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Analysing reforms of social security disability pension provision in a multistage overlapping generations model

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Abstract

This paper introduces a multi-stage Overlapping Generations (OLG) model of workers and retirees, segmented by disability status and eligibility for Disability Pension (DP) benefits. Extending Gertler (1999) with progressive disability stages of distinct productivity and transition probabilities, and calibrated to US data, the model generates DP benefits of 6.78% of GDP in the base case. Sensitivity analyses on replacement ratios, survival, labor-force growth, and differential mortality confirm robustness. We evaluate four reforms: Disability Assistance, employment incentives, benefit reduction, and tighter screening. Benefit reduction and screening are near-substitutes at the macroeconomic level but differ sharply in distributional consequences, while a decomposition of aggregate and per-capita effects shows that the large changes induced by employment incentives are mainly compositional. A two-state benchmark confirms that the multi-stage structure is necessary to capture these distinctions, underscoring the need to balance fiscal sustainability with protection of vulnerable groups.

KEYWORDS

financial decision, overlapping generations model, social disability pension

1 | INTRODUCTION

Disability substantially reduces labor supply and productivity, making it a major driver of poverty and income inequality. According to the Department for Work and Pensions (Department for Work and Pensions, 2025), individuals in families with disabled members are more than twice as likely to live in poverty compared to those without. Disability pensions (DP) form an essential component of social security systems across OECD countries, providing financial protection to individuals whose work capacity is limited due to disability. Over the past five decades, enrollment in DP schemes has grown significantly. In the United States, the share of non-elderly adults receiving disability-related pensions has risen from less than 1% in the late 1950s to nearly 5 percent in recent years, while in several European countries, including the United Kingdom and Norway, DP now cover between 7% and 10% of the working-age population (Autor et al., 2019). As Autor et al. (2019) note, disability-related transfers represent one of the largest components of social spending in OECD nations, often surpassing expenditures on food stamps, traditional cash welfare, or the Earned Income Tax Credit in the United States.

The expansion of DP schemes has generated ongoing debate over their costs and benefits. DP protect against severe income losses, help stabilize living standards, and reduce poverty among vulnerable households (Meyer & Mok, 2019; Stephens, 2001). At the same time, such programs may distort labor supply and human capital decisions while imposing growing fiscal pressures (Autor & Duggan, 2003; Bound, 1989; Chen & Van der Klaauw, 2008; French & Song, 2014). In response, several countries have introduced reforms aimed at strengthening eligibility criteria and modifying benefits to improve fiscal sustainability and encourage employment among individuals with partial work capacity (Autor et al., 2014; Deshpande et al., 2021; Haller et al., 2024). These reforms illustrate the fundamental policy tension between safeguarding the welfare of disabled individuals and preserving the long-term sustainability of public pension systems.

Despite the central role of DP in modern welfare states, a significant gap remains in the literature. Much of the existing research is empirical or micro-focused, analyzing disability as a risk factor within life-cycle models (Chandra & Samwick, 2009; Kitao, 2014; Low & Pistaferri, 2015) or studying its impact on income, consumption, and employment outcomes (Kostøl & Mogstad, 2015; Maestas et al., 2013; Meyer & Mok, 2019). Actuarial approaches have integrated disability and retirement contingencies into solvency assessments of pay-as-you-go pension systems (Garvey et al., 2023; Pérez-Salamero González et al., 2017; Ventura-Marco & Vidal-Meliá, 2014), but these frameworks typically do not model the feedback between disability pension design and household savings or consumption behavior. While these contributions provide valuable insights, macroeconomic analyses of DP remain limited. In particular, there is a lack of macroeconomic frameworks that explicitly model progressive disability transitions across multiple severity levels and evaluate how benefit design affects intergenerational redistribution at the aggregate level.

This paper develops a multi-stage overlapping generations (OLG) model that addresses this gap. The model extends the framework of Gertler (1999) by introducing a progressive disability structure in which workers transition through multiple stages of declining productivity before retirement. Unlike standard OLG models that treat disability as a binary state, our framework distinguishes between moderate and severe disability, allowing each stage to carry distinct productivity levels, benefit entitlements, and transition probabilities. This multi-stage structure is essential for evaluating policies that target specific severity levels, such as disability assistance (DA) for moderately disabled workers or tighter screening for severe disability benefits.

We calibrate the model to US data using three primary sources: the American Community Survey (ACS) 2024 for earnings and employment by disability status (US Census Bureau, 2025b), the ACS 2020–2024 5-year estimates for disability prevalence among the elderly population (US Census Bureau, 2025a), and the Social Security Administration's Annual Statistical Supplement for Social Security Disability Insurance (SSDI) benefit levels and recipient counts (Social Security Administration, 2024). The calibrated model generates a steady-state equilibrium in which disability pension benefits account for 6.78% of GDP, the tax rate is 27.67%, and aggregate consumption is 55.74% of GDP.

Using this framework, we conduct four policy experiments that span two contrasting objectives. Two safety-net policies introduce DA for workers who do not qualify for contribution-based benefits and employment incentives for disabled workers to remain in the labor force. Two fiscal consolidation policies reduce benefit levels and tighten eligibility screening. Each policy is evaluated not only at a single reference intensity but across a continuous range of policy parameters, allowing us to trace the full response surface and identify fiscal tradeoffs. A common Δe (change in benefits as a share of GDP) axis enables direct comparison of reforms that operate through fundamentally different channels.

The analysis yields three key findings. First, benefit reduction and tighter screening produce strikingly similar aggregate effects, suggesting they are near-substitutes as fiscal instruments at the macroeconomic level. However, their distributional consequences differ sharply: benefit cuts lower consumption for DP recipients across the board, while screening concentrates losses on the most severely disabled. Second, DA is nearly Pareto-improving among disabled subgroups, with consumption gains of up to 11.45% for disabled workers at a modest cost of under 1% to healthy workers. Third, employment incentives have the smallest aggregate fiscal footprint and operate through the labor supply channel rather than the transfer channel; a decomposition of aggregate consumption changes shows that their large group-level effects are almost entirely compositional, reflecting the reclassification of individuals across groups, while per-capita consumption changes remain below 0.3% for all groups.

We further demonstrate the robustness of these results through sensitivity analyses along three dimensions: the income replacement ratio, post-retirement survival, and labor force growth. Additionally, we introduce differential mortality across disability groups, showing that the baseline aggregate equilibrium outcomes are robust to empirically motivated mortality gradients, with the tax rate varying by less than 0.07 percentage points across all mortality scenarios.

Two features of the framework shape how these results should be interpreted. Following Gertler (1999), the model achieves analytical tractability through group-level aggregation: assets are pooled within each group and individual asset histories are not tracked, so precautionary saving against idiosyncratic disability risk is not modeled directly. Disability instead affects the equilibrium through group-level productivity, transfers, transition probabilities, and the distribution of assets across groups. The policy experiments should therefore be read as general-equilibrium analyses of how alternative disability-related transfer schemes shape aggregate savings, fiscal balances, and the redistribution of consumption across severity groups and generations, including the compositional dynamics that our decomposition analysis isolates, rather than as a structural treatment of household-level disability insurance design. Within this scope, the multi-stage structure is precisely what makes the policy questions tractable: it distinguishes reforms that operate on benefit levels from reforms that operate on transitions between severity states, a distinction that a binary disability framework cannot represent.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces the model, outlining the demographic structure, individual decision-making, and steady-state equilibrium. Section 4 presents the calibration and base-case results, together with sensitivity analyses and the two-state benchmark comparison. Section 5 discusses the policy reform simulations. Section 6 concludes.

2 | LITERATURE REVIEW

The economic implications of DP have been widely studied, particularly their effects on employment, income adequacy, and public finances. Foundational work (Autor & Duggan, 2003; Benítez-Silva et al., 2004; Bound, 1989; French & Song, 2014; Gruber, 2000; Haveman & Wolfe, 1984a,b; Parsons, 1984; Wachter et al., 2011) shows that disability-related benefits can reduce labor force participation and, in some cases, substitute for other forms of social protection such as unemployment insurance. For example, Bound (1989) finds that fewer than half of rejected applicants return to work, suggesting that disability insurance (DI) receipt discourages employment. Similarly, Autor and Duggan (2003) link the growth in US disability enrollment to falling unemployment, implying substitution effects. Other studies, such as Chen and Van der Klaauw (2008) and French and Song (2014), provide causal evidence that DI receipt significantly lowers employment. These findings motivate modeling disability as a persistent productivity reduction with consequences for both household savings and aggregate fiscal outcomes. Using Austrian reform data, Haller et al. (2024) show that tighter eligibility rules generate larger fiscal savings with smaller insurance losses than benefit reductions, highlighting the importance of distinguishing between these two policy instruments.

Estimates of these employment effects are subject to methodological debate. Parsons (1991) cautions that using denied applicants as a comparison group may overstate DI's disincentive effects, since the application process itself can depress employment. Bound (1991a,b) raise a related concern about the measurement of disability, showing that the choice between self-reported and objective health indicators significantly affects estimated magnitudes. These identification challenges reinforce the value of structural approaches that model disability as an observable productivity state rather than relying on reduced-form estimates of behavioral responses.

Evidence on early retirement is particularly relevant to our framework, which models retirement as a probabilistic transition from any disability state. Haveman and Wolfe (1984b) show that disability transfers accelerated older workers' exit from the labor force, while Hernæs et al. (2024) document that pension reform restricting one early exit route led to increased disability insurance claims in Norway, underscoring the interdependence of retirement and disability pathways that our transition structure is designed to capture. On the targeting side, Benítez-Silva et al. (2004) document substantial misreporting in self-reported disability data, estimating that over 40% of individuals classified as disabled show little evidence of work limitation, motivating policy experiments such as our screening reform that tighten the boundary between disability groups. On the employer side, Aizawa et al. (2025) show that firms may use contract design to screen out disabled workers, providing a complementary equilibrium perspective on the employment incentive simulations examined later in this paper.

While most of the above literature focuses on employment responses, studies such as Stephens (2001) and Meyer and Mok (2019) document persistent declines in income and consumption following disability onset, highlighting the savings and consumption consequences that are the primary focus of our model. OLG models provide a natural framework for analyzing these consequences at the aggregate level.

Early life-cycle applications incorporate disability risk (Chandra & Samwick, 2009; Kitao, 2014). Low and Pistaferri (2015) quantify the insurance incentive trade-off within a structural setting, and Fehr (2024) simulates the interplay between old-age and DP under recent German reforms, showing that expanding disability benefits can offset the fiscal gains of raising the retirement age. At the macroeconomic level, Nishiyama (2015) uses a heterogeneous-agent OLG model with wage and mortality shocks to study fiscal sustainability under population aging, though disability is captured only indirectly. Recent evidence that the mortality effects of DI receipt differ between marginal and inframarginal beneficiaries (Black et al., 2024) further motivates policy frameworks that account for heterogeneity within the disabled population.

A related strand of research examines public pension design using multi-state transition frameworks. Alonso-García and Devolder (2016) derive diversification conditions between PAYG and funded components in mixed pension systems, showing that diversification benefits depend on dynamic efficiency and the correlation of returns across cohorts. Extensions by Alonso-García et al. (2018a,b); Alonso-García and Devolder (2019) emphasize automatic balancing mechanisms, liquidity and solvency rules, and intergenerational fairness. Ventura-Marco and Vidal-Meliá (2016) extend these frameworks by integrating retirement and permanent disability into notional defined contribution (NDC) schemes, highlighting how disability incidence and mortality shape adequacy and sustainability.

Our model takes a different approach by extending the overlapping-generations framework of Gertler (1999) to include multiple stages of work, retirement, and disability. Unlike the pension design frameworks above, which focus on benefit adequacy and solvency, our model embeds disability as a productivity shock within a general equilibrium setting, allowing feedback between pension design and aggregate saving. This multi-stage structure allows us to evaluate policy reforms that target specific severity levels and to identify distributional effects across disability groups that would not be visible in a two-state specification. The productivity differentials across disability stages also connect to Jacobs (2023), who shows that occupations and preference heterogeneity shape the insurance value of DI. This explicit link between disability shocks and productivity within a multi-stage OLG framework is, to the best of our knowledge, new to the literature.

3 | DEVELOPMENT OF A MULTI-STAGE OLG MODEL

Our framework builds on the overlapping-generations model of Gertler (1999), which introduced finite lifespans and heterogeneity between workers and retirees in order to capture the distributional effects of fiscal policy. We extend this setup in two directions. First, we introduce a multi-stage structure of working life, where individuals may experience permanent productivity losses due to disability shocks before transitioning into retirement. This feature allows us to capture the dynamics of earnings heterogeneity and the role of disability-related transfers. Second, we embed a contribution-based social insurance system that involves transfers to both workers and retirees, thereby linking individual income risks to aggregate fiscal policy.

As in Gertler (1999), the model achieves tractability through group-level aggregation. Assets are effectively pooled within each group, and the framework does not track individual asset histories: individual precautionary saving responses to idiosyncratic disability risk are not modeled directly. Instead, disability affects the equilibrium through group-level productivity, benefits, transition probabilities, and the distribution of assets across groups. The risk-adjusted transition probabilities introduced in Section 3.3 capture how workers value income streams across future states, but they do not generate individual-specific buffer-stock saving. This structure is what permits exact aggregation and a transparent characterization of the steady state; its implications for the interpretation of the policy experiments are discussed in the Introduction and the Conclusion.

The remainder of this section develops the model in stages. We begin by describing the demographic structure of workers and retirees, and then specify their decision problems. We next aggregate across groups to characterize the dynamics of consumption and wealth distribution, and derive the steady-state equilibrium. Finally, we describe the pension system, which is central to our analysis since it governs the transfer terms that enter household budgets and the government's budget constraint.

3.1 | Demographic structure

We introduce a model that examines the relationship between disability and labor productivity in an OLG framework. The population progresses through two distinct life stages: work and retirement, with finite lifespans. Workers are classified into M groups, where M denotes the total number of disability stages, based on their labor productivity levels, denoted as $\xi_1, \xi_2, \ldots, \xi_M$, where ξ_1 represents the highest productivity (fully able workers) and ξ_M represents the lowest productivity (workers with severe disability). Figure 1 illustrates this structure and the associated transitions for the case $M = 3$.

New cohorts enter the model as workers with the highest labor productivity ξ_1, representing full ability. Workers with productivity levels $\xi_1, \ldots, \xi_{M-1}$ (nonseverely disabled workers) in the current period face three potential transitions in the subsequent period: maintaining their current productivity level, experiencing productivity decline due to disability onset or worsening, or retiring. Workers who have reached the most severe disability state (productivity level ξ_M) either remain at this productivity level or retire in the following period.

Unlike standard work-life models where retirement is only possible after a stipulated working period, our framework allows retirement from any productivity state. This captures the empirical reality that disability often triggers early retirement, with more severe disability states associated with higher retirement probabilities. In general, $p_{i,j}$ denotes the transition probability from state i to state j in each period: $p_{w,w}$ is the probability of remaining in the current group, $p_{w,w+1}$ is the probability of disability worsening, and $p_{w,r(w)}$ is the probability of retiring from group w. These probabilities vary by disability status, reflecting both voluntary retirement decisions and disability-induced labor force exits.

Mortality during the working stage is not considered. Upon retirement, individuals either continue to live or pass away. Retirees do not re-enter the workforce and are excluded from the labor market.

Workers' groups, determined by their labor productivity ξ_w, are denoted as w, where $w = 1, 2, \ldots, M$. The group of retirees who retired from worker group w is labeled as $r(w)$.

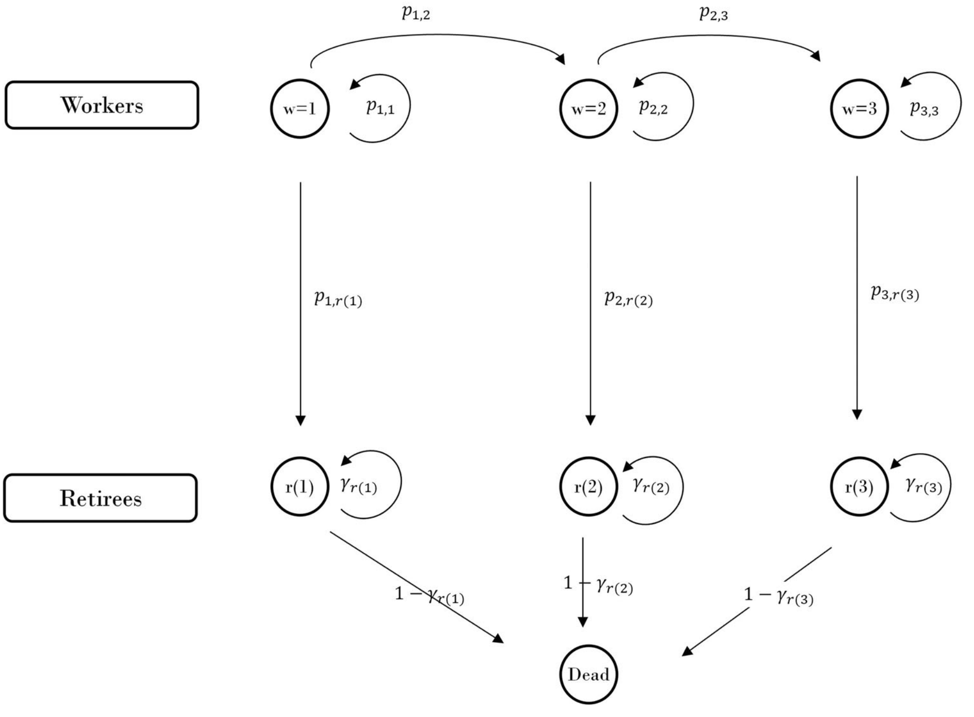


FIGURE 1 Dynamics of the multi-stage OLG model when $M = 3$.

For workers in group $w \in \{1, 2, ..., M\}$, transition probabilities $p_{w,j}$ are non-zero only when: $j = w$ (staying), $j = w + 1$ for $w < M$ (disability worsening), or $j = r(w)$ (retirement). Workers in the terminal group ($w = M$) can only remain at their current productivity level or retire, since there is no further disability stage. The survival probability of retirees in group $r(w)$ is denoted as $\gamma_{r(w)}$. All probabilities are time-invariant. Let N_t^w denote the number of workers in group w at time t. The population of each worker group evolves according to inflows from the preceding group and stayers within the same group:

$$N_{t+1}^w = p_{w-1,w} N_t^{w-1} + p_{w,w} N_t^w, w = 2, 3, ..., M. \tag{1}$$

Similarly, let us denote the number of total retirees at time t as N_t^r, and the number of retirees in group $r(w)$ as $N_t^{r(w)}$, $w = 1, 2, ..., M$. Let n denote the exogenous population growth rate. Then $N_{t+1}^{r(w)} = (1 + n) N_t^{r(w)}$ and $N_t^{r(w)}$ follows the dynamics:

$$N_{t+1}^{r(w)} = \gamma_{r(w)} N_t^{r(w)} + p_{w,r(w)} N_t^w, w = 1, 2, ..., M. \tag{2}$$

Hence, solving the recursive relation yields:

$$N_t^{r(w)} = \frac{p_{w,r(w)}}{1 + n - \gamma_{r(w)}} N_t^w. \tag{3}$$

Therefore, we denote the ratio of the number of retirees in $r(w)$ to that of workers in w as

$$\psi_{r(w)} = \frac{p_{w,r(w)}}{1 + n - \gamma_{r(w)}}. \tag{4}$$

Let $(q^1, q^2, ..., q^M)$ and $(q^{r(1)}, q^{r(2)}, ..., q^{r(M)})$ denote the steady-state population distributions of workers and retirees, respectively. These distributions satisfy the following equilibrium conditions.

First, worker flows must balance in the steady state. New workers enter at rate n into group 1, while existing workers transition across groups according to:

$$(1 + n)q^w = p_{w-1,w}q^{w-1} + p_{w,w}q^w, w = 2, 3, ..., M, \tag{5}$$

where the left-hand side is the size of group w required next period to maintain the steady-state share q^w under population growth at rate n, and the right-hand side is the actual inflow: workers who transition from group $w - 1$ plus those who remain in group w.

Second, the retiree population in group $r(w)$ receives new entrants who retire from worker group w at rate $p_{w,r(w)}$, while existing retirees survive to the next period with probability $\gamma_{r(w)}$. In steady state, requiring $(1 + n)q^{r(w)} = \gamma_{r(w)}q^{r(w)} + p_{w,r(w)}q^w$ and solving for $q^{r(w)}$ yields a fixed ratio between retirees and workers:

$$q^{r(w)} = \psi_{r(w)}q^w, w = 1, 2, ..., M. \tag{6}$$

Finally, the population shares must sum to unity:

$$\sum_{w=1}^{M} (q^w + q^{r(w)}) = 1. \tag{7}$$

Together, Equations (5)–(7) form a system of $2M$ equations that uniquely determine the steady-state distributions.

3.2 | Retirees' decision

Retirees maximize lifetime utility using recursive preferences following Farmer (1990). For retirees in group $r(w)$, the value function is

$$V_t^{r(w)} = \left[\left(C_t^{r(w)}\right)^\rho + \beta\gamma_{r(w)}\left(V_{t+1}^{r(w)}\right)^\rho\right]^{1/\rho} \tag{8}$$

where $V_t^{r(w)}$ is the lifetime value function, $C_t^{r(w)}$ is consumption at time t, ρ is the curvature parameter of the CES aggregator ($\rho < 1$), β is the subjective discount factor, $\sigma = 1/(1 - \rho)$ is the intertemporal elasticity of substitution, and $\gamma_{r(w)}$ is the survival probability.

Each retiree in group $r(w)$ holds financial assets $A_t^{r(w)}$, receives social security benefits $E_t^{r(w)}$, and faces the budget constraint

$$A_{t+1}^{r(w)} = \frac{R_t}{\gamma_{r(w)}}A_t^{r(w)} + E_t^{r(w)} - C_t^{r(w)}, \tag{9}$$

where R_t is the gross rate of return on assets. The term $R_t/\gamma_{r(w)}$ reflects the mortality-adjusted return, as surviving retirees inherit the assets of deceased cohort members, effectively boosting the per-survivor return above the market rate. Solving the retiree's optimization problem (see Appendix B for the full derivation), the optimal consumption function takes the form

$$C_t^{r(w)} = \epsilon_t^{r(w)} \pi_t^w \left(\frac{R_t}{\gamma_{r(w)}} A_t^{r(w)} + S_t^{r(w)} \right), \tag{10}$$

where π_t^w is the marginal propensity to consume out of total wealth, $\epsilon_t^{r(w)}$ is the ratio of a retiree's consumption propensity to that of a worker in the same group, and $S_t^{r(w)}$ is social security wealth, defined as the present value of the entire future benefit stream. The term in parentheses represents the retiree's total effective wealth at time t: mortality-adjusted financial assets plus social security wealth.

3.3 | Workers' decision

Workers face probabilistic transitions across productivity levels and into retirement. The notation follows the same convention as for retirees, with superscript w replacing $r(w)$: C_t^w denotes consumption, A_t^w financial assets, and so forth. A worker in group w maximizes

$$V_t^w = \left[\left(C_t^w \right)^\rho + \beta \left(\sum_j p_{w,j} V_{t+1}^j \right)^\rho \right]^{1/\rho} \tag{11}$$

where the summation is over the feasible destination states: $j \in \{w, w+1, r(w)\}$ for $w < M$ and $j \in \{w, r(w)\}$ for $w = M$.

The optimal consumption function is:

$$C_t^w = \pi_t^w \left(R_t A_t^w + H_t^w + S_t^w \right) \tag{12}$$

subject to the budget constraint

$$A_{t+1}^w = R_t A_t^w + \xi_w W_t + E_t^w - T_t^w - C_t^w, \tag{13}$$

where W_t is the aggregate wage rate, E_t^w is disability pension benefits, and T_t^w is the tax levied on group w. Here H_t^w represents human wealth (the present value of future net labor income) and S_t^w is the expected present value of future social security benefits. The term in parentheses in (12) thus represents total effective wealth: financial assets plus human wealth plus social security wealth.

A key innovation is that workers use risk-adjusted transition probabilities $p_{w,j}^*$ when valuing future income streams. These probabilities weight each transition by the relative marginal utility in the destination state:

$$p^*_{w,w} = \frac{p_{w,w}}{\Omega^w_{t+1}}, \tag{14}$$

$$p^*_{w,w+1} = \frac{p_{w,w+1}\left(\pi^{w+1}_{t+1}/\pi^w_{t+1}\right)^{1/(1-\sigma)}}{\Omega^w_{t+1}}, \tag{15}$$

$$p^*_{w,r(w)} = \frac{p_{w,r(w)}\left(\epsilon^{r(w)}_{t+1}\right)^{1/(1-\sigma)}/\gamma_{r(w)}}{\Omega^w_{t+1}}, \tag{16}$$

where Ω^w_{t+1} is the risk adjustment factor, defined as

$$\Omega^w_{t+1} = p_{w,w} + p_{w,w+1}\left(\pi^{w+1}_{t+1}/\pi^w_{t+1}\right)^{1/(1-\sigma)} + p_{w,r(w)}\left(\epsilon^{r(w)}_{t+1}\right)^{1/(1-\sigma)}/\gamma_{r(w)}. \tag{17}$$

This factor normalizes the risk-adjusted probabilities so that $p^*_{w,w} + p^*_{w,w+1} + p^*_{w,r(w)} = 1$. The risk-adjusted retirement probability accounts for both the consumption drop upon retirement ($\epsilon^{r(w)}_t$) and mortality risk ($\gamma_{r(w)}$), while the disability worsening probability reflects differences in consumption propensities (π^{w+1}_t/π^w_t) across productivity states. See Appendix C for the full derivation.

3.4 | Aggregation decision

Now, we amalgamate individual functions to derive an aggregate function. Concerning the distribution of wealth, denoted for $w = 1, 2, ..., M$, λ^w_t and $\lambda^{r(w)}_t$ signify the proportions of financial assets held by workers in group w and retirees in group $r(w)$, respectively. The entirety of financial assets in the economy is represented as $A^\bullet_t = K_t + B_t$, where the superscript $\bullet$ denotes aggregation across all individuals within a group. This constitutes the combination of capital accumulation and government bonds. The financial assets held by workers in group w can be represented by $\lambda^w_t A^\bullet_t$, while those held by retirees in group $r(w)$ can be delineated as $\lambda^{r(w)}_t A^\bullet_t$. By construction, these shares exhaust total wealth: $\sum_w (\lambda^w_t + \lambda^{r(w)}_t) = 1$.

Subsequently, by summing (10) across individual retirees, the aggregate consumption $C^{r(w)\bullet}_t$ for each retiree group at time t can be expressed as

$$C^{r(w)\bullet}_t = \epsilon^{r(w)}_t \pi^w_t \left(R_t \lambda^{r(w)}_t A^\bullet_t + S^{r(w)\bullet}_t\right), \tag{18}$$

where $S^{r(w)\bullet}_t$ is aggregate social security wealth for retiree group $r(w)$, defined as the present value of current and future aggregate benefit flows $E^{r(w)\bullet}_t$ (see Appendix B for the full recursion). The consumption propensity $\epsilon^{r(w)}_t \pi^w_t$ remains consistent across retirees in group $r(w)$, facilitating the aggregation. Each retiree receives a total return on financial wealth equal to $R_t/\gamma_{r(w)}$, but only the proportion $\gamma_{r(w)}$ of all retirees survive; thus, the total wealth available to retirees in each group at time t is $R_t \lambda^{r(w)}_t A^\bullet_t$.

Similar to retirees, workers within the same group exhibit identical consumption propensities. Consequently, the aggregate consumption for each worker group can be derived by summing (12) across workers, resulting in

$$C_t^{w\bullet} = \pi_t^w\left(R_t\lambda_t^w A_t^\bullet + H_t^{w\bullet} + S_t^{w\bullet}\right), \tag{19}$$

where $H_t^{w\bullet}$ and $S_t^{w\bullet}$ are aggregate human wealth and social security wealth for worker group w, respectively (see Appendix C for definitions, recursions, and the role of population-scaling factors in the aggregation).

The government finances its general fiscal expenditures and social security benefits by issuing 1-year short-term government bonds and levying taxes on workers. The total tax revenue is governed by the following government budget constraint:

$$T_t^\bullet = R_tB_t - B_{t+1} + G_t + E_t^\bullet. \tag{20}$$

This equation outlines the total tax collected, $T_t^\bullet$, which encompasses the redemption of short-term government bonds $(R_tB_t - B_{t+1})$, government spending (G_t), and social security benefits $(E_t^\bullet)$. Subsequently, the total tax levied on the labor income of each worker group is determined.

By combining equations (18) and (19), we derive the aggregate consumption function as follows:

$$\begin{aligned} C_t^\bullet &= \sum_w C_t^{r(w)\bullet} + \sum_w C_t^{w\bullet} \\ &= \sum_w \epsilon_t^{r(w)}\pi_t^w\left(R_t\lambda_t^{r(w)}A_t^\bullet + S_t^{r(w)\bullet}\right) + \sum_w \pi_t^w\left(R_t\lambda_t^w A_t^\bullet + H_t^{w\bullet} + S_t^{w\bullet}\right) \\ &= R_tA_t^\bullet\left[\sum_w \pi_t^w\left(\epsilon_t^{r(w)}\lambda_t^{r(w)} + \lambda_t^w\right)\right] + \sum_w \pi_t^w\left(\epsilon_t^{r(w)}S_t^{r(w)\bullet} + H_t^{w\bullet} + S_t^{w\bullet}\right). \end{aligned} \tag{21}$$

Consequently, the equilibrium output matches aggregate demand

$$Y_t = C_t^\bullet + K_{t+1} - (1-\delta)K_t + G_t, \tag{22}$$

where aggregate supply is determined by a Cobb-Douglas production function

$$Y_t = \left(X_t\sum_w \xi_w N_t^w\right)^\alpha (K_t)^{1-\alpha}, \tag{23}$$

with X_t representing labor-augmenting technology, N_t^w denoting the number of workers in group w, and K_t being the aggregate capital stock. The term $\xi_w N_t^w$ is the effective labor supply of group w, so that $\sum_w \xi_w N_t^w$ is total effective labor in the economy. Capital depreciates at rate δ and the labor income share is α. Under perfect competition, factor prices equal marginal products, giving the real wage per unit of effective labor and the gross rate of return as

$$W_t = \alpha Y_t \Big/ \sum_w \xi_w N_t^w \quad \text{and} \quad R_t = (1-\alpha)Y_t/K_t + (1-\delta). \tag{24}$$

The distribution of wealth evolves endogenously as workers save for retirement and transition across productivity states, while retirees dissave and face mortality risk (see Appendix D for details).

3.5 | Steady-state equilibrium

We now proceed to establish the steady-state equations. In the steady state, the economy experiences continuous growth at a rate of $(1+x)(1+n)$, with technology growth rate x and workforce growth rate n. We standardize all quantity variables by dividing them by the output Y, denoting the resulting normalized steady-state values in lowercase: $c^w = C^{w\bullet}/Y$, $h^w = H^{w\bullet}/Y$, $s^w = S^{w\bullet}/Y$, and so forth. Let us consider an economy characterized by input parameters such as workers' labor productivity $\{\xi_w : w = 1, 2, ..., M\}$, retirees' survival rates $\{\gamma_{r(w)} : w = 1, 2, ..., M\}$, transition probabilities $\{p_{i,j}\}$, subjective discount factor β, intertemporal elasticity of substitution σ, labor income share α, government debt to GDP ratio b, social security benefits to GDP ratio e and government expenditure to GDP ratio g. The steady-state population distributions of workers and retirees can be determined from (5), (6), and (7).

By defining $k = K/Y$ in equation (24) and expressing it in terms of k and R at the steady state, we can deduce the capital intensity as follows:

$$k = (1-\alpha)/[R-(1-\delta)]. \tag{25}$$

From (20), we derive τ, the ratio of total tax to output, as

$$\tau = [R-(1+x)(1+n)]b + g + e. \tag{26}$$

Consequently, the total tax is allocated to each worker group $w = 1, 2, ..., M$ in proportion to their share of labor income:

$$\tau^w = (\alpha^w/\alpha)\tau, \tag{27}$$

where $\alpha^w := (q^w\xi_w/\sum_i q^i\xi_i)\alpha$ represents the income distribution for each worker group.

The consumption propensities for each worker and retiree are expressed by the following equations:

$$\epsilon^{r(w)}\pi^w = 1-(R)^{\sigma-1}(\beta)^\sigma\gamma_{r(w)}, \tag{28}$$

$$\pi^w = 1-(R\Omega^w)^{\sigma-1}(\beta)^\sigma, \tag{29}$$

which correspond to (B1) and (C1), respectively. The steady-state equation governing a retiree's social security wealth, derived from equation (B3), is given by

$$s^{r(w)} = e^{r(w)}/[1-(1+x)\gamma_{r(w)}/R]. \tag{30}$$

From (D2) and (D3), we can derive the human wealth $\{h^w : w = 1, 2, ..., M\}$ and social security wealth $\{s^w : w = 1, 2, ..., M\}$ for workers by solving the following system of linear equations, where $p^*_{w,j}$ denotes the steady-state risk-adjusted transition probabilities defined in Section 3.3:

$$h^w = \begin{cases} \dfrac{R}{R-(1+x)p^*_{w,w}}\left[(\alpha^w - \tau^w) + p^*_{w,w+1}\dfrac{(1+x)}{R}\dfrac{q^w}{q^{w+1}}h^{w+1}\right], & w = 1, 2, \ldots, M-1, \\ \dfrac{R}{R-(1+x)p^*_{w,w}}(\alpha^w - \tau^w), & w = M, \end{cases} \tag{31}$$

$$s^w = \begin{cases} \dfrac{R}{R-(1+x)p^*_{w,w}}\left[e^w + p^*_{w,w+1}\dfrac{(1+x)}{R}\dfrac{q^w}{q^{w+1}}s^{w+1} + p^*_{w,r(w)}\dfrac{(1+x)}{R/\gamma_{r(w)}}\dfrac{q^w}{q^{r(w)}}s^{r(w)}\right], & w = 1, 2, \ldots, M-1, \\ \dfrac{R}{R-(1+x)p^*_{w,w}}\left[e^w + p^*_{w,r(w)}\dfrac{(1+x)}{R/\gamma_{r(w)}}\dfrac{q^w}{q^{r(w)}}s^{r(w)}\right], & w = M, \end{cases} \tag{32}$$

The allocation of financial assets among worker groups $\{\lambda^w : w = 2, \ldots, M\}$ can be determined from the steady-state asset accumulation equations (see Appendix D for the full derivation) by solving the following system of linear equations:

$$\begin{aligned} \lambda^w = & \frac{p_{w,w}[(\alpha^w + e^w - \tau^w) - \pi^w(h^w + s^w)]}{[(1+x)(1+n) - p_{w,w}R(1-\pi^w)](k+b)} \\ & + \frac{p_{w-1,w}[(\alpha^{w-1} + e^{w-1} - \tau^{w-1}) - \pi^{w-1}(h^{w-1} + s^{w-1}) + \lambda^{w-1}R(1-\pi^{w-1})(k+b)]}{[(1+x)(1+n) - p_{w,w}R(1-\pi^w)](k+b)}, \\ & w = 2, \ldots, M. \end{aligned} \tag{33}$$

Similarly, from the retiree asset dynamics derived in Appendix D, the distribution of financial assets among retiree groups $\{\lambda^{r(w)} : w = 2, \ldots, M\}$ satisfies:

$$\begin{aligned} \lambda^{r(w)} = & \frac{(e^{r(w)} - \epsilon^{r(w)}\pi^w s^{r(w)}) + p_{w,r(w)}/p_{w,w}(1+x)(1+n)\lambda^w(k+b)}{[(1+x)(1+n) - (1-\epsilon^{r(w)}\pi^w)R](k+b)} \\ & - \frac{p_{w-1,w}\,p_{w,r(w)}/p_{w,w}[R(1-\pi^{w-1})\lambda^{w-1}(k+b) + (\alpha^{w-1} + e^{w-1} - \tau^{w-1}) - \pi^{w-1}(h^{w-1} + s^{w-1})]}{[(1+x)(1+n) - (1-\epsilon^{r(w)}\pi^w)R](k+b)}, \\ & w = 2, \ldots, M. \end{aligned} \tag{34}$$

and

$$\lambda^{r(1)} = \frac{(e^{r(1)} - \epsilon^{r(1)}\pi^1 s^{r(1)}) + p_{1,r(1)}/p_{1,1}(1+x)(1+n)\lambda^1(k+b)}{[(1+x)(1+n) - (1-\epsilon^{r(1)}\pi^1)R](k+b)} \tag{35}$$

The total financial asset allocation satisfies the constraint

$$\sum_w \lambda^{r(w)} + \sum_w \lambda^w = 1. \tag{36}$$

Thus, we have

$$\lambda^1 = \left(1 - \left(\sum_{w=2,\,\ldots,\,M} \lambda^w + \sum_{w=1,\,\ldots,\,M} \lambda^{r(w)}\right)\right) \tag{37}$$

From (18) and (19), we derive the steady-state equations for consumption as follows:

$$c^{r(w)} = \epsilon^{r(w)}\pi^w[R\lambda^{r(w)}(k + b) + s^{r(w)}], \tag{38}$$

$$c^w = \pi^w[R\lambda^w(k + b) + (h^w + s^w)]. \tag{39}$$

Combining these equations, we obtain the steady-state equation for the total consumption function

$$\begin{aligned} c &= \sum_w c^{r(w)} + \sum_w c^w \\ &= R(k + b)\left[\sum_w \pi^w(\epsilon^{r(w)}\lambda^{r(w)} + \lambda^w)\right] + \sum_w \pi^w[\epsilon^{r(w)}s^{r(w)} + (h^w + s^w)]. \end{aligned} \tag{40}$$

This equation allows us to compute the capital intensity as follows:

$$[(1 + x)(1 + n) - (1 - \delta)]k = 1 - c - g, \tag{41}$$

which can be expressed as

$$\begin{aligned} &[(1 + x)(1 + n) - (1 - \delta)]k \\ &= 1 - g - R(k + b)\left[\sum_w \pi^w(\epsilon^{r(w)}\lambda^{r(w)} + \lambda^w)\right] - \sum_w \pi^w[\epsilon^{r(w)}s^{r(w)} + (h^w + s^w)]. \end{aligned} \tag{42}$$

Hence, we get values for R and $\{\Omega^w : w = 1, 2, \ldots, M\}$, as follows:

$$R = \frac{1 - g - [(1 + x)(1 + n) - (1 - \delta)]k - \sum_w \pi^w[\epsilon^{r(w)}s^{r(w)} + (h^w + s^w)]}{(k + b)\sum_w \pi^w(\epsilon^{r(w)}\lambda^{r(w)} + \lambda^w)}, \tag{43}$$

$$\Omega^w = p_{w,w} + p_{w,w+1}(\pi^{w+1}/\pi^w)^{1/(1-\sigma)} + p_{w,r(w)}\frac{1}{\gamma_{r(w)}}(\epsilon^{r(w)})^{1/(1-\sigma)},\ w = 1, 2, \ldots, M. \tag{44}$$

3.6 | Social security benefits and disability pension

The pension system is a central institution in the model, as it defines the transfer terms E_t^w and $E_t^{r(w)}$ that appear in household budget constraints. These transfers affect individual consumption possibilities, tax burdens, and the aggregate composition of human and social security wealth.

For clarity, we distinguish between time-varying flows E_t^w and $E_t^{r(w)}$, which enter household budgets, and their steady-state counterparts e^w and $e^{r(w)}$, expressed as ratios to output. The pension system consists of two components: DP for active workers and retirement pensions for retirees.

For workers in group $w = 1, \ldots, M$, the disability pension is determined by the replacement ratio ζ^w and the group's wage share α^w:

$$e^w = \zeta^w \alpha^w. \tag{45}$$

This ensures that disability benefits scale with labor productivity and provide an income-replacement floor.

For retirees, pensions are tied to contribution histories. For groups $w < M$, benefits are given by

$$e^{r(w)} = \sum_{i=1}^{w} \Gamma \frac{\iota_i^w \alpha^i}{q^i} q^{r(w)}, \tag{46}$$

where Γ is the accrual rate and ι_i^w denotes years of service in each productivity group. For the most severely disabled group $w = M$, pensions guarantee a minimum by comparing contribution-based benefits with the disability replacement floor:

$$e^{r(M)} = \max\left(\sum_{i=1}^{M} \Gamma \frac{\iota^M{}_i \alpha^i}{q^i} q^{r(M)}, \zeta^M \alpha^M \frac{q^{r(M)}}{q^M} \right). \tag{47}$$

These rules ensure that retirement pensions grow with contribution histories, while DP provide insurance against income loss at any stage. For retirees, disability benefits continue to bind if they exceed contribution-based pensions. Together, these transfers feed directly into steady-state taxes, social security wealth, and consumption propensities, shaping the aggregate equilibrium. The assumption of a benefit floor is crucial because it removes any "incentives" to become or remain disabled (Haberman, 1987).

4 | MODEL CALIBRATION AND NUMERICAL RESULTS

This section presents numerical examples to illustrate the proposed multi-stage OLG model. We first set the structural parameters governing preferences, technology, and fiscal policy, and then calibrate the group-level parameters to U.S. data. Then the steady-state equilibrium is computed by simultaneously solving the system of equations presented in Section 3.5. This system comprises: (i) population distribution equations (5)–(7), (ii) consumption propensity equations (28)–(29), (iii) wealth valuation equations (30)–(32), (iv) asset distribution equations (33)–(37), and (v) market clearing conditions (43)–(44).

For our baseline analysis, we consider a model with three groups: healthy, disabled, and severely disabled ($M = 3$), where only the severely disabled group receives DP. We later compare this with a simplified two-group specification ($M = 2$) to validate the multi-stage structure, and extend to a four-group specification ($M = 4$) in the policy simulations to examine DA.

4.1 | Structural parameters

The model employs eight parameters that are linked to the economic situation and individual preferences: government debt to GDP ratio, government expenditure to GDP ratio, labor income share, labor growth, technical growth, depreciation, subjective discount factor, and intertemporal substitution. Mathematical symbols, descriptions, and their values are displayed in Table 1. The values of these parameters follow Lee et al. (2025).

4.2 | Calibration to US data

The model requires calibration of population shares, transition probabilities, and productivity parameters. Population shares and transition probabilities shape the demographic structure underpinning the policy simulations, while productivity parameters determine the earnings differentials across disability groups. Since the reform experiments in Section 5 operate by perturbing transition probabilities and benefit levels, anchoring these to observed population distributions and earnings data is essential for generating credible policy conclusions. We calibrate the steady-state distributions q^w and $q^{r(w)}$ to match US demographic and disability data, back out the transition probabilities from steady-state flow equations, and set productivity ratios from median earnings by disability status.

We calibrate the model's population structure using three US data sources. Worker counts are drawn from the American Community Survey (ACS) 2024 Table S1811 (US Census

TABLE 1 Descriptions and values of key economic parameters.

Parameter	Description	Value
b	Government debt to GDP	0.2
g	Government expenditure to GDP	0.2
α	Labor income share	0.667
n	Labor growth rate	0.01
x	Technology growth rate	0.01
δ	Depreciation	0.1
β	Subjective discount factor	1
σ	Intertemporal substitution	0.4

Bureau, 2025b), which reports employment and earnings by disability status for the civilian noninstitutionalized population aged 16 and over. Retiree structure is based on ACS 2020–2024 Table S1810 (US Census Bureau, 2025a), which reports disability prevalence by age group. Disability pension parameters are taken from the Social Security Administration's Annual Statistical Supplement 2024 (Social Security Administration, 2024).

4.2.1 | Population structure

Workers are partitioned into three groups: $w = 1$ (healthy), comprising 154.8 million non-disabled employed persons; $w = 2$ (disabled), comprising 12.5 million disabled employed persons; and $w = 3$ (severely disabled), comprising 2.0 million SSDI recipients with marginal work capacity, for a total of 169.3 million workers. These counts are drawn from ACS Table S1811 (US Census Bureau, 2025b). The 2.0 million in $w = 3$ are disabled individuals who report earnings but are not classified as employed, consistent with SSDI recipients engaged in trial work. The SSA reports 7.2 million disabled-worker beneficiaries receiving SSDI in December 2024 (Social Security Administration, 2024), and Liu and Stapleton (2010) find that 28% of the 1996 SSDI award cohort had earned more than \$1000 in at least 1 year within 10 years of entering the rolls. The ratio $2.0/7.2 \approx 28\%$ corroborates this estimate.

Retirees are partitioned into three groups: $r(1)$ (healthy), $r(2)$ (disabled), and $r(3)$ (severely disabled); Table 2 reports the resulting steady-state population shares. ACS Table S1810 (US Census Bureau, 2025a) reports 56.2 million persons aged 65 and over, of whom 18.5 million (32.9%) report a disability. Of the 7.2 million SSDI beneficiaries, 2.0 million are already classified as workers ($w = 3$), leaving 5.2 million non-working beneficiaries whose benefits automatically convert to retired-worker status upon reaching full retirement age (Social Security Administration, 2024). We therefore assign $r(3)$ to these 5.2 million former SSDI beneficiaries, $r(2)$ to the remaining 13.3 million disabled retirees ($18.5 - 5.2$), and $r(1)$ to the 37.7 million non-disabled retirees, satisfying $(q^{r(2)} + q^{r(3)})/(q^{r(1)} + q^{r(2)} + q^{r(3)}) = (13.3 + 5.2)/(37.7 + 13.3 + 5.2) = 32.9\%$.

Workers constitute 75.1% of the model population. The total disabled population (workers plus retirees) equals $(0.0555 + 0.0088 + 0.0589 + 0.0231) = 14.6\%$ of the model population, broadly consistent with the ACS national disability prevalence rate of 13.3% among the civilian non-institutionalized population (US Census Bureau, 2025a). The difference arises because the model's disabled groups include both working-age SSDI beneficiaries (in $w = 3$ and $r(3)$) and retirees with disabilities (in $r(2)$), capturing a broader population than the ACS age-specific prevalence rate.

4.2.2 | Survival, retirement and transition probabilities

Following the population structure of Gertler (1999), each retiree survives to the next period with constant probability $\gamma_{r(w)}$, implying a geometric distribution of post-retirement lifetimes with expected duration $1/(1 - \gamma_{r(w)})$ years.

TABLE 2 Distribution of each group for the base-case scenario.

q^1	q^2	q^3	$q^{r(1)}$	$q^{r(2)}$	$q^{r(3)}$
0.6864	0.0555	0.0088	0.1673	0.0589	0.0231

While disability affects both labor-force exit timing and mortality in practice, we impose equal post-retirement survival probabilities across all retiree groups ($\gamma_{r(1)} = \gamma_{r(2)} = \gamma_{r(3)} = 0.882$), corresponding to an average of 8.5 years in retirement. However, total expected lifespan still differs across groups because disabled workers face higher retirement probabilities ($p_{3,r(3)} = 0.333$ vs $p_{1,r(1)} = 0.031$) and therefore transition to retirement earlier, spending fewer years in the workforce. The equal-γ assumption thus abstracts from differential mortality conditional on retirement, while still capturing the dominant channel through which disability shortens economic lifetimes: earlier exit from the labor force. We relax this assumption and examine differential post-retirement mortality across disability groups as a sensitivity analysis in Section 4.4.1.

Given the population shares $\{q^w, q^{r(w)}\}$ and survival probabilities $\{\gamma_{r(w)}\}$, the transition probabilities $\{p_{i,j}\}$ are backed out from the steady-state population flow equations (5)–(6). These are implied residuals consistent with the steady state, not econometrically estimated from longitudinal microdata. This distinction is important for interpreting reform experiments (Policies 2 and 4) that directly perturb these probabilities: such experiments represent comparative statics across steady states rather than simulations of transitional dynamics. Table 3 reports the calibrated values.

The retirement probability increases with disability severity ($p_{1,r(1)} = 0.031, p_{2,r(2)} = 0.136, p_{3,r(3)} = 0.333$), consistent with the empirical observation that more severe disability is associated with earlier labor force exit.

4.2.3 | Productivity and benefit parameters

Labor productivity parameters are calibrated using data from the American Community Survey and the Social Security Administration. From ACS Table S1811 (US Census Bureau, 2025b), median annual earnings among the civilian noninstitutionalized population aged 16 and over with earnings are \$48,514 for persons without a disability and \$34,114 for persons with a disability (in 2024 inflation-adjusted dollars). Normalizing nondisabled earnings to unity, the relative productivity of the disabled group is $\xi_2 = 34,114/48,514 = 0.703$. For the severely disabled group ($w = 3$), which represents SSDI beneficiaries who retain marginal work capacity, we proxy annual labor productivity using the average monthly SSDI disabled-worker benefit of \$1,537 (Social Security Administration, 2024), annualized to \$18,444, yielding $\xi_3 = 18,444/48,514 = 0.380$. This proxy assumes that the benefit level approximates the economic value of the residual labor these individuals supply, which is a simplification since SSDI benefits are determined by pre-disability earnings histories rather than current productive capacity.

The disability benefit replacement rate is set to $\zeta^3 = 1$, indicating that the severely disabled group receives full DP benefits. The pension benefit parameters ι_j^w reflect differential pension entitlements across groups and contribution histories: healthy workers who retire in group $r(1)$

TABLE 3 Transition and survival probabilities. Transition probabilities out of each origin state sum to one; displayed values may not, due to rounding.

$p_{1,1}$	$p_{1,2}$	$p_{1,r(1)}$	$p_{2,2}$	$p_{2,3}$	$p_{2,r(2)}$	$p_{3,3}$	$p_{3,r(3)}$	$\gamma_{r(1)}$	$\gamma_{r(2)}$	$\gamma_{r(3)}$
0.953	0.016	0.031	0.810	0.055	0.136	0.667	0.333	0.882	0.882	0.882

receive $\iota_1^1 = 30$ reflecting full career contributions, while disabled workers who transition to retirement receive progressively lower pension amounts ($\iota_1^2 = 15, \iota_2^2 = 7$) due to reduced earnings histories. Severely disabled retirees receive the lowest pension entitlements ($\iota_1^3 = 15, \iota_2^3 = 3, \iota_3^3 = 2$), consistent with their limited labor market attachment. The pension accrual rate is set to $\Gamma = 0.01$. These values and all productivity parameters are summarized in Table 4.

4.3 | Base case results

After calibrating the parameters, we compute the steady-state equilibrium by simultaneously solving the system of equations presented in Section 3.5, using a nonlinear equation solver. We employ an iterative fixed-point algorithm that updates the interest rate R and risk adjustment factors Ω^w until convergence is achieved, ensuring all equilibrium conditions are satisfied simultaneously. The results are presented in Table 5.

Taxes paid by all workers amount to approximately 27.67% of GDP, suggesting that 41.48% of labor income is paid in taxes. Additionally, 55.74% of GDP is consumed by both workers and retirees. DP benefits account for 6.78% of GDP. Retirees accumulate more wealth than workers in general, with healthy retirees holding the largest asset share ($\lambda^{r(1)} = 0.199$). This pattern arises because individuals who remain healthy throughout their working lives benefit from full career earnings and uninterrupted saving, while disabled workers face reduced productivity and, in the case of SSDI recipients, earnings constraints that limit wealth accumulation during their working years. Among the disabled groups, those receiving DP benefits accumulate less wealth while working but more upon retirement, reflecting the earnings limits imposed on beneficiaries, a pattern consistent with the literature documenting work disincentives associated with disability benefit receipt.

TABLE 4 Productivity, benefit replacement, and pension input values for the base-case scenario.

Γ	ξ_1	ξ_2	ξ_3	ζ^3	ι_1^1	ι_1^2	ι_2^2	ι_1^3	ι_2^3	ι_3^3
0.01	1	0.703	0.380	1	30	15	7	15	3	2

TABLE 5 Key economic variables for the base-case scenario.

Group	τ	e	h	s	λ	c	q	π ($\epsilon\pi$)	Ω
$w = 1$	0.2606	0.0000	2.3758	0.5815	0.6397	0.3593	0.6864	0.0804	1.0798
$w = 2$	0.0148	0.0000	0.0684	0.0539	0.0548	0.0276	0.0555	0.1095	1.1392
$w = 3$	0.0013	0.0031	0.0039	0.0197	0.0090	0.0057	0.0088	0.1258	1.1748
$r(1)$		0.0459		0.2815	0.1989	0.1130	0.1673	0.1503	
$r(2)$		0.0107		0.0659	0.0684	0.0342	0.0589	0.1503	
$r(3)$		0.0080		0.0492	0.0292	0.0178	0.0231	0.1503	
Total	0.2767	0.0678			1.0000	0.5574	1.0000		
$k = 2.0196, R = 1.0649$									

4.4 | Sensitivity analysis

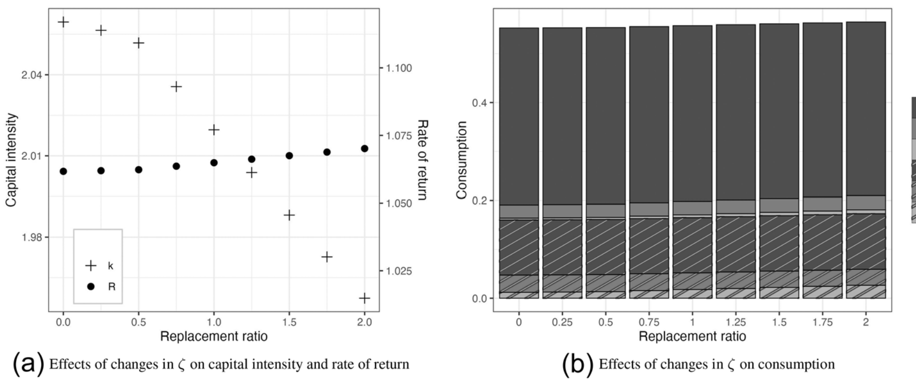
For the sensitivity analysis, we examine three dimensions of the parameter space: the income replacement ratio (ζ), expected post-retirement survival (γ), and labor-force growth (n). Table 6 summarizes the base-case equilibrium and the endpoint values for each sensitivity dimension; the full response surfaces are displayed in Figures 2 and 3.

We first vary the income replacement ratio ζ from 0 (no DP benefits) to 2 (double the base-case level). Figure 2a shows that higher DP benefits reduce capital intensity from $k = 2.060$ at $\zeta = 0$ to $k = 1.957$ at $\zeta = 2$, while raising the rate of return, consistent with the crowding-out of private saving as expanded public transfers substitute for private self-provision. The tax rate rises from 26.88% to 28.89% over this range, reflecting the direct fiscal cost of benefit provision. Figure 2b reveals that increased DP benefits reduce consumption among the healthy group while modestly raising consumption for DP recipients, reflecting the redistributive effect of the tax-transfer system.

Population structure changes are examined along two margins. First, Figure 3a,b display the effects of varying expected postretirement survival years uniformly across all groups. As expected survival rises from 6.5 to 10.5 years, capital intensity increases from $k = 1.714$ to

TABLE 6 Summary of sensitivity analysis: Base-case and endpoint values.

Parameter	Range	k	R	τ	c
Base case		2.020	1.065	0.277	0.557
ζ	0	2.060	1.062	0.269	0.553
	2	1.957	1.070	0.289	0.565
γ	6.5 y	1.714	1.094	0.283	0.594
	10.5 y	2.238	1.049	0.274	0.531
n	−1%	2.385	1.040	0.276	0.562
	+3%	1.745	1.091	0.278	0.555



(a) Effects of changes in ζ on capital intensity and rate of return

(b) Effects of changes in ζ on consumption

FIGURE 2 Effects of changes in income replacement ratio. (a) Effects of changes in ζ on capital intensity and rate of return, (b) Effects of changes in ζ on consumption.

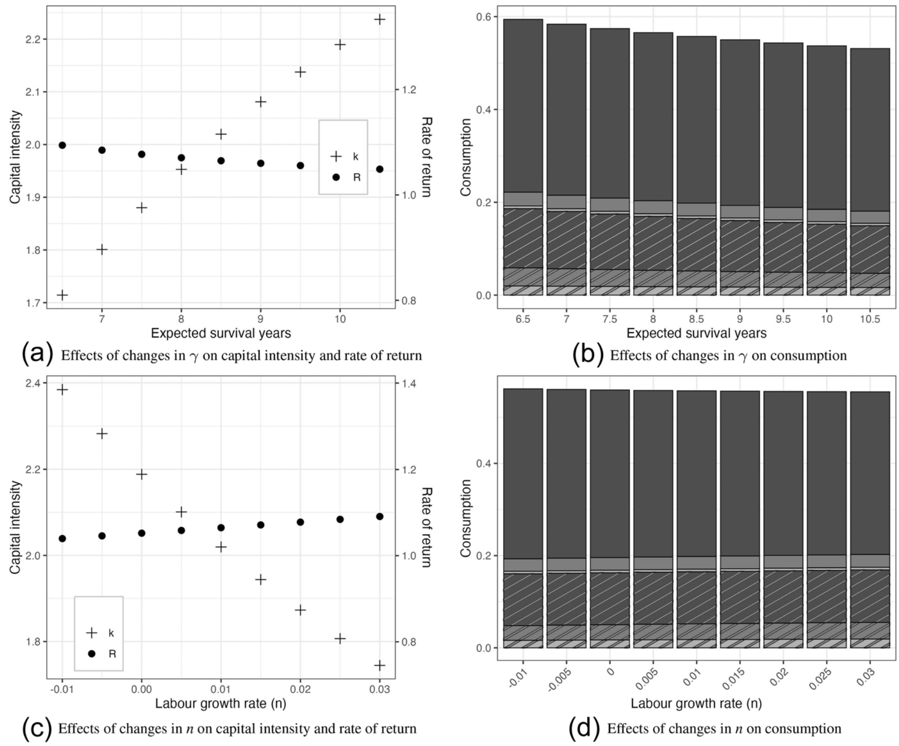


FIGURE 3 Effects of changes in population structure. (a) Effects of changes in γ on capital intensity and rate of return, (b) Effects of changes in γ on consumption, (c) Effects of changes in n on capital intensity and rate of return, (d) Effects of changes in n on consumption.

$k = 2.238$ as individuals save more to finance longer retirements. The rate of return declines correspondingly from $R = 1.094$ to $R = 1.049$ due to higher capital supply. On the consumption side, longer lifespans reduce annual consumption from $c = 0.594$ to $c = 0.531$ as households spread resources over more retirement years. Second, Figure 3c,d present the effects of varying labor-force growth from $n = -1\%$ to $n = 3\%$. A decline in n, representative of societies with low fertility and limited immigration, raises capital intensity from $k = 1.745$ at $n = 3\%$ to $k = 2.385$ at $n = -1\%$ through capital deepening, while the tax rate changes only modestly across the range (27.57% to 27.88%), indicating that labor growth primarily affects capital intensity rather than fiscal pressure directly. Both improved survival and reduced labor growth strain the social security system by increasing the ratio of beneficiaries to contributors, reinforcing the motive for private saving.

4.4.1 | Differential mortality across disability groups

The preceding analyses assume equal survival probabilities across all retiree groups. In practice, individuals with disabilities experience substantially higher mortality. Meseguer (2021) finds that during the most recent study period (2011–2015), new DI disabled-worker beneficiaries faced a life expectancy gap of roughly 14 years at age 35, narrowing to about 10 years at age 50 and 7 years at age 60, relative to the general population. Even the healthiest long-term beneficiaries, those surviving 10 or more years on the rolls, still fell short by around 9 years at age 50, with mortality levels closer to the general population of a century ago than to their contemporary peers. While this gap has narrowed since the 1990s due to a shift toward less deadly diagnoses, qualifying for DI benefits remains associated with losing nearly a decade of expected life. Furthermore, Majer et al. (2011) document a hazard ratio of 7.85 for mortality among individuals with activities of daily living (ADL) disability at age 55; and the SSA reports that roughly 14% of SSDI entrants die within 4 years of initial award (Social Security Administration, 2024).

To assess the sensitivity of our results to these mortality differentials, we construct three gradient scenarios that progressively widen the gap in expected post-retirement survival across disability groups, as summarized in Table 7. This exercise recomputes the baseline steady-state equilibrium under each mortality gradient; the policy reform scenarios of Section 5 are not recomputed under alternative gradients. In each scenario, expected survival years for healthy retirees exceed the base-case value of 8.5 years while those for disabled and severely disabled retirees fall below it, spanning the range of empirical estimates cited above.

Table 8 reports the equilibrium outcomes under each scenario. Introducing differential mortality has modest effects across all gradients: capital intensity varies by less than 2% across the full range of scenarios, while the tax rate and aggregate consumption remain virtually unchanged. Under the low and mid gradients, capital intensity rises slightly (to $k = 2.051$ and $k = 2.058$, respectively) as the longer expected survival of healthy retirees, the largest group, strengthens the aggregate savings motive. Under the high gradient, this effect reverses: the sharply reduced survival of severely disabled retirees ($\gamma_{r(3)} = 0.778$, corresponding to only 4.5 years) lowers their savings sufficiently to offset the increased savings of the healthy group, returning capital intensity close to the baseline level ($k = 2.028$).

These results indicate that the baseline aggregate equilibrium outcomes are robust to empirically motivated mortality differentials. The tax rate is essentially invariant to mortality

TABLE 7 Mortality gradient scenarios for sensitivity analysis.

	Expected survival (years)			Survival probability γ		
Scenario	$\boldsymbol{r}(1)$	$\boldsymbol{r}(2)$	$\boldsymbol{r}(3)$	$\gamma_{r(1)}$	$\gamma_{r(2)}$	$\gamma_{r(3)}$
Baseline	8.5	8.5	8.5	0.882	0.882	0.882
Low gradient	9.5	8.0	6.5	0.895	0.875	0.846
Mid gradient	10.0	7.7	5.9	0.900	0.870	0.831
High gradient	11.0	7.0	4.5	0.909	0.857	0.778

TABLE 8 Equilibrium outcomes under differential mortality scenarios.

Scenario	$\boldsymbol{k}$	$\boldsymbol{R}$	$\boldsymbol{\tau}$	$\boldsymbol{c}$
Baseline	2.0196	1.0649	0.2767	0.5574
Low gradient	2.0508	1.0624	0.2762	0.5537
Mid gradient	2.0581	1.0618	0.2761	0.5528
High gradient	2.0279	1.0642	0.2766	0.5564

assumptions, varying by less than 0.07 percentage points across all scenarios. The primary quantitative effect operates through the savings channel, but even this effect is small and non-monotonic: moderate mortality gradients slightly increase aggregate savings as healthy retirees dominate, while extreme gradients partially offset this through reduced savings by the shorter-lived disabled groups. This provides reassurance that the equal-mortality simplification adopted in the base case does not materially distort the baseline equilibrium. We note, however, that since the reform scenarios have not been recomputed under alternative gradients, this robustness claim applies to the baseline equilibrium rather than to all policy implications; the small magnitude of the baseline effects nonetheless suggests that mortality assumptions are unlikely to overturn the qualitative policy rankings.

4.5 | Two-state benchmark comparison

To clarify the value added by the multi-stage framework, we compare the base-case $M = 3$ model with a simplified $M = 2$ benchmark that collapses all disabled workers into a single group. In the $M = 2$ model, Group 1 comprises healthy workers and Group 2 merges the moderately and severely disabled, using a population-weighted average productivity of $\xi_2^{M=2} = (12.511 \times 0.703 + 1.995 \times 0.380)/(12.511 + 1.995) = 0.659$. The severe fraction ($1.995/14.506 = 13.8\%$) of the merged group receives DP at full replacement, while the remainder receives no benefits. Retirement benefits are calibrated with approximate working-year profiles that match the $M = 3$ aggregates. Table 9 reports the comparison.

The two models produce qualitatively similar macroeconomic outcomes but differ in economically meaningful ways. The $M = 2$ model understates total benefit spending by 4.13% ($e = 0.0650$ vs. 0.0678) because it cannot separately calibrate the benefit structures of moderately and severely disabled groups. This translates into a lower tax rate (-1.16%) and higher capital intensity (+1.33%), as the reduced fiscal burden leaves more income available for savings. Aggregate consumption is 0.57% lower in the $M = 2$ model, reflecting the compositional distortion from pooling groups with very different productivity levels and benefit entitlements.

The key limitation of the two-state model, however, is not quantitative but structural. Without distinguishing moderate from severe disability, the $M = 2$ framework cannot evaluate policies that target specific severity levels. Policy 3 (benefit reduction) targets the benefit levels of specific severity groups, and Policy 4 (tighter screening) operates on the transition between moderate and severe disability; these margins simply do not exist in a two-state world. Similarly, Policy 1 (disability assistance) requires separating DA-eligible from non-DA workers within the disabled population. As the distributional analysis in Section 5 demonstrates, the

TABLE 9 $M = 2$ vs $M = 3$ benchmark comparison.

Variable	$M = 2$	$M = 3$	Difference	% Difference
k	2.0465	2.0196	+0.0269	+1.33%
R	1.0627	1.0649	−0.0022	−0.21%
τ	0.2735	0.2767	−0.0032	−1.16%
c	0.5542	0.5574	−0.0032	−0.57%
e	0.0650	0.0678	−0.0028	−4.13%

group-level consumption effects of these policies differ sharply across severity groups. A two-state model would average these opposing effects into a single muted response, concealing the distributional tradeoffs that are central to policy evaluation. The $M = 3$ framework thus provides the minimum level of heterogeneity needed to capture the policy-relevant distinctions between moderate and severe disability.

5 | POLICY REFORM ANALYSIS

In this section, we analyze the implications of two contrasting policy approaches: enhancing safety nets for vulnerable groups and promoting fiscal soundness, each with two distinct scenarios. The first approach focuses on strengthening safety nets to provide increased support and protection for individuals most at risk of economic insecurity. The scenarios under consideration include implementing DA for individuals who are denied DP (Policy 1) and creating incentives for both employees and employers to support the continued employment of disabled workers (Policy 2).

Conversely, the policies aimed at fiscal soundness focus on maintaining the economic stability of the state. This approach typically involves measures aimed at reducing public spending and may include scenarios such as reducing the levels of DP benefits (Policy 3) and tightening the eligibility criteria for receiving disability benefits (Policy 4). These measures are intended to control costs and ensure the long-term sustainability of public finances, reflecting a strategic balance between supporting vulnerable populations and maintaining fiscal discipline.

Policy 1 (Disability Assistance): Unlike DP, which requires a specific amount and period of contributions, many countries provide disability benefits in the form of DA. We split the original Group 2, separating the 30% of its workers who are DA-eligible into a new Group 3, thereby extending M to 4. We assume that DA is only paid to workers, rather than to both workers and retirees. The level of income replacement provided by the DA benefit is assumed to be 30%, and 30% of the original Group 2 workers are eligible for DA. The working years for Groups 3 and 4 are now defined as follows: $\iota_1^3 = 15$, $\iota_2^3 = 5$, $\iota_3^3 = 2$, $\iota_1^4 = 15$, $\iota_2^4 = 3$, and $\iota_3^4 = 2$. This segmentation allows us to model the effects of DA more precisely within our expanded framework. Full results are summarized in Table 10.

TABLE 10 Key economic variables under Policy 1 (Disability Assistance, 30% coverage).

Group	τ	e	h	s	λ	c	q	π ($\epsilon\pi$)	Ω
$w = 1$	0.2639	0.0000	2.6161	0.6128	0.6449	0.3568	0.6858	0.0749	1.0707
$w = 2$	0.0105	0.0000	0.0434	0.0365	0.0362	0.0219	0.0388	0.1321	1.1908
$w = 3$	0.0045	0.0032	0.0144	0.0240	0.0149	0.0113	0.0166	0.1527	1.2394
$w = 4$	0.0013	0.0031	0.0024	0.0108	0.0077	0.0068	0.0088	0.2155	1.4092
$r(1)$		0.0459		0.3363	0.2134	0.1045	0.1671	0.1238	
$r(2)$		0.0075		0.0405	0.0419	0.0244	0.0412	0.1739	
$r(3)$		0.0034		0.0250	0.0198	0.0089	0.0177	0.1238	
$r(4)$		0.0083		0.0319	0.0213	0.0207	0.0239	0.2504	
Total	0.2801	0.0715			1.0000	0.5553	1.0000		
$k = 2.0378$, $R = 1.0634$									

Policy 2 (Employment incentives): This policy encourages continued employment among disabled workers by increasing the staying probability $p_{2,2}$ and correspondingly decreasing the retirement probability $p_{2,r(2)}$, representing workplace accommodations, wage subsidies, or vocational rehabilitation programs.

Policy 3 (Benefit reduction): This policy reduces DP benefits by scaling the replacement ratio ζ from its base-case value of 1.0. At the reference point, $\zeta = 0.8$ represents a 20% benefit cut.

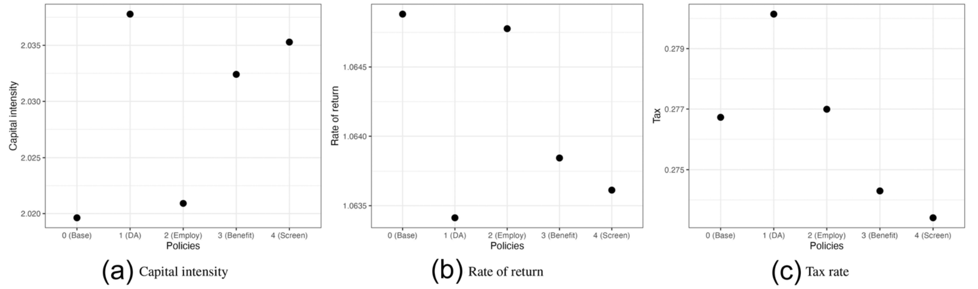
Policy 4 (Tighter screening): This policy tightens eligibility screening by increasing $p_{2,2}$ (staying in the moderately disabled group) and correspondingly decreasing $p_{2,3}$ (transitioning to severe disability), reducing the flow of workers into the DP-eligible group.

5.1 | Macroeconomic effects

Although it is difficult to directly compare the magnitude of changes across policies because the costs and implementation challenges vary, it is possible to analyze the direction of changes in key economic variables relative to the base case (Policy 0). Figure 4a–c compare the macroeconomic variables at representative policy intensities: Policy 1 at 30% DA eligibility, Policy 2 at a 2.5 percentage point employment shock, Policy 3 at a 20% benefit reduction ($\zeta = 0.8$), and Policy 4 at a 2.5 percentage point screening shock. Table 11 reports the percentage changes.

Policies 1 and 2, aimed at aiding vulnerable groups, tend to increase savings (thereby reducing the rate of return) and decrease consumption. This could be attributed to an increased tax burden under Policy 1 (+1.24%), while Policy 2 demonstrates a negligible fiscal footprint ($\Delta\tau = +0.09\%$). Policy 2's encouragement of employment among disabled workers with potentially reduced productivity leads to decreased consumption and a greater need for savings. Therefore, without adjusting the wages of disabled workers, providing incentives to work may have unintended side effects such as reduced consumption.

Policies 3 and 4 aim to reduce the tax burden, and both successfully demonstrate this reduction (Table 11). However, a decrease in tax rates does not necessarily lead to a boost in consumption. In both cases, there has been a slight increase in savings compared to the base case, resulting in a reduction in the rate of return. Under Policy 3 (benefit reduction at $\zeta = 0.8$), the tax rate falls by 0.88% and capital intensity rises by 0.63%. Policy 4 (tighter screening at 2.5pp) achieves a larger tax reduction (−1.20%) and a larger increase in capital intensity



(a) Capital intensity (b) Rate of return (c) Tax rate

FIGURE 4 Key macroeconomic variables at representative policy intensities. (a) Capital intensity, (b) Rate of return, (c) Tax rate.

TABLE 11 Policy comparison: Percentage changes from base case.

Policy	$\%\Delta k$	$\%\Delta R$	$\%\Delta \tau$	$\%\Delta c$
0 (Base)	$k = 2.020, R = 1.065, \tau = 0.277, c = 0.557$			
1 (DA, 30%)	+0.90	−0.14	+1.24	−0.39
2 (Employ, 2.5pp)	+0.06	−0.01	+0.09	−0.03
3 (Benefit, $\zeta = 0.8$)	+0.63	−0.10	−0.88	−0.27
4 (Screen, 2.5pp)	+0.78	−0.12	−1.20	−0.34

(+0.78%). Both policies reduce aggregate consumption, illustrating how policies aimed at fiscal soundness by reducing benefits can impact consumption patterns.

5.2 | Distributional effects

Because Policies 2 and 4 operate by perturbing transition probabilities, they alter the steady-state population distribution across groups. As a result, changes in aggregate group consumption reflect both per-capita consumption effects and compositional shifts in group sizes. When a policy reclassifies individuals across groups, aggregate group consumption changes even if no individual's consumption changes. Table 12 decomposes the distributional effects into aggregate, per-capita, and population components for all four policies; compositional shifts arise only under the two transition-altering policies (Policies 2 and 4).

For Policy 2, the large aggregate changes (up to 13%) are almost entirely compositional: per-capita consumption changes by less than 0.3% for all groups. The apparent intergenerational tradeoff visible in the aggregate figures (workers gain, retirees lose) is largely a compositional artifact: more disabled individuals remain classified as workers rather than transitioning to retirement, without materially changing per-capita consumption.

For Policy 4, the contrast is even more striking. The aggregate consumption of the severe group ($w = 3$) falls by 37.7%, but per-capita consumption actually rises by 0.98%. This apparent paradox is a composition (survivor) effect: tighter screening shrinks the severe group by 38.3%, so aggregate group consumption falls roughly in proportion to group size, while the smaller number of individuals who do reach the severe state each draw on a less crowded benefit pool, leaving per-capita consumption marginally higher. Conversely, the moderate group ($w = 2$) shows an aggregate increase of 12.6% but a per-capita decline of 1.03%, as reclassified workers who would have transitioned to full DP now remain in the non-DP group. This illustrates how policies aimed at fiscal soundness by reducing the number of beneficiaries can have distributional consequences that are obscured by aggregate figures, particularly among vulnerable groups.

For Policy 3, population shares are unchanged because benefit levels do not affect transition probabilities, so aggregate and per-capita effects coincide. Healthy workers gain modestly (+0.21%), while the severely disabled bear the largest losses (−8.50% for $w = 3$, −9.68% for $r(3)$).

For Policy 1, since the DA split only redistributes workers within the existing disabled population ($q^{w=2}_{M=4} + q^{w=3}_{M=4} = q^{w=2}_{M=3}$), aggregate and per-capita consumption changes are equivalent. Per-capita consumption rises by 6.90% for disabled workers ($w = 2 + 3$ combined)

TABLE 12 Aggregate versus per-capita consumption changes at reference policy intensities: P1 at 30% DA, P2 at +2.5pp employment shock, P3 at $\zeta = 0.8$, P4 at +2.5pp screening shock.

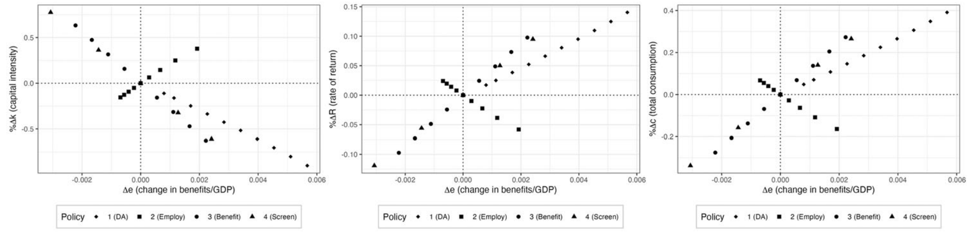
Policy	Group	Aggregate %Δc	Per-capita %Δc	Δq (%)
P1 (DA 30%)	$w = 1$	−0.57	−0.57	0.00
	$w = 2 + 3$	+6.90	+6.90	0.00
	$w = 4$	+4.33	+4.33	0.00
	$r(2 + 3)$	+1.99	+1.99	0.00
	$r(4)$	+4.72	+4.72	0.00
P2 (Employ)	$w = 1$	−0.93	−0.09	−0.84
	$w = 2$	+13.02	−0.25	+13.30
	$w = 3$	+13.45	+0.10	+13.33
	$r(2)$	−7.46	+0.14	−7.59
	$r(3)$	+13.40	+0.08	+13.31
P3 (Benefit)	$w = 1$	+0.21	+0.21	0.00
	$w = 2$	−0.87	−0.87	0.00
	$w = 3$	−8.50	−8.50	0.00
	$r(2)$	+0.85	+0.85	0.00
	$r(3)$	−9.68	−9.68	0.00
P4 (Screen)	$w = 1$	−0.24	+0.18	−0.42
	$w = 2$	+12.62	−1.03	+13.79
	$w = 3$	−37.70	+0.98	−38.31
	$r(2)$	+15.03	+1.10	+13.78
	$r(3)$	−38.01	+0.39	−38.25

and 4.33% for severely disabled workers ($w = 4$) at 30% coverage, with healthy workers bearing a cost of only 0.57%. These gains are uniformly positive across all disabled groups and scale nearly linearly with coverage, reaching +11.45% at 50% DA eligibility.

5.3 | Robustness across policy intensities

To assess the robustness of these findings, each policy is evaluated over a continuous range of intensities: DA eligibility from 0% to 50% (Policy 1), employment shocks from −10 to +10 percentage points (Policy 2), benefit adjustments from $\zeta = 0.8$ to $\zeta = 1.2$ (Policy 3), and screening shocks from −2.5 to +2.5 percentage points (Policy 4). All results are plotted on a common fiscal dimension (Δe, change in total benefits as a share of GDP) to enable direct cross-policy comparison.

Figure 5a–c displays the macroeconomic response surfaces. Policies 1, 3, and 4 trace a common downward-sloping relationship between Δe and $\%\Delta k$ (Figure 5a), reflecting the



(a) $\%\Delta k$ vs Δe (b) $\%\Delta R$ vs Δe (c) $\%\Delta c$ vs Δe

FIGURE 5 Fiscal tradeoff: macroeconomic response surfaces across policy intensities. (a) $\%\Delta k$ vs Δe, (b) $\%\Delta R$ vs Δe, (c) $\%\Delta c$ vs Δe.

standard crowding-out mechanism: higher benefit expenditure reduces private savings and capital accumulation. Among these, Policies 3 and 4 follow nearly overlapping trajectories, confirming that benefit cuts and screening tightening are near-substitutes at the macroeconomic level across the full range of intensities. Policy 1 (DA) traces the same negative slope from the benefit-expansion side.

Policy 2 traces a distinct, positively-sloped trajectory, in contrast to the negative slope shared by the other three policies. This reflects a fundamental difference in the transmission channel. Policies 1, 3, and 4 operate primarily through the transfer channel: changes in benefit generosity crowd out (or crowd in) private savings, producing the standard negative relationship between fiscal expenditure and capital accumulation. Policy 2, by contrast, operates through the labor supply channel. Keeping disabled workers employed longer generates continued labor income and savings, raising capital intensity even as total benefit expenditure increases modestly due to population reshuffling. The positive slope thus reflects the productive contribution of retained workers rather than a transfer effect, suggesting that employment-oriented disability policies can expand the safety net without the capital-crowding costs associated with direct benefit provision.

Figure 6a–d displays the group-level consumption responses over the full range for each policy. The near-linearity of all response surfaces confirms that the single-point results reported in Sections 5.1 and 5.2 are representative and not artifacts of a particular policy calibration.

6 | CONCLUSION

This paper develops a multi-stage OLG model that incorporates DP as a central element of the social security system. By distinguishing between workers and retirees, and further categorizing them by disability status, the model provides a nuanced framework for analyzing how disability, aging, and pension design interact to shape macroeconomic outcomes. The model is calibrated to US data using the American Community Survey and Social Security Administration statistics, with disability pension benefits accounting for 6.78% of GDP in the base case.

Our results show that higher levels of DP benefits reduce private savings and increase capital returns, while also leading individuals to lower their risk adjustment factors. Demographic changes reinforce these dynamics. Improvements in mortality and reductions in labor supply both increase aggregate private saving, but they produce divergent consumption patterns: workers reduce their consumption while retirees maintain stability. These shifts exacerbate wealth inequality, as retirees accumulate a growing share of assets, and they impose

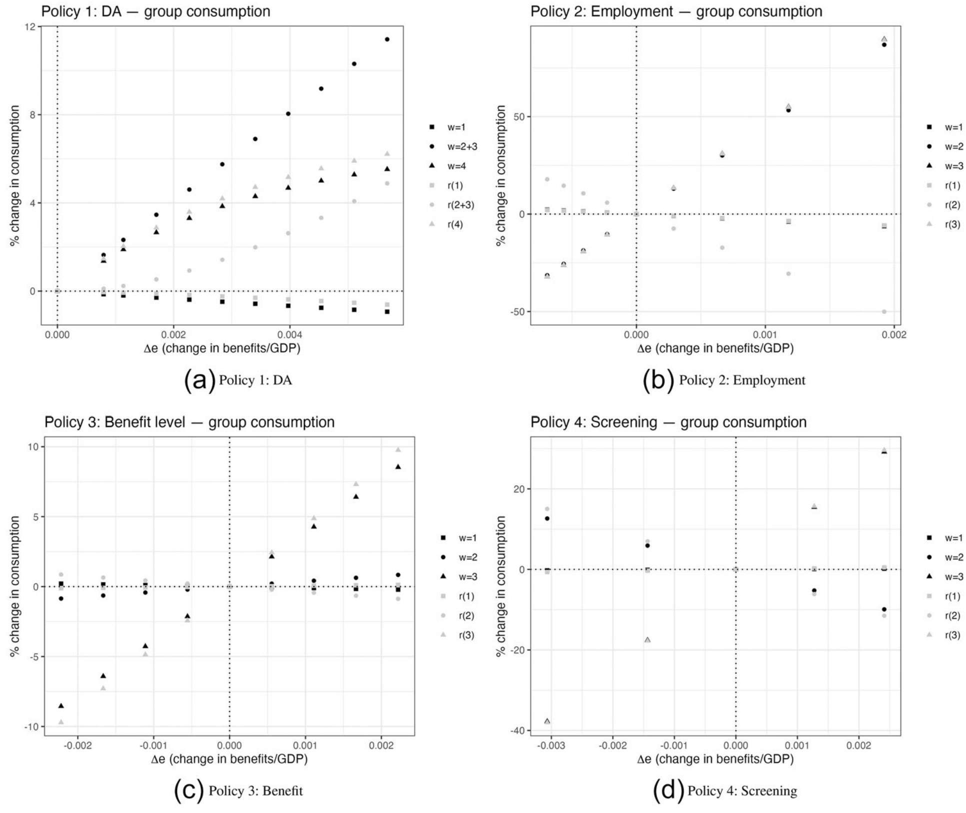


FIGURE 6 Group-level consumption responses across policy intensities. (a) Policy 1: DA, (b) Policy 2: Employment, (c) Policy 3: Benefit, (d) Policy 4: Screening.

additional fiscal burdens on pension systems. Introducing differential mortality across disability groups, motivated by empirical evidence that disabled individuals face substantially higher mortality, leaves the baseline equilibrium outcomes essentially unchanged: the tax rate varies by less than 0.07 percentage points across all mortality scenarios. Consistent with Bullard et al. (2012), our findings highlight the importance of demographic transitions, particularly declining fertility and rising longevity, in driving higher aggregate savings and altering intergenerational risk-sharing.

A formal comparison with a simplified two-state ($M = 2$) benchmark demonstrates that the multi-stage structure is essential for policy analysis. While the two-state model produces similar aggregate outcomes, it understates benefit spending by 4.13% and cannot evaluate policies that target specific disability severity levels.

Policy simulations reveal further complexities. Introducing DA provides an important safety net for individuals with less severe disabilities or limited contribution histories, complementing contributory DP schemes. Per-capita consumption rises by 6.90% for disabled workers and 4.33% for severely disabled workers at 30% DA coverage, with healthy workers bearing a cost of under 0.6%, and the nearly linear relationship between DA coverage and outcomes suggests that such programs can be scaled gradually. Policies that encourage the employment of disabled workers operate through the labor supply channel rather than the transfer channel, raising capital intensity even as benefit expenditure increases modestly; however, the per-capita distributional effects are negligible (under 0.3% for all groups), with the large aggregate changes driven almost entirely by population reshuffling across groups. Measures focused on fiscal soundness, such as reducing benefit levels or tightening eligibility, achieve similar budgetary gains at the macroeconomic level, suggesting they are near-substitutes as fiscal instruments. Their distributional consequences, however, differ sharply: benefit cuts directly reduce per-capita consumption for DP recipients (-8.5% for severely disabled workers at a 20% cut), while tighter screening primarily reshuffles population composition, with per-capita effects no larger than about 1.1% for all groups. These results illustrate the tradeoffs between adequacy, fairness, and sustainability in the design of DP.

Our equilibrium framework complements solvency-oriented assessments of PAYG pension systems that integrate disability and retirement contingencies into actuarial balance sheets (Garvey et al., 2023; Pérez-Salamero González et al., 2017; Ventura-Marco & Vidal-Meliá, 2014). While those approaches focus on long-term fiscal sustainability through actuarial net-worth indicators, our model captures how policy reforms affect the distribution of consumption and wealth across disability groups, a dimension that balance-sheet frameworks typically do not address.

In interpreting these findings, the scope of the framework should be kept in mind. Because the model pools assets at the group level and abstracts from individual precautionary saving against idiosyncratic disability risk, the policy experiments operate through savings, transfers, and the reshuffling of population composition across disability states rather than through household-level insurance mechanisms. The distinctive contribution of this paper is accordingly to show how a tractable Gertler-style multi-stage OLG structure generates redistributional and compositional dynamics under alternative transfer schemes, effects that the aggregate versus per-capita decomposition makes explicit, rather than to provide a structural assessment of disability insurance design at the household level. Questions such as the optimal generosity of disability benefits in the presence of individual savings responses, or the insurance value of disability benefits to risk-averse households facing uninsured idiosyncratic risk, would require heterogeneous-agent extensions that we leave for future work.

Taken together, the analysis demonstrates that DP affect not only the consumption of directly affected individuals but also aggregate savings, consumption, and intergenerational redistribution. By integrating disability states and pension design into a multi-stage OLG framework, this study provides a tractable platform for evaluating the macroeconomic and distributional consequences of disability pension reforms. The findings suggest that policy reforms must carefully balance fiscal sustainability with the protection of vulnerable groups, recognizing that interventions designed to relieve fiscal pressures can generate broader, and sometimes unintended, consequences for the economy. Several limitations suggest directions for future research, including the incorporation of business-cycle effects on disability onset, household structure and intrahousehold risk-sharing, occupational sorting and skill depreciation associated with disability (Jacobs, 2023), and open-economy extensions that permit analysis of how international capital flows interact with national disability pension systems.

AUTHOR CONTRIBUTIONS

Hangsuck Lee contributed to conceptualization, funding acquisition, methodology, and project administration. Steven Haberman contributed to methodology and writing. Byungdoo Kong contributed methodology, and writing. Seung Yeon Jeong contributed to formal analysis, investigation, methodology, and writing. All authors read and approved the final manuscript.

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

The code that support the findings of this study are available from the corresponding author upon reasonable request.

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APPENDIX A: DETAILED POLICY RESULTS

This appendix reports full group-level results for each policy reform at its reference intensity. Tables A1, A2, and A3 correspond to the $M = 3$ model (Policies 2–4).

APPENDIX B: DERIVATION OF RETIREES' DECISIONS

This appendix derives the optimal consumption-savings decisions for retirees. We establish two key results: (i) the consumption propensity that determines the fraction of wealth consumed each period, and (ii) the valuation formula for social security wealth.

Retirees in group $r(w)$ allocate their total wealth between current consumption and future savings. The consumption propensity $\epsilon_t^{r(w)}\pi_t^w$ represents the fraction of total wealth consumed in period t, where $\epsilon_t^{r(w)}$ captures the adjustment factor for retirement relative to a worker's propensity π_t^w. This propensity depends on the interest rate, discount factor, and survival probability.

From the first-order conditions of the retiree's optimization problem, we derive:

$$\epsilon_t^{r(w)}\pi_t^w = 1 - (R_{t+1})^{\sigma-1}(\beta)^{\sigma}\gamma_{r(w)}\frac{\epsilon_t^{r(w)}\pi_t^w}{\epsilon_{t+1}^{r(w)}\pi_{t+1}^w}. \tag{B1}$$

Social security wealth $S_t^{r(w)}$ represents the present value of all future social security benefits, discounted by both the interest rate and mortality risk. This wealth enters the retiree's budget constraint alongside financial assets.

TABLE A1 Policy 2: Employment incentives (2.5 percentage point shock to $p_{2,2}$).

Group	τ	e	h	s	λ	c	q	π $(\epsilon\pi)$	Ω
$w = 1$	0.2587	0.0000	2.3672	0.5805	0.6341	0.3559	0.6807	0.0800	1.0793
$w = 2$	0.0168	0.0000	0.0844	0.0627	0.0622	0.0312	0.0629	0.1059	1.1319
$w = 3$	0.0014	0.0035	0.0044	0.0224	0.0102	0.0064	0.0100	0.1257	1.1749
$r(1)$		0.0455		0.2793	0.1971	0.1120	0.1659	0.1503	
$r(2)$		0.0099		0.0609	0.0633	0.0317	0.0545	0.1503	
$r(3)$		0.0091		0.0558	0.0331	0.0201	0.0261	0.1503	
Total	0.2770	0.0681			1.0000	0.5573	1.0000		
$k = 2.0209$, $R = 1.0648$									

TABLE A2 Policy 3: Reduced DP benefits ($\zeta = 0.8$).

Group	τ	e	h	s	λ	c	q	π $(\epsilon\pi)$	Ω
$w = 1$	0.2583	0.0000	2.3997	0.5807	0.6400	0.3600	0.6864	0.0800	1.0801
$w = 2$	0.0147	0.0000	0.0690	0.0500	0.0555	0.0273	0.0555	0.1091	1.1395
$w = 3$	0.0013	0.0025	0.0039	0.0159	0.0090	0.0052	0.0088	0.1253	1.1750
$r(1)$		0.0459		0.2829	0.1988	0.1128	0.1673	0.1498	
$r(2)$		0.0107		0.0662	0.0691	0.0345	0.0589	0.1498	
$r(3)$		0.0064		0.0396	0.0284	0.0160	0.0231	0.1498	
Total	0.2743	0.0655			1.0000	0.5559	1.0000		
$k = 2.0324$, $R = 1.0638$									

TABLE A3 Policy 4: Tighter screening (2.5 percentage point shock to $p_{2,3}$).

Group	τ	e	h	s	λ	c	q	π $(\epsilon\pi)$	Ω
$w = 1$	0.2560	0.0000	2.3988	0.5763	0.6371	0.3584	0.6836	0.0798	1.0800
$w = 2$	0.0166	0.0000	0.0819	0.0550	0.0634	0.0311	0.0631	0.1080	1.1375
$w = 3$	0.0008	0.0019	0.0024	0.0122	0.0057	0.0035	0.0055	0.1252	1.1750
$r(1)$		0.0457		0.2816	0.1969	0.1122	0.1666	0.1497	
$r(2)$		0.0122		0.0753	0.0789	0.0393	0.0671	0.1497	
$r(3)$		0.0049		0.0305	0.0181	0.0110	0.0142	0.1497	
Total	0.2734	0.0647			1.0000	0.5556	1.0000		
$k = 2.0353$, $R = 1.0636$									

The recursive valuation formula for an individual retiree is:

$$S_t^{r(w)} = E_t^{r(w)} + \frac{1}{R_{t+1}/\gamma_{r(w)}} S_{t+1}^{r(w)}, \tag{B2}$$

where $E_t^{r(w)}$ is the current period benefit and the discount factor $\frac{1}{R_{t+1}/\gamma_{r(w)}}$ adjusts for mortality risk.

Aggregating across all retirees in group $r(w)$ and accounting for population growth at rate n, the aggregate social security wealth satisfies:

$$S_t^{r(w)\bullet} = E_t^{r(w)\bullet} + \frac{1}{1+n}\frac{1}{R_{t+1}/\gamma_{r(w)}} S_{t+1}^{r(w)\bullet}, \tag{B3}$$

where $E_t^{r(w)\bullet}$ denotes total benefits paid to all retirees in group $r(w)$ at time t. The additional $1/(1+n)$ term reflects the dilution of per-capita wealth due to population growth.

B.1 | Consumption propensity

Proof. The optimization of the utility function (8) subject to budget constraints (9) can be expressed as follows, employing the Lagrange multiplier $\mu_t^{r(w)}$:

$$L_t^{r(w)} = V_t^{r(w)} - \mu_t^{r(w)}\left(A_{t+1}^{r(w)} - \frac{R_t}{\gamma_{r(w)}} A_t^{r(w)} - E_t^{r(w)} + C_t^{r(w)}\right). \tag{B4}$$

The first-order conditions for optimization are given by

$$\frac{\partial L_t^{r(w)}}{\partial C_t^{r(w)}} = \left(V_t^{r(w)}\right)^{1-\rho}\left(C_t^{r(w)}\right)^{\rho-1} - \mu_t^{r(w)} = 0, \tag{B5}$$

$$\frac{\partial L_t^{r(w)}}{\partial A_{t+1}^{r(w)}} = \left(V_t^{r(w)}\right)^{1-\rho}\beta\gamma_{r(w)}\frac{\partial V_{t+1}^{r(w)}}{\partial A_{t+1}^{r(w)}}\left(V_{t+1}^{r(w)}\right)^{\rho-1} - \mu_t^{r(w)} = 0. \tag{B6}$$

Hence, we get the following relations:

$$\left(C_t^{r(w)}\right)^{\rho-1} = \beta\gamma_{r(w)}\frac{\partial V_{t+1}^{r(w)}}{\partial A_{t+1}^{r(w)}}\left(V_{t+1}^{r(w)}\right)^{\rho-1}, \tag{B7}$$

$$\mu_t^{r(w)} = \left(V_t^{r(w)}\right)^{1-\rho}\left(C_t^{r(w)}\right)^{\rho-1}. \tag{B8}$$

Utilizing the Envelope theorem on the parameter $A_t^{r(w)}$ yields

$$\frac{\partial V_t^{r(w)}}{\partial A_t^{r(w)}} = \frac{\partial L_t^{r(w)}}{\partial A_t^{r(w)}} = \mu_t^{r(w)} \frac{R_t}{\gamma_{r(w)}} = \left(V_t^{r(w)}\right)^{1-\rho} \left(C_t^{r(w)}\right)^{\rho-1} \frac{R_t}{\gamma_{r(w)}}, \tag{B9}$$

hence, (B7) becomes

$$C_{t+1}^{r(w)} = (R_{t+1}\beta)^{\sigma} C_t^{r(w)}, \tag{B10}$$

where $\sigma = 1/(1-\rho)$.

Let us conjecture the form of the utility function $V_t^{r(w)}$ as follows:

$$V_t^{r(w)} = \left(\epsilon_t^{r(w)} \pi_t^w\right)^{-1/\rho} C_t^{r(w)}. \tag{B11}$$

Then (8) can be rewritten as

$$\left(\epsilon_t^{r(w)} \pi_t^w\right)^{-1} \left(C_t^{r(w)}\right)^{\rho} = \left(C_t^{r(w)}\right)^{\rho} + \beta \gamma_{r(w)} \left(\epsilon_{t+1}^{r(w)} \pi_{t+1}^w\right)^{-1} \left(C_{t+1}^{r(w)}\right)^{\rho}. \tag{B12}$$

With (B10), (B12) can be rearranged as follows:

$$\epsilon_t^{r(w)} \pi_t^w = 1 - (R_{t+1})^{\sigma-1} (\beta)^{\sigma} \gamma_{r(w)} \frac{\epsilon_t^{r(w)} \pi_t^w}{\epsilon_{t+1}^{r(w)} \pi_{t+1}^w}. \tag{B13}$$

Thus, (B1) holds. □

B.2 | Social security wealth valuation

Proof. Using the consumption function (10) and the consumption growth equation (B10), we get

$$\epsilon_{t+1}^{r(w)} \pi_{t+1}^w \left(\frac{R_{t+1}}{\gamma_{r(w)}} A_{t+1}^{r(w)} + S_{t+1}^{r(w)} \right) = (R_{t+1}\beta)^{\sigma} \epsilon_t^{r(w)} \pi_t^w \left(\frac{R_t}{\gamma_{r(w)}} A_t^{r(w)} + S_t^{r(w)} \right). \tag{B14}$$

Since the retiree's consumption propensity (B1) can be rewritten as

$$\epsilon_{t+1}^{r(w)} \pi_{t+1}^w = (R_{t+1})^{\sigma-1} \beta^{\sigma} \gamma_{r(w)} \frac{\epsilon_t^{r(w)} \pi_t^w}{1 - \epsilon_t^{r(w)} \pi_t^w}, \tag{B15}$$

we obtain the following equation by multiplying both sides of (B15) by $R_{t+1}(1 - \epsilon_t^{r(w)} \pi_t^w)\left(\frac{R_t}{\gamma_{r(w)}} A_t^{r(w)} + S_t^{r(w)} \right)$:

$$\epsilon_{t+1}^{r(w)}\pi_{t+1}^{w}\left[R_{t+1}\left(1-\epsilon_t^{r(w)}\pi_t^{w}\right)\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+S_t^{r(w)}\right)\right]=(R_{t+1}\beta)^{\sigma}\gamma_{r(w)}\epsilon_t^{r(w)}\pi_t^{w}\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+S_t^{r(w)}\right). \tag{B16}$$

By (B14), (B16) can be written as

$$\epsilon_{t+1}^{r(w)}\pi_{t+1}^{w}\left[R_{t+1}\left(1-\epsilon_t^{r(w)}\pi_t^{w}\right)\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+S_t^{r(w)}\right)\right]=\gamma_{r(w)}\epsilon_{t+1}^{r(w)}\pi_{t+1}^{w}\left(\frac{R_{t+1}}{\gamma_{r(w)}}A_{t+1}^{r(w)}+S_{t+1}^{r(w)}\right), \tag{B17}$$

and it can be simplified as follows:

$$\left(1-\epsilon_t^{r(w)}\pi_t^{w}\right)\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+S_t^{r(w)}\right)=A_{t+1}^{r(w)}+\frac{\gamma_{r(w)}}{R_{t+1}}S_{t+1}^{r(w)}. \tag{B18}$$

We can rewrite the left-hand side of (B18) as

$$\begin{aligned}\left(1-\epsilon_t^{r(w)}\pi_t^{w}\right)\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+S_t^{r(w)}\right)&=\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+S_t^{r(w)}\right)-\epsilon_t^{r(w)}\pi_t^{w}\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+S_t^{r(w)}\right)\\&=\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+S_t^{r(w)}\right)-C_t^{r(w)}\\&=\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+E_t^{r(w)}-C_t^{r(w)}\right)+\left(S_t^{r(w)}-E_t^{r(w)}\right).\end{aligned} \tag{B19}$$

From the budget constraint (9), the right-hand side of (B18) can be written as

$$\left(\frac{R_t}{\gamma_{r(w)}}A_t^{r(w)}+E_t^{r(w)}-C_t^{r(w)}\right)+\frac{\gamma_{r(w)}}{R_{t+1}}S_{t+1}^{r(w)}, \tag{B20}$$

and hence, (B18) can be simplified as follows:

$$S_t^{r(w)}=E_t^{r(w)}+\frac{\gamma_{r(w)}}{R_{t+1}}S_{t+1}^{r(w)}. \tag{B21}$$

Thus, (B2) holds. □

APPENDIX C: WORKERS' DECISION

This appendix derives the optimal consumption-savings decisions for workers, including the (i) consumption propensity and (ii) risk-adjusted valuation of future income streams.

The worker's consumption propensity π_t^w determines the fraction of total wealth consumed each period. It depends on the interest rate, discount factor, and crucially, a risk adjustment factor that accounts for transition uncertainty:

$$\pi_t^w = 1 - \left(R_{t+1}\Omega_{t+1}^w\right)^{\sigma-1}(\beta)^\sigma \frac{\pi_t^w}{\pi_{t+1}^w} \tag{C1}$$

where the risk adjustment factor is:

$$\Omega_{t+1}^w = p_{w,w} + p_{w,w+1}\left(\pi_{t+1}^{w+1}/\pi_{t+1}^w\right)^{1/(1-\sigma)} + p_{w,r(w)}\frac{1}{\gamma_{r(w)}}\left(\epsilon_{t+1}^{r(w)}\right)^{1/(1-\sigma)} \tag{C2}$$

This adjustment factor reflects both mortality risk after retirement and consumption changes across transitions, allowing workers to properly value lifetime resources under uncertainty.

Workers face uncertainty over future productivity transitions and retirement timing, which affects how they value future income streams. Their total wealth consists of three components: financial assets A_t^w, human wealth H_t^w (present value of future labor income), and social security wealth S_t^w (present value of future benefits).

A key innovation is that workers use risk-adjusted transition probabilities $p_{w,j}^*$ when valuing these future streams. These probabilities weight each possible transition by the marginal utility of consumption in the destination state, ensuring proper valuation across states with different consumption levels.

In the following recursions, the ratio q^w/q^{w+1} is a population-scaling factor that converts per-capita wealth in group $w+1$ to the scale of group w. This adjustment is necessary because workers transitioning from group w to $w+1$ carry claims on future income valued at the destination group's per-capita rate, but the two groups differ in size.

The sum of human wealth and social security wealth satisfies the recursive relation:

$$\begin{aligned} H_t^w + S_t^w = {}& \xi_w W_t + E_t^w - T_t^w \\ & + \left[p_{w,w}^*\frac{1}{R_{t+1}}\left(H_{t+1}^w + S_{t+1}^w\right) + p_{w,w+1}^*\frac{1}{R_{t+1}}\left(H_{t+1}^{w+1} + S_{t+1}^{w+1}\right) + p_{w,r(w)}^*\frac{1}{R_{t+1}/\gamma_{r(w)}}S_{t+1}^{r(w)}\right] \end{aligned} \tag{C3}$$

where current income flows are augmented by the risk-adjusted present value of wealth in all possible future states.

This can be separated into human wealth:

$$H_t^w = \xi_w W_t - T_t^w + \left[p_{w,w}^*\frac{1}{R_{t+1}}H_{t+1}^w + p_{w,w+1}^*\frac{1}{R_{t+1}}H_{t+1}^{w+1}\right] \tag{C4}$$

and social security wealth:

$$S_t^w = E_t^w + \left[p_{w,w}^*\frac{1}{R_{t+1}}S_{t+1}^w + p_{w,w+1}^*\frac{1}{R_{t+1}}S_{t+1}^{w+1} + p_{w,r(w)}^*\frac{1}{R_{t+1}/\gamma_{r(w)}}S_{t+1}^{r(w)}\right] \tag{C5}$$

C.1 | Worker's consumption propensity

Proof. The maximization of the utility function (11) under the constraints of budget (13) can be formulated as follows, utilizing the Lagrange multiplier μ_t^w:

$$L_t^w = V_t^w - \mu_t^w\left(A_{t+1}^w - R_t A_t^w - \xi_w W_t - E_t^w + T_t^w + C_t^w\right). \tag{C6}$$

The first-order conditions for optimization are given by

$$\frac{\partial L_t^w}{\partial C_t^w} = \left(V_t^w\right)^{1-\rho}\left(C_t^w\right)^{\rho-1} - \mu_t^w = 0, \tag{C7}$$

$$\begin{aligned}\frac{\partial L_t^w}{\partial A_{t+1}^w} = &\left(V_t^w\right)^{1-\rho}\left[p_{w,w}V_{t+1}^w + p_{w,w+1}V_{t+1}^{w+1} + p_{w,r(w)}V_{t+1}^{r(w)}\right]^{\rho-1}\\ &\times\beta\left[p_{w,w}\frac{\partial V_{t+1}^w}{\partial A_{t+1}^w} + p_{w,w+1}\frac{\partial V_{t+1}^{w+1}}{\partial A_{t+1}^{w+1}} + p_{w,r(w)}\frac{\partial V_{t+1}^{r(w)}}{\partial A_{t+1}^{r(w)}}\right] - \mu_t^w = 0.\end{aligned} \tag{C8}$$

From the above conditions, we get the following:

$$\begin{aligned}\left(C_t^w\right)^{\rho-1} = &\left[p_{w,w}V_{t+1}^w + p_{w,w+1}V_{t+1}^{w+1} + p_{w,r(w)}V_{t+1}^{r(w)}\right]^{\rho-1}\\ &\times\beta\left[p_{w,w}\frac{\partial V_{t+1}^w}{\partial A_{t+1}^w} + p_{w,w+1}\frac{\partial V_{t+1}^{w+1}}{\partial A_{t+1}^{w+1}} + p_{w,r(w)}\frac{\partial V_{t+1}^{r(w)}}{\partial A_{t+1}^{r(w)}}\right],\end{aligned} \tag{C9}$$

$$\mu_t^w = \left(V_t^w\right)^{1-\rho}\left(C_t^w\right)^{\rho-1}. \tag{C10}$$

Applying the Envelope theorem on the parameter A_t^w, we obtain

$$\frac{\partial V_t^w}{\partial A_t^w} = \frac{\partial L_t^w}{\partial A_t^w} = \mu_t^w R_t = \left(V_t^w\right)^{1-\rho}\left(C_t^w\right)^{\rho-1}R_t. \tag{C11}$$

Hence, with (C11) and (B9), (C9) can be written as follows:

$$\begin{aligned}\left(C_t^w\right)^{\rho-1} = &\left[p_{w,w}V_{t+1}^w + p_{w,w+1}V_{t+1}^{w+1} + p_{w,r(w)}V_{t+1}^{r(w)}\right]^{\rho-1}\\ &\times\beta\Bigg[p_{w,w}\left\{\left(V_{t+1}^w\right)^{1-\rho}\left(C_{t+1}^w\right)^{\rho-1}R_{t+1}\right\} + p_{w,w+1}\\ &\left\{\left(V_{t+1}^{w+1}\right)^{1-\rho}\left(C_{t+1}^{w+1}\right)^{\rho-1}R_{t+1}\right\}\\ &+ p_{w,r(w)}\left\{\left(V_{t+1}^{r(w)}\right)^{1-\rho}\left(C_{t+1}^{r(w)}\right)^{\rho-1}\frac{R_{t+1}}{\gamma_{r(w)}}\right\}\Bigg].\end{aligned} \tag{C12}$$

Let us conjecture the form of the utility function V_t^w as

$$V_t^w = \left(\pi_t^w\right)^{-1/\rho} C_t^w. \tag{C13}$$

Then, (C12) can be rewritten as follows:

$$\begin{aligned}\left(C_t^w\right)^{\rho-1} &= \left[p_{w,w}\left(\pi_{t+1}^w\right)^{-1/\rho} C_{t+1}^w + p_{w,w+1}\left(\pi_{t+1}^{w+1}\right)^{-1/\rho} C_{t+1}^{w+1} + p_{w,r(w)}\left(\epsilon_{t+1}^{r(w)}\pi_{t+1}^w\right)^{-1/\rho} C_{t+1}^{r(w)}\right]^{\rho-1} \\ &\times \beta R_{t+1}\left[p_{w,w}\left(\pi_{t+1}^w\right)^{(\rho-1)/\rho} + p_{w,w+1}\left(\pi_{t+1}^{w+1}\right)^{(\rho-1)/\rho} + p_{w,r(w)}\frac{1}{\gamma_{r(w)}}\left(\epsilon_{t+1}^{r(w)}\pi_{t+1}^w\right)^{(\rho-1)/\rho}\right] \\ &= \left[p_{w,w} C_{t+1}^w + p_{w,w+1}\left(\pi_{t+1}^{w+1}/\pi_{t+1}^w\right)^{-1/\rho} C_{t+1}^{w+1} + p_{w,r(w)}\left(\epsilon_{t+1}^{r(w)}\right)^{-1/\rho} C_{t+1}^{r(w)}\right]^{\rho-1} \times \beta R_{t+1} \times \Omega_{t+1}^w,\end{aligned} \tag{C14}$$

where

$$\Omega_{t+1}^w = p_{w,w} + p_{w,w+1}\left(\pi_{t+1}^{w+1}/\pi_{t+1}^w\right)^{1/(1-\sigma)} + p_{w,r(w)}\frac{1}{\gamma_{r(w)}}\left(\epsilon_{t+1}^{r(w)}\right)^{1/(1-\sigma)}. \tag{C15}$$

Furthermore, the utility function (11) can be expressed as

$$\begin{aligned}\left(\pi_t^w\right)^{-1/\rho} C_t^w = \Bigg[\left(C_t^w\right)^\rho + \beta\Bigg(& p_{w,w}\left(\pi_{t+1}^w\right)^{-1/\rho} C_{t+1}^w + p_{w,w+1}\left(\pi_{t+1}^{w+1}\right)^{-1/\rho} C_{t+1}^{w+1} \\ &+ p_{w,r(w)}\left(\epsilon_{t+1}^{r(w)}\pi_{t+1}^w\right)^{-1/\rho} C_{t+1}^{r(w)}\Bigg)^\rho\Bigg]^{1/\rho}.\end{aligned} \tag{C16}$$

Substituting (C14) for (C16) and rearranging the equation, we obtain

$$\pi_t^w = 1 - \left(R_{t+1}\Omega_{t+1}^w\right)^{\sigma-1}(\beta)^\sigma \frac{\pi_t^w}{\pi_{t+1}^w}. \tag{C17}$$

Thus, (C1) holds.

For workers in the terminal group ($w = M$), the derivation proceeds identically except that there is no further disability stage, eliminating the worsening term. Thus

$$\Omega_{t+1}^M = p_{M,M} + p_{M,r(M)}\frac{1}{\gamma_{r(M)}}\left(\epsilon_{t+1}^{r(M)}\right)^{1/(1-\sigma)} \tag{C18}$$

and the consumption propensity equation (C1) holds with this simplified risk adjustment factor. □

C.2 | Worker's risk-adjusted valuation

Proof. We first rewrite (C14) as follows:

$$\left(\beta R_{t+1}\Omega_{t+1}^{w}\right)^{\sigma}C_t^w = p_{w,w}C_{t+1}^w + p_{w,w+1}\left(\pi_{t+1}^{w+1}/\pi_{t+1}^{w}\right)^{-1/\rho}C_{t+1}^{w+1} + p_{w,r(w)}\left(\epsilon_{t+1}^{r(w)}\right)^{-1/\rho}C_{t+1}^{r(w)}. \tag{C19}$$

From the consumption functions (10) and (12), (C19) can be written as

$$\begin{aligned}
&\left(\beta R_{t+1}\Omega_{t+1}^{w}\right)^{\sigma}\pi_t^w\left(R_tA_t^w + H_t^w + S_t^w\right)\\
&= p_{w,w}\pi_{t+1}^w\left(R_{t+1}A_{t+1}^w + H_{t+1}^w + S_{t+1}^w\right) + p_{w,w+1}\left(\pi_{t+1}^{w+1}/\pi_{t+1}^{w}\right)^{-1/\rho}\pi_{t+1}^{w+1}\left(R_{t+1}A_{t+1}^{w+1} + H_{t+1}^{w+1} + S_{t+1}^{w+1}\right)\\
&\quad + p_{w,r(w)}\left(\epsilon_{t+1}^{r(w)}\right)^{-1/\rho}\epsilon_{t+1}^{r(w)}\pi_{t+1}^w\left(\frac{R_{t+1}}{\gamma_{r(w)}}A_{t+1}^{r(w)} + S_{t+1}^{r(w)}\right).
\end{aligned} \tag{C20}$$

We slightly change the form of (C1) as

$$\pi_{t+1}^w = \left(R_{t+1}\Omega_{t+1}^w\right)^{\sigma-1}(\beta)^{\sigma}\frac{\pi_t^w}{1-\pi_t^w}, \tag{C21}$$

and multiplying both sides by $R_{t+1}\Omega_{t+1}^w(1-\pi_t^w)(R_tA_t^w + H_t^w + S_t^w)$. Then, it becomes

$$\pi_{t+1}^w\left(1-\pi_t^w\right)R_{t+1}\Omega_{t+1}^w\left(R_tA_t^w + H_t^w + S_t^w\right) = \left(\beta R_{t+1}\Omega_{t+1}^w\right)^{\sigma}\pi_t^w\left(R_tA_t^w + H_t^w + S_t^w\right). \tag{C22}$$

Hence, with (C22), (C20) can be written as

$$\begin{aligned}
&\left(1-\pi_t^w\right)\left(R_tA_t^w + H_t^w + S_t^w\right)\\
&= \frac{1}{R_{t+1}}\left[\frac{p_{w,w}}{\Omega_{t+1}^w}\left(R_{t+1}A_{t+1}^w + H_{t+1}^w + S_{t+1}^w\right) + \frac{p_{w,w+1}\left(\pi_{t+1}^{w+1}/\pi_{t+1}^{w}\right)^{1/(1-\sigma)}}{\Omega_{t+1}^w}\left(R_{t+1}A_{t+1}^{w+1} + H_{t+1}^{w+1} + S_{t+1}^{w+1}\right)\right.\\
&\quad\left. + \frac{p_{w,r(w)}\left(\epsilon_{t+1}^{r(w)}\right)^{1/(1-\sigma)}/\gamma_{r(w)}}{\Omega_{t+1}^w}\left(R_{t+1}A_{t+1}^{r(w)} + \gamma_{r(w)}S_{t+1}^{r(w)}\right)\right]\\
&= \frac{1}{R_{t+1}}\left[p_{w,w}^*\left(R_{t+1}A_{t+1}^w + H_{t+1}^w + S_{t+1}^w\right) + p_{w,w+1}^*\left(R_{t+1}A_{t+1}^{w+1} + H_{t+1}^{w+1} + S_{t+1}^{w+1}\right)\right.\\
&\quad\left. + p_{w,r(w)}^*\left(R_{t+1}A_{t+1}^{r(w)} + \gamma_{r(w)}S_{t+1}^{r(w)}\right)\right].
\end{aligned} \tag{C23}$$

We can rewrite the left-hand side of (C23) as

$$\begin{aligned}\left(1-\pi_t^w\right)\left(R_tA_t^w+H_t^w+S_t^w\right)&=\left(R_tA_t^w+H_t^w+S_t^w\right)-\pi_t^w\left(R_tA_t^w+H_t^w+S_t^w\right)\\&=\left(R_tA_t^w+H_t^w+S_t^w\right)-C_t^w\\&=\left(R_tA_t^w+\xi_wW_t+E_t^w-T_t^w-C_t^w\right)+\left(H_t^w-\xi_wW_t-E_t^w+T_t^w\right)+\left(S_t^w-0\right).\end{aligned}\tag{C24}$$

Hence, (C23) can be expressed as follows:

$$\begin{aligned}&\left(R_tA_t^w+\xi_wW_t+E_t^w-T_t^w-C_t^w\right)+\left(H_t^w-\xi_wW_t-E_t^w+T_t^w\right)+\left(S_t^w-0\right)\\&=\frac{1}{R_{t+1}}\Big[p_{w,w}^*\left(R_{t+1}A_{t+1}^w+H_{t+1}^w+S_{t+1}^w\right)+p_{w,w+1}^*\left(R_{t+1}A_{t+1}^{w+1}+H_{t+1}^{w+1}+S_{t+1}^{w+1}\right)\\&\quad+p_{w,r(w)}^*\left(R_{t+1}A_{t+1}^{r(w)}+\gamma_{r(w)}S_{t+1}^{r(w)}\right)\Big],\end{aligned}\tag{C25}$$

suggesting that the worker's wealth at the end of time t equals the risk-adjusted discounted value of her wealth at the end of time $t+1$. It is evident from (13) and $A_{t+1}^w=A_{t+1}^{w+1}=A_{t+1}^{r(w)}$ that

$$R_tA_t^w+\xi_wW_t+E_t^w-T_t^w-C_t^w=p_{w,w}^*A_{t+1}^w+p_{w,w+1}^*A_{t+1}^{w+1}+p_{w,r(w)}^*A_{t+1}^{r(w)},\tag{C26}$$

and hence,

$$\begin{aligned}H_t^w+S_t^w&=\xi_wW_t+E_t^w-T_t^w\\&\quad+\frac{1}{R_{t+1}}\Big[p_{w,w}^*\left(H_{t+1}^w+S_{t+1}^w\right)+p_{w,w+1}^*\left(H_{t+1}^{w+1}+S_{t+1}^{w+1}\right)+p_{w,r(w)}^*\left(\gamma_{r(w)}S_{t+1}^{r(w)}\right)\Big].\end{aligned}\tag{C27}$$

Thus, (C5) holds.

For workers in the terminal group ($w=M$), the valuation equations simplify because there is no further disability stage beyond M, so the terms involving h^{w+1} and s^{w+1} are absent. □

APPENDIX D: AGGREGATION DECISION

Additionally, we define the ratio of the total amount of social security benefits disbursed by the government to GDP as e_t, i.e.,

$$E_t^{\bullet}=\sum_w\left(E_t^{r(w)\bullet}+E_t^{w\bullet}\right)=e_tY_t,\tag{D1}$$

where Y_t signifies the GDP at time t.

Here, the aggregated human wealth and social security wealth are represented as

$$H_t^{w\bullet} = \begin{cases} \xi_w W_t N_t^w - T_t^{w\bullet} + \dfrac{1}{1+n}\dfrac{1}{R_{t+1}} & w = 1, 2, \ldots, M-1, \\ \quad \left[p_{w,w}^* H_{t+1}^{w\bullet} + p_{w,w+1}^* \dfrac{q^w}{q^{w+1}} H_{t+1}^{w+1\bullet}\right], & \\ \xi_w W_t N_t^w - T_t^{w\bullet} + \dfrac{1}{1+n}\dfrac{1}{R_{t+1}} p_{w,w}^* H_{t+1}^{w\bullet}, & w = M, \end{cases} \tag{D2}$$

and

$$S_t^{w\bullet} = \begin{cases} E_t^{w\bullet} + \dfrac{1}{1+n}\dfrac{1}{R_{t+1}} & w = 1, \ldots, M-1, \\ \quad \left[p_{w,w}^* S_{t+1}^{w\bullet} + p_{w,w+1}^* \dfrac{q^w}{q^{w+1}} S_{t+1}^{w+1\bullet} + p_{w,r(w)}^* \dfrac{q^w}{q^{r(w)}} \gamma_{r(w)} S_{t+1}^{r(w)\bullet}\right], & \\ E_t^{w\bullet} + \dfrac{1}{1+n}\dfrac{1}{R_{t+1}}\left[p_{w,w}^* S_{t+1}^{w\bullet} + p_{w,r(w)}^* \dfrac{q^w}{q^{r(w)}} \gamma_{r(w)} S_{t+1}^{r(w)\bullet}\right], & w = M, \end{cases} \tag{D3}$$

respectively.

The evolution of financial wealth distribution for each worker group unfolds through the following mechanisms:

$$\lambda_{t+1}^w A_{t+1}^\bullet = p_{w,w}\left(R_t \lambda_t^w A_t^\bullet + \xi_w W_t N_t^w + E_t^{w\bullet} - T_t^{w\bullet} - C_t^{w\bullet}\right), \quad w = 1, \tag{D4}$$

and

$$\begin{aligned} \lambda_{t+1}^w A_{t+1}^\bullet = {} & p_{w,w}\left(R_t \lambda_t^w A_t^\bullet + \xi_w W_t N_t^w + E_t^{w\bullet} - T_t^{w\bullet} - C_t^{w\bullet}\right) \\ & + p_{w-1,w}\left(R_t \lambda_t^{w-1} A_t^\bullet + \xi_{w-1} W_t N_t^{w-1} + E_t^{w-1\bullet} - T_t^{w-1\bullet} - C_t^{w-1\bullet}\right), w = 2, \ldots, M. \end{aligned} \tag{D5}$$

This equation illustrates the savings behavior of workers for the next period. Meanwhile, the financial assets of each retiree group evolve as follows:

$$\begin{aligned} \lambda_{t+1}^{r(w)} A_{t+1}^\bullet = {} & \left(R_t \lambda_t^{r(w)} A_t^\bullet + E_t^{r(w)\bullet} - C_t^{r(w)\bullet}\right) \\ & + p_{w,r(w)}\left(R_t \lambda_t^w A_t^\bullet + \xi_w W_t N_t^w + E_t^{w\bullet} - T_t^{w\bullet} - C_t^{w\bullet}\right). \end{aligned} \tag{D6}$$

This expression encompasses existing retirement savings along with assets contributed by newly retired workers. To further elucidate, (D4) can be reconfigured as

$$R_t \lambda_t^w A_t^\bullet + \xi_w W_t N_t^w + E_t^{w\bullet} - T_t^{w\bullet} - C_t^{w\bullet} = \frac{\lambda_{t+1}^w A_{t+1}^\bullet}{p_{w,w}}, \quad w = 1, \tag{D7}$$

and (D5) is,

$$R_t\lambda_t^{w}A_t^{\bullet} + \xi_w W_t N_t^{w} + E_t^{w\bullet} - T_t^{w\bullet} - C_t^{w\bullet} \\ = \frac{\lambda_{t+1}^{w}A_{t+1}^{\bullet}}{p_{w,w}} - \frac{p_{w-1,w}}{p_{w,w}}\left(R_t\lambda_t^{w-1}A_t^{\bullet} + \xi_{w-1}W_tN_t^{w-1} + E_t^{w-1\bullet} - T_t^{w-1\bullet} - C_t^{w-1\bullet}\right). \quad w = 2, \ldots, M. \tag{D8}$$

Substituting (D7) into (D6), we derive

$$\lambda_{t+1}^{r(w)}A_{t+1}^{\bullet} = \left(R_t\lambda_t^{r(w)}A_t^{\bullet} + E_t^{r(w)\bullet} - C_t^{r(w)\bullet}\right) + p_{w,r(w)}\frac{\lambda_{t+1}^{w}A_{t+1}^{\bullet}}{p_{w,w}}, \quad w = 1, \tag{D9}$$

also from (D8) and (D6), we get

$$\begin{aligned}\lambda_{t+1}^{r(w)}A_{t+1}^{\bullet} = {} & \left(R_t\lambda_t^{r(w)}A_t^{\bullet} + E_t^{r(w)\bullet} - C_t^{r(w)\bullet}\right) \\ & + p_{w,r(w)}\left[\frac{\lambda_{t+1}^{w}A_{t+1}^{\bullet}}{p_{w,w}} - \frac{p_{w-1,w}}{p_{w,w}}\left(R_t\lambda_t^{w-1}A_t^{\bullet} + \xi_{w-1}W_tN_t^{w-1}\right.\right. \\ & \left.\left. + E_t^{w-1\bullet} - T_t^{w-1\bullet} - C_t^{w-1\bullet}\right)\right]. \quad w = 2, \ldots, M.\end{aligned} \tag{D10}$$

Finally, to determine the proportions of financial assets held by each group relative to the total financial assets, normalization is applied to fulfill the conditions

$$\sum_w \left(\lambda_t^{r(w)} + \lambda_t^{w}\right) = 1. \tag{D11}$$