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RESEARCH ARTICLE

Exact inference from finite market data

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Abstract

We derive conditions for individual choices and ϵ-equilibrium prices to be inferred from a finite set of market data in the following sense: Although potential observations are infinite, inference can be attained and known to have been attained after a finite number of observations. While we first examine the case of observations on an individual's demand at different prices and incomes, our primary focus is on equilibrium, where only profiles of individual endowments and associated equilibrium prices are observable. Our main result is that finite inference is possible in the equilibrium setting if one wants to infer either individual choices or the ϵ-equilibrium correspondence.

Keywords Identification · Finite data · Preferences · Demand · Walrasian equilibrium

JEL Classification D80 · G10

From only a finite number of observations of market data, can we infer what an individual shall decide when faced with choices not previously encountered? Can we infer equilibrium prices and allocations as individual endowments vary? Are the finite data needed to make such inference commonly accessible?

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To clarify our notion of finite inference, consider an abstract setting of an economic model $\mathbf{M}$ that maps exogenous variables $x \in \mathbf{X} \subset \mathbb{R}^n$ into endogenous variables $y \in \mathbf{Y} \subset \mathbb{R}^m$. This map may be a correspondence, with $\mathbf{M}(x) \subset \mathbf{Y}$. A learning algorithm $\mathcal{A}$ takes as input a finite dataset $\mathbf{D} \subset \text{graph}(\mathbf{M})$ and a single exogenous point $x^* \in \mathbf{X}$, and it returns a set $\mathcal{M}(x^*) \subset \mathbf{Y}$, such that $\mathbf{M}(x^*) \cap \mathcal{M}(x^*) \neq \phi$. The (generalization) error of $\mathcal{A}$ at x^* is

$$\epsilon = \sup_{s,s' \in \mathcal{M}(x^*)} \|s - s'\|.$$

Given an infinite sequence $\{x_k, k = 1, \ldots\}$ that becomes dense in the set of exogenous variables $\mathbf{X}$ and finite and nonempty $\mathbf{M}_k \subset \mathbf{M}(x_k)$, $k = 1, \ldots$, the algorithm $\mathcal{A}$ finitely identifies $\mathbf{M}$ if , for any $\epsilon > 0$ and for any exogenous variable x^*, there is some n such that $\mathcal{A}$ generates a generalization error of less than ϵ on the finite dataset $\mathbf{D} = \{(x_k, \mathbf{M}_k), k = 1, \ldots, n\}$. Notice this notion of finite inference only requires inference at *one* specific price and not the infinite correspondence.

Intuitively, our algorithm maps finite data sets to several possible values of the underlying map at the given exogenous point x^*. Our notion of inference requires that for all dense sequences of data, all possible values the algorithm outputs are (eventually) arbitrarily close to each other. As such, finite inference is a joint statement about the set $\mathbf{X}$, the model $\mathbf{M}$, and the learning algorithm $\mathcal{A}$.

Suppose that the underlying model $\mathbf{M}$ is an income Lipschitz demand function. Consider the algorithm $\mathcal{A}$ that, at an exogenous price p^* and for any dataset, returns the set of demands not revealed as preferred at p^*. We show that $\mathcal{A}$ finitely identifies $\mathbf{M}$.

However, if we consider $\mathbf{M}$ to be the equilibrium correspondence that maps profiles of individual endowments to equilibrium prices and allocations, finite inference can generally not be established. This is due to the multiplicity of equilibrium prices and the non-computability of the $\epsilon - \delta$ relationship between excess demand and equilibrium prices (Richter and Wong 1999). In fact, if equilibrium is known to be unique for all profiles of individual endowments or if the $\epsilon - \delta$ relationship can be established, we show that finite inference can be restored for equilibrium prices. Our main result is that, in the equilibrium setting (under conditions on preferences we explain below), finite inference is possible if we require inference to be a statement on the $\epsilon-$equilibrium correspondence. This is true even if only equilibrium prices (but not allocations) are observable.

We consider two classes of data: First, we examine the case of observations on an individual's demand at different prices and incomes. Second, for the case of an exchange economy, we assume that only equilibrium prices and profiles of individual endowments are observable[1]. We consider both the *local* case, where observations lie in some open set of prices and incomes (in the case of individual demand) or some open set of individual endowments, as well as the *global* case, where we allow for all positive prices and incomes, or, in the case of equilibrium, all aggregate endowments

[1] While, in some settings, individual consumption is observable, there are other settings where only partial or no observations are available. We derive our results for the extreme case where there are no observations on individual demand.

and any compact set of income distributions. In both settings, we consider sequences of nested observations that become dense in the chosen domains as in Mas-Colell (1978)[2].

For the case of individual demand, our result follows from a result in Mas-Colell (1977), who showed that if a consumption bundle x is strictly preferred to a bundle y, there must exist prices and incomes such that x is revealed preferred to y.[3] The result can also be derived from Mas-Colell (1978) who proved that any sequence of preferences $\succeq^n$ that rationalize the data for every n must converge to the true preferences. To obtain our results, one needs to show that, for a given n, there is an effective method (as defined, for example, in Hunter (1973)) to verify that any $\succeq^n$ that rationalizes the observations up to n satisfies $x \succeq^n y$. This additional step follows from the existing literature under our assumptions of smoothness and density of observations. These assumptions allow us to construct the required rationalizers, and smoothness allows us to conduct the verification as well as gurantees that our procedure will eventually terminate.

Our primary focus is on equilibrium, where one observes aggregate endowments, incomes, and prices. We first provide sufficient conditions for the following result to hold. If x is strictly preferred to y by some individual, then sufficiently many observations of equilibrium prices and endowment profiles suffice to prove that the individual strictly prefers x to y. We show that, while predictions of exact comparative statics are generally impossible, we can predict approximate equilibrium prices from finite data. This is true for both setups, regardless of whether observations consist of individual choices as prices and incomes vary or equilibrium prices as profiles of individual endowments vary.

Our analysis is heavily based on the extensive literature on testable implications of demand. Afriat (1967) established the generalized axiom of revealed preference as necessary and sufficient for a finite set of demand data to be derived from the maximization of a preference relation or ordinal utility function. Hurwicz et al. (1971) and Mas-Colell (1977) gave necessary and sufficient conditions for the integrability of a demand function, the derivation of the generating ordinal utility function, and for a demand function to identify preferences.

Brown and Matzkin (1996) extended (Afriat 1967) to a finite number of observations on profiles of equilibrium endowments and Walrasian equilibrium prices – that is, observations on the equilibrium manifold. Chiappori et al. (2004) showed that the equilibrium manifold locally identifies individual preferences. Brown and Matzkin (1990), and in particular, Matzkin (2006) proved a global version of the result.

Although these results on identification answer our motivating question affirmatively for an infinite number of, they say nothing for the case of a finite number of observations, and they do not address the question of asymptotics. Mas-Colell (1978)

[2] Alternatively, observations are drawn randomly from a uniform distribution of exogenous variables, and the endogenous data is generated by utility-maximizing individuals. In this case, all our statements are valid only with probability one.

[3] How these revealing prices and incomes are to be found is not addressed. We show that if x is strictly preferred to y, and if prices and incomes are given by arbitrary sequences of nested observations that become dense in the chosen domains, then, after finitely many draws, we must observe prices and incomes so that x is strictly revealed preferred to y.

solved this question by considering a nested, increasing sequence of demand data that, at the limit, covers a dense set of prices and provided sufficient conditions for any associated sequence of preference relations to converge to the unique preference relation that generated demand. In a recent paper, Chambers et al. (2021) considered the case where data sets are collections of choices from pairwise comparisons of alternatives. Convergence is obtained only if the preferences satisfy a condition implied by, but weaker than monotonicity. For the case of a strictly strictly monotone preference relation, convergence follows from Forges and Minelli (2009).

Beigman and Vohar (2006) analyzed the PAC learnability of demand functions in environments similar to ours. They first demonstrated that the fat-shattering dimension of the set of income–Lipschitz demand functions is infinite, implying that such demand functions are not PAC learnable. They further showed that an upper bound on the Lipschitz constant renders the shattering dimension finite and enables uniform learning guarantees. Our definition of learnability is stronger than the notion of PAC learnability as we require the generalization error to be small with probability one rather than in expectation. Hence, even if we restrict the underlying set of models to functions rather than correspondences, any possibility of uniform guarantees for convergence rates is ruled out by their results.

We circumvent this limitation because the dataset size required for finite inference depends on the underlying model. A similar logic applies to the equilibrium correspondence. We make no assumptions on how prices depend on economic fundamentals. Intuitively, Lipschitz bounds are necessary for uniform learning because they limit how rapidly demand can vary with inputs; without such constraints, small input changes could induce arbitrarily large shifts, undermining uniform error guarantees. In our setting, however, the analyst is ex post certain whether the generalization error is below any ϵ; uncertainty lies only in the sample size needed to achieve it. Our framework thus replaces ex ante uniform guarantees with ex post verifiability.

An obvious question concerns possible empirical applications of our results. Blundell (2005) pointed to the advantages of the empirical application of revealed preference theory for measuring consumers' responses to variations in prices and income. In addition to the apparent measurement error problem, applied work should address the fact that a failure of the strong axiom does not help characterize the nature of irrationality or the degree/direction of changing tastes – Blundell et al. (2003) and Blundell et al. (2008).

In addition, none of this work considered observations restricted to aggregate demand or equilibrium prices, and it is unclear how these problems can be addressed in that setting. Therefore, the results in this paper are not directly applicable to empirical work. However, it is important to identify circumstances in which identification is possible in principle. In the absence of identification, empirical non-parametric work is futile.

The rest of the paper is organized as follows. Section 1 outlines some preliminaries. Section 2 considers the case where individual demand is observable, and Section 3 focuses on observations of the equilibrium correspondence. All proofs can be found in the appendix.

1 Preliminaries

We collect the definitions and results needed for our analysis.

1.1 Preferences

An individual has a (complete and transitive) preference relation $\succeq$ over the consumption set $\mathbb{R}^L_+$. The preference relation $\succeq$ is upper semi-continuous if, for every $x \in \mathbb{R}^L_+$, the upper contour set $\mathbf{R}_+(x) = \{y : y \succeq x\}$ is closed. It is continuous if, for every $x \in \mathbb{R}^L_+$, the upper contour set as well as the lower contour set $\mathbf{R}_-(x) = \{y : x \succeq y\}$ are closed. It is strictly monotonically increasing (or simply strictly monotone) if $x > y$ implies $x \succ y$. It is convex if $x \succeq y$ implies that $\lambda x + (1 - \lambda)y \succeq y$ for all $\lambda \in [0, 1]$, and it is strictly convex if $x \succeq y$, $x \neq y$, implies that $\lambda x + (1 - \lambda)y \succ y$ for all $\lambda \in (0, 1)$.

If, for an open set $\mathbf{X} \subset \mathbb{R}^L_+$, $x \succeq y \Leftrightarrow x \succeq' y$, whenever $x, y \in \mathbf{X}$ we write $\succeq =_{\mathbf{X}} \succeq'$.

In the arguments that follow, we assume that the preferences are differentiably strictly monotonic and strictly convex in the sense of Debreu (1972): They can be represented by a utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ that is twice continuously differentiable in $\mathbb{R}^L_{++}$, the interior of the consumption set, with $Du(x) \gg 0$ and $D^2u(x)$ negative definite on the orthogonal complement of the gradient $[Du(x)]^{\perp}$. We refer to such preferences simply as "smooth preferences". The assumption of smooth preferences greatly simplifies many of the arguments, but it should be emphasized that most of our results hold under strictly weaker assumptions. Preferences can be identified from demand functions globally if they are increasing, convex and Lipschitzian following (Mas-Colell 1977). Smooth preferences are always Lipschitzian, but the converse does not hold. Under the assumption of Lipschitzian preferences, inverse demand functions may not exist, and it is unclear how to define appropriate domains for local identification. Local recoverability was shown assuming that preferences generate income Lipschitzian demand in Hurwicz et al. (1971). All smooth preferences satisfy this condition, and all preferences that satisfy this condition are Lipschitzian in the sense of Mas-Colell (1977), but there is no equivalence. For our purposes, income Lipschitzian demand functions are not sufficient because our local analysis relies on the existence of an inverse demand function. Diasakos and Gerasimou (2022) proved that inverse demand exists under strictly weaker assumptions than smoothness, but their conditions on preferences do not imply the Lipschitzian property and are not known to guarantee the identification of demand. To simplify the exposition, we assume smooth preferences.

1.2 Demand

There are L commodities, $l \in \mathbf{L}$. Budget sets are

$$\mathbf{B}(p) = \{x \in \mathbb{R}^L_+ : p \cdot x \leq 1\}, \quad p \gg 0,$$

and the Walrasian demand correspondence is defined by

$$f^{\succeq}(p) = \{x \in \mathbf{B}(p) : x \succeq y, \quad \text{for all } y \in \mathbf{B}(p)\}.$$

For a strictly monotone, strictly convex and continuous preference relation $\succeq$, we use the same notation, $f^{\succeq}(\cdot)$ to denote the (continuous) Walrasian demand function that is generated by $\succeq$. When we want to vary income explicitly, we write

$$d^{\succeq}(p, w) = f^{\succeq}(\frac{p}{w}).$$

Under our assumption of smooth preferences, demand in the interior of the consumption set is characterized by

$$D_x u(x) - \lambda p = 0, \quad p \cdot x = 1, \quad \lambda > 0, \tag{1}$$

and the following properties of demand are easy to derive:

- For $x \in \mathbb{R}^L_{++}$, there is a unique $p \in \mathbb{R}^L_{++}$, such that $x = f^{\succeq}(p)$. That is, there exists an inverse demand function $f^{\succeq -1} : \mathbb{R}^L_{++} \to \mathbb{R}^L_{++}$. From (1), it follows that this function is continuously differentiable.
- For $p = f^{\succeq -1}(x)$, for $x \in \mathbb{R}^L_{++}$, the demand function $f^{\succeq}(p)$ is continuously differentiable and satisfies the Slutsky decomposition

$$D_p d^{\succeq}(p, w) = S^{\succeq}(p, w) - v^{\succeq}(p, w) d^{\succeq}(p, w)', \tag{2}$$

  where the matrix $S^{\succeq}(p, w)$ is symmetric, of rank $(L - 1)$, negative semi-definite and satisfies $pS^{\succeq}(p, w) = 0$, and the vector $v^{\succeq}(p, w) = D_w d^{\succeq}(p, w)$ satisfies $pv^{\succeq}(p, w) = 1$.
- For any open, convex set $\mathbf{X} \subset \mathbb{R}^L_+$

$$\succeq =_{\mathbf{X}} \succeq' \quad \Leftrightarrow \quad f^{\succeq -1}(x) = f^{\succeq' -1}(x), \quad \text{for all } x \in \mathbf{X}.$$

  As shall become apparent below, this local identification result is a necessary condition for most of our results on inference from finite data.

For open sets $\mathbf{P} \subset \mathbb{R}^L_{++}$ and $\mathbf{W} \subset \mathbb{R}_{++}$ we say that a function $f : \mathbf{P} \times \mathbf{W} \to \mathbb{R}^L_+$ is a demand function if it is homogenous of degree zero and if it satisfies Walras' law. We say that it satisfies the Slutsky equation if it is continuously differentiable and (2) holds.

1.3 Equilibrium

We consider a pure exchange economy with H individuals, $h \in \mathbf{H}$. We assume consumption sets are $\mathbb{R}^L_+$, and each individual, h has smooth preferences $\succeq^h$, and endowment $e^h \in \mathbb{R}^L_{++}$. A profile of preferences across individuals is $\succeq^{\mathbf{H}}$, and a profile of endowments across individuals is $e^{\mathbf{H}} \in \mathbb{R}^{HL}_{++}$ and aggregate endowments are $e = \sum_{h \in \mathbf{H}} e^h$.

The equilibrium correspondence is defined by

$$\Pi^{\succeq^{\mathbf{H}}}(e^{\mathbf{H}}) = \{p \in \mathbb{R}^L_{++} : \sum_{h \in \mathbf{H}} (f^{\succeq^h}(\frac{p}{p \cdot e^h}) - e^h) = 0\}.$$

Chiappori et al. (2004) and Matzkin (2006) derived sufficient conditions on preferences that ensure classical identification: for any two preference profiles $\succeq^{\mathbf{H}}$ and $\tilde{\succeq}^{\mathbf{H}}$,

$$\succeq^{\mathbf{H}} = \tilde{\succeq}^{\mathbf{H}} \Leftrightarrow \Pi^{\succeq^{\mathbf{H}}}(\cdot) = \Pi^{\tilde{\succeq}^{\mathbf{H}}}(\cdot).$$

More interestingly, they provided a local version of the result. For our setting of finitely many observations, to state the local result, it is helpful to observe that this problem is identical to the identification of preferences from aggregate demand.

For individual incomes $w^{\mathbf{H}} = (w^1, \ldots, w^H) \in \mathbb{R}^H_{++}$, we define the aggregate demand function

$$d^{\succeq^{\mathbf{H}}}(p, w^{\mathbf{H}}) = \sum_{h \in \mathbf{H}} f^{\succeq^h}(\frac{p}{w^h}).$$

Note that, for any $e \in \mathbb{R}^L_{++}$ and any profile of incomes $w^{\mathbf{H}} = (w^1, \ldots, w^H)$,

$$p \in \Pi^{\succeq^{\mathbf{H}}}(\frac{w^1}{\sum_{h \in \mathbf{H}} w^h} e, \ldots, \frac{w^H}{\sum_{h \in \mathbf{H}} w^h} e) \Leftrightarrow d^{\succeq^{\mathbf{H}}}(p, w^{\mathbf{H}}) = e.$$

Therefore, for two profiles of preference relations $\succeq^{\mathbf{H}}$ and $\tilde{\succeq}^{\mathbf{H}}$,

$$d^{\succeq^{\mathbf{H}}}(\cdot) = d^{\tilde{\succeq}^{\mathbf{H}}}(\cdot) \Leftrightarrow \Pi^{\succeq^{\mathbf{H}}}(\cdot) = \Pi^{\tilde{\succeq}^{\mathbf{H}}}(\cdot). \tag{3}$$

We define $\mathbf{W}^{\mathbf{H}} \times \mathbf{P} = \times_{h \in \mathbf{H}} \mathbf{W}^h \times \mathbf{P} \subset \times_{h \in \mathbf{H}} \mathbb{R}_{++} \times \mathbb{R}^L_{++}$ to be an open set of incomes and prices: incomes and prices are observed locally. As Matzkin (2006) explained, this differentiates crucially the setting from Balasko (2004), who pointed out that, without restrictions on the domain of the equilibrium correspondence, the problem reduces to the individual problem when individual endowments tend to vanish for all but one individual. In this context, it is important to note that even for the case $\mathbf{P} = \mathbb{R}^L_{++}$ the results in Balasko (2004) only apply if individual incomes can be arbitrarily close to zero for all but one individual. In our analysis below, our primary focus will be on the case where all incomes are bounded away from zero (but prices become dense in $\mathbb{R}^L_{++}$).

We make the following high-level assumption on preferences and observations introduced in Matzkin (2006).

Assumption 1 *(heterogeneity of income effects)* For any individual $h \in \mathbf{H}$ and any preference relation $\succeq' \neq \succeq^h$ that generates a demand function $d^{\succeq'}(\cdot)$ on $\mathbf{P} \times \mathbf{W}^h$, there exist $p \in \mathbf{P}$ and $w, w' \in \mathbf{W}^h$, such that

$$d^{\succeq^h}(p, w) - d^{\succeq^h}(p, w') \neq d^{\succeq'}(p, w) - d^{\succeq'}(p, w'). \tag{4}$$

Note that this is a joint assumption on an individual's preferences and on the domain of the demand function $\mathbf{P} \times \mathbf{W}^h$.

The following example shows that local identification may fail if the assumption is violated, as the following example shows.

Example 1 Given any utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ and associated demand function $d^u(p, w)$, define, for $\epsilon \in \mathbb{R}^L$ and $x + \epsilon > 0$,

$$u^\epsilon(x) = u(x_1 + \epsilon_1, \ldots, x_L + \epsilon_L).$$

For $w + \sum_{l=1}^L \epsilon_l p_l > 0$, the associated demand function is

$$d^{u^\epsilon}(p, w) = d^u(p, w + p \cdot \epsilon) - \epsilon.$$

If $u(\cdot)$ represents quasi-homothetic preferences, $D_w d^{\succeq}(p, w)$ is constant in w, and it is easy to see that Assumption 1 fails for the preference profiles $(\succeq^1, \succeq^2)$ and $(\succeq^{1'}, \succeq^{2'})$, where $\succeq^1$ is identical to $\succeq^2$ and represented by $u(\cdot)$ while $\succeq^{1'}$ is represented by $u^\epsilon(\cdot)$ and $\succeq^{2'}$ is represented by $u^{-\epsilon}(\cdot)$, for some $\epsilon \neq 0$, with $w + p\epsilon > 0$, $w - p\epsilon > 0$ for all $p \in \mathbf{P}$ and all $w \in \mathbf{W}$. In this case, preferences cannot be identified from aggregate demand. □

The following lemma is used throughout.

Lemma 1 *Assumption 1 implies that for any* $\succeq^{\mathbf{H}} \neq \succeq'^{\mathbf{H}}$, *there exist prices* $\bar{p} \in \mathbf{P}$ *and incomes* $\bar{w}^{\mathbf{H}} \in \mathbf{W}$, *such that*

$$d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) \neq d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}).$$

By the continuity of the aggregate demand function, the assumption implies that, for different profiles of preferences, there must exist an open neighborhood of prices and incomes where aggregate demand (as a function of prices and profiles of income) differs.

The following proposition gives sufficient conditions for Assumption 1.

Proposition 1 *If preferences are smooth, Assumption 1 holds if, for each* $h \in \mathbf{H}$, *at least one of the following conditions holds:*

a. income can be arbitrarily low or

$$(p, w) \in \mathbf{P} \times \mathbf{W}^h \Rightarrow \lambda w \in \mathbf{W}^h, \textit{for all } \lambda \in (0, 1],$$

b. there are $p \in \mathbf{P}$ *and distinct* $w, w', w'' \in \mathbf{W}^h$, *such that, for all* $\lambda \neq 0$,

$$(v^{\succeq^h}(p, w) - v^{\succeq^h}(p, w')) \neq \lambda(v^{\succeq^h}(p, w) - v^{\succeq^h}(p, w'')). \tag{5}$$

Clearly, when preferences are homothetic, condition (5) never holds. Example 1 shows that quasi-homothetic preferences cannot be identified with local observations.

The condition goes a bit further than ruling out just quasi-homothetic preferences. It is easy to see that Property (5), together with the assumption that the income effect $v_l^h(\cdot)$ is a twice differentiable function of income w, implies the following condition from Chiappori et al. (2004):

1. for every commodity,
$$\frac{\partial v_l^h}{\partial w} \neq 0,$$
while
2. there exist commodities, m and n , such that
$$\frac{\partial}{\partial w}(\ln \frac{\partial v_m^h}{\partial w}) \neq \frac{\partial}{\partial w}(\ln \frac{\partial v_n^h}{\partial w}).$$

Kübler and Polemarchakis (2024) showed that under smoothness, the two conditions are equivalent. To better understand this assumption, it is useful to discuss (Lewbel 1987). This paper gives a full characterization of demand systems that can be written as
$$d(p, w) = a_0(p) + a_1(p)w + a_2(p)h(w)$$
for differentiable functions $a_i : \mathbb{R}^L_{++} \to \mathbb{R}^L$ and a differentiable $h : \mathbb{R}_{++} \to \mathbb{R}$. It is easy to see ((Kübler and Polemarchakis 2024) provided a detailed argument) that this is equivalent to a violation of (5). Remarkably, Lewbel (1987) showed that, to be consistent with utility maximization, the function $h(\cdot)$ must take one of the following three forms. Either $h(x) = x^k$ for some $k = 1, 2, \ldots$, or $h(x) = \log(x)$. For any other demand system, identification is possible.

2 Individual demand

Throughout this section, we assume that we observe choices of a single individual who has smooth preferences $\succeq$ over $\mathbb{R}^L_+$. We consider an infinite sequence of prices $(p_k : k = 1, \ldots)$. We hold the sequence fixed throughout the argument, and we assume that n observations consist of the first n prices of this sequence together with optimal choices, (p_k, x_k), $k = 1, \ldots, n$, where $x_k = f^{\succeq}(p_k)$, for all $k = 1, \ldots, n$. We assume that $\{x_k, k = 1, \ldots\}$ becomes dense in a convex and open set $\mathbf{X} \subset \mathbb{R}^L_+$. Under the assumption that preferences $\succeq$ are smooth, there is an open set of prices $\mathbf{P} = f^{\succeq -1}(\mathbf{X})$, such that $p_k \in \mathbf{P}$ for all k. We consider the "global" case of $\mathbf{X} = \mathbb{R}^L_{++}$ as well as the local case where $\mathbf{X}$ is bounded.

The two questions we pose are as follows.

1. Given arbitrary $x, y \in \mathbf{X}$, with $x \nsim y$, can one be sure that the individual's preferences over the set $\{x, y\}$ will be known after observing some finite number of market choices?
2. Can we construct an algorithm which, given an arbitrary price $p^* \in \mathbf{P}$, for any $\epsilon > 0$, will eventually produce a set of radius less than ϵ after a finite number of observed choices. That is to say, is finite inference possible for individual demand?

The answers to both questions are affirmative. But it is important to point out that, posed slightly differently, the first question can lead to negative conclusions: Fixing the number of observations n, there always exist $\{x, y\}$, $x \succ y$, over which one cannot determine the individual's preference.

It is well known that a finite number of observations can be rationalized by any strictly convex, continuous, and strictly monotone preference relation if and only if they satisfy the Strong Axiom of Revealed Preference. For completeness, it is useful to state the strong axiom.

Definition 1 Observations satisfy the Strong Axiom of Revealed Preference (SARP) if, for every ordered subset $\{i_1, i_2, ..., i_m\} \subset 1, \ldots$ with $x_{i_k} \neq x_{i_j}$, for all k, j, and with

$$p_{i_1} \cdot x_{i_2} \leq p_{i_1} \cdot x_{i_1},$$

$$p_{i_2} \cdot x_{i_3} \leq p_{i_2} \cdot x_{i_2},$$

$$\vdots$$

it must be the case that

$$p_{i_m} \cdot x_{i_1} > p_{i_m} \cdot x_{i_m}.$$

Chiappori and Rochet (1987) introduced a strong version of the strong axiom (SSARP) that requires, in addition to SARP, that

$$p_i \neq p_k \Rightarrow x_i \neq x_k, \quad \text{for all } \; i, k \in \{1, \ldots, N\}. \tag{6}$$

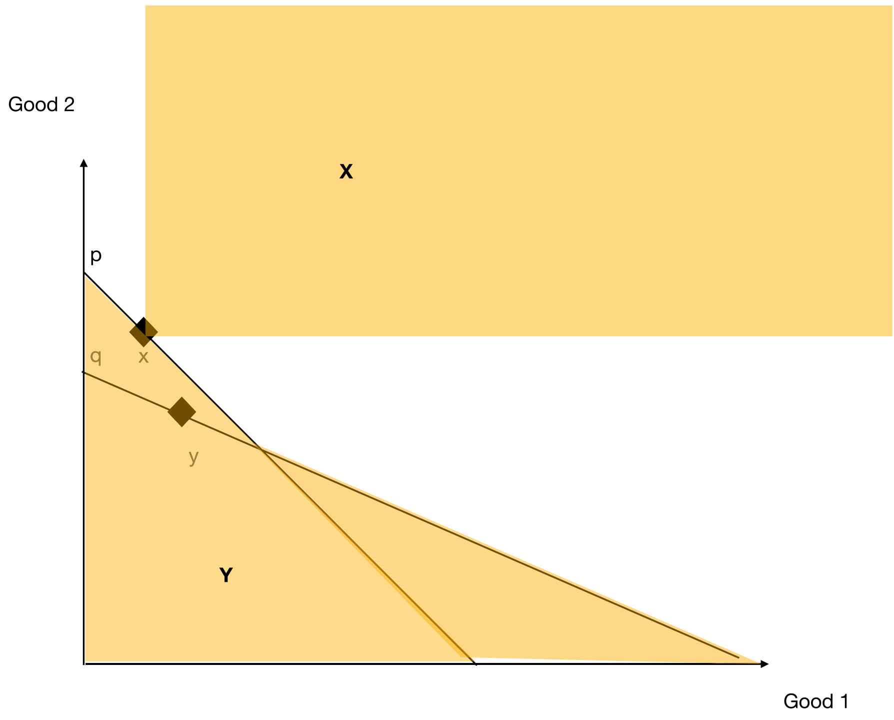
They showed that observations can be rationalized by smooth preferences if they satisfy SSARP.

2.1 Identification of preferences

Varian (1982) showed how to construct partial preferences from a finite number of observations on prices and choices. Figure 1 illustrates the basic idea. If we observe that a bundle x is chosen at some prices p and a bundle y is chosen at prices q, and if y lies below the budget line of x, we can infer that x is strictly preferred to y, and, importantly, that all bundles strictly greater than x are strictly preferred to all bundles in the budget set at prices q Figure 1, 2.

However, the question of whether, given an eventually dense sequence of observations, and any $x \succ y$, there must be some finite number of observations after which x is revealed preferred to y is not directly addressed in Varian (1982).

For $x, y \in \mathbf{X}$, we say that x is strictly revealed preferred to y through n observations, $x \succ^{R_n} y$, if there are k observations indexed by $i_1, \ldots, i_k \in \{1, \ldots, n\}$, such that $x \geq x_{i_1}$, $p_{i_j} \cdot x_{i_{j+1}} < p_{i_j} \cdot x_{i_j}$ for all $j = 1, \ldots k-1$ and $p_{i_k} \cdot x_{i_k} \geq p_{i_k} \cdot y$. We say x is strictly revealed inferior to y if y is strictly revealed preferred to x. Evidently, if $x \succ^{R_n} y$, for some n, then $x \succ y$. Our first result is a converse: if $x \succ y$, then there is an n such that $x \succ^{R_n} y$. The result builds on Mas-Colell (1977), Remark 12, and, in parts, our proof closely follows the argument there. The idea of the proof is to show



$$x = f^{\succeq}(p), y = f^{\succeq}(q) \Rightarrow \text{For all } x' \in \mathbf{X}, y' \in \mathbf{Y} \quad x' \succ y'$$

Fig. 1 From demand to revealed preference

that, for $y \in \mathbf{X}$, the set of $z \in \mathbf{X}$ that are not strictly revealed inferior to y for any n (call this set T_y) is identical to the upper contour set of y – the set of commodity bundles that are weakly preferred to y. To do so, we follow (Mas-Colell 1977) and construct a new preference relation $\succeq'$ that is identical to $\succeq$ precisely when T_y is identical to the upper contour set at y. The key is to show that $\succeq'$ generates the same demand function as $\succeq$. Since $\succeq$ is smooth, this implies that the two preference relations must be identical. A formal proof of the following result can be found in the appendix.

Lemma 2 *If $\succeq$ is smooth and $x, y \in \mathbf{X}$, then $x \succ y$ if and only if there is $n \in \{1, \ldots\}$, such that $x \succ^{R_n} y$.*

Mas-Colell (1977) provided, in Example 1,[4] two distinct, strictly convex, strictly monotone, but not smooth, preference relations that generate identical demand functions. In that example, identification in our sense is impossible: there exist $x, y \in \mathbf{X}$ and two strictly monotone and strictly convex preference relations $\succeq$ and $\succeq'$, such that

$$f^{\succeq}(p) = f^{\succeq'}(p), \quad \text{for all } p \in \mathbf{P},$$

4 A sketch of an example is attributed to Lloyd Shapley and presented as an open problem in Rader (1972).

and $x \succ y$, while $y \succ' x$. Without further assumptions on preferences, identification from market choices is therefore impossible. This seems surprising since, with a finite number of observations, one can always construct smooth preferences if SSARP holds. Lemma 2 shows that the additional condition (6) guarantees that for sufficiently many observations, x is eventually revealed preferred to y if x is strictly preferred to y. As pointed out in Section 1 above, Mas-Colell (1977) shows that the assumption of Lipschitz continuity (which is strictly weaker than smooth preferences) is sufficient for identification.

It is instructive to discuss how the lemma is related to the asymptotic results in Mas-Colell (1978). Suppose $x \succ y$ and take any sequence of preferences $\succeq'_k$ that rationalizes $(p_i, x_i)_{i=1}^k$ and satisfies $y \succeq'_k x$. Since we assume that preferences are smooth, it follows from Mas-Colell (1978), Theorem 4, that $\succeq'_k \to \succeq$ in the topology of closed convergence. When restricted to compact subsets of $\mathbb{R}^L_{++}$, convergence in the topology of closed convergence is equivalent to convergence in the Hausdorff distance (Hildenbrand 1974). But, since $x \succ y$, clearly, $(y, x) \in \succeq'_k$ while $(y, x) \notin \succeq$. Furthermore, there must be an open ϵ neighborhood around (x, y) in $\mathbf{X} \times \mathbf{X}$ so that $x' \succ y'$ in that neighborhood – a contradiction to the definition of convergence. One way to finish an alternative proof of Lemma 2 is to use Theorem 1 from Quah (2014): If $x \succ y$ but there is no n for which $x \succ^{R_n} y$, then Quah (2014) implies that there exist preference relations $\succeq^n$ that rationalize the data with $y \succeq^n x$ for all n, which contradicts the convergence result. Since the proof in Mas-Colell (1978) is rather involved, we prefer the direct argument given in the appendix that is based on Mas-Colell (1977).

While our result focuses on a single point, it can be easily extended to compact sets. If x is strictly revealed preferred to y, there must be an open neighborhood around y such that x is strictly revealed preferred to all points in that neighborhood. If x is strictly preferred to all bundles in some compact set, A, we can, find an open cover and (because of compactness) a finite sub-cover of A consisting of open neighborhoods of points that are strictly revealed inferior to x. This leads to the following result:

Corollary 1 *Suppose that preferences are smooth, $A \subset X$ is compact, and $x \succ y$ for all $y \in A$, then there is a n, such that $x \succ^{R_n} y$ for all $y \in A$.*

For any two bundles between which the individual is not indifferent, a finite number of observations suffice to predict how the individual shall choose. In a market setting, it may be more relevant to ask how the individual shall choose given arbitrary prices and incomes.

2.2 Finite identification of the demand function

For concreteness, we repeat our definition of a learning algorithm and of finite inference for the case of individual demand.

Definition 2 *(Learning Algorithm)*

A learning algorithm $\mathcal{A}$ takes as input one price $p^* \in \mathbf{P}$ and a finite data set (p_k, x_k), $k = 1, \ldots, n$, where $x_k = f^{\succeq}(p_k)$, for all $k = 1, \ldots, n$. It generates a set $\hat{x}(p^*)$ such that $f^{\succeq}(p^*) \in \hat{x}(p^*)$.

Definition 3 *(Finite Inference of Demand)* We say that demand $f^{\succeq}$ is finitely inferred by a learning algorithm $\mathcal{A}$ if, given any **dense** sequence of prices and optimal choices (p_k, x_k), $k = 1, \ldots, n$, where $x_k = f^{\succeq}(p_k)$, for any price p^*, as n approaches infinity, the error bound at p^*,

$$\epsilon = \sup_{x,x' \in \hat{x}(p^*)} ||x - x'||$$

approaches zero.

Given the price $p^* \in \mathcal{P}$, our algorithm computes the set of demands that are consistent with SARP at the price p^* (we call this set $x^{R_n}(p^*)$). We take any element of this set as the candidate demand $x(p^*)$, and the error bound ϵ as the radius of this set. Clearly, $f \succeq (p^*) \in x^{R_n}(p)$. We then show that the radius of this set goes to zero for all $p^* \in \mathbf{P}$, to completing the proof.

Analogously to the analysis above, we can define a revealed demand correspondence as

$$x^{R_n}(p) = \{x \in \mathbf{X} : p \cdot x = 1, \text{ there is no } x' \in \mathbf{X}, p \cdot x' \leq 1, x' \succ^{R_n} x\}.$$

For each p, the set $x^{R_n}(p)$ can be defined by a finite number of linear inequalities that can be computed using Fourier-Motzkin elimination as in Basu et al. (2006).

It is clear that whenever preferences are convex and strictly monotone, and when $p \in f^{\succeq -1}(\mathbf{X})$,

$$f^{\succeq}(p) \in x^{R_n}(p), \quad n = 1, \ldots.$$

It is also clear that, from n observations on demand, one cannot recover the demand function, and $x^{R_n}(p)$ shall not be single-valued. For sufficiently large n, demand can be arbitrarily well approximated by the revealed demand correspondence in the following sense.

Theorem 1 *Suppose preferences are smooth. Given any compact* $\widetilde{\mathbf{P}} \subset \mathbf{P}$ *and any* $\epsilon > 0$*, there exists an* $\bar{n}$ *such that, for all* $p \in \widetilde{\mathbf{P}}$,

$$x^{R_n}(p) \subset \{x \in \mathbf{B}(p) : \| f^{\succeq}(p) - x\| \leq \epsilon\} \textit{ for all } n \geq \bar{n}.$$

The basic idea of the proof (which can be found in the appendix) is as follows: For any bundle z on the budget line at distance at least ε from the optimum $f^{\succeq}(p)$, we construct a finite SARP chain revealing z to be inferior. The key challenge is to obtain a *uniform* threshold $\bar{n}$ that works for all $p \in \widetilde{P}$ simultaneously. We achieve this via compactness: we cover the set of problematic pairs (p, z) with finitely many open sets, each associated with a finite detection threshold. Finite inference is possible in the case of individual demand.

3 Equilibrium

So far, our analysis has focused on the classical demand setting. More generally, economic theory derives relationships between the fundamentals of the economy, some of which may not be observable, and observed individual or aggregate behavior or equilibrium prices. It is then of interest to ask whether what is observed can be used to deduce individual preferences. We tackle this issue in the classical setting of Walrasian equilibrium and assume that individual behavior, that is, individual demand, is not observable. Observations consist only of equilibrium prices and individual endowments or equivalently, as we showed in Section 1, of prices, individual incomes, and the resulting aggregate demand[5].

Throughout the section, we assume that there are H individuals, $h \in \mathbf{H}$ with preferences $\succeq^h$, for each h. Similarly to our analysis above, we are interested in identifying the preferences of the individual h over a convex and open set $\mathbf{X}^h \subset \mathbb{R}^L_+$. However, now we are in a setting where we do not observe individual choices and, therefore, cannot "observe" $\mathbf{X}^h$. We, therefore, assume throughout this section that aggregate demand (or the equilibrium correspondence) is observed at prices and incomes (or profiles of individual endowments) where every individual's demand is strictly positive[6].

We consider a sequence of prices and individual incomes. We assume that the sequence $\{(p_k, w_k^1, \ldots, w_k^H), \quad k = 1, \ldots\}$ becomes dense in $\mathbf{P} \times \mathbf{W}^{\mathbf{H}} \subset \mathbb{R}^L_{++} \times \mathbb{R}^H_{++}$, and we define observations of aggregate demand as

$$D_k = d^{\succeq^{\mathbf{H}}}(p_k, w_k^1, \ldots, w_k^H), \quad k = 1, \ldots .$$

Note that in the context of observations on the equilibrium correspondence, this assumption is much stronger than it appears. While it is true that (3) allows us to consider the aggregate demand function directly, the assumption that observed prices become dense in $\mathbf{P}$ implies that, in the context of multiple equilibria, one observes the *corresponding* selection of the equilibrium correspondence.

Given n observations, it is useful to define the set of consumption allocations that are consistent with the n observations in the sense that they add up to aggregate demand and satisfy the strong axiom:

$$\mathbf{C}_n = \left\{ x \in \mathbb{R}^{nHL}_+ : \begin{array}{c} \sum_{h \in \mathbf{H}} x_k^h = D_k, \quad k = 1, \ldots, n, \\ p_k \cdot x_k^h = w_k^h, \quad k = 1, \ldots, n, \text{ for all } h \in \mathbf{H} \\ (x_k^h, p_k)_{k=1}^n \quad \text{satisfy SARP for all } h \in \mathbf{H} \end{array} \right\}.$$

Note that this is a semi-algebraic set that can be written as the finite union and intersection of sets of the form $\{x \in \mathbb{R}^n : g(x) > 0\}$ or $\{x \in \mathbb{R}^n : f(x) = 0\}$, where f and g are polynomials in x with coefficients in $\mathbb{R}$. As in Basu et al. (2006), quantifier-elimination can be used to compute the sets.

[5] As mentioned in the introduction, one could imagine a setting where individual demand is partially observed. Epstein (1982) examined integrability of individual demand in such a setting (without observing aggregate demand). Clearly, partial observations on demand generally do not allow for identification. Whether or not they would allow us to relax Assumption 1 is subject to further research.

[6] The assumption in Debreu (1972) that guarantees that demand is interior at all prices $p \gg 0$ is that $\mathbf{R}_+(x) \subset \mathbb{R}^L_{++}$, for any $x \in \mathbb{R}^L_{++}$, which is restrictive, but can be relaxed.

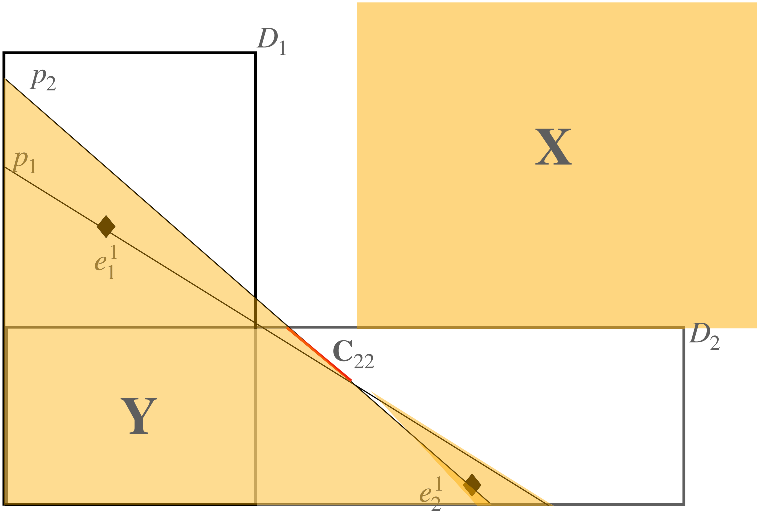


Fig. 2 From aggregate demand to preferences

We denote the projection of $\mathbf{C}_n$ onto the coordinates corresponding to the k'th observation by $\mathbf{C}_{nk} \subset \mathbb{R}_+^{HL}$, $k = 1, \ldots, n$. That is, $\mathbf{C}_{nk}$ is the set of all consumption vectors across the H individuals that are on the budget hyperplane and add up to aggregate consumption, and for which there are consumptions for all other observations $k' \neq k$ that also add up to the relevant aggregate consumptions, are on relevant budget planes and, crucially, altogether satisfy the strong axiom of revealed preference.

We illustrate how observations on aggregate demand, individual endowments, and equilibrium prices limit the set of possible consumption vectors. Figure 2 shows two Edgeworth-boxes, D_1 corresponding to prices p_1 and endowments $(e_1^1, D_1 - e_1^1)$ and D_2 corresponding to prices p_2 and endowments $(e_2^1, D_2 - e_2^1)$. The red line segment, C_{22} on the budget line p_2 corresponds to the possible consumption of individual 1 in the second observation. Since individual 1 must consume inside the first Edgeworth box given prices p_1, the only consumption consistent with the weak axiom lies on the red line segment C_{22}. We can then conclude that any bundle in the set $\mathbf{X}$ is strictly preferred to any bundle in $\mathbf{Y}$.

Importantly, in our context, Assumption 1 implies that, for sufficiently large n, the sets $\mathbf{C}_{nk}$ become *small* in the sense that they contain only a neighborhood around the profile of actual individual demands. The key to this result lies in the fact that if for some k and some h, some $y_k^h \neq f^{\succeq^h}(\frac{p_k}{w_k^h})$ then there must be an n such that $y_k^{\mathbf{H}} \notin \mathbf{C}_{nk}$. If $y_k^{\mathbf{H}} \in \mathbf{C}_{nk}$ for all n, since SARP holds, there must be some preferences $\succeq^{h\prime}$ that rationalize the data (Richter 1966) and this contradicts Assumption 1. The proof of the following lemma can be found in the appendix.

Lemma 3 *Suppose Assumption 1 holds and preferences are smooth. Given any $\epsilon > 0$, and any observation k, there exists an n such that*

$$x^{\mathbf{H}} \in \mathbf{C}_{nk} \Rightarrow \|x^h - f^{\succeq^h}(\frac{p_k}{w_k^h})\| < \epsilon \text{ for all } h \in \mathbf{H}.$$

For a given n, the ϵ can be calculated explicitly since $\mathbf{C}_n$ can be computed explicitly.

Note that this result is consistent with the results in Chiappori et al. (2004) and in particular, in Matzkin (2006). Observation of aggregate demand means that one observes the income effects of each individual's demand. The contribution of our lemma is to show that with finite data, this is approximately true.

3.1 Identification of individual choice

As in Section 2, we want to develop a notion of "equilibrium revealed preferred", which means that, through observations of equilibrium prices, it is revealed that an individual must prefer some bundle x to another bundle y. Given $x, y \in \mathbf{X}$, we say that x is equilibrium revealed preferred to y by individual h given n observations if for all $(x_k^{\mathbf{H}})_{k=1,\ldots,n} \in \mathbf{C}_n$, there are N observations indexed by $i_1, \ldots, i_N \in \{1, \ldots, n\}$, so that $x \geq x_{i_1}^h$, and $p_{i_j} \cdot x_{i_{j+1}}^h < p_{i_j} \cdot x_{i_j}^h$, for all $j = 1, \ldots N$, as well as $x_{i_N}^h \geq y$. If x is equilibrium revealed preferred to y by an individual h, we write $x \succ^{R_n^h} y$.

While for the classical notion of revealed preferred, x is said to be revealed preferred to y if the observed choices reveal that an individual must prefer x to y, in this setting, equilibrium prices reveal that an individual prefers x to y. This is possible because incomes vary and Lemma 1 shows that individual choices can be recovered (approximately) from aggregate demand.

Obviously, if there is a n such that $x \succ^{R_n^h} y$ then $x \succ^h y$. As in Section 2, we show that the converse also holds, for sufficiently large n, $x \succ^{R_n^h} y$ must hold whenever $x \succ^h y$. The following result follows by combining Lemma 2 with Lemma 1.

Theorem 2 *Suppose that individual preferences are smooth and Assumption 1 is satisfied. Given any $h \in \mathbf{H}$, there is an open and convex set $\mathbf{X}^h$, such that, for any $x, y \in \mathbf{X}^h$ with $x \succ^h y$, there exists an n, such that*

$$x \succ^{R_n^h} y.$$

This is one of the main results of our paper. It shows the existence, for each individual h, of an open and convex set $\mathbf{X}^h$ over which strict preference between any two bundles is revealed after finitely many observations[7]. In the global case, when $\mathbf{P} = \mathbb{R}_{++}^L$ the set $\mathbf{X}^h$ is given by $\mathbf{X}^h = \mathbb{R}_{++}^L$.

It follows directly from the above that sufficiently many observations on the equilibrium manifold also allow us to predict the Walrasian demand of individual h at

[7] More precisely, given any two non-indifferent bundles in $\mathbf{X}^h$, there exists a finite n such that the preference over these two bundles are revealed after n observations.

prices $p \in f^{\succeq^h -1}(\mathbf{X}^h)$. Formally, we can define the equilibrium revealed demand correspondence as

$$x^{R_n^h}(p) = \{x \in \mathbf{X}^h : p \cdot x = 1, \text{ there is no } x' \in \mathbf{X}^h, p \cdot x' \leq 1, x' \succ^{R_n^h} x\}.$$

Note that the definition is identical to the definition in Section 2 except that we replace "revealed preferred" with "equilibrium revealed preferred". Note also that the domain of the revealed demand correspondence is only implicitly defined. As mentioned above, it is clear that whenever preferences are convex and strictly monotone, for each agent h and each $p \in f^{-1\succeq^h}(\mathbf{X}^h)$ we must have the following.

$$f^{\succeq^h}(p) \in x^{R_n^h}(p), \quad n = 1, \ldots.$$

The following result shows that, for sufficiently large n, demand can be arbitrarily well approximated by the revealed demand correspondence:

Theorem 3 *Suppose that preferences are smooth and that Assumption 1 holds. Given any individual h and compact $\widetilde{\mathbf{P}} \subset f^{\succeq^h -1}(\mathbf{X}^h)$ and any $\epsilon > 0$ there exists an $n \in \{1, \ldots\}$, such that for all $p \in \widetilde{\mathbf{P}}$,*

$$x^{R_n^h}(p) \subset \{x \in \mathbf{B}(p) : \| f^{\succeq}(p) - x\| \leq \epsilon\}.$$

Given the proof of Theorem 2, the result follows immediately. Sufficiently rich data on the equilibrium correspondence finitely identifies individual demand.

For the case of $\mathbf{P} \neq \mathbb{R}^L_{++}$, the result is a bit unsatisfactory since the domain where demand can be approximated well is not explicitly given.[8] In the last subsection, we therefore focus on the case $\mathbf{P} = \mathbb{R}^L_{++}$.

3.2 Finite inference

The identification of preferences from observed behavior has strong positive and normative implications. The transfer paradox, introduced by Leontief (1936), makes it clear that knowledge of the utility functions is necessary in order to identify the welfare effects of transfers. However, resolving the transfer paradox would also require forecasting equilibrium prices at profiles of endowments not previously encountered. Unfortunately, things then become much more complicated because of the intricate relation between approximate equilibria and exact equilibrium.

While it is true that (under standard assumptions on preferences) for any economy and any $\delta > 0$, there is an $\epsilon > 0$, such that, if the norm of aggregate excess demand is smaller than ϵ, prices are within δ of equilibrium prices (Anderson 1986), there is no known constructive algorithm to determine ϵ. The question of the possible existence of such an algorithm relies on the underlying model of computability, but the general view (Richter and Wong 1999) is that such an algorithm does not exist.

[8] Note, however, that it can be computed from the observations.

Given n observations of prices and incomes, $(p_k, w_k^{\mathbf{H}})_{k=1,\ldots,n}$, with $p_k \in \mathbb{R}^L_{++}$, and $w_k^{\mathbf{H}} \in \mathbf{W}^{\mathbf{H}}$ for all $k = 1, \ldots, n$ we define $\mathbf{P}(e^{\mathbf{H}}) = \{p \in \mathbb{R}^L_{++} : p \cdot e^h \in \mathbf{W}^h$ for all $h \in \mathbf{H}\}$ and we define the revealed approximate equilibrium correspondence as follows:

$$\Pi^{R_n^{\mathbf{H}}}(e^{\mathbf{H}}) = \left\{ p \in \mathbf{P}(e^{\mathbf{H}}) : \exists\, x^{\mathbf{H}} \in \mathbb{R}^{HL}_{++}, \begin{array}{l} x^h \in x^{R_n^h}(\frac{p}{w_h}) \text{ for all } h \in \mathbf{H}, \\ \sum_{h \in \mathbf{H}} (x^h - e^h) = 0, \end{array} \right\}.$$

As above, it is clear that for a given profile of individual endowments, $e^{\mathbf{H}}$, the set of equilibria with prices in $\mathbf{P}(e^{\mathbf{H}})$ is a subset of $\Pi^{R_n^{\mathbf{H}}}(e^{\mathbf{H}})$. It follows directly from Theorem 3 that, for any $\epsilon > 0$, the correspondence describes the set of all ϵ-equilibria with incomes in $\mathbf{W}^{\mathbf{H}}$ and with prices in $\mathbf{P}(e^{\mathbf{H}})$.

Theorem 4 *Suppose $\mathbf{P} = \mathbb{R}^L_{++}$, Assumption 1 holds, and preferences are smooth. For all $e^{\mathbf{H}} \in \mathbb{R}^{HL}_{++}$ and all n,*

$$\Pi^{\succeq^{\mathbf{H}}}(e^{\mathbf{H}}) \cap \mathbf{P}(e^{\mathbf{H}}) \subset \Pi^{R_n^{\mathbf{H}}}(e^{\mathbf{H}}).$$

Furthermore, for any $\epsilon > 0$ and any compact $\mathbf{E} \subset \mathbb{R}^{HL}_{++}$ there exists $n \in \{1, \ldots\}$, such that, for all $e^{\mathbf{H}} \in \mathbf{E}$,

$$\Pi^{R_n^{\mathbf{H}}}(e^{\mathbf{H}}) \subset \{p \in \mathbf{P}(e^{\mathbf{H}}) : \|d^{\succeq^{\mathbf{H}}}(p, p \cdot e^1, \ldots, p \cdot e^H) - e\| < \epsilon\}.$$

We can thus conduct finite inference on $\epsilon-$equilibria, and all ϵ-equilibria converge to exact equilibria. However, this is insufficient for finite inference on exact equilibria. Due to the multiplicity of equilibria, the set $\Pi^{R_n^{\mathbf{H}}}(e^{\mathbf{H}})$ may not shrink to a point. Furthermore, without knowing the epsilon-delta relationship between prices and demands, it is unclear how the analyst can find a smaller set that contains at least one equilibrium price.

Finite inference can be established on equilibrium prices if 1) they are known to be unique for all profiles of individual endowments, or 2) the epsilon delta relationship between endowments and prices is known. In the first case, the revealed approximate equilibrium correspondence shrinks to a point. In the second case, this follows by an algorithm which simply returns a delta ball around the closest observed equilibrium price.

A Appendix: Proofs

Proof of Lemma 1

Suppose not, and for all $p \in \mathbf{P}$, $w \in \mathbf{W}^h$, $h \in \mathbf{H}$,

$$d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) - d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) = 0.$$

We know some agent h has $\succeq^h \neq \succeq^{h'}$. By Assumption 1 there are incomes w, w' and some price p such that

$$d^{\succeq^h}(p, w) - d^{\succeq^h}(p, w') \neq d^{\succeq^{h'}}(p, w) - d^{\succeq^{h'}}(p, w').$$

Consider the exercise of changing only the income of agent h and fixing prices. If the he new aggregate demand is $d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}})$ under preference profile $\succeq^{\mathbf{H}}$ and $d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}})$ under preference profile $\succeq'^{\mathbf{H}}$, then

$$\begin{aligned}
&[d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) - d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}})] - [d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}}) - d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}})] \\
&\quad = [d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) - d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}})] - [d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) - d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}})] \\
&\quad = [d^{\succeq^h}(p, w) - d^{\succeq^h}(p, w')] - [d^{\succeq'^h}(p, w) - d^{\succeq'^h}(p, w')] \neq 0.
\end{aligned}$$

But,

$$[d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) - d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}})] - [d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}}) - d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}})] \neq 0$$

contradicts

$$[d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) - d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}})] = 0 = [d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}}) - d^{\succeq'^{\mathbf{H}}}(\bar{p}, \bar{w}'^{\mathbf{H}})],$$

which proves the lemma.

Proof of Proposition 1

Assumption 1 can only be violated if there is a function $g : \mathbf{P} \to \mathbb{R}^L$, such that, for all $p \in \mathbf{P}$ and $w \in \mathbf{W}$,

$$d^{\succeq}(p, w) = d^{\succeq'}(p, w) + g(p), \quad \text{while} \quad p \cdot g(p) = 0, \tag{7}$$

where the function $g(\cdot)$ is homogeneous of degree zero. If the domain of the demand function satisfies the property that

$$(p, w) \in \mathbf{P} \times \mathbf{W} \Rightarrow \text{for all } \lambda \in (0, 1] : \lambda w \in \mathbf{W}, \tag{8}$$

then Equation (7) can only hold with $g(p) = 0$, and hence Equation (4) must hold automatically. Therefore, it is clear that Assumption 1 must always hold, independently of preferences, when for all individuals h Property (8) is satisfied.

By the Slutsky equation and (7), the condition is violated only if there are $\succeq$ and $\succeq'$ with

$$S^{\succeq}(p, w) = D_p d^{\succeq}(p, w) + v(p, w) d^{\succeq}(p, w)',$$

and

$$S^{\succeq'}(p, w) = D_p(d^{\succeq}(p, w) + g(p)) + v(p, w)(d^{\succeq}(p, w) + g(p))'.$$

Since $S^{\succeq}(p, w) - S^{\succeq'}(p, w)$ is a symmetric matrix, the matrix

$$(S^{\succeq}(p, w) - S^{\succeq'}(p, w)) - (S^{\succeq}(p, w') - S^{\succeq'}(p, w')) = (v(p, w) - v(p, w'))g(p)'$$

must be symmetric for any fixed $p \in \mathbf{P}$ and any $w, w' \in \mathbf{W}$. This implies that, for all w, w', the vector $v(p, w) - v(p, w')$ must be collinear with $g(p)$ for any w, w'.

Proof of Lemma 2

The "if" part is clear.

For the converse, given any $y \in int(\mathbf{X})$, define

$$T_y = \{z \in \mathbf{X} : \text{there is no } n, \text{ such that } y \succ^{R_n} z\},$$

and define a new preference relation $\succeq'$ by

$$u \succeq' v \text{ if } \begin{cases} u \in \text{conv}(T_y \cup \mathbf{R}_+(v)) & \text{if } v \notin T_y \\ u \in T_y \cap \mathbf{R}_+(v) & \text{if } v \in T_y \end{cases},$$

where $\mathbf{R}_+(\cdot)$ denotes the upper contour set under the preference relation $\succeq$.

It can be verified that $\succeq'$ is upper-semi continuous, strictly monotone and convex, and we can define the demand correspondence $f^{\succeq'}$. We shall argue below that it suffices to show that $f^{\succeq}(\cdot) = f^{\succeq'}(\cdot)$. To prove this, note first that $f^{\succeq'}(\cdot)$ is non-empty for all p. Then suppose that for some $p \in \mathbf{P}$, $u \neq v = f^{\succeq}(p)$ but $u \in f^{\succeq'}(p)$. By the definition of $\succeq'$,, this can only be the case if $u \in T_y$ but $v \notin T_y$. By the continuity of $f^{\succeq}(\cdot)$, for sufficiently large number of observations, n, there must be a $j \in \{1, \ldots, n\}$ and a p'_j sufficiently close to p so that $f^{\succeq}(p'_j) \notin T_y$ but $p'_j \cdot f^{\succeq}(p'_j) > p'_j \cdot u$. This is a contradiction to transitivity since we would have that y is revealed preferred to $f^{\succeq}(p'_j)$, which is revealed preferred to u and u cannot be in T_y.

Therefore, $\succeq$ and $\succeq'$ generate the same demand functions on $\mathbf{P}$. This implies that the two preference relations coincide, which is only possible if T_y is equal to the upper contour set of $\succeq$ at y.

Proof of Theorem 1

Define

$$K_\varepsilon := \{(p, x) \in \widetilde{P} \times \mathbb{R}^L_+ : x \in B(p) \text{ and } \|x - f_{\succeq}(p)\| \geq \varepsilon\}.$$

We first note that K_ε is compact. Since $\widetilde{P}$ is compact and $\widetilde{P} \subset \mathbb{R}^L_{++}$, there exists $\delta > 0$ such that $p_\ell \geq \delta$ for every $\ell = 1, \ldots, L$ and every $p \in \widetilde{P}$. Hence, if $x \in B(p)$ for some $p \in \widetilde{P}$, then

$$0 \leq x_\ell \leq \frac{1}{\delta} \qquad \text{for all } \ell = 1, \ldots, L,$$

and the union of the budget sets $\{B(p) : p \in \widetilde{P}\}$ is bounded. Therefore,

$$\{(p, x) \in \widetilde{P} \times \mathbb{R}^L_+ : x \in B(p)\}$$

is compact. Since $f_{\succeq}$ is continuous, the map $(p, x) \mapsto \|x - f_{\succeq}(p)\|$ is continuous, and it follows that K_ε is a closed subset of a compact set. Hence K_ε is compact.

Now fix any $(p, x) \in K_\varepsilon$. Since $x \in B(p)$ and $x \neq f_{\succeq}(p)$, strict monotonicity and strict convexity imply

$$f_{\succeq}(p) \succ x.$$

To ensure the SARP chain is robust to perturbations in both p and x, we insert intermediate bundles with strict vector inequalities. By continuity of preferences, we can choose $\hat{x}, \hat{y} \in \mathbb{R}^L_+$ (choosing $\hat{x}, \hat{y} \in X$, which is possible since X is open) such that

$$f_{\succeq}(p) \gg \hat{x} \succ \hat{y} \gg x.$$

By Lemma 2, there exists an integer $n(p, x)$ such that

$$\hat{x} \succ^R_{n(p,x)} \hat{y}.$$

Thus there are observations indexed by $i_1, \ldots, i_m \leq n(p, x)$ such that

$$\hat{x} \geq x_{i_1}, \qquad p_{i_j} \cdot x_{i_{j+1}} < p_{i_j} \cdot x_{i_j} \quad \text{for } j = 1, \ldots, m - 1,$$

and

$$p_{i_m} \cdot \hat{y} \leq p_{i_m} \cdot x_{i_m}.$$

Because $f_{\succeq}$ is continuous and $f_{\succeq}(p) \gg \hat{x}$, there exists an open neighborhood $U_{p,x}$ of p, such that

$$f_{\succeq}(q) \geq x_{i_1} \qquad \text{for all } q \in U_{p,x}.$$

Likewise, since $\hat{y} \gg x$, there exists an open neighborhood $V_{p,x}$ of x, such that

$$p_{i_m} \cdot z \leq p_{i_m} \cdot x_{i_m} \qquad \text{for all } z \in V_{p,x}.$$

Therefore, for every $(q, z) \in U_{p,x} \times V_{p,x}$, the same finite chain of observations witnesses

$$f_{\succeq}(q) \succ^R_{n(p,x)} z.$$

Hence the family

$$\left\{(U_{p,x} \times V_{p,x}) \cap K_\varepsilon : (p, x) \in K_\varepsilon\right\}$$

is an open cover of the compact set K_ε. We may therefore extract a finite subcover

$$\{(U_r \times V_r) \cap K_\varepsilon\}_{r=1}^R,$$

with associated integers n_r. Let

$$\bar{n} := \max_{1 \leq r \leq R} n_r.$$

To connect this covering result to the revealed demand correspondence, we introduce an auxiliary set. Define

$$x_n^{R,*}(p) := B(p) \setminus \{x \in B(p) : f_{\succeq}(p) \succ_n^R x\}.$$

That is, $x_n^{R,*}(p)$ is the set of affordable bundles that are not algorithmically revealed inferior to the optimum at level n.

Take any $p \in \widetilde{P}$ and any $z \in B(p)$ such that

$$\|z - f_{\succeq}(p)\| \geq \varepsilon.$$

Then $(p, z) \in K_\varepsilon$, and so

$$(p, z) \in (U_r \times V_r) \cap K_\varepsilon$$

for some $r \in \{1, \ldots, R\}$. By construction,

$$f_{\succeq}(p) \succ_{n_r}^R z.$$

Since $n_r \leq \bar{n}$, monotonicity of the revealed-preference relation in n implies

$$f_{\succeq}(p) \succ_{\bar{n}}^R z.$$

Hence,

$$z \notin x_{\bar{n}}^{R,*}(p).$$

We have thus shown that, for every $p \in \widetilde{P}$,

$$x_{\bar{n}}^{R,*}(p) \subseteq \{x \in B(p) : \|x - f_{\succeq}(p)\| \leq \varepsilon\}.$$

Finally, by definition of the revealed demand correspondence,

$$x_n^R(p) = \{x \in X : p \cdot x = 1,\ \text{there is no } x' \in X,\ p \cdot x' \leq 1,\ x' \succ_n^R x\}.$$

Therefore, if $x \notin x_n^{R,*}(p)$, then in particular $f_{\succeq}(p) \succ_n^R x$, with $f_{\succeq}(p) \in X$ and $p \cdot f_{\succeq}(p) = 1$, so $x \notin x_n^R(p)$. Thus,

$$x_n^R(p) \subseteq x_n^{R,*}(p) \qquad \text{for all } p \in \widetilde{P},\ n \in \mathbb{N}.$$

Applying this with $n = \bar{n}$ gives

$$x_{\bar{n}}^R(p) \subseteq \{x \in B(p) : \|x - f_{\succeq}(p)\| \leq \varepsilon\} \qquad \text{for all } p \in \widetilde{P}.$$

Since the relation $\succ_n^R$ is increasing in n, the correspondence $x_n^R(p)$ is decreasing in n, and for every $n \geq \bar{n}$,

$$x_n^R(p) \subseteq x_{\bar{n}}^R(p).$$

Consequently,

$$x_n^R(p) \subseteq \{x \in B(p) : \|x - f_{\succeq}(p)\| \leq \varepsilon\}, \qquad \text{for all } p \in \widetilde{P} \text{ and all } n \geq \bar{n},$$

which proves the theorem.

Proof of Lemma 3

For each observation $\ell \geq 1$, define the set of feasible allocations at ℓ:

$$B_\ell^{\mathbf{H}} := \left\{ x^{\mathbf{H}} \in \mathbb{R}_+^{HL} : \textstyle\sum_{h \in \mathbf{H}} x^h = D_\ell \ \text{ and } \ p_\ell \cdot x^h = w_\ell^h \text{ for all } h \in \mathbf{H} \right\}.$$

Each $B_\ell^{\mathbf{H}}$ is compact (closed and bounded, since $x^h \geq 0$ and $\sum_h x^h = D_\ell$).

Recall the consistency set

$$\mathbf{C}_n = \left\{ (x_1^{\mathbf{H}}, \ldots, x_n^{\mathbf{H}}) \in \textstyle\prod_{\ell=1}^n B_\ell^{\mathbf{H}} (x_\ell^h, p_\ell)_{\ell=1}^n \text{ satisfies SARP for each } h \in \mathbf{H} \right\},$$

and let $\mathbf{C}_{nk}$ denote the projection of $\mathbf{C}_n$ onto the coordinates of observation k. Let $\overline{\mathbf{C}}_{nk}$ denote the closure of $\mathbf{C}_{nk}$ in $B_k^{\mathbf{H}}$. Since $B_k^{\mathbf{H}}$ is compact, $\overline{\mathbf{C}}_{nk}$ is compact. The projections are nested: $\mathbf{C}_{n+1,k} \subseteq \mathbf{C}_{nk}$ and $\overline{\mathbf{C}}_{n+1,k} \subseteq \overline{\mathbf{C}}_{nk}$ for all n, since adding an observation can only further constrain the set of SARP-consistent completions.

Fix $\epsilon > 0$ and an observation k. For each $h \in \mathbf{H}$, write $x_k^{h,*} := f^{\succeq^h}(p_k/w_k^h)$. Define the bad set

$$K_\epsilon := \left\{ x^{\mathbf{H}} \in B_k^{\mathbf{H}} : \|x^h - x_k^{h,*}\| \geq \epsilon \text{ for some } h \in \mathbf{H} \right\}.$$

This is a closed subset of the compact set $B_k^{\mathbf{H}}$, hence compact. The proof has two steps: pointwise exclusion from $\overline{\mathbf{C}}_{nk}$ (Step 1), then a compactness argument to make the exclusion uniform (Step 2). The conclusion for $\mathbf{C}_{nk} \subseteq \overline{\mathbf{C}}_{nk}$ follows immediately.

Step 1 (Pointwise exclusion).

Claim. For every $y_k^{\mathbf{H}} \in B_k^{\mathbf{H}}$ with $y_k^h \neq x_k^{h,*}$ for at least one h, there exists $n(y_k^{\mathbf{H}}) \geq k$ such that

$$y_k^{\mathbf{H}} \notin \overline{\mathbf{C}}_{n(y_k^{\mathbf{H}}),\, k}.$$

Proof of the Claim. Suppose, towards a contradiction, that $y_k^h \neq x_k^{h,*}$ for some h but $y_k^{\mathbf{H}} \in \overline{\mathbf{C}}_{nk}$ for all $n \geq k$.

For each $n \geq k$, pick a sequence in $\mathbf{C}_{nk}$ converging to $y_k^{\mathbf{H}}$. Each element of this sequence can be completed to a full n-tuple in $\mathbf{C}_n$. In particular, for each $m \geq k$ there exists $z^m = (z_1^{\mathbf{H},m}, \ldots, z_m^{\mathbf{H},m}) \in \mathbf{C}_m$ such that $z_k^{\mathbf{H},m}$ is within distance $1/m$ of $y_k^{\mathbf{H}}$ (and $z_k^{\mathbf{H},m} \to y_k^{\mathbf{H}}$ as $m \to \infty$).

Since $z_\ell^{\mathbf{H},m} \in B_\ell^{\mathbf{H}}$ and each $B_\ell^{\mathbf{H}}$ is compact, a standard diagonal argument yields a subsequence $(m_j)_{j\geq 1}$ and a sequence $(y_\ell^{\mathbf{H}})_{\ell \geq 1} \in \prod_{\ell=1}^\infty B_\ell^{\mathbf{H}}$ such that, for every fixed n,

$$(z_1^{\mathbf{H},m_j}, \ldots, z_n^{\mathbf{H},m_j}) \longrightarrow (y_1^{\mathbf{H}}, \ldots, y_n^{\mathbf{H}}) \qquad \text{as } j \to \infty.$$

Since $z_k^{\mathbf{H},m_j} \to y_k^{\mathbf{H}}$ by construction, the kth coordinate of the limit is $y_k^{\mathbf{H}}$.

For each $h \in \mathbf{H}$, the data $(y_\ell^h, p_\ell)_{\ell \geq 1}$ is obtained as the limit of SARP-consistent data: for any fixed n, the approximating prefix $(z_1^{h,m_j}, \ldots, z_n^{h,m_j})$ satisfies SARP for all sufficiently large j (since $m_j \geq n$ implies $z^{m_j} \in \mathbf{C}_{m_j}$, and projection onto the first n coordinates preserves SARP).

Since all preferences under consideration are smooth—represented by a C^2 utility with $Du \gg 0$ and D^2u negative definite on $[Du]^\perp$, generating continuously differentiable demand with a continuously differentiable inverse—the limit data for each agent h is rationalised by a smooth preference $\succeq^{h\prime}$ with invertible demand. This follows from the identification result: smooth preferences are Lipschitzian, so by Mas-Colell (1977), the demand function generated by a smooth preference identifies that preference. The limit data lies on the graph of a demand function (being the pointwise limit of demands generated by smooth preferences), and this demand function is generated by some smooth preference $\succeq^{h\prime}$. In particular,

$$y_\ell^h = f^{\succeq^{h\prime}}\left(\frac{p_\ell}{w_\ell^h}\right), \qquad \ell = 1, 2, \ldots. \tag{9}$$

Let $\succeq^{\prime\mathbf{H}} := (\succeq^{h\prime})_{h \in \mathbf{H}}$. Since the true preference $\succeq^h$ is strictly convex, $x_k^{h,*}$ is the unique maximiser in $B(p_k/w_k^h)$ under $\succeq^h$. Since $y_k^h \neq x_k^{h,*}$ while $y_k^h = f^{\succeq^{h\prime}}(p_k/w_k^h)$, we have $\succeq^{h\prime} \neq \succeq^h$, hence $\succeq^{\prime\mathbf{H}} \neq \succeq^{\mathbf{H}}$.

At every observation ℓ, feasibility gives

$$d^{\succeq^{\prime\mathbf{H}}}(p_\ell, w_\ell^{\mathbf{H}}) = \sum_{h \in \mathbf{H}} y_\ell^h = D_\ell = d^{\succeq^{\mathbf{H}}}(p_\ell, w_\ell^{\mathbf{H}}).$$

Both $d^{\succeq^{\mathbf{H}}}$ and $d^{\succeq^{\prime\mathbf{H}}}$ are continuous, since both profiles consist of smooth preferences with invertible demand. Since two continuous functions that agree on the dense set $\{(p_\ell, w_\ell^{\mathbf{H}})\}_{\ell \geq 1} \subseteq \mathbf{P} \times \mathbf{W}^{\mathbf{H}}$ must agree everywhere,

$$d^{\succeq^{\prime\mathbf{H}}}(p, w^{\mathbf{H}}) = d^{\succeq^{\mathbf{H}}}(p, w^{\mathbf{H}}) \qquad \text{for all } (p, w^{\mathbf{H}}) \in \mathbf{P} \times \mathbf{W}^{\mathbf{H}}.$$

But $\succeq^{\prime\mathbf{H}} \neq \succeq^{\mathbf{H}}$, so Assumption 1 (via Lemma 1) implies that there exist $\bar{p} \in \mathbf{P}$ and $\bar{w}^{\mathbf{H}} \in \mathbf{W}^{\mathbf{H}}$ with $d^{\succeq^{\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}}) \neq d^{\succeq^{\prime\mathbf{H}}}(\bar{p}, \bar{w}^{\mathbf{H}})$, a contradiction. This proves the Claim.

Step 2 (Uniform exclusion by compactness).

Let $y_k^{\mathbf{H}} \in K_\epsilon$. By the Claim, there exists $n(y_k^{\mathbf{H}}) \geq k$ such that $y_k^{\mathbf{H}} \notin \overline{\mathbf{C}}_{n(y_k^{\mathbf{H}}),k}$. Since $\overline{\mathbf{C}}_{n(y_k^{\mathbf{H}}),k}$ is closed, there exists an open neighbourhood $V(y_k^{\mathbf{H}})$ of $y_k^{\mathbf{H}}$ in $B_k^{\mathbf{H}}$ with

$$V(y_k^{\mathbf{H}}) \cap \overline{\mathbf{C}}_{n(y_k^{\mathbf{H}}),k} = \varnothing.$$

The collection $\{V(y_k^{\mathbf{H}}) : y_k^{\mathbf{H}} \in K_\epsilon\}$ is an open cover of the compact set K_ϵ. Extract a finite subcover corresponding to points $y_{k,1}^{\mathbf{H}}, \ldots, y_{k,m}^{\mathbf{H}} \in K_\epsilon$, and set

$$\bar{n} := \max_{1 \leq r \leq m} n(y_{k,r}^{\mathbf{H}}).$$

By nestedness, for every $n \geq \bar{n}$ and every $r = 1, \ldots, m$,

$$\mathbf{C}_{nk} \subseteq \overline{\mathbf{C}}_{nk} \subseteq \overline{\mathbf{C}}_{n(y_{k,r}^{\mathbf{H}}),k},$$

so that $\mathbf{C}_{nk} \cap V(y_{k,r}^{\mathbf{H}}) = \varnothing$ for all r. Therefore

$$\mathbf{C}_{nk} \cap K_\epsilon = \varnothing \qquad \text{for all } n \geq \bar{n}.$$

Equivalently, for all $n \geq \bar{n}$,

$$x^{\mathbf{H}} \in \mathbf{C}_{nk} \implies \|x^h - f^{\succeq^h}(p_k/w_k^h)\| < \epsilon, \quad \text{for all } h \in \mathbf{H}.$$

Proof of Theorem 2

For each agent h, there is an open and convex set $\mathbf{X}^h$ such that $f^{-1}(\mathbf{X}^h) \subset \mathbf{P}$. For any $x, y \in \mathbf{X}^h$, Theorem 2 implies that there is a $\bar{n}$ such that, for all $n > \bar{n}$, there is a $(x_k^{\mathbf{H}})_{k=1,\ldots,n} \in \mathbf{C}_n$ so that the associated (x_k^h) together with (p_k) imply that x is revealed preferred to y by individual h. Lemma 1 implies that for sufficiently large n there can be no other solutions $(\tilde{x}_k^{\mathbf{H}})_{k=1,\ldots,n} \in \mathbf{C}_n$ for which this does not hold.

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References

Afriat, S.: The construction of a utility function from demand data. Int. Econ. Rev. **8**, 67–77 (1967)
Anderson, R.M.: Almost" implies "near. Trans. Am. Math. Soc. **296**(1), 229–237 (1986)
Balasko, Y.: The equilibrium manifold keeps the memory of individual demand functions. Econ. Theor. **24**(3), 493–501 (2004)
Basu, S., Pollack, R., Roy, M.-F.: Algorithms in real algebraic geometry, volume 10. Springer, (2006)
Beigman, E., Vohra, R.: Learning from revealed preference. In Proceedings of the 7th ACM Conference on Electronic Commerce, pages 36–42, (2006)
Blundell, R.: How revealing is revealed preference? J. Eur. Econ. Assoc. **3**, 211–235 (2005)
Blundell, R., Browing, M., Crawford, I.: Nonparametric engel curves and revealed preference. Econometrica **71**, 205–240 (2003)

Blundell, R., Browing, M., Crawford, I.: Best nonparametric bounds on demand responses. Econometrica **76**, 1227–1262 (2008)
Brown, D., Matzkin, R.: Recoverability and estimation of the demand and utility functions of traders when demands are unobservable. Unpublished manuscript, (1990)
Brown, D., Matzkin, R.: Testable restrictions on the equilibrium manifold. Econometrica **64**, 1249–1262 (1996)
Chambers, Ch., Echenique, F., Lambert, N.: Recovering preferences from finite data. Econometrica **89**(4), 1633–1664 (2021)
Chiappori, P.-A., Rochet, J.-C.: Revealed preferences and differentiable demand. Econometrica **55**, 687–691 (1987)
Chiappori, P.-A., Ekeland, I., Kübler, F., Polemarchakis, H.: Testable implications of general equilibrium theory: a differentiable approach. J. Math. Econ. **40**, 105–119 (2004)
Debreu, G.: Smooth preferences. Econometrica **603–615**, 40 (1972)
Diasakos, T., Gerasimou, G.: Preference conditions for invertible demand functions. American Economic Journal: Microeconomics **14**, 118–138 (2022)
Epstein, L.G.: Integrability of incomplete systems of demand functions. Rev. Econ. Stud. **49**(3), 411–425 (1982)
Forges, F., Minelli, E.: Afriat's theorem for general budget sets. Journal of Economic Theory **144**(1), 135–145 (2009)
Hildenbrand, W.: Core and Equilibria of a Large Economy, Princeton University Press (1974)
Hunter, G.: Metalogic: An introduction to the metatheory of standard first order logic, Univ of California Press (1973)
Hurwicz, L., Uzawa, H.: On the integrability of demand functions. In L. Hurwicz J. Chipman and H. Sonnenschein, editors, Preferences, Utility and Demand. Harcourt Brace, Jovanovich, (1971)
Kübler, F., Polemarchakis, H.: Identification in general equilibrium. J. Math. Econ. **113**, 103017 (2024)
Leontief, V.: A note on the pure theory of transfer. In: Exploration in Economics: Notes and Essays in Honor of F. W. Taussig. McGraw Hill, (1936)
Lewbel, A.: Characterizing some gorman engel curves. Econometrica, pages 1451–1459, (1987)
Mas-Colell, A.: On the recoverability of consumers' preferences from demand behavior. Econometrica **45**, 1409–1430 (1977)
Mas-Colell, A.: On revealed preference analysis. Rev. Econ. Stud. **45**, 121–131 (1978)
Matzkin, R.L.: Identification of consumer preferences when their choices are unobservable. In *Rationality and Equilibrium*, pages 195–215. Springer, (2006)
Quah, J.K.-H.: A test for weakly separable preferences. Unpublished manuscript, (2014)
Rader, T.: A Theory of Microeconomics, Academic Press (1972)
Richter, M.K.: Revealed preference theory. Econometrica **34**, 635–645 (1966)
Richter, M.K., Wong, K.-C.: Non-computability of competitive equilibrium. Econ. Theor. **14**(1), 1–27 (1999)
Varian, H.R.: The nonparametric approach to demand analysis. Econometrica **50**, 945–973 (1982)