The Impact of Voluntary Disclosure on a Firm's Investment Policy*

Laura Delaney †
Jacco J.J. Thijssen‡

October 20, 2014

Abstract

In this paper we provide a model which describes how voluntary disclosure impacts on the timing of a firm's investment decisions. A manager chooses a time to invest in a project and a time to disclose the investment return in order to maximise his monetary payoff. We assume that this payoff is linked to the level of the firm's stock price. Prior to investing, the profitability of the project and the market reaction to the disclosure of the investment return are uncertain, but the manager receives signals at random points in time which assist in resolving some of this uncertainty. We find that a manager whose objective can only be achieved through voluntarily disclosing the return is motivated to invest at a time that would be sub-optimal for an identical manager with a profit maximising objective.

Keywords: Economics, Real options, Voluntary disclosure, Sub-optimal investment.

JEL Classification Numbers: C41, C44, C61, D81.

---

*We thank Giulia Iori, Albert Banal-Estanol, and Polina Kovaleva for their much appreciated comments and suggestions.

†(Corresponding author) Department of Economics, City University, Northampton Square, London, EC1V 0HB, UK. Email: laura.delaney.1@city.ac.uk Tel.:+44-(0)207-040-4129.

‡Department of Economics and Related Studies, University of York, Heslington, York YO10 5DD, UK. Email: jacco.thijssen@york.ac.uk

1 Introduction

The purpose of this paper is to investigate the influence of corporate voluntary disclosure on the timing of a firm's investment decision when the manager of the firm has incomplete information regarding the true profitability of the investment and the true market response to the investment strategy. Voluntary disclosures relate to those announcements willingly made by firms outside of their legal and regulatory requirements. We develop a theoretical model of investment whereby the manager of a firm acquires an *option* to disclose the return arising from some investment venture only after the investment has been undertaken. The real options methodology is used to develop the model. This technique has been widely applied to investment decisions (see Dixit and Pindyck [6] for a general presentation of real options and investment), but the use of real options methodology in relation to voluntary disclosure has been relatively scant (see Dempster [5]). This is surprising given that voluntary disclosure decisions share three important characteristics with many investment decisions; i.e., they are irreversible, the payoff is uncertain, and the decision-maker has some leeway over deciding when to disclose.

In our model, the manager of a firm has the option to invest in some risky venture. Once he exercises this investment option, he acquires another separate option which is to voluntarily disclose to the market the return acquired from investing. He only acquires the disclosure option *after* having invested, and the value of his option to invest is dependent upon the value of his option to disclose. So, we view investment and disclosure as a compound option, which is a novel viewpoint. A related paper to ours is Mittendorf [21] who considers the issue of information revelation in a real options framework. However, in that paper the information is revealed via an action taken by the manager which serves as a signal to outsiders about the potential profitability of the project in which the manager chooses to invest. In our paper, we interpret information disclosure as the direct communication to outsiders via some medium such as a press release, the company website, or annual and quarterly reports.

In order for the disclosure option to have value, we assume that the manager's remuneration is dependent on the level of his firm's stock price. This is motivated by the fact that the manager's disclosure of investment returns is unexpected information to market participants (henceforth referred to as the "investors"), who subsequently respond by altering their demand for the firm's shares. This impacts positively or negatively on the firm's pre-disclosure stock price level and, hence, on the manager's compensation. If the manager's remuneration was not linked in some way to the impact from disclosure, then the option to disclose would have no value and the manager's investment policy would be formed with a profit maximising objective. Indeed, this is the benchmark scenario against which we compare our results.

The manner in which we deal with uncertainty differs from standard real options models (for example, Dixit and Pindyck [6] and McDonald and Siegel [20]) where uncertainty is constant over time. In our model, uncertainty is resolved over time because the manager receives signals at irregular intervals about the expected profitability of the investment and the associated market response, similar to the approach of Thijssen et al. [30]. However, their model pertains to a stand-alone investment timing decision whereas our model incorporates the voluntary disclosure option into the optimal stopping problem.

The problem of adjusting the standard real options model of complete information to one of incomplete information has become popular in the operations research literature. For example, Hsu and Lambrecht [16] and Nishihara and Fukushima [23] consider the problem, but in the context of strategic games. Our paper is somewhat related to Shibata [27] who also considers a real options model of incomplete information where uncertainty is resolved over time. However, the purpose of his paper differs from ours in that his aim is to examine the impact of state variable uncertainty on the real options value and its trigger. To achieve this objective, the set-up of his model is more closely related to the standard approach in that he formulates the underlying state variable as a stochastic process, whereas we formulate it as a random variable since this is a more appropriate approach to achieve our objective. In his model, learning occurs via a Kalman filtering procedure whereas in our model, information uncertainty is resolved via the arrival of irregular signals.

Our contribution provides a theoretical framework for the growing body of survey, anecdotal, and empirical evidence which finds that managers of corporations take real economic actions (for example, postpone undertaking profitable investments) which could have negative long-term consequences on firm value in an attempt to manage their reported earnings. For example, Graham et al. [10] survey and interview more than 400 executives and find that 78 percent of their sample admits to sacrificing long-term value to smooth reported earnings, while over half of survey respondents (55.3 percent) state that they would delay starting a new project to meet a reported earnings target, even if such a delay entailed a small sacrifice to value. In support of the evidence provided by Graham et al. [10], Roychowdhury [25] argues that firms overinvest and give sales discounts to meet their reporting targets.

We consider two separate scenarios. In one scenario the market fully observes the investment timing strategy of the manager, but does not observe the investment return. We refer to this as the observable investment decision. In the other scenario, the market does not observe if and when the manager invests and, thus, cannot determine whether or not the manager has undertaken an investment until he opts to disclose the investment return. We refer to this as the unobservable investment decision.

We find that when a disclosure option holds value for a firm's manager his investment strategy can become sub-optimal. In particular, the manager will invest too early relative to an identical profit-maximising manager (i.e., one for whom the disclosure option holds no value) if the positive stock price impact is expected to be high relative to the negative stock price impact, and he will invest too late if the positive stock price impact is expected to be low relative to the negative stock price impact. Furthermore, the manager may even risk investing in a negative net present value (NPV) venture if the expected positive stock price impact is sufficiently high relative to the expected negative stock price impact and if, simultaneously, the signals which the manager receives are not very informative.

Moreover, we show that when the investment decision is unobservable, the manager will invest but withhold disclosing the return acquired until at a later date. A possible motivation for this behaviour is that the manager may consider the investment to be a worthwhile venture for the firm, but expects that if the market participants were to learn of it at the time of investment they may not fully appreciate its potential. Therefore, the investors may need to be prepared for the product before its existence is revealed. That way, when the manager does disclose, the likelihood of a positive stock price impact is greater.[1]

Even though the objective of this paper is to understand the determinants of firms' investment decisions, rather than a firm's disclosure policy, our paper is related at some level to the voluntary disclosure literature. One of the earliest findings in this literature, provided by Grossman and Hart [12] and Grossman [11], has become known as the "unraveling result". If the managers of firms holding private information choose not to disclose their information to outside investors, then the investors will discount the value of the firm to the lowest possible value consistent with whatever firm-specific information they have. Once the managers realise this, they will have an incentive to make full disclosure.

The unraveling argument is relevant to our model when the manager's investment strategy is fully observed by the market. However, when the investment strategy is not observed by the market, the unraveling argument will not hold because we assume that the manager cannot communicate his lack of investment. This implies that when the investment decision is not observed by investors until the manager discloses, the firm is indistinguishable from a firm which has not invested. Since our objective is not to investigate a firm's equilibrium disclosure policies, we do not consider how non-disclosure impacts on investment timing.Therefore, in the unobservable investment case, we as-

[1]The launch of Apple's iPad can provide some anecdotal evidence on this issue. At a technology conference in Los Angeles in June 2010, CEO of Apple, Steve Jobs, admitted that the company had developed the iPad before the iPhone, but the announcement of its development was postponed until almost three years after the iPhone was launched (FoxNews [9]). Jobs's justification for this strategy was that the ideas on which the iPad is based "work just as well on a mobile phone". However, at that time, the iPad was unknown and something that Jobs suspected the market did not realise it had a use for, whereas a mobile phone was something that everybody used.

sume that the investors observing non-disclosure do not infer anything about a firm's investment strategy, and thus, non-disclosure implies that the impact on the stock price is zero. By making this assumption we are able to isolate and identify how disclosure *per se* affects a firm's investment timing policy.

The remainder of the paper is organised as follows: In Section 2, we describe the economic environment from both the manager's perspective and the market's perspective. In Section 3 we focus on the situation whereby the investment strategy is fully observed by the market while in Section 4 we consider the case whereby the investment strategy is not known until the disclosure option is exercised. In Section 5 we present the benchmark model of investment against which our results are compared and in Section 6 we present the results that emerge from our model. Finally, in Section 7 we discuss the implications of these results for corporate policy and outline some possible directions for future research. All proofs are placed in the Appendix.

2 An Embedded Options Model for Investment and Disclosure

2.1 The Manager's Perspective

Consider a risk-neutral manager who has the opportunity to undertake some risky investment. The payoff from the investment is uncertain; it can be high, denoted by U^P, or low, denoted by U^N. We assume without loss of generality that $U^N = 0$. Once the investment option is exercised, its return is assumed to be immediately observed by the manager. We denote the sunk costs of investing by $I > 0$, where it is assumed that $I \leq U^P$.[2]

We assume that the realisation of the investment return is private information to the manager. This implies that upon investment he acquires another option to voluntarily disclose the return to the market. If the disclosure option is then subsequently exercised, the market reacts to the manager's disclosure by altering its demand for the firm's shares which, thus, impacts on the firm's stock price level.

We also assume that the voluntary disclosure is fully credible if the investment option is exercised, and is fully incredulous otherwise. This implies that if a firm invests, but then chooses not to disclose, it is indistinguishable from a

[2]We assume, as is standard in the real options literature, that the manager has all the resources necessary to invest. There is a body of literature which deals with projects where this is not the case. For example, in Sabarwal [26], the manager finances part of the sunk investment cost with debt. However, that paper examines the issue about whether a firm's capital structure affects his investment timing decision. For analytical convenience we use the standard assumption of sufficient funds.

firm that has not invested at all; i.e., the firm cannot credibly communicate its lack of investment. This is because, in our model, the disclosure option is only acquired by the manager subsequent to him having invested. The assumption of fully credible (or truthful) disclosure is very standard in the disclosure literature and the justification for this assumption is the potentially large penalties, for example, reputational damage, of deliberately misinforming the market.

Aside from facing uncertainty over the investment return, the manager also faces uncertainty over the market response to the disclosure: it can be "favourable", leading to a positive discounted impact on the stock price level, or "unfavourable", leading to a negative discounted impact. This impact will be modelled in subsection 2.2. For now, it suffices to remark that the stock price will be impacted either positively or negatively as a result of the disclosure. We further assume that any direct costs associated with disclosure, for example, the costs associated with producing and disseminating the information, are negligible compared to the impact from disclosure and, hence, the sunk costs of disclosure are zero.[3]

The objective of the manager is to adopt (i) an investment and (ii) a disclosure policy such that his own (discounted) expected utility from his compensation is maximised. In order for the disclosure option to be valuable to the manager, we assume that the his compensation is dependent on the firm's stock price level.[4]

2.1.1 The Manager's Information Environment

Since the manager's remuneration is linked to his firm's stock price, he only cares about the investment's return through the channel of stock market perceptions. Hence, the manager's investment decision is based on his *belief* about the market reaction to the subsequent disclosure. We assume that over time the manager receives a stream of signals pertaining to market sentiment towards the firm in case the manager would disclose a positive or a negative return. Such signals could, for example, include forecasts issued by financial analysts. A positive forecast would increase the manager's belief in a favourable response, whereas a negative forecast would lower this belief.

The arrival of these signals follows a Poisson process with parameter $\mu > 0$ which is consistent with the dynamics governing the arrival of signals in the model of Thijssen et al. [30]. In particular, this implies that if we denote by n_t the number of signals the manager has at time t, then

$$dn_t = \begin{cases} 1 & \text{with probability } \mu dt \\ 0 & \text{with probability } 1 - \mu dt, \end{cases} \tag{1}$$

[3]Sunk disclosure costs can easily be introduced but this would be at the expense of parsimony and analytical convenience without leading to any new economic insights.

[4]This is plausible since stock and stock options are very popular forms of compensation in many corporations (Hall and Murphy [14]; Hall [13]).

where $n_0 = 0$. Additionally, we denote by g_t the number of signals the manager has at time t which are indicative of a favourable market response and by $n_t - g_t$ the number of signals indicative of an unfavourable market response. In our set-up, the number of signals indicating a positive market reaction *net* of the number of signals indicating a negative market reaction is a sufficient statistic for the manager's optimal investment policy. At time $t \geq 0$ the net number of signals is denoted by $s_t := g_t - (n_t - g_t) = 2g_t - n_t$.

Each signal gives imperfect information about what the true market reaction will be. We assume that the probability that the signal is correct is given by $\theta \in (1/2, 1)$.[5] A signal is correct if it is indicative of a favourable (unfavourable) response (i.e., it is a positive (negative) signal), and the true market response does indeed turn out to be favourable (unfavourable). A correct signal is a binomially distributed random variable with parameters n and θ.

Whenever a signal is observed, the manager updates his belief in a positive market response in a Bayesian way. We suppose that the manager has a prior (before the investment option is exercised) over the probability of a favourable market reaction, denoted by F, equal to $p_0 \in (0, 1)$; i.e.,

$$P(F) = p_0.$$

The probability that the response will be favourable conditional on having n signals, g of which are positive is given by

$$\begin{aligned}
P(F|n,g) =& \frac{P(n,g|F)P(F)}{P(n,g|F)P(F) + P(n,g|UF)P(UF)} \\
=& \frac{\binom{n}{g}\theta^g(1-\theta)^{n-g}p_0}{\binom{n}{g}\theta^g(1-\theta)^{n-g}p_0 + \binom{n}{g}\theta^{n-g}(1-\theta)^g(1-p_0)} \\
=& \frac{\theta^{2g-n}}{\theta^{2g-n} + \zeta(1-\theta)^{2g-n}} \\
=& \frac{\theta^s}{\theta^s + \zeta(1-\theta)^s},
\end{aligned} \tag{2}$$

where $\zeta = 1 - p_0/p_0$ is the prior odds ratio.

Note that $P(F|n,g)$, denoted by p_t hereafter, is a monotonically increasing function in s_t, and that the inverse function is given by

$$s_t := s(p_t) = \frac{\log\left(\frac{1-p_t}{p_t}\right) - \log(\zeta)}{\log\left(\frac{1-\theta}{\theta}\right)}. \tag{3}$$

[5] This assumption is made without loss of generality because a choice of $\theta = \frac{1}{2}$ implies that the signal is pure noise, since the initial prior is not revised. Furthermore, a choice of $\theta = 0.2$ is as informative as a choice of $\theta = 0.8$ since the same analysis may be carried out for $1 - \theta$.

This implies that the analysis of the solution can apply to either the net number of signals or the posterior belief. In terms of solving for the model and analysing the results that emerge, we use both approaches intermittently, depending on analytical convenience.

At a given point in time, the manager uses both the quality, θ, and quantity, μ, of signals to determine his own valuation of the investment opportunity. The process by which this value is determined will be made clear in subsequent sections.

2.2 Managerial Uncertainty over Market Response

The assumption that the manager is uncertain about the market response to his disclosure about the investment return deserves some explanation. The return from undertaking the investment project is (perfect) private information to the manager. Conversely, investors have their own private information regarding the return of other possible investment opportunities (i.e., besides the opportunity to invest in the firm) which they cannot communicate to the manager.[6] This information advantage over the manager may arise for several reasons. Suijs [29] identifies these reasons as being that some of the investors may have acquired their own costly private information on other investment opportunities or that they may be more sophisticated in analysing financial and other available information than the manager "as investors are specialised in doing just that". Consequently, the manager faces uncertainty over the market response because this response is conditional on each of the investors' private information about their expected utilities from investing in other assets.

In a full Bayesian equilibrium describing the interaction between the manager and the investors, strategies are chosen such that (i) the capital allocation decision of the investors maximise their expected utilities with respect to their posterior beliefs about the expected utility he would derive from investing in the firm relative to investing elsewhere, and (ii) that the investment (and subsequent disclosure) timing strategy of the manager maximises his expected (discounted) utility from his compensation given the investors' capital allocation decisions and their posterior beliefs. We focus on the latter condition by assuming that the capital allocation decision by the investors satisfies the former.

[6]The latter assumption is also made by Suijs [29]. As he points out, "for disclosure to be useful, a large body of investors should disclose their private information *and* the firm should collect and process all of this information. This approach is likely to be very costly and subject to cognitive constraints, so firms remain imperfectly informed about investors' private information."

2.3 The Impact of the Market Response

When the manager discloses his investment's return, the market updates its belief about his expected utility from investing in the firm relative to investing elsewhere and responds by allocating his capital accordingly. If, after disclosure by the manager, an investor's expected utility from investing in the firm is worse than his expected utility from investing elsewhere, he responds unfavourably by selling off some of his existing shares in the firm and invests instead in other assets. Therefore, the impact on the firm's stock price is negative. We assume that the *overall* discounted negative impact of disclosure leads to a decrease in stock price by an amount $S^N := (\gamma - 1)I \leq 0$, for some parameter γ such that $\gamma \leq 1$ by assumption.[7] On the other hand, if, after disclosure by the manager, an investor's expected utility from investing in the firm is better than his expected utility from investing in other assets, he responds favourably by allocating more of his available capital to the firm. This has a positive impact on the firm's stock price by an amount we denote by S^P. In this case, we assume that $S^P := (1+\alpha)\big(U^P - I\big) \geq 0$, for some parameter α such that $\alpha \geq -1$.[8] If $\alpha = \gamma = 0$, stock price maximisation and profit maximization lead to the same policies (see also Section 5).

Since the manager is uncertain about the market response prior to disclosure, he knows that a negative impact on the stock price can ensue when he discloses *either* a positive return or a negative return. It is clear that an investor will respond unfavourably to a negative return, but he will also respond unfavourably to the disclosure of a positive return if, in spite of this, his expected utility from investing in other assets still exceeds his expected utility from investing in the firm. Therefore, if the manager discloses a positive return but the market responds unfavourably, the disclosure effect in this case is given by $\gamma I - U^P$. This occurs because when the market responds, it will initially incorporate the positive return into the stock price, and then there will be an additional disclosure effect. We assume that if the overall impact on the stock price is S^N, then the disclosure effect is $S^N - (U^P - I) = \gamma I - U^P$. We denote the probability (at time $t \geq 0$) of an unfavourable response to the disclosure of a positive return as $P(U^P|UF)P(UF|n,g)$, where $P(UF|n,g)$ is the conditional probability of an unfavourable response (an event we denote by UF). Thus, $P(UF|n,g) = 1 - P(F|n,g)$, where $P(F|n,g)$, abbreviated by p_t, is given by equation (2). Alternatively, the disclosure of a negative return is accompanied by an unfavourable response with probability $P(U^N|UF)(1-p_t) = \big(1 - P(U^P|UF)\big)(1-p_t)$. In this case, the disclosure

[7] It should be noted that these changes in capital allocation take place *after* the investment decision has been taken and they have, therefore, no influence on the availability of funds to pay the sunk costs of investment, I. These funds are assumed to be available.

[8] There is a large body of empirical and anecdotal evidence of stock price over- and under-reactions to various streams of good and bad news (see, for example, Graham et al. [10], Sletten [28], Arya and Mittendorf [1], and Barberis et al. [2]).

effect is γI.

A favourable response will accompany the disclosure of a positive return with probability $P(U^P|F)p_t$. The disclosure effect is captured by $\alpha(U^P - I)$. However, it is difficult to imagine any circumstance where a positive response would accompany the disclosure of a negative return. Therefore, we assume without loss of generality that $P(U^N|F)p_t = 0$. However, since $p_t > 0$, this implies that $P(U^P|F) = 1$.

Given this information, the manager's objective function is given by

$$\begin{aligned}
&\max_{\tau \geq 0} E^{p_0}\left[e^{-r\tau} E^{p_\tau}(U|s_\tau)\right] \\
=&\max_{\tau \geq 0} E^{p_0}\Big[e^{-r\tau}\Big(P(U^P|F)p_\tau(U^P - I + \alpha(U^P - I)) \\
&+ \left(1 - P(U^P|F)\right)p_\tau(-I + U^P + \alpha(U^P - I)) \\
&+ P(U^P|UF)(1 - p_\tau)(U^P - I + \gamma I - U^P) \\
&+ \left(1 - P(U^P|UF)\right)(1 - p_\tau)(-I + \gamma I)\Big)\Big] \\
=&\max_{\tau \geq 0} E^{p_0}\Big[e^{-r\tau}\Big(p_\tau(1 + \alpha)(U^P - I) + (1 - p_\tau)(\gamma - 1)I\Big)\Big],
\end{aligned} \tag{4}$$

subject to (2), where $\tau \geq 0$ is a stopping time and r is the discount rate.

A few additional observations can be made about S^P and S^N. Firstly, we specify these variables in terms of the payoffs and cost of the investment strategy so that the model(s) which we derive in subsequent sections are easily compared with the benchmark case of Thijssen et al. [30] which describes the equilibrium investment behaviour of a manager with a profit maximising objective. This enables us to ascertain clearly from our model the impact of disclosure *per se* on the manager's investment timing strategy. The reasoning for our choice of specifications for S^P and S^N is as follows: Once disclosure has been made, the market revises the price of the firm to incorporate the return of the investment venture. We assume that this impacts on the stock price by an amount equal to the revenue obtained from investment less the sunk investment cost; i.e., by $U^P - I$ or $-I$. While it may appear unrealistic to assume that stock price will change by exactly the amount of revenue gained or lost as a result of the investment, it will, realistically, change by some function of this amount. For ease of exposition, and without loss of generality, we assume that the function is linear and that the proportionality constant is one. Added to this effect, there is then an additional impact from exercising the disclosure option owing to the market's interpretation of the information disclosed. Secondly, it is not necessarily the case that $S^P = |S^N|$ because, as documented in the behavioural finance literature, investor sentiment in the form of over- and under-reaction can lead to a differential response to good and bad news (see, for example, Fama [7], Barberis et al. [2], and Maheu and McCurdy [18]). Furthermore, the specifications of S^P and S^N are not as ad hoc or restrictive as they may initially appear. What is important is that an investor allocates less capital to other assets the higher is his expected utility from investing in the firm, conditional on the manager's disclosure. Therefore, the assumption that U^P, I, α and γ are constant is not crucial, but is made to keep the mathematics tractable. The problem with time-varying coefficients is that time becomes part of the state space. In that case no analytical results can be obtained and one has to resort to numerical methods.

3 Observable Investment Decisions

In this section we consider the case whereby the investment decision made by the manager is fully observed by the market, but the return that is acquired from undertaking such an investment is not. (In the following section we relax the assumption of fully observable investment decisions.) Investors in this sense are considered to be "informed" because they can identify the firm as being one that has invested. They know that firm possesses private information about the investment return and, hence, the manager does not have to disclose for the market to respond to the investment decision. The standard unraveling argument leading to full disclosure applies and, thus, it will never be an optimal strategy to invest and not disclose. This is because once the investors know that the manager has invested, but he fails to disclose the return, they are almost certain to infer that the manager has invested in a venture which has not turned out to be profitable and react through selling off their shares in the firm and allocating their capital elsewhere. Therefore, in essence, the manager holds one option only: to invest and disclose simultaneously, or not to invest and, thus, not disclose at all. His problem is to determine the optimal time at which to exercise this option; i.e., to solve for the optimal stopping problem (4).

Solving for the problem requires obtaining a critical level above which the manager will invest and disclose, and below which the option will not be exercised. This threshold takes the form of a conditional belief in a favourable response to the investment return and is denoted by $p^*_{id} = p(s^*_{id})$, where s^*_{id} is the level of s_t such that the manager is indifferent between investing or not. For $s_t \geq s^*_{id}$, or equivalently, $p_t \geq p^*_{id}$, the manager will invest and disclose the return, otherwise he will wait until enough positive signals have arrived to increase the level of s_t to reach the critical level.

The full solution approach to this problem is outlined in Thijssen et al. [30], but we provide a flavour for the method here. There are three regions to be considered: In the stopping region, where $s_t \geq s^*_{id}$, the option is exercised immediately and the manager's payoff from option exercise is given by $U^0(s_t) = p_t(1+\alpha)(U^P - I) + (1-p_t)(\gamma-1)I$ (cf. equation (4)). There are two continuation regions in which the value of the investment-disclosure option to the manager is denoted by $U^j(s_t)$, for $j = 1, 2$. $U^j(s_t)$ satisfies the Bellman equation

$$rU^j(s_t) = \frac{1}{dt}E(dU^j(s_t)). \tag{5}$$

The first region, $j = 1$, is such that even after a new positive signal arrives, it is still not optimal to exercise the option; i.e., $s_t + 1 < s^*_{id}$. The associated expression for $dU^1(s_t)$ deserves some explanation. Recall that with probability μdt, a new signal arrives. The signal can be positive, in which case $ds_t = +1$, or negative, in which case $ds_t = -1$. The manager's valuation of the option is then updated (via the updating of the posterior belief function) to either $U^1(s_t + 1)$ or $U^1(s_t - 1)$. If no new signal arrives in the interval dt, then the manager's valuation of the option does not change. However, even if a signal is received and it is indicative of a favourable (unfavourable) response, the signal is imperfect and may be incorrect; i.e., the signal may be positive (negative), yet the subsequent response turns out to be unfavourable (favourable). The signal is correct with probability θ. Hence

$$\begin{aligned} dU^1(s_t) = \mu dt\Big[&\theta\left(p_t U^1(s_t+1) + (1-p_t)U^1(s_t-1)\right) \\ &+ (1-\theta)\left(p_t U^1(s_t-1) + (1-p_t)U^1(s_t+1)\right)\Big] \\ &+ (1-\mu dt)U^1(s_t). \end{aligned} \tag{6}$$

The second continuation region for $j = 2$ is such that when there are s_t signals, it is not optimal to exercise the option, but if one more positive signal arrives, it will be optimal to invest; i.e., $s_t < s^*_{id} \leq s_t + 1$. The manager's valuation of the option in this region is $U^2(s_t)$. In this case, if a positive signal arrives in the interval dt, s_t will jump up to the stopping region whereby the option will be exercised and the utility payoff to the manager will be $U^0(s_t+1)$. However, if a negative signal arrives, s_t will jump down to the $j = 1$ region and the option will not be exercised. The valuation of the option to the manager is then $U^1(s_t - 1)$. This implies that $dU^2(s_t)$ is given by

$$\begin{aligned} dU^2(s_t) = \mu dt\Big[&\theta\left(p_t U^0(s_t+1) + (1-p_t)U^1(s_t-1)\right) \\ &+ (1-\theta)\left(p_t U^1(s_t-1) + (1-p_t)U^0(s_t+1)\right)\Big] \\ &+ (1-\mu dt)U^2(s_t). \end{aligned} \tag{7}$$

The boundary condition $\lim_{s_t \to -\infty} U^j(s_t) = 0$, as well as the value matching condition $U^2(s^*_{id}) = U^0(s^*_{id})$ and the continuity condition[9] $U^1(s^*_{id} - 1) =$

[9] Despite the fact that the number of signals, s, is an integer variable, the value-matching and continuity conditions hold because the critical level, s^*_{id} can be take real value. Since the realisations of s are discrete, the option is exercised as soon as $s \geq \lceil s^*_{id} \rceil$, where $\lceil s^*_{id} \rceil := \min\{y \in \mathbb{N} | y \geq s^*_{id}\}$ for $s \in \mathbb{R}$.

$U^2(s_{id}^* - 1)$ together yield the threshold in terms of conditional belief; i.e., p_{id}^*. This is given by

$$p_{id}^* = p(s_{id}^*) = \left[1 + \frac{1+\alpha}{1-\gamma}\left(\frac{U^P}{I} - 1\right)\Psi\right]^{-1}, \tag{8}$$

where

$$\Psi = \frac{(r + \mu(1-\theta))\left[\beta_1(r+\mu) - \mu\theta(1-\theta)\right] - \mu^2\beta_1\theta(1-\theta)}{(r+\mu\theta)\left[\beta_1(r+\mu) - \mu\theta(1-\theta)\right] - \mu^2\beta_1\theta(1-\theta)}, \tag{9}$$

and $\beta_1 > \theta$ is the larger root of the quadratic equation

$$\mathcal{Q}(\beta) = \beta^2 - \left(\frac{r}{\mu} + 1\right)\beta + \theta(1-\theta) = 0. \tag{10}$$

We prove in Appendix A that p_{id}^* is a well defined probability.

This implies that for all $p_t \geq p_{id}^*$, the manager will exercise his option because he is sufficiently convinced, from the signals that he has obtained, that investing and disclosing the return will result in a favourable market response. This would give rise to an increase in the firm's stock price and, hence, an increase in his own remuneration. However, for $p_t < p_{id}^*$ he is more sceptical. Either the signals indicate that the venture will be unprofitable and that investing and disclosing the return will almost surely result in an unfavourable market reaction. Alternatively, the signals may indicate that investing in the venture will be profitable, yet they do not sufficiently convince the manager that having adopted the investment good will be a sufficient incentive to encourage the market to allocate more of its capital to the firm. Thus, the manager will refrain from investing when $p_t < p_{id}$ since he deems the risk that there will be an unfavourable reaction and, consequently, a negative impact on the firm's stock price, as being too high.

4 Unobservable Investment Decisions

When investors do not observe if and when an investment option is exercised, the manager's optimal time to invest and his optimal time to disclose will differ. In particular, the manager will invest at some time $\tau \geq 0$, but refrain from disclosing the return until at some later date, say $\tau + h$, when he has obtained even more (net) positive signals regarding the expected market response. This implies that the disclosure option, which he only acquires after having undertaken the investment, can be exercised almost immediately, some time in the future, or may never be exercised at all.

This behaviour is made possible owing to the fact that the manager cannot communicate his lack of investment. Therefore, when the investment decision is unobservable, the firm is indistinguishable from a firm that has not invested at all. Hence, the investors do not know whether the manager possesses any private information and we assume that investors do not react to non-disclosure when they cannot distinguish between a firm that has invested and a firm that has not. Otherwise, it would not be possible to ascertain the impact of disclosure *per se* on the manager's investment timing strategy.[10] This implies that, unlike in the observable investment case, the unraveling argument leading to full disclosure does not apply. It is reasonable for us to assume, therefore, that if the manager invests in a venture that yields a negative return, he will hide this from the market and never disclose it. Thus, in the unobservable investment case, it is always true that $P(UF|U^N) = 0$.[11]

Intuitively, this means that at the time of investment, τ, the manager deems it more likely that disclosure at time τ will result in an unfavourable market response. However, if the market has been sufficiently convinced by some future time, say $\tau + h$, that it has a use for the good, the likelihood of a positive stock price impact through a favourable market response is high enough to make disclosure optimal. Recall the strategy of Apple with the iPad since this is a relevant example of such a scenario. Thus, he may take the risk of investing at time τ, but protect the effect of a negative response on the stock price and, hence, on his compensation by choosing not to disclose. On the other hand, he will not wait until time $\tau + h$ to invest because, unless the investment return is very low, the value of exercising the investment option always exceeds the value of holding it. This is owing to the fact that when the investment decision is unobservable, $P(UF|U^N) = 0$. Hence, he knows that if the project yields a negative return he can withhold it, and any penalty from investing in an unprofitable project (via his compensation) may never be incurred.

Once the investment has been undertaken, the manager continues to observe signals pertaining to the expected market response that will ensue if he subsequently exercises his disclosure option. As usual, the sunk costs of disclosure are zero.

The optimal disclosure threshold in this case is obtained via the same

[10] If they could react to non-disclosure, the problem becomes one which must solve for the full equilibrium disclosure strategy and the impact of disclosure on investment timing could not be isolated within this larger problem.

[11] Of course, one could argue that any poor investment decision cannot be hidden from the market indefinitely and, therefore, it is not plausible to assert that the market will never realise the negative return. However, if the manager has a very short-term focus because of his own career plans, then by the time the market becomes aware of the poor investment decision, the market reaction may no longer be a relevant concern of his. Alternatively, the manager may act strategically to make the poor investment appear irrelevant by investing in some other venture whose effect far outweighs the poor investment decision to the point that the market fails to react to it at all. But, in our model, non-reaction is equivalent to non-disclosure.

method outlined in Section 3. This is because once the manager invests, the decision over when to optimally disclose is independent of the investment decision, and becomes a stand-alone disclosure option. The fact that $P(UF|U^N) = 0$ has no technical impact on the manager's objective function (4) and, therefore, the disclosure threshold when the investment decision is not observed by the market will be exactly equal to the investment-disclosure threshold when the investment decision is fully observable. For clarity, we denote the "unobservable" disclosure (conditional belief) threshold by p_d^*, but technically, $p_d^* = p_{id}^*$, i.e.

$$p_d^* = p(s_d^*) = \left[1 + \frac{1+\alpha}{1-\gamma}\left(\frac{U^P}{I} - 1\right)\Psi\right]^{-1}. \tag{11}$$

Proposition 1. *When the manager's investment decision is not observable, his optimal investment policy is to invest at, or above, the belief threshold p_i^*, where*

$$p_i^* = p(s_i^*) = \Big[\frac{(\varepsilon + \beta_1\mu\theta\alpha)\left(U^P - I\right)}{\varepsilon(I - U^P) - \beta_1\mu(1-\theta)\left(\gamma I - U^P\right)}\Psi + 1\Big]^{-1}, \tag{12}$$

such that

$$\varepsilon = \beta_1(r + \mu) - \mu\theta(1-\theta), \tag{13}$$

and β_1 and Ψ are as previously defined. Moreover, p_i^* is a well-defined probability for low values of U^P.

Proof. See Appendix B. ■

Figure 1 depicts the unobservable investment scenario graphically. We plot the investment and disclosure thresholds of the manager as a function of the project return for a given value of the option to disclose. In particular we see that the investment threshold always lies below the disclosure threshold implying that the manager will invest and withhold disclosure for some period until he is sufficiently convinced of getting a favourable market response. It is also clear from this graph that $p_i^* > 0$ for only very low levels of the investment return. For higher values of return, p_i^* will not be a well-defined probability (i.e., $p_i^* \to -\infty$ as $U^P/I \to \infty$) and we interpret this as evidence that for a sufficiently high investment return, the manager will *always exercise his investment option immediately* because he knows that if the return ends up being negative, he will not have to disclose it, but if the return is positive, it is high enough to always make investment optimal.

5 A Benchmark Case

Thijssen et al. [30] solve the optimal investment policy when the manager's remuneration is not linked to the firm's stock price. Therefore, the disclosure

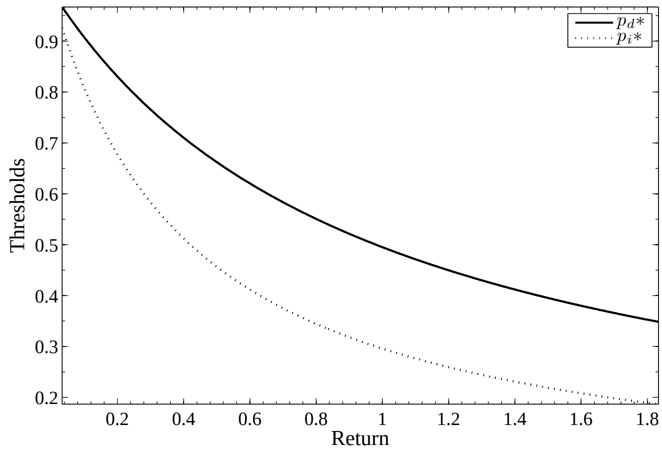

Figure 1: Managerial thresholds as a function of project return.

option holds no value for him and his investment policy is formed with a profit maximising objective. Translated to our model this implies that a manager with a profit maximising objective does not care about any possible over- or under-reaction to the disclosure on the firm's stock-price. He only cares about achieving a positive return which, when disclosed to the market, will be incorporated into the firm's share price. Hence, for a manager with a profit maximising objective, $\alpha = 0 = \gamma$. For such a manager, the objective function (4) becomes

$$\begin{aligned} \max_{\tau \geq 0} E^{p_0}\left[e^{-r\tau} E^{p_\tau}(U|n,g)\right] &= \max_{\tau \geq 0} E^{p_0}\Big[e^{-r\tau}\Big(P(U^P|F)p_\tau(U^P - I) \\ &\quad + \left(1 - P(U^P|F)\right)p_\tau(-I + U^P) \\ &\quad + P(U^P|UF)(1 - p_\tau)(-I) \\ &\quad + (1 - P(U^P|UF))(1 - p_\tau)(-I)\Big)\Big] \\ &= \max_{\tau \geq 0} E^{p_0}\left[e^{-r\tau}\left(p_\tau U^P - I\right)\right]. \end{aligned} \tag{14}$$

Solving for this optimisation problem (via the method presented in Section 3) gives the conditional belief threshold (see also Thijssen et al. [30])

$$\tilde{p}_i^* = p(\tilde{s}_i^*) = \left[1 + \left(\frac{U^P}{I} - 1\right)\Psi\right]^{-1}, \tag{15}$$

where Ψ is given by equation (9). This will be the benchmark against which we compare our results.

Furthermore, the classical net present value (NPV) threshold, denoted by p_{NPV}, is given by

$$p_{NPV} = I/U^P, \tag{16}$$

which is the solution to the equation $p_\tau U^P - I = 0$.

6 Analysis of the Optimal Investment Policy

The impact disclosure has on the investment timing decision of the manager is stated in Proposition 2.

Proposition 2. *If* $|\alpha| \geq |\gamma|$, *then* $p_i^* < p_{id}^* \leq \tilde{p}_i^*$. *Furthermore, if this condition is satisfied with strict inequality, and if* θ *is sufficiently small, then it also holds that* $p_i^* < p_{id}^* \leq p_{NPV}$.

Proof. See Appendix C. ■

It is clear from Proposition 2 that the larger is the positive impact on the stock price resulting from a favourable market reaction to disclosure, α, relative to the size of the negative impact from an unfavourable reaction, γ, the more likely is the manager to invest sub-optimally, *irrespective of whether the investment decision is observable to the market or not.* We refer to a sub-optimal investment as one such that the investment policy of the manager deviates from the benchmark, profit-maximising, investment policy. In particular, a manager behaves sub-optimally when he adheres to any investment threshold which differs from $\tilde{p}_i^*$.

Consider the following table of parameters, the values of which are taken from Thijssen et al. [30]:

Table 1: Parameter values

$\mu = 4$	$r = 0.1$
$p_0 = 0.5$	$\theta = 0.8$
$U^P = 22.5$	$I = 12$

Figure 2 shows the difference between the investment thresholds, s_{id}^* and s_i^*, and the profit-maximising benchmark threshold, $\tilde{s}_i^*$, as a function of $(1+\alpha)/(1-\gamma)$ for the parameter values given in Table 6, and for $\gamma = -2$ and for all $\alpha \in [-0.5, 3]$. Note that in the figure the threshold is given in terms of (net) number of signals, s, rather than in terms of p, but this is only for ease of exposition. For the given values, $\tilde{s}_i^* = 3$ and this does not change with α or γ. Hence, we let this represent the zero line in Figure 2. This figure simply depicts the result that is stated in the first part of Proposition 2 and corresponds with our above discussion. We see that as $|\alpha|$ increases relative to $|\gamma|$, the more likely is the manager to invest too soon relative to the benchmark case (i.e., both s_{id}^* and s_i^* decrease relative to $\tilde{s}_i^*$).

Recall that the manager's compensation is assumed to be dependent on the level of the firm's stock price. The effect on the stock price will be a result of the market reaction to the manager's disclosure of the return acquired through investing in some risky venture. Furthermore, the disclosure option

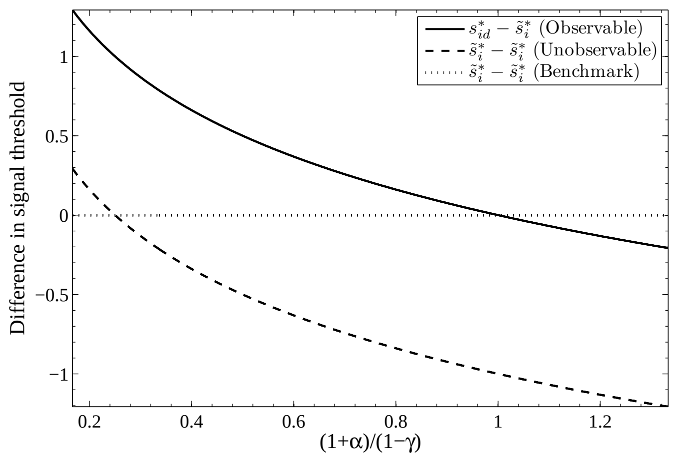

Figure 2: Differences in signal thresholds.

is only acquired after the investment option has been exercised. Therefore, this result implies that if the positive stock price impact from disclosure is sufficiently high relative to the negative stock price impact, the manager will *over-invest* sub-optimally so that he can acquire (and exercise) the disclosure option and realise the benefit to himself through his remuneration package. We refer to over-investment as investment that takes place at a level of p_t such that $p_t < \tilde{p}_i^*$. This implies that a manager who is more concerned with the stock price impact of his investment decision than with the investment's profitability *per se*, will invest after fewer positive signals have been obtained than the number required for an identical manager with a profit-maximising objective. Using these parameter values given in Table 1, but letting $\alpha = 6$ and $\gamma = -2$, the positive stock price impact is $S^P = 73.5$ and the negative impact is $S^N = -36$. This yields a $p_{id}^* \approx 90$ percent and a $p_i^* = 15$ percent. Additionally, the benchmark threshold is found to be $\tilde{p}_i^* = 95$ percent. This highlights our point clearly: When S^P is high relative to $|S^N|$, both in the observable and unobservable scenarios, the manager will invest at a lower threshold than in the benchmark case. This is especially true in the unobservable scenario where he only needs to be 15 percent convinced of a favourable market response.

Of course, sub-optimal investment will not only arise when the manager over-invests, but it will also arise when the manager waits too long before investing relative to the benchmark case; i.e., he invests at some $p_t > \tilde{p}_i^*$. We refer to this sub-optimal behaviour as *under-investment*. The counter-argument to Proposition 2 implies that the more muted will be the positive stock price impact from disclosure relative to the negative impact, the greater the number of positive signals required by the manager before he invests, relative to an identical manager with a profit-maximising objective. This is intuitive in the case where the investment decision is observable since he cannot invest and withhold information. However, when the investment decision is unobservable, he will almost never under-invest (unless in the relatively rare situations where $|\gamma| >> \alpha$).[12] This is also intuitive and is driven by the fact that $P(U^N|S^N) = 0$ in this case. Using these parameter values but letting $\alpha = 6$ and $\gamma = -10$, $S^P = 73.5$ and $S^N = -132$. This yields a $p_{id}^* \approx 97$ percent and $p_i^* = 86$ percent. The benchmark threshold, $\tilde{p}_i^*$, is unaffected and remains 95 percent. This indicates that when $|S^N|$ is high relative to S^P, the manager will under-invest relative to the benchmark case when the investment decision is fully observable, but will still over-invest in the unobservable case. If, however, $\alpha = 6$ but $\gamma = -35$ so that the negative impact is even more pronounced; i.e., $S^N = -432$, then $p_i^* \approx 96$ percent and the manager will then also under-invest relative to $\tilde{p}_i^*$ in the unobservable scenario.

We also find that the quality of the information signals, θ, also plays a part in determining the manager's optimal investment policy. If the signals are not very informative about what the market response to the investment will be, the manager will have little incentive to study them in great depth. Thus, when the quality of the signals is low, but the positive impact of disclosure on the stock price is high relative to the negative impact, the manager will expend little time and effort analysing the signals and just invest (irrespective of whether the investment decision is observable or not). This is because, in this situation, if there is a negative impact from investing on the stock price, it will be relatively contained, whereas if there is a positive impact, it will be relatively large. Thus, the manager has a lot to gain by investing and disclosing, and little to lose. In fact, so much so that if the signal quality is low enough and the positive impact from investing on the firm's stock price is sufficiently high relative to the negative impact, the manager will even opt to take the risk of investing in a negative NPV venture and negate to give any significant consideration to the signals at all. For example, if $\alpha = 10$, $\gamma = -2$, then $S^P = 115.5$ and $S^N = -36$. Let $\theta = 0.55$ (and all other values remain as in Table 1). Then in the benchmark case, $\tilde{p_i^*} = 71$ percent with corresponding net present value as $NPV_{\tilde{p}_i^*} = \tilde{p}_i^* U^P - I = 4.03 > 0$. In the observable scenario, $p_{id}^* = 40$ percent with $NPV_{p_{id}^*} = p_{id}^* U^P - I = -2.93 < 0$ and, in the unobservable scenario, $p_i^* = 32$ percent with $NPV_{p_i^*} = -4.7 < 0$.

Conversely, if the signals are more informative about investor response, the manager will be more inclined to spend the time studying the signals in greater detail. Therefore, the value of waiting to invest will be greater and hence, he will invest only after more positive signals have been obtained; i.e., when the NPV is higher. For example, keeping the above parameter values, but letting $\theta = 0.75$, then $p_{id}^* = 82$ percent with $NPV_{p_{id}^*} = 6.46 > 0$ and $p_i^* = 36$ percent with $NPV_{\tilde{p}_i^*} = -3.86 < 0$.[13] However, in the benchmark case, $\tilde{p}_i^* = 94$ percent

[12]The proof that, unless $|\gamma| >> \alpha$, $p_i^* < \tilde{p}_i^*$ is trivial and is, therefore, omitted.

[13]Recall that in the unobservable case, the manager will almost always over-invest or, in this case, invest in a negative NPV project, unless $|S^N|$ is very high relative to S^P.

with $NPV_{\tilde{p}_i^*} = 9.23$.

These results are depicted in Figure 3 for the observable investment case. A corresponding figure for the unobservable investment threshold is not included as the result is qualitatively the same, but more pronounced. The parameter values are as given above with $\theta \in [0.55, 0.95]$. Note that the dotted zero line depicts the "classical" NPV case which stipulates that investment will occur as soon as the expected benefits from investing equal the expected costs.

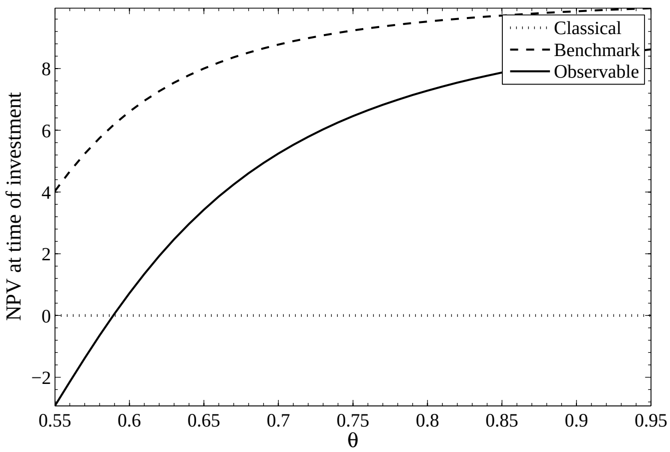

Figure 3: NPV at the time of investment as a function of θ.

The comparative statics for the set of parameters $\{U^P, I, \theta\}$ on the threshold p_{id}^* are qualitatively the same as in Thijssen et al. [30]. They prove that p_{id}^* is increasing in I and θ, and decreasing in U^P. We also find that p_{id}^* decreases in r and increases in μ. This result appears at odds with theirs. However, there is a minor technical error in their proof with respect to the comparative statics for r and μ. In particular, their results suggest that the threshold belief in a good project *increases* with r, which is incorrect. Furthermore, they say that it is not possible to obtain a knife-edged result on the comparative statics with respect to μ. They conclude from simulations that in most cases, the threshold belief increases with μ, which is correct. However, it is not true that a knife-edged result on comparative statics with respect to μ is not possible to obtain. Therefore, for completeness, we prove in Appendix D that p_{id}^* decreases in r and increases in μ.

7 Discussion and Concluding Remarks

The results that emerge from our theoretical model show that when the manager of a firm is compensated via a remuneration package that makes voluntary disclosure valuable to him, his investment strategy can be impacted in a significant way.

In particular, what our results imply is that the investment behaviour of a manager can mitigate investment efficiency[14] when the firm's compensation policy does not encourage him to adopt a profit-maximising objective towards investment. We find that the manager may either over- or under-invest sub-optimally when the investment option is influenced by a valuable option to voluntarily disclose the return acquired from the investment to the market. The stock price linked remuneration package is what gives the disclosure option value in our model. This is because the market reaction to the manager's disclosure impacts on the stock price which, in turn, impacts on the manager's compensation. Therefore, in essence, his compensation is based on the payoff from the disclosure option and thus, he makes his investment timing decision so as to maximise the impact of the associated disclosure on the firm's stock price while eschewing a more forward-looking profit-maximising objective.

This tendency on the part of the manager to act in his own self interest and invest sub-optimally corresponds fundamentally with the definition of myopic managerial behaviour in Cheng et al. [4]. According to Cheng et al. [4], "myopia refers to sub-optimal under-investment in long-term projects for the purpose of meeting short-term goals (for example, Porter [24])". In our paper, under-investment corresponds with waiting too long before investing when the positive impact from disclosure is small relative to the negative impact. However, we also find that managers will over-invest so that they can be more forthcoming with disclosure in order to meet that same short-term goal (i.e., boost their compensation). This arises when the positive impact from disclosure is large relative to the negative impact. Indeed, one of our main results is that if the investment strategy of the manager is not observed by the market, he will *almost always* over-invest relative to a manager with a profit-maximising objective.

Our findings support empirical evidence that myopic behaviour can ensue when firms' incentivisation mechanisms encourage the adoption of a short-term perspective, such as short-term need to raise capital (which has a positive effect on the stock price) (Bhojraj and Libby [3]) and incentive compensation concerns (Matsunaga and Park [19]). Indeed, this tendency for managers to behave myopically is assisted by the fact that, in our model, market participants over-react (both positively and negatively) to firm announcements through excessive buying and selling of firm shares. An effect of this is demonstrated by

[14]Typically, in the corporate finance literature, an inefficient investment policy is one which deviates from the classical zero NPV policy of corporate investment. However, in real options analysis, the zero NPV threshold is shown to be incorrect as it negates to incorporate uncertainty and the value of waiting to invest (Dixit and Pindyck [6]; McDonald and Siegel [20]). Therefore, since we adopt the real options analysis approach in our paper, the inefficiency arises when the investment threshold deviates from the real options profit-maximising investment threshold, $\tilde{p}_i^*$,

Bhojraj and Libby [3] who examine the impact of managerial myopia on capital markets. They show that firms who engage in real actions so as to meet or beat reported earnings targets are able to boost stock price in the short-term but can experience adverse price reversals a few years later.

This demonstrates that other mechanisms ought to be applied in such instances to encourage managers to adopt more long-term profit-maximising strategies for their investment timing decisions. One such approach could be to re-design the manager's compensation contract so that he has no incentive to withhold any of his private information from the market. In that way the disclosure problem would become moot and the manager would have no reason not to adopt a profit-maximising objective. In fact, according to the revelation principle, "any contract can be re-written in a way that induces full revelation of all private information held by the parties to it without affecting the payments they receive" (Myerson [22]). However, such a contract could not be applied to the set-up of our model because once any information is disclosed, the market responds to the disclosure by optimally reallocating its capital between the firm and other assets and, thus, altering the firm's stock price. The only way it could be applied would be if each investor had an enforceable contract with the manager specifying that they would disregard the manager's disclosure in determining their optimal capital allocation strategy. Such contracts are not enforceable.

Another approach that could be applied to our model, however, would be to assume that the shareholder can impose a corporate control challenge on the manager with some positive probability. "Corporate control is the right to determine the management of corporate resources; to hire, fire and set compensation" (see Henderson [15], Jensen and Ruback [17], Fama and Jensen [8]). This approach would be compatible with Henderson [15] who makes the assumption that a corporate control challenge results in dismissal. In her model, the manager faces dismissal if the value-maximising threshold of the shareholder is too far misaligned with the wealth-maximising threshold of the manager. She shows, firstly, that if there is not a well-functioning market for corporate control, the manager will make investment timing decisions which differ markedly from firm value-maximising ones. This is consistent with our findings despite the driving force of our model (the effect of the voluntary disclosure option) being different to hers (incomplete markets). She further shows that when a manager who is faced with idiosyncratic risks is also subject to a corporate control challenge, the risk of a control challenge *always* leads the manager to invest at a threshold closer to the shareholders' value-maximising threshold.

We could apply a similar line of reasoning to our model to ascertain whether the risk of corporate control would be effective in helping to eliminate the opportunistic behaviour of managers for their own personal welfare. This would require him to be more transparent with his private information, particularly in the case where the investment strategy is not observed by the market, so that investors can always ensure that he acts in their best interest by adopting profit-maximising investment strategies. Therefore, we could adapt our model to incorporate the feature of a control challenge resulting in dismissal if the manager is found to be exercising a policy of investment that is not sufficiently transparent for the market. This would allow us to determine the extent to which such a control mechanism would be effective in discouraging the manager from acting in this sub-optimal manner and, in particular, what features of the model are most crucial for achieving this. Such an analysis will be carried out in future research.

References

[1] Arya, A. and B. Mittendorf (2005). Using disclosure to influence herd behaviour and alter competition. *Journal of Accounting and Economics 40*, 231–246.

[2] Barberis, N., A. Shleifer, and R. Vishny (1998). A model of investor sentiment. *Journal of Financial Economics 49*, 307–3343.

[3] Bhojraj, S. and R. Libby (2005). Capital market pressure, disclosure frequency-induced earnings/cash flow conflicts, and managerial myopia. *The Accounting Review 80*, 1–20.

[4] Cheng, M., K. Subramanyam, and Y. Zheng (2005). Earnings guidance and managerial myopia. Working paper, University of Southern California.

[5] Dempster, A. (2006). Valuing announcement options. *European Management Journal 24(2-3)*, 117–127.

[6] Dixit, A. and R. Pindyck (1994). *Investment under uncertainty.* Princeton, NJ: Princeton University Press.

[7] Fama, E. (1998). Market efficiency, long-term returns, and behavioural finance. *Journal of Financial Economics 49*, 283–306.

[8] Fama, E. and M. Jensen (1983a). Separation of ownership and control. *Journal of Law and Economics 26*, 301–325.

[9] FoxNews (2010, June). Jobs says iPad idea came before iPhone. Available at: http://www.foxnews.com/scitech/2010/06/02/jobs-says-ipad-idea-came-iphone/.

[10] Graham, J., C. Harvey, and S. Rajgopal (2005). The economic implications of corporate financial reporting. *Journal of Accounting and Economics 40*, 3–73.

[11] Grossman, S. (1981). The informational law of warranties and private disclosure about product quality. *Journal of Law and Economics 24*, 461–483.

[12] Grossman, S. and O. Hart (1980). Disclosure laws and take-over bids. *Journal of Accounting Research 23*, 123–145.

[13] Hall, B. (1998). The pay to performance incentives of executive stock options. NBER Working Paper 6674.

[14] Hall, B. and K. Murphy (2002). Stock options for undiversified executives. *Journal of Accounting and Economics 33*, 3–42.

[15] Henderson, V. (2010). Is corporate control effective when managers face investment timing decisions in incomplete markets? *Journal of Economic Dynamics and Control 34*, 1062–1076.

[16] Hsu, Y. and B. Lambrecht (2007). Preemptive patenting under uncertainty and asymmetric information. *Annals of Operations Research 151*, 5–28.

[17] Jensen, M. and R. Ruback (1983). The market for corporate control: the scientific evidence. *Journal of Financial Economics 11*, 5–50.

[18] Maheu, J. and T. McCurdy (2004). News arrival, jump dynamics, and volatility components for individual stock returns. *Journal of Finance 59(2)*, 755–768.

[19] Matsunaga, S. and C. Park (2001). The effect of missing a quarterly earnings benchmark on the CEO's annual bonus. *The Accounting Review 76*, 313–332.

[20] McDonald, R. and D. Siegel (1986). The value of waiting to invest. *Quarterly Journal of Economics 101*, 707–728.

[21] Mittendorf, B. (2004). Information revelation, incentives, and the value of a real option. *Management Science 50(12)*, 1638–1645.

[22] Myerson, R. (1979). Incentive compatibility and the bargaining problem. *Econometrica 47*, 61–74.

[23] Nishihara, M. and M. Fukushima (2008). Evaluation of a firm's loss due to incomplete information in real investment decisions. *European Journal of Operational Research 188*, 569–585.

[24] Porter, M. (1992). Capital choices: The causes and cures of business myopia. Research report to the U.S. Government's Council on Competitiveness, Washington D.C.

[25] Roychowdhury, S. (2006). Earnings management through real activities manipulation. *Journal of Accounting and Economics 42(3)*, 335–370.

[26] Sabarwal, T. (2005). The non-neutrality of debt in investment timing: a new NPV rule. *Annals of Finance 1*, 433–445.

[27] Shibata, T. (2008). The impacts of uncertainties in a real options model under incomplete information. *European Journal of Operational Research 187*, 1368–1379.

[28] Sletten, E. (2012). The effect of stock price on discretionary disclosure. *Review of Accounting Studies 17(1)*, 96–133.

[29] Suijs, J. (2007). Voluntary disclosure of information when firms are uncertain of investor response. *Journal of Accounting and Economics 43*, 391–410.

[30] Thijssen, J., K. Huisman, and P. Kort (2004). The effect of information streams on capital budgeting decisions. *European Journal of Operational Research 157*, 759–774.

Appendix

A Proof that $\tilde{p}_{id}^*$ is a well-defined probability

p_{id}^* given by equation (8), is a well-defined probability if, and only if, $0 < p_{id}^* \leq 1$.

$p_{id}^* > 0$ if, and only if, $\Psi \geq 0$, where Ψ is given by equation (9). This is because $(1+\alpha)(U^P - I) \geq 0$ and $(\gamma - 1)I \leq 0$, by assumption. If $r = 0$, from equation (10), $\beta_1 = \theta$, and $\Psi = 0$; i.e., the numerator of (9) is zero. Hence $p_{id}^* = 1 > 0$.

Finding the total derivative of the numerator of Ψ, denoted $n(\Psi)$, with respect to r yields:

$$\begin{aligned}\frac{\partial n(\Psi)}{\partial r} =& \frac{\partial n(\Psi)}{\partial r} + \frac{\partial n(\Psi)}{\partial \beta_1}\frac{\partial \beta_1}{\partial r} \\ =& \Big(\beta_1(r+\mu) - \mu\theta(1-\theta)\Big) + \beta_1\Big(r + \mu(1-\theta)\Big) \\ &+ \frac{\partial \beta_1}{\partial r}\Big((r+\mu)\left(r+\mu(1-\theta)\right) - \mu^2\theta(1-\theta)\Big)\end{aligned}$$

This expression is positive since $r > 0$, $\beta_1 > \theta$, and $\frac{\partial \beta_1}{\partial r} > 0$.

Therefore $n(\Psi) \geq 0$.

On the other hand, when $r = 0$, the denominator of Ψ, denoted $d(\Psi)$, is $\mu^2\theta^2(2\theta - 1) > 0$, since $\theta > \frac{1}{2}$ by assumption. Furthermore

$$\begin{aligned}\frac{\partial d(\Psi)}{\partial r} =& \frac{\partial d(\Psi)}{\partial r} + \frac{\partial d(\Psi)}{\partial \beta_1}\frac{\partial \beta_1}{\partial r} \\ =& \Big(\beta_1(r+\mu) - \mu\theta(1-\theta)\Big) + \beta_1\Big(r + \mu\theta\Big) \\ &+ \frac{\partial \beta_1}{\partial r}\Big((r+\mu)\left(r+\mu\theta\right) - \mu^2\theta(1-\theta)\Big) > 0.\end{aligned}$$

Therefore $d(\Psi) > 0$.

This proves that $\Psi \geq 0$ and $p_{id}^* > 0$.

$p_{id}^* \leq 1$ if, and only if, $\Psi \geq 0$. Indeed, $\Psi \geq 0$, since $r \geq 0$, and thus $p_{id}^* \leq 1$.

Hence, p_{id}^*, given by equation (8), is a well-defined probability. ■

B Proof of Theorem 1

Suppose that at time $t \geq 0$ the net number of signals, s_t, is such that even after a new positive signal arriving, it is still not optimal to invest; i.e., $s_t + 1 <$

s_i^*. It then follows from Thijssen et al. [30] that the value of the investment opportunity, denoted by $V_1(s_t)$, equals

$$V_1(s_t) = \frac{A_1 \beta_1^{s_t}}{\theta^{s_t} + \zeta(1-\theta)^{s_t}},$$

where A_1 is a constant and $\beta_1 > \theta$ is the larger (real) root of the quadratic equation (10).

Alternatively, if the value of s_t is such that it is not optimal to exercise the investment option immediately, but if the manager obtains one more (net) positive signal it will be optimal to invest (i.e., if $s_i^* - 1 \leq s_t < s_i^*$), then Thijssen et al. [30] show that the value of the investment opportunity, denoted by $V_2(s_t)$, equals

$$\begin{aligned} V_2(s_t) = \frac{\mu}{r+\mu}\Big[&\frac{\theta^{s_t+1} + \zeta(1-\theta)^{s_t+1}}{\theta^{s_t} + \zeta(1-\theta)^{s_t}}\Omega(s_t+1) \\ &+ \theta(1-\theta)\frac{A_1\beta_1^{s_t-1}}{\theta^{s_t} + \zeta(1-\theta)^{s_t}}\Big]. \end{aligned} \tag{B.1}$$

Here $\Omega(s_t)$ denotes the *total* value of the undertaking investment for the manager at time t when there are s net positive signals. Note that the total value refers to both the impact from investment and the subsequent acquiring of the disclosure option on the firm's stock price:

$$\begin{aligned} \Omega(s_t) =& P(U^P|S^P)p(s_t)(U^P - I) + P(U^P|S^N)(1-p(s_t))(U^P - I) \\ &+ \text{"Value of Disclosure Option" at } s_t \\ =& U^P - I + \text{"Value of Disclosure Option" at } s_t, \end{aligned} \tag{B.2}$$

since $P(U^P|S^P) = 1$ always and $P(U^N|S^N) = 1 - P(U^P|S^N) = 0$ in the unobservable case.

As soon as the manager invests, he immediately acquires an option to disclose his investment decision to the market. We denote by s_d^* the threshold number of positive over negative signals above which he will opt to disclose and not otherwise. The Bellman equation for an active firm (i.e., one that has invested) in the region where $s_t < s_d^* - 1$ is

$$C(s_t) = e^{-rdt}E[dC(s_t)] \tag{B.3}$$

and E is the expectation operator.

The solution to equation (B.3) is given by

$$C(s_t) = \frac{B_1\beta_1^{s_t}}{\theta^{s} + \zeta(1-\theta)^{s_t}}$$

where B_1 is constant and $\beta_1 > \theta$ is the larger root of the associated quadratic equation, and this is also given by equation (10).

Alternatively, if the net number of signals, s_t, is such that it is not optimal to disclose at time t, but if the manager obtains one more (net) positive signal about the likely market response, then it will be optimal to disclose (i.e., if $s_d^* - 1 \leq s_t < s_d^*$), then the value function in this region becomes:

$$CU(s_t) = \frac{\mu}{r+\mu}\Big[\frac{\theta^{s_t+1} + \zeta(1-\theta)^{s_t+1}}{\theta^{s_t} + \zeta(1-\theta)^{s_t}}\hat{U}(s_t+1) + \theta(1-\theta)\frac{B_1\beta_1^{s_t-1}}{\theta^{s_t} + \zeta(1-\theta)^{s_t}}\Big],$$

where $\hat{U}(s_t)$ denotes the *additional* conditional expected value to the manager from exercising the disclosure option at s_t after the (positive) payoff acquired from the investment has been incorporated into the stock price. This value is given by

$$\begin{aligned}\hat{U}(s_t) =& P(U^P|S^P)p(s_t)\left(S^P - (U^P - I)\right)\\ &+ P(U^P|S^N)(1-p(s_t))\left(S^N - (U^P - I)\right)\\ =&\alpha p(s_t)\left(U^P - I\right) + (1-p(s_t))(\gamma I - U^P).\end{aligned} \tag{B.4}$$

In order to solve for the optimal time to invest, given that through investing the manager acquires an option to disclose his investment decision to the market, we must solve for the following two optimality conditions:

$$V_1(s_i^* - 1) = V_2(s_i^* - 1) \tag{B.5}$$

and

$$V_2(s_i^*) = \Omega(s_i^*). \tag{B.6}$$

Owing to the presence of the disclosure option, $\Omega(s_i^*)$ takes different forms depending on the value of s_i^* in relation to the value of the disclosure threshold s_d^*. If $s_i^* - 1 \leq s_t < s_d^* - 2$, then after one more positive signal, it will be optimal to invest, but not disclose. Even after two more positive signals, it will still not be optimal to disclose. Therefore

$$\begin{aligned}\Omega(s_t) =& U^P - I + C(s_t)\\ =& U^P - I + \frac{B_1\beta_1^{s_t}}{\theta^{s} + \zeta(1-\theta)^{s_t}}.\end{aligned} \tag{B.7}$$

On the other hand, if $s_i^* - 1 \leq s_t < s_d^* - 1$ and if, simultaneously, $s_t \geq s_d^* - 2$, then after one more net positive signal it will be optimal to invest, but not to disclose. However, if there are two more net positive signals it will be optimal to invest and to disclose. This implies that the value of the disclosure option that the manager acquires upon investing is $CU(\cdot)$. Thus

$$\begin{aligned}\Omega(s_t) =& U^P - I + CU(s_t)\\ =& U^P - I + \frac{\mu}{r+\mu}\Big[\frac{\theta^{s_t+1} + \zeta(1-\theta)^{s_t+1}}{\theta^{s_t} + \zeta(1-\theta)^{s_t}}\hat{U}(s_t+1)\\ &+ \theta(1-\theta)\frac{B_1\beta_1^{s_t-1}}{\theta^{s_t} + \zeta(1-\theta)^{s_t}}\Big].\end{aligned} \tag{B.8}$$

Substituting for $V_2(s_i^*)$ and $\Omega(s_i^*)$ in equation (B.6) using equations (B.1) and (B.7) and (B.8), respectively, yields two equations which may be solved simultaneously (after some rather cumbersome, but trivial, algebraic manipulation) for p_i^*. We find that the solution for p_i^* is as follows:

$$p_i^* = \Big[\frac{(\varepsilon + \beta_1\mu\theta\alpha)\big(U^P - I\big)}{\varepsilon(I - U^P) - \beta_1\mu(1-\theta)\left(\gamma I - U^P\right)}\Psi + 1\Big]^{-1}, \tag{B.9}$$

where

$$\varepsilon = \beta_1(r + \mu) - \mu\theta(1-\theta) \tag{B.10}$$

and Ψ is given by equation (9).

B.1 Proof that $p(s_i^*)$ is a well-defined probability for a low investment return

$p(s_i^*) \equiv p_i^*$, given by equation (B.9), is a well-defined if, and only if, $0 < p_i^* \leq 1$.

$p_i^* > 0$ if $\Psi \geq 0$, where Ψ is given by equation (9) and if

$$\frac{\varepsilon + \beta_1\mu\theta\alpha}{\varepsilon(I - U^P) - \beta_1\mu(1-\theta)(\gamma I - U^P)} > 0, \tag{B.11}$$

where ε is given by equation (B.10). This is because $U^P \geq I$, by assumption.

In Appendix A we showed that $\Psi \geq 0$, and thus, it is only necessary for us to show that the condition given by (B.11) holds.

We first show that $\varepsilon > 0$: If $r = 0$, $\beta_1 = \theta$. Thus $\varepsilon = \mu\theta^2 > 0$. $\partial\varepsilon/\partial r = (r+\mu)\partial\beta_1/\partial r + \beta_1 > 0$, since $\partial\beta_1/\partial r > 0$ and $\beta_1 > \theta$. Thus, for $r > 0$, $\varepsilon > \mu\theta^2 > 0$. So $\varepsilon > 0$.

Now, since, $\varepsilon > 0$ and $\gamma I - U^P \leq 0$ (because $U^P \geq I$ by assumption and $\gamma \leq 1$ by definition of S^N), the denominator of (B.11) is more likely to be positive the lower is U^P. Hence, we only need to show that if $\varepsilon + \beta_1\mu\theta\alpha > 0$, the condition will hold when U^P is low. If $r = 0$, the latter equation becomes $\mu\theta^2(1+\alpha) > 0$ since $\alpha \geq -1$ by definition.

$$\frac{\partial}{\partial r}\left(\varepsilon + \beta_1\mu\theta\alpha\right) = \beta_1 + (r + \mu(1+\alpha\theta))\,\frac{\partial\beta_1}{\partial r}$$

which is definitely positive if $\alpha \geq -1/\theta$, since $\partial\beta_1/\partial r > 0$. But since $\alpha \geq -1$ and $\theta \leq 1$, then $\alpha \geq -1/\theta$. Hence, $\varepsilon + \beta_1\mu\theta\alpha > 0$ and condition (B.11) holds when U^P is low.

On the other hand, $p_i^* \leq 1$ if and only if

$$\frac{(\varepsilon + \beta_1\mu\theta\alpha)\big(U^P - I\big)}{\varepsilon(I - U^P) - \beta_1\mu(1-\theta)(\gamma I - U^P)}\Psi \geq 0,$$

which we have just shown to be true when U^P is low.

Thus, p_i^* is a well-defined probability when the investment return is low.

■

C Proof of Proposition 2

The derivation of p_i^*, given by equation (12), is obtained via the assumption that $s_i^* < s_d^*$. Since s_i^* and s_d^* are increasing in p_i^* and p_d^*, respectively, then $p_i^* < p_d^*$. However, as we discuss in Section 4, $p_d^* = p_{id}^*$, where p_{id}^* is given by equation (8). Therefore, it always holds that $p_i^* < p_{id}^*$.

$p_{id}^* \leq \tilde{p}_i^*$ if and only if

$$\begin{aligned}
&\left[1 - \frac{1+\alpha}{\gamma - 1}\left(\frac{U^P - I}{I}\right)\Psi\right]^{-1} \leq \left[1 + \frac{U^P - I}{I}\Psi\right]^{-1} \\
\iff & -(1+\alpha) \leq \gamma - 1 \\
\iff & \alpha \geq -\gamma \\
\iff & \alpha \geq |\gamma|
\end{aligned}$$

$p_{id}^* \leq p_{NPV}$ if and only if

$$\begin{aligned}
&\left[1 - \frac{1+\alpha}{\gamma - 1}\left(\frac{U^P - I}{I}\right)\Psi\right]^{-1} \leq \frac{I}{U^P} \\
\iff & -\frac{1+\alpha}{\gamma - 1}\Psi \geq 1 \\
\iff & (1+\alpha)\Psi \geq 1 - \gamma.
\end{aligned}$$

This latter condition will hold if $\alpha \geq |\gamma|$ and $\theta \approx 1/2$. ■

D Comparative Statics of p_{id}^* w.r.t. r and μ

$$\frac{\partial p_{id}^*}{\partial \chi} > 0, \text{ iff } \frac{\partial \Psi}{\partial \chi} < 0,$$

for $\chi = \{r, \mu\}$.

We re-write equation (9) as

$$\Psi = \frac{(r + \mu(1-\theta))\varepsilon(r,\mu,\theta) - \upsilon(r,\mu,\theta)}{(r + \mu\theta)\varepsilon(r,\mu,\theta) - \upsilon(r,\mu,\theta)},$$

where

$$\varepsilon(r,\theta,\mu) = \beta_1(r+\mu) - \mu\theta(1-\theta)$$

and

$$\upsilon(r,\theta,\mu) = \mu^2\beta_1\theta(1-\theta).$$

Then since $\Psi > 0$ (see Appendix B.1), to determine the sign of the derivative with respect to $\chi = \{r,\mu\}$, we only need to compare the respective derivatives of $(r+\mu(1-\theta))$ and $(r+\mu\theta)$.

$$\frac{\partial}{\partial r}\left(r+\mu(1-\theta)\right) = 1 = \frac{\partial}{\partial r}\left(r+\mu\theta\right).$$

Since $\Psi > 0$ and the derivatives of both $(r+\mu(1-\theta))$ and $(r+\mu\theta)$ with respect to r are equal, then $\frac{\partial\Psi}{\partial r}$ increases in Ψ, and hence, p^*_{id} *decreases* in r.

Finally,

$$\frac{\partial}{\partial\mu}\left(r+\mu(1-\theta)\right) = 1-\theta < \frac{\partial}{\partial\mu}\left(r+\mu\theta\right) = \theta > 0.$$

Since $\frac{\partial}{\partial\mu}\left(r+\mu\theta\right) > \frac{\partial}{\partial\mu}\left(r+\mu(1-\theta)\right)$, the effect on the denominator of Ψ dominates. Therefore,

$$\frac{\partial\Psi}{\partial\mu} < 0,$$

and thus, p^*_{id} *increases* in μ. ■