

STATISTICS

PROGRAM LENGTH	3078	199
COMMON LENGTH	108	8