Price Discrimination Through Transactions Bundling; The Case of Monopsony *

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1 Introduction

The purpose of this paper is the theoretical analysis of price discrimination through the strategy of mixed bundling exercised by a buyer setting prices under conditions of monopsony power. Mixed bundling takes the form of offering either to trade separately in different goods at specific prices, or to trade in a package of goods at an aggregate price which includes a premium.

Mixed bundling is relative to no bundling both profit, for the price setting firm, and trade enhancing.[1] An example of bundling is the case of procurement where contracts are combined by the US government agencies. The result of our analysis is that the US government should ensure that it is of a mixed format.[2] Such an approach is reflected in the recent pieces of legislation concerning defence procurement where it is required that both bundled and unbundled purchasing arrangements have been considered before a decision is reached.[3]

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[1]It is also always weakly profit enhancing relative to pure bundling in which only the bundle is offered. We discuss this briefly in footnote 7.

[2]This is argued at some length in Dassiou - Glycopantis (2006).

[3]Analogous, for the case of monopoly, is the decision in 2004 by the European Commission to force

Mixed bundling can also be used to deal with information asymmetries. If the firm is unable to determine the quality of the goods offered by a trading partner, it may find it profitable to bundle its purchases. This feature introduces what we refer to as *partner preference* (e.g. adverse selection). By bundling its purchases and offering a *premium* for doing so, the firm can reduce adverse selection problems by enhancing its ability to successfully identify trading partners and increase profits.

Mixed purchase bundling by the buyer attracts sellers with low costs in either or both goods, leaving out only firms with high costs in both goods. Hence increasing the volume of trade in each good relative to no bundling. As a result, in the case of US government procurement practices trade will *increase*. This is discussed in Dassiou - Glycopantis (2006).

From the point of view of the price setting buyer, under *transactions bundling* he offers a bundled price which is *higher* than the sum of the two separate prices.[4] In our model package purchasing reduces the buyer's problem which arises from the dispersion in the sellers' individual costs.[5] Offering a combined price which is higher than the sum of the individual prices he attracts also sellers with a high cost in one of the goods but a reasonable overall cost for the bundle. On the other hand, the separate prices are set to extract surplus from those producers who might find it more profitable to trade in only one good.

Microsoft to also sell a version of its Windows Operating System (WOS) without its Windows Media Player (WMP). This is an imposed switch from pure bundling, which was practised for entry deterrence purposes, to mixed bundling (Ayres - Nalebuff (2005)).

[4]We have discussed transaction bundling in its textbook format of a price setting monopolist in Dassiou - Glycopantis (2005b), and in its linked exchange form in Dassiou *et al* (2004).

[5]For an analogous analysis of the monopoly case, see Pepall et al. (2005, p.174) and Nalebuff (2003). Also, Bakos and Brynjolfsson (2000) in a more technical approach show that the customers' valuation of a bundle becomes more predictable as more independent items are added. By averaging out high and low valuations for separate goods, the demand curve for the bundle, in relation to those of the individual goods, becomes more elastic. In the limit, as the number of products in the bundle increases, the monopolist can extract 100% of consumer surplus. Finally as our referee pointed out, there has been also research on the use of bundling in selling multiple objects through auctions. Notably Chakraborty (1999) discusses the case of pure bundling in a Vickrey type auction. In this case the auctioneer bundles together two items and the combined price is set by the bidders. The outcome is that the larger the number of bidders, the more likely it becomes that the second highest bundle price at which the items will be sold is lower than the sum of the individual second highest prices if the goods were offered separately.

Hence, by offering the opportunity of mixed bundling the firm increases its ability to identify trading partners and thus *both* enhances its profits and increases the trade volume. This reason behind purchase bundling by a monopsonist is minimally, if at all, discussed in the literature.

In our model there are no complementarities between the goods in the bundle. The argument is that it is profitable for a monopsonist to offer a bundled purchase price which is higher by a premium than the sum of the individual prices on offer and this does not rely on the existence of complementarities between the goods. If anything the presence of these would add a strategic incentive for the monopsonist to bundle (Nalebuff, 2004), especially if he is faced with potential entrants.

Section 2 discusses the monopsony model, Section 3 proves that the use of mixed bundling is both profit enhancing for the price setting monopsonist and increases the volume of trade in both goods. Section 4 summarises the conclusions of our analysis.

2 The Monopsony Case Model

Our model is loosely related to the bundling models of Adams - Yellen (1976) and the tying model by Whinston (1990). It is closer to the model by McAfee - McMillan - Whinston (1989), (MMW), that deals with mixed bundling offered by a monopolist, and the Dassiou *et al* (2004) model of reciprocal exchange.

There are two goods for potential trade, G1 and G2, between a monopsonistic firm P, and a collection of firms, M, which can be of a number of types. Firms in M attach valuations, equal to the cost of production, θ_i, of Gi and, depending on the prices offered, can decide to sell neither, just one or both goods. The vector $\theta = (\theta_1, \theta_2)$ defines the type of firm as it is endowed by Nature. It is assumed that θ_1 and θ_2 have independent uniform distributions over[6] $[0, 1]$. Every firm in M can supply to P at most one unit of each good.

P, the monopsonist buyer, does not observe the costs; he only knows the probability distribution from which they are drawn. P obtains utility per unit of the good, $S_i^P =$

[6]The case presented can be considered as a limiting case. The unit space is divided into squares of equal area and at every corner there is a firm with valuations given by the coordinates of that point. All valuations are taken to be independent and equally probable. In the limit each square becomes smaller and smaller.

$\alpha_i + \beta_i\theta_i$ where $\alpha_i, \beta_i \geq 0$, which depends on the cost of the good. S_i^P is the valuation of the good bought by the monopsonist. Prices $p = (p_1, p_2)$ and a possible premium $\epsilon > 0$ for selling both goods as a bundle are set and announced by firm P. Because the goods are bought by P we take $p_i \leq 0$ with $p = (p_1, p_2) \geq (-1, -1)$. The prices are set with the objective of maximising his payoff function which is equal to his expected net surplus. For symmetry to the valuations by the suppliers, which lie in $[0, 1]$, we will set $(\alpha_i + \beta_i\theta_i) \in [0, 1]$. The net surplus to the monopsonist per unit of Gi of $S_i^P + p_i$.

We may think of P as someone, (e.g. the government), who is considering to buy a number of flats offering a single price for an unfurnished flat, a single price for a complete set of furniture, or a bundle price with a premium for a furnished flat. We will assume that the buyer uses as a proxy for the quality of the flat and the quality of furniture their costs to the respective sellers. These costs are unobservable to him but he has however sufficient knowledge of the production conditions to know their distribution. He can therefore form an expectation of the surplus that he will receive by buying unfurnished flats, sets of furniture, and furnished flats. By summing up these expectations he obtains a set of prices which he will offer so that the expected surplus from such purchases is maximised.

P's surplus function depends on the type of the firm with which he transacts, and he takes cost as a signal of the quality of the good produced. The importance assigned by the buyer P to cost, i.e. the "cost informativeness", is expressed by the coefficient β_i. We refer to β_i as the degree of P's *partner preference* as it is the only parameter used by the buyer to distinguish between suppliers. The uncovering of the cost of the good is now of double importance for P. It is both relevant for assigning a price to extract the maximum from the seller's surplus, but also for determining P's own valuation. In the case of $\beta_i = 0$, there is no *partner preference*, and the cost of the good is no longer used by P as a proxy to its value.

On the other hand, α_i can be thought of as the component of P's valuation of good Gi, which is independent of the cost of the good. In that sense we refer to it as the *certainty component*, as it does not vary by supplier but only depends on the good. This is the part of the value that P assigns to the good through his use of prior external, market information. It also reflects his individual preference for Gi. Since $S_i^P \leq 1$, in the extreme case where $\alpha_i = 1$, it follows that $\beta_i = 0$ and P's net surplus, $1 + p_i$, will be non-negative.

The prices p_1, p_2 are offered for the single transactions in each good, and $p_b = p_1 + p_2 - \epsilon$, is the price offered for the two purchases bundled together. At prices $p = (p_1, p_2, p_b)$, there are four possibilities. A firm will choose to sell neither good, and we denote by $R_0(p)$ the relevant area (θ_1, θ_2); or sell G1 only, $R_1(p)$; or sell G2 only, $R_2(p)$; or sell both goods, $R_{12}(p)$.

The variables p_1, p_2 and ϵ will be chosen by P to maximize his expected surplus $\mathcal{S}(p_1, p_2, \epsilon)$. We are concerned with local maxima. In the first instance the maximisation of $\mathcal{S}(p_1, p_2, \epsilon)$ is sought with respect to p_1, p_2 for a fixed $\epsilon > 0$, and then we will also look for a local maximum of $\mathcal{S}(p_1, p_2, \epsilon)$ with respect to all three variables. The surprising outcome is that choosing to offer a positive premium is surplus enhancing for the monopsonist, without the need to assume any complementarities in the monopsonist's preferences.

The four possibilities mentioned earlier are defined as:

$$R_0(p) = \{\theta \in [0,1]^2 : 0 > -\theta_1 - p_1,\ -\theta_2 - p_2,\ -\theta_1 - \theta_2 - (p_1 + p_2 - \epsilon)\} \tag{1}$$

$$R_1(p) = \{\theta \in [0,1]^2 : -\theta_1 - p_1 \geq 0, -\theta_1 - p_1 > -\theta_2 - p_2,\ -\theta_1 - \theta_2 - (p_1 + p_2 - \epsilon)\}; \tag{2}$$

$$R_2(p) = \{\theta \in [0,1]^2 : -\theta_2 - p_2 \geq 0,\ -\theta_2 - p_2 > \ -\theta_1 - p_1,\ -\theta_1 - \theta_2 - (p_1 + p_2 - \epsilon)\}; \tag{3}$$

$$R_{12}(p) = \{\theta \in [0,1]^2 : -\theta_1 - \theta_2 - (p_1 + p_2 - \epsilon) \geq 0,\ -\theta_1 - p_1,\ -\theta_2 - p_2\} \tag{4}$$

where $p = (p_1, p_2)$. These four sets partition the unit square.

The expected net surplus is equal to:

$$\mathcal{S} = \int\limits_{R_1(p)} (\alpha_1 + \beta_1\theta_1 + p_1)\, dF(\theta) + \int\limits_{R_2(p)} (\alpha_2 + \beta_2\theta_2 + p_2)\, dF(\theta) +$$

$$\int\limits_{R_{12}(p)} (\alpha_1 + \beta_1\theta_1 + \alpha_2 + \beta_2\theta_2 + p_1 + p_2 - \epsilon)\, dF(\theta) \tag{5}$$

Both θ_1 and θ_2 are *dual role variables* as participation constraints for both M as well as P. In the case of *no bundling* for example, M will not sell good Gi to P unless $-p_i \geq \theta_i$, and P in turn will not purchase the good unless $-p_i \leq \alpha_i + \beta_i\theta_i$.

In the monopoly case discussed by Adams - Yellen (1976, p. 481) the exclusion condition, that "no individual consumes a good if the cost exceeds his reservation price", implies allocative efficiency. In our model of monopsony where θ_1 and θ_2 are dual participation parameters for both M and P, such a condition would require that no company sells in the package a good whose cost exceeds P's valuation. This ensures productive efficiency and it will be shown to hold always.

3 Results

First we are concerned with the change in the local maximum of $\mathcal{S}$ as ϵ varies. This change depends on the values of α_i, β_i and is denoted by ψ. We prove:

Theorem 1 Let ϵ be exogenous. For $\alpha_1, \ \alpha_2 > 0$, the local maximum of $\mathcal{S}$ at $\epsilon = 0$ increases with ϵ, i.e. ψ is positive. This increase (ψ) is a positive function of $\alpha_i, \ \beta_i, \ i = 1, \ 2$, and furthermore for both the special cases (i) $d\alpha_i = -\frac{1}{2}d\beta_i = d\omega$ and (ii) $d\alpha_i = -d\beta_i = d\omega$, the differential $d\psi$ is positive for $d\omega > 0$.

Proof.

It is easier to rewrite (1) to (4) as (6) to (9):

$$R_0(p) = \{\theta \in [0,1]^2 : \theta_1 > -p_1, \theta_2 > -p_2, \theta_1 + \theta_2 > -p_1 - p_2 + \epsilon\}; \tag{6}$$

$$R_1(p) = \{\theta \in [0,1]^2 : \theta_1 \leq -p_1, \theta_2 > -p_2 + \epsilon\}; \tag{7}$$

$$R_2(p) = \{\theta \in [0,1]^2 : \theta_1 > -p_1 + \epsilon, \theta_2 \leq -p_2\}; \tag{8}$$

$$R_{12}(p) = \{\theta \in [0,1]^2 : \theta_1 \leq -p_1 + \epsilon, \theta_2 \leq -p_2 + \epsilon, \theta_1 + \theta_2 \leq -p_1 - p_2 + \epsilon\}. \tag{9}$$

The relations in $R_{12}(p)$ are equivalent to $-(p_1 + p_2 - \epsilon) - \theta_1 - \theta_2 \geq 0$ and $-(p_1 + p_2 - \epsilon) - \theta_1 - \theta_2 \geq -p_i - \theta_i$ for $i = 1, 2$, and therefore suppliers of this type sell the bundle to the price setting buyer P. Note that for P to find it profitable to trade in both goods the gain from trading in only one good, say Gi, rather than the bundle should be smaller or equal to the forgone value of not purchasing the other good, Gj. This means that we require $p_i - (p_1 + p_2 - \epsilon) \leq \alpha_j + \beta_j\theta_j$. Combining this inequality with the one corresponding to $R_{12}(p)$ gives that $\theta_j \leq -p_j + \epsilon \leq \alpha_j + \beta_j\theta_j$ for $j = 1, 2$ and $i \neq j$.[7] This shows that the exclusion condition as defined in the previous section is satisfied.

[7] Note that as mentioned in Adams - Yellen (1976, p. 483) and in MMW (1989, p. 374) mixed bundling is always weakly better than pure bundling for the monopolist. For a textbook analysis on this, see Pepall *et al* (2005, p. 173). In terms of our paper the monopsonist can at worst use a mixed bundling strategy that replicates the pure bundling offer and sets arbitrarily low individual prices by fixing ϵ appropriately.

We place point (p_1, p_2) in the unit square (θ_1, θ_2) in Figure 1. The sum of the four integrals below exhausts the area in which M will be selling one or both goods. The expected net surplus to P, given in (5), becomes more explicitly:

$$\mathcal{S} = \int_{-p_2+\epsilon}^{1} \int_{0}^{-p_1} (\alpha_1 + \beta_1\theta_1 + p_1)\, d\theta_1 d\theta_2 + \int_{-p_1+\epsilon}^{1} \int_{0}^{-p_2} (\alpha_2 + \beta_2\theta_2 + p_2)\, d\theta_2 d\theta_1 +$$

$$\int_{0}^{-p_2+\epsilon} \int_{0}^{-p_1} (\alpha_1 + \beta_1\theta_1 + \alpha_2 + \beta_2\theta_2 + p_b)\, d\theta_1 d\theta_2 +$$

$$\int_{-p_1}^{-p_1+\epsilon} \int_{0}^{-p_1-p_2+\epsilon-\theta_1} (\alpha_1 + \beta_1\theta_1 + \alpha_2 + \beta_2\theta_2 + p_b)\, d\theta_2 d\theta_1. \tag{10}$$

Solving the integrals in (10) we obtain:

$$\begin{aligned}\mathcal{S} = -0.5\left[p_1^2(2-\beta_1) + p_2^2(2-\beta_2) + 2\alpha_1 p_1 + 2\alpha_2 p_2\right] - \\ \left[(3-\beta_1-\beta_2)p_1p_2 + \alpha_1 p_2 + \alpha_2 p_1\right]\epsilon + \\ \frac{1}{2}\left[\alpha_1 + \alpha_2 + (3-\beta_1-\beta_2)(p_1+p_2)\right]\epsilon^2 - \frac{1}{6}(3-\beta_1-\beta_2)\epsilon^3.\end{aligned} \tag{11}$$

For $\epsilon = 0$ we have

$$\mathcal{S}^{\,0} = -0.5[p_1^2(2-\beta_1) + p_2^2(2-\beta_2) + 2\alpha_1 p_1 + 2\alpha_2 p_2] \tag{12}$$

The (locally) optimal $p_1^*,\ p_2^*$ for the two goods are obtained from $\dfrac{\partial S^0}{\partial p_1} = \dfrac{\partial S^0}{\partial p_2} = 0$ of relation (12) which imply:

$$p_1^* = -\frac{\alpha_1}{2-\beta_1}, \quad p_2^* = -\frac{\alpha_2}{2-\beta_2}. \tag{13}$$

For negative prices we require $\beta_1, \beta_2 < 2$ and the function in (12) is strictly concave. Therefore an interior maximum obtained using (13) is a global one. Note that since $|p_1^*|,\ |p_2^*| \leq 1$ the above means that $\alpha_1 + \beta_1 \leq 2$ and $\alpha_2 + \beta_2 \leq 2$.

Next we calculate $\dfrac{\partial S}{\partial \epsilon}\ |_{\epsilon=0,p_1^*,p_2^*}$. This is equal to $\psi = \dfrac{\alpha_1\alpha_2}{(2-\beta_1)(2-\beta_2)} > 0$, implying that the optimal value of ϵ is greater than zero, i.e. bundling is locally optimal. Moreover as $\dfrac{\partial \frac{\alpha_1\alpha_2}{(2-\beta_1)(2-\beta_2)}}{\partial \alpha_i} = \dfrac{\alpha_j}{(2-\beta_1)(2-\beta_2)} > 0$ and $\dfrac{\partial \frac{\alpha_1\alpha_2}{(2-\beta_1)(2-\beta_2)}}{\partial \beta_i} = \dfrac{\alpha_1\alpha_2}{(2-\beta_i)^2(2-\beta_j)} > 0$ for $i \neq j$, i.e. the profitability of bundling is an increasing function of both the certainty component and the degree of partner preference in both goods.

This ensures that the surplus to P from those suppliers that switch from offering the package to selling only one good will either stay the same or increase.

We now consider specific changes in α_i and β_i:

(i) Holding the expected value of P's valuation of G1 constant, i.e. $d[E(S_1^P)] = 0$, implies a parametrisation $d\alpha_1 = -\frac{1}{2}d\beta_1 = d\omega$. Hence,

$$d\psi = \left[\frac{\alpha_2 d\alpha_1}{(2-\beta_1)(2-\beta_2)} + \frac{\alpha_1\alpha_2 d\beta_1}{(2-\beta_1)^2(2-\beta_2)}\right] =$$

$$\left[\frac{\alpha_2}{(2-\beta_1)(2-\beta_2)} - \frac{2\alpha_1\alpha_2}{(2-\beta_1)^2(2-\beta_2)}\right]d\omega \Longleftrightarrow$$

$$d\psi = \frac{\alpha_2}{(2-\beta_1)(2-\beta_2)}\left(1 - \frac{2\alpha_1}{2-\beta_1}\right)d\omega.$$

The term outside the bracket is positive, while the denominator of $\frac{2-\beta_1-2\alpha_1}{2-\beta_1}$ is positive and $2-\beta_1-2\alpha_1 = 2 - \left[\frac{1}{2}\beta_1+\alpha_1\right] - \left[\frac{1}{2}\beta_1+\alpha_1\right]$. As by assumption $0 \leqslant \alpha_1 + \beta_1\theta_1 \leqslant 1$, it follows that $0 \leqslant \alpha_1 + \frac{1}{2}\beta_1 \leqslant 1$. This proves that $d\psi > 0$ for $d\omega > 0$. Analogous results are also obtained for $d[E(S_2^P)] = 0$.

(ii) The impact of the certainty component α_i on profitability dominates that of β_i. We consider this second parametrisation case, where $\alpha_i + \beta_i = 1$, because as it is explained below it gives us the largest percentage improvements in P's net surplus ($\mathcal{S}^*$) from purchase bundling.

Assuming the changes $d\alpha_1 = -d\beta_1 = d\omega$ and totally differentiating ψ:

$$d\psi = \frac{\partial\psi}{\partial\alpha_1}d\alpha_1 + \frac{\partial\psi}{\partial\beta_1}d\beta_1 =$$

$$\left[\frac{\alpha_2}{(2-\beta_1)(2-\beta_2)} - \frac{\alpha_1\alpha_2}{(2-\beta_1)^2(2-\beta_2)}\right]d\omega =$$
$$\frac{\alpha_2}{(2-\beta_1)(2-\beta_2)}\left(1 - \frac{\alpha_1}{2-\beta_1}\right)d\omega$$

which is positive for $d\omega > 0$, given that $\alpha_1 + \beta_1 \leq 2$. An analogous result is obtained for $d\alpha_2 = -d\beta_2$. In other words if α_i is increased (decreased) and β_i is decreased (increased) by the same amount, P's expected net surplus will increase (decrease).

This completes the proof of Theorem 1.

Unless P's ability to ascertain the quality of the goods it considers buying equals zero, bundling the purchases is surplus superior to no bundling. The superior profitability of mixed purchase bundling (relative to no bundling), as expressed by ψ, of a monopsonist is only eliminated if either $\alpha_1 = 0$ or/and $\alpha_2 = 0$.

We now turn our attention to the case in which ϵ is chosen by the firm.

Theorem 2 Let ϵ be endogenous. $\mathcal{S}^*$ can be obtained which is a continuous and differentiable function of ϵ, by substituting p_1 and p_2, as functions of the premium in $\mathcal{S}$.

Proof. We calculate the local maximum through successive maximisation. First we maximise over p_1, p_2 for given of ϵ. Re-substituting we maximize with respect to ϵ.[8]

First order conditions with respect to p_1, p_2 are:

$$
\begin{aligned}
p_1 n - \alpha_1 - \epsilon\alpha_2 + \epsilon p_2 k - \frac{1}{2}\epsilon^2 k = 0 \\
p_2 m - \alpha_2 - \epsilon\alpha_1 + \epsilon p_1 k - \frac{1}{2}\epsilon^2 k = 0
\end{aligned} \tag{14}
$$

for $n = \beta_1 - 2 < 0$, $m = \beta_2 - 2 < 0$, $k = n + m + 1 = \beta_1 + \beta_2 - 3$. The second order conditions are given by the principal minors of the Hessian matrix

$$
\begin{pmatrix} \beta_1 - 2 & -\epsilon(3 - \beta_1 - \beta_2) \\ -\epsilon(3 - \beta_1 - \beta_2) & \beta_2 - 2 \end{pmatrix} \tag{15}
$$

whose determinant is restricted to be positive in order to have an interior maximum solution[9]:

$$
(\beta_1 - 2)(\beta_2 - 2) - \epsilon^2(\beta_1 + \beta_2 - 3)^2 > 0.
$$

Solving the first order conditions in (14) we get:

[8] See Glycopantis - Muir (2004). This approach is based on the maximization of a function on a compact set. The prices are in $[-1, 0]^2$ and ϵ can be taken in an interval around a local maximum. Considering all variables together gives identical results.

[9] The inequality that follows in the text determines the range of values for ϵ which allow for an interior maximum. Specifically, for $\beta_1 = \beta_2 = 0 \Rightarrow \epsilon < 0.67$, for $\beta_i = 1$, $\beta_j = 0 \Longrightarrow \epsilon < 0.7071$, and for $\beta_1 = \beta_2 = 1 \Longrightarrow \epsilon < 1$.

$$p_1 = \frac{1}{2nm - 2\epsilon^2 k^2}\left(2m(\alpha_1 + \epsilon\alpha_2) - 2\epsilon k(\alpha_2 + \epsilon\alpha_1) + \epsilon^2 mk - \epsilon^3 k^2\right), \tag{16}$$

$$p_2 = \frac{1}{2nm - 2\epsilon^2 k^2}\left(2n(\alpha_2 + \epsilon\alpha_1) - 2\epsilon k(\alpha_1 + \epsilon\alpha_2) + \epsilon^2 nk - \epsilon^3 k^2\right) \tag{17}$$

where the denominators in (16) and (17) are non-zero because of the second order conditions above.

Substituting back into (11) gives:

$$\mathcal{S}^* = (2nm - 2\epsilon^2 k^2)^{-1} \times$$

$$\Big(\alpha_1^2(\epsilon^2(2k-n) - m) + 2\alpha_1\alpha_2\epsilon(\epsilon^2 k + k - m - n) + \alpha_1\epsilon^2(k-n)(\epsilon k - m) +$$

$$\alpha_2^2(\epsilon^2(2k-m) - n) + \alpha_2\epsilon^2(k-m)(\epsilon k - n) + \frac{1}{12}k\epsilon^3(2\epsilon^2k^2 - 3\epsilon k(m+n) + 4mn)\Big). \tag{18}$$

Again the denominator is different from zero which guarantees the differentiability of the function in an interval around a locally optimal ϵ. This completes the proof of Theorem 2.

Existence of local optimum and comparative statics results.

The derivatives of $\mathcal{S}^*$ in (18) with respect to ϵ are too complex to allow us to obtain the locally optimal $\mathcal{S}^*$ and then analyse the effect of varying the parameters. For specific values of the parameters $\alpha_1,\ \alpha_2,\ \beta_1,\ \beta_2$, we calculate the locally optimal ϵ and $\mathcal{S}^*$, the corresponding optimal prices, the improvement in the net surplus value and in trade volume.

For $\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$, tables[10] showing the effects of changing α_1 or α_2, one at a time, are presented here[11]. We choose this relationship between the parameters because compared with alternative combinations they give the largest values for $\mathcal{S}^*$, both absolutely and relative to no bundling. For $\alpha_1 = \alpha_2 = 1$, a value of $\mathcal{S}^*$=0.5492 is attained at the premium of $\epsilon = 0.4714$ and the optimal prices are $p_1 = p_2 = -\frac{1}{3}$. This case is illustrated in Figure 2. Note that since $\beta_1 = \beta_2 = 0$, the second order conditions combined with the fact that ϵ is positive mean that $\epsilon < \frac{2}{3}$ as already mentioned[12].

[10] Tables and Figures are in the Appendix.

[11] For the choices of different parameters see Dassiou - Glycopantis (2005a).

[12] A sufficient, though not necessary, condition for a local maximum to exist was that both $\alpha_i + \beta_i \leq 1$ and $\alpha_j + \beta_j \leq 1$.

Numerical calculations[13] confirm that $\mathcal{S}^*$, $|p_1|$, $|p_2|$ and ϵ are increasing with the chosen values α_1 and α_2, (see Tables 1, 2, 3). Table 4 calculates the percentage improvement in P's net surplus from purchase bundling. This is largest for equal values in the certainty components and the degrees of partner preference between the two goods, while it decreases for diverging values between the α's and the $\beta's$. For example the improvement is only 4.5% for $\alpha_i = 0.2$ and $\alpha_j = 1$, and 12.12% for $\alpha_1 = \alpha_2 = 0.2$.

When the degrees of adverse selection of the two goods are identical, it is more profitable to bundle purchase the higher the level of quality uncertainty. Bundling allows P to extract more surplus by enhancing his ability to identify his partners. On the other hand, this surplus improvement for mixed purchase bundling becomes smaller the larger the values of the certainty component, as the problem of identification becomes less acute. These findings are illustrated in Table 4, and they are of additional interest because they contrast to the findings concerning the impact of bundling on the volume of trade. This is discussed below.

For the values of the parameters chosen, the trade volume increases relative to the case of no bundling. P reduces in absolute value both the separate prices he offers to trading partners $(|p_1|, |p_2|)$, relative to their unbundled values $(|p_1^*|, |p_2^*|)$, thus reducing the incentive for a firm M to participate in R_1 and R_2, while offering a premium ϵ for the bundled transaction. This is substantial enough to increase trade in both G1, denoted by $(R_1^B + R_{12}^B)$, and G2, denoted by $(R_2^B + R_{12}^B)$, relative to their unbundled sizes.[14]

Numerically we can check that there is always an increase in the volume of trade in both goods relative to its unbundled size. We substitute for the bundled and unbundled surplus maximising prices and the corresponding optimal value of ϵ and calculate for these the expressions

$$R_1^B + R_{12}^B - R_1^{UB} - R_{12}^{UB} = p_1^* - p_1 - \epsilon p_2 + \frac{\epsilon^2}{2}, \tag{19}$$

and

[13] For a complete description see Dassiou - Glycopantis (2005a).

[14] In Figure 1 we indicate the regions of trade under bundling. They are $R_{12}^B(p) = OIAWJ$, $R_1^B(p) = JWDH$, $R_2^B(p) = IGCA$, $R_0^B(p) = WACO'D$. For the sake of comparison we also indicate the unbundled regions of trade which are $R_{12}^U(p) = OKUL$, $R_1^U(p) = LUFH$, $R_2^U(p) = KUEG$, and $R_0^U(p) = UEO'F$.

$$R_2^B + R_{12}^B - R_2^{UB} - R_{12}^{UB} = p_2^* - p_2 - \epsilon p_1 + \frac{\epsilon^2}{2}, \tag{20}$$

both of which were found to be always positive for the full range of possible values for α_1 and α_2.

Relations (19) and (20) imply that the improvement in the trade for G1 is more (less) substantial the larger (smaller) α_2, and the smaller (larger) α_1 is, while the reverse is true for G2. Through bundling, there is more room for an improvement in the volume of trade in a good with low degree of quality certainty if its purchase is combined with a good of substantially higher quality certainty. For example, compared to no bundling the percentage increase in the trade volume of G1 (G2) is equal to 58.13% (4.44%) for $\alpha_1 = 1 - \beta_1 = 0.2, \alpha_2 = 1 - \beta_2 = 1$ and, symmetrically, equal to 4.44% (58.13%) for $\alpha_1 = 1 - \beta_1 = 1$, $\alpha_2 = 1 - \beta_2 = 0.2$. Table 5 shows an illustration of the improvement in the trade volume for G1 for different values of α_1 and α_2.

The findings in Table 5 contrast with the findings in Table 4 in two important respects: First, the size of the surplus improvement is an increasing function of the degree of quality uncertainty when the latter is equal between the two goods purchased. In contrast, Table 5 indicates that the improvement in the trade volume of either good is a decreasing function of the degrees of quality uncertainty when the latter are equal between the two goods and vary inversely in relation to the $\alpha' s$.

More importantly, in terms of the magnitude of the change, the larger the difference in the degrees of quality uncertainty between two goods is, the smaller the size of the profit enhancement that the price setting firm experiences relative to no bundling. This is in stark contrast to the findings regarding the improvement in the volume of trade under bundling; this is larger, the larger is the gap in the degree of quality uncertainty.

4 Conclusions

We have shown that mixed purchase bundling in a two product market is locally optimal. For specific values of the parameters for quality certainty we determine the optimal value of the premium, prices, the surplus and trade volume. Mixed bundling allows the monopsonist to profitably exploit the producers average willingness to sell by offering the option of a purchase premium for a package sale. This is shown to be profitable, without the need for the existence of any complementarities between the goods, as the problem of the dispersion in the sellers' valuations (costs) of the two goods is reduced.

The volume of trade in goods with high quality uncertainty is increased if the transactions for such goods are packaged. This increase is higher when such a good is bundled with one with a substantially lower degree of quality uncertainty.

A deduction from our model is that if it is thought that pure bundling is trade restricting as in the case of US government procurement, then the right move is a shift towards the use of mixed bundling, rather than banishing bundling altogether.

5 Appendix

Monopsonist bundling premium values, (ϵ), for
$\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$

$\downarrow \alpha_2 \backslash \alpha_1 \longrightarrow$	0	0.2	0.4	0.6	0.8	1
0	0	0	0	0	0	0
0.2	0	0.20	0.25	0.27	0.28	0.29
0.4	0	0.25	0.31	0.35	0.36	0.37
0.6	0	0.27	0.35	0.39	0.41	0.42
0.8	0	0.28	0.36	0.41	0.44	0.45
1	0	0.29	0.37	0.42	0.45	0.47

Table 1

Prices $(p_1, \; p_2)$ charged by the monopsonist for
$\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$

$\downarrow \alpha_2 \backslash \alpha_1 \rightarrow$	0	.2	.4	.6	.8	1
0	$(0, 0)$	$(-.17, 0)$	$(-.29, 0)$	$(-.38, 0)$	$(-.44, 0)$	$(-.50, 0)$
.2	$(0, -.17)$	$(-.14, -.14)$	$(-.25, -.13)$	$(-.33, -.11)$	$(-.40, -.10)$	$(-.45, -.09)$
.4	$(0, -.29)$	$(-.13, -.25)$	$(-.22, -.22)$	$(-.30, -.20)$	$(-.36, -.18)$	$(-.42, -.17)$
.6	$(0, -.38)$	$(-.11, -.33)$	$(-.20, -.30)$	$(-.27, -.27)$	$(-.33, -.25)$	$(-.38, -.23)$
.8	$(0, -.44)$	$(-.10, -.40)$	$(-.18, -.36)$	$(-.25, -.33)$	$(-.31, -.31)$	$(-.36, -.29)$
1	$(0, -.50)$	$(-.09, -.45)$	$(-.17, -.42)$	$(-.23, -.38)$	$(-.29, -.36)$	$(-.33, -.33)$

Table 2

Monopsonist's maximum net surplus
for $\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$

$\alpha_2\backslash\alpha_1$	0	0.2	0.4	0.6	0.8	1
0	0	0.02	0.06	0.11	0.18	0.25
0.2	0.02	0.04	0.08	0.14	0.20	0.28
0.4	0.06	0.08	0.13	0.19	0.26	0.33
0.6	0.11	0.14	0.19	0.25	0.32	0.40
0.8	0.18	0.20	0.26	0.32	0.39	0.47
1	0.25	0.28	0.33	0.40	0.47	0.55

Table 3

Monopsonist's % improvement of maximum net surplus relative to no bundling for $\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$

$\alpha_2\backslash\alpha_1$	0.2	0.4	0.6	0.8	1
0.2	12.12	8.11	7.00	5.67	4.49
0.4	8.11	11.40	10.60	9.40	8.10
0.6	7.00	10.60	11.11	10.70	9.40
0.8	5.67	9.40	10.70	10.40	10.05
1	4.49	8.10	9.40	10.05	9.84

Table 4

% improvement in trade volume for G1 relative to no bundling for $\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$

$\downarrow \alpha_2\backslash\alpha_1 \longrightarrow$	0.2	0.4	0.6	0.8	1
0.2	15.14	9.38	6.78	5.31	4.44
0.4	31.25	19.27	14.61	11.46	9.58
0.6	43.03	27.62	20.82	16.78	14.08
0.8	51.46	32.79	25.27	20.86	17.59
1	58.13	36.79	28.35	23.47	20.26

Table 5

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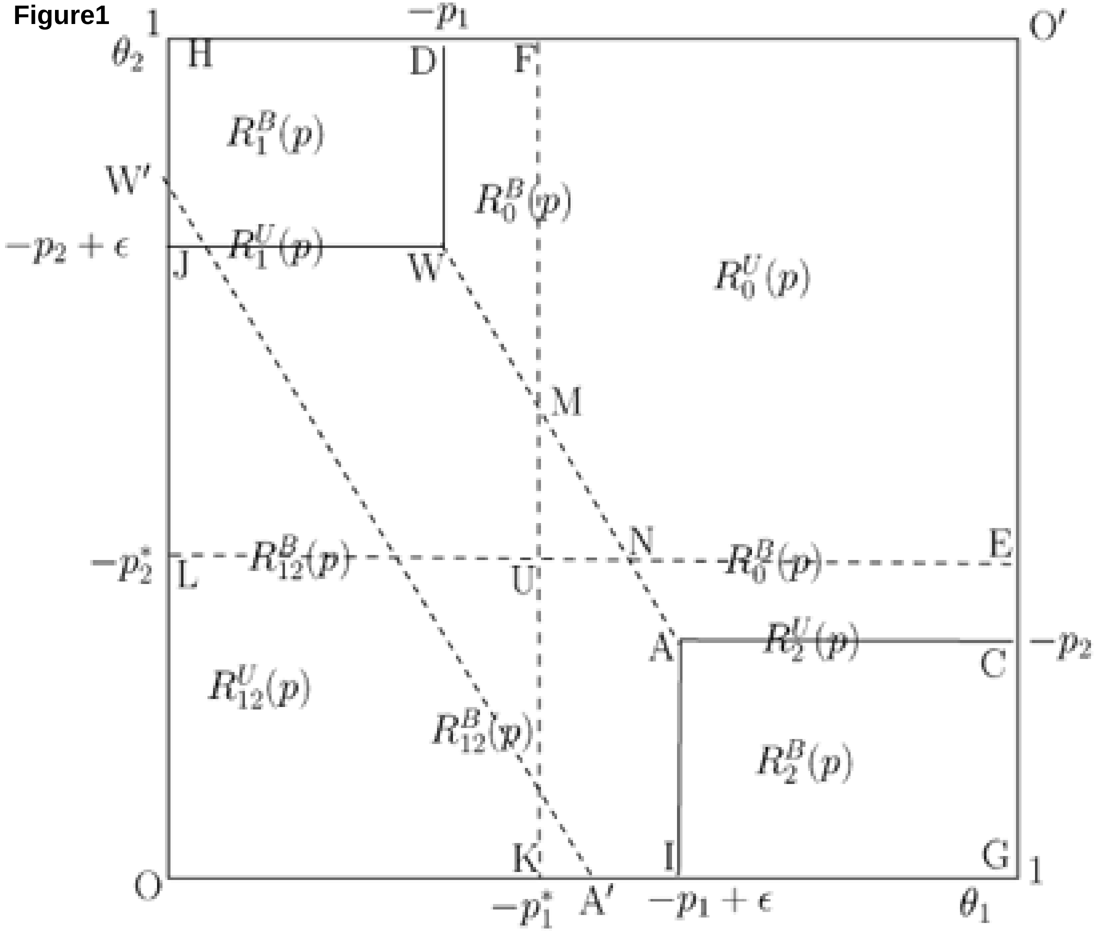
Figure1



Mixed bundling monopsony

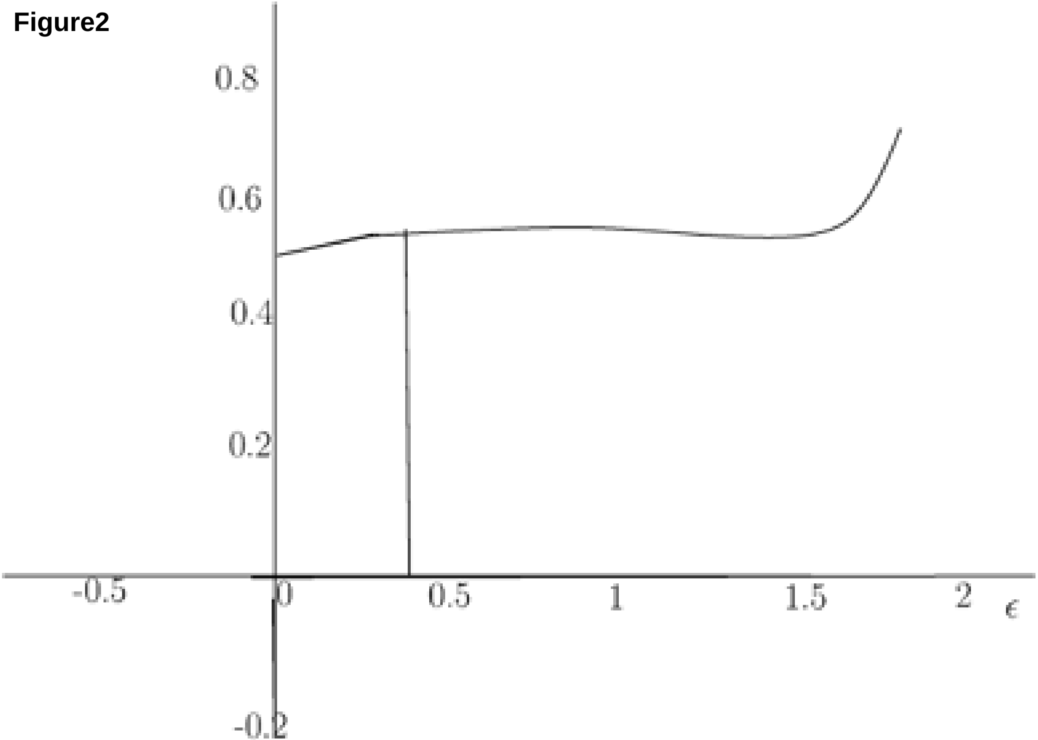
Unbundled prices: $p_1^*,\ p_2^*$; Bundled prices: $p_1,\ p_2$

A′W′: part of $\theta_1+\theta_2=-p_1-p_2$

AW: part of $\theta_1+\theta_2=-p_1-p_2+\epsilon$

The superscript U denotes unbundled and B bundled regions

Figure2



$\alpha_1 = \alpha_2 = 1,\ \beta_1 = \beta_2 = 0$

$\epsilon = 0.47,\ \Sigma = 0.55 \qquad p_1 = p_2 = -0.33$