A STUDY OF COMPETITIVE BIDDING

with particular reference to

THE CONSTRUCTION INDUSTRY

JOHN DEAN WHITTAKER, B.Sc., S. M.

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Abstract

This thesis describes an operational research study applying decision theory and quantitative methods to the problems of competitive bidding. The study was provided with data and information by four English building construction companies. First a preliminary feasibility study was conducted which indicated that the potential for substantial benefits exists. Then the decision problem was formulated in a quantitative manner which allows treatment of the variation due to estimating uncertainty, and of the constraining effect of resources. The Friedman model and some of the published variants were presented and discussed. This led to the development of a General Distribution decision model which incorporates managerial assessment of the competition into a probabilistic framework. This Model, four Friedman variants, and a feedback model were tested with data supplied by the participating companies. The sample was too small for the results to be conclusive but they did indicate that the basic Friedman Model and the General Distribution Model can equal or outperform actual company behaviour. Partial implementation of the General Distribution Model indicated that it may be practicable.

ACKNOWLEDGMENTS

This is to acknowledge and express appreciation to the many individuals who assisted me in this research.
Especially to:

Dr. P. H. Grinyer, who taught me the difference between a specious argument and a valid one;

Mr. A.H. Russell, who taught me that mathematics was more than just symbol manipulation;

Mr. Brian Fine, who always had another alternative.

The Managements of the Sample Companies, who provided me with a sympathetic hearing and access to their confidential files.

Without the help and co-operation of these people, this study would not have been possible.

List of Major Symbols

Except where specified otherwise, the following notation is used.

A_i	-	the arithmetic mean of bids submitted on contract i.
a_{1i}	-	the present value of dispersements associated with contract i.
a_{2i}	-	the present value of the income stream associated with contract i.
b_m	-	the regression coefficients used in Section 3.4.1.
C_{jh}	-	the amount of resource h available in time period j.
d_{ij}	-	the amount of money dispersed on contract i in the j th period.
e_i	-	the variation in value of contract i due to estimate uncertainty.
E( )	-	the expected value function.
F( )	-	the cumulative distribution function of f( ).
G( )	-	the complementary cumulative distribution function of f( ).
i	-	a subscript identifying the contract.

k - the estimated cost.

k' - the corrected estimated cost = $Z(R, n)k$

k_i - the estimated cost of contract i.

K - the vector of estimated costs, k_i

l - the cost of preparing and submitting a bid - a negative quantity.

l_i - the cost of preparing and submitting a bid on contract i - a negative quantity.

L - the vector of costs, l_i

n - the number of competitors on a contract.

N - the number of bidders on a contract. (Also used in Chapter 4 as the number of contracts in the set.)

P - the probability of winning a contract.

$P_i(x_i)$ - the probability of winning contract i with a bid of x_i

q - (Chapter 5) the number of bids in the histogram.

q_{ij} - (Chapter 2) the amount of money received from contract i at the end of the j th period.

r - discounting rate.

R - (no subscript) - the range of estimating accuracy.

R_{ijh} - (Chapter 4) the amount of resource h required by contract i in time period j.

S - the set of opportunities to bid.

t - the length of time period j.

u_i - (Chapter 2) the variation in value due to contract uncertainty.

u_{jh} - (Chapter 4) Kuhn-Tucker multiplier.

V - value.

$V_i(x_i)$ - the value of contract i if it is won with a bid of x_i

w_i - (Chapter 4) Kuhn-Tucker multiplier.

x - bid value.

x_i - the bid value on contract i.

X - the vector of bid values, x_i

y - lowest competitor's bid.

y_i - the lowest competitor's bid on contract i. (Also used in Section 3.4.2 and Appendix 3 as order statistics.)

Y - vector of lowest competitor's bid, y_i

$Z(R, n)$ - the multiplier that corrects the expected error in the estimated cost.

γ - (Chapter 4) the expected remaining resources.

ϕ - a multiplier to convert an estimated cost into a bid.

θ_i - the managerial assessment of the arithmetic mean of the competitors' bids on contract i.

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CHAPTER 1: INTRODUCTION

The 1956 paper of L. Friedman, "A Competitive Bidding Strategy" (11)*, presented the first published probabilistic approach to competitive bidding. Since that time, a considerable volume of operational research literature has dealt with the problems of competitive bidding. This literature has produced a feast of decision models, but a famine of experimental verifications and reported applications.

Competitive bidding is used extensively in the construction industry.

This thesis is an operational research study of competitive bidding in the construction industry.

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* The numbers in brackets refer to similarly numbered entries in the Bibliography.

1.1 PROBLEMS

The general "Management Problem" to which this research is addressed is:

<u>Within the prescribed boundaries, what are the optimal decisions relative to the management objectives?</u>

In other words, which of the alternative choices available should the decision maker select? This problem is called the "Competitive Bidding Problem".

For this study, the Management Problem poses two questions. One is determining the relevant decisions in the situation; the other is determining how to make these decisions. The first question is concerned with the formulation of a general model; the second with the suitability of specific decision models.

1.2 OBJECTIVES

The objectives of this thesis are:

i. To formulate the construction industry Competitive Bidding (Management) Problem in a precise manner.

ii. To devise a method of incorporating managerial judgment into a quantitative decision model.

iii. To develop a quantitative model of the Competitive Bidding Problem so that a mathematical optimum can be calculated.

iv. To evaluate empirically six operational research decision models - four taken from the literature, two developed in the thesis.

1.3 STRUCTURE

Research of this type does not follow in an orderly hypothesis-experiment-conclusion sequence but rather is characterised by cycles with the experiments and conclusions causing redefinition of the hypothesis. The several different objectives also contribute to the disorder. To provide a structure for the logical exposition of the research this thesis is divided into six chapters. Chapter 1 is this Introduction; Chapter 6 is the Summary and Conclusions; Chapters 2, 3, 4, and 5 comprise the body.

The central chapters each commence with an introductory section and conclude with a summary. These sections provide the transition between the chapters and outline the relevance of each to the general problems and objectives.

Chapter 1, this Introduction, states the general problem, delineates the objectives, and outlines the structure of the thesis.

Chapter 2 is the Problem Description where a precise account of the Competitive Bidding Problem is presented.

Chapter 3 deals with methods of accommodating the lowest bid made by competitors. This value is the uncontrollable variable in the decision situation. Several existing decision models are analysed and a new model is developed.

Chapter 4 is a mathematical formulation of the Competitive Bidding Problem. A model for the N contract, resource constrained, sequential bidding case is developed and a solution method proposed.

Chapter 5 investigates the suitability of specific models. Six decision models are tested with historical data and the results evaluated. Partial implementation is also used to evaluate the models.

Chapter 6 summarises the findings of the research and presents the conclusions.

CHAPTER 2: PROBLEM DESCRIPTION

2.1 INTRODUCTION

The purpose of this chapter is to provide a reason and conceptual basis for the research. The title of the research is general - "A Study of Competitive Bidding with particular reference to the Construction Industry" - whereas the research is specific - a study of operational research bidding models based upon information from three construction firms. This chapter provides the transition from the general to the specific.

The Management Problem was stated generally as: Within the prescribed boundaries, what are the optimal decisions relative to the management objectives. This chapter makes this general statement specific by delineating the boundaries, isolating the decisions, and quantifying the objectives.

In addition to the two questions mentioned in Chapter 1, there is a third question - Should the research be done? Since there has been considerable research done on competitive bidding, the possibility that all reasonable approaches have been investigated must be considered. This question is dealt with in section 2.4, Feasibility Study.

The methods employed are varied to suit the subject matter. A descriptive exposition of the industry is used to qualitatively define the boundaries. A schematic model is developed from a description of the decision process. Empirical testing and interviews were used to determine feasibility, and a quantitative objective function is developed from an analysis of the decision variables.

This Chapter provides the foundation - boundaries; decisions; objectives; and feasibility - upon which the research is based.

2.2 STUDY AREA

2.2.1 Construction Industry

The description of the Study Area is a progression from the general to the specific. The starting point therefore is a general definition of the construction industry. This is provided by Standard Industrial Classification Order XVII which covers:

Erecting and repairing bridges of all types. Constructing and repairing roads and bridges; erecting steel and reinforced concrete structures, concrete, other civil engineering works such as laying sewers and gas mains, erecting overhead line supports and aerial masts, open cast coal mining, etc. The building and civil engineering establishments of Defence and other Government Departments are included. Establishments specialising in demolition work or in sections of construction work such as asphalting, electric wiring, flooring, glazing, painting, plastering, plumbing, roofing. The hiring of contractors plant and scaffolding are included.

There are many excellent references (see Bibliography) describing the myriad features of the construction industry and there is little point in duplicating these works or reproducing the many statistics which are available. The following are included only to provide an appreciation of the size and importance of the industry.

- The value of the industry output in the United Kingdom is approximately one-eighth of the Gross National Product. The 1964 figure for total value was £ 3, 614, 000, 000. (6)

- The construction industry of the United Kingdom contains over 80, 000 firms. (6)

- The construction industry employs approximately six percent of the working population. (38)

From these it can be seen that the construction industry is of national importance and is comprised of many firms.

Firms in the construction industry are characterised by diversity and versatility and these factors complicate any attempt at classification. The use of any one base for classification produces anomalies. For example, ranking firms on the basis of assets, or job size, or number of employees, will probably result in three different ranking orders since it is not uncommon for the low assets, one man firm using sub-contractors to be competing for the same job as the larger firm which has several hundred employees. Divisions such as: Civil Engineering; Building; Speciality; and Maintenance, also get confused because the largest firms accommodate several or all of these types of work and several jobs combine them. An example is a power station - Building and Electrical - with extensive site works - Civil Engineering. The movement between classifications also occurs, for example the speciality contractor who becomes a general contractor for one project. Obviously, then any classification or description of a segment of the industry must be a general one based upon an understanding of the norms and not attempting precise identification.

The research is based upon one segment of the construction industry. To delineate the boundaries of this segment all small contractors, specialised and speciality contractors, maintenance contractors, etc., are excluded, and the remaining firms are classified using two characteristics - <u>contractual risk</u> and <u>resource constraints</u>.

A distinction is made in this thesis between contractual risk and uncertainty. Contractual risk relates to the occurrence of low probability events of major consequence. These events usually result in substantial cost increases which must be borne by the contractor. For example: on bridge sub-structure construction the once-in-one-hundred-year flood which swamps the machinery and destroys the formwork is contractual risk;

in tunnelling the unexpected quicksand pocket is contractual risk. Uncertainty, as defined here, relates to the consequence of more probable situations. For example: on some firm price contracts it can be assumed that over the duration of the contract the costs of labour and materials will rise. What is not known is when and by how much they will rise - this is the uncertainty. Thus contractual risk relates to the occurrence of unexpected events, uncertainty to the consequences of situations that can be anticipated. Contractual risk and uncertainty are present in practically all construction work.

Resources, (men, plant, materials, and capital) are utilized in some manner on all contracts. Some contracts and some companies have or cause major resource constraints, others have or do not. The large motorway contract which requires a fleet of earth moving plant constrains the market to those firms which possess or can obtain the plant. The firms that possess the plant are further constrained by its availibility. The building contract which requires only local labour and a tower crane, which can be rented, presents few constraints to the market.

The construction industry, after excluding all the small firms, can be divided into two major groupings - General Building Construction and Civil Engineering Construction. Firms that practice both offer no problem since they tend to adopt a division structure which separates the two and thus can be treated as two firms.

X Civil Engineering Construction usually: is designed by civil engineers; requires the contractor to utilize a quantity of large, expensive, specialised plant; and is constructed in accordance with either the Institute of Civil Engineers, "General Conditions of Contract for Works of Civil Engineering Construction", or the, "General Conditions of Government Contract for Buildings and Civil Engineering Works, Form CCC/Wks/1". Examples of civil engineering work are motorways, dams, harbours, airports, and bridges. There is often a high element of contractual risk in Civil Engineering Construction.

General Building Construction usually: is designed by an architect; requires little specialised plant; and is constructed in accordance with the Royal Institute of British Architects' (RIBA) "Standard Form of Contract". Only minor contractual risk is usually present in Building Construction and when a situation of high risk is apparent, it is often provided for in the contract conditions and the client, not the contractor, bears the cost.

Considering therefore the extreme positions, the characteristics of the division; General Building, Civil Engineering are summarised in the table below.

	GENERAL BUILDING	CIVIL ENGINEERING
RESOURCES	minimal constraint	constrained
CONTRACTUAL RISK	low	high

These differences give rise to two different competitive situations.

The resource requirements of Civil Engineering introduce differences into the valuation of contracts. If a firm has unemployed plant, or specialised plant, or plant located near the proposed work, its situation is obviously different to that of a firm which has not. The resource requirements also make it difficult for firms to enter or leave the market since considerable capital is required to establish, say, a fleet of machines and its disbursement can result in large losses. The high element of contractual risk also makes entry difficult since the firms require skilled personnel and capital to handle the high risk situation and these are often in short supply. The result is a market with a small number of identifiable competitors, each possessing a substantial amount of information about the capabilities and intentions of the others.

The Building Construction Market is the opposite. The minimal resource constraints and low risk enable firms to enter and leave the market with ease. Very little specialised plant is employed and the methods and procedures are generally uniform throughout the industry. The availability of site staff is not the problem it is in Civil Engineering; one company interviewed suggested that the half-life of senior site staff was approximately 2 years. Cost estimates are prepared with the same general procedure by most firms and only some specific company policy such as, for instance, keeping certain staff employed, or doing prestige buildings, can result in one company valuing a contract differently from another. The result is a market with a large number of competitors and only minor valuation differences being placed upon the contracts by these firms.

It is the Building Construction Market that is studied here. That is, a market where there is low risk, and few resource constraints. A market where all competitors tend to place roughly the same value on a contract and where there exist so many competitors that is ~~is~~ uneconomic to obtain data that would be useful for predicting their individual behaviour.

2.2.2 Building Construction Process

Although there is little contractual risk in Building Construction, there is often a great deal of uncertainty. This uncertainty affects the functioning of the competitive bidding process. To determine how and where uncertainty enters the problem, the Building Construction Process is examined.

The Tavistock Report (34) defined the Building Process as:

"The whole series of activities required between the initiating point of a client's need and the production of a building to fulfill that need".

The process can be described as the following set of steps:

CLIENT
|
SPONSOR
|
BRIEF
|
DESIGN
|
BILL OF QUANTITIES
|
SELECTION OF CONTRACTOR
|
CONSTRUCTION

This is not an all inclusive description covering all variations, but is a rough description of the process.

The Client, having determined that he has need of a building, contacts some person or organisation within, or connected to, the construction industry. This person or organisation, who could be an architect, quantity surveyor, speculator, engineer, contractor, etc., is called the Sponsor. The Sponsor, together with the Client, draws up a Brief of the Client's requirements. The Brief may be a set of unrecorded ideas in the Sponsor's head, or it may be a voluminous manuscript that clearly defines the required building down to the last fitting, or it may be anything in between.

After the Brief is prepared, the specialist designers, (soils engineers, services engineers, structural engineers, architects), are employed and the detailed Design work is carried out.

From the Design a Bill of Quantities is prepared. The Bill of Quantities is a description in words of every operation which the contractor will have to carry out to give effect to the plans and specifications. The Bill is prepared in accordance with some predefined method of measurement; for example, the Standard Method of Measurement of the Joint Committee of the R.I.C.S and the N.F.B.T.E.

A Contractor is selected to construct the proposed facility. This selection is usually done by competitive bidding based upon the Bill of Quantities, or the design drawings and specifications, or both. Selection by negotiation is also done by some clients but the industry norm is competitive bidding.

Once the contractor has been selected he proceeds to construct the facility in accordance with the contract instructions.

This brief description of the building process can be compared with the R.I.B.A. Plan of Work reproduced on the following page. There the Client-Sponsor-Brief Phase are seen as Stages A, B, C, and D. The Design Phase, Stages E and F. The Bill of Quantities, Stage G, and the Selection of the Contractor, Stage H. The Construction Phase encompasses Stages J, K, and L.

This is the description of the Building Construction Process that is usually found in the literature. It is an organised process, with specified responsibilities, and little opportunity for uncertainty. This process is called the "Formal Building Process" by the Tavistock Study (34), because:

"The system is formal in that theoretically it is the way in which the control of the building process works. ... It forms the basis of written information about the building process".

The Study then goes on to state:

"The formal system of controls, or directive functions, is not very directly manifested in actual behaviour and, if our information were based only upon the behaviour of the building team on the job, we might never have become aware of the formal system in its true form".

The formal system is how the construction process is supposed to function, not how it does. On actual projects the sequential finality of the described phases is rarely present.

Outline Plan of Work

Stage	Purpose of work and Decisions to be reached	Tasks to be done	People directly involved	Usual Terminology
A. Inception	To prepare general outline of requirements and plan future action.	Set up client organisation for briefing. Consider requirements, appoint architect.	All client interests, architect.	Briefing
B. Feasibility	To provide the client with an appraisal and recommendation in order that he may determine the form in which the project is to proceed, ensuring that it is feasible, functionally, technically and financially.	Carry out studies of user requirements, site conditions, planning, design, and cost, etc., as necessary to reach decisions.	Clients' representatives architects, engineers, and QS according to nature of project.	
C. Outline Proposals	To determine general approach to layout, design and construction in order to obtain authoritative approval of the client on the outline proposals and accompanying report.	Develop the brief further. Carry out studies on user requirements, technical problems, planning, design and costs, as necessary to reach decisions.	All client interests, architects, engineers, QS and specialists as required.	Sketch Plans
D. Scheme Design	To complete the brief and decide on particular proposals, including planning arrangement, appearance, constructional method, outline specification, and cost, and to obtain all approvals.	Final development of the brief, full design of the project by architect, preliminary design by engineers, preparation of cost plan and full explanatory report. Submission of proposals for all approvals.	All client interests, architects, engineers, QS and specialists and all statutory and other approving authorities.	
Brief should not be modified after this point.				
E. Detail Design	To obtain final decision on every matter related to design, specification, construction and cost.	Full design of every part and component of the building by collaboration of all concerned. Complete cost checking of designs.	Architects, QS engineers and specialists, contractor (if appointed).	Working Drawings
Any further change in location, size, shape, or cost after this time will result in abortive work.				
F. Production Information	To prepare production information and make final detailed decisions to carry out work.	Preparation of final production information, i.e. drawings, schedules and specifications.	Architects, engineers and specialists, contractor (if appointed).	
G. Bills of Quantities	To prepare and complete all information and arrangements for obtaining tender.	Preparation of Bills of Quantities and tender documents.	Architects, QS, contractor (if appointed).	
H. Tender Action	Action as recommended in paras. 7–14 inclusive of 'Selective Tendering'*	Action as recommended in paras. 7–14 inclusive of 'Selective Tendering'*	Architects, QS, engineers, contractor, client.	
J. Project Planning	Action in accordance with paras. 5–10 inclusive of 'Project Management'*	Action in accordance with paras. 5–10 inclusive of 'Project Management'*	Contractor, sub-contractors.	Site Operations
K. Operations on Site	Action in accordance with paras. 11–14 inclusive of 'Project Management'*	Action in accordance with paras. 11–14 inclusive of 'Project Management'*	Architects, engineers, contractors, sub-contractors, QS, client.	
L. Completion	Action in accordance with paras. 15–18 inclusive of 'Project Management'*	Action in accordance with paras. 15–18 inclusive of 'Project Management'*	Architects, engineers, contractor, QS, client.	
M. Feed-Back	To analyse the management, construction and performance of the project.	Analysis of job records. Inspection of completed building. Studies of building in use.	Architect, engineers, QS, contractor, client.	

**Publication of National Joint Consultative Council of Architects, Quantity Surveyors and Builders.*

Bills of Quantities are prepared from sketch plans and the building can be completed before the Brief is finalised. An amusing description of a "normal" project can be found in the appendix of the Tavistock publication, Interdependence and Uncertainty (34).

It is in the actual functioning of the process that the uncertainty arises. The ad hoc techniques, and general crisis atmosphere of a construction site, coupled with changes in the specifications by clients, and operative mistakes produce a very uncertain situation.

The uncertainty present in a construction project is classified in this thesis under two categories: Estimating Uncertainty, and Construction Uncertainty. Construction Uncertainty is caused by the variable nature of the project due to client changes, construction errors, personality conflicts, etc. Estimating Uncertainty is caused by information gaps and the subjective nature of estimating. The influence of these uncertainities is discussed in Section 2.5.2. The sources of Estimating Uncertainty are examined in Section 2.3.2.

2.2.3 The Sample Companies

This study was provided with data and assistance by three construction firms. Two of these firms operate a division structure in which the project selection, estimating, and decision on final tender price is the responsibility of divisional management. Since each division is a semi-autonomous decision unit, it can be treated as a separate company. Thus the three firms were able to provide data for four "companies". The firms wish to remain anonymous and so they are referred to as Companies A, B, C, and D.

For each Company there is a data set, which is a "chapter" in the bidding history of a company. A data set consists of a series of contracts which the company has bid on over a period of time. Each set is headed with a brief description of the type of work, and the time period concerned. For each contract in the data set there is the following information.

- a sequence number;
- the sample company's estimated cost;
- the sample company's submitted bid;
- the number of competitors;
- the competitors' bids;

and in some cases, the names of the competitors. Appendix

1, Sample Companies' Data, contains a more detailed description of the data. The contents of the data sets are summarised in the table below.

SAMPLE COMPANY DATA SETS

	Time Period	Number of Contracts Provided	Number of Contracts Usable	Number of different Competitors
Company A	12 mo.	37	34	65
Company B	12 mo.	43	41	81
Company C	9 mo.	41	37	-
Company D	19 mo.	51	41	100

The four sample companies possess the following common characteristics:

i. They are all building construction firms operating in the south of England.

ii. They obtain a large portion of their work load by competitive bidding.

iii. Most of the contracts in the data sets are Bill of Quantities type and use R.I.B.A. terms and conditions.

iv. The contracts in the data set do not call for any specialised techniques, patented process, or specialised plant which would give any firm a competitive advantage.

These common features describe the Study Area. This research is concerned with competitive bidding in the Building Construction Industry where:

i. There are a large number of potential competitors for each project.

ii. There is little product differentiation.

iii. There is little differentiation between companies.

iv. The resource constraints are minimal.

These are the boundaries of the Management Problem.

2.3 MANAGEMENT DECISIONS

2.3.1 Competitive Bidding

Competitive bidding is defined as:

A situation in which a number of contestants (bidders), each submit to a client a price (bid) in return for which they are willing to perform certain specified services. The winning price is determined by some pre-arranged judging procedure (usually the lowest price submitted*) which is known to the client and to the contestants.

The contestants do not know each others bid before the judging.** They may know them after.

Non-price features such as delivery times, client bias, etc. as mentioned by Simmonds (29) are not normally considered in the judging.

Of the many variants of competitive bidding, the two in general use in the construction industry are: Open Competitive Bidding - in which the competition is open to anyone who wishes to compete; and Selective Tendering - where competition is restricted to a few chosen competitors. These two methods

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* But not always, see reference 16.

** This may not necessarily always be true in actual fact. However, this thesis is studying competitive bidding, not colusive bidding; even though it is probably easier to achieve an optimum in the colusive case. For an example of the latter, see The Times, 20 March 1970, news item "Contractors' 'phoney' bids spur inquiry".

have been major subjects in the Banwell (32), Emmerson (33), and Simon (36) reports. Open Bidding being used for reasons of public accountability and Selective Tendering being recommended as conducive to improving the efficiency of the industry. For this thesis, it is irrelevant which of the two variants is used. However, the use of Selective Tendering simplifies the OR problem by controlling the number of competitors and thus allowing their number to be predicted with reasonable accuracy.

Competitive bidding is the normal method of contractor selection. That it is not necessarily a sound process is recognised by the industry. This point of view is illustrated by the following quotations.

From the Building Industry Survey (38);

"One result of the open tendering system has been that successful main contractors drive hard bargains with sub-contractors with the frequent result that the sub-contractor who gets the job is tempted to use inferior materials and lower the quality of his workmanship. Several sub-contractors have told us that they could not make some jobs pay under the present system unless they used inferior materials."

From the Emmerson Report (33);

"Open tendering is still common although this prejudices the firm which maintains a high standard of work and the building owner does not get the best value from the lowest tender."

From "Construction and Professional Management" (26);*

"It is partly the process of awarding contracts to the lowest bidder that accounts for construction contracting having among the lowest gross profit margins of all the industries, about one percent for 1964."

In view of this condemnation, is competitive bidding likely to be replaced by some other process? Reported attitudes and behaviour suggest no. To quote the Banwell Report (32);

"Many clients consider that a building can only be secured at the lowest possible cost if each job is advertised and all contractors are free to quote a price in competition."

Moreover in addition to the attitude of clients, it would appear that the account of cost and quality erosion to which reference has been made may not be generally valid. Judging from the attitudes of the sample companies, competitive bidding does not result in competition based upon costs, or profit margins, but actually produces a lottery in which the inherent uncertainty of the process decides the winner. Evidence to support this contention is the fact that all four companies used almost identical methods of determining the costs and then all used almost the same percentage mark-ups to arrive at their bid prices.

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* American Reference

2.3.2 Estimating and Tendering Procedure

Thus inherent uncertainties appear to be the key to the workings of the competitive bidding process. One source of these uncertainties is the estimating and tendering procedure. The management decisions which are the focus of this research are part of the estimating and tendering procedure. Therefore, to isolate these decisions, and to examine this source of uncertainty, the procedure is investigated.

The estimating and tendering procedure, from receipt of the invitation to tender through to the notification of success or failure, is thoroughly described in the "Code of Estimating Practice", published in 1966 by The Institute of Building (6). The flow chart reproduced on the following page illustrates the major stages and items of the procedure.

The Code also defines the following terms:

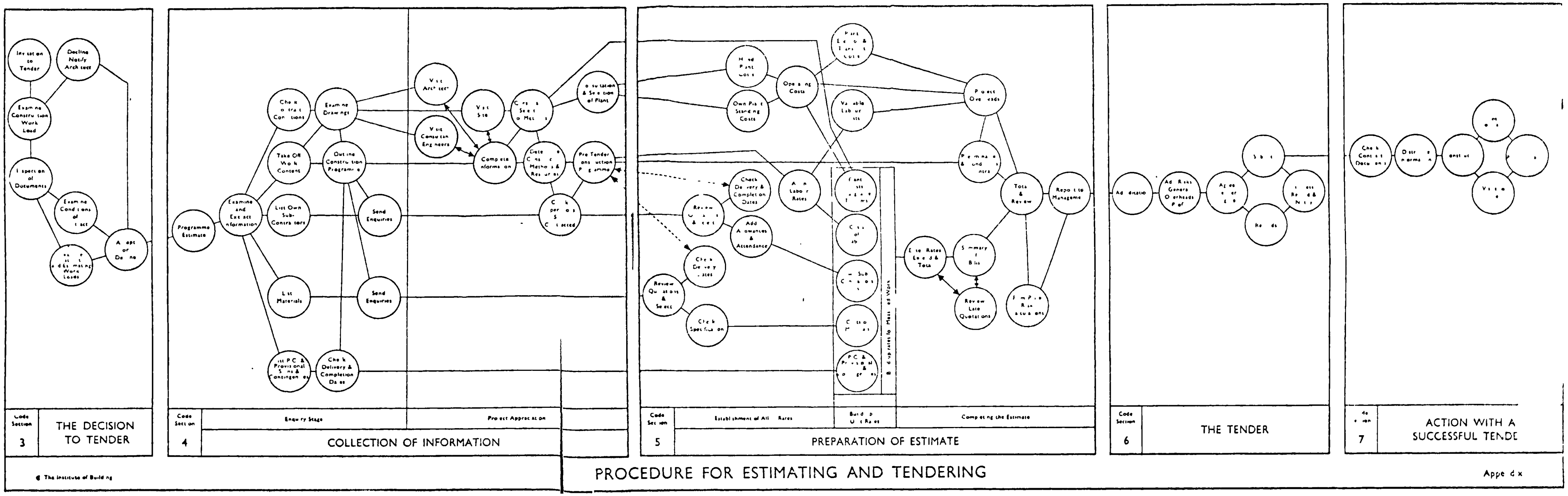
Estimating - is the technical process of predicting cost of construction.

Tendering - is the separate and subsequent commercial function based upon the estimate.

Adjudication - is the action taken by management to convert an estimate into a tender.

APPENDIX D

NETWORK DIAGRAM SHOWING PROCEDURE FOR ESTIMATING AND TENDERING



It is convenient to consider the Procedure as three stages.

I - The Decision to Tender: in which a management decision is made based upon a preliminary examination of the specific contract and consideration of the company's position and market environment.

II - Estimating: which includes sections 4 and 5 of the Code diagram and is the technical process of arriving at an estimated cost figure.

III - Tendering: which can include declining the contract, and is the management decision on the bid (tender) price to submit; based upon an intensive examination of the specific contract, the competition, the company, and the market.

Considering the Estimating Stage first, the Code states:

"An estimate must be prepared in a way that is explicit and consistent and which takes account of methods of construction and all circumstances which may affect the execution of work on the project. It is believed that such a sound estimate can only be achieved when each operation is analysed into its simplest elements and the cost estimated methodically on the basis of factual information."

This presents estimating as a careful, thorough process by which a valid estimated cost is obtained. Yet this is contradicted by P. F. Miller*, who has stated:

"Estimating is the last of the folkcrafts in the construction industry."

Moreover, examination of the workings of the estimation procedure reveals some basis for Miller's statement.

*Statement made while presenting a paper, "Cost Control for Contractors", at the Operational Research Society Construction Industry Study Group Conference at the Loughborough University of Technology, Sept. 17, 1969.

Estimates are usually prepared from a Bill of Quantities. A conventional definition of a Bill of Quantities is found in reference 6:

"A Bill of Quantities is a description in words of every operation which the contractor will have to carry out to give effect to the architect's plans and specifications, prepared in accordance with the Standard Method of Measurement of the Joint Committee of the R.I.C.S and the N.F.B.T.E. or the Scottish Mode of Measurement. Numerical measurements are set against each item and space is left for the builder to set his price against each."

A different, and perhaps more appropriate description of a Bill of Quantities was provided by the Tavistock Institute Report (15) where they described it as a "hypothetical construct".

The extent to which the Bill of Quantities is hypothetical is indicated by the fact that the design is often not completed when the Bill is prepared. Some design details are left until just before construction. The author's limited experience* suggests that the bill is rarely prepared from completed drawings and specifications because these do not exist when the bill is drawn up. Therefore the Bill must contain a number of provisional items and "guess - timates" if it is to describe the desired building even inadequately. Therefore the builders are not estimating some precise, clearly defined and detailed project, but they are competing for the right to build some hypothetical project. The potential for a refined estimate of the construction cost cannot exist in the absence of a completed design.

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*Six months in the employ of an engineering consulting firm in London.

The Bill of Quantities is so used to provide the bidders with a uniform basis for making lump sum offers.

Even if the Bill did represent the building, there are other problems that prevent estimating from being a thorough and objective technique based upon factual information.

i. Time - The process as outlined is time consuming. However the time allowed is not always adequate.

ii. Drawings - The drawings are often incomplete, or drawn at a very small scale, or not available.

iii. Time Delay - The delay by clients in the letting of contracts (i.e. elapsed time between the submission of tenders and the awarding of contract) precludes the assignment of men and equipment at tender time. To quote Banwell (31):

"Public Authorities are said to be particularly slow in notifying the results of tenders and to show undue haste in expecting a physical start once the contract has been let."

iv. Determination of Construction Method - Since the divisions in the bill of quantities bears little relationship to the construction processes involved, and since estimates are built up from the bill of quantities, the relationship between the construction method and the estimated cost appears tenuous. This is illustrated by an example from reference 38.

"Another example was that of a leading architect who designed buildings in such a way as to eliminate the need for scaffolding (although in this case difficulties were encountered since none of the contractors who tendered appreciated this fact.)."

Often there is little communication between the estimator and the site agent so that even if a method is determined at the estimating stage, it is not necessarily the way the project is constructed.

However, despite these problems, it is apparent from an examination of submitted prices that estimators from different companies, working with approximately the same information, arrive at almost the same prices. The average bid range (high bid minus low bid) on the data set contracts being approximately 10 percent (See Figure A-5).

In section 2.2, uncertainty was categorised into Estimating Uncertainty and Contract Uncertainty. It is now possible to make these terms more explicit. Contract Uncertainty is concerned with the difference between the actual structure and the hypothetical one described in the tender documents. Will the firm gain or lose in the transition from one to the other? The Estimating Uncertainty arises in the process discussed above. Does the estimate approximate closely to the cost of the hypothetical project? If the estimate is too high the contract will be lost; if the estimate is too low the contract may be won but the firm is more likely to suffer a loss on the contract. The Uncertainties are separated because they are handled differently in the decision process.

<u>The Stages I and III of the Estimating and Tendering Procedure are the management decisions examined in this thesis:</u>

The Decision to Tender and the Decision on the Tender Price.
In this section some of the relevant elements (those which are listed in the Code) of the decisions are presented. Comment on these elements is reserved until subsequent sections.

Under the Decision to Tender, the Code of Estimating Practice considers the following:

- the organisation's work load in relation to its resources.
- is all necessary information provided?
- is sufficient time allowed for estimating?
- what are the Conditions of Contract?
- are drawings included?
- are operating conditions defined clearly?
- what is the value and extent of the project and what is the main contractors contribution likely to be?
- is the design well developed?
- are the Bills of Quantities standard?
- Is more information available? Where? When? How?
- What is known about:

    the client and consultants?

    the value of the project?

    the Conditions of Contract?
- reconsider the contract in relation to certain and expected construction commitments and the estimating work load.

Under the Tendering Decision are listed:

Matters to be Considered:

- method of construction.
- unusual risks not covered by contract.
- unresolved technical or contractural problems.
- assessment of design process.
- assumptions in preparation of estimate.
- assessment of profitability of project.
- pertinent information on market of industrial conditions.
- need for qualification of tender.
- terms of quotations from sub-contractors.
- time for which tender is to remain open.
- Conditions of Contract.
- contractural risks (including Fixed Price Tender Risk).
- Capital requirement including: work in progress, materials, temporary works, plant investment, retention moneys, and the possibility of under valuation.
- technical and managerial requirements.
- work load.
- the market.
- reputation of client, architect, quantity surveyor, and other consultants.

Additional Factors to be Included:

- Financial implications of items above.
- Risk.
- General Overheads.
- Profit.

2.3.3 A Schematic Decision Model

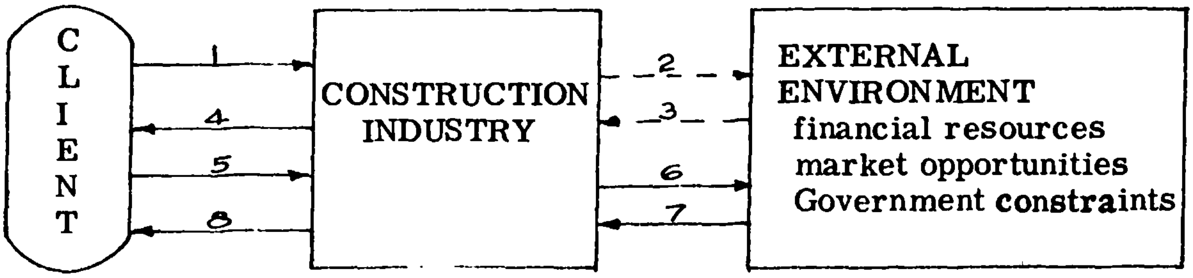
The Decision to Tender (Selection Decision) and the Tender Price (Adjudication) Decision are the focus of this investigation. To highlight the interactions and information flows affecting these decisions a schematic model of the Competitive Bidding Process is developed. This model serves as a definition of the Competitive Bidding Process and is used as a conceptual basis for the following Chapters.

The model is developed by considering a series of Black Box* models, moving from the general to the specific. A simplified view of the construction industry using the Competitive Bidding Process is illustrated below. Dotted arrows are used to indicate information flows; solid arrows indicate material flows.

---

* The Black Box is regarded as a system which is definable in terms of its inputs and outputs, but undefinable in the details of its workings.

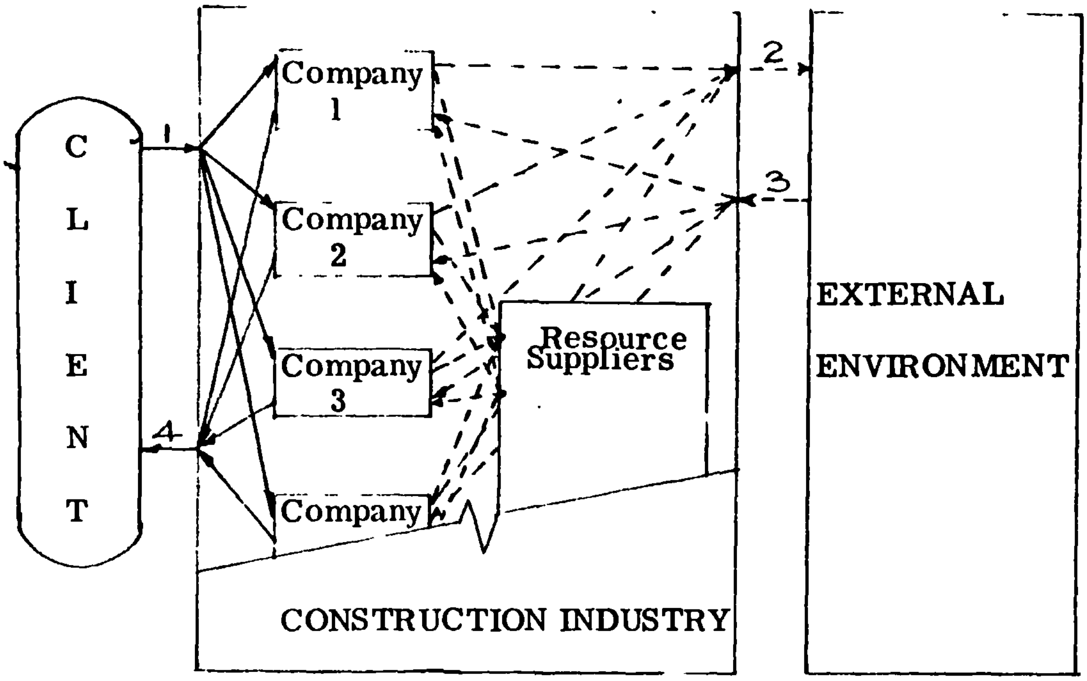
Formal transfers, i.e. a request for tender and a submitted bid, (or refusal) are also indicated with solid arrows.



(1)	- Request for Tender
(2)-(3)	-Information Flows (alternate opportunities, Government Action, financial resources)
(4)	-Response to Tender Request
(5)	-Award of Contract
(6)-(7)	-Flow of resources to and from external environment
(8)	Constructed Building

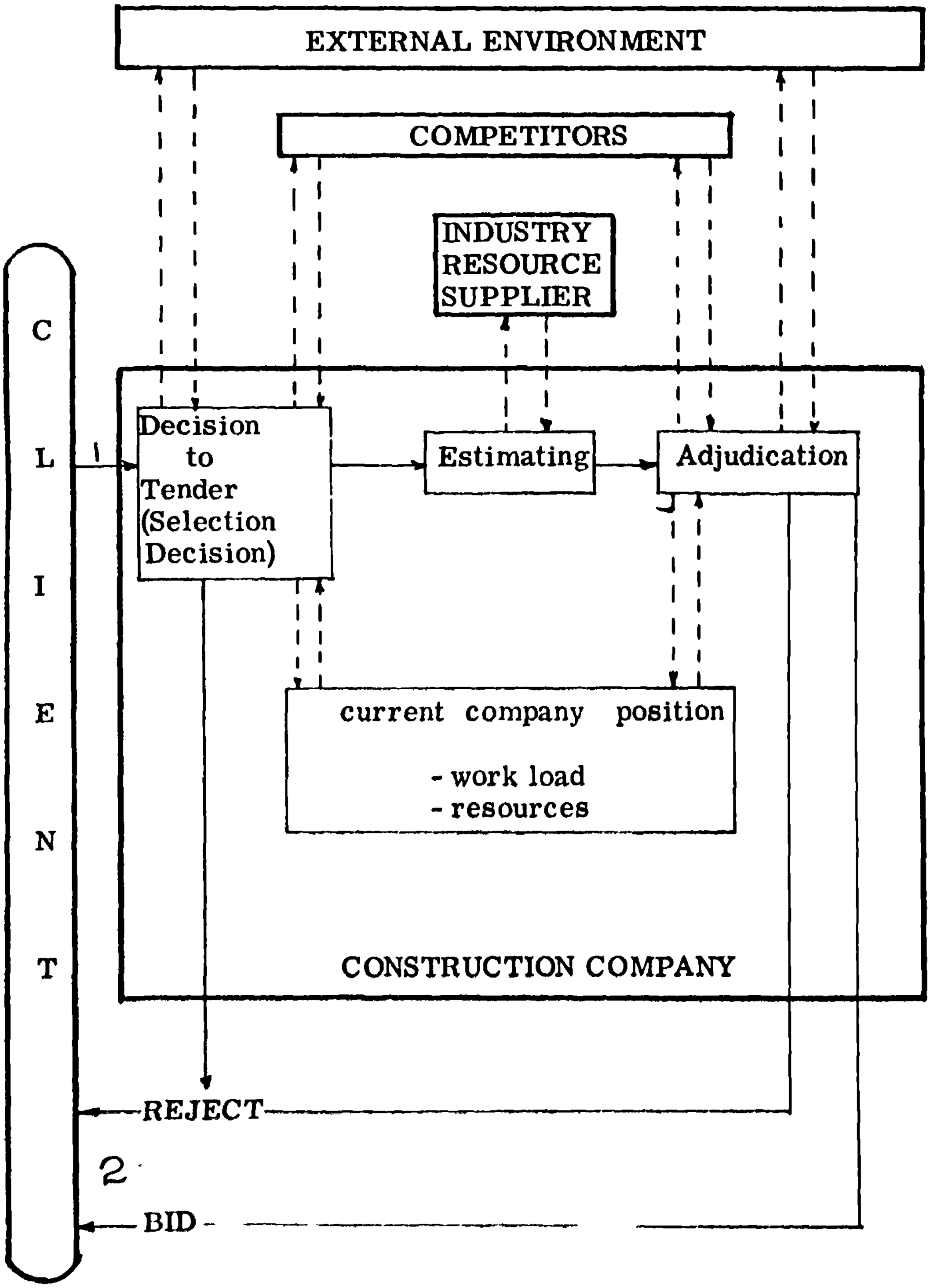
The term, resources is used to encompass men, materials, knowledge, money, etc.

In the diagram, arrows (1), (2), (3) and (4) cover the Competitive Bidding Process. Consider, therefore, only those first four steps. The box labelled Construction Industry contains approximately 80,000 construction companies and their industry resource suppliers. Industry resource suppliers are: building material suppliers, equipment companies, the construction labour market, and the like. Expanding this section of the model -



To follow the information and material flows the model is now reduced to that of a single company.

Using the stages of the Estimating and Tendering procedure discussed in Section 2.3.2, the model now appears as:



This model represents the Competitive Bidding Process that is discussed in this thesis. It is an abstraction, and it is simplified; but it isolates the major decision areas and information flows, and is sufficiently general to be applicable to most bidding situations.

2.4 FEASIBILITY STUDY

A necessary prelude to a research project that is of practical significance is a positive answer to the question; "Is it worthwhile?". Despite contractors' claims that they are losing money, the possibility that they are doing as well as possible within the system must be considered. If this is the case then little practical benefit can result from the study. Also, the subject of competitive bidding has been actively investigated by operational researchers since 1956. Is there anything left to explain or investigate?

To investigate these questions, an initial study of the situation was conducted. This exploratory analysis, as well as demonstrating the need for the study, produced some interesting observations on managerial objectives. Since these observations are used in the study of the process, the analysis is described here in some detail.

2.4.1 Maximize Profits

Defining profits as total receipts minus total cost, the conventional economic objective attributed to a company is to maximize profits.

By considering the contracts offered to an individual company over a specified time period it is possible to aggregate these into a Profit-Volume Opportunity Curve for that Company. The curve is constructed by considering the effects of different policies, e.g. uniform mark-ups of 1%, 2%, . . . , 10%, on total profit and volume. Figure 2.4 is an idealised Profit-Volume Opportunity curve for a firm in the industry.

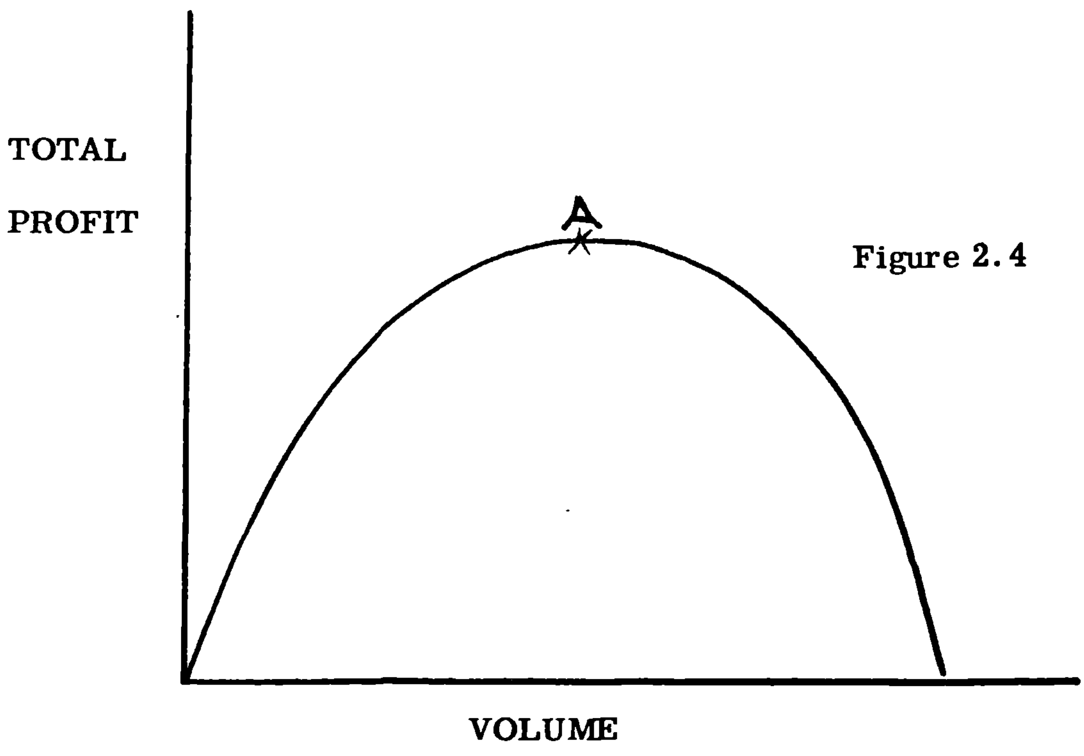


Figure 2.4

The curve is intuitively reasonable since it indicates that low total profits can be obtained by winning a few contracts at a high mark-up or by winning many contracts at a low mark-up, and that an optimal combination of mark-up and contracts exists. A policy designed to maximize profits should result in the company operating at point A on the curve.

The maximum profits operating position when overheads are included on the diagram is a function of the type of overheads. This is illustrated in Figures 2.5 and 2.6.

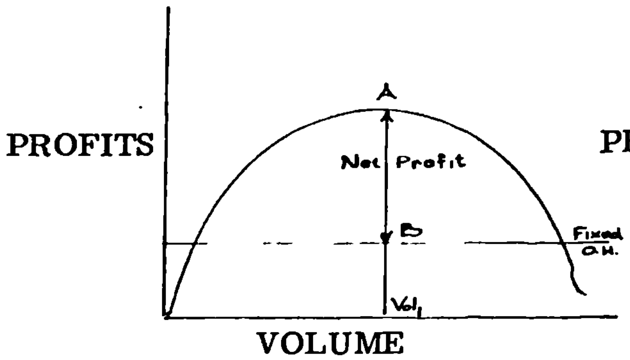


Figure 2.5

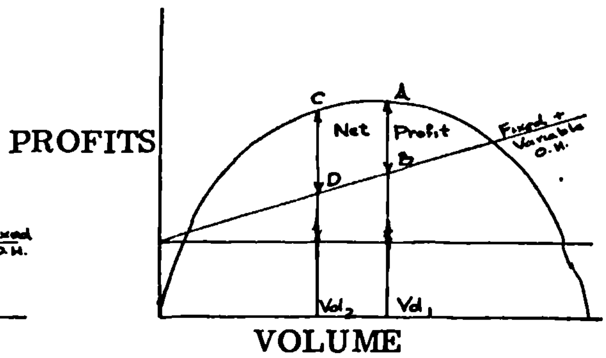


Figure 2.6

If the overhead costs are fixed (i.e. not changing with volume) the maximum profit position is at the peak of the curve (point A) and the corresponding volume is Vol_1. For this case, any policy which maximized gross profits (length A-Vol_1) also maximizes net profits (length A-B), and vice versa. It is the fixed case that one would expect to apply for a construction company. This case because, although the firms discuss overhead as a percentage figure (implying variable overheads) there is a general tendency to keep key staff employed on a year round basis (fixed cost), and items which might normally be variable overheads, for example the installation of site offices, utility costs, permanent staff assigned to a project, equipment maintenance, etc. are usually included in the estimate of site costs.

The second case, Figure 2.6, illustrates the effect of the addition of variable (increasing with volume) overheads to the diagram. Since C-D is longer than A-B, the maximum net profits position has shifted to the left of the maximum gross profits position, from volume Vol_1 to volume Vol_2. Thus the optimum is at that volume at which the first derivatives of the profit and cost curves are equal.

The selection of a profit maximising criterion seems valid in theory; it is in the application that flaws appear. It has been fairly commonplace for researchers to equate the bid price with the receipts and the estimated cost with the true cost and define

$$\text{Gross profit on contract i} = \begin{cases} x_i - k_i & \text{if } x_i < y_i \\ 0 & \text{if } x_i > y_i \end{cases} \tag{2.4.1-1}$$

where x_i = the bid price on contract i

k_i = the estimated site costs for contract i

y_i = the lowest competitor's bid on contract i

ties, $x_i = y_i$ are assumed not to occur.

Then they assume that the variable overheads are included in the estimated cost, k, and that the management's objective is to maximize gross profits.

This definition of gross profits is not completely satisfactory. First it ignores uncertainty, and one of the main points of the previous sections was that the estimated cost is not the true cost. Likewise it was noted that the receipts normally vary from the bid price. The best that the figure (x - k) can be is a prediction of the gross profits, and this assumes that the uncertainty does not introduce any bias. However, this definition does provide an index, since it is reasonable to assume that a contract won with a bid of £x + delta, where delta is a positive quantity, should be more profitable than a contract won with a bid of £x.

A second difficulty with the gross profit criteria is that it ignores the time aspect of contracts. When comparing different contracts, or sets of contracts, this neglect distorts the results. For example, consider the following two contracts:

Contract	Estimated Cost	Bid	Profit	Duration	Profit/ Year
A	£2,000,000	£2,120,000	£120,000	4 yrs	£30,000
B	500,000	530,000	30,000	1 yr	£30,000

Assuming that the cash flows in Contract A in the first year are similar to the flows in Contract B, the two contracts could be roughly equal, considered in terms of the effect on the company's operations in that year. Using a gross profit as a criteria, Contract A is four times more valuable than Contract B. However, it is usually assumed that Contract A is more desirable than Contract B. The use of gross profits as a measure exaggerates this, but does produce the correct ranking order. The opposite, Contract B more valuable than Contract A, could result if more advantageous opportunities occur in the second year. That possibility is not considered in this pilot study. Section 2.5.5 proposes a method of treatment for this aspect.

2.4.2 Decision Rule Simulation

Two decision rules were used to test the potential value of the project. The objective of the earlier researchers, maximize gross profits, was used as the criterion and the current company performance, as exhibited by the data sets, was compared with what the results would have been if:

(a) The company had increased (decreased) all its bids by a uniform percentage amount.

(b) The company had used a policy of uniform percentage mark-up on estimated cost.

A problem in using the (2.4.1-1) definition of gross profits for this purpose is created by the Provisional Cost Allowances in the data set contracts. These sums carry their own profit allowance and this introduce additional variation into the gross profit figure. For an extreme example, assume for a contract that the estimated cost was £100, £50 of which was a Provisional Cost Allowance. A 6 percent mark-up would result in a submitted tender of £103 and a profit, as defined, of £3. If instead, the Allowance was £20, the 6 percent mark-up would result in a submitted tender of £104.8 or profit, if won, of £4.8. The contracts, however, have identical value to the company because they will be paid 6% profit on the work covered by the Provisional Allowance. Theoretically this can be corrected for; in practice the data required to make the corrections was not available, and so it was necessary to ignore the Provisional Allowances. This is equivalent to assuming that the percentage amount of Provisional Allowances is constant for all contracts, and this is not so. However, from interviews it was ascertained that the amount is usually small and does not fluctuate too wildly and so the error introduced by this factor should not be major. In any case, the errors do not affect the terms usefulness as an index, i.e. a contract won at a bid of £x + delta is still more profitable than one won at £x.

To simulate the decision rules, two computer programs, OPTM* and OPT2 were written. Simplified flow charts of these programs are Figures 2.7 and 2.8. The programs were written for use on The City University, I.C.L. 1905 Computer. The data sets from the four sample companies, introduced in section 2.2.3, were used as input.

A series of contracts, such as the data sets, can be manipulated to produce several different profit-volume relationships. There are three basic figures to start with; the estimated cost, the bid, and the lowest competitor's bid, and a different

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* Capitalized names refer to Fortran IV programs.

Flow Chart

OPTM PROGRAM

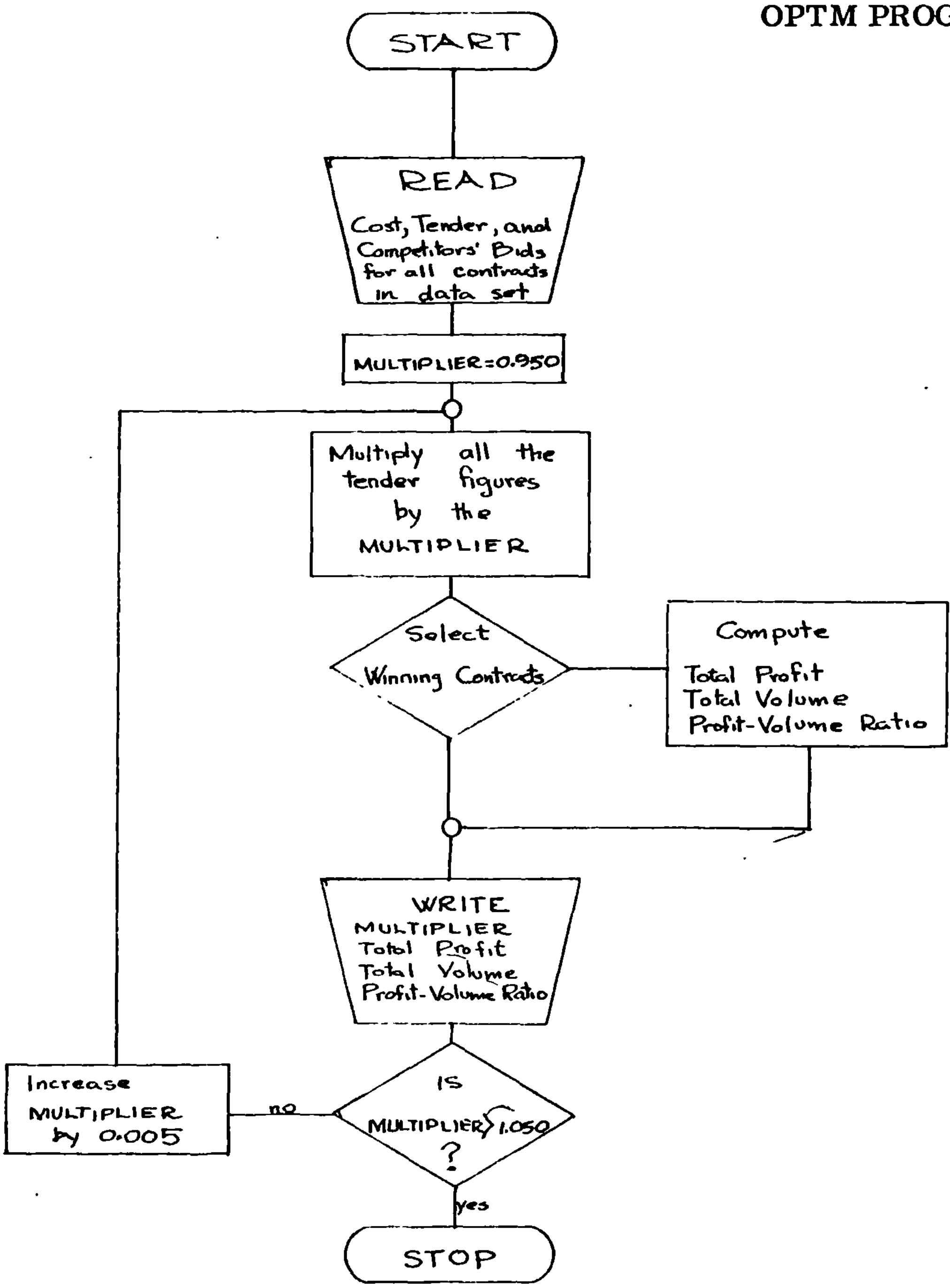


Figure 2.7

Flow Chart

OPT2 PROGRAM

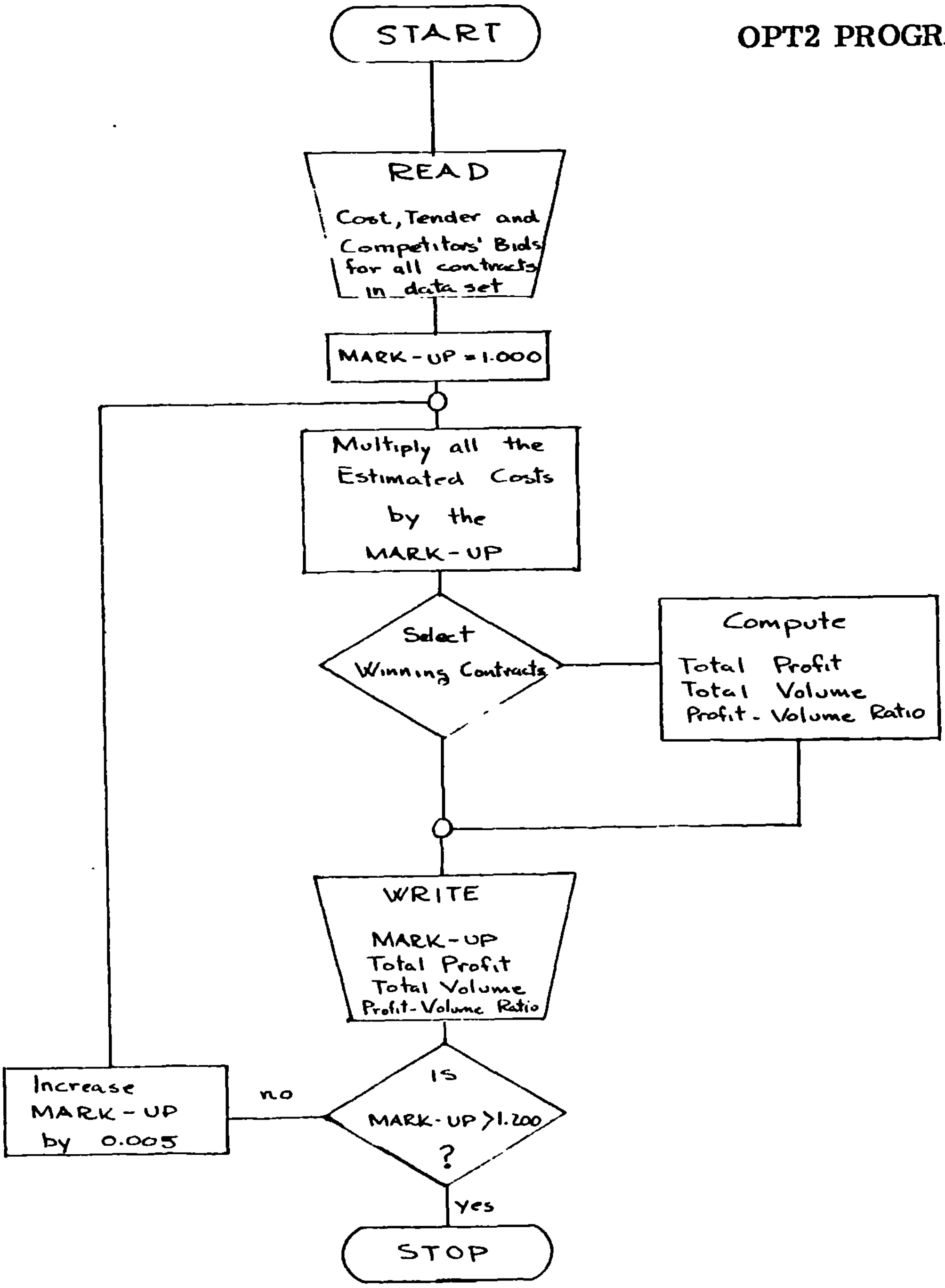


Figure 2. 8

profit-volume relationship can be derived from each.

One approach would be to use the lowest competitor's bid on each contract. Theoretically the maximum profit available to the company would result from incrementally under-bidding the lowest competitor on all contracts which yielded a positive profit (i.e. all contracts for which the lowest competitor's bid is greater than the company's cost estimate). A second approach is to start with the estimated cost figures and evaluate the effect, in terms of profit and volume, of applying the same percentage mark-up to all the contracts in the set. This is done by the OPT2 program. The third approach is to start with the tender figures submitted by the sample company. By making the same incremental change to every bid in the set and calculating the result, a profit-volume relationship is produced.

Only the second and third approaches are used in this thesis. This is because it was felt desirable to stay with results that could realistically be obtained by an operating company. The results that can be achieved by a policy of constant mark-up, or the results that can be achieved by a shift of current bidding policy, are felt to be both practicable and obtainable. The theoretical maximum is not.

Consequently, OPTM evaluates the effect on total profits for the data set of varying all the sample company's bids by a percentage increment. For example, suppose in the data set the company had bid 60 contracts, won 5, and received a total profit of £100,000. The program computes what would have happened, in terms of total profits, if all 60 tenders had been raised (reduced) by 0.5%, 1.0%, 1.5%, ..., 5.0%. The program outputs the results in graphical form.

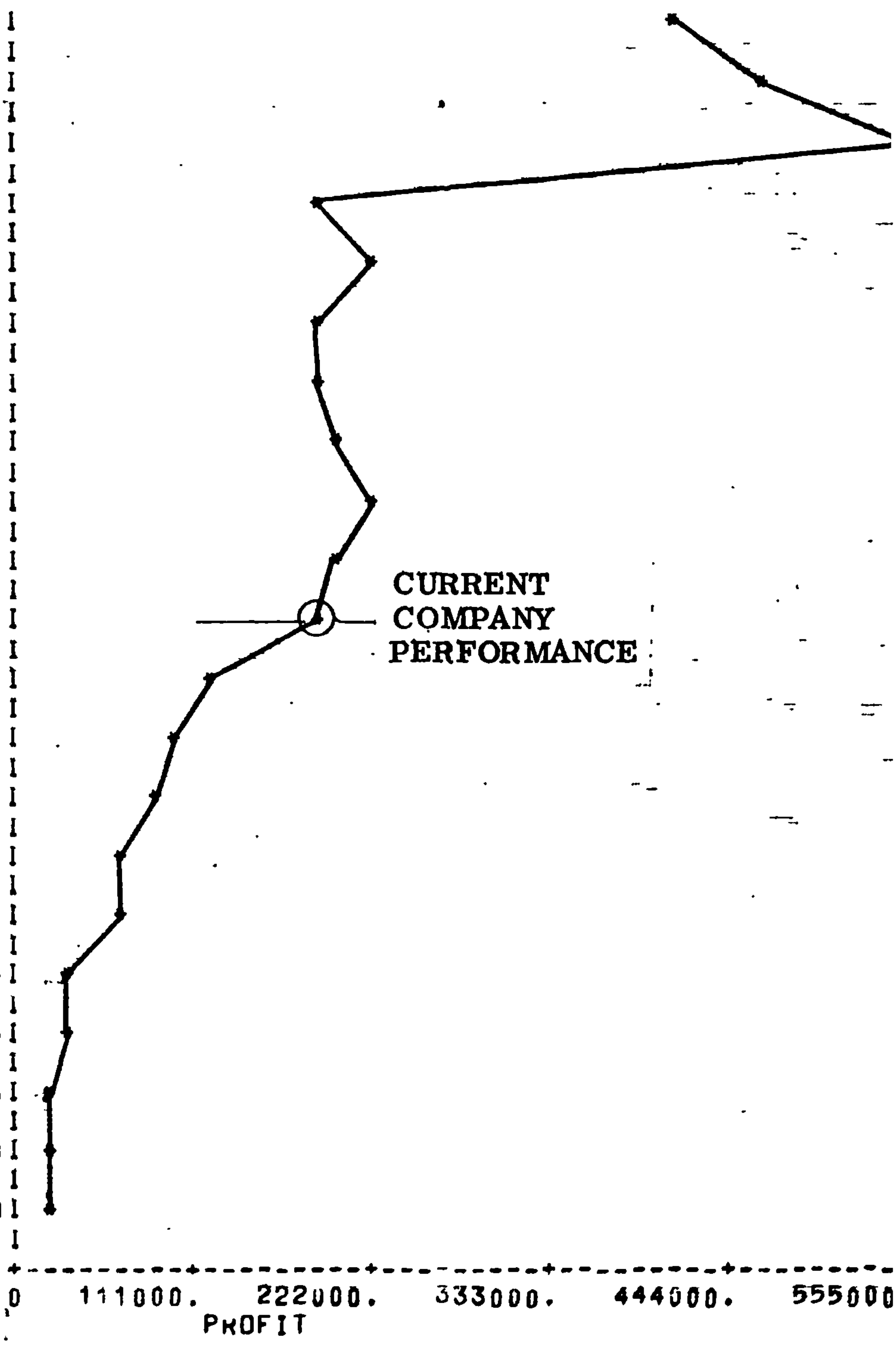
OPT2 is similar to OPTM except that it operates on the estimated cost instead of the tender figure. The program evaluates the effect of a uniform mark-up of 0.5%, 1.0%, 1.5%, ..., 20.0%.

Figures 2.9 and 2.10 are sample output sheets.

To obscure the identities of the sample companies, the data results are presented in percentages. The base figure, 100%, selected for each data set is the maximum figure obtained by using a policy of constant percentage mark-up on the estimated cost for all contracts in the data set. This is the maximum produced by the OPT2 program.

ECT OF VARYING TENDERS
TA SET NUMBER 7
TE 28/01779

LTIPLIER	PROFIT	VOLUME
0.95	410918.	21466561
.955	46145 .	19444653
.96	551984.	18941264
.965	1907 8.	76 8732
0.97	2 9336.	7150831
0.975	1 322 .	5412086
0 98	193853.	4824571
0 985	10943 .	4439197
0.99	222976.	4439197
0.995	2 2487.	3759943
1.0	139884.	3264363
1.0 5	1 7317.	1824260
1.01	1 48 2.	1532453
1.015	36192.	1174433
1.02	64663.	771086
1.025	6876 .	771086
1.03	36248.	354243
1.035	58143.	354243
1.04	24199.	2 5256
1.045	7625.	1390 0
1.05	8374.	1390 0

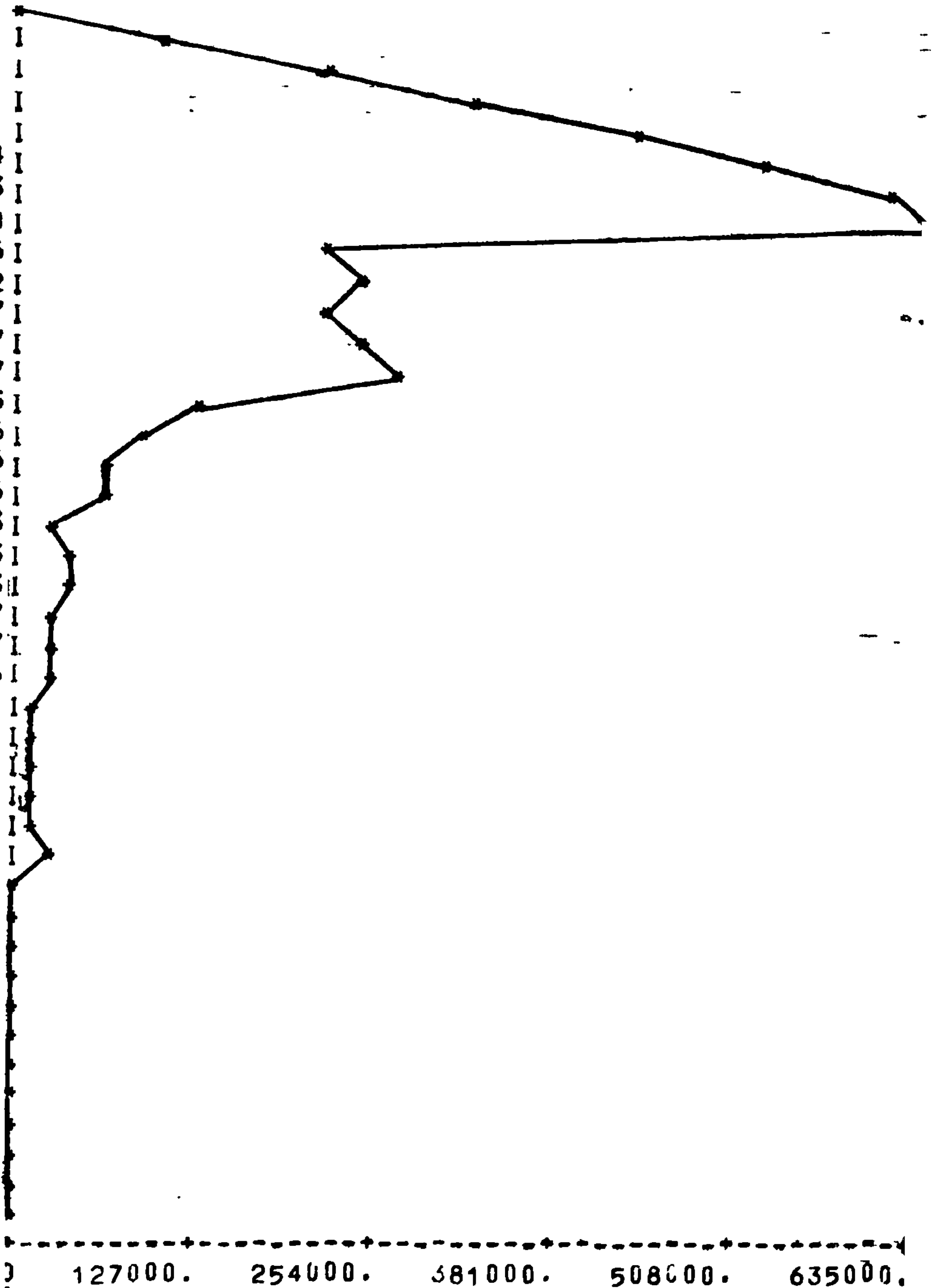


OPTM OUTPUT

Figure 2.9

EFFECT OF VARYING MARK-UP
DATA SET NUMBER 7
DATE 28/01/70

MULTIPLIER	PROFIT	VOLUME
1.0	.	21466561
1.005	107333.	21466561
1.01	214666.	21466561
1.015	321998.	21466561
1.02	429331.	21466561
1.025	514729.	20589154
1.03	603938.	20131253
1.035	638349.	18901410
1.04	216483.	5412086
1.045	235943.	5243122
1.05	221960.	4439197
1.055	244156.	4439197
1.06	266352.	4439197
1.065	128576.	1978095
1.07	90640.	1294856
1.075	57831.	771086
1.08	61687.	771086
1.085	30111.	354243
1.09	31882.	354243
1.095	33653.	354243
1.1	27871.	278797
1.105	29264.	278797
1.11	22578.	205256
1.115	15985.	139000
1.12	16680.	139000
1.125	17375.	139000
1.13	18070.	139000
1.135	18765.	139000
1.14	19460.	139000
1.145	.	0
1.15	.	0
1.155	0.	0
1.16	.	0
1.165	.	0
1.17	.	0
1.175	.	0
1.18	.	0
1.185	.	0
1.19	.	0
1.195	.	0
1.2	.	0



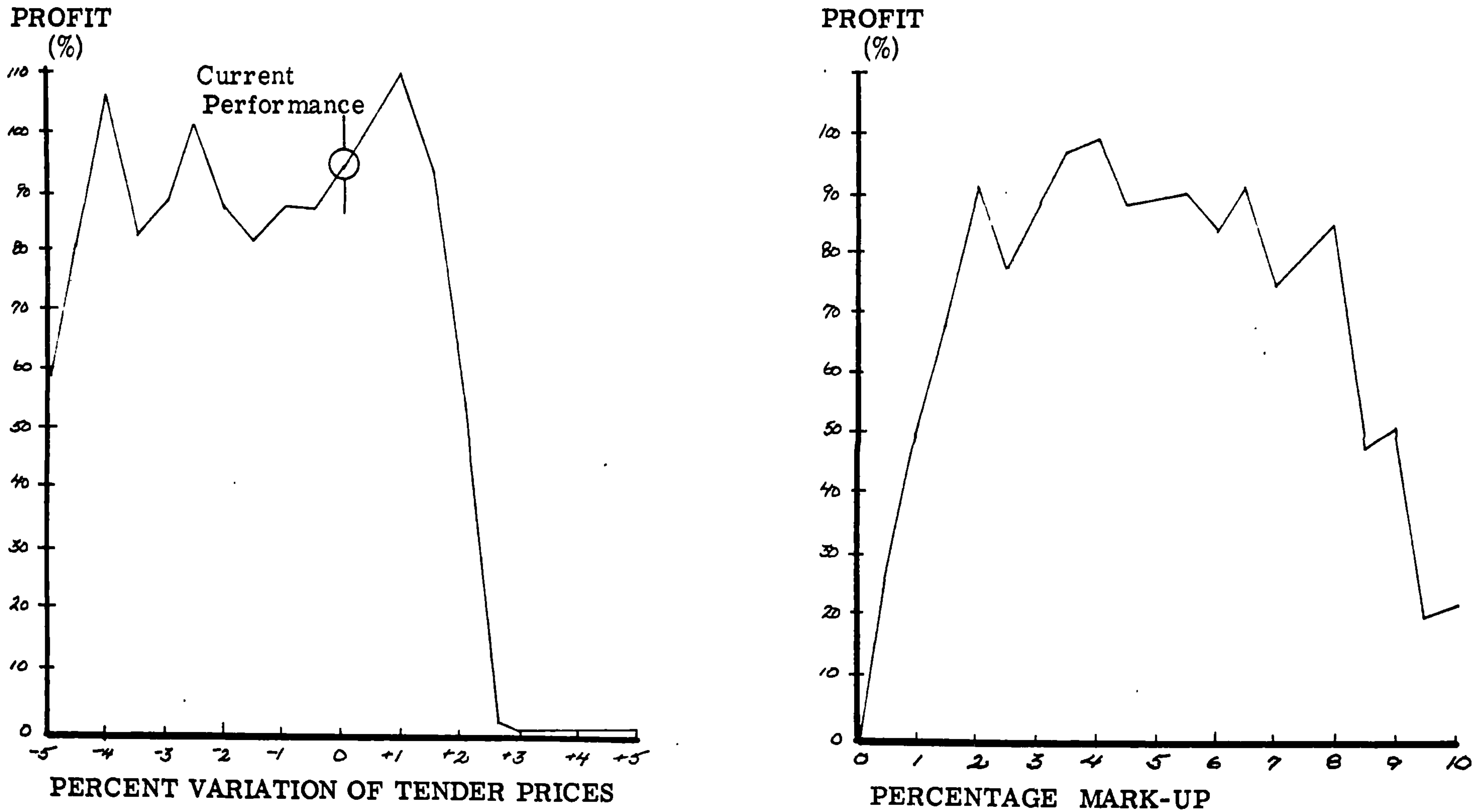
OPT2 OUTPUT

Figure 2. 8

2.4.3 Results and Discussion

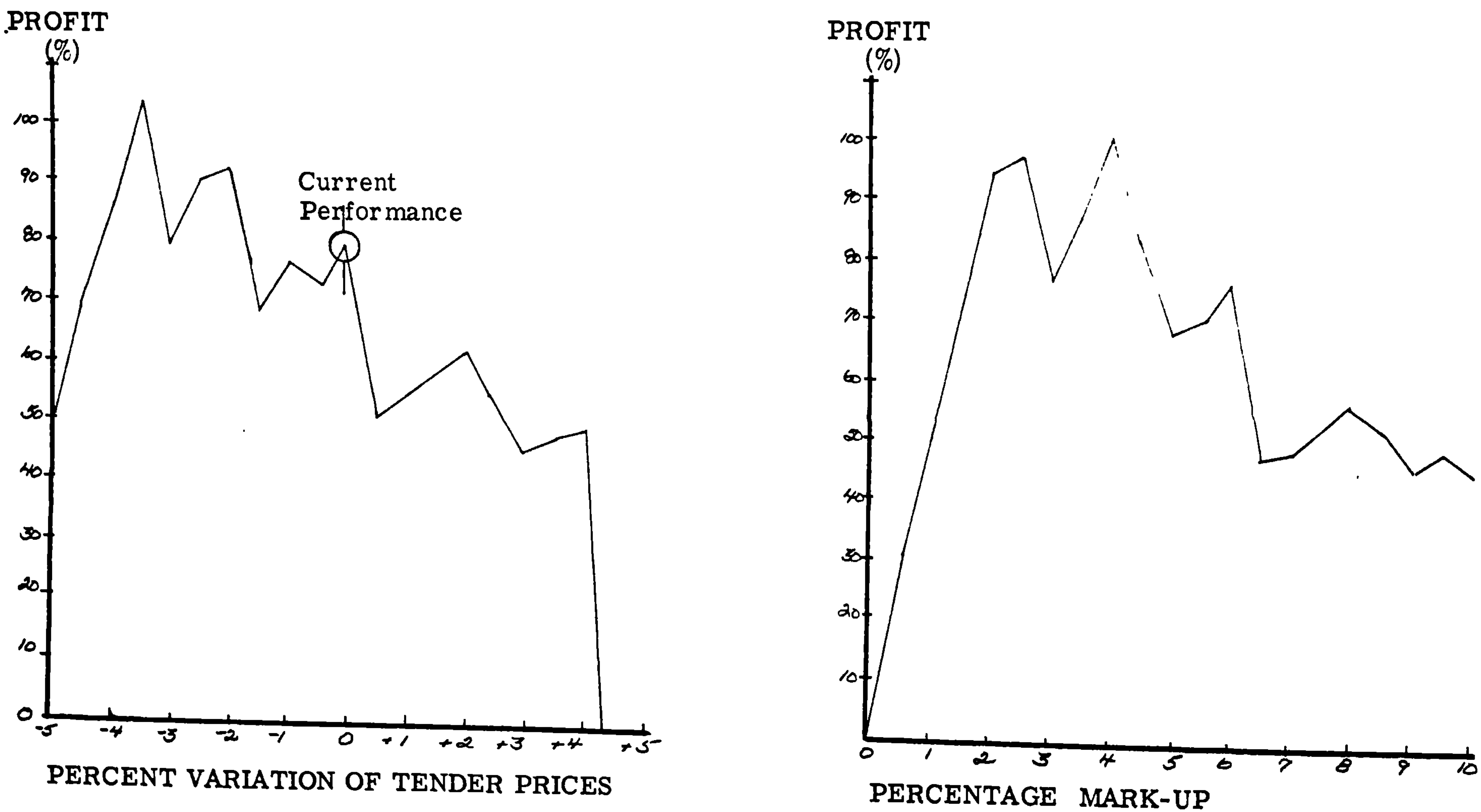
As mentioned above, the OPTM and OPT2 programs were run with the data from the four sample companies. Figures 2.11 to 2.14 are the plotted results. The position of the current company performance is indicated with a circle.

From the graphs it does not appear as if the companies are bidding to maximize profits. The OPT2 graphs indicate that the maximum profit region occurs at mark-ups of $2\frac{1}{2}\%$ to 4%. The companies bid in the region of $6\frac{1}{2}\%$. Considering the OPTM graphs, Company A's bids are 1% below the peak position; Company B's are $3\frac{1}{2}\%$ above; Company C's are 1% above; and Company D's are $3\frac{1}{2}\%$ above the peak. These graphs were discussed with the managements concerned and, as could be expected, Company A agreed that a 1% increase it is bids would have been desirable. On the other hand, Companies B, C, and D flatly rejected any suggestion that they might improve their positions by lowering their bids. To support their point, they revealed that they were losing money on some contracts at the current price level (one contract in three and one in five were the figures quoted), and were certain that a reduction in prices would not improve this situation. A suggestion of bidding in the



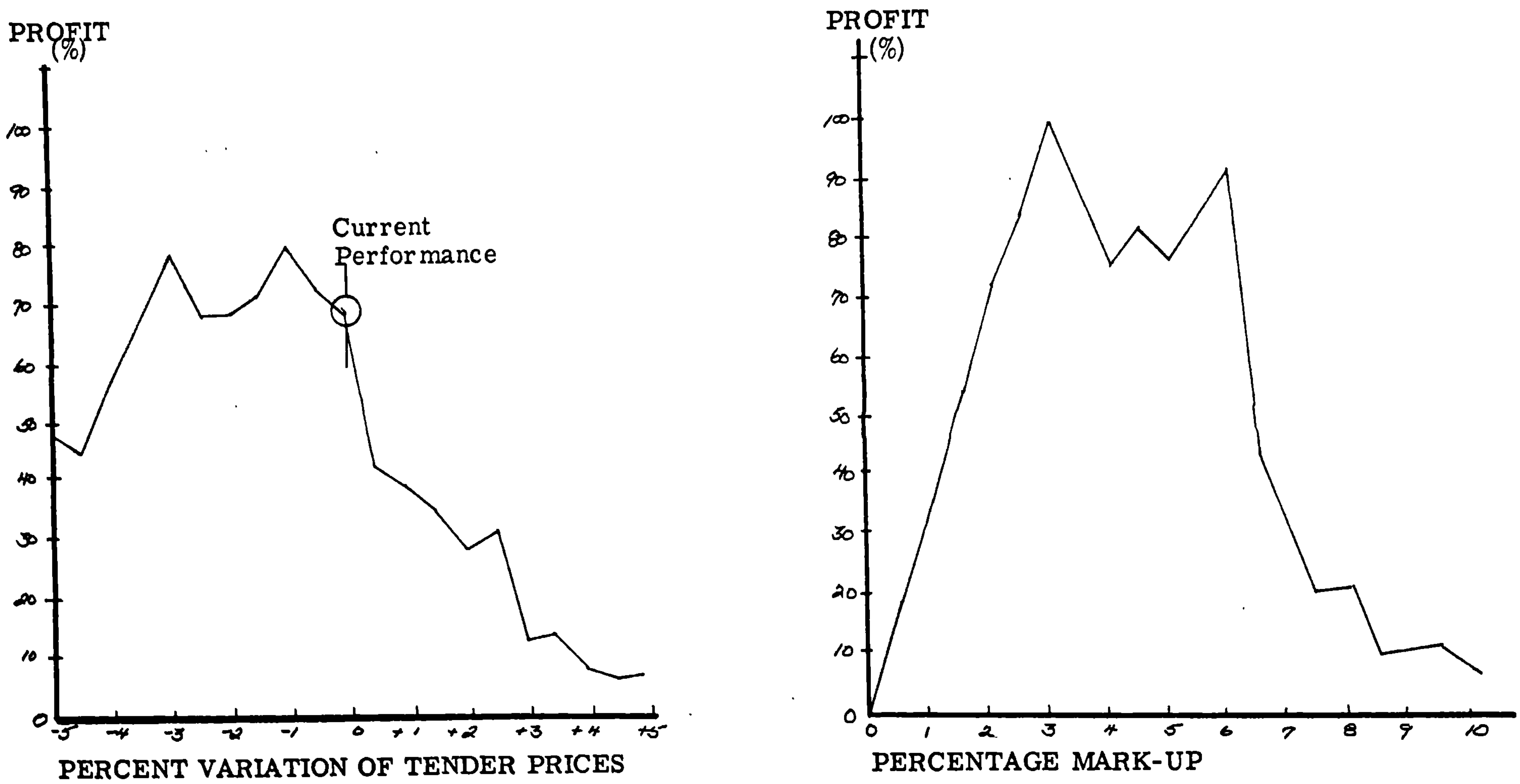
OPTM & OPT2 RESULTS - COMPANY A

Figure 2.11



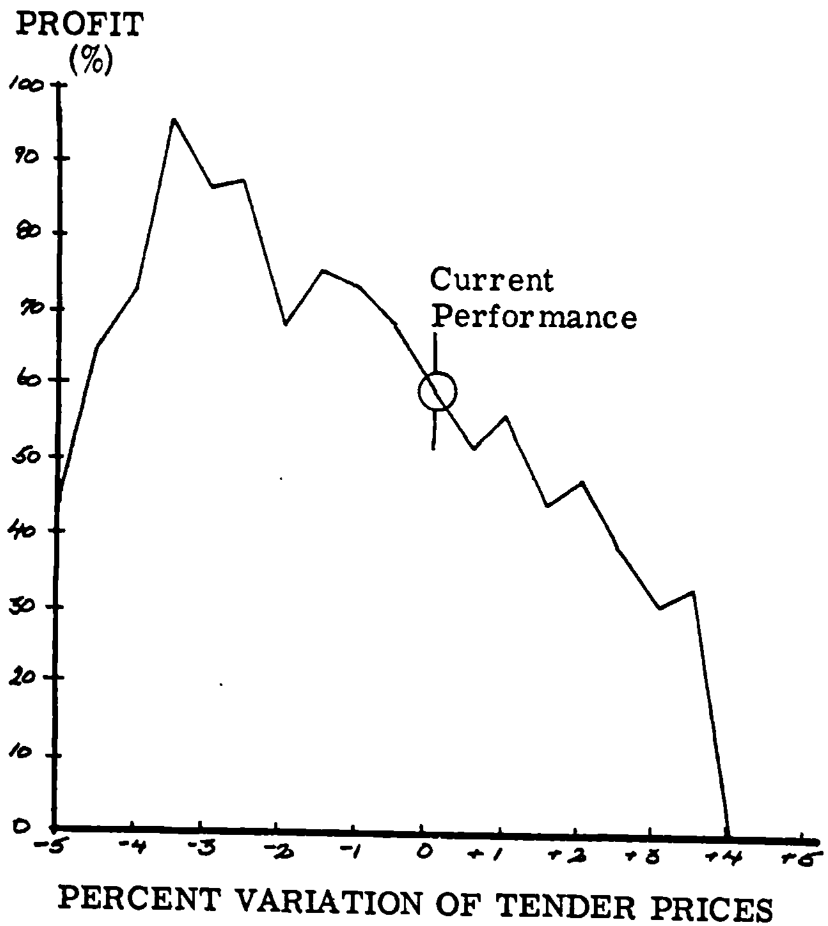
OPTM & OPT2 RESULTS - COMPANY B

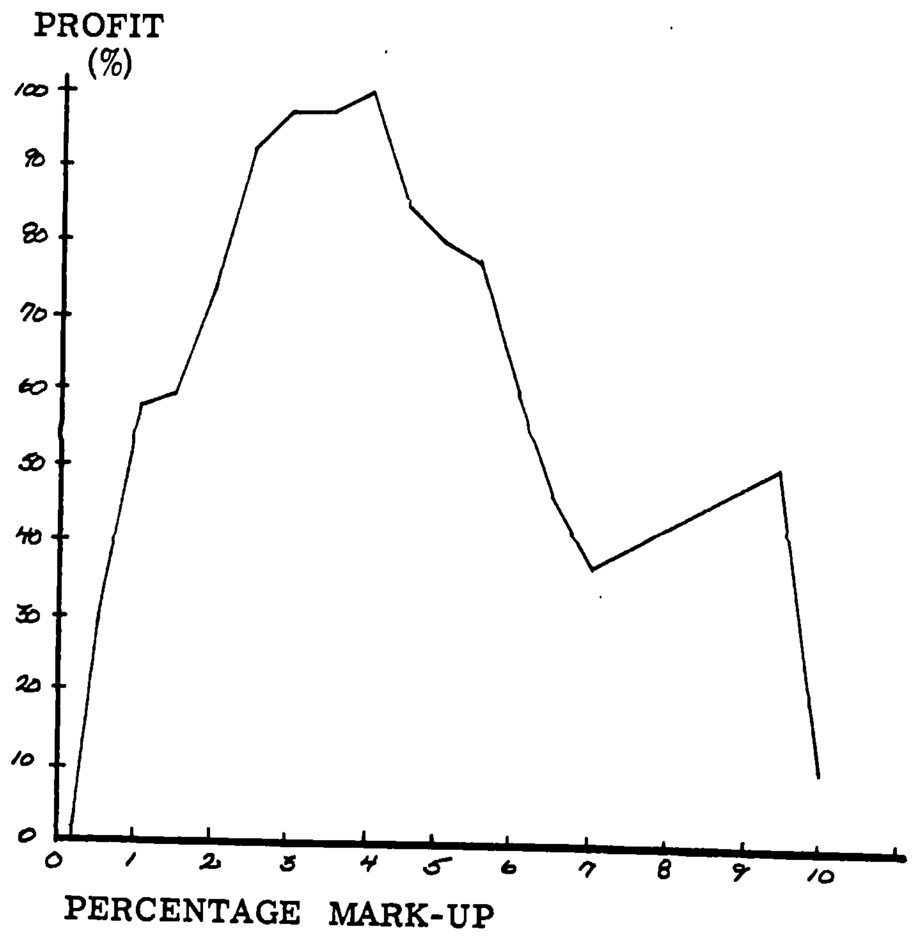
Figure 2.12



OPTM & OPT2 RESULTS - COMPANY C

Figure 2.13





OPTM & OPT2 RESULTS - COMPANY D

Figure 2.14

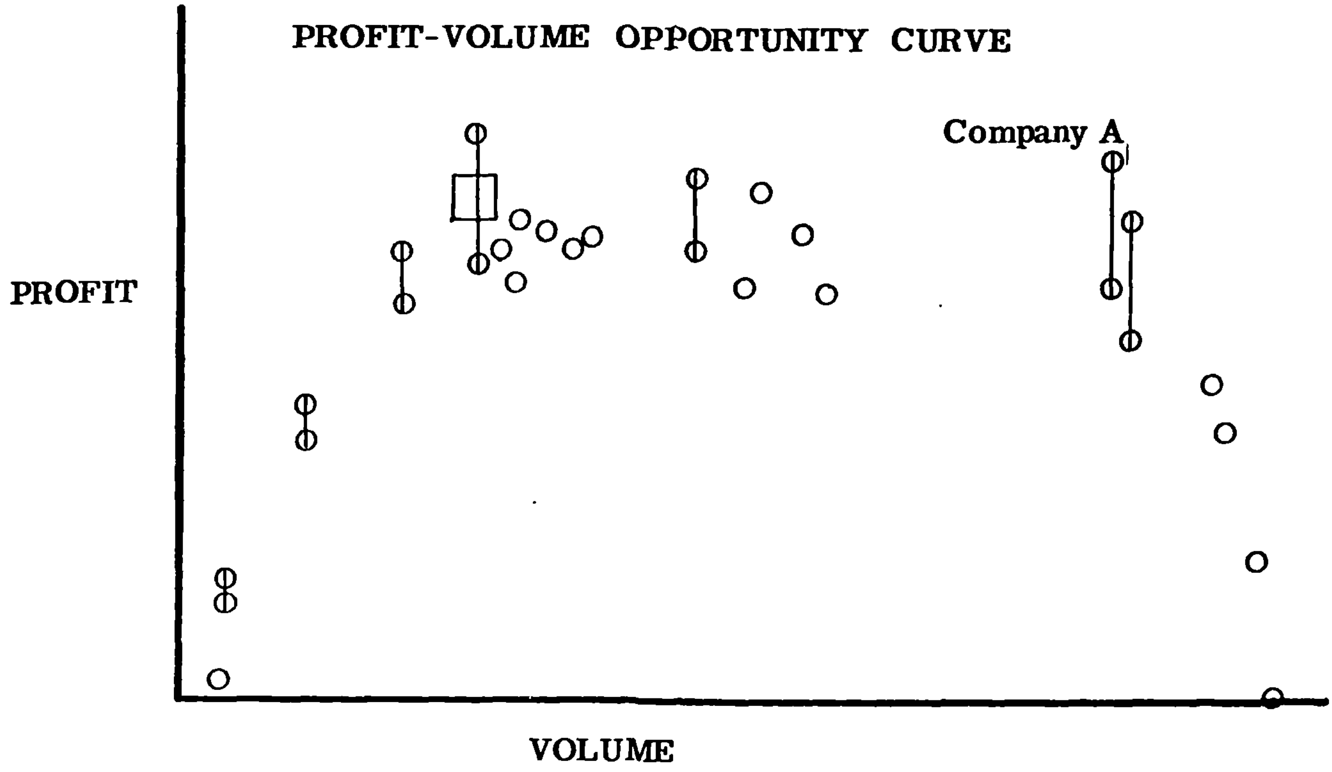
$2\frac{1}{2}\%$ to 4% range was treated with scorn.

Now the sample companies were not in financial difficulties, and all were reasonably happy with their past performance. If it is accepted that their behaviour is rational then either the maximum profits criteria is not applicable to this situation or the assumption of fixed overheads and/or the assumptions in the (x-k) definition of gross profits are distorting the results.

The assumption of fixed overheads was investigated first. From the OPTM and OPT2 output, profit-volume diagrams were prepared. These are Figures 2.15 and 2.16; the squares indicating actual company performance. Smooth curves were fitted by eye to the data and the variable overheads were estimated by calculating the tangent of the curve at the location of actual performance. This procedure produced the following results.

Estimated variable overhead for	Company A	=	19%
	Company B	=	28%
	Company C	=	3.5%
	Company D	=	3.7%

These results are inconclusive. The figures for Companies C


PROFIT-VOLUME OPPORTUNITY CURVE
Company A
PROFIT
VOLUME

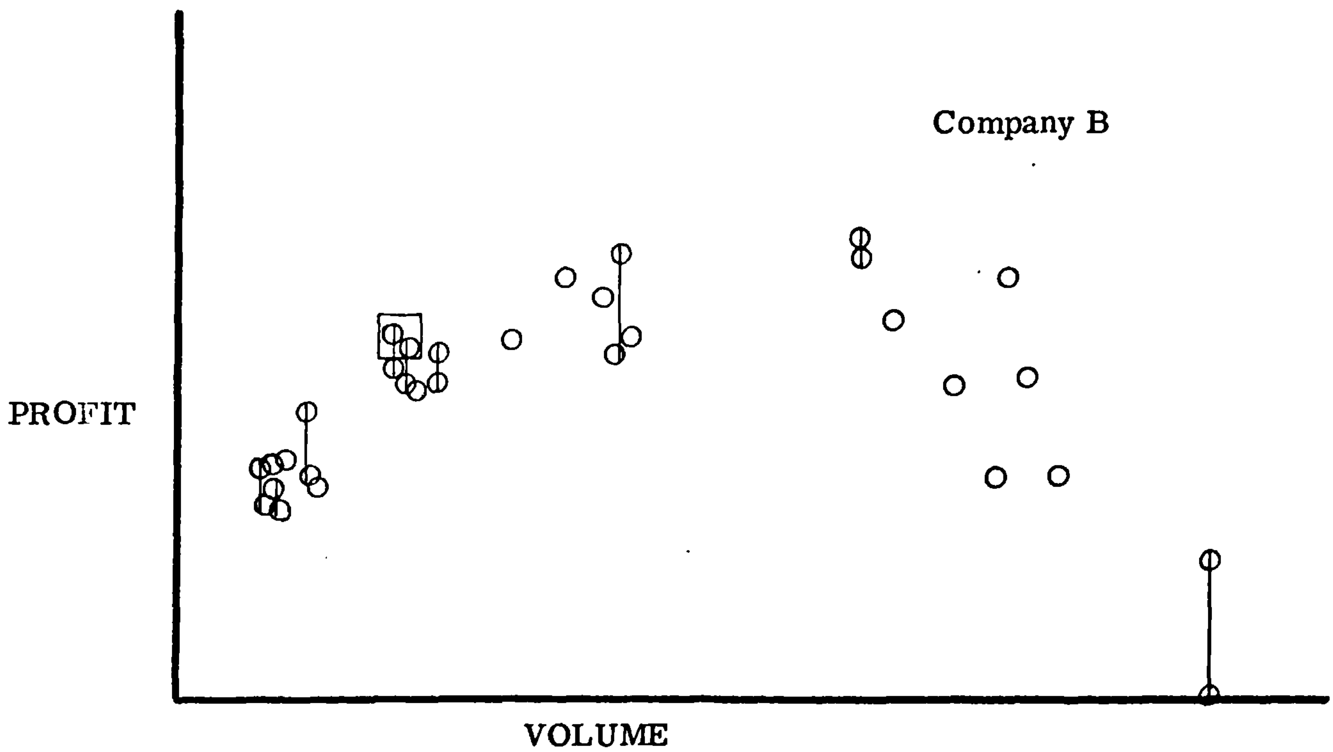
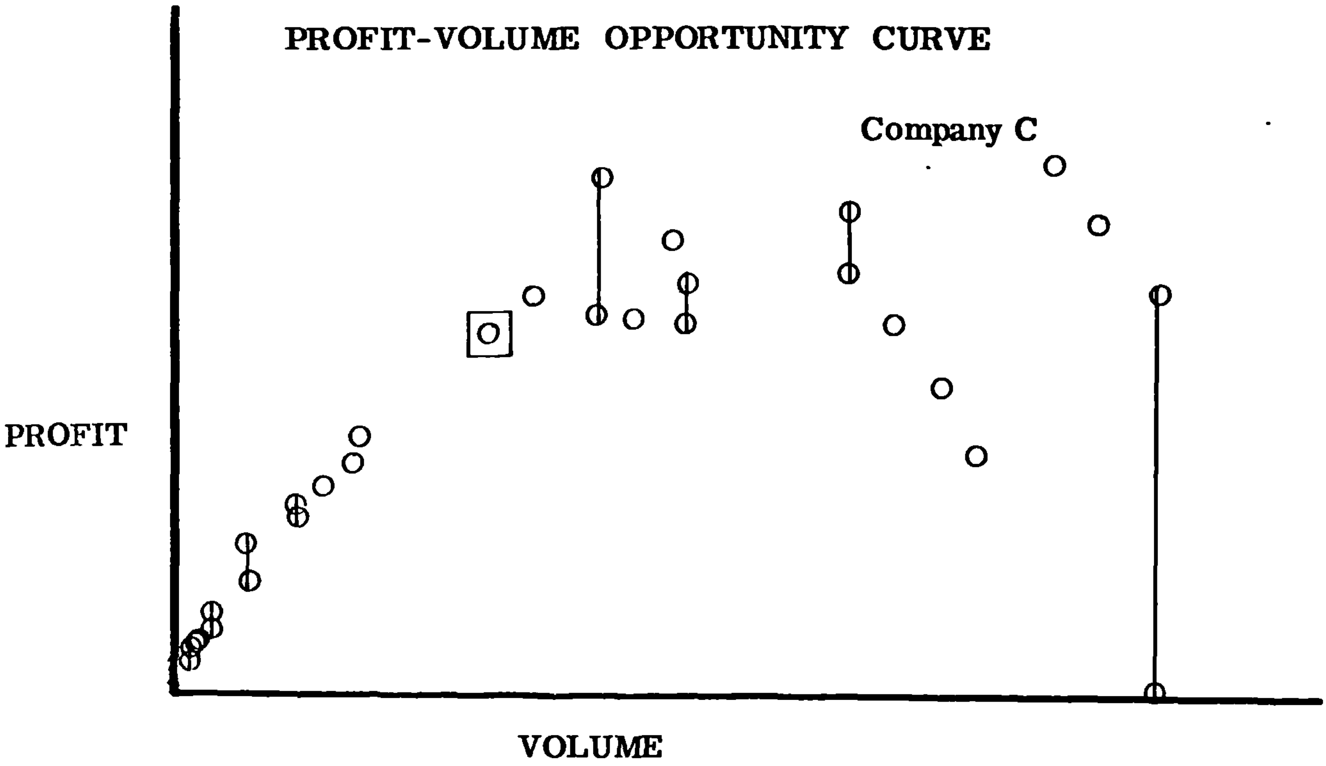

Company B
PROFIT
VOLUME

Figure 2.15


PROFIT-VOLUME OPPORTUNITY CURVE
Company C
PROFIT
VOLUME

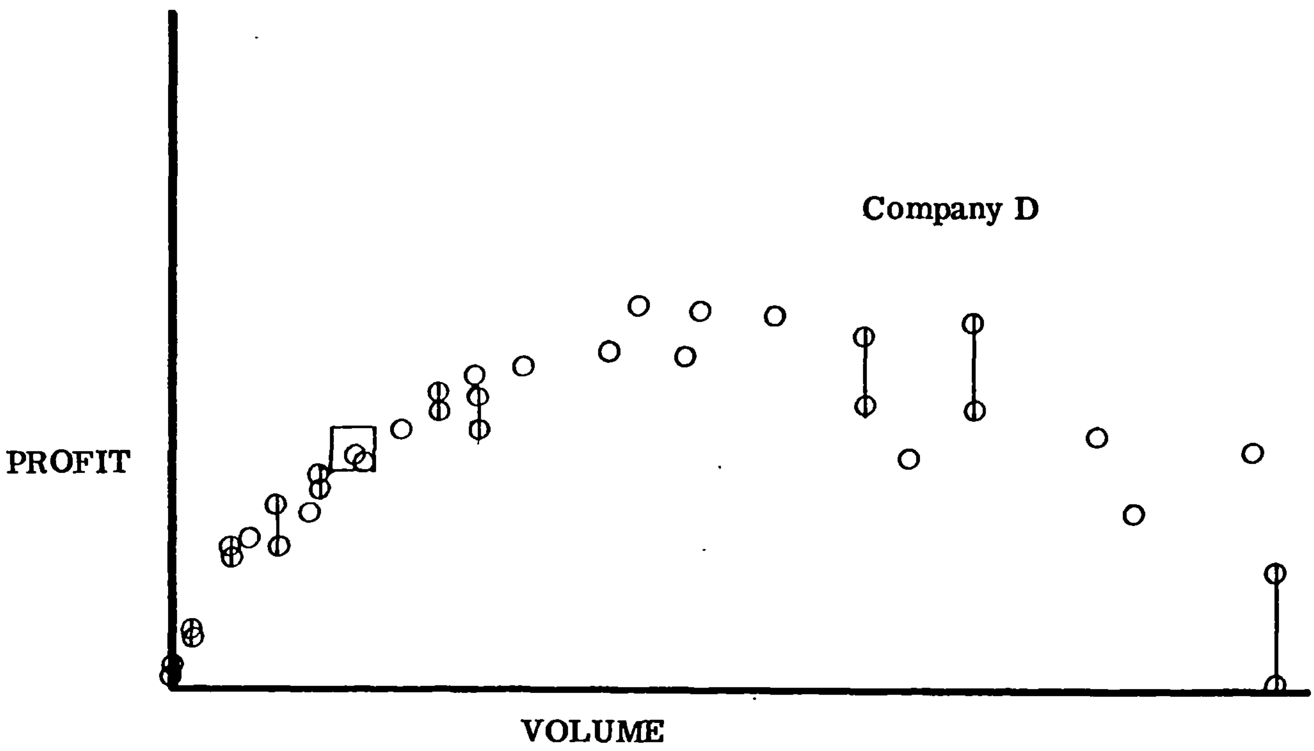

Company D
PROFIT
VOLUME

Figure 2.16

and D could be reasonable, but those for A and B are ridiculous. Considering the approximations and opportunities for error that went into the determination it is reasoned that no definite conclusion can be drawn from this procedure. However, since the companies claim that all variable overheads were included in the estimated costs it is felt that they are not the explanation for the discrepancy between the received theory and practice.

It is apparent from these results that the primitive concept of gross profit maximization, as defined earlier, is at variance with actual behaviour. This is taken as an indication that further research is required in this area. Consequently, this aspect is investigated in the following sections.

A detailed examination of the output from the OPTM and OPT2 programs also revealed the following:-

(a) Company A - If all tenders had been increased by 1%, the company would have achieved a 16% increase in gross profits, a 16% increase in their profit-volume ratio, and exactly the same volume of contracts.

- The uniform mark-up policy in the same region as the current performance would have won exactly the same contracts but with 2% less gross profit.

(b) Company B - An increase of 2% in tender prices would have achieved a 22% decrease in gross profits, a 43% decrease in volume, and a 38% increase in profit-volume ratio.

- A uniform mark-up policy in the same region as the Company's performance would have achieved 4% less gross profits on $2\frac{1}{2}$% greater volume.

(c) Company C - A decrease of 1% in tender prices would have achieved an 18% increase in profits, a 36% increase in volume, and a decrease in the profit-volume ratio of 14%.

- A uniform mark-up policy in the same region as the Company's performance would have achieved a 41% increase in gross profit, a 36% increase in volume, and a 3% increase in the profit-volume ratio.

(d) Company D - An increase of 1% in the tender prices would have achieved a 7% decrease in gross profits, a 20% decrease in volume, and a 16% increase in profit-volume ratio.

- A uniform mark-up policy in the same region as the Company's performance would have achieved a 1% decrease in gross profits, a 1% increase in total volume, and a 2% decrease in the profit-volume ratio.

A general observation is that the decision maker can usually outperform the arbitrary policy. This was the result in three out of four cases, and in the fourth case the management hastened to point out that the sample contained only a few contracts and a single contract can alter the results. It appears that the decision maker's judgment is a significant factor in how well a company performs.

The following specific observations were made for the individual companies.

Company A - an increase of 1% in the tender prices would have produced a more desirable result.

Company B - the desirable policy for the Company is dependent upon the state of the market of the period. If the Company has bid all or most of the contracts available, its current policy was the best. If more jobs were available, (about 50% more), then a 2% increase in tender prices would have more than compensated for the additional estimating that would have been required to achieve the same volume.

Company C - either the current policy or a fixed mark-up policy of 6% are suitable. Any increase or decrease from the region of the current position causes spectacular, but not necessarily desirable results.

Company D - An increase of 1% in all tender prices would have produced a more desirable result.

These observations were discussed with the managements of the companies concerned and general agreement was obtained.

2.4.4 Conclusions

The general conclusion that is drawn from this feasibility study is that competitive bidding in the Construction Industry could be a very fruitful field of investigation. Obvious areas of both academic and practical significance are:

1. The determination of an economic objective function that corresponds to the performance and desires of the sample companies.

2. The development of a system that can realise at least some of the benefits that are revealed by hindsight.

3. The creation of a method of incorporating judgment into an operational research bidding strategy model (since the results indicated that the decision maker's judgment is a relevant factor).

2.5 MANAGEMENT OBJECTIVES

Fundamental to a study of a decision process is a concise definition of objectives. But what are the objectives of a company using the Competitive Bidding Process? When discussing company objectives the lists became long, sometimes confusing, and often contradictory. Some of the possible objectives of a construction company could be:

Maximize Profit,
Maximize Utility,
Minimize Regret,
Minimize the profits of competitors,
Minimize the profits of a particular competitor,
Obtain a certain percentage of the market,
Obtain a certain segment (e.g. prestige buildings) of the market,
Maintain a constant work load,
Achieve a specified return on invested capital,
Make a specified amount of profit,
Obtain a certain volume of work.

The question is which of these, or which combination of these, is appropriate for the Sample Companies? It has already been demonstrated, in section 2.4, that the naive "maximize gross profits" objective is not suitable. An arbitrary selection from other members of the list are equally likely to be unsuitable. Clearly, a more sophisticated approach to determining a quantitative objective function is required.

The managerial objective function proposed for the sample companies is developed from three assumptions.

1. The firm is trying to maximize some value measure.
2. The value measure has an economic basis.
3. The current behaviour of the companies is in the region of the maximum.

The first two assumptions are fairly conventional. It could be argued that the maximizing of an economic value measure is an oversimplification and that satisficing and non-economic considerations also enter the problem. However, some simplification is necessary to reduce the problem to one that can be modelled. Also most non-economic objectives , for example continued employment of loyal staff, can be accommodated as constraints, and satisficing of secondary objectives if often compatible with the maximization of a primary one. The third assumption arises from the fact that the Sample Companies are adaptive, viable, entities that exist in a competitive market and are capable of changing their behaviour if circumstances dictate. The concept, that managerial behaviour is near optimal, was formalised by Bowman in his 1963 paper, "Consistency and Optimalty in Managerial Decision Making" (3). His first two axioms are:

1. Experienced managers are quite aware of and sensitive to the criteria of a system.

2. Experienced managers are aware of the system variables which influences these criteria.

The effect of the third assumption is to provide a means of testing the developed objective function. Data is available for the Adjudication stage of the decision and the search is directed to finding an objective function for that stages that imitates the current performance of the sample companies.

2.5.1 Decision Variables

For the objectives to be relevant to the decision problem they must be related to the variables present in the problem. In the Competitive Bidding Problem, at a specified time, there exist the following variables:

S - The set of opportunites,

X - The vector of tender prices (defined on S),

K - The vector of estimated costs (defined on S),

Y - The vector of lowest competitors' bids (defined on S),

L - The vector of costs of bidding (defined on S).

The resources required by a contract are not considered variables, but are constraints.

The variables can be divided into two groups - - controllable and uncontrollable. The set of opportunities, S, is a controllable variable that can be varied from all the contracts offered to zero by the Decision to Tender. Other methods of controlling this variable are discussed in Section 2.5.4, Market Uncertainty. The tender prices, X, are controlled by the Adjudication Decision. The estimated costs, K, are stochastic variables. The variability can be controlled by the Selection Decision and the bias by correction. The costs of bidding, L, are binary variables; the Decision to Tender determining whether the value is 0 or L. <u>The lowest competitor's bids, Y, are the uncontrollable variables</u> in the decision situation. Chapter 3 is concerned with the statistical treatment of these uncontrollable variables. In this section the value measurements will be formulated as discontinuous functions dependent upon the value of the variables Y. For example:

$$\text{Value of Contract i} = \begin{cases} v_A & \text{if } x_i < y_i \\ v_B & \text{if } x_i > y_i \end{cases} \qquad (2.5.1\text{-}1)$$

2.5.2 Uncertainty

It is assumed that the value measure is basically economic. This means that the value of a contract, if it is won, should be some function of the difference between receipts and costs. Formally

$$\text{Value} = v(\text{ receipts } - \text{ costs }) \qquad (2.5.2\text{-}1)$$

Excluding low probability events such as the client going bankrupt, the following relationships are approximately correct.

Receipts = bid price + change in receipts due to variations

Costs = estimated cost + error due to estimate uncertainty
+ change in costs due to variations.

(2.5.2-2)

For the market being considered contractual risk is not of major significance and so is not included, except where it is implicit in the estimated cost.

Combining the expressions, (2.5.2-2), with equation (2.5.2-1) and combining the costs and receipts from variations into a single term provides

$$\text{Value} = v(\text{bid price} - \text{estimated cost} + \text{estimate uncertainty} + \text{contract uncertainty})$$

If x_i = Bid price on contract i

y_i = lowest competitor's bid on contract i

k_i = estimated cost of contract i

e_i = variation due to estimate uncertainty

u_i = variation due to contract uncertainty

Then

$$\text{Value of Contract i} = \begin{cases} v(x_i - k_i + e_i + u_i) & \text{if } x_i < y_i \\ 0 & \text{if } x_i > y_i \end{cases}$$

(2.5.2-3)

The factor, u_i, is called the <u>contract uncertainty factor</u>. It contains the results of the contract variations: how the job will progress and how the company will fare in the transition from bill of quantities items to constructed structure. Referring back to Section 2.3.2, items such as reputation of client, quantity surveyor, and consultants; design progress; unresolved problems; contract conditions; managerial requirements; etc. are included in u_i. It can normally be regarded <u>as a function of the contract</u> and not of the bid price, therefore it is treated as a parameter of the function v.

The factor e_i is the estimating uncertainty. It is the error resulting from the information gaps in the estimating, the subjective nature of estimating, and computational errors. Estimating departments try to make this error zero; that is they try to make the expected value of the estimated cost equal to what the true cost would be if u_i did not exist. The presence of u_i complicates matters but partial feedback and comparison with other bids permits reasonable control. This error is more of a function of the estimating department than of the contract.

Rewriting equation (2.5.2-3) with u_i as a parameter and using the expected value of the error in k_i, $E(e_i)$ to handle estimate uncertainty

$$\text{Value of Contract } i = \begin{cases} v(u_i\,;\ x_i - k_i + E(e_i)) & \text{if } x_i < y_i \\ 0 & \text{if } x_i > y_i \end{cases}$$

(2.5.2-4)

This is the general formulation of the value function. For clarity of exposition, the cost of bidding, L, is left out of this formulation. It will be introduced later when the Selection Decision is considered.

2.5.3 Interaction Effect

That the estimated cost is a stochastic variable has been recognised by almost all previous investigators. Friedman (11) in 1956 first proposed using the expected value of the estimated cost to handle any possible bias that might exist in the estimate. What has been generally overlooked is that the Competitive Bidding Process introduces a bias into the estimated costs of successful tenders. Simmonds (29) noted this interaction but did not develop it mathematically.

This interaction can de demonstrated for the case where the contract is awarded to the lowest bidder as follows. Assume:

1. That a "true cost" exists. That is, that there is no contract uncertainty, u_i, only estimating uncertainty.
2. That the estimated costs of all bidders on a contract are random samples from a uniform distribution with range

$$\text{true cost} \pm \frac{R}{2}$$

That is, the estimated cost is not biased and the expected value of the estimated cost is the true cost. Moreover estimates are uniformly distributed over the stated range.

3. That the bidder with the lowest estimated cost is the low bidder.

The basis of these assumptions is discussed at the end of this Subsection. First, however, it is demonstrated how these assumptions interact with the Competitive Bidding Process to introduce bias into the cost estimate of the winning bid.

If c is the lowest of N random samples from a distribution h(z); then the frequency distribution of c is

$$f(c)dc = N\left(1 - \int_{-\infty}^{c} h(z)dz\right)^{N-1} h(c)dc \qquad (2.5.3\text{-}3)$$

This formula is derived in Section 3.2.2, Order Statitistic Development. If on a project there are N bidders, all with estimated costs derived according to assumption 2; then if assumption 3 holds the frequency distribution of the winning bidder's estimated cost is again f(c) and

$$f(c)dc = N\left(1 - \int_{\text{true cost} - \frac{R}{2}}^{c} {}^{1}/_{R}\, dz\right)^{N-1} \frac{1}{R}\, dc$$

$$= \frac{N}{R}\left(\frac{1}{2} - \frac{c}{R} + \frac{\text{true cost}}{R}\right)^{N-1} dc$$

(2.5.3-2)

To derive a formula in percentage terms, Let

$$h(z) = 0 \qquad -\infty < z < 100 - R/_2$$

$$h(z) = \frac{1}{R} \qquad 100 - R/_2 \leq z \leq 100 + R/_2$$

$$h(z) = 0 \qquad 100 + \frac{R}{2} < z < +\infty$$

(2.5.3-3)

Where h(z) is the distribution of the estimates, the true cost is 100, and R is the estimating range in percent.

For a specified error range of R percent, the expected percentage amount that the cost estimate of the winning bidder is below the true cost is:

$$E\,(100 - c) = \frac{N}{R} \int_{100 - \frac{R}{2}}^{100 + \frac{R}{2}} (100 - c) \left(\frac{1}{2} - \frac{c}{R} + \frac{100}{R} \right)^{N-1} dc$$

$$= \frac{100\,N}{R} \int_{100 - \frac{R}{2}}^{100 + \frac{R}{2}} \left(\frac{1}{2} - \frac{c}{R} + \frac{100}{R} \right)^{N-1} dc - \frac{N}{R} \int_{100 - \frac{R}{2}}^{100 + \frac{R}{2}} c \left(\frac{1}{2} - \frac{c}{R} + \frac{100}{R} \right)^{N-1} dc$$

$$= R \left\{ \frac{N - 1}{2\,(N + 1)} \right\}$$

(2.5.3-4)

For a specified participant, if n is the number of competitors, then $n = N - 1$, and the expected percentage value that his cost estimate is below the true cost is

$$R \left\{ \frac{n}{2\,(n + 2)} \right\}$$

(2.5.3-5)

if the contract is won.

The results of this interaction can be demonstrated by example. If a company has an estimate range of 10% and is bidding against 5 competitors; the expected value of the estimated cost, if the company wins, is

$$10 \left(\frac{5}{2.7} \right) = 3.57\% \text{ below the true cost.}$$

This calculation partially explains how a contractor can bid his jobs at a 5% mark-up and finish with a gross profit only slightly over 1%.

Let $Z(R, n)$ be a multiplier that corrects the expected error in the estimated cost.

$$Z(R, n) \; E(c) = 100$$

$$Z(R, n) \; (100 - E(100 - c)) = 100$$

$$Z(R, n) = \frac{200(n + 2)}{200(n+2) - Rn} \qquad (2.5.3\text{-}6)$$

Using the earlier example of $R = 10\%$ and $n = 5$;

$$Z(10,5) = \frac{1400}{1400 - 50} = 1.037$$

and the contractor must bid at a 3.7% mark-up to break even.

Inserting the Z multiplier into equation (2.5.2-4), the value function becomes

$$\text{Value of Contract i} = \begin{cases} v(u_i\ ;\ x_i - k_i \cdot Z(R,n)) & \text{if } x_i < y_i \\ 0 & \text{if } x_i > y_i \end{cases}$$

(2.5.3 - 7)

Discussion of Assumptions

Three assumptions were used to create a hypothetical model of the situation from which the interaction error could be calculated. These assumptions are not strictly correct, i.e. contrary examples can be found, but they are reasonably supported by intuition and the empirical evidence available. To the extent that it was possible to check, the hypothetical model can be said to be a reasonable mapping of reality.

The assumption of the existence of a true cost at the estimating stage, assumption 1, is reasonable. However, the assumption must rest on intuitive grounds since subsequent variations make it practically impossible to recreate the conditions and demonstrate its existence.

Assumption 2 is the principle basis for the development of the model. It can be decomposed into three parts:

- That the estimated costs are random samples from a distribution.
- That the distribution is symmetric about the true cost.
- That the distribution is uniform.

The random sample concept is fundamental to the statistical approach. The inherent uncertainties in the estimate and the number of individual items and decisions that go into the determination of the estimated cost combine to produce a result that can be described as a chance or random process. One of the Sample Companies related the following experiment they conducted. They assigned the same job to two different estimators and instructed them to independently arrive at a net estimated cost. The resulting two estimated costs differed by 16%. Thus the idea of an estimated cost being a random sample seems not unreasonable.

A uniform distribution with arbitrary cut-off points may initially appear unrealistic; but no more so than one without cut-offs. Within every contract there exist several figures that define the region of the price. They are the clients budget, the architect-q. s. estimate, and the price of similar structures. There are indications that an estimate deviating substantially (say 15%-20%) from the true cost will be detected and rejected by the management at the adjudication phase. One of these indications is that for all the bids in the Sample Companies' Data Sets, the average value of the range (highest bid on a contract minus the lowest bid on the contract) is approximately 10%. Fine and Hackemer (10) in their simulation generated bids by taking random samples from a uniform distribution. They reported that the distribution of the bids generated compared very favourably with the distribution of bids derived from the company bidding records.

The obvious way to verify the assumption would be by studying the distribution of estimated costs. This information was not available to this study, and it is unlikely that it could be obtained by any study since it requires competitors to disclose their estimated costs. Therefore an alternative method had to be found.

The four Sample Companies were found to be using very similar percentage mark-ups. The average mark-ups of the four companies were all within 0.35% of 6.8%. If this result is generalised to all firms competing, and interviews with personnel from other companies has suggested that it can, then the distribution of estimates should be similar to the distribution of bids. This similarity is examined in Subsection 3.4.2 and the uniform distribution is reasonably coincident with that derived from actual bid values.

The assumption of a symmetrical distribution can be partially substantiated by the empirical evidence. It the mean bid is taken as an estimate of the true cost plus 6.8% (the average mark-up used), then the ratios mean bid to estimated cost should be samples from a distribution with mean 1.068. The hypothesis that the mean of the ratios was not significantly different from 1.068 was tested for each of the four companies with a t test and in each case accepted at a 10% level. The data of the four were combined and the resulting 153 ratios were plotted to form a histogram. The histogram had a symmetrical shape with 75 ratios below 1.068 and 78 above. This symmetry assumption is not necessarily valid for the American-Canadian construction industry where each contractor calculates his own quantities and as Park (24) suggests, "errors of ommission are more likely than errors of commission". The use of the Bill of

Quantities in the English system makes it unlikely that anything can be missed.

The assumption that the lowest estimated cost wins the contract is supported by two figures. The range of the average mark-up used by the four Sample Companies was 0.7%; yet the average amount by which the contracts in the data set were won was 2.8% (low bid minus second low bid). Obviously there are cases where the different mark-ups will cause the second, or third low estimate to win but in general, it appears that the lowest estimated costs results in the lowest bid. This point is further substantiated in Section 5.3.5 where the different models tend to win the same contracts.

As was stated at the start of this discussion, it is not proposed that the assumptions are strictly correct. It is suggested that the assumptions are reasonable and that the resulting model is a fairly realistic mapping of the situation.

2.5.4 Market Uncertainty

In addition to the estimating uncertainty and the contract uncertainty, market uncertainty enters the decision process.

The construction industry is subject to major fluctuations because it is sensitive to the government's methods of regulating the economy. The decision variable S, the set of opportunities, is subject to these fluctuations. However, this does not mean that S is not subject to management control.

Some of the methods of increasing S are:

- pressure on the government to stablize the market. The construction industry represents a major pressure group with organisations such as the R.I.B.A and the N.F.B.T.E., as well as the unions, to voice its complaints.

- diversification into other areas of the field. It is becoming common for major contractors to form alliances with property developers and industrial concerns and thus provide themselves with tied markets. Also package deals and patented systems provide a certain insulation from the major market gyrations.

- promotional activity. This type of activity can range from buying a potential client lunch to bidding a job "at cost" to impress a particular architect. Generally, almost any form of activity other than sitting in the office waiting for invitations to tender can be regarded as promotional activity.

Methods of decreasing S are to decline jobs and to submit cover bids.

The present state of the market and the expectation of the future are the two aspects of the market which influence the bidding decision. These two aspects have differing characteristics. The present state of the market is reflected in S, and this variable is known in detail and subject to control. The future is unknown and the best information available can only assist in predicting trends. The expectation of the future influences the relative value of the contracts in the set S. For example, if it is anticipated that next years market will provide more profitable opportunities, then a one year contract in S will be relatively more desirable than a similar three year contract, since the one year contract will free resources in time to take advantage of the anticipated opportunities.

The proposed method of treating the market in the decision formulation uses the differing characteristics. Aris (2) paraphrases the dynamic programming principle of optimality as:

". . . If you don't do the best you can with what you happen to have got, you'll never do the best you might have done with what you should have had."

and this is the basic concept underlying the treatment.

It is unlikely that the expectation of the future would directly cause the exclusion of a contract from the set S. A contractor would not let men and plant sit idle because he anticipated an improved market in 12 months. The result of expectation is to change the rankings of the contracts within the set, making some more desirable and others less so. Therefore, attention is concentrated on known opportunities and a parameter that will change the relative values is included in the value function. Chapter 4 deals with the treatment of the known set S; the relative value parameter is introduced in Section 2.5.5.

2.5.5 A General Objective Function

A contract value measure should relate:

- The profitability of the contract,
- The uncertainty of estimating,
- The expectation of the future,
- The existing set of opportunities.

The general value function proposed in Section 2.5.3 is:

$$\text{Value of Contract } i = \begin{cases} v(u_i \; ; \; x_i - k_i \cdot Z(R,n)) & \text{if } x_i < y_i \\ 0 & \text{if } x_i > y_i \end{cases}$$

(2.5.5-1)

One factor not explicitly mentioned in the formulation, but implicit in several of the elements, is the time aspect or duration of a contract. A method of treating time in evaluation procedures is by using a discounted cash flow (D.C.F.) procedure. A D.C.F. method is a theoretically sound basis for valuing contracts and is, in fact, very similar to the techniques currently employed by some contractors. The series of papers by Fine (9) at the Building Research Station indicate that a D.C.F. measure is appropriate for building construction contracts. Although the usual measures mentioned in connection with contracts are profits, or profitability, or margin, or turnover, contractors are acutely aware of the importance of the timing of cash flows. This is illustrated by the fact that unbalanced bidding (See Appendix 2) is widely practiced.

Hence a D.C.F. formulation is proposed for contract valuation. Let:

q_{ij} = the amount of money received at the end of the j th. period from contract i

d_{ij} = the amount of money dispersed at the end of the j th. period for contract i

r = the discount rate

a_{1i} = the present value of the dispersements associated with contract i

$$= \sum_{j} \frac{d_{ij}}{(1 + r)^{j}} \qquad (2.5.5\text{-}2)$$

which is a constant for a specified contract.

The R.I.B.A. Contract Conditions, which are the normal ones for building construction, specify progress payments to the contractor at fixed time intervals. These payments are intended to cover work completed up to a specified date, and are based upon measurement and estimate of work in progress. In actual fact these payments tend to be linear with time. This tendency is caused by two factors. One is the difficulty in estimating accurately the amount of work completed, and the second is the fact that the amount of payment is the result of a bargaining session. The only two items that the parties to the bargaining generally agree upon are the original bid price and the scheduled contract completion time; thus if one half of the time has elapsed, the contractor probably has received one half of his bid price.

The present value of the income stream is

$$\sum_{j} \frac{q_{ij}}{(1 + r)^{j}}$$

Now if the income stream is linear with time

$$q_{ij} = q_i = \frac{x_i}{m}$$

where x_i = bid price on contract i

m = the total number of time periods.

The present value of the income stream is

$$\sum_{j=1}^{m} \frac{\frac{x_i}{m}}{(1+r)^j} = Xi \sum_{j=1}^{m} \frac{1}{m(1+r)^j} = Xi \left\{ \frac{(1+r)^m - 1}{mr(1+r)^m} \right\} \qquad (2.5.5\text{-}3)$$

$$= x_i a_{2i}$$

where $a_{2i} = \dfrac{(1+r)^m - 1}{mr(1+r)^m}$

which is a constant for a specified contract i.

The general value function (2.5.5-1) contains the contract uncertainty parameter, u_i. The present value expressions contain a discount rate parameter, r. On a normal contract the positive cash flows, $(q_{ij} > d_{ij})$, do not occur until the later periods. Therefore if the discount rate is increased, the present value of the cash flows would decrease; but the present values of shorter duration contracts would decrease less severely than those of longer duration. Now a contract value measure which accommodates contract profitability and market uncertainty must have some facility for shifting the relative values of the contracts in the set from their bid minus cost valuation.

The proposed value function for a contract on which a tender is submitted is

$$\text{Value of contract } i = \begin{cases} Z(R,n)\cdot\ a_{1i} + a_{2i}\ \ x_i & \text{if } x_i < y_i \\ 0 & \text{if } x_i > y_i \end{cases}$$

$$\text{where}\quad Z(R,n) = \frac{200(\ n+2\ )}{200(n+2) - Rn} \qquad (2.5.5\text{-}4)$$

R = The estimating range in percent

n = The number of competitors

$$a_{1i} = -\sum^{m} \frac{d_{ij}}{(1+r_i)^j}$$

d_{ij} = The amount of money dispersed at the end of the j th period.

$$a_{2i} = \frac{(1+r_i)^m - 1}{m\quad r_i(1+r_i)^m}$$

r_i = The discount rate for contract i

x_i = The bid price for contract i

y_i = The lowest competitor's bid on contract i

m = The total number of time periods, j .

This is the conventional present value function except that the discount rate is varied from contract to contract. Specifically, a base discount rate is determined by expectation of future markets; an optimistic expectation resulting in a high base rate (favouring shorter contracts), a pessimistic outlook would produce a low base rate. The base rate is then varied for the individual contracts on the basis of managerial expectation of the effect of contract uncertainty:

- downward for contracts where the uncertainty is felt to result in advantageous circumstances.
- upwards for contracts where the uncertainty is felt to result in adverse circumstances.

To make this value function applicable to a specific firm, the base rates and limits of contract uncertainty variation would be determined by empirical investigation.

An additional advantage of using a D.C.F. form of valuation is that future, and therefore more uncertain, events are damped and thus play a smaller role in the decision process.

The cost of estimating contract i, l_i, (a negative constant) does not influence the adjudication stage of the decision. It does affect the selection stage and therefore the value function for the two stage decision is

Value of Contract i

$$\text{if it is bid} = \begin{cases} Z(R,n)a_{1i} + a_{2i}x_i + l_i & \text{if } x_i < y_i \\ l_i & \text{if } x_i > y_i \end{cases}$$

$$\text{if it is not bid} = 0$$

(2.5.5-5)

To formulate the management objective function the term, $P_i(x_i)$, the probability of winning contract i with a bid of x_i, is introduced. This term and the related concepts are dealt with in Chapter 3. Using this probability term, the expected value of the contract if it is to be bid is

$$P_i(x_i)\ (Z\ (R,n)\,a_{1i} + a_{2i}\ x_i + l_i) + (1 - P_i(x_i))l_i$$

$$= P_i(x_i)\ (Z\ (R,n)\ a_{1i} + a_{2i}\ x_i\ ) + l_i \qquad (2.5.5\text{-}6)$$

and the expected value of the contract if it is not to be bid is zero.

The management objective for the two stage decision on the Competitive Bidding Process is taken to be <u>maximize the expected value of the decision.</u> Formally

$$\text{Maximize} \sum_{i \in S} \text{Maximum} \left\{ P_i(x_i)\ (Z\,(R,n)\ a_{1i} + a_{2i}\ x_i) + l_i\ ;\ 0 \right\}$$

(2.5.5-7)

Unfortunately data to test this function, and to determine the ranges for r_i, were not available during this study. Therefore, the objective (2.5.5-7) is only a proposed method of accommodating the variables of the decision situation. However, there are data to test the objective formulation for the sub-problem, the Adjudication Decision.

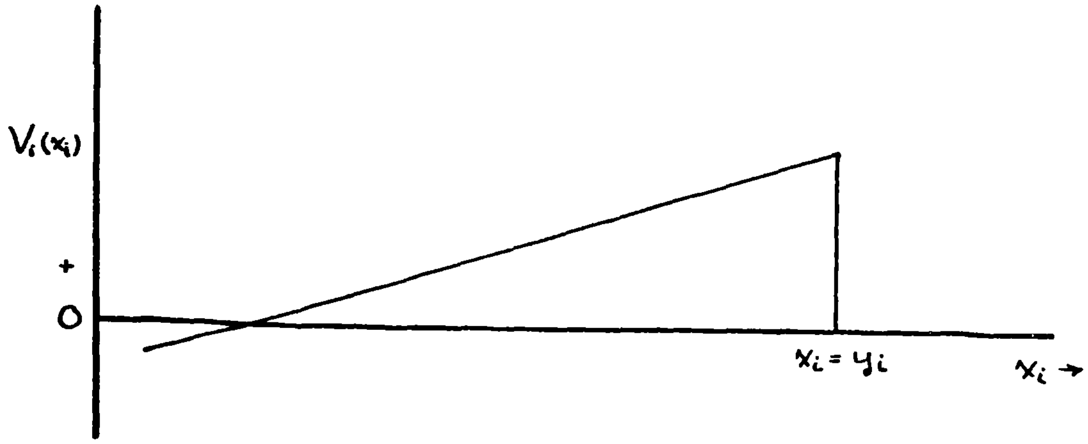
2.5.6 The Adjudication Objective

The contracts in the Sample Company data sets are contracts on which the decision to bid has been made. They all meet the selection criteria and the companies, at the time of bidding, had sufficient resources to undertake them. The managements concerned stated that all the bids were "serious and competitive". In other words, these were contracts that the companies would have liked to win.

Since these contracts have been selected, the elements of the value function that determine selection are irrelevant. The appropriate value function for a contract at this stage is

$$V_i(x_i) = \begin{cases} Z(R,n)\, a_{1i} + a_{2i}\, x_i & \text{if } x_i < y_i \\ 0 & \text{if } x_i > y_i \end{cases} \qquad (2.5.6\text{-}1)$$

The plot of this function is illustrated below. From the plot it is obvious that, if the objective is to



select x_i that maximizes $V_i(x_i)$, as long as the function is increasing with x_i, (a_{2i} is positive), the position of the maximum $V_i(x_i)$ is independent of the values of a_{1i} and a_{2i}. This means that the Adjudication decision is independent of the value of r_i. The data sets contain only the estimated cost, not the timing of the cash flows. This is equivalent to a zero value of r_i. Since the decision is independent of r_i, a zero value can be used and the value function becomes

$$V_i(x_i) = \begin{cases} x_i - Z(R,n)\ k_i & \text{if } x_i < y_i \\ 0 & \text{if } x_i > y_i \end{cases} \tag{2.5.6-2}$$

where x_i = bid price on contract i

$Z(R, n)$ = interaction correction

k_i = the estimated cost of contract i

y_i = the lowest competitor's bid.

This is simply the profit function with the estimated cost corrected for interaction.

The OPTM and OPT2 programs of Section 2.4.2 were used to examine the value measure. In order to conduct the test, it was necessary to determine a parameter R for each of the Sample Companies. Three methods of determining R from the available data were devised. These are presented in Appendix 3. The three methods provided the following values for the Companies.

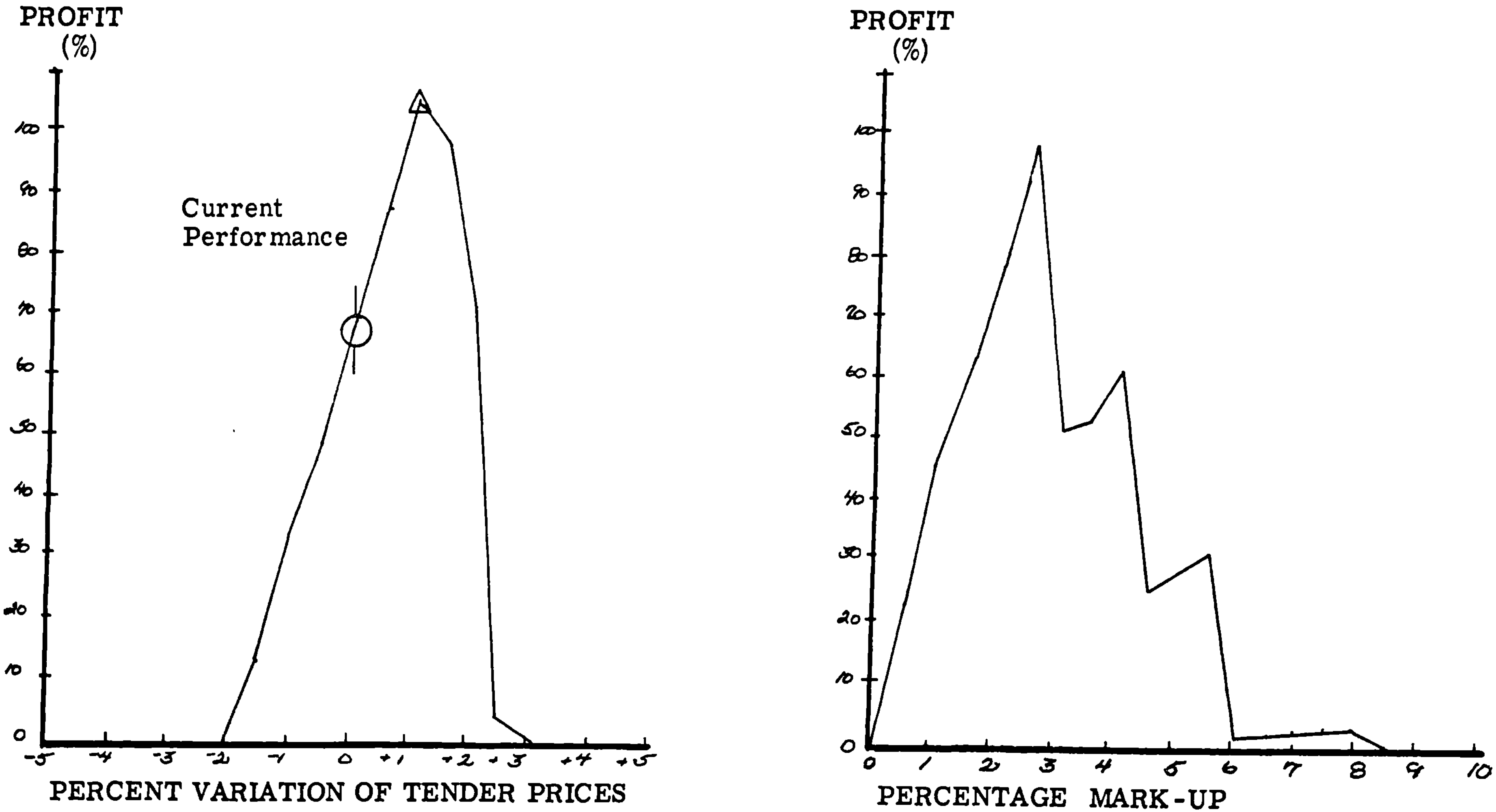
	Method 1	Method 2	Method 3
Company A	12.85%	11.9 %	15.35%
Company B	13.45%	12.9 %	15.5 %
Company C	11.0 %	9.35%	16.65%
Company D	13.1 %	10.7 %	16.95%

These calculated ranges are all within the anticipated region and at this stage no one method was demonstratably more suitable than any other. Therefore the testing was started using all three values for each company.

The Z corrections resulting from the Method 3 range calculations had the effect of reducing the Companies to negligible profit operations. This was felt to be unrealistic and so experimentation with those figures was discontinued. Figures 2.17 to 2.24 are the results of the OPTM and OPT2 programs on the Companies data sets with the estimated costs corrected using the Method 1 and Method 2 ranges. The Company's actual performance position is indicated with a circle; the desired operating position determined in the feasibility study, Section 2.4.3, is marked with a triangle.

For Company A both the Method 1 and Method 2 range estimates produce similar results. The Company is operating 1% below the peak of the graph and the desired position is coincident with the peak.

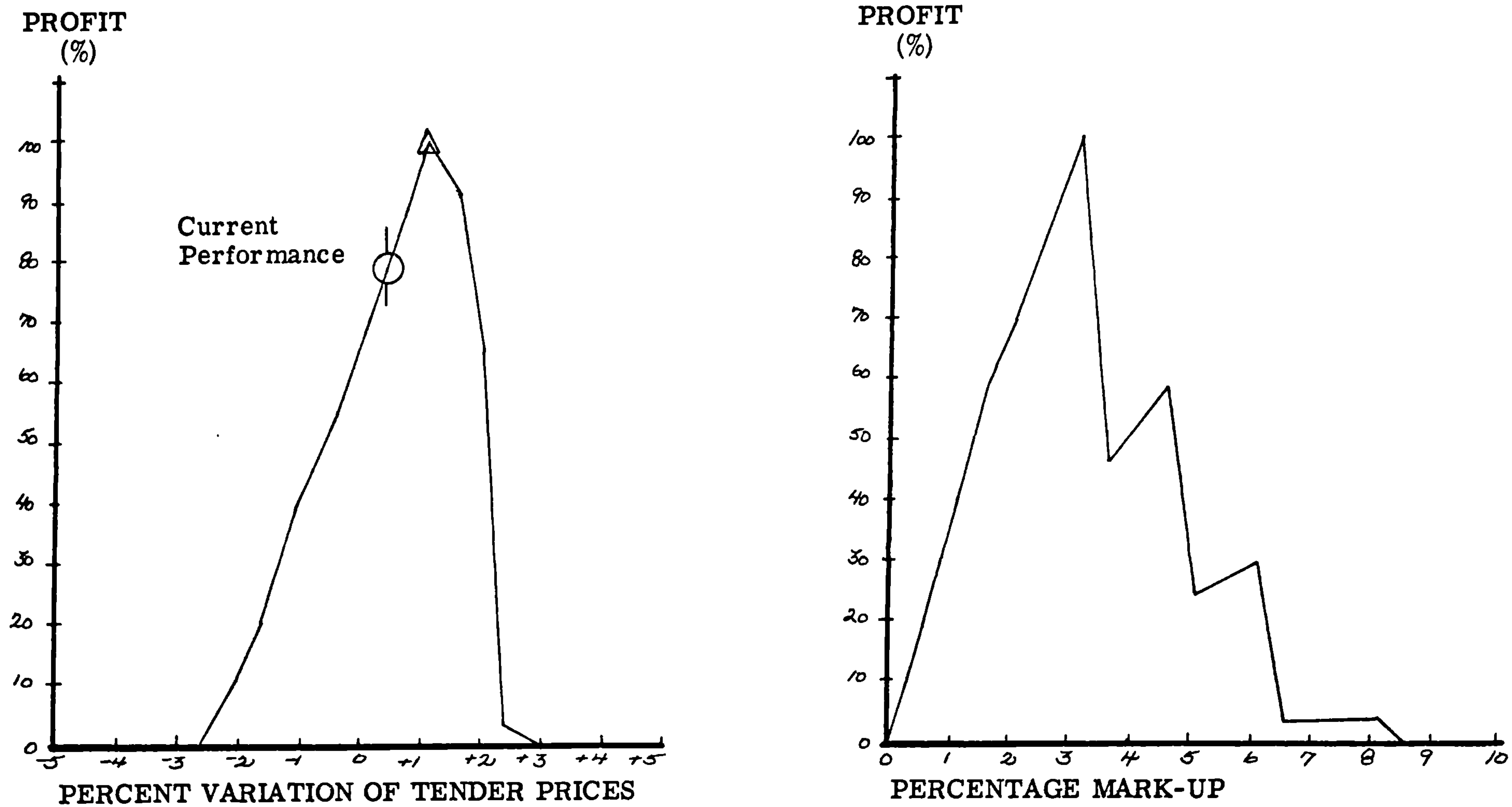
For Company B the Method 2 correction, 12.9% range, appears to be a better indicator of management desires as it produces two, almost equal peaks: one at the current performance, the other at the desired location. Recall from the feasibility study, the alternate location was only desirable if the market would permit the obtaining of additional work. The Method 1 correction produces a definite peak at the desired location.



OPTM & OPT2 RESULTS - COMPANY A

12.85% Range Correction

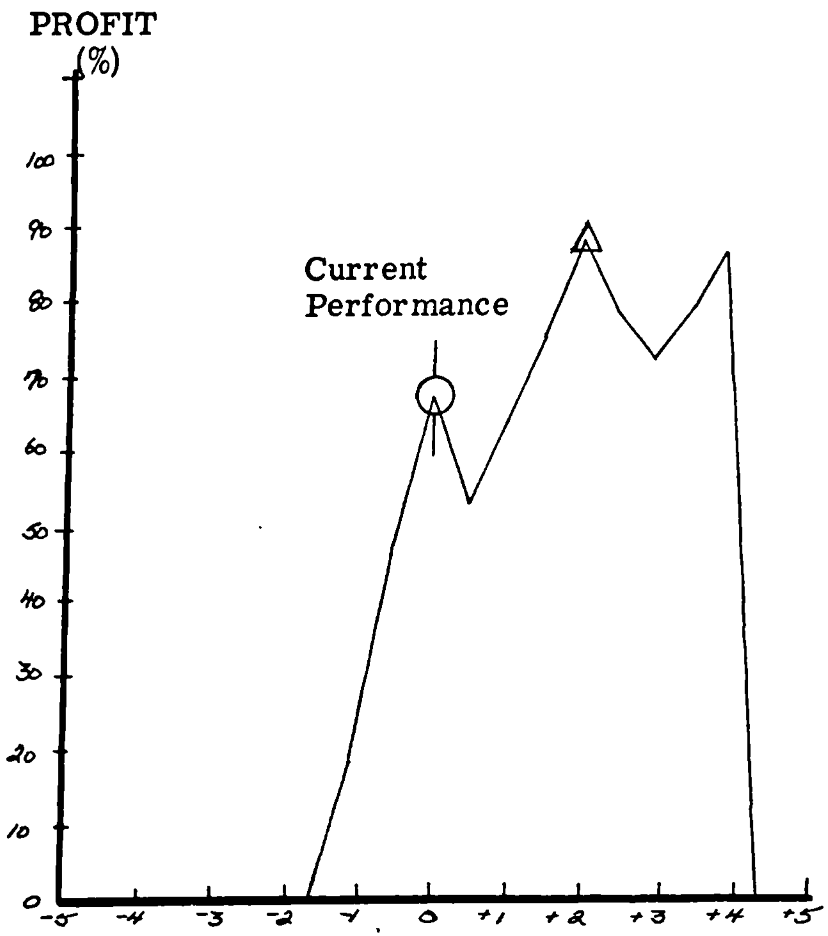
Figure 2.17

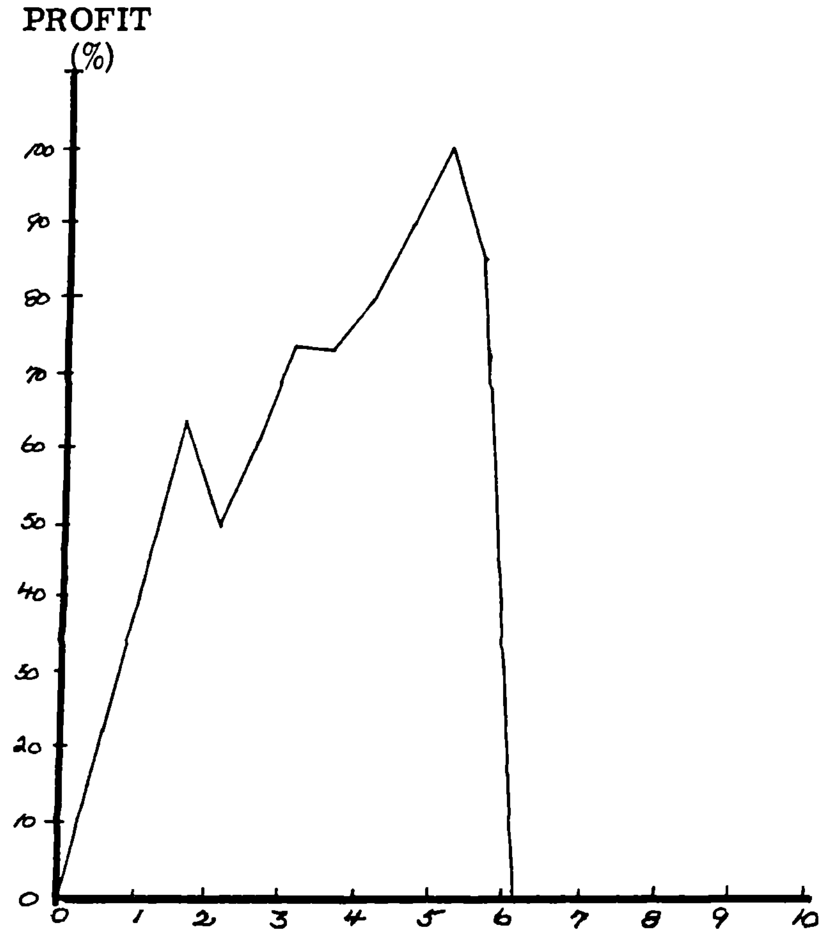


<u>OPTM & OPT2 RESULTS - COMPANY A</u>

11.9% Range Correction

Figure 2.18

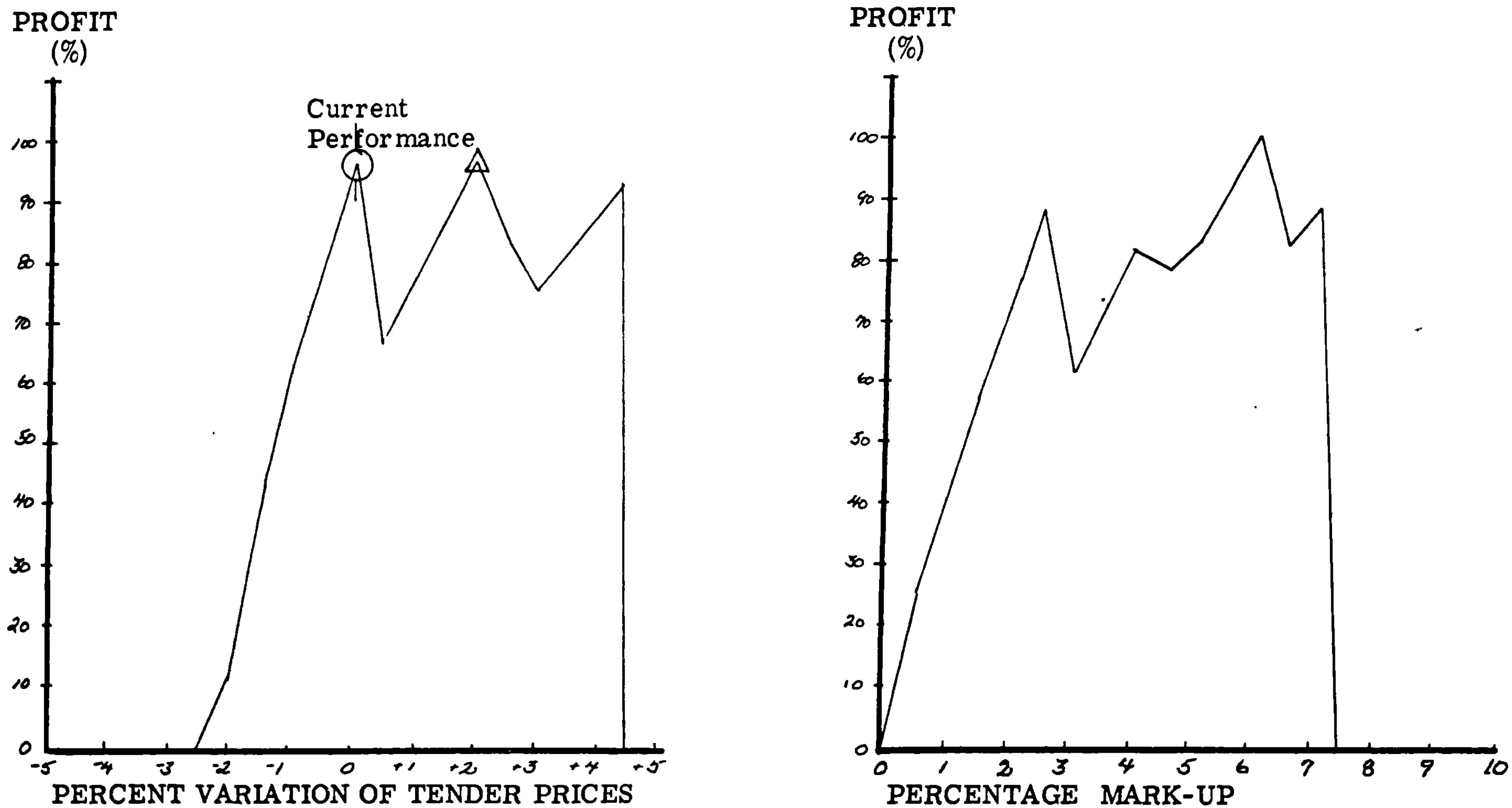




<u>OPTM & OPT2 RESULTS - COMPANY B</u>

13.45% Range Correction

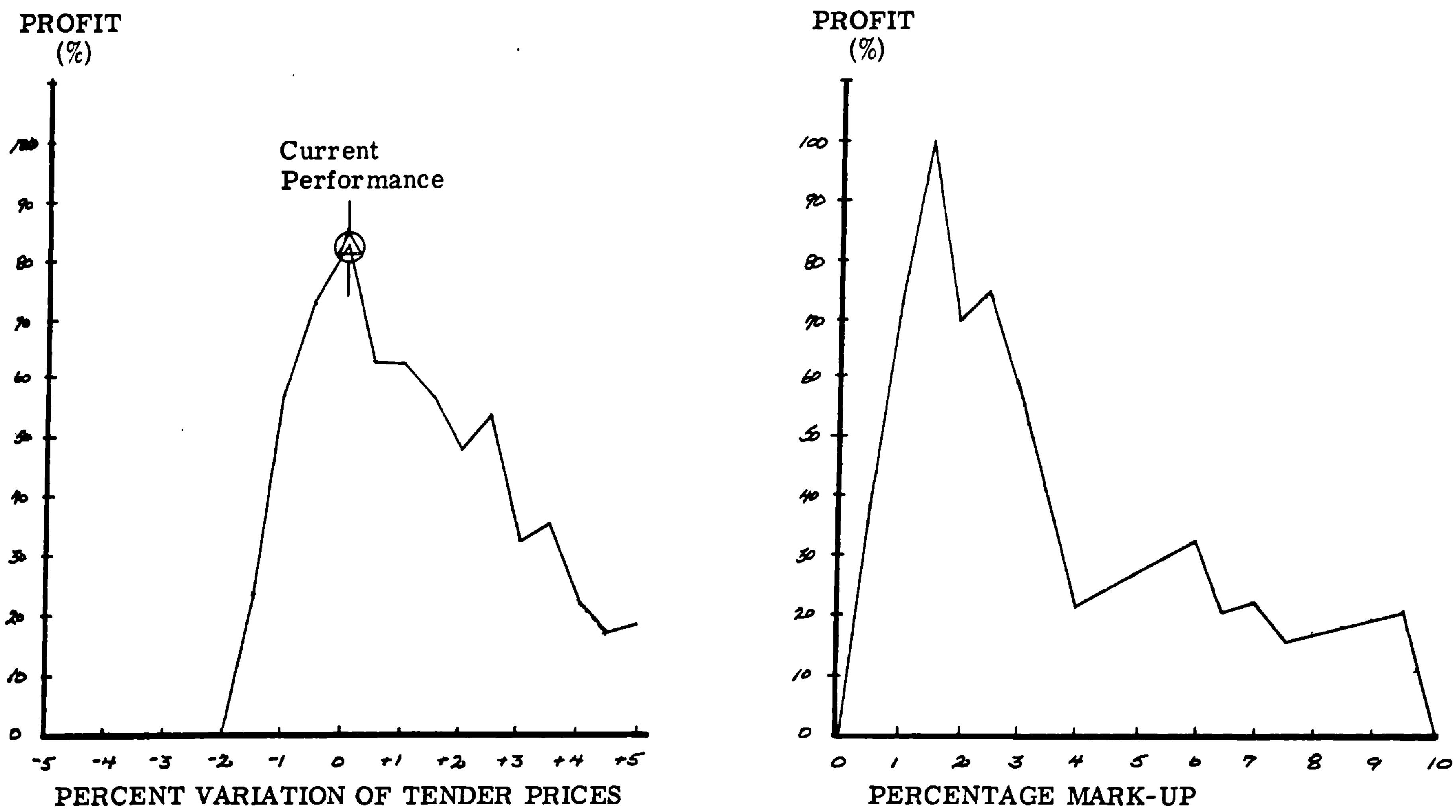
Figure 2.19



<u>OPTM & OPT2 RESULTS - COMPANY B</u>

12.9% Range Correction

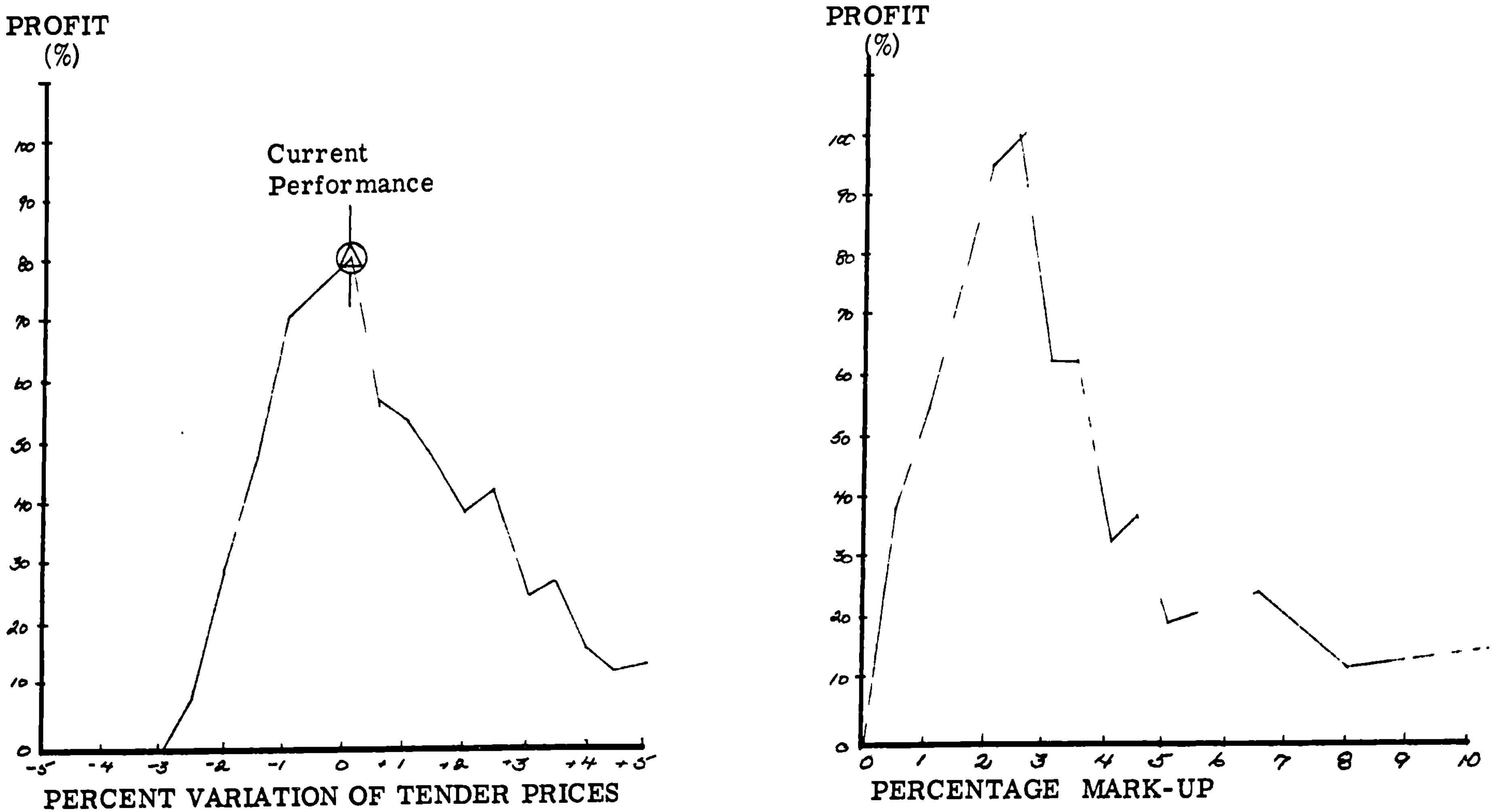
Figure 2.20



OPTM & OPT2 RESULTS - COMPANY C

11.0% Range Correction

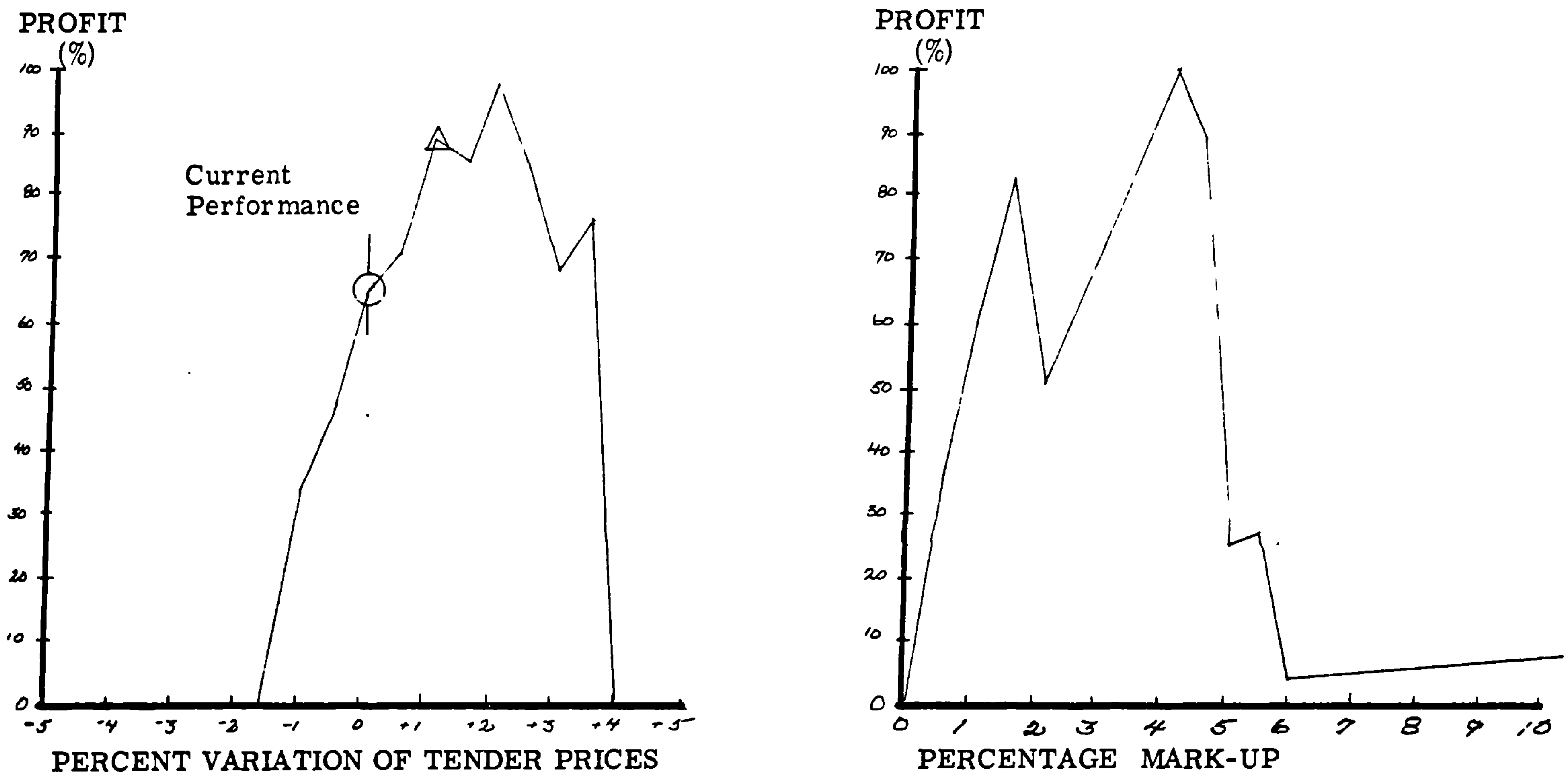
Figure 2.21



OPTM & OPT2 RESULTS - COMPANY C

9.35% Range Correction

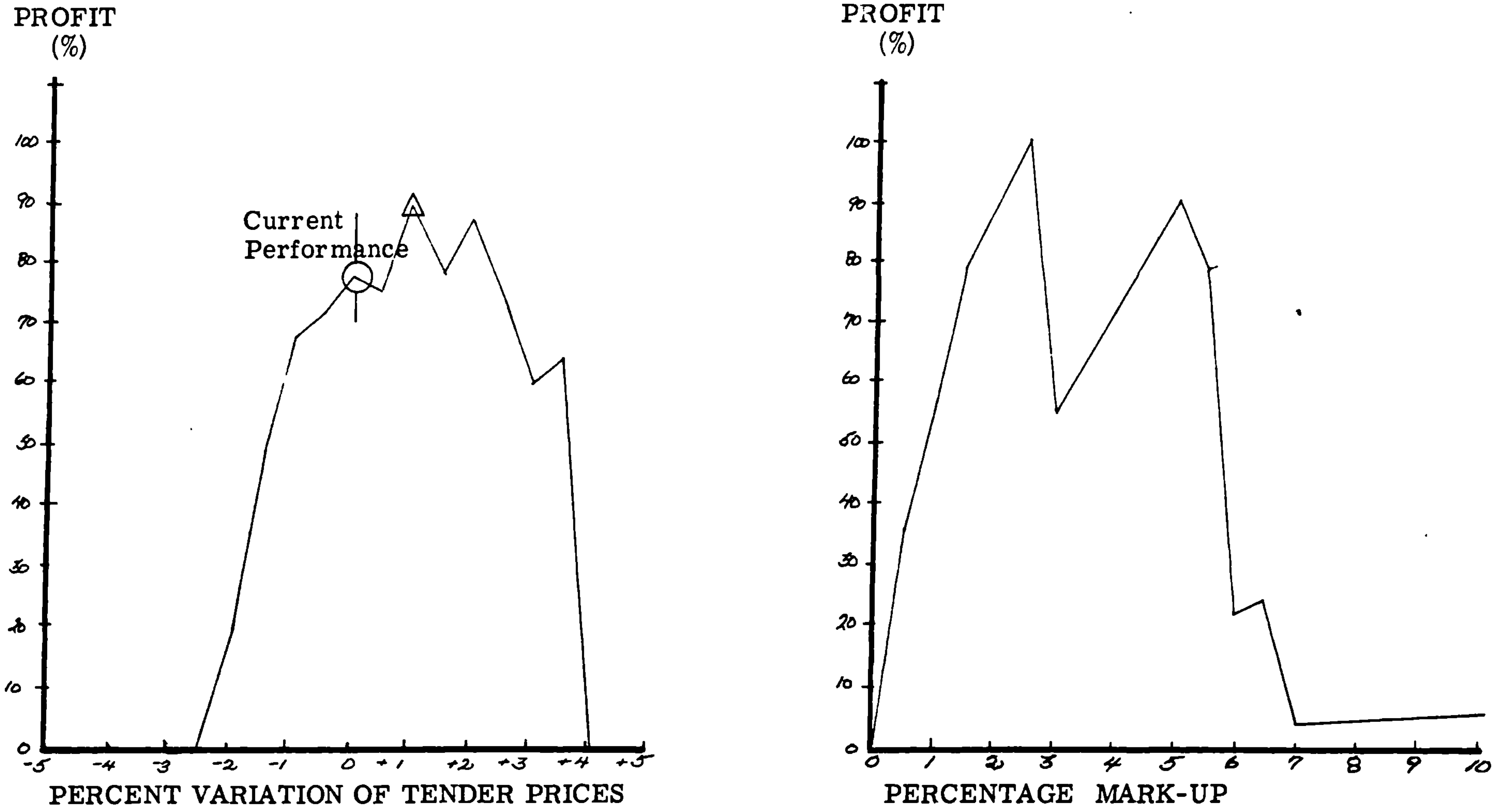
Figure 2.22



OPTM & OPT2 RESULTS - COMPANY C

13.1% Range Correction

Figure 2.23



OPTM & OPT2 RESULTS - COMPANY C

10.7% Range Correction

Figure 2.24

For Company C the two correction methods produce similar results. The actual performance, the desired location, and the peaks of the graph, all coincide.

For Company D the Method 2 correction, 10.7% range, is coincident with the earlier findings; the Method 1 correction is not. In the Method 2 results the desired location determined in the feasibility study occurs at the peak of the graph. In the Method 1 correction results, the peak of the graph is 1% higher than the desired location and 2% higher than the actual performance.

These graphs indicate that the Method 2 evaluation of the estimating accuracy produces the most satisfactory results. Therefore these are the figures that are adopted and the accuracy ranges for the sample

Companies are taken to be:

Company A - R = 11.9 %

Company B - R = 12.9 %

Company C - R = 9.35%

Company D - R = 10.7 %

From the OPTM graphs produced using the Method 2 corrections, the following observations are made.

1. In every case the desired operating position determined from the previous examination of the contract data occurs at the peak of the graph.

2. The actual performance of each sample company is in the region of the peak of its graph.

From the programs' outputs, profit-volume graphs were constructed for the four Companies. These are presented in Figures 2.25 and 2.26. The Company's operating position is indicated with a rectangle, the desired position with a triangle. These plots reinforce the previous observations. The rectangles and triangles occur in the region of the peaks of the curves.

Referring back to the original assumptions of Section 2.5.1, it was assumed that the objective function should maximize some economic measure and coincide with managerial behaviour and desires. The objective function, maximize profits, with profits defined as the bid price minus the corrected estimated costs on contracts won, appears to satisfy the criteria; at least for the four Sample Companies studied. Therefore this is the function that will be used for the remainder of the experimental investigation of this thesis and the term, profit, without qualifying adjectives, will refer to gross profit based upon Z corrected estimated costs.

PROFIT-VOLUME OPPORTUNITY CURVE

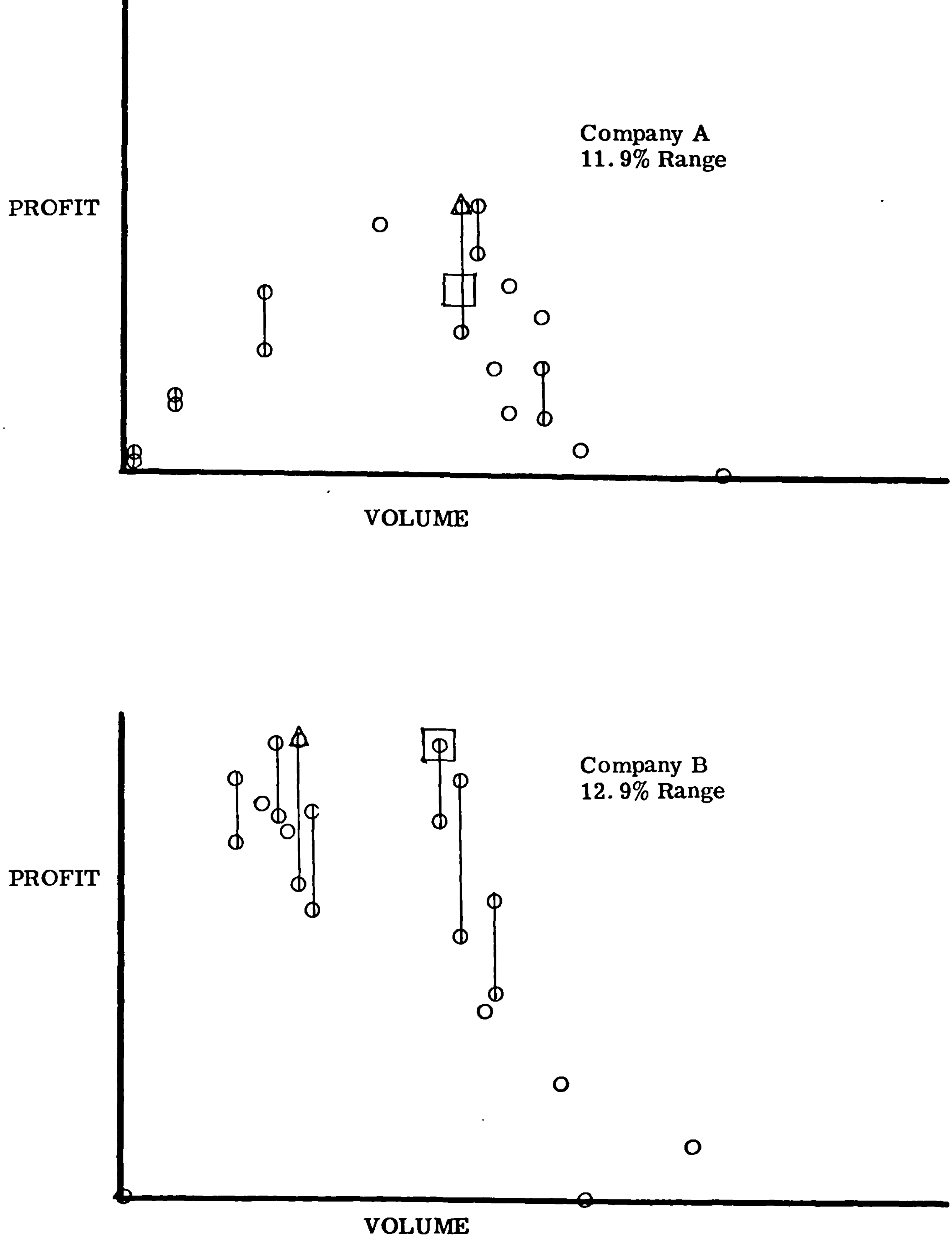


Figure 2.25

PROFIT-VOLUME OPPORTUNITY CURVE

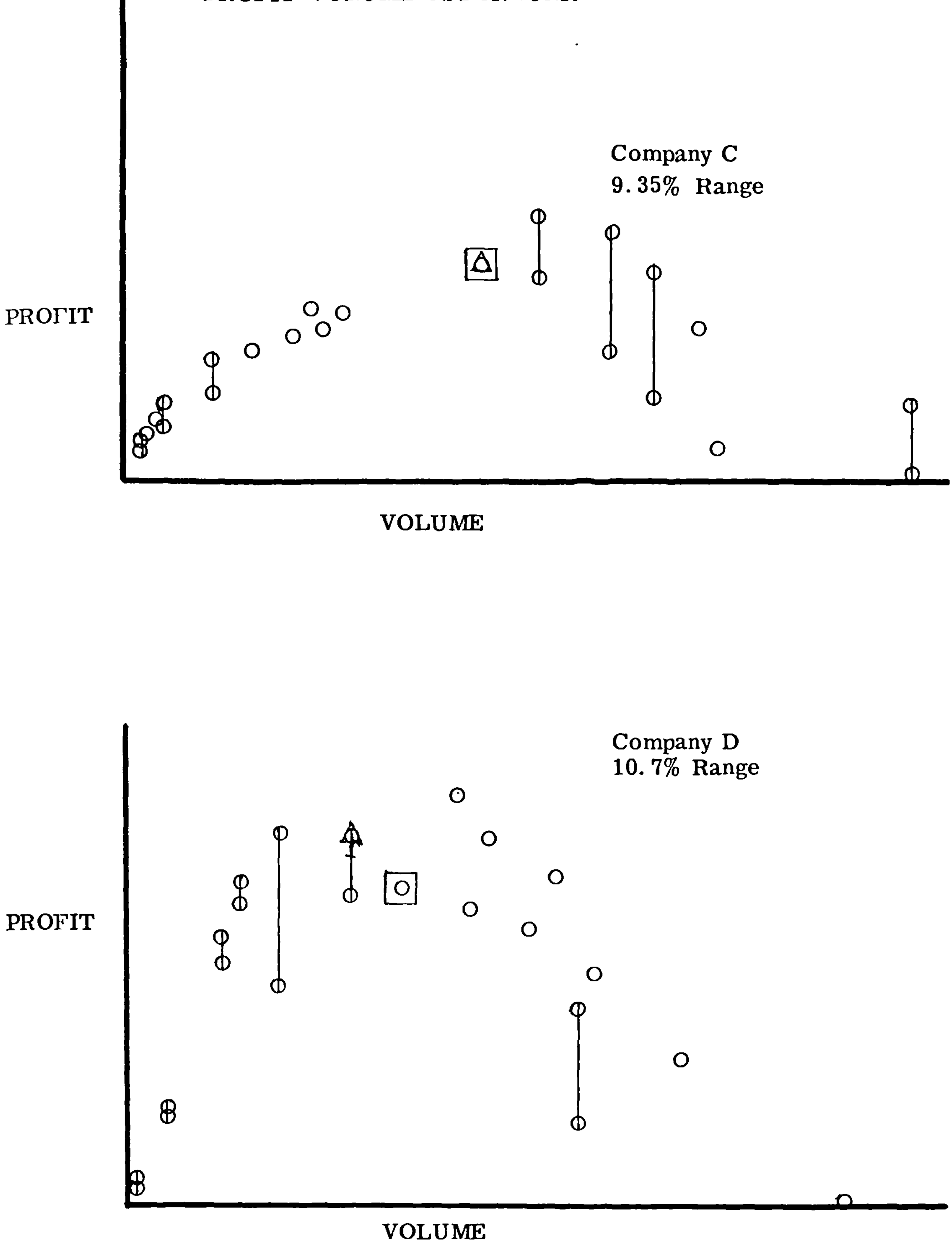


Figure 2.26

It is not proposed that these profit figures necessarily bear any resemblance to the figures that the accounting machinery of the companies produce. A suggestion that they might was accepted by one of the Sample Companies and flatly rejected by another. The one that accepted it stated that they bid projects in the 6% mark-up region and were finishing with profits of 1% to 2%. The one who rejected it stated that they bid in the 6% region and generally showed profits in the 6% plus region. Probably the difference lies in the definitions, the accounting systems, and the site negotiating ability of the firms.

What is proposed is that the function, as defined, imitates management desires and a model programmed to maximize the function should produce results acceptable to management.

Graphs similar to those resulting from the Z corrected data sets could be produced by increasing all the estimated costs by a constant of about 4%. However, no a priori reason could be found for such an action. Any assumption based upon estimating accuracy must include the number of competitors as a parameter and will result in a set of corrections such as were employed. Fixed overhead costs would not cause the peak to shift and variable overheads are already included in the estimates.

The reduction of the objective function to simple profit maximisation leaves unanswered questions about resource utilisation, timing of cash flows, project desirability, etc. This is reasonable given that all these factors are considered in the first decision stage - the Selection of the Contracts. The contracts represented by the data sets already fulfil the requirements of general desirability, resource utilisation, etc. and all that is left is the objective of winning the contract at the maximum possible bid price.

2.6 SUMMARY

To recapitulate briefly, the Management Problem may be stated as, "within the prescribed boundaries, what are the optimal decisions relative to the management objectives?" The principal purpose of this Chapter was to delineate the boundaries, isolate the decisions and quantify the objectives.

- The boundaries are the market of the Sample Companies.
- The decisions are the Selection Decision and the Adjudication Decision.
- The objectives are expressed as

$$\text{Maximize} \sum_{i \in S} \text{Maximum} \left\{ P_i(x_i)\,(Z(R,n).\,a_{1i}+a_{2i}.\,x_i)+\,l_i\;;\;0 \right\}$$

A summary of the development of the Problem description, and objective function formulation follows.

The construction industry is comprised of approximately 80,000 firms and they are characterised by their diversity and versatility. The market which is the focus of this investigation is that segment of the industry referred to as the General Building field. This field is characterised by projects of low contractural risk with minimal resource constraints. It is a market in which there are a large number of potential competitors and there exists little differentiation either in the products offered or in the firms' methods of producing the products. Four Sample Companies that operate in the field are introduced. It is the market of these companies, as exhibited by their bidding histories, that is investigated.

Uncertainty is the key element in the building construction field. It arises in the process of construction - contract uncertainty - and in the method of obtaining contracts - estimating uncertainty. These uncertainties are a natural result of the procedures and practices employed in the industry.

Competitive bidding is the normal method of contract allocation. The manner in which the process functions gives rise to a two stage decision problem. The decision maker is first required to make a Selection Decision (the decision to tender) and then an Adjudication Decision (the decision on a tender price).

The testing of the Sample Companies' data with two decision rules indicated that: there was potential for improvement in the companies' operations; the difference between the bid price and the estimated cost was inadequate as a value measure; managerial judgement appeared to be a relevant variable.

The variables in the decision situation were used as the basis for constructing a quantitative objective function. Using a discounted cash flow basis, an objective function was formulated which could accommodate uncertainty and future expectation. A contracted version of this formulation, the Adjudication objective, was tested and the results indicated that the objective was compatible with situations sought by the sample companies in practice.

In this Chapter only the controllable variables have been considered. However, the measures of value and the objectives are expressed as functions of the uncontrollable variable. The treatment of the uncontrollable variable is the subject of Chapter 3.

CHAPTER 3: THE UNCONTROLLABLE VARIABLE

3.1 INTRODUCTION

The uncontrollable variable in the Competitive Bidding Problem is the value of the lowest competitor's bid (or bids when considering a set of contracts). The relationship between this variable and managerial objectives has been the principal focus of operational research Bidding Theory. Appendix 4 contains a brief review of the Theory.

The probabilistic method has received most attention in the literature and seems to offer the best prospects for application. This approach assumes the existence of a probability of success value $P_i(x_i)$, corresponding to each bid value x_i, and uses expected value concepts in the decision function. The value $P_i(x_i)$ and the expected value concept have already been introduced without explanation, in Section 2.5.5, to formulate the objective function.

This Chapter is concerned with explaining, examining, and extending the probablistic treatment of the Competitive Bidding Problem. The Section titled Probabilistic Basis outlines the assumptions and concepts underlying the approach. Then the Friedman Model and some of the published variants are examined and discussed. Criticisms of the probablistic approach and some of the models is the topic of Section 3. 3. 3. Section 3.4 describes the development of a probability model which incorporates the decision makers subjective assessment of the competition into the bid determination algorithm.

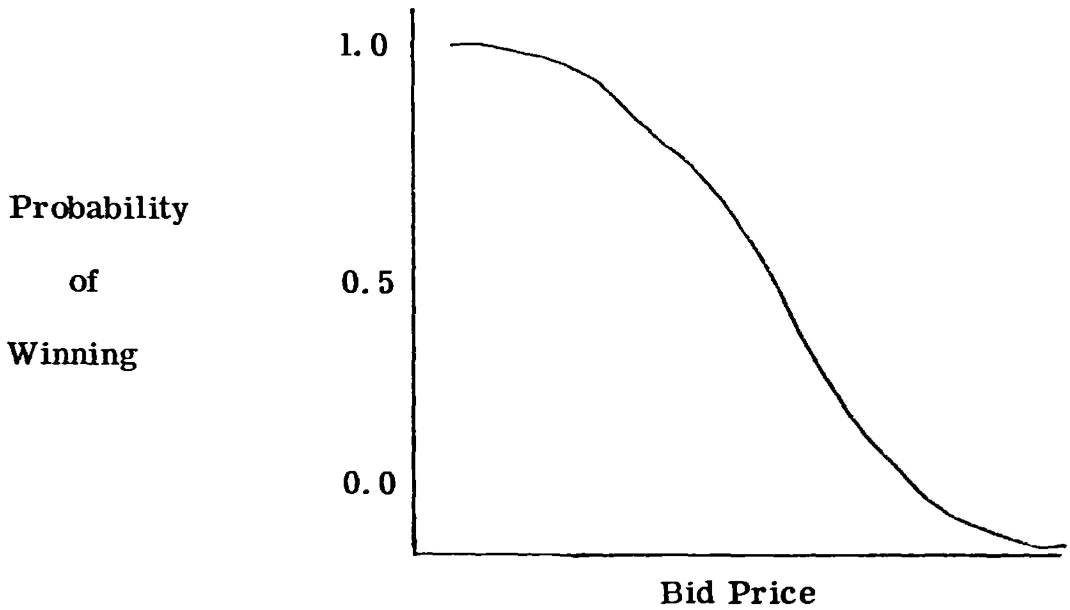
3.2 PROBABILISTIC BASIS

Two basic concepts underly the probablistic method of treating the uncontrollable variable. The first is the assumption of the existence of a continuous function that relates the probability of winning to the value of the bid price. The second is the use of the statistical concept of expected value as a guide to action.

3.2.1 Probability of Winning

It is not the intention of this thesis to enter the subjective -objective probability debate and try to justify the assigning of probabilities to the outcomes of unique events. There exist many precedents in business literature and Bidding Theory for this approach. Therefore, the existence of a continuous function which relates the value of a specified bid to the probability of that bid being the low bid, or winner, on a particular contract is assumed.

Intuitively this assumption seems reasonable. For any contract it is simple to conceive of a high price which could not win (Probability = 0) and a low price that could not lose (Probability = 1). One could normally expect the function between these two points to be a downward sloping curve as illustrated below.



CURVE INDICATING PROBABILITY OF WINNING AS A FUNCTION OF THE BID PRICE

The function is simple to hypothesize, but difficult to define mathematically. The method of distinguishing between the three main probabilistic approaches, Friedmans, Gates, and Edelmans is in the way they evaluate the probability function. A fourth method is developed in Section 3.4.

Marvin Gates (12) builds his probability curve from the assumption that all firms competing for a contract do so on equal terms. Thus the probability of winning equals $1/N$ for each of the N firms competing. By regression on historical bids Gates determined a formula which relates changes in probability to changes from standard bidding position.

Edelman (8) based his curve on the subjective estimates of management and reported very satisfactory results. However Edelman's market contained client bias, and non-price features such as delivery times and thus is not directly applicable to the Management Problem where price alone determines the winner and contracts are not differentiable.

The Friedman approach (11) is based upon the assumption that the competitors bids are random samples from a distribution. Since this assumption is common to the method developed in this thesis it is considered in detail.

3.2.2 Order Statistics Development

Definition - Let $z_1, \ldots, z_n$ be random samples from a distribution. Let $y_1, \ldots, y_n$ be the z_i's arranged in order of magnitude so that

$y_1 < y_2 < \cdots < y_n$. Then the y_i's are

order statistics of the z_i's.

The problem is stated as follows:

Determine for a given contract i, the probability $P_i(x_i)$ that a bid x_i, $(0 \leq x_i \leq \infty)$, will be the lowest bid. There are n competitors, therefore in order for x_i to be the lowest it must be lower than the smallest of the n competitors' bids.

Assume that the n competitors' bids are random samples $(z_1, z_2, \ldots z_n)$ from a density $f(z)$.

Let $y_1, y_2, \ldots, y_n$ be order statistics for the z_i's with $y_1 < y_2 < \cdots < y_n$.

The joint density of the order statistics is*

$$h(y_1, y_2, \ldots, y_n) = n!\ f(y_1)\ f(y_2) \ldots f(y_n)$$

The density function of y_1, $p(y_1)$, is the density function of the lowest of the n bids. $p(y_1)$ is found by integrating y_2 to y_n out of $h(y_1, y_2, \ldots, y_n)$.

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* Reference 21, page 240

$$p(y_1) = \int_{y_1}^{\infty} \cdots \int_{y_1}^{y_4} \int_{y_1}^{y_3} h(y_1, y_2, \ldots, y_n)\, dy_2\, dy_3 \ldots dy_n$$

$$= n!f(y_1) \int_{y_1}^{\infty} \cdots \int_{y_1}^{y_4} \int_{y_1}^{y_3} f(y_2)f(y_3)\ldots f(y_n)\, dy_2 \ldots dy_n$$

And term by term integration gives

$$p(y_1) = n(1 - F(y_1))^{n-1} f(y_1)$$

where $F(y_1) = \int_{-\infty}^{y_1} f(y)dy$

The probability that a bid x is the lowest or winning bid is the probability that y_i is greater than x.

$$P(x) = \int_{x}^{\infty} p(y_1)dy_1 = \int_{x}^{\infty} n(1 - F(y_1))^{n-1} f(y_1)dy_1$$

$$= (1 - F(x))^n$$

(3.2.2-1)

3.2.3 Expected Value

The expected value of an event is defined as the product of the probability of the event occurring and the value of the event if it occurs. For the contract situation if V_a is the value of the contract if it is won; V_b is the value of the contract if it is lost and P is the probability of winning the contract; then the expected value of the contract is

$$(P)\,V_a + (1 - P)\,V_b \qquad (3.2.3\text{-}1)$$

The objective used in this thesis as a criteria for the Management Decision Problem is Maximize Expected Value. The arguments and rationale for this approach have been well developed and the interested reader is referred to, "Probability and Statistics for Business Decisions" by R. Schlaifer (27).

3.3 PUBLISHED MODELS

This Section is concerned with the methods and assumptions of some of the published operational research bidding models. Only the Friedman, and Friedman based models are considered here. This is so because the Edelman model is not applicable to the market studied, and the Gates model is essentially the same as the previously introduced OPTM and OPT2 procedures. The models discussed are all based upon the random sample, order statistic development and all seek to maximize expected value. Where they differ is in their method of deriving the underlying distribution $f(x)$.

3.3.1 Friedman Model

In his 1956 paper (11), Friedman proposed deriving a distribution for each competitor by examining past contracts and building a frequency distribution from the ratios of the competitor's past bids to the company's cost estimates. In this manner he derived a distribution for each competitor. The probability of winning a contract was the product of the probabilities of beating each competitor participating.

This approach is open to criticism in so far as, if the competitors are identifiable, it is probable that far more is known about them than their distribution of bids. For example in the Civil engineering motorway construction field, where the competitors are limited and known, one firm revealed to this study that it keeps very detailed records of its competitors; to the extent that it knows what contracts they have won, the approximate stage of completion of their existing contracts, the location of their equipment, etc. Even without this detailed information, the methods of Mercer and Russell (19) for determining known competitors intentions are likely to provide a better basis for the Management Decision than just the historical distribution of bids.

For the condition where there are so many competitors that it is uneconomic to accumulate data on each one, which is the case in the market being studied, Friedman proposed the concept of an "average competitor". A bidding distribution of the average competitor is derived by combining all previous ratios of an opposition bid to the company's cost estimate into a single distribution function.

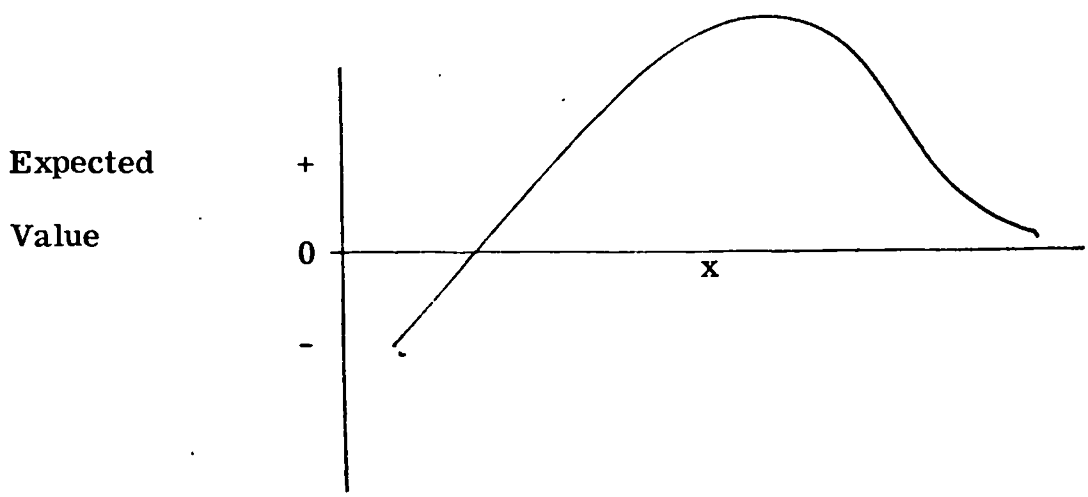
If $f(w)$ is the probability density function of the ratio of the average competitor's bids to the cost estimates; then the probability of a bid of x being lower than one average competitor's bid is

$$\int_{\frac{x}{k}}^{\infty} f(w)\, dw$$

where k is again the estimated cost. The probability of being lower than n average competitors is

$$\left(\int_{\frac{x}{k}}^{\infty} f(w)\, dw \right)^n$$

From an operational viewpoint, this model has much to recommend it. Depending only upon historical data and with an expected value function of a shape illustrated below it is easily adapted to a discrete computer method of solution.



Two published reports of successful application of this model to construction companies are Park (24), and Morin and Clough (20). Park noted that the distribution was fairly stable and that the principal determinant of the optimum bid was the number of competitors. Thus, after determining the distribution for a company, he was able to develop a list of optimum mark-ups, dependent only upon the number of competitors. Morin and Clough used exponential smoothing to introduce the bid-cost ratios into the distribution and reported profit increases in the order of 27%.

3.3.2 Friedman Variants

Hanssmann and Rivett (14) stayed within the basic Friedman concept but replaced the average competitor with the "lowest competitor". A bidding distribution for the lowest competitor is derived by combining all previous ratios of the lowest opposing bid to the company's cost estimate into a single distribution. The method assumes that the firm is only competing against this one super competitor. Whether this is valid is questionable since most of the other determinations regard the number of competitors as a major parameter.

The model proposed in, "Fundamentals of Operations

Research" (1) goes one step further. It assumes that the distribution of the ratios, i. e. lowest competitor bid to company cost estimate, is normal. The Chi-squared test was used to check this assumption for the Sample Companies data. The ratio employed was; lowest competitor bid to Z corrected estimated cost, and the hypothesis that the distribution is normal was accepted in every case at the 10% level of significance.

3.3.3 Criticism of Models

One criticism of the Friedman type models is that they are not applicable if the contracts are differentiable, (i.e. have different values to different firms). This criticism is valid, but not appropriate, since in the market studied the contracts are not significantly differentiable. Appropriate criticism rests on the two assumptions implicit in the use of bid-estimate ratios for deriving the probability functions. The first assumption is <u>that competitors will continue to behave as they have done in the past.</u> The second is <u>that cost estimating does not change.</u>

The first assumption is sufficient to cast doubt on the model. Presumably a competitor's pricing policy is a function of his available capacity and the market condition. As his capacity increases he could be expected to lower his mark-up to obtain additional work. If his capacity is highly utilised, and the market is good, he could be expected to raise his mark-ups. These comments can also apply to the aggregate behaviour of competitors because the major fluctuations in the industry often result in capacity of all firms being highly utilised or all firms being slack. The distribution compiled from the ratios could easily cover more than one market condition and thus indicate a much wider range than is actually present. Equally possible is a case where the distribution could be compiled from a market situation that is the reverse of the one in which the present contract is being bid; with the result that the competitors' behaviour will have changed markedly from that expressed by the distribution.

Friedman proposed his model for the situation where the past bids of the competitors are known but no other information. However, the model was titled, "Realistic Bidding Problems", and it seems improbable that in a realistic bidding problem the only information available is the past history of competitors' bids. Indeed, this information tells which, and how many, contracts the competitors have won. This, coupled with a slight knowledge of their capacities, should indicate whether they are likely to bid high or low. Even without specific knowledge of individual competitor's capacities; one usually possesses some concept of the aggregate capacity of the field. Then, since there is a basis for estimating the volume of work let to date, (from the history of bids), an estimate can be made of the remaining capacity. Reasoning along economic lines it can be assumed that as the remaining capacity of competitors diminishes, there will be an upward trend in prices. This trend will be damped in a model that uses historical data as a basis for computing probabilities. The exponential smoothing approach employed by Morin and Clough (20) should reduce this damping effect. However, even with this device the model has no way of reacting to changes in the situation which are not reflected in past bids, but are known to the decision maker. Therefore some of the available information is suppressed by these models.

The second assumption of constant cost estimation could produce major errors in the bid determination. This could occur in an obvious manner, like the hiring of a new estimator, or it could occur in a less obvious way . Perhaps due to losing a series of contracts the estimator might, consciously or subconsciously, be trying to reduce his cost estimates. It is easy to envisage this sort of situation. In one set of data examined during the research, on 9 out of 19 contracts, the cost estimate was above the winning bid. This situation must obviously result in considerable psychological pressure on the estimators to produce lower estimated costs. The estimating work load can also affect the cost estimates. Presumably an overworked estimating department is more likely to make errors. Again these situations are known to the decision maker but are not incorporated into the models.

The major criticism made of the Friedman, bid-cost ratio approach is that it does not use all the available information. The literature has provided no way of integrating these models with managerial knowledge and expectation of the competitive situation.

3.4 A SUBJECTIVE PROBABILITY APPROACH

The principal criticism of the probability models is, therefore, that they suppress some of the available data. This is significant since the feasibility study, Section 2.4, indicated that these suppressed data, the results of managerial judgement, are relevant in the decision situation. Therefore a model was developed that incorporates managerial judgement in a probabilistic bidding strategy model.

A precedent for this approach is the PERT method of handling uncertain durations. In PERT the problem was to devise a method of scheduling activities of an uncertain or variable duration. The method adopted was to assume that the durations followed a standard distribution, the beta, and use the best information available, managerial estimate, to determine the distribution parameters. In competitive bidding the problem is to determine the uncertain behaviour of the competitors and a method analogous to that of PERT is used.

The initial assumption is that the competitors' bids on a specific contract are random samples from a distribution. This assumption is the same as that of the Friedman approach and the order statistic derivation, Section 3.2.2, of the probability of winning applies. Then it is assumed that there is one general distribution function that applies to all building construction contracts. The parameters of the distribution relate it to a specific contract. That is, there exists a function $f(z; \Theta)$, that is common to all contracts; the Θ being a set of parameters that are unique to a specific contract. If this is the case, and $f(z; \Theta)$ can be determined in terms of the set Θ; then the problem of determing the probability function for a particular contract is reduced to the problem of determing the set of parameters (Θ) that are particular to that contract.

The determination of the parameters is left to the decision maker. His experience, knowledge of the market, intuition, etc. are used to determine the values. In this manner the model incorporates managerial judgement and becomes a subjective probability approach.

3.4.1 Experimental Determination of $f(z; \Theta)$

For computational reasons, it is simpler to determine the cumulative distribution function $F(z; \Theta)$. The determination of $F(\cdot)$ was carried out in the following manner:

1. A general shape was assumed for the function F.
2. The coefficients of the general shape were calculated by multiple regression analysis and tested for significance by means of a t test.

The data for the multiple regression analysis were historical contract bids. These were provided by the Contracts Directorate of the Ministry of Public Buildings and Works. A description of the data is found in Appendix 5. If the original assumption, i.e. that bids are random samples from a distribution, is valid, then when the bids on a given contract are ranked in order of size, from the smallest to the largest, an estimate of the cumulative probability is the rank number divided by the total number of bids plus 1. For example, if N is the total number of bids on the contract, the cumulative probability of the first (lowest) is $\frac{1}{N+1}$, of the second is $\frac{2}{N+1}$, and so on up to the highest which is $\frac{N}{N+1}$. Since the data contained no ties, the problem of resolving ties did not arise.

The multiple regression analysis was done using the ICL Statistical package, XDS3. A Fortran IV program, STAT, was written to generate the input matrix from the data. STAT and XDS3 were run on the City University ICL 1905 Computer.

Trial Number 1

The first assumption made was that the cumulative probability function F had the shape of an S-curve. The equation

$$x = b_1 + b_2F + b_3F^2 + b_4\ln(F) + b_5\ln(1-F)$$

(3.4.1-1)

is a general equation for an S-curve with $0 \leq F \leq 1$.*
Therefore the General Distribution Equation was assumed to be

$$x_i = b_{1i} + b_{2i}F + b_3F^2 + b_4\ln(F) + b_5\ln(1 - F)$$

(3.4.1-2)

where the b_{1i} and b_{2i} are the θ_1 and θ_2 of contract i and the b_3 , b_4 , and b_5 are common to all contracts.
The b_4 and b_5 coefficients control the curvature at the ends of the S-curve and it is unreasonable to assume that they would be constant for all contracts. Therefore the data was stratified and it was assumed that b_3 , b_4 , and b_5 were constant for

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* This equation was provided in a private communication from Mr. A. H. Russell.

contracts within the stratified range, (e.g. for £100,000 $\leq$ x $\leq$ £200,000). The strata boundaries were arbitrarily set at £100,000, £200,000, £300,000, ...

The regression matrix for this trial, where m is the total number of contracts, is

$$
\begin{array}{ccccccccccc}
 & b_{11} & b_{12} & \cdots & b_{1m} & b_{21} & \cdots & b_{2m} & b_3 & b_4 & b_5 \\
x_{11} & 1 & 0 & & 0 & \frac{1}{N_1+1} & & 0 & \left(\frac{1}{N_1+1}\right)^2 & \ln\left(\frac{1}{N_1+1}\right) & \ln\left(1-\frac{1}{N_1+1}\right) \\
x_{12} & 1 & 0 & & 0 & \frac{2}{N_1+1} & & 0 & \left(\frac{2}{N_1+1}\right)^2 & \ln\left(\frac{2}{N_1+1}\right) & \\
\vdots & & & & & & & & & & \\
x_{1N_1} & 1 & 0 & & 0 & \frac{N_1}{N_1+1} & & 0 & \left(\frac{N_1}{N_1+1}\right)^2 & & \\
x_{21} & 0 & 1 & & 0 & 0 & & 0 & & & \\
x_{22} & 0 & 1 & & 0 & 0 & & 0 & & & \\
\vdots & & & & & & & & & & \\
x_{2N_2} & 0 & 1 & & 0 & 0 & & 0 & & & \\
\vdots & & & & & & & & & & \\
x_{m1} & 0 & 0 & & 1 & 0 & & \frac{1}{N_m+1} & & & \\
x_{m2} & 0 & 0 & & 1 & 0 & & \frac{2}{N_m+1} & & & \\
\vdots & & & & & & & & & & \\
x_{mN_m} & 0 & 0 & & 1 & 0 & & \frac{N_m}{N_m+1} & \left(\frac{N_m}{N_m+1}\right)^2 & \ln\left(\frac{N_m}{N_m+1}\right) & \ln\left(1-\frac{N_m}{N_m+1}\right)
\end{array}
$$

A regression analysis was performed on a sample of 19 contracts, ($6 \leq N \leq 10$) ; (£100,000 $\leq x \leq$ £200,000), and the results can best be described as erratic. The b_5 term was not significantly non-zero when tested at a 5% level of significance and only a few of the b_{2i} terms were significant at this level. This indicated that the shape of the distribution was more like

$$x = b_{1i} + b_3F^2 + b_4\ln(F) \qquad (3.4.1\text{-}3)$$

and that the stratification of the data was not sufficient. Therefore the trial was run using normalised bid values. They were normalised by dividing them by the mean of the bids for that contract. That is

$$\text{Norm } x_{ij} = \frac{x_{ij}}{\frac{1}{N_i}\sum_{k=1}^{N_i} x_{ik}} \qquad (3.4.1\text{-}4)$$

When tested at the 5% level this analysis produced non-significant results for b_{1i}, b_{2i}, and b_5 . This reduced the equation to

$$\text{norm } x_i = \text{Const.} + b_3F^2 + b_4\ln(F)$$

(3.4.1-5)

Trial Number 2

Using the normalised data, the next general shape tested was

$$\frac{x_i}{A_i} = b_o + b_3F^2 + \frac{b_6}{F} \qquad (3.4.1\text{-}6)$$

where A_i = the arithmetic mean of the bids on contract i.

Therefore the General Distribution Function was assumed to be

$$x_i = A_i\,(b_o + b_3F^2 + b_6/F) \qquad (3.4.1\text{-}6)$$

with A_i (the mean bid) being the parameter to be estimated and b_o, b_3, and b_6 being common to all contracts. The regression matrix for this trial is

$$\begin{array}{cccc}
 & b_o & b_3 & b_6 \\
\frac{x_{11}}{A_1} & 1 & \left(\frac{1}{N_1+1}\right)^2 & \frac{N_1+1}{1} \\
\vdots & & & \\
\frac{x_{1N_1}}{A_1} & 1 & \left(\frac{N_1}{N_1+1}\right)^2 & \frac{N_1+1}{N_1} \\
\vdots & & & \\
\frac{x_{m1}}{A_m} & 1 & \left(\frac{1}{N_m+1}\right)^2 & \frac{N_m+1}{1} \\
\vdots & & & \\
\frac{x_{mN_m}}{A_m} & 1 & \left(\frac{N_m}{N_m+1}\right)^2 & \frac{N_m+1}{N_m}
\end{array}$$

The regression was performed on a sample of 57 contracts, $7 \leq N \leq 9$. At the 5% level of significance all the variables were included. The results are given in the table below.

TABLE OF REGRESSION ANALYSIS RESULTS

COEFFICIENT	VALUE	STANDARD ERROR
b_o	0.9744490	0.00448
b_3	0.1352319	0.00728
b_6	-0.0055555	0.00076

These results appear reasonable and so the General Distribution is taken to be

$$x_i = A_i(0.974449 + 0.1352319\ F(x_i)^2 - 0.0055555 / F(x_i)) \qquad (3.4.1\text{-}7)$$

where x_i = the bid on contract i

$F(x_i)$ = the cumulative probability distribution

$= \int_{-\infty}^{x_i} f(x)dx$

A_i = the arithmetic mean of the competitors' bids.

3.4.2 Consistency of Distributions

It was argued in Chapter 2 that an estimate was a random sample from a distribution and that this distribution could be approximated by a uniform distribution. Then by assuming that the lowest estimated cost won the auction, and that all competitors had the same estimating accuracy range, the expected value of the winning cost estimate was calculated.

The General Distribution derived from the regression analysis is proposed as the distribution of bids on a contract. Obviously, for this development to be consistent it must be related to the earlier, uniform distribution. The following example is used to indicate the relationship.

Let ϕ_j be the multiplier that converts an estimate k_j into a bid g_j

$$g_j = \phi_j k_j$$

If $f(k) = \frac{1}{R}$; $100-\frac{R}{2} \leq k \leq 100+\frac{R}{2}$

Then $f(g) = \frac{1}{\phi R}$ $\phi(100-\frac{R}{2}) \leq g \leq (100+\frac{R}{2})\phi$

and $F(g) = \displaystyle\int_{\phi(100 - \frac{R}{2})}^{g} \frac{1}{\phi R}\, dy$

$$= \left(\frac{g}{\phi R} - \frac{100}{R} + \frac{1}{2} \right)$$

(3.4.2-1)

Figure 3.1 shows $F(g)$ plotted for an estimating accuracy of $\pm$ 8% (R=16), and ϕ values of 1.05, 1.07, and 1.09. The black dots on the figure are points of the General Distribution equation for a mean bid of 107. The agreement between distributions is taken as an indication that the development is consistent and in reasonable agreement with the data.

It is not proposed that all contracts have the same estimating accuracy, or that the contractor with the lowest mark-up wins, or that the uniform distribution is the best one for estimating accuracy. However, it is argued that the general cumulative frequency distribution for bids and the use of the uniform distribution for cost estimates are consistent and provide adequate description of behaviour of the real system. Hence they may be employed in a prescriptive bidding model.

COMPARISON OF
REGRESSION DISTRIBUTION
AND
UNIFORM DISTRIBUTION

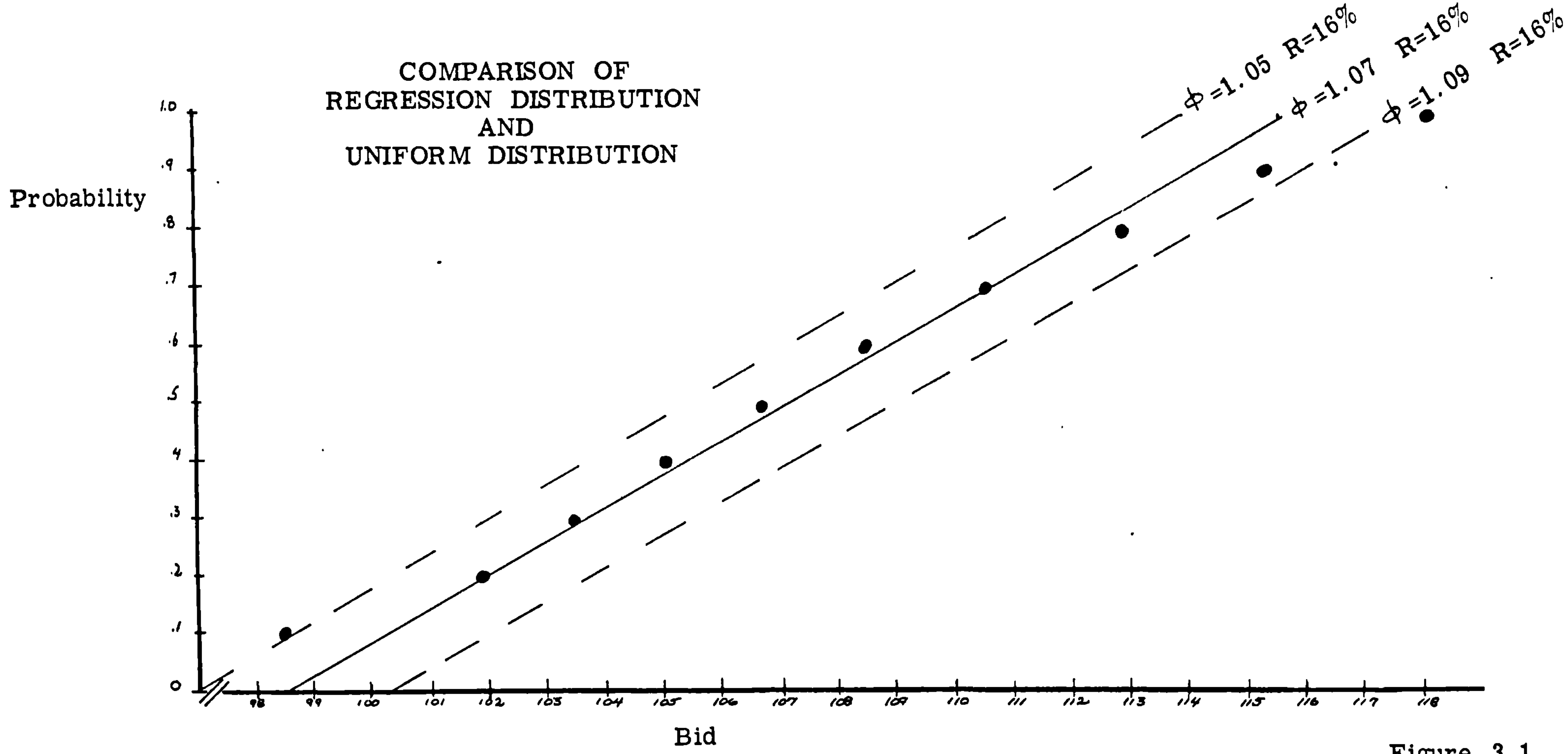


Figure 3.1

3.4.3 The General Distribution Model

It is proposed that the formula (3.4.1-7) be used as the functional relationship between the bid value, x_i, and the probability of winning, $P_i(x_i)$. A managerial estimate is used for the value of A_i. It is an estimate of the arithmetic mean of the competitors' bids and as such is an estimate of the aggregate behaviour of the competitors. Formally, the relationships in the model are

$$\text{Objective} = \text{maximize} \sum_{i \in S} \text{maximum} \left\{ P_i(x_i)\,(Z(R,n)a_{1i} + a_{2i}x_i) + l_i \ ; \ 0 \right\}$$

where

$$P_i(x_i) = (1 - F_i(x_i))^n \qquad (3.4.2\text{-}1)$$

$$x_i = \Theta_i(0.974449 + 0.1352319F_i(x_i)^2 - 0.0055555 / F_i(x_i))$$

Θ_i = managerial assessment of the arithmetic mean of the competitors' bids on contract i.

By incorporating a managerial assessment of the competitive situation into the parameter Θ, this model avoids the information suppression criticism of the previous models.

Since the model implies no fixed relationship between the estimated cost and the level of competitive activity, it is also applicable to situations where the contracts are differentiable, and the competitors unknown. This General Distribution Model is tested in Chapter 5, Empirical Evaluation.

3.5 SUMMARY

This Chapter investigated the relationship between the uncontrollable variable (the lowest competitor's bid) and the managerial objectives. The objectives are expressed in expected value terms and the uncontrollable variable is a subject of statistical prediction.

Assuming that the competitors bids are random samples from a distribution, a distribution of the lowest competing bid is derived. From this the probability of winning can be calculated. The Friedman based bid - cost ratio methods of deriving the underlying distribution are the most popular. However they are liable to criticism as they suppress some of the available information. A method of incorporating the additional information into the probabilistic concept was proposed and the relationship empirically determined. The method is analogous to the PERT method of handling uncertain durations. The result is a model which bases probability determination upon a managerial assessment of the competitive situation and a general distribution of bids about the mean.

CHAPTER 4: OPTIMAL DECISION MODEL

4.1 INTRODUCTION

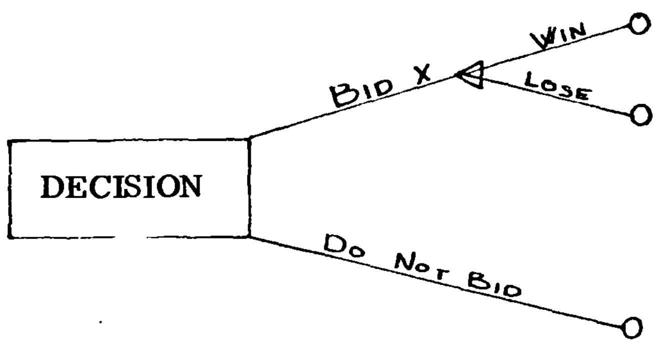
To this point this thesis has concentrated on analysing certain aspects of the Management Problem. An objective function has been proposed and a method of probabilistically treating the predicted behaviour of the competition developed. In this Chapter a mathematical model of the decision situation is developed. Using this formulation it is theoretically possible to calculate a mathematically optimum solution for the Management Problem.

The model is developed by considering a series of cases, proceeding from the simple, single contract unconstrained, case to the complex, N contracts, constrained, sequential bidding case. It is proposed that the complex case is a reasonable mapping of the actual bidding situation.

4.2 MODEL FORMULATION

4.2.1 Decision Situation

The mathematical model is constructed from the single contract decision process presented in Chapter 2. The two parts of the decision, Selection and Adjudication, are combined in this formulation to produce the decision situation illustrated below.



4.2.2 Independence Assumption

In the model the contracts are assumed to be independent. That is there is no inherent interaction between contracts and the right to bid on one is in no way dependent upon the results of action with respect to another. Also the winning of one contract does not provide a competitive advantage, or disadvantage, on another. Internal interactions, i.e. contracts competing for the firms resources, are considered.

This is not an unrealistic assumption. Most of the contract interaction is internal and this is considered. The external interactions that are usually found in building construction are where a contract is one of a series of stage contracts, or where a contract provides opportunities for future work, as in the case of getting on the list of an architect. Both these interactions imply possible future benefits and so can be accommodated by including in the contracts value measure the discounted expected value of the benefit.

The assumption would not be realistic for civil engineering construction where there is specialised plant and wider geographical dispersion of a more limited number of suitably qualified contractors. For a civil engineering contractor, one job in Malaya obviously influences getting another there, and a piece of specialised plant purchased for one contract can produce a competitive advantage on another contract requiring that plant. But building construction has little specialised plant and the companies tend to concentrate their activities in a small area.

4.2.3 Notation

In the formulation, the following notation is used.

i = subscript referring to the individual contracts.

h = subscript referring to resource type.

j = subscript referring to time period.

x_i = amount bid on contract i.

X = vector of contract bids $(x_1, x_2, \dots, x_N)$

$V_i(x_i)$ = value of contract i if it is won with a bid of x_i.

$P_i(x_i)$ = probability of winning contract i with a bid of x_i.

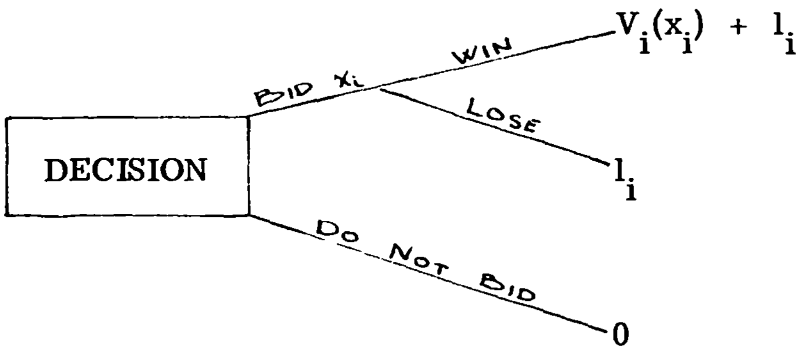
l_i = cost of prearing and submitting a bid for contract i. It is a negative constant.

R_{ijh} = amount of resource h required by contract i in time period j.

C_{jh} = amount of resource h available in time period j.

4.2.4 Expected Value

For each contract i, there is a value attached to each of the possible outcomes of the decision situation. These are illustrated below.



The expected value of the decision, Bid x_i, is

$$P_i(x_i)\,(V_i(x_i) + l_i) + (1 - P_i(x_i))\, l_i$$

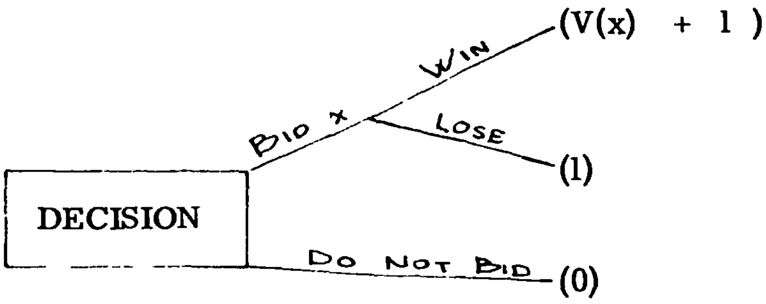
$$= \quad P_i(x_i)\, V_i(x_i) + l_i \qquad (4.2.4\text{-}1)$$

The expected value of the decision, Do Not Bid, is

0

4.2.5 <u>Case 1 - Single Contract: No Constraints</u>

For this case, $x_i = x_1 = x$; $l_i = l_1 = l$ and so on.



The decision tree above illustrates the situation. The expressions in the brackets are the values resulting from the different outcomes.

The expected value of the decision, Bid x

$$= P(x)\,V(x) + 1$$

and the expected value of the decision, Do Not Bid

$$= 0$$

The objective is to determine the maximum expected value decision. The objective function is expressed as

$$\underset{\substack{x \\ 0 \leq P(x) \leq 1}}{\text{MAXIMIZE}} \left\{ P(x)\,V(x) + 1 \;,\quad 0 \right\} \qquad (4.2.5\text{-}1)$$

To determine the x which maximizes the first part of the objective function, differentiate that part of the expression w.r.t. x and set the first derivative equal to zero.

$$\frac{d\,(\,P(x)\,V(x)\;+1\;)}{dx} = 0 \qquad (4.2.5\text{-}2)$$

(4.2.5-2)

$$P(x)\frac{d\,V(x)}{dx} + V(x)\frac{d\,P(x)}{dx} = 0$$

If $(P(x)\ V(x)\ +\ l\ )$ is a concave function in the region of the optimum, the x^* resulting from the solution of (4.2.5-2) is the optimum bid value.

The decision function for Case 1 is therefore:

$$\begin{array}{lll} \text{Bid } x^* & \text{if } P(x^*)\ V(x^*)\ +\ l & > 0 \\ \text{Do Not Bid if} & P(x^*)\ V(x^*)\ +\ l & < 0 \end{array} \qquad (4.2.5\text{-}3)$$

In the rare event that $P(x^*)\ V(x^*)\ +\ l\ =\ 0$, either decision is optimal.

4.2.6 Case 2: N Contracts: Simultaneous Bidding: No Constraints

The decision situation for this Case is the same as that for Case 1 except that here the decision is made simultaneously for all the members of a set of contracts. The expected value is the sum of the expected values of the individual contracts. The objective function for this Case is

$$\underset{\substack{X \\ 0 \leq P_i(x_i) \leq 1}}{\text{MAXIMIZE}} \sum_{i=1}^{N} \left\{ P_i(x_i)\ V_i(x_i)\ +\ l_i, \quad 0 \right\} \qquad (4.2.6\text{-}1)$$

Taking partial derivatives of the first parts of the expression and setting them equal to zero provides N equations.

$$\frac{\partial \sum_{i=1}^{N} (P_i(x_i)V_i(x_i) + 1_i)}{\partial x_i} = P_i(x_i)\frac{\partial V_i(x_i)}{\partial x_i} + V_i(x_i)\frac{\partial P_i(x_i)}{\partial x_i} = 0$$

(4.2.6-2)

for $i = 1, 2, \ldots, N$

which are the same as equation (4.2.5-2) of Case 1. This is obvious as the function is separable. The decision function for each contract in this Case is the same as (4.2.5-3). Thus it can be seen that, in the absence of any constraints, the optimum decision for a given contract i is the same regardless of whether or not it is one of a set of contracts. It is also apparent that, for this Case, the optimum decision is not affected by whether the contracts are bid sequentially, or simultaneously; or by whether you know the results of the first c contracts before you bid on the remaining N-c contracts.

Cases 1 and 2 are essentially the same as the "General Model" developed by Friedman (11).

4.2.7 Case 3: Single Contract: Independent Constraints

An independent constraint is defined as one which is a function of the contract i , but not of the amount bid, x_i . This is the normal type of resource constraint encountered where the men, plant, and materials necessary to fulfill the contract are a function of the contract but independent of the bid price.

With the introduction of constraints, the fact that a contract has a dimension in time must be considered. If the time interval under consideration, i.e. the duration of the contract, is divided into m discrete intervals of length t; then the amount of resource h required in time period j can be expressed as R_{jh} , and the amount of resource h available (capacity) in time period j is C_{jh}.

The decision situation for this Case is the similar to Case 1. The Objective Function for this Case is

$$\underset{\substack{x \\ 0 \leq P(x) \leq 1}}{\text{MAXIMIZE}} \left\{ P(x)\ V(x) + 1', \quad 0 \right\} \qquad (4.2.7\text{-}1)$$

Subject to $\quad R_{jh} \leq C_{jh} \qquad j=1,2,\ldots,m$

$\qquad\qquad\qquad\qquad\qquad h=1,2,\ldots,r$

For the case of a single contract, probabilistic treatment of a constraint is meaningless. It is a binary (yes-no) situation. Either there are sufficient resources to fulfill the contract, in which event the decision function of Case 1 applies; or there are not sufficient resources and the decision is Do Not Bid.

Formally, the decision function for Case 3 is

$$\text{Bid } x^* \qquad \text{if } P(x^*)V(x^*)+1 > 0 \quad \text{and} \quad R_{jh} \leq C_{jh} \quad \begin{array}{l}\text{for } j=1, 2, \ldots, m\\ \phantom{\text{for }} h=1, 2, \ldots, r\end{array}$$

(4.2.7-2)

$$\text{Do Not Bid} \qquad \text{if } P(x^*)V(x^*)+1 < 0 \quad \text{or} \quad R_{jh} > C_{jh} \quad \begin{array}{l}\text{for any } j=1, 2, \ldots, m\\ \phantom{\text{for any }} \text{or } h=1, 2, \ldots, r\end{array}$$

If $P(x^*)V(x^*) + 1 = 0$ and $R_{jh} \leq C_{jh}$ for all j and h, either decision is optimal.

4.2.8 Case 4: N Contracts: Simultaneous Bidding: Independent Constraints

This is the general Case and the other cases can be considered to be this case modified by simplifying assumptions. The case is similar to Case 2 and the objective function is the sum of the individual contract expected values. However, the function is no longer separable as the constraints create an inter-relationship between the contracts.

Since this Case deals with a set of contracts, the statistical concept of expected resource utilisation is employed. The effect of using this concept is best demonstrated by example.

Consider the simple example where a company that possesses one tower crane and cannot acquire another, has the opportunity to bid on two contracts, A and B, each of which would require the crane. Using the expected resource utilisation concept, as long as the sum of the probabilities of winning the two contracts was less than 1, the company would bid both contracts. That is, if

P_A = probability of winning contract A

P_B = probability of winning contract B

R_A = resource requirement for contract A = 1.0

R_B = resource requirement for contract B = 1.0

C = capacity = 1.0

as long as $P_A \cdot R_A + P_B \cdot R_B \leq C$

$$P_A \cdot 1 + P_B \cdot 1 \leq 1.0$$

both contracts will be bid.

This does not prevent the contractor from achieving the embarrassing position of winning two contracts and having only one crane; but it does provide a rationale which permits him to bid both contracts.

For Case 4, the problem is formulated as

$$\underset{\substack{X \\ 0 \leq P_i(x_i) \leq 1}}{\text{MAXIMIZE}} \quad \sum_{i=1}^{N} \left\{ P_i(x_i)\, V_i(x_i) \quad + \quad l_i \ , \quad 0 \right\}$$

$$\text{subject to} \quad \sum_{\text{active } i\text{'s}} P_i(x_i) R_{ijh} \leq C_{jh} \qquad (4.2.8\text{-}1)$$

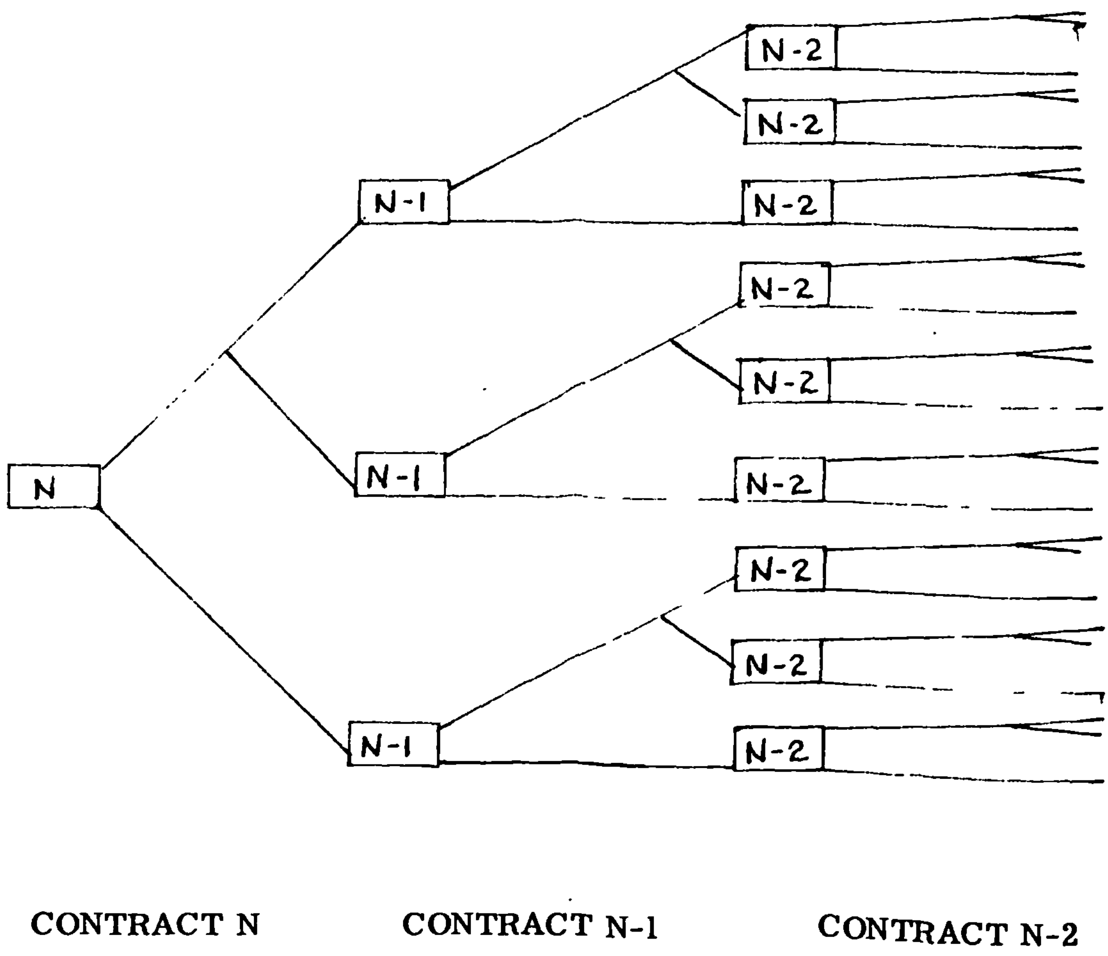
$$\text{for } j = 1, 2, \ldots, m \quad h = 1, 2, \ldots, r$$

$$R_{ijh} \leq C_{jh} \qquad \text{for all active } i\text{'s and for all } j \text{ and } h$$

where the active i's refer to all contracts included in the solution sub-set. Solution procedures for this Case will be considered in Section 4.3.

4.2.9 Case 5: N Contracts: Independent Constraints: Sequential Bidding

In this Case the bids for contracts are made sequentially and the results of the previous auctions are assumed to be known before the next bid is placed. That is, the first contract is bid, the results are made known; then the second contract is bid; and so on. The order in which the contracts are bid is considered in this Case. To formulate the problem the contracts are numbered in the reverse order to the way in which they are bid. First contract N is bid, the results of the auction made known, then contract N-1 is bid, and so on. The decision situation is illustrated diagramatically below.



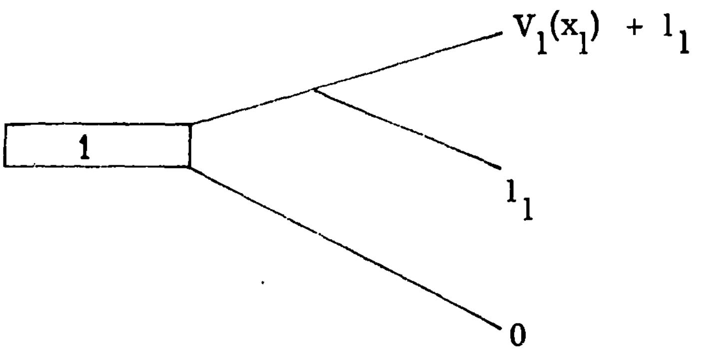
The only decision that must be made is the first one, (Contract N). For that decision there is a total of 3^N possible outcomes.

Since this is a multi-stage decision problem, a dynamic programming approach is used to formulate it. Consider the situation where $j = 1$, $h = 1$; the development for $j, h > 1$ is the same but more complicated. Then considering only those contracts for which $R_i \leq C$, C being the capacity.

Let γ_r be the expected remaining resources when there are $r-1$ decisions left.

$$\gamma_r = C - \sum_{i=r}^{N} P_i(x_i) R_i \qquad (4.2.9.1)$$

When only one decision remains, (i.e. the last contract),



the expected value of the decision is

$$\left\{ P_1(x_1)\, V_1(x_1) + l_1 \;,\; 0 \right\} \qquad (4.2.9\text{-}2)$$

Let $f_1(\gamma_2)$ be the maximum expected value when one decision remains.

$$f_1(\gamma_2) = \underset{x_1}{\text{MAXIMUM}} \left\{ P_1(x_1)\, V_1(x_1) + I_1 \;,\; 0 \right\} \quad (4.2.9\text{-}3)$$

subject to constraint $\qquad P_1(x_1)R_1 \leq \gamma_2$

on the decision to bid.

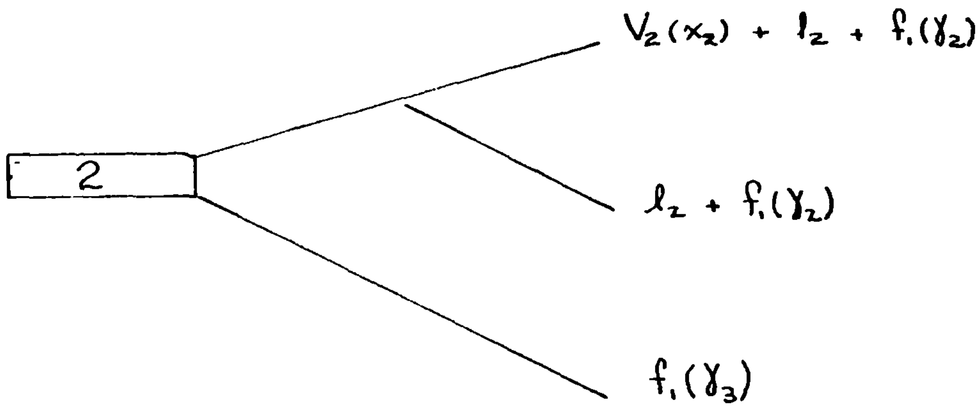
The effect of the concept of expected resource requirement is apparent here. It is possible for $R_1 > \gamma_2$, that is the resource required by contract 1 to be more than the expected remaining capacity, as long as the expected resource requirement, $P_1(x_1)R_1$ is less.

The basis for this approach is that, whilst this is not the way the decision would be made in practice when this decision stage is reached, it permits allowance to be made for future possibilities at the first stage of the decision process.

The two part nature of the objective function results in different constraint conditions applying to the different alternatives. In the situation where there is only one contract left, the constraint

$$P_1(x_1)\, R_1 \leq \gamma_2$$

applies only if the decision to bid results in the maximum value of $f_1(\gamma_2)$. The decision not to bid is not constrained since it requires no resources. This point becomes clearer when the situation of two decisions (contracts) remaining is examined.



$$f_2(\gamma_3) = \mathrm{MAX}\left[P_2(x_2)(V_2(x_2) + l_2 + f_1(\gamma_2)) + (1-P_2(x_2))(l_2 + f_1(\gamma_2)),\ f_1(\gamma_3)\right]$$

$$= \mathrm{MAX}\left[P_2(x_2)V_2(x_2) + l_2 + f_1(\gamma_2),\ f_1(\gamma_3)\right]$$

(4.2.9-4)

subject to constraints

$P_1(x_1)R_1 \leq \gamma_3$ if contract 2 is not bid and contract 1 is bid

$P_2(x_2)R_2 \leq \gamma_3$ if contract 2 is bid and contract 1 is not bid

$P_1(x_1)R_1 + P_2(x_2)R_2 \leq \gamma_3$ if contract 2 is bid and contract 1 is bid.

These constraints can be expressed as

$$\sum_{\substack{\text{active } i\text{'s} \\ \text{from } i=1}}^{2} P_i(x_i)\, R_i \quad \leq \gamma_3$$

where the active i's include only those contracts which are bid.

For the general case, n decisions (contracts) left this becomes

$$f_n(\gamma_{n+1}) = \text{MAX} \left[P_n(x_n)\, V_n(x_n) + l_n + f_{n-1}(\gamma_n),\ f_{n-1}(\gamma_{n+1}) \right]$$

(4.2.9-5)

subject to
$$\sum_{\text{active } i\text{'s}} P_i(x_i)\, R_i \quad \leq \gamma_{n+1}$$

And for the first contract, (N decisions remaining)

$$f_N(\gamma_{N+1}) = \text{MAX} \left[P_N(x_N)\, V_N(x_N) + l_N + f_{N-1}(\gamma_N),\ f_{N-1}(\gamma_{N+1}) \right]$$

(4.2.9-6)

subject to
$$\sum_{\text{active } i\text{'s}} P_i(x_i) R_i \quad \leq \gamma_{N+1}$$

For the first contract the expected remaining resources are the initial starting capacity.

$$\gamma_{N+1} = C$$

Therefore

$$f_n(C) = \text{MAXIMUM}\left[P_N(x_N)\, V_N(x_N) + I_N + f_{N-1}\,(C - P_N(x_N)R_N), \quad f_{N-1}(C) \right]$$

$$\text{subject to} \quad \sum_{\text{active } i\text{'s}} P_i(x_i)R_i \leq C \qquad (4.2.9\text{-}7)$$

which is just another way of writing the equations for the N contract, simultaneous bidding case, (eqn. 4.2.8-1). From this it is concluded that when a contract is a member of a constrained set, if the concept of expected resource utilisation is employed it is necessary to determine the optimum decision for all contracts in the set to obtain the optimum decision for a single contract in the set. The difference between the simultaneous and the sequential cases is that in the simultaneous case all bids are decided, whereas in the sequential case only the decision for the first contract is made. The results of the auction determine the capacity limitations for calculating the optimum decision for the second contract, and so on.

4.3 SOLUTION PROCEDURES

Cases 1, 2, and 3 require to determine the optimum decision that the equation

$$P_i(x_i)\ \frac{dV_i(x_i)}{dx_i} \quad + \quad V_i(x_i)\ \frac{dP_i(x_i)}{dx_i} \quad = \quad 0 \qquad (4.3\text{-}1)$$

be solved for x. The solution procedures adopted for these Cases depend upon the expressions used for P(x) and V(x). In Chapter 5, solution algorithms using the Adjudication Value function from Section 2.5.6 and several different probability functions, are developed and tested.

To solve the problem for the general formulation, Cases 4 and 5, it is necessary to determine the vector of contract bids, X, that

$$\underset{\substack{X \\ 0 \leq P(x) \leq 1}}{\text{MAXIMIZES}} \quad \sum_{i=1}^{N} \left(P_i(x_i)\, V_i(x_i) + 1_i \ , \quad 0 \right)$$

subject to

$$\sum_{\text{active } i\text{'s}} P_i(x_i)\, R_{ijh} \leq C_{jh} \qquad \text{for } j = 1, 2, \ldots, m; \quad h = 1, 2, \ldots, r$$

$$R_{ijh} \leq C_{jh} \qquad \text{for active } i\text{'s and for all } j \text{ and } h$$

Like the real problem it models, the mathematical problem has two parts: first the selection of the optimum sub-set from the given set of N contracts; and second the determination of the optimum bid prices for the constracts in the sub-set. Section 4.3.1 considers the problem of the sub-set N' , where all contracts are bid; and Section 4.3.2 suggests an approach to solution of the combined Selection and Adjudication Problem.

4.3.1 The Sub-set N'

The set of contracts, N, can be divided into two sub-sets. Those which are bid,N', and those which are not. Ignoring, for the moment, how the division is made; the problem of determining the optimum bids for sub-set N' is examined. Since all the contracts are to be estimated and bid, the cost of bidding does not enter the decision. Therefore the problem is

$$\text{MAXIMIZE} \quad \sum_{i=1}^{N'} P_i(x_i)V_i\,(x_i) \tag{4.3.1-1}$$

$$\text{subject to} \quad \sum_{i=1}^{N'} P_i(x_i)R_{ijh} \leq C_{jh}$$

$$R_{ijh} \leq C_{jh}$$

$$R_{ijh} \geq 0 \qquad C_{jh} \geq 0 \qquad \text{for } j = 1, 2, .., m \quad h = 1, 2, ..., r$$

The following changes are made in the notation:

$$P_i = P_i(x_i)$$

$$V_i = V_i(x_i)$$

Eliminate any contract that does not satisfy the absolute capacity constraint: $R_i \leq C$. Then it is possible to formulate the Kuhn-Tucker (17) conditions for optimality for the problem.

For $u_{jh} \geq 0$

$$\frac{\partial \sum_{i=1}^{N'} P_i V_i}{\partial x_i} - \sum_{h=1}^{r} \sum_{j=1}^{m} u_{jh} \frac{\partial \left(\sum_{i=1}^{N'} P_i R_{ijh} - C_{jh} \right)}{\partial x_i} = 0$$

for $i = 1, 2, \ldots, N'$

if $u_{jh} = 0$ then $\sum_{i=1}^{N'} P_i R_{ijh} - C_{jh} \leq 0$

if $u_{jh} > 0$ then $\sum_{i=1}^{N'} P_i R_{ijh} - C_{jh} = 0$

(4.3.1-2)

If $u_{jh} = 0$ for all j and h, then the solution is unconstrained and the optimal x's are the solution to

$$P_i \frac{dV_i}{dx_i} + V_i \frac{dP_i}{dx_i} = 0 \qquad (4.3.1\text{-}3)$$

$i = 1, 2, \ldots, N'$

If $u_{jh} > 0$, for all j and h necessary conditions for optimality are the $N' + (m \cdot r)$ equations:

$$P_i \frac{dV_i}{dx_i} + V_i \frac{dP_i}{dx_i} - \sum_{h=1}^{r} \sum_{j=1}^{m} u_{jh} R_{ijh} \frac{dP_i}{dx_i} = 0 \qquad \text{for } i = 1, 2, \ldots, N'$$

$$\sum_{i=1}^{N'} P_i R_{ijh} - C_{jh} = 0 \qquad \begin{matrix} j = 1, \ldots, m \\ h = 1, \ldots, r \end{matrix}$$

(4.3.1-4)

for the N' unknown x_i's and the $(m \cdot r)$ unkown u's. These are sufficient conditions of optimality if

$$\sum_i P_i V_i$$

is a concave function and the

$$\sum_i P_i R_{ijh} - C_{jh} \qquad \text{for all j and h}$$

are convex functions satisfying the Kuhn-Tucker regularity condition.

The u_{jh}'s that constrain the solution (i.e. those greater than 0) are shadow prices for the resources. They are related to the cost of not having the additional unit of resource h in time period j.

Example: Consider the case where $N = 3$, $m = 1$, $r = 1$.

The Kuhn-Tucker conditions give the following equations.

$$P_1(x_1)\frac{dV_1(x_1)}{dx_1} + V_1(x_1)\frac{dP_1(x_1)}{dx_1} - u_{11}R_{111}\frac{dP_1(x_1)}{dx_1} = 0$$

$$P_2(x_2)\frac{dV_2(x_2)}{dx_2} + V_2(x_2)\frac{dP_2(x_2)}{dx_2} - u_{11}R_{211}\frac{dP_2(x_2)}{dx_2} = 0$$

$$P_3(x_3)\frac{dV_3(x_3)}{dx_3} + V_3(x_3)\frac{dP_3(x_3)}{dx_2} - u_{11}R_{311}\frac{dP_3(x_3)}{dx_3} = 0$$

(4.3.1-5)

$$P_1(x_1)R_{111} + P_2(x_2)R_{211} + P_3(x_3)R_{311} - C_{11} = 0$$

Solving the first three equations for u_{11},

$$u_{11} = \frac{P_1(x_1)\dfrac{dV_1(x_1)}{dx_1} + V_1(x_1)\dfrac{dP_1(x_1)}{dx_1}}{R_{111}\dfrac{dP_1(x_1)}{dx_1}} = \frac{1}{R_{111}}\left\{P_1(x_1)\frac{\dfrac{dV_1(x_1)}{dx_1}}{\dfrac{dP_1(x_1)}{dx_1}} + V_1(x_1)\right\}$$

$$u_{11} = \frac{1}{R_{211}}\left\{P_2(x_2)\frac{\dfrac{dV_2(x_2)}{dx_2}}{\dfrac{dP_2(x_2)}{dx_2}} + V_2(x_2)\right\}$$

$$u_{11} = \frac{1}{R_{311}}\left\{P_3(x_3)\frac{\dfrac{dV_3(x_3)}{dx_3}}{\dfrac{dP_3(x_3)}{dx_3}} + V_3(x_3)\right\}$$

If a feasible solution exists, it can be found by the following gradient procedure.

1. Compute the unconstrained values of x, from eqn. (4.3.1-3) This will provide starting values P_1, P_2, P_3, and $u_{11}^{o} = 0$.

2. If $(P_1R_{111} + P_2R_{211} + P_3R_{311} - C_{11})$

< 0 select a new value $u_{11}^{n+1} = u_{11}^{n} - \delta$; unless

$u_{11}^{n} = 0$ in which case the solution is optimal

$= 0$ this solution is optimal

> 0 select a new value $u_{11}^{n+1} = u_{11}^{n} + \delta$

3. Compute P_1, P_2 and P_3 for the new value of u_{11} and return to step 2.

4.3.2 The Two-Stage Problem

When the Selection phase is introduced the problem becomes more complex. If the number of contracts (N) is small the most satisfactory method of solution may be to try all possible combinations.

A mathematical method of solution is developed as follows:

The Problem:

$$\underset{\substack{X \\ 0 \leq P_i \leq 1}}{\text{MAXIMIZE}} \sum_{i=1}^{N} \left\{ P_i V_i + 1_i \,, \quad 0 \right\} \tag{4.3.2-1}$$

subject to

$$\sum_{\text{active } i\text{'s}} P_i R_{ijh} \leq C_{jh}$$

$$j = 1, \ldots, m$$
$$h = 1, \ldots, r$$

$$R_{ijh} \leq C_{jh}$$

First eliminate all contracts which fail to satisfy the absolute capacity constraint $R_{ijh} \leq C_{jh}$. Then it is necessary to devise a formulation which confines the first constraint to the sub-set of active contracts and requires $P_i V_i + 1_i \geq 0$. Since the 1_i's are independent of the x's; they do not affect the location of the optimum X. Therefore the optimum for the original problem is also the optimum for this equivalent problem.

$$\underset{\substack{X \\ 0 \leq P_i \leq 1.0}}{\text{MAXIMIZE}} \sum_{i} P_i V_i \tag{4.3.2-2}$$

subject to

$$\sum_{i} P_i R_{ijh} \leq C_{jh}$$

and

$$P_i (P_i V_i + 1_i) \geq 0$$

since $0 \leq P_i \leq 1$; the second constraint ensures that either $P_i V_i + l_i \geq 0$ or $P_i = 0$. If $P_i = 0$, contract i does not affect the first constraint.

The Kuhn-Tucker conditions for this formulation are:

for $u_{jh} = 0$ and $w_i = 0$

$$\frac{\partial \sum P_i V_i}{\partial x_i} - \sum_k \sum_j u_{jk} \frac{\partial (P_i R_{ijh} - C_{jh})}{\partial x_i} - \sum_i w_i \frac{\partial (-P_i(P_i V_i + l_i))}{\partial x_i} \tag{4.3.2-3}$$

if $u_{jh} = 0$; then $\sum_i P_i R_{ijh} - C_{jh} \leq 0$

if $u_{jh} > 0$; then $\sum_i P_i R_{ijh} - C_{jh} = 0$

if $w_i = 0$; then $- P_i(P_i V_i + l_i) \leq 0$

if $w_i > 0$; then $- P_i(P_i V_i + l_i) = 0$

The w_i's can be regarded as bid selectors. When $w_i = 0$, the contract i is included in the optimal sub-set; when $w_i > 0$, the contract is excluded. The w's also have "shadow cost implications as they are related to the cost of having to bid when it is not worth doing so.

The model may be extended beyond this point by the inclusion of the relationships derived in Chapters 2 and 3. However, it was decided not to do this, for two reasons. First, it appears that a point of diminishing returns has been achieved. Although the extension of the formulation to include the relationships would be an interesting mathematical exercise; the solution procedures, if they exist, would probably be more complex than the actual problem. As was discussed in Chapter 2, Building Construction contracts are not very resource constrained. In fact, they are even less constrained than an analysis of the resources required for a contract might suggest. This is because of the magnitude of the contracts with respect to the existing free capacity of a firm. The Sample Companies are major companies and yet in the time period covered by the data sets, few contracts are won.

Company A	12 months	5 contracts won
Company B	12 months	5 contracts won
Company C	9 months	10 contracts won
Company D	19 months	6 contracts won

In addition, labour is more mobile between firms than in most industries, and most kinds of building plant may be hired.

The long durations of contracts also affect the problem to the extent that in any one year period a company's bidding activity is only concerned with allocating about 20% of the firms resources. The remaining 80% being already committed from the previous years. These small numbers suggest that a probabilistic procedure is unnecessary since the entire problem can be easily handled in a discrete manner. The current method of integrating resources with bidding is to observe when current projects are scheduled for completion, and intensify bidding activity until contracts commencing at those times are acquired. However, even this time element is not that crucial in most cases because some overlapping of contracts is possible, and alternate interim uses can be found for the site staff who are a major resource.

The functioning of the selection decision in practice is primarily explained by the absolute capacity constraint; $R_{ijh} \leq C_{jh}$. Projects are turned away because of insufficient capacity. This constraint includes the capacity of the estimating department to produce an estimate within the specified time. The formulation indicates that given the absolute capacity, the selection decision hinges on the relationship of the expected value to the cost of estimating. Studies carried out by the Sample Companies indicate that the cost of estimating is approximately

0.1% of the pound volume estimated. This value being so low, it appears unlikely that the formulation would eliminate many contracts for this reason. This agrees with observed practice; i.e. given absolute capacity, few contracts are turned away.

The second reason for not continuing the development of the formulation was that, even assuming that a solution procedure could be devised and that solutions exist, data with which to test the formulation was not available from the Sample Companies within the duration of the investigation.

Hence, for both reasons, it was decided to concentrate further development and testing on the Adjudication Problem, i.e. Cases 1, 2 and 3 of this Chapter. The Sample Companies data sets, to which reference has already been made, permit prescriptive models for this more limited, but significant, problem to be tested.

4.4 SUMMARY

The Competitive Bidding Problem can be described by a decision tree. Using this tree, and expected value concepts, and assuming independent contracts and independent resources, decision functions were developed for a series of competitive bidding situations. It is apparent from the formulations of the different Cases that there are only two basic situations. That is, given sufficient absolute capacity to undertake the contract, the Decision is either constrained by the expected resources utilisation of other potential contracts or it is not. If the contract is not constrained, then the Decision is independent of whether the contract stands alone or is one of a set, and of whether it is bid simultaneously with others or sequentially as one of a series.

The solution procedure for the constrained case involves using mathematical programming and the Kuhn-Tucker optimality conditions. The general structure of an appropriate model was proposed, but it was not developed because, although it is felt to be a reasonable mapping, in the real problem the constraints are not markedly inflexible or critical. Thus attention is concentrated on the unconstrained case which is a close approximation to the real situation and can be evaluated with the available data.

CHAPTER 5: EMPIRICAL EVALUATION

5.1 INTRODUCTION

The preceding chapters have developed the approach to the Competitive Bidding Problem, and a general probabilistic model has been formulated.

The attention of this thesis now shifts to the research question - Which of the alternative methods available should the decision maker use to make his decision? The alternative methods amount to the use of the different probability functions. In this Chapter the different models are empirically tested. All the Friedman variants are included in the testing. This was done because, despite the theoretical criticisms that can be directed at them, there is the possibility that they could perform well, and the success claimed by Park (24), and Morin and Clough (20) cannot be ignored. Also included in the testing are the General

Distribution Model, and a model which arises from trying to use the OPTM procedures as an automatic feedback correction device.

A total of six models are tested. They are:

Model 1A - Single Contract, Unconstrained, Friedman Average Competitor Hypothesis.

This is the basic Friedman model. All historical bid-cost ratios are included in the distribution function with equal weight.

Model 1E - Single Contract, Unconstrained, Friedman Average Competition Hypothesis with exponential smoothing.

This is the Friedman variant proposed by Morin and Clough. Here an exponential weighted average of the bid-cost ratios is used to determine the probability function.

Model 2 - Single Contract, Unconstrained, General Distribution Hypothesis.

This is the model developed by this thesis (equations 3.4.2-1). Since managerial estimates do not exist in the historical data, the model was tested under conditions of perfect information. Subsequently, sensitivity analysis is used to determine the actual information requirement and permissible error ranges.

Model 3 - Single Contract, Unconstrained, Normal Distribution Probability Hypothesis.

This is the model proposed in Fundamentals of Operations Research (1), which assumes that the distribution of the "lowest competitor" is normal.

Model 4 - Single Contract, Unconstrained, Lowest Competitor Bid Hypothesis.

This is the Friedman variant proposed by Hannsmann and Rivett (14), where only the lowest competitor is considered and the number of competitors is not required.

Model 5 - Single Contract, Unconstrained, Drift Model

This model is an outgrowth of the OPTM procedure. The results of the last M contracts are used as a basis for altering the bid on the next contract.

The models are called single contract because they treat each contract individually and ignore interactions between contracts. They are called unconstrained because they ignore any constraints that might exist.

The models are tested by computer simulation with the Sample Companies' data sets. They are all tested with the same data and so it is possible to compare the models with each other, with the sample companies, and with the results obtainable by hindsight.

Another method used to evaluate the models was partial implementation of selected models. Partial because there was insufficient time available to adequately test the models - it was felt that at least a year would be necessary to obtain any meaningful results. Thus while no implementation results are available, observations made during the installation of test procedures have bearing on the usefulness of the models, and these are discussed.

5.2 MODEL ALGORITHMS

Computer programs were written to test the six models with the Study Companies data sets. The objective function used was the adjudication objective of Chapter 2, Maximize V(x)

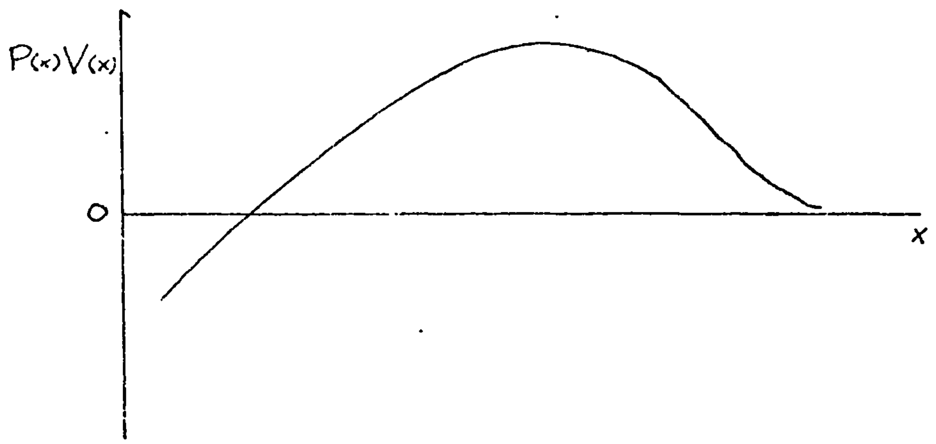
where
$$V(x) = \begin{cases} X - k' & \text{if } x < y \\ 0 & \text{if } x > y \end{cases}$$

and
$$k' = Z(R,n).k \qquad (5.2\text{-}1)$$

The probablistic models, numbers 1 to 4, attempt to maximize the expected value of the objective. This requires determing the bid x^* which satisfies the equation.

$$P(x)\,\frac{dV(x)}{dx} \quad + \quad V(x)\,\frac{dP(x)}{dx} \qquad (5.2.\text{-}2)$$

Since the expected value function has the shape illustrated below, the continuous formulation is easily adapted to an iterative method of solution.



The specific algorithms used to program the models are presented in the following sub-sections.

5.2.1 Model 1A and 1E: Friedman Average Competitor Hypothesis

The Friedman "average bidder" probability density is found by combining all previous ratios of an opposition bid to the company's cost estimate into one distribution function, f(w). Then the probability of a bid of x being lower than one average bidder equals the complementary cumulative distribution

$$G(x) = \int_{\frac{x}{k'}}^{\infty} f(w)\,dw \qquad (5.2.1\text{-}1)$$

and the probability of x being lower than n average bidders is

$$(\,G(x)\,)^{n} = \left(\int_{\frac{x}{k'}}^{\infty} f(w)dw \right)^{n} \qquad (5.2.1\text{-}2)$$

Two methods have been proposed for combining the ratios: the average method, Model 1A, where all previous ratios have equal weights, and the exponential method, Model 1E, where exponential weights are used for the previous ratios. The flow chart, Figure 5.1. shows the computational procedure used for each method.

Both methods are special cases of weighted moving average. However the exponential version is the more general case and given a sufficiently large data base, it can be anticipated that it will be the superior model. First it permits greater weight to be attached to recent observations, and hence allows more fully for recent trends in the ratios of opposition bids to company cost estimates. Secondly, it possesses computational advantages, the storage of data being greatly reduced. Thirdly, as the smoothing coefficient, a, approaches zero the exponentially smoothed value approaches the arithmetic mean, given a sufficiently large number of observations. Hence, given an optimal choice of smoothing coefficient, the exponential model should never be inferior to the average model, but it could well be better. However, since the data base is fairly small, and the simple average version has received much attention (see references 4, 7, 11, and 24) both versions were examined and tested.

Average Method

An array, HIST, was dimensioned in the computer. This array was a discrete representation of the complementary cumulative distribution G(x). It was compiled as follows:

For each competitor's bid, the ratio

h = competitor bid/company's estimated cost

is computed. The value is incorporated in HIST by a loop which

for m = 1, 2, 3, ... 50

if h $\geq$ 0.950 + (m-1) *0.005 add 1 to HIST (m)

FLOW CHART

Friedman
Average Competitor
Hypothesis

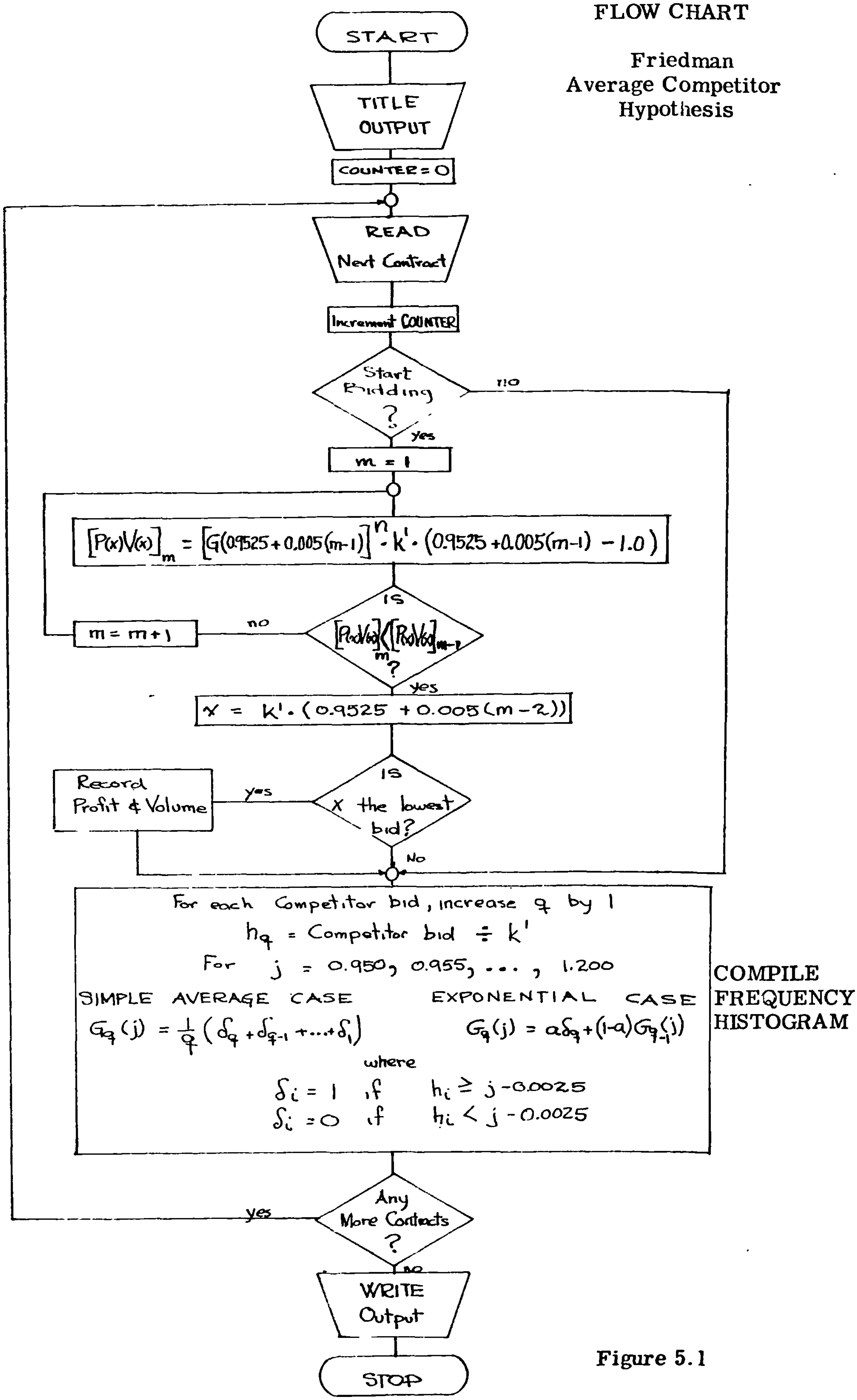
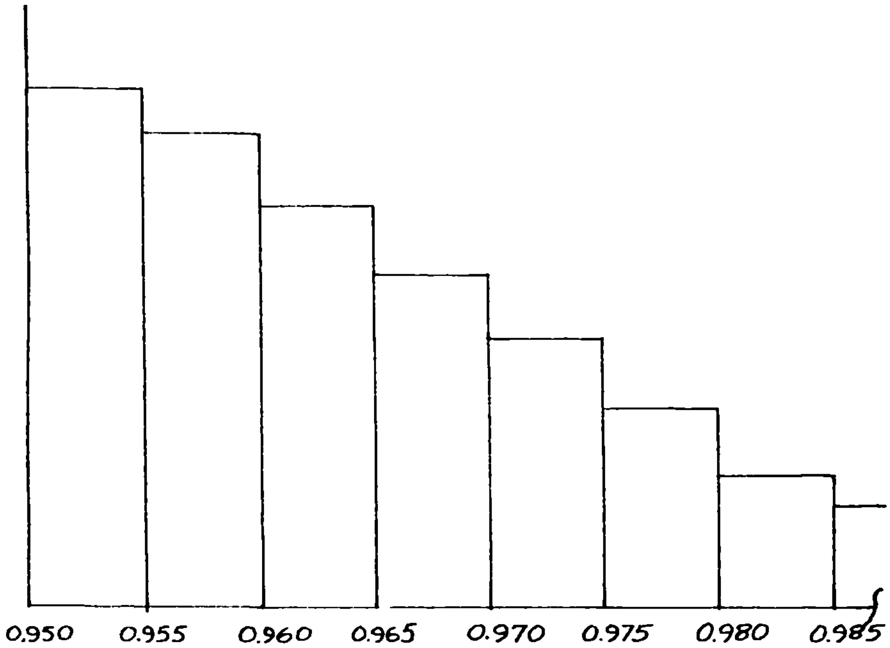


Figure 5.1

The result is illustrated diagrammatically below.



The value of HIST (.) divided by the number of bids used in compiling the histogram to date is the complementary cumulative distribution of the midpoint of the interval.

i.e. $$G_q(0.9525) = \text{HIST}(1) \;/\; q \approx \int_{0.9525}^{\infty} f(w)dw$$

$$G_q(0.9575) = \text{HIST}(2) \;/\; q \approx \int_{0.9575}^{\infty} f(w)dw$$

where q is the total number of competitors bids in the histogram.

To find the maximum expected profit bid, the fact that the objective function has a single optimum is used. Successive expected profits are generated as follows:

For $m = 1, 2, \ldots\ldots\ldots, 50$

$$(P(x)V(x))_m = G(0.9525 + 0.005(m-1))^n * k' * (0.9525 + 0.005(m-1)-1.0)$$

until $(P(x)V(x))_{j+1}$ is less than $(P(x)V(x))_j$. Then $(P(x)V(x))_j$ is taken as the maximum expected profit and the corresponding bid $x = (0.9525 + 0.005\ (j-1))* k'$ is taken as the optimum bid.

In the average method, each probability value is

$$G_q(g) = (\&_q + \&_{q-1} + \ldots + \&_1) \ / \ q$$

where $\&_i = 1$ if $h_i \geq g-0.0025$

$= 0$ if $h_i < g-0.0025$

<u>Exponential Method</u>

An exponential weighted moving average was used in the second version of the model.

Here $$G_q(g) = a\&_q + a(1-a)\&_{q-1} + a(1-a)^2\&_{q-2} + \ldots + a(1-a)^{q-1}\&_1$$

where $\&_i = 1$ if $h_i \geq g-0.0025$

$0 = 0$ if $h_i < g-0.0025$

The updating was done by the formula

$$G_q(g) = a\,\&_q + (1-a)\,G_{q-1}(g) \qquad \text{for all } g$$

The maximum expected profit bid is computed in the same manner as before. The order of the bids is important in this formulation because different competitor bids for the same contract will receive different weights. Although this should not materially affect the results as the differences are very small, to be consistent, the competitors' bids were incorporated in the histogram in descending order.

5.2.2 Model 2: General Distribution Model

This is the model developed in Section 3.4.2, and uses the probability function determined by regression analysis. The computer is programmed to determine, for each contract, the value of x which maximizes $P(x)\ V(x)$

where $V(x) = x - k'$

$$x = \Theta\,(0.974449 + 0.1352319F(x)^2 - 0.0055555/F(x))$$

$$P(x) = (1 - F(x))^n$$

$$0 \leq F(x) \leq 1 \qquad (5.2.2\text{-}1)$$

The equations are used to compute x, $P(x)$, and $V(x)$ for successive values of $F(x)$. The value of x which corresponds to the maximum value of P(x) V(x) is selected at the optimum bid. This procedure is outlined in the flow chart, Figure 5.2.

5.2.3 Model 3: Normal Distribution Probability Hypothesis

The assumption made by this model is that the ratio of the lowest competitor's bid to the Company's cost estimate is a sample from a normal distribution. The method then assumes that you are only competing against one super competitor, the lowest one. Whether this is valid is questionable; especially most of the other determinations consider the number of competitors a major parameter. The assumption does, however, simplify the arithmetic, as the probability function no longer contains the exponent n.

Case 1: Normal Distribution with Constant μ and σ.

For this case a computer program is not necessary. The assumption of a constant mean and standard deviation results in a constant percentage mark-up figure to be applied to all contracts. Thus, the best results possible for this case would be the maximum results obtained by the OPT2 program.

FLOW CHART

General Distribution Hypothesis

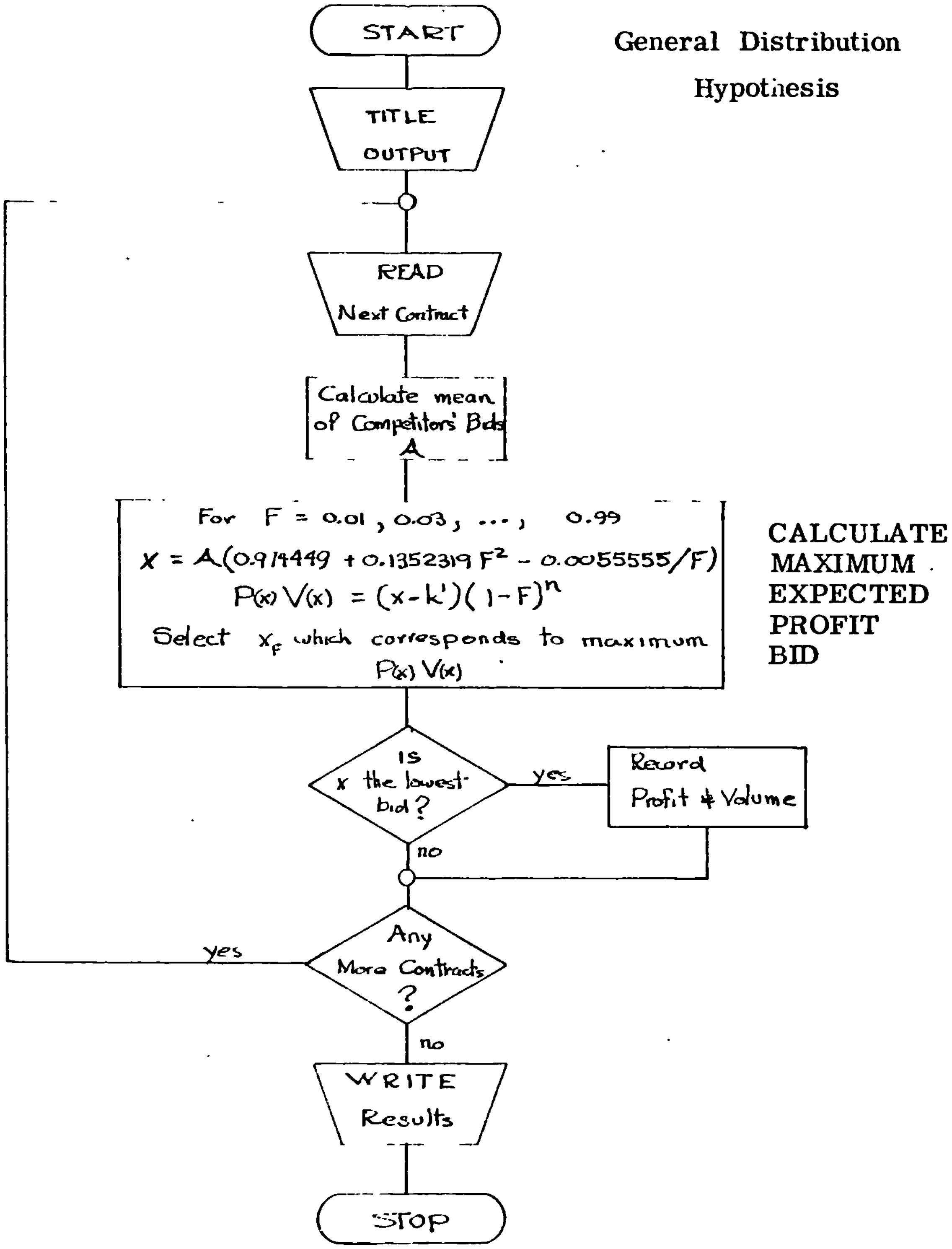


Figure 5.2

Case 2: Normal Distribution with varying μ and σ .

If it is assumed that the mean and standard deviation of the ratio, lowest competitor bid to estimated cost, vary in trends that can be discerned from past bids, it is possible to construct a model. The model seeks to determine the optimum bid by finding the bid x which maximizes

$$P\left(\frac{x}{k'}\right) \; V(x)$$

where $P\left(\frac{x}{k'}\right) = \displaystyle\int_{\frac{x}{k'}}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} \exp - \frac{(s-\mu)^2}{2\sigma^2} \, ds$

the complementary cumulative of the normal distribution.

A computer program was written to compute the mean and standard deviation of the ratios from the first m contracts. These parameters were then used to calculate the optimum bid for the m+1 contract. Then the mean and standard deviation of the ratios for the m+1 contracts were computed. This was used for the m+2 contract and so on. Instead of integrating the normal curve, a table of values of the Cumulative Normal Distribution was included in the program. The program calculates expected profits in the following manner.

for $N = 0.005, 0.001, 0.015, \ldots, r$

calculates $s = \dfrac{(N - \mu)}{\sigma}$

Use s and the table to determine $P(N)$

Then the expected profit is $P(N) . N . k'$

The program continues until

$$\left[P(N). N . k'\right]_m < \left[P(N). N . k'\right]_{m-1}$$

Since the expected profit curve possesses a single optimum, $\left[P(N). N . k'\right]_{m-1}$ is the maximum expected profit and the optimum bid is $x = k' . (1 + N_{m-1})$

The mean μ and standard deviation σ are calculated for all previous contracts by the formula

$$y_q = \frac{\text{lowest competitor on q th contract}}{\text{corrected cost estimate of q th contract}}$$

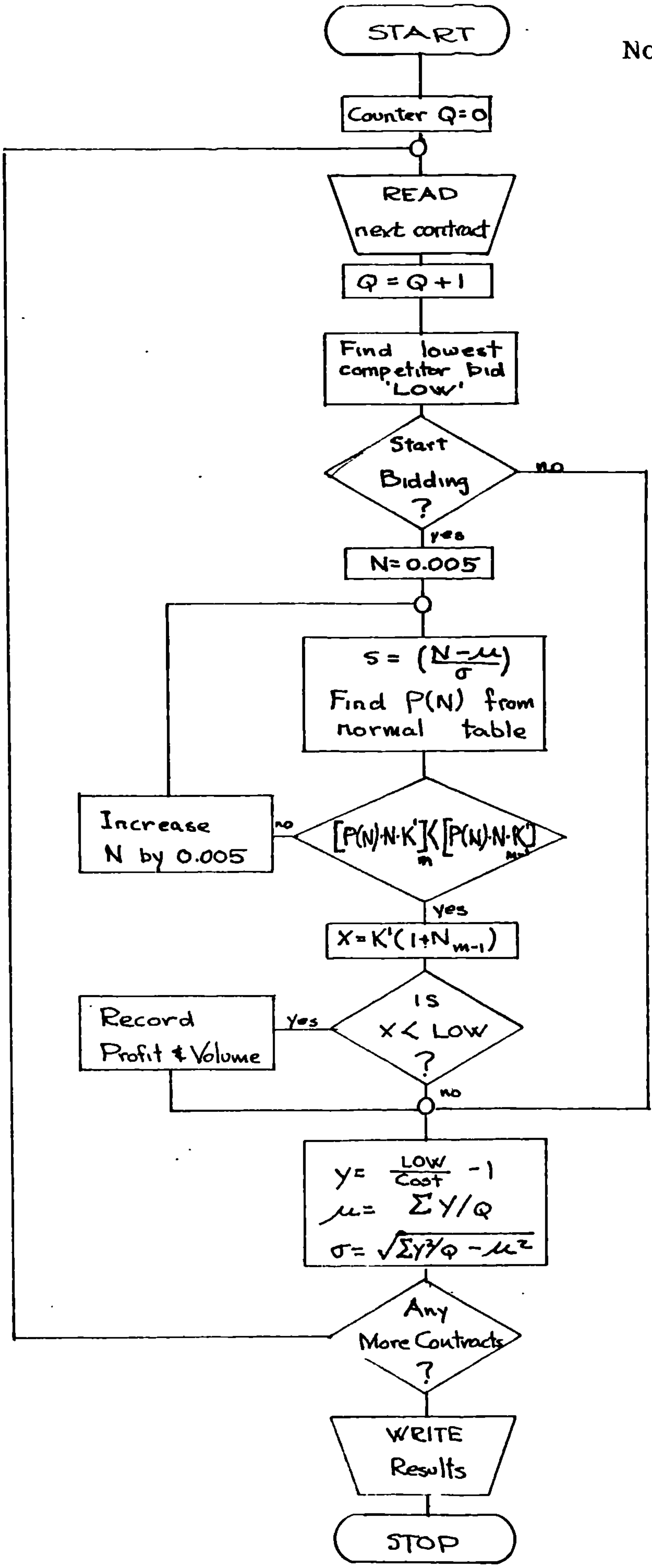
$$\mu = \frac{\sum_q y_q}{q}$$

$$\sigma = \sqrt{\frac{\sum y_q^2}{q} - \mu^2}$$

Figure 5.3 is a flow chart of the program.

FLOW CHART

Normal Distribution Hypothesis



CALCULATE MAXIMUM EXPECTED PROFIT BID

CALCULATE NEW μ & σ

Figure 5. 3

5.2.4 Model 4: Lowest Competitor Bid Hypothesis

This model combines the Friedman historical distribution concept of Model 1 with the Low Competitor concept of Model 3. It assumes that there is a distribution of low or winning bids, and this distribution can be found by combining all the previous ratios of competitor's low bid to Company estimated cost.

The method of calculation is the same as that used in Model 1A. The ratio

$$h_i = \left(\frac{\text{Lowest Competitor bid on contract i}}{\text{Company estimated cost on contract i}} - 1.0\right)$$

is calculated.

$$\text{Frequency } (g) = \frac{1}{q} \left(\&_q + \&_{q-1} + \dots + \&_1\right)$$

$$\text{where } \&_i = 1 \text{ if } h_i \geq g - 0.0025$$
$$= 0 \text{ if } h_i < g - 0.0025$$

For $g = 0.005, 0.010, 0.015, \dots, 0.20$

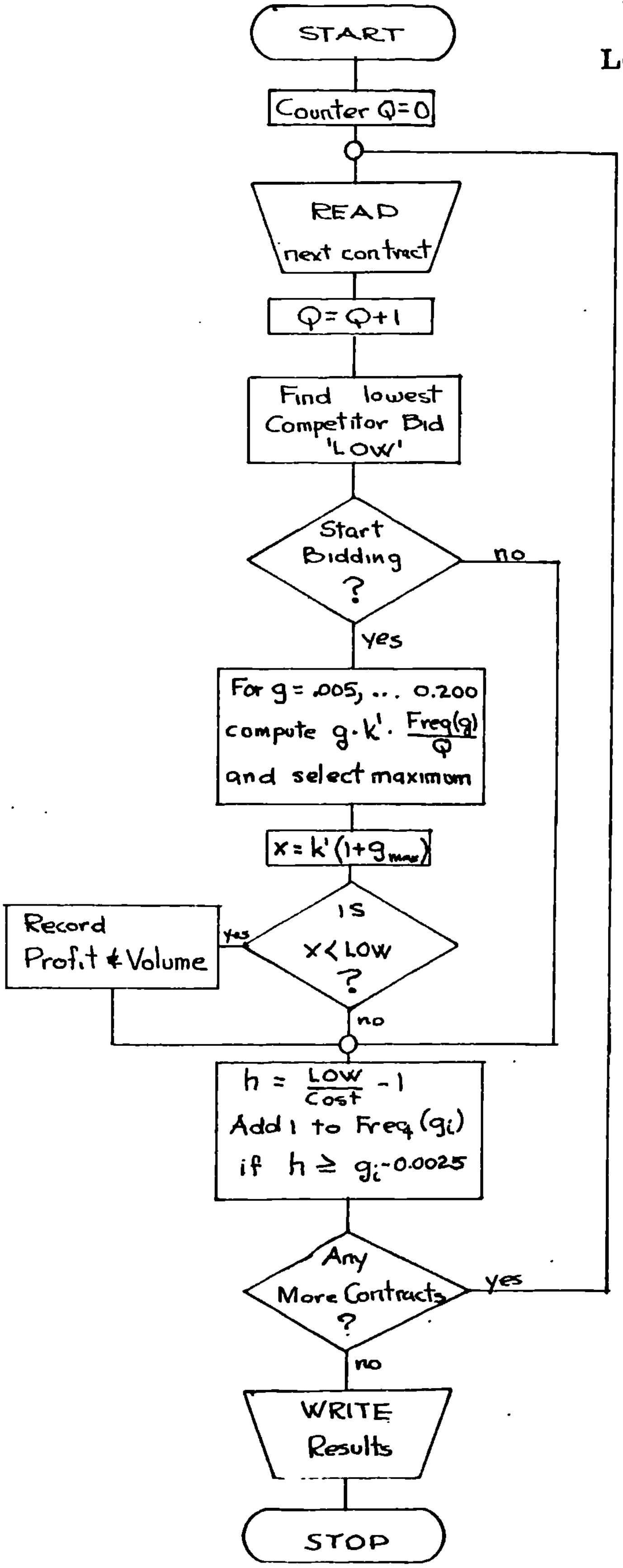
Compute $g \cdot k' \cdot \text{Frequency } (g)$ and select the maximum.

The optimum bid is then $(g_{max} + 1.0) \cdot k = x$

Figure 5.4 shows a flow chart of the program.

FLOW CHART

Lowest Competitor Hypothesis



CALCULATE MAXIMUM EXPECTED PROFIT BID

COMPILE HISTOGRAM OF LOWEST COMPETITOR BIDS

Figure 5.4

5.2.5 Model 5: Drift Model

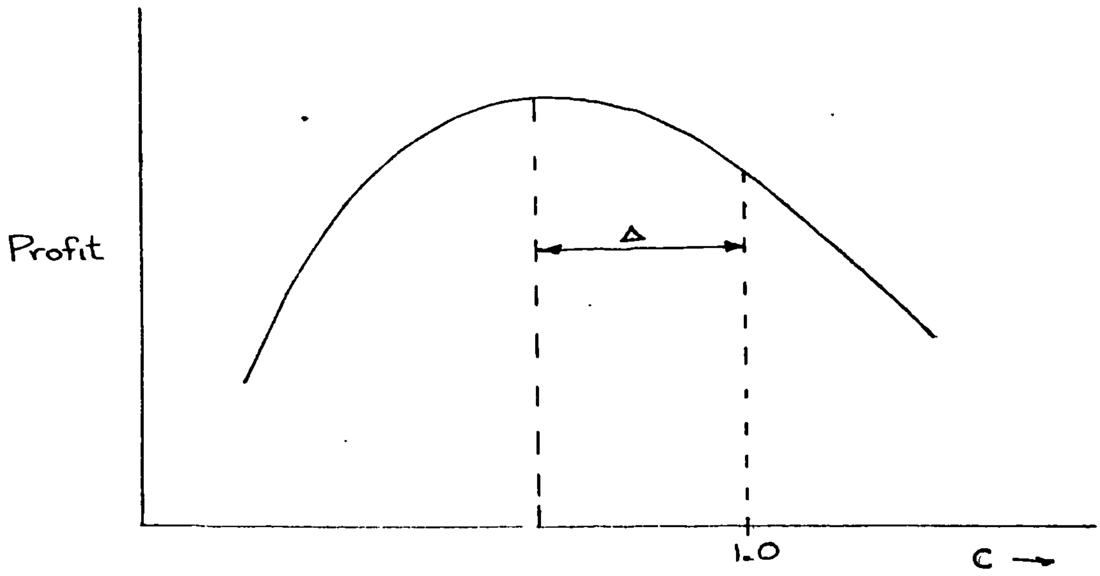
This model was inspired by the OPTM program. The OPTM results indicated that dramatic improvements could sometimes be obtained by incremental varying of the final tender prices. For example, in data set A, an increase of all the tender prices by 1% would have resulted in the winning of exactly the same contracts and an increase in uncorrected profit of 16%. However, this is hindsight. The question is - how can these results be applied to current practice.

Model 5 is an attempt to use the OPTM-program as a tracking device. By using OPTM on the last M contracts bid, and applying the resulting optimum correction to the next contract, the model hopes to eliminate "drifts" away from peak performances.

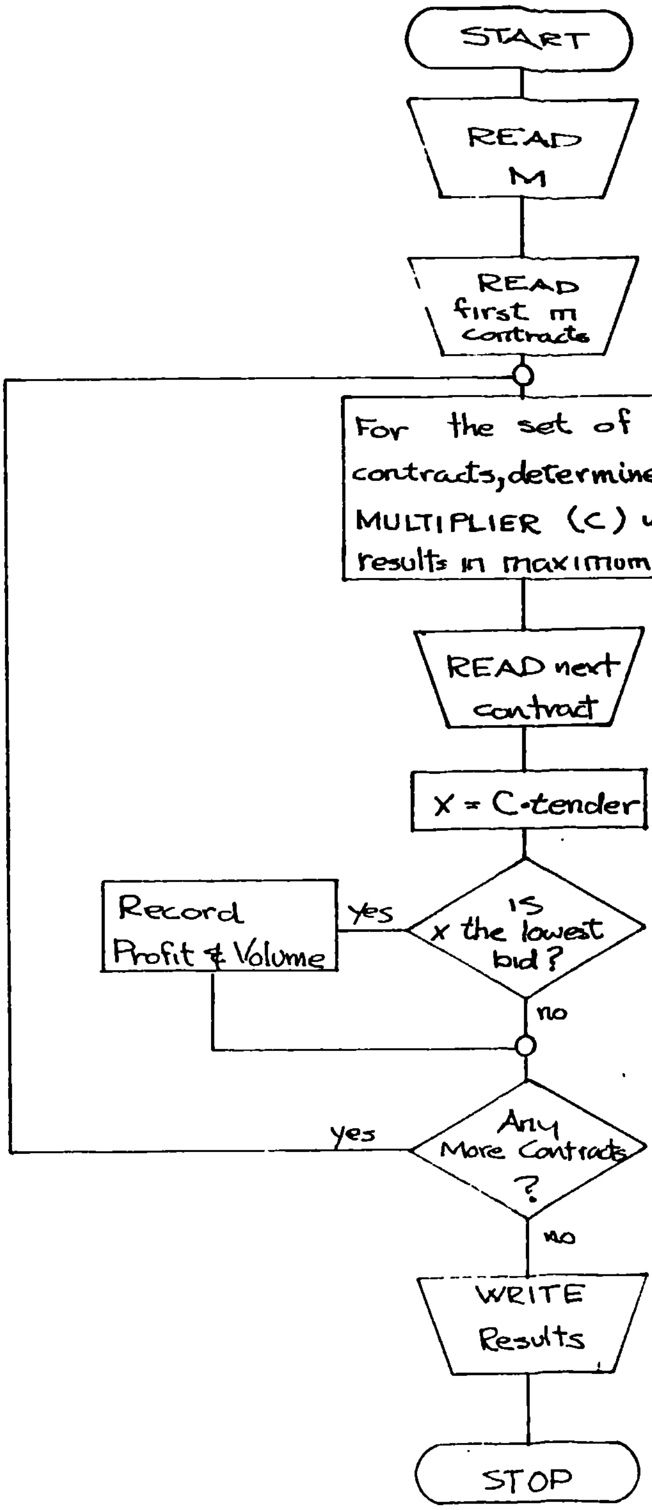
The main problem in setting up the Model is the determination of M, the number of past contracts to examine. This number should be small enough to be sensitive to trends and yet must be large enough that a single large contract will not dominate the results.

The Program operates as follows:

For a set of M contracts, compute the effect on profit of small discrete variations in the tender price. The result will be as illustrated below, where the bid is (c . tender price).



For the M+1 contract the bid submitted is (1.0 - Δ) times original tender price. The first contract is then deleted from the set and the M+1 th. contract added to the set. The process is then repeated to determine the correction for the M+2 nd. contract; and so on. This is a straight forward moving average. Figure 5.5 provides a flow diagram of the Drift Model.



FLOW CHART

Drift Model

Figure 5.5

5.3 SIMULATION TESTING

5.3.1 Testing Program

The five models, the OPTM and OPT2 programs, and a program for graphing the model results on the line printer were incorporated as subroutines in a master program called BIDS. The structure of this program is illustrated diagrammatically in Figure 5.6. The program was stored on The City University ICL 1905 Computer Unit's EDITA tape, from which it can be called by a single instruction.

The card deck for the program is illustrated in Figure 5.7. It consists of:

- two cards to call the program from the EDITA tape
- a model selection card which specifies which models are to be used on the data, which contract to start bidding at, and the exponential smoothing coefficient to use in Model 1.
- the plotting symbols to be used by the graphing programs
- two cards listing the Z correction factors for the data set
- a title card for the data set
- the data set

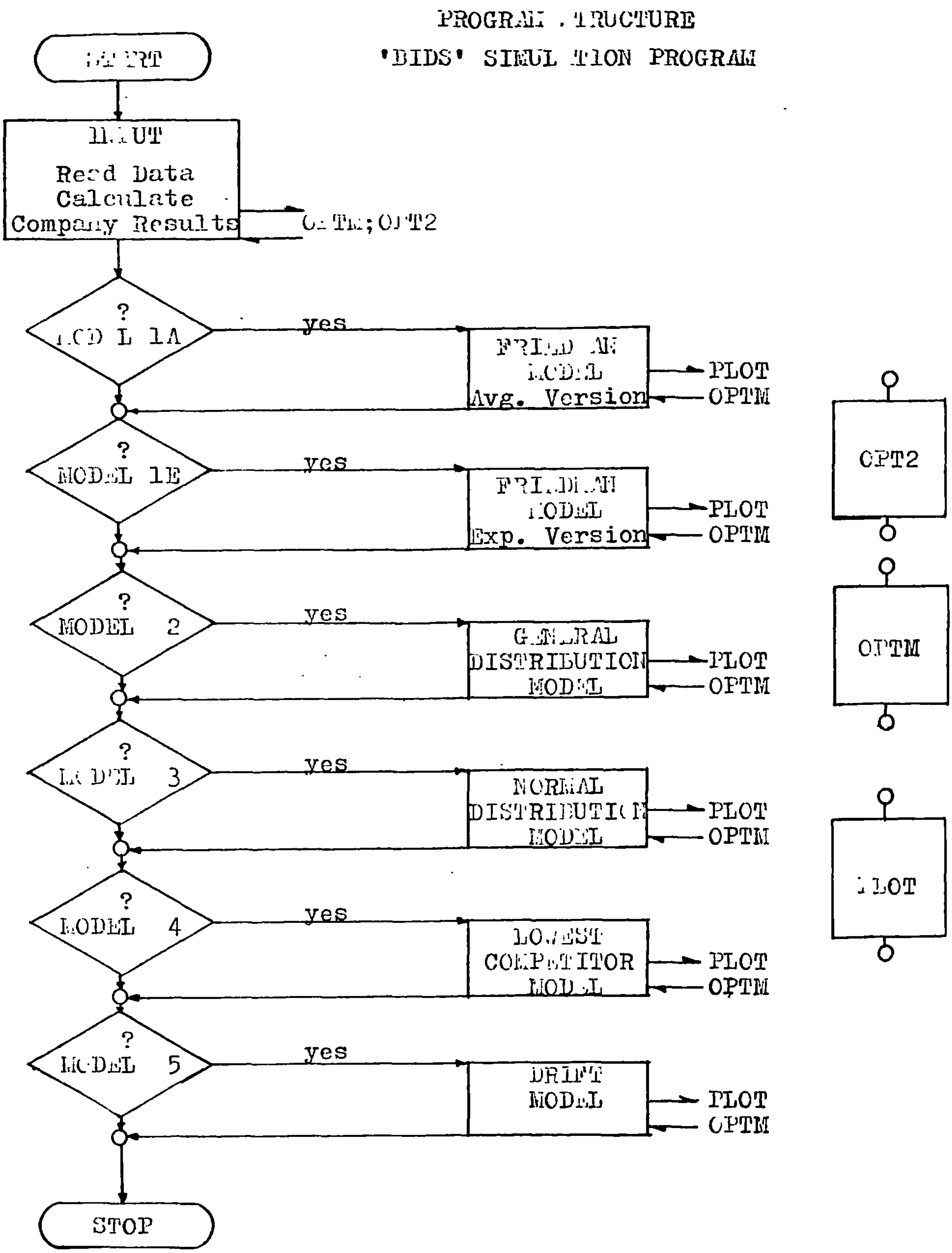

PROGRAM STRUCTURE
'BIDS' SIMULATION PROGRAM
INPUT
Read Data
Calculate
Company Results
OPTM; OPT2
?
MODEL 1A
yes
FRIEDMAN
MODEL
Avg. Version
PLOT
OPTM
?
MODEL 1E
yes
FRIEDMAN
MODEL
Exp. Version
PLOT
OPTM
?
MODEL 2
yes
GENERAL
DISTRIBUTION
MODEL
PLOT
OPTM
?
MODEL 3
yes
NORMAL
DISTRIBUTION
MODEL
PLOT
OPTM
?
MODEL 4
yes
LOWEST
COMPETITOR
MODEL
PLOT
OPTM
?
MODEL 5
yes
DRIFT
MODEL
PLOT
OPTM
STOP
OPT2
OPTM
PLOT

Figure 5.6

BIDS PROGRAM

CARD DECK

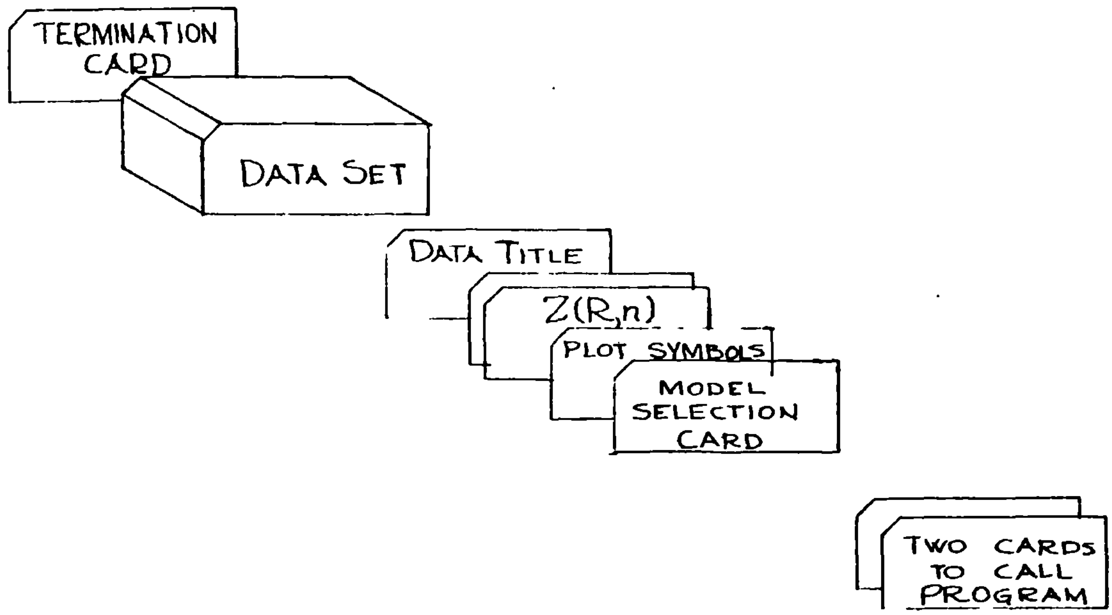


Figure 5.7

- an end card which signals the end of the data set and whether another data set is present.

The INPUT section of the program:

- reads the model selection card
- reads the plotting symbols
- reads the Z factors
- reads the data title
- reads the data cards, applies the Z corrections to the estimated costs, and outputs the corrected data to the line printer and to magnetic tape.
- calculates the Company's performance (Contracts won, Cumulative Profit, Cumulative Volume, Profit-Volume Ratio) and outputs the results on the line printer.
- calls the OPTM and OPT2 programs.

Then it calls the models, specified on the model selection card, to operate on the data.

Each model takes the data stored on the magnetic tape and when it has compiled the specified number of contracts in the cases of Models 1, 3, and 4, or after it has read its starting set size in the case of Model 5, it begins to compute bids for the contracts. The contract number, estimated cost, Model tender, expected profit*, cumulative profit, cumulative expected profit*,

---

* Except for Model 5 where there is no expected profit

cumulative volume and profit-volume ratio are calculated for each contract and output to the line printer. When the Model has completed the data set it calls the PLOT program. This program plots the Model's cumulative profit, the cumulative expected profit, and the Company's cumulative profit over the data set. When the PLOT program is finished, the Model calls the OPTM program to operate on the Model's tenders.

By using the PLOT and OPTM programs on the results of each model, a visual display of each model's effectiveness is provided.

The PLOT graph reveals whether the actual results and the expected results coincide, and whether the model has performed more (or less) effectively than the Company. The OPTM program provides an indication of whether the model is operating in the neighbourhood of its peak.

5.3.2 Program Data

The different models require different information to calculate the optimum bids for the contracts. This information can be divided into two classes -- that which is known and available at the time of tendering; and that which must be estimated or predicted. Information known and available is the past bids of competitors, past cost estimates, and the cost estimate of the contract under consideration. Information that requires determination at the time of tendering is the range of estimating accuracy, the number of competitors, and the assessment of the mean competitors bids.

The ranges calculated by method 2 of Appendix 3 are used in the test. The determination of number of competitors is discussed in Subsection 5.3.3. The problem of mean bid assessment is handled by testing Model 2 under different conditions of information. The initial test assumes perfect information - i.e. the true mean is used as the assessed mean. Then in Section 5.4, an investigation is made of the effect of constant errors, and random errors in the assessment.

5.3.3 The Number of Competitors

The formulation of the effect of estimating accuracy interaction (Chapter 2) indicates that the number of competitors is a major parameter on the competitive bidding situation. The order statistic derivation of probability of winning (Chapter 3) also employs the number of competitors as a major determinant.

Therefore any model which employs either, or both, of these concepts must include some method of determining the number of competitors.

Friedman (11) suggests two methods:

1. Linear regression of plot of the number of competitors vs. cost.
2. Determination of the a priori probability distribution of the number of competitors submitting bids for contracts of a given cost range.

Theoretically pleasing as these methods may be, the plot of number of competitors vs. cost for the four data sets combined, Figure 5. 8, suggests that these methods may not be appropriate for the situation under consideration.

In the construction industry there exists an easier, less sophisticated method of determing this parameter - it is inquiry. The Client will usually, if requested, inform the company which other companies have taken out plans, or expressed an intention to bid. Therefore, by contacting these companies shortly before the tender closing date, and asking them if they are bidding on the contract the number can be determined. If direct inquiry does not sound appealing, the nominated sub-contractors will usually provide the necessary information. Admittedly, these methods

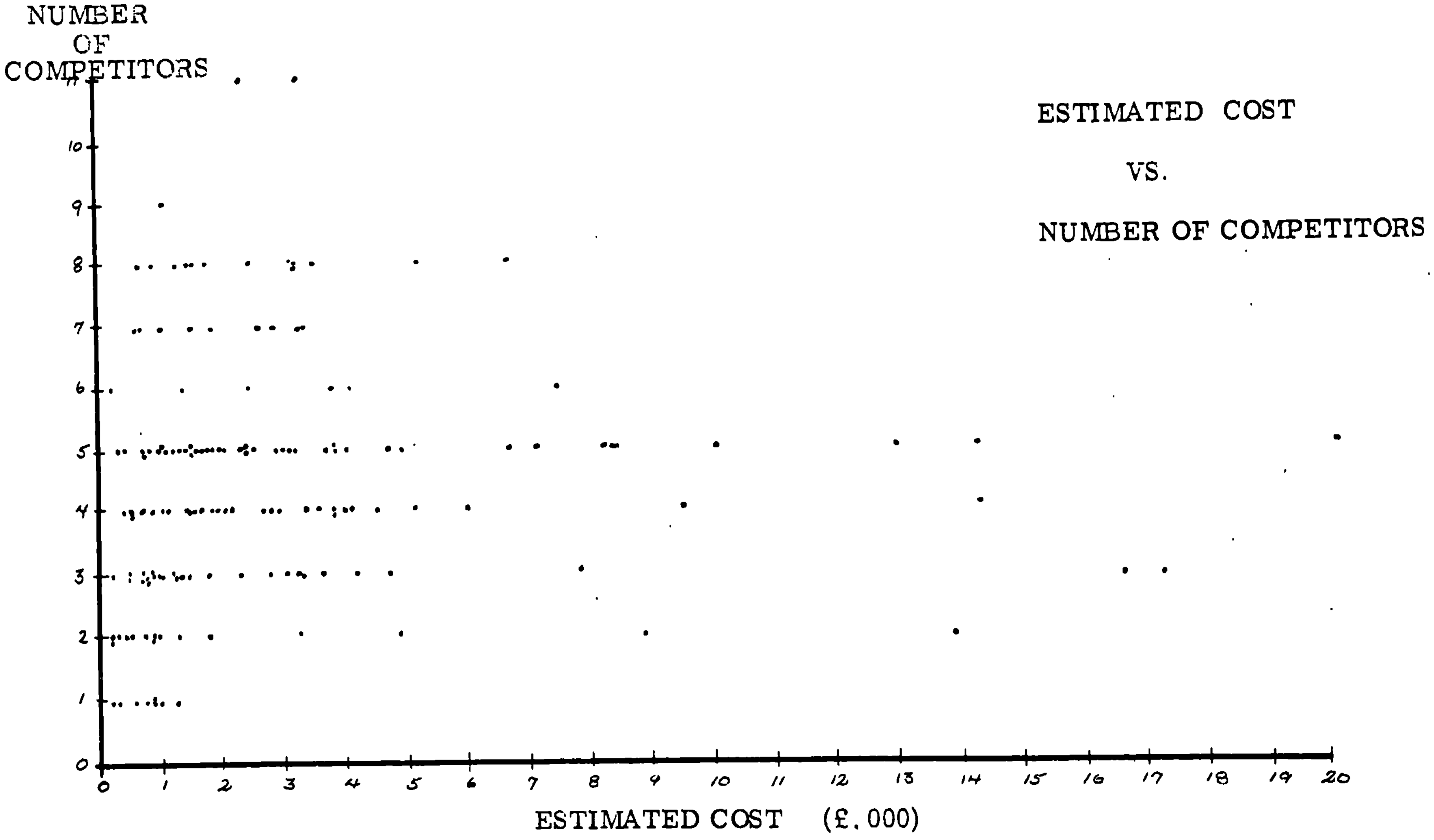

NUMBER OF COMPETITORS
ESTIMATED COST
VS.
NUMBER OF COMPETITORS
ESTIMATED COST (£.000)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Figure 5.8

are not infallible. There will always be the last minute changes of mind, and the unlikely (and difficult to repeat) case of lying. However, this procedure should result in a figure more reliable than can be obtained from any statistical technique. The trend to Selective Tendering should be helpful in this regard. Since the client is endeavouring to get a certain specific number of bids, the possibility of last minute changes of heart, and strange companies materialising are reduced.

The construction personnel interviewed in the course of this research did not regard the determination of the number and identity of their competitors prior to tender date a difficult task. Therefore, in the testing of the models the number of competitors bidding on a contract is taken as a known parameter.

5.3.4 Parameter Determination

There were several test parameters that had to be determined before it was possible to run the simulation. Models 1, 3, and 4 require a certain number of contracts to establish their probability density functions before they can start to compute bids. Model 5 requires a set size to operate with. The exponential smoothing version of Model 1 requires a smoothing coefficient.

To provide a basis for comparing the performance of the different models, it is necessary for them all to start bidding on the same contract. The number of contracts required by Models 1, 3, and 4 to establish their probability functions is not considered to be the governing parameter. This is so because all previous contracts are included in the functions and regardless of whether the model starts bidding at the fifth contract or the fourteenth contract, the bid it will compute for the fifthteenth contract will be the same. Therefore, the set size required by Model 5 was the governing parameter. The earlier contract bids computed by Models 1, 3 and 4 were examined to ensure that they were not abnormal.

Model 5 Set Size

To determine the set size for Model 5, a Fortran program, MARCH, was written. This program is a continued application of Model 5 to the same data but using different set sizes. For example, the program is given a data set and a starting number of 10. It calculates the results of Model 5, starting bidding at the eleventh contract and using a set size of ten. It then bids the same set, starting at the eleventh contract but using a set size of 9, then 8, and so on down to a set size of two.

In the experimental runs, a starting set size of ten was used. This number was selected as the maximum since it represented about three to four months of the Sample Companies' operations, and it was felt that any longer time period would not be sufficiently sensitive to market changes. Also, because of the size of the data sets, any larger number would not leave many contracts for the models to bid.

The results of the MARCH program on the four data sets are summarised in Table 5.1. The figures in the table are the total corrected profits made by the Model for the data set. The maximum total corrected profit values occur at: set sizes 2, 3, 4, and 5 for Company A; set sizes 9 and 10 for Company B; set sizes 3, 4, and 5, for Company C, and set size 3 for Company D.

As the companies are dealing in similar contracts, and are subject to the same market forces, it was felt that one set size should do for all four companies. To determine this compromise set size a ranking procedure was used. The results of each company were assigned a rank number from 1 to 9 in order of decreasing magnitude of total profits. In cases of ties, the arithmetic sum of the affected rank numbers was divided equally among the tied values. Table 5.2 contains the ranking. The rank numbers were summed for each set size and the minimum total, $11\frac{1}{2}$ for set size 3, was used as the selection criteria.

TABLE 5.1

DRIFT MODEL SET SIZE DETERMINATION

SET SIZE	COMPANY A		COMPANY B		COMPANY C		COMPANY D	
	Profit £	Won	Profit £	Won	Profit £	Won	Profit £	Won
10	17,409	2	27,036	2	26,922	6	12,430	4
9	17,409	2	27,036	2	27,777	6	12,430	4
8	17,409	2	17,923	3	22,043	5	12,430	4
7	17,409	2	15,777	3	20,628	5	11,875	4
6	17,409	2	16,851	3	46,280	6	11,875	4
5	24,168	3	16,851	3	52,013	7	16,486	5
4	24,168	3	16,851	3	52,013	7	16,486	5
3	24,168	3	16,851	3	52,013	7	20,053	5
2	24,168	3	16,851	3	26,326	6	15,433	4

TABLE 5.2

RANKING OF SET SIZES

SET SIZE	2	3	4	5	6	7	8	9	10
COMPANY A	2½	2½	2½	2½	7	7	7	7	7
COMPANY B	6	6	6	6	6	9	3	1½	1½
COMPANY C	7	2	2	2	4	9	8	5	6
COMPANY D	4	1	2½	2½	8½	8½	6	6	6
TOTAL	19½	11½	13	13	25½	33½	24	19½	20½

A set size of three was selected and the Models all start bidding at the fourth contract in the data sets.

Exponential Smoothing Coefficient

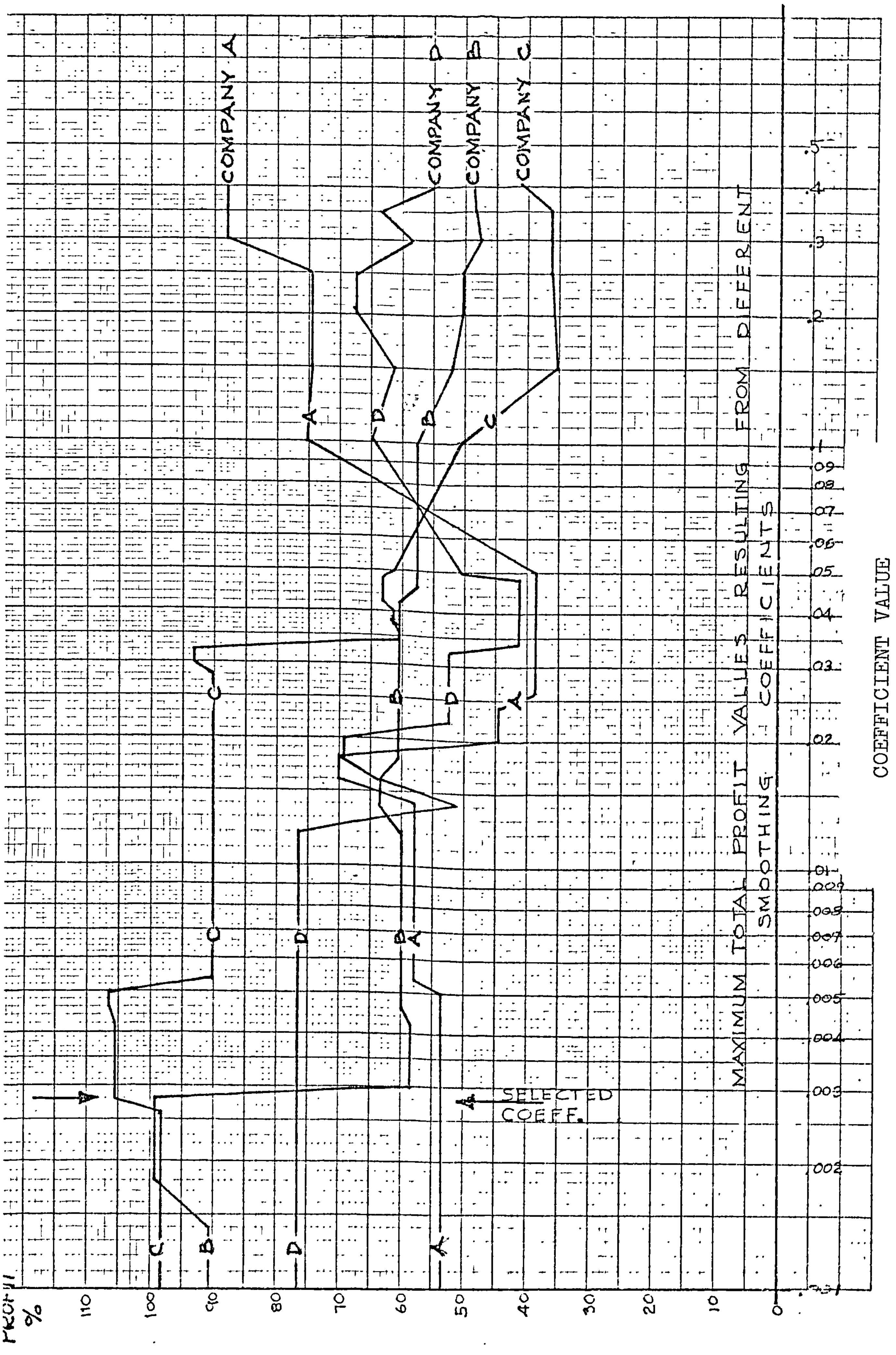
The exponential smoothing coefficient for Model 1 was also determined by experiment.

For the four data sets, Model 1 was tested using exponential coefficients which varied from 0.001 to 0.400. The resulting total profit values are plotted on the graph on page 208. The values have been converted to percentages by using the maximum OPT2 value of the set as 100%.

Since the maximum profit values for the four sets did not occur at the same coefficient value, a compromise value was selected.

If the data sets had been of sufficient length, it would have been possible to use the procedure followed by Morin and Clough (20). This is, splitting the sets and using the first portion to determine the coefficient and the second portion to test. In this manner it would be possible to determine a separate coefficient for each set.

However, there is insufficient data for this procedure and since it is desirable to keep the sets comparable, the coefficient value which yielded the highest percentage sum, value 0.0028, was used in testing all four sets.


MAXIMUM TOTAL PROFIT VALUES RESULTING FROM DIFFERENT
SMOOTHING COEFFICIENTS
COMPANY A
COMPANY D
COMPANY B
COMPANY C
SELECTED
COEFF.
PROFIT
%
110
100
90
80
70
60
50
40
30
20
10
0
COEFFICIENT VALUE
.001
.002
.003
.004
.005
.006
.007
.008
.009
.01
.02
.03
.04
.05
.06
.07
.08
.09
.1
.2
.3
.4
.5

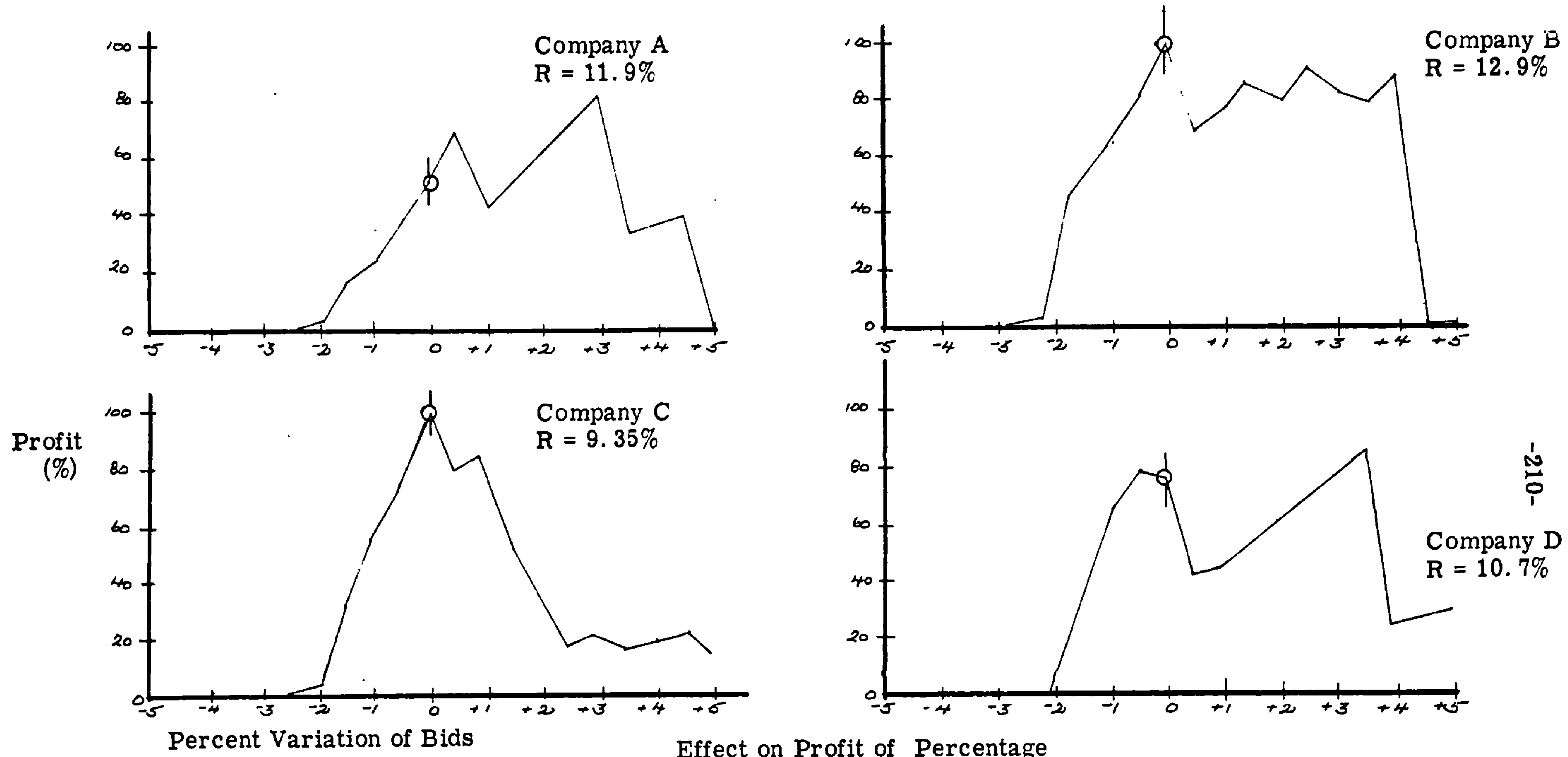
5.3.5 Results and Discussion

The results are discussed under four separate headings. First the performance of each model, as displayed by the OPTM graphs, is discussed. Then the models are compared to each other and to the Sample Companies' performance. The influence of individual contracts on the results is examined and then the theoretical performances, as exhibited by the PLOT graphs, of the models are considered. Only summary results are presented in this section. Figures 5.9 to 5.14 inclusive are the OPTM graphs. Table 5.4 summarises the results of the Models. The PLOT graphs are in Appendix 6.

Model Performance

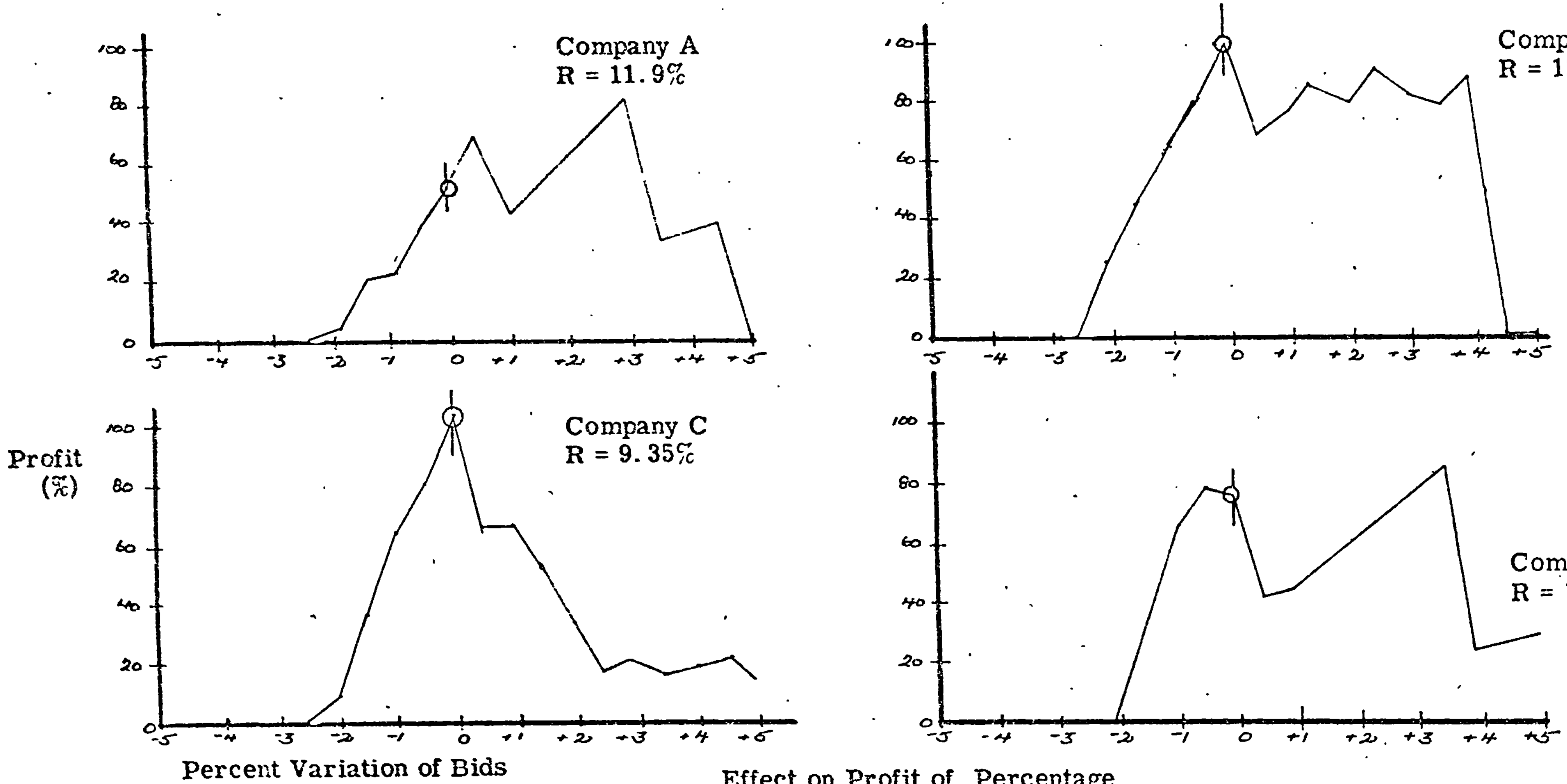
Figures 5.9 to 5.14 inclusive are the OPTM graphs resulting from the program operating upon the tenders computed by the Models. The operating position of the model is indicated on the graphs with a ϕ.

The Table below list the location of the peaks of the graphs.



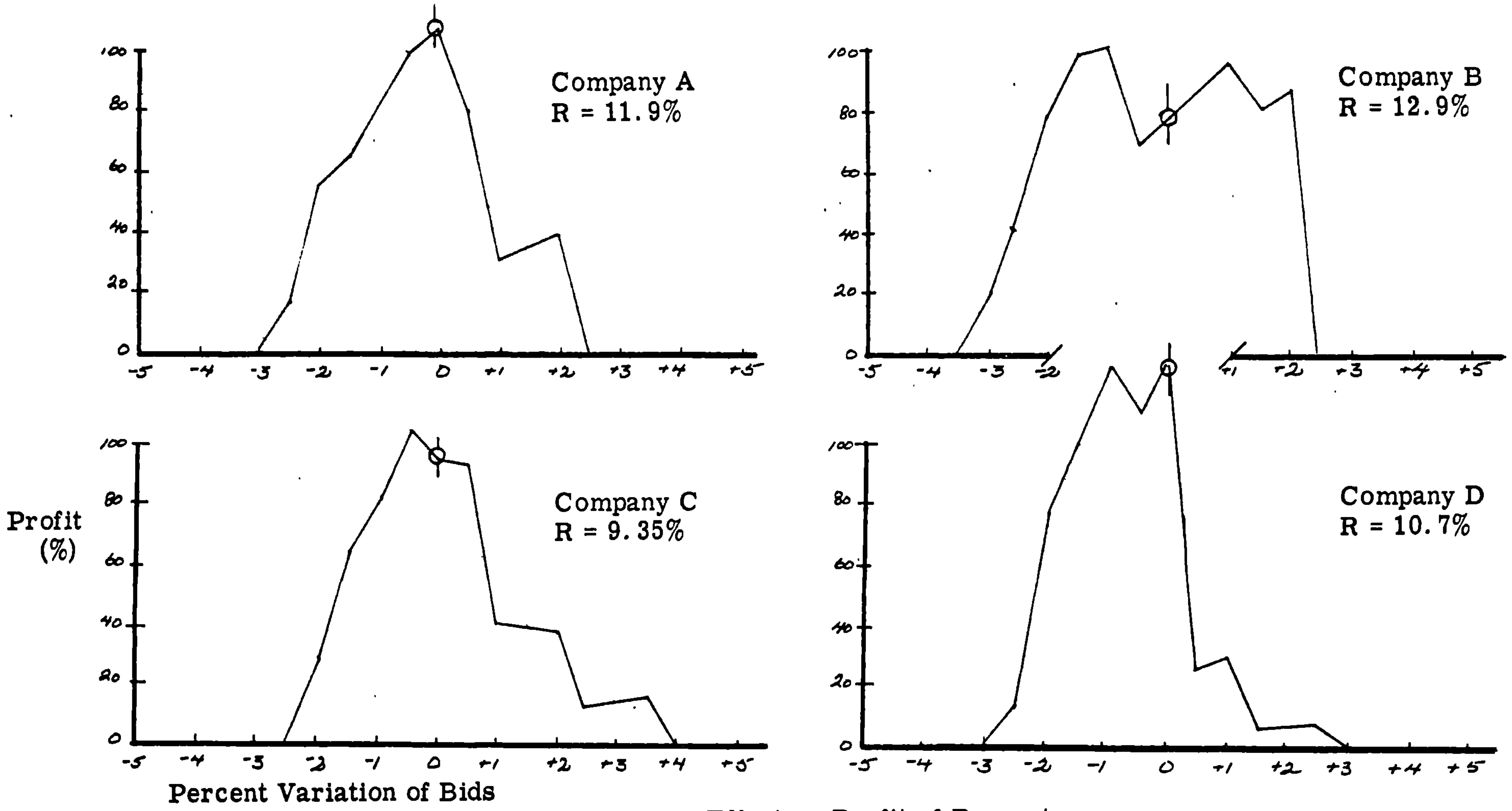
Effect on Profit of Percentage Changes in Model 1A Bids
(starting at the fourth contract)

Figure 5.9



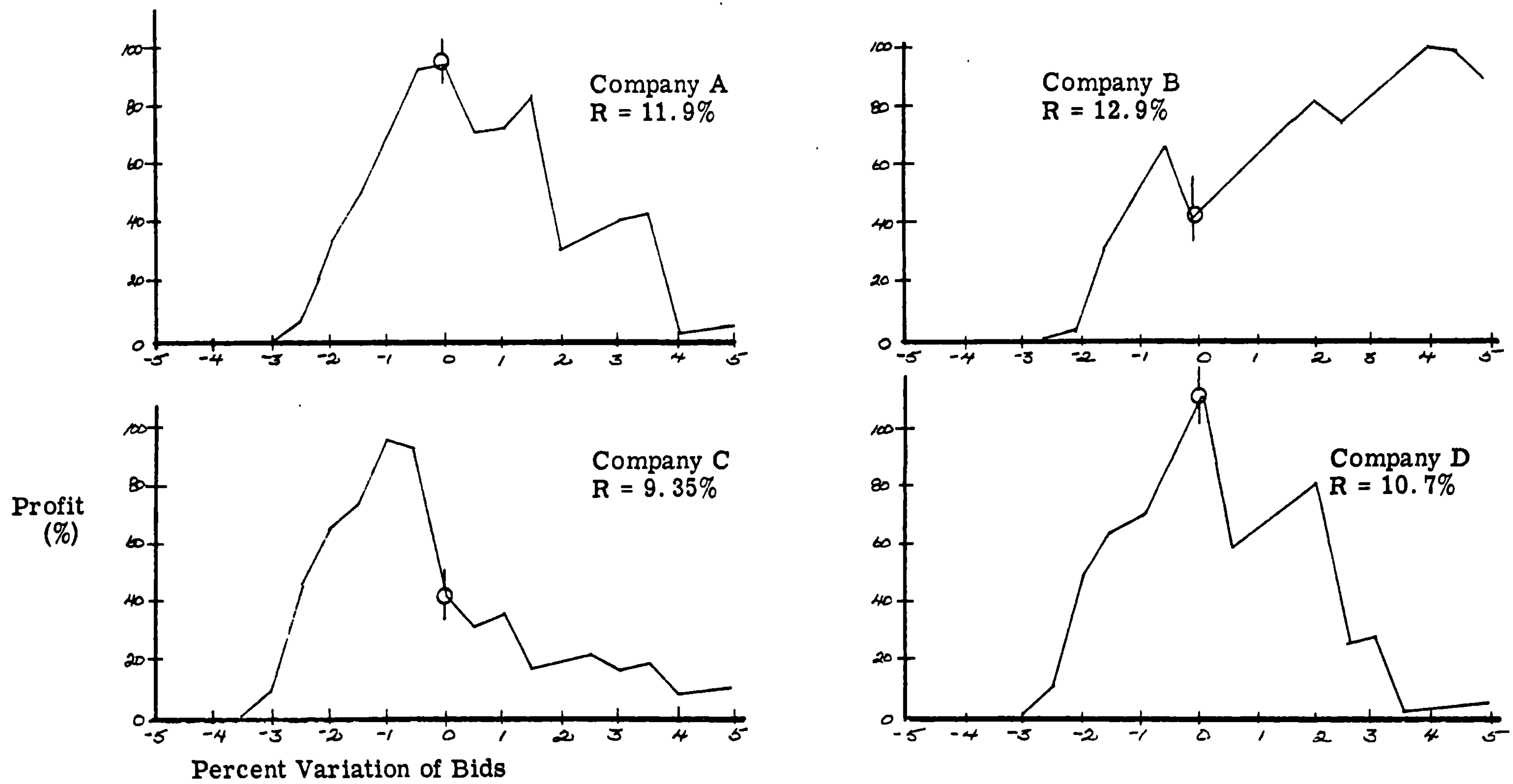
<u>Effect on Profit of Percentage</u>
<u>Changes in Model 1E Bids</u>
(starting at the fourth contract)

Figure 5.10



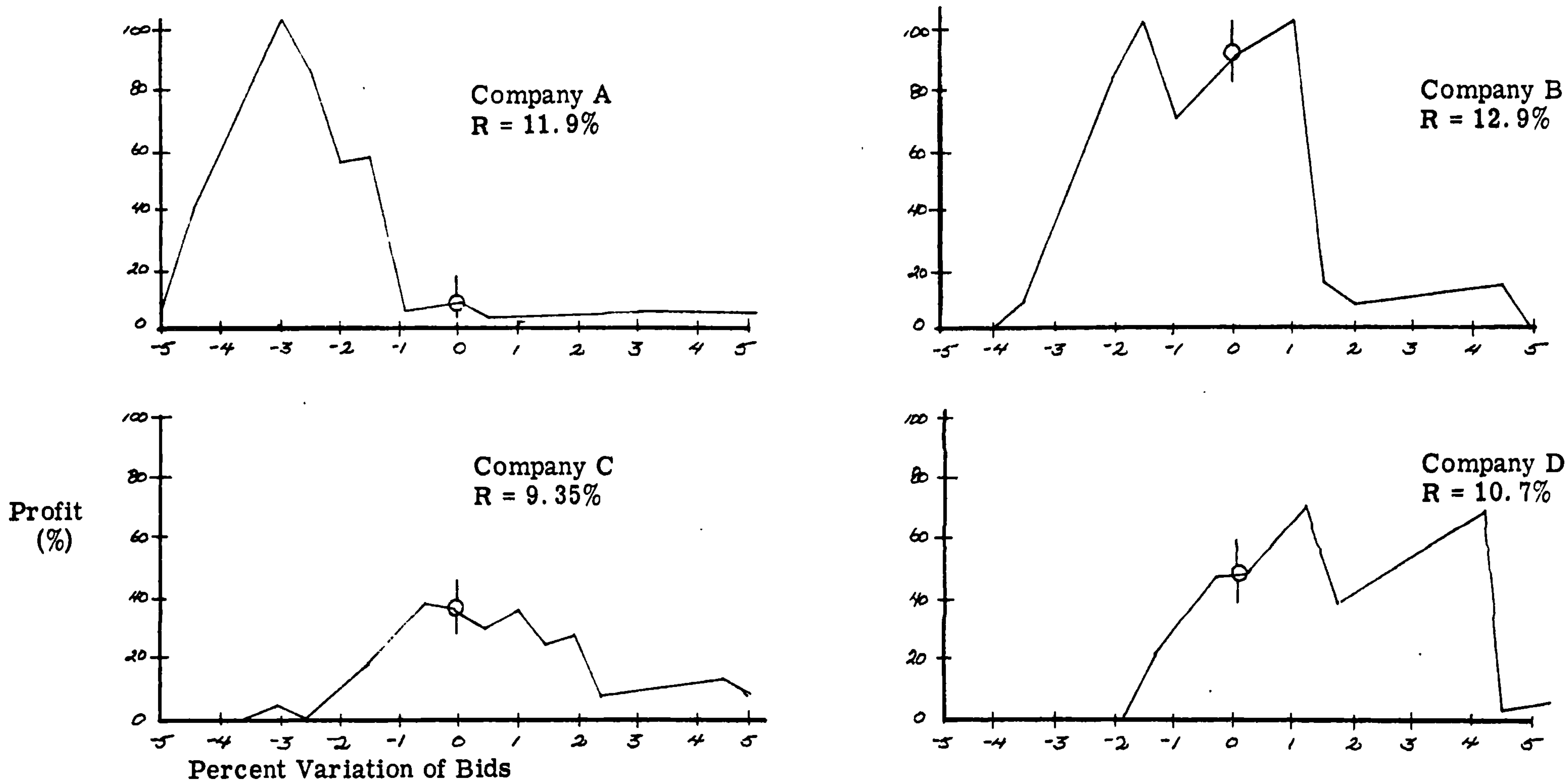
<u>Effect on Profit of Percentage</u>
<u>Changes in Model 2 Bids</u>
(starting at the fourth contract)

Figure 5.11



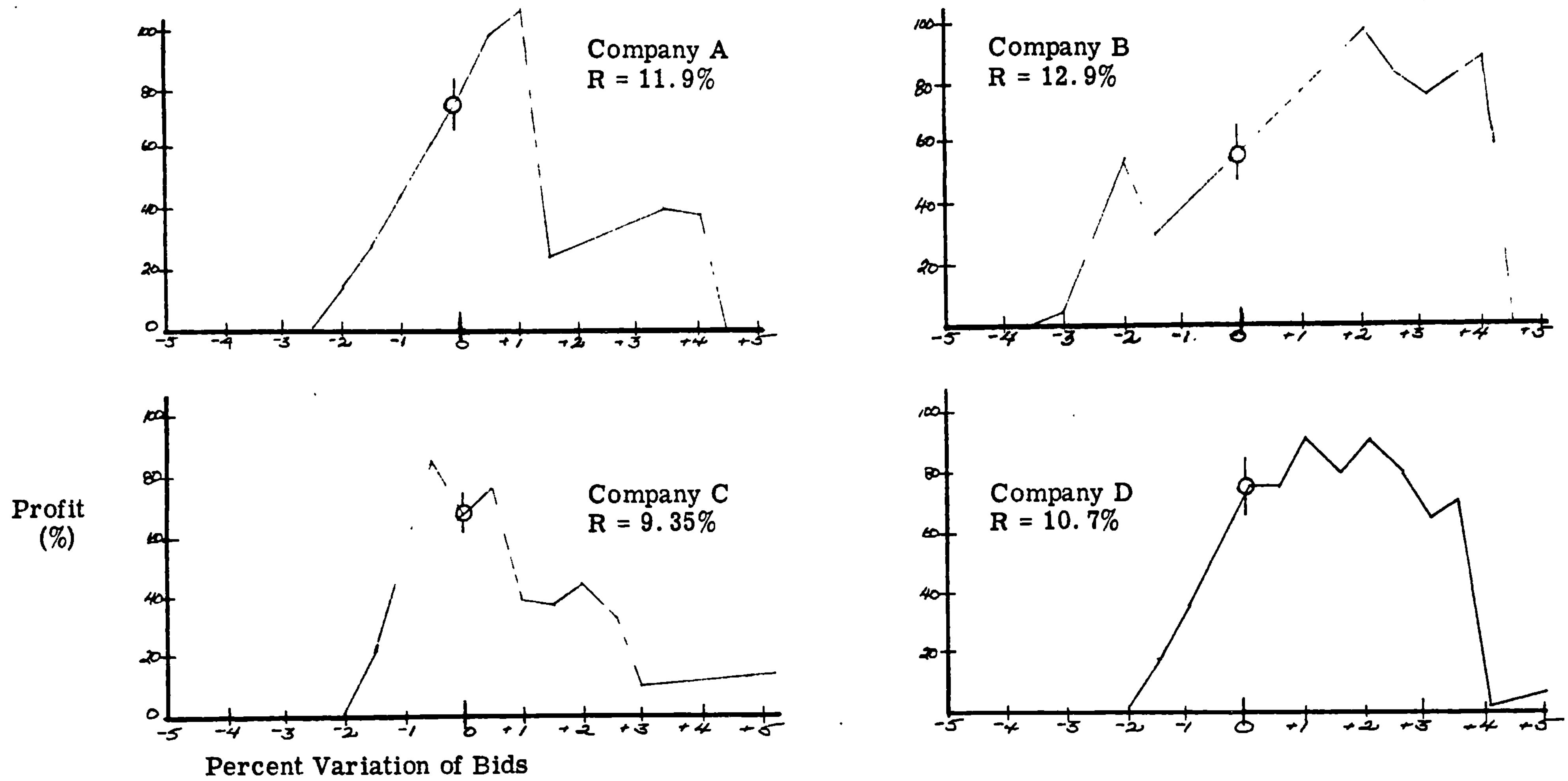
<u>Effect on Profit of Percentage</u>
<u>Changes in Model 3 Bids</u>
(starting at the fourth contract)

Figure 5.12



Effect on Profit of Percentage
Changes in Model 4 Bids
(starting at the fourth contract)

Figure 5.13



Effect on Profit of Percentage
Changes in Model 5 Bids
(starting at the fourth contract)

Figure 5.14

LOCATION OF PEAKS

MODEL	COMPANY A	B	C	D	ROW TOTAL
1(Average)	+3	0	0	$+3\frac{1}{2}$	$+6\frac{1}{2}$
1(Exponent)	+3	0	0	$+3\frac{1}{2}$	$+6\frac{1}{2}$
2	0	−1	$-\frac{1}{2}$	0	$-1\frac{1}{2}$
3	0	+4	−1	0	+3
4	−3	+1	$-\frac{1}{2}$	+1	$-1\frac{1}{2}$
5	+1	+2	$-\frac{1}{2}$	+1	$+3\frac{1}{2}$
COLUMN TOTAL:	+4	+6	$-2\frac{1}{2}$	+9	

This table provides the basis for two lines of investigation. First, it may indicate any consistent bias in a models performance; secondly, it may reveal bias in the data introduced by the selection of the estimating accuracy range.

The row totals, $6\frac{1}{2}$ for Model 1 and for Model 1 Exponential, suggest that these Models may be bidding too low and a uniform percentage increase in all bids may be advantageous.

The column totals, 6 for B, and 9 for D indicate that the range correction for these two Companies may be too large since a lower value of R will shift the peaks to the left.

However, these deviations could be a result of the interaction of both of these possibilities.

The column totals for Companies A, B, C and D excluding Model 1 Average are +1, +6, $-2\frac{1}{2}$, and $+5\frac{1}{2}$; excluding Model 1 Exponential +1, +6, $-2\frac{1}{2}$, and $+5\frac{1}{2}$; and excluding both of them -2, +6, $-2\frac{1}{2}$, and +2.

The column total for Company B remains high suggesting that the selected range of 12.9% is too high. The shape of the Company B graphs, with a large plateau on the positive side supports this notion. Therefore it is concluded that the 12.9% range correction selected for Company B could be too high. A similar conclusion cannot be drawn from the results for Company D. The high column total is primarily due to Model 1 results and this is caused by a low bid on a single contract.

The figures in the Company C column, although not numerically large, are consistent -four being less than zero and two equal to zero. This suggests that the estimating accuracy range of 9.35% selected for Company C may be a bit low.

The general conclusion drawn from these results is that the ranges of estimating accuracy for all four companies are in the region $9.35\% \leq R \leq 12.9\%$, and the ranges selected are approximately correct with Company B, R = 12.9%, possibly a bit high and Company C, R = 9.35%, possibly a bit low.

Considering the OPTM graphs of each Model

Model 1 - Average Version

For companies B and C this Model is operating at the peak. The plots for Companies A and D show that in each case the Models poor performance was due to a low bid on a single contract. In the Company A contract, the Model left 3% on the table, and 3½% on the table in the Company D contract.

Model 1 - Exponential Version

With the exception of Company C, these graphs are almost identical to those of Model 1A. The Company C graphs are also very similar but this version of the Model is able to achieve a higher total profit value.

Model 2

The graph for Company A, Model 2 is the closest of all the graphs plotted to being ideal, and even then it has one contract for which it left 2½% on the table. An ideal graph would have no local peaks, and the model would be operating at the peak. The Model 2 graph demonstrates the effectiveness of this system. It is operating at the peaks of Company A and D, and has attained a high profit location in the regions of the other two peaks, bidding 1% off for Company B and $\frac{1}{2}$% off for Company C.

Model 3

This Model can best be described as inconsistent. It has two very good results, Companies A and D, and two very bad, Companies B and C. It is possible that the bad performances are a result of the wrong range corrections as was discussed earlier. It is also possible that the conclusions arrived at concerning the range corrections are a result of this Model.

Model 4

The performance of this Model is bad. It shifts from the low end of the graph of Company D to the high end for Company A. When it does operate near a peak, Company C, the peak it produces is less than one half the size of that achieved by the other models. Only in one case, Company B, is the Models performance satisfactory.

Model 5

The general impression gained from the Model 5 graphs is that the system is bidding too low. A $\frac{1}{2}$ percent increase in all bids would have increased the profit picture in three of the Companies and resulted in negligible change for the fourth. The Model is operating in the region of the peaks and so its performance is judged to be generally satisfactory.

Comparison of Models

One of the purposes of the simulation was to compare the performances of the Models and to determine whether any one is superior to any other, or to the actual performance of the Companies. The results of the simulation are summarised in Table 5.4. To enable comparisons to be made, a ranking procedure is used with total cumulative profits, taken as the performance criteria. The profits achieved by each Model were ranked, the highest profit figure receiving a rank number of 1, the lowest 7. The ranks are shown in Table 5.5 with the sum of the rank numbers for each Model given in the bottom row.

The profit rank sums suggest the following trial ordering of the Models and Companies.

Model 2

Model 1 Exponential

Model 1 Average

Model 3 – Actual Performance

Model 5

Model 4.

To investigate this ordering, two tests were performed.

TABLE 5.4

SUMMARY OF MODEL RESULTS

COMPANY	MODEL						Actual Perform -ance
	1A	1E	2	3	4	5	
	CUMULATIVE PROFIT %						
A	53.5	53.5	107.1	94.6	8.5	79.2	76.5
B	99.0	99.0	79.5	42.7	91.5	57.2	96.5
C	98.5	105.7	95.0	41.2	36.9	56.0	81.8
D	76.8	76.8	128.5	109.5	47.8	76.0	72.6
CUMULATIVE EXPECTED PROFIT %							
A	67.4	65.6	64.4	102.2	126.0	-	-
B	78.4	77.2	104.2	81.5	122.0	-	-
C	78.5	76.6	56.8	127.0	144.5	-	-
D	146.0	142.0	108.0	145.0	145.5		
NUMBER OF CONTRACTS WON							
A	4	4	4	5	2	5	4
B	6	6	3	6	5	4	5
C	13	13	10	6	6	8	10
D	7	7	6	6	4	6	5
PROFIT VOLUME RATIO							
A	.0174	.0174	.0348	.0284	.0271	.0224	.0249
B	.0281	.0281	.0464	.0186	.0458	.0291	.0281
C	.0208	.0208	.0276	.0330	.0284	.0182	.0235
D	.0196	.0196	.0384	.0275	.0210	.0189	.0220

TABLE 5.5

PROFIT RANKING OF MODELS

COMPANY	MODEL						ACTUAL PERFORM -ANCE
	1A	1E	2	3	4	5	
A	5½	5½	1	2	7	3	4
B	1½	1½	5	7	4	6	3
C	2	1	3	6	7	5	4
D	3½	3½	1	2	7	5	6
TOTAL	12½	11½	10	17	25	19	17

PROFIT VOLUME RATIO RANKING OF MODELS

COMPANY	MODEL						ACTUAL PERFORM -ANCE
	1A	1E	2	3	4	5	
A	6½	6½	1	2	3	5	4
B	4½	4½	1	7	2	3	6
C	6	5	3	1	2	7	4
D	5½	5½	1	2	4	7	3
TOTAL	22½	21½	6	12	11	22	17

The first was the calculation of the Coefficient of Concordance (Reference 22). This coefficient is designed to measure the degree of agreement between the rankings of the companies. It varies from a value of zero, signifying complete randomness in the allocation of rankings, to a value of one, signifying complete agreement. For the rankings of the four companies the Coefficient of Concordance is 0.374.

This Coefficient can be tested for significance with a F test. The F value for the coefficient is 1.76. For the corresponding degrees of freedom the

5% level of F = 2.82

1% level of F = 4.38

Therefore the hypothesis that the rankings are random cannot be rejected at the 5% level.

Second the trial ordering based on the rank sums was compared with the ordering for each company. The following rank correlation coefficients were obtained.

	Rank Correlation Coefficients
Trial Order and Company A	0.574
Trial Order and Company B	0.358
Trial Order and Company C	0.832
Trial Order and Company D	0.784

More contracts determine the total results for Companies C and D (see Table 5.4 and subsection, Influence of Individual Contracts). Therefore, the results of C and D should carry more weight than those of A and B. The high correlation coefficients of C and D are an indication that the trial order may be correct.

The rank correlation coefficients of all couples of companies were computed.

Companies	Rank Correlation Coefficient
A & B	-0.644
A & C	-0.027
A & D	0.714
B & C	0.705
B & D	-0.214
C & D	0.473

No conclusion is drawn from this comparison.

The hypothesis that the ranking of the seven methods is random cannot be rejected. However, although an overall ranking for the six models and the companies cannot be made, it is still possible to make paired comparisons. Therefore the next comparison was between the Actual Companies and each Model. For this a 1, 2 ranking was used.

Data Set	Mod. 1A	Co.	Mod. 1E	Co.	Mod. 2	Co.	Mod. 3	Co.	Mod. 4	Co.	Mod. 5	Co.
A	2	1	2	1	1	2	1	2	2	1	1	2
B	1	2	1	2	2	1	2	1	2	1	2	1
C	1	2	1	2	1	2	2	1	2	1	2	1
D	1	2	1	2	1	2	1	2	2	1	1	2

The chart illustrates that:

Model 2	outperformed	the	Companies	3	out	of	4	times
Model 1E	"	"	"	3	"	"	4	"
Model 1A	"	"	"	3	"	"	4	"
Model 3	"	"	"	2	"	"	4	"
Model 5	"	"	"	2	"	"	4	"
Model 4	"	"	"	0	"	"	4	"

The sample, four companies, is too small to allow any statistically significant conclusions since the hypothesis that all models are equal to the Company cannot be rejected. Even results such as the Model 4 comparison can be expected one sixteenth of the times. However the results suggest that:

1. Models 2 and 1 could improve upon the actual company performance.
2. Models 3 and 5 could equal the actual company performance.
3. Model 4 produces results worse than those of the actual company.

1 - 2 ranking comparison of the performance of Models 2 and 1E indicate that there is little to choose between them.

	Model 2	Model 1E
A	1	2
B	2	1
C	2	1
D	1	2

Influence of Individual Contracts

Figure 5.15 is designed to illustrate the effect of individual contracts on the Models' performance. In the first column is listed the reference number of every contract won, either by one of the models or by the Companies. The six models and the company are listed across the top of the chart. The shaded areas indicate which of the systems (models or Company) won the contract.

A total of 35 contracts were won in the four data sets.

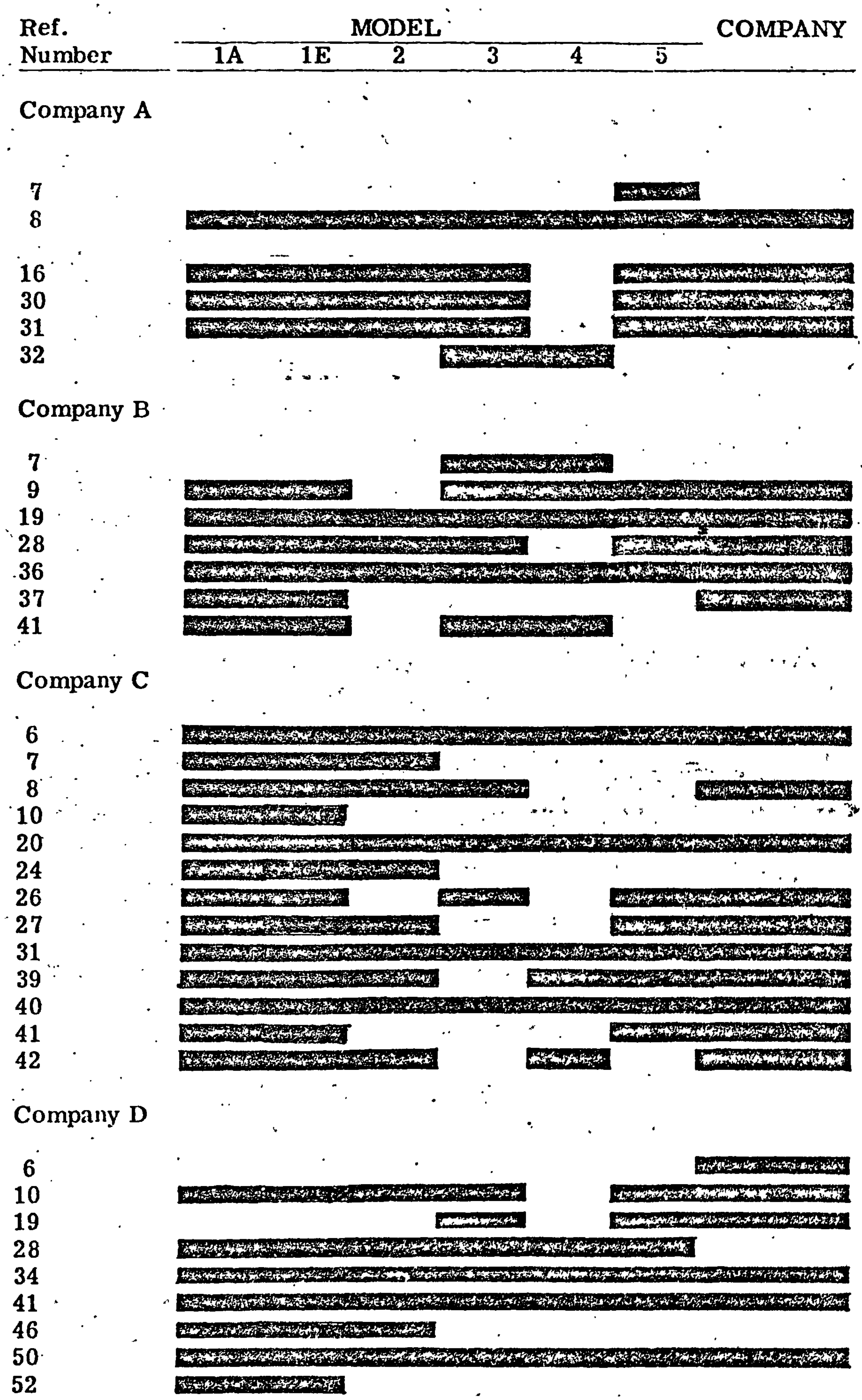

Ref. Number
MODEL
1A
1E
2
3
4
5
COMPANY
Company A
7
8
16
30
31
32
Company B
7
9
19
28
36
37
41
Company C
6
7
8
10
20
24
26
27
31
39
40
41
42
Company D
6
10
19
28
34
41
46
50
52

Figure 5.15

Of these:

10 were won by all 7 systems

8 were won by 6 of the systems

4 were won by 5 of the systems

2 were won by 4 of the systems

5 were won by 3 of the systems

4 were won by 2 of the systems

2 were won by 1 of the systems

The point of Figure 5.15, and the above enumeration is to demonstrate that there are certain contracts that are going to be won, regardless of which system is used. This is not surprising since all the systems operate from the estimated cost figure. Obviously some contracts possess an estimated cost that will result in a winning bid almost irrespective of the system used. Assuming of course that the systems are reasonable and are not making 15% to 20% mark-ups.

The second point to be observed from the chart is the small number of contracts that actually produce the total profit figure.

The second point leads one to question the simple total profits criterion that is being used to judge the models. A model will be judged to be performing well if it is bidding low and manages to win more contracts than the other models, or it is judged to be performing well if it is bidding high and winning the same contracts as the other models but at a higher profit.

This difference is illustrated by the results of models 2 and 1 . Model 2 wins most of the contracts that the companies win but with higher bids - thus with a higher total profit. Model 1 bids lower than the companies and wins 30 contracts to the companies 25; the result is, again, higher total profits. If the companies resources are fully committed by their current bidding behaviour, Model 2 is obviously superior. If, however, the resources are available, and if, as claimed in some quarters, the actual money on which the Company depends for survival is largely made on the site through negotiation and extras, then Model 1 is the superior system.

The fact that the Models and the companies all tend to win the same contracts suggest an answer to the question, what bias was introduced into the data sets by excluding nonstandard and incomplete contracts? Since the excluded contracts were almost all contracts on which the sample Companies were unsuccessful, there is reason to assume that the Models would also be unsuccessful on these contracts. However, the inclusion of these contracts in the data sets would have resulted in higher figures for the cumulative expected profit. Therefore, the plot of expected profit should fall below that of the actual profit is the Model is predicting correctly.

Predictive Performance

The probabilistic Models attempt to maximize expected profits, and an expected profit is calculated for each contract. If the theoretical basis of a model is correct, then there should be some correspondence between the cumulative expected profit and the actual profit obtained. The investigation of this correspondence is complicated by the small number of contracts won, and by the previously mentioned bias in the data which should result in under-prediction. Since the winning of a single contract exerts such a marked effect upon the actual total profit, the comparison of actual profit and expected profit at any one point is not meaningful. Therefore the method of comparison selected was that of visual examination of the cumulative profit graphs. If the plot of cumulative expected profit appears to correspond with the actual profit plot - i.e. has approximately the same slope and does not diverge radically - then it is concluded that the theoretical basis of the model may be valid. The comparison is made more difficult by the different shapes of the plots. The cumulative actual profit being a definite step function whereas the cumulative expected profit is approximately a curve. The cumulative profit graphs are in Appendix 6, PLOT Graphs. They are classified visually into the following classes.

0 - there is no obvious correspondence between the cumulative profit and the cumulative expected profit

? - no classification apparent

+ - there appears to be some agreement between the cumulative profit and the cumulative expected profit.

The results are summarised in the table below.

	COMPANY			
	A	B	C	D
MODEL				
1A	?	+	+	0
1E	?	+	+	0
2	+	+	+	+
3	+	?	0	0
4	0	+	0	0

The results are inconclusive. However, they do suggest that expected profits yielded by Model 2 are associated with actual profits.

5.4 ANALYSIS OF GENERAL DISTRIBUTION MODEL

The experimental testing of the six models indicates that Model 2, the General Distribution Hypothesis, can usually perform as well as, if not better than, the sample companies. In the testing the Model had an advantage over the sample companies because it was operating in conditions of perfect information. That is, it did not rely on a managerial assessment of the mean competitors' bid, it calculated what the mean was. This suggests that it is desirable to investigate the results of Model 2 in greater depth.

5.4.1 Evaluation of Model 2 Performance

Since the Model does not require information from earlier contracts, it is possible for it to bid every contract in the data set. Therefore the Model was tested bidding every contract. Figures 5.16 to 5.19 are the resulting PLOT graphs. Figure 5.20 shows the OPTM graphs, the dashed line is the position of the Sample Company's performance.

To illustrate the performance of the Model, vis-a-vis the sample companies, Table 5.6 was prepared. This table lists all

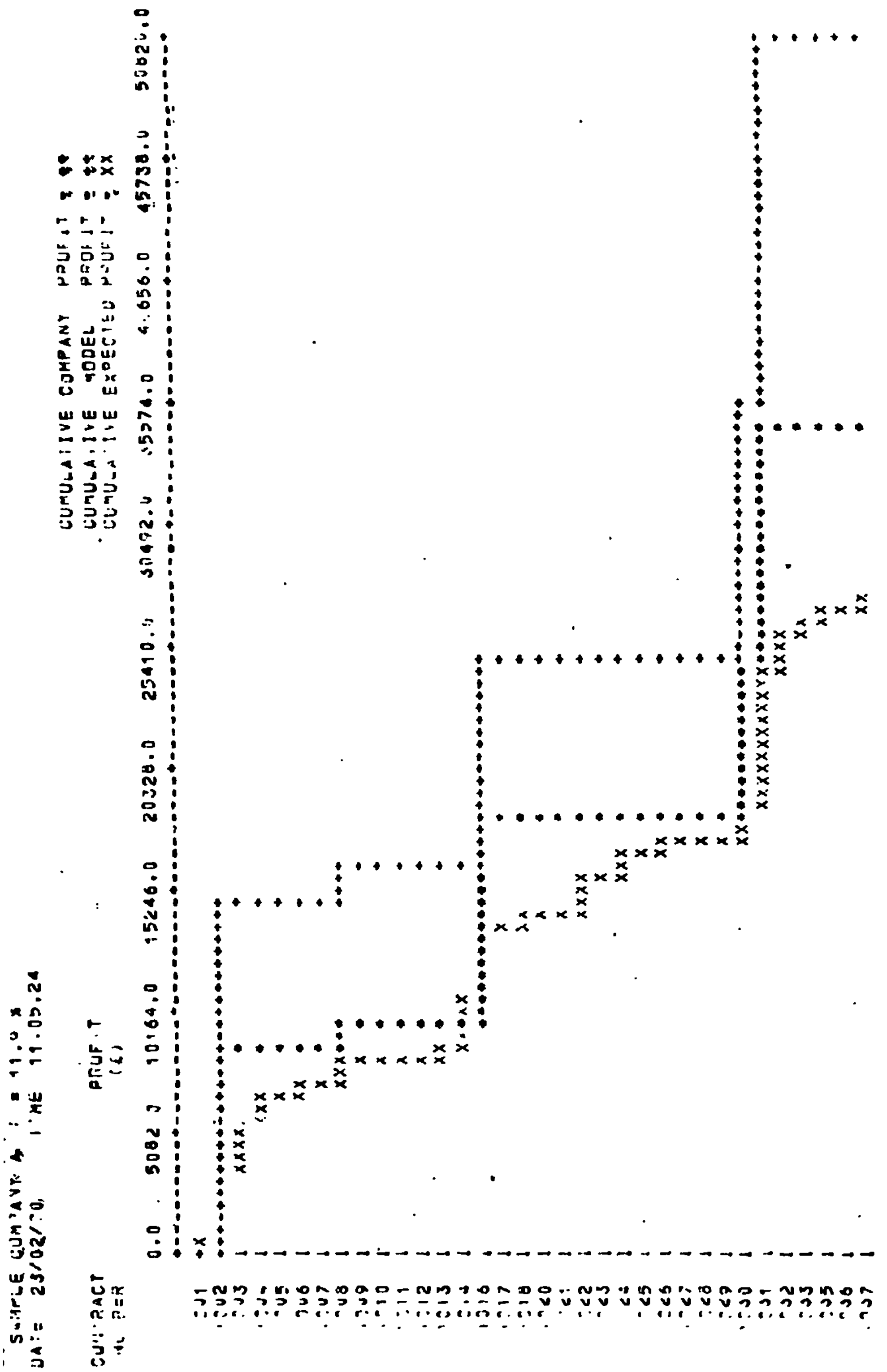

CUMULATIVE COMPANY PROFIT
0.0
10164.0
15246.0
20328.0
25410.0
45738.0

Figure 5.16

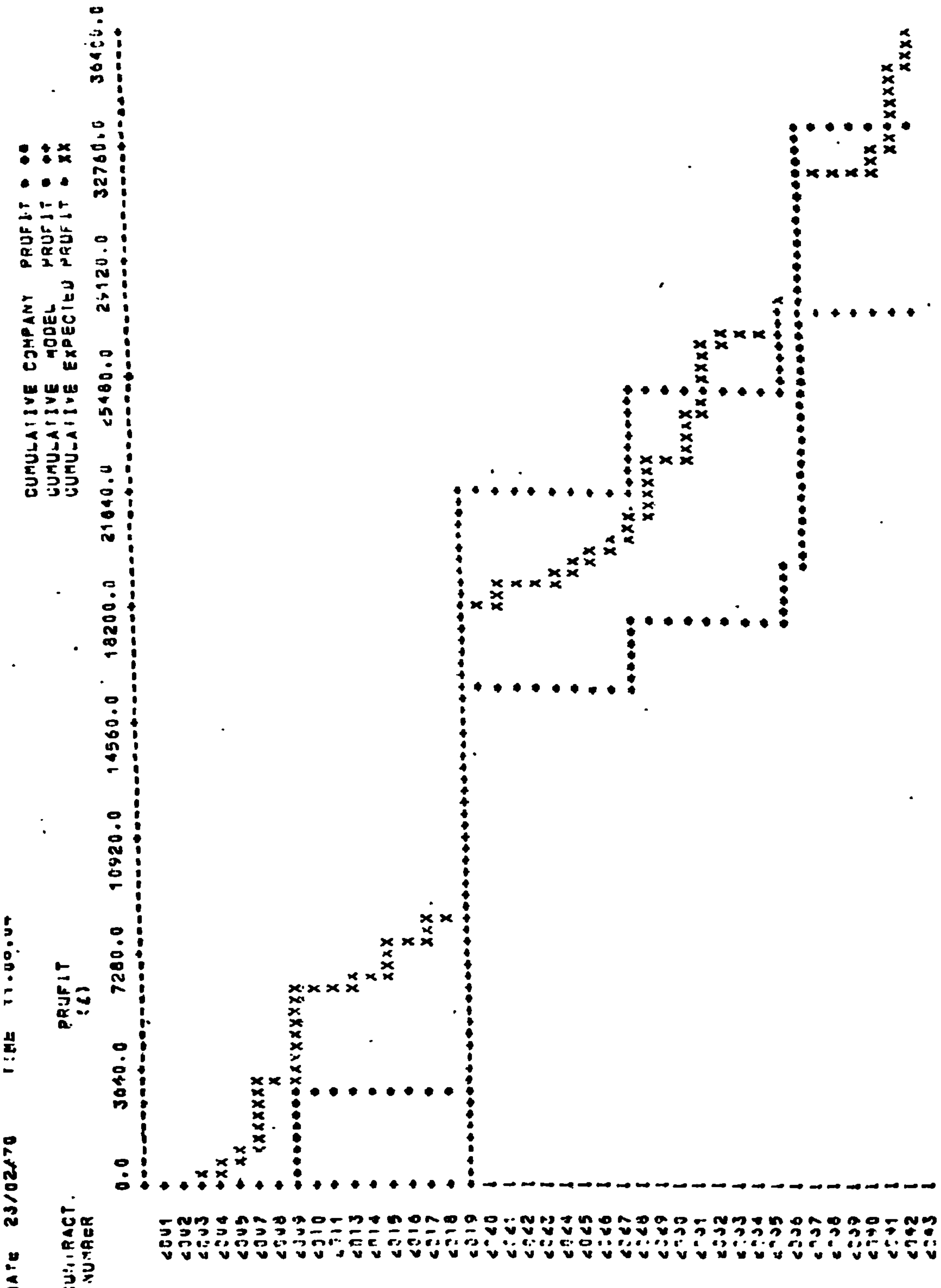

CUMULATIVE COMPANY PROFIT
CUMULATIVE MODEL PROFIT
CUMULATIVE EXPECTED PROFIT = XX
PROFIT
CONTRACT NUMBER

Figure 5.17

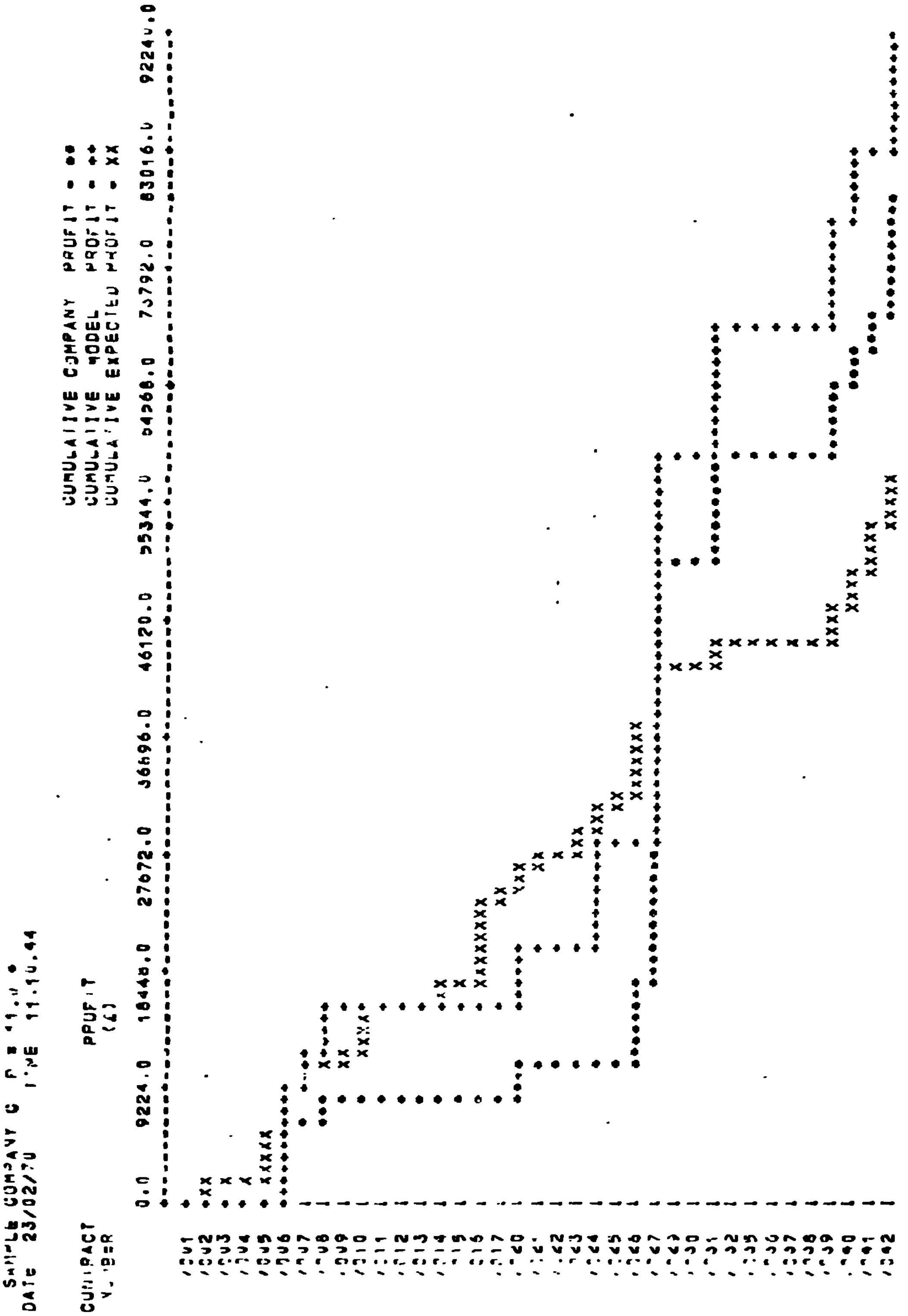

DATE 23/02/70
CONTRACT NUMBER
PROFIT (£)
CUMULATIVE COMPANY PROFIT = **
CUMULATIVE MODEL PROFIT = ++
CUMULATIVE EXPECTED PROFIT = XX
0.0
9224.0
27672.0
46120.0
73792.0

Figure 5.18

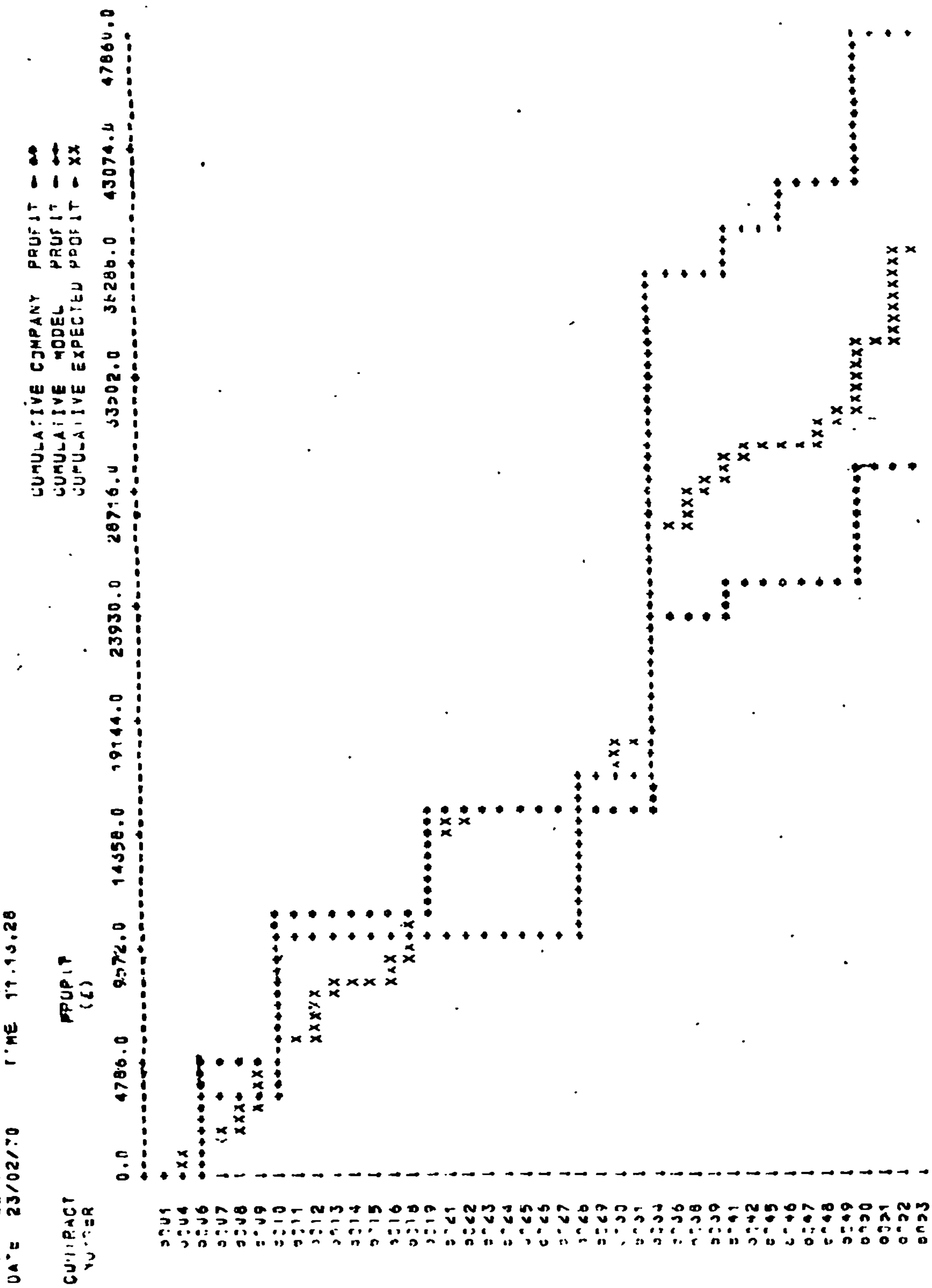
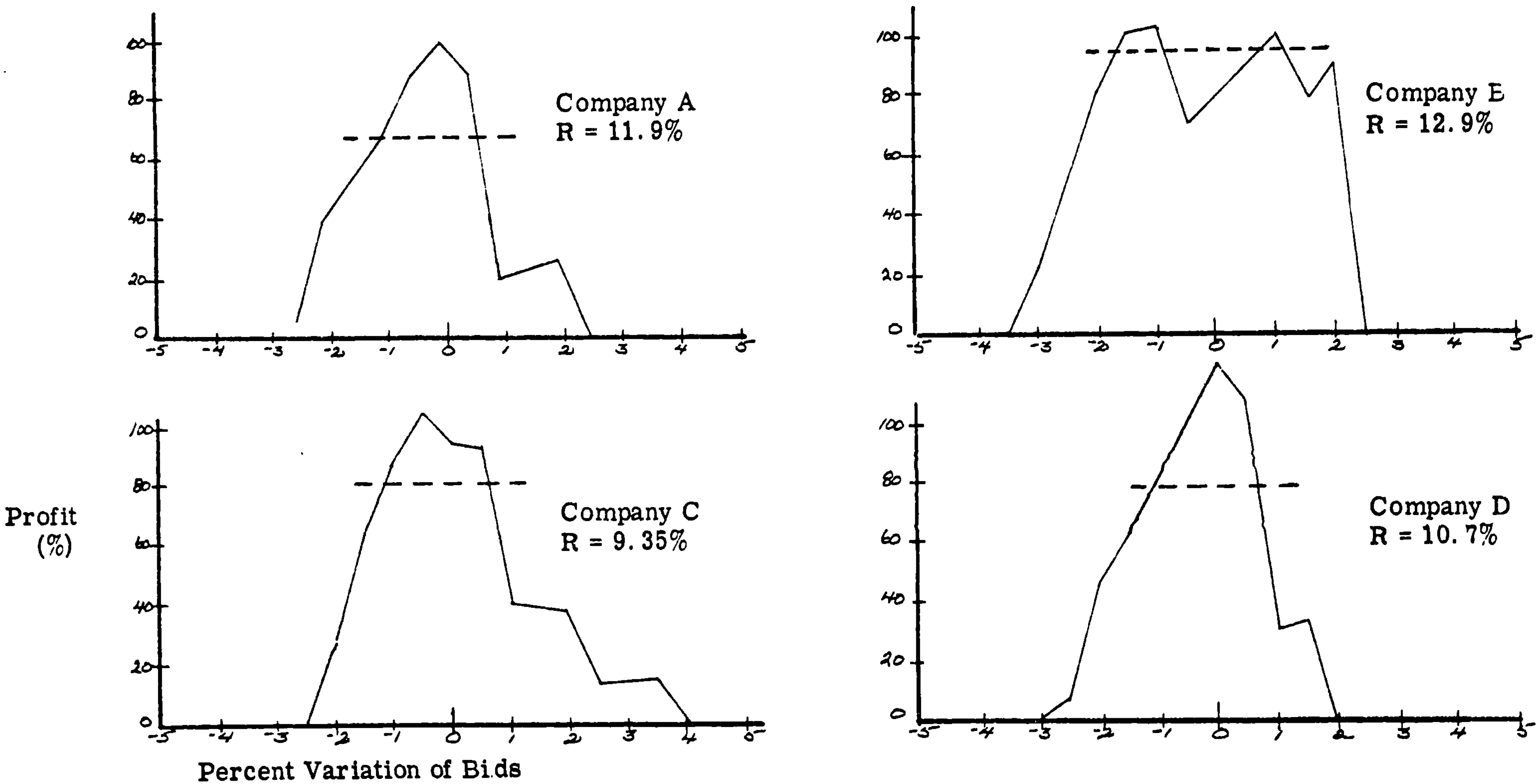

DATE 23/02/70
CUMULATIVE COMPANY PROFIT
CUMULATIVE MODEL PROFIT
CUMULATIVE EXPECTED PROFIT — XX
0.0
4786.0
9572.0
14358.0
19144.0
23930.0
28716.0
33502.0
38288.0
43074.0
47860.0

Figure 5.19



<u>Effect on Profit of Percentage</u>
<u>Changes in Model 2 Bids</u>
(All Contracts Bid)

Figure 5.20

TABLE 5.6

MODEL 2 CONTRACT RESULTS

CONTRACT NUMBER	CONTRACT VALUE £	RESULT		£ CONTRACT PROFIT		£ CUMULATIVE PROFIT*	
		COMPANY	MODEL 2	COMPANY	MODEL 2	COMPANY	MODEL 2
COMPANY A							
2	530,000	WON	WON	35,098	41,880	35,098	41,180
8	20,000	WON	WON	1,525	2,028	36,623	43,208
16	217,000	WON	WON	17,330	17,522	53,953	60,730
30	368,000	WON	WON	20,362	24,106	74,315	84,836
31	393,000	WON	WON	26,780	31,677	101,095	116,513
COMPANY B							
9	86,000	WON	LOST	4,850	0	4,850	0
19	421,000	WON	WON	29,483	38,602	34,288	38,602
28	99,000	WON	WON	6,107	7,132	40,395	45,734
36	50,000	WON	WON	3,744	4,189	44,139	49,923
37	494,000	WON	LOST	29,894	0	74,033	49,923
COMPANY C							
6	139,000	WON	WON	10,880	14,017	10,880	14,017
7	154,000	LOST	WON	0	8,281	10,880	22,298
8	76,000	WON	WON	4,530	5,750	15,410	38,048
20	66,000	WON	WON	4,494	6,127	19,904	44,178
24	342,000	LOST	WON	0	19,365	19,904	63,543
26	403,000	WON	LOST	19,286	0	39,190	63,543
27	1,440,000	WON	WON	82,227	79,730	121,417	143,273
31	417,000	WON	WON	23,402	25,041	144,819	168,314
39	232,000	WON	WON	13,620	16,281	158,439	184,595
40	73,000	WON	WON	4,970	7,503	163,409	192,098
41	126,000	WON	LOST	7,410	0	170,819	192,098
42	292,000	WON	WON	19,065	19,589	189,884	211,687
COMPANY D							
6	144,000	WON	WON	10,834	9,440	10,834	9,440
10	248,000	WON	WON	15,987	16,408	26,821	25,848
19	329,000	WON	LOST	14,819	0	41,640	25,848
28	234,000	LOST	WON	0	13,912	41,640	39,670
34	392,000	WON	WON	23,509	36,878	65,149	76,638
41	18,000	WON	WON	2,216	2,411	67,365	79,049
46	113,000	LOST	WON	0	6,959	67,365	86,008
50	113,000	WON	WON	8,944	10,598	76,309	96,606

* Profit figures not corrected for interaction bias
ie. PROFIT = BID - ESTIMATED COST

the contracts that were won either by the Model or the Companies or both. To make the comparison comprehensible to the Companies' management the results are expressed in uncorrected profit. To render the contracts unidentifiable, the contract value (estimated cost) was rounded off to the nearest £1,000.; and the actual bid prices are not shown. The profits obtained on each contract give the difference, in pounds, between the company's bid and the Model's bid.

The results of all four companies show a total of 30 contracts won by either the companies or the Model or both. The companies won 26 contracts, the Model won 25. On these 30 contracts the Model bid a higher value than the companies on 24 of them, winning 19. The Model bid lower than the companies on 6 contracts; winning four that the companies lost. The majority, 21 of the 30 contracts, were won by both the Model and the companies with the Model submitting a higher bid in 19 of the 21 cases.

The OPTM graphs, Figure 5.20, indicate that for Companies A and D, the Model is bidding at the optimal position. For Company C it is bidding $\frac{1}{2}$% too high; and for Company B it is 1% too low.

The results were also examined on an individual company basis

For Company A, the Model and the Company won exactly the same five contracts; the Model submitting a higher bid in every case.

For Company B, the Company won five contracts, the Model won three of these and lost the other two. One of the contracts which the Company won and the Model lost represented over one third of the total profit obtained on the set. This is the Company that outperformed the Model and the results illustrate the effect that one contract can have when total profit is used as an evaluation criteria. The Model made a higher bid on all five contracts and if only the first four are considered, the Model received a higher profit on the three it won, than the Company did on its four.

Companies C and D with their higher volumes, more contracts won, provide more representative results.

For Company C the Model won two contracts that the Company lost, and vice versa, plus there are eight contracts shared. Here again, a single contract slightly distorts the results. Of the eight contracts won by both, the Model bid higher than the Company on seven of them and lower on one. The seven had an average value of £185,000. The one on which the Model was lower than the Company had a value of £1,440,000.

For Company D the Model won 2 contracts that the Company lost; the Company won one that the Model lost, and there were five contracts which they both won. On the five shared contracts, the Model submitted a higher bid on four of them.

The Table below summarises the results in terms of contracts won

	NUMBER OF CONTRACTS WON			
DATA SET	A	B	C	D
Model	5	3	10	7
Company	5	5	10	6

The following observations are made on the performance of the Model vis-a-vis the companies.

1. The Company and the Model will generally win approximately the same number of contracts. Thus any resource criterion dependent upon the number of projects, for example stationary plant like tower cranes, or batching plants, or site personnel; if it is currently being met by the Company's performance, will also be met by the Models. The fact that the Company and the Model tended to win the same contracts makes this point even more valid.

2. There is a general tendency for the Model to win the contracts at a higher bid that the Company. This would mean higher unit prices in the bill of quantities and higher profit margins on the Provisional Sums. Therefore, regardless of whether the actual profit accruing to the Company is the sum represented by the difference between the estimated cost and the tender sum, or whether it is the result of negotiation based upon submitted unit prices, the final result should be an increase in total profitability if the Model's bids are used.

5.4.2 Sensitivity Analysis of Model 2

The results produced by Model 2 are for a perfect information situation. Since the testing was based upon historical data it was impossible to obtain a managerial assessment of the mean of the competitors' bids, and so the true mean was used. To investigate the effect of this perfect information case, a sensitivity analysis was performed. The operation of the Model is independent of other contracts. Therefore it is possible to group the data sets of the four companies. This provides the advantages of working with a set of 153 contracts, and of damping the effect of any individual contract on the measurement criterion of total profits.

Constant Errors in Assessment

The first item to be investigated was the effect of constant errors. The Model was run with the grouped data set using values for the mean competitors' bid that deviated from the true values by a fixed percentage increment. This was done for increments of - 4.5%, -4.0%, -3.5%, ... +3.5%, +4.0%, +4.5%. The results of these are plotted in Figure 5.21. The zero error value is taken as 100% and the other results expressed in percentage terms. The dashed line at 81% indicates the Companies actual performance position.

The horizontal character on the left hand portion of the graph indicates that the Model's performance is relatively stable over a large range. Comparison with the actual companies' performance suggests that as long as the managerial assessments are within -3.5% to + 1.0%, the Model can outperform the companies.

Random Errors in Assessment

A more realistic investigation of the effect of errors is accomplished by using random errors in the assessment of the mean. A program was written to use the City University Computer Unit's random number generator, RAND, to introduce random errors into the mean bid value. An error range was specified,

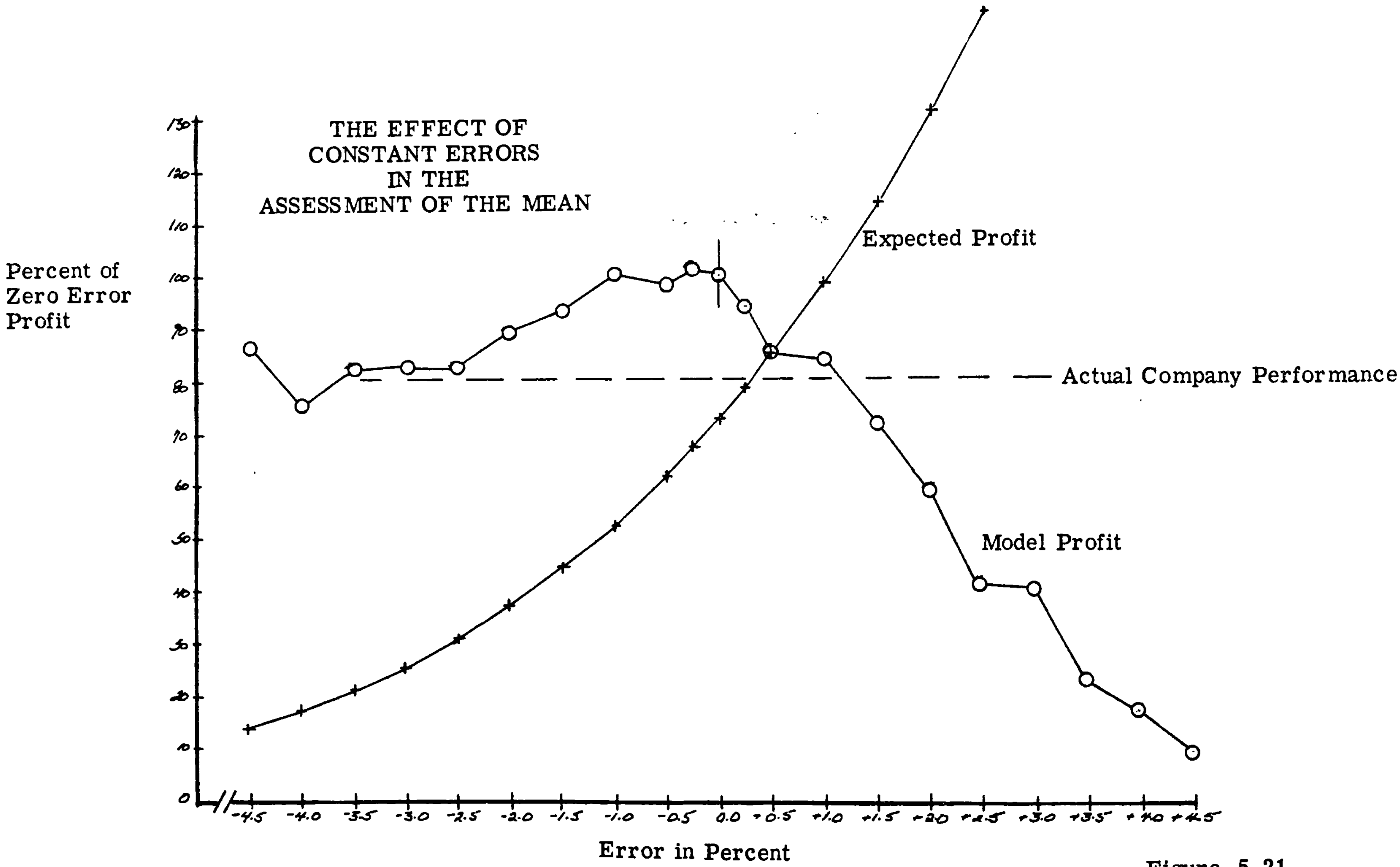

THE EFFECT OF
CONSTANT ERRORS
IN THE
ASSESSMENT OF THE MEAN
Percent of
Zero Error
Profit
130
120
110
100
90
80
70
60
50
40
30
20
10
0
-4.5
-4.0
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
+0.5
+1.0
+1.5
+2.0
+2.5
+3.0
+3.5
+4.0
+4.5
Error in Percent
Expected Profit
Actual Company Performance
Model Profit

Figure 5.21

(true mean $\pm$ e%), the computer would generate a random number between 0.000 and 1.000 and the number would determine where in the range the assessed value would lie. Then this assessed value would be used by the Model to compute a bid.

In addition to computing the bids resulting from the random number stream, the program was also designed to produce the results using the antithetical number stream. That is, if on the first run the mean assessment for contract y is (1.0 + 0.026) times the true value, in the antithetical run contract y's mean assessment would be (1.0 - 0.026) times the true value. The final result is two total set values for each specified error range. These two values represent the opposite extremes and as such, define the limits within which the actual values could be expected.

Three different random number streams were used. Before the program was run the streams were tested with a Chi-square test to see if they were from a uniform distribution. The Chi-square values are:

Random number stream 1	3.8
Random number stream 2	4.2
Random number stream 3	11.0

The five percent significance level for this case is 16.9. Therefore it was accepted that the numbers are random.

The results of the random error runs for the different error ranges are graphed in Figure 5.22. The zero error profit figure is taken as 100%. These graphs bear out the results of the constant error analysis. That is, the Model is reasonably stable and will usually outperform the actual companies if the mean can be assessed within $\pm$ 2% of its true value. The model will produce results similar to that of the companies as long as the mean can be estimated with $\pm$ 3.5%.

Error in Number of Competitors

The number of competitors is the other major parameter in the Model. Therefore, despite the fact that the managements of the sample companies were confident that this number could be predicted with a high degree of accuracy, it is instructive to evaluate the effect of errors in this parameter.

The Model was run with errors of -2, -1, and +1 in the number of competitors and the results are summarised in the table below, the Model Profit for zero error taken as 100%. In cases where the error would reduce the contract to zero competitors, the full error was not applied and the contract was treated as having one competitor.

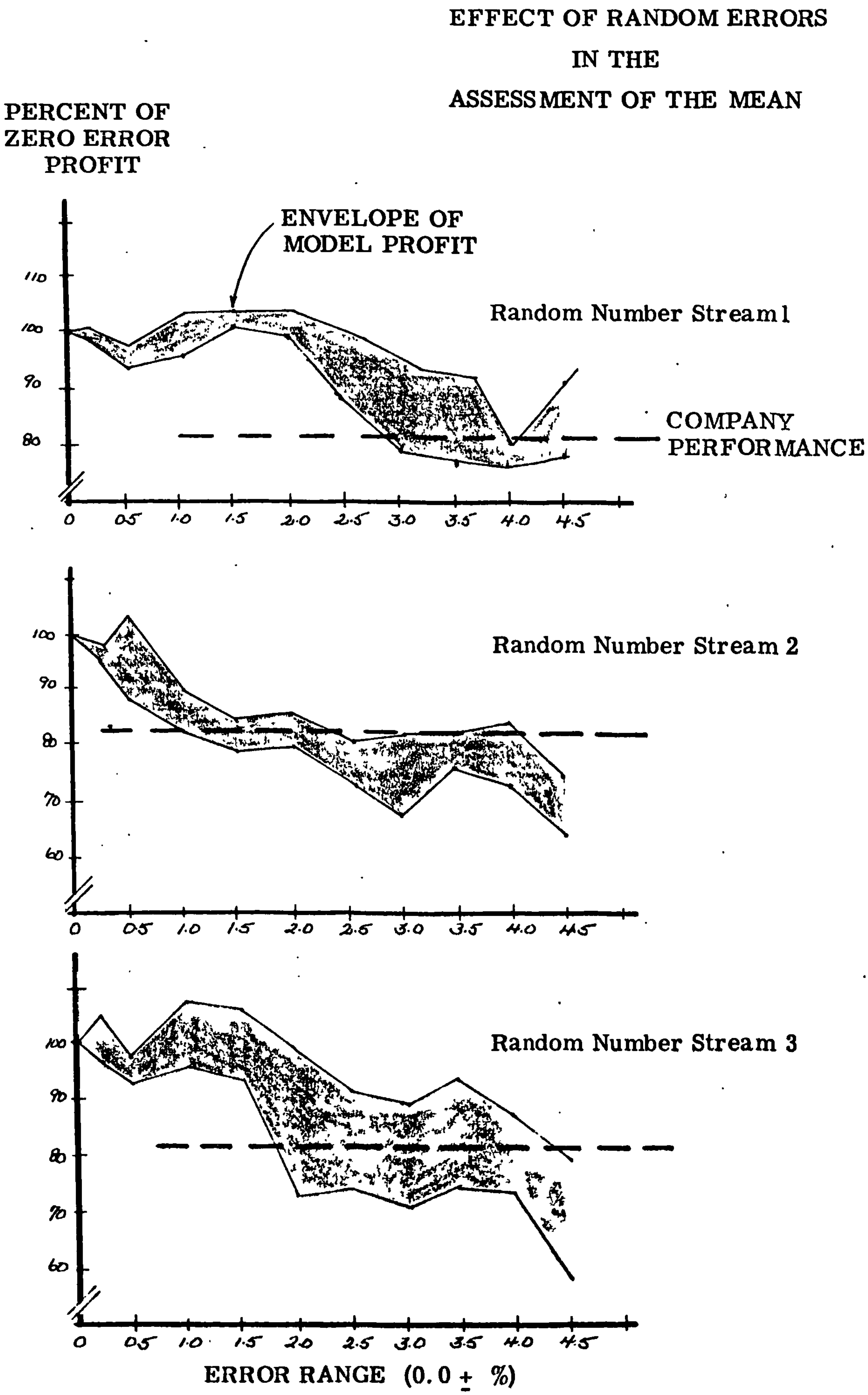

EFFECT OF RANDOM ERRORS
IN THE
ASSESSMENT OF THE MEAN
PERCENT OF
ZERO ERROR
PROFIT
ENVELOPE OF
MODEL PROFIT
Random Number Stream 1
COMPANY
PERFORMANCE
110
100
90
80
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
Random Number Stream 2
100
90
80
70
60
Random Number Stream 3
100
90
80
70
60
ERROR RANGE (0.0 ± %)

Figure 5.22

Error in Number of Competitors	Cumulative Model Profit	Cumulative Expected Profit
n' = n - 2	43.5%	180%
n' = n - 1	78.6%	113%
n' = n	100.0%	73%
n' = n + 1	97.0%	51.5%

This comparison suggests that it is more desirable to over-estimate the number of competitors. This has an advantage from an application point of view since it would be more reasonable to expect last minute "drop outs", than last minute entries.

5.5 PARTIAL IMPLEMENTATION

The simulation test results are on too small a sample to be conclusive but they do indicate that some of the techniques might provide benefits to an operating company. Two of the sample companies were sufficiently interested to wish to pursue the research further and test some of the models in parallel with their normal tendering activity. Therefore work was initiated to implement some of the techniques of the thesis and test some of the others.

Insufficient time has elapsed for these tests to produce any results that may be analysed. Firm conclusions are not expected until the summer of 1971. However, several observations made during the installation of the tests have a bearing on the development of this thesis. Therefore, the work is outlined here and the preliminary observations discussed.

5.5.1 OPTM - A Feedback Device

The OPTM procedure provides a simple, concise method of summarising historical tender information. Although the Model 5 tests indicated it might be of limited benefit as a prescriptive system, the Feasibility Study, Section 2.4, demonstrated that it has value as a reporting system. The information is summarised in such a way that trends and drifts in tendering activity can be readily detected. It is as a reporting system that it was installed.

A standard procedure was developed, together with the necessary forms, computer programs, and instruction manuals, that will produce OPTM reports on a monthly basis covering tendering activity over a specified past time interval. The procedure produces minimum disruption to normal tendering activity, requiring only one form summarising the tender and competitors' bids be completed for each tender submitted. The past tender files are kept up-to-date by the Computer Unit and the program is run once a month. The output is sent directly to the senior decision maker. The output is a single sheet containing three graphs which plot the result of incremental changes in bids in terms of total gross profit, annual gross profit, and number of contracts won.

A secondary benefit of this procedure is that it provides, on cards ready for processing, the tender information required for the other tests.

5.5.2 Parallel Testing

The Friedman Average Model and the General Distribution

Model were selected for further testing. Programs were developed and installed that compute optimum bids from these models. The intention is that these programs should run in parallel with the company's normal estimating and tendering activity so that the decision maker can compare his performance with that of the models. Also, if desired, the models' bids can be made available to the decision maker before he submits his own.

It can be argued that this parallel testing of the Friedman Model is superfluous since the model can be adequately tested with historic data. This may be so but the test is also intended to be a demonstration and the managements are more likely to be impressed with a demonstration based upon current data than one based upon historical data.

The effectiveness of the General Distribution Model, since it relies upon a managerial assessment, can only be properly analysed with a parallel test. The analysis, Section 5.4.2, suggested that the Model would be effective if the decision maker could assess his competition within the prescribed limits. Now it is necessary to test whether the decision maker can. The General Distribution Model is a simple system to install and use. The Model reduces to a Table of Optimum Mark-ups and the determination of the optimum bid requires only two simple calculations. The reduction was accomplished as follows.

The Model's optimum bid is a result of four values: the estimated cost, the estimating accuracy range, the number of competitors, and the assessment of the mean of the competitors' bids. A non-dimensional ratio is obtained by dividing the assessed mean by the estimated cost. Then for a specified estimate accuracy range, the optimum mark-up is a function of this ratio and the number of competitors. Thus it is possible to express the optimum bid as an estimated cost multiplier and incorporate these multipliers for a given estimate range into a single table. A Fortran program was written to solve the Model for a number of mean-estimated cost ratios and an example Table is shown in Figure 5.23.

Using the table, the optimum bid is determined in the following manner.

1. Select the table with the appropriate range R.
2. Use the mean-cost ratio and the number of competitors to determine the optimum multiplier from the table.
3. Optimum Bid = estimated cost • optimum multiplier.

TABLE OF OPTIMUM MARK-UPS
RANGE 12.000

AVERAGE BID / ESTIMATED COST	NUMBER OF COMPETITORS 1	2	3	4	5	6	7	8	9
1.1500	1.1234	1.1025	1.0954	1.0914	1.0889	1.0861	1.0847	1.0831	1.0823
1.1450	1.1210	1.0991	1.0923	1.0884	1.0854	1.0835	1.0821	1.0807	1.0800
1.1400	1.1189	1.0961	1.0895	1.0854	1.0831	1.0813	1.0794	1.0781	1.0774
1.1350	1.1168	1.0934	1.0862	1.0827	1.0801	1.0784	1.0772	1.0759	1.0753
1.1300	1.1148	1.0907	1.0838	1.0800	1.0775	1.0759	1.0747	1.0736	1.0730
1.1250	1.1128	1.0879	1.0812	1.0776	1.0752	1.0737	1.0727	1.0716	1.0711
1.1200	1.1112	1.0859	1.0785	1.0751	1.0733	1.0719	1.0704	1.0699	1.0689
1.1150	1.1092	1.0838	1.0766	1.0729	1.0712	1.0698	1.0689	1.0680	1.0676
1.1100	1.1077	1.0816	1.0746	1.0710	1.0693	1.0681	1.0672	1.0668	1.0664
1.1050	1.1058	1.0802	1.0729	1.0694	1.0678	1.0666	1.0657	1.0653	1.0649
1.1000	1.1040	1.0785	1.0711	1.0680	1.0665	1.0653	1.0649	1.0641	1.0641
1.0950	1.1021	1.0775	1.0701	1.0666	1.0651	1.0643	1.0639	1.0636	1.0632
1.0900	1.1004	1.0761	1.0690	1.0660	1.0644	1.0637	1.0633	1.0629	1.0625
1.0850	1.0986	1.0752	1.0680	1.0649	1.0638	1.0630	1.0626	1.0622	1.0622
1.0800	1.0964	1.0740	1.0670	1.0643	1.0631	1.0623	1.0619	1.0619	1.0619
1.0750	1.0943	1.0732	1.0665	1.0637	1.0625	1.0621	1.0617	1.0617	1.0617
1.0700	1.0926	1.0721	1.0657	1.0632	1.0620	1.0616	1.0612	1.0612	1.0612
1.0650	1.0905	1.0710	1.0649	1.0624	1.0615	1.0611	1.0611	1.0611	1.0611
1.0600	1.0885	1.0696	1.0642	1.0621	1.0612	1.0608	1.0608	1.0608	1.0608

Figure 5.23

5.5.3 Observations

It has been asserted by many of those interviewed during the research that psychological factors have a strong influence on estimating. Although an estimator will tacitly concede that in practice he wins only one job in five, or one in ten; the project he is estimating right now he is estimating to win. Several cases were cited of estimators deliberately juggling their costs because they felt that the mark-ups decided upon by management were too high. Senior estimators know that one of their problems is the young estimator who has not won a contract for a long period.

Psychological considerations are not dealt with in this thesis as the author is not qualified to explain or assess them. However, the personalisation of contracts, and the emotional environment of estimating cannot be ignored. Therefore, the following comment is made.

In order for a system to function it must be accepted by the estimators. If it is not, the attempted introduction will encounter resistance and the installation may be sabotaged. One manager who, as a result of analysing past bids, decided to increase his mark-up wondered how long it would be before his estimators realised his action and started to decrease their cost estimates.

The emotional factors indicate that parallel testing is an essential step in implementation. Only by demonstrating on a continuous basis that the Models can equal or outperform the current methods will acceptance be gained.

One parallel test of the General Distribution Model is, after a month of operation, starting to produce interesting results. This Model was originally developed from the assumption that the decision maker was attuned to his market. This assumption was reinforced by interviews with construction personnel and quotations such as,

"A further disadvantage of competitive tendering from the builders point of view is that cost have little bearing on the quotation which is based upon what he thinks the market will bear. It has been suggested that some companies are "frightened" to quote in any other way." (38)

which appear in the literature.

The experimental tests of the Model indicated that beneficial results could be obtained if the decision maker could assess the mean bid of his competitors to within $\pm$ 2%. The requirement is that the mean be assessed independently of the estimated cost. Independently, because the concept of the Model is to use two figures to arrive at an optimum bid: a market estimate and a cost estimate. If the market estimate is based upon the cost estimate then, as the optimum table shows, the result would just be that of using a constant set of multipliers that vary with the number of competitors, and the market exploitation power of the Model would be lost.

At the time of development is was thought that it would not be too onerous a task for someone who "knows his market" to estimate his market to within ± 2%. However, the Sample Companies that were to participate in the test thought it was. The reason is doubtlessly that construction management is cost, not market oriented. This is not suprising since construction is basically a production industry and the path to management is via the production hierarchy. Thus the main emphasis is on costs and cost control and little attention is paid to marketing.

In view of these doubts, it is interesting that on the one test started, the senior estimator concerned has found, much to his own suprise, that he can estimate his market. The test so far only includes five tenders and on four of these the assessment has fallen within the prescribed ± 2% limits. The estimator concerned feels that, as his learning process continues, he will consistently refine his market assessment ability. He also feels that he will achieve considerable benefit from being forced to look beyond his own company for market indications.

Another firm, not one of the original four, has become interested in the research and has started experimenting with regression analysis to see if the mean can be predicted from contract elements. This work has just started and no results are yet available.

The final observation concerns the use of informative and decision rule models. The OPTM procedure is an informative model. It can indicate, for instance, that an increase of 1% would result in the same acceptances and higher profit. However, the decision makers find it very difficult to apply this result because the tender in front of them is "different". A decision rule model, like the General Distribution Model, which takes an estimate and produces a positive statement, e.g. the optimum multiplier for a specific contract, may be emotionally more acceptable to the decision makers. For him to use its results effectively, however, it is necessary that he should both understand its general structure (and method of operation) and have confidence in its results.

5.6 SUMMARY

Empirical testing and partial implementation were used to evaluate the adjudication decision models. Six models were tested using the data from the four sample companies. The tests were conducted by simulating on a computer, allowing the models to bid against the companies' competitors. The six models tested were:

Model 1A - Friedman Average Competitor Hypothesis
Model 1E - Friedman Average Competitor Hypothesis with exponential smoothing
Model 2 - General Distribution Hypothesis
Model 3 - Normal Distribution Hypothesis
Model 4 - Lowest Competitor Hypothesis
Model 5 - Drift Model.

The results suggested that when total profits is the criterion, both versions of the Friedman Model, and the General Distribution Model will equal or improve upon the current company performance. Generally, when compared with the company; the Friedman Model wins more contracts but at lower bid prices, the General Distribution Model wins the same contracts but at higher prices. The results also indicate the effect of the estimated cost on winning, some contracts having such low estimated costs that all the models won them.

The General Distribution Model was initially tested under conditions of perfect information. Then a sensitivity analysis was conducted to evaluate the effect of errors in information. The result was that the model would equal or outperform the current company performance if the mean of the competitors' bids could be assessed to within $\pm$ 2%.

A partial implementation produced some subjective observations on the usefulness of the models. It is apparent that to be useful the models must be acceptable not just to the management, but also to the estimators. Also there are strong indications that, as the decision maker becomes more adept at estimating his competition, the General Distribution Model will become even more practicable and beneficial.

CHAPTER 6: SUMMARY AND CONCLUSIONS

6.1 SUMMARY

The Management Problem presented in Chapter 1 was:

"Within the prescribed boundaries what are the optimal decisions relative to the management objectives".

This thesis is a study of the Problem as it is manifest in the building construction industry competitive bidding process. This section summarises the central development of the study.

The work progressed generally along the conventional operational research lines of

i. Define problem
ii. Formulate problem in quantitative terms
iii. Develop decision models
iv. Test decision models
v. Implement solutions.

The first concern was to make the Management Problem specific. This was done by delineating the boundaries, describing the decisions , and investigating the objectives. The boundaries of the study are the markets of the four Sample Companies. This is part of the building construction market in the south of England, and is characterised by the following features:

i) There are a large number of potential competitors for each project.

ii) There is little product differentiation.

iii) There is little differentiation between companies.

iv) The resource constraints are minimal.

v) The contractural risk is low.

Black box models were used to isolate the management decisions. The Competitive Bidding Decision was found to consist of two stages: the decision to tender (selection), and the tender price decision (adjudication).

The search for a quantitative objective function was based upon the assumptions that an economic measure existed and that the companies practiced maximizing behaviour. An investigation of the variables in the situation led to consideration of the uncertainty in the process and the explanation of the effect of the estimating uncertainty interacting with the bidding process. A value function was proposed that attempts to integrate the rationale of discounted cash flow procedures with the subjective judgement of management. The Adjudication objective was tested with a simplified version of the function and the results indicated that the measure was suitable.

The problem was modelled in quantitative terms by considering a series of cases from the simple, Single Contract, Unconstrained, situation to the complex, N Contract, Independent Constraints, Sequential Bidding, situation. The complex, N Contract, case, while mathematically intriguing,was felt to be unnecessary for the actual decision situation and so the simple, single contract, case was used.

The development of a decision model began by considering some existing models. These were found to be based upon an assumption of constant competitive behaviour which did not seem appropriate for the situation under consideration. Therefore a model was developed from basic probability concepts that employed a managerial estimate of the present instead of a study of competitors' histories.

The data provided by the four Sample Companies was used to test six different decision models. Although the sample was too small for statistically significant conclusions to be derived, the tests indicated that the Friedman Model and the General Distribution Model could equal, and usually exceed, the current company results. A sensitivity analysis was performed on the General Distribution Model to investigate the effect that errors in the assessment of the mean bid would have on the results.

Implementation revealed two features which affect the potential usefulness of the Models. One is the need for acceptance by the persons (estimators) affected. The other is the indication that the decision maker may be able to assess his competition within the prescribed limits.

6.2 CONCLUSIONS

The main conclusions of this thesis are:

(1) The decision process in the building construction competitive bidding situation is composed of two related management decisions - the selection decision and the adjudication decision. Although it is possible to mathematically model the situation in such a way that the decisions are handled simultaneously, the actual nature of the problem allows the decisions to be handled independently. The independent treatment of the adjudication decision is compatible with the approaches of previous researchers, and with the approach employed by the companies studied.

(2) In the market studied, which is a segment of the building construction market, two important characteristics are apparent.

(a) Different firms attempt to place the same value (estimated cost) on a specified contract. The differences that occur between estimated costs are primarily attributable to uncertainty and not to any competitive advantage possessed by one firm.

(b) The market is comprised of a large number of firms, although in any specified competition only a few of them participate.

The result of these two characteristics is to suggest that statistical techniques which average the behaviour of competitors and aggregate the result of past competitions are the appropriate methods with which to study the situation.

(3) From a study of the decision process, and an analysis of current company performance, it was concluded that a prescriptive decision model, in addition to accommodating the above mentioned characteristics, should provide some method of incorporating managerial judgement of the competitive situation into the decision. The published models examined were unable to include this factor (Friedman), or were inappropriate for the situation (Edelman). Therefore the General Distribution Model which integrates managerial judgement into a probablistic model was developed.

(4) A key factor in the determination of an appropriate objective function was the influence of the uncertainty present in the estimating process. It was concluded that this uncertainty interacts with the competitive bidding mechanism and introduces a bias into the estimates of winning bids. A mathematical model of this interaction was developed and a correction method proposed.

(5) Six prescriptive models were tested and the results were compared between the models, and with the actual performance of the sample companies. The seven methods (six models plus company) were ranked on the basis of total profits achieved, but a rank test indicated that the hypothesis, the ranking is random, could not be rejected. This was felt to be partially due to the large number of methods compared and the small number (four) of tests run.

(6) Comparison of each model's results with those achieved by the companies suggested that the Friedman Model and the General Distribution Model could usually equal or exceed the results achieved by the companies. The sample was too small to permit any statistically significant conclusions, but an examination of the results on an individual contract basis indicates that, depending on the chosen criteria, either model could be superior to current company methods. The Friedman Model tended to win more contracts than the companies and achieved a lower profit volume ratio, higher total profits and a higher volume than the companies. Therefore, if volume is the criterion, the Friedman Model would appear to be suitable. The General Distribution Model tended to win the same number of contracts as the companies, but at higher bid prices and thus achieved a higher profit volume ratio, higher total profits and approximately the same volume.

Therefore, if total profits, or profit volume ratio, is the criterion, the General Distribution Model appears to be the superior method.

(7) The General Distribution Model requires a managerial assessment of the mean of the bids that the competitors will submit. A sensitivity analysis was performed on the model and the results indicate that this assessment need only be within a range of the true value. Specifically, it was concluded from the test results that if the manager can assess the mean within $\pm$ 2% of the true value, the model will equal or exceed current performance. Preliminary trials have indicated that assessment within these limits is possible.

(8) The initial indications are that the General Distribution Model can be used as a prescriptive model by an operating company. This is because, in addition to the test performances mentioned above, the model is emotionally acceptable to the decision maker. Since the model uses managerial judgement as a major input, it is regarded by the decision maker as an aid and not as a potential replacement.

6.3 FURTHER RESEARCH

This thesis is not an isolated work but is part of the continuing study of competitive bidding problems initiated by Friedman in 1956. It is hoped that this study will stimulate still more exploration of the process of competitive bidding. Three obvious extensions of this work are:

(1) The continued implementation and testing of the models of this thesis, and the utilisation of other techniques to complement the models. For example the use of a regression technique such as that of Mercer and Russell (19) for predicting the mean of the competitor's bids.

(2) Empirical research into the relationship between the actual gross profit received and that predicted by the bid minus estimated cost figure. This research could be directed at investigation of the elements of contract uncertainty: client, percentage of sub-contractor's work, etc., it could be an attempt to utilise the proposed general objective function. The latter course, with its rationale for quantifying managerial intuition and preference, could provide the averaging, variance reduction effect sought by Bowmans Theory (3).

(3) The application of the concept of the General Distribution Model to other competitive situations. The idea of an underlying distribution with assessed parameters has worked with PERT, and appears to work here. Possibly it will work in other situations where uncertainty exists and managerial judgement is the best method of assessing it.

BIBLIOGRAPHY

(1) Ackoff, R.L., and Sasieni, M.W., (1968), "Fundamentals of Operations Research", Wiley.

(2) Aris, R., (1964), "Discrete Dynamic Programming", Blaisdell Publishing Company.

(3) Bowman, E.H., (1963), "Consistency and Optimality in Managerial Decision Making", Management Science, Vol. 9.

(4) Casey, J.B., and Shaffer, L.R., (1964), "An Evaluation of Some Competitive Bidding Strategy Models for Some Contractors", University of Illinois.

(5) "Code of Estimating Practice", The Institute of Building, (1966).

(6) Colclough, J.R., (1965), "The Construction Industry of Great Britain", Butterworth and Company (Publishers) Limited.

(7) Dean, B.V., (1965), "Contract Award and Bidding Strategies", IEEE Transactions of Engineering Management.

(8) Edelman, F., (1965), "Art and Science of Competitive Bidding" Harvard Business Review, Vol. 43, July-August 1965.

(9) Fine, Brian, (1966), "On Comparison of Economic Evaluation Criteria", Building Research Station Internal Note 94/66.

(1967), "The Effect of Loans and Internal Rates on the Comparision of Economic Evaluation Criteria", BRS, IN 53/67.

(1967), "The Effect of Inaccurate Estimates on the Comparison of Economic Evaluation Criteria", BRS, IN 54/67.

(10) Fine, B., and Hackemer, G.C., (1970), "A Computer Simulation for Estimating and Bidding Strategy", Paper presented at The City University 'Decision Making for Bidding and Tendering' Conference, May 20 1970.

(11) Friedman, L., (1956), "A Competitive Bidding Strategy", Operations Research, Vol.4., Feb. 1956.

(1957), "Competitive Bidding Strategies", Doctoral Dissertation, Case Institute of Technology.

(12) Gates, M., (1960), "Statistical and Economic Analysis of a Bidding Trend", ASCE Proceedings, Vol.86, NoCO3, November 1960.

(1967), "Bidding Strategies and Probabilities", Vol. 93, NoCO1, ASCE Construction Division Journal, March 1967.

(13) Griesmer, J.H., and Shubic, M., (1963), "Towards a Study of Bidding Processes", Naval Research Logistics Quarterly, Vol. 10, Nos. 1, 2 and 3.

(14) Hansmann, F., and Rivett, B.H.P., (1959), "Competitive Bidding", Operational Research Quarterly, Vol.10, No.1.

(15) Higgins, G., and Jessop, N., (1963), "Communication in the Building Industry", Tavistock Institute of Human Relations.

(16) Hunt, Logan, et. al, (1966), "Contract Award Practices", Vol. 92, NoCO3, ASCE Construction Division Journal, Jan. 1966.

(17) Kuhn, H.W., and Tucker, A.W., (1951), "Nonlinear Programming", in Jerzy. Neymann (ed) Proceedings of the Second Berkeley Symposiom on Mathematical Statistics and Probability, University of California Press, Berkeley.

(18) Luce, R.D., and Raiffa, H., (1957), "Games and Decisions", John Wiley & Sons Inc.

(19) Mercer, A., and Russell, J.L.T., (1969), "Recurrent Competitive Bidding", O.R.Q. Vol. 20, No. 2.

(20) Morin, Thomas, L., and Clough, Richard H., (1969), "OPBID: Competitive Bidding Strategy Model", ASCE Journal of the Construction Division, Vol. 95, NoCOl, July 1969.

(21) Mood, A.M., and Graybill, F.A., (1953), "Introduction to the Theory of Statistics", McGraw-Hill.

(22) Moroney, M.J., (1951), " Facts from Figures", Penguin Books.

(23) Neumann, J., von, and Morgenstern, O., (1944), "Theory of Games and Economic Behaviour", Princeton.

(24) Park, W.R., (1966), "The Strategy of Contracting for Profits", Prentice Hall.

(25) Rothkopf, M.H., (1969), "A Model of Rational Competitive Bidding", Management Science, Vol. 15, No. 7.

(26) Rubey, H., and Milner, W.W., (1966), "Construction and Professional Management". The MacMillan Company.

(27) Schlaifer, R., (1959), "Probability and Statistics for Business Decisions", McGraw-Hill Book Company Inc.

(28) Shortell, G.J., (1966), "Some Aspects of Estimating and Tendering for Building Contracts". Technical Essay, The College of Aeronautics, Cranfield.

(29) Simmonds, K., (1965), "Marketing and Pricing under Competitive Bidding", Ph.d Thesis, London School of Economics.

(1968), "Competitive Bidding", O.R.Q., Vol.19, No.1.

(1968), "Adjusting Bias in Cost Estimates", Viewpoints, O.R.Q., Vol. 19, No.3.

(30) Torgersen, Wyskida, Yarbrough, (1968), "Bidding Work Loading Game", ASCE Construction Division Journal, Vol.94, NoCO2, Oct. 1968.

(31) Turner, J.W.H., (1963), "Construction Management", C.R. Books Limited.

(32) "The Placing and Management of Contracts for Building and Civil Engineering Work", Report of the Committee, Ministry of Public Building and Works, (Chairman, Sir Harold Banwell), HMSO 1964.

(33) "Survey of Problems before the Construction Industries", Ministry of Public Building and Works, (Chairman, Sir Harold Emmerson), HMSO 1962.

(34) Interdependence and Uncertainty - A Study of the Building Industry", Digest of a report from the Tavistock Institute to the Building Industry Communication Research Project, Tavistock Publications (1966).

(35) "Retention of Moneys on Building and Civil Engineering Contracts", Report of the Working Party, (Chairman, E. A. Seal), Ministry of Works, HMSO 1964.

(36) The Placing and Management of Building Contracts", Report of the Central Council for Works and Buildings, (Chairman, Sir Ernest Simon), HMSO 1944.

(37) "The Building Industry and the Public Client", Proceedings of a conference organised by the EDC for Building at the Royal Garden Hotel, 21 March 1968; Published by the National Economic Development Council.

(38) "The Building Industry ... 1962 Onward", A Survey by The Builder Limited (1962).

APPENDIX 1 - SAMPLE COMPANIES' DATA

The investigation was provided with historical data by a number of operating construction companies. These companies are referred to as the Sample Companies. A historical data set consists of a series of contracts which the sample company has bid over a period of time. Each set has a brief description of the type of work, and the time period covered. For each contract in the set the data consists of:

a sequence number

the sample company's estimated cost

the sample company's submitted bid

the number of competitors

the competitors' bids

and in some cases, the names of the competitors. An example data sheet is shown on the following page. For reasons of company security, the data sets cannot be included in the thesis.

Several factors complicated the obtaining of this data.

1. Some of the sample companies do not formally record this data. Therefore, although if exists, it often consists of scribbled notes scattered through their files.

2. The British practice is to not always make available the competitors' prices. This produces gaps in the data.

EXAMPLE DATA SHEET

Page 1 of 6

COMPANY NAME: XYZ Company

TYPE OF WORK: Building Construction

TIME PERIOD: Jan. 1969 to Sept. 1969 inc.

Date (1)	Estimated Cost	Tender	Competitors' Bids	Competitors' Names (2)
1/10/69	£100, 000	£106, 000	£104, 000	W
			109, 000	X
			99, 750	Y
			114, 000	Z

(1) Or chronological sequence number

(2) The Competitors names and bids do not have to match up

At the start of the research, the policy of eliminating all contracts which had incomplete data was followed. It was rapidly discovered that this policy was not reasonable. For example, in Company D this policy reduced the data set from 51 contracts to 30 contracts. Therefore some selectivity was used. For instance, if five of the seven submitted bids on a contract were available, the contract was included; if only the low bid was available, the contract was eliminated. As a result, some of the data contain unknown errors.

3. Very little information is kept by the sample companies on cover bids submitted. However, since these are effectively non-bids, these gaps should not be significant.

4. In cases where the tender documents called for alternates, it is not always recorded which alternate was accepted. However, from the cases where data was available it appears that the rankings of firms are usually the same for the alternatives.

The result of these factors is that the data does not describe the complete bidding histories of the Sample Companies over the periods. Whether these gaps and unknown errors significantly affect the conclusions is unknown. One result of the gaps could be that the computed expected value for the set may be expected to be less than the actual value. The reason for this is discussed in Chapter 5.

Inquiries did not seem to indicate that any information was deliberately withheld, or that the gaps were found to be concentrated on any specific type of contract. The only common factor that they appeared to have was that most of them were contracts on which the company's tender had been unsuccessful. Thus it was felt that there was no deliberate attempt to bias the data.

Certain contracts were eliminated from the data sets by the author. These were contracts that the data indicated were misfits or abnormal contracts. The bases for elimination are outlined below:-

(i) Figure A. 5 was compiled from the original data sets. It shows the frequency of the normalised range of the bids submitted on the contracts.

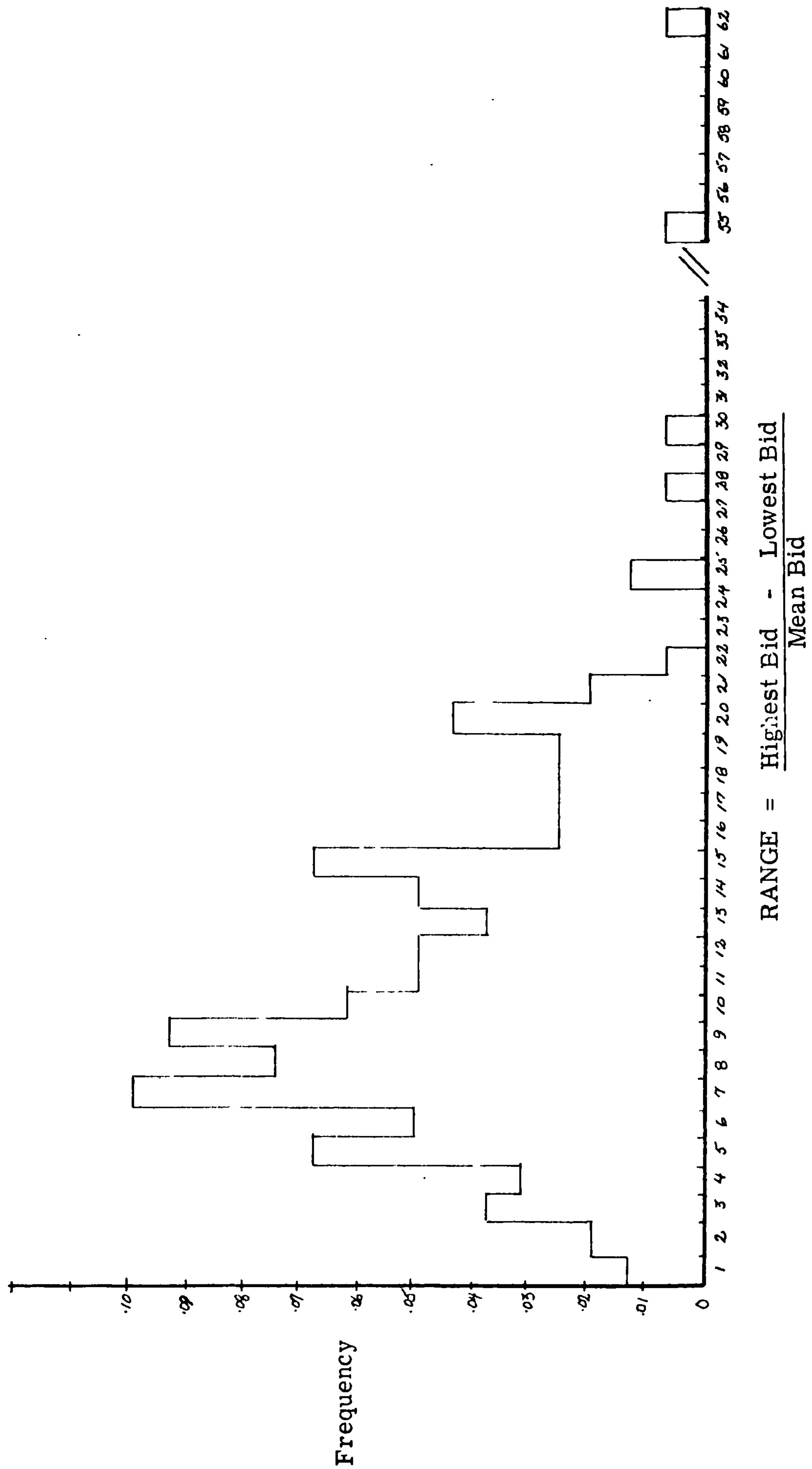
$$\text{Range} = \frac{\text{Highest bid - Lowest bid}}{\text{Mean bid}}$$

Contracts with a bidding range greater than 24% were eliminated.

(ii) Any contracts in which the estimated cost exceeded the average estimated cost of the contracts in the data set by an excessive amount (factors of 6 to 14) were eliminated. This was done for two reasons:

1. The contract would probably be singled out for special attention by management.

2. The presence of these contracts in the data sets distort


Frequency
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34
55 56 57 58 59 60 61 62
RANGE = Highest Bid - Lowest Bid / Mean Bid

the results.

Three contracts were eliminated for this reason. One each from Companies A, C, and D.

(iii) Obviously abnormal contracts were eliminated. For example, in one of Company C's contracts, the estimated cost was 21% higher than the highest competitive bid submitted. This suggests some major estimating error, like the transposing of figures.

The following bid data sets were used in the research.

Company A

Type of Projects................ Building Contracts in England

Time Period .... Jan. 1968 - Dec. 1968 inclusive

Number of Contracts .... 34 ((37))*

Total Number of Different Competitors 65

Company B

Type of Projects................ Building Contracts in England

Time Period .... Jan. 1968 - Dec. 1968 inclusive

Number of Contracts .... 41 ((43))

Total Number of Different Competitors 81

---

*The number in the double brackets is the original number of contracts supplied, before elimination.

Company C

Type of Projects .... Building Contracts in England

Time Period .... Jan. 1969 - Sept. 1969 inclusive

Number of Contracts .... 37 ((41))

Company D

Type of Projects .... Building Contracts in England

Time Period .... May 1968 - Dec. 1969 inclusive

Number of Contracts .... 41 ((51))

Total Number of Different
Competitors 100

APPENDIX 2 - UNBALANCED BIDDING

A device sometimes employed on unit price (Bill of Quantities type) contracts is that of the unbalanced bid. There are two major motives for employing this device:

1. To exploit a mistake on the part of the client organisation.
2. To accelerate the cash flow on a contract.

Unbalancing is best explained by example.

To Exploit a Mistake -

On a unit price contract the following estimated quantities are provided in the contract documents*:

5000 cu. yds solid rock

10000 cu. yds loose rock

40000 cu. yds earth

The estimating department decides that for that site, and equipment, reasonable unit prices are:

£1.25 for solid rock

£0.80 for loose rock

£0.40 for earth

Therefore, the tender would appear as:

5,000	cu. yds solid rock	@ £1.25 =	£6,250
10,000	cu. yds loose rock	@ £0.80 =	8,000
40,000	cu. yds earth	@ £0.40 =	16,000
			£30,250.

---

* The numbers in this example are taken from reference 26 with £ signs used instead of dollar signs.

Now for some reason, (site investigation, previous knowledge), it is assumed that the engineer has made an error in the estimated quantities and that the site actually contains a far higher percentage of rock than is stated. Therefore the bid is unbalanced by increasing the rock prices and decreasing the earth price. The submitted tender appears as:

5,000 cu. yds solid rock	@ £2.0	=	£10,000
10,000 cu. yds loose rock	@ £1.0	=	£10,000
40,000 cu. yds earth	@ £0.2	=	£ 8,000
			£28,000

Now, after the contract has been completed, the surveyor has recorded the following actual quantities.

15,000 cu. yds solid rock	@ £2.0	=	£30,000
20,000 cu. yds loose rock	@ £1.0	=	£20,000
20,000 cu. yds earth	@ £0.2	=	4,000
			£54,000

The advantage is obvious; the company receives £54,000 instead of £32,750 it would have received if the bid had not been unbalanced.

To Accelerate the Cash Flow -

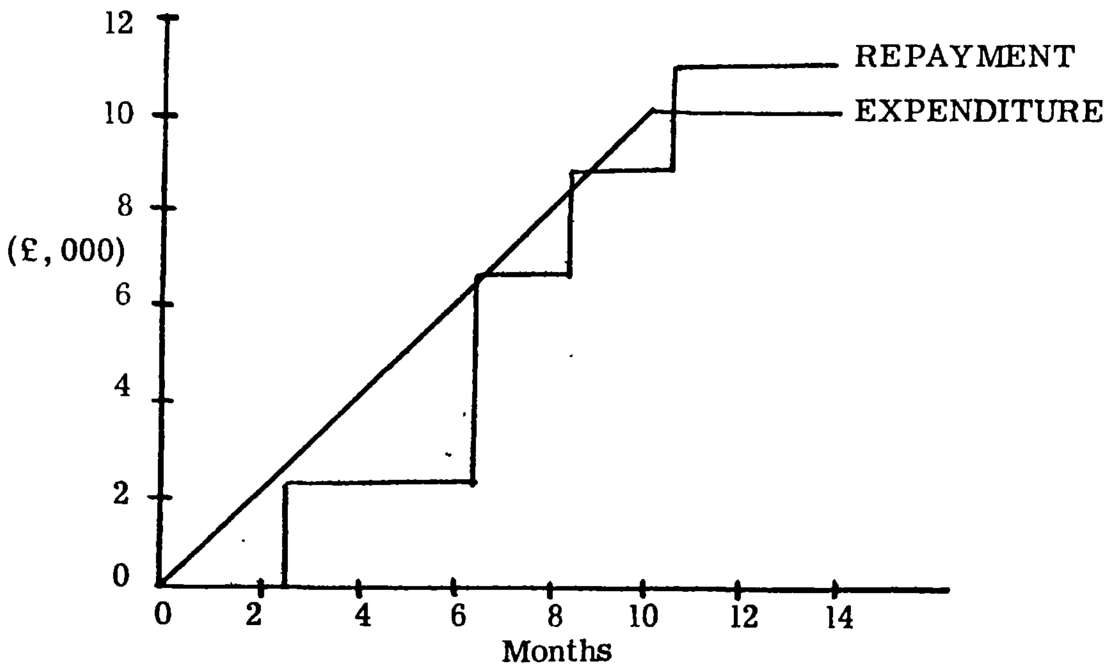
A paving contract specifies payments at the completion of the following stages.

Stage	Estimated Time Months	Estimated Cost £
Curb and gutter	2	2,000
Subgrade excavation	4	4,000
Stabilised base course	2	2,000
Asphalt paving	2	2,000
	12	£10,000

The Company applies a 10% mark-up and so the submitted prices are:

Curb and gutter	£2,200
Subgrade excavation	4,400
Stabilised base	2,200
Asphalt paving	2,200
	£11,000

Assuming a delay of one-half a month between the submission of the certificate and the receipt of payment; the first graph shows the job expenditure and stage repayment plotted against time.

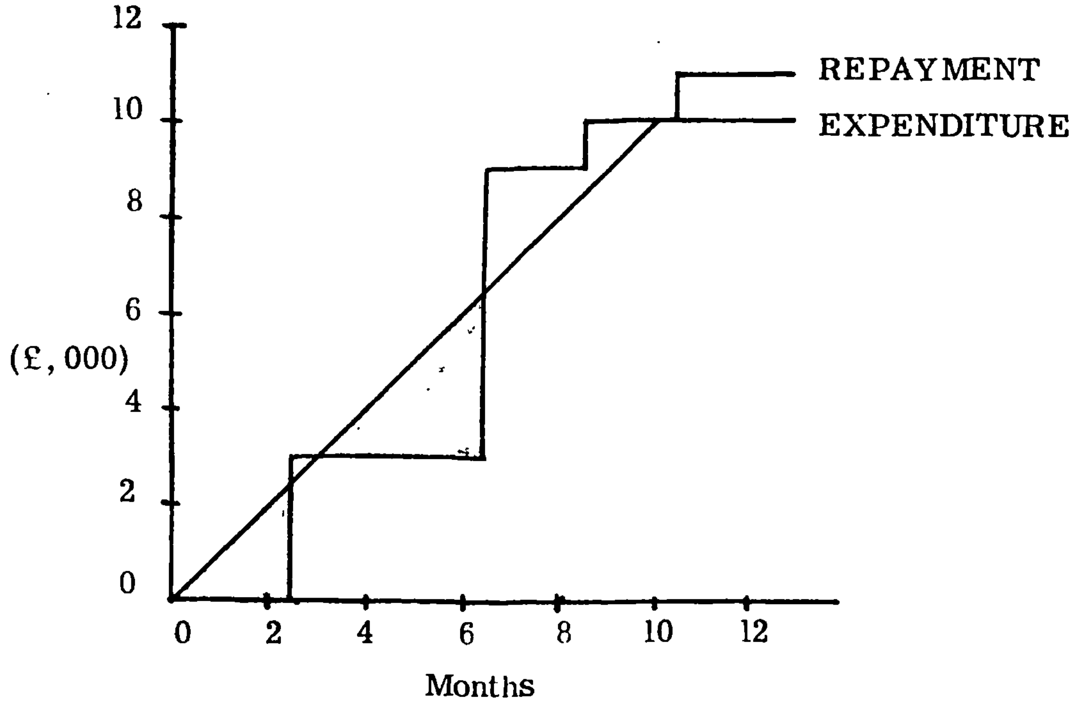


The shaded area is the "job borrowing", i.e. the time the job owes the company money.

Now unbalance the bid to:

Curb and gutter	£3,000
Subgrade excavation	£6,000
Stabilised base	£1,000
Asphalt paving	£1,000
	£11,000

The second graph shows the unbalanced situation



By unbalancing the bidding the borrowing time is reduced and the job rapidly becomes self financing. The only limit on unbalancing appears to be credibility. Unbalancing is frowned upon and so a contractor can only unbalance a bid as far as he thinks he can get away with it.

APPENDIX 3 - ESTIMATE RANGE DETERMINATION

The formulation of the uncertainty interaction correction, Section 2.5.3, requires, to be applicable, the range of estimating accuracy. Now one possible method of determining the range could be from the accounting systems of the companies. If separate job costing is kept by a company, then from the accounting profits received on the projects, the number of competitors, and original bidding information it would be possible to use the formulas of Section 2.5.3 to determine R. This accounting information was not available to this Research. It is often not available to the contractor since bulk buying and general stores complicate the task of determining a cost for a particular project. Variations and extras during the course of a contract further confuse this determination. Therefore three methods were devised for estimating the range from the available data.

Method 1

The first method assumes that on all estimates prepared, a condition of cancelling errors exists. Therefore the average of the ratios of the winning bid to the Company's cost estimate for all the contracts in the set is a measure of the profit potential of the set.

Assume that the Company's average mark-up is the average mark-up of the winning bidders. Therefore

n = the average number of competitors

$$Z = \left(\text{Average Company Mark-up} - \text{Average } \frac{\text{winning bid}}{\text{company cost estimate}}\right)$$

$$= \frac{100 \times R \times n}{n(200-R) \times 400}$$

Solve the formula for R.

Method 2

Method 2 is identical to method 1 except that it assumes that the profit-volume ratio achieved by the Company is the average mark-up of the winning bidders. Thus

$$Z = \left(\text{Company's Profit-Volume Ratio} - \text{Average } \frac{\text{winning bid}}{\text{company cost estimate}}\right)$$

$$= \frac{100 \times R \times n}{n(200-R) \times 400}$$

Solve the formula for R.

Method 3

Let y_1 be the lowest of the n estimated costs

y_n be the highest of n estimated costs

Assuming that the y's are random samples from a distribution, $f(y)$, their joint distribution is

$$P(y_1, y_n)dy_1dy_n = n(n-1) (F(y_n) - F(y_1))^{n-2} f(y_1) f(y_n)dy_1 dy_n$$

where

$$F(y_i) = \int_{-\infty}^{y_i} f(y)\, dy$$

Assume:

1. That all bidders use the same cost multiplier ϕ to determine their bids.

2. That all estimated costs are random samples from

$$f(y) = \begin{cases} \frac{1}{R} & \text{where } 100-\frac{R}{2} \leq y \leq 100+\frac{R}{2} \\ 0 & \text{elsewhere} \end{cases}$$

Make the transformation

$$s_1 = \phi y_1$$
$$s_2 = -\phi y_1 + \phi y_n$$

then

$$s_1 + s_2 = \phi y_n$$
$$y_n = (s_1 + s_2)/\phi$$

and

$$y_1 = s_1/\phi$$

The Jacobian

$$J_s = \begin{vmatrix} \frac{\partial y_1}{\partial s_1} & \frac{\partial y_1}{\partial s_2} \\ \frac{\partial y_n}{\partial s_1} & \frac{\partial y_n}{\partial s_2} \end{vmatrix} = \begin{vmatrix} \frac{1}{\phi} & 0 \\ \frac{1}{\phi} & \frac{1}{\phi} \end{vmatrix} = \frac{1}{\phi^2}$$

$$f(s_1) = \frac{1}{\phi R} \quad \text{where} \quad \phi(100-\frac{R}{2}) \leq s_1 \leq \phi(100+\frac{R}{2})$$

$$f(s_1+s_2) = \frac{1}{\phi R} \quad \text{where} \quad \phi(100-\frac{R}{2}) \leq s_1+s_2 \leq \phi(100+\frac{R}{2})$$

$$F(s_1) = \int_{\phi(100-\frac{R}{2})}^{s_1} \frac{1}{\phi R}\,dy = \frac{1}{\phi R}\left\{ s_1 - 100\phi + \frac{\phi R}{2} \right\}$$

$$F(s_1+s_2) = \int_{\phi(100-\frac{R}{2})}^{s_1+s_2} \frac{1}{\phi R}\,dy = \frac{1}{\phi R}\left\{ s_1+s_2 - 100\phi + \frac{\phi R}{2} \right\}$$

$$p(s_1,s_2)\,ds_1\,ds_2 = n(n-1)\left(F(s_1+s_2) - F(s_1)\right)^{n-2} f(s_1)\, f(s_1+s_2)\, J_s\, ds_1\, ds_2$$

$$= n(n-1)\left(\frac{s_2}{\phi R}\right)^{n-2}\left(\frac{1}{\phi R}\right)^{2}\frac{1}{\phi^2}\, ds_1\, ds_2$$

where $0 \le s_2 \le \phi R$

$$\phi\left(100 - \frac{R}{2}\right) \le s_1 \le \phi\left(100 + \frac{R}{2}\right) - s_2$$

the distribution of s_2 is

$$h(s_2)\,ds_2 = \frac{n(n-1)}{\phi^{n+2} R^n}\int_{(100-\frac{R}{2})\phi}^{(100-\frac{R}{2})\phi - s_2} s_2^{\,n-2}\, ds_1\, ds_2$$

$$= \frac{n(n-1)}{\phi^{n+2} R^n}\, s_2^{\,n-2}\left(R\phi - s_2\right) ds_2$$

the expected value of s_2 is

$$E(s_2) = \int_0^{R\phi} s_2\, h(s_2)\, ds_2$$

$$= \frac{n(n-1)}{\phi^{n+2} R^n}\int_0^{R\phi} s_2^{\,n-1}\left(R\phi - s_2\right) ds_2$$

$$= \frac{n(n-1)}{\phi^{n+2} R^n}\left\{\frac{(R\phi)^{n+1}}{n} - \frac{(R\phi)^{n+1}}{n+1}\right\}$$

$$= \frac{R}{\phi}\left\{\frac{n-1}{n+1}\right\}$$

Now S_2 is the bid range. Given that the assumptions hold, then an estimate of the estimating accuracy can be calculated from the mean value of the bid range.

Example	-	Average bid range	= 10%
		Average number of competitors	= 6
		Average mark-up multiplier ϕ	= 1.06

$$E(S_2) = \frac{R}{\phi} \left\{ \frac{n-1}{n+1} \right\}$$

$$10 = \frac{R}{1.06} \left\{ \frac{5}{7} \right\}$$

$$R = 14.8\%$$

RESULTS AND DISCUSSION

The three different methods for computing the accuracy range for the data sets provide the following results.

	Method 1	Method 2	Method 3
Company A	12.85%	11.9 %	15.35%
Company B	13.45%	12.9 %	15.5 %
Company C	11.0 %	9.35%	16.65%
Company D	13.1 %	10.7 %	16.45%

These ranges are all within the anticipated region and at this stage no one method is demonstratably more suitable than any other.

Using these ranges, three sets of Z correction multipliers were calculated for each company. These multipliers are designed to correct the estimated costs by an amount equal to the expected error. The multipliers calculated for Company C, 11.0% Range are presented below as an example.

Z MULTIPLIERS FOR COMPANY C
Range = 11.0%

Number of Competitors	Z
1	1.019
2	1.028
3	1.034
4	1.038
5	1.041
6	1.043
7	1.045
8	1.046
9	1.047
10	1.048

An assumption implicit in the three methods is that all the companies competing in a data set are doing so on approximately the same terms and in a similar manner.

That they are doing so in a similar manner was illustrated by the fact that the sample companies followed almost identical procedures.

To investigate whether any one company possessed a competitive advantage, a dominance matrix was constructed for Company D. This was possible because Company D's records had the names of the competitors as well as their submitted bids for most of the contracts in the set.

The dominance matrix is constructed by listing all the companies down the left hand side of the matrix. The same companies are listed across the top. The order from top to bottom and from left to right being the same. Then the submitted bids are examined and every time a company listed on the left hand side bids below one listed along the top, one is added to the value of the intersecting cell.

		D	Z	Y	X	W	V	U	T	S	R	Q	P
Company	D		4	3	3	5	2	3	3	1	3	3	5
	Z	6			1			2	2	2		2	1
	Y	4				2					1	3	3
	X	4	2	2		1	1		2		1		1
	W	2											2
	V	4		2					1	1	1		
	U	2								2			
	T	1		1							2		
	S	4	1	1		1							
	R	2					1						
	Q	2				1							1
	P	0	1										

The data is sparse but it is sufficient to illustrate the concept. From the top row it can be seen that Company D bid below Company X three times. From the first column it can be seen that Company X was below Company D four times. If any one Company has an advantage over any other, it will make that portion of the matrix unsymmetrical. For this matrix, there is only sufficient data to look at Company D and a comparison of the first row with the first column indicates that Company D is no better and no worse than these eleven competitors. It could be argued that Company D dominates Company P, but this conclusion would be based upon only five common bidding situations.

The symmetrical appearance of the matrix suggests that no company has any decided advantage over any other, and the assumption of similar behaviour seems reasonable.

Method 3 assumes more than similar behaviour; it assumes identical behaviour. That is, the assumption of all companies using exactly the same mark-up. The assumption was made to try and transform the estimating distribution into the bid distribution and, as the subsequent tests with the derived ranges revealed (Chapter 2), it was inadequate. This point is further discussed in Section 3.4.

APPENDIX 4 - REVIEW OF BIDDING THEORY

The term, Bidding Theory, does not relate to a single unified theory, but is a generic title used to encompass the various management science (operational research) quantitative approaches to the problems of competitive bidding.

The original stimulus was the publication, in 1944, of von Neumann and Morgenstern's book, "Theory of Games and Economic Behaviour" (23). This book introduced an analytic framework for dealing with competitive situations. Following von Neumann and Morgenstern, a number of books and papers concerned with the analytic treatment of competitive situations appeared. However, it was not until 1957 that the problem of competitive bidding was specifically dealt with by Lawrence Friedman in his doctoral dissertation, "Competitive Bidding Strategies" (11).

Friedman approached the problem in two ways. One approach was via the game theoretic, minimax, method of von Neumann and Morgenstern. The other method was based upon probabilistic concepts and used an a priori distribution to arrive at a strategy which maximized the expected value of the objective function. The former method was used primarily for a conceptual understanding of the problem because the games theory framework could not accommodate the complexities of the real situation. The latter method was proposed as a solution to a "realistic bidding problem".

Following Friedman, research on competitive bidding has proceeded, fairly independently, along the two main approaches he started. Since they developed independently, they will be outlined separately. Two new approaches, simulation and operational gamming, have also been used to study competitive bidding. These are discussed under the heading, Other Approaches. Recently the two principal approaches have started to merge. Rothkopf (25) treats the problem as a game with partial information and investigates the question of equilibrium when all bidders maximize.

Games Theory:-

The Theory of Games provides an analytic basis for treatment of competitive problems. However, attempts to apply it often encounter serious conceptual and computational difficulties. This has been the case with competitive bidding.

The original theory developed the two-person, strictly competitive (zero sum), game and introduced the minimax method of solution. A coalition concept was used for considering N-person games. Even in the simpler formulations, the competitive bidding problem becomes an N-person, non zero sum game, with coalitions prohibited and the minimax solution appearing to be too pessimistic.

Several researchers have tried to extend the theory to handle the bidding problem. Friedman, in his thesis, introduced the concept of competitive co-efficients as a suggested method of handling the N-person game. However this does not appear to have been adopted by any subsequent researcher.

The major work done on competitive bidding using a games theory approach was "Towards a Study at Bidding Processes"(13) by Griesmer and Shubic, published in 1963. In their series of three papers they advanced the study of competitive bidding from a "simultaneous move, single shot, two-person" game to one where they could treat "sequential bidding with capacity limitations and varying degrees of information", for the two-person, non-zero sum case. This development is an excellent basis for understanding the interactions of some of the variables of the competitive bidding process but still is not sufficiently developed to provide any obvious operational applications.

Recent work by Rothkopf (25) has extended the theory to consider the uncertainty of the bidder about the value of the subject of the auction to himself; and what happens when each bidder optimizes. The equilibrium strategies devised are based upon assumptions of rational behaviour and, as the author suggests, "It is easy enough to envisage situations that violate them". However, the game theoretic treatment appears to be advancing at such a rate that it may soon pass, or encompass the decision theory competitive bidding models.

Probabilistic Treatment of Competitive Bidding

The 1956 paper, "A Competitive Bidding Strategy" (11) by L. Friedman was the first published probabilistic approach to competitive bidding. In this paper, Friedman hypothesized the existence of an objective function and a probability of winning function - both functions of the bid price. He then determined the bid price which maximised the expected value of the objective function and proposed that this was the optimum bid price.

To determine the probability of winning Friedman utilised the past bidding activity of the company. For all the company's past contracts which Competitor A had bid on, the ratio of Competitor A's bid to the Company's cost estimate was determined. These ratios were combined in a histogram and the resulting curve, $f_A(w)$, was used as a description of Competitor A's bidding behaviour. The probability that a bid-estimated cost ratio of x/k would be below Competitor A's ratio was taken as

$$P\left(\frac{x}{k} < A\right) = \int_{x/k}^{\infty} f(w)\, dw$$

Similar curves were compiled for all the other competitors and the probability of winning a contract was taken as the product of the probability of beating the competitors that were participating.

For the cases where there was not sufficient data to determine each competitor's curve, or where the competitors are unknown, Friedman offered the concept of an "average competitor". The curve for the average competitor was developed by combining all past ratios of competitors bids to company cost estimate into a single histogram. The probability of winning a contract against n competitors was the probability of beating the average competition taken to the n th power.

By studying the cost data of contracts won by the company, Friedman was able to collect ratios of true cost to estimated cost. These he combined into a density function which he used to correct his estimated cost for bias.

Two variants of the basic Friedman model have appeared. Hanssmann and Rivett, 1959, (14) proposed using only the ratio of the lowest competitive bid and Ackoff and Sasieni, 1968, (1), suggested that this ratio followed a normal distribution. Both of these methods eliminate the number of competitors, n, from the probability determination. This simplifies the computation of optimum bid but introduces a possible error since the findings of other investigators (Park (24), Morin and Clough (20), Friedman (11), and this thesis) suggest that this is a significant parameter.

Several attempts have been made to apply the Friedman concepts to construction industry competitive bidding. Two of these, Casey and Shaffer (4), and Shortell (28), were unable to obtain cost data and so produced no results. The work of Park is interesting. He noted that the average competition distribution is relatively stable and the main variation in the optimum bid was introduced by the number of competitors. Thus he was able to develop a series of optimum percentage mark-ups for his study companies, the mark-up varying inversely with the number of competitors. He claims considerable success and supports this claim with case histories.

Morin and Clough (20) also reported successful results, claiming a 27% profit increase. They used a Friedman Average Competition model but employed exponential smoothing instead of the normal averaging technique.

The basic Friedman concepts have stood, relatively unchallenged, until publications by Simmonds, 1968, (29) and Mercer and Russell, 1969, (19). Simmonds noted that the cost bias could not be determined from analysis of contracts won because these contracts constituted a biased set. This point is further developed in this thesis under the heading "Management Objectives". Mercer and Russell observed that the methods of deriving the probability function imply that a contract has the same value to all companies competing. As this is not compatible with 'rational' economic behaviour, this observation casts doubts on the family of Friedman based techniques.

The first major departure from the Friedman method was that of Marvin Gates, 1960, and 1967 (12). Gates uses the expected value concept but derives his probability from a regression analysis of past bids. He concentrates on changes in probability of winning for changes in bid price rather than starting with cost as in the Friedman model. It is suggested that similar results can be obtained from a less involved technique - specifically the OPTM program employed in this thesis.

The next major departure was proposed by Edelman (8) in 1965. Retaining the concept of the maximum expected value bid, he removed the probability function from dependence upon historic data and based it upon subjective management estimates of the competition and the client. His results demonstrated the usefulness of his technique. However, his market was not the construction industry and there are sufficient differences to make his methods not directly applicable.

Up to 1965, researchers were concerned primarily with the static case of a single, unconstrained, contract. In that year Simmonds (29) produced the first major treatment of the effect of a company's internal capacity upon its bidding strategy. He proposed penalty functions for having more, or less, work than the company's capacity and introduced these functions into the bid determination.

The work of Mercer and Russell, 1969 (19) also considers the dynamic nature of the market. They develop a quantitative framework which enables them to interpret individual competitive behaviour over a series of contracts.

The work of this thesis is intended to complement that of Simmonds, and Mercer and Russell. An analytic treatment of the dynamic case is developed and a device for handling the aggregate behaviour of unidentifiable competitors is proposed.

Other Approaches

Two other approaches have also been used to study the problems of competitive bidding. These are Operational Gaming and Simulation.

Although the method of gaming has demonstrated its value as a teaching tool, as a vehicle for research it has three major drawbacks.

1. Assuming that the purpose of the research is to devise a bidding strategy or model, it is necessary for the game to be as realistic as possible. Any assumptions or distribution utilised in constructing the game would also be implicitly contained in the model. The model would be solving the devised problem, i.e. the game, which is not necessarily the real problem.

2. If the purpose of the game is to study bidding behaviour, then it is desirable to have as players actual decision makers from industry. These gentlemen usually have many demands on their time and the problem of getting them is a major one.

3. If the second drawback is overcome, there is still the problem of playing the game long enough to get meaningful results. This, plus the fact that the players do not always take the game seriously compounds the difficulties.

It is not suggested that these problems are insurmountable, or that gaming is not a valid method of research. It is suggested that research effort can presently be more fruitfully employed by more direct studies of the real bidding situation. After more work has been done in analysing the situation, then the data will be available to construct realistic games.

The first drawback of gaming also applies to the method of simulation. The results obtained from a simulation will only be as valid as the degree in which the model maps reality. Simulation studies, for example Hackemer and Fine's (10), highlight some interesting interactions in the process, but it is felt that more basic research on the process is required before this method can be fully exploited.

APPENDIX 5 - M.P.B.W. DATA

The data used to determine the General Distribution was taken from the "Flimsy Summaries" of the Contracts Directorate, Ministry of Public Buildings and Works. The period covered was from January 1, 1967 to February 29, 1968 inclusive.

102 contracts were taken from the year 1967.

20 contracts were taken from the year 1968.

The summaries contain a one line description of the location of the contract, a one line description of the work, the date, and a list of the companies tendering and their tender prices. Also listed were those companies invited to tender but were, "unable to tender", and who, "did not tender".

The summaries contain all contracts handled by the Government ranging from the sale of deer hides and horns to communication towers; therefore selectivity was required in extracting contracts for analysis. Since this thesis is primarily concerned with the building construction industry, only those works which, from the one line description, obviously fell into the domain of the general building contractor were taken. All contracts for mechanical or electrical works, structural steel, prefabricated units, roads and paving, as well as the many supply, renovation, alteration, decorating, cleaning, repair and maintenance, catering, etc.. contracts were excluded.

Only contracts with a value over £50,000 were considered. The contracts usually had one of the following phrases in its one line description:

"Erection and completion, including external services for ....."

or

"Construction of ...."

Contracts with alternates and contracts in foreign countries were excluded.

The types of work included in the data are:

telephone exchanges (new and additions)

quarters for armed services

post offices

buildings on military camps, including schools

gymnasiums, etc.

office buildings

hospitals

prisons and related buildings

airport buildings

mint

The contract values were rounded off to the nearest pound.

APPENDIX 6 - PLOT GRAPHS

[redacted]
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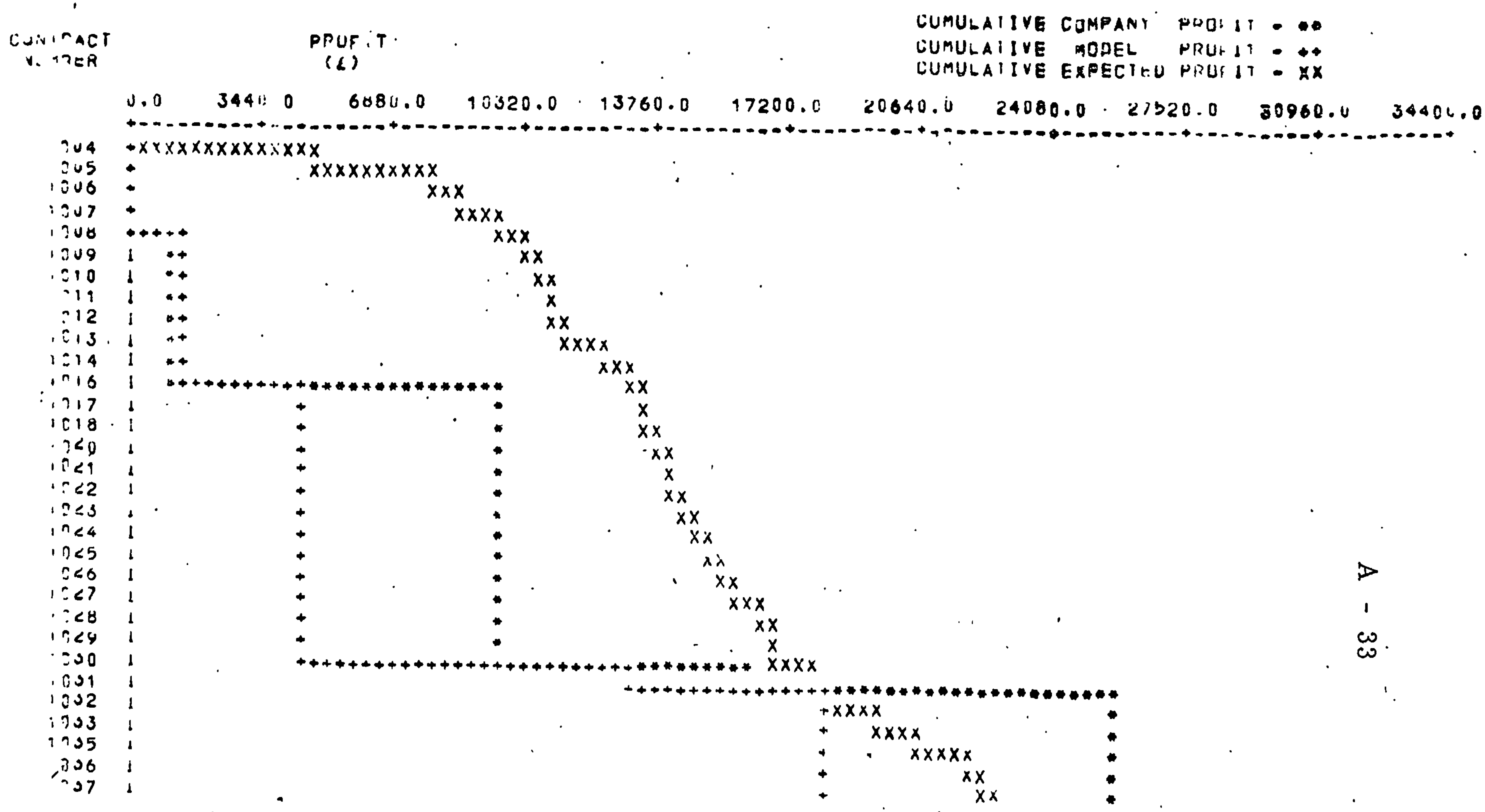
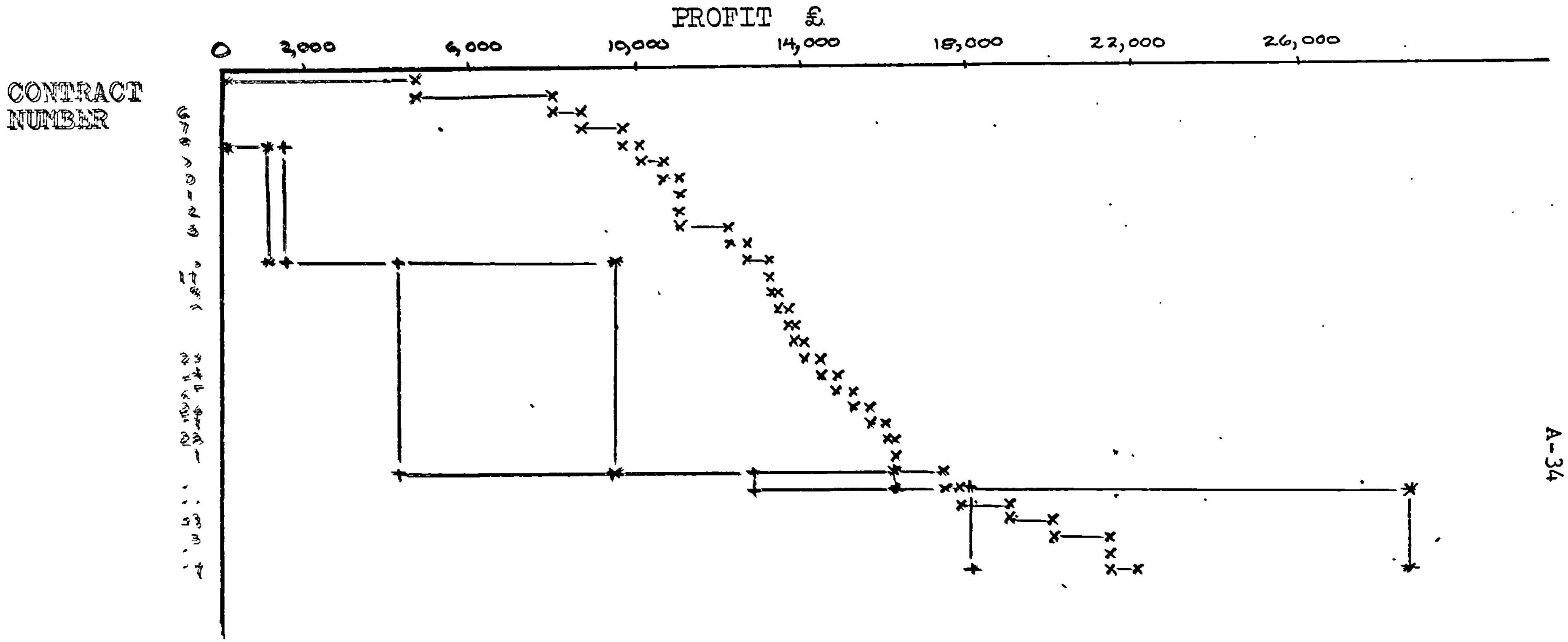
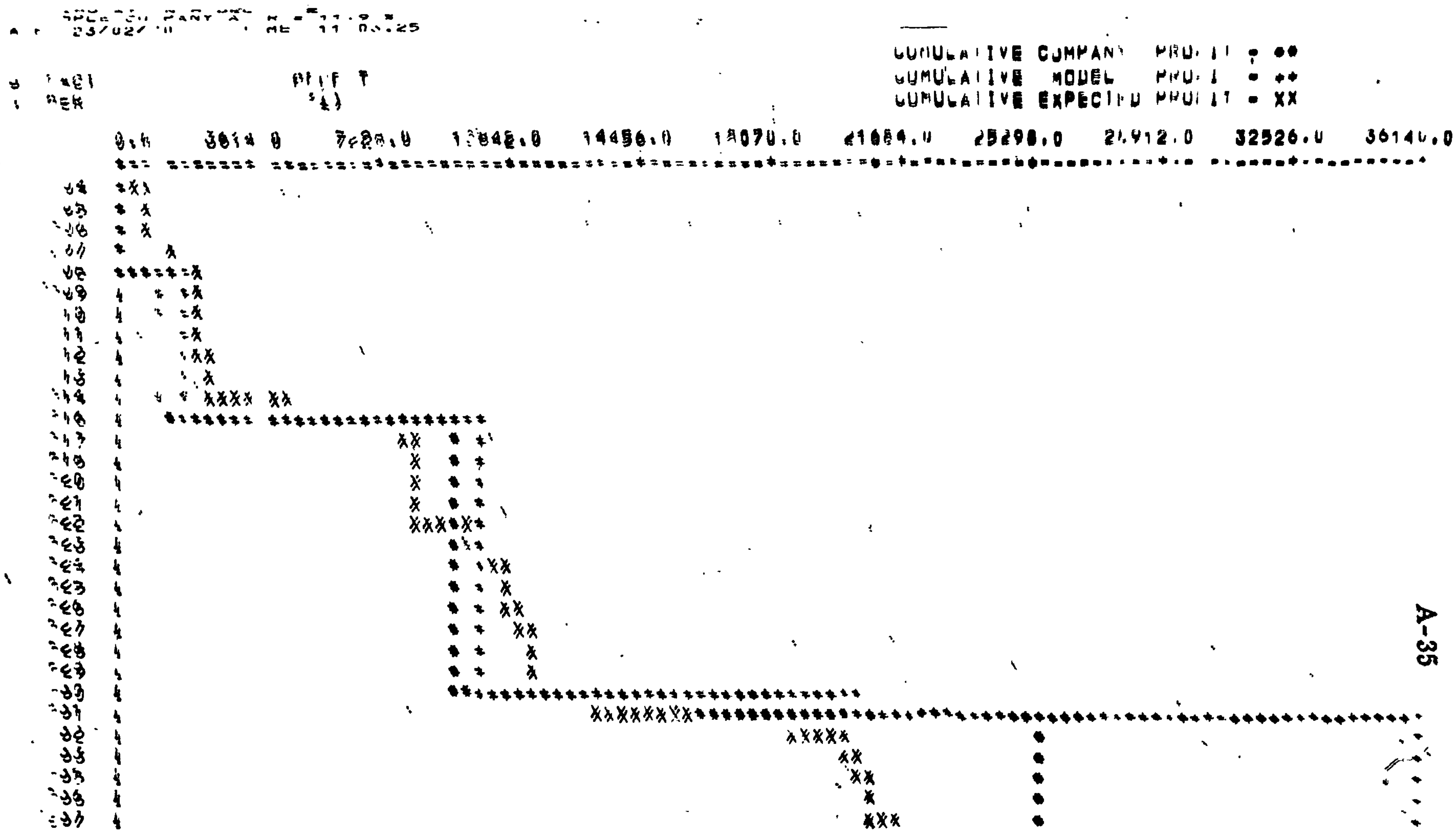


BID SIMULATION MODEL 1E
SAMPLE COMPANY A R = 11.9%

CUMULATIVE COMPANY PROFIT = ✱ ✱
CUMULATIVE MODEL PROFIT – + +
CUMULATIVE EXPECTED PROFIT – x x




CUMULATIVE COMPANY PROFIT = ●●
CUMULATIVE MODEL PROFIT = ++
CUMULATIVE EXPECTED PROFIT = XX
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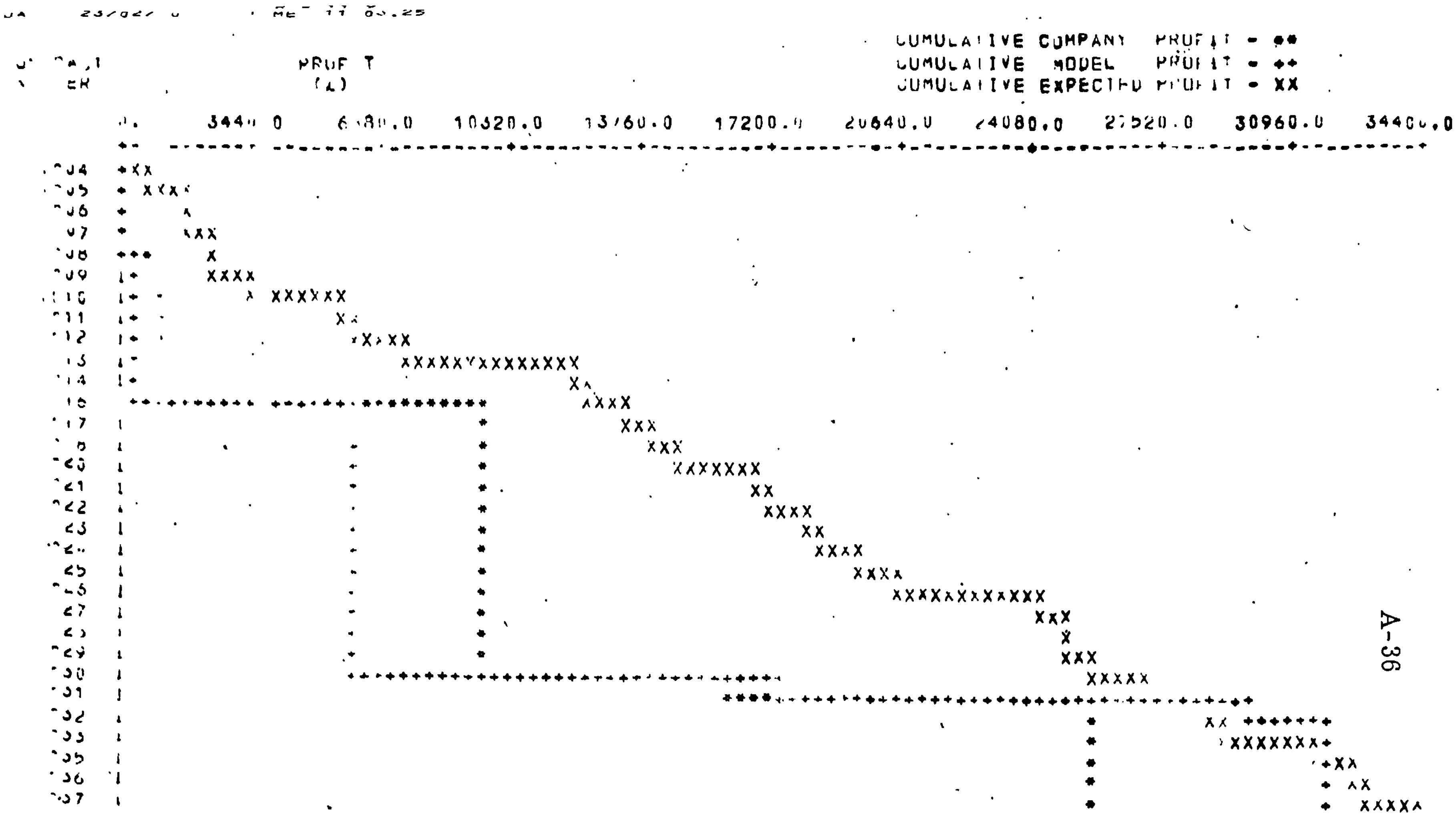
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CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX
PROFIT
(£)
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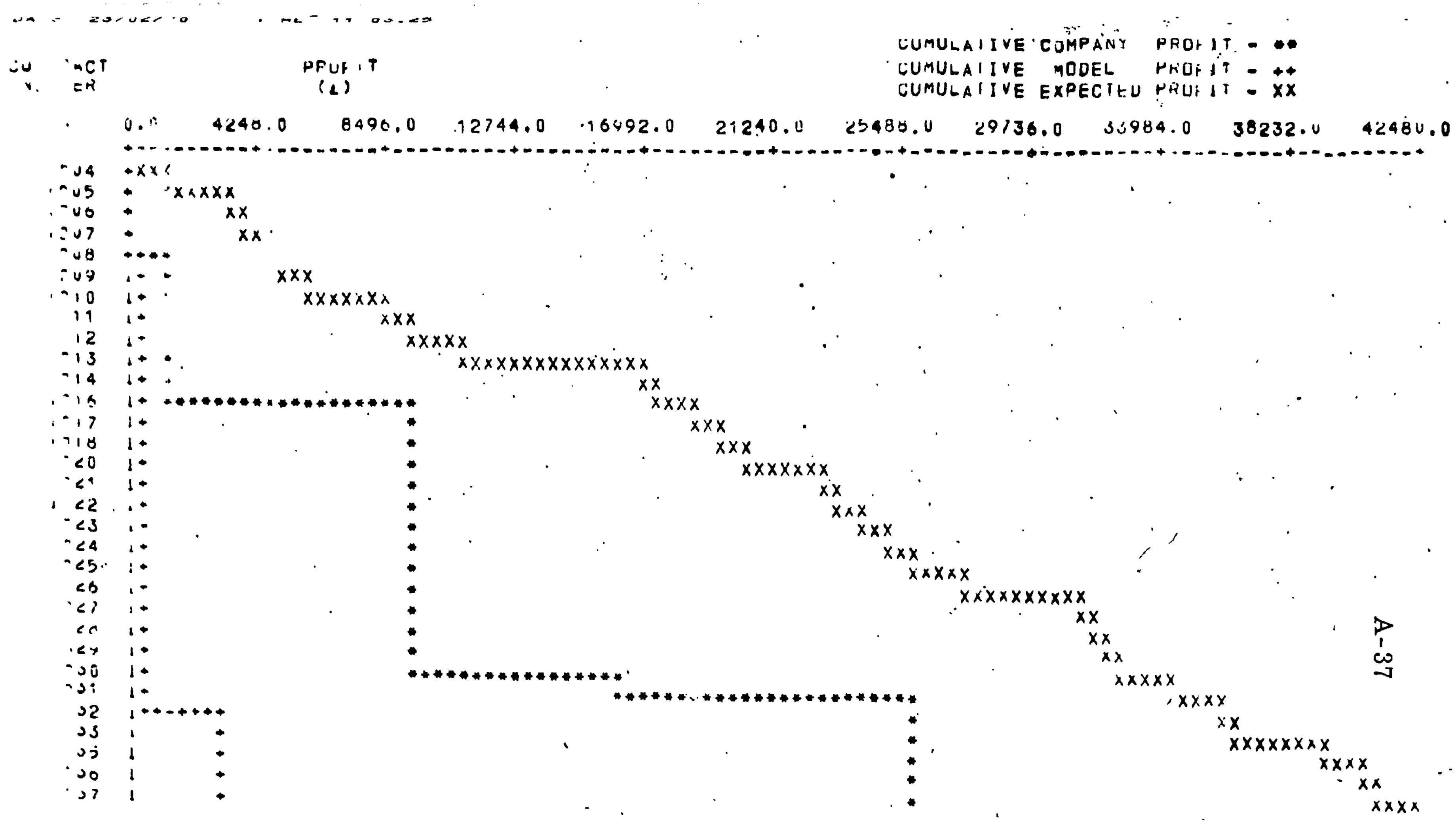
10320.0
13760.0
17200.0
20640.0
24080.0
27520.0
30960.0
34400.0



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CUMULATIVE EXPECTED PROFIT - XX
PROFIT
(£)
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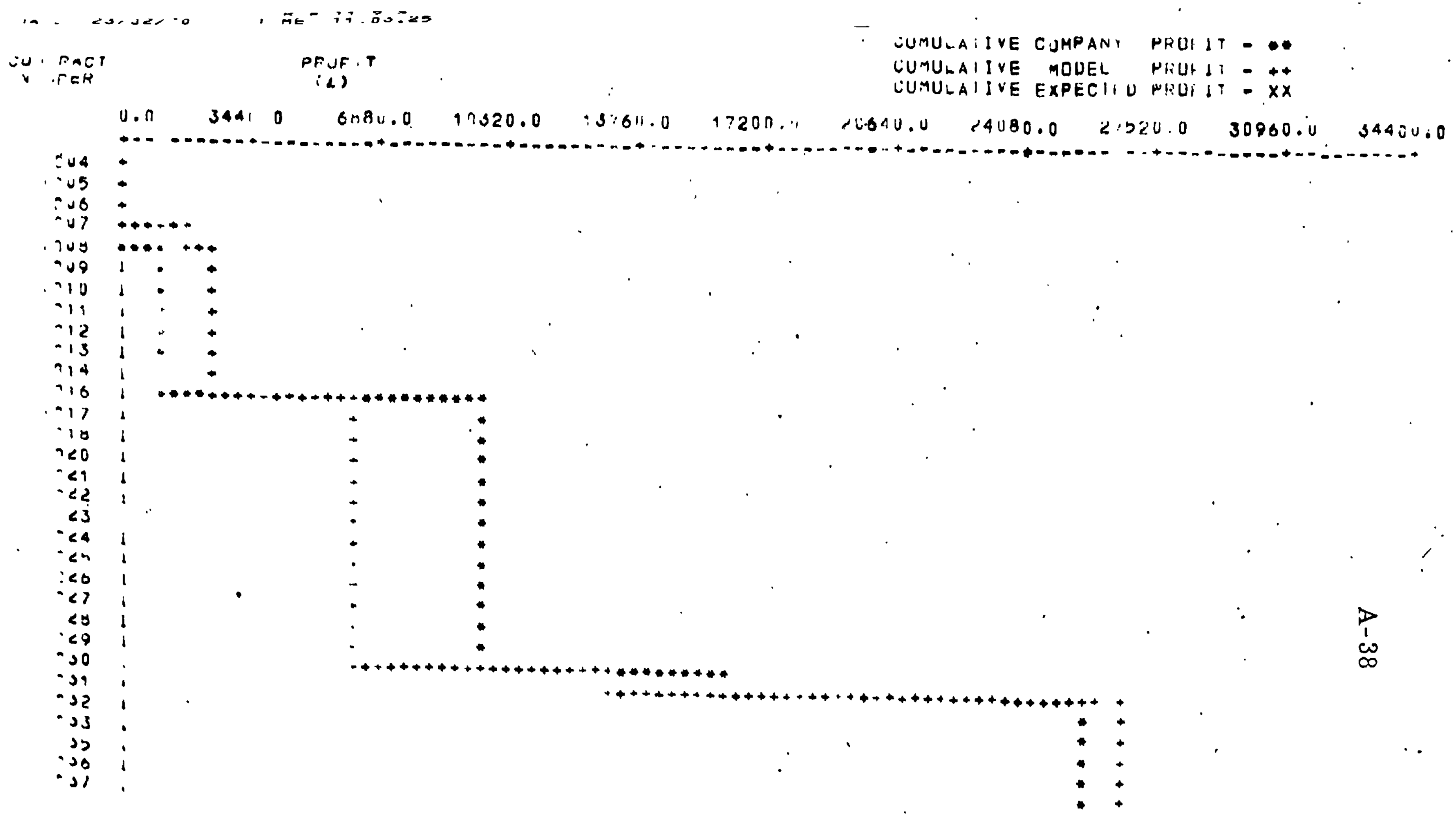
42480.0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CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX
CONTRACT NUMBER
PROFIT ($)
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10320.0
13760.0
17200.0
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24080.0
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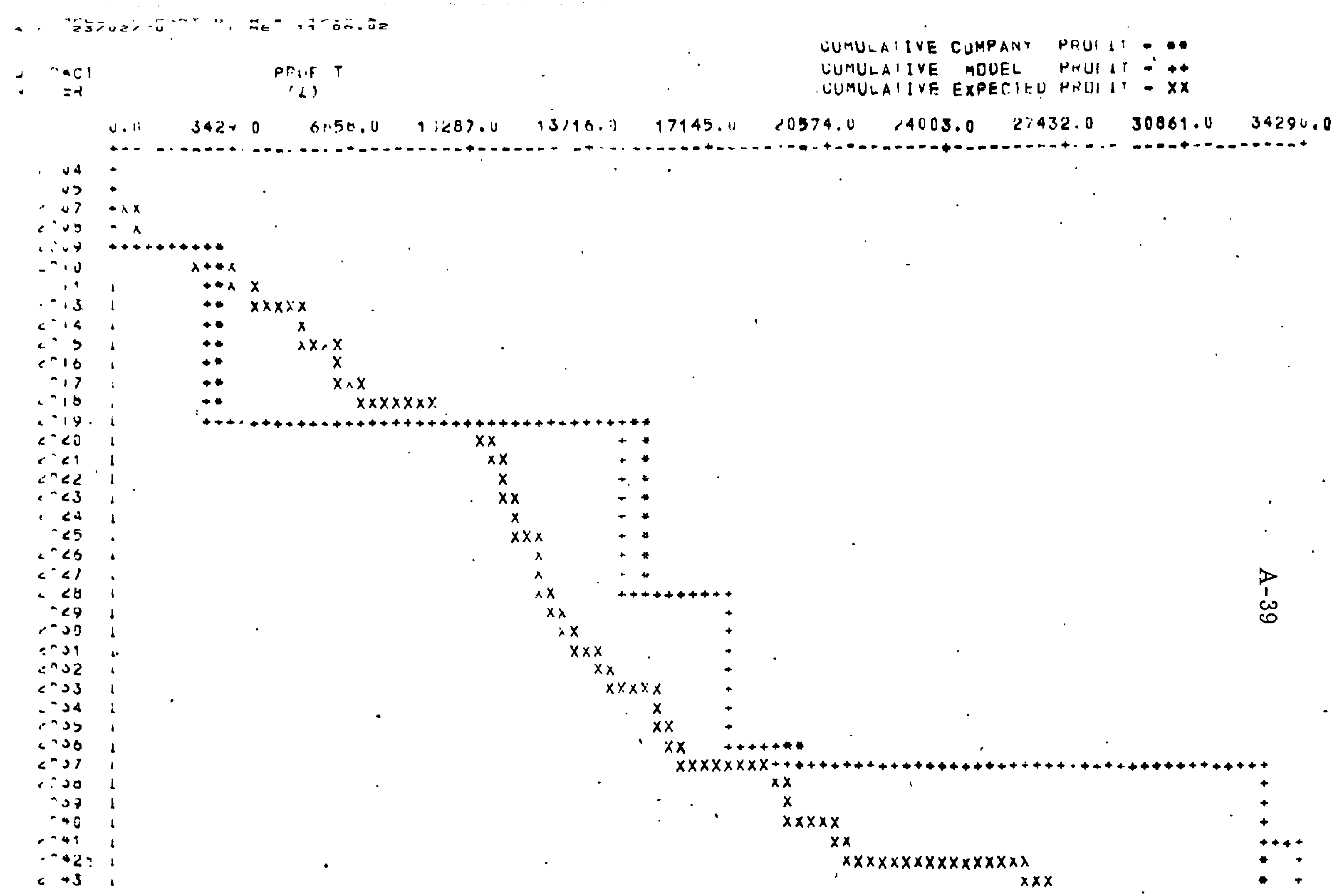
PROFIT
CUMULATIVE COMPANY PROFIT - **
CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX
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20574.0
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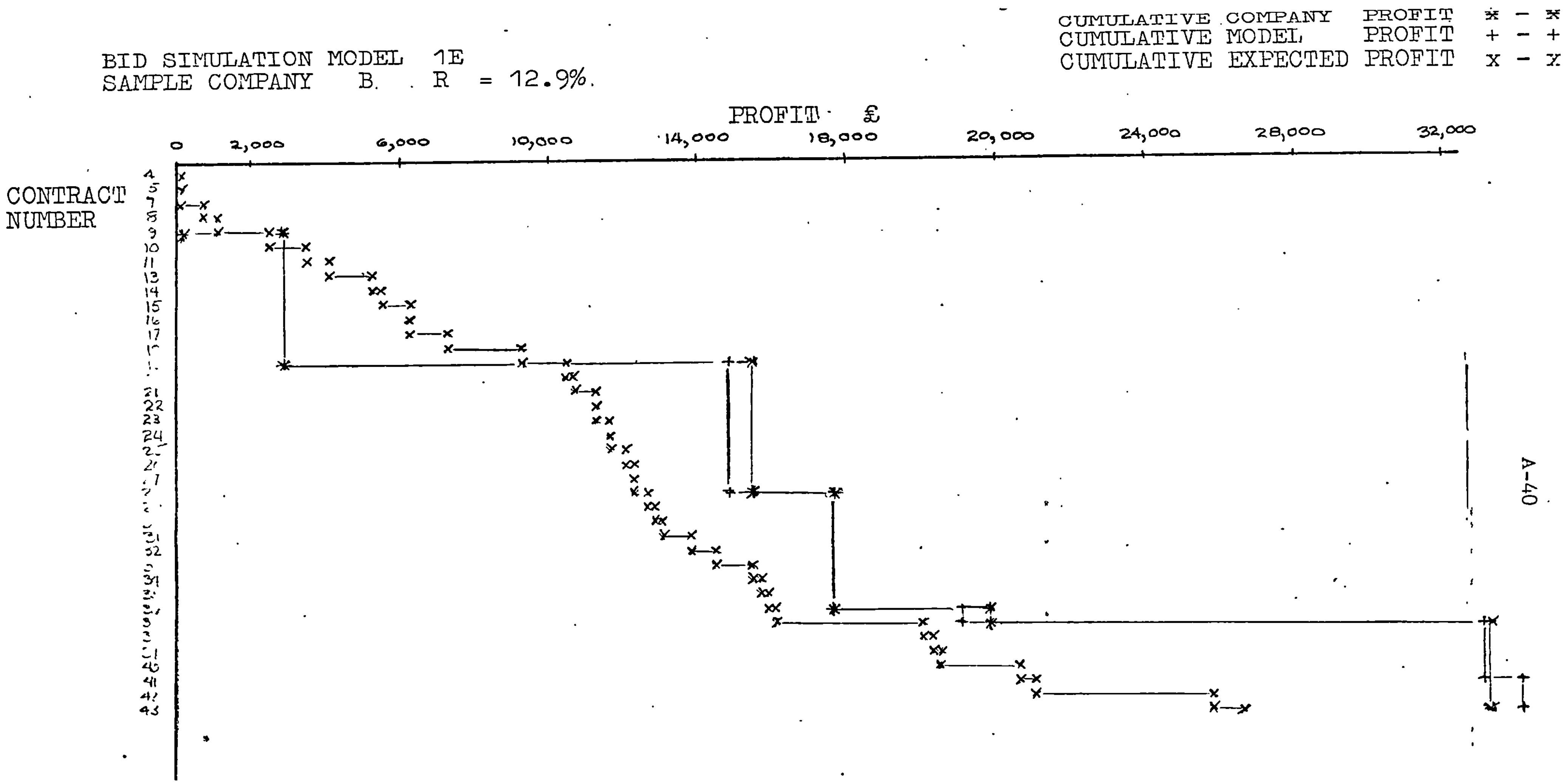
27432.0
30861.0

BID SIMULATION MODEL 1E
SAMPLE COMPANY B. R = 12.9%.

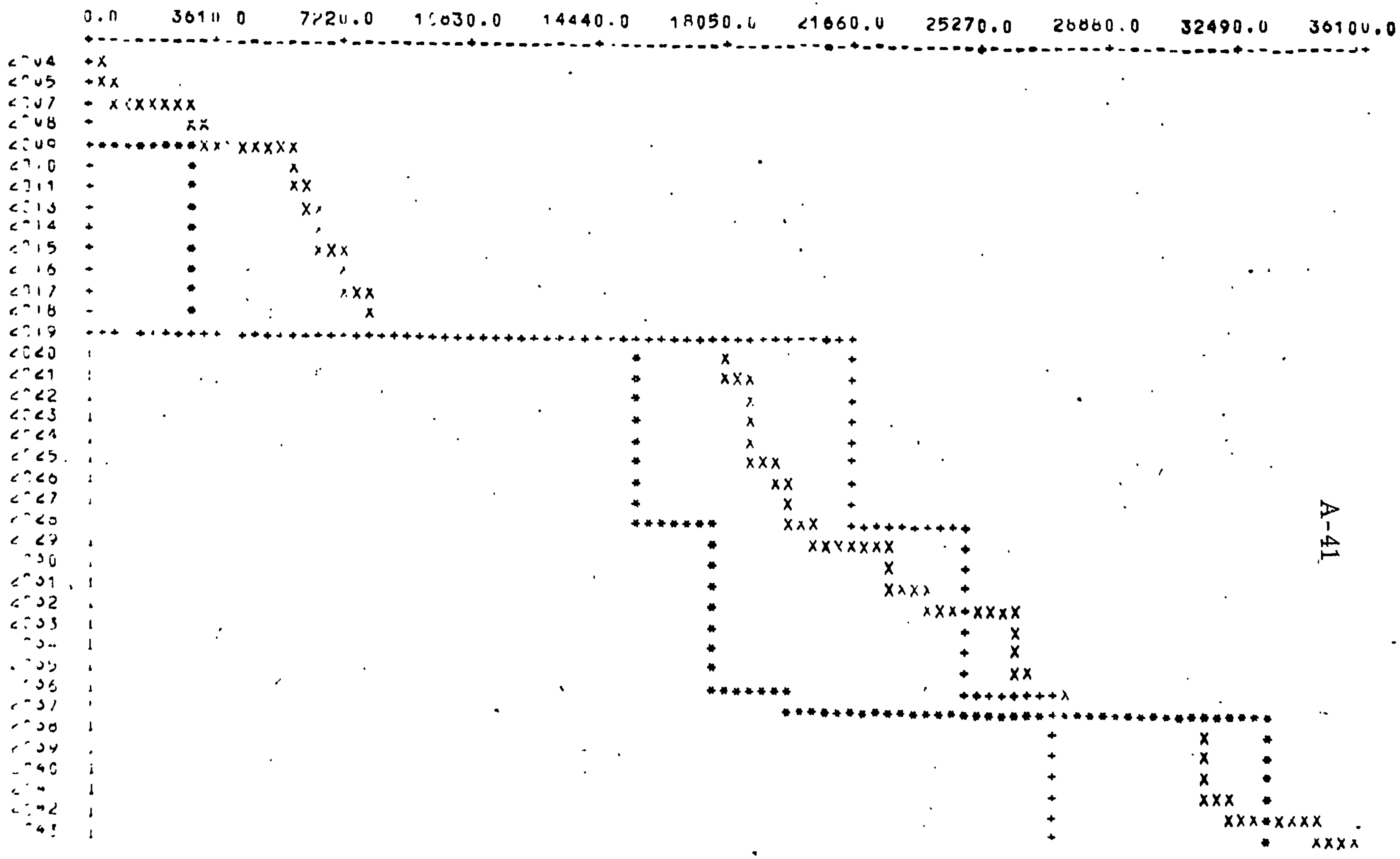


BID SIMULATION MODEL 2
SAMPLE COMPANY B R = 12.9 %
DATE 23/02/70 TIME 11.06.02

CUMULATIVE COMPANY PROFIT = **
CUMULATIVE MODEL PROFIT = ++
CUMULATIVE EXPECTED PROFIT = XX

CO. FACT NUMBER

PROFIT (£)



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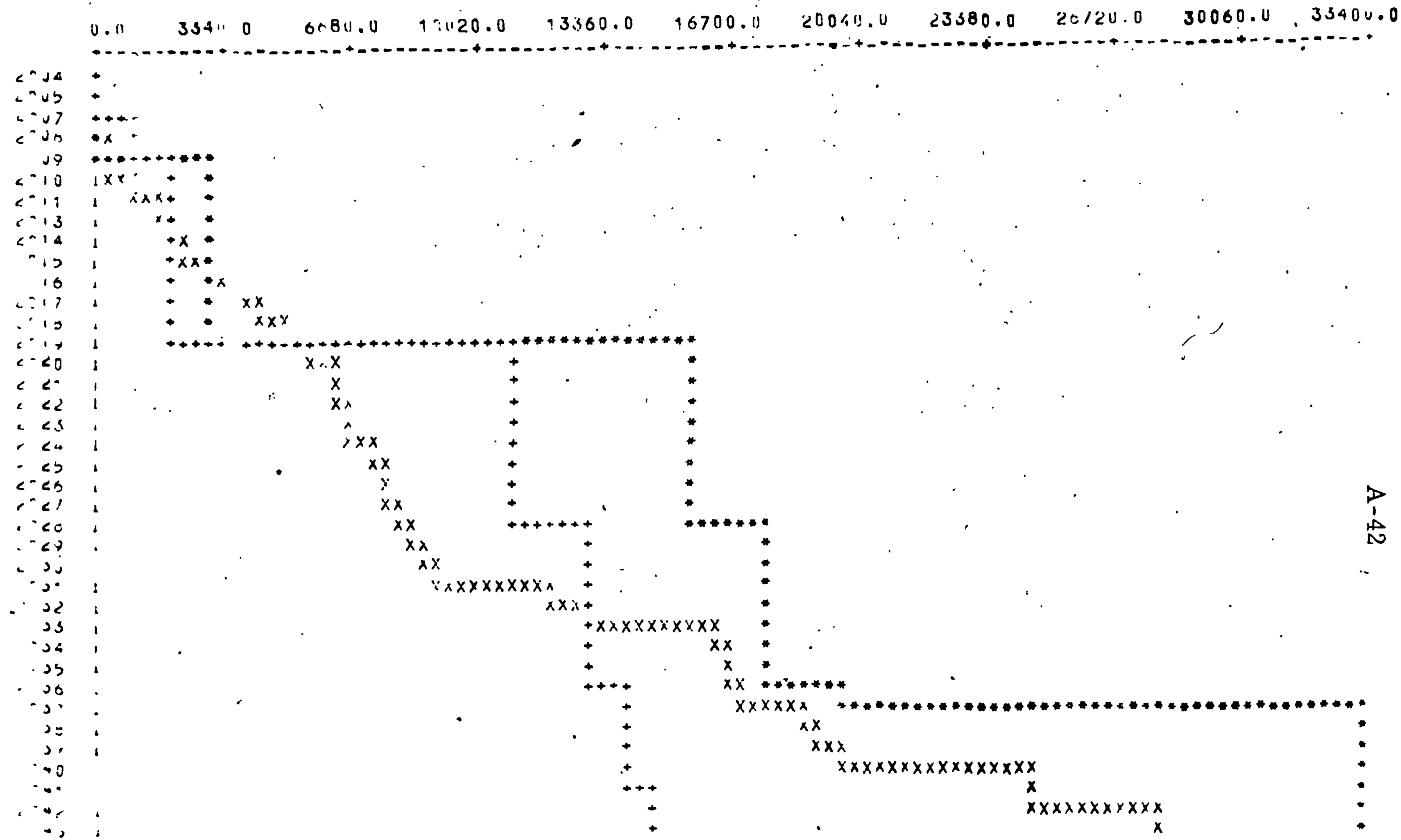
CUMULATIVE COMPANY PROFIT = **
CUMULATIVE MODEL PROFIT = ++
CUMULATIVE EXPECTED PROFIT = XX

CONTRACT NUMBER

PROFIT (£)



DATE 23/02/?0 TIME 11.06.02

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PROFIT (£)

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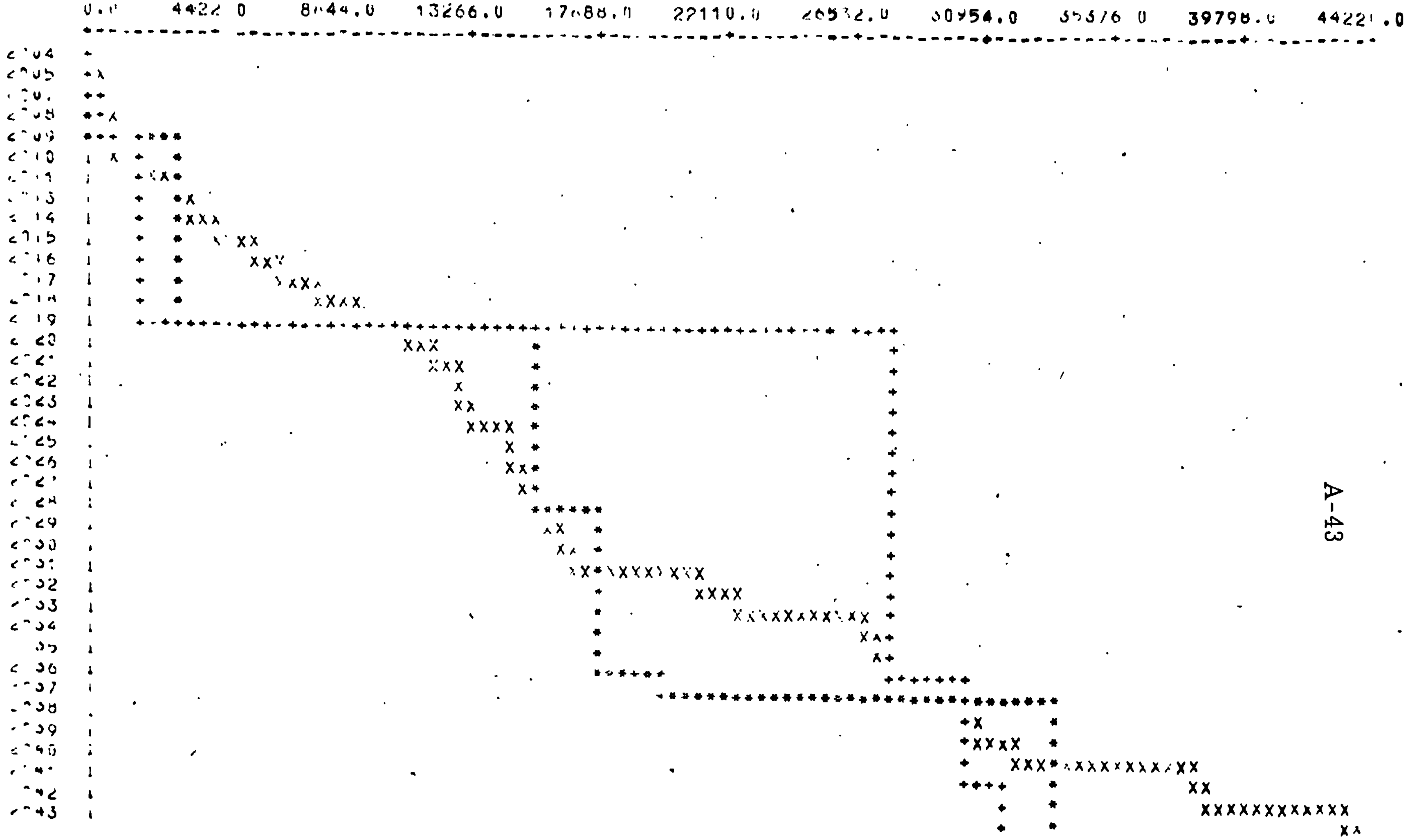
CUMULATIVE MODEL PROFIT = ++
CUMULATIVE EXPECTED PROFIT = XX



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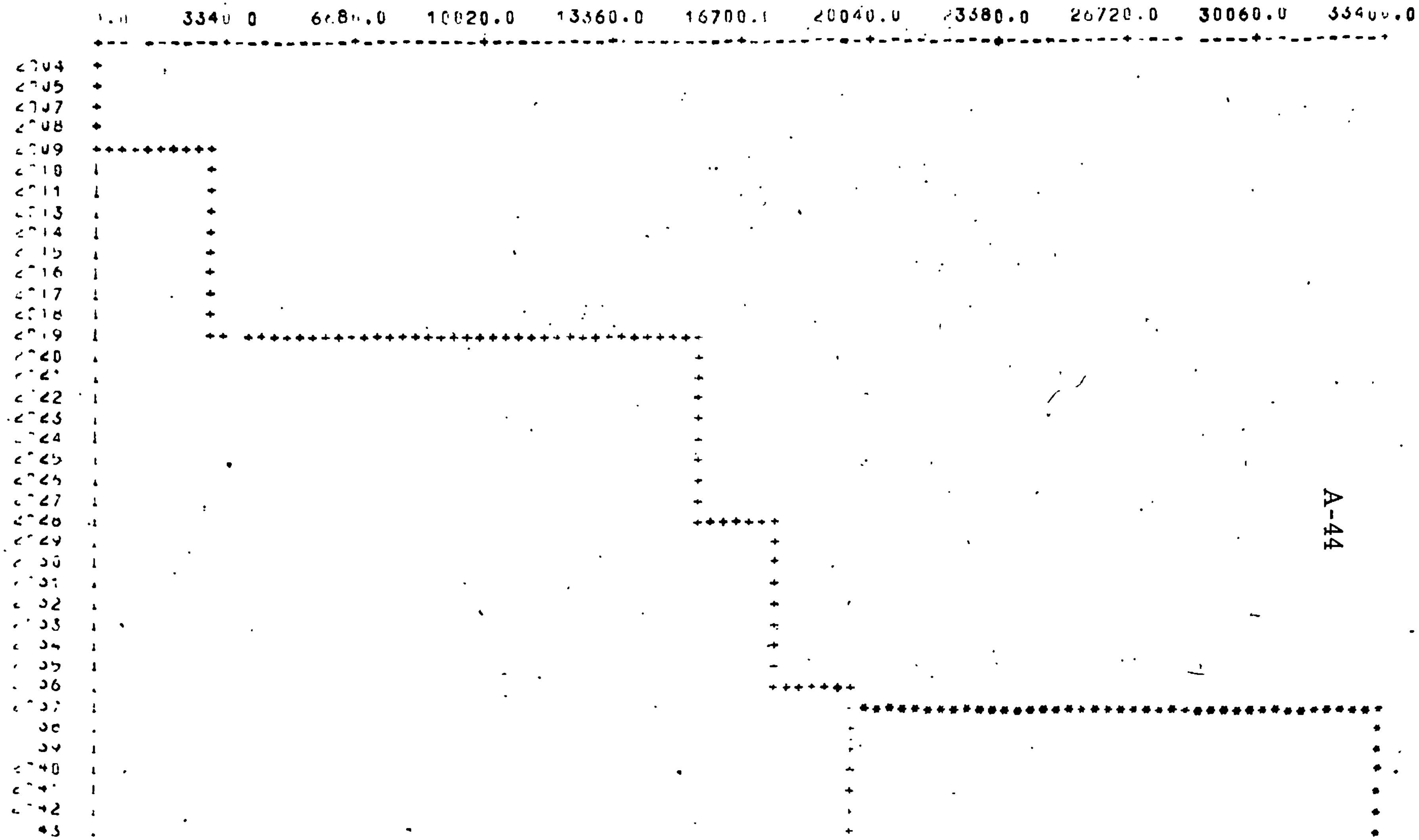
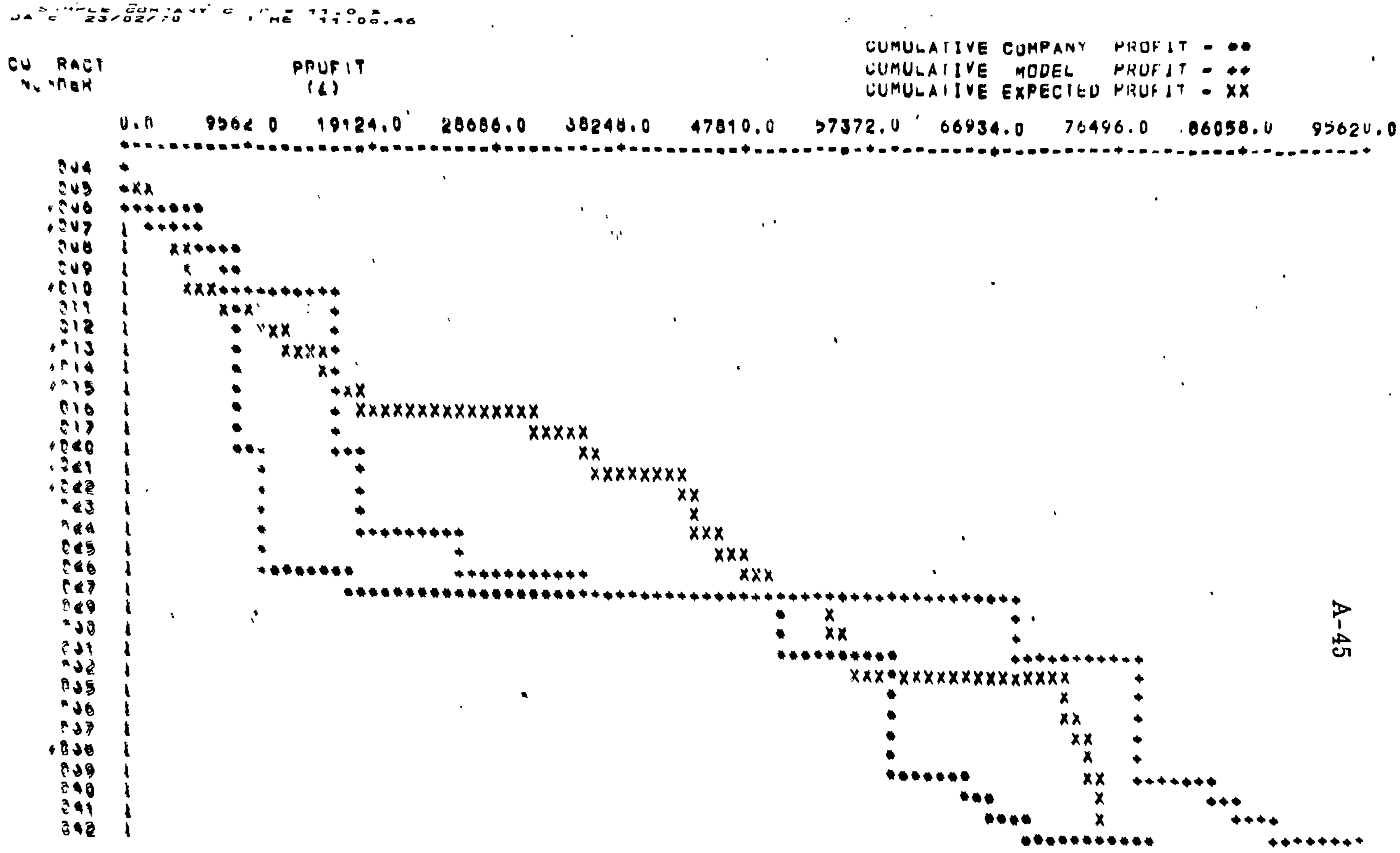
PROFIT (£)




23/02/70
11.00.46
CONTRACT NUMBER
PROFIT (£)
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CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX
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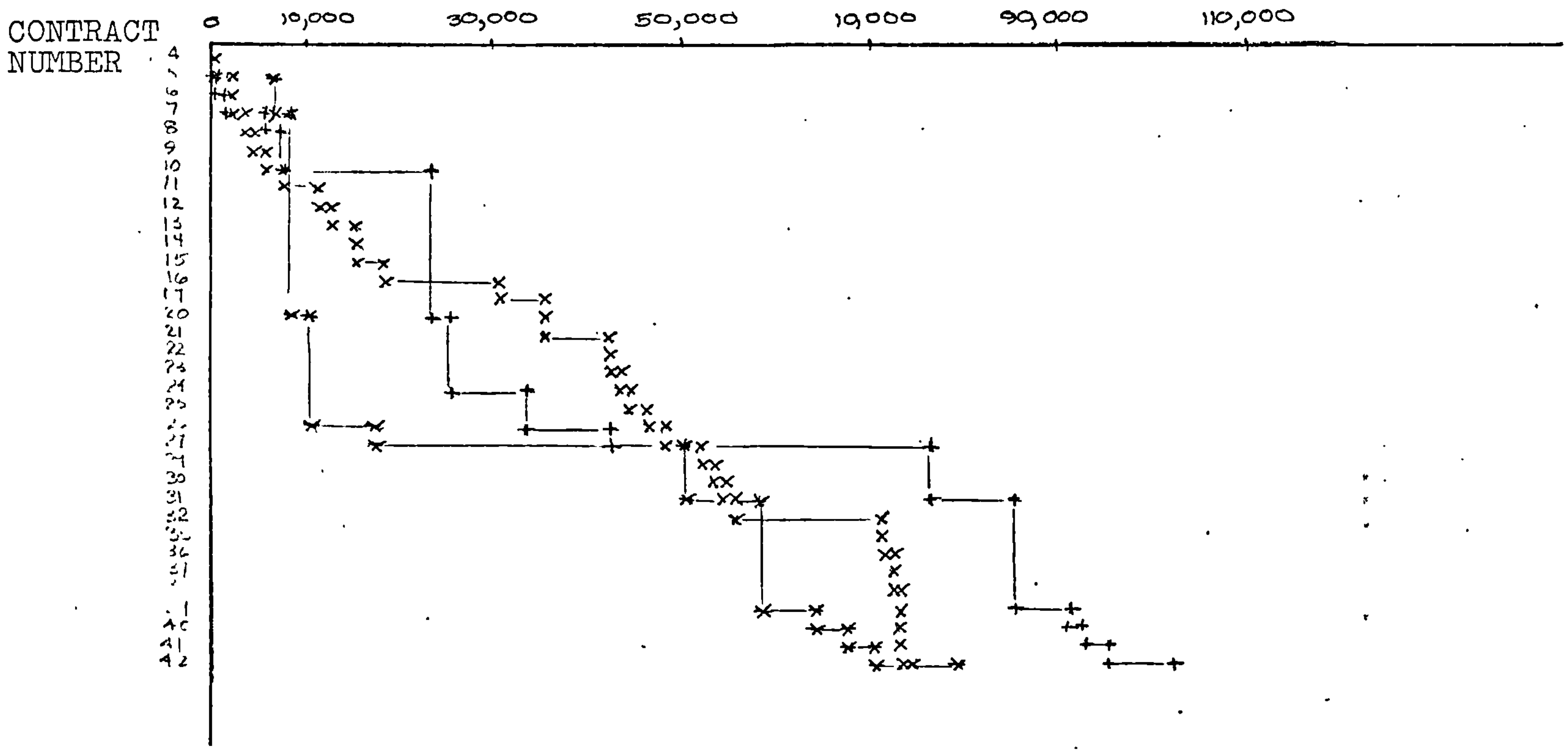
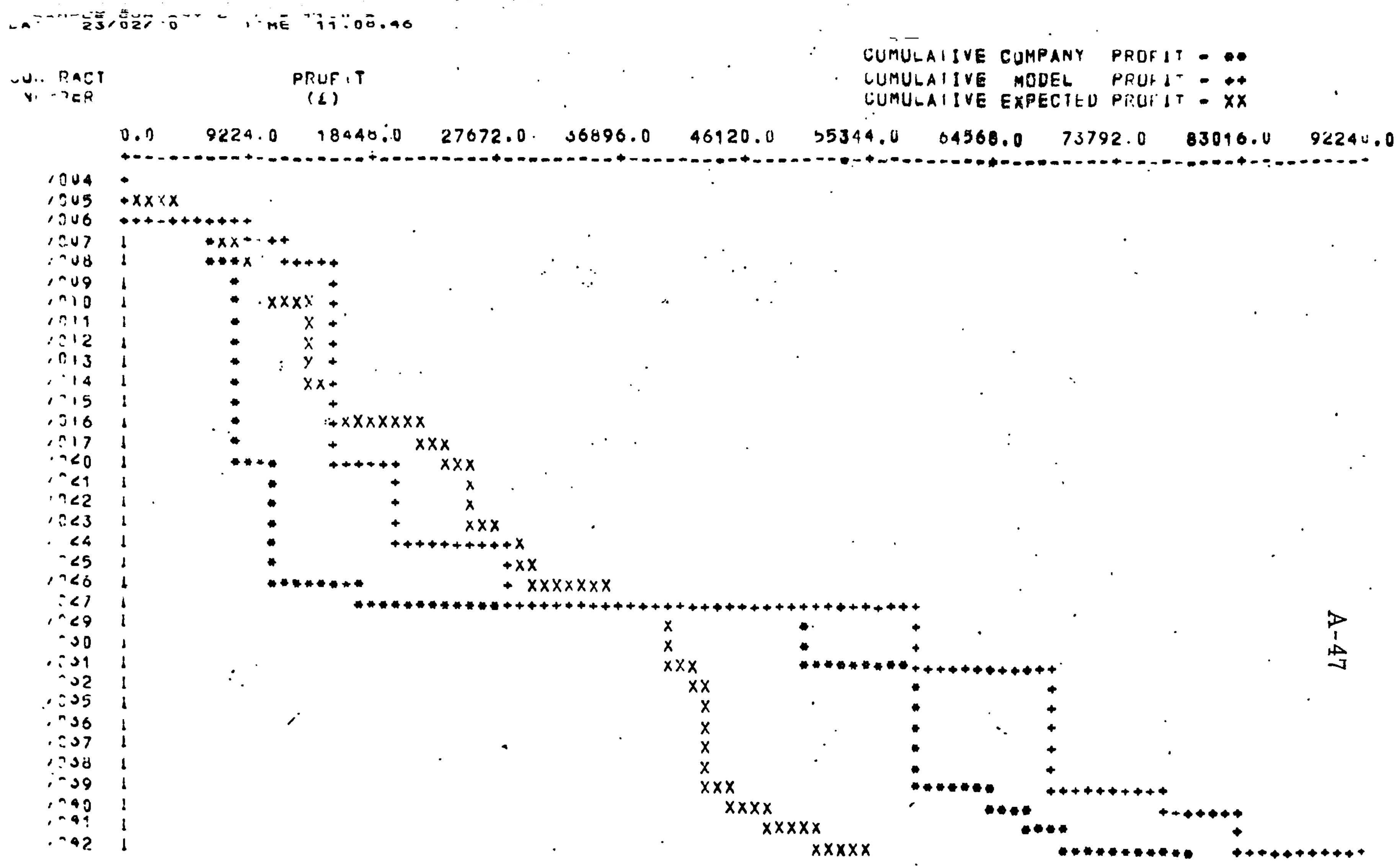
9562.0
19124.0
28686.0
38248.0
47810.0
57372.0
66934.0
76496.0
86058.0
95620.0

BID SIMULATION MODEL 1E
SAMPLE COMPANY C R = 9.35%

CUMULATIVE COMPANY PROFIT * - *
CUMULATIVE MODEL PROFIT + - +
CUMULATIVE EXPECTED PROFIT x - x





TIME 11.08.46
PROFIT
(£)
CUMULATIVE COMPANY PROFIT - **
CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX
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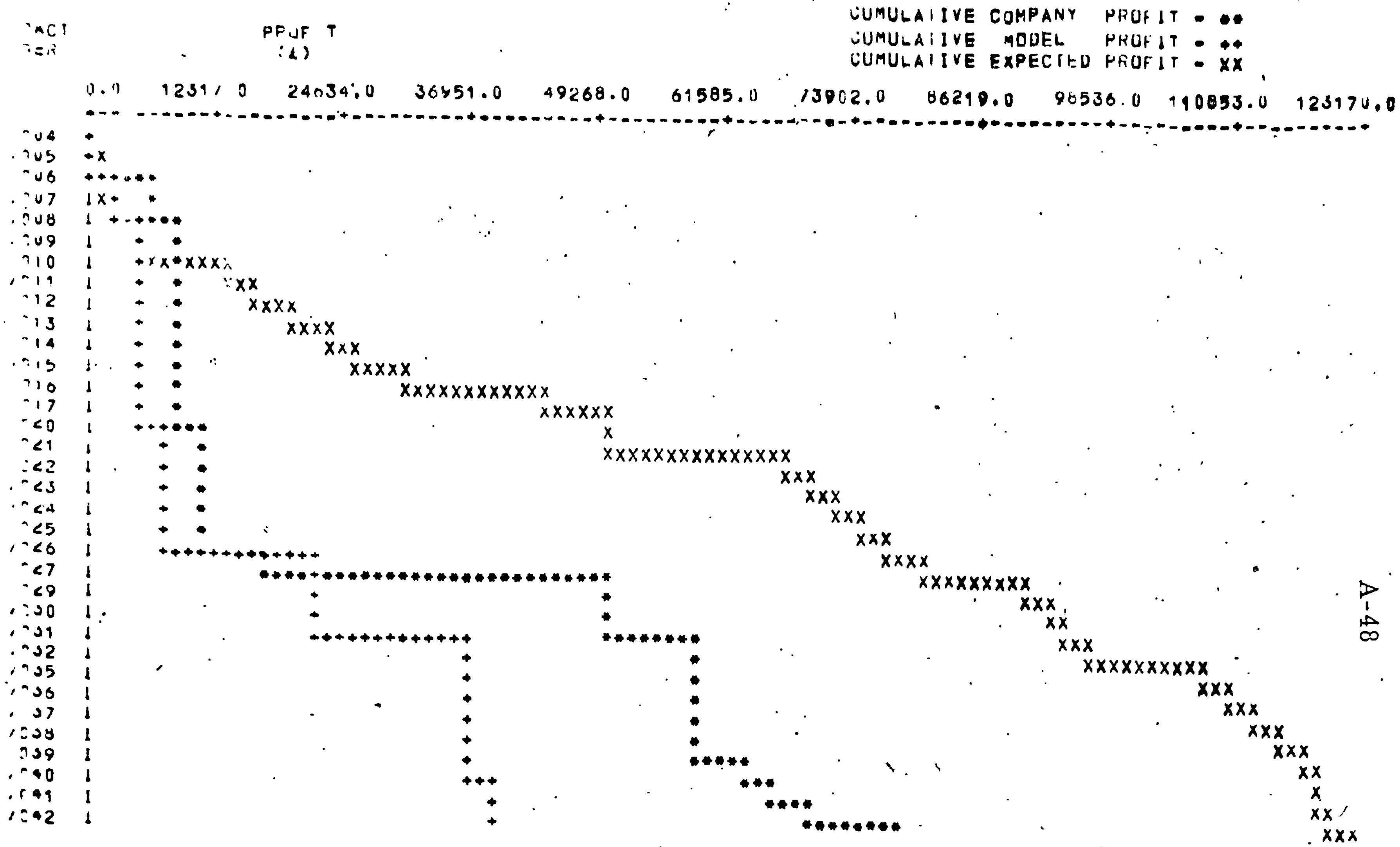
PROF T
(4)
CUMULATIVE COMPANY PROFIT = **
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CUMULATIVE EXPECTED PROFIT = XX
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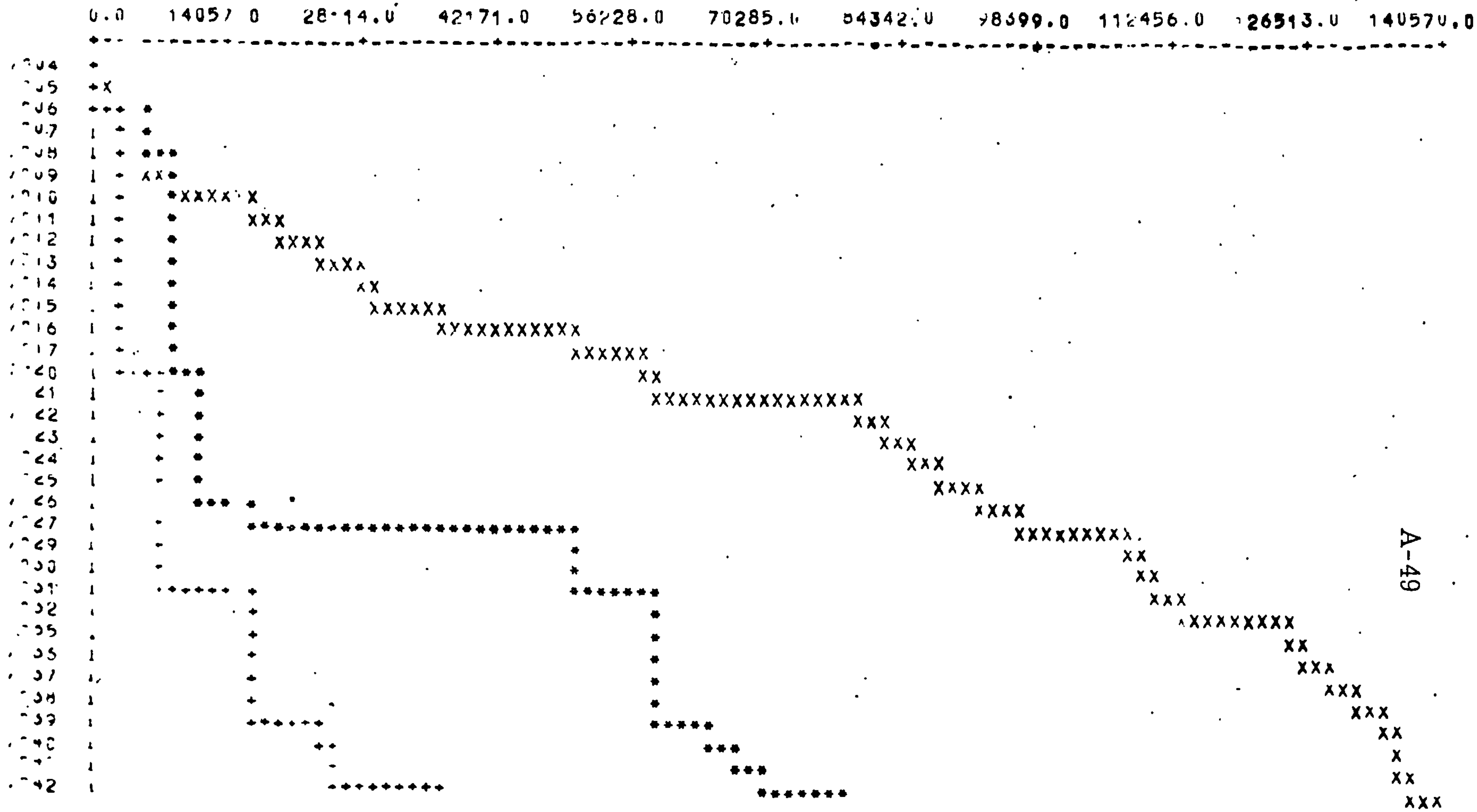
36951.0
49268.0
61585.0
73902.0
86219.0
98536.0
110853.0
123170.0

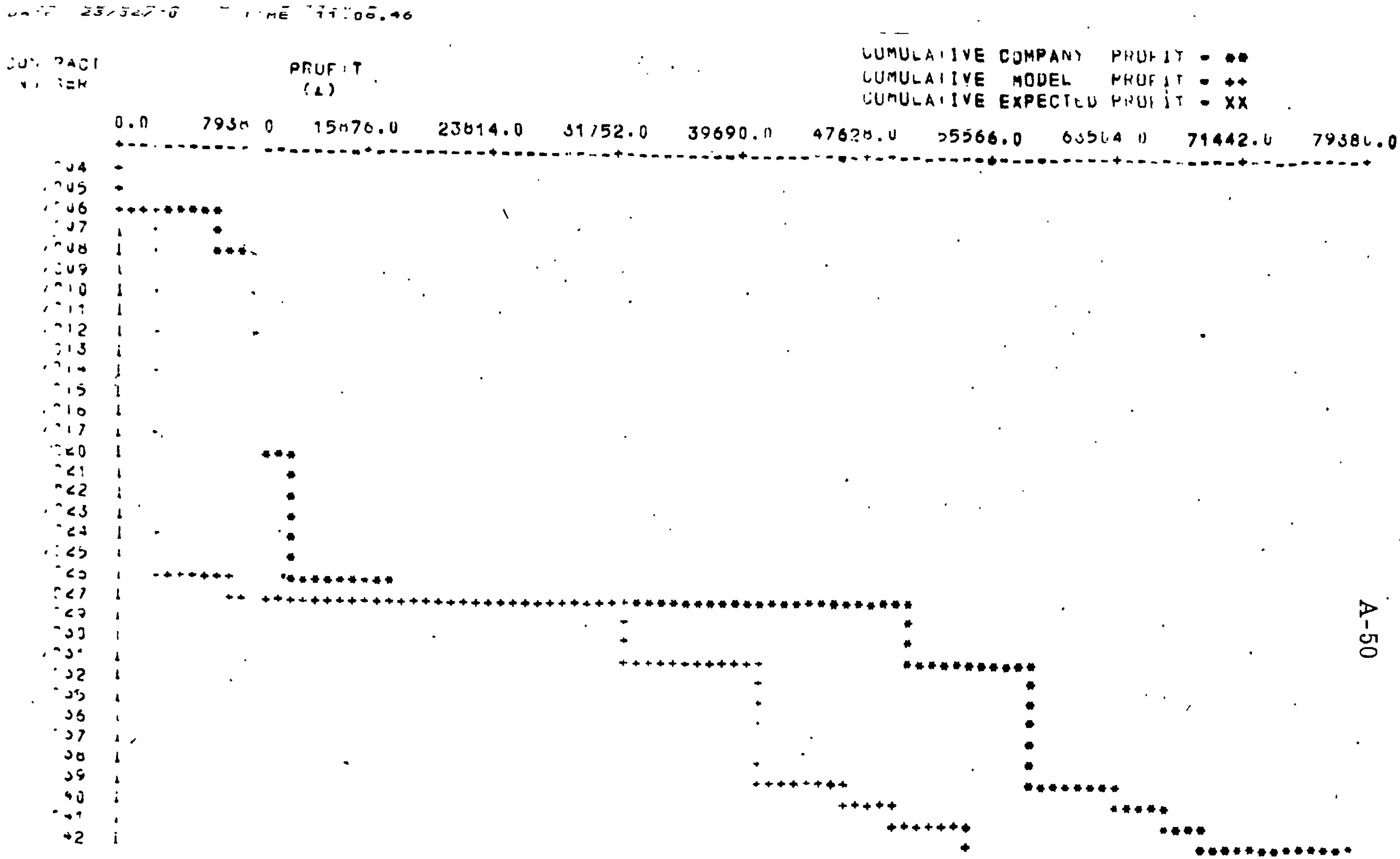
DATE 23/02/ 0 TIME 11.06.46

CUMULATIVE COMPANY PROFIT - **
CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX

CONTRACT NUMBER

PROFIT (£)




PROFIT
(£)
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CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX
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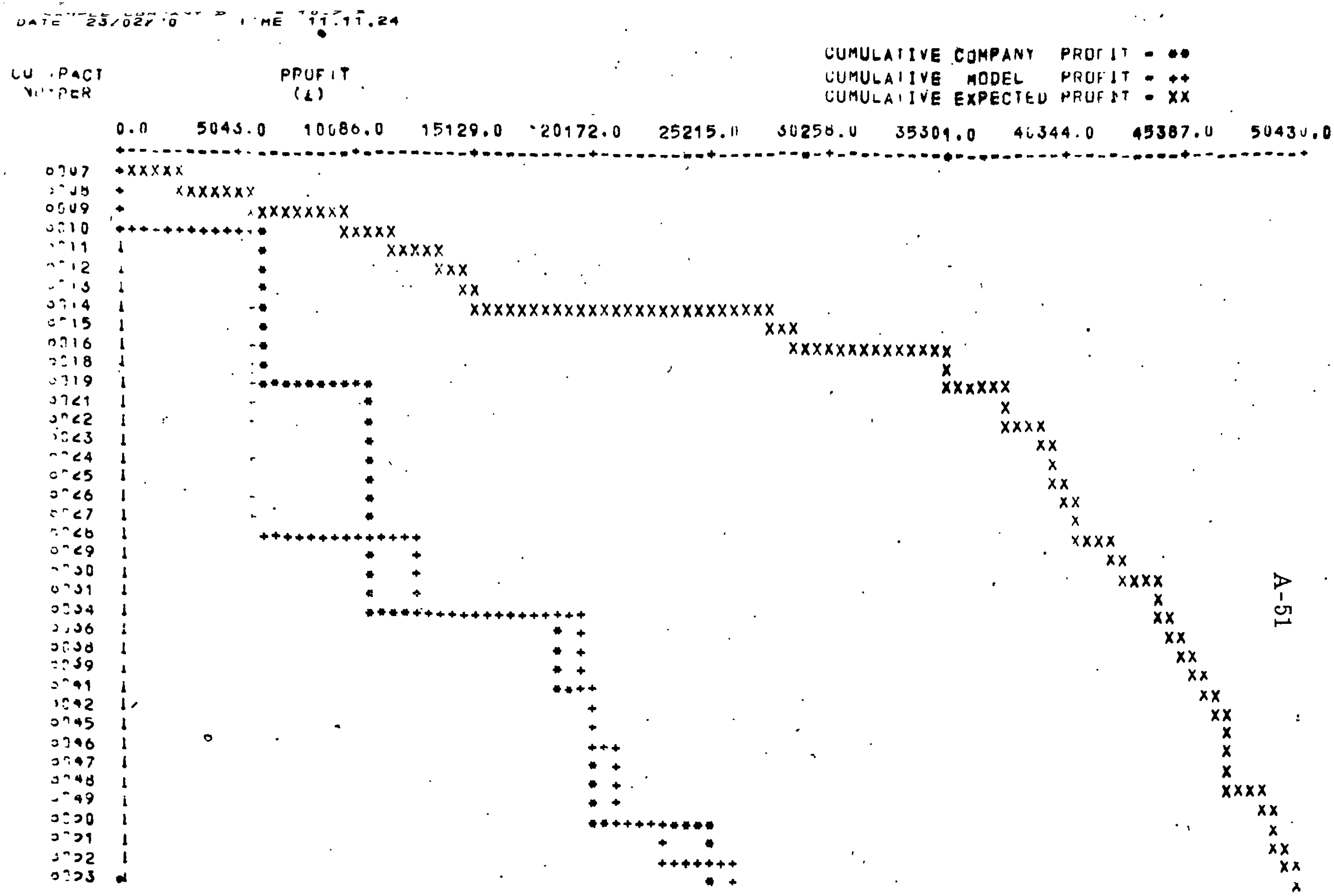
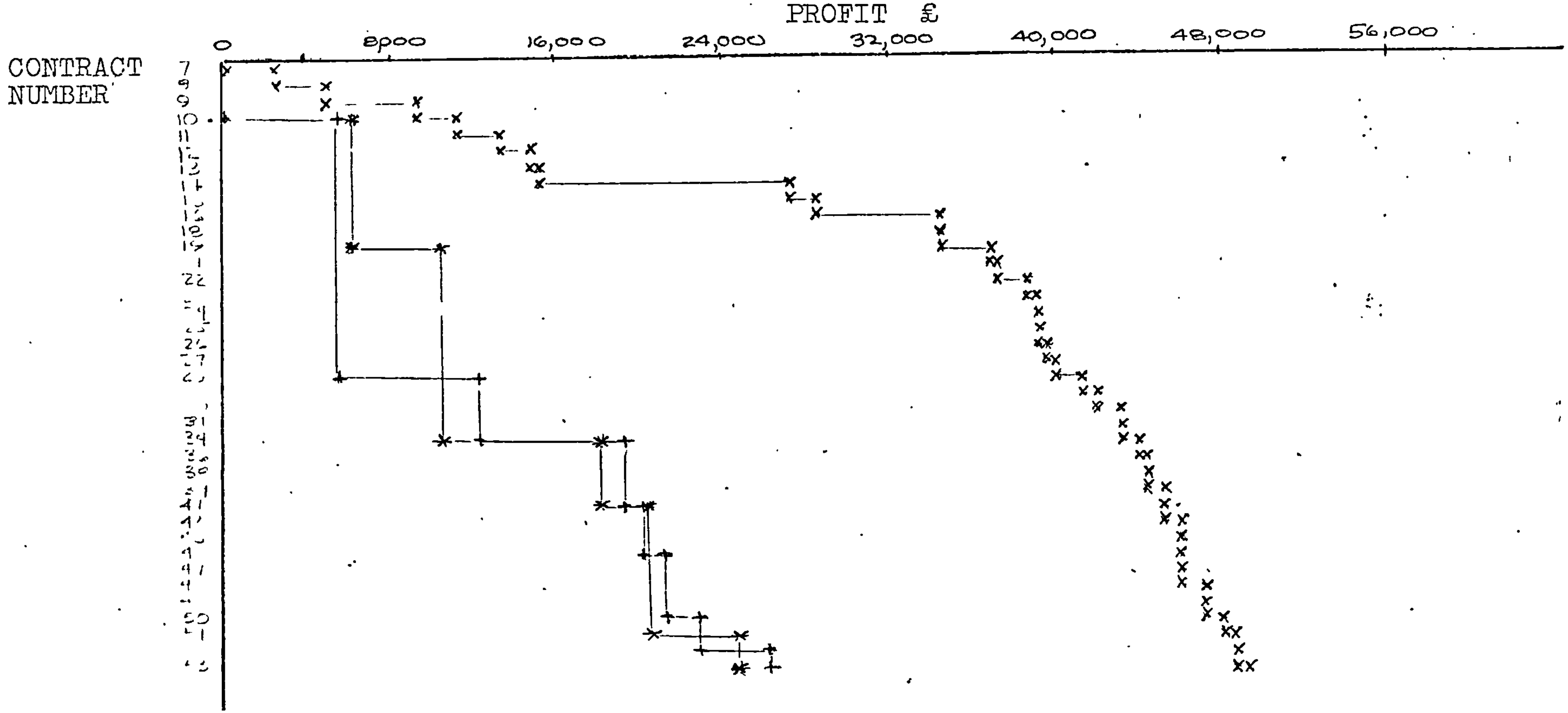
23814.0
31752.0
39690.0
55566.0
71442.0


DATE 23/02/70 TIME 11.11.24
CONTRACT NUMBER
PROFIT (£)
CUMULATIVE COMPANY PROFIT = **
CUMULATIVE MODEL PROFIT = ++
CUMULATIVE EXPECTED PROFIT = XX
0.0
5043.0
10086.0
15129.0
20172.0
25215.0
30258.0
35301.0
40344.0
45387.0
50430.0
0307
0309
0310
0316
0318
0319
0323
0329
0336
0338
0339
0345
0346
0347
0348
0349
0350
0353

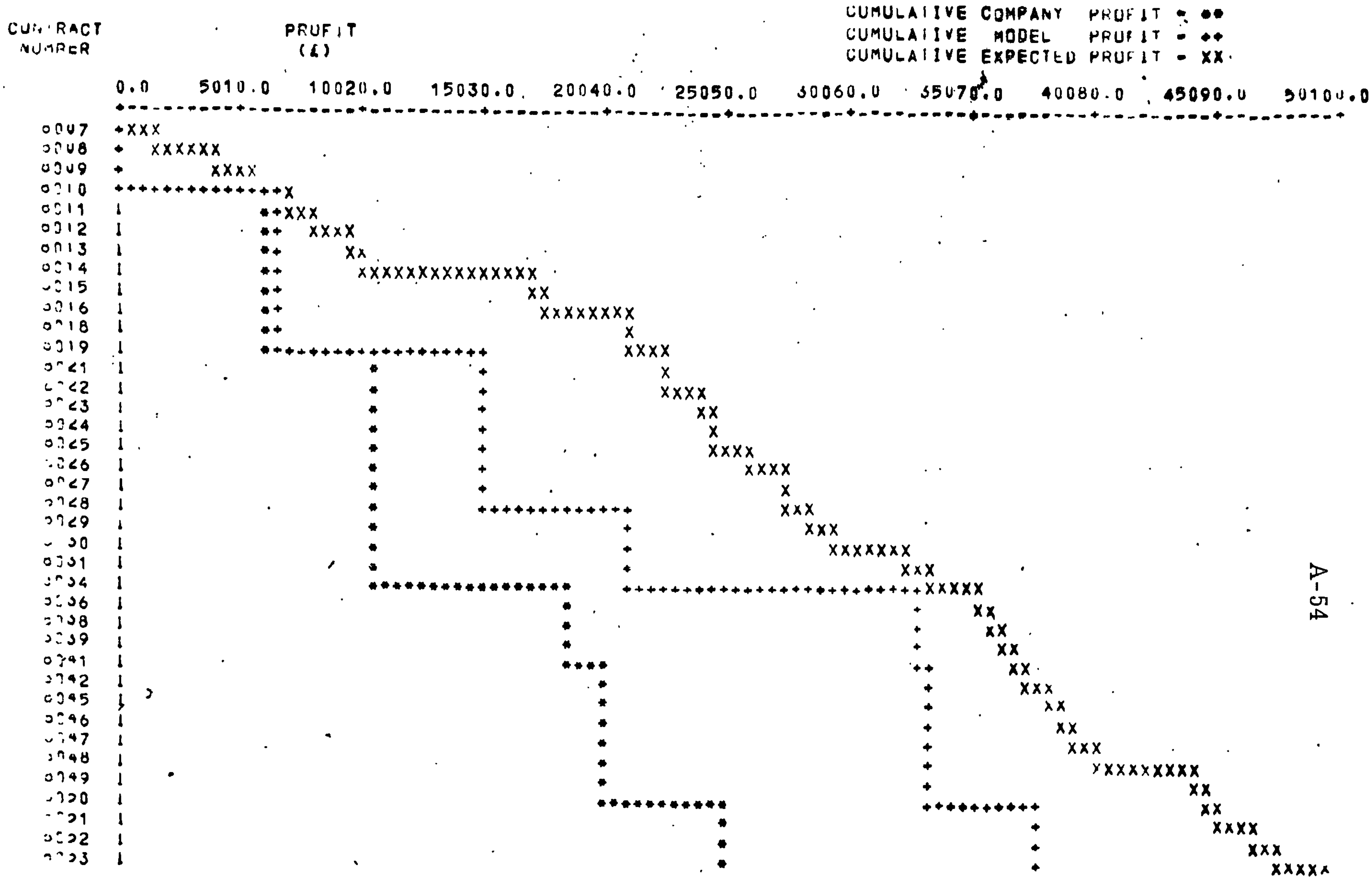
BID SIMULATION MODEL 1E
SAMPLE COMPANY D R = 10.7%

CUMULATIVE COMPANY PROFIT - ✱ - ✱
CUMULATIVE MODEL PROFIT - + - +
CUMULATIVE EXPECTED PROFIT - x - x



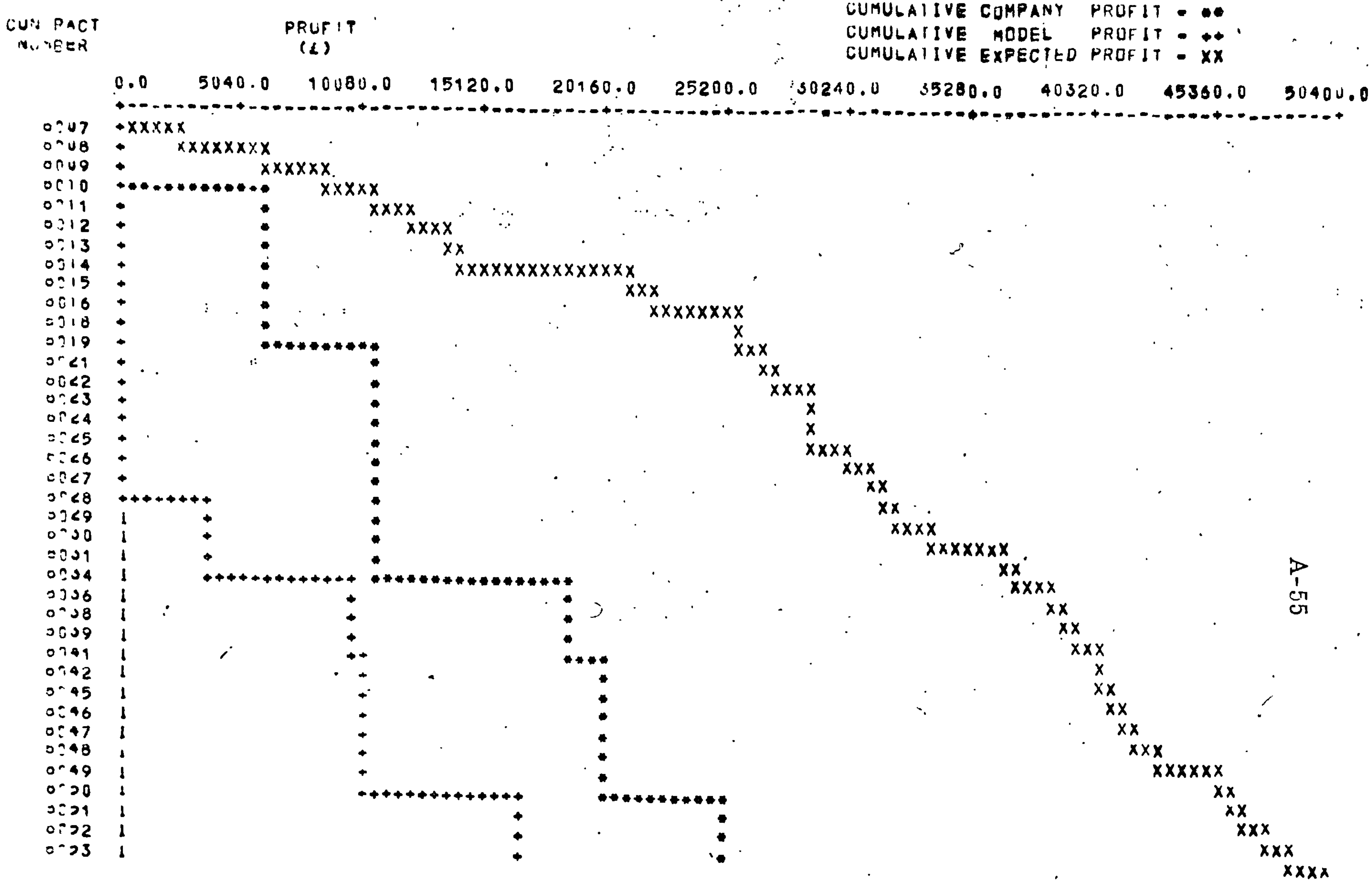

SAMPLE COMPANY
DATE 23/02/70 TIME 11.11.24
CUMULATIVE COMPANY PROFIT - **
CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX
CONTRACT NUMBER
PROFIT (£)
0.0
4449.0
8898.0
13347.0
17796.0
22245.0
26694.0
31143.0
35592.0
40041.0
44490.0
0007
0008
0009
0010
0011
0012
0013
0014
0015
0016
0018
0019
0021
0022
0023
0024
0025
0026
0027
0028
0029
0030
0031
0034
0036
0038
0039
0041
0042
0045
0046
0047
0048
0049
0050
0051
0052
0053

SAMPLE COMPANY
DATE 23/02/70 TIME 11.11.24



SAMPLE COMPANY D/ ? = 10.7 %
DATE 23/02/70 TIME 11.11.24

CUMULATIVE COMPANY PROFIT = ●●
CUMULATIVE MODEL PROFIT = ++
CUMULATIVE EXPECTED PROFIT = XX



SAMPLE COMPANY D = 10.7 %
DATE 23/02/ 0 TIME 11.11.24

CUMULATIVE COMPANY PROFIT - **
CUMULATIVE MODEL PROFIT - ++
CUMULATIVE EXPECTED PROFIT - XX