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CRACK SIZE EVALUATION USING
ULTRASONIC SURFACE WAVE SPECTROSCOPY

LEWIS L. MORGAN

Thesis submitted for the degree of
Doctor of Philosophy

THE CITY UNIVERSITY

1973

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ABSTRACT

The development of broad-band, ultrasonic, Rayleigh wave transducers, and their use in measuring the dimensions of simulated cracks (with dimensions of the order of a millimetre) are described. The method, the generation of the cepstra of signals reflected from the defect, removes the problem of variation in the coupling of the transducer to the testpiece. It also increases the resolution to its limit, this limit being defined by the highest frequency the surface will allow to propagate with acceptable attenuation. It is shown theoretically how the angle of the crack to the surface may be determined, and the possibilities of obtaining other important crack parameters are discussed. With existing transducers the minimum crack depth that can be measured is approximately $1\frac{1}{2}$ mm.

INTRODUCTION

The problem of the fatigue and fracture of component materials in structures has long been the subject of energetic research. The processes of crack initiation and subsequent propagation prior to fracture have proved extremely complex, but the methods of linear fracture mechanics have achieved an analytical treatment of these problems and a complete solution for two-dimensional static cases (see Ref.1). This treatment, however, requires that a great deal be known about the crack. Its depth, width and shape have to be known, together with its position in the sample, its orientation with respect to the applied stress, the stress on the sample and the sample shape. While the sample shape may be known and the applied stress calculated, the other quantities are less easily found. The techniques of Non-Destructive Testing (N.D.T.) have been used with considerable success to determine some of these other quantities, and may, in time, provide them all. When they are available, it will be possible to put more realistic limits on the size of defect which may be accepted, rather than the engineering approximations currently used.

The existence and position of cracks in a sample can be found by a number of methods: dye penetrants, electric and magnetic particle, radiographic, fluoroscopic, xeroradiographic, ultrasonic, eddy-current and thermal methods. These are all well documented; a crack oriented treatment is given in Ref.2. Once a crack has been found, the measurement of its depth poses a considerably more difficult problem. The work described here shows how the size of cracks as small as a few tenths of a millimetre may be measured.

The technique uses the method of ultrasonic reflectography. This method entails exciting a short stress wave in the solid under test. This travels away from the transducer producing the wave and, when it hits a defect in the medium, some of the energy incident on the defect is reflected back and detected by the transducer. The time of travel of the wave gives the distance of the defect from the source, if the velocity is known. To detect small defects the amplitude of the incident wave should be as large as possible so as to reflect a measurable quantity of energy. The duration should also be as short as possible to resolve the reflections from defects close together.

Clearly, measuring the depths of large cracks is not difficult if we use a wave travelling only along the surface, since the reflections from the top and bottom of the cracks can be identified. However, for smaller cracks the signals overlap and this is no longer possible. This may be overcome if broad band Rayleigh waves are used and special signal processing techniques used to treat the reflected signals. This is the method developed and described in this work.

The layout of this thesis follows fairly closely the chronological development of the project. Following a chapter (One) giving the theoretical background to Rayleigh waves, Chapter Two describes experimental and theoretical development of the broad band transducers used in the study. Chapter Three describes how these transducers were used to determine the depth of a slot, together with its other dimensions. Since the practical usefulness of the technique was limited, at that stage, by transducer operation, an account of improvements made, and those projected, to these transducers is detailed in Chapter Four. Chapter Five contains a theoretical description of the reflection from a slot, extended to the case of a slot

angled to the surface. This is followed by further suggestions for ways in which the technique might be extended to quantify other important crack parameters.

CHAPTER ONE

1. THE RAYLEIGH WAVE .
 - 1.1 Introduction
 - 1.2 The Rayleigh Wave - Continuous Wave Analysis
 - 1.3 Transient Rayleigh Wave Analysis
 - 1.3.1 The stress field computation

1. THE RAYLEIGH WAVE

1.1 Introduction

The Rayleigh Wave is just one member of the group of Surface Waves, the most commonly occurring form of which is that generated on water. This wave, however, is maintained by gravity whereas surface waves on solids are maintained by the elastic nature of the medium. Surface waves on solids include:

Love Waves - transverse shear waves 'trapped' or 'guided' by a surface layer (Ref.3),
 Lamb Waves - waves generated in thin plates (Ref.4),
 and the various forms of wave occurring at the interface between two media.

To support a shear stress wave, one medium must be a solid, in which case three types of wave can be described depending on whether the other medium is solid, liquid or gaseous/vacuum.

These waves are:

Stoneley Waves - waves between two solid half spaces (Ref.5),
 Brekhovskikh (or Schult) Waves - waves at a solid/liquid interface (Ref.6),
 Rayleigh Waves - waves at a solid/gaseous or solid/vacuum interface.

This latter form is the surface wave used in the following work.

This wave, first described by Lord Rayleigh in 1885 (Ref.7), is in many ways analogous to the electromagnetic wave confined to the surface of a conducting medium at high frequencies,

described as the skin effect. In both cases the wave amplitude decreases with depth from the surface, and the velocity of propagation is less than that in the unbounded medium. Both, too, have a characteristic skin depth which decreases with increase in frequency. The characteristics are shown clearly in the analysis of the following section. The analysis does not take account of the attenuation of the wave as it travels over the surface. The attenuation of Rayleigh waves is high if the surface is wet, dirty or rough (highest for a roughness amplitude comparable with the wavelength of the wave). Treatment prior to the testing of a surface removes dirt and damp, and, in conventional surface wave reflectometry, the frequency is chosen to give acceptably low attenuation for any given surface roughness (since attenuation is lower at low frequencies). It will be shown that the method of testing developed in this thesis is not sensitive to surface condition (hence attenuation) provided a detectable quantity of energy is propagated at high frequencies (e.g. up to 10 MHz).

1.2 The Rayleigh Wave - Continuous Wave Analysis

Lord Rayleigh showed how the equation describing the propagation of Rayleigh waves might be obtained by solving the bulk wave equations subject to the boundary conditions imposed by the surface.

Without loss of generality one can choose the wave direction to be such that there is no particle motion in the y direction (Fig.1) (assuming propagation in an elastically isotropic medium). This converts it from a three dimensional to a two dimensional problem.

It is assumed that propagation is in the +x direction (Fig.1). The assumption of zero attenuation is vindicated in the section above. Since the problem has been made two dimensional the two wave equations to be satisfied are:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{C_L^2} \frac{\partial^2 \phi}{\partial t^2}$$

and

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{C_S^2} \frac{\partial^2 \psi}{\partial t^2} \quad \dots (1)$$

where ϕ is the longitudinal (or dilatational) displacement potential,

ψ is the shear (or distortional) displacement potential,

C_L is the longitudinal wave velocity,

and C_S is the shear wave velocity.

The associated components of displacement can be derived from the field potentials using the relations:

$$u_x = \left(\frac{\partial \phi}{\partial x} \right) + \left(\frac{\partial \psi}{\partial z} \right) \quad \dots (2)$$

$$u_z = \left(\frac{\partial \phi}{\partial z} \right) - \left(\frac{\partial \psi}{\partial x} \right).$$

Now, since the Rayleigh wave is assumed to be composed of longitudinal and shear components, and since, too, the wave amplitude is known to be a function of depth from the surface, it is reasonable to assume, as the solutions to equations (1), equations of the form:

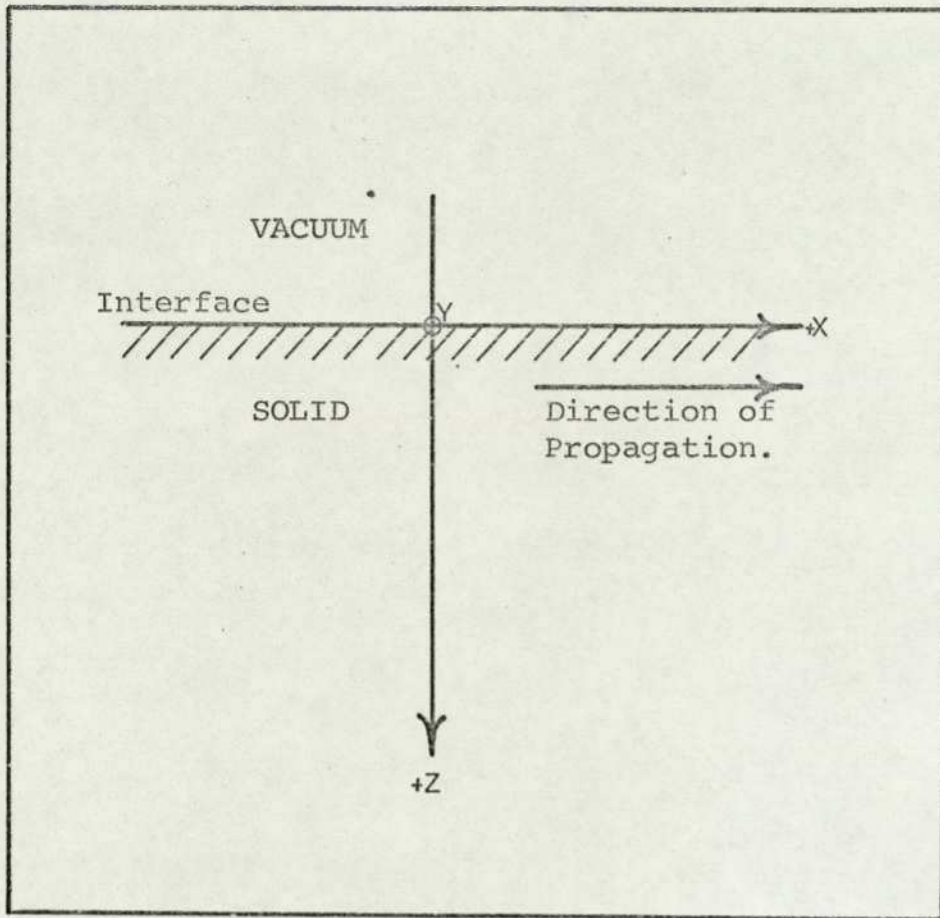


Fig .1. Coordinate system used in the analysis of the Rayleigh wave.

$$\phi = f(z) \exp\{-i(\omega t - kx)\} \quad \dots (3)$$

$$\psi = g(z) \exp\{-i(\omega t - kx)\}$$

where f and g are the functions describing the depth variation of the potentials. They can be obtained by substituting equations (3) into (1), e.g. for ϕ :

$$\begin{aligned} & -k^2 f(z) \exp\{-i(\omega t - kx)\} + f''(z) \exp\{-i(\omega t - kx)\} \\ & = \{f(z)/C_L^2\} \{-\omega^2 \exp(-i(\omega t - kx))\} \end{aligned} \quad \dots (4)$$

where a prime indicates differentiation with respect to the argument. Equation (4) reduces to

$$f''(z) + \{(\omega^2/C_L^2) - k^2\} f(z) = 0 \quad \dots (5)$$

which has the solution:

$$\begin{aligned} f(z) = & A \exp\left\{i\left((\omega^2/C_L^2) - k^2\right)^{\frac{1}{2}}z\right\} \\ & + A_1 \exp\left\{-i\left((\omega^2/C_L^2) - k^2\right)^{\frac{1}{2}}z\right\}. \end{aligned} \quad \dots (6)$$

Since the second term describes a wave whose amplitude increases with increasing z we set this term to zero i.e. $A_1=0$, so that

$$f(z) = A \exp\left\{i\left((\omega^2/C_L^2) - k^2\right)^{\frac{1}{2}}z\right\}. \quad \dots (7)$$

Writing the Rayleigh wave velocity as $C_R = (\omega/k)$ equation (7) may be written as:

$$f(z) = A \exp\left\{ik\left((C_R/C_L)^2 - 1\right)^{\frac{1}{2}}z\right\}. \quad \dots (8)$$

In a similar way, $g(z)$ may be derived as:

$$g(z) = B \exp \left[\left(ik \left((C_R/C_S)^2 - 1 \right)^{\frac{1}{2}} \right) z \right]. \quad \dots (9)$$

Substituting (8) and (9) into (3) gives the field potentials:

$$\begin{aligned} \phi &= A \exp \left[-i \left(\omega t - kx - k \left((C_R/C_L)^2 - 1 \right)^{\frac{1}{2}} z \right) \right] \\ \psi &= B \exp \left[-i \left(\omega t - kx - k \left((C_R/C_S)^2 - 1 \right)^{\frac{1}{2}} z \right) \right]. \end{aligned} \quad \dots (10)$$

It now remains to fit these to the boundary conditions at the surface, namely, that the normal and shear stresses on the surface must be zero. The stress components are given by:

$$\begin{aligned} \sigma_{zx} &= \mu \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) \\ \sigma_{zz} &= (\lambda + 2\mu) \frac{\partial u_z}{\partial z} + \lambda \left(\frac{\partial u_x}{\partial x} \right) \end{aligned} \quad \dots (11)$$

where λ and μ are the Lamé elastic constants of the medium, and where normal tensor subscript notation is used for the stress elements. Combining equations (11) with those of (3) and (10) yields:

$$\begin{aligned} \left\{ \left(2 - (C_R/C_S)^2 \right) A \right\} + \left\{ 2 \left((C_R/C_S)^2 - 1 \right)^{\frac{1}{2}} B \right\} &= 0 \\ \left\{ 2 \left((C_R/C_L)^2 - 1 \right) A \right\} + \left\{ \left(2 - (C_R/C_S)^2 \right) B \right\} &= 0. \end{aligned} \quad \dots (12)$$

The non-zero solutions to these equations are found by putting the determinant of (12) equal to zero, giving

$$\left(2 - (C_R/C_S)^2 \right)^2 = 4 \left\{ \left(1 - (C_R/C_L)^2 \right) \left(1 - (C_R/C_S)^2 \right) \right\}^{\frac{1}{2}}$$

which may be re-written as:

$$\left(\frac{C_R}{C_S}\right)^2 \left\{ \left(\frac{C_R}{C_S}\right)^6 - 8 \left(\frac{C_R}{C_S}\right)^4 + C_R^2 \left\{ \frac{24}{C_S^2} - \frac{16}{C_L^2} \right\} - 16 \left\{ 1 - \left(\frac{C_S^2}{C_L^2}\right) \right\} \right\} = 0$$

.... (13)

this being the 'Rayleigh Equation'. The Rayleigh Wave velocity is seen here to be dependent only on C_L and C_S and hence on the elastic constants of the medium. It is not a function of frequency i.e. the wave is non-dispersive.

Equation (13), being a cubic in C_R^2 , has six roots, the Rayleigh wave corresponding to the root which satisfies the condition $1 > (C_R/C_S) > 0$. For all real media Poissons Ratio, (ν), always lies in the range 0 to 0.5, and this condition ensures that only one root satisfies the condition for the ratio (C_R/C_S) . This can readily be seen by taking, as an example, a medium satisfying Poisson's relation i.e. $\lambda = \mu$, corresponding to a value of Poisson's Ratio of 0.25, typical for many metals. Since the shear and longitudinal wave velocities are related by the equation

$$\frac{C_S}{C_L} = \left\{ \frac{\mu}{\lambda + 2\mu} \right\}^{\frac{1}{2}} = \left\{ \frac{1 - 2\nu}{2 - \nu} \right\}^{\frac{1}{2}}$$

.... (14)

then, in this case

$$C_L = \sqrt{3} C_S.$$

.... (15)

Substituting (15) into (13) gives

$$\left(\frac{C_R}{C_S}\right)^6 - 8 \left(\frac{C_R}{C_S}\right)^4 + \frac{56}{3} \left(\frac{C_R}{C_S}\right)^2 - \frac{32}{3} = 0$$

.... (16)

which has the three real roots

$$\left\{ \frac{C_R}{C_S} \right\}^2 = 4, \quad 2 + \frac{2}{\sqrt{3}}, \quad 2 - \frac{2}{\sqrt{3}}.$$

The first two solutions require that the values of A and B, in equations (12), be imaginary, so cannot be valid. Thus, in this case

$$C_R = C_S \times 0.9194. \quad \dots (17)$$

In general, if $v \neq 0.25$, then an approximate value for the root of equation (13) can be obtained using the equation below, quoted from Reference 18.

$$\left\{ \frac{C_R}{C_S} \right\} = \left\{ \frac{0.87 + 1.12v}{1 + v} \right\} \quad \dots (18)$$

The particle displacements associated with the wave in the medium with $\lambda = \mu$, can be obtained by substitution of (10) into (2), having regard to equation (17). They are:

$$\begin{aligned} u_x &= A k \left(\exp(-0.8475kz) - 0.5773 \exp(-0.3933kz) \right) \sin(\omega t - kx) \\ u_z &= A k \left(1.4679 \exp(-0.3933kz) - 0.8475 \exp(-0.8475kz) \right) \cos(\omega t - kx) \end{aligned} \quad \dots (19)$$

It can be seen from these expressions, that both components decay with depth, in an exponential manner, the rate of fall-off being greater for higher frequencies. The particle motion, being composed of sine and cosine waves at right angles (geometrically), has an elliptical path. Since the sign of u_x changes at a depth of 0.19 of a wavelength, the particle motion also reverses in the way shown in Fig.2 (and Ref.8).

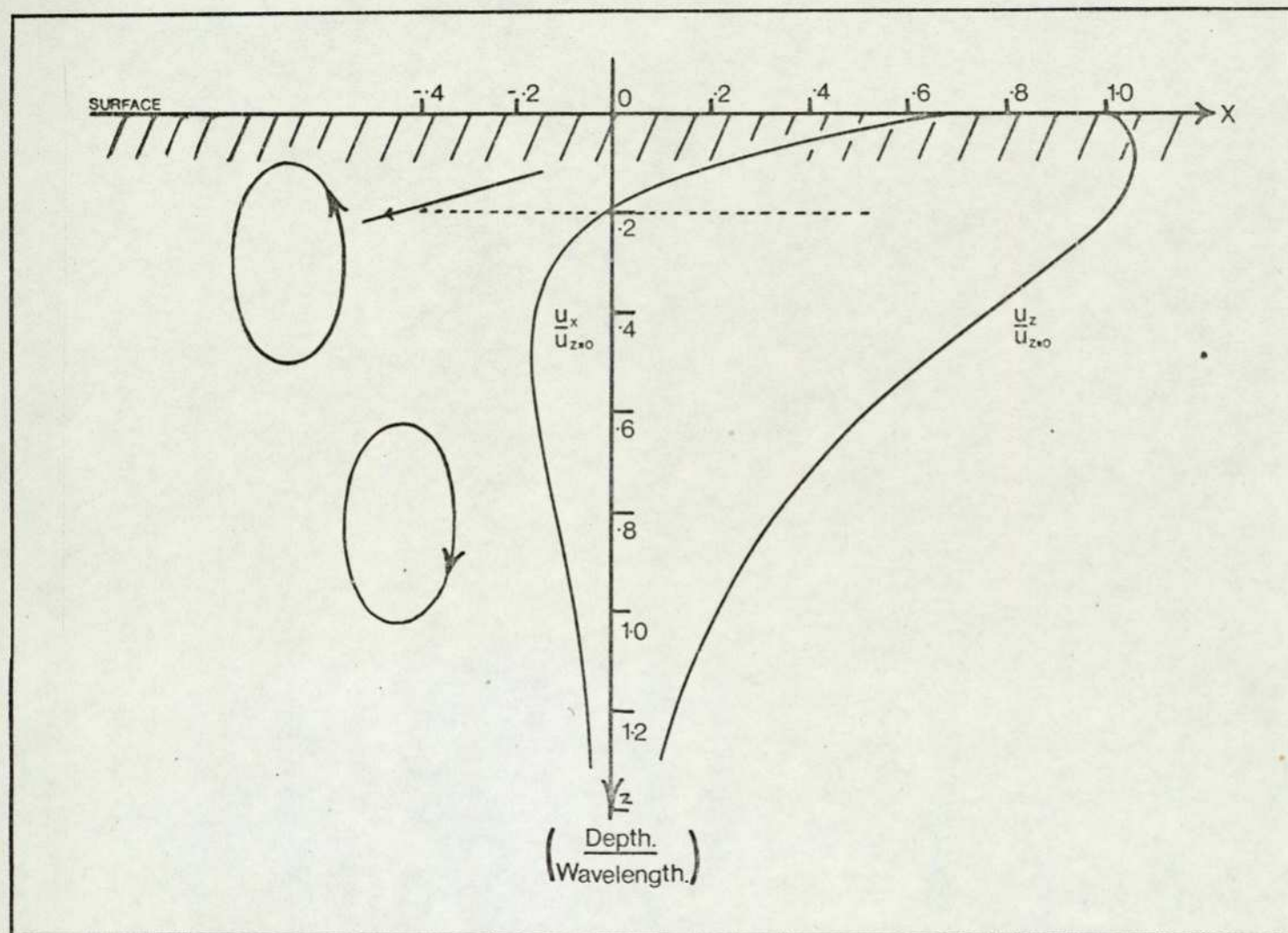


FIG.2. Variation with depth of the normal and shear particle motions of the Rayleigh wave, also showing the particle paths in the two regions above and below a depth of 0.2λ .

Perhaps the most important feature to note in the analysis is that almost the whole of the energy of the wave is contained in a layer next to the surface of thickness only one or two wavelengths. For this reason the Rayleigh wave is ideally suited for use in the detection of surface and near-surface features such as cracks, inclusions, voids etc. by the method of ultrasonic reflectometry.

The analysis provides us with a physical picture of the Rayleigh wave as being a longitudinal and shear wave balanced at the surface to provide the necessary boundary conditions. If such a wave is incident on a defect in the surface, then it is required to satisfy a new set of boundary conditions. This separates the longitudinal and shear components which are scattered separately, some of each back along their original path, recombining, perhaps, to give the reflected Rayleigh wave. Thus, in addition to reflecting some of the incident Rayleigh wave as a reflected Rayleigh wave, a quantity of both longitudinal and shear bulk waves will also be generated. This phenomenon is termed mode conversion, the Rayleigh mode giving the other modes of propagation.

The stress field in the region of the Rayleigh wave can be found from the analysis described above by combining equations (2) and (10) with the equations for stress in the z direction given in (11), together with the corresponding ones for that in the x direction. This procedure will not be elaborated here as the case for which the stress distribution will later be required will be that of a transient Rayleigh wave excited from an explosive source. Adaptation of the continuous wave treatment to

Adaption

the transient case is more complicated than the development of a separate analysis. This analysis is given in the following section.

1.3 Transient Rayleigh Wave Analysis

The continuous wave solution presented so far presents difficulties when applied to a transient wave. Transient wave analysis by this method involves combining the harmonic solutions for each of the spectral components of the transient.

A more direct method has been developed for use with explosively generated surface waves (Ref.9). This procedure will be briefly redeveloped below.

This analysis involves evaluating the stress distribution arising from a time varying line source, as shown in Fig.3, which can be approximated to a two-dimensional plate with a point source at the edge. Since the point of application of the force is chosen as the origin of the coordinate system, then the boundary conditions are:

$$\sigma_{zz}(x,0,t) = -P(t)\delta(x) \text{ for } t > 0$$

$$\text{and } \sigma_{xz}(x,0,t) = 0 \quad \dots (20)$$

where $\delta \equiv$ Dirac delta function

and $P(t)$ is the time varying force applied.

The method of this solution is identical with that in section 1.2 i.e. looking for solutions to (1) of the form

$$\phi(t - (x/c), z) \text{ and } \psi(t - (x/c), z)$$

subject to the boundary conditions. It differs from that in 1.2

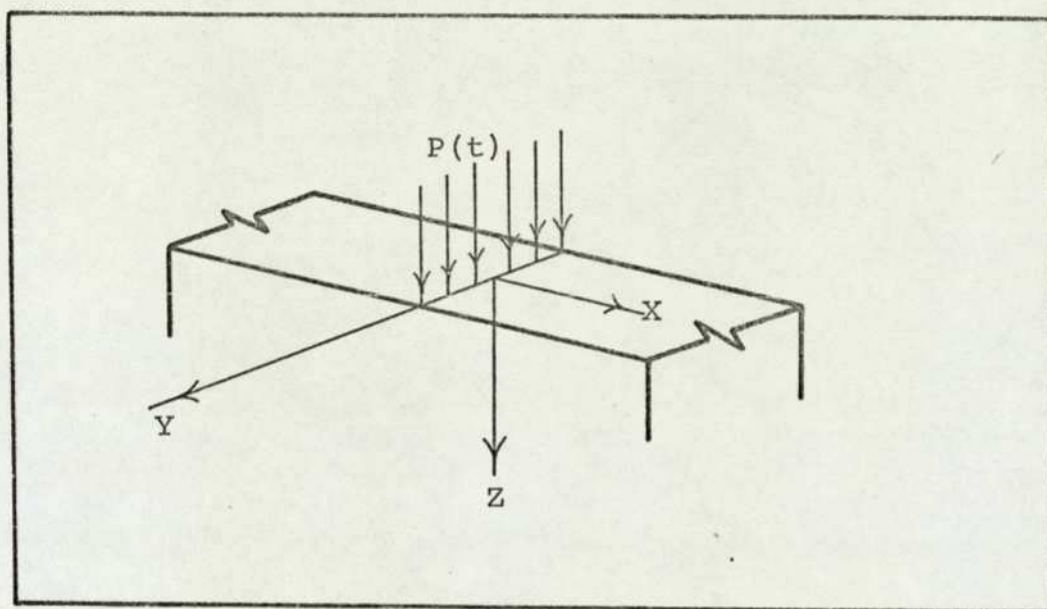


Fig.3. Line source used as a model of an explosive excitation.

in that it proves convenient to convert, from a coordinate system referred to the medium, to one referred to the Rayleigh wavefront. This simplifies the wave equations to Laplacians. That is to say, by converting thus:

$$\xi = t - (x/c), \quad \eta_j = \alpha_j z \quad \dots (21)$$

$$\text{where } \alpha_j = C_R^{-2} - C_j^{-2}$$

(j signifying either L or S, for the longitudinal and shear velocities)

the wave equations, (1), become

$$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta_1^2} = 0$$

and

$$\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta_2^2} = 0 \quad \dots (22)$$

- two Laplace equations, with general real solutions

$$\phi(\xi, \eta_1) = \Re \phi(\zeta_1)$$

$$\psi(\xi, \eta_2) = \Im \psi(\zeta_2) \quad \dots (23)$$

where $\Re \equiv$ Real part of

$\Im \equiv$ Imaginary part of

$$\zeta_j = \xi + i \eta_j$$

and where Φ and Ψ are required to be regular functions of

$$\eta_j > 0.$$

The relationships between these functions, for this case, are determined by substituting the general solutions, (23), into (2) and (11) and requiring conditions (20) be met. This procedure yields:

$$\begin{aligned}
 \Psi(\xi) &= \frac{C_R}{2\alpha_2} \left\{ \alpha_2^2 + \frac{1}{C_R^2} \right\} \Phi''(\xi) \\
 &= \frac{2\alpha_1}{C_R} \left\{ \alpha_2^2 + \frac{1}{C_R^2} \right\}^{-1} \Phi''(\xi) \quad \dots (24)
 \end{aligned}$$

$$\text{so that } (\alpha_2^2 + C_R^{-2})^2 = 4\alpha_1\alpha_2 C_R^{-2} \quad \dots (25)$$

will give the value of C_R . Equation (25) is another form of the Rayleigh equation. The particle displacements and stresses may now be calculated. They are:

$$\begin{aligned}
 u_x &= -C_R \Re(\Phi'(\zeta_1) - \frac{1}{2} C_R^2 (\alpha_2^2 + C_R^{-2}) \Phi'(\zeta_2)) \\
 u_z &= -\alpha_1 \Im(\Phi'(\zeta_1) - 2C_R^{-2} (\alpha_2^2 + C_R^{-2})^{-1} \Phi'(\zeta_2)) \quad \dots (26)
 \end{aligned}$$

and:

$$\sigma_{xx} = \mu \Re((\alpha_2^2 - 2\alpha_1^2 - C_R^{-2}) \Phi''(\zeta_1) + (\alpha_2^2 + C_R^{-2}) \Phi''(\zeta_2)) \quad (a)$$

$$\sigma_{zz} = \mu (\alpha_2^2 + C_R^{-2}) \Re(\Phi''(\zeta_1) - \Phi''(\zeta_2)) \quad (b)$$

$$\sigma_{xz} = 2\mu\alpha_1 C_R^{-1} \Im(\Phi''(\zeta_2) - \Phi''(\zeta_1)) \quad (c)$$

..... (27)

These equations give the stress distribution as a function of Φ , which, for our case, can be obtained from the relation for the tangential stress at the surface derived in Ref.10 i.e.

$$\begin{aligned}
 \sigma_{xx}(z=0) &= \lambda \frac{dP}{dt} (t - (x/C_R)) \\
 &= \lambda P'(\xi) \quad \dots (28)
 \end{aligned}$$

λ being a constant for the medium. Equations 27(a) and (28) give:

$$\mu\Phi'(\xi) = \frac{1}{2}\lambda(\alpha_2^2 - \alpha_1^2)^{-1} P(\xi) \quad \dots (29)$$

with solution:

$$\mu\Phi'(\zeta_j) = i\lambda\{2\pi(\alpha_2^2 - \alpha_1^2)\}^{-1} \int_{-\infty}^{\infty} (\zeta_j - \tau)^{-1} P(\tau) d\tau \quad \dots (30)$$

This, then, provides the function Φ' to use with equations (27) to generate the stress field. It remains merely to insert the time variation of the applied force to yield an explicit solution.

1.3.1 Explosively excited transient

The case in which the Rayleigh Wave is excited by an explosive charge (PbN_6) is now considered. The charge is confined to a semicylinder of a small radius with respect to a wavelength, so that it may be considered a line excitation as in Fig.3. The time variation of such a force can be approximated by the function:

$$P(t) = \frac{1}{2}A \sum_{n=1}^4 C_n (t - t_n)^2 H(t - t_n) \quad \dots (31)$$

where A is a constant,

$$H(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t > 0 \end{cases}$$

and $C_1 = 1$

$$C_2 = -\tilde{t}/(\tilde{t} - t_2)$$

$$C_3 = t_2(t_4 - \tilde{t})/((t_4 - t_3)(\tilde{t} - t_2))$$

$$C_4 = -t_2(t_3 - \tilde{t})/((t_4 - t_3)(\tilde{t} - t_2))$$

$$\tilde{t} = t_3 t_4 / (t_4 + t_3 - t_2)$$

This form has the advantage of having a simple basic shape i.e. the parabola, with subsequent parabolae of differing

parameters being added sequentially to build up the overall shape (Fig.4). Therefore, instead of a complex polynomial we have only to deal with a parabola.

Combining (31) with (30) and introducing the relation

$\lambda A = B\pi c/a$, to nondimensionalise the variables, yields:

$$\mu\Phi'(\zeta_j) = \frac{1}{4}iB(\alpha_2^2 - \alpha_1^2)^{-1} \sum_{n=1}^4 C_n ca^{-1}(\zeta_j - t_n)^2 \ln(\zeta_j - t_n) \quad \dots (32)$$

and since it can be show that

$$\sum_{n=1}^4 C_n t_n^s = 0 \quad \text{for } s = 0, 1 \text{ or } 2$$

then (32) can be written:

$$\mu\Phi''(\zeta_j) = \frac{iB}{2} (\alpha_2^2 - \alpha_1^2)^{-1} \sum_{n=1}^4 C_n ca^{-1}(\zeta_j - t_n) \ln\{ca^{-1}(\zeta_j - t_n)\}. \quad \dots (33)$$

This completes the analysis of Ref.9.

The two results of importance are equations (33) and (27), since they are amenable to programming on an electronic computer for the evaluation of the quantities σ_{xx} , σ_{xy} , σ_{yy} .

1.3.2 Stress field computation

The program written by the author to compute the stresses comprising the explosively generated Rayleigh wave discussed above, is described in Appendix (1). The flow diagram presented there represents the evaluation of the three stress components for any point in the

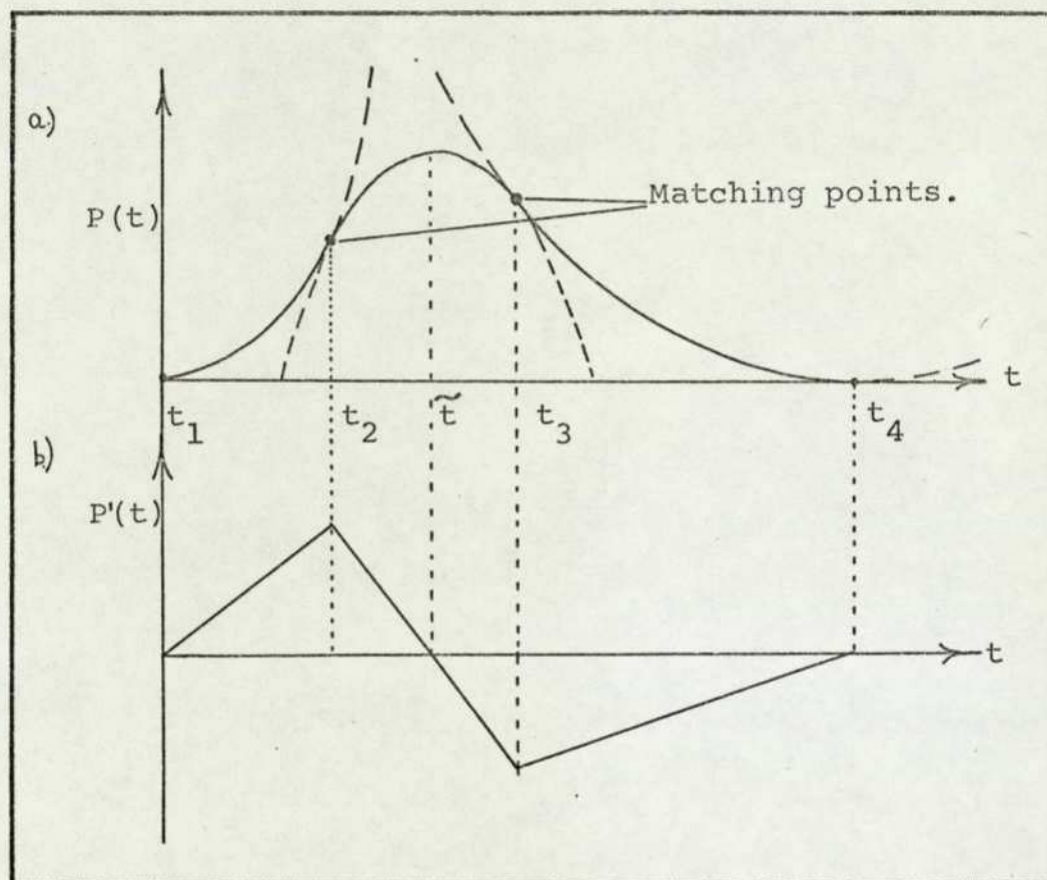
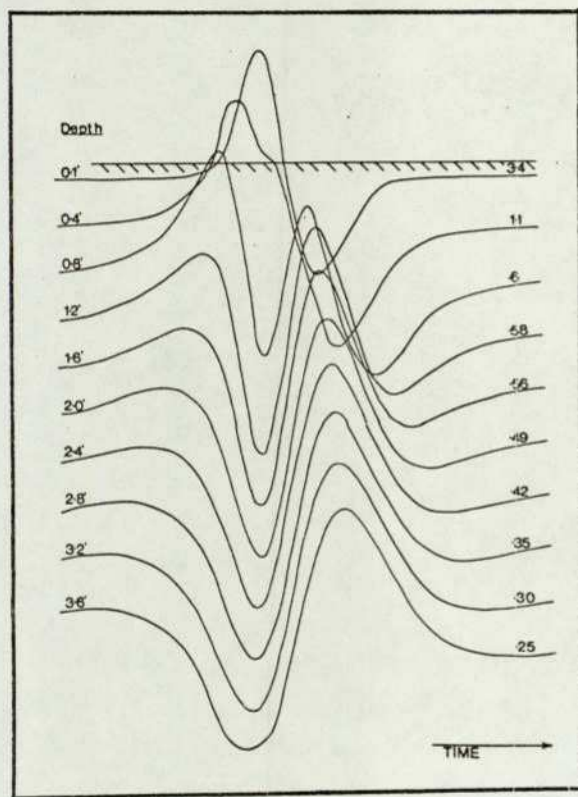


FIG.4. The time variation(a), and its derivative(b), of an idealised explosive excitation.

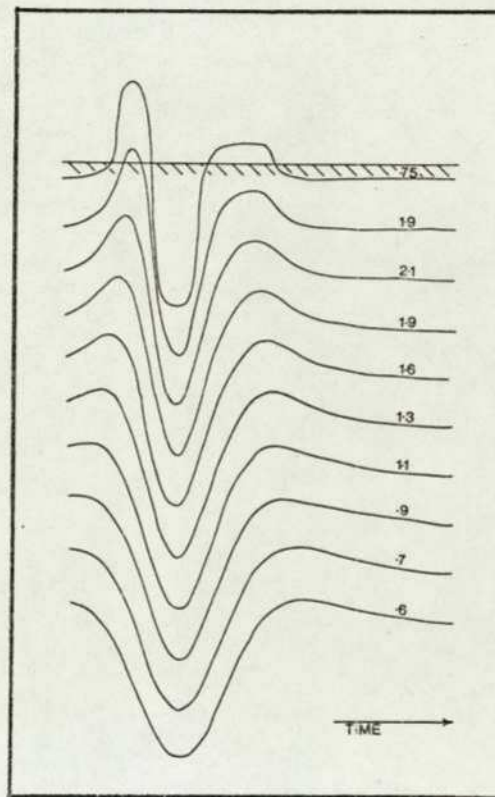
medium defined by the depth, time and distance from the source in the direction of propagation. Since the process must be repeated for many points to generate the time signal at any particular depth, the core of the program (between points (1) and (2) on the diagram) must be as short and efficient as possible. The listing presented represents a routine actually used in this way. Written in 1900 Fortran for use on the I.C.L. 1905 machine of The City University, this program computes the stresses at a point as a function of time for various depths. The program is designed to plot out this information in graphical form together with the spectra of the signals. Since they are of general usefulness, the Fast Fourier Transform Subroutine and plotting subroutine, are also listed (by permission of Mr. E. A. Lloyd and Mr. D. G. Tasker of this University).

The results of this program are given in Figs.(5) and (6). They show the variation of the components of the stress wave with depth. In all cases they are calculated for the far field case ($x \gg \lambda$). The only stress at the surface, σ_{xx} , corresponds to the derivative of the applied stress, shown in Fig.(4).

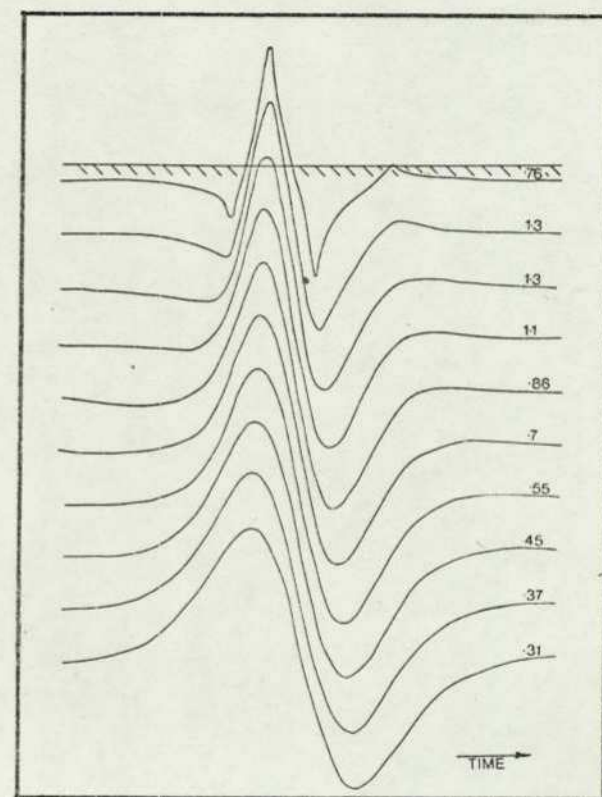
The scales of the horizontal axes of Figs.(5) and (6) are not marked since the two are interrelated and the results are equally applicable for any timescale. To provide an initial point from which the results for any other frequency range may be scaled it can be said that for the values of the timescale used in Ref.9 i.e.



(a)



(b)



(c)

FIG.5. The stresses (a) σ_{xx} , (b) σ_{xy} , (c) σ_{yy} , as functions of time, at the depths indicated.

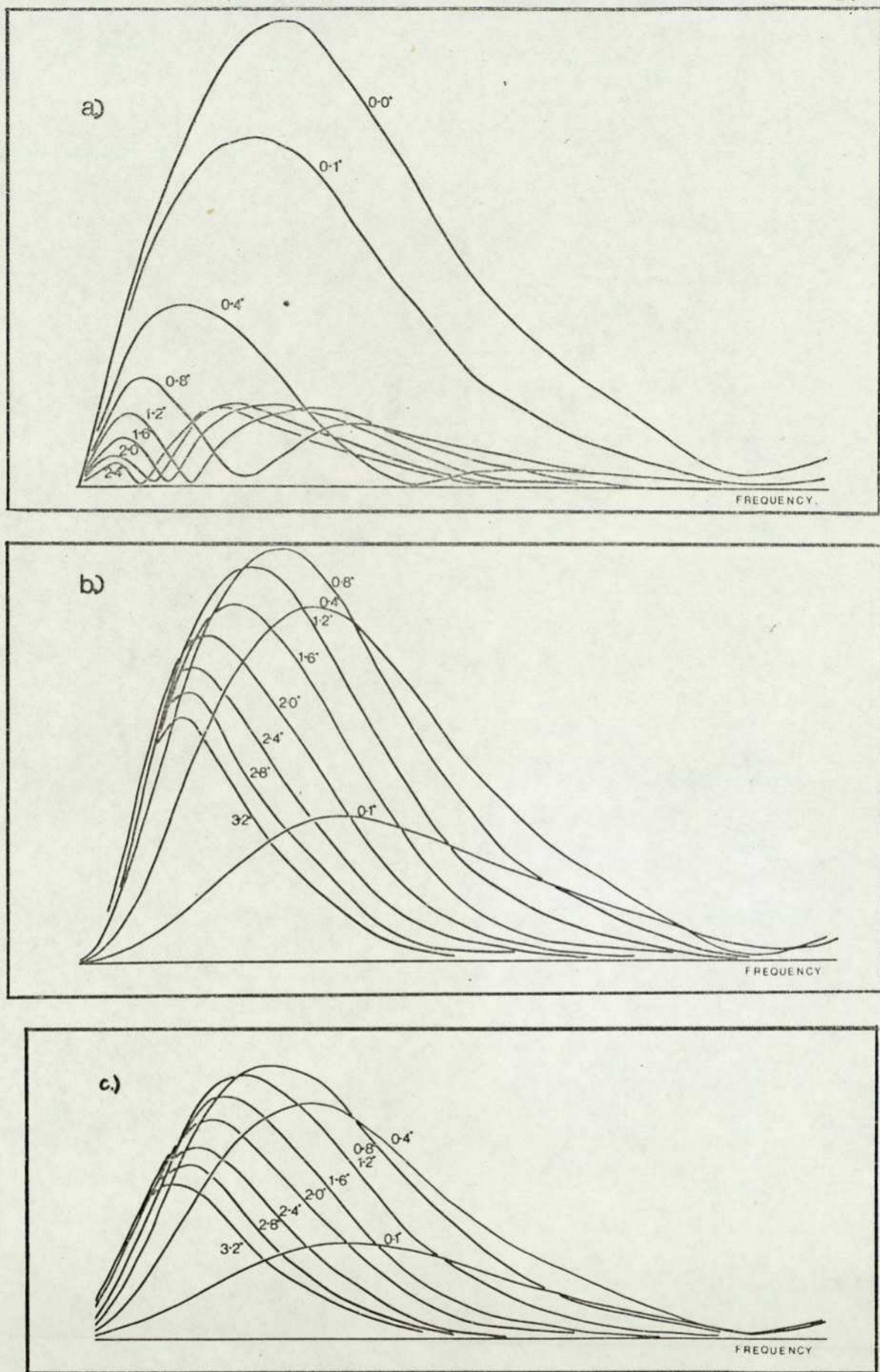


FIG.6. The spectra of the waveforms of Fig.5., showing:
 (a) σ_{xx} (b) σ_{xy} (c) σ_{yy} .

$$t_1 = 0 \quad \text{and} \quad t_4 = 73.4 \mu\text{s}$$

and a Rayleigh velocity of 3,700 inches per second then the maximum in the spectrum for the stress σ_{xx} at $x = 0$, lies at a frequency of 13.2 kHz. The normal stress in the x-direction exhibits a number of interesting features. It can be seen how, at a depth between 0.4 inches and 0.8 inches the wave breaks from a single to a double peak. Since the central frequency of the wave is at a wavelength (λ_0) of about three inches these depths correspond to $0.13\lambda_0$ and $0.26\lambda_0$ respectively. The wave breakup can be attributed to the reversal in sign of the stress at a depth of $0.2\lambda_0$ (see Fig.2). Hence, between the levels $0.13\lambda_0$ and $0.26\lambda_0$ the higher frequencies reverse polarity with respect to the low frequencies, and since the high frequencies are, predominantly, in the leading edge of the time signal and the low frequencies in the tail, the signal breakup shown occurs.

All the signals show a significant lengthening in time as the depth increases. This is due to the loss of the high frequency components in the waves (see following section).

Values to the right of all signals are comparative amplitudes, that for the stress at the surface being 5.8 units. From them it can be seen that there is a subsidiary peak in the overall stress of the waves at a depth of about 0.3λ . This peak was first pointed out by Thau and Dally in Ref.(17). It is important to take the existence of this peak into account when analysing the

reflection of a Rayleigh wave from surface features and not merely to deal with the surface amplitude as many authors have done.

Figure (6) shows the spectra of the time signals of Fig.(5). Fig.6(a) shows the spectrum of the normal stress in the direction of propagation. The null at 0.2λ is clearly shown, progressing to lower frequencies at greater depths.

The growth to a maximum and subsequent gradual fall-off in the peak stresses σ_{xz} , σ_{zz} as we progress deeper into the medium, is exhibited in 6(b) and (c), together with the loss of the high frequency components with increasing depth i.e. the higher the frequency the greater its confinement near the surface.

While the analyses of this chapter give an understanding of the propagation characteristics of Rayleigh waves which will prove useful in the chapters immediately following, the results of the stress distribution calculations are more directly relevant to the work of Chapter 5 where the reflection of a Rayleigh wave from a slot is treated theoretically.

CHAPTER 2

2. PRODUCTION OF BROAD BAND RAYLEIGH WAVES

2.1 Introduction

2.2 Wedge Transducers

2.3 Interdigital Transducers

2.3.1 Characterisation of the spectral responses of the contact mode I.D. transducers

2.4 Theory Of The Two Finger Transducer

2.5 Choice Of Transducer Dimensions

2. PRODUCTION OF BROAD BAND RAYLEIGH WAVES

2.1 Introduction

It was mentioned in the previous section that Rayleigh waves can be produced by an explosion at the surface to be excited. However, this is not usually convenient and, moreover, produces waves of only low frequencies, typically twenty kilohertz. The nondestructive testing (N.D.T.) method of ultrasonic spectroscopy requires that higher frequencies than this be used, for the following reason. The technique involves transmitting an ultrasonic stress wave in the medium under test. Any defect in this medium can then be detected by detecting the portion of the stress wave reflected by the defect. In order to resolve small defects close to each other, short interrogating pulses must be used and this involves the use of high frequencies (up to 20 MHz). To achieve this, piezoelectric effect transducers are used (Ref.11). The stress wave produced by the transducer is introduced into the workpiece either by placing the transducer on its surface and achieving good transmission of the stress via a thin layer of oil (contact coupling mode), or by immersing the workpiece in a bath of liquid and transmitting the stress wave through the liquid (immersion testing). Since the attenuation of Rayleigh waves at a solid/liquid interface is very high, the reflected signals become extremely small. This makes immersion testing impracticable when using Rayleigh waves.

The work in this chapter describes the development of contact transducers for the production of wide-band Rayleigh waves. The immediate advantage of a broad band signal is the greater resolution of the time return from a defect. Narrow band time signals are long compared to wide band ones with the result that two defects close to each other may not be resolved since the signals returned from them overlap. Other advantages of large bandwidth will be described in Chapter 3.

2.2 Wedge Transducers

The most obvious means of obtaining broad band Rayleigh waves is to extend the operation of the wedge transducer (Ref.11). Briefly, this transducer operates by refraction, at the critical angle, of longitudinal waves into the Rayleigh mode (Fig.7). The refraction angle, given by Snell's Law, is

$$\sin \theta = \frac{\text{Longitudinal velocity in wedge}}{\text{Rayleigh velocity in substrate}}$$

and is not a function of frequency. Therefore, the frequency characteristic of the surface wave generated is the same as that of the longitudinal wave transducer indirectly exciting it. So, replacing the narrow band longitudinal wave transducer by one with a large bandwidth yields a broad band Rayleigh wave. At the time of this investigation, however, the broad band longitudinal wave transducer was not available. Considerable advances have been made in this direction since then (Refs.12 and 13) by members of the ultrasonics group, at The City University.

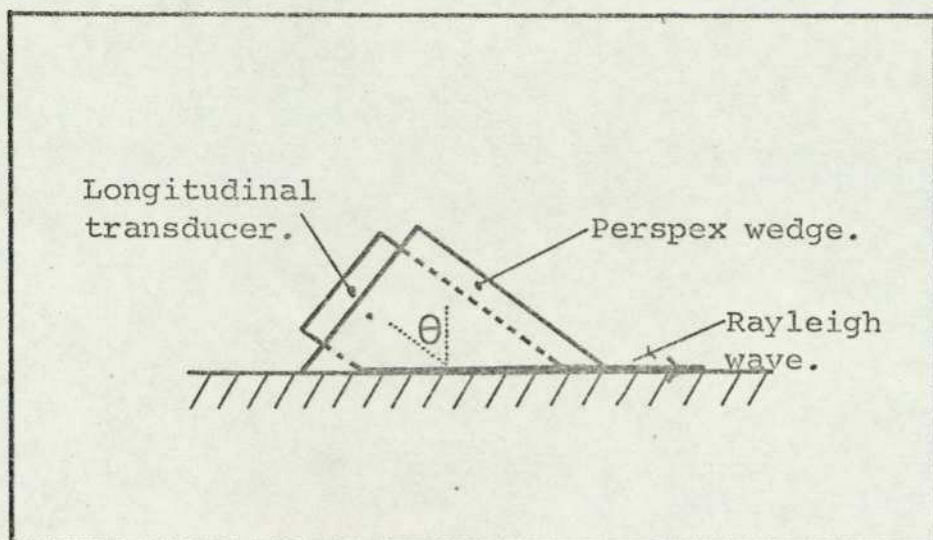


Fig.7. The wedge transducer.

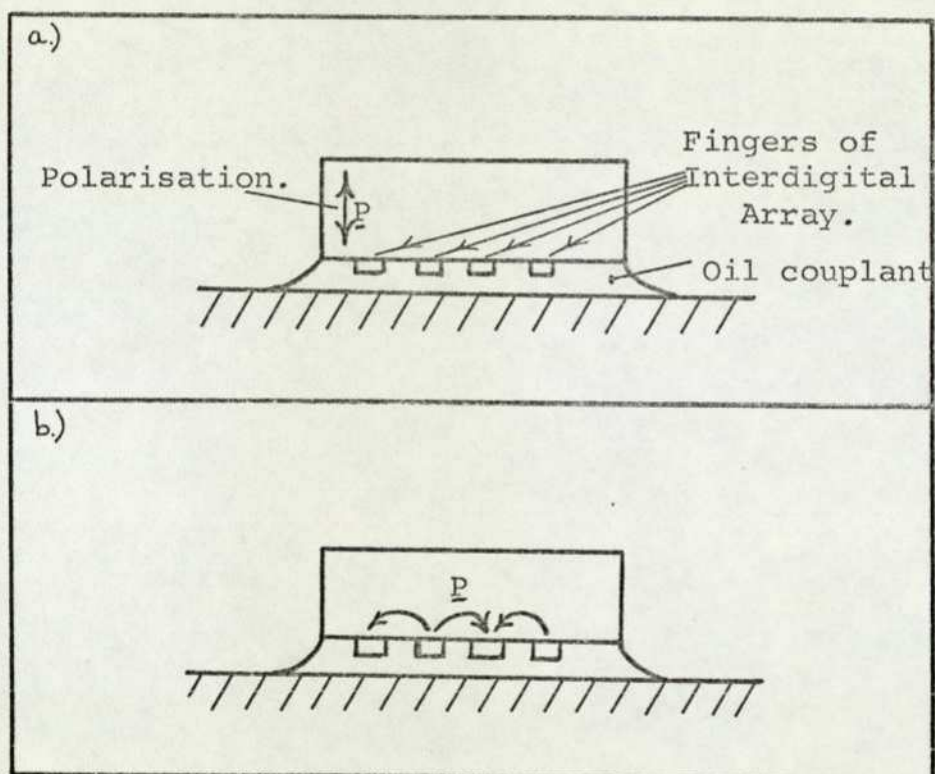


Fig.8. a) Transducer with interdigital array.
b) Transducer polarised so as to increase the output.

2.3 Interdigital Transducer

The interdigital (I.D.) transducer consists of a number of fingers, interleaved, every other finger being connected, giving two sets, these fingers being situated on the surface of a piezoelectric medium.*

The fingers are fabricated by photolithographic techniques (Ref.18). This involves drawing the finger array larger than life size and photographing this drawing. The negative of this design is then obtained. After deposition of a conducting layer (e.g. silver) onto the surface intended to carry the array, a photoresistive material is applied in a thin layer over the conducting layer. This photoresist is then exposed, through the negative of the design, to ultraviolet light, for a short time. Development of this layer leaves a pattern of resist in the design of the original, larger than life, drawing, but to the reduced size. The silver not covered by resist is then etched away and, finally, the resist is removed with a solvent. This leaves the original design, at the reduced size, in the material of the conducting layer, on the piezoelectric substrate.

If a voltage is applied between the two sets of fingers, the stresses generated in the piezoelectric produce a stress wave. Numerous signal processing devices have been constructed using the stress wave produced by this means. Since the topic has expanded rapidly in recent years, the interested reader is referred to the excellent review article of Kino and Matthews (Ref.69).

A similar device may be used for N.D.T. applications. However,

since it is rarely required to test a piezoelectric material, the piezoelectric medium is placed over the testpiece with the I.D. array between (Fig.8). The piezoelectric medium is then excited by the I.D. array, each alternate finger being connected and the signal applied between these sets.

It is found that a useful signal level is achieved by this means, even with the piezoelectric polarisation in the direction indicated in Fig.8(a). If the polarisation were changed to that shown in Fig.8(b) increased output could be expected since the piezoelectric coupling between fingers would be increased.

Two methods of achieving broad bandwidth for this sort of device can be used. The first uses the fact that, as a rule-of-thumb estimate, the bandwidth is inversely proportional to the number of finger pairs, assuming constant finger and gap widths. Use of a single pair of fingers should thus give the broadest bandwidth possible. Alternatively, it is possible to have a number of pairs with the finger and gap spacings varying along the length of the array. This is termed the stagger-tuned array. Results for these designs are shown in Fig.9, together with a block diagram of the apparatus used to obtain these results.

2.3.1 Characterisation of the spectral responses of the contact mode I.D. transducers

2.3.1.1 Apparatus

The apparatus used is shown diagrammatically in Fig.9(a), the results obtained for two array designs being given in Figs.9(b) and (c). Two transducers of the same design are used in each case.

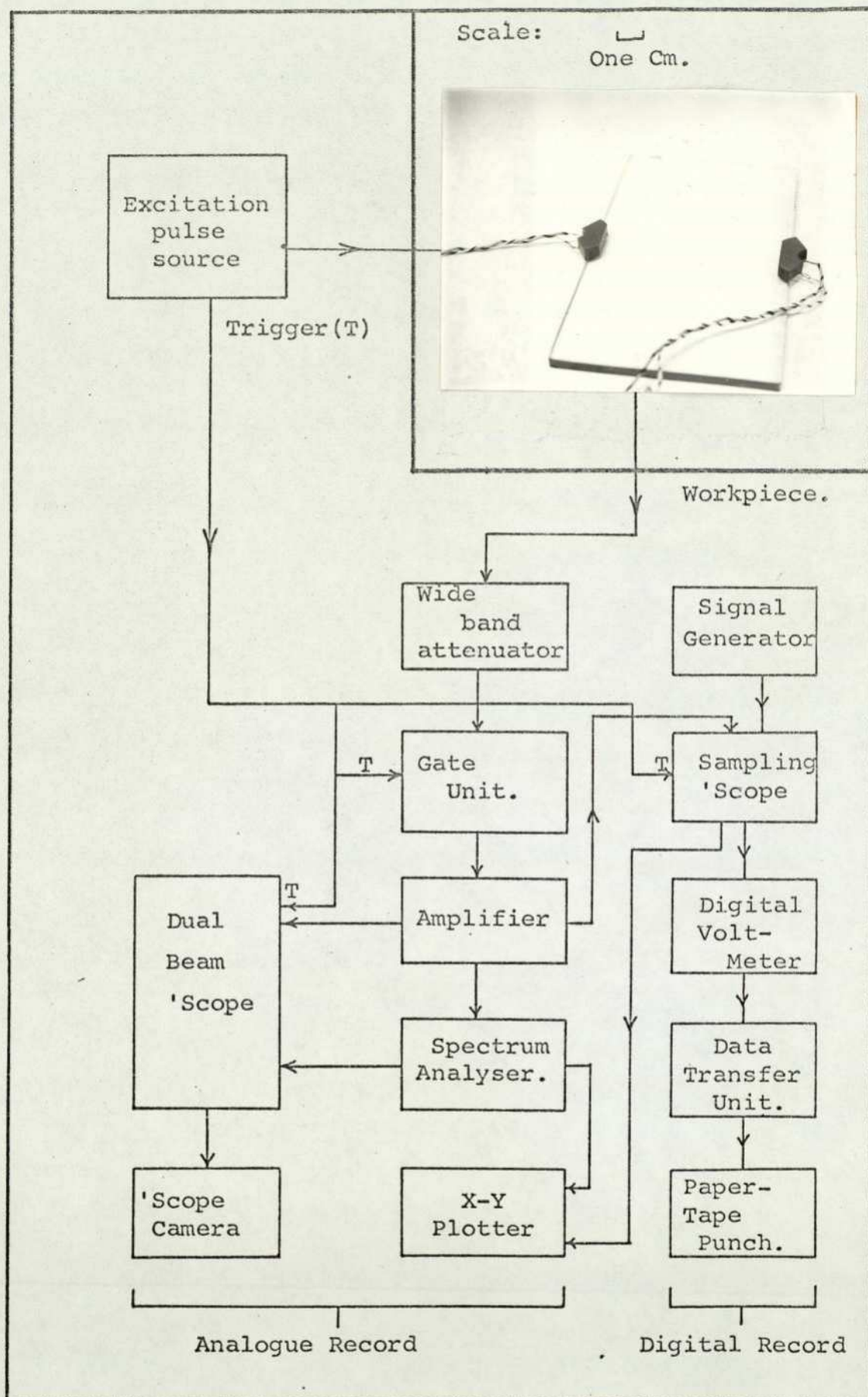


FIG.9(a). Schematic of the system used to characterise the spectral response of the transducers.

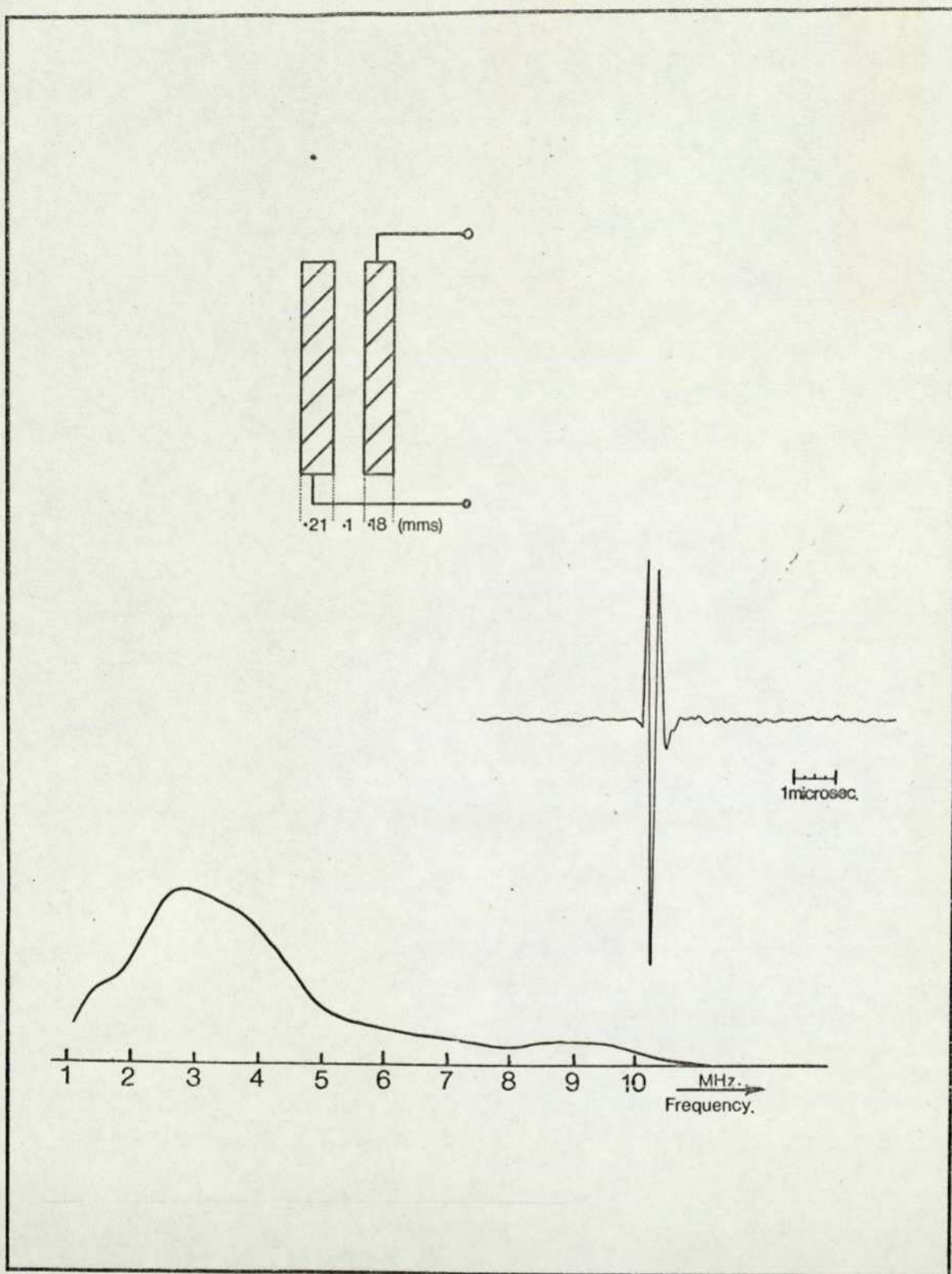


FIG.9(b). Finger geometry of a two-finger transducer;
the time signal obtained from two such
transducers;
&the spectrum of this time signal.

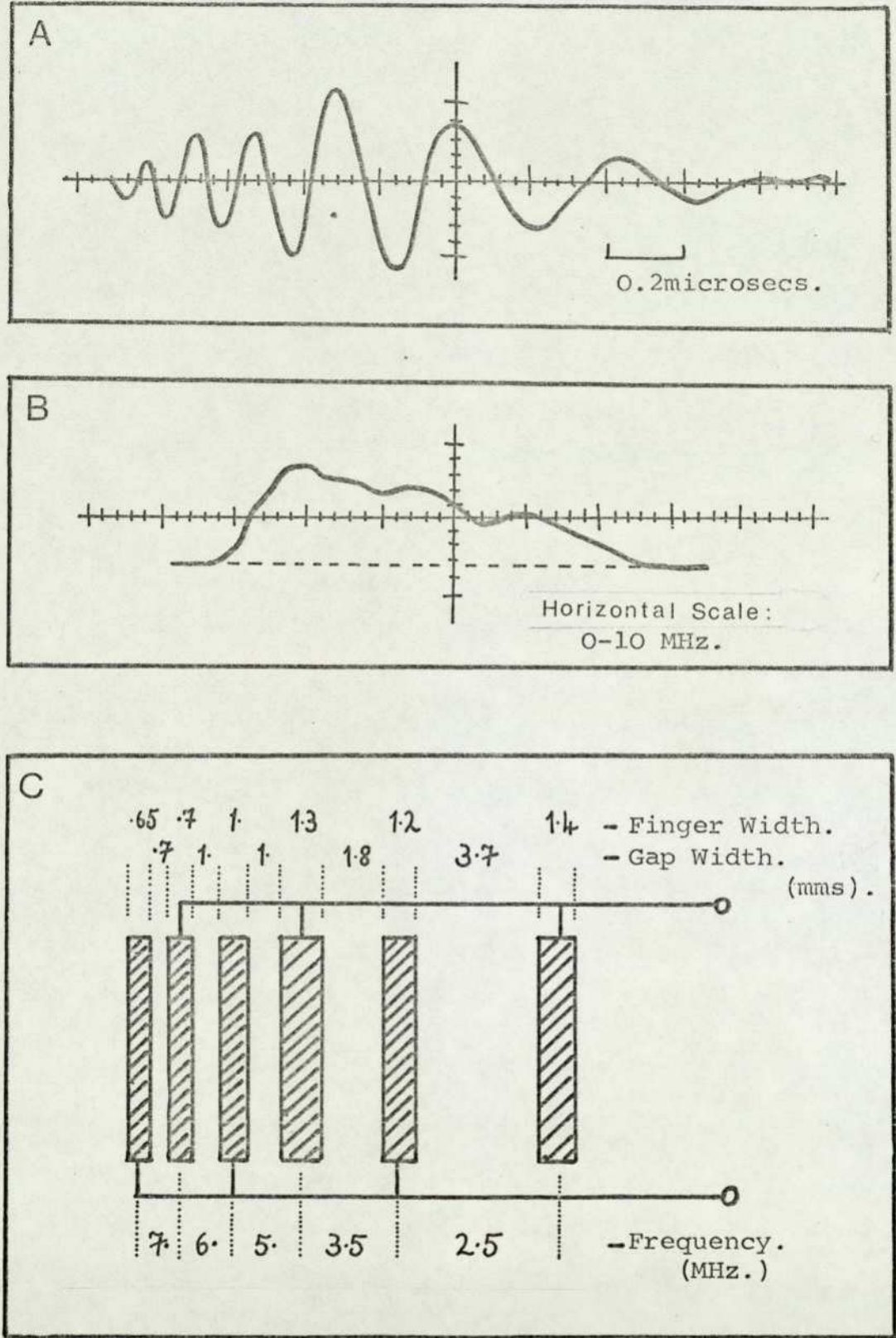


FIG. 9(c). The transmitted pulse (A), and square of the spectrum (B), for the finger geometry (C).

One transducer is excited by a high voltage, broad band, pulse obtained from a Smiths Industries 'Ultrasonic diasonograph'. The second transducer is placed so as to receive the Rayleigh wave from the first, the position being adjusted for a maximum received signal, as measured by the apparatus described in the paragraph below. The transducer axes are then assumed to be aligned.

The output from the second transducer is fed to a wide band attenuator, then to an amplifier and gate. The gate is used to select the period of time of interest in the signal. The attenuator and amplifier are used to allow for the large range of signal amplitudes found in general ultrasonic reflectography work. The attenuators used were Standard Telephones and Cables model 74600, one being variable 0-90 dBs in 10 dBs increments, the other 0-9 dBs in unit increments. The amplifier is designed to provide a gain of up to 63 dBs, operating over the frequency range $\frac{1}{2}$ -20 MHz. The gate is designed to reduce breakthrough outside the gate to less than 160 dBs below that in the gate. Its position, relative to the excitation pulse, is variable between 4.5 μ s and 0.84 ms, and its width variable between 0.6 μ s and 70 μ s.

The amplified time signal is then input to a spectrum analyser which generates the frequency spectrum of the time signal. This device operates over a bandwidth of $\frac{1}{2}$ to 20 MHz, with a linear frequency sweep (to within ± 0.1 MHz) and a resolution limit of 25 kHz. It consists of a double superheterodyne receiver, to achieve the

small resolution required. The output from the spectrum analyser could be recorded on an X-Y plotter or by photography from an oscilloscope. (The gate, amplifier and spectrum analyser were constructed in this laboratory by J. P. Weight. Full details of their construction may be found in his M.Phil thesis - reference 12.)

The time signal could be recorded digitally. Since time signals of less than a microsecond are to be recorded the signal could not be digitised in real time. However, as the signal is repetitive, a sampling unit (Tektronix 1S1 unit attached to a 545B Tektronix Oscilloscope) is used to sample the signal amplitude along its length. This amplitude is read by a digital voltmeter on a command by a data transfer unit, and this value is recorded on paper tape by a tape punch.

2.3.1.2 Results

Fig.9(b) shows the time signal and frequency spectrum, respectively, of the two finger array shown diagrammatically above them. These two signals are photographs of recordings made on an X-Y plotter (see Fig.18).

Fig.9(c), similarly, shows the results for a stagger-tuned array, the dimensions being staggered in a roughly linear fashion. The finger and gap widths are given, in millimetres, together with the synchronous frequency of each finger pair (i.e. the frequency at which the distance between the centres of the fingers correspond to one half of a wavelength). The close agreement between the designed

frequency response and the experimental response should be noted. These results are tracings from oscilloscope photographs.

The length of the time signal is seen to be dependent on the length of the array, there being a cycle of signal per finger. The second method, then, does not give us the short time signals required. The first method does give a fairly short signal, and it is this geometry which is used for the work described in Chapter 3.

The beam width of the two finger transducer was found to be similar, in the plane of the surface, to that of a vibrating piston source in a plane containing the piston axis. The beam of the transducer was probed, as shown in Fig.10(a) with a second similar transducer. The electrode directions were maintained parallel and one transducer moved along this direction. Thus, the devices being reciprocal i.e. having the same characteristics on transmission and on reception, the square of the amplitude of the field over the angle they subtend, was recorded. The square root of this quantity is plotted in Fig.10(b). It is readily verified that the width of this curve is approximately in accordance with the piston source result for the angular width i.e.

$$\sin \theta = 1.22 \lambda/d$$

where d = diameter of the source

λ = wavelength propagated.

The Fresnel distance, given by $(d^2/4\lambda)$, for the transducer described above, is then 26 mm taking the central frequency to be 3 MHz.

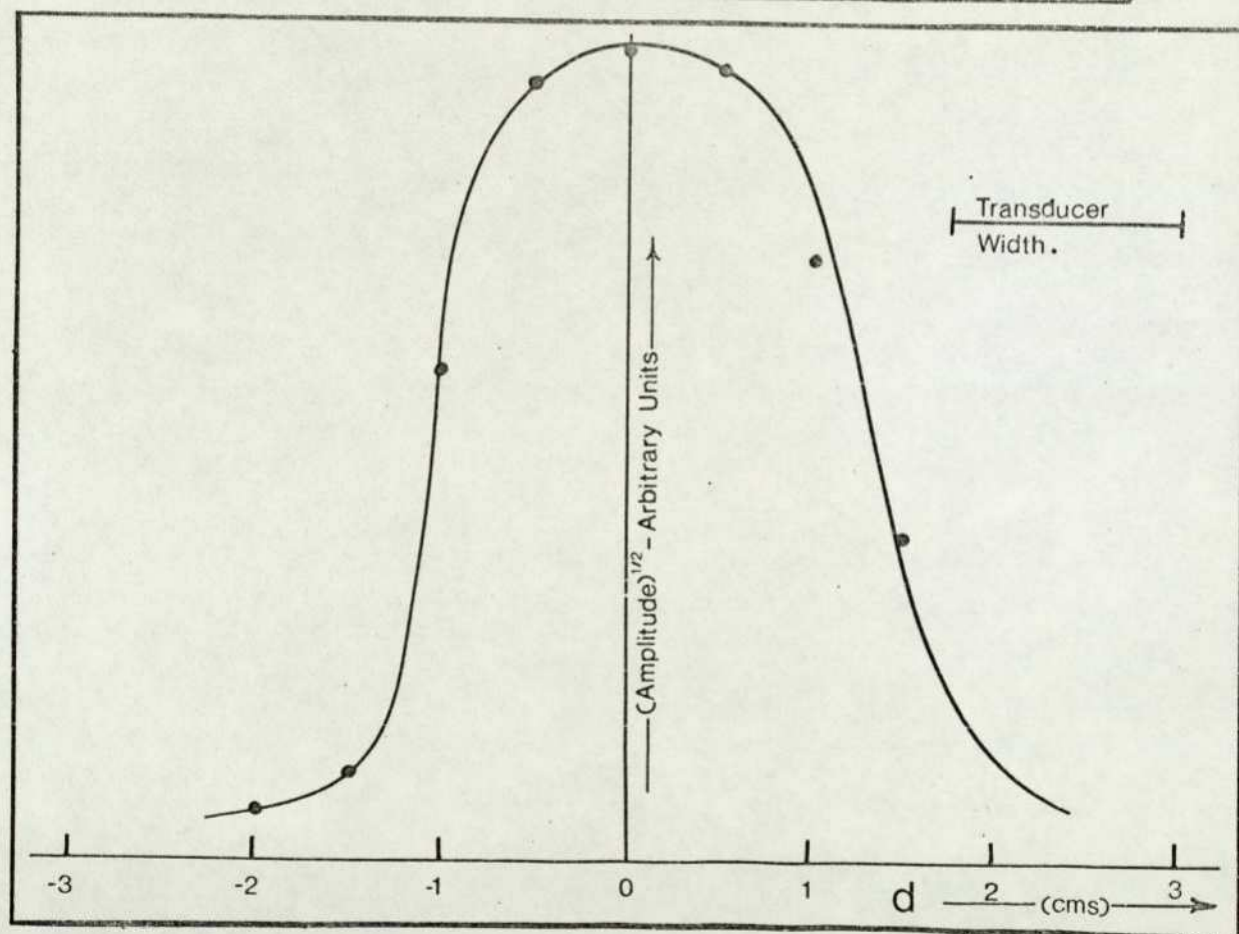
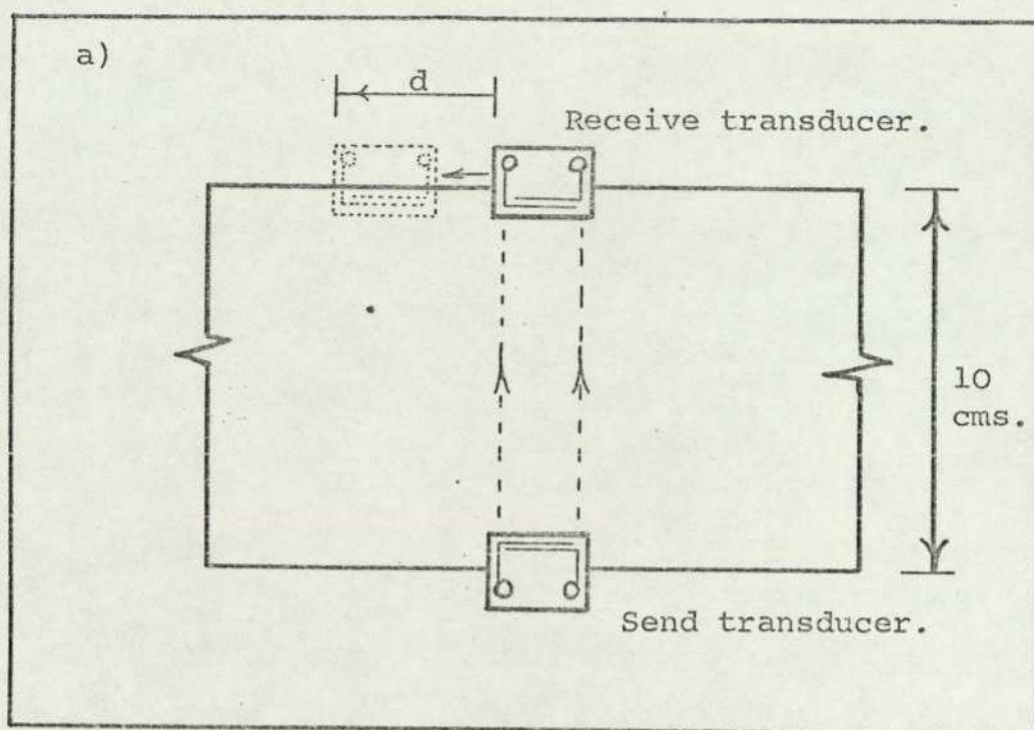


Fig.10. a) The method employed to plot the beam width characteristic.
b) Plot of the beam width characteristic.

Fig.9(c) shows that the stagger-tuned array is particularly suitable for 'tailoring' the output to that required, because of the good agreement between design and practice. It is a little more difficult to tailor the two finger design to a required output owing to the absence of a theoretical model. Such a model is developed in the next section.

2.4 Theory Of The Two Finger Transducer

Previous work on the propagation of surface waves by interdigital electrodes (Refs. 14-16) has been concerned with their generation on piezoelectric substrates. The elastic strains that constitute the surface wave are produced by the electric field within the material, generated by the voltage applied across the fingers of the transducer. The elastic strains and electric field are thus interdependent for this, the piezoelectric case. This makes the analysis very much more complicated than is necessary for a purely elastic disturbance. No allowance can be made for the effect of a coupling medium between the fingers and the substrate. In the case of excitation in non-isotropic media, the various symmetries of the propagation medium are also of importance and further complicate the theory. Rather than simplify these models, it was considered more profitable to generate a simple model from first principles.

Consider the transducer of Fig.8(a), two fingers being excited by a completely broad band signal. The fingers will move in or out of the plane of the transducer surface, depending on the polarity of the pulse and the polarisation of the medium, their displacements being 180° out of phase with each other.

Since only dilatational waves will be transmitted the fingers may be considered in the same way as two slits illuminated by plane waves. Each 'slit' will now be considered separately. The intensity distribution of the wave from the slit on the surface of the medium to be excited is calculable from Fresnel diffraction theory. The geometry of this particular case and of the general case are illustrated in Figs.11(a) and (b) respectively. Since the dimension 'b' is always small, with respect to a wavelength, for contact coupling, we are clearly within the Fresnel region of diffraction. The calculations of the intensity distribution are made lengthy by virtue of the broad bandwidth of the incoming wave. This necessitates an integration (in practice a summation) over all the incident frequencies. For this reason the task was programmed for computer calculation, the flow diagram of which is to be found in Appendix 2, together with an outline of the supporting Fresnel diffraction theory.

The results, for a bandwidth one to ten MHz (the summation being by (1/30)th MHz steps) are shown in Fig.12. No difference was found between this result and that for a summation in $\frac{1}{4}$ MHz steps showing that the summation interval is sufficiently small. Results for various values of coupling distance are shown in Fig.13.

The surface displacements of the transmitted Rayleigh wave can be obtained from the spacial distribution of applied stress, derived above, by an extension of the theory of Ref.10 which states "the Rayleigh wave amplitude is directly proportional to the rate of loading", this being for the case of a time varying

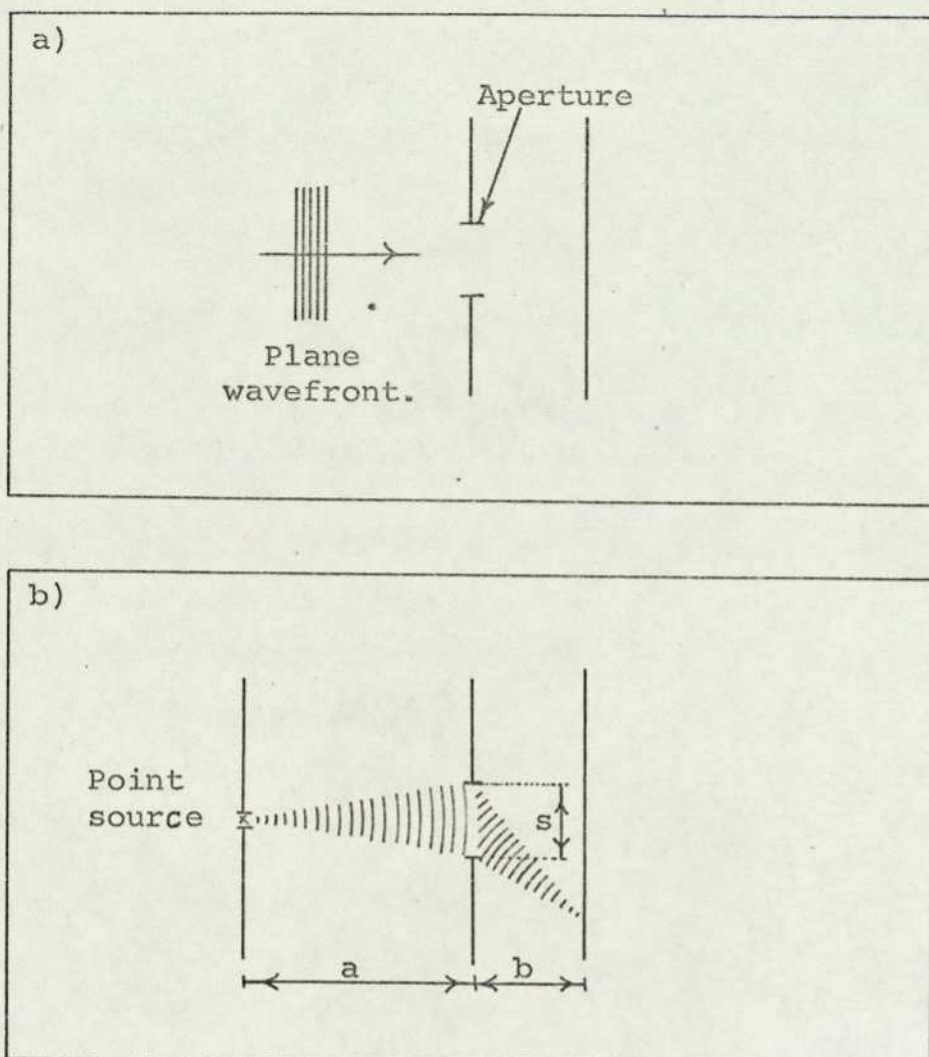


Fig. 11. a) The particular case under discussion.
b) The general Fresnel diffraction case.

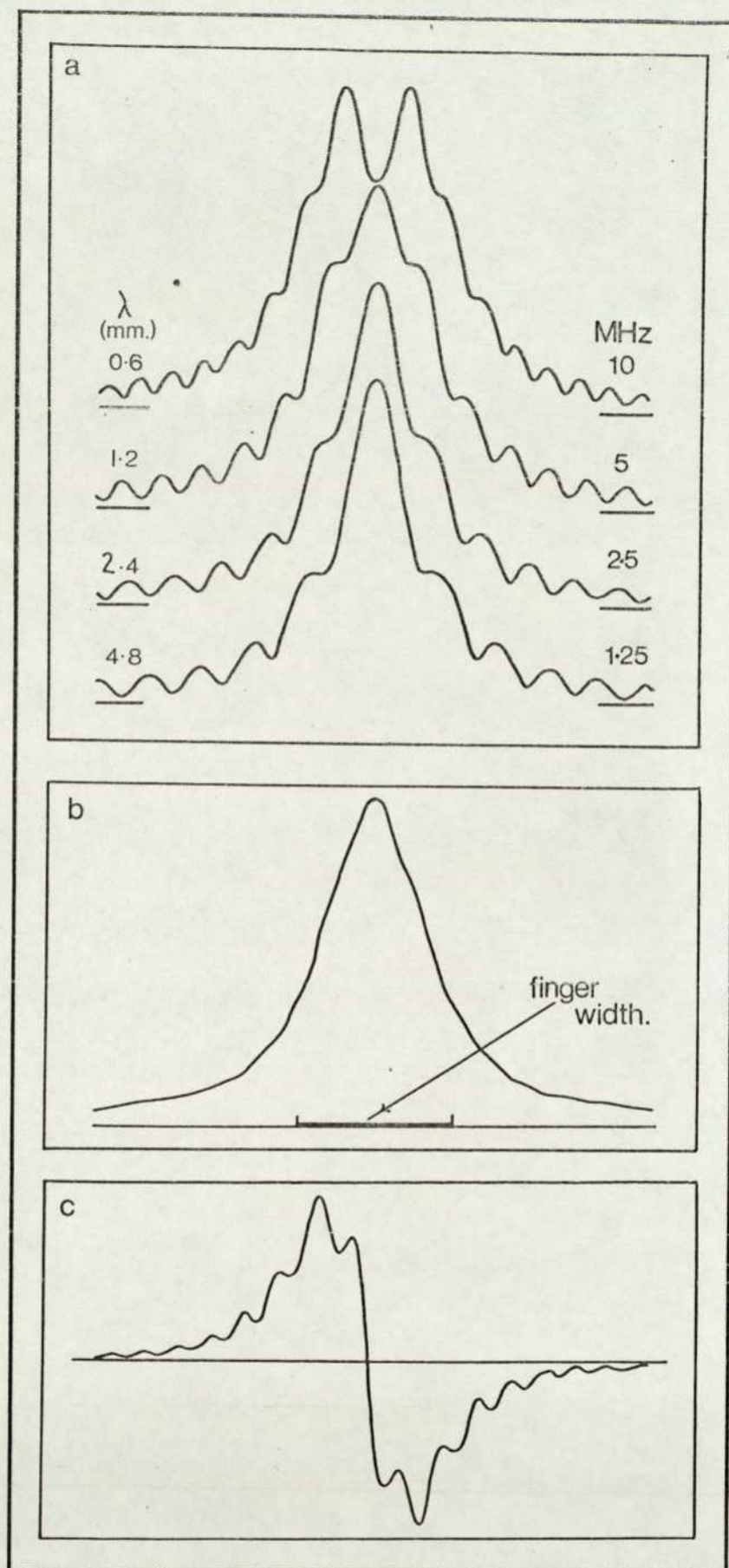


FIG.12(a). Fresnel diffraction patterns at the frequencies indicated, for a slit of width as shown, and a distance from the slit to the plane of .001 inches.

(b) Summation of the diffraction patterns from 1 to 10 MHz. in steps of $(1/30)$ th. MHz.

(c) The differential, with respect to distance, of (b).

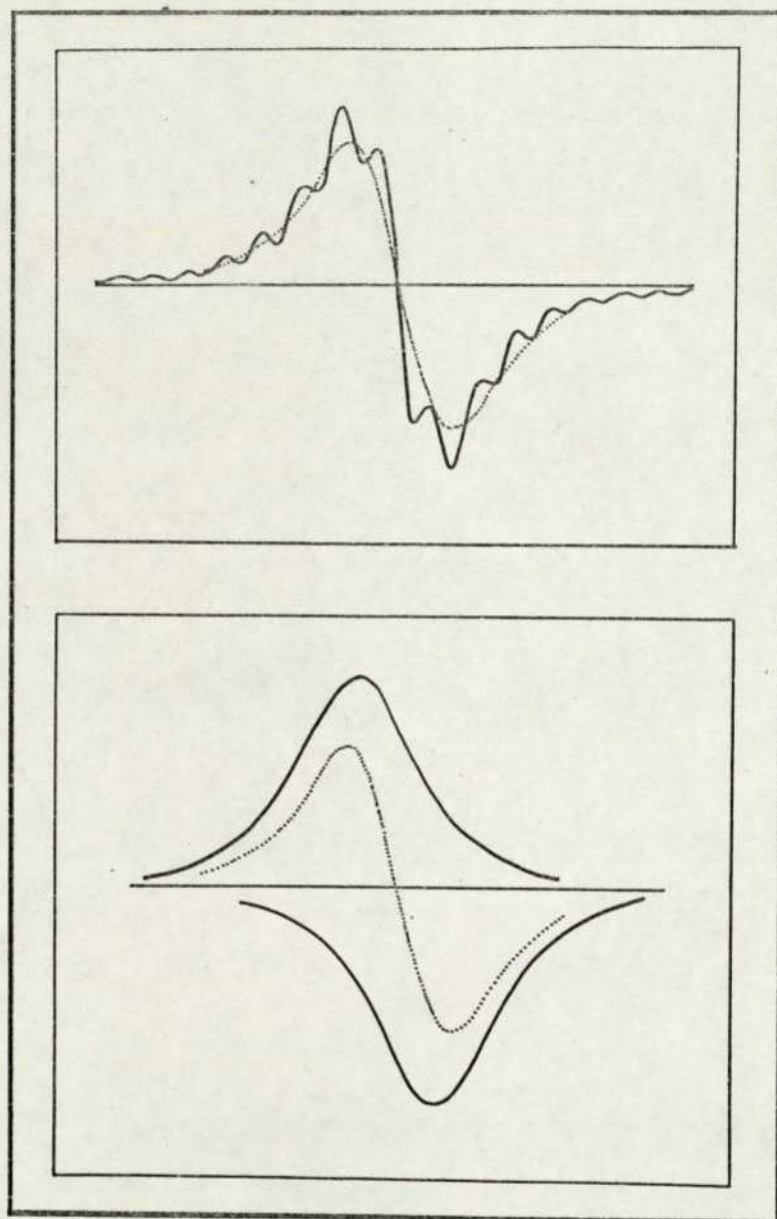


FIG.12(d). The differential of the diffraction pattern is approximated by the dotted curve, which in turn is represented by the summation of the two Gaussians shown.

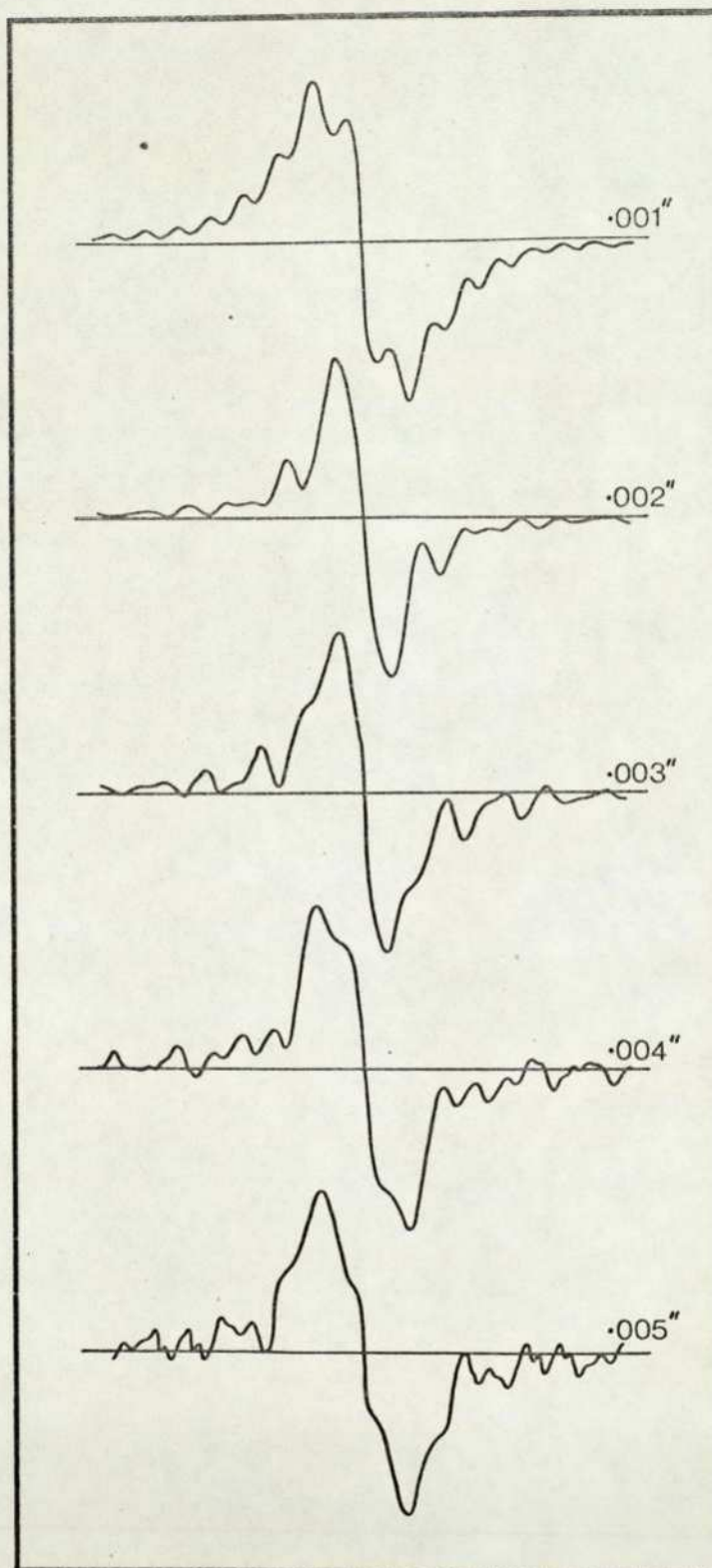


FIG.13. Variation of the differential of the Fresnel diffraction pattern for differing distances (as marked) from the slit to the plane.

load, applied at a point. The equivalence of this case with that which is being considered i.e. a spacially distributed load with impulse excitation, is readily seen if the distributed source is considered as an array of point sources and the time derivatives of the impulses applied to each one are then superposed.

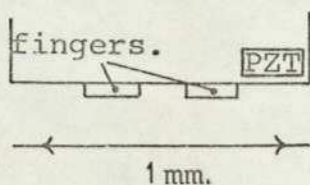
Having, then, the signal from one slit (e.g. Fig.12(c)) we obtain that for the double slit by adding the second signal, being a translation and inversion of the first. The spectrum of this signal then gives the transmission characteristics of the transducer.

In practice, for computational convenience and because the exact coupling distance is unknown in the experimental case, the single slit signal is approximated by a smooth curve which is expressed as the sum of two Gaussians. Similarly for the second slit.

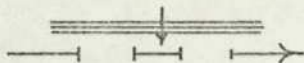
The characteristic is then calculated by convolving the Gaussian with a delta function array, the positions being defined by the edges of the slit (Fig.14). Comparison with experimental results was made by convolving this impulse response with the actual excitation pulse. The convolution was achieved by using a fast Fourier transform routine to generate the spectra, and multiplying them together. The flow diagram of the computer program written to generate the transducer frequency characteristic by this method is given in Appendix 3. The characteristics of transducers with ratios of finger width to half the gap width of values 0.3, 0.5, 0.7 and 0.9 were calculated for a total width ('b' in Fig.15) of 1 mm.

Transducers with the same dimensions as used in the computer calculations were made and their frequency characteristics

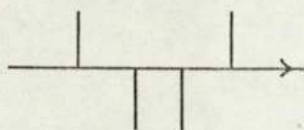
The transducer:



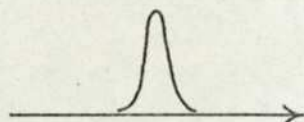
is represented by the double aperture:



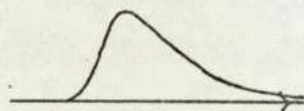
which, in turn, is represented by the array:



This array is first convolved with the gaussian:



and then with the excitation waveform:



to give the overall response of the transducer.

Fig. 14. Representation of the process used to derive the response of the interdigital transducer shown.

determined experimentally. This was done using two transducers of each dimension. One transducer was used to excite Rayleigh waves on the flat surface of a glass slab and the second used to detect this wave. The position of the second was adjusted to give a maximum output, it being assumed that the axes of the two transducers were then aligned. This signal was then analysed and the spectrum taken as the square of the required spectral characteristic, since the wave was being both generated and detected by the transducer.

The theoretical and experimental spectra are compared in Fig.15. The number quoted to the right of each graph is the value of the ratio (finger width/half-gap width). The use of this ratio allows comparison with the piezoelectric substrate case of Ref.17. Since the coupling distances in the experimental cases were not known, the Gaussian width was varied to obtain the best fit. The amplitudes of the experimental and theoretical curves were normalised at their maximum values.

Since the results are in good agreement, it may be concluded that this analysis provides a means of determining the spectra of this sort of transducer, provided all the parameters e.g. coupling distance, excitation pulse shape etc. are known. It is not confined to two fingers only. The example shown in Fig.16 illustrates this. It shows the theoretically predicted decrease in the bandwidth of the fundamental frequency with increase in the number of finger pairs. The dimensions used correspond to the transducer with a finger to half-gap width ratio of 0.7.

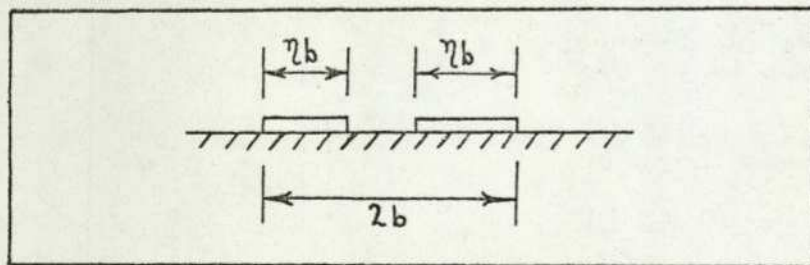
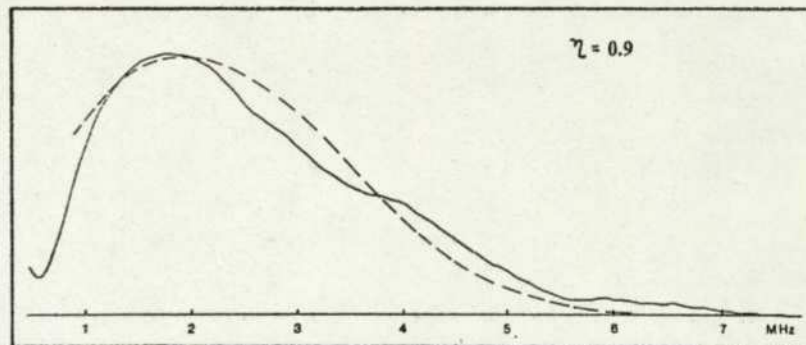
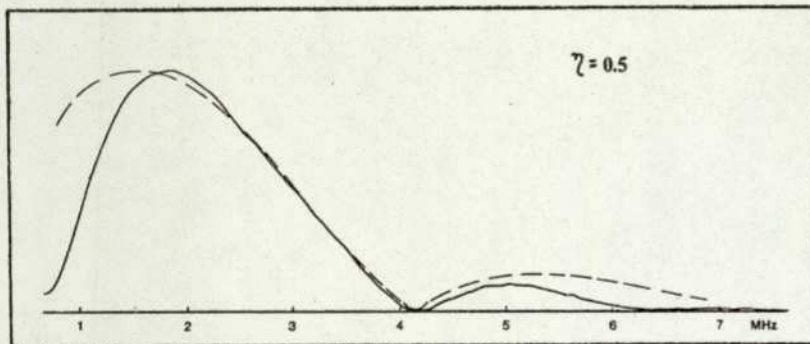
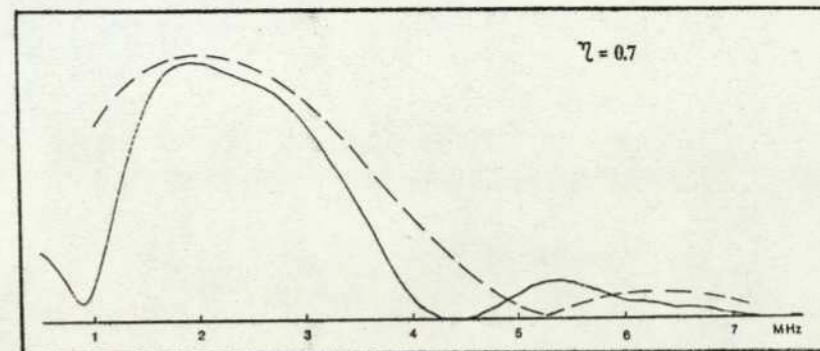
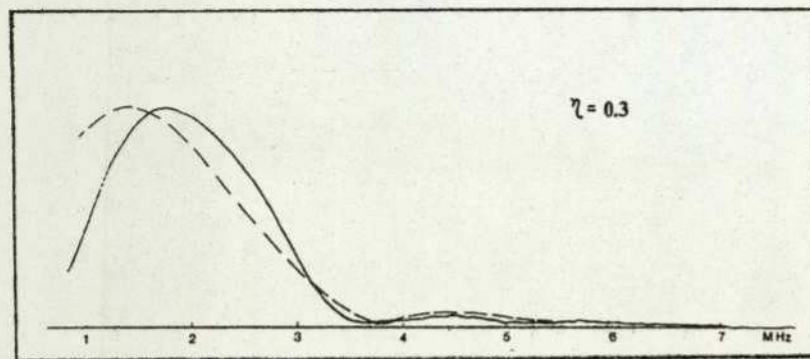


Fig.15. Comparison of experimental (full line) and theoretical (broken line) spectra of transducers, with various finger/gap width ratios, defined by the schematic diagram.

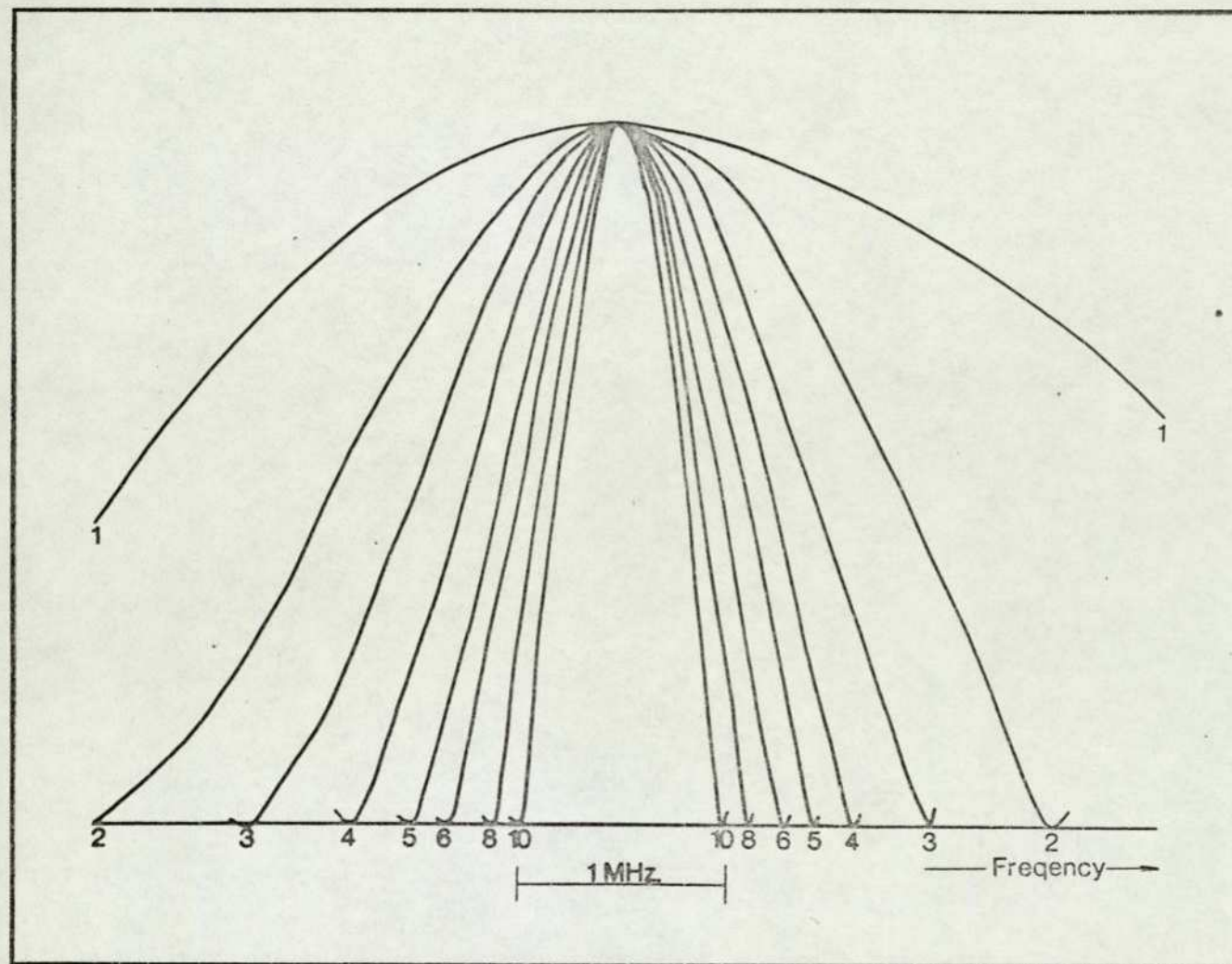


FIG.16. Variation of bandwidth of the interdigital transducer with number of finger pairs.

2.5 Choice Of Transducer Dimensions

While the frequency characteristic of any transducer is of considerable importance, it is not the only consideration to influence the choice of dimensions when making the transducers. If it were a transducer with as small a gap as possible would always be used, to get the best high frequency output. However, to get a large signal amplitude, necessary for the detection of small defects, large voltages must be applied to the transducer (typically 200 volts). In this case, the smaller the gap the greater the tendency to arcing between electrodes, with consequent loss in output and damage to the transducer.

Another consideration lies with the transducer fabrication technique. Photolithography (Ref.18) becomes increasingly difficult as the dimensions are reduced. Fine finger gaps are, therefore, to be avoided.

The compromise dimensions used in most of the following work corresponds to the dimensions of Fig.9(b).

In practice the problem of having a zero in the spectrum (as in the results in Fig.15), would be to limit the use of this transducer, in work described in the next chapter, to frequencies below this zero. However, lack of precision in production can obviate this problem. Imperfect contact between negative and substrate in the photolithographic process gives variation in the ratio (finger/gap width). This results in a mixing of the tightly controlled results of Fig.15, removing the null point.

Alternatively, if tight control is required for good reproducibility, then having each of the two fingers of slightly

different widths achieves the same objective (compare the transducer characteristic in Fig.9(b)).

Another method would be to profile the finger edges to vary between two ratios, along the length of the fingers.

This chapter has shown how broad band Rayleigh wave transducers may be designed and fabricated. The chapter following describes how these may be used to achieve the object i.e. measurement of the depth of a crack.

CHAPTER THREE

3. THE USE OF BROAD BAND RAYLEIGH WAVES TO MEASURE CRACK DEPTHS

- 3.1 General N.D.T. Applications of Rayleigh Waves
- 3.2 Previous Methods of Nondestructively Evaluating The
Depth Of A Crack
- 3.3 A new Method Of Crack Depth Measurement
 - 3.3.1 Discussion of the implications of a result
due to Viktorov
 - 3.3.2 Apparatus used in the new technique
 - 3.3.3 Experimental results
 - 3.3.4 Data processing
 - 3.3.5 Discussion of results
 - 3.3.6 Limitations of the technique
- 3.4 Comparison Of Methods

3. THE USE OF BROAD BAND RAYLEIGH WAVES TO MEASURE CRACK DEPTHS

3.1 General N.D.T. Applications Of Rayleigh Waves

It was noted previously that, since the Rayleigh wave is confined to a layer near the surface, it is particularly well suited to the testing of the surface and near surface for such features as cracks, voids, inclusions, pitting, roughness or surface layers, using the method of ultrasonic reflectography. This suggestion was originally made by Firestone in 1946 (Ref.19) but little use was made of it until the wedge transducer, discussed in Section 2.2 (Page 28), was introduced (Refs.20-22). Since, then, many applications are recorded of the use of Rayleigh waves in the detection of cracks e.g. steel plate (Refs.23, 24) and turbine blades (Refs.25-27). In the turbine blade tests of Ref.27 the ease of testing is commented on. The test did not require any dismantling of the turbine further than that necessary for access to the blades. The reliability of the method is noted in Ref.25, failure of the turbine blades of a military jet engine having been totally eliminated.

Since Rayleigh waves will propagate with negligible loss around a corner of radius large with respect to a wavelength, they may also be used to test round objects for cracks. For example, wires, welding tubing, rotor barrels and springs have

all been tested successfully (Refs.28, 29 & 30, 31, 32 respectively).

To detect fatigue cracks early in life it is necessary to detect them when they are small. At that point they can be ground out to remove the weakness (Ref.25). V-grooves (representing cracks) as small as 50 μm (at 2½ MHz where $\lambda = 1.25 \text{ mm}$) have been reported as detectable (Ref.33). It has been shown (Ref.34) that the process of crack growth can be monitored by surface waves, the reflected amplitude being taken as a measure of the crack depth. The surface wave method was found, in that case, to be considerably more sensitive than the calorimetric method used to standardise it and, not requiring the samples' removal for test, a great deal more convenient.

The growth of cracks may thus be observed and it is possible to predict failure by extrapolation, if the material and its surface condition are known. Reference 35 describes how fatigue cracks in aluminium alloy were detected at approximately 40% of the fatigue life of a sample, this value being higher for poor surface finishes. Better results have been reported in Refs.36 & 37 in the detection of stress corrosion cracking (S.C.C.). In that case, cracks of 70 μm depth were reported to be easily identifiable by reflectography (at a frequency of 4 MHz when $\lambda = 0.7 \text{ mm}$). Using the technique of observing the attenuation across the stressed, and therefore microcracked region, S.C.C. could be detected at only 18% of the normal stress corrosion life.

Less success in defect detection was obtained in Ref.38 when using automated inspection techniques in testing Saturn

rocket skin sections. It was found that there were too many inseperable variables e.g. crack orientation, depth, length, transducer coupling.

Though cracks are of principal interest in surface N.D.T. work, other surface quantities have been investigated using Rayleigh waves. The attenuating effect of a rough surface, mentioned earlier, has been used to measure this roughness (Ref.39). The lay of the roughness can have an unexpected effect, too, producing differing attenuation (up to an 80% difference) over the same path for a Rayleigh wave going either of the two directions 180° to each other (Ref.33). The effect of a thin surface film on a polished surface has also been studied (Ref.40).

Variation in surface properties usually results in a variation in the surface wave velocity and this variation may also be used to study the surface properties. The velocity may be determined in a number of ways. Measurement of time and distance of travel is widely used, with numerous ingenious variations (Ref.41), to measure, for example, the stress at the surface of a sample (Ref.42) or to measure the surface elastic constants. Goniometric methods, measuring the critical angle for conversion of bulk to surface waves (see section 2.2 page 28) have also been used to measure the Rayleigh wave velocity (Refs.43, 44).

The discussion in this section has dealt with the overall application of Rayleigh waves to the N.D.T. field, touching only briefly on the size of cracks that may be detected. The

determination of the crack depth, which fracture mechanics shows to be an extremely important parameter in evaluating the inherent weakness of a crack, has not been included, but will be described in the following section.

3.2 Previous Methods Of Nondestructively Evaluating The Depth Of A Crack

While Rayleigh waves have been found to compare well with other methods of crack detection, such as eddy currents and X-rays (Ref.45), measurement of crack depths has proved undependable (Ref.38) owing to the variables present, particularly in the contact operation of the transducers. This variation was found too in Ref.33, when an attempt was made to correlate the reflected amplitude of the Rayleigh wave to the depth. Close experimental control yielded approximately 20% accuracy, but the method was useful only for cracks of depth less than a wavelength.

As the depth increases to values greater than a wavelength, the reflected energy flattens off. The transmitted wave travels down the first edge of the crack and up the far edge; at each discontinuity some of the wave is reflected back and some transmitted on. There are thus two possibilities for depth measurement. One of these is to detect the transmitted wave. As the crack depth becomes greater the time of travel between source and receiver will become longer. This method has been used by Cook (Ref.46) for the measurement of cracks of depth greater than 1.5 mm. Alternatively, the reflection from the crack might be detected. This should comprise separate components from each edge of the crack, though they might not be

resolved using conventional testing apparatus. This latter is the method developed in following sections of this chapter.

Considerable success has been obtained in measuring crack depths using bulk waves. This method requires that a longitudinal wave be introduced into the sample from the face on which the crack is located. The longitudinal wave is incident on the crack after reflection from the back face of the sample (obviously there will be only a limited number of cases where this geometry is possible). If proper compensation is made for the transducer beam characteristics it has been shown (Ref.47) that the reflected amplitude will give a value of the crack depth accurate to 3 or 4 % (over a range 1 mm to 5 mm depth). This method, naturally, measures the length of the crack normal to the direction of the incident beam, so, unless the angle of the crack relative to the surface, is known, the true crack depth cannot be evaluated immediately. Using a similar amplitude technique, Weil (Ref.48) found that stress corrosion cracks of approximately 25 μm could be evaluated. However, Ref.49 describes how fatigue cracks of less than 1.25 mm could not be reliably detected.

Eddy current methods can also provide a measure of the depth of a crack. An eddy current is excited in a conducting medium to be tested by a coil suspended near it, excited in its turn by an R.F. signal. The eddy current exerts a back e.m.f. on the coil and the two attain an equilibrium condition. If a crack is encountered when the coil is scanned over the medium, the eddy current is disrupted and the equilibrium with the coil upset. This is detected as a change in resistance of the coil

(usually set as one arm of a Wheatstone bridge). Unfortunately, the quantity measured is the component of the depth of the crack perpendicular to the surface of the medium. This method can operate over the range 0.125 mm to 1.75 mm (in aluminium alloy).

The method, previously mentioned, of observing the reflection from a crack will be that employed hereafter, in the development of a new and, hopefully, more reliable method for the measurement of the depth of a crack.

3.3 A New Method Of Crack Depth Measurement

3.3.1 Discussion of the implications of a result due to Viktorov

The most important study, with reference to work described here, is that of I.A. Viktorov, presented in his book - Reference 11. One of the results presented there is reproduced in Fig.17. This is a plot of the signal reflected from a slot, the reflection coefficient being defined as the reflected Rayleigh wave amplitude divided by the incident wave amplitude. The experiment was carried out by cutting slots of progressively greater depths in the flat surfaces of Dural Bars, the slots being normal to the surface and studying the reflection of Rayleigh waves from them. The result is a gradually increasing reflection coefficient with a modulation superimposed, attributable to interference between the reflection from the leading edge of the slot and from the bottom of it.

Here, then, we have the beginnings of a method of measuring the depth of a crack. The horizontal axis of Viktorov's result, it will be noted, is depth normalized to the Rayleigh wavelength i.e. it is proportional to depth multiplied by the frequency of the Rayleigh wave. For a fixed depth, this is proportional only to frequency and is, therefore, the spectrum of the return signal from the slot. Since the return signal is normalized to the input signal, this represents the reflection spectrum for

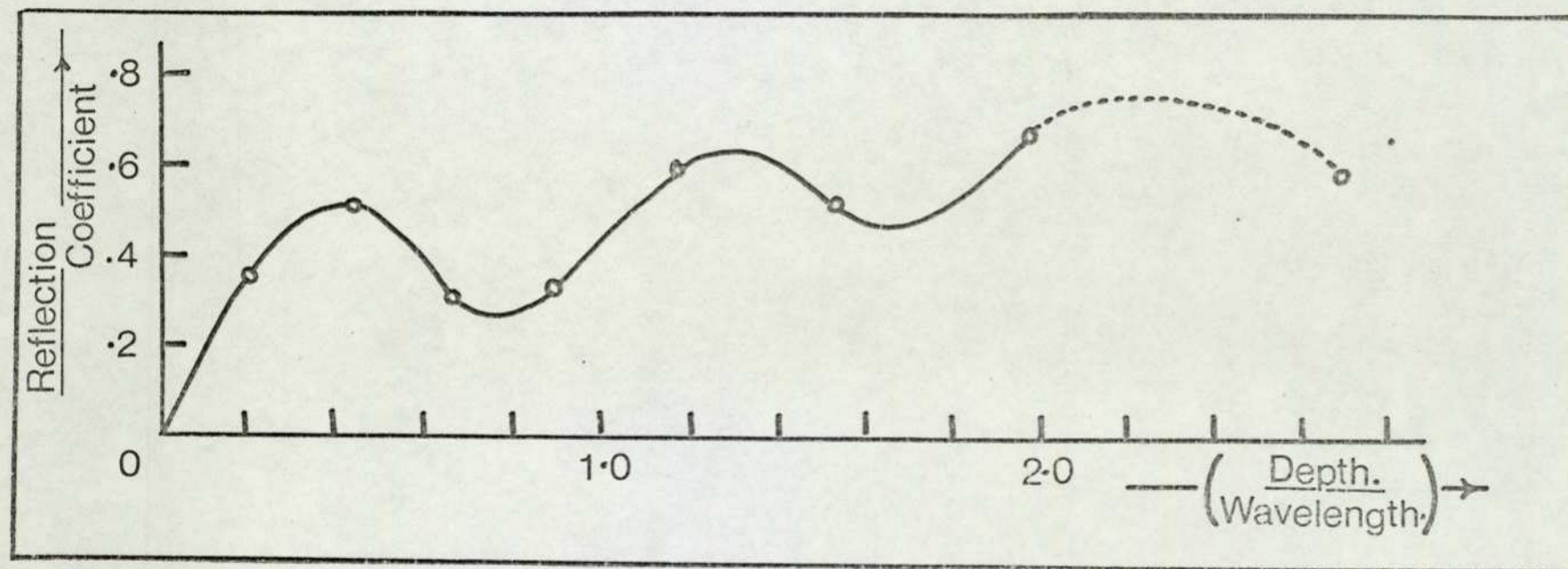


FIG.17. The spectrum of the reflection from a slot as obtained by Viktorov.

an input signal whose spectrum is perfectly uniform.

Thus, by 'illuminating' a crack with wide-band Rayleigh waves and normalising the spectrum of the reflected signal to that of the input signal, a result of the form in Fig.17 would be expected, from which the depth may be evaluated.

3.3.2 The apparatus used

The Rayleigh waves were excited on the polished surface of a block of aluminium alloy of dimensions 4 in x 4 in x 1½ in. A slot of depth 1.4 mm and 0.5 mm width was milled vertical to the square surface and set to run across the centre of it (see Fig.18(b)).

The transducers were of the type described in Chapter 2 i.e. single finger pair, interdigital electrodes (of length 6 mm and finger, gap and finger widths of 0.21, 0.01, 0.18 mm respectively) on a PZT-5A (Lead Zirconate Titanate) block of thickness 9.2 mm. Two of these were attached to the surface using Araldite, care being taken in ensuring their angles, relative to the slot, to be equal (the angle θ of Fig.18(b)), and at the correct distance from the slot to ensure that the wave from one transducer, on reflection from the slot is incident normally on the second transducer.

The distance from the transducer to the slot had also be small enough to preclude the inclusion in the gate (used to select the period of time of interest with respect to the excitation) of any spurious reflections from bulk waves set up in the block. The first of these

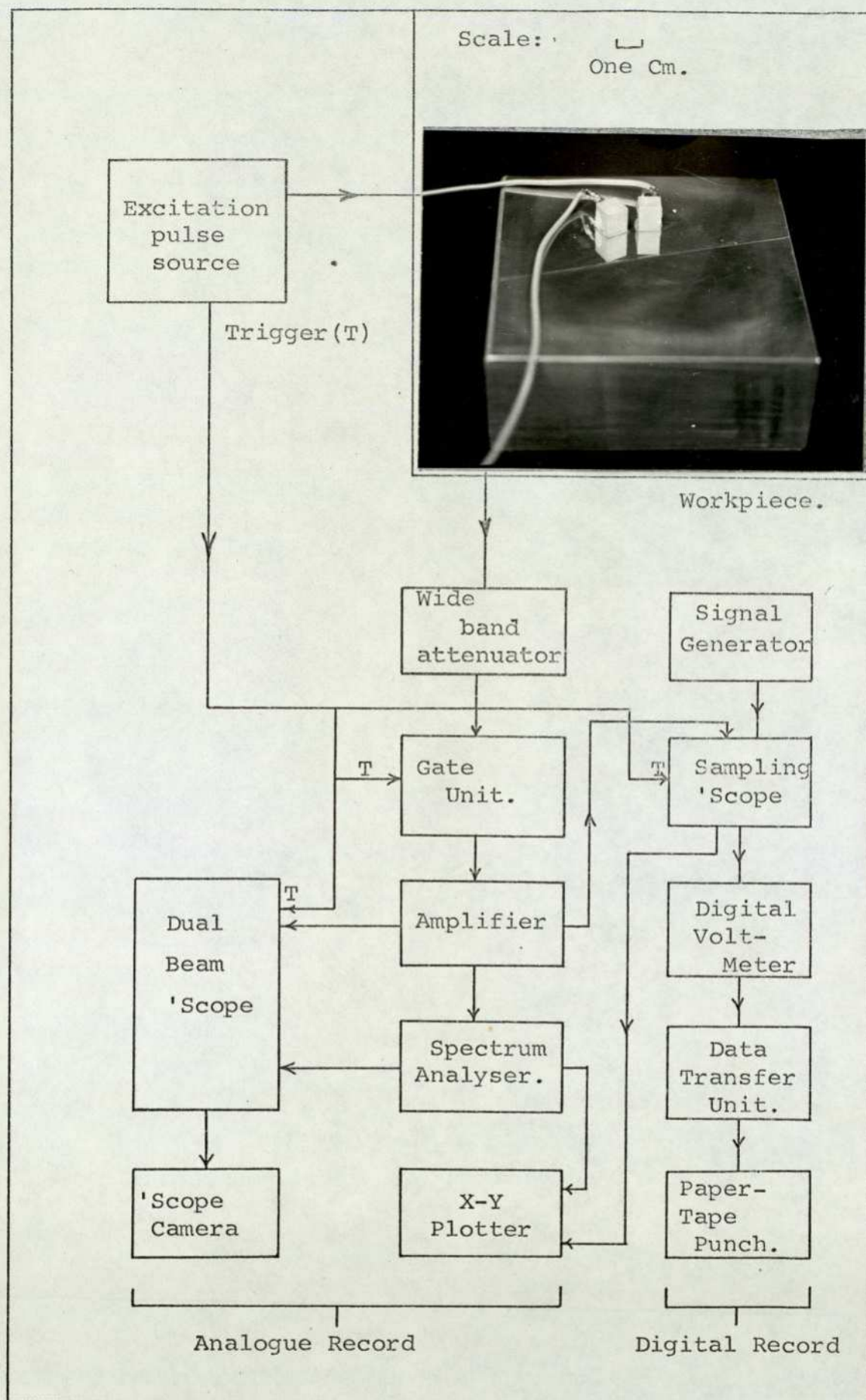


Fig. 18(a) Block diagram of the system used to record the reflection of the rayleigh wave from a slot.

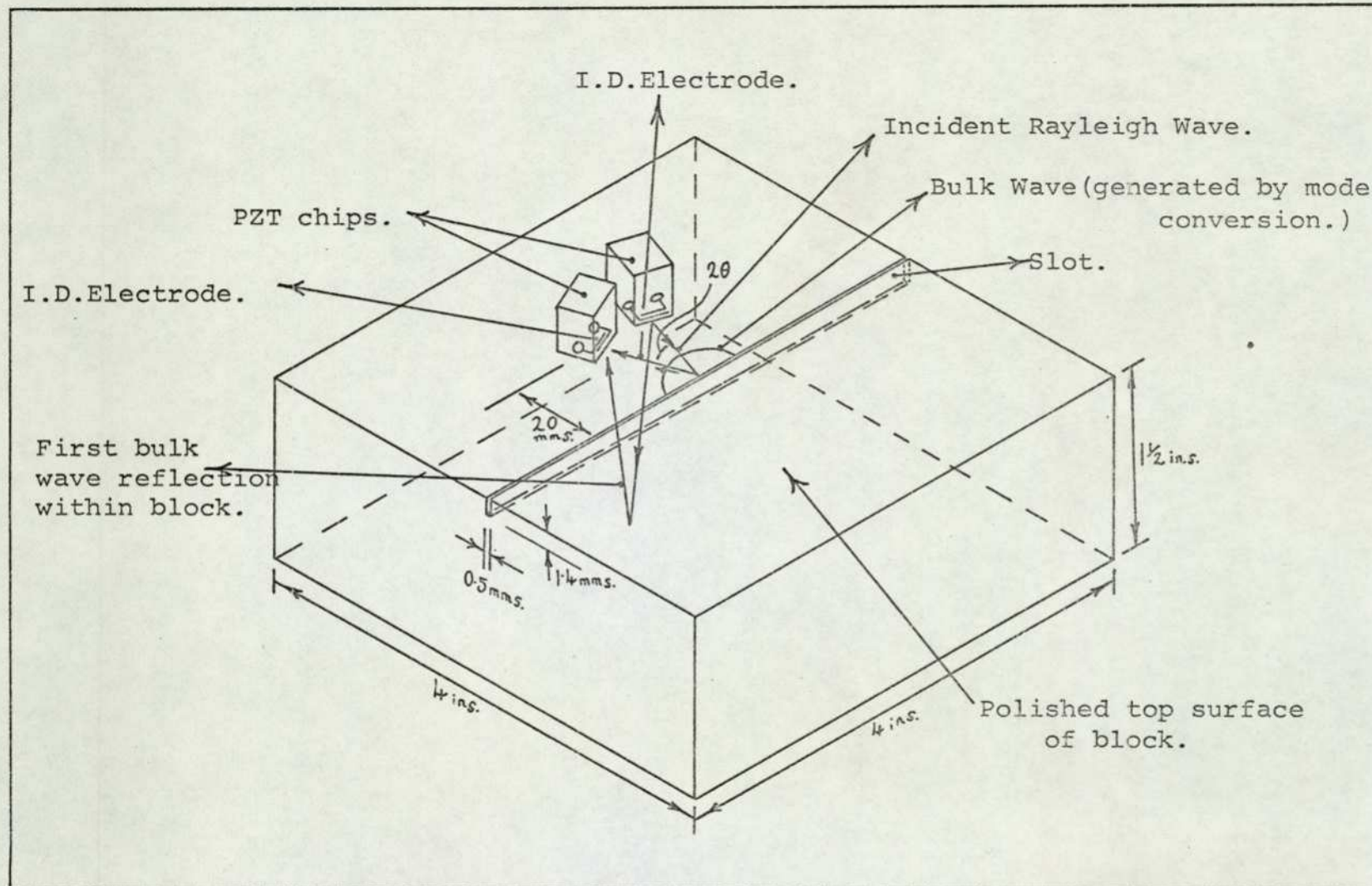


FIG.18(b). The geometry of the transducers on the test block.

would be the longitudinal wave travelling vertically through the block and reflected off the bottom. At a thickness of $1\frac{1}{2}$ in and a velocity of 5,100 metres/sec the gate must stop before 15 microseconds had elapsed after the excitation. With a Rayleigh wave velocity of approximately 3,000 metres/sec the transducer must be at less than 22.5 mm from the slot. They were, in fact, placed at 20 mm. No greater margin was allowed since the distance must also be kept as large as possible to minimise the angle of incidence of the Rayleigh wave to the slot and to maximise the attenuation of the bulk waves generated by mode conversion of the Rayleigh wave at the slot.

The use of the two transducer system was found to be necessary because of its better surface wave signal to background signal ratio. The reverberation signal due to the bulk wave excited in the PZT block of the send transducer, was greater, for any practical Rayleigh wave path length, than the reflected Rayleigh wave amplitude. Thus a second transducer was used.

The transmit transducer was excited by a conventional N.D.T. test set (Smiths "Ultrasonic Diasonograph", type MK7/03) giving a voltage pulse of 160 volts (loaded) at a pulse repetition rate of 750 per second. A block diagram of the set-up used to process the received signal is shown in Fig.18(a). Since the full description of the electronics used was given in section 2.3.1.1, only a brief outline will be repeated below.

The signal is first attenuated (using a wide band attenuator) before being amplified and input to a gate triggered from the excitation pulse. The signal in the gate is then fed to a spectrum analyser which generates the spectrum of the signal. Both the time signal from the gate and the spectrum are displayed on a dual beam oscilloscope, the time signal scan being triggered from the excitation source. This could then be photographed. Alternatively, they can be recorded on an X-Y plotter, the spectrum directly, the time signal (being sub-microsecond) only after sampling by a sampling oscilloscope. The time signal can also be recorded digitally, the output from the sampling oscilloscope, being read by a digital voltmeter and passed, via a data transfer unit, to a tape punch for recording on eight hole paper tape.

3.3.3 Experimental results

The results of the setup described above, as plotted on the X-Y plotter, are shown in Fig.19. The upper trace contains all the signal from the slot. It will be noticed that it is impossible to pick out any features in it. The second trace shows the first part of the reflected signal gated by itself. The sine wave is a 10 MHz signal to provide a time scale. Below these there are the spectra of the two signals above. As expected there is a smooth curve from the second time signal, which corresponds to the reflection from the first edge of the slot. Its shape is basically the frequency characteristic

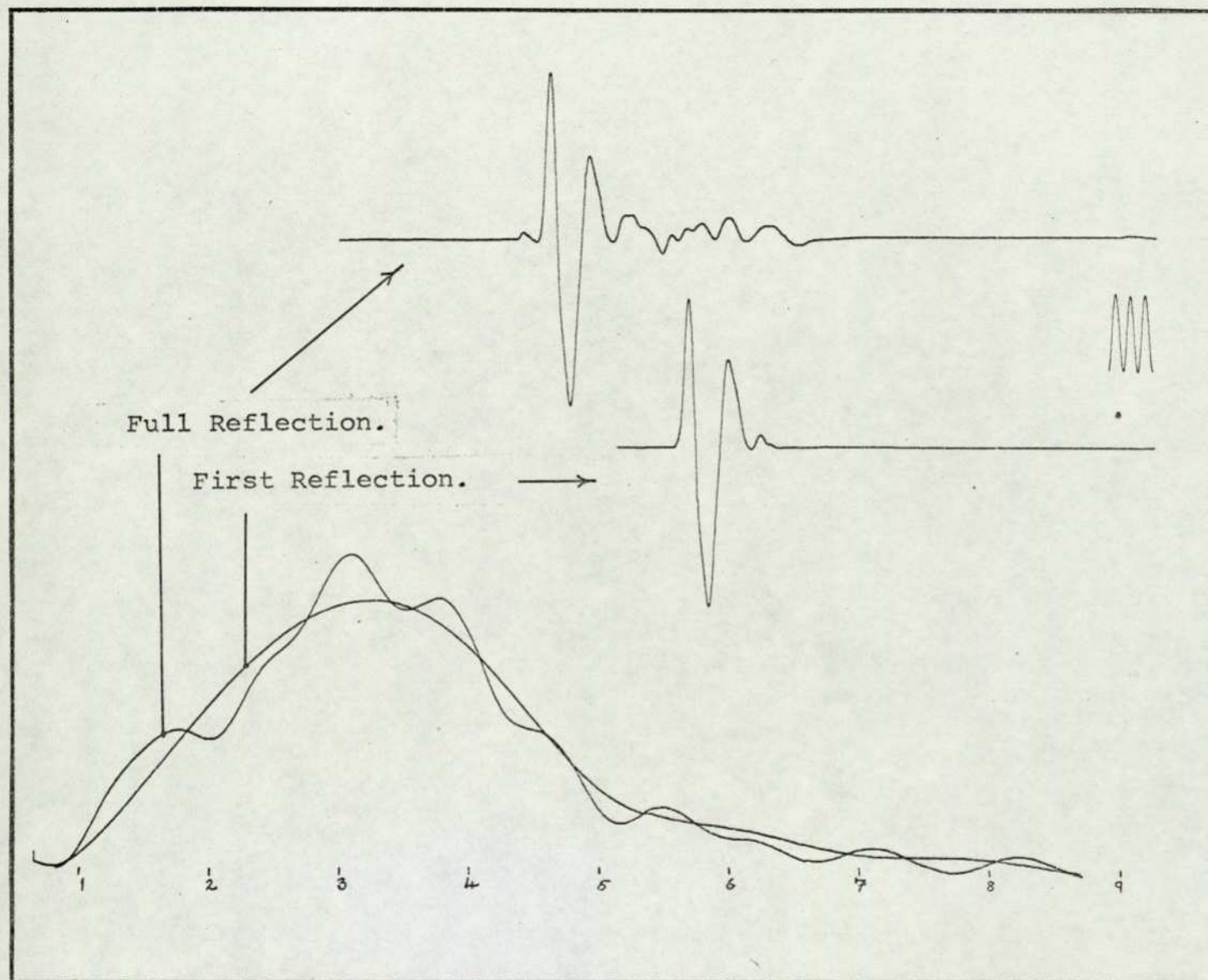


FIG.19. Experimental results (recorded on the X-Y plotter) of the reflection from a slot, showing: the full reflection; the first reflection only; the spectra of these two signals.

of the transducers (i.e. the square of each transducer separately) multiplied by the excitation pulse spectrum, modified by any disturbance to the Rayleigh wave between transducer and slot. The modulated spectrum is that of the whole signal from the slot. The modulation is quite pronounced and suggests agreement with Viktorov's results. However, the modulation corresponds to a wavelength, not of the slot depth, as Viktorov found, but to twice the slot depth. The reason for this will be given in the following sections.

3.3.4 Data processing

The modulation is first separated from the base spectrum in a similar manner to that employed by Viktorov, namely, by dividing, at every frequency, the reflection spectral element by the incident element. However, in this case, this method is neither possible nor desirable. The signal incident on the slot is not available, but the reflection from the first edge is. By normalizing to this reflection the effects of the slot are isolated in the modulation and any dispersion or attenuation effects between transducer and slot are eliminated (assuming the path to be the same to and from the slot).

The results of the computer program designed to do the above process (the flow diagram of which is given in Appendix 4) are shown in Fig.20. The time signal, as digitised by the setup described in Section 3.3.2, is plotted first. The spectra of this time signal with the two gate positions used in Fig.19 are shown below. The

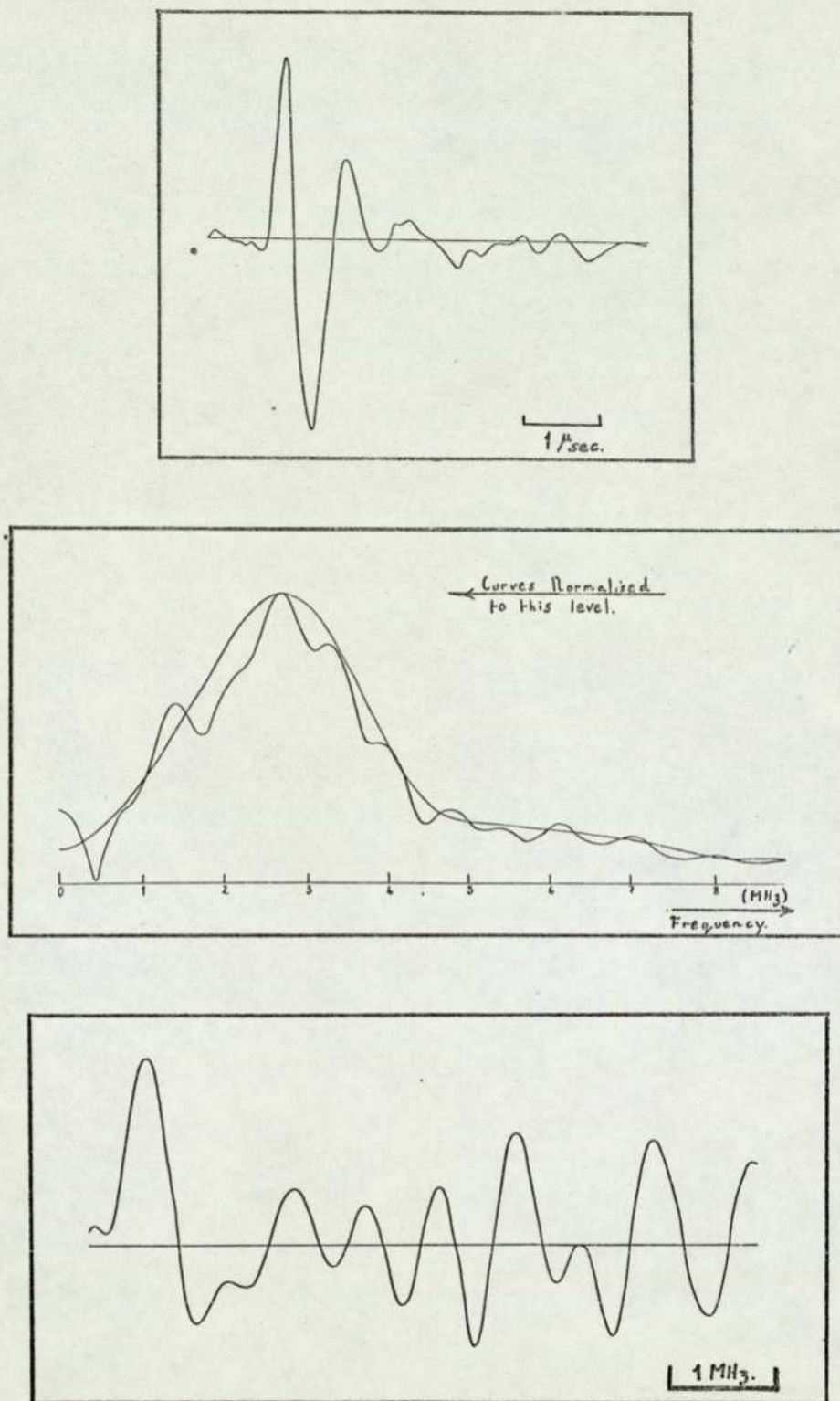


FIG.20(a). (i) The time signal printed from the digitised data.
 (ii) The computed spectra of (i), for the two gate positions shown in fig.19.
 (iii) The ratio of the two spectra minus unity. This is the modulation separated from the base spectrum.

ratio of these two (minus unity to remove the plateau) is shown below that. To determine the exact frequency of the modulation, it seems reasonable to treat this, in its turn, as a time signal and take its spectrum. If this is done, the final graph of the figure is obtained (Fig.20(b)). It is composed of a number of peaks whose significance will now be discussed.

3.3.5 Discussion of results

The result of taking the spectrum of a spectrum is termed the CEPSTRUM of the original time signal, this term having been coined for work in the field of Seismology. The operation of taking a spectrum inverts the time coordinate to give frequency. Taking the spectrum again reinverts the coordinate so as to return to the time domain. It is for this reason that the axis of Fig.20(b) is marked as time.

The peaks on this cepstrum occur at times, relative to the origin, which correspond to the dimensions of features of the slot, in the way indicated on the diagram. The smaller peak between the second and third major maxima corresponds to the depth of a previous shallower mill, the mill cutting the 1.4 mm deep slot not completely covering the previous one, leaving a small shoulder at that point.

The overall process can be thought of, very simply, as being a means of 'cleaning up' a poor time reflection signal to recover information previously lost. The

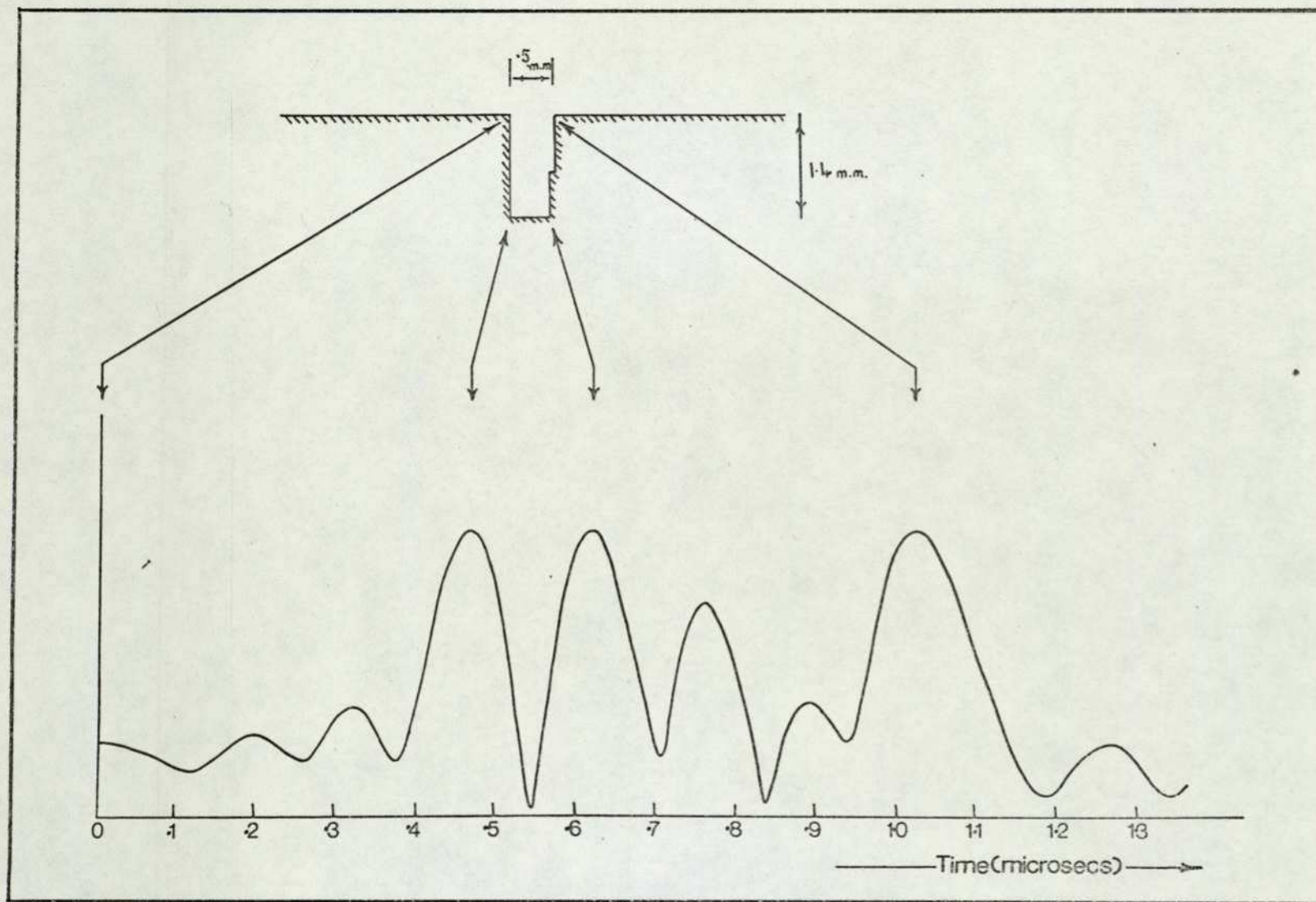


FIG. 20(b) . The Cepstrum of the signal reflected from the slot shown.

normalisation procedure has improved the transducer response to the ideal case of a perfectly broad band signal incident on the crack. The importance of the normalisation can be clearly seen by comparing the cepstrum of the unnormalised spectrum (Fig.21) to Fig.20(b). In Fig.21 the crack features are almost completely obscured by the cepstrum of the smoothed spectrum.

It should be noted that the same result may be effected by smoothing the modulated spectrum, by averaging over the period of modulation, a process which, for an analogue electronic device, would be easier than duplicating the spectrum analysis procedure. The analogy can be understood more easily if one considers the spectrum to be a time signal; then the smoothing acts as a low (frequency) filter, but, since in actuality it is a spectrum, then the filter is a low (time) pass filter i.e. the first reflection has been sampled. Thereafter the process is identical. This process has not been adopted in the computations used here because the gate position is more easily and more sensitively chosen in the time domain.

3.3.6 Limitations of the technique

The care taken in choosing the distance of the transducer from the slot illustrates one of the difficulties. To get information only about features in the slot, the signal isolated in the gate must be only from the slot. If reflections from other sources (e.g.

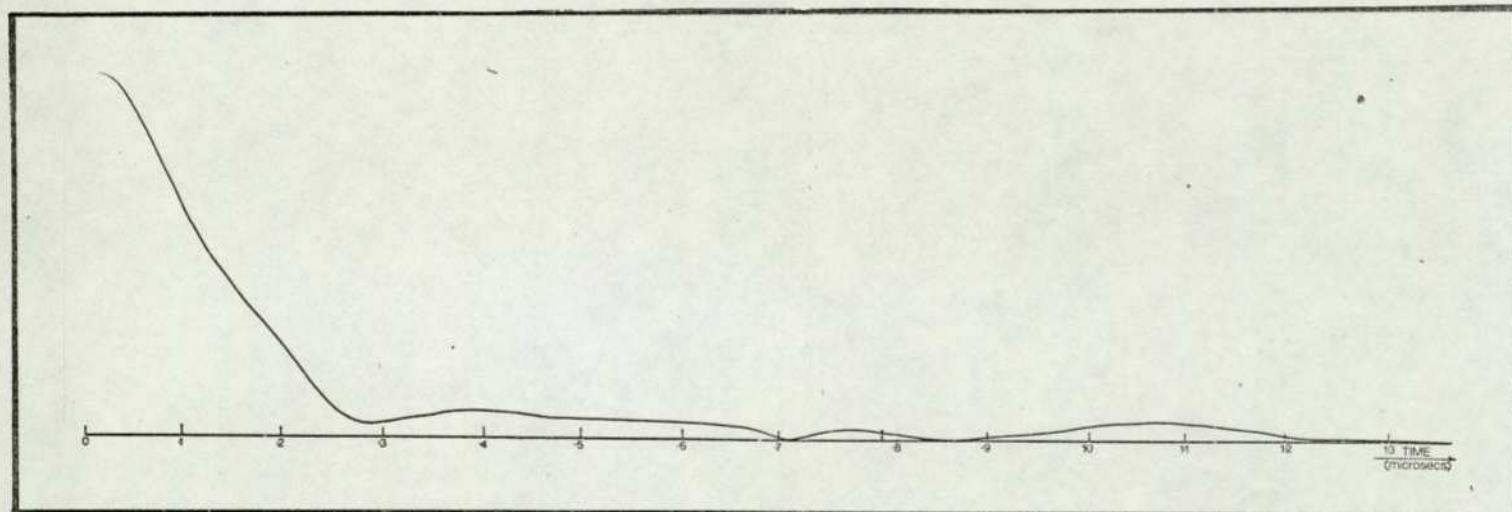


FIG.21. The Cepstrum of the total reflection signal shown in Fig.19., i.e. without subtraction of the first reflection.

longitudinal wave reflections) arrive within the gate they will be registered and recorded as features. For this reason any testing of actual workpieces would have to be in two stages, the first being to locate the crack, the second being the method above with the transducer moved near enough to the crack to eliminate other reflections.

Another limitation is the finite bandwidth that can be generated in practice, being limited by surface roughness and dirtiness. Assuming 0-20 MHz can be propagated with acceptable attenuation, at least a few cycles of modulation on the spectrum is required to resolve the dimension, this corresponds to a distance in aluminium of approximately 0.3 mm, or a limit to the slot depth of 0.15 mm assuming negligible width.

The major limitation to practical utilisation of this method lies with the transducers used. The difficulty of using two transducers is greater for larger slots as shown in Fig.22. Since, for two transducer operation, the Rayleigh wave must approach the slot at an angle, the reflection from the second and subsequent edges of the slot will be spatially displaced with respect to that from the first edge. This makes positioning the second transducer so as to receive reflections in their correct proportions impossible. The difficulty is shown clearly in a repeat of the experiment above, with a slot of approximately (1/10)th inch depth. The position of the

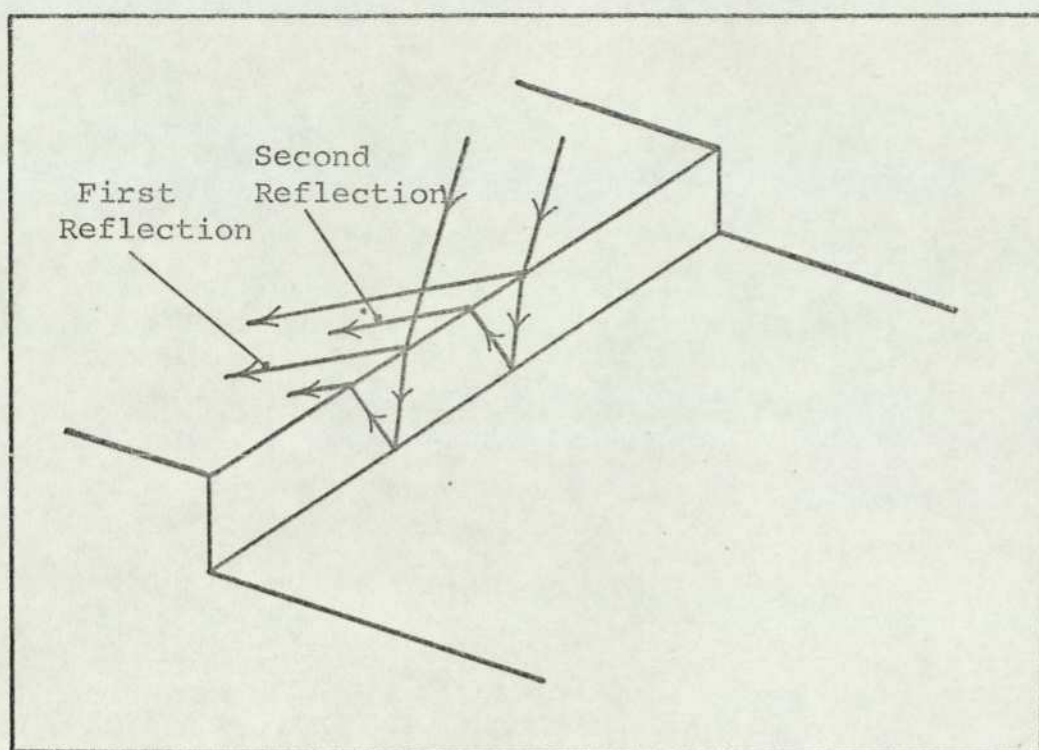


Fig . 22. Illustrating the difficulty of using a two transducer system for evaluating large crack depths.

receive transducer was such as to detect, predominantly, the reflection from the back of the slot (Fig.23), so that depth information is obtained but not information of the crack features.

Therefore, it is important that the difficulties encountered when using a single transducer be overcome, since single transducer operation, naturally, would not carry this limitation.

3.4 Comparison Of Methods

The various methods of surface crack detection and measurement (in Aluminium and Steel cylinders) were investigated and compared by Packman, Pearson, Owens and Young (Ref.49). With regard to the depth of surface cracks they concluded that eddy current determination was dependable over the range 0.125 mm to 1.75 mm and ultrasonic bulk waves above 1.75 mm. Surface waves were not included in the study. The eddy current evaluation must be made under the assumption that the crack lies in a plane perpendicular to the surface, and moreover must be made in a conducting medium. Using surface waves, however, we have shown that ultrasonic depth evaluation should be possible from 0.15 mm upwards, with added information about the shape of the crack. Obviously, a combination of the values for the actual depth of the crack (from surface waves) and of the component of the depth vertical to the surface (eddy currents) will give the angle of the crack to the surface. This may, perhaps, be obtainable directly from the surface wave result (see Chapter 5).

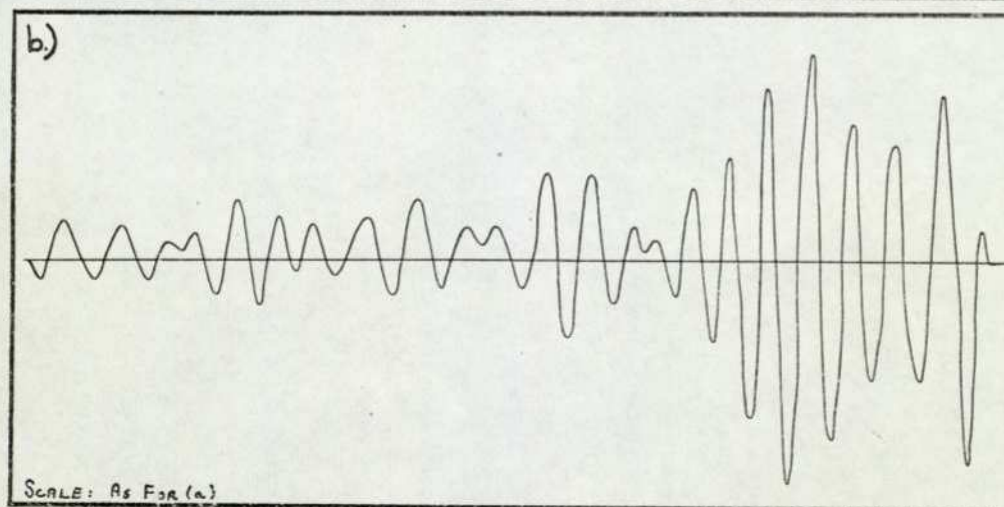
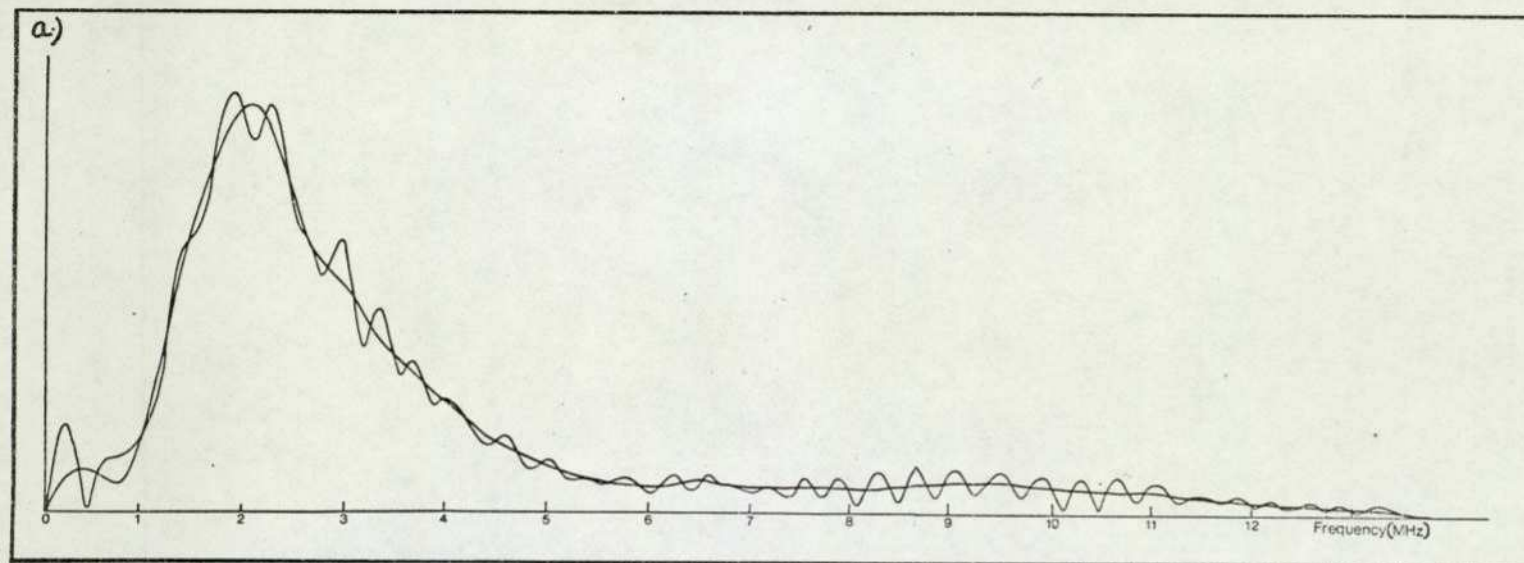


FIG.23.

- (a). Spectra of the total reflection and of the first reflection from a $\sim(1/10)$ inch slot.
- (b). The ratio of the two spectra, over a limited range.

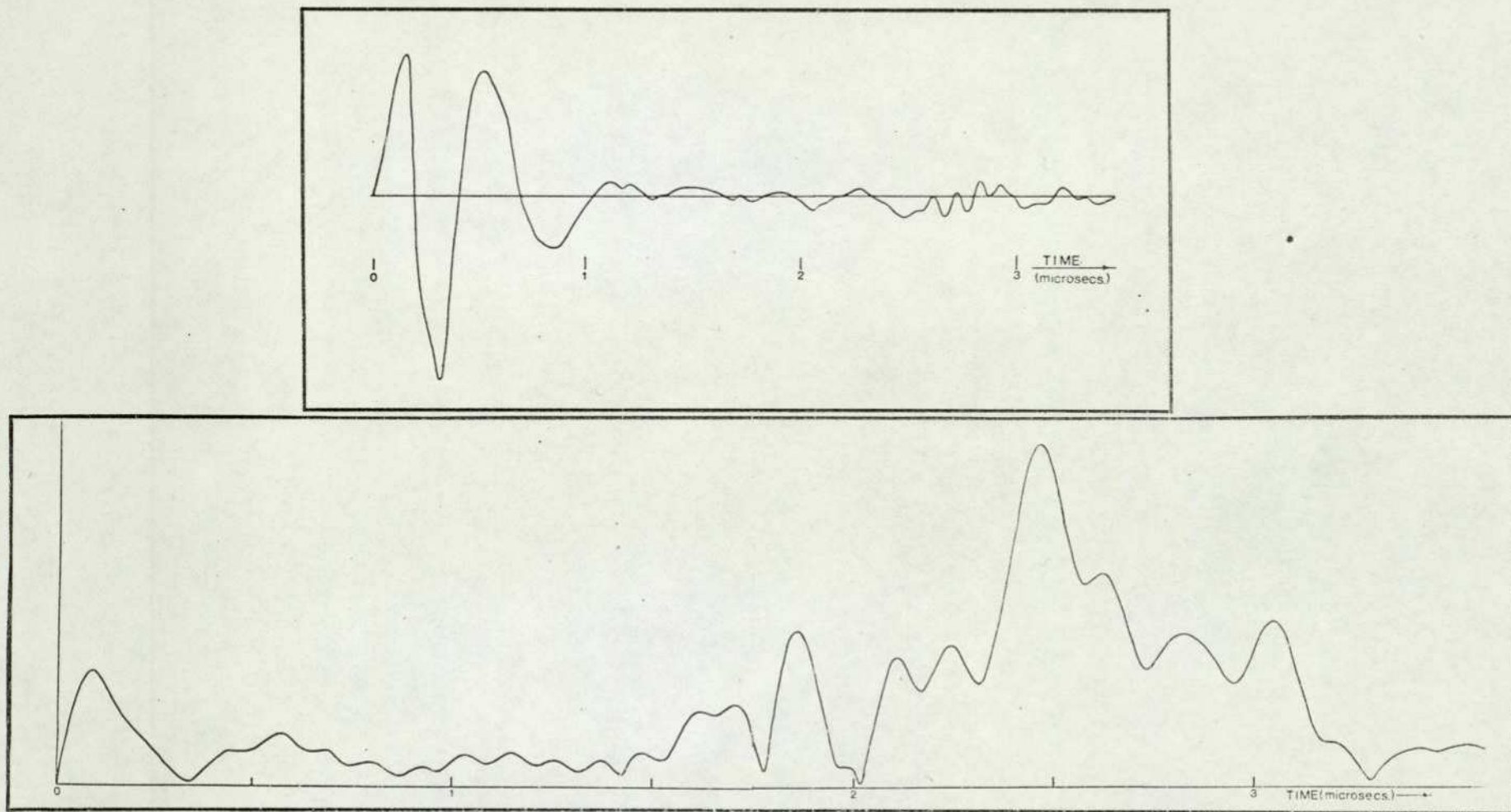


FIG.23. (c) The reflected time signal from a slot of $\sim (1/10)$ th inch.
 (d) The cepstrum of the reflection in (c).

CHAPTER FOUR

4. A TRANSDUCER FOR THE TRANSCEIVER MODE OF OPERATION

4.1 Improvements Achieved

4.2 Projected Improvements

4. A TRANSDUCER FOR THE TRANSCEIVER

MODE OF OPERATION

4.1 Improvements Achieved

In Chapter 3 (page 63), the reason was given for using one transducer to transmit and one to receive, in preference to one fulfilling both functions (i.e. the transceiver mode). This was because the reverberation in the send transducer from the excitation pulse masked the reflected signal to be detected. The reverberation within the send transducer can be substantially reduced, however. A great deal of it is due to the geometric regularity of the PZT chip which bears the I.D. electrodes. The parallel faces provide for coherent reflection in the direction of polarisation and consequent ringing of the chip at its thickness frequency. Roughening this surface serves both to make the reflection non-coherent and to scatter the waves into directions other than that of the polarisation. This then, is the first step.

Secondly, it is desirable to remove the backward generated wave from the PZT into a highly attenuating backing so as to prevent reflection back to the I.D. array for detection. Tungsten loaded Araldite has been found effective in doing this, since it provides a good acoustic match to the PZT making for good transmission of energy from PZT to backing, and also has a high attenuation coefficient.

Lastly, since the block dimensions are large, the frequency at which it rings is low. Therefore, reduction in the low frequency content of the excitation pulse will also reduce the reverberation. A simple high-pass filter comprising a series capacitor and parallel resistance (Fig.24) achieves this. A further advantage of this is that a great deal of the voltage pulse is of low frequency so that reduction of the low frequencies yields a pulse of smaller voltage but with much the same energy in the higher frequencies. This lower voltage results in a longer lifetime for the transducers.

Figure 25 shows the X-Y recorder traces of the excitation pulses and their spectra (unloaded by a transducer), with and without the filter of Fig.24. To allow comparison the traces with the filter are amplified by 15 dB's with respect to those with no filter. The improvement in bandwidth is obvious.

Figure 26 shows only the latter part of the trailing edge of the excitation pulse, this gating allowing amplification of the ringing signal without saturating the amplifier. It shows

- (a) The unfiltered excitation pulse applied to a parallel faced transducer.
- (b) The filtered excitation pulse applied to a parallel faced transducer.
- (c) The unfiltered excitation pulse applied to a transducer with the back face spoiled and the whole encapsulated in tungsten loaded Araldite.
- (d) As for (c) but with a filter on the excitation pulse.

Results (a)-(d) were found to be identical whether loaded or unloaded by the transducer.

The encapsulated transducer is shown in Fig.27. It also

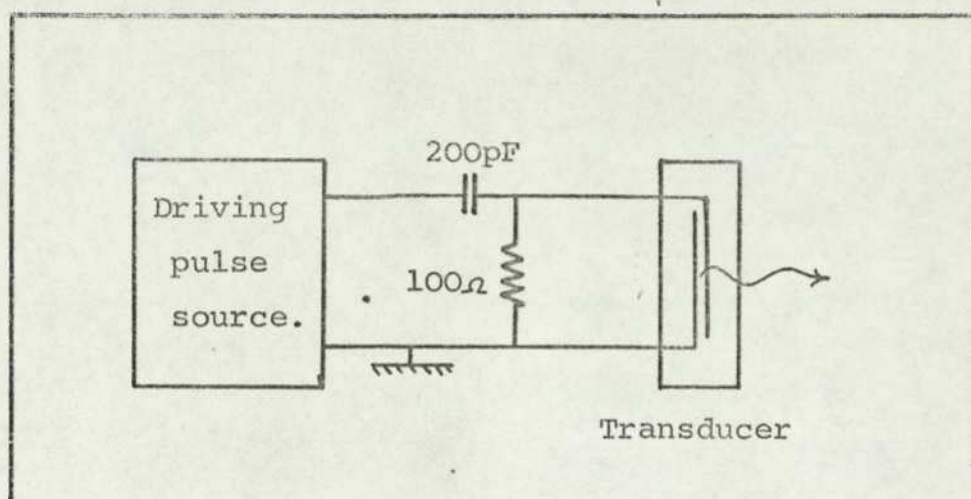


Fig. 24 . High pass filter used to reduce reverberation.

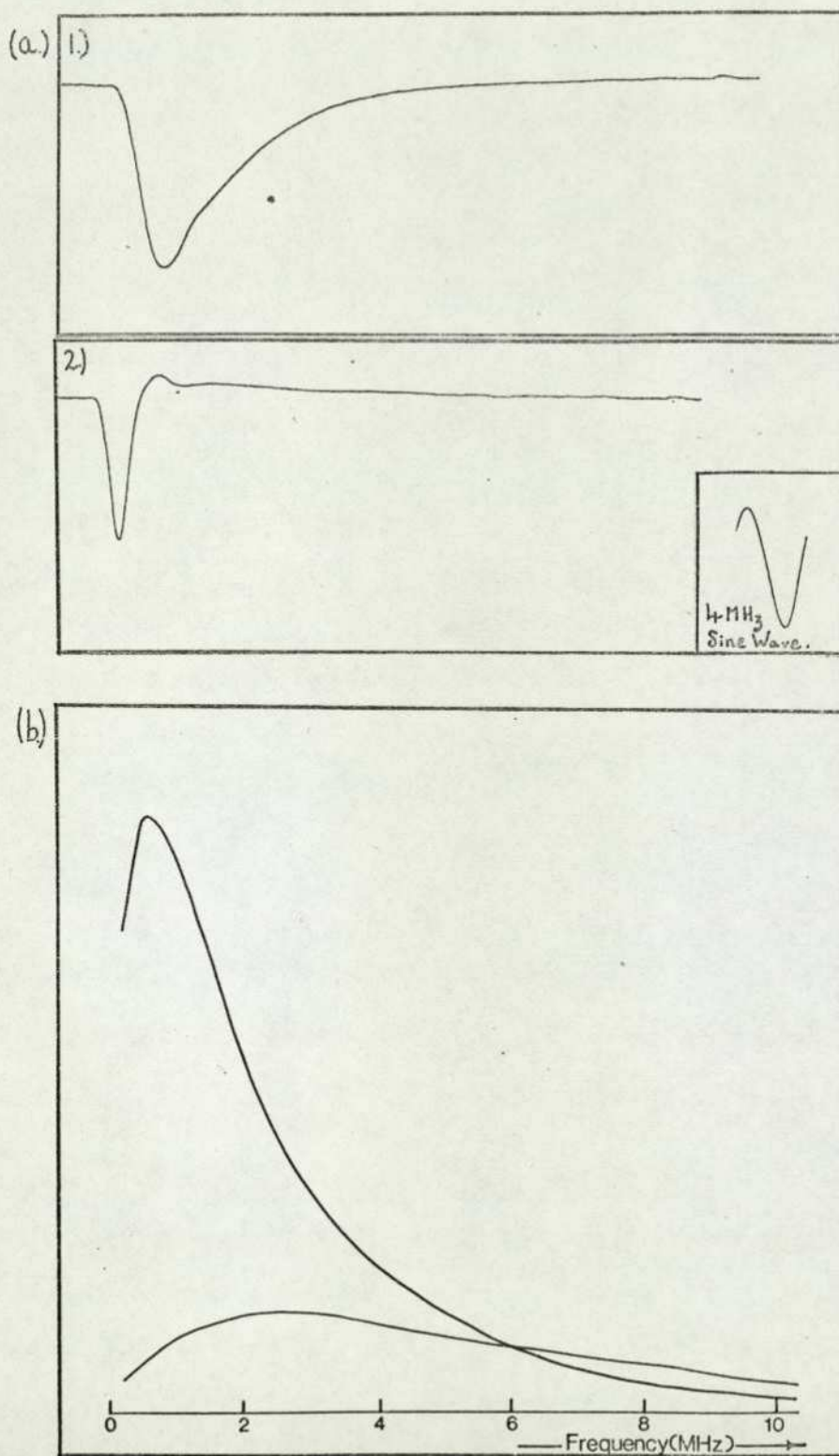


FIG.25. a) 1) Signal from source without filter.
 2) " " " " with " , and
 with 15db's more amplification than (1).
 b) Spectra of these two signals , showing
 how the filter gives a more even distribu-
 -tion of frequencies.

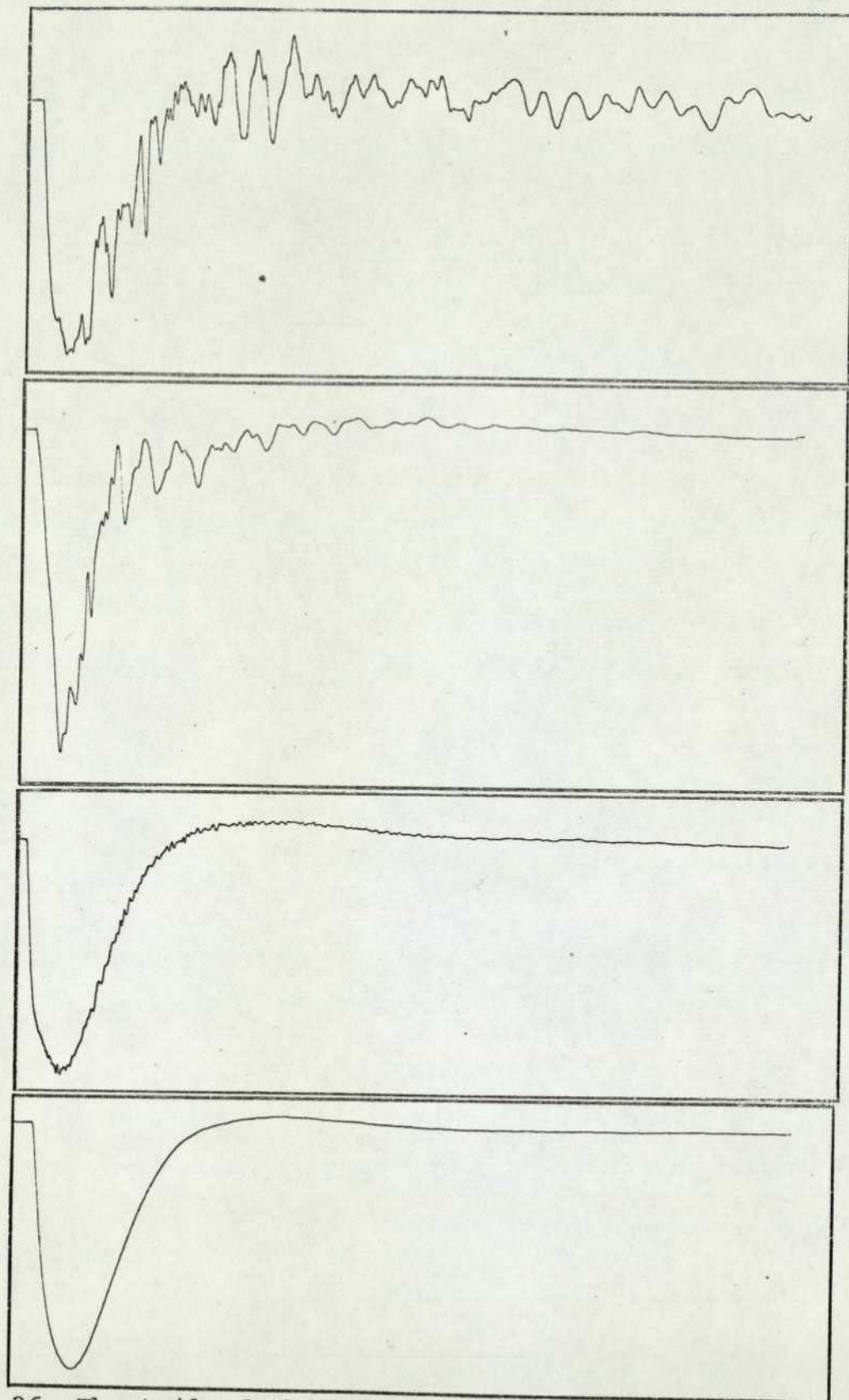


FIG. 26. The tail of the excitation pulse:
 (a) Without filter; Parallel faced transducer.
 (b) With " ; " "
 (c) Without " ; Back " face " spoiled."
 (d) With " ; " " " "
 (Timescale: 9-60microsecs.)

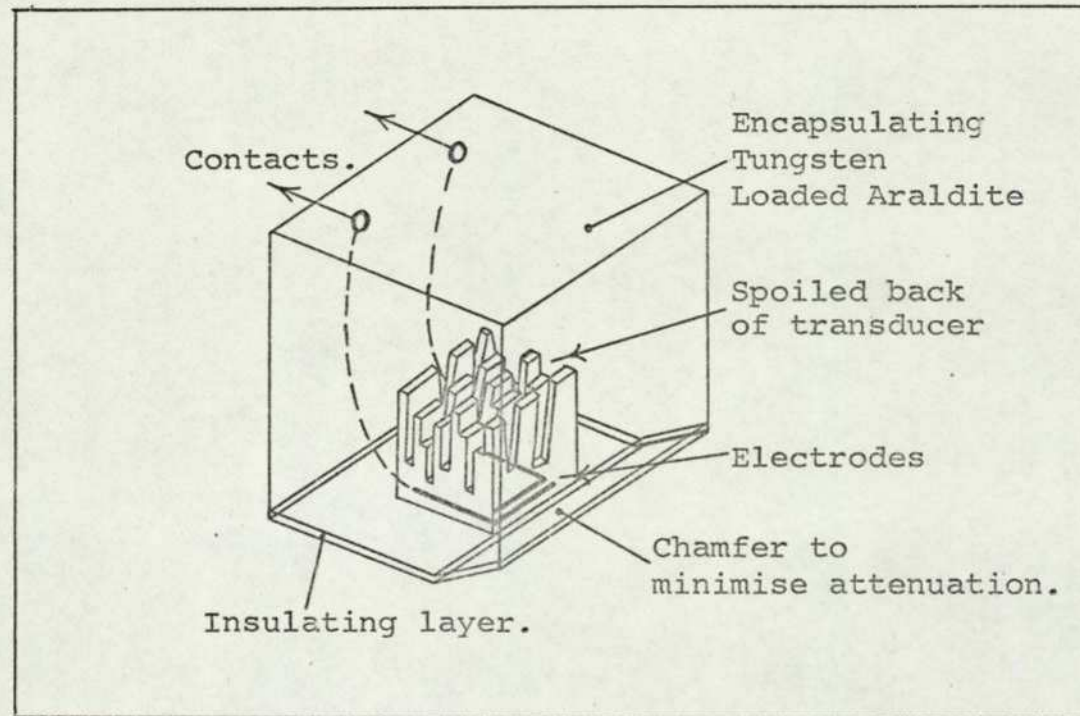


Fig. 27. Transducer for the transceiver mode of operation.

shows the one micron thick layer of electrically insulating polyester, glued onto the active face to insulate the electrodes from any conducting surface.

The usefulness of this transducer is limited by its low sensitivity. That it should be so is unexpected, since one would expect the sensitivity of the send transducer to a reflected wave to be the same as a transducer that had not been previously excited. However, this is not found to be the case. Two transducers of the type in Fig.27 used separately as transceivers required 10 and 14 dB's respectively more amplification to bring the reflections from a slot to that amplitude obtained when they were used together in the two transducer mode of operation. This suggests that the transducers are made insensitive for a short time after the application of an excitation pulse, possibly due to a temporary change of polarisation direction local to the fingers.

The introduction of the high pass filter would have considerably modified the results of the theory presented in Chapter 2, Fig.15, the higher frequencies being relatively enhanced. The theoretical predictions using the pulse shape of Fig.25(a(ii)) are presented in Fig.28, together with the summation of the results for the ratios of finger to half gap widths of 0.5 and 0.7 (normalised to the same maximum as the other curves).

4.2 Projected Improvements

The transducers' sensitivity can be improved by the use of multiple sources. The stagger-tuned array of Fig.9(c) (page 34) is one example of how this may be done. In that case, however,

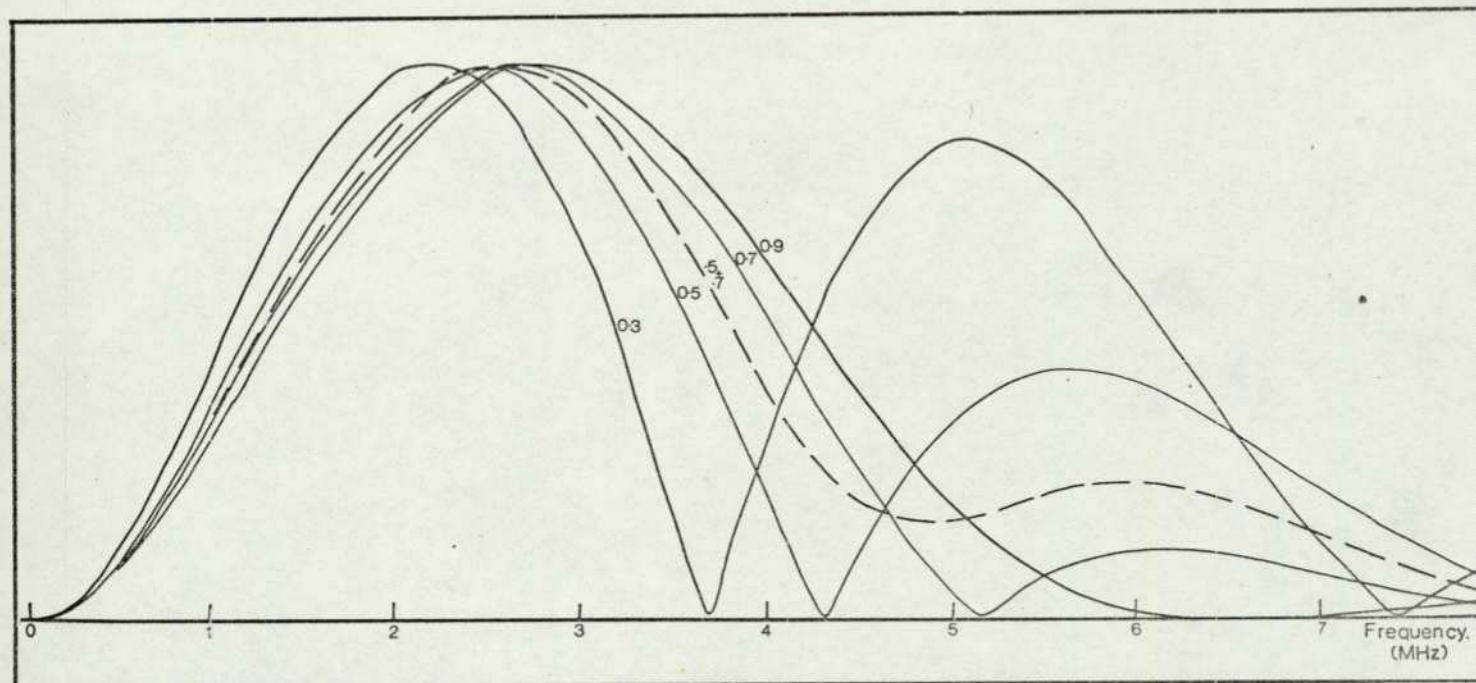


FIG.28. The spectra of two-fingered transducers of various values of the ratio (fingerwidth/ $\frac{1}{2}$ gap width), assuming impulse excitation;-compare with fig.15.

the increased output was spread over a longer period, giving an undesirably long signal. If either of the systems shown in Fig.29 is employed, this problem may be overcome.

Fig.29(i) shows how a large amplitude surface wave might be obtained by a superposition of smaller ones. The surface wave signal from each finger pair travels forward and encounters another finger pair, at the time that this second pair receives an excitation pulse. The surface wave from the second pulse adds to that arriving, giving a surface wave of greater amplitude but still of short duration. The same principle applies in reverse on reception so that the action of the multiple fingers would be doubly effective, greatly increasing the output.

As shown in the figure the piezoelectric elements should be separated physically to minimise the piezoelectric coupling between elements, or each excitation will have considerable effects on the others. This is because the longitudinal wave velocity in the PZT is much higher than the surface wave velocity in any material, so the bulk wave generated at the first excitation will tend to pre-empt the subsequent excitations.

In practice it would be simpler to have two delays per finger pair. One would be a simple delay on the trigger to a high voltage transistor operating in breakdown (Ref.12) to provide delayed excitation pulses. The second would be a delay on reception. This would have to be broad band and would operate, in this case, at a much lower power level than the scheme in the figure.

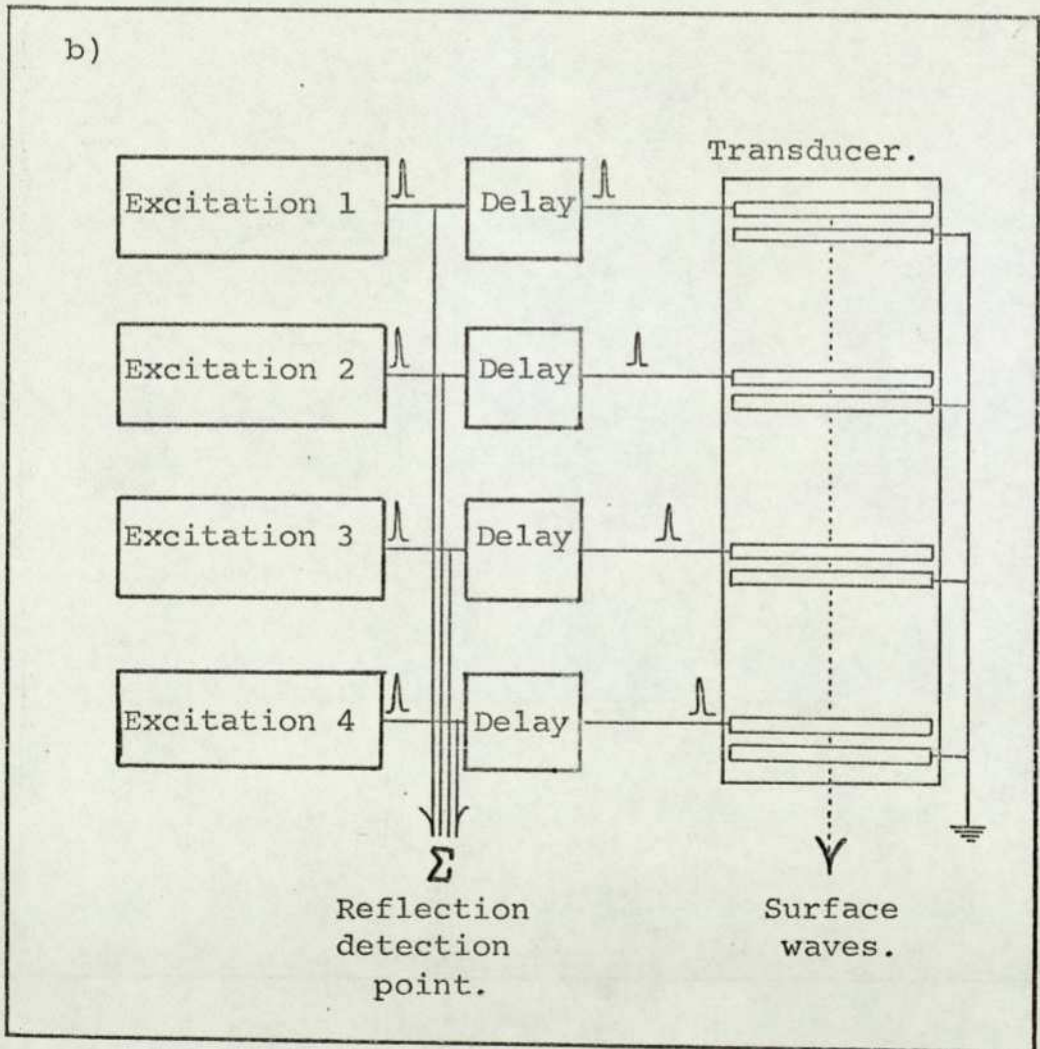
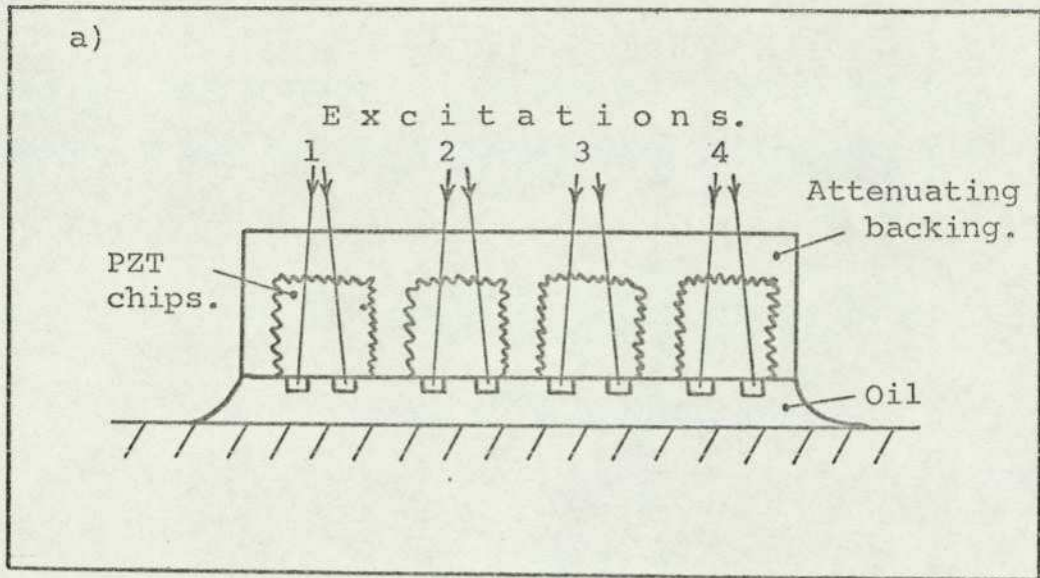


Fig. 29. (i). Schematic of sequentially excited transducer(a), and of its operation(b).

Perhaps a more subtle method of achieving the object i.e. increasing the sensitivity by using multiple sources, is a more direct extension of the stagger-tuned transducer of Fig.9(c). The objection to this transducer, it will be remembered, was that it produced a long 'chirp' signal when excited by a voltage spike. However, if it is excited by a chirp signal it can produce a spike output (in one direction only). Therefore, if one stagger-tuned transducer is excited by a spike and the chirp signal, obtained by receiving this surface wave by a two finger transducer, is amplified and applied to an identical stagger-tuned transducer, a short, broad band spike will be obtained (Fig.29(ii)). This short signal, on reflection from a defect can be detected by a two finger transducer. In practice this could be the first two fingers of the stagger-tuned transducer, with an electronic switch used to connect the two fingers to the others for excitation, and to isolate them for reception.

The advantages of this method are:

- (a) No broad band delays are required, only one broad band amplifier.
- (b) Since the signal applied to the amplifier has each frequency separated in time, the voltage output at any particular frequency will be much higher than that frequency component of an impulse at the same voltage. Typically, impulses of a kilovolt are used on commercial equipment, while with the system described above, the voltage level at any one frequency may be as much as 200 volts. Thus the power output should be much higher.

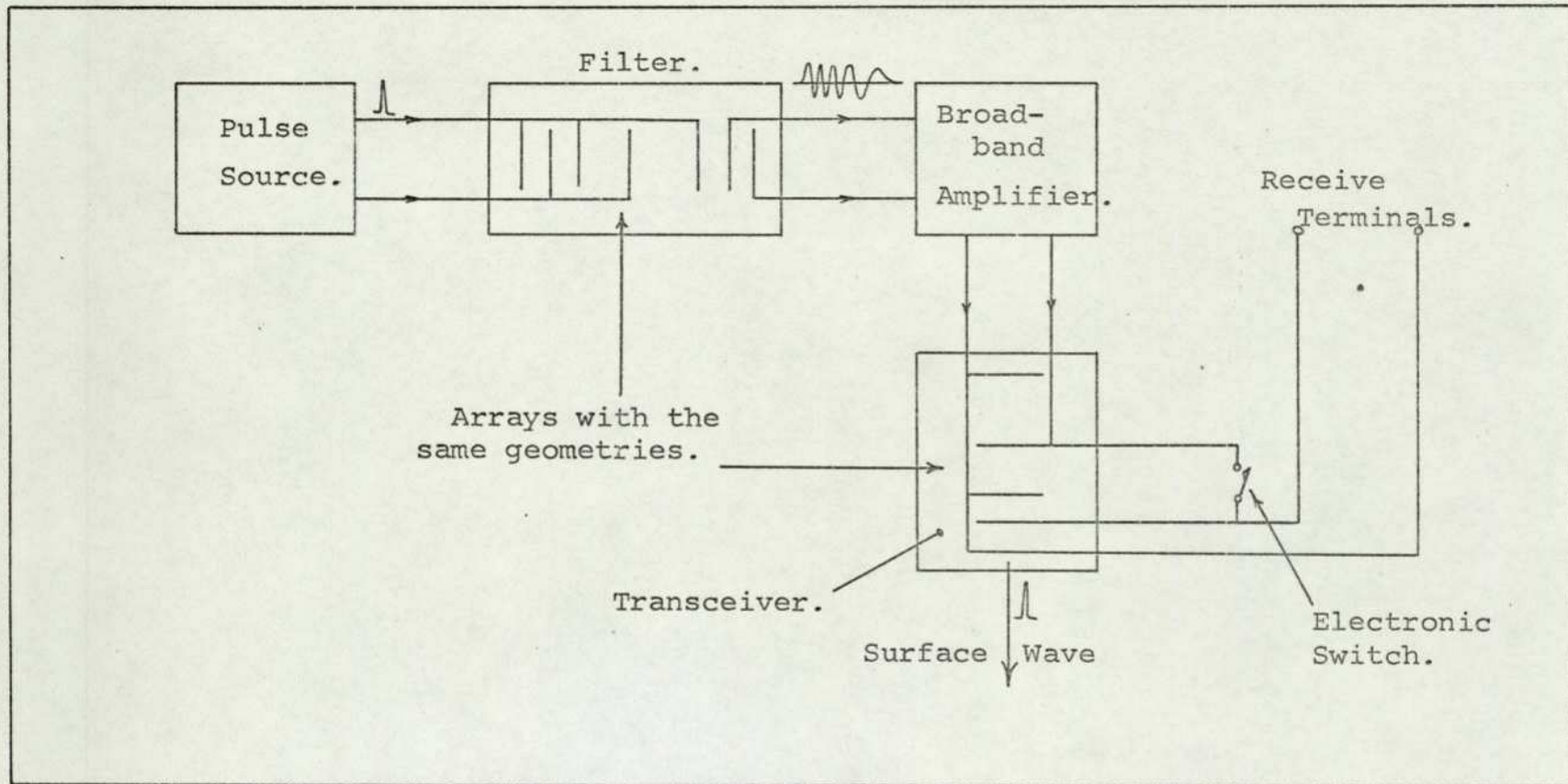


FIG.29.(ii) A method of achieving a short,broad-band,surface wave pulse,using two matched transducers.

Both the methods described above are amenable to design for achieving an output with a nearly flat spectrum. It is important to realise that the attainment of a perfectly broad band output is still very desirable. Though the device of comparing the spectra of the first reflection and of the whole reflection does effectively produce the perfect broad band response, it would be unnecessary to carry out this operation if the spectrum of the first reflection were perfectly broad band to start with. Of course, the generation of a perfectly broad band reflection signal does not ensure that the first reflection will be so, but, having regard to effects such as diffraction, the slot reflection characteristic, and attenuation, it should be possible to generate an output signal giving a roughly broad band first reflection for a range of situations. The use of a manually operated filter array would, perhaps, extend this range.

Operation of a device incorporating the features described above would involve, first, the location of the defect, then, the gating of the first reflection from it. After adjustment of the filter to give the most even spectrum for this signal, the gate is extended to include all reflections, whereupon the features of the defect will be indicated by peaks on the cepstrum.

CHAPTER FIVE

5. THEORETICAL TREATMENT OF THE REFLECTION FROM A SLOT

5.1 Introduction

5.2 Previous Work

5.3 Numerical Treatment And Comparison With Experiment

5.4 Slot Response

5.4.1 Angled slot response

5. THEORETICAL TREATMENT OF THE REFLECTION FROM A SLOT

5.1 Introduction

The work in preceeding chapters described what has, so far, been achieved experimentally. The only requirement of the method of generating the cepstrum, was that every feature of the target should reflect some of the incident Rayleigh wave.

In this chapter a theoretical model is developed to describe this reflection and use it as a basis for investigating possible avenues of development.

5.2 Previous Work

No satisfactory model for the reflection from a slot exists. As a first step towards one, work done on the transmission and reflection characteristics of the wedge, particularly the 90° wedge, or quarter plane as it is called (see Fig.30), may be considered. Experimental work using transducers attached to the edges of two dimensional models (i.e. propagation parallel to the x axis of Fig.30) has been carried out (Refs.11 and 50 to 52) as well as a photoelastic study (Refs.53 and 54). Theoretical treatments, too, have been plentiful (Refs.55 to 63). However, agreement between the theories and experiment has not been good. The chief difficulty is the presence of the geometrical singularity at the wedge apex. Using a ray model it can be seen that this geometry leads to an infinite series of reflections

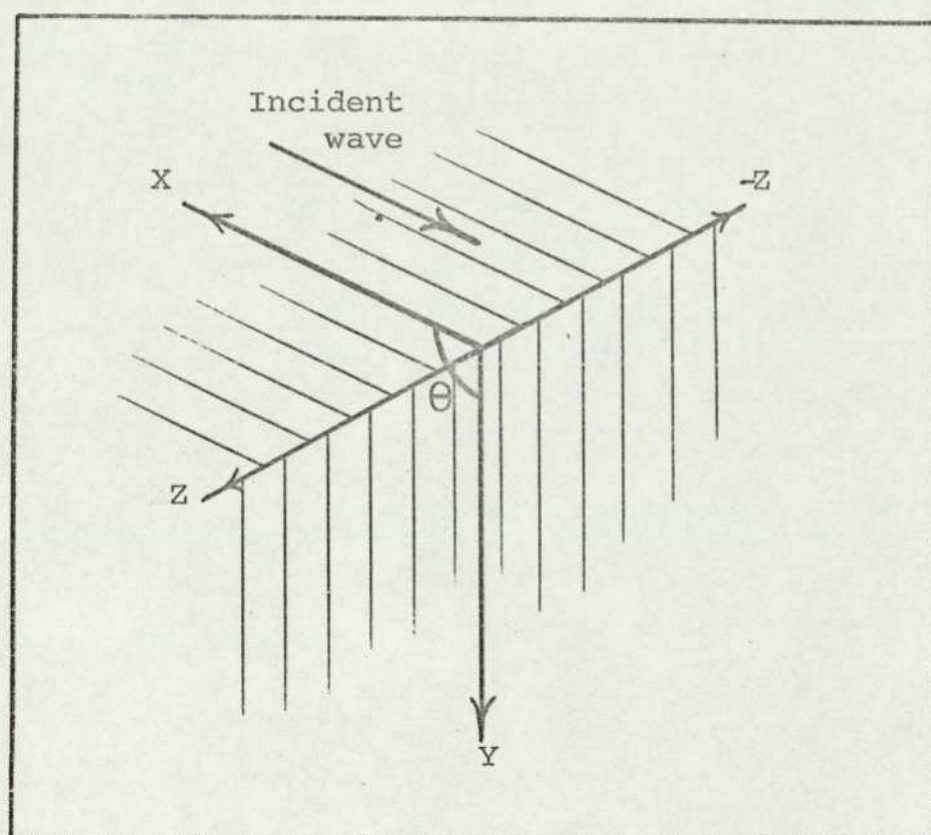


Fig. 30.

Wedge geometry:

$$0 < X < \infty$$

$$0 < Y < \infty$$

$$-\infty < Z < \infty$$

-and when $\theta=90^\circ$ the wedge is termed a 'quarter plane'.

directed towards the apex with mode conversion and mode coupling along the interface. Best agreement with experiment has been achieved by Mal and Knopoff (Ref.59) using a Green's function method of approximation to obtain a pair of coupled integral equations derived by a representational method. This approach was designed to include the multiple reflections. Even this method, however, does not produce good quantitative agreement for angles less than about 120° . The analytical solution of the 90° wedge, described by Ottaviani (Ref.64) as presenting 'almost insurmountable difficulties', does not encourage the extension of this pursuit to the more complicated case of the slot. Numerical methods have been suggested by Alterman and Rotenburg (Ref.62) and it is shown in the following section that a very simple numerical approach can give useful agreement with experiment, in the case of the slot.

5.3 Numerical Treatment-Comparison With Experiment

The aim is to understand how the incident Rayleigh wave energy is reflected from a slot. The energy/depth relationship for a travelling Rayleigh wave can be computed from particle displacements in the same way as Cook and Valkenburg (Ref.65) have done. But this is difficult in the region of the slot, where the Rayleigh wave is considerably modified. The only 'a priori' information, at that point, is in the reduction of the stresses over the surface of the slot to zero. In Fig.31 the sum of the squares of the components of particle displacement and of the stresses producing them in directions normal and shear to the direction of propagation, as functions of depth, are compared.

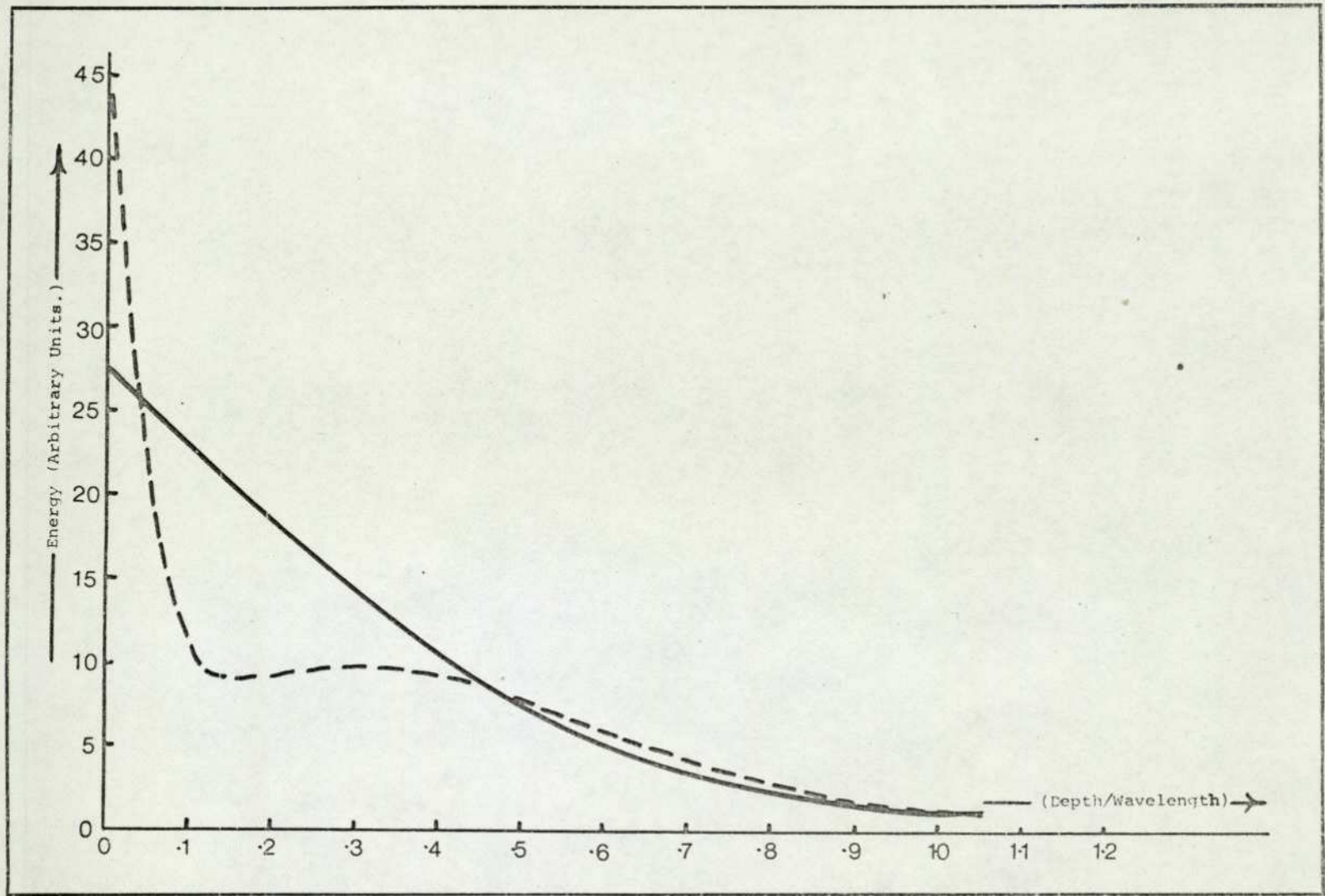


FIG.31. Comparison of the energy/depth relation computed from particle displacements (full line), and the approximate relation from the stress data (dotted line).

These two curves are normalised at a depth equal to half a wavelength. The stresses were obtained by selecting one frequency from the spectra shown in Fig.6 to give the results for a plane harmonic wave. The particle displacements were taken from the result in Kolsky's book (Ref.8), reproduced in Fig.2. As might be expected, agreement is poor near the surface where the boundary conditions are rapidly varying. This deviation near the surface could be reduced if the difference between the actual surface pulse and that of the stress field calculation were corrected for. It can be seen that the waveform at the surface, used in the model {Fig.4(b)} is discontinuous. This gives it a larger high frequency content than the experimental waveform, so that the energy in the model is unrealistically high near the surface.

However, the agreement, as it stands, is good enough to use the stress distribution to give a very rough measure of the energy distribution. If it is now postulated that the energy reflected from the slot is directly proportional to that incident upon it, a plot of reflected energy with respect to slot depth may be obtained. Conveniently, this relationship has already been determined experimentally (Ref.66) for the same pulse waveform as was used in the stress field analysis of Section 1.3.

The theoretical values are found by integrating the spectra of Fig.6 and adding the squares of the two principal stresses. The results are compared in Fig.32 (normalised, again, at a depth of $(\lambda/2)$). They compare well, except for a slight divergence near the surface. This agreement is probably better than that which would be obtained by comparing results for a harmonic disturbance, since the broad bandwidth of the signal

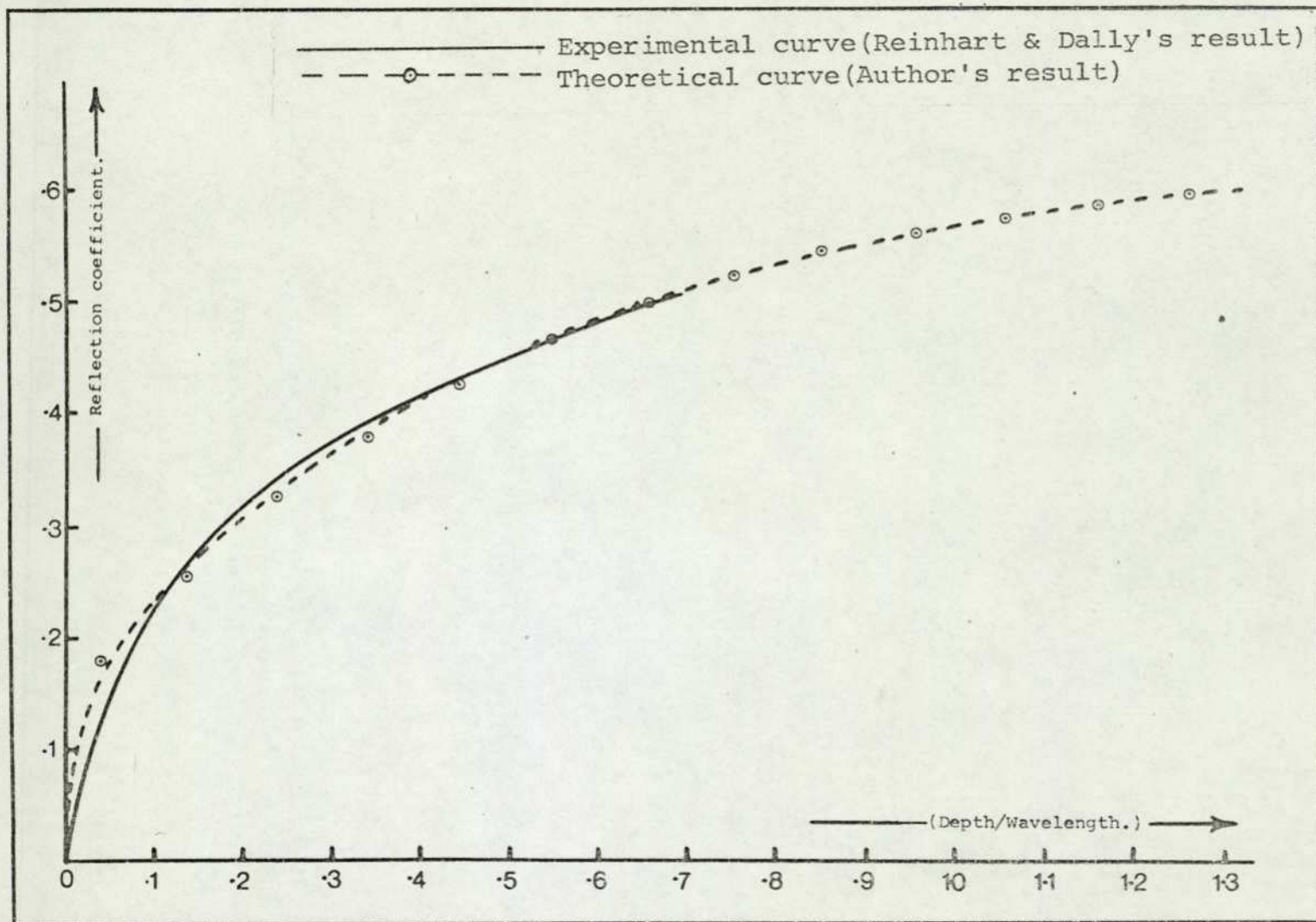


FIG. 32. Comparison of experimental and theoretical curves for the reflection from a slot of a Rayleigh wave generated by an explosive source.

will smooth out the effects of the deviation of the stress derived energy distribution from the more correct, particle motion derived, curve. Clearly this result of reflection versus depth, for the harmonic case will be proportional to the inverse of the function in Fig.31.

5.4 Slot Response

The theoretical response of a slot to a plane harmonic Rayleigh wave (i.e. the inverse of Fig.31) is shown in Fig.33. The horizontal axis is (depth/wavelength) so that, as well as being the plot of the narrow band response of a slot of varying depth, it also represents the broad band response of a slot of fixed depth. A time signal corresponding to this spectrum is shown in Fig.34. This represents, then, the time reflection signal from a slot, for an incident Rayleigh wave of perfectly broad bandwidth up to the limit of the spectrum of Fig.33. The scale of the time signal corresponds to a limit of 10 MHz.

It is to be remembered that this signal represents only the first reflection from the slot, since it was assumed that no transmitted energy is subsequently reflected. This possibility is taken into account by adding the reflections from other edges of the slot, in the time domain, multiplied by the appropriate reflection and transmission coefficients given in the Geophysical literature, in particular Ref.52. The absence of data for obtuse angles makes it necessary to choose values for the 270° reflex angle at the bottom of the slot, to obtain the best agreement, when taking the cepstrum of the final time signal, with the experimental results of Chapter 3, Fig.20(b). This corresponds

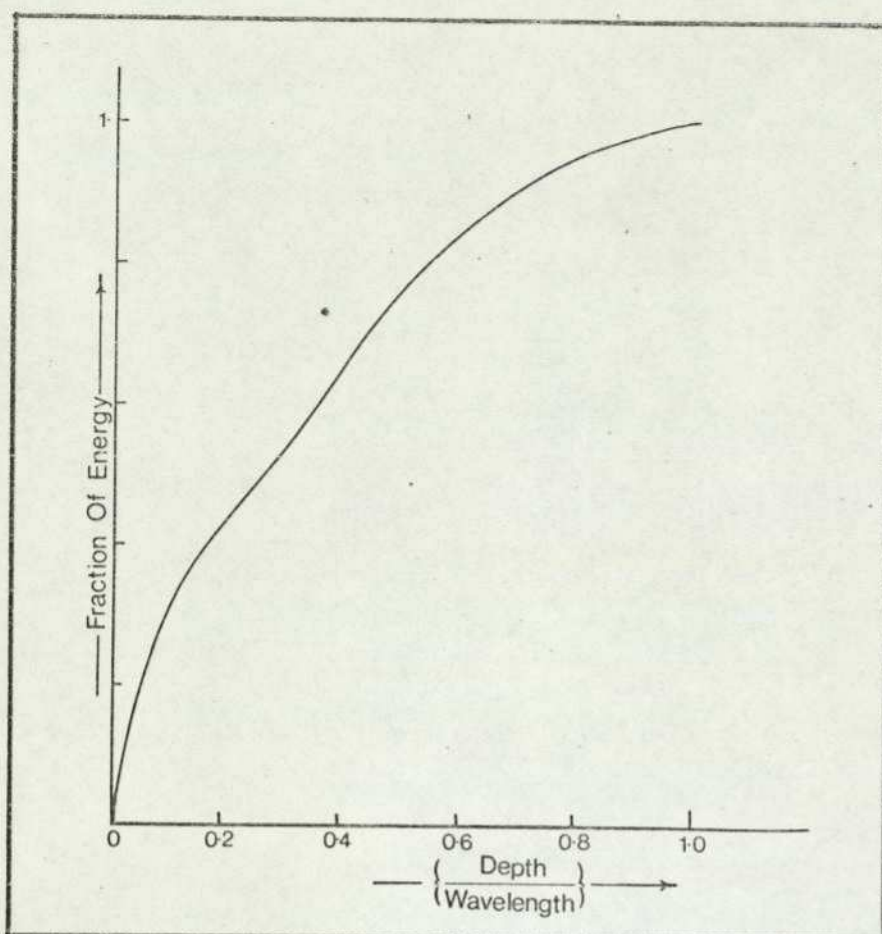


FIG.33. The theoretical, broad-band, reflection characteristic of a slot.

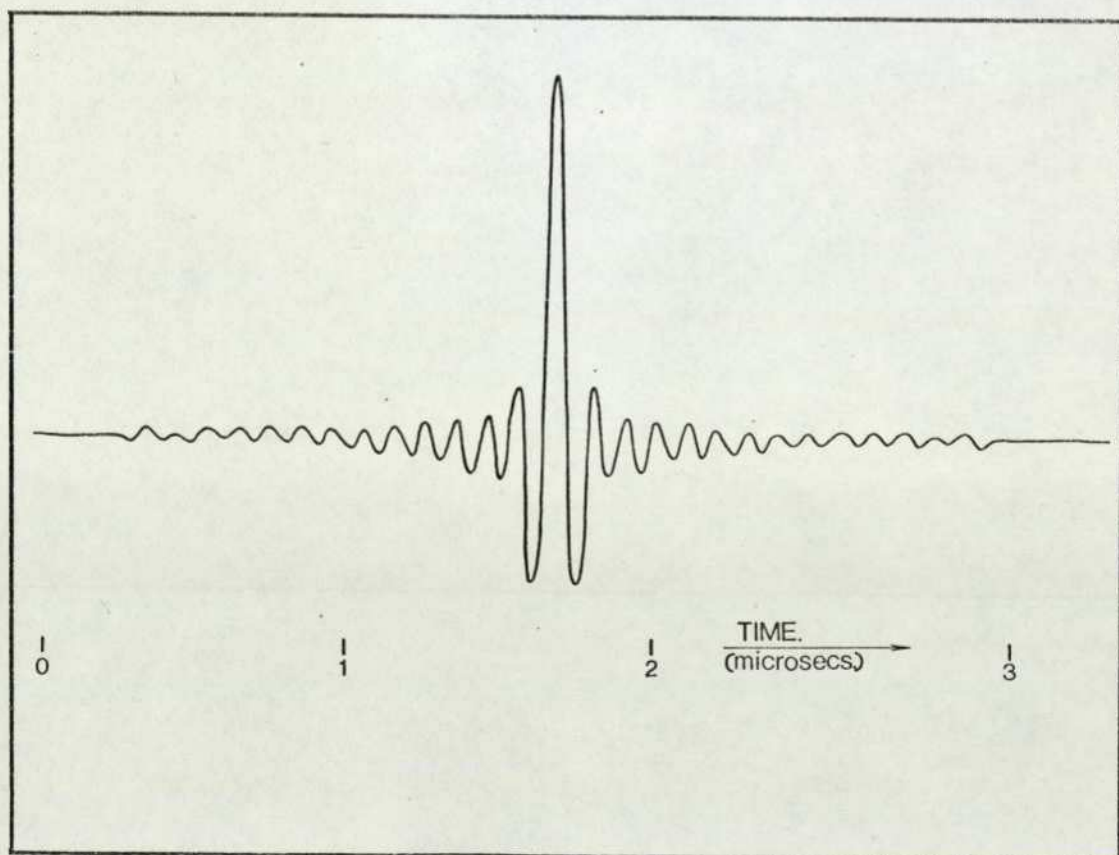


FIG.34. A time signal having the modular spectrum of Fig.33.

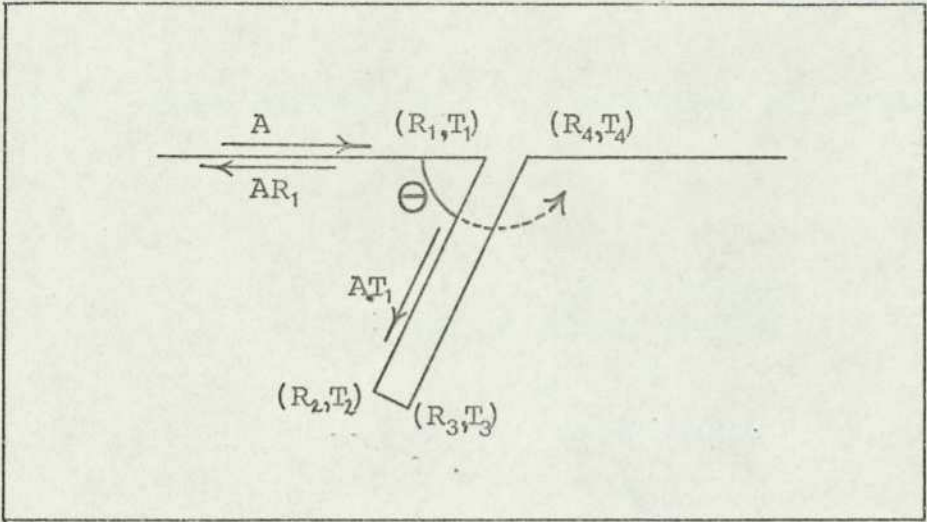
to values of the transmission and reflection coefficients of 0.67 and 0.05 respectively. The transmission coefficient is that for the acute angle case, and the reflection coefficient is a reasonable value following from the statement of Knopoff and Gangi (Ref.51) that 'the reflected energy for wedge angles greater than 180° is less than one per cent of the incident energy'. A one per cent reflection of incident energy gives a reflection coefficient of 0.1.

Lapwood (Ref.55) has shown theoretically that the surface wave changes shape on transmission. However, Pilant et al (Ref.52) have shown experimentally that the phase shift on transmission is frequency independent. This latter conclusion is adopted i.e. we assume no change in pulse shape.

The overall reflection signal from a slot can now be synthesised and this model used to explore the possibility of using the method of Chapter 3 to determine the angle of the slot to the surface.

5.4.1 Angled slots

The angle to be varied, θ , is shown in Fig.35 together with the tabulated values of transmission and reflection data used for these angles. The time signal, synthesised in the previous section, is treated in the same way as the experimental time signals in Chapter 3 to generate the cepstrum, with a slight modification as shown in Fig.36. Let us recap on the process. The time signal, (a), for an angle $\theta = 50^\circ$, has a spectrum (b), the first reflection having the spectrum (c). The ratio of these, (d), is Fourier transformed a second time to give



θ	R_1	R_4	T_1
50°	.428	.161	.446
60°	.529	.261	.197
70°	.382	.251	.647
80°	.067	.365	.780
90°	.252	.252	.670
100°	.365	.067	.549
110°	.251	.382	.489
120°	.261	.529	.468
130°	.161	.428	.532
$T_2 = T_3 = 0.67$			
$R_2 = R_3 = 0.05$			

1st Reflection= AR_1
2nd Reflection= $AT_1^2 R_2$
3rd Reflection= $AT_1^2 T_2^2 R_3$
4th Reflection= $AT_1^2 T_2^2 T_3^2 R_4$

Fig. 35. Data used to obtain the cepstra
for various angles, θ , of the
slot shown. (mainly from Ref.52)

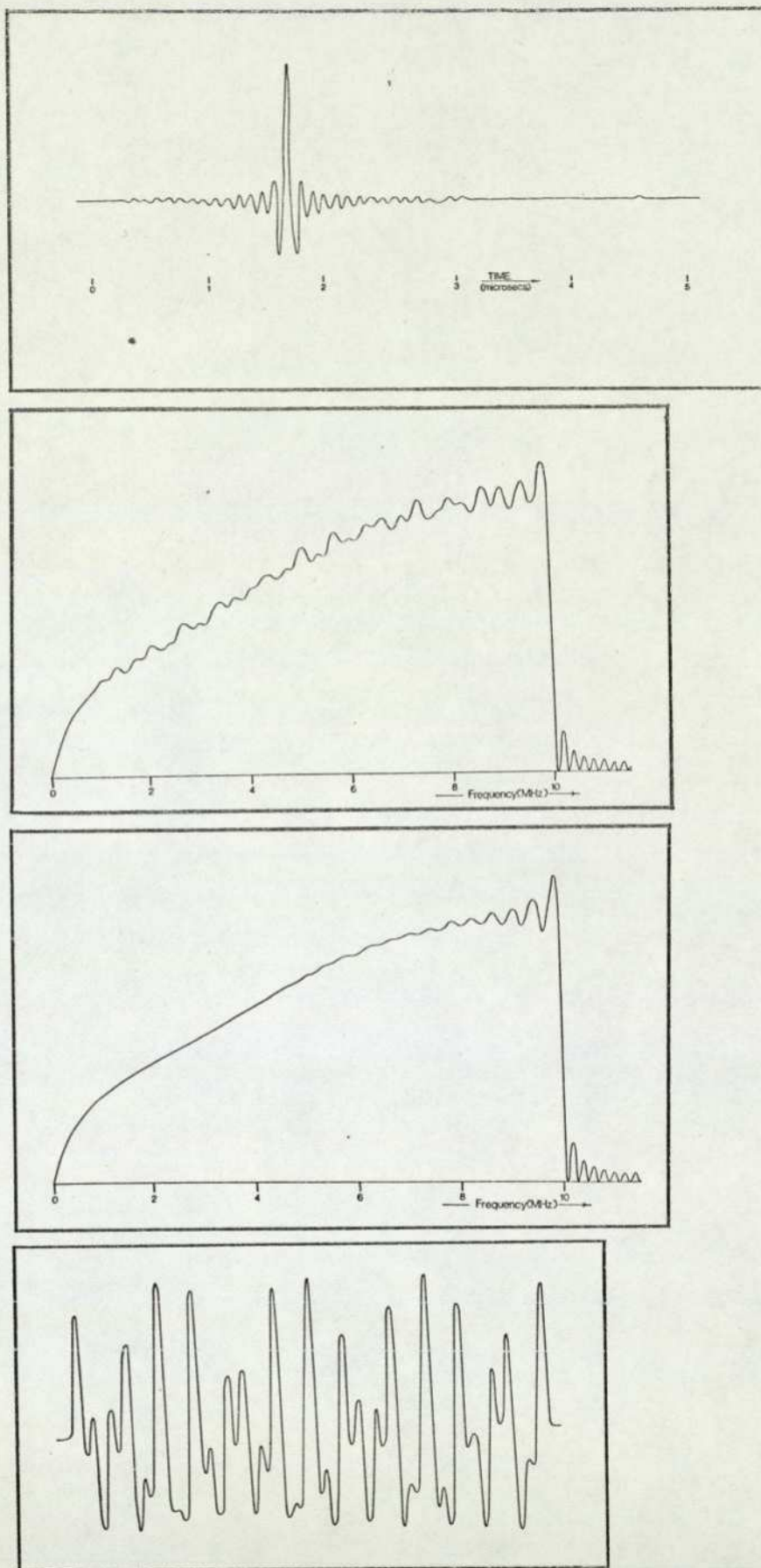


FIG.36. a) Theoretical time signal for 50° angled
 b) The spectrum of (a). [crack.
 c) The spectrum of the first reflection.
 d) The ratio of (b) and (c), being the
 modulation on (b).

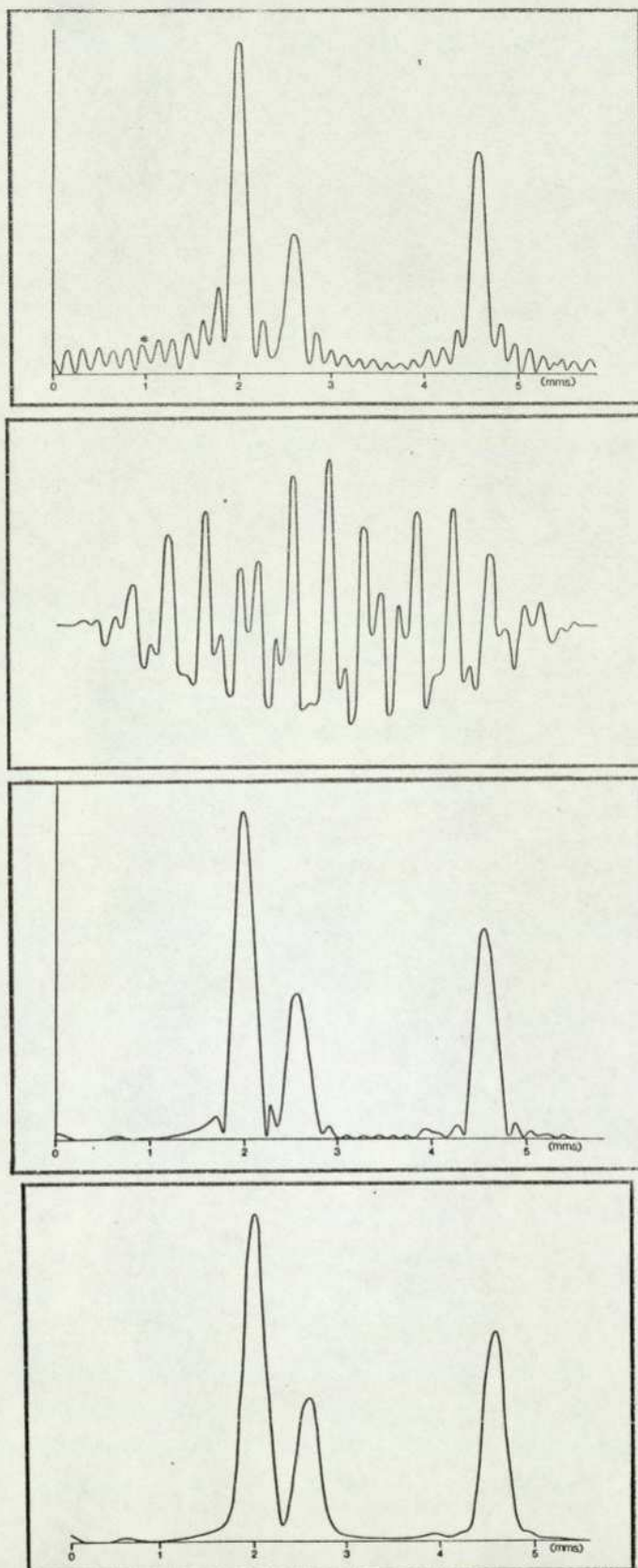


FIG.36.e)The cepstrum of (d).
 i.e.the Fourier transform of (d).
 f)The modulation,(d),multiplied by a
 half period of a sine wave.
 g)The cepstrum of (f).
 h)The cepstrum obtained by averaging
 over a cycle of the modulation of (e).

the cepstrum (e). This cepstrum is modulated at a frequency characteristic of the limited bandwidth of the system. It is the product of the cepstrum required and the transform of the effective frequency 'gate' used. This 'gate' modulation can be reduced by using half a cycle of a sine wave rather than the top hat function as the gate. This gating is shown in (f) and the effect on the cepstrum shown in (g). The modulation can be removed altogether by averaging the cepstrum over the period of modulation. This is shown in (h). It is this plot that is shown in Fig.37.

The flow diagram of the computer program achieving the whole of this process, from the data of Fig.31, is given in Appendix 5. It is designed to repeat the process for angles between 50° and 130° , in 10° steps.

The results are presented in Fig.37, all traces being drawn to the same maximum amplitude, the values at the maximum being shown to the right of each trace. It can be seen that, as the angle changes, the cepstrum changes too, each one being different. Therefore it appears likely that the slot angle may be uniquely characterised by the cepstrum.

Due to the approximate nature of the basis of this analysis and the uncertainty in the reflection and transmission coefficients, these cepstra should not be viewed as anything more than an indication that an experimental investigation should prove productive.

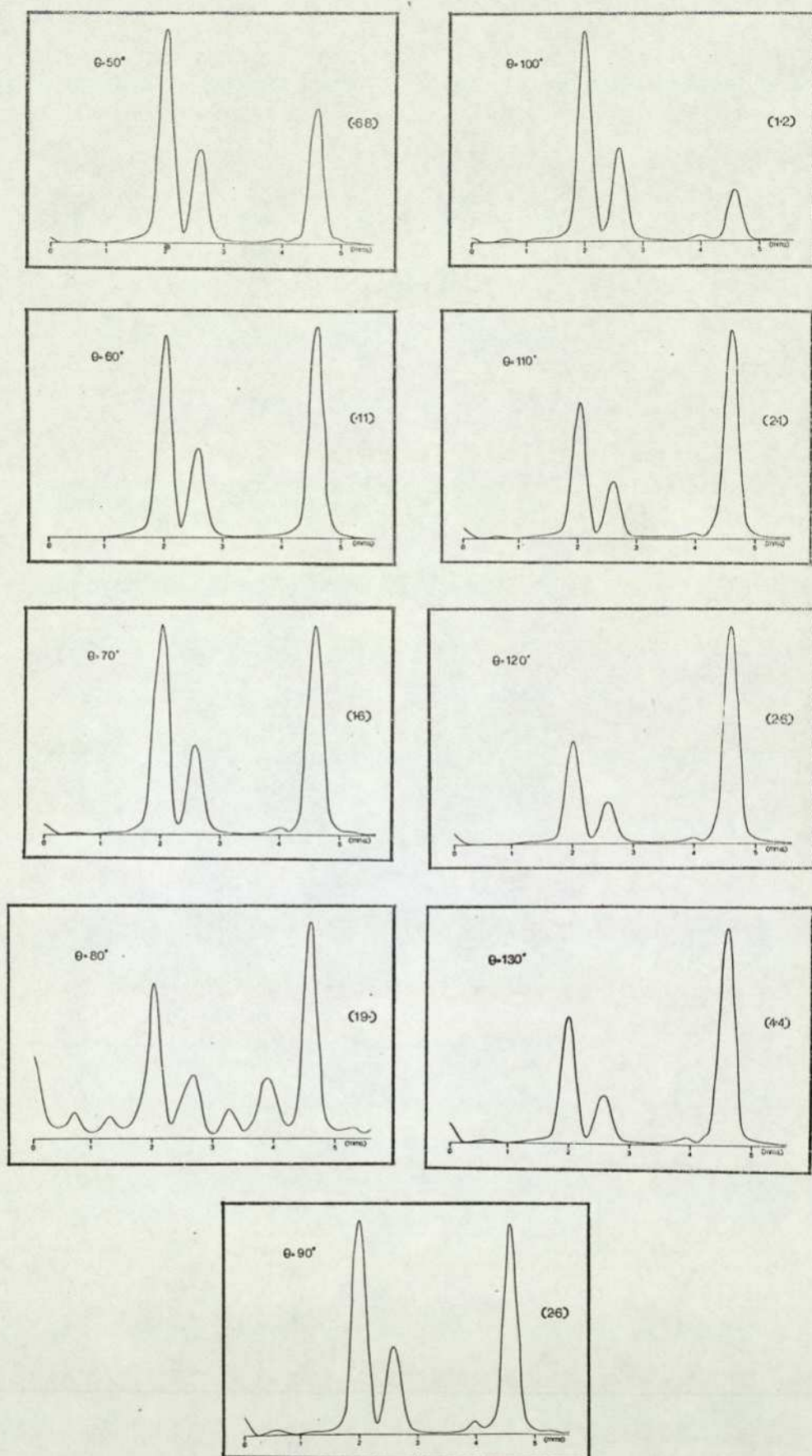


FIG.37.
Variation of the Cepstra from slots of different
angles θ .

CHAPTER SIX

6. FURTHER DEVELOPMENTS PROPOSED

- 6.1 Evaluation Of The Stress At A Crack Tip
- 6.2 Utilising Bulk Wave Conversion
- 6.3 Determination Of Crack Orientation

6. FURTHER DEVELOPMENTS PROPOSED

6.1 Evaluation Of The Stress At A Crack Tip

If the reflection and transmission coefficients can be determined with sufficient accuracy, it may be possible to correlate the crack tip coefficients with the stress distribution about the tip. These coefficients must be functions of the acoustic impedance of the medium into which the wave is propagating. The acoustic impedance, in its turn, is a function of the density of the medium and the velocity.

The sensitivity needed for this sort of measurement can be estimated roughly. Longitudinal waves are relatively unaffected by stress (Ref.66), the velocity change amounting to only slightly more than 0.01% for a tensile stress of 3 tons/sq.in in steel. Shear waves on the other hand experience a 0.05% velocity change at a stress of 5 tons/sq.in (Ref.67), again in steel. Benson (Ref.42) found that in aluminium alloys the shear wave velocity change was higher, typically 0.3% for a 5 tons/sq.in load, but that the surface wave velocity changed much less, typically 0.1% for the same load. As a rough guide we can say that the stress at a crack tip will build up to a value in the region of the materials yield strength before a crack will propagate. The yield strength for many aluminium alloys is in the region of 20 tons/sq.in so that, in order to

monitor the stress level up to this value we would need to resolve approximately 2 tons/sq.in increments or velocity changes of 0.04%. This would mean differences in the amplitude of the peaks in the cepstrum of that amount. Since the main limitation of transducer coupling variation has been removed this might be possible.

The effect might, indeed, be greater than the rough estimate made above because of a further effect. The birefringence of the Rayleigh wave as it passes through a stressed region will tend to separate the longitudinal and shear components. However, if the components were to become out of phase this would disturb the condition of zero normal stress at the boundary. Since this cannot be envisaged, it seems reasonable that some mode conversion would take place, this effect too contributing to a variation in the reflected amplitude. However, it is difficult to estimate the magnitude of such an effect.

6.2 Utilising Bulk Wave Conversion

Reinhart and Dally (Ref.68) note that at slot depths greater than $(\lambda/4)$, approximately 75-80% of the incident energy suffers mode conversion into bulk waves. This energy is not so quickly attenuated in the bulk as may be thought, since it only propagates into a fairly small angle. Fig.38 is a tracing of the bulk wave shown, but not commented on in Ref.59. It shows the longitudinal wave produced at the bottom of the slot, two wavefronts being visible, one direct from the source and one by reflection off the surface. This double signal would allow

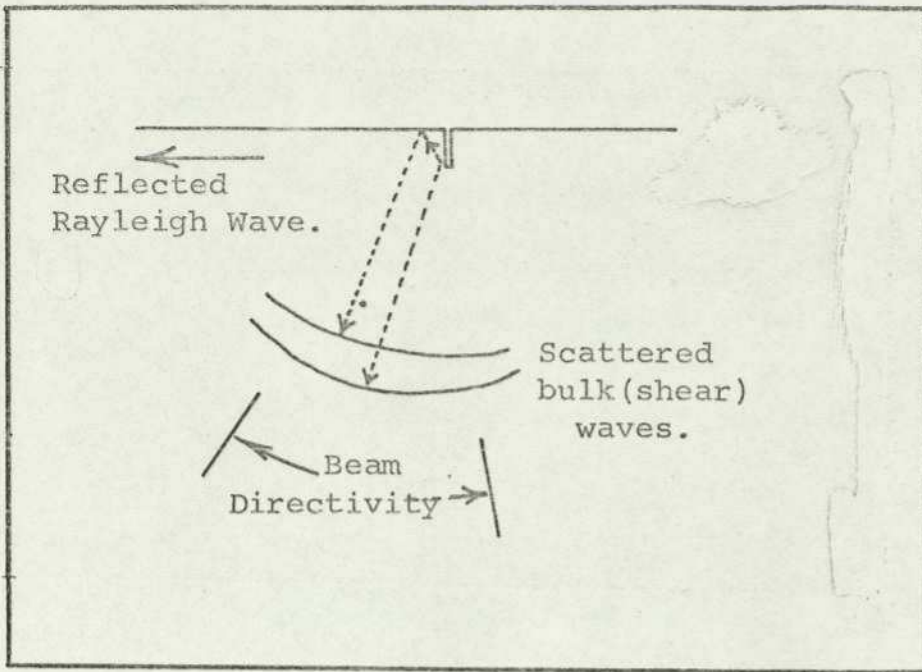


Fig.38. Tracing of the result obtained using a photoelastic method, for the reflection of a Rayleigh wave from a slot.

the cepstrum of the bulk wave received below a crack to be taken, to give the crack depth. Because of its directionality and large energy content, this promises to be a sensitive method. It would also provide a means of examining internal surfaces by bulk wave conversion to Rayleigh waves on the internal surface and subsequent search for bulk wave emissions as described (see Fig.39).

6.3 Crack Orientation

It would be an advantage not to have to adjust the position of a transceiver normal to a crack (i.e. by maximising the reflected amplitude) in order to determine the orientation of the crack in the surface. Use of a broad band transducer might facilitate this. It is assumed here, that most cracks would have a sufficiently rough surface to reflect a significant part of the incident energy back along the direction to the transducer. An angled crack acts as a low pass filter, the pass band getting smaller as the angle increases. Thus a knowledge of the input characteristic and of the filtering characteristic mentioned would enable the crack orientation to be read from the cut-off frequency (only the first reflection being used here of course).

The difficulty of this idea lies in the variation of the input spectrum, due to coupling variation. This can be up to 20% maximum variation at any particular frequency. The design of a shoe to standardise the transducer to working surface distance would probably reduce this variation to a value low enough for the technique described above to be possible.

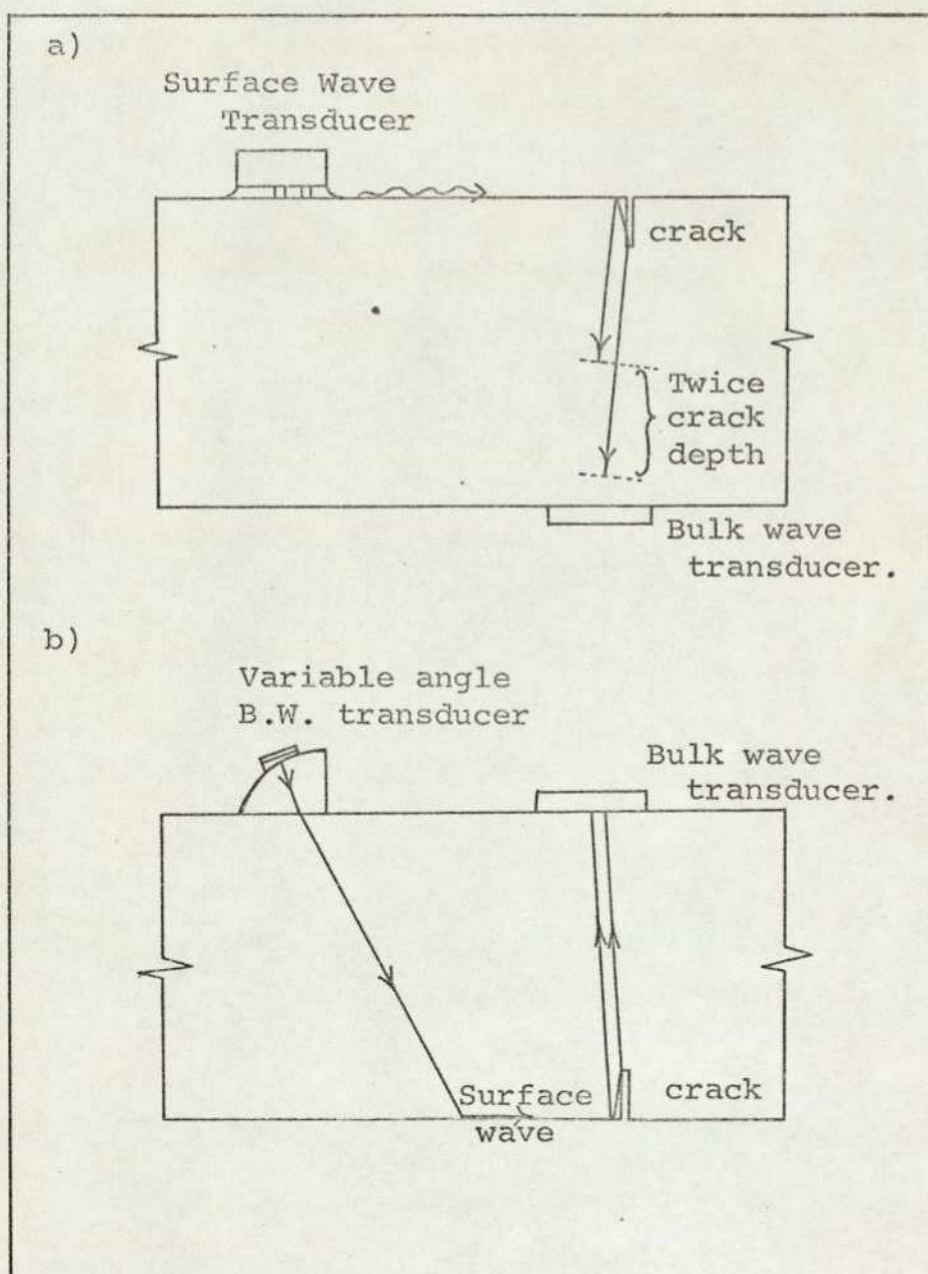


Fig.39. a) Crack depth evaluation using a combination of surface and bulk waves (B.W.)
 b) Method for measuring internal flaws.

Conclusion

The method of generating the cepstra of reflected broad band Rayleigh waves from surface defects has been demonstrated. The presence of discontinuities in the profile of the defect has been shown to give peaks in the cepstum, the positions of which give the defect dimensions. The amplitude of these peaks has yet to be correlated experimentally with such quantities as crack angle or stress distribution, but it has been shown that there is good reason to believe that this may be achieved.

Through the device of comparing the total reflection to the reflection from the first edge of the defect, all variations but those at the crack can be eliminated so that the problem of transducer coupling variation is obviated and the surface condition between the transducer and defect becomes of much reduced importance. The transducers developed for, and used in, this study are capable of considerable improvement and this should lead to some simplification in the method, and reduction in the lower limit of the crack size that may be measured. This value is presently limited by the duration of the surface wave generated by the transducer to 1.4 mm due to the gating system employed. Reduction in the pulse length would reduce this value.

APPENDIX 1

The derivation of the stress distribution in the Rayleigh wave propagating from an explosive source might appear of limited usefulness. However it is capable of extension to give the field for any input waveform. This is done by multiplying the spectrum of the stress field of the explosive source at the point of interest, by the quotient of the spectra of the input and explosive waveforms at the surface. Inverse Fourier transformation of this product gives the waveform required.

Since, then, this may be of general usefulness in reflection problems, in the way described in chapter.5., a listing of the computer program giving the stress field for the explosive source is given below, followed by a simplified flow diagram.

```

MASTER STRESSPLOT
COMPLEX ZETA1,ZETA2,STORE1,STORE2,P,FLOGP,PLOG
1,SIG,UNIT,FMUFI1,FMUFI2,FMUFID,FMUFIN,DOG,COT,
2FISH,UTIT,A(1024)
REAL T(4),C(4),STRSXX(120),STRSXY(120),STRSYY(120)
C
C =====
C INITIAL QUANTITIES
C
  PI=3.14159265
  AA=7.5
  R=37000
  DIS=39600
  DIL=70720
  B=11.5
  READ (1,100) (T(IM) ,IM=1,4 )
100  FORMAT(4F0.0)
  X=3000.
  DT=2.0E-06
C
  -----

```

```

C -----
C CALCULATE INTERMEDIATE VALUES.
C
  RA=R/AA
  RSQ=R*R
  A1=SQRT(1./RSQ-1./{DIL*DIL})
  A2=SQRT(1./RSQ-1./{DIS*DIS})
  C(1)=1.0
  TI=(T(3)*T(4))/(T(4)+T(3)-T(2))
  C(2)=-TI/(TI-T(2))
  C(3)=(+T(2)*(T(4)-TI))/(T(4)-T(3))*(TI-T(2))
  C(4)=(-T(2)*(T(3)-TI))/(T(4)-T(3))*(TI-T(2))
  TIN=T(4)/10.
  ULIT=2.*(A1/R)
  A1SQ=A1*A1
  A2SQ=A2*A2
  PLEB=A2SQ-2.*A1SQ-1./RSQ
  CAT=A2SQ+1./RSQ
  D=1./(A2SQ-A1SQ)
  UNIT=(0.,1.)
  UTIT=UNIT*B*D
  ZO=8.1E-02
  ZA=2.0E-06
C
C =====
C SET THE DEPTH.
C
  DO 14 NA=1,11
  Y=FLOAT (NA-1)*0.1
  IF (Y-0.1) 11,11,0
  Y=Y*4.-0.4
11 CONTINUE
C =====
  FNEW1=A1*Y
  FNEW2=A2*Y
C =====
C SET THE TIME.
C
  DO 1 NO=1,120
  TO=ZO+(FLOAT(NO)*ZA)
C
C =====
C
C 1
C
C =====

```

```

=====
1 .
=====
CALCULATE THE STRESSES.

ETA= TO-X/R
ZETA1=CMPLX(ETA,FNEW1)
ZETA2=CMPLX(ETA,FNEW2)
STORE1=(0.0,0.0)
STORE2=(0.0,0.0)
DO 2 N=1,4
P=ZETA1-CMPLX(T(N),0.0)
P=P*RA
FLOGP=CLOG(P)
PLOG=P*FLOGP
SIG=C(N)*PLOG
STORE1=STORE1+SIG
FMUFI1=UTIT*STORE1
DO 3 N=1,4
P=ZETA2-CMPLX(T(N),0.0)
P=P*RA
FLOGP=CLOG(P)
PLOG=P*FLOGP
SIG=C(N)*PLOG
STORE2=STORE2+SIG
FMUFI2=UTIT*STORE2
DOG=(0.0,0.0)
DOG=DOG+PLEB
DOG=DOG*FMUFI1
COT=CAT*FMUFI2
FISH=DOG+COT
FMUFID=FMUFI1-FMUFI2
FMUFIN=FMUFI2-FMUFI1
STRSXX(NO)=REAL(FISH)
STRSXY(NO)=ULIT*AIMAG(FMUFIN)
STRSYY(NO)=CAT*REAL(FMUFID)
=====
2
=====

```

```

C =====
C
C      2
C
C
C =====
C PLOT THE STRESSES AND THEIR SPECTRA.
C
      DO 4 I=1,120
4      A(I)=CMPLX(STRSXX(I),0.)
      DO 5 I=121,1024
5      A(I)=(0.,0.)
      CALL LOMPLOT (A,120,0.)
      CALL FFTX (10,A,-1.0,1024)
      DO 6 I=1,100
6      A(I)=CMPLX(CABS(A(I)),0.)
      CALL LOMPLOT (A,100,1.)
      IF (Y.EQ.0.) GO TO 14
      DO 7 I=1,120
7      A(I)=CMPLX(STRSXY(I),0.)
      DO 8 I=121,1024
8      A(I)=(0.,0.)
      CALL LOMPLOT (A,120,0.)
      CALL FFTX (10,A,-1.0,1024)
      DO 9 I=1,100
9      A(I)=CMPLX(CABS(A(I)),0.)
      CALL LOMPLOT (A,100,1.)
      DO 10 I=1,120
10     A(I)=CMPLX(STRSYY(I),0.)
      DO 12 I=121,1024
12     A(I)=(0.,0.)
      CALL LOMPLOT (A,120,0.)
      CALL FFTX (10,A,-1.0,1024)
      DO 13 I=1,100
13     A(I)=CMPLX(CABS(A(I)),0.)
      CALL LOMPLOT (A,100,1.)
14     CONTINUE
C
C =====
C      STOP
C      END

```

```

SUBROUTINE LOMPLOT (Z,M,ZERO )
COMPLEX Z(M),A ,Z10
DATA SP/1H /,AS/1H*/ ,CH/1H|/,AJ/1HJ-,AXIS/1H/
Y = AMAX (Z,M)
WRITE (2,3)
POWER = ALOG10(Y)
IPOWER = INT(POWER)
IF (POWER .LT. 0.) IPOWER = IPOWER - 1
P10 = 10. **IPOWER
WRITE (2,4) P10
AK = 1. + ZERO
BK = 50.*ZERO
CK = 50./Y
DO 12 I=1,M
A = Z(I)*CK
NUM = NINT(AK*REAL(A) - BK)
NUMJ = NINT(AK*AIMAG(A) - BK)
Z10 = Z(I) /P10
IF (ZERO) 0,0,10
/ WRITE (2,1) I,Z10 ,(CH,J=1,5),AXIS,(CH,J=1,5)
GO TO 11
10 WRITE (2,1) I,Z10,AXIS,(CH,J =1,10)
11 WRITE (2,2) ((SP,J=1,NUM+51),AS)
12 WRITE (2,2) ((SP,J=1,NUMJ+51),AJ)
1 FORMAT (1H ,I3,2F7.4,5(A1,9X),A1,5(9X,A1))
2 FORMAT (1H+, 16X,102A1)
3 FORMAT (1H1,50X,20HCOMPLEX OUTPUT PLOT.,//,2HI ,
1 9HREAL PART,1X,9HIMAG PART)
4 FORMAT (1H0,1X,30HMULTIPLY ALL READINGS BELOW BY,1X,1PE8.1,
1 1X, 24HTO OBTAIN TRUE READINGS. )
RETURN
END

```

```

FUNCTION AMAX(A,N)
COMPLEX A(N)
AMAX = 0.0
DO 1 I = 1,N
AA = ABS(REAL(A(I)))
BB = ABS(AIMAG(A(I)))
IF( AA .GT. AMAX) AMAX = AA
1 IF ( BB .GT. AMAX) AMAX = BB
RETURN
END

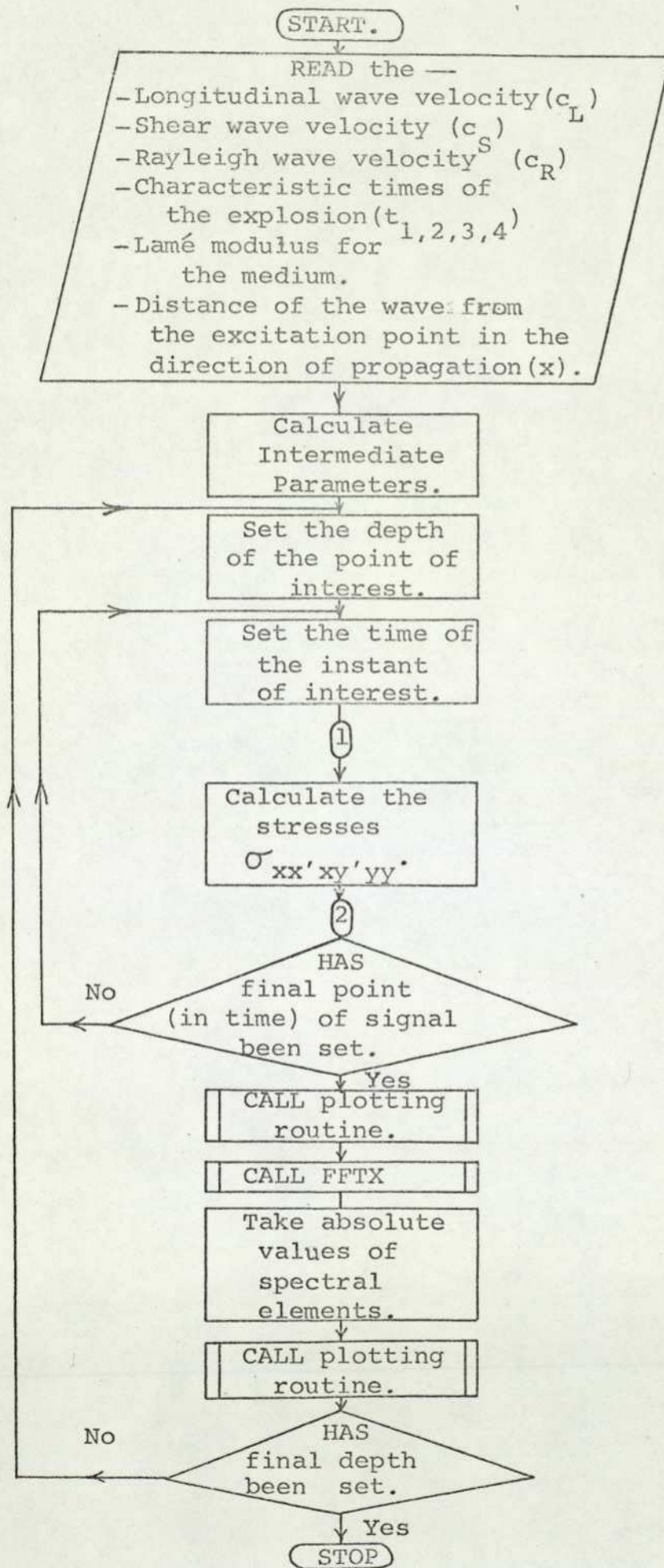
```

```

SUBROUTINE FFTX(N,X,POM,LX)
COMPLEX X,WK,HOLD,Q
DIMENSION M(15),X(LX)
SIGN = POM
FLX = LX
DO 1 I=1,N
1  M(I) = 2**(N-I)
DO 4 L=1,N
NBLOC = 2**(L-1)
LBLOC = LX / NBLOC
LBARF = LBLOC / 2
K = 0
DO 4 IBLOC = 1,NBLOC
FK = K
V = SIGN * 6.2831853 * FK / FLX
WK = CEXP(CMPLX(0. , V))
ISTART = LBLOC * (IBLOC - 1 )
DO 2 I= 1, LBARF
J = ISTART + I
JH = J + LBARF
Q = X(JH) * WK
X(JH) = X(J) - Q
X(J) = X(J) + Q
2  CONTINUE
DO 3 I = 2, N
II = I
IF (K .LT. M(I)) GO TO 4
3  K = K - M(I)
4  K = K + M(II)
K = 0
DO 7 J = 1, LX
IF (K .LT. J) GO TO 5
HOLD = X(J)
X(J) = X(K+1)
X(K+1) = HOLD
5  DO 6 I = 1, N
II = I
IF (K .LT. M(I)) GO TO 7
6  K = K - M(I)
7  K = K + M(II)
IF ( SIGN .LT. 0.0 ) RETURN
DO 8 I = 1, LX
8  X(I) = X(I) / FLX
RETURN
END

```

Appendix 1. (continued.)



APPENDIX 2

FRESNEL DIFFRACTION

When considering diffraction from a slit onto a plane separated from the slit by a distance comparable with a wavelength of the illuminating radiation, we must include the effect of variation in the distance of various points in the slit to the plane, with consequent phase differences at the plane. This effect is incorporated in the Fresnel theory of diffraction.

If the slit is split up into small elementary strips, and each element is represented by a phasor, then, joining the phasors end to end in the order of the strips and allowing the strip widths to become infinitesimally small yields the curve shown in Fig.40 called Cornu's Spiral.

The spiral can be shown, from geometrical considerations, to be represented by the equation:

$$\delta = \left(\frac{\pi}{2} \right) v^2$$

$$\text{where } v = s \left(\frac{2(a+b)}{ab\lambda} \right)^{\frac{1}{2}}.$$

The parameters δ , v , s , a and b are shown on the diagram, and λ is the wavelength of the radiation.

As with any phasor diagram the amplitude due to any section of the curve is given by the length of the chord to the curve. Therefore the energy distribution over the diffraction plane may be found by first fixing the length of the arc from the dimensions of the problem

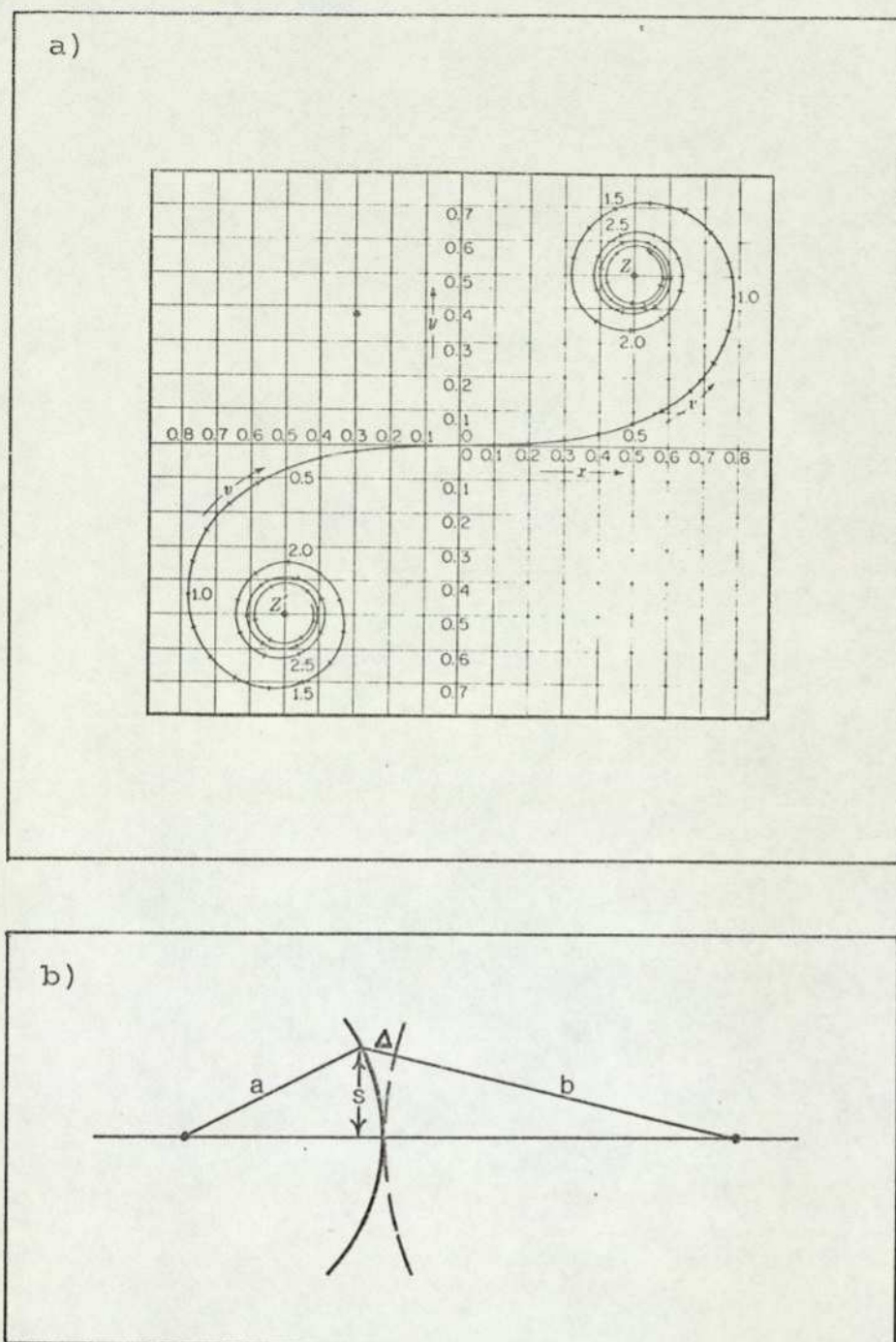


FIG.40.

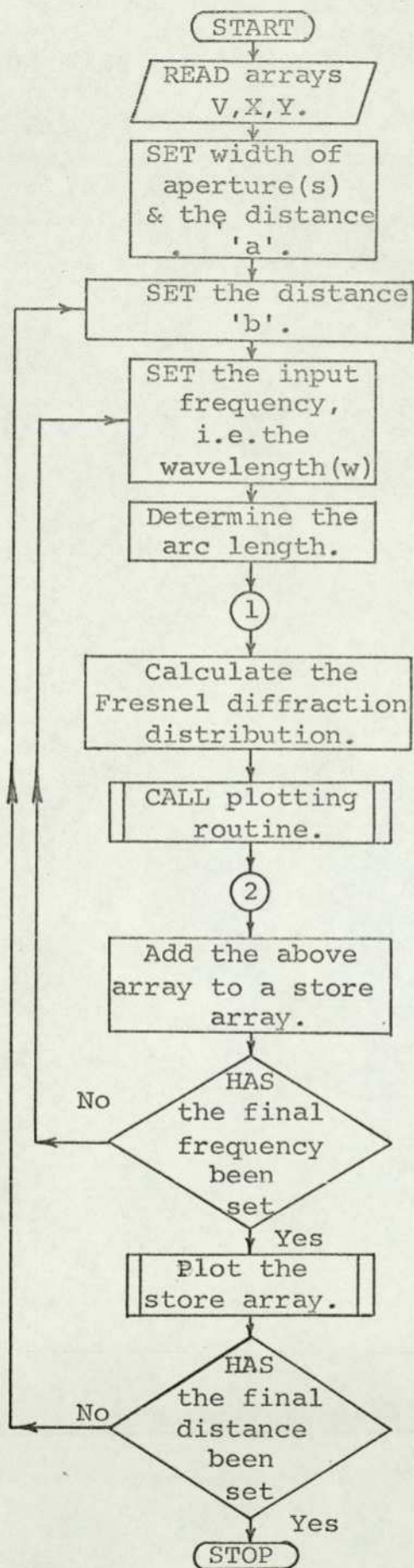
a) Cornu's Spiral.

b) Specification of the parameters
used in the Fresnel diffraction
theory.

(i.e. a , b and λ , using the equation for v above) and moving this arc along the spiral, recording the chord length and assigning this as the energy at the point of the centre of the arc.

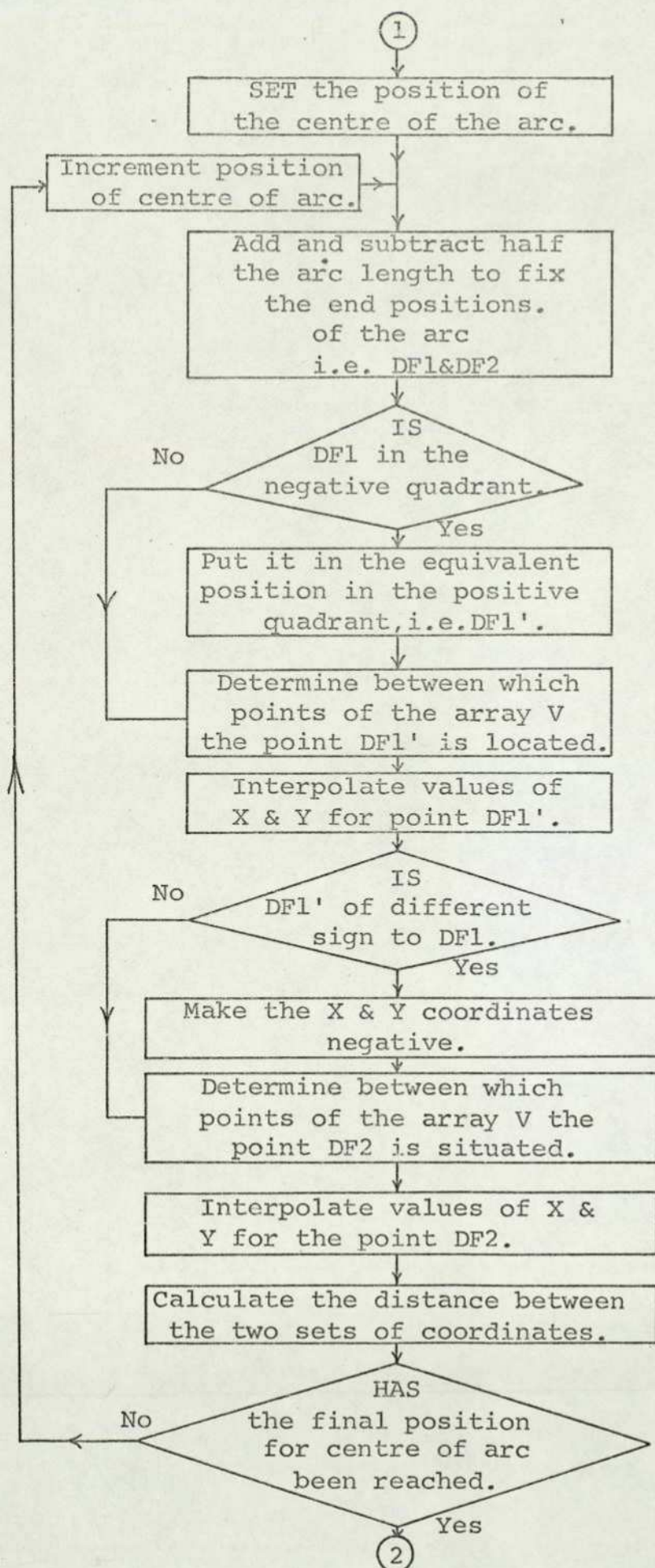
This process is shown in the following flow diagram. The program, rather than evaluating the Fresnel integral for every point, reads the tabulated values of Reference 70 and interpolates from them.

APPENDIX 2 (continued)

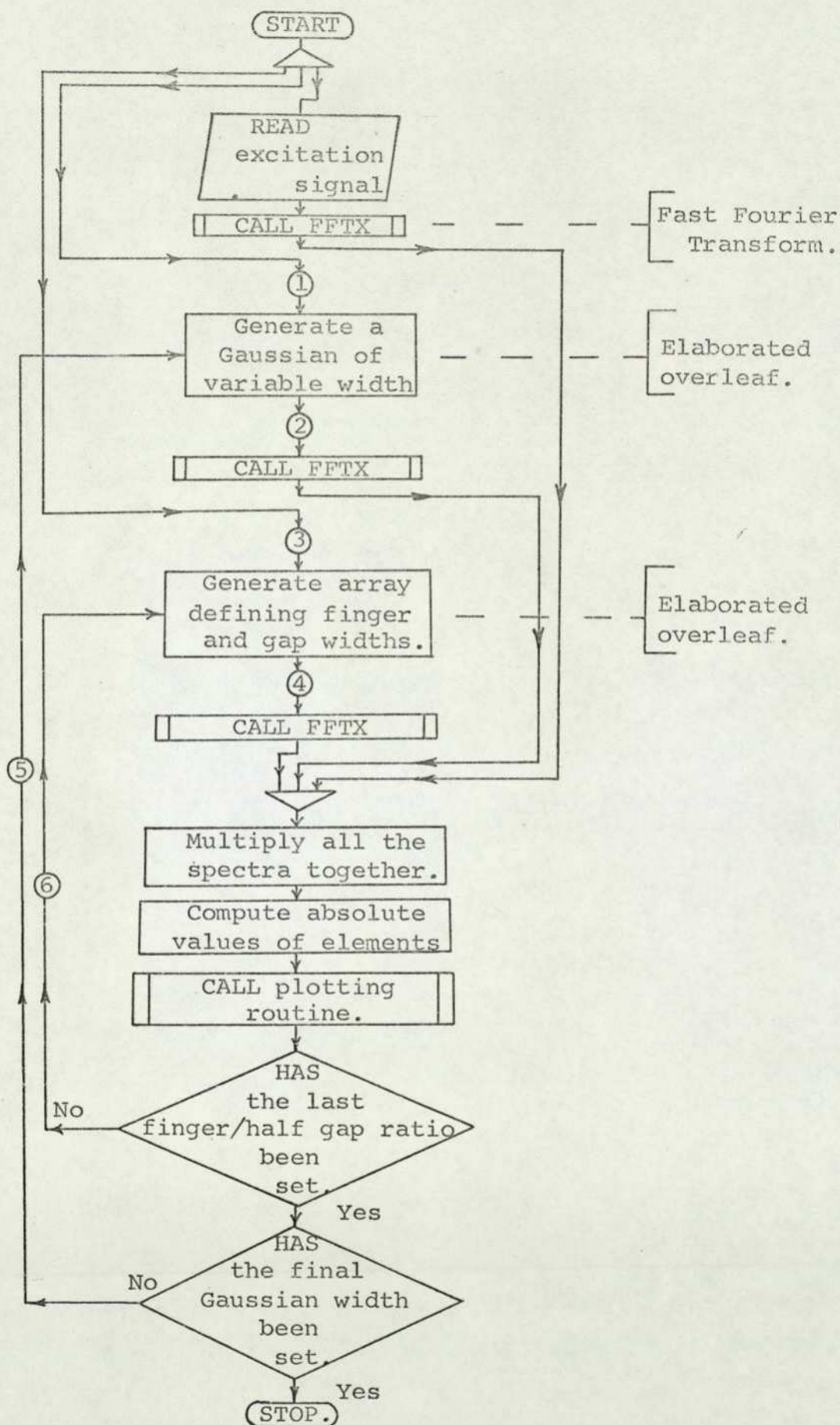


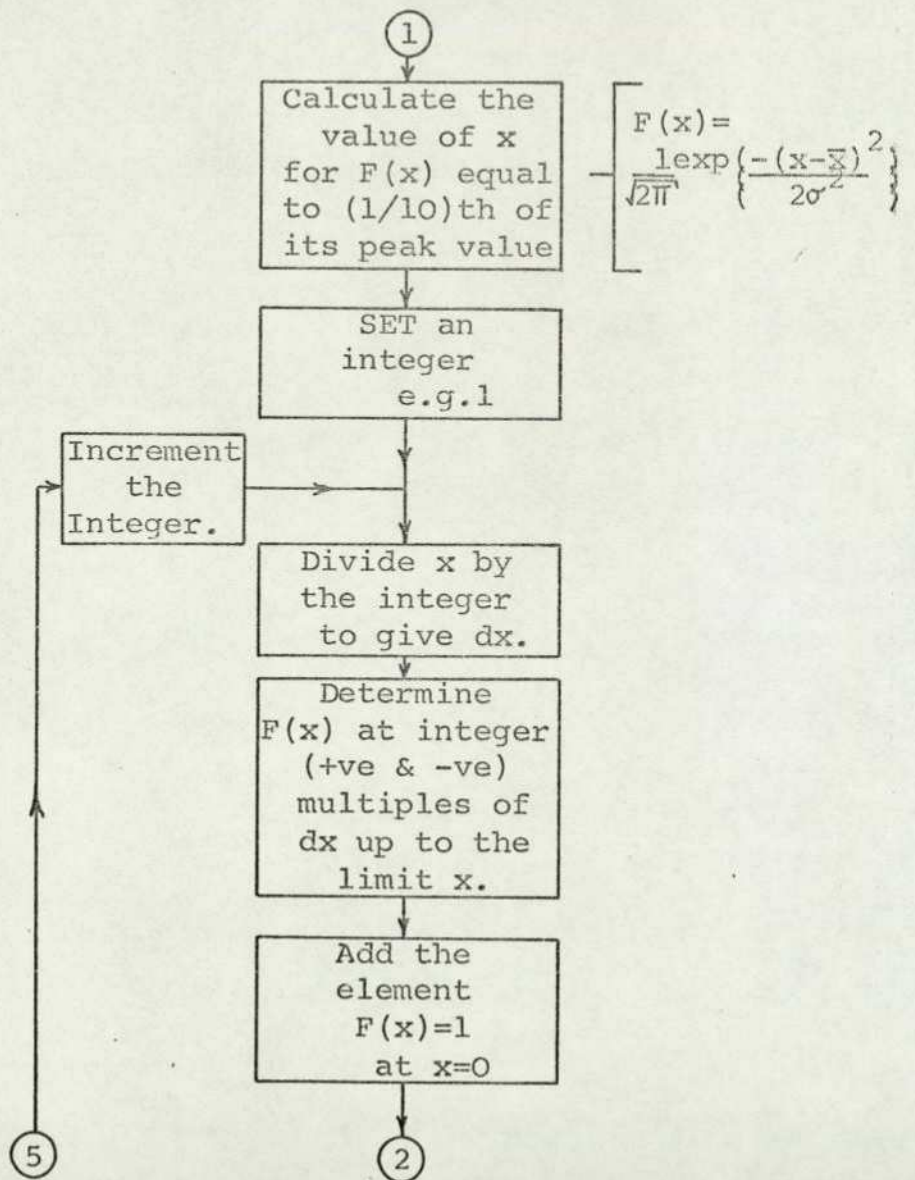
Page 367
of Ref.53.

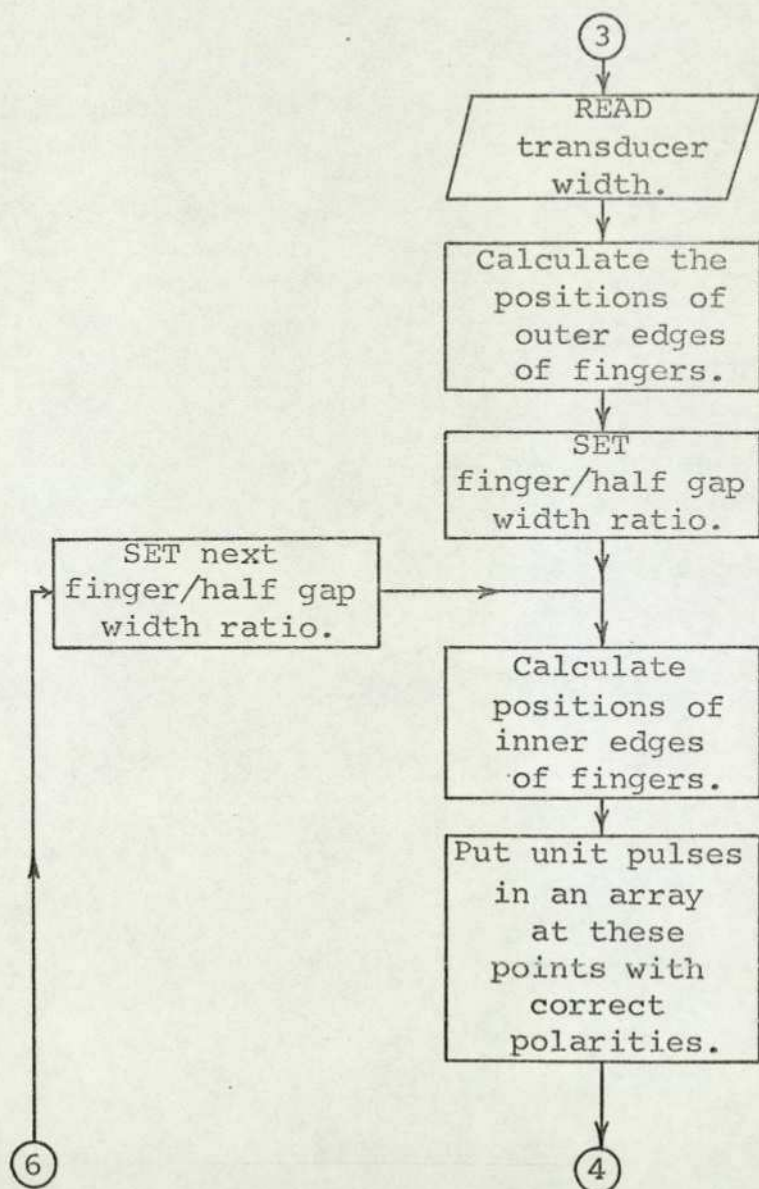
Enlarged
Overleaf.

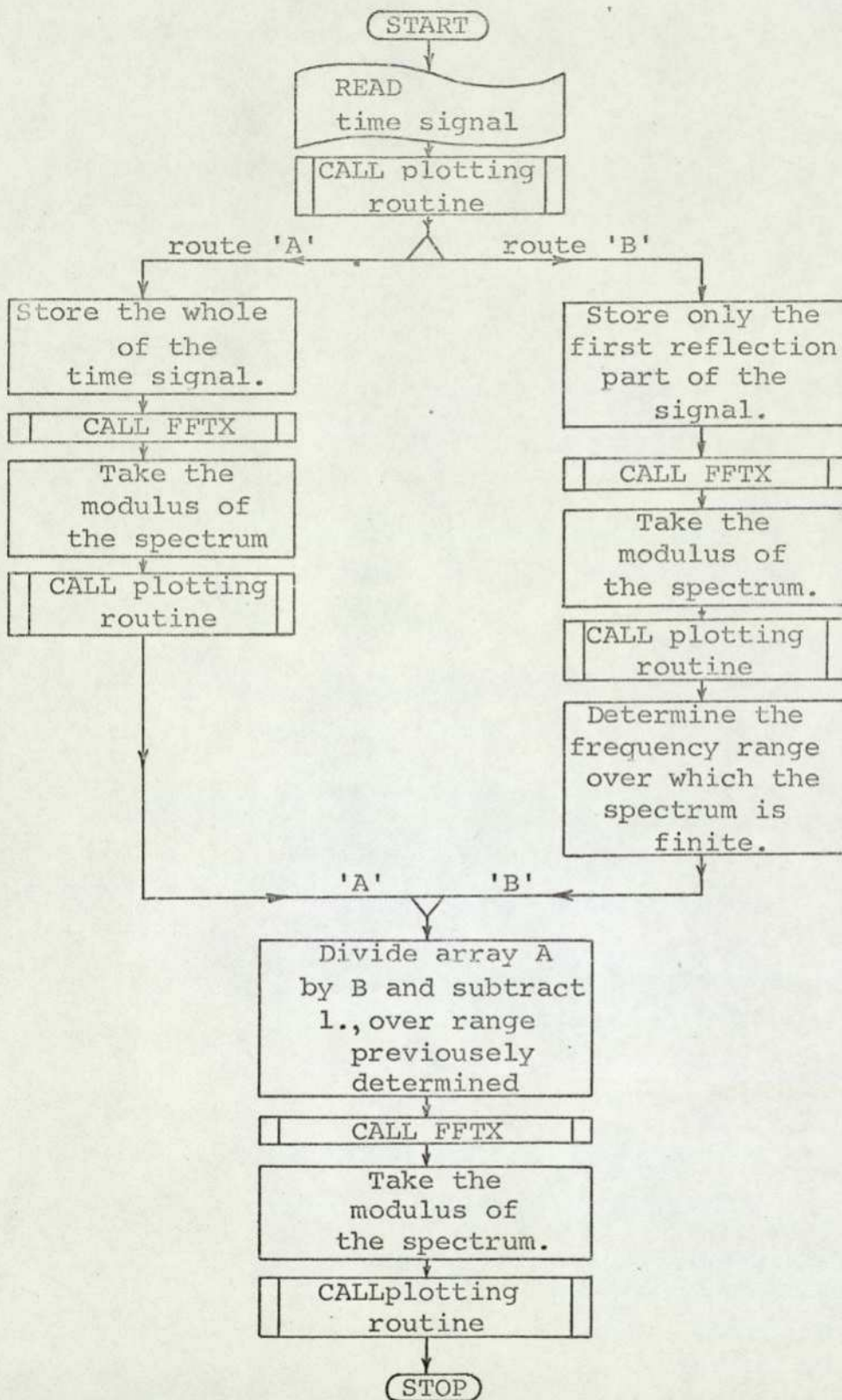


APPENDIX 3

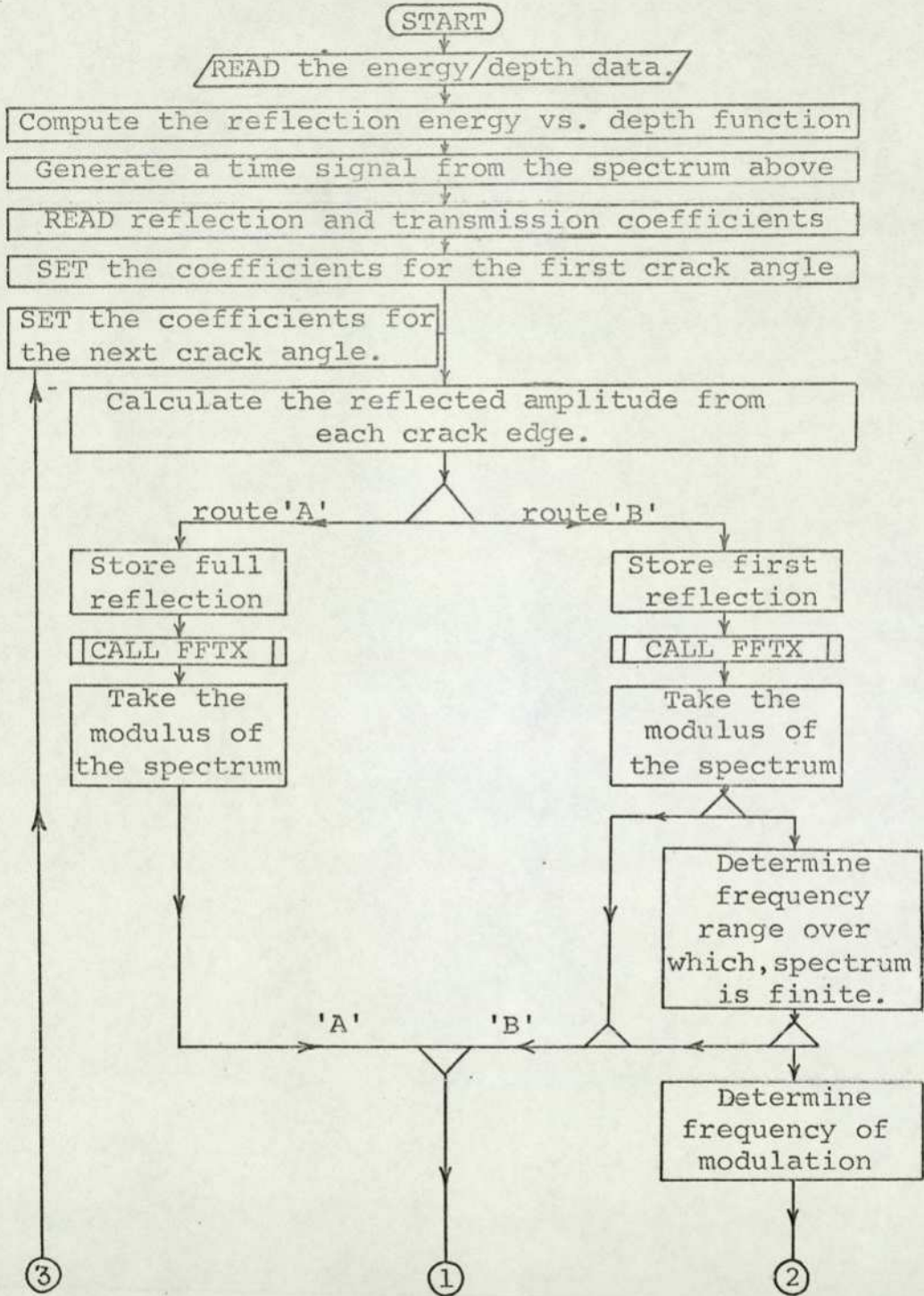


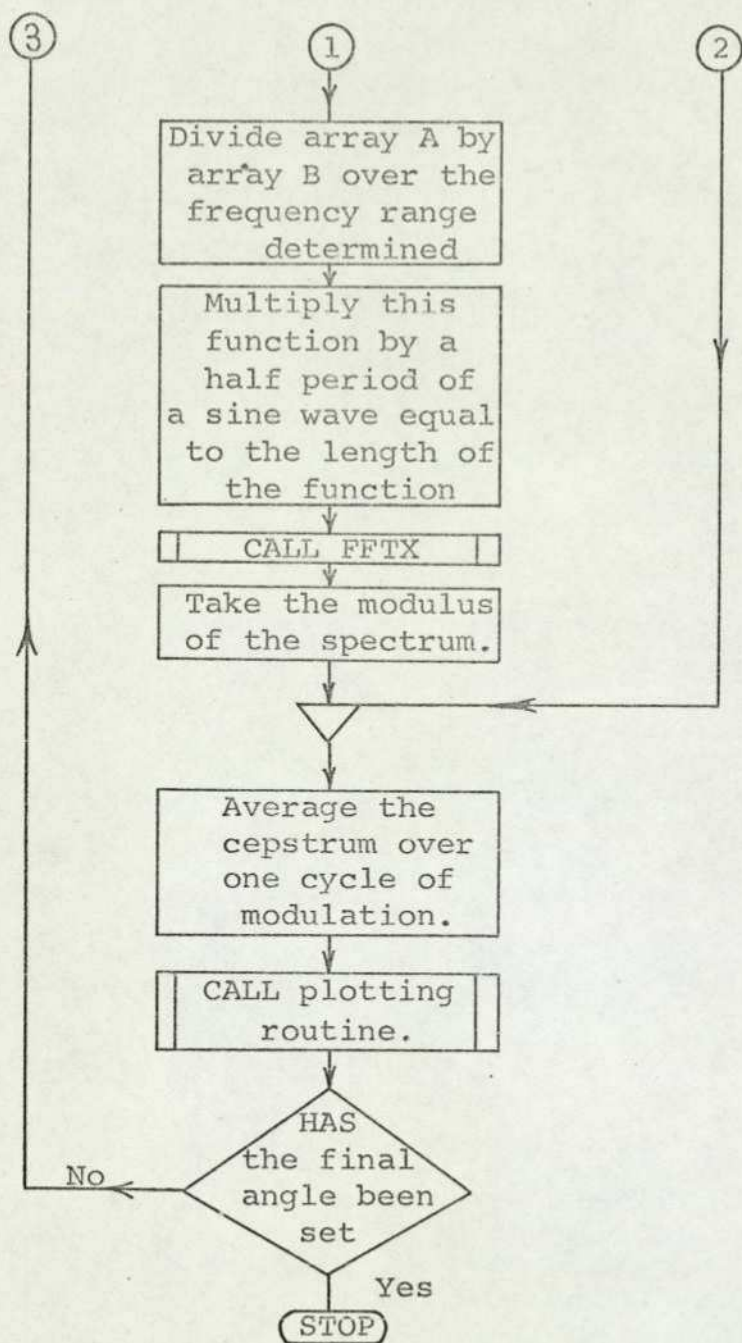






APPENDIX 5





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