English Auctions with Resale:

An Experimental Study

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November 2010

Abstract

I design and test a simple English auction and two English auctions with resale, but with different informational backgrounds. All three treatments theoretically have the same equilibrium. I find, however, that the possibility of resale alters behavior significantly. In the two treatments with resale, subjects deviated from both the Nash prediction and the common results about bidding behavior in English auctions. Subjects tend to overbid, when they are certain they can reap the whole surplus in the resale market. I employ different models like QRE and levels of reasoning and conclude that overbidding can be explained as a rational response to the noisy environment in markets with human participants, that is, as rational decision making when anticipating others to make errors. When the outcome of the resale market is not certain, there is significant signaling behavior and auction prices tend to be lower than the the Nash prediction.

JEL classification: D44, C90

*Bonn Graduate School of Economics, University of Bonn. My thanks go to Reinhard Selten, Rosemarie Nagel, John Kagel, Benny Moldovanu and Joerg Oechssler for their comments and support. I would like to acknowledge the financial support of Spain's Ministry of Education under grant nr. BEC 2000-1029, of the Chair of Economic Theory II and Bonn Econ Lab at the University of Bonn. Additionally I want to thank three anonymous referees, Steffen Altmann for his very thorough proofreading, Nagore Iriberri for very fruitful discussions, Dan Levin, Thomas Palfrey, Vincent Crawford and Charles Holt for useful comments, Michal Paluch and Lars Koch for their technical support, Noemi Sanchez for translating the instructions and many participants at the Econometric Society NA Summer Meeting at Duke, the ES European Meeting in Budapest, the ESA World Meeting in Amsterdam, the EDP Jamboree in Louvain and the Bonn-Cologne Experimental Workshop. The experiment was programmed and conducted with the software z-Tree (Fischbacher 1999).

1 Introduction

Auctions are very often followed by a resale opportunity. For instance, after virtually every durable good auction, the winner can choose to resell the good to the competing bidders or other third parties. Even when resale is explicitly prohibited, ways can be found to get around the prohibition. Consider the case of mobile phone and wireless spectrum licences where "use-it-or-lose-it" conditions to prevent resale are often imposed. Such restrictions can still be circumvented. The company holding the licence can be bought and there have even been cases where special companies were used to buy and resell the licence.

Resale was not studied in auction models until relatively recently. However the work of Haile (1999, 2000, 2001, 2003) has shown that the existence of resale opportunities is important and can change many results of auction theory. In order for resale to be meaningful, the outcome of the initial auction must be inefficient with a positive probability. In Haile's paper (2003) the highest value player does not necessarily win in the initial auction because he does not perfectly know his value or because he is not participating. Other ways to induce resale in equilibrium can be asymmetries in the values of the bidders (Hafalir and Krishna 2008, Garratt and Troeger 2006) or new participants arriving in the resale stage (Haile 1999).

In this paper I test auctions with resale in the laboratory based on a simple model from Haile (1999) and find an alternative reason for resale, namely noisy or erroneous decision making. Noise and errors have not yet been considered in the abstract theoretical models, but are plausible when human players are involved. People make mistakes and anticipate others to make mistakes. This can lead players to deviate from theoretical predictions in systematic ways. To see how it can induce resale, consider the following example.

Suppose you are participating in an English auction for works of art, which in the absence of resale possibilities is often regarded to be a textbook example of private values. Suppose that the current price for the Picasso painting under sale is 10 euros. Even if you do not have a taste for cubism and thus your private value is zero you might still want to participate in the auction, expecting to resell the painting for a higher price. Thus resale introduces a common value element to the valuations of bidders and can induce overbidding. We could use a similar argument in the markets for real estate, bonds, operating licences and more.

Thinking in line with standard models one could note that in such a simple setting, bidding one's value is still a symmetric equilibrium. A strategy of overbidding expecting to resell is not consistent with this equilibrium. If others bid their values, no profitable resale is possible, as winning with a bid higher than your value can only result in zero or negative payoffs. Crucially however, this is only true if bidders never make mistakes, as is usually assumed.

From the economic literature in the lab but also casual observation in the field, we know humans are prone to making mistakes. Expecting high value bidders to make mistakes can make it in turn optimal for low value bidders to bid more than their values. In response, high value bidders have an incentive to bid less, expecting to buy cheaper in the resale stage. Thus resale opportunities can be exploited even if standard theory predicts they will not and can give rise to richer bidding strategies than theoretically expected. Let it be emphasized that this deviation from standard models is quite natural. There is no need for restrictive assumptions on the structure of markets or the private information of bidders to induce resale in real life situations. A sufficient condition, as will be shown, is the presence of a small amount of noise. Such noise exists in many markets, even in financial markets where stakes are very high (see Shleifer and Summers 1990). It can stem from experimentation, lack of experience or misunderstanding of the rules, false transmission of information and mistakes in the execution of orders, liquidity constraints or other exogenous reasons that are not adequately modeled in theory but whose presence in real markets cannot be easily dismissed.

To examine the importance of resale opportunities and the effect of noise in a controlled environment, I designed and ran two experimental treatments of English auctions with resale. Although treatments had different informational backgrounds, they had the same equilibrium bidding functions, prescribing that players bid their values. Subjects exhibited significantly different behavior with respect to both the theory and previous auction experiments without resale. Instead of bidding their values in both treatments, they overbid relative to equilibrium when they can be certain they can reap all the rents in the resale markets, and they tend to underbid when the resale outcomes are uncertain. Moreover this result should not be attributed simply to irrational behavior in the laboratory, but seems to have a rational explanation. Subjects do try to maximize their profits. But while doing so, they anticipate the possibility of others making mistakes and they use this knowledge more or less optimally. In that sense, this paper presents a previously unstudied example of a more general class of games where the anticipation of noise drastically changes players' best responses[1]. In such cases, standard game theory loses much of its predictive power and concepts of bounded rationality, such as a Quantal Response Equilibrium (McKelvey and Palfrey 1995) and levels of reasoning (Nagel 1995, Stahl 1995, Camerer 2004, Crawford and Iriberri 2008), perform much better.

The experimental economics literature has not focused on auctions with resale yet, for the same reasons that there were precious few theoretical models of resale until recently. To my knowledge there exist four other experimental papers on auctions with resale. Two of these analyze symmetric auctions in the spirit of Haile (2003); Georganas (2003) on which part of the present paper is based and independent work by Lange, List and Price (2004). Their experimental treatments are similar to the ones in this paper, but they differ in important ways: first they used first-price sealed-bid auctions and secondly they gave players noisy signals about their private values. They found deviations from equilibrium predictions, which they attribute to risk aversion. However risk aversion alone does not change the equilibrium in the present study's games, so it cannot explain the data. Jabs-Saral (2009) has conducted experiments in English auctions with resale, however the division of the surplus in the resale market is different to the other papers in the literature. Subsequent to the present paper, Georganas and Kagel (forthcoming) analyze asymmetric first price auctions with resale and find support for the equilibrium that predicts weak players bidding more aggressively than without resale, although this result depends on the magnitude of the asymmetry.

Even though the possibility of resale and its potential importance has been recognized in the theoretical literature (Milgrom and Weber, 1982 and Milgrom 1987 with the first models of auctions with resale) there has been a striking absence of formal models featuring resale until recently. A frequent argument has been that resale is covered by the assumption of common values. However, as shown in Haile (2003) players in the initial auction have common values when there is a possibility of resale but, importantly, valuations are endogenously determined

[1] Games with this property include the guessing game, the centipede game and the traveler's dilemma.

and equilibrium strategies are not the same as in the simple common value model. Moreover Revenue Equivalence holds under some assumptions although it does not in the common value case. In Haile (2003) bidders have noisy signals about their values in the initial auction. Noisy signals work in a similar way as the noisy bids in this paper, as they lead to inefficient outcomes and profitable resale.

There exist other possibilities, besides noise, to make resale potentially profitable. Haile (1999) assumes that an a priori known number of bidders is added to the bidder pool in the second period. These new subjects arriving in the resale auction can have higher private values than the winner of the first auction, opening up resale possibilities.

One can alternatively construct models with asymmetric equilibria. Resale seems intuitively plausible in such an equilibrium, as the asymmetry in players' strategies implies that the highest value player will not necessarily be the highest bidder in the initial auction. This option is explored in Garratt, Troeger (2006). In a setup similar to mine they include speculators with zero valuations and find asymmetric equilibria where the speculator wins with positive probability. Gupta and Lebrun (1999) and Hafalir and Krishna (2008), on which the aforementioned Georganas and Kagel paper is based, have bidders with potentially positive use values, which however are asymmetrically distributed. This setup also gives rise to inefficient outcomes and subsequent resale.

Finally, other models including some flavor of resale are Ausubel and Cramton (1999), McAfee (1998) and Jehiel and Moldovanu (1999), although their setups are not directly related to the present paper.

This paper is structured as follows. The experimental procedure is introduced in section 2. Section 3 presents the equilibrium predictions and the results are presented in section 4. Models of bounded rationality involving some flavor of noisy decision making are presented in section 5. Ideas for future work are discussed in section 6 and section 7 concludes.

2 Experimental design

There are two stages in the game. In the first stage four bidders $i = 1, 2, 3, 4$ bid in an English auction for one unit of an indivisible object. Each bidder has a use value v_i, which is identically independently drawn from a discrete uniform distribution with support [0,100]. The distribution of the use values is common knowledge, but the actual use values are private knowledge. We have to emphasize the distinction between a bidder's *use value*, i.e. the value a bidder places on owning the object ignoring any resale possibilities and the bidder's *valuation*, which is the value she places on winning the auction and which is determined endogenously, taking account of the resale opportunity.

For the auction we use an ascending clock design (see Kagel et al. 1987). There is a clock on each computer screen, starting simultaneously from zero and synchronously rising every second in steps of one unit. Each subject can exit the auction at any time by pressing a button. Once out of the auction no reentry is possible. The other bidders can observe the price at which one exits. After three bidders have left the auction, the last remaining bidder obtains the good and has to pay the price p_1 at which the last one left. This concludes the first stage.

In the second stage there is the possibility of resale. This is done through an English auction, where the seller can choose a reservation price. The difference between the two resale treatments, lies in the informational background of the second stage. As discussed there are many ways to model the resale stage. I chose two extremes with a big span between them, to test for a wide range of possibilities. In the first, incomplete information treatment (hence INC), the only information the bidders get about the others' values is through the bids in the initial auction. The seller decides about the reservation price r and then the remaining bidders can see the reserve price and decide simultaneously if they want to participate in the resale auction or not[2]. If no bidder chooses to participate then the ownership of the good and the payoffs remain the same as in the first stage. If only one bidder participates, then she obtains the good and pays the reservation price to the owner. Thus the final payoffs are $r - p_1$ for the first stage winner and $v_i - r$ for the second stage winner. If more than one bidder decides to participate we have an English auction like in the first stage, with the difference that this time the clock starts at the reserve price. Again when only one bidder remains, she obtains the good and has to pay the price p_2 where the last bidder left the auction. The

[2]Sellers did not have the explicit choice not to put the good up for resale, however they were advised to set a reservation price of 100 if they did not want to resell.

following payoffs are then communicated to the subjects: $p_2 - p_1$ for the first stage winner, $v_i - p_2$ for the winner in the second stage and zero for the others. In the same screen they can see the price of the initial auction, the reserve price, the number of participants and the price in the resale auction (zero if there was no resale auction), the highest private value and information about past periods. The information feedback was so rich in order to facilitate learning, as otherwise bidders would be getting too few experiences of winning and thus learning chances. Note that there are 30 periods in each experiment and subjects win on average only 1/4 of the time, which means that they get to win 7 or 8 times on average.

In the second treatment with complete information (COMP), after the first stage bidders get to know the use values of the others as in Gupta and Lebrun (1999). Thus in a subgame perfect equilibrium they will ask for a reservation price equal to the highest private value. This amounts to a take-it-or-leave it offer to the person with the highest private value equal to his private value. It is a well known fact however that subjects in experiments very often deviate from the equilibrium in the direction of a 50-50 split of the surplus, probably because of fairness considerations (see Chapter 4 from Kagel and Roth 1995 for a review). As it is not the subject of this paper to treat bargaining games, I force the winner of the first auction to automatically resell the object in the second stage to the bidder with the highest value, as well as requiring the highest value bidder to purchase at that price. She then received as payoff the highest private value minus the price she paid in the first auction. The rest of the players, including the person who obtains the good after resale, have a payoff of zero. As in INC, after each auction players can see their payoffs, private values, the auction price, the private value of the winner, the highest private value and information about past periods.

The resale treatments are to be compared with the bidding behavior in English clock auctions without resale. Such auctions have been extensively discussed in the literature (see for example Kagel et al. 1987), however it is important to make sure that the unexpected results in COMP and INC are not due to some kind of framing effect or an unusual subject pool. Thus, I also ran a standard English auction (ENG), with IPV drawn from a uniform distribution [0,100]. The experimental mechanism was in all other respects the same as the one used in COMP and INC, so ENG can be directly compared to them. As the results in ENG are very similar to previous studies, this treatment will not be discussed on its own,

Session	Treatment	Exchange rate	Paying Periods	Players	Location
1	COMP	20	30	16	UPF
2	COMP	20	30	16	UPF
3	INC	25	30	16	UPF
4	INC	25	35	16	UPF
5	ENG	20	30	16	UPF
6	COMP	20	30	16	Bonn
7	COMP	20	30	16	Bonn
8	INC	25	30	16	Bonn
9	INC	25	30	16	Bonn
10	ENG	20	30	16	Bonn

Table 1: Summary of sessions

but only in comparison with the other treatments.

Observe that the use of the English auction in all treatments does not allow us to observe the intended bid of the winner, but only a lower bound. One could possibly argue that a second-price sealed-bid auction in the first stage would suit our purpose better. With this configuration the unobservable final bid problem is avoided. However behavior in sealed bid auctions usually presents large deviations from equilibrium, even without resale (see Kagel 1995 for a survey). Thus it is not feasible to separate the effect of resale from the other factors which push behavior away from equilibrium. On the other hand the English auction is widely studied and subjects seem to understand the Nash equilibrium and follow the predicted strategies quite closely.

The experiment involved 10 experimental sessions, with 16 participants in each. For the first 5 sessions, subjects were undergraduate students, mainly from the faculties of law and economics, at the Universitat Pompeu Fabra in Barcelona. The next five sessions were conducted at the University of Bonn, with subjects from many faculties. The analysis finds no consistent differences between the two groups (p-value>0.05, Mann-Whitney U test), so the data are pooled together.

At the beginning of the experiment the participants were divided in two subgroups[3] of 8 and then the players in every subgroup were randomly rematched every period in groups of 4. Subjects did not know what group they had been assigned to or who were the other

[3] This was done for statistical purposes, in order to have two independent observations in every session. Still the subjects did not know this and they thought they were being rematched with another 15 players. So the probability they will try to induce cooperating behaviour and the interperiod effect should remain small.

members of the group. There were 31 periods in almost[4] every experimental session. The first period was a practice period that did not count for the players' payoffs and was not used in the statistical analysis of the data. Subsequent to this period, subjects received an initial capital of 150 units of our experimental currency, the drachma. In the following periods subjects were rewarded according to their success and their profits or losses were added to the initial capital. Despite the sometimes quite aggressive bidding, there were no bankruptcies, although two subjects came close. After the end of each session the experimental currency was transformed to euros in a ratio of 25 drachmas per euro in COMP and 20 drachmas per euro in INC. The reason for this difference is that INC is more complicated. Sessions lasted about thirty minutes longer than COMP and we wished to keep average profits per hour constant. Thus average profit in COMP was 10.56 euro and in INC 15.5 euro. Naturally, this difference is not only due to the different exchange rate but due to the different bidding behavior too.

3 Equilibrium predictions

3.1 Complete information - COMP

In this section I compute the symmetric equilibrium of COMP. An important difference with respect to usual auction models, is that in the presence of resale, players have a value v_i (their exogenous private value) for the good and a valuation u_i , the value she places on winning the auction. The valuation u_i is determined endogenously, as it depends on the outcome in the resale market too. Use values v_i are drawn from a discrete uniform distribution with support $[0, 100]$. Consider the two-stage game COMP played by 4 risk-neutral players for a single indivisible object as described above. Let $y_1^i = \max\{v_j | j \neq i\}$, $i, j \in \{1, 2, 3, 4\}$ denote the highest use value among a given bidder's opponents and let v_{-i} denote the vector of the use values of all players, except i. Let f denote the final price of the game, which is equal to

[4] There was one session with two practice periods, but they did not seem necessary so subsequent sessions had only one. It does not matter for the analysis, as we always use observations after the 9th paying period.

$$f = \begin{cases} p_2 & \text{, if there was a resale auction} \\ r & \text{, if exactly one bidder participates in the resale auction} \\ p_1 & \text{, if no bidder participates in the resale auction} \end{cases}$$

The following proposition, equivalent to Theorem 1 in Haile (1999), describes the equilibrium in the first stage.

Proposition 1 *The symmetric bid your value equilibrium for an English auction without resale is also a Perfect Bayesian Equilibrium bidding strategy when the same auction is followed by a resale opportunity, where the private values of the bidders are publicly announced.*

Proof. Suppose bidder i with use value v_i deviates to a bid $\tilde{v} > v_i$, while all other bidders follow the proposed equilibrium strategy and bid their use values. This would change i's payoff only in the event that $\tilde{v} > y_1^i > v_i$. However if this is the case, i would have to pay y_1^i for the object but could only resell it for some price p_2 in the interval $[r, y_1^i]$. In equilibrium the reseller will set $r = y_1^i$ under complete information in the resale market, but this still leaves him with nonpositive expected profit. By bidding v_i, i would have received zero profit with certainty. A similar argument shows that bidder i would not profit by bidding less than v_i. ■

This proposition provides the risk-neutral symmetric Nash equilibrium[5] under complete information in the resale market, but as we shall see the theorem remains valid under risk aversion and incomplete information. Therefore I will refer to this equilibrium as symmetric[6] Nash equilibrium (SNE). Also, this equilibrium covers the special case of the automated resale market that was actually used in the lab.

A characteristic of the equilibrium that should be noted is, that unlike the simple English auction, bidding your (use) value is not a weakly dominant strategy in the presence of resale

[5]Note that we treat the game as a second price sealed bid auction. The equilibrium we find is the equilibrium for the last stage of the English auction too, where only two bidders remain, as the previous 2 exits do not carry any important information that alters the players' strategies.

[6]It is worth noting here that in discrete value English auctions there exists an asymmetric equilibrium where one player bids his value and the other bids her value minus one increment. In the questions we asked after our experiments, on average 2.3 subjects out of 16 in each session reported that they used such strategies. However, this was scarcely detected in their bidding (only one person in each treatment was consistently bidding one unit below value) and a few players employing such a strategy do not significantly influence the analysis.

possibilities. If the person with the highest use value were to deviate from equilibrium and bid less than their value, it is clear that the best response for the others would be to bid up to this highest value (see Section 5 for an extensive discussion). However, the proof can be generalized to show that it is a best response for a bidder i to bid their *valuation* u_i of the good, where this valuation is determined endogenously by the strategies of his opponents.

Remark 1 *This equilibrium is unique among subgame perfect equilibria in weakly undominated strategies.*

First, note that in any subgame perfect equilibrium, the reserve price in the second stage is set equal to the highest use value and the first stage winner extracts all rents. This means only the first stage winner can make positive profits. Then, value bidding weakly dominates bidding less than value, for any type, since it only lowers their probability of winning in the first stage in the beneficial cases that the price is lower than their value and it does not affect their second stage profits, conditional on losing. But then value bidding also dominates overbidding, since there is no way that the auction price in the first stage will be lower than the highest value in the second. Thus there is no opportunity for profitable resale and bidding above value raises the probability of winning for a player only in the unappealing case where the price is higher than the highest value of all players.

3.2 Incomplete Information - INC

In treatment INC the theoretical prediction is the same as in COMP. The argument is similar to the one above. The only difference is that since private values do not become common knowledge in the resale stage, the reserve price has to be calculated in a more complicated way using the information produced by the signals (bids) in the initial auction. However, minimal rationality implies that independent of these signals the reserve price has to be higher than the private value of the first stage winner. This makes sure that in equilibrium we do not have resale and thus bidding one's value remains an equilibrium strategy in the initial auction. More formally:

Proposition 2 *Let bidders in INC have following pure strategy:*

i) In the first stage player i bids her value, ie $b_i = v_i$

ii) if i wins in the first stage she sets a reserve price $r_i \in [v_i, 100)$

iii) if i loses in the first stage she participates in the second iff $v_i \geq r$

iv) bidders in the second stage bid their values, ie $b_i = v_i$

All strategies of this kind are equilibrium strategies in INC.

Proof. In the second stage it is optimal for the players to enter the auction if the reserve price is less than or equal to their values and bid their values, as it reduces to a simple English auction. Note that given the first period strategies the winner is always the bidder with the highest value. Due to this and given the second stage strategies of the other bidders, any reserve price in the second stage that is equal to or higher than the winner's use value is optimal and leads to the same payoffs. In the first stage, given such a reserve price and that other players bid their values, similar to Proposition 1, there is no profitable deviation for any player. A player i who does not win in the first stage can not expect to buy in the second stage for a price lower or equal to the highest value among all other bidders. But this is actually the price she would have to pay if she won in the first stage. Thus players have no incentive to bid less than their values. Bidding more than their values can not be profitable either, as similar to Proposition 1, a winning bidder i in the first stage pays a price that is equal to the highest use value among the other bidders y_1^i and can never expect to get more than y_1^i in the second stage. ■

Note that in treatment INC, unlike in COMP, losing bidders can make positive profits in the second stage in principle. This makes analyzing the uniqueness of equilibrium more complex. Still in a monotonic symmetric separating equilibrium this will not be the case[7], winners always extract all rents. Thus, for this class of equilibria, we have the same result as in treatment COMP.

Remark 2 *This equilibrium is unique among subgame perfect monotonic symmetric separating equilibria in weakly undominated strategies.*

[7] Recall that the first stage winner observes the highest losing bid. In a monotonic symmetric separating equilibrium the winner can invert the bid function and deduce the highest value, which allows him to extract all the rents.

The argumentation is similar to Remark 1.

Another result is also possible, if we do not require monotonicity and separation. Define as underbidding, a player employing a bid function in the first stage that involves bidding less than value for at least one possible use value and value bidding for all others. For example, a bid function equal to zero for all values amounts to underbidding according to this definition. Equivalently denote as overbidding bidding more than value for at least one possible use value and value bidding for all others.

Remark 3 *This equilibrium is unique among subgame perfect symmetric equilibria in weakly undominated strategies where bidders in the first stage can either overbid, bid their values or underbid.*

Consider the case where players underbid, in the sense described above. Underbidding is weakly dominated by value bidding for the player with the highest value, since he can win the object for sure and pay less than the highest value among his opponents. Thus he avoids having to buy in the second stage where the minimum offer he will get will be at least as high as the highest value among his opponents. For the players who do not have the highest value, underbidding is obviously weakly dominated by value bidding.

Now, consider the case where players overbid. Overbidding by the highest value player is weakly dominated by value bidding, since it increases the probability of winning only for those cases that the price exceeds his value. On the other hand, in case he loses, a lower bid will lead to a weakly lower resale offer, as the optimal reserve price in the second stage rises in the first stage price. For all other players, overbidding is also weakly dominated by value bidding since it can only lead to a win when they bid more than the highest value of any player in the auction and thus negative expected payoffs.

4 Experimental Results

In the following I present the general results and in the subsequent sections I offer explanations for the data. When making the statistical analysis of the results I will start with period 10 unless otherwise stated, to abstract from any learning/adjustment processes.[8]

The main question posed, is if resale possibilities alters behavior in auctions. The answer from the data is a definite *yes*. Figure 1 compares bidding in the first stage of the three treatments, using boxplots which include all but the winning bids.[9]

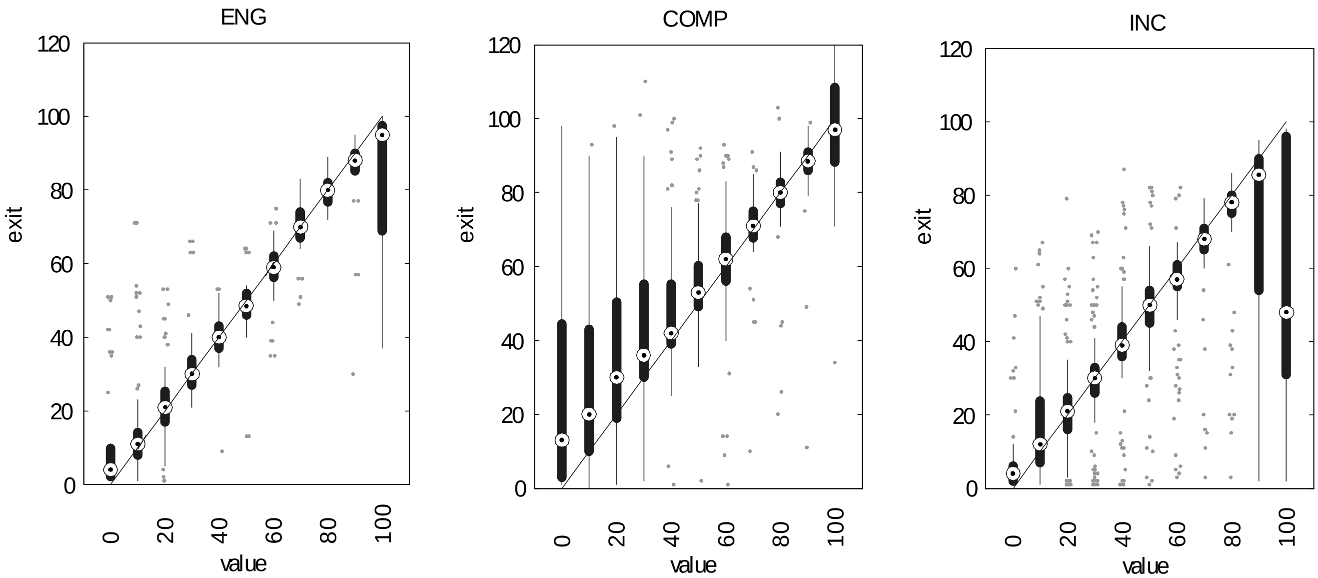

Figure 1: Series of boxplots of private values vs exits in the various treatments. Each box drawn represents the distribution of the bids for a block of values. The circle in the box is the median.The length of the box represents the interquartile range (IQR). The whisker extends from the box to the most extreme data value within 1.5 times the IQR. The dots represent outliers beyond that range.

We can see that although the three games have symmetric equilibria with the same bidding functions in the first stage, actual bidding behavior is quite different. The mere presence of a resale market makes subjects deviate systematically from the equilibrium, in marked contrast to their behavior in the simple English auction.[10] Table 2 also shows that

[8]See also Kagel et al (1987), p. 1286 where the authors claim "subjects' adjusting to experimental conditions argue for throwing out the first three auction periods" or Fehr/Schmidt (1999) who only use last period values.

[9]Keep in mind that the exit price of the last bidder is not equal to the maximum bid he was prepared to make, because he exits automatically once the last-but-one bidder exits the auction. As a consequence we only have a lower bound on the actual bidding strategy of these players. For the graphs and other statistics we exclude the winning bids. Although it leads to some bias, including them leads to an even greater bias. Techniques such as censored regressions do not completely eliminate this problem and they introduce new ones, e.g. they would rely on the restrictive assumptions that bidding is symmetric and follows some particular functional form.

[10]Bidding in COMP is significantly different from the bidding in the treatment ENG (without resale). Comparing INC with ENG we do not always have statistic significance. This can be attributed to several reasons. First, we do not have many observations for the simple English auction. Most previous experiments

bids in simple English auctions are in some cases significantly different from the predicted equilibrium. So, it is not just the significance that is impressive in the case of the treatments with resale (COMP and INC), as much as the systematic deviations that are large and significant, which is obvious from Figure 1. This result shows that studies of auctions should take resale possibilities explicitly into account.

That is not the only interesting result. The specific structure of the resale market makes a difference for the bidding strategies. We see in Figure 1 that in COMP, when subjects have common knowledge of the private values before the second stage, resale gives the low value types an incentive to overbid.[11] On the other hand, low value types bid close to their values in INC and ENG. High value types bid close to their values in COMP and ENG but not in INC. In general, bids in COMP are highly significantly different from bids in INC for all possible values.

Note that the underbidding of the high types in INC is not a spurious phenomenon due to the censoring of winning bids or to a presence of extreme observations driving the average down. Among bidders with use values greater than 50, the percentage of bids lower than 20 in INC are 5.5% versus 1.8% in COMP (one sided Fisher exact test p value $= 0.007$). Comparing high value bidders who bid less than 50 we get an even higher difference with 17.6% versus 6.23% respectively (p value<0.000).

Theoretically the only difference between the two treatments is in the informational structure of the resale stage. Naturally it is possible that differences in the bidding strategies of the subjects are not only due to the theoretical difference, but also due to the different mechanisms used in practice. In particular there is evidence from bargaining games where subjects do not behave "rationally" and split the surplus in ways that do not follow the Nash prediction. In COMP I did not allow subjects to deviate, enforcing on them exogenously the predicted outcome of the second stage. In INC this was not possible and as a consequence

however, have found bidding which is very close to the bid-your-value equlibrium, and including these experiments we would get a significant difference between ENG and INC. The second problem is that for high values, where bidding differs most from the equilibrium , in INC, we have a strong problem of unobservable bids. We tried to control for this by running a censored regression of bids on values including data from ENG and INC. The dummy and interaction terms were highly significant, which supports the hypothesis that bidding in ENG and INC was indeed different.

[11] I use the term overbidding/underbidding loosely, to describe bids higher/lower than a subject's use value, even when such bids are not necessarily irrational.

	Values				
Treatments	0-20	21-40	41-60	61-80	81-100
COMP - SNE	-20.23	-13.80	–5.69	-1.12	1.91
	(0.000)	(0.000)	(0.006)	(0.083)	(1)
INC - SNE	-4.47	-0.99	2.16	7.97	20.25
	(0.000)	(0.404)	(0.0404)	(0.000)	(0.000)
ENG - SNE	-8.03	-2.70	-0.27	-0.40	5.73
	(0.004)	(0.004)	(0.150)	(1)	(0.000)
COMP - INC	-15.75	-12.81	-7.85	-9.09	-18.33
	(0.003)	(0.001)	(0.015)	(0.003)	(0.008)
COMP - ENG	-12.20	-11.10	-5.42	–0.71	–3.81
	(0.072)	(0.008)	(0.072)	(0.808)	(0.153)
INC - ENG	3.55	1.71	2.43	8,38	14.52
	(0.682)	(0.153)	(0.808)	(0.004)	(0.153)

Table 2: Differences in average deviations (private values minus bids), calculated excluding the censored observations. The numbers in parentheses are the p values of a Mann Whitney U test.

subjects were allowed to play the resale game themselves. This difference in the mechanisms used could be a problem, however the data about the rationality in the choice of reserve prices and in the choice of participation presented in Section 4.2 indicates that subject's behavior in the second stage was fairly rational and competitive, even though we cannot show that they were completely following the Nash prediction.[12]

The difference in strategies between treatments translates into different prices in the auction. As we can see in Table 3, average prices in COMP were almost 18% higher than in INC (p-value=0.002, Mann Whitney U-test comparing the 8 independent observations of each treatment) and 15% higher than in ENG (p-value=0.008, U-test using the 4 independent observations in ENG), whereas the average private values were very similar, as happened with the average equilibrium prices too. This difference is not only highly significant but also quite large and economically important. The average highest value in every auction was about 80 so revenues in COMP were almost halfway between the Nash prediction and the maximum rents the seller could possibly appropriate. Prices in ENG were slightly higher than in INC but the difference is not so big and not significant. It should be noted that prices in both are a bit lower than the predicted ones though[13].

[12] There is evidence that subjects cannot calculate difficult equilibria. Setting an optimal reserve price given your beliefs is a fairly complex task even for theorists. Also, Davis, Katok and Kwasnica (2009) investigate reserve price setting experimentally and find frequent and significant deviations from the theory, especially regarding the sellers' response to a different number of buyers.

[13] The results regarding the simple English auction should be received a bit carefully as we do not have many observations. I did not run many experiments, as there exists already a very large literature on simple

	COMP	INC	ENG
Average Observed Price	67.04	56.85	57.97
Average Equilibrium Price	60.41	60.99	60.68
Average Private Value	50.37	51.34	50.68

Table 3: Average Prices, Equilibrium Predictions and Private Values. The Difference between COMP/INC and COMP/ENG are highly significant

In the following I analyze these results in more depth individually for every treatment and I compare a variety of models of bounded rationality that could explain them.

4.1 Complete Information - COMP

Figure 2 graphs average[14] prices in the initial auction, average resale prices[15] p_2 and SNE predictions - which as shown in Proposition 1 are equal to second highest values in the groups- over time, for the pooled data of all sessions of COMP. There were differences between individual sessions but the general tendency to overbid was the same in all of them, so it is not necessary to present individual session data. Table 2 reports mean deviations from the SNE predictions pooled over all sessions of treatment COMP.

In treatment COMP some underbidding is observed in the first few periods. As explained above, we can view these periods as adaptation periods. In the next periods mean prices in the initial auction lie always above the theoretical prediction, sometimes substantially so. Nonetheless, resale occurred in about 25.6% of the cases and mean resale prices are still higher than the initial auction prices, so the winners in the initial auction realize positive profits on average.

Mean overbidding over all subjects and for all values was about 8.4 units. Overbidding was strong on the individual level too. In COMP, 40 out of 64 players bid on average five units higher than their value or more. Setting the bar at 10 units, 31 out of 64 subjects still overbid even higher than that. For a closer view of individual bidding behavior depending on the drawn private value, the box plot of values versus exits in Figure 1 is very informative. Note that under the SNE prediction, all bids should lie on the 45-degree line through the

English auctions. Thus, for comparison purposes I refer to these older results too.

[14] In every period we take the average over the four groups that were formed in every experiment.

[15] Recall the resale price of the good is automatically equal to the highest value in group. Thus, the profit of the initial auction winner is just the difference between the highest value and the initial auction price.

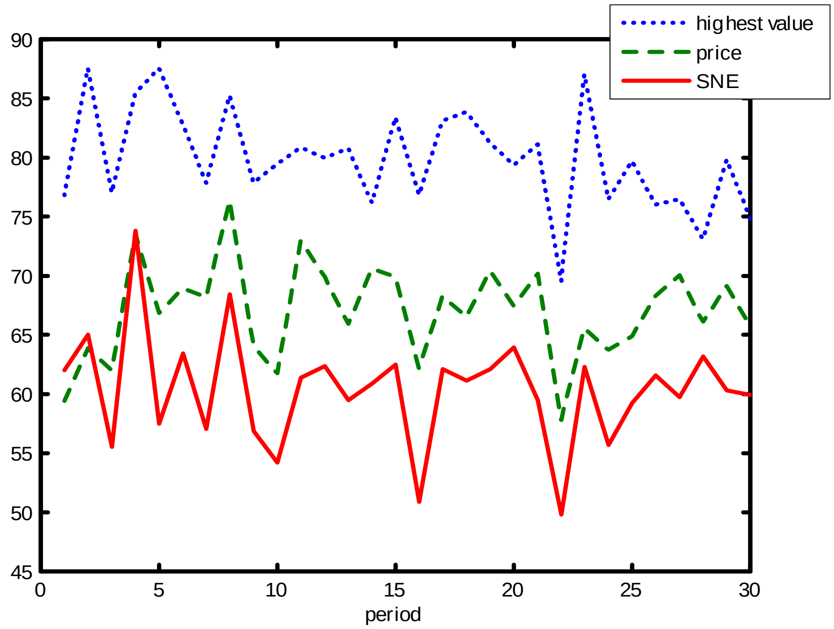

Figure 2: Mean prices, highest values and SNE predictions over time in first stage of treatment COMP.

origin. In this plot the overbidding is even clearer than if we only look at auction prices, especially if we compare bids in this plot with the bidding in ENG or INC. We also see that the high auction prices come almost entirely from the overbidding of the low value players. In fact low value players overbid 40% of the time and in about 83% of the cases where they do so, the highest value bidder does not win the auction. But since there were 4 bidders in every group, overbidding did not necessarily lead to winning. Thus, players who did not have the highest value could keep overbidding without obvious punishment, as they did not win the auction very often and when they won their profits were not very low[16].

The persistent excess of bids and prices above the equilibrium predictions has to be compared with the results of ENG and the previous results in the literature, like the English auction experiment in Kagel et al. (1987). In that study the fast learning and eventual convergence to the equilibrium predictions was attributed partly to the negative profits of subjects who started by overbidding in the first periods. This effect, pushing subjects towards equilibrium behavior does not exist in sessions 1 and 6 and was very weak in sessions 2 and 7.

[16]Mean profit of bidders who did not have the highest value but won was 0.77 over all periods and -0.20 in the last 15.

Thus subjects were not always best responding to the other bidders, but they were still choosing strategies that yielded payoffs close to their best response payoffs. Section 5 presents models that allow for this kind of behavior, by assuming that subjects do not play pure best responses but have a mixed strategy, choosing a probability for an action depending on its expected payoff. I show that such a model rationalizes overbidding as a response to high value players underbidding with a positive probability and explains behavior better than the SNE.

4.2 Incomplete Information - INC

Figure 3 graphs average prices in the initial auction, average highest values and SNE predictions over time, for treatment INC. Table 3 reports mean deviations from the SNE model's predictions pooled for all sessions of treatment INC.

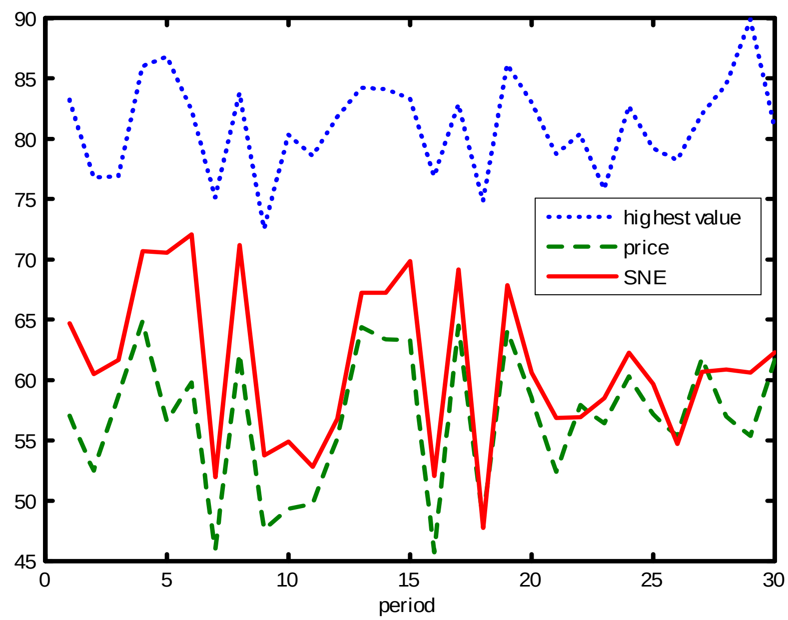

Figure 3: Mean prices, highest values and SNE predictions over time in first stafe of treatment INC.

Similar to treatment COMP we observe some learning in the first periods, as prices are systematically below those predicted under the SNE, after which behavior tends to stabilize. We do not observe any big differences from session to session of treatment INC. It has to be noted that in this treatment the asymmetric information in the resale stage makes richer strategic behavior possible. In particular signaling can be expected to play a significant role, so that looking just at prices or at aggregate values is less informative and a closer look on individual bidding behavior should be more revealing. Still there are some important facts to notice in Figure 3. The most obvious is that the overbidding from COMP has virtually disappeared, replaced by underbidding in most periods. This underbidding is substantial for higher valued bidders and as a result the highest value player loses the initial auction in about 13.4% of the cases. In the resale stage, prices are always higher than the prices in the initial auction, but still sometimes lower than the second highest value. In these cases it has to be that the subject with the second highest value does not participate in the resale auction or that she exits this auction before her use value has been reached. Measuring the rationality of subjects' behavior in the second stage is warranted.

To this end I have prepared a rationality index, RatR, which is a measure of the optimality of the reserve prices in the resale auction. The optimal reserve price depends on the beliefs of the subjects and the beliefs depend on the signals from the initial auction, so it is impossible for us to calculate the optimal reserve price and deviations from it without knowing the subjects' beliefs. However we can expect that when all subjects are rational, the seller has to set a reserve price that is weakly higher than her use value for the good.[17] RatR measures the fraction of sellers who choose a reserve price $r > v_i - 3$.[18] Averaged over periods 10-30 and all sessions, mean RatR was equal to 0.89. There was some variation in the four sessions. In the first session of INC it ranged from 0.5 to 1 with no trend to disappear over time. In session 2, RatR was higher and time had an effect. While in the first 10 periods it mostly ranges from 0.5 to 0.75, in subsequent periods it is always between 0.75 and 1 with an overall average of 0.88. In sessions 3 and 4 RatR was quite high, at 0.97 after the 10th period in both treatments. It is not clear why some subjects set reserve prices with such errors. The seller has her use value as an outside option and she should ask for this value at least.

[17] Note that if one of the subjects is not rational setting a reserve price becomes even more complex. The reasons for such irrationality vary. For instance it is possible and it was observed in some extreme cases that subjects do not participate in the auction when the reserve price is 10 units lower, or less, than their use value. This can be due to fairness considerations. Subjects may find an offer unfair if it gives them a small part of the available rent.

[18] Some bidders might set reserve prices slightly lower than their value due to rounding. I choose a difference of three units between r and v as a threshold but the result is robust with other thresholds too.

However as noted above there are subjects who do not participate in an auction where the starting price is below their value but very close to it, so maybe setting these reserve prices was a rational response to this behavior.[19] A second explanation is that subjects just did not understand that when calculating the optimal reserve price they should think about their use values.[20] In an experiment such as this one, which was arguably more complex than usual auction experiments, such mistakes could occur, so one might think more learning periods are necessary. However, this argument is contradicted by the apparent rationality of players choosing to participate in the resale auction.

Apart from a few mistakes in the early periods, the percentage of subjects who chose to participate in the resale auction, when their use value was higher than the reserve price, was almost 100%. This result is encouraging and suggests that probably the low RatR figures are also not due to miscalculation of the profits, but deliberate choices.

Looking at the boxplots of values versus bids in Figure 1, the stark contrast to COMP becomes clear. Low value players bid close to their values with a small tendency to overbid, while high value players greatly underbid. Furthermore there are some cases, many more than in COMP, where subjects bid 0 or very close to it. These characteristics of the bids can also be explained with the anticipation of noisy bidding or signalling, as we shall see in the next section.

5 Bounded rationality and noisy decision making

In this section I present a variety of models of bounded rationality which are prominent in the literature, to explain subjects' behavior. The explanations differ in the consistency of the beliefs they require of the players, but have in common that they consider noise.

[19] This is related to the phenomenon of "auction fever". In the field auctioneers sometimes start the auction at prices lower than the bidders' values on purpose. Bidders have more time to get excited and this results in more aggressive bidding.

[20] In the short quiz that followed the reading of the instructions, a number of subjects had answered the questions about second stage profits wrongly. Instructors took great care to make these points clear after observing these mistakes. However, it could be the case that some players mistakenly thought the profit from the first period to be their outside option and set a reserve price that was just higher than this number but possibly lower than their value.

5.1 Complete information in the resale market - COMP

Let us start from the basic observation that the Nash equilibrium of the games we tested is not robust to noisy behavior, as will be shown. That is, it is not robust to small perturbations of the bidding strategies. Human players make mistakes and anticipate *others* to make mistakes. In general, as has been shown for example in Goeree et al. (2002), adding noise to the equilibrium bids can shift subjects' best responses quite radically. It remains to be seen if the same effect can be found in the present experiment. In the complete information treatment, if the other players use the SNE strategy, the expected payoff functions of a subject contemplating a deviation from this strategy are broadly the same as in a simple English auction with no resale. However, I will show that if there exists some kind of noisy behavior, which means that subjects make errors when choosing their bids, the payoff functions are quite different.

The following graphs in Figure 4 plot expected profits, depending on one's bid, in the case of a simple English auction (ENG) and an auction with resale (COMP). There are 4 curves plotted in every figure, representing expected profits calculated for use values of, 20, 30, 40 and 50. The upper left graph represents expected profits in ENG when three opponents bid their values without any noise, averaged over every possible value of the opponents. Notice that a bid is a best response given a use value, if it lies at the point where the expected profits reach their maximum value. In this case, payoff is maximized when a player bids her value. For example, the curve drawn for a use value of 50 reaches its highest value exactly for a bid of 50. In the upper right figure I calculate the expected profits, again given that other three subjects bid according to the Nash equilibrium but adding a normally distributed noise to these bids. This means that an opponent with a value of v is assumed to bid $v + \varepsilon$, where $\varepsilon\ \tilde{}\ N(0, \sigma^2)$ and $\sigma = 9$. I proceed to calculate by numerical simulation the expected profit functions of a player facing three opponents employing this noisy Nash bidding[21]. Bid-your-

[21] This is calculated by independently drawing 2 million sextuples of private values v and errors ε for the three opponents. For every opponent I obtain the noisy bid $\tilde{b} = v + \varepsilon$. I then calculate for every possible bid b_i of player i the winning frequency given this bid and the mean highest bid and highest value of her opponents, conditional on the highest bid $\max\{\tilde{b}_{-i}\}$ being lower than b_i. A player's expected profit is then caclulated for any given private value v_i as

$\Pi_i = prob\{b_i > max\{\tilde{b}_{-i}\}\}E[\max\{v_i, v_{-i}\} - \max\{b_{-i}\}|b_i > max\{\tilde{b}_{-i}\}]$

The numerical simulation is helpful because we can calculate these functions for any noise specification value is still a best response. This is to be expected, as bid-your-value is a weakly dominant strategy in English auctions, i.e. an optimal strategy regardless what others do.

The lower graphs depict the same for COMP. Starting from a bid of zero in the x axis, the utility functions without noise look the same as in ENG, up to the point where the bid equals the use value for which the line is drawn. From that point on expected utility is constant. For any value there now is a lower bound equal to zero. Winning bidders never pay more than the highest value in the auction, given their opponents are value-bidding, but receive exactly this amount in the second stage, so that a negative payoff is not possible. Still, there is a maximum at exactly the same points as in ENG. This corresponds to Proposition 1, which states that bidding-your-value is the equilibrium strategy in COMP. However when we add noise as described previously, the curves change dramatically, as is evident in the lower right figure.[22] For every use value there is a new maximum and its exact position depends on one's value. For all use values however, it is now optimal to bid very high (approximately 90).

A new hump arises[23], even when opponents bid with small errors, and the peak of this hump is in the upper region of the bid interval. This means that best responses change discontinuously with noise; optimal bids jump from being equal to player's value to a bid that is significantly higher than a bidder's value, especially when this bidder's value is low. The best response discontinuity exists for many different specifications regarding the functional

and the accuracy of the method is very high, as I have verified in the cases where an algebraic solution is straightforward to obtain. Note that when noise is added, modelling the English auction as a second price auction is not necessarily valid. However given the numerical complexity of the simulations, any more complex model would not be feasible.

[22] The intuition of how mistakes can make overbidding with a low value profitable, is as follows. There are two cases of possible mistakes, opponents can (A) underbid or (B) overbid. In A there is a chance of winning and reselling at a profit. On the other hand in case B defeating opponents who have accidentally overbid is a bad idea, as there will be no profitable resale. Overbidding is a best response because, conditional on winning, case A is more likely than case B.

[23] It is interesting to note how the emergence of the new maximum is the result of the resale opportunity. The expected profit of a first stage bidder is a maximum of two values, expected utility if she consumes the good now and expected profit if she resells it in the second stage. The utility functions graphed are thus the maximum of these two utilities. In the right part of these curves, the resale effect dominates. In the left part, the usual utility enjoyed when she consumes the good herself is dominant. Without noise the utility from resale is zero, as the expected revenue in the second stage equals the expected price in the first auction (both equal the highest value among the other bidders). With noise however this not true anymore, as the expected price in the first stage becomes smaller than expected revenue in the second. This difference is maximized for a bid of around 90 (which is actually higher than the unconditional expected highest value, 75), depending on the size of the errors.

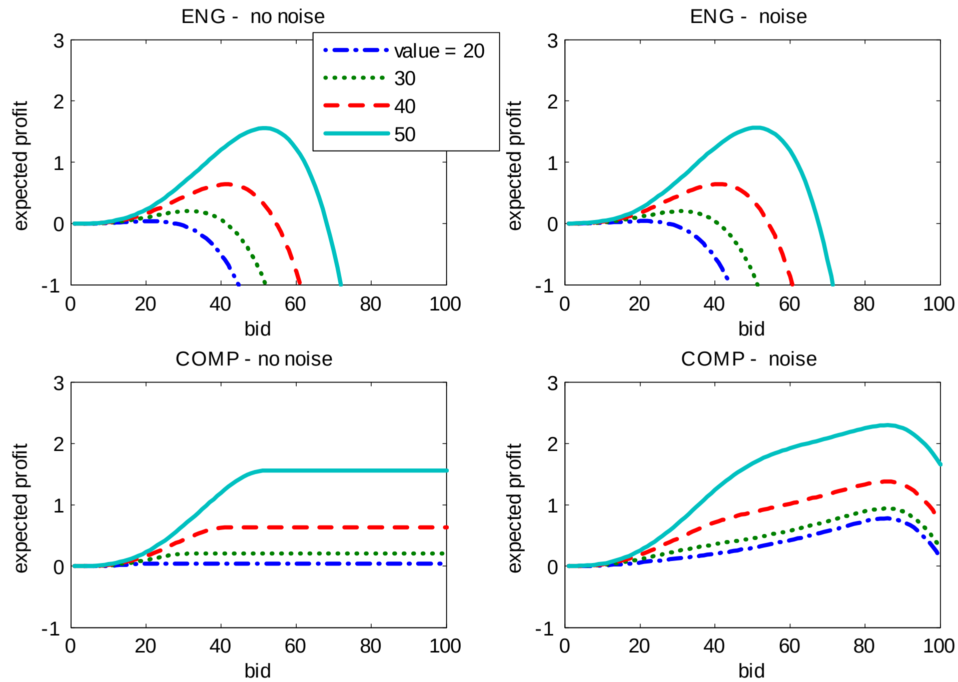

Figure 4: Expected utilites in ENG (upper two figures) and COMP (lower two) without and with noise (normally distriubuted with a σ of 9). The curves are drawn for private use value signals equal to 20, 30, 40 and 50. In the lower left panel utility is very flat but still maximized at a bid equal to value.

form of the noise distribution (e.g. triangle, logistic, uniform, Laplace) and even when the standard deviation is very small (a standard deviation of $\sigma = 1$ is enough in the case of the normal distribution). The intuition is that the hump rises higher the more noisy the bids, but its position on the x axis does not change much. Even with small amounts of noise that lead to a low hump, it will still be higher than the utility derived from a bid equal to or lower than one's value and its peak will be positioned in the upper part of the interval $[0, 100]$. In that way, even the lowest value players should overbid massively in the presence of small amounts of noise.

To test in a systematic way if the characteristics of the game discussed above are indeed influencing the bidders' behavior we consider the following model. Suppose a player believes her 3 opponents want to bid their values but make small errors, distributed normally[24] with

[24] I choose here a σ that is lower than the minimum of the actual estimated standard deviation σ of players' bids in the various sessions of COMP, assuming errors are distributed normally, as can be seen in Table 4. I also tried other distributions and the result was robust to these variations.

$\sigma = 15$. Then this player's best response[25] given such beliefs is approximated by:

$$BR_{naive} = 0.000057v^3 - 0.0046v^2 + 0.098v + 79$$

This is a concave bidding function that starts at 79 for a value of zero and reaches approximately 100 for a value of 100. An alternative to this model is to calculate the best response to the *actual bidding distribution* (and not to the one predicted by the theory). It predicts serious overbidding of approximately[26] the following form:

$$BR_{act} = \begin{cases} 47, v \leq 39 \\ 0.87(v - 39) + 47, v > 39 \end{cases}$$

The hypothesis that subjects were responding optimally to the actual play of the others can be tested using the BR_{act} model. The fit of both these models is presented in table 4.

Levels of reasoning A model that has been found to explain many anomalies in experiments is a level of reasoning model (see for example Nagel 1995, Stahl 1995, Camerer 2004). In specific I will use the level-k version (Crawford and Iriberri, 2008). The idea is simple and rather intuitive. There exist k types of players, varying in their degree of sophistication. Level 0 (L0) players bid randomly with a uniform distribution. Their bids can be interpreted as pure noise, given that values do not correlate with bids at all. In this way this version of the levels of reasoning model used in the literature has built-in the idea of noisy behavior. Level 1 players believe they are playing against L0 players and play a best response, Level 2 players play a best response to Level 1 and so on. I first derive the strategy for a Level 1 player best responding to N players who bid randomly. Her expected profit will equal

[25] The BR and other alternative models we present will be under the assumption that bidders do not update their beliefs after they observe the exits of other players. It does not change the results by much but it greatly simplifies the calculations. Additionally it is confirmed by the data, the main determinant of a player's bid was her use value and the unconditional distribution of the other player's values. Actual observed exits were not a significant factor.

[26] To calculate this best response I first estimated a joint bid-value distribution using the data from the experiments. Then I find by numerical simulation the bidding function that maximises a player's expected payoff when playing against 3 opponents who are employing this empirical bidding strategy. The best response is not exactly piecewise linear, but well approximated by the given function. Note that I use only non censored data when calculating this model and fitting the other models. This can be a problem for high bids, where excluding censored bids can introduce a bias. However the bias when including them can be even stronger.

the maximum of her value and the expected highest value among the opponents minus the expected highest bid, given that the latter is lower than her own bid.

$$\Pi_i = \int_0^{b_i} (\max\{v_i, E[\max\{v_{-i}\}]\} - x)NF(x)^{N-1}dx$$

Note that opponents' values and bids are not correlated. Rearranging and taking first order conditions (see appendix) leads us to following strategy for a level 1 player when we have 4 bidders in total and values are uniform in [0,100]

$$b_{L1} = 25(v_i/100)^4 + 75$$

Thus, Level 1 players bid an increasing concave function of their values, from a bid of 75 for a zero value type to a bid of 100 for a bidder with value 100. Level 2 types best respond to Level 1. It is simple to show that this results in a bid your value strategy. Given that the opponents all bid at least equal to their values there is no opportunity for profitable resale, thus the game reduces to a simple English auction with no resale and bid your value is a best response.[27] However the expected profit curves are not the same as in the Nash equilibrium, as the probability of winning with a bid lower than 75 is zero. This will be important when fitting the model to the data using logistic errors, as they depend on the exact structure of the expected payoff curves. L3 is then exactly equal to the Nash equilibrium, with the same expected payoff functions.

The model is fit assuming that each observed bid is a draw from a common distribution over the three types. The frequency of L1 players in the population is χ_1, χ_2 is the frequency of L2 players and the remaining $1 - \chi_1 - \chi_2$ is the frequency of L3. L0 just exists in the mind of L1, as has been found in Crawford and Iriberri's (CI) work. In CI there is also an alternative specification of the model with truthful bidding as an L0 starting point. Truthful bidding is not useful here, as it would lead to the L1 type (and all higher levels) bidding their value and thus no difference with the SNE.

[27] The best response to a level one player is actually a correspondence and not a function, for private values under 75. Any bid up to 75 is in principle part of the best response, however bid your value is the obvious focal part of this best response. I accordingly expect L2 players to bid their values. Note this is only important for the calculation of L3 as a response to L2. For the actual fitting of L2, the errors are logistic, which means that every action yielding the same payoff is treated as equally likely, thus every action in the best response correspondence is treated the same.

For each type we can calculate expected utility for every possible action, given the beliefs of this type. I assume that a player of a certain type makes errors with a frequency that depends on the expected utility of each action, according to a logit specification. This means the probability for a subject i playing a particular action j out of all actions J is calculated in the following way:

$$p_{ij}(\lambda) = \frac{e^{\lambda U_{ij}}}{\sum\limits_{k=1}^{J} e^{\lambda U_{ik}}}$$

The numerator is the utility from each action transformed by an exponential function, in the denominator we have the sum of all these exponential weights as a scaling factor, so the probabilities add up to one. The parameter λ determines how sensitive errors are to payoff differences. Bids become uniform as $\lambda \to 0$ and errors are eliminated when $\lambda \to \infty$.

QRE The last model I calculate is a Quantal Response Equilibrium which captures the idea of noisy behavior but predicts that players' deviations will be systematic. Similar to the level-k model, I use a logit specification that has been found to give intuitive theoretical predictions in auctions (see Anderson et al. 1998) and to fit experimental data well (see Goeree et al. 2002). Bidders with a given use value have a probability distribution over every possible bid which depends on their payoff sensitivity parameter λ and the actual payoffs.

Players correctly anticipate the bidding distributions of their opponents and all choose the probabilities according to the rule above. Thus, a best response will be played with a higher probability, but not with certainty. The equilibrium is a fixed point of a mapping from choice probabilities to choice probabilities. Note, that although a QRE approaches a Nash equilibrium in the limit when the noise parameter tends to infinity, it can be far away from it for intermediate values of the parameter.

Calculating a QRE with such a large strategy space is a daunting task. With the usual differential equations approach (used for example by Goeree et al. 2002) it is even considered numerically impossible, to the best of my knowledge, as it involves solving a system of 101 simultaneous non-linear differential equations. Therefore I use a less frequently used method to calculate the QRE, namely a Cournot iterative process.[28] Starting with a random bidding function, the expected utility of a player facing three bidders employing such a bidding function is calculated. I proceed to calculate a quantal response by weighing the utility, to get choice probabilities according to the formula above. This process is then iterated until the quantal responses converge to a stable state. Convergence is usually reached after about 15 iterations and does not depend much on the initial bid function.

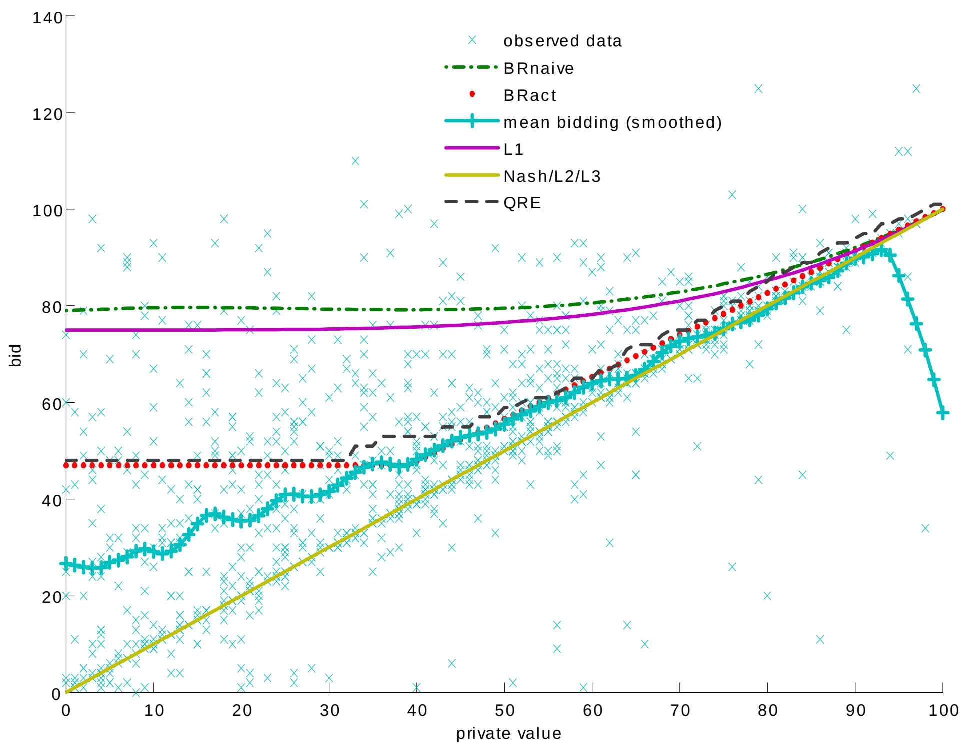

Figure 5: Comparison of the different models for treatment COMP. The QRE predicts a distribution of bids for every use value, so the mean of these bids is presented. Keep in mind however, the model with the best fit is not the one closest to the mean actual behaviour but the one where the whole predicted distribution is closest to the actual one.

Note that in a QRE, different values of λ can lead to radically different predictions.[29] I

[28]To my knowledge an iterative method has been used once before to calculate a QRE in coordination games and the traveller's dilemma using a simple spreadsheet (see chapters 25-26 in Holt 1996). Note that the iterative method reveals an interesting relationship to LOR models. Every iteration of the process corresponds to a level of reasoning. A somewhat similar analysis is done in Goeree and Holt (2004), where they propose a model of noisy introspection using logit response functions, but relaxing the equilibrium assumption. Their model can be seen as an alternative to the level-k model (which it includes as a special case), where there is more noise associated with players' beliefs about higher levels of iterated expectations.

[29]Haile et. al (2006) note that a QRE with two parameters, suitably chosen, can be used to fit any data.

find that the values estimated in other experiments, including auctions, gave a very good fit for our data too[30], which indicates that the QRE is an appropriate model to explain behavior in a wide range of auction experiments.

Comparison of models The different models calculated above for treatment COMP are fit to the data in this section and compared with the same models for treatment ENG. The predictions of the various models in the simple English auction are straightforward. The strategy is actually equal to bid your value for all models except QRE, as this is the weakly dominant strategy in simple English auctions with IPV. Note that unlike in treatment COMP, L0 is exactly equivalent to bid your value, as the L1 type's payoff is not influenced any more by the values of his opponents but just by their bids, which are uniform in both L0 and Nash. The QRE predictions are calculated by simulation, as in the case of COMP.

Results are shown in Figure 5 and the goodness of fit can be found in Table 4. Maximized log likelihood values for each model are presented in the first row. In the case of the pure strategy models (Nash and BR), where no dispersion is predicted by theory, I allowed for normally distributed errors and the estimated σ is shown in brackets. The QRE predicts a dispersion according to the logit formula presented above, so there was no need for additional errors. For the levels of reasoning models I posit logistic errors as described above and calculate them numerically. I assume that λ is independent of subject or type.[31] Thus in total the model has three parameters, the common precision λ, χ_1 and χ_2.

Lastly I estimate a Nash model but with logistic instead of normal errors. This yields a fairer comparison to the QRE and level-k. Note that Nash+logit is equivalent to the L3 model fitted with logistic errors. Since the models are in general not nested I use the

This critique does not apply to this analysis, as with the logit structure of the errors, the payoff perturbations are i.i.d. See Goeree, Holt and Palfrey (2005) for a discussion.

[30] Note that λ depends on the payoff space and has to be adjusted accordingly. For example, in Goeree et al (2002), where the private values have a support of [0,11], λ is found to be on average 10 (actually they use a parameter μ which is equivalent to $1/\lambda$ and they find $\mu = 0.1$). Then for this auction where values are in $[0, 100]$, the values of λ should be close to 1.

[31] In Crawford and Iriberri's study they compared such a model to models where precisions can be type-specific or subject-specific. Forcing subjects to be of a single type in COMP leads to a very large increase in the number of parameters, without adding much to the fit. Also the estimated population frequencies do not change much. If we assume type-specific precisions we get a LL of 4199.6, BIC=8433.8, a frequency of 4.3% L1 types with precision $\lambda = 0.039$, 33.3% L2 with $\lambda > 18$ and 62.4% L3 with $\lambda = 12.7$.

Bayesian Info Criterion (BIC) for model selection. It punishes the level-k model for having more free parameters than all other models, which only have one. Recall that we have more observations for treatment COMP than ENG so the respective likelihoods are not directly comparable.

Model	**Nash**	**BR**$_{naive}$	**BR**$_{act}$	**L1**	**L2**	**L1+L2+L3**	**QRE**	**L3/Nash+logit**
LL COMP	-4469.2	-5044.5	-4391.5	-4555.8	-4492.8	-4312.9	-4207.7	-4367.5
BIC	8945.3	10095.9	8789.9	9118.6	8992.6	8646.6	8422.3	8741.9
Est. λ or σ	20.9	37.1	19.4	0.1	0.22	1.84	1.23	0.87
LL ENG	-2706.2	-2706.2	-2706.2	-2732.4	-2732.4	-2732.4	-2715.1	-2732.4
BIC	5419	5419	5419	5485.4	5485.4	5485.4	5436.8	5485.4
Est. λ or σ	10.38	10.38	10.38	1.13	1.13	1.13	1.1	1.13

Table 4: Goodness of fit of different models for treatments COMP and ENG. LL is the maximised log likelihood. The Bayesian Information Criterion (BIC) is used for selection; the lower its magnitude, the better the fit.

The model that performs best in explaining the results in COMP, as indicated by the BIC, is the QRE model, followed by the mixed levels of reasoning model. A Vuong test rejects that the two models fit equally well at the 0.01 level, in favor of the QRE. The estimated type frequencies for the mixed levels of reasoning (LOR) model was 1% for L1, 24.8% for L2 and 74.2% for L3. Nash with normal errors does not fit the data well, while BR$_{act}$ (the best response to actual behavior model) performs better. There is a leap in the likelihood when we estimate the Nash model with logistic errors instead of normal. Other symmetric error specifications (triangle, uniform) under-perform similarly with respect to logit. Still, the QRE with logistic errors performs even better than Nash+logit which shows that logistic errors are not enough to explain the subjects' behavior. Players not only make errors in a systematic way as is modelled through the logistic distribution, but anticipate others to make errors too, which leads them away from the Nash prediction and towards a quantal response equilibrium. These results are reinforced by the fact that the estimated error parameter λ was quite similar for both treatments and for both models, Nash and QRE.

In ENG all models exhibit a similar performance, which is not surprising given that their predictions are very similar and the greatest difference stems from the different distributions of the errors (logit vs normal). The Nash and BR models have the lowest log likelihood with the QRE performing a bit worse (the difference not being significant according to a Vuong test) and the mixed LOR and Nash+logit with significantly higher LL (p-value<0.01, Vuong test). Overall however the great improvement in fit given by the last three models in COMP means that the total predictive power of these models is higher. If we use the average performance in the two treatments as a selection criterion, the levels of reasoning model and QRE emerge as clear winners, while Nash+logit is close but significantly worse (a likelihood ratio test of LOR with the nested L3/Nash+logit model yields $p<0.001$).

5.2 Incomplete information in the resale market - INC

As in treatment COMP, the anticipation of noisy behavior can be used to explain the data in INC. Very high value players know they will win with a very high probability. But if they try to win in the first stage they would possibly have to pay a price higher than the second highest value in the group because some low value players can be (relatively costlessly) overbidding. They prefer to signal low values[32] and wait for the second stage auction where they know that overbidding for the low value players is exactly as costly as in a simple English auction and will thus be avoided. Given actual behavior such a strategy would be more profitable than the Nash prediction.

Note that this logic is exactly captured by the logistic errors which allow for the fact that players do not always best respond, but still try to avoid the most costly mistakes. Low value types can costlessly overbid in the first stage but avoid overbidding in the second stage. High value types will not avoid underbidding in INC as much as in COMP or ENG, since in case of losing in the initial auction they can still make some profit in the second stage. The question that arises is which of the previously discussed models employing logistic errors will fit the actual behavior better. The QRE assumes that subjects correctly anticipate the logistic errors of their opponents and arrive at an equilibrium where subjects play noisy best responses to each other. On the other hand the level-k model assumes bidders do not think past a limited number of iterated best responses. They intend to play a best response to their opponents, given the beliefs that correspond to their level of reasoning, but make logistic

[32] Suppose there are two biders. Imagine a bidder with value 50 believes the other player is playing the bid your value equilibrium with small symmetric mistakes of maximum magnitude 10, as described in the previous section. He will then bid more than his value, say 60, expecting to resell. In that case, his opponent will have an incentive to bid much less, if he has a high value, say 90. For example if he bids 40 he will lead the winner to believe that he has a value of maximally 50 and he will thus get the good in the second stage for a price of 50. See example 1 in Hafalir and Krishna (2008) for a similar argumentation.

errors. Finally the Nash+logit model just assumes players intend to play the Nash strategies, but make logistic errors.

Due to the interdependence of the two stages and the additional complexity, it is not straightforward to calculate the logistic errors and the QRE and level-k models for treatment INC. One has to use a shortcut, building a reduced form of the game to make it tractable. I therefore assume that the reserve price in the second stage is equal to the use value of the seller and that players who have a higher value than the reserve, do participate in the auction. I also assume that players who have decided to participate in the second stage auction proceed to play exactly as in a simple English auction, bidding their values. These assumptions are largely consistent with actual behavior and partly with theoretical arguments too[33]. I then plug the expected continuation payoffs from the second stage subgame in the first stage payoffs. The resulting game allows the various models' predictions to be calculated as in treatment COMP, with the exception of $\mathbf{BR}_{act}$ where the number of observations in the second stage was not adequate to estimate the empirical distribution of reserve prices and participation strategies. The main difference of this reduced version of INC with COMP is that the winner of the first stage can only expect to get the second highest value among the other bidders in the second, but the losers now have a chance to win in the second stage and appropriate a part of the rent (for example in case they have the highest value, they will get the difference between this value and the second highest). Note that in this reduced game, bid your value is still a Nash equilibrium.

Level 1 play, meaning a best response against opponents who are bidding randomly, results in a bidding function that starts at $(N-1)/(N+1)$ for a value of zero (see appendix). It rises monotonically to $N/(N+1)$ for a value of 100. Level 2 players bid their values[34], up to a maximum bid of $N/(N+1)$ and level 3 do the same, but have different expected payoff functions and thus different error distributions.

[33] The bidding behaviour prescribed for the second stage bidders is rational. For the second stage seller on the other hand, setting a reserve price equal to her use value is not an optimal choice. However, when the number of bidders is high enough, the reserve price becomes irrelevant. When selling to 3 bidders as in our experiments, with values uniformly distributed in [0,1] the expected revenue under the optimal reserve price is around 0.53 and the second highest value (which is the expected revenue without a reserve price) is 1/2; the reserve price enhances revenues by not more than 6%.

[34] As with treatment COMP, the best response to level 1 is a correspondence. Where multiple bids are possible for a given value, bid your value is chosen as the obvious focal one.

Model	Nash	L1+L2+L3	QRE	Nash+logit
LL INC_{red}	-4659.8	-4559.3	-4216	-45863
BIC	9326.6	9152	8466	9179.6
Est. λ or σ	18.1	0.65	1.1	0.73

Table 5: Goodness of fit of different models for the reduced form of treatment INC. LL is the maximised log likelihood, BIC is the Bayesian Information Criterion for model selection.

The estimated frequencies for the mixed levels of reasoning mode are 31.5% for L1, 1% for L2 and 67.5% for L3, relatively close to the values estimated in the previous section for COMP. The simple Nash model with symmetric normal errors performs once more very badly[35]. All the models using logistic errors fit the data better. As in treatment COMP, the QRE outperforms the mixed levels of reasoning model and the Vuong test statistic again rejects the hypothesis that the models fit the data equally well at the 0.01 level.

5.3 Alternative explanations

Several models that have been used to explain overbidding in other auction experiments, like first-price auctions (see Cox et al. 1992), cannot explain the data in this study fully. Consider "joy of winning", meaning that a player's utility is increased by a fixed amount if they manage to get the object and realize profits in the auction. A pure joy-of-winning model predicts the same absolute value of overbidding for all private values, unless the joy of winning is somehow correlated with use values (see Georganas et al. 2009). However, as we saw in the previous section, low value bidders bid much higher than their values whereas high value players' bids are very close to their values. More evidence against this hypothesis is that in the simple English auction no systematic overbidding is observed - after the initial learning periods.

A second explanation, used for example in Cox et al. (1985), is risk aversion. If subjects are risk averse they could value the higher probability of winning, when bidding above their values, more than the loss in their expected profit. In English auctions with no resale the equilibrium is, as noted before, in weakly dominant strategies, so risk aversion does not induce

[35] Note that the existence of better fitting asymmetric Perfect Bayesian equilibria in INC is possible. But the symmetric equilibrium, which is the usual suspect in most applications in the literature, does not perform well in this case.

different behavior. In COMP and INC, risk aversion *alone* does not change the equilibrium predictions. However risk aversion *combined* with some noise in the bidding could be a factor influencing the results.

Another motive for low value players overbidding is spite, as has been found for example in Andreoni et al. (2007). The authors gave subjects information about other bidders' values in second price private value auctions. When players have a low chance of winning due to a low value, but know that some other player has a high value they sometimes tend to overbid in order to lower the winner's earnings. The authors observe that when subjects get more information about others' value, this behavior becomes less risky and overbidding is more frequent. Note that if we model the auction as a series of stages (see Milgrom and Weber 1982), entering a new stage every time a bidder leaves the auction, this behavior is compatible with individual rationality. The SNE equilibrium described in Section 3 is unique only in the last stage of the auction when there are only two bidders left, while in the earlier stages other equilibria are possible, which all lead to the same outcome regarding the identity of the winner and the auction price.

While this explanation is plausible, it cannot account for the entire amount of overbidding observed. The first reason is that in simple English auctions with no resale, the extent of overbidding in early stages is much lower although the risk from overbidding is in theory exactly the same as in INC and COMP. Secondly, in the last stage of the initial auction in COMP when only two bidders are left, overbidding is indeed less risky than in ENG, but unlike the previous stages it is still not part of any symmetric equilibrium strategy and can lead to negative profits[36].

The QRE and level-k models improve upon Nash in INC mainly by predicting some underbidding for the high types. An alternative reasoning for very low bids is reported in Kamecke (1994). In this study it has been found that some subjects tended to bid very low when they thought they did not have a good chance of winning in order to raise the profits of the winner. In Cox et al. (1982) this tendency for low value holders to throw away bids was argued to make economic sense, once one accounts for subjective costs of calculating a

[36] In fact it can be part of an asymmetric equilibrium, where everyone bids their value except for one player who overbids. This strategy however is very risky. If for example there are two spiteful players with low values and they both overbid, the winner of the two will suffer a serious loss.

more meaningful bid under the circumstances. A low chance of winning with a bid close to one's value, however, cannot explain why so many players were underbidding when they had high values. Additionally, there is no obvious reason why the effect of calculation cost and low winning chances should be different in INC than in COMP, so it cannot account for the significantly smaller frequency of low bids in INC.

A model that has recently been found to explain non Nash behavior in auctions is anticipated regret, which losers feel when they fail to obtain the item although the price was favorable (Ozbay and Ozbay 2007). The authors show such considerations lead to overbidding in first-price auctions, although there is no effect in standard English auctions and it is unclear what the effect would be if we allow for resale. Other-regarding preferences have also been used extensively to explain experimental resutls (e.g. Bolton and Ockenfels 2000, Fehr and Schmidt 1999) and might be able to explain part of the behavior in the second stage of the game. A full formal analysis using these models is however outside the scope of this paper.

6 Discussion and extensions

All the models except Nash predict overbidding in the resale treatment COMP. Thus there is no way to separate them based on the qualitative predictions. They differ however in their predictions when we vary the number of bidders in the auction. While Nash predicts no effect, LOR clearly predicts a monotonic rise in the magnitude of overbidding while the QRE predicts an initial rise up to 4 bidders and then a slight fall in parts of the bidding function[37]. In specific, for middle-of-the-range use values, the QRE prediction falls when there are many bidders. Thus an experiment with COMP and 5 or more bidders would allow a neat separation of the models.

As an extension of the present study, in order to test the hypothesis of subjects an-

[37]The reason for the fall has to do with the feature of the QRE, where strategies with a payoff of zero are played with a positive probability. As the number of players grows the probability of winning with a low to middle bid falls dramatically. Thus the part of the bidding function that gives an expected payoff of zero grows and the bidding distribution for a given value comes close to uniform. Thus while for $n = 3$ the QRE predicts a bidder with a value of 70 to bid on average close to 60, for $n = 8$ she will bid close to the average of the uniform distribution in [0,100] which equals 50.

ticipating mistakes, one could run an experiment where human players face computerised opponents. As computerised opponents do not make mistakes, we should expect very similar behavior in all three treatments. On the other hand it is questionable whether players' behavior when playing against machines allows us useful predictions of how they will play against real humans.

A promising idea for future research is the explicit inclusion of a speculator in the game as in the Garratt and Troeger (2006) paper. This experiment will be very useful to compare with INC and will give us valuable insights to the source of the asymmetric behavior in our data.

Finally, experiments with sealed bid auctions would allow a test of the theoretical result of revenue equivalence of the English and second-price sealed-bid auctions under complete information in the resale stage (see Haile 2003). It would be useful additionally, to design these experiments in a way that makes the results comparable with the results of the empirical study in Haile (2001), which has found evidence of the effect of resale markets on US Forest Service timber auctions. As already mentioned, independent work of List et al. (2004) has run first-price sealed bid experiments and compared them with these timber auctions. They seem to have found a significant presence of risk aversion in the data. While this seems like a plausible explanation, it is very likely that the combination of risk aversion with noisy behavior can enhance their results.

7 Conclusions

In the resale treatment under complete information we have a case similar to the "ten little treasures" in Goeree and Holt (2001). The simple English auction represents the "treasure treatment", where Nash theory seems to work perfectly, predicting subjects' behavior with a very high accuracy. When we change the game a bit, adding the resale opportunity, the Nash equilibrium is still valid, prescribing that players bid their values. Nonetheless, subjects seem to see a difference where theory does not see one. Players significantly overbid in the presence of a resale opportunity, under complete information in the resale market, and this overbidding does not tend to fade away with the passage of time and the effect of learning.

However when there is no complete information in the resale market, the results are quite different. Subjects with low values tend to bid a bit more than their values, whereas high value bidders bid much less than their values. This indicates that instead of the usual separating equilibria there is pooling similar to Haile (2000); high value players pretend to have smaller values and expect to get a better offer in the resale market.

What is common in both treatments, comparing to common results in simple English auctions, is that the addition of the resale opportunity alters the strategic behavior of the subjects significantly. The resulting change in the bidding behavior leads to substantially different revenues for the initial seller. Thus, the presence of a resale opportunity is an important feature of an auction environment and has to be included in the analysis.

The second and more general result of this paper stresses the importance of considering noisy decision making and looking at the exact form of expected payoff functions.[38] The three treatments that were tested had the same Nash equilibrium, but subjects' behavior was quite different in each one of them. It was argued that the reason for this is the presence of errors (even small ones suffice) on behalf of some players. Such apparent errors can be attributed to experimentation with different strategies, trembling or idiosyncratic preferences. In auctions in the field, not adequately modelled liquidity constraints can also lead to behavior that looks noisy. Errors and noise are present even in the most important financial markets where the stakes are very high (see Shleifer and Summers 1990). Although errors exist, they should not be thought of as being entirely random. Whatever the reason for making errors, the present study finds that subjects systematically try to avoid the most costly ones, thus the shape of the payoff functions is a good indicator for the empirical distribution of players' errors.

The reaction to errors can be more important than the errors themselves. In cases where the anticipation of errors on behalf of some players does not alter best responses by much, the Nash prediction can be valid, at least qualitatively. However, in cases such as the present experiments, where best responses are sensitive even to small amounts of noise, the anticipation of errors rationally leads human subjects far away from the Nash equilibrium strategies

[38]This feature is demonstrated very strongly in a recent analysis of second price sealed bid auctions by Georganas et al. (2009). The authors use a manipulation of the payoffs that leaves the weakly dominant strategy of bid-your-value unchanged. However this manipulation is used, with different parameters, to lead from extreme overbidding to virtually no overbidding at all.

in a way that can even invalidate comparative statics, e.g. regarding the number of players.

For policy prescription purposes these findings should be taken carefully into account. While some features of a laboratory experiment will probably not apply in real markets (for instance it is not clear that real-life investors will have fairness concerns or display altruistic behavior), others like noisy behavior and the anticipation thereof are surely present and of significant importance. Thus the results of this study must possess some external validity and imply that models of noisy behavior (including QRE and level-k) can yield more realistic results than simple Nash equilibrium analysis.

A Appendix

A.1 Derivation of level 1 bids in treatment COMP

All values are scaled to be in the interval [0,1], N is the number of opponents.

$$\Pi_i = \int_0^{b_i} (E[\max\{v_i, v_{-i}\}] - x) N F(x)^{N-1} dx$$
$$= \int_0^{b_i} (E[\max\{v_i, v_{-i}\}] - x) N x^{N-1} dx = \int_0^{b_i} N(E[\max\{v_i, v_{-i}\}] x^{N-1} - x^N) dx$$
$$= [N(E[\max\{v_i, v_{-i}\}] \tfrac{1}{N} x^N - \tfrac{1}{N+1} x^{N+1})]_0^{b_i} = N(E[\max\{v_i, v_{-i}\}] \tfrac{1}{N} b_i^N - \tfrac{1}{N+1} b_i^{N+1})$$

Taking first order conditions:

$$N(E[\max\{v_i, v_{-i}\}] b_i^{N-1} - b_i^N) = 0 \rightarrow b_i = E[\max\{v_i, v_{-i}\}]$$

$$E[\max\{v_i, v_{-i}\}] = prob(v_i > \max\{v_{-i}\}) v_i + (1 - prob(v_i > \max\{v_{-i}\})) E[\max\{v_{-i}\} | v_i < \max\{v_{-i}\}]$$
$$= v_i^N v_i + (1 - v_i^N) \int_{v_i}^{1} x \frac{N x^{N-1}}{1 - v_i^N} dx = v_i^{N+1} + (1 - v_i^N) \int_{v_i}^{1} \frac{N x^N}{1 - v_i^N} dx$$
$$= v_i^{N+1} + (1 - v_i^N) [\tfrac{N x^{N+1}}{(N+1)(1 - v_i^N)}]_{v_i}^1 = v_i^{N+1} + \frac{N(1 - v_i^{N+1})}{N+1} = \frac{N + v_i^{N+1}}{N+1}$$

In the case of $N = 3$, as in the experiments, we have $b_i = \frac{1}{4} v_i^4 + \frac{3}{4}$

A.2 Derivation of level 1 bids in treatment INC

(for $N > 1$)

$$\Pi_i = \int_0^{b_i} (E[\max\{v_i, \sec\max\{v_{-i}\}\}] - x) N x^{N-1} dx + prob(b_i < \max\{b_{-i}\}) \int_0^{v_i} (v_i - x) N x^{N-1} dx$$
$$= \int_0^{b_i} (E[\max\{v_i, \sec\max\{v_{-i}\}\}] - x) N x^{N-1} dx + (1 - b_i^N) \int_0^{v_i} (v_i - x) N x^{N-1} dx$$

Taking first order conditions:

$$N(E[\max\{v_i, \sec\max\{v_{-i}\}\}] b_i^{N-1} - b_i^N) - N b_i^{N-1} \int_0^{v_i} (v_i - x) N x^{N-1} dx$$

$$b = E[\max\{v_i, \sec\max\{v_{-i}\}\}] - \int_0^{v_i} (v_i - x) N x^{N-1} dx$$

Thus we have the familiar result, bids are equal to the value of winning in the first stage minus the value of winning in the second.

Lemma 1: The value of winning in the first stage equals $E[\max\{v_i, \sec\max\{v_{-i}\}\}]$

$= prob(v_i > \sec\max\{v_{-i}\}) v_i + (1 - prob(v_i > \sec\max\{v_{-i}\})) E[\sec\max\{v_{-i}\} | v_i < \sec\max\{v_{-i}\}]$

omit index i for v_i and let $p = prob(v_i > \sec\max\{v_{-i}\}) = [v^N + N(v^{N-1} - v^N)]$

$$= pv + (1-p) \int_v^1 \frac{xN(N-1)x^{N-2}(1-x)}{1 - v^N - n(v^{N-1} - v^N)} dx = pv + (1-p) \frac{N(N-1)}{1 - v^N - n(v^{N-1} - v^N)} \int_v^1 x^{N-1} - x^N dx$$
$$= pv + (1-p) \frac{N(N-1)}{1 - v^N - n(v^{N-1} - v^N)} [x^N/N - \tfrac{x^{N+1}}{(N+1)}]_v^1 =$$
$$pv + (1-p) \frac{N(N-1)}{1 - v^N - n(v^{N-1} - v^N)} (1/N - 1/(N+1) - v^N/N + \tfrac{v^{N+1}}{(N+1)})$$
$$= (v^N + N(v^{N-1} - v^N)) v + (1 - v^N - N(v^{N-1} - v^N)) \frac{N(N-1)}{1 - v^N - n(v^{N-1} - v^N)} (1/(N^2 + N) - v^N/N + \tfrac{v^{N+1}}{(N+1)})$$

In the case of $N = 3$, as in the experiments, we have

$$E[\max\{v_i, \sec\max\{v_{-i}\}\}] = 2v^4 + 3v^3 + (1 - 4v^3 - 3v^2)\frac{6}{1-4v3-3v^2}(1/12 - v^3/3 + \frac{v^4}{4})$$

Lemma 2: The value of losing in the first stage auction equals

$$N\int_0^{v_i} v_i x^{N-1} - x^N dx = [v_i x^N - \frac{N}{N+1}x^{N+1}]_0^{v_i} = v_i^{N+1} - \frac{N}{N+1}v_i^{N+1} = (1 - \frac{N}{N+1})v_i^{N+1} = \frac{v_i^{N+1}}{N+1}$$

When $N = 3$ this becomes $v_i^4/4$

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