Tiered Refunds and Return Restoration for Seasonal Products: Profitability and Environmental Impacts

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Abstract: The surge in product returns poses significant financial and environmental challenges, particularly for seasonal goods, as retailers must often discard or liquidate returned items at the end of a short selling season. To address these challenges, we propose a *tiered-refund policy* integrated with *return restoration*. Under this policy, the retailer offers full refunds for early returns and only partial refunds for late returns, creating a supply of early returns that can be restored and resold as new. By employing a two-period model to capture the essential dynamic of early and late returns, and accounting for consumer valuation uncertainty and the disutility of returning early, we characterize the retailer's optimal late-return refund, initial inventory level, and restoration quantities. We investigate the impact of the proposed tiered-refund policy with return restoration relative to two benchmarks: a restrictive short-return-window policy and a lenient full-refund policy. Our analytical results show that, even without restoration, the tiered-refund policy dominates the short-return-window policy, and outperforms the full-refund policy unless consumer valuations are low. When restoration is feasible, the tiered-refund policy becomes even more attractive compared to the full-refund policy for lower consumer valuations and strictly dominates if restoration costs are not prohibitively high. We also find that the operational flexibility created by restoration leads the retailer to lower the optimal late-return refund in order to incentivize early returns. Numerical experiments using practice-calibrated parameters show that the tiered-refund policy with restoration increases profits by an average of 11% relative to the short-return-window policy and 7% relative to the full-refund policy, with maximum gains reaching 38%. Furthermore, the policy reduces total environmental impact by an average of 1.5% and 5% against the respective benchmarks, with reductions of up to 13%. These sustainability gains are driven by converting otherwise discarded returns into usable products and by reducing initial inventory requirements through restoration. Finally, we find that tiered refunds and restoration are complementary in improving profitability.

Key words: Inventory, consumer returns, return restoration, tiered refund

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1 Introduction

Over the past few decades, retailers have offered generous return policies to enhance consumer satisfaction and stimulate demand, leading to a surge in product returns. According to the National Retail Federation (2023a[1]), U.S. merchandise returns reached \$743 billion in 2023, representing 14.5% of total sales. Returns can have particularly severe consequences for seasonal products such as fashion or holiday items, which are often salvaged or discarded at the end of the selling season.[2] During the 2022 holiday season, U.S. retailers recorded \$171 billion in returns (18% of holiday sales) (National Retail Federation 2023b). Returns have

[1] For brevity, all web references are provided in the E-companion.

[2] We use the term "seasonal products" to refer to products with a short selling season, for which the retailer cannot place replenishment orders once the season begins (e.g., Gurnani and Tang 1999).

become a major financial challenge for retailers due to lost revenue and the high costs of managing returns: processing a return on a $100 online order can cost companies around $27 (The Wall Street Journal 2023). Due to the high costs and retailers' lack of resources for sorting, inspecting, and processing returns, many returned items may end up in landfills, creating substantial negative environmental impacts. U.S. returns generate up to 9.5 billion pounds of landfill waste each year, as much as 10,000 fully loaded Boeing 747s (The Guardian 2023).

Retailers face a fundamental trade-off in setting return policies: lenient policies increase sales but also raise return-related costs and environmental impact. Despite these financial and environmental drawbacks, many retailers continue to offer full refunds on all returns. However, some retailers have started tightening return policies by shortening return windows. For example, in the 2023 holiday season, Macy's shortened its return period for toys from 90 to 30 days. Although stricter policies can reduce retailers' financial burden from returns, they risk discouraging purchases. According to Blue Yonder's Consumer Retail Returns Survey (2024), 69% of consumers stated that stricter return policies deter them from making purchases. Thus, the overall effectiveness of shortening the return window remains questionable.

Some retailers, on the other hand, have introduced return policies that offer consumers greater flexibility in return timing, albeit with less generous terms for late returns. For example, Amazon Canada refunds 80% of the item price for returns made beyond the standard window (Amazon.ca 2025). Patagonia and Ulta Beauty accept late returns but restrict the refund only to store credit (Patagonia 2025, Ulta Beauty 2025). Another approach requires membership to access an extended return window, e.g., Target's RedCard and Best Buy's My Best Buy Plus (Target 2025 and Best Buy 2025). In these instances, late returns are permitted but impose a cost on the consumer through reduced refunds, less flexible payout forms, or membership fees.

Inspired by these practices, we propose a *tiered-refund policy* (Policy T), under which the refund amount is contingent on the timing of the return. Specifically, consumers who return an unfit product early receive a full refund, whereas those who return late receive only a partial refund (hereafter the *late-return refund*). This structure yields a total demand that falls between that of offering short return windows and lenient full-refunds, while enabling the retailer to influence the composition of early versus late returns. Furthermore, implementing a tiered refund introduces flexibility on the supply side: the retailer can process, restore, and resell early-returned items at full price before the selling season ends. We refer to this practice as *return restoration*. Despite its potential benefits, return restoration is not yet widely adopted, largely due to high operational costs and the limited resources of many retailers to sort, inspect, and process returned products. Consequently, even items that retain near-full value such as unworn clothing or small appliances are frequently discarded, despite needing only minor inspection or rework to be resalable (BBC Earth 2021). According to the Pallite Group (2022), return rates in the fashion industry average approximately 30% for online purchases, with roughly 80% of those items being potentially resalable as new. Generally, a returned item is only resalable if it is collected, inspected, and restocked before the selling season concludes, a process that is both costly and time-sensitive (Mostard and Teunter 2006, Vlachos and Dekker 2003). This hurdle underscores the time value of returns, as a retailer can only leverage returns that arrive early enough to satisfy demand within the remaining selling season.

In this paper, we evaluate the effectiveness of the tiered-refund policy against two standard benchmarks: the full-refund and short-return-window policies, taking into account the potential capability of return restoration. Specifically, we address the following research questions: (1) In the absence of return restoration, does a tiered-refund policy (Policy T) yield profitability gains relative to the full-refund (Policy F) and short-return-window (Policy S) benchmarks? If so, how should the retailer structure the late-return refund to align consumer purchase and return-timing decisions? (2) When return restoration is feasible, how should the retailer determine the optimal initial inventory and restoration quantities, as well as adjust the late-return refund rate? In addition, how does the restoration capability alter the comparative advantage of the tiered-refund policy? (3) What is the environmental impact of the tiered-refund policy relative to the benchmarks, and how does the introduction of restoration influence the policy's overall sustainability performance?

To address these questions, we consider a retailer selling a single product over two periods. Consumers face valuation uncertainty at the time of purchase and return the product if it proves unfit. We assume consumers may experience a hassle when returning a product early (hereafter referred to as *early-return disutility*) and decide on their return timing. If feasible, returns received by the end of the first period can be restored and resold in the second period. The retailer determines the optimal late-return refund, initial inventory level, and restoration quantities.

First, we analyze the case of deterministic demand in the absence of restoration. We show that an optimally structured tiered-refund policy (Policy T) always dominates the short-return-window benchmark (Policy S) and can outperform the full-refund benchmark (Policy F) unless consumer valuations are low. Under a linear disutility function, we fully characterize the optimal late-return refund as a function of the return rate and overall consumer valuations. Next, we examine the supply-side benefits of tiered refunds through return restoration. Under deterministic demand, we characterize the optimal initial inventory and restoration quantities, demonstrating that restoration encourages the retailer to adopt a more stringent return policy (i.e., a lower late-return refund) to incentivize early returns. Overall, the tiered-refund policy with restoration becomes more attractive compared to the full-refund policy and strictly dominates if restoration costs are not prohibitively high. Extending to stochastic demand, we show that optimal restoration follows a constrained restore-up-to structure. We establish that the optimal initial inventory is lower under return restoration, and confirm that a lower late-return refund remains preferable when restoration is feasible.

Furthermore, we evaluate the environmental implications of the tiered-refund policy and return restoration through a life cycle assessment framework. For the deterministic case, we analytically show that without restoration, a strict short-return-window policy yields a lower environmental impact (primarily due to demand dampening) compared to the tiered-refund and the full-refund policies. However, when the tiered-refund policy incorporates return restoration, it may achieve the lowest overall environmental impact, provided that the restoration process has a smaller environmental footprint relative to initial production and disposal. In the stochastic demand setting, we further investigate the impact due to unconsumed products. We show that restoring early-returned units can reduce both the total environmental footprint and the impact of unconsumed units by displacing the need for new production and lowering the required initial inventory.

Finally, we perform numerical experiments calibrated to practice to quantify the economic and environmental gains of the proposed policy. We find that the tiered-refund policy combined with return restoration increases profits by an average of 11% relative to the short-return-window policy and 7% relative to the full-refund policy, with maximum gains reaching 38%. Furthermore, the policy reduces total environmental impact by an average of 1.5% and 5% against these respective benchmarks, with reductions of up to 13%. These sustainability gains are driven by the ability to convert otherwise discarded returns into usable products and by the subsequent reduction in initial inventory requirements. Notably, we also find that tiered refunds and return restoration are complementary in improving profitability.

This paper makes several key contributions to both theory and practice. Theoretically, it advances the literature on refund policies by proposing a tiered-refund policy based on return timing, in contrast to most existing studies, which focus primarily on uniform refund policies (full, partial, or none) applied to all returns. We also endogenize consumers' return-timing decision by incorporating heterogeneity in product valuation and early-return disutility. Moreover, while prior research has largely examined refund policies before returns occur, it has paid limited attention to retailers' post-return actions. Our proposed return restoration policy addresses this gap by providing a framework for post-return decision-making. Managerially, we offer practical guidelines to retailers facing profitability and sustainability pressures caused by product returns. Specifically, we recommend that retailers consider tiered-refund policies based on return timing and invest in restoring returned items for resale. This approach yields dual benefits. On the demand side, it provides retailers with flexibility in balancing product refunds and consumer satisfaction. On the supply side, it incentivizes timely returns, enabling restoration and full-price resale within the limited selling season. As a result, retailers can reduce initial inventory levels and salvage quantities, thereby lowering environmental impact. In summary, our proposed tiered-refund and restoration policy increases profit, reduces environmental impact, mitigates consumer dissatisfaction from stricter policies, and remains straightforward to implement.

The remainder of the paper is organized as follows. Section 2 provides an overview of related literature. Section 3 presents the model framework and market segmentation. Section 4 analyzes the optimal return policy under deterministic demand in the absence of restoration. We then incorporate return restoration, investigating the joint optimization of restoration quantities, initial inventory, and the late-return refund under deterministic demand in Section 5 and stochastic demand in Section 6. Section 7 evaluates the environmental implications of the proposed policy, while Section 8 presents numerical studies that quantify its profitability and sustainability benefits. Finally, Section 9 concludes the paper and offers directions for future research. In the E-companion, Section EC.1 offers extensions and preliminary explorations to our core model: (i) a fraction of returned items that are resalable without incurring restoration costs, (ii) a consumer segment that does not prefer late returns, (iii) partially resolved fit uncertainty, (iv) valuation-dependent match probabilities, and (v) the joint optimization of price and the late-return refund.

2 Literature Review

Our paper is mainly related to and contributes to two streams of literature: (i) inventory decision with product returns, and (ii) return policy leniency.

2.1 Inventory Decisions with Product Returns

Firms' inventory decisions have been studied extensively in the operations and supply chain management literature. Our work is particularly related to inventory models that incorporate consumer returns. Table 1 summarizes the key commonalities and differences between the existing literature and this study. Within this stream, Su (2009) explores the impact of refund policies on supply chain performance under both consumer valuation and aggregate demand uncertainties. Ülkü and Gürler (2018) and Altug et al. (2021) analyze the newsvendor framework in the presence of opportunistic consumer returns. Following the approach of Su (2009) and Ülkü and Gürler (2018), our model incorporates both demand and valuation uncertainties. Our work also relates to the stream of literature on inventory management with resalable returns. While most studies focus on manufacturer-led collection, refurbishment, and remanufacturing (e.g., Hu et al. 2023, Liu et al. 2021), research addressing retailer-level handling and reselling remains comparatively limited. Within the retailer-focused stream, some studies treat returned items as open-box products with lower valuations than new goods, making the two imperfect substitutes. For example, He et al. (2020) investigate an omnichannel retailer's inventory and pricing decisions when returns are sold as refurbished units. Similarly, Akçay et al. (2013) examine inventory, pricing, and refund decisions when a retailer restocks returns as open-box items. They examine three scenarios: no money-back guarantee (base case), money-back guarantee without restocking, and money-back guarantee with restocking. They find that, relative to the base case, the retailer stocks more initial inventory under a money-back guarantee without restocking. However, compared to the non-restocking scenario, the initial inventory may either increase or decrease once restocking is introduced. This ambiguity is driven by two competing effects. On the one hand, as the retailer decides to restock, they offer a higher refund, leading to possible higher sales, which provides an incentive for the retailer to stock more. On the other hand, an opportunity to resell returned items tends to lower the initial inventory. We also observe a similar dynamic in our model, where a higher late-return refund drives up sales and initial inventory, while the restoration acts to reduce the required initial inventory.

Our model considers returned items that are resold as new following restoration, making them perfect substitutes for original inventory. Fisher et al. (2001) examine a two-period newsvendor model where a fixed fraction of products is returned and reused, while Vlachos and Dekker (2003) extend the classic newsvendor framework to incorporate resalable returns. Specifically, Vlachos and Dekker (2003) analyze four return handling options: secondary market sales, resale without recovery cost, and resale with either partial (cost is incurred only for returned items that are reused) or full (cost is incurred for all returned items) recovery costs. Following the above literature, we assume that a fixed percentage of sold products is eventually returned and that items can be resold at most once. However, while Vlachos and Dekker (2003) treat the "serviceable return rate" – the fraction of returns arriving in time for resale as exogenous, we endogenize this rate. We build on the literature by examining the time value of returns and demonstrating how a retailer can influence the serviceable return rate by adjusting the refund offered to late returns. In our framework, although the total return volume is a fixed percentage of sales (i.e., consumers who find the product a misfit will return the product), the proportion of returns that are usable for sales in the next period is strategically determined by the retailer's tiered-refund structure.

In a setting similar to ours, Reimann (2016) studies a retailer that may resell returned items or procure new units at a higher cost. Our work differs from theirs in three primary ways: (i) we allow for partial refunds, (ii) we incorporate consumer valuation uncertainty, and (iii) we jointly optimize the late-return refund, initial inventory, and restoration quantity, whereas Reimann (2016) treats initial inventory as the sole decision variable. Furthermore, while their study assumes demand uncertainty is fully resolved before refurbishing, our model considers restoration decisions both in a stochastic setting where the retailer must make restoration decisions while still facing demand uncertainty.

2.2 Return Leniency

The second stream of related literature studies consumer returns through the lens of return policy leniency. For comprehensive reviews of this field, we refer the reader to Abdulla et al. (2019) and Duong et al. (2022). Return policy leniency primarily influences a retailer's profitability by impacting consumers' initial purchase and subsequent return decisions. As conceptualized by Janakiraman et al. (2016), return leniency comprises five distinct dimensions: time leniency (the duration of the return window) (e.g., Ertekin and

Papers	Demand Uncertainty	Valuation uncertainty	Inventory	Resale as open-box or refurbished goods	restoration and resale as new	restoration as a decision variable	Newsvendor
Altug et al. (2021)	✓	—	✓	—	—	—	✓
Ülkü and Gürler (2018)	✓	✓	✓	—	—	—	✓
Su (2009)	✓	✓	✓	—	—	—	✓
He et al. (2020)	✓	—	✓	✓	—	—	✓
Akçay et al. (2013)	✓	✓	✓	✓	—	—	✓
Fisher et al. (2001)	✓	—	✓	—	✓	—	✓
Vlachos and Dekker (2003)	✓	—	✓	—	✓	—	✓
Reimann (2016)	✓	—	✓	—	✓	—	✓
Our paper	✓	✓	✓	—	✓	✓	✓

Table 1 Summary of Literature on Inventory Decisions with Product Returns

Agrawal 2021, Ma et al. 2020, Shang et al. 2019), money leniency (the refund amount or restocking fees) (e.g., Esenduran et al. 2022, Ren et al. 2026), effort leniency (consumer effort required to execute returns) (e.g., Davis et al. 1998, Hsiao and Chen 2014, Zhang et al. 2022), scope leniency (the range of eligible products) (e.g., Yang et al. 2017), and exchange leniency (the choice between cash refunds and store credit) (e.g., Fan et al. 2022, Wagner and Martínez-de Albéniz 2020). In a meta-analysis of 21 studies, Janakiraman et al. (2016) find that these dimensions exert varying degrees of influence on consumer behavior, noting that leniency generally has a more significant impact on purchase stage than on return stage. Furthering this analysis, Abdulla et al. (2022) identify monetary leniency as the most effective dimension for driving purchase intentions by positively influencing consumers' perceptions.

Our work focuses on the intersection of monetary and time leniency. While refund policies have been extensively studied, the literature offers mixed results regarding the optimality of full refunds. For instance, while some studies highlight the positive impact of full refunds on purchase decisions (e.g., Suwelack et al. 2011, Zhang et al. 2017), Su (2009) suggests that optimal profits are often higher under partial refunds. Most research in this stream assumes a uniform refund policy, where a single refund amount applies to all returns regardless of return timing. However, in short-season contexts (e.g., holiday season), the timing of returns is critical as early returns can be repurposed to satisfy future demand before the selling season ends.

While time leniency literature typically focuses on duration of the return window (e.g., Ma et al. 2020, Chatterjee et al. 2024), we deviate from this approach by capturing the time effect through a monetary lever alone, without explicitly restricting the window. Specifically, we investigate a tiered-refund policy where the refund amount varies based on the timing of the return. To the best of our knowledge, this is the first study to jointly investigate a retailer's inventory and restoration decisions under a tiered-refund structure and to evaluate how such policies affect both profitability and sustainability by influencing return timing.

3 The Tiered-Refund Policy and Market Segmentation

In this section, we introduce the *tiered-refund policy* and characterize the resulting market segmentation. This segmentation provides the basis for evaluating the tiered-refund policy against benchmark policies of full-refund and short-return-window in the following sections.

3.1 Model Framework

We consider a setting where a firm sells a single product in two periods at an exogenous price p. Market sizes are d_1 and d_2, respectively, in the two periods, with $d = d_1 + d_2$. We consider deterministic demand in Sections 4 and 5 and stochastic demand in Section 6. Each consumer purchases at most one unit of the product. Consumers are heterogeneous in their valuations, v, which is assumed to be uniformly distributed in $[0, \bar{v}]$, with $\bar{v} > p$. We assume that consumers are unable to determine whether a product is a good match at the time of purchase and that the uncertainty about product fit is resolved post-purchase. As in Chen and Bell (2009, 2012), Difrancesco et al. (2018) and Wang et al. (2019), we assume the probability or fraction of consumers returning a product is exogenous. Specifically, with probability θ, a consumer finds the product a good match and receives a utility corresponding to their valuation, v, and with probability $(1-\theta)$, they find it a misfit, return it, and receive a refund that may vary based on return timing, as we describe subsequently.

The retailer implements a *tiered-refund* policy. Specifically, consumers who make *early* returns will receive a full refund that equals the original purchase price p, while those who make *late* returns will receive a *late-return refund* of $r \leq p$. To aid tractability, we define *early* returns as returns that are made by the end of the period in which the item is purchased and *late* returns as the items that are returned in the next period.

Consequently, consumers can decide on the timing of their return. As many consumers prefer a longer return window to a shorter return window (see, for example, d'Astous and Guèvremont 2008, Suwelack et al. 2011), it is suggested that at least some consumers experience an additional loss of utility (i.e., a hassle cost) when making early returns. Such a disutility from having to return early may arise due to several factors such as (i) an additional inconvenience for arranging quick returns (e.g., Rintamäki et al. 2021), where higher valuation consumers may incur a higher hassle cost of returning, potentially arising from such consumers' higher time values (e.g., Hsiao and Chen 2012), or (ii) losing the flexibility of being able to make a decision in a longer time frame (e.g., Lyu 2026), where the additional time of experiencing the product may also be more valuable for higher-valued products. To capture such factors, we will therefore assume an early return disutility function $h(v)$ that is continuous, differentiable, and increasing in consumer valuation, v. We will subsequently introduce some additional assumptions on $h(v)$ to provide richer insights.

Consumers make purchase and return decisions based on their expected utilities. Specifically, for a consumer with valuation v, the expected utility of buying and returning early (in the event of a misfit) is given by $EU_e(v) = -p + \theta v + (1-\theta)(p - h(v))^+$, where the plus sign indicates that consumers are not forced to return early if their disutility exceeds refund, and the expected utility of buying and making a late return (in the event of a misfit) is given by $EU_l(v) = -p + \theta v + (1-\theta)r$. That is, their overall expected utility from purchasing is $EU(v) = -p + \theta v + (1-\theta)\max\{(p-h(v))^+, r\}$. (We would like to note that, in extension Section EC.1.2 as well as in our subsequent numerical studies, we also consider a potential additional segment of consumers who always return early regardless of the late-return refund amount, possibly because they experience no disutility from early returns (e.g., consumers who are highly organized, constrained by deadlines, or inclined to make quick return decisions). As we later show, the existence of such a segment does not impact our main qualitative findings.)

We assume that the disutility function, $h(v)$, has the property $0 < h'(v) \leq \theta/(1-\theta)$, where $h'(v) > 0$ indicates that the disutility from early returns is increasing in consumer valuation, v, as described above. The upper bound $h'(v) \leq \theta/(1-\theta)$ ensures that the expected utility from purchase for consumers who would return early is increasing in their valuation.

At the beginning of Period 1, the retailer determines the late-return refund r and an initial inventory quantity I_0, which is procured at unit cost $c < p$ and can be sold in both periods. All unsold inventory, as well as early and late returns, can be salvaged at value $s < c$. In Section 5 and the subsequent analysis, we further assume that the early returns from Period 1 can be restored at unit cost $c_r < c$ and sold as new, and the retailer determines a restoration quantity x.

3.2 Market Segmentation

We first characterize the market segmentation for a given late-return refund amount r. Consumers evaluate purchase and return decisions by comparing their expected utilities for early and late returns in the event of a product misfit. As stated earlier, the expected utility for purchasing and potentially returning early is given by $EU_e(v) = -p + \theta v + (1-\theta)(p - h(v))^+$, whereas for a late return it is $EU_l(v) = -p + \theta v + (1-\theta)r$. In case of indifference, we assume the consumer chooses late return. Consequently, a consumer with valuation v will buy and make early returns (if the product is a misfit) if $EU_e(v) > EU_l(v)$ and $EU_e(v) \geq 0$, and buy and make late returns if $EU_l(v) \geq EU_e(v)$ and $EU_l(v) \geq 0$. Hence, the value of r, as well as a consumer's early-return disutility, $h(v)$, significantly influence the timing of returns. Let $v_e(r)$ and $v_l(r)$ denote the fraction of consumers who purchase the product and intend to return early, and respectively, late in the event of a misfit. Therefore, the total purchase proportion is reflected by $v_e(r) + v_l(r)$.

Proposition 1 characterizes the segmentation of the market for the full spectrum of r. Depending on the late-return refund amount, the market settles into one of three outcomes: (i) early returns only, (ii) simultaneous existence of early and late returns, or (iii) late returns only. We first consider instances where the early-return disutilities are not excessively high and thus all three outcomes may arise depending on r, i.e., they do not exceed the product's price ($h(\bar{v}) \leq p$) and the early-return option is a viable alternative for at least some consumers, ensured by $h(\bar{v}) \leq \theta(\bar{v} - p)/(1-\theta)$, followed by a brief description of the more restricted market outcomes in Remark 1.

PROPOSITION 1. *Suppose $h(\bar{v}) \leq p$ and $h(\bar{v}) \leq \theta(\bar{v} - p)/(1-\theta)$.*
(a) The market segmentation outcomes induced by the late-return refund amount r are as follows:
Define z_e as the unique solution to $EU_e(v) = 0$. Further, let $\underline{r} = p - h(\bar{v})$ and let $\bar{r}$ be the unique value in $[\underline{r}, p]$ satisfying $\bar{r} = p - h\left(\min\left(\frac{p-(1-\theta)\bar{r}}{\theta}, \bar{v}\right)\right)$. Let $h^{-1}(\cdot)$ denote the inverse function of $h(v)$. Then, the market segmentation can be characterized as follows:

(i) If $r \leq \underline{r}$, then $v_e(r) = \frac{1}{\bar{v}}(\bar{v} - z_e)$ and $v_l(r) = 0$.

(ii) If $\underline{r} < r \leq \bar{r}$, then $v_e(r) = \frac{1}{\bar{v}}\left(h^{-1}(p-r) - z_e\right)$ and $v_l(r) = \frac{1}{\bar{v}}\left(\bar{v} - h^{-1}(p-r)\right)$.

(iii) If $r > \bar{r}$, then $v_e(r) = 0$ and $v_l(r) = \frac{1}{\bar{v}}\left(\bar{v} - \frac{p-(1-\theta)r}{\theta}\right)^+$.

(b) Furthermore, $v_e(r)$ is non-increasing, $v_l(r)$ is non-decreasing, the total purchase proportion $v_e(r) + v_l(r)$ is non-decreasing, and the expected return proportion $(1-\theta)(v_e(r) + v_l(r))$ is non-decreasing in r.

Proposition 1 Part (a) shows that, under the maintained assumptions on disutility ($h(\bar{v}) \leq p$ and $h(\bar{v}) \leq \theta(\bar{v} - p)/(1-\theta)$), three market segmentation outcomes arise as the late-return refund r varies: (i) only early-return purchasers, (ii) coexistence of both early- and late-return purchasers, or (iii) only late-return purchasers. Specifically, as indicated in case (i), when the late-return refund amount r is very low such that even the consumer with the highest early-return disutility prefers to return early rather than late, all purchasers return early upon misfit. Here, z_e denotes the threshold valuation above which consumers purchase with the intention to return early if misfit. Conversely, when r exceeds $\bar{r}$ (case (iii)), all purchasing consumers prefer to return late. The threshold $\bar{r}$ is the late-return refund at which the marginal consumer, with valuation $v = (p-(1-\theta)\bar{r})/\theta$, is simultaneously indifferent between early and late returns and between purchasing and not purchasing. Any higher refund therefore induces all purchasers to prefer late returns. For intermediate r (case (ii)), consumers are split into early- and late-return segments, with the indifference valuation $h^{-1}(p-r)$ separating those who prefer early returns (lower v) from those who prefer late returns (higher v). Part (b) summarizes the monotonicity of the segment sizes with respect to r. Specifically, in case (i), the fraction that purchases (all early-returns) is independent of r, as the purchase threshold valuation z_e does not depend on r. In case (ii), the early-return segment $v_e(r)$ decreases in r while the late-return segment $v_l(r)$ increases in r, but total purchases and total returns remain constant. In case (iii), the purchase fraction $v_l(r)$ increases in r. Collectively, part (b) shows that raising the late-return refund r (weakly) reduces early returns, (weakly) increases late returns, and (weakly) increases both total purchases and total returns. Thus, although we study a tiered-refund policy rather than a uniform refund policy commonly examined in the literature (e.g., Liu et al. 2014, Su 2009), we still find that a lenient return policy has a positive impact on the total sales at the expense of inducing more returns.

We conclude our discussion on market segmentation in Remark 1 by mapping the outcomes for cases where extreme disutility limits the retailer's ability to influence return timing, ensuring a comprehensive analysis of the decision space.

REMARK 1. We now consider relaxed assumptions on the disutility function $h(\cdot)$.
(a) Suppose $h(\bar{v}) \leq \theta(\bar{v}-p)/(1-\theta)$ but $h(\bar{v}) > p$. In this case, $h(\bar{v}) > p$ implies that even with $r = 0$ there still exist some high-valuation consumers who prefer late to early returns, so case (i) does not arise. Moreover, when $r = 0$, $v = \frac{p}{\theta}$ represents the minimum valuation required for purchase with the intention of late returns. We therefore examine whether such a consumer derives positive utility from early return to assess whether case (ii) can arise. If $h\left(\frac{p}{\theta}\right) < p$, they derive positive utility, so case (ii) (coexistence) arises for $r \leq \bar{r}$ and case (iii) (late returns only) arises for $r > \bar{r}$. If $h\left(\frac{p}{\theta}\right) \geq p$, then only case (iii) arises.
(b) Suppose instead that $h(\bar{v}) > \theta(\bar{v}-p)/(1-\theta)$ (i.e., early-return expected utility is negative or otherwise dominated by the late-return option for all consumers), then the early return option is always dominated and only case (iii) arises for all r.

In the following sections, we characterize the retailer's optimal late-return refund r under the tiered-refund policy (Policy T) and evaluate its performance relative to two boundary cases serving as *benchmark policies*.

Specifically, the first benchmark is the *full-refund* policy (Policy F), under which the late-return refund is set at $r = p$. The second benchmark is the *short-return-window* policy (Policy S) where the late-return refund satisfies $r \leq \underline{r}$. As shown in Proposition 1, under such a low refund level, late returns do not occur, making this policy effectively equivalent to allowing returns only within one period of purchase.

We conduct the evaluation in three settings: tiered refunds in the absence of restoration under deterministic demand (Section 4), tiered refunds with restoration under deterministic demand (Section 5), tiered refunds with restoration under stochastic demand (Section 6). This step-by-step approach enables us to isolate and assess the distinct benefits of the tiered-refund structure and the restoration option.

4 Optimal Policy without Restoration under Deterministic Demand

In this section, we evaluate the tiered-refund policy against the full-refund and short-return-window policies under *deterministic demand* in the absence of restoration. This setting isolates the value of tiered refunds even if a retailer lacks return restoration capabilities. Consequently, the restoration quantity is $x = 0$ and the retailer's initial inventory is set at $I_0(r) = \big(v_e(r) + v_l(r)\big)d$. We maintain the viability condition $c < \theta p + (1-\theta)s$ throughout the paper (necessary in this section and sufficient for the remainder of the analysis).

Incorporating the market segmentation characterized in Proposition 1, the firm's profit function is:

$$\pi(r) = \big(p - c - (1-\theta)(p-s)\big)v_e(r)d + \big(p - c - (1-\theta)(r-s)\big)v_l(r)d \tag{1}$$

$$= \begin{cases} \big(p - c - (1-\theta)(p-s)\big)\big(1 - \frac{z_e}{\bar{v}}\big)d & \text{if } r \leq \underline{r} \\ \big(p - c - (1-\theta)(p-s)\big)\dfrac{1}{\bar{v}}\big(h^{-1}(p-r) - z_e\big)d & \\ \qquad + \big(p - c - (1-\theta)(r-s)\big)\dfrac{1}{\bar{v}}\left(\bar{v} - h^{-1}(p-r)\right)d & \text{if } \underline{r} < r \leq \bar{r} \\ \big(p - c - (1-\theta)(r-s)\big)\dfrac{1}{\bar{v}}\left(\bar{v} - \dfrac{p-(1-\theta)r}{\theta}\right)^{+} d & \text{if } r > \bar{r} \end{cases} \tag{2}$$

In equation (1), $p - c - (1-\theta)(p-s)$ represents the expected margin per unit sold for early returns, where each unit sold generates revenue p and incurs cost c; with probability $(1-\theta)$, the unit is returned early, in which case the retailer refunds p but recovers salvage value s. The expected margin for late returns is analogous, except that the refund amount is r instead of p. The exact expressions for $v_e(r)$ and $v_l(r)$ are derived in Proposition 1. We first establish below the uniqueness of the optimal late-return refund under reasonable conditions on the early-return disutility function, $h(v)$. We adopt the subscript NR to represent the no-restoration setting and denote the optimal late-return refund by r^*_{NR}.

PROPOSITION 2. *If $h(v)/v$ is decreasing and $h''(v) \leq 0$ for all v, then the profit is unimodal in r and the refund policy has a unique optimal r^*_{NR}.*

Next, we compare the tiered-refund policy against the short-return-window and full-refund policies.

PROPOSITION 3. *The tiered-refund policy always dominates the short-return-window policy (i.e., $r^*_{NR} > \underline{r}$), and it dominates the full-refund policy (i.e., $r^*_{NR} < p$) if $\bar{v} \geq \frac{1}{\theta}(2\theta p - c + (1-\theta)s)$.*

Proposition 3 establishes the optimality of a tiered-refund policy under a general functional form of early-return disutility, which increases with valuation (as described in Section 3). The first key insight is that forcing all consumers to return early is always suboptimal. Offering a modest late-return refund encourages some consumers to return late, allowing the firm to refund only r instead of p to these consumers. At the margin, the first such consumer generates a net profit in proportion to the refund difference of $p-r$, thus benefiting the firm. The second insight relates to the full-refund benchmark. Relative to a full-refund policy, the tiered-refund policy strikes a balance between sales expansion and profit margin. As shown in Equation (1), the late-return refund influences profit in two ways: it shifts consumers' demand and return timing decisions and impacts the profit generated from late returns. Compared to a full refund, a partial refund for late returns reduces demand but increases profit by $p-r$ per late return. Proposition 3 shows that when the maximum product valuation $\bar{v}$ is sufficiently high, the saving of $p-r$ per late return outweighs the reduction in sales volume. In such markets, the tiered-refund policy dominates the full-refund benchmark because the retailer gains more from margin recovery than it loses from demand reduction. It can also be shown that the tiered-refund policy is more likely to be optimal when the match probability θ is lower as the firm deals with a higher volume of returns, making the $p-r$ savings from tiered refunds a critical driver of profitability. Lastly, we note that Proposition 3 continues to hold within the expanded parameter space of Remark 1.

To further derive a closed-form solution for the optimal late-return refund r when tiered-refund policy is optimal, we assume a linear disutility function: $h(v)=a+bv$, where $a\geq 0$ and $b>0$. The constant a reflects the portion of disutility that is independent of the product's valuation, while the coefficient b captures the sensitivity of disutility to the product's valuation. This functional form allows us to characterize how the optimal refund varies with the valuation distribution and product match probability. We maintain this linear form for the disutility function throughout the remainder of the paper to ensure a holistic set of results. [1] Note that by Proposition 3, the full-refund policy is optimal if $\bar{v}<\frac{1}{\theta}(2\theta p-c+(1-\theta)s)$. This bound is at most $2p-c$ for $\theta\in[0,1]$. If we denote $\bar{v}^*=2p-c$ and $\theta_3=\frac{c-s}{2p-s-\bar{v}}$, then the condition is equivalent to $\bar{v}<\bar{v}^*$ and $\theta>\theta_3$. For $\bar{v}$ and θ outside this region, the tiered-refund policy is optimal, and the optimal late-return refund can take one of three forms, as characterized by Proposition 4.

PROPOSITION 4. *When the tiered-refund policy (Policy T) is optimal, there exist thresholds $\theta_1,\theta_2,\tilde{\theta}_2$, $\bar{v}^{**}$, and $\bar{v}^{***}$ (explicit expressions provided in the E-companion) such that the optimal late-return refund r^*_{NR} takes one of the following three forms:*

(i) (Tiered-EL:) Both early and late returns occur. The optimal late-return refund is given by $r^*_{NR}=p-\frac{b\bar{v}+a}{2}$. This is optimal when $\bar{v}\geq\bar{v}^{***}$ and $\theta>\theta_1$.

(ii) (Tiered-Indifferent:) The retailer is indifferent between having both early and late returns and having only late returns. The optimal late-return refund is given by $r^*_{NR}=\frac{(\theta-b)p-\theta a}{\theta-(1-\theta)b}$. This is optimal when $\bar{v}\geq\bar{v}^*$ and $\theta_2<\theta\leq\theta_1$, or when $\bar{v}^{**}\leq\bar{v}<\bar{v}^*$ and $\theta_2<\theta\leq\tilde{\theta}_2$.

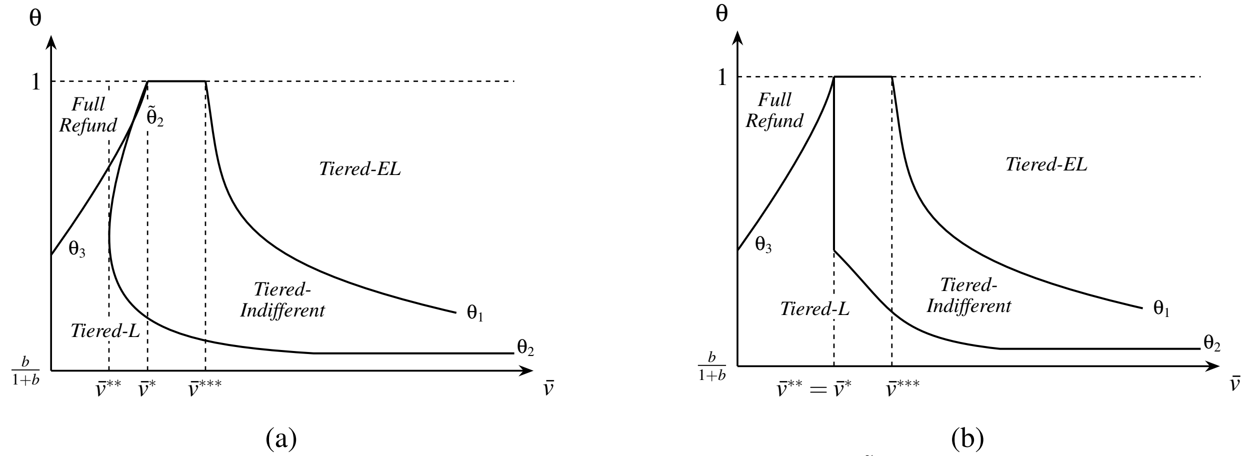

Figure 1 Partition of the $(\theta, \bar{v})$ space by thresholds θ_1, θ_2, $\tilde{\theta}_2$, and θ_3.

(iii) (Tiered-L:) Only late returns occur. The optimal late-return refund is given by $r^*_{NR} = \frac{(2p-\theta\bar{v}-c+(1-\theta)s)}{2(1-\theta)}$*. This is optimal when* $\bar{v} \geq \bar{v}^{**}$ *and* $\theta \leq \theta_2$*, or when* $\bar{v}^{**} \leq \bar{v} < \bar{v}^{*}$ *and* $\tilde{\theta}_2 < \theta \leq \theta_3$*, or when* $\bar{v} < \bar{v}^{**}$ *and* $\theta \leq \theta_3$*.*

Furthermore, we have $\theta_1 \geq \theta_2$ *with both thresholds decreasing in* $\bar{v}$*, and* $\tilde{\theta}_2 \leq \theta_3$ *with both thresholds increasing in* $\bar{v}$*.*

Figure 1 illustrates the regions where different outcomes under Policy T and Policy F are optimal, as characterized in Propositions 3 and 4. Market characteristics, specifically the match probability θ and product valuation (i.e., the upper bound $\bar{v}$), determine which return policy is optimal. Although the regions in which the Tiered-Indifferent outcome applies differ in shape across the two panels, the qualitative patterns are similar. Indeed, panel (b) is simply the degenerate case of panel (a) when $\bar{v}^{**} = \bar{v}^{*}$. A full-refund policy (Policy F) is optimal for products with relatively low valuation and high match probability (top-left region). In all other regions, the tiered-refund policy (Policy T) is optimal. Depending on parameter values, the optimal characterization of Policy T is one of three forms: (i) Tiered-EL (low late-return refund induces some early returns, top-right region corresponding to high valuation and high match probability), (ii) Tiered-Indifferent (intermediate late-return refund makes the retailer indifferent between early/late segmentation and late-only returns, middle region), or (iii) Tiered-L (high late-return refund leads to late returns only, bottom-left region). As only late returns occur, the Tiered-L outcome is effectively equivalent to the partial-refund setting studied in the literature (e.g., Su (2009)). We show that the partial-refund case emerges as a special case of our model and that, the Tiered-L region being optimal means, it yields strictly higher profit than Policy F by balancing sales expansion and per-unit margin. Generally speaking, moving from the bottom-left to the top-right corner on Figure 1, the optimal late-return refund under Policy T becomes progressively lower (i.e., the tiered policy becomes more stringent) as the product valuation and match probability increase.

Having demonstrated the demand-side advantages of tiered refunds, we next examine how the retailer can leverage early returns as a secondary supply source through restoration.

5 Optimal Policy with Return Restoration under Deterministic Demand

In this section, we extend the deterministic demand analysis of Section 4 to incorporate return restoration. Early returns from Period 1 can be restored at unit cost $c_r < c$ and resold as new in Period 2, and all other returns are salvaged at s (e.g., Ketzenberg and Zuidwijk 2009, Chen and Bell 2009, Mostard and Teunter 2006, Vlachos and Dekker 2003). (We comment on an instance where only a fraction of early returns can be restored, with the remainder salvaged, in Section 6.1 after presenting our main results.) Treating these returns as a secondary supply source enables the retailer to reduce its initial inventory requirement. We first characterize the optimal initial inventory I_0^* and restoration quantity x^* for any given late-return refund r. We then evaluate the optimality of the tiered-refund policy with restoration against the two benchmarks: full-refund and short-return-window policies. Finally, we examine how restoration capability affects the optimal late-return refund. Within the first two subsections, we also discuss the implications of incorporating restoration into the short-return-window policy, noting that the full-refund benchmark does not generate early returns and thus cannot facilitate restoration.

5.1 Optimal Inventory and Restoration Decisions

For a given late-return refund r, which induces segments $v_e(r)$ and $v_l(r)$, the firm's optimal initial inventory I_0^* and restoration quantity x^* are determined by solving the following problem (where d_1 denotes first-period demand and d represents the total demand across both periods):

$$\begin{aligned} \max_{I_0,x} \pi(r) &= -cI_0 - (c_r+s)x + \big(p-(1-\theta)(p-s)\big)v_e(r)d + \big(p-(1-\theta)(r-s)\big)v_l(r)d \\ s.t. \quad & 0 \le x \le (1-\theta)v_e(r)d_1, \quad I_0 \ge (v_e(r)+v_l(r))d_1, \quad I_0 + x = (v_e(r)+v_l(r))d \end{aligned} \tag{3}$$

Equation (3) is similar to Equation (1), except that I_0, rather than $(v_e(r)+v_l(r))d$, units of initial inventory are purchased, and restoration incurs a unit cost c_r and forgoes salvage value s. The first constraint ensures that the restoration quantity does not exceed Period-1 early returns. The second constraint requires the initial inventory to cover Period-1 demand. The third constraint ensures that total supply, comprising initial inventory and restored units, matches total demand, consistent with our deterministic demand assumption.

Restoration is feasible only when the late-return refund r is sufficiently low, $r < \bar{r}$, to induce early returns. For higher r, i.e., the regions corresponding to the late-return outcomes of Policy T (and including the boundary $r = p$ (Policy F)), all misfit consumers return late, resulting in no units available for restoration.

PROPOSITION 5. *For a given late-return refund r, the optimal restoration quantity $x^*(r)$ is given by:*

(a) If $c_r \ge c - s$, restoration is not beneficial and $x^*(r) = 0$.

(b) If $c_r < c - s$, restoration is beneficial and $x^*(r) = \min\{\big(v_e(r)+v_l(r)\big)d_2, (1-\theta)v_e(r)d_1\}$. The optimal initial inventory $I_0^*(r) = \big(v_e(r)+v_l(r)\big)d - x^*(r)$, where $x^*(r)$ is decreasing and $I_0^*(r)$ is increasing in r.

(c) Furthermore, the initial inventory is lowest for Policy T where early- and late-returns coexist, followed by Policy S, Policy T when it induces only late returns, and finally Policy F.

Proposition 5 characterizes the optimal initial inventory and restoration quantity for a given late-return refund r. Restoring a unit from Period 1 incurs the restoration cost c_r and forgoes the salvage value s that the unit would have generated if left unrestored, but reduces initial inventory by a unit and saves the procurement cost c. Thus, the marginal value of restoration is $c - s - c_r$. If this value is nonpositive, restoration is not beneficial; all demand should be met through initial inventory and the optimal refund policy is as characterized in Section 4. If the value is positive, restoration is beneficial, and it cannot exceed either Period-2 demand or Period-1 early returns. Therefore, when Period-1 early returns can fully cover Period-2 demand, the retailer orders only enough to meet Period-1 demand. Otherwise, all early returns are restored, and the initial order covers Period-1 demand plus any residual Period-2 demand. Proposition 5 further establishes that a lower late-return refund r results in a higher restoration quantity. Furthermore, the initial inventory requirement is lowest under the tiered-refund policy when early and late returns coexist, which facilitates restoration. While the benchmark short-return-window policy generates the same total demand in this case (see Proposition 1), the absence of restoration necessitates a higher inventory. Finally, the tiered-refund policy when only late returns exist and the full-refund policy induce progressively higher sales but fail to generate any early returns. This lack of restoration, coupled with higher demand, results in higher initial inventory requirements.

We conclude this subsection by evaluating the implications of incorporating restoration capability into the short-return-window policy. We note that the formulation in Equation (3) remains applicable to this case, and parts (a) and (b) of Proposition 5 continue to hold. Regarding part (c), while the monotonicity of $x^*(r)$ is preserved, we find that the initial inventory also becomes monotonic, specifically, the short-return-window policy with restoration now requires the lowest initial inventory (as the short-return-window generates more early returns from the same total demand as the Tiered-EL outcome of the tiered-refund policy), followed by the tiered-refund and full-refund policies where all returns are late. [2]

5.2 Optimal Policy Choice with Restoration

We now evaluate the optimal refund policy when restoration is feasible. When $c_r \geq c - s$, restoration is not beneficial (Proposition 5 part (a)), so the optimal refund characterization remains exactly as in Section 4. When $c_r < c - s$, restoration is beneficial and the following proposition characterizes the optimal policy, extending the policy dominance results of Proposition 3.

PROPOSITION 6. *There exists a threshold $\bar{c}_r$ (dependent on $\bar{v}$ and θ) such that the optimal policy is characterized as follows:*

(a) If $c_r \leq \bar{c}_r$, Policy T is optimal (with the Tiered-EL outcome).

(b) If $c_r > \bar{c}_r$, the optimal policy further depends on the consumer valuation $\bar{v}$: (i) Policy T is optimal (with the Tiered-L outcome) if $\bar{v} \geq \frac{1}{\theta}(2\theta p - c + (1-\theta)s)$, and (ii) Policy F is optimal if $\bar{v} < \frac{1}{\theta}(2\theta p - c + (1-\theta)s)$.

Proposition 6 shows that restoration expands the optimality region of tiered refunds relative to the baseline case without restoration in Section 4. Because the full-refund policy (Policy F) generates no early returns, its profit remains identical to that in the no-restoration setting. In contrast, the tiered-refund policy (Policy T) leverages restoration to lower the required initial inventory, thereby increasing profits compared to its no-restoration counterpart. Consequently, when restoration is not prohibitively expensive, the optimal policy is to implement a tiered-refund policy that induces early returns (the Tiered-EL outcome). When restoration costs are higher, the firm either maintains Policy T with a refund below full refund but still induces only late returns, or shifts to Policy F if consumer valuations are low. (Finally, we note that Proposition 6 continues to hold under the expanded parameter space of Remark 1(a) when case (ii) exists. Conversely, under Remark 1(a) when case (ii) does not exist, as well as under Remark 1(b), Proposition 6 reduces to Proposition 3.)

We note that the short-return-window policy remains suboptimal as implied by Proposition 6, similar to our earlier finding in Proposition 3. However, introducing restoration capabilities to the short-return-window policy can enhance its performance, making it worthwhile to examine the conditions under which it may emerge as the optimal strategy. Remark 2 characterizes these conditions.

REMARK 2. There exists a threshold $\underline{c}_r$ (where $\underline{c}_r \leq \bar{c}_r$), such that the optimal policy is characterized as follows: (a) If $c_r \geq \underline{c}_r$, the optimal policy is as given in Proposition 6. (b) If $c_r < \underline{c}_r$, Policy S with restoration is optimal if $(1-\theta)d_1 < d_2$. Otherwise, Policy T is optimal (with the Tiered-EL outcome).

Remark 2 demonstrates that while Policy S is always dominated in the absence of restoration, it can become optimal when restoration is feasible and sufficiently inexpensive ($c_r < \underline{c}_r$), and when early returns are insufficient to satisfy Period-2 demand, i.e., $(1-\theta)d_1 \leq d_2$, creating a stronger incentive to encourage early returns, compared to the tiered-refund policy.

5.3 The Impact of Restoration on the Optimal Tiered Refund

Finally, we examine the optimal late-return refund r^* in the presence of restoration. Proposition 7 compares the optimal late-return refund in cases with and without restoration.

PROPOSITION 7. *The optimal late-return refund under optimal restoration is weakly lower than the optimal refund without restoration.*

Proposition 7 demonstrates that restoration reduces the optimal late-return refund relative to the no-restoration setting in Section 4. Because restoration does not change the profits under Policy F or Policy T's Tiered-L outcome, but enhances profitability under the Tiered-EL outcome, the optimal policy can shift from full refunds or late-return only regimes toward Tiered-EL where both early and late returns coexist. Consequently, within the $(\bar{v}, \theta)$ parameter space, restoration expands the optimality region of the Tiered-EL outcome while shrinking the region where only late returns occur. Furthermore, the optimal late-return refund under tiered-refund policy when both types of returns coexist decreases when restoration is feasible. This occurs because the retailer gains an additional incentive to elicit early returns (particularly when restoration costs are low) since restored units mitigate the initial inventory requirement. Overall, the operational flexibility created by restoration leads the retailer to lower the optimal late-return refund r in order to incentivize early returns.

6 Optimal Policy under Demand Uncertainty

We now extend the analysis to stochastic demand. Under demand uncertainty, the pool of early returns available for restoration depends on realized Period-1 demand, and the restoration quantity must be chosen at the end of Period 1 after observing early returns but before Period-2 demand is realized. We assume that market sizes in the two periods are independent random variables, denoted by D_t ($t = 1, 2$), with cumulative distribution functions $F_t(\cdot)$ and density functions $f_t(\cdot)$. The realized market sizes are denoted by d_t.

The decision sequence is as follows: At the beginning of Period 1, the retailer sets the late-return refund r and the initial inventory level I_0 (at procurement cost c). At the end of Period 1, the retailer observes the realized Period-1 market size d_1, and consequently the resulting remaining inventory I_1, and early returns R_1. Before Period 2 begins, the retailer determines the restoration quantity $x \leq R_1$ at cost c_r. We solve the retailer's problem using backward induction, first characterizing the optimal restoration quantity (Section 6.1), followed by the initial inventory (Section 6.2), and the optimal late-return refund (Section 6.3).

6.1 Firm's Optimal Restoration Decision

We first characterize the retailer's optimal restoration decision at the beginning of Period 2, after observing realized Period-1 market size d_1. At this point, the retailer's state is defined by the remaining new inventory, $I_1(r, I_0, d_1) = I_0 - \min\big((v_e(r) + v_l(r))d_1, I_0\big)$, and the early returns, $R_1(r, I_0, d_1) = \frac{v_e(r)}{v_e(r)+v_l(r)}(1 - \theta)\min\big((v_e(r) + v_l(r))d_1, I_0\big)$. While I_1 and R_1 are both determined by d_1, we use both in our formulation as the state at the end of Period 1 as it is natural to characterize the optimal restoration in terms of both remaining inventory and available returns, which form an upper bound on the restoration quantity. Further, let $\mathbb{E}[S_2(x)] = E_{D_2}\big[\min\big(\big(v_e(r) + v_l(r)\big)D_2, I_1 + x\big)\big]$ denote the expected Period-2 sales for a given restoration quantity x. We let $\pi_2^*(r, I_0, d_1)$ denote the optimal Period-2 profit for a given late-return refund r, initial inventory I_0, and observed period-1 market size d_1. Since Period-1 procurement costs and revenues are sunk, the firm solves: $\pi_2^*(r, I_0, d_1) = \max_{0 \leq x \leq R_1} \pi_2\big(I_1, R_1, x\big)$, where the expected Period-2 profit for remaining inventory I_1, early returns R_1, and a particular restoration quantity x is given by:

$$\pi_2\big(I_1, R_1, x\big) = p\,\theta\,\mathbb{E}[S_2(x)] + s\Bigg(\underbrace{R_1 - x}_{\substack{\text{Unrestored}\\ \text{Period-1 early returns}}} + \underbrace{I_1 + x - \mathbb{E}[S_2(x)]}_{\substack{\text{Unsold}\\ \text{in Period 2}}} + \underbrace{\frac{v_e(r)}{v_e(r) + v_l(r)}(1-\theta)\mathbb{E}[S_2(x)]}_{\substack{\text{Period-2 early}\\ \text{returns}}}\Bigg)$$
$$+ (s + p - r)\Bigg(\underbrace{\frac{v_l(r)}{v_e(r) + v_l(r)}(1-\theta)(I_0 - I_1 + \mathbb{E}[S_2(x)]}_{\substack{\text{Period-1 and Period-2}\\ \text{late returns}}}\Bigg) - c_r\,x \tag{4}$$

The first term in Equation (4) is the expected Period-2 revenue from consumers who keep the product.

The second term (multiplied by s) represents the salvage value of all items that are either returned early and not restored or any unsold inventory. The third term captures the salvage plus the additional $(p-r)$ margin obtained from late returns across both periods, where $(I_0 - I_1)$ represents realized sales from Period 1 whereas $\mathbb{E}[S_2(x)]$ is the expected sales for Period 2 if x units are restored. The final term is the total restoration cost.

Similar to Section 5 under the deterministic setting, restoration is not feasible under Policy F and the late-return only outcome of Policy T (region (iii) of Proposition 1(a)) (both resulting in $v_e(r) = 0$). Hence, for these cases, the supply of restorable units is zero ($R_1 = 0$), which forces the restoration quantity to $x^* = 0$. Consequently, the optimal restoration policy characterized in Proposition 8 below applies to the Tiered-EL outcome for Policy T, where early and late returns coexist. We note that while our baseline short-return-window benchmark does not incorporate restoration, this same framework would also apply directly to a short-return-window policy with restoration capabilities.

PROPOSITION 8. *The optimal restoration policy is characterized by a restore-up-to level Q^* satisfying*

$$Q^* = (v_e(r) + v_l(r))F_2^{-1}\left(\frac{\theta p + (1-\theta)\frac{v_l(r)}{v_e(r)+v_l(r)}(p-r) - \theta s - c_r}{\theta p + (1-\theta)\frac{v_l(r)}{v_e(r)+v_l(r)}(p-r) - \theta s}\right) \tag{5}$$

The optimal restoration quantity x^* is then obtained as follows:

$$x^* = \begin{cases} 0 & \text{if} \quad I_1 \geq Q^*, \\ Q^* - I_1 & \text{if} \quad I_1 < Q^* \leq I_1 + R_1 \\ R_1 & \text{if} \quad I_1 + R_1 < Q^* \end{cases} \tag{6}$$

Proposition 8 shows that optimal restoration follows a capacitated *restore-up-to* policy. As specified in Equation (5), the optimal restore-up-to target level Q^* is the product of the purchasing consumer fraction, $v_e(r) + v_l(r)$, and the inverse cumulative distribution function of the Period-2 market size, evaluated at a specific critical fractile. The actual optimal restoration quantity x^* is governed by the relationship between the Period-1 ending inventory (I_1), the volume of early returns (R_1), which acts as the restoration capacity, and the target level (Q^*) as detailed in Equation (6). Specifically, the firm opts for no restoration if $I_1 \geq Q^*$. If the total available supply ($I_1 + R_1$) is less than Q^*, the firm restores all available early returns. Otherwise, the firm restores only the amount required to reach the target Q^*.

We refer to the term $\big(\theta p + (1-\theta)\frac{v_l(r)}{v_e(r)+v_l(r)}(p-r) - \theta s - c_r\big)/\big(\theta p + (1-\theta)\frac{v_l(r)}{v_e(r)+v_l(r)}(p-r) - \theta s\big)$ in Equation (5) as the critical fractile, as commonly used in the newsvendor literature. To interpret the critical fractile, we analyze the overage cost c_o and underage cost c_u, recalling that the classical newsvendor critical fractile is $c_u/(c_u + c_o)$. In our setting, the overage cost is the marginal cost of restoring an additional unit not sold by the end of Period 2. Since such a unit is salvaged whether restored or not, we have $c_o = c_r$. The underage cost is more complex. For an additional unit that could have been restored for c_r and sold, with probability θ, the consumer keeps it, generating revenue θp. With probability $1-\theta$, the consumer returns it due to misfit, receiving a refund of p for early returns (yielding no revenue) or r for late returns (yielding $p-r$). Thus, the expected revenue from returns is $(1-\theta)\big(\frac{v_l(r)}{v_e(r)+v_l(r)}(p-r)\big)$. Regarding salvage value, a restored unit returned due to misfit (with probability $1-\theta$) is salvaged at s, but a non-restored unit is also salvaged at s. Thus, the net salvage impact is $(1-\theta)s-s=-\theta s$. Combining these, the underage cost is $\theta p+(1-\theta)\frac{v_l(r)}{v_e(r)+v_l(r)}(p-r)-\theta s-c_r$. To ensure that restoration is economically viable, the numerator of the critical fractile must be positive (the denominator is guaranteed to be positive). This condition is satisfied for any r, provided that $c_r<c-s$ (the condition for restoration to be beneficial under deterministic demand) and $c<\theta p+(1-\theta)s$ (the assumption laid out in Section 4). We also note that, if $c_r=0$, the critical fractile reaches one and hence $x^*=R_1$, i.e., all early returns should be restored.

We next examine the monotonicity of the optimal restore-up-to level, Q^*.

PROPOSITION 9. *The optimal restore-up-to level Q^* (i) decreases with restoration cost c_r and salvage value s, and (ii) increases with late-return refund r when $r<p-\frac{b\bar{v}+a}{2}$, and decreases with r otherwise.*

Proposition 9 part (i) is intuitive as a higher restoration cost or salvage value reduces the incentive to restore. Part (ii) follows because an increase in r simultaneously raises the fraction of late returns (which are more profitable) and lowers the margin $(p-r)$. When r is sufficiently low, the former effect dominates and for higher r, the latter dominates. We also note that the dependence of the optimal restore-up-to level Q^* on the product price p is complex, as price influences not only the profit margin, as in the standard newsvendor model, but also the distribution of early versus late returns. Higher prices increase profit margin, but also reduce total demand, which scales the critical fractile in Equation (5). On the other hand, a higher price might increase or decrease the fraction of late returns among all returns and consequently may increase or decrease the underage cost. The combined effects lead to Q^* being non-monotonic in price p.

Finally, we also note that the same structure holds when only a fraction $\beta\in(0,1]$ of Period-1 early returns can be restored to resalable condition. In that case, the upper bound on x becomes βR_1 while Q^* and the functional form of x^* remain unchanged but R_1 is replaced with βR_1.

6.2 Firm's Initial Inventory Decision

Having characterized the optimal restoration policy, we now analyze the initial inventory decision at the beginning of Period 1. For the benchmark full-refund and short-return-window policies, as well as Policy T's late-return-only outcome, the absence of restoration reduces the inventory decision to a standard newsvendor problem with returns based on cumulative two-period demand (omitted for brevity). In contrast, under Policy T's Tiered-EL outcome (where early and late returns coexist), subsequent return restoration can lower the initial inventory requirement. Here, an important trade-off emerges. While a higher initial inventory I_0 mitigates the need for restoration of returned items, a lower I_0 increases reliance on restoration. However, lower I_0 may also restrict initial sales, naturally yielding fewer returns and limiting restoration opportunities. Thus, determining the optimal initial inventory level is critical.

The retailer determines the optimal initial inventory I_0 by solving the following problem: $\max_{I_0 \geq 0} \pi(r, I_0) = E_{D_1}[p\theta \min\left(I_0, (v_e(r) + v_l(r))D_1\right) + \pi_2^*(r, I_0, D_1)] - c I_0$, which represents the expected revenue from Period-1 unreturned sales, plus the optimal expected Period-2 profit, less the procurement cost. Note that early returns have no immediate Period-1 profit implications because the full purchase price is refunded and the operational impact of early returns (via restoration) and the financial implications of late returns are captured within the expected Period-2 Profit, see Equation (4) in Section 6.1. We find that the objective function is piecewise due to the different impacts of I_0 on restoration capacity R_1. For low I_0, Period-1 ending inventory can be sufficiently small that all early-returned items may be restored, hence increasing I_0 can potentially increase the restoration capacity. In contrast, for high I_0, only a portion of early returns is expected to be restored, so having more initial inventory does not benefit restoration. Letting I_0^{l*} and I_0^{r*} denote the optimal initial inventory levels for each respective region, Proposition 10 characterizes the optimal initial inventory policy.

PROPOSITION 10. *The expected total profit $\pi(r, I_0)$ is concave in the initial inventory I_0. Furthermore, there exists a unit procurement cost threshold $\bar{c} > 0$ such that the optimal initial inventory is characterized by: $I_0^* = I_0^{r*}$ if $c \leq \bar{c}$, and $I_0^* = I_0^{l*}$ if $c > \bar{c}$, where $I_0^{l*} < I_0^{r*}$. In addition, the optimal initial inventory with optimal restoration is weakly lower than that without restoration.*

Proposition 10 demonstrates that despite the piecewise structure of the Period-2 profit function, there exists a unique optimal initial inventory level I_0^*, which falls into one of two regimes. When the unit procurement cost c is low, the firm sets a higher initial inventory I_0^{r*} and expects to rely only partially on early-return restoration. Conversely, when c is high, the firm chooses the lower I_0^{l*}, potentially utilizing all Period-1 early returns to reach the optimal restore-up-to level Q^*. In addition, the availability of restoration weakly reduces the optimal initial inventory level.

Unlike the deterministic demand case in Proposition 5, comparing optimal initial inventory and restoration quantities across return policies becomes considerably more complex under stochastic demand. For instance, while Policy T yields lower demand than Policy F, it results in a higher service level due to a larger underage cost arising from the additional $p - r$ revenue from late returns. Consequently, the net impact remains ambiguous and we defer the comparison to the numerical analysis.

6.3 Optimal Late-Return Refund

Finally, we optimize the late-return refund r itself, completing the backward induction analysis of the retailer's decisions. Consistent with the deterministic insights from Propositions 6 and 7, we find that the ability to restore returns allows the retailer to implement a more stringent refund policy (i.e., a lower r).

PROPOSITION 11. *Under demand uncertainty, the optimal late-return refund when restoration is feasible is weakly lower than the optimal refund without restoration.*

Proposition 11 confirms that the insights obtained under deterministic demand continue to hold under demand uncertainty. Because restoration enhances the profitability of Policy T without affecting the full-refund and short-return-window benchmarks, it can trigger a policy shift (from Policy F to Policy T). Specifically, when Policy T is optimal, restoration lowers the optimal refund level. Without restoration, the refund simply balances the late-return margin $p-r$ against the proportion of late returns, $v_l(r)/(v_e(r)+v_l(r))$. With restoration, however, the refund r serves a dual purpose: it generates late return margins while regulating the supply of early returns for Period 2. Although restoring items incurs costs, the retailer's optimal quantity selection ensures early returns do not become a burden and may in fact help satisfy Period-2 demand. By lowering the refund, the retailer incentivizes consumers to return items earlier, securing a 'real option' to restore inventory. Consequently, the optimal refund r is lower than it would be without restoration.

7 Environmental Impacts of Tiered-Refund Policy with Restoration

Having evaluated the profit implications of the tiered-refund policy, we now turn to its environmental impact. We employ a Life Cycle Assessment (LCA) framework to capture environmental effects across the product's entire life cycle. Following the literature (e.g., Agrawal et al. 2012, Alptekinoğlu and Örsdemir 2022, Atasu and Souza 2013, Tuna and Swinney 2023, Raz et al. 2013), we model these impacts across multiple phases of a product's life cycle. Generally, three phases are considered in the literature: production, use, and disposal. We follow Alptekinoğlu and Örsdemir (2022) and explicitly incorporate the inventory waste resulting from demand uncertainty. Furthermore, we account for the return processing and restoration phases, which are central to our model. We denote the per-unit environmental impact of the six life cycle phases as follows: production (e_p), use (e_u), consumer/retailer disposal (e_d), inventory waste/overordering disposal (e_o), return processing (e_r), and restoration (e_{rs}). The total environmental impact is the sum of the impacts from each phase, calculated by multiplying the number of units involved by their corresponding per-unit impact. The *production impact* is based on all units produced, determined by the retailer's initial inventory: e_pI_0. Let TS denote the expected total sales (with the explicit expression provided when needed in Section 7.2.2); the *use impact* is then $e_u\theta\, TS$, representing the impact from units purchased and kept by consumers. The *disposal impact* arises from two sources: consumer disposal after use, with impact $e_d\theta\, TS$, and retailer disposal of returned but unrestored items, with impact $e_d\big((1-\theta)\, TS-x\big)$. Therefore, the overall disposal impact is $e_d(TS-x)$. We allow a potentially different disposal impact due to overordering: unsold initial inventory and restored items that are not sold in the second period, given by $e_o(I_0+x-\ TS)$. Note that while the impact of disposing of an unrestored returned product is captured by e_d and the impact of disposing of an unsold or restored product is captured by e_o, in practice they are likely similar in magnitude. Finally, *return processing* and *restoration impacts* correspond to the number of units returned and restored, given by $e_r(1-\theta)\, TS$ and $e_{rs}x$, respectively. The total environmental impact (EI_T) is thus given by:

$$EI_T = e_pI_0 + e_u\theta\, TS + e_d(TS-x) + e_o(I_0+x-TS) + e_r(1-\theta)\, TS + e_{rs}x.$$

Consistent with our profit analysis, we first evaluate the impacts under deterministic demand, before extending the analysis to the stochastic demand case. Since demand uncertainty may result in leftover inventory when demand is stochastic, we also evaluate the expected leftover inventory and the leftover-to-sales ratio.

7.1 Environmental Impact under Deterministic Demand

We first evaluate the environmental impacts under deterministic demand.

PROPOSITION 12. *Under deterministic demand, (a) when restoration is not available, the total life-cycle environmental impact under Policy S is lower than that under Policy T without restoration, which is in turn lower than that under Policy F. (b) Under Policy T, implementing restoration reduces the total life-cycle impact if $e_{rs} < e_p + e_d$. (c) If restoration is feasible and $e_{rs} < e_p + e_d$, the total life-cycle impact is lowest for Policy T when early- and late-returns coexist (the Tiered-EL outcome), followed by Policy S, Policy T when it induces only late returns, and finally Policy F.*

Proposition 12 demonstrates that under deterministic demand when restoration is not available, a more lenient return policy increases the total environmental impact due to the demand-expanding effect. When restoration is introduced under the tiered-refund policy, each restored unit incurs an environmental impact of e_{rs} while avoiding both the disposal of a returned product (an impact of e_d) and the production of a new unit of initial inventory (an impact of e_p). Therefore, restoration reduces the total impact provided that $e_{rs} < e_p + e_d$, a condition typically satisfied in practice. Part (c) of the proposition further shows that the policy ranking of the optimal initial inventory carries over to the total environmental impact. Examining the individual components of EI_T, the production impact follows this exact sequence. The use-phase impact mirrors this behavior as well: total sales are identical under Policy T (when early and late returns coexist) and Policy S, and increase progressively under Policy T's late-return-only outcome and Policy F. The comparisons for disposal, return, and restoration impacts follow a similar pattern, but when restoration is feasible under Policy T with both early and late returns, the sum of these three impacts becomes the lowest provided that $e_{rs} < e_p + e_d$. (We note that overordering impact is not relevant in the deterministic setting.)

As a final note, we again examine the possibility of restoration under the short-return-window policy. Under Policy S, introducing restoration reduces the total life-cycle environmental impact under the same condition established in Proposition 12(b), namely $e_{rs} < e_p + e_d$. Furthermore, the structural ranking of the initial inventory carries over to the environmental impact under this condition: Policy S with restoration yields the lowest total life-cycle impact, followed by Policy T and finally Policy F (as shown in the proof of Proposition 12 in the E-companion).

7.2 Environmental Impact under Stochastic Demand

Next, we evaluate the environmental impact under demand uncertainty. Given the analytical complexity of this setting, we defer the comparison across return policies to the numerical analysis in Section 8. Instead, this subsection focuses on the environmental implications of return restoration. We first examine its effect on total life-cycle impact and then assess its impact at the end of the selling season by analyzing expected leftover inventory and the leftover-to-sales ratio.

7.2.1 Life Cycle Assessment (LCA)

In this section, in addition to the total environmental impact (EI_T) which aggregates the burden across all six life cycle phases, we also consider the impact of unconsumed products (EI_U). This measure isolates the environmental burden associated with products that are produced but not ultimately consumed by subtracting the production and disposal impacts of items kept and consumed by consumers, namely $e_p\theta TS$ and $e_d\theta TS$, respectively. Its expression is thus given by $EI_U = EI_T - (e_p + e_d)\theta\, TS$. This distinction allows us to separate the overall environmental footprint from the specific inefficiencies driven by overordering and returns. To address the model's analytical complexity, Propositions 13 - 15 characterize the impact of restoring a marginal unit for a given initial inventory and late-return refund, and a specific sample realization of demand. Specifically, Proposition 13 identifies the condition under which restoration mitigates the total impact, while Proposition 14 identifies when it reduces the impact of unconsumed products. Finally, Proposition 15 evaluates the environmental gains from reducing initial inventory, a reduction made possible through the use of restoration. In the propositions below, we denote Δ_{TS} as the incremental realized total sales due to a marginal unit of restoration.

PROPOSITION 13. *For a given initial inventory I_0 and late-return refund r, and for a specific sample path realization of demand (d_1, d_2), restoring a marginal unit of Period-1 early returns reduces the total environmental impact EI_T if $e_{rs} < e_d - e_o - \big(e_u\theta + e_d + e_r(1-\theta) - e_o\big)\Delta_{TS}$.*

Proposition 13 indicates that for a given initial inventory and late-return refund, and for a specific demand realization, restoration reduces environmental impact as long as the restoration process is sufficiently environmentally friendly. Nevertheless, since $\Delta_{TS} \geq 0$ and e_o is often comparable in magnitude to e_d, this condition may not hold, suggesting that restoration may actually increase the total environmental footprint. However, such an increase is primarily a byproduct of how restoration expands total sales, which in turn amplifies the impacts associated with consumption and subsequent consumer disposal. While consumption contributes to the total footprint, it also generates consumer utility and retailer profit. Thus, its environmental impact is qualitatively distinct from the waste associated with units that are produced but never consumed. To isolate these dynamics, we next examine the environmental impact specifically associated with unconsumed products.

PROPOSITION 14. *For a given initial inventory I_0 and late-return refund r, and for a specific sample path realization of demand (d_1, d_2), restoring a marginal unit of Period-1 early returns reduces the impact of unconsumed products EI_U if $e_{rs} < e_d - e_o + \big(\theta e_p - (1-\theta)(e_d + e_r) + e_o\big)\Delta_{TS}$.*

Similar to Proposition 13, Proposition 14 shows that restoration also reduces the environmental impact of unconsumed products if the restoration burden is sufficiently low. Given that e_d and e_o are of similar magnitude, this threshold condition is met when the production impact outweighs the return and restoration impacts, which is highly likely in practical settings.

If the retailer can lower the initial inventory by a marginal unit by leveraging restored items to satisfy future demand, the *total* environmental impact EI_T can also be reduced. This benefit of restoration is formally presented in Proposition 15.

PROPOSITION 15. *For a given late-return refund r and a specific sample path realization of demand (d_1, d_2), substituting a marginal unit of initial inventory I_0 with a restored Period-1 return reduces the total environmental impact EI_T if $e_{rs} < e_p$ and $e_{rs} < e_p - e_o + \theta e_u + e_d + (1-\theta)e_r$.*

When $e_o \approx e_d$, the condition $e_{rs} < e_p$ is sufficient to ensure that restoration reduces the total environmental impact. This inequality implies that restoring a returned item incurs a lower environmental footprint than producing a new one, which aligns with practical expectations. Notably, Proposition 13 assumes no reduction in initial inventory whereas Proposition 15 assumes a one-for-one reduction with restoration. Under demand uncertainty, the retailer's inventory reduction lies between these two cases, and so does the corresponding threshold for restoration to reduce environmental impact.

Propositions 14 and 15 help us identify two key mechanisms by which restoration reduces environmental impact: (1) it converts products that would otherwise be disposed of into actual sales, thereby reducing the impact of unconsumed products; and (2) it enables a reduction in initial inventory, lowering the environmental footprint in the first place.

7.2.2 Impact of Leftover Inventory

In this subsection, we assess the environmental impact by focusing on items salvaged at the end of the selling season (e.g., Choi and Chiu 2012). Specifically, we define the total quantity of leftover products as the sum of unsold inventory and unrestored returns, both of which are salvaged by the retailer. To provide a more balanced view of consumer utility versus waste, we also calculate the *leftover-to-sales ratio* (*LSR*). This relative measure captures not only the absolute level of salvage but also how it compares to sales. For a given initial inventory I_0 and late-return refund r, the expected total sales (TS), expected leftover (L), and leftover-to-sales ratio (*LSR*) are given by Equations (7), (8), and (9), respectively. (The nonnegativity of L is guaranteed as shown in the proof of Proposition 16.) Similar metrics are employed by Choi and Chiu (2012), where lower values of L and *LSR* indicate a reduced environmental impact.

$$TS = E_{D_1,D_2}\Big[\underbrace{\min\Big(I_0, \big(v_e(r)+v_l(r)\big)D_1\Big)}_{\text{Period-1 total sales}} + \underbrace{\min\Big(\big(I_0 - \big(v_e(r)+v_l(r)\big)D_1\big)^+ + x(D_1), \big(v_e(r)+v_l(r)\big)D_2\Big)}_{\text{Period-2 total sales}}\Big] \tag{7}$$

$$L = \underbrace{I_0 + E_{D_1}x(D_1) - TS}_{\text{Unsold inventory and restored items}} + \underbrace{(1-\theta)TS - E_{D_1}x(D_1)}_{\text{Unrestored returns}} = I_0 - \underbrace{\theta \cdot TS}_{\text{Consumed items}} \tag{8}$$

$$LSR = \frac{L}{TS} = \frac{I_0}{TS} - \theta \tag{9}$$

where $x(D_1)$ is shorthand for the restoration quantity x evaluated at any particular realization of D_1. We find that expected total sales increase with the restoration quantity x, while expected leftover decreases with x. These monotonic relationships suggest that increasing restoration can reduce leftover inventory waste. Proposition 16 summarizes the key insights on how restoration affects these environmental metrics.

PROPOSITION 16. *(i) For a fixed initial inventory I_0, the restoration of early-returned items reduces the environmental impact associated with salvaged products, i.e., by reducing the expected leftover (L) and the leftover-to-sales ratio (LSR), for any particular or optimal late-return refund r.(ii) This environmental benefit is further amplified when the initial inventory I_0 is also optimized to account for restoration capabilities.*

Proposition 16 demonstrates that restoration of early-returned items reduces inventory leftover waste regardless of whether the retailer can adjust the late-return refund or the initial inventory, that is, even in settings where a firm lacks the flexibility to revise its refund policy or inventory decisions. Moreover, when the retailer can optimize initial inventory, the reduction in waste becomes even more significant as the ability to restore items allows the retailer to lower the initial inventory. These results highlight restoration as a powerful tool for achieving better economic and environmental objectives.

8 Numerical Studies

In this section, we conduct numerical studies to quantify the profit and environmental implications of tiered refunds and return restoration. We begin with a comprehensive comparison of Policy T, both with and without restoration, against the two benchmark policies. We then examine the sensitivity of profit and environmental outcomes to key parameters and, finally, investigate whether tiered refunds and return restoration are complementary in improving firm performance.

8.1 Parameter Choices

We select parameter values representative of a mid-market apparel item, specifically a T-shirt. The unit price is set to $p \in \{18, 24, 30\}$, reflecting realistic pricing in this segment. Following industry-standard apparel markups of 2.2x to 2.5x (Vogue 2020), we set the unit procurement cost $c \in \{8, 10, 12\}$. Upper valuation bounds $\bar{v} \in \{40, 50, 60\}$ are chosen to range from approximately 1.5 to 3 times the price p. To reflect the low salvage values typical of seasonal products, we set $s \in \{0, 0.1p, 0.15p, 0.2p, 0.3p\}$. The product match probability $\theta \in \{0.7, 0.8, 0.9\}$, leads to return rates of 10%, 20%, and 30%, which align with observed U.S. apparel return averages, approximately 17% overall and up to 25% for e-commerce (National Retail Federation 2024, Richter 2025, ICSC 2024). Due to limited empirical data regarding consumer disutility specifically associated with early returns, we follow Anderson et al. (2009) and model early return hassle as a percentage of the product price. We vary the baseline early-return disutility $a \in \{0.75, 1.5, 2.25, 3, 3.75\}$, and the valuation sensitivity parameter $b \in \{0.03, 0.06, 0.09, 0.12, 0.15\}$. These combinations will lead to an early-return disutility of an average of one fifth of the product's price. Restoration costs for early returns, comprising of labor and repackaging estimated at \$3.6 and \$0.3, respectively by F. Curtis Barry & Company (2024), are set as $c_r \in \{0, 1, 2, 3, 4, 5\}$. Market sizes (a fraction of which turns to demand) in both periods is assumed to follow a normal distribution with mean $\mu = 50$ and standard deviation $\sigma \in \{5, 10, 15, 20, 25\}$ with positive support. Environmental impact parameters are based on life cycle data for a typical cotton T-shirt (Sciarrone 2024) with values set as ($e_p = 5.5$, $e_u = 1.5$, $e_d = 0.5$, $e_o = 0.5$, $e_r = 1$, $e_{rs} = 0$). In our numerical study, we consider all combinations of price p and valuation bound $\bar{v}$. For all other parameters, we consider $c = 10$, $s = 0.15p$, $c_r = 2$, $\theta = 0.8$, $a = 1.5$, $b = 0.06$, $\sigma = 15$ as the base case and vary each parameter individually across its specified range.

8.2 Comparison of Policy Performance

We perform a numerical study to assess the effectiveness of the proposed tiered-refund policy (Policy T) relative to the two benchmark policies: the lenient full-refund policy (Policy F) and the restrictive short-return-window policy (Policy S). Under each policy, the retailer optimizes the initial inventory level (I_0), the late-return refund (r, where applicable), and the restoration quantity (x, where applicable). We evaluate performance based on the improvement in profit and the change in total environmental impact (kgCO_2-eq). We provide representative subsets of the results from the comprehensive numerical study across various parameter combinations in Tables EC.1 and EC.2 (E-companion Section EC.2). First, we observe that the optimal refund $r^* = p$ when $\bar{v}$ is lower in Table EC.2, while $r^* < p$ when $\bar{v}$ is higher in Table EC.1. This shows that the main insight from our analytical findings on the optimal level of r under the deterministic setting in Propositions 3 and 4 also extend to stochastic demand. Next, we numerically address the initial inventory comparison, deferred from Section 6.2. Recall that under deterministic demand, the initial inventory requirement does not always increase monotonically with return-policy leniency. Our numerical results confirm that this non-monotonic pattern also holds for the stochastic setting. As shown in Table EC.1, Policy T with restoration has a lower average inventory requirement of 43 units compared to the benchmark policies of Policy F (45.5) and Policy S (43.4). This finding highlights that a firm can leverage a relatively lenient return policy to capture greater demand while maintaining a lower initial inventory than required under a stricter short-return-window policy.

A summary of the comprehensive numerical study is provided in Table 2. The results demonstrate that the tiered-refund policy outperforms both benchmark policies. In the absence of restoration, tiered refunds improve profits by an average of 4.98% (up to 23%) relative to the full-refund policy and 9.25% (up to 26%) relative to the short-return-window policy. When return restoration is integrated, the advantage of the

Table 2 Summary of Percentage Improvements under Tiered-Refund Policy

	Profit Gain (%)			Total Impact Change (%)		
Policies	Min	Max	Avg	Min	Max	Avg
Tiered Refunds without Restoration						
Policy T vs S	2.02	26.37	9.25	0	16.89	2.02
Policy T vs F	0	22.96	4.98	-7.41	0.74	-1.98
Tiered Refunds with Restoration						
Policy T with restoration vs S	2.06	37.65	11.08	-8.77	16.89	-1.52
Policy T with restoration vs F	0	35.49	6.83	-13.07	0	-5.40
Policy T with vs without restoration	0	14.44	1.63	-10.84	0.00	-3.49
Additional Comparisons						
Policy T with restoration vs S with restoration	0	12.28	2.24	0	33.24	6.79
Policy S with vs without restoration	0.24	37.52	8.58	-12.28	-2.67	-7.76
Policy S vs F	-15.58	-0.69	-3.89	-14.45	-0.59	-3.86

tiered-refund policy is further amplified and it yields average profit gains of 6.83% (up to 35%) against the full-refund policy and 11.08% (up to 38%) against the short-return-window policy. Even when restoration is integrated into the short-return-window policy, the tiered-refund remains superior, yielding average profit gains of 2.24% (up to 12%). (We note that the full-refund policy precludes return restoration as all returns arrive late.) Zero values in the table indicate instances where the tiered policy reduces to a benchmark.

The tiered-refund policy also remains attractive from an environmental perspective. In the absence of restoration, the tiered policy reduces the total environmental impact by an average of 1.98% relative to the full-refund policy. While it results in an average 2.02% higher impact than the short-return-window policy, this is primarily because the latter's restrictive terms suppress overall sales, thereby reducing production, usage, and disposal. However, when restoration is integrated, the tiered-refund policy reduces the total environmental impact by an average of 1.52% compared to the standard short-return-window policy.

Our results further quantify the value of the restoration capability. Integrating restoration into the tiered-refund policy increases profits by an average of 1.63% (up to 14%) while reducing environmental impact by an average of 3.49% (up to 11%).

Overall, these results suggest that the tiered-refund policy achieves a superior balance between profitability and sustainability, yielding consistent economic gains alongside favorable environmental outcomes.

8.3 Sensitivity Analysis of Profit and Environmental Impact

To provide further insights into the conditions under which the tiered-refund policy is most advantageous, we examine how key parameters affect the profit and environmental performance of the tiered-refund policy with restoration against the two benchmark policies. Environmental impacts can vary significantly by product type: some generate a high per-unit impact during production (e.g., outdoor jackets), while others are more impactful during the use phase (e.g., T-shirt). Because such a distinction can affect the total environmental impact under different return policies, we examine both types of products in our sensitivity analysis. Following Vedantam et al. (2021), we classify products with a relatively high e_p/e_u ratio as *production dominant* and those with a relatively high e_u/e_p ratio as *use-dominant*. As profit is not related to whether the product is use-dominant or production-dominant, the profit patterns are the same across the two environmental scenarios. Figure EC.2 in the E-companion presents an illustrative sensitivity analysis based on the numerical setting introduced in Section 8.1, showing how the profit and total environmental impact of the three policies vary with key parameters of match probability θ, the baseline early-return disutility a, the valuation sensitivity of early-return disutility b, and the restoration cost c_r.

Overall, the tiered-refund policy demonstrates robust performance across a wide range of parameter values. However, its environmental performance relative to the short-return-window policy is contingent upon the product's life-cycle impact profile, specifically, whether it is production- or use-dominant. In terms of profitability, Figure EC.2 illustrates a sensitivity analysis across key parameters, visualizing the broader patterns summarized in Table 2. These plots show that the short-return-window policy is consistently dominated. Furthermore, within the illustrated parameter space (which specifically satisfies the conditions under which Policy T dominates Policy F in the deterministic setting (cf. Proposition 6(b)), Policy T yields higher profit than Policy F. The profit advantage is most pronounced when θ or c_r is low, and it remains robust across variations in a and b.

Environmentally, Policy F consistently yields the highest impact. Between Policies T and S, Policy T achieves a lower impact when early-return disutility (a or b) or restoration costs are low, though this ranking may reverse as these frictions increase. This dynamic is driven by two main factors: Policy T expands demand (increasing use-phase impacts) but induces restorable early returns (mitigating upfront inventory and production-phase impacts). At low friction levels, high restoration effectiveness allows Policy T's inventory-savings to dominate. At high levels, Policy S's demand-reducing effect becomes the primary driver. The overall impact depends on whether the product is production- or use-dominant. In production-dominant settings, where initial inventory carries more weight, Policy T exhibits a clear advantage under low frictions. In use-dominant settings, this gap narrows considerably or may reverse as higher usage impacts largely offset Policy T's inventory savings.

In summary, compared to Policy F, Policy T generates strictly higher profit (except when θ is sufficiently high) while consistently reducing environmental impact, thereby achieving a win-win outcome. Relative to Policy S, Policy T achieves a win-win outcome when early-return disutility or restoration costs are relatively low, particularly in production-dominant settings.

8.4 Complementarity Between Tiered Refunds and Restoration

Having demonstrated the profitability benefits of the tiered-refund policy and return restoration, we lastly investigate whether their combined use yields a complementary effect. To isolate the impact of restoration in the absence of a tiered-refund incentive, we introduce a parameter $q_E \in [0,1]$, representing the fraction of consumers who always return a product early upon misfit. In this setup, $q_E = 0$ corresponds to our main model where return timing is entirely strategic, while $q_E > 0$ provides a baseline supply of early returns regardless of the refund policy. (We also briefly discuss the robustness of our main qualitative insights under this extension in the E-companion Section EC.1.2.)

We evaluate profits across four scenarios: (1) full refund (baseline), (2) tiered refund only, (3) restoration only, and (4) tiered refund with restoration. Table 3 illustrates the complementary effect. We define the *complementary profit gain* as the additional profit generated by combining both tiered-refund and restoration beyond the sum of their individual contributions, calculated as: Profit(tiered refund with restoration) – Profit(tiered refund only) – Profit(restoration only) + Profit(full refund). As shown in Table 3, both tiered refund and restoration individually outperform the full refund baseline across all values of q_E. However, their effectiveness varies inversely with q_E. As q_E increases, the effectiveness of the tiered-refund policy gradually declines because a larger portion of the market already returns items early, leaving fewer consumers to be influenced by the financial incentive. Conversely, the effectiveness of restoration grows with q_E, as the exogenous increase in early returns provides more units for restoration and resale.

Notably, the joint implementation of tiered refunds and restoration consistently yields the highest profit and exhibits a complementary effect, where the combined profit gain exceeds the sum of the individual contributions of the two policies. This synergy arises because the tiered-refund policy actively expands the pool of early returns when the retailer has the capability to restore them. The complementarity is most pronounced at lower values of q_E, where the tiered-refund policy has the greatest influence on shifting consumer behavior. When $q_E = 1$, the tiered refund becomes redundant as all returns are already early, and the complementary gain naturally drops to zero.

9 Conclusion

The prevalence of high return rates in modern retail imposes substantial financial and environmental burdens. Retailers face a persistent challenge in balancing the costs associated with returns while recognizing that strict return policies may drive consumers away. This issue is particularly relevant for seasonal products with short selling windows, where returned items often need to be salvaged. Motivated by these dynamics, we propose a novel *tiered-refund policy* integrated with *return restoration*. By offering full refunds for early returns and only partial refunds for late returns, and subsequently restoring early-returned items for resale as new, retailers can enhance both profitability and environmental performance.

Our study offers several important theoretical and managerial contributions. Theoretically, we extend the literature on return leniency and inventory management by analyzing a novel tiered-refund policy combined with return restoration, whereas prior research has largely focused on full, partial, or no-refund settings. By endogenizing return timing and incorporating consumer heterogeneity, we uncover new insights into the interaction between refund design and consumer behavior. We also address the underexplored issue of post-return operational decisions, shifting attention from pre-return policies to restoration strategies after returns are received. Managerially, our findings provide actionable guidance for retailers balancing profitability and sustainability. We show that adopting a tiered-refund policy and investing in restoration (even at nontrivial

Table 3 Complementary Effect Between Tiered-Refund and Restoration on Profitability ($\bar{v} = 50$, $c = 10$, $\theta = 0.8$, $p = 24$, $a = 1.5$, $b = 0.06$, $s = 3.6$, $c_r = 2$)

q_E	Full Refund	Tiered Refund Only	Restoration Only	Tiered Refund with Restoration	Complementary Profit Gain
0.0	446.92	462.37	446.92	466.52	4.14
0.2	446.92	459.28	452.96	468.55	3.23
0.4	446.92	456.19	458.94	470.56	2.36
0.6	446.92	453.10	464.86	472.56	1.53
0.8	446.92	450.01	470.71	474.54	0.74
1.0	446.92	446.92	476.51	476.51	0.00

cost) can improve profitability, reduce waste and landfill disposal, and mitigate consumer dissatisfaction, while remaining practical to implement.

Although our work is based on a two-period model for seasonal products, the tiered-refund policy coupled with return restoration can also be applied to regular products held in inventory over multiple periods. Tiered refunds continue to balance sales growth without eroding margins, and return restoration further increases sales while reducing initial inventory needs. Nevertheless, the complementary benefits may be of a smaller magnitude, as the urgency to encourage early returns is less critical when the selling season extends over a longer period. An interesting extension of this research would be to explore a policy where the retailer provides a menu of multiple refund options tied to the timing of the return.

Endnotes

[1] The results in Propositions 5, 8, 10, 12-16 and Remark 2 also extend to the general disutility function described in Section 3.

[2] See E-companion EC.3 for the formal derivation of these rankings.

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E-Companion for: Tiered Refunds and Return Restoration for Seasonal Products: Profitability and Environmental Impacts

EC.1 Extensions

EC.1.1 A Fraction of Returned Items Are Resalable Without Additional Costs

In this section, we examine the case where some of the early-returned items are resalable with no additional costs incurred and thus will be automatically added to the beginning inventory of Period 2. We assume that among the early-returned items, a fraction of α requires some degree of processing such as repackaging, with a unit restoration cost of c_r. The rest, $(1-\alpha)$ fraction, can be resold directly at no cost and therefore will be automatically added to the Period-2 beginning inventory. Consequently, different from the main model, the restoration quantity x now has an upper bound of αR_1 instead of R_1. With a restoration quantity of x, the available inventory at the beginning of Period 2 becomes $I_1+(1-\alpha)R_1+x$. The Period-2 profit function is then given by Equation (EC.15) in the E-companion. Proposition EC.1 demonstrates the optimal Period-2 target inventory and the corresponding optimal restoration quantities. Note that the optimal restore-up-to level Q^* is the same as that of the main model but the optimal restoration quantity x^* is lower since a fraction will be automatically added to the Period-2 inventory. All qualitative results in Section 6.1 remain.

PROPOSITION EC.1. *The optimal restoration policy is characterized by a restore-up-to level Q^* satisfying*

$$Q^* = (v_e+v_l)F_2^{-1}\left(\frac{\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s-c_r}{\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s}\right) \tag{EC.1}$$

The optimal restoration quantity x^* is then obtained as follows:

$$x^* = \begin{cases} 0 & \text{if } I_1+(1-\alpha)R_1 \geq Q^*, \\ Q^*-I_1-(1-\alpha)R_1 & \text{if } I_1+(1-\alpha)R_1 < Q^* \text{ and } I_1+R_1 \geq Q^*, \\ \alpha R_1 & \text{if } I_1+(1-\alpha)R_1 < Q^* \text{ and } I_1+R_1 < Q^* \end{cases} \tag{EC.2}$$

EC.1.2 A Fraction of Consumers Do Not Prefer Late Returns

In the main text, we have set q_E, the fraction of consumers who do not experience disutility from returning a product early, to be zero. In this extension, we extend the model by considering a positive $q_E > 0$. Since these consumers always return early, the late-return refund has no impact on their behavior. Therefore, the resulting profit functions can be expressed as a linear combination of the original profit (scaled by $(1-q_E)$) plus a constant term arising from the q_E segment. As a result, the structure of the optimization problem is preserved, and the optimal late-return refund remains unchanged.

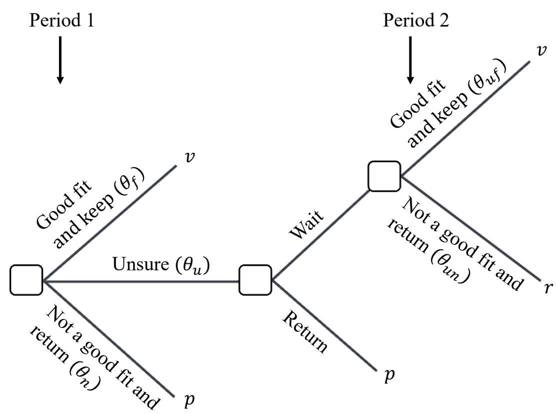

Figure EC.1 Consumer decision tree after purchase when fit uncertainty is only partially resolved

EC.1.3 Fit Uncertainty is Only Partially Resolved in Period 1

In this extension, we consider the case where consumers may not fully resolve their uncertainty about product fit in Period 1. As in the model in Section 3, the tiered-refund policy offers a full refund (p) for returns made in Period 1 and a partial refund ($r < p$) for returns made in Period 2. In Period 1, a consumer discovers the product is a good match with probability θ_f, a poor fit with probability θ_n, or remains uncertain with probability θ_u. If uncertain, the consumer can either return the product immediately for a full refund (p), or keep the product and defer the decision to Period 2. In the latter case, the consumer will eventually resolve the uncertainty, discovering the product to be a good match with probability θ_{uf} or a poor fit with probability θ_{un}. If the product is a poor fit, the consumer returns it and receives the lower refund amount ($r < p$). The sequence of events and corresponding payoffs are illustrated in Figure EC.1. The results are consistent with those of the main model, as presented in Proposition 3. The tiered-refund policy performs the best when the upper bound of consumer valuation is relatively high, whereas the full-refund policy is optimal when the upper bound is relatively low, and the short-return-window policy is never optimal. Proposition EC.2 summarizes the findings.

PROPOSITION EC.2. *The optimal return policy is summarized as follows:*

(i) the short-return-window policy is never optimal;

(ii) If $\bar{v} > 2p - s$*, the tiered-refund policy is optimal. The optimal late-return refund is given by* $r^*_{NR} = \frac{2p - \theta_{uf}\bar{v} - s(1-\theta_{un})}{2\theta_{un}}$*;*

(iii) if $\bar{v} \leq 2p - s$*, the full-refund policy is optimal.*

EC.1.4 Probability of a Good Match Depends on Product Valuation

In the main model, we assume that the probability of a good match, θ, is independent of the product valuation v. We now relax this assumption by allowing θ to depend on v, where $\theta(v) \in [0, 1]$ is continuous and differentiable. We do not impose any monotonicity assumption on $\theta(v)$ (i.e., $\theta'(v)$ may be either positive or negative), as the literature suggests both increasing and decreasing relationships are possible. We show that, as long as the probability of a good match is not overly sensitive to product valuation, the results are similar to those in Section 4. That is, the short-return-window policy is never optimal, and the full-refund policy is dominated by the tiered-refund policy when the valuation upper bound is sufficiently large compared to price.

EC.1.5 The Retailer Optimizes Both Price and Late-Return Refund Amount

In the main model, we consider a fixed price p to isolate the impact of the tiered-refund policy. We now extend the model in Section 4 by optimizing both the price p and the late-return refund r. We show in the proofs that the tiered-refund policy is always at least as profitable as the full-refund policy. The optimal price is any p in the interval $p \in \left[(1-\frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1-\frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta},\ \ \frac{\theta\bar{v}+c-(1-\theta)s}{2\theta}\right]$, with the corresponding optimal late-return refund given by $r^*_{NR} = \frac{2p-\theta\bar{v}-c+(1-\theta)s}{2(1-\theta)}$. The resulting optimal profit equals the profit of the full-refund policy when $p^* = r^*_{NR} = \frac{\theta\bar{v}+c-(1-\theta)s}{2\theta}$. This result implies that tiered-refund policy achieves the global optimum across a continuum of price-refund combinations, whereas the full-refund policy matches this optimum only at the point $p^* = r^*_{NR} = \frac{\theta\bar{v}+c-(1-\theta)s}{2\theta}$. Consumers may, in some cases, prefer lower-priced products paired with modest refund amounts over higher-priced products with more generous refunds. The tiered-refund policy therefore, provides retailers with greater flexibility to select prices and refund levels that better align with consumer preferences.

EC.2 Figures and Tables

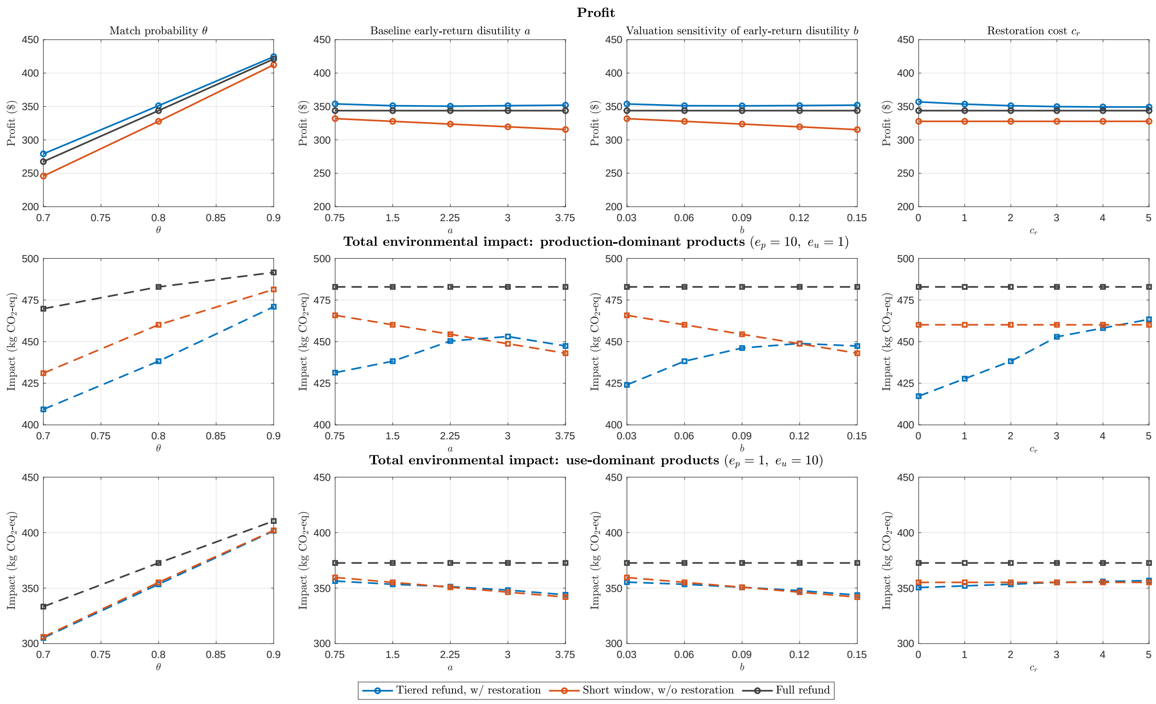

Figure EC.2 Sensitivity of Profit and Environmental Impact to Key Parameters ($\bar{v} = 40$, $p = 24$, $c = 10$, $s = 3.6$)

Table EC.1 Numerical results for $e_p = 5.5, e_u = 1.5, \bar{v} = 50, p = 30$

	Optimal I_0					Optimal r		Optimal profit					Total environmental impact				
Scenario	S	SR	T	TR	F	T	TR	S	SR	T	TR	F	S	SR	T	TR	F
Base	43.3	39.2	43.7	42.6	45.2	26.7	26.4	516.5	535.4	541.2	541.6	539.0	311.6	287.2	314.2	307.5	325.3
$c = 8$	46.0	41.9	48.0	48.0	48.0	30.0	30.0	605.7	616.3	632.1	632.1	632.1	328.6	304.2	342.9	342.9	342.9
$c = 12$	41.0	37.1	41.4	38.6	42.8	26.7	25.9	432.1	459.2	456.4	460.3	451.0	296.8	273.7	299.4	282.7	309.8
$\theta = 0.7$	41.1	35.1	41.6	40.8	44.4	26.7	26.5	408.9	435.3	445.1	445.4	440.9	294.6	258.9	297.9	293.0	318.2
$\theta = 0.9$	45.1	43.0	45.2	44.7	46.0	26.7	26.4	626.0	636.1	638.6	638.9	637.8	325.8	313.3	326.5	323.5	332.3
$a = 0.75$	43.7	39.6	43.8	41.7	45.2	27.5	26.8	521.6	540.7	540.9	543.2	539.0	314.5	290.1	315.2	302.7	325.3
$a = 2.25$	42.9	38.8	43.2	43.2	45.2	25.9	25.9	511.3	530.1	541.5	541.5	539.0	308.7	284.3	310.6	310.6	325.3
$a = 3$	42.4	38.4	42.8	42.8	45.2	25.2	25.2	506.2	524.8	541.6	541.6	539.0	305.1	281.4	307.7	307.7	325.3
$a = 3.75$	42.0	38.0	42.7	42.7	45.2	24.9	24.9	501.1	519.4	541.6	541.6	539.0	302.2	278.4	306.9	306.9	325.3
$b = 0.03$	43.8	39.7	43.9	41.2	45.2	27.6	27.2	522.7	541.9	540.8	542.7	539.0	315.2	290.8	315.9	299.8	325.3
$b = 0.09$	42.8	38.7	43.2	43.1	45.2	25.8	25.7	510.1	528.8	541.5	541.5	539.0	308.0	283.6	310.6	309.9	325.3
$b = 0.12$	42.2	38.2	42.7	42.7	45.2	24.9	24.9	503.6	522.1	541.6	541.6	539.0	303.7	279.9	306.9	306.9	325.3
$b = 0.15$	41.7	37.7	42.7	42.7	45.2	24.9	24.9	497.0	515.3	541.6	541.6	539.0	300.1	276.2	306.9	306.9	325.3
$s = 0.1p$	42.1	37.9	42.2	39.9	44.0	26.7	26.0	496.7	522.0	521.2	523.9	518.4	303.9	278.9	304.6	290.9	317.6
$s = 0.2p$	44.9	40.8	45.2	45.2	46.8	26.7	26.7	538.0	550.5	563.0	563.0	561.5	321.7	297.3	323.6	323.6	335.4
$s = 0.3p$	51.0	48.5	52.3	52.3	53.2	28.6	28.6	589.5	590.9	615.4	615.4	615.2	359.4	344.4	368.5	368.5	374.9
$c_r = 0$	43.3	39.0	43.7	41.0	45.2	26.7	26.0	516.5	542.7	541.2	544.3	539.0	491.6	446.6	496.0	467.6	513.2
$c_r = 1$	43.3	39.1	43.7	41.8	45.2	26.7	26.2	516.5	538.9	541.2	542.5	539.0	311.6	286.6	314.2	302.7	325.3
$c_r = 3$	43.3	39.4	43.7	43.4	45.2	26.7	26.6	516.5	532.1	541.2	541.3	539.0	311.6	288.4	314.2	312.3	325.3
$c_r = 4$	43.3	39.7	43.7	43.7	45.2	26.7	26.7	516.5	529.2	541.2	541.2	539.0	311.6	290.3	314.2	314.2	325.3
$c_r = 5$	43.3	40.1	43.7	43.7	45.2	26.7	26.7	516.5	526.6	541.2	541.2	539.0	311.6	292.7	314.2	314.2	325.3
$\sigma = 5$	39.9	36.1	40.2	40.2	41.6	26.7	26.7	552.7	568.1	578.1	578.1	576.9	292.4	269.7	294.3	294.3	304.9
$\sigma = 10$	41.7	37.7	41.8	41.3	43.6	26.7	26.5	534.5	551.7	559.6	559.7	557.9	302.6	278.8	303.3	300.3	316.3
$\sigma = 20$	44.8	40.5	45.2	43.7	46.8	26.7	26.3	500.9	521.3	525.5	526.2	522.8	320.1	294.5	322.7	313.6	334.3
$\sigma = 25$	46.0	41.6	46.4	44.8	48.0	26.7	26.3	490.0	511.3	514.4	515.5	511.4	327.0	300.9	329.6	319.9	341.2
$e_r = 3.5$	43.3	39.2	43.7	42.6	45.2	26.7	26.4	516.5	535.4	541.2	541.6	539.0	330.1	305.8	332.7	326.0	344.6
Average	43.4	39.4	43.9	43.0	45.5	26.6	26.4	517.8	535.5	545.0	545.7	542.7	319.6	295.3	322.8	317.3	335.2

S = Policy S; SR = Policy S with restoration; T = Policy T; TR = Policy T with restoration; F = Policy F. Base case: $c = 10$, $\theta = 0.8$, $a = 1.5$, $b = 0.06$, $s = 0.15p$, $c_r = 2$, $\sigma = 15$, $e_r = 1$.

Table EC.2 Numerical results for $e_p = 5.5, e_u = 1.5, \bar{v} = 40, p = 30$

	Optimal I_0					Optimal r		Optimal profit					Total environmental impact				
Scenario	S	SR	T	TR	F	T	TR	S	SR	T	TR	F	S	SR	T	TR	F
Base	25.9	23.4	28.3	28.3	28.3	30.0	30.0	308.7	320.0	336.9	336.9	336.9	186.4	171.5	203.6	203.6	203.6
$c = 8$	27.5	25.1	30.0	30.0	30.0	30.0	30.0	362.0	368.4	395.1	395.1	395.1	196.4	182.1	214.3	214.3	214.3
$c = 12$	24.5	22.1	26.8	26.8	26.8	30.0	30.0	258.3	274.5	281.9	281.9	281.9	177.3	163.1	193.9	193.9	193.9
$\theta = 0.7$	23.7	20.2	27.7	27.7	27.7	30.0	30.0	235.6	250.8	275.6	275.6	275.6	169.9	149.0	198.6	198.6	198.6
$\theta = 0.9$	27.7	26.4	28.7	28.7	28.7	30.0	30.0	383.9	390.1	398.6	398.6	398.6	200.1	192.3	207.4	207.4	207.4
$a = 0.75$	25.9	23.4	28.3	28.3	28.3	30.0	30.0	308.7	320.0	336.9	336.9	336.9	186.4	171.5	203.6	203.6	203.6
$a = 2.25$	26.4	23.9	28.3	28.3	28.3	30.0	30.0	315.1	326.6	336.9	336.9	336.9	190.0	175.1	203.6	203.6	203.6
$a = 3$	25.3	22.9	28.3	28.3	28.3	30.0	30.0	302.3	313.3	336.9	336.9	336.9	182.1	167.8	203.6	203.6	203.6
$a = 3.75$	24.8	22.4	28.3	28.3	28.3	30.0	30.0	295.9	306.7	336.9	336.9	336.9	178.5	164.2	203.6	203.6	203.6
$b = 0.03$	26.5	24.0	28.3	28.3	28.3	30.0	30.0	316.5	328.1	336.9	336.9	336.9	190.7	175.9	203.6	203.6	203.6
$b = 0.09$	25.2	22.8	28.3	28.3	28.3	30.0	30.0	300.7	311.7	336.9	336.9	336.9	181.3	167.1	203.6	203.6	203.6
$b = 0.12$	24.5	22.2	28.3	28.3	28.3	30.0	30.0	292.6	303.3	336.9	336.9	336.9	176.3	162.7	203.6	203.6	203.6
$b = 0.15$	24.3	22.0	28.3	28.3	28.3	30.0	30.0	289.4	300.1	336.9	336.9	336.9	174.8	161.2	203.6	203.6	203.6
$s = 0.1p$	25.2	22.7	27.5	27.5	27.5	30.0	30.0	296.8	312.0	324.0	324.0	324.0	181.9	167.0	198.5	198.5	198.5
$s = 0.2p$	26.8	24.4	29.3	29.3	29.3	30.0	30.0	321.5	329.0	350.9	350.9	350.9	192.1	177.8	209.9	209.9	209.9
$s = 0.3p$	30.5	29.0	33.2	33.2	33.2	30.0	30.0	352.3	353.2	384.5	384.5	384.5	214.9	205.9	234.0	234.0	234.0
$c_r = 0$	25.9	23.3	28.3	28.3	28.3	30.0	30.0	308.7	324.4	336.9	336.9	336.9	294.1	266.8	321.3	321.3	321.3
$c_r = 1$	26.0	23.3	28.3	28.3	28.3	30.0	30.0	308.7	322.1	336.9	336.9	336.9	187.0	170.9	203.6	203.6	203.6
$c_r = 3$	26.0	23.6	28.3	28.3	28.3	30.0	30.0	308.7	318.0	336.9	336.9	336.9	187.0	172.7	203.6	203.6	203.6
$c_r = 4$	26.0	23.7	28.3	28.3	28.3	30.0	30.0	308.7	316.3	336.9	336.9	336.9	187.0	173.3	203.6	203.6	203.6
$c_r = 5$	25.9	25.0	28.3	28.3	28.3	30.0	30.0	308.7	314.1	336.9	336.9	336.9	186.4	181.4	203.6	203.6	203.6
$\sigma = 5$	23.9	21.6	26.0	26.0	26.0	30.0	30.0	330.4	339.6	360.6	360.6	360.6	175.1	161.3	190.6	190.6	190.6
$\sigma = 10$	24.9	22.5	27.2	27.2	27.2	30.0	30.0	319.5	329.8	348.7	348.7	348.7	180.7	166.4	197.4	197.4	197.4
$\sigma = 20$	26.8	24.2	29.2	29.2	29.2	30.0	30.0	299.4	311.6	326.8	326.8	326.8	191.5	176.0	208.6	208.6	208.6
$\sigma = 25$	27.5	24.8	30.0	30.0	30.0	30.0	30.0	292.9	305.6	319.6	319.6	319.6	195.5	179.4	213.3	213.3	213.3
$e_r = 3.5$	25.9	23.4	28.3	28.3	28.3	30.0	30.0	308.7	320.0	336.9	336.9	336.9	197.4	182.6	215.7	215.7	215.7
Average	25.9	23.5	28.5	28.5	28.5	30.0	30.0	309.0	319.6	339.2	339.2	339.2	190.8	176.3	209.6	209.6	209.6

S = Policy S; SR = Policy S with restoration; T = Policy T; TR = Policy T with restoration; F = Policy F. Base case: $c = 10$, $\theta = 0.8$, $a = 1.5$, $b = 0.06$, $s = 0.15p$, $c_r = 2$, $\sigma = 15$, $e_r = 1$.

EC.3 Proofs

Proof of Proposition 1 Part (a): As indicated in the main text, the expected utility for purchasing and potentially returning early is given by $EU_e(v) = -p + \theta v + (1-\theta)(p - h(v))^+$, where $h(v)$ is the disutility from early returns. Since $h(v)$ is increasing, and $h(\bar{v}) \le p$ by the assumptions of Proposition 1, we have $p - h(v) \ge 0$ for all v so $EU_e(v) = -p + \theta v + (1-\theta)(p - h(v))$. For a late return the expected utility for purchasing and potentially returning late is $EU_l(v) = -p + \theta v + (1-\theta)r$, where r represents the refund amount for late returns. Consequently, a consumer with valuation v will buy and make early returns (if the product is a misfit) if $EU_e(v) > EU_l(v)$ and $EU_e(v) \ge 0$, and buy and make late returns if $EU_l(v) \ge EU_e(v)$ and $EU_l(v) \ge 0$.

Under the assumption $0 < h'(v) \le \theta/(1-\theta)$ stated in Section 3.1, we have $\frac{\partial EU_e(v)}{\partial v} = \theta - (1-\theta)h'(v) \ge 0$. Further, $\frac{\partial EU_l(v)}{\partial v} = \theta \ge 0$, and $\frac{\partial (EU_e(v) - EU_l(v))}{\partial v} = -(1-\theta)h'(v) \le 0$. Therefore, both $EU_e(v)$ and $EU_l(v)$ are increasing in v, and $EU_l(v)$ increases faster. First, we compare EU_e with 0. Denote z_e as the unique solution to $EU_e(v) = -p + \theta v + (1-\theta)(p - h(v))^+ = 0$, which is independent of r. As $h(\bar{v}) \le p$, it is equivalent to the solution to $-p + \theta v + (1-\theta)(p - h(v)) = 0$. Since $EU_e(0) = -\theta p - (1-\theta)h(0) \le 0$, and by the condition of Proposition 1, $h(\bar{v}) \le \theta(\bar{v} - p)/(1-\theta)$, and thus $EU_e(\bar{v}) \ge 0$, we have $0 \le z_e \le \bar{v}$. Similarly, we compare EU_l with 0. The solution to $EU_l = 0$ is $\frac{p-(1-\theta)r}{\theta} \ge 0$ (because $r \le p$), thus $EU_l(v) \ge 0$ iff $v \ge \frac{p-(1-\theta)r}{\theta}$. Finally, we compare EU_e with EU_l. If $r \le p - h(\bar{v})$, then $h(\bar{v}) \le p - r$, so $EU_l(v) - EU_e(v) \le EU_l(\bar{v}) - EU_e(\bar{v}) = (1-\theta)\big(r - p + h(\bar{v})\big) \le 0$. Otherwise, if $r > p - h(\bar{v})$, denote $h^{-1}(\cdot)$ as the inverse function of h, then the solution to $EU_e(v) = EU_l(v)$ is $v = h^{-1}(p-r)$, so $EU_e(v) \le EU_l(v)$ iff $v \ge h^{-1}(p-r)$. The market segmentation then depends on the relative orders of $z_e, \frac{p-(1-\theta)r}{\theta}, h^{-1}(p-r)$, and $\bar{v}$.

Case (i): If $r \le p - h(\bar{v}) \triangleq \underline{r}$, as shown above, $EU_e(v) \ge EU_l(v)$ holds for all consumers (i.e., buying and returning a misfit product late is always dominated). Hence, consumers whose valuation $v \in [z_e, \bar{v}]$ buy and return early, while others do not buy. We can then derive the fraction of consumers who buy and return a misfit product early as $v_e(r) = \frac{1}{\bar{v}}\big(\bar{v} - z_e\big)$, and the fraction of consumers who buy and return a misfit product late as $v_l(r) = 0$. We refer to this case as Case (i). Note that Case (i) is effectively equivalent to the short-return-window policy since no consumer returns late.

If $r > \underline{r} = p - h(\bar{v})$, then $\bar{v} > h^{-1}(p-r)$. There further exist two cases depending on the relative order of $\frac{p-(1-\theta)r}{\theta}$ and $h^{-1}(p-r)$. When $\frac{p-(1-\theta)r}{\theta} \le h^{-1}(p-r)$, there are three segments - no purchase, purchase and return early, purchase and return late. Otherwise, there are only two segments - no purchase, purchase and return late. The details of these two scenarios are provided in Cases (ii) and (iii) below. Now we examine the condition that distinguishes the two cases. When $p < \theta\bar{v}$, $\frac{p-(1-\theta)r}{\theta} \le \bar{v}$ for all r, thus $h(\frac{p-(1-\theta)r}{\theta})$ is well defined for all r, so $\frac{p-(1-\theta)r}{\theta} \le h^{-1}(p-r)$ can be equivalently written as $p - r - h\Big(\frac{p-(1-\theta)r}{\theta}\Big) \ge 0$. Taking the first derivative w.r.t r on the LHS, we get $-1 + \frac{(1-\theta)}{\theta}h'(\frac{p-(1-\theta)r}{\theta}) \le -1 + \frac{(1-\theta)}{\theta} \cdot \frac{\theta}{(1-\theta)} = 0$ since $h'(\cdot) \le \frac{\theta}{1-\theta}$. Thus there exists a unique $\bar{r}$ satisfying $\bar{r} = p - h\Big(\dfrac{p-(1-\theta)\bar{r}}{\theta}\Big)$ such that $\frac{p-(1-\theta)r}{\theta} \le h^{-1}(p-r)$ and there are three segments iff $r \leq \bar{r}$. When $p > \theta\bar{v}$, $h(\frac{p-(1-\theta)r}{\theta})$ is not well defined for some r. To resolve this issue, we can define $\bar{r}$ as the unique solution to $p - r - h(\min\{\frac{p-(1-\theta)r}{\theta}, \bar{v}\}) = 0$. Note that the LHS is equal to $p - \bar{r} - h(\bar{v})$ for $r < \frac{p-\theta\bar{v}}{1-\theta}$ and $p - r - h\left(\frac{p-(1-\theta)r}{\theta}\right)$ otherwise. Both decrease in r, and at $r = \frac{p-\theta\bar{v}}{1-\theta}$, the LHS is equal to $p - \frac{p-\theta\bar{v}}{1-\theta} - h(\bar{v}) = \frac{-(1-\theta)h(\bar{v})+\theta(\bar{v}-p)}{1-\theta} \geq 0$ based on the assumption of Proposition 1. Therefore, the r that makes the LHS equal to 0 must be greater than $\frac{p-\theta\bar{v}}{1-\theta}$, under which the LHS $= p - r - h(\frac{p-(1-\theta)r}{\theta})$. Thus there is a unique solution $\bar{r}$, and at $\bar{r}$, we have $p - \bar{r} - h\left(\frac{p-(1-\theta)\bar{r}}{\theta}\right) = 0$. Having characterized the condition that distinguishes case (ii) and (iii), we now provide the details of the two cases.

Case (ii): If $\underline{r} < r \leq \bar{r}$, then $\frac{p-(1-\theta)r}{\theta} \leq h^{-1}(p-r)$. Since $EU_e(v) = EU_l(v)$ at $v = h^{-1}(p-r)$, and $EU_l(v)$ increases faster, we must have $EU_e(v) \geq EU_l(v)$ for $v \leq h^{-1}(p-r)$. Also since $EU_e(v)$ is increasing and $EU_e(z_e) = 0$, we have $z_e \leq h^{-1}(p-r)$ and $EU_e(v) \geq 0$ for $v \in [z_e, h^{-1}(p-r)]$. Therefore, assuming that consumers choose to buy and return late whenever being indifferent, consumers whose valuations are in the range of $v \in [z_e, h^{-1}(p-r)]$ buy and return a misfit product early. Similarly, since $EU_e(v) \geq 0$ for $v \geq z_e$ and $h^{-1}(p-r) \geq z_e$, we have $EU_l(v) \geq EU_e(v) \geq 0$ for $v \in [h^{-1}(p-r), \bar{v}]$. Therefore, consumers whose valuations are in the range $v \in [h^{-1}(p-r), \bar{v}]$ buy and return a misfit product late. The rest of the consumers do not buy. In summary, the fraction of consumers who buy and return a misfit product early is given by $v_e(r) = \frac{1}{\bar{v}}\left(h^{-1}(p-r) - z_e\right)$, and the fraction of consumers who buy and return a misfit product by $v_l(r) = \frac{1}{\bar{v}}\left(\bar{v} - h^{-1}(p-r)\right)$.

Case (iii): If $r > \bar{r}$, then $\frac{p-(1-\theta)r}{\theta} > h^{-1}(p-r)$. For $v \in [\frac{p-(1-\theta)r}{\theta}, \bar{v}]$, $EU_l(v) \geq 0$ and $EU_l(v) \geq EU_e(v)$. For $v \in [0, \frac{p-(1-\theta)r}{\theta})$, $EU_e(v) \leq 0$ and $EU_l(v) \leq 0$. So consumers whose valuations are in the range $v \in [\frac{p-(1-\theta)r}{\theta}, \bar{v}]$ buy and return a misfit product late if $EU_l(v) \geq 0$, and the rest do not buy. Hence, $v_e(r) = 0$ and $v_l(r) = \frac{1}{\bar{v}}\left(\bar{v} - \frac{p-(1-\theta)r}{\theta}\right)^+$.

In summary, the three possible market segmentation outcomes are as follows:

Case (i): If $r \leq \underline{r}$, then $v_e(r) = \frac{1}{\bar{v}}\left(\bar{v} - z_e\right)$, $v_l(r) = 0$;

Case (ii): If $\underline{r} < r \leq \bar{r}$, then $v_e(r) = \frac{1}{\bar{v}}\left(h^{-1}(p-r) - z_e\right)$, $v_l(r) = \frac{1}{\bar{v}}\left(\bar{v} - h^{-1}(p-r)\right)$;

Case (iii): If $r > \bar{r}$, then $v_e(r) = 0$, $v_l(r) = \frac{1}{\bar{v}}\left(\bar{v} - \frac{p-(1-\theta)r}{\theta}\right)^+$.

Note that the resulting market segmentation is continuous in r.

Lastly, we can show that $\underline{r} \leq \bar{r} \leq p$. At $r = \underline{r} = p - h(\bar{v})$ we have $EU_e(\bar{v}) = EU_l(\bar{v})$, and since we assume $EU_e(\bar{v}) \geq 0$, it follows that $EU_l(\bar{v}) \geq 0$. Because $EU_l(v)$ is increasing in v and $EU_l\left(\frac{p-(1-\theta)r}{\theta}\right) = 0$, we must have $\frac{p-(1-\theta)r}{\theta} \leq \bar{v}$. At $r = p - h(\bar{v})$, $p - r - h\left(\frac{p-(1-\theta)r}{\theta}\right) = h(\bar{v}) - h\left(\frac{p-(1-\theta)r}{\theta}\right)$, which is positive because $h(\cdot)$ is increasing and $\frac{p-(1-\theta)r}{\theta} \leq \bar{v}$. And since we show that it is decreasing in r and equals 0 at $\bar{r}$, we have $p - h(\bar{v}) \leq \bar{r}$. Regarding $\bar{r} \leq p$, we have shown that $p - r - h\left(\frac{p-(1-\theta)r}{\theta}\right)$ is decreasing in r. At $r = p$, $p - p - h\left(\frac{p-(1-\theta)p}{\theta}\right) < 0$ holds. Again, since $p - r - h\left(\frac{p-(1-\theta)r}{\theta}\right)$ decreases in r, and at $r = \bar{r}$, $p - r - h\left(\frac{p-(1-\theta)r}{\theta}\right) = 0$, it follows that $\bar{r} \leq p$.

Part (b): As r increases, we transition from Case (i) to (ii) to (iii). In Case (i), $v_e(r) = \frac{1}{\bar{v}}\big(\bar{v} - z_e\big)$, and as z_e is independent of r, $v_e(r)$ is independent of r. Further, as $v_l(r) = 0$, it is also independent of r. In Case (ii), $v_e(r) = \frac{1}{\bar{v}}\big(h^{-1}(p-r) - z_e\big)$, $v_l(r) = \frac{1}{\bar{v}}\big(\bar{v} - h^{-1}(p-r)\big)$. Since $h(\cdot)$ is non-decreasing, $h^{-1}(\cdot)$ is also non-decreasing, therefore $v_e(r)$ is non-increasing in r and $v_l(r)$ is non-decreasing in r. In this case, total purchases, $v_e(r) + v_l(r) = 1 - \frac{z_e}{\bar{v}}$ as well as total returns, which is a fraction $1-\theta$ of total sales, are also independent of r. In Case (iii), $v_e(r) = 0$, $v_l(r) = \frac{1}{\bar{v}}\big(\bar{v} - \frac{p-(1-\theta)r}{\theta}\big)^+$ which is increasing in r. Therefore, total purchases and total returns in this case are also increasing in r. ■

Proof of Remark 1. (a) If $p - h(\bar{v}) < 0$, $\underline{r} < 0$ so case (i) disappears.

If $h(\bar{v}) \leq \theta(\bar{v} - p)/(1-\theta)$, then $-p + \theta\bar{v} + (1-\theta)(p - h(\bar{v})) \geq 0$. Since $p - h(\bar{v}) < 0$, we must have $-p + \theta\bar{v} \geq 0$, thus $\frac{p-(1-\theta)r}{\theta} \leq \bar{v}$ for any r. So $p - r - h(\min(\frac{p-(1-\theta)r}{\theta}), \bar{v}) = p - r - h(\frac{p-(1-\theta)r}{\theta})$. There are two scenarios depending on the comparison between p and $h(\frac{p}{\theta})$. If $p < h(\frac{p}{\theta})$, then at $r = 0$, $p - r - h(\frac{p-(1-\theta)r}{\theta}) = p - h(\frac{p}{\theta}) < 0$. As we have shown that $p - r - h(\frac{p-(1-\theta)r}{\theta})$ decreases in r, only a negative r can make it equal to zero, thus $\bar{r} < 0$ and only case (iii) exists. Otherwise, $\bar{r} \geq 0$ so both cases (ii) and (iii) exist. Note that in this case, z_e is still the solution to $-p + \theta v + (1-\theta)(p - h(v)) = 0$, since $z_e < \frac{p-(1-\theta)r}{\theta} \leq \frac{p}{\theta}$, we have $h(z_e) < h(\frac{p}{\theta}) < p$, so at $v = z_e$, $EU_e(v) = -p + \theta v + (1-\theta)(p - h(v))^+ = -p + \theta v + (1-\theta)(p - h(v))$.

(b) If $h(\bar{v}) > \theta(\bar{v} - p)/(1-\theta)$, then $-p + \theta\bar{v} + (1-\theta)(p - h(\bar{v})) < 0$. For any $v \in [0, \bar{v}]$, there are two possibilities. First, suppose $h(v) \leq p$, then $EU_e(v) = -p + \theta v + (1-\theta)(p - h(v)) \leq EU_e(\bar{v}) < 0$. Second, suppose $h(v) > p$, then $(p - h(v))^+ = 0$, and thus $EU_e(v) = -p + \theta v < -p + \theta v + (1-\theta)r = EU_l(v)$. Therefore, for any $v \in [0, \bar{v}]$, returning early is always dominated. Therefore, we only need to compare EU_l and 0, consistent with Case (iii) in Proposition 1. ■

Note: For ease of exposition, beginning with the proof of Proposition 2, we suppress the dependence on r and write $v_e(r)$ and $v_l(r)$ as v_e and v_l, respectively.

Proof of Proposition 2. We will prove the case where $\bar{r} \geq \underline{r} > 0$ since the others are degenerative cases. As v_e and v_l are continuous, π is also continuous. We first show that the profit function π (See Equation (1)) is piecewise concave. It is constant in Case (i). In Case (ii):

$$\frac{\partial^2 \pi}{\partial r^2} = (1-\theta)\frac{\partial h^{-1}(p-r)}{\partial r}\frac{d}{\bar{v}} - (1-\theta)\left(-\frac{\partial h^{-1}(p-r)}{\partial r}\right)\frac{d}{\bar{v}} + (p-r)(1-\theta)(-1)\frac{\partial^2 h^{-1}(p-r)}{\partial r^2}\frac{d}{\bar{v}}.$$

Note that $\frac{\partial^2 h^{-1}(p-r)}{\partial r^2} = -\frac{h''\left(h^{-1}(p-r)\right)}{\left(h'\left(h^{-1}(p-r)\right)\right)^3} d \geq 0$ if h is concave. Thus, $\frac{\partial^2 \pi}{\partial r^2} \leq 0$.

In Case (iii), $\frac{\partial^2 \pi^2}{\partial r^2} = \frac{(1-\theta)(-2(1-\theta))}{\theta\bar{v}} d < 0$ so it is also concave.

Furthermore, we can show that if $\frac{h(v)}{v}$ is decreasing, and $h''(v) \leq 0$, for all v, then π is unimodal in [0, p] and thus the tiered-refund policy has a unique optimal r^*_{NR} . Denote the profit function in Case (ii) by π^B and the profit function in Case (iii) by π^C. Since the profit function is constant in Case (i) and piece-wise concave in Case (ii) and (iii), a sufficient condition for a unique optimal is $\left(\frac{\partial\pi^B}{\partial r}-\frac{\partial\pi^C}{\partial r}\right)\Big|_{r=\bar{r}}\geq 0$. For Case (ii), $\pi^B=\Big((\theta p-c+(1-\theta)s)(\bar{v}-z_e)+(p-r)(1-\theta)(\bar{v}-h^{-1}(p-r))\Big)\frac{d}{\bar{v}}$ and $\frac{\partial\pi^B}{\partial r}=\Big(0-(1-\theta)(\bar{v}-h^{-1}(p-r))+(1-\theta)(p-r)\frac{1}{h'(h^{-1}(p-r))}\Big)\frac{d}{\bar{v}}$. For Case (iii), $\pi^C=\Big((\theta p-c+(1-\theta)s)(\bar{v}-\frac{p-(1-\theta)r}{\theta})+(p-r)(1-\theta)(\bar{v}-\frac{p-(1-\theta)r}{\theta})\Big)\frac{d}{\bar{v}}$ and $\frac{\partial\pi^C}{\partial r}=\Big((\theta p-c+(1-\theta)s)\frac{1-\theta}{\theta}-(1-\theta)\big(\bar{v}-\frac{p-(1-\theta)r}{\theta}\big)+(1-\theta)(p-r)\frac{1-\theta}{\theta}\Big)\frac{d}{\bar{v}}$. At $r=\bar{r}$, $r=p-h\Big(\frac{p-(1-\theta)r}{\theta}\Big)\iff p-r=h\Big(\frac{p-(1-\theta)r}{\theta}\Big)\iff h^{-1}(p-r)=\frac{p-(1-\theta)r}{\theta}$. Therefore, $\left(\frac{\partial\pi^B}{\partial r}-\frac{\partial\pi^C}{\partial r}\right)\Big|_{r=\bar{r}}=\Big((1-\theta)(p-r)\frac{1}{h'(h^{-1}(p-r))}-(\theta p-c+(1-\theta)s)\frac{1-\theta}{\theta}-(p-r)(1-\theta)\frac{1-\theta}{\theta}\Big)\frac{d}{\bar{v}}=\Bigg((1-\theta)\frac{h\big(\frac{p-(1-\theta)r}{\theta}\big)}{h'\big(\frac{p-(1-\theta)r}{\theta}\big)}-\frac{1-\theta}{\theta}\big(p-c+(1-\theta)s-(1-\theta)r\big)\Bigg)\frac{d}{\bar{v}}=\Bigg((1-\theta)\frac{h\big(\frac{p-(1-\theta)r}{\theta}\big)}{h'\big(\frac{p-(1-\theta)r}{\theta}\big)}-\frac{1-\theta}{\theta}\big(p-(1-\theta)r\big)+\frac{1-\theta}{\theta}(c-(1-\theta)s)\Bigg)\frac{d}{\bar{v}}\geq(1-\theta)\Bigg(\frac{h\big(\frac{p-(1-\theta)r}{\theta}\big)}{h'\big(\frac{p-(1-\theta)r}{\theta}\big)}-\frac{p-(1-\theta)r}{\theta}\Bigg)\frac{d}{\bar{v}}$. Since $\frac{h(v)}{v}$ is decreasing, the derivative $\frac{vh'(v)-h(v)}{v^2}\leq 0$. Since $h'(v)>0$ we have $\frac{h(v)}{h'(v)}-v\geq 0$. Thus $\frac{h\big(\frac{p-(1-\theta)r}{\theta}\big)}{h'\big(\frac{p-(1-\theta)r}{\theta}\big)}-\frac{p-(1-\theta)r}{\theta}\geq 0$, and therefore $\left(\frac{\partial\pi^B}{\partial r}-\frac{\partial\pi^C}{\partial r}\right)\Big|_{r=\bar{r}}\geq 0$. Thus $\pi(r)$ is unimodal with a unique r^*_{NR}. ■

Proof of Proposition 3. The short-return-window policy (Case (i)) only exists when $\underline{r}>0$, i.e., when $p>h(\bar{v})$, thus we compare the short-return-window policy with the tiered-refund policy under this condition. We know that Case (i) profit is a constant and is effectively equivalent to the short-return-window policy and Case (ii) is the tiered-refund policy, to show the tiered-refund policy dominates, it is sufficient to show $\frac{\partial\pi^B}{\partial r}>0$ at the boundary $\underline{r}=p-h(\bar{v})$. Note that at $r=\underline{r}=p-h(\bar{v})$, $h(\bar{v})=p-r$, and $\bar{v}=h^{-1}(p-r)$. Hence, since $h'(\cdot)>0$ and $r<p$, we have

$$\frac{\partial\pi^B}{\partial r}\bigg|_{r=p-h(\bar{v})}=-(1-\theta)\Big(\bar{v}-h^{-1}(p-r)\Big)\frac{d}{\bar{v}}+(p-r)(1-\theta)\left(\frac{1}{h'(h^{-1}(p-r))}\right)\frac{1}{\bar{v}}\bigg|_{r=p-h(\bar{v})}$$
$$=0+(p-r)(1-\theta)\left(\frac{1}{h'(h^{-1}(p-r))}\right)\frac{d}{\bar{v}}>0,$$

In Case (iii), $\pi^C=\big(p-c-(1-\theta)(r-s)\big)v_ld=\big(p-c-(1-\theta)r+(1-\theta)s\big)\Big(\bar{v}-\frac{p-(1-\theta)r}{\theta}\Big)^+\frac{d}{\bar{v}}$, At $r=p$, $\bar{v}-\frac{p-(1-\theta)r}{\theta}>0$, $\frac{\partial\pi^C}{\partial r}=\frac{(1-\theta)d}{\theta\bar{v}}\big(2p-c+(1-\theta)s-\theta\bar{v}-2(1-\theta)r\big)$, and $\frac{\partial\pi^C}{\partial r}\Big|_{r=p}=\frac{(1-\theta)d}{\theta\bar{v}}\big(2\theta p-c+(1-\theta)s-\theta\bar{v}\big)$. Since $r=p$ indicates a full-refund policy, to show that the tiered-refund policy dominates, it is sufficient to show $\frac{\partial\pi^C}{\partial r}\leq 0$ at $r=p$. Thus, if $p\leq\frac{\theta\bar{v}+c-(1-\theta)s}{2\theta}$, or equivalently, $\bar{v}\geq\frac{2\theta p-c+(1-\theta)s}{\theta}$, we have $\frac{\partial\pi^C}{\partial r}\Big|_{r=p}\leq 0$, and the tiered-refund policy outperforms the full-refund policy. ■

Proof of Proposition 4. Assuming $h=a+bv$, we calculate the z_e, $\underline{r}$, $h^{-1}(p-r)$ and $\bar{r}$ in Proposition 1 to obtain: $z_e=\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}$, $\underline{r}=p-b\bar{v}-a$, $h^{-1}(p-r)=\frac{p-r-a}{b}$, and $\bar{r}=\frac{\theta p-\theta a-bp}{\theta-(1-\theta)b}$.

The condition of $h'(v)\leq\frac{\theta}{1-\theta}$ becomes $\theta-(1-\theta)b\geq 0$, and Proposition 1 becomes:

- Case (i): If $r<p-b\bar{v}-a$, then $v_e=\big(\bar{v}-\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}\big)\frac{d}{\bar{v}}$, $v_l=0$;
- Case (ii): If $p-b\bar{v}-a\leq r\leq\frac{(\theta-b)p-\theta a}{\theta-(1-\theta)b}$, then $v_e=\frac{1}{\bar{v}}\big(\frac{p-r-a}{b}-\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}\big)$, $v_l=\frac{1}{\bar{v}}\big(\bar{v}-\frac{p-r-a}{b}\big)$. The profit function is given by $\pi^B=(\theta p-c+(1-\theta)s)\big(\bar{v}-\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}\big)\frac{d}{\bar{v}}+(p-r)(1-\theta)\big(\bar{v}-\frac{p-r-a}{b}\big)\frac{d}{\bar{v}}$, $\frac{\partial\pi^B}{\partial r}=(1-\theta)\frac{d}{\bar{v}}\left(-\bar{v}+\frac{p-r-a}{b}+\frac{p-r}{b}\right)$, and the optimal solution is $r^*_{NR}=(p-\frac{b\bar{v}+a}{2})$.

- Case (iii): If $r > \frac{(\theta-b)p-\theta a}{\theta-(1-\theta)b}$, then $v_e = 0$, $v_l = \left(\bar{v} - \frac{p-(1-\theta)r}{\theta}\right)^+ \frac{d}{\bar{v}}$. The profit function is given by $\pi^C = (p-c+(1-\theta)(s-r))\left(\bar{v} - \frac{p-(1-\theta)r}{\theta}\right)^+ \frac{d}{\bar{v}}$. When the positive sign does not take effect, $\frac{\partial \pi^C}{\partial r} = (1-\theta)\frac{d}{\bar{v}}\left(-\bar{v} + \frac{p-(1-\theta)r}{\theta}\right) + \frac{d}{\bar{v}}\left(p-c+(1-\theta)(s-r)\frac{1-\theta}{\theta}\right)$, and the optimal solution is $r^*_{NR} = \frac{(2p-\theta\bar{v}-c+(1-\theta)s)}{2(1-\theta)}$.

By Proposition 3, the short-return-window policy, i.e., Case (i), is always dominated by tiered-refund policy. In addition, full refund, i.e., $r = p$, is optimal if $\bar{v} < \frac{2\theta p - c + (1-\theta)s}{\theta}$ (or equivalently, $p > \frac{\bar{v}}{2} + \frac{c-(1-\theta)s}{2\theta}$). Denote $\bar{v}^* = 2p - c$ and $\theta_3 = \frac{c-s}{2p-s-\bar{v}}$, then the condition is equivalent to $\bar{v} < \bar{v}^*$ and $\theta > \theta_3$. When θ and $\bar{v}$ are outside this region, tiered-refund policy is optimal, and we further characterize the optimal solutions below.

Since $h''(v) = 0$ and $\frac{h(v)}{v} = \frac{a+bv}{v} = \frac{a}{v} + b$ is decreasing in v, by Proposition 2, the profit function is unimodal. Therefore, assuming that Case (ii) exists (we will discuss the conditions for that at the end of the proof), there are only three possibilities: (1) π^B is decreasing at $\bar{r}$, which is ensured by $p < (1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - (1 + \frac{1-\theta}{\theta}b)\frac{a}{2b}$ then case (ii) is optimal. We refer to this outcome as Tiered-EL outcome; (2) If π^B is increasing at $\bar{r}$, and π^C is decreasing at $\bar{r}$, which is ensured by ensured by $(1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - (1 + \frac{1-\theta}{\theta}b)\frac{a}{2b} \le p < (1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1 - \frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta}$, then $\bar{r}$ is optimal. We refer to this outcome as Tiered-Indifferent outcome; (3) Otherwise, case (iii) is optimal. We refer to this as Tiered-L outcome.

Comparing the two thresholds above, we have $(1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - (1 + \frac{1-\theta}{\theta}b)\frac{a}{2b} - \left((1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (\theta - (1-\theta)b)\frac{c-(1-\theta)s}{2\theta^2}\right) \le \frac{a}{2}(\frac{1-\theta}{\theta} - \frac{1}{b}) - (1 - \frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta} \le 0$ (by the assumption $\theta - (1-\theta)b \ge 0$). Comparing the second threshold with $\frac{\bar{v}}{2} + \frac{c-(1-\theta)s}{2\theta}$, we have $(1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1 - \frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta} - (\frac{\bar{v}}{2} + \frac{c-(1-\theta)s}{2\theta}) = -\frac{1-\theta}{\theta}b\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a - \frac{(1-\theta)}{\theta}b\frac{c-(1-\theta)s}{2\theta} \le 0$.

In summary, when Policy T is optimal, the optimal outcome is as follows:

1. Tiered-EL outcome: If $p < (1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - (1 + \frac{1-\theta}{\theta}b)\frac{a}{2b}$, (i.e., if $\bar{v} > \frac{2\theta bp + \left(\theta + (1-\theta)b\right)a}{\left(\theta - (1-\theta)b\right)b}$), then Case (ii) is optimal, with $r^*_{NR} = \left(p - \frac{b\bar{v}+a}{2}\right)$;
2. Tiered-Indifferent outcome: If $(1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - (1 + \frac{1-\theta}{\theta}b)\frac{a}{2b} \le p < (1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1 - \frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta}$, (i.e., if $\frac{2\theta p + 2(1-\theta)a}{\theta-(1-\theta)b} - \frac{1}{\theta}(c - (1-\theta)s) < \bar{v} \le \frac{2\theta bp + \left(\theta + (1-\theta)b\right)a}{\left(\theta - (1-\theta)b\right)b}$), the retailer is indifferent between Case (ii) and Case (iii), $r^*_{NR} = \frac{(\theta-b)p-\theta a}{\theta-(1-\theta)b}$;
3. Tiered-L outcome: If $(1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1 - \frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta} \le p \le \frac{\bar{v}}{2} + \frac{c-(1-\theta)s}{2\theta}$, (i.e., if $2p - \frac{c-(1-\theta)s}{\theta} \le \bar{v} \le \frac{2\theta p + 2(1-\theta)a}{\theta-(1-\theta)b} - \frac{1}{\theta}(c - (1-\theta)s)$, Case (iii) is optimal, with $r^*_{NR} = \frac{(2p-\theta\bar{v}-c+(1-\theta)s)}{2(1-\theta)}$;

We can further convert these results into conditions based on θ:

- Note that the condition of Tiered-EL outcome, $p < (1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - (1 + \frac{1-\theta}{\theta}b)\frac{a}{2b}$, is equivalent to $\frac{1-\theta}{\theta} < \frac{b\bar{v}-2bp-a}{(a+b\bar{v})b}$. When $\bar{v} \le 2p + \frac{a}{b}$, $\frac{b\bar{v}-2bp-a}{(a+b\bar{v})b} \le 0$, so this condition is never satisfied and Tiered-EL outcome is never optimal. When $\bar{v} > 2p + \frac{a}{b} \triangleq \bar{v}^{***}$, the condition is equivalent to $\theta > \frac{(a+b\bar{v})b}{(b\bar{v}-2bp-a)+(a+b\bar{v})b} \triangleq \theta_1$, which is decreasing in $\bar{v}$. For $\bar{v} \le \bar{v}^{***}$, define $\theta_1 = 1$. Therefore, Tiered-EL outcome is optimal if $\bar{v} \ge \bar{v}^{***}$ and $\theta > \theta_1$.
- The left boundary of Tiered-Indifferent outcome is thus $\theta \le \theta_1$. The right boundary of Tiered-Indifferent outcome, $p < (1 - \frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1 - \frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta}$, is equivalent to $b(c-s)(\frac{1-\theta}{\theta})^2 +$

$[b(\bar{v}+c)-(c-s)+2a]\frac{1-\theta}{\theta}+2p-c-\bar{v}<0$. Denote the LHS as $H(\frac{1-\theta}{\theta})$, which is a quadratic function with a positive coefficient in the squared term so it is negative between two roots. By the assumption in Proposition 1, $\frac{1-\theta}{\theta}\in[0,\frac{1}{b}]$.

— If $\bar{v}\geq\bar{v}^*=2p-c$, then we know that at $\theta=1$, i.e., at $\frac{1-\theta}{\theta}=0$, $H(0)=2p-c-\bar{v}\leq 0$. Therefore there is at most one root (the larger one) between $[0,\frac{1}{b}]$, which can be solved as $\frac{-[b(\bar{v}+c)-(c-s)+2a]+\sqrt{[b(\bar{v}+c)-(c-s)+2a]^2-4b(c-s)(2p-c-\bar{v})}}{2b(c-s)}$. If this root is larger than $\frac{1}{b}$ then the condition holds for all θ. $\frac{1-\theta}{\theta}<\frac{-[b(\bar{v}+c)-(c-s)+2a]+\sqrt{[b(\bar{v}+c)-(c-s)+2a]^2-4b(c-s)(2p-c-\bar{v})}}{2b(c-s)}$ can be written as $\theta>\frac{2b(c-s)}{2b(c-s)-[b(\bar{v}+c)-(c-s)+2a]+\sqrt{[b(\bar{v}+c)-(c-s)+2a]^2-4b(c-s)(2p-c-\bar{v})}}$, and $\frac{1-\theta}{\theta}\leq\frac{1}{b}$ can be written as $\theta\geq\frac{b}{1+b}$. Denote
$\theta_2\triangleq\max\left[\frac{b}{1+b},\frac{2b(c-s)}{2b(c-s)-[b(\bar{v}+c)-(c-s)+2a]+\sqrt{[b(\bar{v}+c)-(c-s)+2a]^2-4b(c-s)(2p-c-\bar{v})}}\right]$,
then the right condition of Tiered-Indifferent outcome is satisfied if $\theta>\theta_2$. We can show that θ_2 decreases in $\bar{v}$. Since $\frac{1-\theta_2}{\theta_2}$ is the larger root, we have $\frac{\partial H}{\partial(\frac{1-\theta}{\theta})}|_{\theta=\theta_2}\geq 0$. On the other hand, $\frac{\partial H}{\partial\bar{v}}=b\frac{1-\theta}{\theta}-1\leq 0$. Therefore $\frac{\partial(\frac{1-\theta_2}{\theta_2})}{\partial\bar{v}}=-\frac{\frac{\partial H}{\partial\bar{v}}}{\frac{\partial H}{\partial(\frac{1-\theta}{\theta})}}|_{\theta=\theta_2}\geq 0$. Therefore $\frac{1-\theta_2}{\theta_2}$ increases in $\bar{v}$ and thus θ_2 decreases in $\bar{v}$. We can also show that $\theta_2\leq\theta_1$ as follows. As we know that if $p<(1-\frac{1-\theta}{\theta}b)\frac{\bar{v}}{2}-(1+\frac{1-\theta}{\theta}b)\frac{a}{2b}$ then $p<(1-\frac{1-\theta}{\theta}b)\frac{\bar{v}}{2}-\frac{1-\theta}{\theta}a+(1-\frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta}$, it is equivalent to claim that if $\theta>\theta_1$ then $\theta>\theta_2$. Therefore we must have $\theta_2\leq\theta_1$. In summary, in this case, Tiered-Indifferent outcome is optimal if $\bar{v}\geq\bar{v}^*$ and $\theta_2<\theta\leq\theta_1$.

— If $\bar{v}<\bar{v}^*=2p-c$ (which is also less than $\bar{v}^{***}=2p+\frac{a}{b}$), we know that Tiered-EL outcome is never optimal. Recall that the right boundary of Tiered-Indifferent outcome is equivalent to $H(\frac{1-\theta}{\theta})\leq 0$. Denote $\Delta\triangleq[b(\bar{v}+c)-(c-s)+2a]^2-4b(c-s)(2p-c-\bar{v})=b^2(\bar{v}+c)^2-2b(\bar{v}+c)[(c-s)-2a]+[(c-s)-2a]^2-4b(c-s)*2p+4b(c-s)(c+\bar{v})=b^2(\bar{v}+c)^2+[2b(c-s)+4ab](\bar{v}+c)+[(c-s)-2a]^2-4b(c-s)*2p$. For any $\bar{v}+c\geq 0$, $\frac{\partial\Delta}{\partial(\bar{v}+c)}=2b^2(\bar{v}+c)+2b(c-s)+4ab\geq 0$, thus there exists a $\bar{v}^{**}\geq 0$, such that $\Delta<0$ if $\bar{v}<\bar{v}^{**}$ and $\Delta\geq 0$ otherwise.

  * If $\frac{c-s-2a}{b}-c\leq 2p-c$, we can further show that $\frac{c-s-2a}{b}-c<\bar{v}^{**}$, since at $\bar{v}=\frac{c-s-2a}{b}-c$, $\Delta=0^2-4b(c-s)(2p-c-\frac{c-s-2a}{b}+c)<0$. As Δ is increasing in $\bar{v}$, this implies that $\frac{c-s-2a}{b}-c<\bar{v}^{**}$. Then for $\bar{v}\leq\frac{c-s-2a}{b}-c$, we have $\bar{v}\leq\bar{v}^{**}$, thus $\Delta<0$, which means that H does not have roots and is always positive, so the right boundary of Tiered-Indifferent is never satisfied and case (ii) does not exist. For $\frac{c-s-2a}{b}-c\leq\bar{v}<2p-c$, at $\frac{1-\theta}{\theta}=0$, We have $H(0)=2p-c-\bar{v}>0$ and $H'(0)=b(\bar{v}+c)-(c-s)+2a\geq 0$ since $\bar{v}\geq\frac{c-s-2a}{b}-c$. Therefore H is positive for all $\frac{1-\theta}{\theta}\in[0,\frac{1}{b}]$, so Tiered-Indifferent outcome is never optimal in this region. In summary, if $\frac{c-s-2a}{b}-c\leq 2p-c$, Tiered-Indifferent outcome is never optimal for $\bar{v}<\bar{v}^*=2p-c$. If we redefine $\bar{v}^{**}=\bar{v}^*$ then we can say that Tiered-Indifferent outcome is optimal if $\bar{v}^{**}<\bar{v}<\bar{v}^*$ and if any condition on θ holds.

* If $\frac{c-s-2a}{b}-c > 2p-c$, for $\bar{v} \le \bar{v}^{**}$, $\Delta < 0$ and thus case (ii) does not exist. For $\bar{v}^{**} < \bar{v} < 2p-c < \frac{c-s-2a}{b}-c$, at $\frac{1-\theta}{\theta}=0$, We have $H(0) = 2p-c-\bar{v} > 0$ and $H'(0) = b(\bar{v}+c)-(c-s)+2a < 0$ and $\Delta \ge 0$. So there can be two positive roots and the condition is satisfied when $\frac{1-\theta}{\theta}$ is in between, i.e., $\frac{-[b(\bar{v}+c)-(c-s)+2a]-\sqrt{[b(\bar{v}+c)-(c-s)+2a]^2-4b(c-s)(2p-c-\bar{v})}}{2b(c-s)} \le \frac{1-\theta}{\theta} \le \frac{-[b(\bar{v}+c)-(c-s)+2a]+\sqrt{[b(\bar{v}+c)-(c-s)+2a]^2-4b(c-s)(2p-c-\bar{v})}}{2b(c-s)}$. If the lower root is larger than $\frac{1}{b}$, then Tiered-Indifferent outcome is never optimal. Define θ_2 the same way as before, and denote
Denote $\tilde{\theta}_2 \triangleq \max\left\{\frac{b}{1+b}, \frac{2b(c-s)}{2b(c-s)-[b(\bar{v}+c)-(c-s)+2a]-\sqrt{[b(\bar{v}+c)-(c-s)+2a]^2-4b(c-s)(2p-c-\bar{v})}}\right\}$,
then the right boundary of Tiered-Indifferent outcome is satisfied if $\theta_2 < \theta \le \tilde{\theta}_2$. We can also show that $\tilde{\theta}_2$ increases in $\bar{v}$. Since $\frac{1-\tilde{\theta}_2}{\tilde{\theta}_2}$ is the smaller root, we have $\frac{\partial H}{\partial(\frac{1-\theta}{\theta})}|_{\theta=\tilde{\theta}_2} \le 0$. As before, $\frac{\partial H}{\partial \bar{v}} = b\frac{1-\theta}{\theta} - 1 \le 0$. Therefore $\frac{\partial(\frac{1-\theta_2}{\theta_2})}{\partial \bar{v}} = -\frac{\frac{\partial H}{\partial \bar{v}}}{\frac{\partial H}{\partial(\frac{1-\theta}{\theta})}}|_{\theta=\tilde{\theta}_2} \le 0$. So $\frac{1-\tilde{\theta}_2}{\tilde{\theta}_2}$ decreases in $\bar{v}$ and $\tilde{\theta}_2$ increases in $\bar{v}$. In summary, if $\frac{c-s-2a}{b}-c > 2p-c$, then Tiered-Indifferent outcome is optimal if $\bar{v}^{**} \le v < \bar{v}^{*}$ and $\theta_2 < \theta \le \tilde{\theta}_2$.

- Next, for Tiered-L outcome:
  - If $\bar{v} \ge \bar{v}^* = 2p-c$: The left boundary of Tiered-L outcome is thus $\theta \le \theta_2$. The right boundary of Tiered-L outcome, $p \le \frac{\bar{v}}{2} + \frac{c-(1-\theta)s}{2\theta}$, is equivalent to $\frac{1-\theta}{\theta} \ge \frac{2p-c-\bar{v}}{c-s}$. If $\bar{v} \ge 2p-c$, this condition is always satisfied, and thus the condition is satisfied for any $\theta \le \theta_2$. In summary, Tiered-L outcome is optimal if $\bar{v} \ge \bar{v}^*$ and $\theta \le \theta_2$.
  - If $\bar{v} < \bar{v}^* = 2p-c$, the left boundary is:
    * If $\frac{c-s-2a}{b}-c \le 2p-c$, the left boundary of Tiered-L outcome is always satisfied as shown above. Keep in mind that we redefine $\bar{v}^{**} = \bar{v}^*$ in this case. so combining it with the condition of Tiered-Refund policy being optimal, we have that the Tiered-L outcome is optimal if $\bar{v} < \bar{v}^{**}$ and $\theta \le \theta_3$.
    * If $\frac{c-s-2a}{b}-c > 2p-c$, the left boundary is $\theta \le \theta_2$ or $\theta > \tilde{\theta}_2$ when $\bar{v}^{**} \le \bar{v} < \bar{v}^*$, or no boundary if $\bar{v} \le \bar{v}^{**}$. Since we know that if $p < (1-\frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1-\frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta}$ then $p < \frac{\bar{v}}{2} + \frac{c-(1-\theta)s}{2\theta}$, it is equivalent to claim that if $\theta_2 < \theta < \tilde{\theta}_2$ then $\theta < \theta_3$, so we must have $\tilde{\theta}_2 \le \theta_3$. In summary, combining the conditions with the condition of Policy T being optimal, we have that the Tiered-L outcome is optimal if $\bar{v} < \bar{v}^{**}$ and $\theta \le \theta_3$, or if $\bar{v}^{**} \le \bar{v} < \bar{v}^*$, and $\theta \le \theta_2$ or $\tilde{\theta}_2 < \theta \le \theta_3$.

Summarizing the findings above, we have:

- The Tiered-EL outcome is optimal if $\bar{v} \ge \bar{v}^{***}$ and $\theta > \theta_1$.
- The Tiered-Indifferent outcome is optimal if $\bar{v} \ge \bar{v}^*$ and $\theta_2 < \theta \le \theta_1$, or if $\bar{v}^{**} \le \bar{v} < \bar{v}^*$ and $\theta_2 < \theta \le \tilde{\theta}_2$.
- The Tiered-L outcome is optimal if $\bar{v} \ge \bar{v}^*$ and $\theta \le \theta_2$, or if $\bar{v} \le \bar{v}^{**}$ and $\theta \le \theta_3$, or if $\bar{v}^{**} \le \bar{v} < \bar{v}^*$ and $\theta \le \theta_2$ or $\tilde{\theta}_2 < \theta \le \theta_3$. Alternatively, this can be rewritten as the Tiered-L outcome being optimal if $\bar{v} \ge \bar{v}^{**}$ and $\theta \le \theta_2$, or if $\bar{v} < \bar{v}^{**}$ and $\theta \le \theta_3$, or if $\bar{v}^{**} \le \bar{v} < \bar{v}^*$ and $\tilde{\theta}_2 \le \theta \le \theta_3$.

In addition, for Tiered-EL and Tiered-Indifferent outcomes to be optimal, it is also necessary for Case (ii) to exist. From the proof of Proposition 1, we need (1) $p \leq \bar{v} - \frac{1-\theta}{\theta}h(\bar{v})$, and (2) $p \geq h(\bar{v})$, or $p < h(\bar{v})$ and $p \geq h(\frac{p}{\theta})$. The first condition is equivalent to $\theta \geq \frac{a+b\bar{v}}{a+b\bar{v}+\bar{v}-p}$. For the second condition, $p \geq h(\bar{v})$ is equivalent to $\bar{v} \leq \frac{p-a}{b}$. Since $\bar{v} \geq p$, when $\frac{p-a}{b} < p$, or equivalently, $(1-b)p < a$ this is not possible. On the other hand, $p \geq h(\frac{p}{\theta})$ is equivalent to $p \geq a + b\frac{p}{\theta}$. When $p < a$ then it is also not possible. Otherwise, it is equivalent to $\theta \geq \frac{bp}{p-a}$. Since $\theta \leq 1$, if $\frac{bp}{p-a} > 1$, or equivalently, $(1-b)p < a$, then it is not possible.

At $\bar{v} = \frac{p-a}{b}$, we can show that $\theta = \frac{a+b\bar{v}}{a+b\bar{v}+\bar{v}-p} = \frac{bp}{p-a}$, therefore, the three lines, $\theta = \frac{a+b\bar{v}}{a+b\bar{v}+\bar{v}-p}$, $\bar{v} = \frac{p-a}{b}$, and $\theta = \frac{bp}{p-a}$ intersect at the same point $(\bar{v} = \frac{p-a}{b}, \theta = \frac{bp}{p-a})$. Since $\theta = \frac{a+b\bar{v}}{a+b\bar{v}+\bar{v}-p}$ decreases in $\bar{v}$ and $\theta = \frac{bp}{p-a}$ is constant with respect to $\bar{v}$, we must have $\frac{bp}{p-a} \geq \frac{a+b\bar{v}}{a+b\bar{v}+\bar{v}-p}$ for $\bar{v} > \frac{p-a}{b}$. Therefore, Case (ii) may exist when $p \geq a$, and $\theta \geq \frac{a+b\bar{v}}{a+b\bar{v}+\bar{v}-p}$ if $\bar{v} \leq \frac{p-a}{b}$, $\theta \geq \frac{bp}{p-a}$ if $\bar{v} > \frac{p-a}{b}$. We can then redefine the thresholds of θ_1 and θ_2 as follows:

If $(1-b)p < a$, redefine $\theta_1 = \theta_2 = 1$; Otherwise, redefine $\theta_1 \triangleq \max\{\theta_1, \frac{a+b\bar{v}}{a+b\bar{v}+\bar{v}-p}\}$ when $\bar{v} \leq \frac{p-a}{b}$, and $\theta_1 \triangleq \max\{\theta_1, \frac{bp}{p-a}\}$ when $\bar{v} > \frac{p-a}{b}$. Similarly, redefine $\theta_2 \triangleq \max\{\theta_2, \frac{a+b\bar{v}}{a+b\bar{v}+\bar{v}-p}\}$ when $\bar{v} \leq \frac{p-a}{b}$, and $\theta_2 \triangleq \max\{\theta_2, \frac{bp}{p-a}\}$ when $\bar{v} > \frac{p-a}{b}$. Then the optimal policy structure in Proposition 4 holds. ■

Proof of Proposition 5. Since $\theta p + (1-\theta)s > c$, and demand is deterministic, the total supply (initial inventory plus restored units) matches demand. Thus the profit maximization problem is given by:

$$\begin{aligned}
\max_{I_0,x} \pi = \; & -cI_0 - c_r x + s[(1-\theta)(v_e+v_l)d - x] + \theta p(v_e+v_l)d + (p-r)(1-\theta)v_l d \\
s.t \quad & 0 \leq x \leq (1-\theta)v_e d_1 \\
& I_0 \geq (v_e+v_l)d_1 \\
& I_0 + x = (v_e+v_l)d
\end{aligned}$$

Substituting $I_0 = (v_e+v_l)d - x$ in the objective function, we have $\max_x \pi = -c((v_e+v_l)d - x) - c_r x + s[(1-\theta)(v_e+v_l)d - x] + \theta p(v_e+v_l)d + (p-r)(1-\theta)v_l d$. The first derivative with respect to x is $c - c_r - s$. Therefore, if $c - c_r - s < 0$, $x^* = 0$, and $I_0^* = (v_e+v_l)d - x^* = (v_e+v_l)d$. If $c - c_r - s \geq 0$, then x^* is the maximum x that satisfies $x \leq (1-\theta)v_e d_1$ and $I_0 = (v_e+v_l)d - x \geq (v_e+v_l)d_1$, or equivalently, $x \leq (v_e+v_l)d_2$. Therefore, $x^* = \min((1-\theta)v_e d_1, (v_e+v_l)d_2)$, and $I_0^* = (v_e+v_l)d - \min\{(v_e+v_l)d_2, (1-\theta)v_e d_1\}$. Since (v_e+v_l) is constant in r and v_e decreases in r, we have x^* decreases and I_0^* increases in r.

Notice that $I_0^* = (v_e+v_l)d - \min\{(v_e+v_l)d_2, (1-\theta)v_e d_1\}$ under the Tiered-EL outcome of Policy T, while $I_0^* = (v_e+v_l)d$ under Policy S, the Tiered-L outcome of Policy T, and Policy F. We have shown in Proposition 1 that $(v_e+v_l)d$ are identical under Policy S and the Tiered-EL outcome of Policy T, while it is increasing as the policy moves to the Tiered-L outcome of Policy T, and then to Policy F. It then follows that I_0^* is the lowest under the Tiered-EL outcome of Policy T, and increases progressively under Policy S, the tiered-L outcome of Policy T, and Policy F.

Proof of results when restoration is also feasible under the short-return-window policy: Under both Policy S and T with Tiered-EL outcome, $v_e + v_l = 1 - \frac{z_e}{\bar{v}}$, but v_e under Policy S is larger than v_e under Tiered-EL, thus x^* is weakly larger. And $x^* = 0$ for the other policies since $v_e = 0$.

We have shown that $v_e + v_l$ weakly increases in r in Proposition 1. Since we know x^* gets weakly lower from Policy S to T to F, $I_0^* = (v_e + v_l)d - x^*$ gets weakly higher from Policy S to T to F. ■

Proof of Proposition 6. When restoration is feasible, we refer to the tiered-refund policy with T-EL outcome and restoration as Policy TR-EL. All the other policies, including S, F, and T with T-L outcome, do not generate early returns and thus make restoration infeasible. As a result, their profits remain unchanged. For brevity we also refer to the Tiered-L outcome of Policy T as Policy T-L.

Note that regardless of the inventory and restoration policies, consumer segmentation still follows Proposition 1. Therefore, Policy S applies for $r \in [0, \underline{r}]$, Policy TR-EL applies for $r \in (\underline{r}, \bar{r}]$, Policy T-L applies for $r \in (\bar{r}, p)$, and Policy F is when $r = p$. The profit function is given by:

$$\pi(r) = -cI_0^* - c_r x^* + \theta p(v_e + v_l)d + s[(1-\theta)(v_e + v_l)d - x^*] + (p - r)(1-\theta)v_l d.$$

By Proposition 5, the initial inventory and restoration quantity are given by: $x^* = \min((v_e + v_l)d_2, (1-\theta)v_e d_1)$ and $I_0^* = (v_e + v_l)d - x^*$. Therefore, under Policy TR-EL, $v_e > 0$ and $v_l > 0$, and the initial inventory and restoration quantity are given by Proposition 5. If $h(v) = a + bv$, the condition $(1-\theta)v_e d_1 > (v_e + v_l)d_2$ is equivalent to $(1-\theta)[\frac{p-r-a}{b} - \frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}]\frac{1}{\bar{v}}d_1 > [\bar{v} - \frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}]\frac{1}{\bar{v}}d_2$, which can be rewritten as $r < p - a - b\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b} - b[\bar{v} - \frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}]\frac{d_2}{(1-\theta)d_1} \triangleq \hat{r}$. Therefore, if $\underline{r} \le r < \hat{r}$, $I_0^* = (v_e + v_l)d_1$, $x^* = (v_e + v_l)d_2$; if $\hat{r} \le r \le \bar{r}$, $I_0^* = (v_e + v_l)d - (1-\theta)v_e d_1$, $x^* = (1-\theta)v_e d_1$. Note that $\hat{r}$ can be lower than $\underline{r}$, making the first phase not exist. Specifically, when $(1-\theta)d_1 \le d_2$, $\hat{r} - \underline{r} = p - a - b\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b} - b[\bar{v} - \frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}]\frac{d_2}{(1-\theta)d_1} - (p - a - b\bar{v}) = b[\bar{v} - \frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}][1 - \frac{d_2}{(1-\theta)d_1}] \le 0$, so $\hat{r} \le \underline{r}$. The profit function can be written as:

If $\underline{r} \le r < \hat{r}$:

$$\begin{aligned}
\pi^{TR-EL}(r) &= -c(v_e + v_l)d_1 - c_r(v_e + v_l)d_2 + \theta p(v_e + v_l)d \\
&\quad + s[(1-\theta)(v_e + v_l)d - (v_e + v_l)d_2] + (p - r)(1-\theta)v_l d, \\
\frac{\partial \pi^{TR-EL}}{\partial r} &= (1-\theta)d\frac{\partial}{\partial r}[(p - r)(\bar{v} - \frac{p-r-a}{b})],
\end{aligned}$$

since $v_e + v_l$ is independent of r.

If $\hat{r} \le r \le \bar{r}$:

$$\begin{aligned}
\pi^{TR-EL}(r) &= -c[(v_e + v_l)d - (1-\theta)v_e d_1] - c_r(1-\theta)v_e d_1 + \theta p(v_e + v_l)d \\
&\quad + s[(1-\theta)(v_e + v_l)d - (1-\theta)v_e d_1] + (p - r)(1-\theta)v_l d, \\
\frac{\partial \pi^{TR-EL}}{\partial r} &= (c - s - c_r)(1-\theta)d_1\frac{\partial v_e}{\partial r} + (1-\theta)d\frac{\partial}{\partial r}[(p - r)(\bar{v} - \frac{p-r-a}{b})] \\
&= (c - s - c_r)(1-\theta)d_1(-\frac{1}{b}) + (1-\theta)d\frac{\partial}{\partial r}[(p - r)(\bar{v} - \frac{p-r-a}{b})] \\
&\le (1-\theta)d\frac{\partial}{\partial r}[(p - r)(\bar{v} - \frac{p-r-a}{b})],
\end{aligned}$$

since $c-s-c_r \geq 0$.

Therefore, $\pi^{TR-EL}(r)$ is quadratic in r and the first derivative decreases at $\hat{r}$, thus it is unimodal in $r \in [\underline{r}, \bar{r}]$.

Under Policy T-L, $v_e = 0$, and restoration is not feasible, so the profit is the same as that in Section 4.

$$
\begin{aligned}
\pi^{T-L} &= (\theta p - c + s(1-\theta))v_l d + (p-r)(1-\theta)v_l d \\
\frac{\partial \pi^{T-L}}{\partial r} &= (\theta p - c + s(1-\theta))\tfrac{1}{b}d + (1-\theta)d\tfrac{\partial}{\partial r}[(p-r)(\bar{v} - \tfrac{p-(1-\theta)r)}{\theta})] \\
&\geq (1-\theta)d\tfrac{\partial}{\partial r}[(p-r)(\bar{v} - \tfrac{p-(1-\theta)r)}{\theta})]
\end{aligned}
$$

Therefore at $r = \bar{r}$, when the profit function switches from π^{TR-EL} into π^{T-L}, the first derivative increases, implying that profit can be bimodal.

Now we compare the profits under different policies. We notice that the profits under Policy T-L and F are independent of c_r, thus the comparison between them also does not rely on c_r. Policy F can outperform T-L if $\frac{\partial \pi^{T-L}}{\partial r}|_{r=p} > 0$, which is equivalent to $\bar{v} < \frac{1}{\theta}(2\theta p - c + (1-\theta)s)$, same as in Proposition 4. Also, since $x = 0$ is a feasible solution and x^* is the optimal restoration, we know the profit under TR-EL is higher than that under T-EL, and we have shown in Proposition 3 that Policy S is dominated by T-EL, then Policy S is also dominated by TR-EL.

Finally, it is clear that π^{TR-EL} decreases in c_r. Since the profits under Policy F and T-L are constant in c_r, there exists a $\bar{c}_r$ such that TR-EL is optimal if $c_r \leq \bar{c}_r$, and Policy F or T-L is optimal if $c_r > \bar{c}_r$. ■

Proof of Remark 2: When restoration is feasible for Policy S, we refer to it as Policy SR. By Proposition 5, the initial inventory and restoration quantity are given by: $x^* = \min((v_e + v_l)d_2, (1-\theta)v_e d_1)$ and $I_0^* = (v_e + v_l)d - x^*$. Under Policy SR, $v_l = 0$. Therefore, if $(1-\theta)v_e d_1 > (v_e + v_l)d_2$, or equivalently, if $(1-\theta)d_1 > d_2$, $I_0^* = v_e d_1$, $x^* = v_e d_2$, and the profit is given by: $\pi^{SR}(r) = -cv_e d_1 - c_r v_e d_2 + \theta p v_e d + s[(1-\theta)v_e d - v_e d_2]$. If $(1-\theta)d_1 \leq d_2$, $I_0^* = \theta v_e d_1 + v_e d_2$, $x^* = (1-\theta)v_e d_1$, and the profit is given by: $\pi^{SR}(r) = -c(\theta v_e d_1 + v_e d_2) - c_r(1-\theta)v_e d_1 + \theta p v_e d + s(1-\theta)v_e d_2$. Similar to Policy S, the profit under Policy SR, π^{SR}, is independent of r.

To further compare the policies, we will consider two scenarios. First, if $(1-\theta)d_1 > d_2$, then $\hat{r} > \underline{r}$, so at $r = \underline{r} = p - h(\bar{v}) = p - a - b\bar{v}$, $v_l = 0$, and we have $\frac{\partial \pi^{TR-EL}}{\partial r}|_{r=\underline{r}} = (1-\theta)d\frac{\partial}{\partial r}[(p-r)(\bar{v} - \frac{p-r-a}{b})]|_{r=\underline{r}} = (1-\theta)d(a+b\bar{v})\frac{1}{b} \geq 0$, therefore Policy SR is dominated by Policy TR-EL. As Policy TR-EL profit decreases in c_r while Policy T-L or F profit is independent of c_r, there exists a threshold $\bar{c}_r$ such that Policy T-L or F is optimal if $c_r > \bar{c}_r$.

If $(1-\theta)d_1 \leq d_2$, then $\hat{r} \leq \underline{r}$, so at $r = \underline{r} = p - h(\bar{v}) = p - a - b\bar{v}$, $v_l = 0$, and we have $\frac{\partial \pi^{TR-EL}}{\partial r}|_{r=\underline{r}} = (c-s-c_r)(1-\theta)(-\frac{1}{b})d_1 + (1-\theta)d\frac{\partial}{\partial r}[(p-r)(\bar{v} - \frac{p-r-a}{b})]|_{r=\underline{r}} = (c-s-c_r)(1-\theta)(-\frac{1}{b})d_1 + (1-\theta)d(a+b\bar{v})\frac{1}{b}$. This is negative if and only if $(c-s-c_r)d_1 > (a+b\bar{v})d$, or $c_r < c - s - (a+b\bar{v})\frac{d}{d_1}$. When this condition holds, Policy SR dominates Policy TR-EL. Note that this condition is more likely to hold (i.e., Policy SR is more likely to dominate) when a, b, s, c_r are smaller or when c is larger, which is later confirmed by numerical studies. Furthermore, we can show next that SR profit decreases faster than that under the optimal TR-EL policy. Under Policy SR, $x^* = (1-\theta)v_e d_1$, and $\frac{\partial \pi^{SR}}{\partial c_r} = -(1-\theta)v_e^{SR} d_1 < 0$. Under the optimal Policy TR-EL, since $(1-\theta)d_1 \leq d_2$, we know that $\hat{r} \leq \underline{r}$, thus $\frac{\partial \pi^{TR-EL*}}{\partial c_r} = -(1-\theta)v_e(r^{TR-EL*})d_1$. Since $r^{TR-EL*} \in [\underline{r}, \bar{r}]$ and $v_e(r)$ decreases in r, we have $v_e(r^{TR-EL*}) \leq v_e(\underline{r}) = v_e^{SR}$, therefore, $\frac{\partial \pi^{SR}}{\partial c_r} \leq \frac{\partial \pi^{TR-EL*}}{\partial c_r} \leq \frac{\partial \pi^{T-L*}}{\partial c_r} = \frac{\partial \pi^{F}}{\partial c_r} = 0$. Since both Policy SR profit and Policy TR-EL profit decrease, and Policy SR profit decreases faster, while Policy T-L or F profit is a constant in c_r, there exist two thresholds $\underline{c}_r \leq \bar{c}_r$ such that Policy SR is optimal when $c_r < \underline{c_r}$, Policy TR-EL is optimal when $\underline{c}_r \leq c_r \leq \bar{c}_r$, and Policy T-L or F is optimal when $c_r > \bar{c}_r$. Note that at $\bar{c}_r$ the profit under TR-EL is the same as that under T-L or F, therefore it is the same $\bar{c}_r$ as in Proposition 6.

To summarize the findings above, we have Policy F is optimal only when $\bar{v} < \frac{1}{\theta}(2\theta p - c + (1-\theta)s)$ and $c_r > \bar{c}_r$. Policy SR is optimal only when $(1-\theta)d_1 \leq d_2$ and $c_r < \underline{c_r}$. Otherwise, Policy T (i.e., either TR-EL or T-L) is optimal. Specifically, TR-EL dominates T-L if $c_r \leq \bar{c}_r$. ∎

Proof of Proposition 7. Recall from Proposition 3 that $\underline{r} < r^*_{NR} \leq p$, and note that the profits under Policy S, F and T-L are not changed by restoration, while the profit under Policy TR-EL is higher than that under Policy Tiered-EL. Therefore, when $r^*_{NR} > \bar{r}$, r^* either stays the same or becomes smaller. When $\underline{r} < r^*_{NR} \leq \bar{r}$, r^* remains in this region, and we can show that the first derivative becomes smaller and thus r^* is smaller. From the proof of Proposition 4, we have $\frac{\partial \pi_{NR}}{\partial r} = (1-\theta)d\frac{\partial}{\partial r}[(p-r)(\bar{v} - \frac{p-r-a}{b})]$. From the proof of Proposition 6 we know that $\frac{\partial \pi^{TR-EL}}{\partial r} = (1-\theta)d\frac{\partial}{\partial r}[(p-r)(\bar{v} - \frac{p-r-a}{b})]$ if $r \in [\underline{r}, \hat{r})$ and $\frac{\partial \pi^{TR-EL}}{\partial r} = (c-s-c_r)(1-\theta)d_1(-\frac{1}{b}) + (1-\theta)d\frac{\partial}{\partial r}[(p-r)(\bar{v} - \frac{p-r-a}{b})]$ if $r \in [\hat{r}, \bar{r}]$. Therefore, we have $\frac{\partial \pi^{TR-EL}}{\partial r} \leq \frac{\partial \pi_{NR}}{\partial r}$, and thus the optimal $r^{TR-EL*} \leq r^*_{NR}$.

∎

Proof of Proposition 8.

$$\begin{aligned}
\pi_2\big(I_1, R_1, x\big) &= p\,\theta\,\mathbb{E}[S_2(x)] + s\Bigg(\underbrace{R_1 - x}_{\substack{\text{Unrestored}\\ \text{Period-1 early returns}}} + \underbrace{I_1 + x - \mathbb{E}[S_2(x)]}_{\substack{\text{Unsold}\\ \text{in Period-2}}} + \underbrace{\frac{v_e}{v_e+v_l}(1-\theta)\mathbb{E}[S_2(x)]}_{\substack{\text{Period-2 early}\\ \text{returns}}}\Bigg) \\
&\quad + (s+p-r)\Bigg(\underbrace{\frac{v_l}{v_e+v_l}(1-\theta)(I_0 - I_1 + \mathbb{E}[S_2(x)])}_{\substack{\text{Period-1 and Period-2}\\ \text{late returns}}}\Bigg) - c_r x \\
&= [\theta p + (1-\theta)\tfrac{v_l}{v_e+v_l}(p-r) - \theta s]\mathbb{E}[S_2(x)] + s(R_1+I_1) + (s+p-r)(1-\theta)\tfrac{v_l}{v_e+v_l}(I_0 - I_1) - c_r x
\end{aligned} \tag{EC.3}$$

Define $Q = I_1 + x$, then $\mathbb{E}[S_2(x)]$ becomes $\mathbb{E}[S_2(Q)] = E_{D_2}\min((v_e+v_l)D_2, Q)$. Thus the problem becomes:

$$\begin{aligned}
\max_{I_1 \leq Q \leq R_1 + I_1} \pi_2(I_1, R_1, Q) \;=\; & [\theta p + (1-\theta)\tfrac{v_l}{v_e+v_l}(p-r) - \theta s]E_{D_2}\min((v_e+v_l)D_2, Q) \\
& + s(R_1+I_1) + (s+p-r)(1-\theta)\tfrac{v_l}{v_e+v_l}(I_0 - I_1) - c_r(Q - I_1).
\end{aligned}$$

It is clear that $\pi_2(I_1, R_1, Q)$ is concave. Taking the first-order derivative of $\pi_2(I_1, R_1, Q)$ w.r.t. Q:

$$\frac{\partial \pi_2(I_1, R_1, Q)}{\partial Q} = \theta p + (1-\theta)\frac{v_l}{v_e + v_l}(p-r) - \theta s - c_r - \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big) F_2\Big(\frac{Q}{v_e+v_l}\Big).$$

By setting the first-order derivative to zero, we have:

$$\frac{Q^*}{v_e+v_l} = F_2^{-1}\Big(\frac{\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r}{\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s}\Big).$$

Considering the constraint $I_1 \leq Q \leq R_1 + I_1$, the retailer chooses the restoration quantity x^* so that the total available inventory after restoration, $I_1 + x^*$, is as close as possible to the unconstrained optimal level Q^*. Specifically, if $Q^* < I_1$, no restoration is needed, so $x^* = 0$ and $I_1 + x^* = I_1$. If $I_1 \leq Q^* \leq R_1 + I_1$, the retailer restores just enough units to reach Q^*, so $x^* = Q^* - I_1$ and $I_1 + x^* = Q^*$. If $Q^* > R_1 + I_1$, all available returns are restored, so $x^* = R_1$ and $I_1 + x^* = R_1 + I_1$. ■

Proof of Proposition 9. By Proposition 8, the optimal restore-up-to level is given by $Q^* = (v_e + v_l)F_2^{-1}\left(\frac{\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s-c_r}{\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s}\right)$.

(i) Note that v_e and v_l are independent of s and c_r. As $\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s$ decreases with s, the critical fractile, and thus the restore-up-to level Q^*, decreases with s. Similarly, As $\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r$ decreases with c_r, Q^* decreases with c_r.

(ii) From the proofs of Propositions 1 and 4, we know that in Case (ii), $v_e + v_l$ is independent of r, and $v_l = \frac{1}{\bar{v}}\big(\bar{v} - \frac{p-r-a}{b}\big)$. The term $v_l(p-r) = \big(\bar{v} - \frac{p-r-a}{b}\big)\frac{1}{\bar{v}}(p-r)$ is a quadratic function of r, with the maximum taken at $\tilde{r} = p - \frac{b\bar{v}+a}{2}$. Therefore, the critical fractile $\frac{\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s-c_r}{\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s}$ is increasing-decreasing with r, and thus Q^* is increasing-decreasing with r with the maximum taken at $\tilde{r}$.

■

Proof of Proposition 10. From Proposition 8, we know that

$$x^* = \begin{cases} 0 & \text{if } I_1 \geq Q^*, \\ Q^* - I_1 & \text{if } I_1 < Q^* \leq I_1 + R_1, \\ R_1 & \text{if } I_1 + R_1 < Q^* \end{cases}$$

We now substitute x^* back to the Period-2 expected profit function as shown in Equation (EC.3):

- If $I_1 \geq Q^*$, then $x^* = 0$, and

$$\begin{aligned}\pi_2(r, I_1, R_1) = &\Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) + (1-\theta)s\Big) I_1 \\ &- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{I_1}{v_e+v_l}} \big(I_1 - (v_e+v_l)d_2\big) f(d_2) dd_2 \\ &+ \Big((1+\frac{v_l}{v_e})s + \frac{v_l}{v_e}(p-r)\Big) R_1\end{aligned}$$

- If $I_1 < Q^* \le I_1 + R_1$, then $x^* = Q^* - I_1$, and

$$\begin{aligned}\pi_2(r, I_1, R_1) = &\Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r\Big)Q^* \\ &- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{Q^*}{v_e+v_l}} \big(Q^* - (v_e+v_l)d_2\big)f(d_2)dd_2 \\ &+ (c_r+s)I_1 + \Big((1+\frac{v_l}{v_e})s + \frac{v_l}{v_e}(p-r)\Big)R_1\end{aligned}$$

- If $I_1 + R_1 < Q^*$, then $x^* = R_1$, and

$$\begin{aligned}\pi_2(r, I_1, R_1) = &\Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) + (1-\theta)s\Big)I_1 \\ &- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{I_1+R_1}{v_e+v_l}} \big(I_1 + R_1 - (v_e+v_l)d_2\big)f(d_2)dd_2 \\ &+ \Big((\frac{\theta v_e + v_l}{v_e+v_l} + \frac{v_l}{v_e})p + (1+\frac{v_l}{v_e} - \theta)s - (\frac{(1-\theta)v_l}{v_e+v_l} + \frac{v_l}{v_e})r - c_r\Big)R_1\end{aligned}$$

Moreover, Period-1 sales, Period-1 remaining inventory I_1, and Period-1 early returns R_1 are given by $\min\big((v_e+v_l)d_1, I_0\big)$, $I_0 - \min\big((v_e+v_l)d_1, I_0\big)$, and $\frac{v_e}{v_e+v_l}(1-\theta)\min\big((v_e+v_l)d_1, I_0\big)$, respectively. Subsequently, we have that, if $(v_e+v_l)d_1 \le I_0$, $I_1 = I_0 - (v_e+v_l)d_1$, and $R_1 = v_e(1-\theta)d_1$; if $(v_e+v_l)d_1 > I_0$, $I_1 = 0$, and $R_1 = \frac{v_e}{v_e+v_l}(1-\theta)I_0$. We can now rewrite the Period-2 profit as a function of I_0 and d_1. We start with the first scenario where $d_1 \le \frac{I_0}{v_e+v_l}$:

1) If $I_1 \ge Q^*$ (i.e., if $I_0 - (v_e+v_l)d_1 \ge Q^*$, or equivalently, if $d_1 \le \frac{I_0 - Q^*}{v_e+v_l}$), we have

$$\begin{aligned}\pi_2^*(r, I_0, d_1) = &\Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) + (1-\theta)s\Big)I_0 - \theta(v_e+v_l)pd_1 \\ &- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{I_0-(v_e+v_l)d_1}{v_e+v_l}} \Big(I_0 - (v_e+v_l)d_1 - (v_e+v_l)d_2\Big)f(d_2)dd_2\end{aligned} \tag{EC.4}$$

2) If $I_1 < Q^*$ and $I_1 + R_1 \ge Q^*$, (i.e., if $I_0 - (v_e+v_l)d_1 < Q^*$ and $I_0 - (v_e+v_l)d_1 + (1-\theta)v_e d_1 \ge Q^*$, or equivalently, if $\frac{I_0-Q^*}{v_e+v_l} < d_1 \le \frac{I_0-Q^*}{\theta v_e+v_l}$), we have

$$\begin{aligned}\pi_2^*(r, I_0, d_1) = &(c_r+s)I_0 + \Big(v_l(1-\theta)(p-r) - \theta(v_e+v_l)s - (v_e+v_l)c_r\Big)d_1 \\ &- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{Q^*}{v_e+v_l}} \Big(Q^* - (v_e+v_l)d_2\Big)f(d_2)dd_2 \\ &+ \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r\Big)Q^*\end{aligned} \tag{EC.5}$$

3) If $I_1 < Q^*$ and $I_1 + R_1 < Q^*$, (i.e., if $I_0 - (v_e+v_l)d_1 < Q^*$ and $I_0 - (v_e+v_l)d_1 + (1-\theta)v_e d_1 < Q^*$, or equivalently, if $d_1 > \frac{I_0-Q^*}{\theta v_e+v_l}$), we have

$$\begin{aligned}\pi_2^*(r, I_0, d_1) = &\Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) + (1-\theta)s\Big)I_0 \\ &+ \Big(\big(-\theta(v_e+v_l) + v_e(1-\theta)\frac{\theta v_e + v_l}{v_e+v_l}\big)p - \theta v_e(1-\theta)s - \frac{(1-\theta)^2 v_e v_l}{v_e+v_l}r - (1-\theta)v_e c_r\Big)d_1 \\ &- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{I_0-(\theta v_e+v_l)d_1}{v_e+v_l}} \Big(I_0 - (\theta v_e+v_l)d_1 - (v_e+v_l)d_2\Big)f(d_2)dd_2\end{aligned} \tag{EC.6}$$

Note that cases 1) and 2) always exist for $d_1 \leq \frac{I_0}{v_e+v_l}$. However, case 3) does not necessarily exist. For case 3) to exist, $\frac{I_0-Q^*}{\theta v_e+v_l} \leq \frac{I_0}{v_e+v_l}$ needs to hold, i.e., $I_0 \leq \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$ needs to hold. We now follow the same logic and study the second scenario where $d_1 > \frac{I_0}{v_e+v_l}$:

1) If $I_1 \geq Q^*$ (i.e., if $0 \geq Q^*$) exists only if Q^* equals zero.
2) If $I_1 < Q^*$ and $I_1+R_1 \geq Q^*$, (i.e., if $0 < Q^*$ and $0+(1-\theta)\frac{v_e}{v_e+v_l}I_0 \geq Q^*$, or equivalently, if $I_0 \geq \frac{Q^*(v_e+v_l)}{(1-\theta)v_e}$), we have

$$\begin{aligned}\pi_2^*(r,I_0,d_1) = &\Big(\frac{(1-\theta)v_l}{v_e+v_l}(p-r)+(1-\theta)s\Big)I_0 \\ &-\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s\Big)\int_0^{\frac{Q^*}{v_e+v_l}}\Big(Q^*-(v_e+v_l)d_2\Big)f(d_2)dd_2 \\ &+\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s-c_r\Big)Q^*\end{aligned} \tag{EC.7}$$

3) If $I_1 < Q^*$ and $I_1+R_1 < Q^*$, (i.e., if $0 < Q^*$ and $0+(1-\theta)\frac{v_e}{v_e+v_l}I_0 < Q^*$, or equivalently, if $I_0 < \frac{Q^*(v_e+v_l)}{(1-\theta)v_e}$), we have

$$\begin{aligned}\pi_2^*(r,I_0,d_1) = &\left(\frac{v_e(1-\theta)\Big(\big(\frac{\theta v_e+v_l}{v_e+v_l}+\frac{v_l}{v_e}\big)p+\big(\frac{v_e+v_l}{v_e}-\theta\big)s-\big(\frac{(1-\theta)v_l}{v_e+v_l}+\frac{v_l}{v_e}\big)r-c_r\Big)}{v_e+v_l}\right)I_0 \\ &-\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s\Big)\int_0^{\frac{(1-\theta)v_eI_0}{(v_e+v_l)^2}}\Big(\frac{(1-\theta)v_e}{v_e+v_l}I_0-(v_e+v_l)d_2\Big)f(d_2)dd_2\end{aligned} \tag{EC.8}$$

From the analysis above, we can see how the profit function transitions among different cases. Omitting r below, we denote the Period-2 profit function for $I_0 < \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$ as $\pi_2^l(I_0,d_1)$ and that for $I_0 \geq \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$ as $\pi_2^r(I_0,d_1)$. To summarize,

If $I_0 < \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$,

$$\pi_2(I_0,d_1)=\pi_2^l(I_0,d_1)=\begin{cases} V_2(I_0,d_1) & \text{if } d_1 \leq \frac{I_0-Q^*}{v_e+v_l}, \\ V_3(I_0,d_1) & \text{if } \frac{I_0-Q^*}{v_e+v_l} < d_1 \leq \frac{I_0-Q^*}{\theta v_e+v_l}, \\ V_4(I_0,d_1) & \text{if } \frac{I_0-Q^*}{\theta v_e+v_l} < d_1 \leq \frac{I_0}{v_e+v_l} \\ V_6(I_0) & \text{if } d_1 > \frac{I_0}{v_e+v_l} \end{cases}$$

If $I_0 \geq \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$,

$$\pi_2(I_0,d_1)=\pi_2^r(I_0,d_1)=\begin{cases} V_2(I_0,d_1) & \text{if } d_1 \leq \frac{I_0-Q^*}{v_e+v_l}, \\ V_3(I_0,d_1) & \text{if } \frac{I_0-Q^*}{v_e+v_l} < d_1 \leq \frac{I_0}{v_e+v_l}, \\ V_5(I_0) & \text{if } d_1 > \frac{I_0}{v_e+v_l} \end{cases}$$

where the expressions of $V_2(I_0,d_1)$, $V_3(I_0,d_1)$, $V_4(I_0,d_1)$, $V_5(I_0)$, and $V_6(I_0)$ are given by Equations (EC.4), (EC.5), (EC.6), (EC.7), and (EC.8), respectively. Consequently, let $\pi_2(I_0)$, $\pi_2^l(I_0)$, and $\pi_2^r(I_0)$ be the Period-2 expected profit, the expectation of $\pi_2^l(I_0,d_1)$, and the expectation of $\pi_2^r(I_0,d_1)$, respectively. We then have:

If $I_0 < \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$,

$$\pi_2(I_0)=\pi_2^l(I_0) = \int_0^{\frac{I_0-Q^*}{v_e+v_l}} V_2(I_0,d_1)f(d_1)\,dd_1 + \int_{\frac{I_0-Q^*}{v_e+v_l}}^{\frac{I_0-Q^*}{\theta v_e+v_l}} V_3(I_0,d_1)f(d_1)\,dd_1$$

$$+\int_{\frac{I_0-Q^*}{\theta v_e+v_l}}^{\frac{I_0}{v_e+v_l}} V_4(I_0,d_1)f(d_1)\,dd_1+\int_{\frac{I_0}{v_e+v_l}}^{\infty} V_6(I_0)f(d_1)\,dd_1$$

If $I_0 \geq \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$,

$$\begin{aligned}\pi_2(I_0)=\pi_2^r(I_0) &= \int_0^{\frac{I_0-Q^*}{v_e+v_l}} V_2(I_0,d_1)f(d_1)\,dd_1+\int_{\frac{I_0-Q^*}{v_e+v_l}}^{\frac{I_0}{v_e+v_l}} V_3(I_0,d_1)f(d_1)\,dd_1\\ &+\int_{\frac{I_0}{v_e+v_l}}^{\infty} V_5(I_0)f(d_1)\,dd_1\end{aligned}$$

Hence, the firm is faced with two different profit functions for Period 1. Similarly, we denote the expected total profit function for $I_0 < \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$ and that for $I_0 \geq \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$ as $\pi^l(I_0)$, and $\pi^r(I_0)$, respectively. The firm's expected total profit function then becomes: If $I_0 < \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$,

$$\begin{aligned}\pi(I_0)=\pi^l(I_0) &= p\theta\Big(\int_0^{\frac{I_0}{v_e+v_l}}(v_e+v_l)d_1f(d_1)\,dd_1+\int_{\frac{I_0}{v_e+v_l}}^{\infty} I_0f(d_1)\,dd_1\Big)-cI_0+\pi_2^l(I_0)\\
&= p\theta\int_0^{\frac{I_0}{v_e+v_l}}(v_e+v_l)d_1f(d_1)dd_1+p\theta\int_{\frac{I_0}{v_e+v_l}}^{\infty} I_0f(d_1)dd_1-cI_0\\
&+\int_0^{\frac{I_0-Q^*}{v_e+v_l}}\Bigg(\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)+(1-\theta)s\Big)I_0-\theta(v_e+v_l)pd_1\\
&-\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s\Big)\int_0^{\frac{I_0-(v_e+v_l)d_1}{v_e+v_l}}\Big(I_0-(v_e+v_l)d_1-(v_e+v_l)d_2\Big)f(d_2)dd_2\Bigg)f(d_1)dd_1\\
&+\int_{\frac{I_0-Q^*}{v_e+v_l}}^{\frac{I_0-Q^*}{\theta v_e+v_l}}\Bigg((c_r+s)I_0+\Big(v_l(1-\theta)(p-r)-\theta(v_e+v_l)s-(v_e+v_l)c_r\Big)d_1\\
&-\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s\Big)\int_0^{\frac{Q^*}{v_e+v_l}}\Big(Q^*-(v_e+v_l)d_2\Big)f(d_2)dd_2\\
&+\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s-c_r\Big)Q^*\Bigg)f(d_1)dd_1\\
&+\int_{\frac{I_0-Q^*}{\theta v_e+v_l}}^{\frac{I_0}{v_e+v_l}}\Bigg(\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)+(1-\theta)s\Big)I_0\\
&+\Big(\big(-\theta(v_e+v_l)+v_e(1-\theta)\frac{\theta v_e+v_l}{v_e+v_l}\big)p-\theta v_e(1-\theta)s-\frac{(1-\theta)^2v_ev_l}{v_e+v_l}r-(1-\theta)v_ec_r\Big)D_1\\
&-\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s\Big)\int_0^{\frac{I_0-(\theta v_e+v_l)D_1}{v_e+v_l}}\Big(I_0-(\theta v_e+v_l)D_1-(v_e+v_l)d_2\Big)f(d_2)dd_2\Bigg)f(d_1)dd_1\\
&+\int_{\frac{I_0}{v_e+v_l}}^{\infty}\Bigg(\Bigg(\frac{v_e(1-\theta)\Big(\big(\frac{\theta v_e+v_l}{v_e+v_l}+\frac{v_l}{v_e}\big)p+\big(\frac{v_e+v_l}{v_e}-\theta\big)s-\big(\frac{(1-\theta)v_l}{v_e+v_l}+\frac{v_l}{v_e}\big)r-c_r\Big)}{v_e+v_l}\Bigg)I_0\\
&-\Big(\theta p+(1-\theta)\frac{v_l}{v_e+v_l}(p-r)-\theta s\Big)\int_0^{\frac{(1-\theta)v_eI_0}{(v_e+v_l)^2}}\Big(\frac{(1-\theta)v_e}{v_e+v_l}I_0-(v_e+v_l)d_2\Big)f(d_2)dd_2\Bigg)f(d_1)dd_1\end{aligned} \tag{EC.9}$$

If $I_0 \geq \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$,

$$
\begin{aligned}
\pi(I_0) = \pi^r(I_0) &= p\theta\Big(\int_0^{\frac{I_0}{v_e+v_l}} (v_e+v_l)d_1 f(d_1)\,dd_1 + \int_{\frac{I_0}{v_e+v_l}}^{\infty} I_0 f(d_1)\,dd_1\Big) - cI_0 + \pi_2^r(I_0) \\
&= p\theta\int_0^{\frac{I_0}{v_e+v_l}} (v_e+v_l)d_1 f(d_1)dd_1 + p\theta\int_{\frac{I_0}{v_e+v_l}}^{\infty} I_0 f(d_1)dd_1 - cI_0 \\
&+ \int_0^{\frac{I_0-Q^*}{v_e+v_l}} \Bigg(\Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) + (1-\theta)s\Big)I_0 - \theta(v_e+v_l)pd_1 \\
&- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{I_0-(v_e+v_l)d_1}{v_e+v_l}} \Big(I_0 - (v_e+v_l)d_1 - (v_e+v_l)d_2\Big)f(d_2)dd_2\Bigg)f(d_1)dd_1 \\
&+ \int_{\frac{I_0-Q^*}{v_e+v_l}}^{\frac{I_0}{v_e+v_l}} \Bigg((c_r+s)I_0 + \Big(v_l(1-\theta)(p-r) - \theta(v_e+v_l)s - (v_e+v_l)c_r\Big)d_1 \\
&- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{Q^*}{v_e+v_l}} \Big(Q^* - (v_e+v_l)d_2\Big)f(d_2)dd_2 \\
&+ \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r\Big)Q^*\Bigg)f(d_1)dd_1 \\
&+ \int_{\frac{I_0}{v_e+v_l}}^{\infty} \Bigg(\Big(\frac{(1-\theta)v_l}{v_e+v_l}(p-r) + (1-\theta)s\Big)I_0 \\
&- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{Q^*}{v_e+v_l}} \Big(Q^* - (v_e+v_l)d_2\Big)f(d_2)dd_2 \\
&+ \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r\Big)Q^*\Bigg)f(d_1)dd_1
\end{aligned} \tag{EC.10}
$$

We find that $\pi(I_0)$ is continuous at the boundary $I_0 = \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$, and the first-order derivatives of $\pi^l(I_0)$ and $\pi^r(I_0)$ are the same at that point. Additionally, the second-order conditions for both $\pi^l(I_0)$ and $\pi^r(I_0)$ are satisfied so both are concave. We denote the unique first-order solution for $\pi^l(I_0)$ as I_0^{l*} and for $\pi^r(I_0)$ as I_0^{r*}. To summarize,

$$
I_0^* = \begin{cases} I_0^{l*} & \text{if } \frac{\partial \pi_1^r(I_0)}{\partial I_0} \leq 0 \text{ at } I_0 = \frac{(v_e+v_l)Q^*}{(1-\theta)v_e} \\ I_0^{r*} & \text{if } \frac{\partial \pi_1^r(I_0)}{\partial I_0} > 0 \text{ at } I_0 = \frac{(v_e+v_l)Q^*}{(1-\theta)v_e} \end{cases} \tag{EC.11}
$$

In addition, we have that, at $I_0 = \frac{(v_e+v_l)Q^*}{(1-\theta)v_e}$,

$$
\begin{aligned}
\frac{\partial \pi_1^r(I_0)}{\partial I_0} &= \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) + (1-\theta)s - c\Big) \\
&- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r\Big)\Big(F_1\big(\frac{Q^*}{(1-\theta)v_e}\big) - F_1\big(\frac{(\theta v_e+v_l)Q^*}{(1-\theta)v_e(v_e+v_l)}\big)\Big) \\
&- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{(\theta v_e+v_l)Q^*}{(1-\theta)v_e(v_e+v_l)}} \int_0^{\frac{Q^*-(1-\theta)v_e d_1}{(1-\theta)v_e}} f(d_2)dd_2 f(d_1)dd_1
\end{aligned} \tag{EC.12}
$$

Note that the terms in both the second and third lines of Equation (EC.12) are positive. Hence, for $\frac{\partial \pi_1^r(I_0)}{\partial I_0} \geq 0$ to hold, we have $c \leq \bar{c}$, where

$$\bar{c} = \theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) + (1-\theta)s$$
$$- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r\Big)\left[F_1\Big(\frac{Q^*}{(1-\theta)v_e}\Big) - F_1\Big(\frac{(\theta v_e+v_l)Q^*}{(1-\theta)v_e(v_e+v_l)}\Big)\right] \tag{EC.13}$$
$$- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)\int_0^{\frac{(\theta v_e+v_l)Q^*}{(1-\theta)v_e(v_e+v_l)}}\int_0^{\frac{Q^*-(1-\theta)v_e d_1}{(1-\theta)v_e}} f(d_2)\,dd_2\,f(d_1)\,dd_1$$
$$> 0$$

Hence, Equation (EC.11) becomes

$$I_0^* = \begin{cases} I_0^{r*} & \text{if } c \le \bar{c} \\ I_0^{l*} & \text{if } c > \bar{c} \end{cases}$$

Next we show that restoration reduces the optimal initial inventory. For a given I_0 and realized demand d_1, $I_1 = \max\{I_0-(v_e+v_l)d_1, 0\}$ and $R_1 = \frac{v_e}{v_e+v_l}\min\{I_0,(v_e+v_l)d_1\}$. With optimal restoration, by the proof of Proposition 8, the profit function can be written as

$$\begin{aligned}\pi(I_0,d_1) &= p\theta\min\{I_0,(v_e+v_l)d_1\} - cI_0 + \pi_2^*(I_1,R_1,x^*)\\ &= p\theta\min\{I_0,(v_e+v_l)d_1\} - cI_0 + s(R_1+I_1)\\ &\quad + (s+p-r)(1-\theta)\frac{v_l}{v_e+v_l}(I_0-I_1)\\ &\quad + \left[\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\right]\mathbb{E}_{D_2}\min\{(v_e+v_l)D_2, I_1+x^*\}\\ &\quad - c_r(I_1+x^*) + c_r I_1.\end{aligned}$$

Denote $Y(x) = \left[\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\right]\mathbb{E}_{D_2}\min\{(v_e+v_l)D_2, x\} - c_r x$, then

$$\begin{aligned}\pi(I_0,d_1) = p\theta\min\{I_0,(v_e+v_l)d_1\} - cI_0 + s(R_1+I_1) + (s+p-r)(1-\theta)\frac{v_l}{v_e+v_l}(I_0-I_1)\\ + c_r I_1 + Y(I_1+x^*).\end{aligned}$$

Without restoration, we have

$$\begin{aligned}\pi^{NR}(I_0,d_1) = p\theta\min\{I_0,(v_e+v_l)d_1\} - cI_0 + s(R_1+I_1) + (s+p-r)(1-\theta)\frac{v_l}{v_e+v_l}(I_0-I_1)\\ + c_r I_1 + Y(I_1).\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial\pi(I_0,d_1)}{\partial I_0} - \frac{\partial\pi^{NR}(I_0,d_1)}{\partial I_0} &= \frac{\partial Y(I_1+x^*)}{\partial I_0} - \frac{\partial Y(I_1)}{\partial I_0}\\ &= \left(\frac{\partial Y(I_1+x^*)}{\partial I_1} - \frac{\partial Y(I_1)}{\partial I_1}\right)\frac{\partial I_1}{\partial I_0}.\end{aligned}$$

Note that π_2 is concave in Q and therefore $Y(I_1)$ is also concave since it contains all the terms related to Q. Moreover, $Y'(Q^*) = 0$ and I_1 increases in I_0.

By Proposition 8, we have

1) If $R_1 + I_1 < Q^*$, then $x^* = R_1$ and $Y(I_1 + x^*) = Y(I_1 + R_1)$. Since $I_1 + R_1 \geq I_1$ and Y is concave, $\frac{\partial Y(I_1+x^*)}{\partial I_1} \leq \frac{\partial Y(I_1)}{\partial I_1}$.
2) If $I_1 \leq Q^* \leq R_1 + I_1$, then $I_1 + x^* = Q^*$ and $\frac{\partial Y(I_1+x^*)}{\partial I_1} = 0$. Since $I_1 \leq Q^*$, Y is concave and $Y'(Q^*) = 0$, $\frac{\partial Y(I_1)}{\partial I_1} \geq 0$. Thus, $\frac{\partial Y(I_1+x^*)}{\partial I_1} \leq \frac{\partial Y(I_1)}{\partial I_1}$.
3) If $Q^* < I_1$, then $x^* = 0$ and $\frac{\partial Y(I_1+x^*)}{\partial I_1} = \frac{\partial Y(I_1)}{\partial I_1}$.

In summary, $\frac{\partial Y(I_1+x^*)}{\partial I_1} \leq \frac{\partial Y(I_1)}{\partial I_1}$. Thus,

$$\frac{\partial \pi(I_0, d_1)}{\partial I_0} - \frac{\partial \pi^{NR}(I_0, d_1)}{\partial I_0} = \left(\frac{\partial Y(I_1 + x^*)}{\partial I_1} - \frac{\partial Y(I_1)}{\partial I_1} \right) \frac{\partial I_1}{\partial I_0} \leq 0.$$

After taking expectation over D_1, this relationship is preserved. Thus, $\frac{\partial \pi}{\partial I_0} \leq \frac{\partial \pi^{NR}}{\partial I_0}$. Since π is concave in I_0 as shown above , we have $I_0^* \leq I_{0,NR}^*$.

■

LEMMA EC.1. *If the Tiered-EL outcome is optimal under stochastic demand without restoration, then the optimal late-return refund is given by* $r_{NR}^* = p - \frac{b\bar{v}+a}{2}$, *which is the same as the case under deterministic demand.*

Proof of Lemma EC.1 Note that mathematically optimizing r and then I_0 is equivalent to optimizing I_0 and then r, since there is no other decision in between. For a given I_0, the expected total profit when no inventory is restored is given by Equation (EC.14) below.

$$\begin{aligned}
\pi_{NR}(I_0) = p\Bigg(& \underbrace{\theta E \min\big(I_0, (v_e+v_l)D_1\big)}_{\text{Period-1 effective sales}} + \underbrace{\theta E \min\big(I_0 - E\min\big(I_0,(v_e+v_l)D_1\big), (v_e+v_l)D_2\big)}_{\text{Period-2 effective sales}}\Bigg) \\
& + (p-r)\Bigg(\underbrace{\frac{v_l}{v_e+v_l}(1-\theta)E\min\big(I_0,(v_e+v_l)D_1\big)}_{\text{Period-1 late returns}}\Bigg) \\
& + (p-r)\Bigg(\underbrace{\frac{v_l}{v_e+v_l}(1-\theta)E\min\big(I_0 - E\min\big(I_0,(v_e+v_l)D_1\big),(v_e+v_l)D_2\big)}_{\text{Period-2 late returns}}\Bigg) \\
& + s\Bigg(\underbrace{I_0 - E\min\big(I_0,(v_e+v_l)D_1\big) - E\min\big(I_0 - E\min\big(I_0,(v_e+v_l)D_1\big),(v_e+v_l)D_2\big)}_{\text{Period-2 unsold items}}\Bigg) \\
& + s\Bigg(\underbrace{(1-\theta)\Big(E\min\big(I_0,(v_e+v_l)D_1\big) + E\min\big(I_0 - E\min\big(I_0,(v_e+v_l)D_1\big),(v_e+v_l)D_2\big)\Big)}_{\text{Total returns from both periods}}\Bigg) \\
& - \underbrace{cI_0}_{\substack{\text{Initial}\\ \text{inventory}}}
\end{aligned} \tag{EC.14}$$

Recall that in Case (ii), $\frac{\partial v_e}{\partial r} = -\frac{1}{b\bar{v}} < 0$, $\frac{\partial v_l}{\partial r} = \frac{1}{b\bar{v}} > 0$, and $\frac{\partial (v_e+v_l)}{\partial r} = 0$. Examining Eq (EC.14), we note that lines 1, 4, 5, and 6 are independent of r, therefore,

$$\begin{aligned}\frac{\partial \pi_{NR}(I_0)}{\partial r} &= \frac{1-\theta}{v_e+v_l} E \min\Big(I_0, (v_e+v_l)D_1\Big)\Big(\frac{\partial v_l}{\partial r}(p-r) - v_l\Big) \\ &+ \frac{1-\theta}{v_e+v_l} E \min\Big(I_0 - E\min\big(I_0, (v_e+v_l)D_1\big), (v_e+v_l)D_2\Big)\Big(\frac{\partial v_l}{\partial r}(p-r) - v_l\Big) \\ &= \Big(\frac{\partial v_l}{\partial r}(p-r) - v_l\Big)\cdot A \\ &= (2p - 2r - b\bar{v} - a)\frac{1}{b\bar{v}} \cdot A\end{aligned}$$

where $A = \frac{1-\theta}{v_e+v_l}\Big(E\min\big(I_0, (v_e+v_l)D_1\big) + E\min\big(I_0 - E\min(I_0, (v_e+v_l)D_1), (v_e+v_l)D_2\big)\Big)$ is independent of r. Setting the first order derivative above to zero and solving for r, we have $r^*_{NR} = p - \frac{b\bar{v}+a}{2}$. Since this optimal r is independent of I_0, after optimizing on I_0 it is still optimal. ■

Proof of Proposition 11. Note that the profits under Policy S, F and the Tiered-L outcome of Policy T are not affected by restoration, while the profit under Tiered-EL is increased by restoration. Therefore, when the optimal $r^*_{NR} \geq \bar{r}$ without restoration, indicating Policy F or T-L outcome is optimal, under restoration the optimal refund either stays the same or shift to a level below $\bar{r}$, that is, $r^* < \bar{r}$. On the other hand, when the optimal $r^*_{NR} \in [\underline{r}, \bar{r})$ without restoration, it stays in this region with restoration. Furthermore, under the Tiered-EL outcome of Policy T, we can further show that the first derivative of profit with respect to r is reduced by restoration, and thus the optimal late-return refund is also reduced. Since Lemma EC.1 shows that the optimal r is unchanged under stochastic demand, it is sufficient to show that $r^* < r^*_{NR}$. Note that mathematically optimizing r and then I_0 is equivalent to optimizing I_0 and then r, since there is no other decision in between. For any given D_1 and I_0, we have $I_1(D_1) = \big(I_0 - (v_e+v_l)D_1\big)^+$, $R_1(D_1) = \frac{v_e(1-\theta)}{v_e+v_l}\min\big(I_0, (v_e+v_l)D_1\big)$. Note that $I_1(D_1)$ is independent of r, and $R_1(D_1)$ decreases in r. Let $y(D_1) = I_1(D_1) + x(D_1)$, we can then write the profit function for optimal restoration as follows:

$$\begin{aligned}\pi^*(I_0, D_1, y(D_1)) &= p\theta E\min\big(I_0, (v_e+v_l)D_1\big) + p\theta E\min\big(y(D_1), (v_e+v_l)D_2\big) \\ &+ (p-r)\frac{v_l}{v_e+v_l}(1-\theta)E\min\big(I_0, (v_e+v_l)D_1\big) \\ &+ (p-r)\frac{v_l}{v_e+v_l}(1-\theta)E\min\big(y(D_1), (v_e+v_l)D_2\big) \\ &+ sE\big(y(D_1) - (v_e+v_l)D_2\big)^+ \\ &+ s(1-\theta)E\min\big(y(D_1), (v_e+v_l)D_2\big) \\ &+ s\frac{v_l}{v_e+v_l}(1-\theta)E\min\big(I_0, (v_e+v_l)D_1\big) \\ &+ s\Big(\big(I_0 - (v_e+v_l)D_1\big)^+ + \frac{v_e(1-\theta)}{v_e+v_l}\min\big(I_0, (v_e+v_l)D_1\big) - y(D_1)\Big) \\ &- c_r\big(y(D_1) - I_1(D_1)\big) - cI_0\end{aligned}$$

Note that the second and third lines from the bottom can be written as $s\Big(\big(I_0-(v_e+v_l)D_1\big)^+ + (1-\theta)\min\big(I_0,(v_e+v_l)D_1\big)\Big) - sy(D_1)$. From Proposition 8, we know that

$$x^* = \begin{cases} 0 & \text{if } I_1 \geq Q^*, \\ Q^* - I_1 & \text{if } I_1 < Q^* \text{ and } I_1 + R_1 \geq Q^*, \\ R_1 & \text{if } I_1 < Q^* \text{ and } I_1 + R_1 < Q^* \end{cases}$$

Equivalently,

$$y^*(D_1) = I_1(D_1) + x^*(D_1) = \begin{cases} I_1(D_1), & \text{and } \frac{\partial \pi^{OR}}{\partial y}\Big|_{y=y^*(D_1)} < 0 \text{ if } I_1(D_1) \geq Q^*, \\ Q^*, & \text{and } \frac{\partial \pi^{OR}}{\partial y}\Big|_{y=y^*(D_1)} = 0 \text{ if } I_1(D_1) < Q^* \text{ and } I_1(D_1) + R_1(D_1) \geq Q^*, \\ I_1(D_1) + R_1(D_1), & \text{and } \frac{\partial \pi^{OR}}{\partial y}\Big|_{y=y^*(D_1)} > 0 \text{ if } I_1(D_1) < Q^* \text{ and } I_1(D_1) + R_1(D_1) < Q^*. \end{cases}$$

Therefore, we have

$$\frac{\partial \pi^*(I_0, D_1, y(D_1))}{\partial r}\Big|_{y=y^*(D_1)} = \frac{\partial (p-r)v_l}{\partial r}\Big|_{y=y^*(D_1)} \cdot C + \frac{\partial \pi^*}{\partial y}\Big|_{y=y^*(D_1)} \cdot \frac{\partial y^*(D_1)}{\partial r},$$

where C is a constant. The first term equals zero at $r = r^*_{NR}$. Hence,
if $I_1(D_1) \geq Q^*$, then $y^*(D_1) = I_1(D_1)$, and thus $\frac{\partial y^*(D_1)}{\partial r} = 0$;
if $I_1(D_1) < Q^*$ and $I_1(D_1) + R_1(D_1) \geq Q^*$, then $y^*(D_1) = Q^*$, and thus $\frac{\partial \pi^*}{\partial y}\Big|_{y^*} = 0$;
If $I_1(D_1) < Q^*$ and $I_1(D_1) + R_1(D_1) < Q^*$, then $y^*(D_1) = I_1(D_1) + R_1(D_1)$ and $\frac{\partial \pi^*}{\partial y}\Big|_{y^*} > 0$. And $\frac{\partial\big(I_1(D_1)+R_1(D_1)\big)}{\partial r} = \frac{\partial v_e}{\partial r} \cdot \text{constant} < 0$. Thus, $\frac{\partial \pi^*}{\partial y}\Big|_{y^*(D_1)} \cdot \frac{\partial y^*(D_1)}{\partial r} < 0$.

Hence, overall, $\frac{\partial \pi^*}{\partial y}\Big|_{y^*(D_1)} \cdot \frac{\partial y^*(D_1)}{\partial r} \leq 0$ holds in all cases. Therefore,

$$\frac{\partial \pi^*\big(I_0, D_1, y(D_1)\big)}{\partial r}\Bigg|_{y^*(D_1)} \leq \frac{\partial\big((p-r)v_l\big)}{\partial r}\Bigg|_{y^*(D_1)} \cdot \text{constant holds.}$$

At $r = r^*_{NR}$, we have

$$\frac{\partial \pi^*\big(I_0, D_1, y(D_1)\big)}{\partial r}\Bigg|_{y^*(D_1), r=r^*_{NR}} \leq \frac{\partial\big((p-r)v_l\big)}{\partial r}\Bigg|_{y^*(D_1), r=r^*_{NR}} = 0.$$

Note that after taking the expectation over D_1, this relationship still holds. If the profit is unimodal, then the optimal $r^* \leq r^*_{NR}$. While we cannot show the unimodality due to its complexity, extensive numerical studies do not find a violating occasion. Therefore, for optimal I_0^* we still have $r^* \leq r^*_{NR}$. ■

Proof of Proposition 12. (a) Under deterministic demand without restoration, $x^* = 0$ and $TS = I_0 = (v_e + v_l)d$, so the total life-cycle impact is given by $\big(e_p + e_u\theta + e_d + e_r(1-\theta)\big)(v_e + v_l)d$. Since we have shown in Proposition 1 that $v_e + v_l$ increases in r, the total life-cycle impact increases as the policy moves from S to T to F.

(b) Under Policy T, we compare the total life-cycle impacts of with and without restoration. We know from Proposition 5 that $I_0^* = (v_e + v_l)d - x^*$, so the total life-cycle impact with restoration can be written as $\big(e_p + e_u\theta + e_d + e_r(1-\theta)\big)(v_e + v_l)d - (e_p + e_d - e_{rs})x$. Comparing the life-cycle impacts with and without restoration given in Part (a), we find that restoration reduces impact if $e_p + e_d - e_{rs} > 0$.

(c) We have shown in Proposition 1 that $(v_e + v_l)d$ are identical under Policy S and T-EL, and increases progressively under T-L and F. Therefore, if $e_p + e_d - e_{rs} > 0$, the term $(e_p + e_d - e_{rs})x$ in EI_T for T-EL is positive, and thus the total life-cycle impact increases as the policy moves from T-EL with restoration to S, T-L, and F.

If restoration is also possible under Policy S, then we have shown in Propositions 1 and 5 that $(v_e + v_l)d$ increases and x^* decreases as we move from Policy S to T to F. Therefore, if $e_p + e_d - e_{rs} > 0$, then the total life-cycle impact increases as we move from Policy S to T to F.

■

Proof of Proposition 13. We denote TS_{NR} as the realized total sales without restoration. $TS_{NR} = \min(I_0, (v_e + v_l)d_1) + \min((I_0 - (v_e + v_l)d_1)^+, (v_e + v_l)d_2)$, $TS_{NR} + \Delta_{TS} = \min(I_0, (v_e + v_l)d_1) + \min((I_0 - (v_e + v_l)d_1)^+ + 1, (v_e + v_l)d_2)$. We have $0 \le \Delta_{TS} \le 1$. Table EC.3 summarizes each type of environmental impact for the two policies. The total impact difference between with restoration and without restoration is given by $(e_u\theta + e_d + e_r(1-\theta) - e_o)\Delta_{TS} + (e_{rs} + e_o - e_d)$. Hence, for fixed I_0 and r, restoration reduces the total impact if and only if $\big(e_u\theta + e_d + e_r(1-\theta) - e_o\big)\Delta_{TS} + (e_{rs} + e_o - e_d) < 0$. ■

Table EC.3 Total Environmental Impacts with and without Restoration

Impact Type	Unit impact	Without restoration	With restoration	Difference
Production	e_p	I_0	I_0	0
Use	e_u	θTS_{NR}	$\theta(TS_{NR} + \Delta_{TS})$	$\theta\Delta_{TS}$
Disposal	e_d	TS_{NR}	$TS_{NR} + \Delta_{TS} - 1$	$\Delta_{TS} - 1$
Return processing	e_r	$(1-\theta)TS_{NR}$	$(1-\theta)(TS_{NR} + \Delta_{TS})$	$(1-\theta)\Delta_{TS}$
Overordering	e_o	$I_0 - TS_{NR}$	$I_0 - (TS_{NR} + \Delta_{TS}) + 1$	$-\Delta_{TS} + 1$
Restoration	e_{rs}	0	1	1

Proof of Proposition 14. The environmental impact of unconsumed products is summarized in Table EC.4. Total difference is given by $(-\theta e_p + (1-\theta)(e_d + e_r) - e_o)\Delta_{TS} + (e_{rs} + e_o - e_d)$. ■

Table EC.4 Environmental Impact of Unconsumed Products with and without Restoration

Impact Type	Unit impact	Without restoration	With restoration	Difference
Production	e_p	$I_0 - \theta TS_{NR}$	$I_0 - \theta(TS_{NR} + \Delta_{TS})$	$-\theta\Delta_{TS}$
Retailer disposal	e_d	$(1-\theta)TS_{NR}$	$(1-\theta)(TS_{NR} + \Delta_{TS}) - 1$	$(1-\theta)\Delta_{TS} - 1$
Return processing	e_r	$(1-\theta)TS_{NR}$	$(1-\theta)(TS_{NR} + \Delta_{TS})$	$(1-\theta)\Delta_{TS}$
Overordering	e_o	$I_0 - TS_{NR}$	$I_0 - (TS_{NR} + \Delta_{TS}) + 1$	$-\Delta_{TS} + 1$
Restoration	e_{rs}	0	1	1

Proof of Proposition 15. Next we show that the total impact can be reduced if I_0 is adjusted lower. For given r, D_1, and D_2, consider two options: 1) Option 1: order I_0, no restoration; and 2) Option 2: order (I_0-1), restore 1 unit. Recall that $TS_{NR} = \min(I_0, (v_e+v_l)D_1) + \min((I_0-(v_e+v_l)D_1)^+,\ (v_e+v_l)D_2)$, and $TS_{NR}+\Delta_{TS} = \min(I_0-1, (v_e+v_l)D_1) + \min((I_0-1-(v_e+v_l)D_1)^+ + 1,\ (v_e+v_l)D_2)$. We now show that $-1 \le \Delta_{TS} \le 0$ holds:

Case (i): If $I_0 - 1 > (v_e + v_l)D_1$, then $I_0 > (v_e + v_l)D_1$. $TS_{NR} = TS_{NR} + \Delta_{TS} = (v_e + v_l)D_1 + \min((I_0-(v_e+v_l)D_1)^+,\ (v_e+v_l)D_2)$. Thus $\Delta_{TS} = 0$.

Case (ii): If $I_0 \le (v_e+v_l)D_1$, then $I_0 - 1 \le (v_e+v_l)D_1$. $TS_{NR} = I_0$. $TS_{NR}+\Delta_{TS} = I_0 - 1 + \min(1,\ (v_e + v_l)D_2)$. $\Delta_{TS} = \min(1,\ (v_e+v_l)D_2) - 1 \in [-1,0]$.

Case (iii): If $I_0 - 1 < (v_e+v_l)D_1 < I_0$, then $TS_{NR} = (v_e+v_l)D_1 + \min(I_0-(v_e+v_l)D_1,\ (v_e+v_l)D_2) = \min(I_0,\ (v_e+v_l)(D_1+D_2))$. $TS_{NR}+\Delta_{TS} = I_0 - 1 + \min(1,\ (v_e+v_l)D_2) = \min(I_0,\ I_0 - 1 + (v_e+v_l)D_2)$. Since $I_0 - 1 < (v_e+v_l)D_1$, we have $\Delta_{TS} \le 0$. And depending on the case, Δ_{TS} equals either 0, $I_0 - 1 - (v_e + v_l)D_1$ (which ≥ -1), or $-1 + (v_e+v_l)D_2$ (which ≥ -1). Therefore $-1 \le \Delta_{TS} \le 0$.

The environmental impact is summarized in Table EC.5. The difference is negative if $(e_{rs}-e_p) + (\theta e_u + e_d + (1-\theta)e_r - e_o)\Delta_{TS} < 0$. If $\theta e_u + e_d + (1-\theta)e_r - e_o \ge 0$, since $\Delta_{TS} \le 0$, it is sufficient to require $e_{rs} - e_p < 0$. If $\theta e_u + e_d + (1-\theta)e_r - e_o < 0$, since $\Delta_{TS} \ge -1$, we have $(e_{rs}-e_p) + (\theta e_u + e_d + (1-\theta)e_r - e_o)\Delta_{TS} \le (e_{rs}-e_p) + (\theta e_u + e_d + (1-\theta)e_r - e_o)(-1) = (e_{rs} - e_p - \theta e_u - e_d - (1-\theta)e_r + e_o)$, thus it is sufficient to require $e_{rs} - e_p - \theta e_u - e_d - (1-\theta)e_r + e_o < 0$. This condition also implies that $e_{rs} - e_p < 0$. Therefore, in summary, if $e_{rs} - e_p < 0$ and $e_{rs} - e_p - \theta e_u - e_d - (1-\theta)e_r + e_o < 0$, then the total environmental impact is reduced. ■

Table EC.5 Environmental impacts with reduced initial inventory $(I_0 - 1)$

Impact Type	Unit Impact	Without Restoration	With Restoration	Difference
Production	e_p	I_0	$I_0 - 1$	-1
Use	e_u	θTS_{NR}	$\theta(TS_{NR}+\Delta_{TS})$	$\theta\Delta_{TS}$
Consumer & retailer disposal	e_d	TS_{NR}	$(TS_{NR}+\Delta_{TS})$	Δ_{TS}
Return processing	e_r	$(1-\theta)TS_{NR}$	$(1-\theta)(TS_{NR}+\Delta_{TS})$	$(1-\theta)\Delta_{TS}$
Overordering	e_o	$I_0 - TS_{NR}$	$I_0 - 1 - (TS_{NR}+\Delta_{TS}) + 1$	$-\Delta_{TS}$
Restoration	e_{rs}	0	1	1

Proof of Proposition 16. We adopt the two measures, expected leftover L and leftover-to-sales ratio LSR, from the literature. For given I_0, r, d_1, and d_2, let x be a feasible restoration quantity under this demand realization, then the total sales is given by $TS(x,d_1,d_2) = \min\left((I_0-(v_e+v_l)d_1)^+ + x,\ (v_e+v_l)d_2\right) + \min\left(I_0,\ (v_e+v_l)d_1\right)$. The expected total sales is $TS(x) = E_{D_1,D_2}TS(x,D_1,D_2)$, the expected leftover is $L = I_0 - \theta TS$, and the Leftover-to-sales ratio is $LSR = \frac{I_0-\theta TS}{TS} = \frac{I_0}{TS} - \theta$. We can show that L is always nonnegative because $\theta TS \le \theta(I_0+x) = \theta(I_0 + (1-\theta)\min(I_0,d_1)) \le \theta(I_0+(1-\theta)I_0) \le \theta I_0 + (1-\theta)I_0 = I_0$. We use superscript R for restoration and N for no restoration.

Part (i): Since $TS(x)$ increases in x, $TS(x) \ge TS(0)$ for any positive x. After taking expectation on D_1, D_2 we still have $TS^R = E_{D_1,D_2}TS(x) \ge TS^N = E_{D_1,D_2}TS(0)$. Therefore $L^R = I_0 - \theta TS^R \le I_0 - \theta TS^N = L^N$, and $LSR^R = \frac{I_0}{TS^R} - \theta \le \frac{I_0}{TS^N} - \theta = LSR^N$. Therefore restoration reduces both L and LSR.

If r is adjusted: note that $v_e + v_l$ does not depend on r, so $TS(x)$ remains the same, and so does the environmental impact of restoration.

Part (ii): If I_0 is adjusted: Proposition 10 shows that restoration leads to lower I_0 so $I_0^R \le I_0^N$. Expected leftover $L = I_0 - \theta TS$ increases in I_0, therefore restoration still reduces the environmental impact of disposing of leftover and returns, and even more significantly if I_0 is lowered to the optimal value.

For LSR, we examine how $\frac{TS(I_0,x)}{I_0} = \frac{\min\left((I_0-(v_e+v_l)D_1)^+ + x,\ (v_e+v_l)D_2\right) + \min(I_0,(v_e+v_l)D_1)}{I_0}$ changes with I_0. If $I_0 < (v_e+v_l)D_1$, then $\frac{TS(I_0,x)}{I_0} = \frac{\min(x,(v_e+v_l)D_2)+I_0}{I_0}$, which is decreasing in I_0. If $I_0 \ge (v_e+v_l)D_1$ and $I_0 - (v_e+v_l)D_1 + x < (v_e+v_l)D_2$, then $\frac{TS(I_0,x)}{I_0} = \frac{I_0+x}{I_0}$, which is decreasing in I_0. If $I_0 - (v_e+v_l)D_1 + x \ge (v_e+v_l)D_2$ then $\frac{TS(I_0,x)}{I_0} = \frac{(v_e+v_l)(D_1+D_2)}{I_0}$ which is decreasing in I_0. Therefore, for restoration ($x^R \ge 0$) vs. no restoration ($x^N = 0$): for a fix $x = x^R$, by Proposition 10, $I_0^R \le I_0^N$ and $\frac{TS(I_0,x^R)}{I_0}$ decreases in I_0, hence $\frac{TS(I_0^R,x^R)}{I_0^R} \ge \frac{TS(I_0^N,x^R)}{I_0^N}$. Since $\frac{TS(I_0^N,x)}{I_0^N}$ is nondecreasing in x, we have $\frac{TS(I_0^N,x^R)}{I_0^N} \ge \frac{TS(I_0^N,0)}{I_0^N}$. Hence, $\frac{TS^R}{I_0^R} \ge \frac{TS^N}{I_0^N}$, so $LSR^R = \frac{I_0^R}{TS^R} - \theta \le \frac{I_0^N}{TS^N} - \theta = LSR^N$. So, similarly, the environmental impact of restoration, represented by LSR, is also more significant if I_0 is adjusted to the optimal value. ■

Proof of Proposition EC.1. We now find the optimal restore-up-to level for the case where a fraction α of the Period-1 early returns requires restoration with a unit cost c_r and the rest can be resold with no cost incurred. The Period-2 profit function is given as

$$\begin{aligned}
\pi_2\big(I_1,R_1,x\big) &= p\,\theta\,\mathbb{E}[S_2(x)] + s\Big(\underbrace{\alpha R_1 - x}_{\substack{\text{Unrestored}\\ \text{Period-1 early returns}}} + \underbrace{I_1 + (1-\alpha)R_1 + x - \mathbb{E}[S_2(x)]}_{\substack{\text{Unsold}\\ \text{in Period-2}}} + \underbrace{\frac{v_e}{v_e+v_l}(1-\theta)\mathbb{E}[S_2(x)]}_{\substack{\text{Period-2 early}\\ \text{returns}}}\Big) \\
&\quad + (s+p-r)\Big(\underbrace{\frac{v_l}{v_e+v_l}(1-\theta)(I_0 - I_1 + \mathbb{E}[S_2(x)]}_{\substack{\text{Period-1 and Period-2}\\ \text{late returns}}}\Big) - c_r x \\
&= [\theta p + (1-\theta)\tfrac{v_l}{v_e+v_l}(p-r) - \theta s]\mathbb{E}[S_2(x)] + s(R_1+I_1) + (s+p-r)(1-\theta)\tfrac{v_l}{v_e+v_l}(I_0-I_1) - c_r x
\end{aligned} \tag{EC.15}$$

Similar to Proposition 8, define $Q = I_1 + (1-\alpha)R_1 + x$, then $\mathbb{E}[S_2(x)]$ becomes $\mathbb{E}[S_2(Q)] = E_{D_2}\min((v_e + v_l)D_2, Q)$. Thus the problem becomes:

$$\begin{aligned}\max_{I_1+(1-\alpha)R_1\le Q\le R_1+I_1} \pi_2(I_1,R_1,Q) = &\ [\theta p + (1-\theta)\tfrac{v_l}{v_e+v_l}(p-r) - \theta s]E_{D_2}\min((v_e+v_l)D_2, Q)\\ &+ s(R_1+I_1) + (s+p-r)(1-\theta)\tfrac{v_l}{v_e+v_l}(I_0 - I_1) - c_r(Q - I_1 - (1-\alpha)R_1).\end{aligned}$$

Take the first-order derivative w.r.t. Q:

$$\begin{aligned}\frac{\partial \pi_2(I_1,R_1,Q)}{\partial Q} = &\ \theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r\\ &- \Big(\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s\Big)F_2\Big(\frac{Q}{v_e+v_l}\Big)\end{aligned}$$

By setting the first-order derivative to zero, we have:

$$\frac{Q^*}{v_e+v_l} = F_2^{-1}\Big(\frac{\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s - c_r}{\theta p + (1-\theta)\frac{v_l}{v_e+v_l}(p-r) - \theta s}\Big)$$

Considering the constraint $I_1 + (1-\alpha)R_1 \le Q \le R_1 + I_1$, the retailer chooses the restoration quantity x^* so that the total available inventory after restoration, $I_1 + (1-\alpha)R_1 + x^*$, is as close as possible to the unconstrained optimal level Q^*. Specifically, if $Q^* < I_1 + (1-\alpha)R_1$, no additional restoration is needed, so $x^* = 0$ and $I_1 + (1-\alpha)R_1 + x^* = I_1 + (1-\alpha)R_1$. If $I_1 + (1-\alpha)R_1 \le Q^* \le R_1 + I_1$, the retailer restores just enough units to reach Q^*, so $x^* = Q^* - I_1 - (1-\alpha)R_1$ and $I_1 + (1-\alpha)R_1 + x^* = Q^*$. If $Q^* > R_1 + I_1$, all restorable returns are restored, so $x^* = \alpha R_1$ and $I_1 + (1-\alpha)R_1 + x^* = R_1 + I_1$. ■

Proof of Extension EC.1.2. For the fraction q_E of consumers whose $h = 0$, their expected utility of buying and returning a misfit product early is given by $EU_e = -p + \theta v + (1-\theta)(p-0) = \theta(v-p)$. Therefore, consumers with valuation $v \in [p, \bar{v}]$ will buy and return early. The profit functions can be modified as follows. Cases (i), (ii), and (iii) correspond to those in Proposition 1.

Case (i): $\pi = (\theta p - c + (1-\theta)s)q_E(\bar{v} - p)\frac{d}{\bar{v}} + (\theta p - c + (1-\theta)s)(1-q_E)\left(\bar{v} - \frac{\theta p + (1-\theta)a}{\theta - (1-\theta)b}\right)\frac{d}{\bar{v}}$

Case (ii): $\pi = (\theta p - c + (1-\theta)s)q_E(\bar{v} - p)\frac{d}{\bar{v}} + (1-q_E)\Big((\theta p - c + (1-\theta)s)\big(\bar{v} - \frac{\theta p + (1-\theta)a}{\theta-(1-\theta)b}\big)\frac{d}{\bar{v}} + (p-r)(1-\theta)\big(\bar{v} - \frac{p-r-a}{b}\big)\frac{d}{\bar{v}}\Big)$

Case (iii): $\pi = (\theta p - c + (1-\theta)s)q_E(\bar{v} - p)\frac{d}{\bar{v}} + (1-q_E)\Big((\theta p - c + (1-\theta)s)\big(\bar{v} - \frac{p-(1-\theta)r}{\theta}\big)\frac{d}{\bar{v}} + (p-r)(1-\theta)\big(\bar{v} - \frac{p-(1-\theta)r}{\theta}\big)\frac{d}{\bar{v}}\Big)$

It is observed that all the profit functions are multiplied by a constant scalar $(1-q_E)$ and then increased by a constant term $(\theta p - c + (1-\theta)s)q_E(\bar{v} - p)\frac{d}{\bar{v}}$. Therefore, the optimal late-return refund r remains unchanged. ■

Proof of Proposition EC.2. The consumer decision process is illustrated in Fig EC.1. We assume: $\theta_f + \theta_u + \theta_n = 1$ and $\theta_{uf} + \theta_{un} = 1$. Let U_e be the expected utility of purchasing and, if unsure, returning in Period

1: $U_e = -p + \theta_f \cdot v + (\theta_u + \theta_n)p$. Let U_l be the expected utility of purchasing and, if unsure, waiting until Period 2 to decide: $U_l = -p + \theta_f \cdot v + \theta_u(\theta_{uf} \cdot v + \theta_{un} \cdot r) + \theta_n p$. Therefore, $U_e > U_l$ if $p > \theta_{uf} \cdot v + \theta_{un} \cdot r$, i.e., $v < \frac{p-\theta_{un} r}{\theta_{uf}}$. In addition, $U_e > 0$ if $-p + \theta_f \cdot v + (\theta_u + \theta_n)p > 0$, i.e., $v > p$. And $U_l > 0$ if $-p + \theta_f \cdot v + \theta_u(\theta_{uf} \cdot v + \theta_{un} \cdot r) + \theta_n \cdot p > 0$, i.e., $v > \frac{(1-\theta_n)p - \theta_u\theta_{un} r}{\theta_f + \theta_u\theta_{uf}}$.

Similar to Proposition 1, we now derive the three cases:

Case (i): If $\bar{v} < \frac{p-\theta_{un} r}{\theta_{uf}}$, i.e., $r < \frac{p-\theta_{uf}\bar{v}}{\theta_{un}}$, then $U_e > U_l$ for all $v \in [0, \bar{v}]$. All unsure consumers will return early. Thus $v_l = 0$, $v_e = (\bar{v} - p)\frac{1}{\bar{v}}$.

Case (ii): If $r \geq \frac{p-\theta_{uf}\bar{v}}{\theta_{un}}$ and $p < \frac{(1-\theta_n)p-\theta_u\theta_{un} r}{\theta_f+\theta_u\theta_{uf}}$, then $\left(\theta_f + \theta_u\theta_{uf}\right)p < (1-\theta_n)p - \theta_u\theta_{un} r$, i.e., $r < p$. Thus if $\frac{p-\theta_{uf}\bar{v}}{\theta_{un}} < r < p$, then both early and late return segments exist. $v_e = \left(\frac{p-\theta_{un} r}{\theta_{uf}} - p\right)\frac{1}{\bar{v}}$, and $v_l = \left(\bar{v} - \frac{p-\theta_{un} r}{\theta_{uf}}\right)\frac{1}{\bar{v}}$. Note that consistent with the main model, v_e decreasing in r, v_l increasing in r, and $v_e + v_l$ being independent of r still holds.

Case (iii): Case (iii) exists if $p \geq \frac{(1-\theta_n)p-\theta_u\theta_{un} r}{\theta_f+\theta_u\theta_{uf}}$, i.e., $p \leq r$. However, this case cannot exist since we require $r < p$.

We now examine the profit functions:

In Case (i), $\pi = (p-c)(v_e + v_l)d - (p-s)v_e(\theta_n + \theta_u)d = \frac{d}{\bar{v}}(p - c - (p-s)(\theta_n + \theta_u))(\bar{v} - p)$, which is independent of r.

In Case (ii), $\pi = (p-c)(v_e+v_l)d - (p-s)v_e(\theta_n+\theta_u)d - (r-s)v_l\theta_u\theta_{un}d - (p-s)v_l\theta_n d = (p-c)(\bar{v} - p)\frac{d}{\bar{v}} - (p-s)(\theta_n+\theta_u)v_e d - ((r-s)\theta_u\theta_{un} + (p-s)\theta_n)v_l d$.

We next solve for the optimal r for Case (ii) We compute the derivative of profit w.r.t. r:

$$\frac{\partial \pi}{\partial r} = -(p-s)d(\theta_u+\theta_n)\frac{\partial v_e}{\partial r} - \theta_u\theta_{un}v_l d - ((r-s)\theta_u\theta_{un} + (p-s)\theta_n)\frac{\partial v_l}{\partial r}d.$$

Since $\frac{\partial v_e}{\partial r} = -\frac{\theta_{un}}{\theta_{uf}\bar{v}}$ and $\frac{\partial v_l}{\partial r} = \frac{\theta_{un}}{\theta_{uf}\bar{v}}$, this becomes

$$\frac{\partial \pi}{\partial r} = -(p-s)(\theta_u+\theta_n)\left(-\frac{\theta_{un}}{\theta_{uf}}\right)\frac{d}{\bar{v}} - \theta_u\theta_{un}\left(\bar{v} - \frac{p-\theta_{un} r}{\theta_{uf}}\right)\frac{d}{\bar{v}} - ((r-s)\theta_u\theta_{un} + (p-s)\theta_n)\left(\frac{\theta_{un}}{\theta_{uf}}\right)\frac{d}{\bar{v}}.$$

Simplifying yields

$$\frac{\partial \pi}{\partial r} = \frac{\theta_u\theta_{un}}{\theta_{uf}}\left(2p - \bar{v}\theta_{uf} - s(1-\theta_{un}) - 2\theta_{un} r\right)\frac{d}{\bar{v}}.$$

Setting $\frac{\partial \pi}{\partial r} = 0$ gives the optimal refund

$$r^*_{NR} = \frac{2p - \bar{v}\theta_{uf} - s(1-\theta_{un})}{2\theta_{un}}.$$

Comparing r^*_{NR} with the boundary between Case (i) and (ii), we have Case (ii), or the tiered-refund policy, dominates Case (i), or short-return-window policy, if $r^*_{NR} = \frac{2p-\bar{v}\theta_{uf}-s(1-\theta_{un})}{2\theta_{un}} > \frac{p-\theta_{uf}\bar{v}}{\theta_{un}}$, which yields the condition $\bar{v} > \frac{1-\theta_{un}}{\theta_{uf}}s$. Since $1-\theta_{un} = \theta_{uf}$ and $\bar{v} > p > s$, this condition always holds.

On the other hand, the tiered-refund policy dominates the full-refund policy if $r^*_{NR} < p$, yielding $\bar{v} > \frac{(1-\theta_{un})(2p-s)}{\theta_{uf}}$. Since $1-\theta_{un} = \theta_{uf}$, this yields $\bar{v} > 2p - s$. ■

Proof of Extension EC.1.4. We will focus on the case where $\bar{r} \geq \underline{r} > 0$ since the others are special cases. Suppose θ depends on v, and $\theta(v)$ is continuous and differentiable and defined on $[0,1]$. Note that $\theta'(v)$ can be either positive or negative. The expected utilities are given by $EU_e = -p + \theta(v) \cdot v + (1-\theta(v))(p - bv - a)$, and $EU_l = -p + \theta(v) \cdot v + (1-\theta(v))r$. We make the following two assumptions - Assumption 1: $\frac{\partial EU_e}{\partial v} > 0$, which means higher-value consumers have higher expected utility, and Assumption 2: $-\frac{1}{v} \leq \frac{\theta'(v)}{\theta(v)} \leq \frac{1}{p}$, which means $\theta'(v)$ is not too high or too low. Under these assumptions, we know EU_e increases in v. We next show that EU_l also increases in v: By Assumption 2, we have $\theta(v) - p\theta'(v) \geq 0$. We also know that $r \leq p$, therefore, if $\theta'(v) \geq 0$, then $\frac{\partial EU_l}{\partial v} = \theta'(v)v + \theta(v) - r\theta'(v) \geq \theta'(v)v + \theta(v) - p\theta'(v) \geq 0$. On the other hand, if $\theta'(v) < 0$, since Assumption 2 implies $\theta'(v)v + \theta(v) \geq 0$, we have $\frac{\partial EU_l}{\partial v} = \theta'(v)v + \theta(v) - r\theta'(v) \geq 0$. In both cases $EU_l(v)$ increases in v.

Also, when $EU_e = EU_l$, $p - bv - a = r$, i.e., $v = \frac{p-a-r}{b}$, which is unique. At $v = \frac{p-a-r}{b}$, we have $\frac{\partial(EU_e - EU_l)}{\partial v} = \frac{\partial}{\partial v}\Big((1-\theta(v))(p - bv - a - r)\Big) = -b(1-\theta(v)) - \theta'(v)(p - bv - a - r)\Big|_{v=\frac{p-a-r}{b}} = -b\Big(1 - \theta\left(\frac{p-a-r}{b}\right)\Big) < 0$. Therefore, EU_e and EU_l only intersect once, and EU_l increases in v faster than EU_e does when they intersect.

Similar to the main model, we now analyze the three cases:

Case (i): If $p - b\bar{v} - a > r$, then $EU_e > EU_l$ holds for all consumers, i.e., only early returns exist. For $EU_e > 0$ to hold, we need $-p + \theta(v) \cdot v + (1-\theta(v))(p - bv - a) > 0$. By Assumption 1, there exists a unique $v \triangleq z_e$ such that $EU_e(z_e) = 0$. Then consumers whose valuations are in the range $v \in [z_e, \bar{v}]$ buy and return early, while the rest do not buy. Note that z_e does not depend on r.

If $p - b\bar{v} - a < r$, then since both EU_e and EU_l are increasing in v and EU_l and EU_e only intersect once, and EU_l increases faster than EU_e at the intersection, we can only have two cases:

Case (ii): consumers whose valuations are in the range $v \in \left[z_e,\ \frac{p-a-r}{b}\right]$ buy and return a misfit product early, while consumers whose valuations are in the range $\left[\frac{p-a-r}{b},\ \bar{v}\right]$ buy and return a misfit product late.

Case (iii): Let $z_l(r)$ be the valuation v such that $EU_l(v) = 0$. It depends on r. Then consumers whose valuations are in the range $v \in [z_l(r),\ \bar{v}]$ will buy and return a misfit product late, while others do not buy.

It is unclear whether $\frac{\partial(EU_l - EU_e)}{\partial v}$ is monotonic in r. Therefore, Case (ii) and Case (iii) may alternate in $r \in [p - b\bar{v} - a,\ p]$. However, we will now show that $r = p - b\bar{v} - a$ falls into Case (ii) and $r = p$ falls into Case (iii). At $r = p - b\bar{v} - a$, $EU_l\left(\frac{p-a-r}{b}\right) = EU_e(\bar{v}) \geq 0$. Thus it is case (ii). At $r = p$, $EU_e \leq EU_l$ for all $v \in [0, \bar{v}]$, so it is case (iii).

We will show that at $r = p - b\bar{v} - a$, profit is increasing, and at $r = p$, profit is decreasing under certain conditions. This then proves that the tiered-refund policy is always better than the short-return-window policy, and is better than the full-refund policy under certain conditions.

In case (i), profit $\pi = (p-c)(\bar{v} - z_e)\frac{d}{\bar{v}} - (p-s)\frac{d}{\bar{v}}\int_{z_e}^{\bar{v}}(1-\theta(v))\,dv$, which is a constant independent of r.

In case (ii), profit $\pi = (p-c)(\bar{v} - z_e)\frac{d}{\bar{v}} - (p-s)\frac{d}{\bar{v}}\int_{z_e}^{\frac{p-a-r}{b}}(1-\theta(v))\,dv - (r-s)\frac{d}{\bar{v}}\int_{\frac{p-a-r}{b}}^{\bar{v}}(1-\theta(v))\,dv$.

$$\frac{\partial \pi}{\partial r}\bigg|_{r=p-b\bar{v}-a} = 0 - (p-s)\Big(1 - \theta(\frac{p-a-r}{b})\Big)(-\frac{d}{\bar{v}b} + (r-s)\Big(1 - \theta(\frac{p-a-r}{b})\Big)(-\frac{d}{b\bar{v}}) - d\frac{1}{\bar{v}}\int_{\frac{p-a-r}{b}}^{\bar{v}}(1-\theta(v))\,dv\bigg|_{r=p-b\bar{v}-a}$$

$$= (p-r)(1-\theta(\bar{v}))\frac{d}{b\bar{v}} \geq 0$$

Therefore, the short-return-window policy is always dominated by the tiered-refund policy.

In case (iii), profit $\pi = d\frac{1}{\bar{v}}\int_{z_l(r)}^{\bar{v}} \big(p-c-(1-\theta(v))(r-s)\big)dv$.

$$\frac{\partial \pi}{\partial r} = d\frac{1}{\bar{v}}\int_{z_l(r)}^{\bar{v}} -(1-\theta(v))\,dv + \Big(p-c+\big(1-\theta(z_l(r))\big)(r-s)\Big)\cdot\left(-\frac{\partial z_l(r)}{\partial r}\right).$$

The first term is negative. However, $\frac{\partial z_l(r)}{\partial r} < 0$ (shown below), so the second term is positive.

$$\frac{\partial z_l(r)}{\partial r} = -\frac{\frac{\partial EU_l}{\partial r}}{\frac{\partial EU_l}{\partial v}}\bigg|_{v=z_l(r)} = -\frac{1-\theta(v)}{\frac{\partial EU_l}{\partial v}}\bigg|_{v=z_l(r)} < 0 \quad \text{since } \frac{\partial EU_l}{\partial v} > 0 \text{ as shown before.}$$

If $\frac{\partial \pi}{\partial r}\big|_{r=p} < 0$ then the tiered-refund policy is better than the full-refund policy, i.e., if

$$\frac{d}{\bar{v}}\int_{z_l(p)}^{\bar{v}} -(1-\theta(v))\,dv + \Big(p-c-\big(1-\theta(z_L(p))\big)\cdot(p-s)\Big)\left(-\frac{\partial z_l(r)}{\partial r}\bigg|_{r=p}\right) < 0.$$

We will show that $z_l(p) = p$: Note that $z_l(p)$ is the v such that $EU_l(v) = 0$ at $r = p$. When $r = p$, $EU_L = -p+\theta(v)\cdot v+(1-\theta(v))\cdot p = \theta(v)(v-p)$. At $v = p$, $EU_l(v) = 0$. Therefore $z_l(p) = p$.

So the condition becomes: if

$$\frac{d}{\bar{v}}\int_{p}^{\bar{v}} -(1-\theta(v))\,dv + (p\cdot\theta(p)-c+(1-\theta(p))s)\cdot\left(-\frac{\partial z_l(r)}{\partial r}\bigg|_{r=p}\right) < 0.$$

Since

$$\frac{\partial z_l(r)}{\partial r}\bigg|_{r=p} = -\frac{1-\theta(v)}{\frac{\partial EU_l}{\partial v}}\bigg|_{r=p,\,v=z_l(r)} = -\frac{1-\theta(v)}{\theta(v)}\bigg|_{v=z_l(r)} = -\frac{1-\theta(p)}{\theta(p)},$$

the condition becomes: if

$$\frac{d}{\bar{v}}\int_{p}^{\bar{v}} -(1-\theta(v))\,dv + \Big(p\theta(p)-c+(1-\theta(p))s\Big)\frac{1-\theta(p)}{\theta(p)} < 0 \tag{EC.16}$$

then the tiered-refund policy is better than the full-refund policy. The condition is equivalent to $\int_p^{\bar{v}}(1-\theta(v))\,dv > \Big(p\theta(p)-c+(1-\theta(p))s\Big)\frac{1-\theta(p)}{\theta(p)}$, so as in the main model, the tiered-refund policy is better than the full-refund policy if $\bar{v}$ is large enough relative to p.

Furthermore, we now show that there exist common forms of $\theta(v)$ that satisfy Assumptions 1 and 2. Specifically, we examine three possible form of $\theta(v)$: $\theta(v) = \theta$, $\theta(v) = k+\delta v$, and $\theta(v) = k-\delta v$, with $k, \delta \geq 0$.

(1) If $\theta(v) = \theta$, then $\frac{\theta'(v)}{\theta(v)} = 0$, so Assumption 2 is satisfied. Assumption 1 is satisfied under the assumptions in the main model.

(2) If $\theta(v) = k+\delta v$, then $\frac{\theta'(v)}{\theta(v)} = \frac{\delta}{k+\delta v} \leq \frac{\delta}{k}$. So a sufficient condition to satisfy Assumption 2 is $\frac{\delta}{k} \leq \frac{1}{p}$, i.e., $\delta \leq \frac{k}{p}$. For Assumption 1, plugging in $\theta(v) = k+\delta v$, we have $EU_e = -p+(k+\delta v)v+(1-k-\delta v)(p-$

$bv-a)$, so $\frac{\partial EU_e}{\partial v}=k+2\delta v-b(1-k-\delta v)-\delta(p-bv-a)=2(1+b)\delta v+k(1+b)-b-\delta(p-a)\geq k(1+b)-b-\delta(\bar{v}-a)$ since $p\leq\bar{v}$. Therefore, Assumption 1 is satisfied as long as $\delta\leq\frac{k(1+b)-b}{\bar{v}-a}$.

(3) If $\theta(v)=k-\delta v$, then $\frac{\theta'(v)}{\theta(v)}=-\frac{\delta}{k-\delta v}$. For Assumption 2 to hold we need $-\frac{\delta}{k-\delta v}\geq-\frac{1}{v}$, i.e., $\delta\leq\frac{k}{2v}$. So a sufficient condition is $\delta\leq\frac{k}{2\bar{v}}$. For Assumption 1, plugging in $\theta(v)=k-\delta v$, we have $EU_e=-p+(k-\delta v)v+(1-k+\delta v)(p-bv-a)$, so $\frac{\partial EU_e}{\partial v}=k-2\delta v-b(1-k+\delta v)+\delta(p-bv-a)=k(1+b)-b+\delta(p-a)-2(1+b)\delta v\geq k(1+b)-b-2(1+b)\delta v$. Therefore, a sufficient condition for Assumption 1 to hold is $k(1+b)-b-2(1+b)\delta\bar{v}\geq 0$, i.e., $\delta\leq\frac{k(1+b)-b}{2(1+b)\bar{v}}$.

In summary, for both $\theta(v)=k+\delta v$ and $\theta(v)=k-\delta v$, as long as δ is not too large, i.e., when θ is not overly sensitive to v , Assumptions 1 and 2 can be satisfied. ■

Proof of Extension EC.1.5. Based on the results in Propositions 3 and 4, we have:

(Tiered-EL outcome) If $\bar{v}>\frac{2\theta bp+\left(\theta+(1-\theta)b\right)a}{\left(\theta-(1-\theta)b\right)b}$, i.e., if $p<\frac{\bar{v}\left(\theta-(1-\theta)b\right)b-\left(\theta+(1-\theta)b\right)a}{2\theta b}$, then $r^*_{NR}=p-\frac{b\bar{v}+a}{2}$.

$$\begin{aligned}\pi(p,r^*_{NR}(p))&=(\theta p-c+(1-\theta)s)\Big(\bar{v}-\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}\Big)\frac{d}{\bar{v}}+(p-r)(1-\theta)\Big(\bar{v}-\frac{p-r-a}{b}\Big)\frac{d}{\bar{v}}\\&=(\theta p-c+(1-\theta)s)\Big(\bar{v}-\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}\Big)\frac{d}{\bar{v}}+(p-p+\frac{b\bar{v}+a}{2})(1-\theta)\Big(\bar{v}-\frac{p-p+(b\bar{v}+a)/2-a}{b}\Big)\frac{d}{\bar{v}}\\&=(\theta p-c+(1-\theta)s)\Big(\bar{v}-\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}\Big)\frac{d}{\bar{v}}+(1-\theta)\frac{(b\bar{v}+a)^2}{4b}\frac{d}{\bar{v}}\end{aligned}$$

Setting $\frac{\partial\pi}{\partial p}=\theta\Big(\bar{v}-\frac{\theta p+(1-\theta)a}{\theta-(1-\theta)b}\Big)\frac{d}{\bar{v}}-\frac{(\theta p-c+(1-\theta)s)\theta}{\theta-(1-\theta)b}\frac{d}{\bar{v}}=\frac{\theta\Big(\bar{v}(\theta-(1-\theta)b)-2\theta p-(1-\theta)a+c-(1-\theta)s\Big)}{\theta-(1-\theta)b}\frac{d}{\bar{v}}=0$ and solve for p, we have $p^*=\frac{\bar{v}(\theta-(1-\theta)b)-(1-\theta)a+c-(1-\theta)s}{2\theta}$. We now compare p^* with the boundary:

$$\frac{\bar{v}(\theta-(1-\theta)b)-(1-\theta)a+c-(1-\theta)s}{2\theta}-\frac{\bar{v}(\theta-(1-\theta)b)b-(\theta+(1-\theta)b)a}{2\theta b}=$$
$$\frac{-(1-\theta)ab+(\theta+(1-\theta)b)a+cb-(1-\theta)sb}{2\theta b}>\frac{a}{2b}>0$$

Thus π is increasing in this region.

(Tiered-Indifferent outcome) If $\frac{2\theta p+2(1-\theta)a}{\theta-(1-\theta)b}-\frac{1}{\theta}(c-(1-\theta)s)<\bar{v}\leq\frac{2\theta bp+\left(\theta+(1-\theta)b\right)(a)}{\left(\theta-(1-\theta)b\right)b}$ (i.e., $(1-\frac{1-\theta}{\theta}b)\frac{\bar{v}}{2}-(1+\frac{1-\theta}{\theta}b)\frac{a}{2b}<p\leq(1-\frac{1-\theta}{\theta}b)\frac{\bar{v}}{2}-\frac{1-\theta}{\theta}a)+(1-\frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta}$), then

$$r^*_{NR}=\frac{(\theta-b)p-\theta a}{\theta-(1-\theta)b}.$$

$$\begin{aligned}\pi&=\big(p-c-(1-\theta)(r-s)\big)\Big(\bar{v}-\frac{p-(1-\theta)r}{\theta}\Big)\frac{d}{\bar{v}}\\&=\Big(p-c+(1-\theta)s-(1-\theta)\frac{(\theta-b)p-\theta a}{\theta-(1-\theta)b}\Big)\Big(\bar{v}-\frac{p-(1-\theta)\frac{(\theta-b)p-\theta a}{\theta-(1-\theta)b}}{\theta}\Big)\frac{d}{\bar{v}}\end{aligned}$$

Setting

$$\frac{\partial\pi}{\partial p}=\left(\bar{v}-\frac{2}{\theta}\Big(p-(1-\theta)\frac{(\theta-b)p-\theta a}{\theta-(1-\theta)b}\Big)+c-(1-\theta)s\right)\cdot\frac{\partial}{\partial p}\Big(p-(1-\theta)r^*(p)\Big)\frac{d}{\bar{v}}=0,$$

we have $p^* = \frac{\left(\bar{v}+c-(1-\theta)s\right)\left(\theta-(1-\theta)b\right)-2(1-\theta)a}{2\theta}$, which equals to the boundary. Thus π is increasing in this region.

(Tiered-L outcome) If $2p - \frac{c-(1-\theta)s}{\theta} < \bar{v} < \frac{2\theta p+2(1-\theta)a}{\theta-(1-\theta)b} - \frac{1}{\theta}(c-(1-\theta)s)$, i.e., $(1-\frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1-\frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta} < p < \frac{\theta\bar{v}+c-(1-\theta)s}{2\theta}$, then $r^*_{NR} = \frac{2p-\theta\bar{v}-c+(1-\theta)s}{2(1-\theta)}$. The profit is given by

$$\begin{aligned}
\pi &= \left(p-c-(1-\theta)(r-s)\right)\left(\bar{v}-\frac{p-(1-\theta)r}{\theta}\right)\frac{d}{\bar{v}} \\
&= \left(p-c+(1-\theta)s-(1-\theta)\frac{2p-\theta\bar{v}-c+(1-\theta)s}{2(1-\theta)}\right)\left(\bar{v}-\frac{1}{\theta}\left(p-(1-\theta)\frac{2p-\theta\bar{v}-c+(1-\theta)s}{2(1-\theta)}\right)\right)\frac{d}{\bar{v}} \\
&= \frac{(\theta\bar{v}+c-(1-\theta)s)^2}{4\theta}\frac{d}{\bar{v}},
\end{aligned}$$

which is a constant.

(Full-Refund policy) If $\bar{v} \le 2p - \frac{c-(1-\theta s)}{\theta}$, i.e., $p \ge \frac{\theta\bar{v}+c-(1-\theta)s}{2\theta}$, then $r^*_{NR} = p$ and

$$\pi = \left(p-c-(1-\theta)(p-s)\right)\left(\bar{v}-\frac{p-(1-\theta)p}{\theta}\right)\frac{d}{\bar{v}} = \left(\theta p - c + (1-\theta)s\right)(\bar{v}-p)\frac{d}{\bar{v}}.$$

The optimal price is $p^* = \frac{\theta\bar{v}+c-(1-\theta)s}{2\theta}$. Thus π is decreasing in this region.

In summary, π is increasing – constant – decreasing. The optimal price is any $p \in \left[(1-\frac{1-\theta}{\theta}b)\frac{\bar{v}}{2} - \frac{1-\theta}{\theta}a + (1-\frac{(1-\theta)}{\theta}b)\frac{c-(1-\theta)s}{2\theta},\ \ \frac{\theta\bar{v}+c-(1-\theta)s}{2\theta}\right]$ and $r(p) = \frac{2p-\theta\bar{v}-c+(1-\theta)s}{2(1-\theta)}$. The optimal profit is $\frac{(\theta\bar{v}+c-(1-\theta)s)^2}{4\theta}\frac{d}{\bar{v}}$. Note that this optimal profit is the same as that under the full-refund policy. ■

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