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A re-evaluation of the Fourier descriptor approach to quantifying sand particle geometry

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ABSTRACT: This paper describes an evaluation of the use of Fourier descriptors to characterize soil particle shape. The approach proposed by Bowman et al (2001) was implemented in a MATLAB code, taking advantage of the MATLAB Image Processing Toolbox. The method uses a discrete Fourier transform, to obtain a series of descriptors. Bowman et al proposed that the first four descriptors be used to quantify elongation, triangularity, squareness and asymmetry; altogether a measure of particle shape or 'morphology'. In the current study, this approach was first fundamentally examined using simple, ideal geometries. Two granular quartz soils, Ham River Sand and Toyoura Sand, with mean particle sizes of 170μm and 300μm respectively, were then analysed. Attention was paid to determining a minimum population of particles to obtain consistent data. The results indicated a significant shortcoming in this approach to quantify particle geometry. While the elongation descriptor was in agreement with measurements of particle elongation obtained from other approaches, the remaining descriptors exhibited significant scatter. Based upon the particle types considered, it was concluded that this approach has limited potential for use in soil shape characterization. Further research is needed to establish whether the method may indeed be useful for quantifying roughness.

1 INTRODUCTION

It is widely known that particle shape affects the mechanical behaviour of granular soils. Previous studies had highlighted the influence of particle angularity and elongation on soil strengths and stress-strain characteristics. Angular grains are more likely to exhibit higher shear resistance than rounder ones, also the increase in the particle irregularity, i.e. angularity and eccentricity, leads to an increase in void ratio and a decrease in soil stiffness (Cho et al 2006). Matsushima (2001) in a series of 2D DEM simulations showed that irregularly shaped grains exhibit higher shear strength in comparison with circular particles. It has also been shown that the range of void ratios (e_{min}-e_{max}) tends to decrease with increasing sphericity and roundness (Cho et al 2006, Miura et al 1997). Sukumaran and Ashmawy (2001) found that the large strain drained friction angle of sand varies as a function both of particle shape and particle angularity.

The term shape describes the entire particle morphology comprising the form, roundness and surface texture (Barret, 1980). Traditionally, the measurement of particle morphology has relied upon visual comparison of particle shapes with standard charts. For example, Krumbein and Sloss (1963) presented a chart to quantify roundness and sphericity. Nowadays the development of computer image processing enables more accurate evaluations. For example, Sukumaran and Ashmawy (2001) approximated 2D projections of particles using equivalent polygons and formulated two new parameters: a shape factor and an angularity factor. Bowman et al (2001) presented an approach that uses Fourier descriptors to quantify geometry.

An evaluation of the Fourier descriptor approach for particle shape characterization is presented here. This paper firstly presents an overview of the Fourier Descriptor approach. We then discuss the implementation of this approach using the MATLAB image processing toolbox. Results for analysis of ideal geometries and real sands are then presented.

2 FOURIER DESCRIPTOR SHAPE ANALYSIS

Shape analysis of objects and particles is important across a wide range of disciplines, consequently many methods have been proposed in the literature. One approach, that has become much easier to implement as digital images and image analysis software has become more readily available, is Fourier descriptor analysis. In sedimentology the importance of quantifying particle morphology has long been recognised, with much interesting work being completed in the 1950s, and sedimentologists have proposed various shape characterization techniques (Clark, 1981).

A procedure for quantifying the shape of particles based on Fourier series analysis was introduced by Schwarcz and Shane (1969) and Ehrlich and Weinberg (1970) from a sedimentology perspective. This technique makes use of 2D particle outline plotted in polar coordinates r and θ where the origin is the grain centroid, as illustrated in Figure 1. A harmonic analysis is made of the function r(θ) by measurements of equally spaced points along the curve. Using Fourier analysis, the shape is then represented by a series of independent coefficients. This methodology proved to be effective; however it suffered from two major shortcomings. Firstly it is difficult to accurately locate the particle centroid for complex natural particles. Secondly, for non convex particles the polar radius can cross the edge more than once, as illustrated in Figure 1.

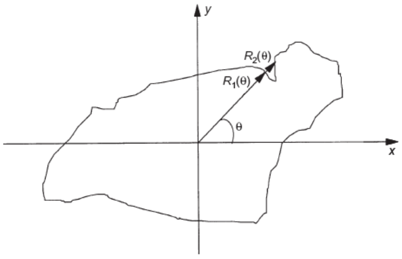


Figure 1 – Difficulties associated with re-entrance angle, which may produce two different values of R for the θ same value (Bowman et al 2001).

Despite these shortcomings, the Fourier descriptor method was initially applied by researchers in sedimentology, though typically not to non-convex grain shapes. Grain shape features including roundness, angularity and texture were related to the lower frequency harmonics of the Fourier series.

With continued application of the method, a number of researchers suggested refinements to this Fourier descriptor approach. The representation of the particle outline in the complex plane and interpolation between digitized boundary points was introduced (Clark, 1987). Employing boundary information only (i.e. the centroidal coordinates are no longer needed), Thomas et al (1995) used the fast Fourier transform (FFT) algorithm to produce a rapid analysis of regular and highly irregular grain profiles. This approach overcomes both of the limitations of the original algorithm noted above.

To implement the approach, the grain outline must be considered in the complex plane. The boundary of the particle is then circumnavigated with a uniform velocity to generate a series of complex numbers. The value of velocity is chosen to achieve 2^k equally spaced points where k is a user specified integer (e.g. for k=7, we obtain 128 points). Consequently determination of the centroid is unnecessary and the problems associated with the re-entrant values disappear as the boundaries coordinates are uniquely defined in this way (Thomas et al 1995). Wallace and Wintz (1980) proposed that all the Fourier descriptors obtained be normalized by the largest descriptor. The descriptor values obtained are then invariant to position, size, and orientation.

3 IMPLEMENTATION OF IMAGE PROCESSING/ANALYSIS

3.1 *Methodology*

Digital images of sand particles (without overlap) were considered in this study. These images were obtained using a high quality digital camera (7 mega pixels). These digital images are considered to be maps of intensity-levels were each pixel can have a value between 0-256, with zero representing black and 256 representing white. A series of image processing techniques were then applied in order to obtain the particle outline. The first step is to perform the image segmentation i.e. to separate the particles from the background. Therefore the intensity distribution is plotted as a histogram and a threshold value is selected. This threshold value (*T*) separates the dominant modes in the histogram so that intensity values lesser than *T* will be labelled as object and greater that *T* will be labelled as background. Therefore, a binary image can be obtained where white pixels correspond to objects and black pixels to background (or vice versa). Filters can then be used to enhance the image quality and reduce noise, i.e. very small particles or dust was deleted from the image at this point.

Matlab’s Image Processing Toolbox offers a number of edge detection algorithms and all of these were explored to determine the optimum approach. As illustrated in Figure 2, the Prewitt and Sobel methods tend to produce noisy results. While the Laplacian and Canny methods produce smoother results, the Canny method was chosen as this method uses two thresholds to detect “strong” and “weak” edges and is less likely to detect spurious edges. Further details on these algorithms are available at Mathworks, (2007)

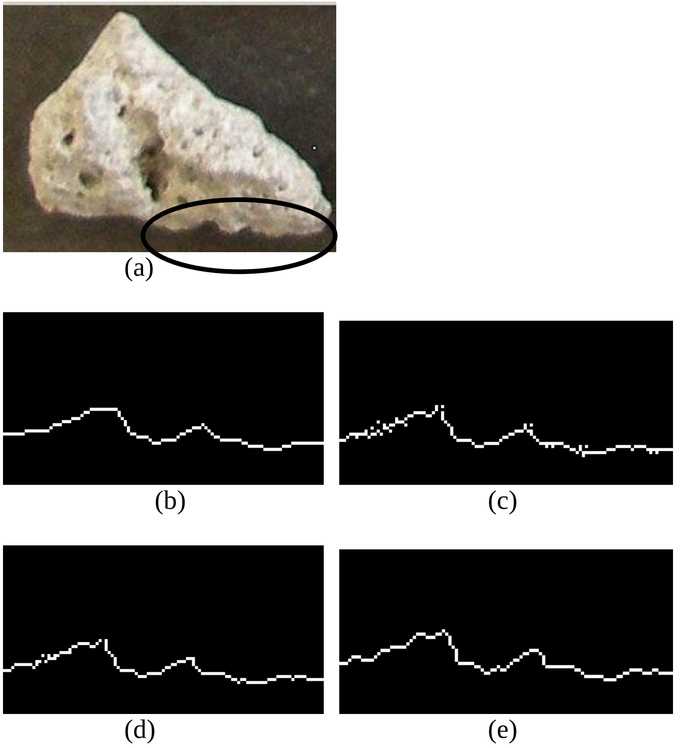


Figure 2 – Examples of outlines obtained using different edge detection methods. (a) Original particle (b) Canny (c) Prewitt (d) Sobel (e) Laplacian

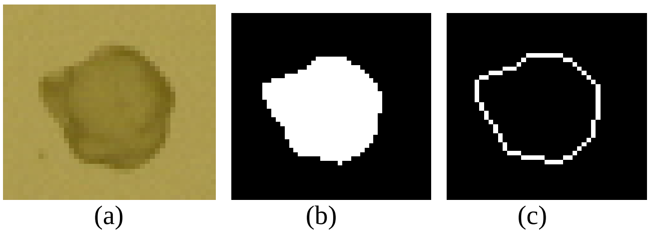


Figure 3 – Results of the application of image processing algorithm (a) Original particle (b) Binary image (c) Particle boundary

The output from the edge detection function is the boundary coordinates and these are stored in an array. These coordinates tend to be randomly spaced depending on the shape and roughness variation around the grain, i.e. a rough edge is defined using more points than a smooth one. In order to obtain equally spaced points along the boundary, the perimeter of the particle is calculated, and then divided in n segments. The number of points, n, chosen to represent the outline dictates the number of descriptors gained from the Fourier analysis and therefore the level of detail described. The coordinates of the ends of these segments are calculated using linear interpolation. The Cartesian coordinates (x_i,y_i) of these points, are converted into complex numbers to obtain a vector of complex numbers (x_i+iy_i) for input into the FFT function.

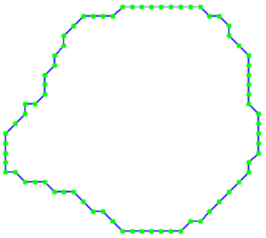


Figure 4 – Original digitalised outline of the particle.

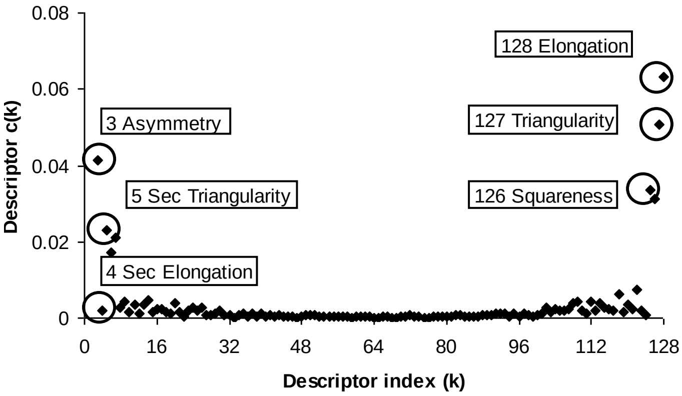


Figure 5 – Fourier descriptors obtained for the particles represented above.

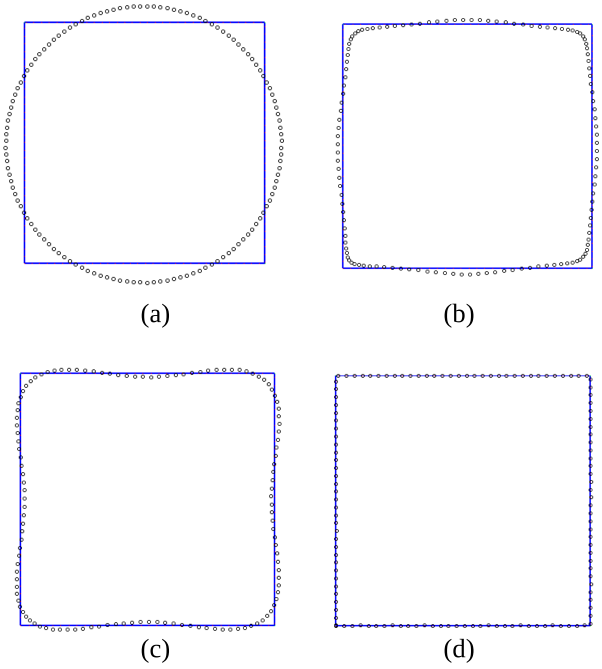
The output from the FFT, is itself a vector of n complex numbers, f(k), and the complex modulus of each of these numbers gives the magnitude of the Fourier descriptors, i.e. a vector $c(k)=|f(k)|$. By considering only the magnitudes of the FFT, we achieve rotation invariance, as the phase information is ignored. To obtain scale invariant descriptors, we then divide each Fourier descriptor by the magnitude of the second Fourier descriptor, $c(k)=c(k)/c(2)$.

These descriptors c(k) describe the frequency contents of the curve representing the boundary coordinates. Figure 5 illustrates the 128 Fourier descriptors from an analysis of the particle illustrated in Figure 4. As can be seen by reference to Figure 5, the most significant descriptors are those close to the points k=1 and k=n(=128). In fact, a value of k close to one or close to n describes low frequency information, i.e. an approximate shape, and the higher frequencies will describe finer details of the shape. We are therefore most interested in the lower order descriptors c(1), c(2),……,c(i) and c(n), c(n-1),……, c(n-i).

The first descriptor (k=1) represents the position of the grain centre of gravity and is not useful for shape description. In order to make the description invariant to translation of the shape, this descriptor is set to zero. The second descriptor, c(2), describes the size of the particle.

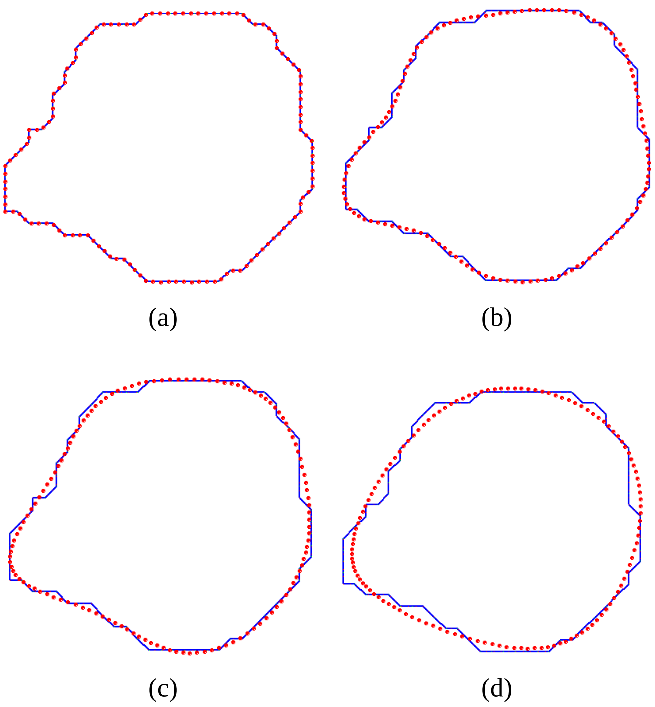
We can reconstruct the geometry for the particle using an inverse FFT. The input to the inverse FFT will be a select number of the descriptor terms f(k) (i.e. direct output of original FFT). We can assess the contribution of each descriptor, c(k), by including the corresponding complex number f(k) in the inversion. For example, if we create an array of n complex numbers, g(k), setting g(2)=c(2), and all other terms equal to 0, we will obtain a circle, with size similar to the original particle. As we gradually include more of the low frequency terms (i.e. g(3)=c(3), g(4)=c(4), and g(127)=c(127), g(128)=c(128), etc), we refine our approximation of the geometry. By increasing the number of components in the description, higher frequency features are also rendered, and sharp curves or details can be generated.

The level of detail captured by increasing the number of descriptors in the inversion can be appreciated by reference to Figure 6, which considers a simple shape (i.e. a square). It is clear that using only a small number of descriptors the general shape is captured (e.g. Figure 6(b)), however to accurately represent corners, we need to include a greater number of descriptors. A similar exercise is illustrated in Figure 7 considering a real soil particle. Again to capture information about features such as surface roughness a large number of descriptors are needed.


(a) (b)

(c) (d)

Figure 6 - The importance of high order Fourier descriptors in the reconstruction of a simple shape, a square (a) c(1) and c(2) only (b) c(1) + c(2) and c(121) to c(128) only (c) c(1) to c(7) and c(126) to c(128) only (d) all 128 descriptors


(a) (b)

(c) (d)

Figure 7 – Representation of a sand particle using different number of Fourier descriptors. (a) all 128 descriptors (b) c(1) to c(8) and c(121) to c(128) only (c) c(1) to c(4) and c(125) to c(128) only (d) c(1) to c(2) and c(127) to c(128) only

3.2 *Geometrical Meaning of Descriptors (Bowman et al approach)*

Figures 6 and 7 illustrate a link between the descriptor values and the particle shape. The precise nature of this link must be established so that the descriptors values can be used for particle characterization.

Diepenbroek et al (1992) proposed a geometrical interpretation of the spectra resulting from the Fourier analysis to derive roundness information for grains. By analysing the harmonics resulting from a Fourier series analysis Diepenbroek et al proposed that the zeroth harmonic produces a circle. This zeroth harmonic is our c(2) coefficient as illustrated, in Figure 6, and the circle has a diameter equal to the mean diameter of the grain outline. The second harmonic adds a component of elongation, this is is our c(n) descriptor. The third harmonic adds a triangular component, and is represented by our c(n-1) descriptor. The fourth harmonic is represented by the c(n-2) descriptor here and adds a square component. In general as k increases and we consider the terms c(k) and c(n-k) we are considering higher frequency sinusoidal waves with increasing frequency and smaller magnitude.

Bowman used standard shapes to extract features of the descriptors. The six shapes were a circle, an ellipse, a rectangle, a square, an equilateral triangle and isosceles triangle. From the analysis of the standard shapes particular features or signature descriptors were obtained. For example, Figure 8 illustrates the output from Bowman et al's analysis for a typical particle. Note that Bowman et al used a slightly different convention to index their Fourier descriptors. Their descriptor $c^B(-1)$ is equivalent to our descriptor c(n), $c^B(-2)$ is equivalent to our descriptor c(n-1), etc. The positive descriptors $c^B(0)$ equals our descriptor c(1), $c^B(1)$ equals c(2), etc.

The c(2) gives the radius or particle size, the descriptor c(n) ($c^B(-1)$) gives the elongation, the descriptor c(n-1) ($c^B(-2)$) gives a measure of triangularity, c(n-2) ($c^B(-3)$) quantifies squareness. The descriptor c(3) ($c^B(+1)$) gives a measure of asymmetry or irregularity such that the regular shapes with the centre of gravity equidistant from any corner have values of 0.00. The second order elongation and triangularity are given by c(4) and c(5) respectively ($c^B(+2)$, $c^B(+3)$).

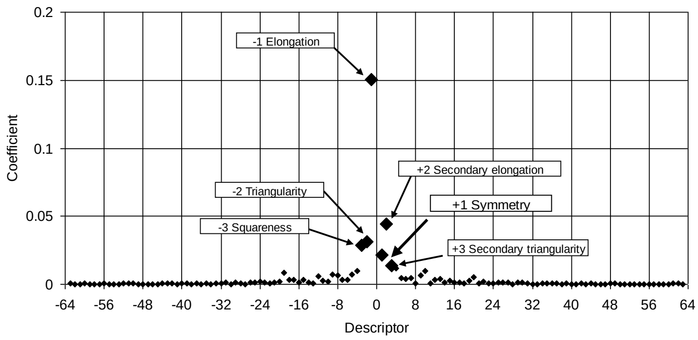


Figure 8– Fourier descriptors output from Bowman et al analysis applied to a Ham River sand particle

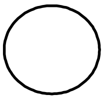
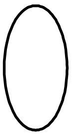
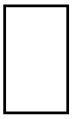
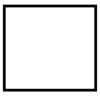
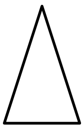
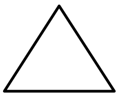
3.3 *Validation- Application to ideal particles*

The six standard shapes considered by Bowman et al were used to validate the MATLAB implementation of this approach. The coefficients obtained for the first seven descriptors were found to be in agreement with the one obtained by Bowman et al (2001), as indicated in Table 1.

Table 1 also provides information to relate the magnitude of the descriptors to the particle geometry. For example referring to the shapes presented in Table 1 elongation is quantified only for in the ellipse, the rectangle and the isosceles triangle. The triangularity descriptor is non-zero only for the triangle shapes. The triangularity is greater in magnitude for the equilateral and decreases with increasing elongation of the triangle. The squareness descriptor has a maximum value for the square, however it does have a non-zero value for the rectangle, the ellipse and the isosceles triangle. The asymmetry is only non-zero for the isosceles triangle; all the other shapes are considered symmetric or regular. Secondary elongation describes the same shapes as elongation although with inferior coefficients, also it has been found that in a triangle with rough faces the coefficient becomes non zero. In a similar way the secondary triangularity has non zero values for the triangles and the rough shapes.

Based on these results Bowman proposed the four signature descriptors c(n), c(n-1), c(n-2) and c(3) are enough to describe the particle morphology. In fact the rough shapes show a slight decrease in these descriptors and an increase of the higher order ones; although the signature morphology is not affected.

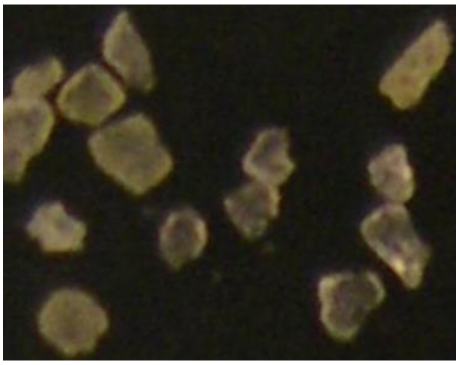
Table 1. General shapes used to investigate Fourier descriptors for morphological characterization.

Shape	**Coefficient (k)**	**Descriptor (Bowman)**	**Descriptor n=128**	**Descriptor n=512**
 (a)	n 3 n-1 4 n-2 5	0.0000 0.0000 0.0000 0.0000 0.0000 0.0000	0.0004 0.0004 0.0004 0.0004 0.0004 0.0004	0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
 (b)	n 3 n-1 4 n-2 5	0.2607 0.0000 0.0000 0.0749 0.0137 0.0000	0.2645 0.0051 0.0014 0.0720 0.0150 0.0032	0.2641 0.0012 0.0003 0.0744 0.0140 0.0008
 (c)	n 3 n-1 4 n-2 5	0.2800 0.0000 0.0000 0.0814 0.0814 0.0000	0.2691 0.0055 0.0055 0.0797 0.0838 0.0028	0.2681 0.0014 0.0014 0.0081 0.0820 0.0007
 (d)	n 3 n-1 4 n-2 5	0.0000 0.0000 0.0000 0.0000 0.1113 0.0000	0.0062 0.0062 0.0062 0.0021 0.1138 0.0000	0.0015 0.0015 0.0015 0.0005 0.1118 0.0000
 (e)	n 3 n-1 4 n-2 5	0.2341 0.0346 0.2286 0.0467 0.0281 0.0496	0.2343 0.0324 0.2319 0.0469 0.0299 0.0489	0.2369 0.0346 0.2286 0.0478 0.0287 0.0489
 (f)	n 3 n-1 4 n-2 5	0.0001 0.0001 0.2500 0.0001 0.0001 0.0625	0.0095 0.0047 0.2535 0.0000 0.0000 0.0615	0.0024 0.0012 0.2509 0.0000 0.0000 0.0623
 (g)	n 3 n-1 4 n-2 5	- - - - - -	0.0966 0.0081 0.1669 0.0167 0.0113 0.0624	- - - - - -
 (h)	n 3 n-1 4 n-2 5	- - - - - -	0.1655 0.0719 0.2655 0.0205 0.0565 0.0468	- - - - - -

4 APPLICATION: RESULTS AND DISCUSSION

4.1 *Application to real sand*

The effectiveness of the methodology for characterizing particles of real soil was tested considering two sands. Toyoura sand (TSand), it is a widely studied Japanese quartz sand. This sand is typically qualitatively described as having sub-rounded to sub-angular grains. The sand has a mean particle diameter of 190μm. The transparent nature of most of Toyoura sand grains poses problems for segmentation as the contrast between the particle and the background is reduced. Ham River sand (HRSand) from the Thames Valley in the UK is a silica sand with a d_{50}=300μm. The grains are sub-rounded as a consequence of erosive and transport processes. Grains of a limestone aggregate weathered (wL) and non-weathered (nwL) were tested with the aim of investigating how image analysis can capture the weathered effect and at the same time explore the use of the same procedures to particles with bigger size than sand grains. The particle size of these aggregates varies from 5mm to 20mm. The numbers of particles considered in this study were 478 Ham River sand grains, 556 Toyoura sand grains, 186 weathered limestone particles, and 122 non-weathered limestone particles. Digital images of the different particles are shown in Figure 9.


(a)


(b)


(c)


(d)

Figure 9 – Digital images of the soil particles used in this project (a) TSand, magnification 50x (b) HRSand, 20x (c) wL, 0.5x (d) nwL, 0.5x

4.2 *Results*

Table 2 summarizes the overall results of the study, giving the mean values of the signature descriptors for each soil type considered. The standard deviation for each descriptor was calculated to evaluate whether these mean values can be considered representative of the overall material. As illustrated in Table 3, in all cases the standard deviations were very large relative to the mean descriptor values.

Table 2. General shapes used to investigate Fourier descriptors for morphological characterization.

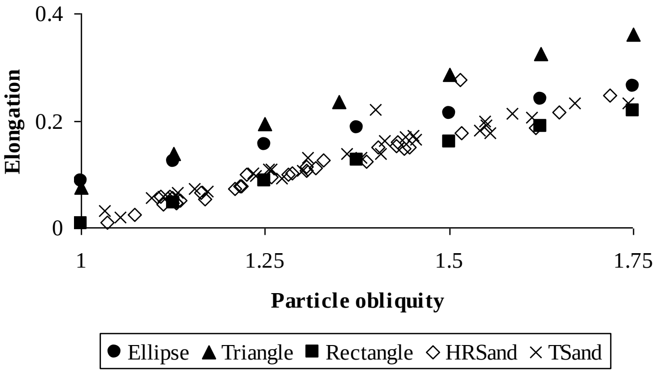
Soil type	**Mean values for the signature descriptors**			
	n Elongation	**n-1 Triangularity**	**n-2 Squareness**	**3 Asymmetry**
HRSand	**0.1190**	**0.0546**	**0.0302**	**0.0286**
TSand	**0.1502**	**0.0638**	**0.0413**	**0.0422**
wL	**0.1597**	**0.0615**	**0.0304**	**0.0260**
nwL	**0.1771**	**0.0647**	**0.0332**	**0.0226**

Table 3. General shapes used to investigate Fourier descriptors for morphological characterization

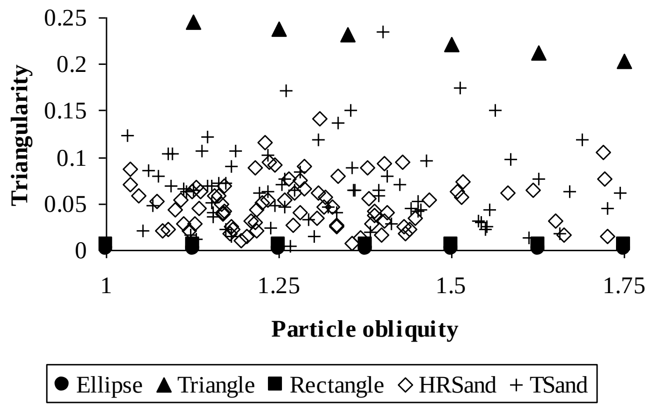
Soil type	**Standard deviation values for the signature descriptors**			
	n Elongation	**n-1 Triangularity**	**n-2 Squareness**	**3 Asymmetry**
HRSand	**0.0702**	**0.0302**	**0.0274**	**0.0630**
TSand	**0.0916**	**0.0420**	**0.0283**	**0.0586**
wL	**0.0856**	**0.0340**	**0.0168**	**0.0150**
nwL	**0.0991**	**0.0309**	**0.0161**	**0.0148**

As illustrated in Figure 10(a) we also considered the variation in the signature descriptor values as a function of particle obliquity. The particle obliquity was measured as the ratio between the longest and shortest axes of and ellipse that has the same normalized second central moments as the particle (Mathworks, 2007). As we would expect, there is a clear correlation between the particle obliquity and the elongation descriptor, however the elongation descriptor does not equal the obliquity of the particle. While we would not necessarily expect any strong correlation between the particle obliquity and the other signature descriptors, we are also including Figures 10(b), (c) and (d) to illustrate the large degree of scatter associated with these parameters for a given soil type. We noted that for triangularity and squareness the standard triangle and rectangular shapes provided an upper bound to the measured data.

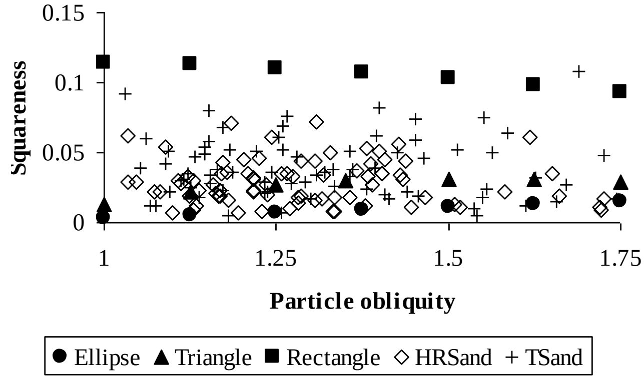
We also carried out a parametric study to assess how many sand particles we need to analyse to obtain statistically representative data. Representative results from this study are illustrated in Figure 11. Figure 11 shows the mean data obtained for Toyoura sand were consistent for samples of about 100 particles or more. Similar results were found for the other materials.



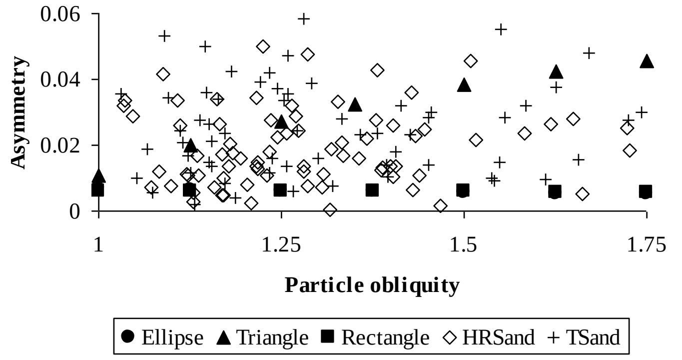
(a)



(b)



(c)



(d)

Figure 10 – Plots of descriptors vs. particle obliquity for sand particles and ideal shapes. (a) Elongation (b) Triangularity (c) Squareness (d) Asymmetry

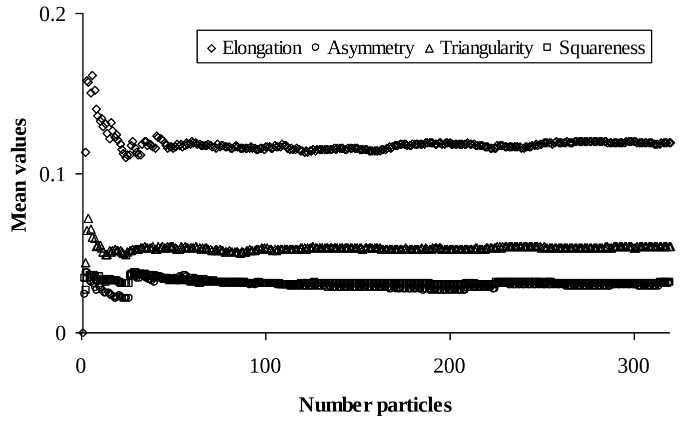


Figure 11 – Evolution of mean values against number of particles for HRSand descriptors

CONCLUSIONS

Digital images of soil grains can be now be acquired easily using optical microscopes or high resolution digital cameras, making quantitative shape analysis a more viable tool for soil mechanics research. This paper has provided details on the implementation of Fourier descriptor analysis in a MATLAB code using the Image Processing Toolbox, following earlier studies including Bowman et al (2001). Data for four different granular materials were analyzed using the Fourier descriptor method, using Bowman et al approach that considered four ‘signature descriptors’, to describe the particle morphology. The elongation descriptor clearly did give a measure of the particle elongation. It is more difficult to relate the other three descriptors to conventional characterization of soil particle geometry and these descriptors exhibited very large scatter for the materials considered. Following our analysis of this method we agree that this method is not likely to be widely applied for general soil characterization work. However, it certainly has potential to give an assessment of detailed features of particle morphology including texture and roughness, and more research is needed to explore this in detail.

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