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The role of communication and imitation in limit order markets

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Abstract. In this paper we develop an order driver market model with heterogeneous traders that imitate each other on different network structures. We assess how imitations among otherway noise traders, can give rise to well known stylized facts such as fat tails and volatility clustering. We examine the impact of communication and imitation on the statistical properties of prices and order flows when changing the networks' structure, and show that the imitation of a given, fixed agent, called "guru", can generate clustering of volatility in the model. We also find a positive correlation between volatility and bid-ask spread, and between fat-tailed fluctuations in asset prices and gap sizes in the order book.

PACS. -0.2.50.-r Probability theory, stochastic processes and statistics – -0.5.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – -89.65.Gh Economics, econophysics, financial markets, business and management – -89.65.-s Social and economic system

1 Introduction

The main purpose of this paper is to understand if and how herding effects may be responsible for the persistence of asset price volatility. While stocks returns themselves are relatively uncorrelated, the squares or absolute values of returns are autocorrelated, reflecting a tendency for markets to move from relative quiet periods to more turbulent ones. Significant positive autocorrelation of absolute stock returns survives for a year or more, and decays at a rate which is slower than exponential. A number of econometric time series techniques have been introduced for the modelling of time varying variances and covariances[1]. However, statistical analysis alone is not sufficient to understand the presence or absence of long-range memory in volatility, and economic mechanisms that can explain the origin of this phenomena are needed. Some insights into volatility clustering have been provided by agent-based models. Following the pioneering Santa Fe Artificial Stock Market, SF-ASM[1], several artificial financial markets have been developed over the last 15 years. In particular understanding how sophisticated agents may be able to reproduce volatility clustering has been widely studied in the economic literature. For instance ([2], [3], [4], [5], [6]) are examples of agent based models able to explain the influence of traders' behaviours on the persistence of asset price volatility. As argued by [7], some mechanisms of behavioural switching can generate persistent return volatility, even though they are not sufficient to create long range dependence in absolute returns[2].

The models cited above have looked at the effect of coordination of traders strategies via market mediated interactions (for example when agents follow common chartist trading rules). Collective behaviour nonetheless could reflects the phenomenon known as herding which occurs when agents take actions on the basis of directly imitating each other ([8], [9], [10], [11], [12], [13], [14]). Most of the studies on herding effects have focused on how herding can lead to large price fluctuation but only a few papers have investigated its role as a source of volatility clustering ([14], [15], [16]). As pointed out by the model of [14] of which this is, in a way, an extension, local communication and imitation among traders, if dynamically evolving, can induce not only volatility clustering but multiscaling as well. The originality of this work as regards [14] is the mechanism of price formation. We introduce here an order-driven market where limit orders are stored in the order book and executed using time priority at a given price and price priority across prices. The advantage of an auction market respect to a dealer market (as in [14]) is that the price is determined through the trading mechanism itself without ad-hoc pricing rules or unrealistic tatonnement mechanism for reaching an equilibrium price. Moreover, while in [14] agents were placed on

[1] The first instrument developed in 1982 by Engle for analyzing the variation in higher order moments is the AutoRegressive Conditional Heteroskedastic (ARCH) model.

[2] A short memory process has autocorrelations decaying to zero exponentially as the lag increases. In contrast, the autocorrela- tion of the long memory process decays to zero at a hyperbolic rate with an exponent smaller than 1.

a regular grid and could only interact with their nearest neighbours, here we introduce a random communication structure that can evolve dynamically. The communication network is taken as exogenous. In this setting, we compare the effects of random interaction versus the imitation of a given, fixed agent, that we name the guru.

Agents in our model imitate the expectations of the Guru and not its actions. In an order driven market, it is not necessarily the case that this kind of imitation leads to larger coordination and in turn to larger price fluctuations. In fact, even if the guru expects a price increase/decrease, he himself and/or the agents that follow his advice may submit limit orders to buy/sell instead of market orders, and the immediate impact of their trades may be negligible. The long term impact of these imitation lead trades is also not clear a priori. For example, if imitation results in an accumulation of limit orders near the bid/ask they may dampen future price fluctuations instead of amplifying them.

In the last part of our study we investigate the correlation between herding, volatility, bid-ask spread and order flows. While the availability of order book data has made possible a number of empirical studies on limit order flows, only a few papers [5] have analyzed these features is simulation studies. Empirical analysis has showed fat tails in the distribution of limit order arrivals ([17] and [18], [19]) and fat tail decay of order distribution in the limit order book. [20] has also shown that fat tails in the distribution of returns emerge when large gaps are present between the best price and the price at the next best quote (and are not linked, as suggested by [21], to large market orders arrival). We show that our model is capable of reproducing these stylized facts.

The rest of the paper is organized as follows. In section 2 we describe the model; in section 3 we present the results of the simulations; section 4 concludes.

2 The mathematical structure of the model

2.1 The market

The market in our model is order-driven. A population of N traders can either place market orders, which are immediately executed at the current best listed price, or they can place limit orders. Limit orders are stored in the exchange's book and executed using time priority at a given price and price priority across prices. A transaction occurs when a market order hits a quote on the opposite side of the market.

Trading happens over a number of periods t_k, with $k = 1, \cdots T$. At the beginning of each period, traders make expectations about the price at the end of a given time horizon τ (that we take to be the same for all traders). The future price expected at time $t_k + \tau$ by agent i is given by

$$\hat{p}^i_{t_k,t_k+\tau} = p_{t_k} e^{\hat{r}^i_{t_k,t_k+\tau}\sqrt{\tau}} \tag{1}$$

where $\hat{r}^i_{t_k,t_k+\tau}$ is the agent's expectation on the spot return which, as we will see later, may be affected by the expectation of other agents and p_{t_k} is the reference price observed by all agents at the beginning of each period.

After expectations are made, N_t trades are submitted each period. Agents enter the market, sequentially and in a random order, determine a limit price and an order size, and submit their orders. If an agent expects a future price increase (decrease) he decides to buy (sell) at a price b^i_t (a^i_t) lower (higher) than his expected future price $\hat{p}^i_{t_k,t_k+\tau}$. Bids and asks are uniformly distributed in an interval $(p_{\min}, p_{\max})$ around the current price, calculated according to the following rules

$$b^i_t \sim U(p^b_{\min}, p^b_{\max}),\ p^b_{\min} = p_t(1-\gamma^1_t),\ p^b_{\max} = \hat{p}^i_{t_k,t_k+\tau} \tag{2}$$

$$a^i_t \sim U(p^a_{\min}, p^a_{\max}),\ p^a_{\min} = \hat{p}^i_{t_k,t_k+\tau},\ p^a_{max} = p_t(1+\gamma^2_t) \tag{3}$$

where $\gamma^{1,2}_t$ are random variables uniformly distributed in the interval $(0,1)$ and p_t is the price at the time the order is submitted. The price is recalculated as trading goes on as follows: p_t is given by the price at which a transaction occurs, if any; if no new transaction occurs, a proxy for p_t is given by the average of the quoted ask a^q_t (the lowest ask listed in the book) and the quoted bid b^q_t (the highest bid listed in the book), $p_t = (a^q_t + b^q_t)/2$; if no bids or asks are listed in the book a proxy for p_t is given by the previous traded or quoted price. Bids, asks and prices are positive and investors can submit limit orders at any price on a prespecified grid, defined by the tick size Δ[3].

We assume that agents have a random demand function and that the size of their order is bounded only by budget constraints. Agents hold a finite amount of cash C^i_t and stocks S^i_t in their portfolio. The size s^i of agent's i order is determined as follows:

- if the agent expects a price decrease he sells a random fraction of his assets $s^i = \xi_t S^i$
- if the agent expects a price increase he invests a random fraction of his cash in the assets equal to
  $s^i = \xi_t C^i_t / a^q_t$ if he submits a market order
  $s^i = \xi_t C^i_t / b^i_t$ if he submits a limit order

where ξ_t is a random variable uniformly distributed on the interval (0, 1).

If $b^i_t \leq a^t_q$ or $a^i_t > b^t_q$ the agent place a limit order at its limit price. If $b^i_t \geq a^t_q$, then the agent places a market order to buy at the current quoted ask a^t_q. If the supply available on the book at the ask price a^t_q is not sufficiently large to fulfil the order, the agent buys all the quantity

[3] The tick is the smallest possible change of p_t.

available at the ask and then moves on to check the second best ask price, iterating the process until he has no more stocks to buy or there are no more sell orders in the book at a price smaller than b_t^i. Symmetrically, if $a_t^i < b_q^t$, then the agent places a market order to sell at a price b_q^t. If the demand available on the book at this price is not sufficiently large, he fills the available demand at the bid and then moves on to check the second best bid price, iterating the process until the agent has no more stock to sell or there are no more buy orders in the book at a price greater than a_t^i. If the order can only be partially filled, the agent places its unmatched demand/supply as a limit order. If the limit order is still unmatched at time $t + \tau$, it is removed from the book.

When agents place a market order, their cash and stocks positions are updated accordingly. When agents place a limit order, the cash they commit to buy and the stocks they commit to sell are also temporarily removed from their portfolios (even if a limit order does not comport an immediate transaction). In this way agents can not spend money or sell stocks that have already committed in the book. If the order is cancelled, the stocks and cash that were tied down in the orders are returned to the traders who had submitted them.

2.2 The interaction network

To model how agents' decision are influenced by their mutual interaction we introduce a communication structure in which nodes represents agents and the hedges are the connective links between them. Links are directional and go from the agent that requests advice to the agent that provides advice.

In general local interaction models, agent interacts directly with a finite number of others in the population. The set of nodes with whom a node is linked is referred to as its neighbourhoods. In our model the number of outgoing links is constrained to be one. The reason being that in a highly connected random network synchronisation could be achieved via indirect links. The effects of direct imitation are easier to be tested in a diluted network where indirect synchronisation is less likely to arise.

We start with the case where each agent i makes a unique link with a randomly chosen agent j. In this case the probability that any two agents (i, j) are linked is independent of both i and j. We call this model the random attachment case. We then move to the case where each agent i chooses to link, with a probability λ (independent of i), to a fixed agent j and with probability $1 - \lambda$ to a randomly chosen agent. We call this fixed agent j, who attracts the largest number of in-coming links, the "guru". The random attachment case is recovered in the limit $\lambda = 0$. When λ is equal to one all agents are linked to the guru and the network becomes star like.

We subsequently introduce a time dependent probability of attachment that evolves as

$$\frac{d\lambda_t}{\lambda_t} = \sigma_\lambda \epsilon_t. \tag{4}$$

The probability λ_t is constrained to take values in the interval $(0, 1)$ and is reflected at the boundaries.

2.3 The expectation formation mechanism

At the beginning of each trading period t_k, agents make idiosyncratic expectations about the spot return, $\hat{r}^i_{t_k,t_k+\tau}$ in the interval $(t_k, t_k + \tau)$. We assume that agents are not informed and have random expectation of future returns. We also assume that agents are heterogeneous in that they have different forecasts of the returns' volatility, σ_t^i. Expected returns are thus given by

$$\hat{r}^i_{t_k,t_k+\tau} = \sigma^i_{t_k} \epsilon_{t_k} \tag{5}$$

where $\sigma^i_{t_k}$ is a positive, agent specific, constant and $\epsilon_t \sim N(0, 1)$ is a normal noise.

After individual expectation are generated, a consultation round starts during which agents sequentially, and in a random order, revise their expectation. The revised expected return is obtained by weighing agent i's own expectation with that of agent j to which i is linked to

$$r^i_{t_k,t_k+\tau} = w\hat{r}^i_{t_k,t_k+\tau} + (1 - w)\hat{r}^j_{t_k,t_k+\tau}. \tag{6}$$

When w is equal to zero, i trusts completely the opinions of j, while when w is equal to one i considers exclusively his own opinion and agents decisions are fully independent from each other. At the beginning of each period t_k agents expectations are reset to random values. We stress that in the model imitation is purely expectation based, and agents do not imitate the actions of others. This choice is motivated by the fact that in a real market normally the order book is not fully visible to traders, and the order submission is anonymous.

While agents are noise traders in our model, we assume that they correctly anticipate the impact of herding on asset prices. In particular if an agent has several incoming links, and w is small (in which case the agent expects to be able to influence the decisions of others), he forecast a larger price volatility. This is incorporated in the model by assuming that the volatility of forcasted returns is proportional to the number of incoming links and to the weights w, such that

$$\sigma^i_{t_k} = \sigma^i_0 (A + l^{\%}_{i,t_k}(1 - w)) \tag{7}$$

where $l^{\%}_{i,t_k}$ in the percentage of existing links that point to agent i at time t_k and A is a constant parameter. The values of σ^i_0 are chosen, with uniform probability, in the interval $(0, \sigma_0)$. The effect of this rule is to favour market order submission over limit order submission from agents who have many incoming links. This happens because the interval over which the limit price is chosen is wider when $\sigma^i_{t_k}$ is larger and the selected limit order are more likely to be immediately marketable. Not only the guru is more likely to submit a market order in this way, but also his followers are, because of the expectation formation mechanism in equation (6). A sequence of market orders in the same direction will in turn generate large price fluctuations fulfilling the original expectation of the guru.

3 Simulations and results

The model is studied numerically for different values of the parameters λ and w. We focus the analysis on the statistical properties of the probability distribution of stock returns and on the auto-correlation of market volatility.
We also analyse some properties of the order submission strategies such as the spread, the distance between best price (bid/ask) and the price at the next best quote and the trading volume.
In the simulations the number of traders is set at $N = 100$. Each agent is initially given a random amount of stocks, uniformly distributed in the interval $S_0 \in [0, 100]$ and an amount of cash uniformly distributed in the interval $C_0 \in [0, 100]$. The initial stock price is chosen at $p_0 = 1000$. We fix $\tau = 200, \Delta = 0.01$, $n_0 = 0.01$, $\sigma_\lambda = 0.01$. The results reported here are the outcome of simulations of $T = 2000$ periods and $N_t = 200$ trades per periods. With about 200 transactions per period, a trading period would be of the order of few minutes to few hours, depending on the the liquidity of the market. Simulations are repeated $M = 100$ times with a different random seed. While 100 agents may appear quite small, the number of agents in real markets, who monitor each other expectations is not large. Adding a number of pure noise traders to the system, may make the simulations more realistic but would not change qualitatively the results, because noise traders transactions would, on average, offset each others.

3.1 Returns and volatility

We first analyse the statistical properties of returns in a network with a fixed guru as we increase the (constant) probability of attachment λ.
Figure (1) displays a sample path of the return time series for $\lambda = 0$, $\lambda = 0.5$ and $\lambda = 1$ when $w = 0.1$.

Figure (2) shows the return time series when the probability of attachment follows equation (4), for different values of w.

A more precise measure of fat tails is provided by the Hill exponent. In figure (3)(left side) we plot the hill exponent as a function of λ when w=0.5, and in the right side we plot it as a function of w for $\lambda = 0$, $\lambda = 1$, and λ time dependent.

Empirically the tail exponent is found to take values between 2 and 4. Changing the parameters of the model our simulations generates values for the Hill exponent in the same range. When $w = 1$ or $\lambda = 0$, that is in absence of imitation, the tail exponent approaches the normal value of 4. Nonetheless this limit value is never reached, because of the budged constrain.

It is clear from the figures that the introduction of a fixed guru in the model leads to fat tails but is not capable of generating volatility clustering. This is due to the fact that synchronisation effects become more and more important when λ increases and this can induce large fluctuations at all times. Volatility clustering instead reflects the transition from relatively quite trading period to more turbulent ones. To generate volatility clustering in our model we need to have a probability of attachment λ that increases and decreases over time.
A simple way to check if volatility is persistent is to measure the autocorrelation function of absolute returns for different time lags. Empirically it is observed that absolute returns are autocorrelated over lags of several weeks and decay slowly to zero. In fact, several authors (see [23], [24] and[25], [26], [27], [28] and [7] for evidence), have shown that the autocorrelation function of absolute returns, decreases hyperbolically with the time lag τ:

$$|C(\tau)| \sim \frac{B}{\tau^{\alpha}} \tag{8}$$

with $\alpha \in [0.2, 0.4]$ and is B a positive constant called the tail or scale parameter. In figure (4) we plot the autocorrelation of absolute returns (left) and of return (center) for w=0.1 and $\lambda = 0$ (black line), $\lambda = 1$ (red line) and time dependent λ (green line). We note that while the autocorrelation of returns is insignificant in all cases, a positive and slowly decaying autocorrelation of absolute returns is present in the case of time dependent λ. The autocorrelation function of absolute returns is well fitted by a power law with $\beta = 0.32$ as seen in figure (4) (left side).

A more precise measure of long term memory is provided by the R/S statistics. In the modified R/S analysis the parameter that allows to discriminate between short and long term memory is the exponent β. If only short memory is present β converges to 1/2 while with long memory, it converges to a value larger than 1/2. In figure (5) we plot the modified R/S exponent β for absolute returns as a function of w. We observe that, only for time dependent λ, β becomes significantly larger that 0.5. In this case we also observe that β increases when w decreases reflecting that the memory is longer when the imitation is stronger.

The attachment mechanism in equation (4) shows that the dynamic of creating, sustaining and destroying the guru is a plausible source of volatility clustering and long memory. The λ in our model evolves according to a reflected Geometric Brownian Motion. The exact dynamic of λ is not crucial but volatility clustering only arises if λ does not change too quickly[4]. Here λ is updated every

[4] In the endogenous version of the model oscillations on the number of incoming links to the guru are generated by allow-

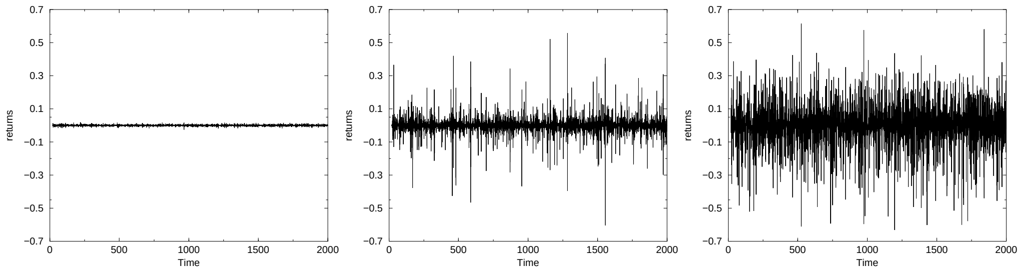

Fig. 1. Return time series for $\lambda = 0$ (left) and $\lambda = .5$ (center) and $\lambda = 1$ (right) and *w=0.1*

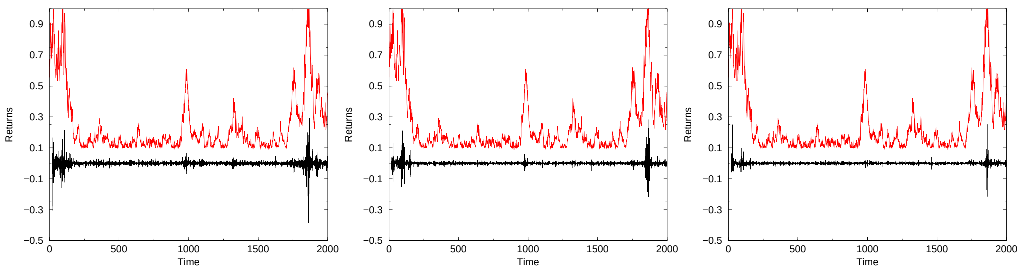

Fig. 2. Returns time series with a time dependent λ and w=0.2 (left), w=0.5 (center) and w=0.8 (right). The evolution of p is described by the red line.

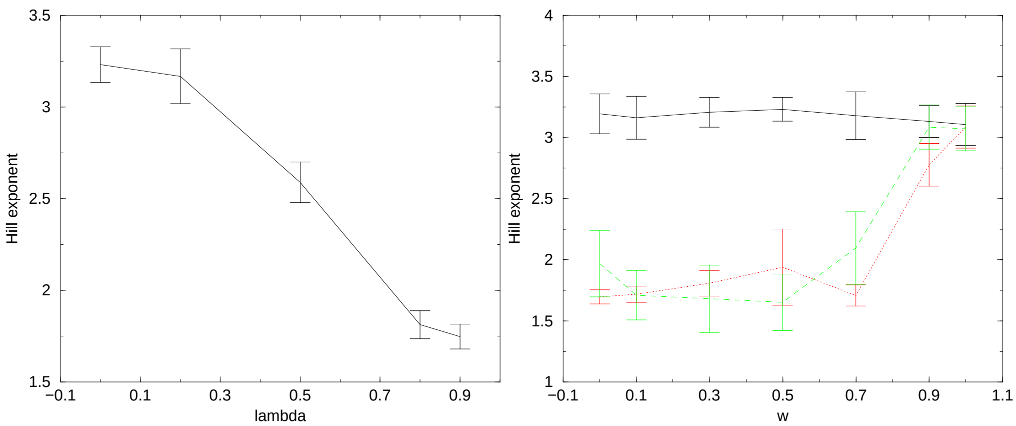

Fig. 3. Hill exponents of the returns distribution as as a function of constant λ for $w = 0.5$ (left side). Hill exponents of the returns distribution as a function of w for $\lambda = 0$, $\lambda = 1$ and λ time dependent, respectively black, red and green lines (right side).

trading period, but the value of σ_λ used for the simulations is small ($\sigma_\lambda = 0.01$). Figure (2) indeed shows that significative changes in λ, and the burst of volatility, occur over a few hundreds periods that correspond, approximately, to periods of months. A more precise fine tuning of the exponent β could be achieved by optimizing the ratio between the time scale at which the information network evolves and the time scale at which the trading decision is updated. A different ratio between these two time scales could explain the different values of β observed empirically in different markets.

ing agents to choose to whom to link. In fact, we implement an algorithm where agents link to the most successful agent, and not to a fixed guru. In that contest we introduce a switching probability smaller than one, to avoid that a new guru substitutes himself too quickly to the previous one.

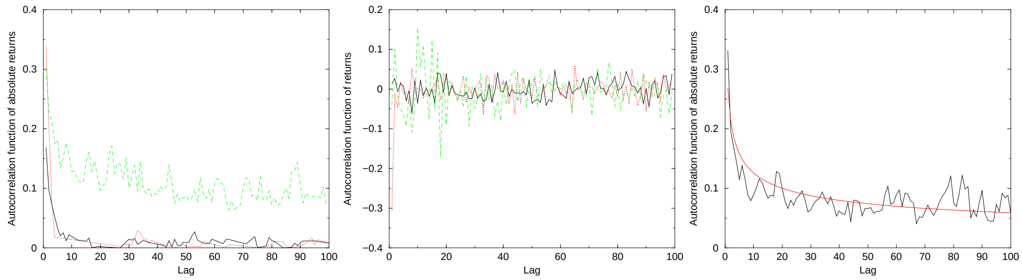

Fig. 4. Autocorrelation of absolute returns (left) and of returns (center) for w=0.1 and $\lambda = 0$ (black line), $\lambda = 1$ (red line) and time dependent λ (green line). Autocorrelation of absolute returns for w=0.5 and time dependent λ (black line) and the power law best fit (red line) (left side).

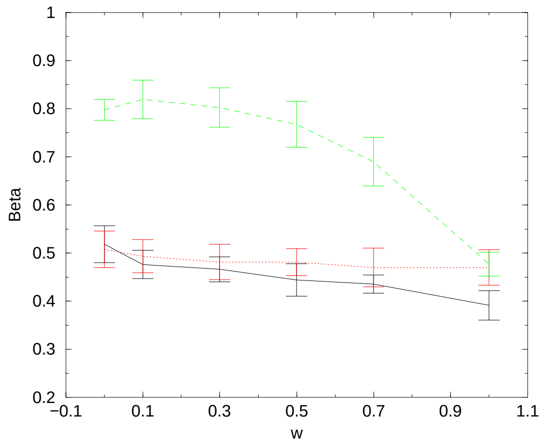

Fig. 5. R/S exponent for absolute returns as function of w for $\lambda = 0$ (black line), $\lambda = 1$ (red line) and the time dependent case (green line).

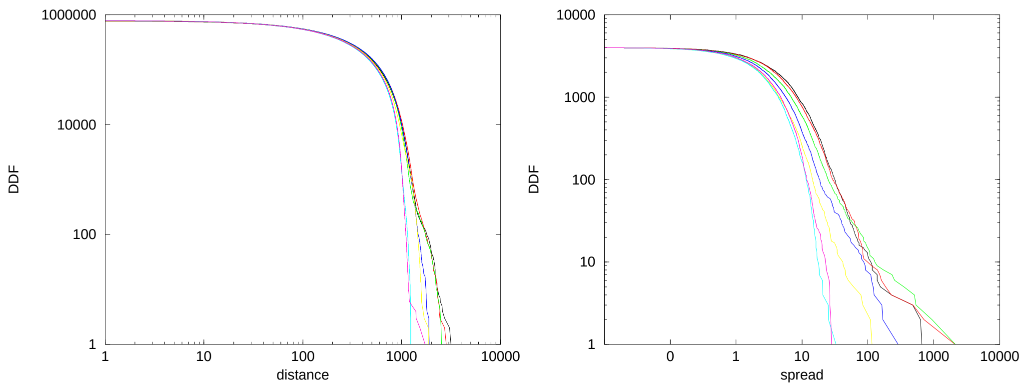

Fig. 6. DDF of limit order placement distance from the midpoint, respectively for w equal to 0 (black line), 0.1 (red line), 0.3 (green line), 0.5 (blue line), 0.7 (yellow line), 0.9 (cyan line) and 1 (magenta line) (right side). DDF of bid-ask spread, respectively for w equal to 0 (black line), 0.1 (red line), 0.3 (green line), 0.5 (blue line), 0.7 (yellow line), 0.9 (cyan line) and 1 (magenta line) (left side).

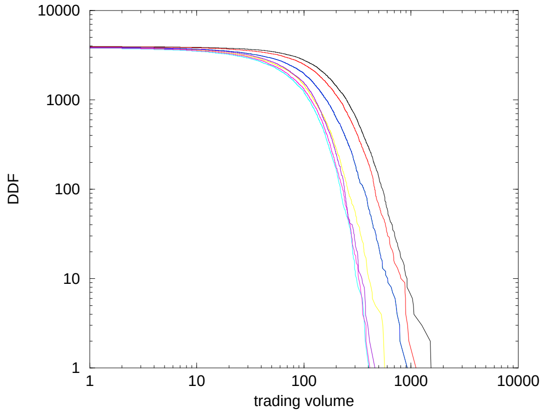

Fig. 7. DDF of trading volume, respectively for w equal to 0 (black line), 0.1 (red line), 0.3 (green line), 0.5 (blue line), 0.7 (yellow line), 0.9 (cyan line) and 1 (magenta line).

3.2 Book and order flows

In this subsection we investigate the impact of the imitation on the order flows when the probability of attachment follows equation (4) (the case that leads to volatility clustering).
Figure (6) (left side) displays the decumulative distribution function (DDF) of the limit order placement distance from the midpoint for different values of w. We observe that, increasing imitation, limit orders are placed at large distance from the midpoint. Figure (6) (right side) shows the DDF of the bid-ask spread when changing w. For small values of w we observe that the DDF becomes power-law, in agreement with the empirical observation that spread and volatility are positively correlated. These results are consistent with the empirical analyses of [19] and [18].

Next, we investigate the relationship between fat-taild fluctuations in asset prices and trading volume and gap sizes. According to [21], power-law tails in price fluctuation are driven by power-low fluctuations in the volume of transactions. The power-law decay of trading volume nonetheless is not well establishes empirically and varies from exchange to exchange. [20] have suggested an alternative explanation for the large fluctuation in returns. They show empirically that order submission typically results in a large price change, not when the order is large but when a large gap is present between the best price (ask- bid) and the price at the next best quote. Figure (7) displays the DDF of the trading volume in our model for different values of w. We observe that, when imitation increases, trading volume also increases, but the DDF does not change shape to become more fat tailed, thus contradicting the argument of [21]. Figure (8) shows the DDF of the first gap on the ask side (left) and bid side (right) in our model as a function of w. We observe that the gap distribution does become power law as imitation increases. This result indicates that the formation of large gaps is the leading mechanism generating large price fluctuations in the model. Large gaps in our model are created when imitation is large because limit orders are placed at a wider range of distances from the midpoint (as seen in Figure (6)). This in turn is due to the larger volatility predicted by the Guru, via equation (7), which affects both the choice of $\hat{p}^i_{t_k,t_k+\tau}$ and of the interval $(p_{\min}, p_{\max})$.

4 Conclusion

In this paper we have investigated the impact of imitating a fixed guru on the statistical properties of asset prices. We have shown that the model can reproduce a number of stylised facts observed in real financial time series.

The model proposed in this paper is a toy model that relies on a number of ad-hoc exogenously imposed rules whose purpose is not to describe the behaviour of real agents in real markets, but to identifying the main conditions under which imitiation leads to fat tails and volatility clustering in a limit order market. In particular imitation of believes in itself does not generate trading synchronisation and large price fluctuations unless the guru is capable of anticipating the impact of herding on the asset price. Furthermore for clustering to appear the popularity of the guru need to change, slowly, over time.

The main limitation of this study is in that the comunication structure is imposed exogenous on the system. In a forthcoming paper [22] we extend the analysis allowing imitation to arise endogenoulsy via a fitness mechanism based on agents wealth. In this more general setting we also study under which conditions a guru may endogenously rise and fall over time, and how imitation affects the distribution of agents wealth. Furthermore we introduce a proper utility function from which to derive agent's demand, like the one in [5] and [29] and we perform a more careful study of how the results depend on the system size and on the ratio between the time scale at which the information network evolves and the time scale the trading decision is updated.

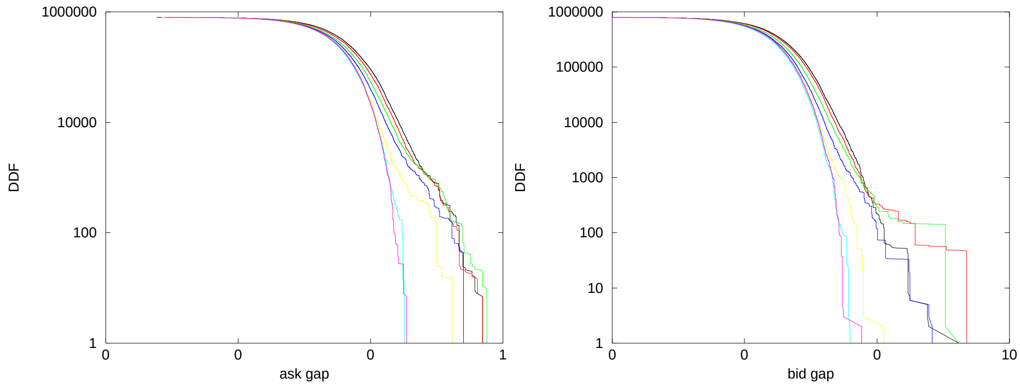

Fig. 8. DDF of gap sizes for ask (left) and bid (right) side, respectively for w equal to 0 (black line), 0.1 (red line), 0.3 (green line), 0.5 (blue line), 0.7 (yellow line), 0.9 (cyan line) and 1 (magenta line).

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