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A Homogeneous Model for Heat Transfer
to a Turbulently Flowing Solid-Liquid
Suspension

by

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A Thesis Submitted for the Degree of
Doctor of Philosophy

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Abstract

The problem of forced convective heat transfer to a suspension of small solid particles in a liquid is treated by developing a homogeneous equilibrium flow model. Suitable average properties are determined for the mixture which is then treated as a pseudofluid, obeying the usual equations of single-component flow.

The evaluation of suspension effective thermal conductivity from component values is considered first. Following a critical examination of the approach assuming a unidirectional heat flux, two digital simulation models are developed which eliminate this restriction and the requirement for a rigid geometric arrangement of phases. Thermal conductivity values are determined through solving a three-dimensional Laplace equation, using a finite difference technique, to give a steady state temperature distribution for boundary conditions derived from consideration of the Fourier rate equation. Effective thermal conductivities of several aqueous metallic powder suspensions have been experimentally determined using the relative plane horizontal layer method. Overall mean errors in prediction of 2.55 and 6.64 per cent were obtained for the two models. Theoretical and experimental results from previous studies have been used in a comprehensive examination of prediction methods.

The second section develops the homogeneous flow model. An existing numerical solution technique for the boundary layer equations is adapted for use with two-component fluids through the identification of effective two-component property relationships. An eddy diffusivity model based on

the mixing length concept, which is developed and shown to be valid for single component flows, is modified to extend its use to the two-component case. Evaluation of the technique, using the experimental results of Orr and Dalla Valle indicates an overall solution prediction error of 5.5 per cent. Further examination of results for individual suspensions indicates the range of suspension solids for which the model is strictly valid.

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ERRATA

Page	
xii	definition of subscript y : read 'yield value'
8	line 3 : for 'conductivity' read 'shape factor'
31	line 24 : for '3' read 'III'
129	line 26 : add 'fluid' between 'independent' and 'quantity'
136	last line : after 'data' add 'was very similar in appearance to Newtonian heat transfer data'.
263	reference/27/: read MYERS, G.E.
270	definition of V_E : read $V_E = \overline{E^2} - \bar{E}^2$
271	definition of V_R : read $V_R = \overline{R^2} - \bar{R}^2$
272	line 4 : for 'filling' read 'falling'.

NomenclatureUsed in Section A

A	:	cross-sectional area of node domain normal to grid line, m^2
D	:	depth of experimental cell, m
F_1, F_2	:	functions in equation 1.6
h	:	length of grid line, m
K	:	thermal conductivity, $W/m \text{ } ^\circ C$
L	:	length of model side, m
m	:	unit cell parameter defined by equation 2.1
N	:	model division, nodes per co-ordinate direction
n	:	particle shape factor in equation 1.5
No	:	number of basic units assigned in random placement model
P	:	directional normal
P_1	:	one-dimensional porosity
P_2	:	two-dimensional porosity
q	:	heat flux, W/m^2
r, t	:	parameters of binomial distribution equation 1.17
S	:	number of nodes adjacent to node considered
T	:	temperature, $^\circ C$
u, v, w	:	general grid spacing in co-ordinate directions, m
$\bar{W}$	:	relaxation factor
X	:	particle shape factor in equation 1.5
x, y, z	:	cartesian co-ordinate directions
θ	:	pure component conductivity ratio
μ	:	mean value
σ	:	standard deviation

ν	:	coefficient of variation
φ	:	volume fraction solid, concentration
ψ	:	particle sphericity

Subscripts

A	:	refers to component 'A' of Jefferson model
c	:	continuous phase property
d	:	discontinuous phase property
i	:	node adjacent to node under consideration
i,n	:	property of grid line between nodes i and n
L	:	lower test cell
n	:	the n^{th} node, node under consideration
S	:	suspension property
U	:	upper test cell
p,q,r	:	upper, centre and lower test cell discs
in, out	:	model boundary conditions
+	:	updated estimate of temperature

used in section B

A_j, B_j, C_j, D_j	:	coefficients in finite difference equation
C_p	:	specific heat, $J/K_g \text{ } ^\circ C$
D	:	pipe diameter, m
d	:	particle diameter, m
f	:	fanning friction factor
G	:	Mass Flux, $K_g/s. m^2$
g	:	acceleration of gravity, m/s^2

H	:	enthalpy, J
h	:	convection coefficient, $W/m^2\ ^\circ C$
K	:	thermal conductivity $W/m\ ^\circ C$
K'	:	fluid consistency index, $N\ S^n /m^2$
K^*	:	von-Karman constant
l	:	mixing length, m
m	:	mass flow rate, Kg/s
n'	:	fluid behaviour index
P	:	pressure, N/m^2
q	:	heat flux, W/m^2
r	:	radial co-ordinate
T	:	temperature, $^\circ C$
u	:	axial velocity, m/s
v	:	radial velocity, m/s
w	:	weight fraction solid
x	:	axial co-ordinate
y	:	transverse co-ordinate
ϵ_H	:	eddy diffusivity for heat
ϵ_M	:	eddy diffusivity for momentum
η	:	coefficient of rigidity, Kg/ms
μ	:	dynamic viscosity, Kg/ms
ν	:	Kinematic viscosity, m^2/s
ρ	:	density, Kg/m^3
τ	:	shear stress, N/m^2
φ	:	volume fraction solid, concentration

Subscripts

b	:	bulk property
c	:	continuous, liquid phase property
d	:	discontinuous, solid phase property
eff	:	effective property value
i	:	axial index
j	:	radial index
o	:	evaluated at initial conditions
s	:	suspension property
t	:	settling property
w	:	evaluated at wall conditions
y	:	yield at wall conditions

Dimensionless Groups

Nu	:	Nusselt number
Pr	:	Prandtl number
Re	:	Reynolds number
St	:	Stanton number

Dimensionless Terms

$\bar{u}$	=	$u/u_{b,0}$
$\bar{v}$	=	$v/u_{b,0}$
$\bar{r}$	=	r/r_w
$\bar{l}$	=	l/r_w
$\bar{x}$	=	$x/2r_w$
$\bar{y}$	=	y/r_w

$$\bar{p} = p/p_o$$

$$\bar{c}_p = c_p/c_{po}$$

$$\bar{p} = (p_o - p) / p_o u_{b,o}^2$$

$$\bar{h} = (h - h_o) / c_{po} T_o$$

$$\bar{\mu}_{eff} = \mu_{eff} / \mu_o$$

$$\bar{k}_{eff} = k_{eff} / k_o$$

$$q^+ = q_w (p_o, u_{b,o}, T_o, c_{po})$$

$$u^+ = u / u^*$$

$$y^+ = y u^* / \nu$$

$$u^* = \sqrt{\tau / \rho}$$

Preface

Many of the previous studies concerned with the problem of heat transfer to a turbulently flowing two-phase system, where the components are an insoluble solid suspended in a liquid, have attempted to correlate experimental data in terms of the variables used in analysing single component flows. The standard relationships applied to the flow of liquids with heat transfer are fitted to the suspension data; this procedure requires knowledge of equivalent thermodynamic and transport properties for the solid-liquid mixture. Although these suspension properties do not always conform to rigorous definitions it has been found that they are a useful concept in applying experimental evidence to practical situations and there have been some investigations conducted into the prediction of such properties from a knowledge of the relevant values for the constituent components. However, useful as they may be as a starting point, correlations of experimental data are the least sophisticated of techniques available for analysing two-phase flows and although they have the virtue of being easy to use they provide little insight into the basic phenomena nor into ways in which accuracy can be improved. Since the correlations based upon the concept of suspension properties have been fairly successful this suggests the possibility of taking the use of such properties a step further by utilising them in a simple analytical model, with the expectation that an improvement in the prediction of design parameters will be forthcoming.

The use of suspension effective properties, by implication, states that the model will be of a homogeneous type in which the components are treated as a pseudo-fluid with average properties and without the

requirement for a detailed description of the flow pattern. This in turn implies restrictions on the applicability of the technique. If, in addition, the average values of temperature and velocity are the same as the values for each component then we may term the analysis a homogeneous equilibrium flow model. For this model to be successfully applied to the situation of heat transfer to a turbulently flowing solid-liquid suspension requires the careful identification of the pseudo-fluid properties. Of these suspension viscosity and thermal conductivity are not readily determinable.

A detailed examination of the effective thermal conductivity of suspensions was therefore a necessary pre-requisite to the development of the proposed model; the examination conducted included an evaluation of previously proposed prediction techniques, the development of an analytical model which removed certain restrictions on previous work and an experimental investigation. A similar investigation into the viscosity of a suspension whilst certainly required was considered to form a sufficiently detailed project of itself and in consequence could not be attempted during this investigation. Sufficient information on this topic was available to be able to utilise the homogeneous flow model and to examine its validity whilst the incorporation of the results of any subsequent investigations into the viscosity of suspensions or the actual rheological constants measured experimentally would be expected to improve further the accuracy and applicability of the model.

This work therefore is in two distinct sections, the first being an investigation of the effective thermal conductivity of suspensions with the second being the development of a model for predicting the

heat transfer performance of a suspension of small solid particles in a liquid during turbulent flow through a circular tube. Each section would stand as a separate investigation but the input requirements of a homogeneous flow heat transfer model for information such as that provided by the thermal conductivity investigation link the two sections closely.

Section A

The Thermal Conductivity of Solid-Liquid Suspensions.

1. LITERATURE SURVEY

1.1 Introduction

The problem of designing convective heat transfer equipment for use in situations where a mixture of a solid and a liquid may be employed, is frequently encountered, especially in the chemical engineering industry. A commonly employed solution procedure involves what may be termed a homogeneous flow model in which equivalent transport properties for the suspension are determined from the individual component properties. The suspension may then be treated as a pseudo-fluid with the mean properties being utilised in conventional single phase correlations. It is therefore necessary to be able to accurately determine suspension properties especially in those cases where the value for the mixture is not simply a weighted average of the constituent properties. Such a parameter is thermal conductivity, which whilst not rigorously fulfilling the criteria of the defining equation, Fourier's Law, and therefore not being a property in the true thermodynamic sense, proves to be an extremely useful concept in practice.

In spite of a number of studies, both theoretical and experimental, the ability of proposed solutions accurately to predict the effective thermal conductivity of two component systems has been widely held to be limited to low volume fractions of the discontinuous phase and to small values of the ratio of the conductivities of the constituents. With increasing values of these two parameters, in general, the predictions become unreliable.

The purposes of this study are then, firstly, to remove some of the restrictions made in the development of previous prediction methods and to examine the effect on accuracy that this extension produces and,

secondly, to obtain experimental results for the thermal conductivity of representative suspensions in order to assess the models developed and also to re-examine several previous models.

1.2 Previous Theoretical Work

Many models exist for producing an estimate of the thermal conductivity of a given suspension; most of these consider a simplified system in order to facilitate a solution. In general attention is focused upon a particle of the discontinuous phase which is surrounded by a field of the continuous phase and the conductivity of this unit is taken as representative of the value for the whole system. It is possible to divide most previous work into two categories, those employing the so called self-consistent scheme and those using a unidirectional heat flow assumption.

1.2.1 The Self-Consistent Models

These schemes are developed from the assumption that a typical basic element of a heterogeneous medium can be regarded as being embedded in an equivalent homogeneous medium whose conductivity is then the unknown to be calculated.

This method finds its first expression in the equation developed by Maxwell /1/ for determining the electrical conductivity of a medium which had spherical inclusions of uniform size. A restriction, that the inclusions must be at such distances from each other that their effect in disturbing the course of the current may be taken as independent of each other, limited the applicability of the equation to small values of inclusion volume fraction. Tareev /2/, noting the similarity between electrical and thermal fields, proposed the application of the equation

for thermal conductivity purposes, when the suspension conductivity will be given by:-

$$K_s = K_c \left[\frac{2K_c + K_d - 2\phi(K_c - K_d)}{2K_c + K_d + \phi(K_c - K_d)} \right] \quad (1.1)$$

Rayleigh /3/ considered the case of an otherwise uniform medium interrupted by a cubic array of spherical particles. An expression was derived involving the ratio of two infinite series where the higher order terms become more important as the concentration increases. All but the first four terms in each series may be neglected, when the conductivity of the suspension is given by:-

$$K_s = K_c \left[\frac{2K_c + K_d - 2\phi(K_c - K_d) - 0.523(K_c - K_d) \left[\frac{3K_c - 3K_d}{4K_c + 3K_d} \right] \phi^{10/3}}{2K_c + K_d + \phi(K_c - K_d) - 0.523(K_c - K_d) \left[\frac{3K_c - K_d}{4K_c + 3K_d} \right] \phi^{10/3}} \right] \quad (1.2)$$

This is a second approximation to the expression; the first approximation containing the first three terms and being equivalent to Maxwell's equation, thus the extra terms in the Rayleigh expression may be interpreted as being corrections for interference between adjacent spheres.

The Rayleigh equation was also used by Meredith and Tobias /4/ who included an extra term in both the numerator and denominator so that the effect of particle interaction at higher concentrations may be better represented, this gave:-

$$K_s = K_c \left[\frac{2K_c + K_d - 2\varphi(K_c - K_d) + 0.409(K_c - K_d) \left[\frac{6K_c + 3K_d}{4K_c + 3K_d} \right] \varphi^{7/3}}{2K_c + K_d + \varphi(K_c - K_d) + 0.409(K_c - K_d) \left[\frac{6K_c - 3K_d}{4K_c - 3K_d} \right] \varphi^{7/3}} \right. \\ \left. - \frac{2.133(K_c - K_d) \left[\frac{3K_c - 3K_d}{4K_c + 3K_d} \right] \varphi^{10/3}}{0.906(K_c - K_d) \left[\frac{3K_c - 3K_d}{4K_c + 3K_d} \right] \varphi^{10/3}} \right] \quad (1.3)$$

At low values of volume fraction solid the higher order terms become negligible and the above equation produces substantially the same results as the Maxwell and Rayleigh equations.

Fricke /5/ attempted to account for the influence of particle shape on system conductivity by representing the particles as spheroids. The Maxwell equation was modified by inclusion of a parameter, X , which is a function of the ratio of constituent conductivities, θ , and the ratio of the axis of symmetry of the spheroid to the other axes, a/b . For the case of spherical particles $x = 2$ and the equation again reduces to Maxwell's expression, but the determination of X being complicated for other particle shapes curves are given showing the variation of X with θ and a/b . The conductivity of the suspension is determined from:-

$$\frac{\frac{K_s}{K_c} - 1}{\frac{K_s}{K_c} - X} = \varphi \left[\frac{\frac{K_d}{K_c} - 1}{\frac{K_d}{K_c} - X} \right] \quad (1.4)$$

Fricke further concluded that for a constant volume concentration the conductivity of the suspension is independent of the size of the suspended particles and also nearly independent of the form of the particles when the difference between the conductivities of the constituents was not large.

Hamilton and Crosser /6/ also considered the effect of particle shape on suspension conductivity and concluded that, since for spherical particles the conductivity depended on neither the size of the particles nor on the relative size of the conductivities of the two phases, then other particle shapes should be referred to the case of spheres. Maxwell's equation was used, including an empirical constant n , which must be determined experimentally for mixtures containing particles of arbitrary shapes:

$$K_s = K_c \left[\frac{K_d + (n-1)K_c - (n-1)\varphi(K_c - K_d)}{K_d + (n-1)K_c + \varphi(K_c - K_d)} \right] \quad (1.5)$$

In terms of Fricke's 'shape factor' $n = X + 1$ where the particles are spherical.

Hamilton and Crosser found that where mixture components have nearly the same conductivity then the value of $n = 3$, which is the spherical case, could be used for all particle shapes, but when the ratio of component conductivities is greater than 100 there is a significant increase in mixture conductivity where the particles are non-spherical. It was considered that all the experimental data used could be correlated using $n = \frac{3}{\psi}$, where ψ is the sphericity of the particles and is defined as the ratio of the surface area of a sphere with a volume equal to the particle, to the surface area of the particle. Hamilton and Crosser also found that their 'shape factor' was not in agreement with that given by Fricke where the particles were non-spherical.

Strickler /7/ proposed the use of Maxwell's equation with coefficients, F_1 and F_2 , being functions of surface area and conductivity of the solid material, when the conductivity of the suspension would be given by:-

$$K_s = K_c F_1 \left[\frac{K_d(3 - 2\varphi) + 2\varphi K_c}{K_c(3 - \varphi) + \varphi K_d} \right] + F_2 \quad (1.6)$$

No information was, however, forthcoming on how these functions were to be determined.

Bruggeman /8/ sought to extend the equation of Maxwell for use where the conductivity of suspensions of high concentration is required. An arbitrary dilute suspension was taken as being a pseudo-continuous phase whose conductivity would be adequately represented by Maxwell's equation. Considering the addition of a further small quantity of the discontinuous phase, the conductivity of the new suspension could again be found using Maxwell's equation, and again this would be taken as a pseudo-continuous phase. For repeated applications of this equation the conductivity of the suspension approaches a limit given by:-

$$\left[\frac{K_s}{K_c} \right]^{1/3} (1 - \varphi) = \frac{K_d - K_s}{K_d - K_c} \quad (1.7)$$

This equation may be much simplified if $K_d \gg K_c$ when:

$$K_s = \frac{K_c}{(1 - \varphi)^3} \quad (1.8)$$

Hashin /8/ has conducted a thorough analysis of the self-consistent scheme and has pointed out the doubt that exists as to what constitutes the basic element of a heterogeneous medium. A typical inclusion may be considered as the basic element, this leads to the Maxwell equation, or an inclusion surrounded by a matrix shell may be taken as the basic element. The results obtained by the two methods are different and it is difficult to say which is the more reliable. Hashin provides a self-consistent scheme approximation which includes both methods as special cases and concludes that rigorous values of thermal conductivity depend upon the entire phase geometry and not just the volume fraction of the discontinuous phase.

1.2.2 Unidirectional Heat Flux Models.

A second general approach to the determination of the thermal conductivity of suspensions involves the assumption of a geometrical arrangement of the phases and a principal direction of heat transfer, there being no heat flux components normal to this direction. This means, simply, that the method becomes one of combining the phases in a series or parallel network with the problem being the designation of this combination.

The two extreme cases of this method correspond to a combination of the constituent phases of the mixture in either a series or a parallel configuration. These give respectively the minimum and the maximum values of effective conductivity that can be obtained with given components at a given volume fraction. With the two phases combined in series with respect to the direction of heat flow, the overall conductivity is given by:-

$$K_s = \frac{K_c \cdot K_d}{\phi K_c + (1 - \phi) K_d}$$

This corresponds also to a weighted harmonic mean of the conductivities of the two phases.

If the phases are combined in parallel with respect to the direction of heat flow then:-

$$K_s = (1 - \phi)K_c + \phi K_d \quad (1.10)$$

which is the result also of a weighted arithmetic mean of the two phase conductivities.

Of necessity, all other combinations of the phases must give a result that lies between the values given by these two combinations.

Woodside and Messmer /10/ give an expression that yields an intermediate value between these two extremes, being based on a weighted geometric mean of the phase conductivities:-

$$K_s = K_d^\phi \cdot K_c^{1-\phi} \quad (1.11)$$

This may also be expressed as a weighted arithmetic mean of the logarithms of the individual conductivities:-

$$\ln K_s = \phi \ln K_d + (1 - \phi) \ln K_c \quad (1.12)$$

Therefore this equation predicts a linear relationship, on a log-log plot, between the effective conductivity of a two-phase system and the conductivity of the discontinuous phase.

Jefferson et.al. /11/ based their model on a cubic array of spherical particles and took a representative sphere surrounded by a cube of the continuous phase as a basic unit whose conductivity was to be assessed. The particular combination of the phases that was used is comprehensively examined in the following section. The result is expressed as a set of equations which, in the case of the component conductivities differing widely, may be combined to give the following explicit equation for the effective thermal conductivity of a suspension:-

$$K_s = K_c \left[1 - 1.21\phi^{2/3} + 0.4875\phi^{1/3} \left[\frac{\ln \frac{K_d}{K_c} - 1}{0.25 + (0.403\phi^{-1/3} - 0.5)(\ln \frac{K_d}{K_c} - 1)} \right] \right] \quad (1.13)$$

Tsao /12/ considered that bulk porosity alone was not enough to describe the characteristics of the material with regard to its effective thermal conductivity and proposed the additional parameters of one-dimensional porosity P_1 and two-dimensional porosity P_2 . P_1 is the fraction of the linear space occupied by the discontinuous phase parallel to the direction of heat flow, while P_2 is the fraction of the area normal to the heat flow direction that is occupied by the discontinuous phase. The model was divided into slices normal to the heat flow

direction with the discontinuous phase in each layer being re-arranged into a rectangle. The layers were then re-ordered to give a decrease in P_2 along the axis of heat flow. This manipulation of the phases within the model is consistent with the assumption of a unidirectional heat flux. The effective conductivity of the model would then be given by:-

$$K_s = \frac{1}{\int_0^1 \frac{dP_1}{K_c + (K_d - K_c) P_2}} \quad (1.14)$$

A solution requires the definition of a functional relationship between P_1 and P_2 ; and Tsao adopted a probabilistic approach to this problem by using the binominal theorem and assuming that the binominal distribution approached the normal distribution for this case. Thus:

$$P_2 = \int_{P_1}^1 \frac{1}{\sigma \sqrt{2\pi}} e^{-1/2} \left[\frac{P_1 - \mu}{\sigma} \right]^2 dP_1 \quad (1.15)$$

where μ is the mean of P_1 and σ the standard deviation of P_1 . For any given two-phase heterogeneous material these parameters have to be determined experimentally and are therefore not readily obtainable.

This approach has been criticised by Warren and Messmer /13/ who claimed that there was a fundamental deficiency in the mathematical model employed in that it must contain a discontinuity to satisfy the physical conditions of the problem exactly. Additionally they concluded that Tsao did not prove that two independent parameters P_2 and σ are

necessary to characterise the effective thermal conductivity of a disordered two-phase medium and further that the use of a stochastic model implies the assumption that its behaviour is governed solely by its bulk properties.

Cheng and Vachon /14/ extended the model of Tsao by circumventing the necessity of experimentally determining μ and σ , since these parameters are not required. The proposed technique was based upon assuming a parabolic distribution of the discontinuous phase in the continuous phase. As with Tsao's model the discontinuous phase is re-arranged within a series of slices, with the layers again being re-ordered but in a different sequence which gives a parabolic distribution. An expression for the effective thermal conductivity of the model is derived in terms of the constants of the parabola and the thermal conductivity of the constituents. The parabola constants are determined by reducing the normal distribution assumed for the discontinuous phase volume fraction to a parabolic function thus enabling the distribution to be expressed in terms of constants dependant on the proportion of the constituents. This gives the following expression for effective conductivity:-

$$\frac{1}{K_s} = \frac{B^{1/2}}{2[(K_d - K_c)(K_c + B(K_d - K_c))]^{1/2}} \cdot \ln \frac{[K_c + B(K_d - K_c)]^{1/2} + [B(K_d - K_c)]^{1/2}}{[K_c + B(K_d - K_c)]^{1/2} - [B(K_d - K_c)]^{1/2}} + \frac{1 - B}{K_c} \quad (1.16)$$

where $B = \left[\frac{3\phi}{2} \right]^{1/2}$ and $K_d > K_c$

Deshpande and Couper /15/ have also considered Tsao's approach to the provision of some account for the spatial distribution of particles in the prediction of an effective thermal conductivity for suspensions. Claiming that the normal distribution involves a density function which does not integrate to unity over its sample space they proposed the use of the beta distribution, which is the limiting case of the point binomial distribution, for relating P_1 and P_2 . This gave the expression:-

$$P_2 = \frac{\Gamma(r+t)}{\Gamma(r)\Gamma(t)} \int_{P_1}^1 x^{r-1} (1-x)^{t-1} dx \quad (1.17)$$

which could be used in conjunction with equation (1.14) to determine effective conductivity. The r , t parameters of the binomial distribution were obtained from a study by Leek et.al. /16/, who determined the spatial distribution of particles in a solid liquid suspension, using a special spectrophotometer to measure P_1 . Three methods for the evaluation of equation (1.17) were also proposed.

1.2.3 Further Models.

All the prediction methods considered so far lead to an explicit equation for the thermal conductivity of suspensions but it is possible to relax the restriction of a unidirectional heat flow by employing a numerical solution technique on a given model; in this case the whole solution procedure must be followed through for every system studied.

Deissler and Eian /17/ used a numerical relaxation procedure in applying a two-dimensional heat conduction situation to a rectangular array of long cylinders. The results were extended by approximation to the case of a cubic array of spheres.

During the course of this present work two further methods were published, both of which applied a tridirectional heat flux to a particular model. Baxley et.al. /18/, /19/ used a random simulation technique for approximating the two-phase system whilst Fidelle and Kirk /20/ based their work upon the unit cell obtained from the proposal of Jefferson et.al. /11/. These two methods and their relation to the present work are discussed fully in succeeding chapters of this report.

1.3 Previous Experimental Work.

Several experimental studies of the effective thermal conductivity of two-component systems have been conducted for the purpose of evaluating various prediction methods. Lees /21/ determined the conductivity of various liquid-liquid mixtures and also such solid-solid systems as marble-vaseline and iron or zinc sulphate in lard. McAlister and Orr /22/ performed experiments on mixtures of Aluminium and Zinc powders in grease. Johnson /23/ considered principally aqueous thoria suspensions but also investigated copper and aluminium powders suspended in water. Jefferson et.al. /11/ determined the conductivity of graphite powder suspended in either water or silicone oil; Hamilton and Crosser /6/ used balsa or aluminium in rubbers as a two-component medium. Baxley and Couper /18/ considered systems composed of aluminium

oxide-polypropylene glycol, aluminium-polypropylene glycol and selenium-polypropylene glycol. The experimental work of Fidelle and Kirk /20/ was based on carborundum-plaster, aluminium-plaster and alumina-polyester systems; they also considered the effect of system to particle dimensions and concluded that there was a critical ratio of about 10 below which the particulate composite can no longer be called macroscopically homogeneous.

A review article by Gorrington and Churchill /24/ considers the results of several experimental studies and concludes that for spherical particles Maxwell's equation represents the data adequately.

2. ANALYSIS OF THE UNIDIRECTIONAL HEAT FLUX ASSUMPTION

2.1 Introduction.

Previous solutions proposed for the calculation of the thermal conductivity of two-component systems have been based, in general, on a model having a regular arrangement of particles of the discontinuous phase from which an effective conductivity can be determined using the parameters of volume fraction solid and component conductivities. Those solutions that are not confined to an assumption of a unidirectional heat flux lead to extremely complicated mathematics and are simplified by assuming that interaction of the fields surrounding each particle does not occur. For this condition to be satisfied the volume fraction solid must be low and the component conductivities not differ greatly; thus a severe restriction is placed upon the applicability of the model.

The use of a model in which the effective thermal conductivity is assessed following the assumption of a geometrical arrangement of spherical particles and a unidirectional heat flux through the system has considerable appeal, being simple in concept and leading to solutions that may be readily evaluated. One such model is that proposed by Jefferson et.al. /11/ and in the development of a suitable prediction method a detailed examination of this model and the validity of the assumptions made was first undertaken. This prediction method is widely used and it has been suggested by McAlister and Orr /22/ that although some discrepancy exists between experimental and predicted conductivities from this method the arguments put forward in its derivation are apparently valid. Fidelle and Kirk /20/ maintain that the assumption of a unidirectional heat flux is not very restrictive.

However it can be shown that not only does the assumption of a unidirectional heat flux lead to an inherent inconsistency but also to the requirement of reference to experimental evidence in the formulation of an explicit equation for the prediction of the thermal conductivity of two-component systems. Although the details of the analysis refer only to the one model, the conclusions are nonetheless valid for the case of other solutions that depend upon the same assumptions.

The models of Tsao /12/ and Cheng and Vachon /14/ whilst apparently differing considerably from the Jefferson model in reality differ only in respect of their system dimensions. This arises from the assumption of statistical distribution functions for the phases whilst with the Jefferson model the distribution function is implicit in the use of a cubic array of particles. Differences occur in the derivation of an explicit expression for the effective thermal conductivity of each model, as will be shown this is purely arbitrary. However given the method used by both Tsao and Cheng and Vachon, of dividing the model into elementary strips perpendicular to the direction of heat flow, the subsequent re-ordering of these strips will have no effect on the prediction given the assumption of a unidirectional heat flux. This re-ordering of elements is merely a device to facilitate the derivation of an explicit equation but does not in practice alter the proposed models and their affinity to the Jefferson model.

2.2 Jefferson's Prediction Model.

The model proposed by Jefferson considered a cubic array of spherical particles and divided the system into a number of cubic blocks each having a sphere at its centre. A representative block was then taken from the field and its thermal conductivity assessed making the following assumptions:

- 1) The solid particles are spherical and uniform in size and are distributed in an orderly manner.
- 2) Conductivities and densities of both components are constant.
- 3) The entire solid-liquid suspension sample is confined between two constant temperature planes perpendicular to the direction of heat flow. Thus it follows by symmetry that any other normal plane which separates any two adjacent layers of cubes will be isothermal.
- 4) Heat flow is in one direction only.

2.2.1 Inconsistency of Assumptions.

If the heat flow through the model is in one direction only then isothermal planes must not merely be those normal planes separating any two adjacent layers of cubes but all planes perpendicular to the direction of heat flow. Any departure from this condition would involve a temperature gradient being set up along the normal plane and therefore the presence of a heat flow in that direction. The assumption of a one-dimensional heat flow also implies that, in the portion of the model where the continuous and discontinuous phases are adjacent the heat flux is not uniform through the model cross-section. This is a necessary requirement to allow the temperature gradient to be similar in both phases and prevent a temperature gradient in the normal direction to heat flow. Were such a temperature gradient to exist then the heat flow would not be unidirectional. However, in the portion of the model where there is only the continuous phase present an opposite requirement applies. A uniform heat flux through the model cross-section is necessary for the existence of isothermal planes normal to the heat

flow direction. The assumption of one dimensional heat flow therefore leads to the requirement of a heat flux varying in value with position in the model but this in turn requires that the heat flow be multi-directional. Hence there are two necessary and conflicting requirements following from this assumption. This inconsistency cannot be resolved but requires the removal of the unidirectional heat flux condition.

2.2.2. Jefferson's Derivation of an Explicit Equation.

The derivation of an explicit equation that will predict the thermal conductivity of a two-component system is facilitated by dividing the model of the system into a number of components. Following the assumption of a unidirectional heat flux, the model conductivity is assumed to be the overall conductivity of a network of the components combined in series and in parallel. Jefferson divided his model into a half-cube and then subdivided this unit into three components; A, B and C as shown in figure 2.1. Component A consisted of the half sphere with containing cylinder of liquid of the same radius and length equal to the radius. Component B was the liquid half cube with the centre removed as a cylinder.

Component C was the cylinder of liquid between the end of the sphere and the edge of the cube.

The dimensions of the model were determined by considering the volume fraction solid of the system. With the solid sphere taken as being of unit diameter then the side of the cube is of length $m + 1$ where m is given by:

$$m = 0.806 \cdot \phi^{-1/3} - 1.0 \quad (2.1)$$

and corresponds to twice the length of component C.

The arrangement of the network of components was to take A and C in series, with the resultant in parallel with B.

The determination of the effective conductivity of component A had also to be considered since it consisted of both the continuous and discontinuous phases. An expression for the conductivity of this component was obtained by dividing it into elementary strips parallel to the direction of heat flow and containing the two phases in series. Taking the heat flow to be through an infinite number of these strips in parallel and summing over the model cross-sectional area gave an equation for the component conductivity:

$$K_A = K_d K_c \left[\frac{2K_d}{(K_d - K_c)^2} \cdot \ln \frac{K_d}{K_c} - \frac{2}{(K_d - K_c)} \right] \quad (2.2)$$

This result is used together with the dimensions of components B and C, both of which consist entirely of the continuous phase, to produce an overall conductivity prediction for the proposed combination of components in a series/parallel network. The resulting conductivity is then taken as being representative of the effective thermal conductivity of a solid-liquid suspension. Combining the three components in the order stated above Jefferson obtained the following explicit equation for the network conductivity:

$$K_s = \frac{\pi}{4(m+1)^2} \left[\frac{K_A K_c (m+1)}{K_A m + K_c} - K_c \right] + K_c \quad (2.3)$$

Together with the value for m as given by equation (2.1) and that for K_A given by equation (2.2) this expression constitutes the prediction method for the thermal conductivity of two-component systems given by Jefferson.

2.3 Alternative Predictions Derived from Jefferson's Model.

2.3.1 'Component A' Conductivity.

Consideration of Jefferson's method for deriving an expression for the thermal conductivity of component A of his model indicates that an alternative arrangement of elements within the component may be proposed. Such an alternative would be equally consistent with the assumption of a unidirectional heat flux and would provide a different expression for the evaluation of component conductivity. Comparison of the predictions produced by two alternative methods that do not contradict the assumptions made in their development may be expected to give some indication as to the validity of those assumptions.

The proposed method considered division of the component into elementary strips perpendicular to the direction of heat flow and which therefore contained the two phases in parallel. Figure 2.2 shows the division of the component. Heat flow is, hence, through an infinite number of these strips in series enabling an explicit expression for the effective conductivity of this component to be obtained by integrating over the component lengths.

The volume fraction solid content of an elementary strip, referring to figure 2.2, is:

$$\varphi = \frac{(0.25 - x^2) \pi}{0.25 \pi} \quad (2.4)$$

and therefore the conductivity of a strip is given by the standard

expression for two dissimilar materials in parallel:

$$K_{\text{strip}} = K_c + \frac{(0.25 - x^2)(K_d - K_c)}{0.25} \quad (2.5)$$

Summing over the length of the component to give the equivalent conductivity of all the strips in series:

$$K_A = \frac{0.5}{\int_0^{0.5} \frac{dx}{K_c + \left[\frac{0.25 - x^2}{0.25} \right] (K_d - K_c)}} \quad (2.6)$$

Let:

$$A = \frac{K_d - K_c}{0.25} \quad (2.7)$$

then substituting:

$$K_A = \frac{0.5}{\int_0^{0.5} \frac{dx}{K_c + (0.5\sqrt{A})^2 - (x\sqrt{A})^2}} \quad (2.8)$$

$$\text{Let: } B^2 = K_c + (0.5\sqrt{A})^2 \quad (2.9)$$

then:

$$K_A = \frac{0.5}{\int_0^{0.5} \frac{dx}{B^2 - (x\sqrt{A})^2}} \quad (2.10)$$

thus:

$$K_A = \frac{0.5}{\frac{1}{\sqrt{A}} \cdot \int_0^{0.5} \frac{\sqrt{A} dx}{B^2 - (x\sqrt{A})^2}} \quad (2.11)$$

integrating:

$$K_A = \frac{0.5}{\left[\frac{1}{2B\sqrt{A}} \cdot \ln \frac{B + x\sqrt{A}}{B - x\sqrt{A}} \right]_0^{0.5}} \quad (2.12)$$

$$K_A = \frac{0.5}{\frac{1}{2B\sqrt{A}} \cdot \ln \frac{B + 0.5\sqrt{A}}{B - 0.5\sqrt{A}}} \quad (2.13)$$

where:

$$A = 4(K_d - K_c) \quad (2.14)$$

$$\begin{aligned} B &= (K_c - 0.25A)^{1/2} \\ &= K_d^{1/2} \end{aligned} \quad (2.15)$$

therefore the conductivity of component A may be determined from:

$$K_A = \frac{2[K_d(K_d - K_c)]^{1/2}}{\ln \frac{K_d^{1/2} + (K_d - K_c)^{1/2}}{K_d^{1/2} - (K_d - K_c)^{1/2}}} \quad (2.16)$$

This equation provides an alternative to equation (2.2) for determination of the conductivity of component A and may be used in

the calculation of the overall effective thermal conductivity of the suspension by substituting the result in equation (2.3).

In applying the expression developed above and that developed by Jefferson to the determination of the component conductivity for a given solid-liquid system it is found that a wide variation exists between the two predictions. Table 2.1 lists the results obtained from the two expressions for a range of conductivity ratio and for purposes of comparison also details the results from a simple series or parallel combination of phases as given by equations (1.9) and (1.10). The two theoretical expressions both produce values that lie within the lower and upper limits for a combination of phases, which are given by the series and parallel configurations respectively, so that they both fulfil this basic requirement. It is, then, impossible to distinguish between the two expressions on grounds of validity, both appearing to be perfectly consistent within the assumption of a unidirectional heat flux, but it is clear that the system is not adequately represented by this approach. It is apparent that the one-dimensional heat flux treatment cannot account for conduction within a simple system that is geometrically well defined and regular, a situation that throws doubt upon its ability to be extended to the full case of a suspension. It is concluded that a unidirectional heat flux approach is not valid for any system where the component dimensions are not in-variant in any direction excepting the direction of heat flow.

2.3.2. Suspension Conductivity.

The method of utilising alternative combinations of constituents which was applied in the case of the assessment of the conductivity of component 'A' may also be employed in the evaluation of the

conductivity of the suspension model from the component values. Any change in the order of combination of the components into which the model is divided will not violate the original assumptions made by Jefferson and should therefore produce a similar result to the previous combination. It is necessary to modify slightly the division of the model into components from that given by Jefferson with the result that:

Component A remains unchanged.

Component B becomes that part of the liquid half-cube adjacent to A only.

Component C becomes the remaining portion of the half-cube between the end of the sphere and the edge of the cube. The rearranged model division is shown in figure 2.3. As in the previous model division, components B and C are composed entirely of the continuous phase. If a series-parallel network of components is now formed, with components A and B in parallel with their resultant effective conductivity being considered in a series arrangement with component C, all relative to the direction of heat flow, then an explicit equation may be derived which will give the overall conductivity. Using the standard expression which gives the overall conductivity of two dissimilar materials which are in parallel relative to the direction of heat flow we obtain for the combination of components A and B:-

$$K_{AB} = \frac{\pi (K_A - K_c)}{4(m+1)^2} + K_c \quad (2.17)$$

From the expression giving the overall conductivity of two dissimilar materials in series relative to the direction of heat flow, the combination of component C with the resultant of the component A and B combination yields the overall conductivity of the model:-

$$K_s = \frac{K_{AB} K_c (m + 1)}{K_{AB} m + K_c} \quad (2.18)$$

Substituting equation (2.17) in (2.18) eliminates the composite conductivity K_{AB} , and rearranging gives the equation for the effective conductivity of a suspension as:

$$K_s = \frac{(m + 1) K_c}{m + \frac{K_c}{\frac{\pi(K_A - K_c)}{4(m + 1)^2} + K_c}} \quad (2.19)$$

where m is given by equation (2.1) and K_A by either (2.2) or (2.16).

We have then, two expressions for the prediction of the thermal conductivity of component A and two alternative combinations of the components into which the model is divided. It is therefore possible using these four equations to propose four different methods by which a prediction of the thermal conductivity of a suspension may be produced from the basic concepts. These may be designated as follows:-

Model I - uses equations (2.2) and (2.3). This produces the prediction presented by Jefferson.

- Model II - equations (2.16) and (2.3)
- Model III - equations (2.2) and (2.19)
- Model IV - equations (2.16) and (2.19)

Analogues for the models in terms of equivalent electrical circuits are shown in figure (2.4).

It would, of course be possible to devise many other divisions of the suspension model and subsequent order of combination of the components; the four expressions presented being used merely to illustrate the deficiencies of this approach to the problem.

2.4 Discussion of Unidirectional Heat Flux Predictions.

The four models developed in the previous sections may be tested by applying them to the same system of a two-component mixture over a range of volume fraction solids content. Figures (2.5) to (2.7) show the results for system pure component conductivity ratios ranging from 10 to 1000 and for volume fraction solids content up to 40 per cent.

It can be seen that there is no agreement between the predictions produced by all four models and as may be expected the divergence becomes greater as the volume fraction solid or the pure component conductivity ratios are increased. Since none of the predictions are in conflict with the assumptions made in their development each one could be taken to be valid; hence there is no basis for deciding which of them gives the most accurate results for any given two-component system. Consequently the proposal or utilisation of any particular model must be the result of a purely arbitrary decision unless reference is made to the actual experimental results for the suspension under consideration. This gives rise to the paradoxical situation where the actual effective thermal conductivity of a two-component

system must be known before the theoretical prediction of this value can be made with any certainty of accuracy if the unidirectional heat flux assumption is made. Even in this event there is no guarantee that the model providing the best prediction for a given case will necessarily produce the most accurate result in a different case, even for varying volume fraction solids content of the same two-component mixture.

The performance of all four models when used for the prediction of the thermal conductivities of the two-component systems studied experimentally by Jefferson et.al /11/ are shown graphically in figures (2.8) and (2.9). The models numbered II and III can be seen to produce substantially more accurate predictions of the experimental results than does the model presented by Jefferson and numbered I in this report. For the graphite-water suspension an overall mean error of prediction of 28.0% was obtained by Jefferson as against 12.2% and 9.2% for models II and III respectively. In the case of the graphite-silicone oil suspension the overall mean error of 27.9% obtained by Jefferson was reduced to 24.1% and 12.2% through the use of models II and III. It is apparent that a significant change in prediction accuracy results merely from consideration of various combinations of the constituent parts of a single model; neither this nor the initial assumptions under which the solution proceeds need be changed. This situation has received no comment from the proposers of unidirectional heat flux models and suggests that the methods employed in deriving explicit prediction equations are the result of arbitrary decisions.

Notwithstanding the deficiencies of the unidirectional heat flux approach to this problem as detailed above, it may still be worthwhile examining the performance of the proposed models in predicting conductivities for various two-component systems. The resulting expressions are easily evaluated and, provided that a loss of accuracy can be tolerated, a quick solution to a given problem can be obtained by choosing the most appropriate model. Unfortunately, it is not possible to formulate a general rule to aid this selection, but guidelines may be given which will indicate areas of suitability for each model.

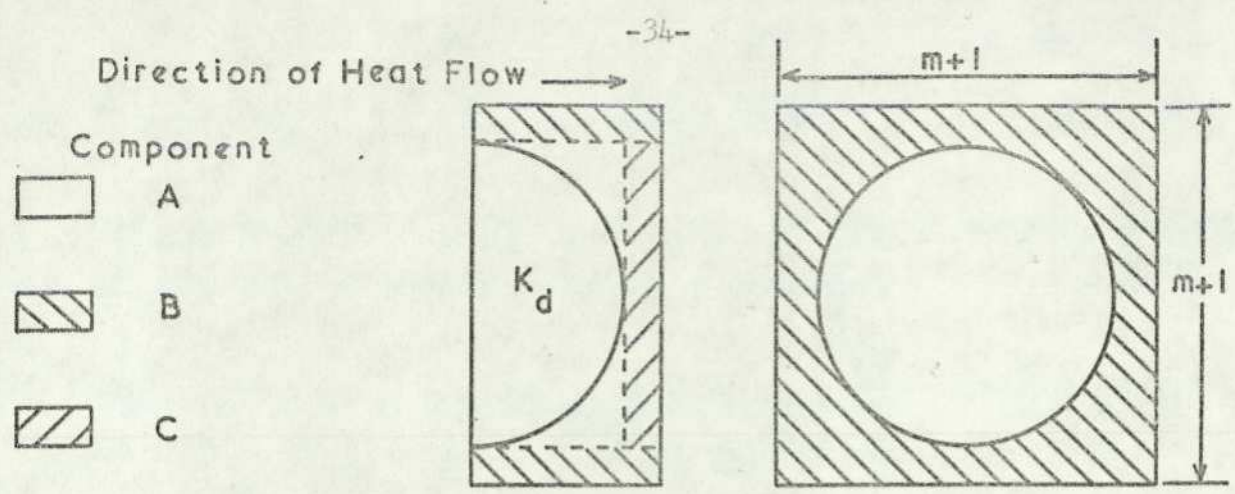
The ability of the four given models to predict the authors experimental results is shown graphically in figures (2.10) to (2.12). It can be seen that Model II provides the most accurate overall prediction with a mean error, considering all cases of 5.75%. If the results of other experimental studies are also included /11/, /20/, /23/ then overall mean errors are obtained as Model I 13.8%, Model II 12.7%, Model III 10.9%, with the fourth model proving extremely inaccurate for every case studied. Considering the accuracy of prediction for individual suspensions the three models provide the best prediction in the approximate ratio of 1 : 6 : 4 for Models, I, II and III respectively. There is, however, no range of system pure component conductivity ratio for which either Model II or III proves to be exclusively superior. If the total range of experimental results is considered with reference only to the system volume fraction solid then it is found that the best predictions given by Model II are concentrated mainly in the range 0 - 0.1, whilst model 3 is in general of volume fraction solid from 0.1 to 0.4. The few best predictions produced by Jefferson's equation, designated Model I in this treatment, occurred in the range of volume fraction solid from 0.2 to 0.3.

It would appear then that if this approach is to be used, a choice between Models II and III is the best under the circumstances but on the understanding that large errors could occur for certain individual suspensions.

Table 2.1

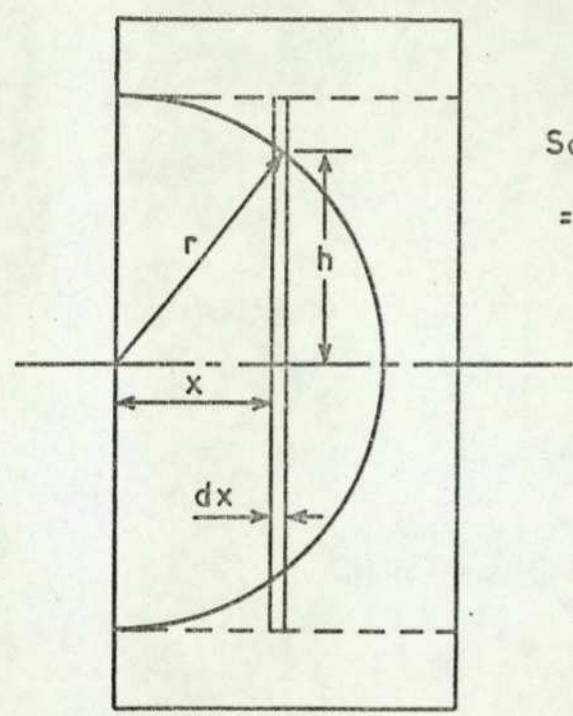
Comparison of Alternative Expressions for the Thermal
Conductivity of Component A.

$\frac{K_d}{K_c}$	K_A/K_C			
	Equation 2.2	Equation 2.16	Phases in Series	Phases in Parallel
10.0	3.46	5.32	1.89	5.71
30.0	5.21	12.37	2.02	16.18
50.0	6.11	18.72	2.05	26.66
80.0	6.96	27.59	2.07	42.36
100.0	7.38	33.24	2.08	52.84
200.0	8.69	59.71	2.09	105.2
300.0	9.48	84.5	2.09	157.56
500.0	10.48	131.45	2.09	262.28
700.0	11.14	176.27	2.1	367.0
1000.0	11.84	241.03	2.1	524.08



JEFFERSON DIVISION OF UNIT CELL MODEL

Figure 2.1

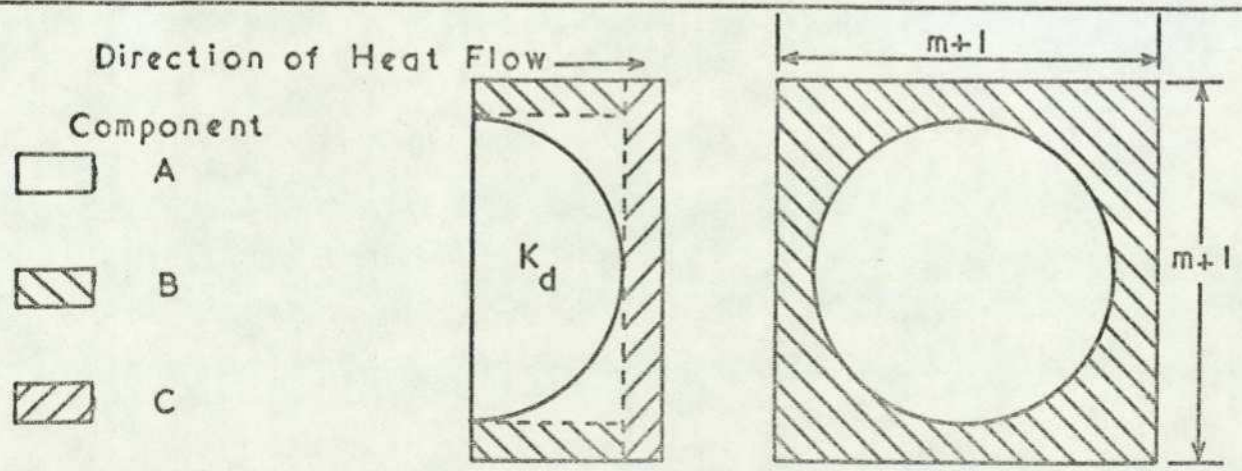


Solid Phase Cross Sectional Area

$$= \pi h^2 = \pi [r^2 - x^2]$$

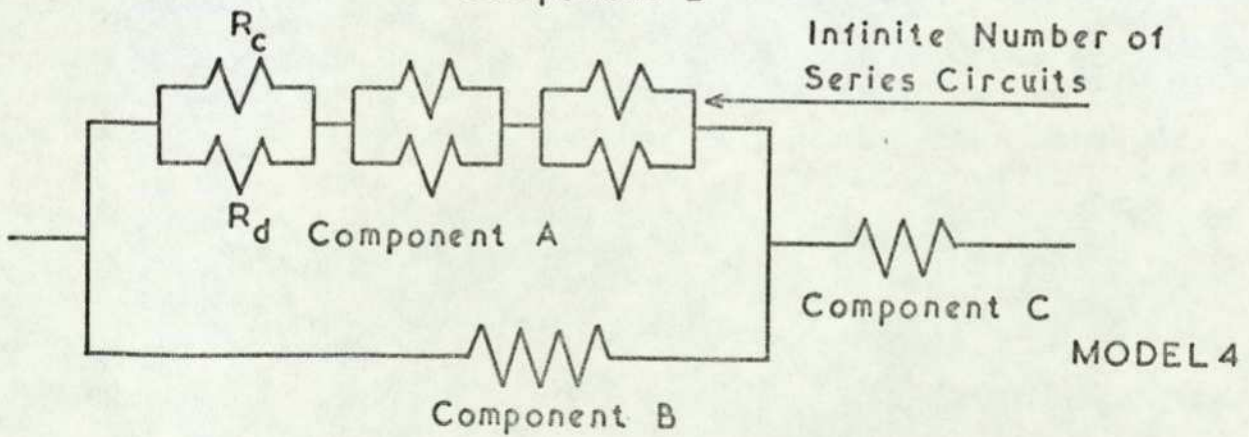
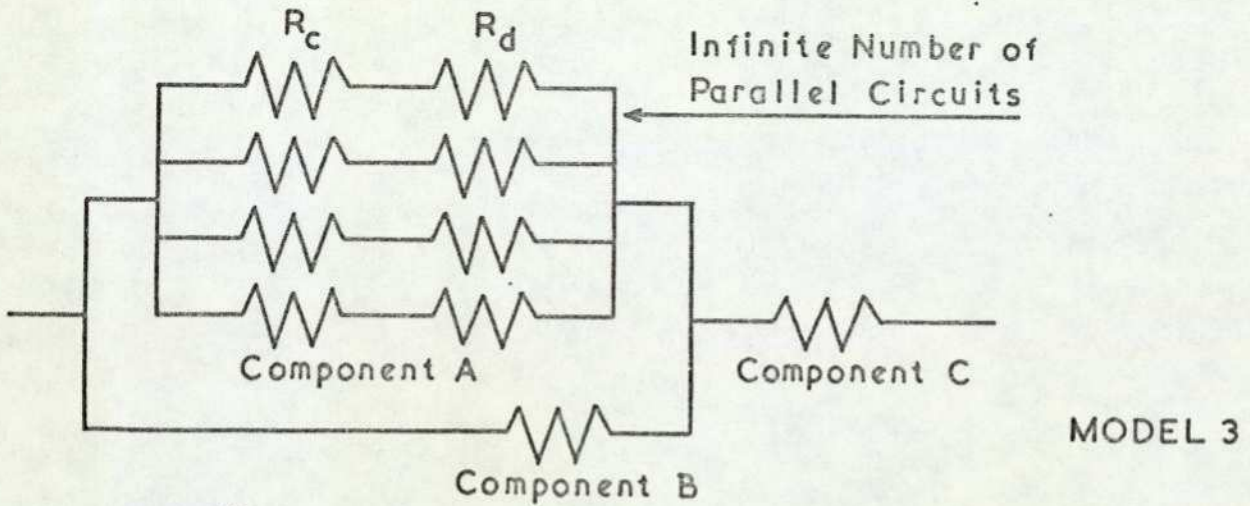
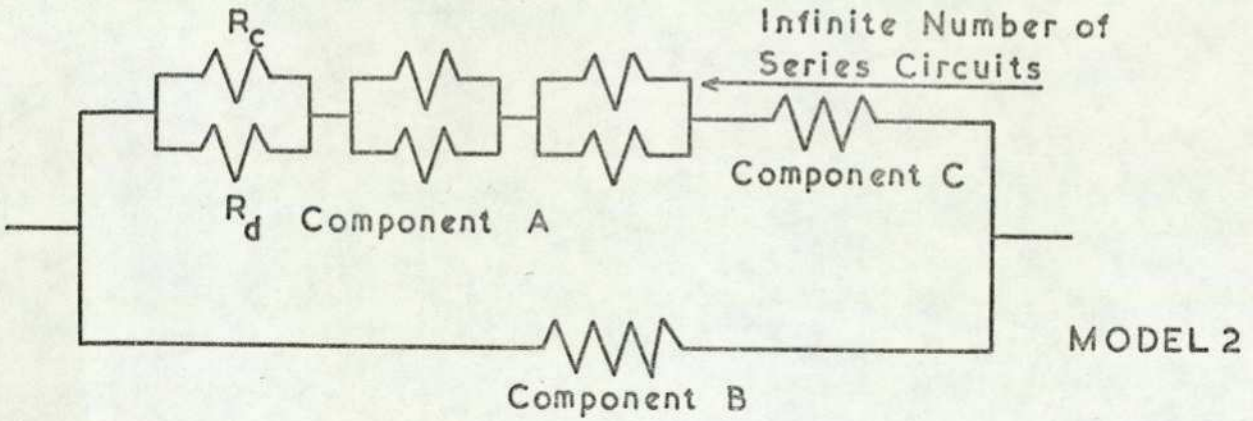
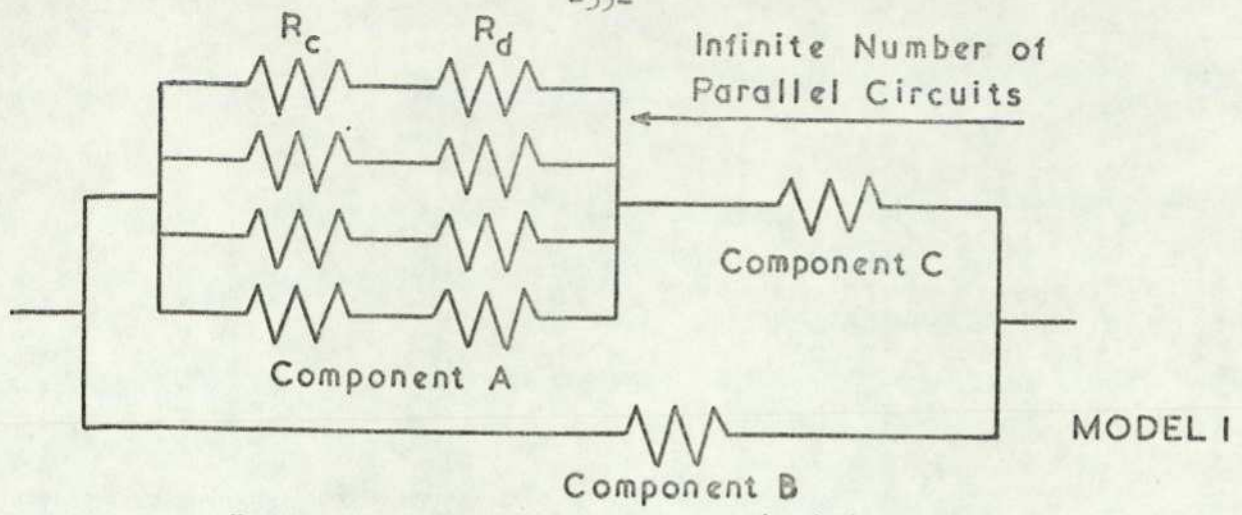
ALTERNATIVE DIVISION OF COMPONENT 'A' FOR THERMAL CONDUCTIVITY DETERMINATION

Figure 2.2



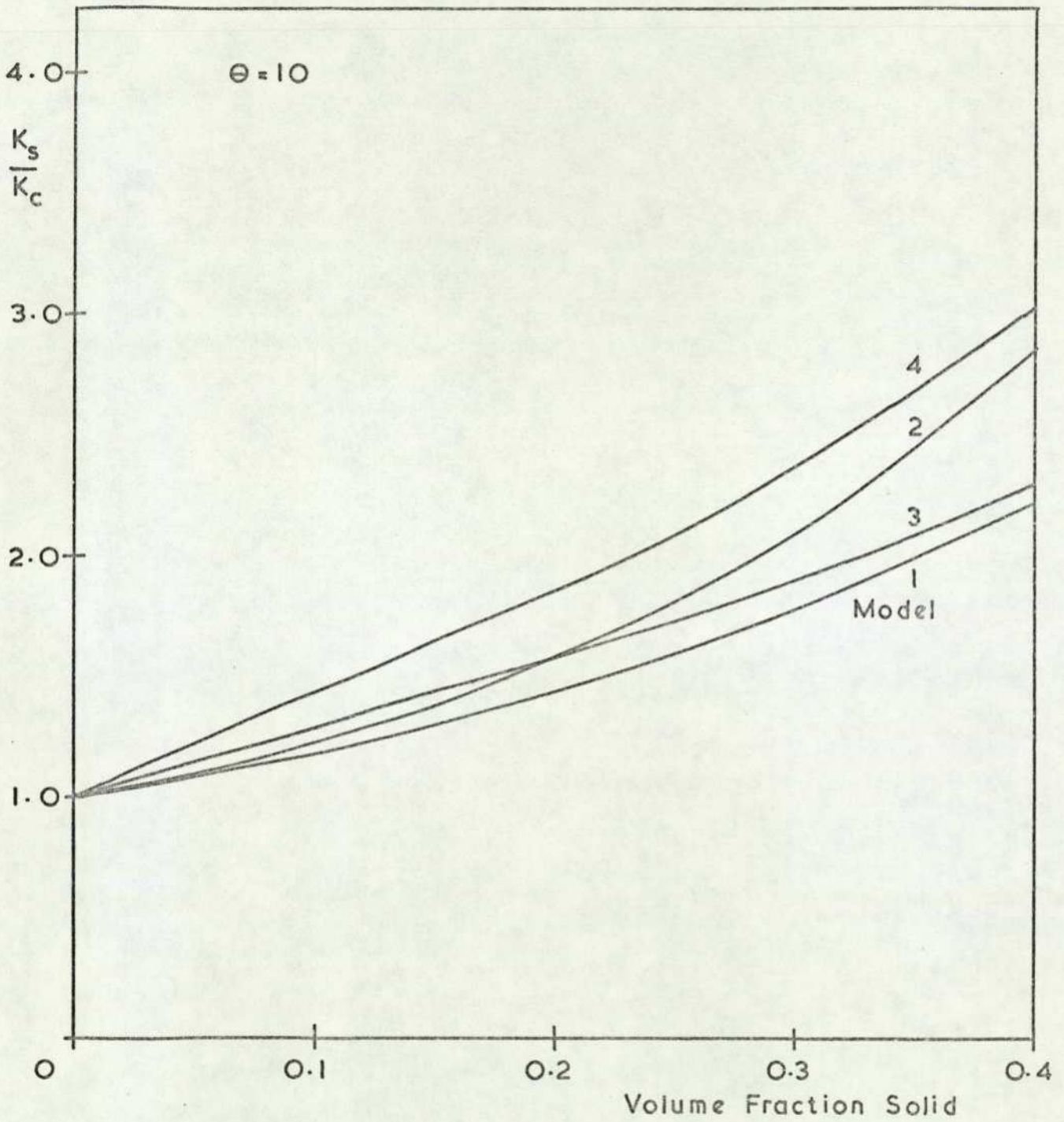
ALTERNATIVE DIVISION OF UNIT CELL MODEL

Figure 2.3



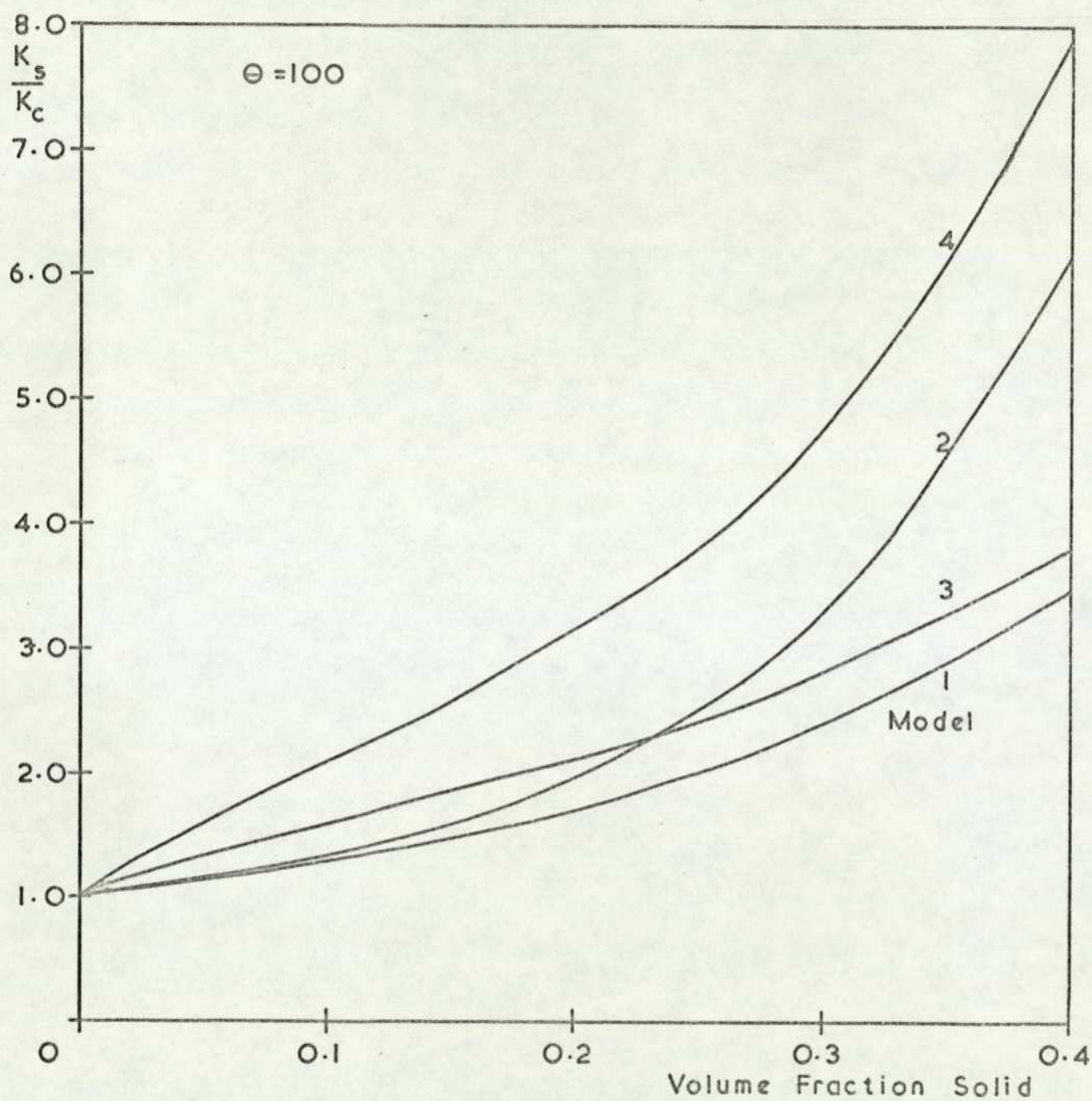
ELECTRICAL ANALOGUE OF UNIDIRECTIONAL HEAT FLUX MODELS

Figure 2.4



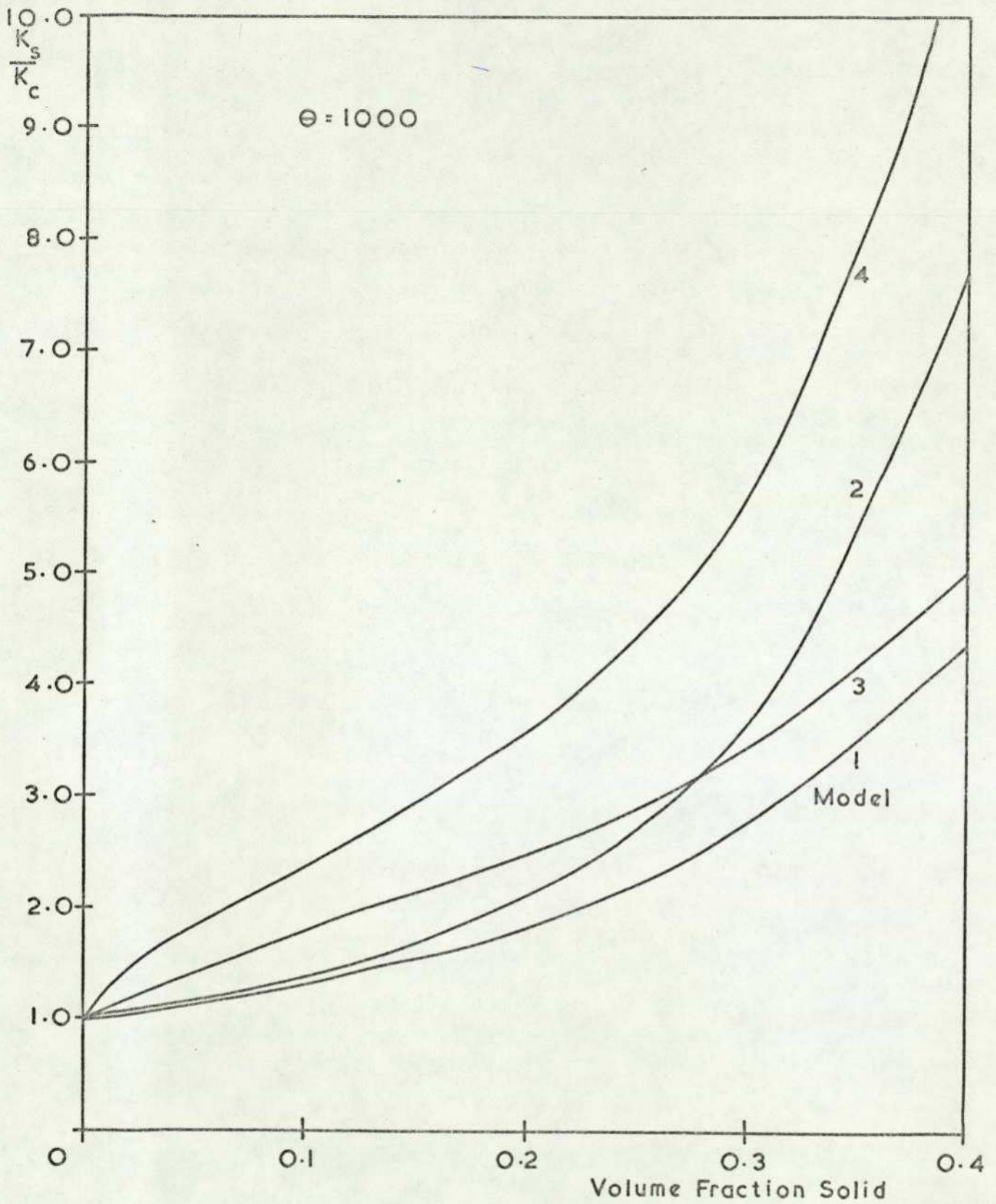
VARIATION OF PREDICTION FOR TWO COMPONENT SYSTEM
THERMAL CONDUCTIVITY USING UNIDIRECTIONAL HEAT
FLUX MODELS

Figure 2.5



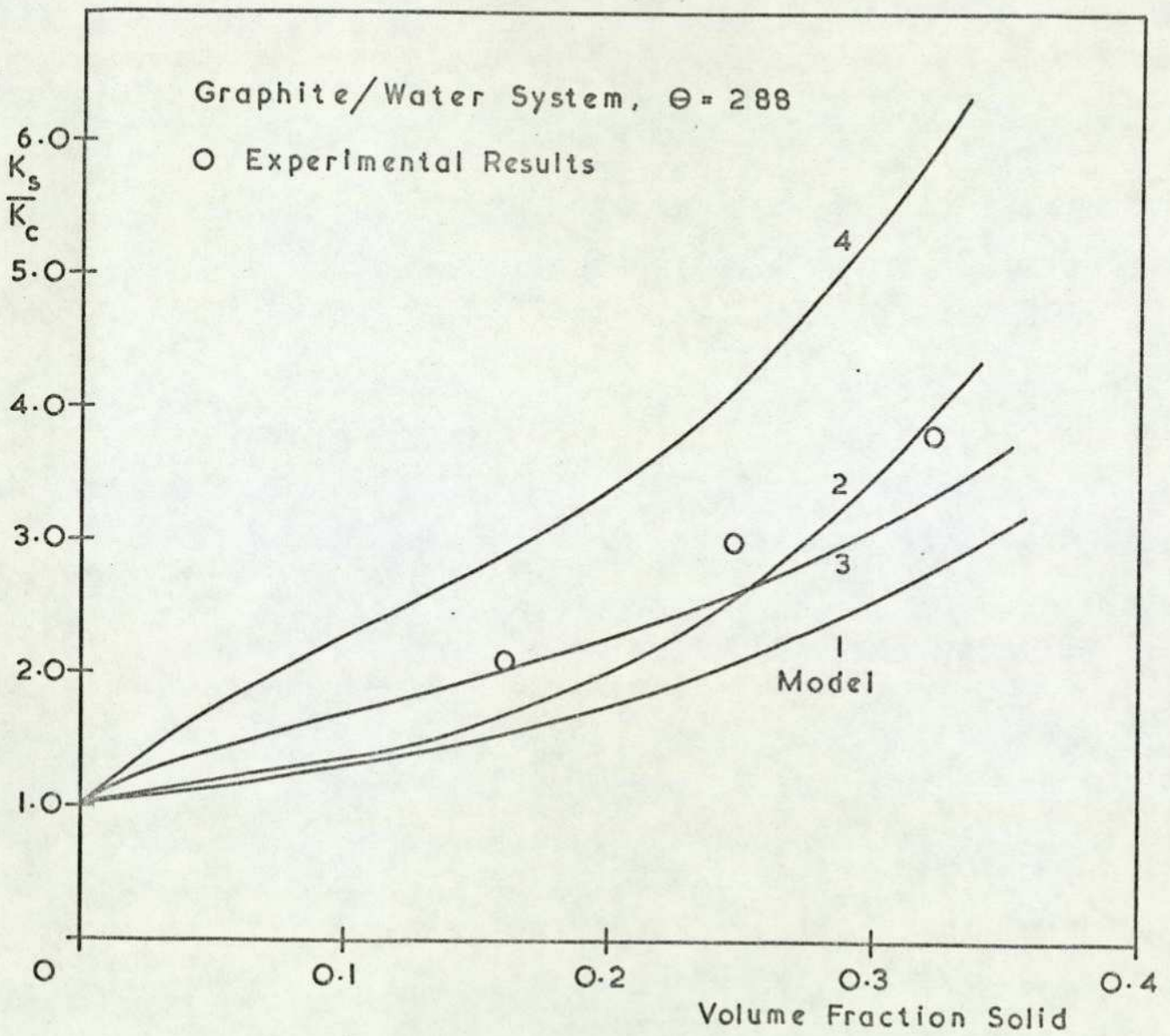
VARIATION OF PREDICTION FOR TWO COMPONENT SYSTEM
THERMAL CONDUCTIVITY USING UNIDIRECTIONAL HEAT
FLUX MODELS

Figure 2.6



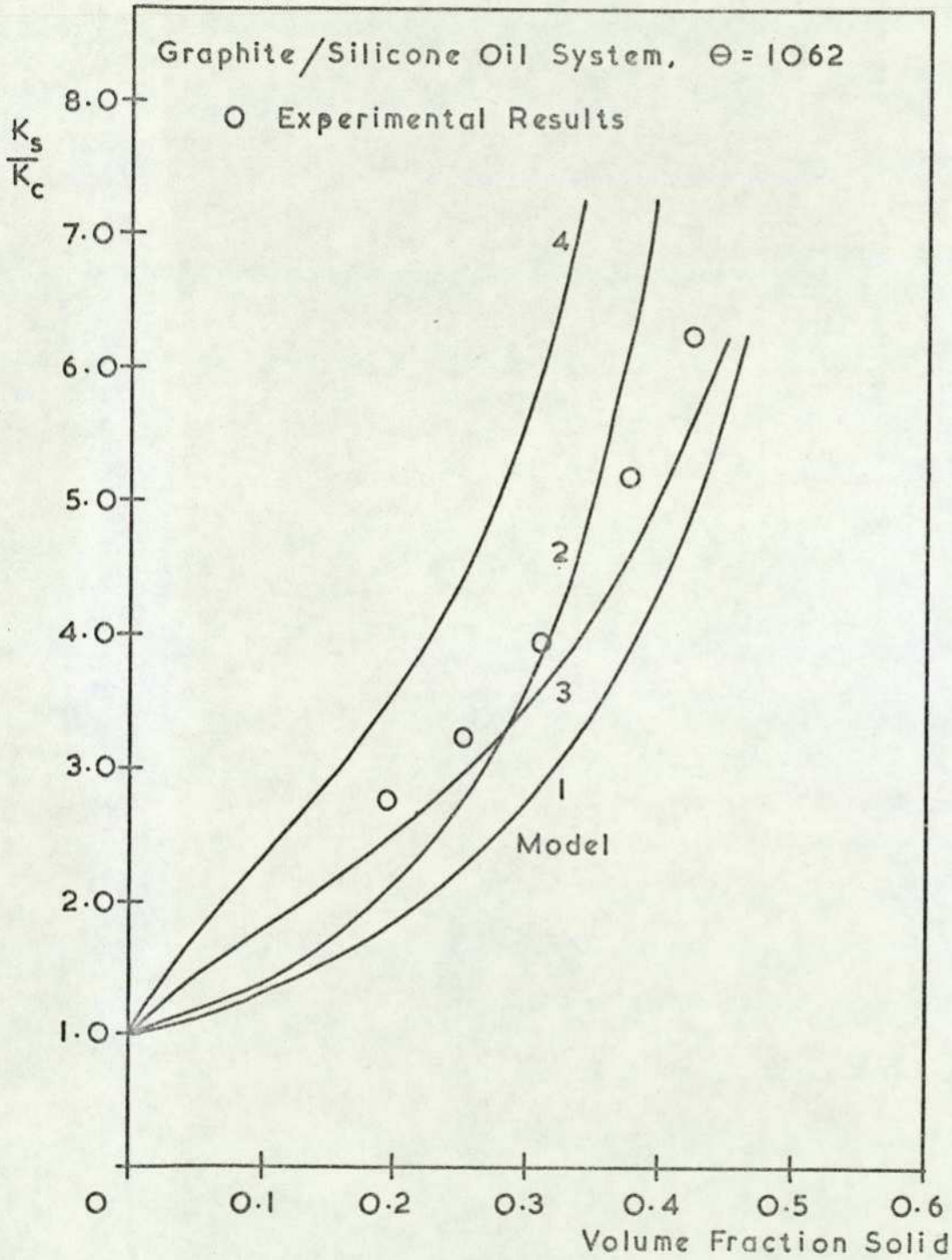
VARIATION OF PREDICTION FOR TWO-COMPONENT SYSTEM
THERMAL CONDUCTIVITY USING UNIDIRECTIONAL HEAT
FLUX MODELS

Figure 2.7



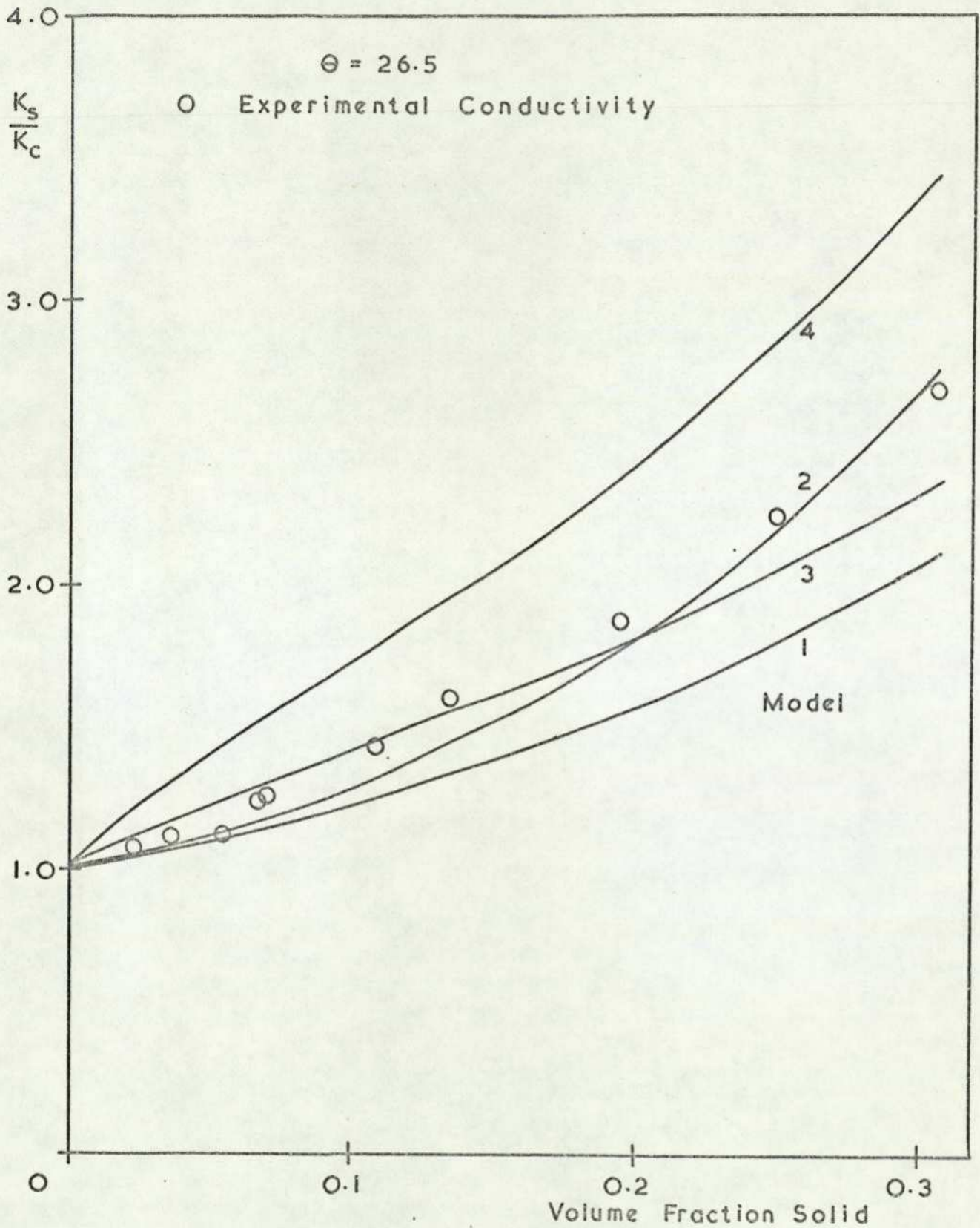
PREDICTION OF JEFFERSON'S EXPERIMENTAL RESULTS
BY UNIDIRECTIONAL HEAT FLUX MODELS

Figure 2.8



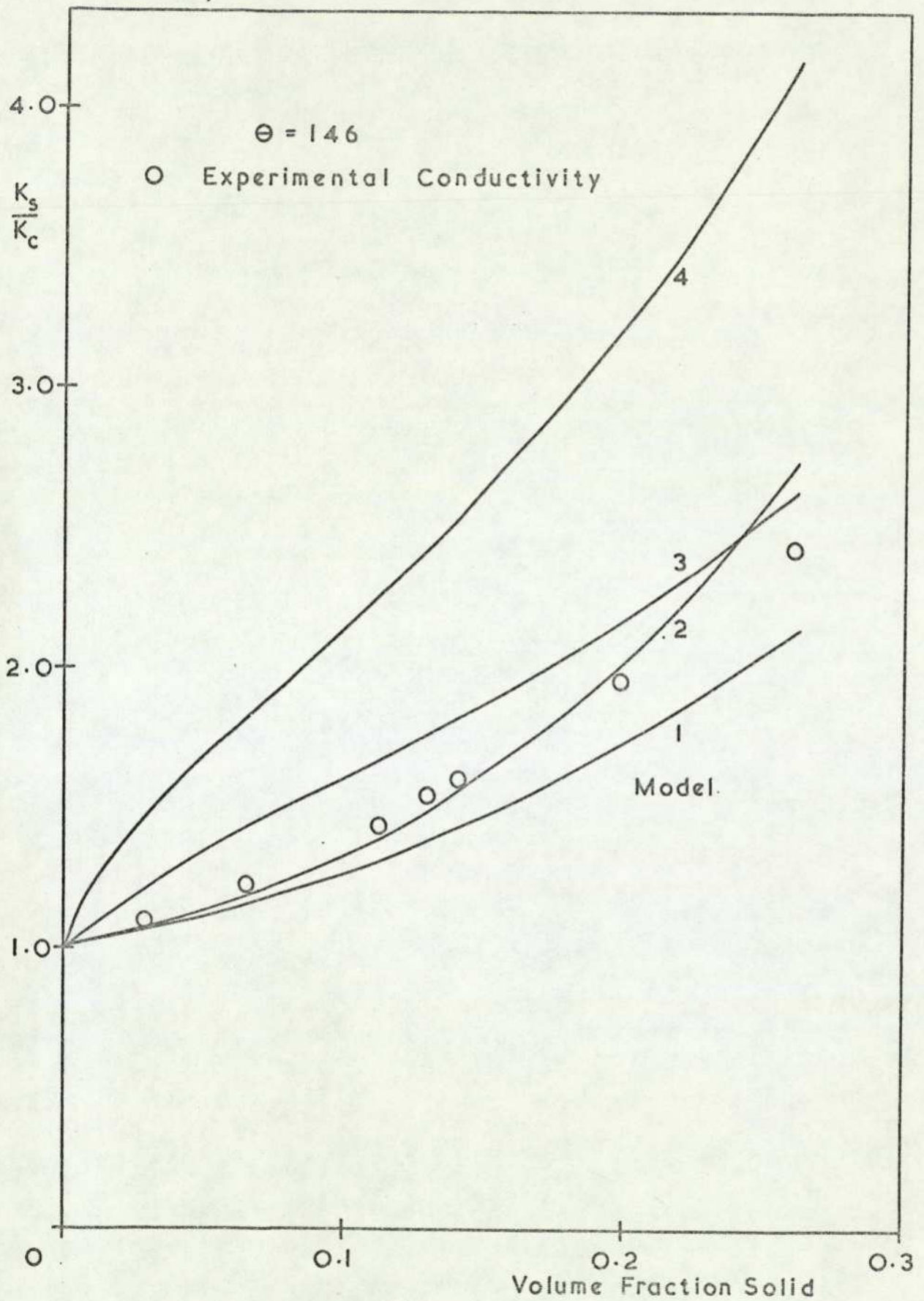
PREDICTION OF JEFFERSON'S EXPERIMENTAL RESULTS
BY UNIDIRECTIONAL HEAT FLUX MODELS

Figure 2.9



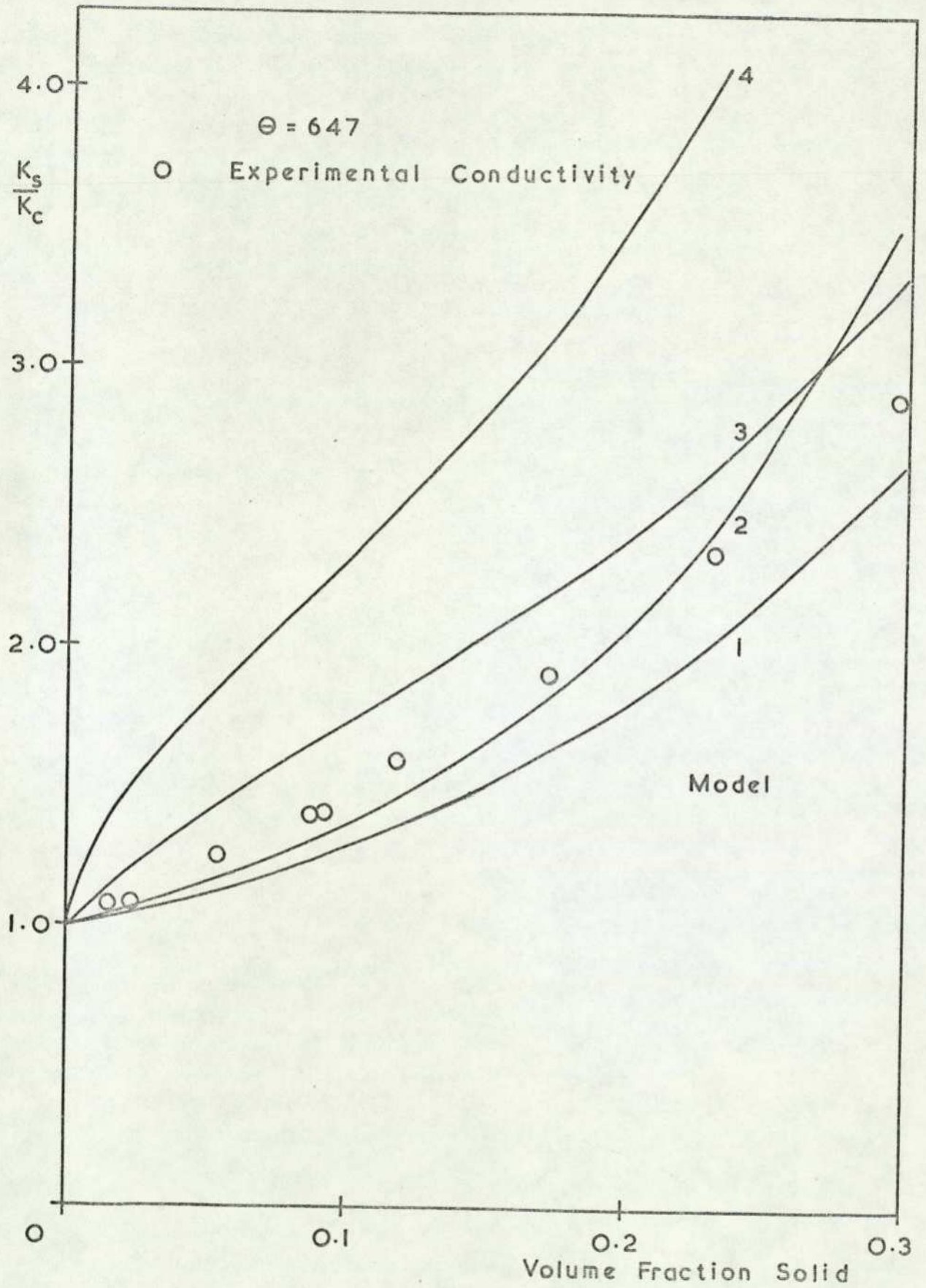
UNIDIRECTIONAL HEAT FLUX MODEL PREDICTIONS OF STAINLESS STEEL-WATER SYSTEM CONDUCTIVITY

Figure 2.10



UNIDIRECTIONAL HEAT FLUX MODEL PREDICTIONS
OF NICKEL-WATER SYSTEM CONDUCTIVITY

Figure 2.11



UNIDIRECTIONAL HEAT FLUX MODEL PREDICTIONS
OF COPPER-WATER SYSTEM CONDUCTIVITY

Figure 2.12

3. DEVELOPMENT OF PREDICTION TECHNIQUE

3.1 Introduction.

The inherent inconsistency and the uncertainty of prediction technique detailed in the previous section follows directly from the over-simplification of the model for a two-component system through the assumption of a unidirectional heat flux.

The first stage in producing a valid prediction method for the effective thermal conductivity of suspensions was the reconsideration of the model proposed by Jefferson /11/ with this restriction removed. Consequently a tridirectional heat flux situation within the model was considered and the conductivity determined under this condition.

Following from this model, which incorporated the simulation of a sphere in a rectangular co-ordinate system, a further model was developed in which this simulation formed a basic unit to be distributed in a random fashion through a field of the continuous phase. Again the concept of a tridirectional heat flux was utilised in the evaluation of the system conductivity.

Consideration of the following parameters could therefore apply in the prediction of an effective thermal conductivity using either model:-

- 1) The thermal conductivities of the constituent phases.
- 2) The volume concentration of the constituent phases.
- 3) The distribution of the phases in the system.

For the case of the extension of Jefferson's model this latter parameter would be idealised as a regular geometric arrangement of the solid phase but for the second model a random distribution of this constituent is permitted.

The following assumptions are made in deriving the models:-

- 1) The particles of the solid phase are spherical.
- 2) There is no contact resistance between phases.
- 3) The ratio of particle size to system size is of negligible effect.
- 4) Radiation heat transfer is negligible.

3.2 Application of Tridirectional Heat Flux.

The same governing equations and method of solution apply to both models and will be detailed before the individual models are discussed.

The effective thermal conductivity of the system is determined by imposing upon it the conditions implied in Fourier's 'Law of Conduction'. An overall one-dimensional heat flux results from the application of an arbitrary temperature gradient between two parallel faces of a cubic model with the remaining faces being considered to be perfectly insulated. This heat flux together with the system dimensions are then substituted into Fourier's Law to give the required conductivity. Within the model the allowance of a tri-directional heat flux situation requires the determination of the system temperature distribution in order that the mean heat flux may be evaluated. This is only possible through the use of a digital computer to execute a suitable solution procedure, since an explicit prediction equation cannot be derived for this situation.

If the conductivity of each phase is assumed to be independent of temperature then the steady state temperature distribution in each phase is given by Laplace's equation:-

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad : \quad T = T_c, T_d \quad (3.1)$$

The model boundary conditions are:-

$$z = 0, \quad T = T_{in} \quad (3.2)$$

$$z = L, \quad T = T_{out} \quad (3.3)$$

$$\left. \begin{array}{l} x = 0 \\ x = L \end{array} \right\} \frac{\partial T}{\partial x} = 0 \quad (3.4)$$

$$\left. \begin{array}{l} y = 0 \\ y = L \end{array} \right\} \frac{\partial T}{\partial y} = 0 \quad (3.5)$$

In addition the following conditions must be satisfied at a phase interface:-

$$T_c = T_d \quad (3.6)$$

$$K_c \frac{\partial T_c}{\partial p} = K_d \frac{\partial T_d}{\partial p} \quad (3.7)$$

where p is the directional normal to the phase interface.

3.3 Solution Procedure.

Laplace's equation is a partial differential equation of elliptic form and hence the domain of integration for a three-dimensional equation is always a volume bounded by a closed surface. The boundary conditions as detailed specify the value of the function or the value of its normal derivative at every point on the surface and the solution must satisfy the partial differential equation at every point inside the closed surface. Only a limited number of special types of elliptic equations have been solved analytically and so the solution of the equations defining the prediction models was approached through the use of a numerical approximation method. In particular a finite difference technique was employed.

The method used divides the volume of integration by a three-dimensional grid network formed by three sets of equally spaced lines, each set being parallel to one of the three cartesian co-ordinate axes. The intersection points of the grid lines are called nodes and the domain of each node is the cube of the size of the grid spacing which has the node at its centre. Grid lines must form the edge of the model so that the nodes that lie on these lines may be treated in such a way as is specified by the boundary conditions. Since the domain of a node may only lie within the model, the domain of those nodes that lie on the boundary surface will not be the full cube. For a node located on a side of the model the domain will be half-size; if located on an edge it will be a quarter-size and for a node at a corner it will be one-eighth of the full domain. An approximate solution to the partial differential equation is found at each nodal point and is considered to be constant over the whole of the node domain.

The solution procedure requires that at each node the partial differential equation is replaced by a finite difference approximation in terms of the values of the temperature at the node and surrounding nodes.

3.3.1 Derivation of Finite Difference Approximation.

When a function T and its derivatives are single valued, finite and continuous functions of T then by Taylor's Theorem, assuming a grid spacing of u, v, w in the x, y, z co-ordinate directions:-

$$\begin{aligned} T(x+u, y, z) = T(x, y, z) + U T'(x, y, z) + \frac{1}{2} U^2 T''(x, y, z) \\ + \frac{1}{6} U^3 T'''(x, y, z) + \dots \end{aligned} \quad (3.8)$$

and:

$$\begin{aligned} T(x-u, y, z) = T(x, y, z) - U T'(x, y, z) + \frac{1}{2} U^2 T''(x, y, z) \\ - \frac{1}{6} U^3 T'''(x, y, z) + \dots \end{aligned} \quad (3.9)$$

adding these expansions:

$$T(x+u, y, z) + T(x-u, y, z) = 2T(x, y, z) + U^2 T''(x, y, z) + O(U^4) \quad (3.10)$$

Assuming that the fourth and higher powers of u are negligible in comparison with the lower powers then:

$$T''(x, y, z) = \left[\frac{d^2 T}{dx^2} \right]_{x, y, z} = \frac{1}{U^2} \left[T(x+u, y, z) - 2T(x, y, z) + T(x-u, y, z) \right] \quad (3.11)$$

similarly:

$$\left[\frac{d^2 T}{dy^2} \right]_{x, y, z} = \frac{1}{V^2} \left[T(x, y+v, z) - 2T(x, y, z) + T(x, y-v, z) \right] \quad (3.12)$$

$$\left[\frac{\partial^2 T}{\partial z^2} \right]_{x,y,z} = \frac{1}{W^2} \left[T(x,y,z+w) - 2T(x,y,z) + T(x,y,z-w) \right] \quad (3.13)$$

hence substituting equations (3.11), (3.12) and (3.13) into equation

$$(3.1): \frac{1}{U^2} \left[T(x+u,y,z) - 2T(x,y,z) + T(x-u,y,z) \right] + \\ \frac{1}{V^2} \left[T(x,y+v,z) - 2T(x,y,z) + T(x,y-v,z) \right] + \\ \frac{1}{W^2} \left[T(x,y,z+w) - 2T(x,y,z) + T(x,y,z-w) \right] = 0 \quad (3.14)$$

multiplying by u, v, w .

$$\frac{VW}{U} \left[T(x+u,y,z) - 2T(x,y,z) + T(x-u,y,z) \right] + \\ \frac{UW}{V} \left[T(x,y+v,z) - 2T(x,y,z) + T(x,y-v,z) \right] + \\ \frac{UV}{W} \left[T(x,y,z+w) - 2T(x,y,z) + T(x,y,z-w) \right] = 0 \quad (3.15)$$

if the grid system is cubic, i.e. if $u = v = w$, then the equation reduces to the standard seven point formula:

$$T(x+u,y,z) + T(x-u,y,z) + T(x,y+v,z) + T(x,y-v,z) + \\ T(x,y,z+w) + T(x,y,z-w) - 6T(x,y,z) = 0 \quad (3.16)$$

3.3.2 Incorporation of Interfacial Conditions.

Where adjacent nodes lie on opposite sides of a phase boundary then the finite difference approximation must also satisfy equations (3.6) and (3.7). If the nodal domains each consist of one phase only then the phase boundaries will always lie on a boundary between node domains. Denoting the temperature at the interface by T_b we can replace the interfacial condition given by equation (3.7) by a finite difference approximation.

Considering a boundary between phases that lies on a $y - z$ plane:

$$q = \frac{K(x,y,z)}{U/2} [T(x,y,z) - T_b] = \frac{K(x+u,y,z)}{U/2} [T_b - T(x+u,y,z)] \quad (3.17)$$

Let $K_{i,n}$ be an overall equivalent conductivity between the adjacent nodes so that equation (3.17) will also be equivalent to:

$$q = \frac{K_{i,n}}{U} [T(x,y,z) - T(x+u,y,z)] \quad (3.18)$$

Therefore:

$$T(x,y,z) - T_b = \frac{K_{i,n}}{K(x,y,z)} \cdot \frac{1}{2} [T(x,y,z) - T(x+u,y,z)] \quad (3.19)$$

$$T_b - T(x+u,y,z) = \frac{K_{i,n}}{K(x+u,y,z)} \cdot \frac{1}{2} [T(x,y,z) - T(x+u,y,z)] \quad (3.20)$$

adding equations (3.19) and (3.20):

$$T(x,y,z) - T(x+u,y,z) = K_{i,n} \left[\frac{1}{2K(x,y,z)} + \frac{1}{2K(x+u,y,z)} \right] [T(x,y,z) - T(x+u,y,z)] \quad (3.21)$$

which gives:

$$K_{i,n} = \frac{2K(x,y,z) \cdot K(x+u,y,z)}{K(x,y,z) + K(x+u,y,z)} \quad (3.22)$$

It can be seen then that in order to satisfy the interfacial conditions the finite difference approximation must be modified by multiplying the temperature difference between adjacent nodes by the overall conductivity of the grid line connecting these nodes. This has the effect of incorporating equation (3.22) into the finite difference approximation and although it is strictly only necessary to apply this condition when writing the finite difference equation for nodes that are adjacent to a phase boundary, in practice it is more convenient to consider the one equation that will be universal in application. Therefore combining equations (3.15) and (3.22) a general finite difference approximation may be written in terms of the properties at a node and its surrounding nodes:

$$\sum_{i=1}^S \frac{A_{i,n}}{h_{i,n}} \cdot K_{i,n} (T_i - T_n) = 0 \quad (3.23)$$

where:

S is the number of nodes surrounding the n^{th} node, which has a temperature of T_n

$A_{i,n}$ is the cross-sectional area of the mesh normal to the grid line from node i to node n and $h_{i,n}$ is the grid line length.

$K_{i,n}$ is the overall conductivity of the grid line and is given by equation (3.22). Hence when adjacent nodes are both of the discontinuous phase:

$$K_{i,n} = K_d \quad (3.24)$$

When adjacent nodes are both of the continuous phase:

$$K_{i,n} = K_c \quad (3.25)$$

and when adjacent nodes are of different phases:

$$K_{i,n} = \frac{2 K_c K_d}{K_c + K_d} \quad (3.26)$$

In physical terms equation (3.23) is a heat balance written around each node, with no heat generation and with heat flow being confined to grid lines.

3.3.3 Treatment of Finite Difference Equation at Model Boundary.

For a node that lies within the model there will be a total of six adjacent nodes and each node domain cross-sectional area will be of full size. In the case of nodes on the boundary surface special treatment of the general finite difference equation is required; the modification incurred will depend both upon the specification of the boundary condition at that point and the position of the node. At those nodes where the boundary condition is specified as a temperature a finite difference equation will not be written since the solution is known. The other boundary condition requires that the temperature

gradient normal to the model surface is zero; that is the boundary surface is perfectly insulated. At these nodes the number of terms in the heat balance will depend upon the number of adjacent nodes since they only exist within the model. The position of the node on the boundary surface will also determine the respective cross-sectional area of node domains to be used in the heat balance.

The modifications to the general finite difference approximations may be generalised for the various boundary node locations as follows. If a node lies on a surface of the model it will have five adjacent nodes and hence five terms in the heat balance equation, with the cross-sectional area of the domain being half of the full size for heat flow to those adjacent nodes that also lie on a surface. A node on an edge of the model will have four adjacent nodes and in this case the domain cross-sectional area will be one-quarter full size for heat flow to those nodes on the edge and one-half full size for those nodes that lie on a surface. If a node is located at the corner of the model in this case it will be subject to the boundary condition of a temperature specification and hence no finite difference approximation is required.

3.4 Determination of Model Thermal Conductivity.

The finite difference equation written at each node is unique and taken together form a set of equations whose simultaneous solution will yield the steady state temperature at each node. With an overall one-dimensional heat flux being imposed upon the model through the establishment of a temperature gradient between two parallel faces, the net heat flux between any two adjacent layers of nodes normal to the direction of heat flow will be invariant.

The assumption that heat flow is confined to the mesh lines is implicit in the solution procedure and therefore the overall heat flow between adjacent layers of nodes is obtained by considering Fourier's 'Law of Conduction' between each pair of nodes in the given direction and summation of the individual heat flows. It is most convenient to consider the heat flow across the boundary surfaces since the boundary node temperatures are constant over the plane and are not subject to solution errors.

The heat flow into the model enables the overall heat flux to be determined from:-

$$q_1 = -\frac{1}{h_{1,N}^2} \sum_{n=1}^{N^2} \frac{A_{1,n}}{h_{1,n}} \cdot K_{1,n} (T_n - T_{in}) \quad (3.27)$$

Using the heat flow out of the model gives the overall heat flux as:-

$$q_2 = -\frac{1}{h_{1,N}^2} \sum_{n=1}^{N^2} \frac{A_{2,n}}{h_{2,n}} \cdot K_{2,n} (T_{out} - T_n) \quad (3.28)$$

Comparison of the two results affords a useful check on the solution accuracy.

With the mean heat flux through the model being known Fourier's conduction equation may be applied to the whole model and upon rearrangement the thermal conductivity of the simulation is given by:-

$$K_s = 0.5(q_1 + q_2) \frac{h_{1,N}}{(T_{in} - T_{out})} \quad (3.29)$$

This procedure was applied to the two proposed models of two-component systems.

4. UNIT CELL MODEL

4.1 Introduction.

This prediction method takes the basic unit that Jefferson proposed as representing a sample of the two-component mixture and extends its validity by applying the condition of a tridirectional heat flux in assessing the model thermal conductivity. The remaining assumptions made by Jefferson in the development of his prediction will still be applicable.

1. The solid particles are spherical and uniform in size and are arranged in a cubic array.
2. Conductivities and densities of both components are constant.
3. The entire sample is confined between two constant temperature planes perpendicular to the overall direction of heat flow.

The unit cell model will therefore consist of a sphere of the discontinuous phase at the centre of a cube of the continuous phase. If the sphere is considered to be of unit diameter then the dimensions of the cube may be determined from the volume fraction solid content of the system:-

$$L = 0.806 \varphi^{-1/3} \quad (4.1)$$

4.2 Simulation of Unit Cell.

For determination of the effective thermal conductivity of the model the prediction technique outlined in the preceding section was utilised. Under this scheme where the solution region is divided by a grid network the simulation of the model must allow the assignment

of the respective regions of continuous and discontinuous phase to accord with the solution technique. Any interface between phases will lie along the border between node domains and not along grid lines, in accordance with the modifications made to the finite difference equation. With a rectangular grid system and a model consisting of a sphere within a cube obviously the simulation could not be exact, it being necessary to approximate the sphere by an array of cubes. The degree to which this approximation could be achieved was of necessity restricted by the limitations of computer storage capacity and of computing time requirements.

A sphere was simulated by taking a cube that just enclosed it and dividing this into a block of $5 \times 5 \times 5$ nodal domains. The edge cubes of this block were considered to consist of the continuous phase, whilst the remaining 81 cubes were considered to be the discontinuous phase and to approximate to a sphere. Whilst the simulation was undoubtedly far from exact it was thought to be sufficient for the purpose and in the subsequent light of the satisfactory predictions produced it was felt that no overall benefit would ensue from an attempt to improve it.

Within this block of cubes the grid spacing was invariant over all systems and was determined from the assumption of a unit diameter sphere, whereupon the nodes are 0.2 units apart. The remaining portion of the model between the central block and the boundary was divided into two nodal domains with the outer grid lines co-incident with the boundary surface. These outer domains consisted entirely of the continuous phase and the width of this layer was made variable and dependant on the volume fraction solid content of the system. Hence

the grid spacing in the outer layer is not constant but is given for any system by $L/3$ units. The division of the model into nodal domains and their respective sizes is shown in figure (4.1). Symmetry of the model requires that only one-eighth of the unit cell need be considered, thus there are 5 grid lines in each co-ordinate direction giving a total of 125 nodal points. Grid lines must lie along the axes of symmetry, which for the purpose of the finite difference approximation are treated in exactly the same way as are insulated surfaces, provided that they are not subject to a boundary temperature specification.

There are a total of 50 nodal points that lie on either of the two parallel planes where a temperature is specified as a boundary condition, thus giving a remainder of 75 nodes for each of which a finite difference equation must be written. Hence, we have a set of 75 equations which when solved simultaneously provide the steady state temperature distribution through the model.

4.3 Solution Technique.

The solution of large sets of linear simultaneous equations requires the use of a digital computer, there being in general two techniques available; a direct or iterative method. In this case the number of equations in the set was sufficiently small to allow the use of a direct method in a program run on an I.C.L. 1905 computer. The following conductivity prediction model, which derives from this one, utilises an iterative scheme and thus allows of comparisons between the techniques when used for this purpose.

This direct method of solution requires that the finite difference equations given by equation (3.23) are re-arranged in matrix form. If the nodes for which the equation is written are numbered consecutively from 1 to 75, then the set of equations may be represented as:-

$$\begin{bmatrix}
 B_1 & I_{1,2} & 0 & 0 & 0 & I_{1,6} & 0 & \dots & I_{1,26} & \dots & \\
 I_{2,1} & B_2 & I_{2,3} & 0 & 0 & \dots & I_{2,7} & 0 & \dots & \dots & \\
 0 & I_{3,2} & B_3 & I_{3,4} & 0 & \dots & \dots & \dots & \dots & \dots & \\
 \vdots & \vdots & \vdots & \vdots & \vdots & & & & & & \\
 0 & \dots & \dots & \dots & \dots & \dots & I_{25,24} & B_{25} & I_{25,26} & \dots & \\
 \vdots & & & & & & \vdots & \vdots & \vdots & & \\
 & & & & & & & & & & \\
 \vdots & & & & & & & & & & \\
 0 & \dots & & & & & \vdots & \vdots & \vdots & & \\
 0 & \dots & & & & & \dots & 0 & I_{74,73} & B_{74} & I_{74,75} \\
 0 & \dots & & & & & \dots & 0 & I_{75,74} & B_{75} & \\
 \vdots & & & & & & & & & &
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 \vdots \\
 T_{25} \\
 T_{26} \\
 \vdots \\
 T_{50} \\
 T_{51} \\
 \vdots \\
 T_{75}
 \end{bmatrix}
 =
 \begin{bmatrix}
 \frac{K_{1,f} A_{1,f}}{h_{1,f}} T_{in} \\
 \frac{K_{2,f} A_{2,f}}{h_{2,f}} T_{in} \\
 \vdots \\
 \frac{K_{25,f} A_{25,f}}{h_{25,f}} T_{in} \\
 0 \\
 \vdots \\
 0 \\
 \frac{K_{51,f} A_{51,f}}{h_{51,f}} T_{out} \\
 \vdots \\
 \frac{K_{75,f} A_{75,f}}{h_{75,f}} T_{out}
 \end{bmatrix} \quad (4.2)$$

where:

$$B_n = \sum_{i=j}^m \frac{A_{n,i} K_{n,i}}{h_{n,i}} \quad (4.3)$$

with j to m being the nodes adjacent to the n^{th} node and:

$$I_{n,i} = \frac{A_{n,i} K_{n,i}}{h_{n,i}} \quad (4.4)$$

$A_{n,i}$, $K_{n,i}$ and $h_{n,i}$ are respectively the cross-sectional area of the node domain normal to the grid line between nodes n and i, the overall grid line conductivity between these nodes and the grid spacing for this case.

$A_{n,f}$, $K_{n,f}$ and $h_{n,f}$ refer to the same variables between the n^{th} node and an adjacent node on the boundary face where the temperature is specified in the problem definition.

Because the grid spacing is not constant throughout the model the overall grid line conductivity $K_{n,i}$ will depend not only on the conductivity of adjacent node domains but also upon the grid length where the adjacent node domains are of different size. For this case equation (3.26) giving the grid conductivity will become:

$$K_{n,i} = \frac{4 K_n K_i}{h_n K_i + h_i K_n} \quad (4.5)$$

The matrix equation may be solved to yield the model temperature distribution by any of a number of standard techniques suitable for use with a digital computer. For this study the method of Gaussian Elimination with pivotal condensation was used, but being utilised as a subroutine to the main computer program could be replaced by any other method. With the temperature distribution determined, the overall mean heat flux and hence the effective thermal conductivity of the two-component simulation may be evaluated as described in the previous section.

It is worth noting that the solution procedure is quite independent of the boundary temperatures T_{IN} and T_{OUT} which may therefore be chosen arbitrarily. The effect of the overall temperature gradient on the mean heat flux is negated by their use in the Fourier Equation. Also the method is independent of the system of units in which the pure component conductivities are specified since the prediction will simply be in the same system. This being the case it is more convenient to employ non-dimensionalised conductivities by using the

ratio of component values since this allows of easy comparison between different studies and systems.

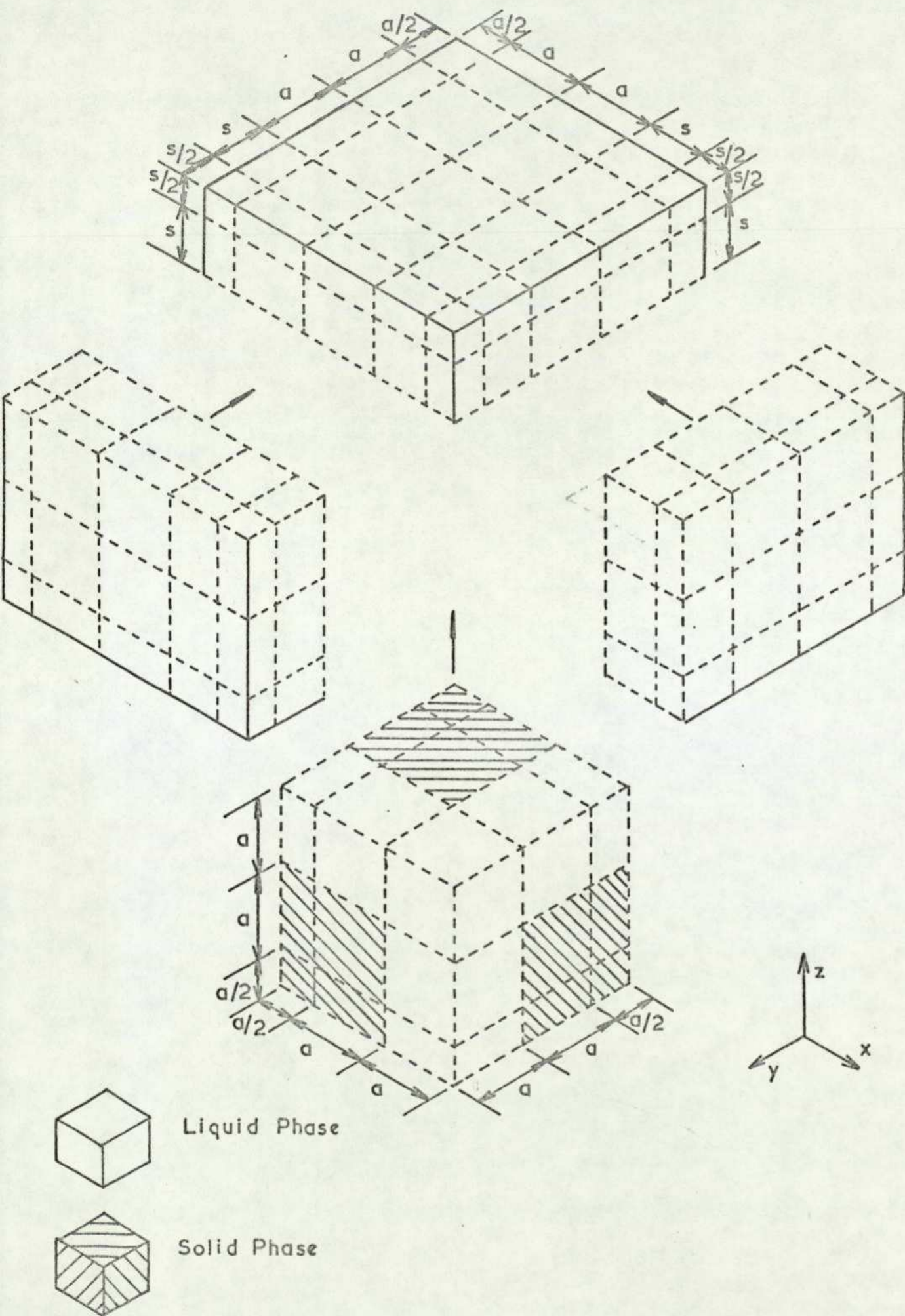
4.4 Comparison with Fidelle and Kirk's Model.

Fidelle and Kirk /20/ have also presented a method for determining the thermal conductivity of a two-component system that is based upon the unit cell model. As with the authors prediction a tridirectional heat flux situation was envisaged in deriving a solution but the simulation of the model differed in concept from the present work. The model of Fidelle and Kirk was divided throughout by a grid of fixed spacing and allowance made for various values of volume fraction solid content by assigning the number of nodes within each phase to correspond with this. Therefore with this simulation as the volume fraction solid content of the system under consideration was increased the number of nodal points within the continuous phase would decrease. It is believed that the present method, although more complicated to program, offers an advantage in that firstly, the simulation of the model remains the same for all values of volume fraction solid hence making comparisons more valid. More importantly as the volume fraction solid increases and the prediction consequently becomes more difficult, the grid spacing in the critical interfacial region automatically becomes smaller thus allowing greater accuracy.

In figures 4.2 to 4.4 are shown the performance of Fidelle and Kirk's model and the present work in predicting the experimental results obtained by Fidelle and Kirk for various two-component systems. The simulation of Fidelle and Kirk employed an array of $12 \times 12 \times 12$ nodal points whilst a $5 \times 5 \times 5$ array was used by the author, but it can be seen that the latter method does not suffer by this. Comparing the overall mean errors in prediction for the three systems combined, that for Fidelle and Kirk's model is 12.3% as against 9.2% for the

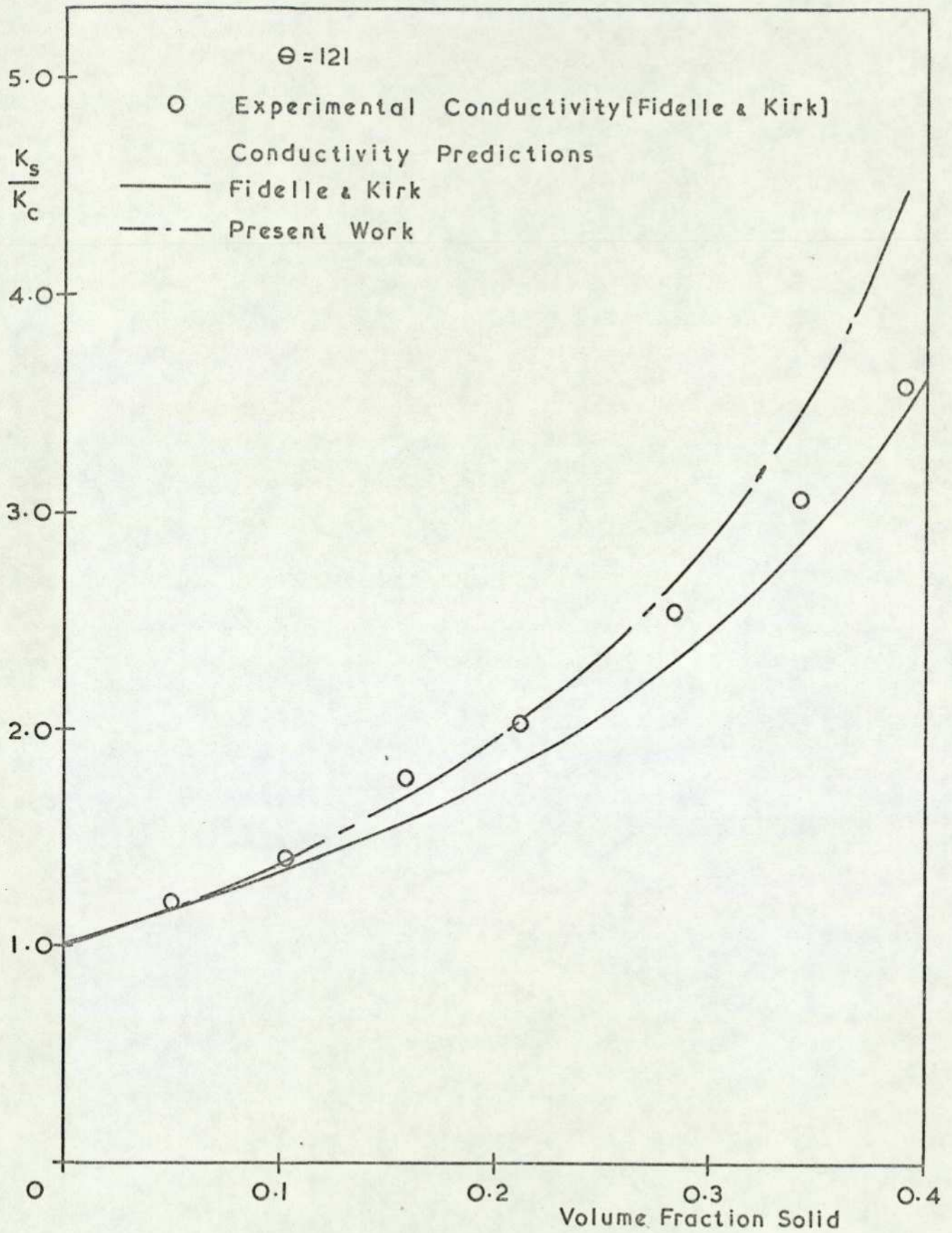
present work. This improvement in prediction accuracy combined with the coarser grid spacing and consequent saving in computer storage and execution time requirements are sufficient to outweigh the disadvantages associated with a variable grid spacing and indicate the usefulness of the simulation.

It is, however, possible that the solution technique, employed by Fidelle and Kirk on the set of simultaneous equations formed from the finite difference approximation, contributed to this disparity in prediction accuracy. This subject is discussed fully in the following chapter in conjunction with a further development of this model.



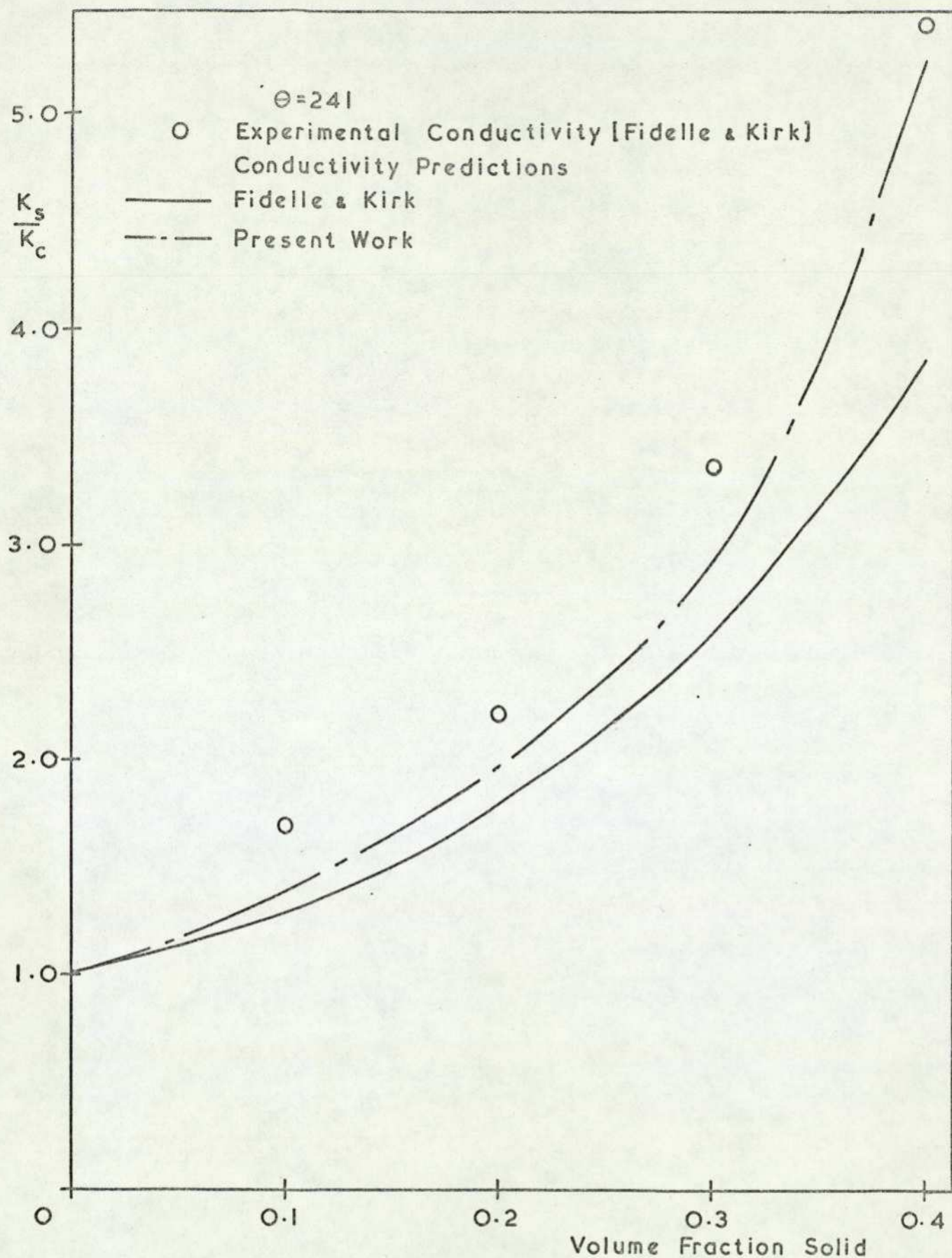
DIVISION OF UNIT CELL MODEL INTO
NODAL DOMAINS

Figure 4.1



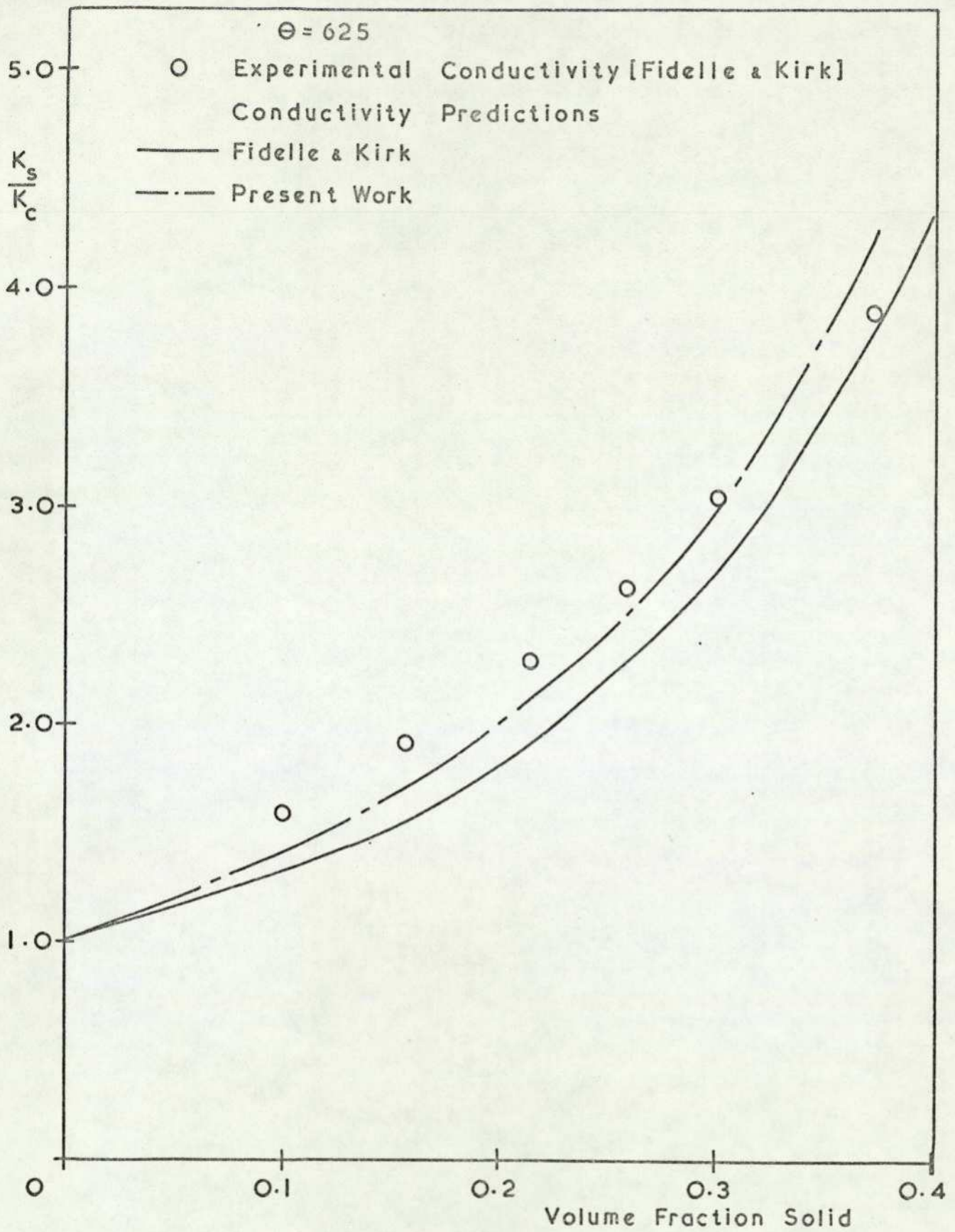
COMPARISON OF UNIT CELL MODEL CONDUCTIVITY PREDICTIONS FOR ALUMINA-PLASTER SYSTEM

Figure 4.2



COMPARISON OF UNIT CELL MODEL CONDUCTIVITY PREDICTIONS FOR ALUMINA-POLYESTER SYSTEM

Figure 4.3



COMPARISON OF UNIT CELL MODEL CONDUCTIVITY
PREDICTIONS FOR CARBORUNDUM-PLASTER SYSTEM

Figure 4.4

5 RANDOM PLACEMENT MODEL

5.1 Definition of Model.

The concept of a unit cell which incorporates the simulation of a spherical particle within a rectangular co-ordinate system was utilised in the development of a further model in which the restriction imposed by the assumption of a regular arrangement of phases was relaxed. This, to some degree, allows the distribution of phases in a two-component system to be more nearly approximated by a digital simulation. The remaining basic assumptions made in the development of the general solution technique and the unit cell model apply to this model.

This model is based upon the designation of a unit cell as a basic unit which is to be placed in a random fashion into a field of the continuous phase. In this way, the requirement of simulating each spherical particle within the rectangular co-ordinate system of the field in the modelling of a sample of a system is overcome without the necessity to approximate the particles as simple cubes as in the method of Baxley and Couper /18/. If the thermal conductivity of the basic unit is first assessed then they may be treated as if they were a homogeneous entity possessing an equivalent conductivity. The simulation is built up by assigning sufficient of the units into the continuous field to give the required volume fraction solid for the system, so that the model consists of a two-component mixture that has been modified from the original designation. Instead of the specified continuous and discontinuous phases there are the continuous phase as specified and a discontinuous phase given by the basic unit which may be termed a pseudo-phase. The thermal conductivity of the simulation may then be determined by applying to it the previously developed prediction which incorporates a tridirectional heat flux.

5.2 Simulation of Two-Component System.

The number of basic units placed within the field of the continuous phase is determined by both the system volume fraction solid and the relative proportions of phases within the unit cell. Initially it was proposed to use a cube that just enclosed the solid sphere thus giving a volume fraction solid of 0.5236 for the cell; but noting that the prediction accuracy of the previous model was greatest for lower volume fraction solid values an increase in cube size was made. This was found to improve the prediction by, firstly giving a greater accuracy for the unit cell conductivity and secondly allowing a wider dispersion of the basic unit throughout the field and minimising the accumulation of units in any one portion of the model. However, the volume fraction solid of the basic unit also defines the upper limit for the system which will be reached when the field is full of the unit cells, therefore the relative sizes of sphere and enclosing cube must represent a compromise between the applicability of the model and its accuracy. A value of 0.446 for the unit cell volume fraction solid was chosen for this study, it having also the advantage of suiting computational requirements.

5.2.1 Placement of Basic Unit.

In the general solution technique and unit cell model previously developed it was assumed that the division of the model into separate phases would follow the division into nodal domains. It is however, possible using a random placement scheme to allow the phases to coincide with the actual grid lines which divide the solution region with a three-dimensional cubic network. This latter method would follow the

treatment adopted by Schenk /25/ for systems having material of more than one conductivity value. The solution procedure using this placement method would follow that outlined in the preceding section with the exception that instead of determining the overall grid line conductivity between nodes by equation (3.26) this would be given by taking the mean of the conductivities of the cubes surrounding the relevant grid line. This in effect would be the effective conductivity of the number of cubes in a parallel arrangement. The general finite difference equation (3.23) will then be the same for both placement methods and both will give a valid solution to Laplace's equation since the energy balance will be correctly formed at each node for both cases. Comparison of sample temperature profiles through a two-component simulation obtained using both basic unit placement methods and given in table 5.1 indicate the similarity of the solutions; it is not possible to use exactly the same simulation for both (although the basic unit is of the same conductivity). However, the estimation of model thermal conductivity requires the assumption that heat flow is confined to grid lines and, hence, a correct estimate of grid line conductivity. The method of placement of basic units co-incident with grid lines produces an overestimate of grid line conductivity which in turn leads to an overestimate of mean heat flux and hence of effective system conductivity. Although, therefore, the method used by Schenk appears valid as a solution technique for temperature distribution it cannot be used for the assessment of effective thermal conductivity.

In consequence the method adopted involved the placement of the basic units co-incident with nodal domains and although this entailed the use of half units at the model boundary where the nodal domains are

reduced in size in the event this proved to give finer control over the simulation volume fraction solid value.

5.2.2. Assignment of Basic Unit Co-ordinates.

The basic unit distribution through the three-dimensional field was controlled by describing their position with co-ordinates obtained using a random number generator. Strictly, the unit co-ordinates were assigned from pseudo-random numbers, the multiplicative congruential method being used thus supplying random variables with a uniform probability density function. It is shown in /26/ that frequency and serial tests indicate that the random numbers so supplied are uncorrelated and uniformly distributed.

A 10 x 10 x 10 nodal array divided the two-component system simulation, which therefore required the number of basic units assigned for any problem to be given by:

$$N_o = \frac{9^3 \varphi}{0.446} \quad (5.1)$$

The basic unit was also divided by a nodal array of similar size so that the same solution procedure could be used in evaluating the conductivity of both this and the system. Prediction of the system thermal conductivity was thus in two stages with the conductivity of the basic unit being first determined and then fed directly into the system simulation as a discontinuous phase value. In effect then, this method has the effect of concentrating a much finer grid network in the region in close proximity to the discontinuous phase inclusions and allowing a coarser network in remoter regions, whilst avoiding the

necessity of either matching up differing grid sizes or repeating the process for each sphere in the system model. The use of a finer grid, in general, improves the accuracy of a finite difference solution.

The simulation of the basic unit was accomplished in a simpler manner than for the unit cell model since it was of a fixed volume fraction solid, and this was done by allowing the nodal domains to consist of the phase that contained their respective node in the geometric arrangement of sphere and cube.

5.3 Solution Procedure.

A boundary temperature was specified for the nodes on two parallel faces of the model leaving a total of 800 nodes for which the finite difference equation was written. With a large set of simultaneous equations an indirect method of solution was required both from considerations of computer storage and in order to take advantage of the large number of zero components in the matrix representation of these equations.

The general finite difference equation (3.23) may be rearranged to set up a Gauss-Siedal iterative scheme:

$$T_{n+} = \frac{\sum_{i=1}^s K_{i,n} \frac{A_{i,n}}{h_{i,n}} T_i}{\sum_{i=1}^s K_{i,n} \frac{A_{i,n}}{h_{i,n}}} \quad (5.2)$$

where T_{n+} is the improved estimate of the temperature at the n^{th} node and with the latest iterates being used as soon as they are all available. In general the order in which the node points are scanned

will determine the rate of convergence to the solution; the method used being the scanning of successive grid lines in the same direction on each consecutive plane.

An accelerated rate of convergence of the iterative process is obtained by using the Extrapolated Liebmann method given by:

$$T_{n+} = \bar{W} (T_{n+} - T_n) + T_n \quad (5.3)$$

where T_{n+} is the estimate given by the Gauss-Siedal scheme and the relaxation fraction $\bar{W}$ lies between 1 and 2. The optimum value of the relaxation factor will depend upon the particular problem and is determined largely by experience. For this study it was found to be 1.8. Convergence rate is also improved by assigning a reasonable initial estimate of temperature to each node; this was accomplished by taking a linear temperature gradient between the non-insulated boundary surfaces and allowing the normal planes to be isothermal.

The computation process is much simplified by using the nodal code scheme outlined by Schenk /25/ in which each node is given a code depending upon the relevant number of terms in equation (5.3) and the orientation of the grid lines to adjacent nodes. At each iteration the code directs the program to the correct equation and therefore avoids the need to write a separate equation for each node or the use of imaginary nodes outside the boundary surface. The nodal code layout for the random placement model is shown in figure 5.1.

Convergence of the iterative scheme to a solution was assumed when successive estimates of temperature at each node differed by less than 0.01 per cent. Comparison of the heat fluxes across both non-insulated boundary surfaces indicate that this was a valid criterion for both the unit cell and the random placement system.

5.4 Comparison with Baxley and Couper's Model.

Baxley and Couper /18/ have developed a model for the prediction of the thermal conductivity of two-component systems that also involves a simulation using a random placement of the discontinuous phase and a tridirectional heat flux. The placement of the discontinuous phase was directly as cubes within the field of the continuous phase, with no attempt at modelling the shape of the particle being made. A wide variance placement for any given system resulted from this technique and the variance increased as the ratio of the conductivity of the discontinuous phase to that of the continuous increased. In order to overcome this disadvantage and produce a reasonable prediction, the mean of nine or ten individual predictions was taken by Baxley and Couper as representing the system conductivity.

The present work, which involves the modelling of the particle shape and consequently producing a finer dispersion of the phases, was found to produce a much lower variance in prediction and it was considered that this was sufficiently reduced to allow the use of any one prediction as an accurate solution. In table 5.2 the performance of the present model and Baxley and Couper's model, when applied to the most testing of the latter authors' experimental systems, are compared. Ten individual predictions were obtained with both models, then a mean value and the variance were calculated using standard formula. Since the numerical value of the variance is dependant on the units used for the parameter under examination a better comparison is given by the coefficient of variation, defined as:

$$v = \frac{\sigma(K)}{\mu(K)} 100\%$$

(5.4)

It can readily be seen that the present work produces a more consistent and reproduceable result.

A further criticism of the method proposed by Baxley and Couper arises from consideration of their solution technique whereby they obtain the steady state temperature distribution through the model of the suspension. The solution procedure is not defined, in their report, in terms of a general solution technique for elliptic partial differential equations although, in dividing the model into cubes which are designated to be of either phase at random, by implication this procedure is adopted. Each cube is considered to be isothermal, thus it follows that a node lies at the centre of each cube and that grid lines will bisect the cube; further, the cubes must therefore coincide with the nodal domains. A general finite difference equation is written for all except the nodes where a boundary temperature is specified; the cubes at the edge of the model are treated by reducing the number of terms in this equation, so that in physical terms the heat balance for such a node only includes heat exchange with a reduced number of adjacent nodes. The expedient by which this boundary treatment is accomplished is to propose imaginary cubes outside the model, each having zero conductivity; hence the grid line conductivity will be zero in this direction thus removing the relevant terms from the finite difference equation. However, this is not a sufficient treatment of the boundary condition since:

1. The boundary must coincide with a grid line, this follows because the solution of elliptic partial differential equations requires that the boundary condition must be specified at all points round a closed surface which, in

the numerical approximation method, reduces to a specification at each node on the surface. This means that the boundary of the model is actually located along the centre line of each boundary cube, reducing the simulation dimensions by one cube length in each direction.

2. The finite difference approximation written for each boundary node must be modified to take account of the reduced area for heat conduction to adjacent nodes also on the boundary, as detailed by Meyers /27/ whereas the method of Baxley and Couper does not differentiate between these nodes and those in the centre of the model.

It is difficult to estimate the overall effect of these deviations from the standard solution technique for this class of equation since:

1. The temperature distribution will not be correct although it probably will not be much changed.
2. The heat flux determined from the temperature distribution will be incorrect because of the above reason and also because the area of heat flow between boundary nodes has been taken as the full nodal cross-sectional area.
3. The model dimensions used in evaluating the effective thermal conductivity using Fourier's Equation do not coincide with the boundary of the solution volume. This will be the largest error in the method in all probability and will underestimate the conductivity by a factor of 539/640.

The cumulative effect appears not to be great but there is nonetheless a conceptual error in the solution technique which must cast doubts upon its validity.

This same solution procedure is also used by Fidelle and Kirk /20/, in their digital simulation of a unit cell model, without any comment being made as to its validity. If, as it appears, the method has been used as proposed by Baxley and Couper then the foregoing comments must apply also to their work.

RANDOM PLACEMENT MODEL TEMPERATURE PROFILES
FOR TWO VARIATIONS OF BASIC UNIT POSITION

TEMPERATURE PROFILES THROUGH MODELS																			
UNIT PLACEMENT COINCIDENT WITH NODAL DOMAINS										UNIT PLACEMENT COINCIDENT WITH GRID LINES									
KD/KC = 647.15 VF = 0.297 KS/KC = 2.728										KD/KC = 647.15 VF = 0.297 KS/KC = 3.189									
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	X	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
8.63	8.70	8.99	8.82	8.89	8.88	9.05	9.10	8.95	8.88	X	9.33	9.22	9.13	9.05	8.85	8.89	9.13	9.30	9.20
8.64	8.69	8.76	8.78	8.70	8.81	8.83	9.13	8.97	9.13	X	9.16	9.16	9.16	9.03	8.84	8.90	9.07	9.20	9.12
8.71	8.78	8.87	8.80	8.73	8.70	8.93	9.04	9.12	9.05	X	9.03	9.11	9.24	9.03	8.82	8.88	8.98	9.07	9.06
8.65	8.82	8.85	8.95	9.13	9.07	9.16	9.00	8.88	9.00	X	8.98	9.02	9.03	8.86	8.76	8.87	8.96	8.97	8.95
8.84	8.98	8.84	8.80	8.96	9.16	9.21	9.07	8.99	8.85	X	8.92	8.93	8.86	8.68	8.73	8.90	9.00	9.07	9.10
8.76	8.76	8.60	8.54	8.92	8.99	9.04	8.94	8.90	8.74	X	8.89	8.90	8.84	8.74	8.89	9.00	9.06	9.20	9.22
8.59	8.73	8.70	8.57	8.92	9.00	9.21	9.09	9.13	9.13	X	8.95	8.99	8.93	8.92	9.09	9.10	9.07	9.12	9.13
8.71	8.69	8.80	8.83	9.17	9.14	9.15	9.26	9.20	9.09	X	9.00	9.06	9.02	9.03	9.16	9.13	9.07	9.08	9.05
8.83	8.94	8.84	8.95	9.22	9.22	9.06	9.04	8.88	8.86	X	9.09	9.07	8.97	8.99	9.06	9.09	9.08	9.03	8.90
8.92	9.08	8.77	8.92	9.07	9.07	8.91	8.98	8.94	8.95	X	9.24	9.07	8.89	8.88	8.91	9.01	9.10	9.01	8.83
7.14	7.18	7.42	7.55	7.54	7.70	7.78	7.88	7.88	7.85	X	7.85	8.10	8.22	8.23	7.96	7.96	8.08	7.46	7.29
7.21	7.25	7.25	7.50	7.60	7.72	7.81	7.98	7.89	7.96	X	7.78	7.80	7.93	8.11	7.93	7.93	8.04	7.72	7.58
7.38	7.53	7.43	7.56	7.54	7.74	7.87	8.06	7.91	7.90	X	7.85	7.70	7.49	7.65	7.79	7.93	8.09	8.04	7.98
7.60	7.64	7.69	7.55	7.81	7.81	7.84	7.88	8.02	7.86	X	7.90	7.85	7.73	7.66	7.72	7.99	8.10	8.01	7.92
7.68	7.67	7.63	7.64	7.72	7.83	7.95	7.67	7.66	7.53	X	7.81	7.85	7.89	7.79	7.79	8.02	8.05	7.89	7.72
7.61	7.45	7.42	7.55	7.63	7.79	7.82	7.62	7.72	7.74	X	7.69	7.76	7.84	7.81	7.82	8.04	8.06	7.86	7.81
7.59	7.60	7.47	7.61	7.90	7.74	8.01	7.78	7.89	7.95	X	7.76	7.75	7.87	7.99	8.07	8.08	8.05	8.04	8.11
7.46	7.57	7.68	7.55	7.88	8.06	8.24	8.07	8.16	8.12	X	7.84	7.75	7.84	8.07	8.21	8.10	8.08	8.19	8.11
7.61	7.56	7.71	7.76	7.96	8.10	8.05	8.05	8.07	8.09	X	7.66	7.69	7.85	7.99	7.98	7.97	8.01	8.11	7.97
7.68	7.80	7.74	7.53	7.95	7.99	7.98	8.01	8.00	8.09	X	7.48	7.68	7.90	8.00	7.95	7.88	7.92	8.08	7.98
6.41	6.41	6.30	6.35	6.41	6.52	6.44	6.55	6.85	6.66	X	6.00	6.01	5.99	6.01	6.32	6.39	6.27	6.41	6.41
6.50	6.58	6.47	6.27	6.47	6.67	6.77	6.46	6.67	6.53	X	6.26	6.26	6.16	6.09	6.41	6.58	6.44	6.49	6.55
6.56	6.64	6.34	6.43	6.72	6.83	6.82	6.61	6.45	6.72	X	6.37	6.41	6.37	6.37	6.60	6.72	6.46	6.53	6.93
6.61	6.60	6.40	6.33	6.53	6.58	6.42	6.54	6.67	6.65	X	6.49	6.50	6.47	6.51	6.65	6.63	6.45	6.60	7.00
6.68	6.62	6.58	6.51	6.53	6.59	6.59	6.48	6.57	6.64	X	6.65	6.66	6.68	6.67	6.71	6.59	6.43	6.51	6.71
6.75	6.61	6.57	6.52	6.40	6.64	6.72	6.77	6.68	6.88	X	6.67	6.73	6.89	6.90	6.74	6.50	6.46	6.51	6.52
6.66	6.78	6.74	6.62	6.62	6.84	6.69	6.74	6.68	6.91	X	6.57	6.68	6.88	6.98	6.75	6.60	6.67	6.65	6.50
6.53	6.53	6.55	6.77	6.73	6.69	6.84	6.86	7.00	7.21	X	6.57	6.65	6.75	6.78	6.66	6.69	6.75	6.73	6.61
6.49	6.38	6.60	6.70	6.78	6.74	6.87	7.04	7.28	7.32	X	6.66	6.70	6.73	6.77	6.85	6.81	6.84	6.82	6.73
6.60	6.52	6.64	6.74	6.93	6.89	6.90	7.02	7.18	7.32	X	6.65	6.70	6.69	6.77	6.97	6.96	6.90	6.84	6.79
5.51	5.44	5.36	5.31	5.24	5.22	5.24	5.09	5.17	5.20	X	5.28	5.28	5.26	5.25	5.29	5.26	5.22	5.24	5.26
5.66	5.49	5.46	5.30	5.18	5.49	5.27	4.94	5.06	5.17	X	5.36	5.38	5.34	5.29	5.28	5.25	5.26	5.26	5.27
5.58	5.58	5.39	5.38	5.33	5.38	5.32	5.26	5.43	5.28	X	5.46	5.52	5.48	5.41	5.34	5.28	5.41	5.28	5.13
5.63	5.55	5.57	5.51	5.25	5.22	5.28	5.32	5.19	5.33	X	5.51	5.49	5.46	5.52	5.48	5.29	5.38	5.26	5.10
5.73	5.56	5.62	5.48	5.41	5.41	5.45	5.56	5.37	5.45	X	5.61	5.50	5.38	5.44	5.43	5.32	5.37	5.33	5.41
5.57	5.52	5.53	5.46	5.40	5.52	5.69	5.50	5.44	5.51	X	5.63	5.51	5.32	5.33	5.39	5.43	5.43	5.40	5.55
5.56	5.62	5.61	5.45	5.24	5.46	5.55	5.45	5.48	5.61	X	5.55	5.51	5.40	5.20	5.38	5.45	5.50	5.51	5.56
5.51	5.40	5.41	5.49	5.42	5.54	5.58	5.54	5.57	5.60	X	5.60	5.65	5.67	5.54	5.52	5.64	5.78	5.73	5.56
5.50	5.51	5.66	5.46	5.44	5.63	5.67	5.58	5.81	5.68	X	5.67	5.77	5.82	5.80	5.81	5.83	5.97	5.91	5.78
5.36	5.40	5.59	5.58	5.59	5.82	5.76	5.95	5.91	5.90	X	5.68	5.80	5.85	5.88	6.01	5.91	6.01	6.10	6.06

X																				
4.41	4.44	4.34	4.27	4.12	4.19	4.07	3.81	3.63	3.82	X	4.38	4.35	4.35	4.37	4.32	4.19	4.00	4.00	4.12	4.14
4.37	4.22	4.32	4.29	4.20	4.17	3.89	3.87	3.65	3.84	X	4.40	4.34	4.31	4.35	4.34	4.18	4.00	3.99	4.06	4.11
4.47	4.33	4.36	4.30	4.05	3.98	3.90	4.06	3.84	3.63	X	4.50	4.41	4.30	4.28	4.30	4.17	4.00	4.00	3.89	3.78
4.68	4.59	4.41	4.31	4.04	4.09	4.05	4.01	3.72	3.80	X	4.50	4.45	4.40	4.26	4.13	4.14	4.19	4.15	3.93	3.72
4.40	4.26	4.40	4.37	4.28	4.24	4.01	4.02	3.72	3.95	X	4.33	4.36	4.41	4.34	4.23	4.24	4.31	4.26	4.05	3.81
4.27	4.36	4.45	4.41	4.34	4.41	4.19	4.05	4.04	4.07	X	4.37	4.39	4.41	4.37	4.35	4.46	4.41	4.33	4.17	3.96
4.46	4.37	4.48	4.40	4.23	4.08	4.12	3.90	4.16	4.03	X	4.49	4.48	4.43	4.36	4.33	4.50	4.50	4.34	4.31	4.33
4.68	4.69	4.66	4.38	4.08	4.20	4.25	4.08	3.97	3.99	X	4.51	4.49	4.50	4.40	4.41	4.43	4.36	4.28	4.40	4.52
4.51	4.65	4.60	4.34	4.06	4.29	4.52	4.04	3.91	3.79	X	4.52	4.48	4.55	4.61	4.50	4.52	4.42	4.25	4.35	4.47
4.28	4.34	4.45	4.49	4.13	4.38	4.53	4.36	3.97	3.98	X	4.53	4.52	4.54	4.57	4.59	4.80	4.75	4.14	4.16	4.43
X																				
3.29	3.32	3.29	3.30	3.31	3.38	3.23	2.84	2.79	2.81	X	3.48	3.43	3.47	3.62	3.39	3.17	3.08	3.15	3.19	3.11
3.22	3.40	3.34	3.34	3.39	3.28	3.12	2.93	2.79	2.72	X	3.47	3.42	3.40	3.50	3.39	3.15	3.08	3.14	3.19	3.10
3.31	3.22	3.38	3.42	3.26	3.29	3.11	2.94	2.68	2.73	X	3.35	3.33	3.30	3.30	3.19	3.10	3.05	3.05	3.08	3.04
3.36	3.41	3.37	3.21	3.19	3.11	3.00	2.94	2.81	2.69	X	3.24	3.24	3.21	3.18	3.11	3.13	3.08	3.03	3.06	3.02
3.15	3.25	3.27	3.32	3.26	3.16	2.76	2.76	2.81	2.66	X	3.31	3.32	3.33	3.33	3.17	3.07	3.10	3.19	3.16	3.07
3.40	3.30	3.51	3.44	3.31	3.09	2.74	2.95	2.87	2.83	X	3.45	3.44	3.52	3.61	3.29	3.02	3.15	3.38	3.30	3.16
3.40	3.12	3.38	3.52	3.23	3.04	2.81	2.93	2.90	2.89	X	3.55	3.56	3.63	3.65	3.30	3.04	3.11	3.39	3.39	3.26
3.34	3.29	3.46	3.39	3.12	3.02	2.87	2.86	2.82	2.92	X	3.49	3.55	3.66	3.58	3.22	3.14	3.20	3.36	3.41	3.32
3.33	3.42	3.32	3.24	3.10	2.96	3.18	2.91	2.67	2.75	X	3.43	3.39	3.31	3.26	3.18	3.25	3.24	3.22	3.29	3.28
3.30	3.29	3.29	3.37	3.27	3.16	3.41	2.92	2.60	2.72	X	3.42	3.29	3.18	3.21	3.15	3.17	2.98	3.03	3.14	3.22
X																				
2.30	2.07	2.20	2.26	2.26	2.29	2.15	1.98	1.91	1.88	X	2.73	2.46	2.15	1.88	1.97	2.05	2.02	1.88	1.83	1.92
2.26	2.31	2.24	2.30	2.23	2.18	2.31	2.04	1.90	1.78	X	2.73	2.53	2.23	1.99	1.94	2.04	2.04	1.90	1.82	1.92
2.39	2.30	2.54	2.42	2.22	2.17	2.29	1.96	1.88	1.94	X	2.62	2.43	2.21	2.11	1.99	1.97	2.01	1.98	2.03	2.20
2.25	2.20	2.54	2.33	2.21	1.98	2.07	2.13	1.93	1.78	X	2.31	2.22	2.14	2.13	2.07	1.96	1.99	2.12	2.16	2.19
2.18	2.23	2.49	2.39	2.26	1.96	1.96	2.01	2.04	1.84	X	2.20	2.23	2.21	2.08	2.05	2.02	2.11	2.20	2.11	2.10
2.15	2.17	2.45	2.55	2.36	1.94	1.93	1.97	1.82	1.61	X	2.10	2.18	2.22	1.97	2.04	2.09	2.12	2.04	2.05	2.09
2.07	2.05	2.19	2.37	2.17	2.04	2.01	2.12	1.77	1.74	X	2.19	2.12	1.98	1.81	2.01	2.13	2.16	1.99	1.99	2.07
2.00	1.86	1.95	2.04	2.09	2.11	2.15	2.03	1.86	1.83	X	2.34	2.20	1.81	1.69	1.85	1.98	2.01	1.90	1.91	2.03
2.13	1.98	1.94	2.08	2.05	1.99	1.95	1.93	1.83	1.82	X	2.38	2.26	1.98	1.75	1.69	1.76	1.84	1.89	1.97	2.05
2.28	2.14	2.09	2.08	2.07	2.09	2.01	1.91	1.81	1.75	X	2.54	2.29	2.06	1.82	1.56	1.61	1.88	1.94	2.00	2.06
X																				
1.11	1.07	1.14	1.20	1.23	0.93	0.85	1.01	1.00	1.08	X	0.75	0.94	1.07	1.15	1.16	1.05	1.01	0.96	0.91	0.90
1.11	1.05	1.19	1.30	1.00	0.88	1.11	1.13	0.99	0.97	X	0.81	0.93	1.09	1.18	1.15	1.05	1.02	0.99	0.88	0.82
1.13	1.10	1.32	1.24	1.12	1.07	1.06	0.87	0.94	0.95	X	0.85	0.95	1.14	1.21	1.14	1.04	1.00	0.99	0.91	0.79
1.04	0.93	1.30	1.26	1.30	1.19	1.07	1.04	0.96	0.92	X	0.94	1.00	1.10	1.20	1.19	1.06	0.94	0.90	0.95	1.01
1.03	1.06	1.24	1.41	1.18	0.95	1.03	1.19	1.03	0.90	X	1.02	0.99	0.96	1.05	1.15	1.06	0.90	0.87	1.01	1.05
0.88	1.03	1.17	1.35	1.14	0.99	0.98	0.89	0.88	0.75	X	1.00	0.97	0.86	0.89	1.09	1.03	0.89	0.91	1.06	1.13
0.83	0.97	1.03	1.02	1.05	1.01	1.00	0.95	0.66	0.63	X	0.93	1.00	1.01	0.97	1.12	1.03	0.87	0.91	0.94	1.00
0.95	0.95	0.94	0.99	1.02	0.95	1.10	1.05	0.84	0.77	X	0.91	0.95	1.00	0.97	1.01	1.03	0.98	1.00	0.97	0.91
1.01	0.95	0.82	0.96	0.99	0.94	0.93	0.95	0.90	0.90	X	0.89	0.91	0.99	0.97	0.90	0.90	0.93	1.00	1.03	1.00
1.05	1.00	0.94	0.90	0.95	0.81	0.79	0.98	0.91	0.90	X	0.75	0.91	1.07	1.01	0.86	0.79	0.85	0.97	1.00	1.01
X																				
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	X	-0.00	-0.00	-0.00	-0.00	-0.00</					

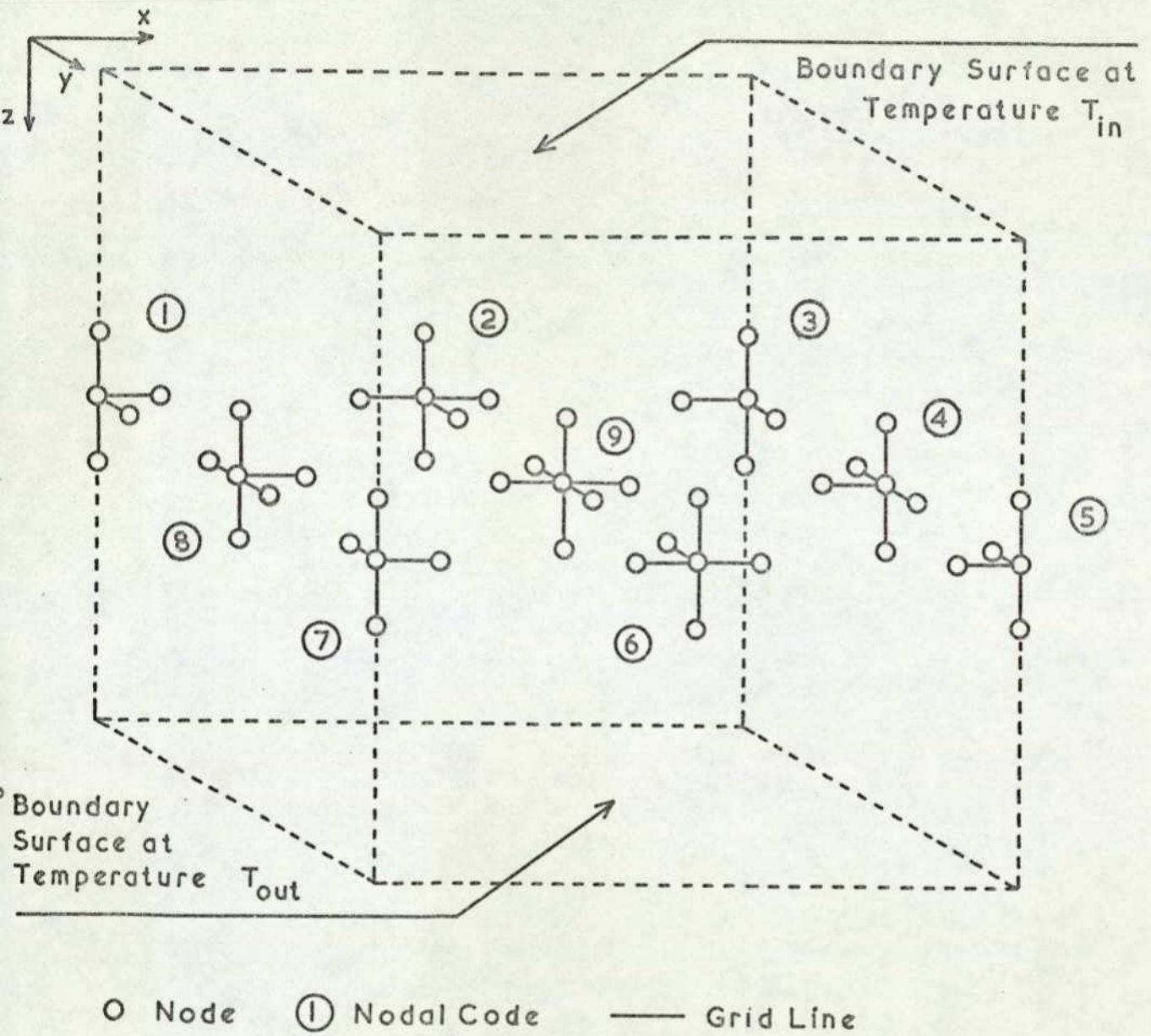
UNIT PLACEMENT COINCIDENT WITH NODAL DOMAINS

UNIT PLACEMENT COINCIDENT WITH GRID LINES

Comparison of the Repeatability of Random Prediction Models.

Prediction Method	Volume Fraction Solid	Mean Conductivity K_s/K_c	Variance in Prediction	Coefficient of Variance Per Cent
Baxley and Couper	0.3	4.456	25.74	113.8
Baxley and Couper	0.4	12.552	90.93	75.97
Random Placement	0.3	2.739	0.038	7.11
Random Placement	0.4	4.034	0.0002	0.339

Suspension - Aluminium/Polypropylene Glycol System.



NODAL CODE LAYOUT FOR RANDOM PLACEMENT MODEL

Figure 5.1

6 EXPERIMENTAL DETERMINATION OF SUSPENSION THERMAL CONDUCTIVITIES

6.1 Scope of Experiments.

Experimental thermal conductivity values were obtained for suspensions of four different metallic powders in water; thereby providing a wide range of pure component conductivity ratios for prediction evaluation. Three of these powders contained particles that were approximately spherical, in accordance with the assumptions made in developing the theoretical models; however a powder containing irregular shaped particles was also investigated for purposes of comparison. Pure component properties were obtained from published sources and are detailed in table 6.1. Particle size distributions in the copper, aluminium and stainless steel powders were obtained by a conventional sieving method, and are represented in figure 6.1. The particle size of the nickel powder was quoted by the manufacturer as being between extremely close limits. Particle shapes for each of the powders are illustrated by the photographs in figure 6.2. The aluminium powder particles can be seen to be far removed from spherical in shape, tending toward ellipsoidal or needle-like in form. With the stainless steel powder agglomeration tends to obscure the particle shape in some areas but those individual particles that can be clearly defined indicate that although their shape cannot be said to be regular yet they tend towards a spherical outline which would, therefore, be a reasonable first approximation. Agglomeration is again a problem with the finer copper powder but, again, it is sufficiently clear that an assumption of spherical particle shape is reasonable. The details of the nickel powder are supplied by the manufacturer and indicate that the particles are regular and close to spheres in shape.

6.2 Theory of Experimental Method.

A choice of experimental method to be used in this investigation, from the multiplicity of techniques which have been proposed for thermal conductivity determination, was made with two governing requirements in mind. The apparatus had to be suitable for use with a suspension, it being possible to prevent any settling out of the solid component during the duration of the test and this factor being subject to check. Secondly, the suspension sample must be of a sufficient size to enable it to be treated as a homogeneous entity. These two requirements seemed to rule against an apparatus of the co-axial cylinder type whilst horizontal flat plate equipment of the absolute type, owing to high secondary heat flows, demands a high experimental skill and sophistication of design to yield precise results. Consequently, a comparative steady state method was selected as being suitable for use with a suspension and being simple in design and concept whilst affording acceptable accuracy of results.

The precise experimental technique employed was the well established relative plane horizontal layer method, first proposed in 1881 by Christiansen /28/ and since used in many experimental studies of the thermal conductivities of gases and liquids, including two-component systems /21/, /22/, /23/, /31/.

The procedure involves passing a steady state heat flux consecutively through two plane layers of two substances, one of which is a reference standard of known conductivity. Temperature gradients will be set up that are inversely proportional to the conductivities of the two substances. Radiation heat transfer may be neglected for liquids and the arrangement of the apparatus minimises convection (as detailed

later); therefore heat transfer across both substances can be taken to be by conduction only and we may write:

$$q = \frac{K_U}{D_U} (T_p - T_q) = \frac{K_L}{D_L} (T_q - T_r) \quad (6.1)$$

hence for the test sample in the lower layer:

$$K_L = K_U \cdot \frac{D_L (T_p - T_q)}{D_U (T_q - T_r)} \quad (6.2)$$

The thermal conductivity of the unknown substance can be calculated from this equation using the experimental value of the temperature drop in each layer when the conductivity of the standard substance and the thicknesses of both layers are known. Using this method, then, removes the need to measure heat flux and only the temperature differences have to be measured during experimentation. Accurate values of the thermal conductivity of the reference substance must be available.

It is not necessary to measure the thickness of both layers in order to determine the value of the thickness ratio $\frac{D_U}{D_L}$, since by performing the experiment with both layers filled with the reference substance the ratio is obtained directly from:

$$\frac{D_U}{D_L} = \frac{K_U (T_p - T_q)}{K_L (T_q - T_r)} \quad (6.3)$$

Consequently by repeating the experiment several times the mean value can be obtained with a reasonably close accuracy. The problem of thermal conductivity evaluation thus reduces to an exact determination of temperature differences.

6.3 Experimental Apparatus.

The working chambers containing the reference and experimental substances were formed from two rigid P.T.F.E. rings of 7 cm. inside diameter, held between three copper discs which were chromium plated to prevent any corrosive effects and to minimise radiation heat transfer. A sindanio cover plate for the upper disc contained a heater formed from a close wound ferry resistance wire of 32 s.w.g. and having a total resistance of approximately 76 ohms. This heater was held in a spiral slot cut in the cover plate face and was insulated from the upper disc by means of a thin layer of mica; a 30 watt Farnell stabilised power supply unit was used to control the input to the heater. With the upper disc made substantially thicker than the other two it acted as a heat spreader and ensured a uniform temperature boundary for the upper cell.

Into each disc was inserted three 32 s.w.g. Copper/~~Copper~~/constantan thermocouples, which were threaded through stainless steel hypodermic tubes to facilitate removal and replacement. These thermocouples were located in radially drilled holes, one terminating at the centre of each disc and the others at radii of 1.3 cm. and 2.5 cm. respectively. Provision for expansion in each cell was made by means of a small hole in the upper and centre discs connecting the working cells with short vertical tubes open to atmosphere. The unit was clamped together by four side rods which tied the lower disc to a yoke fitted over the

cover plate. The lower disc formed the centre section of the base of a brass container of 15 cm. diameter to which it was fastened by four set bolts and by extensions of the side rods; a rubber joint was held between the faces of the disc and container. Insulation material was packed in the space between the cell unit and the container wall and also over the assembly. The container was suspended in a water bath which thus constituted a heat sink for the conductivity measurement unit, since the lower disc was exposed to the water. Close control of the temperature of the water in the bath was obtained by operating two tubular immersion heaters through a Jackson mercury thermoregulator. Agitation of the water was provided by an electrically driven stirring rotor. The sensitivity claimed for this arrangement indicated an attainable control of bath temperature within the limits of $\pm 0.01^{\circ}\text{C}$.

Details of the assembled equipment are shown in figures (6.3) to (6.5).

By regulating the temperature of the water bath and the heat input to the upper cell disc the temperature gradients across the cells and their mean temperatures could be set as desired.

Water was used as the reference substance and was contained in the upper cell, with the aqueous suspension under test being contained in the lower. To prevent settling out of the solid phase during the test the water phase of the suspension was gelled by the addition of 1% by weight of agar.

The copper discs being of high thermal conductivity could be regarded as isothermal between the thermocouple locations and the disc surface; the distance involved was small. This allowed the cell

temperature gradients to be obtained directly from the thermocouple temperature differences. The thermocouple e.m.f.s' were measured, in relation to a standard melting ice cold junction, using a Croydon Instruments thermocouple potentiometer.

6.4 Calibration of Apparatus.

The thermocouple calibration was carried out against a standard melting ice cold junction using a thermostatically controlled oil bath whose temperature was determined using an N.P.L. calibrated mercury-in-glass thermometer. From the tabulated values of temperature and thermocouple e.m.f.s', an explicit equation was derived using a Lagrange interpolation method which reduced the tabulated values to a second order polynomial. It was considered that the errors involved were less than those encountered in calibrating the thermocouples.

A series of preliminary experiments using water in both cells of the equipment enabled the thickness ratio of the cells D_u/D_L to be accurately determined. From ten experiments the mean value was found to be $1.1603 \pm \begin{smallmatrix} 0.01525 \\ 0.009 \end{smallmatrix}$ with the analysis of variation as detailed in table (6.2). The experimental errors involved in this calibration were calculated to be within the bounds of $\pm 1.8\%$.

The experiments were repeated with the lower cell being filled with a gell formed by adding 1% by weight of agar to water. The ratio of the conductivity of gelled water to that of pure water was found to be 0.984 ± 0.015 and this effect of a slight suppression of the conductivity of water by a gelling agent would seem to be supported by the work of Boggs and Sibbitt /29/. They did not perform tests on

agar gels, but in their work found that gelatine lowered the thermal conductivity of water by about 0.5% for each weight per cent added; methocil produced about the same effect whilst various other commercial gelling agents did not materially affect the conductivity of water, with a maximum decrease of 1% being observed. The decrease in conductivity therefore could be considered insignificant compared with the experimental error involved in thermal conductivity measurement and which therefore enabled the pure water values to be used for calculation purposes.

The thermal conductivity values for water were obtained from the expression presented by Powell et.al. /30/, where the results of numerous experimental studies were represented by:-

$$K = [-5821.871 + 63.613T - 0.0797158T^2].10^{-4} \quad (6.4)$$

with a mean deviation of 0.21% and where T is the temperature in degrees Kelvin.

The temperatures recorded at the centre of each disc were used in the calculation of conductivity because at that position the heat flux would be identical in both the upper and lower cells; this results from the circular design of the equipment which gives an area for radial heat flow that approaches zero as the centre of the cell is approached and therefore tends to an infinite resistance for radial heat loss. The remaining temperature measurements formed a check on radial heat loss, and there being no discernable difference between the two

innermost temperatures at each disc it was concluded that the above expectation was confirmed. This supports the similar finding of Kapustinsky and Ruzarvin /31/ who used an identical technique.

6.5 Effect of Convection on Conductivity Measurements.

The arrangement of horizontal working chambers with a heat flux directed downwards, together with the cell thicknesses and temperature gradients being kept to a minimum, following the recommendations of Tsederberg /32/, gives an inherently stable system thus minimising the possibility of convection currents being set up in the substances contained in the apparatus.

If, however, we consider the possibility that convection within the cells, could have occurred, then a measure of its maximum effect can be provided by considering the experiments conducted to determine the thermal conductivity of the water/agar gel. In such experiments, obviously, no convection could occur in the firm gel that a 1% agar solution produces and the effect of any convection in the reference substance, which was contained in the upper cell, would be to give an underestimate of conductivity for the test substance. If we accept the finding of Boggs and Sibbitt /29/ that the addition of a gelling agent would not increase the conductivity of water and assume further that it has no discernable effect, then the experimental indication of the lowering of the conductivity of water by agar would be solely due to convection in the upper cell. Since, however, the indicated conductivity of the gel is lower than the value for water by a mean difference of only 1.6%; on the basis of this argument this will be the maximum effect that convection can have for this equipment operating under these test conditions and is considerably less than the calculated error bound for the experiments.

This is to take the worst possible case of error due to convection, but, in fact, the absence of any effect from this cause can be shown from the consistency of the results of the experiments on agar gels over the range of test conditions. The temperature gradients across the cells were varied by a factor of three and the mean cell temperatures by a factor of two, but all the results agreed within the bound of + 1.541% and - 1.068%, which again is well within the expected experimental error limits. If convection had been a factor in the heat transmission across the experimental equipment then an increase in the cell temperature gradients by so large a factor would be expected to produce a correspondingly large divergence in the indicated conductivities.

6.6 Experimental Procedure.

The test specimen of metallic particles suspended in an agar/water gel was formed by casting in a lower cell wall ring which was held between two perspex plates. During the setting of the gel the mould was subject to continuous agitation to prevent settling out of the discontinuous phase. A visual check on the uniformity of the suspension was made before it was transferred to the conductivity measurement apparatus. The apparatus was assembled with the upper cell being filled with distilled water and the cell rings sealed against leakage by the application of silicon grease.

With the assembly suspended in the water bath the upper disc and bath temperatures were set to give the required thermal gradients and the apparatus allowed to reach steady state conditions. This normally took about three hours and was subject to the check of three consecutive readings of the disc temperatures being constant; the readings being

taken at intervals of approximately half an hour. The final disc temperatures were recorded for calculation purposes and upon dismantling the apparatus the centre core of the test specimen was removed for determination of solid content. By weighing before and after evaporation of the gel the weight fraction and hence volume fraction solid value of the suspension was evaluated.

Experimental results were obtained for suspensions of pure component solid to liquid conductivity ratios between 26.5 and 646 to 1 and of up to 30 per cent of solid phase by volume, at a mean temperature of 30°C.

The maximum error involved in experimentation was calculated to be $\pm 3.8\%$.

Table 6.1

Metal Powder Properties

Material	Supplier	Particle Shape	Nominal Particle Size	at 30°C	
				Specific Gravity	θ
Copper	Powder Metallurgy Ltd	Spherical	-300 mesh	8.92	647
Stainless Steel	B.S.A. Metal Powders Ltd	Spherical	-140 mesh	7.82	26.5
Nickel	International Nickel Ltd	Spherical	7 - 9 μ	8.9	146
Aluminium	Metals and Alloys (Birmingham) Ltd	Non-spherical	-120 mesh	2.71	386

Table 6.2

Calibration of Thermal Conductivity Measurement Equipment and
Comparison of Conductivities of Water and 1 per cent Agar Gel.

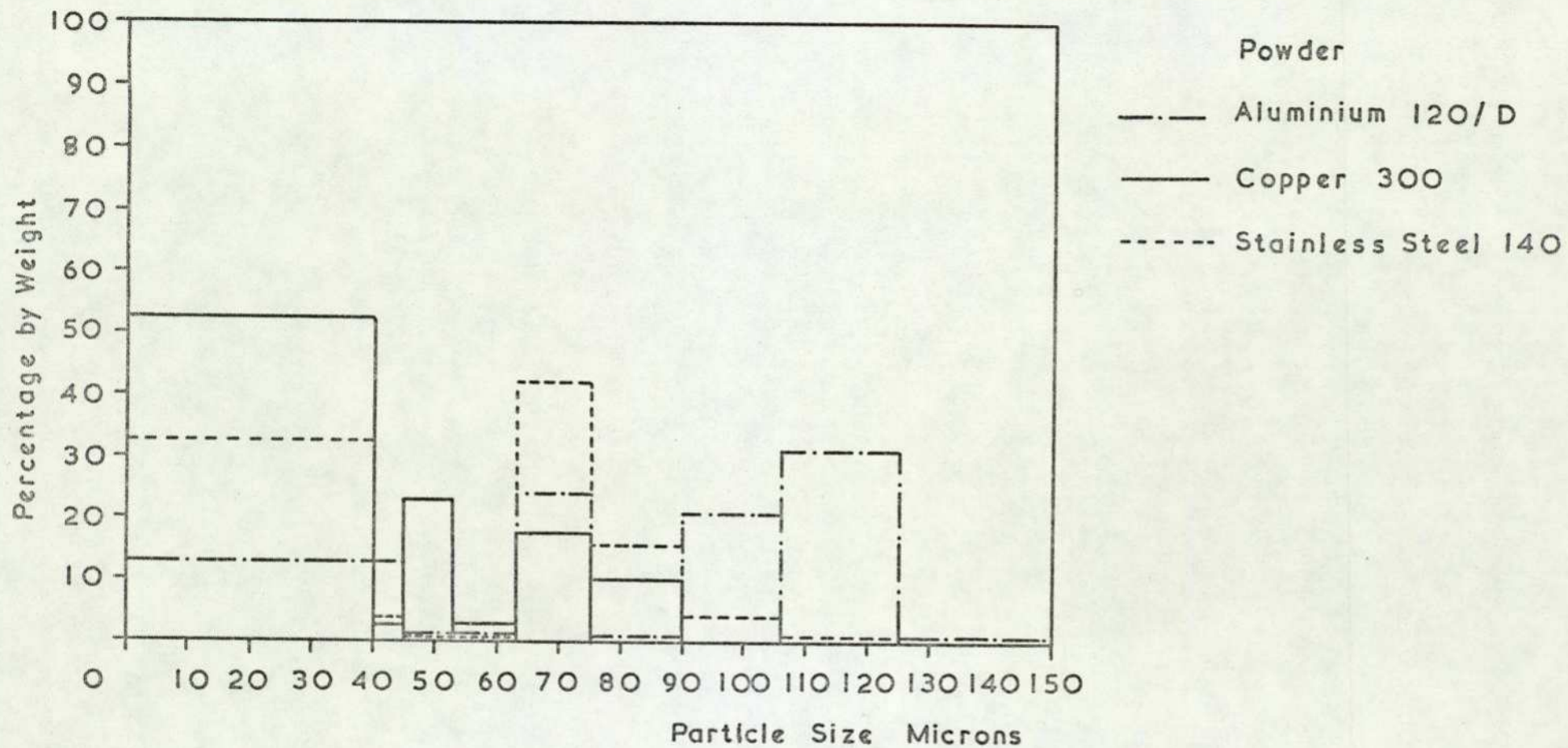
Mean Ratio of Cell Thickness	1.1603
Maximum Experimental Deviation	+1.525%, -0.905%
Standard Deviation	0.009
Coefficient of Variance	0.772%

Measurements to Determine Gel Conductivity:-

Apparent Mean Ratio of Cell Thickness	1.1421
Maximum Experimental Deviation	+1.541%, -1.068%
Standard Deviation	0.013
Coefficient of Variance	1.113%

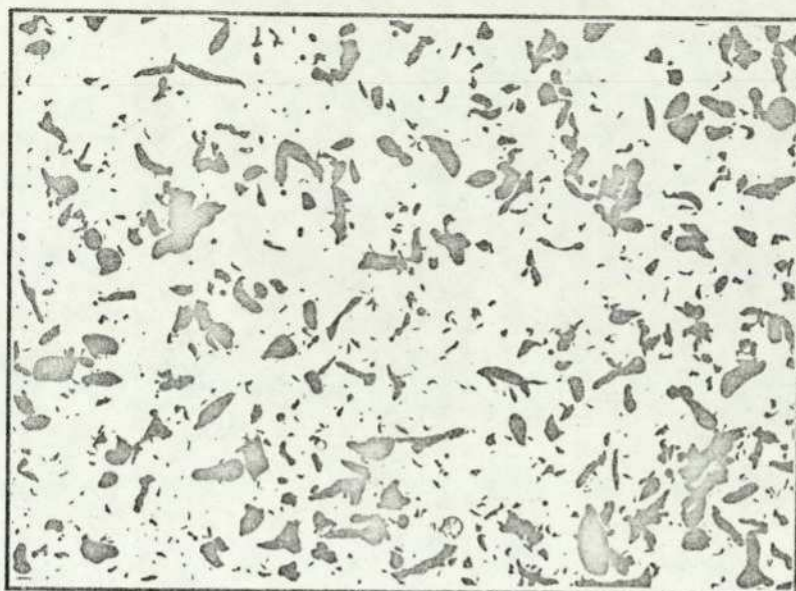
Deviation of Gel Conductivity from Water Conductivity:-

Maximum	- 2.62%
Minimum	- 0.052%
Mean	- 1.568%



DISTRIBUTION OF PARTICLE SIZE IN METALLIC POWDERS

Figure 6.1



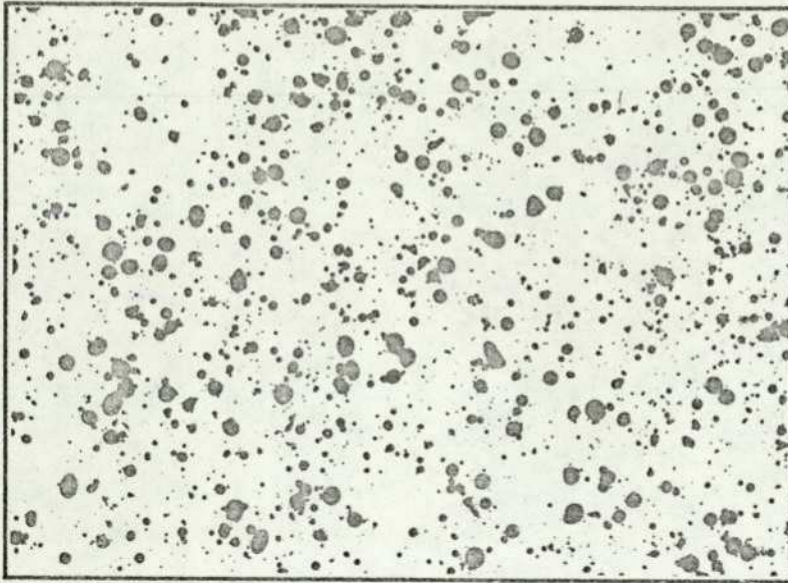
(1) ALUMINIUM POWDER, -120 MESH



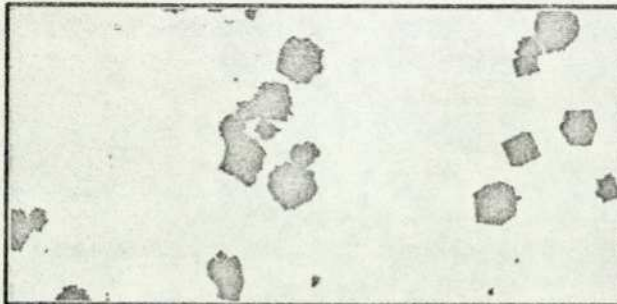
(2) STAINLESS STEEL POWDER, -140 MESH

METAL POWDER PARTICLE SHAPES

Figure 6.2



(3) COPPER POWDER , -300 MESH

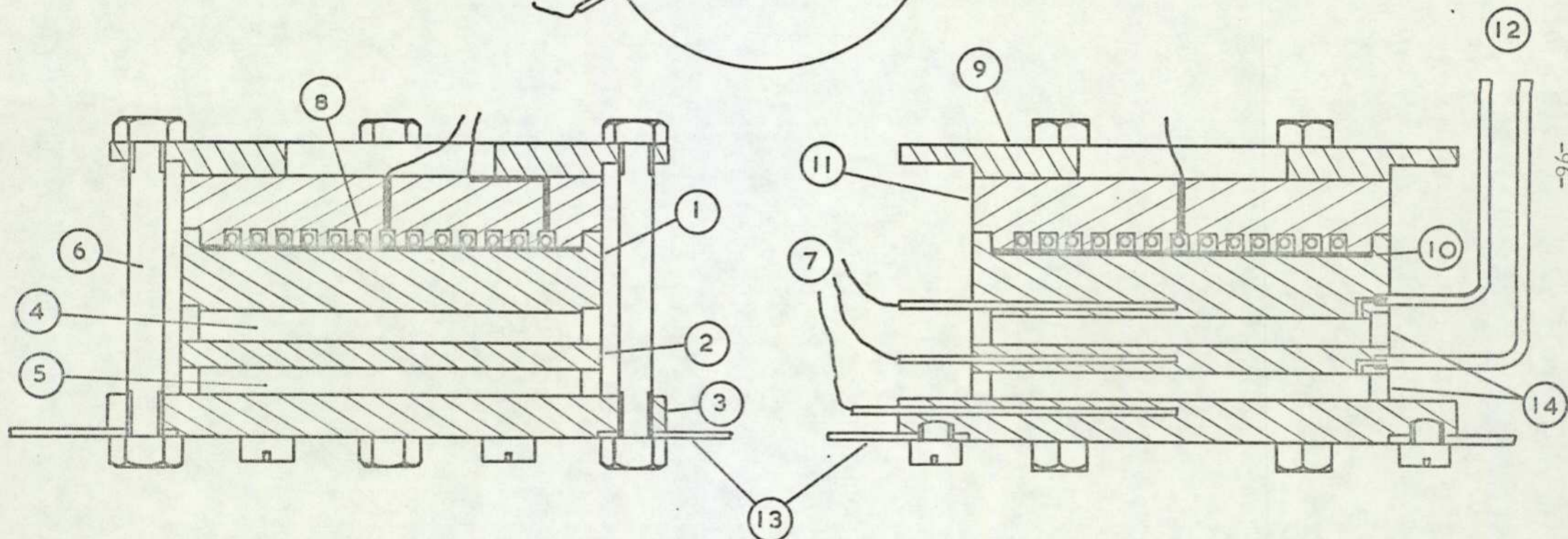
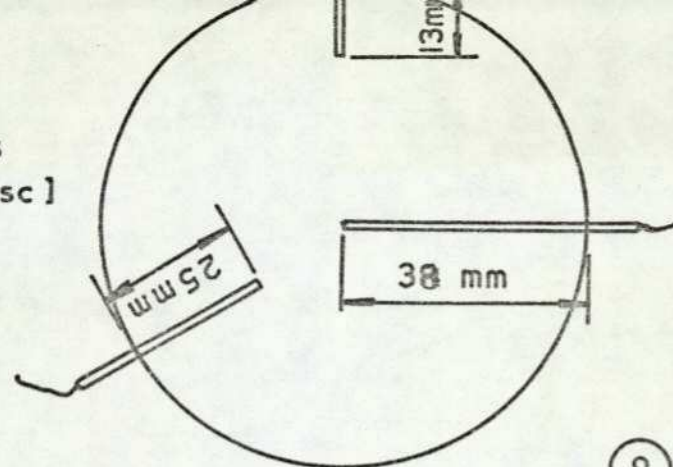


(4) NICKEL POWDER, 7-9 MICRONS

METAL POWDER PARTICLE SHAPES

Figure 6.2

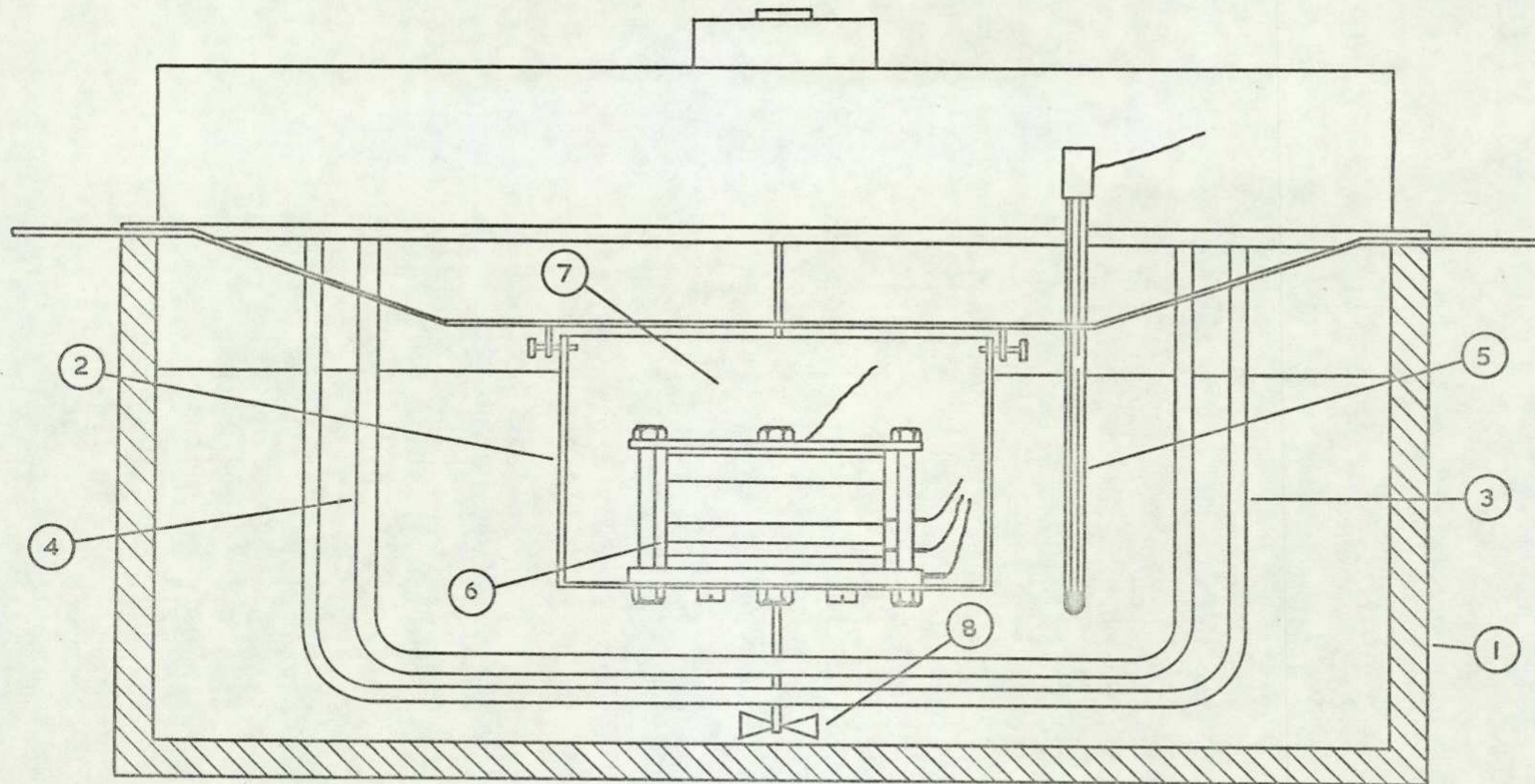
Thermocouple
Location in Discs
[+13mm in Bottom Disc]



1	Top Disc	2	Centre Disc	3	Bottom Disc	4	Upper Test Cell	5	Lower Test Cell
6	Side Rods	7	Thermocouples	8	Heater Winding	9	Yoke	10	Mica Insulation
11	Sindanio Cover Plate	12	Cell Vents	13	Outer Container	14	PTFE Test Cell Walls		

THERMAL CONDUCTIVITY TEST CELL ASSEMBLY

Figure 6.3

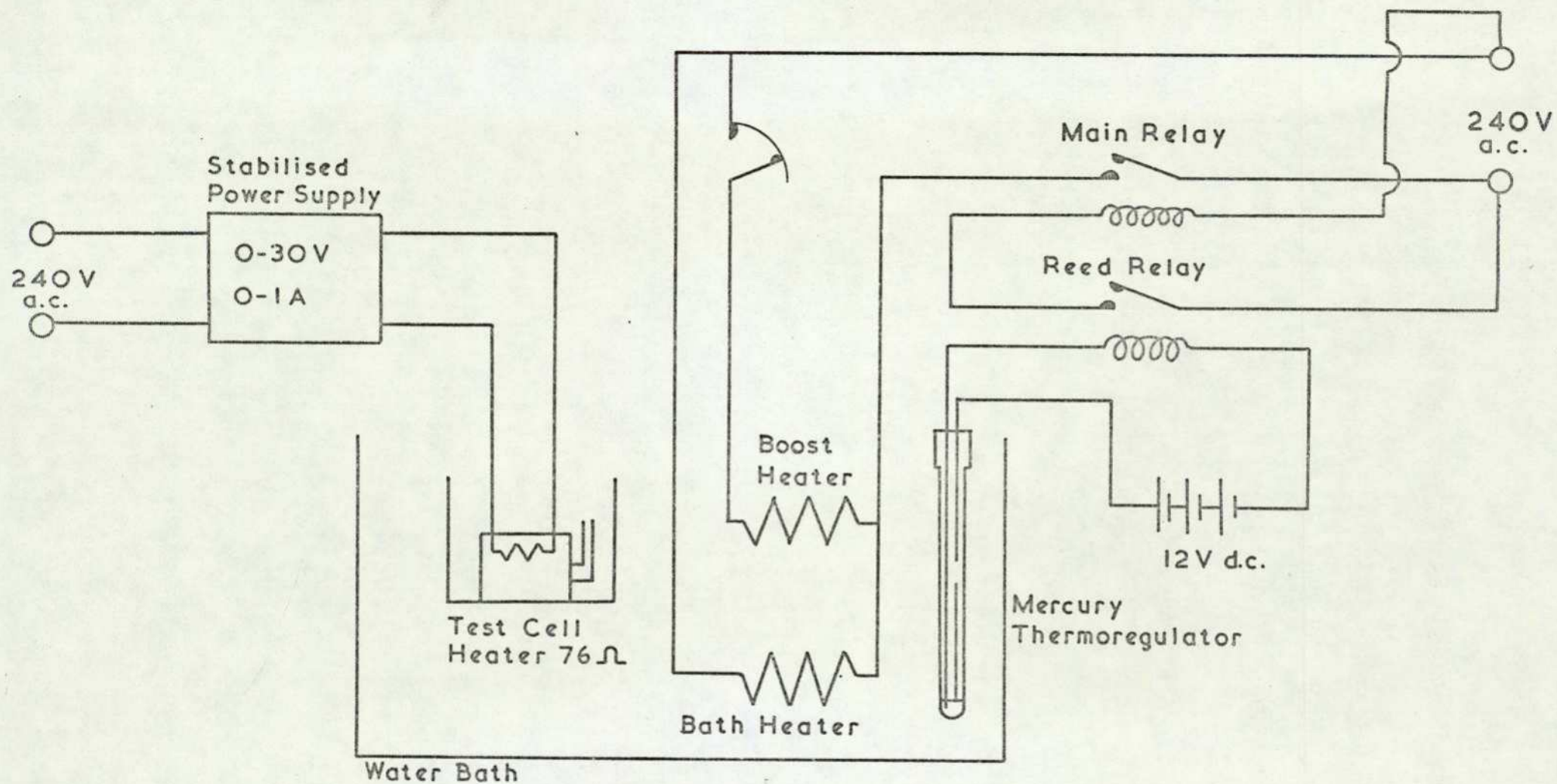


-97-

- | | | | | | | | | | |
|---|------------|---|-----------------|---|---------|---|--------------|---|-----------------|
| 1 | Water Bath | 2 | Outer Container | 3 | Heater | 4 | Boost Heater | 5 | Thermoregulator |
| 6 | Test Cell | 7 | Insulation | 8 | Stirrer | | | | |

THERMAL CONDUCTIVITY MEASUREMENT EQUIPMENT

Figure 6.4



TEST CELL TEMPERATURE CONTROL SYSTEM

Figure 6.5

7 RESULTS AND DISCUSSION

7.1 Effect of Component Conductivities on Suspension Value.

The results of the experimental determination of the thermal conductivity of suspensions are presented in graphical form in figures (7.1) to (7.5) and tabulated in table (7.1). The experimentation was performed with the primary purpose of assessing the accuracy of the prediction methods developed in this study but several general points concerning the conductivity of suspensions may be made.

The experiments demonstrate the result, noted by Metzner /33/ using Maxwell's equation, that the thermal conductivity of a liquid cannot be increased by more than a fewfold even by the addition of considerable quantities of a material of high conductivity. From figure (7.5) the conductivity of the discontinuous component can be seen to have little effect on the conductivity of a suspension provided that the component particles are of a similar shape. The conductivity of a suspension having a component conductivity ratio of 26.5 is very little below that for a suspension with a conductivity ratio of 647. Figure (7.5) however, demonstrates that factors other than the conductivity of the discontinuous phase have a far greater effect on suspension conductivity; a suspension containing irregular shaped particles has a greater conductivity than one containing spherical particles even though in the latter case the component conductivity ratio is greater. Obviously then any departure from sphericity in the discontinuous phase will have a severe effect upon the accuracy of any prediction using the models developed, since the assumption of spherical particles is seen to be critical and cannot be overlooked in extending the methods outside the terms on which they are based.

The assertion of Baxley and Couper /18/ that an increase in the conductivity of the continuous phase produces an even greater percentage increase in the effective conductivity of the suspension was not upheld by this study. This conclusion was reached by Baxley and Couper on the basis of comparing the experimental conductivities obtained from two studies on suspensions containing the same discontinuous phase material, being the work of McAlister and Orr /22/ with an aluminium - grease suspension and Johnson /23/ who studies an aluminium - water suspension. A suspension from each study with similar volume fraction solid value was compared; both works stated that the aluminium particles were irregular in shape. However, we may also compare suspensions on the basis of component conductivity ratio, a procedure just as meaningful since suspensions having the same conductivity ratio must of necessity exhibit the same heat flow pattern and therefore the same normalised suspension effective conductivity value, always providing that the simplifying assumptions regarding phase interfacial effects and radiation heat transfer can still be made. This being so it does not signify if a change is made in the conductivity of either the continuous or discontinuous phase, the only parameter to consider is the change in component conductivity ratio and this will give the measure of the effect on the suspension conductivity. Hence, we can see that for the assertion of Baxley and Couper to hold would require the effect of the discontinuous phase on the conductivity of the suspension to become greater as the component conductivity ratio was reduced, whereas figure (7.5) indicates that a slight reduction in the effect of this phase occurs in that the normalised suspension conductivity is lowered by a small amount.

In fact, the use of the experimental results of McAlister and Orr in this comparison would account for this erroneous conclusion since the author believes that there must be considerable doubt as to the validity of these results. It appears that the experimental conductivity values obtained by McAlister and Orr are considerably below those that would be reasonably expected considering other studies. To illustrate this point we may use as an example the comparison between the experimental results and the predicted values obtained using Maxwell's equation. It has been shown in this work and in other studies /10/, /18/, /20/ that Maxwell's equation consistently produces a prediction that is an underestimate of suspension conductivity for the case of spherical particles and therefore would be expected to be biased even more below experimental results for the case of non-spherical particles. However, the results of McAlister and Orr appear to be the only ones in the literature for which the experimental results are, with one exception, all below the values to be obtained from the Maxwell equation, even for the case of irregular particles. Obviously, an explanation of this phenomenon cannot be advanced but the use of grease as the continuous phase in the experiments would suggest that air entrainment in the mixture may well have been a problem.

7.2 Comparison of Experimental and Predicted Conductivities.

The two models developed in this study were used to predict the conductivities of the suspensions containing spherical particles that were used in the experimental work; the results are given in table (7.2) together with the predictions obtained from the use of several other methods. The other methods used were those due to Maxwell /1/,

Bruggeman /8/, Cheng and Vachon /14/, Jefferson /11/ together with the variations on this unidirectional heat flux model outlined in section 2 and also the resulting conductivity from taking the two phases in a series combination. The error of each prediction, expressed as a percentage, for the individual results is given together with the mean prediction error for the case of each combination of components and an overall mean error for all the experimental results considered. In the error analysis for each suspension and for the overall case, the mean error is the average percentage deviation of the prediction for the experimental value. The variance of the error indicates the distribution of the errors about the mean error and is therefore a measure of the consistency of the prediction, whilst the variance of prediction denotes the distribution of the prediction about the experimental values thus giving a measure of the accuracy of the prediction. The average bias of the error denotes the amount by which the prediction, on the average, exceeds or underestimates the experimental value.

As shown by figures (7.1) to (7.4) and table (7.2) the author's predictions give good agreement with the experimental results where the suspensions conform with the assumption of spherical particles. The unit cell model predicts the author's experimental values with an overall mean error of 2.55 per cent. This compares favourably with the calculated limits of error that may be attributed to the experimental method which were ± 3.8 per cent. The random placement model predicts the same results with an overall mean error of 6.64 per cent and is biased below the experimental values by 6.53 per cent. In the latter case the model is subject both to the error in evaluating the basic

unit conductivity and the simulated two-component system conductivity. The individual errors of the unit cell model prediction were fairly constant throughout the range of volume fraction solids considered, while those of the random placement model tended to increase as the discontinuous phase content increased.

Bruggeman's prediction method also proved extremely accurate with an overall mean error of 3.371 per cent and also exhibited the same tendency for the error to rise as the volume fraction solid. The models of Maxwell and Cheng and Vachon performed almost equally well, with the former proving more accurate at low volume fractions whilst the accuracy of the latter increased with the volume fraction solid value. Jefferson's model and the variations derived from this have been fully dealt with in section 2. The models all produce an underestimate of conductivity on the average with the one exception of Cheng and Vachon, disregarding the unidirectional heat flux variations.

The various prediction methods have also been tested against the experimental results obtained from other studies /11/, /20/, /23/, with only the results for suspensions containing spherical particles being considered. The results of McAlister and Orr are not included for the reasons stated above, although some previous investigators have included them in testing their predictions. Table (7.3) shows the results of the comparison for all the experimental work to hand. The unit cell model again provided the best overall prediction record with a mean error of 6.561 per cent whilst the random placement model had a mean error of 10.69 per cent. Bruggeman's model again proved accurate, somewhat surprisingly in view of the previous neglect of this prediction

in favour of Maxwell's equation, which however did not give nearly such good results. The model of Cheng and Vachon had an overall mean error of 9.264 per cent and was again overall slightly overestimating the conductivities. The results of Jefferson's model and the variations of this technique again demonstrate the uncertainties associated with the assumption of a unidirectional heat flux. One particular prediction method may, in the case of an individual two-component system, produce a result that is inconsistent with its performance in the case of other systems; this may be due to the effect of particle shape. For example, if there is a departure from particle sphericity and a consequent increase in system conductivity, then Cheng and Vachons' model, which tends to overestimate conductivity for spherical particle suspensions, will be likely to produce the most accurate prediction. This may be seen in table (7.4) where comparisons are made between the various prediction models and known irregular shaped particle suspensions.

In fact several studies have reported their work on the basis of component and suspension conductivities being given in various systems of units; this method does not allow of easy comparison between different systems and may lead to the overlooking of discrepancies which may be attributed to the departure from particle sphericity. The effect of this sensitive parameter may best be determined by presenting conductivities in a normalised form.

7.3 Use of Direct or Indirect Simultaneous Equation Solution Method for this Problem.

A comparison may be made for the use of an indirect or direct method of solving the sets of simultaneous equations that result from the application of a tridirectional heat flux to this problem.

An indirect method is decidedly more economical in computer storage requirements and will also prove more efficient regarding execution time providing a suitable relaxation factor is chosen. As a consequence it is possible to utilise a finer grid on a given computer and thereby enhance accuracy, with the added advantage that the method allows of easy modification of the computer program to change grid characteristics. Although based on iteration the indirect methods are less prone to the accumulation of rounding errors than the direct methods since only those occurring on the last iteration are of consequence.

A direct method will offer only the advantages of not requiring any decision on acceptable convergence levels or relaxation factors. In general, although suitable for small sets of simultaneous equations, it is more difficult to program and once it is programed it requires to be completely rewritten to effect any basic change.

The indirect methods also offer the advantage of probably being easier to visualise in physical terms and are therefore closer to producing a model in the full sense of the word.

Both methods appear satisfactory as regards any mathematical consideration when used for this problem so that the indirect methods appear to offer overriding advantages.

7.4 Conclusion.

Prediction methods for the thermal conductivity of suspensions that are based upon the assumption of a unidirectional heat flux contain an inherent inconsistency and require a solution procedure that is entirely arbitrary in nature.

Two prediction techniques have been developed which overcome some of the restrictions placed on previous solutions. The unit cell method extends Jefferson's model giving a considerably improved prediction and can be used to produce accurate results where the greater outlay in time and effort is justified. The random placement method, whilst not proving so accurate a predictor as the former, goes some way to providing a closer model of actual systems and is capable of being extended to the case of suspensions containing irregular shaped particles by allowing anisotropic conductivity values in the basic unit.

Quick and accurate predictions can also be obtained using the explicit equation developed by Bruggeman /8/ which proved the best of previous solutions.

Table 7.1

Experimental Thermal Conductivity Results

1. System: Copper - Water Suspension, $\Theta = 647$
Temperature 30°C

Volume Fraction Solid	K_s/K_c
0.015	1.079
0.023	1.081
0.053	1.256
0.087	1.399
0.092	1.406
0.118	1.605
0.173	1.912
0.231	2.344
0.297	2.891

2. System: Nickel - Water Suspension, $\Theta = 146$
Temperature 30°C

Volume Fraction Solid	K_s/K_c
0.029	1.090
0.065	1.234
0.113	1.436
0.130	1.554
0.141	1.619
0.199	1.957
0.263	2.416

Table 7.1 continued.

3. System: Stainless Steel - Water Suspension, $\Theta = 26.5$

Temperature 30°C

Volume Fraction Solid	K_s/K_c
0.024	1.079
0.037	1.123
0.056	1.128
0.068	1.246
0.070	1.261
0.109	1.435
0.135	1.615
0.195	1.883
0.251	2.251
0.307	2.697

4. System: Aluminium - Water Suspension, $\Theta = 386$

Temperature 30°C

Volume Fraction Solid	K_s/K_c
0.042	1.248
0.042	1.253
0.095	1.692
0.095	1.687
0.133	2.066
0.133	2.068
0.165	2.469
0.218	2.859
0.286	3.753
0.299	3.773
0.299	3.770

COMPARISON OF AUTHORS EXPERIMENTAL RESULTS
FOR SPHERICAL PARTICLE SUSPENSIONS WITH
VARIOUS THEORETICAL PREDICTIONS

EXPERIMENTAL RESULTS, THEORETICAL PREDICTIONS AND ERROR ANALYSIS FOR THERMAL CONDUCTIVITY OF SUSPENSIONS

COPPER-WATER SYSTEM, BUTTERFIELD
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
647.1540	0.015	1.0790	1.1454	1.0420	1.0455	1.0464	1.1701	1.0283	1.0321	1.1488	1.3938	1.0152
647.1540	0.023	1.0812	1.0738	1.0636	1.0703	1.0723	1.2210	1.0462	1.0528	1.2111	1.4972	1.0235
647.1540	0.053	1.2557	1.1922	1.1601	1.1671	1.1775	1.3829	1.1754	1.1472	1.4157	1.8180	1.0559
647.1540	0.087	1.3993	1.3444	1.2795	1.2844	1.3140	1.5527	1.2372	1.2860	1.6304	2.1552	1.0951
647.1540	0.092	1.4059	1.3483	1.2785	1.3024	1.3358	1.5777	1.2556	1.3095	1.6618	2.2054	1.1011
647.1540	0.118	1.6048	1.4091	1.4152	1.3993	1.4575	1.7103	1.3601	1.4453	1.8271	2.4756	1.1336
647.1540	0.173	1.9121	1.8178	1.7084	1.6241	1.7680	2.0156	1.6349	1.8247	2.2014	3.1279	1.2088
647.1540	0.231	2.3442	2.2405	2.1146	1.8958	2.1990	2.3980	2.0228	2.4158	2.6596	4.0198	1.2998
647.1540	0.297	2.8911	2.9130	2.7261	2.2591	2.8783	2.9559	2.6362	3.4996	3.3107	5.5111	1.4215

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
647.154	0.015	1.079	3.114	3.429	3.107	3.023	8.447	4.695	4.342	6.471	29.172	5.912
647.154	0.023	1.081	0.684	1.628	1.009	0.823	12.928	3.739	2.625	12.014	38.479	5.736
647.154	0.053	1.256	5.057	7.613	7.057	6.230	10.128	10.577	8.637	12.744	44.782	15.913
647.154	0.087	1.399	3.923	8.561	8.210	6.098	10.963	11.587	8.094	16.514	54.017	21.737
647.154	0.092	1.406	2.674	7.639	7.361	4.986	12.222	10.691	6.857	18.201	56.871	21.677
647.154	0.118	1.605	6.586	11.939	12.808	9.182	6.575	15.246	9.936	13.854	54.265	29.345
647.154	0.173	1.912	4.932	10.653	15.064	7.536	5.411	14.497	4.573	15.130	63.584	36.782
647.154	0.231	2.344	4.424	9.794	19.130	6.195	2.293	13.712	3.054	13.454	71.480	44.553
647.154	0.297	2.891	0.757	5.638	21.860	0.443	2.242	8.817	21.046	14.512	90.624	50.830
MEAN ERROR			3.573	7.433	10.623	4.946	7.912	10.318	7.685	13.655	55.919	25.790
VARIANCE OF ERROR			0.000	0.001	0.004	0.001	0.001	0.002	0.003	0.001	0.030	0.023
AVERAGE BIAS OF ERROR			-3.404	-7.433	-10.623	-4.946	7.912	-10.318	-2.329	13.655	55.919	-25.790
VARIANCE OF PREDICTION			0.005	0.022	0.084	0.009	0.014	0.042	0.049	0.064	1.523	0.467

NICKEL-WATER SYSTEM, BUTTERFIELD
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
146.3415	0.029	1.0897	1.0946	1.0809	1.0877	1.0923	1.2380	1.0573	1.0679	1.2057	1.4681	1.0297
146.3415	0.065	1.2336	1.2395	1.1940	1.2041	1.2234	1.4175	1.1519	1.1863	1.4082	1.8108	1.0690
146.3415	0.113	1.4360	1.4635	1.3776	1.3735	1.4329	1.6474	1.3133	1.4013	1.6674	2.2520	1.1264
146.3415	0.130	1.5542	1.5516	1.4506	1.4379	1.5186	1.7320	1.3807	1.4953	1.7613	2.4176	1.1483
146.3415	0.141	1.6192	1.6113	1.4972	1.4809	1.5777	1.7882	1.4273	1.5618	1.8233	2.5291	1.1628
146.3415	0.199	1.9572	1.9694	1.8385	1.7266	1.9458	2.1125	1.7166	2.0022	2.1731	3.1968	1.2463
146.3415	0.263	2.4155	2.4879	2.3199	2.0414	2.4980	2.5503	2.1405	2.7355	2.6258	4.1798	1.3536

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
146.341	0.029	1.090	0.450	0.808	0.180	0.239	13.613	2.976	2.004	10.642	34.725	5.510
146.341	0.065	1.234	0.478	3.210	2.395	0.628	14.904	6.624	3.834	14.156	46.789	13.342
146.341	0.113	1.436	1.915	4.067	4.353	0.213	14.722	8.542	2.416	16.116	56.827	21.559
146.341	0.130	1.554	0.167	6.666	7.484	2.291	11.438	11.166	3.788	13.327	55.556	26.119
146.341	0.141	1.619	0.488	7.535	8.545	2.544	10.438	11.854	3.542	12.605	56.196	28.184
146.341	0.199	1.957	0.623	6.065	11.782	0.581	7.937	12.283	2.297	11.029	63.335	36.321
146.341	0.263	2.416	2.997	5.958	15.438	3.417	5.580	11.383	13.249	8.707	73.041	43.964
MEAN ERROR			1.017	4.615	7.175	1.447	11.233	9.261	4.447	12.369	55.210	25.000
VARIANCE OF ERROR			0.000	0.000	0.002	0.000	0.001	0.001	0.001	0.001	0.013	0.015
AVERAGE BIAS OF ERROR			0.830	-4.615	-7.175	-0.403	11.233	-9.261	-0.005	12.369	55.210	-25.000
VARIANCE OF PREDICTION			0.001	0.008	0.033	0.001	0.029	0.032	0.016	0.039	1.052	0.305

STAINLESS STEEL-WATER SYSTEM, BUTTERFIELD
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
26.5040	0.024	1.0785	1.0697	1.0573	1.0658	1.0756	1.1563	1.0404	1.0486	1.1145	1.2213	1.0236
26.5040	0.037	1.1228	1.1136	1.0926	1.1027	1.1198	1.2142	1.0664	1.0611	1.1689	1.3139	1.0369
26.5040	0.056	1.1283	1.1821	1.1442	1.1582	1.1887	1.2925	1.1084	1.1348	1.2450	1.4398	1.0570
26.5040	0.068	1.2464	1.2276	1.1785	1.1944	1.2352	1.3402	1.1372	1.1724	1.2920	1.5166	1.0700
26.5040	0.070	1.2613	1.2353	1.1902	1.2005	1.2432	1.3681	1.1422	1.1790	1.2998	1.5293	1.0722
26.5040	0.109	1.4554	1.3959	1.3105	1.3242	1.4137	1.5012	1.2488	1.3236	1.4515	1.7770	1.1172
26.5040	0.135	1.6146	1.5134	1.4088	1.4122	1.5451	1.6054	1.3302	1.4388	1.5539	1.9476	1.1493
26.5040	0.195	1.8827	1.8255	1.6783	1.6341	1.9170	1.8665	1.5515	1.7728	1.8037	2.3792	1.2310
26.5040	0.251	2.2514	2.1808	1.9893	1.8689	2.3799	2.1677	1.8059	2.1940	2.0622	2.8584	1.3184
26.5040	0.307	2.6966	2.6472	2.3639	2.1362	3.0047	2.4832	2.1154	2.7644	2.3561	3.4532	1.4193

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
26.504	0.024	1.079	0.816	1.966	1.174	0.269	7.210	3.536	2.768	3.334	13.244	5.087
26.504	0.037	1.123	0.819	2.690	1.789	0.272	8.145	5.027	3.715	4.103	17.022	7.649
26.504	0.056	1.128	4.768	1.409	2.654	5.356	14.556	1.766	0.574	10.341	27.605	6.323
26.504	0.068	1.246	1.508	5.448	4.176	0.895	7.524	8.760	5.934	3.657	21.675	14.151
26.504	0.070	1.261	2.061	5.637	4.824	1.433	6.878	9.444	6.525	3.050	21.244	14.991
26.504	0.109	1.435	2.752	8.701	7.747	1.510	4.581	13.000	7.789	1.108	23.801	22.170
26.504	0.135	1.615	6.268	12.746	12.538	4.306	0.545	17.617	10.887	3.758	20.624	28.816
26.504	0.195	1.883	3.155	10.857	13.207	1.820	0.859	17.591	5.836	4.198	26.371	34.616
26.504	0.251	2.251	3.136	11.642	16.990	5.707	4.608	19.789	2.549	8.406	26.961	41.439
26.504	0.307	2.697	1.832	12.338	20.783	11.425	7.915	21.554	2.515	12.629	28.057	47.368

	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
MEAN ERROR	2.712	7.343	8.588	3.299	6.282	11.808	4.909	5.458	22.661	22.261
VARIANCE OF ERROR	0.000	0.002	0.004	0.001	0.001	0.005	0.001	0.001	0.002	0.021
AVERAGE BIAS OF ERROR	-1.758	-7.061	-8.057	1.562	3.497	-11.808	-4.291	-0.340	22.661	-22.261
VARIANCE OF PREDICTION	0.003	0.029	0.058	0.012	0.012	0.079	0.008	0.018	0.171	0.333

OVERALL ERROR ANALYSIS

	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
MEAN ERROR	2.553	6.640	8.912	3.371	8.179	10.607	5.746	10.156	42.936	24.220
VARIANCE OF ERROR	0.000	0.001	0.004	0.001	0.002	0.003	0.002	0.002	0.040	0.020
AVERAGE BIAS OF ERROR	-1.631	-6.531	-8.708	-1.220	7.108	-10.607	-2.458	7.926	42.936	-24.220
VARIANCE OF PREDICTION	0.003	0.021	0.060	0.008	0.017	0.053	0.025	0.040	0.876	0.372

COPPER-WATER SYSTEM, BUTTERFIELD
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
647.1540	0.015	1.0790	1.0454	1.0420	1.0455	1.0464	1.1701	1.0283	1.0321	1.1488	1.3938	1.0152
647.1540	0.023	1.0812	1.0738	1.0636	1.0703	1.0723	1.2210	1.0462	1.0528	1.2111	1.4972	1.0235
647.1540	0.053	1.2557	1.1922	1.1601	1.1671	1.1775	1.3829	1.1254	1.1472	1.4157	1.8180	1.0559
647.1540	0.087	1.3993	1.3444	1.2795	1.2844	1.3140	1.5527	1.2372	1.2860	1.6304	2.1552	1.0951
647.1540	0.092	1.4059	1.3463	1.2985	1.3024	1.3358	1.5777	1.2556	1.3095	1.6618	2.2054	1.1011
647.1540	0.118	1.6048	1.4091	1.4132	1.3993	1.4575	1.7103	1.3601	1.4453	1.8271	2.4756	1.1336
647.1540	0.173	1.9121	1.8178	1.7084	1.6241	1.7680	2.0156	1.6349	1.8247	2.2014	3.1279	1.2088
647.1540	0.231	2.3442	2.2405	2.1146	1.8958	2.1940	2.3980	2.0228	2.4158	2.6596	4.0198	1.2998
647.1540	0.297	2.8911	2.9130	2.7281	2.2591	2.8783	2.9559	2.6362	3.4996	3.3107	5.5111	1.4215

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
647.154	0.015	1.079	3.114	3.429	3.107	3.023	8.447	4.695	4.342	6.471	29.172	5.912
647.154	0.023	1.081	0.684	1.628	1.009	0.823	12.928	3.239	2.625	12.014	38.479	5.336
647.154	0.053	1.256	5.057	7.613	7.057	6.230	10.128	10.377	8.637	12.744	44.782	15.913
647.154	0.087	1.399	3.923	8.561	8.210	6.098	10.963	11.587	8.094	16.514	54.017	21.737
647.154	0.092	1.406	2.674	7.639	7.361	4.986	12.222	10.691	6.857	18.201	56.871	21.477
647.154	0.118	1.605	6.586	11.939	12.808	9.182	6.575	15.246	9.936	13.854	54.265	29.365
647.154	0.173	1.912	4.932	10.653	15.064	7.536	5.411	14.497	4.573	15.130	63.584	36.782
647.154	0.231	2.344	4.424	9.794	19.130	6.195	2.293	13.712	3.054	13.454	71.480	44.553
647.154	0.297	2.891	0.757	5.638	21.860	0.443	2.242	8.817	21.046	14.512	90.624	50.830

	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
MEAN ERROR	3.573	7.433	10.623	4.946	7.912	10.318	7.685	13.655	55.919	25.790
VARIANCE OF ERROR	0.000	0.001	0.004	0.001	0.001	0.002	0.003	0.001	0.030	0.023
AVERAGE BIAS OF ERROR	-3.404	-7.433	-10.623	-4.946	7.912	-10.318	-2.329	13.655	55.919	-25.790
VARIANCE OF PREDICTION	0.005	0.022	0.084	0.009	0.014	0.042	0.049	0.064	1.523	0.467

NICKEL-WATER SYSTEM, BUTTERFIELD
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
146.3415	0.029	1.0897	1.0946	1.0809	1.0877	1.0923	1.2380	1.0573	1.0679	1.2057	1.4681	1.0297
146.3415	0.065	1.2336	1.2395	1.1940	1.2041	1.2234	1.4175	1.1519	1.1863	1.4082	1.8108	1.0690
146.3415	0.113	1.4360	1.4635	1.3776	1.3735	1.4329	1.6474	1.3133	1.4013	1.6674	2.2520	1.1264
146.3415	0.130	1.5542	1.5516	1.4506	1.4379	1.5186	1.7320	1.3807	1.4953	1.7613	2.4176	1.1483
146.3415	0.161	1.6192	1.6113	1.4972	1.4809	1.5777	1.7882	1.4273	1.5618	1.8233	2.5291	1.1628
146.3415	0.199	1.9572	1.9694	1.8385	1.7266	1.9458	2.1125	1.7166	2.0022	2.1731	3.1968	1.2463
146.3415	0.263	2.4155	2.4879	2.3199	2.0414	2.4980	2.5503	2.1405	2.7355	2.6258	4.1798	1.3536

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES	
146.341	0.029	1.090	0.450	0.808	0.180	0.239	13.613	2.976	2.004	10.642	34.725	5.510	
146.341	0.065	1.234	0.478	3.210	2.395	0.828	14.904	6.624	3.834	14.156	46.789	13.342	
146.341	0.113	1.436	1.915	4.067	4.353	0.213	14.722	8.547	2.416	16.116	56.827	21.559	
146.341	0.130	1.554	0.167	6.866	7.484	2.291	11.438	11.166	3.788	13.327	55.556	26.119	
146.341	0.141	1.619	0.488	7.535	8.543	2.564	10.438	11.854	3.542	12.605	56.196	26.184	
146.341	0.199	1.957	0.623	6.065	11.782	0.581	7.937	12.283	2.297	11.029	63.335	36.321	
146.341	0.263	2.416	2.997	5.956	15.438	3.417	5.580	11.383	13.249	8.707	73.041	43.964	
MEAN ERROR			UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES	
				1.017	4.615	7.175	1.447	11.233	9.261	4.447	12.369	55.210	25.000
VARIANCE OF ERROR				0.000	0.000	0.002	0.000	0.001	0.001	0.001	0.013	0.015	
AVERAGE BIAS OF ERROR				0.830	-4.615	-7.175	-0.403	11.233	-9.261	-0.005	12.369	55.210	-25.000
VARIANCE OF PREDICTION				0.001	0.008	0.033	0.001	0.029	0.032	0.016	0.039	1.052	0.305

Table 7.3

-III-

EXPERIMENTAL AND PREDICTED CONDUCTIVITIES
FOR SPHERICAL PARTICLE SUSPENSIONS

STAINLESS STEEL-WATER SYSTEM, BUTTERFIELD
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
26.5040	0.024	1.0785	1.0697	1.0573	1.0658	1.0756	1.1563	1.0404	1.0486	1.1145	1.2213	1.0236
26.5040	0.037	1.1226	1.1136	1.0926	1.1027	1.1193	1.2142	1.0664	1.0811	1.1689	1.3139	1.0369
26.5040	0.056	1.1283	1.1821	1.1442	1.1582	1.1867	1.2925	1.1084	1.1348	1.2450	1.4398	1.0570
26.5040	0.068	1.2464	1.2274	1.1785	1.1944	1.2352	1.3402	1.1372	1.1724	1.2920	1.5166	1.0700
26.5040	0.070	1.2613	1.2553	1.1902	1.2005	1.2432	1.3481	1.1422	1.1790	1.2998	1.5293	1.0722
26.5040	0.109	1.4354	1.3959	1.3105	1.3242	1.4137	1.5012	1.2488	1.3236	1.4513	1.7770	1.1172
26.5040	0.135	1.6146	1.5134	1.4088	1.4122	1.5451	1.6058	1.3302	1.4388	1.5539	1.9476	1.1493
26.5040	0.195	1.8827	1.8233	1.6783	1.6341	1.9170	1.8665	1.5515	1.7728	1.8037	2.3792	1.2310
26.5040	0.251	2.2514	2.1808	1.9893	1.8689	2.3799	2.1477	1.6059	2.1940	2.0622	2.8584	1.3184
26.5040	0.307	2.6966	2.6472	2.3839	2.1382	3.0047	2.4832	2.1154	2.7644	2.3561	3.4532	1.4193

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
26.504	0.024	1.079	0.816	1.966	1.174	0.269	7.210	3.536	2.768	3.334	13.244	5.087
26.504	0.037	1.123	0.819	2.690	1.789	0.272	8.145	5.027	3.715	4.103	17.022	7.649
26.504	0.056	1.128	4.768	1.409	2.654	5.356	14.556	1.766	0.574	10.341	27.605	6.323
26.504	0.068	1.246	1.508	5.448	4.176	0.895	7.524	8.760	5.934	3.657	21.675	14.151
26.504	0.070	1.261	2.061	5.637	4.824	1.433	8.878	9.444	6.525	3.050	21.244	14.991
26.504	0.109	1.435	2.752	8.701	7.747	1.510	4.581	13.000	7.729	1.108	23.801	22.170
26.504	0.135	1.615	6.268	12.746	12.538	4.306	0.545	17.617	10.867	3.758	20.624	28.818
26.504	0.195	1.883	3.135	10.857	13.207	1.820	0.859	17.591	5.836	4.198	26.371	34.616
26.504	0.251	2.251	3.136	11.642	16.990	5.707	4.608	19.789	2.549	8.406	26.961	41.439
26.504	0.307	2.697	1.832	12.338	20.763	11.425	7.915	21.554	2.515	12.629	28.057	47.368

	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
MEAN ERROR	2.712	7.343	8.588	3.299	6.282	11.806	4.909	5.458	22.661	22.261
VARIANCE OF ERROR	0.000	0.002	0.004	0.001	0.001	0.005	0.001	0.001	0.002	0.021
AVERAGE BIAS OF ERROR	-1.758	-7.061	-8.057	1.562	3.497	-11.806	-4.291	-0.340	22.661	-22.261
VARIANCE OF PREDICTION	0.003	0.029	0.058	0.012	0.012	0.079	0.008	0.018	0.171	0.333

GRAPHITE-SILICONE OIL SYSTEM, JEFFERSON
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
1062.8600	0.194	2.7543	1.9610	1.8631	1.7194	1.9098	2.1543	1.7805	2.0129	2.4172	3.4645	1.2404
1062.8600	0.251	3.2229	2.4212	2.3139	2.0016	2.3799	2.5634	2.2205	2.7020	2.9235	4.4706	1.3347
1062.8600	0.310	3.9314	3.0934	2.9042	2.3423	3.0441	3.1071	2.8495	3.8326	3.5842	6.0172	1.4487
1062.8600	0.375	5.1770	4.3486	3.6345	2.7919	4.0960	3.9402	3.8637	6.1529	4.5745	9.0236	1.5991
1062.8600	0.423	6.2400	6.2117	4.2276	3.1886	5.2056	4.8246	4.9712	9.7498	5.6003	13.5229	1.7319

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
1062.860	0.194	2.754	20.802	32.357	37.568	30.600	21.786	35.357	26.700	12.240	25.785	54.964
1062.860	0.251	3.223	24.875	28.204	37.896	26.157	20.463	31.102	16.163	9.291	38.715	58.587
1062.860	0.310	3.931	21.316	26.128	40.420	22.571	20.967	27.520	2.514	8.832	53.054	63.151
1062.860	0.375	5.177	15.002	29.793	46.071	20.881	23.891	25.366	18.850	11.638	74.303	69.112
1062.860	0.423	6.240	0.454	32.250	48.901	16.577	22.682	20.333	56.247	10.252	116.714	72.245

	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
MEAN ERROR	18.290	29.747	42.171	23.369	21.958	27.936	24.095	10.451	61.714	63.612
VARIANCE OF ERROR	0.010	0.101	0.002	0.002	0.000	0.003	0.032	0.000	0.162	0.004
AVERAGE BIAS OF ERROR	-18.290	-29.747	-42.171	-23.369	-21.958	-27.936	5.944	-10.451	61.714	-63.612
VARIANCE OF PREDICTION	0.552	1.821	4.017	0.890	1.001	1.292	2.819	0.219	14.850	9.029

GRAPHITE-WATER SYSTEM, JEFFERSON
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
287.9300	0.161	2.1080	1.7365	1.6245	1.5686	1.6932	1.9246	1.5439	1.7159	2.0262	2.8767	1.1911
287.9300	0.247	3.0090	2.3656	2.2151	1.9706	2.3422	2.4833	2.0881	2.5843	2.6506	4.1446	1.3265
287.9300	0.321	3.8060	3.2154	2.9160	2.3948	3.1944	3.1475	2.7869	3.9575	3.3567	5.9301	1.4703

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
287.930	0.161	2.108	17.623	22.936	25.588	19.676	8.699	26.758	18.601	3.880	36.465	43.496
287.930	0.247	3.009	21.383	26.584	34.510	22.162	17.472	30.605	14.114	11.912	37.742	55.915
287.930	0.321	3.806	15.512	23.647	37.027	16.069	17.302	26.775	3.980	11.806	55.808	61.368
MEAN ERROR			18.173	24.322	32.375	19.302	14.491	28.046	12.231	9.200	43.338	53.593
VARIANCE OF ERROR			0.001	0.000	0.002	0.001	0.002	0.000	0.004	0.001	0.008	0.006
AVERAGE BIAS OF ERROR			-18.173	-24.322	-32.375	-19.302	-14.491	-28.046	-9.578	-9.200	43.338	-53.593
VARIANCE OF PREDICTION			0.300	0.558	1.118	0.330	0.248	0.735	0.119	0.112	2.131	3.042

COPPER-WATER SYSTEM, JOHNSON
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
700.0000	0.065	1.2100	1.2440	1.2010	1.2076	1.2234	1.4439	1.1626	1.1924	1.4960	1.9408	1.0694
700.0000	0.097	1.4000	1.3924	1.3217	1.3207	1.3581	1.6038	1.2753	1.3341	1.6984	2.2606	1.1072
700.0000	0.107	1.4100	1.4426	1.3600	1.3577	1.4043	1.6546	1.3150	1.3853	1.7621	2.3640	1.1196
700.0000	0.152	1.7000	1.6829	1.5809	1.5350	1.6399	1.8952	1.5225	1.6639	2.0607	2.8686	1.1789
700.0000	0.218	2.1300	2.1362	2.0079	1.8318	2.0911	2.3072	1.9295	2.2631	2.5607	3.8046	1.2783
700.0000	0.244	2.3100	2.3535	2.2300	1.9628	2.3144	2.4979	2.1523	2.5891	2.7682	4.2751	1.3221
700.0000	0.300	2.9200	2.9526	2.7583	2.2779	2.9155	2.9889	2.6793	3.5708	3.3636	5.6170	1.4277

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
700.000	0.065	1.210	2.310	0.744	0.198	1.107	19.328	3.918	1.453	23.638	60.393	11.619
700.000	0.097	1.400	0.514	5.593	5.662	2.992	14.554	8.904	4.708	21.311	61.472	20.911
700.000	0.107	1.410	2.312	3.546	3.706	0.408	17.345	6.736	1.752	24.973	67.656	20.594
700.000	0.152	1.700	0.653	7.006	9.704	3.536	11.483	10.442	2.126	21.220	68.738	30.650
700.000	0.218	2.130	0.291	5.732	14.002	1.825	8.318	9.415	6.248	20.221	78.620	39.988
700.000	0.244	2.310	1.883	3.463	15.031	0.190	8.135	7.693	12.084	20.700	85.068	42.764
700.000	0.300	2.920	1.116	5.538	21.990	0.156	2.361	8.244	22.286	15.190	92.363	51.106
MEAN ERROR			1.369	4.517	10.042	1.459	11.646	7.907	7.237	21.036	73.473	31.090
VARIANCE OF ERROR			0.000	0.000	0.005	0.000	0.003	0.000	0.005	0.001	0.013	0.017
AVERAGE BIAS OF ERROR			1.035	-4.517	-10.042	-1.089	11.646	-7.907	4.368	21.036	73.473	-31.090
VARIANCE OF PREDICTION			0.001	0.010	0.094	0.001	0.038	0.027	0.075	0.148	2.499	0.622

ALUMINA-PLASTER SYSTEM, FIDELLE AND KIRK
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
121.0000	0.050	1.2070	1.1757	1.1463	1.1538	1.1664	1.3401	1.1085	1.1318	1.5155	1.6500	1.0522
121.0000	0.103	1.4140	1.4107	1.3286	1.3551	1.3856	1.5909	1.2726	1.3475	1.5952	2.1241	1.1138
121.0000	0.159	1.7920	1.7128	1.5931	1.5508	1.6812	1.8719	1.5009	1.6737	1.9009	2.6724	1.1872
121.0000	0.211	2.0330	2.0475	1.8996	1.7776	2.0360	2.1711	1.7725	2.1006	2.2154	3.2890	1.2646
121.0000	0.283	2.5530	2.6775	2.4752	2.1442	2.7130	2.6862	2.2718	3.0058	2.7324	4.4540	1.3902
121.0000	0.342	3.0700	3.4708	3.0271	2.5022	3.5101	3.2470	2.8275	4.2352	3.2834	5.9083	1.5133
121.0000	0.389	3.6000	4.5029	3.5156	2.8349	4.3841	3.8370	3.4046	5.8414	3.7899	7.7059	1.6281

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
121.000	0.050	1.207	2.593	5.029	4.404	3.368	11.025	8.160	6.231	9.012	36.706	12.827
121.000	0.103	1.414	0.233	6.040	5.577	2.012	12.509	9.999	4.702	12.815	50.222	21.233
121.000	0.159	1.792	4.420	11.099	13.459	6.185	4.459	16.246	6.604	6.075	49.129	33.750
121.000	0.211	2.033	0.713	6.562	12.561	0.146	6.791	12.813	3.326	8.973	61.781	37.795
121.000	0.283	2.553	4.877	3.047	16.012	6.265	5.216	11.013	17.736	7.027	74.462	45.548
121.000	0.342	3.070	13.055	1.397	18.495	14.336	5.765	7.897	37.953	6.299	92.451	50.708
121.000	0.389	3.600	25.081	2.344	21.253	21.779	6.584	5.427	62.261	5.275	114.052	54.775
MEAN ERROR			7.282	5.074	13.169	7.727	7.479	10.222	19.830	7.925	68.400	36.662
VARIANCE OF ERROR			0.007	0.001	0.003	0.005	0.001	0.001	0.043	0.001	0.064	0.020
AVERAGE BIAS OF ERROR			5.211	-5.074	-13.109	4.423	7.479	-10.222	14.820	7.925	68.400	-36.662
VARIANCE OF PREDICTION			0.143	0.012	0.172	0.121	0.026	0.051	0.945	0.028	4.512	1.248

CARBORUNDUM-PLASTER SYSTEM, FIDELLE AND KIRK
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
625.0000	0.100	1.6000	1.4073	1.3504	1.3316	1.3717	1.6176	1.2858	1.3486	1.7099	2.2848	1.1109
625.0000	0.157	1.9200	1.7182	1.7142	1.5556	1.6692	1.9216	1.5459	1.6991	2.0844	2.9205	1.1859
625.0000	0.215	2.2870	2.1120	1.9441	1.8167	2.0672	2.2835	1.9022	2.2266	2.5197	3.7408	1.2733
625.0000	0.259	2.6330	2.4923	2.3509	2.0418	2.4578	2.6138	2.2537	2.8041	2.9086	4.5641	1.3488
625.0000	0.301	3.0530	2.9634	2.7582	2.2830	2.9280	2.9940	2.6768	3.5841	3.3477	5.6131	1.4296
625.0000	0.373	3.9000	4.2758	3.5300	2.7711	4.0569	3.8755	3.0943	5.9465	4.3349	8.6410	1.5934
625.0000	0.411	4.5670	5.5691	4.0169	3.0764	4.8939	4.5271	4.4575	8.3602	5.0393	11.6153	1.6959

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
625.000	0.100	1.600	12.044	15.600	16.777	14.256	1.100	19.637	15.712	6.868	42.797	30.568
625.000	0.157	1.920	10.510	10.719	18.982	13.051	0.084	14.486	11.506	8.561	52.107	38.235
625.000	0.215	2.287	7.652	14.993	20.566	9.609	0.151	16.827	2.642	10.175	63.569	44.323
625.000	0.259	2.633	5.344	10.714	22.453	6.654	0.729	14.467	6.499	10.467	73.342	48.774
625.000	0.301	3.053	2.935	10.311	25.220	4.095	1.934	12.324	17.397	9.653	84.020	53.173
625.000	0.373	3.900	9.636	9.487	28.946	4.024	0.629	5.274	52.474	11.151	121.565	59.144
625.000	0.411	4.567	21.942	12.045	32.638	7.158	0.874	2.598	83.057	10.342	154.330	62.866
MEAN ERROR			10.009	11.981	23.455	8.410	0.786	12.908	27.041	9.602	84.533	48.155
VARIANCE OF ERROR			0.003	0.000	0.003	0.001	0.000	0.004	0.075	0.000	0.137	0.011
AVERAGE BIAS OF ERROR			-0.987	-11.981	-23.655	-5.215	-0.448	-12.908	18.509	9.602	84.533	-48.155
VARIANCE OF PREDICTION			0.183	0.120	0.695	0.049	0.001	0.104	2.715	0.095	12.293	2.808

ALUMINA-POLYESTER SYSTEM-FIDELLE AND KIRK
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
241.0000	0.100	1.6960	1.4034	1.3246	1.3268	1.3717	1.6001	1.2737	1.3421	1.6394	2.2006	1.1106
241.0000	0.200	2.2200	1.9886	1.8657	1.7385	1.9531	2.1492	1.7529	2.0387	2.2627	3.3451	1.2487
241.0000	0.300	3.3650	2.9192	2.6871	2.2632	2.9155	2.9187	2.5332	3.4480	3.0842	5.2354	1.4260
241.0000	0.400	5.1740	5.0075	3.7590	2.9592	4.6296	4.1815	3.8583	6.0852	4.3230	9.4767	1.6621

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
241.000	0.100	1.696	17.252	21.899	21.653	19.119	5.657	24.903	20.867	3.339	29.753	34.517
241.000	0.200	2.220	10.423	15.959	21.691	12.021	3.189	21.042	8.165	1.921	50.680	43.752
241.000	0.300	3.365	13.248	20.146	32.744	13.360	13.263	24.718	2.466	8.346	55.585	57.622
241.000	0.400	5.174	3.218	27.348	42.807	10.521	19.182	25.429	35.006	16.409	83.161	67.877
MEAN ERROR			UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
VARIANCE OF ERROR			11.035	21.338	29.724	13.755	10.323	24.023	16.626	7.506	54.795	50.942
AVERAGE BIAS OF ERROR			0.003	0.002	0.008	0.001	0.004	0.000	0.016	0.003	0.036	0.016
VARIANCE OF PREDICTION			-11.035	-21.338	-29.724	-13.755	-10.323	-24.023	2.110	-6.543	54.795	-50.942
			0.091	0.681	1.622	0.169	0.300	0.705	0.861	0.201	5.883	4.345

OVERALL ERROR ANALYSIS

	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
MEAN ERROR	6.561	10.690	16.716	7.467	0.264	13.779	12.743	10.913	56.931	35.994
VARIANCE OF ERROR	0.006	0.007	0.016	0.006	0.004	0.007	0.026	0.003	0.078	0.034
AVERAGE BIAS OF ERROR	-3.317	-10.642	-16.626	-4.655	2.051	-13.779	3.549	6.271	56.931	-35.994
VARIANCE OF PREDICTION	0.107	0.255	0.648	0.128	0.133	-0.240	0.757	0.088	4.442	1.934

EXPERIMENTAL RESULTS, THEORETICAL PREDICTIONS AND ERROR ANALYSIS FOR THERMAL CONDUCTIVITY OF SUSPENSIONS

ALUMINIUM-WATER SYSTEM, BUTTERFIELD
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
386.1789	0.042	1.2479	1.1464	1.1204	1.1305	1.1374	1.3215	1.0926	1.1090	1.3238	1.6798	1.0437
386.1789	0.042	1.2527	1.1464	1.1204	1.1305	1.1374	1.3215	1.0926	1.1090	1.3238	1.6798	1.0437
386.1789	0.045	1.6918	1.3813	1.3255	1.3122	1.3491	1.5854	1.2614	1.3214	1.6458	2.1995	1.1047
386.1789	0.045	1.6866	1.3813	1.3255	1.3122	1.3491	1.5854	1.2614	1.3214	1.6458	2.1995	1.1047
386.1789	0.133	2.0662	1.5775	1.4972	1.4561	1.5344	1.7800	1.4173	1.5303	1.8790	2.5948	1.1529
386.1789	0.133	2.0682	1.5775	1.4972	1.4561	1.5344	1.7800	1.4173	1.5303	1.8790	2.5948	1.1529
386.1789	0.165	2.4690	1.7638	1.6412	1.5873	1.7177	1.9565	1.5744	1.7523	2.0872	2.9677	1.1970
386.1789	0.218	2.8588	2.1297	2.0062	1.8281	2.0911	2.2886	1.8967	2.2443	2.4706	3.7138	1.2778
386.1789	0.286	3.7528	2.7687	2.5508	2.1887	2.7473	2.8250	2.4595	3.2335	3.0701	5.0710	1.3991
386.1789	0.299	3.7733	2.9255	2.6665	2.2655	2.9030	2.9473	2.5919	3.4934	3.2037	5.4116	1.4250
386.1789	0.299	3.7700	2.9255	2.6665	2.2655	2.9030	2.9473	2.5919	3.4934	3.2037	5.4116	1.4250

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
386.179	0.042	1.248	8.134	10.217	9.411	8.857	5.900	12.442	11.134	6.086	34.609	16.362
386.179	0.042	1.253	8.486	10.561	9.758	9.206	5.494	12.777	11.474	5.679	34.093	16.682
386.179	0.045	1.692	18.353	21.651	22.436	20.255	6.288	25.443	21.893	2.720	30.007	34.704
386.179	0.045	1.687	18.102	21.410	22.197	20.009	5.999	25.213	21.652	2.420	30.408	34.503
386.179	0.133	2.066	23.652	27.538	29.527	25.737	13.853	31.404	25.935	9.059	25.582	44.200
386.179	0.133	2.068	23.726	27.609	29.595	25.809	13.936	31.470	26.006	9.147	25.461	44.254
386.179	0.165	2.469	28.562	33.528	35.709	30.430	26.756	36.232	29.028	15.463	20.196	51.519
386.179	0.218	2.859	25.504	29.824	36.055	26.853	19.945	33.652	21.694	13.579	29.906	55.301
386.179	0.286	3.753	26.223	32.079	41.678	26.793	24.723	34.462	13.837	18.193	35.126	62.718
386.179	0.299	3.773	22.468	29.332	39.959	23.065	21.890	31.304	7.417	15.096	43.418	62.236
386.179	0.299	3.770	22.401	29.271	39.906	22.998	21.822	31.248	7.336	15.022	43.544	62.203
MEAN ERROR			20.510	24.816	28.748	21.819	14.601	27.786	17.928	10.274	32.032	44.062
VARIANCE OF ERROR			0.004	0.006	0.012	0.004	0.005	0.006	0.006	0.003	0.005	0.026
AVERAGE BIAS OF ERROR			-20.510	-24.816	-28.748	-21.819	-12.529	-27.786	-17.928	-8.085	32.032	-44.062
VARIANCE OF PREDICTION			0.375	0.568	0.898	0.409	0.273	0.677	0.200	0.136	0.868	2.101

ALUMINIUM OXIDE-POLYPROPYLENE GLYCOL SYSTEM, BAXLEY
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
1582.0000	0.100	1.6910	1.4088	1.3404	1.3320	1.3717	1.6256	1.2947	1.3515	1.7671	2.3248	1.1110
1582.0000	0.200	3.4050	2.0052	1.9253	1.7482	1.9531	2.1985	1.8347	2.0826	2.5111	3.5814	1.2099
1582.0000	0.300	6.4300	4.4570	2.8355	2.2822	2.9155	3.0137	2.7687	3.6145	3.5446	5.7601	1.4282
1582.0000	0.400	11.5700	5.1620	4.0307	2.9437	4.6296	4.3842	4.5119	7.7769	5.2438	11.1572	1.6660

PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGEMAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
1582.000	0.100	1.691	16.688	20.733	21.193	18.880	3.869	21.439	20.078	4.501	37.480	34.297
1582.000	0.200	3.405	41.110	43.457	48.657	42.640	35.433	46.118	38.836	26.252	5.181	43.295
1582.000	0.300	6.430	30.684	55.933	64.506	54.659	53.131	56.941	43.787	44.875	10.418	77.789
1582.000	0.400	11.570	55.385	55.162	74.125	59.984	62.107	61.004	32.784	54.678	3.568	85.601
MEAN ERROR			35.967	46.321	52.120	44.041	38.635	46.875	33.871	32.576	14.162	65.246
VARIANCE OF ERROR			0.920	0.028	0.040	0.025	0.049	0.021	0.008	0.037	0.019	0.038
AVERAGE BIAS OF ERROR			-35.967	-46.321	-52.120	-44.041	-38.635	-46.875	-33.871	-30.326	7.169	-45.246
VARIANCE OF PREDICTION			11.749	18.022	23.408	15.633	16.192	10.461	6.045	12.288	0.263	32.022

Table 7.4 [Cont.]

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ALUMINIUM-POLYPROPYLENE GLYCOL SYSTEM, BAXLEY
EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS FOR K-SUSP/K-CONT

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGENAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
277.8000	0.100	1.4150	1.4042	1.3257	1.3294	1.3717	1.6036	1.2757	1.3434	1.6508	2.2170	1.1107
277.8000	0.200	3.0630	1.9912	1.8699	1.7400	1.9531	2.1560	1.7607	2.0448	2.2848	3.3764	1.2489
277.8000	0.300	5.2880	2.9258	2.6973	2.2661	2.9155	2.9318	2.5550	3.4706	3.1245	5.3036	1.4264
277.8000	0.400	8.5140	5.0409	5.2773	2.9645	4.6296	4.2091	5.9153	7.0874	4.4023	9.6832	1.6627

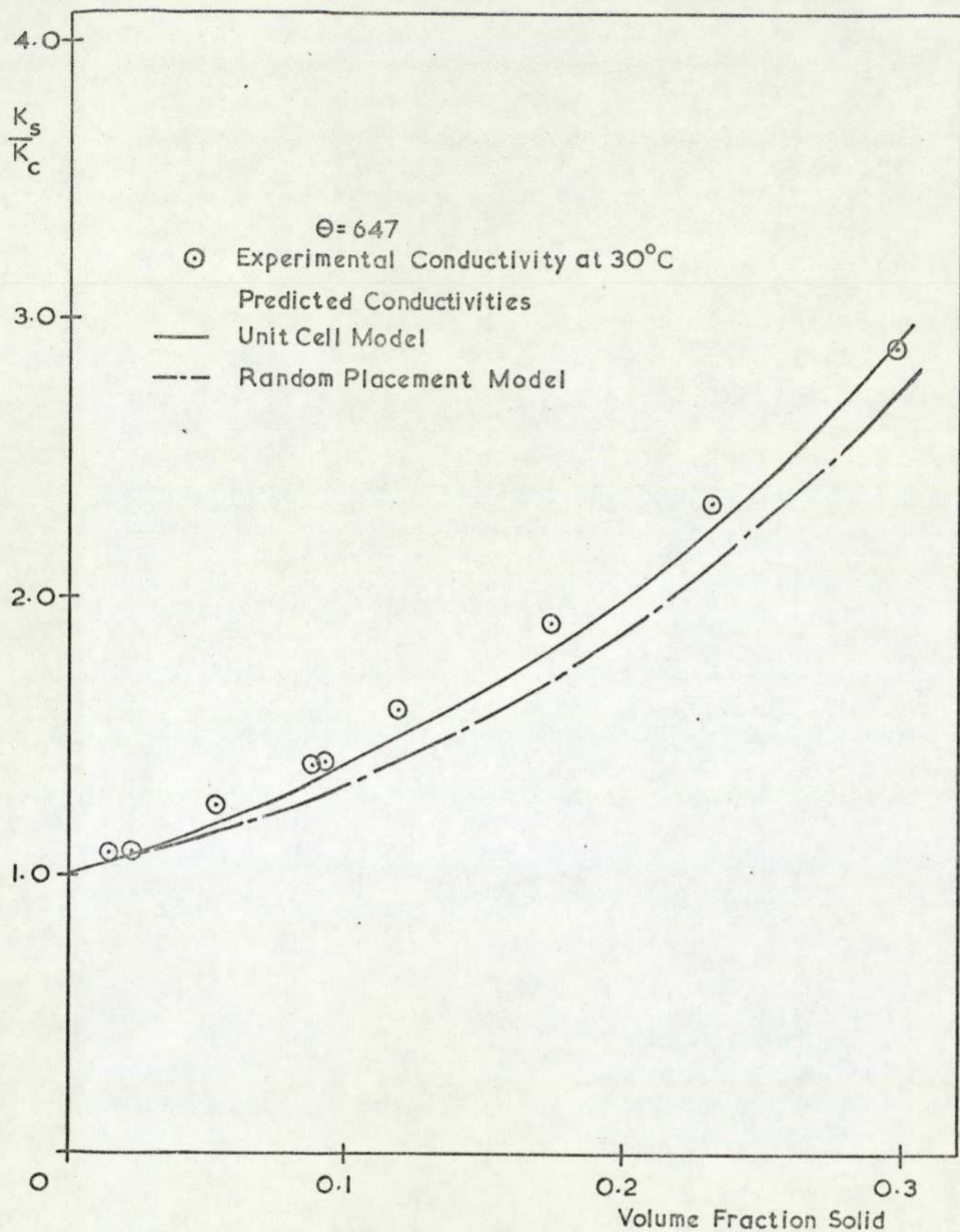
PERCENTAGE ERRORS BETWEEN THEORETICAL AND EXPERIMENTAL THERMAL CONDUCTIVITY

KD/KC	VF	K-EXP	UNIT CELL	RANDOM	MAXWELL	BRUGGENAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
277.800	0.100	1.415	0.763	0.511	6.052	3.057	13.327	9.843	5.059	16.661	56.679	21.508
277.800	0.200	3.063	34.992	38.952	43.194	36.235	29.611	42.517	33.241	25.405	10.231	59.227
277.800	0.300	5.288	44.671	44.992	57.146	44.867	44.558	51.683	34.369	40.913	0.295	73.026
277.800	0.400	8.514	40.910	55.628	65.181	45.623	50.562	54.014	16.756	48.293	13.733	80.471

	UNIT CELL	RANDOM	MAXWELL	BRUGGENAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
MEAN ERROR	30.334	37.471	42.893	32.445	34.515	39.515	22.356	32.818	20.235	58.558
VARIANCE OF ERROR	0.030	0.036	0.051	0.030	0.021	0.031	0.015	0.016	0.047	0.052
AVERAGE BIAS OF ERROR	-30.334	-37.471	-42.893	-32.445	-27.851	-39.515	-22.356	-24.488	20.235	-58.558
VARIANCE OF PREDICTION	4.715	7.544	10.422	5.488	6.236	7.583	1.595	5.562	0.527	16.309

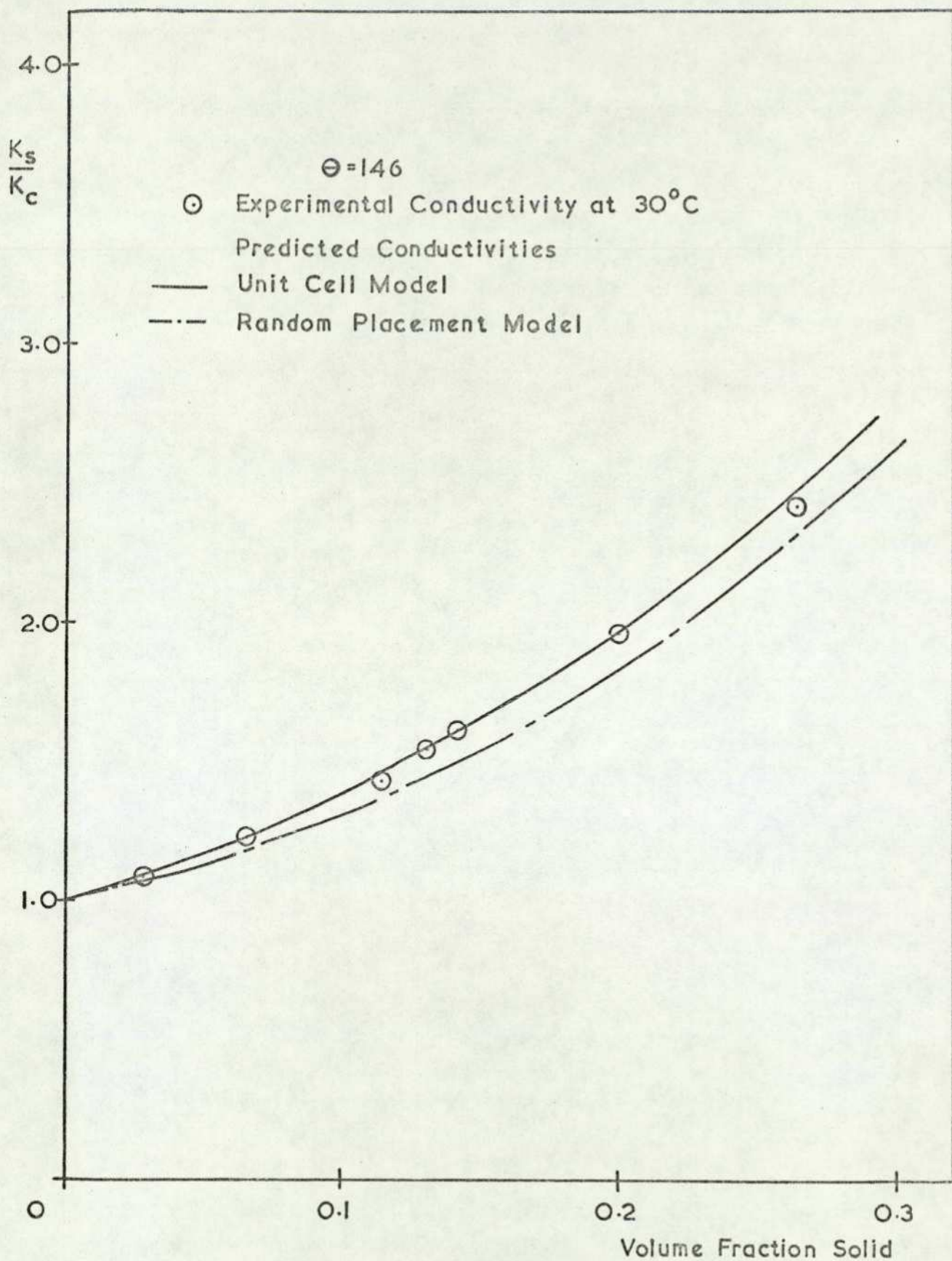
OVERALL ERROR ANALYSIS

	UNIT CELL	RANDOM	MAXWELL	BRUGGENAN	CH. & VA.	JEFFERSON	MODEL 2	MODEL 3	MODEL 4	SERIES
MEAN ERROR	25.832	32.007	36.647	28.735	23.853	34.274	22.217	19.684	25.786	51.573
VARIANCE OF ERROR	0.017	0.025	0.036	0.022	0.030	0.021	0.012	0.025	0.022	0.042
AVERAGE BIAS OF ERROR	-25.832	-32.007	-36.647	-28.735	-21.251	-34.274	-22.217	-16.220	24.314	-51.573
VARIANCE OF PREDICTION	3.683	5.732	7.642	4.694	4.880	5.454	1.724	3.836	0.669	11.391



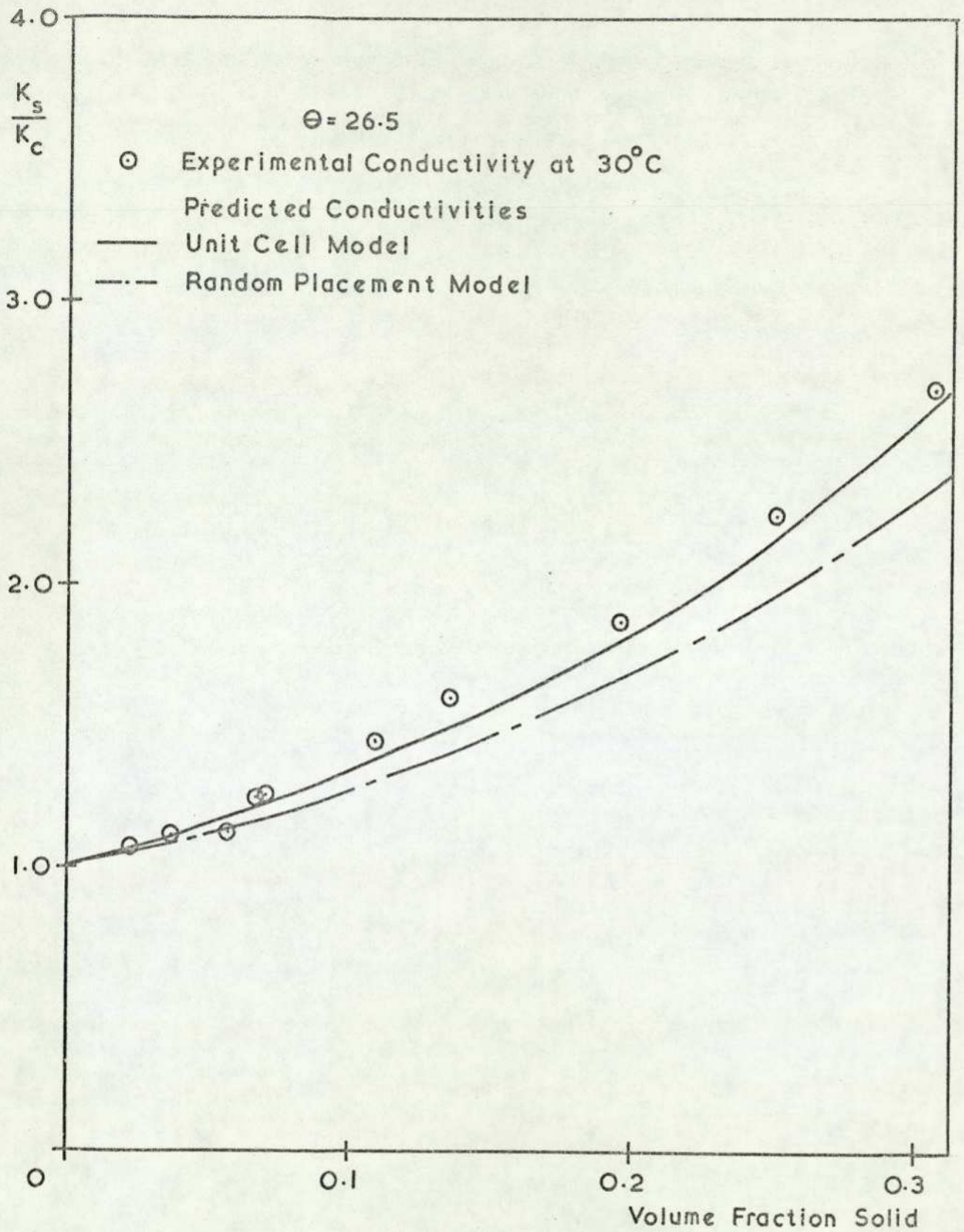
AUTHORS EXPERIMENTAL AND PREDICTED THERMAL CONDUCTIVITIES FOR COPPER WATER SYSTEM

Figure 7.1



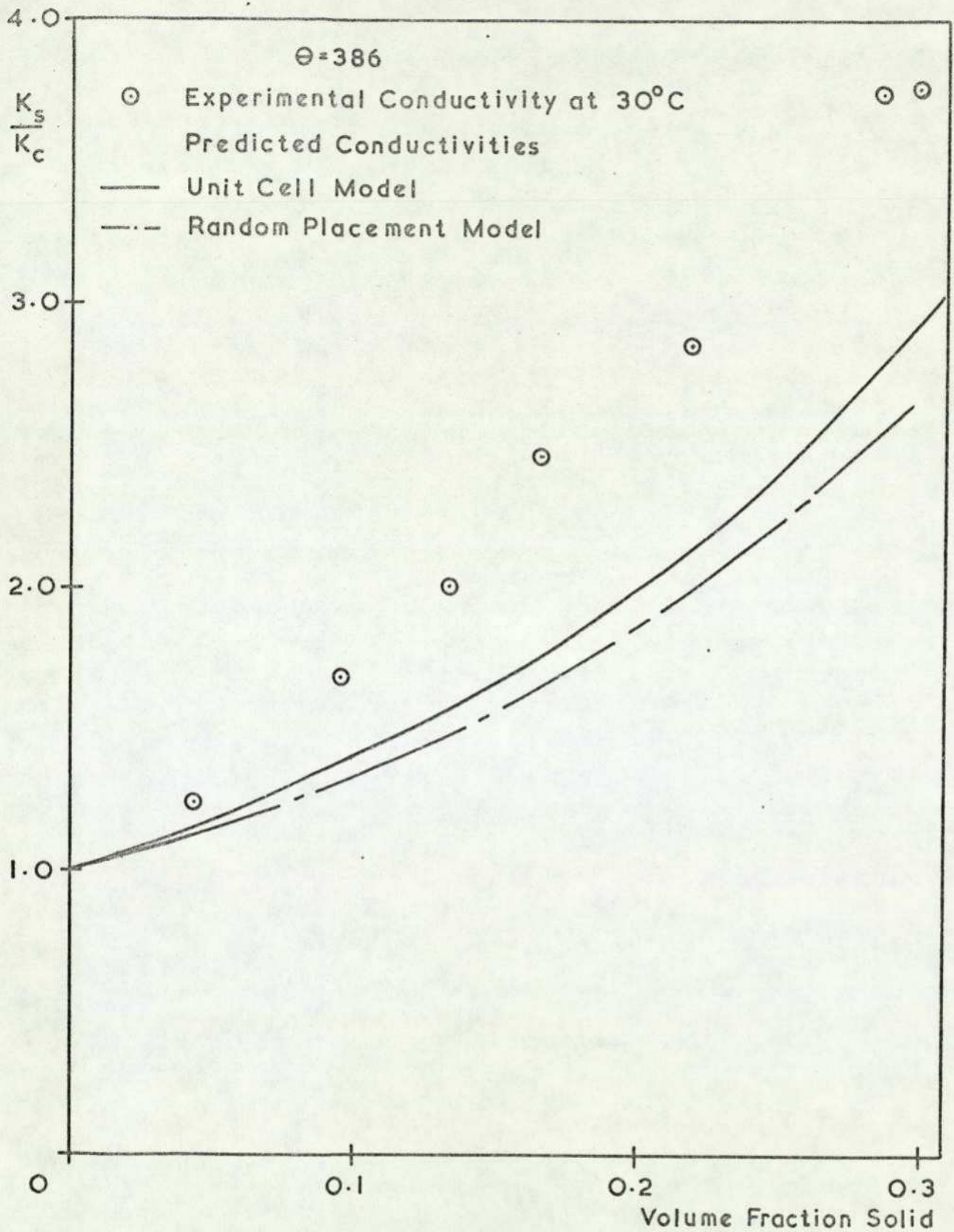
AUTHORS EXPERIMENTAL AND PREDICTED THERMAL
CONDUCTIVITIES FOR NICKEL WATER SYSTEM

Figure 7.2

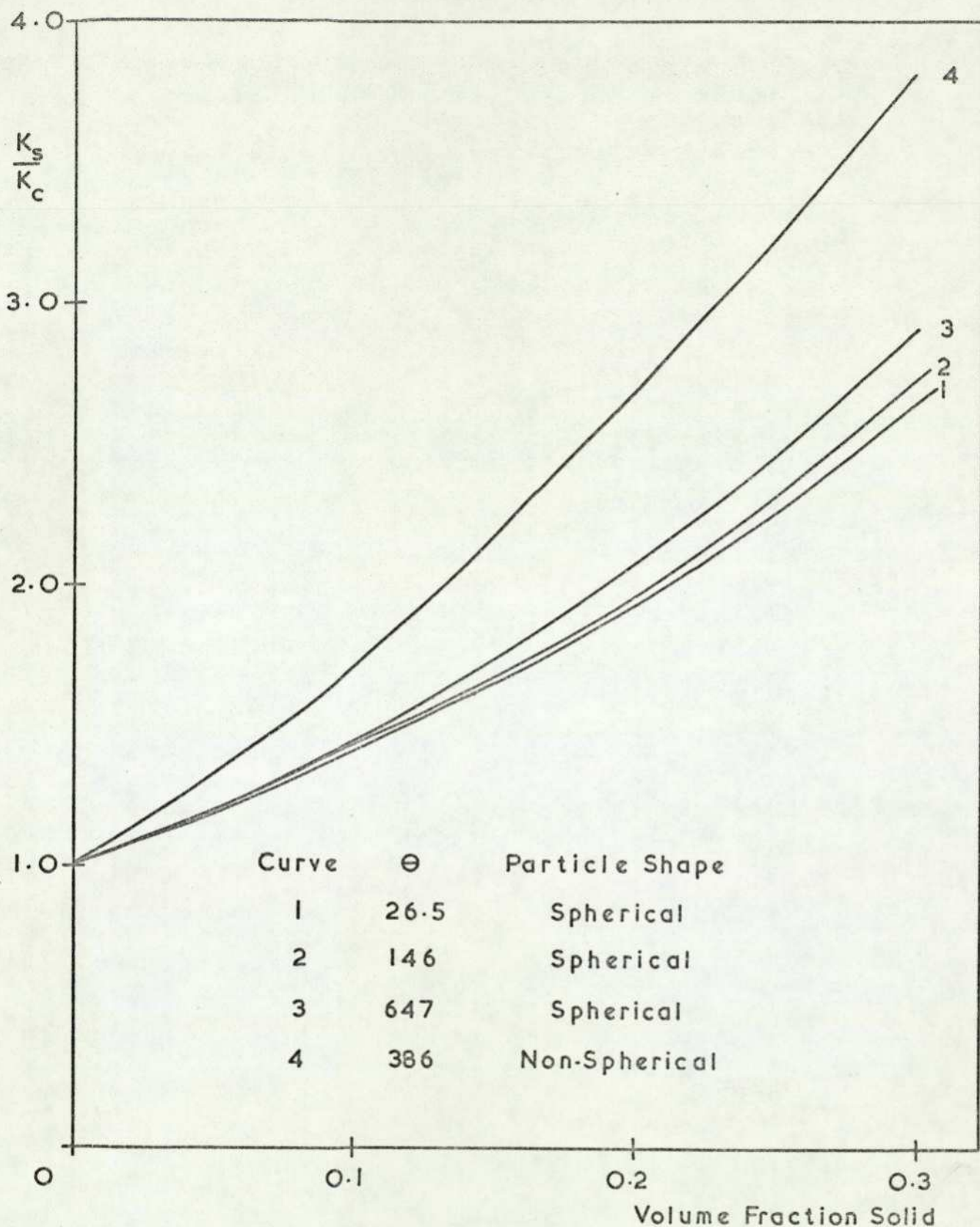


AUTHORS EXPERIMENTAL AND PREDICTED THERMAL CONDUCTIVITIES FOR STAINLESS STEEL WATER SYSTEM

Figure 7.3



AUTHORS EXPERIMENTAL AND PREDICTED THERMAL
CONDUCTIVITIES FOR NON-SPHERICAL ALUMINIUM
WATER SYSTEM
Figure 7.4



EFFECT OF COMPONENT CONDUCTIVITY RATIO ON
EXPERIMENTAL THERMAL CONDUCTIVITIES

Figure 7.5

Section B

Turbulent Heat Transfer to Solid-Liquid Suspensions

8 LITERATURE SURVEY

8.1 Introduction.

Two-component flows are frequently encountered in engineering, typical applications of solid-liquid suspensions are found in hydraulic conveying and flow of slurries, for example in nuclear reactors. The term two-component is sometimes used to describe flows in which the phases do not consist of the same chemical substance, while the term two-phase is applied to the opposite case. Since, however, the system descriptions are identical the two expressions are generally treated as synonyms, as in this study.

A general approach to the study of the dynamics of multiphase systems, particularly suited to engineering design problems, has been through modifying the continuum mechanics of single phase fluids in such a way as to account for the presence of particles. Initial work in the field of solid-liquid systems used an approach based on the assumption of Newtonian behaviour since it was found that deviations will be small for suspensions of inert solids at sufficiently low concentrations. Later studies were concerned with extending relationships to fluids that exhibited appreciable non-Newtonian characteristics, of which concentrated solid-liquid suspensions formed merely a sub-set. A major consideration has been the definition of a suitable viscosity term for use in dimensionless groups, the selection of a viscosity term for heat transfer correlations has usually followed from studies of turbulent flow pressure drop characteristics.

8.2 Prior Heat Transfer Correlations for Two-Component Suspensions.

8.2.1 Early Studies.

Bonilla et.al. /34/ investigated turbulent flow heat transfer to slurries of precipitated chalk in water and derived a correlation of

experimental results based upon mean suspension properties. The density and specific heat of the suspensions were taken as weighted averages of the component properties, whilst the thermal conductivity was accepted as that of the suspending medium. Various theoretical and empirical equations for predicting the viscosity of a suspension were considered, as well as the possibility of using the viscosity of the suspending medium, in order to remove the need for rheological measurements in design procedures. The choice of viscosity correlation was resolved by choosing that which enabled the best correlation of the experimental heat transfer data to be obtained; this proved to be the expression due to Hatschek /35/ who gave:

$$\mu_s = \frac{\mu_c}{(1 - \phi^{1/3})} \quad (8.1)$$

In treating the heat transfer data it was found that a plot of $N_u \cdot P_r^{-1/3}$ against Re, with the percentage of solid by weight as a parameter, showed that the value of $N_u \cdot P_r^{-1/3}$ decreased with an approximate linearity as the solid content of the mixture was increased and that the effect was more pronounced at lower Reynolds numbers. The empirical equation developed was:

$$\left[\frac{hD}{K_c} \right]_f \left[\frac{\mu_s C_{p_s}}{K_c} \right]_f^{-1/3} = 0.0438 \left[\frac{\rho_s U_b D}{\mu_s} \right]_f^{0.758} - 555W \quad (8.2)$$

where the subscript f refers to the mean film temperature; the mean of wall and bulk temperatures. This correlation was found to agree with the experimental data with an average error of 10 per cent. McAdams /88/ has pointed out that equation (8.2) may lead to the prediction of zero or even negative heat transfer coefficients within the range of the authors' data.

Orr and Dalla Valle /36/ considered that a correlation such as equation (8.2), differing only in the values of the constants, could probably be obtained for any apparatus and liquid-solid suspension, but proposed the more universal situation of a correlation equally applicable to all suspensions within a given range. The results of an experimental investigation of turbulent heat transfer to mixtures covering an extensive range of solid materials suspended in water and ethylene glycol and with particle sizes ranging from 1 to 260 microns, were correlated using the conventional Sieder-Tate equation for single-component fluids by specifying mean suspension properties.

Suspension thermal conductivities were evaluated using the Maxwell relationship /1/ following an experimental investigation which confirmed that this expression would satisfactorily represent the data for the mixtures. A tube viscometer expressly designed for use with suspensions was used to obtain laminar flow curves for the various fluids; they were taken to demonstrate approximate Newtonian characteristics and, further, that curves for each suspension plotted on the basis of fluidity as a function of concentration were of the same general shape. By normalising the variables it was found possible to obtain an approximate expression which covered all suspension viscosities in the form of the Newtonian relationship:

$$\mu_s = \frac{\mu_c}{\left[1 - \varphi/\varphi_{bed}\right]^{1.8}} \quad (8.3)$$

where φ_{bed} is the volume fraction of solid particles in a gravity sedimented bed, evaluated after prolonged standing of the mixture. Equation (8.3), although considered adequate for the purpose of correlating heat transfer data of suspensions was not recommended for pipeline design or for suspensions differing radically from Newtonian characteristics.

Using the suspension properties as defined, the heat transfer data was correlated by the equation:

$$\frac{h D}{K_s} = 0.027 \left[\frac{D u_b \rho_s}{\mu_s} \right]^{0.8} \left[\frac{\mu_s C_{p_s}}{K_s} \right]^{1/3} \left[\frac{\mu_c}{\mu_{c,w}} \right]^{0.14} \quad (8.4)$$

which is not recommended for use below Reynolds numbers of 10^4 . The authors do not give limits of accuracy for this equation but Salamone and Newman /37/ state that this expression correlates the data within approximately 20 per cent.

The use of a turbulent viscosity as suggested by Alves et.al. /38/ and Binder and Busher /39/ was adopted in the study by Salamone and Newman /37/, who utilised a pipeline viscometer to measure the viscosities employed in the correlation of heat transfer data. This viscometer was of the same diameter as the heat transfer pipe and was in series with it so

that the slurry flow rate was exactly the same in both sections of the apparatus. From the pressure drop across the viscometer and the suspension density a friction factor was calculated and a Reynolds number read off from a calibration curve. An apparent viscosity was then obtained from the Reynolds number. The apparent viscosity obtained in this manner was shown to decrease with increasing flow rate of slurry.

Following a dimensional analysis an empirical equation was obtained with constants evaluated from arbitrary sets of data embracing copper, carbon, silica and chalk suspensions in water, and with particle sizes ranging from 1.5 to 56 microns. The expression derived was:

$$\frac{hD}{K_c} = 0.131 \left[\frac{DU_b \rho_s}{\mu} \right]^{0.62} \left[\frac{\mu_d C_{Pc}}{K_c} \right]^{0.72} \left[\frac{K_d}{K_c} \right]^{0.05} \left[\frac{D}{d} \right]^{0.05} \left[\frac{C_{Pd}}{C_{Pc}} \right]^{0.35} \quad (8.5)$$

Thus the equation contains a Nusselt, Prandtl and Reynolds number based upon an apparent viscosity and suspension density; the last three terms are regarded as correction factors accounting for the presence of solids with the ratio of component specific heats being shown to be the most significant. The low exponents of the diameter and conductivity ratios indicate that under these conditions the effect of these groups is small.

The average deviation of the experimental data from the correlating equation was given as being within the calculated experimental precision of 10 per cent.

A disadvantage in the use of a turbulent viscosity is that in being a function of pipe size and shear rate pilot plant data is necessary for scale-up and design.

Metzner /33/ has severely criticised equation (8.5) since no term is included in it which might account for the relative quantities of the two phases present, except in the viscosity and density terms. Obviously the specific heat and thermal conductivity terms must also depend on the relative quantities of the two phases, as well as on the corresponding values of the individual constituents. The limitations of this approach have placed an undue and perhaps unrealistic importance on the dimensionless groups containing density and viscosity. Equation (8.5) is considered untenable therefore in two respects:

1. The unusually high value of the exponent on the Prandtl number, which is indicated to be incorrect by all prior work on both suspensions and single-phase systems.
2. The predicted net effect of viscosity on the convection coefficient is to make those for viscous fluids greater than those for less viscous fluids in contradiction to the behaviour usually observed,

8.2.2 The Generalised Dimensionless Group Approach

A more rigorous approach which includes not only the case of dilute suspensions but is also applicable for fluids which exhibit highly non-Newtonian behaviour was provided following the definition by Metzner and Reed /41/ of a generalised Reynolds number based on tube viscometer data. The Mooney-Rabinowitsch equation for flow in circular tubes may be used to show that for any time-independent the quantity $(8U_b/D)$ is some unique function of the wall shear stress only:

$$\tau_w = K' \left[\frac{8 U_b}{D} \right]^{n'} \quad (8.6)$$

where the fluid consistency index K' and the flow behaviour index n' are, in general, not constants but vary with $8U_b/D$.

From the usual definition of the fanning friction factor and by forcing the fluid to obey the standard laminar Newtonian relationship between friction factor and Reynolds number, Metzner and Reed /41/ defined a generalised Reynolds number as:

$$Re' = \frac{D^{n'} U_b^{2-n'} \rho}{K' 8^{n'-1}} \quad (8.7)$$

Metzner, Vaughn and Houghton /42/ considered that the dimensionless groups used in Newtonian heat transfer correlations could be generalised for application to non-Newtonian fluids if a judiciously chosen apparent viscosity is used in place of the Newtonian viscosity. Since the generalised Reynolds number had been shown to be successful in correlating pressure drop data, it was suggested that by equating the Newtonian Reynolds number to the generalised one, a suitable apparent viscosity could be determined. Hence:

$$\frac{D U_b \rho}{\mu_a} = \frac{D^{n'} U_b^{2-n'} \rho}{\gamma} \quad (8.8)$$

where $\gamma = K' 8^{n'-1}$

This gives:

$$\mu = \gamma \left[\frac{U_b}{D} \right]^{n'-1} \quad (8.9)$$

substitution into the conventional Prandtl number gives:

$$Pr' = \frac{C_P}{K} \gamma \left[\frac{U_b}{D} \right]^{n'-1} \quad (8.10)$$

and a dimensionless consistency ratio analogous to the Sieder-Tate viscosity correction factor is suggested as γ/γ_w

It was proposed that if the generalisations were both valid and adequate then the functional relationships between the dimensionless groups must be the same as for Newtonian heat transfer. Hence for turbulent heat transfer to suspensions:

$$\frac{h D}{K_s} = 0.023 \left[\frac{D^{n'} U^{2-n'} \rho_s}{\gamma} \right]^{0.8} \left[\frac{C_{Ps}}{K_s} \gamma \left[\frac{U_b}{D} \right]^{n'-1} \right]^{0.4} \left[\frac{\gamma}{\gamma_w} \right]^{0.14} \quad (8.11)$$

This expression has not been subject to extensive experimental evaluation.

A difficulty arising in practical application of a method involving generalised variable groups is that laminar flow data must be obtained over the complete range of shear rates of interest but limitations to the minimum size of capillary tube that may be used in a viscometer arise from consideration of the phenomenon of 'effective slip' near the tube wall.

The generalised Reynolds number concept was used by Dodge and Metzner /43/ in extending the Newtonian logarithmic resistance formula of Von Karman to the case of fluids of the power law type and thereby providing one of the necessary inputs for an analogy between heat and momentum transfer for such fluids developed by Metzner and Friend /44/.

In developing the resistance law for non-Newtonian fluids, Dodge and Metzner utilised the two constant power law model to represent the rheological properties of the fluid and a theoretical analysis analogous to that performed for Newtonian fluids yielded a general equation. The unknown coefficients in this equation were determined empirically by experiment using aqueous polymer solutions and slurries of Attasol. The final expression was:

$$\sqrt{\frac{1}{f}} = \frac{4.0}{(n')^{0.75}} \ln \left[Re' (f)^{1-n'/2} \right] - \frac{0.4}{(n')^{1.2}} \quad (8.12)$$

Although the analysis was based on the assumption of a power law function describing fluid laminar flow properties the result can be used for fluids that do not conform to this relationship by using the property parameters n' and K' . It is considered that these parameters should

be evaluated at the existing value of the wall shear stress.

Metzner and Friend /44/ developed a turbulent heat and momentum transfer analogy for purely viscous non-Newtonian fluids by extending the relationship due to Friend /40/ for Newtonian fluids in which an allowance is made for slight turbulence in the sub-layer adjacent to the wall. The assumptions under which the relation was developed were:

1. Heat flux is constant along the heat exchanger.
2. Heat flux is a linear function of tube radius.
3. Eddy diffusivities of heat and momentum are equal at all radii.
4. The fluid is time independent and free from viscoelasticity.
5. The fluid Prandtl number is 'high' (> 1.0).

The empirical two constant power law was again used to represent rheological properties and following experiments made using polymer solutions, Attagel slurries and corn syrup it was found that using re-defined parameters the equation previously developed for the Newtonian case would represent the heat transfer data adequately. This involved using the Prandtl number evaluated at the wall and the friction factor for non-Newtonian fluids given by Dodge and Metzner, which are functions of the flow behaviour index of the fluid.

Hence:

$$St = \frac{f/2}{1.20 + 11.8 \sqrt{f/2} (Pr_w - 1)(Pr_w)^{-0.33}} \quad (8.13)$$

The relationship between the Prandtl number evaluated using the apparent viscosity at the wall and the generalised Prandtl number is given as:

$$Pr_w = \frac{Pr'}{\left[\frac{16f}{Re'} \right]^{(n'-1)/n'} \frac{(3n'+1)}{4n'}} \quad (8.14)$$

The expression derived represented the experimental data with an average deviation of 17.6 per cent and a standard deviation of 23.6 per cent and was shown to be a significant improvement over the the relationship proposed by Metzner et.al. /42/.

Since the development of equation (8.14) was confined to an 'isothermal' coefficient of heat transfer it was suggested that correction to actual flow conditions could be obtained through multiplication by factors analogous to that of Sieder-Tate $(\gamma/\gamma_w)^{0.14}$ or Krieth-Summerfield $(\gamma/\gamma_w)^{0.1}$

Again although the power law was used to develop equation (8.14) the authors regard it as applicable to all time-independent non-elastic fluids since the second term in the denominator is usually much greater than the quantity 1.20 and because the friction factor may be evaluated for these fluids as previously outlined.

Peterson and Christensen /45/ extended the heat and momentum transfer analogy of Metzner and Friend to non-isothermal flow by:

1. Use of the Krieth-Summerfield correction $(\mu/\mu_w)^{0.1}$
2. Evaluation of the Prandtl number at the bulk fluid temperature.
3. Use of the friction factor that would be obtained if the flow were isothermal at the bulk fluid temperature f_i , where:

$$f_i = f(\mu_w/\mu)^{-0.1} \quad (8.15)$$

4. Correction of the data for a trend with flow behaviour index by normalising the critical Reynolds number for non-Newtonian flow to 2100, which is the value for Newtonian flow. Hence a more characteristic Prandtl number is yielded:

$$Pr = Pr_b \left[\frac{Re_{w.c.}}{2100.0} \right] \quad (8.16)$$

where the subscript W.C. denotes evaluation at the wall shear stress and critical flow rate.

Hence the correlation is amended to:

$$St = \frac{f_i \left[\frac{\mu_b}{\mu_w} \right]^{0.1}}{1.2 + 11.8 \sqrt{\frac{f_i}{2} \frac{[Pr - 1]}{[Pr]^{0.33}}}} \quad (8.17)$$

The authors claim that the standard deviation of Metzner and Friend, 23.6 per cent, is reduced to 16.2 per cent by this correlation.

In addition, Petersen and Christiansen /45/ have presented an extension of this analogy to the transitional flow region with a similar accuracy in prediction of experimental data.

8.2.3 Later Studies.

Thomas /46/ in studying the heat and momentum transport characteristics of aqueous thorium oxide suspensions found that the Bingham plastic model would adequately represent the laminar flow data obtained in a capillary tube viscometer. This model gives:

$$\tau = \tau_y + \eta \frac{du}{dr} : \tau > \tau_y$$

$$\frac{du}{dr} = 0 : \tau < \tau_y$$

(8.18)

Consideration was given to various definitions of non-Newtonian viscosity terms; friction factor and heat transfer data were plotted, for a particular slurry, as a function of a Reynolds number calculated using two different viscosity terms given by:

1. The effective viscosity corresponding to the particular value of the wall shear stress.
2. The limiting viscosity at high rates of shear; for the Bingham plastic model this is simply the coefficient of rigidity, η .

Good agreement was found, using both viscosities, between the laminar flow pressure drop in various tubes and the theoretical laminar curves. In the turbulent region it was shown that turbulent heat transfer and pressure drop data for a given suspension can be correlated uniquely, independent of tube diameter, provided the experimentally determined limiting viscosity at high shear rate is used in calculating the Reynolds and Prandtl numbers. The use of the effective viscosity was found to give a pronounced diameter effect in turbulent flow correlation.

Except for the displacement in critical Reynolds number due to the non-Newtonian laminar characteristics the thorium oxide suspension heat transfer data. Thus the Sieder-Tate equation was considered satisfactory

in correlating the experimental results, provided that the Reynolds number is at least four times the critical value.

$$j = \left[\frac{h}{C_{Ps} U_b \rho_s} \right] \left[\frac{C_{Ps} \eta}{K_s} \right]^{2/3} \left[\frac{\eta_w}{\eta} \right]^{0.14} = 0.027 \left[\frac{D U_b \rho_s}{\eta} \right]^{-0.2} \quad (8.19)$$

The slurry heat capacity was considered to be an additive function of the pure material heat capacities and thermal conductivity was calculated from the pure material values by the use of the Maxwell equation.

The data for fully developed flow heat transfer was also in substantial agreement with the Colburn, Prandtl, Von Karman, Metzner and Friend and Martinelli heat and momentum transfer analogies. The best fit was observed with the Martinelli analogy which correlated the experimental data within + 17 per cent and - 36 per cent. It may be written as:

$$j = \frac{\frac{\epsilon_H}{\epsilon_M} \sqrt{\frac{f}{2}} [Pr]^{2/3} \frac{T_w - T_c}{T_w - T_b}}{5 \left[\frac{\epsilon_H}{\epsilon_M} + \ln \left[1 + 5 \frac{\epsilon_H}{\epsilon_M} Pr \right] + 0.5 F_1 \ln \frac{Re}{60} \sqrt{\frac{f}{2}} \right]} \quad (8.20)$$

Considerable simplification is afforded by using the tabulated values of parameters given by Knudsen and Katz /47/. The ratio of eddy diffusivities was taken as unity.

The Von Karman analogy was also recommended as the scatter of the data probably did not warrant the use of a complex equation. This gives:

$$j = \frac{Pr^{2/3} \sqrt{\frac{f}{2}}}{1 + 5 \sqrt{\frac{f}{2}} \left[(Pr - 1) + \ln \left(1 + \frac{5}{6} (Pr - 1) \right) \right]} \quad (8.21)$$

Friction factors are obtained through a correlation of turbulent flow data with a Blasius relationship:

$$f = B Re^{-b}$$

where

$$B = 0.079 \left[\frac{\mu_c}{\eta} \right]^{0.48}$$

$$b = 0.25 \left[\frac{\mu_c}{\eta} \right]^{0.15} \quad (8.22)$$

An additional term is included in the expressions for the Blasius coefficient and exponent for slurries having a high yield stress but for moderate values this term contributes less than 5 per cent and can be neglected.

Mitzushima and Kuriwaki /48/ considered turbulent heat transfer to suspensions whose rheological properties were represented by either the Bingham plastic model or a power law. Using a similar argument to Metzner and Friend /44/ they assumed that almost all the resistance for momentum and heat transfer in the turbulent flow of highly viscous

fluids exists in the laminar film and so the apparent viscosity evaluated at the wall shearing stress may be the characteristic property. Therefore, if the viscosity term of the Reynolds and Prandtl numbers in the equations for Newtonian fluids is replaced by the apparent viscosity at the wall those equations may be applied to non-Newtonian fluids.

The Newtonian friction factor equation of Koo was taken to be a sufficiently accurate approximation after consideration had been given to the expression developed by Dodge and Metzner. Assuming the Chilton-Colburn analogy to be valid for non-Newtonian fluids then for power law fluids the following expression was derived:

$$Nu = 0.023 \, Re^{5(n+2)/[3(4n+1)]} \, Pr^{4(n+2)/[3(4n+1)]} \, Pr^{1/3} \quad (8.23)$$

where the Reynolds and Prandtl numbers are equivalent to the generalised terms defined in equations (8.7) and (8.10). The power law flow behaviour index n is defined by the rheological model:

$$\tau = K \left[\frac{du}{dy} \right]^n \quad (8.24)$$

For fluids represented by the Bingham plastic model the equation derived was:

$$\frac{hD}{K_s} = 0.023 \left[1 - \frac{\tau_y}{\tau_w} \right]^{7/15} \left[\frac{DU_b \rho_s}{\eta} \right]^{0.8} \left[\frac{C_{Ps} \eta}{K_s} \right]^{1/3} \quad (8.25)$$

These equations are claimed to represent the experimental data, obtained from studies using polymer solutions and cement slurries, better than the expression given by Metzner and Friend, equation (8.13).

Thermal and momentum diffusivity profiles were also experimentally determined for non-Newtonian fluids from measurements in a rectangular duct. It was found that eddy diffusivities at any transverse position were proportional to Reynolds number. It was also concluded that eddy diffusivity profiles in non-Newtonian flow are the same as those in Newtonian flow if the Reynolds numbers, as defined in the study, are equal.

Crewe and Simons /49/ have compared the heat transfer characteristics of flocculated Kaolin suspensions and non-flocculated coal suspensions in water. Both slurry types were found to be best fitted with the Bingham plastic flow model to represent rheological data. Following Thomas, the coefficient of rigidity was used as the characteristic viscosity term in the dimensionless groups and an empirical equation was derived expressing this variable in terms of the volume fraction solid and the viscosity of the suspending medium.

$$\eta = \mu_c \exp(B\phi) \quad (8.26)$$

where B is a constant ideally obtained from laminar flow experiments in an appropriate tubular viscometer. On this basis the slurry pressure drop characteristics were found to agree with the Thomas friction factor plot given by equation (8.22).

Differences between the heat transfer performances of the two slurries were resolved by empirical means. For flocculated suspensions the expression derived was:

$$j = \left[\frac{h}{C_{Ps}G} \right] \left[\frac{C_{Ps} \mu_c \exp(B\varphi)}{K_s} \right]^{2/3} = 0.027 \left[\frac{D U_b \rho_s}{\mu_c \exp(B\varphi)} \right]^{-0.2} \quad (8.27)$$

which correlated the experimental data within + 19.3 per cent and - 17.8 per cent. For the non-flocculated suspensions:

$$\left[\frac{h}{C_{Ps}G} \right] \left[\frac{C_{Ps} \mu_c \exp(B\varphi)}{K_s} \right]^{2/3} \exp \left[-\frac{B\varphi}{4} \right] = 0.0285 \left[\frac{D U_b \rho_s}{\mu_c \exp(B\varphi)} \right]^{0.2} \quad (8.28)$$

correlated the data within + 17.8 per cent and - 11.1 per cent.

Experimental data was obtained for both heating and cooling of the fluid but no difference could be determined between the respective points.

The remaining suspension properties were obtained using Maxwell's equation to determine thermal conductivity with the usual weighted averages for specific heat and density.

As the equations show, a significant difference was observed between the heat transfer characteristics of the two slurry types. However no significant difference was observed between the turbulent flow measurements in approximately the same wall shear stress range as the heat transfer

experiments. The exact mechanistic difference between the heat transfer characteristics of the two suspensions was not determined and although several explanations appeared possible experimental verification was lacking. Skelland in a private communication to the authors proposed that turbulence damping in the flocculated slurries had a greater effect on heat transfer than on momentum transfer.

Kiyohashi, Kawashima and Ishihama /50/ have recently conducted experimental work in cooling heat transfer from aqueous limestone slurries in circular tubes. Particle sizes varied from 25 to 130 microns and mass concentrations ranged up to 50 per cent. Laminar flow measurements taken using a tube viscometer indicated that all of the slurries, except in low concentrations, exhibited the characteristics of Bingham plastic fluids; considerable scatter in the data, however, precluded the determination of a relation between the concentration and viscosity. Hence the slurry viscosity was assessed following an equation attributed to Taylor /51/:

$$\mu_s = \mu_c (1 + 22.4 \varphi) \quad (8.29)$$

This expression has also been reported by Michiyoshi et.al /52/ but it is worth noting that there does not appear to be any justification for its use in the original work quoted.

Friction factors for suspension flow, except for the largest particles at lower Reynolds numbers, were found to be slightly lower than for Newtonian fluids, confirming the result of Thomas.

Thermal conductivities of the slurries were determined by the Maxwell equation and specific heats were measured experimentally and fitted with an empirical relationship independent of particle size. The experimental heat transfer data of the study was found to be reasonably correlated for a range of mass flow concentration and particle size by the equation:

$$\frac{h D}{K_s} = 0.00567 \left[\frac{D U_b \rho_s}{\mu_s} \right]^{0.987} \left[\frac{C_{Ps} \mu_s}{K_s} \right]^{0.4} \quad (8.30)$$

Of interest is the use made of the correlation determined convection coefficients in performing calculations concerned with the prevention of freezing of pipeline slurries which may be important in the transport of solid particles by hydraulic methods.

8.3 Purpose of this Study.

There are many areas of study in the field of solid-liquid suspensions and the problems can be approached from different viewpoints and with varying degrees of rigour of treatment. This study is concerned with obtaining quantitative relationships useful in engineering applications in terms of measureable properties of the fluid in question.

It is clear from the foregoing that there exists a need for a method of determining heat transfer information of suspensions that does not depend upon the definition of one particular viscosity model. Such a method would be equally applicable to Newtonian or non-Newtonian fluids and could be used with any suitable relationship determining laminar flow

properties from the constituent component properties. If such a relationship is available then the laminar flow curve may be related to the flow conditions existing at all points throughout the fluid in turbulent heat transfer.

9 THE HOMOGENEOUS EQUILIBRIUM FLOW MODEL

9.1 Introduction.

A fundamental problem in the analysis of two-component flows is the evaluation of the forces exerted on the individual solid particles by the fluid and the relation of these forces to the actual particle behaviour and the resultant energy losses. These forces are a consequence of the motion of the fluid surrounding each particle and arise in particular from fluctuations in velocity due to turbulence. The problem is further complicated since the presence of suspended particles affects the fluid turbulence.

A model based upon the assumption of homogeneous flow of a solid-liquid suspension provides the simplest technique for the analysis of such phenomena. If the flow of suspension approximates to the regime prescribed by the simplifying assumptions and provided that suitable average properties may be determined, the mixture can be treated as a pseudo-fluid that obeys all the usual equations of single component flow. Differences in velocity and temperature between the phases will promote momentum and heat transfer; if the processes proceed very rapidly, as when one phase is finely dispersed in the other, it can be assumed that equilibrium is reached. Thus the average values of velocity and temperature are the same as the values for each component and we have homogeneous equilibrium flow. The standard methods of fluid mechanics can then be applied; in particular the problem of a single-component fluid in turbulent flow through circular tubes having been successfully approached by the use of the boundary layer approximations in differential form with solutions being obtained by numerical techniques, it may be expected that the assumption of homogeneous equilibrium flow enables the

solution for a two-component fluid to be approached in the same manner. It is implicit in the homogeneous flow model that the effects of interaction between the phases are contained within the relationships for effective suspension properties. The average properties that are required in this case are density, viscosity, thermal conductivity and specific heat together with a model that represents the turbulent transport properties of the two-component mixture.

For the techniques used in analysing single-component turbulent flows to be applied to two-component flows it is sufficient that the flow is locally homogeneous; one velocity and temperature profile is determined for both components together. To solve the equations it is necessary that the suspension properties be determined as a function of position. Usually these properties are given as functions of volume fraction solid, thus the definition of a flow regime in terms of a concentration profile and the properties of the components enables the required suspension properties to be calculated everywhere. The turbulent transport model presents more difficulty because comprehensive empirical expressions for two-component flows have not yet been established.

Difficulties in the simple definition of a two-component flow regime may occur because:-

1. The concentration is not always an adequate means of describing the local two-phase properties, there may be significant differences in the behaviour of various sizes of particles within a suspension.

2. The concentration of a particular phase may be neither uniform nor symmetrical.
3. There may be relative motion between the phases on a scale which is larger than the distance at which a continuum description is meaningful; two velocity profiles would therefore exist, one for each phase.

The presence of any of these effects would put the problem outside the scope of this treatment but in a large number of cases useful information can be obtained by idealising the flow in accordance with the following assumptions.

9.2 Assumptions and Limitations of the Model.

The assumptions made in this study may be divided into two groups:-

- a. Intrinsic assumptions which contain constraints inherent in the homogeneous equilibrium flow model.
- b. Dependant assumptions which relate to the conditions under which the effective suspension properties are evaluated from the constituent component properties.

The first group of assumptions are:-

- 1) Each phase has identical profiles of velocity and temperature and velocity and temperature gradients.
- 2) The solid particles follow the fine structure of turbulence.
- 3) Heat and momentum transfer due to particle collision with the pipe wall or with each other is negligible.
- 4) Each solid particle is small and maintains a uniform temperature because of its high thermal conductivity.

- 5) Radiation heat transfer is negligible.
- 6) Virtual or apparent mass effects are neglected.

The dependant assumptions are:

- 7) The particles are approximately spherical.
- 8) The solid particles are uniformly in suspension throughout the pipe cross-section.
- 9) Variations in particle size are of no significance.

The dependant assumptions may be varied when the model is used with any suitable description of flow regime or suspension property relationship and are not confined to the simplest case of a uniform concentration of spherical particles considered in this investigation. As further information becomes available it can readily be incorporated into the computational scheme that is set up.

Practical limitations to the use of the homogeneous flow model are founded upon consideration of the physics of the flow and the particle characteristics as criteria for determining the point of departure from a fully suspended transport regime. Factors which must be taken into consideration include particle size and density, concentration, fluid velocity and density, flow orientation and pipe diameter. Following the concepts introduced by Wilson /53/ the distinction commonly made in the literature is to divide the flow of solid particles in a liquid into homogeneous and heterogeneous regimes. The former occurs when the solid particles are finely divided and the settling tendency is so small that the mixtures may be regarded as non-settling. According to Graf /54/ these are characterised by a settling velocity below 0.002 to 0.005 feet/sec., the settling velocity depending on the concentration as well

as on the usual parameters. Heterogeneous flow is characterised by larger particles and covers a range of transport regimes from fully suspended flow to sliding bed transport; if the fluid velocity is sufficiently high the particles may be uniformly distributed, this regime having been termed pseudo homogeneous. It is necessary to distinguish between the common definition of homogeneous flow and a homogeneous flow model, since if the level of turbulence is such as to keep all the particles suspended so that the flow is locally homogeneous and the required property relationships can be specified along with a concentration description, then it may be possible to treat settling suspensions using the model described in this study. However, settling suspensions exhibiting marked concentration profiles in horizontal flow are likely to fail to satisfy the homogeneous flow model intrinsic assumptions as, for example, the relative velocity between the phases will be significant. Again, for settling suspensions in vertical flow although the concentration and velocity profiles may be axisymmetric and sensibly uniform, the model assumptions may not be satisfied especially for larger dense particles.

Durand /55/ studied the flow of slurries through horizontal pipes and classified suspensions according to the solid phase particle size. He considered particles up to 25 microns diameter gave rise to suspensions behaving as homogeneous fluids. An intermediate range of particle size from 25 to 50 microns and a sub-division of the heterogeneous transport regime with particles less than 200 microns and obeying Stoke's Law of settling may also be expected to be suitable for treatment with a homogeneous flow model under certain conditions.

Newitt et.al. /56/ stated that homogeneous flow was characteristic of suspensions of particle size less than 30 microns and the following relationship, which should hold for the flow to be homogeneous, was given:

$$\frac{1800 \text{ g D U}_t}{U_b^3} < 1 \quad (9.1)$$

Empirical relations are presented by Spels /57/ giving the mean minimum and standard liquid velocities for settling suspensions. The minimum velocity is that at which the particles will all be just in suspension with a concentration gradient in horizontal flow. At the standard velocity the flow becomes homogeneous.

Cairns et.al. /58/ measured concentration profiles in dilute suspensions of small particles (< 16 microns) and considered that velocities greater than 13.4 ft/s (Re > 161 000) were required to produce a homogeneous suspension. This result is in marked contrast to the much lower values given by other workers. It was also found that the concentration in vertical flow was substantially uniform after a distance of four feet from the start of the vertical section.

Thomas /59/, /60/ has presented relationships for the minimum transport velocity of flocculated and non-flocculated suspensions in horizontal pipe flow covering a range of particle size from 1 to 310 microns diameter. Suspension regimes are derived in terms of the relative size of particle and pipe flow zones of laminar sublayer and buffer region, with transport relations being given in terms of the ratio of particle settling velocity to friction velocity.

Through consideration of the correlations for flow regimes it is possible to determine if the homogeneous flow model will be appropriate for a given suspension problem. In the case of vertical flow a conservative estimate would consider the limitations as for horizontal flow but in assessing the validity of the model it is hoped to give some further guide to its limitations in this situation.

9.3 The Basic Equations.

The homogeneous equilibrium flow model is utilised by solving the boundary layer equations in differential form for internal flow as applied to the case of pseudofluid obtained by the idealisation of the two-component mixture. Some of the advantages over a correlation of experimental results using average suspension properties are:-

1. An analytical treatment allows the boundary conditions for a given problem to be varied and hence obviates the need for them either to conform to experimental conditions or be an approximation.
2. Extensions to problems outside the range of experimental evidence have greater validity.
3. Information on velocity and temperature profiles, friction factors, local Nusselt numbers, etc. may be obtained from the one treatment.
4. Updated correlations for average properties of suspensions may be simply included in the treatment if they become available and their effect may be readily determined.

The boundary layer approximations for steady turbulent flow are reductions of the Navier-Stokes and energy equations, with the usual time average terms, to a form that can readily be solved. Simplification of

the basic equations using an order of magnitude evaluation is detailed in most standard textbooks, of which reference /61/ may be quoted as an example. Where flows have a predominant direction, such that cross-stream gradients are much larger than streamwise gradients the truncation of the Navier-Stokes and energy equations are valid. Turbulent terms must be included in the relation of shear stress and heat flux to velocity and temperature profiles; an effective viscosity model is employed to represent the transport properties by a sum of molecular and eddy exchange coefficients.

The solution procedure adopted allows the fluid properties, viscosity, thermal conductivity and specific heat to be temperature dependant and in addition provides for the fluid density to be dependant on temperature and pressure. Obviously a suspension of a solid in a liquid may be treated as an incompressible fluid, where the density is assumed to be constant, but since the solution procedure allows a general variable property treatment it was felt worthwhile retaining this feature.

As is usual with forced convection body forces are neglected, also viscous dissipation will be small and may be omitted from the energy equation; hence the boundary layer approximations may be written:

a. continuity

$$\frac{\partial(u\rho)}{\partial x} + \frac{1}{r} \frac{\partial(v\rho r)}{\partial r} = 0 \quad (9.2)$$

b. momentum in streamwise (x) direction

$$\frac{1}{r} \frac{\partial}{\partial r} \left[\mu_{eff} r \frac{\partial u}{\partial r} \right] = \frac{\partial p}{\partial x} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} \quad (9.3)$$

c. energy

$$\frac{1}{r} \frac{d}{dr} \left[\frac{k_{eff}}{c_p} r \frac{dH}{dr} \right] = \rho u \frac{dH}{dx} + \rho v \frac{dH}{dr} \quad (9.4)$$

The equations may be normalised by dividing variables by their inlet values as defined in the nomenclature and become:

$$\frac{\partial(\bar{\rho} \bar{u})}{\partial \bar{x}} + \frac{2}{\bar{r}} \frac{\partial(\bar{\rho} \bar{v} \bar{r})}{\partial \bar{r}} = 0 \quad (9.5)$$

$$\frac{4}{Re_o \bar{r}} \frac{d}{d\bar{r}} \left[\bar{\mu}_{eff} \bar{r} \frac{d\bar{u}}{d\bar{r}} \right] = \frac{d\bar{P}}{d\bar{x}} + \bar{\rho} \bar{u} \frac{d\bar{u}}{d\bar{x}} + 2\bar{\rho} \bar{v} \frac{d\bar{u}}{d\bar{r}} \quad (9.6)$$

$$\frac{4}{Re_o Pr_o \bar{r}} \frac{d}{d\bar{r}} \left[\frac{\bar{k}_{eff}}{\bar{c}_p} \bar{r} \frac{d\bar{H}}{d\bar{r}} \right] = \bar{\rho} \bar{u} \frac{d\bar{H}}{d\bar{x}} + 2\bar{\rho} \bar{v} \frac{d\bar{H}}{d\bar{r}} \quad (9.7)$$

In addition the integral continuity equation is required for internal flows to provide sufficient equations to enable the pressure gradient to be calculated.

$$\int_0^1 \bar{\rho} \bar{u} \bar{r} d\bar{r} = \frac{1}{2} \quad (9.8)$$

The boundary conditions at the wall are the usual no-slip and impermeable wall conditions together with either a specified wall heat flux condition or a specified wall temperature. At the tube centre-line the conditions are zero axial velocity and temperature gradients together with zero radial velocity component:

$$\begin{aligned}\bar{U}(\bar{X}, 1) &= 0, \quad \bar{V}(\bar{X}, 1) = 0 \\ \frac{\partial \bar{U}}{\partial \bar{r}}(\bar{X}, 0) &= 0, \quad \frac{\partial \bar{H}}{\partial \bar{r}}(\bar{X}, 0) = 0, \quad \bar{V}(\bar{X}, 0) = 0\end{aligned}$$

for specified wall heat flux:

$$\frac{\partial \bar{H}}{\partial \bar{r}}(\bar{X}, 1) = \frac{q^+(\bar{X}) Re_o Pr_o \bar{C}_{P_w}}{2 \bar{K}_w}$$

and for specified wall temperature:

$$\bar{H}(\bar{X}, 1) = \bar{C}_{P_w} (\bar{T}_w(\bar{X}) - 1) \quad (9.9)$$

The initial conditions are a specified velocity and temperature distribution for the fluid, with heating being initiated at any point of the flow and free to vary arbitrarily with position.

Hence:

$$\begin{aligned}\bar{U}(0, \bar{r}) &= \bar{U}_o(\bar{r}), \quad \bar{V}(0, \bar{r}) = \bar{V}_o(\bar{r}) \\ \bar{p}(0, \bar{r}) &= \bar{p}_o(\bar{r}), \quad \bar{H}(0, \bar{r}) = \bar{H}_o(\bar{r})\end{aligned} \quad (9.10)$$

The relationships of the fluid properties with temperature are not specified as part of the solution procedure and may be supplied to suit any particular situation. This allows the specification of the properties of a real fluid or of a pseudo-fluid when considering either one or two-component flow; in the simplest analysis this will entail consideration of the volume fraction solid and component properties as additional parameters:

$$\begin{aligned}
 \bar{\rho} &= f_1(\bar{T}, \varphi, \bar{\rho}_c, \bar{\rho}_d) \\
 \bar{\mu} &= f_2(\bar{T}, \varphi, \bar{\mu}_c) \\
 \bar{K} &= f_3(\bar{T}, \varphi, \bar{K}_c, \bar{K}_d) \\
 \bar{C}_p &= f_4(\bar{T}, \varphi, \bar{C}_{pc}, \bar{C}_{pd}) \\
 \bar{H} &= f_5(\bar{T}, \varphi, \bar{H}_c, \bar{H}_d)
 \end{aligned}
 \tag{9.11}$$

The effective viscosity and thermal conductivity are to be evaluated so as to include the effects of molecular exchange and the exchange due to turbulent eddy motion. It is necessary for empirical information to be introduced since no rigorous theory is as yet available to describe this exchange. The simple linear combination using eddy diffusivities for momentum and heat are used as follows:

$$\bar{\mu}_{eff} = \bar{\mu} \left[1 + \frac{\epsilon_M}{V} \right]
 \tag{9.12a}$$

$$\bar{K}_{eff} = \bar{K} \left[1 + Pr_o \frac{\bar{\mu} \bar{C}_p}{\bar{K}} \frac{\epsilon_H}{V} \right]
 \tag{9.12b}$$

Where the eddy diffusivities are generally functions of position and dynamic variables; any suitable method for calculating effective transport properties could be employed in the treatment but as a starting point an evaluation of eddy diffusivity for momentum will be made using a mixing length model. Prandtl's mixing length hypothesis provides a useful, if limited, method of evaluating the required relationships and has been found to give satisfactory results for a large number of problems. The eddy diffusivity of momentum is given by an expression of the form:

$$\epsilon_M = l^2 \left| \frac{\partial u}{\partial r} \right|$$

(9.13)

Where the distribution of mixing length is obtained from empirical relationships; the form of this relationship for one and two-component flows will be discussed later.

9.4 Solution Procedure.

The boundary layer assumptions change the nature of the momentum and energy equations from elliptic to parabolic partial differential equations and are therefore tractable to numerical solution through a variety of methods, the more especially with the use of high speed digital computers. Rather than develop and program a solution procedure in its entirety it was felt to be more realistic to take an established method that had been utilised for a number of single-component flow problems and adapt this to the homogeneous equilibrium flow model for

two-component flow. The finite difference method developed by Bankston and McEligot /62/ has been shown to provide accurate results when applied to a variety of heated turbulent flow situations /63/, /64/, /65/ and in addition is applicable to a wide range of conditions, economical of computer time and capable of being readily assimilated by any user. In consequence this numerical procedure was adopted for the present work and, since details of the development of the technique and validation data are given in the above references, it is necessary only to describe the method in brief with only the additions made in this work being fully documented.

A finite difference grid is defined along the axis of the tube on any rx plane with the intersection of lines of constant r and constant x becoming the nodes. Finite control volumes are designated as the volume enclosed between surfaces of constant r and constant x and are ring elements with radial surfaces being located half way between adjacent nodes. Radial location of the nodes is adjusted so that grid distances become successively smaller as the wall is approached; these locations are then kept fixed throughout the solution although the number of radial nodes may be varied as desired. Solution of the equations is by a marching procedure where the equations are solved in the cross-stream direction at one particular axial station before moving downstream to the next station. Axial step lengths are varied from an initial specification as the boundary layer thickness varies or the rate of convergence diminishes.

The finite difference equations were obtained by essentially rederiving the governing equations for the macroscopic control volumes; this procedure is equivalent to integrating the governing equations over the control surfaces with the assumption that the dependent variables vary linearly in the axial direction and are uniform in the radial direction. This method produces finite difference equations that are conservative, whereas derivations based on Taylor series expansions of the partial differential equations sometimes fail in this respect. The basic situation considered in developing the energy and momentum equations are shown in figure 9.1. Control over the centring of the finite difference representation of the diffusion terms with respect to the grid is provided by a weighting factor. The basic equations are detailed in reference /62/ and so will not be repeated here.

The solution procedure involves rearranging the equations so that both energy and momentum equations are linear and implicit. To do this all the known values are grouped into the coefficients of recurrence relations, so that the energy balance becomes:

$$A_j [\bar{H}_{i+1,j+1}] + B_j [\bar{H}_{i+1,j}] + A_{j-1} [\bar{H}_{i+1,j-1}] + D_j = 0$$

(9.14)

The momentum equation becomes:

$$A_j [\bar{U}_{i+1,j+1}] + B_j [\bar{U}_{i+1,j}] + A_{j-1} [\bar{U}_{i+1,j-1}] + C_j \bar{P}_{i+1} + D_j = 0$$

(9.15)

where the coefficients are not the same as those in the energy equation. It is noted also that the momentum equation is linearised by the use of a predicted velocity in the coefficient of the axial outflow term. Only the transverse diffusion terms and the axial convective terms are used in the coefficients of the unknown quantities to ensure stability in the recursive solution.

The continuity equation is defined as:

$$\dot{m}_{r,j-1/2} = \dot{m}_{r,j+1/2} + (\bar{\rho}_{i+1,j} \bar{u}_{i+1,j} - \bar{\rho}_{i,j} \bar{u}_{i,j})(\bar{r}_{j+1/2}^2 - \bar{r}_{j-1/2}^2) \quad (9.16)$$

and the integral continuity equation is:

$$\dot{m} = \sum_1^{N+1} \bar{\rho}_{i+1,j} [\bar{u}_{i+1,j}] (\bar{r}_{j+1/2}^2 - \bar{r}_{j-1/2}^2) = 1 \quad (9.17)$$

Boundary conditions are either formed by the appropriate balance in the same manner as the equations for the interior control volumes, or the unknown will be specified directly.

The governing equations are solved sequentially at each axial step and are iterated until the solutions converge. From the predicted values of the variables, effective viscosity and thermal conductivity values are

calculated at the control surfaces using a given turbulence model. The energy equation is solved for $\bar{H}(\bar{r})$ by applying the recurrence relations presented by Richtmyer /77/. The x - momentum and the integral continuity equation are solved simultaneously for $\bar{U}(\bar{r})$ and $\bar{P}$. For the integral continuity equation $\bar{\rho}(\bar{r})$ is calculated from $\bar{H}(\bar{r})$ found with the energy equation. Finally the continuity equation is solved from the wall to the tube centre to obtain $\bar{\rho}\bar{V}(\bar{r})$; the most recent values of $\bar{\rho}(\bar{r})$ and $\bar{U}(\bar{r})$ are used. Predicted values of the variables, necessary in the coefficients are revised at each iteration. Once converged the results for step $i+1$ are taken for the first set of predicted values for the next step. The energy balance and the rate of change of pressure gradient are used as convergence tests during iteration at each step; in addition a maximum number of iterations is specified.

Correct selection of grid parameters appears to be a matter of experience and compromise; no general rules have been formulated for their selection. A balance must be drawn between computer requirements of storage and execution time and the solution accuracy; some indication of useful starting values is given by Bankston and McEligot /62/.

9.5 Additions to the Solution Procedure.

The numerical solution method outlined above was implemented by Bankston and McEligot in a computer program written using the FORTRAN IV language for execution on a C.D.C. 6600 computer. Adaptation of this method consisted in the main of the provision or re-writing of peripheral routines which input values to the general solution procedure.

These included interpolation and curve fitting routines together with a number of turbulence models; in addition several redundant segments were removed from the program. A major change was the inclusion of routines providing average suspension properties when the boundary layer equations are to be solved for a pseudofluid idealisation; single component flow was made a subset of the general solution technique, having a zero volume fraction solid. The changes necessary to utilise the homogeneous flow concept and the selection of property correlations will be covered in subsequent chapters.

Apart from minor programming modifications the solution procedure was changed in only one respect, extending it in allowing for the specification of a wall temperature boundary condition in addition to a wall heat flux specification, since this would allow easier comparison with experimental results as well as being more flexible.

Specification of a wall temperature simplifies the solution procedure in that it removes the necessity for forming an energy balance for the control volume adjacent to the wall. The given value may be substituted directly into the recurrence relationship and the enthalpy field at the axial station, obtained directly by successive substitution starting at the wall. Values of wall heat flux and bulk enthalpy must now be calculated using this condition. To evaluate the wall heat flux the thermal gradient at the wall is required; methods by which this may be approximated include:

1. assuming that the thermal gradient is given by the average gradient over the half-shell adjacent to the wall.

2. using a divided difference technique incorporating as many nodes from the wall as desired.
3. fitting a quadratic through the wall and two adjacent nodal temperatures.

Whilst the second method is most accurate it introduces unnecessary complications and the first method is an oversimplification. Using the third condition:

$$\frac{d\bar{T}}{d\bar{r}} = \frac{\bar{y}_{n-1}^2 (\bar{T}_{i+1,n+1} - \bar{T}_{i+1,n}) - \bar{y}_n^2 (\bar{T}_{i+1,n+1} - \bar{T}_{i+1,n-1})}{\bar{y}_n \bar{y}_{n-1} (\bar{y}_n - \bar{y}_{n-1})} \quad (9.18)$$

when the wall heat flux will be given by:

$$q_{w_{i+1}}^+ = \frac{2 \bar{K}_{i+1,n+1}}{Re_o Pr_o} \left[\frac{d\bar{T}}{d\bar{r}} \right]_{w_{i+1}} \quad (9.19)$$

If the wall boundary condition is a specified heat flux then the fluid bulk enthalpy may be simply obtained by considering the energy added at the wall by integrating over the boundary surface for the axial step. With a specified wall temperature the average heat flux over the step length could be assumed to be the mean of the heat fluxes at the two boundary axial stations but a more accurate method of obtaining

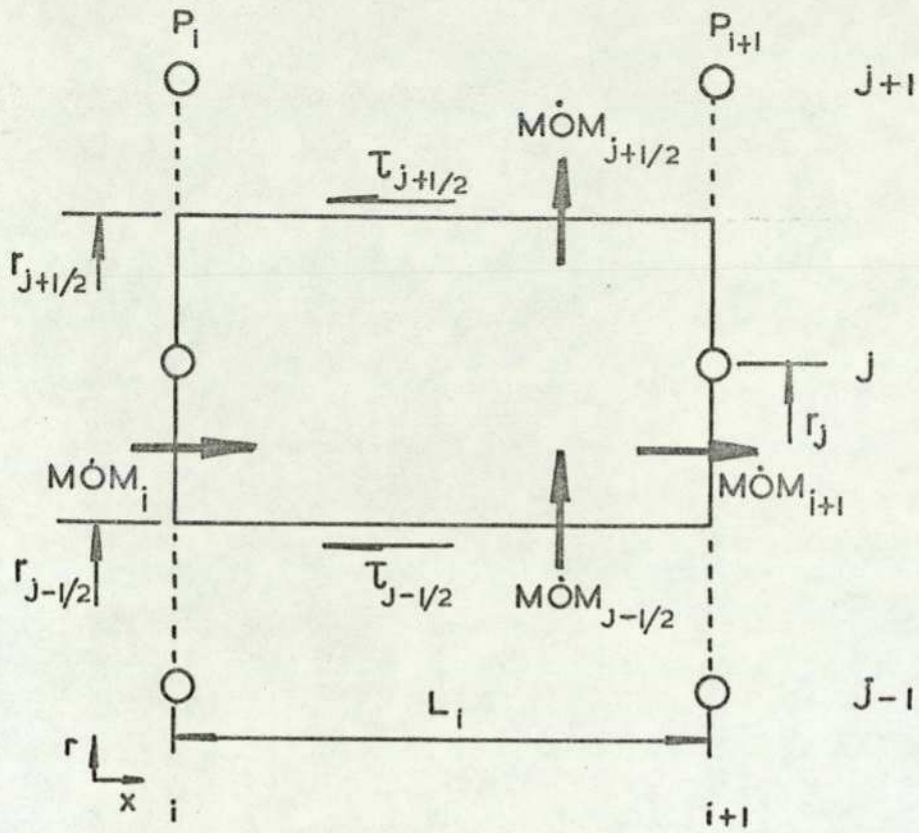
the bulk enthalpy is to use the definition of 'mixing cup' properties;
hence:

$$\bar{H}_b = \frac{\int_0^1 \bar{\rho} \bar{U} \bar{H} (2\pi \bar{r}) d\bar{r}}{\int_0^1 \bar{\rho} \bar{U} (2\pi \bar{r}) d\bar{r}} \quad (9.20)$$

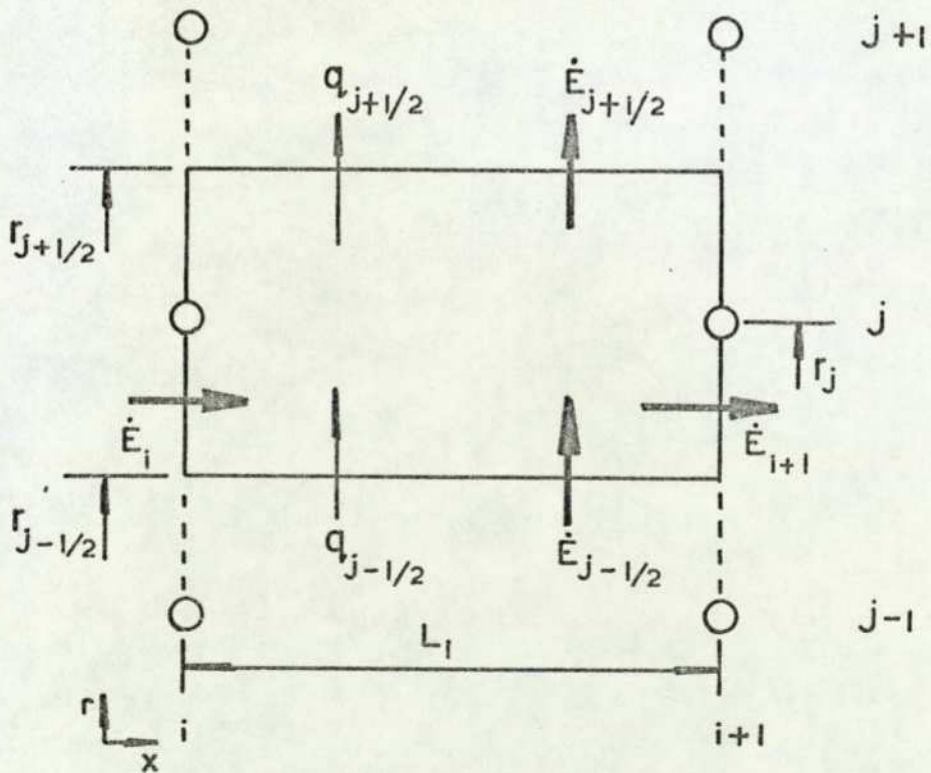
which becomes:

$$\bar{H}_{b_{i+1}} = \frac{\sum_{j=1}^N \dot{m}_j \bar{H}_{i+1,j}}{\dot{m}} \quad (9.21)$$

It is noted that the use of this boundary condition removes one of the tests for convergence, namely the energy balance, so that the pressure criterion becomes controlling. In practice this was always satisfactory and could be made more stringent if required.



ENERGY EQUATION



MACROSCOPIC CONTROL VOLUMES FOR DERIVATION OF FINITE DIFFERENCE EQUATIONS

Figure 9.1

10 SOLUTION EVALUATION WITH SINGLE-COMPONENT FLUIDS

10.1 Introduction.

Before the solution procedure outlined above was applied to the case of heated flow of a solid-liquid suspension it was necessary, firstly, to develop the required auxiliary relationships, in particular a turbulence model and, secondly, to test the method against established relationships and experimental evidence for single-component flows. Having established the validity of the technique for the isothermal turbulent flow of a liquid, consideration could then be given to its use in predicting heat transfer performance of both single and two-component systems with, in the latter case, any auxiliary relationships suitably modified for use with pseudofluid properties.

An experimental test circuit was constructed for use in the evaluation of the homogeneous flow model and although not ultimately used for this purpose it was used in developing relationships and assessing the method performance for the flow of water.

10.2 The Experimental Circuit.

Since the experimental circuit was not used for heat transfer tests its details are only briefly described. In design the apparatus was similar to that used in many other heat transfer studies /36/, /37/, /46/ being a recirculating system with heating and cooling sections to enable steady state measurements to be made. An outline diagram is shown as figure 10.1.

The material used for the circuit was drawn copper tube of one inch nominal bore and 20 s.w.g. The working fluid was passed through the system

by a Safran single stage centrifugal pump with a rated water head of 0/50 m at a delivery rate of 600/230 litres per min. Suction was from a reservoir tank which was fitted with a paddle stirrer and baffles in order to maintain sufficient agitation to ensure a uniform suspension. The pump discharge was controlled by a bypass line returning to the reservoir and a butterfly valve in the main discharge line. A metric series Rotameter type 65Q was fitted to enable the flow rate of pure liquids to be determined and an estimate made of the magnitude of the flow of a suspension. The heated section was preceded by a calming length of 100 diameters at the entrance to which was a fluid mixing chamber designed to mix the fluid thoroughly so that its bulk temperature could be measured by a thermocouple incorporated therein. A heat transfer section of 120 tube diameters was constructed by enclosing the test circuit within a steel pipe of $3\frac{1}{2}$ inches diameter to form a steam heating chamber. Saturated steam at a supply pressure of 13 bar was admitted to the heater through a needle valve, providing control over the quantity admitted by throttling. A throttling calorimeter was fitted to the steam supply line. Condensate passed from the heater through a float controlled steam trap, which incorporated a thermocouple, a multi-tube cooler to prevent 'flashing' into steam and then discharged either to drain or to a weigh tank.

From the heating section the fluid again entered a mixing chamber for bulk temperature determination. The fluid was then cooled in a single pass multi-tube heat exchanger with water used as the cooling medium and was then returned to the reservoir or, for flow measurement, diverted to a weigh tank. Glass fibre insulation was used on the heated

section and calming length to cut down heat loss.

The wall temperature of the test circuit within the heated section was determined by means of thermocouples located at intervals along the outer surface and with the copper tube itself forming one side of the thermocouple junction. Two closely spaced slots were made circumferentially round the tube at each measuring station and a constantan wire of 26 s.w.g. was wound into one of these; the ridge between the slots was peened over to secure the wire and the slots filled with high temperature solder to the original tube surface. The constantan leads were protected by silicon tubing and led out of the steam space through a gland. A copper wire affixed to the test circuit outside the heater completed the high temperature side of the thermocouple circuit. All the constantan leads of the thermocouples in the circuit were led to a Cropico SP6 selector switch, to enable each reading to be obtained in reference to a single 'melting ice' cold junction. A solartron IM 1420.2 digital voltmeter was used to measure the thermocouple e.m.f.

At each end of the heated section was fitted a pitot tube traverse for velocity profile measurement. Details of the pitot, which incorporated a wall static pressure tapping, are shown in figure 10.2. A micrometer traverse enabled the pitot radial location to be accurately determined whilst the fluid dynamic pressure was measured using a Honeywell 1028 diaphragm type inductive differential pressure transducer of range 0 to 35 KN/m^2 . The transducer output was amplified by a carrier amplifier and the reading obtained on a Solartron digital voltmeter. A linear relationship between pressure differential and transducer output allowed the instrument calibration to be expressed by a simple regression

line obtained by the conventional analysis. Pressure loss across the test length was measured using a mercury manometer connected between the static pressure tappings.

Velocity profiles were measured for the flow of water and for the dilute flow of metallic powder suspensions. For the latter case it was found only possible to consider very dilute flows ($\phi < 0.03$) before problems of tube blockage became pronounced, in spite of the incorporation of sediment traps and the adoption of backflushing. As will be discussed later, no significant difference between these velocity profiles and those for water could be determined under these conditions.

As stated, it was intended to perform heat transfer tests for the evaluation of the proposed homogeneous flow model using this equipment. For this purpose it was necessary to have accurate readings of wall temperature in the heated section but upon commencing tests, it was found that the measured temperature at each axial station fluctuated rapidly by amounts of up to 30°C and that mean values between adjacent stations could vary by up to the same amount. This was especially noticeable at those thermocouple locations removed from the steam entry point. No faults could be found in the thermocouple circuits or construction, so it was concluded that condensate either built up to such a level, in the steam space, that it covered the lower surface of the tube, or, being agitated by the passage of steam, formed waves which alternately covered and uncovered the tube. It was noted that for those thermocouples which recorded the lower mean temperatures the constantan wire encountered the copper tube initially on the lower tube surface and that the thermocouples that provided the steadiest readings were on the upper surface.

It was clear that sufficient space had not been provided for the condensate and although a modification was made within the limitations of the existing equipment, a significant improvement could not be effected without substantial alteration. Limitations of time and manpower necessitated the abandonment of these heat transfer tests.

10.3 Solution for the Isothermal Flow of Water.

Velocity profiles were measured for the fully developed flow of water through the test circuit at Reynolds numbers from 7.9×10^4 to 2.1×10^5 . These profiles were first compared with the conventional logarithmic velocity profile relationship due to Nikuradse:

$$u^+ = 2.5 \ln y^+ + 5.5 \quad (10.1)$$

and were found to be substantially in agreement as shown in figure 10.3, having regard to the report of Brinkworth and Smith /66/ that this expression predicts core velocities up to 4% lower than those actually measured.

The boundary layer equations were now solved for the isothermal flow of water under the test conditions for purposes of comparison. Fully developed flow at entry to the test section was simulated by solving for an entry length prior to the actual solution under consideration. An entry length of 45 diameters was found to be adequate both for uniform velocity or approximate fully developed initial conditions. The required property relationships were provided by the input of tabular data for water from those given by Mayhew and Rogers /67/. Property values at

inlet to the test section were determined using a Lagrange interpolation routine and the tabular values were normalised by division by the inlet value and then expressed as a polynomial function of temperature for use with the solution procedure. Only for heated flow, obviously, would they be required.

At every axial station in the solution the radial velocity gradient is required across the section for use with an eddy diffusivity model of the mixing length type. The profile obtained from the solution was differentiated using a cubic spline function and the required end conditions of specified slope were obtained using a divided difference routine based on as many nodal points as desired; in general four were found to give satisfactory results.

The mixing length model used initially was that recommended by Bankston and McEligot /62/ and was the Van Driest /68/ modification of the original Prandtl mixing length assumption which provides for a damping of the turbulent fluctuations near to a wall. The mixing length is given by:

$$l = K^* y (1.0 - \exp[-y^*/A_*]) \quad (10.2)$$

where K^* is the Von Karman constant 0.4 and A_* is usually taken as a constant with value typically 26.0. Further from the wall the mixing length approaches Prandtl's assumption of linear variation with distance.

The suggestion of Patankar and Spalding /69/ was adopted in using the local value of shear stress instead of the wall value in evaluating the dimensionless distance coefficient in the exponential term since local damping is more likely to be affected by this. Hence:

$$y^+ = \frac{y}{\mu} \sqrt{\tau(y) \rho} \quad (10.3)$$

Using this turbulence model the solution procedure produced an accurate estimate for experimental friction factor but when tested against the criterion of velocity profile was found to be in error by a considerable amount as the example in figure 10.6 shows.

The mixing length model recommended by Escudier /70/ and again used by Patankar and Spalding /69/ was considered; this gives:

$$l = \kappa^* y : 0 < y < \lambda y_1 / \kappa^* \quad (10.4a)$$

$$l = \lambda y_1 : \lambda y_1 / \kappa^* < y \quad (10.4b)$$

where y_1 is a representative distance and λ lies between 0.07 and 0.12. The value of K^* , traditionally 0.4, is given as a range from 0.35 to 0.6. A modification was made in using the Van Driest damping factor in place of the simple linear relationship of equation (10.4a). In spite of an improvement in prediction of experimental velocity profiles, as shown in figure 10.6, it was not found possible to vary the model constants to give a good agreement.

It was felt necessary to obtain agreement of experiment and prediction for the two criteria of pressure loss and velocity profile in isothermal flow of a single-component fluid before considering the heat transfer problem. Schlichting /71/ has noted that since the simple assumption of a linear relation for mixing length does not seem to be suitable for the whole pipe diameter it appears preferable to deduce the dependence of mixing length on distance directly from experiment. This procedure was now followed.

10.4 Experimental Determination of Mixing Length Distribution.

For fully developed pipe flow, inertial terms being zero, the radial shear stress distribution is linear:

$$\frac{\tau}{\tau_w} = \left[1 - \frac{y}{r_w} \right]$$

(10.5)

and by definition:

$$\tau = \left[\mu + \rho l^2 \left| \frac{du}{dy} \right| \right] \frac{du}{dy} \quad (10.6)$$

Combining equations (10.5) and (10.6) we obtain an expression for the variation in mixing length:

$$l = \left[\frac{\tau_w (1 - \bar{y})}{\rho \left[\frac{du}{dy} \right]^2} - \frac{v}{\frac{du}{dy}} \right]^{1/2} \quad (10.7)$$

The mixing length distribution may therefore be determined from the measured velocity distribution in isothermal flow and the pressure drop over the test length, from which the apparent shear stress at the wall is obtained by a simple momentum balance:

$$\tau_w = \frac{\Delta P r_w}{2 L} \quad (10.8)$$

Differentiation of the experimental velocity profile could not be accomplished with the cubic spline routine used in the computer solution since this constrained the curve to pass through the given set of tabular co-ordinates and, since these were subject to experimental error, could result in considerable inaccuracies in the estimation of radial velocity gradients. The method adopted was to fit a curve of the form:

$$\bar{U} = [C_1 + C_3(1 - \bar{y})^2](2\bar{y} - \bar{y}^2)^{C_2} \quad (10.9)$$

to the set of experimental readings $(\bar{y}_i, \bar{U}_i)$ for each velocity profile. A generalised Newtons method was used to find the coefficients $\underline{C}$ such that:

$$\sum_{i=1}^N [f(\underline{C}_i, \bar{y}_i) - \bar{U}_i] \quad \text{is a minimum.}$$

The procedure was iterative and the significance of the determined coefficients was ascertained by consideration of their standard error estimates. In all cases this was less than one-tenth of the value of the coefficient, sample curves are shown in figure 10.14. The velocity gradient may now be expressed as:

$$\frac{\partial U}{\partial y} = \left[C_2(C_1 + C_3(1 - \bar{y})^2)(2 - 2\bar{y})(2\bar{y} - \bar{y})^{C_2-1} - 2C_3(1 - \bar{y})(2\bar{y} - \bar{y}^2)^{C_2} \right] \frac{U_{b,o}}{r_w} \quad (10.10)$$

and used with equations (10.7) and (10.8) to determine the mixing length distribution for each experimental profile. The relation of mixing length to radial position was expressed as a polynomial for each experimental profile; the coefficients being evaluated using the same generalised Newton's method and with the wall boundary condition of zero mixing length constraining the equation by forcing a zero constant term. It was found that a third order equation would adequately represent the data, indeed a fourth order expression proved less successful. As the coefficients of each equation obtained appeared to vary only between limits that could be attributed to experimental error and as there did not appear to be any effect due to the flow Reynolds number over the experimental range, one universal distribution equation was obtained by taking the mean of the coefficients. This gave:

$$\bar{l} = 0.416\bar{y} - 0.799\bar{y}^2 + 0.654\bar{y}^3 \quad (10.11)$$

This equation and typical experimental points are shown in figure (10.4), while table 10.1 enumerates the maximum and standard deviations for each coefficient and gives confidence limits for their mean values. The agreement of this expression with the Prandtl assumption of $\bar{l} = 0.4\bar{y}$ in the wall region is shown in figure 10.5.

When this expression is used to give the effective viscosity values for use in the boundary layer solution for the isothermal flow of water it produces velocity profile predictions in good agreement with the experimentally measured profiles. This was hardly surprising, since the

mixing length distribution was derived from those same profiles but it does confirm the validity of the basic equations and the solution technique. It was noted, however, that the pressure drop characteristics were not accurately predicted and this was considered to be due to the failure of the distribution to take account of the effect of Kinematic viscosity on the turbulence near the wall. Consequently the Van Driest damping factor was incorporated into the mixing length correlation to give:

$$\bar{l} = (1 - \exp[-y^+/26.0])(0.416\bar{y} - 0.799\bar{y}^2 + 0.654\bar{y}^3) \quad (10.12)$$

where the local shear stress is used in y^+ .

Comparison of the experimental and predicted velocity profiles obtained using this distribution are shown in figures (10.6) and (10.7).

Also shown on figures (10.4) and (10.5) are the experimental mixing length distributions due to Nikuradse /72/ who gave:

$$\bar{l} = 0.4\bar{y} - 0.44\bar{y}^2 + 0.24\bar{y}^3 - 0.06\bar{y}^4 \quad (10.13)$$

and that obtained by Maise and McDonald /73/:

$$\bar{y} < 0.1 : \bar{l} = 0.41(1 - \exp[-y^+/26.0])\bar{y} \quad (10.14a)$$

$$\begin{aligned} 0.1 \leq \bar{y} \leq 0.6 : \bar{l} = & 0.41(1 - \exp[-y^+/26.0])\bar{y} - 1.53506(\bar{y} - 0.1)^2 \\ & + 2.75625(\bar{y} - 0.1)^3 - 1.88425(\bar{y} - 0.1)^4 \end{aligned} \quad (10.14b)$$

$$0.6 \leq \bar{y} : \bar{l} = 0.089 \quad (10.14c)$$

Agreement between the various distributions is good in the wall region but considerable differences occur in the central part of the duct. In fact, the effect of such variation on the velocity distribution in the region $\bar{y} > 0.6$ has been considered to be slight /74/, whilst Nikuradse has stated that in this region the velocity gradients produced from the velocity measurements were too small to permit much reliance to be placed on the calculated values. Although the experimental mixing length distribution produced in this study was derived for use with the heat transfer circuit measurements, it was subsequently found to be at least as satisfactory for other flow situations as the other distributions and consequently it was retained as a feature of the solution procedure.

10.5 Validity of Mixing Length Distribution Outside Experimental Range.

The mixing length distribution given by equation (10.12) was derived from experimental evidence in the Reynolds number range of 7.9×10^4 to 2.1×10^5 . Later consideration of flow outside this range led to the requirement of evaluating the validity of extending its use to these situations. This was done by solving the boundary layer equations for isothermal single-component flows outside the experimental range of Reynolds number and comparing the velocity profile and friction factor predictions with established values. Velocity profiles for the flows having the lowest Reynolds number considered are shown in figure (10.8) compared with the standard power law profiles of Nikuradse. A test at a very high Reynolds number is shown in figure (10.9) with again a power law profile for comparison. Velocity profiles in the wall region are shown in figures (10.10) to (10.12) compared with the universal velocity profile curves for the laminar sublayer, buffer zone and turbulent core.

Friction factor predictions over a wide range of Reynolds number from 10^4 to 10^6 are given in figure (10.13) together with the curve obtained by Nikuradse:

$$\frac{1}{\sqrt{4f}} = 0.86 \ln Re \sqrt{4f} - 0.8 \quad (10.15)$$

These results show that the turbulence model derived in this study leads to acceptable results when used in a numerical solution for Reynolds numbers outside the experimental range. There is, however, some indication of a fall in the accuracy of prediction at lower Reynolds numbers which would tend to support the conclusion of Nikuradse, who found that his mixing length correlation was independent of Reynolds number effects for values greater than 10^5 .

It is considered that it is justifiable to utilise the given equation in heat transfer studies for turbulent flows over the range of Reynolds number 10^4 to 10^6 .

It remains only to define the relationship of the eddy diffusivity for momentum to that for heat to complete the relationships for single-component fluid flow. Considerable experimental uncertainty and conflicting evidence exists on this point with, in general, the ratio of diffusivities being taken as a constant value between 0.8 and 1.0. In calculation procedures the ratio is often taken as unity since in the light of uncertain evidence the simplest case has that one advantage;

since satisfactory results have been claimed using this ratio it was adopted in the numerical solution of this study. Confirmation that this was in fact substantially correct has been supplied by Brinkworth and Smith /75/ who determined experimentally the ratio of diffusivities for the turbulent flow of water at a Reynolds number of 10^5 and gave the value as 0.977 throughout the core region. Hughmark /76/ has taken the velocity profile model of the previous authors and reconsidered the experimental data of other workers to show that their results also indicate equal diffusivities of momentum and heat for turbulent pipe flow of liquids of moderate molecular Prandtl number. The assumption of a turbulent Prandtl number of unity for water and water based suspensions in this study would seem therefore to be appropriate.

Table 10.1

Analysis of Coefficients of Experimental Mixing Length

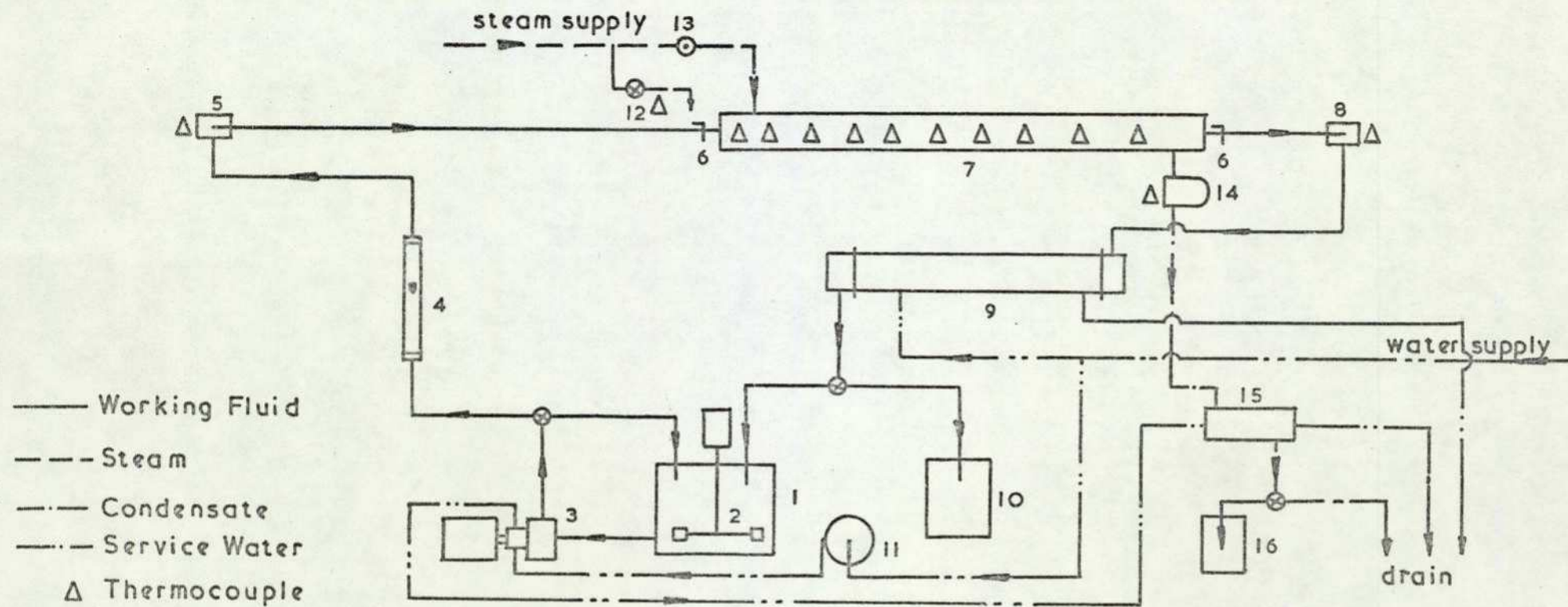
Correlation.

Correlation: $\bar{I} = c_1 \bar{y} + c_2 \bar{y}^2 + c_3 \bar{y}^3$

Coefficient C_1 :	Value	=	0.416
	Maximum Deviations.	=	+ 6.0%, -3.37%
	Standard Deviation.	=	0.013
	Limits at 95% Confidence Level.	=	0.402, 0.430

Coefficient C_2 :	Value	=	-0.799
	Maximum Deviations	=	+ 9.89%, -4.88%
	Standard Deviation	=	0.043
	Limits at 95% Confidence Level	=	0.848, 0.750.

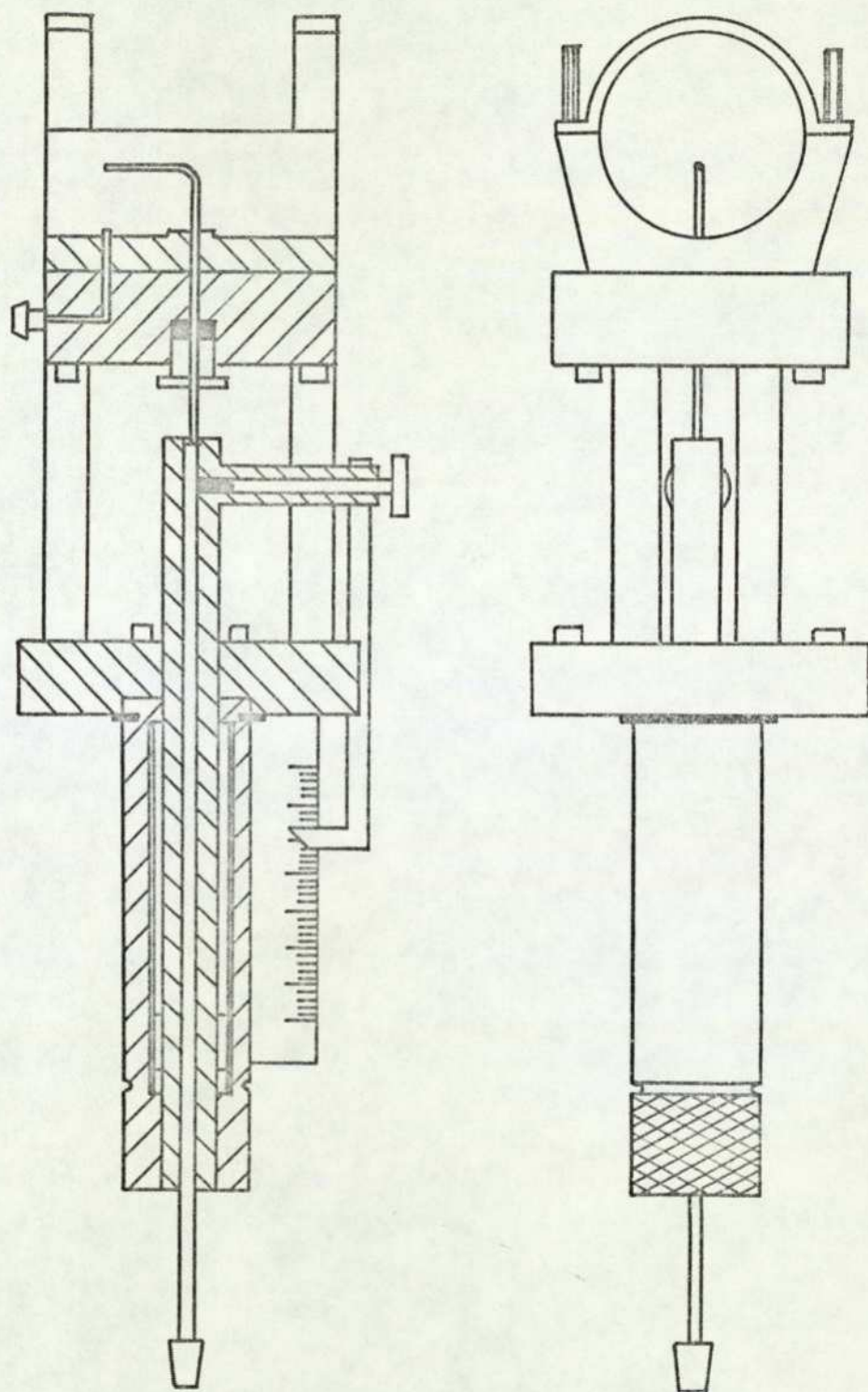
Coefficient C_3 :	Value	=	0.654
	Maximum Deviations =	=	+8.1%, -4.28%
	Standard Deviation	=	0.028
	Limits at 95% Confidence Level	=	0.686, 0.622



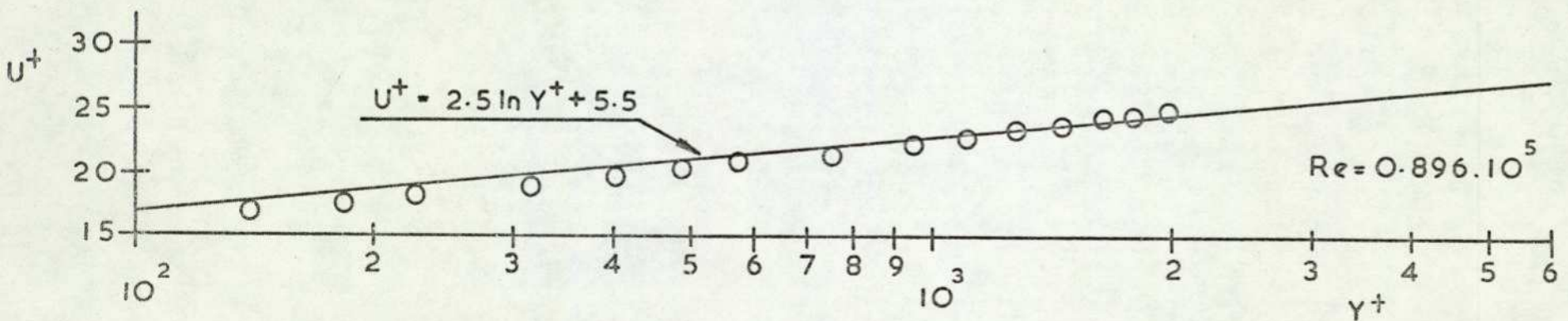
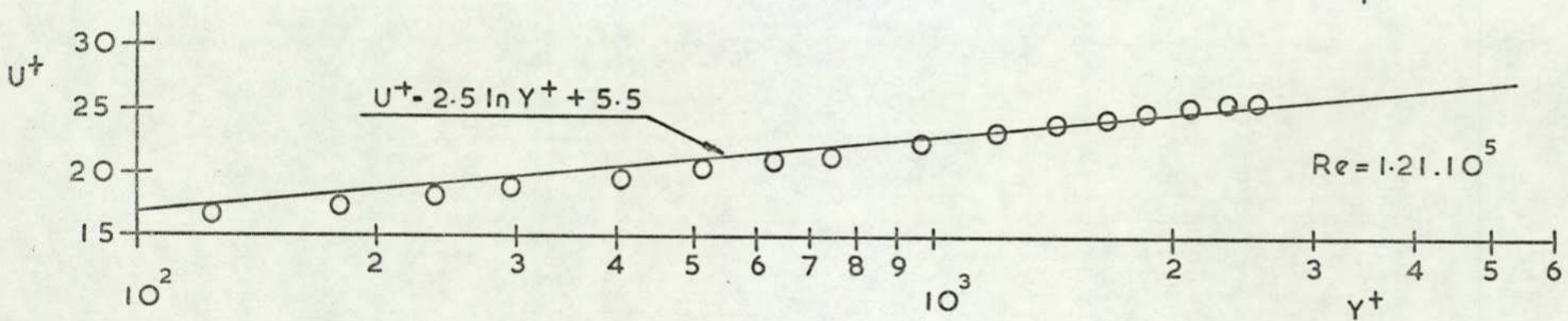
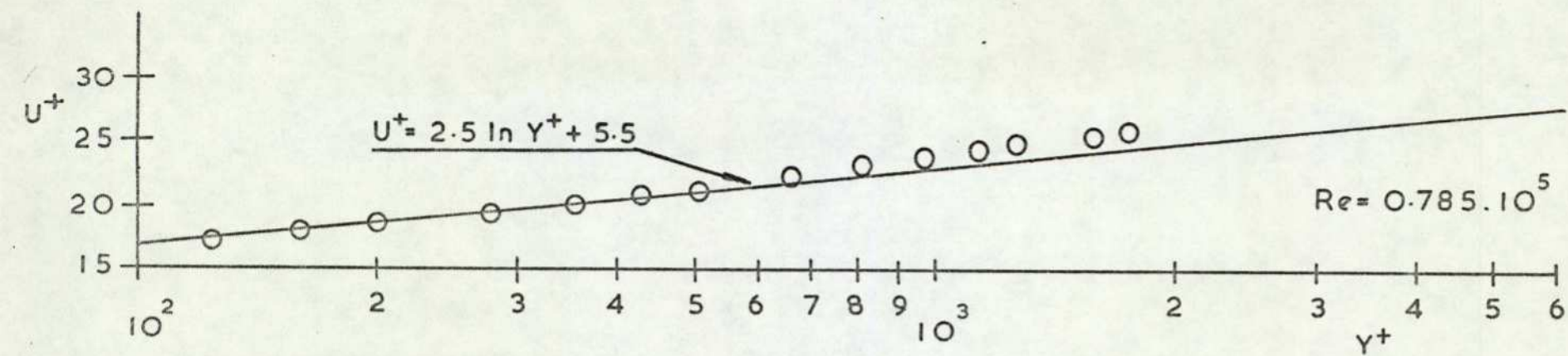
1 Reservoir	2 Agitator	3 Suspension Pump	4 Rotameter	5 Fluid Mixer - Inlet Bulk Temp.
6 Pitot Static Probe	7 Heater	8 Fluid Mixer - Outlet Bulk Temp.	9 Cooler	10 Weigh Tank
11 Gland Sealing Pump	12 Throttling Calorimeter	13 Needle Valve	14 Steam Trap	
15 Condensate Cooler	16 Condensate Weigh Tank			

OUTLINE DIAGRAM OF EXPERIMENTAL CIRCUIT

Figure 10-1

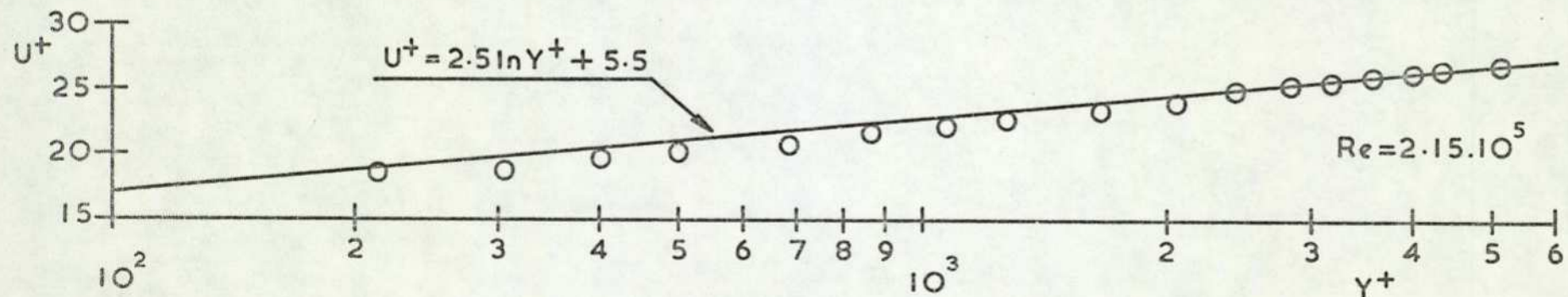
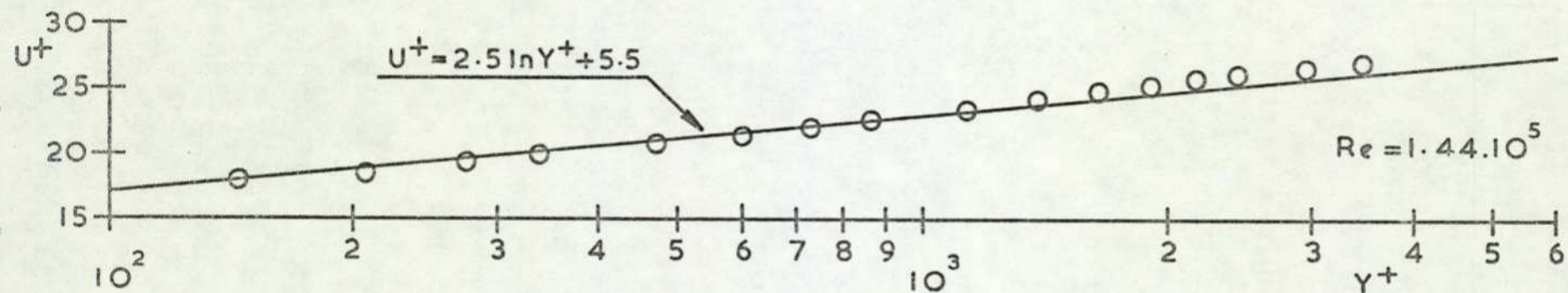
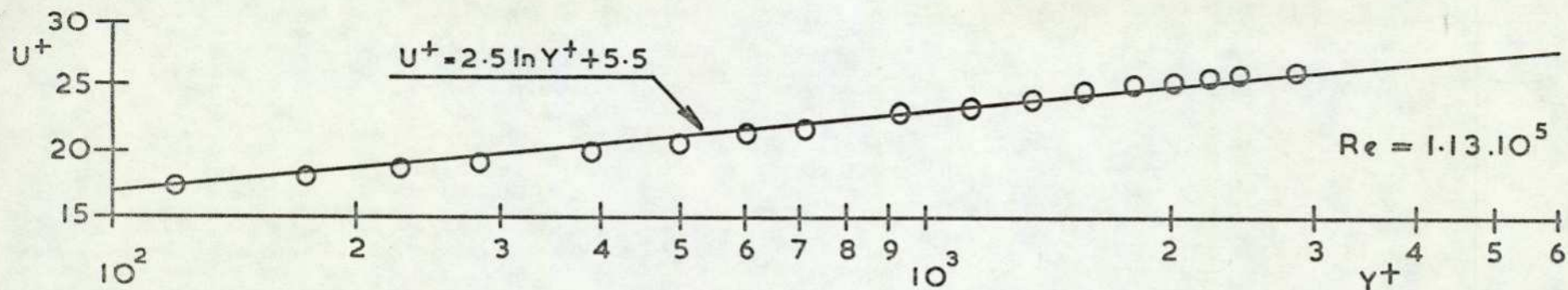


PITOT-STATIC PROBE IN EXPERIMENTAL CIRCUIT



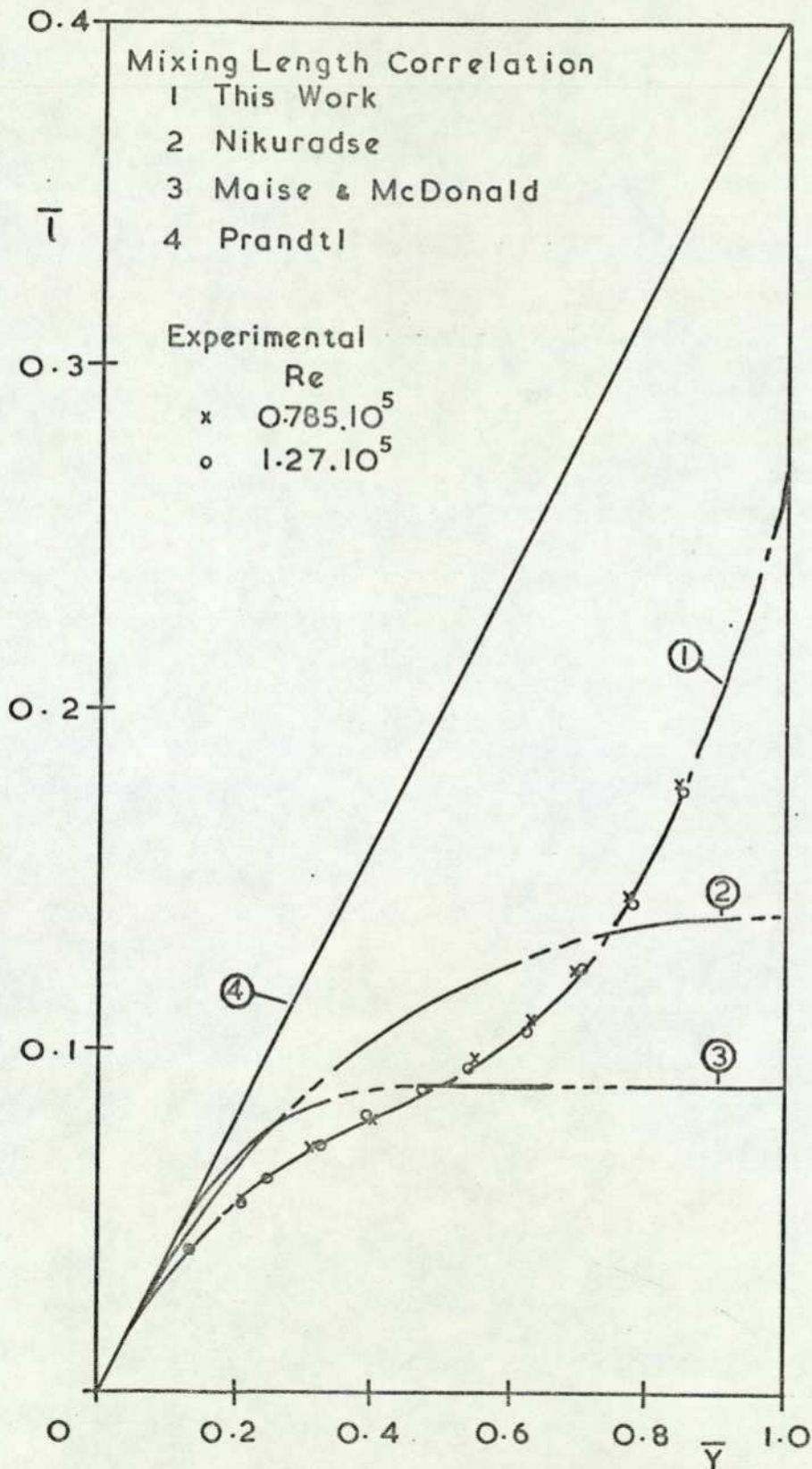
NON DIMENSIONALISED EXPERIMENTAL VELOCITY PROFILES

Figure 10.3



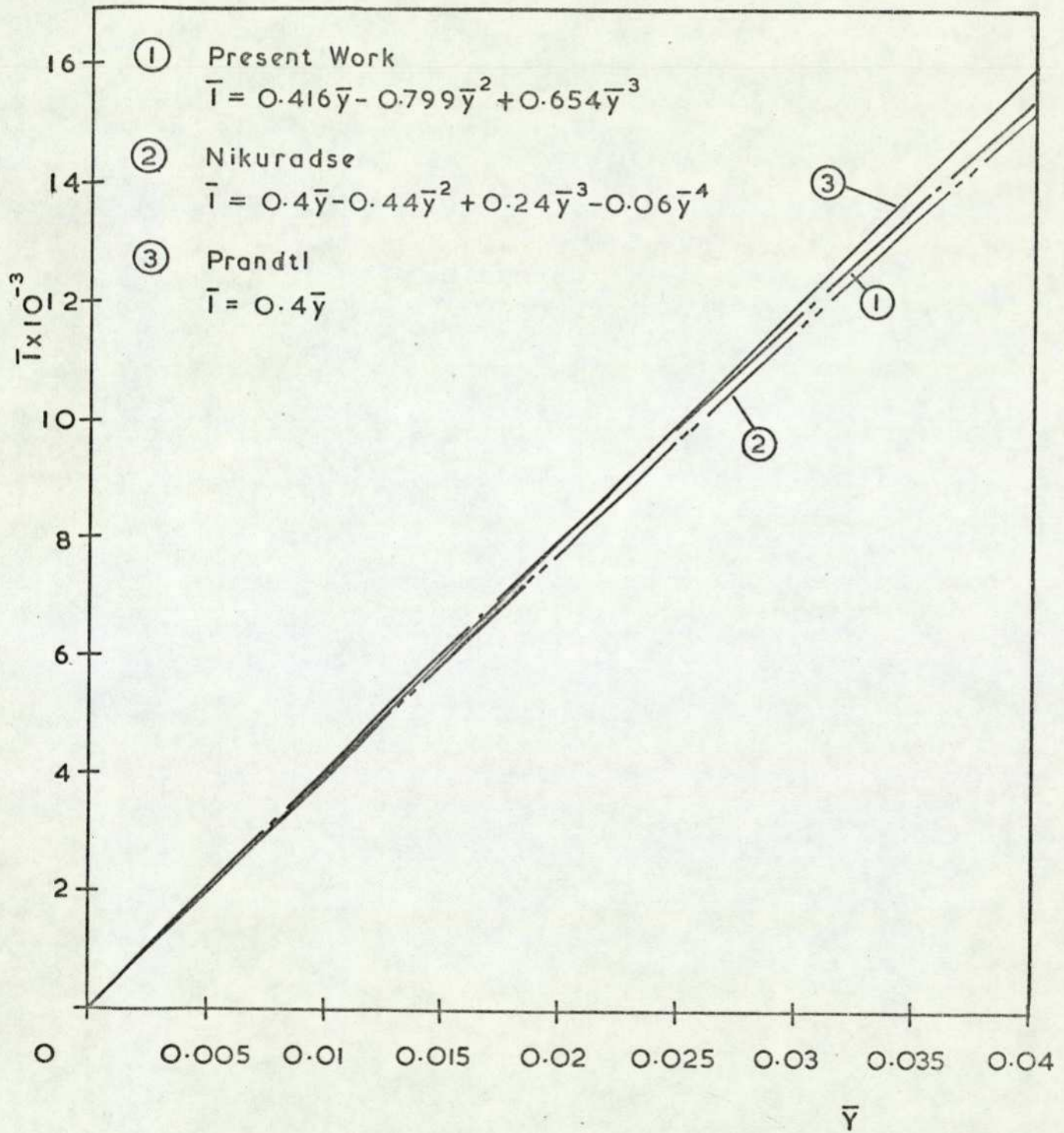
NON-DIMENSIONALISED EXPERIMENTAL VELOCITY PROFILES

Figure 10.3 Cont.



COMPARISON OF MIXING LENGTH CORRELATIONS

Figure 10.4



MIXING LENGTH PREDICTIONS : WALL REGION

Figure 10.5

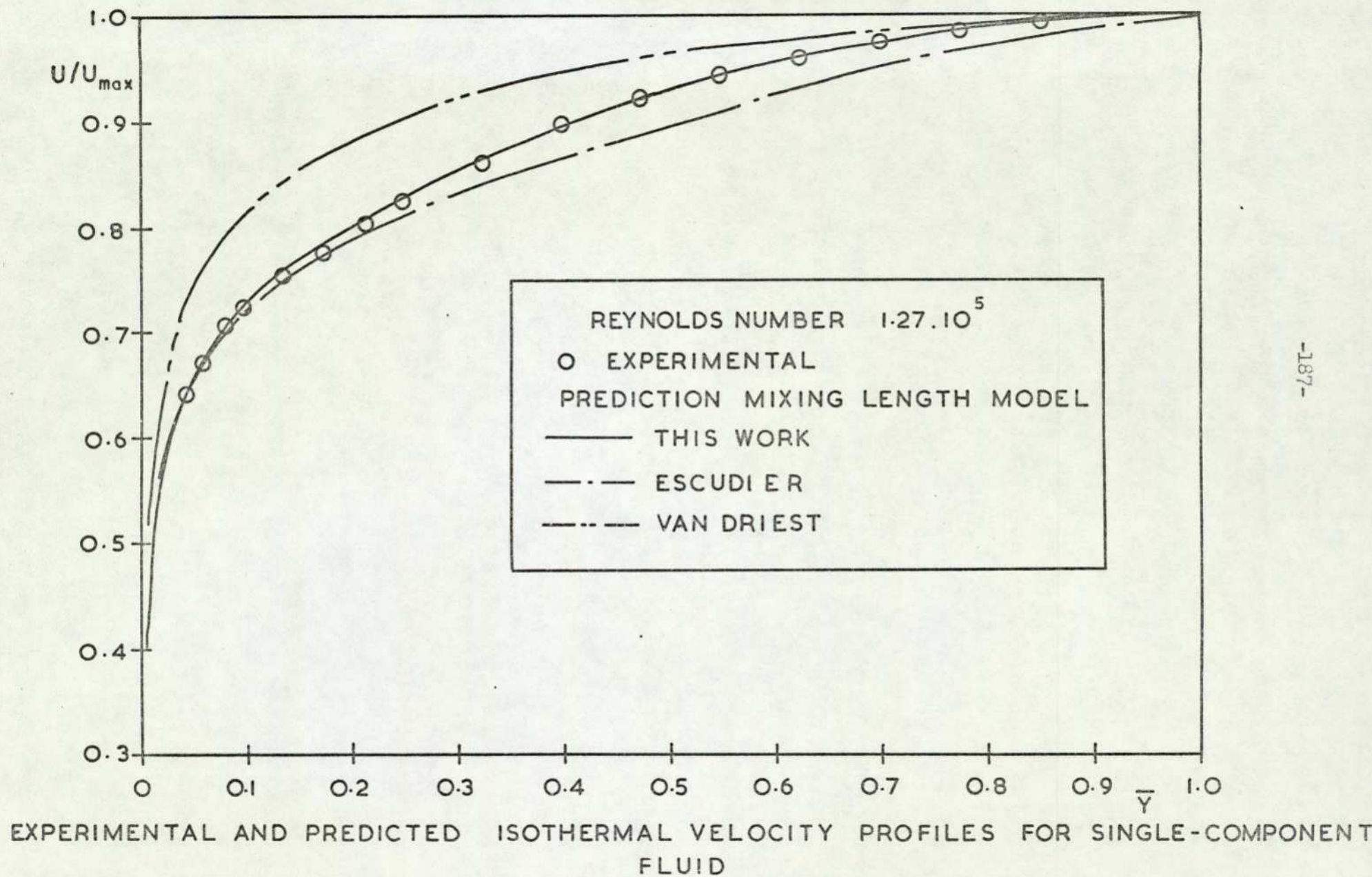


Figure 10.6

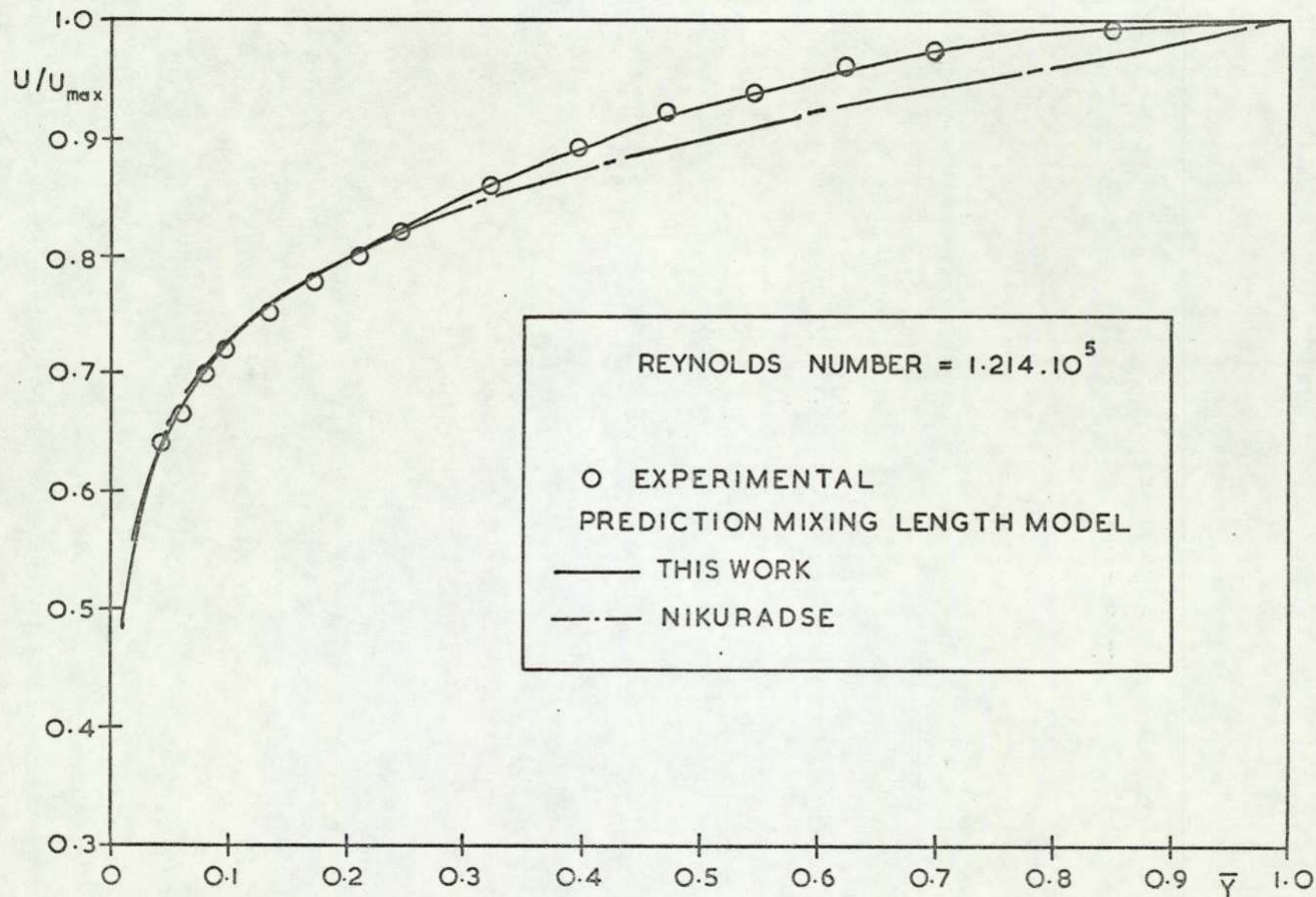
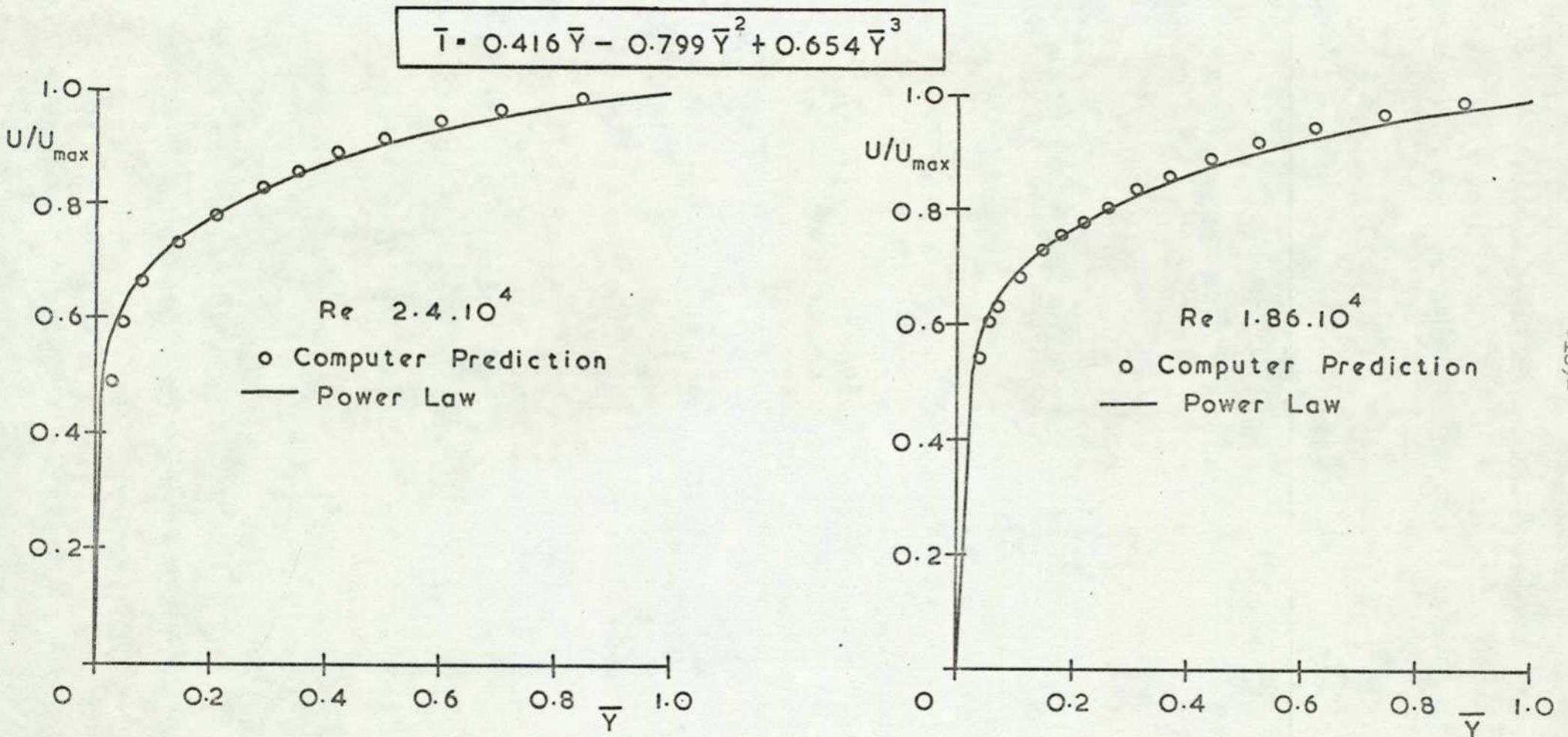
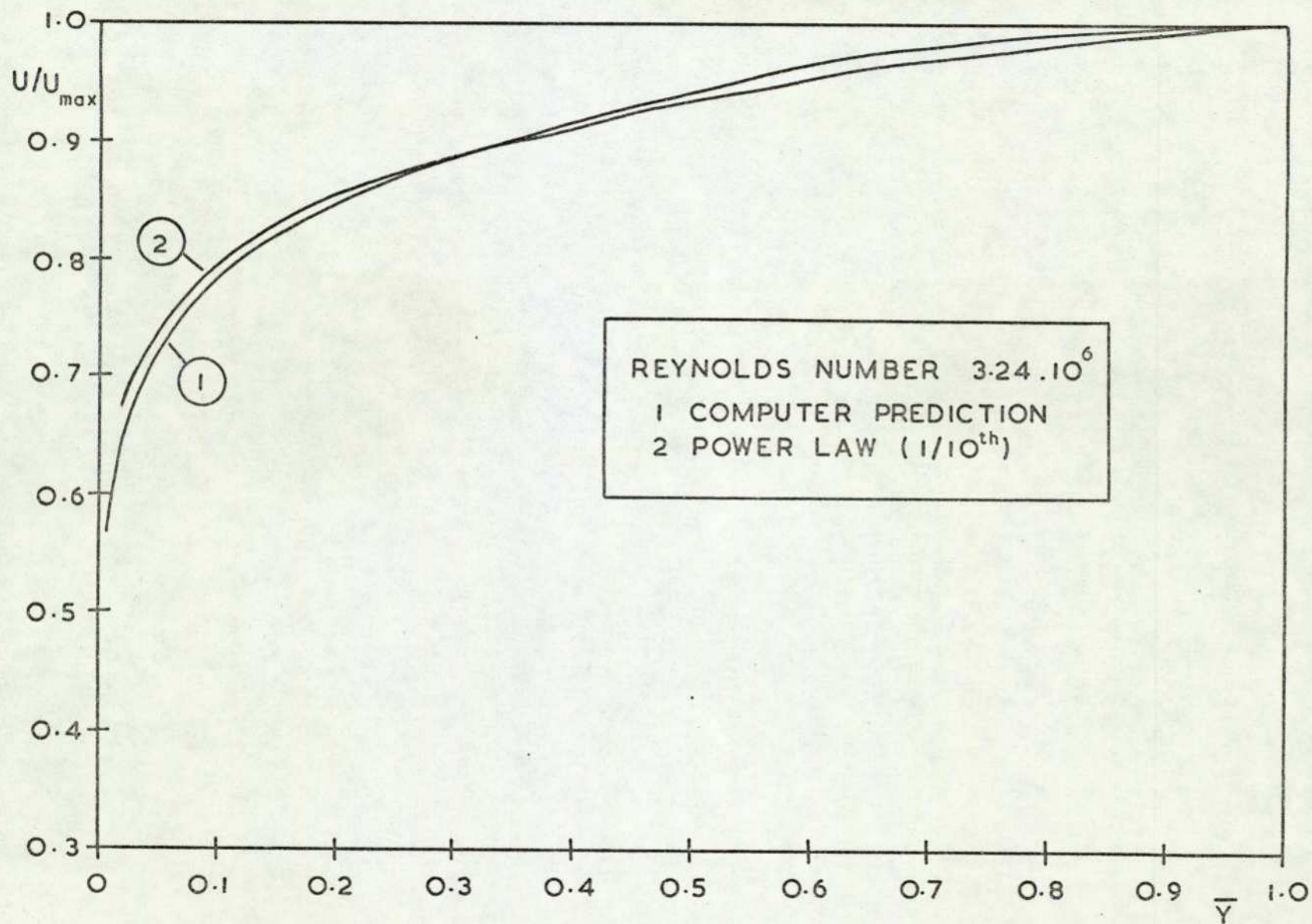


Figure 10-7 EXPERIMENTAL AND PREDICTED ISOTHERMAL VELOCITY PROFILES FOR SINGLE-COMPONENT FLUID



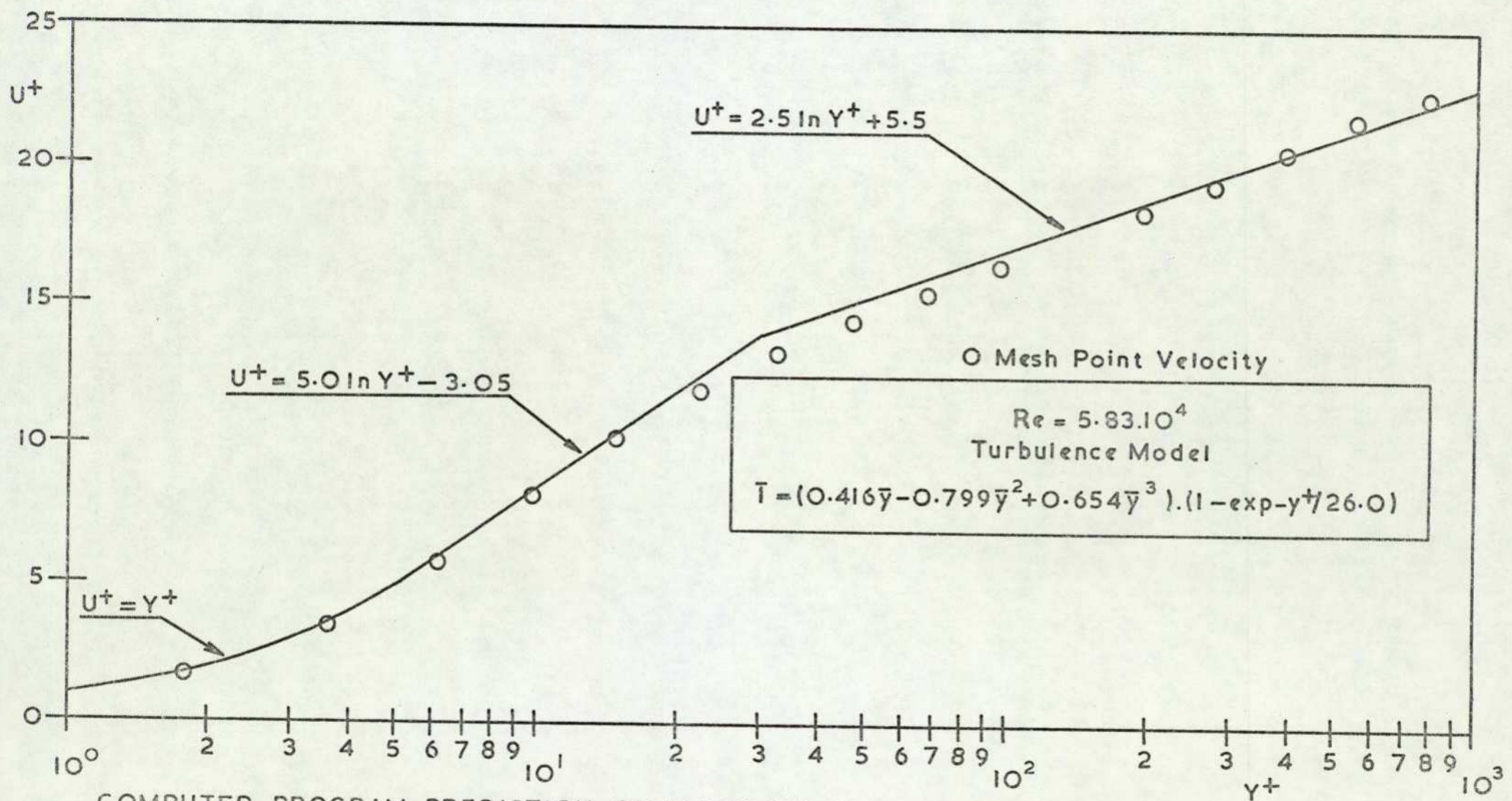
VELOCITY PROFILES FROM EXTENSION OF MIXING LENGTH CORRELATION TO LOWER REYNOLDS NUMBER SINGLE COMPONENT FLOWS

Figure 10-8



VELOCITY PROFILE FROM EXTENSION OF MIXING LENGTH CORRELATION TO HIGHER
 REYNOLDS NUMBER SINGLE-COMPONENT FLOW

Figure 10.9



COMPUTER PROGRAM PREDICTION OF VELOCITY PROFILE IN WALL
REGION FOR SINGLE PHASE FLUID

Figure 10-10

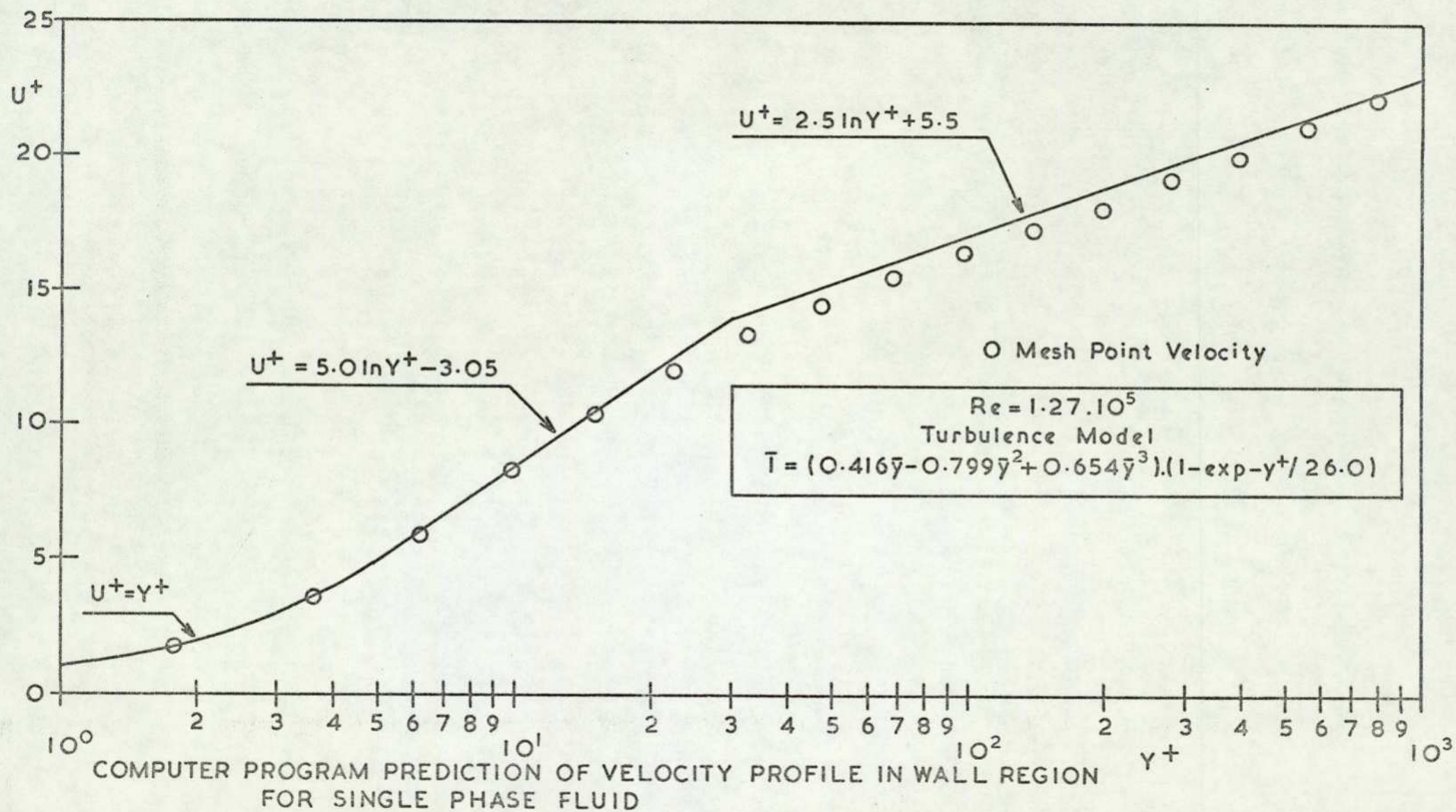
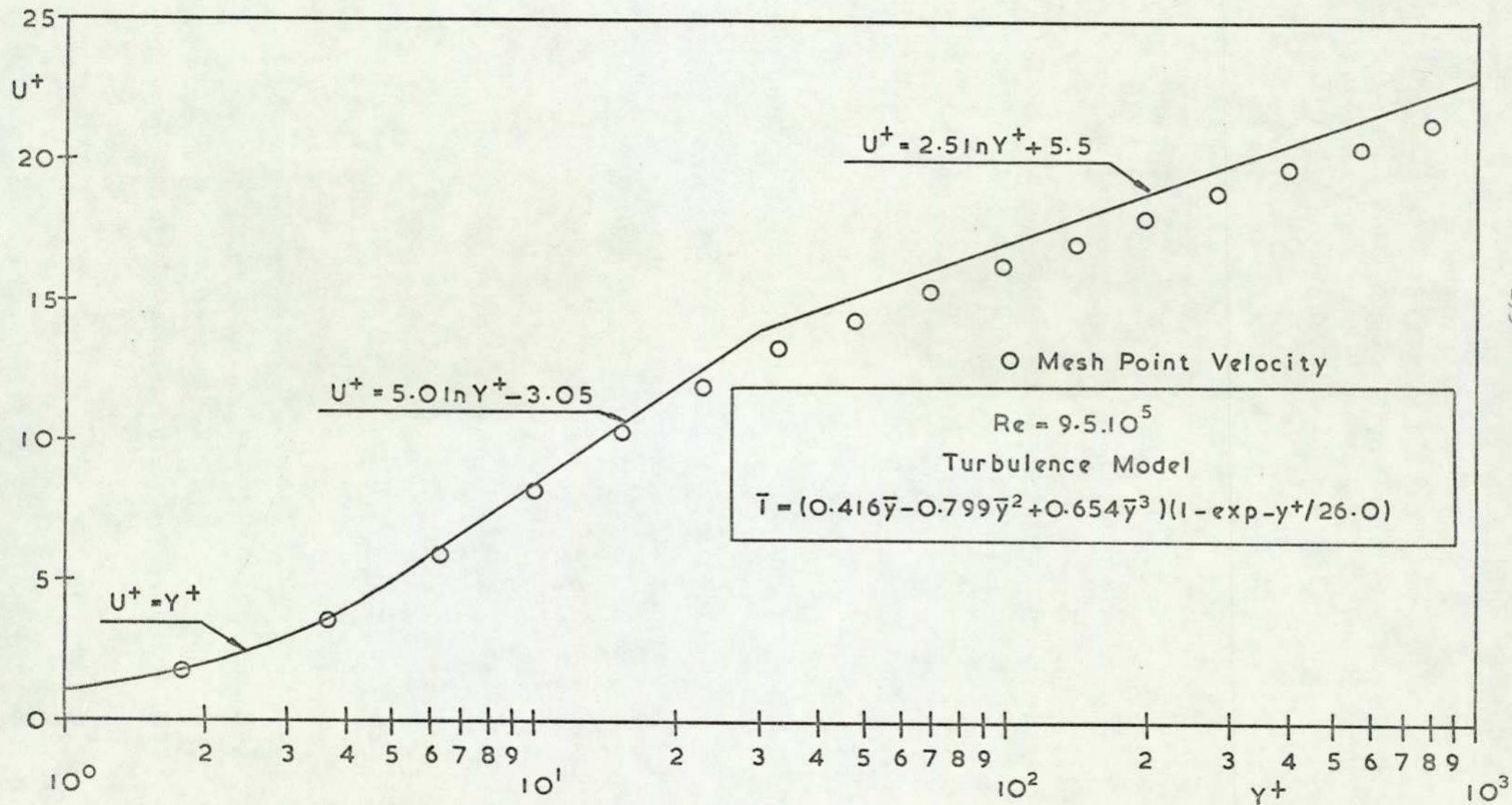
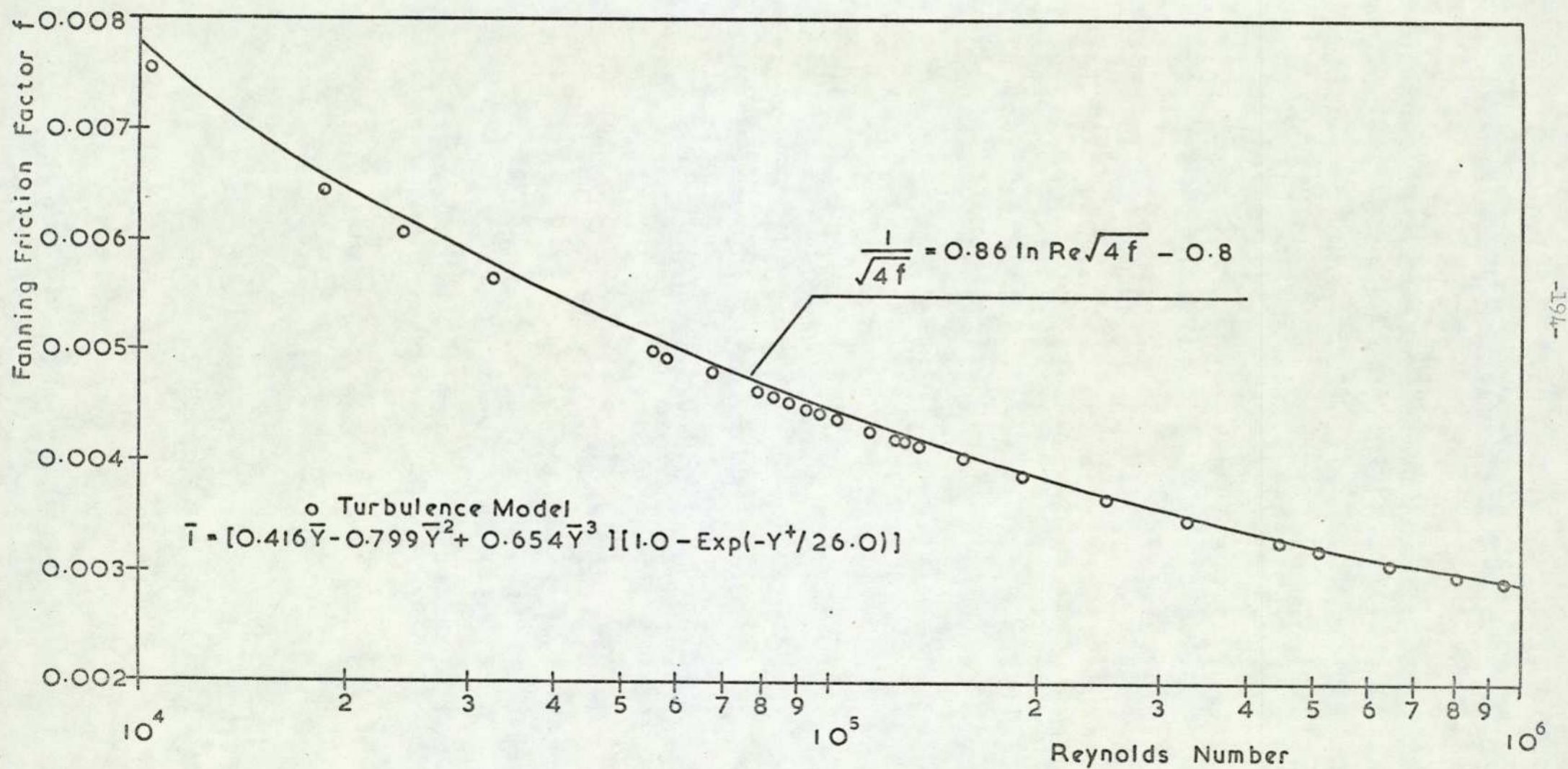


Figure 10-11



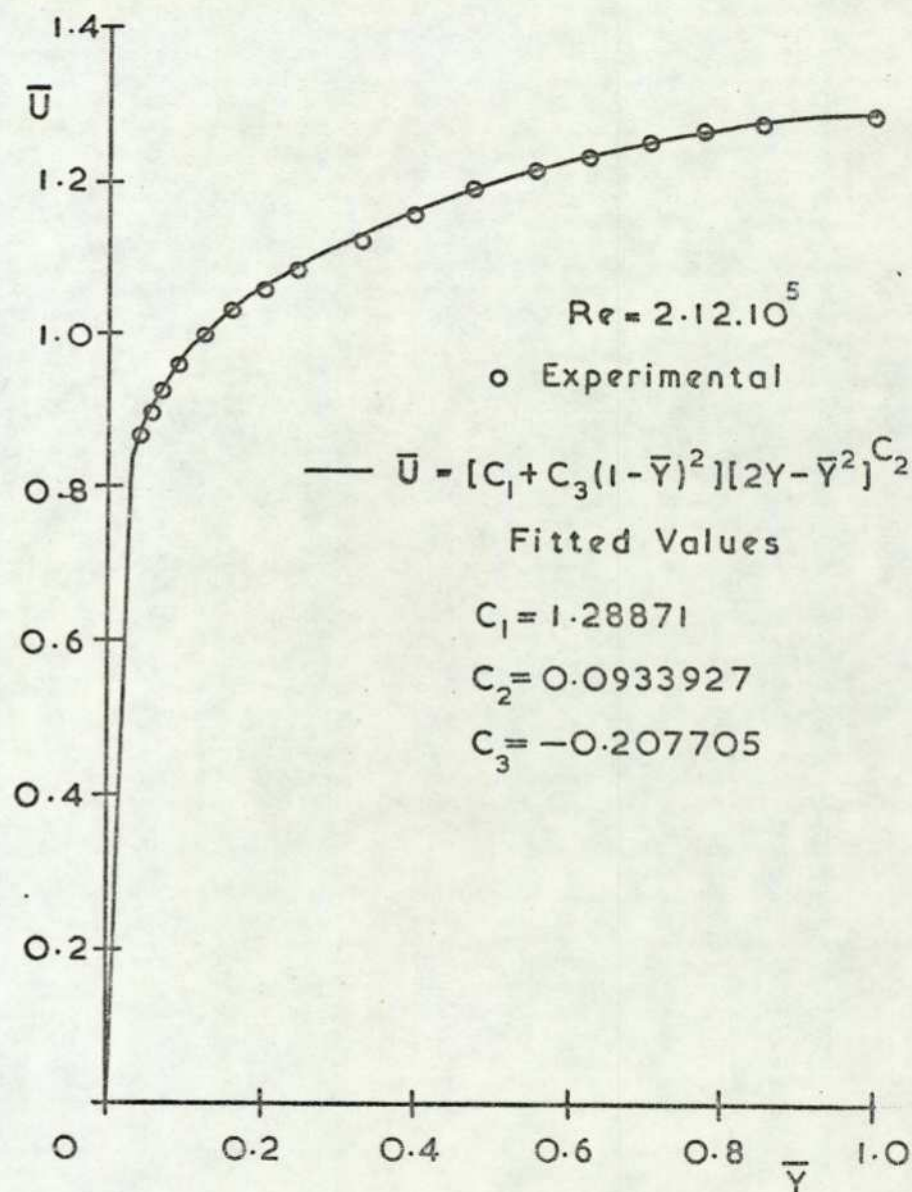
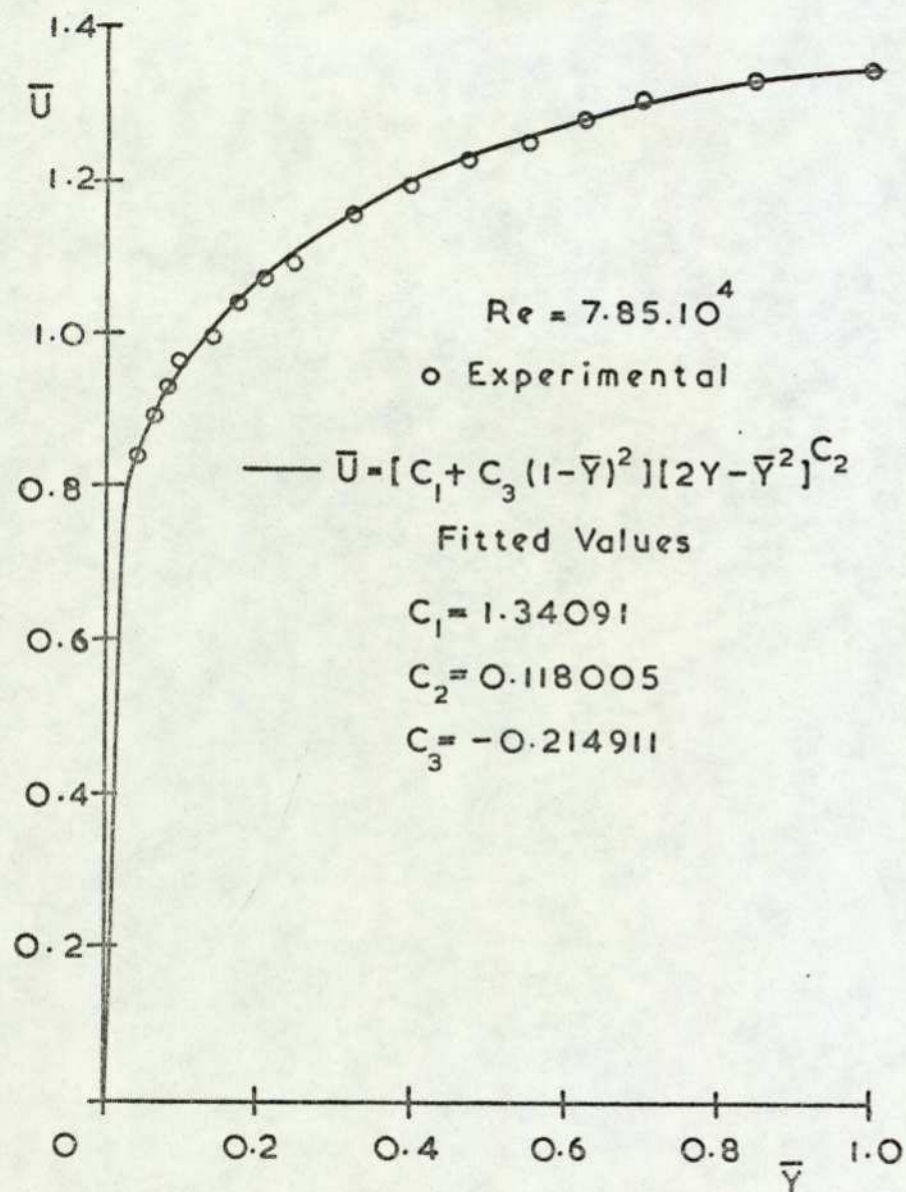
COMPUTER PROGRAM PREDICTION OF VELOCITY PROFILE IN WALL REGION
FOR SINGLE PHASE FLUID

Figure 10-12



FRICTION FACTOR PREDICTION FOR SINGLE-COMPONENT FLOW

Figure 10.13



CURVE FITTED TO EXPERIMENTAL VELOCITY PROFILES FOR DIFFERENTIATION
 IN MIXING LENGTH DETERMINATION

Figure 10.14

11 EXTENSION OF SOLUTION TO TWO-COMPONENT FLOWS

11.1 Effective Property Specification.

11.1.1 Density and Specific Heat.

Previous studies concerned with solid-liquid suspensions in which mean suspension properties are specified invariably take the mixture specific heat and density to be weighted averages of the constituent component properties if the solid phase is truly inert. On this basis the suspension density may be taken as a function of component densities and volume fraction solid:

$$\rho_s = \varphi \rho_d + (1 - \varphi) \rho_c \quad (11.1)$$

The suspension specific heat is obtained from the weight fraction of the solid and the component specific heats:

$$C_{P_s} = W C_{P_d} + (1 - W) C_{P_c} \quad (11.2)$$

Volume fraction solid and weight fraction solid are linked simply by the component densities:

$$\varphi = \frac{w \rho_c}{(1 - w) \rho_d + w \rho_c} \quad (11.3)$$

11.1.2 Thermal Conductivity.

The study of the thermal conductivity of solid-liquid suspensions given in the first section of this thesis indicates that accurate predictions may be made by assuming a simple model for the system and solving the governing equations by numerical techniques. However the study also showed that the increase in the conductivity of a fluid to be obtained by the addition of even large quantities of a highly conducting solid is of a small order and, since the influence of conductivity on the heat transfer performance of a suspension has been shown in previous studies to be of secondary importance, the use of a complex prediction method may not be justified. In the present instance the computer time requirements for the homogeneous flow model solution could well be increased by up to 50 per cent by using the methods developed in the first section. Obviously such an increase could not be justified, especially as the other inputs to the model are of a lesser order of accuracy; a simple explicit relation for conductivity would be more applicable for this purpose. The equation due to Maxwell /1/ has been much used in previous work, although subject to comments on its tendency to under predict the true value, and it has been shown in the present study that the expression given by Bruggeman /8/, and written as equation (1.8), produced a consistently more accurate result. Consequently, this expression was used to derive suspension conductivity values for use in the homogeneous flow model of this study.

11.1.3 Suspension Viscosity.

Previous studies concerned with the determination of the heat and momentum characteristics of solid-liquid suspensions have treated the problem of defining an effective suspension viscosity in a number of different ways. These have ranged from assuming that such mixtures exhibit

behaviour that is approximately Newtonian to fitting experimental data with a mathematical model that described an idealised non-Newtonian fluid.

Graf /54/ has written that information is scarce on solid-liquid suspensions that behave as non-Newtonian liquids and points out that even the available literature is often contradictory, with no definite criteria set as yet.

A problem is encountered in extending the correlations proposed by various authors to suspensions other than those used in the actual experiments, indeed the range of materials used in determining an experimental relation often encompass a spectrum of suspension behaviour. Although a large number of studies are in existence in which the viscosity of a dilute suspension of spherical particles is predicted from a Newtonian relationship, with the exception of a study by Thomas /78/, no work has been performed which will permit the behaviour of a suspension to be predicted using a non-Newtonian model and the physical properties of the constituents. Recourse must generally be made to rheological experiment.

Since this study does not include any experimental work on suspension viscosity and since the range of suspensions to be used in evaluating the heat transfer model cover a range of classification which probably extends from non-settling to pseudohomogeneous and even to the heterogeneous regime, a relation that will be approximate for most of the materials is taken.

Thomas /78/ has presented relationships which fit the laminar flow curves of suspensions with the Bingham plastic model, the coefficients being derived from consideration of the constituent properties. Experimental evidence was obtained from suspensions of thorium oxide, titanium oxide and Kaolin in water and with a particle size range of 0.74 to 13 microns. It was shown that the ratio of the coefficient of rigidity to the viscosity of the suspending medium was directly proportional to the

volume fraction solid with, in addition, for the smaller particles a dependency on particle diameter; for the larger particles the ratio was almost independent of particle size. An expression was derived that asymptotically approached the theoretical viscosity relationship of Einstein for large particle size:

$$\frac{\eta}{\mu_c} = \exp(k_1 \psi)$$
$$k_1 = 2.5 + \frac{14}{\sqrt{d}} \psi \quad (11.4)$$

where the particle shape factor ψ may be taken as unity for spherical particles. For large particles and dilute suspensions the expression reduces to the Einstein /79/ equation:

$$\eta = \mu_c (1 + 2.5 \psi) \quad (11.5)$$

Thus the Bingham coefficient of rigidity, which is also the limiting viscosity at high rates of shear, is reported to approach the theoretical Newtonian viscosity for a suspension of large spherical particles.

Thomas /46/ has shown that the limiting viscosity is most appropriate in defining a characteristic Reynolds number for Bingham plastic flows and since the regions of most importance in heat and momentum transfer are regions of high shear, it may well be a sufficient first approximation to take the limiting viscosity as a characteristic viscosity. Therefore

the same relationship may be used to cover Newtonian and non-Newtonian fluids so that the laminar flow properties do not have to be determined experimentally. This approach neglects any value of fluid yield stress but except in the centre of the tube its effect may be considered negligible and this region, it is proposed, will not be of significance in the overall fluid heat transfer performance. It may also be remarked that some doubts have been expressed as to the existence of a true yield stress as required by the Bingham plastic model.

It has, however, been widely reported that Einstein's equation does tend to underestimate effective viscosity and several authors have produced different expressions which, also, are claimed to be appropriate for higher concentrations. These include the equation proposed by Roscoe /80/ and Brinkman /81/:

$$\mu_s = \mu_c (1 - \varphi)^{-2.5} \quad (11.6)$$

and that due to Happel /82/:

$$\mu_s = \mu_c (1 + 5.5 \varphi) \quad (11.7)$$

Equation (11.7) was found to produce slightly better predictions when used in the homogeneous flow model, so that this study assumes as a first approximation and until further relationships become available:

$$\eta = \mu_s = \mu_c (1 + 5.5 \varphi) \quad (11.8)$$

11.1.4 Suspension Mixing Length.

Comprehensive empirical equations for mixing length in two-component flow have not yet been established. Experimental data on the change in pressure drop characteristics of a suspension compared with a single-component fluid in turbulent flow have been interpreted to show either a thickening of the laminar sublayer and buffer layer of the flow near the wall or the damping of turbulence being manifested in a decrease in the Prandtl mixing length constant. For purposes of calculation it is most convenient to consider the latter approach with the effect of the suspended particles being derived from turbulent flow correlations following the method used by Wasp, Aude, Seiter and Thompson /83/ and in accordance with the assumption of Soo /84/ and Julian and Dukler /85/ who proposed that the eddy diffusivity of a suspension be unchanged in form by the presence of particles but merely altered in magnitude. This method is based on the assumption that within the range of interest the mean velocity distribution is unaffected by the presence of solid particles and attributes the lower pressure drop over a given length of pipe to reduced mixing length due to dissipation by the particles. Since the effect of solid parameters on damping the mixing length is not known the result must be deduced from flow data correlations.

Wasp et.al. expressed this as a damping of the von Karman constant, typically a coal suspension in water at 50 per cent weight fraction gave a 15 per cent reduction. The fanning friction factor is directly related to the von Karman constant by the Colebrook function:

$$\frac{1}{\sqrt{f}} = \frac{1.52}{K^*} \ln(Re \sqrt{f}) - \frac{0.152}{K^*}$$

so that by comparing the friction factors for one and two-component flows we have an estimate of the reduction in the Von Karman constant for any given suspension. An explicit expression for the friction factor of solid-liquid suspensions is provided by Thomas /86/ who gave this modification of the Blasius equation:

$$f_s = B Re^{-b}$$

where

$$B = 0.079 \left[\frac{\mu_c}{\eta} \right]^{0.48}$$
$$b = 0.25 \left[\frac{\mu}{\eta} \right]^{0.15}$$

(11.10)

It may be noted that Hanks and Dadia /87/ have produced a theoretical friction factor chart for suspensions in which the Hedstrom number is a parameter in the turbulent region when the limiting viscosity is used in defining the Reynolds number. Such a result would contradict the finding of Thomas, that the use of the limiting viscosity would enable all the data for a given suspension to be correlated by a single curve, independent of tube diameter. It is suggested by Hanks and Dadia that scatter in the results of Thomas could have justified his assumption but, as no experimental evidence is offered for the proposed Hedstrom number dependance, the simpler

result of Thomas will be used in the present work.

Substituting the viscosity correlation in equation (11.10):

$$f_s = 0.079 (1 + 5.5 \phi)^{-0.48} Re^{-0.25 / (1 + 5.5 \phi)^{0.15}} \quad (11.11)$$

The Blasius equation is used to give the friction factor for a single-component fluid:

$$f_c = 0.079 Re^{-0.25} \quad (11.12)$$

Hence by rearranging the Colebrook equation and expressing the results for single and two-component flows as a ratio:

$$\frac{K_s^*}{K_c^*} = \sqrt{\frac{f_s}{f_c} \left[\frac{1.52 \ln_{10} Re \sqrt{f_s} - 0.152}{1.52 \ln_{10} Re \sqrt{f_c} - 0.152} \right]} \quad (11.13)$$

This gives the Von Karman constant for a solid-liquid suspension expressed as a fraction of the single-component value.

If the mixing length correlation for single-phase fluids expressed in equation (10.12) is rearranged in the form $\bar{l} = K^* \bar{y} (1 - \exp -y^+ / 26.0)$ then:

$$K_c^* = 0.416 - 0.799 \bar{y} + 0.654 \bar{y}^2 \quad (11.14)$$

Substituting for K_c^* in equation (11.13) and obtaining friction factors from equations (11.12) and (11.11) the Von Karman constant and hence the mixing length for two-component mixtures is obtained:

$$\bar{l}_s = (0.416\bar{y} - 0.799\bar{y}^2 + 0.654\bar{y}^3)(1 - \exp - y^*/26.0).$$

$$\left[\sqrt{\frac{f_s}{f_c}} \cdot \frac{1.52 \ln_{10} Re \sqrt{f_s} - 0.152}{1.52 \ln_{10} Re \sqrt{f_c} - 0.152} \right]$$

(11.15)

This mixing length correlation is used to derive eddy diffusivity profiles for turbulent solid-liquid flows in the homogeneous flow model solution procedure. The reduction in the Von Karman constant through the introduction of solid particles into the flow, as calculated by this method, is illustrated in figure (11.1).

11.2 Computer Program Modification for Two-Component Flow.

The use of the boundary layer solution procedure for two-component flow is accomplished simply by the addition to the computer program of a subroutine which supplies mean suspension properties to the main program. Solid component property values are given to the routine in tabular form and are combined with the relevant fluid properties, using the specified relationships, to give tabular suspension property values. The suspension property relationships may easily be changed in order that any alternatives may be incorporated and evaluated.

Suspension properties are used in the place of pure fluid properties in the solution procedure and are normalised and expressed as polynomial functions of temperature as described for single-component flow. The necessary relation of enthalpy with temperature for the suspension solution is obtained taking a mean value based on the weight fraction of each phase, the solid component values being based on specific heat and temperature increment and the liquid phase values being taken from published sources.

Specification of a two-component flow also brings into use the modification of mixing length in the routine which provides effective viscosity and conductivity values in turbulent flow.

11.3 Solution Evaluation for Two-Component Flows.

The computer program, listed in appendix 3, and embodying the homogeneous flow model was used to predict the experimental heat transfer performance of various solid-liquid suspensions. Since it was found impossible to provide the required information using the equipment constructed for this purpose, the solution procedure was evaluated against the experimental results obtained by Orr and Dalla Valle /36/. These results are obtained from heat transfer experiments in vertical flow and are comprehensive in covering a wide range of materials and particle sizes. In addition they include the details of the experimental boundary conditions which are required by the solution procedure. The relevant details of the solid constituents are shown in table 11.1; the property values of the solids were obtained from the reference sources given by Orr and Dalla Valle.

A comparison of the predicted and experimental heat transfer performance of the suspensions and of water only was effected by comparing

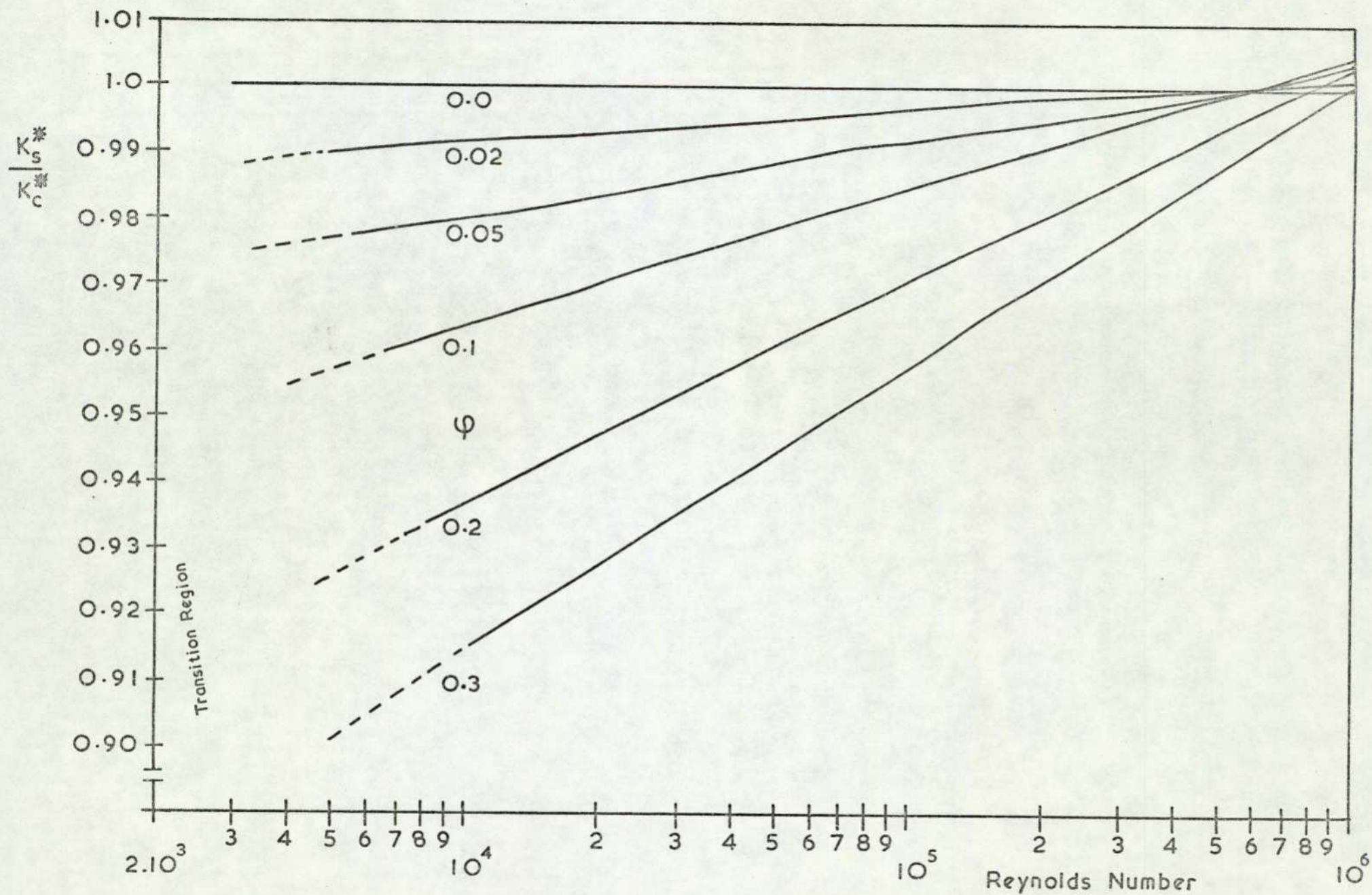
the bulk temperatures obtained at outlet from the heated section as experimentally determined and as calculated. This, in effect, compares the predicted and experimental energy inputs to the fluid and is a more severe test than comparison on the basis of conventional dimensionless groups. With the latter method the bulk outlet temperatures would be combined with other reference temperatures, which being equal for both calculation and experiment, serves to moderate any discrepancies between the results.

The values compared are tabulated and discussed in the succeeding chapter.

Table 11.1

Solid Particle Physical Property Values.

Material	Density Kg/m ³	Specific Heat KJ/Kg °C	Thermal Conductivity KW/m °C	Particle size.
Copper	8323.6	McAdams /88/	McAdams /88/	3.3 microns
Attapulgis Clay	2564.5	0.754(37.8°C) 0.921(148.8°C)	0.013	1-2 microns
No. 18 Glass Bead	2891.7	0.7536	0.0017	35 microns
No. 9 Glass Bead	2891.7	0.7536	0.0017	260 microns
Acheson Graphite	2012.0	Perry /89/	Powell /90/	200 mesh (75 microns)
Aluminium	2700.7	McAdams /88/	McAdams /88/	325 mesh (50 microns)



VARIATION OF VON KARMAN CONSTANT WITH TWO-COMPONENT FLUIDS

Figure 11-1

12 RESULTS AND DISCUSSION

12.1 Treatment of Results

The boundary layer equations were solved, using the procedure previously outlined, with parameters obtained from the experimental turbulent heat transfer results given by Orr and Dalla Valle /36/. The experimental conditions of a fully-developed flow at entry to the heat transfer section, which had a length to diameter ratio of 187, the fluid bulk entry temperature and a wall temperature expressed as a linear function of axial distance, were simulated in the solution. A variety of suspensions of solid particles in water were used in the experimental work and for the prediction of these flows the governing equations were solved using the mean suspension property relationships developed in this study; a number of the results were for the flow of water only and these provided a datum for the analysis of the solutions for suspension flow.

Table 12.1 shows details of the results of the comparison between predicted and experimental bulk temperature increments for each flow considered and is subdivided into groups of individual one or two-component fluid. The results for the suspensions containing either attapulugus clay or copper powder were obtained using a slight modification to the solution which improved the accuracy; this will be fully discussed later. Also given for each sample group are estimates of mean error, error bias and mean temperature increment ratio, which is the ratio of the predicted to the experimental rise in bulk temperature, together with the standard deviations for each value. The methods by which these values were evaluated are detailed in appendix 1.

The data was subject to a screening process to remove the possibility of 'rogue' experimental points exerting an undue influence on individual sample parameters. This is always possible with heat transfer experiments and especially significant where the sample sizes are small. Basing the screening procedure on the initial sample estimates of mean temperature increment ratio and standard deviation, the criteria for neglecting any result was that it lay outside the region given by $m \pm 2.33.S$. The possibility of neglecting a significant result using this criteria is 2 per cent. In total 5 such results were neglected out of the 73 experimental flows considered, these were comprised of one result each for water only, copper suspension and aluminium suspension and two results for graphite suspensions. Revised estimates of the sample parameters were obtained based on the reduced number of results.

The effect of the screening procedure on the overall error of the solution was negligible since not only are a large number of results considered but those neglected were almost equally distributed as to the sign of their error.

In table 12.2a the overall accuracy of the solution procedure is shown; considering all the heat transfer results the experimental values of outlet bulk temperature were predicted with an overall error of 7.52 per cent and the overall mean temperature increment ratio was 1.055. This in itself represents a significant improvement over the correlation given by Orr and Dalla Valle, but the analysis of the results on the basis of temperature increment ratio for each sample group provides further information on the validity of the solution.

12.2 Prediction and Validity of the Solution

12.2.1 Results for Single-Component Fluid

The solution of the boundary layer equations for a single-component fluid would be expected to produce accurate results if the method is to be extended to the two-component case. The results given in table 12.1a show that the solution predicts outlet bulk temperatures that are consistently some 1 to 12 per cent above the experimental values; a mean temperature increment ratio of 1.058 was obtained. Whilst this figure will be well within any expected experimental error, it can be shown that the effect of experimental inaccuracies will be to produce an underestimate of experimental bulk temperature increment for a given fluid.

Measurement of fluid bulk temperature at inlet to the test section was accomplished prior to a calming section which, in turn, preceded the heated section of the circuit. Thus, this value was determined at some distance from the actual entry point and since the fluid was above ambient temperature, the effect of any heat loss from the fluid would be an overestimate of this parameter. Similarly, at exit from the heated section any heat loss occurring before the mixing chamber would lead to an underestimate of outlet bulk temperature. The net effect would be an underestimate of temperature rise in the heated section and this error would be expected to increase as the conditions for heat loss become more favourable. Figure 12.1 shows the error between the experimental and predicted temperature increments plotted against fluid inlet and outlet bulk temperatures and also against the flow Reynolds number for the results using water only. A definite trend of increase in the error

can be seen with an increase in these parameters; the less pronounced trend in the case of fluid outlet temperature is probably because of the reduced distance available for heat loss.

It is therefore suggested that the predicted values of temperature increment, for the case of the flow of water, are closer to the actual increases than the tabulated results would indicate. Further, since the same trend may be expected for suspension flows, the validity of the homogeneous flow model may be more fairly assessed in a comparison between the solution accuracy for single and two-component fluids.

12.2.2 Significance of Results for Sample Groups.

As can be seen from table 12.1a - g, mean prediction accuracy varies by a considerable degree between sample groups. It is to be expected that the homogeneous flow model would only be appropriate for those situations where the flow rates are sufficiently high and particle size sufficiently small for the flows to be considered after the manner of a homogeneous fluid, where the lack of a detailed description of the flow pattern is not of importance and where the suspension property relationships are adequate.

An analysis based on statistical methods, which are outlined in appendix 1, was performed to determine whether the differences between the prediction performance for each suspension could be attributed to experimental variation or whether the result indicated that the homogeneous flow model was not applicable for any particular case. The difference between the sample mean temperature increment ratios was investigated by means of the 't' test. Since this test is based upon

the assumption that the two samples compared are drawn from populations identical as to variance, the 'F' test was also employed to ascertain the validity of assuming that any two sample variances are independent estimates of the same population variance.

The results of these two tests, carried out between all the sample groups, are shown in table 12.3, with the terms 'probably significant', 'significant' and 'highly significant' referring to a degree of probability of 0.95, 0.99 and 0.999 respectively that the difference between the two values arises from causes other than chance.

Only three of the pairings of suspension groups were found to exhibit any significance in their difference of variance and then only at the lowest level; this indicates that the following 't' test on mean temperature increment ratios will be a valid comparison. The result of 'probably significant' for the comparison of the variance for water and No. 18 glass bead results whilst not invaliding any further conclusion does point out that this cannot be firm and must be used judiciously. In considering the significance of the difference in mean temperature increment ratio between samples, prime importance will be attached to the results obtained between water only and the various suspensions. Since it is known that the method is valid for a single-component fluid in the Prandtl number range of water, then the implication can be drawn that it will also be valid for a two-component fluid if the prediction accuracies are comparable.

On the basis given above the homogeneous flow model can be seen to be inappropriate for the aluminium powder/water suspensions since the difference between the predictions for this fluid and water is 'highly significant'. This result was to be expected since it was known that the aluminium particles would probably be highly irregular in shape and therefore several of the suspension property relationships would not be valid. The thermal conductivity study of this report indicates the large effect of particle shape on this suspension property and other studies have shown a similar situation to exist with suspension viscosity. Thomas /91/ has stated that particle shape apparently affects turbulent flow data in a way which cannot be predicted from simple laminar flow measurements. From consideration of the other results it is not likely that particle size, being less than 50 microns, would be a factor in the model failing to represent this suspension.

A 'significant' difference between the predictions for water and for No. 9 glass beads may, on the other hand, be ascribed to the large size of the particles. With a particle diameter of 260 microns the relative velocity and temperature between the phases would assume some importance and it would also not be expected that the particles would follow the turbulent fluctuations in the manner of smaller particles. Thus, at least for Reynolds numbers below 10^5 , the assumption of homogeneous flow for suspensions containing particles of this size has been shown to be inadequate.

The prediction method, when employed for the flows of graphite and No. 18 glass bead suspensions, can be seen to give results that do not differ significantly in accuracy from those for water alone. In the one

case the particle size was less than 75 microns and in the other the mean particle diameter was 35 microns. The mean temperature increment ratio for the graphite suspension is 0.38 per cent below that for water and for the glass bead suspension it is 1.71 per cent lower. It is recognised that the result of the variance test for the latter suspension does not allow a firm conclusion on the point of comparison of accuracy but, since the result fits into the expected pattern of conformity with the homogeneous flow model, it appears reasonable to accept it. It is therefore concluded that for particles of this size range (35 to 75 microns) the homogeneous flow model is both valid and productive of accurate results.

For the attapulugus clay suspensions the 'highly significant' difference indicates that the homogeneous flow model will not be applicable. Metzner /33/ has commented that this material is not truly inert, as required by the property relationships; the clay tending to be swelled or solvated by the liquid phase. In addition, with the particle size being in the range 1 - 2 microns there may have been a tendency for the particles to form agglomerations, which would again be a factor in the failure of the model to represent the data.

Flocculation may also be responsible for the 'significant' difference between the mean temperature increment ratios of copper suspensions and of water. The mean particle size was 3.3 microns; Thomas /91/ gives a particle size of 5 to 10 microns as being a point where the forces responsible for the distinctive behaviour of colloidal materials are of importance and notes that flocculation has been observed with particle sizes of up to 20 microns. Orr and Dalla Valle /36/ in reporting their

experimental results do not give any consideration to deviations in the flow patterns for individual suspensions, so that there is no evidence to support this explanation.

As noted by Thomas /78/, particle size will be a factor in the viscosity correlation when considering particles in the above mentioned range. A modification to the solution procedure, to attempt to allow for this factor and the agglomeration of particles, was incorporated into the program and used to obtain the results for copper and attapulugus clay suspensions. As shown in the following passage this does not give a sufficient improvement in prediction. However, the mean temperature increment ratio obtained for the copper suspensions is only 4.45 per cent below that for water, so that the effect of discrepancies is not as great as in the case of clay suspensions. This being so, although the homogeneous flow model does not strictly represent the data for this suspension, the prediction is sufficiently accurate to allow the results to be used in providing data for an investigation into trends in accuracy with Reynolds number and volume fraction solid.

The effect of the ratio of solid to liquid density could not be investigated with the experimental evidence given since results for suspensions containing particles of similar size but varying densities were not available. In the suspensions used for the experimental investigation the dense materials were present in the form of the smallest particles so that the density ratio alone would not appear as a factor in prediction discrepancies. The mass of an individual particle of the

copper powder, for example, was considerably less than that of one of the smallest glass beads. In general the criteria for determination of a given suspension will be adequately represented by the homogeneous flow model, on the basis of density ratio, must come from consideration of the flow regime correlations previously discussed. It would appear that for commonly used materials suspended in water a particle size of up to 50 microns will not produce any significant deviation from the model assumptions.

Considering the suspensions containing graphite, No. 18 glass beads and copper, with also the results for water, then for 50 experiments:

1. 58 per cent of predictions are accurate within 5 per cent.
2. 88 per cent of predictions are accurate within 10 per cent.
3. 96 per cent of predictions are accurate within 15 per cent.

The mean error is shown in table 12.2a to be 5.39 per cent and the mean temperature increment ratio is 1.034. The results for the three suspensions are, overall, 3.41 per cent below the mean prediction for water.

Confidence limits for the mean temperature increment ratios of all the samples and of the overall values are given in table 12.2b and indicate the limits within which the actual values can be taken to lie with the probabilities of 0.95 and 0.99. A comparison between the predicted and experimental bulk temperature at outlet from the heat transfer section is shown in figure 12.2.

12.3 Solution Modification for Small Particle Suspensions

The finding of Crewe and Simons /49/ of a significant difference in the heat transfer characteristics of flocculated and non-flocculated suspensions would appear to be substantiated by the results of this study. The homogeneous flow model together with the derived property relationships satisfactorily represents the heat transfer performance of several suspensions but fails for the case of a suspension containing particles of 3.3 microns diameter. This indicates that the property relationships are not adequate for this case or the existence of a fundamental difference between the two flows.

The results for the copper and attapulugus clay suspensions obtained using the unmodified homogeneous flow model solution are given in table 12.4, while the results considered previously were those from a modified procedure.

The effect of particle size on suspension viscosity was accounted for by using the correlation given by Thomas /78/ and written as equation (11.4). However, the net result of using this viscosity relationship is to significantly worsen the prediction performance of the model so it is clear that a consideration of this factor alone will not account for the flow of this type of suspension. Crewe and Simons have also reported that no significant difference was noted between the two types of flow when comparing friction factor characteristics. Following from this finding, then, the turbulence damping factor is taken to be the same magnitude in both flocculated and non-flocculated flows; this improves the prediction slightly but not above the original figures for the unmodified solution.

Consideration must also be given to the flow pattern where the particles are likely to form agglomerates. Michaels and Bolger /92/ describe two flow regimes for flocculated suspensions; at low shear rates the particles are assumed to be contained in aggregates formed by mutual attraction, at high rates of shear these aggregates are broken down completely with the shear forces pulling the clusters apart as quickly as they form. Since in tube heat transfer the fluid properties in the region of high shear adjacent to the wall are of controlling importance, the fluid properties must be evaluated under the non-flocculated conditions prevailing in this section of the flow. Without the necessary description of flow regimes, the modification made to the solution consists in evaluation of the Prandtl number using the non-flocculated relationship for viscosity given by equation (11.8). Since the Reynolds number evaluated using a flocculated viscosity relationship has been shown to give a correlation for pressure drop and with this parameter controlling the momentum exchange characteristics the value given by this method was used.

The modification is seen to be a simple and crude compromise but does give an improved prediction for the copper particle suspension flows without, however, making the homogeneous flow model strictly applicable for representing heat transfer to this fluid.

A note is also made of the apparent anomaly in the results for the copper suspensions of very low volume fraction solid (less than 0.15 per cent), which would be expected to have a prediction accuracy similar to

that for water only. In fact, table 12.5 indicates that the mean temperature increment ratios for samples divided into low and high volume fraction suspensions are not significantly different. This suggests that a minute quantity of solid material has, in this case, a significant effect on heat transfer processes. The result of a 'significant' difference between the predictions for water and for the low volume fraction solids suspensions also suggests that the effect cannot be attributed merely to experimental error. There is no indication of what the mechanistic difference between the fluids may be nor can one be suggested; the indications are probabilities based on the available evidence and requires further investigation.

12.4 Prediction Trend with Reynolds Number and Concentration.

An investigation to determine if the prediction demonstrated any trends that could be taken as factors of Reynolds number or volume fraction solid used the results for those fluids for which the homogeneous flow model has been shown to be satisfactory. These were graphite, No. 18 glass bead and copper suspensions, together with the results for water alone. For each test the results were divided into groups covering a range of Reynolds number or concentration, each range being defined arbitrarily to give a reasonable sample size.

The mean temperature increment ratios for each sample group divided on the basis of volume fraction solid are shown in table 12.6, while the results given in table 12.7 demonstrate that there is no significant difference between any of the values for the individual groups nor between their variance. Therefore, no difference in the accuracy of prediction of the model exists within the range of volume fraction solid from zero to 19 per cent.

Sample division on the basis of Reynolds number gives the results presented in table 12.8, with the significance tests tabulated in table 12.9. A 'highly significant' result was found for the difference in mean temperature increment ratio between the lowest Reynolds number range and several higher groups. This result could be due to the inbuilt tendency of the experimental equipment, as noted for the flow of water, in which the higher Reynolds number flows are liable to a larger discrepancy between experimental and predicted bulk temperature increase because of the underestimate of the experimental value through increased heat losses. This result could also be due to the assumptions made in the homogeneous flow model not being satisfied at Reynolds numbers lower than $5.0 \cdot 10^4$. However, there is no reason to suppose this to have been the case for vertical flows and since the experimental circuit trend has been definitely identified it must be accounted the more likely explanation.

The fluctuation in individual temperature increment ratios with Reynolds number and with volume fraction solid for the fluids considered is shown in figures 12.3 and 12.4; the analysis performed indicates that there is no significance in these variations.

12.5 Comparisons of Single and Two-Component Fluids.

The computer program solution of the boundary layer equations has been employed in comparing the predicted heat transfer performance of the aqueous suspensions used in some of the experimental tests with the expected performance of water only. No attempt has been made to identify

and codify the controlling factors of any change in heat transfer characteristics. There is no obvious basis for a comparison; in this case the conditions of equality of volume flow rate, Reynolds number and mass flow rate have been used. Previous authors, in making such a comparison, have used the overall convective heat coefficient as the basis but in this instance the energy transferred in each of the simulated heat transfer tests will form the means for the evaluation of comparative characteristics. It is suggested that this gives a more valid comparison, although it must be noted that as the various fluid bulk outlet temperatures are of the same magnitude for a given set of boundary conditions then the comparison on the basis of convection coefficients will be very similar to that presented.

The ratios between the suspension and water energy transfer rates were evaluated using the relationships given in appendix I. Comparisons were also made of pressure drop characteristics under the isothermal conditions prevailing at the fluid entry. Table 12.10 gives the results of the comparisons.

At similar volume flow rates water is shown to be a slightly better heat transfer medium than all the suspensions except for the one containing the densest solid phase. With equality of volume flow rate the greater suspension density ensures a greater mass flow rate but since the specific heat of water is much larger than that of the solid material the net effect is near similarity of energy transfer rates. Suspensions with lower volume fraction solids and denser solid phase are the most effective.

With the basis of comparison is equal Reynolds number the greater suspension viscosity means that the mass flow rate of suspension must be much higher than for the pure fluid and will be sufficiently increased to compensate for the reduction in suspension heat capacity. This comparison indicates that the suspensions are better heat transfer mediums than water, with the higher volume fractions and denser materials producing the greatest enhancement.

At equivalent mass flow rates the greater specific heat of water ensures that the energy transfer to the fluid will be considerably greater than for the suspension; the lower volume fraction and less dense solid phases perform better on this basis. Metzner /33/ arrived at a similar conclusion for a comparison on an equivalent mass flow basis from a consideration of the experimental data of Salamone and Newman /37/.

Orr and Dalla Valle /36/ gave a comparison between an aqueous copper suspension and water based on their correlation of experimental results. They concluded that at equal volume flow rates the suspension proved the better heat transfer medium by about 22 per cent, while at equal Reynolds numbers water was slightly better. On repeating this comparison, using the figures given by the authors, it was found that the correlation shows the suspension to be better medium in both cases, with a 34 per cent improvement at equal volume flow rates and a 12 per cent at equal Reynolds number. The contradiction between the comparison made using the equation given by Orr and Dalla Valle and that made using the homogeneous flow model is partially due to the different viscosity models employed and partially

because the above authors include results that are not adequately represented by an effective suspension property approach.

From table 12.10 it is also noted that the ratio of suspension to water pressure drop characteristics is in all cases greater than the respective energy transfer ratio.

Velocity profile predictions for turbulent flow may also be produced by the computer solution and those for some of the test suspensions and for a single component fluid are shown in figure 12.5. Figure 12.6 shows the velocity profiles for the flow of various concentrations of a given suspension at the same Reynolds number. A comparison of the velocity profiles for one and two-component fluids shows that the curves given agree with the findings of previous investigators, notwithstanding the fact that no account is taken of yield stress values for those suspensions which may be fitted to the Bingham plastic model. Brogue and Metzner /92/ measured turbulent core profiles for various non-Newtonian fluids and found that they were essentially the same as for Newtonian fluids when normalised with respect to the mean velocity. Neither a Reynolds number nor a non-Newtonian effect was discernable. An examination of the curves for suspension profiles presented by these authors, however, does indicate that at the centre of the pipe the experimental points lie above the standard Newtonian curve and fall below this line nearer the wall; this is the trend shown by figure 12.5, although being slight was probably within experimental error and so not accounted significant.

Graf /54/ has written that the turbulent velocity profiles for non-Newtonian fluids are similar to Newtonian ones, the latter being slightly flatter; this description fits the curves shown in figure 12.6.

The computer program can also be used to give predictions of various flow parameters at any intermediate point in the solution; figures 12.7 and 12.8 show local Nusselt numbers and bulk fluid temperatures for suspensions of varying concentration at a constant Reynolds number and with the same arbitrary boundary conditions. Increasing the amount of solid in the suspension can be seen to reduce the Nusselt number and slightly increase the bulk temperature at any axial position. This may be interpreted to indicate that the increase in film coefficient due to the addition of particles is not as large as the increase in thermal conductivity; this may be attributed to the damping of turbulence by the particles. However, these curves have been included to demonstrate how the use of a homogeneous flow model for suspensions incorporated in a numerical solution of the governing equations can produce design information that is more comprehensive than that obtained from simple correlations. By implication if the technique is suitable in predicting certain parameters then the other information produced can be accepted although no experimental proof of this has been obtained.

12.6 Conclusions.

1. The results show that the homogeneous flow model will provide a valid solution technique for turbulent heat transfer to suspensions containing spherical particles in a size range from 20 to at least 75 microns in diameter and over the range of

density ratio from 2 to 8.5. It can also be used for certain suspensions that tend to flocculate with a reasonable accuracy but is not strictly valid for these cases. The model will not be appropriate for irregular particle suspensions, particle sizes above 260 microns and for solids that are not inert.

2. Suitable relationships for the evaluation of effective solid-liquid suspension properties from the properties of their constituents have been identified.
3. Comparisons of the effectiveness of single and two-component fluids as heat transfer media may be made using the given solution procedure. The indications are that only small beneficial effects are possible through the addition of solids to liquids and these are outweighed by the associated difficulties.

12.7 Recommendations for Further Work.

A major handicap in studies involving solid-liquid suspensions is the need for experimental evidence in determining the rheological properties of each individual fluid. There is a pressing need for the establishment of relationships that will enable the laminar flow properties to be predicted from parameters associated with the constituent components. Such relationships could well be the idealised mathematical models which have been commonly used in correlating experimental results. A detailed distinction between suspensions exhibiting Newtonian or non-Newtonian behaviour remains to be given. The identification of flow regimes in flows where the particles form agglomerates, together with the associated

properties would enable the simple homogeneous flow model to be modified to accommodate such fluids.

Where so much of the fundamental nature of turbulence in single-component flow is, as yet, not fully understood a rigorous approach to the basic mechanisms of turbulence in two-component flows is not practicable. A proposal worthy of consideration is that made by Eissenberg /93/ who suggests determining what measurable intrinsic property of non-Newtonian fluids is related to the damping of turbulence.

The specification of suspension property relationships where the particles shapes are irregular would enable the homogeneous flow model to encompass such mixtures. A possible method of extending the numerical simulation model for thermal conductivity evaluation has been suggested in section A.

The computer solution embodying the homogeneous flow model may be used to investigate the effect of various component property ratios and parameters on the heat transfer performance of suspensions and therefore to identify the controlling factors without recourse to experiment.

PREDICTED TEMPERATURE INCREMENTS COMPARED WITH THE EXPERIMENTAL RESULTS OF ORR AND DALLA VALLE

TABLE 12.1-A
FLUID - WATER ONLY

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
1	92506.0	0.000	0.000	1.564	1.591	4.787	1.048
6	116255.0	0.000	0.000	1.475	1.503	5.895	1.059
7	58269.0	0.000	0.000	1.738	1.764	3.525	1.035
8	102223.0	0.000	0.000	1.513	1.541	5.458	1.055
29	55974.0	0.000	0.000	1.617	1.654	5.997	1.060
42	69319.0	0.000	0.000	1.536	1.576	7.463	1.075
71	97316.0	0.000	0.000	1.435	1.485	11.494	1.115
72	68369.0	0.000	0.000	1.524	1.562	7.252	1.073
73	18548.0	0.000	0.000	2.250	2.259	0.720	1.007

MEAN ERROR = 5.84% , ERROR BIAS = 5.84%
STANDARD DEVIATION OF ERROR: SAMPLE = 2.78% , POPULATION = 2.95%
MEAN TEMPERATURE INCREMENT RATIO = 1.058
STANDARD DEVIATION: SAMPLE = 0.028 , POPULATION = 0.030

TABLE 12.1-B
 FLUID - ATTAPULGUS CLAY/WATER SUSPENSION
 PARTICLE SIZE = 1-2 MICRONS

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
2	66037.0	0.308	0.790	1.661	1.634	4.085	0.959
3	54602.0	1.325	3.340	1.709	1.678	4.372	0.956
4	51443.0	2.098	5.230	1.684	1.640	6.433	0.936
5	47677.0	2.912	7.180	1.599	1.572	4.508	0.955

MEAN ERROR = 4.85% , ERROR BIAS = -4.85%

STANDARD DEVIATION OF ERROR: SAMPLE = 0.93% , POPULATION = 1.07%

MEAN TEMPERATURE INCREMENT RATIO = 0.952

STANDARD DEVIATION: SAMPLE = 0.009 , POPULATION = 0.011

TABLE 12.1-C
FLUID - COPPER POWDER/WATER SUSPENSION
PARTICLE SIZE = 3.3 MICRONS

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
9	106210.0	0.086	0.710	1.522	1.532	1.916	1.019
10	42656.0	0.086	0.710	1.896	1.895	0.112	0.999
11	106776.0	0.085	0.710	1.581	1.602	3.614	1.036
12	37000.0	0.101	0.830	2.104	2.004	9.058	0.909
13	69902.0	0.100	0.830	1.750	1.747	0.400	0.996
14	145819.0	0.099	0.830	1.380	1.394	3.684	1.037
15	151268.0	0.155	1.260	1.370	1.371	0.270	1.003
16	68486.0	0.153	1.260	1.615	1.632	2.764	1.028
17	123927.0	1.995	14.600	1.449	1.463	3.118	1.031
18	80555.0	1.931	14.200	1.465	1.453	2.581	0.974
19	142670.0	1.851	13.700	1.317	1.341	7.571	1.076
20	147993.0	1.848	13.700	1.395	1.360	8.861	0.911
21	118398.0	1.852	13.700	1.430	1.413	3.953	0.960
23	93689.0	3.984	25.800	1.412	1.424	2.913	1.029
24	87011.0	1.946	14.400	1.358	1.358	0.000	1.000
25	120164.0	5.829	34.300	1.349	1.359	2.865	1.029
26	51984.0	6.314	36.000	1.649	1.691	6.471	1.065
27	104437.0	7.119	39.200	1.396	1.431	8.838	1.088
28	26081.0	7.000	38.600	1.664	1.644	3.012	0.970

MEAN ERROR = 3.79% , ERROR BIAS = 0.84%
STANDARD DEVIATION OF ERROR: SAMPLE = 2.91% , POPULATION = 2.99%
MEAN TEMPERATURE INCREMENT RATIO = 1.008
STANDARD DEVIATION: SAMPLE = 0.047 , POPULATION = 0.048

TABLE 12.1-D
 FLUID - NO 18 GLASS BEAD/WATER SUSPENSION
 PARTICLE SIZE = 35 MICRONS

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
30	50035.0	3.066	8.390	1.682	1.656	3.812	0.962
31	124725.0	2.130	5.980	1.388	1.400	3.093	1.031
32	109641.0	6.333	16.500	1.398	1.407	2.261	1.023
33	55014.0	6.679	17.200	1.651	1.664	1.997	1.020
34	31082.0	5.527	14.500	1.780	1.763	2.179	0.978
35	93865.0	10.310	25.100	1.435	1.511	17.471	1.175
36	52979.0	9.638	23.600	1.649	1.695	7.088	1.071

MEAN ERROR = 5.41% , ERROR BIAS = 3.70%
 STANDARD DEVIATION OF ERROR: SAMPLE = 4.19% , POPULATION = 4.69%
 MEAN TEMPERATURE INCREMENT RATIO = 1.037
 STANDARD DEVIATION: SAMPLE = 0.065 , POPULATION = 0.070

TABLE 12.1-E

FLUID - NO 9 GLASS BEAD/WATER SUSPENSION

PARTICLE SIZE = 260 MICRONS

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
37	99345.0	9.696	23.900	1.356	1.390	9.551	1.096
38	58551.0	8.626	21.600	1.485	1.514	5.979	1.060
39	68730.0	21.156	43.900	1.448	1.530	18.304	1.183
40	37914.0	15.314	34.500	1.615	1.700	13.821	1.138
41	82701.0	22.227	45.700	1.226	1.257	13.717	1.137

MEAN ERROR =12.27% , ERROR BIAS =12.27%

STANDARD DEVIATION OF ERROR: SAMPLE = 4.19% , POPULATION = 4.69%

MEAN TEMPERATURE INCREMENT RATIO = 1.123

STANDARD DEVIATION: SAMPLE = 0.042 , POPULATION = 0.047

TABLE 12.1-F
 FLUID - ACHESON GRAPHITE/WATER SUSPENSION
 PARTICLE SIZE = -200 MESH

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
43	66406.0	1.866	3.700	1.534	1.567	6.180	1.062
44	62869.0	4.645	9.010	1.384	1.404	5.208	1.052
45	118535.0	4.619	9.010	1.238	1.263	10.504	1.105
46	94693.0	4.663	9.010	1.471	1.507	7.643	1.076
47	88832.0	7.042	13.300	1.501	1.522	4.192	1.042
48	55623.0	7.095	13.400	1.507	1.525	3.550	1.036
49	25948.0	7.151	13.400	1.933	1.909	2.572	0.974
50	101006.0	8.862	16.500	1.456	1.425	6.798	0.932
51	119600.0	8.814	16.500	1.228	1.254	11.404	1.114
52	69006.0	10.582	19.400	1.389	1.407	4.627	1.046
53	32396.0	10.627	19.400	1.520	1.512	1.538	0.985
54	67568.0	13.887	24.700	1.342	1.357	4.386	1.044
55	86816.0	13.874	24.700	1.418	1.401	4.067	0.959
56	77007.0	16.782	29.100	1.350	1.398	13.714	1.137
58	69796.0	18.663	31.800	1.339	1.409	20.649	1.206

MEAN ERROR = 7.14% , ERROR BIAS = 5.14%
 STANDARD DEVIATION OF ERROR: SAMPLE = 4.88% , POPULATION = 5.05%
 MEAN TEMPERATURE INCREMENT RATIO = 1.051
 STANDARD DEVIATION: SAMPLE = 0.070 , POPULATION = 0.072

TABLE 12.1-G
FLUID - ALUMINIUM POWDER/WATER SUSPENSION
PARTICLE SIZE = -325 MESH

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
61	96521.0	1.491	4.010	1.350	1.402	14.857	1.149
62	117742.0	1.489	4.000	1.423	1.483	14.184	1.142
64	121303.0	2.098	5.570	1.348	1.382	9.770	1.098
65	109840.0	3.835	9.900	1.355	1.422	18.873	1.189
66	61658.0	3.853	9.900	1.489	1.573	17.226	1.172
67	105403.0	3.478	9.030	1.333	1.400	20.084	1.201
68	48889.0	3.493	9.030	1.518	1.595	14.865	1.149
69	93602.0	5.321	13.400	1.328	1.410	25.000	1.250
70	41788.0	5.340	13.400	1.480	1.606	26.250	1.263

MEAN ERROR =17.90% , ERROR BIAS =17.90%

STANDARD DEVIATION OF ERROR: SAMPLE = 5.00% , POPULATION = 5.30%

MEAN TEMPERATURE INCREMENT RATIO = 1.179

STANDARD DEVIATION: SAMPLE = 0.050 , POPULATION = 0.053

Table 12.2a

Overall Error Analysis Comparing Predicted and
Experimental Temperature Increments

a. All Samples :-
Mean Error = 7.52 %, Error Bias = 5.51 %
Standard Deviation of Error = 9.68 %
Mean Temperature Increment Ratio = 1.055
Standard Deviation = 0.08
b. Water, Copper Powder, No. 18 Glass Bead and Acheson Graphite Samples.
Mean Error = 5.39 %, Error Bias = 3.43 %
Standard Deviation of Error = 4.2 %
Mean Temperature Increment Ratio = 1.034
Standard Deviation = 0.059
c. Copper Powder, No. 18 Glass Bead and Acheson Graphite Samples
Mean Error = 5.29 %, Error Bias = 2.9 %
Standard Deviation of Error = 4.42 %
Mean Temperature Increment Ratio = 1.029
Standard Deviation = 0.063
Mean Error for Samples Compared to Prediction
Accuracy for Water only = 3.41 %
Error Bias = -3.41 %

Table 12.2 b

Confidence Limits for Sample and Overall Mean
Temperature Increment Ratios

Sample	Mean Temperature Increment Ratio	Confidence Limits	
		95%	99%
Water	1.058	1.079 1.037	1.089 1.027
Attapulgiis Clay/ Water	0.952	0.967 0.937	0.979 0.925
Copper Powder/Water	1.008	1.031 0.985	1.039 0.977
No. 18 Glass Bead/ Water	1.037	1.097 0.976	1.128 0.946
No. 9 Glass Bead/ Water	1.123	1.175 1.07	1.209 1.037
Acheson Graphite/ Water	1.051	1.089 1.013	1.104 0.998
Aluminium Powder/ Water	1.179	1.217 1.141	1.235 1.123
Overall (all samples)	1.055	1.074 1.036	1.081 1.029
Copper Powder, No. 18 Glass Bead, Acheson Graphite Suspensions	1.029	1.059 0.999	1.068 0.990

Table 12.3

Significance Tests on Differences Between Sample
Variance and Mean Temperature Ratios.

Sample Groups : (1) Water only (2) Attapulgis Clay
 (3) Copper Powder (4) No. 18 Glass Bead
 (5) No. 9 Glass Bead (6) Acheson Graphite
 (7) Aluminium Powder

a. Significance of Difference in Variance

Sample Groups	Degrees of Freedom		Variance Ratio F	Significance
	N ₁	N ₂		
1/2	8	3	7.601	not significant
1/3	18	8	2.681	not significant
1/4	6	8	5.695	probably significant
1/5	4	8	2.528	not significant
1/6	14	8	5.957	significant
1/7	8	8	3.228	not significant
2/3	18	3	20.376	probably significant
2/4	6	3	43.289	significant
2/5	4	3	19.212	probably significant
2/6	14	3	45.279	significant
2/7	8	3	24.539	probably significant
3/4	6	18	2.124	not significant
3/5	18	4	1.061	not significant
3/6	14	18	2.222	not significant

Table 12.3 Cont.

Sample Groups	Degrees of Freedom		Variance Ratio F	Significance
	N ₁	N ₂		
3/7	8	18	1.204	not significant
4/5	6	4	2.253	not significant
4/6	14	6	1.046	not significant
4/7	6	8	1.764	not significant
5/6	14	4	2.357	not significant
5/7	8	4	1.277	not significant
6/7	14	8	1.845	not significant

b. Significance of Difference in Mean Temperature Increment Ratio

Sample Groups	Degree of Freedom	Estimate of Pop ^{ln} Standard Deviation	Student 't'	Significance
1/2	11	0.0257	6.9119	highly significant
1/3	26	0.0434	2.8461	significant
1/4	14	0.051	0.83	not significant
1/5	12	0.036	3.184	significant
1/6	22	0.06	0.278	not significant
1/7	16	0.043	5.966	highly significant
2/3	21	0.0449	2.3042	probably signif- icant.
2/4	9	0.0578	2.3597	probably signif- icant.
2/5	7	0.0361	7.0679	highly significant
2/6	17	0.0655	2.7114	probably signif- icant.

Table 12.3 Cont.

Sample Groups	Degree of Freedom	Estimate of Pop ^{ln.} Standard Deviation	Student 't'	Significance
2/7	11	0.0455	8.3136	highly significant
3/4	24	0.055	1.1820	not significant
3/5	22	0.048	4.7315	highly significant
3/6	32	0.06	2.0781	probably significant
3/7	26	0.05	8.4630	highly significant
4/5	10	0.062	2.358	probably significant
4/6	20	0.072	0.439	not significant
4/7	14	0.061	4.613	highly significant
5/6	18	0.067	2.056	not significant
5/7	12	0.051	1.977	not significant
6/7	22	0.066	4.607	highly significant

Mean Temperature Increment Ratio :-

Water only = 1.058, Standard Deviation = 0.028

Attapulugus Clay/Water Suspension = 0.94, Standard Deviation = 0.014

Copper Powder/Water Suspension = 0.998, Standard Deviation = 0.044.

Significance of Differences Between Prediction for Water and Suspensions:-

Sample Groups 1) Water 2) Attapulugus Clay/Water
 3) Copper Powder/Water

a) Significance of Difference in Variance

Sample Groups	Degrees of Freedom		Variance Ratio F	Significance
	N ₁	N ₂		
1/2	8	3	3.54	not significant
1/3	18	8	2.358	not significant

b) Significance of Difference in Mean Temperature Increment Ratio

Sample Groups	Degree of Freedom	Estimate of Pop ^{ln.} Standard Deviation	Student 't'	Significance
1/2	11	0.026	7.457	highly significant
1/3	26	0.041	3.612	Significant.

TABLE 12.4 CONTINUED INDIVIDUAL PREDICTION RESULTS
 (A) FLUID - ATTAPULGUS CLAY/WATER SUSPENSION
 PARTICLE SIZE = 1-2 MICRONS

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
2	67785.0	0.308	0.790	1.661	1.633	4.236	0.958
3	61209.0	1.325	3.340	1.709	1.671	5.360	0.946
4	61776.0	2.098	5.230	1.684	1.630	7.895	0.921
5	61657.0	2.912	7.180	1.599	1.560	6.511	0.935

MEAN ERROR = 6.00% , ERROR BIAS = -6.00%

STANDARD DEVIATION OF ERROR: SAMPLE = 1.36% , POPULATION = 1.57%

MEAN TEMPERATURE INCREMENT RATIO = 0.940

STANDARD DEVIATION: SAMPLE = 0.014 , POPULATION = 0.016

TABLE 12.4 CONTINUED INDIVIDUAL PREDICTION RESULTS
(B) FLUID COPPER POWDER/WATER SUSPENSION
PARTICLE SIZE = 3.3 MICRONS

TEST NO	REYNOLDS NUMBER	VOLUME FRACTION %	WEIGHT FRACTION %	NORMALISED OUTLET TEMPERATURE		ERROR %	TEMPERATURE INCREMENT RATIO
				EXPERIMENTAL	PREDICTED		
9	106640.0	0.086	0.710	1.522	1.532	1.916	1.019
10	42829.0	0.086	0.710	1.896	1.895	0.112	0.999
11	107208.0	0.085	0.710	1.581	1.602	3.614	1.036
12	37176.0	0.101	0.830	2.104	2.004	9.058	0.909
13	70234.0	0.100	0.830	1.750	1.747	0.400	0.996
14	146506.0	0.099	0.830	1.380	1.394	3.684	1.037
15	152354.0	0.155	1.260	1.370	1.371	0.270	1.003
16	68982.0	0.153	1.260	1.615	1.632	2.764	1.028
17	136897.0	1.995	14.600	1.449	1.459	2.227	1.022
18	88683.0	1.931	14.200	1.465	1.450	3.226	0.968
19	156413.0	1.851	13.700	1.317	1.338	6.625	1.066
20	162224.0	1.848	13.700	1.395	1.357	9.620	0.904
21	129811.0	1.852	13.700	1.430	1.410	4.651	0.953
23	115411.0	3.984	25.800	1.412	1.414	0.485	1.005
24	95867.0	1.946	14.400	1.358	1.355	0.838	0.992
25	164966.0	5.829	34.300	1.349	1.350	0.287	1.003
26	73502.0	6.314	36.000	1.649	1.672	3.544	1.035
27	155208.0	7.119	39.200	1.396	1.417	5.303	1.053
28	38474.0	7.000	38.600	1.664	1.625	5.873	0.941

MEAN ERROR = 3.39% , ERROR BIAS = -0.16%
STANDARD DEVIATION OF ERROR: SAMPLE = 2.82% , POPULATION = 2.90%
MEAN TEMPERATURE INCREMENT RATIO = 0.998
STANDARD DEVIATION: SAMPLE = 0.044 , POPULATION = 0.045

Table 12.5

Prediction Comparison for High and Low Volume
Fraction Solid Copper Powder Suspensions.

1) High Volume Fraction (1.848 - 7.119 per cent)

Mean Error = 4.56%, Error Bias = 1.22%

Standard Deviation = 2.77%

Mean Temperature Increment Ratio = 1.012

Standard Deviation = 0.052

2) Low Volume Fraction (0.085 - 0.153 per cent)

Mean Error = 2.73%, Error Bias = 0.33%

Standard Deviation = 2.75%

Mean Temperature Increment Ratio = 1.003

Standard Deviation = 0.039

Significance Tests on Differences Between Mean Temperature
 Increment Ratios:

Samples	Degree of Freedom	Student 't'	Significance
Water/High Vol. Frac.	18	2.28	probably significant
Water/Low Vol. Frac.	15	3.2	highly significant
High/Low Vol. Frac.	17	0.38	not significant

Table 12.6

Comparison of Experimental and Predicted Temperature
Increment Ratios on the Basis of Volume Fraction Solid.

a) Volume Fraction solid between 0 and 0.015 Mean error = 2.73%, Error bias = 0.33% Standard deviation of error = 2.76 % Mean Temperature Increment Ratio = 1.003 Standard deviation = 0.039
b) Volume Fraction Solid between 0.015 and 0.03 Mean error = 4.61%, Error bias = 0.21 % Standard deviation of error = 2.86 % Mean temperature increment ratio = 1.002 Standard deviation = 0.054
c) Volume Fraction Solid between 0.03 and 0.06 Mean error = 4.78 %, Error bias = 3.28% Standard deviation of error = 2.71% Mean temperature increment ratio = 1.033 Standard deviation = 0.044
d) Volume Fraction Solid between 0.06 and 0.08 Mean error = 4.11 %, Error bias = 2.72 % Standard deviation of error = 2.22% Mean temperature increment ratio = 1.027 Standard deviation = 0.038

Table 12.6 cont.

e) Volume Fraction Solid between 0.08 and 0.11 Mean error = 8.15%, Error bias = 5.38% Standard deviation of error = 5.11% Mean temperature increment ratio = 1.054 Standard deviation = 0.08
f) Volume Fraction Solid between 0.11 and 0.19 Mean error = 10.7%, Error bias = 8.67% Standard deviation of error = 6.93% Mean temperature increment ratio = 1.087 Standard deviation = 0.094
g) Overall Mean error = 5.29%, Error bias = 2.9% Standard deviation of error = 4.42% Mean temperature increment ratio = 1.029 Standard deviation = 0.063

Table 12.7

Significance Tests on Differences of Variance and Mean
Temperature Increment Ratio Between Suspension Sample
Groups Divided on the Basis of Volume Fraction Solid.

Sample Group Volume Fraction Solid

- (1) 0.0 - 0.015 (2) 0.015 - 0.03 (3) 0.03 - 0.06
(4) 0.06 - 0.08 (5) 0.08 - 0.11 (6) 0.11 - 0.19

a) Significance of Difference Variance

Sample Groups	Degrees of Freedom		Variance Ratio F	Significance
	N ₁	N ₂		
1/2	6	7	2.013	not significant
1/3	7	7	1.3	not significant
1/4	7	7	1.0297	not significant
1/5	5	7	4.4784	not significant
1/6	3	7	6.823	not significant
2/3	6	7	1.548	not significant
2/4	6	7	2.073	not significant
2/5	5	6	2.224	not significant
2/6	3	6	3.389	not significant
3/4	7	7	1.339	not significant
3/5	5	7	3.443	not significant
3/6	3	7	5.246	not significant
4/5	5	7	4.611	not significant
4/6	3	7	7.026	not significant
5/6	3	5	1.524	not significant

Table 12.7 cont.

b. Significance of Difference in Mean Temperature Increment Ratio

Sample Groups	Degree of Freedom	Estimate of Pop ^{ln.} Standard Deviation	Student 't'	Significance
1/2	13	0.05	0.0481	not significant
1/3	14	0.044	1.3293	not significant
1/4	14	0.041	1.1614	not significant
1/5	12	0.065	1.4438	not significant
1/6	10	0.069	1.9879	not significant
2/3	13	0.053	1.125	not significant
2/4	13	0.05	0.9731	not significant
2/5	11	0.073	1.2699	not significant
2/6	9	0.079	1.7184	not significant
3/4	14	0.044	0.2561	not significant
3/5	12	0.067	0.5799	not significant
3/6	10	0.071	1.239	not significant
4/5	12	0.064	0.7644	not significant
4/6	10	0.068	1.4253	not significant
5/6	8	0.096	0.5338	not significant

Table 12.8

Comparison of Experimental and Predicted Temperature
Increment Ratio on the Basis of Reynolds Number

a) Reynolds Number between $1.0 \cdot 10^4$ and $5.0 \cdot 10^4$ Mean Error = 2.74%, Error Bias = -2.51% Standard deviation of error = 2.75 % Mean temperature increment ratio = 0.975 Standard deviation = 0.029
b) Reynolds Number between $5.0 \cdot 10^4$ and $7.0 \cdot 10^4$ Mean Error = 5.66 %, Error bias = 5.1 % Standard deviation of error = 4.45 % Mean Temperature increment ratio = 1.051 Standard deviation 0.051
c) Reynolds Number between $7.0 \cdot 10^4$ and $9.0 \cdot 10^4$ Mean Error = 5.17 %, Error bias = 2.95 % Standard deviation of error = 4.29 % Mean temperature increment ratio = 1.03 Standard deviation = 0.06
d) Reynolds Number between $9.0 \cdot 10^4$ and $1.1 \cdot 10^5$ Mean Error = 6.65 %, Error bias = 5.42 % Standard deviation of error = 4.43 % Mean temperature increment ratio = 1.054 Standard deviation = 0.059

Table 12.8 cont.

e) Reynolds Number between $1.1 \cdot 10^5$ and $1.3 \cdot 10^5$ Mean Error = 5.83 %, Error bias = 4.7 % Standard deviation of error = 3.38% Mean temperature increment ratio = 1.047 Standard deviation = 0.048
f) Reynolds Number between $1.3 \cdot 10^5$ and $1.5 \cdot 10^5$ Mean Error = 5.1 %, Error bias = 0.57 % Standard deviation of error = 3.38% Mean temperature increment ratio = 1.007 Standard deviation = 0.061
g) Overall Mean Error = 5.39 %, Error Bias = 3.43% Standard deviation of error = 4.18 % Mean temperature increment ratio = 1.034 Standard deviation = 0.059

Table 12.9

Significance Tests on Differences of Variance and Mean
Temperature Increment Ratio Between Sample Groups
Divided on the Basis of Reynolds Number

Sample Group Reynolds Number

(1) $1.5 \cdot 10^4 - 5.0 \cdot 10^4$ (2) $5.0 \cdot 10^4 - 7.0 \cdot 10^4$ (3) $7.0 \cdot 10^4 - 9.0 \cdot 10^4$
(4) $9.0 \cdot 10^4 - 1.1 \cdot 10^5$ (5) $1.1 \cdot 10^5 - 1.3 \cdot 10^5$ (6) $1.3 \cdot 10^5 - 1.6 \cdot 10^5$

a. Significance of Difference in Variance

Sample Groups	Degrees of Freedom		Variance Ratio F	Significance
	N_1	N_2		
1/2	14	6	2.764	not significant
1/3	5	6	4.348	not significant
1/4	10	6	3.788	not significant
1/5	6	6	2.712	not significant
1/6	3	6	4.904	probably significant
2/3	5	14	1.573	not significant
2/4	10	14	1.371	not significant
2/5	14	6	1.019	not significant
2/6	3	14	1.774	not significant
3/4	5	10	1.148	not significant
3/5	5	6	1.603	not significant
3/6	3	5	1.128	not significant
4/5	10	6	1.397	not significant
4/6	3	10	1.294	not significant
5/6	3	6	1.809	not significant

Table 12.9 cont.

b. Significance of Difference in Mean Temperature Increment Ratio

Sample Groups	Degree of Freedom	Estimate of Pop ^{ln.} Standard Deviation	Student 't'	Significance
1/2	20	0.047	3.521	highly significant
1/3	11	0.05	1.961	not significant
1/4	16	0.053	3.135	highly significant
1/5	12	0.043	3.137	highly significant
1/6	9	0.048	1.063	not significant
2/3	19	0.057	0.785	not significant
2/4	24	0.057	0.142	not significant
2/5	20	0.053	0.164	not significant
2/6	17	0.056	1.403	not significant
3/4	15	0.063	0.768	not significant
3/5	11	0.059	0.534	not significant
3/6	8	0.068	0.524	not significant
4/5	16	0.058	0.254	not significant
4/6	13	0.064	1.277	not significant
5/6	9	0.059	1.096	not significant

Table 12.10

Comparison between Single and Two-Component Fluids.

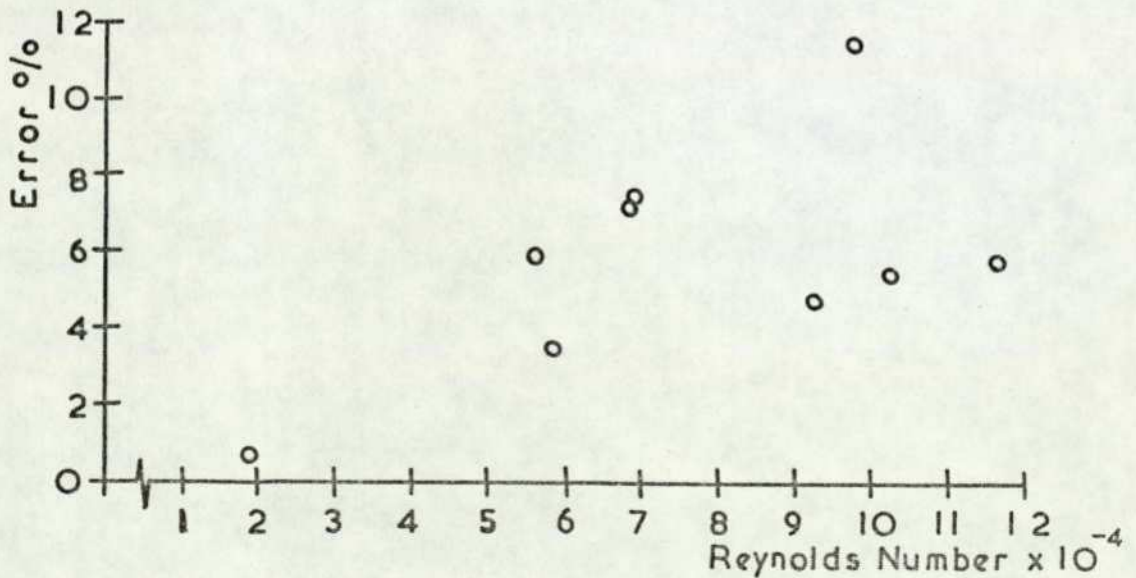
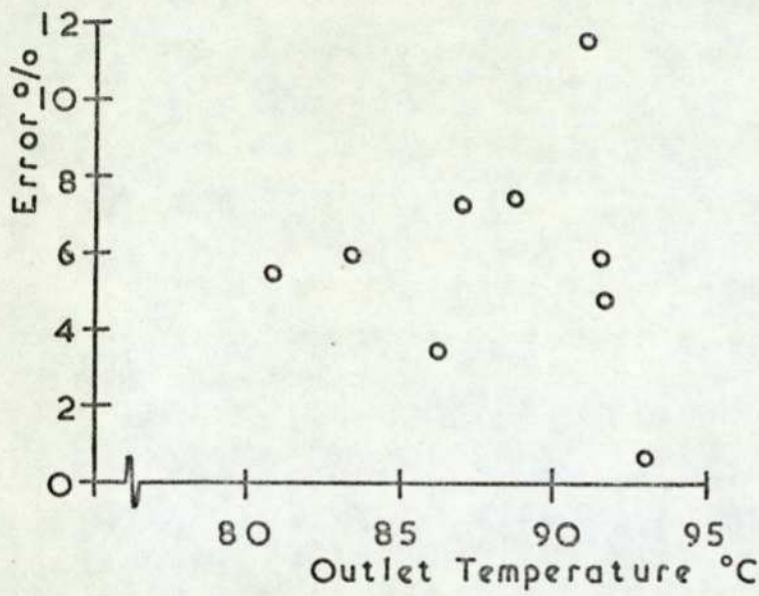
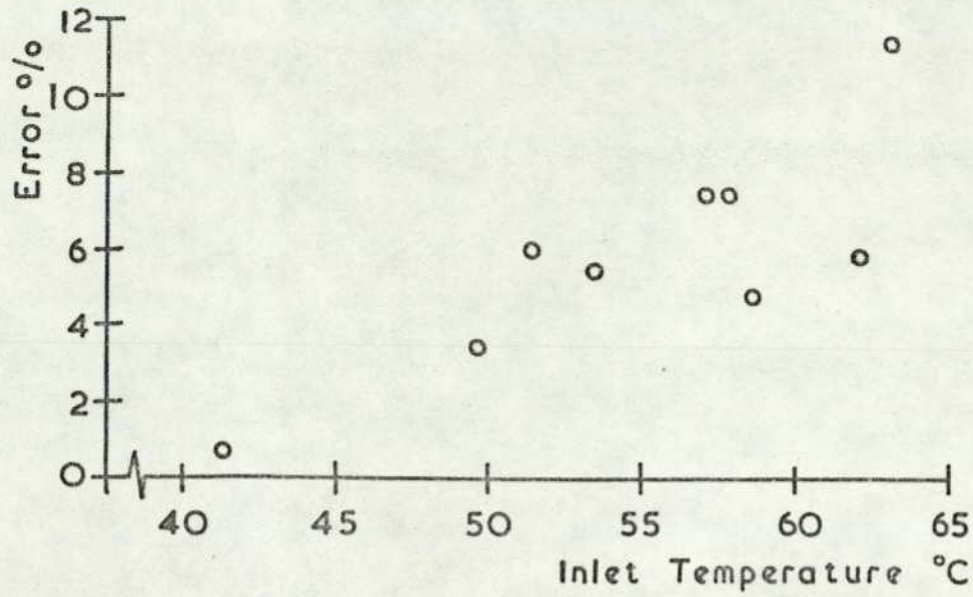
1. Energy Transfer

Test No.	Solid Material	ϕ per cent	Equal Volume Flow Rate	Equal Reynolds Number	Equal Mass Flow Rate
27	Copper	7.12	1.02	1.34	0.67
36	Glass	9.64	0.97	1.24	0.82
41	Glass	22.2	0.92	1.41	0.64
55	Graphite	13.9	0.93	1.42	0.82
59	Graphite	18.78	0.92	1.53	0.77

2. Pressure Drop

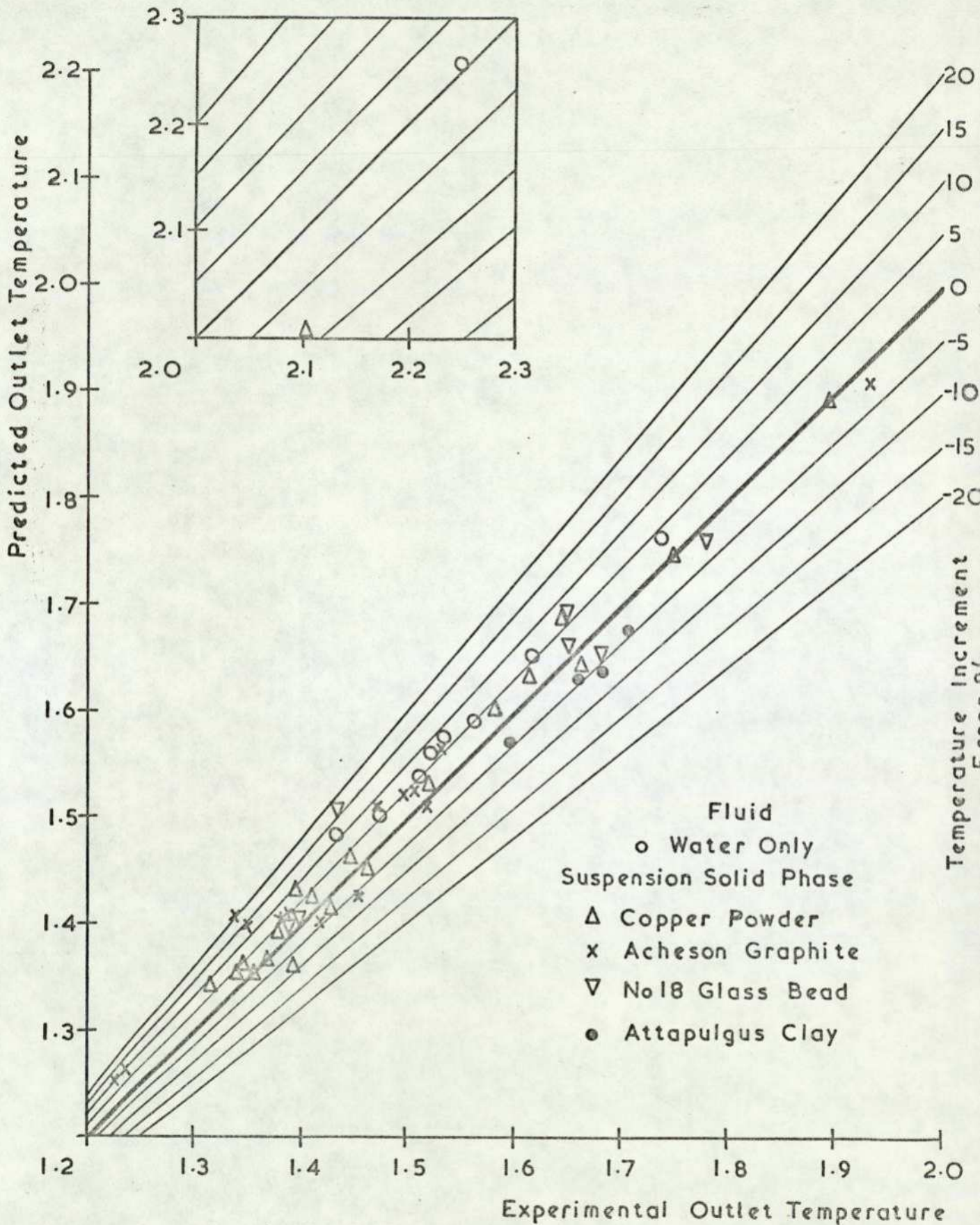
27	Copper	7.12	1.61	2.70	0.74
36	Glass	9.64	1.23	1.94	0.91
41	Glass	22.2	1.51	3.29	0.78
55	Graphite	13.9	1.22	2.64	0.95
59	Graphite	18.78	1.29	3.30	0.93

Where the value given is the ratio between the performance of the suspension and the performance of water alone.



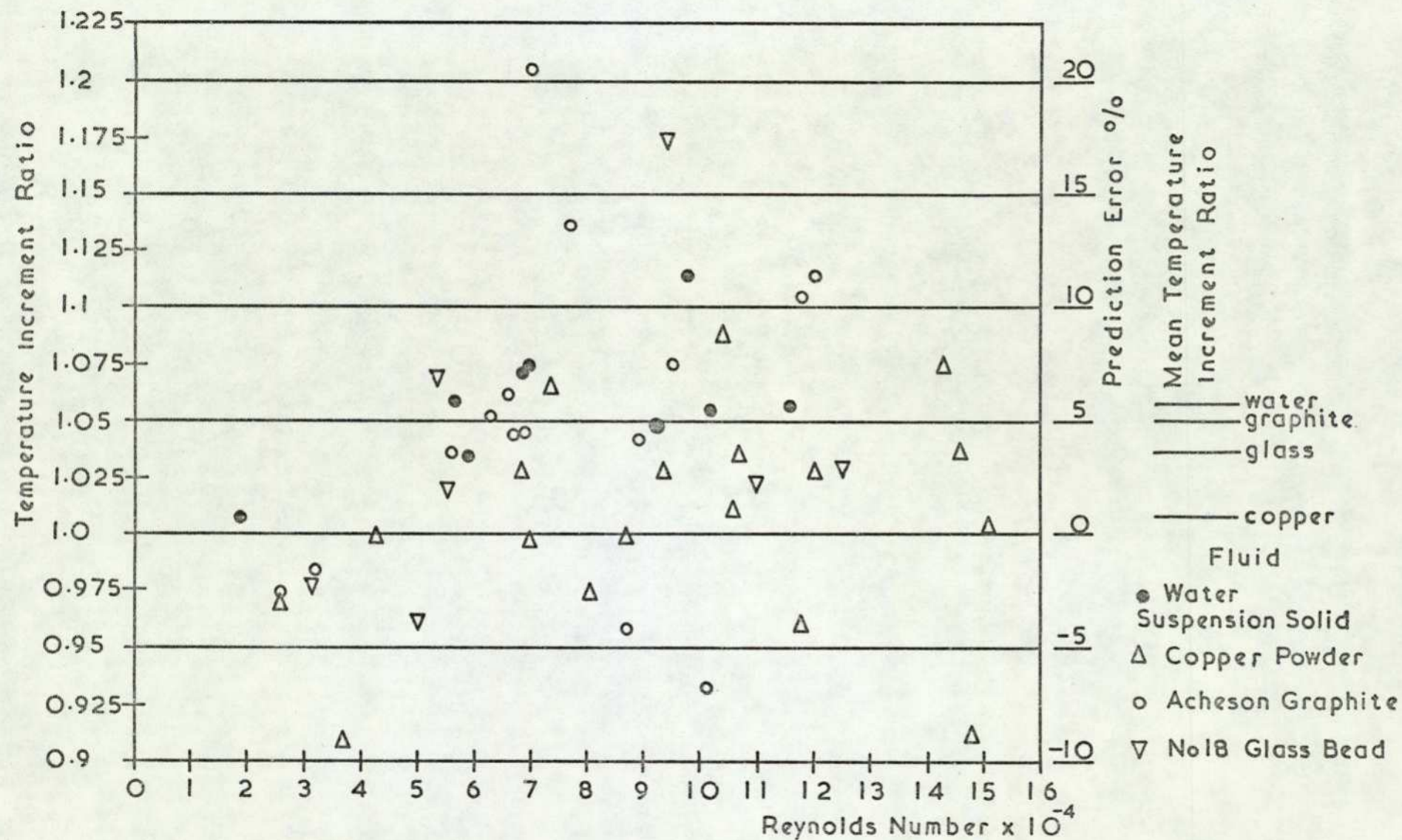
TREND OF ERROR BETWEEN EXPERIMENTAL AND PREDICTED TEMPERATURE INCREMENTS WITH INLET AND OUTLET TEMPERATURES AND INLET REYNOLDS NUMBER FOR SINGLE-COMPONENT FLUID (ORR AND DALLAVALLE RESULTS)

Figure 12.1

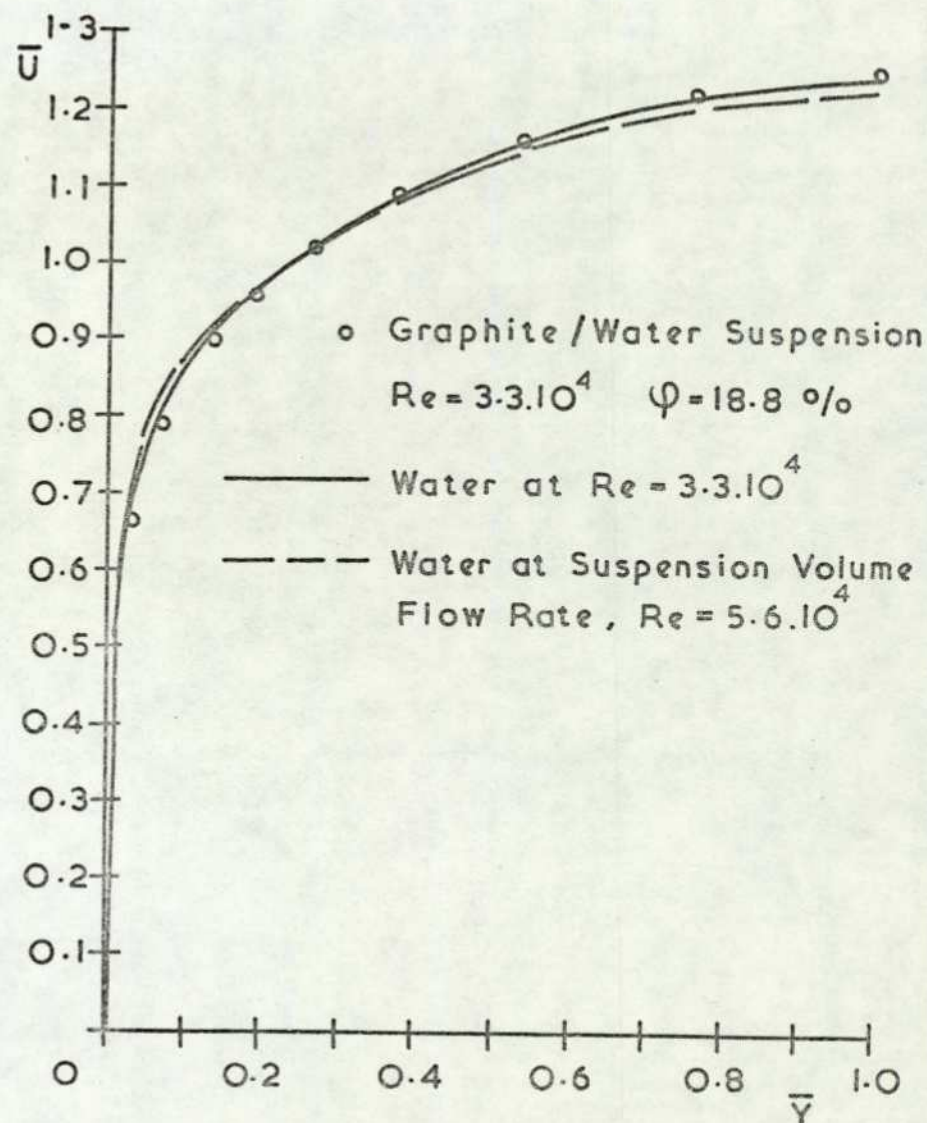
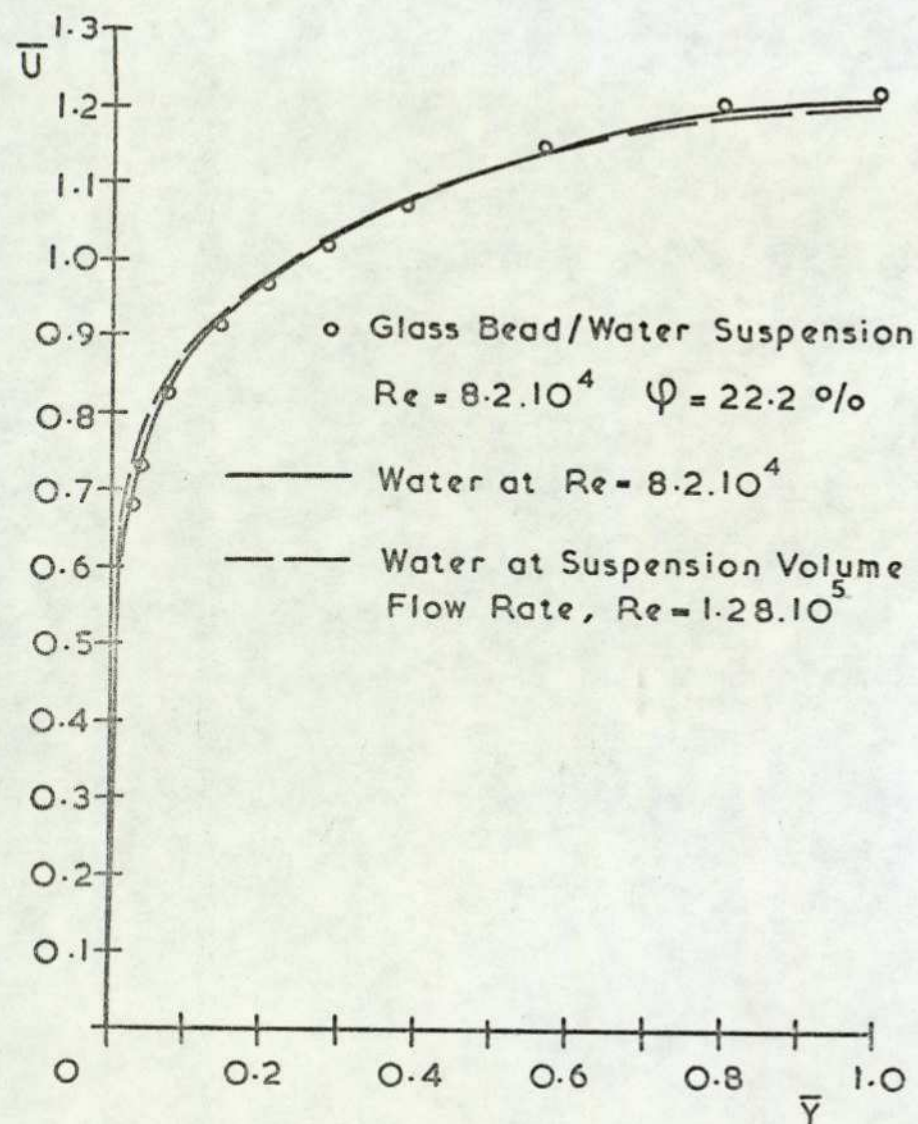


COMPARISON OF EXPERIMENTAL AND PREDICTED
NORMALISED HEATER OUTLET TEMPERATURES

Figure 12.2

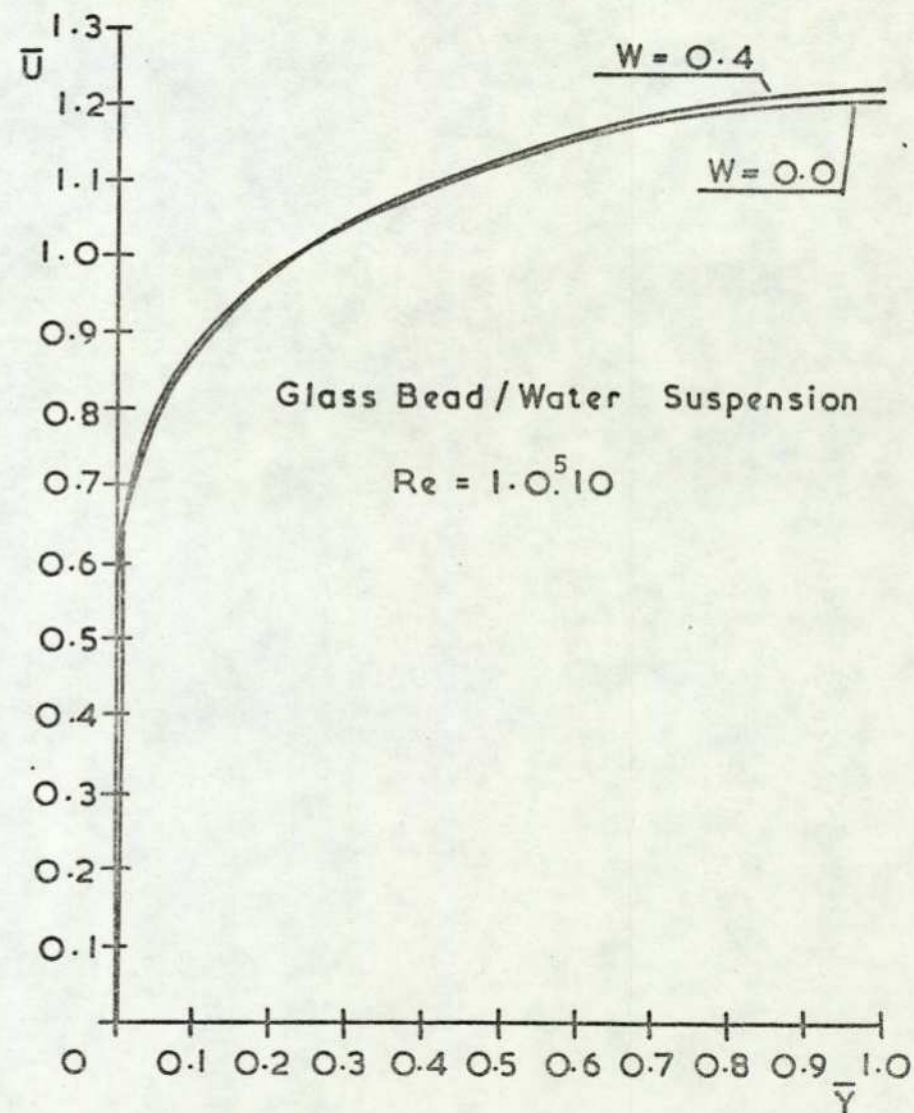
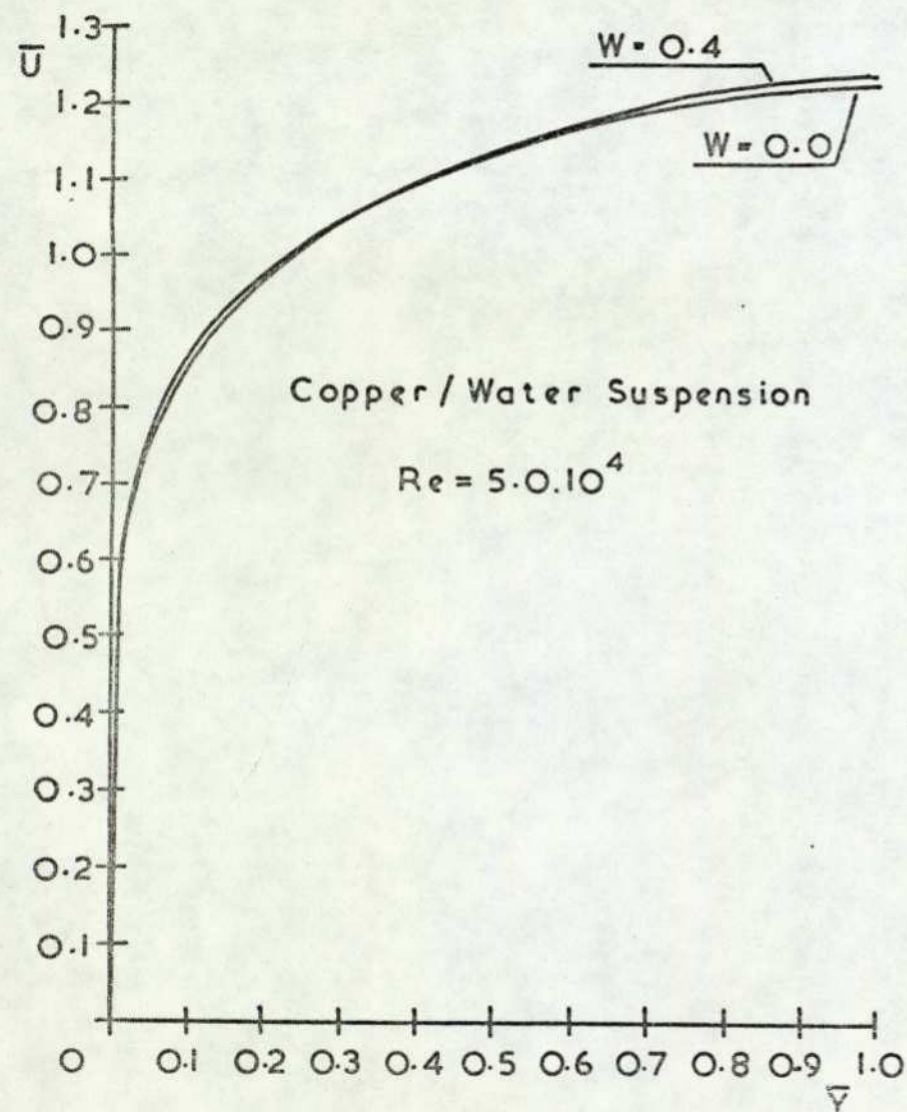


VARIATION IN TEMPERATURE INCREMENT RATIO WITH REYNOLDS NUMBER
Figure 12.4



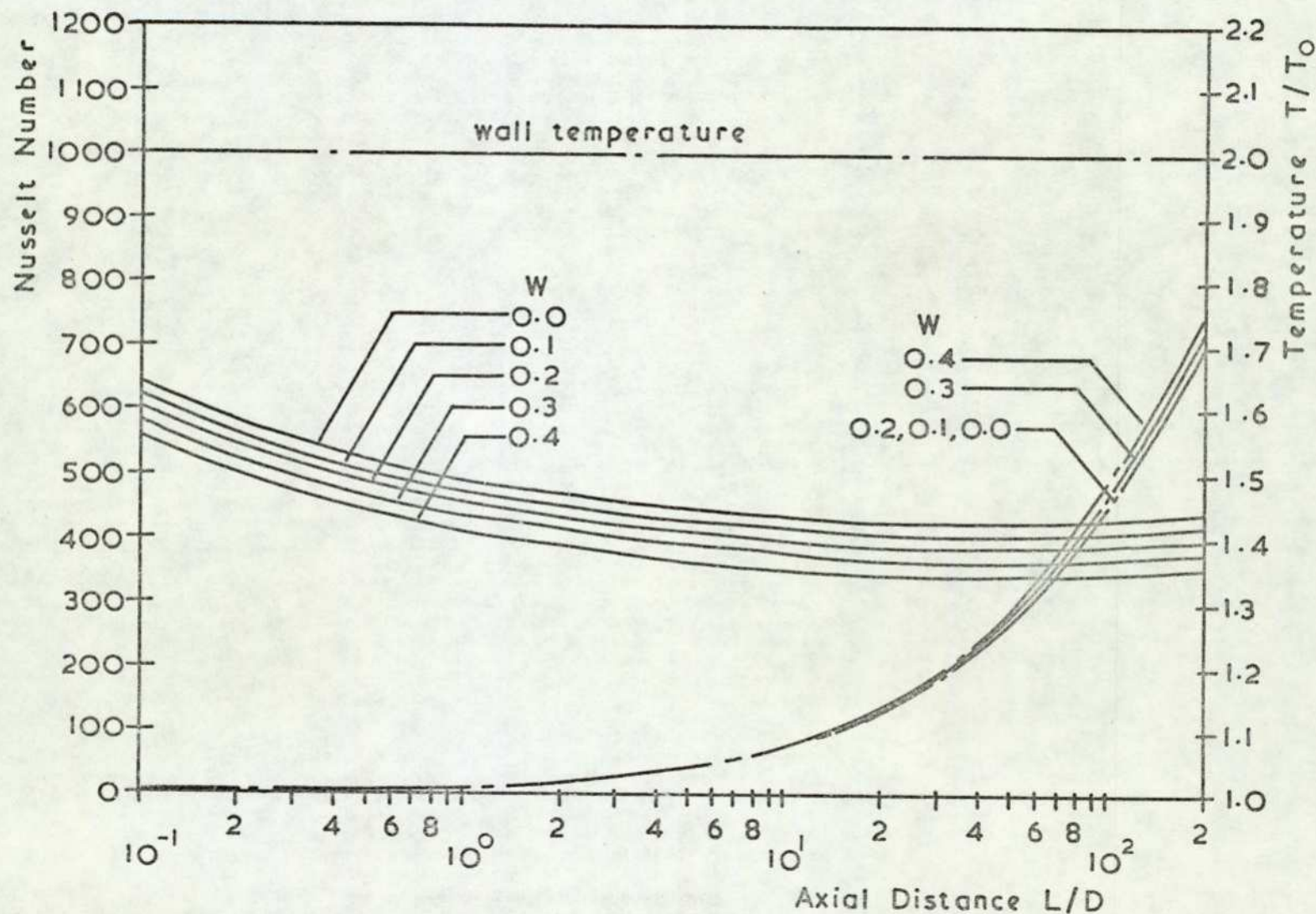
COMPARISON OF PREDICTED ISOTHERMAL VELOCITY PROFILES FOR ONE AND TWO COMPONENT FLOW

Figure 12.5



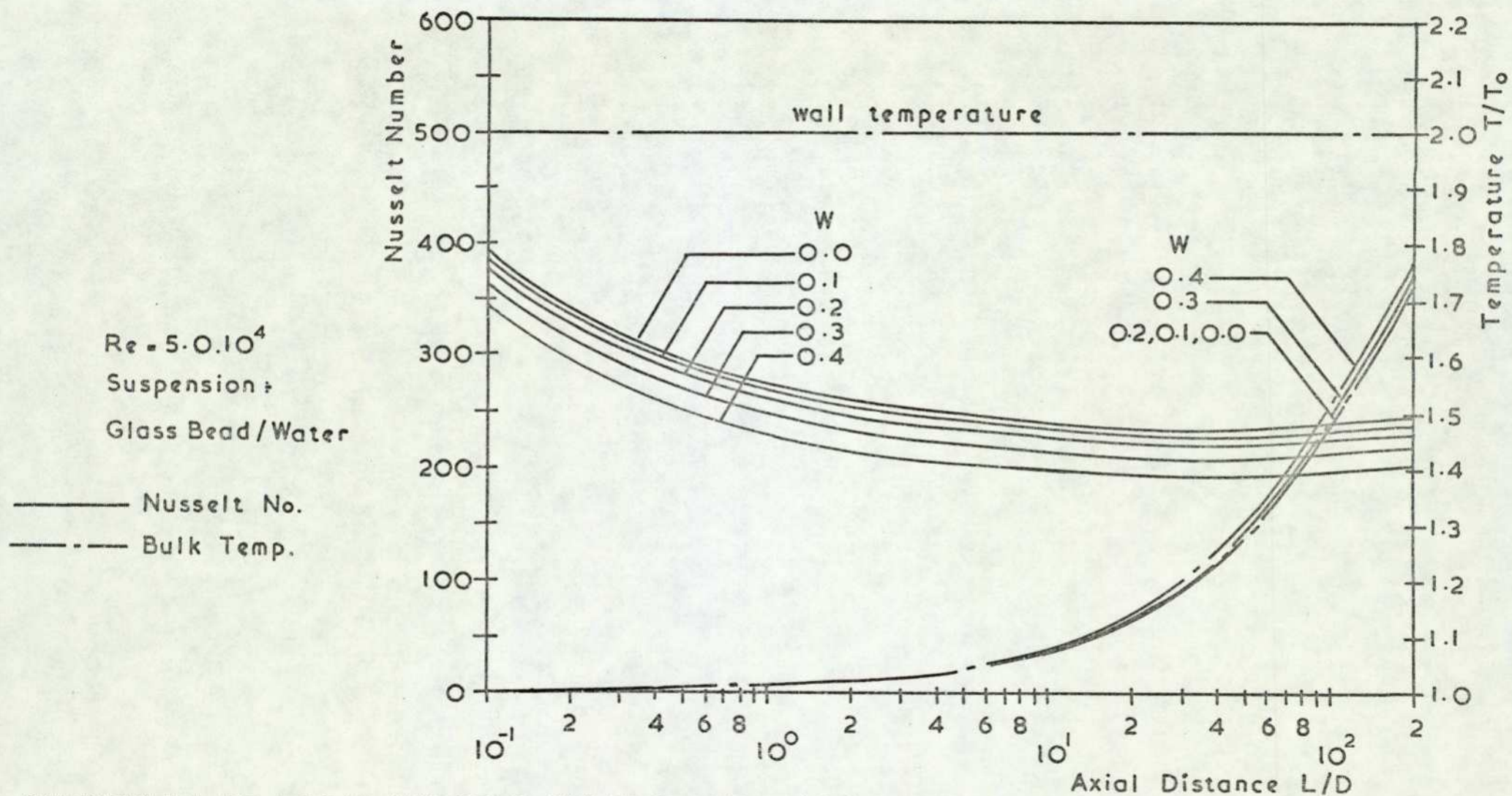
PREDICTED ISOTHERMAL VELOCITY PROFILES FOR TWO-COMPONENT FLUIDS

Figure 12.6



PREDICTION OF LOCAL NUSSELT NUMBER AND BULK TEMPERATURE FOR TWO-COMPONENT FLOW

Figure 12.7



PREDICTION OF LOCAL NUSSELT NUMBER AND BULK TEMPERATURE FOR TWO-COMPONENT FLOW

Figure 12.8

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Appendix I : Analysis of Heat Transfer Results.

a. Definition of Terms.

Let:

T_i = bulk temperature at inlet to heater

T_o = bulk temperature at outlet from heater

E = error in prediction as fraction of temperature increment

R = mean temperature increment ratio.

and where subscript:

p denotes predicted value

e denotes experimental value

then:

$$E = \frac{\left| \left[\frac{T_o}{T_i} \right]_p - \left[\frac{T_o}{T_i} \right]_e \right|}{\left[\frac{T_o - T_i}{T_i} \right]_e}$$

$$R = \frac{\left[\frac{T_o - T_i}{T_i} \right]_p}{\left[\frac{T_o - T_i}{T_i} \right]_e}$$

$$\text{Variance of error} = \frac{1}{n} \sum (E - \bar{E})^2$$

hence

$$V_E = \bar{E}^2 - \overline{E^2}$$

Variance of mean temperature increment ratio

$$V_R = \bar{R}^2 - \overline{R^2}$$

b. Significance Tests

- i) Significance of difference between sample mean temperature increment ratios:

The student 't' test is used to determine whether the difference between the mean values of two samples drawn from different sources is significant of a real difference between the parent sources. The best estimate of the population variance is obtained from a pooled estimate on the null hypothesis that the two samples are drawn from populations identical as to mean and variance.

$$\hat{\sigma}^2 = \frac{1}{n_1 + n_2 - 2} \left[n_1 V_{R_1} + n_2 V_{R_2} \right]$$

n_1, n_2 are number of observations in the sample.

The best estimate of the standard error for the difference of the means of two samples of this size is:

$$\sigma_{n_1 - n_2} = \hat{\sigma} \left[\frac{1}{n_1} + \frac{1}{n_2} \right]^{0.5}$$

Hence students t :

$$t = \frac{\bar{R}_1 - \bar{R}_2}{\hat{\sigma} \left[\frac{1}{n_1} + \frac{1}{n_2} \right]^{0.5}}$$

where the number of degrees of freedom is $n_1 + n_2 - 2$.

The level of significance is then obtained from tables of the distribution of 't', which gives the values of 't' corresponding to various values of the probability of a random value falling outside the limits $\pm t$.

ii) Significance of differences in sample variance:

A logical precursor to 't' test which determines if the sample variances are sufficiently alike to warrant assuming that they are independent estimates of the same population variance is given by the 'F' test.

The variance ratio 'F' is defined as:

$$F = \frac{\text{greater estimate of variance of the population}}{\text{lesser estimate of variance of the population}}$$

The estimates are the best estimates arrived at by applying Bessel's correction to the sample variances:

$$\hat{\sigma}^2 = \left[\frac{n}{n-1} \right] s^2$$

The degrees of freedom are $(n_1 - 1)$ and $(n_2 - 1)$, with the significance being determined from variance ratio tables.

iii) Confidence levels for mean values:

If the values for the population, of mean and standard deviation, are unknown the sample mean, m , provides an estimate of population mean and the sample standard deviation, s , provides an estimate of the population variance.

The magnitude of $S/\sqrt{n}$ provides a measure of uncertainty in the value of m as an estimate of the population value. This uncertainty can most simply be expressed by means of a confidence interval:

$$P[m - t(p) S/\sqrt{n} \leq \mu \leq m + t(p) S/\sqrt{n}] = (100 - p) \%$$

This expression states that, with a confidence expressed as a probability of $(100 - p)$ per cent, the unknown mean value lies within the given interval.

c. Comparison of Energy Transfer Rates.

1. Suspension and pure fluid having the same volume flow rate:

$$\frac{\dot{e}_s}{\dot{e}_c} = \left[\varphi \left[\frac{\rho_d}{\rho_c} - 1 \right] + 1 \right] \cdot \left[W \left[\frac{C_{P_d}}{C_{P_c}} - 1 \right] + 1 \right] \cdot \frac{\Delta T_s}{\Delta T_c}$$

2. Suspension and fluid having the same Reynolds number:

$$\frac{\dot{e}_s}{\dot{e}_c} = (1 + 5.5 \varphi) \cdot \left[W \left[\frac{C_{P_d}}{C_{P_c}} - 1 \right] + 1 \right] \cdot \frac{\Delta T_s}{\Delta T_c}$$

3. Suspension and fluid having the same mass flow rate:

$$\frac{\dot{e}_s}{\dot{e}_c} = \left[W \left[\frac{C_{P_d}}{C_{P_c}} - 1 \right] + 1 \right] \cdot \frac{\Delta T_s}{\Delta T_c}$$

Appendix II

Computer Program Documentation for Prediction of the Thermal Conductivity of Suspensions.

This appendix details the computer programs used for the evaluation of the thermal conductivity of suspensions using either the unit cell model, developed in chapter 4 of this report, or the random placement model which was developed in chapter 5. Both models, employ the general finite difference approximation technique for solving Laplace's Equation in three dimensions; this follows the procedure described in chapter 3.

Apart from the program listings it is not intended to give a full description of the programming logic of each implementation since the programs do not contain any novel features. Sufficient information will be given to enable anyone desiring to do so to follow the program listings or without any need for study of the programs to utilise them as they stand.

The programming language used is FORTRAN IV and the computer used was an I.C.L. 1905; some features of I.C.L. software are used, notably free format input of data, but this may easily be adapted to conform with any other FORTRAN implementation.

1. UNIT CELL MODEL

Program name MESH, subroutine name SØLV

a) Program Run Time Requirements.

The program may be contained on about 240 cards. Using an I.C.L. 1905 computer the core storage requirement is in the region of 18,000 words and for each suspension conductivity value calculated an execution time in the region of 100 seconds (real time) is necessary.

b) Program Input

Data input to the program is on cards with the format statements being of free form.

The cards required are:-

1st and $(n + 2)^{\text{th}}$ card contain the values of KI, TP

where KI is an integer control value

if $KI > 0$ program execution continues

$KI \leq 0$ program stops

TP is the value of the component conductivity ratio
for the system under test.

n cards each containing one value of MI, V

where MI is an integer control value

if $MI > 0$ program execution continues

$MI \leq 0$ control returns to input of $(n + 2)^{\text{th}}$ card

V is the volume fraction solid of the system under
consideration.

c) Program Output

Line printer output gives values of TP, TS, V, QIN, QØUT under
relevant headings

where TP is component conductivity ratio

TS is normalised suspension conductivity

V is volume fraction solid for the system

QIN is the heat flux input to the model

QOUT is the heat flux output from the model.

d) Parameters Defined in DATA Statement.

TL : will be equal to unity if TP is the component conductivity ratio. If absolute conductivity values are used TL will be the conductivity of the continuous phase.

TC, TF : are model boundary temperatures and may be chosen arbitrarily to give any desired temperature gradient across the model

A : is the grid spacing at the centre of the model and must be set to a value of 0.2 units.

e) Solution Procedure.

The division of the unit cell model into a grid network is permanently fixed to give a 5 x 5 x 5 nodal array. The problem of the steady state temperature distribution within the model then reduces to a set of 75 simultaneous equations. Being written as a matrix equation, $\underline{A} \underline{T} = \underline{B}$, a solution is obtained using a direct method; Gauss elimination with pivotal consideration being the particular technique employed.

Within the master segment of the program the grid spacing in the outer section of the model is determined from consideration of the system volume fraction solid and the coefficients of the matrices $\underline{A}$ and $\underline{B}$ are assigned according to the general finite difference equation (3.23). The two matrices are combined to give a new matrix $\underline{C}$ where last column consists of $\underline{B}$. In this form it is transferred to the

subroutine $S\phi LV$ where the solution procedure is performed. This subroutine may therefore be replaced by any suitable procedure for solving matrix equations if desired. The solution for temperature distribution is returned to the master segment as the last column of $\underline{C}$. The heat flux across both boundary surfaces is now determined using equations (3.27) and (3.28) and the overall system conductivity obtained from equation (3.29).

2. RANDOM PLACEMENT MODEL.

Program name RACØ

Subroutine names SETK, TNC, SØLVE

Function name FPMCRV

a) Program Run Time Requirements

The program may be contained on about 240 cards

Core storage requirement is approximately 11,000 words with an execution time of about 250 seconds (real time) being necessary for each suspension conductivity calculated using an I.C.L. 1905 computer with the given nodal division.

The random number generator is provided by the function segment FPMCRV which is a unit of the I.C.L. scientific subroutine package. If access to this routine is not available a substitute must be provided, preferably one using the multiplicative congruential method for pseudo-random number generation.

b) Program Input

Data input to the program is on cards using a free format.

The cards required are:

1st and $(n + 2)^{th}$ card contain the values of MC, KS

where MC if positive is an integer giving the number of nodes per co-ordinate direction.

if zero - program execution ceases

KS - real; value of the component conductivity ratio for the system under consideration.

n cards each containing one value of M, VF

where M is an integer control value

if M is non-zero, program execution continues

if $M = 0$; control returns to input of $(n + 2)^{\text{th}}$ card

VF is the system volume fraction solid

c) Program Output

Output from the program is by line printer and is in two sections

- i) Details of the basic unit conductivity; values of KS, KCUBE, CVØL, IT, QI, QØ are given

where KS : is a real and is the value of the component conductivity ratio

KCUBE : a real, normalised conductivity of the basic unit

CVØL is the basic unit volume fraction solid

IT is the number of iterations performed in reaching a solution.

QI, QØ are the heat fluxes across the model boundary surfaces.

- ii) Details of the suspension conductivity; values of KS, KSUS, VF, SVF, IT, QI, QØ are given

where KS is the component conductivity ratio

KSUS is the normalised suspension conductivity

VF is the system volume fraction solid specified

SVF is the system volume fraction actually assigned

IT is the number of iterations performed in reaching a solution

QI, QØ are the heat fluxes across the model boundary surfaces.

d) Parameters Defined in Data Statements

In the master segment:

TIN, TØUT : model boundary temperatures; may be chosen arbitrarily to give a temperature gradient across the model

KL : real value equal to unity if conductivities are in normalised form; if actual conductivities are used

KL is the continuous phase value.

In subroutine SØLVE

ACF : iteration relaxation factor, chosen on the basis of
experience.

TØL : maximum acceptable difference between successive estimates
at each node for iterative system assumed to have
converged to a solution.

e) Solution Procedure

In this implementation of the random placement model the division of both the basic unit and the suspension system into a grid network may be made as coarse or fine as desired, with the proviso that an increase in the nodal division over 10 per co-ordinate direction will require an increase in core storage, run time and a re-definition of the dimension statements contained in the program listing.

The program master segment controls the evaluation of the suspension conductivity. Simulation of the model is achieved, in the case of the basic unit by assigning nodal domain conductivities according to the node position within the unit cell and for the case of the suspension system by assigning basic units within the field of the continuous phase according to a random fashion. The remainder of the method being identical for the assessment of both the basic unit and suspension conductivities is therefore supplied as subroutine to avoid duplication. Subroutine SETK sets the conductivity of the whole field initially to the continuous phase value.

Subroutine TNC assigns an initial temperature to each node based upon a linear temperature gradient between the model boundary faces and gives a nodal code value to each node which is dependant upon its position in the model.

Subroutine *SOLVE*, as the name implies, solves a given set of simultaneous equations to give the steady state temperature distribution throughout the model and hence determines the heat fluxes across both boundary faces and the effective thermal conductivity of either the basic unit or suspension system. The solution of the simultaneous equations involves the use of an indirect method; a Gauss-Siedal iterative scheme with Liebmann extrapolation being utilised. The iteration process scans the nodal field sequentially and for each node determines the grid line conductivities to adjacent nodes, the nodal code is used to direct control of the program to the relevant finite difference equation, which is solved in terms of node temperature following equation (5.2). The iteration process is assumed to have converged to a solution when successive estimates of temperature at each node agree within a specified amount. Equations (3.27) and (3.28) are used to give the heat fluxes and equation (3.29) gives the effective thermal conductivity.

UNIT CELL MODEL PROGRAM

(a) MASTER SEGMENT

```

MASTER MESH
C *****
C PROGRAM FOR DETERMINING EFFECTIVE THERMAL CONDUCTIVITY OF
C TWO-COMPONENT MIXTURES WHICH CONTAIN SPHERICAL PARTICLES
C USING UNIT CELL METHOD DETAILED IN RESEARCH MEMORANDUM ML-62
C THE CITY UNIVERSITY, LONDON
C *****
C DIMENSION C(75,76)
C *****
C TC,TF ARE ARBITRARY BOUNDARY TEMPERATURES
C *****
C DATA TL,TC,TF,A/1.0,10.0,0.0,0.0,0.2/
30 FORMAT(1H1,30H THERMAL CONDUCTIVITY RATIOS,7X15HVOLUME FRACTION,
14X9HHEAT FLUX,4X9HHEAT FLUX/1X12HSOLID/LIQUID,4X17HSUSPENSION/LIQU
21D,9X5HSOLID,13X2HIN,10X3HOUT)
40 FORMAT(1H ,F11.4,9XF12.4,6XF11.3,2XF13.4,F11.4)
70 FORMAT(10,F0.0)
C *****
C TP IS COMPONENT CONDUCTIVITY RATIO
C *****
15 READ(1,70) KI,TP
IF(KI) 11,11,19
19 WRITE(2,30)
C *****
C V IS SYSTEM VOLUME FRACTION SOLID
C *****
195 READ(1,70) MI,V
IF(MI) 15,15,16
16 DO 100 M=1,75
DO 101 N=1,75
C(M,N)=0.0
101 CONTINUE
100 CONTINUE
C *****
C DETERMINE SYSTEM DIMENSIONS
C *****
S=(0.80594/(V**((1.0/3.0))-1.0))/3.0
DS=S*S/(S+1)
DA=A*A/(S+1)
D=S*A/(S+A)
SA=S*S/A
SA2=SA/2.0
SA4=SA/4.0
AS=A*A/S
AS2=AS/2.0
AS4=AS/4.0
A2=A/2.0
A4=A/4.0
S2=S/2.0
S4=S/4.0
AAK=A*A*TL*TP/(TP*S+TL*A)
AK=A*TL*TP/(TP+TL)

```


UNIT CELL MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

C *****
C PROBLEM WRITTEN IN MATRIX FORM C*T=B WHERE T IS COLUMN MATRIX OF
C NODAL TEMPERATURES
C ASSIGN COEFFICIENTS OF C
C *****
C(13,14),C(13,18),C(14,19),C(14,13),C(14,23),C(15,13),C(15,15),C(1
15,19),C(18,6),C(18,2),C(20,7),C(21,7),C(16,18),C(33,42),C(33,62),C
2(34,67),C(34,37),C(34,47),C(37,34),C(37,58),C(42,33),C(42,35),C(42
3,59),C(17,14),C(35,42),C(47,34),C(62,37),C(62,67),C(63,59),C(63,33
4)=S
C(2,17),C(1,12),C(4,8),C(5,9),C(9,11),C(10,16),C(11,3),C(12,4),C(2
17,32),C(31,32),C(32,27),C(38,39),C(38,63),C(38,43),C(52,57),C(53,5
27),C(61,56),C(61,52),C(32,31)=-A
C(1,36),C(2,41),C(3,23),C(3,47),C(4,28),C(5,29),C(6,35),C(6,15),C(
116,22),C(17,10),C(27,36),C(28,11),C(28,31),C(29,16),C(31,28),C(35,
222),C(36,27),C(36,3),C(41,4),C(47,10),C(52,61),C(53,53),C(57,52),C
3(54,56)=-D
C(1,16),C(2,11),C(2,21),C(4,4),C(5,3),C(5,5),C(31,52),C(52,31),C(5
13,27),C(27,56),C(7,16),C(8,4)=-SA2
C(1,6),C(4,2),C(20,31),C(20,11),C(21,3),C(21,27),C(27,6),C(31,2)=-
1DS
C(3,21),C(6,5),C(7,22),C(8,10),C(26,27),C(26,31),C(27,26),C(31,26)
1,C(51,52),C(51,56),C(52,51),C(53,51)=-A2
C(7,46),C(8,30),C(30,21),C(46,5)=-D/2.0
C(3,17),C(6,9),C(9,17),C(10,12),C(10,22),C(11,9),C(12,8),C(12,10),
10C(32,57),C(61,32)=-SA
C(9,13),C(9,37),C(10,18),C(10,42),C(11,33),C(11,13),C(12,34),C(12,
114),C(13,12),C(13,8),C(14,17),C(37,32),C(45,9),C(32,33),C(33,12),C
2(34,17),C(37,8),C(42,9),C(61,58),C(62,57),C(63,57),C(32,37),C(61,6
32),C(33,32)=-2.0*D
C(9,7),C(11,7),C(18,8),C(18,32),C(18,12),C(32,7)=-2.0*DS
C(13,38),C(14,43),C(15,39),C(33,38),C(34,43),C(37,38),C(38,37),C(3
18,33),C(38,13),C(42,39),C(62,63),C(63,63)=-2.0*DA
C(16,24),C(17,20),C(19,2),C(19,6),C(20,1),C(21,1),C(58,55),C(28,61
1),C(28,41),C(29,66),C(29,36),C(29,46),C(35,72),C(36,29),C(36,53),C
2(41,28),C(41,30),C(41,54),C(47,60),C(54,36),C(54,66),C(55,41),C(55
3,61),C(55,71),C(57,54),C(57,28),C(58,53),C(58,29),C(30,41),C(46,29
4),C(59,66),C(60,54)=-S2
C(16,48),C(17,40),C(35,48),C(47,40)=-DA
C(19,26),C(26,1)=-DS/2.0
C(26,51),C(51,26)=-SA4
C(28,37),C(29,42),C(33,36),C(34,41),C(36,33),C(37,28),C(41,34),C(4
12,29),C(54,62),C(55,67),C(57,58),C(58,59),C(62,61),C(63,53)=-AS
C(30,47),C(35,46),C(46,35),C(59,72),C(60,60),C(47,30)=-AS2
C(30,71),C(46,55),C(59,46),C(60,30)=-S4
C(24,25),C(56,47),C(67,35),C(23,23),C(23,25),C(24,15),C(25,24),C(2
15,20)=-TL*S2
C(22,44),C(44,19),C(64,68),C(65,64),C(68,67),C(69,59)=-AAK*2.0
C(23,49),C(24,45),C(45,24),C(49,20),C(56,73),C(67,65),C(70,72),C(7
11,60)=-AAK
C(25,50),C(50,25)=-AAK/2.0
C(39,42),C(39,18),C(43,14),C(43,34),C(66,62),C(66,58)=-TL*2.0*DA
C(39,44),C(39,68),C(43,64),C(43,44),C(44,43),C(44,39),C(66,64),C(6
16,68),C(68,43),C(68,63),C(69,63),C(69,39)=-2.0*AK
C(39,38),C(39,48),C(43,38),C(43,40),C(66,38),C(40,43),C(48,39)=-TL
1*A

```


UNIT CELL MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

C(40,47),C(40,23),C(48,15),C(48,35)=-TL*DA
C(40,49),C(40,73),C(45,48),C(48,65),C(48,45),C(49,40),C(70,48),C(7
11,40)=-AK
C(44,45),C(44,69),C(44,49),C(45,44),C(49,44),C(68,69),C(69,69),C(6
19,65),C(68,73),C(70,68),C(71,64)=-TP*A
C(45,50),C(45,74),C(49,70),C(49,50),C(70,74),C(71,70),C(50,49),C(5
10,45)=-TP*A2
C(50,75)=-TP*A4
C(56,71),C(67,55)=-TL*AS2
C(64,66),C(65,54)=-AS*TL
C(22,18),C(22,20),C(22,14),C(22,24),C(23,19),C(24,19),C(64,42),C(6
14,62),C(64,72),C(65,58),C(65,60),C(65,34),C(67,59),C(56,67)=-TL*S
C(22,19),C(64,67),C(65,59)=TL*(4.0*S+AS)+2.0*AAK
C(1,11),C(4,3),C(27,31),C(31,27)=3.0*A2+S+SA2
C(3,22),C(6,10)=SA+2.0*D+A
C(7,21),C(8,5)=3.0*A4+SA2+D/2.0
C(9,12)=2.0*(A+DS)+4.0*D+SA
C(10,17),C(12,9),C(61,57)=2.0*(A+SA)+4.0*D
C(11,8),C(32,32)=2.0*(A+S+D)+SA
C(13,13),C(33,37),C(37,33)=2.0*(A+S+D)+AS
C(14,18),C(15,14),C(34,42),C(42,34),C(62,62),C(63,58)=2.0*A+3.0*S+
1AS
C(2,16),C(5,4),C(52,56),C(53,52)=3.0*A2+D+SA
C(23,24),C(24,20),C(56,72),C(67,60)=TL*(2.0*S+AS2)+AAK
C(25,25)=TL*(S+AS4)+AAK/2.0
C(39,43),C(66,63)=TL*(2.0*A+4.0*DA)+4.0*AK
C(40,48),C(48,40)=TL*(2.0*DA+A)+2.0*AK
C(43,39)=TL*(4.0*DA+2.0*A)+4.0*AK
C(69,64),C(44,44),C(68,68)=TP*3.0*A+4.0*AK+2.0*AAK
C(45,49),C(49,45),C(70,73),C(71,65)=TP*2.0*A+AK+AAK
C(50,50)=5.0*TP*A4+AAK/2.0
C(72,68),C(72,70),C(72,44),C(72,64),C(72,74),C(74,69),C(73,69)=-1.
10
C(75,50)=-0.25
C(74,75),C(74,65),C(74,45),C(73,73),C(73,75),C(73,49)=-0.5
C(16,23),C(17,15),C(35,47),C(47,35)=3.0*S2+AS2+A
C(18,7)=3.0*S+6.0*DS
C(20,6),C(21,2)=2.0*(S+DS)
C(26,26)=A+DS/2.0+SA4
C(28,36),C(36,28)=AS+2.0*D+S
C(30,46),C(46,30)=3.0*S4+AS2+D/2.0
C(38,38)=6.0*DA+3.0*A
C(41,29),C(29,41),C(54,61),C(57,53)=3.0*S2+AS+D
C(51,51)=A+SA2
C(55,66),C(58,54)=2.0*S+AS
C(59,71),C(60,55)=S+AS2
C(72,69)=6.0
C(73,74),C(74,70)=3.0
C(75,74),C(75,70)=-0.5
C(75,75)=1.5
C(19,1)=5.0*S4+DS/2.0
C
*****
C
ASSIGN COEFFICIENTS OF B      THEY FORM LAST COLUMN OF C
C
*****
C(1,76),C(2,76),C(3,76),C(4,76),C(5,76),C(6,76)=TF*A2

```


UNIT CELL MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

C(7,76),C(8,76)=TF*A4
C(9,76),C(10,76),C(11,76),C(12,76)=A*TF
C(13,76),C(14,76),C(15,76)=TF*AS
C(16,76),C(17,76)=TF*AS2
C(18,76)=TF*S
C(19,76)=TF*S4
C(20,76),C(21,76)=TF*S2
C(22,76)=TF*TL*AS
C(23,76),C(24,76)=TF*TL*AS2
C(25,76)=TF*TL*AS4
DO 200 I=26,50
200 C(I,76)=0.0
C(51,76)=TC*SA4
C(52,76),C(53,76)=TC*SA2
C(54,76),C(55,76),C(57,76),C(58,76)=TC*S2
C(59,76),C(60,76)=TC*S4
C(61,76)=TC*SA
C(62,76),C(63,76)=TC*S
C(64,76),C(65,76)=TC*TL*S
C(66,76)=TC*TL*A
C(56,76),C(67,76)=TC*TL*S2
C(68,76),C(69,76)=TC*TP*A
C(70,76),C(71,76)=TC*TP*A2
C(72,76)=TC
C(73,76),C(74,76)=TC*0.5
C(75,76)=TC*0.25
C *****
C COEFFICIENT ASSIGNMENT COMPLETE
C SOLVE USING GAUSSIAN ELIMINATION WITH PIVOTAL CONDENSATION
C *****
C J=76
C I=75
C CALL SOLV(C,I,J)
C *****
C DETERMINE HEAT FLUX INPUT AND OUTPUT
C *****
QIN=((C(1,76)-TC)*S*S4+(2.0*TC-C(52,76)-C(56,76))*S*S2+(TC-C(57,
176))*S*S+(6.0*TC-C(53,76)-C(54,76)-C(60,76)-C(61,76)-C(66,76)-C(72
2,76))*A*S2+(2.0*TC-C(55,76)-C(71,76))*A*S4+(4.0*TC-C(58,76)-C(59,7
36)-C(62,76)-C(67,76))*A*S+(TC-C(63,76))*A*A)*TL+((3.0*TC-C(64,76)-
4C(68,76)-C(69,76))*A*A+(4.0*TC-C(65,76)-C(70,76)-C(73,76)-C(74,76)
5)*A*A2+(TC-C(75,76))*A*A2)*TP)/A
QOUT=((C(1,76)-TF)*S*S4+(C(4,76)+C(3,76)+C(10,76)+C(11,76)+C(16,76
1)+C(22,76)-6.0*TF)*S*A2+(C(13,76)+C(14,76)+C(18,76)+C(19,76)-4.0*T
2F)*A*A+(C(15,76)+C(20,76)+C(23,76)+C(24,76)-4.0*TF)*A*A2+(C(5,76)+
3C(21,76)-2.0*TF)*S*A4+(C(8,76)+C(9,76)+C(12,76)+C(17,76)-4.0*TF)*A
4*S+(C(7,76)-TF)*S*S+(C(2,76)+C(6,76)-2.0*TF)*S*S2+(C(25,76)-TF)*A*
5A4)*TI/S
QMEAN=(QIN+QOUT)/2.0
TS=QMEAN/((2.5*A+1.5*S)*(TC-TF))
WRITE(2,40)TP,TS,V,QIN,QOUT
GO TO 195
11 STOP
END

```


UNIT CELL MODEL PROGRAM

(b) SUBROUTINE SOLV

```

SUBROUTINE SOLV(A,N,M)
DIMENSION A(N,M)
*****
C TO SOLVE AX=B: M=N+1 THE FIRST N ROWS AND COLUMNS OF A CONTAIN
C THE ELEMENTS OF THE INPUT MATRIX A, THE REMAINING COLUMNS CONTAIN B
C *****
NM1=N-1
MM1=M-1
*****
C FIND PIVOTS
C *****
DO 1 I=1,NM1
ELMAX=0.0
DO 2 J=I,N
TEST=ABS(A(J,I))
IF(ELMAX.GE.TEST) GO TO 2
IL=J
ELMAX=TEST
2 CONTINUE
*****
C IL IS PIVOT ROW NUMBER
C INTERCHANGE ROWS I AND IL
C *****
DO 3 J=I,M
W=A(IL,J)
A(IL,J)=A(I,J)
3 A(I,J)=W
*****
C DO THE ELIMINATION
C *****
IP1=I+1
DO 1 J=IP1,N
IF(ABS(A(I,I)).LT.1E-20) GO TO 6
FAC=A(J,I)/A(I,I)
DO 1 IJ=1,M
1 A(J,IJ)=A(J,IJ)-FAC*A(I,IJ)
*****
C GAUSS ELIMINATION COMPLETE
C BACK SUBSTITUTION
C *****
DO 4 I=1,N
FAC=A(I,I)
DO 4 J=1,M
4 A(I,J)=A(I,J)/FAC
DO 5 K=1,NM1
KK=N-K
K1=KK+1
DO 5 I=1,KK
FAC=A(I,K1)
DO 5 J=K1,M
5 A(I,J)=A(I,J)-FAC*A(K1,J)
RETURN
6 WRITE(2,7)
7 FORMAT(10X18HMATRIX IS SINGULAR)
STOP
RETURN
END

```


RANDOM PLACEMENT MODEL PROGRAM

(a) MASTER SEGMENT

```

MASTER RACO
C *****
C PROGRAM FOR DETERMINING THE EFFECTIVE THERMAL CONDUCTIVITY OF
C TWO-COMPONENT MIXTURES WHICH CONTAIN SPHERICAL PARTICLES USING
C THE RANDOM PLACEMENT MODEL DETAILED IN RESEARCH MEMORANDUM ML-62
C THE CITY UNIVERSITY, LONDON
C *****
C REAL KS, KL, KCUBE, KSUS
C COMMON T(10,10,10), C(10,10,10), NC(10,10)
C *****
C TIN, TOUT ARE ARBITRARY BOUNDARY TEMPERATURES
C *****
C DATA TIN, TOUT, KL/10, 0, 0, 0, 1, 0/
220 FORMAT(10, 2F0.0)
230 FORMAT(1H0, 30H THERMAL CONDUCTIVITY RATIOS, 7X15HVOLUME FRACTION,
14X15HVOLUME FRACTION, 4X10HITERATIONS, 4X9HHEAT FLUX, 4X9HHEAT FLUX/1
2X12HSOLID/LIQUID, 4X17HSUSPENSION/LIQUID, 4X15HSOLID SPECIFIED, 6X12H
3SOLID ACTUAL, 23X2HIN, 10X3HOUT)
240 FORMAT(1H , F11.4, 9XF12.4, 6XF11.3, 7XF11.3, 12XI4, F13.4, F11.4)
250 FORMAT(1H0, 30H THERMAL CONDUCTIVITY RATIOS, 7X15HVOLUME FRACTION,
14X10HITERATIONS, 4X9HHEAT FLUX, 4X9HHEAT FLUX/1X12HSOLID/LIQUID, 4X17
2HSUSPENSION/LIQUID, 9X5HSOLID, 27X2HIN, 10X3HOUT)
260 FORMAT(1H , F11.4, 9XF12.4, 6XF11.3, 12XI4, F13.4, F11.4)
270 FORMAT(1H1, 39X42HDETAILS OF BASIC CUBE THERMAL CONDUCTIVITY)
280 FORMAT(1H0, 39X41HPREDICTED SUSPENSION THERMAL CONDUCTIVITY)
C *****
C MC IS BASIC UNIT NODAL DIVISION
C KS IS COMPONENT CONDUCTIVITY RATIO
C *****
11 READ(1, 220) MC, KS
IF(MC.EQ.0) CALL EXIT
JJ=0
C *****
C **** STEP 1 ****
C DETERMINE THERMAL CONDUCTIVITY OF BASIC UNIT
C *****
CALL SETK(MC, KL)
MC1=MC-1
W=0.5/(FLOAT(MC1)+0.5)-0.0005
SOL=0.0
X=0.0
DO 43 I=1, MC1
Y=0.0
DO 44 J=1, MC1
Z=0.0
DO 45 K=1, MC1
R=X+Y+Z
IF(R.GT.0.25) GO TO 47
C(I, J, K)=KS
ZN=1.0
IF(I.EQ.1.OR.I.EQ.MC) ZN=ZN*0.5
IF(J.EQ.1.OR.J.EQ.MC) ZN=ZN*0.5
IF(K.EQ.1.OR.K.EQ.MC) ZN=ZN*0.5
SOL=SOL+ZN
47 Z=Z+W
45 CONTINUE

```


RANDOM PLACEMENT MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

Y=Y+W
44 CONTINUE
X=X+W
43 CONTINUE
CVOL=VOL/(FLOAT(MC1+MC1+MC1))
CALL TNC(MC,TIN,TOUT)
CALL SOLVE(MC,TIN,TOUT,IT,KCUBE,Q1,Q0)
WRITE(2,270)
WRITE(2,250)
WRITE(2,260) KS,KCUBE,CVOL,IT,Q1,Q0
WRITE(2,280)
WRITE(2,230)
C *****
C ** STEP 1 COMPLETED **
C *****
X=0.1
C *****
C M IS SYSTEM NODAL DIVISION
C VF IS SYSTEM VOLUME FRACTION SOLID
C *****
12 READ(1,220) M,VF
IF(M.EQ.0) GO TO 11
M1=M-1
JJ=JJ+1
XN=FLOAT(M)
CALL SETK(M,KL)
IF(JJ.EQ.1) CALL TNC(M,TIN,TOUT)
SVF=0.0
SVOL=CVOL/(FLOAT(M1*M1*M1))
C *****
C **** STEP 2 ****
C PLACEMENT OF BASIC UNIT IN FIELD USING RANDOM NUMBER GENERATOR
C *****
DO 35 IN=1,5000
I=FPMCRV(X)*XN
IF(I.EQ.0) I=M
J=FPMCRV(X)*XN
IF(J.EQ.0) J=M
K=FPMCRV(X)*XN
IF(K.EQ.0) K=M
IF(ABS(C(I,J,K)-KCUBE).LT.1.0E-10) GO TO 35
C(I,J,K)=KCUBE
ZN=1.0
IF(I.EQ.1.OR.I.EQ.M) ZN=ZN*0.5
IF(J.EQ.1.OR.J.EQ.M) ZN=ZN*0.5
IF(K.EQ.1.OR.K.EQ.M) ZN=ZN*0.5
SVF=SVF+SVOL*ZN
IF(ABS(SVF-VF)=SVOL*0.5.LT.1.0E-10) GO TO 40
35 CONTINUE
C *****
C ** STEP 2 COMPLETED **
C SOLVE FOR THERMAL CONDUCTIVITY
C *****
40 CALL SOLVE(M,TIN,TOUT,IT,KSUS,Q1,Q0)
WRITE(2,240) KS,KSUS,VF,SVF,IT,Q1,Q0
GO TO 12
END

```


RANDOM PLACEMENT MODEL PROGRAM

(b) SUBROUTINE SETK

```

SUBROUTINE SETK(N,KL)
C *****
C THIS SUBROUTINE SETS THE CONDUCTIVITY OF THE WHOLE FIELD INITIALLY
C TO THE VALUE OF THE CONTINUOUS PHASE
C *****
PEAL KL
COMMON T(10,10,10),C(10,10,10),NC(10,10)
DO 20 I=1,N
DO 15 K=1,N
DO 10 J=1,N
C(I,J,K)=KL
10 CONTINUE
15 CONTINUE
20 CONTINUE
RETURN
END

```

(c) SUBROUTINE TNC

```

SUBROUTINE TNC(N,TIN,TOUT)
C *****
C THIS SUBROUTINE SETS THE INITIAL TEMPERATURE FIELD AND
C ASSIGNS NODAL CODES
C *****
COMMON T(10,10,10),C(10,10,10),NC(10,10)
N1=N-1
C *****
C SET INITIAL TEMPERATURE FIELD
C *****
TEMP=10.0
DT=(TIN-TOUT)/FLOAT(N1)
DO 60 K=1,N
DO 55 J=1,N
DO 65 I=1,N
T(I,J,K)=TEMP
65 CONTINUE
55 CONTINUE
TEMP=TEMP-DT
60 CONTINUE
C *****
C SET NODAL CODES
C *****
NC(1,1)=1
NC(N,1)=3
NC(N,N)=5
NC(1,N)=7
DO 70 I=2,N1
NC(I,1)=2
NC(N,I)=4
NC(1,N)=6
NC(I,I)=8
DO 75 J=2,N1
NC(I,J)=9
75 CONTINUE
70 CONTINUE
RETURN
END

```

RANDOM PLACEMENT MODEL PROGRAM

(d) SUBROUTINE SOLVE

```

SUBROUTINE SOLVE(N,TIN,TOUT,IT,KSUSN,Q1,Q0)
C *****
C THIS SUBROUTINE SOLVES FOR THE TEMPERATURE DISTRIBUTION AND
C DETERMINES THE HEAT FLUX AND MEAN THERMAL CONDUCTIVITY OF THE FIELD
C *****
REAL KSUSN
COMMON T(10,10,10),C(10,10,10),NC(10,10)
C *****
C ACF IS RELAXATION FACTOR
C TOL IS CONVERGENCE LEVEL-MAXIMUM ACCEPTABLE DIFFERENCE BETWEEN
C SUCCESSIVE ESTIMATES AT EACH NODE
C *****
DATA ACF,TOL/1.8,0.001/
N1=N-1
LL=N*N*(N-2)
IT=0
C *****
C SOLUTION BY GAUSS-STIEDEL ITERATIVE SCHEME WITH LIEBMANN EXTRAPOLATION
C *****
90 L=0
DO 100 K=2,N1
DO 105 I=1,N
DO 110 J=1,N
C7=2.0*C(I,J,K)
C8=C(I,J,K)
IF(I.EQ.N) GO TO 115
C1=C7*C(I+1,J,K)/(C8+C(I+1,J,K))
115 IF(I.EQ.1) GO TO 120
C2=C7*C(I-1,J,K)/(C8+C(I-1,J,K))
120 IF(J.EQ.N) GO TO 125
C3=C7*C(I,J+1,K)/(C8+C(I,J+1,K))
125 IF(J.EQ.1) GO TO 130
C4=C7*C(I,J-1,K)/(C8+C(I,J-1,K))
130 C5=C7*C(I,J,K+1)/(C8+C(I,J,K+1))
C6=C7*C(I,J,K-1)/(C8+C(I,J,K-1))
IG0=NC(I,J)
GO TO(1,2,3,4,5,6,7,8,9),IG0
1 TEMP=((T(I+1,J,K)*C1+T(I,J+1,K)*C3)*2.0+T(I,J,K+1)*C5+T(I,J,K-1)*C
16)/((C1+C3)*2.0+C5+C6)
GO TO 135
2 TEMP=(T(I+1,J,K)*C1+T(I-1,J,K)*C2+T(I,J+1,K)*C3*2.0+T(I,J,K+1)*C5+
BT(I,J,K-1)*C6)/(C1+C2+C3*2.0+C5+C6)
GO TO 135
3 TEMP=((T(I-1,J,K)*C2+T(I,J+1,K)*C3)*2.0+T(I,J,K+1)*C5+T(I,J,K-1)*C
C6)/((C2+C3)*2.0+C5+C6)
GO TO 135
4 TEMP=(T(I-1,J,K)*C2+2.0+T(I,J+1,K)*C3+T(I,J-1,K)*C4+T(I,J,K+1)*C5+
DT(I,J,K-1)*C6)/(C2*2.0+C3+C4+C5+C6)

```


RANDOM PLACEMENT MODEL PROGRAM

(d) SUBROUTINE SOLVE [Cont.]

```

      GO TO 135
5  TEMP=((T(I=1,J,K)*C2+T(I,J=1,K)*C4)*2.0+T(I,J,K+1)*C5+T(I,J,K-1)*C
      E6)/((C2+C4)*2.0+C5+C6)
      GO TO 135
6  TEMP=(T(I+1,J,K)*C1+T(I-1,J,K)*C2+T(I,J=1,K)*C4*2.0+T(I,J,K+1)*C5+
      FT(I,J,K-1)*C6)/(C1+C2+C4*2.0+C5+C6)
      GO TO 135
7  TEMP=((T(I+1,J,K)*C1+T(I,J=1,K)*C4)*2.0+T(I,J,K+1)*C5+T(I,J,K-1)*C
      G6)/((C1+C4)*2.0+C5+C6)
      GO TO 135
8  TEMP=(T(I+1,J,K)*C1*2.0+T(I,J+1,K)*C3+T(I,J-1,K)*C4+T(I,J,K+1)*C5+
      HT(I,J,K-1)*C6)/(C1*2.0+C3+C4+C5+C6)
      GO TO 135
9  TEMP=(T(I+1,J,K)*C1+T(I-1,J,K)*C2+T(I,J+1,K)*C3+T(I,J-1,K)*C4+T(I,J
      IJ,K+1)*C5+T(I,J,K-1)*C6)/(C1+C2+C3+C4+C5+C6)
135 Z=ABS(T(I,J,K)-TEMP)
      IF(Z.GT.TOL) GO TO 140
      L=L+1
140 T(I,J,K)=T(I,J,K)+ACF*(TEMP-T(I,J,K))
110 CONTINUE
105 CONTINUE
100 CONTINUE
      IT=IT+1
      IF(LL.NE.L) GO TO 90
      =====
      CALCULATE HEAT FLUX IN K DIRECTION
      =====
      QI=0.0
      QO=0.0
      DO 145 I=1,N
      DO 150 J=1,N
      C1=C(I,J,1)*C(I,J,2)*2.0/(C(I,J,1)+C(I,J,2))
      DQI=C1*(TIN-T(I,J,2))
      C2=C(I,J,N1)*C(I,J,N)*2.0/(C(I,J,N1)+C(I,J,N))
      DQO=C2*(T(I,J,N1)-TOUT)
      ZN=1.0
      IF(I.EQ.1.OR.I.EQ.N) ZN=ZN*0.5
      IF(J.EQ.1.OR.J.EQ.N) ZN=ZN*0.5
      QI=QI+DQI*ZN
      QO=QO+DQO*ZN
150 CONTINUE
145 CONTINUE
      QMEAN=(QI+QO)*0.5
      KSUSN=QMEAN/((TIN-TOUT)*FLOAT(N1))
      RETURN
      END

```

Appendix III

Computer Program Documentation for Homogeneous Flow Model.

This appendix details the computer program, which incorporates the homogeneous equilibrium flow concept in the solution of the boundary layer equations, for the prediction of the heat transfer characteristics of solid-liquid suspensions. It is impossible to present a complete description of all its features; sufficient information will be given to enable the program to be utilised as it stands. If it is desired to change any of the parameters other than the input data, the programming logic should be reasonably accessible through a study of the program listing in conjunction with the information given in reference /62/

The programming language is FORTRAN IV; as it stands the program is designed for execution on a C.D.C. 6400, 6600 or 7600 computer and contains some features of C.D.C. software. However, a version of the program was adapted for use with an I.C.L. 1905 computer without difficulty.

Program name HTFLØ

Subroutines ØUTLET, SUMMARY, TRØPS, DSFI, DSF2, TTPF, TPØS,
PØLI, LAGR, ENTPY, TEMP, DENS, VISC, CØND, SPEC,
VFS

Additional Segment BLØCK DATA

a) Program Run Time Requirements

The program may be contained on about 1350 cards. Using a C.D.C. 7600 computer an execution time in the region of 15 seconds (real time)

is necessary for each problem. Core storage requirement on an I.C.L. 1905 computer may be kept in the region of 19500 words by using the overlay system; the run time in this case will be about 500 seconds.

b) Program Input

Data input to the program is on cards, the format being a field width specification.

Card 1 - HEAD : a general heading for the output

Card 2 - THEAD: a title giving details of the individual run

Cards 3 - 14 : 12 cards each containing a value of CHEAD, JCØL

where:- CHEAD : column heading for a parameter in the output
table

JCØL : parameter identification number from an array of
the choices in the result vector, RV

Card 15 - TD, YPI, XI, CA, XL, XLH

TD : node spacing multiplier

YPI : non-dimensional distance from the wall to the nearest
grid line

XI : initial axial step length

CA : boundary layer thickness multiplier

XL : length of heated tube

XLH : length of unheated entry section.

Typical values for the first four variables are

4.0, 0.5, 0.01 and 5.0 respectively

Card 16 - TO : fluid bulk entry temperature

Card 17 - IC : a six element control vector

IC (1) : not in use

IC (2) : controls calculation of transverse
velocity = $V \times IC (2)$

IC (3) : maximum number of intermediate print outputs;
between 0 and 50

IC (4) : controls print options

IC (5) : controls transverse velocity wall boundary
condition; zero gives impermeable wall.

IC (6) : single component flow (1) or two component flow (2)
Typical values for the first five elements are 0,
1, 25, 2, 0 respectively.

Card 18 - WF : weight fraction solid; only required if IC (6) = 2.

Card 19 - NIV, NWALL, NVAR, NDIR, RE, QW

NIV : specifies form of initial velocity profile

- 1 fully developed
- 2 uniform profile
- 3 tabular input

NWALL : specifies type of thermal boundary condition

- 1 wall heat flux or isothermal flow where $QW = 0.0$
- 2 wall temperature

NVAR : specifies form of thermal boundary condition

- 1 second order polynomial
- 2 tabular input

NDIR : specifies heating (1) or cooling (2)

RE : entry Reynolds number

QW : heat flux parameter

Card 20 - QW1, QW2, QW3 : coefficients of a polynomial thermal boundary condition.

Cards specifying tabular velocity or thermal boundary profiles follow if required.

c) Parameters Defined in DATA Statements

Various error messages, control parameters, convergence tolerance factors and headings are specified in DATA Statements. In general, these should be satisfactory as given in the program listing but may readily be changed if desired.

Other variables must be specified for each particular liquid and solid phase considered.

In BLOCK DATA segment:

TT : array of temperatures at which liquid phase properties are specified

TCP, TH, TR ϕ , TC, TMU : tabular liquid properties; respectively specific heat, enthalpy, density, thermal conductivity and viscosity.

NR ϕ , NMU, NC, NCP, NT, NVF : specify the degree of polynomial relationship representing fluid properties

XOUT : array of axial stations for output of data.

In subroutine TP ϕ S:

TS : array of temperatures at which solid phase properties are specified

TSR ϕ , TS ϕ CP, TSC : tabular solid properties; respectively density, specific heat and thermal conductivity.

d) Program Output

Output from the program is by line printer. At each axial station specified a single page of parameters is output. These include non-dimensional results, dependant variables with velocity and temperature profiles in tabular form. On completion of the solution a summary table is printed which lists the twelve variables chosen, together with a heading of initial and boundary conditions with grid control parameters.

e) Solution Procedure.

The solution of the boundary layer equations, as described in chapter 9, is performed in the main segment of the program. Additionally all the input variables are read into this segment, the grid spacing is determined, the problem specified and the variables initialised before the iterative solution procedure is entered. Output variables and property values are evaluated in subprograms. The subroutine OUTLET transforms the calculated dependant variables into a form suitable for output; these are printed at any desired point in the solution. In addition 12 selected integral quantities are stored for entry in the final table printed on completion of the program. These are output by subroutine SUMMARY.

Effective transport properties of the fluid are calculated in the subroutine TPROPS using the given mixing length model. Fluid properties are normalised in subroutine TTPF; for two component flows TPØS produces effective suspension properties. PØLY fits a polynomial relationship to tabular property values. A cubic spline function is fitted to sets of tabular values by DSF1 whilst DSF2 provides the associated interpolation routine. Subroutine LAGR contains a standard Lagrange interpolation

routine. The remaining subroutines interpolate for various values of fluid properties which are required by the solution procedure.

HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT

```

PROGRAM HTFLO(INPUT,OUTPUT,TAPE1=INPUT,TAPE2)
C *****
C THIS PROGRAM SOLVES THE BOUNDARY LAYER EQUATIONS FOR ONE AND TWO-
C COMPONENT FLOW OF FLUIDS IN CIRCULAR TUBES ASSUMING A HOMOGENEOUS
C FLOW MODEL FOR THE TWO-COMPONENT CASE.
C *****
REAL MTOL,MOTOL,MBAL,MBALK,MOBAL,MOBALK,NORM,MDOT
COMMON IC(6),HEAD(8),RESULT(12,100),U(100),UI(100),UP(100),H(100),
AHI(100),HP(100),S(100),SI(100),RHOVA(100),RHOVA(100),E(100),Z(100)
A,R(100),Y(100),RDR(100),AREA(100),DCM(100),DCE(100),TAU(100)
COMMON /IN1/ QWM,QW1,QW2,QW3,NVAR,NWALL,NDIR,XLH,XL,
B NIV,RE,PR,EC,TD,YP1,N1,CA,X1
COMMON /OUT/ ELI,ENTH,ENERG,HB,HBI,HW,HWI,IOUT,ITT,JJ,N,P,PI,QH,QW
1I,RET,X,XMASS,XMOM,XMOMI,FSR1,FBR1,NG,MDOT,EBAL,MBAL,MOBAL,DPDX,
2XKE,XKEI,FA,FAI,FB,FBI,FS,FSI
COMMON /IN2/ NQ,XQ(50),QWALL(50),D2QDX(50)
COMMON /PRINT/ XOUT(50)
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1 NRO,NMU,NC,NCP,NH,NT
COMMON /TAB/ JCOL(12),CHEAD(12),THEAD(8),NRV(12)
COMMON /TRANSP/ BL,DELTA,MODEL,YPL,CAPPA,TMODEL(8)
COMMON /SPLI/ AJ(100),BJ(100),CJ(100),DJ(100),FJ(100),HJ(100),
1PJ(100),QJ(100),RJ(100)
DIMENSION QTAB(3),MESSAGE(3)
1001 FORMAT(8F10.3)
1002 FORMAT(6I2)
1003 FORMAT(8A10)
1005 FORMAT(I2)
1006 FORMAT(4I2,F12.2,F10.1,F10.7,F10.3)
1010 FORMAT(A10,I2)
1020 FORMAT(/I5,5(1PE15.7))
1022 FORMAT(4(1PE20.12))
2002 FORMAT(1H,5(1PE20.7))
2003 FORMAT(1H0)
2062 FORMAT(27H1 ENTHALPY NEGATIVE AT J = ,I3)
2063 FORMAT(27H VELOCITY NEGATIVE AT J = ,I3,14H, VELOCITY = ,E12.4)
3007 FORMAT(2H,3HIT=,I2,A10,5H, EB=,F8.4,6H, MOB=,F8.6,4H, P=,1PE12.4
1,7H, DPDX=,1PE12.4,5H, FB=,1PE12.6,5H, TR=,1PE12.6,5H, NU=,
2 1PE12.6)
3046 FORMAT(1H,45H ELI LESS THAN DXMIN, PROBLEM TERMINATED ****)
3048 FORMAT(1H0,4HJJ =,I4,4H, X=,1PE12.6,5H, DX=,1PE8.2,5H, HT=,1PE8.2,
1 5H, UT=,1PE8.2,6H, BLT=,1PE8.2,5H, CA=,1PE8.2,5H, VB=,
2 1PE12.6,5H, HB=,1PE12.6)
DATA MESSAGE/10H PRESSURE ,10H ENERGY R ,10H ENERGY A /
C INITIAL VALUES OF PROGRAM PAPAMERERES
DATA WGT,AVG,FRAC,ETOL,XMTOL,XMOTOL,PTOL,ITMAX1,ITMAX2,ITMIN,DXMAX
1,DXMIN,SHRINK,GROW/0.75,1.0,0.5,0.0001,0.00001,0.01,0.0001,10,20,2
2,2.0,0.25,0.5,1.1/
JOB=0
MDOT=1.0
IT=0
TR=0.0
XNU=0.0
NBL=1
BLT=0.0

```


HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

C      WF=0.0
C      *****
C      INPUT HEADINGS
C      *****
C      READ(1,1003) HEAD
C      READ(1,1003) THEAD
C      DO 10 J=1,12
C      READ(1,1010) CHEAD(J),JCOL(J)
10  CONTINUE
C      *****
C      SPECIFY GRID CONSTANTS
C      *****
C      READ(1,1001) TD,YP1,X1,CA,XL,XLH
C      *****
C      SPECIFY PROBLEM
C      *****
C      READ(1,1001) TO
C      READ(1,1002) (IC(K),K=1,6)
C      IF(IC(6).EQ.2) READ(1,1001) WF
C      READ(1,1006) NIV,NWALL,NVAR,NDIR,RE,EC,QW
C      READ(1,1001) QW1,QW2,QW3
C      NPH=IC(6)
C      CALL TTPF(NPH,TO,WF,PR)
C      PIN=1.0
C      TIN=1.0
C      DELTA=0.0
C      BL=0.0
C      HIN=0.0
C      GAMAN2=EC
C      IF(NIV.EQ.2) BL=1.0
C      IF(NIV.EQ.4) BL=2.0
C      *****
C      INITIALISE FLAGS AND COUNTERS
C      *****
C      NG=1
C      IOUT=1
C      LPRE=1
C      JJ=0
C      JJJ=0
C      MOTOL=XMOTOL
C      MTOL=XMTOL
C      CAZERO=CA
C      *****
C      CALCULATE NUMBER OF TRANSVERSE NODES
C      *****
C      EFF=0.0014+0.125/RE**0.32
C      YPC=0.5*RE*SQRT(0.5*EFF)
C      U(1)=0.0
C      U(2)=YP1/YPC
C      Y1=U(2)
C      DY=Y1
C      C=2.0**((1.0/TD)
C      DO 510 K=3,200
C      Y2=Y1+C*DY
C      U(K)=Y2

```

HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

Y1=Y2
DY=C*DY
IF(Y2.GT.1.0) GO TO 512
510 CONTINUE
512 N1=K
N=N1-1
C *****
C CALCULATE NODE PARAMETERS AND SET ARRAYS
C *****
U(N1)=1.0
DO 520 J=1,N1
K=N1+1-J
Y(J)=U(K)
R(J)=1.0-Y(J)
H(J)=0.0
HI(J)=0.0
HP(J)=0.0
S(J)=1.0
SI(J)=1.0
RHOVA(J)=0.0
E(J)=1.0
Z(J)=1.0
520 CONTINUE
YI=Y(1)
AREA(N1)=0.0
U(N1)=0.0
DO 522 J=1,N
YO=0.5*(Y(J)+Y(J+1))
AREA(J)=(YI-YO)*(2.0-YI-YO)
YI=YO
TAREA=2.0*(R(J+1)+R(J))
ELJ=R(J+1)-R(J)
RDR(J)=2.0*TAREA/ELJ
522 CONTINUE
C *****
C SET INITIAL VALUES OF VARIABLES
C *****
HB=0.0
HBI=0.0
HW=0.0
HWI=0.0
TB=1.0
RHOB=1.0
RHOB1=1.0
VB=1.0
QWI=0.0
QWM=QW
P=0.0
PI=0.0
PLAST=PIN
DPDX=-0.5
DPLAST=-0.5
ENERG=0.0
EBAL=0.0
EBALK=0.0

```


HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

MBAL=0.0
MBALK=0.0
MOBAL=0.0
MOBALK=0.0
DXMIN=0.25*RE*Y(N)**2
X=0.0
ELI=X1
REDX=WGT*ELI/RE
REDX1=(1.0-WGT)*ELI/RE
GO TO 1440
540 IF(NVAR.NE.2) GO TO 545
C *****
C WALL BOUNDARY CONDITION SPECIFIED BY CARD INPUT
C *****
READ(1,1005) NQ
DO 541 I=1,NQ
1007 READ(1,1007) XQ(I),QWALL(I)
541 FORMAT(2F15.11)
541 CONTINUE
NS=4
NF=4
CALL DSF1(NQ,XQ,QWALL,D2QDX,NS,NF,AJ,BJ,CJ,DJ,PJ,FJ,QJ,HJ)
545 QW=0.0
GO TO (550,620,550) NIV
C *****
C VELOCITY PROFILES SPECIFIED BY CARD INPUT
C *****
550 READ(1,1020) N1,RE,XMOM,FA,FB,FS
READ(1,1022) (U(J),J=1,N1)
LPRE=1
GO TO 690
555 LPRE=3
XKE=0.0
DO 560 J=1,N1
UI(J)=U(J)
UP(J)=U(J)
RHOUA(J)=S(J)*U(J)*AREA(J)
XKE=XKE+U(J)*U(J)*RHOUA(J)
560 CONTINUE
CALL TEMP(H(N1),PI,TW)
CALL VISC(TW,PI,XMU)
SHEAR=2.0*XMU*U(N)/(RE*Y(N))*MDOT**2
SHEARI=SHEAR
XKEI=XKE
XMOMI=XMOM
XMASS=1.0
FAI=FA
FBI=FB
FSI=FS
GO TO 1400
C *****
C UNIFORM INITIAL VELOCITY PROFILE
C *****
600 LPRE=2
XMOM=0.0

```

HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

XKE=0.0
YJ=1.0
YI=1.0
Y1=0.5*Y(N)
U1=1.0/(1.0-2.0*Y1+Y1*Y1)
UP(N1)=0.0
UI(N1)=0.0
RHOUA(N1)=0.0
DO 620 J=1,N
U(J)=U1
UP(J)=U1
UI(J)=U1
RHOUA(J)=S(J)*U(J)*AREA(J)
XMON=XMON+U(J)*RHOUA(J)
XKE=XKE+U(J)*U(J)*RHOUA(J)
620 CONTINUE
C *****
C ESIMATE F FROM EXTERNAL BOUNDARY LAYER SOLUTION
C *****
FS=0.059/((RE**0.2)*(X1**0.2))
FBI=0.664/SQRT(0.01*RE*X1)
GO TO 690
630 FA=FS
FAI=FBI
FB=FBI
FSI=FS
LPRE=3
CALL TEMP(H(N1),PI,TW)
CALL VISC(TW,PI,XMU)
SHEAR=2.0*XMU*U(N)/(RE*Y(N))*MDOT**2
SHEARI=SHEAR
XKEI=XKE
XMONI=XMON
XMASS=1.0
GO TO 1400
C *****
C CALCULATE AN APPROXIMATE FULLY DEVELOPED VELOCITY PROFILE
C *****
650 IF(RE.GT.RET) GO TO 660
FS=16.0/RE
DO 655 J= 1,N1
U(J)=2.0*(1.0-R(J)**2)
655 CONTINUE
GO TO 670
662 FS=0.0014+0.125/RE**0.32
FSK=FS
K=0
662 FS=0.25*(1.0/(2.0*ALOG10(RE*SQRT(4.0*FSK))-0.8)**2)
IF(ABS((FS-FSK)/FS).LT.1.0E-4) GO TO 664
K=K+1
FSK=FS
IF(K.LT.10) GO TO 662
664 DO 665 J=1,N1
U(J)=Y(J)*RE*SQRT(0.125*FS)
IF(U(J).LT.11.6) GO TO 665

```


HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

        U(J)=5.5+5.75*ALOG10(U(J))
665  CONTINUE
670  XMASS=0.0
        XMOMI=0.0
        XKE=0.0
        XMOM=0.0
        DO 672 J=1,N
        XMASS=XMASS+AREA(J)*S(J)*U(J)
672  CONTINUE
        DO 673 J=1,N1
        U(J)=U(J)/XMASS
        RHOVA(J)=S(J)*U(J)*AREA(J)
        XMOM=XMOM+U(J)*RHOVA(J)
        XKE=XKE+U(J)*U(J)*RHOVA(J)
        UP(J)=U(J)
        UJ(J)=U(J)
673  CONTINUE
        FA=FS
        FB=FS
C      *****
C      TRANSPORT PROPERTIES ARE CALCULATED BY TPROPS OR BY
C      SPECIAL PROGRAMMING INSERTED HERE
C      *****
690  CALL TPROPS
        GO TO(555,630,700),LPRE
C      SOLVE ENERGY EQUATION FOR H(I+1,J)
700  IF(X,LT,XLH,OR,ABS(X-XLH).LT.1.0E-10) GO TO 800
        IF(ABS(QNM).LT.1.0E-10,AND,NWALL,EQ.1) GO TO 800
        HIN=0.25*(HP(2)+HP(1)+HI(1)+HI(2))
        AJ(1)=-REDX*RDR(1)*Z(1)
        AJ1=AJ(1)
        BJ(1)=RHOVA(1)-AJ(1)
        NORM=-1.0/BJ(1)
        DJ(1)=DCE(1)+RHOVA(1)*HIN
        QJ(1)=DJ(1)*NORM
        PJ(1)=NORM*AJ(1)
        DO 730 J=2,N
        AJ(J)=-REDX*RDR(J)*Z(J)
        BJ(J)=RHOVA(J)-AJ(J)-AJ1
        HO=0.25*(HP(J+1)+HP(J)+HI(J+1)+HI(J))
        DJ(J)=DCE(J)+RHOVA(J)*HO-RHOVA(J-1)*HIN
        HIN=HO
C      *****
C      TRANSFORM COEFFICIENTS
C      *****
        NORM=-1.0/BJ(J)
        AJ1N=AJ1*NORM
        AJ1=AJ(J)
        A1RN=1.0/(1.0-AJ1N*PJ(J-1))
        PJ(J)=NORM*A1RN*AJ(J)
        QJ(J)=A1RN*(AJ1N*QJ(J-1)+NORM*DJ(J))
730  CONTINUE
C      *****
C      APPLY WALL BOUNDARY CONDITIONS
C      *****

```


HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

GO TO (740,746) NWALL
740 DJ(N1)=(HBI-HB)-RHOVA(N)*HIN+DCE(N1)
   BJ(N1)=REDX*RDR(N)*Z(N)
   H(N1)=(QJ(N)-DJ(N1)/BJ(N1))/(1.0-PJ(N))
   HW=H(N1)
   IF(H(N1).GT.-1.0) GO TO 745
   WRITE(2,2062) N1
   GO TO 1532
745 CALL TEMP(H(N1),P,TW)
746 CALL DENS(TW,P,S(N1))
   HW=H(N1)
   DO 780 K=1,N
   J=N+1-K
   H(J)=PJ(J)*H(J+1)+QJ(J)
   IF(H(J).GT.-0.0001) GO TO 775
   WRITE(2,2062) J
   GO TO 1532
775 CALL TEMP(H(J),P,T1)
   CALL DENS(T1,P,S(J))
780 CONTINUE
C *****
C SOLVE THE MOMENTUM AND INTEGRAL CONTINUITY EQUATIONS
C SIMULTANEOUSLY FOR U(I+1,J) AND P
C *****
800 UIN=0.25*(UP(2)+UP(1)+UI(2)+UI(1))
   AJ(1)=-REDX*RDR(1)*E(1)
   AJ1=AJ(1)
   BJ(1)=RHOVA(1)-AJ(1)
   NORM=-1.0/BJ(1)
   CJ(1)=-AREA(1)
   DJ(1)=DCM(1)+RHOVA(1)*UIN
   PJ(1)=NORM*AJ(1)
   QJ(1)=NORM*DJ(1)
   RJ(1)=NORM*CJ(1)
   DO 830 J=2,N
   UO=0.25*(UP(J+1)+UP(J)+UI(J+1)+UI(J))
   AJ(J)=-REDX*RDR(J)*E(J)
   BJ(J)=RHOVA(J)-AJ(J)-AJ1
   CJ(J)=-AREA(J)
   DJ(J)=DCM(J)+RHOVA(J)*UO-RHOVA(J-1)*UIN
   UIN=UO
C *****
C TRANSFORM COEFFICIENTS
C *****
   NORM=-1.0/BJ(J)
   AJ1N=AJ1*NORM
   AJ1=AJ(J)
   A1=1.0-AJ1N*PJ(J-1)
   A1RN=1.0/A1
   PJ(J)=NORM*A1RN*AJ(J)
   QJ(J)=A1RN*(AJ1N*QJ(J-1)+NORM*DJ(J))
   RJ(J)=A1RN*(AJ1N*RJ(J-1)+NORM*CJ(J))
830 CONTINUE
C *****
C SOLVE FOR PRESSURE DEFECT

```


HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

C      *****
      FJ(N1)=0.0
      FJ(N)=QJ(N)
      HJ(N1)=0.0
      HJ(N)=RJ(N)
      DO 840 K=2,N
      J=N-K+1
      FJ(J)=PJ(J)*FJ(J+1)+QJ(J)
      HJ(J)=PJ(J)*HJ(J+1)+RJ(J)
840  CONTINUE
      SUM1=0.0
      SUM2=0.0
      DO 850 J=1,N
      RHOA=S(J)*AREA(J)
      SUM1=SUM1+RHOA*FJ(J)
      SUM2=SUM2+RHOA*HJ(J)
850  CONTINUE
      P=(MDOT-SUM1)/SUM2
C      *****
C      SOLVE FOR AXIAL VELOCITY
C      *****
      XMASS=0.0
      XMOM=0.0
      XKE=0.0
      ENTH=0.0
      DO 860 J=1,N
      U(J)=FJ(J)+HJ(J)*P
      IF(U(J).GT.-0.00001) GO TO 855
      WRITE(2,2053) J,U(J)
      GO TO 1404
855  DMASS=S(J)*U(J)*AREA(J)
      XMASS=XMASS+DMASS
      XMOM=XMOM+RHOA(J)*U(J)
      XKE=XKE+U(J)*U(J)*RHOA(J)
      ENTH=ENTH+DMASS*H(J)
      AJ(J)=DMASS-SI(J)*UI(J)*AREA(J)
860  CONTINUE
C      *****
C      SOLVE CONTINUITY EQUATION FOR V
C      *****
      IF(IC(2).EQ.3) GO TO 898
      RHOVA(N1)=FLOAT(IC(5))*ELI/XL
      AJ(N1)=0.0
      DO 870 K=1,N
      J=N+1-K
      RHOVA(J)=RHOVA(J+1)+AJ(J+1)
870  CONTINUE
898  GO TO(899,900) NWALL
899  CALL TEMP(H(N1),P,TW)
      GO TO 901
900  CALL ENTPY(TW,P,H(N1))
      HB=ENTH
901  CALL VISC(TW,P,XMU)
      DPDX=(PI-P)/ELI
      SHEAR=2.0*XMU*U(N)/(RE*Y(N))*MDOT**2

```

HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

MBAL=(XMASS-MDOT)/MDOT
EBAL=0.0
IF(ABS(HB).GT.1.0E-10) EBAL=(ENTH-HB)/HB
SHEARM=0.5*(SHEAR+SHEARI)
MOBAL=((XMOM-XNOMI)+4.0*ELI*SHEARM-(P-PI))/XMOM
C *****
C TEST FOR ABSOLUTE AND RELATIVE CONVERGENCE
C *****
1400 KM=1
IR=0
IF(ABS(DPDX).LT.1.0E-10) GO TO 1401
IF(ABS((DPDX-DPLAST)/DPDX).GT.PTOL) GO TO 1402
1401 KM=2
IF(ABS(EBAL-EBALK).GT.0.1*ETOL) GO TO 1402
KM=3
IF(ABS(MBAL-MBALK).GT.XMTOL) GO TO 1402
IF(ABS(EBAL).GT.ETOL.AND.IT.LT.ITMAX1) GO TO 1402
C *****
C RELATIVE CONVERGENCE HAS BEEN ACHIEVED
C *****
GO TO 1408
1402 IR=-1
IF(IC(4).LT.0) GO TO 1408
14021 IF(IT.GT.ITMAX2) GO TO 1404
C *****
C RESET VARIABLES AND ARRAYS FOR NEXT ITERATION
C *****
IT=IT+1
EBALK=EBAL
MBALK=MBAL
MOBALK=MOBAL
PLAST=P
DPLAST=DPDX
COMP=1.0-AVG
DO 1403 J=1,N1
UP(J)=AVG*U(J)+COMP*UP(J)
HP(J)=AVG*H(J)+COMP*HP(J)
RHOUA(J)=AVG*S(J)*U(J)*AREA(J)+COMP*RHOUA(J)
1403 CONTINUE
GO TO 200
C *****
C CONVERGENCE HAS NOT BEEN ACHIEVED SO ELI IS REDUCED
C *****
1404 WRITE(2,3027) IT,MESAGE(KM),EBAL,MOBAL,P,DPDX,FB,TR,XNU
IF(ELI*SHRINK.GT.OXMIN) GO TO 1406
WRITE(2,3046)
JOB=1
GO TO 1532
1405 CA=CA*SHRINK
P=PI
X=X+(SHRINK-1.0)*ELI
ELI=ELI*SHRINK
DO 1405 J=1,N1
UP(J)=UI(J)
HP(J)=HI(J)

```


HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

      RHOUA(J)=SI(J)*UI(J)*AREA(J)
      U(J)=UI(J)
      H(J)=HI(J)
      S(J)=SI(J)
1405  CONTINUE
      GO TO 1415
1408  IF(JJJ.EQ.0) GO TO 14081
      GO TO(14008,14009) NWALL
14009  CALL TEMP(H(N),P,TN)
      CALL TEMP(H(N-1),P,TN1)
      CALL COND(TW,P,EW)
      DTDW=(Y(N-1)*Y(N-1)*(TW-TN)-Y(N)*Y(N)*(TW-TN1))/(Y(N)*Y(N-1)*(Y(N)
1=Y(N-1)))
      QW=2.0*EW*DTDW/(RE*PR)
14008  CALL TEMP(HB,P,TB)
      CALL DENS(TB,P,RHOB)
      CALL COND(TB,P,E1)
      D1=0.5*(P-PI)/ELI
      FB=RHOB*(D1-0.5*(XMOM-XMOMI)/ELI)
      FA=RHOB*(D1+0.5*(1.0/RHOB1-1.0/RHOB)/ELI)
      XNU=0.0
      IF(ABS(QW).GT.1.0E-10) XNU=RE*PR*QW/(E1*(TB-TW))
      TR=TW/TB
      VB=MDOT/RHOB
      FS=SHEAR*2.0*RHOB
      IF(IC(4).LT.2) WRITE(2,3007) IT,MESAGE(KM),EBAL,MOBAL,P,DPDX,FB,TR
1,XNU
      IF(IR) 14021,14081,14081
14081  XO=X
      IF(X.GE.XLH) XO=X-XLH
      IF((XO-ELI).GT.XL.AND.X.GE.XLH) GO TO 1440
C      *****
C      OPTION TO ENTER EVERY STEP IN RESULT TABLE
C      *****
      IF(IC(4).NE.4) GO TO 14005
      IC(4)=3
      CALL OUTLET(1.0,XO)
      IC(4)=4
14005  ELII=ELI
      I=IOUT
      IF((XO.GE.(XOUT(I)-1.0E-10)).AND.((XO-ELI).LT.XOUT(I)))GO TO 14001
      GO TO 14002
14001  IOUT=IOUT+1
      FACTOR=(XOUT(I)-XO+ELI)/ELI
      IF(ABS(FLOAT(JJJ)+XLH).GT.1.0E-10) CALL OUTLET(FACTOR,XOUT(I))
      IF(ABS(X-XLH).GT.1.0E-10) GO TO 14003
      IOUT=1
      NG=1
      JJ1=JJ+1
      CALL OUTLET(1.0,0.0)
      CA=CAZERO
      ELI=X1
      IOUT=IOUT+1
      IF(RE.GT.RET.AND.XLH.LT.30.0) GO TO 1414
      IF(RE.LT.RET.AND.XLH.LT.200.0) GO TO 1414

```


HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER SEGMENT [Cont.]

```

DII=0.0
DO 14010 J=1,N
DII=DII+SQRT((U(J)+U(J+1))/Z(J))*(R(J+1)-R(J))
14010 CONTINUE
DI=DII/SQRT(PR*VB)
GO TO 1414
14003 I=IOUT
IF((XO.GE.(XOUT(I)-1.0E-10)).AND.((XO-ELI).LT.XOUT(I)))GO TO 14001
14002 IF(JJJ.EQ.0) GO TO 1414
IF(JJ.NE.100) GO TO 14006
CALL SUMMARY
JJ=0
14006 IF(BLT.LT.0.5) GO TO 14004
IF(IT.LT.ITMIN.AND.ELI*GROW,LE.DXMAX) CA=CA*GROW
C *****
C      CALCULATE BOUNDARY LAYER THICKNESSES AND NEXT STEP SIZE
C *****
14004 BLT=1.0
IF(NBL.EQ.2) GO TO 1413
HBLT=1.0
UBLT=1.0
DO 1409 J=2,N
IF(U(J)/U(1).LE.0.98) GO TO 1410
1409 CONTINUE
1410 UBLT=Y(J-1)
NBLU=J
IF(ABS(HB).LT.1.0E-10) GO TO 1413
DELH=HW-H(1)
DO 1411 J=2,N
IF((H(J)-H(1))/DELH.GE.0.02) GO TO 1412
1411 CONTINUE
1412 HBLT=Y(J-1)
NBL=J
1413 BLT=HBLT
IF(UBLT.LT.BLT) BLT=UBLT
IF(UBLT.LT.BLT) NBL=NBLU
ELI=CA*0.5*BLT
IF(ELI.GT.0.5*DXMAX) ELI=0.5*DXMAX
IF(X.GE.XLH) GO TO 1414
IF(X+ELI.LT.XLH) GO TO 1414
ELI=XLH-X
IOUT=1
1414 X=X+ELI
XKEI=XKE
XMOMI=XMOM
PI=P
RHOBI=RHOB
FBI=FB
FAI=FA
FSI=FS
HWI=HW
HBI=HB
QWI=QW
SHEARI=SHEAR
JJJ=JJJ+1

```


(a) MASTER SEGMENT [Cont.]

```

1415 MDOT=1.0-FLOAT(IC(5))*X/XL
C *****
C CALCULATE WALL BOUNDARY CONDITION AND ENTHALPIES
C *****
IF(X.LE.XLH) GO TO 1418
XT=X-XLH
XTI=XT-ELI
GO TO(11416,11415) NWALL
11415 IF(NVAR.NE.2) GO TO 1417
CALL DSF2(NQ,XQ,QWALL,D2QDX,XT,QTAB,AJ)
TW=QTAB(1)
GO TO 11419
1417 TW=QW1+QW2*XT+QW3*XT*XT
GO TO 11419
11416 IF(NVAR.NE.2) GO TO 1416
CALL DSF2(NQ,XQ,QWALL,D2QDX,XT,QTAB,AJ)
QW=QWM*QTAB(1)
HB=HBI-4.0*QWM*ELI/MDOT*(QTAB(1)-0.5*ELI*QTAB(2))
GO TO 1418
1416 QW=QWM*(1.0+QW1+QW2*XT+QW3*XT**2)
HB=HBI-4.0*QWM*(XT-XTI)*((1.0+QW1)+QW2/2.0*(XT+XTI)+QW3/3.0*(XT**2
1+XT*XTI+XTI**2))/MDOT
1418 CALL TEMP(HB,P,TB)
CALL DENS(TB,P,RHOB)
VBP=MDOT/RHOB
11419 IT=1
IF(IC(4).LT.2) WRITE(2,3048) JJJ,X,ELI,HBLT,UBLT,BLT,CA,VBP,HP
REDX=WGT*ELI/RE
REDX1=(1.0-WGT)*ELI/RE
C *****
C RESET VARIABLES AND ARRAYS FOR NEXT STEP
C *****
DCM(1)=-RHOUA(1)*U(1)+P*AREA(1)-REDX1*RDR(1)*E(1)*(U(2)-U(1))
DCE(1)=-RHOUA(1)*H(1)-REDX1*RDR(1)*Z(1)*(H(2)-H(1))
DO 1420 J=1,N1
UI(J)=U(J)
HI(J)=H(J)
SI(J)=S(J)
IF(J.EQ.1) GO TO 1419
IF(J.EQ.N1) GO TO 1419
DCM(J)=-RHOUA(J)*U(J)-REDX1*(RDR(J)*E(J)*(U(J+1)-U(J))-RDR(J-1)*E(
1J=1)*(U(J)-U(J-1)))+P*AREA(J)
DCE(J)=-RHOUA(J)*H(J)-REDX1*(RDR(J)*Z(J)*(H(J+1)-H(J))-RDR(J-1)*Z(
1J=1)*(H(J)-H(J-1)))
1419 RHOUA(J)=S(J)*U(J)*AREA(J)
RHOUA(J)=0.0
UP(J)=U(J)
HP(J)=H(J)
1420 CONTINUE
DCE(N1)=REDX1*RDR(N)*Z(N)*(H(N1)-H(N))
IF(NWALL.EQ.2) CALL ENTPY(TW,P,H(N1))
C *****
C BEGIN CALCULATIONS FOR NEXT STEP
C *****
GO TO 690

```


HOMOGENEOUS FLOW MODEL PROGRAM

(a) MASTER-SEGMENT [Cont.]

```

1440 CALL SUMMARY
      IF(JJ) 540,540,999
1532 WRITE(2,2003)
      WRITE(2,2002) AVG,P,PI,RHOB,RHOB1
      IF(JOB.EQ.1) GO TO 1440
      GO TO 1404
999  STOP
      END
  
```

(b) SUBROUTINE OUTLET

```

C      SUBROUTINE OUTLET(FRACC,XPRINT)
C      *****
C      THIS SUBROUTINE PRINTS A TABLE OF RESULTS AT
C      ANY DESIRED AXIAL POSITION
C      *****
      REAL MBAL,MOBAL,MDOT
      COMMON IC(6),HEAD(8),RESULT(12,100),U(100),UI(100),UP(100),H(100),
V      HI(100),HP(100),S(100),SI(100),RHOVA(100),RHOVA(100),
V      E(100),Z(100),R(100),Y(100),RDR(100),AREA(100),DCM(100),
V      DCE(100),TAU(100)
      COMMON /IN1/ QWH,QW1,QW2,QW3,NVAR,NWALL,NDIR,XLH,XL,NIV,RE,PR,EC,
W      TD,YP1,N1,CA,X1
      COMMON /IN2/ NQ,XQ(50),QWALL(50),DZQDX(50)
      COMMON /OUT/ ELI,ENTH,ENERG,HB,HBI,HW,HWI,ICUT,ITT,JJ,N,P,PI,QW,
X      QWI,RET,X,XMASS,XMOM,XMOMI,FSR1,FBR1,NG,MDOT,EBAL,
X      MBAL,MOBAL,DPDX,XKE,XKEI,FA,FAI,FB,FBI,FS,FSI
      COMMON /PRINT/ XOUT(50)
      COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
      COMMON /TAB/ JCOL(12),CHEAD(12),THEAD(8),NRV(12)
      COMMON /TRANSP/ BL,DELTA,MODEL,YPL,CAPPA,TMODEL(8)
      DIMENSION SAVE(10),RV(50)
      DATA GZ,GZI/.0,0.0/
2003 FORMAT(1H)
3052 FORMAT(1H ,8H      R/RO,10X4HU/UB,10X4HV/VB,8X12H(TW-T)/THETA)
3054 FORMAT(1H ,4(1PE14.4))
3056 FORMAT(1H+,62X2HY+,12X2HU+,12X2HT+,10X5HMUEFF,10X4HKEFF)
3058 FORMAT(1H+,56X5(1PE14.4))
3060 FORMAT(1H+,9H  X/D      =,11X10H,  RE      =,11X10H,  PR      =,11X10H,
      1Q+      =,11X10H,  RER      =)
3062 FORMAT(1H+,9H  X+      =,11X10H,  GZ      =,11X10H,  X+,X      =,11X10H,
      1Q+      =,11X10H,  RER      =)
3064 FORMAT(1H+,9H  P        =,11X10H,  DP/DX =,11X10H,  XMOM =,11X10H,
      1K.E. =,11X10H,  ALPHA =,11X10H,  BETA  =)
3066 FORMAT(1H+,9H  NU       =,11X10H,  ST      =,11X10H,  TW/TB =,11X10H,
      1FA      =,11X10H,  FB      =,11X10H,  FB*RE =)
3068 FORMAT(1H,22X10H,  FREF = ,1PE11.4,10H,  RATIO =,1PE11.4)
3069 FORMAT(1H+,9H  RO       =,11X10H,  UB      =,11X10H,  TB      =,11X10H,
      1TW      =,11X10H,  RHOV =,11X10H,  TAUW =)
  
```


HOMOGENEOUS FLOW MODEL PROGRAM

(b) SUBROUTINE OUTLET [Cont.]

```

3070 FORMAT(6(10X1PE11.4))
3072 FORMAT(1H0,39X30HINDEPENDENT SIMILARITY NUMBERS/)
3074 FORMAT(1H0,39X16HINTEGRAL RESULTS/)
3076 FORMAT(1H0,39X28HDEPENDENT SIMILARITY NUMBERS/)
3078 FORMAT(1H0,39X18HNCNORMALIZED RESULTS/)
3079 FORMAT(1H0,39X48HNCNDIMENSIONAL VELOCITY AND TEMPERATURE PROFILES/
1)
3090 FORMAT(1H1)
      FRAC=FRACC
      SAVE(1)=HB
      SAVE(2)=HW
      SAVE(3)=QW
      SAVE(4)=XMOM
      SAVE(5)=X
      SAVE(6)=P
      SAVE(7)=ELI
      SAVE(8)=FA
      SAVE(9)=FB
      SAVE(10)=FS
      HB=HBI+FRAC*(HB-HBI)
      HW=HWI+FRAC*(HW-HWI)
      QW=QWI+FRAC*(QW-QWI)
      XMOM=XMOMI+FRAC*(XMOM-XMOMI)
      X=XPRINT
      IF (ABS(SAVE(5)-XLH).LT.1.0E-10.AND.NG.NE.1) X=SAVE(5)
      IF (ABS(MDOT-1.0).GT.1.0E-10) MDOT=1.0-X/XL*FLOAT(IC(5))
      P=PI+FRAC*(P-PI)
      ELI=FRAC*ELI
      CONT=XMASS-1.0
      CALL TEMP(HB,P,TB)
      CALL DENS(TB,P,RHOB)
      CALL TEMP(HBI,PI,TBI)
      CALL DENS(TBI,PI,RHOB1)
      CALL TEMP(HW,P,TW)
      CALL VISC(TB,P,XMU)
      CALL COND(TB,P,E1)
      CALL SPEC(TB,P,S1)
      IF (ABS(QW).LT.1.0E-10.OR.ABS(HB).LT.1.0E-10) TB=TW+1.0E-10
      T1=TW/TB
      REX=RE*MDOT/XMU
      PRX=PR*S1*XMU/E1
      XNU=RE*PR*QW/(E1*(TB-TW))
      ST=QW/(S1*(TB-TW))
      PAR1=XNU/(PRX**0.4)
      PAR2=PAR1/(0.021*(REX**0.8))
      QB=0.5*QW*RE*PR/(E1*TB)
      XPI=2.0*X/(RE*PR)
      XP=2.0*X/(REX*PRX)
      IF (X.LT.1.0E-10) GO TO 410
      GZ=2.0/XP
      GZI=2.0/XPI

```

HOMOGENEOUS FLOW MODEL PROGRAM

(b) SUBROUTINE OUTLET [Cont.]

```

410 FA=FAI+FRAC*(FA-FAI)
   FB=FBI+FRAC*(FB-FBI)
   FS=FSI+FRAC*(FS-FSI)
   FR=FS/(0.0014+(0.125/REX**0.32))
   VB=MDOT/RHOB
   ECX=EC*VB*VB/TB
   S1=S(N+1)
   G1=SQRT(2.0*RHOB*S1/FS)
   B1=QW*SQRT(2.0*RHOB/(FS*S1))
   JJ=JJ+1
   FREF=16.0/REX
   IF (RE.GT.RET) FREF=0.0014+0.125/REX**0.32
   FREQ=FB*REX
   FRES=FS*REX
   QPLUS=-0.5*QW*RE*PR
   RER=RE*RHOVA(N1)/(4.0*SAVE(7))
   BETA=XMOM/(RHOB*VB*VB)
   ALPHA=(XKEI+FRAC*(XKE-XKEI))/(RHOB*VB**3)
   FRATIO=FB/FREF
   *****
C      DEFINE MEMBERS OF RV (RESULT VECTOR)
C      *****
RV(1)=CONT
RV(2)=ECX
RV(3)=EBAL
RV(4)=FA
RV(5)=TW/TB
RV(6)=FB
RV(7)=FR
RV(8)=FREQ
RV(9)=FRES
RV(10)=FS
RV(11)=GZ
RV(12)=GZI
RV(13)=HB
RV(14)=HW
RV(15)=P
RV(16)=PAR1
RV(17)=PAR2
RV(18)=PRX
RV(19)=-QB
RV(20)=-QW
RV(21)=REX
RV(22)=RHOB
RV(23)=ST
RV(24)=U.U
RV(25)=TB
RV(26)=TW
RV(27)=VB
RV(28)=X
RV(29)=XMASS
RV(30)=XMOM
RV(31)=XP
RV(32)=XFI
RV(33)=XNU

```


HOMOGENEOUS FLOW MODEL PROGRAM

(b) SUBROUTINE OUTLET [Cont.]

```

RV(34)=J.0
RV(35)=0.0
RV(36)=FREF
RV(37)=FRATIO
RV(38)=DPDX
RV(39)=RER
RV(40)=BETA
RV(41)=ALPHA
DO 430 J=1,12
DO 420 I=1,42
IF(JCOL(J).NE.I) GO TO 420
RESULT(J,JJ)=RV(I)
NRV(J)=I
GO TO 430
420 CONTINUE
430 CONTINUE
IF((IC(3).EQ.0).OR.(NG.GE.IC(3))) GO TO 470
432 IF(ABS(X-XOUT(NG)).LT.1.0E-10) GO TO 434
IF(X.LT.XOUT(NG)) GO TO 470
NG=1+NG
GO TO 432
C *****
C CALCULATIONS FOR NON-DIMENSIONAL TURBULENT PROFILES
C *****
434 CALL VISC(TW,P,XMUW)
CALL DENS(TW,P,RHOW)
CALL SPEC(TW,P,CPW)
TAUW=0.5*FS/RHOB
CY=0.5*RE*SQRT(RHOW*TAUW)/XMUW
CU=VB/SQRT(TAUW/RHOW)
CT=0.0
IF(ABS(QW).GT.1.0E-10) CT=CPW*TW*SQRT(TAUW/RHOW)/QW
RI=R(2)*0.5
IF(IC(4).EQ.3) GO TO 393
WRITE(2,3090)
WRITE(2,3072)
IF(RE.LT.RET) GO TO 390
WRITE(2,3070) X,REX,PRX,QPLUS,RER
WRITE(2,3060)
GO TO 392
390 WRITE(2,3070) XPI,GZ,XP,QPLUS,RER
WRITE(2,3062)
392 WRITE(2,3074)
WRITE(2,3070) P,DPDX,XMOM,XKE,ALPHA,BETA
WRITE(2,3064)
WRITE(2,3076)
WRITE(2,3070) XNU,ST,RV(5),FA,FB,FREB
WRITE(2,3066)
WRITE(2,3078)
WRITE(2,3068) FREF,FRATIO
WRITE(2,3079)
WRITE(2,3070) R(N1),VB,TB,TW,RHOW,TAUW
WRITE(2,3069)
WRITE(2,2003)
WRITE(2,3052)

```

HOMOGENEOUS FLOW MODEL PROGRAM

(b) SUBROUTINE OUTLET [Cont.]

```

IF(RE.GT.RET) WRITE(2,3056)
WRITE(2,2003)
393 NNT=1
IF(N1.GT.34) NNT=2
IF(N1.GT.68) NNT=3
DO 460 I=1,N1
J=I
IF(I.EQ.1) V1=0.0
IF(I.EQ.N1) V1=0.25*RHOVA(I)/(SAVE(7)*(FRAC*S(I)+(1.0-FRAC)*SI(I))
1)
IF((I.EQ.1).OR.(I.EQ.N1)) GO TO 457
RHOM=SI(J)+FRAC*(S(J)-SI(J))
RO=0.5*(R(J+1)+R(J))
FLUX=RHOVA(J-1)/RI+(RHOVA(J)/RO-RHOVA(J-1)/RI)*(R(J)-R(J-1))/(R(J+
11)-R(J-1))
RI=RO
EP=E(J-1)+(E(J)-E(J-1))*(R(J)-R(J-1))/(R(J+1)-R(J-1))
ZP=Z(J-1)+(Z(J)-Z(J-1))*(R(J)-R(J-1))/(R(J+1)-R(J-1))
V1=FLUX/(RHOM*4.0*SAVE(7))
457 H1=HI(I)+FRAC*(H(I)-HI(I))
CALL TEMP(H1,P,T1)
U1=(UI(I)+FRAC*(U(I)-UI(I)))/VB
IF(I.NE.1) GO TO 458
EP=E(1)
ZP=Z(1)
458 YPLUS=Y(I)*CY
UPLUS=U1*CU
TPLUS=(1.0-T1/TW)*CT
T1=(TW-T1)/(TW-TB)
IF(I.EQ.N1) GO TO 459
IF(MOD(I-1,NNT).NE.0) GO TO 460
459 IF(IC(4).EQ.3) GO TO 460
WRITE(2,3054) R(I),U1,V1,T1
IF(RE.GT.RET) WRITE(2,3058) YPLUS,UPLUS,TPLUS,EP,ZP
460 CONTINUE
NG=NG+1
470 HB=SAVE(1)
HW=SAVE(2)
QW=SAVE(3)
XMOM=SAVE(4)
X=SAVE(5)
P=SAVE(6)
ELI=SAVE(7)
FA=SAVE(8)
FB=SAVE(9)
FS=SAVE(10)
RETURN
END

```


HOMOGENEOUS FLOW MODEL PROGRAM

(c) SUBROUTINE SUMMARY

```

SUBROUTINE SUMMARY
REAL MBAL,MOBAL,MDOT
COMMON IC(6),HEAD(8),RESULT(12,100),U(100),UI(100),UP(100),
1      H(100),HI(100),HP(100),S(100),SI(100),RHOVA(100),RHOVA(100)
2      ,E(100),Z(100),R(100),Y(100),RDR(100),AREA(100),DCM(100),
3      DCE(100),TAU(100)
COMMON /IN1/ QWM,QW1,QW2,QW3,NVAR,NWALL,NDIR,XLH,XL,NIV,RE,PR,
X      EC,TD,YP1,N1,CA,X1
COMMON /IN2/ NQ,XQ(50),QWALL(50),D2QDX(50)
COMMON /OUT/ ELI,ENTH,ENERG,HB,HBI,HV,HVI,IOUT,ITT,JJ,N,P,PI,
Z      QW,QWI,RET,X,XMASS,XMOM,XMOMI,FSR1,FBR1,NG,MDOT,
Z      EBAL,MBAL,MOBAL,DPDX,XKE,XKEI,FA,FAI,FB,FBI,FS,FSI
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
COMMON /TAB/ JCOL(12),CHEAD(12),THEAD(8),NRV(12)
COMMON /TRANSP/ BL,DELTA,MODEL,YPL,CAPPA,TMODEL(8)
2003 FORMAT(1H1)
3021 FORMAT(1H //1X12A11)
3022 FORMAT(1H ,I7,11I11,/)
3024 FORMAT(1H ,12(1PE11.3)/1X)
3026 FORMAT(1H0,24X8A10)
3030 FORMAT(1H ,18X7HEXPA = ,G9.4,9H, EXPB = ,G9.4,9H, EXPC = ,G9.4,9H,
1      EXPD = ,G9.4)
3032 FORMAT(1H ,29HTURBULENT TRANSPORT MODEL** ,8A10)
3034 FORMAT(1H ,30HBOUNDARY CONDITIONS** NVAR = ,I2,10H, NWALL = ,I2,
19H, NDIR = ,I2,7H, QM = ,G9.3,8H, QW1 = ,G9.3,8H, QW2 = ,G9.3,8H,
2QW3 = ,G9.3)
3036 FORMAT(1H ,28HINITIAL CONDITIONS** NIV = ,I2,8H, REI = ,G9.4,8H,
1      PRI = ,G9.4,10H, MACHI = ,G9.4)
3038 FORMAT(1H ,17HGEOMETRY** L/D = ,G11.4,9H, LE/D = ,G9.4)
3040 FORMAT(1H ,22HGRID CONSTANTS** TD = ,F5.1,7H, N1 = ,I3,7H, Y1 = ,G
19.4,7H, X1 = ,G9.4,8H, BLF = ,G9.4)
3042 FORMAT(1H ,23HCONTROL VECTOR** IC = ,6(I2,2H, ))
WRITE(2,2003)
WRITE(2,3026) HEAD
WRITE(2,3026) THEAD
IF(RE.GT.RET) WRITE(2,3032) (TMODEL(K),K=1,8)
WRITE(2,3034) NVAR,NWALL,NDIR,QWM,QW1,QW2,QW3
WRITE(2,3036) NIV,RE,PR,EC
WRITE(2,3038) XL,XLH
WRITE(2,3040) TD,N1,Y(N),X1,CA
WRITE(2,3042) IC
IF(JJ.EQ.0) RETURN
WRITE(2,3021) CHEAD
WRITE(2,3022) (NRV(J),J=1,12)
IF(JJ.GT.40) GO TO 1441
WRITE(2,3024) ((RESULT(I,J),I=1,12),J=1,JJ)
GO TO 1442
1441 WRITE(2,3024) ((RESULT(I,J),I=1,12),J=1,40)
WRITE(2,3021) CHEAD
WRITE(2,3022) (NRV(J),J=1,12)
WRITE(2,3024) ((RESULT(I,J),I=1,12),J=41,JJ)
1442 RETURN
END

```


HOMOGENEOUS FLOW MODEL PROGRAM

(d) SUBROUTINE TPROPS

SUBROUTINE TPROPS

THIS PROGRAM CALCULATES TRANSPORT PROPERTIES

REAL MDOT,MBAL,MOBAL,LAMDA

COMMON IC(6),HEAD(8),RESULT(12,100),U(100),UI(100),UP(100),H(100),

X HI(100),HP(100),S(100),SI(100),RHOVA(100),RHOVA(100),E(100)

X ,Z(100),R(100),Y(100),RDR(100),AREA(100),DCM(100),DCE(100),

X TAU(100)

COMMON /IN1/ QWM,QW1,QW2,QW3,NVAR,NWALL,NDIR,XLH,XL,NIV,RE,PR,EC,

Y TD,YP1,N1,CA,X1

COMMON /OUT/ ELI,ENTH,ENERG,HB,HBI,HW,HWI,IOUT,ITT,JJ,N,P,PI,QW,

1 QWI,RET,X,XMASS,XMOM,XMOMI,FSR1,FBR1,NG,MDOT,EBAL,

2 MBAL,MOBAL,DPDX,XKE,XKEI,FA,FAI,FB,FBI,FS,FSI

COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)

1 ,NRO,NMU,NC,NCP,NH,NT

COMMON /TRANSP/ BL,DELTA,MODEL,YPL,CAPPA,TMODEL(8)

COMMON /SPLI/ AJ(100),BJ(100),CJ(100),DJ(100),FJ(100),HJ(100),

1PJ(100),QJ(100),RJ(100)

DIMENSION UDR(3)

IF(RE.GT.RET) GO TO 200

100 DO 120 J=1,N

HJT=0.5*(H(J)+H(J+1))

CALL TEMP(HJT,P,TJ)

CALL VISC(TJ,P,E(J))

CALL COND(TJ,P,Z1)

CALL SPEC(TJ,P,CP)

Z(J)=Z1/(CP*PR)

120 CONTINUE

RETURN

200 JDELTA=1

IF(ABS(BL).LT.1.0E-10) GO TO 300

DO 220 J=2,N

IF(U(1)-U(J).LT.0.0001) GO TO 220

IF(1.0-R(J).GE.DELTA) JDELTA=J

IF(JDELTA.EQ.2) GO TO 210

DELTA=1.0-R(JDELTA)

IF(DELTA.GT.720.0/RE) BL=1.0

IF(ABS(BL-2.0).LT.1.0E-10) GO TO 100

GO TO 300

210 BL=0.0

DELTA=1.0

JDELTA=1

GO TO 300

220 CONTINUE

IF(ABS(DELTA).LT.1.0E-10) GO TO 100

300 NBLP=N1-JDELTA+1

NS=4

NF=4

CALL DSF1(NBLP,R(JDELTA),U(JDELTA),PJ(JDELTA),NS,NF,AJ,BJ,CJ,DJ,QJ

1,FJ,RJ,HJ)

CALL DSF2(NBLP,R(JDELTA),U(JDELTA),PJ(JDELTA),R(N1),UDR,AJ)

CALL TEMP(HBI,P,TB)

CALL TEMP(H(N1),P,TW)

CALL VISC(TW,P,XMUW)

HOMOGENEOUS FLOW MODEL PROGRAM

(d) SUBROUTINE TPROPS [Cont.]

```

CALL DENS(TW,P,RHOW)
CALL DENS(TB,P,RHOB)
IF(IC(6).NE.2) GO TO 404
CALL VISC(TB,P,XMU)
REX=RE*MDOT/XMU
CALL VFS(TB,P,VF)
FW=0.079/(REX**0.25)
FSL=0.079/(((1.0+5.5*VF)**0.48*REX**0.25/((1.0+5.5*VF)**0.15)))
VKC=SQRT(FSL/FW)*(1.52*ALOG10(REX*SQRT(FSL))-0.152)/(1.52*ALOG10(R
1EX*SQRT(FW))-0.152)
404 IF(IC(6).NE.2) VKC=1.0
TAUW=-UDR(2)*XMUW
IF(TAUW.LT.0.0) TAUW=-TAUW*U(N)/(Y(N)*UDR(2))
IF(JDELTA.LT.N) FS=4.0*RHOB*TAUW/RE
DUDRJ=0.0
DO 320 J=JDELTA,N
CALL DSF2(NBLP,R(JDELTA),U(JDELTA),PJ(JDELTA),R(J+1),UDR,AJ)
IF(UDR(2).GT.0.0) UDR(2)=(U(J+1)-U(J))/(R(J+1)-R(J))
DUDY=-0.5*(UDR(2)+DUDRJ)
DUDRJ=UDR(2)
TAU(J)=E(J)*DUDY/TAUW
320 CONTINUE
TAU(N1)=1.0
NJ=N+JDELTA
DO 405 JR=JDELTA,N
J=NJ-JR
HJT=0.5*(H(J)+H(J+1))
CALL TEMP(HJT,P,TJ)
CALL VISC(TJ,P,XMU)
CALL COND(TJ,P,Z1)
CALL DENS(TJ,P,RHO)
CALL SPEC(TJ,P,CP)
DUDY=ABS(TAUW*TAU(J)/E(J))
RJV=0.5*(R(J)+R(J+1))
YJ=1.0-RJV
401 FSS=FS*TAU(J)
C2=0.5*RE/XMU*SQRT(0.5*FSS*RHO/RHOB)
YP=YJ*C2
A1=1.0-EXP(-YP/YPL)
EDDY=0.5*RE*((0.416*YJ-0.799*YJ*YJ+0.654*YJ*YJ*YJ)*A1*VKC)**2*DUDY
402 E(J)=XMU+EDDY*RHO
Z(J)=Z1/(CP*PR)+EDDY*RHO
405 CONTINUE
IF(JDELTA.EQ.1) RETURN
J1=JDELTA-1
DO 407 J=1,J1
HJT=0.5*(H(J)+H(J+1))
CALL TEMP(HJT,P,TJ)
CALL VISC(TJ,P,E(J))
CALL COND(TJ,P,Z1)
CALL SPEC(TJ,P,CP)
Z(J)=Z1/(CP*PR)
407 CONTINUE
RETURN
END

```


HOMOGENEOUS FLOW MODEL PROGRAM

(e) SUBROUTINE TTPF

```

C      SUBROUTINE TTPF(NPH,TO,WF,PRT)
C      *****
C      THIS SUBROUTINE NORMALISES THE FLUID TRANSPORT AND
C      THERMODYNAMIC PROPERTIES
C      *****
      REAL MU
      COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
      DIMENSION HH(6),HT(6),HCP(6),HC(6),HMU(6),HRO(6)
      DIMENSION TH2(15)
      NTP=6
      M=6
      NTPM1=NTP-1
      IF(NPH.EQ.2) CALL TPOS(NTP,WF)
      CALL LAGR(NTP,TT,TH,TO,H)
      CALL LAGR(NTP,TT,TRO,TO,RO)
      CALL LAGR(NTP,TT,TMU,TO,MU)
      CALL LAGR(NTP,TT,TC,TO,C)
      CALL LAGR(NTP,TT,TCP,TO,CP)
      PRT=CP*MU/C
      TL=TO
      DT=(TT(NTP)-TO)/FLOAT(NTPM1)
      DO 30 J=1,NTP
      CALL LAGR(NTP,TT,TH,TL,HL)
      CALL LAGR(NTP,TT,TRO,TL,ROL)
      CALL LAGR(NTP,TT,TCP,TL,CPL)
      CALL LAGR(NTP,TT,TC,TL,CL)
      CALL LAGR(NTP,TT,TMU,TL,XMUL)
      HH(J)=HL
      HT(J)=TL
      HC(J)=CL
      HCP(J)=CPL
      HMU(J)=XMUL
      HRO(J)=ROL
      TL=TL+DT
30  CONTINUE
      DO 40 J=1,NTP
      TT(J)=HT(J)/TO
      TRO(J)=HRO(J)/RO
      TMU(J)=HMU(J)/MU
      TC(J)=HC(J)/C
      TCP(J)=HCP(J)/CP
      TH(J)=(HH(J)-H)/(CP*TO)
      TH2(J)=TH(J)
40  CONTINUE
      CALL POLY(M,NRO,TT,TRO)
      CALL POLY(M,NMU,TT,TMU)
      CALL POLY(M,NC,TT,TC)
      CALL POLY(M,NCP,TT,TCP)
      CALL POLY(M,NH,TT,TH)
      CALL POLY(M,NT,TH2,TT)
      RETURN
      END

```


HOMOGENEOUS FLOW MODEL PROGRAM

(f) SUBROUTINE TPOS

```

SUBROUTINE TPOS(NTP,WF)
C *****
C THIS SUBROUTINE GIVES FLUID PROPERTIES FOR A TWO-COMPONENT MIXTURE
C *****
COMMON /PROPS/TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
COMMON/VOLF/ TVF(6),NVF
DIMENSION TSC(6),TSCP(6),TSRO(6),TS(6)
DATA TSC/6*0.01298/
DATA TSRO/6*2564.46/
DATA TSCP/0.6966,0.7419,0.7871,0.8323,0.8775,0.9228/
DATA TS/0.0,30.0,60.0,90.0,120.0,150.0/
K=6
HL=0.0
TL=0.0
DO 10 I=1,NTP
T=TT(I)
CALL LAGR(K,TS,TSRO,T,RHOS)
CALL LAGR(K,TS,TSC,T,CONDS)
CALL LAGR(K,TS,TSCP,T,CPS)
VF=WF*TRO(I)/((1.0-WF)*RHOS+WF*TRO(I))
TCP(I)=WF*CPS+(1.0-WF)*TCP(I)
TMU(I)=TMU(I)*(1.0+5.5*VF)
TVF(I)=VF
TRO(I)=VF*RHOS+(1.0-VF)*TRO(I)
TC(I)=TC(I)/(1.0-VF)**3
HS=CPS*(T-TL)+HL
TH(I)=WF*HS+(1.0-WF)*TH(I)
HL=HS
TL=T
10 CONTINUE
CALL POLY(K,NVF,TT,TVF)
RETURN
END

```

(g) SUBROUTINE LAGR

```

SUBROUTINE LAGR(N,X,Y,XP,YP)
C *****
C LAGRANGE INTERPOLATION ROUTINE
C *****
DIMENSION X(N),Y(N)
YP=0.0
DO 505 K=1,N
XPLK=1.0
DO 510 I=1,N
IF(I.EQ.K) GO TO 510
XPLK=XPLK*(XP-X(I))/(X(K)-X(I))
510 CONTINUE
YP=YP+XPLK*Y(K)
505 CONTINUE
RETURN
END

```


HOMOGENEOUS FLOW MODEL PROGRAM

(h) SUBROUTINE DSFI

```

C      SUBROUTINE DSFI(NO,X,Y,M,KS,KN,H,LAMDA,MU,D,Q,U,DD,DDN)
C      *****
C      THIS SUBROUTINE FITS A CUBIC SPLINE FUNCTION TO
C      A SET OF TABULATED DATA
C      *****
      REAL MU,M,LAMDA
      DIMENSION X(NO),Y(NO),M(NO),H(NO),LAMDA(NO),MU(NO),D(NO),Q(NO),
1      U(NO),DD(KS),DDN(KN)
      N=NO
      DO 5 I=2,N
      H(I)=X(I)-X(I-1)
5      CONTINUE
      N1=N-1
      DO 15 J=2,N1
      LAMDA(I)=H(I+1)/(H(I)+H(I+1))
      MU(I)=1.0-LAMDA(I)
      D(I)=6.0*((Y(I+1)-Y(I))/H(I+1)-(Y(I)-Y(I-1))/H(I))/(H(I)+H(I+1))
15      CONTINUE
      KS1=KS-1
      KN1=KN-1
      DO 105 I=1,KS1
      DD(I)=(Y(I+1)-Y(I))/H(I+1)
105      CONTINUE
      DO 106 I=1,KN1
      DDN(I)=(Y(N+1-I)-Y(N-I))/H(N+1-I)
106      CONTINUE
      SUM=DD(1)
      SUMN=DDN(1)
      DO 115 MN=2,KS1
      J=KS-MN
      DO 125 I=1,J
      DD(I)=(DD(I+1)-DD(I))/(X(I+MN)-X(I))
125      CONTINUE
      ADD=DD(1)
      DO 135 L=2,MN
      ADD=ADD*(X(1)-X(L))
135      CONTINUE
      SUM=SUM+ADD
115      CONTINUE
      DO 116 MN=2,KN1
      J=KN-MN
      DO 126 I=1,J
      DDN(I)=(DDN(I)-DDN(I+1))/(X(N+1-I)-X(N+1-I-MN))
126      CONTINUE
      ADDN=DDN(1)
      DO 136 L=2,MN
      ADDN=ADDN*(X(N)-X(N+1-L))
136      CONTINUE
      SUMN=SUMN+ADDN

```


HOMOGENEOUS FLOW MODEL PROGRAM

(h) SUBROUTINE DSFI [Cont.]

```

116 CONTINUE
    LAMDA(N)=0.0
    MU(N)=1.0
    D(1)=6.0/H(2)*((Y(2)-Y(1))/H(2)-SUM)
    D(N)=6.0/H(N)*(SUMN-(Y(N)-Y(N-1))/H(N))
    Q(1)=-0.5
    U(1)=D(1)/2.0
    DO 25 I=2,N
        P=MU(I)*Q(I-1)+2.0
        Q(I)=-LAMDA(I)/P
        U(I)=(D(I)-MU(I)*U(I-1))/P
25 CONTINUE
    M(N)=U(N)
    DO 35 I=1,N1
        J=N-I
        M(J)=Q(J)*M(J+1)+U(J)
35 CONTINUE
    RETURN
    END
    
```

(i) SUBROUTINE DSF2

```

C SUBROUTINE DSF2(N,X,Y,M,XINT,F,H)
C *****
C THIS SUBROUTINE INTERPOLATES WITHIN A SET OF
C DATA USING THE RESULTS FROM DSF1
C *****
    REAL M
    DIMENSION X(N),Y(N),M(N),H(N),F(3)
    XI=XINT
    IF(XI.LT.X(1).OR.XI.GT.X(N)) GO TO 999
    DO 5 I=2,N
        H(I)=X(I)-X(I-1)
        IF(XI.GE.X(I-1).AND.XI.LE.X(I)) GO TO 15
5 CONTINUE
15 F(1)=M(I-1)/(6.0*H(I))*(X(I)-XI)**3+M(I)/(6.0*H(I))*(XI-X(I-1))**3
    1+(Y(I-1)-M(I-1)*H(I)*H(I)/6.0)*(X(I)-XI)/H(I)+(Y(I)-M(I)*H(I)*H(I)
    2/6.0)*(XI-X(I-1))/H(I)
    F(2)=M(I)/(2.0*H(I))*(XI-X(I-1))**2-M(I-1)/(2.0*H(I))*(X(I)-XI)**2
    1+(Y(I)-Y(I-1))/H(I)-(M(I)-M(I-1))*H(I)/6.0
    F(3)=M(I-1)*(X(I)-XI)/H(I)+M(I)*(XI-X(I-1))/H(I)
    RETURN
999 WRITE(2,300)
300 FORMAT(1H ,50H*****EXTRAPOLATION HAS BEEN ATTEMPTED IN DSF2-*****)
    RETURN
    END
    
```

HOMOGENEOUS FLOW MODEL PROGRAM

(j) SUBROUTINE POLY

```

SUBROUTINE POLY(NU,M,X,Y)
C *****
C THIS SUBROUTINE FITS A POLYNOMIAL TO A SET OF TABULATED DATA
C *****
  DIMENSION X(NU),Y(NU),A(11,11),B(11),P(20)
  MX2=M*2
  DO 15 I=1,MX2
    P(I)=0.0
  DO 13 J=1,NU
    P(I)=P(I)+X(J)**I
13 CONTINUE
15 CONTINUE
  N=M+1
  DO 30 I=1,N
    DO 35 J=1,N
      K=I+J-2
      IF(K) 29,29,28
28 A(I,J)=P(K)
      GO TO 35
29 A(1,1)=NU
35 CONTINUE
30 CONTINUE
  B(1)=0.0
  DO 21 J=1,NU
    B(1)=B(1)+Y(J)
21 CONTINUE
  DO 22 I=2,N
    B(I)=0.0
    DO 23 J=1,NU
      B(I)=B(I)+Y(J)*X(J)**(I-1)
23 CONTINUE
22 CONTINUE
  NM1=N-1
  DO 300 K=1,NM1
    KP1=K+1
    L=K
    DO 400 I=KP1,N
      IF(ABS(A(I,K))-ABS(A(L,K))) 400,400,401
401 L=1
400 CONTINUE
    IF(L=K) 500,500,405
405 DO 410 J=K,N
      TEMP=A(K,J)
      A(K,J)=A(L,J)
      A(L,J)=TEMP
410 CONTINUE
    TEMP=B(K)
    B(K)=B(L)
    B(L)=TEMP
500 DO 300 I=KP1,N
      FACTOR=A(I,K)/A(K,K)
      A(I,K)=0.0
      DO 301 J=KP1,N
        A(I,J)=A(I,J)-FACTOR*A(K,J)
301 CONTINUE

```


HOMOGENEOUS FLOW MODEL PROGRAM

(j) SUBROUTINE POLY [Cont.]

```
B(I)=B(I)-FACTOR*B(K)
300 CONTINUE
Y(N)=B(N)/A(N,N)
I=NM1
710 IP1=I+1
SUM=0.0
DO 700 J=IP1,N
SUM=SUM+A(I,J)*Y(J)
700 CONTINUE
Y(I)=(B(I)-SUM)/A(I,I)
I=I-1
IF(I) 800,800,710
800 CONTINUE
RETURN
END
```

(k) SUBROUTINE TEMP

```
SUBROUTINE TEMP(H1,P,T1)
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
T1=TT(1)
DO 10 I=1,NT
T1=T1+TT(I+1)*H1**I
10 CONTINUE
RETURN
END
```

(l) SUBROUTINE ENTPY

```
SUBROUTINE ENTPY(T1,P,H1)
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
H1=TH(1)
DO 10 I=1,NH
H1=H1+TH(I+1)*T1**I
10 CONTINUE
RETURN
END
```

HOMOGENEOUS FLOW MODEL PROGRAM

(m) SUBROUTINE DENS

```
SUBROUTINE DENS(T1,P,RHO)
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),C(6)
1      ,NRO,NMU,NC,NCP,NH,NT
RHO=TRO(1)
DO 10 I=1,NRO
RHO=RHO+TRO(I+1)*T1**I
10 CONTINUE
RETURN
END
```

(n) SUBROUTINE VFS

```
SUBROUTINE VFS(T,P,VF)
COMMON/VOLF/TVF(6),NVF
VF=TVF(1)
DO 10 I=1,NVF
VF=VF+TVF(I+1)*T**I
10 CONTINUE
RETURN
END
```

(o) SUBROUTINE SPEC

```
SUBROUTINE SPEC(T1,P,CP)
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
CP=TCP(1)
DO 10 I=1,NCP
CP=CP+TCP(I+1)*T1**I
10 CONTINUE
RETURN
END
```

(p) SUBROUTINE VISC

```
SUBROUTINE VISC(T1,P,XMU)
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
XMU=TMU(1)
DO 10 I=1,NMU
XMU=XMU+TMU(I+1)*T1**I
10 CONTINUE
RETURN
END
```


HOMOGENEOUS FLOW MODEL PROGRAM

(q) SUBROUTINE COND

```

SUBROUTINE COND(T1,P,E1)
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
E1=TC(1)
DO 10 I=1,NC
E1=E1+TC(I+1)*T1**I
10 CONTINUE
RETURN
END
    
```

(r) BLOCK DATA SEGMENT

```

BLOCK DATA
COMMON /OUT/ ELI,ENTH,ENERG,HB,HBI,HW,HWI,ICUT,ITT,JJ,N,P,PI,QW,QW
11,RET,X,XMASS,XMOM,XMOMI,FSR1,FBR1,NG,MDOT,EBAL,MBAL,MOBAL,DPDX,
2XKE,XKEI,FA,FAI,FB,FBI,FS,FSI
COMMON /PRINT/ XOUT(50)
COMMON /PROPS/ TT(6),TH(6),TRO(6),TMU(6),TCP(6),TC(6)
1      ,NRO,NMU,NC,NCP,NH,NT
COMMON/VCLF/ TVF(6),NVF
COMMON /TRANSP/ BL,DELTA,MODEL,YPL,CAPPA,TMODEL(8)
DATA CAPPA,YPL,BL,RET/0.416,26.0,0.0,2100.0/
DATA (TMODEL(K),K=1,8) /10H D.B. MIXI,10HNG LENGTH,10H CORRELATIO,
110HN WITH VAN,10H DRIEST DA,10HMPING FACT,10HOR + SPALD,10HING MOD
2. /
DATA(XOUT(K),K=1,10) /0.2,0.1,0.5,2.0,10.0,30.0,60.0,100.0,187.0,2
150.0/
DATA TT/20.0,40.0,65.0,95.0,130.0,180.0/
DATA TCP/4.183,4.179,4.188,4.213,4.27,4.42/
DATA TH/83.9,167.5,272.0,398.0,546.0,763.15/
DATA TRO/998.2,992.2,980.4,961.5,934.6,886.5/
DATA TC/0.000603,0.000632,0.000658,0.000678,0.000688,0.000676/
DATA TMU/0.001002,0.000551,0.00043,0.000294,0.000211,0.000149/
DATA NRO,NMU,NC,NCP,NH,NT/5,5,5,5,5,5/
DATA NVF/5/
END
    
```