EFFECTS OF INSIDER TRADING UNDER DIFFERENT MARKET STRUCTURES

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ABSTRACT:

In this paper, we study the relationship between the market structure in the real sector and the effects of insider trading. Specifically, we analyze two models, one in which the insider is a price-choosing monopolist in the real sector and the other in which he is a Cournot duopolist. The aim is to study the effects of different market structures in the real sector on the real and financial effects of insider trading by the manager. We find that the market structure in the real sector matters. When the monopolist insider chooses the price of the real good rather than the output, insider trading increases the price rather than the quantity. When the insider competes with another firm in the real sector, and chooses quantity, the output increases due to insider trading but by less than in monopoly models. In addition, the stock price is more informative than in monopoly models. Finally, the competition with another firm in the real sector reduces the insider's profits from financial transactions below that in monopoly models.

1. INTRODUCTION:

The debate on the effect, as well as the value, of trading on financial markets by agents who have inside information has a long history. Much of the debate has centered on two issues. The first is fairness: should an individual who has inside information about the activities of a firm be able to trade on that information at the expense of individuals without that information? The second deals with the dissemination of information. For efficient markets it is necessary that all information be disseminated and then evaluated by all agents. It is argued that the role of the insider is to help disseminate information and therefore the gain of the insider is merely a payoff to releasing this private information. Indeed, this informational effect is precisely what has been captured by Kyle (1985) in his seminal work on insider trading. However, there is another effect of insider trading, namely, the relationship between the financial decisions made by the insider and the 'real' activities of the firm. In particular, the insider's ability to make real decisions affects the financial markets just as insider trading affects the real output and the price of the good.

The question of the relationship between the real and financial effects of insider trading has been studied recently in Jain and Mirman (2000) (henceforth, JM) in the context of Kyle's model of insider trading. In Kyle's model, the insider is assumed to know the value of the firm (which is drawn from a normal distribution) but has no effect on the decisions of the firm. In JM, the insider is modelled as a manager, as well as a trader in the stock, of the firm. In this way, the manager can influence the real decisions of the firm changing the value of the firm in the financial markets, while maximizing his own profits on the financial market. This manipulation has an effect on the firm' profit and thus on its 'real' value. The market valuation of the firm is determined by a perfectly competitive market maker, who in Kyle's model sees only the order flow of the insider and the noise traders. However, in JM, the market maker also sees other sources of information, e.g. the firm's price in the real sector may be public information and contain valuable signals for pricing the stock. In JM, the insider is a quantity-setting monopolist and sets a quantity higher (and average price lower) than would a monopolist without insider trading.

Despite the fact that there is an important relationship between real and financial market variables in JM, the result that output is higher due to insider trading must be interpreted carefully. In JM, there is an important relationship between the 'real' signal (a price observation in JM) and the incentive for the firm to produce more in order to signal a lower value of the firm and therefore a lower stock price, enabling the insider to make higher profits. Indeed, it is the purpose of this paper to show that the market structure and the 'real' signal observed by the market maker are important in determining the real and financial effects of insider trading.

We refine the results obtained in JM by examining the role of the market structure in the real sector. In particular, we show that the real effect of insider trading is sensitive to the type of market structure in which the firm operates and the signals available to the market maker from the real sector. We also show that increased competition in the real sector (for example, duopoly) changes the stock pricing function significantly and makes the stock price even more informative than in the monopoly models (of Kyle and JM). Finally, the increased information release has the effect of lowering the profits of the insider compared to JM and models in which the market maker sees only the total order flow (henceforth, the Kyle-type models). The comparison with JM is interesting since it implies that the competition in real sector leads to lower profits from the financial sector, thus emphasising the informational link between the two sectors. The comparison with Kyle-type models reinforces the result in JM that the insider's inability to manipulate the signal from the real sector leads to a higher information revelation and lower profits.

In this paper, we present two models of insider trading based on two different market structures in the real sector. The purpose is to study the effect of market structure on how insider trading affects the real as well as the financial variables. In the first model, the insider is the manager of the firm whose stock he trades. As manager, he chooses the price of the real good that the firm produces, rather than the output as in JM. The firm is assumed to be monopolistic. In the financial market, the market maker sets the price of the stock based on the prior distribution of the value of the firm as well as two signals that he observes. One signal is provided by the total stock order flow, made up of the manager's order and the noise trade. The other signal is observed in the real sector, which is a noisy observation of the quantity produced by the manager. This is also in contrast to JM where the real sector signal is a noisy observation of the price of the real good.

We find that the average equilibrium output is lower and the price of the real good higher, a result opposite to JM. Thus it matters what the monopolist chooses and what the market maker sees and thus the welfare implications are to be drawn depending on the market structure and the informational structure.[3]

In the second model, the firm, managed by the insider, competes with another firm in the real sector and the insider chooses the quantity to be produced. Thus there is Cournot duopoly in the real sector. The market maker observes the total order flow and a noisy market price of the real good as in JM. Interestingly, we find that the competition in the real sector influences the equilibrium values of the variables in the financial market. Specifically, the stock pricing function is different in Cournot duopoly in the real sector compared to the monopoly models. However, the amount of insider trading remains unchanged. The informativeness of the stock price is higher under Cournot duopoly than in either Kyle or JM. Finally, profits of the insider are lower compared to JM and a Kyle-type model in which the market maker only observes the total order flow. Thus the market structure in the real sector affects the outcomes in the financial sector and the profits of the insider that are derived from trading in the financial markets. This result is similar in spirit to the results of Eaton-Mirman (1991) and Jain-Mirman (2001) in which segmented markets are related through the process of information gathering.

Most of the theoretical literature on insider trading, until recently, focusses on the financial market only. Some recent work (See Dow and Rahi (1997), Leland (1992) and Manove (1989)[4]) incorporates real as well as financial sectors in their models of insider trading. However, these papers are more interested in the issue of fairness than in the question of the relationship between the real and financial variables due to insider trading. In particular, the role of the insider in making real decisions and thus leading to an interdependence between the real and financial decisions has not been analyzed.

The rest of the paper is organized as follows: in Section 2, we present the price-choosing monopolist model of insider trading; in Section 3, we present the Cournot duopoly model; we conclude in Section 4.

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(3) It may seem that allowing the market maker to see the price of the real good will restore the results of JM. However, while a noisy price of the good is a sensible signal when the monopolist chooses output, it is no longer so when the monopolist chooses price of the good, as in this paper. This is because the choice variable is deterministic and can be inferred whereas the outcome is a result of the interaction with the random market demand and thus contains valuable information for the market maker. Indeed, one of the points made in this paper is to show that the equivalence observed in micro theory between the price- choosing monopolist and the output-choosing monopolist no longer holds when financial decisions are integrated. See section 2 for details.

(4) Ausubel (1990) also studies insider trading but without financial markets.

2. MODEL I: PRICE-CHOOSING MONOPOLIST

In this section, we analyze a model of insider trading in which the insider chooses the price of the real good, produced by the firm, and trades in the firm's stock. The market maker observes noisy output. We compare the results with the JM model in which the insider chooses output in the real sector and the market maker observes the noisy price of the good. A crucial difference emerges. In our model, insider trading by the manager leads to a higher price of the real good and thus a lower real output on average. Although in other respects, the overall flavor of the results is the same, this difference calls for caution in drawing welfare inferences from any given model. The market structure and thus the nature of the information available to the market maker is important in determining how insider trading affects real variables in the economy.

Following JM, we assume one real good and one financial asset in the economy. The real good is produced by a monopolistic firm managed by the insider and the financial asset is its stock that is publicly traded. We also assume that the cost of production is zero, for convenience. The firm is owned and managed by two different agents. The owner has no decisions to make in the model and thus does not enter the analysis explicitly.

The insider chooses the price of the real good, denoted by q. On the basis of his inside information, he also trades in the financial market and thus chooses the stock order. Thus the insider makes two decisions in this model, a real decision and a financial decision.

The demand function for the real good is given by,

$$y' = (a - bq)z,$$

where z is assumed to be the private information of the insider/manager and is normally distributed with mean $\bar{z}$ and variance σ_z^2, and y' is the quantity demanded of the real good.[(5)]

The value of the firm per share is,

$$v = (a - bq)qz - A\bar{z}, \tag{1}$$

[(5)] Due to the normality of z, y' varies from negative infinity to positive infinity. While this may seem implausible at first glance, the results, namely the equilibrium levels of the price of the real good and the average real output are consistent with the results of the output-choosing market model. The price-choosing model of this paper is a straightforward analogue of the output-choosing model in an economy where demand is stochastic. The negativity of demand must be accepted in order to work within the linear-normal paradigm of Kyle and obtain simple, intuitive solutions, linking the real and the financial sectors.

where the first term is simply the profit per share and the second term $A\bar{z}$ is the compensation of the manager per unit of stock. [6] A is a positive constant (taken as given by the insider and the market maker) to be determined endogenously to ensure the existence of equilibrium.[7] Thus the insider is rewarded for buying the stock and penalized for selling it. The effect of this scheme, intuitively, is to align the interests of the insider/manager and the owners of the firm. In the absence of this scheme, for 'small', positive values of z, the insider has an incentive to short-sell the stock and produce infinite amount of output. (See JM for more details.)

Information Structure: The insider is assumed to know the realization of z before making decisions. The market maker knows the distribution of z and observes two signals correlated with z. In the spirit of Kyle (1985), the market maker observes the total order flow η, i.e.,

$$\eta = x + u,$$

where the insider's stock order is denoted by x and the noise trade is denoted by u. Noise trade u is assumed to be independent of z and normally distributed with mean 0 and variance σ_u^2.

In addition to η, the market maker also observes a noisy signal from the real sector, in the spirit of JM. We denote this second signal by y. Specifically, y is given by,

$$y = (a - bq)(z + \epsilon),$$

where ϵ is a random variable distributed normally with mean 0 and variance σ_ϵ^2 and is independent of the random variables z and u. This signal is to be interpreted as a noisy observation of the quantity produced by the monopolist. We assume that the market maker does not observe anything else except the two signals specified above. We also assume that the insider does not observe either the signal y or the signal η, following Kyle and JM. This assumption is convenient but not necessary. (For example see Rochet and Vila, 1994.)

Setting the stock price function:

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[6] The normalization of value of the firm on a per share basis is done in the spirit of Kyle (1985). In contrast to Kyle, in our model, the value of the firm is endogenous.

[7] JM show that a compensation scheme of this form is needed to ensure the existence of a linear-normal equilibrium when the real decisions of the insider are analyzed. The same argument applies here. In the absence of the compensation scheme, the second order condition is not satisfied for all z and the equilibrium exists if and only if the mean value of the firm is 0.

The market maker sets the stock price competitively, as in Kyle. Denoting the stock price by p, the zero-expected-profit condition reduces to,

$$p = E(v/y, \eta).$$

We conjecture that the stock price is linear in both signals and then verify that the conjecture is valid. Thus let,

$$E(v/y, \eta) = p = \mu_0 + \mu_1 y + \mu_2 \eta. \tag{2}$$

The insider's net profits are,

$$\Pi = E_\epsilon E_u[(v - p)x + A\bar{z}x].$$

We assume that this is what the insider maximizes. That is, the insider is risk neutral. Substituting for v from (1) and p from (2) into the profit function, we obtain,

$$\Pi = ((a - bq)qz - A\bar{z} - \mu_0 - \mu_1(a - bq)z - \mu_2 x)x + A\bar{z}x.$$

This reduces to,

$$\Pi = ((a - bq)qz - \mu_0 - \mu_1(a - bq)z - \mu_2 x)x.$$

The insider maximizes these profits by choosing x and q, i.e. the stock trade as well as the price of the real good.

Solving for Equilibrium

We now determine the profit-maximizing stock trade and the price of the real good chosen by the insider, given the stock pricing rule (2). Then we determine the stock pricing rule and discuss the comparative statics.

The first order conditions of the insider's maximization problem are,

$$(a - 2bq + b\mu_1)xz = 0,$$

and,

$$(a - bq)qz - \mu_0 - \mu_1(a - bq)z - 2\mu_2 x = 0.$$

Simplifying yields,

$$q = \frac{a + b\mu_1}{2b}, \tag{3}$$

and,

$$x = \frac{(a - bq)(q - \mu_1)z - \mu_0}{2\mu_2}. \tag{4}$$

From the linearity of x and non-randomness of q, the following lemma follows.[(8)]

Lemma 1: The random variables v, y and η are jointly normally distributed.

Set,

$$A = \frac{(a - bq)(q - \mu_1)}{2}.$$

Thus the compensation of the manager becomes,

$$C \equiv \frac{(a - bq)(q - \mu_1)}{2}\bar{z}x.$$

It will be shown below that A is positive.

We now determine the stock pricing rule set by the market maker by solving for μ_0, μ_1 and μ_2. By Theorem 3.10 of Graybill (1961), it is straightforward to solve for these coefficients. The results are summarized in Lemma 2 below.

Lemma 2:

(i)

$$\mu_0 = 0,$$

(ii)

$$\mu_1 = \frac{a\sigma_z^2}{b(\sigma_z^2 + 4\sigma_\epsilon^2)},$$

(8) The conditions for joint normality can be easily verified.

(iii)

$$\mu_2 = \frac{a^2\sigma_\epsilon}{4b\sigma_u}\sqrt{(1-k)^3 k},$$

where

$$k = \frac{\sigma_z^2}{\sigma_z^2 + 4\sigma_\epsilon^2}.$$

Proof: See the Appendix.

Clearly μ_1 is positive and less than $\frac{a}{b}$. We select the positive root for μ_2 to satisfy the second order conditions for the insider's maximization problem. From Lemma 2, it also follows that the compensation paid to the insider per unit of the stock traded, namely, A, is positive (since μ_1 is less than $\frac{a}{b}$). Thus the insider gets rewarded for buying the stock and penalized for selling it. A is chosen in order to make μ_0 zero. This ensures that the second order conditions of the insider's maximization problem are satisfied. This condition essentially ensures that the insider does not run the firm down by choosing an infinite price of the good and short-selling the stock. In other words, making μ_0 zero has the effect of aligning the interests of the insider/manager and the owners of the firm. (See JM for details.)

Note that the change in the real decision variable from output (as in JM) to price, as here, does not change the stock pricing function at all, except that the real demand parameters a and b are parameters of the direct demand function rather than the inverse demand function. This implies that as long as the relationship between the choice variable of the monopolist and the source of information to the market maker in the real sector remains unchanged, the financial variables do not change (given monopoly in the real sector).

The next Proposition presents the linear equilibrium of the model. It is straightforward to verify that the second order conditions of the insider's maximization problem are satisfied.

Proposition 1: A linear equilibrium exists. The equilibrium is unique and is characterized by the following values of variables x, q and p,

$$q = \frac{a + b\mu_1}{2b},$$

$$x = \frac{(a - bq)(q - \mu_1)z}{2\mu_2},$$

$$p = \mu_1 y + \mu_2 \eta$$

and

$$C = \frac{(a - bq)(q - \mu_1)}{2}\bar{z}x,$$

(9)

where for

$$k = \frac{\sigma_z^2}{\sigma_z^2 + 4\sigma_\epsilon^2},$$

$$\mu_1 = \frac{ak}{b},$$

$$\mu_2 = \frac{a^2\sigma_\epsilon}{4b\sigma_u}\sqrt{(1-k)^3 k}.$$

$\mu_1 \in (0, \frac{a}{b})$ and $\mu_2 > 0$.

Discussion of the Equilibrium: It is useful to recall the results of the output-choosing monopolist model of JM. There, it is shown that real decisions and financial decisions made by the insider are interrelated. Specifically, the real output increases due to insider trading and the price of the stock varies with the real sector parameters.

Interestingly, while the general flavor of the results is the same in the price-choosing monopolist model presented here, the effect on the real price and the real output is reversed. Since μ_1 is positive and less than $\frac{a}{b}$, the price of the real good, q, is higher than it would be without the financial decisions that the insider makes. Thus insider trading by the manager has the effect of increasing the price of the real good. The intuition is that a higher price of the real good leads to a lower signal y, given the shocks z and ϵ. This leads to a lower stock price when z is positive and a higher stock price when z is negative, given the stock price function. In JM, the same intuition leads to a higher real output. This difference is due to the fact that in both models, the relationship between what the insider chooses in the real sector and what the market maker observes in the real sector is exactly the same. Thus the variable chosen by the insider in the real sector changes

(9) Note that the coefficient of C is a function of q, which in turn is a function of the exogenously given parameters of the demand function and the endogenously determined constant μ_1.

in the same direction in the two models. However, the economic effect of this is important. The analysis of a price-choosing monopolist shows that depending on what the market maker observes, and what the insider chooses, the real price or the real quantity could be higher. Thus one must be careful when drawing welfare inferences from these models.

However, interestingly, the stock pricing function and the stock price and the level of insider trading are related to the real sector variables in exactly the same way as in the model where the monopolist chooses output. Thus the effect on the financial variables of real decisions made by the insider is exactly the same regardless of the choice variable.

Except for the effects on the real output and the price of the real good (which are reversed), the comparative statics in this model are identical to those in JM and thus we omit that discussion. However, it is worth noting that the stock price in the price-choosing monopolist model presented here continues to be more informative than in Kyle (1985) and Rochet and Vila (1994). Specifically, the variance of the value of the firm conditional on the two signals observed by the market maker is,

$$\mathrm{Var}(v/q,\eta) = \frac{(a-b\mu_1)\sigma_v^2}{2(a+b\mu_1)}.$$

Note that the coefficient of σ_v^2 is less than half. Now, substituting for μ_1 from Lemma 2, we get,

$$\mathrm{Var}(v/q,\eta) = \frac{\sigma_\epsilon^2}{\sigma_z^2+2\sigma_\epsilon^2}\sigma_v^2.$$

Note that the fraction of the unexplained variance varies negatively with the underlying variability σ_z^2 and positively with the variability σ_ϵ^2 in contrast to the Kyle model where the amount of information revealed is exactly half of the information possessed by the insider.

Finally, the profits of the insider are lower than profits in Kyle-type models (models in which the market maker sees only the total order flow). Specifically, the insider's expected profits in equilibrium equal,

$$\Pi' = \frac{((a-by)(y-\mu_1)z)^2}{4\mu_2}.$$

Substituting for q, μ_1 and μ_2 from Proposition 1, into Π', we obtain,

$$\Pi' = \frac{2(az\sigma_\epsilon^2)^2\sigma_u}{b\sigma_z(\sigma_z^2+4\sigma_\epsilon^2)^2}. \tag{5}$$

Thus as in JM, the inability of the insider to manipulate the real sector signal prevents him from offseting the negative effect of the greater information release on his profits.

In the next section, we discuss Cournot duopoly in the real sector and examine its effects on the real and financial variables when the manager of one firm trades in its stock. We show that competition with another firm in the real sector not only affects the real decisions but also the financial decisions.

3. COURNOT DUOPOLY IN THE REAL SECTOR

In this section, we introduce another firm that competes with the insider in the real sector. For the sake of convenience, let the insider-managed firm be firm 1 and the competitor be firm 2. For simplicity, firm 2 is assumed to be a standard neoclassical firm (privately held) with its financial decisions ignored. The aim is to examine the effect of competition in the real sector on how insider trading by the manager of firm 1 affects the real price and quantity produced and the stock price, stock pricing rule and its informativeness. We also want to examine the effect on the insider's profits.

We assume that firms 1 and 2 compete in the real sector by choosing output. Thus the insider now chooses real output to be produced by firm 1 as well as the amount of the stock of firm 1 that he wants to buy or sell, based on his private informtion about firm 1's value.

Denoting firm i's output by y_i, the inverse demand function for the real good can be written as:[10]

$$q' = (a - b(y_1 + y_2))z. \tag{6}$$

where z is a normally distributed variable with mean $\bar{z}$ (assumed to be strictly positive) and variance σ_z^2 as in the price-choosing monopolist model above.

[10] For convenience, we continue to use symbols a and b to denote the intercept and the slope of the demand function in the monopoly and duopoly models. Other notation is also kept the same wherever possible.

Firm 2 maximizes its expected profits by choosing y_2. The expected profits are[11]

$$\Pi_2 = (a - b(y_1 + y_2))y_2\bar{z}.$$

While firm 2 maximizes its profits by choosing how much to produce, the manager of firm 1, that is, the insider, maximizes his profits by choosing how much to produce and how much stock to trade. The insider's profits therefore are

$$\Pi = E_u E_\epsilon((v - p)x) + B\bar{z}x.$$

where

$$v = (a - b(y_1 + y_2))y_1 z - B\bar{z}, \tag{7}$$

and $B\bar{z}$ is the manager's compensation received from the firm's owner per unit of the stock traded, B being the anlagoue of A from the monopoly model of the previous section.[12]

In the financial market, the stock of firm 1, managed by the insider, is traded. The price of this stock, p is set by a market maker conditional on his information and is taken as given by the insider. The structure of the financial market is the same as assumed in the monopoly models. That is, the market maker sets the stock price given his information, to make zero expected profits.

Information Structure: We assume that z is assumed to be privately known to the manager of firm 1. The market maker and the rival firm only know the distribution of z. Further, the market maker gets his information from observing the total order flow made up as before of the insider's stock order and of the noise trade. We denote this by η. However, instead of the noisy observation of output produced, as in the monopoly model of section 2, we assume that the market maker sees a noisy observation, denoted by $\bar{q}$[13] of the price of the real good, q'.

The signal $\bar{q}$ is defined as follows:

[11] If $\bar{z} = 0$, firm 2's profits are identically zero and the model reduces to the monopoly model.

[12] It is easy to verify that this form of the compensation scheme for the manager/insider continues to be indispensable in order to ensure the existence of an equilibrium. The scheme ensures that a second order condition for the existence of maximum is satisfied.

[13] Thus this model differs from the output-choosing monopoly model of JM only in one respect, namely the market structure in the real sector. In particular, there is no difference in the types of signals available to the market maker. However, we will see that the difference in market structure has significant implications for informational efficiency of the stock price and the profits of the insider.

$$\bar{q} = (a - b(y_1 + y_2))(z + \epsilon), \tag{8}$$

where ϵ is normally distriubted with mean 0 and variance σ_ϵ^2 and is independent of z and u. We assume that the insider does not see ϵ or $z + \epsilon$.

Setting the Stock Price Function:

The zero profit condition for the market maker then reduces to

$$p = E(v/\bar{q}, \eta).$$

We again look for a linear equilibrium and conjecture that the stock price p is set to be linear in the signals:

$$p = E(v/\bar{q}, \eta) = \mu_0 + \mu_1 \bar{q} + \mu_2 \eta. \tag{9}$$

Substituting for v from (7) and p from (9), in the insider's profit function, we can write down his maximization problem as follows:

Maximize

$$\Pi_1 = ((a - b(y_1 + y_2))y_1 z - B\bar{z} - \mu_0 + \mu_1(a - b(y_1 + y_2))z - \mu_2 x)x + B\bar{z}x$$

by choosing y_1 and x.

Solving for Equilibrium

The first order condition for firm 2's maximization problem is:

$$\bar{z}(a - by_1 - 2by_2) = 0.$$

This yields

$$y_2 = \frac{a - by_1}{2b}. \tag{10}$$

The first order conditions for the insider's maximization problem are:

$$zx(a - by_2 - 2by_1 + \mu_1 b) = 0$$

and

$$(a - b(y_1 + y_2))(y_1 - \mu_1)z - \mu_0 - 2\mu_2 x = 0.$$

Simplifying leads to

$$y_1 = \frac{a - by_2 + \mu_1 b}{2b} \tag{11}$$

and

$$x = \frac{(a - b(y_1 + y_2))(y_1 - \mu_1)z - \mu_0}{2\mu_2}. \tag{12}$$

Substituting for y_2 from (10) into (11), we obtain

$$y_1 = \frac{a + 2\mu_1 b}{3b}. \tag{13}$$

Substituting this back into (10), we obtain

$$y_2 = \frac{a - \mu_1 b}{3b}. \tag{14}$$

Note that y_1 and y_2 are deterministic and therefore x is linear in z. Thus we have the following counterpart of lemma 1.

Lemma 1′: The random variables v, q and η are jointly normally distributed.

Also note that the total output produced by the two firms equals

$$Y \equiv y_1 + y_2 = \frac{2a + b\mu_1}{3b}. \tag{15}$$

Next we need to solve for the stock pricing function. Using probability theory (Theorem 3.10 of Graybill (1961), as in section 2), we can easily solve for the coefficients μ_0, μ_1 and μ_2. First note that setting the compensation scheme coefficient $B = \frac{(a-b(y_1+y_2))(y_1-\mu_1)}{2}$ yields $\mu_0 = 0$. This is necessary for the existence of equilibrium (as in section 2) since one of the second order conditions for the insider's maximization problem requires that

$$zx > 0.$$

Substituting $\mu_0 = 0$ into (12) and then substituting for x in the above second order condition yields

$$z\frac{(a - b(y_1 + y_2))(y_1 - \mu_1)z}{2\mu_2} > 0. \tag{16}$$

We will show that μ_1 and μ_2 are such that this second order condition is satisfied.

The following lemma (the counterpart of Lemma 2 of section 2) presents the solution for the stock price function:

Lemma $2'$:

(i)

$$\mu_1 = \frac{a\sigma_z^2}{b(\sigma_z^2 + 3\sigma_\epsilon^2)}$$

(ii)

$$\mu_2 = \frac{a^2\sigma_\epsilon}{3\sqrt{3}b\sigma_u}\sqrt{(1-k)^3 k}$$

where

$$k = \frac{\sigma_z^2}{\sigma_z^2 + 3\sigma_\epsilon^2}.$$

The proof is similar to that of Lemma 2 (in the appendix) and is thus omitted.

Note that $\mu_1 \in (0, \frac{a}{b})$ and $\mu_2 > 0$ and thus (16) is satisfied. Thus the same type of compensation scheme that we used in the monopolistic models is sufficient in the duopoly model as well to ensure the existence of an equilibrium. The next proposition presents the unique linear equilibrium of the model.

Proposition 2: A linear equilibrium exists. The equilibrium is unique and is characterized by the following values of variables x, y_1, y_2 and p:

$$y_1 = \frac{a + 2b\mu_1}{3b}$$

$$y_2 = \frac{a - b\mu_1}{3b}$$

$$x = \frac{(a - b(y_1 + y_2))(y_1 - \mu_1)z}{2\mu_2}$$

$$p = \mu_1 q + \mu_2 \eta$$

where for

$$k = \frac{\sigma_z^2}{\sigma_z^2 + 3\sigma_\epsilon^2},$$

$$\mu_1 = \frac{ak}{b}$$

and

$$\mu_2 = \frac{a^2 \sigma_\epsilon}{3\sqrt{3} b \sigma_u} \sqrt{(1-k)^3 k}$$

Corollary 1: For m denoting the monopoly model of JM,

(i) $\mu_1 > \mu_1^m$ and $\mu_2 < \mu_2^m$,

(ii) $y_1 < y$,

(iii) $Y > y$ and

(iv) $x = x^m$.

Proof: Recall (from section 2, Proposition 1) that[14]

$$\mu_1^m = \frac{a\sigma_z^2}{b(\sigma_z^2 + 4\sigma_\epsilon^2)}.$$

Comparing this value with μ_1 given by Proposition 2, it follows that $\mu_1 < \mu_1^m$. A similar comparison of μ_2^m from section 2, Proposition 1, results in (i).

For (ii), we reproduce the JM result:

$$y = \frac{a + b\mu_1^m}{2b}.$$

Comparing y_1 given by Proposition 2 and using (i) yields (ii). Similarly, comparing Y given by (15) with the expression for y above, and using (i) yields (iii). Finally, for (iv), substituting for y_1, Y, μ_1 and μ_2 from Proposition 2 into the expression for x given in the same proposition yields

[14] This is the same value as in JM, except that the coefficients a and b are the parameters of the direct demand function. For the purpose of comparison with Cournot duopoly outcomes, the coefficients are to be interpreted as the parameters of the inverse demand function.

$$x = \frac{\sigma_u z}{\sigma_z}, \tag{17}$$

which is the same level as obtained in JM as well as in the monopoly model of section 2. ∎

Corollary 2: The output increase in Cournot duopoly due to insider trading by one firm is less than the output increase in the output-choosing monopoly model of JM.

Proof: The increased output in the monopoly model of JM due to insider trading equals (see the expression for y in corollary 1),

$$\frac{b\mu_1^m}{2b}.$$

The increase in total output under Cournot duopoly is (see (15)),

$$\frac{b\mu_1}{3b}.$$

The result follows from Corollary 1(i). ∎

Discussion of the Equilibrium: First of all, note that the total output produced by the two firms is more than what is produced in a Cournot duopoly without insider trading. This is a similar result to what we obtained in the output-choosing monopoly model (and different from the monopoly model of section 2 for reasons discussed in that section). However the increase in output is less than the increase in output when there is monopoly in the real sector. The intuition is that the insider is less able to influence the market price of the real good (and thus the signal available to the market maker) by increasing his firm's output. For each added unit to the output, firm 2 reduces its output, resulting in a relatively less lower price than in the output-choosing monopoly model.

Secondly, note that the stock price function is affected by competition in the real sector, even though the structure of the financial market continues to be the same as in the monopoly models. Specifically, the response of the market maker to the signal from the real sector (that is μ_1) is higher and the response to the total order flow signal (that is μ_2) is lower in Cournot duopoly (see

Corollary 1(i)) than in JM. This result is in contrast to the monopoly model of section 2, where the financial variables remained completely unchanged despite the change of the choice variable. The change in the stock price function reinforces the interrelationship between the real sector and the financial sector, as establihed in JM.

Third, the level of insider trading remains the same. Intuitively, this is due to the fact that the effect of increased output (as a result of an additional firm in the real sector) on the two signals is 'predictable' for the market maker (since the outputs are deterministic) and thus is correctly incorporated in the stock price function (through lower weights on the two signals). Thus the insider's optimal response remains unchanged.

The general properties of the stock pricing function are the same as in the output choosing monopolist model of JM. For instance, the stock pricing coefficients for the two signals are positive. The comparative statics of the Cournot duopoly model with respect to the real demand parameters and the variances of the random variables z, ϵ and u, are also very similar. However, there is a significant respect in which the duopoly model differs from the monopoly models. The informativeness of the stock price is greater in the duopoly model. We also show that the profits of the insider are lower in the duopoly model than in JM, a result that emphasises the informational link between the otherwise segmented markets. Finally, we confirm the JM result that the profits of the insider are lower in the duopoly model with insider trading than in a Kyle-type model where only the total order flow is observed by the market maker. These results are discussed below.

Informativeness of Stock Price: We show below that the stock price reveals more information when there is Cournot duopoly in the real sector than when there is monopoly in the real sector. In JM, we showed that even in the monopoly model, adding another signal from the real sector for the market maker had the effect of increasing information revelation more than in the Kyle-type models (See for example Kyle (1985) and Rochet-Vila (1994), where the information revealed is exactly half of what the insider knows.). The competition in the real sector makes the stock price even more informative. Further, as in JM, the amount of information revealed continues to vary with some of the underlying parameters, in contrast to Kyle, for instance. This reflects the inability of the insider to manipulate the signal from the real sector, in contrast to the total order flow.

A measure of informativeness when we deal with multivariate normal distributions is the conditional variance of the value of the firm given the information of the market maker. The lower the conditional variance, the higher the information content of the stock price. In our set-up, this can be written down as: (See Graybill (1961).)

$$\mathrm{Var}(v/q,\eta) = \sigma_v^2 - \mu_1\sigma_{vq} - \mu_2\sigma_{v\eta}.$$

This reduces to

$$\frac{(a-b\mu_1)\sigma_v^2}{2(a+2b\mu_1)}. \tag{18}$$

Note that the coefficient of σ_v^2 is less than half. Thus the stock price here reveals more information than in Kyle (1985) and Rochet and Vila (1994). We show below that the information revelation is even greater than in the monopoly models of insider trading with a real sector (as in JM and the model of section 2).

Proposition 3: The stock price reveals more information when there is Cournot duopoly in the real sector than when there is monopoly in the real sector.

Proof: Recall that the conditional variance of value of the firm in the monopoly model of JM or equivalently in the monopoly model of section 2,[15] equals

$$\frac{(a-b\mu_1^m)\sigma_v^{2m}}{2(a+b\mu_1^m)}$$

where, as earlier, the superscript m denotes monopoly. Comparing the coefficient of σ_v^{2m} in this expression with the coefficient of σ_v^2 in (18), gives us the result.[16] ∎

Thus competition in the real sector makes the stock price more revealing. This is intuitive. The insider does not have complete control over the choice of real output and the price of the real good. Thus he is not able to manipulate the signal $\bar{q}$ as effectively as in the monopoly models of JM and of section 2.

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[15] Since the informativeness of the stock price is equal in the price-choosing monopoly model presented in section 2 and in the output-choosing monopoly model of JM, we confine the reference to the JM model.

[16] Obviously, the value of the firm as well as the value of the coefficient μ_1 are different in the two models. But that is irrelevant for the proposition. We are interested in the fraction of variance explained by the stock price.

Also note in (18) that the coefficient of σ_v^2 depends on μ_1 which in turn depends negatively on σ_ϵ^2 and positively on σ_z^2 (see Proposition 2). This implies that the amount of information revealed varies with σ_ϵ^2 and σ_z^2, a result similar to JM and in contrast to Kyle.

Insider's Profits:

Now we present two properties of the insider's profits. First, we show that the insider's profits under the Cournot duopoly model are less than those under the monopoly model of JM. Note that this is different from the standard result of lower Cournot duopoly profits compared to monopoly profits. In our context, the comparison is not in the market where the market structure differs. Instead, we are comparing the profits of the insider, which are purely financial, under two different market structures in the real sector. Thus our result is that even when the structure of the financial market continues to be the same, a different market structure in another good has an effect on the outcome of the financial trading by the insider. This result is similar in spirit to other results that show that even when markets are segmented, if one firm operates in all of those markets, the outcomes are interrelated. (See Jain and Mirman (1999) and Eaton and Mirman (1991).)

Then we show that the insider's profits under Cournot duopoly as obtained in our model (with two correlated signals of value for the market maker) are less than the insider's profits under Cournot duopoly but with only the total order flow as the signal for the market maker. This is similar to what we showed in JM. Thus regardless of the informativeness of the signal from the real sector for the market maker, the profits of the insider are less than in a Kyle-type model with only one signal (namely, the total order flow). As mentioned earlier, this is due to the insider's inability to manipulate the signal from the real sector and thus a greater release of information through the stock price.

In order to prove these two properties, we first calculate the profits of the insider under Cournot duopoly (with two signals for the market maker) in equilibrium. Substituting for y_1, y_2, x and μ_0 from Proposition 2 in the insider's profit function, we obtain

$$\Pi = \frac{(a - b\mu_1)^4 z^2}{324 b^2 \mu_2}.$$

Now substituting for μ_1 and μ_2, these profits become

$$\Pi = \frac{a^2 \sigma_u (\sigma_\epsilon^2)^2 z^2}{3b\sigma_z(\sigma_z^2 + 3\sigma_\epsilon^2)^2}.$$

Denote the insider's profits in the monopoly equilibrium of JM and of section 2, by Π_m. Then

$$\Pi_m = \frac{2a^2 \sigma_u (\sigma_\epsilon^2)^2 z^2}{b\sigma_z(\sigma_z^2 + 4\sigma_\epsilon^2)^2}.$$

A comparison of Π and Π_m yields the following result:

Proposition 4: $\Pi < \Pi_m$.

Next, we compute the profits of the insider in a modified version of our Cournot duopoly model. We assume that now the market maker only observes the total order flow. Everything else remains unchanged in the model. We consider this a Kyle-type model since only one source of information is allowed to the market maker and thus there is no effect of financial trading on the real decisions.

Firm 2's problem continues to be the same as earlier in the paper. Also, the insider still maximizes the following profit function:

$$\Pi = E[(v - p)x + B'\bar{z}x].$$

where $v = (a - b(y_1 + y_2))y_1 z - B'\bar{z}$. As in our main model, a compensation scheme is needed to ensure the existence of a linear equilibrium.(17) This model is the same as the duopoly model presented in this paper. However, the stock price p is now set as follows:

$$p = E(v/\eta) = \mu_o + \mu_2\eta.$$

assuming linearity.

We skip the details of the analysis since they are very similar to Kyle (1985) and simply report the profits (denoted by Π_k) in equilibrium:

(17) Even when the price is not a source of information, our compensation scheme is required for the existence of an equilibrium.

$$\Pi_k = \frac{a^2 z^2 \sigma_u}{18 b \sigma_z}.$$

Comparing it with Π yields the following proposition:

Proposition 5: $\Pi < \Pi_k$

Thus as one would expect, the profits of the insider continue to be lower than when the market maker only observes the total order flow. That is, a higher information revelation, as shown in Proposition 3 (since the information revealed in the benchmark Kyle-type model is also exactly half), leads to lower profits under Cournot duopoly in the real sector, a result similar to the monopoly models.

4. CONCLUSION:

In this paper, we have shown that the market structure in the real sector matters in how insider trading affects the informativeness of the stock price as well as the real variables. Market structure in the real sector also has significant effects on the formation of stock prices and the level of insider profits. These effects are important ingredients in determining the social desirability of insider trading.

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APPENDIX

Proof of Lemma 2

We use Theorem 3.10, Graybill to determine the coefficients μ_0, μ_1 and μ_2.

First, note that,

$$\mu_0 = \bar{v} - \mu_1 \bar{y} - \mu_2(\bar{x} + 0).$$

Substituting for v from (1), $y = (a - bq)(z + \epsilon)$, $A = \frac{(a-bq)(q-\mu_1)}{2}$ and x from (4), we obtain

$$\mu_0 = 0.$$

The coefficients μ_1 and μ_2 are given by the following equation:

$$(\mu_1 \qquad \mu_2) = (\sigma_{vy} \qquad \sigma_{v\eta}) \begin{pmatrix} \sigma_y^2 & \sigma_{y\eta} \\ \sigma_{y\eta} & \sigma_\eta^2 \end{pmatrix}^{-1}.$$

Multiplying the matrices yields,

$$\mu_1 = \frac{\sigma_{vy}\sigma_\eta^2 - \sigma_{v\eta}\sigma_{y\eta}}{\sigma_y^2\sigma_\eta^2 - (\sigma_{y\eta})^2},$$

$$\mu_2 = \frac{\sigma_{v\eta}\sigma_y^2 - \sigma_{vy}\sigma_{y\eta}}{\sigma_y^2\sigma_\eta^2 - (\sigma_{y\eta})^2}.$$

Substituting for the various variances and covariances, we obtain the following two equations:

$$\mu_1 = \frac{(a - bq)^2 q \sigma_z^2 \sigma_u^2}{D}, \tag{19}$$

$$\mu_2 = \frac{(a - bq)^4 q (q - \mu_1) \sigma_z^2 \sigma_\epsilon^2}{2\mu_2 D}, \tag{20}$$

where

$$D = \mathrm{Det} \begin{pmatrix} \sigma_y^2 & \sigma_{y\eta} \\ \sigma_{y\eta} & \sigma_\eta^2 \end{pmatrix}.$$

Substituting for the variances and covariances in this matrix and simplifying the expression for the determinant yields,

$$D = (a-bq)^2(\sigma_u^2(\sigma_z^2+\sigma_\epsilon^2)) + \frac{(a-bq)^4(q-\mu_1)^2\sigma_z^2\sigma_\epsilon^2}{4\mu_2^2}. \tag{21}$$

Simplifying (19) and (20) yields,

$$2\mu_2^2 = \frac{(a-bq)^2(q-\mu_1)\sigma_\epsilon^2\mu_1}{\sigma_u^2}. \tag{22}$$

Substituting for D from (21) into (20) and then simplifying, we obtain,

$$2\mu_2^2 = \frac{(a-bq)^2 q(q-\mu_1)\sigma_\epsilon^2\sigma_z^2}{\sigma_z^2\sigma_u^2 + \sigma_\epsilon^2(\frac{(a-bq)^2(q-\mu_1)^2\sigma_z^2}{4\mu_2^2} + \sigma_u^2)}.$$

Simplifying further yields,

$$2\mu_2^2 = \frac{(a-bq)^2(q-\mu_1)\sigma_\epsilon^2\sigma_z^2(q+\mu_1)}{2\sigma_u^2(\sigma_z^2+\sigma_\epsilon^2)}. \tag{23}$$

Solving (22) and (23), we get,

$$2(\sigma_z^2+\sigma_\epsilon^2)\mu_1 = \sigma_z^2(q+\mu_1).$$

Substituting for y and simplifying further to solve for μ_1, we obtain,

$$\mu_1 = \frac{a\sigma_z^2}{b(\sigma_z^2+4\sigma_\epsilon^2)}.$$

Substituting this back into (22) and taking the positive square root, we get,

$$\mu_2 = \frac{a^2\sigma_\epsilon}{4b\sigma_u}\sqrt{(1-k)^3k},$$

where

$$k = \frac{\sigma_z^2}{\sigma_z^2+4\sigma_\epsilon^2}.$$