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REVERSIBLE RELAY COUNTERS

a thesis submitted for
the degree of

Doctor of Philosophy
of the
University of London

by

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vi + 168 pages

ABSTRACT

This work is concerned with the design of counters built on electromagnetic relays, having two input switches, and indicating the difference between the numbers of times the input switches have been operated, modulo some number, the scale of the counter. The methods used are those for the design of 'sequential switching circuits' introduced by Huffman and subsequently developed by him and others.

Several kinds of design are presented. These are:-

(a) a routine design using the minimum number of secondary relays, that is, five. In the course of this the number of four-digit Gray codes of length ten has been determined.

(b) designs using a single transfer contact for each input, in which

(i) the relays are paired so as to form distinct flip-flops, and

(ii) the relays are not so paired.

All the above designs use series-control only.

(c) designs using τ-flip-flops having combined series- and shunt-control. The design-process for a counter of any length has been reduced to a simple exercise in combinational logic.

Finally, some comments are offered on the problems likely to be met in the practical realisation of the designs, and the steps to be taken to avoid hazards.

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1 INTRODUCTION - ANALYSIS AND DESIGN OF RELAY CIRCUITS

1.1 Contact Networks

1.1.1 Relation of the algebra of contact networks to the calculus of propositions. Historical note.

In this investigation into the design of Reversible Relay Counters use has been made of the techniques of Logic Design which have been developed over a period of a little more than thirty years. The beginning of this development is frequently credited to C. E. Shannon, who, in 1938, published his now classic paper, "A Symbolic Analysis of Relay and Switching Circuits",(1) though several authors draw attention to the work of A. Nakashima(2) at the same period and M. Gavrilov(3) also mentions the work of V. I. Shestakov.(4) Gavrilov also quotes P. Ehrenfest, who in a review dated 1910(5) of a book on the algebra of logic had clearly pointed out the correspondence between a proposition being "true" or "false" and a switch being "on" or "off" and suggested that the calculus of propositions might be applicable to networks of switches. No steps were taken to work out the details of this correspondence however, until the three authors mentioned published the results of their investigations.

This correspondence is not complete. Shannon(1) writes that the calculus of switching circuits is equivalent to "certain elementary parts of the calculus of propositions" (italics ours). In the calculus of propositions a proposition A may imply a proposition B, but the converse may not be the case. In a contact network it may be that if a certain switch is closed there will be a transmission path between a certain pair of terminals, but there may be an alternative transmission path if that switch is open. So far the correspondence is complete. The difference is that in the case of the contact network it will be possible to say in exactly what circumstances there is a transmission path between the two terminals in question. In the case of propositions it may not be possible to deduce the truth of proposition A from the truth or falsity of other propositions, and statements which imply a degree of ignorance may nevertheless be useful.

In the preceding paragraph was mentioned the first of the problems with which switching circuit theory is concerned, the expression of the transmission between two terminals of a contact network

as an algebraic function of the variables which describe the positions of the switches, and the manipulation and simplification of these expressions. No details will be given here since they are to be found in the paper by Shannon, and in many textbooks, of which Caldwell[6] has been found an excellent exposition of the work done up to the time of its publication. There is however a number of points which are worthy of comment.

1.1.2 Binary variables.

The variables are binary - two-valued. Switches are open or closed, there is apath between two terminals or there is not. It is customary to denote these two values by '0' and '1', but these are not to be identified with the cardinal numbers 0 and 1, and alternatives have been used, such as 0 and ∞.[2] A view of the relation between the two symbols and the cardinal numbers has been suggested by the present author.[7] Because the meaning of the symbols is different the rules for their manipulation are different. The algebra is "Boolean." Nevertheless, many of the identities of conventional algebra are still true.

1.1.3 Hindrance and Transmission Functions.

Shannon makes use of the "hindrance" function which is defined to have the value '1' when there is an open circuit between two terminals and '0' when there is a closed path between them, rather than the transmission, which has the values '0' and '1' for an open and a closed circuit respectively. He does so on the ground that this corresponds more closely with the concept of impedance in linear circuits, which, he says, is more familiar than that of admittance. This statement would not be true nowadays, at least of those who are concerned with linear network synthesis, and in the investigations which follow the transmission concept has been preferred as is now more usual.

1.1.4 Manipulation of contact networks.

Corresponding to the rules for manipulating linear networks (such as the delta-star and star-delta transformations) there are rules for manipulating contact networks. Shannon[1] writes that (for contact networks) there is no unique delta corresponding to a given star, and gives the simplest. He contrasts this with the case for linear networks where, he says, there is a unique circuit. This is not justified. In linear networks there are unique frequency functions, but to each of

these may correspond several networks (according to whether partial or continued fraction expansions, or a mixture of the two are used) and there may be none of these which uses only positive elements.

1.1.5 A note on Terminology.

Some of the terminology appropriate to the calculus of propositions has been used in dealing with switching circuit theory where it is not so appropriate. Thus what Shannon and others term "negation" is here termed "complementation" following Caldwell,[6] and others. Similarly, the expression "Table of Combinations" has been used in preference to "Truth Table," and for Boolean multiplication and addition have been used, respectively the dot and the plus sign (+) rather than the alternatives.

1.1.6 Design of series parallel networks. The Karnaugh map. Non-series parallel networks.

It was said in section 1.1.1 that switching circuit theory is concerned with the expression of the transmission between two terminals as an algebraic function of the variables which describe the positions of the switches, and the manipulation and simplification of these expressions. The more usual practical problem is one of synthesis, however - to devise a circuit which has the required transmission for each set of values of the variables. This information is given in the form of a list which as already mentioned, will be referred to as the "Table of Combinations." It may be that some sets of values of the variables will never occur, or if they do occur, that the value of the transmission for them does not matter. The entries in the table of combinations are then said to be "optional" and denoted by 'ϕ' which may be regarded as a combination of '0' and '1'. Alternatively these states are referred to as "don't care" conditions, and denoted by 'd'.

A possible way of deriving a network from the table of combinations is as follows. An algebraic function consisting of a sum of products, each product corresponding to a '1' value for the transmission, or a product of sums, each factor corresponding to a '0' value for the transmission, is formed. Terms or factors, as the case may be, are combined in pairs or larger groups, as may be possible, so as to reduce their number, so that a mimimum sum of products or product of sums is formed. When the number of variables is small a useful aid is the Karnaugh map[8] which displays the information in the table of

combinations in such a way that it is easily seen which terms may be combined. The Karnaugh map is easy to use with up to four variables, and is still useful with five or six. For larger numbers of variables proposals have been made for extending the map method (for example by A. Jakobschuk[9]) and systematic procedures for pairing terms have been devised by McCluskey.[10] The next step for a sum of products is to group the terms so that from each group common factors may be extracted, the object being to minimise the number of "literals" (letters, primed or not). Here again the Karnaugh map is useful for small numbers of variables. Usually the two steps carried out in succession will produce an expression with the smallest number of literals. Examples are known however, where the grouping and factorisation of an expression which is not a minimum sum of products, produces an expression which contains fewer literals than is obtained by grouping terms and factorising the minimum sum of products.[11] This situation, in which the division of a minimisation process into two parts does not produce an overall minimum is not unusual in synthesis problems, but the uncertainties have to be accepted, in order to keep the number of possibilities to be examined within reasonable bounds.

Having obtained an expression containing what, it is hoped, is the minimum number of literals, it is a routine process to draw a contact network with one contact to each literal. Such a network will be series-parallel. However it may be manipulated in various ways to reduce the number of contacts and to these manipulations of the network there are no corresponding algebraic manipulations.

Thus the resulting network is likely to be "fairly economical" in contacts, but there will be no certainty that a more economical network cannot be found with the expenditure of more effort, and in practice the saving of cost of manufacture has to be balanced against expenditure on the designer's time. In some cases however it has been proved that a circuit is minimal, and considerable effort has been made to determine lower bounds for the number of contacts required to realise a given class of functions, for example, by Shannon[12], and Povarov.[13]

1.1.7 Multiterminal networks.

The previous discussion concerned the design of a two-terminal network

such as might be used to control a single relay, by being connected in series with it and the source of power. If several relays have to be controlled, the transmission functions being different functions of the same set of variables an obvious solution would be to use a separate two-terminal network in series with each, but even if the two-terminal networks were separately minimal the set of them would almost certainly not be a minimal solution of the problem. A single network with one terminal grounded and a number of other terminals, each connected to the coil of one of the relays (assumed to have one coil each) might be more economical. The remaining terminals of the coils would be connected together and to the live terminal of the power source. Even this might not be the best solution. A single network containing contacts and relay coils, neither terminal of each of the latter being necessarily connected to either terminal of the power source or to any other coil terminal might be even more economical. This type of circuit has been considered by P. Calingaert.(14) His procedure will be considered in some detail, in order to explain, first, why more use has not been made of a procedure which, prima facie, would seem to be ideally suited to our purpose, and second, in what respects it seems now, in retrospect, more use might have been made of it than was made. Briefly, his method is as follows.

1.1.7.1 Calingaert's procedure - an assessment.

The problem is to control r relays, relay i to be operated when a function F_i of m variables $x_1, x_2, \ldots\ldots x_m$

$$F_i = f_i(x_1, x_2 \ldots\ldots x_m) = 1$$

A trivial solution of the type referred to previously would be to use separate networks to control each relay. If in a network subdivided in this way, each relay coil F_i were replaced by a contact F_i, the F_i now being regarded as independent variables, the transmission of this network would be

$$\sum_{i=1}^{i=r} F_i f_i(x_1, x_2 \ldots\ldots x_m)$$

Calingaert now designs a network to realise this function and to satisfy certain specified conditions, but without any restriction on where the F_i contacts may appear in the network. Finally he replaces the F_i contacts by coils. If there were several contacts F_i (with the same

suffix i) the i_{th} relay will require several coils.

Calingaert claims to have devised "a systematic procedure for the synthesis of multiple-output relay switching circuits." This claim seems too sweeping, since for the final circuit to be more economical the derived circuit must necessarily be non-series-parallel and it is precisely this type of circuit for which systematic design procedures are lacking. Also, when showing that it is always possible to find a derived circuit satisfying the required conditions, he simply calls attention to the trivial circuit on which he undertook to improve. If it is objected (in defence of Calingaert's claim) that the collapsed tree networks to be described later may be designed by such systematic procedures this may be admitted. However when A-derivable circuits are designed as collapsed trees as part of the whole Calingaert procedure the resulting circuits form a subset having special features which make it possible to derive them in a much simpler way. It is necessary for the present purpose to consider tree circuits quite apart from their possible use as part of the Calingaert procedure, and this will be done next.

1.1.7.2 Tree networks - Scheinmann's procedure.

In discussing tree networks we will use the terms used by Lawler[15] to distinguish the various kinds of tree. Tree circuits are implicit in Shannon's expansion theorem,[1] which states that

$$f(x_1,x_2.....x_m) = x_1 f(1,x_2.....x_m) + x_1' f(0,x_2.....x_m)$$

The function $f(x_1,x_2.....x_m)$ may thus be realised by a transfer contact on relay X_1 with networks realising $f(1,x_2.....x_m)$ and $f(0,x_2.....x_m)$ connected to the make and break respectively. This process may be continued, expanding the functions $f(1,x_2.....x_m)$ and $f(0,x_2.....x_m)$ about some other variable, and continuing until all the variables have been used. The ends of all the branches of the tree* (leaves in Lawler's terminology) are then joined. It is not necessary that the functions $f(1,x_2.....x_m)$, $f(0,x_2.....x_m)$ be expanded about the same variable, and the same applies to the

*The expression "branches" is here used in the common rather than the technical sense.

expansion of the sets of functions obtained at later stages. If at each stage of the expansion (level of the tree) all functions are expanded about the same variable the tree will be ordered. It may be "strongly-" or "weakly-ordered." It will be "strongly-ordered" if at every level in every path there is a contact. If however at some levels in some paths there is a short circuit (because it has been found that $f(\phi_1,\phi_2,....1,.....x_m) = f(\phi_1,\phi_2,.....0,.....x_m)$, where ϕ stands for 0 or 1 and the order of the variables is the order in which the expansions have been performed, the tree is "weakly-ordered." If it is not ordered it is "disordered." If at some level of the tree two of the functions produced by the expansion of two <u>different</u> functions from the preceding level are identical the two contacts from which they spring may be joined. The result is a "collapsed" tree. The networks for symmetric functions described by Shannon are much collapsed trees, and "folding" is a further process of collapsing.

It is not necessary however to construct the function $f(x_1,.....x_m)$ from the table of combinations. It is possible to work directly from the table of combinations, as has been shown by Scheinmann[16] who expresses those states for which transmission is required in decimal form (octal form is even more convenient as he points out) and uses these in a numerical-graphical procedure. The allocation of decimal numbers to the states presupposes a decision about the ordering of the variables in the tree. The resulting tree is at least weakly-ordered, but subsequent manipulation to reduce the number of contacts may cause it to become disordered.

It may be that when the number of contacts in a tree has been reduced as far as possible the contact load in one or some relays is excessive. In that case it may be possible to redistribute the contact load using the process described by Shannon.[12] He shows that in a complete decoding tree it is possible to produce an "almost uniform" distribution of contacts. It does not seem possible to make any general statement about what may be done to redistribute contact load in collapsed trees.

It was pointed out that the application of Scheinmann's technique requires a prior decision about the ordering of the variables. If there are n variables there are n! alternative orderings and one or some of these may lead to a more economical circuit than the rest.

If n is large however, it is impracticable to examine them all, and Lawler[15] showed that this was unnecessary. Briefly, Lawler makes use of the fact that if any sub-tree whose leaves are the leaves of the complete tree may be replaced by a more economical sub-tree with the variables in a different order, all those trees in which the less economical sub-tree appears may be eliminated. This reduces the amount of computation considerably, but it is still large.

So far, apart from a brief reference to Calingaert, to which we must return, only two-terminal networks have been considered. Scheinmann, in a second paper[17] showed how to extend his method to multiple-output networks, such as would be required to control several relays. He gave two methods. The first is applicable to disjunctive networks (where only one output must be grounded at a time) but may be extended to networks which are not disjunctive. The second is applicable to all networks, disjunctive or not, without modification.

In the first method, extra digits are added to the (decimal) or octal) number indicating those states for which transmission is required. The extra digits indicate to which output transmission is required for that state. It is not necessary to add an extra binary digit for each additional output, though this impression may inadvertently be obtained from Scheinmann. An extra octal digit will cater for up to eight outputs and two extra octal digits for up to sixty-four.

When this method is used for a non-disjunctive network, a disjunctive network is first designed, providing transmission to only one of the output terminals to which it is required. Additional networks are then added between output terminals, to connect them for those states for which transmission is required to them simultaneously.

In Scheinmann's second method the design process begins at the output terminals as if separate networks were being designed, but as the design proceeds possibilities of "collapsing" appear as a result of which the trees merge and finish at a single root node.

1.1.7.3 Use of tree networks in Calingaert's procedure.

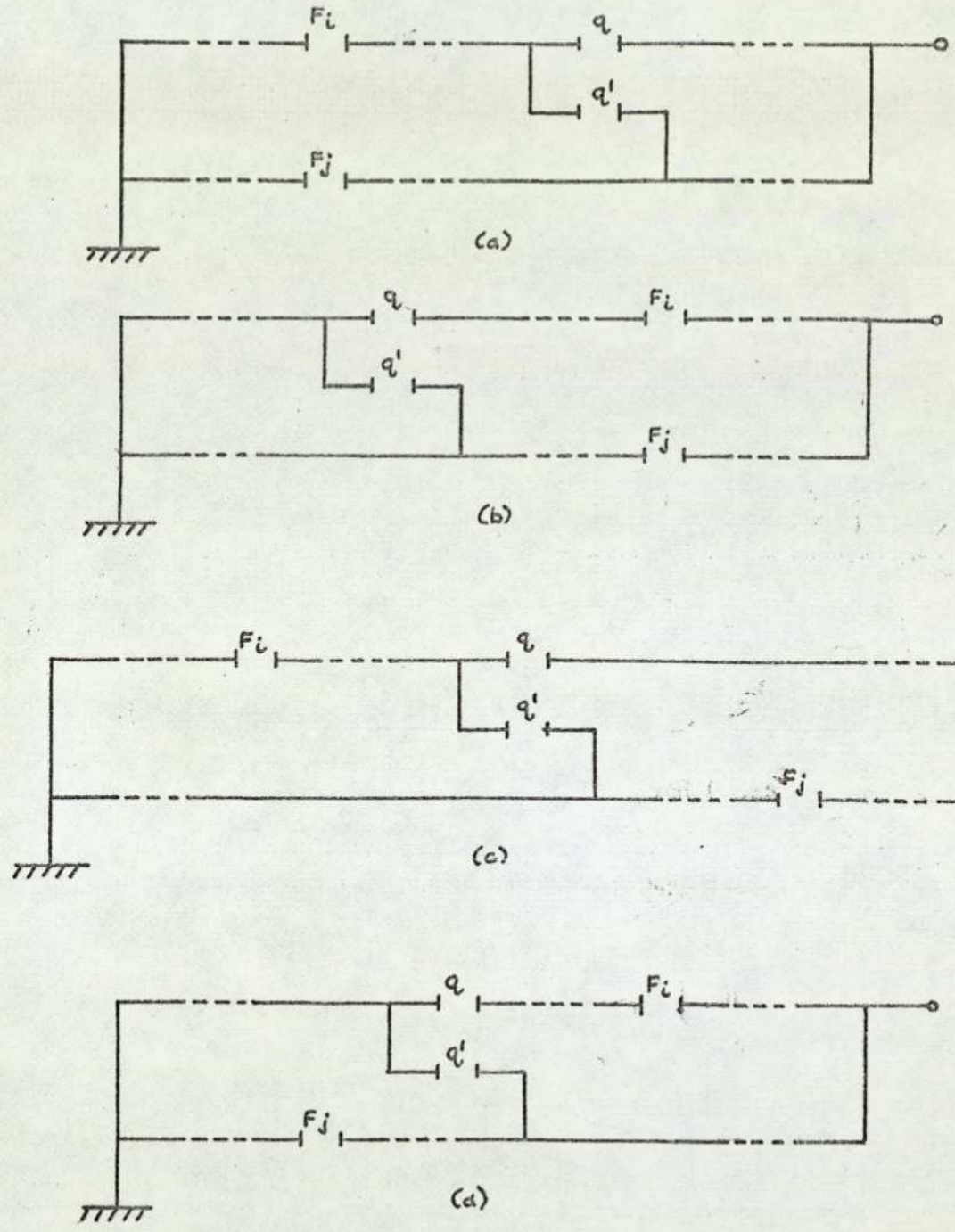
Next will be considered the situation which arises if a collapsed tree is used as the A-derivable network in Calingaert's procedure. In general, in an A-derivable network, it may be possible to trace a path through the network, from one terminal to the other, which does

not contain an F_i (coil substitute) contact, but all such paths must contain at least one pair of complementary contacts, otherwise the source would be short-circuited for some state or states. If a path is traced through a tree network, proceeding from each level to the next highest level there cannot be more than one contact on each relay and so there cannot be a pair of complementary contacts in such a path. Every such path must therefore contain an F_i contact. (It is not meant that such trees are impossible - such a circuit has been used in one instance (sec. 5.7) - but that they are not produced by the standard design procedures). Any path which contains complementary contacts must return from a higher to a lower level before continuing to the highest level, crossing, in so doing, from one monotonic path to another. Now each of these monotonic paths must contain an F_i contact and these may lie on the same side of the crossing as at Fig. 1.1, (a) and (b), or opposite sides as at (c) and (d).

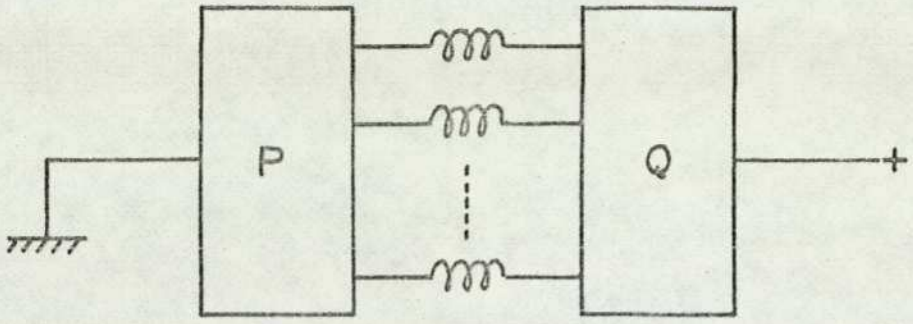
Now (c) contains a path through the network with two coils in series, and (d) contains a short-circuiting path through the network, so neither is A-derivable. Now since every monotonic path through the network contains an F_i contact, and since no two of these F_i contacts may be on opposite sides of a branch joining monotonic paths, the F_i contacts must form a cut-set, and the two-terminal network consists of two separate multiterminal networks joined by F_i contacts, or in the final network, by coils, as shown in Fig. 1.1, (e). A network of this form is not necessarily A-derivable. A sufficient though not necessary condition for A-derivability is that one of the networks, P or Q, should be disjunctive.[12] Calingaert's Fig. 6, is of the form of Fig. 1.1, (e). Neither P or Q is disjunctive, but the whole network is A-derivable.

If the relay coils are excited via a network of this type the excitation functions are products of two factors, one of which is the transmission to the terminal in question from the common node of P, the other the corresponding function for Q. The first step in design would be to separate the excitation functions into two suitable factors. Often, though not necessarily, this will involve separating the variables into two sub-sets, the first appearing in the P-functions, the second in the Q-functions.

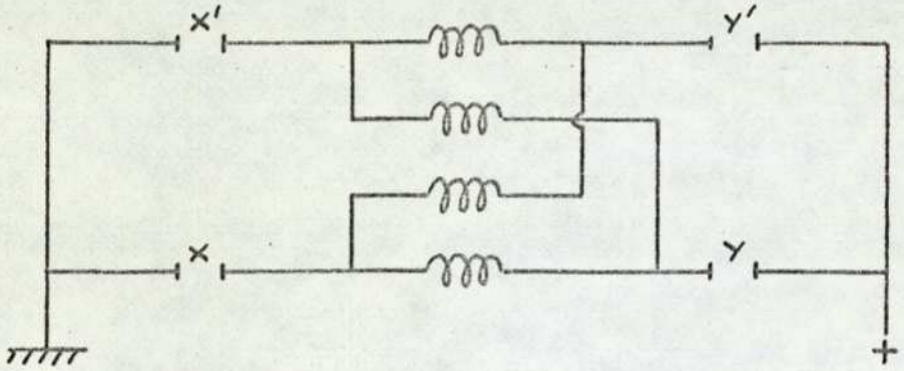
At one stage it was thought that networks of this kind might be useful certainly as output networks and possibly also as secondary



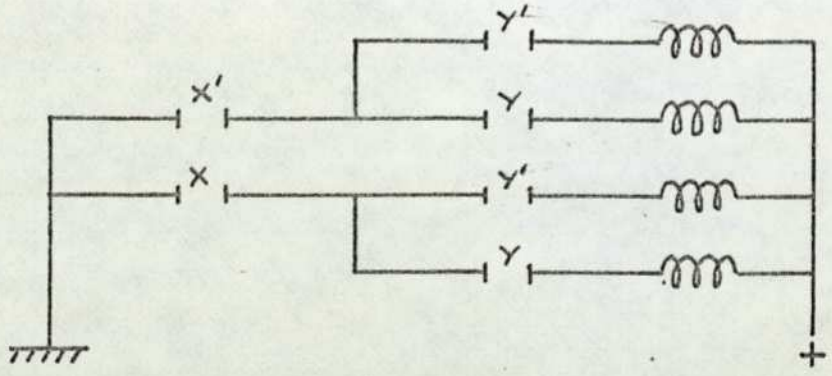
Figs. 1.1, (a - d)



(e)



(f)



(g)

Figs. 1.1, (e - g)

excitation networks. Suppose, for example, an output network for a two binary digit counter is required, the network of Fig. 1.1, (f) uses two transfer contacts as compared with three in that of Fig. 1.1, (g). However, when the intended current flows through one of the coils in (a) a reduced current flows through the other three, which are in series. This might sometimes be permissible in practice. Whether this is so depends on device characteristics which it is outside the scope of this work to consider.

In the counter designs which are given later extensive use has been made of tree networks. Most of these are disjunctive. In those cases when they are not we have used Scheinmann's method of modifying a disjunctive network. It has not been found necessary to use the formal procedures devised by Scheinmann and Lawler: instead it has been found possible to work directly from the Karnaugh map. The ad hoc procedure we have sometimes used in order to select the best variable for the highest level in a tree seems to be closely related to Lawler's procedure.

This completes the survey of the methods of designing contact networks. To say that it is not exhaustive would be an understatement. It is intended to indicate the sources of the concepts and modes of thinking which have been used, and it is hoped that this has been achieved.

1.2 Sequential Circuits

1.2.1 Feedback.

A contact network is a means of controlling a device so that it is operated when a set of inputs have certain sets of values and not operated when they have other sets of values. If this set of inputs is unaffected by the device itself we need only to know what these inputs are to know the state of the device. If however one of the inputs is derived from the device itself we cannot (in general) deduce the state of the device unless we already know it. A relay controlled by one of its own make -contacts in series with its coil is stable in the operated or not-operated state. To know its present state we need to know its history. Such a circuit could be part of a sequential circuit, for which we need to know the sequence of previous inputs in order to know the present state.

1.2.2 Analysis - construction of the flow-table.

In general a sequential circuit consists of a number of

devices (secondary devices) the present state of each of which depends in part on the present states of them all and in part on independent inputs. A relay sequential circuit consists of a number of secondary relays each controlled by a contact network made up in part of their own contacts and in part of contacts on other relays or switches. A systematic method of analysing and synthesising sequential circuits was devised by Huffman.(18)

1.2.2.1 Stable states.

Suppose that the values of the input variables are known and that the armatures of the secondary relays are wedged in the operated or non-operated condition. It will be possible to determine which secondary relay coils are energised and which are not. If the coils of those relays whose armatures are wedged in the operated position are energised and the coils of those relays whose armatures are wedged in the non-operated condition are not energised, the wedges may be removed and the armatures will not move. The circuit is stable.

1.2.2.2 Transitions.

Suppose however that there is one relay whose armature is wedged in the operated position but whose coil is not energised, and that the wedges are removed one by one, the wedge being removed from this relay last. When this is done the armature moves to the non-operated position, the contacts on this relay (which control it and the other relays) change state and it is necessary to ask whether the new state is stable or not. Similar remarks apply if there is a relay whose armature is wedged in the non-operated condition but whose coil is energised.

1.2.2.3 Races.- critical and non-critical.

Suppose next there are two relays whose armatures are wedged in the non-operated position but whose coils are energised or vice versa. When the wedges are removed both armatures move, and in general one will change the state of its own contacts before the other - there is a "race." It is then necessary to consider whether the new state is stable or not. In due course it may be that a stable state will be reached (but not necessarily: a circuit may exhibit cyclic behaviour). If the other relay had responded first the circuit might also in due course have reached a stable state. If the final state is the same whichever relay responded first the race is said to be non-critical. Otherwise it is critical. In practice, of course, relays in sequential circuits do not have their armatures wedged in position; they

enter unstable states because of a change in the input variables.

Beside the network controlling the secondary relays there will be a second network - the output network - having contacts on the secondary relays and possibly primary contacts as well. This network may be combined with the secondary excitation network to form a larger multiple-output network. At any time the outputs will be related to the present and earlier inputs in the way specified when the machine was designed.

1.2.2.4 The Flow-table.

The behaviour of a sequential circuit may be described by means of the flow table introduced by Huffman.[18] This is a matrix. The columns correspond to the input states and the rows to the secondary states. For each input state there will usually (though not always) be some secondary state or states which are stable. These are numbered and ringed. Those states which are unstable, but change to a stable state with the change of one secondary variable are given the number of the stable state to which they go, but it is not ringed. To describe the behaviour when an unstable state is followed by changes in more than one secondary variable, vertical arrows have to be used. An alternative way of displaying the same information has been proposed by Tsetlin[19] and a very similar method by Aufenkamp[20] in considering the problem of simplifying flow tables referred to later.

1.2.3 Synthesis.

1.2.3.1 Flow table from specification.

The usual problem, however, is not to analyse a given circuit, but to design a sequential circuit with prescribed behaviour. The starting point is a statement in words of the desired behaviour and the first stage the construction, from this statement, of the flow-table in its "primitive" form. In the "primitive flow table" one row is allocated to each stable secondary state. It is not possible at this stage to say how many secondary states will be required. All that is possible is to prepare some flow table which describes the required behaviour. With each secondary stable state is associated an output state, so that there is an output state corresponding to each row of the flow table.

1.2.3.2 Minimisation of flow-table.

The next stage is to reduce the flow-table to a simpler form. This is taken to be the form with the minimum number of rows, it being

supposed that this will lead to the simplest circuit. Huffman[18] describes three procedures for manipulating flow tables. The first is the identification of equivalent states. If two states have identical outputs and lead to identical outputs for every possible input sequence they are equivalent and may be replaced by a single state (a single row in the flow-table).

The second process is that of merging of rows. If two rows have identical number patterns and outputs they may be merged to form a single row. Identical number patterns may include corresponding blank entries, or a blank entry in one row corresponding to a specific entry in the other. In the latter case, however, if several mergers are possible, but not all are possible simultaneously, it is a matter of some difficulty to choose those which lead to a minimum-row flow-table.

A third process mentioned by Huffman is that of row splitting. This involves the insertion of additional rows in the flow table. The need for this usually appears at a later stage when the problem of "secondary assignment," referred to later, is under consideration, but the manipulations used by Paull and Unger in the three examples they use to introduce their treatment of the problem of flow-table minimisation[21] may be viewed as the splitting of a row followed by two mergers.

1.2.3.3 Modified presentation.

As the problem of minimising flow-tables began to be considered in more detail, the presentation of the flow-table was changed in a number of ways.[21] In the first place, whereas Huffmann had regarded the coding of the input states as given, in later treatments the input states were merely listed, it being unncessary to consider the problem of encoding them till after the flow-table minimisation was complete. The same applies to the output states which were (or could be) specified not merely for every row of the flow-table but for every cell in the matrix. Thus the flow-table could be used to show whether an output change took place immediately, as a consequence of an input change, or later and indirectly, as a consequence of the secondary change which followed the input change. Another change was that the use of rings to indicate stable states was abandoned and the rows were numbered, so that a stable state was indicated by the equality of the number of the row and an entry in that row. The flow-table thus consisted of two matrices superimposed: for each total state,

that is for each combination of input state and secondary state there were two entries, the first giving the next state and the second the present output. The use of arrows to indicate some secondary transitions was made superfluous, while at the same time the notation was uniform whether a given secondary state was stable under one or several inputs or none.

At the same time the processes of merging and the identification of equivalent states, which in Huffman[18] appear to be distinct, were given a more unified treatment. In considering whether two rows could be replaced by a single row, the answer would be an unconditional "yes" if corresponding entries were the same, the numbers of the two rows being regarded as the same for this purpose, but it might be a conditional "yes" - they could be replaced by a single row if one or several other pairs of rows could be replaced by single rows. The possibilities of combining rows and their implications concerning the combinations of other rows could be set out systematically in the "implication table" described by Paull and Unger.[21] The procedures worked out by Paull and Unger considerably reduce the amount of work involved in arriving at a flow-table with the minimum number of rows, even though at the last step they have to resort to enumeration of the various possibilities.

1.2.3.4 Secondary assignment.

When the minimum-row flow-table (or some other acceptable flow-table) has been discovered, the next step is to determine the number of secondary devices required. If this is s and n is the number of rows in the merged flow-table, s will not be less than the least integer greater than $\log_2 n$ and may be greater. The 2^s secondary states available, or some of them, must now be assigned to the rows of the flow-table, possibly with the addition of extra rows. This is the problem of secondary assignment, and it is one of the least susceptible to systematic solution in the whole field of logical design. Three methods of making a secondary assignment may be distinguished.

1.2.3.4.1 Transition diagram.

The first is intuitive, making use of graphical aids. The transition diagram[6] is such an aid. It has a number of nodes, one corresponding to each row of the merged flow-table. When a transition is required between two rows of the flow-table the corresponding nodes are joined. An attempt is now made to fit this transition diagram into

a Karnaugh map. Non-critical races may be used, if several secondary relays change state in making a transition, provided that all the intermediate states on all the alternative paths are not otherwise used. Alternatively additional rows may be introduced into the flow-table and a transition involving a change in several secondary variables made by arranging that they change one at a time in a predetermined sequence. An analytical approach to the problem of fitting the transition diagram into the Karnaugh map has been put forward by Enomoto and Ichikawa.(22) It would seem that if an analytical method is used to solve a graphical problem, which is used in turn to solve an analytical problem, the analytical method might be applicable to the analytical problem directly.

1.2.3.4.2 Universal assignments.

The second method of secondary assignment is to use universal assignments, so devised that for a given number of secondary states transitions are possible between any two of them. These universal assignments fall into two categories. In the first, described by Huffman(18) a set of states is assigned to each row of the flow-table, so arranged that it is possible to reach any member of the set from any other, either by the change of a single variable or by a non-critical race, and that for any two rows of the flow-table there is at least one state assigned to the first row which is adjacent to one state assigned to the second. "Adjacent" here means adjacent in the Karnaugh map sense, that is, differing in a single variable. Transitions then take place in two stages, a movement within a row - set to the appropriate state, which may involve a change in one or more variables, or may not be necessary at all, followed by a change in a single variable to move into the destination set.

The other type of universal assignment is the single-transition-time (STT) assignment. One state is assigned to each row in such a way that each transition involves either a change in a single variable, or a non-critical race in which none of the intermediate states involved is used in any other non-critical race. Work has been directed towards determining the number of secondary variables required to give universal STT assignments between a given number of secondary states.(23,24,25).

1.2.3.4.3 Partitions with substitution property and partition pairs.

The third approach to the problem of secondary assignment is that of Hartmanis and Stearns.(26,27) In general the next state of

any one secondary device may be dependent on the present state of all of them and of the inputs. Hartmanis and Stearns endeavour to find secondary assignments which make the next state of a secondary device dependent on the inputs and on the states of a small subset of the secondary devices. In their first paper, which made use of partition of the secondary states having what they called the "substitution property" the next values of a subset of the secondary variables are made dependent on the inputs and the present values of the same subset of secondary variables. In their second paper, which makes use of "partition pairs" this restriction (to the same subset) is removed. In their papers, Hartmanis and Stearns, as the title implies, regard their work as a contribution to the secondary assignment problem. In their book,[28] published later, the emphasis is somewhat changed. They regard their work as being concerned with "machine decomposition," the breaking down of a sequential machine into a number of simpler sequential machines interconnected in a number of different ways, and the relevance to secondary assignment as a part of the "spin-off." It should be noted that making the next state variables dependent on a small subset of the present state variables has not been proved to lead to simpler secondary excitation networks. However, it seems inherently probable that in a majority of cases it will, and this has not been contradicted by experience.

The methods of Hartmanis and Stearns, as presented by them, are applicable to synchronous machines, such as have been postulated by Moore[29] and Mealy,[30] that is machines in which changes in secondary devices take place only when a clock pulse is received. After the secondary devices have responded to a clock pulse the gates which determine which secondary devices shall respond to the next clock pulse are set to their new condition comparatively slowly, so that no question of races, critical or non-critical, arises. The requirement that critical races are to be avoided places restrictions on secondary assignment of which the methods of Hartmanis and Stearns take no account.

1.2.3.4. Methods used in the present work.

It has not therefore been possible to make use of their methods in this work, and neither has it been possible to make use of the universal assignments because the transitions required were required between only a small proportion of the pairs of states. This left the transition

diagram, which has been found adequate to deal with the problems met with.

1.2.3.5 Hazards - consideration postponed.

There is one further topic to which it will be necessary to refer; this is the problem of hazards. These are of several kinds, and we shall have occasion to refer to the work of Unger, Huffman and others.[31,32,33] It seems preferable however to postpone this until it is possible to discuss particular cases, and this has been done.

1.3 Relay Counters - Some other designs.

1.3.1 Reversible Relay Counters - scope of term.

We come now to the specific topic with which this research is concerned, the logical design of reversible relay counters. By this is meant a device that will indicate the algebraic difference between the number of times two input switches have been operated and released, modulo some number, the scale of the counter. It is assumed that both switches will not be operated simultaneously, and that the maximum rate at which they will be operated will be limited to some figure dependent on the counter itself. The counter will consist of relays only (apart from the devices used to display the state of the count) and no steps will be taken to limit the speed of operation of the relays, such as the use of slugs. Rectifiers will not be used in the contact networks. In some cases only series control of the relays will be used: in others combined series and shunt control. Provision will be made for carry and borrow outputs, so as to operate further decades or units of other scale. It will be regarded as desirable, though not essential, that the change in output of these higher decades be simultaneous with that of the first.

1.3.2 Designs using logic design procedures.

Such counters appear to have received little attention. Huffman[18] showed how to design a scale-of-four reversible counter to illustrate the application of his methods of designing sequential circuits. Klir[34] gave the transition diagram which has been derived in a later section, and completed the design of a reversible scale-of-four counter in a conventional way.

1.3.3 Some ad-hoc designs.

Apart from these two authors who made use of the techniques

have beendescribed, Aschmoneit[35] describes a reversible relay counter of the twisted-ring or "Johnson" type, and unidirectional counters designed by ad hoc procedures have been described by several authors of whom Poljak[36] may be mentioned. Aschmoneit's reversible counter uses a number of relays equal to the scale arranged in pairs to form a type of set-reset flip flop. An unusual feature of these flip-flops is that the set input has to be connected to one terminal of the power supply, while the reset terminal has to be connected to the other. As usual in a twisted-ring counter the flip-flops, initially reset, are all set, one by one, and then all reset one by one, in counting in a given direction, that is. The flip-flops are remarkably simple, and in spite of the feature mentioned, it might well be possible to develop counters using sequences other than the twisted ring sequence, in the way described later using a different type of flip-flop. Poljak describes three counters having scales of 10, 12 and 11 respectively. They are not reversible. The second and third types have interesting features. In the second five relays are used, the first two operating in a two-digit (four-step) reflected binary code, and the remaining three in a three digit (eight-step) reflected binary code. A relay of the latter group changes state, however, only at every third change of a relay of the former group. The third counter is of an essentially different type. The information about the state of the count is stored in four relays, each having a coil in series with one of its own make contacts. Each of these relays has a second coil through which a capacitor may be discharged in either direction when the input switch is operated, depending on whether it is desired to operate or release that particular relay.

1.3.4 Advantages of designs to be presented.

The designs which follow have advantages over all the designs mentioned. In the first place the response is more rapid, since either only one relay is required to change state when the input changes, or, if two change, they change simultaneously and not in sequence. With Poljak's first and second circuits at some steps in the count two relays change state in sequence, while in his third circuit the need to recharge the capacitor after each input pulse may be expected to set a limit to the recovery time. Apart from this, it is not certain these designs can be made reversible. Compared with Aschmoneit's ring counter, in

which the number of relays increases proportionately to the scale, in those presented here the number of relays required increases proportionately to the logarithm of the scale. Finally the design process ultimately evolved can be used for any scale, in the sense that only an elementary knowledge of combinational circuits is needed to produce an acceptable design.

2 SCALE-OF-TEN COUNTER ON FIVE RELAYS

The first design is for a scale-of-ten counter on five relays, a straightforward application of the principles of sequential circuit design set out in the papers referred to.

2.1 Flow tables, Primitive and Merged. Transition diagram.

2.1.1 Primitive flow table.

Inputs X_1X_2	00	01	11	10
	(0)	10		20
	(1)	11		21
X_1 "forward" or	(2)	12		22
"add" input	(3)	13		23
	(4)	14		24
X_2 "reverse" or	(5)	15		25
"subtract"	(6)	16		26
input	(7)	17		27
	(8)	18		28
	(9)	19		29
	9	(10)		
	0	(11)		
	1	(12)		
	2	(13)		
	3	(14)		
	4	(15)		
	5	(16)		
	6	(17)		
	7	(18)		
	8	(19)		
	1			(20)
	2			(21)
	3			(22)
	4			(23)
	5			(24)
	6			(25)
	7			(26)
	8			(27)
	9			(28)
	0			(29)

Fig. 2.1 (a)

2.1.2 Merged flow table.

The mergers possible are so simple and symmetrical that no merger diagramis required. The rows containing 11 and 29 may be merged and nine other such mergers are possible. The merged flow table is shown in fig. 2.1, (b).

X_1X_2	00	01	11	10	
	⓪	10		20	a
	①	11		21	b
	②	12		22	c
	③	13		23	d
	④	14		24	e
	⑤	15		25	f
	⑥	16		26	g
	⑦	17		27	h
	⑧	18		28	j
	⑨	19		29	k
	0	⑪		㉙	l
	1	⑫		⑳	m
	2	⑬		㉑	n
	3	⑭		㉒	o
	4	⑮		㉓	p
	5	⑯		㉔	q
	6	⑰		㉕	r
	7	⑱		㉖	s
	8	⑲		㉗	t
	9	⑩		㉘	u

Fig. 2.1, (b)

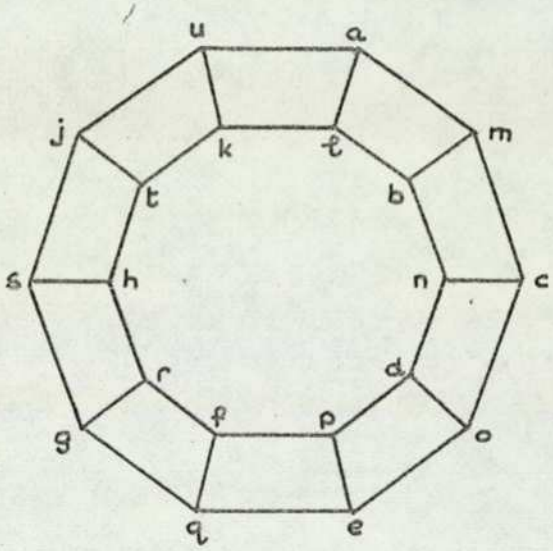
2.1.3 Transition diagram.

Next the transition diagram is constructed. After some trial it was found it could be drawn as shown in fig. 2.1, (c). This diagram was previously given by Klir.(34)

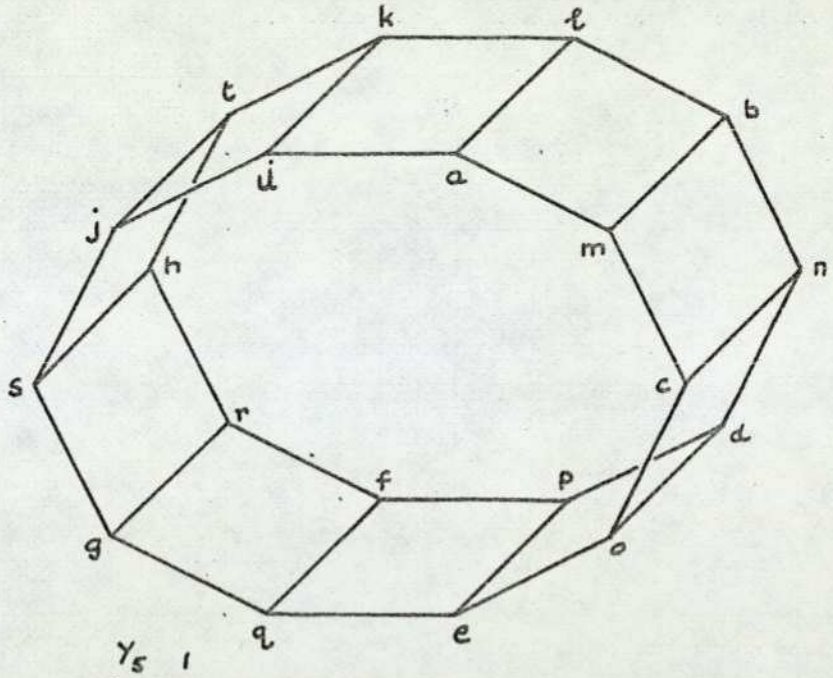
2.2 Secondary Assignment.

2.2.1 Enumeration of possible sequences.

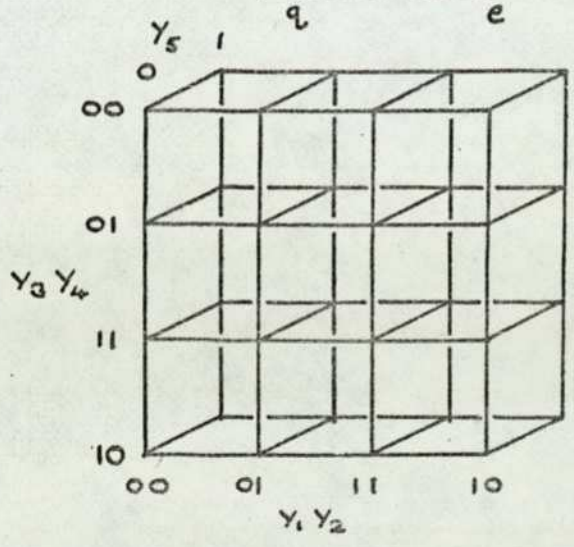
To the 20 secondary states must now be assigned states of a group of relays. Five relays are required, 5 being the smallest value of n for which 2^n is greater than 20. To see how this might be done



(c)



(d)



(e)

Figs. 2.1, (c-e)

it was found convenient to redraw the transition diagram in the form of a chain as shown in fig. 2.1, (d). This chain could then be fitted into the three dimensional Karnaugh map shown in fig. 2.1, (e), where the vertices represent the states. It will be clear how the transition diagram may be fitted into this map with the states a, m, c, o etc. on the front ($Y_5 = 0$) surface and the states l, b, n, d etc. on the back ($Y_5 = 1$) surface. Then the relay which changes state when either input is released is always Y_5. When either input is operated the relay which changes state may be any of the remaining four. In order to determine how the states on the "front" surface are to be allocated to the vertices on the "front" surface it is necessary to determine first in what sequence the relays should change. This is done in the next section.

Suppose one input is operated ten times in succession: the circuit must return to its original state. Each relay of the four must change state an even number of times since each relay must return to its original state, at least two, otherwise it would be redundant. This accounts for eight changes so one of the four relays must change state four times to make up the ten steps. No relay may change twice in succession, so that we may begin with a sequence in which the relay which changes state four times does so on alternate steps and then insert the two additional changes required into the sequence. These may be inserted (a) in the same gap, or

(b) in adjacent gaps, or

(c) in non-adjacent gaps,

giving the following distributions of the changes of this relay throughout the sequence, it having been numbered '1' which may be done without loss of generality.

(a) 1 - 1 - 1 - 1 - - -

(b) 1 - 1 - 1 - - 1 - -

(c) 1 - 1 - - 1 - 1 - -

The relay which changes second may arbitrarily be numbered '2', and the relay which changes fourth may not be '2' (for otherwise the counter would return to its original state after four steps) so let it be '3.' For a similar reason the relay which changes state last may only be '3' or '4', so that for the sequences as so far constructed there are six alternatives.

(a) 1 2 1 3 1 - 1 - - 3

(b) 1 2 1 3 1 - 1 - - 4

(c) 1 2 1 3 1 - - 1 - 3

(d) 1 2 1 3 1 - - 1 - 4

(e) 1 2 1 3 - 1 - 1 - 3

(f) 1 2 1 3 - 1 - 1 - 4

The sequences may now be completed as follows. In sequence (a) two changes of relay 4 have to be inserted, as well as one further change of relay 2. The two changes of relay 4 may not be inserted in the same gap so one must be inserted in the single gap giving

1 2 1 3 1 4 1 - - 3

The remaining change of relay 4 may not be inserted first in the remaining gap, for this would give a sub-sequence, of 1 4 1 4 so that the counter could operate in a scale of four, so it must be inserted in the second space. '2' is entered in the remaining space, giving

1 2 1 3 1 4 1 2 4 3 A

The remaining part-completed sequences are completed in the same way, care being taken to avoid sub-sequences which contain each number an even number of times. It is only necessary to examine subsequences of length 4, since if an impermissible subsequence of length 6 is present, the complementary sequence of length 4 will also be impermissible. The sequences obtained are:-

from (a)	1 2 1 3 1 4 1 2 4 3	A
from (b)	1 2 1 3 1 4 1 2 3 4	B
	1 2 1 3 1 2 1 4 3 4	C
	1 2 1 3 1 4 1 3 2 4	D
from (c)	1 2 1 3 1 2 4 1 4 3	E
	1 2 1 3 1 4 2 1 4 3	F
from (d)	1 2 1 3 1 4 2 1 3 4	G
	1 2 1 3 1 2 4 1 3 4	H
	1 2 1 3 1 4 3 1 2 4	J
from (e)	1 2 1 3 4 1 2 1 4 3	K
from (f)	1 2 1 3 2 1 4 1 3 4	L
	1 2 1 3 4 1 2 1 3 4	M
	1 2 1 3 4 1 3 1 2 4	N

In this way thirteen sequences are obtained, but they are not necessarily all distinct. It has to be borne in mind that the counter i s to be <u>reversible</u>, so that if when a sequence is reversed it is seen to be equivalent to another sequence (after a permutation of the numbers) one of the two must be discarded.

It will be clear that when a sequence is reversed the pattern of '1's is not changed, so that sequences have to be compared only within each of the three groups. Reversing the sequences of the first

group (A - D) gives

1 4 1 3 1 2 1 3 4 2	A'
1 4 1 3 1 2 1 4 3 2	B'
1 2 1 3 1 2 1 4 3 4	C'
1 4 1 3 1 2 1 4 2 3	D'

Interchanging 2 and 4 except in C' we obtain

1 2 1 3 1 4 1 3 2 4	A"
1 2 1 3 1 4 1 2 3 4	B"
1 2 1 3 1 2 1 4 3 4	C" = C'
1 2 1 3 1 4 1 2 4 3	D"

Now A" is the same as D, so D must be discarded. B and B" are the same as are C and C", and it is convenient to note at this stage that when reversed their form is unchanged. D" is the same as A, which it must be if A" is the same as D.

The next group of sequences E - J is treated in exactly the same way. Reversed, they become:-

1 3 1 2 1 3 4 1 4 2	E'
1 3 1 2 1 3 4 1 2 4	F'
1 3 1 2 1 4 3 1 2 4	G'
1 3 1 2 1 4 3 1 4 2	H'
1 3 1 2 1 4 2 1 3 4	J'

and interchanging 2 and 3 we get:-

1 2 1 3 1 2 4 1 4 3	E"
1 2 1 3 1 2 4 1 3 4	F"
1 2 1 3 1 4 2 1 3 4	G"
1 2 1 3 1 4 2 1 4 3	H"
1 2 1 3 1 4 3 1 2 4	J"

E" is the same as E, G" and J" as G and J respectively. F" and H" are the same as H and F respectively, so H must be discarded.

With the third group another circumstance has to be taken into account: the pattern of '1's is the same if the sequence is begun halfway through. Rewriting the sequences starting in the middle we get:-

1 2 1 4 3 1 2 1 3 4
1 4 1 3 4 1 2 1 3 2
1 2 1 3 4 1 2 1 3 4
1 3 1 2 4 1 2 1 3 4

Permuting the numbers where necessary to make the sequences begin with 1 2 1 3 we get:-

1 2 1 3 4 1 2 1 4 3
1 2 1 3 2 1 4 1 3 4
1 2 1 3 4 1 2 1 3 4
1 2 1 3 4 1 3 1 2 4

All these sequences are the same as the ones from which they were derived. Now reversing them we obtain

1 2 1 4 3 1 2 1 3 4	K'
1 4 1 2 3 1 2 1 4 3	L'
1 2 1 4 3 1 2 1 4 3	M'
1 3 1 4 3 1 2 1 4 2	N'

Permuting the numbers so that the sequences begin with 12 13 we get:-

1 2 1 3 4 1 2 1 4 3	K"
1 2 1 3 4 1 3 1 2 4	L"
1 2 1 3 4 1 2 1 3 4	M"
1 2 1 3 2 1 4 1 3 4	N"

K", M" are the same as K, M respectively, L", N" are the same as N, L respectively, so N may be discarded. We are thus left with 10 sequences: (fig. 2.1, (f))

1 2 1 3 1 4 1 2 4 3	A	*
1 2 1 3 1 4 1 2 3 4	B	
1 2 1 3 1 2 1 4 3 4	C	
1 2 1 3 1 2 4 1 4 3	E	
1 2 1 3 1 4 2 1 4 3	F	*
1 2 1 3 1 4 2 1 3 4	G	
1 2 1 3 1 4 3 1 2 4	J	
1 2 1 3 4 1 2 1 4 3	K	
1 2 1 3 2 1 4 1 3 4	L	*
1 2 1 3 4 1 2 1 3 4	M	

Fig. 2.1, (f)

Most of these sequences when reversed give a sequence of identical form but those marked with an asterisk do not.

If the secondary devices to be used were symmetrical, so that the labelling of the states was arbitrary, these ten sequences would give ten possible codes, but as it is intended to use relays which have on and off states distinguished in a non-arbitrary way, there are more. There are four relays concerned and the sequence A can start with these

in any one of sixteen states so that sixteen sequences are possible. The sequence B can start on any one of sixteen states, but this sequence when reversed and the variables, appropriately permuted, becomes itself and the sixteen sequences are not all distinct. Consider the sequence which starts at P with 0000 and the sequence which has 0000 at Q

P 1 2 1 3 1 4 1	Q 2 3 4
0000	0111
1000	1111
1100	1011
0100	0011
0110	0001
1110	1001
1111	1000
0111	0000
0011	0100
0001	0110

Now interchange digits 2 and 4 in the second sequence and it becomes the first reversed. The remaining fourteen sequences may be paired in the same way so that in this case there are only eight distinct sequences. This applies to C, E, G and J, whereas F is like A.

The final group of sequences K, L and M again present a special problem. Dealing first with sequence L, the second half of the sequence is like the first but with 2 replaced by 4. Consider the sequence beginning at D with 0000 and that beginning at Q with 0000

P 1 2 1 3 2 Q 1 4 1 3 4

These are:-

0000	0010
1000	1010
1100	1110
0100	0110
0110	0100
0010	0000
1010	1000
1011	1001
0011	0001
0001	0011

Interchanging digits 2 and 4 in the second sequence makes it identical with the first. Thus there are eight distinct assignments derived from L. Sequence M partakes of the natures of both L and B and here there

are only four alternatives. The same is true of K, but since this is less obvious the sequences starting on four states are given

0000	0111	0100	0011
1000	1111	1100	1011
1100	1011	1000	1111
0100	0011	0000	0111
0110	0001	0010	0101
0111	0000	0011	0100
1111	1000	1011	1100
1011	1100	1111	1000
0011	0100	0111	0000
0010	0101	0110	0001
I	II	III	IV

Of these II is obtained from I by interchanging digits 3 and 4, III is obtained by reversing I and IV by reversing II.

There are thus two sequences of changes giving 16 assignments each, six sequences giving 8 assignments and two giving 4 assignments, a total of 88 assignments. This result may be expressed by saying that there are 88 Gray codes of length 10 on the 4-cube, codes which become identical when one is reversed being considered the same, those which can be made identical by permutation of the variables being considered the same, while those which can be made identical only by complementation of one or more variables are considered distinct.

2.2.2 Choice of a sequence.

Each of these 88 assignments leads to a set of excitation functions, or rather to two sets, since Y_5 may be assigned in two different ways. It is obviously impossible to consider them all, so one has been selected, bearing in mind

(a) the behaviour of the counter when first switched on, and

(b) the power requirements, which it is desirable should be as low and uniform as possible.

(a) It is desirable that when the counter is first switched on it should go automatically into the '0' state. There are two ways in which this may be achieved. One is to use a sequence which includes the state $Y_1Y_2Y_3Y_4 = 0000$ and to use this as the '0' state, making $Y_5 = 0$ also. The alternative is to use a sequence not containing the $Y_1Y_2Y_3Y_4 = 0000$ state, choose some other state as the '0' state, and arrange that when the counter is switched on the relays operated in

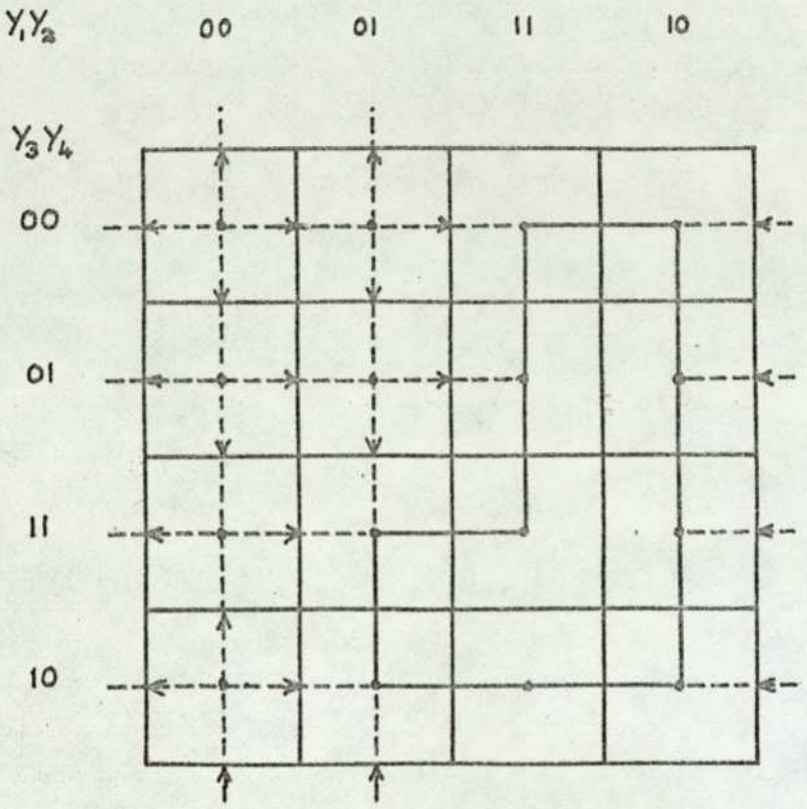


Fig. 2.2, (a)

the '0' state are excited in some sequence, such that they never go through a state forming part of the counting sequence. Clearly this restricts the states which may be used as the '0' state, for example, in fig. 2.2, (a) the states 1111 and 1110 may not be so used because all the adjacent states are part of the counting sequence. All the other states of the s equence may be used, and if the states 0101 or 0011 are passed through these may be reached from 0000 by a non-critical race. A non-critical race may also be used to reach the state 0110.

Of the two alternatives the first has been chosen on the ground that there is at present no reason to suppose that the choice of one sequence rather than another will lead to a more complex network, whereas to require that a counter shall enter as well as operate in a given sequence is likely to make the contact networks more complex.

(b) Power requirements: The second requirement was that the power consumption should be as low and uniform as possible. It has already been decided on other grounds that the sequence shall contain the 0000 state. There are four states in which one relay is operated and six in which two are operated, so that there are sufficient states in which no more than two relays (of the group Y_1 - Y_4) are operated to form the sequence of ten. It can be shown, however, that it is not possible to arrange ten of these eleven states in a sequence in which only one relay changes at a time. At least five of the six states in which two relays are operated must be used, and each of these five states must be entered from and left for a state in which one relay is operated. Thus there must be ten steps which end or begin on a state in which one relay is operated, but there are only four such states, and each may be entered and left only once in a sequence. It follows that any sequence must contain at least one state in which three of the four relays are operated.

Finally it will reduce the length of the design process (though not necessarily the complexity of the contact networks) if the sequence chosen exhibits some symmetry.

These requirements do not point inexorably to any one sequence, and the work involved in an exhaustive examination of all the possibilities was considered prohibitive, so a less rigorous approach was used. The requirement of symmetry points to sequence K which remains the same sequence if 3 and 4 are interchanged, a property it shares only with sequence L (2 and 4 are interchanged). Also it is the same sequence when reversed, a property possessed by no other sequence (though

sequence M becomes the same if it is reversed and 3 and 4 interchanged). The state 0000 may be placed only between 3 and 4 or 4 and 3. If it is placed elsewhere it is followed four steps later or preceded four steps earlier by the state 1111 which it is desired to avoid in the interests of low and uniform power requirements. The two sequences obtained (with the two positions of the state 0000) become the same if 3 and 4 are interchanged so there is only one distinct sequence. It is shown in fig. 2.2, (b). (For values assigned to Y_5 see below).

$Y_1Y_2Y_3Y_4Y_5$
0 1 1 0 1
1 1 1 0 0
1 0 1 0 1
0 0 1 0 0
0 0 0 0 1
0 0 0 1 0
1 0 0 1 1
1 1 0 1 0
0 1 0 1 1
0 1 0 0 0

Fig. 2.2, (b)

Next it is necessary to assign values to Y_5 in the various states. There are only two ways of doing this. Considering only the requirement that the counter shall enter the '0' state when first switched on, the simplest way is to make $Y_5 = 0$ when $Y_1Y_2Y_3Y_4 = 0000$. If this is done then there are some states stable in the absence of an input in which four relays are operated. This is inevitable since two relays (Y_5 and one of the relays $Y_1 - Y_4$) change state at each step so that Y_5 is operated whenever an odd number of the relays $Y_1 - Y_4$ is operated. On the other hand, if the value '1' is assigned to Y_5 when $Y_1Y_2Y_3Y_4 = 0000$ then the total number of relays operated in the absence of an input never exceeds three. If this is done it isnecessary to consider afresh what happens when the counter is switched on, and in particular, whether there will be a critical race between Y_5 and one or more of the other relays. Clearly the state in which no relay is operated must be unstable in the absence of an input, but it is one which occurs in the normal operation of the counter when the '0' state is approached in either direction. When the input is removed Y_5 becomes energised and operates, so that it will also operate when the counter

is switched on. Values were assigned to Y_5 in this way and have been added to the foregoing table.

These secondary states are now assigned to the row in which the stable states ⓪- ⑨ occur in the merged flow table, the state 00001 being assigned to the row containing state 0 and the others in sequence. Then because Y_5 is to change state whenever an input is released, the excitation matrix may be completed in the usual way. This has been done in fig. 2.2, (c).

X_1X_2 / $y_1y_2y_3y_4y_5$	00	01	11	10
00001	00001	00101		00011
00010	00010	00000		10010
11010	11010	10010		01010
01011	01011	11011		01001
01000	01000	01010		01100
01101	01101	01001		11101
11100	11100	01100		10100
10101	10101	11101		00101
00100	00100	10100		00000
00000	00001	00000		00000
00011	00010	00011		00011
10010	10011	10010		10010
11011	11010	11011		11011
01010	01011	01010		01010
01001	01000	01001		01001
01100	01101	01100		01100
11101	11100	11101		11101
10100	10101	10100		10100
00101	00100	00101		00101

Fig. 2.2, (c)

2.3 Design of the Secondary Excitation Network.

The next step is to design the secondary excitation network. On examining the excitation matrix it is found that for all the relays except Y_5 and for any one state of operation of the relays including Y_5 the only functions of X_1X_2 which occur are shown in fig. 2.3, (a).

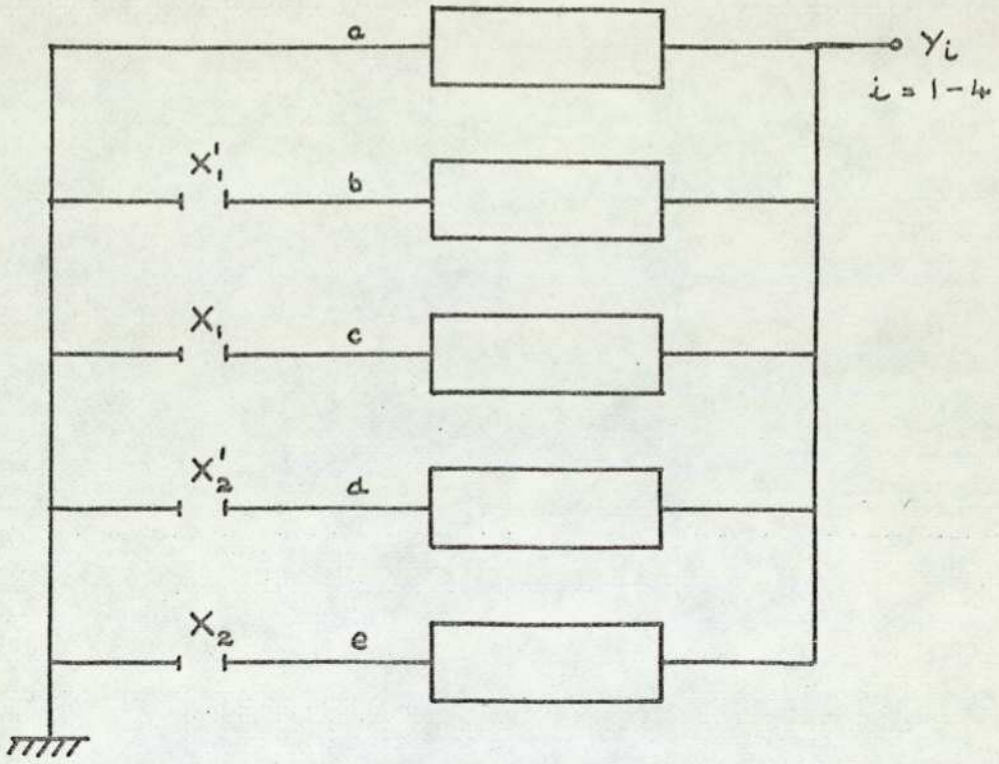


Fig. 2.3, (b)

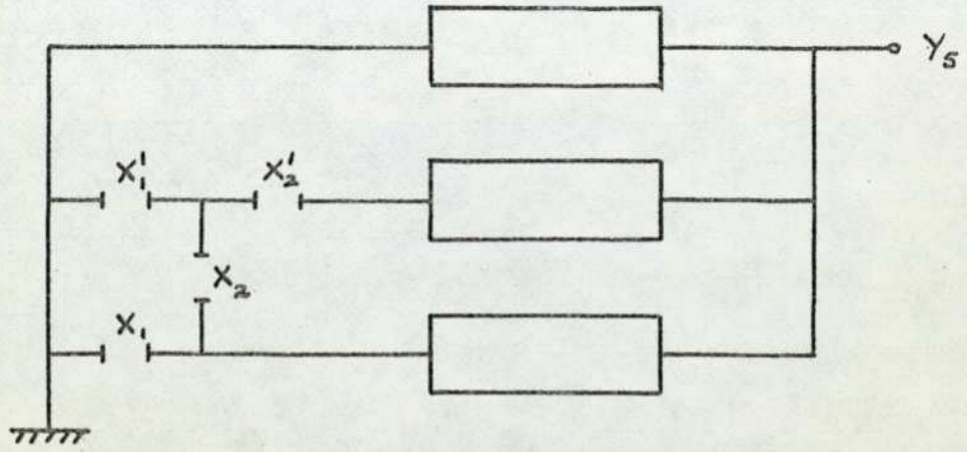


Fig. 2.3, (c)

X_1X_2	00	01	11	10	for example, for Y_1 when $y_1y_2y_3y_4y_5$ =	Function
	0	0	ø	0	00001	0
	0	0	ø	1	00010	X_1
	0	1	ø	0	00100	X_2
	1	0	ø	1	10011	X_2'
	1	1	ø	0	11010	X_1'
	1	1	ø	1	10010	1

Fig. 2.3, (a)

The excitation networks for each of the relays Y_1 - Y_4 may therefore be constructed in the form shown in fig. 2.3, (b) in which there are no contacts on X_1 and X_2 except those shown explicitly.
Further, since for any secondary state there is a path to Y_i from only one of the points b, c, d, e no two of them are ever connected, and X_1', X_1, X_2' and X_2 may be made common to all four networks without sneak paths occurring. Moreover, when this has been done, there are never paths from one of the points b, c, d, e to two of the relays, since on the closure or release of the corresponding primary contact two relays would be excited or released, and there would be a race, and races have been avoided in the secondary assignment.

Y_5 cannot be included with Y_1 - Y_4 in the above arrangement: the functions of X_1X_2 which occur are:-

X_1X_2	00	01	11	10	for example when $y_1y_2y_3y_4y_5$ =	Function
	0	0	ø	0	00010	0
	0	1	ø	1	00011	$X_1 + X_2$
	1	0	ø	0	00000	X_1X_2
	1	1	ø	1	00001	1

The excitation network for Y_5 may be constructed in the form shown in fig. 2.3, (c). The fact that two transfer contacts are necessary on both X_1 and X_2 is inconvenient, particularly when several decades are cascaded and the input variables for the more significant digits are functions of the input to the system and the secondary variables associated with the less significant digits, so it is appropriate to ask why Y_5 has this peculiar feature. This question may be answered as follows.

Any of the states stable in the absence of an input may be approached from either direction and when the input is removed (whether it is X_1 or X_2) the effect is to energise or de-energise Y_5.

If Y_5 is de-energised it must be because in the state existing before the input was removed it was energised via a path containing an X_1 or an X_2 contact, which must therefore be in parallel. If Y_5 is energised by the removal of the input it must be via a path containing both X_1' and X_2' contacts in series. If however the release of X_1 caused the energisation or de-energisation of a different relay from the one which is energised or de-energised by the release of X_2, this would no longer be the case. This in turn would require that each state stable in the absence of an input be approached via a different secondary state according to whether it was approached in the forward or reverse direction, and this would further require that the mergers made in the flow table should not be made, and that the design be started again almost from the beginning. This will be done in due course, but as the present design has proceeded so far, it willbe completed, even though its defects have been recognized. The first step is to list the contacts (X_1', X_1, X_2', X_2 or 1) to which each relay (Y_1, Y_2, Y_3 or Y_4) must be connected, for each secondary state which hasbeen assigned. This is done in fig. 2.3, (d), to which a further column has been added for Y_5, where $A = X_1 + X_2$, so that $A' = X_1'X_2'$.

$y_1y_2y_3y_4y_5$	$Y_1Y_2Y_3Y_4Y_5$
0 0 0 0 1	0 0 $X_2X_1$1
0 0 0 1 0	$X_1$0 0 X_1'0
1 0 0 1 1	$X_2'X_1$0 1 1
1 1 0 1 0	$X_1'X_2'$0 1 0
0 1 0 1 1	$X_2$1 0 X_1'1
0 1 0 0 0	0 1 $X_1X_2$0
0 1 1 0 1	$X_1$1 X_2'0 1
1 1 1 0 0	$X_2'X_1'$1 0 0
1 0 1 0 1	$X_1'X_2$1 0 1
0 0 1 0 0	$X_2$0 X_1'0 0
0 0 0 0 0	0 0 0 0 A'
0 0 0 1 1	0 0 0 1 A
1 0 0 1 0	1 0 0 1 A'
1 1 0 1 1	1 1 0 1 A
0 1 0 1 0	0 1 0 1 A'
0 1 0 0 1	0 1 0 0 A
0 1 1 0 0	0 1 1 0 A'
1 1 1 0 1	1 1 1 0 A
1 0 1 0 0	1 0 1 0 A'
0 0 1 0 1	0 0 1 0 A

Fig. 2.3, (d)

Next, the information in this table is set out on Karnaugh maps, (fig. 2.3, (e).

y_1y_2 / $y_3y_4y_5$	00	01	11	10	00	01	11	10
000	0	0			0	1		
001	0	0			0	1		
011	0	X_2	1	X_2'	0	1	1	X_1
010	X_1	0	X_1'	1	0	1	X_2'	0
110								
111								
101	0	X_1	1	X_1'	0	1	1	X_2
100	X_2	0	X_2'	1	0	1	X_1'	0
		Y_1					Y_2	
000	0	X_1			0	X_2		
001	X_2	0			X_1	0		
011	0	0	0	0	1	X_1'	1	1
010	0	0	0	0	X_2'	1	1	1
110								
111								
101	1	X_2'	1	1	0	0	0	0
100	X_1'	1	1	1	0	0	0	0
		Y_3					Y_4	
000	A'	0						
001	1	A						
011	A	1	A	1				
010	0	A'	0	A'				
110								
111								
101	A	1	A	1				
100	0	A'	0	A'				
		Y_5						

Fig. 2.3, (e)

From these maps the excitation functions are next written down: where there are alternative simplest forms these are given, the alternative

variables being shown one below the other. These functions are:-

$$
\begin{aligned}
Y_1 &= y_1(y_2'y_5' + y_2y_5) \\
&+ X_1'(y_1y_2'\,{}^{y_3}_{y_4'}\,y_5 + y_1y_2\,{}^{y_3'}_{y_4}\,y_5') \\
&+ X_2'(y_1y_2\,{}^{y_3'}_{y_4}\,y_5 + y_1y_2\,{}^{y_3}_{y_4'}\,y_5) \\
&+ X_1(y_1'y_2'y_4'y_5' + y_1'y_2y_3y_5) \\
&+ X_2(y_1'y_2'y_3'y_5' + y_1'y_2y_4y_5) \\
Y_2 &= y_2(y_1' + y_5) \\
&+ X_1'\,y_1y_2\,{}^{y_3}_{y_4'}\,y_5' \\
&+ X_2'\,y_1y_2\,{}^{y_3'}_{y_4}\,y_5' \\
&+ X_1\,y_1y_2'\,{}^{y_3'}_{y_4}\,y_5 \\
&+ X_2\,y_1y_2'\,{}^{y_3}_{y_4'}\,y_5 \\
Y_3 &= y_1\,{}^{y_3}_{y_4'} + y_3(y_2'y_5 + y_2y_5') \\
&+ X_1'y_1'y_2'y_3y_5' \\
&+ X_2'y_1'y_2y_3y_5 \\
&+ X_1y_2y_3'y_4'y_5' \\
&+ X_2y_2'y_3'y_4'y_5 \\
Y_4 &= y_1\,{}^{y_3'}_{y_4} + y_4(y_2'y_5 + y_2y_5') \\
&+ X_1'y_1'y_2y_4y_5 \\
&+ X_2'y_1y_2y_4y_5' \\
&+ X_1y_2'y_3'y_4'y_5 \\
&+ X_2y_2y_3'y_4'y_5'
\end{aligned}
$$

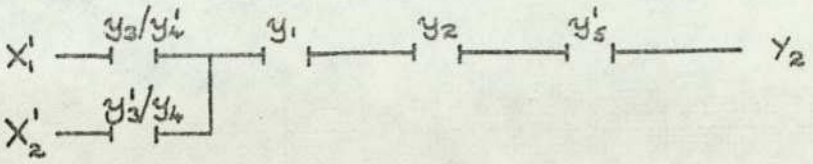


Fig. 2.3, (f)

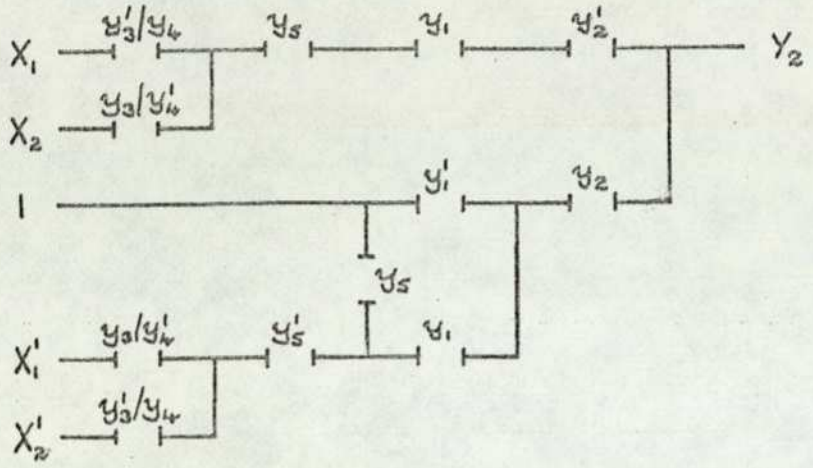


Fig. 2.3, (g)

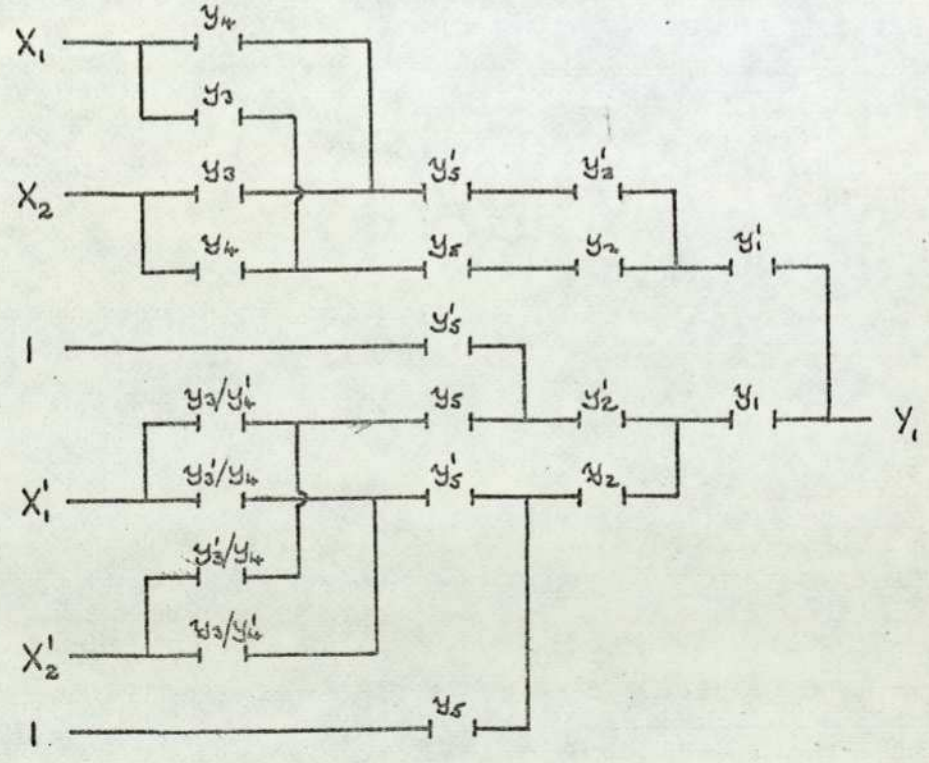
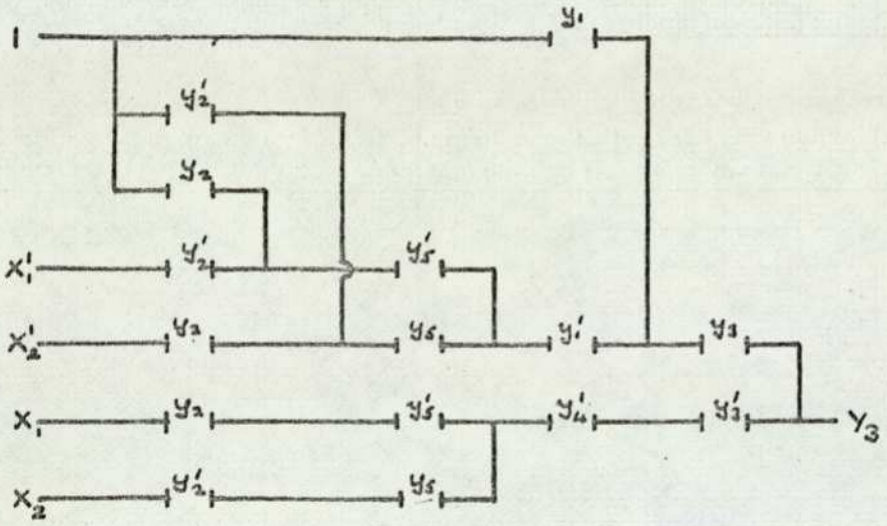


Fig. 2.3, (h)


1
y_1
y_2'
y_2
x_1'
y_2'
y_5'
x_2'
y_2
y_5
y_1'
y_3
x_1
y_2
y_5'
y_4'
y_3'
Y_3
x_2
y_2'
y_5

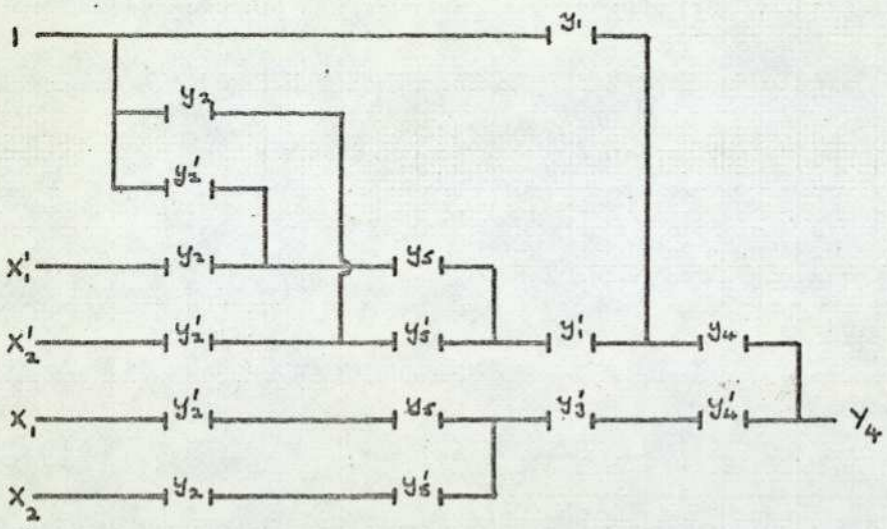

1
y_1
y_2
y_2'
x_1'
y_2
y_5
x_2'
y_2'
y_5'
y_1'
y_4
x_1
y_2'
y_5
y_3'
y_4'
Y_4
x_2
y_2
y_5'

Fig. 2.3, (j)

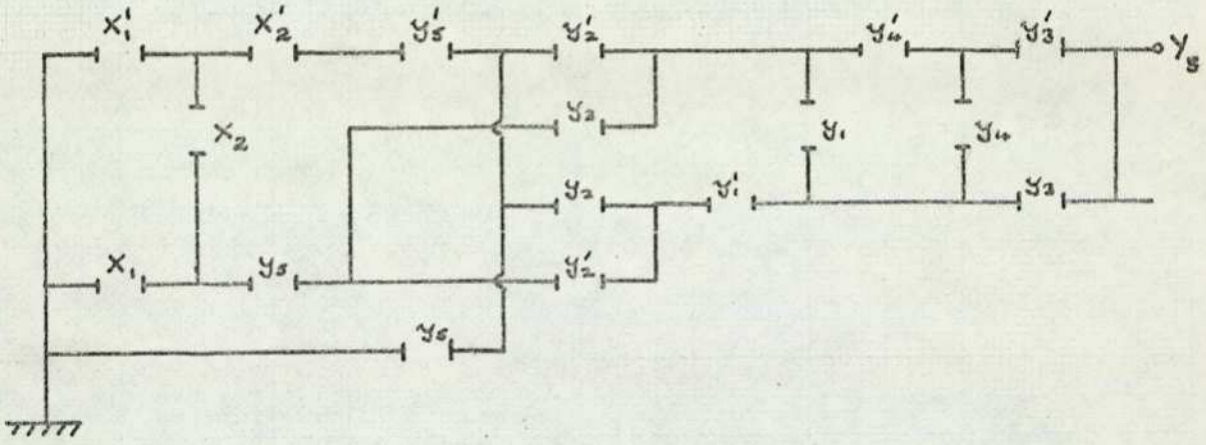


Fig. 2.3, (k)

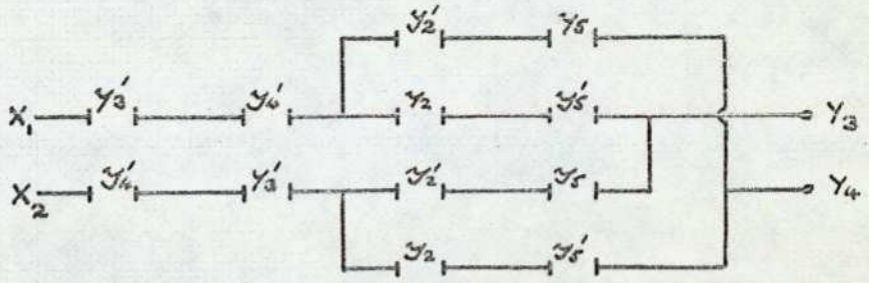


Fig. 2.3, (l)

$$Y_5 = y_2'y_3'y_4'y_5 + y_1'y_2y_4y_5 + y_1'y_2y_3y_5 + y_1y_2'y_5$$
$$+ X_1'X_2'(y_2'y_3'y_4'y_5 + y_1'y_2y_4y_5' + y_1'y_2y_3y_5' + y_1y_2'y_5')$$
$$+ (X_1 + X_2)(y_2y_3'y_4'y_5 + y_1'y_2'y_4y_5 + y_1'y_2'y_3y_5 + y_1y_2y_5)$$
$$= y_2'y_5(y_1 + y_3'y_4') + y_1'y_2y_5(y_3 + y_4)$$
$$+ X_1'X_2'\, y_2'y_5'(y_1 + y_3'y_4') + y_1'y_2y_5'(y_3 + y_4)$$
$$+ (X_1 + X_2)\, y_2y_5(y_1 + y_3'y_4') + y_1'y_2'y_5(y_3 + y_4)$$

Next the contact networks must be drawn. This will provide paths from ground and from each of the input contacts X_1, X_1', X_2 and X_2' to each of the four relays Y_1 - Y_4, besides the paths from $X_1' + X_2'$, X_1X_2 to Y_5. These paths need not be separate, though this forms a convenient starting point for the design. Taking Y_2 as an example the paths from X_1' and X_2' have three factors in common, and we have to ask whether duplicate contacts are necessary. The result of using a common path is shown in fig. 2.3, (f) and this is permissible provided that there is never transmission between X_1' and X_2'. The transmission between X_1' and X_2' remains zero for three of the four possible choices. In two of these the path contains complementary contacts. In another it contains y_3 and y_4 contacts in series and these are never closed at the same time. The only choice which is not permissible is y_4' and y_3'. Further application of this principle leads to the network of fig. 2.3, (g). Where alternatives are shown they are subject to the restriction mentioned. Also, use has been made of the method described by Caldwell[6] for obtaining two complementary transmissions to enable two transfer contacts to be used in obtaining the functions $y_1' + y_5$, y_1y_5'. The network for Y_1 is obtained similarly. (It is shown in fig. 2.3, (h). The y_3y_4 paths between X_1 and X_2 are permissible as for Y_2, and the position with regard to the alternatives on X_1' and X_2' is similar to that for the Y_2 network. The excitation networks for Y_3 and Y_4 are similar, one being obtained from the other by the interchange of y_3 and y_4 and the complementation of y_2 and y_5. They are shown in fig. 2.3, (j). Finally, the network for Y_5 is given in fig. 2.3, (k) where again use has been made of Caldwell's method for complementary transmissions. It was found that it was more economical to have the contacts on Y_5 next to the input contacts rather than those on Y_2.

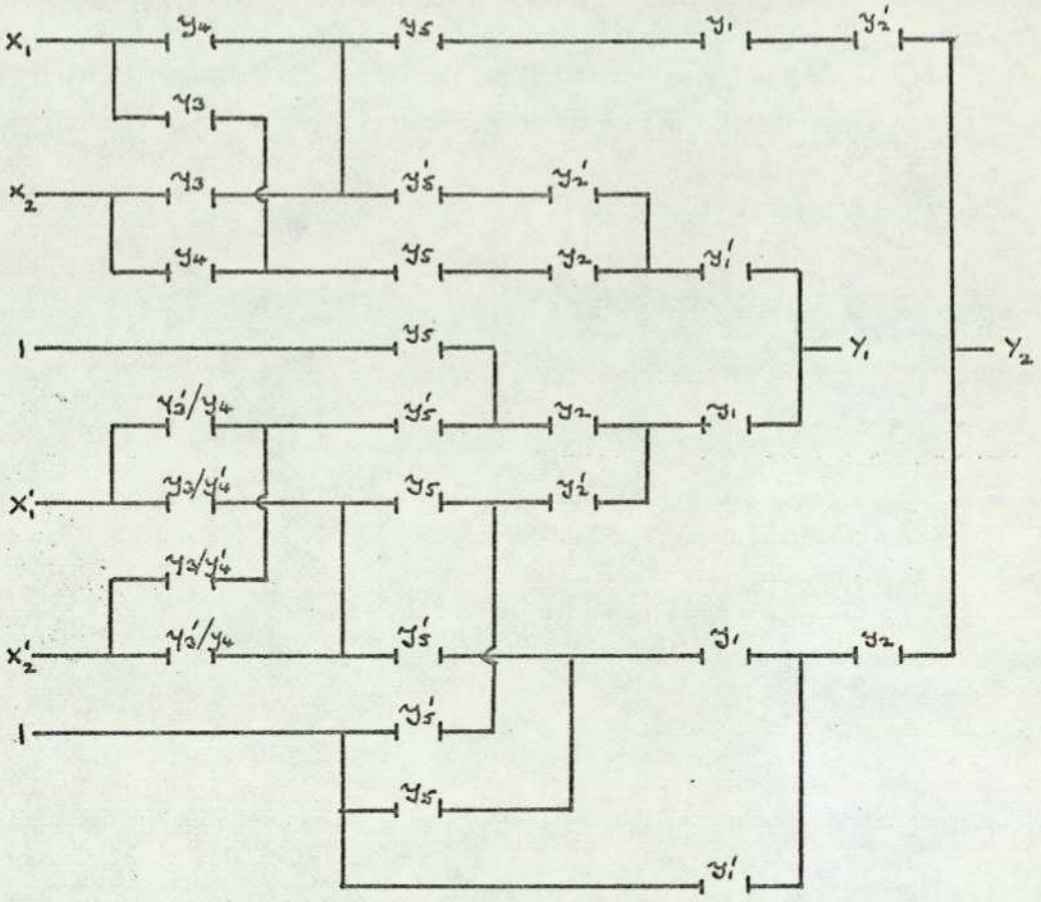

x1
y4
y5
y1
y2'
y3
x2
y3
y5'
y2'
y4
y5
y2
y1'
1
y5
Y1
Y2
y3/y4
y5'
y2
y1
x1'
y3/y4'
y5
y2'
y3'/y4'
x2'
y3'/y4
y5'
y1
y2
1
y5'
y5
y1'

Fig. 2.3, (m)

These networks may now be compared to see whether there is any further unnecessary duplication of contacts. Examining first Y_1 and Y_2 it is seen that in Y_1 between X_1 and X_2 there is a y_4y_3 path, and that this is possible for Y_2. If these contacts are combined the transmission between Y_1 and Y_2 must not be affected. It is not affected, since the path contains complementary contacts on Y_5 In the same way the (y_3/y_4') , (y_3'/y_4) contacts on X_1' and X_2' need not be duplicated for Y_2. In spite of the close similarity between the networks for Y_3 and Y_4 no simplification is possible. The possibility of economy by altering the order of the contacts in the paths to X_1 and X_2 was considered, the $y_3'y_4'$ contacts being placed next to X_1 and X_2. The revised version of this part of the network is shown in fig. 2.3, (ℓ). The number of contacts is unchanged. The y_3' and y_4' contacts previously combined with make contacts at Y_3 and Y_4, may now be combined with the corresponding make contacts which form part of the Y_1Y_2 network. However, the original scheme is preferable, since all the contacts on Y_2 may be combined in pairs to form transfer contacts. The two transfer contacts on Y_2 to ground cannot be combined: the result of doing so would be to connect X_1' and X_2' permanently in parallel. Finally the transfer contact on X_1 in the Y_5 network may not be combined with the transfer contact on X_1 in the other networks since this would place an X_2 contact across X_1 and X_1'.

The combined network for Y_1 and Y_2 has been drawn in fig. 2.3, (m), in view of the extensive combination of the two networks.

The numbers of contacts required are set out in the following table.

Excitation networks for:-	Contacts on:-					
	Y_1	Y_2	Y_3	Y_3/Y_4	Y_4	Y_5
Y_1Y_2	2T, 1M	3T	2M	2T	2M	4T, 1M
Y_3Y_4	2T	6T	1T, 1B	1T, 1B		4T
Y_5	1T	2T	1T	1T		1T, 1M
All	4T, 1M, 1B	11T	2T, 2M, 1B. 2T	2T, 2M, 1B. 2T		9T, 2M

M = make B = break T = transfer

Fig. 2.3, (n).

It will be seen that the number of contacts is very large, particularly on Y_2 and Y_5, and though the right hand part of the tree in the network for Y_1Y_2 may be arranged to transfer a transfer contact from Y_2 to Y_1, this is the only place where this can be done. The circuit could be built on commercially available relays, but its complexity does not commend it.

3 TWISTED-RING AND RELATED COUNTERS

3.1 Scale-of-four counters on four relays.

An undesirable feature of the previous design was that additional contacts were required on X_1 and X_2 to control Y_5, and it was shown that this could be avoided by refraining from making the possible mergers in the flow table. It was thought that to design a scale-of-ten counter might prove rather difficult, and that some light might be thrown on the problems involved by designing a scale-of-four counter first. Huffman used a scale-of-four reversible counter as an illustration in his classic paper referrred to previously[18] and the flow table is taken directly from that paper in its unmerged form, except that the states have been numbered 0 - 7 instead of 1 - 8. It is shown in fig. 3.1, (a). The transition diagram is shown in fig. 3.1, (b).

3.1.1 Flow table and transition diagram.

X_1X_2	00	01	11	10	
	(0)	4		8	a
	(1)	5		9	b
	(2)	6		10	c
	(3)	7		11	d
	3	(4)			e
	0	(5)			f
	1	(6)			g
	2	(7)			h
	1			(8)	j
	2			(9)	k
	3			(10)	ℓ
	0			(11)	m

Fig. 3.1, (a)

3.1.2 Enumeration of possible sequences; secondary assignment.

It will be seen that a pair of variables changes in going from 'a' to 'b' and the same pair must change in going from 'b' to 'a', in the same order. Similarly two variables must change in going from 'b' to 'c' and from 'c' to 'b'. All these four variables must be distinct, otherwise 'e' or 'm' would coincide with 'f' or 'j'. Hence the sequence in which the variables change in counting in a given direction consists of pairs of variables, no two successive

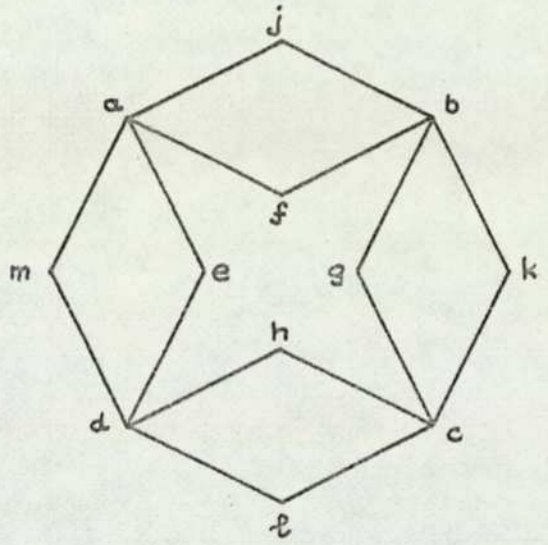


Fig. 3.1, (b)

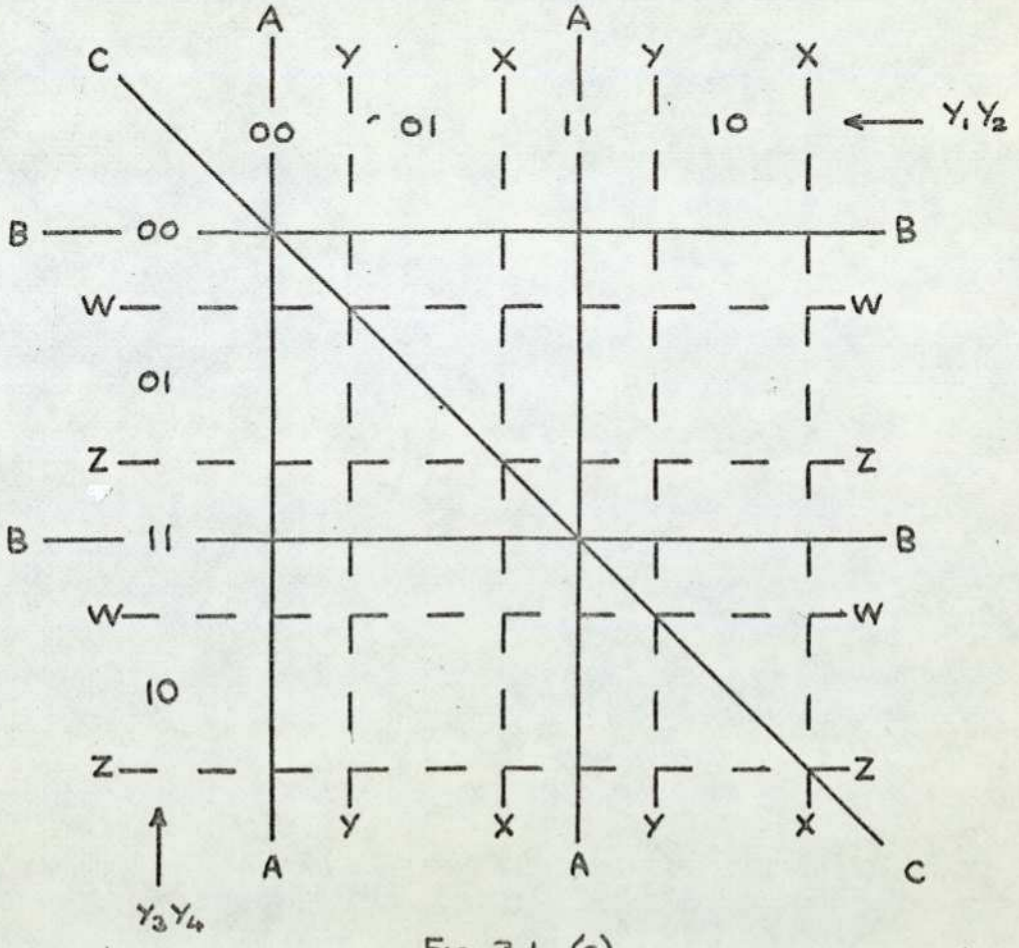


Fig. 3.1, (c)

pairs of which intersect. Hence there must be at least four variables for a counter of this type of any scale. For a scale- of-four counter four variables suffice. They must change in the order (1,2), (3,4), (1,2), (3,4), the order within the pairs being as yet undetermined. The first four variables to change when counting in the forward direction may be called 1,2,3,4 respectively. The sequence may the be completed in one of three ways:-

1 2 3 4 1 2 3 4 (a)

1 2 3 4 2 1 3 4 (b)

1 2 3 4 2 1 4 3 (c)

The sequence 1 2 3 4 1 2 4 3

is the same as (b) if 1 and 2 are interchanged with 3 and 4, respectively.

Each of these sequences of changes of variables leads to several assignments, depending on the state from which they start. These different assignments may be supposed derived from a particular assignment by complementation of one or more variables. Not every such set of complementations produces a distinctly new assignment however. Several assignments may be identical apart from a permutation of the variables.

It is convenient to visualise these operations of complementation and permutation by means of a Karnaugh map repeated indefinitely in all directions, a device used by Caldwell to test for symmetrical functions[37] so that each state is represented by a set of congruent spaces in a square pattern. Part of such a map is shown in fig. 3.1, (c). From this figure it can be seen that -

complementation of Y_1 is represented by 180° rotation about XX

" " Y_2 " " " " " " YY

" " Y_3 " " " " " " ZZ

" " Y_4 " " " " " " WW

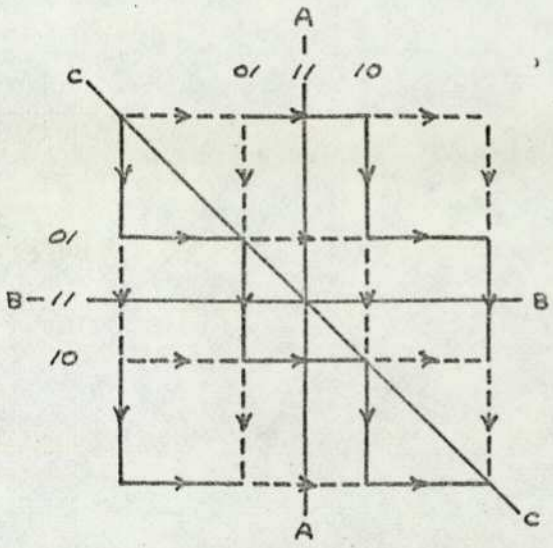
The rotations are rotations of the path relative to the map. Rotations about any of the lines marked XX are congruent, and so are those about YY, ZZ and WW. It may also be seen that

rotation about AA through 180° interchanges Y_1 and Y_2

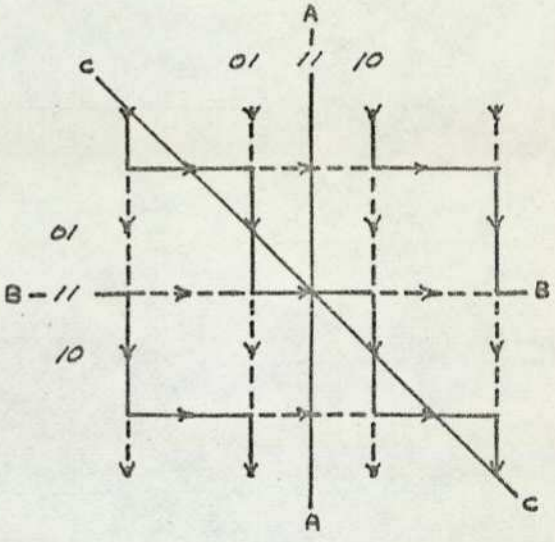
" " BB " " " Y_3 and Y_4, and

" " CC " " " Y_1 and Y_3, and Y_2 and Y_4.

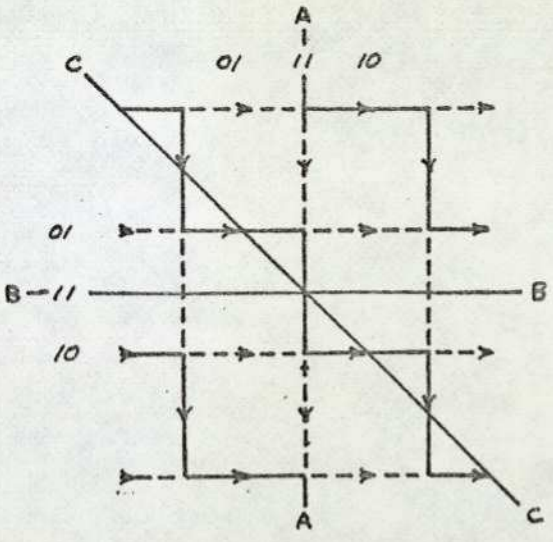
All permutations which do not involve splitting the pairs may



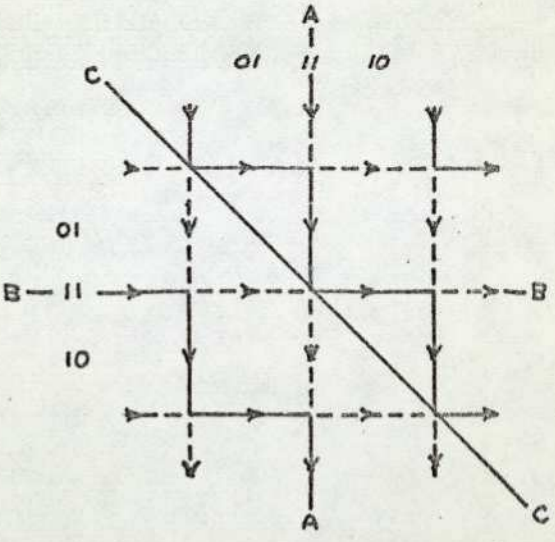
A



B

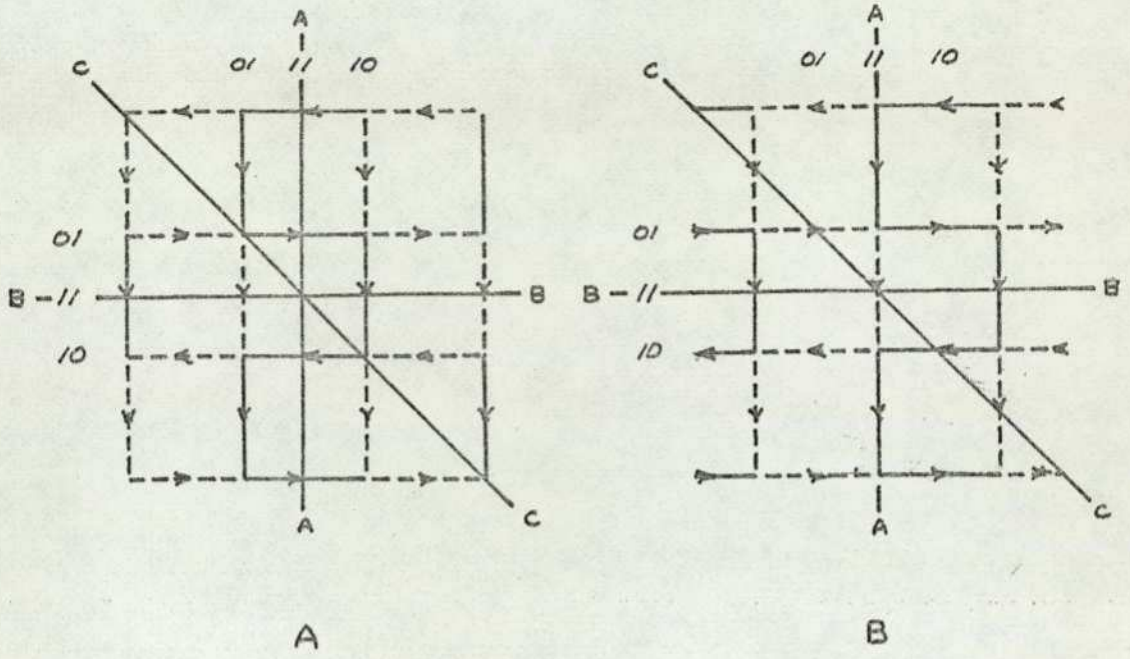


C



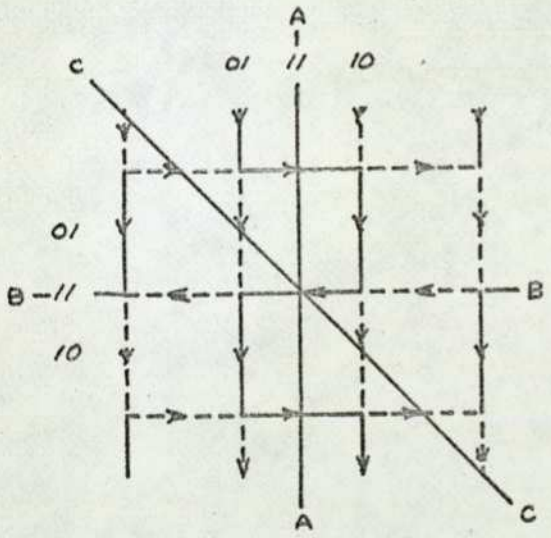
D

Fig. 3.1, (d) (A - D)

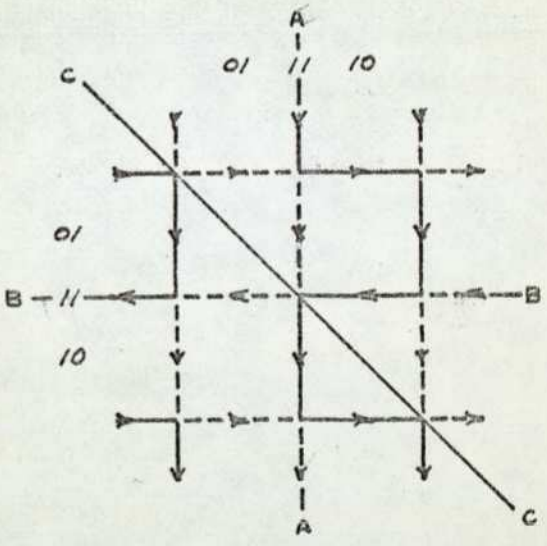


A

B



C



D

Fig. 3.1, (e) (A - D)

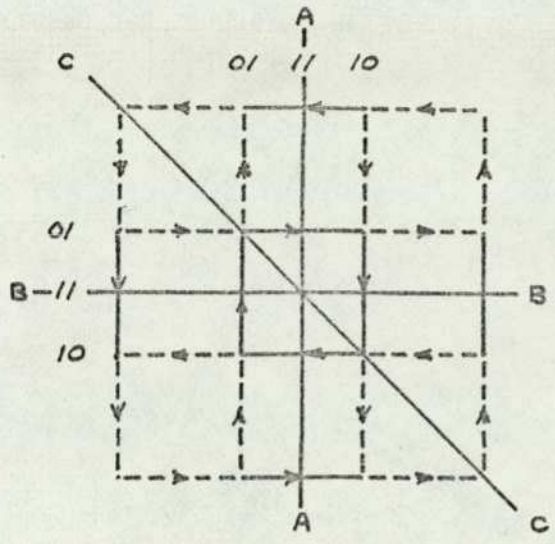
be formed from combinations of the three permutations above.

It is convenient to combine each complementation with a permutation, for example, a rotation about WW followed by a rotation about BB is equivalent to moving the paths upward one space, so that this represents the complementation of Y_4 followed by the interchange of Y_3 and Y_4. The other complementations may similarly be combined with permutations and the problem becomes on e of examining those paths obtained by movements of 0,1,2 and 3 spaces vertically together with movements of 0,1, 2 and 3 spaces horizontally in orde r to divide them into groups (equivalence classes) any member of which may be obtained from any other by operations representing permutation only. For this purpose the labels "forward" and "reverse" on the two paths are of no significance. The number of equivalence classes is the number of distinct assignments. It is now necessary to examine the three types of path separately.

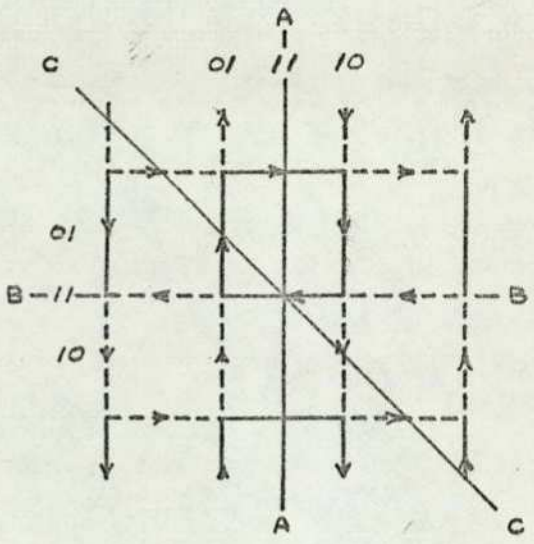
Paths of type (a), in which the variables change in the sequence 12341234 may be represented in fig. 3.1, (d) (A - D). Forward and reverse paths are shown full and dotted, with arrows.

Taking A as the starting point, moving the paths upwards one space gives B, which cannot be brought into coincidence with A since any rotation about AA, BB, or CC does not bring the "vertices" of the path onto the line CC. Hence B represents a distinct assignment. Moving the paths upwards from A two spaces merely interchanges the forward and reverse paths so no new assignment is obtained. Again starting with A, moving the paths one space horizontally gives C. After rotation through 180° about CC, the paths of C coincide with those of B so no new assignment results. At D is shown the result of moving the paths of A one space upwards and one space to the right, but although the "vertices" of the paths are on the line CC as are those of A, rotations about AA, BB, CC leave a "vertex" at the point 1111. Hence this assignment is distinct, and there are three assignments for the sequence of changes (a).

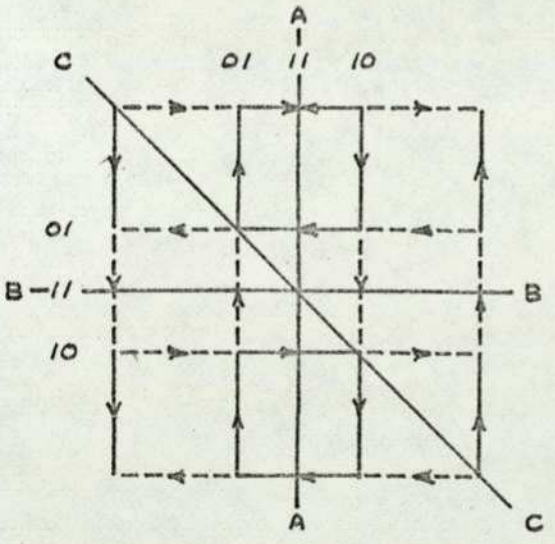
Paths of type (b) in which the variables change in the sequence 12342134 may be represented as in fig. 3.1, (e) (A - D). It may be noted first that moving the paths of A two spaces to the right leads to B in which the paths have been merely interchanged. Moving the paths upwards two spaces and then rotating them about AA leaves them unchanged. Hence it is only necessary to consider a horizontal move of one space, a vertical move of one space, and a



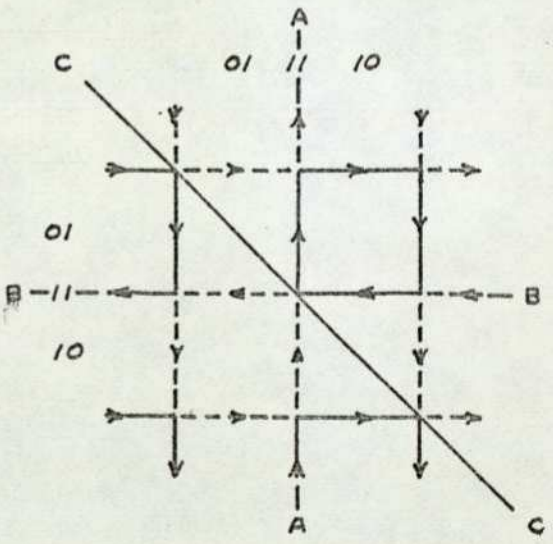
A



B



C



D

Fig. 3.1,(f) (A – D)

horizontal and vertical move combined. All these operations produce new assignments, since the point 1111 is in a different position relative to the path in each case, and this cannot be altered by any of the permuting rotations. These moves are illustrated at B, C and D respectively. There are thus four distinct assignments of this type.

Paths of type (c) in which the variables change in the sequence 12342143 are shown in fig. 3.1, (f) (A - D). First it may be noted that a movement to the right is equivalent to an upwards movement of the same number of spaces, since a move to the right followed by rotations about AA and CC has the same effect on a move upwards. A vertical movement of one space produces a new assignment as does a vertical movement of two spaces. These are shown at B and C. A movement upwards of three spaces followed by rotations about AA and BB has the same effect as a movement upwards of one space. A movement upwards of one space followed by a movement to the right of one space produces a new assignment as shown at D. A vertical move of two spaces and a horizontal move of one space followed by rotation about AA, BB, or CC produces the original path. Hence there are four distinct assignments of this type.

Altogether then there are eleven distinct secondary assignments. To investigate them all would be a tedious, and, it is thought, a not very profitable task. We first reject all those assignments which have forward and reverse paths which remain distinct after any permutation of the variables. This leaves (a)A, (a)D, (c)C and (c)D. Next we note that in two of these cases the states stable in the absence of an input all have two relays operated while in the other two cases this is not so, and choose the former for further investigation. They are (a)A, and (c)C. The secondary assignments are set out against the flow table in fig. 3.1, (g).

X_1X_2	00	01	11	10	(a) A	(c) C
	⓪	4		8	0101	0101
	①	5		9	1001	1001
	②	6		10	1010	1010
	③	7		11	0110	0110
	3	④			0111	0100
	0	⑤			0001	1101
	1	⑥			1000	1000
	2	⑦			1110	1110
	1			⑧	1101	0001
	2			⑨	1011	1011
	3			⑩	0010	0010
	0			⑪	0100	0111

Fig. 3.1, (g)

3.1.3 Secondary excitation matrices and networks.

The next step for each assignment is to draw up the excitation matrix and design the excitation network. This is done in the same way as for the relays 1 to 4 in the scale-of-10 counter, except that there are alternatives in some cases where there are entries in only two columns of the flow-table. For this reason the table of fig. 2.3, (a) must be expanded, by adding the following rows, (fig. 3.1, (h)).

X_1X_2	00	01	11	10	Function
	0	0	ø	ø	0 or X_1
	0	1	ø	ø	X_2
	1	1	ø	ø	1 or X_1'
	1	0	ø	ø	X_2'
	0	ø	ø	0	0 or X_2
	0	ø	ø	1	X_1
	1	ø	ø	1	1 or X_2'
	1	ø	ø	0	X_1'

Fig. 3.1, (h)

Where there are alternatives and one of them is '0' this is indicated as an optional entry on the appropriate map. Where one of the alternatives is '1' this has been indicated by inserting X_1' or X_2' on the '1' map, and 1 on the X_1' or X_2' map, as the case may be.

In figs. 3.1, (j) and (k) are shown the excitation maps for the assignments (a)A and (c)C respectively, together with the input contact or ground to which transmission is required from each secondary relay for each secondary state. The maps for the transmissions of the networks connected to X_1', X_1, X_2' X_2 and 1 respectively, are shown in fig. 3.1, (ℓ) for assignment (a)A and in fig. 3.1, (m) for assignment (c)C. The excitation functions are next obtained from the maps. Besides making the best use of conventional optional entries it is necessary also to choose between the entries on the '1' map and those on the X_1' or X_2' maps, as the case may be, where alternatives exist. The way in which this has been done has been indicated thus: '/' for the entry used, '×' for the entry not used. The functions thus obtained for assignment (a)A are:-

$$Y_1 = X_1'y_1y_4' + X_1y_1'y_4 + X_2'y_1y_4 + X_2y_1'y_4'$$

$$Y_2 = y_1'y_2 + X_1'y_1'y_2\overset{y_3}{\underset{y_4'}{}} + X_1y_1y_2\overset{y_3'}{\underset{y_4'}{}} + X_2'y_1'y_2\overset{y_3'}{\underset{y_4'}{}} + X_2y_1y_2\overset{y_3}{\underset{y_4'}{}}$$

$$Y_3 = X_1'y_2y_3 + X_1y_2'y_3' + X_2'y_2'y_3 + X_2y_2y_3'$$

$$Y_4 = y_3'y_4 + X_1'\overset{y_1'}{\underset{y_2}{}}y_3'y_4' + X_1\overset{y_1}{\underset{y_2'}{}}y_3y_4 + X_2'\overset{y_1}{\underset{y_2'}{}}y_3'y_4' + X_2\overset{y_1'}{\underset{y_2}{}}y_3y_4$$

These functions are sufficiently simple for the networks to be drawn by inspection. The networks for Y_1 and Y_2 and for Y_3 and Y_4 may conveniently be combined, if the appropriate factors are chosen where a choice exists (the alternatives are shown vertically). They are shown in fig. 3.1, (n).

The excitation functions for assignment (c)C are:-

$$Y_1 = y_1y_2'y_4 + X_1'(y_1y_4' + y_1'y_2'\overset{y_3'}{\underset{y_4}{}}) + X_2(y_1'y_3 + y_1y_2\overset{y_3'}{\underset{y_4}{}})$$

$$Y_2 = y_1'y_2y_3 + X_1'(y_2y_3' + y_1'y_2'\overset{y_3}{\underset{y_4'}{}}) + X_2(y_2'y_4 + y_1y_2\overset{y_3}{\underset{y_4'}{}})$$

$$Y_3 = y_2y_3y_4' + X_1(y_1y_3' + \overset{y_1'}{\underset{y_2}{}}y_3y_4) + X_2(y_2y_3 + \overset{y_1'}{\underset{y_2}{}}y_3y_4)$$

$$Y_4 = y_1y_3'y_4 + X_1(y_2y_4' + \overset{y_1}{\underset{y_2'}{}}y_3y_4) + X_2(y_1'y_4 + \overset{y_1}{\underset{y_2'}{}}y_3'y_4')$$

$y_1y_2y_3y_4$	X_1X_2	00	01	11	10	Y_1	Y_2	Y_3	Y_4
0101		0101	0111		1101	X_1	1	X_2	1
1001		1001	0001		1011	X_2'	0	X_1	1
1010		1010	1000		0010	X_1'	0	X_2'	0
0110		0110	1110		0100	X_2	1	X_1'	0
0111		0110	0111			$0/X_1$	$1/X_1'$	$1/X_1'$	X_2
0001		0101	0001			$0/X_1$	X_2'	$0/X_1$	$1/X_1'$
1000		1001	1000			$1/X_1'$	$0/X_1$	$0/X_1$	X_2'
1110		1010	1110			$1/X_1'$	X_2	$1/X_1'$	$0/X_1$
1101		1001			1101	$1/X_2'$	X_1	$0/X_2$	$1/X_2'$
1011		1010			1011	$1/X_2'$	$0/X_2$	$1/X_2'$	X_1
0010		0110			0010	$0/X_2$	X_1'	$1/X_2'$	$0/X_2$
0100		0101			0100	$0/X_2$	$1/X_2'$	$0/X_2$	X_1'

Fig. 3.1, (j)

$y_1y_2y_3y_4$	X_1X_2	00	01	11	10	Y_1	Y_2	Y_3	Y_4
0101		0101	0100		0001	0	X_1'	0	X_2'
1001		1001	1101		1011	1	X_2	X_1	1
1010		1010	1000		0010	X_1'	0	X_2'	0
0110		0110	1110		0111	X_2	1	1	X_1
0100		0110	0100			$0/X_1$	$1/X_1'$	X_2'	$0/X_1$
1101		0101	1101			X_2	$1/X_1'$	$0/X_1$	$1/X_1'$
1000		1001	1000			$1/X_1'$	$0/X_1$	$0/X_1$	X_2'
1110		1010	1110			$1/X_1'$	X_2	$1/X_1'$	$0/X_1$
0001		1001			0001	X_1'	$0/X_2$	$0/X_2$	$1/X_2'$
1011		1010			1011	$1/X_2'$	$0/X_2$	$1/X_2'$	X_1
0010		0110			0010	$0/X_2$	X_1'	$1/X_2'$	$0/X_2$
0111		0101			0111	$0/X_2$	$1/X_2'$	X_1	$1/X_2'$

Fig. 3.1, (k)

		Y_1				Y_2				Y_3				Y_4				
	y_1y_2	00	01	11	10	00	01	11	10	00	01	11	10	00	01	11	10	
	y_3y_4																	
	00	ø	0	ø	$\overset{\times}{X'_1}$	ø	$\overset{\checkmark}{X'_2}$	ø	0	ø	0	ø	0	ø	0	ø	0	
1	01	0	0	$\overset{\times}{X'_2}$	0	0	1	0	0	0	0	0	0	$\overset{\checkmark}{X'_1}$	1	$\overset{\checkmark}{X'_2}$	1	1
	11	ø	0	ø	$\overset{\times}{X'_2}$	ø	$\overset{\checkmark}{X'_1}$	ø	0	ø	$\overset{\times}{X'_1}$	ø	$\overset{\times}{X'_2}$	ø	0	ø	0	
	10	0	0	$\overset{\times}{X'}$	0	0	1	0	0	$\overset{\times}{X'}$	0	$\overset{\times}{X'}$	0	0	0	0	0	
	00	ø	0	ø	$\overset{\checkmark}{(1)}$	ø	0	ø	0	ø	0	ø	0	ø	1	ø	0	
	01	0	0	0	0	0	0	0	0	0	0	0	0	$\overset{\times}{(1)}$	0	0	0	
X'_1	11	ø	0	ø	0	ø	$\overset{\times}{(1)}$	ø	0	ø	$\overset{\checkmark}{(1)}$	ø	0	ø	0	ø	0	X'_1
	10	0	0	$\overset{\checkmark}{(1)}$	1	1	0	0	0	0	1	$\overset{\checkmark}{(1)}$	0	0	0	0	0	
	00	ø	0	ø	0	ø	0	ø	ø	ø	0	ø	ø	ø	0	ø	0	
X_1	01	ø	1	0	0	0	0	1	0	ø	0	0	1	0	0	0	0	X_1
	11	ø	ø	ø	0	ø	0	ø	0	ø	0	ø	0	ø	0	ø	1	
	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	ø	0	
	00	ø	0	ø	0	ø	$\overset{\times}{(1)}$	ø	0	ø	0	ø	0	ø	0	ø	1	
X'_2	01	0	0	$\overset{\checkmark}{(1)}$	1	1	0	0	0	0	0	0	0	0	0	$\overset{\times}{(1)}$	0	X'_2
	11	ø	0	ø	$\overset{\checkmark}{(1)}$	ø	0	ø	0	ø	0	ø	$\overset{\checkmark}{(1)}$	ø	0	ø	0	
	10	0	0	0	0	0	0	0	0	$\overset{\checkmark}{(1)}$	0	0	1	0	0	0	0	
	00	ø	ø	ø	0	ø	0	ø	0	ø	ø	ø	0	ø	0	ø	0	
X_2	01	0	0	0	0	0	0	0	0	0	1	ø	0	ø	0	0	0	X_2
	11	ø	0	ø	0	ø	0	ø	ø	ø	0	ø	0	ø	1	ø	0	
	10	ø	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	

Fig. 3.1, (1)

		Y_1				Y_2				Y_3				Y_4				
	y_1y_2	00	01	11	10	00	01	11	10	00	01	11	10	00	01	11	10	
	y_3y_4																	
	00	ø	0	ø	$\overset{\times}{X'_1}$	ø	$\overset{\times}{X'_1}$	ø	0	ø	0	ø	0	ø	0	ø	0	
1	01	0	0	0	1	0	0	$\overset{\times}{X'_1}$	0	0	0	0	0	$\overset{\times}{X'_2}$	0	$\overset{\checkmark}{X'_1}$	1	1
	11	ø	0	ø	$\overset{\checkmark}{X'_2}$	ø	$\overset{\checkmark}{X'_2}$	ø	0	ø	0	ø	$\overset{\times}{X'_2}$	ø	$\overset{\times}{X'_2}$	ø	0	
	10	0	0	$\overset{\times}{X'_1}$	0	0	1	0	0	$\overset{\times}{X'_2}$	1	$\overset{\checkmark}{X'_1}$	0	0	0	0	0	
	00	ø	0	ø	$\overset{\checkmark}{(1)}$	ø	$\overset{\checkmark}{(1)}$	ø	0	ø	0	ø	0	ø	0	ø	0	
X'_1	01	1	0	0	0	0	1	$\overset{\checkmark}{(1)}$	0	0	0	0	0	0	0	$\overset{\times}{(1)}$	0	X'_1
	11	ø	0	ø	0	ø	0	ø	0	ø	0	ø	0	ø	0	ø	0	
	10	0	0	$\overset{\checkmark}{(1)}$	1	1	0	0	0	0	0	$\overset{\times}{(1)}$	0	0	0	0	0	
	00	ø	ø	ø	0	ø	0	ø	ø	ø	0	ø	ø	ø	ø	ø	0	
X_1	01	0	0	0	0	0	0	0	0	0	0	ø	1	0	0	0	0	X_1
	11	ø	0	ø	0	ø	0	ø	0	ø	1	ø	0	ø	0	ø	1	
	10	0	0	0	0	0	0	0	0	0	0	0	0	0	1	ø	0	
	00	ø	0	ø	0	ø	0	ø	0	ø	1	ø	0	ø	0	ø	1	
X'_2	01	0	0	0	0	0	0	0	0	0	0	0	0	$\overset{\checkmark}{(1)}$	1	0	0	X'_2
	11	ø	0	ø	$\overset{\times}{(1)}$	ø	$\overset{\times}{(1)}$	ø	0	ø	0	ø	$\overset{\checkmark}{(1)}$	ø	$\overset{\checkmark}{(1)}$	ø	0	
	10	0	0	0	0	0	0	0	0	$\overset{\checkmark}{(1)}$	0	0	1	0	0	0	0	
	00	ø	0	ø	0	ø	0	ø	0	ø	0	ø	0	ø	0	ø	0	
X_2	01	0	0	1	0	ø	0	0	1	ø	0	0	0	0	0	0	0	X_2
	11	ø	ø	ø	0	ø	0	ø	ø	ø	0	ø	0	ø	0	ø	0	
	10	ø	1	0	0	0	0	1	0	0	0	0	0	ø	0	0	0	

Fig. 3.1, (m)

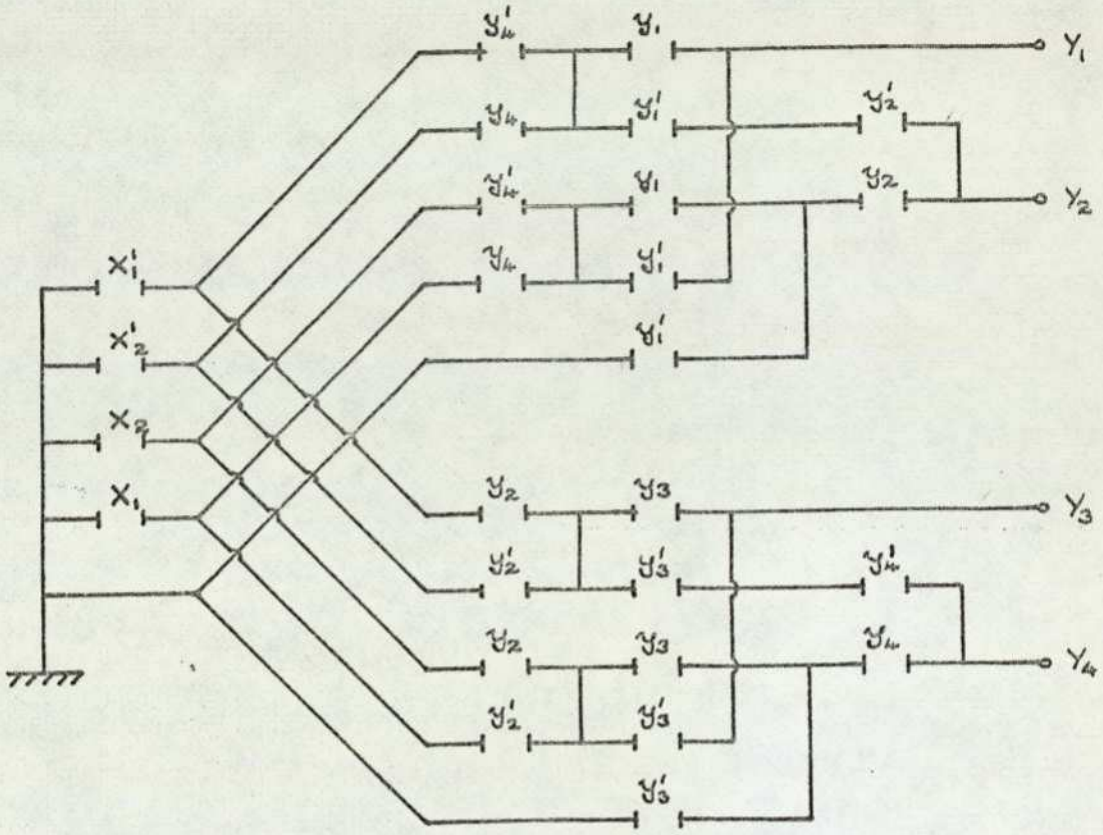


Fig. 3.1, (n)

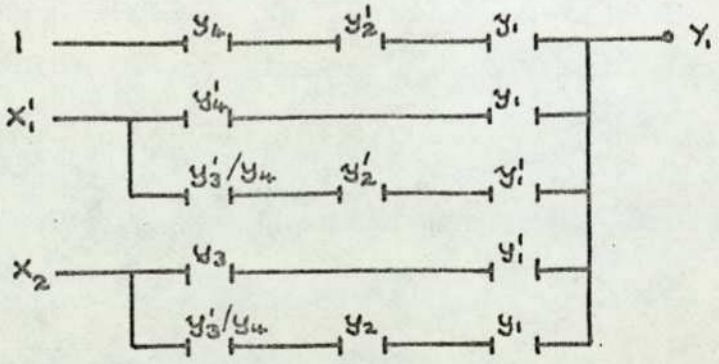


Fig. 3.1, (p)

Here, as before, alternative factors are shown vertically. For this assignment there are no simple networks as obvious as there were for the previous one. The steps by which a fairly simple network for Y_1 was found will be set out. The networks for the other relays are then obtained by permutations of the variables.

A network with a one-to-one correspondence between contacts and literals is shown in fig. 3.1, (p). Alternatives are shown where permissible. The upper and lower y_1 contacts may now be combined since the path in parallel with X_2 which is formed contains complementary contacts on Y_2. If in the lower path y_4 is chosen rather than y_3', the two y_4 contacts may be moved next to y_1 and combined. The remaining y_1 contact may now be combined since the path in parallel with X_1' which is formed contains complementary contacts on y_4. The y_1' contacts may now be combined provided that y_3' is chosen rather than y_4. The resulting network is shown in fig. 3.1, (q). The excitation networks for the other relays are the same in form, but the variables are permuted in accordance with the following scheme.

Y_1	Y_2	Y_3	Y_4
y_1	y_2	y_3	y_4
y_2	y_1	y_4	y_3
y_3	y_4	y_1	y_2
y_4	y_3	y_2	y_1
X_1	X_1	X_2	X_2
X_2	X_2	X_1	X_1

It is now necessary to inquire whether any combination of contacts is possible. If the adjacent y_2' and y_3' contacts in the Y_1 network are interchanged and the corresponding change is made in the Y_2 network, then in both networks there are y_3' and y_4' contacts adjacent to the X_1' contact. This involves the sacrifice of a transfer contact in each case, so that even if some advantage were to be gained this fact might well offset it. To make the discussion easier to follow the two networks modified in the way suggested have been drawn out (fig. 3.1, (r). It is necessary to examine whether when the y_3' or y_4' contact are combined, a path is placed in parallel with X_1' or X_2, or between them, for any state used by the circuit. If the y_3' contacts are combined the path in parallel with X_2 is $y_3y_2'y_3y_1' = y_1'y_2'y_3$ which is closed for a state used by the counter. Similarly if the y_4' contacts are combined the path placed in parallel with X_2 is $y_4y_1'y_4y_2' = y_1'y_2'y_4$ which again is closed for a state used by the counter.

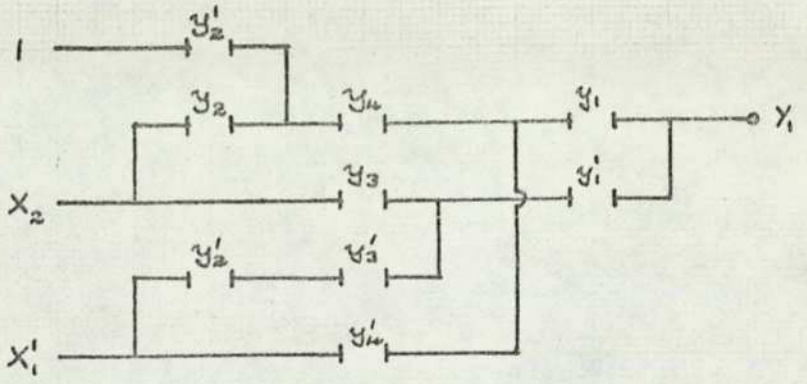


Fig. 3.1, (q)

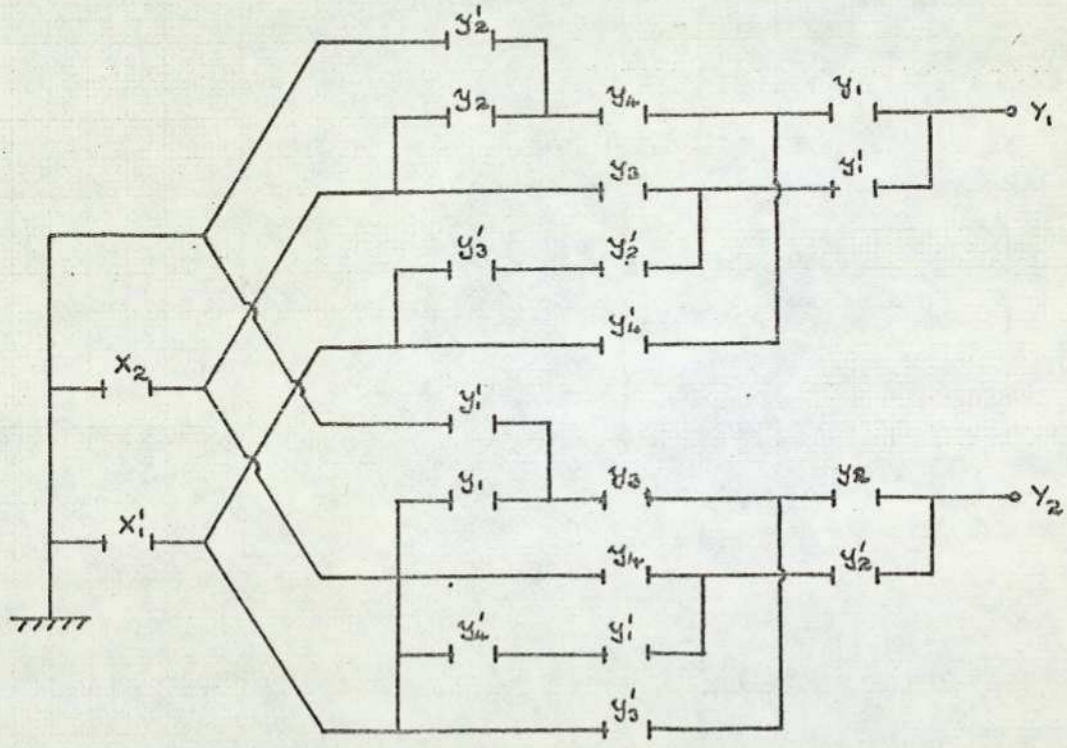


Fig. 3.1, (r)

The networks cannot therefore be combined in this way. It is always possible to design a single multiterminal networks using Scheinmann's technique but this has not been thought worthwhile because the simplest individual networks all have the variables in different orders and it seems improbable that the additional complexity introduced by altering the orders of the variables in three of the networks will be offset by the economy of combining them, at least not to the extent of making the combined network more economical than the combined network for sequence (a)A shown in fig. 3.1, (n).

Output networks have not been drawn. For the counter using sequence (a)A the output networks are very simple, using a total of three transfer contacts on Y_1 and Y_3. A new output appears whenever an input switch is operated, delayed only by the operate or release time of a single relay. In the counter using sequence (c)C, the position is much more involved, since any one of the four relays may change its state on the application of an input pulse. If then a new output is to appear whenever an input switch is operated the output network will have contacts on all four relays.

3.2 Extension of the design method.

The purpose of designing the scale-of-four reversible counter was to throw some light on the more difficult problem of designing a scale- of-ten counter. There are two different ways in which the scale-of-four counter may be generalized to higher scales. Both start from the assignment (a)A; the first is as follows.

The scale-of-four counter with assignment (a)A may be regarded as a pair of bistable units or flip-flops. One of these, formed by relays Y_1 and Y_2, responds to input pulses X_1 and X_2 in a way determined by the state of the other, indicated by y_4. The other, formed by Y_3 and Y_4 responds in a way determined by y_2. For scales greater than 4 the number of flip-flops must exceed 2, and the response of a given flip-flop to the inputs in general depends on the states of more than one of the remaining flip-flops.

The alternative method of generalization may be understood by comparing the scale-of-four counter with assignment (a)A, with the scale-of-ten circuit designed previously. In the latter, the relay which changed on the release of an input was always the same, Y_5; in the former it could be Y_2 or Y_4, and, when counting continuously in one direction was Y_2 or Y_4 alternately. In generalizing to higher scales this feature is retained, namely, a group of two relays, one

of which always changes on the release of an input, but the group consisting of the remaining relays, one of which changes when an input is applied, is enlarged. Of these two methods of generalization, only the former will be presented here. The latter is applicable only to counters which have scales which are multiples of 4.

3.2.1 Scale-of-six counter on six relays.

3.2.1.1 Possible sequences.

The scale-of-six counter will consist of three pairs of relays, each pair forming a flip-flop, the response of any pair to an input pulse being determined by the states of the other two pairs. Numbering the flip-flops 1 - 3 and the steps in the sequence 1 - 6 (the starting point being arbitrary), let flip-flop 1 change at step 1. It may change again at step 3 or 4 or 5. In the latter case let steps 1 and 5 be renumbered 3 and 1 respectively. Let the flip-flop which changes at step 2 be number 2. The sequences of changes so far are

1 2 1 - - - and 1 2 - 1 - -
(a) (b)

In sequence (a) the flip-flop which changes at step 4 must be 3 and this flip-flop must also change at step 6. This sequence can therefore be completed in only one way, namely

1 2 1 3 2 3
(c)

In sequence (b) the flip-flop which changes at step 3 must be number 3. Flip-flops 2 and 3 may then change at step 5 and 6 in either order giving sequences (d) and (e).

1 2 3 1 2 3 1 2 3 1 3 2
(d) (e)

However, if in (e) 1 and 2 are interchanged the result is (c) so that there are only two possibilities, (c) and (d). Of these two sequences (c) remains the same if '1' and '3' are interchanged or if it is reversed; (d) remains the same for any cyclic permutation of the flip-flops and for the interchange of any two and reversal. Since (d) exhibits the greater symmetry (c) has not been investigated further.

When sequence (d) has been chosen there is still a further choice to be made. Of the eight states of three flip-flops only six are used - two are omitted, and since, when counting in a given direction for three successive steps, the three flip-flops each change state once, it follows that a state PQR is omitted, P'Q'R' is also omitted. If then the state 000 is omitted, the state 111 will also be omitted

and the states used will be those in which either one or two flip-flops are set. If any other pair of states is omitted then the sequence will consist of a mixture of states in which none, one, two or three flip-flops are set. Clearly the former choice is more symmetrical, but both will be investigated as it involves little more work, and the less symmetrical arrangement will be seen to have advantages.

If the states 000, 111 are included there is only one possible sequence, and there is only one if they are not included. These sequences are given in fig. 3.2, (a).

$Y_1Y_2Y_3Y_4Y_5Y_6$	$Y_1Y_2Y_3Y_4Y_5Y_6$
0 1 0 1 0 1	0 1 1 0 0 1
1 0 0 1 0 1	1 0 1 0 0 1
1 0 1 0 0 1	1 0 0 1 0 1
1 0 1 0 1 0	1 0 0 1 1 0
0 1 1 0 1 0	0 1 0 1 1 0
0 1 0 1 1 0	0 1 1 0 1 0
(0 1 0 1 0 1)	(0 1 1 0 0 1)
(a)	(b)

Fig. 3.2, (a)

The states listed are those stable in the absence of an input. In any transition Y_1 Y_3 or Y_5 changes on the application of an input and Y_2, Y_4 or Y_6 on its removal. The next step is to derive the excitation networks. It is only necessary to derive the excitation networks for one flip-flop for sequence (a). The remaining excitation networks for both sequence (a) and sequence (b) may be obtained by permutation and complementation of the variables.

3.2.1.2 Excitation matrix, functions, and networks for one pair of relays.

The excitation networks for Y_1 and Y_2 in sequence (a) will be derived. X_1, X_2, y_4 and y_6 are regarded as input variables; y_1 and y_2 are the secondary variables. The flow table, with the secondary assignment, is shown in fig. 3.2, (b) and the excitation map in fig. 3.2, (c).

$X_1X_2y_4y_6$ / y_1y_2	0000	0100	1000	0001	0101	1001	0011	0111	1011	0010	0110	1010
00	1		⑤				13	⑰				
01	①	3	②				⑬	⑭	15	⑦	⑧	⑨
11	4	③					16		⑮			
10	④	⑥	5	⑩	⑪	⑫	⑯	17	⑱			

Fig. 3.2, (b)

$X_1X_2y_4y_6$ / y_1y_2	0000	0100	1000	0001	0101	1001	0011	0111	1011	0010	0110	1010
00	01		00				01	00				
01	01	11	01				01	01	11	01	01	01
11	10	11					10		11			
10	10	10	00	10	10	10	10	00	10			

Fig. 3.2, (c)

This map is now rearranged as in the scale-of-four case. The only difference is that it has already been recognized that the functions need not involve y_3 or y_5, while on the other hand they are now depending on variables from two flip-flops rather than only one. (Fig. 3.2, (d))

		Y_1				Y_2			
	y_4y_6 / y_1y_2	00	01	11	10	00	01	11	10
	00	0	ø	0	ø	0	ø	0	ø
1	01	0	ø	0	0	1	ø	1	1
	11	$\overset{\times}{X}'_1$	ø	$\overset{\times}{X}'_2$	ø	0	ø	0	ø
	10	0	1	0	ø	0	0	0	ø
	00	0	ø	0	ø	1	ø	0	ø
X'_1	01	0	ø	0	0	0	ø	0	0
	11	(1)	ø	0	ø	0	ø	0	ø
	10	1	0	0	ø	0	0	0	ø
	00	0	ø	ø	ø	0	ø	0	ø
X_1	01	0	ø	1	0	0	ø	0	0
	11	0	ø	0	ø	0	ø	1	ø
	10	0	0	0	ø	0	0	0	ø
	00	0	ø	0	ø	0	ø	1	ø
X'_2	01	0	ø	0	0	0	ø	0	0
	11	0	ø	(1)	ø	0	ø	0	ø
	10	0	0	1	ø	0	0	0	ø
	00	ø	ø	0	ø	0	ø	0	ø
X_2	01	1	ø	0	0	0	ø	0	0
	11	0	ø	0	ø	1	ø	0	ø
	10	0	0	0	ø	0	0	0	ø

Fig. 3.2, (d)

The resulting excitation functions, with optional and alternative factors are:-

$$Y_1 = (y_1y_2')y_4'y_6 + X_1'y_1(y_4')y_6' + X_1y_1'(y_2y_4)y_6 + X_2'y_1y_4(y_6) + X_2y_1'(y_2)y_4'(y_6')$$

$$Y_2 = y_1'y_2 + X_1'y_1'y_2'\frac{y_4'}{y_6'} + X_1y_1y_2\frac{y_4}{y_6} + X_2'y_1'y_2'\frac{y_4}{y_6} + X_2y_1y_2\frac{y_4'}{y_6'}$$

A network, believed to be as simple as possible is shown at fig. 3.2, (e). Three points may be noted.

(a) The paths between X_1' and X_2' on the one hand, and X_1 and X_2 on the other of necessity contain complementary contacts. Between X_1' and X_2' it is necessary to insert a y_4' or y_6 contact, making use of optional factors, and similarly between X_1 and X_2 a y_4 or y_6' contact.

(b) If, for the foregoing reason, a y_4' contact is inserted in series with X_1', the first term in the expression for Y_1 may be provided by adding a y_6 contact only, making use of the optional y_1 factor. This introduces a $y_1'y_2'y_4'y_6$ path to ground for Y_2, which is permissible, this term being optional, the state 0001 never occurring. (It should be noticed that in fig. 3.2, (d) y_1y_2 are plotted vertically and y_4y_6 horizontally).

(c) The order of the contacts on Y_1 and Y_2, in the excitation network for Y_2, may be reversed as shown in fig. 3.2, (f). This exchanges a y_1' contact for a y_2 contact. If no further development of the networks for the whole counter is intended, this will almost certainly be undesirable, since contacts on Y_2 will occur in the excitation networks for the other flip-flops, while contacts on Y_1 do not. On the other hand, if this is done, it introduces the possibility of designing a single multi-output contact network on Y_2, Y_4 and Y_6 to replace the separate contact networks.

The excitation networks for the other relays for sequence (a) are identical in form but require permutation of the variables. To work out the permutations required parts of the sequence of states shown in fig. 3.2, (a) has been rearranged, in fig. 3.2, (g).

$Y_1Y_2Y_3Y_4Y_5Y_6$	$Y_3Y_4Y_5Y_6Y_1Y_2$	$Y_5Y_6Y_1Y_2Y_3Y_4$
01 10 10	01 10 01	01 01 01
01 01 10	01 01 01	01 10 01
01 01 01	01 01 10	01 10 10

Fig. 3.2, (g)

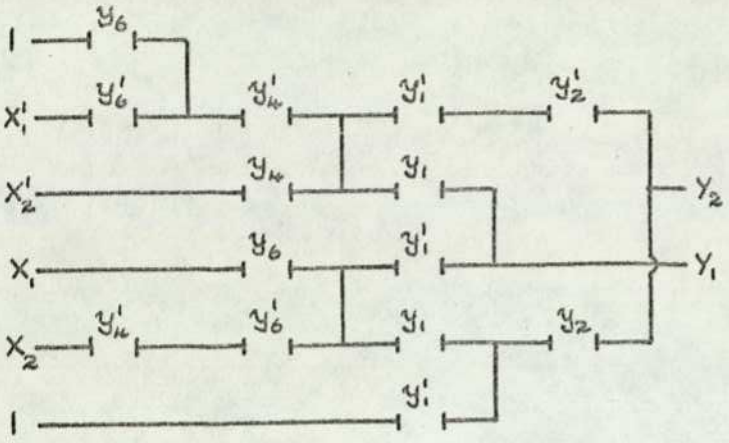


Fig. 3.2, (e)

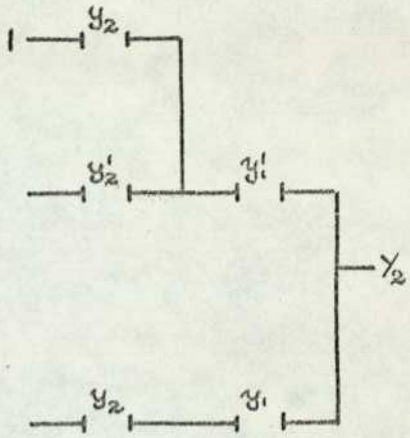


Fig. 3.2, (f)

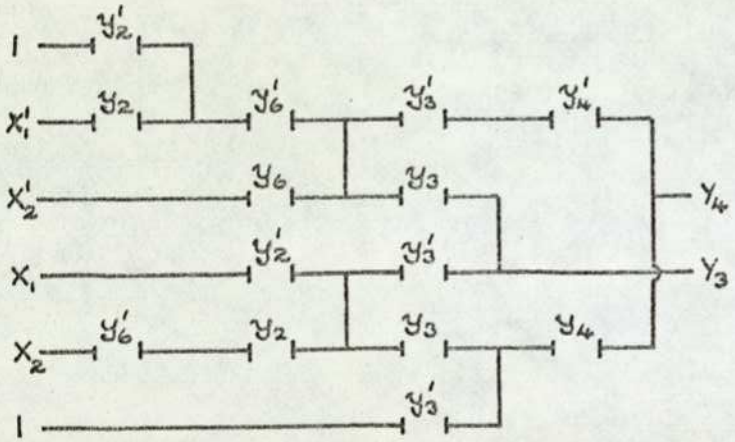


Fig. 3.2, (h)

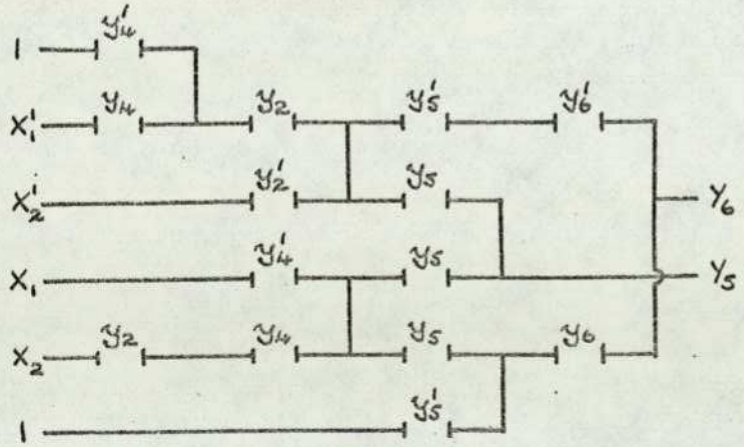


Fig. 3.2, (j)

The required permutations are then seen to be

Y_1Y_2		Y_3Y_4		Y_5Y_6
y_4	→	y_6	→	y'_2
y_6		y'_2		y'_4

Although the configuration is exactly the same, these networks have been drawn out as it will be convenient to be able to refer to them in the next section. The X'_1, X'_2, X_1 and X_2 contacts are common to the three networks. (Figs. 3.2, (h,j)).

The excitation networks for sequence (b) may similarly be derived by rewriting the relevant parts of the sequences as follows:

$Y_1Y_2Y_3Y_4Y_5Y_6$ (a)	$Y_1Y_2Y_3Y_4Y_5Y_6$ (b)	$Y_3Y_4Y_5Y_6Y_1Y_2$ (b)	$Y_5Y_6Y_1Y_2Y_3Y_4$ (b)
0 1 1 0 1 0	0 1 0 1 1 0	0 1 0 1 1 0	0 1 0 1 1 0
0 1 0 1 1 0	0 1 1 0 1 0	0 1 1 0 1 0	0 1 1 0 1 0
0 1 0 1 0 1	0 1 1 0 0 1	0 1 1 0 0 1	0 1 1 0 0 1

The permutations required are then seen to be

Y_1Y_2 (a)		Y_1Y_2 (b)		Y_3Y_4 (b)		Y_5Y_6 (b)
y_4 (a)	→	y'_4 (b)	→	y'_6 (b)	→	y'_2 (b)
y_6 (a)		y_6 (b)		y_2 (b)		y_4 (b)

These excitation networks have not been drawn out.

3.2.1.3 Behaviour when first switched on.

It will be assumed that when the equipment is first switched on, neither X_1 nor X_2 is operated. In that case the excitation networks for Y_1 and Y_2 (for either sequence) reduce to the circuit of fig. 3.2, (k). The excitation networks for the remaining flip-flops are similar. Thus on switching on, Y_2, Y_4 and Y_6 operate and Y_1 Y_3 and Y_5 remain unoperated. The state 010101 implied is part of sequence (a), fig. 3.2, (a) which is satisfactory, but not of sequence (b). It is also clear that, in the absence of an input the two relays of a flip-flop cannot both remain operated or both released.

3.2.1.4 Carry and Borrow networks.

Since the counter enters the state 010101 on switching on and is easily reset to this state, this will be regarded as the '0' state. State '5' is then the state preceding this when counting forward, namely 010110. These and the other states are conveniently shown on a Karnaugh map (fig. 3.2, (ℓ)).

y_2y_4	00	01	11	10
y_6 0	3	-	5	4
1	2	1	0	-

Fig. 3.2, (ℓ)

'Carry', which will be designated C is required to appear with '5' and X_1. It is therefore given by $C = X_1y_4y_6'$. Its complement C' is also required; $C' = X_1' + y_4' + y_6$. 'Borrow', designated by B, must appear with '0' and X_2. It is therefore given by $B = X_2y_2y_6$, and its complement $B' = X_2' + y_2' + y_6'$.

Without the carry and borrow contacts the counter is symmetrical. It remains unchanged if y_1 and y_5, y_2 and y_6, X_1 and X_2 are interchanged. The carry and borrow contacts are not symmetrical since they operate for transitions between 010101 and 010110 which are not symmetrical in the above sense. Hence the carry and borrow networks must be designed separately.

The contact networks already contain on the even-numbered relays 4 transfers and either a make or break contact. It was made an objective to add the carry and borrow contacts in such a way that the counter could be built on relays with no more than six transfer contacts. This was achieved in the following way.

(a) $C' = X_1' + y_4' + y_6$ may be obtained by adding a transfer contact on Y_4 to the Y_1Y_2 network as shown in fig. 3.2, (m). The presence of the y_4 contact ensures the absence of sneak paths.

(b) $X_2' + y_2' + y_6'$ may be obtained in a similar way but first it is necessary to modify the contact network for Y_3Y_4. It will be recalled that it was necessary to ensure that there was never a path between X_1' and X_2' and that there were two alternative ways of doing this. It is necessary to choose the alternative way to that chosen earlier and then add a transfer contact on Y_2. This has been done in fig. 3.2, (n). Again, the presence of a y_2 contact ensures the absence of sneak paths.

(c) Provision of the outputs $X_1y_4y_6'$ and $X_2y_2y_6$ requires considerable revision of the networks already designed, because of the limit it has been decided to impose on the number of contacts per relay. It is necessary to consider afresh that part of the network between the contacts X_1 and X_2 on the one hand and the points B on the other (see fig. 3.2, (e, h, j), and to make use of the optional factors in the X_1 and X_2 terms of the excitation functions. The transmissions required for the three flip-flops to the points B are as follows:-

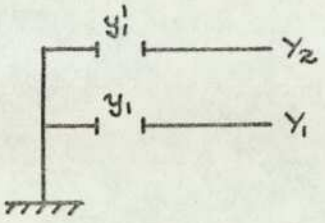


Fig. 3.2,(k)

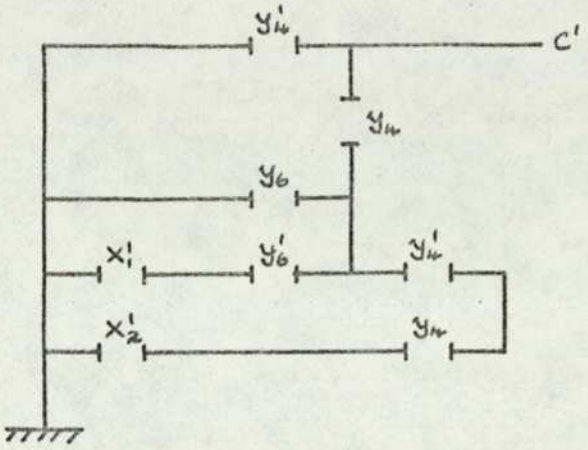


Fig. 3.2, (m)

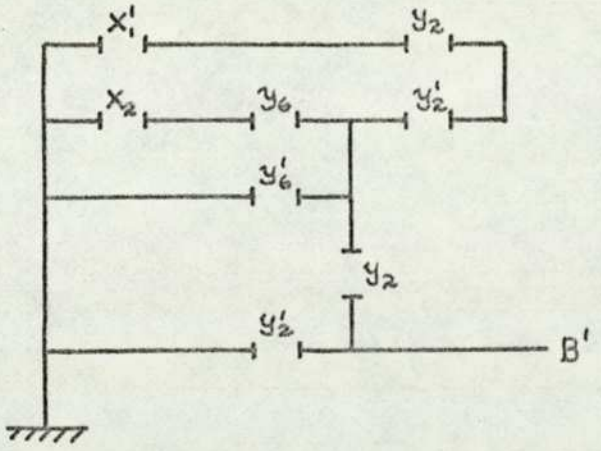


Fig. 3.2, (n)

	Y_1Y_2	Y_3Y_4	Y_5Y_6	C
X_1	$(y_2y_4)y_6$	$(y_4y_6)y_2'$	$(y_6y_2')y_4'$	$(y_2)y_4y_6'$
				B
X_2	$(y_2y_6')y_4'$	$(y_4y_2)y_6'$	$(y_6y_4)y_2$	$y_2y_6(y_4)$

As before optional factors are included in brackets. The lack of symmetry between the two sets of functions (for X_1 and X_2) is evident, for example, the "borrow" function and the Y_5Y_6 function on X_2 become the same if all optional factors are used. The "carry" function cannot be made the same as any of the functions for X_1 however the optional terms are used. The networks have to be designed in such a way that the transmission between X_1 and X_2 is always zero. They will be designed without reference to this requirement in the first place, and subsequently examined to see whether it has been met.

Considering first the X_1 network, it may be noted that the factor y_4' occurs only once, in the Y_5Y_6 path and that there it is the only essential term. The tree may therefore begin with a transfer contact on Y_4, and nothing further need be connected to the y_4' contact. In the remaining terms y_6' occurs only in the "carry" so that the network may be continued with a transfer contact on Y_6 connected to Y_4. Nothing further need be connected to y_6' and a transfer contact on Y_2 connected to y_6 completes the network, which is shown in fig. 3.2, (p).

The network from X_2 is built up in a similar but not identical manner. All the terms contain the factor y_2 so the network may begin with a y_2 contact. The rest of the network is a tree on Y_4 and Y_6, the $y_4'y_6$ term not being required. It may begin with a transfer contact on Y_4 or Y_6 after which two contacts (counting a transfer contact as one), are required on the other relay. In those parts of the carry and borrow networks designed so far, two transfer contacts on Y_2, two on Y_4 and one on Y_6 have been used. Together with those on the flip-flops this makes the numbers of contacts on Y_2, Y_4 and Y_6 5, 5 and 4 respectively. The two contacts should therefore be on Y_6, and the resulting network is shown in fig. 3.2, (q).

Next, it is necessary to examine whether, when the two networks are combined, a path ever exists between X_1 and X_2. It is found that this is not so, so that in this respect the networks are satisfactory. It is also necessary to ask whether there is a path to C other than via X_1 and a path to B, other than via X_2. There is no other path to C but there is a path to B via X_1 and y_4'. The transmission required

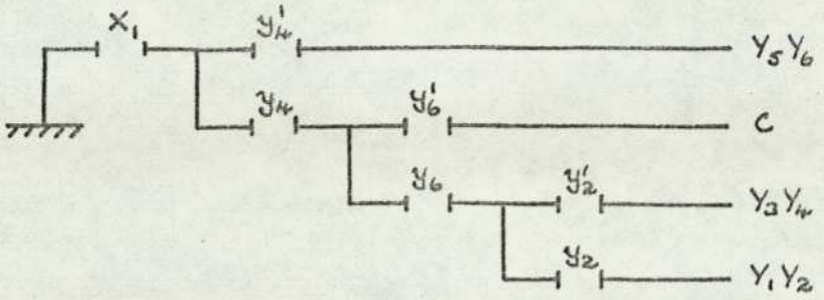


Fig. 3.2, (p)

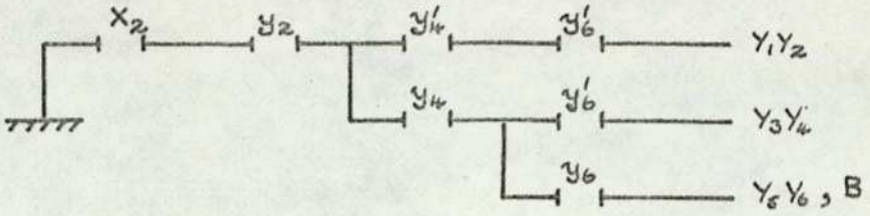


Fig. 3.2, (q)

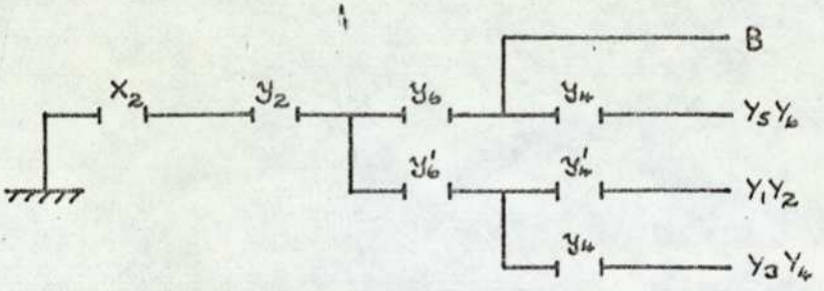


Fig. 3.2, (r)

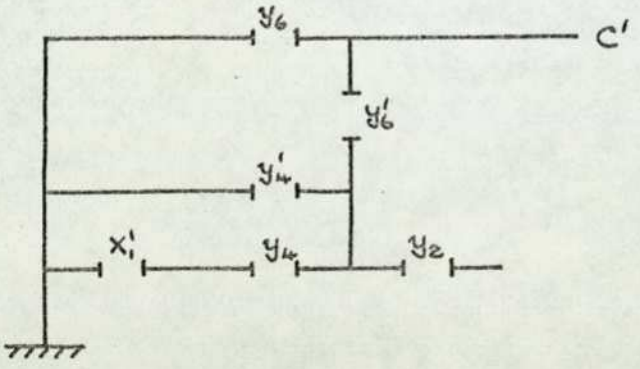


Fig. 3.2, (s)

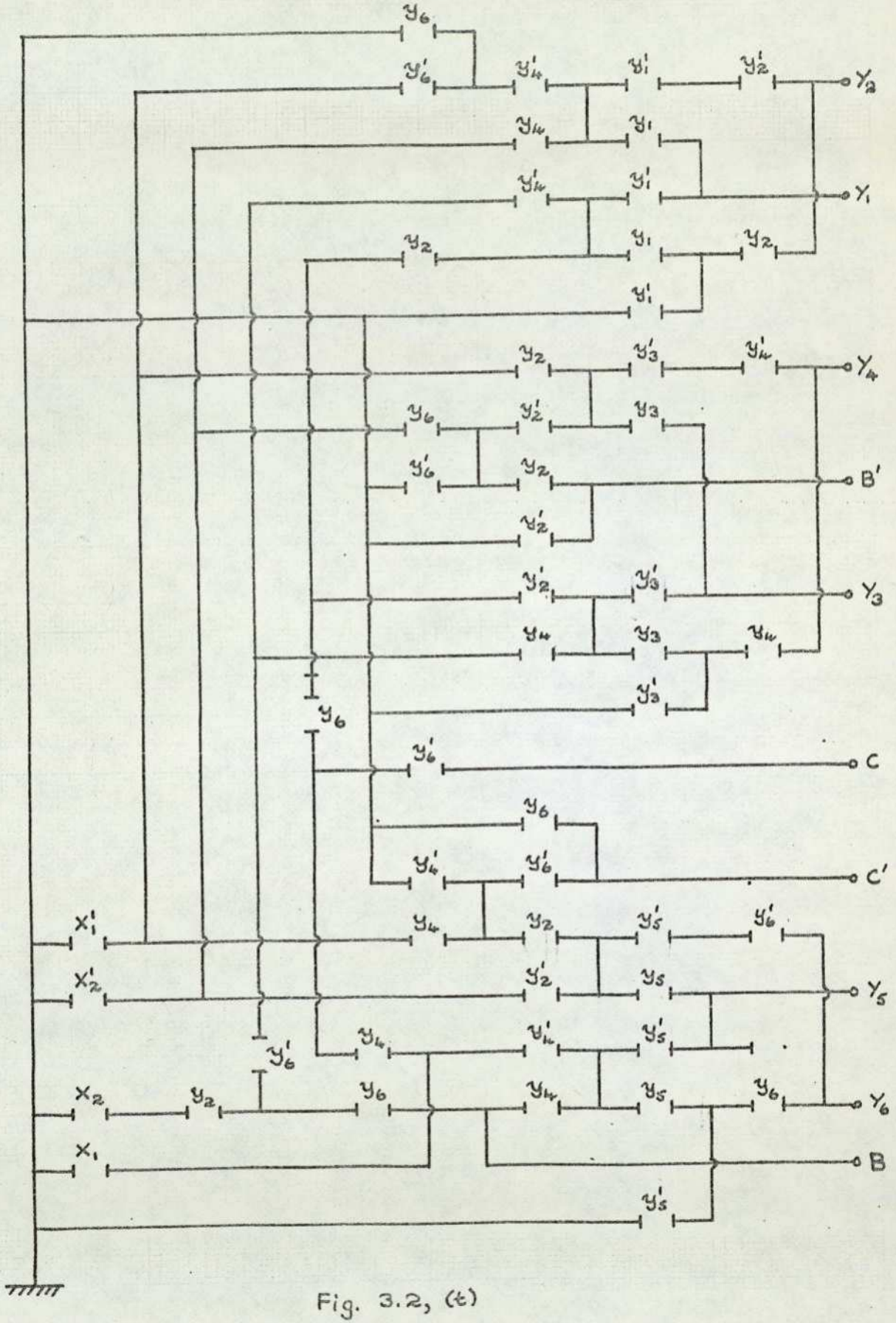

y_6
y'_6
y'_4
y'_1
y'_2
Y_2
y_4
y_1
y'_4
y'_1
Y_1
y_2
y_1
y_2
y'_1
y_2
y'_3
y'_4
Y_4
y_6
y'_2
y_3
y'_6
y_2
B'
y'_2
y'_2
y'_3
Y_3
y_4
y_3
y_4
y'_3
y_6
y'_6
C
y_6
y'_4
y'_6
C'
X'_1
y_4
y_2
y'_5
y'_6
X'_2
y'_2
y_5
Y_5
y'_6
y_4
y'_4
y'_5
X_2
y_2
y_6
y_4
y_5
y_6
Y_6
X_1
B
y'_5

Fig. 3.2, (t)

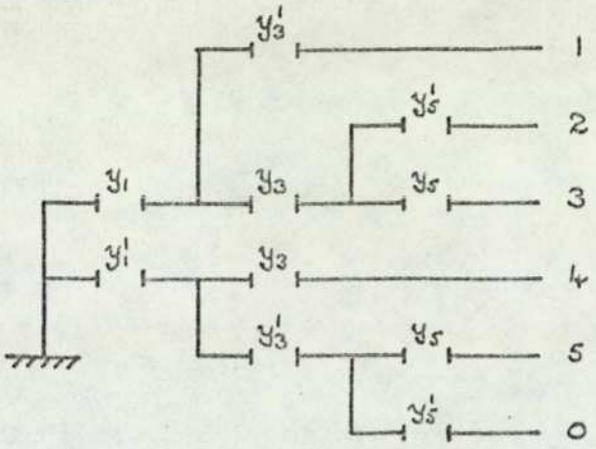


Fig. 3.2, (a)

for B is y_2y_6, with y_4 as an optional factor. If the tree following y_2 is developed in the alternative order and B taken as shown below, the network would be satisfactory. This leads to the network of fig. 3.2, (r).

With this arrangement however the number of contacts on Y_4 is now 7 while the number on Y_6 is only 5. In order to redress the balance the $X_1' + y_4' + y_6$ term for "carry" may be obtained by adding a transfer contact on Y_6 to the Y_5Y_6 network, rather than a transfer contact on Y_4 to the Y_1Y_2 network. The relevant part of the Y_5Y_6 network is as shown in fig. 3.2, (s). This completes the secondary excitation network, but the problem of races when two hexades are cascaded has still to be considered.

If two "hexades" are considered as a single counter the requirement that only one flip-flop should change at a time has been relaxed, and two relays change state when the input is applied and two more when it is removed. There are races and it is necessary to determine whether they are critical. On the application of an input pulse only the odd-numbered relays change state and since these have no contacts in the other flip-flops they cannot affect the excitation of other relays. The first race is therefore non-critical. The relays of the second hexade have no contacts in the networks of the first and hence if the flip-flop in the second hexade "wins the second race" the first hexade is unaffected. If the flip-flop in the first hexade "wins the second race" the only effect could be to remove the carry or borrow from the second hexade, but this has already been done so no further effect is possible.

If the above conclusion is correct it follows that the requirement that one flip-flop should change state at each step may be relaxed. To what extent this may be done is a question we have not considered.

This completes the design of the scale-of-six counter except for the output network, the design of which is straightforward. It consists of a 3-variable tree, with two of the outputs not required and combined with adjacent ones. The complete circuit is shown in figs. 3.2, (t) and (u).

3.2.1.5 Longer twisted-rings.

Similar counters with even scales may be designed by adding extra flip-flops, the scale being twice the number of flip-flops. The next state of any flip-flop may be determined if its present state and

the states of only the two adjacent ones are known. If there are n flip-flops in the chain the excitation networks of the r th flip-flop may be obtained from fig. 3.2, (h) by writing 2r-2, 2r-1, 2r and 2r+2 for 2, 3, 4 and 6 respectively. For the two relays of the first flip-flop the excitation networks are obtained by writing 2n for 6 in the circuit of fig. 3.2, (e). For the two relays of the last flip-flop the excitation networks are obtained by writing 2n-2, 2n-1, 2n, for 4, 5, and 6 in the circuit of fig. 3.2, (j).

4 SCALE -OF-TEN COUNTER ON SIX RELAYS

4.1 Possible Sequences - necessity for six relays.

In discussing the scale-of-four counter with only one transfer contact on each input it was shown that four relays were sufficient and necessary. It will also be evident that onfour relays no longer sequence is possible (with the given restriction on input contacts) since at no point in the sequence (of the scale-of-four counter) is any choice possible beyond the order in which the next pair of relays change, and the counter returns inevitably to its original state after four steps, notwithstanding the fact that on four relays 16 states are available and in the scale-of-four counter only 12 are used. For a scale-of-ten counter 30 states are required, and though on 5 relays 32 states are available it does not follow that a scale-of-ten counter may be constructed on 5 relays. In fact it cannot, for the following reason.

Suppose that in one of the states stable in the absence of an input an even number of relays is operated. Then since two relays change state in passing to the next similar state, in that state also an even number of relays will be operated, so that in all states stable in the absence of an input an even number of relays will be operated. In the intermediate states an odd number of relays will be operated. If in one of the states stable in the absence of an input an odd number of relays is operated the converse holds. Now for a counter of scale n, 2n intermediate states are required, and in half the states available on S relays, an even number of relays is operated, and in the remaining half an odd number is operated. Hence on S relays the maximum scale is 2^{S-2}. For a scale-of-ten counter the number of relays must be at least six. It should benoted that it has not been proved that six are sufficient, only that fewer are insufficient.

As it seems probable that the number of sequences on six relays will be large attention will be confined to those sequences which consist of two identical subsequences. With or without this restriction it may be noted that -

No relay may change six times in counting 10 input pulses of the same sign. If it did it would change on successive pulses at least once, which is not permissible.

Every relay must change at least twice for otherwise it would be redundant, and it has been shown that 5 relays are not sufficient.

Since twenty changes altogether are required, four relays must change four times and two twice.

With the restriction proposed it follows that in each subsequence of 10 changes (5 pairs) four relays must change twice and two once. The possible sequences in which the relays change state may now be determined.

It is convenient to draw up a table with five columns, one for each step, and six rows, one for each relay. Each entry in the table indicates that the relay corresponding to that row changes at the step corresponding to that column. In each of the first four rows are two entries which may not be adjacent. In each of the last two rows is one entry. In each column there are two entries.

It will be shown first that the single entries in rows 5 and 6 must not be in the same column. If they were, the remaining four columns would contain two entries for each of relays 1 - 4, and after four steps the counter would return to its starting point.

Next, the single entries in rows 5 and 6 must not be in adjacent columns. If they are, part of the transition diagram will be as shown in fig. 4.1, (a).

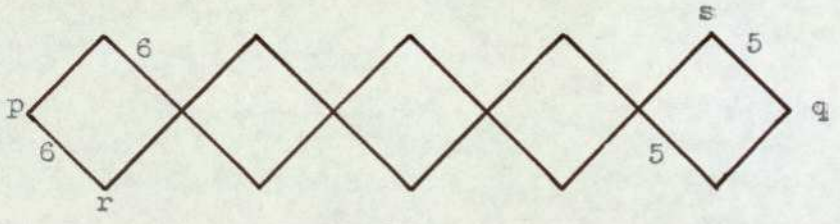


Fig. 4.1, (a)

The steps at which relays 5 and 6 change are adjacent to p and q. In passing from p to q relays 1 - 4 each change twice and relays 5 and 6 once. In passing from r to s (it is not implied that this can necessarily happen when the counter is operated), relays 1 - 4 each change twice, so that these two states would be the same, which is not permissible.

It follows that 5 and 6 must change on different and non-adjacent steps.

Next it will be shown that the relay which changes at the same step as 5 cannot be the same as that which changes at the same step as 6, that is, the pattern of entries shown in fig. 4.1, (b) is not permissible.

1	x	-	x	-	-
2	·	·	·	·	·
3	·	·	·	·	·
4	·	·	·	·	·
5	x	-	-	-	-
6	-	-	x	-	-

Fig. 4.1, (b)

The remaining six entries have to be inserted in the three remaining columns. Now two of the three rows may not be identical, for, if they were, the two entries in the remaining row would have to be inserted in the remaining column. The three remaining rows must therefore all be different. But there are only three ways of inserting two entries into three columns, and in one of these two entries a re adjacent. The arrangement shown is therefore not permissible.

Now there must be some row with an entry in the same column as the single entry in row 5. The second entry in this row may not be adjacent to the first, nor in the same column as the entry in row 6. There is therefore only one possible position. The same applies to the row having an entry in the same column as that in row 6. This leads to the table shown in fig. 4.1, (c).

1	x	-	-	x	-
2	-	-	x	-	x
3	·	·	·	·	·
4	·	·	·	·	·
5	x	-	-	-	-
6	-	-	x	-	-

Fig. 4.1, (c)

1	x	-	-	x	-
2	-	-	x	-	x
3	-	x	-	x	-
4	-	x	-	-	x
5	x	-	-	-	-
6	-	-	x	-	-

Fig. 4.1, (d)

Both the remaining rows must now have an entry in column 2, and the remaining two entries go into columns 4 and 5, the completed table being shown in fig. 4.1, (d). It is now necessary to check that no closed sequences exist. This is easily done. Considering the whole sequence of ten steps, no sequence containing 5 or 6 only once can be closed. If there is a closed sequence containing 5 twice there is also a complementary sequence not containing 5 at all which will be closed: similarly for 6. It is therefore only necessary to examine those sequences not including 5 or 6. These are 1,34,2 and 2,13,24,1 neither of which is closed.

Finally, the table of fig. 4.1, (d) has been rearranged, by making column 2 the first column and rearranging the rows, (fig. 4.1, (e)).

This is symmetrical about AA', the sequences remaining unchanged if Y_1, Y_2, Y_3 are interchanged with Y_4, Y_5, Y_6 and the directions of counting interchanged.

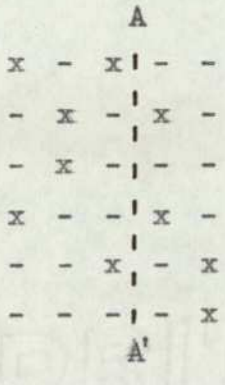


Fig. 4.1, (e)

The secondary assignment is not complete until a specific secondary state has been assigned to one point in the sequence. This determines the rest of the states stable in the absence of an input. It still remains to decide, for each step of the count, which relay shall change first. This second question will be deferred. There are 64 states which may be assigned to any s tate in the sequence, but since the sequence of relay changes is repeated, not more than 32 distinct assignments are possible. The total number is less than 32 since some sequences may be obtained from others by interchanging the directions of counting and permuting the variables. However, since this process performed on some of the sequences may produce the same sequence, the number of sequences is greater than 16, and too great for exhaustive investigation. No way of selecting one of these in the expectation that it will produce the simplest contact networks is evident, so a choice has been made using a different criterion. It will be noted that each of the relays 1 - 4 is operated at 4 steps in the count and released at the remaining 6, or vice versa. We choose the former alternative for all of them. This will reduce the average power consumption. Relays 5 and 6 are each operated at five steps and released at five steps so their choice should be that which will reduce the peak power consumption. There are two pairs of successive states in which only one of the relays 1 - 4 is operated and if it is arranged that both Y_5 and Y_6 are operated in one of these pairs a particularly uniform power requirement is obtained. The sequence of states obtained is shown in fig. 4.1, (f). The number of relays operated is 3 at 8 of the steps and 1 at the remaining 2. It is convenient to allow "carry" and "borrow" to occur in passing between these two steps since the power requirement of other decades will sometimes increase in passing between them.

1 0 0 0 0 0	0
1 1 1 0 0 0	1
0 1 1 0 1 0	2
0 0 1 1 1 0	3
0 0 1 1 0 1	4
1 0 1 0 0 1	5
1 1 0 0 0 1	6
0 1 0 0 1 1	7
0 0 0 1 1 1	8
0 0 0 1 0 0	9

Fig. 4.1, (f)

4.2 Excitation matrix, functions and networks.

The ten states have been plotted on a 6-variable Karnaugh map in fig. 4.2, (a). The states used in passing between them have been indicated by dots: there are twenty of these. The remaining 34 states which are never used, have been indicated by 'Ø'.

$y_1y_2y_3$ / $y_4y_5y_6$	000	001	011	010	110	111	101	100
000	·	Ø	·	Ø	·	①	·	⓪
001	Ø	·	Ø	·	⑥	·	⑤	·
011	·	Ø	Ø	⑦	·	Ø	Ø	Ø
010	Ø	·	②	Ø	Ø	·	Ø	Ø
110	·	③	·	Ø	Ø	Ø	Ø	Ø
111	⑧	·	Ø	·	Ø	Ø	Ø	Ø
101	·	④	Ø	Ø	Ø	Ø	·	Ø
100	⑨	·	Ø	Ø	Ø	Ø	Ø	·

Fig. 4.2, (a)

4.2.1 Conflicting choices of order at each step of the count.

The order in which the two variables of any pair change may be determined so as to simplify the contact networks. However, there are six excitation functions to be determined and the order which makes one function simple may make another complex. The total number of alternatives is very large but as choices are made for successive functions, the number of alternatives for the remaining functions is progressively reduced. As a first step, the alternatives for Y_1 will be set out. The alternatives for Y_4 are similar (after permutation of

the variables) but as there are two steps at which both Y_1 and Y_4 change, and they cannot both change first (or second) it would appear unlikely that both functions may be made as simple as possible simultaneously.

For each of the relays Y_1 to Y_6 we have to determine the excitation function as the sum of five terms in 1, X_1', X_2', X_1 and X_2 respectively. To do this by writing out the excitation matrix in full for each possible choice of order for each number pair would be excessively time-consuming, and the labour may be reduced by considering the excitation of one relay at a time, for each state stable in the absence of an input and the two such adjacent states. In these three states stable in the absence of an input, any one variable may have any one of 8 possible sequences of values. Of these 010 and 101 have already been purposely excluded. Of the remaining six 000 and 111 imply that no changes take place. This leaves four cases in which a relay changes state, and it may do so on the appearance of the input or its removal. This gives ten distinct cases. For each of these ten cases extracts from the excitation matrix have been prepared, corresponding to the extract from the flow table (fig. 2.1, (a), shown in fig. 4.2, (b). (The order of the rows has been varied).

X_1X_2	00	01	11	10
	(n)			
	n	(n+1)(mod 10)+10		
	(n+1)(mod 10)	(n+1)(mod 10)+10		(n+1)(mod 10)+20
	(n+2)(mod 10)			(n+1)(mod 10)+20
	(n+2)(mod 10)			

Fig. 4.2, (b)

Each row of the flow table corresponds to a particular state of the secondary relays Y_1 - Y_6, and the extracts from the excitation matrix shown in fig. 4.2, (c) indicate whether the path through the contacts on Y_1 - Y_6 should go to the X_1', X_1, X_2', X_2 contact or direct to ground ('1').

We now make use of these extracts from the excitation matrix to fill in a map showing the various alternatives for the excitation function Y_1, (fig. 4.2, (d)).

Consider first the entry for the state '0' - 100000. This is the middle state of the sequence (9)-(0)-(1) in which the variable y_1 takes successively the values 011. We have therefore to refer to the

X_1X_2	00	01	11	10	Path to
	0				
	0	0			0 or X_1
	0	0		0	0
	0			0	0 or X_2
	0				

(i)

	00	01	11	10	Path to
	1				
	1	1			1 or X_1'
	1	1		1	1
	1			1	1 or X_2'
	1				

(ii)

X_1X_2	00	01	11	10	Path to
	0				
	0	0			0 or X_1
	0	0		1	X_1
	1			1	1 or X_2'
	1				

This variable changes first

(iii)

	00	01	11	10	Path to
	0				
	0	0			0 or X_1
	0	0		0	0
	1			0	X_1'
	1				

second

(iv)

X_1X_2	00	01	11	10	Path to
	0				
	0	0			0 or X_1
	1	0		1	X_2'
	1			1	1 or X_2'
	1				

This variable changes first

(v)

	00	01	11	10	Path to
	0				
	0	1			X_2
	1	1		1	1
	1			1	1 or X_2'
	1				

second

(vi)

X_1X_2	00	01	11	10	Path to
	1				
	1	1			X_1' or 1
	0	1		0	X_2
	0			0	X_2 or 0
	0				

This variable changes first

(vii)

	00	01	11	10	Path to
	1				
	1	0			X_2'
	0	0		0	0
	0			0	0 or X_2
	0				

second

(viii)

Fig. 4.2, (c), (i - viii)

X_1X_2	00	01	11	10	Path to	00	01	11	10	Path to
	1					1				
	1	1			1 or X_1'	1	1			1 or X_1'
	1	1		0	X_1'	1	1		1	1
	0			0	0 or X_2	0			1	X_1
	0					0				
This variable changes				first					second	
				(ix)					(x)	

Fig. 4.2, (c), (ix,x)

extracts from the excitation matrix (v) and (vi) which indicate that the entry must be X_2' or 1 depending on whether, in moving between states ⑨ and ⓪, y_1 changes first and y_4 second, or vice versa. These alternatives have been separated by a vertical line, to indicate that they are contingent onthe choice of the sequence in which the relays change, and the choice which corresponds to the relay with the smaller suffix changing first has been placed on the left. If, in changing from ⓪ to ⑨ Y_1 changes state first, the counter goes first to the state 000000, and fig. 4.2, (c)(v) indicates that the entry may be 0 or X_1. These have been separated by a horizontal line, to indicate that the choice is free, that is, not contingent on a decision about order. If, instead, Y_1 changes second, the counter goes first to state 100100 and fig. 4.a, (c)(vi) indicates that the entry must be X_2. If Y_1 changes second, the state 000000 is passed through in going from ⑨ to ⓪ and to determine the entry it is necessary to consider the three states ⑧ ⑨ ⓪ in which Y_1 has the values 001 respectively, and refer to fig. 4.2, (c)(iv) which indicates that the entry must then by X_1'. This is separated from the other two alternatives by a vertical line, because the choice of alternatives is dependent on a decision about order, and it is placed on the right because it occurs when the lowest numbered relay changes second. In the same way the alternative entry at 100100 is X_2' or 1, the choice being free, and it is placed on the left. The entry at 000100 is X_1 or 0 depending on the order in which the relays change in passing between ⑨ and ⓪. The choices between the alternatives separated by vertical lines are not independent, and to indicate this they have been joined by lines. Since Y_1 goes through the same sequence of five values twice, the entries for the second time are the same as for the first, and the only difference is that both y_3 and y_6 are complemented.

In passing between ⑧ and ⑨ y_1 does not change: y_5 and y_6 do. If in going from ⑨ to ⑧ y_5 changes first 000110 is the state passed through and the entry is X_1 or 0, the choice being free. If y_6 changes first the entry is the same but it is entered in 000101. In that case 000110 is passed through in going from ⑧ to ⑨ and reference to fig. 4.2, (c)(i) shows the entry to be X_2 or 0, the choice being free. If y_5 changes first, this is the alternative entry in 000110. Here again, the two choices are not independent, and this has been indicated by joining the two entries with a line. The rest of the entries are determined in the same way, the only entries for which there is no choice

being at 001110 and 000111, for which Y_1 does not change when counting up or down.

Examination of the map shows that it is possible to make Y_1 independent of y_3 and y_6, because it is always possible to make the entries within a sub-cube y_1y_2- y_4y_5- the same for all values of y_1, y_2, y_4 and y_5. To do so, however, implies some decisions about order.

Consider first the sub-cube 00-11- : two of the entries (001110, 000111) are 0, there being no alternatives, so the two remaining entries (000110, 001111) must be the same. Making them so implies no decision about order.

Next, consider the sub-cube 00-10- . Here all entries may be 0 or all may be X_1. Making all the entries 0 implies no decision concerning the order in which the relays 5 and 6 change in passing between states ③ (001110) and ④ (001101), and ⑧ (000111) and ⑨ (000100) respectively. (The rings have been used to emphasise that these numbers refer to states of the count, not to relays). The choice of 0 does imply, however, that when relays 1 and 4 change in going between states ④ (001101) and ⑤ (101001), and ⑨ (000100) and ⓪ (100000) respectively, relay 4 changes first. It further implies that in the sub-cubes 00-00- , 10-10- and 10-00- the entries X_1', X_2 and 1, respectively, must be chosen. In the first two of these sub-cubes the remaining entries are entirely optional, and in the last, although the alternatives for the two remaining entries (101000, 100001) are correlated with other entries (110000, 111001) these entries are not further restricted and no decision about order is implied.

If, however, (in the sub-cube 00-10-) X_1 is chosen, this implies, first, that in passing between states ③ and ④ , and ⑧ and ⑨ , respectively, relay 6 changes before relay 5. This further implies that X_1 cannot be chosen at 000110 and 001111, but as this alternative has already been rejected, this is not an additional restriction. The choice of X_1 at 000100 and 001101 implies that in passing between states ④ and ⑤, and ⑨ and ⓪ , respectively, relay 1 changes before relay 4, and it restricts the alternatives in sub-cube 00-00- to X_1 and 0, and in sub-cube 10-10- to X_2' and 1. In sub-cube 10-00- the only possible choice is X_2' and since this must therefore be the entry in 101000 and 100001 relay 3 must change before relay 2 in passing between states ⓪ (100000) and ① (111000) and ⑤ (101001) and ⑥ (110001) respectively. Now consider the sub-cube 11-00- :

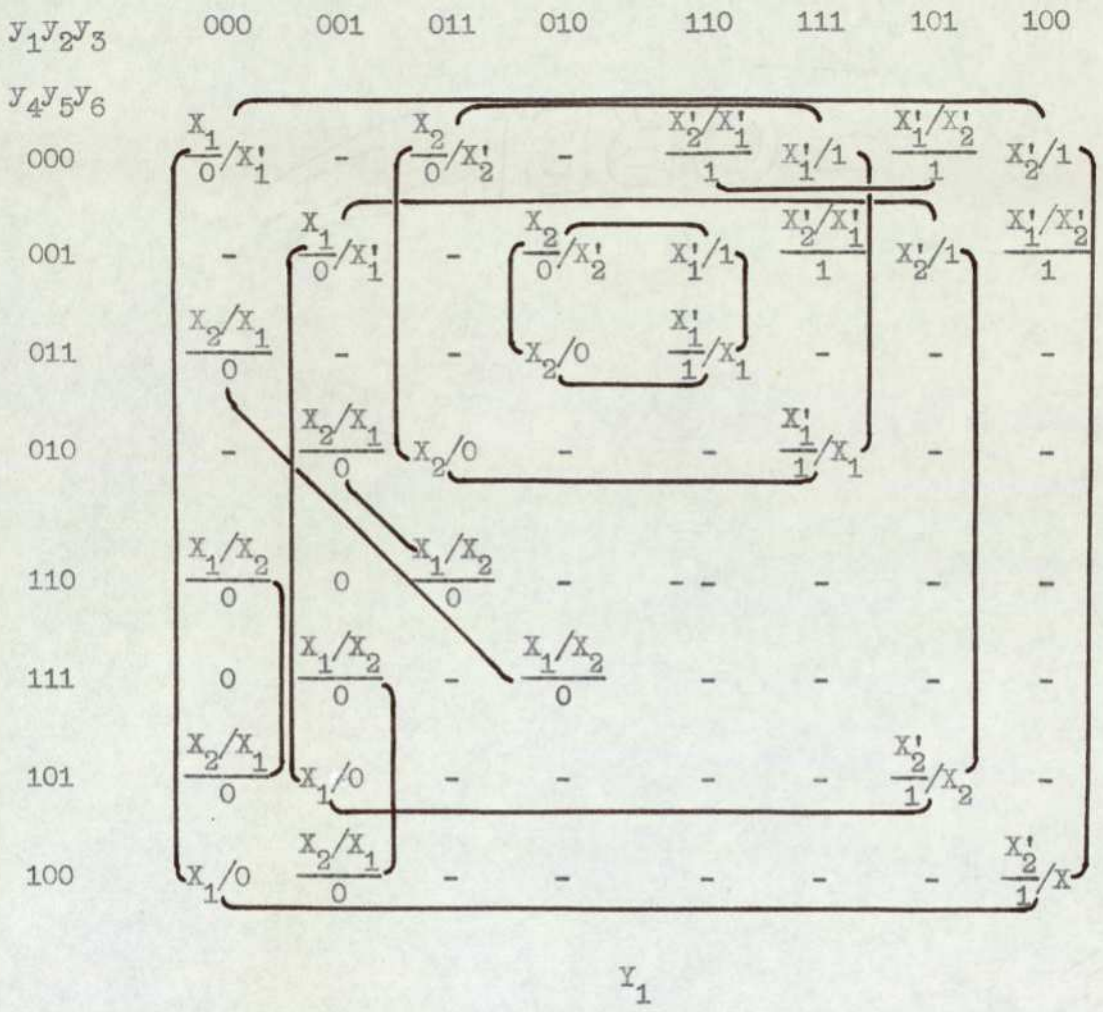

$y_1y_2y_3$
000
001
011
010
110
111
101
100
$y_4y_5y_6$
000
001
011
010
110
111
101
100
Y_1

Fig. 4.2, (d)

the entries in 110000 and 111001 may not be X_2', but since the entries in 111000 and 110001 may not be X_2' either, no further restriction is implied. If however, in the sub-cube 11-00- X_1' is chosen at 111000 and 110001 so that relay 1 changes before relay 5, then X_1' must also be chosen at 110000 and 111001 so thatrelay 3 must change before relay 2.

The conclusion is, therefore, that if Y_1 is to be independent of y_3 and y_6 either relay 4 must change before relay 1 or, if relay 1 changes before relay 4 then relay 3 must change before relay 2 and relay 6 before relay 5, and also, if relay 1 changes before relay 5, relay 3 must change before relay 2.

Next, the excitation map for Y_2 is constructed in the same way. It may be noted that, since the sequence of values of Y_2, beginning at state (1) is the same as the sequence of values of Y_1 beginning at state (0) , the entries on the Y_2 map for states $\overline{(n+1)(\text{mod } 10)}$ will be the same as those on the Y_1 map for states (n). The map is shown in fig. 4.2, (e).

Like Y_1, Y_2 may be made independent of y_3 and y_6, and the various alternatives may be examined in the same way.

Consider first the sub-cube 10-00- . Although each entry may be 0 all 4 entries may not be 0 simultaneously. The only possible choice is X_1, and this implies that relay 2 changes before relay 3. Similarly X_2' is the only possible choice for the sub-cube 11-00- . In the sub-cube 00-10- the only possible choice is 0. This does not imply any decision about the order in which relays 5 and 6 change, and does not affect any other choice. In the sub-cube 00-11- there are two alternatives, 0 and X_2. If 0 is chosen relay 4 changes before relay 2 and the entries in the sub-cubes 00-01- , 01-00- and 01-11- must be X_2', 1 and X_1 respectively. If X_2 is chosen relay 2 changes before relay 4, relay 6 changes before relay 5 and the entries in 00-01- , 01-00- and 01-11- must be X_2 or 0, X_1' and X_1' or 1 respectively.

The conclusion follows that if Y_2 is to be independent of y_3 and y_6 relay 2 must change before relay 3 and, in addition, if relay 2 changes before relay 4, relay 6 must change before relay 5.

Without drawing up the map, it may be seen that Y_3 cannot be independent of y_3 for if it were each y_3 and y_3' contact could be replaced by a network of contacts on the remaining relays and relay 3 would become superfluous. (A delay would be eliminated but this would not affect the operation of the circuit). The conditions for Y_4 and Y_5

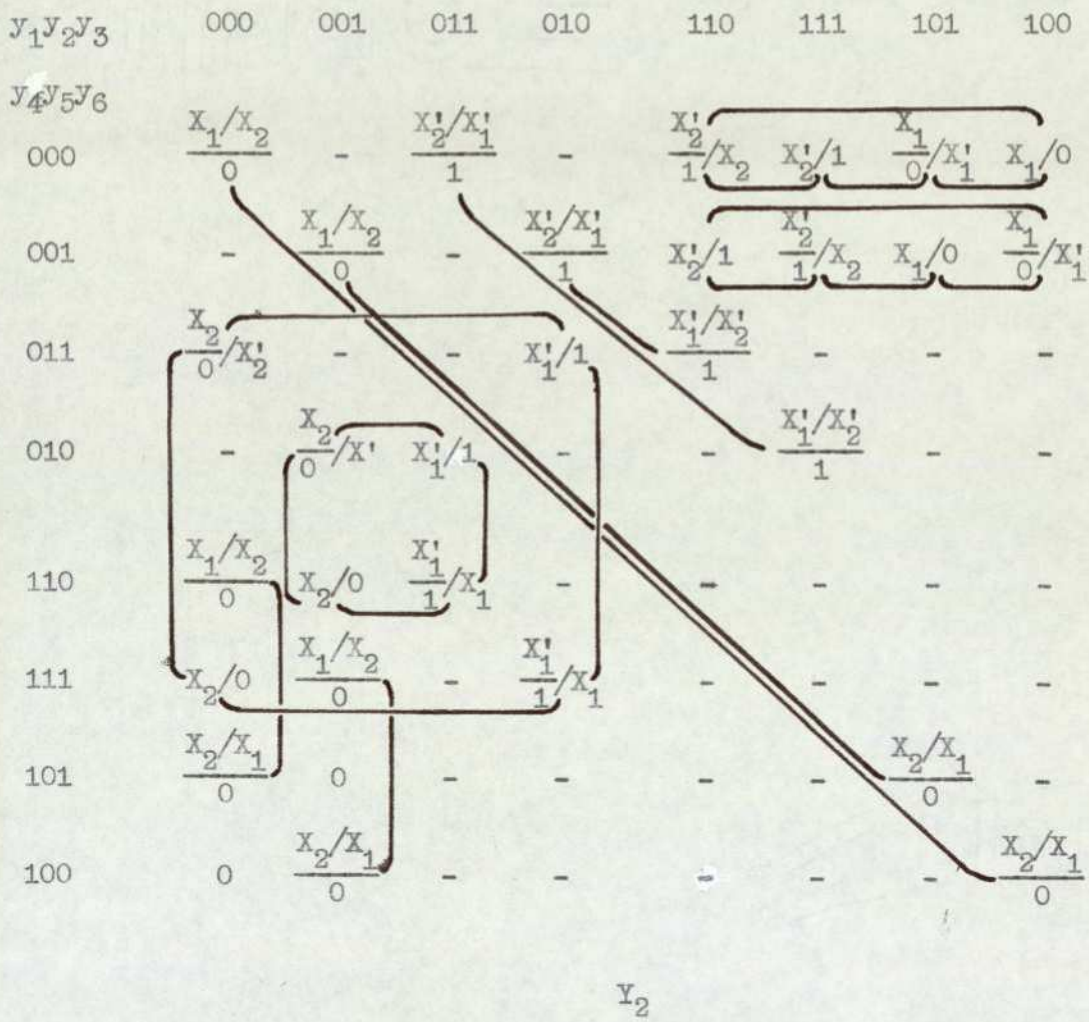

$y_1y_2y_3$
000
001
011
010
110
111
101
100
$y_4y_5y_6$
000
001
011
010
110
111
101
100
Y_2

Fig. 4.2, (e)

to be independent of y_3 and y_6 may be found from those for Y_1 and Y_2 respectively, by symmetry. Y_6 cannot be independent of y_6, just as Y_3 cannot be independent of y_3.

4.2.2 Formal determination of limits of choice.

In order to state these results concisely and facilitate subsequent manipulation the following symbolism will be introduced:

(PbQ) is a binary variable. (PbQ) = 1 means that relay P changes before relay Q. Then clearly (QbP) = (PbQ)'

$(Y_r, y_1 \ldots y_t)$ is a binary variable. $(Y_r, y_1 \ldots y_t) = 1$ means that Y_r may be independent of all the variables $y_1 \ldots y_t$. Using this notation the foregoing results become:

$$\begin{aligned}(Y_1, y_3 y_6) &= \{(4b1) + (3b2)(6b5)\}\ \{(5b1) + (3b2)\} \\ &= (4b1)(5b1) + (4b1)(3b2) + (3b2)(6b5)(5b1) + (3b2)(6b5) \\ &= (4b1)(5b1) + (4b1)(3b2) + (3b2)(6b5)\end{aligned}$$

$$(Y_2, y_3 y_6) = (2b3)\ \{(4b2) + (6b5)\} \qquad 4.2.2.1$$

The corresponding results for Y_4 and Y_5 may be obtained by symmetry. They are:

$$(Y_4, y_3 y_6) = (1b4)(2b4) + (1b4)(6b5) + (3b2)(6b5)$$

$$(Y_5, y_3 y_6) = (5b6)\ \{(1b5) + (3b2)\} \qquad 4.2.2.2$$

The negative results obtained for Y_3 and Y_6 may be written:

$$(Y_3, y_3 y_6) = 0 \qquad (Y_6, y_3 y_6) = 0 \qquad 4.2.2.3$$

We next ask whether two or more of the variables Y_1, Y_2, Y_4 and Y_5 may be simultaneously independent of y_3 and y_6. They cannot all be independent of y_3 and y_6, for if they were, this group of four relays would form an independent counter. In that case one relay from this group would have to change state when an input is applied and one would have to change state when an input was released. But the counter h as been designed so that only one relay changes state in either of these circumstances, so that relays 3 and 6 could never change state a nd would be redundant. It follows that all four variables Y_1, Y_2, Y_4 and Y_5 cannot be independent of y_3 and y_6 so the next question must be which pairs of them may be simultaneously independent of y_3 and y_6, and on what conditions. These may be derived by formal manipulation of the results previously obtained, for example, the condition that Y_1 and Y_2 shall simultaneously be independent of y_3 and y_6 is derived as follows:

$$(Y_1, y_3 y_6)(Y_2, y_3 y_6) = (4b1)(5b1)(2b3)\ \{(4b2) + 6b5)\} \qquad 4.2.2.4$$

Similarly

$$(Y_1,y_3y_6)(Y_4,y_3y_6) = (3b2)(6b5) \qquad 4.2.2.5$$

and

$$(Y_1,y_3y_6)(Y_5,y_3y_6) = \{(4b1)(5b1) + (4b1)(3b2) + (3b2)(6b5)\} \times (5b6)\{(1b5) + (3b2)\}$$
$$= (4b1)(5b6)(3b2) \qquad 4.2.2.6$$

From $(Y_1,y_3y_6)(Y_5,y_3y_6)$ by symmetry may be obtained

$$(Y_2,y_3y_6)(Y_4,y_3y_6) = (1b4)(2b3)(6b5) \qquad 4.2.2.7$$

Next

$$(Y_2,y_3y_6)(Y_5,y_3y_6) = (2b3)(4b2)(5b6)(1b5) \qquad 4.2.2.8$$

Finally, from $(Y_1,y_3y_6)(Y_2,y_3y_6)$ may be obtained by symmetry -

$$(Y_4,y_3y_6)(Y_5,y_3y_6) = (1b4)(2b4)(5b6)\{(1b5) + (3b2)\} \qquad 4.2.2.9$$

Next, bearing in mind that

$$\{(Y_1,y_3y_6)(Y_2,y_3y_6)\}\{(Y_2,y_3y_6)(Y_4,y_3y_6)\} = (Y_1,y_3y_6)(Y_2,y_3y_6)(Y_4,y_3y_6), \text{ etc.}$$

the conditions that three of the variables Y_1, Y_2, Y_4 and Y_5 shall be independent of y_3 and y_6 may be determined. There are four possibilities (corresponding to the omission of each of the four variables) and only two of these need be examined, the results for the remaining two following from symmetry. It is easily seen that

$$(Y_1,y_3y_6)(Y_2,y_3y_6)(Y_4,y_3y_6) = 0 \qquad 4.2.2.10$$

and

$$(Y_1,y_3y_6)(Y_2,y_3y_6)(Y_5,y_3y_6) = 0 \qquad 4.2.2.11$$

from which it follows that no more than two of the four variables may be simultaneously independent of y_3 and y_6.

The six alternative pairs of variables occus in symmetrical pairs, except for Y_1Y_4 and Y_2Y_5, which are themselves symmetrical, so that there are only four alternatives to consider. Of these four alternatives, the combination of Y_1 and Y_4 places fewest restrictions on the orders in which relays change, even when allowance is made for the fact that this is a symmetrical choice, so that, whether relay 1 changes before relay 4 or not, the resulting networks will be of equal complexity. However, before concluding that this is the best choice, the maps for Y_3 and Y_6 will be examined. The map for Y_3 is shown in fig. 4.2, (f).

The map for Y_6 may be obtained by symmetry. It will be found that suitable choices of order can reduce the numbers of variables on which Y_3 and Y_6 depend.

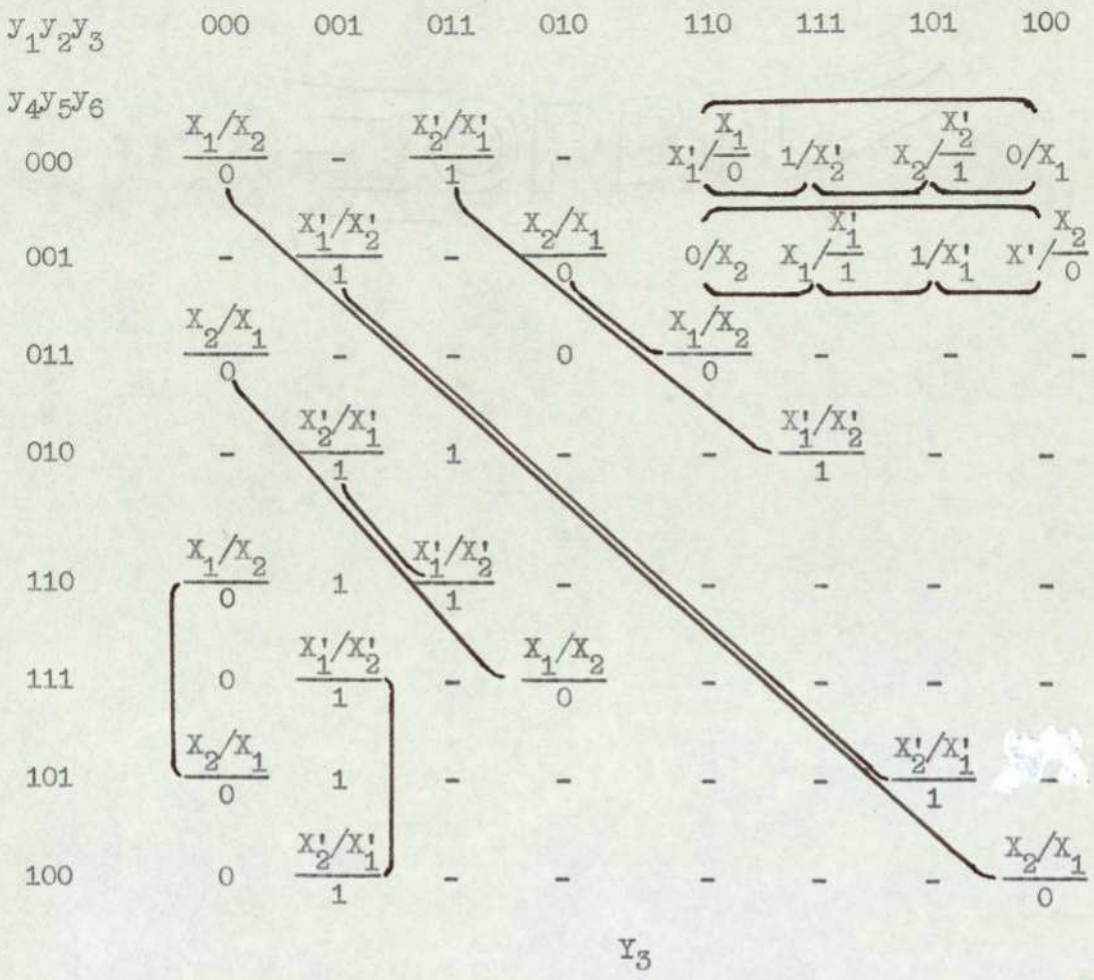

$y_1y_2y_3$
000
001
011
010
110
111
101
100
$y_4y_5y_6$
000
001
011
010
110
111
101
100
Y_3

Fig. 4.2, (f)

To make Y_3 independent of y_1, it is necessary that for every combination of the variables $y_2y_3y_4y_5y_6$ the entries for $y_1 = 0$ and $y_1 = 1$ may be made the same by suitable choices from the alternatives. In many cases one or both of these entries are entirely optional, but there are some cases where the choice between alternative entries implies a decision about order. For example, in 000000 and 100000 the entries may both be 0, if relay 2 changes before relay 3, or X_1, if relay 3 changes before relay 2 and relay 1 before relay 4. Also, if relay 3 changes before relay 2, the entry in 011000 must be the same as that in 111000, namely X_2', and relay 1 must change before relay 5. Thus the only condition that Y_3 should be independent of y_1 is that either relay 2 changes before relay 3, or relay 1 changes before relay 4 and relay 1 before relay 5. In the same way the conditions that Y_3 should be independent of the other variables may be obtained. They are

$$\begin{aligned}(Y_3,y_1) &= (2b3) + (1b4)(1b5)\\ (Y_3,y_2) &= (3b2)\\ (Y_3,y_4) &= (2b3) + (4b1)\\ (Y_3,y_5) &= (2b3) + (5b1)\end{aligned} \qquad 4.2.2.12$$

Y_3 cannot be made independent of y_3, for then it would be a slave relay, and any contact on it could be replaced by a network of contacts on the remaining relays. Neither can Y_3 be independent of y_6, for if we so far relaxed the restriction that the complete sequence of states shall consist of two identical subsequences (arbitrarily imposed) to allow relay 3 to change before relay 2 in going between (0) and (1), and relay 2 to change before relay 3 in going between (5) and (6), so making the entries in 110000 and 110001 both 0, the entries in 111000 and 111001 would be X_2' and X_1' respectively.

The next step is to determine those combinations of variables of which Y_3 may be independent, and on what conditions. Clearly, if Y_3 is to be independent of both y_1 and y_2 (for example) the conditions that Y_3 shall be independent of y_1 and y_2 separately, must both be satisfied. However, though these conditions are necessary, they are not sufficient. Suppose that one sub-cube from the map is as shown in fig. 4.2, (g) and that the alternatives for P and Q do not permit them

y_1		0	1
y_2	0	Ø	P
	1	Q	Ø

Fig. 4.2, (g)

to be made the same. Then the function may be independent of y_1 or y_2 but not of both simultaneously. This requires that for all values of y_3, y_4, y_5 and y_6 the entries for y_1y_2 = 00, 01, 11, 10 may be made identical.

Examination of the map shows that for the sub-cube --0000 the only possible choice is X_1, and that this requires that (1b4) = 1 and (3b2) = 1. Similarly for the sub-cube --1000 X_2' is the only possible choice. This requires that (1b5) = 1 and (3b2) = 1. Examination of the remaining sub-cubes shows that these lead to no additional requirements, so that

$$(Y_3,y_1y_2) = (1b4)(1b5)(3b2)$$

In this particular case therefore,

$$(Y_3,y_1y_2) = (Y_3,y_1)(Y_3,y_2)$$

even though this is not generally true.

To determine whether Y_3 may be independent of y_1 and y_4 simultaneously, the sub-cube -00-00 may be first examined. Here the only possible entry is 0, and this requires that (2b3) = 1. The entry in 101000 must then be X_2, which makes it impossible for the four entries in the sub-cube -01-00 to be the same. Hence Y_3 cannot be independent of y_1 and y_4.

Since $(Y_3,y_1)(Y_3,y_5)$ = (2b3), it is a necessary though not sufficient condition for (Y_3,y_1y_5) = 1 that (2b3) = 1. This leads to X_2 in 101000, and since it is not possible to have X_2 in 001010, (Y_3,y_1y_5) = 0.

Since $(Y_3,y_2)(Y_3,y_4)$ = (3b2)(4b1), it is a necessary condition for (Y_3,y_2y_4) = 1 that (3b2)(4b1) = 1. In this case examination of the map shows that it is also sufficient.

Similarly, it is a necessary condition for (Y_3,y_2y_5) = 1 that (3b2)(5b1) = 1, and again examination of the map shows that this is also sufficient.

Finally, $(Y_3,y_4)(Y_3,y_5)$ = (2b3) + (4b1)(5b1) and examination of the map shows that (2b3) = 1 and (4b1)(5b1) = 1 are each, separately, sufficient conditions for (Y_3,y_4y_5) = 1.

Summarizing these results

$$(Y_3, y_1y_2) = (1b4)(1b5)(3b2)$$

$$(Y_3, y_1y_4) = 0$$

$$(Y_3, y_1y_5) = 0$$

$$(Y_3, y_2y_4) = (3b2)(4b1)$$

$$(Y_3, y_2y_5) = (3b2)(5b1)$$

$$(Y_3, y_4y_5) = (2b3) + (4b1)(5b1) \qquad 4.2.2.13$$

Next we enquire whether Y_3 may be independent of three variables simultaneously. If it may, and the three variables are y_A, y_B and y_C, $(Y_3, y_Ay_By_C)$ is not identically zero, and neither are (Y_3, y_Ay_B) (Y_3, y_By_C) and (Y_3, y_Ay_C). It implies further that these three functions may have the value 1 simultaneously, so that $(Y_3, y_Ay_B)(Y_3, y_By_C)(Y_3, y_Ay_C)$ is not identically zero. Examination of the results summarized immediately above shows that the only function of this type which is not identically zero is

$$(Y_3, y_2y_4)(Y_3, y_4y_5)(Y_3, y_2y_5) = (3b2)(4b1)(5b1) \qquad 4.2.2.14$$

so that, if Y_3 may be independent of three variables these must be y_2, y_4 and y_5. For this to be so it is a necessary condition that $(3b2)(4b1)(5b1) = 1$, and examination of the map shows that it is also sufficient.

Thus

$$(Y_3, y_2y_4y_5) = (3b2)(4b1)(5b1) \qquad 4.2.2.15$$

while all the other similar functions involving Y_3 are zero. By symmetry the corresponding result for Y_6 is

$$(Y_6, y_1y_2y_5) = (6b5)(1b4)(2b4) \qquad 4.2.2.16$$

from which it follows that Y_3 and Y_6 cannot be simultaneously independent of three variables.

It follows then, that only one of the functions Y_3 and Y_6 may be independent of three variables, and in view of the symmetry there is no loss of generality in supposing Y_3 to be so independent. Comparison of equation 4.2.2.15 with equations 4.2.2.4-9 shows that at the same time Y_1 and either Y_4 or Y_5 may be independent of y_3 and y_6, but that this is not true of any other pair of functions. The conditions are

$$(Y_3, y_2y_4y_5)(Y_1, y_3y_6)(Y_4, y_3y_6) = (3b2)(4b1)(5b1)(6b5) \qquad 4.2.2.17$$

and

$$(Y_3, y_2y_4y_5)(Y_1, y_3y_6)(Y_5, y_3y_6) = (4b1)(5b6)(3b2)(5b1) \qquad 4.2.2.18$$

in either case the choice of the value of (2b4) is still open.

Referring again to equations 4.2.2.13 and adding the right-hand sides, the condition that Y_3 shall be independent of some two variables is

$$(3b2) + (4b1)(5b1) = 1,$$

and by symmetry the condition that Y_6 shall be independent of some two variables is

$$(6b5) + (1b4)(2b4) = 1$$

This is consistent with 4.2.2.17 but not with 4.2.2.8. The value of (2b4) is still open.

It appears then, that it is possible to make Y_3 independent of three variables and Y_6 independent of two variables and at the same time to make Y_1 and Y_4 both independent of y_3 and y_6. This determines the orders in which the relays of a pair change at each step of the count, except in the case of relays 2 and 4, in which the choice is still open. These decisions are implied by equation 4.2.2.17. It is proposed therefore to complete the design, noting in each case the alternative excitation functions and networks.

4.2.3 Excitation networks: alternatives.

The map for Y_1 is obtained from the complete map (fig. 4.2, (d)) by replacing each sub-cube corresponding to particular values of $y_1y_2y_4y_5$ and the four possible values of y_3y_6, by a single entry, taking into account the decisions about order implied in equation 4.2.2.17. This leads to the map of fig. 4.2, (h).

y_1y_2 / y_4y_5	00	01	11	10
00	X_1'	X_2'	1	1
01	$\frac{X_2/X_1}{0}$	0	X_1	.
11	0	$\frac{X_1/X_2}{0}$	.	.
10	0	.	.	X_2

Y_1

Fig. 4.2, (h)

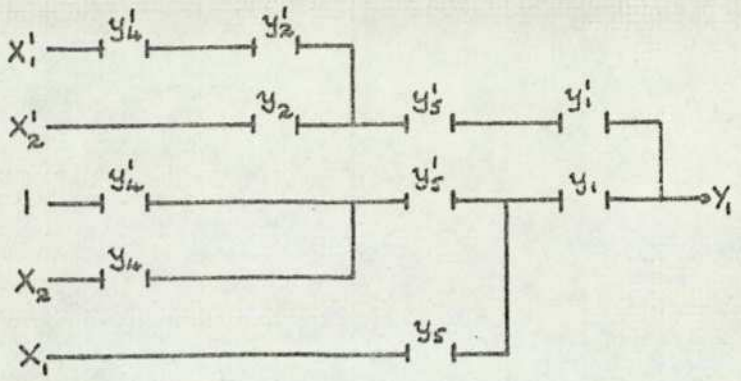


Fig. 4.2,(j)

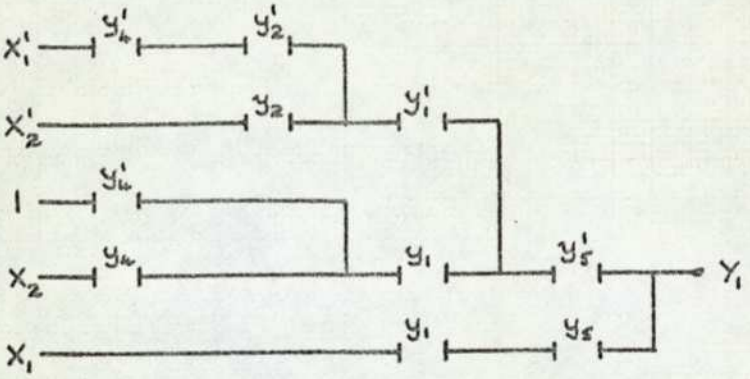


Fig. 4.2,(k)

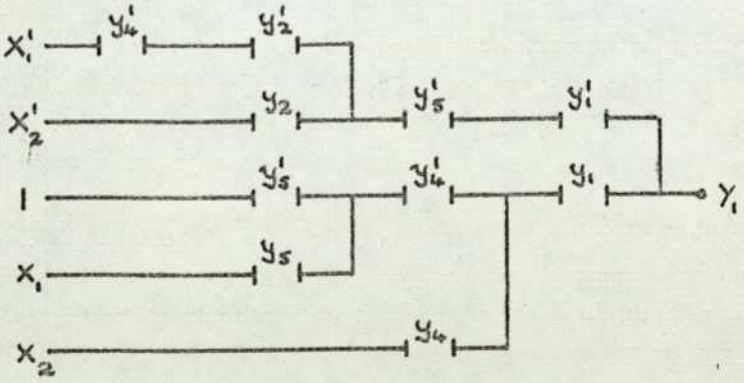


Fig. 4.2,(ℓ)

Two possible excitation functions are

$$Y_1 = y_1'y_2'y_4'y_5'X_1' + y_1'y_2'y_5'X_2' + y_1y_4'y_5' + y_1y_5X_1 + y_1y_4y_5'X_2$$

and

$$Y_1 = y_1'y_2'y_4'y_5'X_1' + y_1'y_2y_5'X_2' + y_1y_4'y_5' + y_1y_4'y_5X_1 + y_1y_4X_2$$

The value of (4b2) makes no difference to these functions.

In developing the tree the first transfer contact should provide a contact in each path. For the first function the choice is between contacts on Y_1 and Y_5, while there is no alternative to Y_1 for the second function. The continued application of the same principle leads to the three alternative networks shown in Figs. 4.2, (j - ℓ). These are of equal complexity.

Y_2 is not independent of y_3 and y_6, but in every case the entries for $y_1y_3 = 00$ and 11 are the same, and so are the entries for $y_1y_3 = 01$ and 10. The network may therefore begin with the complementary functions $y_3'y_6' + y_3y_6$, $y_3y_6' + y_3'y_6$ and then continue with two further subnetworks with contacts on the other four relays whose maps are obtained from fig. 4.2, (e) and given in fig. 4.2, (m).

y_1y_2	00	01	11	10	00	01	11	10
y_4y_5								
00	$\frac{X_2}{0}$	·	X_2	0	·	$\frac{X_1'}{1}$	1	X_1'
01	·	·	·	·	$\frac{X_2}{0}/X_2'$ — $X_1'/1$		$\frac{X_2'}{1}$	·
11	$\frac{X_2}{0}$	·	·	·	$X_2/0$ — $\frac{X_1'}{1}/X_1$		·	·
10	0	·	·	$\frac{X_1}{0}$	$\frac{X_1}{0}$	·	·	·
	$y_3'y_6' + y_3y_6$				$y_3'y_6 + y_3y_6'$			

Fig. 4.2, (m)

The excitation functions are

$$Y_2 = (y_3'y_6' + y_3y_6)y_2X_2 + (y_3'y_6 + y_3y_6')\{X_2y_1'y_2'y_5 + X_1'(y_1'y_2 + y_1y_2') + y_1y_2\}$$

if (2b4) = 1, and

$$Y_2 = (y_3'y_6' + y_3y_6)y_2X_2 + (y_3'y_6 + y_3y_6')\{X_2'y_1'y_2'y_4' + y_2y_4' + X_1y_2y_4 + X_1'y_1y_2'\}$$

if (4b2) = 1

From these excitation functions the networks shown in fig. 4.2, (n) and (p) respectively, result. Both these networks require 6 transfer contacts, one make and one break contact. In either network the levels

of the contacts on Y_3 and Y_6 may be interchanged.

It has been shown that Y_3 may be made independent of y_2y_4 and y_5 with the order choices made. The map for Y_3 then becomes as shown in fig. 4.2, (q).

y_1y_3 / y_6	00	01	11	10
0	0	1	X_2'	X_1
1	0	1	X_1'	X_2

Fig. 4.2, (q)

and the excitation function

$$Y_3 = y_1'y_3 + y_1y_3'y_6'X_1 + y_1y_3y_6X_2 + y_1y_3y_6'X_2' + y_1y_3y_6X_1'$$

which gives the network shown in fig. 4.2, (r), which requires four transfer contacts and one make contact. Two other similar networks are possible, the first developed with the contacts on Y_1 and Y_3 in the reverse order, the second with the contacts on Y_3 and Y_6 in the reverse order.

The networks for Y_4 will next be obtained. In order to make use of the symmetry between Y_4 and Y_1 it is necessary to go back to the map for Y_1, fig. 4.2, (d) since the subsequent choices of order are not symmetrical. We require a network for Y_4 for the choices of order implied in (3b2)(4b1)(5b1)(6b5) = 1. This may be derived by symmetry from a network for Y_1 for the choices of order implied in (6b5)(1b4)(2b4)(3b2) = 1. From the map of fig. 4.2, (d) the variables Y_3 and Y_6 may then be eliminated, giving the map of fig. 4.2, (s).

y_1y_2 / y_4y_5	00	01	11	10
00	$\frac{X_1}{0}$	$\frac{X_2}{0}/X_2'$	$X_1'/1$	X_2'
01	$\frac{X_2}{0}$	$X_2/0$	$\frac{X_1'}{1}/X_1$	.
11	0	$\frac{X_1}{0}$	.	.
10	X_1	.	.	$\frac{X_2'}{1}$

Fig. 4.2, (s)

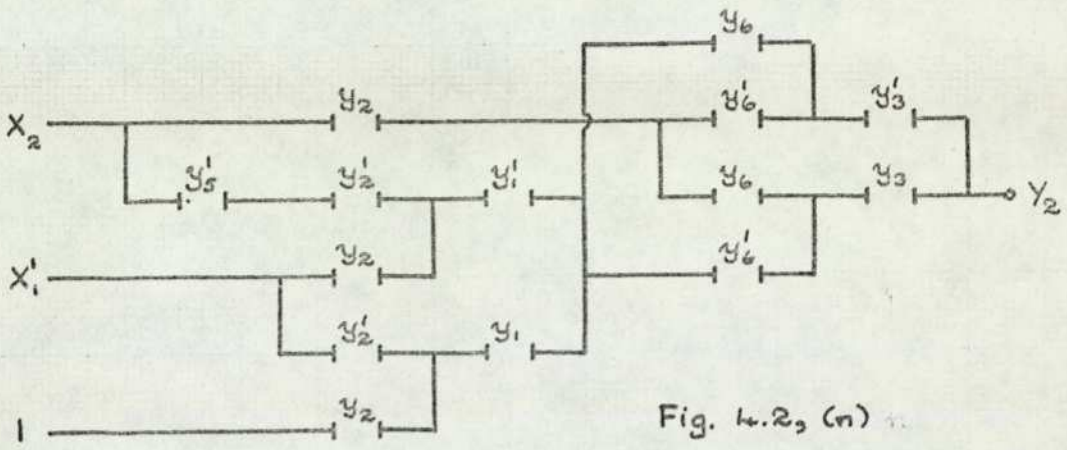


Fig. 4.2, (n)

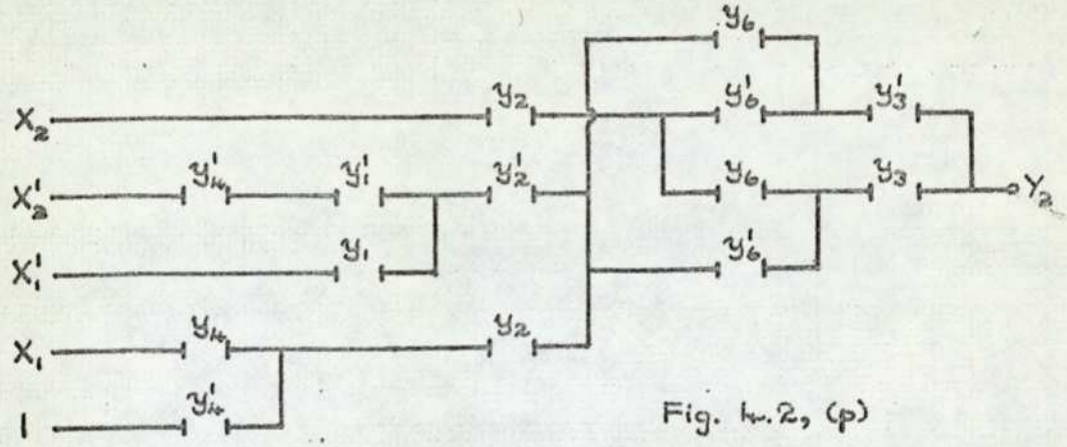


Fig. 4.2, (p)

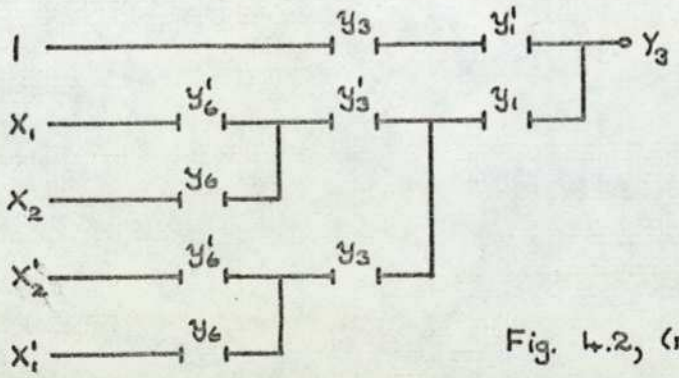


Fig. 4.2, (r)

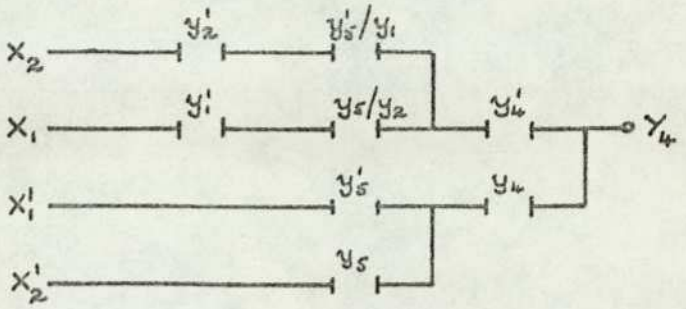


Fig. 4.2, (t)

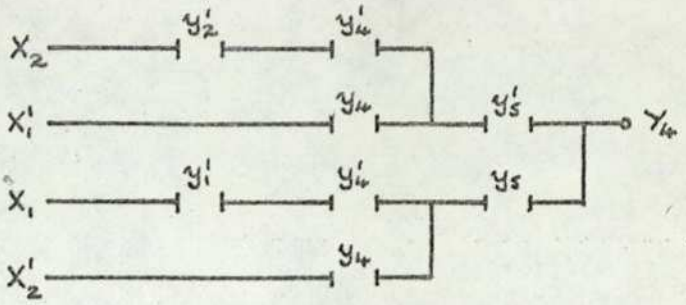


Fig. 4.2, (u)

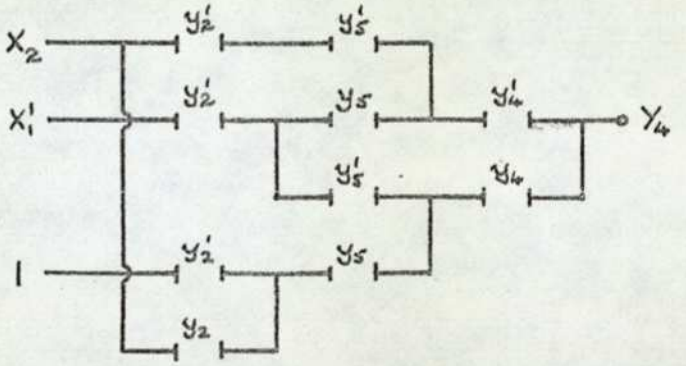


Fig. 4.2, (v)

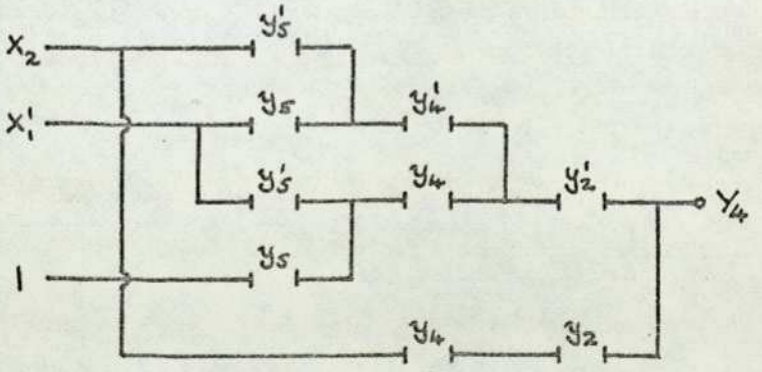


Fig. 4.2, (w)

For (1b5) = 1 the excitation function, with alternatives, is

$$Y_1 = y_1'y_2'y_5X_1 + y_1'\overset{y_2}{\underset{y_4}{y_5}}y_4'X_2 + y_1y_2X_1' + y_1y_2'X_2'$$

and for (5b1) = 1

$$Y_1 = y_1'y_2'y_5'X_1 + (y_1y_2' + y_1'y_2)y_5'X_2' + y_1y_2y_5' + y_1(y_2)y_5X_1$$

By symmetry, for (4b2) = 1

$$Y_4 = y_2'y_4'\overset{y_1}{\underset{y_5}{y_5'}}X_2 + y_1'y_4'y_5X_1 + y_4y_5X_2' + y_4y_5'X_1'$$

and for (2b4) = 1

$$Y_4 = y_2'y_4'y_5'X_2 + y_2'(y_4'y_5 + y_4y_5')X_1' + y_2'y_4y_5 + y_2y_4(y_5)X_2$$

Possible contact networks for (4b2) = 1 are shown in figs. 4.2, (t) and (u) and for (2b4) = 1 in figs. 4.2, (v) and (w).

The derivation of the excitation functions and networks for Y_5 combines features of the derivation of those for Y_2 and Y_4. First functions for Y_2 when (6b5)(1b4)(2b4)(3b2) = 1 are derived. The map for Y_2 with these choices made is obtained from fig. 4.2, (e), as shown in fig. 4.2, (x). Writing $Y_2 = (y_3'y_6' + y_3y_6)Y_{2A} + (y_3'y_6 + y_3y_6')Y_{2B}$ the maps for Y_{2A} and Y_{2B} are as shown in fig. 4.2, (y).

Then $Y_{2A} = y_2X_2$

and for (5b1) = 1

$$Y_{2B} = y_1'y_2'y_5X_2 + (y_1'y_2 + y_1y_2')X_1' + y_1y_2$$

while for (1b5) = 1

$$Y_{2B} = y_1'y_2'y_5X_2 + (y_2y_5 + y_1y_2'y_5')X_1' + y_2y_5'$$

where the factor y_1' in the coefficient of X_2 or the factor y_5' in the coefficient of X_1' may be omitted.

These results may be combined to give the complete expressions for Y_2, and the expressions for Y_5 may then be derived by symmetry. They are for (2b4) = 1

$$Y_5 = (y_3'y_6' + y_3y_6)y_5X_1 + (y_3'y_6 + y_3y_6')\ x$$
$$\{y_2y_4'y_5'X_1 + (y_4'y_5 + y_4y_5')X_2' + y_4y_5\}$$

and for (4b2) = 1

$$Y_5 = (y_3'y_6' + y_3y_6)y_5X_1 + (y_3'y_6 + y_3y_6')\ x$$
$$\{y_2y_4'y_5'X_1 + (y_2y_5 + y_2'y_4y_5')X_2' + y_2'y_5\}$$

where the factor y_4' in the coefficient of X_1 or the factor y_2' in the coefficient of X_2' may be omitted.

If (2b4) = 1 a contact network is shown in fig. 4.2, (z). The levels of y_3 contacts on Y_3 and Y_6 may be interchanged as may also

$y_1y_2y_3$ / $y_4y_5y_6$	000	001	011	010	110	111	101	100
000	$\frac{X_1}{0}$	-	$\frac{X'_2/X'_1}{1}$	-	X_2	1	X'_1	0
001	-	$\frac{X_1}{0}$	-	$\frac{X'_2/X'_1}{1}$	1	X_2	0	X'_1
011	$\frac{X_2}{0}$	-	-	X'_1	$\frac{X'_1/X'_2}{1}$	-	-	-
010	-	$\frac{X_2}{0}$	X'_1	-	-	$\frac{X'_1/X'_2}{1}$	-	-
110	$\frac{X_2}{0}$	X_2	$\frac{X'_1}{1}$	-	-	-	-	-
111	X_2	$\frac{X_2}{0}$	-	$\frac{X'_1}{1}$	-	-	-	-
101	$\frac{X_1}{0}$	0	-	-	-	-	$\frac{X_2}{0}$	-
100	0	$\frac{X_1}{0}$	-	-	-	-	-	$\frac{X_2}{0}$

Fig. 4.2, (x)

y_1y_2 / y_4y_5	00	01	11	10	00	01	11	10
00	$\frac{X_1}{0}$	-	X_2	0	-	$\frac{X'_2/X'_1}{1}$	1	X'_1
01	-	-	-	-	$\frac{X_2}{0}$	X'_1	$\frac{X'_1/X'_2}{1}$	-
11	$\frac{X_2}{0}$	-	-	-	X_2	$\frac{X'_1}{1}$	-	-
10	0	-	-	$\frac{X_2}{0}$	$\frac{X_1}{0}$	-	-	-
		Y_{2A}				Y_{2B}		

Fig. 4.2, (y)

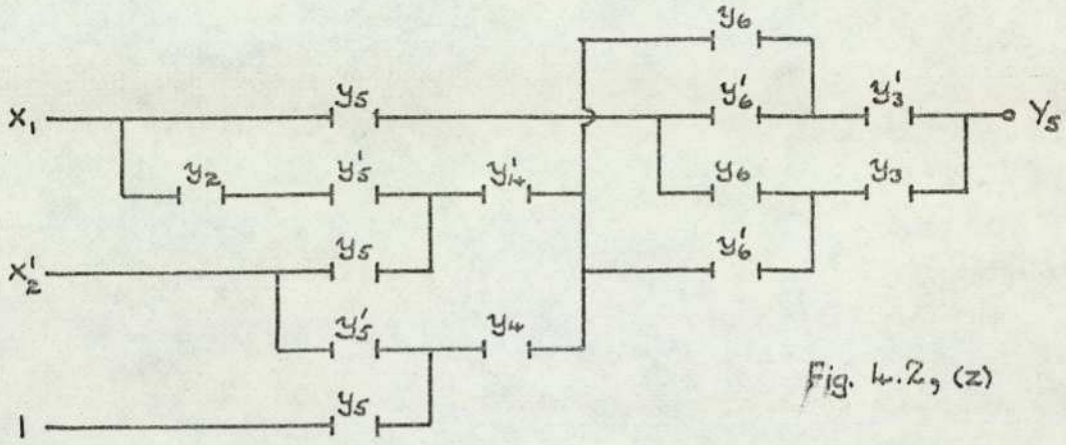


Fig. 4.2, (z)

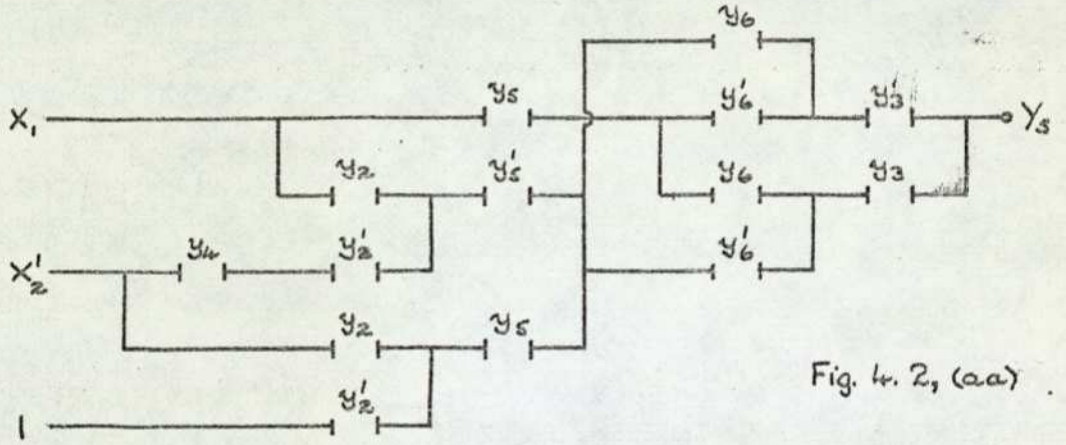


Fig. 4. 2, (aa)

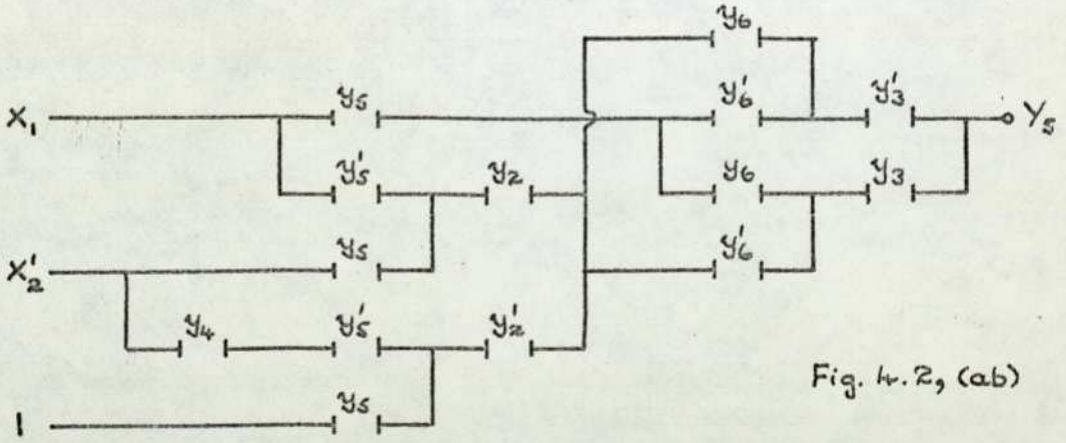


Fig. 4. 2, (ab)

those on Y_4 and Y_5. The total number of contacts remains unchanged at 6 transfers and 2 make contacts.

If (4b2) = 1 two possible contact networks are those shown in figs. 4.2, (aa) and (ab). Again the levels of the contacts on Y_3 and Y_6 may be interchanged as may also those on Y_2 and Y_5. The total number of contacts is unaltered at 6 transfers and 2 make contacts.

It has been shown that, with the choices of order already made, Y_6 may be made independent of two, and not more than two, variables. The conditions that Y_6 shall be independent of two variables may be obtained by symmetry from those for Y_3 (equations 4.2.2.13). They are:

$$(Y_6, y_1y_2) = (5b6) + (1b4)(2b4)$$
$$(Y_6, y_1y_4) = 0$$
$$(Y_6, y_1y_5) = (6b5)(1b4)$$
$$(Y_6, y_2y_4) = 0$$
$$(Y_6, y_2y_5) = (6b5)(2b4)$$
$$(Y_6, y_4y_5) = (4b1)(4b2)(6b5)$$

Of these conditions only the last two may be satisfied if (3b2)(4b1)(5b1)(6b5) = 1, which expresses the choices of order already made. The choice of the first requires that, in addition, (2b4) = 1 and of the second, that (4b2) = 1.

The excitation functions and networks for Y_6 for these two cases may be obtained from the symmetrical functions for Y_3 with the symmetrical conditions that
(6b5)(1b4)(2b4)(3b2) = 1 and either (5b1) = 1 or (1b5) = 1.
If (5b1) = 1 the variables y_2 and y_5 may be eliminated from the Y_3 map, (fig. 4.2, (f)) and it becomes as shown in fig. 4.2, (ac).

y_1y_3 / y_4y_6	00	01	11	10
00	$\frac{X_1}{0}$	1	X_2'	X_1
01	0	$\frac{X_1'}{1}$	X_1'	X_2
11	0	1	$\frac{X_2'}{1}$	.
10	0	1	.	$\frac{X_2}{0}$

Fig. 4.2, (ac)

The simplest function which may be obtained from this map ("simplest" meaning "having the smallest number of literals") is:-

$$Y_3 = y_3(y_1' + y_4) + X_1'y_1y_3y_4y_6 + X_1y_3'y_4'y_6' + X_2'y_1y_3y_4'y_6' + X_2y_1y_3'y_6$$

That this is the simplest function may be shown by an exhaustive consideration of the alternatives. The details of this will not be given because there is no certainty that the simplest function in this sense will lead to the simplest network, meaning that with the smallestnumber of contacts, because there is no one-to-one correspondence between literals and contacts. Recalling that this is not the required function for Y_3 but the function which is symmetrical with the required function for Y_6, the latter becomes

$$Y_6 = y_6(y_4' + y_1) + X_2'y_1y_3y_4y_6 + X_2y_1'y_3'y_6' + X_1'y_1'y_3'y_4y_6 + X_1y_3y_4y_6'$$

A network derived from this function is shown in fig. 4.2, (ad).

The functions and networks for Y_6 when it is made independent of y_4 and y_5 are derived in the same way. The map for Y_3 when (1b5) = 1 is as shown in fig. 4.2, (ae), the function being independent of y_1 and y_2.

y_3	0	1
$y_4y_5y_6$		
000	X_1	X_2'
001	X_2	X_1'
011	0	·
010	·	1
110	$\frac{X_2}{0}$	1
111	0	$\frac{X_2'}{1}$
101	$\frac{X_1}{0}$	1
100	0	$\frac{X_1'}{1}$

Fig. 4.2, (ae)

A resulting function is

$$Y_3 = y_3(y_4 + y_5) + X_1'y_3y_4'y_5'y_6 + X_1y_3'y_4'y_5'y_6' + X_2'y_3y_4'y_5'y_6' + X_2y_3'y_4'y_5'y_6$$

Although the factor y_5' may be omitted from the term in X_1 it is better to retain it since then the first term contains a factor $y_4 + y_5$ and all the others a complementary factor $y_4'y_5'$.

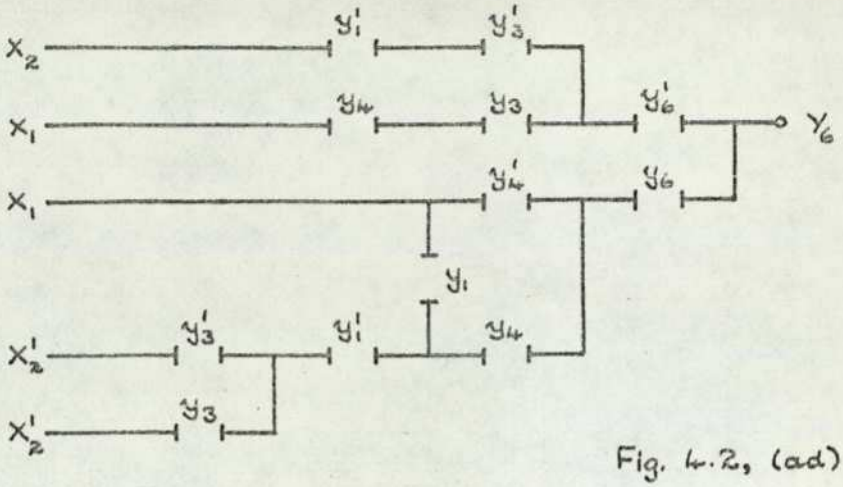


Fig. 4.2, (ad)

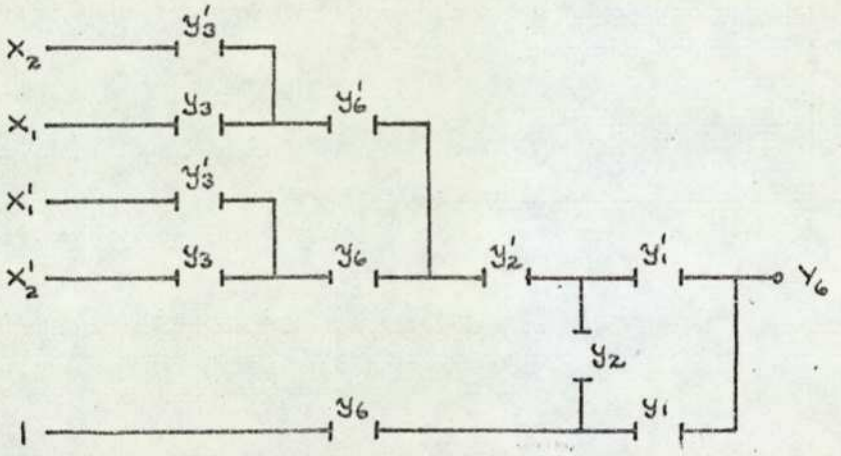


Fig. 4.2, (af)

The required function for Y_6 is therefore

$$Y_6 = y_6(y_1 + y_2) + X_2'y_1'y_2'y_3y_6 + X_2y_1'y_2'y_3'y_6' + X_1'y_1'y_2'y_3y_6 + X_1y_1'y_2'y_3'y_6'$$

The fact that the first term contains the factor $(y_1 + y_2)$ andall the others its complement $y_1'y_2'$ suggests the realisation shown in fig. 4.2, (af). The levels of the contacts on Y_3 and Y_6 may be interchanged.

4.3 Output, and carry and borrow networks. Completion of design.

4.3.1 Output network.

The information required for the design of the output network is contained in the map of fig. 4.2, (a), on which the sequence of states is shown, and equation 4.2.2.17, which implies the choices made about order. One choice, the value of (4b2) remains to be made. It will be assumed that it is intended that a new output shall appear as soon as possible after the input which causes it. Accordingly there have been entered on a map for the six secondary variables (fig. 4.2, (ag)), first, the number of the output for those states stable in the absence of an input and, second, the number of the output for those states via which they are approached for the two directions of counting. The second step cannot be taken where a choice of order has not been made. In all the other cases there are three states having a given output. For these three states four of the variables are the same, while the other two take on three of the four possible values. It will simplify the networks if they may also take on the fourth value. The map has been plotted thus far in fig. 4.3, (a) and it may be seen that in every case the state required for the fourth value is not used, and the desired output value has been entered in it.

There are two steps at which relays 2 and 4 change, those at which the output changes between (2) and (3), and (7) and (8). Taking the first as an example, if Y_4 changes before Y_2, 2 is entered in 001010 and 3 in 011110. 2 may now be entered in 101010 and 3 in 011100. If Y_2 changes before Y_4, 2 is entered in 011110 and 3 in 001010. 2 may now be entered in 111110 and 3 may be entered in 001000. This may be done notwithstanding the fact that 1 has already been entered in 001000 without affecting the behaviour of the counter since this state is not used. It will however make the output network non-disjunctive and more difficult to design. This was confirmed by some tentative attempts which were made. Accordingly it was decided that Y_4 should change

before Y_2, and the map completed accordingly (fig. 4.3, (b).

$y_1y_2y_3$ / $y_4y_5y_6$	000	001	011	010	110	111	101	100
000	0	1	1	0	0	1	1	0
001	6	4		6	6	5	5	6
011		4		7	7			
010			2			2		
110	9	3						9
111	8	4						
101	8	4				5	5	
100	9	3						9

Fig. 4.3, (a)

$y_1y_2y_3$ / $y_4y_5y_6$	000	001	011	010	110	111	101	100
000	0	1	1	0	0	1	1	0
001	6	4	4	6	6	5	5	6
011	7	4	4	7	7	5	5	7
010	0	2	2	0	0	2	2	0
110	9	3	3	9	9	3	3	9
111	8	4	4	8	8	5	5	8
101	8	4	4	8	8	5	5	8
100	9	3	3	9	9	3	3	9

Fig. 4.3, (b)

In many cases it was possible to include the 4-member sub-cubes in a larger sub-cube. In only one case was a choice available; the sub-cube --0010 could be combined with 0, 2 or 7. It was decided to combine it with 0, a particularly simple and symmetrical network resulting. This is shown in fig. 4.3, (c).

4.3.2 Carry and borrow networks.

Referring again to fig. 4.2, (a), a carry input to the second decade must appear when the first decade has been indicating '9' and a forward input is applied changing the indication to '0', that is when the first decade is in the state 000000. This carry input to the second decade must disappear when the counter input is released, for if it did not, and a reverse input was applied next, the input to the second decade (corresponding to X_1 and X_2) would change directly from

10 to 01, and the counter has not been designed to respond to such changes. (It could be that although the counter has not been designed to do so it might respond correctly to such an input change. This would seem unlikely, but is worth investigating). Hence of the states used by the counter the carry output must appear only for $y_1y_2y_3y_4y_5y_6$ = 000000. It may also appear for 001000 or 000001, these being adjacent unused states, so that the forward input to the second decade corresponding to X_1 may be $y_1'y_2'y_4'y_5'y_6'$ or $y_1'y_2'y_3'y_4'y_5'$. The complementary function corresponding to X_1' is also required, $y_1+y_2+y_4+y_5+y_6$ or $y_1+y_2y_3y_4y_5$. In the same way the borrow outputs corresponding to X_2 and X_2' may be $y_1y_2'y_4y_5'y_6'$ and $y_1'+y_2+y_4'+y_5+y_6$ or $y_1y_2'y_3'y_4y_5'$ and $y_1'+y_2+y_3+y_4'+y_5$ respectively. These several functions may be realised by the networks of fig. 4.3, (d) either or both of which y_6 may be substituted for y_3.

4.3.3 Completion of the design.

The total numbers of contacts required on each of the six relays may now be determined. It will be remembered that in designing the output network it was found advantageous to allow Y_4 to change before Y_2, and the corresponding secondary excitation networks have been used. The numbers of contacts required on each of the relays are shown in fig. 4.3, (e). The numbers given are for the circuits as drawn, without subsequent manipulation or combining of make and break contacts to form transfer contacts. The make and break contacts on Y_4 which form part of the excitation networks for Y_2 and Y_5 may be combined to form a single transfer contact. The numbers of moving springs on the various relays then varies between 8 and 10. While such heavily loaded relays are commercially available their use is not common.

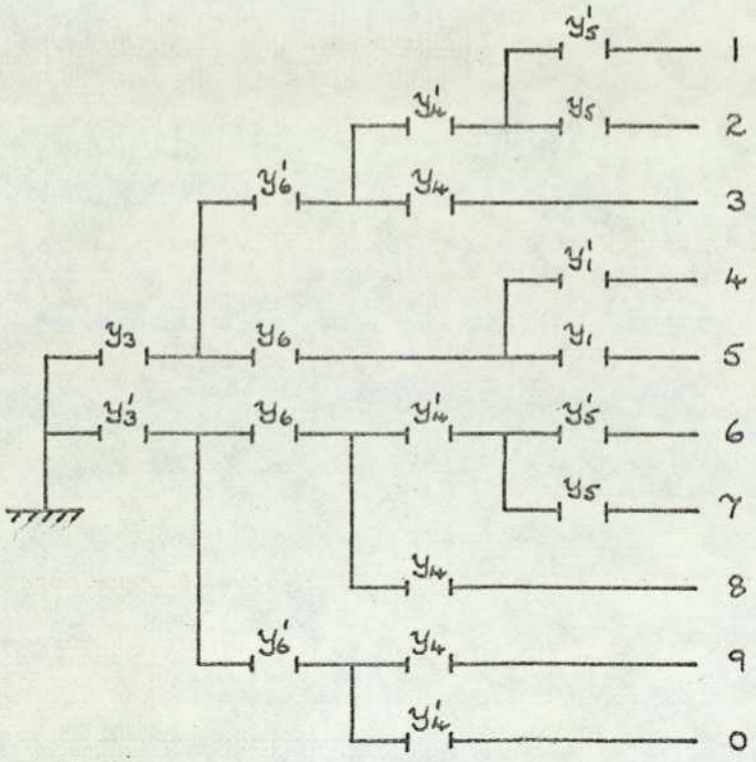


Fig. 4.3, (c)

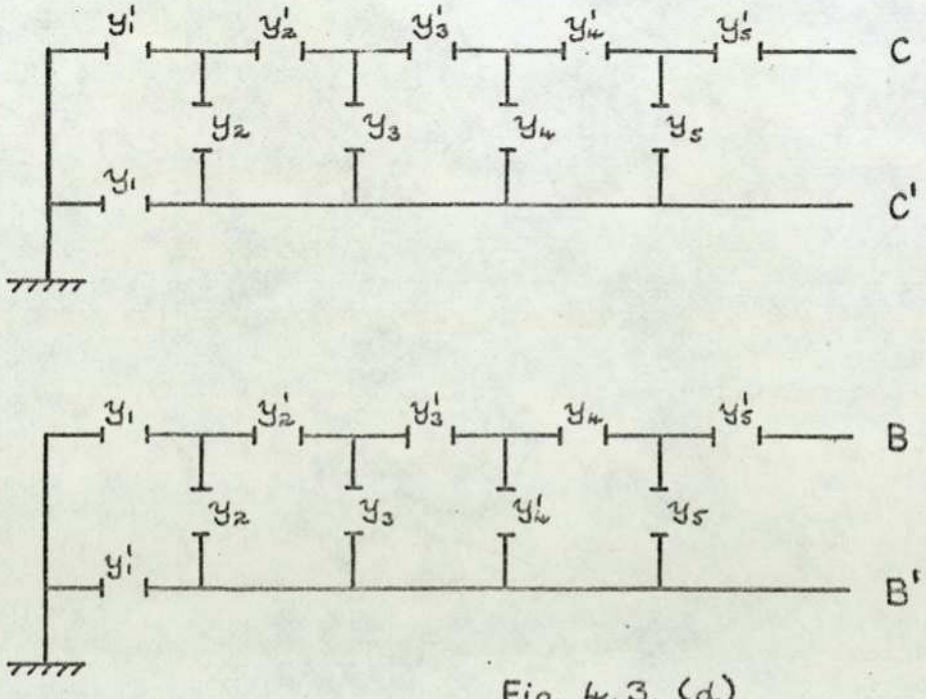


Fig. 4.3, (d)

Contacts on relay -	Y_1			Y_2			Y_3			Y_4			Y_5			Y_6			From fig.
Network -	T	M	B	T	M	B	T	M	B	T	M	B	T	M	B	T	M	B	
Y_1	1	1	-	1	-	-	-	-	-	1	-	1	1	-	-	-	-	-	4.2, (k)
Y_2	1	-	-	1	1	-	1	-	-	1	-	1	-	-	-	2	-	-	4.2, (p)
Y_3	1	-	-	-	-	-	1	1	-	-	-	-	-	-	-	2	-	-	4.2, (r)
Y_4	-	-	1	-	-	1	-	-	-	1	-	-	2	-	-	-	-	-	4.2, (t)
Y_5	-	-	-	2	-	-	1	-	-	-	1	-	1	1	-	2	-	-	4.2, (aa)
Y_6	1	-	-	1	-	-	2	-	-	-	-	-	-	-	-	1	1	-	4.2, (af)
All sec. excitation	4	1	1	5	1	1	5	1	-	3	1	2	4	1	-	7	1	-	
Output	1	-	-	-	-	-	1	-	-	3	-	-	2	-	-	2	-	-	4.3, (c)
Carry and borrow	2	-	-	2	-	-	2	-	-	2	-	-	2	-	-	-	-	-	4.3, (d)
Total	7	1	1	7	1	1	7	1	-	8	1	2	8	1	-	9	1	-	

Fig. 4.3, (e)

5 COUNTERS USING τ-FLIP-FLOPS

5.1 Introduction.

The previous counter was complex in design and the contact loads were somewhat heavy though not impossibly so. In the following sections will be presented a method of design in counters of any scale which is straightforward, and requires a larger number of relays with contact loads of a more acceptable size.

This design uses a number of τ-flip-flops, that is, bistable circuits which change state when the single input terminal is grounded and released. Each input switch consists of a single make contact connected to the input terminals of the τ-flip-flops via a tree network, so that when an input switch is operated the input terminal of one (or sometimes two) of the τ-flip-flops is grounded, and it changes state. Of the pair of relays which form a flip-flop, one changes state when the input switch is operated and the other when it is released. The output network is made up of contacts on the former and the secondary-excitation networks of contacts on the latter.

5.2 Design of the τ-flip-flop.

The flow-table is constructed in the standard way and shown in fig. 5.2, (a). ① and ③ are the two states stable in the absence of an input. X is the input variable.

X	0	1	
	①	2	p
	3	②	q
	③	4	r
	1	④	s

Fig. 5.2, (a)

The transition diagram is given in fig. 5.2, (b), and using two relays there are only two distinct secondary assignments, because of the symmetry between states ① and ③ , and ② and ④. These assignments and the excitation matrices to which they give rise are shown in fig. 5.2, (c).

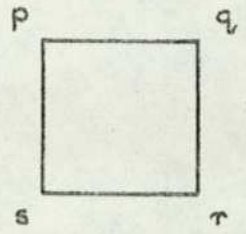


Fig. 5.2, (b)

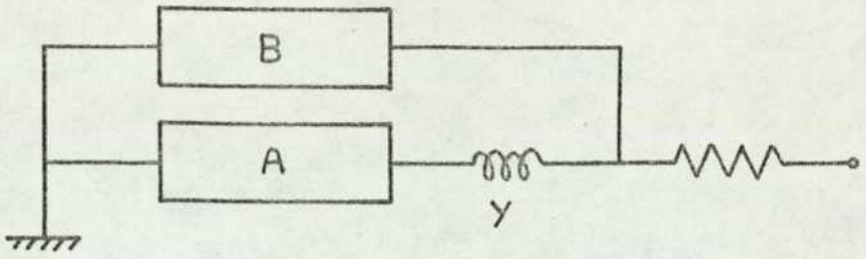


Fig. 5.2, (d)

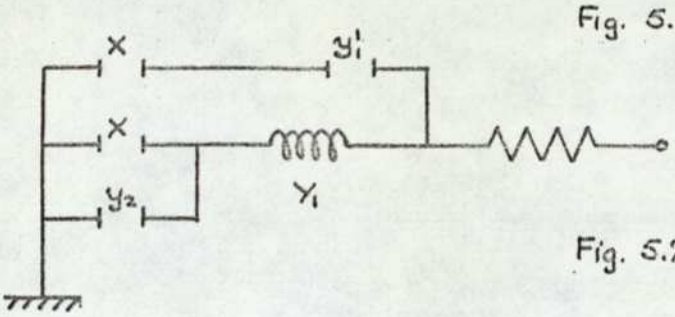


Fig. 5.2, (e)

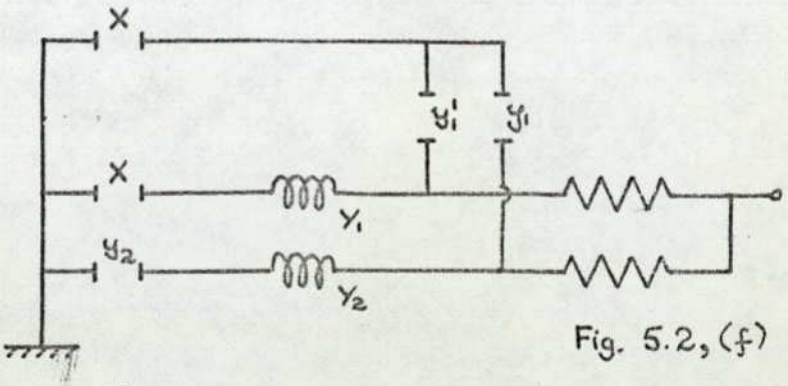


Fig. 5.2, (f)

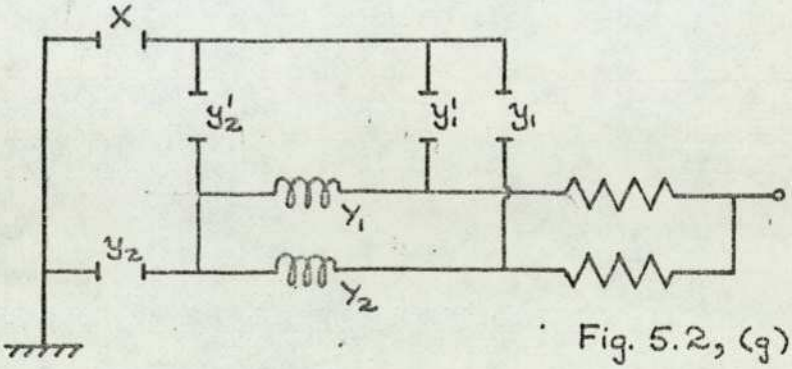


Fig. 5.2, (g)

X	0	1	y_1y_2	0	1	y_1y_2
	00	01	00	01	11	01
	11	01	01	10	11	11
	11	10	11	10	00	10
	00	10	10	01	00	00
	Assignment I			Assignment II		

Fig. 5.2, (c)

The excitation functions obtained directly from these matrices are

$$Y_1 = X'y_2 + Xy_1$$
$$Y_2 = X'y_2 + Xy_1'$$

for assignment I, and

$$Y_1 = X'y_1 + Xy_2$$
$$Y_2 = X'y_1' + Xy_2$$

for assignment II. Contact networks having these transmissions cannot be realised with a single make contact on the input switch (an X contact) only. However, the excitation function may be realised by using combined series and shunt control. Fig. 5.2, (d) shows a relay Y controlled by series and shunt networks A and B respectively. The excitation is given by $Y = AB'$. To obtain Y_1 and Y_2 in this form one first writes down the expression in product of sums form and then complements the factor containing X'. Thus, for assignment I

$$Y_1 = (X + y_2)(X' + y_1)$$
$$= (X + y_2)(Xy_1')$$

and

$$Y_2 = (X + y_2)(X' + y_1')$$
$$= (X + y_2)(Xy_1)'$$

the two functions differing only in the complementation of y_1. The network for Y_1 is shown in fig. 5.2, (e). The network for Y_2 is similar, the y_1 contact being replaced by a y_1' contact. The networks for the two relays may be combined as in fig. 5.2, (f). No sneak paths are introduced, the path between the live sides of the relay coils containing complementary contacts. The two X contacts may now be combined provided the lower one is first isolated to avoid a sneak path. The resulting network is shown in fig. 5.2, (g). Assignment II does not lead to a circuit containing only one X contact. The circuit of fig. 5.2, (g) is given by Frost.(38)

5.3 Scale-of-four counter on two flip-flops.

Before proceeding with the design of a scale-of-10 counter a scale- of-4 counter will first be designed. This is a less complex problem and throws some useful light on the best way to choose a sequence of states. Four states will require two bistable relay-pairs and the problem is to design the network which will replace the X contacts in the T-flip-flops. This will consist of make contacts on each of the input switches, designated X_1 and X_2, with a tree network connected to each through which the input terminal of the bistable flip-flop which is required to change state will be grounded when the input switch is operated. The two trees will be combined to an extent to be determined.

The flow-table (not to be confused with the flow table for a single bistable flip-flop) is shown in fig. 5.3, (a). The two flip-flops are designated B_1 and B_2 and the columns headed b_1b_2 represent their successive states, the sequence being that in which one flip-flop changes at a time. The columns headed B_1 indicate the states for which the input terminal of flip-flop B_1 is required to be grounded; B_{1f} (forward counting) indicates that it is to be grounded via X_1 and B_{1f} (reverse counting) that it is to be grounded via X_2. Thus if the counter is in the state b_1b_2 = 01 and the forward counting input switch X_1 is operated, flip-flop B_1 is required to change state, so its input terminal must be grounded via X_1; the entry in column B_{1f}, row b_1b_2 = 01 is therefore 1. The other entries in the column B_{1f} and also in the columns B_{1r}, B_{2f}, B_{2r} are similarly determined.

	b_1b_2	B_{1f}	B_{1r}	B_{2f}	B_{2r}	
forward ↓	00	0	1	1	0	↑ reverse
	01	1	0	0	1	
	11	0	1	1	0	
	10	1	0	0	1	

Fig. 5.3, (a)

The resulting functions are

$$B_{1f} = b_1'b_2 + b_1b_2' = B_{2r}$$

$$B_{2f} = b_1'b_2' + b_1b_2 = B_{1r}$$

These may be realised by the networks of fig. 5.3, (b) which may then be combined to form the network of fig. 5.3, (c). There is

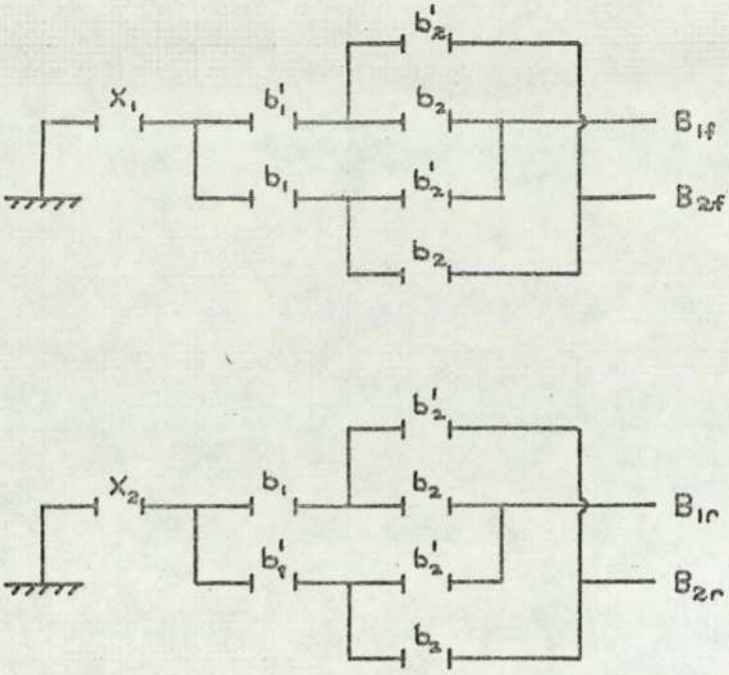


Fig. 5.3, (b)

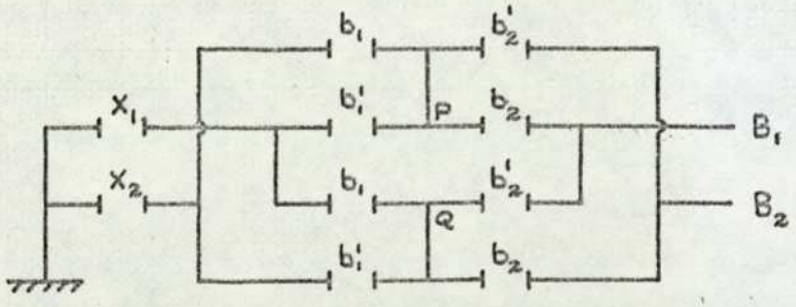


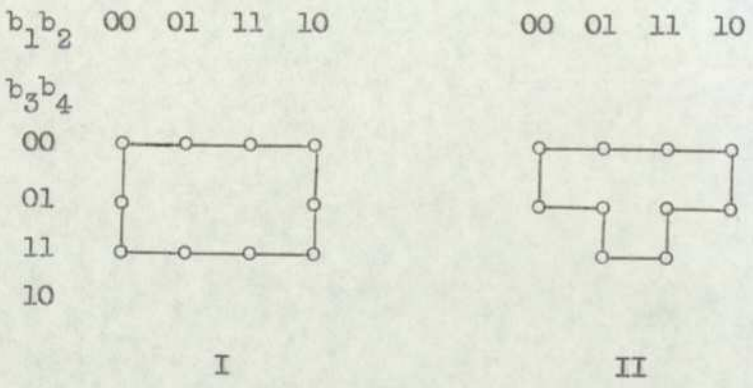
Fig. 5.3, (c)

no path through the network between X_1 and X_2 since the flip-flop which must change for forward counting is never the same as that which must change for reverse counting. There is an alternative similar network with the contacts on B_1 and B_2 in the opposite order.

It will be noted that the two networks for B_1 and B_2 may be combined apart from the transfer contacts on B_1 connected to X_1 and X_2. This is clearly an economical feature, and one to be retained, if possible, in the scale-of-ten counter. Referring to fig. 5.3, (c) it will be seen that P is grounded if $X_1 b_1' + X_2 b_1 = 1$ while Q is grounded if $X_1 b_1 + X_2 b_1' = 1$. Both these functions are unchanged if X_1 and X_2 are interchanged and b_1 is complemented. This implies that if for a certain state in the count a given flip-flop changes state when X_1 is grounded, the same flip-flop must change state for the state in the count for which b_1 has the complementary value when X_2 is grounded. Thus the sequence must be chosen in such a way that if b_1 is complemented it remains the same sequence, but the directions of counting are interchanged.

5.4 Scale-of-ten counter on four flip-flops.

It is easy to find sequences satisfying the condition derived in the previous section. This may be done semi-intuitively using the Karnaugh map. There are only two fundamentally different sequences. These are shown in fig. 5.4, (a).



I II

Fig. 5.4, (a)

The same conclusion may be arrived at by examining the sequences of section 2. Those required are the ones which are unaltered by reversing the direction of counting. These are C and K, corresponding to II and I respectively after permutation of the variables. In each case the state 0000 has been included in the sequence since this is the state into which the counter will go when first switched on. It

should be noted however, that if the state of a flip-flop is indicated by a transfer contact on one of its relays the outward indication (as distinct from the internal operation) is completely symmetrical, the two fixed springs being alternately connected to the moving spring as the input contact is grounded (or released).

The procedure for obtaining the excitation functions is the same as for the scale-of-four counter, but since four variables are involved, maps have been prepared rather than tables. These are for sequence I and are shown in fig. 5.4, (b).

		Forward (Clockwise on map)				Reverse (Anticlockwise on map)			
	b_1b_2 / b_3b_4	00	01	11	10	00	01	11	10
B_1	00	0	1	0	0	0	0	1	0
	01	0	·	·	0	0	·	·	0
	11	0	0	1	0	0	1	0	0
	10	·	·	·	·	·	·	·	·
B_2	00	1	0	1	0	0	1	0	1
	01	0	·	·	0	0	·	·	0
	11	0	1	0	1	1	0	1	0
	10	·	·	·	·	·	·	·	·
B_3	00	0	0	0	0	0	0	0	0
	01	0	·	·	1	1	·	·	0
	11	1	0	0	0	0	0	0	1
	10	·	·	·	·	·	·	·	·
B_4	00	0	0	0	1	1	0	0	0
	01	1	·	·	0	0	·	·	1
	11	0	0	0	0	0	0	0	0
	10	·	·	·	·	·	·	·	·

Fig. 5.4, (b)

It has already been decided that the network will contain two transfer contacts on B_1, one connected to X_1 the other to X_2 and joined as in fig. 5.3, (c). The rest of the network may be considered in two parts, one joined to P, the other to Q. Each of these networks may be considered to be derived from a complete decoding tree for 3 variables having 7 transfer-contacts by a

process of simplification. It may be noted that

(a) the elimination of an output for an unused state eliminates a make or break contact, but no moving springs, unless it differs from a wanted output only in the variable which is last in that part of the tree, in which case a transfer contact may be eliminated.

(b) The only function which contains two '1's in the part of the network connected to $X_1 b_1' + X_2 b_1$ is B_2. These two '1's differ in every variable so they cannot be combined so as to eliminate any contacts.

(c) The tree network must have five outputs before the two on B_2 are joined. Since each transfer contact joins two outputs or common terminals of transfer contacts, it cannot contain less than four transfer contacts. If a network can be found containing four transfer contacts, it is minimal.

(d) The tree may be exhaustive (in Lawler's sense[15]), but need not be. Considering now the separate functions, if the 1 in B_1 connected to $X_1 b_1' + X_2 b_1$ is to be combined with a '1' in an adjacent unused state, the last transfer contacts in the network must be on B_3 or B_4. The corresponding results for all the functions are tabulated below.

Function	Transfers last in tree
B_1	B_3 or B_4
B_2 (000)	B_3
B_2 (111)	B_3 or B_4
B_3	B_4
B_4	B_2

It is evident that the transfers on B_3 or B_4 ought to be last in the tree. The two alternatives lead to the networks of figs. 5.4, (c) and (d).

In each case the levels of contacts on B_2 and on B_3 and B_4 respectively may be interchanged, leaving the total number of contacts unaltered. All four alternative networks contain four transfer contacts, so they are all minimal.

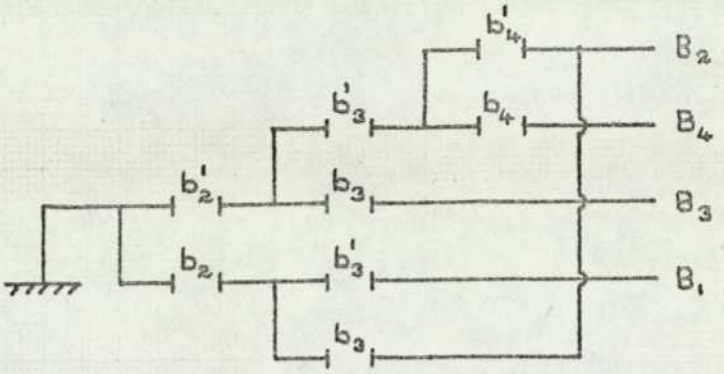


Fig. 5.4, (c)

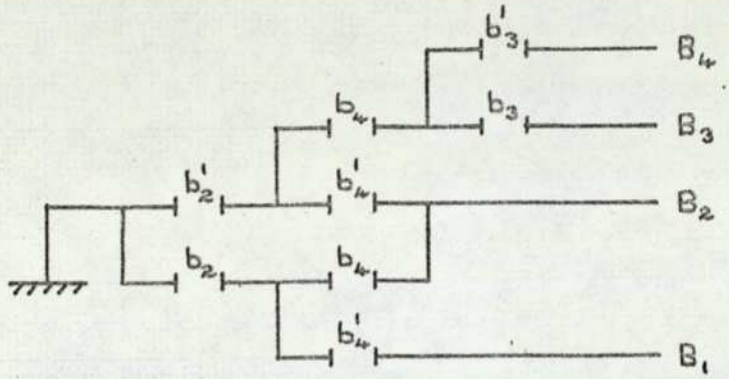


Fig. 5.4, (d)

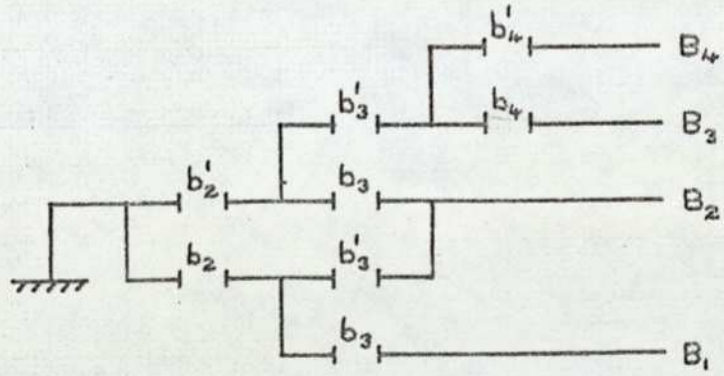


Fig. 5.4, (e)

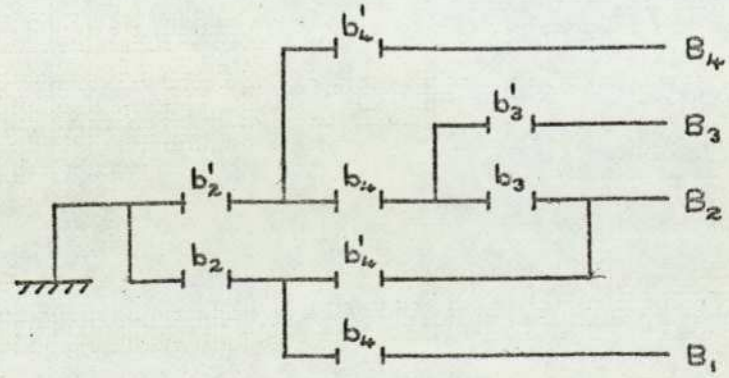


Fig. 5.4, (f)

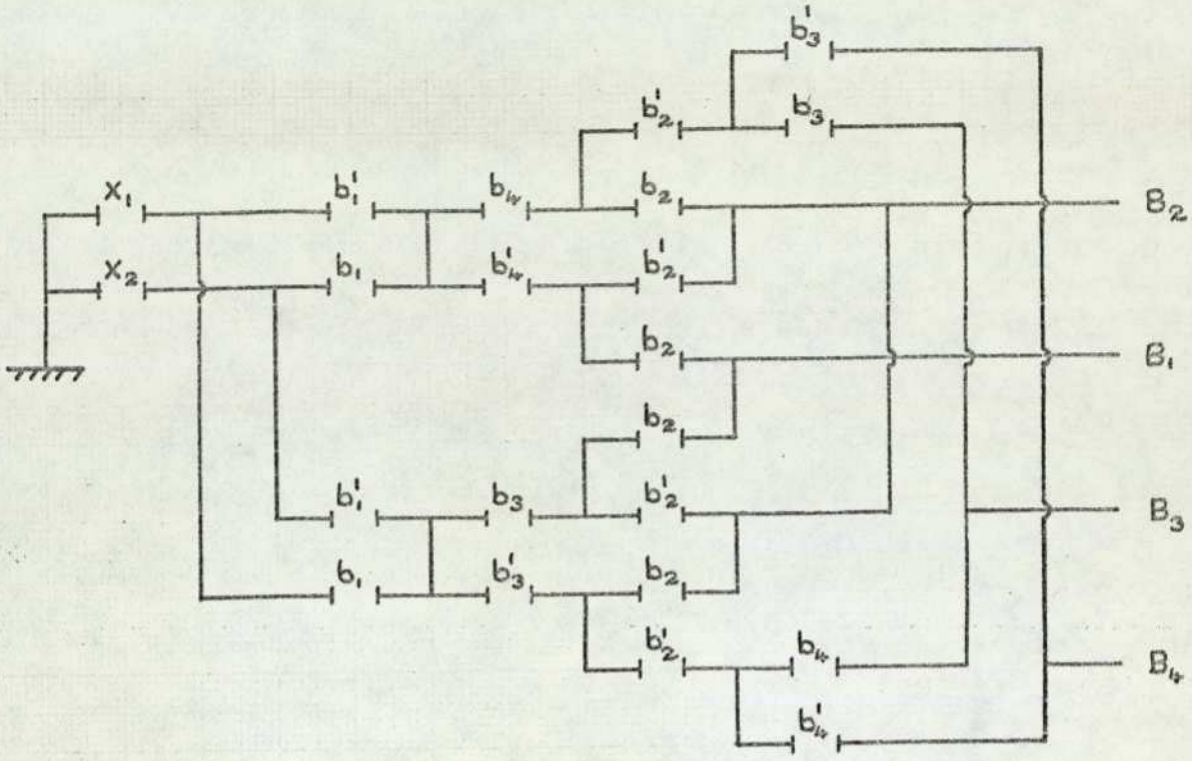


Fig. 5.4, (g)

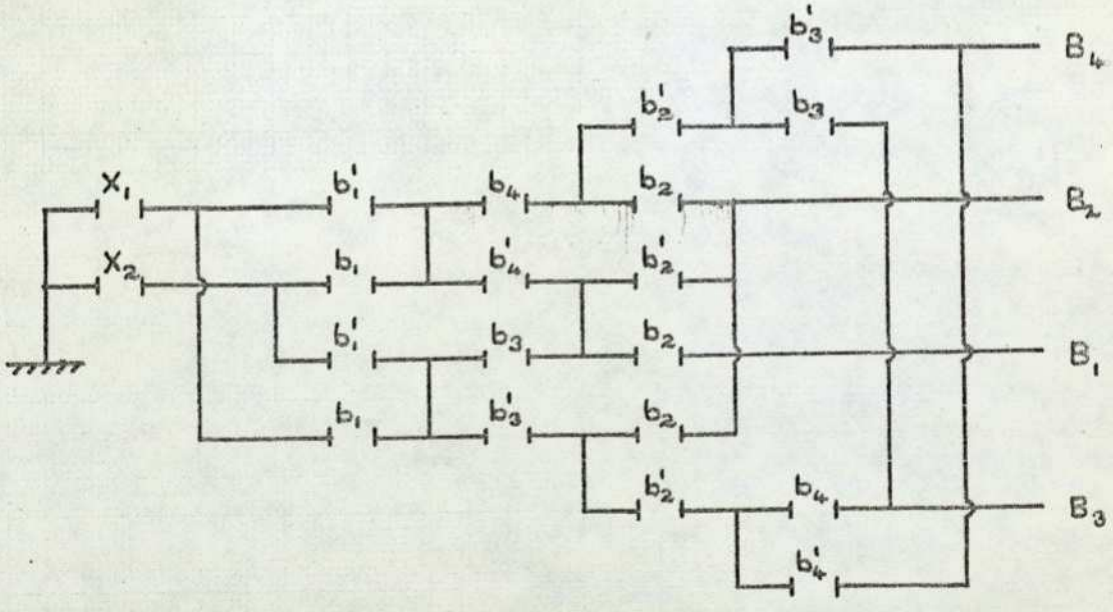


Fig. 5.4, (h)

The corresponding results for the other half of the network connected to $X_1b_1 + X_2b_1'$ are

Function	Transfers last in tree
B_1	B_3 or B_4
B_2 (100)	B_3 or B_4
B_2 (011)	B_4
B_3	B_2
B_4	B_3

The situation is very similar to that for the first half of the network. The alternatives for the second half of the network are shown in figs. 5.4, (e) and (f). Again the levels of the contacts on B_2 and on B_4 and B_3 respectively may be interchanged, leaving the total number of contacts unaltered, and all four alternative networks contain four transfer contacts, so they are all minimal.

One of the networks of figs. 5.4, (c) and (d) must now be combined with one of the networks of figs. 5.4, (e) and (f), two transfer contacts on B_1, and the make input contacts X_1 and X_2 to form the complete secondary excitation network. If the network of fig. 5.4, (d) and the network of fig. 5.4, (e) are chosen and in each case the contacts on B_2 are transferred to the intermediate level, the network of fig. 5.4, (g) is the result.

It will be noted that between outputs B_1 and B_2 there are two transfer contacts on B_2, and it is necessary to ask whether they may be combined. This is the same as joining nodes B and C, but these nodes are always joined either via two b_2' contacts or via two b_2 contacts, so that it is permissible to replace the two transfer contacts by a single one. The same is not true of nodes A and D. These are joined only for $b_2 + b_3b_4 = 1$ as the network stands. To join them introduces a $b_3'b_4$ path between X_1 and X_2 which includes two states in the sequence of ten, and so is not permissible. When the permissible modifications have been made the result is the network of fig. 5.4, (h).

5.4.1 Carry and borrow networks.

The next problem to be considered is the provision of carry and borrow outputs when two or more decades are cascaded. If we insist that changes in the second decade must take place at the same time as those in the first (without so much delay as the operating or release time of a relay) the closure of the input contact must

ground the input terminal of two flip-flops, one in the first and one in the second decade, simultaneously. In that case, when the two decades are viewed as a whole the condition that only one flip-flop shall change state at a time has been relaxed. If this may be done with cascaded decades there would appear to be no reason why it should not be done in the design of a single decade or counter of other scale. This is important, since it would permit the design of counters with odd scales. If only one flip-flop may change at a time, since every flip-flop must change state an even number of times in order to return to the initial state, only even scales would be possible.

If two flip-flops change state simultaneously, their input terminals must be connected together before they are grounded via the input contact. We inquire therefore, whether two flip-flops whose input terminals are connected together interfere with each other's operation in any way. Two such flip-flops are shown in fig. 5.4, (j). The action of the top flip-flop will be affected by the bottom one only if the latter adds a sneak path in parallel with X. Examination shows that it adds no short circuit path in parallel with X, but it does add a path containing resistance - $y_1'y_2$ in series with the coil of Y_1 and y_1y_2 in series with the coil of Y_2, altogether a path of transmission y_2 with some resistance.

This sneak path can only affect the operation of the circuit when it is closed and X is open. This situation arises when y_1y_2 = 01 and X has just been released. In that case either y_3y_4 = 01 or y_3y_4 = 10. If y_3y_4 = 01 P rises to a potential determined by the series and coil resistances and Y_1 and Y_3 are energised and operated. When Y_1 is operated P is disconnected from the junction of R_1 and Y_1 and connected to the junction of R_2 and Y_2, but its potential is affected by onoy a small amount depending on resistance tolerances. The same applies to Y_3. Since the potential of P is unaffected neither flip-flop interferes with the operation of the other.

If y_3y_4 = 10, when X is released Y_1 is operated. The path through R_3, Y_3 y_4' and X is broken but current may continue to flow through Y_3 through the sneak path previously pointed out, that is, via y_4', then first through y_1' and Y_1 afterwards via y_1 and Y_2, the path being completed by y_2. The current through Y_3 will be reduced, though not to zero. The ratio in which the current is reduced depends on the ratio of the coil and series resistances. If these are equal

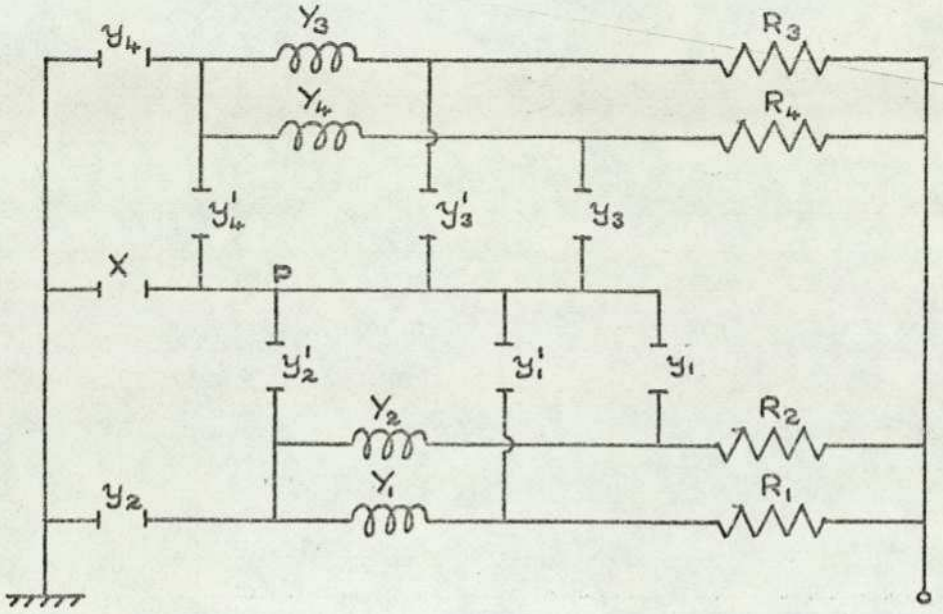


Fig. 5.4, (j)

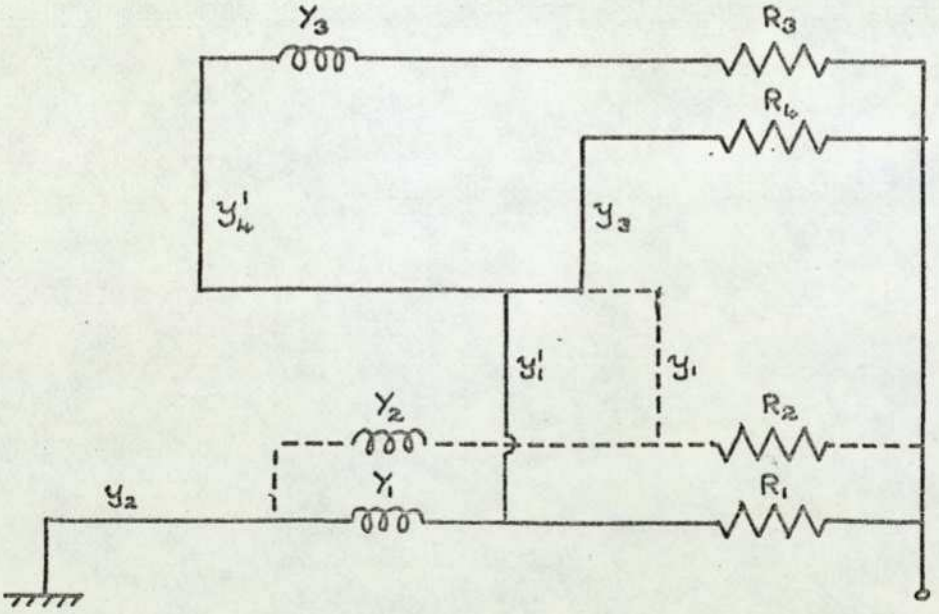


Fig. 5.4, (k)

it will be reduced in the ratio $\frac{2}{1+\frac{2}{5}} \times \frac{2}{5} = \frac{2}{7}$ (see fig. 5.4, (k)).

Whether Y_3 will be released depends on the particular characteristics of the relays used. In a test circuit using BPO type 3000 relays it was found that it was not released. To insure the release of Y_3 the current ratio may be reduced by decreasing the series resistance but this will increase the dissipation in the series resistance when a relay is prevented from being energized because its shunt network is a short circuit. It seems unlikely that in many cases the increased dissipation will be an acceptable price to pay for the removal of delay in the operation of the higher decades, but the necessary "carry" and "borrow" networks will be determined for the case in which it is acceptable and for the case in which it is not.

5.4.1.1 Carry and borrow networks for simultaneous operation of all decades.

Referring to the map showing the sequence (fig. 5.4, (a),(I) if $b_1b_2b_3b_4 = 0000$ is the '0' state and forward counting proceeds clockwise round the map $b_1b_2b_3b_4 = 0001$ is the '9' state. The "carry" output terminal of the first decade, that is, the forward input terminal of the second decade is required to be grounded when B_4 flip-flop input terminal is grounded to change the first decade from the '9'- to the '0'-state. However, B_4 input terminal is also grounded when B_4 flip-flop changes state a second time when counting in the forward direction (when the first decade changes from the '3'- to the '4'-state) and also on two occasions when counting in the reverse direction, and in none of these three occasions is the forward input terminal of the second decade required to be grounded. Of the four states in which the input terminal of B_4 flip-flop is grounded the state in which the forward input terminal of the second decade is required to be grounded is distinguished by having $b_1 = 0$, $b_4 = 1$. Hence the "forward" input to the second decade may be obtained by connecting a $b_1'b_4$ path to the input terminal of the B_4 flip-flop in the first decade. In a similar way the "reverse" input to the second decade may be obtained by connecting a $b_1'b_4'$ path to the input terminal of the B_4 flip-flop in the first decade. The b_1' contact may be made common to both networks, so that the contacts to be added are as shown in fig. 5.4, (ℓ). It is to be noted that the b_4 and b_4' contacts must remain closed, respectively, until the input contact is released. Hence they must be on that relay forming part of the B_4

flip-flop which changes state second, that is, the one corresponding to Y_1 in the circuit of fig. 5.2, (g). On the other hand, since the changes of state of B_1 flip-flop are remote from those during which inputs to the second decade are required, the contact labelled b_1' may be on either of the relays forming part of the B_1 flip-flop.

5.4.1.2 Carry and borrow networks for reduced dissipation, but with propagation delay.

The alternative to grounding the input terminals of the second decade via the input terminals of the first, with the increased dissipation necessary to ensure that flip-flops in one decade do not interfere with flip-flops in another is to ground them via contacts on the relays of the first decade only, without making use of the input contacts to the first decade. It is now necessary to distinguish the two relays which make up each of the flip-flops B_1 - B_4, calling these Y_1 - Y_8 respectively, and letting the even-numbered relays be those which change state when the input terminal is grounded, corresponding to Y_2 in fig. 5.2, (g). The forward input terminal of the second decade must be grounded when B_4 is changing state and $y_7y_8 = 10$. However, $y_7y_8 = 10$ when counting in the reverse direction and going from '4' to '3'. In the former case $b_1b_2b_3 = 000$ and in the latter $b_1b_2b_3 = 100$. The former case is distinguished by having $b_1 = 0$, so that the path required is $b_1'y_7y_8'$. Similarly the reverse input terminal of the second decade must be grounded via a $b_1'y_7'y_8$ path. The b_1' contact may be made common to both paths giving either of the networks shown in fig. 5.4, (n). As before, the b_1' contact may be on either of the relays forming the flip-flop B_1.

The choice between the two networks of fig. 5.4, (n) and the allocation of the b_1' contact to either Y_1 or Y_2 depend on the availability of contacts on the various relays. This in turn depends on the design of the output networks which for the earliest response must be placed on the even-numbered relays, which change state when an input is grounded, in contrast to the secondary excitation network, which must be built of contacts on the odd numbered relays.

5.4.1.3 Output networks: Completion of the design.

The output network will be a tree and if one thinks of this as being built up from the root, the first transfer contact enables one to distinguish between two outputs and each additional transfer contact enables one to distinguish an additional output. Hence the

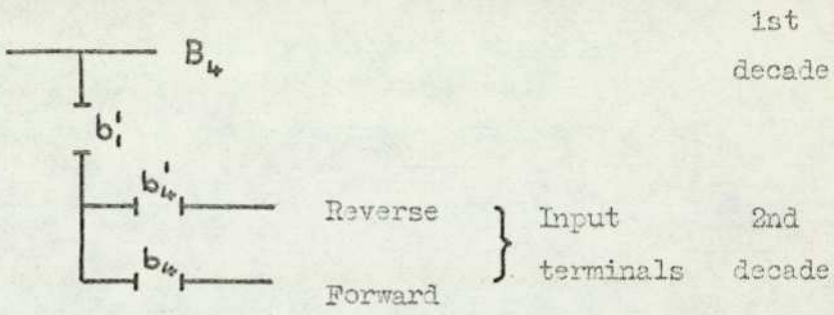


Fig. 5.4, (ℓ)

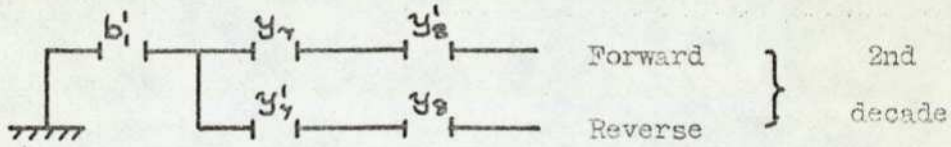


Fig. 5.4, (m)

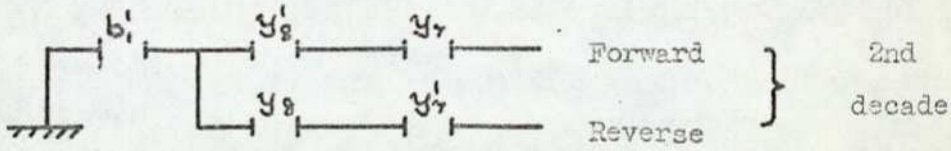


Fig. 5.4, (n)

number of contacts necessary is one less than the number of outputs. It appears that this number is also sufficient. If this is so the number of transfer contacts will be nine. Of these one relay can have only one, and the remaining eight may be distributed over the remaining three relays in an "almost uniform" manner,[12] so that there will be three transfer contacts on each of two of the three relays and two on the one remaining.

We may now list the numbers of transfer-contacts on the various relays in the various component networks, (fig. 5.4, (q)). It will be our aim to build the counter on standard relays with four transfer contacts. The flip-flops contain a transfer contact on each relay. The secondary excitation network is shown in fig. 5.4, (h), from which are taken the figures in the third row of the table. Separate make and break contacts are not available on Y_7 so carry and borrow networks of fig. 5.4, (m) must be used, requiring a single transfer contact on Y_7 and two contacts on Y_8. The break contact on B_1 may be on Y_1 since no other contacts are required on this relay. These figures are entered in the fourth row in the table. We now have available for the output network one transfer contact on Y_8 and three each on Y_2, Y_4 and Y_6. This is sufficient with two to spare, provided a satisfactory distribution may be achieved.

The single transfer contact must be on Y_8 (B_4) as only one transfer contact is available on this relay. For all the states for which $Y_8 = 0$ the value of Y_6 (B_3) is immaterial so this part of the tree is completed by three transfer contacts, one on Y_2 and two on Y_4 or vice versa. For the other part of the tree, for which $Y_8 = 1$ we may begin with three contacts (altogether) on Y_2 and Y_4 but transfer contacts on Y_6 must be added to the $y_8y_2'y_4'$ and $y_8y_2y_4'$ paths. The completed tree is shown in fig. 5.4, (p). This completes the design, for the counter with propagation delay and reduced dissipation. The counter without propagation delay differs only in the carry network. Since this requires fewer contacts than the counter with propagation delay, the rest of the circuit may be the same. The numbers of transfer contacts required in the output network have been entered in the fifth row of fig. 5.4, (q), and the totals on each relay in the final row.

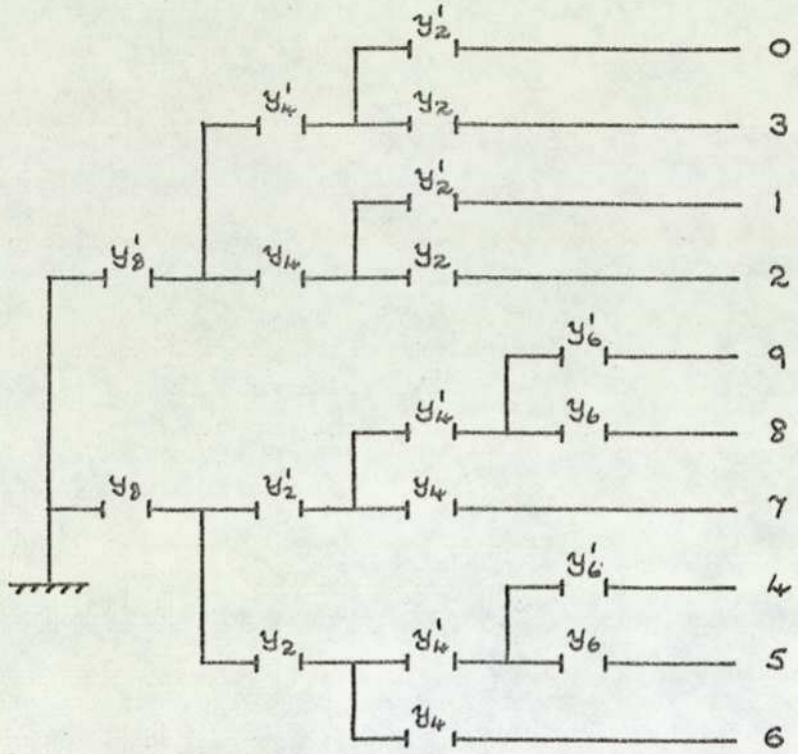


Fig. 5.4(p)

Contacts on Relay	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8
Flip-flops	1	1	1	1	1	1	1	1
Secondary excitation	2		3		2		2	
Carry and borrow	1						1	2
Output		3		3		2		1
Total	4	4	4	4	3	3	4	4

Fig. 5.4, (q).

5.5 Counters of other scales, including odd scales.

It was pointed out in section 5.4.1 that if more than one flip-flop may change state at the same time the restriction to even scales is removed. It was later concluded that two flip-flops may change state simultaneously in the same direction although difficulties arise if they are required to change state simultaneously in opposite directions. It is sufficient to remove the restriction to even scales if two flip-flops may change state simultaneously in the same direction. However, there are restrictions on the sequence of changes.

5.5.1 Reversible scale-of-three counter on two flip-flops, and scale-of-seven on three, not possible.

Suppose it is proposed to build a scale-of-three counter on two flip-flops B_1 and B_2. At one step in the count both flip-flops must change state simultaneously from the '0'-state to the '1'-state, and vice versa in the opposite direction. The complete sequence is shown in fig. 5.5,(a). An apparently different sequence is obtained by using B_1B_2 = 01 as the third state but this is merely an interchanging of B_1 and B_2.

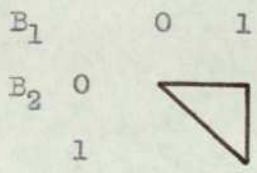


Fig. 5.5, (a)

Let the forward direction of counting be clockwise on the map. Suppose the counter is in the state $B_1B_2 = 00$. A reverse input would send the counter to the 11 state so that before the reverse input terminal is grounded the reverse input terminals of both flip-flops must be connected together by the reverse secondary excitation network. Buf if so the grounding of the forward input terminal, which ought to ground the input terminal of B_1 only, would also ground the input terminal of B_2, and the counter would go to the state 11. Thus a flip-flop which must change state when leaving a state of the counter in the forward direction must not change state when leaving the same state in the reverse direction and vice versa. This makes it impossible to build a scale-of-three counter on two flip-flops.

This same rule makes it impossible to build a scale-of-seven counter on three flip-flops. Such a counter if it could be built, could be thought of as derived from a scale-of-eight counter with two steps coalesced into one. But in the scale-of-eight counter one flip-flop must change state four times and since it cannot change state on successive steps it must change state on alternate steps. If then two steps are combined, this flip-flop must change state when leaving one state in either direction, which, as has been shown, is not permissible.

5.5.2 Scale-of-five counter on three flip-flops. Choice of sequence.

It is possible to construct a scale-of-five counter on three flip-flops. Numbering these 123, suppose 2 and 3 change state simultaneously at one step. At the adjacent steps flip-flop 1 must change state. At the two remaining steps 2 and 3 must change state. The order does not matter since 2 and 3 have not been distinguished, and in any case the alternative order is followed in the reverse direction. For this reason the secondary excitation networks for the reverse and forward directions may be obtained from each other by interchanging 2 and 3. It is a matter for choice whether B_1 is in the '0'- or the '1'-state when B_2 and B_3 change simultaneously between the '0'- and the '1'-state. This makesno difference to the secondary excitation networks (except for the complementation of b_1) but it does affect the position of the '0'-state in the sequence, this being taken as the state into which the counter goes when switched on. This turn affects the design of the carry and borrow networks where these

are required, since the step at which two relays change state is in a different relation to the step at which carry and borrow outputs are required to appear. The two possibilities are shown in fig. 5.5, (b).

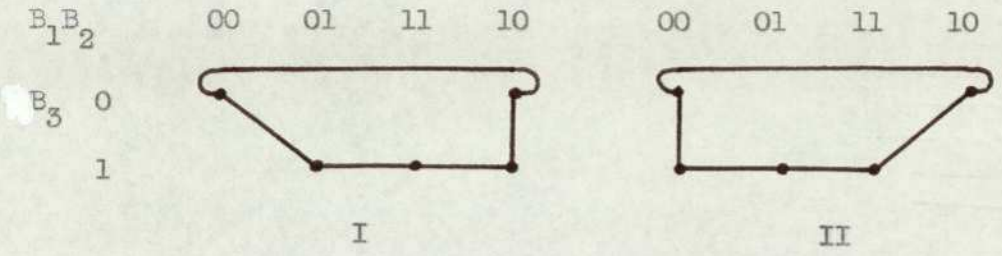


Fig. 5.5, (b)

In I the step at which the carry and borrow outputs are to appear may be the step at which two flip-flops change state simultaneously, if the forward direction of counting is clockwise on the map, or adjacent to it, if the forward direction is anti-clockwise. In II the step at which two flip-flops change state simultaneously is remote from that at which the carry and borrow outputs are required to appear, and this is the one chosen for further development. The forward direction has been taken as clockwise.

5.5.2.1 Secondary excitation network.

The construction of the secondary excitation maps is straightforward, the procedure being the same as for the scale-of-ten counter. They are given in fig. 5.5, (c).

		B_1				B_2				B_3			
	b_1b_2	00	01	11	10	00	01	11	10	00	01	11	10
Forward	b_3 0	1	·	·	0	0	·	·	1	0	·	·	1
	1	0	0	1	·	0	1	0	·	1	0	0	·
Reverse	b_3 0	0	·	·	1	0	·	·	0	1	·	·	0
	1	0	1	0	·	1	0	1	·	0	0	1	·

Fig. 5.5, (c)

In deriving secondary excitation networks from these maps it may first be noted that the functions are not disjunctive. This is a necessary consequence of the fact that at one step in the count two flip-flops are required to change state simultaneously and therefore their input terminals must be grounded simultaneously. It follows that Scheinmann's first method of designing the networks cannot be used except in its modified form.[17] This requires that, when two (or more) output terminals are required to be grounded simultaneously,

one (any one) of them shallbe grounded directly via the tree and the other (or others) shall be connected to it by an additional network (or networks) added to the tree.

In this particular problem B_2 and B_3 outputs must be connected when $b_1b_2b_3 = 100$ and X_1 is grounded. They may be connected when $b_1b_2b_3 = 100$ and X_2 is grounded, since neither B_2 nor B_3 is required to be grounded in this condition. For a similar reason B_2 and B_3 outputs must be connected when $b_1b_2b_3 = 111$. They may also be connected when $b_1b_2b_3 = 110$ and 101 since these states never occur, and so for all the states for which $b_1 = 1$. A b_1 contact between B_2 and B_3 outputs is therefore required.

The next step is to design the trees. It may be noted from the forward secondary excitation maps, that of the four two-entry sub-cubes not containing b_3 or b_3' as a factor, only one, $b_1b_2 = 00$, contains different entries for the two values of b_3. Hence, if, in the development of the tree b_3 is made the last variable (variable furthest from the root), only one transfer contact on B_3 will be required. A similar conclusion may be drawn with respect to the number of transfer contacts on B_2. Of the four two-entry sub-cubes not containing b_1 or b_1' as a factor, two contain different entries for the two values of b_1 so that, if b_1 is made the last variable in the tree, two transfer contacts on B_1 will be required. Now every state forming part of the sequence must contain an entry '1' on at least one of the forward secondary excitation maps (for otherwise the counter could never leave that state in the forward direction) and hence every two-entry sub-cube contains an entry '1' on at least one of the maps, except that sub-cube which contains two states not forming part of the sequence, namely $b_1b_2b_3 = -00$. Hence, if b_1 is made the last variable in the development of the tree the $b_2'b_3'$ path through the higher levels of the tree will not be required and either a b_2' or a b_3' contact may be omitted. The number of contacts required in the tree will be two transfer-contacts on B_1, one transfer-contact on B_2 or B_3 and a transfer-contact and a make-contact on B_3 or B_2, respectively, four transfer-contacts and a make-contact in all. On the other hand, if b_2 or b_3 is made the last variable in the tree, three transfer-contacts are needed in the two higher levels, four transfer-contacts in all.

Exactly the same conclusion may be arrived at with reference to the reverse secondary excitation network. This may be done

either by repetition of the foregoing argument or from symmetry. The sequence in which the flip-flops change state when counting in the forward direction is 3, 1, 23, 1, 2. If this sequence is reversed and 2 and 3 are interchanged there is no nett change. It must not be concluded, however, that the reverse excitation network may be obtained from the forward merely by interchanging b_2 and b_3. The state in the reverse sequence which corresponds to $b_1b_2b_3 = 100$ in the forward sequence is $b_1b_2b_3 = 111$. Hence b_2 and b_3 must not only be interchanged but complemented. That reversing the forward sequence, interchanging b_2 and b_3 and complementing them yields the original forward sequence is easily verified. The details are given in fig. 5.5, (d).

Forward sequence						
000		011		011		000
100		111		111		100
111	⟶	100	⟶	100	⟶	111
011	Reverse	000	Interchange	000	Complement	011
001		001	b_2 and b_3	010	b_2 and b_3	001

Fig. 5.5, (d)

The reverse secondary excitation network is therefore to be obtained from the forward by interchanging and complementing b_2 and b_3, as well as substituting X_2 for X_1. B_2 and B_3 also require to be interchanged.

The forward and reverse secondary excitation networks may now be drawn, as shown in fig. 5.5, (e) with the intention of examining whether they may in any degree be combined. This will be most evident if the order of the contacts in the tree is the same for both. This is possible and convenient since it has already been concluded that networks of equal and minimum complexity will be obtained in each case if b_2 or b_3 occupies the highest level in the trees. The choice, still open, of connecting the $X_1b_1b'_2b'_3$ and $X_2b_1b_2b_3$ paths to either B_2 or B_3 is indicated, but the b_1 contact between B_2 and B_3 has not been shown. b_3 has been placed at the highest level. If b_2 is placed at the highest level the variables are permuted and complemented, but the problem is otherwise unaltered.

Examination of these networks did not reveal any instances

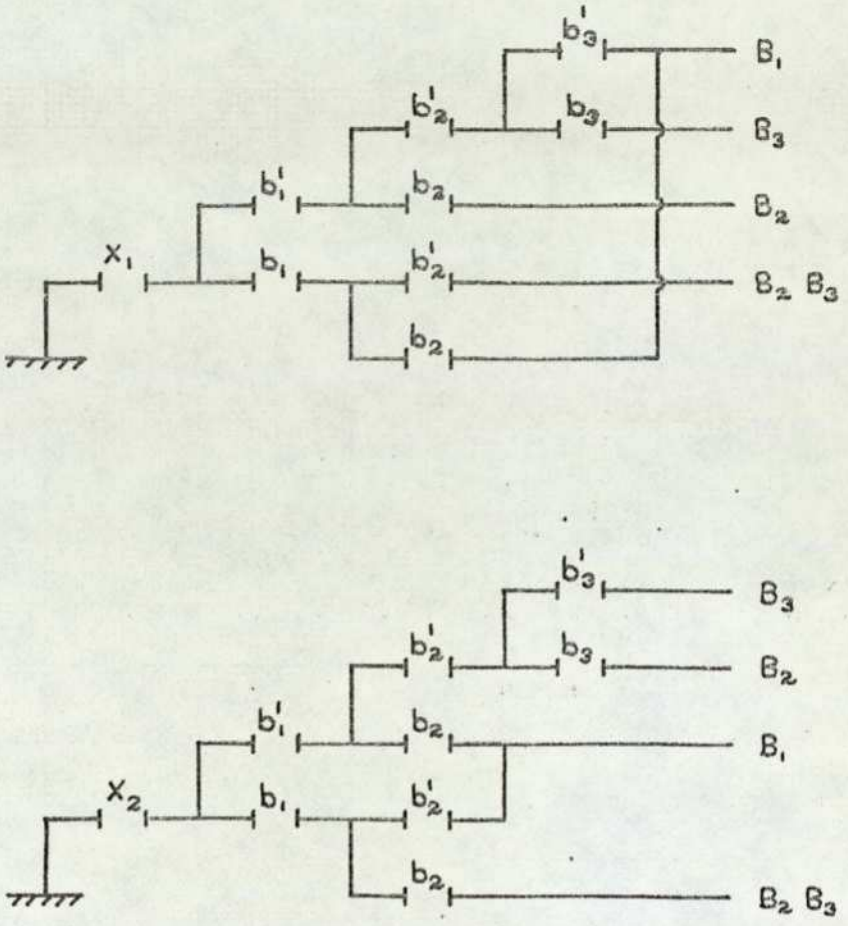


Fig. 5.5, (e)

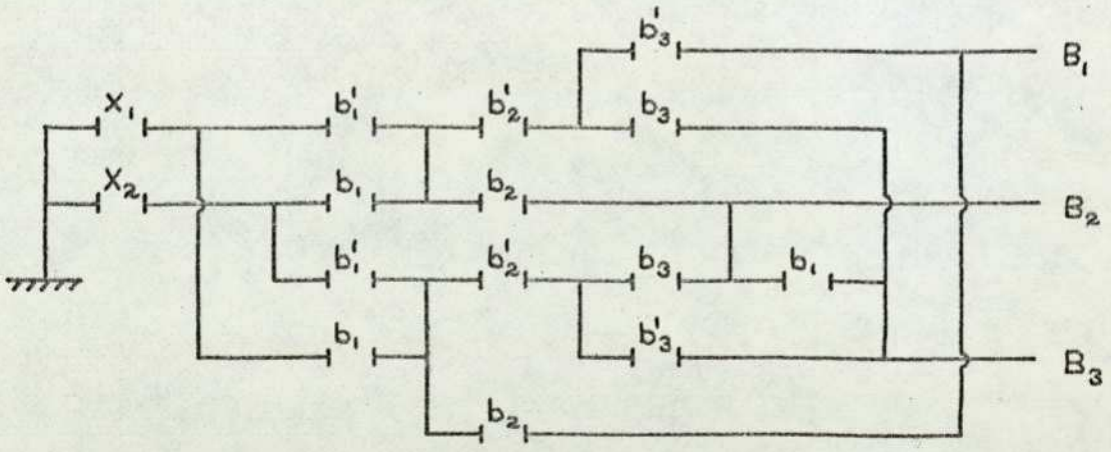


Fig. 5.5, (h)

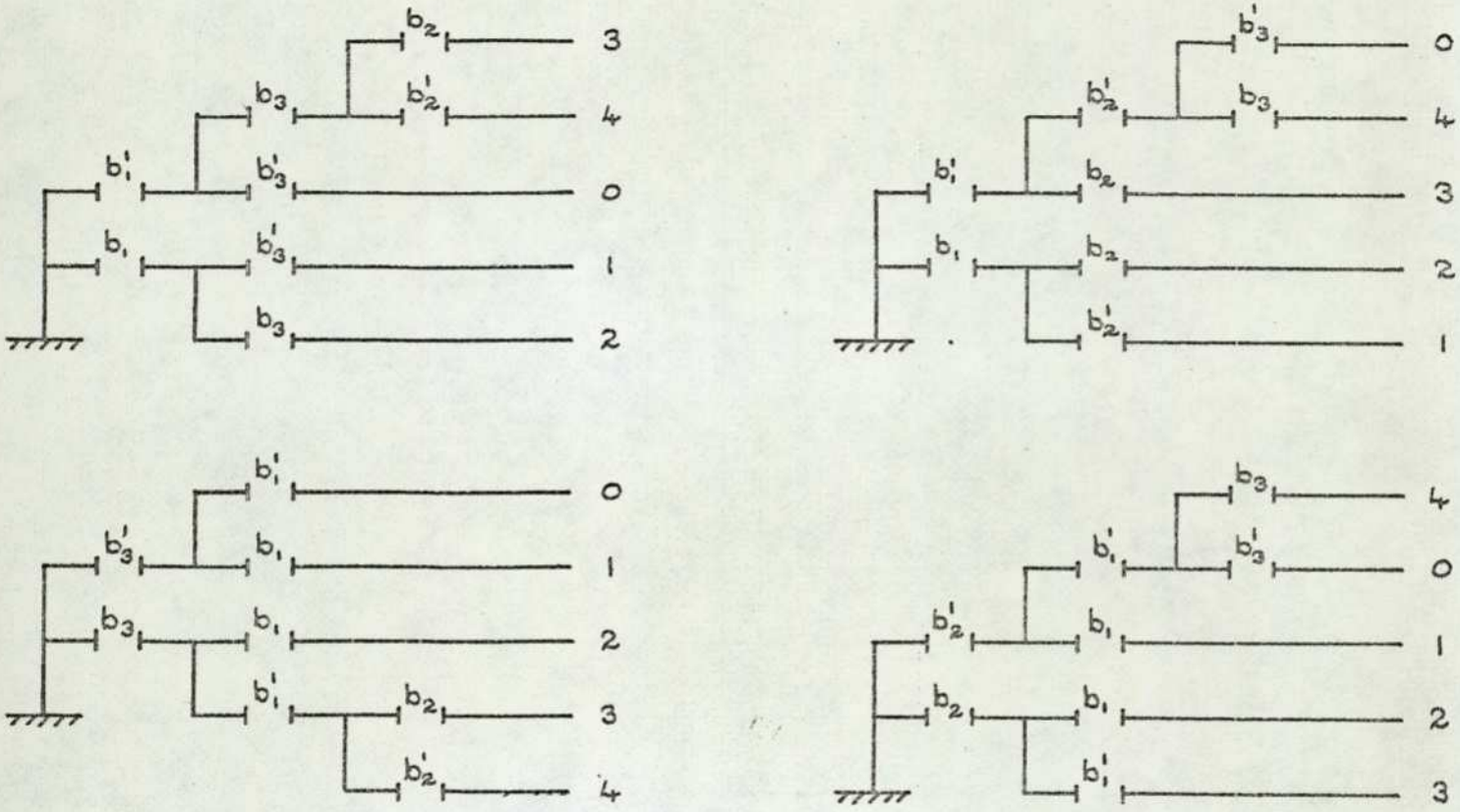

b2
3
b3
b'2
4
b'1
b'3
0
b1
b'3
1
b3
2
b'3
0
b'2
b3
4
b'1
b2
3
b1
b2
2
b'2
1
b'1
0
b'3
b1
1
b3
b1
2
b'1
b2
3
b'2
4
b3
4
b'1
b'3
0
b'2
b1
1
b2
b1
2
b'1
3

Fig. 5.5, (j)

in which a contact could be made common to both, but it did suggest an alternative approach which, it subsequently appeared, led to a more economical combined network. In both networks B_1 output is connected to a b_2 contact which, in the forward network, forms part of a $b_1'b_2$ path and in the reverse network forms part of a b_1b_2 path. Also, in the forward network B_1 is connected to X_1, via a $b_1'b_2'b_3'$ path, and in the reverse network B_1 is connected to X_2 via a b_1b_2' path which reference to the maps shows could be a $b_1b_2'b_3'$ path. These paths differ only in the complementation of b_1. This suggested the possibility of a combined network like that for the scale-of-ten counter, (fig. 5.4, (h)) in which only the two transfer-contacts on B_1 may be identified with the forward and reverse modes of operation. By complementing b_1, however, the scale-of-five sequence is altered, and not merely reversed, as the scale-of-ten sequence was. It can, however, be regarded as derived from a scale-of-six counter by taking two steps at once. This scale-of-six counter is obtained from the scale-of-five counter by adding the state $b_1b_2b_3 = 101$ (not 110) and may be reversed by complementing b_1 so that it may use a combined secondary excitation network similar to that for the scale-of-ten counter. Then the addition of a b_1 contact between the input terminals of flip-flops B_2 and B_3 (the need for which in a scale-of-five counter has already been established) will ensure that whenever $b_1 = 1$, flip-flops B_2 and B_3 will change state simultaneously.

The sequence for the scale-of-six counter is shown in fig. 5.5, (f), and the forward secondary excitation maps in fig. 5.5, (g). As the reverse sequence is obtained from the forward by complementing b_1, the reverse secondary excitation networks are obtained similarly. The derivation of the complete secondary excitation network is straightforward; it is shown in fig. 5.5, (h) and includes the b_1 contact between the input terminals of flip-flops B_2 and B_3 necessary to eliminate the extra state and produce a scale-of-five counter.

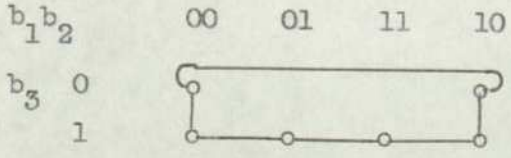


Fig. 5.5, (f)

b_1b_2	00	01	11	10
b_3 0	1	·	·	0
1	0	0	1	0

B_1

b_1b_2	00	01	11	10
b_3 0	0	·	·	0
1	0	1	0	1

B_2

b_1b_2	00	01	11	10
b_3 0	0	·	·	1
1	1	0	0	0

B_3

Fig. 5.5, (g)

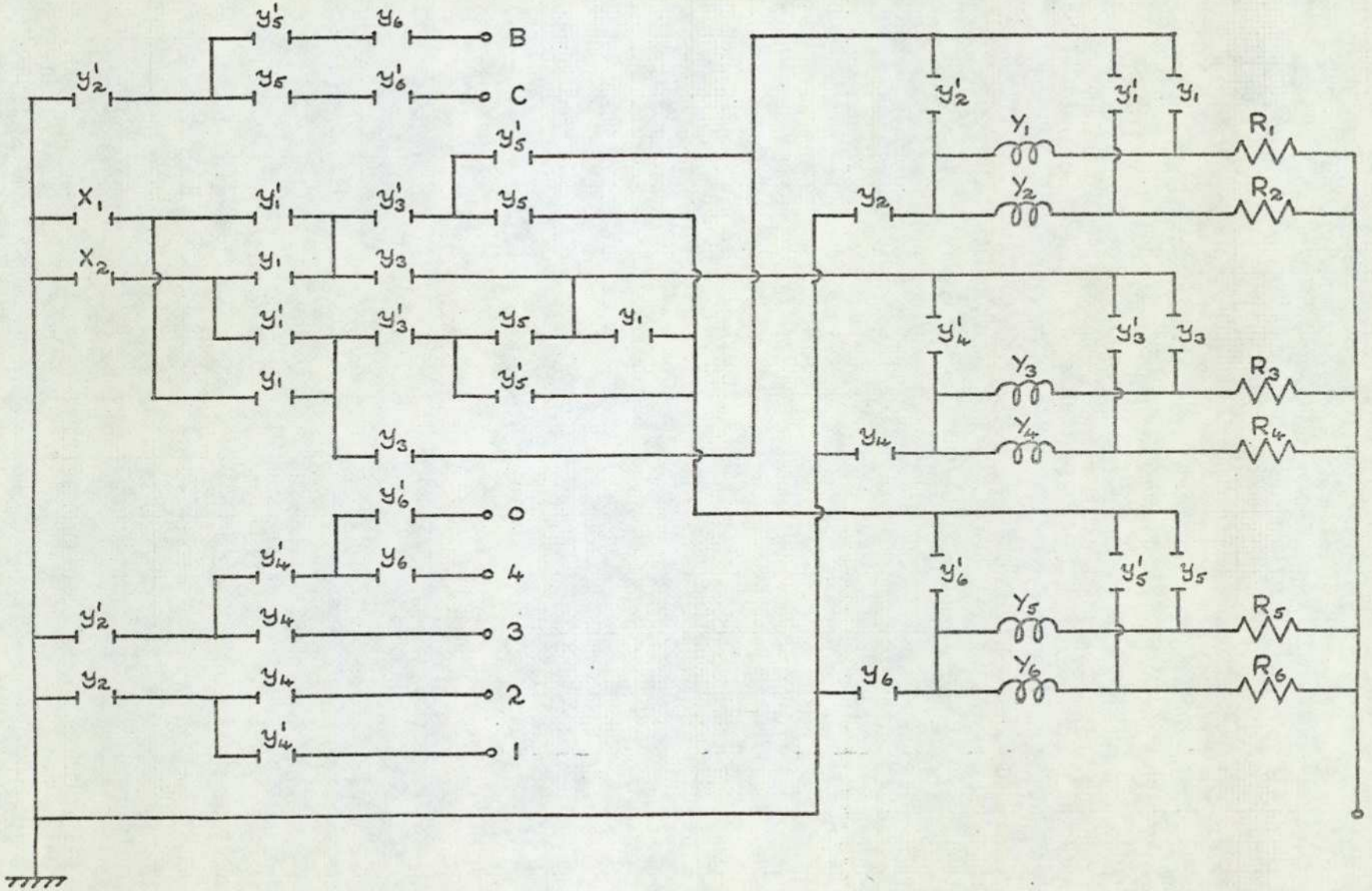


Fig. 5.5, (ℓ)

5.5.2.2 Output networks.

The design of the output networks is also straightforward. Since 5 outputs will be required, four transfer-contacts will be necessary (see 5.4.1.3). Examination of the map showing the sequence of states (fig. 5.5, (b)) shows that if contacts on B_2 or B_3 are made last in the tree (furthest from the root) only one transfer-contact will be necessary at this level whereas if contacts on B_1 are made last, two will be required. Choosing either of the first two alternatives, contacts on the first two levels must form a complete tree, and either may be made the first level. There are thus four alternative trees each requiring four transfer-contacts. These are shown in fig. 5.5, (j).

5.5.2.3 Carry and borrow networks.

These are designed as for the scale-of-ten counter: there are no new principles involved. If delays in the appearance of the carry and borrow are accepted in order to avoid increased dissipation in the series resistors a combined network is shown in fig. 5.5, (k), where the six relays forming the three flip-flops B_1 - B_3 have been designated Y_1 - Y_6. The complete circuit of the counter is given in fig. 5.5, (ℓ).

5.6 General application of the method.

5.6.1 Choice of sequences.

The method used to design counters with scales of ten and five may be used to design counters of any scale, with one possible exception, and it is proposed now to offer some comments on the choice of sequences.

The exception referred to is the scale-of-three counter. It was shown that it is impossible to build a scale-of-three counter on two flip-flops and the argument may be simply extended to show that it is not possible to build a scale-of-three counter with one secondary state for each state of the count on any number of flip-flops. For since the scale is odd, at some step two flip-flops must change state. In order that the counter may return to its initial state these two flip-flops must each change state a second time, and since each step is adjacent to both other steps, this would have to take place at an adjacent step which, as has been shown, in not permissible. The only way to build a scale-of-three counter of this type is to build a scale-of-six and pair the secondary states.

5.6.1.1 Sequences derived from reflected binary codes.

For five digits (flip-flops) or less (scales up to 32), suitable unit distance codes having thesymmetry necessary to enable the forward and reverse networks to be combined may be found easily using the Karnaugh map. For longer sequences the reflected binary codes form a convenient starting point, and these may be used for short sequences as well as long. They are formed as follows. Given a reflected binary sequence of order n the sequence of order n+1 is formed by repeating the sequence of order n, but in the reverse (reflected) order. The first half of the new sequence (that is the original sequence) is now prefixed by '0's and second (reflected) half by '1's. The sequence of order 1 is 0,1. The reflected binary sequences of orders 1 - 4 are shown in fig. 5.6, (a).

	0	00	000	0000
	1	01	001	0001
		11	011	0011
		10	010	0010
			110	0110
			111	0111
			101	0101
			100	0100
				1100
				1101
				1111
				1110
				1010
				1011
				1001
				1000
Digits	1	21	321	4321
	(1)	(2)	(3)	(4)

Reflected binary sequences of orders 1 - 4

Fig. 5.6, (a)

It is easily seen that the unit distance property is preserved if two adjacent states having the same right hand digit are omitted, for in three steps the right hand digit changes twice and some other digit once so that the newly-adjacent states differ only

in this other digit. In this way the length of the sequence may be reduced by any multiple of two, so that a sequence of any even length may be obtained. It should be noted however that in order to preserve the symmetry of the sequence, states must be removed symmetrically, for example, to obtain a sequence of length 14 = 16 - 2, the states omitted from the sequence of fig. 5.6, (a),(4) must be either 0000, 1000 or 0100, 1100, while to obtain a sequence of length 12, 4 states such as 0110, 0111, 1111, 1110 must be removed. It may be noted that by including the state 1111 in those removed the maximum power required is reduced.*

5.6.1.2 Modified procedure.

To produce a sequence of odd length it is necessary to omit from an even sequence one state. This cannot always be done. For example, if it is desired to form a sequence of length $2^n - 1$ this cannot be done by omitting a state from a reflected binary sequence of length 2^n because in such a sequence the right-hand digit changes at every alternate step, and if a state was omitted this digit would change at two successive steps. However, by the omission of a state from a sequence of length 2^n obtained in a modified way, a sequence of length $2^n - 1$ may be obtained. This will be illustrated by deriving a sequence of length 15.

Starting from the reflected binary sequence of length 8 (fig. 5.6, (a)(3)) we note that the process of reflection and prefixing an extra digit may be carried out whichever state is taken as the starting point. For example, if we start with the state 001, the sequence of length 16 derived is shown in fig. 5.6, (b), column (1). In column (2) are given the numbers of those flip-flops which change at each step, and in column (3) are marked those states which may be omitted so that two flip-flops change state simultaneously. If the state 0011 is omitted, the digits which change in the sequence 1001, 0001, 0010, 0110, are 4, (12), 3 which is permissible, but when the digits 1 and 2 change state simultaneously, they change in opposite directions, giving rise to the difficulties to which

*A method of generating sequences starting from a reflected binary sequence has been described by Sander.[39] He shows how to return to the starting point from any state in a reflected binary sequence by the shortest route and without using any state a second time.

attention was drawn in section 5.4.1. However, if the state 0100 is omitted, digits change in the sequence 2, (13), 4, and when digits 1 and 3 change simultaneously, they do so in the same direction, so that the omission of this state yields an acceptable sequence.

0001		
0011	2	x
0010	1	
0110	3	
0111	1	
0101	2	
0100	1	x
0000	3	
1000	4	
1100	3	x
1101	1	
1111	2	
1110	1	
1010	3	
1011	1	x
1001	2	
(0001)	4	
(1)	(2)	(3)

Fig. 5.6, (b)

The fact that the foregoing procedure would lead to an acceptable sequence was not evident when it was begun, so next it will be shown how to direct the procedure so as to produce an acceptable sequence with certainty.

000	0		
001	1	x	1
011	2		2
010	1		1
110	2		3
111	3		1
101	2	x	2
100	1	x	1
(000)	(0)		3
(1)	(2)	(3)	(4)

Fig. 5.6, (c)

Our starting point is again the reflected binary sequence of length 8 which is reproduced in fig. 5.6, (c) column (1). Column (2) gives the number of flip-flops whose relays are operated at each step. Using this column we then pick out those states in which the number of operated flip-flops in the preceding and following states is different; these are marked with a cross in column (3). These are the states which, when prefixed by an additional digit, may be omitted from the sequence of length 16. In column (4) are given the numbers of the flip-flops which change state at each step, the numbering, as previously, being from right to left. Suppose now we wish to omit from the sequence of length 16 the state 0001. Then state 0000 will be followed by state 0011 which cannot then be followed by 0010, since flip-flop 1 will change state on successive steps. The sequence must therefore be broken at this point and 010 made the starting point before reflection. This has been done in fig. 5.6, (d), column (1).

(1)	(2)	(3)
010	0010	
110	0110	
111	0111	
101	0101	
100	0100	
000	0000	3
001	0001	1
011	0011	2
	1011	4
	1001	
	1000	
	1100	
	1101	
	1111	
	1110	
	1010	

Fig. 5.6, (d)

In column (2) this sequence has been prefixed by '0's and reversed and prefixed by '1's. This sequence of length 16 may then be shortened to 15 steps by the omission of the state 0001, as was

intended. The digits which change in the three successive steps 0100 - 0000 - 0011 - 1011 are 3, (12), 4, as indicated in column (3). This is permissible.

The procedure described in the preceding section may be used to obtain a sequence of length 2^n from which one state may be omitted to give a sequence of length $2^n - 1$ when $n \geqslant 4$. It has already been shown that a sequence of length 3 cannot be obtained by the omission of a state from a sequence of length 4; this is the case $n = 2$. The process also fails when $n = 3$, so that a sequence of length 7 cannot be obtained by this method. This may be proved as follows.

The starting point would be a two-digit sequence of length 4. In such a sequence the two digits change alternately. If two steps are combined the two digits change simultaneously, and one or other of these digits changes at the two adjacent steps. The process described in the previous section enables us to change the digit which changes at one of these adjacent steps, but not both. Alternatively, we may reason as follows. In a three-digit sequence of length 8 one of the digits must change four times, and since this digit cannot change on adjacent steps it must change on alternate steps. When two steps are combined this digit will change on adjacent steps, which is not permissible. A counter of this type with scale seven cannot therefore be built on three flip-flops. It can however be built on four flip-flops by omitting one state other than 0000 or 1111, from the familiar "twisted ring" sequence shown in fig. 5.6, (e) (I) or, preferably, by the omission of any of the states 0001, 0011, 1011, 1001 from the sequence of fig. 5.6, (e)(II) which possesses the necessary symmetry for the forward and reverse secondary excitation networks to be combined.

0000	0000
0001	0001
0011	0011
0111	0111
1111	1111
1110	1011
1100	1001
1000	1000
(I)	(II)

Fig. 5.6, (e)

The processes described thus enable a suitable sequence of any length except three to be found.

5.6.1.3 A possible difficulty, illustrated by the design of a scale-of-fifteen counter.

In designing the tree networks via which the input terminals of the flip-flops are grounded, a question arises which has not yet been considered. The scale-of-five counter was derived from a scale-of-six counter by connecting the input terminals of the two flip-flops which were required to change simultaneously, when they were required to change simultaneously. The same procedure may be adopted for the scale-of-fifteen counter. The scale-of-sixteen counter from which it is derived, has the sequence represented in fig. 5.6, (f). The state to be omitted is 0001, so that the scale- of-fifteen counter takes the path of the dotted line, b_2 and b_1 changing simultaneously. The input terminals of flip-flops 1 and 2 must therefore be connected when they are required to change simultaneously and not connected when they are required to change independently. The partially completed map for the transmission between the two input terminals is shown in fig. 5.6, (g).

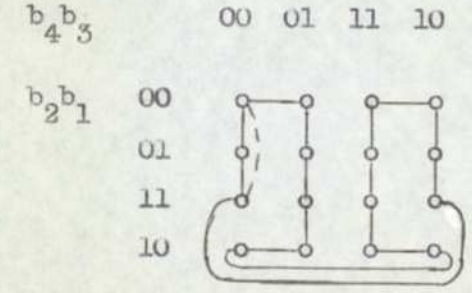


Fig. 5.6, (f)

b_4b_3 / b_2b_1	00	01	11	10
00	1	0	0	0
01		0	0	0
11	1	0	0	0
10		0	0	

Fig. 5.6, (g)

It would simplify the network if the two input terminals could be connected in the states 0001. The state 0001 is never entered except perhaps transiently when the input terminal of the

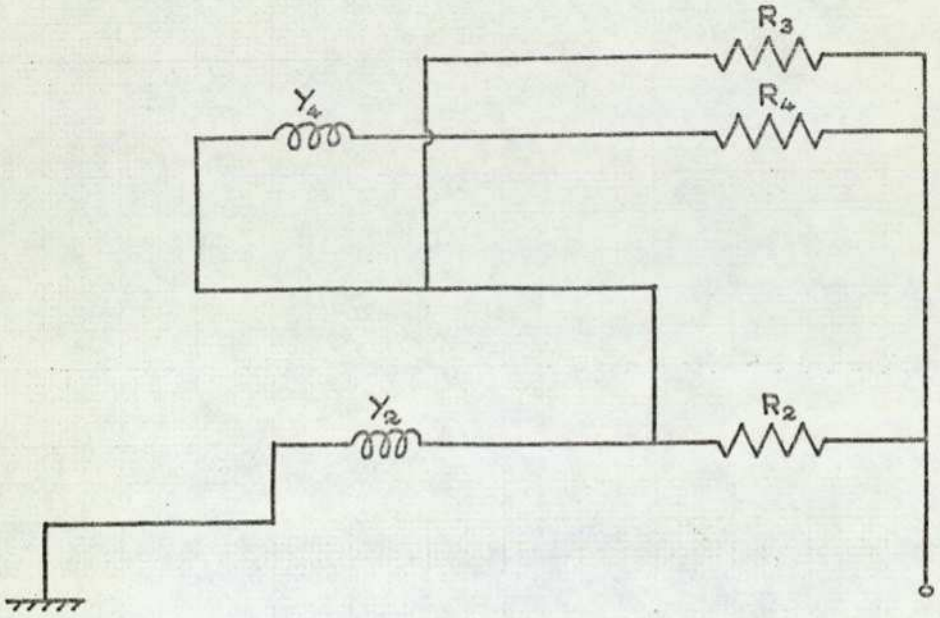


Fig. 5.6, (h)

counter is ungrounded if the two flip-flops change state with unequal speed. The state 0010 is entered in the normal operation of the counter, but the input terminals of flip-flops 1 and 2 will never be grounded in this state since it is entered and left by the changing of state of flip-flops 3 and 4. Next it should be noted that in this state flip-flops 1 and 2 are in opposite conditions, and necessarily so, since the adjacent states in which they are in the same condition are those between which the counter moves directly. It is necessary to ask whether the temporary connection between the input terminals will cause them to enter the same state. The problem is related to that considered in section 5.4.1 but is distinct from it. Referring to fig. 5.4, (j), suppose that Y_1Y_2 correspond to flip-flop B_2, operated in the state 0010, and Y_3Y_4 correspond to flip-flop B_1, released in this state. Current flows through the coil Y_4 via the y'_4 y_1 and y_2 contacts and the coil Y_2, the paths being shown in fig. 5.6, (h) which is drawn with the same configuration as fig. 5.4, (k). The current through Y_4 is less than its normal value. If the coil and series resistances are equal, it is reduced in the ratio $\frac{2}{7}$. The question now is whether this reduced current will cause Y_4 to operate and not, as in section 5.4, whether it will allow it to release. This is a much less stringent requirement and it was not expected to cause difficulty. In the counter constructed it caused none, the transmission between the input terminals of flip-flops 1 and 2 having been made $b'_3b'_4$.

5.7 Excitation networks for various designs.

5.7.1 Scale-of-fifteen counter.

All the principles involved in the completion of the design have now been explained, so no further comment is necessary.

From fig. 5.6, (f), (which shows the sequence of states on a Karnaugh map), the secondary excitation maps shown in fig. 5.7, (a) are derived.

b_4b_3 / b_2b_1	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	0
10	0	0	1	0

B_1

b_4b_3 / b_2b_1	00	01	11	10
00	0	0	0	0
01	0	1	0	1
11	1	0	1	0
10	0	0	0	0

B_2

b_4b_3 / b_2b_1	00	01	11	10
00	1	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	1	0	1

B_3

b_4b_3 / b_2b_1	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	1
10	1	0	0	0

B_4

Fig. 5.7, (a)

It is evident from the maps that (in the notation of Shannon[1])

$$B_1 + B_4 = S_{1,3}\,(b_1b_2b_3b_4)$$

$$B_2 + B_3 = S_{0,2,4}\,(b_1b_2b_3b_4)$$

so that a network producing these two functions could form the basis of the secondary excitation network. The method of designing this network has been described by Shannon[1], and the result is shown in fig. 5.7, (b).

Now $B_2 = b_1\, S_{0,2,4}\,(b_1b_2b_3b_4)$

and $B_3 = b_1'\, S_{0,2,4}\,(b_1b_2b_3b_4)$

and there are b_1' and b_1 contacts connected to the $S_{0,2,4}\,(b_1b_2b_3b_4)$ output. Hence these have only to be separated to obtain the required functions B_2 and B_3.

Also $B_1 = (b_3 + b_2')\, S_{1,3}\,(b_1b_2b_3b_4)$

and $B_4 = b_3'b_2\, S_{1,3}\,(b_1b_2b_3b_4)$

so that these functions may be obtained by adding to the $S_{1,3}$ output a network producing the two complementary functions $b_3 + b_2'$ and $b_3'b_2$. The resulting network is shown in fig. 5.7, (c). This is the forward network. The reverse network is the same, except that the transfer on B_4 is reversed. The rest of the network is common. This

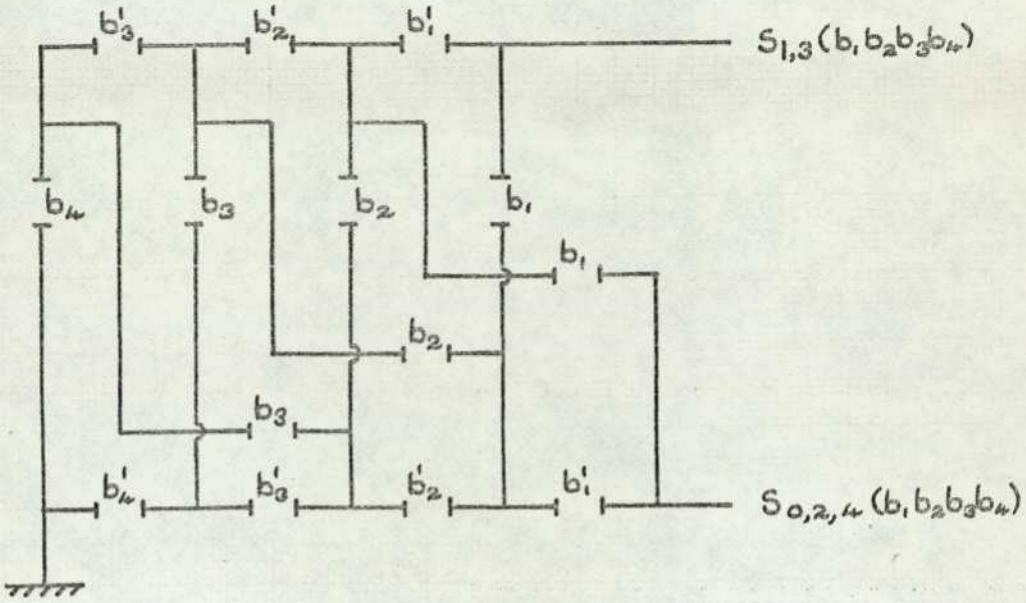


Fig. 5.7, (b)

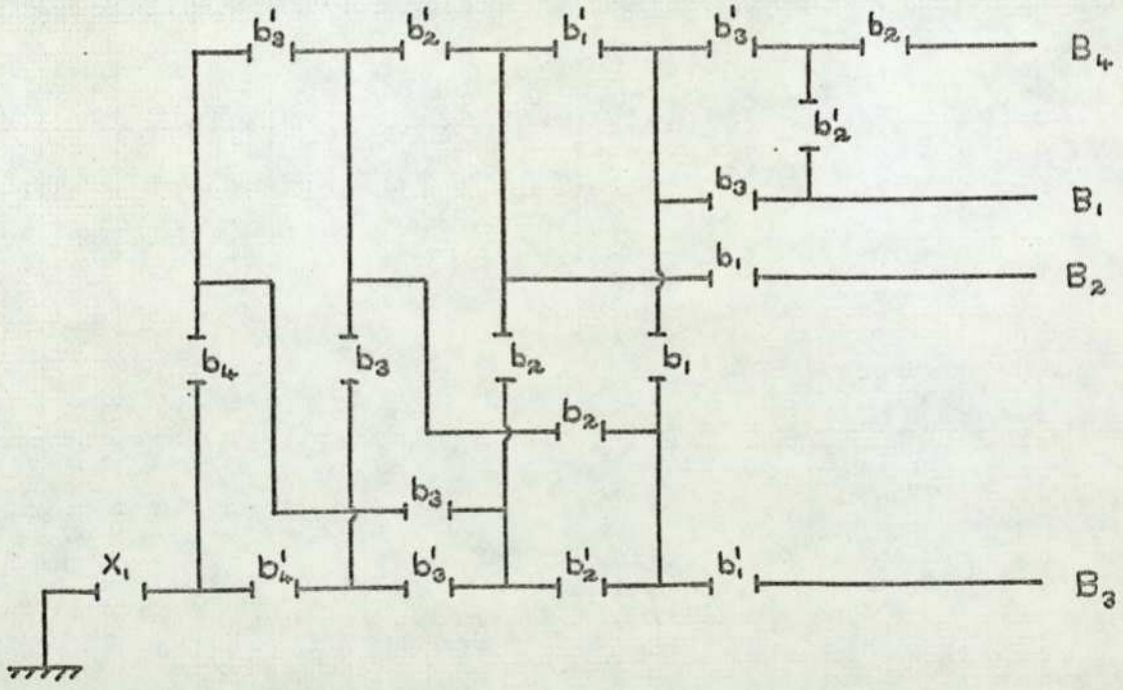


Fig. 5.7, (c)

network produces a s cale-of-sixteen counter. To secure the omission of the state 0001 a $b_3'b_4'$ path between the input terminals of flip-flops B_1 and B_2 is required. The complete secondary excitation network is shown in fig. 5.7, (d).

5.7.2 Counters of scale 2^n, operating in reflected binary code.

The unexpected economy of the secondary excitation networks for the scale-of-16 counter just designed (from which the design for the scale-of-15 counter was obtained), prompted an investigation of counters using reflected binary code. The sequence and derived forward excitation maps and functions for a scale-of-8 counter are given in fig. 5.7, (e).

$B_3B_2B_1$

000
001
011
010
110
111
101
100

B_1

b_3b_2	00	01	11	10
b_1 0	1	0	1	0
1	0	1	0	1

B_2

	00	01	11	10
	0	0	0	0
	1	0	1	0

B_3

b_1				
0	0	1	0	1
1	0	0	0	0

$$B_1 = S_{0,2}(b_1b_2b_3)$$
$$B_2 = b_1 S_{1,3}(b_1b_2b_3)$$
$$B_3 = b_1' S_{1,3}(b_1b_2b_3)$$

Fig. 5.7, (e)

The two symmetric functions are obtained by the methods described by Shannon( ), mentioned previously. In the network obtained, the $S_{1,3}$ output terminal is connected to b_1 and b_1' contacts. If these are separated, the functions B_2 and B_3 are obtained. The reverse network is obtained by complementing b_3, apart from which the forward and reverse networks are identical and may be combined. The resulting complete secondary excitation network is shown in fig. 5.7, (f).

The sequence, maps, functions and network for a scale-of-16 counter on 4 flip-flops are shown in figs. 5.7, (g)(i - iii) and (h).

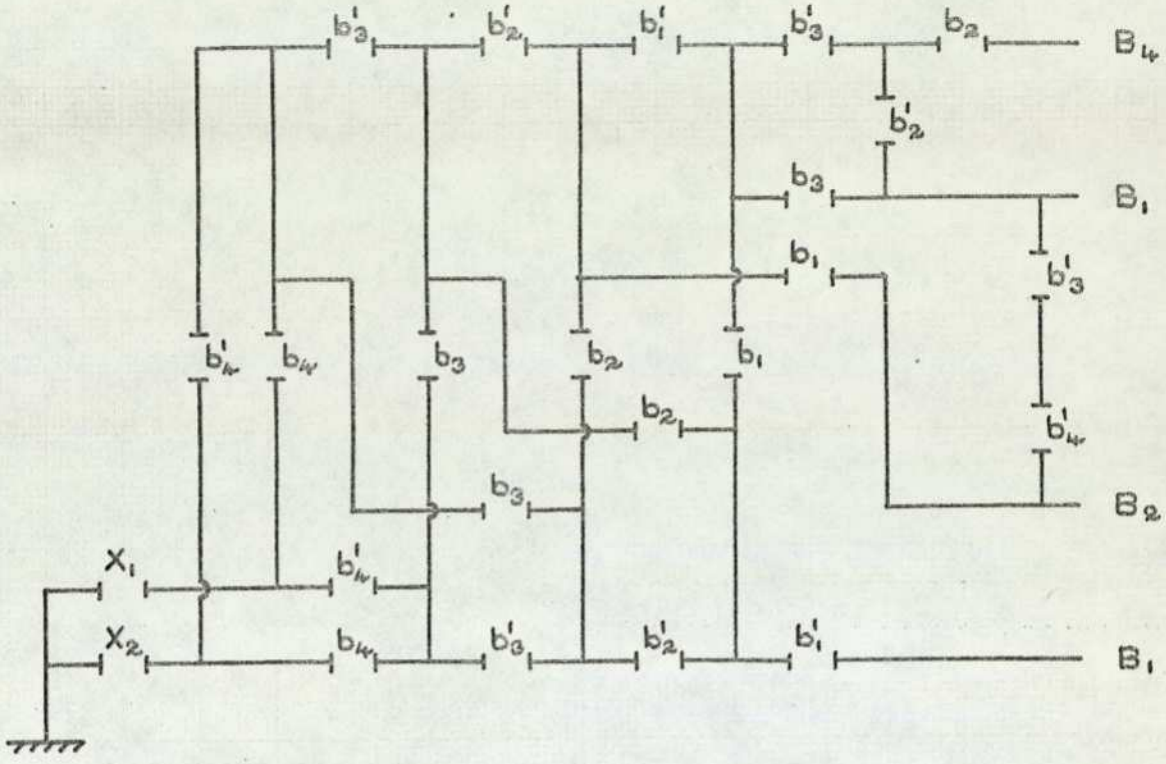


Fig. 5.7, (d)

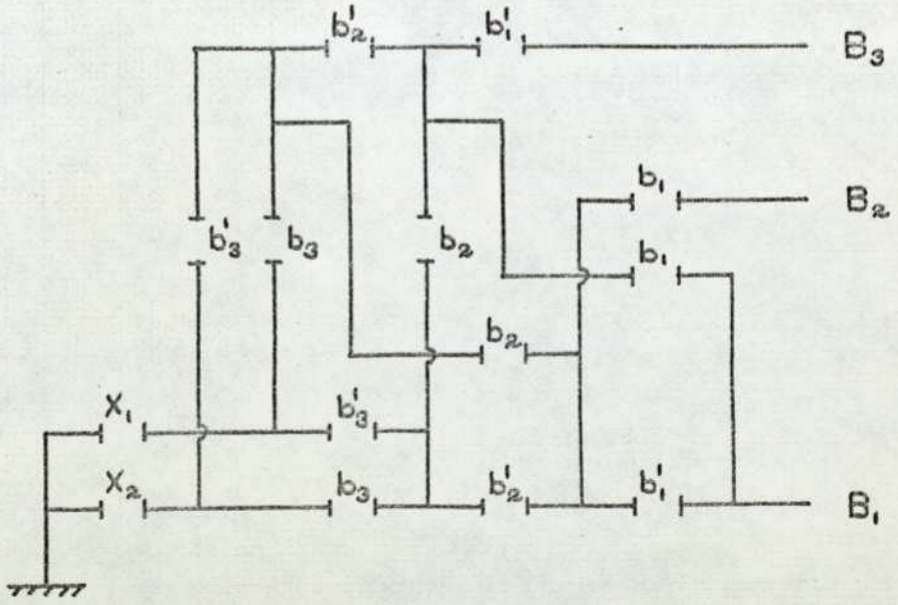


Fig. 5.7 (f)

The development is the same as for the scale-of-8 except for the last step. The b_2' and b_2 contacts cannot be separated to produce the B_3 and B_4 functions, for this would interfere with the generation of the other symmetric function. The B_3 and B_4 functions are produced by adding a transfer on B_2 to the network.

$B_4B_3B_2B_1$ — b_4b_3

b_2b_1

	00	01	11	10
00	1	0	1	0
01	0	1	0	1
11	1	0	1	0
10	0	1	0	1

B_1

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

B_2

	00	01	11	10
00	0	1	0	1
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

B_3

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

B_4

0000
0001
0011
0010
0110
0111
0101
0100
1100
1101
1111
1110
1010
1011
1001
1000

(i)

(ii)

$$B_1 = S_{0,2,4}\,(b_1b_2b_3b_4)$$

$$B_2 = b_1\,S_{1,3}\,(b_1b_2b_3b_4)$$

$$B_3 = b_1'b_2\,S_{1,3}\,(b_1b_2b_3b_4)$$

$$B_4 = b_1'b_2'\,S_{1,3}\,(b_1b_2b_3b_4)$$

(iii)

Fig. 5.7, (g)

The pattern is now evident, and it is straightforward to draw the secondary excitation network for a scale-of-2^n counter, whatever the value of n. That for a counter of scale $2^5 = 32$ is shown in fig. 5.7, (j).

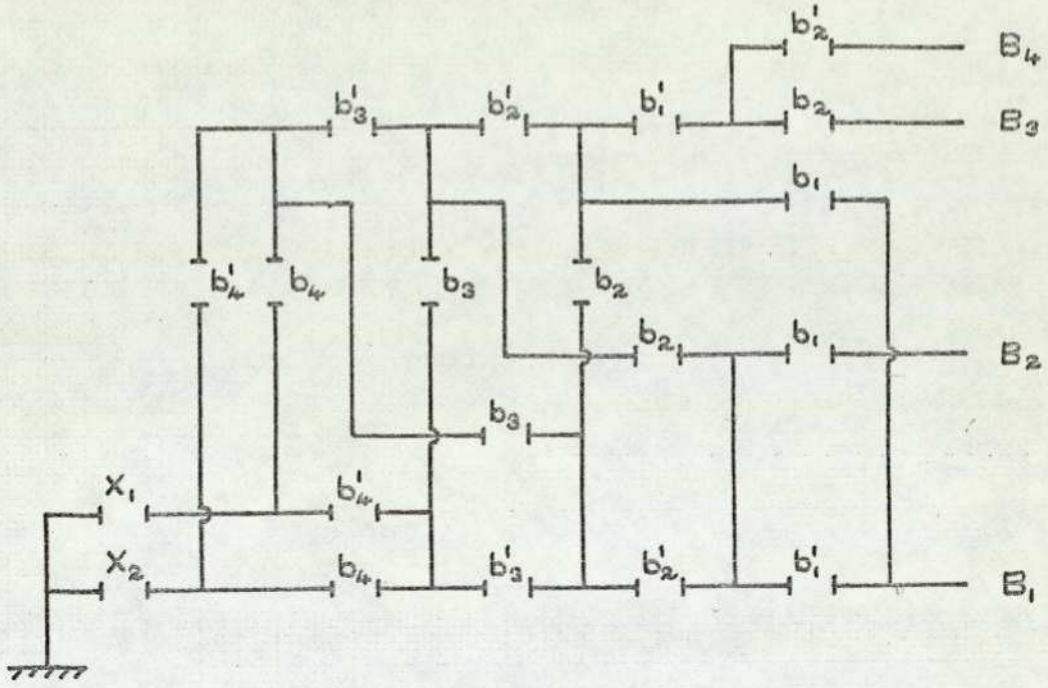


Fig. 5.7, (h)

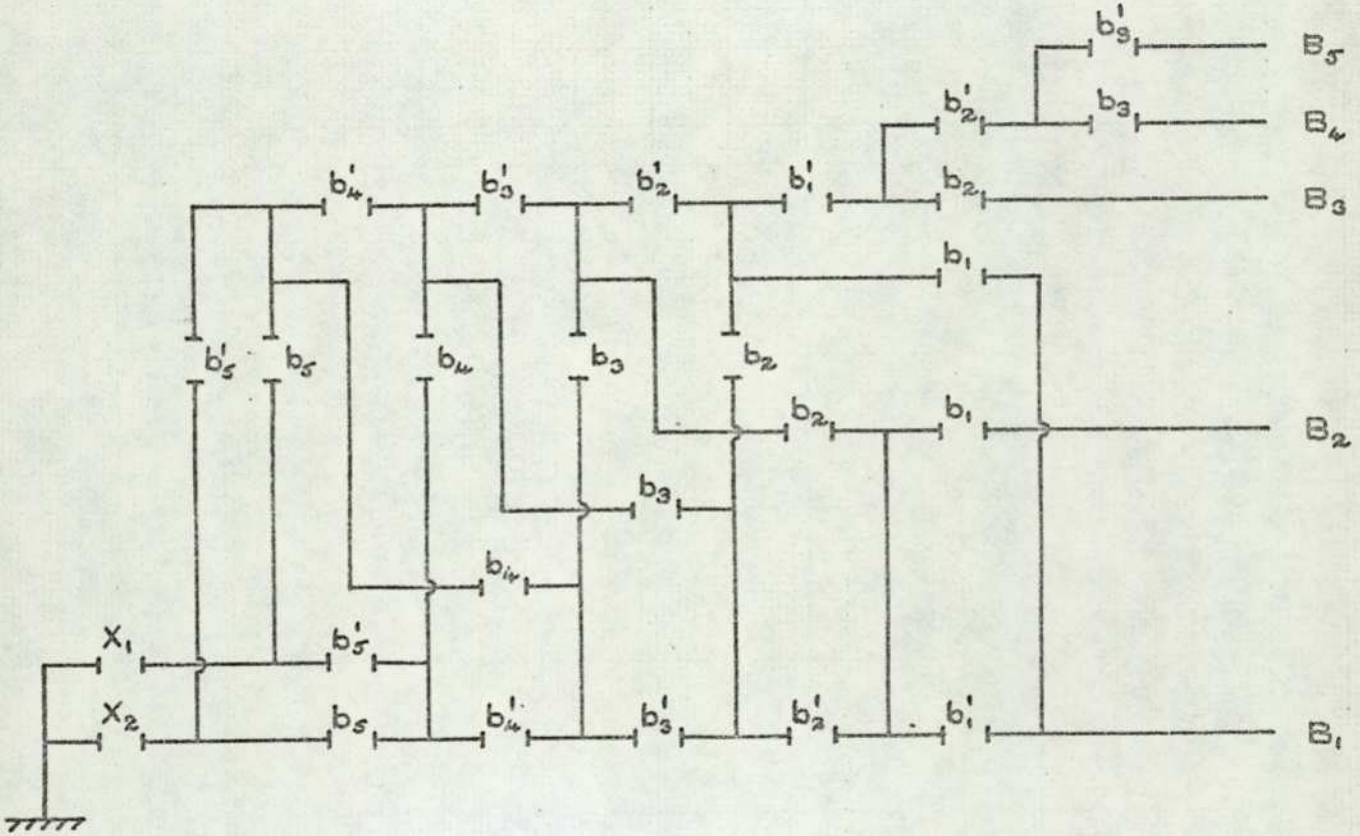

b'_5
B_5
b'_4
b_3
B_4
b'_3
b'_2
b'_1
b_2
B_3
b_1
b'_5
b_5
b_4
b_3
b_2
b_2
b_1
B_2
b_3
b_4
X_1
b'_5
X_2
b_5
b'_4
b'_3
b'_2
b'_1
B_1

Fig. 5.7, (j)

6 PRACTICAL REALISATION - HAZARDS

6.1 Experience with counters built.

It was never the intention that this work should include an investigation of the particular characteristics which the relays of which the counters were built would require to have, but on the other hand, it was intended that this work should be relevant to the problem of building reversible relay counters, an intention which would not have been fulfilled if it should have proved impossible to build reversible counters operating in the sequences and with the contact networks determined. Accordingly, three counters were constructed after some preliminary tests had been carried out. These have been referred to previously, briefly.

The preliminary tests were to determine the behaviour of two flip-flops connected in parallel. It was suspected that one might interfere with the other if they were required to change state in opposite directions and this was found to be the case, the explanation being as given in section 5.4.1.

Next the scale-of-four counter on two flip-flops described in section 5.3 was constructed, but without output networks. This worked in the way in which it had been designed to work.

Then the scale-of-fifteen counter on four flip-flops, which embodies all the principles discussed, was constructed, but again without output networks, and this too was found to work as designed.

Finally a counter using series control only was constructed. The example chosen was the simplest, the scale-of-four counter on four relays described in section 3.1. This did not work as designed. When switched on the even numbered relays are energised; for example, in fig. 3.1, (p), Y_2 will be energised via the path $y_2'y_1'y_4'X_1'$. Y_2 should operate and continue to be energised via the y_2y_1' path. However, the path containing y_2' is broken before the path containing y_2 is made, and the armature falls back. Y_2 is again energised via the $y_2'y_1'y_4'X_1'$ path, the armature begins to move towards the operated position and oscillates irregularly. A test with a lightly loaded relay in a similar situation (fig. 6.1, (a)), showed that it operated and was held as expected. A further test on a heavily loaded relay which failed to operate when a break-before-make transfer-contact was used, showed that if a continuity-transfer-contact was used it operated and was held as intended. A further test showed that in

a circuit of the type shown in fig. 6.1, (b) where a' and a are formed by a break-before-make transfer-contact, the operation of relay A did not cause relay B to be released, although the circuit was temporarily broken. The counter was therefore rebuilt using a continuity transfer-contact on each of the four relays, with the common springs connected directly to the coils, and the circuit was then found to operate as designed. In general therefore it would seem that break-before-make transfer-contacts may be used, except when they form part of the excitation network of the relay to which they belong, when continuity transfer-contacts are required.

It might be wondered, in view of this, why the scale-of-four counter on two flip-flops described in section 5.3 worked as designed when the excitation networks of the even-numbered relays contain a continuity transfer-contact on themselves, respectively, but it may be remembered that this counter was built without output networks and that the relays in question were therefore very lightly loaded, so that their armatures move fast and the interval between break and make is very small.

6.2 Theoretical treatment of hazards.

The initial failure of one of the counters built to function as intended was due to the failure of the relays to operate in the ideal way which has been assumed, in that make-contacts (in this case the make half of a transfer-contact) did not make at the same instant as break-contacts on the same relay broke the circuit. This failure of devices to operate in an ideal way is said to create "hazards" and these have been studied and classified by Huffman[32] and Unger[31].

6.2.1 Static and Dynamic Hazards.

Huffman studied static and dynamic hazards which may be detected by examining a contact network, or very simply, as he showed, the Karnaugh maps, if these were used to design it. They will be absent from a network designed on a tie-set basis, (that is, from an expression in SP form), if every change of a variable which is not intended to cause the transmission to change from '1' takes place within a tie-set, rather than involving stepping from one tie-set to another. The hazards which caused the malfunction of the scale-of-four counter referred to in section 6.1 as first built were of

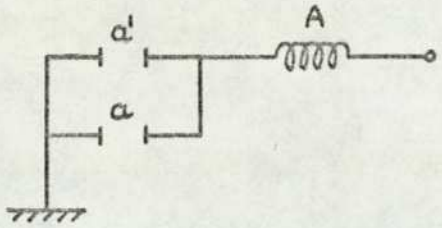


Fig. 6.1, (a)

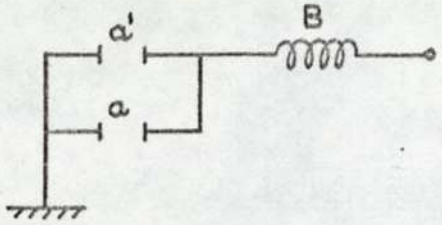


Fig. 6.1, (b)

this type. They were overcome by the use of continuity transfer-contacts in appropriate places. Huffman[32] showed how to overcome them by using additional tie-sets (adding additional contacts) to the network, but to modify our networks in this way would involve a complete redesign and make irrelevant the procedures we have used. Huffman also describes a dynamic hazard, which occurs when a function whose value is intended to change its value once, changes it three times before reaching a steady value.

6.2.2 Essential Hazards.

A more involved type of hazard is the "essential hazard" studied by Unger.[31] These occur in sequential circuits when three successive changes of an input variable cause the circuit to arrive in a state different from that arrived at as the consequence of a single change in that input variable. They may be detected from the flow diagram and are an essential feature of counter circuits. They are due to the different delays between the change of an input variable and its effect on different secondary variables.

No attempt will be made to repeat or even summarize the general treatment which Unger gives. Instead, we will consider the counter on four relays with series control only, with the circuit shown in fig. 3.1, (n) and show how, if it had been designed differently, essential hazards might have occurred, for it appears that having designed the circuit to use only single-transfer input contacts, essential hazards have been eliminated.

Suppose then, first, that $Y_1Y_2Y_3Y_4 = 0101$ and $X_1X_2 = 00$, and that X_1 now changes from 0 to 1. Y_1 is excited via the $y_1'y_4X_1$ path and then via the $y_1y_4X_2'$ path. Y_2 which was excited via the y_2y_1' path is now excited via the $y_1y_4X_1$ path. Suppose however that the $X_1y_4y_1y_2$ path to Y_2 and the $X_1y_4y_1'$ path to Y_1 had been separate and that the X_1 contact in the former was still open after the Y_1 had operated. Y_2 would have been released and not subsequently re-energised, when this X_1 contact finally closed. However, Y_3 would then have been energised and operated.

Suppose next that this hazard had not materialised, that Y_1 had operated correctly and Y_2 had not been released. X_1 now returns the val ue 0. The path $X_1y_4y_1y_2$ is broken and Y_2 is released. Suppose however that the paths $X_1y_4y_1y_2$ to Y_2 and $X_1y_2'y_3'$ to Y_3 used separate X_1 contacts, and that that in the path to Y_3 was still closed

after Y_2 had been released, Y_3 would be energised via $X_1y'_2y'_3$ it would operate and be held via the $y_3y'_2X'_2$ path. At the same time the y'_3y_4 path to Y_4 would be broken and Y_4 would be released.

Now suppose that $Y_1Y_2Y_3Y_4 = 1010$ and that X_1 is operated. Y_1, which was energised via the $y_1y'_4X'_1$ path, is released. However, if the $X'_1y'_4y_1$ path to Y_1 and $X'_1y_4y'_1y'_2$ path to Y_2 were separate, and if the X'_1 contact in the path to Y_2 were still closed when Y_1 was released Y_2 would be energised via the $X'_1y'_4y'_1y'_2$ path, it would operate, and then be held via the $y_2y'_1$ path to ground. The $X'_1y_2y_3$ path to Y_3 would eventually be broken and Y_3 would be released. There is a further hazard when, supposing the counter to have correctly entered the 0010 state when X_1 was operated, X_1 is subsequently released, and a corresponding set of hazards when the X_2 input is used.

Even in a circuit with separate input contacts in the various paths it is unlikely that the hazards described will prove troublesome. Unger appears to assume that essential hazards do not occur in relay circuits at all, but Marcus gives examples. It is worth noting that in Marcus' examples[33] the secondary relay is always released (that is, never operated) and he recommends shunt control as a remedy for the hazard. This would appear to be just one method of delaying the release of the relay concerned. However, in our circuit the use of single input contacts common to every path seems to eliminate the possibility of essential hazards, so that a feature which was intended to simplify the carry and borrow circuits in a counter with several units in cascade, and perhaps to speed up the operation of the counter, has conferred an unexpected benefit. It may be that there is a general principle here which is capable of wider application.

7 SUGGESTIONS FOR FURTHER WORK

7.1 Practical value of the designs worked out.

In the preceding sections several types of reversible relay counter have been described. Some of these are unlikely to be of practical value because of the large contact loads required. Of the rest, it seems that the circuits using τ-flip-flops, described in section 5, will be of practical use because of the straightforward method of design and the comparatively light contact loads. The twisted-ring counters described in section 3 may be ofuse when the increased dissipation due to combined series and shunt control is considered a disadvantage.

7.2 Other designs worth investigation.

Excluding the counter described in section 2, that is, considering only those in which no relay changes state on successive steps, the counters considered may be regarded as selected from eight categories distinguished by the following features:

(a) whether combined series and shunt control or series control only is used,

(b) whether the pairs of relays which change state at the different steps in the count are disjunct (as in sections 3 and 5), or not, (as in section 4),

(c) whether the sequences may be reversed by complementing one of the variables.

This classification suggests other types of counters which might be considered, and of these we think the following are worth investigating.

(a) Counters using series control only where the sequence maybe reversed by complementing one of the variables, and the relay pairs are disjunct. The scale-of-four counter of fig. 3.1, (n) is the simplest member of this class. The next simplest would use the sequence (c) of section 3.2.1.1 which it was decided, perhaps mistakenly, not to investigate further.

(b) Counters using the sequence of section 4 but with combined series and shunt control.

(c) Counters using a sequence similar to that of section 4 but in which the second half of the sequence is obtained by reversing the first half, so that the sequence in which the relays change would be

14, 23, 15, 24, 56, 14, 56, 24, 15, 23.

These might use combined series and shunt control or series control only.

For all the counters designed or suggested so far it has been assumed that two inputs are never present simultaneously. If this restriction is removed the design problem is radically altered. The preparation of the flow-table and transition diagram is straightforward, but there is no obvious solution to the problem of secondary assignment. This requires a systematic approach, for which the methods of Tracey[24] seem appropriate.

In the wider field of contact network design a topic which seems worth investigating is the following. We have on occasion used tree networks in which the number of levels was greater than the number of variables. Most obviously this was so in the circuits of figs. 5.7, (h) and (j). We think it likely that these circuits are simpler than any which can be designed using one level per variable. It seems worthwhile to examine whether this is so, and in what circumstances, if at all, tree circuits with more levels than variables are more economical.

Less obviously, the secondary excitation network of fig. 5.5, (h) may be regarded as a tree with more levels than variables, and this network, unlike those in figs. 5.7, (h) and (j) is not disjunctive. It might be possible to devise a method of designing non-disjunctive multiple output networks using trees with increased numbers of levels. This could probably be regarded as an extension of Scheinmann's first method of designing non-disjunctive multiple-output networks.[17]

In the still wider field of sequential circuit design there seems to be a notable similarity between the initial steps of Unger's procedure[21] for determining the two-member compatibles and the initial steps of the Hartmanis and Stearn's procedure[27,28] for determining the partition pairs. It might well be worth investigating whether this is significant and whether the reduction of the flow-table and the determination of a good secondary assignment might be combined to form a single process.

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