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Effects of particle morphology and contact curvature on particle breakage behaviour

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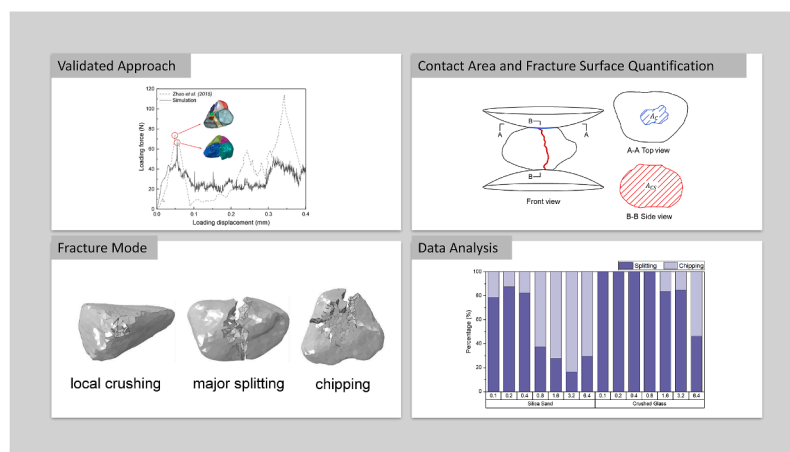
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HIGHLIGHTS

- Particle shape controls fracture force variability and strength estimation.
- μ FE model with CIEs accurately simulates particle breakage mechanisms.
- New method normalises fracture force by contact area for consistency in strength measurement.
- Normalised contact area converges across indenter sizes despite variations.
- Smaller indenters cause splitting; larger ones promote chipping behaviour.

GRAPHICAL ABSTRACT



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ABSTRACT

Understanding the particle strength is crucial for improving the efficiency, sustainability, and overall performance of processes involving particle breakage. While extensive laboratory testing has provided valuable insights, a consistent method for estimating tensile strength remains elusive. This challenge arises primarily from the complex interplay between geometric and material anisotropy, as well as the inherent heterogeneity of particles, which complicates the standardisation and reproducibility of testing procedures. In this contribution, a point loading test with varying indenter sizes is employed to investigate the effect of contact area on particle tensile strength. A micro finite element (μ FE) model is employed to capture the morphology (i.e. shape and size) of real granular materials, incorporating cohesive interface elements to simulate particle breakage. It is observed that the normalised contact area remains independent of indenter size, which motivates the introduction of a correction factor to reconcile variations in tensile strength estimate for the same particle tested with different indenters. The influence of contact area on crushing force, morphological characteristics, indenter size, and fracture mode is systematically analysed. This study provides a framework for optimising tensile strength

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measurements, which are challenging to achieve through experimental methods alone, and underscores the advantages of the μ FE approach in geotechnical research.

Nomenclature

a	Parameter used for estimating the compressive and tensile strength	N	Total number of particles
a_t	Theoretical contact radius	P_s	Survival probability
A	Surface area of a particle	r	Radius of the equivalent volume sphere of a particle
A_c	Contact area	R	Radius of a sphere
A_{cs}	Cross-sectional area	R_1	Radius of an indenter
A_t	Theoretical contact area	R_2	Half of the longest dimension of a particle
d	Diameter of a disc sample	R_e	Effective radius
$\bar{d}$	Average diameter of a particle	t	Thickness of a disc sample
d_1	Longest dimension of a particle	t_n	Nominal traction stress in the normal direction
d_2	Intermediate dimension of a particle	t_n^0	Initial nominal traction stress in the normal direction
d_3	Loading distance/shortest dimension of a particle	t_s	Nominal traction stress in the shear direction
d_a	Area equivalent circle diameter	t_s^0	Initial nominal traction stress in the shear direction
d_i	Indentation depth	V	Volume of a particle
E	Young's modulus	V_{CH}	Volume of convex hull
F	Normal force	δ_n	Crack opening displacement
F_f	Fracture force	δ_s	Crack sliding displacement
G^c	Critical mixed-mode fracture energy	η	Semi-empirical criterion exponent
G_n	Mode I fracture energy	θ	Contact area in degrees
G_n^c	Critical mode I fracture energy	μ	Friction coefficient
G_s	Mode II fracture energy	ρ	Density
G_s^c	Critical mode II fracture energy	σ	Stress
G_t	Mode III fracture energy	σ_0	Characteristic tensile strength
k_n^0	Initial normal stiffness	σ_c	Compressive strength
k_s^0	Initial first shear stiffness	σ_t	Tensile strength
m	Weibull modulus	χ	Empirical parameter to consider heterogeneity
n	Number of particles	ν	Poisson's ratio

1. Introduction

From large boulders in coastal protection to fine sand in oil and gas production, accurate measurement of tensile strength is critical for understanding and predicting particulate behaviour under various loading conditions. However, measuring the strength of individual irregular particles present a challenge. The variability in particle size and shape makes sample preparation for laboratory testing challenging. Additionally, even when using the ideal morphology, the internal structure and mineralogy of each particle differ, requiring a significant number of tests to quantify tensile strength (Zhang et al., 2020). This further exacerbates the already challenging situation, adding to the complexity.

Fairhurst (1964) used the Brazilian test to indirectly estimate the tensile strength σ_t of disc-shaped rock samples:

$$\sigma_t = \frac{2F_f}{\pi dt} \quad (1)$$

where F_f is the fracture force; d and t are the diameter and thickness of the disc sample, respectively. Later, Hiramatsu and Oka (1966) advanced this approach for irregular rock samples by considering the loading distance d_3 between two loading platens:

$$\sigma_t = 0.9 \frac{F_f}{d_3^2} \quad (2)$$

Note that d_3 is also the smallest dimension of the sample, accounting for the size effect in irregular materials.

Lee (1992) performed diametrical compression of particles using two flat platens. They defined the fracture force F_f of a particle and calculated the corresponding tensile strength σ_t as follows:

$$\sigma_t = \frac{F_f}{\bar{d}^2} \quad (3)$$

where $\bar{d}$ is the average diameter of the particle. Furthermore, using Christensen's multiaxial failure criterion (Christensen, 2000), Russell and Muir Wood (2009) proposed an approximate expression for the compressive and tensile strength of an elastic sphere under compression:

$$\sigma_c = \frac{aF_f}{\pi R^2 \sin^2 \theta}, \sigma_t = \frac{-aF_f}{\chi \pi R^2 \sin^2 \theta} \quad (4)$$

where σ_c is the compressive strength, R is the radius of the sphere, θ is the contact area in degrees, and χ is an empirical parameter to consider heterogeneity. The parameter a can be defined, and the θ can be derived by using the Hertzian contact theory as follows:

$$a = \frac{3 \left(\frac{3}{32} + \frac{\sqrt{2}}{24} + \left(\frac{\sqrt{2}}{12} - \frac{1}{4} \right) \mu + \left(\frac{1}{2} - \frac{\sqrt{2}}{3} \right) \mu^2 \right)}{(2 - \sqrt{2})(1 + \mu)}, \theta = 2 \cos^{-1} \left(1 - \left(\frac{3F(1 - \nu^2)}{4ER} \right)^{2/3} \right) \quad (5)$$

where μ is the coefficient of friction, F is the normal force, E is the elastic modulus, and ν is the Poisson ratio. A few more recent studies improve the estimation of tensile strength by considering the shape effect. Cavarretta et al. (2017) mathematically examined Eq. (3) by replacing d and t in Eq. (1) with $\bar{d}$, finding that Lee (1992) overestimated the maximum tensile stress by 60% and suggesting that their calculated result represents a nominal strength. To address this, they proposed incorporating the intermediate dimension d_2 in Eq. (2) to better capture the shape effect:

$$\sigma_t = 0.9 \frac{F_f}{d_2 d_3} \quad (6)$$

indicating that the maximum tensile stress should act on the smallest vertical cross section of the particle, represented by d_2 and d_3 . More recently, Liu et al. (2023) critiqued Eqs. (2) and (6), arguing that considering only one or two principal dimensions is insufficient to account for the irregular particles. They proposed an equivalent diameter d_a as an improvement on Eq. (1) to represent overall particle morphology:

$$\sigma_t = \frac{F_f}{d_a^2} \quad (7)$$

where d_a is the diameter of a circle with an area equivalent to the particle's projected area, averaged from 10 to 20 perspectives. These perspectives were captured as the particle fell freely by gravity and rotated. This method shows expected correlations between fracture forces and nominal tensile strengths across particles of various morphologies within a similar size range.

However, the contact between the particle and loading platens is complex and may lead to varied mechanical behaviours, resulting in different fracture forces and breakage patterns, which complicates the estimation of tensile strength. By analysing the silica sand after the oedometer test, Nakata, Hyodo, et al. (2001) defined the particle breakage behaviours in five classes: (1) no visible damage, (2) single abrasion, (3) multi abrasions, (4) major splitting, and (5) further fragmentation of sub-particles. Later, Cavarretta and O'Sullivan (2012) investigated the breakage of irregular single particles under uniaxial compression and found that it involves five main stages to crush the sample: (1) initial rotation, (2) damage, (3) elastic response, (4) fragmentation, and (5) crushing. With the advancement of technology, Wang and Coop (2016) successfully captured real-time images of crack

initiation and propagation on a single particle using a high-speed microscope camera. Based on the rapidity of failure and associated fragments, they categorised the particle failure mode into four types: (1) splitting, (2) explosive, (3) explosive splitting, and (4) chipping. They proposed a local roundness parameter to evaluate the contact between the particle and loading platens. They suggested that high local roundness (*i.e.*, flatter contact area with flat platens) tends to cause explosive breakage with higher strength, whereas low local roundness triggers splitting breakage. Following this conclusion, they replaced the flat platens with rigid balls to minimise the effect of loading apparatus (Wang & Coop, 2018). They found that the point loading test provides slightly more consistent strength measurements by minimising the effects of contact topology. However, they also noted that the point loading test is more difficult to perform and requires frequent replacement of the loading balls.

Alternatively, numerical simulation has been proven to be a reliable approach to overcome the difficulties faced in experiments. Tang et al. (2004) numerically investigated the particle breakage under point-to-point, multipoint, point-to-plane, and plane-to-plane loading in 2D. They found that the tensile strength increases sequentially from point-to-point to plane-to-plane loading. Later, Zhu and Zhao (2019) extended the breakage analysis in 3D by using a spherical particle. They concluded that the number of fragments generated from a crushed particle does not exhibit any significant correlation with the coordination number (*i.e.*, number of contact points between particles). By considering particle morphology, Liu et al. (2020) conducted point loading tests using a sphere-based discrete element method (DEM) representation. They observed two crack zones developing from the compressive contact points and propagating toward each other until complete breakage occurred, resulting in the particle splitting into several pieces. In addition, bonded-cell DEM approaches using polyhedral particles have also been developed to model the breakage behaviour of irregularly shaped particles. Hu et al. (2023) employed polyhedral particles discretised by Voronoi tessellation together with a cohesive zone model to investigate the effects of particle size and morphology on crushing strength. However, sphere- and polyhedron-based DEM approaches are still limited in their ability to fully represent realistic particle morphologies and complex contact topologies, which directly influence stress distribution and particle deformation (Druckrey et al., 2016; Fonseca et al., 2013). With the emergence of the micro-computed tomography (μ CT) approach, Nadimi and Fonseca (2017, 2018, 2019) adopted the micro finite element (μ FE) model from biomedical applications to investigate particle-to-particle interactions under loading. They highlighted its ability to compute the map of stress distribution inside each grain. Recent advances in image-based particle characterisation have enabled more accurate quantification of particle morphology. For example, Gong, Liu, et al. (2025) developed a deep-learning-based framework for the automated characterisation of particle size and shape in stacked gravelly soils, providing an efficient approach for extracting morphology descriptors from large image datasets. In addition, Gong, Wang, et al. (2025) investigated the influence of particle-size non-uniformity on the stress–force–fabric relationship in granular materials, highlighting the important role of particle-scale characteristics in micromechanical and macroscopic responses. Furthermore, Wei et al. (2019) applied the μ FE approach, coupling it with cohesive interface elements (CIEs) to create a method for numerically reproducing the point loading test. They compressed particles along their three principal dimensions and observed that particles generally break along the smaller dimensions without experiencing chipping. They also concluded that traditional compression tests using platens tend to overestimate the breakage stresses.

While tensile strength measured from traditional compression tests is often overestimated, no universally accepted criterion has been established for conducting tensile strength tests to achieve consistent results. According to observations of particle breakage, before major breakage, the particle cracks during point loading and crushes under platen

loading (Tang et al., 2004). Thus, a transition likely exists between these two behaviours, which can be linked by altering the contact topologies. Additionally, according to fracture mechanics, the crack should initiate at the sample centre and propagate towards the loading ends to best estimate the tensile strength (Fairhurst, 1964), whereas point loading induces crush at the loading ends and platen loading triggers off-centre breakage due to chipping and local roughness. Therefore, altering the radius of the indenters during the compression test enables this study to observe variations in the breakage behaviour of particles. By utilising the μ FE model enriched with CIEs, the same particle is compressed under various indenters, a scenario that cannot be studied experimentally, to uncover the relationship between contact curvature and fracture behaviour. Forty-two particles are tested using seven different indenters, resulting in 294 simulations. A novel equation is proposed to provide a geometry-corrected estimate of particle tensile strength by reducing the influence of particle morphology and contact curvature on tensile strength estimate. Recent studies have also highlighted the coupled evolution of particle size, shape, and strength during particle breakage and granular deformation (Jiang et al., 2026; Li et al., 2025, 2026). These studies demonstrate that particle morphology not only influences breakage behaviour but also evolves systematically during fragmentation. Furthermore, morphology analyses are carried out both before and after particle breakage to provide deeper insights into the influence of particle shape on its breakage behaviour.

2. Model set-up

The μ FE model provides a high-fidelity representation of particle morphology and deformation using a finite element formulation (Nadimi & Fonseca, 2018). Particle breakage under external loading is captured through element-to-element interfaces where CIEs are introduced. Once fracture initiates, each fragment is treated as a separate body. The indenter-to-fragment and fragment-to-fragment interactions are resolved using hard contact in the normal direction and frictional contact in the tangential direction. This consistent treatment ensures realistic force transfer and stable post-breakage behaviour in the simulation. While the framework combines finite element and discrete element concepts similar to the combined finite–discrete element method (FDEM) (Munjiza, 2004), the present μ FE framework provides an effective approach for detailed analysis of internal stress distribution associated with particle morphology and contact curvature.

To improve reproducibility, the full computational workflow is outlined in Fig. 1. Particle geometries were reconstructed from μ CT images and meshed using an open-source software package, after which an in-house code was employed to insert CIEs at all element boundaries via node duplication. The resulting mesh was exported into Abaqus/Explicit, which employs an explicit central-difference integration scheme (Nadimi & Fonseca, 2018). The solver advances the solution in small stable time increments without iterative equilibrium checks, and stability was verified by ensuring that kinetic energy remained below 10% of the total energy throughout the simulations. CIE parameters, including interface stiffnesses, tensile strength, and fracture energy etc., were adopted from previous study (Zhang et al., 2023), in which they were calibrated and validated. Post-processing involved extracting fracture force, contact area, and cross-sectional area of the fracture plane to enable tensile strength estimate using in-house code.

2.1. Image acquisition and processing

In this study, two samples were prepared for image scanning. The first is silica sand (British rail sand) obtained from Network Rail, United Kingdom. The second is recycled crushed glass, produced by washing and crushing various types of glass bottles, then sieved using mesh size of 1.18 mm. The μ CT system, SkyScan 1176, located at the Preclinical *in-Vivo* Imaging Facility at Newcastle University Medical School was employed to capture the particle morphology. Scans were performed

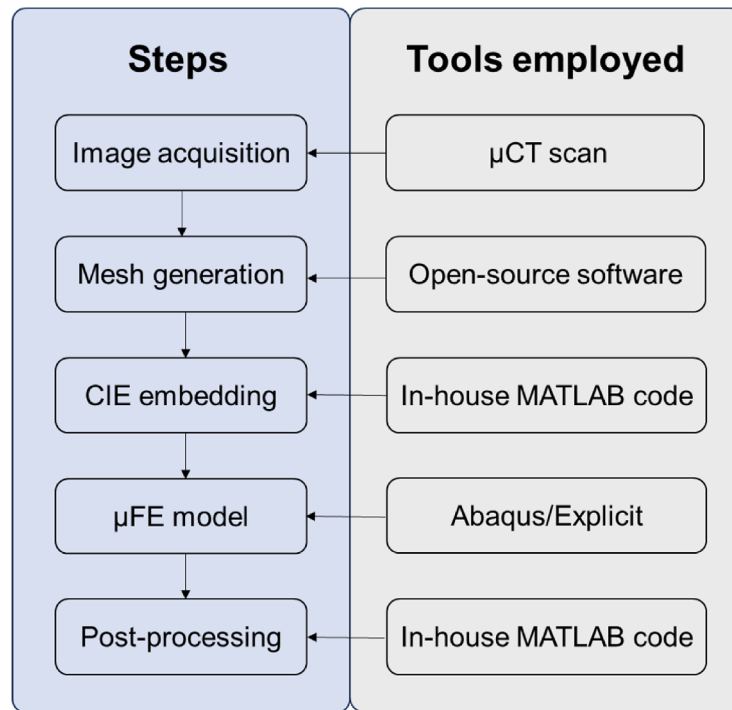


Fig. 1. Computational workflow.

with a source current of 357 μA and a voltage of 70 kV. The 3D shape of the particles was obtained by reconstructing the μCT images into grey-scale cross-sectional slices, achieving a voxel resolution of 8.81 μm . This resulted in 3D images with dimensions of approximately $7444 \times 7444 \times 7117$ voxels. To improve computational efficiency during image processing, the μCT images were resized to reduce the 3D matrix dimensions while preserving the particles' size and shape characteristics (Maramizonouz et al., 2023).

To accurately represent the complex and irregular shapes of particles while maintaining repeatability, an advanced surface reconstruction algorithm is employed, utilising the open-source Computational Geometry Algorithms Library (Rineau & Yvinec, 2012) through a Matlab (The MathWorks Inc., 2020) based workflow comprising two stages. The first stage focuses on surface mesh extraction, where triangular iso-surfaces are generated from 3D segmented images using a refinement of the constrained Delaunay triangulation (Shewchuk, 2002). Preset values for density and the smallest angle are applied to control the size and number of triangles, allowing fine meshes to detail angular features while larger triangles to approximate flat surfaces. In the second stage, the sub-volumes bounded by these isosurfaces are filled with tetrahedral elements to create a volumetric mesh. This integrated approach ensures that the volumetric mesh preserves the original boundaries of the particles while optimising the element size according to geometric constraints. It overcomes the limitations of voxel-based meshes, such as stepped boundaries, and produces high-quality elements that enhance computational efficiency and solver performance, even for complex topologies.

2.2. Damage model

The breakage behaviour of particles is simulated by enriching the μFE model with CIEs, which follow the traction–separation law that includes two stages: damage initiation and damage evolution.

2.2.1. Cohesive interface elements

CIEs are embedded between adjacent elements without altering the overall mesh morphology due to their zero geometric thickness. How-

ever, as no existing code was available for embedding CIEs in 3D, an in-house code is developed based on the framework proposed by Zhang et al. (2023) to implement CIEs in a volumetric mesh. The general process includes three main steps as illustrated in Fig. 2. First, every pair of tetrahedral elements in the targeted region is identified. Each pair, by definition, shares a common face consisting of three nodes. To ensure that no faces share the same nodes, the nodes of the shared face are duplicated, and the element connectivity matrix is updated accordingly (Fig. 2(b)). The CIE is then inserted between the adjacent tetrahedral elements, using the nodes of the duplicated face, as shown in Fig. 2(c). Due to the definition used in Abaqus, a correction algorithm is applied to check the connectivity of CIE as described below:

- 1) Compute the normal of CIE based on current connectivity. For example, in Fig. 2(c): $n = \text{Edge}_{9,10} \times \text{Edge}_{9,11}$
- 2) Compute the centre of each element in the pair (e.g., C_1 and C_2).
- 3) Compute $n \cdot C_1 C_2$ (scalar product):

$n \cdot C_1 C_2 > 0$, the current connectivity is consistent with the definition in Abaqus.

$n \cdot C_1 C_2 < 0$, the current connectivity is incorrect, switch the position of the 2nd and 5th nodes with the 3rd and 6th nodes, receptively. For example, the new connectivity of CIE would be {9 11 10 6 8 7}.

2.2.2. Damage initiation

The failure mechanism of CIE, considering the deformation in both normal (δ_n) and shear (δ_s) directions (Fig. 3), has two components: a damage initiation criterion, and a damage evolution law.

Damage initiation marks the onset of material degradation, which begins at a specific point within the material. Degradation starts when the stresses along the CIE meet the specified damage initiation criteria. In this study, the quadratic nominal stress criterion is adopted, as it is widely used for mixed-mode fracture and allows the interaction of normal and shear tractions to drive damage initiation, rather than requiring either mode to reach its maximum value independently. Damage initiation occurs when the following interaction function reaches unity:

during the simulation. The material properties of the LBS particles were directly adopted from the calibrated quartz sand parameters reported in Zhang et al. (2023), as shown in Table 1. In that study, the adopted CIE formulation was subjected to systematic mesh sensitivity and parametric analyses, which quantified the influence of interface stiffness, tensile strength, and fracture energy on the fracture response and established the parameter ranges used in the present study. The same parameter set was therefore adopted here to isolate the effects of particle morphology and contact geometry on particle breakage and tensile strength estimate.

The simulation results, including the fracture pattern and force–displacement curve, are presented in Fig. 4 for comparison with the laboratory tests. Both the simulated and experimental results exhibit an initial rapid increase in loading force followed by a sudden force drop associated with fracture initiation at a loading displacement of approximately 0.05 mm. In addition, both cases show a chipping-dominated failure pattern characterised by the formation of a major fragment accompanied by several smaller detached fragments. Although differences are observed in the post-peak response, the simulation reasonably captures the overall mechanical response and major fracture characteristics observed experimentally. These results indicate that the selected physical parameters and the proposed μ FE framework are suitable for subsequent simulations investigating the influence of contact curvature on breakage behaviour.

2.4. Modified point loading test

Tensile strength estimated from traditional compression tests with flat platens is often higher than that obtained from point load tests, as demonstrated both experimentally and numerically. In the following study, the breakage behaviour of single particles under different contact curvatures is investigated, bridging the gap between point loading and platen loading. As illustrated in Fig. 5, the radius of indenters is increasing from 0.1 mm to 0.2 mm, 0.4 mm, and up to 6.4 mm. The curvature of 0.2 mm matches the point ends used by Wang and Coop (2018), while the curvature of 6.4 mm approximates a flat contact, given the relatively small size of the tested particle. For comparison, all tested particles were manually rotated about their geometric centre in Abaqus to achieve a stable initial contact with the smallest indenter, minimising the tendency for further particle rotation under the initial loading condition. This orientation was then preserved for subsequent tests using larger indenters to ensure that the observed differences in breakage behaviour could be attributed primarily to changes in contact geometry rather than particle orientation. It should be noted that the conclusions presented in this study are therefore associated with the selected particle orientation. The adopted orientation was selected to represent a stable particle placement condition prior to loading, which is consistent with typical laboratory particle crushing procedures.

Table 1
Material parameters of LBS particle.

Solid elements			
Density	ρ (kg/m ³)	2500	
Young's modulus	E (GPa)	63	
Poisson's ratio	ν	0.22	
Cohesive interface elements (CIEs)			
Normal stiffness	k_n (N/mm ²)	63000	
First shear stiffness	k_s (N/mm ²)	31500	
Second shear stiffness	k_t (N/mm ²)	31500	
Tensile strength	N_{max} (MPa)	25	
First shear strength	S_{max} (MPa)	12	
Second shear strength	T_{max} (MPa)	12	
Mode I fracture energy	G_n (N/mm)	0.1	
Mode II fracture energy	G_s (N/mm)	0.2	
Mode III fracture energy	G_t (N/mm)	0.2	
Semi-empirical criterion exponent	η	2	
Contact law			
Friction coefficient	μ	0.5	

Two granular materials are investigated in this study: silica sand and crushed glass. Given that tensile strength is significantly influenced by particle size (Nakata, Kato, et al., 2001), the size of each particle was adjusted during reconstruction to match the surface area of a reference sphere with a 0.5 mm radius. This surface area normalisation was adopted to reduce the influence of particle size on breakage behaviour and allow a more consistent investigation of morphology and contact curvature related effects. To further isolate the influence of particle morphology and contact curvature on breakage behaviour, differences in material microstructure and intrinsic mechanical properties between silica sand and crushed glass are deliberately disregarded. Instead, both materials were assigned identical material properties, as listed in Table 1. This simplification is justified by the study's objective: to understand how morphological characteristics govern the breakage mechanisms, independent of material composition. By controlling for size and material, the observed differences in breakage behaviour can be more confidently attributed to variations in particle morphology and contact curvature.

In order to quantify the particle morphology, four characteristics listed in Table 2 have been measured via an open source software SHAPE (Angelidakis et al., 2021). The results are plotted in Fig. 6 using the empirical cumulative distribution function (eCDF). It highlights the distinct differences in shape characteristics between the two groups, with crushed glass generally exhibiting more elongated shapes compared to the more spherical and convex silica sand.

3. Results and discussion

This section focuses on the statistical analysis and discussion of the results, exploring how contact curvature correlates with breakage behaviour by considering particle morphology.

3.1. Fracture force

Fig. 7 presents the typical load–strain curves of silica sand and crushed glass. Overall, the larger loading distance of silica sand, owing to its more compact shape, requires a higher fracture force compared with crushed glass, which has a smaller loading distance due to its elongated shape and therefore requires a lower fracture force. A convergence trend of the loading forces with increasing indenter size was observed for silica sand in Fig. 7(a), owing to the compacted shape that results in consistent fracture planes across indenters. In contrast, no clear convergence is observed for crushed glass, as shown in Fig. 7(b). The elongated particle morphology leads to changes in the first point of contact and fracture plane with indenter size, so the peak load continues to vary, and this variability is later reflected in the tensile strength results.

For the purpose of the present study, the fracture force was defined as the load corresponding to fracture initiation, identified by the first occurrence of fracture in the simulation together with the associated local peak in the load–strain response. In the case of silica sand in Fig. 7 (a), this fracture force generally coincides with the global peak load, as indicated by a substantial drop in load following the peak. This fracture event is then followed, beyond $\sim 5\%$ strain, by post-fracture interactions between the indenter and the newly formed fragments. Larger indenters are able to maintain contact with multiple fragments after breakage, contributing to higher post-peak load values and sustained oscillations, whereas smaller indenters tend to slip into the gaps between fragments, causing the load to drop rapidly toward zero.

For crushed glass in Fig. 7(b), a load drop is more noticeable with smaller indenters, similar to silica sand, and can be attributed to the loss of contact between the newly formed fragments and the indenter. However, compared to silica sand, this drop is less pronounced, which is likely due to the elongated morphology of crushed glass fragments that still maintain partial contact even after breakage. When larger indenters are used, no sharp load drop is observed; instead, the load transitions more smoothly and continues to increase. Consequently, the fracture

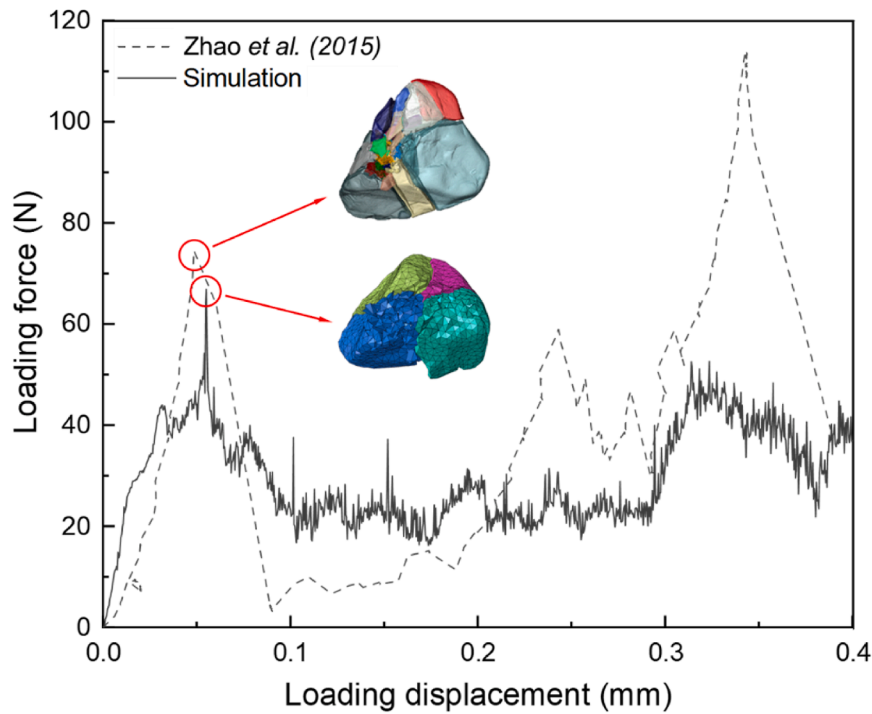


Fig. 4. Validation of μFE model against experimental data.

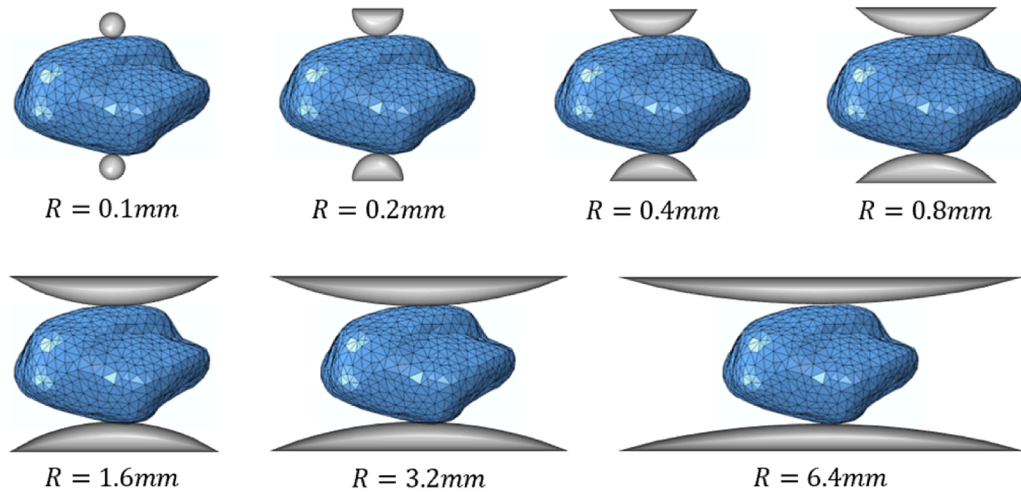


Fig. 5. Modified point loading test using different indenters.

Table 2
Four key characteristics of particle morphology.

	Formula	Range	Parameters
Flatness	$\frac{d_2^2}{d_1 \cdot d_3 + d_2^2} - \frac{d_3}{d_1 + d_3}$	(0,1]	d_1 : Longest dimension d_2 : Intermediate dimension d_3 : Shortest dimension
Elongation	$\frac{d_1 \cdot d_3}{d_1 \cdot d_3 + d_2^2} - \frac{d_3}{d_1 + d_3}$	(0,1]	
Sphericity	$\frac{\sqrt[3]{36\pi V^2}}{A}$	(0,1]	V: Volume; A: Surface area
Convexity	$\frac{V}{V_{CH}}$	(0,1]	V_{CH} : Volume of convex hull

force does not necessarily coincide with the global peak load for crushed glass particles subjected to larger indenters. This behaviour arises because the elongated fragments remain in contact with the broader surface of the larger indenters, sustaining the load transfer despite

significant fragmentation, as illustrated in the inset of Fig. 7(b). These contrasting behaviours highlight how particle morphology and indenter size jointly govern the post-fracture load response.

Fig. 8 summarises the fracture forces at the onset of major breakage. Results obtained from 21 particles within each morphology category under seven different indenter sizes are presented. The white markers represent individual particle results. The central line indicates the median value, the box denotes the interquartile range, and the whiskers represent the minimum and maximum values. All data points, including extreme values, are retained in the analysis. For both silica sand and crushed glass, the fracture force increases with indenter size. Data scatter is relatively tight for small indenters but becomes more dispersed as the indenter radius grows, reflecting increasingly complex particle-to-indenter interactions where local surface roughness plays a stronger role. This increase in scatter is more pronounced for crushed glass, whose elongated morphology leads to greater variability in contact geometry and fracture

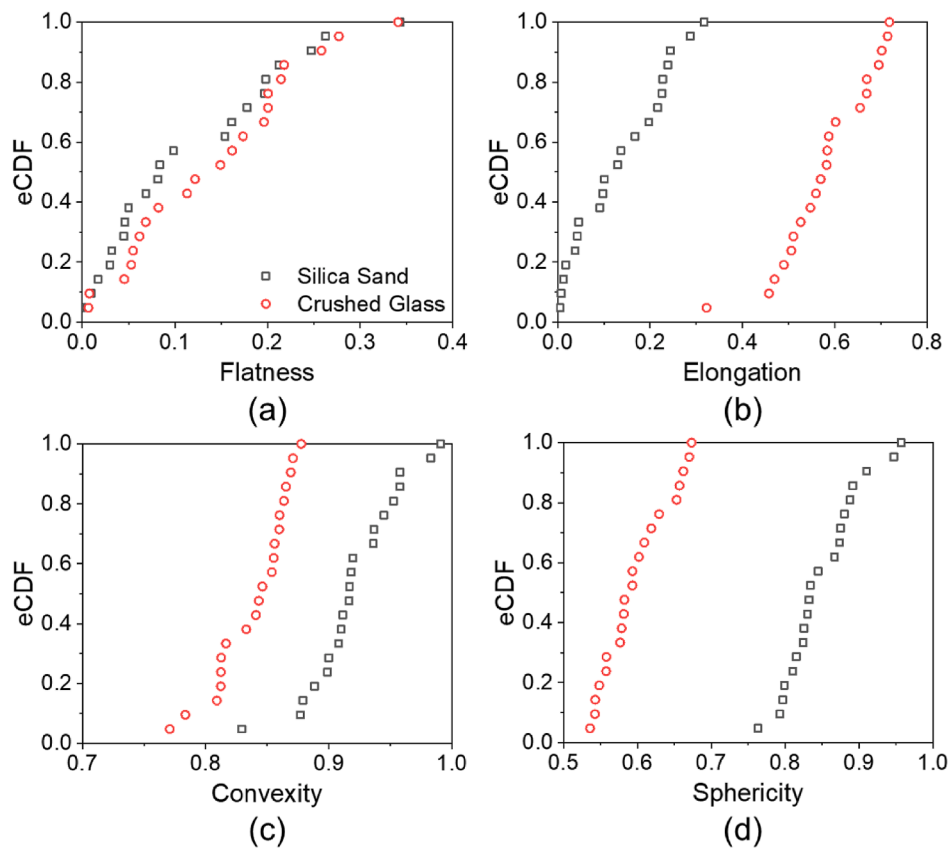


Fig. 6. Empirical cumulative distribution function of the morphological characteristics of two samples.

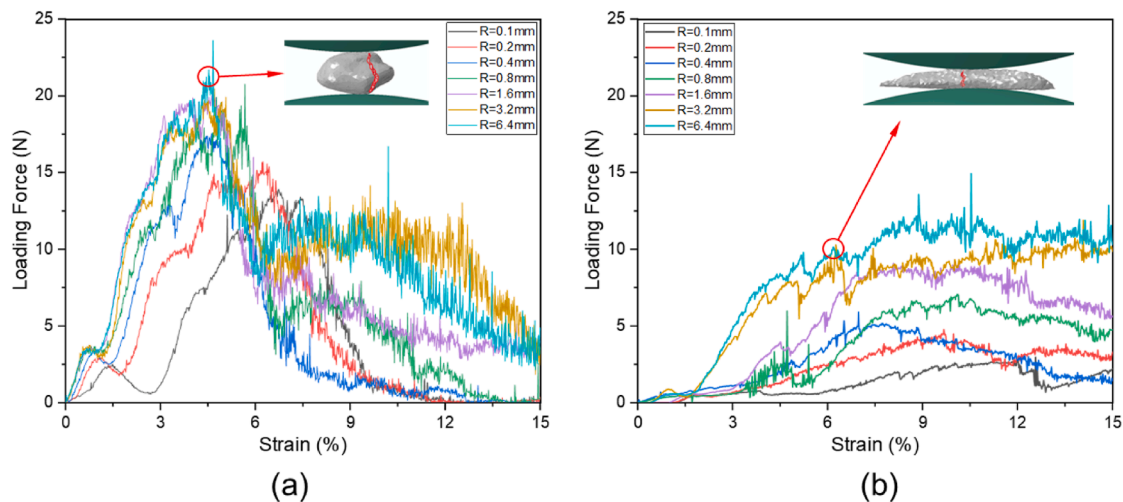


Fig. 7. Typical load-strain curves: (a) silica sand and (b) crushed glass.

plane development under different indenter sizes. Silica sand consistently shows higher fracture forces than crushed glass, which is attributed to its more compact particle shape and the resulting greater effective loading distance. Overall, the results indicate that fracture force is governed primarily by contact geometry and particle morphology, with local surface roughness contributing to the observed scatter.

3.2. Contact area

In the present study, the contact area was calculated from the upper particle-to-indenter contact only, as local particle morphology and

progressive breakage result in different contact areas at the upper and lower contacts during loading. Using the upper particle-indenter contact as a single reference therefore avoids ambiguity associated with averaging or combining two evolving contact areas. The contact area was obtained directly from the Abaqus contact output at the moment of fracture. Since the particles were modelled as deformable bodies, the contact area evolved continuously during loading in response to particle deformation and fracture development. All simulations employed identical loading conditions and contact settings; therefore, the observed differences in contact area primarily reflect the influence of particle morphology, breakage evolution, and indenter geometry. To

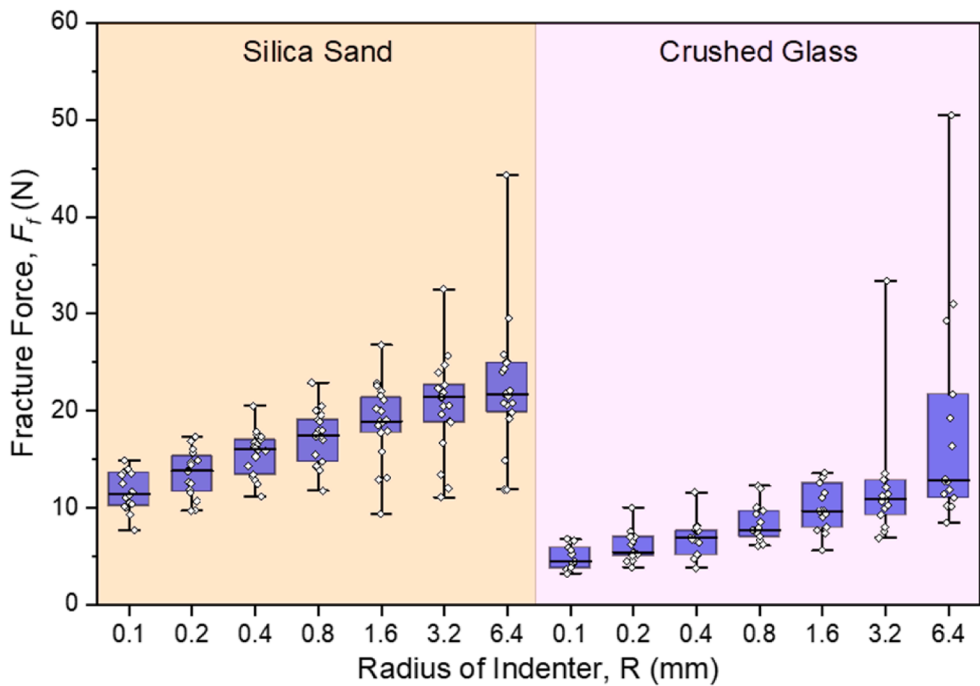


Fig. 8. Fracture force of silica sand and crushed glass.

investigate the role of contact area in particle breakage behaviour, the relationships between contact area, fracture force, particle morphology, and indenter size are examined.

3.2.1. Effect of contact area on fracture force

When the particle experiences major breakage, the associated contact area has been captured and plotted against its fracture force, as shown in Fig. 9. When the tested particle is exposed to the same contact area, silica sand usually requires a higher force to trigger breakage

process comparing to crushed glass. This is because the stress in silica sand, owing to its compacted shape, could be distributed more evenly across the contact area, whereas the crushed glass tends to experience higher stress concentrations at the points of contact due to its irregular geometry. Thus, for the same contact area, crushed glass breaks at lower forces compared to silica sand. A linear regression was fitted to the combined dataset of silica sand and crushed glass particles. The positive trend indicates that larger contact areas generally require higher fracture forces to initiate breakage. However, the coefficient of

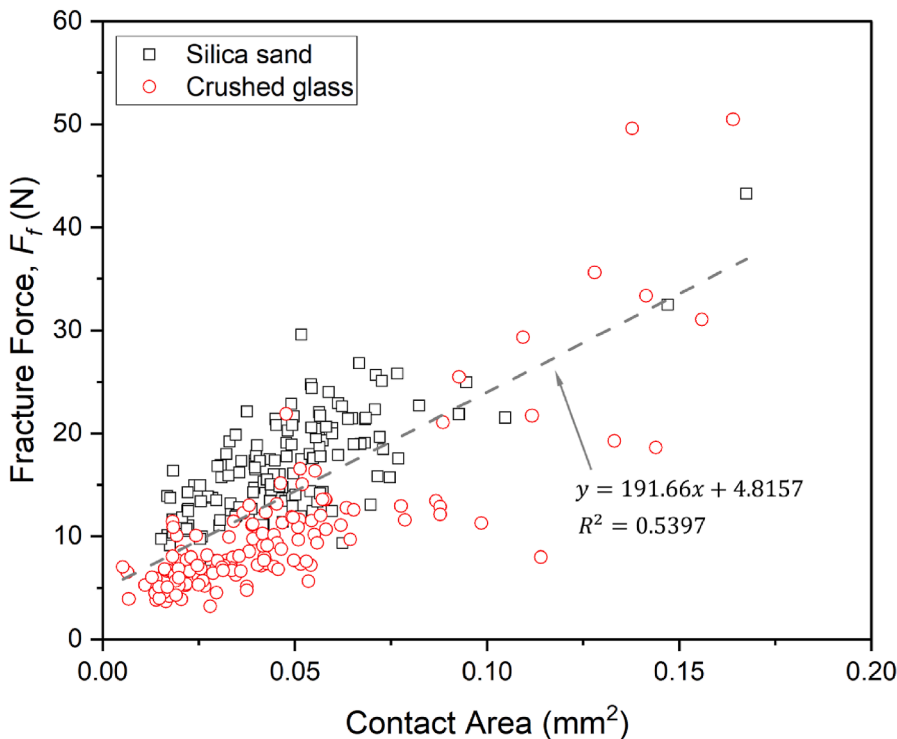


Fig. 9. Relationship between contact area and fracture force.

determination suggests that contact area alone does not fully explain the observed variability, with particle morphology and local contact geometry also contributing to the fracture response.

3.2.2. Effect of indenter size on contact area

The contact area between the particle and indenter throughout the simulation is visualised for silica sand in Fig. 10(a) and crushed glass in Fig. 10(b). A total of 21 silica sand particles and 21 crushed glass particles were tested, each subjected to simulations with the different indenter sizes. The solid lines represent the mean contact area for each indenter size, while the shaded regions denote the corresponding standard deviation. As expected, larger indenters generate a larger contact area for both materials, although the trends differ between the two.

To validate the accuracy, the contact area immediately after initial contact with a smallest non-zero value of about 0.002 mm^2 has been treated as an empirical lower bound on discretisation. At fracture initiation, the contact areas range from a lower bound of 0.02 mm^2 , with a median of 0.04 mm^2 and an upper bound of 0.17 mm^2 . Using 0.002 mm^2 as the reference increment, the corresponding relative resolutions are therefore approximately 10% for the lower bound, 5% for the median, and 1% for the upper bound. These values confirm that the contact areas captured at fracture are resolved with sufficient numerical resolution.

For silica sand in Fig. 10(a), as the indenter size increases, the slope of the contact area curve increases, peaking at a later strain stage. Convergence is observed for larger indenters (i.e., 3.2 mm and 6.4 mm), confirming that no larger radii are required. In contrast, crushed glass in

Fig. 10(b) shows no peak in the contact area curve, with the slope gradually increasing as the indenter size grows. This trend is attributed to the elongated shape of the crushed glass, allowing larger indenters to create a more extensive contact area.

To better understand the relationship between indenter size and contact area, the extracted contact area values were normalised using a theoretical geometric contact approximation. For two bodies in contact, the theoretical contact radius a_t is estimated as:

$$a_t = \sqrt{R_e d_i} \quad (10)$$

where R_e is the effective radius and d_i is the indentation depth. The effective radius is defined as:

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} \quad (11)$$

where R_1 and R_2 represents the radius of two spheres in contact. The theoretical contact area A_t is then expressed as:

$$A_t = \pi a_t^2 = \pi d_i \left(\frac{R_1 R_2}{R_1 + R_2} \right) \quad (12)$$

In this study, R_1 is the radius of indenter and R_2 is half of the length d_1 (largest dimension) of the particle. After normalisation, the contact areas associated with different indenters become less sensitive to indenter size, as shown in Fig. 10(c) for silica sand and Fig. 10(d) for crushed glass. The normalised contact area for smaller indenters

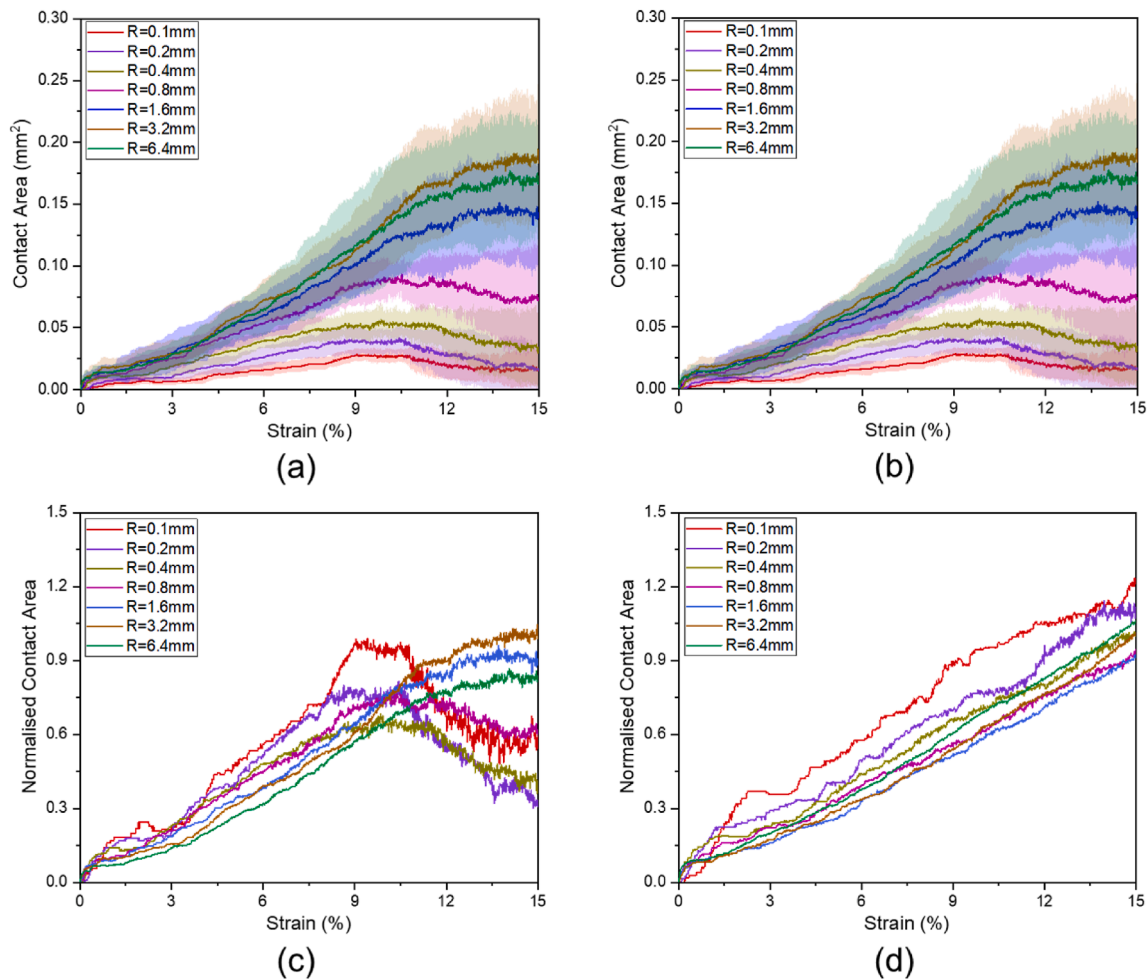


Fig. 10. Relationship between contact area and indenter size: (a) silica sand, (b) crushed glass, (c) silica sand after normalisation and (d) crushed glass after normalisation.

becomes more pronounced compared to larger indenters, with a decreasing trend as the radius increases. Throughout the simulation, most data exhibit an increasing trend, except for the peaks observed with smaller indenters, particularly in Fig. 10(c). This is due to the smaller indenter surfaces not being large enough to maintain contact with the fragments during particle breakage, causing significant drops in the plots. It should be noted that, based on strain, the loading force may have already dropped to zero while the normalised contact area curves still show a residual value. This is because contact area is defined geometrically as the total surface of all particle–indenter contacts, irrespective of load transmission. After breakage, some fragments remain in physical contact with the indenter surface, even though they no longer carry significant force. This effect is more pronounced for larger indenters, which can maintain surface contact with multiple fragments after breakage.

Despite these fluctuations, the overall converging trend remains consistent for all tested particles before breakage. This suggests a potential method for estimating tensile strength, independent of the specific loading methods used, which will be further discussed in the following section.

3.3. Tensile strength

3.3.1. Determination of tensile strength

The tensile strength of the silica sand and crushed glass under different indenters is first calculated using the method of Hiramatsu and Oka (1966). Results obtained from 21 particles within each morphology category under seven different indenter sizes are presented in Fig. 11(a), and all data points, including extreme values, are retained in the analysis. The box-plot convention follows that described for Fig. 8. Since each particle geometry was analysed under multiple indenter sizes, the

resulting dataset contains multiple loading conditions applied to the same particle geometry. The observed scatter therefore includes both particle-to-particle variability arising from differences in particle morphology and within-particle variability associated with changes in indenter size. The box plots presented here are intended primarily to illustrate comparative trends in tensile strength response under varying loading conditions rather than strictly independent population statistics.

In Fig. 11(a), the estimated tensile strength for silica sand across different indenters shows a consistent median value and a similar range of variation. However, the data for crushed glass display a wider distribution, with median values increasing as the indenter size grows. This indicates that the loading distance d_3 in Eq. (2) used by Hiramatsu and Oka (1966) can capture the size effect but has limited capability to account for the shape effect. Several extreme values are observed in the distributions and were retained in both the graphical presentation of the data and the subsequent statistical analyses to fully reflect the simulation outcomes under all investigated loading conditions. Although these values increase the scatter of the distributions, the primary conclusions are drawn from the dominant trends observed across the dataset rather than from individual extreme cases.

To better quantify the shape effect, Eq. (6) provided by Cavarretta et al. (2017) incorporating the measurements of the two principal axes of the particle is used and the results are shown in Fig. 11(b). By considering the shape effect in two dimensions, the variation range of the data and the differences between median values for crushed glass are reduced. This result is further validated in Fig. 11(c), where the tensile strength is calculated directly using the cross-sectional area where the crack initiated and propagated.

Both Fig. 11(b) and (c) show an increase in data variation and median values as the indenter size increases. This agrees with experimental observations, where traditional compression tests using flat platens

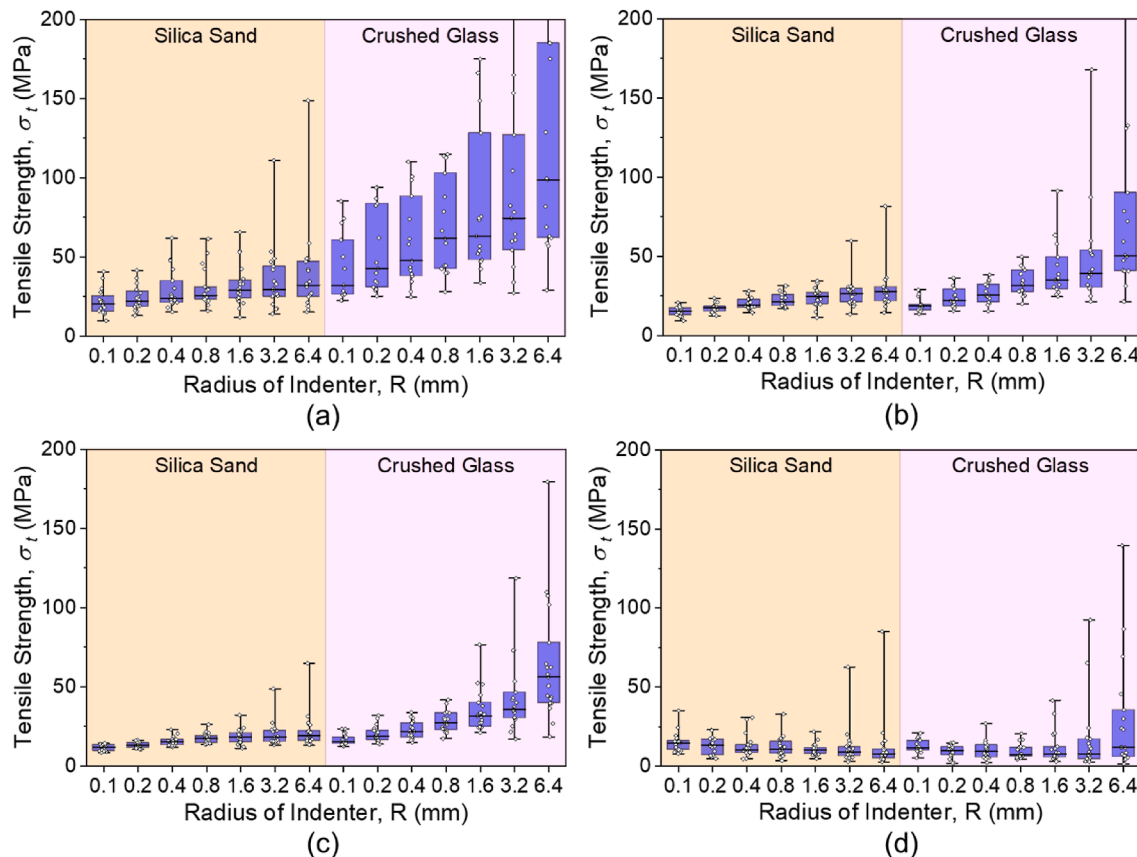


Fig. 11. Estimated tensile strength using different methods: (a) Hiramatsu and Oka (1966), (b) Cavarretta et al. (2017), (c) using cross-sectional area, and (d) present study.

(with a radius approaching infinity) tend to overestimate the tensile strength, while point load tests with smaller indenters provide more consistent strength measurements (Wang & Coop, 2018). However, because tensile strength is an inherent material property, the estimated values for the same particle should be independent of indenter size.

To minimise the influence of loading geometry, the fracture force is normalised by the ratio of the measured contact area to the theoretical contact area. The rationale is that compression-based tests measure not only the energy required to break the particle in tension, but also the additional energy dissipated in local crushing at the loading ends. By applying this normalisation, the effect of contact area is reduced, providing a more unified estimate of tensile strength across indenter sizes.

Using the contact area correction to reduce the influence of loading curvature, the fracture force was normalised to estimate particle tensile strength. A physically motivated equation is proposed:

$$\sigma_t = F_f \times \frac{A_c}{A_t} \times \frac{1}{A_{cs}} \times \frac{2r}{d_1} \quad (13)$$

where A_t denotes the theoretical contact area estimated from Eq. (12), and A_c denotes the measured contact area at the moment of breakage. The ratio A_c/A_t was introduced to account for the deviation between the measured contact area and the theoretical contact area during particle loading. The cross-sectional area A_{cs} corresponds to the fracture plane generated under tension (see Fig. 12) and can be determined experimentally using optical imaging or ultrasonic sensing. The term $1/A_{cs}$ converts the fracture force into a nominal tensile stress acting on the fracture surface. The factor $2r/d_1$ was introduced as a geometry based correction term to account for deviations of the particle shape from its equivalent volume sphere, where r is the radius of the equivalent volume sphere and d_1 is the largest particle dimension. This term therefore links the proposed correction to the principal particle dimension used to describe particle morphology. Eq. (13) is intended for irregular particles subjected to finite contact areas and actual fracture surfaces, rather than as a direct replacement for the classical Brazilian disc solution derived for circular discs under idealised loading conditions. The application of Eq. (13) is limited to uniaxial particle crushing tests, where the geometry of the contact and the failure cross section can be clearly defined. In multiaxial loading or complex shear conditions, the presence of multiple principal stresses and stress rotation leads to heterogeneous stress fields and non-unique fracture planes, making Eq. (13) inapplicable without further modification. In such cases, the fracture process is further influenced by confinement, stress path, and crack propagation mechanisms, which would require alternative formulations or numerical modelling approaches.

Based on Eq. (13), the results are presented in Fig. 11(d). The proposed normalisation provides a more consistent geometry-corrected tensile strength estimate under the investigated loading conditions. For silica sand, the relatively compact morphology leads to more consistent fracture planes across different indenters, resulting in relatively limited dispersion in the estimated tensile strength. For crushed glass, however, the elongated particle morphology leads to greater variations in fracture plane development with increasing indenter size, resulting in larger scatter, particularly under larger indenters. These results suggest that particle morphology remains an important source of variability in the estimated tensile strength, with the remaining variability primarily associated with morphology-dependent differences in fracture plane development, particularly for the more elongated crushed glass particles.

To quantify the scatter of the estimated tensile strengths, the mean value, standard deviation (SD), and interquartile range (IQR) obtained using the four calculation methods are summarised in Table 3. Compared with the methods of Hiramatsu and Oka (1966) and Cavarretta et al. (2017), both the cross-sectional area method and the present method reduce the overall scatter. For geometries derived from silica sand, the dispersion obtained using the present method remains of the same order as that obtained using the cross-sectional area method, although the latter gives slightly lower SD and IQR values. For geometries derived from crushed glass, incorporating the contact area correction leads to a pronounced reduction in the IQR.

3.3.2. Distribution of tensile strength

The survival probability P_s is used to analyse the estimated tensile strength according to the equation below proposed by Nakata et al. (1999):

Table 3

Statistical summary of the tensile strength estimate obtained using different calculation methods.

Method	Particle morphology	Mean (MPa)	SD (MPa)	IQR (MPa)
Hiramatsu and Oka (1966)	Silica sand	30.68	17.46	13.46
	Crushed glass	82.18	83.83	50.23
Cavarretta et al. (2017)	Silica sand	22.58	8.02	8.92
	Crushed glass	37.14	34.90	24.33
Cross-sectional area method	Silica sand	16.84	5.02	5.90
	Crushed glass	33.09	24.67	18.74
Present study	Silica sand	12.04	8.09	7.96
	Crushed glass	12.79	15.79	8.42

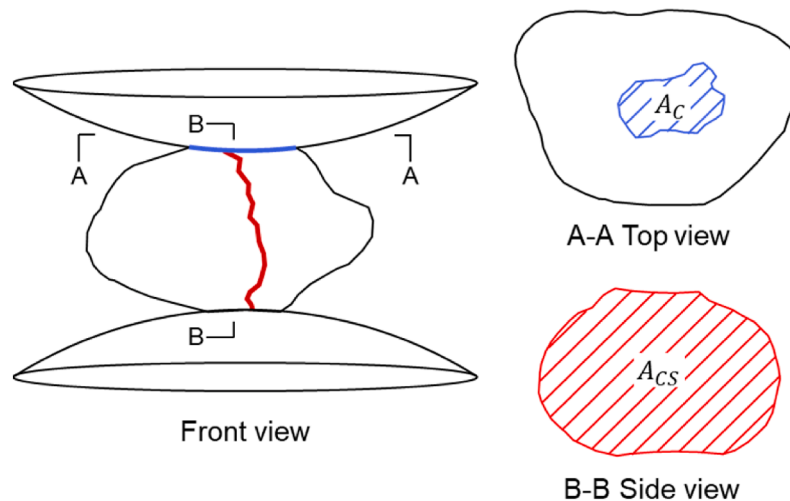


Fig. 12. Schematic diagram of the moment when particle breakage occurs.

$$P_s = \frac{n(\sigma_t \geq \sigma)}{N} \quad (14)$$

where n is the number of particles with σ_t equal or greater than a given σ , and N is the total number of particles. The results shown in Fig. 13(a) indicate that silica sand behaves similarly to crushed glass, with a slightly higher survival probability at higher tensile strength. Most of the computed tensile strength values fall within the range of 0–20 MPa. These high values have been retained to reflect the full output of simulations, but they are not representative of the overall distribution. In practice, such extreme values may be due to rare contact configurations or localised mesh artefacts.

Compared with experimental values commonly reported in the literature (i.e., 30–40 MPa), the simulated strength values are generally lower. This discrepancy may stem from differences in material properties, loading configurations, contact conditions, fracture force definitions, and the equations used to convert fracture force into apparent tensile strength. Additionally, the results from the numerical model may be influenced by the selected material parameters, such as stiffness, tensile/shear strength, and fracture energy, as well as modelling assumptions, including the friction coefficient and boundary conditions. Nevertheless, it should be noted that the tensile strength in the present study is calculated using Eq. (13), which accounts for the contact area and thereby provides a geometry-corrected estimate of tensile strength.

Furthermore, the Weibull function (Weibull, 1951) is used to characterise the relationship between P_s and σ_t :

$$P_s = \exp \left[- \left(\frac{\sigma_t}{\sigma_0} \right)^m \right] \quad (15)$$

where σ_0 is the characteristic tensile strength and corresponds to σ_t at $P_s = 37\%$. The Weibull modulus m is a shape parameter that describes the distribution, with a higher value indicating less variation in tensile strength. By taking the natural logarithm twice on both sides of Eq. (15), the Weibull function transforms into a linear relationship:

$$\ln \left[\ln \left(\frac{1}{P_s} \right) \right] = m \ln \frac{\sigma_t}{\sigma_0} \quad (16)$$

According to Eq. (16), the Weibull distribution is plotted in Fig. 13(b). Linear relationships are observed for both silica sand and crushed glass, indicating that the estimated tensile strengths of both materials conform to the Weibull distribution. Smaller m of crushed glass suggests a wider dispersion in tensile strength compared to silica sand. The Weibull analyses presented here should be interpreted primarily as comparative trend analyses of tensile strength variability under different

loading conditions rather than as strictly independent population statistics.

3.4. Particle fracture pattern

In this study, three fracture modes—local crushing, major splitting, and chipping—were commonly observed during particle breakage with different indenters, as illustrated in Fig. 14. The local crushing mode occurs when the indenter penetrates the particle, causing localised breakage and generating small fragments (see Fig. 14(a)). The major splitting mode is characterised by the particle breaking into two nearly equal major fragments (see Fig. 14(b)). The chipping mode is defined when the particle produces two or more fragments, with at least one major fragment and one or more minor fragments (see Fig. 14(c)).

According to the observations, when the indenter size is small, particle breakage often initiates with local crushing and then progresses to major splitting. However, when the indenter is larger, the increased contact area minimises local stress concentration and leads to the chipping mode of fracture. Although local crushing is frequently observed during the early stages of loading, it generally acts as a precursor to the subsequent major splitting or chipping failure modes. The final fracture pattern therefore depends strongly on the contact geometry imposed by the indenter size. These observations provide indirect evidence of the changing stress concentration mechanisms associated with different contact geometries, although a detailed stress field analysis is beyond the scope of the present study.

To quantify the fracture modes of major splitting and chipping across all simulations, an in-house code was developed to capture the fragments generated during particle breakage. During the preprocessing stage, when CIEs were inserted, all node information of the particle mesh was stored. After the simulation, the updated nodal data were compared with the initial nodal data and the connectivity matrix, enabling the newly generated fragments to be identified along with their nodal coordinates. The volume of each fragment was then calculated from its constituent elements. The volume of element on the surface ($\sim 0.00002 \text{ mm}^3$), based on the selected element size, is sufficiently small relative to the particle volume ($\sim 0.5 \text{ mm}^3$) to enable precise quantification of each fragment's volume and thereby ensure reliable identification of fracture modes. This mesh resolution also defines the lower bound of detectable fragment size in the present study. Fragments smaller than the characteristic element size cannot be explicitly resolved. However, the selected mesh provides sufficient resolution for the fracture mode classification and fragment morphology analyses considered herein. While a globally finer mesh might further reduce local discretisation error, the use of CIEs requires a uniform element size

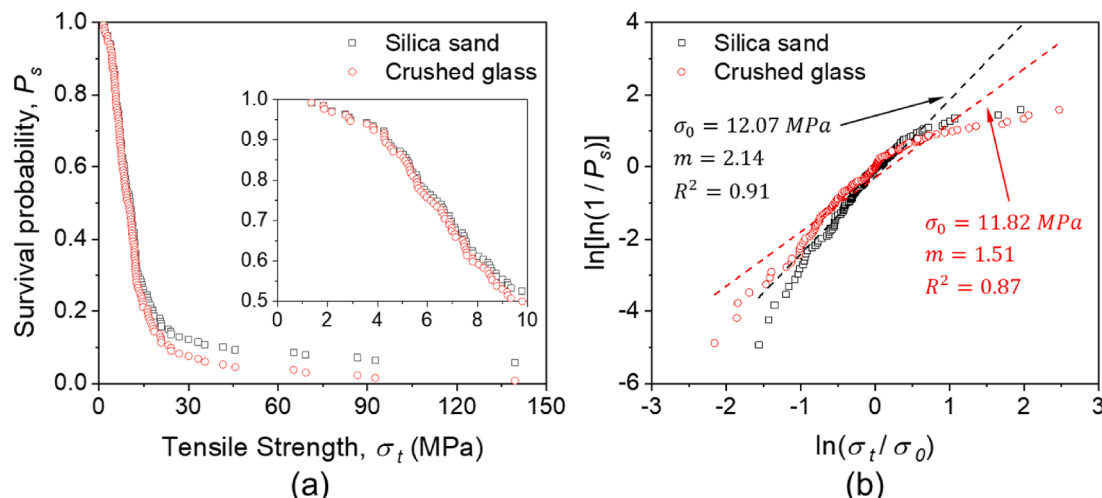


Fig. 13. (a) Particle survival probability and (b) Weibull distribution of tensile strength.

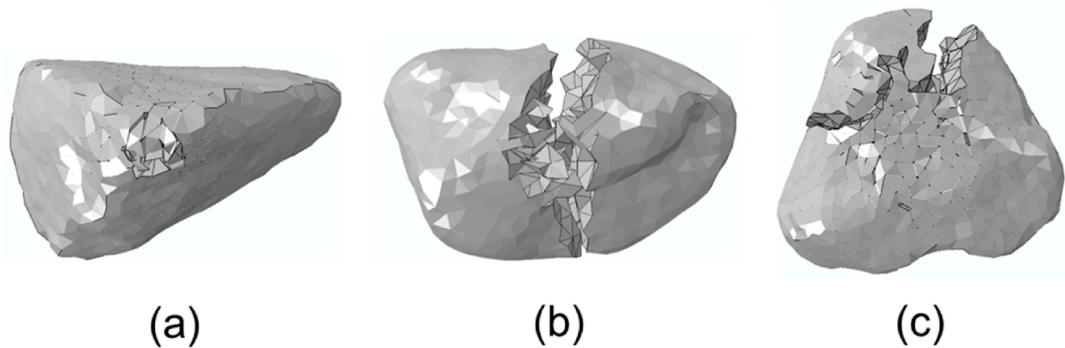


Fig. 14. Different fracture modes captured during this study: (a) local crushing, (b) major splitting, and (c) chipping.

across the entire particle to allow cracks to nucleate and propagate randomly. Adopting a much finer global mesh would therefore lead to a prohibitively large number of elements, with excessive computational time and potential convergence difficulties. The selected mesh size was adopted based on the mesh sensitivity study reported in Zhang et al. (2023), which demonstrated convergence of the predicted fracture behaviour with mesh refinement. It therefore provides a practical balance between computational efficiency and the resolution required for fracture simulation, contact area measurement, and fragment identification.

For each test, the ratio of the largest fragment volume to the particle volume was calculated. A representative subset of simulations was then visually inspected, showing that the major splitting mode (two fragments of comparable size) yielded ratios of 0.4–0.6, whereas the chipping mode (one dominant fragment with minor chips) produced ratios exceeding 0.8. Accordingly, a threshold of 0.7 was adopted to classify all tests numerically, providing an objective criterion when visual identification was ambiguous. This value is intended as a practical screening threshold rather than a strict physical boundary, and all cases with ratios in the intermediate 0.6–0.8 range were manually examined using the full three-dimensional fragment geometry to ensure correct classification. Therefore, the final fracture mode classification was not determined solely by the adopted threshold, and small variations in the threshold value are not expected to affect the overall fracture mode trends presented in Fig. 15. Using this threshold, the fracture modes for

all particles are summarised in Fig. 15. The results align with the mentioned observations: the dominant fracture mode is major splitting with smaller indenters, transitioning to chipping as the indenter size increases. This transition is more pronounced in silica sand compared to crushed glass, likely due to its more compact shape. Although further fragmentation may occur if loading continues after the onset of major breakage, the present study focuses on the primary fracture mode associated with fracture initiation and tensile strength estimate. A detailed investigation of the subsequent fragmentation process is beyond the scope of the current work.

3.5. Characterisation of fragment morphology

Furthermore, the morphological characteristics of fragments are plotted in Fig. 16 to compare with their parent particles. Based on the results, the fragmentation patterns of silica sand and crushed glass differ significantly. The fragments of silica sand tend to become more angular after breakage, showing lower convexity and sphericity values, indicating more irregular breakage. In contrast, the fragments of crushed glass maintain more self-similar shapes, displaying a more uniform fragmentation pattern, particularly with reduced elongation.

The radius of indenters plays a key role in influencing convexity and sphericity: larger indenters generally produce fragments with smaller convexity and sphericity values, leading to more irregular fragmentation. This trend is particularly pronounced in silica sand. However, an

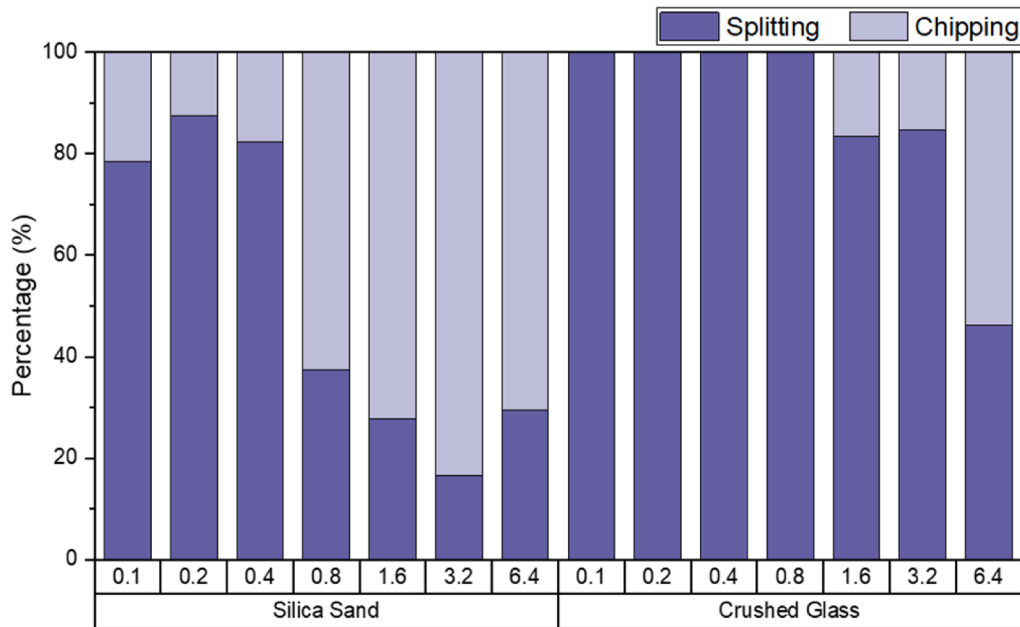


Fig. 15. Percentage histogram of fracture mode using different indenters.

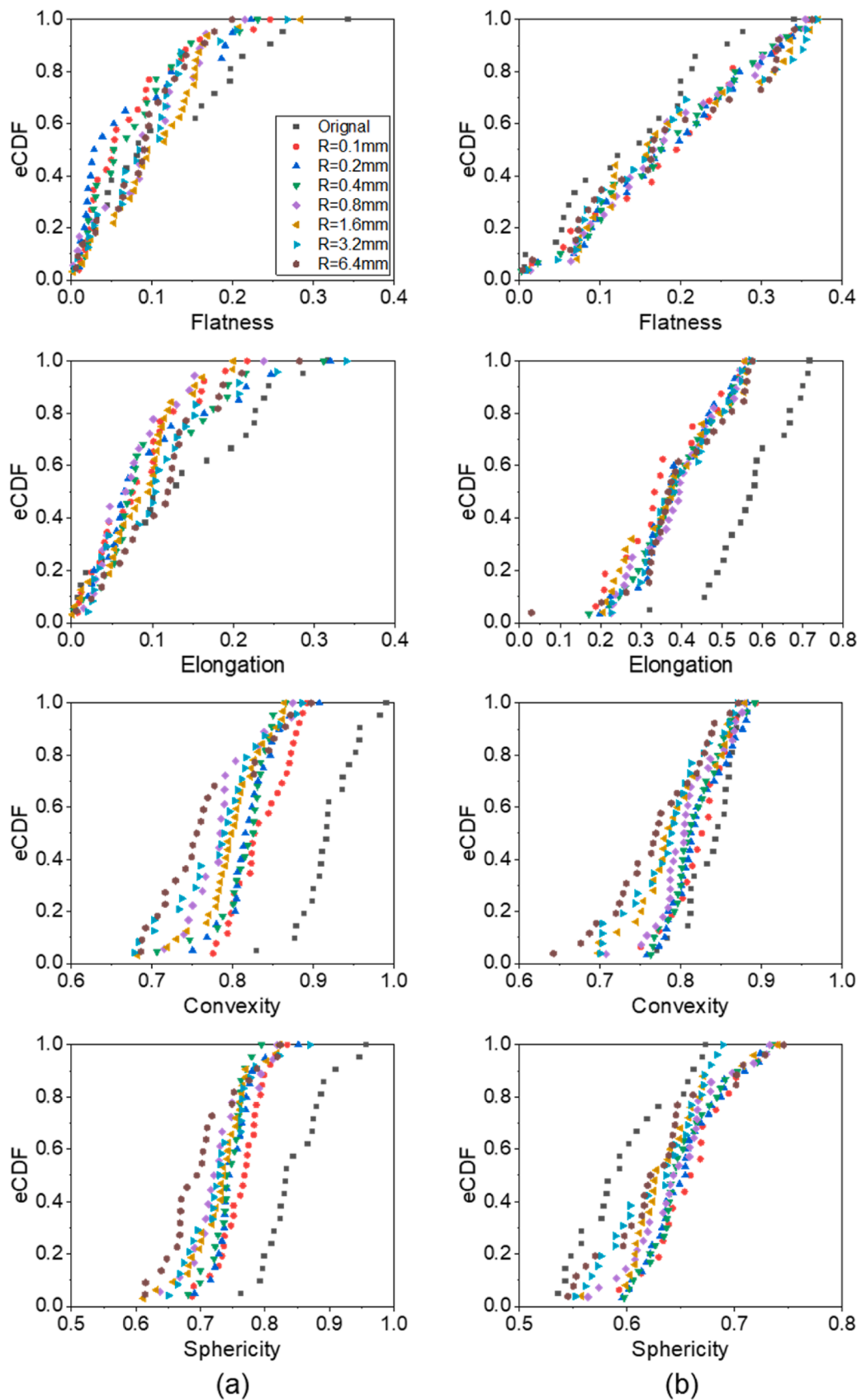


Fig. 16. Empirical cumulative distribution function of the morphological characteristics of parent and child particles: (a) silica sand and (b) crushed glass.

exception is observed in crushed glass, where sphericity actually increases after breakage, suggesting a more uniform fragmentation process. Overall, crushed glass, due to its elongated particle shape, retains more consistent fragment shapes across different indenter sizes, making the fragmentation behaviour more predictable. Although flatness, elongation, and sphericity provide useful measures of global particle shape, particle breakage is also strongly influenced by local contact geometry and fracture plane development, which cannot be fully characterised by these descriptors alone. Consequently, establishing direct predictive relationships between individual morphology descriptors and fracture force, tensile strength, or fracture mode remains challenging.

4. Conclusion

This study investigated the influence of contact area on the breakage behaviour of particles with different morphologies, namely compact silica sand and elongated crushed glass, through point loading tests using indenters with radii ranging from 0.1 mm to 6.4 mm. The key findings are summarised as follows:

- (1) The μ FE approach coupled with CIEs effectively simulates particle breakage behaviour under various loading conditions.
- (2) The load–strain responses of silica sand and crushed glass show distinct characteristics. Compact particles exhibit a sudden post-peak load drop upon breakage, while elongated particles demonstrate a smoother transition due to continued contact between elongated fragments and the indenter, especially at larger radii.
- (3) A new method for estimating particle tensile strength was proposed. By normalising the fracture force with the actual contact area and cross-sectional area at breakage, this approach yields more consistent tensile strength estimate across different indenter sizes and minimises the influence of loading curvature.
- (4) The dominant fracture mode is influenced by indenter size. Smaller indenters tend to produce major splitting, whereas larger indenters result in chipping due to broader contact areas and reduced local stress concentration.
- (5) Fragment morphology differs between compact and elongated particles. Compact particles tend to generate more angular and irregular fragments with lower convexity and sphericity values. In contrast, fragments of elongated particles tend to retain more self-similar shapes and exhibit lower elongation values.

It should be noted that the present study isolates the influence of particle morphology and contact curvature on breakage behaviour by assigning identical material properties to all particles. Therefore, the observed differences in breakage behaviour are primarily attributed to geometric effects rather than material-dependent properties. The conclusions are limited to single-particle breakage under quasi-static uniaxial loading conditions and do not account for the effects of internal microstructure, mineral composition, confining pressure, loading rate, or multiparticle interactions.

Although this study focuses on single-particle breakage, the insights gained may inform future studies on larger granular systems. They provide refinements for experimental methods aimed at more reliable particle tensile strength measurement, offer micromechanical understanding that can be incorporated into discrete element method (DEM) simulations, and support future extensions of the current μ FE model to bulk multiparticle analyses, such as triaxial compression where different breakage modes often occur simultaneously at multiple locations. Such developments would enable investigation of macroscopic strength and deformation while retaining particle-scale resolution, thereby bridging microscale and macroscale mechanical behaviour.

CRedit authorship contribution statement

Bin Zhang: Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Joana Fonseca:** Writing – review & editing, Supervision, Conceptualization. **Sadegh Nadimi:** Writing – review & editing, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization.

Data availability

Additional details regarding the numerical implementation and post-processing procedures are documented in the appendix of the corresponding doctoral thesis (Zhang, 2025).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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