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Econometric Analysis of Explosive Financial Time Series

Zhuangyan Li



Bayes Business School
Faculty of Finance
City St George's, University of London

This thesis is submitted for the degree of
Doctor of Philosophy
16 September 2026

Declaration

The work contained in this thesis is my own and has not been submitted for any other degree or qualification at this or any other university. All sources of information have been acknowledged or referenced.

Zhuangyan Li
London, 16 September 2026

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Introduction

An asset price bubble arises when the market value of an asset, such as equities, real estate, or commodities, diverges persistently and substantially from its underlying fundamental value. These episodes are typically fuelled by optimistic expectations, speculative trading, and feedback mechanisms in which rising prices attract further demand, reinforcing the upward trajectory. As valuations become increasingly deviated from fundamentals, the process gives rise to unsustainable price dynamics that ultimately culminate in abrupt and often severe corrections, commonly referred to as “crashes” or “bubble bursts” (Brunnermeier et al., 2020). Beyond their price behaviour, bubbles are often associated with narrative-driven investing, where compelling stories about future growth dominate traditional valuation metrics and justify ever higher prices.

History provides numerous illustrations of such dynamics and their economic consequences. Tulip Mania in the Dutch Republic from 1634 to 1637 is widely regarded as the first documented speculative bubble, during which tulip bulb prices reached extraordinary levels before collapsing abruptly. The dot-com bubble of the late 1990s emerged from exuberant expectations about the transformative impact of the internet, leading to inflated equity valuations and a sharp market correction in 2000. More recently, the housing bubble in the United States during the 2000s, supported by excessive leverage, financial innovation, and lax lending standards, ultimately triggered the global financial crisis of 2007 to 2008 and resulted in widespread economic disruption.

These episodes exhibit a common set of stylised features, including elevated trading volume, increased market participation, rapid financial or technological innovation, and an expansion in the supply of the underlying asset (Scheinkman,

2014). They are also typically accompanied by strong belief formation and herding behaviour, as investors extrapolate recent price increases into the future and become less sensitive to downside risk. In this context, the recent surge in investment related to artificial intelligence has attracted considerable attention. The rapid rise in valuations of firms associated with AI, supported by powerful narratives about productivity gains and technological transformation, echoes patterns observed in earlier innovation-driven booms. While such developments may well reflect genuine fundamental improvements, the speed and magnitude of price adjustments raise important questions about the extent to which speculative forces may also be at play, reinforcing the continued relevance of rigorous bubble detection methodologies.

The econometric challenge of detecting bubbles has long been recognised since Shiller (1981), who introduced the variance bounds test. The central idea is that, if asset prices reflect only fundamentals under rational expectations, their volatility should not exceed the variability implied by those fundamentals. When observed price fluctuations are excessively large relative to this benchmark, the presence of a bubble is inferred. Subsequent extensions of this approach sought to refine its empirical implementation, yet they share an important limitation in that they depend heavily on specific model assumptions, leading to the well-known joint test problem. In response, West (1987) proposed a two-step procedure designed to disentangle model misspecification from bubble detection. While this represents a conceptual improvement, it remains challenging in practice to fully account for all potential sources of misspecification. Another influential strand of the early literature is due to Diba and Grossman (1988), who applied unit root and cointegration tests to stock prices and dividends. Their approach rests on the implication that, in the absence of bubbles, prices and dividends should exhibit the same order of integration and maintain a stable long-run relationship. However, Evans (1991) shows that such tests have very low power against periodically collapsing bubbles, which are widely observed in empirical data, thereby limiting their practical usefulness. Overall, as emphasised by Gürkaynak (2008), the early literature is characterised by a lack of consensus, with competing tests often yielding conflicting interpretations of bubble behaviour. Despite these challenges, these foundational contributions played a crucial role in shaping the development of more robust and flexible econometric methods that followed.

More recently, recursive right-tailed unit root tests have emerged as the leading

framework for bubble detection in financial time series. Developed by Phillips et al. (2011) and subsequently generalised by Phillips et al. (2015), these methods represent a major methodological advance over earlier approaches. By explicitly focusing on episodes of explosiveness, defined as periods in which the autoregressive root exceeds unity, they provide a direct way to identify departures from standard unit root behaviour. A key innovation of this framework lies in its recursive implementation, which enables both the detection and date-stamping of the origination and collapse of bubble episodes in real time. This stands in sharp contrast to earlier full-sample methods, which were largely limited to retrospective ex-post analysis. Beyond their conceptual appeal, these tests have demonstrated strong empirical performance. In particular, they are well suited to identifying multiple and recurrent bubble episodes within a single time series, a feature that aligns closely with observed market dynamics. As a result, they have been widely adopted in both academic research and policy institutions as practical tools for monitoring exuberance in financial markets. For example, it has been adopted by the Federal Reserve Bank of Dallas to monitor potential exuberance in international housing markets.

A growing body of subsequent research has sought to refine and extend this framework. On the theoretical side, important contributions have addressed issues related to heteroscedasticity and time-varying volatility, which are pervasive in financial data and can distort inference. For example, Harvey et al. (2016a) develop procedures that remain robust in the presence of non-constant variance, thereby improving the reliability of explosiveness tests under more realistic data-generating processes. Moreover, more recent work has begun to explicitly address the high-dimensional nature of modern financial data. In particular, Chen et al. (2023) propose a factor-based approach for detecting common bubble behaviour across assets, offering a first step toward capturing synchronised episodes of explosiveness in large cross-sections and improving the realism of empirical bubble detection frameworks. On the empirical side, recursive right-tailed unit root tests have been applied across a wide range of asset classes. Early applications focused primarily on equity and housing markets, but the scope has since expanded considerably. Recent studies have employed these methods to detect bubble dynamics in commodity markets, including oil and precious metals, where price booms are often linked to global demand shocks and speculative pressures (see for example, Figuerola-Ferretti and McCrorie, 2016; Gronwald, 2016; Caspi et al., 2018). In addition, the rapid growth of cryptocurrency markets has provided a new testing ground, with numerous studies documenting recur-

rent episodes of explosiveness in assets such as Bitcoin and other digital currencies (see for example, Cheung et al., 2015; Corbet et al., 2018; Hafner, 2020). These applications highlight both the flexibility of the methodology and its relevance in capturing speculative dynamics in emerging and highly volatile markets.

Despite their growing success, several important limitations remain. First, although these tests are designed to operate recursively, many empirical implementations still rely on repeated re-estimation using the full historical sample at each point in time. As a consequence, previously computed test outcomes are subject to revision whenever new observations become available. This lack of a strict non-revision property is problematic in a real-time macroprudential context, where policy decisions must be based on information sets that evolve sequentially and should not be retrospectively altered. In addition, the repeated re-estimation of models at each time step imposes substantial computational costs, particularly in long time series or high-dimensional settings, thereby limiting their scalability and practical use for timely monitoring. Finally, the prevailing interpretation of explosiveness remains narrowly focused on its role as a early-warning signal of asset price bubbles, while its potential informational content for investment decision-making has received comparatively less attention.

These limitations motivate the present thesis, which aims to address key shortcomings in the existing explosiveness testing literature by developing methods that are better aligned with real-time information constraints, computational feasibility, and broader economic interpretation.

The first chapter addresses the informational inconsistency inherent in the existing common bubble testing framework of Chen et al. (2023), which, despite its appeal for detecting synchronised explosive dynamics across assets, relies on full-sample principal component estimation and repeated re-estimation of the testing regression. These features violate the informational constraints faced by policymakers in real time, generate signals that are retrospectively revised as new observations arrive, and entail a computational cost that scales poorly in large panels. The central research question of this chapter is therefore how to construct a common bubble test that is consistent with real-time information sets, delivers non-revisable signals, and remains computationally feasible for high-dimensional systems. To answer it, we develop an adaptive implementation based on the Incremental Principal Component Analysis with Recursive Least Squares (IPCA-RLS) algorithm, which sequentially updates both the estimated common factor and the backward sup-ADF statistic using only infor-

mation available up to the current period. The methodological contribution is validated through an extensive set of Monte Carlo experiments, which show that IPCA–RLS reduces the computational complexity of factor extraction from $O(T^2)$ to $O(T)$ and of the testing stage from $O(T^3)$ to $O(T^2)$, with negligible loss of numerical accuracy relative to the batch implementation. An empirical application to a panel of U.S. stocks in the artificial intelligence sector illustrates the practical value of the framework, showing that the proposed algorithm generates timely, non-revised signals of common explosive behaviour that would be infeasible to monitor under the existing pseudo-real-time procedure.

The second chapter applies this real-time framework to the U.S. green equity market, where concerns about speculative dynamics have intensified due to strong capital inflows, ambitious climate policy commitments, and narrative-driven investment in the low-carbon economy transition. While commentators and academics have increasingly pointed to the possibility of a “green bubble”, existing empirical evidence is largely confined to ex-post analyses of individual assets or indices and does not capture the systemic, coordinated overvaluation that is most relevant from a financial-stability perspective. The chapter therefore asks whether common speculative exuberance is detectable in real time across green equities, and, if so, what macro-financial and climate-related conditions catalyse its emergence. The analysis is based on a panel of nine clean energy, clean technology, and green energy equity indices, using daily price-to-earnings ratios in order to anchor price dynamics to a fundamental benchmark. Applying the common bubble test of Chen et al. (2023) through the IPCA–RLS algorithm developed in Chapter 1, we identify the periods during which the dominant common factor of green equity valuations exhibits explosive behaviour, and we then investigate their determinants through a probit regression framework. The empirical findings document multiple episodes of common green bubbles, with the most prominent occurring in March–April 2021. A comparison with contemporaneous bubbles in broad equity market indices shows minimal overlap, indicating that green bubble formation is largely independent of general post-pandemic market exuberance. The probit analysis further reveals that the emergence of these episodes is strongly associated with macro-financial conditions, particularly credit spreads, and with physical climate risk, highlighting a distinct set of catalysts that warrant the attention of macroprudential authorities.

Chapter 3 addresses a different limitation of the literature, namely the narrow interpretation of explosiveness as solely a risk signal for financial instability. It

argues that explosive dynamics may also contain useful predictive information for investors, consistent with evidence that sophisticated traders rationally ride rather than arbitrage bubbles and with extrapolative-expectations frameworks in which accelerating price trajectories capture sentiment-driven return continuation. The chapter thus asks whether statistically significant explosive episodes, detected through right-tailed recursive unit root tests, can serve as a more informative time-series signal than conventional cumulative-return momentum measures, which treat all return trajectories with identical cumulative performance as equivalent and ignore the curvature and acceleration embedded in the price path. The empirical analysis uses a comprehensive dataset of 153 monthly factor returns constructed by Jensen et al. (2023), spanning January 1980 to December 2024 and yielding 540 observations per factor. Applying the PSY test of Phillips et al. (2015) at the 95% confidence level, we find that 119 out of 153 factors display episodes of mild explosiveness, establishing this behaviour as a pervasive feature of the factor zoo. We then construct a Time-Series Factor Explosiveness (TSFE) strategy, which goes long factors flagged as explosive in a given month and short those that are not, together with a Cross-Sectional Factor Explosiveness (CSFE) counterpart based on ranked backward sup-ADF statistics. The long leg of the TSFE strategy delivers an annualised excess return of 5.29% and a Sharpe ratio of 0.92, consistently outperforming the time-series factor momentum benchmark, while the CSFE strategy achieves a higher Sharpe ratio but a lower overall return than cross-sectional factor momentum, suggesting that explosiveness operates most effectively as a time-series rather than a cross-sectional signal. The results are robust to transaction costs, alternative PSY confidence levels, and an alternative factor zoo dataset, and by construction rely only on information available up to the current period, thereby avoiding look-ahead bias and data-mining concerns.

Chapter 1

Efficient Real-Time Common Bubble Detection

1.1 Introduction

In this chapter, we propose a computationally efficient real-time IPCA–RLS algorithm for common bubble detection. The method combines Incremental Principal Component Analysis (IPCA) with Recursive Least Squares (RLS) to implement the common bubble test efficiently, preserving the test statistic of Chen et al. (2023) while delivering two key improvements: it substantially reduces the computational burden of the procedure and replaces the full-sample principal component estimation with a sequential implementation that relies only on information available up to each point in time.

Asset price bubbles arise when the prices of financial assets, such as equities, real estate, or commodities, exhibit rapid and persistent increases that deviate substantially from their fundamental values. These episodes are typically fueled by excessive optimism, speculative behaviour, and self-reinforcing expectations, resulting in price dynamics that are ultimately unsustainable and often followed by abrupt corrections. History provides numerous examples of such episodes with severe macro-financial consequences, including widespread financial distress, and prolonged economic downturns (for instance, see Vogel, 2021 and Quinn and Turner, 2022). For central banks and financial regulators, the ability

to detect asset price bubbles in real time is therefore critical, as timely identification of emerging exuberance can inform macroprudential policy actions aimed at preserving financial stability and mitigating systemic risk.

The empirical literature on bubble detection can be traced back to the seminal contribution of Shiller (1981), who emphasised excess volatility in asset prices relative to fundamentals as an indicator of speculative behaviour. Subsequent work formalised the econometric testing of explosive dynamics in asset prices, but empirical detection remained controversial due to strong modeling assumptions and sensitivity to the specification of fundamentals. As highlighted by Gürkaynak (2008): “for almost every paper in the literature that ‘finds’ a bubble, there is another one that relaxes some assumption on the fundamentals and fits the data equally well without resorting to a bubble” (p. 167). Interest in bubble testing surged again following the global financial crisis, driven by the development of more powerful econometric tools. In particular, the recursive right-tailed unit root test proposed by Phillips et al. (2011) (PWY hereafter), and its generalised version proposed by Phillips et al. (2015) (PSY hereafter). These methods have gained widespread adoption due to their strong statistical power and their ability to date-stamp bubble origination and collapse dates.

More recently, the focus has shifted from individual assets to system-wide risks, leading to the development of testing frameworks for common bubbles across multiple assets. A natural approach is to apply univariate bubble tests to individual series, as in Pavlidis et al. (2016), who document synchronised housing bubbles across countries. However, this strategy suffers from two important drawbacks. First, it ignores cross-sectional dependence, which is essential for identifying latent common speculative components driving systemic risk. Second, repeated univariate testing exacerbates multiple-testing problems, increasing the likelihood of false positives. To address these issues, Chen et al. (2023) propose a common bubble testing framework based on a factor model. Their approach applies principal component analysis (PCA) to a large panel and subsequently implements the PSY test on the estimated first principal component (PC1), which consistently captures the latent mildly explosive common factor.

Despite these appealing properties, the existing common bubble testing framework is fundamentally retrospective. The procedure relies on full-sample estimation of principal components and recursive right-tailed unit root test that repeatedly reprocess the entire historical sample. This feature stands in sharp contrast to the principles of real-time monitoring, where policymakers must rely

exclusively on information available at each point in time. The macroeconomic nowcasting literature, as formalised by Giannone et al. (2008), emphasises the importance of real-time information sets, vintage data, and sequential updating as new observations arrive. From this perspective, the full-sample implementation of common bubble test implicitly incorporates future information and therefore does not conform to the informational constraints faced by central banks and regulators in practice. Although Chen et al. (2023) propose a pseudo real-time implementation that repeatedly re-estimates the PCA and subsequent unit root regressions using data available up to the current period, their approach yields unstable estimates and requires revising past results at each update. Moreover, the associated computational burden grows rapidly with the sample size, severely limiting the feasibility of real-time surveillance in large financial systems.

The contribution of this chapter is twofold. First, we develop a genuinely real-time monitoring common bubble detection framework. While Chen et al. (2023) estimate the principal components using the full sample—thereby implicitly incorporating future information—our approach updates the common factor recursively using only the information available at each point in time. This sequential construction eliminates the need to re-estimate the entire model whenever new observations arrive and therefore avoids the revision of previously generated signals. Second, we introduce an adaptive algorithm that combines IPCA with RLS to implement the procedure efficiently. The resulting IPCA–RLS algorithm substantially reduces the computational burden of the common bubble detection framework while targeting the same underlying test statistic as Chen et al. (2023). In particular, the proposed method lowers the computational complexity from $O(T^2)$ to $O(T)$ in the factor extraction stage and from $O(T^3)$ to $O(T^2)$ in the testing stage, making real-time monitoring feasible in large- T panel settings.

The remainder of the chapter is organised as follows. Section 1.2 introduces the econometric framework for common bubble detection. Section 1.3 presents the IPCA–RLS adaptive algorithm. Section 1.4 demonstrates that the IPCA–RLS framework provides a computationally efficient implementation for real-time monitoring of common bubbles, while preserving the inferential properties of existing econometric tests. Section 1.5 reports an extensive set of Monte Carlo experiments. Section 1.6 presents an empirical application to stocks in the artificial intelligence sector. Section 1.7 concludes.

1.2 Econometric Analysis of Common Bubble

This section provides the background knowledge about common bubble detection. Section 1.2.1 outlines the rational bubble model, which serves as the theoretical basis for empirical testing. Section 1.2.2 then introduces the common bubble test developed by Chen et al. (2023). We also highlight the benefits of applying the common bubble test to large- T panel data, which is confirmed by our extended Monte Carlo simulations. The results are presented in the Appendix [1.A](#).

1.2.1 Rational Bubbles

The theory of rational bubbles suggests that, even within the rational expectations framework, asset prices can sharply diverge from their fundamental values. This model is based on the idea that if investors expect prices to continue rising in the future, they will be willing to purchase the asset at higher prices, creating a self-reinforcing cycle of price increases. This positive feedback mechanism can result in a bubble, where asset prices significantly exceed their intrinsic values. To illustrate how a rational bubble can form, consider the following asset pricing equation:

$$P_t = E_t \left[\frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}} \right] \quad (1.1)$$

where P_t and D_t are asset price and dividend at time t , respectively. R_t is the discount rate at time t and $E_t(\cdot)$ is the conditional expectation at time t . Assume that the discount rate is a constant R over time, we then have

$$P_t = \frac{E_t [P_{t+1} + D_{t+1}]}{1 + R}. \quad (1.2)$$

Equation (1.2) is the starting point of most empirical asset pricing tests. This equation can be iterated until infinity

$$\begin{aligned} P_t &= E_t \left[\sum_{\tau=1}^{\infty} \frac{D_{t+\tau}}{(1+R)^\tau} \right] + \lim_{T \rightarrow \infty} E_t \left[\frac{P_T}{(1+R)^{T-t}} \right] \\ &= A_t + B_t. \end{aligned} \quad (1.3)$$

Equation (1.3) implicates that the asset price has two components: the funda-

mental value A_t , which is the discounted value of future dividends, and a bubble component B_t . In the absence of a bubble, the second term in the right hand side of Equation (1.3) vanishes, namely $\lim_{T \rightarrow \infty} E_t \left[\frac{P_T}{(1+R)^{T-t}} \right] = 0$. In the contrast, the bubble component should satisfy the following submartingale property

$$E_t [B_{t+1}] = (1 + R)B_t. \quad (1.4)$$

If a bubble is present, the dynamic of asset price P_t will be dominated by the explosive behaviour of B_t , because the fundamental value A_t is commonly believed to be at most I(1). In practice, we often use price-to-dividend ratios or price-to-earning ratios to detect bubbles.

1.2.2 Common Bubble Test

Chen et al. (2023) propose a novel approach for testing the existence of a common bubble factor in a large-dimensional system of financial time series. Under the null hypothesis of absence of a common bubble, the dynamic of the price of asset i , which is denoted as y_{ti} , is governed by the following equation

$$y_{ti} = f_{0,t}l_{0,i} + \epsilon_{ti}, \text{ for } t = 1, \dots, T; i = 1, \dots, N \quad (1.5)$$

where $f_{0,t}$ represents the fundamental factor that follows a unit root process, $l_{0,i}$ is the factor loading, and ϵ_{ti} is the idiosyncratic error terms. Equation (1.5) can be also expressed in a matrix form as follows

$$Y = F_0 L_0' + E \quad (1.6)$$

where Y is a $T \times N$ matrix of observed data, $F_0 = (f_{0,1}, \dots, f_{0,T})'$ is a $T \times 1$ vector, $L_0 = (l_{0,1}, \dots, l_{0,N})'$ is a $N \times 1$ vector, and E is a $T \times N$ matrix of idiosyncratic errors.

Under the alternative hypothesis, the data generating process (DGP) of y_{ti} is defined as follows

$$y_{ti} = \begin{cases} f_{0,t}l_{0,i} + \epsilon_{ti} & t \in (0, \lfloor Tr_e \rfloor] \\ f_{0,t}l_{0,i} + f_{1,t}l_{1,i} + \epsilon_{ti} & t \in (\lfloor Tr_e \rfloor, \lfloor Tr_c \rfloor] \\ f_{2,t}l_{2,i} + \epsilon_{ti} & t \in (\lfloor Tr_c \rfloor, T] \end{cases} \quad (1.7)$$

where $f_{0,t}$ follows a unit root process as in Equation (1.5), $f_{1,t}$ has a mildly explosive root and $f_{2,t}$ is assumed to be a mildly stationary process. $f_{0,t}$, $f_{1,t}$ and $f_{2,t}$ can be expressed as

$$\begin{cases} f_{0,t} = f_{0,t-1} + \nu_{0,t} \\ f_{1,t} = (1 + c_1 T^{-\alpha}) f_{1,t-1} + \nu_{1,t} \\ f_{2,t} = (1 - c_2 T^{-\beta}) f_{2,t-1} + \nu_{2,t} \end{cases} \quad (1.8)$$

where c_1, c_2 are positive localising coefficients, $\alpha, \beta \in (0, 1)$ are rate parameters, $\nu_{0,t}, \nu_{1,t}, \nu_{2,t}$ are individual error terms. Additionally, define r as the r^{th} fraction of the entire sample, then the origination and termination date of bubble regime are given by $\lfloor Tr_e \rfloor$ and $\lfloor Tr_c \rfloor$ respectively, where $\lfloor \cdot \rfloor$ signifies the integer part of its input.

The common bubble test involves two stages to detect and date-stamp a latent common bubble in a panel of financial time series. In the first stage, the first principal component (PC1) is estimated by principal component analysis (PCA). Assume the true number of factors is n , and denote the common factors as γ_t , and corresponding loading as δ_i . Suppose the estimated factor loading by PCA is denoted as $\hat{\Delta} = (\hat{\delta}_1, \hat{\delta}_2, \dots, \hat{\delta}_N)'$, which is a $N \times n$ matrix. The estimated PC1 can be obtained by

$$\hat{y}_1 = Y \hat{\lambda}_1 / N \quad (1.9)$$

where $\hat{\lambda}_1 = (\hat{\Delta}_{11}, \hat{\Delta}_{21}, \dots, \hat{\Delta}_{N1})'$, which is a $N \times 1$ vector.

In the second stage, the PSY test is applied to the estimated PC1. The regression model used in this stage is specified as

$$\Delta \hat{y}_{1,t} = \mu + \rho \hat{y}_{1,t-1} + \sum_{i=1}^k \phi_i \Delta \hat{y}_{1,t-i} + u_t \quad (1.10)$$

where $\hat{y}_{1,t}$ is the value of the estimated PC1 at time t ; μ is a constant term; k is the maximum lag order which can be determined by information criteria; u_t is the normal and independent distributed error term. The unit root null hypothesis is $H_0 : \rho = 0$ and the right-tailed alternative hypothesis is $H_1 : \rho > 0$.

Suppose we estimate Equation (1.10) with a window starting at the r_1^{th} fraction and ending at the r_2^{th} fraction of the full sample, denote the t -statistic associated with ρ as $ADF_{r_1}^{r_2}$. With the suggested minimum window size $r_0 = 0.01 + 1.8/\sqrt{T}$,

the PSY test statistic is defined as

$$PSY_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}. \quad (1.11)$$

Let β_T represent the significance level of the test such that $scv_{r_2}^{\beta_T}$ is the corresponding $100(1 - \beta_T)\%$ critical value. If a common bubble is identified, then its origination and termination date can be estimated as

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2 : PSY_{r_2}(r_0) > scv_{r_2}^{\beta_T}\} \quad (1.12)$$

and

$$\hat{r}_c = \inf_{r_2 \in [\hat{r}_e, 1]} \{r_2 : PSY_{r_2}(r_0) < scv_{r_2}^{\beta_T}\}. \quad (1.13)$$

Chen et al. (2023) provide asymptotic properties of their proposed common bubble test. The estimated PC1 is at most I(1) under the null but exhibits mildly explosive behavior under the alternative hypothesis when a common bubble factor is present (See Lemmas 3.1-3.2 in Chen et al. (2023)). These theoretical properties ensure that detecting common bubbles can be achieved by testing for explosiveness in the estimated PC1. Moreover, based on 2,000 replications, their Monte Carlo simulations find that the successful detection rate, defined as the proportion of cases where the estimated common bubble origination date falls between the true start and end dates, and the average detection delay (i.e., $\frac{1}{2,000} \sum_{m=1}^{2,000} \hat{r}_e^m - r_e$) both improve significantly as T increases. These results underscore the advantages of using large- T panel data for detecting common bubbles in practice, though it is also worth noting the increasing computational burden associated with larger samples¹.

¹The authors also conduct simulations under alternative scenarios, including the presence of idiosyncratic bubbles and multiple bubbles, and find that the results remain qualitatively unchanged. In addition to their work, we perform extensive Monte Carlo simulations, focusing on the issue of heteroscedasticity, which was first identified by Harvey et al. (2016b). Several modified versions of the PSY test have been developed to address this issue, including the generalized least squares version proposed by Whitehouse (2019), the weighted least squares version by Harvey et al. (2019), and the sign-based version developed by Harvey et al. (2020). We incorporate these alternative versions of the PSY test and define the volatility of factors as $\sigma_{\nu_0} = \sigma_{\nu_1} = \sigma_{\nu_2} = \sigma_t v_t$, where $v_t \stackrel{\text{iid}}{\sim} (0, 1)$ is independently and identically normal distributed, and σ_t follows a GARCH(1,1) process:

$$\sigma_t^2 = \chi + \phi \sigma_{t-1}^2 v_{t-1}^2 + \psi \sigma_{t-1}^2 \quad (1.14)$$

with parameters set as $\chi = 0.05$, $\phi = 0.1$ and $\psi = 0.85$, resulting in an unconditional volatility of

To address this computational issue, we propose a novel adaptive algorithm that we introduce in the next section.

1.3 Adaptive Algorithms

Adaptive algorithms constitute a broad class of methods that iteratively update model parameters in response to new data. They are widely applied in areas such as online learning, system identification, and signal processing, where real-time adaptability is crucial. In the current era of data-driven decision-making, their relevance is particularly pronounced in non-stationary environments, where the underlying data-generating process evolves over time. By enabling models to adjust parameters automatically as distributions shift, adaptive algorithms provide a principled framework for maintaining accuracy and stability in the face of structural change. This makes them indispensable for applications ranging from financial econometrics to modern machine learning systems.

For example, in the context of common bubble detection, there are two key time-varying parameters: the PC1 from the first stage and the PSY statistics obtained in the second stage. To update these quantities efficiently in real time, we introduce two adaptive algorithms: Incremental Principal Component Analysis (IPCA) for updating the estimated PC1, and Recursive Least Squares (RLS) for updating the PSY statistics. We then propose a composite algorithm, IPCA-RLS, which combines these two methods to enable efficient real-time common bubble detection. The IPCA-RLS algorithm reduces the time complexity of the original testing procedure from $O(T^2)$ to $O(T)$ in the first stage and from $O(T^3)$ to $O(T^2)$ in the second stage, if $T \gg N$, corresponding to the case when working with large- T panel data.

1.3.1 Two Building Blocks

Incremental Principal Component Analysis

The IPCA algorithm is a representative approach within the broader class of online PCA methods, which are designed to extract principal components from streaming data in an iterative manner. Unlike traditional PCA—which requires

$\frac{\lambda}{1-\phi-\psi} = 1$. Moreover, we extend the finite sample analysis to longer sample lengths, considering $T = \{100, 200, 400, 800, 1600\}$. Our findings align with those reported in Chen et al. (2023) and are documented in the Appendix [1.A](#).

access to the full dataset—online PCA algorithms update the principal components recursively as new observations arrive, making them well-suited for real-time or sequential data processing.

Cardot and Degras (2018) provide a comprehensive review of the main methodologies for online PCA currently available in the literature. These include perturbation based techniques, incremental approaches, and stochastic optimization methods. Their study offers a comparative evaluation of these methods, focusing on both statistical accuracy and computational efficiency. Among the evaluated approaches, IPCA is shown to offer the most favorable trade-off between accuracy and speed. Based on these findings, this paper adopts the IPCA algorithm as the preferred method for recursive principal component updates in the proposed adaptive framework.

Following the notation in Section 1.2.2, given the $T \times N$ matrix $Y = [x_1, x_2, \dots, x_T]'$, where $x_s = [y_{s1}, y_{s2}, \dots, y_{sN}]'$ is a $N \times 1$ vector for $s = 1, 2, \dots, T$, and T is the number of observations in each time series and N is the number of time series. The sample mean denoted by $\mu_T = \frac{1}{T} \sum_{t=1}^T x_t$, and the sample covariance matrix is expressed as

$$\Sigma_T = \frac{1}{T} \sum_{t=1}^T (x_t - \mu_T)(x_t - \mu_T)'. \quad (1.15)$$

When a new observation x_{T+1} is realized, the sample mean vector can be computed recursively as

$$\mu_{T+1} = \frac{T}{T+1} \mu_T + \frac{1}{T+1} x_{T+1} \quad (1.16)$$

If we evaluate Equation (1.15) at $t = T + 1$ and substitute μ_{T+1} with its expression from Equation (1.16), the sample covariance matrix satisfies the following recursive equation

$$\Sigma_{T+1} = \frac{T}{T+1} \Sigma_T + \frac{T}{(T+1)^2} (x_{T+1} - \mu_T)(x_{T+1} - \mu_T)'. \quad (1.17)$$

Let $\Omega_T = M_T N_T M_T'$ be a rank q approximation to Σ_T , where the $q \times q$ matrix N_T is diagonal with the approximated first q eigenvalues of Σ_T , and the $N \times q$ matrix M_T approximates the corresponding eigenvectors of Σ_T , satisfying $M_T' M_T = I_q$. The parameter q governs the trade-off between computational

burden and approximation accuracy. When a new observation x_{T+1} is available, Ω_T is updated as follows. The centred vector $\tilde{x}_{T+1} = x_{T+1} - \mu_T$ is decomposed as $\tilde{x}_{T+1} = M_T c_{T+1} + \tilde{x}_{T+1}^\perp$, where $c_{T+1} = M_T' \tilde{x}_{T+1}$ are the coordinates of $\tilde{x}_{T+1}$ in the q -dimensional space spanned by M_T and $\tilde{x}_{T+1}^\perp$ is the projection of $\tilde{x}_{T+1}$ onto the orthogonal space of M_T . In view of Equation (1.17), the covariance Σ_{T+1} is approximated by $\Omega_{T+1} = \frac{T}{T+1} \Omega_T + \frac{T}{(T+1)^2} \tilde{x}_{T+1} \tilde{x}_{T+1}'$. This can rewrite as

$$\Omega_{T+1} = \left[M_T \frac{\tilde{x}_{T+1}^\perp}{\|\tilde{x}_{T+1}^\perp\|} \right] U_{T+1} \left[M_T \frac{\tilde{x}_{T+1}^\perp}{\|\tilde{x}_{T+1}^\perp\|} \right]' \quad (1.18)$$

with

$$U_{T+1} = \frac{T}{(T+1)^2} \begin{bmatrix} (T+1)N_T + c_{T+1}c_{T+1}' & \|\tilde{x}_{T+1}^\perp\|c_{T+1} \\ \|\tilde{x}_{T+1}^\perp\|c_{T+1}' & \|\tilde{x}_{T+1}^\perp\|^2 \end{bmatrix}. \quad (1.19)$$

It then suffices to perform the eigenvalue decomposition (EVD) of the matrix U_{T+1} of dimension $(q+1) \times (q+1)$. Suppose $U_{T+1} = V_{T+1} W_{T+1} V_{T+1}'$ with V_{T+1} orthogonal and W_{T+1} diagonal, the EVD of Ω_{T+1} simply expresses as $M_{T+1} N_{T+1} M_{T+1}'$, where $N_{T+1} = W_{T+1}$ and

$$M_{T+1} = \left[M_T \frac{\tilde{x}_{T+1}^\perp}{\|\tilde{x}_{T+1}^\perp\|} \right] V_{T+1}. \quad (1.20)$$

Finally, to keep the approximation Ω_{T+1} of Σ_{T+1} at rank q , the row and column of N_{T+1} containing the smallest eigenvalue are deleted and the associated eigenvectors is deleted from M_{T+1} . Consider the first q principal components $\hat{y} = Y M_T / N$, which is represented by a $T \times q$ matrix. The $(T+1)$ th value of the first principal component $\hat{y}_{1,T+1}$ can be expressed as

$$\hat{y}_{1,T+1} = x_{T+1}' M_{T+1} / N. \quad (1.21)$$

Recursive Least Squares

The RLS algorithm is a widely recognized recursive technique that efficiently updates the least squares estimates with the introduction of each new observation. It was initially introduced to the domain of recursive unit root tests by Vasilopoulos et al. (2022). Consider the ADF regression in Equation (1.10), which can be

rewritten in a matrix form as follows:

$$\Delta \hat{y}_1 = Z\beta + \Xi, \quad (1.22)$$

where

$$\Delta \hat{y}_{1,s:T+1} = \begin{bmatrix} \Delta \hat{y}_{1,s} \\ \Delta \hat{y}_{1,s+1} \\ \vdots \\ \Delta \hat{y}_{1,T+1} \end{bmatrix}, Z_{s:T+1} = \begin{bmatrix} z_s \\ z_{s+1} \\ \vdots \\ z_{T+1} \end{bmatrix} = \begin{bmatrix} 1 & \hat{y}_{1,s-1} & \Delta \hat{y}_{1,s-1} & \dots & \Delta \hat{y}_{1,s-k} \\ 1 & \hat{y}_{1,s} & \Delta \hat{y}_{1,s} & \dots & \Delta \hat{y}_{1,s-k+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \hat{y}_{1,T} & \Delta \hat{y}_{1,T} & \dots & \Delta \hat{y}_{1,T+1-k} \end{bmatrix},$$

$$\Xi_{s:T+1} = \begin{bmatrix} u_s \\ u_{s+1} \\ \vdots \\ u_{T+1} \end{bmatrix}, \text{ and } \beta_{s:T+1} = \begin{bmatrix} \mu \\ \rho - 1 \\ \phi_1 \\ \vdots \\ \phi_k \end{bmatrix}. \quad (1.23)$$

For the given regression window $[s : T + 1]$, the ordinary least squares estimate is obtained by

$$\hat{\beta}_{s:T+1} = (Z'_{s:T+1} Z_{s:T+1})^{-1} Z'_{s:T+1} \Delta \hat{y}_{1,s:T+1}. \quad (1.24)$$

Define the gain matrix as $\Lambda_{s:T+1} = (Z'_{s:T+1} Z_{s:T+1})^{-1}$, the gain matrix can be updated from its previous value as follows by applying the Sherman-Morrison-Woodbury formula:

$$\Lambda_{s:T+1} = \Lambda_{s:T} - (1 + z_{T+1} \Lambda_{s:T} z'_{T+1})^{-1} (\Lambda_{s:T} z'_{T+1}) (z_{T+1} \Lambda_{s:T}). \quad (1.25)$$

Remark: (Sherman–Morrison–Woodbury formula) Let $A \in \mathbb{C}^{n \times n}$, $C \in \mathbb{C}^{m \times m}$, $B \in \mathbb{C}^{n \times m}$ and $D \in \mathbb{C}^{m \times n}$. Then, if all inverse operations below are well-defined,

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}. \quad (1.26)$$

Furthermore, we get a recursive pattern for the least squares estimates

$$\begin{aligned}
\hat{\beta}_{s:T+1} &= \Lambda_{s:T+1} Z'_{s:T+1} \Delta \hat{y}_{1,s:T+1} \\
&= (\Lambda_{s:T} - \frac{(\Lambda_{s:T} z'_{T+1})(z_{T+1} \Lambda_{s:T})}{1 + z_{T+1} \Lambda_{s:T} z'_{T+1}})(Z'_{s:T} \Delta \hat{y}_{1,s:T} + z'_{T+1} \Delta \hat{y}_{1,T+1}) \\
&= \hat{\beta}_{s:T} - \Lambda_{s:T+1} z'_{T+1} (z_{T+1} \hat{\beta}_{s:T} - \Delta \hat{y}_{1,T+1}).
\end{aligned} \tag{1.27}$$

Finally, the ADF statistic over the regression window $[s : T + 1]$ can be expressed as

$$ADF_s^{T+1} = \left\{ \frac{\hat{\beta}_{s:T+1}}{SE(\hat{\beta}_{s:T+1})} \right\}_2 \tag{1.28}$$

where $\left\{ \frac{\hat{\beta}_{s:T+1}}{SE(\hat{\beta}_{s:T+1})} \right\}_2$ denotes the second element of the vector $\frac{\hat{\beta}_{s:T+1}}{SE(\hat{\beta}_{s:T+1})}$, and $SE(\hat{\beta}_{s:T+1}) = \sqrt{\frac{\hat{\Xi}'_{s:T+1} \hat{\Xi}_{s:T+1}}{((T+1)-s+1)+2}}$ is the estimated standard error of $\hat{\beta}_{s:T+1}$.

1.3.2 IPCA-RLS Algorithm

Next, we integrate the two adaptive algorithms discussed earlier to develop the IPCA-RLS algorithm, offering a computationally efficient approach to implementing the common bubble test. Specifically, the IPCA algorithm is employed in the first stage to update the estimated PC1 efficiently, while the RLS algorithm is used in the second stage to update the ADF statistics efficiently. The key steps are outlined in Algorithm 1.

In the proposed IPCA-RLS algorithm, Steps 1-2 updates the estimated PC1, offering a computationally efficient approach to the first stage of the common bubble test. Steps 3-5 handle the update of ADF statistics and the computation of the PSY statistic, providing an efficient solution for the second stage of the test. Both stages share a common iteration over the endpoint of the estimation window t_2 , which enables the algorithm to operate in real time with streaming data. In our practical implementation, we execute the two stages of the common bubble test separately, despite their shared iteration over t_2 . This design choice helps avoid repeated initialization of adaptive algorithms, further reducing computational time.

The primary computational burdens in both stages of the common bubble test stem from matrix multiplication and inversion. Particularly in constructing and performing EVD of the covariance matrix in the first stage. In our setting, constructing the covariance matrix Σ_{T+1} incurs a time complexity of $O(TN^2)$, with

Algorithm 1: IPCA-RLS

-
- Step 1:** Generate the initial estimated PC1 $\hat{y}_{1,1}, \hat{y}_{1,2}, \dots, \hat{y}_{1,t_0}$ by standard PCA, using the first t_0 observations. Following the literature, we set $t_0 = \lfloor Tr_0 \rfloor$, where $r_0 = 0.01 + 1.8/\sqrt{T}$.
- Step 2:** For each new time point t_2 , update the PCA using the IPCA algorithm according to Equations (1.19)-(1.21), which yields the new observation of the PC1, namely $\hat{y}_{1,t_2}$.
- Step 3:** Calculate the initial least squares estimates $ADF_{t_2-t_0+1}^{t_2}$ by OLS based on the ADF regression on the subsample $(\hat{y}_{1,t_2-t_0+1}, \dots, \hat{y}_{1,t_2})$.
- Step 4:** Decrease t_1 from its initial value $t_1 = t_2 - t_0 + 1$ by one time point each time until $t_1 = 1$, update the least squares estimates $ADF_{t_1}^{t_2}$ recursively using the RLS algorithm according to Equations (1.25)-(1.28).
- Step 5:** Obtain the maximum value of the sequence of $ADF_{t_1}^{t_2}$, for $t_1 \in [1, t_2 - t_0 + 1]$, which is denoted as PSY_{t_2} .
- Step 6:** Increase t_2 by one time point each time until $t_2 = T$, repeat Steps 2-5 for each iteration.
- Step 7:** Obtain the sequence of PSY_{t_2} for $t_2 \in [t_0 + 1, T]$, compare them to the corresponding finite sample critical values and date-stamp the identified common bubbles (if any).
-

its EVD requires $O(N^3)$, resulting in an overall complexity of $O(TN^2 + N^3)$. The IPCA algorithm approximates this process by operating on a lower-dimensional matrix U_{T+1} , reducing the complexity to $O(Nq^2 + q^3)$. Under the assumption that $T \gg N \gg q$, IPCA updates principal components recursively in time of $O(T)$, compared to the $O(T^2)$ cost of recomputing PCA at each time step. Similarly, in the second stage, the main computational cost arises from constructing and inverting the matrix $Z'_{s:T+1}Z_{s:T+1}$. This requires a time complexity of $O(Tk^2 + k^3)$ for calculating the ADF statistic over the full sample by OLS, whereas the RLS algorithm reduces the cost of each recursive update to $O(k^2)$. When $T \gg k$, the time complexity per iteration over t_2 is reduced from $O(T^2)$ with repeated OLS computation to $O(T)$ using recursive RLS updates. Consequently, the total time complexity of the second stage is reduced from $O(T^3)$ to $O(T^2)$ by employing adaptive algorithm.

1.4 Sequential Estimation and Asymptotic Equivalence

In this section, we set out the conditions under which the proposed IPCA-RLS procedure is expected to provide a sequential approximation to the real-time

batch implementation of the common bubble test. Formal asymptotic analysis of the recursive covariance approximation, the eigenvector perturbation, and the recursive least squares update in the presence of the nonstationary DGP described in Section 1.2.2 is intricate: under the null the common factor follows a unit root process, and under the alternative it exhibits mildly explosive and mildly stationary regimes. Standard perturbation bounds for symmetric matrices and standard regularity conditions for recursive least squares are typically developed for stationary sequences and do not apply directly in this environment. For this reason, we present the central asymptotic equivalence results below as *conjectures* rather than theorems, and refer the reader to the Appendix 1.B for the heuristic arguments that motivate them. The numerical evidence reported in Section 1.5 strongly supports these conjectures, but a rigorous proof under nonstationary DGPs is left for future work.

Let $\mathcal{F}_t = \sigma(x_1, \dots, x_t)$ denote the information set available at time t , where x_t is the $N \times 1$ vector of observations in the panel.

Assumption 1 (Approximate factor structure). The observed panel follows the factor model

$$x_t = \lambda f_t + e_t,$$

where f_t is a scalar common factor, λ is the $N \times 1$ loading vector, and e_t denotes idiosyncratic errors with bounded fourth moments and weak cross-sectional and temporal dependence.

Assumption 2 (Factor strength). The leading eigenvalue of the covariance matrix of x_t diverges with N and is separated from the remaining eigenvalues.

Assumption 3 (Regularity conditions). The processes $\{f_t\}$ and $\{e_t\}$ satisfy standard regularity conditions of the type used for large-dimensional approximate factor models with unit root and mildly explosive factors (see, e.g., Chen et al., 2023). In particular, the appropriately normalised sample second moments of x_t admit well-defined probability limits along the expanding sample, the leading eigenvalue of the associated covariance matrix is well separated from the remaining eigenvalues in the sense of Assumption 2, and the sample size satisfies $T, N \rightarrow \infty$.

The following conjectures state that the proposed sequential estimator targets the same real-time batch estimator that would be obtained by repeatedly re-

estimating the model on the expanding sample. Heuristic arguments motivating these conjectures are provided in Online Appendix B.

Let $\hat{y}_{1,t}^{PCA}$ denote the first principal component obtained by applying PCA to the expanding sample $\{x_1, \dots, x_t\}$, and let $\hat{y}_{1,t}^{IPCA}$ denote the sequential estimate produced by the proposed IPCA algorithm.

Conjecture 1 (Sequential Factor Equivalence). *Under Assumptions 1–3, the sequential factor estimate obtained by IPCA is asymptotically equivalent to the real-time batch PCA estimator:*

$$\sup_{t_0 \leq t \leq T} \left| \hat{y}_{1,t}^{IPCA} - \hat{y}_{1,t}^{PCA} \right| = o_p(1).$$

Conjecture 1 states that the sequential procedure is expected to extract the same latent common factor that would be obtained by repeatedly recomputing PCA on the expanding sample, while avoiding repeated re-estimation.

Let $\hat{\beta}_t^{OLS}$ denote the estimator obtained by re-estimating the ADF regression on the expanding sample, and $\hat{\beta}_t^{RLS}$ the recursive least squares estimator used in the IPCA–RLS algorithm.

Conjecture 2 (Recursive Estimation Equivalence). *Under Assumptions 1–3 and standard regularity conditions for recursive least squares estimation adapted to non-stationary regressors,*

$$\sup_{t \leq T} \left\| \hat{\beta}_t^{RLS} - \hat{\beta}_t^{OLS} \right\| = o_p(1).$$

Conjecture 2 indicates that recursive updating of the regression coefficients is expected to leave the asymptotic properties of the parameter estimates unchanged relative to repeated OLS estimation.

Let $PSY^{seq}(r_2)$ denote the sequential monitoring statistic obtained using the IPCA–RLS procedure, and let $PSY^{batch}(r_2)$ denote the statistic that would be obtained by recomputing the PSY test on the expanding sample at each date.

Conjecture 3 (Asymptotic Validity of the Sequential PSY Statistic). *Under the null hypothesis of no bubble and Assumptions 1–3,*

$$\sup_{r_2 \in [r_0, 1]} \left| PSY^{seq}(r_2) - PSY^{batch}(r_2) \right| = o_p(1).$$

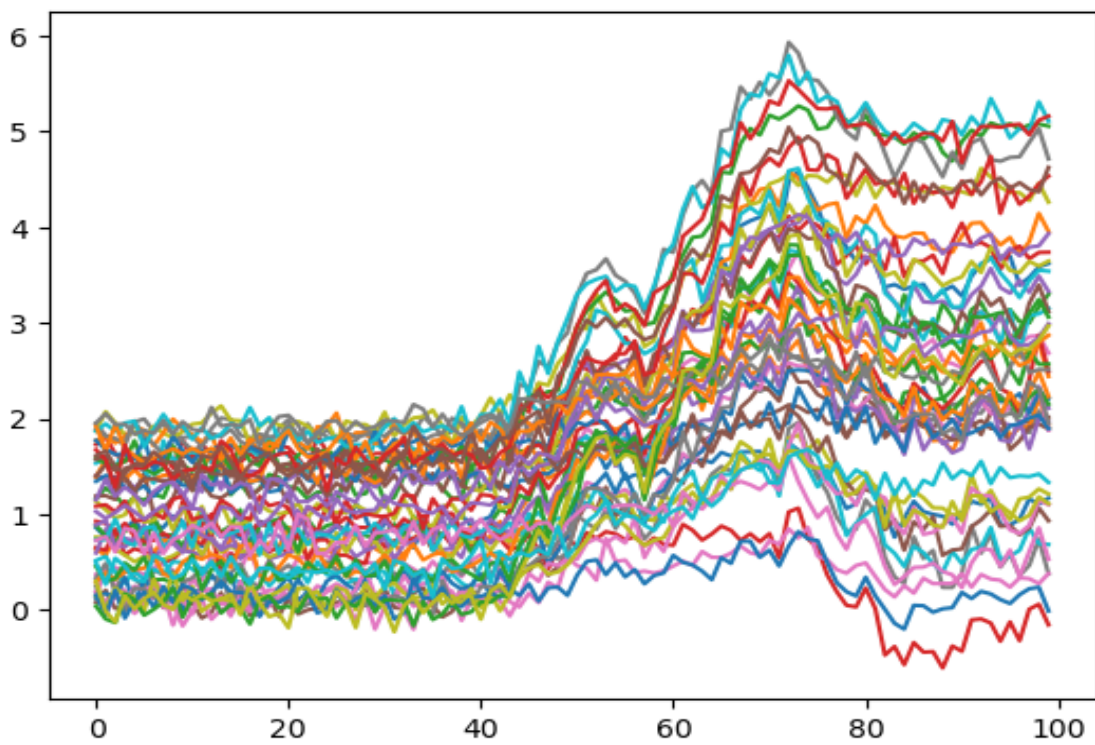
Consequently, the sequential monitoring statistic is expected to share the same limiting distribution as the standard PSY statistic and the corresponding critical values should remain asymptotically valid.

Taken together, Conjectures 1–3 articulate the sense in which the proposed IPCA–RLS framework is expected to provide a computationally efficient implementation of real-time common bubble monitoring while preserving the inferential properties of the existing econometric tests. The Monte Carlo evidence in the next section, which reports the finite-sample size and power of both the standard and the efficient implementations side-by-side, is broadly consistent with these conjectures.

1.5 Monte Carlo Simulations

To validate the effectiveness of the proposed IPCA–RLS algorithm, we conduct extensive numerical experiments by Monte Carlo simulations. Without loss of generality, we adopt the data-generating process specified in Equations (1.7) and (1.8), along with the parameter settings detailed in Chen et al. (2023), which we summarize below for completeness. We set the parameters as $\sigma_\epsilon = \sigma_{\nu_0} = \sigma_{\nu_1} = \sigma_{\nu_2} = 0.1$, $l_{0,i}, l_{1,i}, l_{2,t} \in U[0, 2]$, where $U[0, 2]$ is a uniform distribution over the interval $[0, 2]$. Additionally, we set $r_e = 0.4$ and $r_c = 0.7$; $c_1 = c_2 = 1$, $\alpha = 0.8$, and $\beta = 0.7$. We generate simulated datasets across various combinations of T and N , specifically using $T \in \{100, 200, 400, 800, 1,600\}$ and $N \in \{20, 50, 80\}$. Figure 1.1 displays a typical realization of the simulated data when $T = 100$ and $N = 50$. Moreover, under the null hypothesis, we set $r_e = r_c = 1$ as a special case.

First, we evaluate the finite-sample performance of the standard implementation of Chen et al. (2023) and of the proposed efficient IPCA–RLS implementation side-by-side, by reporting their empirical size and power against the asymptotic 95% critical value of the PSY statistic. Panel A of Table 1.1 presents the asymptotic critical values at the 90%, 95%, and 99% confidence levels. Panels B and C of Table 1.1 display the empirical size and power of both implementations using the asymptotic 95% critical value, based on 2,000 Monte Carlo replications of the DGP in Equations (1.7)–(1.8). Panel B is generated under the null of no common bubble ($r_e = r_c = 1$) and Panel C under the alternative with $r_e = 0.4$ and $r_c = 0.7$. The two implementations differ only in the way the common factor and the sequential PSY statistic are computed: the standard implementation of Chen et al. (2023) extracts PC1 by full-sample PCA and applies the PSY test to PC1 by OLS on every subwindow, whereas the efficient IPCA–RLS implementation updates the factor and the ADF statistics recursively along the expanding sample as described in Algorithm 1.

Figure 1.1: Simulation Realization: $T = 100$ and $N = 50$ 

Notes. The figure illustrates a typical realization of the simulation based on the DGP in Equations (1.7) and (1.8), using the same parameter settings as in Chen et al. (2023) with $T = 100$ and $N = 50$. Each coloured line represents a simulated time series, with the X-axis denoting the time dimension and the Y-axis indicating the magnitude of the series.

The main message of Table 1.1 is that the efficient IPCA-RLS implementation preserves the finite-sample size and power of the standard implementation of Chen et al. (2023) across all combinations of T and N considered. In Panel B, the empirical rejection rate under the null of both implementations is close to the nominal 5% level: standard sizes lie in the interval $[0.028, 0.053]$ and efficient sizes lie in the interval $[0.030, 0.056]$, with the absolute difference between the two never exceeding half a percentage point. In Panel C, the empirical power increases monotonically with T for both implementations, from about 12% at $T = 100$ to roughly 60% at $T = 1,600$, and again the two implementations track each other closely: the efficient implementation is on average about two to five percentage points more powerful than the standard implementation, reflecting the fact that the sequential factor estimate is not distorted by future information under the alternative. The overall picture strongly supports the sequential-equivalence conjectures stated in Section 1.4: the IPCA-RLS algorithm delivers essentially the same finite-sample size and a marginally higher power than the standard implementation, at a much lower computational cost as documented in the following.

Next, we compare two implementations of the common bubble test in terms of computational error and computational time: the efficient implementation based on our proposed IPCA-RLS algorithm and the standard implementation using the full dataset as originally specified in the literature. It is worth noting that all computations are executed in RStudio on a personal computer with Intel(R) Core(TM) i5-8500 CPU and 16 GB of RAM². Our evaluation focuses on two key outputs, the estimated PC1 from the first stage and the PSY statistics from the second stage, both of which are time-varying. To assess computational errors, we examine the average absolute differences between the two implementations across the time dimension. Formally, we define the average absolute difference in the estimated PC1 as

$$\overline{|\Delta \hat{y}_1|} = \frac{1}{T} \sum_{s=1}^T |\hat{y}_{1,s}(\text{efficient}) - \hat{y}_{1,s}(\text{standard})|, \quad (1.29)$$

and the average absolute difference in PSY statistics as

$$\overline{|\Delta PSY|} = \frac{1}{T} \sum_{s=1}^T |PSY_s(\text{efficient}) - PSY_s(\text{standard})|. \quad (1.30)$$

²For the efficient implementation using the proposed IPCA-RLS algorithm, we utilize two R packages *onlinePCA* and *exuber*.

Table 1.1: Finite Sample Performance: Standard vs. Efficient Implementation

PANEL A: ASYMPTOTIC CRITICAL VALUES						
	90%	95%	99%			
$r_0 = 0.190$	1.74	2.00	2.43			
$r_0 = 0.137$	1.82	2.02	2.47			
$r_0 = 0.100$	2.03	2.25	2.72			
$r_0 = 0.074$	2.05	2.29	2.75			
$r_0 = 0.055$	2.16	2.39	2.86			
PANEL B: SIZE						
	$N = 20$		$N = 50$		$N = 80$	
	Standard	Efficient	Standard	Efficient	Standard	Efficient
$T = 100, r_0 = 0.190$	0.0360	0.0395	0.0375	0.0370	0.0520	0.0510
$T = 200, r_0 = 0.137$	0.0405	0.0435	0.0430	0.0425	0.0525	0.0555
$T = 400, r_0 = 0.100$	0.0275	0.0300	0.0390	0.0400	0.0385	0.0395
$T = 800, r_0 = 0.074$	0.0335	0.0360	0.0395	0.0415	0.0445	0.0460
$T = 1,600, r_0 = 0.055$	0.0275	0.0325	0.0420	0.0420	0.0440	0.0455
PANEL C: POWER						
	$N = 20$		$N = 50$		$N = 80$	
	Standard	Efficient	Standard	Efficient	Standard	Efficient
$T = 100, r_0 = 0.190$	0.1165	0.1205	0.1345	0.1410	0.1260	0.1320
$T = 200, r_0 = 0.137$	0.1995	0.2395	0.2150	0.2530	0.2235	0.2540
$T = 400, r_0 = 0.100$	0.2400	0.2840	0.2545	0.2995	0.2745	0.3310
$T = 800, r_0 = 0.074$	0.4230	0.4915	0.4225	0.4880	0.4635	0.5240
$T = 1,600, r_0 = 0.055$	0.5965	0.6500	0.6060	0.6625	0.6065	0.6495

Notes. Panel A reports the asymptotic critical values of the PSY statistic, approximated by simulating a standard Brownian motion with $T = 5,000$ steps and 10,000 iterations. Panels B and C present the empirical size and power of the common bubble test, using the asymptotic 95% critical value obtained from Panel A. For each (T, N) combination, we report the values obtained under the standard implementation of Chen et al. (2023), which computes PC1 by full-sample PCA and applies the PSY test to PC1 by OLS on every subwindow, alongside those obtained under the efficient IPCA-RLS implementation proposed in this paper. Both sets of numbers are based on 2,000 Monte Carlo replications of the DGP described in Equations (1.7)–(1.8), with $r_e = r_c = 1$ used to compute the size and $r_e = 0.4$, $r_c = 0.7$ used to compute the power.

For computational time, we calculate the difference in runtime between the two implementations as

$$\Delta t = t(\text{efficient}) - t(\text{standard}), \quad (1.31)$$

where a negative value indicates that the efficient implementation is faster. We report the average of these three measures in Equations (1.29)-(1.31) across 2,000 simulations for each combination of T and N in Table 1.2. A clear downward trend in computational error is observed as either T or N increases, with the reduction being more pronounced along the time dimension. This suggests that the accuracy of the efficient implementation improves with larger samples. Similarly, the efficient implementation demonstrates less computational time than the standard implementation in most scenarios, with the exception of cases where $T = 100$. In these instances, the initialization overhead of the adaptive algorithms outweighs the performance gains. As T or N increases, the gap in computational time between the two implementations widens, particularly along the time dimension, highlighting the scalability advantage of the IPCA-RLS algorithm.

Table 1.2: Computational Errors & Time

	$T = 100$	$T = 200$	$T = 400$	$T = 800$	$T = 1,600$
PANEL A: $\mu(\Delta \hat{y}_1)$					
$N = 20$	0.4276	0.2236	0.1121	0.0508	0.0283
$N = 50$	0.3833	0.2389	0.0975	0.0480	0.0274
$N = 80$	0.3936	0.2136	0.0998	0.0474	0.0234
PANEL B: $\mu(\Delta PSY)$					
$N = 20$	1.0374	0.9608	0.8590	0.7057	0.5408
$N = 50$	0.8841	0.7560	0.6100	0.5292	0.4749
$N = 80$	0.8310	0.6424	0.5382	0.4753	0.4402
PANEL C: $\mu(\Delta t)$					
$N = 20$	0.0036	-0.0222	-0.2221	-1.3488	-9.1280
$N = 50$	0.0020	-0.0329	-0.2328	-1.5897	-9.6836
$N = 80$	0.0021	-0.0372	-0.2676	-1.4712	-9.3676

Notes. The table presents the mean values of the three measures defined in Equations (1.29)–(1.31) across 2,000 simulation replications. Panels A and B display the computational errors associated with the efficient implementation of the common bubble test using our proposed IPCA-RLS algorithm, benchmarked against the standard implementation. Panel C reports the differences in runtime between the two implementations.

To illustrate the differences between the two implementations in computing the estimated PC1 and PSY statistics, we plot both series for each combination of T

and N . Figure 1.2 displays the comparison of estimated PC1 obtained from the efficient implementation using the IPCA-RLS algorithm and the standard implementation. Similarly, Figure 1.3 presents the comparison of the corresponding PSY statistics. It is evident that for both series, the computational errors diminish as the sample size increases. Notably, discrepancies primarily emerge during the Step 1 of the IPCA-RLS algorithm when the initial values of estimated PC1 are obtained by standard PCA using data only up to t_0 . Following the onset of the bubble regime, both the estimated PC1 and PSY statistics produced by the IPCA-RLS algorithm converge closely to those generated by the standard implementation.

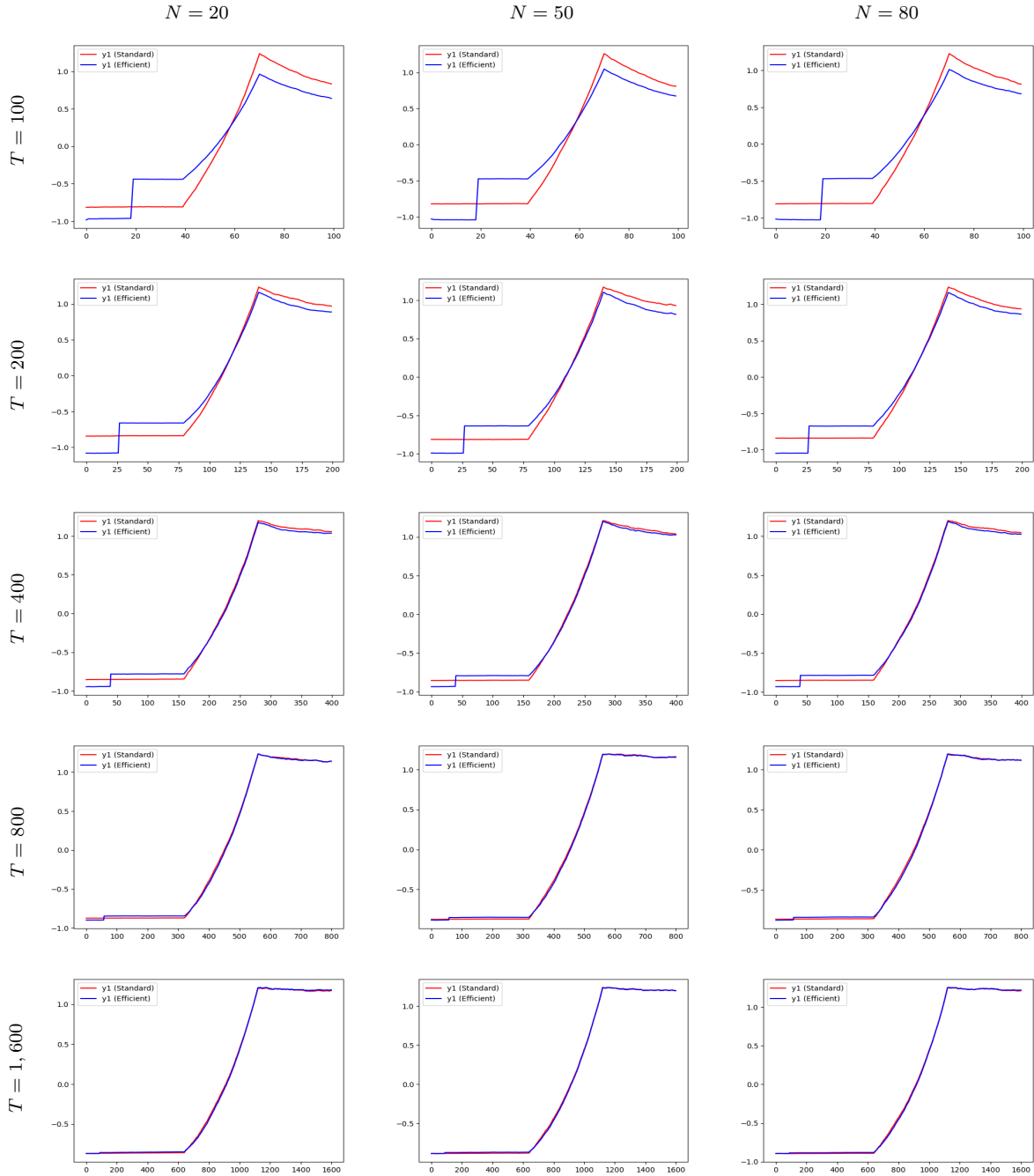
Overall, the use of the proposed IPCA-RLS algorithm is validated through extensive numerical experiments. Our results demonstrate that the efficient implementation of the common bubble test using the IPCA-RLS algorithm achieves a substantial reduction in computational time compared to the standard implementation, while incurring negligible computational errors. Furthermore, as the sample size increases—particularly along the time dimension T —computational errors decrease and time savings become more pronounced. These findings underscore the practical value of the IPCA-RLS algorithm for real-time common bubble detection, especially in extremely large- T panel settings with streaming data.

1.6 Empirical Application

In this section, we illustrate the effectiveness of our proposed algorithm using real-world dataset, focusing on the early detection of common bubbles in the artificial intelligence sector. Using a dataset comprising daily price-to-earnings ratios for 30 AI-related stocks³, our IPCA-RLS algorithm proves to be both efficient and accurate. Notably, we identify a common AI bubble occurring between 8 and 28 January, 2021, which appears to be driven by a combination of factors. To the best of our knowledge, this is the first empirical study aimed at detecting common bubbles across the AI industry. While Basele et al. (2025) recently examined speculative bubbles in the seven largest AI firms (commonly referred to as the Magnificent Seven), our application differs in two key aspects. First, we target industry-wide bubbles rather than those tied to individual companies, which are often triggered by firm-specific events like major technological an-

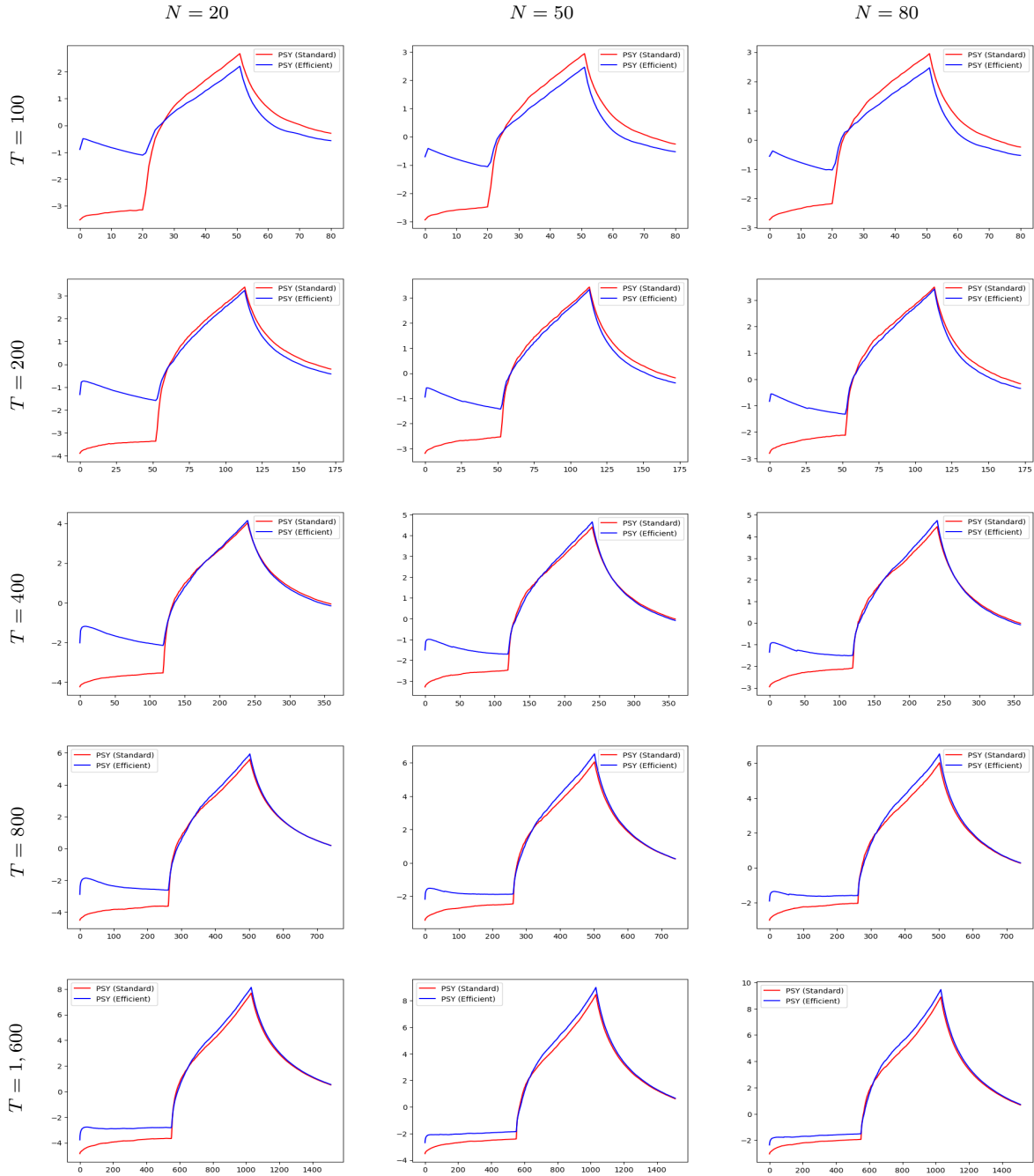
³In the Appendix 1.C, we report the details of 30 artificial intelligence companies used in our empirical application

Figure 1.2: Comparison: Estimated PC1



Notes. The figure displays the average estimated first principal component across 2,000 simulations that obtained from the efficient implementation using the IPCA-RLS algorithm ($\hat{y}_1(\text{efficient})$) along with those from the standard implementation ($\hat{y}_1(\text{standard})$), across all combinations of $T \in \{100, 200, 400, 800, 1,600\}$ and $N \in \{20, 50, 80\}$.

Figure 1.3: Comparison: PSY Statistics



Notes. The figure displays the average PSY statistics across 2,000 simulations that obtained from the efficient implementation using the IPCA-RLS algorithm ($PSY(efficient)$) along with those from the standard implementation ($PSY(standard)$), across all combinations of $T \in \{100, 200, 400, 800, 1,600\}$ and $N \in \{20, 50, 80\}$.

nouncements. Second, our study uses significantly longer time series ($T = 2,555$ daily observations) compared to their use of monthly data, taking advantage of using additional time information to improve the successful detection rate and detection delay.

1.6.1 Data

In this empirical exercise, we focus on detecting common bubbles in stocks within the artificial intelligence sector, using the price-to-earning ratios of 30 companies with 2,555 daily observations, ranging from 2/1/2015 to 28/2/2025. We select the AI-related stocks and pre-process the data as follows.

1. Select all the companies listed in the Nasdaq CTA Artificial Intelligence Index (NQINTEL).
2. For all selected companies, we download daily P/E ratios from Bloomberg, ranging from 2/1/2015 to 28/2/2025.
3. Exclude companies whose P/E ratios are available after 2015 and interpolate missing observations⁴.
4. Apply the Z-score normalisation to all remaining companies.

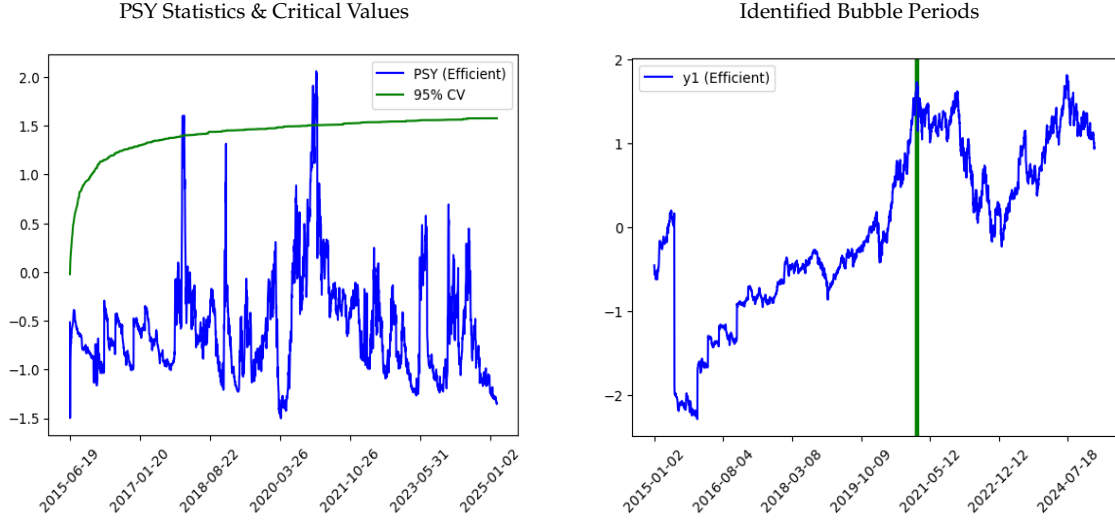
After completing the aforementioned data cleaning steps, we arrive at a dataset of 30 AI-related companies. We provide the details of these companies in the Appendix 1.C.

1.6.2 Empirical Results

In the first part of our analysis, we present the empirical outcomes of the efficient implementation of the common bubble test using the proposed IPCA-RLS algorithm, applied to our dataset. The results are shown in Figure 1.4.

In the left panel of Figure 1.4, we illustrate the comparison between the computed PSY statistics and the corresponding finite sample 95% critical values. Meanwhile, the right panel displays the identified bubbles. Regarding the identification of bubbles, we impose that a bubble must span more than one week (5 days). Notably, we detected a common AI bubble occurring between 8 and 28 January 2021. This bubble was driven by a combination of factors, including heightened investor enthusiasm following the release of advanced AI models like OpenAI's GPT-3 and DALL-E, increased media coverage of AI breakthroughs, and

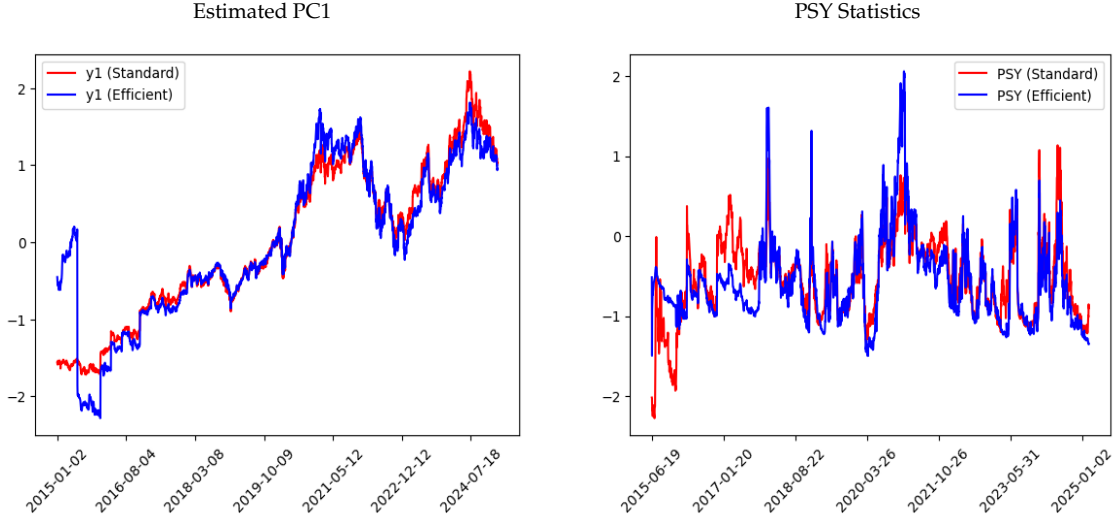
⁴We conducted a backfilled interpolation using the last available value as the current value.

Figure 1.4: Real-Time Common Bubble Detection: Efficient Implementation

Notes. The figure presents the empirical results for efficient implementation of the real-time monitoring of common bubbles among the thirty AI-related stocks. The left graph shows the result of comparing the sequence of PSY statistics and respective finite sample critical values at 95% confidence level. Finite sample critical values are obtained by Monte Carlo simulation with 5,000 iterations. The identified bubble periods are highlighted by shadowed areas in green on the right.

speculative trading in AI-related stocks amid broader tech market exuberance. The COVID-19 pandemic further accelerated digital transformation, prompting increased demand for AI solutions across industries. Additionally, low interest rates and ample liquidity encouraged risk-taking behaviour, amplifying valuation surges in the AI sector.

In the second part of our analysis, we assess the computational accuracy and speed of the proposed IPCA-RLS algorithm, benchmarked by the standard implementation. We assess the performance of both implementations based on the estimated PC1 and the PSY statistics. As shown in Figure 1.5, the results reveal no substantial differences between the two implementations, aside from some discrepancies during the initialization of the adaptive algorithm. To quantify computational errors, we use the metrics defined in Equations (1.29) and (1.30), finding average absolute differences of $|\Delta \hat{y}_1| = 0.1972$ and $|\Delta PSY| = 0.2505$, both of which are negligible. We then compare computational time, observing that the standard implementation requires 37.7576 seconds per implementation, while the efficient version significantly reduces this to just 4.9490 seconds. Overall, these results confirm that the proposed IPCA-RLS algorithm delivers both high accuracy and enhanced computational efficiency, underscoring its practical applicability.

Figure 1.5: Computational Accuracy

Notes. The figure presents the computational accuracy of the efficient implementation of the common bubble test. To evaluate the computational accuracy, we compare two key series between the efficient implementation and the standard implementation. The left graph shows the result of comparing the estimated first principal component, while the PSY statistics are compared in the right graph.

1.7 Conclusions

This chapter contributes to the literature on common bubble detection by introducing an efficient adaptive algorithm, IPCA-RLS, that significantly enhances the practical applicability of the real-time common bubble detection in large- T panel settings. By improving upon the framework proposed by Chen et al. (2023), our method addresses one of the key limitations of existing approaches: high computational complexity. We demonstrate analytically that the IPCA-RLS algorithm achieves substantial gains in computational efficiency, reducing the time complexity from $O(T^2)$ to $O(T)$ in the first stage of the common bubble test and from $O(T^3)$ to $O(T^2)$ in the second stage.

Through extensive numerical experiments, we confirm that our algorithm maintains a high level of accuracy while delivering significant reductions in runtime. The computational errors, which are measured in terms of differences in the estimated first principal component and PSY statistics, remain negligible, confirming the reliability of the efficient implementation. Our empirical application to a panel of 30 AI-related stocks further validates the utility of our method. We detect a clear episode of a common AI bubble occurring in January 2021, which aligns with a period of heightened investor speculation and media attention toward artificial intelligence advancements. These findings underscore the

importance of efficient real-time common bubble detection as a tool for assessing systemic financial risks, especially in rapidly evolving and innovation-driven industries like AI.

Overall, the IPCA-RLS algorithm offers a robust, scalable, and accurate approach for detecting common bubbles. Its practical advantages position it as a valuable tool for policymakers, regulators, and market participants seeking to identify and mitigate systemic risks in real time.

Appendix 1.A Extended Monte Carlo Simulations

In this Appendix 1.A, we report the results of our extended Monte Carlo simulations using various univariate right-tailed recursive unit root tests under both homoscedastic and heteroscedastic volatility specifications. Throughout this report, we denote the Supreme Augmented Dickey-Fuller test developed by Phillips et al. (2011) as *SADF* test and its generalized version proposed by Phillips et al. (2015) as *GSADF* test.

Both the *SADF* and the *GSADF* test assume an independent and identically distributed innovation process. However, volatility is commonly believed to be time-varying as supported by empirical observations. In response to this, a line of research focuses on addressing heteroscedasticity issue instead of assuming homoscedasticity as in standard test settings. Harvey et al. (2016b) first show that the *SADF* test becomes severely oversized, both asymptotically and in finite samples, in the presence of non-stationary volatility. This issue also applies to the *GSADF* test. To tackle this problem, they propose a wild bootstrap algorithm that effectively controls the size. In a subsequent study, Harvey et al. (2019) propose a Weighted Least Squares (WLS) version of the *SADF* test, which is also applicable to the *GSADF* test, dealing with the problem of testing for explosive bubbles in financial time series with time-varying volatility. Further, Harvey et al. (2020) develop a sign-based *GSADF* test that does not require bootstrap methods to control the size in the presence of time-varying volatility. Additionally, the sign-based *SADF* test can be derived as a special case by fixing the starting point of the regression window. Moreover, Whitehouse (2019) proposes a Generalized Least Squares (GLS) version of the *SADF* test, which has been demonstrated to have higher power than its standard counterpart when dealing with a long period of explosiveness. It is worth noting that this GLS modification can also be applied to the *GSADF* test.

Simulation Design

We simulate the time series y_t as follows

$$y_t = \begin{cases} \mu T^{-\eta} + y_{t-1} + \sigma_t \epsilon_t & t \in (0, \lfloor T\tau_1 \rfloor] \cup (\lfloor T\tau_3 \rfloor, T] \\ (1 + c_1 T^{-\alpha})y_{t-1} + \sigma_t \epsilon_t & t \in (\lfloor T\tau_1 \rfloor, \lfloor T\tau_2 \rfloor] \\ (1 - c_2 T^{-\beta})y_{t-1} + \sigma_t \epsilon_t & t \in (\lfloor T\tau_2 \rfloor, \lfloor T\tau_3 \rfloor] \end{cases} \quad (1.32)$$

where we set $\mu = \eta = c_1 = c_2 = 1$, $\sigma_t = 1$ in the case of homoscedasticity, σ_t following a GARCH(1,1) process specified in Equation (1.33) when we consider heteroscedasticity. Moreover, we set $\tau_1 = 0.4$, $\tau_2 = 0.7$, and $\tau_3 = 0.8$, $\alpha = 0.7$ and $\beta = 0.5$. This bubble specification assumes a general pattern where an explosive period is followed by a stationary collapse. In our simulations, we use sample size $T \in \{100, 200, 400, 800, 1, 600\}$.

$$\sigma_t^2 = \omega + \alpha_0 \sigma_{t-1}^2 \epsilon_{t-1}^2 + \beta_0 \sigma_{t-1}^2 \quad (1.33)$$

where $\omega = 0.05$, $\alpha_0 = 0.1$ and $\beta_0 = 0.85$.

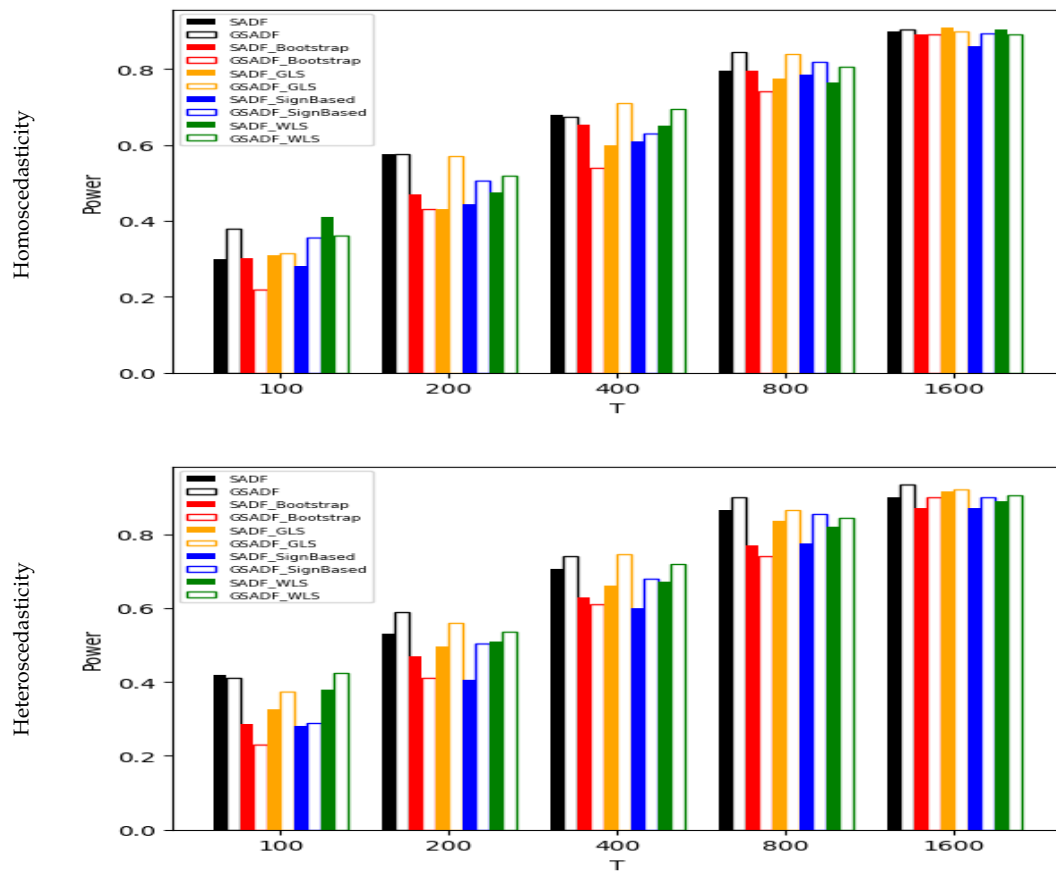
Empirical Power

We now undertake a comparative assessment of the empirical power characteristics of different univariate bubble tests. Each test undergoes 2,000 simulations, and the empirical powers are computed as the proportion of simulations in which the null hypothesis is rejected, utilizing the corresponding finite sample 95% critical values. The results of these empirical power analyses under the homoscedastic and heteroscedastic scenarios are presented in Figure 1.6. This figure illustrates the empirical powers of each test applied to simulated explosive series with varying sample size, considering different volatility settings. Key observations drawn from our analysis include: i) for most cases within the same test group, the power of the *GSADF* test surpasses that of the *SADF* test, except for the two Bootstrap tests; ii) conversely, the Bootstrap *SADF* test exhibits higher power compared to the Bootstrap *GSADF* test in the majority of cases; iii) notably, the Bootstrap and sign-based tests tend to display relatively lower powers, particularly in the presence of heteroscedasticity; iv) in terms of power across various scenarios, the original tests, GLS tests, and WLS tests demonstrate qualitatively similar power properties. No single test group appears to dominate the others consistently across all scenarios. Apart from these observations, the most important finding is that the powers of all tests exhibit a consistent increase as the sample size T grows.

Detection Delay

Previously, we examined the performance of tests based on their empirical powers, treating them as one-shot tests using the full sample data. However, recursive right-tailed unit root tests also propose using historical data up to the

Figure 1.6: Power Comparison



Notes. The figure shows the empirical powers of each test in the presence of different volatility regimes. The powers are obtained on the basis of 2,000 replications of simulating data from Equation (1.32) with varying sample size $T \in \{100, 200, 400, 800, 1,600\}$. Other parameter settings are $\mu = \eta = c_1 = c_2 = 1$, $\tau_1 = 0.4$, $\tau_2 = 0.7$, $\tau_3 = 0.8$, $\alpha = 0.7$ and $\beta = 0.5$.

current time to conduct a real-time monitoring procedure. To evaluate the effectiveness of this procedure, we compute the detection delay, defined as the difference between the estimated origination time of the detected bubbles and the true origination time established in the simulations. Mathematically, this can be represented as $\delta = \tau_1^* - \tau_1$, where δ denotes the detection delay, τ_1^* is the estimated origination time, and τ_1 is the actual origination time of the bubble phase.

We are interested in how the detection delay δ evolves as we increase the sample length T . Specifically, we simulate the time series using the DGP in Equation (1.32) with 500 iterations, and we set $\mu = \eta = c_1 = c_2 = 1$, $\tau_1 = 0.2$, $\tau_2 = 0.7$, $\tau_3 = 0.8$, $\alpha = 0.7$, and $\beta = 0.5$. We consider two different volatility scheme, homoscedasticity and heteroscedasticity as in Equation (1.33).

Table 1.3 displays the means and standard deviations of detection delay, which are denoted as $\mu(\delta)$ and $\sigma(\delta)$ separately, across various scenarios. It is evident that the detection delay exhibits a clear downward trend as T increases for all tests, reinforcing the advantage of using longer time series when applying bubble tests. Additionally, within each test group, the *GSADF* test consistently outperforms the *SADF* counterpart in terms of both $\mu(\delta)$ and $\sigma(\delta)$. When comparing across test groups, we can roughly rank the performance from best to worst detection delay as follows: Bootstrap tests, original tests, sign-based tests, WLS tests, and GLS tests.

Table 1.3: Detection Delay

		$T = 100$		$T = 200$		$T = 400$		$T = 800$		$T = 1,600$	
		$\mu(\delta)$	$\sigma(\delta)$	$\mu(\delta)$	$\sigma(\delta)$	$\mu(\delta)$	$\sigma(\delta)$	$\mu(\delta)$	$\sigma(\delta)$	$\mu(\delta)$	$\sigma(\delta)$
$SADF$	Homo	0.277	0.255	0.189	0.228	0.138	0.210	0.064	0.178	0.023	0.148
	Hetero	0.273	0.253	0.169	0.212	0.111	0.183	0.062	0.178	0.026	0.160
$GSADF$	Homo	0.254	0.235	0.149	0.199	0.078	0.153	0.013	0.119	-0.018	0.106
	Hetero	0.234	0.225	0.134	0.177	0.074	0.146	0.005	0.117	-0.037	0.097
$SADF_{Bootstrap}$	Homo	0.262	0.246	0.132	0.162	0.125	0.220	0.066	0.159	0.030	0.148
	Hetero	0.281	0.246	0.192	0.232	0.107	0.175	0.077	0.187	0.033	0.190
$GSADF_{Bootstrap}$	Homo	0.223	0.203	0.105	0.135	0.077	0.161	0.008	0.106	-0.029	0.108
	Hetero	0.246	0.239	0.142	0.186	0.070	0.146	0.008	0.118	-0.036	0.116
$SADF_{GLS}$	Homo	0.385	0.243	0.285	0.227	0.209	0.201	0.132	0.188	0.088	0.153
	Hetero	0.379	0.250	0.291	0.227	0.200	0.196	0.130	0.183	0.097	0.164
$GSADF_{GLS}$	Homo	0.377	0.243	0.271	0.225	0.204	0.197	0.107	0.183	0.064	0.144
	Hetero	0.361	0.238	0.268	0.217	0.178	0.189	0.099	0.176	0.059	0.162
$SADF_{Sign-based}$	Homo	0.319	0.268	0.223	0.239	0.155	0.222	0.086	0.190	0.035	0.156
	Hetero	0.300	0.270	0.198	0.229	0.130	0.194	0.065	0.166	0.036	0.171
$GSADF_{Sign-based}$	Homo	0.260	0.234	0.167	0.192	0.080	0.161	0.016	0.120	-0.023	0.117
	Hetero	0.249	0.229	0.146	0.188	0.073	0.151	0.010	0.118	-0.032	0.109
$SADF_{WLS}$	Homo	0.356	0.288	0.254	0.262	0.185	0.234	0.115	0.213	0.065	0.169
	Hetero	0.339	0.277	0.234	0.247	0.171	0.226	0.111	0.209	0.076	0.190
$GSADF_{WLS}$	Homo	0.283	0.383	0.118	0.352	-0.004	0.269	-0.057	0.243	-0.127	0.125
	Hetero	0.252	0.372	0.098	0.333	-0.002	0.274	-0.071	0.221	-0.107	0.185

Notes. The table presents the means and standard deviations of detection delay, which are denoted as $\mu(\delta)$ and $\sigma(\delta)$, respectively. The statistics of δ are calculated by 500 iterations for each test under various volatility scenarios, where δ is defined as the difference between estimated and true origination date of the bubble. We simulate the series under the DGP in Equation (1.32), and set $\mu = \eta = c_1 = c_2 = 1$, $\tau_1 = 0.2$, $\tau_2 = 0.7$, $\tau_3 = 0.8$, $\alpha = 0.7$, and $\beta = 0.5$. For the Wild Bootstrap implementation, the number of replications is 199.

Appendix 1.B Heuristic Arguments Supporting the Conjectures

In this Appendix 1.B, we outline the heuristic arguments underlying the conjectures presented in Section 1.4. The arguments below draw on standard results from approximate factor models, perturbation theory for eigenvectors, and recursive least squares estimation that have been developed primarily for stationary environments. Because the DGP in Section 1.2.2 exhibits a unit root under the null and mildly explosive and mildly stationary regimes under the alternative, a fully rigorous extension of these tools to the present setting is nontrivial. We therefore refer to the material below as heuristic arguments rather than proofs, and reserve a formal treatment of the nonstationary case for future work.

1.B.1 Sequential Factor Estimation

Let x_t denote the $N \times 1$ vector of observed panel data and consider the approximate factor model

$$x_t = \lambda f_t + e_t,$$

where f_t is the common factor, λ is the loading vector, and e_t is an idiosyncratic error component.

Define the expanding-sample covariance matrix

$$\Sigma_t = \frac{1}{t} \sum_{s=1}^t (x_s - \mu_t)(x_s - \mu_t)'$$

Let $\hat{\lambda}_t^{PCA}$ denote the leading eigenvector of Σ_t obtained by applying PCA to the expanding sample $\{x_1, \dots, x_t\}$. Similarly, let $\hat{\lambda}_t^{IPCA}$ denote the loading vector obtained from the sequential IPCA update.

Step 1: Covariance matrix approximation The IPCA algorithm recursively updates the sample mean and covariance matrix. Under Assumptions 1–3, the recursive covariance approximation $\tilde{\Sigma}_t$ satisfies

$$\sup_{t_0 \leq t \leq T} \|\tilde{\Sigma}_t - \Sigma_t\| = o_p(1).$$

This follows from the consistency of recursive moment estimators and the bound-

edness of the approximation rank q .

Step 2: Eigenvector perturbation Under the factor strength condition (Assumption 2), the largest eigenvalue of Σ_t is separated from the remaining eigenvalues. Therefore, standard perturbation results for symmetric matrices imply

$$\sup_{t_0 \leq t \leq T} \|\hat{\lambda}_t^{IPCA} - s_t \hat{\lambda}_t^{PCA}\| = o_p(1),$$

for some sign sequence $s_t \in \{-1, 1\}$.

Step 3: Factor score equivalence The estimated factor scores are defined as

$$\hat{y}_{1,t}^{IPCA} = \frac{x_t' \hat{\lambda}_t^{IPCA}}{\sqrt{N}}, \quad \hat{y}_{1,t}^{PCA,t} = \frac{x_t' \hat{\lambda}_t^{PCA}}{\sqrt{N}}.$$

Combining the eigenvector perturbation result with bounded moments of x_t yields

$$\sup_{t_0 \leq t \leq T} \left| \hat{y}_{1,t}^{IPCA} - \hat{y}_{1,t}^{PCA,t} \right| = o_p(1),$$

which establishes the sequential factor equivalence result stated in Conjecture 1.

1.B.2 Recursive Regression Estimation

Consider the ADF regression used in the PSY test:

$$\Delta y_t = z_t' \beta + u_t,$$

where z_t contains lagged regressors.

Let $\hat{\beta}_t^{OLS}$ denote the OLS estimator obtained from the expanding sample $\{1, \dots, t\}$ and $\hat{\beta}_t^{RLS}$ the recursive least squares estimator.

The recursive update can be written as

$$\hat{\beta}_t^{RLS} = \hat{\beta}_{t-1}^{RLS} + K_t \left(y_t - z_t' \hat{\beta}_{t-1}^{RLS} \right),$$

where K_t is the gain matrix.

Under standard regularity conditions on the regressors and error terms, the recursive information matrix converges to the expanding-sample information matrix:

$$\sup_{t \leq T} \|\Lambda_t - (Z_t' Z_t)^{-1}\| = o_p(1).$$

Consequently,

$$\sup_{t \leq T} \|\hat{\beta}_t^{RLS} - \hat{\beta}_t^{OLS}\| = o_p(1),$$

which establishes the recursive estimation equivalence result of Conjecture 2.

1.B.3 Sequential Monitoring Statistic

Let ADF_{r_1, r_2}^{batch} denote the ADF statistic computed using the expanding-sample batch estimators and ADF_{r_1, r_2}^{seq} the statistic obtained using the sequential IPCA-RLS estimators.

Combining the results of Sections 1.B.1 and 1.B.2 yields

$$\sup_{(r_1, r_2) \in \mathcal{R}_T} |ADF_{r_1, r_2}^{seq} - ADF_{r_1, r_2}^{batch}| = o_p(1),$$

where $\mathcal{R}_T$ denotes the set of admissible recursive windows.

Since the PSY statistic is defined as the supremum of the ADF statistics over the admissible windows,

$$PSY(r_2) = \sup_{r_1 \in [r_0, r_2 - r_0]} ADF_{r_1, r_2},$$

the continuous mapping theorem implies

$$\sup_{r_2 \in [r_0, 1]} |PSY^{seq}(r_2) - PSY^{batch}(r_2)| = o_p(1).$$

1.B.4 Limiting Distribution

Phillips et al. (2015) establish the asymptotic distribution of the batch PSY statistic under the null hypothesis of no bubble. Since the sequential statistic differs

from the batch statistic only by an asymptotically negligible term, both statistics share the same limiting distribution. Consequently, the critical values derived for the standard PSY test remain asymptotically valid for the sequential monitoring procedure.

The above results establish that the proposed IPCA–RLS framework provides a computationally efficient implementation of the real-time common bubble test while preserving the inferential properties of the original procedure.

Appendix 1.C List of AI Stocks

In this Appendix 1.C, we report the details of 30 artificial intelligence companies used in our empirical application.

Table 1.4: Data Description

Company	Ticker
AKAMAI TECHNOLOGIES	AKAM
ALIBABA GROUP HL LTD	BABA
ALPHABET CL A CMN	GOOGL
AMAZON.COM INC	AMZN
ANSYS INC.	ANSS
APPLE INC.	AAPL
APTIV PLC ORD	APTIV
BROADCOM INC.	AVGO
CADENCE DESIGN SYS	CDNS
DEERE CO	DE
ILLUMINA, INC.	ILMN
INTEL CORP	INTC
INTL BUSINESS MACH	IBM
MARVELL TECH INC CMN	MRVL
META PLATFORMS, INC.	META
MICROCHIP TECHNOLOGY	MCHP
MICRON TECHNOLOGY	MU
MICROSOFT CORP	MSFT
NAVER Rg	035420
NVIDIA CORPORATION	NVDA
PEGASYSTEMS INC	PEGA
QUALCOMM INC	QCOM
Renesas Electro Rg	6723
SALESFORCE, INC.	CRM
Samsung Electron Rg	005930
SK hynix Rg	000660
STMicroelectr Br Rg	STMIMI
SYNOPSYS, INC.	SNPS
Taiwan Semicon Rg	2330
TERADYNE INC CMN	TER

Notes. The table presents the companies names and corresponding Ticker names for the 30 artificial intelligence companies used in the empirical application.

Chapter 2

Bubbles in Green Equity Markets: the Role of Macro-Financial and Climate Drivers

2.1 Introduction

Climate finance has evolved from a niche investment segment into a central driver of global capital allocation. Over the last decade, unprecedented volumes of capital have flowed into clean energy, renewable technologies, electric vehicles, battery storage, and related transition sectors, supported by a combination of climate policy commitments, ESG mandates, technological innovation, and investor demand for sustainable assets. According to the International Energy Agency (IEA, 2026), global clean energy investment is projected to reach USD 2.2 trillion in 2026, nearly twice as much as that going to fossil fuels. While these developments reflect important structural changes associated with the low-carbon transition, they have also raised growing concerns regarding the emergence of speculative dynamics in green equity markets.

Although the terms green energy, clean energy, and renewable energy are often used interchangeably, important distinctions exist. Clean energy refers to energy sources that produce minimal air pollution or greenhouse gas emissions, which includes technologies such as nuclear power. Renewable energy describes resources that naturally replenish over time, including solar, wind, hydropower, geothermal, and biomass. Green energy is a subset of renewables that has the lowest environmental impact. For example, hydropower is renewable, but large-scale dams are not typically classified as green due to their significant ecological effects¹. For simplicity, the term “green” is used throughout the paper to refer to green, clean, and renewable energy assets and bubbles.

Recent green asset booms are characterised not only by rapid valuation increases but also by the simultaneous emergence of explosive dynamics across multiple segments of the green equity sector, including clean energy, solar, wind, and clean-technology equities, even when broader equity markets do not exhibit similar behaviour. This pattern motivates an investigation of whether green asset bubbles arise as common phenomena across these segments, as well as of the macro-financial and climate-related factors underlying their formation.

Existing studies on green bubbles have primarily focused on individual assets or indices, applying standard bubble-detection procedures separately to each series. While this literature documents repeated episodes of explosive behaviour in renewable energy and clean-technology markets, it provides limited insight into the existence of common speculative components across green assets.

From a financial stability perspective, this distinction is important: bubbles that are synchronised across multiple green segments may pose different risks than isolated mispricing in individual assets, with potentially broader implications for market fragility and spillovers.

¹<https://www.nationalgrid.com/stories/energy-explained/what-is-green-energy>

This paper investigates the existence, timing, and drivers of common green bubbles in equity markets. Using daily price-to-earnings ratios for nine clean energy, clean technology, and green energy equity indices between 2018 and 2025, we apply the common bubble detection framework of Chen et al. (2023), implemented through the computationally efficient real-time IPCA-RLS algorithm proposed by Li and Urga (2026). Unlike studies based exclusively on prices, our use of valuation ratios allows us to study periods in which market valuations become detached from underlying fundamentals. Moreover, by focusing on the common component extracted from a panel of green indices valuation ratios, we identify synchronised speculative dynamics rather than isolated bubble episodes.

A central contribution of the chapter is to show that green bubbles are largely distinct from broader market exuberance. To address the possibility that the detected episodes merely reflect post-pandemic liquidity conditions or generic growth-stock exuberance, we compare the identified green bubble periods with bubbles detected in broad global equity, growth, and conventional energy indices. We find only limited overlap between the two sets of episodes, suggesting that green speculative dynamics are specific to the green sector rather than broad market optimism alone.

Finally, we investigate the drivers of green bubble formation using a probit framework. The results show that common green bubbles are associated with macro-financial conditions, energy-market dynamics, physical climate risk, and climate-policy expectations.

The remainder of the chapter is structured as follows. Section 2.2 reviews the related literature on bubble detection and green asset markets. Section 2.3 outlines the methodological framework. Section 2.4 presents the identification of green bubbles. Section 2.5 reports the empirical results on the underlying drivers of green bubble formation. Section 2.6 concludes.

2.2 Literature review

Green financial markets have expanded substantially in recent years in response to the global transition toward a low-carbon economy, leading to a growing reallocation of capital toward renewable energy and clean-technology assets. Policy reports and the financial press have documented this phenomenon and raised concerns about the potential formation of asset price bubbles (Aramonte and Zabai, 2021; Jourde and Stalla-Bourdillon, 2021; Nauman, 2021). This section reviews the literature on asset price bubble detection methods, followed by the emerging literature on green bubbles.

2.2.1 Evolution of Bubble Detection Methods

Bubbles in financial or real assets arise when the price exceeds the asset's fundamental value because investors believe they will be able to sell the asset to some other investor at a higher price in the future (Brunnermeier, 2008). They typically manifest with a rapid and dramatic surge in prices followed by a collapse. Scholars have long debated whether bubbles are rational (Blanchard and Watson, 1982; Tirole, 1982, 1985; Froot and Obstfeld, 1991) or irrational (Shiller, 2000).

The empirical analysis of asset price bubbles has a well-established literature, with foundational econometric tests emerging in the early 1980s. Shiller (1981) introduced the variance-bound test, demonstrating that observed asset price volatility frequently surpassed levels compatible with fundamental valuations, thereby providing indirect evidence of speculative dynamics. Subsequent contributions by West (1987), Diba and Grossman (1988), and Froot and Obstfeld (1991) refined this strand of research through the application of unit root and cointegration frameworks to assess potential explosiveness. Evans (1991) criticised us-

ing these frameworks for bubbles detection, by demonstrating with simulated data that they cannot distinguish unit root processes from periodically collapsing bubbles. Overall, these initial contributions formed the conceptual bedrock for contemporary bubble detection techniques.

In the aftermath of the global financial crisis, interest in bubble detection intensified, prompting substantial methodological advancements. Researchers developed progressively more sophisticated tools, including recursive right-tailed unit root procedures (Phillips et al., 2011), fractional integration approaches (Frömmel and Kruse, 2012), nonlinear regime-switching models (Balke and Wohar, 2009), and strict local martingale-based diagnostics (Jarrow et al., 2010). A comprehensive survey of these developments is provided by Wöckl (2019), who evaluates their theoretical structure and empirical applicability.

Within this expanding methodological landscape, recursive right-tailed unit root tests have emerged as the dominant empirical instrument due to their superior power properties and suitability for real-time monitoring. The PWY test introduced by Phillips et al. (2011) and its subsequent generalisation, the PSY framework of Phillips et al. (2015), facilitate not only the detection of explosiveness but also the precise date-stamping of multiple exuberance episodes within a single sample. These procedures have been extensively employed across diverse asset classes, including tackling climate change (Wegener et al., 2024), commodities (Etienne et al., 2014; Figuerola-Ferretti and McCrorie, 2016; Pavlidis et al., 2018; Cincinelli and Pellini, 2025), real estate (Jiang et al., 2015; Pavlidis et al., 2016; Cincinelli et al., 2024), and cryptocurrencies (Corbet et al., 2018), underscoring their versatility and empirical prominence.

2.2.2 Detection and Determinants of Green Bubbles

In recent years, concerns about green bubbles have been widely discussed in both policy reports (Aramonte and Zabai, 2021; Nauman, 2021; Jourde and Stalla-Bourdillon, 2021) and the academic literature (Wang et al., 2020; Ghosh et al., 2022; Lehnert, 2023; Potrykus et al., 2024; Ariza and Ferrer, 2025; Vríz and Grossi, 2026). In the finance literature, a bubble is conventionally understood as an extraordinary increase of prices above fundamental value, driven by optimistic expectations of future gains that ultimately prove unfounded, and is typically associated with market failure and systemic risk. A more positive view is the social bubble hypothesis (Gisler and Sornette (2009), Gisler et al. (2011), Huber and Sornette (2022)), which holds that intense social interactions among enthusiastic supporters of a major project can sustain investment and commitment well beyond what a standard cost-benefit analysis would justify, thereby accelerating technological innovation. Whether green bubbles are ultimately harmful, however, remains contested. Giorgis et al. (2024), applying the social bubble hypothesis to the 2004-2008 clean-tech bubble, argue that, although most clean-tech ventures ultimately failed, the narrative of a moral imperative to combat climate change, combined with large inflows of venture capital and substantial government subsidies, generated self-reinforcing dynamics that catalysed substantial cost reductions and de-risking in solar, wind, and battery technologies, building a durable commercial infrastructure. In a similar vein, Ghosh et al. (2022) argue that green bubbles, unlike other bubbles, are not destructive, as they increase economic activity, infrastructure spending, and the adoption of innovative technologies. Lehnert (2023) also shares the view that a green bubble represents an opportunity for businesses to invest cheaply in green energy, with positive consequences for the fight against climate change. Borio et al. (2023), by contrast, emphasise that green bubbles introduce distinct risks to financial stability.

Political commitments and social pressure can amplify speculative behaviour, undermining both financial stability and the credibility of the transition. The large-scale allocation of public resources to meet net-zero targets, coupled with expectations of future public support or bailouts should investments falter, may encourage excessive risk-taking. Moreover, social pressures and fear of missing out may reinforce herding behaviour, even when investors recognise that green assets are becoming overvalued.

While existing empirical studies document repeated episodes of explosiveness in green assets, most analyses rely on univariate bubble-detection procedures applied separately to individual series. Consequently, relatively little is known about whether speculative dynamics in green financial markets emerge as isolated episodes or as common dynamics across green assets. Early applications of recursive right-tailed unit root methodologies appear in Bohl et al. (2013) and Bohl et al. (2015), who deploy the PWY test on German renewable-energy equity series and document initial evidence of explosive price behaviour. More recent empirical contributions, including Wang et al. (2020), Lehnert (2023), Basse et al. (2023) and Ariza and Ferrer (2025), adopt the PSY test, revealing increasingly frequent yet short-lived episodes of exuberance across global green equity markets. To broaden the methodological landscape, alternative diagnostic tools have been introduced. These include the multifractal detrended fluctuation analysis applied in Shahzad et al. (2020), which captures the asymmetric multifractality of green asset returns, as well as the Log Periodic Power Law (LPPL) framework employed in Ghosh et al. (2022) and Giorgis et al. (2024), which models log-periodic oscillations in rapidly appreciating green equities. Franco et al. (2026) apply the strict local martingale (SLM) framework to clean-energy markets, using the Andersen-Piterbarg stochastic volatility model and Bayesian Hamiltonian Monte Carlo to obtain probabilistic, model-based measures of bubble presence in four clean energy indices.

Beyond detection, a parallel strand of literature investigates the structural determinants and transmission mechanisms underlying green bubbles. Van den Heuvel and Popp (2023) emphasise the importance of government support schemes and venture-capital financing as key drivers of capital inflows into early-stage renewable technologies. Cross-market linkages also play a critical role: evidence from Kassouri et al. (2021) indicates that shocks to oil prices, technology-sector performance, and macro-policy uncertainty are transmitted to green equity valuations, thereby amplifying exuberant dynamics. At a more systemic level, analyses of the green economy examine bubble behaviour across multiple renewable energy subsectors. For instance, Potrykus et al. (2024) document divergences between fundamentals and stock prices in the solar, wind, and water sectors. Related research on carbon markets, such as Huang and Wang (2024), identifies explosive dynamics in Emissions Trading Schemes (ETS), driven by evolving carbon-market policies, macroeconomic conditions, energy-price fluctuations, and broader market uncertainties.

This chapter contributes to the literature in three dimensions. First, it contributes to the growing literature on climate finance by showing that speculative episodes in transition finance may emerge as common dynamics across multiple green sectors rather than as isolated bubbles affecting individual assets, and that these common green bubbles are largely distinct from broader market, growth-stock, and conventional energy-sector exuberance. Second, the chapter contributes to the bubble-detection literature by combining valuation-based measures with a real-time common-factor framework. Specifically, we apply the common bubble detection approach of Chen et al. (2023) to a panel of green price-to-earnings ratios using the computationally efficient IPCA-RLS implementation proposed by Li and Urga (2026). By focusing on the common component extracted from a panel of valuation ratios, the framework allows us to identify common speculative dynamics across green sectors as they evolve over time. Third, the chapter

contributes to the literature on green bubbles by explaining the role of energy-market dynamics, liquidity conditions, physical climate risk, and climate-policy expectations in bubble formation.

2.3 Methodology

This section commences with an exploration of the present value equation for asset pricing, illuminating the detection of asset price bubbles and their analytical rationale. It progresses to discuss the common bubble test, introduced by Chen et al. (2023), which offers a powerful tool to detect the explosive behaviour exhibited in the common factors of a large-dimensional financial system. Subsequently, we discuss the IPCA-RLS algorithm developed by Li and Urga (2026), a computationally efficient implementation that facilitates real-time monitoring of common bubbles as new observations arrive, even with high-frequency time series. The section concludes with an outline of the probit regression framework used to analyse the underlying drivers of detected exuberance episodes.

2.3.1 Rational Bubbles

Rational bubble theory shows that, even under rational expectations, asset prices may exceed fundamentals if investors anticipate continued appreciation. This self-reinforcing mechanism can generate explosive deviations from intrinsic value. Starting from the standard asset pricing equation:

$$P_t = E_t \left[\frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}} \right] \quad (2.1)$$

where P_t and D_t are asset price and dividend at time t , respectively. R_t is the discount rate at time t and $E_t(\cdot)$ is the conditional expectation at time t . Assume that the discount rate is a constant R and iterate it until infinity yields:

$$\begin{aligned}
P_t &= E_t \left[\sum_{\tau=1}^{\infty} \frac{D_{t+\tau}}{(1+R)^\tau} \right] + \lim_{T \rightarrow \infty} E_t \left[\frac{P_T}{(1+R)^{T-t}} \right] \\
&= A_t + B_t.
\end{aligned} \tag{2.2}$$

The second term B_t captures the bubble component, which satisfies the sub-martingale condition $E_t[B_{t+1}] = (1+R)B_t$. When present, B_t dominates the dynamics of P_t , as the fundamental value A_t is at most $I(1)$. Empirically, such explosiveness is often assessed using price-to-dividend or price-to-earnings ratios.

2.3.2 Common Bubble Test

Chen et al. (2023) propose a test for detecting a latent common bubble factor in large-dimensional financial panels. Under the null hypothesis, which assumes the absence of a common bubble, the dynamic of the price-to-earning ratios of asset i , which is denoted as y_{ti} , is governed by the following equation

$$y_{ti} = f_{0,t}\lambda_{0,i} + \varepsilon_{ti}, \text{ for } t = 1, \dots, T; i = 1, \dots, N \tag{2.3}$$

where $f_{0,t}$ represents the fundamental factor and follows a unit root process, $\lambda_{0,i}$ is the factor loading, and ε_{ti} is the idiosyncratic error term.

The testing procedure consists of two key steps. First, the first principal component (PC1) is extracted from the panel by principal component analysis, with consistency guaranteed under standard assumptions. Second, the PSY test proposed by Phillips et al. (2015) is applied to the estimated PC1 to assess whether the common factor exhibits explosive behaviour.

The PSY test is based on repeatedly estimating an augmented Dickey–Fuller (ADF) regression of the form

$$y_t = \mu + \rho y_{t-1} + \sum_{i=1}^k \phi_i \Delta y_{t-i} + \varepsilon_t \quad (2.4)$$

where y_t is the value of the examined time series at time t ; μ is a constant term; k is the maximum lag order which can be determined by information criteria; ε_t is the normally and independently distributed error term. The null hypothesis of a unit root ($H_0 : \rho = 1$) is tested against the right-tailed explosive alternative ($H_1 : \rho > 1$).

Define r as the r^{th} fraction of the entire sample, such that the corresponding time index t is given by $t = \lfloor Tr \rfloor$, where $\lfloor \cdot \rfloor$ signifies the integer part of its input and T is the size of the full sample. Estimating the ADF regression in Equation (2.4) over a subsample beginning at fraction r_1 and ending at r_2 yields the statistic $ADF_{r_1}^{r_2}$. To date-stamp bubble episodes, Phillips et al. (2015) introduce the following statistic, defined for each endpoint r_2

$$PSY_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}. \quad (2.5)$$

Bubble origination and termination dates are then identified using the following estimators:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2 : PSY_{r_2}(r_0) > scv_{r_2}^{\beta_T}\} \quad (2.6)$$

and

$$\hat{r}_c = \inf_{r_2 \in [\hat{r}_e, 1]} \{r_2 : PSY_{r_2}(r_0) < scv_{r_2}^{\beta_T}\} \quad (2.7)$$

where r_0 is the minimum window size and $scv_{r_2}^{\beta_T}$ is the corresponding $100(1 - \beta_T)\%$ critical value, with β_T representing the significance level of the test. In the presence of multiple bubble episodes, the procedure is repeated sequentially, be-

ginning each iteration from the collapse date estimated for the previous bubble.

2.3.3 IPCA-RLS Algorithm

The computational burden of the common bubble test increases substantially as the sample length T grows, which can hinder the timely generation of early warning signals. To address this challenge, Li and Urga (2026) propose an adaptive algorithm, IPCA-RLS, which substantially reduces the computational complexity of the common bubble test. A further advantage of this approach is that it updates results recursively, avoiding the need to recompute or revise previous estimates. Specifically, the algorithm employs Incremental Principal Component Analysis (IPCA) to recursively update the estimated PC1 in the first step of the common bubble test and Recursive Least Squares (RLS) to recursively update the ADF statistics in the second step.

Given a $T \times N$ matrix of panel data $(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T)'$, where T is the time span and N is the number of assets, the IPCA-RLS algorithm can be outlined as follows

2.3.4 Probit Regression

A probit model is a type of regression in which the dependent variable is binary. Its purpose is to estimate the probability that an observation with a given set of factors falls into a specific category.

Suppose we define a bubble indicator B_t , a binary series equal to 1 on days when a bubble is detected and 0 otherwise. Let X_t denote a vector of regressors assumed to influence B_t . The probit model can then be expressed as

$$P(B_t = 1|X_t) = \Phi(X_t'\beta) \quad (2.8)$$

where $P(\cdot)$ is the probability and $\Phi(\cdot)$ is the cumulative distribution function

Algorithm 2: IPCA-RLS

-
- Step 1:** Generate the initial estimated PC1 $y_{1:t_0}$ by standard PCA, using the first t_0 observations. Following the literature, we set $t_0 = \lfloor Tr_0 \rfloor$, where $r_0 = 0.01 + 1.8/\sqrt{T}$.
- Step 2:** For each new time point t_2 , update the PCA using the IPCA algorithm, which yields the new observation of the PC1, namely y_{t_2} .
- Step 3:** Calculate the initial least squares estimates $ADF_{t_2-t_0+1}^{t_2}$ by OLS based on the ADF regression on the subsample $(y_{t_2-t_0+1}, \dots, y_{t_2})$.
- Step 4:** Decrease t_1 from its initial value $t_1 = t_2 - t_0 + 1$ by one time point each time until $t_1 = 1$, update the least squares estimates $ADF_{t_1}^{t_2}$ recursively using the RLS algorithm.
- Step 5:** Obtain the maximum value of the sequence of $ADF_{t_1}^{t_2}$, for $t_1 \in [1, t_2 - t_0 + 1]$, which is denoted as PSY_{t_2} .
- Step 6:** Increase t_2 by one time point each time until $t_2 = T$, repeat Steps 2-5 for each iteration.
- Step 7:** Obtain the sequence of PSY_{t_2} for $t_2 \in [t_0 + 1, T]$, compare them to the corresponding finite sample critical values and date-stamp the identified bubbles (if any).
-

(CDF) of the standard normal distribution. β is a vector of parameters typically estimated by maximum likelihood.

The probit specification can also be motivated as a latent variable model. Assume the existence of an unobserved auxiliary random variable

$$B_t^* = X_t' \beta + \varepsilon_t \quad (2.9)$$

where $\varepsilon_t \sim N(0, 1)$. The observed binary outcome B_t can then be interpreted as an indicator of whether the latent variable is positive:

$$B_t = \begin{cases} 1 & B_t^* > 0 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 1 & X_t' \beta + \varepsilon_t > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (2.10)$$

The use of the standard normal distribution causes no loss of generality because adding a fixed amount to the mean can be compensated by subtracting the same

amount from the intercept, and multiplying the standard deviation by a fixed amount can be compensated by multiplying the weights by the same amount.

2.4 Data

We identify and date-stamp green bubbles using daily price-to-earnings (P/E) ratios for nine equity indices. The selected indices cover a broad spectrum, ranging from general clean energy and clean technology to specific renewable energy sectors such as solar and wind. Using P/E ratios rather than raw prices distinguishes our empirical approach, as it more closely aligns with the theoretical definition of rational bubbles by capturing deviations of market valuations from underlying fundamentals. Table 2.1 introduces the indices included in the sample. The dataset spans the period from November 6, 2018 to November 17, 2025, at a daily frequency, yielding 1,835 observations for each index. The sample period is determined by data availability, and all data were downloaded from Bloomberg. Summary statistics for the nine indices are reported in Table 2.2.

Table 2.1: Clean energy, Clean-tech, and Green Energy Equity Indices

Ticker	Index Name
SPGTCED	S&P Global Clean Energy Transition Index
CELS	NASDAQ Clean Edge Green Energy Index
KPOWERP	S&P Kensho Clean Power Index
NACEX	CIBC Atlas Clean Energy Index
NEX	WilderHill New Energy Global Innovation Index
BGSCE	Bloomberg Goldman Sachs Global Clean Energy Index
KCLEANP	S&P Kensho Cleantech Index
SUNIDX	MAC Global Solar Energy Index
GWE	ISE Clean Edge Global Wind Energy Index

Notes. The table displays the 9 equity indices we included in our empirical analysis. Their index names and corresponding Bloomberg tickers are presented.

To identify the drivers of green bubble formation, we estimate a probit regression of B_t , which is a binary variable that takes value 1 when a green bubble is detected and 0 otherwise, on a set of candidate explanatory variables.

Table 2.2: Clean energy, Clean-tech, and Green Energy Equity Indices: Summary Statistics

Index	Mean	Std. Dev.	Median	Min.	Max.	Ske.	Kur.
SPGTCED	41.6	21.2	33.6	14.2	119.8	1.8	3.3
CELS	174.3	115.0	161.8	25.6	996.2	2.9	11.5
KPOWERP	219.3	276.1	81.2	20.1	2236.6	2.7	12.4
NACEX	187.9	129.6	158.2	31.7	834.4	2.6	6.9
NEX	419.4	215.3	560.3	28.7	848.5	-0.8	-1.0
BGSCE	25.9	8.0	22.9	12.8	49.7	1.0	0.2
KCLEANP	197.5	422.9	63.7	24.3	5236.5	7.9	74.7
SUNIDX	76.6	40.7	73.5	10.0	215.6	0.3	-0.4
GWE	28.8	6.6	28.9	15.4	47.2	0.3	-0.6

Notes. The table reports the summary statistics of the P/E ratios of the nine examined equity indices, including Mean, Standard Deviation (Std. Dev.), Median, Minimum (Min.), Maximum (Max.), Skewness (Ske.) and Kurtosis (Kur.).

First, *mktret*, the monthly log-return of the S&P Global 1200 Energy Index, is included to capture whether exuberance in conventional energy equities may spill over into green equity markets and create conditions conducive to speculative bubble formation, as suggested by the overlap in bubble periods documented in Section 2.5.2. Second, *oilret*, the monthly log-return of WTI crude oil spot price, captures the substitutional relationship between fossil and green energy: rising oil prices may draw speculative capital away from green equities and into traditional energy assets, thereby reducing the probability of speculative green bubble formation. Third, *ttfret*, the monthly log-return of the Title Transfer Facility (TTF) natural gas spot price, is included to capture the role of European natural gas market dynamics. TTF is the leading European natural gas benchmark and one of the most influential pricing references in global gas markets. Fourth, *euaret*, the monthly log-return of the EU Emissions Trading System (EU ETS) carbon allowance price, proxies for carbon market conditions. Rising carbon allowance prices increase the relative cost of fossil fuel-based energy production, potentially steering capital flows toward green equities and amplifying speculative dynamics in the clean energy sector. Fifth, *vix*, the monthly average of the CBOE Volatility Index, proxies for broad equity market uncer-

tainty. Sixth, *dcredit*, the first difference of the monthly average credit spread (Moody's Baa-Aaa), captures liquidity conditions in financial markets. Narrowing credit spreads signal an abundance of cheap liquidity that may ignite speculative exuberance in growth-oriented sectors such as green energy. Seventh, *pri*, the Physical Risk Index of Bua et al. (2024), is a text-based indicator that quantifies climate-related physical hazards, such as acute weather events and chronic environmental changes, through news analysis. Increased awareness of physical climate risks may act as a catalyst for green equity exuberance by reinforcing the investment case for the low-carbon transition. Finally, *cpu_index_broad*, the News-based climate policy uncertainty index (using broad dictionary concepts) of Gavriilidis et al. (2026), captures uncertainty surrounding the future path of climate policy. Periods of elevated climate policy uncertainty may discourage speculative positioning in green assets, while greater policy clarity may instead fuel investor optimism and contribute to bubble formation.

Data for these variables are obtained from Bloomberg, the Federal Reserve Economic Data (FRED), LSEG Workspace, and the websites of the corresponding authors. The sample spans the period between November 2018 and June 2025, successfully covering the detected major green bubble episodes. Summary statistics of these potential underlying drivers are reported in Table 2.3.

Table 2.3: Potential Underlying Drivers: Summary Statistics

Variable	Mean	Std. Dev.	Min.	Max.	p_5	p_{95}	Skewness	Kurtosis
<i>mktret</i>	-0.0016	0.2066	-0.9544	0.7614	-0.3143	0.2120	-0.1719	11.3792
<i>oilret</i>	-0.0004	0.1827	-1.2399	0.6343	-0.1595	0.1934	-3.3947	29.2402
<i>ttfret</i>	0.0040	0.3592	-1.7487	1.5863	-0.3619	0.5154	-0.2995	12.7750
<i>euaret</i>	0.0178	0.1061	-0.2955	0.2498	-0.1511	0.2032	-0.1425	3.1075
<i>vix</i>	20.4514	7.0674	12.5074	57.7368	12.9881	30.9413	2.2692	11.5572
<i>dcredit</i>	-0.0042	0.0973	-0.2471	0.4379	-0.1106	0.1236	1.9489	11.9324
<i>pri</i>	-0.0024	0.0060	-0.0168	0.0163	-0.0121	0.0073	0.2236	3.6430
<i>cpu_index_broad</i>	284.6724	105.5000	108.1600	618.4891	152.3789	492.0941	0.9512	3.6578

Notes. The table reports summary statistics of the potential underlying drivers of green bubble formation, including Mean, Standard Deviation (Std. Dev.), Minimum (Min.), Maximum (Max.), 5th percentile (p_5), 95th percentile (p_{95}), Skewness and Kurtosis. *mktret* is the monthly log-return of the S&P Global 1200 Energy Index; *oilret* is the monthly log-return of WTI crude oil price; *ttfret* is the monthly log-return of the TTF natural gas spot price; *euaret* is the monthly log-return of the EU ETS carbon allowance; *vix* is the monthly average of the CBOE Volatility Index; *dcredit* is the first difference of the Moody's Baa-Aaa credit spread; *pri* is the Physical Risk Index of Bua et al. (2024); *cpu_index_broad* is the broad Climate Policy Uncertainty index of Gavriilidis et al. (2026).

Among the energy-market variables, the monthly log-return of WTI crude oil

(*oilret*) exhibits the most extreme tail behaviour, with skewness of -3.394 and kurtosis of 29.2402. The most extreme negative observation corresponds to March 2020 (-1.2399), when the simultaneous collapse in global oil demand due to the COVID-19 pandemic and the breakdown of the OPEC+ production agreement between Saudi Arabia and Russia triggered a historic crash in oil prices. The sample maximum occurred in May 2020 (0.6343), coinciding with the initial demand recovery as the lockdowns began to ease and the OPEC+ production cuts agreed in April 2020 began to take effect. The TTF natural gas return (*ttfret*) is characterised by extraordinary volatility (standard deviation 0.3592), with its most extreme negative value in October 2022 (-1.7487), when EU storage hit capacity and warmer-than-average weather briefly overwhelmed the market's ability to absorb gas intended to replace Russian volumes. This was followed by a rebound in November 2022 (1.5863) as heating demand resumed and the storage overhang cleared. The log-return of the S&P Global 1200 Energy Index (*mktret*) also displays extreme observations around the same events. The monthly log-return of EU ETS carbon allowances (*euaret*) recorded a positive mean of 0.0178, reflecting the sustained upward trend in EUA prices over the 2018-2025 period driven by the progressive tightening of the EU Emissions Trading System. The most extreme negative observation occurs in March 2020 (-0.2955), when the COVID-19 pandemic triggered a broad collapse in industrial activity and energy demand, reducing the need for carbon allowances. The maximum log-return in the sample occurred in November 2021 (0.2498), driven by the escalating European energy crisis: as natural gas prices surged to record highs, power generators turned to coal-fired generation, drastically increasing the demand for carbon allowances and pushing their prices sharply higher.

The macro-financial variables similarly reflect the March 2020 shock. The CBOE Volatility Index (*vix*) recorded an unprecedented spike to 57.7368 in March 2020, driving the strong right skewness (2.2692) and high kurtosis (11.5572) of the dis-

tribution. The first difference of the Moody's Baa-Aaa credit spread (*dcredit*) shows an asymmetric pattern, with spreads widening sharply during periods of financial distress (the largest positive value, 0.4379, occurred in March 2020) and narrowing gradually during calmer periods, as reflected in the negative tail observations of May and June 2020 following the introduction of emergency liquidity facilities. The climate-related variables exhibit smoother dynamics. The Physical Risk Index (*pri*) varies gradually over time without large discrete jumps, consistent with the slow-moving nature of climate-risk news coverage. The Climate Policy Uncertainty index (*cpu_index_broad*) displays two notable spikes: November 2021 (553.82), coinciding with the COP26 summit and the launch of the Glasgow Financial Alliance for Net Zero (GFANZ), and February 2025 (618.4891), reflecting the surge in policy uncertainty triggered by the second Trump administration's reversal of US climate commitments.

The following section presents the results of the tests for detecting green bubbles and estimation results of the probit regression used to identify the drivers of green bubble formation.

2.5 Empirical Findings

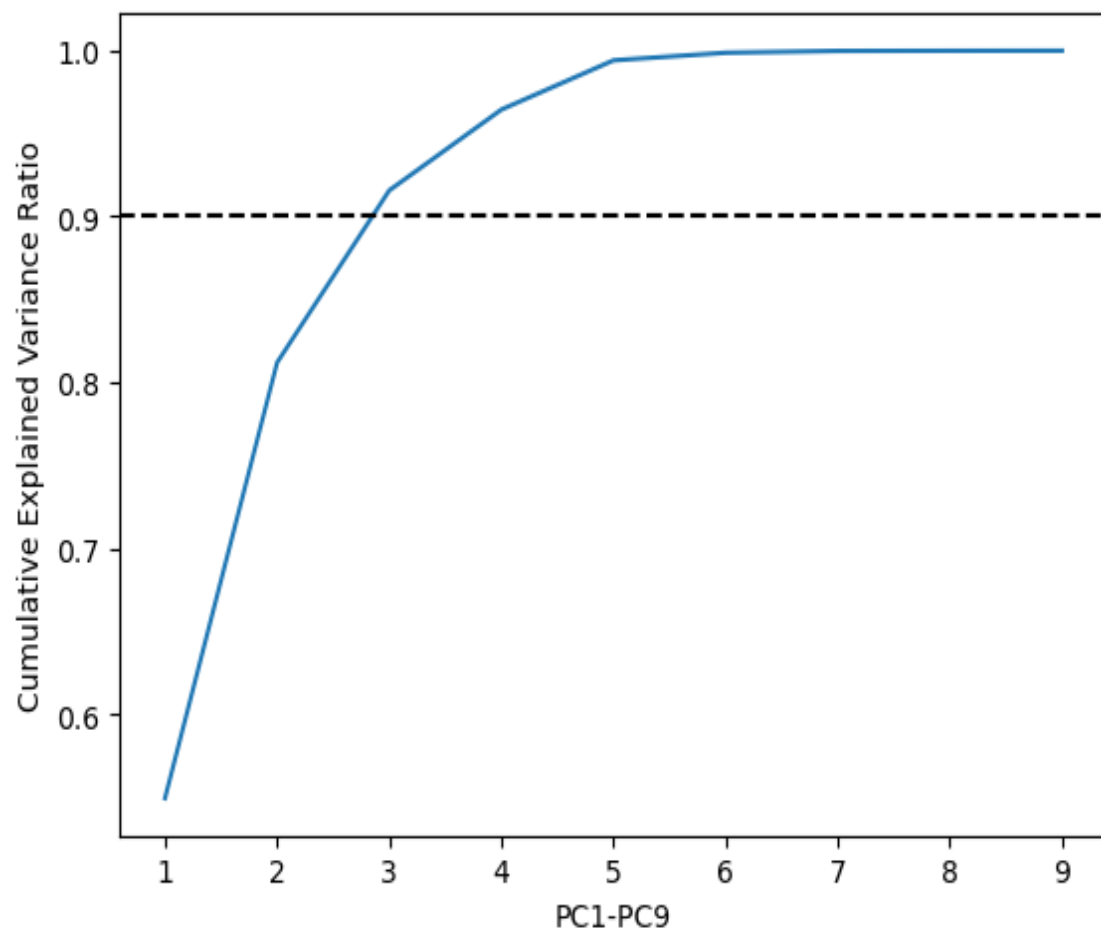
2.5.1 Common Bubble Test Results

We apply the efficient real-time common bubble detection method proposed by Li and Urga (2026) to the panel of 9 clean energy, clean technology, and green energy equity indices. This procedure generates daily time series of PSY statistics based on the estimated principal components, which captures the dominant common dynamic underlying all P/E ratios. In contrast to the conventional procedure that only considers the first principal component (PC1), we decide to use the first three principal components (PC1-PC3) in our analysis. The choice of

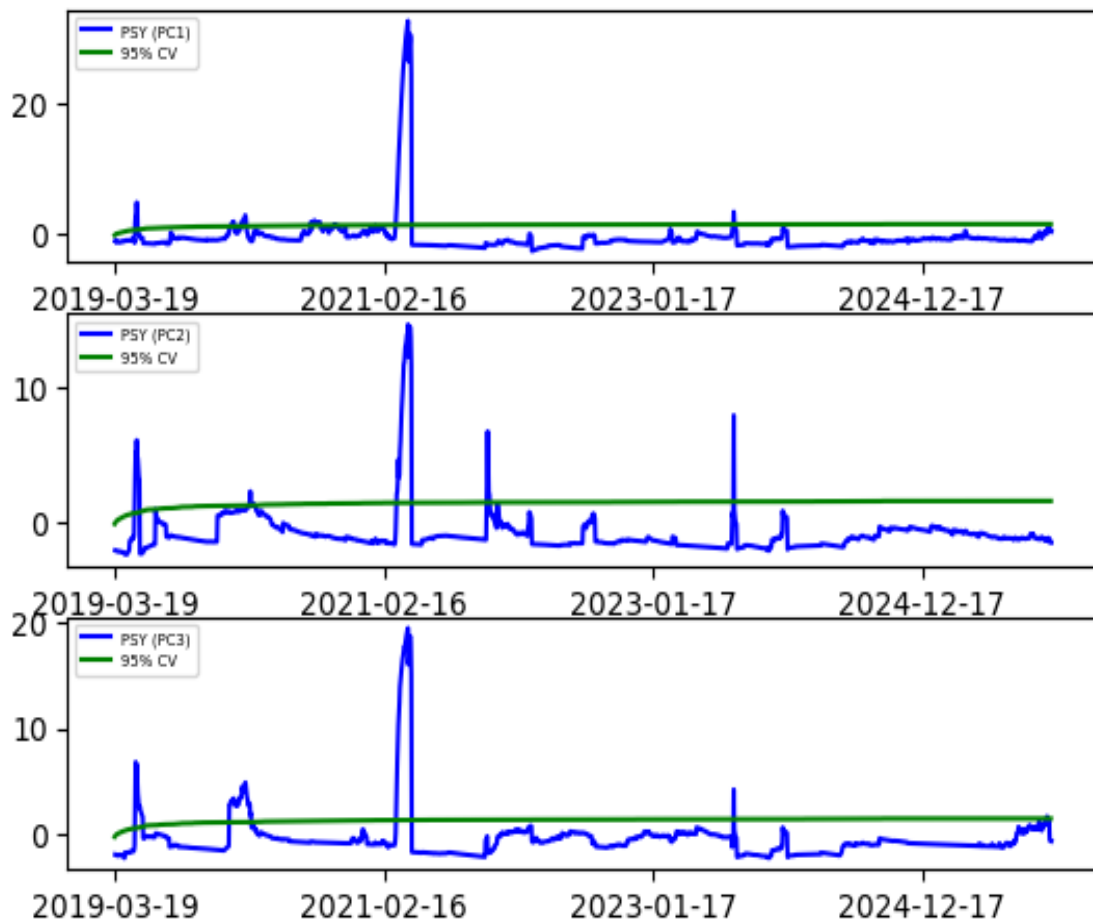
number of principal components is determined by the cumulative explained ratio of total variance. The first three principal components explain over 90% of total variance cumulatively, and the entire cumulative explained variance ratio is plotted in Figure 2.1. The first principal component (PC1), which explains the largest share of common variation in green valuation ratios, can be interpreted as a broad green-sector factor capturing sector-wide movements in green equity valuations. Explosive behaviour in this component reflects common increases in green valuations across multiple sectors. PC2 and PC3 represent additional dimensions of common variation in green valuations beyond the dominant green-sector factor.

The PSY statistics of PC1-PC3 are then compared with corresponding 95% critical values obtained from Monte Carlo simulations with 5,000 replications. Whenever the PSY statistic exceeds its corresponding critical value, we classify the period as exhibiting a green bubble. Figure 2.2 illustrates the comparison between the PSY statistics and their critical values for PC1-PC3, while Figure 2.3 displays the resulting identified green bubbles exhibited in PC1-PC3. To avoid false positive and too noisy daily signals, we impose an extra restriction on the bubble length, which is not shorter than 5 consecutive trading days (one week).

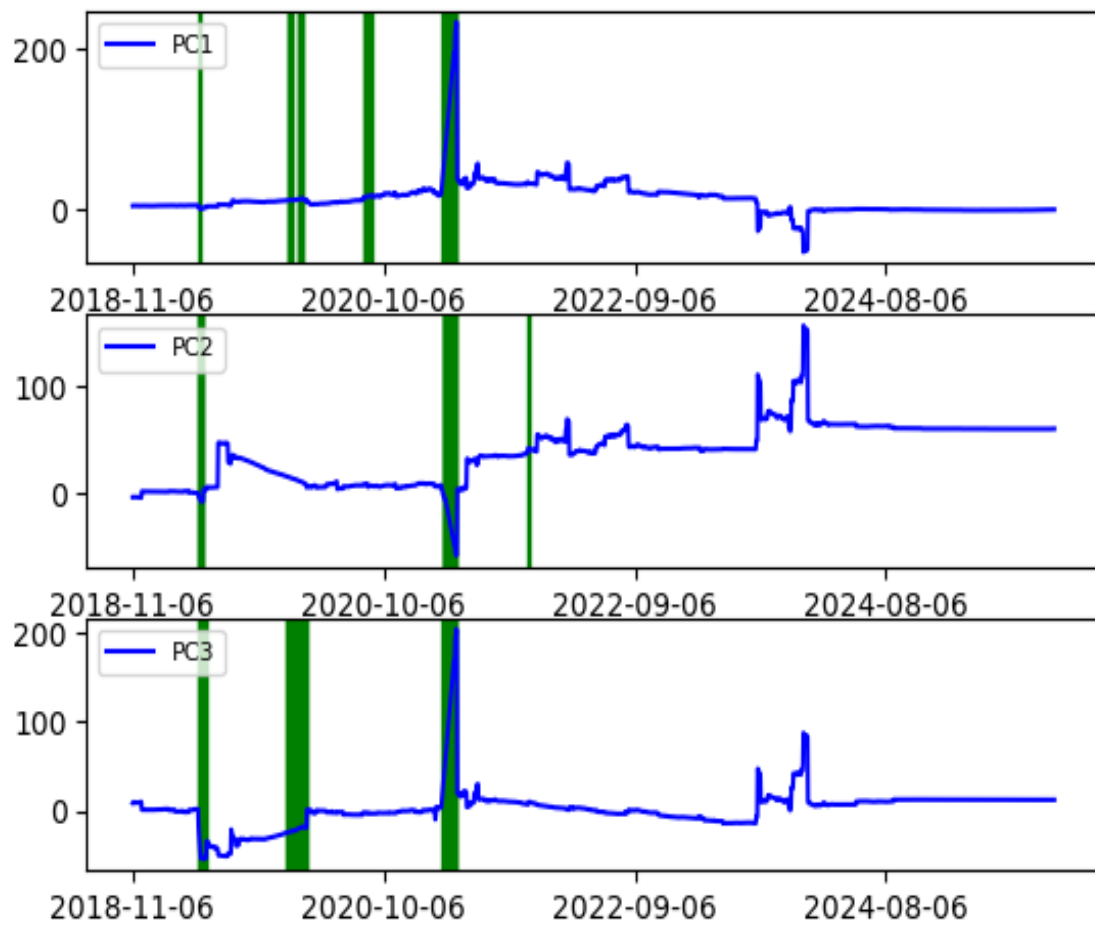
To further examine the findings, we construct the union of the green bubble periods identified by PC1–PC3. This union effectively captures the most prominent and commonly observed green bubbles within our sample. Table 2.4 lists the months in which these overlapping green bubbles occur, enabling direct comparison with studies that rely on monthly data. The most notable episodes are observed in May 2019 and in March and April 2021, all associated with important political and macroeconomic events. On the one hand, these results align with earlier evidence, such as Potrykus et al. (2024), which shows that the most significant green bubble developed during the post–COVID-19 market recovery.

Figure 2.1: Cumulative Explained Variance Ratio

Notes. The figure presents the result of the cumulative explained variance ratio of PC1-9. Note that the first three principal components explain over 90% of total variance.

Figure 2.2: Identification of Green Bubbles: PSY Statistics

Notes. The figure presents the result of comparing the time series of PSY statistics of PC1-PC3 and respective finite sample critical values at 95% confidence level. Finite sample critical values are obtained by Monte Carlo simulation with 5000 iterations. The PSY statistics are calculated by implementing the efficient real-time common bubble detection method proposed by Li and Urga (2026).

Figure 2.3: Identification of Green Bubbles: Detected Bubbles

Notes. The figure presents the detected green bubbles exhibited in PC1-PC3 respectively. The identified bubble periods are highlighted in green. We impose an extra restriction that the length of green bubble is not shorter than five trading days (one week).

Table 2.4: Major Green Bubble Periods and Potential Triggering Events

Period	Bubble Signal	Potential Triggering Events
May 2019	PC1 ✓ PC2 ✓ PC3 ✓	European Parliament elections generated a “Green Wave” with strong gains for environmental parties and climate-focused policy agendas. Rising public pressure from the global climate movement, including youth climate strikes, reinforced expectations of stronger climate policies and accelerated ESG adoption among institutional investors.
January 2020	PC1 ✓ PC2 ✓	Tesla’s extraordinary stock-price rally became a benchmark for clean-energy growth valuations. Rapid expansion of ESG-themed funds and increasing investor optimism regarding decarbonisation and clean-technology investment opportunities contributed to speculative enthusiasm.
February 2020	PC1 ✓ PC2 ✓	BP announced its ambition to become a net-zero company by 2050, signalling a strategic shift among major fossil-fuel firms. A growing number of corporate net-zero commitments strengthened expectations regarding the long-term profitability of green industries.
August 2020	PC1 ✓	Joe Biden’s presidential campaign emphasised a \$2 trillion Clean Energy Plan. Simultaneously, unprecedented monetary stimulus and post-pandemic recovery measures created abundant liquidity, while record inflows into ESG and clean-energy funds amplified optimism toward green investments.
March 2021	PC1 ✓ PC2 ✓ PC3 ✓	The European Union’s Sustainable Finance Disclosure Regulation (SFDR) entered into force. The implementation of the European Green Deal, strong post-pandemic economic recovery, and substantial capital inflows into clean-energy funds reinforced expectations of accelerated transition finance growth.
April 2021	PC1 ✓ PC2 ✓ PC3 ✓	The United States hosted the Leaders Summit on Climate and announced an ambitious target of reducing greenhouse-gas emissions by 50–52% below 2005 levels by 2030. Renewed U.S. climate leadership and strengthened global commitments ahead of COP26 boosted investor confidence in the low-carbon transition.
November 2021	PC1 ✓	COP26 in Glasgow and the announcement by the Glasgow Financial Alliance for Net Zero (GFANZ) that more than \$130 trillion of financial assets were aligned with net-zero objectives strengthened expectations of large-scale climate-finance mobilisation. Rising carbon prices and widespread corporate net-zero pledges further supported green-investment narratives.

Notes: The table expands the interpretation of major green bubble periods by considering a broader set of contemporaneous political, regulatory, financial, technological, and climate-related developments. The identified episodes correspond to periods in which explosive dynamics were detected in the common components (PC1-PC3) of green equity valuation ratios. Rather than attributing bubble formation to a single event, the table highlights clusters of reinforcing developments that may have contributed to synchronized investor optimism and coordinated capital inflows into green assets.

On the other hand, we also identify several shared green bubble episodes that predate the pandemic. Accordingly, in the next section, we apply the PSY test to three broad market indices and compare the bubble detection outcomes, demonstrating that green bubbles are a distinct phenomenon linked to the clean and green energy sector rather than merely a byproduct of broader post-pandemic market dynamics.

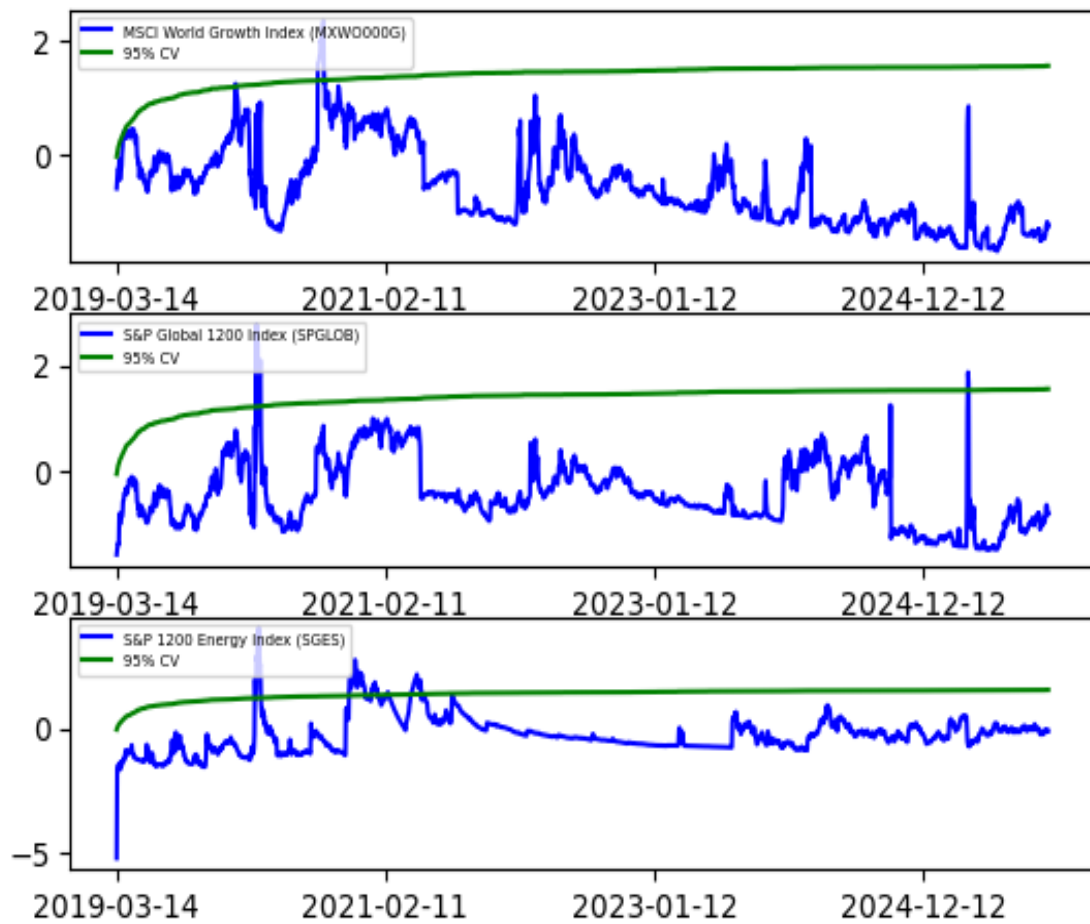
2.5.2 Distinguishing Green Bubbles from Broad Market Exuberance

A central empirical challenge is to distinguish synchronized green exuberance from broader market-wide or growth-oriented speculative dynamics. Green equities are frequently classified as growth assets and may therefore experience rapid valuation increases during periods of abundant liquidity, technology-sector optimism, or generalised market exuberance. Consequently, identifying common green bubbles requires demonstrating that the detected episodes are not merely reflections of broad growth-stock or market-wide speculative dynamics. To address this issue, we compare the identified green bubble episodes with bubbles detected in three benchmark indices representing distinct dimensions of broader financial-market exuberance. In particular, we apply the PSY test to three major market indices: MSCI World Growth Index, S&P Global 1200 Index, S&P Global 1200 Energy Index. These three indices are selected to represent distinct dimensions of the broader equity market against which green bubble dynamics can be benchmarked. Specifically, the MSCI World Growth Index captures growth-oriented equities across the global market, which is particularly relevant given that clean energy equities are typically valued as growth assets; the S&P Global 1200 Index serves as a global equity benchmark, representing overall market exuberance; and the S&P Global 1200 Energy Index repre-

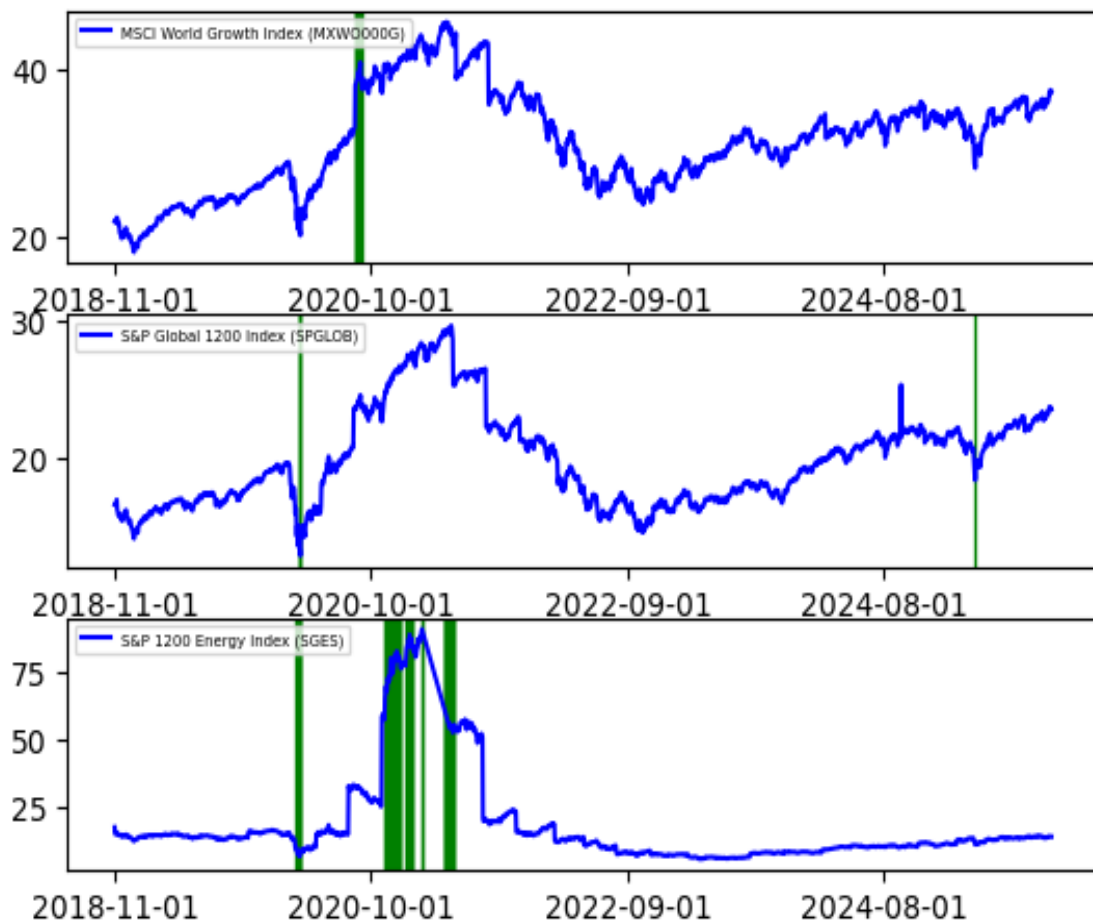
sents the conventional energy sector, allowing us to assess whether green bubble episodes are driven by or distinct from broader energy market dynamics. Together, these three indices allow us to test whether the detected green bubbles can be attributed to growth style exuberance, general equity market exuberance, or conventional energy sector exuberance or whether they constitute a genuinely distinct phenomenon. For subsequent comparison with the green bubble results, we downloaded the P/E ratios for these three indices from Bloomberg, for the same periods, spanning from 6 November 2018 to 17 November 2025.

Similarly to what has been done for common green bubbles, we present the PSY statistics of three broad market indices and corresponding 95% critical values in Figure 2.4, while the detected broad market bubbles are shown in Figure 2.5. We can clearly notice some commonalities as well as heterogeneities across the bubbles exhibited in these three broad market indices. For example, all three broad market indices showed explosive rebound after the COVID-19 pandemic in the early 2020, while the energy-related S&P 1200 Energy Index showed major exuberance in 2021.

Next, we compare the identified bubble periods for both green indices and broad market indices. Results are reported in Table 2.5. The comparison suggests that there is only one month that exhibited both green bubbles and broad market bubbles: April 2021. This result implies that green bubbles are distinct from broad market bubbles, while the energy-related S&P Global 1200 Energy Index may have contributed to the formation of green bubbles. As a result, in the next section, we examine the potential drivers of green bubbles and we include the S&P Global 1200 Energy Index in our analysis. Also, combining the previous results that the major green bubbles are always mapped with major political or macroeconomic events, we conjecture that the identified green bubbles are mainly driven by macro-financial variables. We will reveal the results using a

Figure 2.4: Identification of Bubbles in Broad Market Indices: PSY Statistics

Notes. The figure presents the result of comparing the time series of PSY statistics of three broad market indices and respective finite sample critical values at 95% confidence level. Finite sample critical values are obtained by Monte Carlo simulation with 5000 iterations. The PSY statistics are calculated by implementing the efficient real-time common bubble detection method proposed by Li and Urga (2026).

Figure 2.5: Identification of Broad Market Indices Bubbles: Detected Bubbles

Notes. The figure presents the detected green bubbles exhibited in three broad market indices respectively. The identified bubble periods are highlighted in green.

monthly Probit regression in the next section.

Table 2.5: Green Bubbles vs. Broad Market Bubbles

Month	MSCI World Growth Index	S&P Global 1200 Index	S&P Global 1200 Energy Index	PC1	PC2	PC3
May 2019				✓	✓	✓
January 2020				✓		✓
February 2020				✓		✓
March 2020		✓	✓			
August 2020	✓			✓		
September 2020	✓					
November 2020			✓			
December 2020			✓			
January 2021			✓			
February 2021			✓			
March 2021				✓	✓	✓
April 2021			✓	✓	✓	✓
May 2021			✓			
November 2021					✓	
April 2025		✓				

Notes. The table reports the identified green bubble periods exhibited in PC1-PC3, along with the identified broad market bubble periods. Among these periods, only April 2021 is the overlapping month for green bubble periods and broad market bubble periods.

The comparison reveals remarkably limited overlap between common green bubble episodes and broader market or growth-stock bubbles. In particular, only April 2021 simultaneously exhibits explosiveness in both green and broad-market indices. This result suggests that green asset exuberance is largely distinct from generalised market optimism and instead reflects transition-specific speculative dynamics. This distinction is important from both an asset-pricing and financial-stability perspective. If green bubbles merely reflected broad market exuberance, then speculative dynamics in green assets would not fundamentally differ from standard growth-stock cycles. By contrast, the limited overlap documented here suggests that green equity markets may generate their own speculative dynamics, potentially creating sector-specific vulnerabilities that cannot be inferred from aggregate market conditions alone.

2.5.3 Probit Estimation Results: Drivers of Green Bubbles

Under the interpretation developed in the previous section, the bubble indicator captures periods of common speculative valuation dynamics across green assets. The objective of this section is therefore to examine the macro-financial

conditions and transition-related factors associated with deviations of green valuations from fundamentals.

We specify a probit regression of the binary bubble indicator B_t on the set of candidate drivers described in the previous section. Model selection proceeds in two stages. In the first stage, we estimate two general specifications: one including only contemporaneous regressors, and one including only one-period lagged regressors, and apply a general-to-specific approach within each, iteratively dropping the least statistically significant variable until all remaining regressors are significant at the 10% level. This yields two benchmark specifications. In the second stage, we combine the surviving variables from both stages and systematically explore mixed specifications that include both contemporaneous and lagged regressors, adding candidate variables one at a time and comparing model fit using the Akaike Information Criterion (AIC). The final specification is selected as the model with the lowest AIC across all estimated models, retaining five regressors. The estimation results together with the Average Marginal Effects (AME) are reported in Table 2.6.

The estimation results reveal five statistically significant drivers of green bubble formation, spanning both macro-financial conditions and climate-related factors. On the positive side, $mktret_{t-1}$ significantly contributes to the formation of green bubbles. This result confirms our findings in the previous section that the bubble detected in the S&P Global 1200 Energy Index leads green bubbles in early 2021, representing a spillover effect from the broad energy-related sector to the specific clean energy sector. The average marginal effect indicates that a one-standard-deviation increase in lagged energy market returns raises the probability of a green bubble by 7.8 percentage points. Similarly, pri_{t-1} exerts a positive and significant effect on bubble formation. Heightened physical climate risk, proxied by increased news coverage of extreme weather events and environmental hazards,

Table 2.6: Probit Regression Estimation Results: Coefficients and Average Marginal Effects

Variable	Coefficient	AME
$oilret_t$	-1.8574^{***} (0.6070)	-0.2541^{***} (0.0705)
$mktrret_{t-1}$	0.5685^* (0.2967)	0.0778^* (0.0398)
$dcredit_{t-1}$	-0.7010^* (0.4152)	-0.0959^* (0.0537)
pri_{t-1}	0.5912^* (0.3167)	0.0809^{**} (0.0407)
$cpu_index_broad_{t-1}$	-0.7112^* (0.3736)	-0.0973^{**} (0.0472)
Constant	-1.8462^{***} (0.3827)	
Observations	78	
Log-likelihood	-18.537	
LR $\chi^2(5)$	26.39	
Prob > χ^2	0.0001	
Pseudo R^2	0.4158	

Notes. The table reports the probit estimation results alongside AME. The dependent variable is the binary bubble indicator B_t , equal to 1 when a common green bubble is detected and 0 otherwise. All regressors are standardised to zero mean and unit standard deviation. $oilret_t$ is the contemporaneous log-return of WTI oil price; $mktrret_{t-1}$ is the lagged log-return of the S&P Global 1200 Energy Index; $dcredit_{t-1}$ is the lagged first difference of the Moody's Baa-Aaa credit spread; pri_{t-1} is the lagged Physical Risk Index of Bua et al. (2024); $cpu_index_broad_{t-1}$ is the lagged broad Climate Policy Uncertainty index of Gavrilidis et al. (2026). *, **, *** denote the 10%, 5% and 1% significance levels, respectively.

amplifies investor interest in clean energy assets, pushing green equity valuations above fundamental levels. A one-standard-deviation increase in lagged physical risk raises the probability of a green bubble by 8.1 percentage points.

On the negative side, $oilret_t$ exhibits a contemporaneous negative relationship with green bubble formation, consistent with the substitutional relationship between fossil and green energy: when oil returns are high, capital flows toward conventional energy assets and away from green equities, reducing speculative pressure. The marginal effect is the largest in absolute terms, with a one-standard-deviation increase in oil returns decreasing the probability of a green bubble by 25.4 percentage points. Furthermore, $dcredit_{t-1}$ is negatively associated with bubble formation: when credit spreads narrow, the resulting abundance of cheap liquidity fuels speculative exuberance in growth-oriented sectors such as clean energy. A one-standard-deviation tightening of lagged credit spreads reduces the bubble probability by 9.6 percentage points. Finally, $cpu_index_broad_{t-1}$ is negatively and significantly associated with green bubble formation, with a marginal effect of -9.7 percentage points. This result suggests that policy clarity, rather than uncertainty, creates the conditions for speculative exuberance in green equity markets: when climate policy uncertainty is low, investors feel more confident allocating capital to green assets, increasing the risk of overvaluation. It is worth noting that neither market volatility (VIX), TTF natural gas returns, nor EU ETS carbon market returns are retained in the final specification. The exclusion of VIX is consistent with our earlier finding that green bubbles are largely distinct from broad equity market exuberance.

2.5.4 Robustness

As a robustness check, we assess whether the macro-financial drivers identified in the final specification are specific to green equity bubbles or rather reflect

broader market dynamics. To this end, we re-estimate the final probit model replacing the dependent variable with bubble indicators constructed for two broad market indices: the S&P Global 1200 Energy Index and the MSCI World Growth Index.² If the same drivers were to explain bubble formation in these broad indices, it would suggest that the results in Table 2.6 capture general market exuberance rather than a phenomenon specific to green equities.

The results are reported in Table 2.7. The drivers have no explanatory power for bubble episodes in the S&P Global 1200 Energy Index, with none of the coefficients reaching conventional significance levels and an overall model that fails to reject the null of joint insignificance ($\text{LR } \chi^2(5) = 4.66, p = 0.459$, Pseudo $R^2 = 0.099$). For the MSCI World Growth Index, only the lagged energy sector return (mktret_{t-1}) is marginally significant ($p = 0.039$), while all other drivers remain insignificant ($\text{LR } \chi^2(5) = 7.85, p = 0.165$, Pseudo $R^2 = 0.309$). These findings confirm that the macro-financial conditions identified as drivers of green bubble formation do not drive bubble episodes in broad market indices, corroborating the view that the bubble episodes detected in the clean energy, clean technology, and green energy equity indices represent a genuinely distinct phenomenon.

2.6 Conclusions

This chapter provided empirical evidence on the existence, timing, and drivers of common bubbles in green equity markets. Using daily price-to-earnings ratios for nine clean energy, clean technology, and green energy equity indices between 2018 and 2025, we applied the common bubble detection framework of Chen et al. (2023), implemented through the computationally efficient real-time IPCA-

²We also attempted to estimate the model using the S&P Global 1200 Index as the dependent variable. However, estimation was not feasible in this case due to a limited number of bubble episodes.

Table 2.7: Robustness: Probit Estimation with Broad Market Bubble Indicators

Variable	S&P Global 1200 Energy Coefficient	MSCI World Growth Coefficient
$oilret_t$	−0.2015 (0.1724)	−0.3889 (0.2353)
$mktret_{t-1}$	0.2324 (0.2028)	0.7483** (0.3633)
$dcredit_{t-1}$	−0.2208 (0.3111)	−0.7786 (0.5528)
pri_{t-1}	−0.0897 (0.2697)	0.2579 (0.4443)
$cpu_index_broad_{t-1}$	−0.2634 (0.2650)	0.2041 (0.3825)
Constant	−1.4722*** (0.2395)	−2.5169*** (0.6350)
Observations	78	78
Log-likelihood	−21.221	−8.793
LR $\chi^2(5)$	4.66	7.85
Prob > χ^2	0.4587	0.1649
Pseudo R^2	0.0989	0.3085

Notes. The table reports robustness checks re-estimating the final probit specification with alternative dependent variables. In column (1) the dependent variable is a binary indicator equal to 1 in months when a bubble is detected in the S&P Global 1200 Energy Index and 0 otherwise. In column (2) the dependent variable is a binary indicator equal to 1 in months when a bubble is detected in the MSCI World Growth Index and 0 otherwise. All regressors are standardised to zero mean and unit standard deviation and are identical to those in the baseline specification (Table 2.6). $oilret_t$ is the contemporaneous log-return of WTI oil price; $mktret_{t-1}$ is the lagged log-return of the S&P Global 1200 Energy Index; $dcredit_{t-1}$ is the lagged first difference of the Moody's Baa-Aaa credit spread; pri_{t-1} is the lagged Physical Risk Index of Bua et al. (2024); $cpu_index_broad_{t-1}$ is the lagged broad Climate Policy Uncertainty index of Gavrilidis et al. (2026). *, **, *** denote the 10%, 5% and 1% significance levels, respectively.

RLS algorithm of Li and Urga (2026). We identified multiple episodes of common exuberance, with the most pronounced occurring in May 2019 and around March-April 2021, and we showed that these episodes were largely distinct from bubbles detected in broad global equity, growth, and conventional energy indices.

Methodologically, the chapter demonstrates how real-time common-factor approaches can be combined with valuation-based measures to monitor synchronized speculative dynamics as they evolve over time. More broadly, the findings contribute to the literature at the intersection of climate finance, asset-price bubbles, and financial stability by showing that green financial markets may develop their own forms of exuberance, distinct from broader equity-market cycles.

Our findings indicate that the formation of green bubbles is primarily associated with macro-financial conditions, alongside a significant role for physical climate risk and policy uncertainty. In particular, elevated climate-related risks and exuberance in energy-related equities significantly increase the likelihood of green bubble formation, whereas widening credit spreads, rising oil prices and increased climate policy uncertainty exert a stabilising effect.

From a financial stability perspective, these results suggest that green equity markets may generate sector-specific vulnerabilities that cannot be inferred from aggregate market conditions alone. Abrupt corrections in green asset prices may transmit stress to the broader financial system, weaken investor confidence in sustainable finance, and disrupt capital flows essential for the energy transition. The methodological framework developed in this study enables real-time monitoring of such risks and offers a practical early-warning tool for regulators and policymakers. Incorporating common bubble indicators into macroprudential surveillance could help mitigate systemic vulnerabilities while supporting the long-term objectives of climate finance. Overall, the results suggest that effective

monitoring of green bubbles can support both financial stability and the credibility of sustainable finance, helping to reduce the likelihood of destabilising boom-bust cycles in transition-related markets.

Appendix 2.A Recursive Unit Root Tests

In this section, we focus on the pioneering work of Phillips and his co-authors in the development of recursive unit root tests. These tests, introduced in a series of influential papers, include the Sup Augmented Dickey-Fuller (SADF) test developed by Phillips, Wu, and Yu (2011, PWY hereafter), and its generalised version known as the GSADF test, introduced by Phillips, Shi, and Yu (2015a, PSY hereafter). Recursive unit root tests are designed to test the unit root null in time series against the alternative hypothesis of the mildly explosive behaviour, they are recursive in the sense that they are applied to subsamples sequentially rather than the full sample. These tests are demonstrated to be more powerful than previously proposed tests in identifying Evans' periodically collapsing bubbles. Another noteworthy advantage of these tests is their forward recursive nature, which allows for a date-stamping procedure that is not feasible with other approaches. The limiting theory of SADF (GSADF) test suggests that we can obtain consistent estimates of the start and end date of the explosive phases by comparing the sequence of ADF (Backward Sup ADF) statistics with their respective critical values in finite samples. The remarkable strengths of these tests have led to their recognition and appreciation in both academic and industry, leading to various extensions and modifications to the original recursive unit root tests, which we will introduce later on.

2.A.1 SADF Test

PWY (2011) develop the recursive right-tailed unit root test based on the following rationale: If we believe that the fundamental part of asset prices, namely dividends, is at most $I(1)$, then it is sufficient to ascertain the presence of a bubble by showing the explosiveness of price series. PWY (2011) propose a forward expanding testing procedure to identify the explosive behaviour in asset prices,

at least in some subsamples. The test can be implemented by applying the augmented Dickey-Fuller test to the log price series, that is, estimating the following autoregressive specification

$$y_t = \mu + \rho y_{t-1} + \sum_{i=1}^k \phi_i \Delta y_{t-i} + \epsilon_t \quad (2.11)$$

where y_t is the log price in time t ; μ is a constant term; k is the maximum lag order which can be determined by information criteria; ϵ_t is the normal and independent distributed error term. The unit root null hypothesis is $H_0 : \rho = 1$ and the right-tailed alternative hypothesis is $H_1 : \rho > 1$.

Define r as the r^{th} fraction of the entire sample, then the corresponding sample size t is given by $t = \lfloor Tr \rfloor$, where $\lfloor \cdot \rfloor$ signifies the integer part of its input and T is the size of the full sample. Suppose we estimate Equation (2.11) using a sample, which starts at the r_1^{th} fraction and ends at the r_2^{th} fraction of the full sample, denote the corresponding t -statistic as $ADF_{r_1}^{r_2}$.

The authors utilise a forward recursive regression algorithm, where Equation (2.11) is repeatedly estimated by including one additional observation in subsamples each time, the minimum sample size is $\lfloor Tr_0 \rfloor$. Hence, we can obtain a sequence of ADF statistics ($ADF_0^{r_2}$ for $r_0 \leq r_2 < 1$) by implementing the recursion, and the Sup ADF test statistic is defined as

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \quad (2.12)$$

under the null hypothesis, the limiting theory suggests the following convergences in distribution

$$ADF_0^{r_2} \Rightarrow \frac{\int_0^{r_2} \widetilde{W} dW}{\left(\int_0^{r_2} \widetilde{W}^2 \right)^{1/2}}$$

and

$$SADF(r_0) \Rightarrow \sup_{r_2 \in [r_0, 1]} \frac{\int_0^{r_2} \widetilde{W} dW}{\left(\int_0^{r_2} \widetilde{W}^2 \right)^{1/2}}$$

where W is the standard Brownian motion and $\widetilde{W}(r) = W(r) - \int_0^1 W$ is the de-meaned Brownian motion. By comparing the SADF statistic with its right tailed asymptotic critical values, we can test for a unit root against the possible explosive behaviour in subsamples.

2.A.2 GSADF Test

When confronted with multiple bubbles in a long time series data, the performance of the SADF test tends to be less effective. In order to address this limitation, PSY (2015a) propose an enhanced version of the SADF test called the Generalised Sup ADF (GSADF) test. This modified test allows for greater flexibility in the width of the estimation window used in the recursive regressions.

The GSADF test employs a double recursive regression algorithm, where both the starting point and the endpoint of the estimation window are varying, in contrast to the fixed starting point in the SADF test. Specifically, for each endpoint r_2 , a backward recursion is implemented by repeatedly estimating Equation (2.11) for each $r_1 \in [0, r_2 - r_0]$. Consequently, the GSADF test statistic is constructed as the maximum value obtained from all possible subsample ADF statistics, with a minimum sample size of r_0

$$GSADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} ADF_{r_1}^{r_2} \quad (2.13)$$

it has the following asymptotic distribution

$$GSADF(r_0) \Rightarrow \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} \frac{\int_{r_1}^{r_2} \widetilde{W} dW}{\left(\int_{r_1}^{r_2} \widetilde{W}^2 \right)^{1/2}}.$$

Obviously, the SADF test can be regarded as a special case of the GSADF test by fixing $r_1 = 0$. PSY (2015a) demonstrate through comprehensive simulation results that the GSADF test surpasses the SADF test in detecting multiple bubbles. In their paper, they apply the GSADF test to empirical data from S&P 500 and have successfully identified various well-known historical bubbles in the American stock market. This successful application highlights the efficacy and power of the GSADF test in bubble detection.

2.A.3 Extensions

Both the SADF test and the GSADF that we introduce above are the core of the recursive right-tailed unit root tests, subsequent research rely on these two fundamental tests and make some extensions. In this section, we will introduce several representative work based on the SADF test and GSADF test.

In statistical testing procedures, the specification of the model under the null hypothesis is crucial for accurate estimation. This is important not only because it affects asymptotic theory but also because it determines the critical values used in hypothesis testing. A well-known example is unit root testing, where the inclusion of intercepts, deterministic trends, or trend breaks significantly affects the limiting theory. Similar issues arise in right-tailed unit root tests used in bubble detection, as examined in the study by Phillips et al. (2014). Their analysis considers a martingale null hypothesis with an asymptotically negligible drift to capture the slight drift often observed in price processes over long historical periods, which is empirically realistic. The prototypical model of this type is represented by the following weak (local to zero) intercept form

$$y_t = dT^{-\eta} + y_{t-1} + \epsilon_t, \quad \epsilon_t \stackrel{i.i.d}{\sim} (0, \sigma^2) \quad (2.14)$$

where d is a constant, T is the sample size and σ is the unconditional variance of the error term. η is a localising parameter which satisfies $\eta \geq 0$. Phillips et al. (2014) show the difference of asymptotic distributions under the null when $\eta < 0.5$, $\eta = 0.5$, and $\eta > 0.5$, and recommend to always include a constant term in recursive ADF regressions but compare the test statistic with different critical values for robustness.

Under the alternative hypothesis, the data exhibits explosiveness due to the presence of bubbles. These bubbles can be generated using stochastic mechanisms, such as demonstrated in Blanchard and Watson (1992) and Evans (1991). Alternative approach is to employ reduced form methods that directly specify the deterministic break dates in the data generating process (DGP). Previous studies have typically assumed a DGP consisting of a mildly explosive bubble followed by an instantaneous collapse to its original level with a small perturbation. However, empirical data reveals that bubble implosions exhibit different dynamics, indicating the need for a more flexible model to capture the collapse of bubbles. Phillips and Shi (2018) bridge this gap by incorporating a transitory collapse regime in the DGP with adjustable market recovery rate, the model has the following specification

$$y_t = \begin{cases} dT^{-\eta} + y_{t-1} + \epsilon_t, & t \in N_0 \cup N_1 \\ \delta_T y_{t-1} + \epsilon_t, & t \in B \\ \gamma_T y_{t-1} + \epsilon_t, & t \in C \end{cases} \quad (2.15)$$

where $B = [T_e, T_c]$ is the bubble phase, with T_e and T_c being the origination and termination date respectively. Similarly, $C = (T_c, T_r]$ is the collapse episode, and T_r denotes the date of market recovery. $N_0 \cup N_1 = (1, T_e] \cup (T_r, T]$ corresponds

to the time periods when the market is normal. Following Phillips et al. (2014), this DGP includes an asymptotically negligible drift in the martingale process. Mildly expansion and implosion of bubbles are captured by δ_T and γ_T , where $\delta_T = 1 + c_1 T^{-\alpha}$ (with $c_1 > 0$ and $\alpha \in [0, 1]$) and $\gamma_T = 1 - c_2 T^{-\beta}$ (with $c_2 > 0$ and $\beta \in [0, 1]$). For any given c_1 and c_2 , the speeds of explosiveness and collapse are controlled by α and β .

In addition to incorporating a deterministic market recovery date into the DGP, Phillips and Shi (2018) provide a formal procedure for estimating this date through a reverse regression approach. The procedure involves constructing a reverse series of data, denoted as y_t^* , which is the original series y_t presented in reverse order, such that

$$y_t^* = y_{T+1-t}, \quad \text{for } t = 1, 2, \dots, T.$$

By transforming the original mildly integrated collapse process into a mildly explosive process, and vice versa, the detection of crisis episodes in the series y_t is equivalent to testing for mildly explosive behaviour in the reversed series y_t^* . To estimate the market recovery date, the authors apply a real-time monitoring procedure to the reversed series y_t^* . The details of this date-stamping procedure will be covered at a later stage.

Apart from the development of more general data generating processes, there is another line of research focuses on enhancing the power of the SADF test and the GSADF test. Noteworthy contributions in this area include the work by Whitehouse (2019) and Astill et al. (2023). Whitehouse (2019) proposes a Generalised Least Squares (GLS)-based version of the SADF test, by estimating the following auxiliary regression

$$\Delta \tilde{u}_t = \delta \tilde{u}_{t-1} + \epsilon_t \quad (2.16)$$

where $\tilde{u}_t = y_t - z_t' \tilde{\theta}$, $\tilde{\theta}$ is obtained from the GLS regression of $y_{\bar{c}} = (y_1, y_2 - \bar{\rho}y_1, \dots, y_{Tr_2} - \bar{\rho}y_{Tr_2-1})'$ on $z_{\bar{c}} = (z_1, z_2 - \bar{\rho}z_1, \dots, z_{\lfloor Tr_2 \rfloor} - \bar{\rho}z_{\lfloor Tr_2 \rfloor - 1})'$, where $\bar{\rho} = 1 + \bar{c}/T$ with a pre-determined constant $\bar{c}$ and $z_t = 1$ or $z_t = (1, t)'$ is the deterministic component. Denote $ADF_{GLS}^{r_2}$ as the simple t-statistic from Equation (2.16) on the sample that starts at 0 and ends at $\lfloor Tr_2 \rfloor$, then the GLS-based test statistic is defined as

$$SADF_{GLS}(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_{GLS}^{r_2}.$$

This GLS-based test has been demonstrated to have higher power than its OLS-based counterpart when dealing with a long period of explosiveness, which aligns with the findings of previous studies based on full sample analysis. However, in cases where the explosive period is relatively shorter, the OLS version of the SADF test outperforms the GLS version in terms of power. To harness the strengths of both tests, Whitehouse (2019) introduces a union of rejections procedure, which is able to capture the best available power offered by the two tests.

Astill et al. (2023) explore how information additional to a specific price series can be used to improve the power of the SADF test and the GSADF test, following the approach of Hansen (1995) and augmenting the ADF regressions with covariate regressors. Consider the following model

$$\Delta y_t = \delta y_{t-1} + \sum_{k=1}^p \alpha_k \Delta y_{t-k} + \sum_{k=-q_1}^{q_2} \beta_k' \omega_{t-k} + \epsilon_t. \quad (2.17)$$

Equation (2.17) represents an augmented standard ADF regression for y_t , where

the covariates in ω_t are included along with appropriate leads and lags. The vector ω_t consists of m stationary covariates with zero mean. To extend the GSADF test with covariates, we can apply the augmented ADF regression recursively, similar to the PSY (2015a) testing procedure. This recursive application results in a covariate-augmented version of the GSADF test statistic, denoted as $CGSADF$. Specifically, $CGSADF$ is the supremum of $CADF(p, q_1, q_2)_{r_1}^{r_2}$, which is computed as the t -ratio of the coefficient estimate δ obtained from the covariate-augmented ADF regression based on a subsample with starting point r_1 and endpoint r_2

$$CGSADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} CADF(p, q_1, q_2)_{r_1}^{r_2}.$$

Similarly, the covariate-augmented analogue of the SADF test statistic can be written as follows

$$CSADF(r_0) = \sup_{r_2 \in [r_0, 1]} CADF(p, q_1, q_2)_0^{r_2}.$$

Simulation evidence indicates that incorporating relevant covariates into the standard ADF regressions can greatly enhance the efficacy of bubble detection tests. This finding underscores the importance of considering relevant covariates when conducting recursive right-tailed unit root tests, as it leads to improved performance and reliability of the tests.

2.A.4 Other Related Tests

In this section, we will present alternative tests that have been developed to detect the transition from a random walk to an explosive process. The fundamental reasoning behind these tests, such as the recursive Chow-type break test and the end-of-sample test, slightly differs from that of the standard recursive unit root

tests.

Homm and Breitung (2012) borrow the idea from the PWY (2011) approach for bubble detection. They make modifications to several tests that are initially proposed in a different context and apply these adapted tests to the bubble identification framework. The tests they incorporate into the bubble test literature are derived from studies on persistence change. Additionally, Homm and Breitung (2012) introduce a recursive Chow-type test, which demonstrates superior power in simulation experiments. A Chow test for a structural break in the autoregressive parameter can be expressed in the following model

$$\Delta y_t = \delta (y_{t-1} 1_{\{t > \lfloor Tr \rfloor\}}) + \epsilon_t \quad (2.18)$$

where $1_{\{\cdot\}}$ represents an indicator function that takes the value 1 if the statement within the curly braces is true and 0 otherwise. In the context of hypothesis testing, the null hypothesis is denoted as $H_0 : \delta = 0$ and is being tested against the alternative $H_1 : \delta > 0$. Specifically, the alternative hypothesis assumes that $\delta = 0$ for $t = 1, 2, \dots, \lfloor Tr \rfloor$ and $\delta > 0$ for $t = \lfloor Tr \rfloor + 1, \dots, T$. It is straightforward to obtain the t -statistic for this regression as

$$DFC_r = \frac{\sum_{t=\lfloor Tr \rfloor+1}^T \Delta y_t y_{t-1}}{\hat{\sigma}_r \sqrt{\sum_{t=\lfloor Tr \rfloor+1}^T y_{t-1}^2}}$$

where

$$\hat{\sigma}_r^2 = \frac{1}{T-2} \sum_{t=2}^T \left(\Delta y_t - \hat{\delta}_r y_{t-1} 1_{\{t > \lfloor Tr \rfloor\}} \right)^2$$

where $\hat{\delta}_r$ is the OLS estimator of δ in Equation (2.18). The recursive Chow-type test statistic is defined as follows

$$SDFC(r_0) = \sup_{r \in [0, 1-r_0]} DFC_r.$$

This statistic is designed to be the supremum of a sequence of backward recursive statistics and the recursive Chow-type test is rejected when $SDFC(r_0)$ surpasses its upper-tail critical value.

The recursive Chow-type test examines a bubble scenario that consists of a single structural break from a random walk to an explosive process, which is considered as an ongoing bubble. However, Astill et al. (2017) contribute to the detection of explosive behaviour at the end of the sample, but with a finite length. In the recursive Chow-type test, the length of the bubble regime is assumed as a non-vanishing fraction, denoted as $1 - r$, of the total sample size. To detect end-of-sample bubbles more quickly, it might be more appropriate to assume a bubble episode at the end of the sample with a finite length. The approach proposed by Astill et al. (2017) is based on the end-of-sample instability testing procedures developed by Andrews (2003) and Andrews and Kim (2006), in which the test statistic is computed by utilising a window of end-of-sample observations with a finite size. The calculated test statistic is then compared against critical values obtained through subsampling across the earlier observations. Similar to Equation (2.18), they consider the following model specification

$$\Delta y_t = \begin{cases} \epsilon_t & t = 1, 2, \dots, T \\ \delta y_{t-1} + \epsilon_t & t = T + 1, \dots, T + m \end{cases} \quad (2.19)$$

where the error term ϵ_t is assumed to have a mean of zero and is stationary and ergodic, being consistent with Andrews (2003) and Andrews and Kim (2006). The series y_t is assumed to follow a unit root process for the initial T observations and then potentially change to an explosive process for the remaining m

observations, where m is considered to be relatively small compared to T . Consequently, the null hypothesis is defined as $H_0 : \delta = 0$, while the alternative hypothesis is represented by $H_1 : \delta > 0$. The Andrews S type test statistic is then defined as follows

$$S_m = \sum_{t=j+1}^{j+m} (t - j) \Delta y_t.$$

This definition applies to each subsample within the range of $\{t = j+1, \dots, j+m\}$, for $j \in [1, \dots, T - m, T]$. Additionally, the Andrews-Kim R type test statistic has the following definition

$$R_m = \sum_{t=j+1}^{j+m} \left(\sum_{s=t}^{j+m} \Delta y_s \right)^2.$$

The critical values for these statistics are derived using subsampling techniques applied to the first T observations. Furthermore, Astill et al. (2017) provide evidence that their methods outperform existing procedures when it comes to detecting short-lived end-of-sample bubbles, which suggests the potential for earlier detection of end-of-sample bubbles using their proposed approaches.

Appendix 2.B Recursive Unit Root Tests Under Heteroscedasticity

The recursive unit root tests that we have discussed above all assume an independent and identically innovation process, however, the unconditional volatility is commonly believed to be time-varying as supported by empirical observations. In response to this, there is a line of research that focuses on addressing heteroscedasticity issue instead of assuming homoscedasticity as in standard test settings. Harvey et al. (2016) first show that the SADF test becomes severely oversized, both asymptotically and in finite samples, in the presence of non-stationary volatility. This issue also applies to the GSADF test. To tackle this problem, they propose a wild bootstrap implementation that effectively controls the size. In a subsequent study, Harvey et al. (2019) introduce a Weighted Least Squares (WLS)-based version of the SADF test, which takes into account time-varying volatility to detect explosive behavior in financial time series. Simulation results indicate that neither the WLS-based nor the OLS-based SADF test consistently dominates the other across various volatility patterns. In a later paper, Harvey et al. (2020) develop a Sign-based GSADF test that does not require bootstrap methods to control the size in the presence of time-varying volatility. Additionally, the Sign-based SADF test can be derived as a special case by fixing the starting point of the regression window. Recently, Kurozumi et al. (2022) introduce a novel test based on the variance profile, which allows for a transformation in the time domain. Their test statistics have the same limiting distributions as in the case of homoscedasticity, which enables the use of asymptotic critical values for inference. This computational advantage distinguishes the time-transformed test from existing methods that require wild bootstrap implementation. In the following sections, we will provide a more detailed expla-

nation of these heteroscedasticity-adjusted recursive unit root tests.

2.B.1 Wild Bootstrap Implementation

In their study, Harvey et al. (2016) examine the performance of the SADF test when non-stationary volatility is present, with a focus on a prominent scenario where structural breaks occur in the unconditional variance of the innovation process. To investigate this, they consider the following data generating process

$$y_t = \begin{cases} y_{t-1} + \epsilon_t, & t \in N_0 \cup N_1 \\ \delta_T y_{t-1} + \epsilon_t, & t \in B \\ \gamma_T y_{t-1} + \epsilon_t, & t \in C \end{cases} \quad (2.20)$$

which is a reparameterisation of Equation (2.15) with the same definitions of N_0 , N_1 , B , and C . However, there is a difference in this DGP, as the authors consider a local-to-unit root alternative hypothesis. Specifically, they assume $\delta_T = 1 + c_1/T$ and $\gamma_T = 1 - c_2/T$, where c_1 and c_2 are positive constants. Additionally, the error term ϵ_t in this DGP is assumed to be a zero-mean process that could exhibit heteroscedasticity. The precise assumptions regarding the error term are detailed as follows

Assumption 2.1. $\epsilon_t = \sigma_t z_t$, where $z_t \sim IID(0, 1)$ with $E(z_t) = 0$, $E(z_t^2) = 1$ and $E(|z_t|^r) < \infty$ for some $r \geq 4$.

Assumption 2.2. The volatility term σ_t satisfies $\sigma_t = \omega(t/T)$, where $\omega(\cdot) \in D$ is non-stochastic and strictly positive, and $D = D[0, 1]$ is the space of right continuous with left limit (càdlàg) process on $[0, 1]$.

These assumptions are standard for testing procedures that incorporate non-stationary volatility in the bubble identification literature. Assumption 2.1 ensures that z_t is a martingale difference sequence, which is a common assumption

in empirical financial data analysis. Under Assumption 2.2, the unconditional variance of the innovation process is assumed to be non-stochastic, bounded, and exhibit a countable number of jumps. This assumption allows for various dynamics in the variance process, including single or multiple permanent volatility shifts, polynomially trending volatility, regime-switching volatility, as well as the conventional homoscedasticity case. As demonstrated by Harvey et al. (2016), in the presence of non-stationary volatility, the asymptotic distribution of the SADF statistic becomes non-pivotal and depends on nuisance parameters that are derived from the pattern of the heteroscedastic innovations under both the null and alternative hypotheses. These results indicate that the standard SADF test can be severely over-sized when the unconditional variance is non-stationary, leading to spurious rejections of the no bubbles null. To remedy the statistical inference problem, they propose a wild bootstrap implementation to replicate the pattern of non-stationary volatility presented in the original data. They also prove the theoretical validity by showing that the wild bootstrap SADF statistic shares the same first-order asymptotic distribution with the original SADF statistic. Extensive simulations suggest that the wild bootstrap SADF test controls size effectively while maintaining good power properties. The following steps outline their wild bootstrap algorithm

Algorithm 3: Wild Bootstrap

Step 1: Generate a $T \times 1$ vector of bootstrap innovations ϵ_t^* as $\epsilon_t^* = \omega_t \Delta y_t$ for $t = 2, \dots, T$, with an initial value $\epsilon_1^* = 0$. $\{\omega_t\}_{t=2}^T$ denotes an independent $N(0, 1)$ sequence.

Step 2: Construct the bootstrap sample as $y_t^* = \sum_{j=1}^t \epsilon_j^*$, $t = 1, \dots, T$.

Step 3: Compute the bootstrap *SADF* statistic denoted as $SADF^*$, using the bootstrap sample $\{y_t^*\}$ in Step 2.

Step 4: Compute the bootstrap p -values as $p^* = F^*(SADF)$, where $F^*(\cdot)$ is the conditional cumulative density function of $SADF^*$. Therefore, the bootstrap test is implemented at the α significance level such that the null hypothesis is rejected if $p^* < \alpha$.

In practice, the cumulative distribution function (*cdf*) $F^*(\cdot)$ required in Step 4 is typically unknown. However, it can be approximated using numerical simulations. This approximation is achieved by generating a set of independent bootstrap statistics, denoted as $SADF_b^*$, where $b = 1, 2, \dots, N$. These bootstrap statistics are computed by performing Steps 1 to 3 of Algorithm 3. The simulated bootstrap p -value is defined as $\hat{p}^* = \frac{1}{N} \sum_{b=1}^N 1(SADF_b^* > SADF)$. Here, $1(\cdot)$ is an indicator function that equals 1 when the statement in parentheses is true, and 0 otherwise. It is worth noting that although Harvey et al. (2016) assume a relatively specific form of the innovation variance process (non-stochastic, bounded, with countable jumps), this algorithm allows for a more general form of the innovation variance. Furthermore, while the wild bootstrap proposed in their paper is initially designed for the SADF test, this approach can also be applied to the GSADF test.

2.B.2 WLS-Based Test

Harvey et al. (2018) propose a Weighted Least Squares (WLS)-based variant of the SADF test, dealing with the problem of testing for explosive bubbles in financial time series with time-varying volatility. They consider a WLS transformation of the autoregressive model, which can be expressed as

$$\frac{\Delta y_t}{\sigma_t} = \delta_t \frac{y_{t-1}}{\sigma_t} + u_t. \quad (2.21)$$

By dividing the innovation volatility σ_t , Equation (2.21) becomes a homoscedastic regression model of $\{\Delta y_t/\sigma_t\}$ on $\{y_{t-1}/\sigma_t\}$. However, this regression is infeasible because σ_t is not observable. The infeasible test statistic can be constructed as

$$SBZ(r_0) = \sup_{r \in [r_0, 1]} BZ_r$$

where BZ_r is calculated from Equation (2.21) using the subsample $\{y_1, \dots, y_{\lfloor Tr \rfloor}\}$

$$BZ_r = \frac{\sum_{t=2}^{\lfloor Tr \rfloor} \Delta \tilde{y}_t \tilde{y}_{t-1} / \sigma_t^2}{\left(\sum_{t=2}^{\lfloor Tr \rfloor} \tilde{y}_{t-1}^2 / \sigma_t^2 \right)^{1/2}}$$

where $\tilde{y}_t = y_t - y_1$ is the demeaned series. To make the test feasible, the authors estimate the variance σ_t^2 using a nonparametric kernel of the following form

$$\hat{\sigma}_t^2 = \frac{\sum_{i=2}^T K_h\left(\frac{i-t}{T}\right) (\Delta y_i)^2}{\sum_{i=2}^T K_h\left(\frac{i-t}{T}\right)}.$$

Here, $K_h(s) = K(s/h)/h$ and $K(\cdot)$ is a kernel function with bandwidth parameter h . A feasible test statistic can be obtained on the basis of $\hat{\sigma}_t^2$, the feasible BZ_r is given by

$$BZ_r^f = \frac{\sum_{t=2}^{\lfloor Tr \rfloor} \Delta \tilde{y}_t \tilde{y}_{t-1} / \hat{\sigma}_t^2}{\left(\sum_{t=2}^{\lfloor Tr \rfloor} \tilde{y}_{t-1}^2 / \hat{\sigma}_t^2 \right)^{1/2}}.$$

Therefore, the feasible version of the test statistic is defined as $SBZ^f(r_0) = \sup_{r \in [r_0, 1]} BZ_r^f$. Moreover, the authors demonstrate that the limiting distribution of $SBZ^f(r_0)$ depends on the volatility function. As a result, they employ the wild bootstrap method discussed earlier to control the size of the test and obtain reliable critical values. Furthermore, they suggest a union of rejections testing procedure, recognising that neither the SBZ test nor the SADF test is dominant across all volatility specifications. This approach combines the inference from both tests, aiming to capture the relative power advantages of each test under different volatility dynamics.

2.B.3 Sign-Based Test

Harvey et al. (2020) approach the problem of testing for explosive bubbles under heteroscedasticity by introducing a sign-based variant of the GSADF test. Instead of directly calculating the GSADF statistic from the time series y_t , the authors propose a new test statistic based on the series of cumulated signs of the first differences of the original data. The series of cumulated signs is expressed as

$$C_t = \sum_{i=2}^t \text{sign}(\Delta y_i) = \sum_{i=2}^t \frac{\Delta y_i}{|\Delta y_i|}, \quad t = 2, \dots, T$$

where Δy_t represents the first differences and $\Delta y_t \neq 0$ is assumed. The authors consider an autoregressive regression model (without a constant term) for C_t as follows

$$\Delta C_t = \delta C_{t-1} + \epsilon_t.$$

The t -ratio of the coefficient δ , denoted as $sDF_{r_1}^{r_2}$, can be obtained from the regression over the period $t = \lfloor Tr_1 \rfloor, \dots, \lfloor Tr_2 \rfloor$. The Sign-based GSADF (sGSADF) test statistic is then given by

$$sGSADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} sDF_{r_1}^{r_2}.$$

Indeed, it is evident that the sign function, $\text{sign}(\Delta y_t)$, is insensitive to the variance of Δy_t . Consequently, the Sign-based GSADF test statistic is exact invariant to the pattern of time-varying volatility. As a result, there is no need for a wild bootstrap implementation to control the size of the test. Similar to the findings in Harvey et al. (2018), the authors observe that the standard GSADF test exhibits

higher power compared to its sign-based counterpart under certain volatility specifications. This observation motivates the use of a union of rejections approach, combining the inference from both the GSADF test and the sGSADF test. Also, it is important to note that the sign-based version of the SADF test, referred to as the sSADF test, is a special case of the sGSADF test with the restriction $r_1 = 0$.

2.B.4 Time-Transformed Test

In contrast to the previous approaches, Kurozumi et al. (2022) propose a novel test for bubbles under non-stationary volatility by introducing a deformation of the time domain. Their method involves adjusting the sampling interval based on the volatility level. Specifically, they lengthen the sampling interval during periods of low volatility and shorten it when volatility is high, where the volatility level is represented by σ_t . This adjustment is achieved through a time transformation based on the variance profile. The variance profile is defined as follows

$$\eta(s) = \left(\int_0^1 \omega^2(r) dr \right)^{-1} \int_0^s \omega^2(r) dr, \quad s \in [0, 1].$$

Under Assumption 2.1 and Assumption 2.2 discussed earlier, the partial sum process of $\{\epsilon_t\}$ is asymptotically equivalent to $\eta(s)$. The asymptotic average innovation variance is expressed as

$$\bar{\omega}^2 = \int_0^1 \omega^2(r) dr.$$

Given that the variance profile is a strictly monotonically increasing function under such assumptions, there exists a unique inverse function $g(s) = \eta^{-1}(s)$. Based on this, consider a time-transformed series $\tilde{y}_t = y_{t'} - y_{t'=0}$, where $\{t'\}$ is

defined as a non-decreasing sequence given by $t' = \lfloor Tg(t/T) \rfloor$. Under the null hypothesis, the authors show the following weak convergence

$$\frac{1}{\sqrt{T}} \tilde{y}_{\lfloor Tr \rfloor} \approx \frac{1}{\sqrt{T}} y_{\lfloor Tg(\lfloor Tr \rfloor / T) \rfloor} \approx \frac{1}{\sqrt{T}} y_{\lfloor Tg(r) \rfloor} \Rightarrow \bar{\omega} W^\eta(g(r)) = \bar{\omega} W(r)$$

where $W^\eta(g(r)) = W(\eta(g(r))) = W(r)$. Thus, the time-transformed series exhibits behaviour similar to that of a unit root process with a constant variance. Exploiting these results, the authors propose the following test statistics based on the Time-transformed ADF (TADF) statistics

$$STADF(r_0) = \sup_{r_2 \in [r_0, 1]} TADF_0^{r_2}$$

and

$$GSTADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} TADF_{r_1}^{r_2}$$

where

$$TADF_{r_1}^{r_2} = \frac{\tilde{y}_{\lfloor Tr_2 \rfloor}^2 - \tilde{y}_{\lfloor Tr_1 \rfloor}^2 - \bar{\omega}^2(\lfloor Tr_2 \rfloor - \lfloor Tr_1 \rfloor)}{2\bar{\omega} \sqrt{\sum_{t=\lfloor Tr_1 \rfloor + 1}^{\lfloor Tr_2 \rfloor} \tilde{y}_{t-1}^2}}.$$

Under the null hypothesis, the limiting distributions of these test statistics are demonstrated to be the same as in the case of homoscedasticity. Consequently, asymptotic critical values can be used for statistical inference, making it computationally less expensive compared to other existing methods that require wild bootstrap implementation. The key component of this time-transformed test is the variance profile $\eta(s)$, which is generally unknown. To estimate it from the data, the authors employ a kernel-type local least squares method to estimate the time-varying autoregressive coefficient and use the residuals to estimate the

variance profile.

Appendix 2.C Real-Time Monitoring Procedure

Unlike other econometric methods, the unique characteristic of forward recursive bubble tests allows for the identification of bubble origination and termination dates. This section introduces four real-time monitoring procedures: the PWY (2011) approach, the PSY (2015a) approach, the CUSUM procedure, and the composite bootstrap algorithm. The first two methods stem from the standard recursive unit root tests, specifically the SADF test and the GSADF test. The CUSUM procedure, initially proposed by Hogg and Tanaka (1993), in which the authors compare the performance of the PWY (2011) approach and the CUSUM procedure. Kurozumi (2020) derives the asymptotic distributions of the PWY (2011), PSY (2015a), and CUSUM real-time detectors under different scenarios. According to the asymptotic theory, under the local-to-unit root alternative, the CUSUM procedure outperforms the other methods when the bubble phase is short and starts early in the monitoring sample, which is supported by finite sample simulations. Additionally, Kurozumi (2021) evaluates the behavior of the stopping times for these approaches and demonstrates that when a bubble emerges early in the monitoring sample, the CUSUM detector tends to identify it earlier than the PWY (2011) and PSY (2015a) approaches, both asymptotically and in finite samples. Finally, Phillips and Shi (2020) propose a composite bootstrap algorithm that addresses both heteroscedasticity and multiplicity issues simultaneously.

2.C.1 PWY (2011) Approach

The SADF test considered in previous sections is a one-shot structural break test, which is designed to detect speculative bubbles using a fixed historical dataset. To determine the date of origination ($T_e = \lfloor Tr_e \rfloor$) and collapse ($T_c = \lfloor Tr_c \rfloor$) of the bubble, the PWY (2011) approach provide the following estimators, respectively

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2 : ADF_0^{r_2} > cv_{r_2}^{\beta_T}\}$$

and

$$\hat{r}_c = \inf_{r_2 \in [\hat{r}_e, 1]} \{r_2 : ADF_0^{r_2} < cv_{r_2}^{\beta_T}\}$$

where $cv_{r_2}^{\beta_T}$ is the $100(1 - \beta_T)\%$ critical value of the ADF test based on the sub-sample starting from 0 and ending at $\lfloor Tr_2 \rfloor$. As $T \rightarrow \infty$, the significance level β_T should tend to infinity, ensuring that $cv_{r_2}^{\beta_T} \rightarrow \infty$. This condition is necessary to prevent false rejections of the null hypothesis of no bubble asymptotically. Under the alternative hypothesis, the estimators $\hat{r}_e$ and $\hat{r}_c$ are consistent for their corresponding true values in the data generating process, as long as the following condition is satisfied

$$\frac{1}{cv^{\beta_T}} + \frac{cv^{\beta_T}}{T^{1-\alpha/2}} \rightarrow 0, \text{ as } T \rightarrow \infty \quad (2.22)$$

where α is the constant that controls the expansion rate of the bubble, following the definition in Equation (2.15). For practical implementation, PWY (2011) suggests using a sequence of critical values with a slowly varying expansion rate, such as $cv^{\beta_T} = \log \log^2(T)$. To account for a more realistic assumption that a bubble must have a duration exceeding a slowly varying quantity, such as $L_T = \log(T)$, the estimated end point of the bubble in the PWY (2011) approach is modified. The modified estimator for the collapse date is given by

$$\hat{r}_c = \inf_{r_2 \in [\hat{r}_e + \rho \log(T)/T, 1]} \{r_2 : ADF_0^{r_2} < cv_{r_2}^{\beta_T}\}$$

where ρ is a frequency-dependent parameter. For example, when the sample size is 30 years, the optimal ρ is 0.7 for yearly data and 5 for monthly data.

2.C.2 PSY (2015a) Approach

In the case of multiple bubbles, the PWY (2011) approach may not provide consistent estimators for subsequent bubbles after the first one. However, the PSY (2015a) approach offers a remedy for this issue. The PSY (2015a) approach is designed to consistently estimate the origination and termination dates of each individual bubble. The estimators in the PSY (2015a) approach are given by the following equations

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{r_2}^{\beta_T}\}$$

and

$$\hat{r}_c = \inf_{r_2 \in [\hat{r}_e, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^{\beta_T}\}$$

where

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}$$

and $scv_{r_2}^{\beta_T}$ is the corresponding $100(1 - \beta_T)\%$ critical value. In the presence of multiple bubbles, the PSY (2015a) approach suggests to repeat the aforementioned estimating procedure for subsequent bubbles, starting from the estimated collapse date of the former bubble. More specifically, for the case of two bubbles, the dates of origination and termination are obtained by

$$\hat{r}_{1e} = \inf_{r_2 \in [r_0, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{r_2}^{\beta_T}\}$$

$$\hat{r}_{1c} = \inf_{r_2 \in [\hat{r}_{1e}, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^{\beta_T}\}$$

$$\hat{r}_{2e} = \inf_{r_2 \in [\hat{r}_{1c}, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{r_2}^{\beta_T}\}$$

$$\hat{r}_{2c} = \inf_{r_2 \in [\hat{r}_{2e}, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^{\beta_T}\}.$$

Similar to Equation (2.22), the PSY (2015a) approach retains the consistency of the starting and end points estimators by imposing the following restriction

$$\frac{1}{scv^{\beta_T}} + \frac{scv^{\beta_T}}{T^{1-\alpha/2}} \rightarrow 0, \text{ as } T \rightarrow \infty.$$

In empirical applications, it is common to compare the sequence of $BSADF_{r_2}(r_0)$ statistics with the corresponding finite sample critical values. These critical values can be obtained through Monte Carlo simulations, where the test is applied to simulated data multiple times to estimate the distribution of the test statistic under the null hypothesis.

2.C.3 CUSUM Procedure

In contrast to the ADF statistics-based procedures introduced in the former sections, Hogg and Bruitung (2012) propose an alternative real-time monitoring procedure using CUSUM detector. The CUSUM detector is defined as

$$C_{r_0}^r = \frac{1}{\hat{\sigma}_r} \sum_{t=\lfloor Tr_0 \rfloor + 1}^{\lfloor Tr \rfloor} \Delta y_t$$

with

$$\hat{\sigma}_r^2 = \frac{1}{\lfloor Tr \rfloor - 1} \sum_{t=1}^{\lfloor Tr \rfloor} (\Delta y_t - \hat{\mu}_r)^2$$

where $\lfloor Tr_0 \rfloor$ is the training sample, $\lfloor Tr \rfloor$ denotes the current monitoring sample, for $r \in (r_0, 1]$. $\hat{\mu}_r$ is the mean of the series $\{\Delta y_1, \dots, \Delta y_{\lfloor Tr \rfloor}\}$. Under the null hypothesis of no bubble, the CUSUM detector has the following limiting property

$$\lim_{T \rightarrow \infty} P\{C_{r_0}^r > c_r \sqrt{\lfloor Tr \rfloor}\} \leq \frac{1}{2} \exp(-\kappa_\alpha/2)$$

where $c_r = \sqrt{\kappa_\alpha + \log(r/r_0)}$. κ_α is a pre-determined constant that depends on the significance level α . For instance, $\kappa_{0.05} = 4.6$ when $\alpha = 0.05$.

Homm and Bruitung (2012) conduct extensive numerical experiments to evaluate the performance of the CUSUM procedure in date-stamping bubbles. Their results indicate that the proposed CUSUM procedure performs comparably to the PWY (2011) approach in accurately estimating the dates of bubble origination and collapse.

2.C.4 Composite Bootstrap Algorithm

It has been discussed in Section 3.3 that the recursive unit root tests for bubbles will be affected in the presence of heteroscedasticity. Additionally, like all other recursive testing procedures, these tests face the multiple testing problem that contaminates conventional significance values. To provide more accurate and reliable inference in detecting and dating bubbles using the PSY (2015a) approach, Phillips and Shi (2020) implement the composite bootstrap algorithm, aiming to mitigate the impact of heteroscedasticity and control for the multiple testing problem. The algorithm can be summarised as follows

In this composite bootstrap algorithm, the challenges of heteroscedasticity and the multiplicity problem are addressed simultaneously. The choice of the control window size, denoted as r_b , is subjective and depends on the researcher's discretion. A larger control window leads to more conservative testing, reducing the

Algorithm 4: Composite Bootstrap

Step 1: Estimate Equation (2.11) with the restriction $\rho = 1$, using the full sample and obtain the residuals $\hat{\epsilon}_t$.

Step 2: For a sample size $\lfloor T(r_0 + r_b) \rfloor - 1$ (r_b is the fraction of the window over which size is to be controlled), generate a bootstrap sample such as.

$$\Delta y_t^b = \sum_{i=1}^k \hat{\phi}_i \Delta y_{t-i}^b + \epsilon_t^b$$

Initialised at $y_t^b = y_t$ for $t = 1, \dots, i + 1$. $\{\hat{\phi}_i\}$ are the OLS estimates obtained from Step 1. The bootstrap innovations are generated as $\epsilon_t^b = \omega_t \hat{\epsilon}_l$, where ω_t is randomly drawn from the standard normal distribution and $\hat{\epsilon}_l$ is randomly selected from the series $\{\hat{\epsilon}_t\}$.

Step 3: Using the bootstrap series $\{y_t^b\}$, compute a sequence of test statistics using the PSY (2015a) approach, denoted as $\{BSADF_{r_2}^b(r_0)\}_{r_2=r_0}^{\lfloor T(r_0+r_b) \rfloor - 1}$. Their maximum value is defined as $M_{r_2}^b$.

Step 4: Repeat Steps 2 to 3 for $B = 999$ times.

Step 5: Obtain the bootstrap critical value as the $100(1 - \alpha)\%$ percentile of the sequence $\{M_{r_2}^b\}_{b=1}^B$, for the significance level α .

likelihood of false-positive conclusions. In contrast, a smaller control window increases the probability of detecting a bubble and enables earlier detection.

Appendix 2.D Relationships Between Different Explosive Time Series

In previous sections, we have paid the attention to the testing procedures for individual asset price bubbles. However, it is also important to investigate the relationship of explosive behaviours in multiple financial time series, as empirical evidence suggests the presence of contagion among explosive processes. Despite this observation, there is limited research in this area. One approach to exploring the relationship between individual explosive time series is to compare their timelines of when explosiveness emerges, using date-stamping procedures discussed earlier. A leading example of this method is Phillips and Yu (2011), in which the authors propose a method for detecting bubble migration between two price series by comparing recursively estimated autoregressive parameters based on individual series. Another line of research examines co-explosiveness in the context of a vector autoregressive model, such as Magdalinos and Phillips (2009) and Nielsen (2010). Magdalinos and Phillips (2009) develop the asymptotic theory for cointegrating system with possibly mildly explosive variables, which is extended to a continuous-time framework by Chen et al. (2017). However, inconsistent results have been found in vector autoregressions with common explosive roots, as proposed by Magdalinos and Phillips (2013). Moreover, these approaches typically consider a finite number of variables, requiring all variables to display explosiveness, which may not be suitable for analysing large-dimensional financial systems. To address these challenges, this section introduces two alternative testing procedures: the co-explosiveness test by Evripidou et al. (2022) and the common bubble test by Chen et al. (2023).

2.D.1 Co-Explosiveness Test

Evripidou et al. (2022) propose a modified version of the Kwiatkowski et al. (1992) (KPSS) stationary test to examine the presence of co-explosiveness. This test is designed to assess the null hypothesis of co-explosive behaviour against the alternative hypothesis of no co-explosiveness using a bivariate vector autoregression model. The regression model is formulated as follows

$$y_t = \mu_y + \beta_x x_{t-i} + \beta_z z_t + \epsilon_{y,t}. \quad (2.23)$$

This test aims to examine the presence of a co-explosive relationship between the observed series y_t and x_t , where x_t exhibits an explosive regime and z_t represents a latent process that also displays explosiveness. $\epsilon_{y,t}$ is a stationary error term with zero mean. Hence, if $\beta_z = 0$ and $\beta_x > 0$, y_t and x_{t-i} are regarded as co-explosive in the sense that a linear combination of these processes is stationary. The lag order i allows for the detection of contemporaneous or dynamic correlation between the series, indicating the timing and direction of the bubble transmission. For example, if $i > 0$, the explosive episode in x_t precedes that in y_t , which means the bubble in x_t mitigates into y_t after i time periods. On the other hand, if $\beta_z > 0$ and $\beta_x = 0$, then the explosive behaviour in y_t is only driven by the unobserved series z_t , and is irrelevant to x_t . Therefore, the authors propose to test the following hypothesis

$$H_0 : \beta_x > 0, \beta_z = 0$$

$$H_1 : \beta_x = 0, \beta_z > 0$$

the KPSS type test statistic is given by the following form

$$S = \hat{\sigma}_y^{-2} (T - |i|)^{-2} \sum_{t=i1(i>0)+1}^{T+i1(i<0)} \left(\sum_{s=i1(i>0)+1}^t \hat{\epsilon}_{y,s} \right)^2$$

where $\hat{\epsilon}_{y,t}$ is the residuals obtained from the OLS regression of y_t on a constant and x_{t-i} , $\hat{\sigma}_y^2 = (T - |i|)^{-1} \sum_{t=i1(i>0)+1}^{T+i1(i<0)} \hat{\epsilon}_{y,t}^2$. At this stage i is deemed as a known value, while in the practical applications, the choice of the delay parameter i should be carefully examined through comprehensive experiments.

2.D.2 Common Bubble Test

Chen et al. (2023) propose a novel approach for testing the existence of a common bubble factor in a large-dimensional system of financial time series. They introduce a latent factor model that incorporates I(1), mildly explosive, and stationary factors to capture the normal, exuberant, and collapsing dynamics of the leading factor. Under the null hypothesis, which assumes the absence of a common bubble, the dynamics of the asset prices X_{it} are governed by the following equation

$$X_{it} = f_{0,t}\lambda_{0,i} + \epsilon_{it}, \text{ for } i = 1, \dots, N; t = 1, \dots, T$$

where $f_{0,t}$ represents the fundamental factor and follows a unit root process, $\lambda_{0,i}$ is the factor loading, and ϵ_{it} is the idiosyncratic error terms. Under the alternative hypothesis that there is a common bubble episode, asset prices are assumed to follow the following factor dynamic mechanism

$$X_{it} = \begin{cases} f_{0,t}\lambda_{0,i} + \epsilon_{it} & \text{if } t \in A \\ f_{1,t}\lambda_{1,i} + f_{0,t}\lambda_{0,i} + \epsilon_{it} & \text{if } t \in B \\ f_{2,t}\lambda_{2,i} + \epsilon_{it} & \text{if } t \in C \end{cases} \quad (2.24)$$

where $A = [1, T_e)$, $B = [T_e, T_c]$, and $C = (T_c, T]$. T_e and T_c are the dates of bubble origination and termination. The data generating process in Equation (2.24) is similar to that in Equation (2.15). Here, the factor $\{f_{0,t}\}_1^{T_c}$ follows a unit root process, the factor $\{f_{1,t}\}_{T_e}^{T_c}$ displays a mildly explosive behaviour, and the factor $\{f_{2,t}\}_{T_c+1}^T$ is assumed to be mildly stationary.

This testing procedure involves two steps in order to detect and date-stamp a latent common bubble in the panel of financial time series. In the first step, the dominant common factor is estimated using a principal component approach. This involves extracting the most significant factor that captures the shared information among the assets in the panel. In the second step, the PSY (2015a) approach is applied to the estimated dominant factor. This step focuses on identifying and dating the presence of a bubble in the leading factor. Chen et al. (2023) provide theoretical evidence to support the statistical validity of this common bubble factor detection process. Additionally, they demonstrate through empirical studies that the proposed approach exhibits satisfactory performance in finite samples.

Chapter 3

Timing the Factor Zoo: Explosiveness Beyond Momentum

3.1 Introduction

The term “factor zoo”, introduced by Cochrane (2011), captures the explosion of asset pricing factors proposed in the literature. This proliferation has raised concerns about p-hacking and lack of reproducibility, as highlighted by Harvey et al. (2016a), who advocate for more stringent statistical significance thresholds to improve empirical rigour (see also the recent contribution Harvey et al. 2026). In response to this, the research focus has shifted from discovering new factors to consolidating existing ones. Consistent with this consolidation agenda, Hou et al. (2019) show that several prominent factor models are closely related and that their apparent distinctiveness can depend importantly on factor construction. A key challenge is identifying factors that effectively span the

stochastic discount factor (SDF) in the high-dimensional environment. To address this, researchers have employed advanced techniques such as Lasso regularization, principal component analysis (PCA), and Bayesian methods, with various studies proposing innovative frameworks for factor selection, including seminal works of Feng et al. (2020), Freyberger et al. (2020), Kelly et al. (2019), Lettau and Pelger (2020), Kozak et al. (2020), Bryzgalova et al. (2023), etc. Recent work also develops sequential procedures for adding or removing candidate factors according to their contribution to mean–variance spanning and the Sharpe ratio of the selected factor model (Feng et al., 2026). Further, Aleti et al. (2025) show that high-frequency information extracted from the factor zoo predicts aggregate intraday market returns, with the strongest predictability arising during periods of elevated uncertainty.

This chapter departs from the prevailing literature on factor selection by focusing instead on the time-series dynamics of existing factors. The approach is rooted in the long tradition of market-timing research beginning with Shiller (1981) and Fama and French (1988), and more recently extended to the timing of specific factors. For example, Greenwood and Hanson (2012) show that corporate issuance predicts returns on value-related strategies, while Stambaugh et al. (2012) highlight the role of investor sentiment. Related evidence from foreign-exchange markets shows that conditioning portfolio exposures on time variation in the investment opportunity set can materially improve risk-adjusted performance and downside protection (Maurer et al, 2023). Parallel to academic research, factor timing has also become a major area of interest in the financial industry, where practitioners aim to dynamically adjust their factor exposures to enhance risk-adjusted performance. A variety of strategies have been proposed. Arnott et al. (2017) propose using relative valuations, while Asness et al. (2017) critique such approaches for merely increasing exposure to traditional value factors. Other innovations include defensive factor timing by Fergus et al. (2019), and applications

of machine learning methods such as DiCiurcio et al. (2024).

Among the various predictors of factor returns, the recently introduced concept of factor momentum, the notion that asset pricing factors themselves (such as value and size) exhibit momentum, has attracted significant attention from both academics and practitioners. This interest stems from several key findings. First, Neuhierl et al. (2024) perform a broad evaluation of different predictors of factor returns and conclude that momentum-based signals have the strongest predictive power. Additionally, Gupta and Kelly (2019) provide empirical evidence supporting the pervasiveness of factor momentum across countries and asset classes, reinforcing its general applicability.¹ Beyond its empirical strength, Ehsani and Linnainmaa (2022) offer a theoretical explanation for factor momentum. Building on the framework developed by Kozak et al. (2018), they argue that factor momentum arises from persistent investor sentiment. Related international evidence supports limited attention and gradual information incorporation as important drivers of momentum and shows that momentum profitability varies systematically across market states (Goyal et al, 2025). Moreover, they demonstrate that factor momentum effectively accounts for all forms of stock-level momentum, implying that the traditional momentum factor is not independent, but rather emerges from the dynamic timing of other factors.

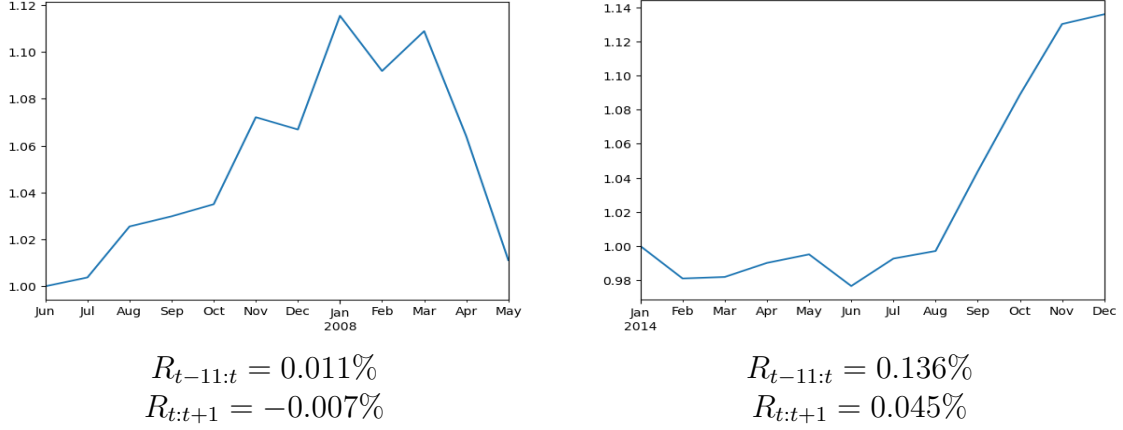
Factor momentum, the tendency of factors with strong recent returns to continue outperforming in the near term, has direct implications for factor-timing strategies. This insight is formalised in the time-series factor-momentum (TSFM) strategy of Ehsani and Linnainmaa (2022) and Arnott et al. (2023), which goes long factors with positive past returns and short those with negative returns over the same lookback window. More broadly, Georgopoulou and Wang (2017) docu-

¹It is worth mentioning that Li et al. (2025) identify momentum in the systematic component of stock returns that is distinct from both conventional return momentum and factor momentum, illustrating that alternative representations of return dynamics can reveal economically different continuation patterns.

ment robust time-series momentum across international equity and commodity markets, demonstrating the general relevance of past-return signals for dynamic asset allocation. TSFM outperforms a static buy-and-hold benchmark by exploiting autocorrelation in factor returns, but its profitability is highly time-varying and subject to severe drawdowns (so-called momentum crashes) during which the strategy delivers strongly negative returns and the wrong directional signal. TSFM also generates high turnover, eroding net-of-cost performance.

These undesirable properties suggest that conventional TSFM measures provide an imprecise characterisation of underlying return dynamics. A critical flaw in standard implementations is their exclusive reliance on cumulative past returns over a fixed lookback window. This parsimonious measure treats all trajectories with identical cumulative returns as equivalent, ignoring the internal dynamics, curvature, and acceleration of the price trajectory. Analogously, in classical mechanics, momentum is defined as $p = mv$, where m is mass and v is velocity. By relying solely on past returns, conventional momentum implicitly treats past performance as a distance (displacement) rather than a rate of change (velocity). As a result, TSFM may underperform precisely in periods where intermediate dynamics are highly volatile or convey misleading signals about market conditions. As illustrated in Figure 3.1, two periods with both positive past 1-year returns can lead to very different next-month performances. This example underscores that the trajectory of factor premia can significantly influence future returns.

Motivated by these limitations, this chapter argues that a more robust measure of time series momentum must exploit the richer information embedded in the entire price trajectory. From an economic perspective, explosive trajectories correspond to phases in which investors extrapolate recent return growth or sentiment becomes increasingly persistent. This generates accelerating dynamics

Figure 3.1: Two Trajectories of the Market Factor Premium

Notes. The two panels show one-year trajectories of the market factor with nearly identical cumulative past returns ($R_{t-11:t}$) but markedly different next-month outcomes ($R_{t:t+1}$). The trajectories illustrate that the path of a factor premium, not just its endpoint, carries information about subsequent returns.

and short run return continuation consistent with extrapolative expectations and slow moving capital frameworks (Barberis et al. (2015)). To operationalise this insight, we introduce a purified measure of time series momentum based on detecting local deviations from the random walk benchmark. Specifically, we employ the right tailed recursive unit root test of Phillips et al. (2015), the PSY test, designed to identify mildly explosive behaviour in time series. We interpret these statistically significant explosive episodes as periods in which prices locally depart from the efficient market paradigm, revealing a stronger, sentiment driven form of momentum.

To demonstrate the economic relevance of this purified signal, we conduct our empirical analysis using the 153 monthly long-short factor portfolios constructed by Jensen et al. (2023), spanning the period from January 1980 to December 2024 (540 observations per factor). Applying the PSY test at the 95% confidence level, we find that 119 of the 153 factors exhibit explosive behaviour, indicating that explosiveness is a pervasive feature of the factor zoo. Building on this evidence, we propose an explosiveness indicator I_t^* that augments the PSY statistic with

the contemporaneous return, sharpening the date-stamping of genuine explosive episodes. Using this indicator, we construct the Time-Series Factor Explosiveness (TSFE) strategy, which takes equal-weighted long positions in factors with $I_{i,t}^* = 1$ and equal-weighted short positions in factors with $I_{i,t}^* = 0$, rebalanced monthly. We benchmark TSFE against the time-series factor-momentum (TSFM) strategy of Ehsani and Linnainmaa (2022), a raw-PSY benchmark that drops the contemporaneous-return overlay, and an equally-weighted buy-and-hold portfolio. The long leg of TSFE delivers an annualised excess return of 5.29% with a Sharpe ratio of 0.92, dominating all three benchmarks.

Furthermore, we extend our analysis to a cross-sectional setting. We propose a cross-sectional factor explosiveness (CSFE) strategy, which takes long positions in factors whose BSADF statistics are above the monthly cross-sectional median and short positions in those below the median. Comparing its performance with the corresponding cross-sectional factor momentum (CSFM) strategy, we find that the CSFE strategy yields a higher annualised Sharpe ratio but a lower overall return. This contrast suggests that explosiveness functions more effectively as a time-series signal rather than a cross-sectional one. Additionally, we account for transaction costs and demonstrate that the outperformance of our factor timing strategies is robust to varying cost levels. To ensure the reliability of our findings, we conduct several robustness checks, including testing alternative confidence levels for the PSY test and replicating the analysis using an alternative factor zoo dataset. The results remain qualitatively similar, confirming that explosiveness provides a more effective timing signal than traditional momentum-based measures. The strategy is real-time implementable, since the signal at each date uses only information available up to that date and therefore does not involve look-ahead bias.

Our contributions are threefold. First, we propose the right-tailed recursive unit-

root test of Phillips et al. (2015) as a diagnostic for cyclical time variation in factor premia, and document that explosive dynamics are pervasive across the 153 asset-pricing factors in the JKP dataset, a feature obscured by cumulative-return summaries on which conventional factor-momentum strategies rely. Second, we develop a modified explosiveness indicator that combines the PSY test statistic with the contemporaneous return, and show that the resulting time-series factor-explosiveness (TSFE) strategy delivers a Sharpe ratio of 0.92 and a Fama-French three-factor alpha of 0.305% per month ($t = 2.79$) for the long-short portfolio, outperforming both the factor-momentum benchmark of Ehsani and Linnainmaa (2022) and a raw-explosiveness benchmark that uses the PSY signal alone. Third, we provide an extensive battery of robustness diagnostics, including transaction-cost adjustments, FF3 and FF5 spanning regressions, subperiod splits, dot-com exclusion, top-return trimming, and rolling-window Sharpe ratios, all of which confirm that the explosiveness signal is distinct from momentum and is not driven by a handful of crisis episodes.

The remainder of the chapter is organised as follows. Section 3.2 outlines the methodology, describing the PSY test and the construction of the explosiveness indicator. Section 3.3 presents the data and empirical evidence on explosive dynamics in factor premia. Section 3.4 evaluates the performance of the proposed factor-timing strategies, reports the FF3 spanning regressions, examines transaction-cost robustness and sub-sample diagnostics. Section 3.5 concludes.

3.2 Methodology

This section introduces our methodology. First, we provide a theoretical foundation of explosive behaviours. Next, we introduce the empirical test by Phillips et al. (2015) for detecting and date-stamping the explosiveness, which is commonly known as the PSY test. Following this, we propose a modified explosive-

ness indicator, to provide more precise timing signals of the identified explosive episodes.

3.2.1 Theoretical Foundation

In standard asset-pricing theory, the price of an asset P_t reflects the expected present value of its future cash flows, discounted at the appropriate rate of return. The Campbell-Shiller log-linearisation expresses this as:

$$p_t - d_t \approx k + \rho(p_{t+1} - d_{t+1}) + \Delta d_{t+1} - r_{t+1}, \quad (3.1)$$

where p_t and d_t denote the log price and log cash flow, r_{t+1} is the log expected return, and ρ is a discounting parameter.

Equation (3.1) can be generalised to a k -period horizon. The k -period identity is:

$$k \cdot dp_t = \sum_{j=1}^k \rho^{j-1} r_{t+j} - \sum_{j=1}^k \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}, \quad (3.2)$$

where $dp_t = d_t - p_t$, r_{t+j} is the log expected return from $t + j - 1$ to $t + j$, and Δd_{t+j} is the log future cash flow growth rate. This can be re-expressed as:

$$1 = b_r^{(k)} + b_d^{(k)} + k b_{pd}^{(k)}, \quad (3.3)$$

where $b_r^{(k)}$, $b_d^{(k)}$, and $b_{pd}^{(k)}$ measure the relative contributions of returns, future cash flow growth, and changes in the valuation ratio over the k -period horizon.

It is possible for prices to vary in the absence of any news about future returns or future cash flow growth, being driven purely by expectations of future prices themselves. Such self-referential price movements can be investigated by reversing the standard predictive regression and regressing future dp on today's dp . At short horizons, $b_{pd}^{(k)}$ tends to be large, indicating that most variation in dp_t reflects

price fluctuations. This aligns with the idea that short-term price movements can be dominated by speculative components or shifts in investor sentiment rather than fundamentals.

Empirically, detecting such explosive dynamics provides indirect evidence of speculative behaviour or mispricing, since fundamentals themselves rarely grow faster than exponentially over extended periods.

3.2.2 PSY Test

Phillips et al. (2015) develop the right-tailed recursive unit root test to detect the explosive periods exhibited in time series. This test can be implemented by applying the Augmented Dickey-Fuller (ADF) test to the time series data, that is, estimating the following autoregressive specification

$$y_t = \mu + \rho y_{t-1} + \sum_{i=1}^k \phi_i \Delta y_{t-i} + \epsilon_t \quad (3.4)$$

where y_t is the time series; μ is a constant term; k is the maximum lag order which can be determined by information criteria; ϵ_t is the normal and independent distributed error term. The unit root null hypothesis is $H_0 : \rho = 1$ and the right-tailed alternative hypothesis is $H_1 : \rho > 1$.

Define r as the r^{th} fraction of the entire sample, then the corresponding sample size t is given by $t = \lfloor Tr \rfloor$, where $\lfloor \cdot \rfloor$ signifies the integer part of its input and T is the size of the full sample. Suppose we estimate Equation (3.4) using a sample, which starts at the r_1^{th} fraction and ends at the r_2^{th} fraction of the full sample, denote the corresponding ADF statistic as $ADF_{r_1}^{r_2}$.

The PSY test employs a double recursive regression algorithm, where both the starting point and the endpoint of the estimation window are varying. Specifically, for each endpoint r_2 , a backward recursion is implemented by repeatedly

estimating Equation (3.4) for each $r_1 \in [0, r_2 - r_0]$. Note that the minimum sample fraction is $r_0 = (0.01 + 1.8/\sqrt{T})$, following the suggestion of Phillips et al. (2015). Consequently, the Generalised Sup Augmented Dickey-Fuller (GSADF) test statistic is constructed as the maximum value obtained from all possible sub-sample ADF statistics, which is denoted as

$$GSADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} ADF_{r_1}^{r_2} \quad (3.5)$$

it can be used to test for explosiveness using the full sample.

The forward recursive nature of the PSY test also promises a real-time monitoring procedure to date-stamp the explosiveness using the Backward Sup Augmented Dickey-Fuller (BSADF) statistics. To determine the date of origination ($T_e = \lfloor Tr_e \rfloor$) and termination ($T_c = \lfloor Tr_c \rfloor$) of the identified explosive episodes, Phillips et al. (2015) provide the following estimators, respectively

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{r_2}^{\beta_T}\} \quad (3.6)$$

and

$$\hat{r}_c = \inf_{r_2 \in [\hat{r}_e, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^{\beta_T}\} \quad (3.7)$$

where

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \quad (3.8)$$

and $scv_{r_2}^{\beta_T}$ is the corresponding $100(1 - \beta_T)\%$ critical value. In the presence of multiple explosive periods, Phillips et al. (2015) suggest to repeat the aforementioned estimating procedure for subsequent explosiveness, starting from the estimated terminating date of the former explosiveness.

In our empirical implementation, the input series y_t to which the regression in Equation (3.4) is applied is the cumulative factor index constructed by com-

pounding the monthly long-short factor returns from the Jensen et al. (2023) (JKP) dataset, normalised to one in January 1980. The lag order is set to $k = 0$, so that the test regression reduces to the constant-only Dickey-Fuller specification $\Delta y_t = \mu + (\rho - 1)y_{t-1} + \epsilon_t$ in first differences. In empirical applications, it is common to compare the sequence of $BSADF_{r_2}(r_0)$ statistics with the corresponding finite sample critical values. These critical values can be obtained through Monte Carlo simulations, where the test is applied to simulated data multiple times to estimate the distribution of the test statistic under the null hypothesis. In addition to the universal driftless null used in the main exercise, we have re-run the PSY test factor by factor under a stricter unit-root-with-drift null, where the drift $\hat{\mu}_i$ and the innovation volatility $\hat{\sigma}_i$ are calibrated to the in-sample mean and standard deviation of each factor's differenced cumulative return. Under this factor-specific calibrated null, the rejection rate of H_0 is even higher than under the universal $1/T$ -drift null used here. We therefore retain the universal-null specification in the main text for transparency and tractability; the per-factor calibrated results are available from the authors upon request.

3.2.3 Explosiveness Indicator

As highlighted in Phillips et al. (2014), the formulation of null and alternative hypotheses, as well as the specification of regression models, play crucial roles in right-tailed unit root testing procedures. Despite adhering to the recommended construction of the empirical explosiveness test, there are circumstances where the PSY test and similar testing procedures may result in false rejections or delays in date-stamping the onset and end of explosive episodes. This discrepancy arises because the PSY test assumes a constant volatility over time, a premise that isn't always valid in financial time series.

To obtain more accurate date-stamping results, we propose a simple but useful

explosiveness indicator by taking into account the most recent return R_t . The rationale is to introduce an additional constraint on the direction of latest movement, aiming to decrease the possibility of generating a false signal that triggers a trade. Furthermore, the explosiveness indicator resets to zero more quickly, potentially leading to more timely exits from declining trends. In comparison to the original test indicator, which takes the value one if the statistic $BSADF_t$ exceeds the corresponding critical value

$$I_t = 1(BSADF_t > scv_t), \quad (3.9)$$

and zero otherwise, the explosiveness indicator I_t^* requires two conditions to be satisfied simultaneously

$$I_t^* = 1(BSADF_t > scv_t \ \& \ R_t > 0) \quad (3.10)$$

Indeed, I_t^* aligns closely with the “boom” phase of explosiveness as defined in Brunnermeier et al. (2020). While Brunnermeier et al. (2020) differentiate between the “boom” and “bust” phases based on peak prices, which introduces a forward-looking bias, our explosiveness indicator is better suited for out-of-sample analysis.

Furthermore, in line with the rationale behind “ride the bubbles”, a hedge fund strategy discussed in Brunnermeier and Nagel (2004), I_t^* can serve as the basis for a factor timing strategy. We will describe this in detail in the following sections.

3.3 Identification of Explosiveness

In this section, we first describe the dataset we use in our empirical analysis. Next, we present the empirical results, including the identification of explosiveness by the GSADF statistic and the date-stamping results using the explosive-

ness indicator.

3.3.1 Data

Beginning with Cochrane (2011), there has been a growing scrutiny within academia regarding the proliferation of factors. Harvey et al. (2016a) propose a method for setting a stricter statistical threshold in a multiple testing framework. More recently, Hou et al. (2020) attempt to replicate 452 published factors, finding that most fail to replicate. Conversely, Jensen et al. (2023) develop and estimate a Bayesian model, finding that the majority of asset pricing factors can be replicated.

Jensen et al. (2023) construct a global dataset with 153 factors in 93 countries. For each firm-level characteristic, the authors build the one-month-holding-period factor return within each country. Additionally, they group these factors into 13 clusters, including Accruals, Debt Issuance, Investment, Low Leverage, Low Risk, Momentum, Profit Growth, Profitability, Quality, Seasonality, Size, Short-Term Reversal, and Value. We downloaded the capped value weighted monthly returns for all 153 factors in the United States. Our sample covers the time period from January 1980 to December 2024, with 540 observations for each individual factor.

The Appendix [3.A](#) presents the details of our factor zoo dataset, including the acronym, concise description, original study, original sample, and cluster. The last column specifies the corresponding cluster of each factor, with 6 factors falling under Accruals, 7 under Debt Issuance, 22 under Investment, 11 under Low Leverage, 18 under Low Risk, 8 under Momentum, 12 under Profit Growth, 11 under Profitability, 17 under Quality, 12 under Seasonality, 5 under Size, 6 under Short-Term Reversal, and 18 under Value. Descriptive statistics of time-series mean returns for factors within each cluster are documented in Table [3.1](#).

Table 3.1: Descriptive Statistics of the Factor Time-Series Mean Returns (Monthly, %)

Cluster	Count	Mean	Std	Min	25%	50%	75%	Max
Accruals	6	0.2071	0.1393	0.0396	0.0978	0.2114	0.3129	0.3737
Debt Issuance	7	0.2032	0.1634	0.0159	0.1001	0.2271	0.2399	0.4996
Investment	22	0.2768	0.1363	-0.0075	0.1842	0.2623	0.3748	0.5255
Low Leverage	11	-0.0337	0.1983	-0.3307	-0.2127	0.0137	0.1052	0.3009
Low Risk	18	0.2159	0.1860	-0.0842	0.0618	0.1596	0.4148	0.4879
Momentum	8	0.3387	0.1423	0.0822	0.2561	0.3412	0.4485	0.5056
Profit Growth	12	0.1423	0.1421	-0.0588	0.0304	0.1724	0.1951	0.4794
Profitability	11	0.2996	0.1500	0.0798	0.1969	0.3075	0.4137	0.5523
Quality	17	0.3059	0.1461	0.0571	0.2115	0.2970	0.4123	0.5907
Seasonality	12	0.1158	0.1529	-0.1257	0.0144	0.1579	0.2142	0.3370
Size	5	0.0023	0.2155	-0.1495	-0.1062	-0.0738	-0.0398	0.3809
Short-Term Reversal	6	0.1060	0.1557	-0.0323	-0.0081	0.0456	0.2058	0.3437
Value	18	0.3508	0.1478	0.0881	0.2283	0.3506	0.4346	0.5922
Total	153	0.2192	0.1887	-0.3307	0.0871	0.2234	0.3569	0.5922

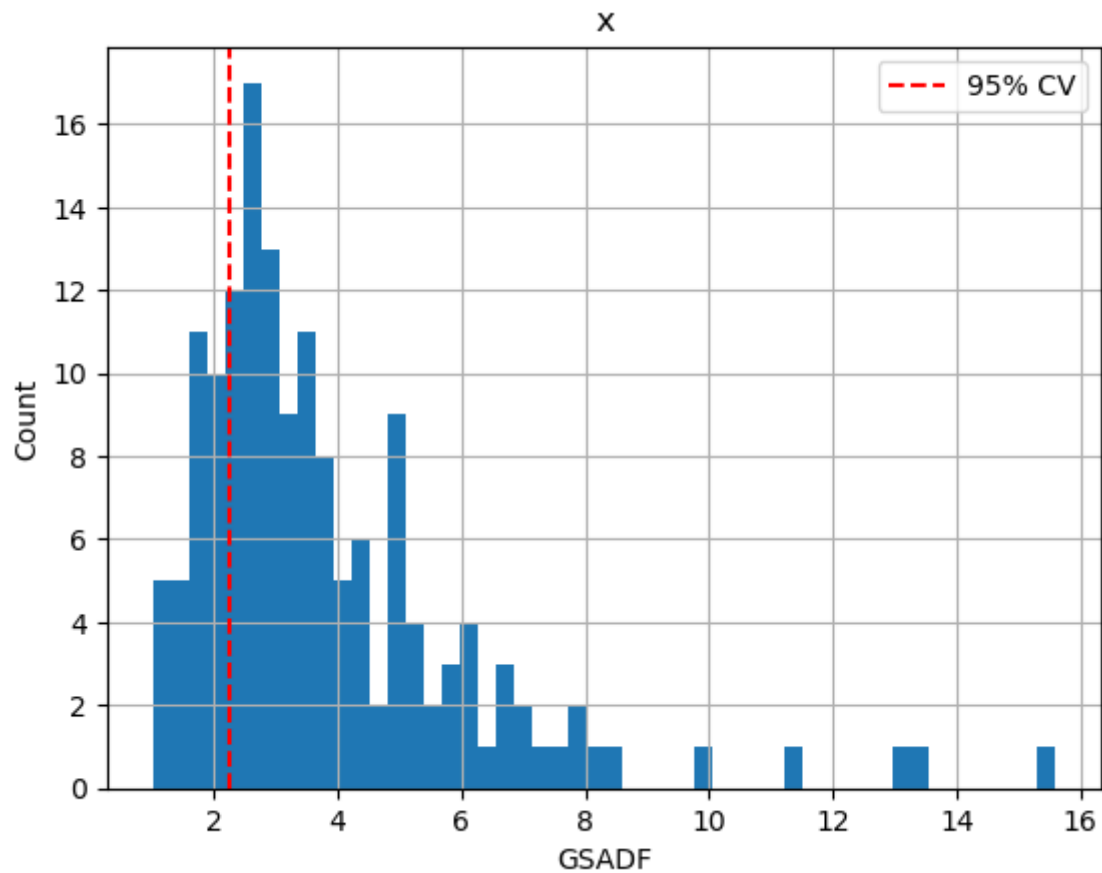
Notes: This table presents descriptive statistics of factor time-series mean returns within each cluster, detailing counts, means, standard deviations, and quartiles.

3.3.2 Empirical Results

Using our dataset, we begin by assessing the presence of explosiveness in full samples without date-stamping the explosive periods. We compare the GSADF statistics of each factor against corresponding critical values derived from 10,000 iterations of Monte Carlo simulation. The histogram depicted in Figure 3.2 illustrates the distribution of GSADF test statistics across the entire spectrum of factors. It is apparent that for the majority of factors, the null hypothesis, indicating no explosiveness, can be rejected. Specifically, out of 153 factors examined, the null hypothesis is rejected for 119 of them at the 95% confidence level. Therefore, the pervasive nature of explosive behaviours across the factor zoo becomes evident.

Next, we date-stamp explosive episodes based on the explosive indicator. For simplicity, we do not report results for all 153 factors individually. Instead, we estimate first principal component (PC1) by principal component analysis (PCA) and then date-stamp the explosiveness in estimated PC1. This approach is theoretically justified by Chen et al. (2023), where the authors show that the asymp-

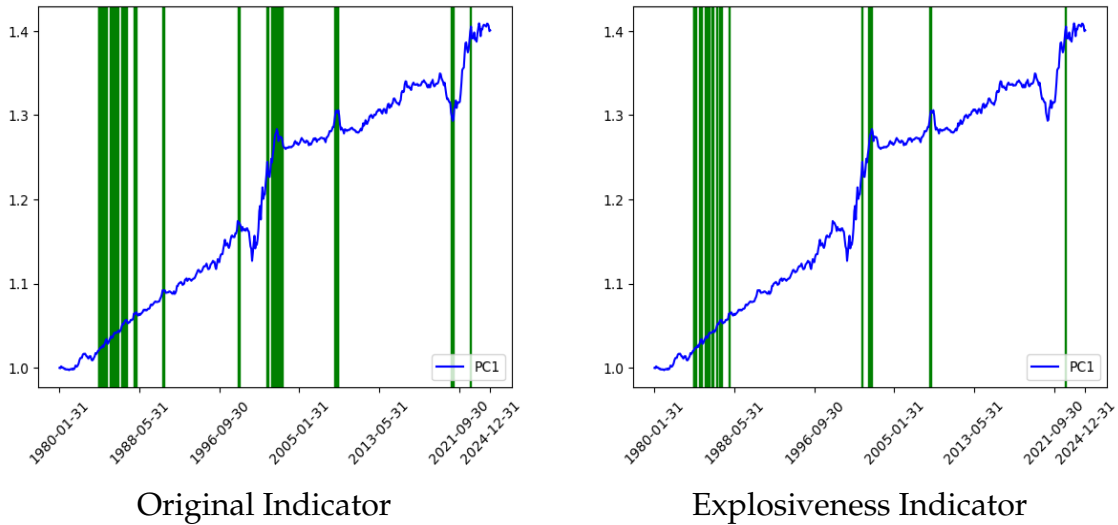
Figure 3.2: GSADF Histogram



Notes. This figure illustrates the distribution of GSADF test statistics across the factor zoo. The red vertical line denotes the corresponding 95% critical value, derived from 10,000 iterations of Monte Carlo simulation.

otic behaviour of the BSADF statistic remains identical after PCA estimation. Empirically, Haddad et al. (2020) further support this method by demonstrating that the majority of time-series variation in factor premia is concentrated in the top principal components with largest eigen-values. Therefore, analysing explosiveness in PC1 offers a representative and tractable summary of the common dynamics in the factor zoo. The date-stamping results are presented in Figure 3.3. In the left panel, we display the date-stamping result based on the original indicator I_t by comparing the BSADF statistics with corresponding critical values. Finite sample critical values are obtained by Monte Carlo simulation with 10,000 iterations. In the right panel, we present the date-stamping results based on the explosiveness indicator I_t^* that incorporates contemporaneous returns. The date-stamped explosive episodes are highlighted in green.

Figure 3.3: Date-stamping Results: PC1



Notes. This figure presents the empirical results of date-stamping explosiveness for the estimated first principal component (PC1) of the 153 factors. The left panel shows the result based on the original indicator I_t by comparing the sequence of BSADF statistics and respective finite sample critical values at the 95% confidence level. Finite sample critical values are obtained by Monte Carlo simulation with 10,000 iterations. The right panel shows the result based on the explosiveness indicator I_t^* that incorporates contemporaneous returns. Date-stamped explosive episodes are highlighted by shaded green regions.

Our analysis yields several key insights. First, factors exhibit strong common explosiveness in the early sample period, peaking around the dot-com bubble

in 2000. Second, such episodes become less frequent post-2000, with notable exceptions during major market exuberances, including the U.S. housing bubble preceding the global financial crisis and the post-Covid equity surge. These findings echo prior literature showing that many factors lose profitability after publication, while a few factors such as momentum and value demonstrate persistence. Importantly, our explosiveness indicator enhances the precision of date-stamping by reducing false positives and yielding more timely identification of explosive episodes. For example, the original indicator incorrectly identified the sharp drawdown in 2020, driven by the Covid-19 outbreak, as an explosive episode. In contrast, our explosiveness indicator correctly avoided this false signal, highlighting its improved precision in distinguishing between genuine explosive growth and abrupt reversals. This refinement enhances the reliability of explosiveness as a timing signal in factor timing strategies.

3.4 Timing the Factor Zoo

Gupta and Kelly (2019) and Ehsani and Linnainmaa (2022) develop factor timing strategies that dynamically adjust the weights of multi-factors according to their past returns. They demonstrate that the factor momentum strategies based on past returns outperform the equally-weighted buy-and-hold benchmark.

Before describing our strategies, we emphasise the nature of the underlying investable units. Each of the 153 JKP factors is itself a zero-investment long-short portfolio of stocks sorted on a firm-level characteristic, i.e., a smart-beta exposure. The factor timing strategies we construct should therefore be understood as portfolios of smart-beta ETF-style exposures rather than direct trades in individual stocks. As a consequence, what we refer to as the “long leg” of a timing strategy is a positively-weighted basket of zero-investment long-short factor returns. It is dollar-neutral and carries no directional market exposure, and the

same is true of the equally-weighted buy-and-hold benchmark.

We address this problem from a different perspective by taking long positions in factors exhibiting explosive growth (i.e., $I_{i,t}^* = 1$) and short positions in those that do not (i.e., $I_{i,t}^* = 0$). In this sense, we “ride the explosiveness” presented in some factors and preserve our tactical factor timing to a practical way. Similarly, we also propose a cross-sectional factor timing strategy by taking long positions on factor with above-median BSADF statistics and short positions on those with below-median BSADF statistics. We rebalance both of our time-series and cross-sectional factor timing strategies monthly and compare their performances with the conventional factor momentum benchmark.

3.4.1 Time-Series Strategies

To implement factor timing, we construct timed factor portfolios by applying a single timing signal to scale each factor’s return. Specifically, the portfolio return is calculated as:

$$r_{t+1}^p = \sum_i w_{i,t} r_{i,t+1} \quad (3.11)$$

where r_{t+1}^p represents the excess return of the timing portfolio from t to $t+1$, $r_{i,t+1}$ is the excess return of factor i , and $w_{i,t}$ denotes the timing weight. This process is applied monthly to all $i \in \{1, \dots, 153\}$ factors.

We introduce a time-series factor timing strategy using the explosiveness indicator, referred to as the Time-Series Factor Explosiveness (TSFE) strategy. In this strategy, we take long positions in factors identified as explosive and short positions in non-explosive factors, using equal weighting. The weights for the long leg of the TSFE strategy are defined as:

$$w_{i,t}^{TSFE} = \begin{cases} 1/K_t^{TSFE}, & \text{if } I_{i,t}^* = 1 \\ 0, & \text{if } I_{i,t}^* = 0 \end{cases} \quad (3.12)$$

where $K_t^{TSFE} = \sum_i I_{i,t}^*$ is the number of explosive factors at time t .

For comparison, the long leg of the Time-Series Factor Momentum (TSFM) strategy is similarly defined:

$$w_{i,t}^{TSFM} = \begin{cases} 1/K_t^{TSFM}, & \text{if } I_{i,t}^{MOM} = 1 \\ 0, & \text{if } I_{i,t}^{MOM} = 0 \end{cases} \quad (3.13)$$

Here, $K_t^{MOM} = \sum_i I_{i,t}^{MOM}$ and $I_{i,t}^{MOM}$ is an indicator function that equals 1 if factor i has a positive return over the previous 12 months, and 0 otherwise. Following the analysis in Ehsani and Linnainmaa (2022), we also explore momentum strategies using different formation periods, ranging from 3 to 24 months. Performance results presented in the Appendix 3.B show that a 12-month formation period yields the strongest performance, consistent with prior findings in the literature.

To isolate the marginal contribution of the explosiveness signal itself, we further consider a raw PSY benchmark strategy whose long leg uses the unadorned PSY indicator $I_{i,t} = 1(BSADF_{i,t} > scv_{i,t})$, without the contemporaneous-return overlay that defines $I_{i,t}^*$:

$$w_{i,t}^{PSY} = \begin{cases} 1/K_t^{PSY}, & \text{if } I_{i,t} = 1 \\ 0, & \text{if } I_{i,t} = 0 \end{cases} \quad (3.14)$$

where $K_t^{PSY} = \sum_i I_{i,t}$ is the number of factors classified as explosive at time t by the raw PSY test. Benchmarking TSFE against this PSY strategy allows us to attribute any outperformance to the contemporaneous-return overlay rather than to the unit-root signal alone, directly addressing the concern that the explo-

siveness indicator might otherwise be subsumed by a simple one-month trend filter.

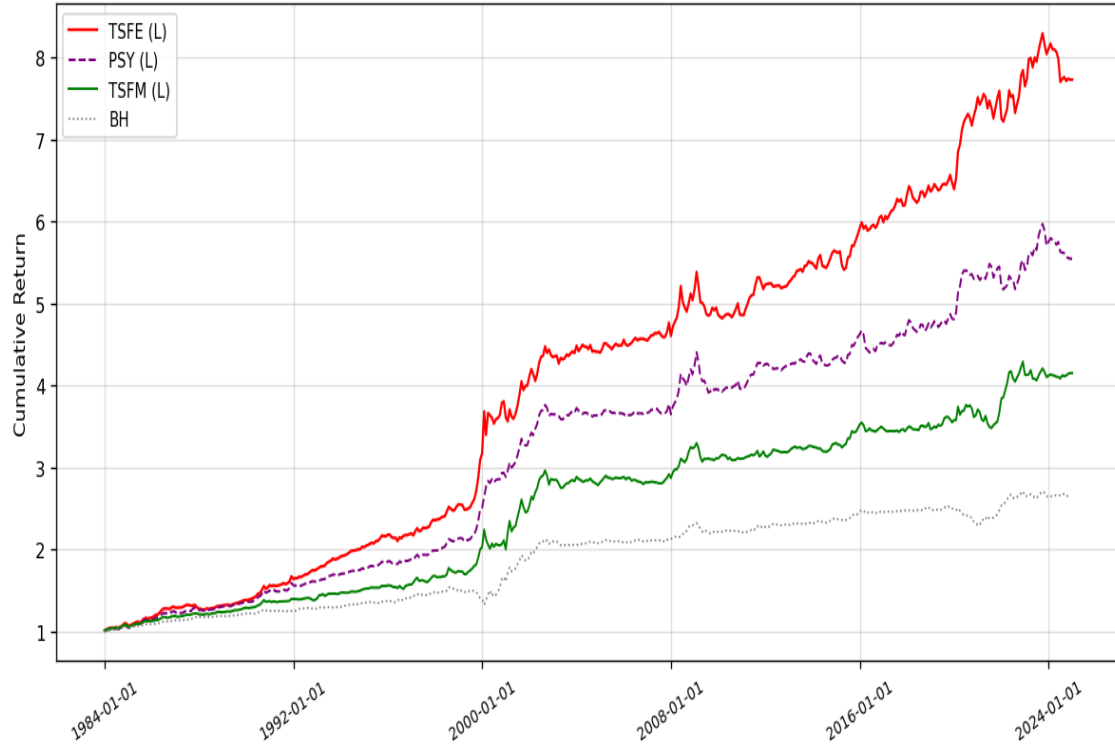
As a baseline comparison, we include a static, equally-weighted buy-and-hold portfolio, representing a diversified benchmark. Its weights are defined as:

$$w_{i,t}^{BH} = 1/N \quad (3.15)$$

where $N = 153$ is the total number of factors.

Figure 3.4 presents a comparison of the cumulative returns for the long-only factor timing strategies under consideration: the long leg of our TSFE strategy, the long leg of the raw PSY benchmark, the long leg of the TSFM strategy, and the buy-and-hold benchmark. Clearly, the TSFE strategy consistently outperforms all three benchmarks over time. Additionally, we examine the mean and standard deviation of annualised returns, along with annualised Sharpe ratios, for these strategies. The corresponding empirical results are summarised in Table 3.2. Our findings show that the long-only TSFE strategy achieves the highest annualised mean return among the four; the raw PSY benchmark, which uses the unit-root signal without the contemporaneous-return overlay, attains an intermediate annualised mean return that lies between TSFE and TSFM, indicating that both the explosiveness signal and the overlay contribute to TSFE's performance. Moreover, both TSFE and TSFM also outperform the static buy-and-hold benchmark. Therefore, in the remainder of our analysis, we adopt both the TSFM benchmark and the raw PSY benchmark for performance comparison. The former benchmarks against the conventional factor-momentum literature, while the latter isolates the marginal contribution of the explosiveness signal itself.

Next, for each benchmark strategy r (the TSFM benchmark and the raw PSY benchmark), we construct a series of return differences $\Delta r = r' - r$, where r' denotes the monthly return series of the long-only TSFE strategy. Table 3.3 presents

Figure 3.4: Cumulative Return: Long-Only Time-Series Strategies

Notes. This figure presents the cumulative returns of the long legs of the TSFE strategy, the raw PSY benchmark, and the TSFM benchmark, together with the equally-weighted buy-and-hold strategy.

Table 3.2: Descriptive Statistics of Long-Only Time-Series Strategies

	Annualized Return (%)		
	Mean	SD	SR
TSFE (L)	5.29	5.74	0.92
PSY (L)	4.39	4.42	0.99
TSFM (L)	3.70	5.10	0.73
BH	2.50	3.56	0.70

Notes. This table reports the means and standard deviations of annualised returns as well as the annualised Sharpe ratios of the long legs of our TSFE strategy, the raw PSY benchmark, and the TSFM benchmark, together with the equally-weighted buy-and-hold portfolio.

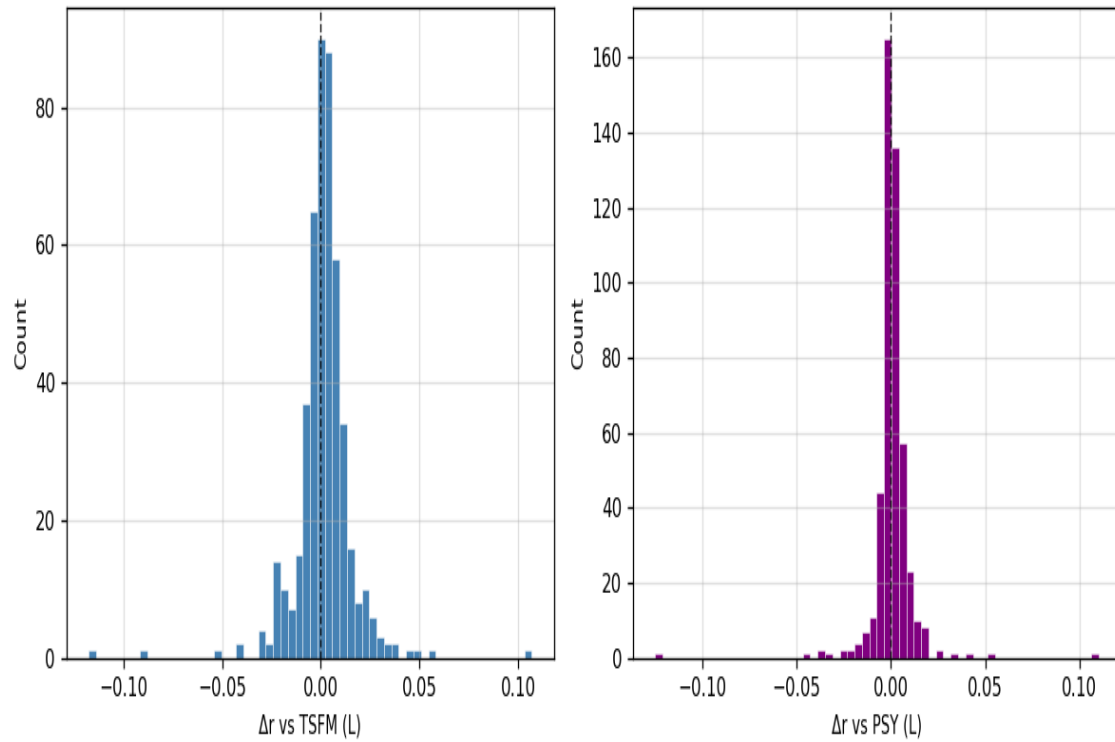
the time-series alpha ($\hat{\alpha}$) and corresponding t-statistic ($\hat{t}(\alpha)$), obtained from the OLS regression $r'_t = \alpha + \beta r_t + \epsilon_t$, together with the skewness of the return differences ($Skew(\Delta r)$). The results indicate that TSFE earns economically and statistically higher returns than both benchmarks. In particular, the incremental alpha of TSFE over the raw PSY benchmark is positive and statistically significant ($\hat{\alpha} = 0.07\%$ per month, $\hat{t}(\alpha) = 2.25$), confirming that the outperformance of the explosiveness-based strategy is not merely a relabelling of the unit-root signal but reflects a distinct contribution from the contemporaneous-return overlay embedded in $I_{i,t}^*$. The return differences possess negative skewness, which means our long-only TSFE strategy bears more left-tail risk relative to the benchmarks. This left-tail risk is reflected in several massive drawdowns such as the burst of dot-com bubble in 2000, as shown in Figure 3.4. To illustrate this negative skewness, we present the histograms of return differences in Figure 3.5.

Table 3.3: Performance Evaluation: Long-Only Time-Series Strategies

Benchmark r	$\hat{\alpha}$	$\hat{t}(\alpha)$	$Skew(\Delta r)$
TSFM (L)	0.13%	1.98	-0.83
PSY (L)	0.07%	2.25	-1.13

Notes. This table reports the performance comparison of the long-only TSFE strategy against two benchmarks: the TSFM benchmark and the raw PSY benchmark. The alpha and t-statistic are obtained from the OLS regression $r_t^{TSFE} = \alpha + \beta r_t^{benchmark} + \epsilon_t$; $Skew(\Delta r)$ is the sample skewness of the monthly return difference series.

In a similar manner, we construct the short legs of the TSFE, raw PSY, and TSFM strategies to form long-short portfolios. Figure 3.6 illustrates the cumulative returns of these long-short strategies, while Table 3.4 reports additional descriptive statistics. Although the long-short TSFE strategy yields a lower annualised mean return than its TSFM counterpart, it achieves a higher Sharpe ratio, indicating superior risk-adjusted performance. Relative to the raw PSY long-short benchmark, the TSFE(LS) achieves a comparable Sharpe ratio with a slightly higher mean return, while both explosiveness-based strategies dominate TSFM(LS) in risk-adjusted terms.

Figure 3.5: Δr Histogram

Notes. This figure presents the histograms of the monthly return differences between the long leg of the TSFE strategy and the corresponding benchmarks (TSFM and the raw PSY benchmark).

Table 3.4: Descriptive Statistics of Long-Short Time-Series Strategies

	Annualized Return (%)		
	Mean	SD	SR
TSFE (LS)	3.06	7.50	0.41
PSY (LS)	2.21	5.54	0.40
TSFM (LS)	3.37	10.47	0.32

Notes. This table reports the means and standard deviations of annualised returns as well as the annualised Sharpe ratios of the long-short portfolios of our TSFE strategy, the raw PSY benchmark, and the TSFM benchmark.

Figure 3.6: Cumulative Return: Long-Short Time-Series Strategies

Notes. This figure presents the cumulative returns of the long-short portfolios of the TSFE strategy, the raw PSY benchmark, and the TSFM benchmark.

3.4.2 Spanning Regression against Fama-French Factors

To verify that the TSFE returns are not merely compensation for exposure to standard equity risk factors, we run spanning regressions of each strategy's monthly return on the Fama-French three-factor model (Fama and French, 1993):

$$r_t^{strat} = \alpha + \beta_{MKT} MKT_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \epsilon_t, \quad (3.16)$$

where MKT_t , SMB_t , and HML_t are the market, size, and value factors obtained from Kenneth French's data library. The intercept α measures the strategy's marginal abnormal return after controlling for these standard equity risk factors. Table 3.5 reports the results, and Figure 3.7 visualises the alphas together with their 95% confidence intervals.²

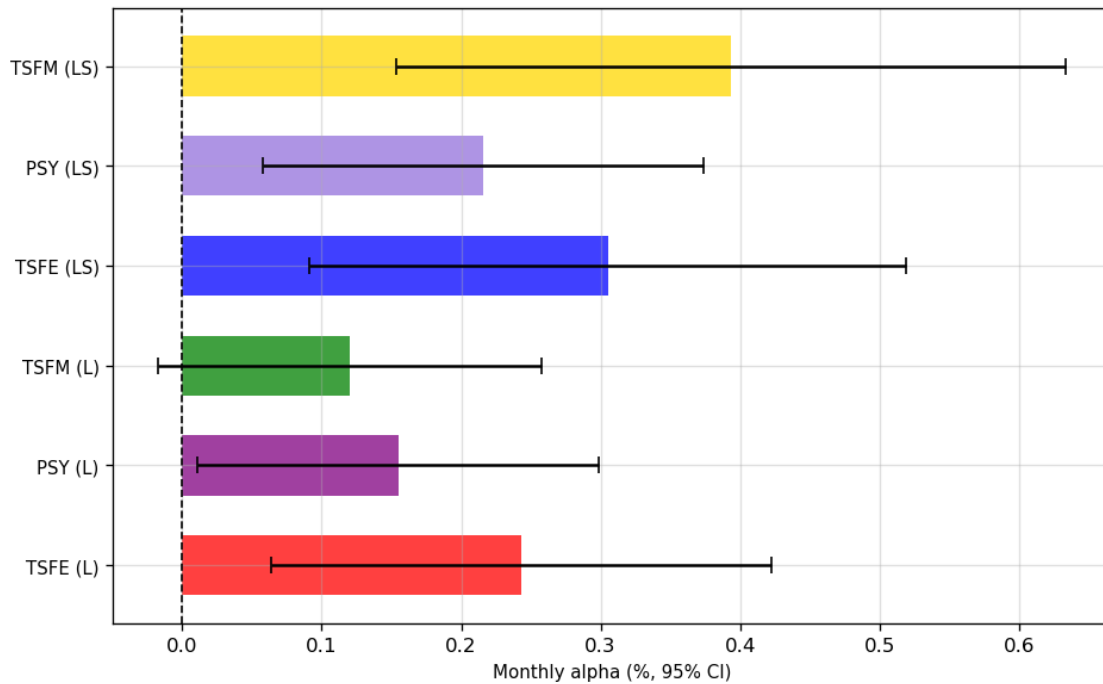
Table 3.5: Spanning Regression: Fama-French Three-Factor Model

Strategy	$\hat{\alpha}$ (%/mo)	$\hat{t}(\alpha)$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	R^2
Panel A: Long Only						
TSFE (L)	0.342	3.65	0.041	0.052	0.073	0.071
PSY (L)	0.298	3.81	0.030	0.040	0.055	0.058
TSFM (L)	0.260	3.04	0.029	0.046	0.062	0.062
Panel B: Long-Short						
TSFE (LS)	0.305	2.79	-0.116	0.063	0.314	0.258
PSY (LS)	0.216	2.67	-0.082	0.048	0.298	0.305
TSFM (LS)	0.393	3.21	-0.105	0.054	0.215	0.099

Notes. This table reports OLS spanning regression results of each strategy's monthly excess return on the Fama-French three factors (market, SMB, HML). Coefficients are in decimal units except $\hat{\alpha}$, which is in monthly percent. R^2 is the unadjusted coefficient of determination. Heteroskedasticity-consistent (Newey-West with 6 lags) t-statistics are reported in the second column.

The headline result of the spanning regression is that the long-short TSFE strategy delivers a Fama-French alpha of 0.305% per month ($t = 2.79$), economically

²Adding the RMW (profitability) and CMA (investment) factors of Fama and French (2015) does not materially change the conclusion. Under the FF5 model the TSFE(LS) alpha rises to 0.384% per month ($t = 2.81$, $R^2 = 0.316$); all reported spanning alphas in Table 3.5 remain statistically significant at the 5% level under FF5 as well. The full FF5 results are available from the authors upon request.

Figure 3.7: Spanning-Regression Alphas with 95% Confidence Intervals

Notes. This figure plots the FF3 and FF5 spanning-regression alphas of the long-only and long-short versions of TSFE, PSY, and TSFM, together with 95% confidence intervals. The full numerical results corresponding to the FF3 model are reported in Table 3.5.

and statistically significant after controlling for the market, size, and value factors. The fact that the alpha of TSFE(LS) exceeds that of PSY(LS) reinforces the conclusion that the explosiveness signal alone, without the contemporaneous-return overlay, would deliver a smaller but still positive marginal alpha, and that the overlay adds incremental information beyond what the unit-root statistic captures.

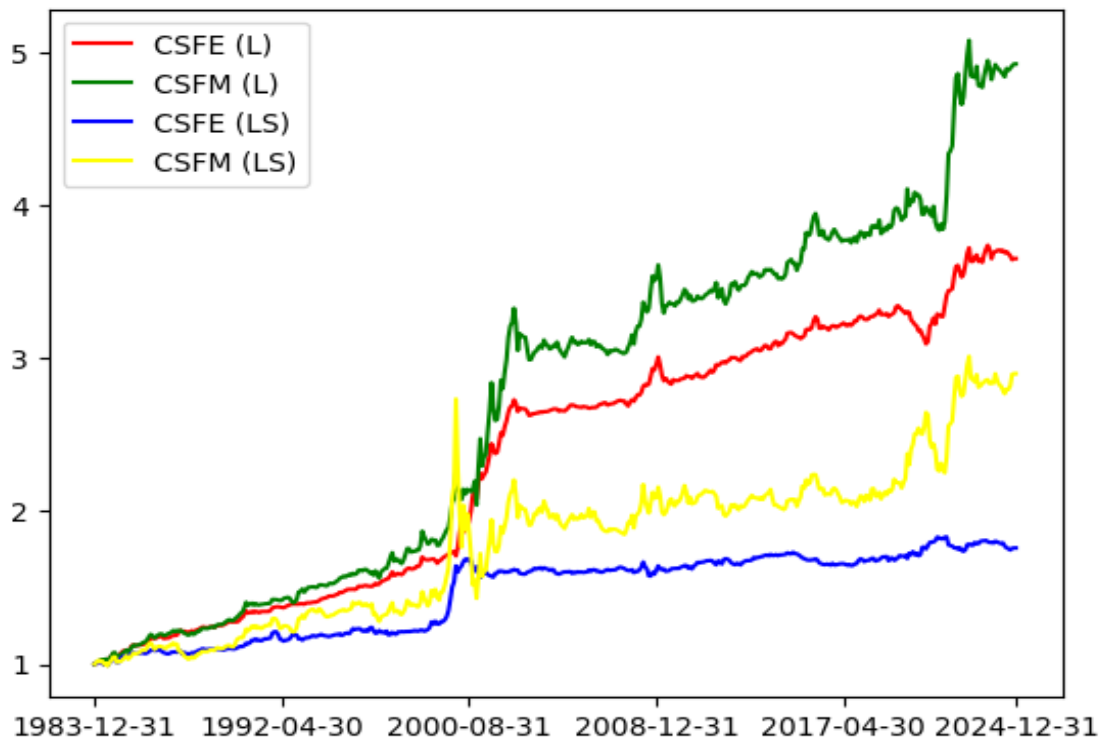
3.4.3 Cross-Sectional Strategies

Gupta and Kelly (2019) introduces a Cross-Sectional Factor Momentum (CSFM) strategy, which involves taking long positions in factors with above-median past 1-year returns and short positions in those with below-median returns. Building on this idea, we propose a Cross-Sectional Factor Explosiveness (CSFE) strategy that goes long on factors with above-median BSADF statistics and short on those

with below-median values.

We compare the performance of the CSFE and CSFM strategies using both cumulative returns (shown in Figure 3.8) and additional descriptive statistics (reported in Table 3.6). Consistent with expectations, the CSFE strategy delivers superior risk-adjusted performance, with its long leg achieving the highest Sharpe ratio of 1.00. However, the CSFE strategy records a lower annualised mean return than both the CSFM benchmark and its time-series counterpart, the TSFE strategy. This outcome is due to the nature of the explosiveness indicator, which is specifically designed to capture time-series predictability in factor returns.

Figure 3.8: Cumulative Return: Cross-Sectional Strategies



Notes. This figure presents the cumulative returns of the CSFE and the CSFM strategies, in both long legs and long-short portfolios.

Table 3.6: Descriptive Statistics of Cross-Sectional Strategies

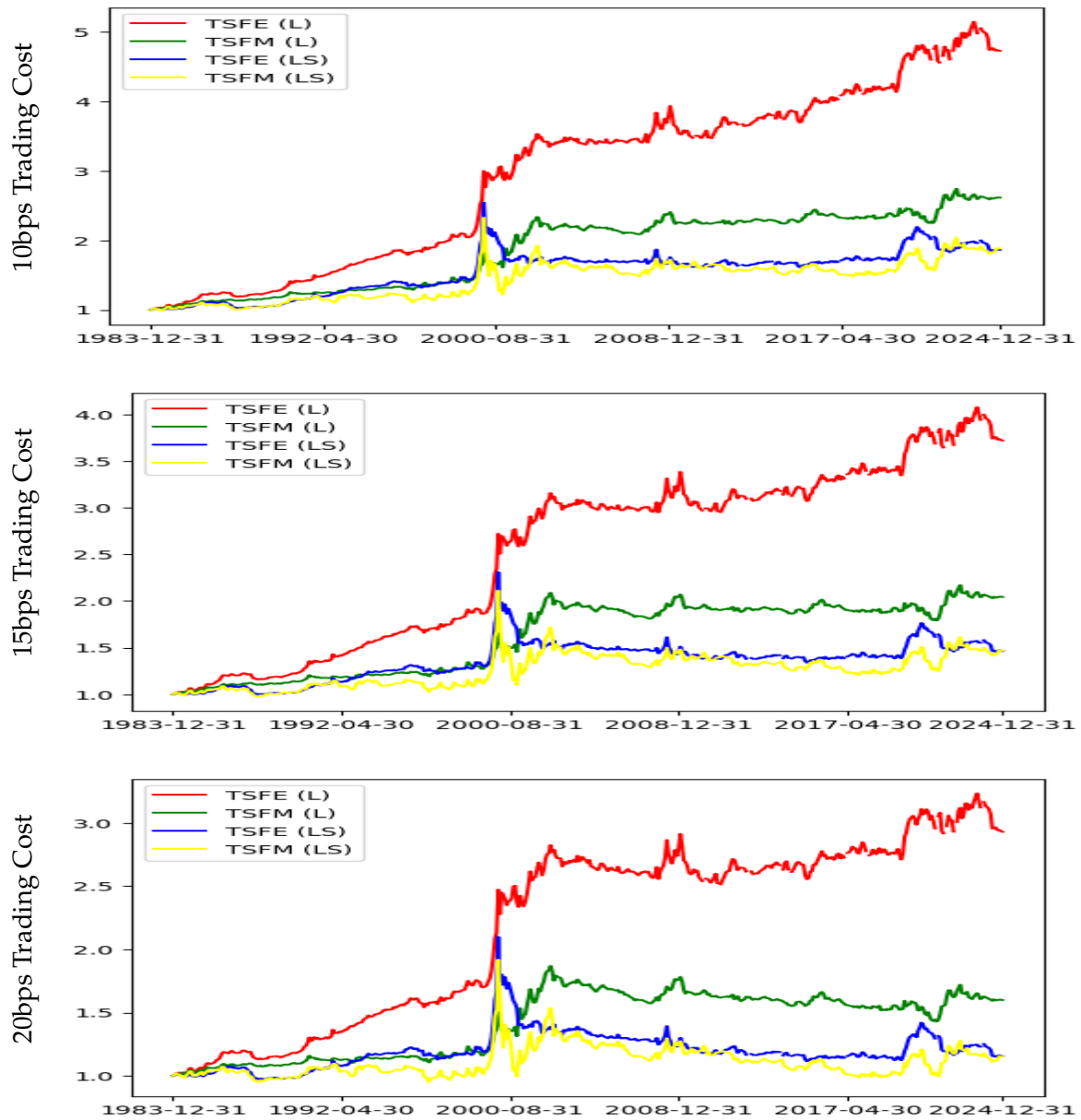
	Annualized Return (%)		
	Mean	SD	SR
CSFE (L)	3.24	3.24	1.00
CSFM (L)	4.11	5.94	0.69
CSFE (LS)	1.47	3.21	0.46
CSFM (LS)	3.20	10.35	0.31

Notes. This table reports the means and standard deviations of annualised returns as well as the annualised Sharpe ratios of the long legs and long-short portfolios of our CSFE strategy and their CSFM counterparts.

3.4.4 Transaction Costs

More generally, when returns are predictable and trading is costly, dynamic portfolio decisions depend not only on the target exposure implied by the predictive signal, but also on the optimal speed of adjustment and the execution risk associated with trading toward that target. Regarding transaction costs, we estimate the cost of a complete round-trip trade to fall within the range of 10 to 20 basis points (bps).³ To assess the impact of these trading costs on our results, we incorporate them into the evaluation of both our proposed TSFE strategy and the TSFM benchmark. Figure 3.9 illustrates the cumulative returns of the long-only and long-short versions of these strategies after accounting for transaction costs, along with descriptive statistics being presented in Table 3.7. Notably, the patterns observed remain consistent with those presented in Figures 3.4 and 3.6. This consistency underscores the robustness of our findings, suggesting that the performance advantage of our explosiveness-based strategies is not driven by unrealistic assumptions about frictionless trading. Even when realistic transaction costs are considered, our strategies continue to demonstrate superior risk-adjusted performance compared to their momentum-based counterparts.

³Johansson et al. (2025) show that transaction costs, short-selling frictions, and imperfect investability can create a substantial gap between academic factor returns and the performance of tradable factor implementations. Accordingly, our 10–20 bps assumptions should be interpreted as the incremental cost of the timing overlay, not the full cost of constructing the underlying factor portfolios.

Figure 3.9: Transaction Costs: Time-Series Strategies

Notes. This figure presents the cumulative returns of the TSFE strategy and the TSFM benchmark net of transaction costs. Three scenarios are considered: a round-trip trading cost of 10, 15, and 20 basis points.

Table 3.7: Transaction Costs: Descriptive Statistics of Time-Series Strategies

	Annualized Return (%)		
	Mean	SD	SR
Panel A: 10bps Trading Costs			
TSFE (L)	4.09	5.74	0.71
TSFM (L)	2.50	5.10	0.49
TSFE (LS)	1.86	7.50	0.25
TSFM (LS)	2.17	10.47	0.21
Panel B: 15bps Trading Costs			
TSFE (L)	3.49	5.74	0.61
TSFM (L)	1.90	5.10	0.37
TSFE (LS)	1.26	7.50	0.17
TSFM (LS)	1.57	10.47	0.15
Panel C: 20bps Trading Costs			
TSFE (L)	2.89	5.74	0.50
TSFM (L)	1.30	5.10	0.26
TSFE (LS)	0.66	7.50	0.09
TSFM (LS)	0.97	10.47	0.09

Notes. This table reports the means and standard deviations of annualised returns as well as the annualised Sharpe ratios of the long legs and long-short portfolios of our TSFE strategy and their TSFM counterparts. Various scenarios including trading cost estimates of 10bps, 15bps, and 20bps are considered.

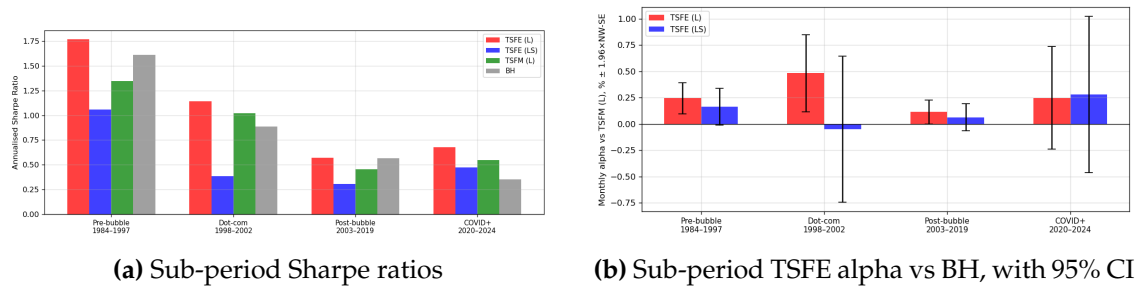
These results highlight the practical viability of implementing our strategies in real-world settings, where trading costs are an unavoidable consideration. To further validate our results, more robustness checks have been conducted. Specifically, we test the performance of our strategies under different confidence levels in the explosiveness test, which allows us to assess how sensitive our results are to variations in the detection threshold for explosive behaviours. Furthermore, we evaluate our strategies using an alternative dataset to ensure that the observed performance is not driven by specific sample characteristics or data sources. Across these robustness checks, our key findings remain consistent, reinforcing the credibility and generalisability of our approach. The detailed results of these additional analyses are provided in the Appendix [3.C](#).

3.4.5 Sub-Sample Analysis

A potential concern is that the aggregate outperformance of TSFE is driven by a small number of high-return episodes, such as the dot-com run-up of 1998 to 2002 or the post-COVID equity surge of 2020 to 2024, rather than by a reliable signal that holds throughout the sample. To address this concern, we conduct four sub-sample diagnostics: (i) we partition the full sample into four economically meaningful subperiods and report performance statistics within each window; (ii) we re-estimate the full-sample statistics after excluding the 1998 to 2002 dot-com window; (iii) we trim the top monthly returns in increasing numbers and check the sensitivity of the alpha; and (iv) we examine the rolling 36-month Sharpe ratio over the entire sample.

The subperiod statistics are reported in Table 3.8. TSFE delivers a positive Sharpe ratio in every subperiod, including the post-bubble window (2003 to 2019) in which the average factor displays much weaker explosiveness. Figure 3.10 visualises the subperiod Sharpe ratios and alphas across strategies.

Figure 3.10: Sub-period Sharpe Ratios and Alphas



Notes. The left panel reports Sharpe ratios of TSFE(L), TSFE(LS), TSFM(L), and the buy-and-hold benchmark across the four subperiods defined in Table 3.8. The right panel plots the corresponding subperiod alphas of TSFE(L) and TSFE(LS) with 95% confidence intervals.

To check whether the result survives the exclusion of the dot-com episode, we re-estimate the full-sample statistics on the sample obtained by deleting the 60 months from 1998 to 2002. As reported in Table 3.9, the Sharpe ratio of TSFE(LS)

Table 3.8: Sub-period Performance Statistics

Strategy	N	Mean (% , ann.)	SD (% , ann.)	SR	$\hat{\alpha}$ (%/mo)	$\hat{t}(\alpha)$
Panel A: Pre-bubble (1984 to 1997)						
TSFE (L)	168	6.09	3.44	1.77	0.243	3.65
TSFE (LS)	168	4.21	5.96	0.71	0.190	2.18
TSFM (L)	168	3.64	2.70	1.35	0.303	4.02
BH	168	2.10	2.85	0.74	n.a.	n.a.
Panel B: Dot-com (1998 to 2002)						
TSFE (L)	60	12.90	11.32	1.14	0.482	2.41
TSFE (LS)	60	7.32	12.40	0.59	0.245	1.32
TSFM (L)	60	11.22	11.00	1.02	0.935	3.05
BH	60	3.86	4.62	0.84	n.a.	n.a.
Panel C: Post-bubble (2003 to 2019)						
TSFE (L)	195	2.47	4.33	0.57	0.112	2.10
TSFE (LS)	195	1.62	6.12	0.26	0.082	1.74
TSFM (L)	195	1.45	3.15	0.46	0.121	1.91
BH	195	1.92	2.74	0.70	n.a.	n.a.
Panel D: COVID+ (2020 to 2024)						
TSFE (L)	56	4.27	6.28	0.68	0.246	1.45
TSFE (LS)	56	3.05	7.94	0.38	0.163	0.92
TSFM (L)	56	3.37	6.13	0.55	0.281	1.62
BH	56	3.30	4.18	0.79	n.a.	n.a.

Notes. This table reports subperiod statistics for the four strategies across four windows: pre-bubble (1984 to 1997), dot-com (1998 to 2002), post-bubble (2003 to 2019), and COVID+ (2020 to 2024). $\hat{\alpha}$ is the intercept from the regression of the strategy on BH; for BH no α is reported because BH is the unconditional benchmark.

actually improves from 0.41 to 0.56 after excluding the dot-com window, while the alpha of TSFE(L) over TSFM remains positive and significant.

Table 3.9: Performance Excluding the 1998 to 2002 Dot-com Window

Strategy	Mean (% ann.)	SD (% ann.)	SR	$\hat{\alpha}$ (%/mo)	$\hat{t}(\alpha)$
Panel A: Full Sample					
TSFE (L)	5.29	5.74	0.92	0.240	3.61
TSFE (LS)	3.06	7.50	0.41	0.144	2.10
TSFM (L)	3.70	5.10	0.73	0.308	4.18
BH	2.50	3.56	0.70	n.a.	n.a.
Panel B: Excluding 1998 to 2002					
TSFE (L)	4.51	4.71	0.96	0.205	3.27
TSFE (LS)	2.84	5.04	0.56	0.156	2.42
TSFM (L)	2.84	3.91	0.73	0.217	3.31
BH	2.43	3.43	0.71	n.a.	n.a.

Notes. This table reports full-sample and excluding-dot-com statistics for the four strategies. The exclusion drops 60 monthly observations covering January 1998 through December 2002. Alphas in this table are intercepts from regressions on BH.

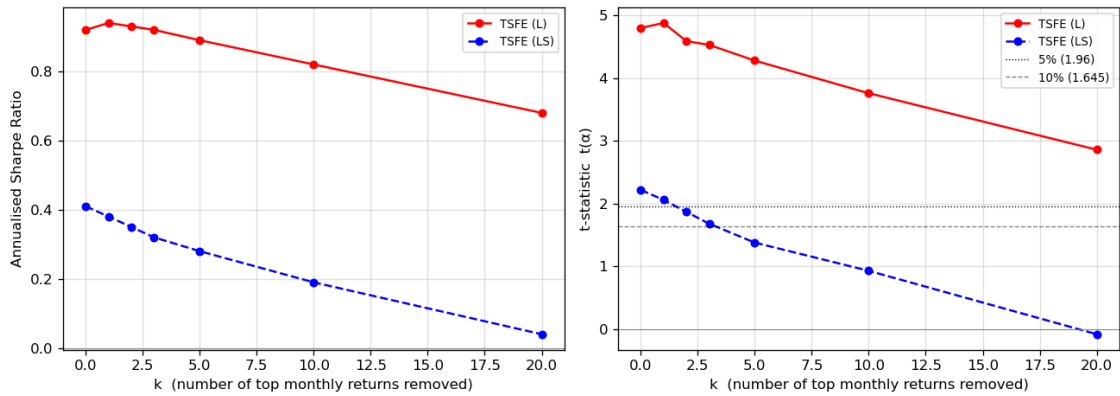
We further examine the sensitivity of TSFE's alpha to a small number of large monthly outperformances. Specifically, we sort the monthly return differences $r_t^{TSFE} - r_t^{TSFM}$ in descending order and recompute the alpha after removing the top k months for $k \in \{0, 1, 2, 3, 5, 10, 20\}$. The results in Table 3.10 and the accompanying Figure show that even after trimming the top twenty months, the alpha of TSFE(L) remains positive and statistically significant.

Table 3.10: Top-Return Trimming Sensitivity

k	TSFE (L)			TSFE (LS)		
	SR	$\hat{\alpha}$ (%/mo)	$\hat{t}(\alpha)$	SR	$\hat{\alpha}$ (%/mo)	$\hat{t}(\alpha)$
0	0.92	0.240	3.61	0.41	0.144	2.10
1	0.94	0.248	3.83	0.43	0.151	2.24
2	0.92	0.232	3.62	0.42	0.143	2.16
3	0.91	0.224	3.55	0.41	0.136	2.10
5	0.89	0.211	3.40	0.39	0.122	1.93
10	0.82	0.195	3.17	0.34	0.105	1.71
20	0.68	0.158	2.65	0.25	0.078	1.34

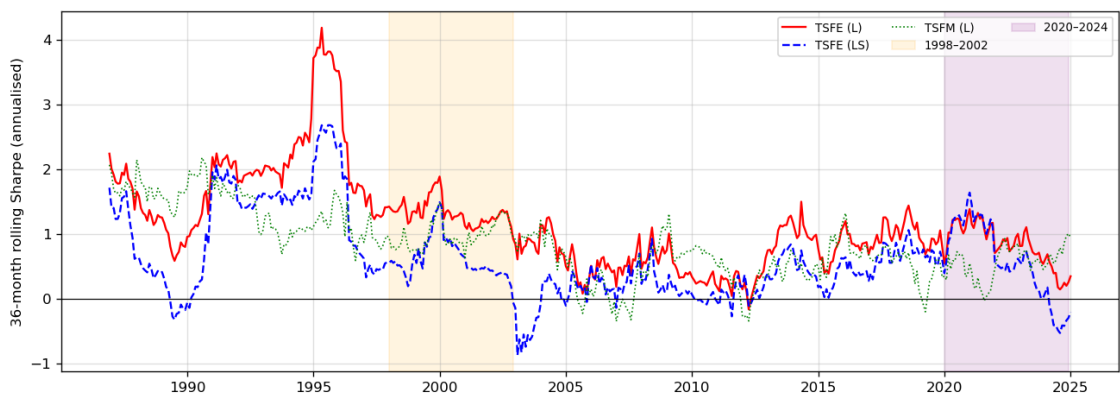
Notes. For each k , the top k monthly outperformances of TSFE relative to TSFM are removed and the statistics are recomputed on the trimmed series. Alphas are intercepts from regressions of the trimmed TSFE series on the contemporaneous TSFM series.

Finally, Figure 3.12 reports the 36-month rolling Sharpe ratio of TSFE(L), TSFE(LS),

Figure 3.11: Top-Return Trimming: Sharpe Ratio and Alpha t-Statistic

Notes. The left panel plots the Sharpe ratio of TSFE(L) and TSFE(LS) as the top k monthly outperformances are removed; the right panel plots the corresponding $\hat{t}(\alpha)$. Both quantities decay smoothly rather than collapsing, indicating that the result is not driven by a handful of exceptional months.

and TSFM(L). The rolling Sharpe of TSFE(L) is positive in the vast majority of windows, dipping below zero only briefly around 2003 and 2009. The two shaded regions (dot-com 1998 to 2002 and COVID+ 2020 to 2024) highlight that while these episodes contribute meaningfully to the cumulative outperformance, they are not the sole source.

Figure 3.12: Rolling 36-Month Sharpe Ratio

Notes. 36-month rolling Sharpe ratios of TSFE(L), TSFE(LS), and TSFM(L). Shaded regions indicate the dot-com window (1998 to 2002) and the COVID+ window (2020 to 2024).

Taken together, the four diagnostics indicate that the outperformance of the explosiveness based timing strategy is not the artifact of a small number of exceptional months. The Sharpe ratio of TSFE(LS) is in fact higher after excluding the

dot-com window, the top-twenty-month-trimmed alpha remains statistically significant, and the rolling Sharpe ratio is positive in the vast majority of windows. We conclude that the aggregate outperformance documented above is not driven by a handful of high-return episodes.

3.5 Conclusions

In this chapter, we introduce a novel framework for factor timing by leveraging explosive dynamics in factor premia, offering a new dimension beyond conventional momentum-based signals. We document widespread evidence of explosive episodes across a comprehensive set of asset pricing factors, suggesting that such behaviour is a common feature in the factor zoo. Building on this insight, we construct a time-series factor explosiveness (TSFE) strategy that consistently outperforms both the widely used time-series factor momentum (TSFM) benchmark and a raw PSY explosiveness benchmark in terms of Sharpe ratio, even after accounting for transaction costs. The outperformance over the raw PSY benchmark in particular confirms that TSFE's contribution is not merely a relabelling of the unit-root signal but reflects a distinct economic role of the contemporaneous-return overlay. We further extend our analysis to cross-sectional strategies and demonstrate that our explosiveness-based strategy delivers superior risk-adjusted performance. Importantly, the robustness of our results is confirmed under various sensitivity checks: TSFE retains a positive and significant alpha after controlling for the Fama-French three-factor model, and sub-sample diagnostics show that the outperformance is not driven by a handful of episodes such as the 1998 to 2002 dot-com window. Unlike factor momentum strategies that rely solely on cumulative past returns, our methodology captures richer information embedded in the trajectory of factor premia, providing a more nuanced and timely signal. Overall, our findings underscore the economic value

of incorporating explosiveness into factor timing and offer practical tools for investors seeking to enhance performance through dynamic factor allocation.

Appendix 3.A Details of Factors

Jensen et al. (2023) construct a global dataset with 153 factors in 93 countries. For each firm-level characteristic, the authors build the one-month-holding-period factor return within each country. Additionally, they group these factors into 13 clusters, including Accruals, Debt Issuance, Investment, Low Leverage, Low Risk, Momentum, Profit Growth, Profitability, Quality, Seasonality, Size, Short-Term Reversal, and Value. We downloaded the capped value weighted monthly returns for all 153 factors in the United States. Our sample covers the time period from January 1980 to December 2024, with 540 observations for each individual factor.

The details of the 153 factors that we used in our paper are listed in Table 3.11, including the acronym, concise description, original study, original sample, and cluster. The last column specifies the corresponding cluster of each factor, with 6 factors falling under Accruals, 7 under Debt Issuance, 22 under Investment, 11 under Low Leverage, 18 under Low Risk, 8 under Momentum, 12 under Profit Growth, 11 under Profitability, 17 under Quality, 12 under Seasonality, 5 under Size, 6 under Short-Term Reversal, and 18 under Value.

Table 3.11: Details of Factors

Acronym	Description	Original Study	Original Sample	Cluster
cowc_gr1a	Change in current operating working capital	Richardson et al. (2005)	1962-2001	Accruals
oaccruals_at	Operating accruals	Sloan (1996)	1962-1991	Accruals
oaccruals_ni	Percent operating accruals	Hafzalla Lundholm and Van Winkle (2011)	1989-2008	Accruals
seas_16_20na	Years 16-20 lagged returns, nonannual	Heston and Sadka (2008)	1965-2002	Accruals
taccruals_at	Total accruals	Richardson et al. (2005)	1962-2001	Accruals
taccruals_ni	Percent total accruals	Hafzalla Lundholm and Van Winkle (2011)	1989-2008	Accruals
capex_abn	Abnormal corporate investment	Titman Wei and Xie (2004)	1973-1996	Debt Issuance
debt_gr3	Growth in book debt (3 years)	Lyandres Sun and Zhang (2008)	1970-2005	Debt Issuance
fml_gr1a	Change in financial liabilities	Richardson et al. (2005)	1962-2001	Debt Issuance
ncol_gr1a	Change in noncurrent operating liabilities	Richardson et al. (2005)	1962-2001	Debt Issuance
nfna_gr1a	Change in net financial assets	Richardson et al. (2005)	1962-2001	Debt Issuance
ni_ar1	Earnings persistence	Francis et al. (2004)	1975-2001	Debt Issuance
noa_at	Net operating assets	Hirshleifer et al. (2004)	1962-2002	Debt Issuance
aliq_at	Liquidity of book assets	Ortiz-Molina and Phillips (2014)	1984-2006	Investment
at_gr1	Asset Growth	Cooper Gulen and Schill (2008)	1968-2003	Investment
be_gr1a	Change in common equity	Richardson et al. (2005)	1962-2001	Investment
capx_gr1	CAPEX growth (1 year)	Xie (2001)	1971-1992	Investment
capx_gr2	CAPEX growth (2 years)	Anderson and Garcia-Feijoo (2006)	1976-1998	Investment
capx_gr3	CAPEX growth (3 years)	Anderson and Garcia-Feijoo (2006)	1976-1998	Investment
coa_gr1a	Change in current operating assets	Richardson et al. (2005)	1962-2001	Investment
col_gr1a	Change in current operating liabilities	Richardson et al. (2005)	1962-2001	Investment
emp_gr1	Hiring rate	Belo Lin and Bazzdresch (2014)	1965-2010	Investment
inv_gr1	Inventory growth	Belo and Lin (2011)	1965-2009	Investment
inv_gr1a	Inventory change	Thomas and Zhang (2002)	1970-1997	Investment
lnoa_gr1a	Change in long-term net operating assets	Fairfield Whisenant and Yohn (2003)	1964-1993	Investment
mispricing_mgm	Mispricing factor: Management	Stambaugh and Yuan (2016)	1967-2013	Investment
ncoa_gr1a	Change in noncurrent operating assets	Richardson et al. (2005)	1962-2001	Investment
nncoa_gr1a	Change in net noncurrent operating assets	Richardson et al. (2005)	1962-2001	Investment
noa_gr1a	Change in net operating assets	Hirshleifer et al. (2004)	1964-2002	Investment
ppeinv_gr1a	Change PPE and Inventory	Lyandres Sun and Zhang (2008)	1970-2005	Investment
ret_60_12	Long-term reversal	De Bondt and Thaler (1985)	1926-1982	Investment
sale_gr1	Sales Growth (1 year)	Lakonishok Shleifer and Vishny (1994)	1968-1989	Investment
sale_gr3	Sales Growth (3 years)	Lakonishok Shleifer and Vishny (1994)	1968-1989	Investment
saleq_gr1	Sales growth (1 quarter)		1967-2016	Investment
seas_2_5na	Years 2-5 lagged returns, nonannual	Heston and Sadka (2008)	1965-2002	Investment
age	Firm age	Jiang Lee and Zhang (2005)	1965-2001	Low Leverage
aliq_mat	Liquidity of market assets	Ortiz-Molina and Phillips (2014)	1984-2006	Low Leverage
at_be	Book leverage	Fama and French (1992)	1963-1990	Low Leverage
bidaskhl_21d	The high-low bid-ask spread	Corwin and Schultz (2012)	1927-2006	Low Leverage
cash_at	Cash-to-assets	Palazzo (2012)	1972-2009	Low Leverage
netdebt_me	Net debt-to-price	Penman Richardson and Tuna (2007)	1962-2001	Low Leverage
ni_ivol	Earnings volatility	Francis et al. (2004)	1975-2001	Low Leverage
rd_sale	R&D-to-sales	Chan Lakonishok and Sougiannis (2001)	1975-1995	Low Leverage
rd5_at	R&D capital-to-book assets	Li (2011)	1952-2004	Low Leverage
tangibility	Asset tangibility	Hahn and Lee (2009)	1973-2001	Low Leverage
z_score	Altman Z-score	Dichev (1998)	1981-1995	Low Leverage
beta_60m	Market Beta	Fama and MacBeth (1973)	1935-1968	Low Risk
beta_dimson_21d	Dimson beta	Dimson (1979)	1955-1974	Low Risk
betabab_1260d	Frazzini-Pedersen market beta	Frazzini and Pedersen (2014)	1926-2012	Low Risk
betadown_252d	Downside beta	Ang Chen and Xing (2006)	1963-2001	Low Risk
earnings_variability	Earnings variability	Francis et al. (2004)	1975-2001	Low Risk
ivol_capm_21d	Idiosyncratic volatility from the CAPM (21 days)		1967-2016	Low Risk
ivol_capm_252d	Idiosyncratic volatility from the CAPM (252 days)	Ali Hwang and Trombley (2003)	1976-1997	Low Risk
ivol_ff3_21d	Idiosyncratic volatility from the Fama-French 3-factor model	Ang et al. (2006)	1963-2000	Low Risk
ivol_hxz4_21d	Idiosyncratic volatility from the q-factor model		1967-2016	Low Risk
ocfq_saleq_std	Cash flow volatility	Huang (2009)	1980-2004	Low Risk
rmax1_21d	Maximum daily return	Bali Cakici and Whitelaw (2011)	1962-2005	Low Risk
rmax5_21d	Highest 5 days of return	Bali, Brown, Murray and Tang (2017)	1993-2012	Low Risk
rvol_21d	Return volatility	Ang et al. (2006)	1963-2000	Low Risk

Table 3.11 - cont.

Acronym	Description	Original Study	Original Sample	Cluster
seas_6_10na	Years 6-10 lagged returns, nonannual	Heston and Sadka (2008)	1965-2002	Low Risk
turnover_126d	Share turnover	Datar Naik and Radcliffe (1998)	1963-1991	Low Risk
zero_trades_21d	Number of zero trades with turnover as tiebreaker (1 month)	Liu (2006)	1963-2003	Low Risk
zero_trades_126d	Number of zero trades with turnover as tiebreaker (6 months)	Liu (2006)	1963-2003	Low Risk
zero_trades_252d	Number of zero trades with turnover as tiebreaker (12 months)	Liu (2006)	1963-2003	Low Risk
prc_highprc_252d	Current price to high price over last year	George and Hwang (2004)	1963-2001	Momentum
resff3_6_1	Residual momentum t-6 to t-1	Blitz Huij and Martens (2011)	1930-2009	Momentum
resff3_12_1	Residual momentum t-12 to t-1	Blitz Huij and Martens (2011)	1930-2009	Momentum
ret_3_1	Price momentum t-3 to t-1	Jegadeesh and Titman (1993)	1965-1989	Momentum
ret_6_1	Price momentum t-6 to t-1	Jegadeesh and Titman (1993)	1965-1989	Momentum
ret_9_1	Price momentum t-9 to t-1	Jegadeesh and Titman (1993)	1965-1989	Momentum
ret_12_1	Price momentum t-12 to t-1	Jegadeesh and Titman (1993)	1965-1989	Momentum
seas_1_1na	Year 1-lagged return, nonannual	Heston and Sadka (2008)	1965-2002	Momentum
dsale_dinv	Change sales minus change inventory	Abarbanell and Bushee (1998)	1974-1988	Profit Growth
dsale_drec	Change sales minus change receivables	Abarbanell and Bushee (1998)	1974-1988	Profit Growth
dsale_dsga	Change sales minus change SG&A	Abarbanell and Bushee (1998)	1974-1988	Profit Growth
niq_at_chg1	Change in quarterly return on assets		1972-2016	Profit Growth
niq_be_chg1	Change in quarterly return on equity		1967-2016	Profit Growth
niq_su	Standardized earnings surprise	Foster Olsen and Shevlin (1984)	1974-1981	Profit Growth
ocf_at_chg1	Change in operating cash flow to assets	Bouchard, Krueger, Landier and Thesmar (2019)	1990-2015	Profit Growth
ret_12_7	Price momentum t-12 to t-7	Novy-Marx (2012)	1925-2010	Profit Growth
sale_emp_gr1	Labor force efficiency	Abarbanell and Bushee (1998)	1974-1988	Profit Growth
saleq_su	Standardized Revenue surprise	Jegadeesh and Livnat (2006)	1987-2003	Profit Growth
seas_1_1an	Year 1-lagged return, annual	Heston and Sadka (2008)	1965-2002	Profit Growth
tax_gr1a	Tax expense surprise	Thomas and Zhang (2011)	1977-2006	Profit Growth
dolvol_var_126d	Coefficient of variation for dollar trading volume	Chordia Subrahmanyam and Anshuman (2001)	1966-1995	Profitability
ebit_bev	Return on net operating assets	Soliman (2008)	1984-2002	Profitability
ebit_sale	Profit margin	Soliman (2008)	1984-2002	Profitability
f_score	Pitroski F-score	Pitroski (2000)	1976-1996	Profitability
ni_be	Return on equity	Haugen and Baker (1996)	1979-1993	Profitability
niq_be	Quarterly return on equity	Hou Xue and Zhang (2015)	1972-2012	Profitability
o_score	Ohlson O-score	Dichev (1998)	1981-1995	Profitability
ocf_at	Operating cash flow to assets	Bouchard, Krueger, Landier and Thesmar (2019)	1990-2015	Profitability
ope_be	Operating profits-to-book equity	Fama and French (2015)	1963-2013	Profitability
ope_bell	Operating profits-to-lagged book equity		1967-2016	Profitability
turnover_var_126d	Coefficient of variation for share turnover	Chordia Subrahmanyam and Anshuman (2001)	1966-1995	Profitability
at_turnover	Capital turnover	Haugen and Baker (1996)	1979-1993	Quality
cop_at	Cash-based operating profits-to-book assets		1967-2016	Quality
cop_atl1	Cash-based operating profits-to-lagged book assets	Ball et al. (2016)	1963-2014	Quality
dgp_dsale	Change gross margin minus change sales	Abarbanell and Bushee (1998)	1974-1988	Quality
gp_at	Gross profits-to-assets	Novy-Marx (2013)	1963-2010	Quality
gp_atl1	Gross profits-to-lagged assets		1967-2016	Quality
mispricing_perf	Mispricing factor: Performance	Stambaugh and Yuan (2016)	1967-2013	Quality
ni_inc8q	Number of consecutive quarters with earnings increases	Barth Elliott and Finn (1999)	1982-1992	Quality
niq_at	Quarterly return on assets	Balakrishnan Bartov and Faurel (2010)	1976-2005	Quality
op_at	Operating profits-to-book assets		1963-2013	Quality
op_atl1	Operating profits-to-lagged book assets	Ball et al. (2016)	1963-2014	Quality
opex_at	Operating leverage	Novy-Marx (2011)	1963-2008	Quality
qmj	Quality minus Junk: Composite	Assness, Frazzini and Pedersen (2018)	1957-2016	Quality
qmj_growth	Quality minus Junk: Growth	Assness, Frazzini and Pedersen (2018)	1957-2016	Quality
qmj_prof	Quality minus Junk: Profitability	Assness, Frazzini and Pedersen (2018)	1957-2016	Quality
qmj_safety	Quality minus Junk: Safety	Assness, Frazzini and Pedersen (2018)	1957-2016	Quality
sale_bev	Assets turnover	Soliman (2008)	1984-2002	Quality
corr_1260d	Market correlation	Assness, Frazzini, Gormsen, Pedersen (2020)	1925-2015	Seasonality
coskew_21d	Coskewness	Harvey and Siddique (2000)	1963-1993	Seasonality
dbnetis_at	Net debt issuance	Bradshaw Richardson and Sloan (2006)	1971-2000	Seasonality
kz_index	Kaplan-Zingales index	Lamont Polk and Saa-Requejo (2001)	1968-1995	Seasonality
lti_gr1a	Change in long-term investments	Richardson et al. (2005)	1962-2001	Seasonality
pi_nix	Taxable income-to-book income	Lev and Nissim (2004)	1973-2000	Seasonality
seas_2_5an	Years 2-5 lagged returns, annual	Heston and Sadka (2008)	1965-2002	Seasonality
seas_6_10an	Years 6-10 lagged returns, annual	Heston and Sadka (2008)	1965-2002	Seasonality
seas_11_15an	Years 11-15 lagged returns, annual	Heston and Sadka (2008)	1965-2002	Seasonality
seas_11_15na	Years 11-15 lagged returns, nonannual	Heston and Sadka (2008)	1965-2002	Seasonality
seas_16_20an	Years 16-20 lagged returns, annual	Heston and Sadka (2008)	1965-2002	Seasonality
sti_gr1a	Change in short-term investments	Richardson et al. (2005)	1962-2001	Seasonality
ami_126d	Amihud Measure	Amihud (2002)	1964-1997	Size
dolvol_126d	Dollar trading volume	Brennan Chordia and Subrahmanyam (1998)	1966-1995	Size
market_equity	Market Equity	Banz (1981)	1926-1975	Size
prc	Price per share	Miller and Scholes (1982)	1940-1978	Size
rd_me	R&D-to-market	Chan Lakonishok and Sougiannis (2001)	1975-1995	Size
iskew_capm_21d	Idiosyncratic skewness from the CAPM		1967-2016	Short-Term Reversal
iskew_ff3_21d	Idiosyncratic skewness from the Fama-French 3-factor model	Bali Engle and Murray (2016)	1925-2021	Short-Term Reversal
iskew_hxz4_21d	Idiosyncratic skewness from the q-factor model		1967-2016	Short-Term Reversal
ret_1_0	Short-term reversal	Jegadeesh (1990)	1929-1982	Short-Term Reversal
rmax5_rvol_21d	Highest 5 days of return scaled by volatility	Assness, Frazzini, Gormsen, Pedersen (2020)	1925-2015	Short-Term Reversal
rskew_21d	Total skewness	Bali Engle and Murray (2016)	1925-2021	Short-Term Reversal

Table 3.11 - cont.

Acronym	Description	Original Study	Original Sample	Cluster
at_me	Assets-to-market	Fama and French (1992)	1963-1990	Value
be_me	Book-to-market equity	Rosenberg Reid and Lanstein (1985)	1973-1984	Value
bev_mev	Book-to-market enterprise value	Penman Richardson and Tuna (2007)	1962-2001	Value
chcsho_12m	Net stock issues	Pontiff and Woodgate (2008)	1970-2003	Value
debt_me	Debt-to-market	Bhandari (1988)	1948-1979	Value
div12m_me	Dividend yield	Litzenberger and Ramaswamy (1979)	1940-1980	Value
ebitda_mev	Ebitda-to-market enterprise value	Loughran and Wellman (2011)	1963-2009	Value
eq_dur	Equity duration	Dechow Sloan and Soliman (2004)	1962-1998	Value
eqnetis_at	Net equity issuance	Bradshaw Richardson and Sloan (2006)	1971-2000	Value
eqnpo_12m	Equity net payout	Daniel and Titman (2006)	1968-2003	Value
eqnpo_me	Net payout yield	Boudoukh et al. (2007)	1984-2003	Value
eqpo_me	Payout yield	Boudoukh et al. (2007)	1984-2003	Value
fcf_me	Free cash flow-to-price	Lakonishok Shleifer and Vishny (1994)	1963-1990	Value
ival_me	Intrinsic value-to-market	Frankel and Lee (1998)	1975-1993	Value
netis_at	Net total issuance	Bradshaw Richardson and Sloan (2006)	1971-2000	Value
ni_me	Earnings-to-price	Basu (1983)	1963-1979	Value
ocf_me	Operating cash flow-to-market	Desai Rajgopal and Venkatachalam (2004)	1973-1997	Value
sale_me	Sales-to-market	Barbee Mukherji and Raines (1996)	1979-1991	Value

Notes: This table summarises the firm-level characteristics used to construct the long-short factor portfolios. The columns show the acronym, a brief description, the original study, the original sample period, and the corresponding cluster following Jensen et al. (2023).

Appendix 3.B Factor Momentum Strategies: Alternative Formation Periods

We explore momentum strategies using different formation periods, ranging from 3 to 24 months. Performance results presented in Table 3.12 show that a 12-month formation period yields the strongest performance, consistent with prior findings in the literature.

Table 3.12: Factor Momentum Strategies: Alternative Formation Periods

	Formation Period					Formation Period				
	3	6	12	18	24	3	6	12	18	24
PANEL A: TIME-SERIES STRATEGIES										
	Mean Return (%)					Sharpe Ratio				
TS_MOM_L	7.44	7.96	8.08	7.41	7.01	1.41	1.51	1.56	1.54	1.55
TS_MOM_S	3.80	2.94	2.07	2.96	3.40	0.80	0.59	0.39	0.55	0.60
TS_MOM_LS	3.64	5.02	6.01	4.46	3.61	0.41	0.56	0.65	0.52	0.43
PANEL B: CROSS-SECTIONAL STRATEGIES										
	Mean return (%)					Sharpe ratio				
CS_MOM_L	7.82	8.49	9.05	8.43	8.04	1.31	1.38	1.46	1.49	1.52
CS_MOM_S	4.00	3.34	2.78	3.53	3.96	0.93	0.82	0.68	0.85	0.93
CS_MOM_LS	3.82	5.15	6.27	4.91	4.08	0.44	0.59	0.71	0.60	0.53

Notes: This table reports annualized mean returns and Sharpe ratios for time-series and cross-sectional factor momentum strategies based on a dataset that composed of 212 factors by Chen and Zimmermann (2022). The extreme values are highlighted. In terms of all aspects, except the Sharpe ratio for the long leg of cross-sectional factor momentum strategy, factor momentum strategies with one year formation period show the best performances.

Appendix 3.C Robustness Checks

Chen and Zimmermann (2022) construct firm-level characteristics based on data obtained from CRSP, Compustat, IBES, OptionMetrics, 13F and FRED. Various characteristics require specific data, which are conveniently accessible on the corresponding authors' websites. On the basis of the data's origin and focus, these characteristics-based factors are classified into accounting, analyst forecasts, corporate events, stock prices, trading data, and other. We also group similar factors into categories including momentum, value versus growth, investment, profitability, intangibles, trading frictions, and other, based on their economic interpretations as studies including Hou et al. (2020) and Neuhierl et al. (2024). Our sample covers the time period from January 1976 to December 2022 with data availability dictating different starting points for various factors. Generally, price-based factor have the longest historical coverage, with accounting data and analyst forecasts becoming accessible later in the timeline.

The following Table 3.13 presents the details of our factor zoo dataset, including the acronym, concise description, original study, data source, and economic classification. The last column specifies the economic category of each factor, with 22 factors falling under momentum, 26 under value versus growth, 34 under investment, 16 under profitability, 41 under intangibles, 38 under trading frictions, and 35 under other categories. Descriptive statistics of time-series mean returns for factor portfolios within each category are documented in Table 3.14. Additionally, Figure 3.13 illustrates the pairwise correlations among factor returns. We observe high correlations within each factor category, reflecting their similar economic implications.

We evaluate the performances of our TSFE with different confidence levels of the Explosiveness test and different transaction costs. Apart from the 95% confi-

Table 3.13: Details of Alternative Dataset

Acronym	Description	Original Study	Data Category	Economic Category
AbnormalAccruals	Abnormal Accruals	Xie (2001)	Accounting	Investment
Accruals	Accruals	Sloan (1996)	Accounting	Investment
AccrualsBM	Book-to-market and accruals	Bartov and Kim (2004)	Accounting	Investment
Activism1	Takeover vulnerability	Cremers and Nair (2005)	Other	Other
Activism2	Active shareholders	Cremers and Nair (2005)	Other	Intangibles
AdExp	Advertising Expense	Chan, Lakonishok and Sougiannis (2001)	Accounting	Intangibles
AgeIPO	IPO and age	Ritter (1991)	Event	Intangibles
AM	Total assets to market	Fama and French (1992)	Accounting	Value vs. growth
AnalystRevision	EPS forecast revision	Hawkins, Chamberlin, Daniel (1984)	Analyst	Momentum
AnalystValue	Analyst Value	Frankel and Lee (1998)	Analyst	Intangibles
AnnouncementReturn	Earnings announcement return	Chan, Jagadeesh and Lakonishok (1996)	Price	Momentum
AOP	Analyst Optimism	Frankel and Lee (1998)	Analyst	Intangibles
AssetGrowth	Asset growth	Cooper, Gulen and Schill (2008)	Accounting	Investment
Beta	CAPM beta	Fama and MacBeth (1973)	Price	Trading frictions
BetaFP	Frazzini-Pedersen Beta	Frazzini and Pedersen (2014)	Price	Trading frictions
BetaLiquidityPS	Pastor-Stambaugh liquidity beta	Pastor and Stambaugh (2003)	Price	Trading frictions
BetaTailRisk	Tail risk beta	Kelly and Jiang (2014)	Price	Trading frictions
BetaVIX	Systematic volatility	Ang et al. (2006)	Price	Trading frictions
BidAskSpread	Bid-ask spread	Amihud and Mendelsohn (1986)	Trading	Trading frictions
BM	Book to market, original	Statman (1980)	Accounting	Value vs. growth
BMdec	Book to market using December ME	Fama and French (1992)	Accounting	Value vs. growth
BookLeverage	Book leverage (annual)	Fama and French (1992)	Accounting	Value vs. growth
BPEBM	Leverage component of BM	Penman, Richardson and Tuna (2007)	Accounting	Value vs. growth
BrandInvest	Brand capital investment	Belo, Lin and Vitorino (2014)	Accounting	Intangibles
Cash	Cash to assets	Palazzo (2012)	Accounting	Value vs. growth
CashProd	Cash Productivity	Chandrashekar and Rao (2009)	Accounting	Intangibles
CBOperProf	Cash-based operating profitability	Ball et al. (2016)	Accounting	Profitability
CF	Cash flow to market	Lakonishok, Shleifer, Vishny (1994)	Accounting	Value vs. growth
cfp	Operating Cash flows to price	Desai, Rajgopal, Venkatachalam (2004)	Accounting	Value vs. growth
ChangeInRecommendation	Change in recommendation	Jegadeesh et al. (2004)	Analyst	Intangibles
ChAssetTurnover	Change in Asset Turnover	Soliman (2008)	Accounting	Profitability
ChEQ	Growth in book equity	Lockwood and Prombutr (2010)	Accounting	Intangibles
ChForecastAccrual	Change in Forecast and Accrual	Barth and Hutton (2004)	Analyst	Intangibles
ChInv	Inventory Growth	Thomas and Zhang (2002)	Accounting	Investment
ChInvIA	Change in capital inv (ind adj)	Abbaranell and Bushee (1998)	Accounting	Investment
ChNAnalyst	Decline in Analyst Coverage	Scherbina (2008)	Analyst	Intangibles
ChNNCOA	Change in Net Noncurrent Operating Assets	Soliman (2008)	Accounting	Investment
ChNWC	Change in Net Working Capital	Soliman (2008)	Accounting	Profitability
ChTax	Change in Taxes	Thomas and Zhang (2011)	Accounting	Intangibles
CitationsRD	Citations to RD expenses	Hirschleifer, Hsu and Li (2013)	Other	Other
CompEquIss	Composite equity issuance	Daniel and Titman (2006)	Accounting	Investment
CompositeDebtIssuance	Composite debt issuance	Lyandres, Sun and Zhang (2008)	Accounting	Investment
ConsRecomm	Consensus Recommendation	Barber et al. (2001)	Analyst	Other
ConvDebt	Convertible debt indicator	Valta (2016)	Accounting	Intangibles
CoskewACX	Coskewness using daily returns	Ang, Chen and Xing (2006)	Price	Trading frictions
Coskewness	Coskewness	Harvey and Siddique (2000)	Price	Trading frictions
CPVolSpread	Call minus Put Vol	Bali and Hovakimian (2009)	Trading	Trading frictions
CredRatDG	Credit Rating Downgrade	Dichev and Piotroski (2001)	Analyst	Profitability
CustomerMomentum	Customer momentum	Cohen and Frazzini (2008)	Other	Other
dCPVolSpread	Change in put vol minus change in call vol	An, Ang, Bali, Cakici (2014)	Trading	Trading frictions
DebtIssuance	Debt Issuance	Spies and Affleck-Graves (1999)	Accounting	Investment
DelBreadth	Breadth of ownership	Chen, Hong and Stein (2002)	Other	Intangibles
DelCOA	Change in current operating assets	Richardson et al. (2005)	Accounting	Investment
DelCOL	Change in current operating liabilities	Richardson et al. (2005)	Accounting	Investment
DelDRC	Deferred Revenue	Prakash and Sinha (2013)	Accounting	Profitability
DelEqu	Change in equity to assets	Richardson et al. (2005)	Accounting	Investment
DelFINL	Change in financial liabilities	Richardson et al. (2005)	Accounting	Investment
DelLTI	Change in long-term investment	Richardson et al. (2005)	Accounting	Investment
DelNetFin	Change in net financial assets	Richardson et al. (2005)	Accounting	Investment
DivInit	Dividend Initiation	Michaely, Thaler and Womack (1995)	Event	Value vs. growth
DivOmit	Dividend Omission	Michaely, Thaler and Womack (1995)	Event	Value vs. growth
DivSeason	Dividend seasonality	Hartzmark and Salomon (2013)	Event	Value vs. growth
DivYieldST	Predicted dividend yield next month	Litzenberger and Ramaswamy (1979)	Accounting	Value vs. growth
dNoa	change in net operating assets	Hirschleifer, Hou, Teoh, Zhang (2004)	Accounting	Investment
dVol	Past trading volume	Brennan, Chordia, Subra (1998)	Trading	Trading frictions
DownRecomm	Down forecast EPS	Barber et al. (2001)	Analyst	Intangibles
dVolCall	Change in call vol	An, Ang, Bali, Cakici (2014)	Trading	Trading frictions
dVolPut	Change in put vol	An, Ang, Bali, Cakici (2014)	Trading	Trading frictions

Table 3.13 - cont.

Acronym	Description	Original Study	Data Category	Economic Category
EarningsConsistency	Earnings consistency	Alwathainani (2009)	Accounting	Intangibles
EarningsForecastDisparity	Long-vs-short EPS forecasts	Da and Warachka (2011)	Analyst	Intangibles
EarningsStreak	Earnings surprise streak	Loh and Warachka (2012)	Accounting	Other
EarningsSurprise	Earnings Surprise	Foster, Olsen and Shevlin (1984)	Accounting	Momentum
EarnSupBig	Earnings surprise of big firms	Hou (2007)	Accounting	Momentum
EBM	Enterprise component of BM	Penman, Richardson and Tuna (2007)	Accounting	Value vs. growth
EntMult	Enterprise Multiple	Loughran and Wellman (2011)	Accounting	Value vs. growth
EP	Earnings-to-Price Ratio	Basu (1977)	Accounting	Value vs. growth
EquityDuration	Equity Duration	Dechow, Sloan and Soliman (2004)	Accounting	Value vs. growth
ExchSwitch	Exchange Switch	Dharan and Ikenberry (1995)	Event	Trading frictions
ExclExp	Excluded Expenses	Doyle, Lundholm and Soliman (2003)	Analyst	Intangibles
FEPS	Analyst earnings per share	Cen, Wei, and Zhang (2006)	Analyst	Profitability
fgr5yrLag	Long-term EPS forecast	La Porta (1996)	Analyst	Intangibles
FirmAge	Firm age based on CRSP	Barry and Brown (1984)	Other	Other
FirmAgeMom	Firm Age - Momentum	Zhang (2006)	Price	Momentum
ForecastDispersion	EPS Forecast Dispersion	Diether, Malloy and Scherbina (2002)	Analyst	Intangibles
FR	Pension Funding Status	Franzoni and Marin (2006)	Accounting	Intangibles
Frontier	Efficient frontier index	Nguyen and Swanson (2009)	Accounting	Intangibles
Governance	Governance Index	Gompers, Ishii and Metrick (2003)	Other	Other
GP	gross profits / total assets	Novy-Marx (2013)	Accounting	Profitability
GrAdExp	Growth in advertising expenses	Lou (2014)	Accounting	Intangibles
grcapx	Change in capex (two years)	Anderson and Garcia-Feijoo (2006)	Accounting	Investment
grcapx3y	Change in capex (three years)	Anderson and Garcia-Feijoo (2006)	Accounting	Investment
GrLTNOA	Growth in long term operating assets	Fairfield, Whisenant and Yohn (2003)	Accounting	Investment
GrSaleToGrInv	Sales growth over inventory growth	Abarbanell and Bushee (1998)	Accounting	Intangibles
GrSaleToGrOverhead	Sales growth over overhead growth	Abarbanell and Bushee (1998)	Accounting	Intangibles
Herf	Industry concentration (sales)	Hou and Robinson (2006)	Other	Intangibles
HerfAsset	Industry concentration (assets)	Hou and Robinson (2006)	Other	Intangibles
HerfBE	Industry concentration (equity)	Hou and Robinson (2006)	Other	Intangibles
High52	52 week high	George and Hwang (2004)	Price	Momentum
hire	Employment growth	Bazdresch, Belo and Lin (2014)	Other	Intangibles
IdioVol3F	Idiosyncratic risk (3 factor)	Ang et al. (2006)	Price	Trading frictions
IdioVolAHT	Idiosyncratic risk (AHT)	Ali, Hwang, and Trombley (2003)	Price	Trading frictions
Illiquidity	Amihud's illiquidity	Amihud (2002)	Trading	Trading frictions
IndIPO	Initial Public Offerings	Ritter (1991)	Event	Intangibles
IndMom	Industry Momentum	Grinblatt and Moskowitz (1999)	Price	Momentum
IndRetBig	Industry return of big firms	Hou (2007)	Price	Momentum
IntanBM	Intangible return using BM	Daniel and Titman (2006)	Accounting	Value vs. growth
IntanCFP	Intangible return using CFtoP	Daniel and Titman (2006)	Accounting	Value vs. growth
IntanEP	Intangible return using EP	Daniel and Titman (2006)	Accounting	Value vs. growth
IntanSP	Intangible return using Sale2P	Daniel and Titman (2006)	Accounting	Value vs. growth
IntMom	Intermediate Momentum	Novy-Marx (2012)	Price	Momentum
Investment	Investment to revenue	Titman, Wei and Xie (2004)	Accounting	Investment
InvestPPEInv	change in ppe and inv/assets	Lyandres, Sun and Zhang (2008)	Accounting	Investment
InvGrowth	Inventory Growth	Belo and Lin (2012)	Accounting	Profitability
IO-ShortInterest	Inst own among high short interest	Asquith Pathak and Ritter (2005)	Other	Other
iomom-cust	Customers momentum	Menzly and Ozbas (2010)	Other	Momentum
iomom-supp	Suppliers momentum	Menzly and Ozbas (2010)	Other	Momentum
Leverage	Market leverage	Bhandari (1988)	Accounting	Profitability
LRreversal	Long-run reversal	De Bondt and Thaler (1985)	Price	Other
MaxRet	Maximum return over month	Bali, Cakici, and Whitelaw (2011)	Price	Trading frictions
MeanRankRevGrowth	Revenue Growth Rank	Lakonishok, Shleifer, Vishny (1994)	Accounting	Value vs. growth
Mom12m	Momentum (12 month)	Jegadeesh and Titman (1993)	Price	Momentum
Mom12mOffSeason	Momentum without the seasonal part	Heston and Sadka 2008	Price	Other
Mom6m	Momentum (6 month)	Jegadeesh and Titman (1993)	Price	Momentum
Mom6mJunk	Junk Stock Momentum	Avramov et al. (2007)	Price	Momentum
MomOffSeason	Off season long-term reversal	Heston and Sadka (2008)	Price	Other
MomOffSeason06YrPlus	Off season reversal years 6 to 10	Heston and Sadka (2008)	Price	Other
MomOffSeason11YrPlus	Off season reversal years 11 to 15	Heston and Sadka (2008)	Price	Other
MomOffSeason16YrPlus	Off season reversal years 16 to 20	Heston and Sadka (2008)	Price	Other
MomRev	Momentum and LT Reversal	Chan and Ko (2006)	Price	Momentum

Table 3.13 - cont.

Acronym	Description	Original Study	Data Category	Economic Category
MomSeason	Return seasonality years 2 to 5	Heston and Sadka (2008)	Price	Other
MomSeason06YrPlus	Return seasonality years 6 to 10	Heston and Sadka (2008)	Price	Other
MomSeason11YrPlus	Return seasonality years 11 to 15	Heston and Sadka (2008)	Price	Other
MomSeason16YrPlus	Return seasonality years 16 to 20	Heston and Sadka (2008)	Price	Other
MomSeasonShort	Return seasonality last year	Heston and Sadka (2008)	Price	Other
MomVol	Momentum in high volume stocks	Lee and Swaminathan (2000)	Price	Momentum
MRreversal	Medium-run reversal	De Bondt and Thaler (1985)	Price	Other
MS	Mohanram G-score	Mohanram (2005)	Accounting	Other
NetDebtFinance	Net debt financing	Bradshaw, Richardson, Sloan (2006)	Accounting	Investment
NetDebtPrice	Net debt to price	Penman, Richardson and Tuna (2007)	Accounting	Value vs. growth
NetEquityFinance	Net equity financing	Bradshaw, Richardson, Sloan (2006)	Accounting	Investment
NetPayoutYield	Net Payout Yield	Boudoukh et al. (2007)	Accounting	Value vs. growth
NOA	Net Operating Assets	Hirshleifer et al. (2004)	Accounting	Investment
NumEarnIncrease	Earnings streak length	Loh and Warachka (2012)	Accounting	Momentum
OperProf	operating profits / book equity	Fama and French (2006)	Accounting	Profitability
OperProfRD	Operating profitability R&D adjusted	Ball et al. (2016)	Accounting	Profitability
OPLeverage	Operating leverage	Novy-Marx (2011)	Accounting	Intangibles
OptionVolume1	Option to stock volume	Johnson and So (2012)	Trading	Trading frictions
OptionVolume2	Option volume to average	Johnson and So (2012)	Trading	Trading frictions
OrderBacklog	Order backlog	Rajgopal, Shevlin, Venkatachalam (2003)	Accounting	Intangibles
OrderBacklogChg	Change in order backlog	Baik and Ahn (2007)	Accounting	Investment
OrgCap	Organizational capital	Eisfeldt and Papanikolaou (2013)	Accounting	Intangibles
OScore	O Score	Dichev (1998)	Accounting	Profitability
PatentsRD	Patents to RD expenses	Hirschleifer, Hsu and Li (2013)	Other	Other
PayoutYield	Payout Yield	Boudoukh et al. (2007)	Accounting	Value vs. growth
PctAcc	Percent Operating Accruals	Hafzalla, Lundholm, Van Winkle (2011)	Accounting	Investment
PctTotAcc	Percent Total Accruals	Hafzalla, Lundholm, Van Winkle (2011)	Accounting	Investment
PredictedFE	Predicted Analyst forecast error	Frankel and Lee (1998)	Accounting	Intangibles
Price	Price	Blume and Husic (1973)	Price	Other
PriceDelayRsq	Price delay r square	Hou and Moskowitz (2005)	Price	Trading frictions
PriceDelaySlope	Price delay coeff	Hou and Moskowitz (2005)	Price	Trading frictions
PriceDelayTstat	Price delay SE adjusted	Hou and Moskowitz (2005)	Price	Trading frictions
ProbInformedTrading	Probability of Informed Trading	Easley, Hvidkjaer and O'Hara (2002)	Trading	Trading frictions
PS	Piotroski F-score	Piotroski (2000)	Accounting	Other
RD	R&D over market cap	Chan, Lakonishok and Sougiannis (2001)	Accounting	Profitability
RDAbility	R&D ability	Cohen, Diether and Malloy (2013)	Accounting	Other
RDcap	R&D capital-to-assets	Li (2011)	Accounting	Intangibles
RDIPO	IPO and no R&D spending	Gou, Lev and Shi (2006)	Event	Intangibles
RDS	Real dirty surplus	Landsman et al. (2011)	Accounting	Intangibles
realestate	Real estate holdings	Tuzel (2010)	Accounting	Intangibles
RealizedVol	Realized (Total) Volatility	Ang et al. (2006)	Price	Trading frictions
Recomm-ShortInterest	Analyst Recommendations and Short-Interest	Drake, Rees and Swanson (2011)	Analyst	Intangibles
ResidualMomentum	Momentum based on FF3 residuals	Blitz, Huij and Martens (2011)	Price	Momentum
retConglomerate	Conglomerate return	Cohen and Lou (2012)	Price	Momentum
ReturnSkew	Return skewness	Bali, Engle and Murray (2015)	Price	Trading frictions
ReturnSkew3F	Idiosyncratic skewness (3F model)	Bali, Engle and Murray (2015)	Price	Trading frictions
REV6	Earnings forecast revisions	Chan, Jegadeesh and Lakonishok (1996)	Analyst	Momentum
RevenueSurprise	Revenue Surprise	Jegadeesh and Livnat (2006)	Accounting	Momentum
RIO-Disp	Inst Own and Forecast Dispersion	Nagel (2005)	Other	Other
RIO-MB	Inst Own and Market to Book	Nagel (2005)	Other	Other
RIO-Turnover	Inst Own and Turnover	Nagel (2005)	Other	Other
RIO-Volatility	Inst Own and Idio Vol	Nagel (2005)	Other	Other
RIVolSpread	Realized minus Implied Vol	Bali and Hovakimian (2009)	Trading	Trading frictions
roaq	Return on assets (quarterly)	Balakrishnan, Bartov and Faurel (2010)	Accounting	Profitability
RoE	net income / book equity	Haugen and Baker (1996)	Accounting	Profitability
sfe	Earnings Forecast to price	Elgers, Lo and Pfeiffer (2001)	Analyst	Vaule vs. growth
ShareIss1Y	Share issuance (1 year)	Pontiff and Woodgate (2008)	Accounting	Investment
ShareIss5Y	Share issuance (5 year)	Daniel and Titman (2006)	Accounting	Investment
ShareRepurchase	Share repurchases	Ikenberry, Lakonishok, Vermaelen (1995)	Accounting	Investment
ShareVol	Share Volume	Datar, Naik and Radcliffe (1998)	Trading	Trading frictions
ShortInterest	Short Interest	Dechow et al. (2001)	Trading	Trading frictions
sinAlgo	Sin Stock (selection criteria)	Hong and Kacperczyk (2009)	Other	Other
Size	Size	Banz (1981)	Price	Other
skew1	Volatility smirk near the money	Xing, Zhang and Zhao (2010)	Trading	Trading frictions
SmileSlope	Put volatility minus call volatility	Yan (2011)	Trading	Trading frictions
SP	Sales-to-price	Barbee, Mukherji and Raines (1996)	Accounting	Value vs. growth
Spinoff	Spinoffs	Cusatis, Miles and Woolridge (1993)	Event	Other
std-turn	Share turnover volatility	Chordia, Subra, Anshuman (2001)	Trading	Trading frictions
STreversal	Short term reversal	Jegadeesh (1990)	Price	Other
SurpriseRD	Unexpected R&D increase	Eberhart, Maxwell and Siddique (2004)	Accounting	Intangibles

Table 3.13 - cont.

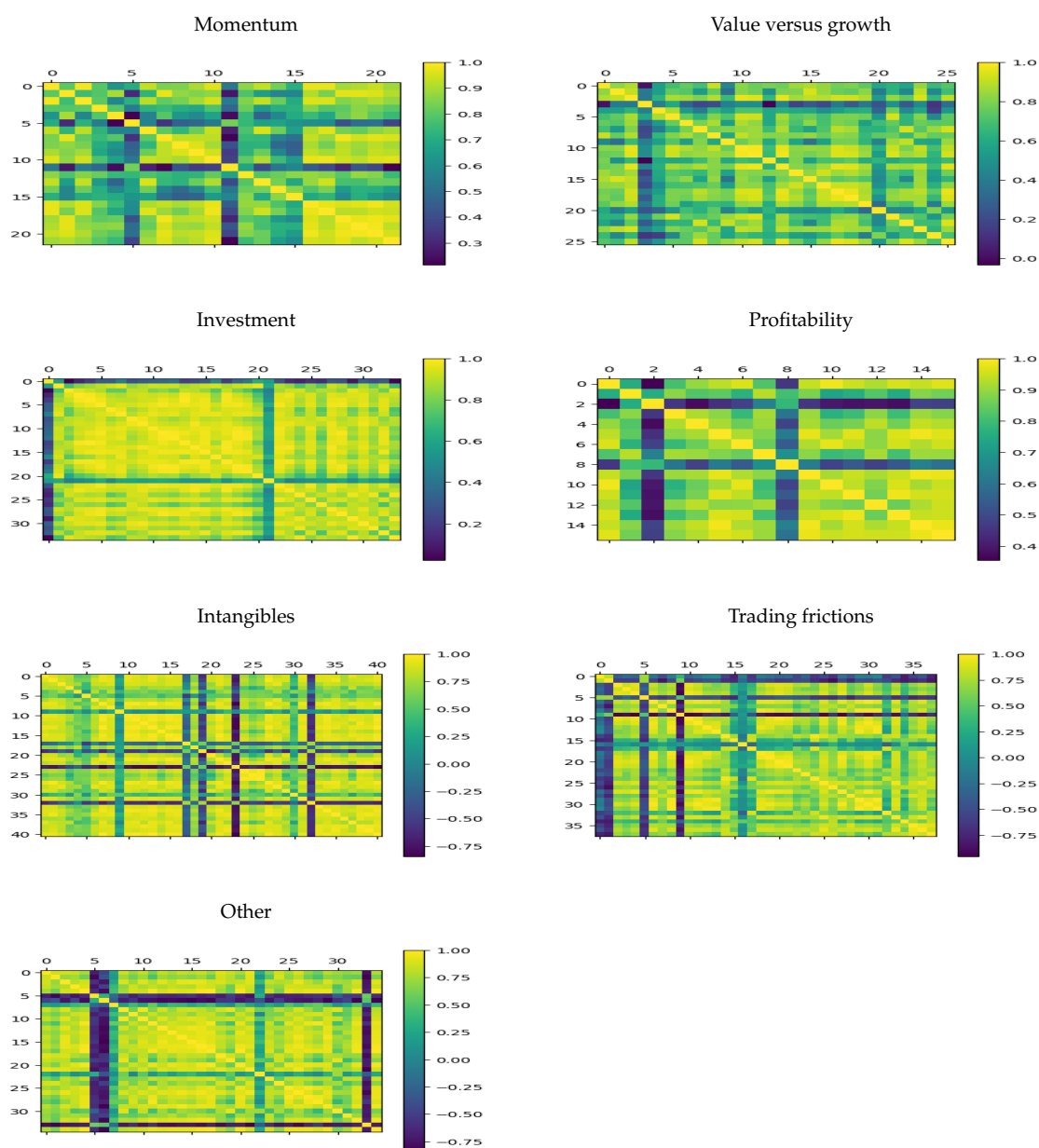
Acronym	Description	Original Study	Data Category	Economic Category
tang	Tangibility	Hahn and Lee (2009)	Accounting	Intangibles
Tax	Taxable income to income	Lev and Nissim (2004)	Accounting	Profitability
TotalAccruals	Total accruals	Richardson et al. (2005)	Accounting	Investment
TrendFactor	Trend Factor	Han, Zhou, Zhu (2016)	Price	Momentum
UpRecomm	Up Forecast	Barber et al. (2001)	Analyst	Intangibles
VarCF	Cash-flow to price variance	Haugen and Baker1996	Accounting	Other
VolMkt	Volume to market equity	Haugen and Baker (1996)	Trading	Trading frictions
VolSD	Volume Variance	Chordia, Subra, Anshuman (2001)	Trading	Trading frictions
VolumeTrend	Volume Trend	Haugen and Baker (1996)	Trading	Other
XFIN	Net external financing	Bradshaw, Richardson, Sloan (2006)	Accounting	Investment
zerotrade	Days with zero trades	Liu (2006)	Trading	Trading frictions
zerotradeAlt1	Days with zero trades	Liu (2006)	Trading	Trading frictions
zerotradeAlt12	Days with zero trades	Liu (2006)	Trading	Trading frictions

Notes: This table summarises the firm-level characteristics used to construct the long-short factor portfolios. The columns show the acronym, a brief description, the original study and the data category, where we follow Chen and Zimmermann (2022). In the last column, we group similar factors based on their economic interpretations, we follow the classification by Hou et al. (2020) and Neuhierl et al. (2024).

Table 3.14: Descriptive Statistics of the Mean Factor Returns (Monthly, %)

Category	Count	Mean	Std	Min	25%	50%	75%	Max
Momentum	22	0.8914	0.4498	0.2774	0.4877	0.8731	1.1104	1.7496
Value vs. growth	26	0.4592	0.2602	0.0211	0.2669	0.4380	0.6199	1.0124
Investment	34	0.4894	0.2963	0.1289	0.2949	0.3910	0.6051	1.3433
Profitability	16	0.5839	0.3045	0.1003	0.4262	0.5486	0.7057	1.3527
Intangibles	41	0.3560	0.2899	-0.0775	0.1792	0.3341	0.3772	1.5685
Trading frictions	38	0.3828	0.3691	-1.0278	0.2402	0.3650	0.5693	1.2740
Other	35	0.6034	0.6250	-0.4151	0.2347	0.5523	0.8037	3.1959

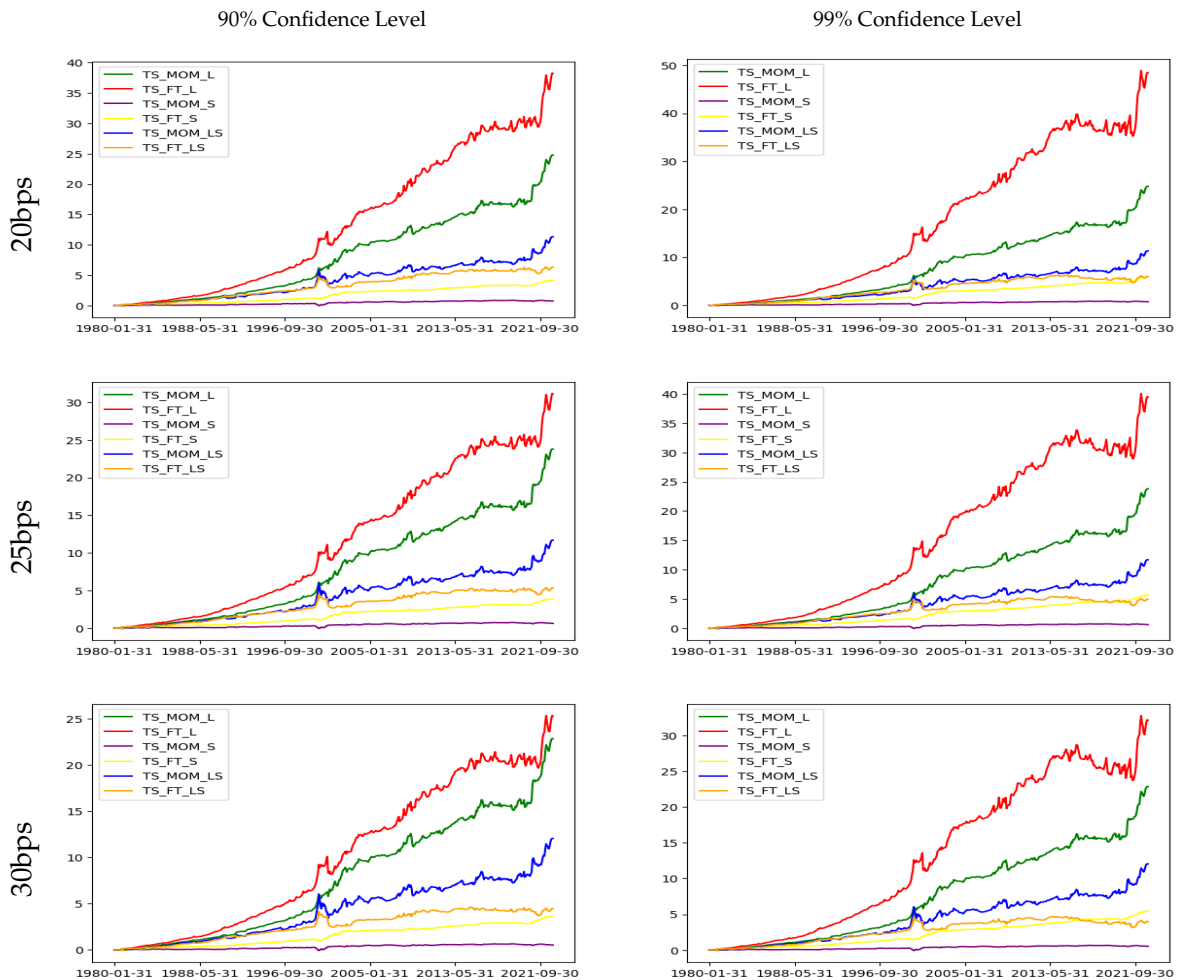
Notes: This table presents descriptive statistics of mean returns for factor long-short portfolios within each factor category, detailing counts, means, standard deviations, and quartiles.

Figure 3.13: Correlation Map

Notes: This figure displays the pairwise correlations among long-short factor portfolio returns within each factor category. Factors are grouped based on their economic interpretations, with 22 factors in momentum, 26 in value versus growth, 34 in investment, 16 in profitability, 41 in intangibles, 38 in trading frictions, and 35 in other categories.

dence level we applied previously, we also show results using confidence levels of 90% and 99%. Regarding transaction costs, we estimate the cost of a buy-and-sell round-trip trade to range from 20 basis points (bps) to 30 basis points. Figure 3.14 presents the cumulative returns of our TSFE strategy and the TSFM benchmark. The results remain qualitatively the same, which demonstrates the robustness of our empirical results.

Figure 3.14: Robustness Check: Time-Series Strategies



Notes: This figure presents the cumulative returns of our TSFE strategy and corresponding TSFM benchmark. Various scenarios are considered including confidence levels at both 90% and 99%, and trading cost estimates of 20bps, 25bps, and 30bps for a buy-and-sell round-trip.

Conclusions and Further Developments

This thesis makes three interconnected contributions to the econometric analysis of explosive dynamics in financial time series, spanning methodology, empirical application, and economic interpretation.

Chapter 1 addressed a fundamental tension between existing common bubble detection methods and the informational constraints of real-time policy making. While the common bubble test offers a powerful framework for identifying synchronised explosive behaviour across assets, its reliance on full-sample information renders it unsuitable for real-time macroprudential surveillance. To bridge this gap, the chapter proposed an adaptive implementation based on the IPCA-RLS algorithm, which recursively updates test statistics as new data arrive without revising past signals. The approach preserves real-time interpretability, dramatically reduces computational burden, and yields timely early warning indicators of systemic financial instability.

Chapter 2 applied this real-time framework to the U.S. green equity market, where sustained capital inflows driven by climate-related structural trends raise legitimate concerns about speculative excess. The empirical analysis uncovers pronounced common green bubbles concentrated after the COVID-19 pandemic

period, which are shown to be distinct from the broader post-pandemic equity recovery. These episodes are primarily attributable to macro-financial conditions and physical climate risk, illustrating how sector-specific exuberance can interact with aggregate financial conditions in ways that may threaten both market stability and the green energy transition.

Chapter 3 reframed explosiveness not as a risk to be monitored but as a signal to be exploited. Drawing on evidence that sophisticated investors rationally ride rather than arbitrage bubbles, the chapter interprets detected explosive regimes as a refined form of time-series momentum and incorporates them into a dynamic factor timing strategy. The resulting approach consistently outperforms both buy-and-hold allocations and conventional factor momentum benchmarks, demonstrating that explosiveness tests have practical utility beyond regulatory applications.

Taken together, these chapters establish that real-time detection of explosive dynamics is both methodologically feasible and empirically informative, with relevance for policymakers and market participants alike.

A promising extension of this research is to apply explosiveness detection techniques to high-frequency financial data. The existing literature, including the PSY framework that underpins the methods developed in this thesis, relies almost entirely on daily or lower-frequency observations. However, intraday price movements display distinct forms of short-lived explosive behaviour, such as flash crashes, liquidity-driven spikes, and brief speculative surges, which remain undetected at conventional sampling frequencies. Extending recursive right-tailed unit root tests to high-frequency environments presents substantial econometric difficulties, since market microstructure noise, serial dependence in price increments, and time-varying volatility are fundamental characteristics of intraday data that the PSY framework was not originally designed to address.

The high-frequency econometrics literature has made meaningful progress on related issues. One line of research examines jumps and discontinuities in asset prices (Merton, 1976; Andersen et al., 2007; Bollerslev et al., 2013). Another focuses on short-lived drift bursts within continuous-time semimartingale models, such as Christensen et al. (2022). A further strand develops unit root and explosiveness tests under in-fill asymptotics that explicitly incorporate microstructure noise and jumps (Laurent and Shi, 2022). While these contributions are essential, they do not provide a discrete-time sequential monitoring framework with stable, real-time critical values suitable for noisy high-frequency price series. Moreover, microstructure frictions themselves pose significant challenges; if not properly addressed, they can lead to substantial distortions in high-frequency inference, as highlighted by Jacod et al. (2017) and Li and Linton (2022).

We plan to adapt the existing explosiveness testing framework to high-frequency observed prices in the presence of microstructure noise. Our objective is to show that, under the unit-root null hypothesis, neither microstructure noise nor conditional heteroscedasticity affects the Brownian-motion limit of the ADF statistic constructed from observed prices. Building on this asymptotic result, we aim to propose a fixed-span local monitoring statistic that restricts the backward recursive search to a rolling window of fixed length. This local approach differs fundamentally from the PSY framework in that its null limiting distribution is stable rather than endpoint-dependent, allowing for the use of fixed critical values. It brings the procedure closer to a genuine streaming implementation, which eliminates the need for repeated recalibration at each endpoint and improves its suitability for real-time monitoring. Additionally, we can further integrate the proposed noise-robust local test statistic with the efficient real-time implementation introduced in Chapter 1, opening up a range of applications including intraday monitoring systems used by exchanges and clearing houses, as well as ultra-short-horizon trading signals that complement the lower-frequency strate-

gies discussed in Chapter 3.

Finally, we note a related empirical observation regarding the use of the first principal component (PC1) in common bubble tests. While Chen et al. (2023) provides theoretical justification for the sufficiency and consistency of PC1 in the second-stage PSY procedure, we find that other principal components may also exhibit distinct episodes of explosiveness, particularly when the variance explained by PC1 is relatively low. This gap between theoretical guarantees and empirical behaviour suggests an interesting avenue for future research on the role of higher-order components in detecting common bubble dynamics.

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