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SYSTEMS AND CONTROL PROBLEMS IN EARLY SYSTEMS DESIGN

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Επειδή ξέρω ότι με βλέπεις και χαμογελάς από εκεί ψηλά,

αυτό είναι για σένα.....

To the loving memory of my father

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DECLARATION

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ABSTRACT

This thesis is concerned with the evaluation of properties of early design models, the control structure selection and the representation and properties of composite system models. The work is motivated by the need to introduce a Systems and Control Theory based framework for early design stages of the overall system design of engineering processes, and in particular chemical processes, such as process synthesis and control structure selection. The overall spirit of the work is that engineering design is an evolutionary process, the different stages of which shape the structure of the resulting system models and precondition the potential for design at the next stage. The work identifies a number of key problems in the overall design, which are of a generic, systemic character, and then deals with working out solutions for such problems. The results contribute in the development of a framework for systems integration using as criteria and tools, those provided by Systems and Control Theory. The work aims to provide a control theoretic dimension to the rules and practices currently used in the specific application areas.

The thesis contributes in the development of a Systems and Control conceptual and tools framework for integrated design of engineering processes by providing results in the following areas:

- Specification of a number of generic problems in the field of integrated design and identification of relevant control theoretic concepts and tools.
- Study of Model Orientation for linear implicit state-space models and definition of classes of oriented realisations.
- Development of solutions to two problems of Structural Identification for uncertain early process models related to infinite zero structure and McMillan degree.
- Development of a generic representation of composite systems that allows the study of transition from the aggregate to composite system properties, as a generalised Control Design and characterisation of some key system properties.

- Specification of a framework for integrated Control Structure Selection and development of software for many approaches of the "interaction analysis" indicators.

The results contribute in the area of development of the systems and control ideas for the problems of systems integration and early design. The work emphasises the strong links between Modelling, Selection of Control Structures and Control Structural methodologies.

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NOTATION - ABBREVIATIONS

GPI:	Global Process Instrumentation
MOP:	Model Orientation Problems
MPP:	Model Projection Problems
MEP:	Model Expansion Problems
MCP:	Model Composition Problems
GCT:	Global Co-ordination Theory
EPD:	Early Process Design
CT:	Control Theory
CTD:	Control Theory and Design
SSM:	State-Space Models
ESSM:	Extended State-Space Models
D-MPP:	Dimensional Model Projection Problems
GS-MPP:	Graph Structural Model Projection Problems
IS-MPP:	Invariant Structural Model Projection Problems
PO-MPP:	Performance Optimisation Model Projection Problems
FDI:	Fault Detection and Isolation
PI:	Property Indicators
SI:	System Invariants
LIS:	Lyapunov Internally Stable
IS:	Internal Stability
BIBO:	Bounded Input Bounded Output
TS:	Totally Stable
MFD:	Matrix Fraction Description
WCP:	Well Conditioning Problem
OT:	Orientation Transformation
GS:	General Singular
I-ORP:	Input-Output Reduction Problem
CCR:	Canonical Composite Representation
LWCA:	Local Well Connectedness Assumption

LWSA:	Local Well Structured Assumption
GWFA:	Global Well Formedness Assumption
LWFP:	Local Well Formedness Problem
LWSP:	Local Well Structuring Problem
GWFP:	Global Well Formedness Problem
DMPP:	Decentralised Model Projection Problem
STFM:	Structured Transfer Function Matrices
IPM:	Internal Progenitor Model
EPM:	External Progenitor Models
GSTFM:	Generic Structured Transfer Function Matrices
RGA:	Relative Gain Array
D-RGA:	Dynamic Relative Gain Array
PRGA:	Performance Relative Gain Array
D-PRGA:	Dynamic Performance Relative Gain Array
SGM:	Scaled Gain Matrix
CLDG:	Closed Loop Disturbance Gain
MIC:	Morari Index of Controllability
SVD:	Singular Value Decomposition
SSV:	Structured Singular Value
CSTR:	Continuous Stirred Tank Reactor
cmi:	column minimal indices
rmi:	row minimal indices
e.d.	elementary divisors
f.e.d.	finite elementary divisors

Chapter 1

INTRODUCTION

Chapter 1

INTRODUCTION

Increasing demands for efficient operations and utilisation of energy and raw materials in chemical processes drive the need for more integrated processes. This increasing integration is expressed in terms of heat and material recycles and produces processes with greater coupling, which leads to behaviours determined by the properties of the resulting system rather than the individual dynamics of the subprocesses. This integration, also, has a significant impact on the dynamics, control and operation of process plants. The process of synthesis in the chemical process area is predominantly characterised by the chemical engineering practice, which involves conceptual design, optimisation, hazard and operability studies, process controllability etc. These activities are dominated by rules of the area, heuristics and some use of control indicators. The field can benefit by deploying system theoretic approaches, which in fields like network synthesis have benefited the design of electric networks and electronics. However, such techniques have not been deployed in any systematic way for chemical processes. One of the main objectives of this thesis is to develop the framework for composite structure evolution in the process synthesis area by developing a generic representation that will allow the systematic use of control theoretic tools, which can complement the current technology and practices.

Process control provides a way of satisfying the need for more efficient and reliable operation of processes. Traditional process control deals with the design of simple control schemes (single-loop design of PID controllers by providing rules for tuning) and multivariable schemes are designed, nowadays, using Model Predictive Control (MPC) methodologies. However, it is also providing useful tools in the earlier design stages and control structure selection by supplying heuristics and simple indicators for selecting control

structures. Furthermore, process control has been active by providing semi-heuristic approaches in the earlier stages of selecting input, output schemes and evaluating alternatives in process synthesis, in terms of operability and process controllability studies. Such areas are using some of the modern tools of linear and nonlinear control theory but they lack a coherent systematic framework for their development. An important contribution of the thesis is in the area of development of theory and tests, which may assist in the enrichment of such fields with a systemic framework and control theory based analysis.

Control structure design is one of the major areas of concern in this thesis and it is defined as the structural decisions involved in control system design, including the following tasks [Nish. et al, 1], [Skog. & Postl., 1]:

(i) Selection of control objectives
(ii) Selection of controlled variables
(iii) Selection of measured variables
(iv) Selection of manipulated variables
(v) Selection of a structure interconnecting the measured and manipulated variables
(vi) Selection of desirable dynamics for the defined structure schemes

Unfortunately, there is no systematic procedure available for translating the results from process design into the specifications needed for synthesising a control system. In practice, most problems are solved without any concise theoretical tools. Advances have been made in control theory and in the formulation of tools for analysing certain properties regarding the controllability of a plant. These tools can be used in screening and proposing possible control structures, but a systematic method for generating promising alternative structures has not been proposed. One major reason for this, is the fact that until recently most process plants were already designed and constructed before the control structure selection phase. This imposes serious limitations on the ability of the designer to explore all possible alternatives for the control structure. Physical aspects of the system, such as process units size and location, economic and environmental restrictions etc., affect the freedom of choice between alternative control schemes. Another major reason, is the difficulty in identifying and defining mathematically the control structure design problem, due to its size and its resulting complexity and the large cost involved in making a precise problem definition, including for example, a detailed steady-state and dynamic model of the plant. In practice, problems are resolved by facing each operation individually and not in a

plantwide manner. An alternative way, to avoid this is the use of suitable heuristic rules based on experience and understanding of the whole process.

The need for improvement of process synthesis and control structure selection becomes more urgent in the area of engineering processes. In fact, processes have been designed in the past based on a set of given criteria and requirements. Changes in demands and requirements imply that existing processes may not be suitable for the new demands. Certain improvements may be introduced by retuning or redesigning the controllers. However, this may not be adequate for many systems. Reengineering of such processes may require a more drastic intervention in earlier stages such as control structure selection (improved new structures) and possibly intervention in the structuring of the interconnection between subprocesses, which implies an intervention in process synthesis. The general framework for intervention in control structure selection and process synthesis can be also beneficial for this more restricted, and more difficult, problem of reengineering.

One of the major aims here is to examine the general issues arising in the selection of systems of actuation and measurement variables and then provide a system and control theory based framework for global instrumentation. Global instrumentation is seen as a model structure shaping, design stage as far as the characteristics of the final model are concerned [Karc., 8]. Taking into account that the structure of the model determines what can be achieved under compensation, global instrumentation is intimately linked to control design. The problems of control design and overall selection of input, output structures for a process have been considered within the area of control structure selection, rather extensively, in terms of examples, in the process control area. However, no systematic attempt has been made towards the development of a unifying framework for the selection of systems of actuation and measurement variables for processes, where the model shaping role of global instrumentation is the central feature. The general analysis of the effect of selection of inputs and outputs [Karc., 8] is further developed here by establishing a link between process synthesis, control structure selection and well structuring of early process models.

This thesis focuses on the problems regarding the development and properties of early process models. In order to move towards a more integrated view of the structure of the final model that can be used for design, problems need to be identified and addressed as early as possible. The definition of process variables and their subsequent classification into controlled, measured manipulated, disturbances etc. plays a very important role in the design of a control scheme. Failure to identify and classify them correctly can lead to very

difficult or even impossible requirements from the control structure. Progenitor models, which are models that correspond to all possible inputs and all possible outputs, are examined. The fact that they are derived using heuristics and physical arguments regarding the process may lead them to having very large dimensions and be ill-defined. This investigation takes a step forward by defining procedures that may lead to the derivation of effective models of the models, which are subsets of the original progenitor model that have favourable structural characteristics.

A vast array of indicators have been developed, in order to aid the selection of control structures. These indicators assist the designer to select control pairings between inputs and outputs that have favourable properties and dismiss alternatives that will create problems in the closed-loop plant. The indicators focus on different properties and aspects of the control structure and can tackle a steady-state or a dynamic version of the plant. The problem in the area of process control is that there is not an integrated framework, in which these indicators can provide conclusive results regarding alternative control structures. Furthermore, the indicators can provide conflicting results and, since there is not a weighting procedure giving precedence of some indicators over others, the selection can by no means, be completely straightforward. The general aspects of a framework for control structure selection are introduced here.

The main objective of the thesis is to contribute in the development of a system and control framework for integrated design. This is achieved by developing results in the following areas:

- Development of a systems based framework for the role of selection of inputs and outputs in the shaping of the model that is used for control design.
- Study of system problems in the development of early process models and, in particular, the study of the Model Orientation Problem (MOP) in a state-space set-up and the problem of well conditioning of large early dynamic models.
- Development of diagnostics for large-scale early process models linked to two structural identification problems; the McMillan degree and the infinite zero structure identification
- Development of a framework for representation of composite systems based on the notion of completeness and physical streams interconnection and the development of some results for describing properties of systems deviating from completeness.

- Reviewing and development of software for indicators linked to interaction analysis, as part of an integrated methodology for control structure selection.

The above areas of work define elements of a systems theoretic framework, which is seen as complementary to the current chemical engineering practices. A reviewing of the fundamentals of control theory results and concepts and of the key problems and concepts in chemical process synthesis and process control design, provide directions for the further development of this work and clearly suggest that the field requires more extensive consideration.

The structure of the thesis and the main results are described below:

Chapter 2 examines the area of chemical process design from early stages and issues of process control and aims to extract the generic problems where systems and control theory can intervene by developing the required framework and tools. As such, this is used as the main motivation of the problems considered in this thesis.

Chapter 3 is a first step in the development of the systems approach by considering the system issues in the selection of sets of inputs and outputs for large chemical processes and it is referred to as framework for Global Process Instrumentation. This term is used as a counterpart to the traditional instrumentation of physical variables, which deals with techniques for measuring physical variables (sensing) and acting upon physical variables (actuation). In this area, the model-shaping role of selection of input, output schemes is developed and clusters of control theoretic problems are developed. The work builds upon previous results [Karc., 8] and defines two of the problems subsequently studied in detail in the thesis. These are the Model Orientation Problem (MOP) and the effect of local input, output selection for the evolution of properties of composite processes.

Chapter 4 is a systematic review of the background of linear systems and control theory, which provides the basic concepts and tools for the problems of generalised design, identified in the previous chapter. The results are based on linear systems and deal with the system structure expressed in terms of invariants and a large set of property indicators currently used in traditional control design. The spirit of the work is to present the above as notions, which evolve (are assigned in the early stages of system design). As such, this chapter is the essential background required for structure and system properties evolution in the early design stages.

Chapter 5 deals with the problem of classifying implicit variables in early systems modelling by deciding which are suitable for control inputs or outputs based on control

theory criteria and the resulting structure of the model. The set-up for this study is matrix pencils and a state-space framework; however, these results may be also developed for more general classes, such as the autoregressive models.

Chapter 6 examines the problem of evaluation of suitability of early models, derived on the basis of heuristics and physical arguments, which may not have the required qualities for control design. This problem may be thought of as part of the process controllability studies and it has a clearly systemic character. The main issue addressed is the selection of effective subsets of physical inputs and physical outputs, such that the resulting model is well behaved as far as the required properties for control design are concerned.

Chapter 7 investigates alternative aspects of the study of early process models, which are characterised by dominant dynamics (coming from the simple subsystem models) and uncertainty about the values of other parameters. As part of the process controllability studies, it is important to be able to evaluate in an approximate but fast way, values of structural characteristics, such as the McMillan degree and the infinite zero structure. The study of these problems in large dimension early models with parameter uncertainty is referred to here as Structural Identification. The work extends previous results by introducing effective computational procedures and by unifying two different problems.

Chapter 8 focuses on the development of a systems approach for the representation of composite structures and the evolution of basic system properties of the structural type (zeros, indices etc.) and properties such as controllability, observability, stability etc. from the subsystem level to the composite system level. The work expands previous results on the notion of completeness [Karc., 10] by characterising the notion of fixed dynamics from the aggregate to the complete system, describing the implications of lack of completeness and by considering special cases of total loss of inputs or outputs at the subsystem level. The modelling of lack of completeness as decentralised input, output squaring down and the interconnection rule as output feedback establishes an important framework and tools for intervening with system theoretical tools in process synthesis.

Chapters 9 and **10** deal with control structure selection with traditional methodologies based on interaction analysis. The development of software and the specification of a general framework for control structure selection, provide the means for using such results and a number of typical studies, and have a design character.

Chapter 11 summarises the work and defines open issues and problems for future research.

Chapter 2

SYSTEMS AND CONTROL CONCEPTS IN CHEMICAL PROCESS SYNTHESIS AND PROCESS CONTROL DESIGN APPROACHES

Chapter 2

SYSTEMS AND CONTROL CONCEPTS IN CHEMICAL PROCESS SYNTHESIS AND PROCESS CONTROL DESIGN APPROACHES

2.1. INTRODUCTION

The current desire for greater flexibility, higher efficiency, cost reduction and shorter cycle times together with concern for the environment, quality and safety, demands an integrated approach encompassing all types of activity from high level strategy to plant operation. Business level strategies cannot be accepted as feasible unless their realisation on the different operational layers is first considered; similarly, operational strategies are not acceptable unless their implementability on a given system, process is evaluated. The increased requirements for efficient, safe and environmentally friendly operations process plants can be met provided that they have been considered already at the early stages of plant design. Furthermore, modern plants are composed of units with smaller size and medium capacities, but with extensive use of recycles and increased degree of energy integration, which make the plant operation more sensitive to disturbances, and possibly lead to inherently unstable behaviour. Therefore, to design plants that are safe, easy to operate and cost-efficient, a new approach is required that will transcend the traditional separation and sequencing among the activities of process design, such as chemical process engineering (or other process nature dependent discipline), process optimisation and economic appraisal, instrumentation engineering, and control analysis and design. Designing plants, which can perform well throughout their life cycle, is difficult. Issues of redesign of existing systems frequently arise when the original operational assumptions are not valid anymore. Integrating operations and design is a formidable scientific and

technological challenge. The close integration of business, operational and design issues has not been considered so far in any systematic way and this has been the source of difficulties in implementing effectively business level strategies on industrial processes. The setting up of operations and design activities are supported by databases and software systems, which however are usually dedicated to the particular activity. Integration of software systems and data structures is an important issue, which heavily depends on adopting common standards. However, such issues are not considered here.

The current practice of treating every design issue independently, without taking into account the existing interactions and relying on testing for the evaluation of alternatives, is time consuming, expensive and rarely leads to good results. The need for an integrated approach that breaks the traditional boundaries between technical and managerial disciplines, between operational and design issues, as well as between software and data supporting individual activities is becoming very strong. Global enterprises have to be able to respond to sudden changes in market demands and this implies that they have to be able to propagate high level decisions throughout the organisation down to the lowest level and in turn be able to perceive and react to changes at the lowest level. The responsiveness of the plant to such requirements implies that operational requirements have to be interpretable in design terms and these should be considered at the design stage; otherwise, the problem of plant redesign has to be considered, which by no means is a simple matter. The natural hierarchical organisation of operations and tasks defines a multimodel environment where understanding the role of interfaces becomes a critical issue. Hybrid systems are naturally linked to the problem of understanding behaviours based on multimodel interconnected processes, whereas global control and measurement issues arise due to the hierarchical form of organisation.

2.2. INTEGRATING CONTROL INTO PLANT DESIGN

Traditionally, process design and control system design have been separate activities and process control aspects have little or no influence on plant design. The design of the control system is undertaken after the process flowsheet has been synthesised and designed or even after major pieces of equipment have been ordered and placed. This does not allow us to have important information regarding the process, such as what units are in the plant and their sizes, the way they are interconnected, the range of operating conditions, possible disturbances, available measurements and manipulations and what problems may

arise during start-up or shutdown of the plant. The above information is necessary for the design of an effective control scheme. On the other hand, it poses serious limitations, since the plant design determines, to a considerable degree, the process dynamics of the plant as well as the operability of it. So working with a largely specified system, the task to design a control scheme that ensures satisfactory operation of the plant is not easy. This task may not be always possible because the process may not possess a sufficient number of manipulated variables, strong interactions may exist amongst the processing units and it may not be possible to cope with all external disturbances. Also, the time lags may be significant or the process gains too low or to high or the process may be inherently unstable and in extreme situations, the plant may be uncontrollable even though the process design appears satisfactory from a steady-state point of view. The above observations demonstrate that the problems emerging have to be considered in the context of Complex Systems.

The term Complex Systems, is a generic term used to describe some of the major challenges in Science and its applications, Engineering, Business, Society, Environment, etc. This term refers to problems which may be of large or small scale, centralised or distributed, have a composite nature (in terms of simpler subproblems), high degree of interaction between subsystems, manifest a multi-facet behaviour (in terms of particular aspects), have possibly an internal organisation and require a multidisciplinary approach for their study. It is thus clear that complexity has many different dimensions and gaining understanding for each of these dimensions is critical in developing approaches for complex systems. The nature of complexity implies that there is need for division of the overall problem into subproblems, which may be more easily handled by teams of specialists. Such solutions are usually worked out by teams of experts with little knowledge on the issues of the other areas; furthermore, there is no global co-ordination and understanding of the interactions of the alternative aspects of complexity and this makes the development of acceptable global solutions a major challenge. Systems Integration emerges as the general task that can co-ordinate the activities in the particular subproblem areas to produce solutions, which are meaningful and optimal (in some sense) for the whole. The development of a systemic approach for such complex problems is the essence of integration. This requires ability to specialise the set of global objectives to the level of the subsystem, it needs methods to work out solutions which are locally and globally feasible and in a sense optimal, as well as understanding of interactions between the subsystems and alternative aspects of the overall problem. Systems integration is a multi-task, multidisciplinary problem that embodies major technological challenges.

The problem of system integration in process systems is examined here from the viewpoint considered in [Karc., 5]; this problem, however, is perceived by different people in different areas from entirely different viewpoints. The current dominant trend is to treat the problem as a software problem and neglect the multidisciplinary nature of the task and the very many different technological aspects of the problem, apart from those of software and data. The practical significance of integration has created some urgency in working out solutions to difficult problems and this has led to the development of interdisciplinary teams empowered with the task to create such solutions. Bringing together people from different areas is clearly necessary, but not sufficient in producing solutions with acceptable performance. The key issue here is the lack of methodology that bridges disciplines and provides a framework for studying problems in the interface of particular tasks. Recent developments in the area of hybrid systems, new developments in the area of organisation and overall architectures contribute in the emergence of elements of such a methodology. There are, however, many more aspects in the effort to develop a framework of integration, which are currently missing. This chapter deals with the examination of the basic technology areas and the approaches developed within them for handling the difficult problems of integrated design. As such, it provides the basis for identifying the systems type problems, which are essential for development of a holistic approach. The latter are addressed in the following chapter.

2.3. EARLY-LATE DESIGN AND THE MODEL ENVIRONMENT

Early design begins when a new possible activity is discovered or invented, usually a business opportunity, sometimes a social necessity (i.e. waste disposal). The significance of the formulation of an early model of a process with inherent "good potential" for the final control design, will be outlined in this section. Failure to do so, will result in an ill-defined model, which has inherent bad control properties, thus making the development of a control structure and control design very difficult or even impossible.

The design of a process in an early stage involves as a fundamental stage the problem of conceptual modelling, which transforms requirements and objectives to sets of preliminary designs referred to here as conceptual process flowsheets. The procedure of forming such a model is illustrated in Figure (2.1):

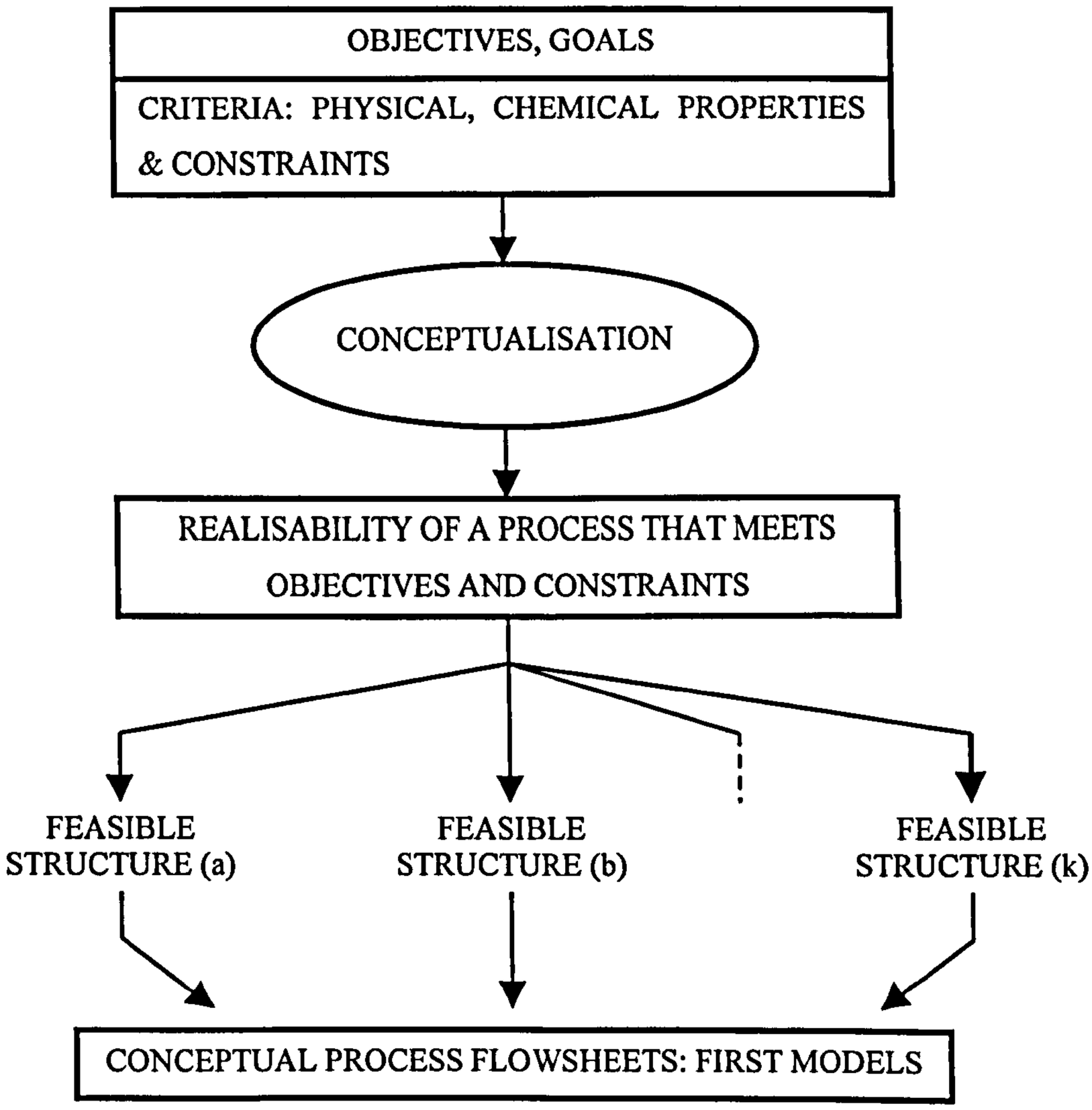

Figure (2.1): Summary of Development of Conceptual Process Flowsheet

The development of such models, in practice, is done generally by experienced engineers and leads to a family of conceptual process flowsheets, which are the first models available and are denoted by M_i^c. The overall set of such models is denoted by $M \overset{\Delta}{=} \{M_i^c, i = 1,2,....,k\}$. The most important issue in this generation of process flowsheets is the identification of generic aspects, which may have some impact on other application domains, and simple ways of formulating such a conceptual family of models. Some research has been made in this area [Steph., 1], [Doug., 1] and this conceptual viewpoint will be examined closely later in this chapter.

Another very important issue is to characterise mathematically the structure of the resulting conceptual process flowsheet models M_i^c, in terms of the general interconnection rules and the associated graph, as well as the early description of subprocesses in terms of simple models. The resulting graph will contain the fundamental variables linked to the physical interconnection. As design progresses the dimensionality of physical

interconnection streams may change, as the model becomes more detailed. In this section, we focus on the models derived in the early stages of design and their properties.

In the stages of early design, one can derive simple models of the process. As process and control design advances to later stages the process models become more complex and more precise. A mathematical model [Rijn., 2] corresponds to a part, or the whole system and it is linked to a goal-oriented process. The mathematical terms can be categorised as:

- **Structure:** The choice of variables and a notion of the existence of dependencies between them, as they may be specified by a digraph diagram or a Boolean matrix.
- **Functions:** The mathematical specification of the functional form of each dependency.
- **Parameter Values:** The values of all parameters in the functional forms are given.

An ill-defined model is a description of the system that may be regarded as a precursor of, or a substitute for, a mathematical model. A model presented as being mathematical may be an ill-defined one because of lack of completeness or precision of the specification. When using such models it is not possible to find accurate values for the parameters during the stages of early design and some of the assumptions lead to an oversimplification of the model. As design progresses, the model is the subject of continuous evolution. During this progression, choices have to be made constantly, for example on different types of equipment, operation modes etc. For this purpose, the most likely alternatives have to be elaborated in sufficient detail to enable a decision to be made. Quite often the requirements specification is not sufficiently detailed to allow making a rational choice, in which case the requirements specification is provided with more detail, in order to help the decision-making process. It is very common, in chemical process practice, to make this decision in an informal way, primarily based on heuristics and industrial common practice. This decision making process is not restricted to the early stages of design, but applies to the whole design procedure.

The alternative reasoning of waiting until the final stages of process design when more precise knowledge is available for finding the optimum operating conditions and then defining the control scheme is not advantageous at all. Common practice, as mentioned before, shows a trend to choose control structures and instrumentation systems in a heuristic and qualitative way, mainly supported by an intuitive feel for process behaviour. There are

several reasons for an integrated approach of the process design and the control design. Firstly, the inherent control structural properties need to be considered thoroughly in the early stages of design. In these early stages there are more alternatives for structuring the process and process operability and controllability need to be carefully evaluated. If for example, many recycles are introduced then process operation is more susceptible to disturbances entering the process and may possess inherent instabilities. It is obvious, that at the last stages, when a very complicated model has been structured, intuition and common sense cannot be enough to drive the decision-making. Another important reason is that at the early stages, it is easier to identify plant section that can pose difficulties in the control and therefore need more attention than other parts of the plant that can be manipulated in a more straightforward way. Finally, the increasingly stricter safety and environmental regulations require a thorough analysis in the area of Hazard and Operability (HAZOP) studies. It is, therefore, easier to improve the quality of these studies by utilising a more integrated and systematic approach from an early stage.

2.4. SYSTEM AND CONTROL DESIGN PROBLEMS IN THE INTEGRATED DESIGN OF CHEMICAL PROCESSES

2.4.1. General Issues

The leading trend for higher efficiency and performance in the operation of chemical process plants, while at the same time considering the need to comply with strict environmental restrictions, high quality requirements and improved safety operational conditions implies that the different aspects of process operations have to be considered in an integrated way. The increased requirements on performance of process plants necessitate their translation into the language of the different operational modes and finally into process design terms; in fact, strict requirements can be met provided that they have been considered at the early stages of process plant design. It is an important new trend that modern plants are composed of units with smaller size and medium capacities but with extensive use of recycles and increased degree of energy integration, which make the plant operation more sensitive to disturbances, and possibly lead to inherently unstable behaviour [Steph., 1], [Doug., 1], [Marlin, 1]. Consequently, a different approach needs to be proposed, in order to deal with processes that need to be safe, operated easily and cost efficient. This approach will take into account the traditional approach of separation and sequencing among the

activities of process design, process optimisation and economic appraisal, control analysis and design and overall process instrumentation. The need for integration of the design stages is considered together with the need to integrate the different aspects of global process operations. The need for flexibility in the mode of plan operations also generates additional requirements on the design of new plants, or the redesign of existing ones.

The construction of control systems for a complete continuous process needs to address and satisfy a large variety of issues, such as:

a) Regulation of the production and product quality.

b) Satisfaction of environmental regulations.

c) Provision of safe and desirable operations.

d) Achievement of optimum economic operation.

e) Reduction of utilities consumption.

f) Improvement of flexibility.

The diversity of high-level goals makes the process of designing control systems for complete plants a very cumbersome activity. Control Theory and Design is well developed for handling traditional control problems, but has not developed a concise and integrated methodology in order to handle the many issues of integrated process design and other operational issues. The operation of modern chemical plants is dominantly driven by the need for higher efficiency and performance. At the same time, restrictions such as environmental regulations, high quality requirements and safety operational conditions need to be satisfied. Considering that the general trend is to move to flexible plants that can operate in many modes, can satisfy a variety of products, demands and regulations, can move from one mode to another with the minimum effort and can be easily controlled at all modes, two key points can be made considering the design of these processes:

a) The different aspects of operation modes have to be considered in an integrated manner rather than the traditional where the issues of design are considered separately and little exchange of information between the different operational modes exists.

b) All the operational requirements, however strict they are, can be satisfied provided that they have been considered already at the early stages of plant design.

The need for high efficiency and the requirement to satisfy increasingly strict economic objectives have led to very complex modern designs. These, frequently contradict with older traditions on which a lot of knowledge exists. Integrating both operational and design issues has been recently recognised, as one of the most significant technological challenges. This need has raised a number of new problems for study and provides major challenges in areas such as modelling, system and control methodologies, software design, economic appraisal etc. In this section we examine the approaches followed in the early–late design of Chemical processes connected with the overall control design problem. The design of a control scheme for a Chemical plant is a topic that has received a lot of attention and it is well developed [Maciej., 1], [Marlin, 1], [Doug., 1], [Steph., 1], [Ng & Steph., 1]. Traditionally, design was based around the assumption that the system model is fixed, i.e. it has a given set of inputs and outputs. Another basic assumption is that the structure of the controller is being assumed that it is given, as a certain way of coupling inputs and outputs. Additionally, the order of controller dynamics is being assumed that it is fixed. Hence, traditional process control design is essentially a problem of tuning the parameters of the given structure and possibly specifying the dynamic complexity of the controller, to satisfy certain control design objectives. Seeing the problem of selection of inputs, outputs, as well as the structuring of the controller and the selection of its dynamic complexity as part of the overall control design, is what we refer to as Total Control Design and it is a topic that has been addressed within the area of Process Control. So far, there has been no systematic methodology for tackling all the issues involved.

Many industrial problems have been investigated in an effort to suggest a systematic method for synthesising control schemes for complete plants. The inability, generally, to address the problems in an integrated way can be attributed to some key elements of the chemical process plants:

i) Chemical processes are generally, highly non-linear and multiple couplings among variables can be made.

ii) The measurement and manipulation of process variables is limited to a relatively small number of variables.

iii) The control objectives may not be clearly stated (or even known) at the beginning of the control system design.

iv) Evaluation of the control system is based on a number of different objectives, such as:

- Safety
- Reliability
- Stability
- Range of control
- Ease of start-up and shutdown
- Cost of the control system
- Ease of operation of the system

v) The process structure may need changes in order to improve control.

vi) There may be considerable uncertainty in the prediction of process behaviour.

One of the important subjects of chemical process control is to develop a dynamic structure of measured and manipulated variables so that certain processing objectives are satisfied. Difficulties arise because, in certain cases, a variable will be both manipulated and controlled (e.g., ratio control of input streams). This implies that the various feasible sets of controlled, measured and manipulated variables and the interconnecting structure cannot be selected independently but should be considered simultaneously. Additionally, the optimal operating conditions change as a function of the external disturbances. [Maar. & Rijn., 1] have demonstrated that the optimum operation of a plant switches discontinuously from one process constraint to another. Experience from industrial problems suggests that such operational policy is quite common and economically sound. It is clear though, that switching the operation of a plant from one given set of constraints to another implies a change in the plant's regulation structure.

Within this context, a very important property that has to be taken into account is the ability of the system to move from one operating point to another, according to the economic feasibility of the move, in a smooth, safe and reliable way. This conflict can be resolved by systematically formulating the regulatory structure and simultaneously optimising the control structure. Additionally, if it is technically and economically feasible all the controlled variables will be measured. Otherwise, secondary measurements will be chosen, in conjunction with estimation techniques, to infer the value of the unmeasured control objectives. The estimator will be part of the structure interconnecting the measurements and the manipulated variables. The development of preliminary control structures, which are feasible from an engineering and system structure point of view and

based on simple models, takes place, followed by an evaluation where more detailed static or dynamic models are required. The complexity of the encountered physicochemical systems makes checks for interaction and effects of nonlinearity necessary.

2.4.2. Process Controllability

The design of a control system for a chemical plant is steered by the desire to maximise an objective, usually measured by terms of profit, generated by transforming raw materials into useful products, while satisfying product specifications, safety and operational constraints and environmental regulations. Each of these constraints needs special consideration. In order to satisfy the product specifications and the subsequent customer expectations, it is very important to meet very strict specifications regarding product quality and production rate. These facts have been the driving forces for the implementation of on-line, optimal control in the chemical industry. The process also has to be operated safely at all times, in order to protect the plant personnel and nearby communities. There are other constraints, which are often not associated directly with safety, that impose regulations in the operational use of the process equipment. Finally, there exist environmental constraints that require that the process should comply with air and water quality specifications and waste disposal.

In the design of a process control system, it is common to view the process in terms of its input and output variables. Process outputs are usually associated with streams leaving the process or with measurements inside a process vessel, a subset of which are selected as variables to be controlled, i.e. controlled variables. Process inputs are those independent variables that affect the process and its outputs. These can be divided in two subgroups: manipulated or controlled variables, and those controlled by the external environment, i.e. disturbance variables.

Generally, it is not always feasible to control all the output variables for three main reasons:

1. It may not be possible or economical to measure all of the outputs, especially compositions.
2. There may not be enough manipulated variables (degrees of freedom) to control all of the outputs.

3. Potential control loops may be impractical because of slow dynamics, low sensitivity to the available manipulated variables or interactions with other control loops.

In general, controlled variables are measured on-line and the measurements are used for feedback control. However, it is possible to control a process variable that is not measured by using a mathematical model of the process to calculate the value of the unmeasured controlled variable.

As mentioned before, the criteria used in the selection of process flowsheets are usually driven by economics, with little consideration to the inherent transient behaviour of the final process. Once the detailed process flowsheet has been selected, a plant control system is to be implemented so that the process can achieve some desired closed-loop characteristics. The achievable closed-loop performance of the control system is closely associated with controller configuration, controller algorithm used, as well as the process that has been designed. It is widely acknowledged, in practice, that an improperly designed process with inherent hard dynamic characteristics can lead to a difficult control problem.

The term controllability as used in process context, relates to the ability of the system to accomplish the dynamic transition between the operating states in an acceptable manner. [Zieg. & Nich., 1] first pointed out the relationship between process design and process controllability. There is a need to differentiate this overall notion of process controllability to the precise system theoretic notion of state controllability [Kalman, 1], output controllability [Chen, 1]. The quality of a closed-loop control system depends on:

- The control strategies applied. If the system is controlled by a fully multivariable controller or by some specific single-input, single-output pairings.
- The controller algorithm used. The level of sophistication of the controller, as well as its tuning parameters.
- The modelability of the process. Whether a model can be developed for control purposes.
- The process itself. The size and interconnection of units in the process.

If the process is inherently hard to control, even employing the best control strategies, the most sophisticated controller algorithm and the most accurate process model, control performance may still be unsatisfactory.

2.4.3. Process Flexibility

Additional issues arise with respect to this need for integration. The notions of plant flexibility, simultaneous design and redesign have to be addressed in the integrated plant design. In an era, where all chemical plants are market-driven and there is huge economic pressure, it is highly desirable for a process to be able to incorporate these notions, something that cannot be achieved by traditional control design. The flexibility of the process relates to the ability to produce a variety of products with a wide range of quality specifications and the ability to switch smoothly between the different operating regimes. Much research has been focused in this area, but mainly with scheduling as the main tool and there has not been any attempt to investigate it in an integrated manner with respect to control design. Simultaneous design relates to the notion of designing the control structure of the complete plant not for a fixed plant configuration but for a broader range of operation and possible configurations. This notion contradicts the industrial practice, where the complete plant is designed and physically constructed taking into account very little control considerations and then attempting to design the control structure. The notion of redesign corresponds to the capability to alter the control regime in order to respond to any possible changes concerning the operation of the process. Redesign is also contradictory to general practice since it cannot be easily applied if control properties have not been taken into account in the early stages of the design. Thus, it will be economically inefficient for a fitted control structure in a process to be redesigned, especially if the process possesses inherent difficult control properties.

2.4.4. Overview of General Issues in Integration

The need to interpret operational and design goals into the level of benefits (financial etc.) of the industrial enterprise, is an additional driving force behind the integration effort; in fact, the need for integration has also been expressed at the business operational layer. Evidence for such needs is the emergence of topics in Engineering, Management Studies etc., known as Integrated Design and Operations, Concurrent Engineering etc., which deal with software and engineering aspects of the integration problem and most of the time are linked to the specifics of the application area. So far, there has been no systematic effort to establish links between the alternative mathematical modelling tools used in the problem of global process operations and there is no form of

system theory that allows the integration of concepts and model based properties. The lack of any significant progress in this area is mainly due to that the area is not well defined and there has been no effort to study and formulate the issues in a form that lead to the development of a generic, unifying methodology. By the nature of the problem, such a methodology has to be of the systems type, since it has to deal with the many different operational and design aspects of the overall system. Identifying the requirements for the development of this methodological framework involves:

i) Specification of the problem area.
ii) Identification of the major issues in the problem area.
iii) Evaluation of methodologies used in the study of the particular problem area.
iv) Specification of the requirements for the generic methodology and characterisation of the possible building blocks.

One of the aims of this chapter is to specify the requirements for integration of process operations and overall design for a complete plant. Many important emerging issues concerning the operation of chemical process plants, as described above, are unresolved with respect to a Control Theory and Design perspective. In this sense, it is important to identify the key elements of a systems and control approach that goes beyond the current state of development and the establishment of the basis for an integrated methodology.

2.5. APPROACHES FOR PROCESS SYSTEMS SYNTHESIS

2.5.1. Introduction: Classification of Methods

This section provides an insight into the trends in chemical process design and the reasons that drive the need for an integrated methodology from an early stage, where control design has to evolve simultaneously with process design. The problem of synthesis of chemical processes is an issue that heavily depends on Chemical Engineering theory and practice. The structural implications of the development of the overall system structure in process synthesis, as well as the selection of control structures related to process synthesis will be examined here briefly. [Nish. et al, 1] have classified the techniques used for the systematic synthesis of entire chemical processes (including reactors, separators, energy-transfer equipment etc) into the following cases:

(i) Approaches without an initial structure.

(ii) Structural parameter or integrated approaches.

2.5.2. Approaches Without an Initial Structure

A computer program called AIDES (Adaptive Initial Design Synthesiser) [Siir. et al, 1] has been developed, which utilises systematic heuristic procedures for process synthesis. AIDES performs the stream source/destination matching for the entire flowsheet in one step. It separately considers the flow of each species within the flowsheet, developing for each a scoring function, which rates each possible source stream/destination stream match. The scoring attempts to account for potential separation costs, which might result, if the match is made. After scoring matches for all species, the entire stream matching is done in a single "parallel" step by solving a linear program to optimise the sum of match scores. A procedure has been proposed for the synthesis of promising initial designs of chemical processing systems using the techniques employed for mechanical theorem proving. Underlying this method is the resolution principle, where the designer attempts to derive conflicts among a set of facts (premises and axioms of chemical processing systems) and the desired goals (desired feasible flowsheet). The procedure begins with the consideration of production goals (desired product streams) one at a time, and ends with a process flowsheet, which is feasible, in terms of mass and energy balances. The above are discussed in [Nist., 1] in a more detailed way. Using a sequential depth-first procedure, the following structural rules have been developed:

a) Use the compositionally most similar source process streams to generate product streams.

b) Give first preference to by-product streams already generated.

c) Reduce the mass load on separation sequences. Earlier work by many researchers, has been mainly aimed at the early stages of process development, to select the optimal equipment configuration to transform given raw material streams into desired product streams using a mixture of dynamic programming and branch and bound arguments.

2.5.3. Integrated Approaches

These approaches can be divided into three subcategories. A more detailed description is given in [Nist., 1]:

a) The analytic and algorithmic methods, which employ the necessary condition for the optimal system and then develop a specific algorithm on the basis of necessary conditions.

b) The decomposition and/or transformation methods, which decompose or transform the synthesis problem into smaller problems so that the smaller problems are solved separately and their solution, co-ordinated in some way to assure the final solution of the individual problems, coincides with that of the overall problem.

c) The direct application of optimisation techniques of non-linear programming. Necessary conditions have been derived for the optimal system using the structural parameter approach. An evolutionary search for the optimal structure (ESOS) was developed, starting from a simple feasible structure.

Decomposition techniques may be one possible way to solve the structural parameter synthesis problem. To ease the difficulty of computations for structure optimisation problems, several authors have proposed decomposition techniques. A method, which has been used, is an infeasible two-level technique in conjunction with the structural parameter approach. Their method was applied to the synthesis problem of a simple reactor-separator synthesis problem. Stephanopoulos and Westerberg [Steph. & West., 1] has developed a two-level method, into which Hestene's method of multipliers was incorporated. A penalty term is used to guarantee the success of the method in the presence of functional non-convexities, often encountered in chemical process design. A feasible two-level method has been proposed in [Gov. & Pow., 1], which consists of the first-level and the second-level problem. Several authors have used non-linear optimisation techniques to solve various synthesis problems of chemical engineering interest. [Umeda et al, 1] used a direct search technique namely Box's Complex method to synthesise a chemical process system consisting of two reactors, two distillation columns and several heat exchangers. Process synthesis methodologies generate the process flowsheet and thus specify the first feasible set of control structure, based entirely on process synthesis criteria. In this sense, they provide the basis for the consideration of the overall control structure selection.

A widely accepted notion, concerning the performance of a control scheme, is that the choice of measured and manipulated variables employed in a control scheme can have a strong effect on the performance of it. Systematic methods to select the economically optimal control structure of a process, without designing the process controller, while maintaining good controllability characteristics, have been examined by a number of researchers. Examination of the effects of process dynamics on process economics and how changes in the control structure alter these economics have been examined. These studies have been limited to selecting economically optimal square regulatory feedback control structures for processes, whose operation is dominated by steady state aspects. In these studies much attention [Maar. & Rijn., 1], [Mor. & Steph., 1], [Prett & Garcia, 1], [Marlin et al., 1] has been focused on the role played by constraints in limiting the steady state performance of the plant. The presence of disturbances moves the actual behaviour of the plant from the optimum point. The main concern is to be able to keep the operation in a feasible region around and close to the nominal optimum point. On this point, the choice by the designer of the control scheme becomes very significant. Different control structures can possibly lead to the same end result, but with the difference in their dynamic behaviour and other inherent control structural properties. The major factor in the decision made by the designer is the economic objective function based on the system model; a trade-off between instrumentation costs and operating benefits has to be introduced in order to distinguish the best control structure.

Traditionally, in the early design of control structures for chemical processes a nominal set of optimum operating points, subject to various equality and inequality constraints, is chosen. These constraints correspond to any regulations concerning the plant, such as safety and environmental regulations, quality criteria etc. The values of the system corresponding to these points are the ones that optimise the control objective function of the plant. This objective function is formulated in economic terms, taking into account the operating costs, the cost of implementing and operating the control scheme, the gross return etc. The next important issue that arises is the use of an optimal steady state process design. The use of it usually results in plant operation on the operational constraints, something unrealistic in practice. The presence of disturbances in the process will cause a different dynamic behaviour of the system. These disturbances can be either of an economic nature, such as i.e. fluctuation in the prices of raw materials or of a purely chemical engineering nature, such as i.e. the inability to have raw materials that have, all the time, exactly the same composition. Thus, it is necessary to introduce a back off from the nominal optimum

point and thus move the steady state operating point sufficiently far into the feasible region. By this, we can assure that most possible disturbances will be adequately rejected when entering the process and no constraints are violated, especially those concerning the safety of the plant. It is obvious, that this deviation from the steady state nominal optimum will incur an economic penalty, since even when no disturbances are present the plant is not operating at the optimum. Many researchers, i.e. [Narr. et al, 1], [Narr., 1], [De Hen., 1], [Loeb., 1] have investigated different types of plants using this type of analysis. It is also obvious that the economic performance will depend on the type of controller implemented. The area of economic appraisal is still in its early stages of development. Although the control structure plays an important role in the overall shaping of the design cost, the analysis should also take into account the overall process synthesis and optimisation. The fact that all aspects have to be considered together makes the problem of economic appraisal rather difficult.

2.6. CLASSIFICATION OF PROCESS VARIABLES

With a process we may associate a large number of variables, which may have a physical origin or may be of mathematical nature. For purposes of Control and Monitoring of the process, we have to classify the physical variables and here we consider procedures that have been developed within the process field. Newell and Lee [New. & Lee, 1] suggested qualitative criteria, in order to classify the process variables and assist the selection of controlled, manipulated and measured variables that are suitable for the early analysis in the design of a plantwide control system.

2.6.1. Selection of Controlled Variables

The consideration of plant and control objectives has led to a number of suggested guidelines for the selection of controlled variables from the available output variables:

- **Select variables that are non-self-regulating.** A non-self-regulating process is a process that is described by a state equation of the form $\dot{x} = f\{x,u\}$, in which the state variable, x, does not appear in the function $f\{x,u\}$; i.e., $\dot{x} = f\{u\}$. As a result, changes in the input, u, affect the process output as a pure integrator. When the process is

unstable in open loop (that is, in the absence of feedback control), a change in the input causes the system to go unstable. These processes are in contrast to self-regulating processes, in which changes in the input cause the process to move to another stable steady state. Clearly, process outputs that are non-self-regulating must be selected as controlled variables. A common example is the liquid level in a storage vessel with a pump in the exit line.

- **Choose output variables that may exceed equipment and operating constraints without control.** It is obvious, that when safety and operational constraints are imposed, it is important to measure and control these outputs to comply with the constraints. Usually, such variables are temperatures, pressures and compositions.
- **Select output variables that are a direct measure of the product quality or that strongly affect it.** This guideline helps the control system to ensure that the product specifications are regulated and met. Examples of variables that are a direct measure of the product quality are the composition and the refractive index, whereas those that strongly affect it are the temperature and pressure.
- **Choose output variables that seriously interact with other controlled variables.** Plantwide control must handle the potential interactions in the process. Improved closed-loop performance is achieved by stabilising output variables that interact significantly with each other. The steam header pressure for a plant boiler that supplies several downstream units is an example of this type of output variable.
- **Choose output variables that have favourable dynamic and static characteristics.** Ideally, there should be at least one manipulated variable that has a significant, direct and rapid effect on each controlled variable.

These five guidelines should not be considered to be hard and fast rules. Also, for a particular application the guidelines may be inconsistent and thus result in a conflict. As an example of their use, an output variable such as temperature must be kept within limits (second guideline). Temperature could also affect other output variables (e.g., composition, pressure) and thus also should be selected according to the fourth guideline. If there were a conflict, the second guideline would be the overriding concern in this situation.

2.6.2. Selection of Manipulated Variables

Based on the plant and control objectives, a number of guidelines have been proposed for the selection of manipulated variables from among the input variables:

- **Select inputs that have large effects on the controlled variables.** For each control loop, select an input with as large a steady-state gain as possible and sufficient range to adjust the controlled variable. For example, when a distillation column operates with a large reflux ratio, it is much easier to control the level in the reflux drum using the reflux flow rate rather than using the distillate flow rate, since the reflux flow rate is five times larger. However, the effect of this choice on the product compositions must also be considered in making the final decision.
- **Choose inputs that rapidly affect the controlled variables.** It is desirable for a manipulated variable to affect the corresponding controlled variable as quickly as possible. Thus, any time delays or time constants that are associated with the manipulated variable should be small relative to the dominant process time constant.
- **The manipulated variables should affect the controlled variables directly rather than indirectly.** Compliance with this guideline usually results in a control loop with favourable static and dynamic characteristics. For example, when appropriate for the design of an exothermic reactor, it is preferable to inject a coolant directly rather than use a cooling jacket.
- **Avoid recycling disturbances.** It is preferable not to manipulate an inlet stream or a recycle stream, because disturbances tend to be propagated forward or recycled back to the process. This problem can be avoided by manipulating a utility stream to absorb disturbances or an exit stream that allows the disturbances to be passed downstream, provided that the exit stream changes do not upset downstream process units.

These guidelines may be in conflict. For example, a comparison of the effects of two inputs on a single controlled variable may indicate that one has a larger steady-state gain but slower dynamics. In this situation, a trade-off between static and dynamic considerations must be made in selecting the appropriate manipulated variable from the two input candidates.

2.6.3. Selection of Measured Variables

The safe and efficient operation of a processing plant is made possible by the on-line measurement of key process variables. It is obvious, that output variables that are used as controlled variables should be measured. Other output variables are also measured in order to provide additional information to the plant operators or for use in model-based control schemes such as supervisory control or inferential control. It is also desirable to measure selected input variables as well as output variables, since recorded measurements of manipulated inputs provide useful information for tuning controllers and troubleshooting control loops. Also, measurements of disturbance inputs can be used in feedforward control schemes. In choosing, which outputs to measure and in locating measurement points, both static and dynamic considerations are important.

- **Reliable, accurate measurements are essential for good control.** There are plenty of examples in literature that show that inadequate measurements are a key contributor to poor control. Examples of poorly designed measurements include orifices with insufficient straight piping, saturated liquids that flash in the orifice etc.

- **Select measurement points that have an adequate degree of sensitivity.** For example, in distillation columns a product composition is often controlled indirectly by regulating a temperature near the end of the column if an analyser is not available. However, for high purity separations the location of the temperature measurement point can be important. If a tray near the end of the column is selected, the tray temperature tends to be insensitive, since the tray composition can vary significantly even though the tray temperature changes very little. By contrast, if the temperature measurement point is moved closer to the feed tray, the temperature sensitivity is improved, but disturbances entering the column at the ends (e.g. condenser, reboiler) are not sensed as quickly.

- **Select measurement points that minimise time delays and time constants.** Large time delays and dynamic lags in the process limit the achievable closed-loop performance. These should be reduced, whenever possible, in the process design and the selection of measurements.

2.7. HIERARCHICAL APPROACH FOR DEVELOPMENT OF PLANT CONTROL STRATEGIES

2.7.1. Hierarchy of Plant Representations

A large-scale chemical plant is an ordered complex system. Simon [Simon, 1] stated that an ordered complex system "... is one that is made up of a large number of parts that interact in a non simple way. In such systems the whole is more than the sum of the parts,..., given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole". To make the control system design for a complex system more tractable, we can perform a change of representation on the process and transform it into a hierarchical system. A hierarchical system is one that is composed of interrelated subsystems, each of the latter being hierarchical in structure until we reach some lowest level of elementary subsystem [Simon, 1]. In recent years, there has been a growing recognition of the importance of various dynamics and control issues in the early plant design [Steph., 1], [Doug., 1]. Nowadays, plants are constructed in a very complex and highly integrated manner, in order to incorporate rising costs in energy and raw materials, strong competition in the market, stricter safety and pollution standards, tighter performance specifications and strong interactions between the processing units of the plant. Very significant, also, is the fact that if the flowsheet has been realised by ordering or placing any important processing units, the modifications proposed by a control scheme afterwards, may introduce high costs for rearranging or replacing them. All the above, highlight the importance of the interaction between process design and control.

It has to be noted that judging alternative designs, only from an economic basis perspective, without taking into account dynamic performance criteria, can be problematic. It may lead to elimination of slightly less economical alternatives, which are easy to control, in favour of slightly more economical designs that may be extremely difficult to control. Thus, the decision making relating to alternative plant designs can not be solely based on steady-state economics, because the resulting plants are often difficult to control, resulting in products which do not satisfy their specifications, excessive use of energy and resources and associated profitability losses. The first step in realising in an integrated manner the control strategy for a process plant is to approach it in a hierarchical way. An approach for involving control strategies to plant design has been proposed by Ng and Stephanopoulos [Ng & Steph., 1]. This approach takes into consideration every aspect of the plant in an

order of ascending detail and is very useful, as it will be explained later in this chapter, for the creation of an early design model. The design of a plant control strategy is made more manageable if the plant is decomposed into a hierarchy of representations and the plant is viewed at different stages of abstraction. These plant representations are very similar to the ones used in the conceptual design of chemical plants, as proposed by Douglas [Doug., 1] and mentioned before. Beginning with an abstract viewpoint of the plant, the representation can be progressively refined into new viewpoints each of which contains an increasing level of details about the plant. It is obvious that every viewpoint corresponds to a particular range of characteristics of the plant. The example of an HDA plant can be used as a useful example to be decomposed into a hierarchy of plant representations, in order to illustrate the particulars of the proposed method. The hierarchy of the plant representations can be divided in four levels of increasing level of details:

Level 1: Input-Output Representation of the Plant

The input-output representation is the most abstract viewpoint of the plant (Figure (2.2)). It represents in a unique way the overall scope of the plant, which is to produce the desired products by converting the feed streams and using the available utilities. In this manner, the control strategy is focused on the overall plant objectives and the economic aspect of the decision making that influence the interactions between the plant and the environment. Issues and objectives that are important in this level of representation include the overall material and energy balances of the process, the production rate and the quality control management of the plant.

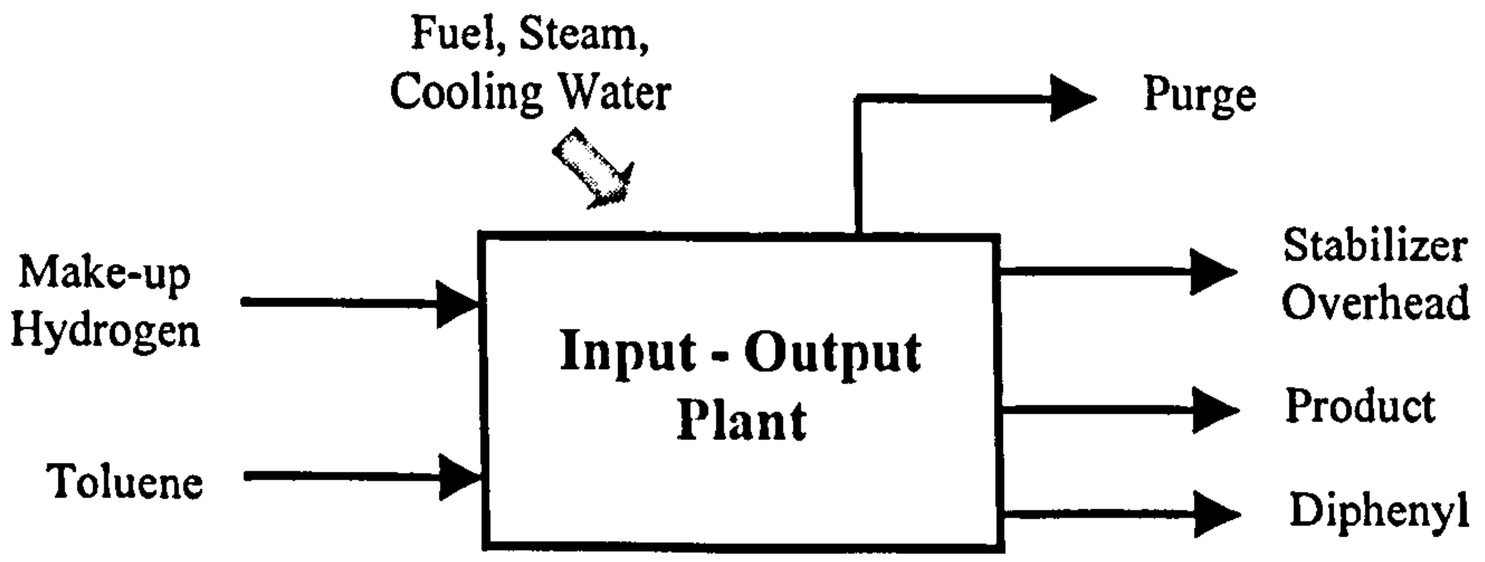

Figure (2.2): Level 1: Input-Output Plant

Level 2: Recycle Structure of the Plant

At this next level, the main block created in the first level, the input-output block is decomposed into a number of sub-blocks by grouping the activities, which are dynamically similar to each other. In the example of the HDA plant, one block represents the generalised reaction unit and the other the generalised separation system (Figure 2.3). By using this decomposition the focus of the control strategy design is shifted to the effect of the variations of the recycle flows to the overall process. This level of representations can reveal many potential problems in the control of the plant. Lyuben [Lyuben, 1] has studied the dynamics and control of many recycle systems. In these systems, it has been shown that some particular combinations of control strategies can have undesirable effects in the closed-loop behaviour of the plant. An unsuitable decision regarding the control structure may create the "snowball effect" [Lyuben, 1]. By this term, the case of a small increase in the feed streams that causes an extremely large change in the recycle flows is described. Additionally, the same studies have shown that the behaviour of a recycle system depends strongly on the recycle loop gain and less strongly on the dynamics of the individual process units in the recycle loop, thus this effect is a steady-state phenomenon. This level representation supplies a way so these problems can be accommodated.

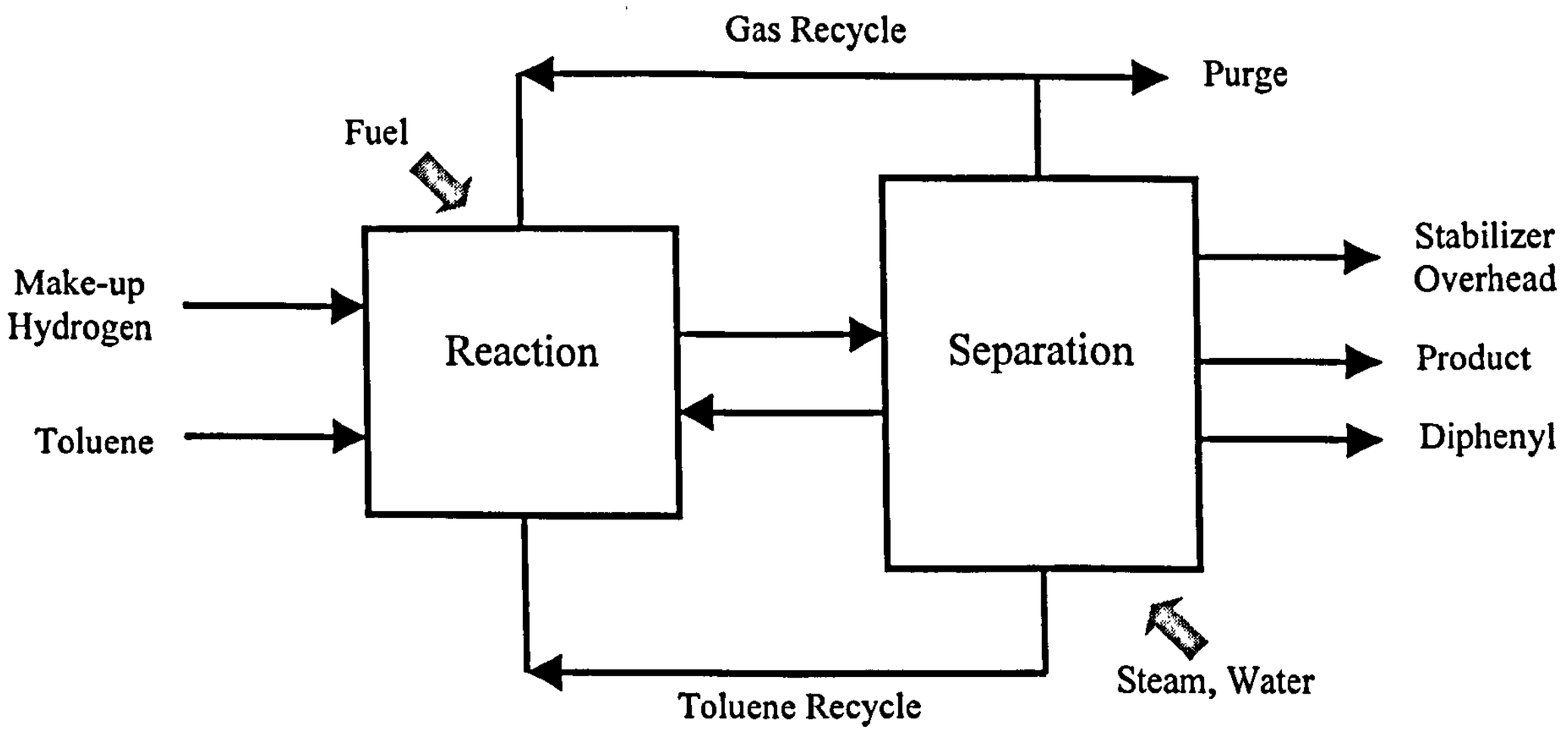

Figure (2.3): Level 2: Recycle Structure

Level 3: Refined Representation of the Plant

At Level 3, the sub-blocks created in Level 2 are expanded in stages through a series of refined representations of the plant. The role of the process units is being systematically evaluated in these viewpoints. For example, at Level 3a (Figure (2.4a)), we can examine how the division of materials in the generalised phase-separation unit plays a role in the process. At Level 3b (Figure (2.4b)), we can take a closer look at the reaction section. At Level 3c (Figure (2.4c)), the interaction between the separation system and the rest of the plant is emphasised. As the representations become more refined, more details about the behaviour of the individual process units and the interaction between them become available. The issues and objectives, which are important to this level of representations, are the issues and objectives corresponding to the individual units.

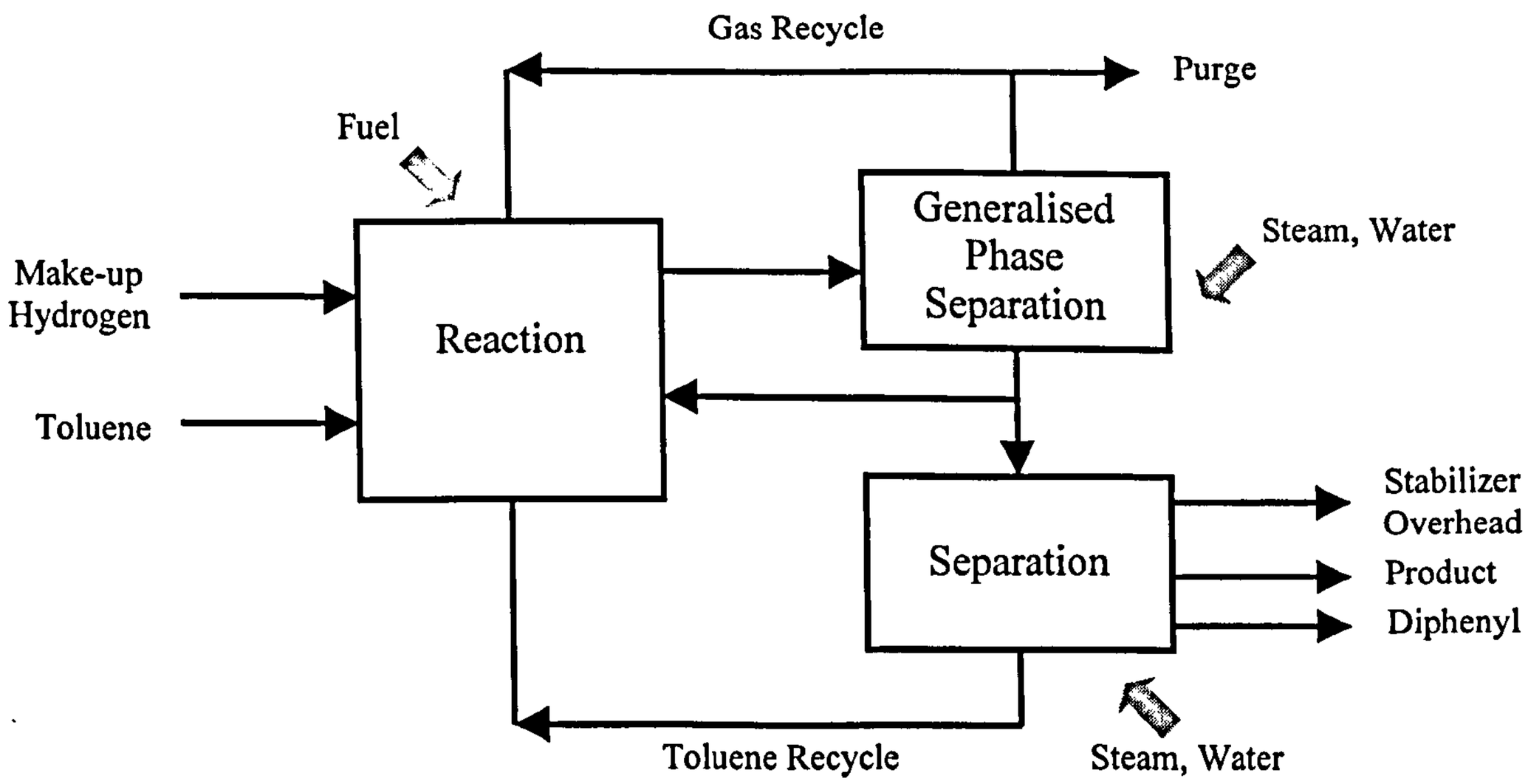

Figure (2.4a): Level 3a: Generalised Reaction – Expanded Separation

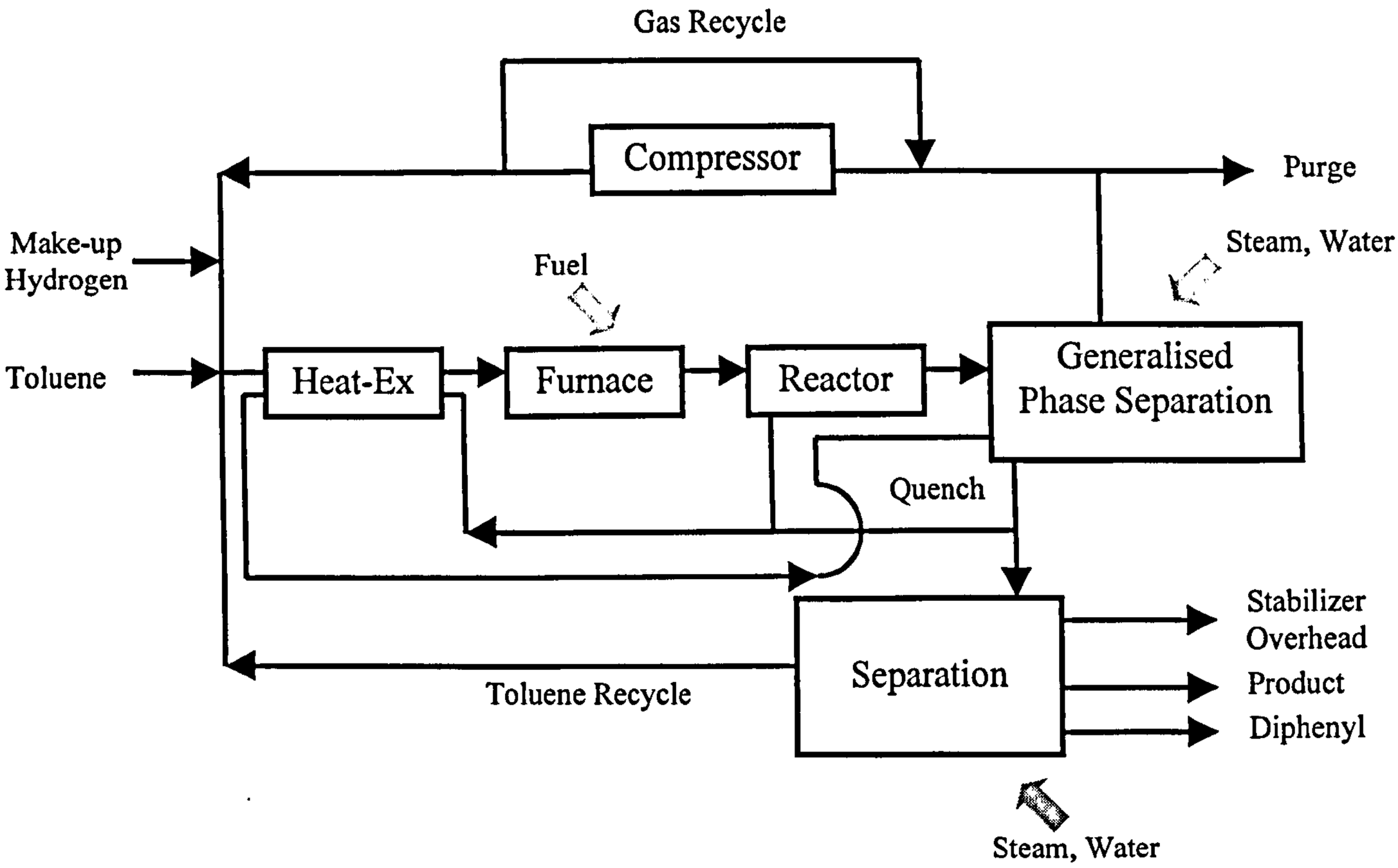

Figure (2.4b): Level 3b: Detailed Reaction – Generalised Separation System

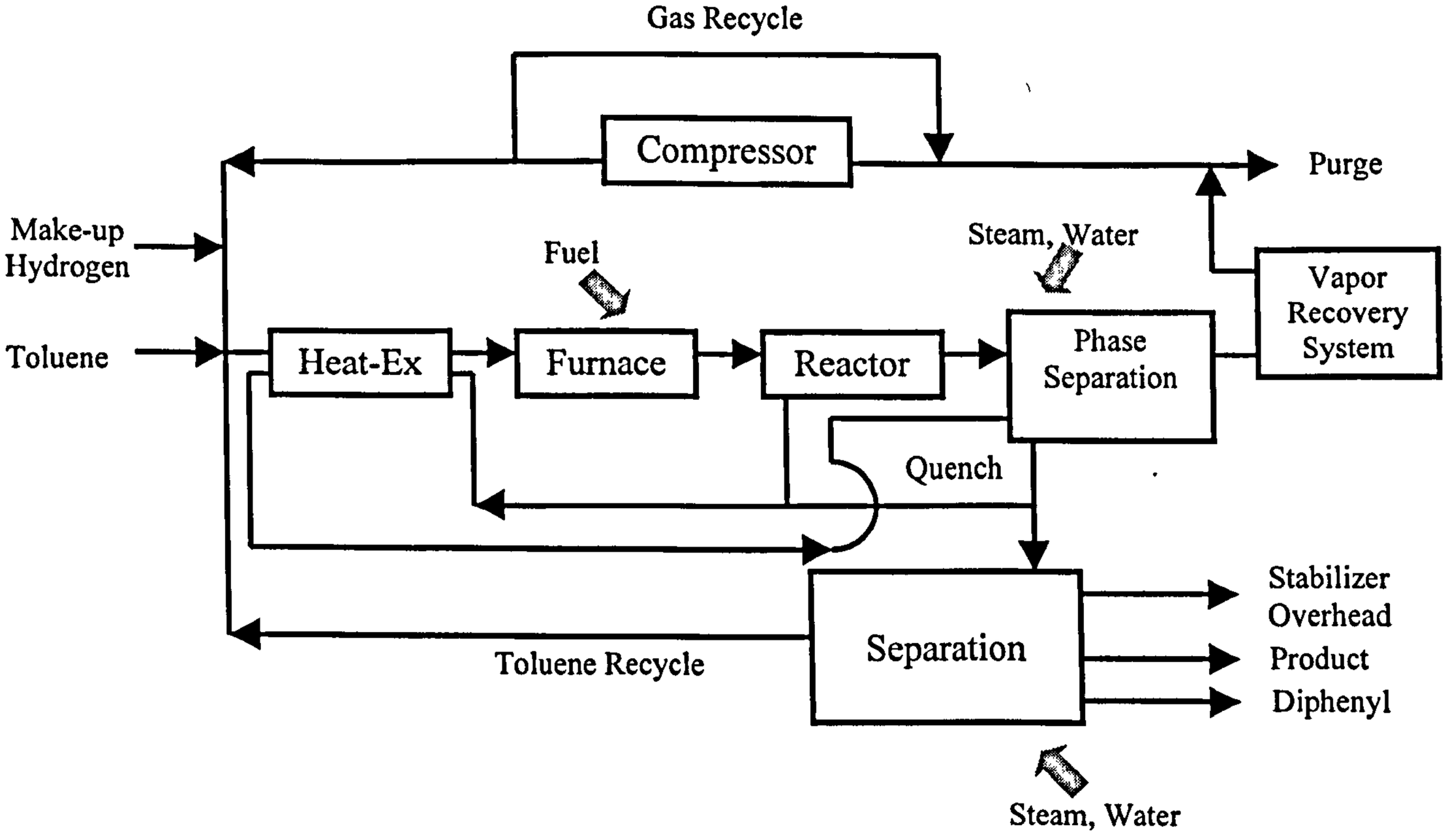

Figure (2.4c): Level 3c: Detailed Reaction – Expanded Separation

Level 4: Detailed Representation of the Plant

In this level of representation (Figure (2.5)), the individual unit-operations are the basic blocks in the representation. The analysis focuses on the effect of changes in the different processing units. The focus in this detailed level is localised in the process behaviour of each unit-operation.

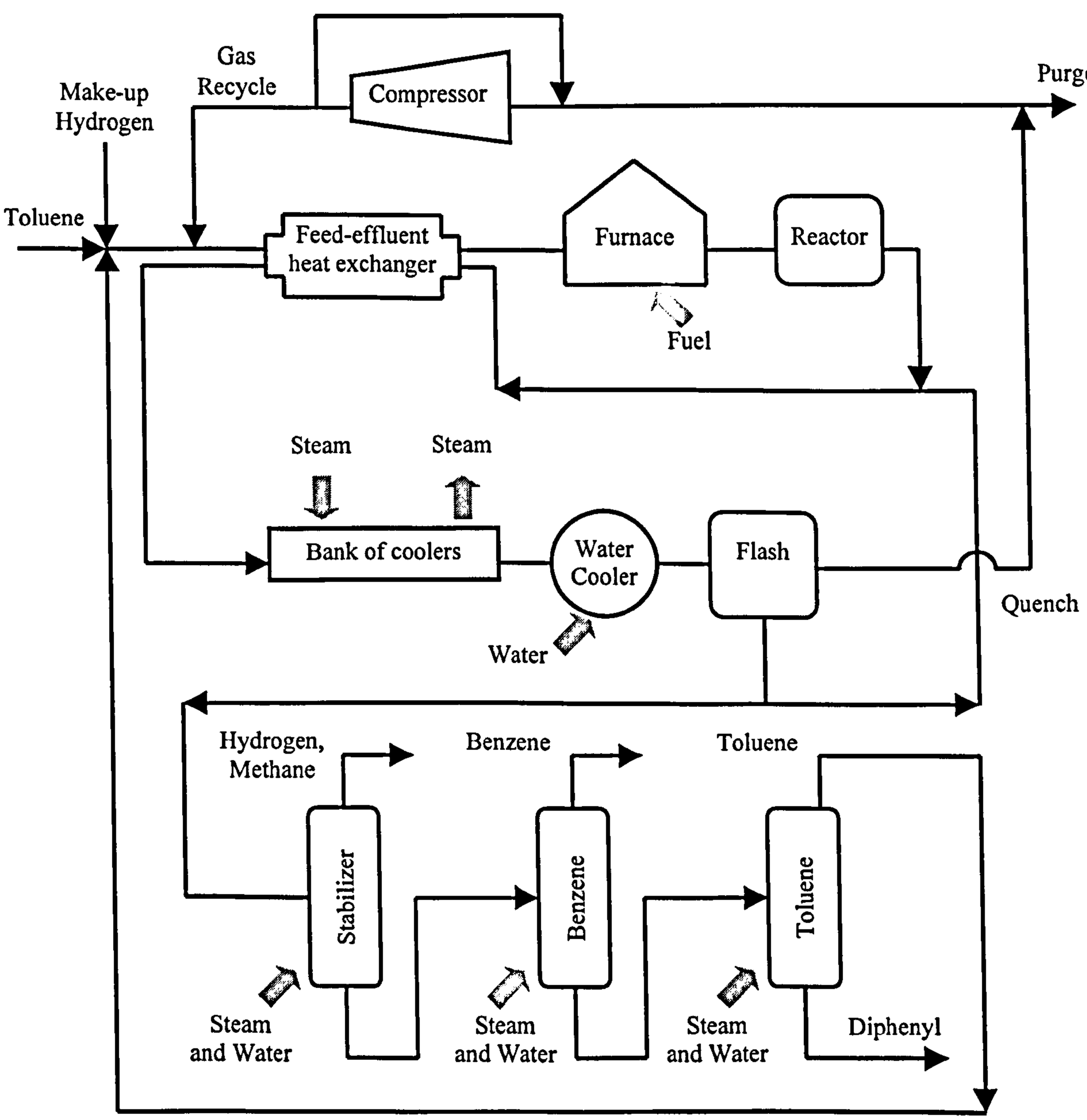

Figure (2.5): Level 4: Detailed Reaction – Detailed Separation

These four levels of representation of the plant can suitably address both the global (implicit) objectives and the local (explicit) objectives. At the most abstract level (Level 1),

the focus is on objectives which specify the overall process behaviour and those which deal with the interactions of the plant with the external environment. At the next level, the process objectives are being translated to the new level. In this manner, the objectives at a high level constrain the behaviour at a lower level. Consequently, consistency among the hierarchy of viewpoints is ensured. It is possible for new objectives to be observed at a later level as more details of the plant are being exposed. Objectives translated from Level 1 can be refined or spawned to reflect the added details in the new viewpoint. This procedure is then repeated for any subsequent level. Using this process, the focus of the design is systematically shifted from a global viewpoint to a more local one. Thus, the complexity of the design problem can be reduced and a plant control system can be generated, which accounts for both the peculiarity of the unit-operations and the desired global behaviour. The process in which the control tasks at each representation are generated is illustrated in Figure (2.6).

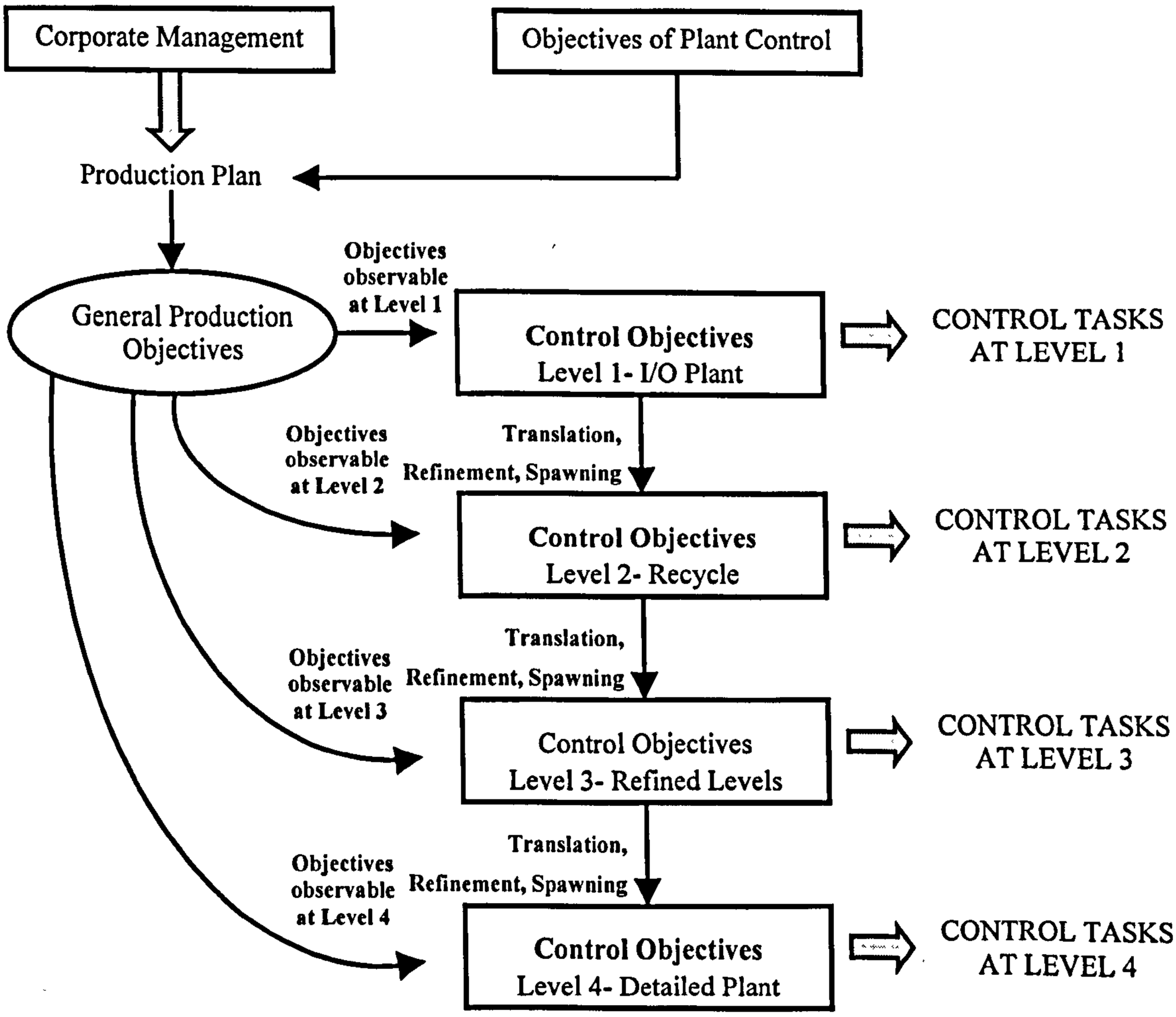

Figure (2.6): Generation of Control Tasks

A further step towards a more integrated consideration of the plant is discussed in the section below.

2.7.2. Hierarchy and time-horizon of control tasks

In the development of a plant control system, there is a range of process phenomena that need to be addressed. Although, a significant criterion of process control is the ability to initiate fast response of the system in order to accommodate any setpoint changes or the effect of any other process disturbance, it is only one out of several others. The ability to respond in a fast manner is only useful for controlling variables, which are dominant and are critical specifications in the process [Arbel et al, 1]. On the other hand, slow response is introduced in the cases where modifications of the more long-range process behaviour are needed. The benefit of constructing the hierarchy of viewpoints is that the higher-frequency dynamics (or those effects which are important in the short time-horizon) involving the internal structure of the components have been separated from the lower-frequency dynamics (or those effects which are only important in the long time-horizon) involving interactions among component at the more abstract levels [Simon, 1]. Thus, the process phenomena have been divided according to their associated time-horizons, each of which is characterised only by one of the representations in the previously discussed hierarchy of plant viewpoints.

At Level 1, the input-output model represents the longest time-scale of operation and the focus is on the slowest dynamics of the plant. Disturbances that have a low frequency variation, such as changes in the operating points or persistent exogenous process disturbances are considered important here. In this viewpoint, the long-term *static feasibility* of the process is evaluated. In this stage and in order to design a control system for a chemical process, it is important to consider how the different production levels are to be met, how the overall material and energy balances can be achieved, or how the operation at the cost optimal steady-state can be maintained. The control tasks that address these steady-state considerations and behaviour can be developed using the input-output plant.

Level 2 represents a more refined representation of the plant. In this level, the characteristic time-scales of operation of the individual blocks are smaller than that of the overall process. The difference in time-scales between the Level 1 and Level 2 representations could be of an order of magnitude. In this level, the disturbances that vary at higher frequencies become important. As we move down the hierarchy, the planning

horizon decreases in time and space. The viewpoints become more refined and each individual block represents yet shorter time-scales of operation. Within these more refined representations, it is possible to study how the individual units interact to bring about changes in the overall process and the control tasks that are required to ensure that these changes can be achieved smoothly.

The more detailed level (Level 4) reveals the higher order dynamics of the overall process (such as inverse responses, capacitors in series). At this level, the dynamic operability of both the overall process and the individual processing units can be studied. This viewpoint also exposes how the high frequency external disturbances and changes of manipulated variables within the process affect the local process variables such as flow rates, compositions, temperatures, pressures or tank levels. Control strategies for direct process regulation can therefore be derived from this representation. The hierarchy of process representation provides a framework in which sets of consistent control tasks are displayed in viewpoints, which match their relevant time-scales. The frequency of the disturbances and the corresponding types of control tasks are shown in the following figure. Within this time resolution the disturbances of the system, have been partitioned according to their frequencies of variations.

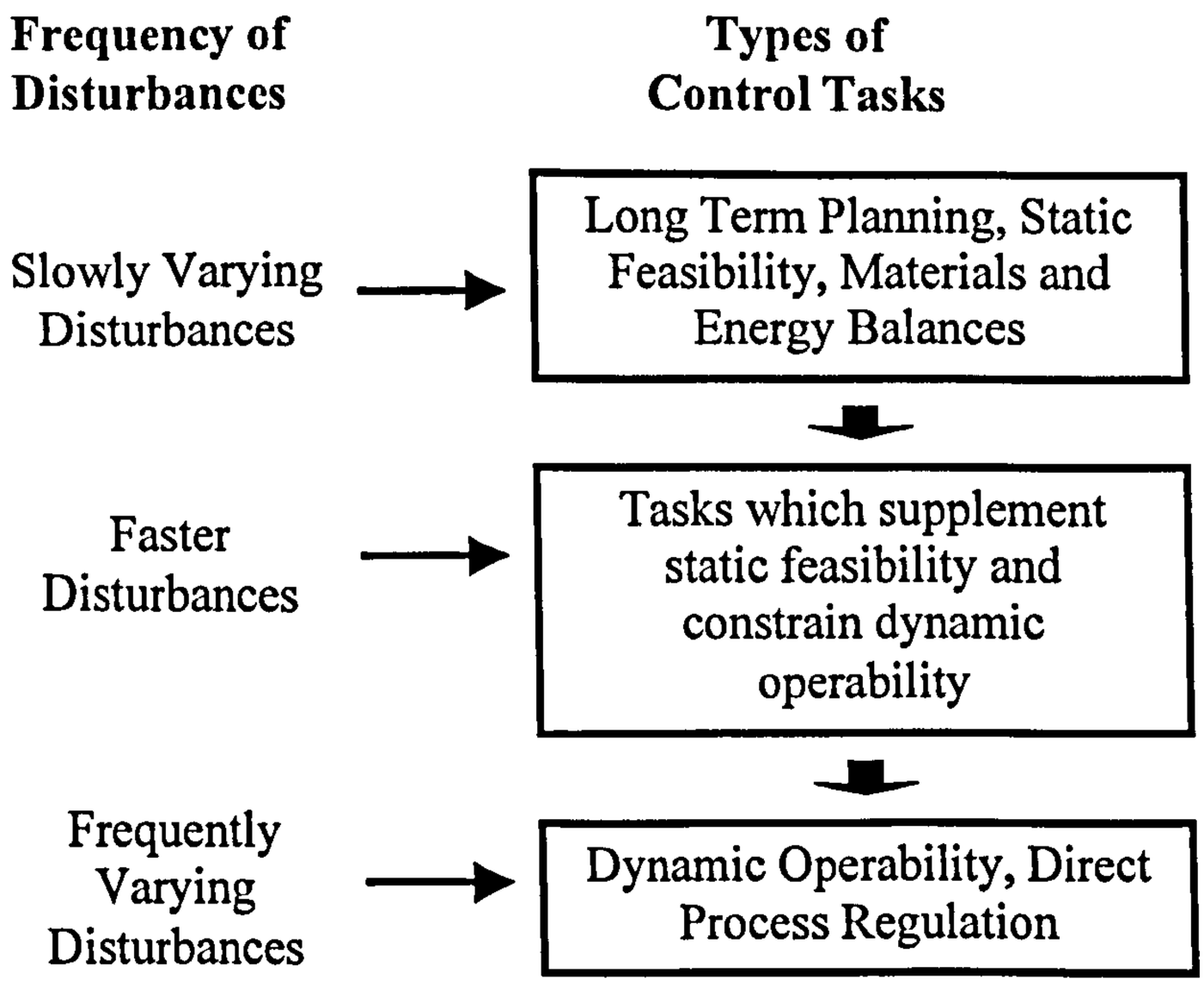

Figure (2.6): Hierarchical View of Control Tasks

In the same manner that the above procedure is proposed, a conceptual hierarchical approach of a process is discussed, that encompasses all the various viewpoints and approaches in a conceptual manner.

2.8. CONCEPTUAL HIERARCHICAL APPROACH FOR DEVELOPMENT OF CONTROL STRUCTURES

Fischer, Doherty and Douglas [Fisch. et al, 1] proposed a conceptual hierarchical approach to synthesise a control system for a complete process. The steps of this hierarchy are discussed below:

I. Steady-state considerations. Identify and eliminate by process redesign hard control problems based on steady-state models (which are much simpler than the dynamic models), thus minimising the design effort.

A. Identify the significant disturbances that affect the system and the locations in the process flowsheet, where they appear. In particular, specify (identify):

1. Disturbances that affect the process constraints.

2. Disturbances that affect the operating costs.

3. Disturbances that do not have a significant effect on either of the above and thus they can be ignored and the problem may be simplified.

B. Check that the manipulative variables available in the flowsheet are adequate (both in number and sensitivity) to be able to satisfy the process constraints and to optimise the operating variables over the complete range of the disturbances. Specifically:

1. If the number of manipulative values is not adequate, then the process is not controllable, since all possible disturbances entering the process can not be rejected or compensated.

2. In order to restore controllability, three possibilities are proposed:

a. Modify the flowsheet to introduce more manipulative variables.

b. Modify the equipment designs so that some constraints do not become active over the complete (reasonable) range of disturbances.

c. Neglect the least important optimisation variables.

C. Check whether any equipment constraints are encountered that prevent the changes in the manipulative variables from satisfying the process constraints or optimising the operating variables over the complete (reasonable) range of the disturbances. In particular:

1. If the process constraints cannot be satisfied, the constrained equipment must be overdesigned, to restore the operability of the process.

2. If the process is operable when there are equipment constraints, the savings in operating costs by introducing equipment overdesign in order to remove equipment constraints might be economically justified.

D. Use heuristics to select the controlled variables such that the steady-state behaviour of the process will be close to the optimum steady-state performance.

E. Select pairings of the manipulative and controlled variables for single-loop controllers. This involves selection of criteria and procedures for evaluation of pairings. Such a methodology involves:

1. Criteria based on:

a. High sensitivities.

b. Small dead times (the pair of the variables should be as physically close as possible within the plant).

2. Evaluation of pairings using methodologies such as:

a. Relative gain array.

b. Singular value decomposition.

3. Elimination of pairings with large interactions.

4. Development of several alternative control systems.

II. **Normal dynamic response:** For small perturbations and linear process dynamics, the analysis continues (is repeated) using dynamic models of a simple character. This process involves:

A. Development of requirements to build dynamic models. This process includes particular tasks such as:

1. All equipment capacities must be specified, i.e., the holdup in the tubes and the shells of each heat exchanger, the holdup on the trays in a distillation column, etc.

2. The sizes of reflux drums, column sumps, flash drums, intermediate storage vessels, etc., must be specified.

B. Assumption of perfect level control in any unit where there are two-phase mixtures.

C. Evaluation of the stability of the controlled and uncontrolled processes.

D. Use linear dynamic models to evaluate the steady-state plant control systems having the fewest interactions. This involves the specific tasks:

1. Use the difference between the total operating cost of the optimum steady-state control response and the dynamic response of the controlled plant as a performance measure to compare control system alternatives for an assumed pattern of disturbances and then, check the sensitivity of the results to the disturbance pattern.
2. Evaluate the robustness of the control system.
3. If the dynamic response is not satisfactory, then consider the problem of redesign that may involve:
   a. Change of the control system.
   b. Modification of the flowsheet.

E. Design the level controllers and recheck the performance.

III. Abnormal dynamic operation – Large perturbations and nonlinear dynamic response have special requirements for the control design. The important areas requiring attention are:

A. Start-up and shutdown of the process. This involves development of:

1. A flowsheet showing all intermediate storage, which is used as a starting point for the analysis.
2. The generated flowsheet should be checked and modified to correspond to the start-up strategy of the plant.
3. The control systems required for plant start-up and shutdown are different from the controls used for normal operation and have to be developed.

B. Failures in the process have to be considered. This requires:

1. A failure analysis of the flowsheet needs to be undertaken.
2. Special control systems to handle failures might be needed.

IV. Implementation of the control. Important questions which have to be answered are:

A. Should distributed control be used, and if yes, how?

B. What kind of computer control – human interface is required?

If the process can not be controlled, the start-up or shutdown is very difficult, or if the process becomes unsafe because of a failure in one or more equipment pieces, then it may be necessary to alter the flowsheet or even to abandon the project. Taking into account that flowsheet modifications are generally very expensive, it is desirable to identify any potential control problems as early as possible in the development of a design. It should be noted that the procedure described above could be undertaken as soon as a conceptual design has been completed and a proposed flowsheet is available.

2.9. CONCEPTUAL DESIGN OF PLANTWIDE CONTROL

Ng and Stephanopoulos [Ng & Steph., 1] suggests an alternative methodology for the conceptual design of plantwide control systems, which can be combined with the guidelines (proposed by Newell and Lee [New. & Lee, 1]) for the selection of the manipulated and controlled variables. This approach consists of the following steps:

Step 1. Divide the process into separate subsystems. Each subsystem consists of one or more processing units with a common processing goal. Thus, for example, a subsystem containing a distillation column should also include its condenser and reboiler, and may include its feed preheater.

Step 2. Determine the structural features of the control scheme. This involves defining the degrees of freedom and the number of controlled and manipulated variables for each subsystem. The computation of the degrees of freedom is based on the formula:

$$N_{Manipulated} = N_{Variables} - N_{Externally\ Defined} - N_{Equations}$$

Step 3. Determine all feasible loop configurations for each subsystem. This involves either using the qualitative guidelines mentioned before or using quantitative criteria, like the Relative Gain Array (RGA) etc.

Step 4. Recombine the subsystems with their loop configurations.

Step 5. Eliminate conflicts among the control systems for the various subsystems. This step is necessary to resolve the overspecification of manipulated variables, resulting from having defined control configurations for each subsystem separately. When necessary, such conflicts are resolved by the elimination of superfluous loops.

Step 6. Improve the control configuration generated in Step 5. By their nature, Steps 1-5 generate decentralised control systems. In some situations, it is preferable to select pairings between the manipulated variables and the controlled variables taking into account the interactions between the subsystems. This can be accomplished using the quantitative analysis mentioned before.

2.10. THE CONTROL STRUCTURE SELECTION

The problem of control structure selection is within the overall area of development of control schemes for the overall plant, but it is different than the control design, since it is between Instrumentation and Control. This problem has been studied extensively in the area of Chemical Processes and a large variety of criteria and heuristics have been proposed and used; however there is up to now no systematic methodology for synthesising control structures for the complete plant. The main issues addressed here, can be considered to be part of the extended control design problem which consists of the following main parts:

i) Selection of controlled and manipulated variables.

ii) Selection of coupling of controlled and manipulated variables, as well as specification of controller dynamics.

iii) Design of the control scheme, incorporating the coupling specified in (ii) with a variety of control performance objectives and criteria.

So far, Control Design has addressed predominantly the area (iii) and has assumed that both the input-output structure and the coupling of variables have been previously decided. In the process control area, the issues in (ii) have been previously considered, but there is no systematic methodology emerging yet that covers all different aspects. Area (ii) has also been addressed within Control Theory and Design, but not as a design of decentralisation schemes. Part (i) has been considered extensively earlier in this chapter.

The need for an integrated design methodology implies that issues related to the design of input-output structures for processes have to be seen together with the selection of the control structure itself. The evolution of the control structure properties has to be monitored through the different design stages of the design, especially in the early stages of design and if possible be the driving force in the continuing evolution. Important problems that arise within the context of control structure selection are:

- Classification of the operating regimes (which include start-up, shut-down, emergencies and smooth transition from one operating condition to another) for which we require alternative solutions for (i), (ii), (iii) problem areas.
- Simultaneous, robust design of either of the (i), (ii), (iii) areas, when common solutions are feasible for groupings of operating points.
- When more than one grouping of operating regimes emerges, which implies switching, there is a need for an appropriate supervisory strategy for running effectively and safely such schemes.
- Taking into account operational criteria (i.e. optimisation on plant level, total quality) in the design or redesign of sections of the process and evaluating the impact of local designs on the general performance indices referred to sections, or whole plant.

2.11. CONCLUSIONS

The fundamental issues of integrated design in the context of chemical processes have been considered and a number of approaches and methodologies, which aim at handling the fundamental issues have been considered. The area is mature as far as process based methodologies are concerned, but an overall system theoretic framework that may provide advanced methodologies based on generic modelling tools is missing. An attempt to extract and formulate system and control problems from such a problem environment is made in the following chapter, where the fundamentals of a generic approach, as originally conceived within the EPIC [Karc., 2], [Karc., 5] and SESDIP projects [Karc., 3], [Karc., 4] and further developed here is presented.

Chapter 3

GLOBAL INSTRUMENTATION OF A PROCESS: A SYSTEMS AND CONTROL THEORY FRAMEWORK

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GLOBAL INSTRUMENTATION OF A PROCESS: A SYSTEMS AND CONTROL THEORY FRAMEWORK

3.1. INTRODUCTION

The overall design of large-scale industrial processes has as integral parts the following main stages:

- Process Synthesis
- Global Instrumentation
- Total Control Design

The first is dominated by the specifics of the particular domain, whereas the last involves the study of control problems on systems, which have been already shaped. We focus here on the subject of Global Instrumentation, which is seen as a major task that shapes the model on which control design is based. Control Design is considered in the current context as the area that provides the desirable features of the model on which design is performed.

The instrumentation of a process, that is the selection of measurement variables (outputs) and actuation variables (inputs), has a "micro" (local), as well as a "macro" (global) aspect. The "micro" role of instrumentation has been well-developed [Fink. & Grat., 1] and deals with the problem of measurement, or implementation of action upon given physical variables; instrumentation theory and practice deals almost exclusively with the latter problems. The "macro" aspects [Karc., 8] of instrumentation stem from that

designing an instrumentation scheme for a given process (classification and selection of input and output variables) expresses the attempt of the "observer" (designer) to build bridges with the "internal mechanism" of the process in order to observe it and/or act upon it. What is considered as the final system, on which Control Systems Design is to be performed, is the object obtained by the interaction of the "internal mechanism" and the specification of the overall instrumentation scheme. Difficulties in control of the final system may be assessed in terms of certain structural characteristics of the final system model. These structural characteristics are formed through the various stages, which the design goes through; however, the process of formation of such structural characteristics, as well as the link between their types, values and nature to control problems is not yet well understood. The aim of this chapter is to examine the general issues arising in the selection of systems of actuation and measurement variables and then provide a system and control theory/design based framework for global instrumentation; this is achieved by examining a number of problems, associated with the selection of input, output schemes of a process and by highlighting their control theory and design context. From the systems viewpoint, global instrumentation is seen as a model structure shaping, design stage as far as the characteristics of the final model. Given that the structure of the model determines in a sense what can be achieved under compensation, global instrumentation is intimately linked to control design. The problems of control design and overall selection of input, output schemes for a process, referred to here as Global Process Instrumentation (GPI), are strongly interrelated and this has been specially recognised in the Process Control area, where issues of selection of input, output schemes have been considered within the area of control structure selection [Mor. et al, 2], [Gov. & Pow., 1], [Geor. & Fl., 1]. So far, however, there has been no systematic attempt to develop a unifying framework for selection of systems of measurement and actuation variables for processes, where the model structure shaping role of Global Instrumentation is the central feature. Our attempt here reflects the view that instrumentation and control cannot be seen as independent activities, but as interrelated tasks within an integrated methodology. Of course, Global Instrumentation has many more aspects, than the model structure shaping role considered here; it is believed that the system character of the problems addressed here has the potential to provide bridges with aspects such as traditional instrumentation ("micro aspects"), signal processing, communication (field bus technology), artificial intelligence (smart sensors) and neuro fuzzy modelling (soft sensors). This chapter contributes to the further development of the area by providing a classification of the fundamental system type problems arising in

GPI into four main clusters of problems of the Systems and Control Theory type. Such problems may be tackled with tools from the latter two areas and within a framework where system structure formation plays a predominant role.

The problem of selection of input, output schemes for a process, is part of the overall design of the process, which is of cascade nature and has as main stages, Process Synthesis, Global Process instrumentation and Control System design. It has been argued [Karc., 8] that there is a correspondence between the successive design stage decisions of the cascade design process and the evaluation of structural characteristics of process models. Global Instrumentation plays a crucial role in the shaping of structural characteristics and it has the advantage that very frequently it has many degrees of freedom, which may be used for design purposes. The central characteristic of this approach is that we view GPI as a process of shaping further the inherited structure from the process synthesis stage; furthermore, the term structure is viewed here as a linear graph and as system invariants of the underlined model. The structural methodology adopted here centers around the study of four clusters of problems. These are [Karc., 7]: (i) Model Orientation Problems (MOP), (ii) Model Projection Problems (MPP), (iii) Model Expansion Problems (MEP), and (iv) Model Composition Problems (MCP). These problems belong to the general area of Control Theory and Design, but they have not been properly recognised and addressed there as model structure shaping problems. Their study is essential in the effort to develop conceptual and design tools for Global Process Instrumentation, which may supplement the application area dependent methods and heuristics. The chapter focuses on reviewing the issues, problems and the general methodology rather than discussing in detail each of the problems introduced. Each of the problems described is the subject of a separate investigation undertaken at the moment. The development of a systems based framework for GPI generates new requirements for developments in Systems and Control. These issues are discussed here and are issues for parallel activities in these fields.

The chapter is organised as follows: In Section (2) we summarise the issues in the general field of Instrumentation and in Section (3) we examine the cascade nature of the Engineering Design and the general requirements for integration. The special characteristics of Global Instrumentation, and in particular its model shaping role are considered in Section (4). The nature and role of models in Global Instrumentation is discussed in Section (5). This provides the basis for the classification of the different types of the System and Control Problems of GPI presented in Section (6). Section (7) deals with the new control type requirements emerging from the classification of GPI problems, and finally Section (8)

provides a description of long-term issues and problems, which are integral parts of the overall problem area.

3.2. GENERAL ASPECTS OF THE FIELD OF INSTRUMENTATION

3.2.1. Classification of Issues

The selection of the set of inputs and outputs in systems is a problem that is intimately linked to process modelling and has many aspects, which make it a clear multidisciplinary study area. In this section, we briefly examine the different aspects involved in the general problem and which are potential contributors into the development of an integrated methodology to Global Process Instrumentation. A diagram describing the overall area is given in Figure (3.1) below:

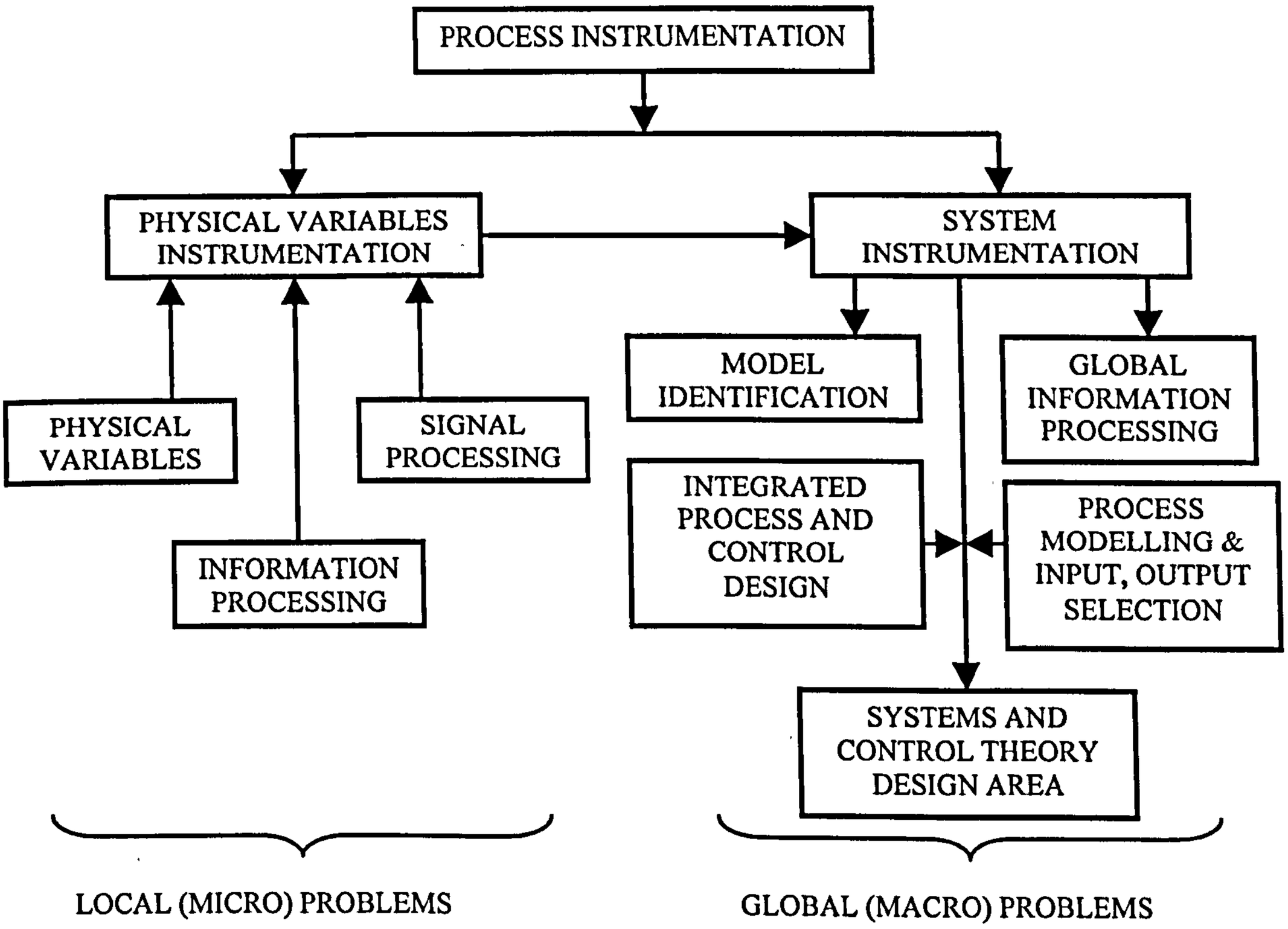

Figure (3.1): The area of Global Process Instrumentation

Modern Instrumentation [Fink. & Grat., 1], is the integration of many different technologies. They range from the technology of information and knowledge processing systems to the engineering of sensors and actuators involving the advanced use of physical

and chemical effects and the application of optical, sonic and semiconductor device technology. The primary concern of this technology is the engineering of devices and systems to meet a need, or to exploit advances in enabling means. The overall area of Process Instrumentation is divided into two main categories representing complementary and mostly disjoint aspects of the overall field. These are:

(i) Physical Variables Instrumentation

(ii) System Instrumentation

The first category deals with issues related to measurement and actuation of physical variables and this is what is traditionally recognised as Instrumentation. The emphasis is on the single physical variables and questions related on the effect of such actions on the overall system and shaping of its properties are not considered. It is for this reason that this area will be referred to as "Physical-", "Micro-", or "Local-Instrumentation". The physical and chemical properties are central in the modelling of sensing and actuation devices. Such devices deal with information and signals and thus the overall area heavily relies on methodologies and tools from Signals, Information Processing and more recently Knowledge engineering (intelligent sensors etc.) [Fink. & Grat., 1]. What we are concerned here is the second area, which is referred to as System Instrumentation, and deals with the classification of process variables and the definition of effective systems of process inputs, outputs from the many different alternatives on a given process. It is clear that Physical Instrumentation plays a crucial role in specifying and classifying the different alternatives. It is assumed however here, that the particular aspects of physical instrumentation are addressed after the classification and input, output structure are specified out of the given alternatives. The dominant feature in System Instrumentation is that we examine the effect of variable classification and input, output selection on the features, properties and quality of the resulting model. Thus, it is the shaping of system properties rather than our ability to measure, or act upon physical variables, which is the theme of the System Instrumentation area. Issues related to System Instrumentation have started to emerge in areas such as Process Control, Flexible Space Structures etc., but there has been no systematic effort so far to identify generic problems (application area independent) and develop a Systems and Control Theory/Design Methodology for this area. Even within the specific application areas the available results and techniques are partial and rather weak.

The development of a Systems and Control framework and approach for System Instrumentation has a unifying, integrating role for all other aspects of the topic. The overall area is clearly multidisciplinary and apart from the Physical Instrumentation issues, important additional aspects, which relate to the problem, are:

(a) Model Identification

(b) Global Information Processing

(c) Process Modelling and Input-Output Selection

It is because we address the overall system aspects, rather than the individual physical variables, that we may refer to System Instrumentation also as "Global" Instrumentation. The input-output selection in the context of system modelling is closely related to our tasks here and is briefly considered below.

3.2.2. Process Modelling and Input-Output Selection

The selection and classification of process variables is an integral part of the overall exercise and it is influenced by (a) the purpose, which the model is to serve, and (b) the boundaries of the system to be modelled. The purpose of the model clearly influences the choice of relevant variables to be included in the model, the detail and accuracy desired of the model and the procedures necessary to derive it. Given the purpose of the model, the next step is to specify the boundaries of the system, which is to be modelled. For example, we may be concerned with developing a mathematical model of an entire corporation, of a refinery or an integrated plant, of a processing system, of a unit process such as an individual heat exchanger, or we may desire a model of the flow pattern in the elbow joint of a pipe; each of these is an appropriate subject for modelling. The location of boundaries determines the particular variables, which must be taken into consideration, as well as their status as independent and dependent quantities in the model. The above two factors are instrumental in the overall classification of variables and are considered as external modelling factors in the classification process.

From the point of view of control, the typical process can be looked upon as a multivariable system with a number of input and output variables. The inputs are the independent variables of the process; they may be considered as casual factors, in the sense that the dependent variables, the outputs may be considered as effects, or responses to

inputs. A diagram summarising the classification of variables is shown in Figure (3.2) [Savas, 1], where the independent and dependent variables are classified further into controlled-uncontrolled and performance-intermediate respectively. Such a classification is intimately related to the purpose and boundaries of the modelling exercise. The classification of process variables is discussed below:

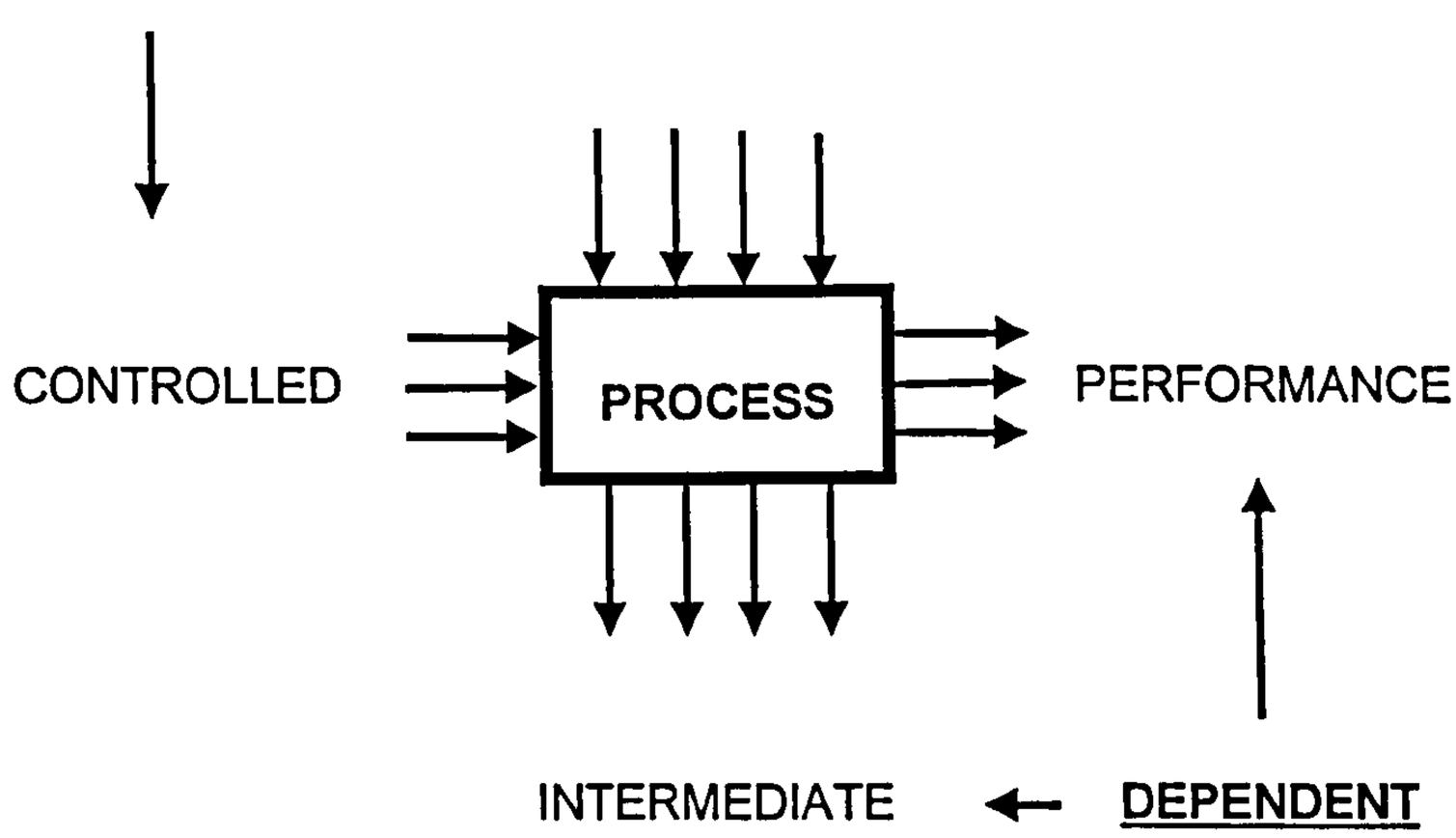

Figure (3.2): Classification of Model Variables

(a) **Uncontrolled Variables:** An uncontrolled variable, also called a disturbance, is a quantity which affects the process operation, but over which the operator has no direct control; its value is often determined by some known, or unknown agency external to the process boundary. Uncontrolled variables may be classified into five categories, those with **(1)** raw materials, **(2)** ambient conditions, **(3)** equipment condition, **(4)** economic factors and **(5)** loading effects.

Raw material variables are an omnipresent source of process disturbances and affect virtually all processes. The particular disturbance variable or set of variables associated with raw materials express variations from nominal conditions and clearly depend on the application area. Such disturbances may be less severe if the material entering the process is an intermediate product derived from a prior process where some control action has been taken to smooth the effect of variations. Ambient conditions constitute a second major category of process upsets. Variables such as the temperature, humidity and sometimes pressure have frequently to be considered, since their nominal values may change. The changing state of equipment condition and materials is the third source of disturbances and

many of the more obscure disturbances originate here. Special modelling effort is needed to include such changes as appropriate variables in the model. Another class of disturbances may be termed economic, or managerial disturbances and they are associated with raw-material costs and product prices variations with time, as a result of which a different mode of process operation is required. Although very frequently the latter changes are modelled as disturbances, there is nowadays the tendency to view them as discrete event type inputs coming from process recipe setting area and thus hybrid modelling and control techniques emerge as important in this area. Finally it should be mentioned that the specification of modelling boundaries, turns a number of variables linked to the interconnection of the system to its environment, into loading disturbances; this implies that design on the given boundary system must try to compensate the effect of such variables. In a larger boundary model the nature of such variables changes.

In reality, it is never possible even to identify all the disturbance variables of the process, let alone measure them. The best procedure is to include in the model the major disturbances, which afflict the process and ignore factors involving second-order effects. The latter approximation will render the minor disturbances indistinguishable from model errors and thus they may be handled by robust control methodology. Special disturbance rejection methodologies may then be used to handle the major process disturbances.

(b) **Controlled Variables:** These are variables over which the operator can exercise control. Such variables may be classified as basic control variables and transformed control variables. The first are primary variables which the operator can handle. Usually, in analysing the process variables and formulating a model for control purposes, it is more convenient to think not in terms of the basic control variables, but rather in terms of a set of transformed control variables linked to some fundamental properties of the process. Clearly, the transformed independent controlled variables do not form a unique set, but depend on the preferences and approach of the designer. However, each model must be internally consistent and the number of transformed independent variables must be equal to the number of basic control variables.

Given the past and present values of the independent variables, the dependent quantities are completely determined. Dependent variables enter the model for two reasons; either they are directly related to process performance or they arise as intermediate variables, which indirectly affect the operation of the process. Thus, we distinguish:

(c) **Performance Variables**: Performance variables are those which serve to evaluate directly the performance, or condition of the process. In practise, these are the variables, which the operator should constantly bear in mind while running the process. We may classify these variables into: **1)** Economic variables, **2)** Constrained variables. The first family includes those which provide a direct measure of the economic performance of the process. According to the nature of the process and the management policy, a number of such variables are specified. The second category, the constrained variables, includes quantities, which are restricted, or limited to a certain range of values. Constrained variables are further classified to physical and managerial types. Physical constraints are imposed principally by capacity, safety etc., considerations, whereas managerial constraints relate to policy decisions. In the latter family we distinguish those related to product quality and size of production. In general, these are many quality and quantity constraints on process and their nature is limited to the particular physical and operational characteristics of them.

(d) **Intermediate Variables:** Intermediate dependent variables constitute the remainder of the pertinent process variables. They are not of direct, immediate, or explicit use in evaluating the performance and conditions of the process, in the sense that they do not have direct economic impact, nor are they explicitly constrained. Their role however may be significant in the overall control of the process, as well as the development of advanced schemes for evaluating key quality variables, which cannot be directly measured.

3.3. THE CASCADE DESIGN NATURE OF ENGINEERING DESIGN

3.3.1. Integrated Design and its Requirements

The general problem of Global Instrumentation has a number of aspects, which are inherited by the fact that this activity is one of many stages in the overall design of the process. Some of the general features of this embedding are considered here. The dominant trend in the synthesis of large complex processes is the division of the problem into subproblems; then, each subproblem becomes the task of a group of specialists. The process synthesis/design is characterised in general by the following general steps [Shigley, 1]:

(1) Recognition of need, General objectives

(2) Definition of problem, Specifications

(3) Synthesis

(4) Analysis and optimisation

(5) Evaluation

(6) Presentation

Such a procedure is clearly iterative and it is the result of the technological complexity and possibly large-scale nature of the engineering task. The complexity of large engineering processes implies that a combination of different skills and expertise are required for the solution of the problem and this results in a division of labour. This is manifested by the existence of divisions in a modern engineering firm, each one of them dealing with the particular aspects of the overall problem. The process synthesis/design problem has a cascade nature with feedback loops between the various substages or blocks. The present aim is to investigate some important problems arising due to the cascade nature of design, which hinders the process of achieving the final task; this will demonstrate the need for a new design philosophy that may help to overcome the difficulties associated with the current practices. A representative diagram of the cascade procedure for the design process is given in Figure (3.3).

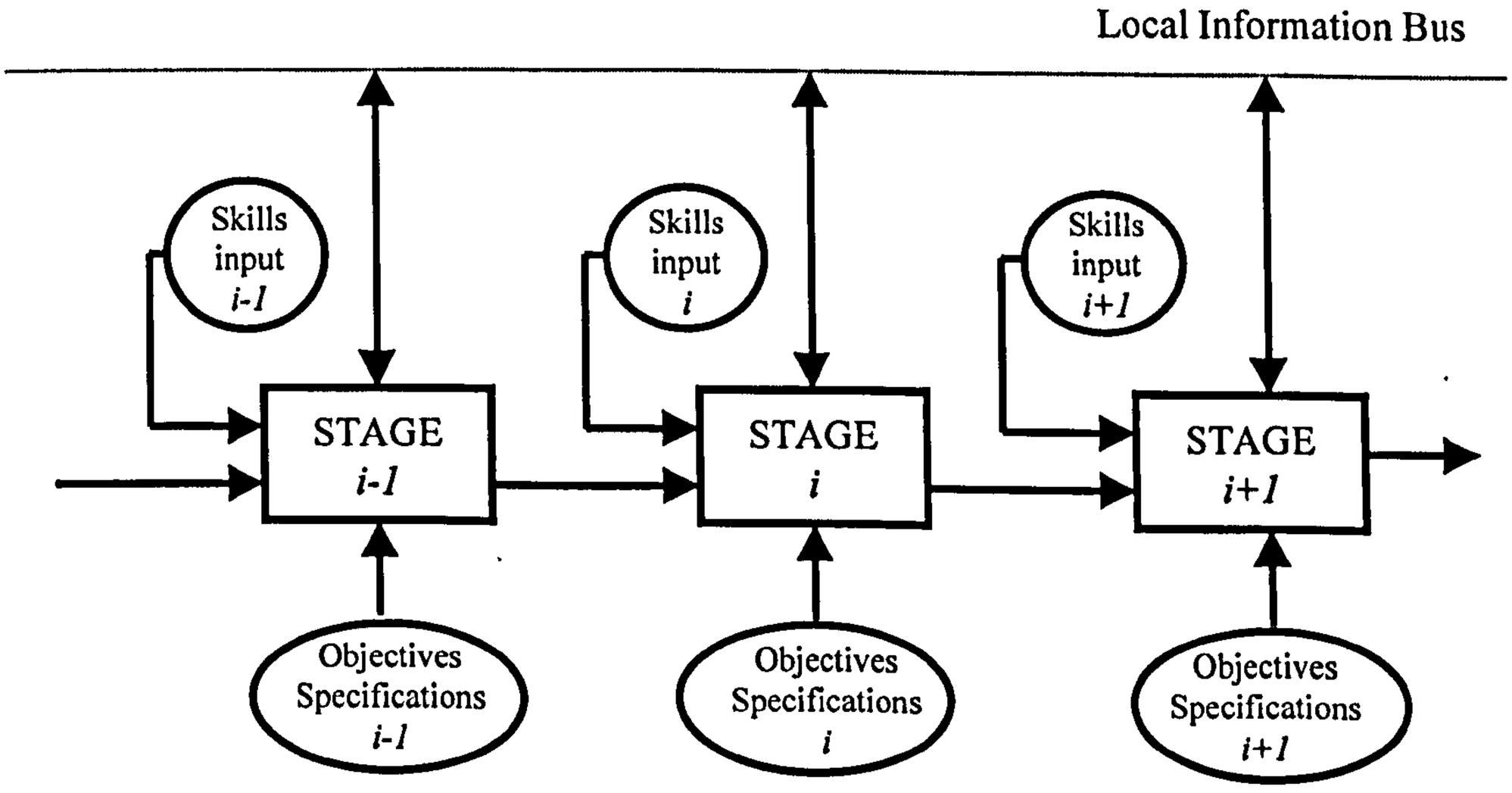

Figure (3.3): Cascade Design Process

The main inputs at every design stage are the special skills, body of knowledge, the local objectives and specification and the final result, model of the previous design stage.

Secondary inputs are provided by the exchange of information between the given stage and the other design stages, whenever they exist. The secondary inputs express the iterative, trial and error nature of the overall design process. In fact, this exchange of information is mostly empirical and decisions taken at previous stages are evaluated by means of simulations at the successive design stages. For most cases, there is no "a priori" knowledge of the implications of the decisions taken on stages..., (i-2), (i-1), on the nature of possible results that may be achieved at stages (i), (i+1),... without going through a complete design exercise at the successive local stage. Defining "a priori" a tight set of specifications for every local design stage is also difficult, since what is the best that may be achieved locally is not clear for the above mentioned reasons.

The trial and error nature of the overall design procedure is time consuming, uneconomic and very frequently not possible, as far as major design alterations in the previous stages. Experience on similar nature designs, is always of immense value; however, it might create some inertia that hinders the testing of new ideas. For a large number of process designs, previous experience may not be highly relevant, since even changes on the size of a given process may lead to a drastic change in the original dynamics and thus may require an entirely new design and use of technology. The cascade design procedure is dynamic in the sense that what it is feasible to achieve at a given stage is influenced by the decisions taken at the previous design stages. It is thus a characteristic feature of the cascade design process that decisions taken at one stage, which may been seen as technically reasonable and economically sound according to local criteria, may not necessarily be good as far as the overall design process. This is due to the fact that the overall system tends to display behaviour that is not an aggregate of partial behaviours.

3.3.2. Global Co-ordination in Integrated Design

The problems associated with the cascade mode of design is the price which has to be paid by moving from the "artisan design mode" to the "modern design mode" dominated by division of the overall design to a cascade structure of subdesigns. The characteristic of the "artisan design model" or "small scale integrated design" is that the designer has overall control and knowledge of the whole process; knowledge of the overall process may be empirical, or scientific, and it helps to overcome problems that may arise in the various substages of design by enabling the designer to foresee the impact of decisions taken at a time on the final design. The global empirical or scientific body of knowledge of the small-

scale designer, or craftsman (artisan) is indispensable; this holistic understanding of the overall design process will be referred to as global co-ordination knowledge [Karc., 7].

A global understanding by a single agent of the "cascade", or "modern design" of most of large-scale processes is missing. The main reasons for this are: (i) the high complexity of modern engineering designs, (ii) the need for use of highly specialised engineers (lacking most of the time a global understanding and view of the process) and, (iii) probably the most important of all, the fact that the simple empirical rules of the small scale designer cannot be readily extended to large scale complex designs. The lack of a global body of knowledge and rules for the overall design is due to the fact that knowledge of a whole process synthesis/design is not arrived at through the particular sciences, bodies of knowledge. Such knowledge has to be above the specialised features of the particular science, it must be general enough and detached from the particular characteristics of the subprocess under study, capable of accepting specification and design constraints in a unifying manner and organised in the form of a theory that would allow global and local co-ordination and direction of the overall process synthesis/design task. The holistic, global nature of such a theory indicates that it has to be system theoretic in character. We shall refer to such a theory as Global Co-ordination Theory (GCT). The present work aspires to contribute in the long term in the formulation and development of such a theory and associated methodologies for the design of new, or redesign of industrial processes. The process instrumentation is the area of special interest in this project. Areas such as process synthesis and integration of higher-level control activities with lower level ones may be also considered within this framework. The desirable, general characteristics of a co-ordination theory for the process synthesis/design procedure should include [Karc., 10]:

(a) Ability to interpret tools, concepts, decisions, specifications, constraints and results associated with the particular discipline associated with a given design stage in an abstract language which may be shared by all design stages. This is necessary for the exchange of information between the various design stages. In particular, it is necessary to formalise the final result of a design in the form of a model that may be understood by all other stages of design.

(b) Ability to evaluate the effects of local decisions (at a given stage) on the resulting local model of the particular design stage (local model evaluation property).

(c) Ability to interpret the implications of the structure of local model (expressing the cumulative effect of local decisions at a design stage) on the range of possible

structural characteristics, compatible specifications, operability features and achievable limits of performance of following design stages. In more general terms, this implies ability to interpret local decision making in the context of global decisions and somehow predict their implications for the decision making of successive design stages.

(d) Ability to formulate global criteria, which may be interpreted at the local level and then used for the evaluation of local decision making. This is necessary to enable the formation of a "good" sequence of local decisions, based on local, as well as global criteria.

In diagrammatic terms we may illustrate the role of global co-ordination theory as in Figure (3.4):

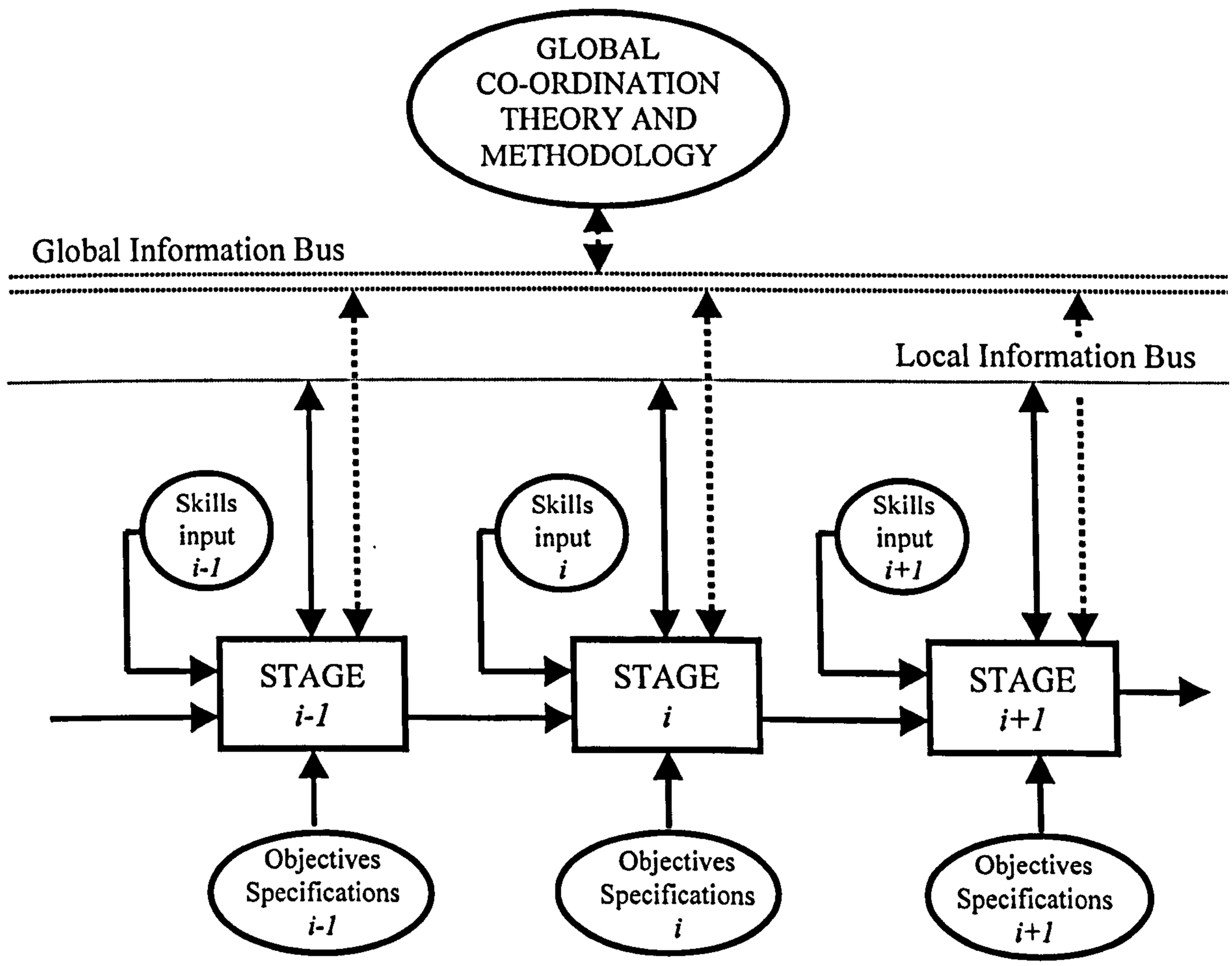

Figure (3.4): Coordinated Cascade Design

The high level nature of Global Co-ordination Theory (GCT) implies that its language is that of the model, and thus it has to be of system theoretic character. The aim of GCT is to assist the designer in taking local decisions with local, as well as global criteria. Thus, the language of GCT has to be of some mathematical nature, which allows computations predicting the effect of local decisions on the resulting structure of models of successive design stages.

3.3.3. Model Structure Evolution in Integrated Design

In the following, the term design stage model is understood as the given purpose mathematical model, expressing our understanding of the interactive compositions of specifications, local and global criteria, and design decisions for all previous stages up to the one we consider. Such a model is used as an input to the next design stage, as shown in Figure (3.5) below, and demonstrates the dynamic nature of the cascade design process (past decisions affect the range of options for the present decision making).

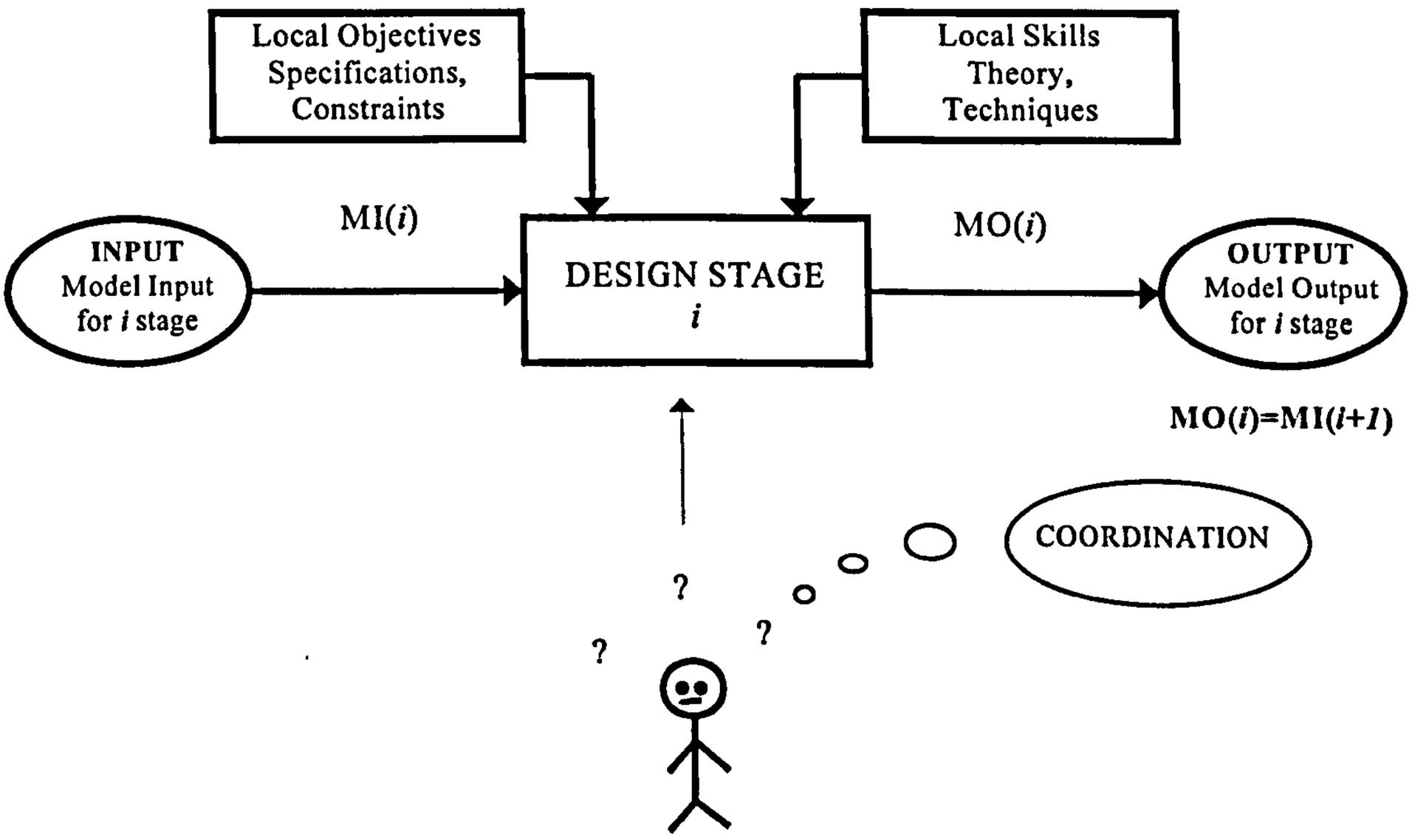

Figure (3.5): Cascade Design and Model Evolution [Karc.,10]

A crucial importance, desirable feature of GCT is its predictive character; being able to interpret local decisions on the level of design stage model, which allow the classification of them into those having a structural impact on the model, and those having a parameter adjusting effect on the model. The first will be referred to as structural decisions, whereas the latter as parameter tuning decisions. Since structural decisions precondition the

characteristics of design models of the following design stages, they have a broad horizon and qualifying them is essential in the development of the predictive character of GCT. Parameter tuning decisions, on the other hand, may be altered by next design stages structural decisions and thus usually have a short horizon. However, understanding such decisions is essential, especially at the last design stage, when the exact values of the model parameters will determine the quantitative properties of the final mode.

The formation of structural characteristics of the overall process is reminiscent of an evolution process [Karc., 10]. The first stage synthesis, acts as the parent gene and thus predetermines a possible range of central characteristics of the final process. Structural decisions on the successive design stages contribute to the gradual shaping of the final structural characteristics, however, within a range of possible options; such decisions correspond to the sequence of successive mutations. Structural properties evolve, but not in a simple additive, or multiplicative manner. An essential difference between gene evolution and process model evolution is that in the latter case it might be possible to go back and alter a previous stage design; very frequently, however, only minor modifications may be feasible. Ideally, we would like to have assigned certain desirable characteristics to the model of every single design stage and thus finally guarantee the shaping of a process with fine tuned properties. This requires perfect control of the model evolution process, which is not feasible. In fact, such a task requires immense resources in modelling. However, not all activities may be modelled with the required accuracy; furthermore, what is desirable as final design is impossible to predict at the beginning and thus to interpret it as partial prespecified objectives for each design stage. A feasible design philosophy, which may be adopted, is that of directing the model evolution process towards final designs that may possess desirable properties with high probability. This implies that in early design stages we have to make structural decisions that exclude undesirable properties. Naturally, whenever the possibility of assigning desirable properties arises, this should not be missed. The main effort of assigning the desirable properties to the final design is then left to the final design stage. The main role of GCT is thus to provide the concepts, tools and techniques that may direct the overall design along good branches of the model evolution tree. GCT has also an important role to play in the evolution of compatible specifications for the final design. It should be emphasised that understanding issues of composite structure formation and model evolution in cascade design are prerequisite to the development of methodologies for concurrent engineering [Pars. & Sull., 1]. The above methodological framework is specialised next to the engineering design stage of instrumentation.

3.4. GLOBAL INSTRUMENTATION WITHIN THE FIELD OF INTEGRATED DESIGN

The specific role of the selection of measurement and actuation variables in the context of overall process design is examined in this section. This also serves to illustrate the general philosophical approach on integrated design.

3.4.1. The Main Design Stages and the Need for Integration

The selection of systems of actuation, sensor variables, referred to here as Global Instrumentation is part of three main engineering stages represented in Figure (3.6). The general features of the technological stages are briefly considered first, before we focus on the significance of Global Instrumentation.

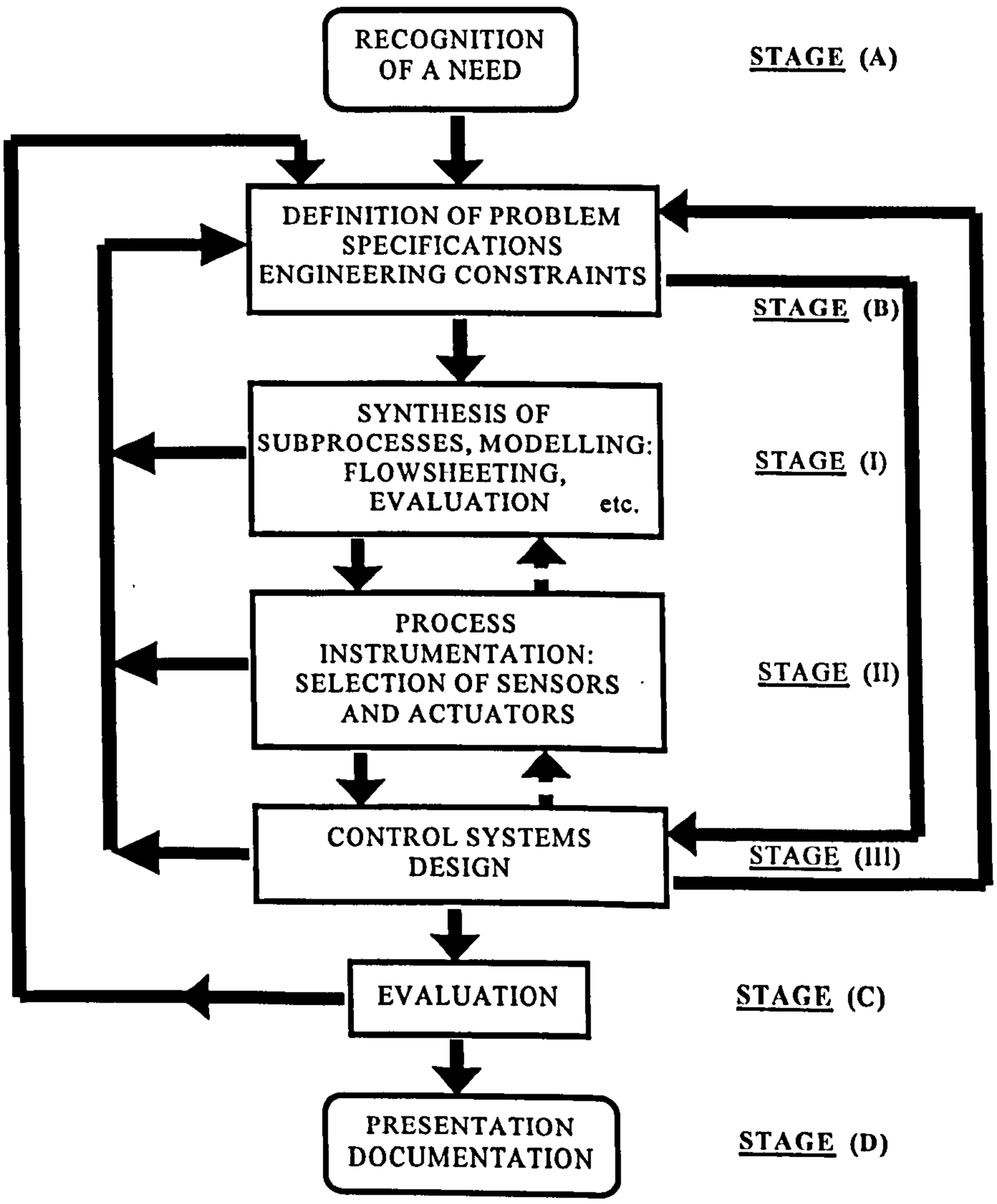

Figure (3.6): Simplified Form of Engineering Design Process [Karc., 10]

Our attention is focused on the purely technological nature stages of design, that is:

STAGE (I): Process Synthesis

STAGE (II): Process Instrumentation

STAGE (III): Process Control.

Each of the above stages operates under a set of engineering specifications and constraints, which together with the economic constraints define the boundaries of the local decision making. Experience from building similar processes provides rules, guidelines of what you can do and what you should avoid. This body of knowledge is indispensable, but not sufficient for the fulfilment of the original task, that is deriving final designs with the minimal effort and economic cost and which have desirable performance characteristics. In fact, we may view this empirical knowledge and rules as an intermediate stage co-ordination layer with a rather short prediction horizon. GCT aims at enlarging the knowledge required for an improved process synthesis by introducing system and control based criteria, rules and techniques.

The development of GCT for the above design stages is a very long-term task, which requires considerable effort. The raw material for such a theory may be found in the areas of Systems, Control Theory and Design, Information Theory etc., since they deal with the properties of system models. However, the problem of evolution of model structure through the successive design stages has not been considered before. Over specialisation and division of tasks in engineering enterprises has implied that process engineers have no understanding of the effect of their decisions on model structure shaping and control engineers have assumed that the system is already formed and have not examined the mechanisms of model structure formation. There is no understanding in any significant degree of the mechanisms of model structure formation in the early stages of design and this defines a new challenging problem. Building GCT is thus equivalent to introducing Control Theory and Design concepts and tools at early design stages. This is a new challenging role for Control Theory and Design, which has to develop within the framework of traditional process and instrumentation design concepts and methodologies. This latter aspect is an important additional dimension of the "conceptual framework" of GCT. A diagram summarising the overall structure of GCT is given below:

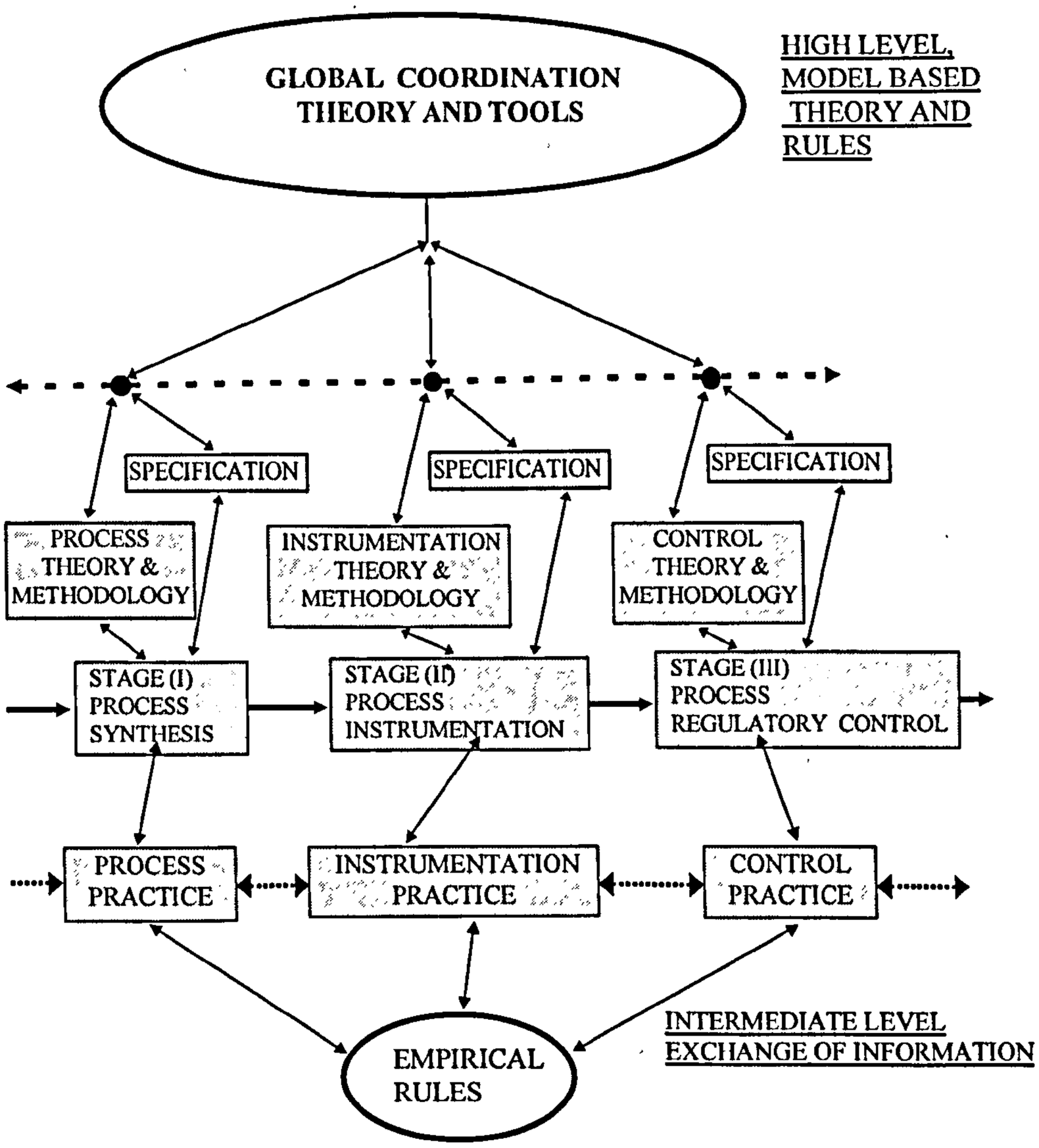

Figure (3.7): General view of GCT

3.4.2. Global Process Instrumentation and its Model Shaping Role

The traditional role of instrumentation, referred to here as the "micro" role, is well developed and deals with the problem of measurements or implementation of action upon given physical variables [Fink. & Grat., 1]. This is closely related to the physics of the particular problem and issues related to signal processing are also crucial. The focus point in traditional instrumentation is the particular variable, whereas the effect, significance of such selection on the shaping of the overall process model is not considered. It has been noted [MacF. & Karc., 1], [Kouv. & MacF., 1], [Rosen. & Power, 1] that the selection of sensors and actuators (their location, as well as the way we measure, implement action) plays a decisive role in the formation of the characteristics of the final design and it is this role, which will be referred to as "macro" (global) role of instrumentation.

The internal process characteristics, dynamics (result of the process synthesis design stage) are essential, since they determine the progenitor basic characteristics of the final design; however, the manner we observe and try to act upon the variables of the progenitor system, determines the final characteristics of the system. The final system model is the product of interaction of the internal dynamics and its environment. The role of instrumentation (both micro and macro aspects) is in the building of bridges between the internal mechanism of the process and the system environment and a simple diagram illustrating this process is that of Fig. (3.8).

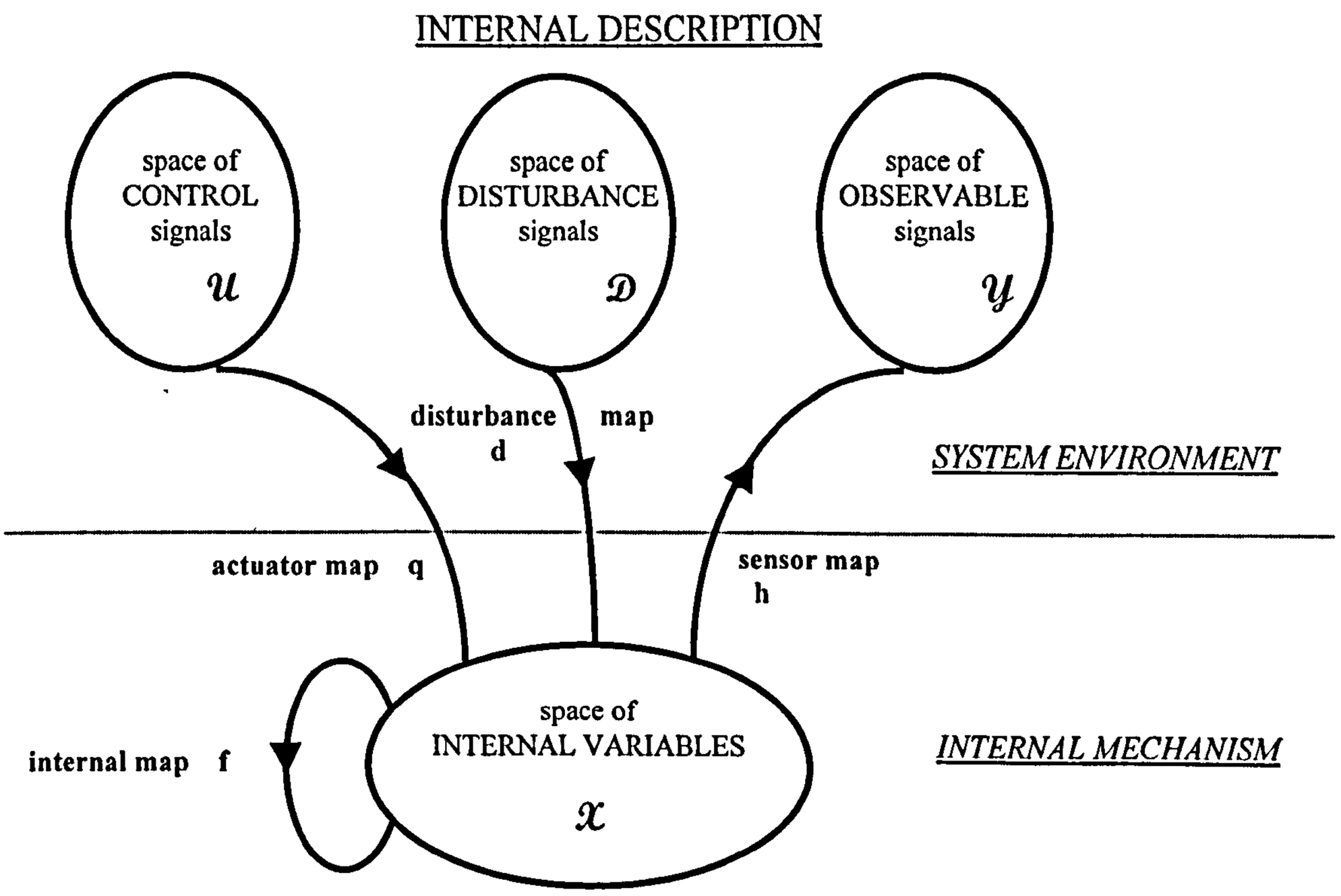

Figure (3.8): System Model Structure and Instrumentation Maps

What the "observer" understands as a system is the set $\Sigma = (\mathcal{Z}, \mathcal{U}\ \mathcal{Y}, \mathbf{f}, \mathbf{q}, \mathbf{h})$ of signal spaces and inter-relationships between them. Clearly, the properties of Σ express the cumulative effect of the composition of the **f, q, h** maps and thus their formation is affected by the selection of the actuator, sensor maps constructed by the "observer". The map **f** expresses the cumulative effect of process synthesis, whereas **d** expresses the linking of disturbances to internal dynamics. The maps **q, h** are those which may thus be considered here as design parameters in the shaping of characteristics of Σ, viewed as an information processing device or object to be controlled. The "macro" aspects of instrumentation have to do with the design of the **q, h** maps using global criteria and techniques, stemming from the information processing, control capabilities of the resulting system. Clearly, the "macro"

aspects of instrumentation have to develop within the framework of constraints imposed by the traditional instrumentation practice.

The main tasks involved in the development of concepts, methodology and tools for Global Instrumentation are:

(i) Characterising the desirable and undesirable performance characteristics of the overall system and the limits of what best can be achieved under compensation.

(ii) Relate the best achievable performance, or undesirable performance characteristics to the system model structural type characteristics and their values.

(iii) Establishing the functional relations between model parameters and structural characteristics.

(iv) Formulating and solving suitable structure formation problems.

The overall problem which is considered is an attempt to shape the final characteristics of the process model that emerges from the process synthesis and process instrumentation stage, and thus make the final control design problems as simple as possible with natural consequences on costs, operability, safety etc. of the final process. The formation of structural characteristics of the overall process is reminiscent of an evolution process. In fact, each design stage starts with a model (parent gene) and decisions taken there contribute to the gradual shaping of the final structural characteristics, however, within a range of possible options; structural properties, characteristics, however, thus evolve, but not in a simple manner. The main objective is to drive the model evolution along paths avoiding the formation of undesirable structural characteristics and where possible to assign desirable ones. The main issues raised in (i) - (iv) may be discussed within the general area of Control Theory and Design (CTD); however, only (i) has been the main theme within CTD and to lesser extent (ii) (see for instance [Kailath, 1], [Maciej., 1] and references therein). The topics addressed in (iii), (iv) have been considered in the particular case of zero assignment [Kouv. & MacF., 1], [Rosen. & Power, 1], [Karc. & Gian., 1], whereas structural methodologies of the graph type have been deployed in the search for feasible control structures [Mor. & Steph., 1], diagnosis of control difficulties in early design stages [Russ. & Perk., 1], classification of internal variables into input, outputs [Georg. & Fl., 1] etc. With the exception of the zero assignment, most of the other approaches deal with diagnostics, rather than try to define a synthesis methodology based on both aspects of structure (graph and parameter dependent invariants).

3.5. THE MODEL ENVIRONMENT OF GLOBAL INSTRUMENTATION

The characteristics and nature of Global Instrumentation depend on the type of available possible models, used to describe the system; this is referred to as the "Model Environment" of the problem. Depending on the nature of the process and the modelling approach, which is used, we distinguish the following three classes of models:

(i) Internal Models (Ims)

(ii) External Models (Ems)

(iii) Composite Models (Cms)

(i) **Internal Models:** These are described in terms of ordinary nonlinear differential equations and if they are first order, they are called state space models (SSMs). The system dynamics of SSMs are represented by:

$$\dot{\underline{x}} = r(\underline{x},\underline{u}),\ \underline{y} = h(\underline{x}) \tag{3.1}$$

where h, r are vector valued functions representing the sensors, composite internal, actuator maps respectively. In the case of linear systems, (3.1) becomes

$$S(A,B,C):\ \dot{\underline{x}} = A\underline{x} + B\underline{u},\ \ \underline{y} = C\underline{x} \tag{3.2}$$

and the $n\times n$, $n\times \ell$, $m\times n$ matrices A, B, C represent the internal, actuator, sensor maps respectively. If all physically possible variables, that may be acted upon and measured are included, then the model is referred to as extended SSM (ESSM). A more general family of internal models that frequently arise are the Implicit Systems [Lewis, 1], [Karc. & Kalog., 2], which in the linear case may be represented as:

$$F\dot{\underline{\xi}} = G\underline{\xi} \tag{3.3}$$

where F, G are $r\times k$ matrices and $\underline{\xi}$ is a vector of all possible internal variables, where there is no distinction between controlled and measured variables. This description is close to the spirit of the behaviour based approach to systems [Will., 1].

(ii) **External models:** If $\mathcal{V}$, $\mathcal{Z}$ denote the spaces of all potential inputs, measurements, referred to as extended input, output spaces respectively and v, z are the corresponding p, q-dimensional vectors, then the internal map f is a vector values function f: $\mathcal{V} \rightarrow \mathcal{Z}$ where $\underline{z} = \mathbf{f}(\underline{v})$. For the case of linear, time invariant systems f is a convolution function, or it is represented by the qxp rational transfer function matrix F(s), for which:

$$\underline{z}(s) = F(s)\,\underline{v}(s) \tag{3.4}$$

Note that $\mathcal{V}$, $\mathcal{Z}$ denote the potential input, output spaces and not the effective ones, which are denoted by $\mathcal{U}$, $\mathcal{Y}$ and have corresponding dimensions ℓ, m. If $\underline{u}$, $\underline{y}$ are the effective input, output vectors, then the corresponding model is illustrated in Fig. (3.9) and is defined by

$$\underline{y}(s) = W(s)\,u(s)\;W(s) = H(s)\,f(S)\,Q(s) \tag{3.5}$$

where H(s), Q(s) are the transfer function matrix representations of the sensor, actuator maps respectively. We shall refer to the system models S(A): $\dot{x} = A\,x$, F(s), as internal - external-progenitor models respectively; in fact, S(A), F(s) act as parents for S(A, B, C), W(s) descriptions and (B, Q(s)), (C, H(s)) should be treated as design parameters in Global Instrumentation.

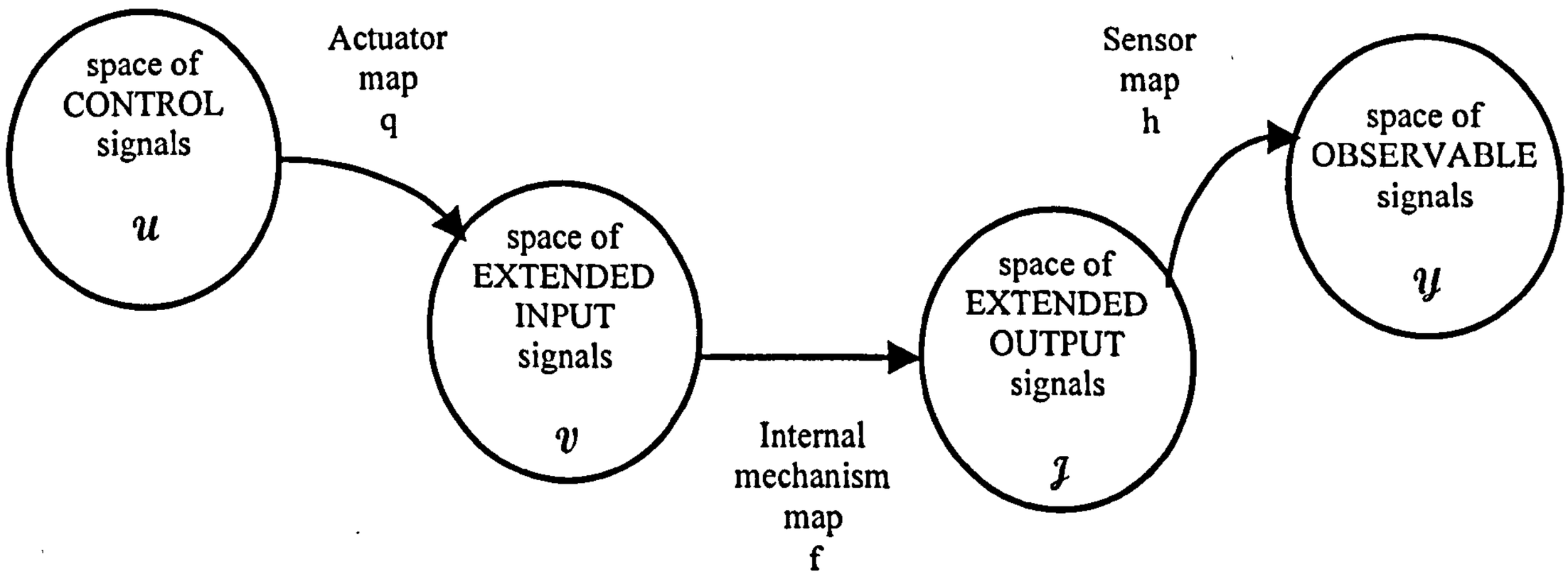

Figure (3.9): External Progenitor Models

(iii) **Composite Models:** A large process is always synthesised by connecting subprocesses and the two fundamental ingredients of the composite system model are: (a) The topology (graph) of system interconnections $\Im$ and (b) The family τ of subsystem models which may be of any of the types discussed before. For simplicity we may assume that each subsystem is represented by a transfer function $G_k(s)$, or by a state space model Σ (A,B,C,D). If $\Im = \{\Sigma_i, i=1, 2,...,\mu\}$ is the set of subsystems, then the system defined as a direct sum of their input, state, output spaces respectively $\Sigma_a = \{\Sigma_1;...; \Sigma_\mu\}$ is called the system aggregate. The interconnection rule $\Im$ (expressing the act of synthesis) applied on Σ_a defines a system:

$$\Sigma_c = \Sigma_a * F = (\Sigma_i, i = 1,...,\mu; F) \tag{3.6}$$

which is referred to as the composite system. Modelling composite systems involves specifying the subsystem model, in terms of the link vectors (input, output), describe the relationship between them and then the overall interconnection rule F. The interconnection rule may be expressed as a vector graph, where the nodes are the subsystem inputs, outputs. Shaping composite systems depends heavily on the selection of subsystem inputs, outputs; although the general characteristics of the interconnection rule are defined by the process synthesis, the dimensionality of the vector transmittances depends on the degree of the subsystem modelling and the selection of subsystem inputs, outputs. The role of subsystem input, output selection in the shaping of the composite system properties is one of the fundamental problems of GPI.

3.6. Variable Complexity Modelling and Early Design

An issue that arises in the overall selection of input, output schemes is the use of models of different type and of variable dynamic complexity and accuracy at the different stages of design. Thus at very early stages, simple, graph type models may be used and progressively at the later stages simple, multiple lag, full order lines and finally non-linear models. This implies that the central issues in the selection of input, output schemes have to be addressed within the context of variable complexity modelling, implying use of graph, simple and progressively more complex and more detailed models.

Starting from a conceptual model Σ^c, the evolution of the models, which preserve the generic structure of the interconnection rule, leads to the following nested set of models:

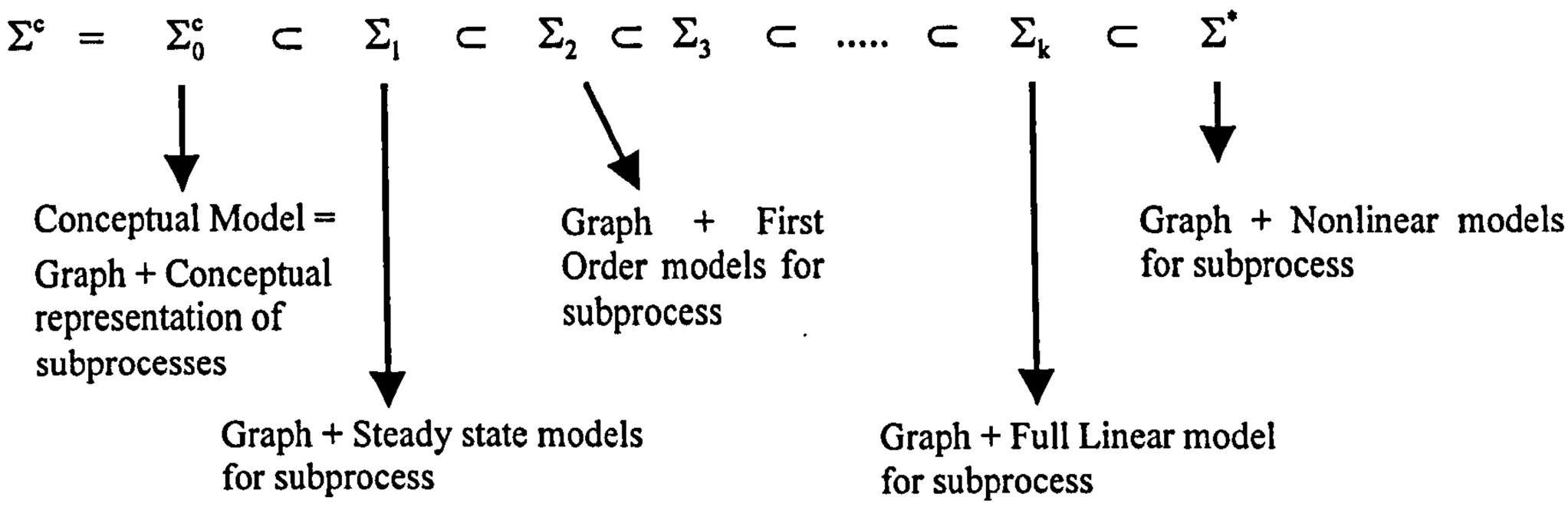

Figure (3.10): Nested Set of Conceptual Models

The model building process can proceed beyond the construction of Σ^*, which can be considered as the simplest nonlinear model that can be used for the evaluation of designs (through simulation). This nesting describes an evolution of the overall system model, which is due to the evolution of Dynamic Richness of the subsystem models and it is due to the time dimension (Early-Late) of the design process.

If $\Im$ denotes the Graph of Σ^c and we denote by $\{\Sigma_i^a, i=1,...,\mu\}$ the aggregate of the simple models of the a-stage, we can denote by Σ^a, the model defined as: $\Sigma^a = \Im * \text{diag}\{\Sigma_i^a : i=1,...,\mu\}$. Note that as model complexity for subsystems increases, we may also consider issues of dimensional expansion and/or evolutionary expansion of the corresponding graph. Instead of assuming a fixed $\Im$ as above we may have that: $M^a = \Im^a * \text{diag}\{M_i^a : i=1,...,\mu\}$, where also the set $\{\Im^a\}$ is ordered in an evolutionary manner i.e. $\Im_0 \subseteq \Im_1 \subseteq \Im_2 \subseteq .... \subseteq \Im_k \subseteq \Im^*$

The above nesting expresses the progressive enrichment of the initial graph that may be due to either model detailed description of the physical interconnection streams (dimensional expansion of graph branches), or addition of new subprocesses (graph growth). Such changes express distinct forms of evolution in the overall model and raise important new issues in the context of systems that may be referred to as Evolutionary Systems (Graph evolution). The family of models generated by both local complexity evolution and possibly graph evolution may be represented as:

EVOLUTION OF MODELS IN EARLY DESIGN

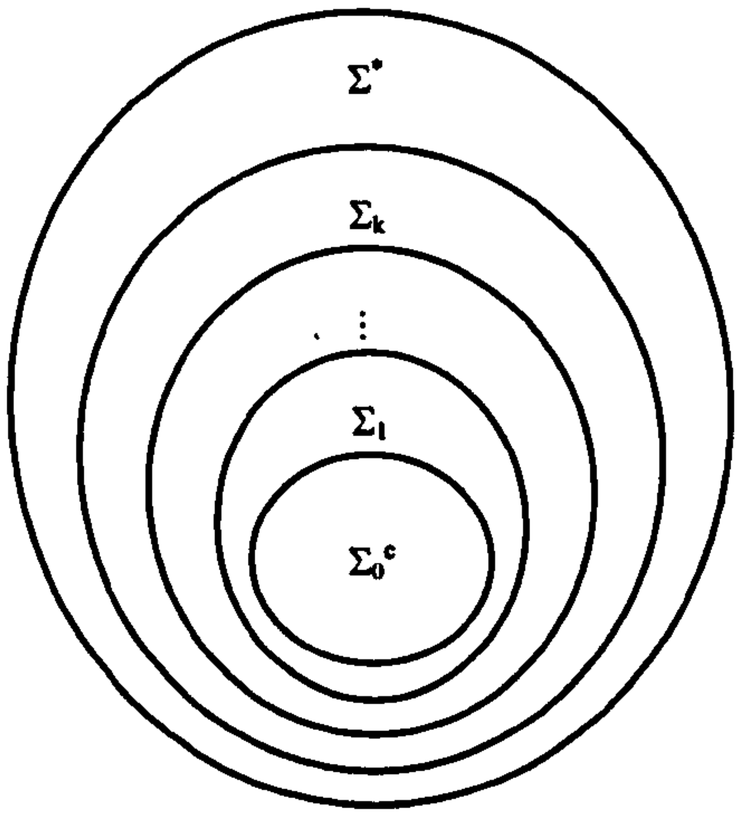

Figure (3.11): Model Embedding Process

A very important issue is to evaluate the mechanisms of this nesting from the modelling viewpoint by studying the effects of subsystem model complexity evolution and graph evolution of physical interconnections on corresponding system structure and properties. This problem can be approached by being split into two stages. Firstly, a procedure for simplification of description of subsystem models is adopted, by using model reduction and other simplification techniques, while we preserve the graph structure as constant. The latter implies a fixed input, output structure for the subsystems. Secondly, we fix the subsystem models and examine the graph variability. By adopting the input-output model and preserving the interconnection graph structure, we may define descriptions using simplifications for each of the subsystem models. This way, we can generate for each of the (a) subsystem the sequences of models:

$$\Sigma_0^a \subset \Sigma_1^a \subset \Sigma_2^a \subset \Sigma_3^a \subset \ldots\ldots \subset \Sigma_k^a \subset \Sigma_{k+1}^a \subset \Sigma_{k+2}^a \subset \ldots\ldots$$

where Σ_1^a is the linear steady state model, Σ_0^a is a simplification of Σ_1^a, Σ_2^a is first order dynamics, Σ_{k+1}^a the nonlinear model with simple Voltera description, Σ_{k+2}^a the nonlinear model with double Voltera description etc.

Finally, a very important point is the reversibility of the Model Complexity Evolution and the Model Simplification Approach, as shown in Figure (3.12). Model Evolution and Model Reduction may become completely reverse processes, if we use fixed input, output subsystem structures and interconnection graphs. This will be referred to as Duality between Model Reduction and Model Complexity Evolution.

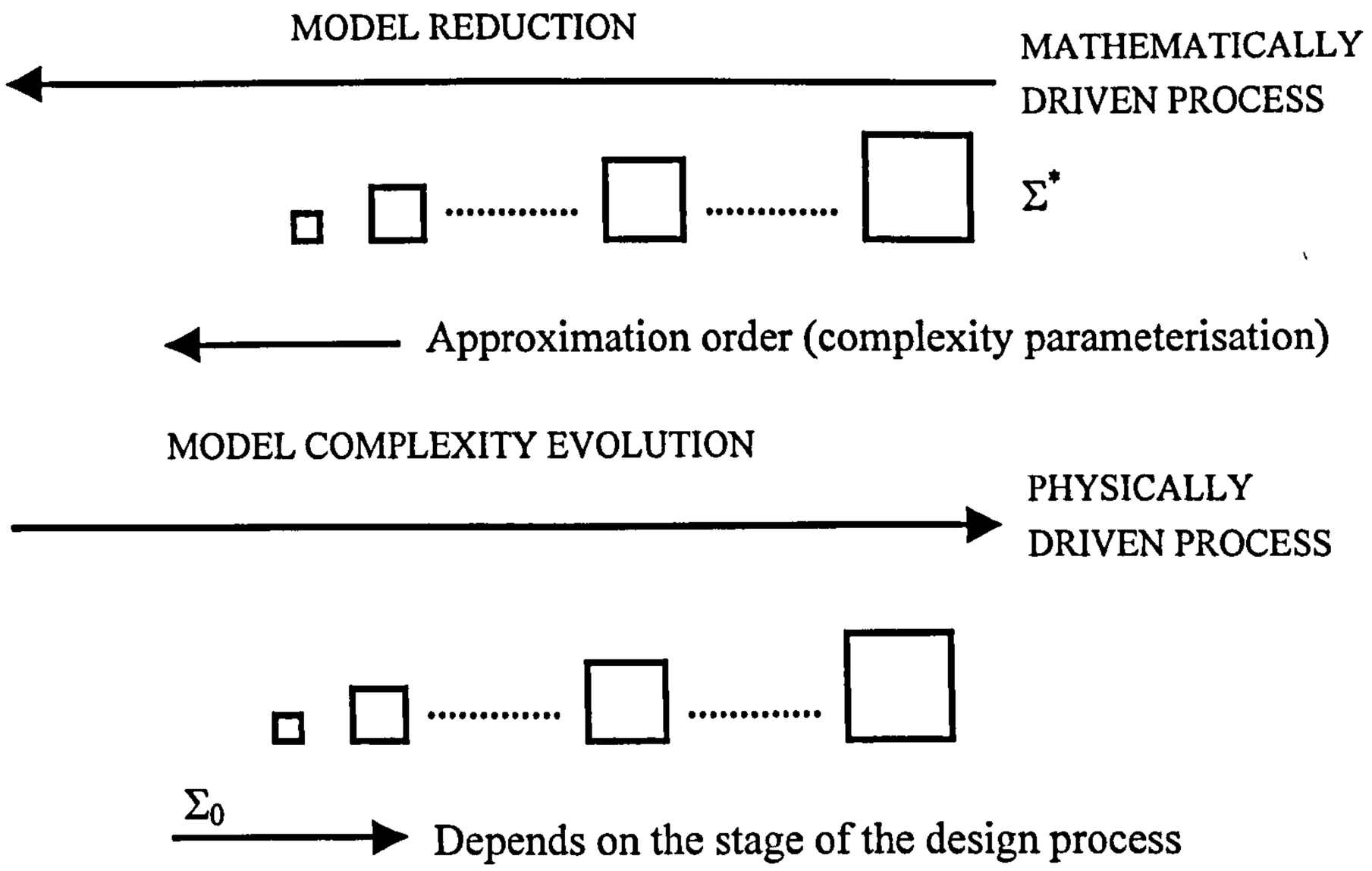

FIGURE (3.12): Comparison of Model Reduction and Model Complexity Evolution

3.7. Fundamental System and Control Problems in Global Instrumentation

In general, we may view instrumentation as the body of knowledge and techniques that allows the selection of input, output schemes for a given process and with certain objectives, criteria. In view of the crucial role, which input/output (i/o) structures play in control and signal processing, a refined classification of the issues involved in the i/o selection is essential. We may classify the issues, problems as follows [Karc., 10]:

(i) Model Orientation Problems (MOP)
(ii) Model Projection Problems (MPP)
(iii) Model Composition Problems (MCP)
(iv) Model Expansion Problems (MEP)

The above classification is primarily based on the nature of issues, which are involved rather than the tools needed for their study. A brief discussion of the main issues involved in each of the above clusters of problems is given below:

3.7.1. Model Orientation Problems

The general problem of the classification of system variables as inputs and outputs is referred to as model orientation. In many systems, the orientation is not known, or that

depending on the use of the system the orientation changes. Questions such as, when is a set of variables implied, or not anticipated by another, or when is it free, have to be answered, if model orientation criteria based on the nature of the process are to be derived; the specific use of the system may provide additional model orientation criteria. It may happen, that the above two types of criteria do not provide a unique solution to model orientation; note that for each alternative orientation we have a different i/o model and thus criteria based on the resulting model characteristics have to be used for the final evaluation, selection. Problems of this type have been recently realised [Karc., 8] and their solution, as far as techniques are concerned, is in its early stages. The need for nonoriented models [Karc. & Hayt., 1], [Apl., 1], [Will., 1] has been realised for many years; however, they have been seen more as unifying descriptions, rather than as objects on which we pose structure assignment problems by selection of input, output variables. In practice, the problem of model orientation is in constrained form, since certain variables have been already assigned the role of input, or output; for the sake of simplicity of the presentation we consider here the general free problem, where all internal variables are free, as far as becoming either input, or outputs.

We consider implicit descriptions of state space type (3.3) or more generally autoregressive forms of the type

$$H(p)\underline{\zeta} = 0, H(p) \in \Re^{vx\mu}[p] \tag{3.7}$$

Descriptions of the type (3.3), (3.7) naturally arise in many practical situations [Apl., 1]; the vectors $\underline{\xi}, \underline{\zeta}$ are referred to as <u>implicit</u> vectors and contain all variables of importance to the structure of the system without making a distinction between control, observation, internal dynamic variables and without making any assumption on their independence. Although the study of dynamics may be carried out on implicit nonoriented forms [Karc. & Hayt., 1], [Apl., 1], [Will., 1], when it comes to observing, controlling, or trying to connect the process as part of a composite structure, this classification is essential. The free version of MOP is defined as follows:

<u>Definition (3.1)</u>: Given the matrix pencil implicit model of equation (3.3) define a transformation $Q: \underline{\xi} = Q\hat{\underline{\xi}}, Q \in \Re^{kxk}, |Q| \neq 0$, such that (3.3) is equivalent to:

$$\begin{bmatrix} p\hat{E}-\hat{A} & -\hat{B} \\ -\hat{C} & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ u(t) \end{bmatrix}=\begin{bmatrix} 0 \\ -y(t) \end{bmatrix} \tag{3.8}$$

and where $\underline{x} \in \Re^n$, $\underline{u} \in \Re^t$, $\underline{y} \in \Re^\ell$. The system $S(E,\hat{A},\hat{B},\hat{C})$ is called an orientation of the (pF-G) and $\Sigma(F,G)$ denotes the family of all such systems.

Given the polynomial implicit model of equation (3.7) define a transformation $R(p):\xi=R(p)\,\hat{\xi}$ $R(p)\in\Re^{\mu x\mu}[p]$, $|R(p)| = c\neq 0$ such as (3.7) is equivalent to:

$$\begin{bmatrix} T(p) & U(p) \\ -V(p) & W(p) \end{bmatrix}\begin{bmatrix} w(t) \\ u(t) \end{bmatrix}=\begin{bmatrix} 0 \\ -y(t) \end{bmatrix} \tag{3.9}$$

and $\hat{\xi}=\left[\underline{w}^t,\underline{u}^t,\underline{y}^t\right]^t$, where $\underline{x} \in \Re^n$, $\underline{u} \in \Re^t$, $\underline{y} \in \Re^m$ and $\mu = \nu + \ell + m$. The system described by the system matrix [Rosen., 1] in equation (3.9) is called (ν, ℓ, m)-Rosenbrock orientation and will be denoted by $\mathcal{L}(T,U,V,W)$ and the family of such models will be denoted by $\Sigma(H)$.

The families $\Sigma(F,G)$ or $\Sigma(H)$ contain more than one solution. Such solutions may be classified according to the invariant structural characteristics of the corresponding orientation, as well as the input, output type properties of the resulting oriented model. Furthermore, we might have a variety of solutions due to the variability of the number of inputs, outputs we specify, as well as the selection of alternative sets. For the case of polynomial implicit descriptions, the current definition of orientation is based on equivalence that preserves only the smooth space of solutions of the original and oriented model. Alternative orientation problems may also be defined, which preserve also impulsive behaviour. An important issue in selecting oriented models is the issue of model minimality [Kuij. & Schum., 1], [Bon. & Mal., 1] that is equivalent to selecting a minimal number of internal variables. Issues of minimality, as well as assignment of desirable structural characteristics are important criteria which have to be used in the parameterisation of the $\Sigma(F,G)$ or $\Sigma(H)$ families. The different issues of MOP described above have not been addressed before as design problems. Whenever orientation issues have emerged, they have been tackled using rules and heuristics dependent on the particular application. The main issues on which the model based approach for MOP is based are:

a) Exploit the underlying structure of the implicit non-oriented model to define families of oriented solutions of given input, output dimension.

b) Define the conditions for the selection of the orientation, which are needed to guarantee properties for the oriented system such as: minimality, causality, controllability, observability, invertibility etc.

c) Use the analysis on parameter redundancy to develop an approach for selection of independent variables as an alternative to the graph approach.

The study of MOP in the context of the implicit state-space description will be considered again in a following chapter.

3.7.2. Model Projection Problems

For many systems the number of potential control variables and potential measurements, which ideally may be used can become very large. In an ideal design, unconstrained by resources and effort all possible inputs and outputs should be used; economic and technical reasons, however, force us frequently to select a subset of the potential inputs, outputs as effective, operational inputs, outputs. Engineering specifications and past experience with similar designs provide some guidance in how to select the effective ℓ-inputs and effective m-outputs, but they do not specify a solution uniquely. Developing criteria and techniques for selection of an effective input, output scheme, as projections of the extended input, output vectors respectively, is what we call Model Projection Problems (MPP). This problem has always to be discussed within the framework of engineering specification and constraints. For linear systems, where orientation has already been decided, and represented by an external progenitor model $(\mathcal{V}, \mathcal{Z}, \mathcal{F})$ (or a q x p rational matrix F(s)) the MPP is equivalent to selecting the sensor, actuator maps h, g (or m x q, p x ℓ rational matrices H(s), Q(s), $m \leq q$, $\ell \leq p$) such that the transfer function:

$$W(s) = H(s)F(s)Q(s) \qquad (3.10)$$

has certain desirable properties. Clearly, the problem as stated above is in the form of a generalised two parameter Model Matching. The design parameters H(s), Q(s) may be assumed in the first instance to be constant; in the case, where they are considered as dynamic, their order is not free, since they represent the dynamics of sensors and actuators. The overall MPP involves reduction of the input, output variables and the resulting model

has structural features, which evolve from the progenitor model. The nature of desirable properties of the reduced model W(s) depends on its use and the properties of the F(s) model. Some of the key issues in the selection of the effective sets of inputs, outputs, or the maps q, h are:

(i) Define the lowest bounds for the number of effective inputs, output, which are needed for certain control scheme, or family of alternative control schemes.

(ii) Define the best location of effective inputs, outputs, as well as the structure of actuator, sensor maps, which may guarantee structural controllability, and observability and other graph-related structural properties.

(iii) Evaluation of effect of selection of a given sensor, actuation scheme on the formation of parameter dependent structural characteristics, that is the resulting system invariants.

(iv) Evaluation of effect of a selected input, output scheme on the features of the resulting performance indicators, which characterise the different aspects of control quality of the resulting model.

The above sets of questions, as well as their order, express an underlying structural philosophy that system properties stem from the problem dimensionality, underlying graph structure, system invariants and they are finally measured by the characteristics of the property indicators. Each one of the above issues defines a cluster of MPPs, which are briefly described below [Karc., 10]:

(a) **Dimensional MPPs (D-MPP):** Such problems are defined on any type of progenitor models and aim at using conditions, for generic solvability of control problems, or generic system properties to define the least required numbers of effective inputs, outputs needed to guarantee certain structural properties. Early results [Karc., 9] are based on indicators, such as the Segre index to define certain bounds of effective inputs, outputs. The desire to guarantee solvability of control problems such as pole assignment, stabilisation etc. with different control schemes introduces alternative criteria, which also involve the McMillan degree and the control complexity. The latter requires the development of methodologies for robust McMillan degree identification on early process design models. In terms of the two-parameter scheme associated with MPP, the overall objective here is to determine the least number of columns of Q and number of rows of H matrices.

(b) **Graph Structural MPPs (GS-MPP):** On early process design models of the graph type (state space formulation) or structured transfer function type there is frequently the need to define a subset from all possible inputs, outputs of the progenitor model, or appropriate structural combinations of them. The criteria for such selection stem from that we would like to guarantee structural properties such as controllability, observability, disturbance rejection etc. Issues related to robustness under fault conditions may be also used as criteria here. In terms of the two parameter scheme associated with MPP, the problem here is to expand the results of D-MPP by defining the simplest required Boolean structure of the Q and H matrices.

(c) **Invariant Structural MPPs (IS-MPP):** On linear progenitor models of the transfer function, or state space type, the study of effect of selection of Q, H matrices may proceed (after some preliminary study of D-MPP GS-MPP) to the specification of numerical values for the Q and H matrices. The selection of constant Q, or H matrices leads to new models where the invariant structure is obtained by appropriate transformation of the progenitor model invariant structure. The transformation of one set of invariants to another is a challenging problem not fully understood; certain results in relationship to decoupling have been established in [Lois., 1], whereas in [Karc. & D. Vaf., 1] it has been shown that one-sided MPPs are equivalent to generalised cover problems of geometric theory. A special case of IS-MPP is the zero assignment by squaring down [Kouv. & MacF., 1], [Karc. & Gian., 1]; a two parameter version of squaring down (design of both Q and H) such that the resulting transfer function W(s) is square and has a given zero structure is discussed in [Karc. & Lev., 1]. The family of IS-MPP is rich, even when Q, H are constant matrices. The overall philosophy is to design Q, H such that the resulting model has a given desirable invariant structure or avoids having undesirable structural characteristics. Two different classes of problems within this category will be considered elsewhere.

(d) **Performance Optimisation - MPPs (PO-MPPs):** For a linear progenitor model F(s), an alternative family of MPPs may be defined to that of IS-MPP type, where system structure is assigned. We may pose problems where we avoid the formation of certain undesirable structural characteristics (such as right half plane zeros, high order infinite zeros etc) and at the same time optimise the values of certain key indicators, such as singular value properties of controllability, observability Grammians, condition number, singular values of resulting transfer function etc. Within this class we may also consider the problems where the selection of Q, H aim at minimising some form

of uncertainty of the progenitor model. The overall approach here is to utilise the degrees of freedom in Q, H matrices, which exist when avoidance of structural features rather than assignment of them is the central objective, to optimise certain key performance, or control structure indicator. Such problems are of the optimisation type and it seems that techniques from H-∞ optimisation are relevant here. Such issues are not considered in this thesis.

Although the above four classes of problems have been stated as independent, they are highly interrelated and in practice, mixed forms of them have to be addressed.

3.7.3. Model Composition Problems

The area of Composite Systems is one of the important aspects of Large Scale Systems Theory. The composite nature of the problem implies that the system is formed as a synthesis of subprocesses according to some interconnection rule, frequently referred to as process flowsheet (layout) and which acts as a progenitor of the underlined graph (derived when we specify the subprocess models). It is the composite nature that makes the overall process model, not an amorphous input-output model, but one reflecting the nature of subprocesses and the process flowsheet. The latter provides the first of the two aspects of the model system structure, the second being those referred to as structural invariants. The study of relationships between subprocesses and overall processes on the level of models and system properties has been an area of interest for many application areas. The way subprocesses enter the composite structure, in terms of interconnecting local variables (subsystem connecting inputs, outputs and effective control inputs and measured outputs) affects drastically the overall properties of the composite system. Most of the previous work [Saeks & DeC., 1], [Vid., 1] deal with the study of properties of composite systems without seeing the interconnection scheme and the selection of local input, output structure as design parameters. A first attempt to link model composition to feedback was made in [Cal. & Des., 1] and subsequently developed in [Karc., 9]. Problems connected to the local input, output structure selection are referred to as Model Composition Problems (MCP). A general scheme that addresses interconnection rules and local selection of inputs, outputs simultaneously and which treats both issues, as an equivalent control problem will be presented later in this thesis. The overall emphasis is to address issues of subsystem input, output selection, which are linked to the given interconnection rule.

3.7.4. Model Expansion Problems

Measurement and actuation of a process is closely linked to modelling of the process. In fact, defining input test signals and selecting appropriate measurements is an integral part of the identification, modelling exercise. Defining appropriate input, output schemes, as well as excitation signals for model identification, or additional inputs, outputs, which may be used for reconstructing unmeasured internal variables, lead to an enhancement of the existing model. These problems are of the opposite nature to those described before, where progenitor model information was used as the starting point of MOP, MPP and MCP investigations. By Model Expansion Problem (MOP), we mean any problem where by additional actuation, measurement we aim at enhancing the properties of a given model, or reconstruct unmeasured internal variables. Question related to the nature of test signals, or properties of the measured signals are also important here, on top of more general questions related to the structure of the I/O scheme; the latter gives a distinct signal processing flavour to MEP. Some distinct problem areas are:

(a) **Additional Measurements for Estimation of Variables:** Frequently in process control some important variables are not available for measurement. Secondary measurements have to be selected and used in conjunction with estimations to infer the value of unmeasurable variables. The proper selection of secondary measurements is a task of paramount importance for the synthesis of control schemes. The various aspects for the problem are discussed within the area of state estimation (deterministic, of stochastic); this area is well developed and a good account may be found in [Astrom, 1], [Shigley, 1].

(b) **Input, Output schemes for System Identification:** The selection of input test signals and output measurements is an integral part of the setting up of model identification experiments. In fact, the identified model is always a function of the way the system is excited and observed, i.e., of the way the system is embedded in its experimental environment. Most of the work so far has concentrated on SISO identification techniques and on the effect of test signal characteristics on the identification aspects of the model [Eykh., 1]. The study of effect of location of the group of excitation signals and corresponding group of extracted measurements on the identification problem has not been properly examined so far and its proper study is long overdue. Issues such as how and whether additional signals and extracted measurements may

enhance the scope and accuracy of identifiable models are important. Within the same category we may include methods for Fault Detection and Fault Isolation (FDI) referred to as functionally-redundant schemes [Patton et al, 1], which are based on the design and optimal placement of sensor systems for FDI purposes.

3.8. CONTROL THEORY AND DESIGN REQUIREMENTS

Control Theory (CT) is the backbone of Control Systems Design (CSD) since it provides the conceptual framework (concepts and tools) as well as the algorithms on which CSD philosophies strategies and techniques are based. CT and CSD are well developed especially in the context of linear systems, however, the development of CT has been almost entirely based on the assumption that the system model is given and fixed. There are few examples, where the fixed structure of the model is disputed, such as, the zero assignment problem (selection of output matrix, or squaring down compensator with zero assignment criteria) [Kouv. & MacF., 1], [Rosen. & Power, 1], [Karc. & Gian., 1]. Despite the fact that the formation of structure has not been properly addressed within CT, the basic concepts, tools and techniques needed for GI originate within CT. In fact, the need for development of GI, defines new tasks, or emphasises the role of existing areas of CT. These are:

(i) **Control Quality Criteria:** Characterisation of shapes, or values of Property Indicators (PI) [Maciej., 1] and System Invariants (SI) [Kailath, 1], which may ease, or make difficult the Control Synthesis Design Problem. Integral parts of this task are:

(a) Establishment of links between the limits in compensation of the various PIs and the relevant SIs.

(b) Further development of the solvability conditions of control synthesis problems, in terms of SIs and PIs.

The above problems are expected to lead to a classification of desirable or undesirable values of SIs and PIs (parameterisation issues) and are essential inputs for the following task.

(ii) **Structure Synthesis Problems:** Development of a methodology for shaping the instrumentation maps **q, h** with control based criteria. Crucial aspects of this task are:

(a) Understanding the mechanisms of formation of values of PIs and SIs as functions of the model parameters.

(b) Derivation of techniques for designing of **q**, **h** maps such that we avoid the formation of undesirable properties in the model and if possible we assign desirable model characteristics.

Clearly, such techniques should operate within the constraints, specifications and traditional instrumentation practices. The task in **(ii b)** is rather new for CT and arises mainly from the new role of CT as an intervention tool in GI. The tasks **(i a)**, **(i b)**, **(ii a)** are traditional CT task and a lot of results may be found in the Control literature, but not always in a suitable and accessible form. The area of work described in **(i a)** is still in its early stages of development. CT is the body of concepts, tools and techniques, which deals with the qualitative, quantitative properties of a system model and the methods for solving a variety of control problems. The nature of CT, clearly depends on the type of model, which is used to describe the system. Linear System Theory is the most developed and thus the first effort to develop GI has to be based on it. Issues related to model uncertainty, are integral parts of the overall study and have to be addressed also within the same framework.

3.9. CONCLUSIONS

The results from the applications area of process systems have provided the motivation for the definition of a number of important system and control theory problems which are generic and thus independent from the given application area. The emphasis here is on the system theoretic aspects of Global Instrumentation. In the following chapters we consider a number of generic Systems and Control problems, which have been specified in this chapter. The role of Control Theory is crucial in the study of generalised systems design problems, which will be considered, the following chapter provides a basis for such studies by reviewing the fundamentals of Control Theory, which will be used as the enabling instrument in these challenging tasks.

Chapter 4

SYSTEM PROPERTIES, PROPERTY INDICATORS AND SYSTEM INVARIANTS: THE BACKBONE OF THE STRUCTURAL APPROACH TO INTEGRATED SYSTEM DESIGN

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4.1. INTRODUCTION

The problem of addressing the design of all fundamental stages of large engineering processes, such as the chemical processes, is multidisciplinary and has a clear complex nature. Complexity is a consequence of the multidisciplinarity, the large dimension of processes, the cascade-evolutionary nature of the design process, the early-late design evolution of requirements, the uncertainty due to not well clearly specified goals, model uncertainty and uncertainty in the design environment (disturbances). The dominant trend in the development of methodology and approaches for integrated design is to specialise the study within a specific discipline and rely on the particulars of the sector and the process dependent heuristics in answering some of the fundamental challenging questions. The essence of the work in this thesis is to demonstrate that a systems and control theory based framework has a significant role to play in the development of a conceptual and design framework for tackling problems of overall systems design. This chapter aims at examining the fundamentals of the systems theory based framework, and thus underpins the developments in the following chapters. The following review is based on the results of the SESDIP project ([Karc., 3], [Karc., 4], [Karc., 7]).

Control Theory, is the backbone of Control Systems Design and involves:

i. Study of systems properties

ii. Classification of systems

iii. Characterisation of solvability conditions of control problems

iv. Synthesis methods for control problems

The above four aspects are central to the development of design strategies and techniques, since they provide the tools for analysis and development of compensation techniques. Control Theory is model dependent and the richest part of it is that dealing with linear, time invariant, finite dimensional systems. Such simple models seem to be appropriate for the Early Process Design (EPD) environment, where there is neither the scope, nor the possibility for detailed modelling. In the final design stage (control systems design) more elaborate models, taking into account effects of nonlinearities, delays etc., have to be used.

The central theme in every science is the classification of objects, which are under study. Control Theory studies the system models and the classification of systems into various families is achieved using a variety of criteria. It is the aim of this chapter to examine the criteria that allow such a classification. The key concepts are those of property indicators and system invariants. Property indicators express the state, the value of a certain system property, which, however, may change under compensation. System invariants on the other hand are functions, defined on the model, which remain the same under certain types of transformations; thus, they characterise not only a single model but a whole family (equivalence class). It is because of the latter property that invariants emerge as constraints in the shaping of property indicators under compensation.

The problem of classification, parameterisation of system models according to system invariants may seem to be of academic interest only. However, invariants express the system's potential for achieving certain type of performance and thus, if we are aiming at selecting systems structures with "good" potential for control and operability, it is essential to understand their role in control design (relationships to property indicators), as well as the mechanisms of their formation during EPD and in particular global instrumentation (GI). The classification of system models to those with "good" (desirable) and those with "bad" (undesirable) control characteristics is an ongoing research topic in Control Theory, far from the stage of full development, even for simple models. The study of mechanisms of formation of certain control characteristics is a topic which has been partially addressed before in [Rosen. & Power, 1], [Kouv. & MacF., 1] [Karc. & Gian., 1] and it is a major research area that requires special attention. Our aim here is to examine

systematically the most relevant of the background theory linked to the structural methodology that is used in this thesis and thus provide the means for their transformation as intervention tools in both Control Design and development of methodologies for Global Instrumentation. The work here builds upon the results of the ESPRIT project SESDIP [Karc., 7]. The chapter is structured as follows: In section 2, a rather general definition and classification of system properties, property indicators and system invariants is given. The different system properties and indicators are examined in section 3, whereas system invariants are discussed in section 4. Finally, in section 5, a first evaluation of the different concepts and tools, as far as their relevance to GI and control systems design is presented.

4.2. SYSTEM PROPERTIES, PROPERTY INDICATORS AND INVARIANTS: GENERAL ISSUES

4.2.1. System Properties and Property Indicators: Definitions and Classification

In this section we introduce the notions of system properties, property indicators and systems invariants in more precise mathematical terms and on a given family of models $\mathcal{M}$, which are not made precise in the first instance. The motivation behind this is to provide a framework for discussion of such concepts in an environment of variable nature and variable complexity models, such as those emerging in both integrated design and global process operations. Let $\mathcal{M}$, be the family of system models (internal or external): $\mathcal{M}$ will be referred to as the model set. By $\mathcal{A}$ we shall denote the set of all possible attributes (characteristics), that may be associated with every model $M \in \mathcal{M}$, and shall be referred to as the model attributes set. We denote by $\mathcal{B}$ a general set with elements, numbers, graphical statements, criteria etc., called the criteria set.

Definition (4.1): A system property is a function $p: \mathcal{M} \to \mathcal{A}$. If $p(\mathcal{M})$ is the image of $\mathcal{M}$ under p, then a p-property test is a function $g: p(\mathcal{M}) \to \mathcal{B}$ and the composition $f: \mathcal{M} \to \mathcal{B}$ defined by $f \triangleq g \circ p$ will be called a p-property indicator.

□

We may illustrate diagrammatically the above definition as shown in Figure (4.1). In simple terms, a property indicator is a function defined on the system model and whose values characterise the property. Depending on whether the model is internal, or external,

the property will be referred to as internal or external respectively. If the attribute associated with the property expresses a qualitative property of dynamic behaviour of the system, which may be defined on a general family of models, then it will be called qualitative (examples of such properties are stability, controllability, existence of periodic motions etc.). The criteria set for such properties are of a binary nature (the model has, or does not have the property). If the attribute associated with the property has a quantitative character, that is numerical values are involved in its definition then it will be called quantitative. For such properties the criteria set is not of a binary nature but it may contain a range of values, which express a "degree" of possessness of the property by the model.

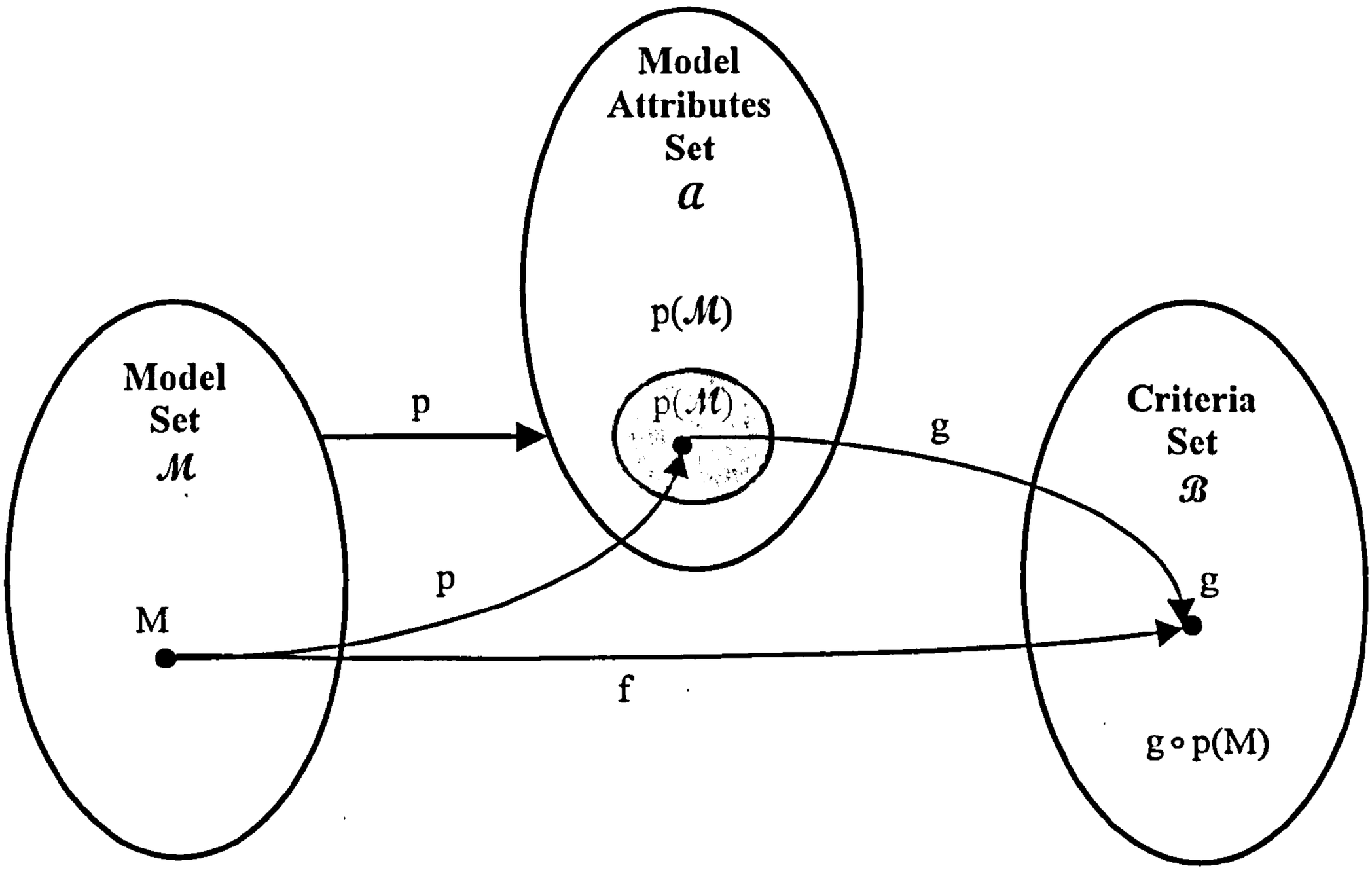

Figure (4.1): Properties, Property indicators

A further classification of properties is in terms of the notions of genericity and robustness. If $\mathcal{M}$ is a family of models characterised by a common fixed structure (for instance a given linear graph), but with otherwise arbitrary parameters, then with every model $M \in \mathcal{M}$ we may associate a parameter vector $\alpha(M)$ in the parameter space $\mathbf{R}^N$. A property is called generic, if it holds true for almost all $M \in \mathcal{M}$; otherwise, the subset $\mathcal{M}'$ of $\mathcal{M}$ for which the property does not hold true have parameter vectors $\alpha(M)$ which belong to a proper variety $\mathbf{V}$ of the parameter space $\mathbf{R}^N$, ([Wonham, 1], [Hirsch & Smale, 1]. The property that is valid on a proper variety of $\mathbf{R}^N$ is called nongeneric. For the set of nxn real

matrices, the property of having distinct eigenvalues is generic, whereas having repeated eigenvalues is a nongeneric property. A property that holds true not only for an M$\in\mathcal{M}$, but for some neighbourhood $\mathcal{R}$(M) of models around $\mathcal{M}$ is called well posed. If the neighbourhood of models $\mathcal{R}$(M) is large, the property is called robust, otherwise nonrobust. Robustness, is thus connected to the size of permitted perturbations on the nominal model parameters before the property, that holds true on the nominal model, is violated. A generic property may also be referred to as structural. A property depending on the internal mechanism model will be called prime and if it depends on the interaction of internal mechanism and environment it will be called composite (internal stability is a simple property, but controllability is a composite property). A property indicator that is used for assessing a single property will be called simple; if many different properties are assessed through the same indicator, then it will be called multiple. If a property indicator is an explicit, implicit function of the model parameters, then the indicator will be called explicit, implicit respectively (the controllability matrix is an explicit indicator for controllability, the Nyquist diagrams are implicit indicators for closed-loop stability). For a given property we may use two alternative indicators; such indicators used for evaluation of the same property are called equivalent (the controllability matrix and the controllability pencils are equivalent indicators, as far as assessing controllability). The above classification of properties and indicators is important, since it is related to the problem of shaping models in EPD. Generic properties are important in an environment of "ill defined" EPD models. Explicit indicators are easier to shape in EPD, than implicit indicators. Prime indicators are shaped in the Process synthesis stage, whereas composite indicators (properties) are the result of interaction of process synthesis and instrumentation stages. Internal properties, (indicators) have a more explicit relationship to model parameters, than external properties (indicators).

4.2.2. System Invariants: Definitions and Classification

The compensation theory of linear systems aims at producing new systems with desirable set of properties. Both representation and compensation theory of linear systems, deals with families of models, which are interrelated by certain types of transformations (representation, or compensator type). The classification of properties of such families is thus essential in understanding what is the backbone of system (essential structure that is not affected by the transformations) and what are the limitations of given compensation scheme,

which is deployed to alter certain undesirable properties of a system. The classification of families of models for a given system is intimately related to the notions of equivalence, invariants and canonical forms. Some general definitions and classification issues for invariants are examined next. Let $\mathcal{M}$ be a family of linear models, $\mathcal{E}$ an equivalence relation defined on $\mathcal{M}$, $\mathcal{E}(M)$ the equivalence class of $M \in \mathcal{M}$ and let $\mathcal{M}/\mathcal{E}$ be quotient set or orbit (set of all equivalence classes). We may define [MacL. & Bir., 1]:

Definition (4.2): Let $\mathcal{M}$ be a model family, $\mathcal{I}$ a set, $\mathcal{E}$ an equivalence relation defined on $\mathcal{X}$.

(i) A function $\mathbf{f}: \mathcal{M} \rightarrow \mathcal{I}$ is called an invariant of $\mathcal{E}$, when $M_1 \mathcal{E} M_2$ implies $\mathbf{f}(M_1) = \mathbf{f}(M_2)$. $\mathbf{f}$ is called a complete invariant for $\mathcal{E}$, when $\mathbf{f}(M_1) = \mathbf{f}(M_2)$ implies $M_1 \mathcal{E} M_2$.

(ii) A set of invariants $\{f_i: f_i: M \rightarrow I_i, i = 1,2,...,k\}$ is a complete set of $\mathcal{E}$ on $\mathcal{M}$, if the map f defined by: $\mathbf{f}: \mathcal{M} \rightarrow \underset{i=1}{\overset{k}{\times}} \mathcal{I}_i : M \rightarrow \mathbf{f}(M) \underline{\Delta} \{f_1(M),.., f_k(M)\}$ is a complete invariant for $\mathcal{E}$ on $\mathcal{X}$. The complete set of invariants is called independent, if there is no subset, which is also complete.

□

Note that a complete invariant defines an one-to-one correspondence between $\mathcal{E}(\mathcal{M})$ equivalence classes and image of $\mathbf{f}$ in $\mathcal{I}$. The notion of independence is essential in the minimal parameterisation of $\mathcal{E}(\mathcal{M})$ by invariants. An important issue for system identification and control analysis is that of canonical form for $\mathcal{E}(\mathcal{M})$. If $\mathbf{f}: \mathcal{M} \rightarrow \underset{i=1}{\overset{k}{\times}} \mathcal{I}_i$ is a complete and independent invariant for $\mathcal{E}$ on $\mathcal{M}$, by specialising the invariant $\mathbf{f}$ such that its image $\mathbf{C}$ is in $\mathcal{M}$, we define a canonical element, or a canonical form.

Definition (4.3): A set of canonical forms, $\mathcal{C}$, for $\mathcal{E}$ equivalence an $\mathcal{M}$, is a subset of $\mathcal{M}$ such that for every $M \in \mathcal{M}$ there exists a unique $C \in \mathcal{C}$ for which $M \in \mathcal{C}$.

□

Canonical forms are uniquely defined elements of $\mathcal{M}$, which have the simplest possible structure (least number of parameters) and which describe the invariant in the language of the model (in terms of a simple model). Canonical forms, are often used as analysis tools and describe the simplest possible type of model that may be defined under the set of transformations defining the equivalence relation. The set of canonical forms

provides a system of canonical distinct representatives of $\mathcal{M} / \mathcal{E}$. The diagram in Fig. (4.2) illustrates the notion of canonical form.

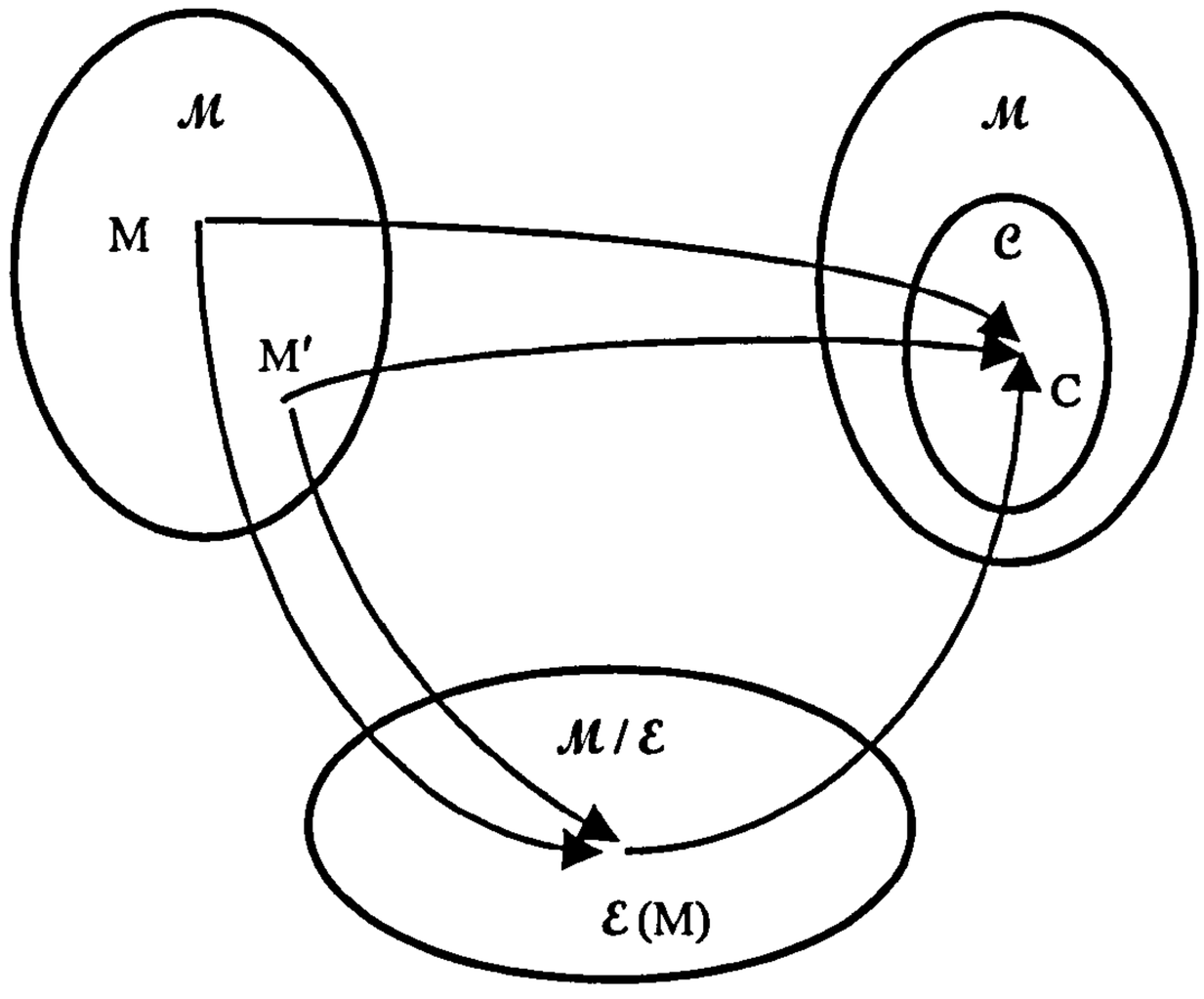

Figure (4.2): Canonical forms

The classification of invariants to internal / external, simple / composite, explicit / implicit is the same to that given for properties. An invariant will be called global, if it takes nontrivial values for all $M \in \mathcal{M}$, otherwise, if it takes nontrivial values only on a proper variety of the model parameter space $\mathbf{R}^N$, it will be called local. The value of a global invariant will be called generic, if it is constant for almost all $M \in \mathcal{M}$. That is the models for which the value may differ from the constant is a proper variety of $\mathbf{R}^N$; such values will be called nongeneric. An invariant of representation transformations will be referred to as a representation invariant, whereas those of compensation transformations will be called a compensation invariant. An invariant will be called strong, or weak, if it is preserved, or not preserved under more general types of transformation. The above terminology will be clarified with the concrete examples that will be presented in the following sections.

4.3. SYSTEM PROPERTIES AND PROPERTY INDICATORS

4.3.1. Introduction

In this section we shall examine some of the fundamental properties of a linear system model (internal, or external) which are essential in understanding the dynamic behaviour and performance characteristics of control systems.

4.3.2. Fundamental concepts and properties for state space descriptions

We consider a linear system described by the state-space or by the transfer function model:

$$S(A,B,C,D) = \begin{cases} \dot{\underline{x}} = A\underline{x} + B\underline{u},\ A \in R^{n\times n},\ B \in R^{n\times p},\ \rho(B) = \rho \\ \underline{y} = C\underline{x} + D\underline{u},\ C \in R^{m\times n},\ D \in R^{m\times p} \end{cases} \tag{4.1}$$

$$G(s) = C(sI - A)^{-1}B + D = N_r(s)D_r(s)^{-1} = D_\ell(s)^{-1}N_\ell(s) \tag{4.2}$$

where $N_r(s),\ N_\ell(s) \in R^{mxp}[s],\ D_r(s) \in R^{pxp}[s], D_\ell(s) \in R^{mxm}[s]$ characterise right, left coprime MFDs and coprimeness assumption implies that the matrices:

$$T_r(s) = \begin{bmatrix} N_r(s) \\ D_r(s) \end{bmatrix},\quad T_\ell(s) = [N_\ell(s), D_\ell(s)] \tag{4.3}$$

have no zeros (full rank for all $s \in \mathcal{C}$). By a state trajectory we shall mean the solution of (1a) obtained for a given pair of initial condition x(0) and input u(t); the resulting output y(t) (defined by (4.1b)) will be referred to as output trajectory. The basic concepts of poles and zeros are considered first.

(a) Eigenvalues, Poles and Eigenframes

Classical control design techniques are based on the concepts of poles and zeros of a rational function. Dynamically, poles are associated with "resonance" phenomena (explosion of the gain) and zeros with "antiresonance" phenomena (vanishing of the gain). In this sense, they are dual concepts and this type of duality carries over to their dynamic characterisation (in terms of trajectories having certain properties)

Definition (4.4): The set of eigenvalues of the matrix A (roots of the characteristic polynomial) will be called the system internal poles, or system eigenvalues, whereas the roots of the pole polynomial of G(s), defined by the Smith-Macmillan form of G(s), will be called external poles, or simply system poles.

□

With every eigenvalue λ of A, we have two eigenvalue-eigenvector problems:

$$A\underline{u} = \lambda \underline{u}, \quad \underline{\upsilon}' A = \lambda \underline{\upsilon}', \quad \underline{\upsilon}' \ \underline{u} = I \tag{4.4}$$

where $\underline{u}$ is a right - and $\underline{\upsilon}^t$ a left-eigenvector. The triple $(\lambda, \ \underline{u}, \underline{\upsilon}')$ is called a system mode. If $\phi(A)$ is the set of distinct eigenvalues then the structure of $\lambda \in \phi(A)$ is defined by λ-Segre characteristic, $S(\lambda) \underline{\underline{\Delta}} \{\nu_i i \in \underset{\sim}{q}\}$ that is the dimensions of λ-Jordan blocks in the Jordan form of A. Alternatively, ρ(λ) is defined by the set of degrees of the $(s-\lambda)^{\nu}$ type ed of the Smith form of sI_n -A, $\sum_{i=1}^{a} \nu_i \underline{\underline{\Delta}} \rho$ is called the algebraic multiplicity and q the geometric multiplicity of λ. System poles are defined through the Smith McMillan form of G(s) and have a structure which is defined similarly by the ed associated with a given pole λ (these are defined by factorising the elementary pole polynomials $p_i(s)$). The poles may be computed independently from the Smith McMillan form as described below:

Result (4.1) [MacF. & Karc., 1]: The pole polynomial p(s) and G(s) may be computed as the least common multiple, of all order minors of G(s).

□

Result (4.2) [Kailath, 1]: The pole polynomial $p(s) = |D_r(s)| = |D_\ell(s)|$, where $D_r(s)$, $D_\ell(s)$ are right, left denominators to coprime MFDs.

□

If $\mathcal{P}_s$, $\mathcal{P}_G$ are the sets of internal, external poles, then $\mathcal{P}_G \subseteq \mathcal{P}_s$ and for a generic system $\mathcal{P}_G = \mathcal{P}_s$. Internal poles are defined entirely by the internal mechanism map A, whereas external poles are generally a subset of the internal poles. The instrumentation maps acts as "selectors" of the subset. This issue is connected to controllability and observability of the system. If $\delta_M = \partial[p(s)]$ ($\partial\{.\}$ denotes degree of polynomials) is the McMillan degree of G(s), then δ_M defines the minimal dimension of state space models which have G(s) as transfer function. The eigenvalues with their corresponding structure (as well as the poles) are primary response indicators. The role of eigenframes is important in sensitivity analysis [Wilk., 1] and thus plays a key role in the design of robust state space

schemes. For physical internal variables, the eigenframe is also an indicator for transient behaviour. The notion of normal eigenframe seems to be crucial for the latter properties.

Result (4.3) [Gant.,1]: $Q \in \mathcal{C}^{nxm}$ is normal, iff Q has a complete orthonormal system of eigenvectors, i.e. the eigenvalue - eigenvector decomposition of Q is $Q = W \Lambda W^*$, $\Lambda = \text{diag}\{\lambda_1 ,.., \lambda_m\}$ and W a unitary matrix.

□

Result (4.4) [Wilk., 1]: If Q is normal with λ_i eigenvalues and $Q(I+\Delta)$ is a perturbed matrix with λ eigenvalues, then the λ eigenvalues are in discs around λ_i , which are defined by:

$$|\lambda - \lambda_i| \leq |\lambda_i| . \|\Delta\|_2 \leq \|Q\|_2 . \|\Delta\|_2 \qquad (4.5)$$

□

The insensitivity of spectrum of a normal matrix is relevant for both state space analysis and frequency domain method (Nyquist theory). Although normal matrices have nice spectral properties, they constitute a relatively small set among general matrices. For each normal matrix, there is a whole neighbourhood of approximately normal matrices. Measuring the departure from normality of a matrix, or the skewness of eigenframes is also important, in the context of frequency response indicators.

Remark (4.1): A measure of skewness of the eigenframe of A provides an indicator of sensitivity of eigenvalues to perturbations of the elements of A. If the eigenframe is close to an orthogonal frame the eigenvalues are insensitive to parameter variations. It should be also stressed that the concept of normality makes sense in physical variable models on which constraints on the variables may be defined, and variations of the model parameters have a physical origin.

□

Eigenvalues and poles may be dynamically characterised by zero input rectilinear motions problems [MacF. & Karc., 1]. That is for a zero input and appropriate initial condition, the state and output are rectilinear trajectories (simple exponential), where the frequency is defined by the eigenvalue and the initial condition is defined by the corresponding eigenvector. This problem is illustrated by the following diagram and has motivated together with the classical pole zero duality the dynamic characterisation of zeros.

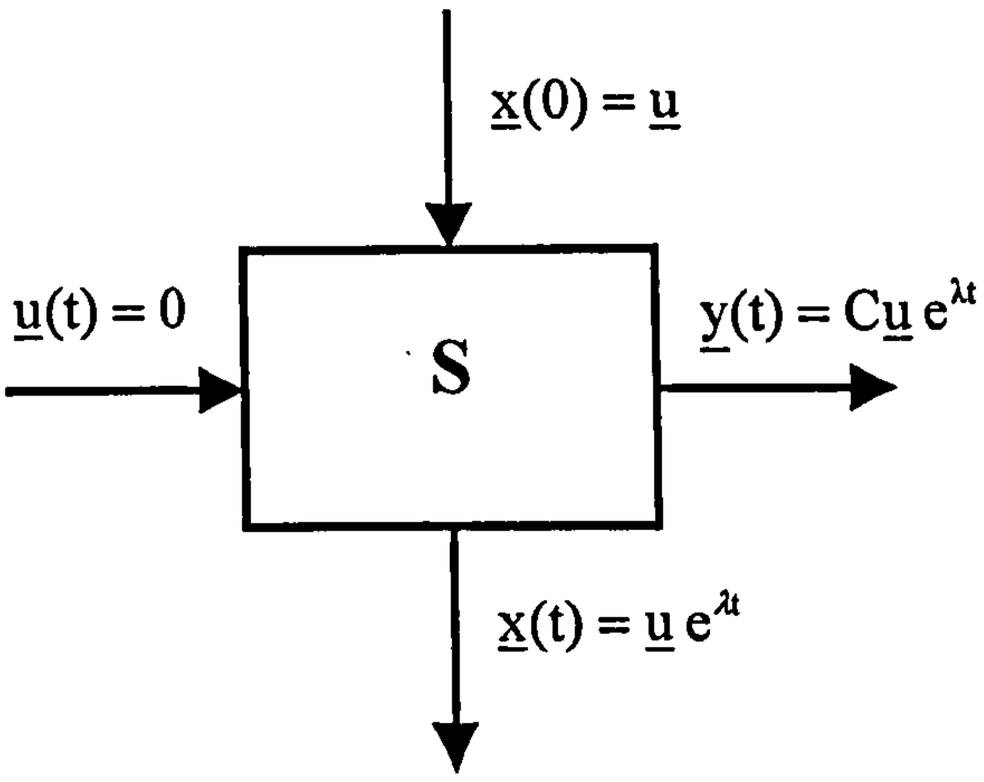

Figure (3): Zero input problem

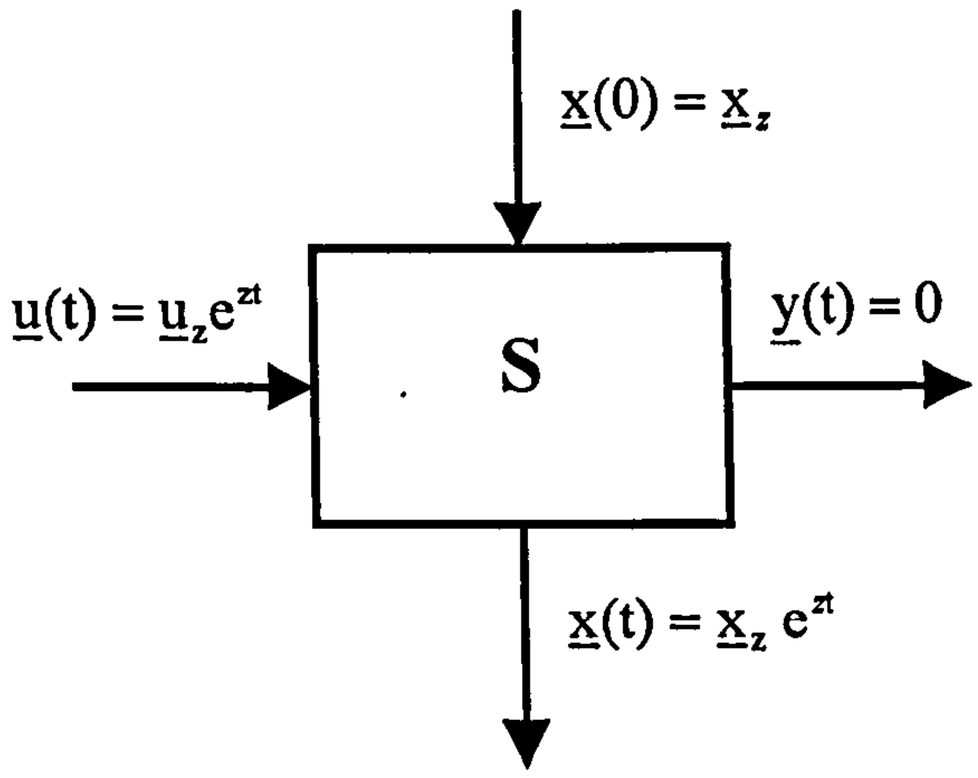

Figure (4): Zero output problem

(b) Zeros and Zero directions

The study of simple rectilinear motions of the type $\underline{x}e^{st}$, $s \in \mathcal{C}$ is central in the understanding of propagation of more general signals. The generation of rectilinear motions is characterised by the following results.

Result (4.5) [MacF. & Karc., 1]: For all $s_o \in \mathcal{C}$, there exist rectilinear motions of the type $\underline{u}(t) = \underline{u}_o e^{s_o t}$, $\underline{x}(t) = \underline{x}_o e^{s_o t}$, $\underline{y}(t) = \underline{y} e^{s_o t}$ $t \geq 0$, iff:

$$\begin{bmatrix} s_o I - A & -B \\ -C & -D \end{bmatrix} \begin{bmatrix} \underline{x}_z \\ \underline{u}_z \end{bmatrix} = \begin{bmatrix} \underline{o} \\ -\underline{y}_o \end{bmatrix} \Leftrightarrow \begin{cases} (s_o I - A)\underline{x}_o = B\underline{u}_o \\ \underline{y}_o = C\underline{x}_o + D\underline{u}_o \end{cases} \tag{4.6}$$

□

The first of conditions (4.6) defines a generalised eigenvalue-eigenvector problem for ($\underline{s}_o$, $\underline{x}_o$, $\underline{u}_o$)and (s_o, x_o) are potential closed-loop eigenvalue, eigenvectors under a state feedback satisfying $L\underline{x}_o = \underline{u}_o$. Result (4.5) describes the "Simple Frequency Transmission Problem" and a special case of this is the "Transmission Blocking Problem" defined below.

Result (4.6) [MacF. & Karc., 1]: Let $m \geq p$ and assume $\rho\,(G(s)) = p$. Necessary and sufficient condition for the block of the rectilinear transmission of a frequency $z \in \mathcal{C}$ at the system output ($\underline{y}(t) \equiv 0$, all $t \geq 0$) is that

$$\begin{bmatrix} zI - A & -B \\ -C & -D \end{bmatrix} \begin{bmatrix} \underline{x}_z \\ \underline{u}_z \end{bmatrix} = 0 \Leftrightarrow P(z) \begin{bmatrix} \underline{x}_z \\ \underline{u}_z \end{bmatrix} = \underline{0} \tag{4.7}$$

□

Definition (4.5): The set of frequencies $z \in \mathcal{C}$, $\mathcal{Z}_p$, for which P(z) loses rank below its normal rank, are defined as internal transmission zeros and the vectors $\underline{x}_z$, $\underline{u}_z$ as state, input-zero directions.

□

The set of zeros may be defined as the roots of the invariant factors of the Smith form of P(s). A simpler method for computing the zeros is:

Result (4.7) [Karc. & Kouv., 1]: If D=0, then the set of zeros is defined as the finite eigenvalues of the zero pencil Z(s) = sNM-NAM where N, M are left, right annihilators of B, C matrices.

□

Remark (4.2): The zeros are byproducts of the selection of actuators and sensors (selection of B and C matrices) and they are formed at the instrumentation stage of the process design. They express the overall interaction of the A, B, C matrices.

□

Definition (4.6) [Rosen., 1]: The set of zeros of the elementary zero, polynomials $\varepsilon_i(s)$ of the Smith-McMillan form of G(s), $\mathcal{Z}_g$ are defined as the external transmission zeros and $z(s) = \Pi\varepsilon_i(s)$ is defined as the external zero polynomial.

□

Result (4.8) [MacF. & Karc., 1]: Let p(s) be the pole polynomial of G(s) (computed as in Result (4.1). The zero polynomial z(s) may be computed as the greatest common divisor of the numerators of all maximal order minors of G(s), which have been previously adjusted to have p(s) as common denominator.

□

Result (4.9) [Kailath, 1]: If $N_r(s)$, $N_l(s)$ are any right, left numerators of comprime MFDs of G(s), then the external transmission zeros of G(s) are the same as the zeros of $N_r(s)$, $N_\ell(s)$ (defined by the corresponding Smith forms).

□

The set of external transmission zeros $\mathcal{Z}_G$ is a subset of the internal zeros $\mathcal{Z}_p$ and for a generic system the two sets coincide.

(c) Zeros at infinity

A general linear system (not necessarily proper) may have poles and zeros at ∞, which indicate that elements of G(s) tend to ∞, and thus G(∞) loses rank. For proper systems (which are of interest here) we have no poles at infinity, but possibly zeros at infinity. According to the system description which is used we may have different definitions for zeros at infinity. Dynamically, infinite zeros are associated with the blocking of impulsive inputs. A number of algebraic definitions are given below.

Definition (4.7) [Vard. et al, 1], [Dion & Com., 1]: If diag $\{s^{-q_i}: q_i > 0\}$ is the essential part of the Smith McMillan form of s=∞ of a proper G(s), then the numbers q_i, $q_i > 0$ are defined as the orders of external infinite zeros, or as the orders of transfer function infinite zeros.

Definition (4.8) [Karc. & Hayt., 1], [Mal., 1]: For a state space model with associated system matrix pencil P(s), and if strictly proper, with a zero pencil Z(s), we define as:

(i) Divisors at infinity, the set of i-ed of P(s) and as restricted divisors at infinity, the set of i-ed of Z(s).

(ii) Orders of internal infinite zeros, the orders of infinite zeros of the Smith-McMillan form at ∞ of P(s).

□

The relationships between the different definitions of zeros at infinity is discussed in [Karc. & Hayt., 1] and this issue will be re-examined in the section on invariants. The most frequently used definition is that based on transfer function. An alternative definition of infinite zeros has emerged in the Root-Locus Analysis [Kouv. & Shaked, 1]. This is related to the complex analysis treatment of the problem of infinite zeros and is discussed in [Smith, 1]. Generically the two sets coincide.

(d) Internal, External and Total Stability

For linear time invariant systems the notions of stability, which are more frequently used are defined below (i.e. [Chen, 1]). We consider stability of equilibrium points, whereas

stability of motion is always reduced to the previous case. Note that the origin ($\underline{x}=\underline{0}$) is always an equilibrium point for S(A,B,C,D) models.

Definition (4.9): The state space model S(A,B,C,D) will be called:

(i) Internally stable in the sense of Lyapunov (LIS), if for any initial $\underline{x}(0)$ the zero input response (free motion, $\underline{u}(t)=0$) remains bounded for all $t\geq 0$.

(ii) Asymptotically internally stable, if for any initial state $\underline{x}(0)$ the zero input response remains bounded for all $t \geq 0$ and tends to zero as $t\to\infty$. This property will be referred to in short as internal stability (IS).

(iii) Bounded Input Bounded Output (BIBO) stable, if for any bounded input the zero state output response ($x(0)=0$) is bounded.

(iv) Totally stable (TS) if for any initial state $\underline{x}(0)$ and any bounded input $\underline{u}(t)$, the output, as well as all state variables are bounded.

□

The notion of BIBO stability refers to the transfer function description and may also be called as external stability. A number of criteria for the above properties, based on eigenvalues and poles are summarised below [Chen, 1].

Result (4.10): Consider the system S(A,B,C,D) with G(s) transfer function and let $\{\lambda_i = \sigma_i + j\omega_i,\ i\varepsilon \underset{\sim}{n}\}$, $\{p_j = \overline{\sigma}_j + j\overline{\omega}_j, j\varepsilon \underset{\sim}{\upsilon}\}$ be the sets of eigenvalues, poles respectively. The system is:

(i) Lyapunov internally stable, iff $\sigma_i \leq 0$ for all $i \in \underset{\sim}{n}$, and those with $\sigma_i = 0$ have a simple structure (algebraic multiplicity is equal to the geometric multiplicity).

(ii) Asymptotically internally stable, iff $\sigma_i > 0$, all $i \in \underset{\sim}{n}$.

(iii) BIBO stable, iff $\overline{\sigma}_j < 0$ for all $i \in \underset{\sim}{\upsilon}$.

(iv) Totally stable, if it is Lyapunov internally stable and BIBO stable.

□

Note that IS implies BIBO-S and thus TS. BIBO stability does not always imply IS, since transfer function and state space are not always equivalent. If the two

representations are equivalent (when system is both controllable and observable), then BIBO-stability is equivalent to IS and thus TS.

Remark (4.3): Eigenvalues and poles are indicators of stability. Equivalent tests for stability, without computing the eigenvalues, poles are defined on the characteristic, pole polynomial by the Routh-Hurwitz conditions and equivalent tests.

□

(e) Controllability, Observability

Some of the most fundamental concepts characterising the coupling of internal mechanism to its environment are those of controllability and observability [Kalman, 1]. These concepts dominate control theory and they are defined below.

Definition (4.10) [Kalman, 1]: Consider the state space mode S(A,B,C,D) and let $\mathfrak{X}$ be its state space ($\mathbf{R}^N$). Then the system is called:

(i) State controllable, or simple controllable, if there exists a finite time T>0, such that for initial state $\underline{x}(0) \in \mathfrak{X}$ and any $x_1 \in \mathfrak{X}$, there exists an input ($\underline{u}(t)$), defined on [0, T] that will transfer $\underline{x}(0)$ to $\underline{x}_1$ at time T ($\underline{x}(T) = \underline{x}_1$). Otherwise, is called uncontrollable.

(ii) State observable, or simple observable, if there exists a finite time T>0, such that for initial state x(0), the knowledge of the input u(t) and output over the time interval [0, T] suffices to determine the state $\underline{x}(0)$; otherwise, the system is called unobservable.

(iii) Output controllable, if for any output vector y_1 there exists T>0 and an input $\underline{u}(t)$ defined over [0,T] that transfers the output y(0)=0 to $y(T)=y_1$. Furthermore, if it is output controllable and the output can be steered over any interval of time on a preassigned curve, it will be called output function controllable.

□

Result (4.11) (Controllability, Observability criteria): The state space model S(A,B,C,D) with n, m, p number of states, outputs, inputs is:

(a) State controllable, iff either of the following equivalent conditions hold true:

(i) All rows of $e^{At} B$ are linearly independent on [0 ∞] over $\mathbb{C}^N$.

(ii) All rows of $(sI-A)^{-1} B$ are linearly independent over $\mathbb{C}^N$.

(iii) The controllability Grammian $W_c \underline{\Delta} \int_0^T e^{A\tau} BB^t e^{A^t\tau} d\tau$ is nonsingular for any T>0.

(iv) The nx(np) controllability matrix $Q_c \underline{\Delta}$ [B,AB,...,A^{n-1} B] has rank n.

(v) The controllability pencil, $P_c(s) \underline{\Delta}$[sI-A, -B] has rank n for all s∈C, or equivalently it has no f-ed.

(vi) The restricted controllability pencil, $R_c(s) \underline{\Delta}$sN-NA (N is a left annihilator of B) has rank n for all s∈C, or equivalently, it has no f-ed.

(b) State observable, iff either of the following equivalent conditions hold true:

(i) All columns of Ce^{At} are linearly independent on [0 ∞] over $\mathbb{C}^N$.

(ii) All columns of $C(sI-A)^{-1}$ are linearly independent over $\mathbb{C}^N$.

(iii) The observability Grammian $W_o \underline{\Delta} \int_0^T e^{A^t\tau} C^t C e^{A\tau} d\tau$ is nonsingular for any T>0.

(iv) The (nm)xn observability matrix, $Q_o = [C^t, A^t C^t, ..., (A^t)^{n-1} C^t]^t$ has rank n.

(v) The observability pencil, $P_o(s) = [sI-A^t, -C^t]^t$ has rank n for all s∈C, or equivalently, it has no f-ed.

(vi) The restricted observability pencil, $R_o(s)$=sM-AM (M is a right annihilator of C), has rank n for all s∈C, or equivalently, it has no f-ed.

(c) Output controllable, iff either of the equivalent conditions hold true:

(i) All rows of G(s) are linearly independent over $\mathbb{C}$.

(ii) The matrix Q_{oc} = [D, CB, CAB,...,CA^{n-1} B] has rank m.

(d) Output function controllable, iff rank of G(s) is equal to m, over **R**(s).

□

The above tests define equivalent indicators for the controllability, observability properties and demonstrate that both properties express the interaction of internal mechanism with the environment represented by the inputs, outputs. Thus controllability, observability properties are shaped at the instrumentation stage of the process design.

Remark (4.4): A generic system is always state controllable and observable. Thus, if all parameters in S(A,B,C,D) with fixed dimensions, are free the cases of uncontrollability, unobservability are nongeneric. Note however, that because of the process interconnections,

we frequently deal with S(A,B,C,D) families with a fixed underlined graph and it is then the graph that determines the controllability, observability properties. These graph based notions of controllability, observability are referred to as "structural" and are essential in the selection stage of process input, outputs. These issues are also examined elsewhere in this thesis.

□

Remark (4.5): Controllability and observability tests are based on the notions of rank of matrices, which generically is full. The degree of nonsingularity, singularity, measured by the smallest singular value, or the condition number is important indicators of "how well" the system is controllable, observable.

□

The questions of controllability and observability may be equivalently interpreted as questions of controlling, or observing the system eigenvalues. Using the Jordan decomposition of state equations (A is in Jordan canonical form) alternative tests for controllability, observability may be stated and they may be found, for example, in [Chen, 1]. Some important implications of these tests are:

Result (4.12): Let q be the maximal geometric multiplicity of the eigenvalues of A and let Σ be the family of n, m, p fixed dimension systems, having the same q, but with otherwise arbitrary parameters.

(i) Necessary condition for every S(A,B,C,D) $\in\Sigma$ to the controllable and observable is that $p \geq q$ and $m \geq q$ respectively.

(ii) Every system in Σ for which $p<q$, $m<q$ is uncontrollable, unobservable respectively.

□

Note that the value of q is frequently a property that may be inferred from the structure (graph) of the process and the nature of subprocesses and thus q may serve as a prime indicator on the necessary minimum number of inputs and outputs. The concepts of controllability, observability are essential in the establishment of the relationships between internal and external descriptions. This is illustrated by the following result [Kalman, 1].

Result (4.13): For the system S(A,B,C,D), there is a coordinate transformation $\underline{x}'=U\underline{x}$ such that the corresponding state space description S'(A',B',C',D') has the form known as Kalman decomposition i.e.

$$\begin{bmatrix} \dot{\underline{x}}'_{c\bar{o}} \\ \dot{\underline{x}}'_{co} \\ \dot{\underline{x}}'_{\bar{o}\bar{o}} \\ \dot{\underline{x}}'_{\bar{c}\bar{o}} \end{bmatrix} = \begin{bmatrix} A'_{\bar{c}o} & A'_{12} & A'_{13} & A'_{14} \\ 0 & A'_{oo} & A'_{23} & 0 \\ 0 & 0 & A'_{\bar{c}o} & 0 \\ 0 & 0 & 0 & A'_{\bar{c}\bar{o}} \end{bmatrix} \begin{bmatrix} \underline{x}'_{c\bar{o}} \\ \underline{x}'_{co} \\ \underline{x}'_{\bar{c}o} \\ \underline{x}'_{\bar{c}\bar{o}} \end{bmatrix} + \begin{bmatrix} B'_{c\bar{o}} \\ B'_{co} \\ 0 \\ 0 \end{bmatrix} \underline{u}, \underline{y} = [0, C'_{co,}, C'_{\bar{c}o}, 0]\, \underline{x}' + D\underline{u} \quad (4.8)$$

where $\underline{x}'_{c\bar{o}}$, $\underline{x}'_{co}$, $\underline{x}'_{\bar{o}\bar{o}}$, $\underline{x}'_{\bar{c}\bar{o}}$ are controllable-unobservable, controllable-observable, uncontrollable-observable and uncontrollable-unobservable states. Furthermore, S(A,B,C,D) and S'(A',B',C',D') have the same transfer function $G(s) = C'_{co}(sI - A'_{co})^{-1}B'_{co} + D$.

□

Remark (4.6): The transfer function and the state space descriptions are completely equivalent, iff the system is both controllable and observable. The McMillan degree $\delta_M(G)$ defines the dimension of the observable and controllable subsystems of any realisation (S,(A,B,C,D)) model of G(s). The transfer function represents only the controllable and observable subsystem, but not in general the whole S(A,B,C,D) system.

□

Definition (4.11): The system S(A,B,C,D) will be called:

(i) Stabilisable, if the unstable eigenspace of A is contained in the controllable subspace of the system.

(ii) Detectable, if the unobservable subspace of the system is contained in the stable eigenspace A.

□

Result (4.14): The systems S(A,B,C,D) is:

(i) Stabilisable, iff its uncontrollable eigenvalues (in the modal sense) are stable.

(ii) Detectable, iff its unobservable eigenvalues (in the modal sense) are stable.

□

If the system is both stabilisable and detectable, then the transfer function may be used for feedback design, but not otherwise. The uncontrollable, unobservable, uncontrollable and unobservable eigenvalues are also referred to as input - output, input-output decoupling zeros (idz, odz, i-odz) [Rosen., 1] and the corresponding sets, including multiplicities, will be denoted by $\mathcal{Z}_{ID}$, $\mathcal{Z}_{OD}$, $\mathcal{Z}_{IOD}$ respectively. These sets are computed by the following property [Rosen., 1], [Karc. & MacB., 1].

Result (4.15): (i)

(i) $\mathcal{Z}_{ID}$ is defined by the roots of the f-ed of $P_C(s) = [sI-A,-B]$, sN-NA.

(ii) $\mathcal{Z}_{OD}$ is defined by the roots of f-ed of $P_0(s)= [sI-A^t-C^t]^t$, or equivalently, $R_o(s)$=sM-AM.

(iii) $\mathcal{Z}_{IIOD} = \mathcal{Z}_{ID} \cap \mathcal{Z}_{OD}$.

□

Remark (4.7): The system is stabilizable and detectable iff the f-ed of $P_c(s)$ (or $R_c(s)$) and $P_o(s)$ (or $R_o(s)$) are stable.

□

Output controllability is only concerned with getting to a $y(T) = \underline{y}_i$ final value, but nothing in the definition states that the output will stay at the $\underline{y}(t) = \underline{y}_1$ value, or track a specified function $\underline{c}(t)$. Output function controllability addresses this property, but the required $\underline{u}(t)$ might contain impulses. Output function controllability with a smooth input $\underline{u}(t)$ will be referred to as smooth output controllability.

Result (4.16) [Skel., 1]: For the system S(A,B,C,D) we have:

(i) The system cannot smoothly track an output function $\underline{c}(t)$ with more than n-1 independent derivatives.

(ii) The system can track smoothly the vector function $\underline{c}(t)$ up to its first $r \leq n-1$ derivatives, if $\underline{c}(t)$ is sufficiently smooth to have r derivatives and the matrix M_r has rank (r+1)m, where

$$M_r = \begin{bmatrix} D & 0 & \cdots & 0 & 0 \\ CB & D & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ CA^{r-1}B & CA^{r-2}B & \cdots & CB & D \end{bmatrix} \tag{4.9}$$

□

Note that for strictly proper system (D=0) the above conditions can never be satisfied. In this case $\underline{c}(t)$ cannot be matched exactly, but if $\rho(M_r)= rm$, then tracking occurs with a constant offset ($\underline{y}(t) - \underline{c}(t) =$ constant). The above conditions reveal that M_r is an open-loop tracking indicator, that tracking properties are functions of n and they are linked with the structure at infinity of transfer functions [Karc., & Hayt., 1]. A similar class of problems deals with disturbances. A typical internal type disturbance model is obtained by expanding S(A,B,C,D) as:

$$S(A,B,E,C,D): \quad \begin{aligned} \dot{\underline{x}} &= A\underline{x} + B\underline{u} + E\underline{w} \\ \underline{y} &= C\underline{u} + D\underline{u} \end{aligned} \tag{4.10}$$

where $\underline{w} \in R^d$, $E \in R^{nxd}$ are the disturbance-vector and disturbance matrix respectively (the rest of dimensions are as before). By output disturbability we shall mean the same as in Definition (4.10) part (iii), except that the "input" $\underline{w}(t)$ is considered now to be a "disturbance". This allows the definition of the following property: The system totally rejects the disturbance, or that we have complete disturbance rejection, if $\underline{y}(t)$ is governed by its undisturbed response regardless of the disturbance $\underline{w}(t)$.

Result (4.17) [Skel., 1]: For an arbitrary disturbance $\underline{w}(t)$ we have complete disturbance rejection iff $Q_d \underline{\Delta}[CE, \text{CAE},\ldots,\text{CA}^{n-1}E] = 0$.

□

The open-loop tracking and complete disturbance rejection are idealised (exact) problems. In most of the applications we are interested in satisfying the above properties in the closed-loop (using feedback) and in an approximate sense. It is worth pointing out here that the matrix Q_d may be used as a disturbance localisation indicator, in the sense that if $\|Q_d\|_2$ is very small and the system is BIBO stable from $\underline{w}(t)$, then the effect of disturbance on the total response is small for all bounded disturbances. The controllability, observability

indicators Q_C W_C, Q_O, W_O may be used for systems having "physical internal variables" as indicators of relative controllability and observability. For systems, however, where the states have no concrete physical meaning the magnitudes for neither $\|Q_c\|$, $\|W_C\|$ nor $\|Q_0\|$, $\|W_O\|$ have meaning in an absolute sense. We may always, however, normalise a controllable and observable system such that is "equally" controllable and observable in some sense. By defying an appropriate coordinator transformation the controllability and observability Grammians may become equal and diagonal, so that each state variable is equally controllable and observable. Such description is frequently referred to as balanced realisation [Moore, 1].

Controllability, observability are concepts essential for state feedback design. Two more indicators, playing a key role in state space design are the Controllability, Observability-Plücker matrices, P(A,B), P(A,C) [Karc. & Gian., 2] defined as follows:

$$C_n([sI-A,-B]) = \underline{e}_n^t(s)P(A,B),\ P(A,B):(n+1)x\binom{n+p}{n} \tag{4.11a}$$

$$C_n\begin{bmatrix} sI-B \\ -C \end{bmatrix} = P(A,C)\underline{e}_{-n}(s), P(A,C):\binom{n+m}{n}x(n+1) \tag{4.11b}$$

when $\underline{e}_n(s) = [1,2,...,s^n]^t$ and $C_p(.)$ denotes the p-th compound [Marcus & Minc, 1]. In terms of P(A,B), P(A,C), we have:

Result (4.18) [Karc. & Gian, 2]: The system is controllable, iff P(A,B) has full rank and observable, iff P(A,C) has full rank.

□

4.3.3. Singular Value and Polar decomposition of transfer function matrices

The most important indicator of system performance in the frequency domain for the Control Design Configuration is provided by the singular value decomposition (SVD) and the polar decomposition of transfer function matrices. The linear system $G(s) \in R(s)^{mxp}$ is a matrix-valued function of the complex variable s. If we evaluate G(s) at each $s \in \mathcal{C}$ then the $G(j\omega)$ we may define the SVD and polar decomposition as summarised below [MacF. & Scot, 1].

The Singular Value Decomposition: Let $G(j\omega) \in \mathcal{C}^{mxp}$, σ_i, $i \in \underset{\sim}{r}$ the singular values of G, $\sigma_r \geq ... \geq \sigma_1 \geq 0, \Gamma = \text{diag}\{\sigma_r,..,\sigma_1\}$ (r = p if m ≥ p and r=m of m ≤ p), then G is expressed as:

$$G = Y\Gamma U^* \tag{4.12}$$

(i) If m≥p: then $Y \in C^{mxp}$, $\Gamma \in R^{pxp}$, $U \in C^{pxp}$, $Y^*Y = I_p$, $U^*U = I_p = UU^*$

(ii) If m≤p: then $Y \in C^{mxm}$, $\Gamma \in R^{mxm}$, $U \in C^{mxp}$, $Y^*Y = I_m = YY^*$, $U^*U = I_m$

The above decomposition is known as the singular value Decomposition (SVD), Y, U are referred to as output, input singular-vector frame matrices and Σ the principal gain matrix.

□

The Polar Decomposition: Let $G \underline{\Delta} G(j\omega) \in \mathcal{C}^{mxp}$ and consider this SVD of G as in (4.12). Then G may be expressed as:

$$G = (Y\Sigma Y^*)(YU^*) = M_l\Phi = (YU^*)(U\Sigma U^*) = \Phi M_r \tag{4.13}$$

where Φ, M_l, M_r are referred to as Phase, Left, Right-modulus matrices, and the above as polar decomposition. If G is square, YU* is unitary and its characteristic decomposition is expressed by:

$$YU^* = \Phi = P\theta P^*, \ \theta = \text{diag}\{e^{j\theta_i}, i = 1,2,...\} \tag{4.14}$$

where P is unitary and the set of angles θ_i are defined as principle phases.

□

For details on those two decompositions see [MacF. & Post., 1] and references therein. Since G(s) is analytic, the plots of singular values are continuous functions [Hung & MacF., 1], which will be denoted by $\sigma_i(j\omega)$ and called the principal gain functions. For the case m=p, the principal phases are also functions of $j\omega$, will be denoted by $\theta_i(j\omega)$ and called principal phases. The plots of $\sigma_i(j\omega)$ defined the multivariable amplitude Bode diagrams. An important concept in analysis and design, which is related to the SVD of G(s)

is that of vector gain. Thus, if $y(s) = G(s)\,u(s)$, $s=j\omega$ then one may define the vector gain of G(s) for input u(s) as gain: $G|\ u = \|\underline{y}(s)\|_2 / \|\underline{u}(s)\|_2$ where $\|\bullet\|_2$ denotes the standard Euclidean norm. If, $\sigma(j\omega)$, $\bar{\sigma}(j\omega)$ denote the minimal and maximal singular values of $G(j\omega)$, then:

$$\underline{\sigma}(j\omega) \le \|\underline{y}(j\omega)\| / \|\underline{u}(j\omega\| \le \bar{\sigma}(j\omega), \text{ for all } \underline{u}(j\omega) \tag{4.15}$$

The above property is known as Min-Max Theorem and indicates that the important indicator for performance are the plots of $\underline{\sigma}(j\omega)$, $\bar{\sigma}(j\omega)$,for every ω and all other gains are sandwiched between those two plots. The plots $\underline{\sigma}(j\omega)$, $\bar{\sigma}(j\omega)$ are called primary gain plots and a typical illustration is shown below.

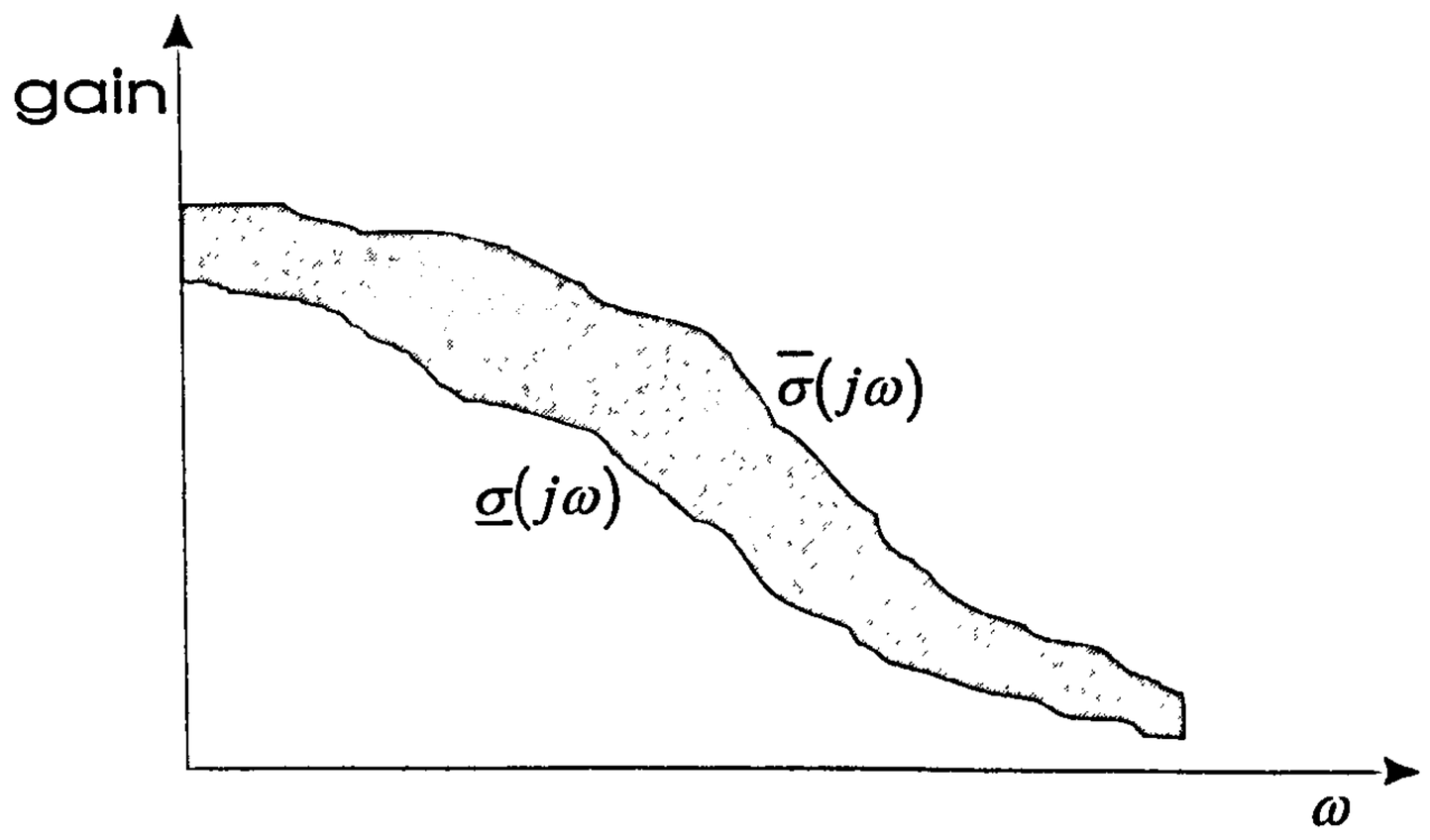

Figure (4.4): Typical primary gain plots

For every $s = j\omega$, $\bar{\sigma}(j\omega)$ defines $\|G(j\omega)\|_2$, the 2-norm of $G(j\omega)$ and $c(j\omega) = \bar{\sigma}(j\omega) / \underline{\sigma}(j\omega)$ the condition number of $G(j\omega)$. $c(j\omega)$ is an indicator of the relative spread of the gain at the corresponding frequency. Principal gain decompositions, or the primary gain plots are performance indicators for Control Design and they provide specific criteria for tracking, disturbance rejection, robust stability etc. Note that the gains defined by the characteristic gain decomposition are special types of vector gain. Some interesting relationships between characteristic gain loci and principal gain, phases are summarised below [Post. et al, 1].

Result (4.19): Let $G(j\omega) \in C^{nxn}, g_i(j\omega), i \in \underset{\sim}{m}$ be the characteristic gains $\underline{\sigma}(j\omega), \bar{\sigma}(j\omega)$ the primary gains and $\underline{\theta}(j\omega)$, $\bar{\theta}(j\omega)$ the minimum, maximum principal phases. Then

(i) $\underline{\sigma}(j\omega) \leq g_i(j\omega) \leq \bar{\sigma}(j\omega)$, , all i=1,...,m

(ii) If the principal phases have a spread of less than π radians, then $\underline{\theta}(jw) \leq \arg\{g_i(jw)\} \leq \bar{\theta}(jw)$, all $i = 1,...m$

□

Using a combination of principal gains and phases a Nyquist type indicator for robust stability may be developed based on ideas of gain and phase margin [Post. et al, 1]. Characteristic gains, despite their useful role in closed-loop stability analysis, do not always give an adequate description of the gain behaviour of an operator. If a transfer function is normal, the characteristic gains completely define the principal gains and phases. If the transfer function is approximately normal, then also the Nyquist diagrams may be used to describe the system performance. Indicators measuring the "departure from normality" are essential not only for assessing when Nyquist provides a good indicator for performance, but also for robustness of performance analysis. Such indicators express the skewness of vector frames and are based on the Schur Triangular Decomposition, Grammian etc.

4.4. SYSTEM INVARIANTS

4.4.1. Introduction

On a given system we may apply different types of transformations, some of them corresponding to a change of representation and some others having a compensator, or feedback interpretation. The theory of system invariants is important for control theory and design since they describe structural characteristics, which remain unaffected under the transformations. Their importance for EPD is due to that for certain types of them it might be possible to assign desirable values by EPD decisions. Here we try to summarise the basic invariants and where possible indicate their significance. This section is structured as follows: We first discuss the effect of transformations on the fundamental system properties and then we discuss the theory of invariants for state space models and finally for transfer

function models. The topic on system invariants and canonical forms is quite extensive. Here we attempt to summarise the basic aspects of the theory.

4.4.2. System transformations and Fundamental system properties

On a state space model S(A,B,C,D) we may apply coordinate and feedback transformations thus we consider the following cases:

(a) System properties under coordinate transformations.

(b) Systems properties under coordinate transformations and feedback.

Coordinate transformations are of the type $\underline{x} = Q\,\underline{x}'$ $\underline{y}' = T\underline{y}$, $\underline{u} = R\,\underline{u}'$, where Q, T, R are square nonsingular matrices. The effect of these transformations on system properties is summarised below.

Result (4.20): If Q, T, R are state, output, input coordinate transformations, then:

(i) The characteristic polynomial $\phi(\lambda) = |\lambda I - A|$, the eigenvalues and associated Segre characteristics, are invariants under all (Q, T, R) transformations.

(ii) The controllability, observability; stabilisability, detectability, output function controllability are invariants under all (Q,T,R) transformations.

(iii) The transfer function matrix and Markov parameters are invariant under all Q transformations.

□

Thus, the fundamental characteristics of the free motion, as well as stability properties, controllability properties may be inferred from any model obtained from S(A,B,C,D) and Q transformations. The eigenvectors, however, are functions of Q and their description changes with the changing of Q. The eigenframe is important, when we deal with a coordinate frame characterising physical states. Under the feedback transformations L,K,F expressing state, output feedback and output injection respectively, we have:

Result (4.21): If L, K, F denote state feedback, output feedback and output injection matrices respectively, then:

(i) Controllability and stabilisability are invariant under all L, K.

(ii) Controllability is invariant under all F, iff the system has no zeros. For any system, controllability is invariant under a generic F.

(iii) If the system is stabilisable then stabilisability is invariant under all F iff the system has no right half plane zeros. For any stabilisable system, stabilisability is invariant under a generic F.

(iv) Observability and detectability are invariants under all F, K.

(v) Observability is invariant under all L iff the system has no zeros. For any system, observability is invariant under a generic L.

(vi) If the system is detectable, then detectability is invariant under all L, iff the system has no right half plane zeros. For any detectable system, detectability is invariant under a generic L.

□

The presence of finite zeros implies that for certain families of output injection we lose controllability and for certain families of state feedback we lose observability [Shaked & Karc., 1]. The presence of right half plane zeros has corresponding implications to loss of stabilizability, detectability under certain families of output injections, output feedback correspondingly. More general types of transformations, which preserve the transfer function and certain properties of PMDs are discussed in [Rosen., 1]. The notion of coordinate transformations for state space models has its equivalent in the transfer function matrix concept, which is that of unimodular matrices. According to what sort of fractional description we consider G(s) we have the $\mathcal{K}$-unimodular matrices, $\mathcal{U}(m, \mathcal{K})$, where m denotes the dimension of the square matrix with elements from $\mathcal{K}$($\mathcal{K}$ is $\mathbf{R}[s]$, $\mathbf{R}_{pr}(s)$ or $\mathbf{R}_p(s)$) and if $Q \in \mathcal{U}(m, \mathcal{K})$, then $|Q|$ a unit of $\mathcal{K}$. The role of coordinate transformations in the system representation is emphasised by the following results.

Result (4.22): Let $G(s) \in \mathbf{R}_{pr}^{m \times p}(s)$. Then,

(i) [Kalman, 1] The minimal state space models S_i, $i = 1, 2$ have the same transfer function G(s), iff they are related by a state coordinate transformation.

(ii) [Kailath, 1] The left, right $\mathcal{K}$-coprime MFD pairs (A_{1i}, B_{1i}), (B_{2i}, A_{2i}), $i = 1, 2$, ($\mathcal{K}$ is $\mathbf{R}[s]$, $\mathbf{R}_{pr}(s)$, or $\mathbf{R}_p(s)$) have the same transfer function G(s), iff

$$[A_{12}, B_{12}] = L[A_{11}, B_{11}],\ L \in U(m,\mathcal{K}), \begin{bmatrix} B_{22} \\ A_{22} \end{bmatrix} = \begin{bmatrix} B_{21} \\ A_{21} \end{bmatrix} R,\ R \in U(p,\mathcal{K}) \qquad (4.16)$$

□

Remark (4.8): State feedback and output injection affect the closed-loop eigenvalues, but never shift them to infinity if their norm is bounded. Bounded norm output feedback, however, may result in a not well posed feedback system, which corresponds to the fact that certain eigenvalues may become arbitrarily large. Similarly, use of state, output derivative feedback may shift some of the eigenvalues to infinity, i.e., the resulting system may become singular.

□

4.4.3. State space invariants

On state space models we may apply different types of representation, compensation transformations and thus a variety of invariants and canonical forms are defined. Summarising the most fundamental types of state space invariants, is the aim of this section. Central to the definition and computation of most of the invariants is the theory of Kronecker invariants (and associated canonical form) of matrix pencils [Gant., 1]. The most general types of transformations that may be applied on the S(A,B,C,D) system are those defined by Q, T, R state, output, input coordinate transformations, state feedback L and output injection F. Based on the Q, T, R, L, F transformation we may define the following ordered sets of transformations.

$$\mathcal{H}_k \triangleq \{\mathcal{H}_k : \mathcal{H}_k \triangleq (Q,T,R;L,F)\},\ \mathcal{H}_B^r \triangleq \{\mathcal{H}_B^r : \mathcal{H}_B^r = Q,R;L)\},\ \mathcal{H}_B^l \triangleq \{H_B^l : H_B^l = (Q,T;F)\} \qquad (4.17a)$$

$$\mathcal{H}_c \triangleq \{\mathcal{H}_c : \mathcal{H}_c = (Q,T,R;0,0) \triangleq (Q,T,R)\},\ \mathcal{H}_c^{is} \triangleq \{\mathcal{H}_c^{is} : \mathcal{H}_c^{is} = (Q,0,R) \triangleq (Q,R)\} \qquad (4.17b)$$

$$\mathcal{H}_c^{os} \triangleq \{\mathcal{H}_c^{os} : \mathcal{H}_c^{os} = (Q,T,0) \triangleq (Q,T)\},\ \mathcal{H}_c^{s} \triangleq \{\mathcal{H}_c^{s} : \mathcal{H}_c^{s} = (Q,0,0) \triangleq (Q)\} \qquad (4.17c)$$

These transformations form groups (under a standard composition rule) $\mathcal{H}_k$, $\mathcal{H}_B^r$, $\mathcal{H}_B^l$ will be referred to as the Kronecker, Right-, Left-Brunovsky groups and $\mathcal{H}_c^{is}$, $\mathcal{H}_c^{os}$, $\mathcal{H}_c^{s}$ as general-input-state-, state-output-, state-co-ordinate groups respectively. The action of these groups on the system may be expressed as action on pencils associated with the corresponding type of system, which is considered. Thus,

(i) Action of $\mathcal{H}_k$, $\mathcal{H}_c$ on S'(A,B,C,D) is defined by:

$$\begin{bmatrix} sI-A' & -B' \\ -C & -D' \end{bmatrix} \triangleq \begin{bmatrix} Q^{-1} & F \\ 0 & T \end{bmatrix} \begin{bmatrix} sI-A & -B \\ -C & -D \end{bmatrix} \begin{bmatrix} Q & 0 \\ L & R \end{bmatrix} \tag{4.18a}$$

(ii) Action of $\mathcal{H}_B^r$, $\mathcal{H}_C^{is}$ on S (A,B) is defined by:

$$[sI-A, -B] = Q^{-1}\,[sI-A, -B] \begin{bmatrix} Q & 0 \\ L & R \end{bmatrix} \tag{4.18b}$$

(iii) Action of $\mathcal{H}_B^{\ell}$, $\mathcal{H}_c^{os}$ on S(A,C) is defined by:

$$\begin{bmatrix} sI-A' \\ -C' \end{bmatrix} \triangleq \begin{bmatrix} Q^{-1} & F \\ 0 & T \end{bmatrix} \begin{bmatrix} sI-A \\ -C \end{bmatrix} Q \tag{4.18c}$$

(iv) Action of $\mathcal{H}_c^s$ on S(A) is defined by: $sI-A' \triangleq Q^{-1}(sI-A)Q$

□

We consider next the types of invariants and canonical forms that may be defined on state space models under the different groups.

(I) Invariants and canonical forms under co-ordinate transformations

(a) State coordinate transformations on S(A)

For the system S(A): $\dot{\underline{x}} = A\underline{x}$, co-ordinate transformations are known also as similarity transformations. The structure of eigenvalues defines the invariants and canonical form [Gant., 1].

Result (4.23): If ϕ (A) is the root range of A, and $S(A,\lambda) = \{\nu_1 \leq \ldots \leq \nu_q\}$ is the Segre characteristic for every $\lambda \in \phi(A)$, then the set $\{\phi(A);\ S(A,\lambda), \text{ all } \lambda \in \phi(A)\}$ is a complete invariant for similarity equivalence on S(A). If $J_k(\lambda) \triangleq \lambda I_k + H_k$ is a typical kxk λ-Jordan block, then the corresponding canonical form is the Jordan canonical form

$$J(A) = \text{diag}\{...; J(\lambda); ...\},\ J(\lambda) = \text{diag}(J_{v1}(\lambda); ..., J_{vq}(\lambda)\} \tag{4.19}$$

□

The invariants and canonical form may be computed algebraically by use of the Smith form of sI-A (Computation of set of eds), or by alternative means based on sequences of numbers [Karc., 2]. The maximum of the geometric multiplicities of eigenvalues is denoted by ρ and referred to as the Segre index. If $\nu(\lambda)$ is the maximal value in $S(A,\lambda)$, then $n=\Sigma\ \nu(\lambda)$ defines the degree of the minimal polynomial of A. Alternative canonical forms, such as those of the companion type may be found in [Gant., 1].

Remark (4.9): The similarity invariants define the nature of elementary motions of S(A) and characterise stability properties. For eigenvalues on the imaginary axis it is essential to compute the corresponding Segre characteristic since this defines the difference between Lyapunov stability and instability. The Segre index ρ (max of q for all eigenvalues) defines the minimum number of inputs, outputs that are needed for controllability, observability, when inputs and outputs are selected.

□

(b) State, input coordinate transformations on S(A,B)

Throughout this section we assume that S(A,B) has n states, p inputs and $\rho(B)=p$. For the pair (A,B) we define the sequence of matrices, $Q_{c,k} \overset{\Delta}{=} [B,AB,....., A^KB]$, k=0,1,2.... where $Q_{c,n-1} \underline{\Delta}\ Q_c$ is the controllability matrix and $\rho(Q_{c,k}) \leq \rho(Q_{c,k+1})$.

Definition (4.12) [Kailath, 1]: The smallest integer μ for which $\rho(Q_{c,k}) = \rho(Q_{c,\mu+\nu})$ is defined as the controllability intex of S(A,B). If we assume that the linearly independent columns of Q_c in order from left to right here have been found and rearrange these independent columns as $\{\underline{b}_1, A\underline{b}_1, ..., A^{\mu-1}\underline{b}_1, ..., \underline{b}_p\ Ab_1, ..., A^{m\ p-1}\underline{b}_p\}$ then set of indices $\{\mu_i, i \in p\}$ are called the controllability indices to S(A,B).

□

Some important properties of these indices are summarised by the following result [Kailath, 1], [Chen, 1], [Karc. & MacB., 1]. Note $\mu_i \geq 1$, for all i=1, 2,.... and the zero value appears, only when $\rho(B) < p$ (which is not considered here).

Result (4.24): For the set $\mathcal{J}_c = \{\mu_I, \text{ie} \underset{\sim}{p}\}$ of controllability indices of S(A,B) we have:

(i) The controllability index $\mu = \max\{\mu_1,\mu_2,....,\mu_l\}$

(ii) If n is the degree of the minimal polynomial of A, then $n/p \leq \mu \leq \min(\bar{n}, n-p+1) \leq n-p+1$

(iii) $\mu_1+\mu_2+....+\mu_p \leq n$ and equality holds iff the system is controllable. Furthermore, $\sum_{i=1}^{\ell} \mu_i \underline{\underline{\Delta}} n_c$ is the dimension of the controllable space of the systems and $n-n_c$ defines the total number of uncontrollable modes.

(iv) The controllability indices are invariant under state, input coordinate transformations and state feedback.

(v) The set $\mathcal{J}_c$ is the same with the set of column minimal indices of the pencil $P_c(s) = [sI-A, -B]$.

(vi) The set $\mathcal{J}_c = \{\mu_i, \text{iep}\}$ defines the set of column minimal indices $\{\mu_i\}$ of the pencil $R_c(s)=sN-NA$ by the rule $\tilde{\mu}_i = \mu_i - 1$, i=1,2,...p.

(vii) If $G(s) = N(s)\, D(s)^{-1}$ is any R[s] - right coprime MFD with D(s) column reduced and S(A,B,C) is a minimal realization of G(s) (assume G(s) strictly proper), then the column degrees of D(s) define the controllability indices of S(A,B).

□

Remark (4.10): The set of controllability indices and the set of fed of $P_c(s)$ pencil are invariant under $\mathcal{H}_C^{IS}$ group, but they are not complete, that is, more invariants are needed to define a complete set.

□

Defining a complete set of invariants for S(A,B) under $\mathcal{H}_C^{S}$, $\mathcal{H}_C^{IS}$ groups is related to the theory of the Popov canonical form [Kailath,1]. The Popov canonical form [Popov,1], is a unique form under similarity for S(A,B) and contains all additional information about the new invariants, which are now a set of real numbers.

(c) State-Output Coordinate transformations on S(A,C)

Note that the definitions and results presented for (A,B) pairs have their equivalents for the case of (A,C) by using "transposed duality" arguments, that is (A^t, C^t) is first seen as a state, input pair and by transposition and use of the changes: controllability ↔

observability, right MFD, ↔ input ↔ output etc. all definitions and results may be stated for state, output pairs (A,C). The set of observability indices is denoted by $\mathcal{J}_o = \{\theta_i, i \in \underset{\sim}{m}\}$, where m is the number of outputs and θ denotes the observability index which now satisfies the inequality in (4.20), where $\bar{n}$ is the degree of the minimal polynomial.

- $$\frac{n}{m} \leq \theta \leq \min(\bar{n}, n-m+1) \leq n-m+1 \qquad (4.20)$$

(d) State Coordinate transformations of (A,B,C)

For systems S(A,B,C), the theory of invariants and canonical forms is richer than that of S(A,B), S(A,C) systems, since both aspects of the above two subsystems are involved. The sets of controllability, observability indices, are invariants, as well as the sets of input, output decoupling zeros and finite, infinite zeros. Note that the additional invariants, which will be defined under the Kronecher group $\mathcal{H}_k$, are also invariant under $\mathcal{H}_c^s$ since $\mathcal{H}_c^s$ is a subgroup of $\mathcal{H}_{c\,k}^s$. The canonical forms, which have been defined in the literature do not always demonstrate the structure of all of these invariants. If Q is a transformation that brings (A,B), to the Popov form (A_c,B_c) defined before, then the output $C_c=CQ$ is uniquely defined and (A_c,B_c,C_C) is an input based canonical form. Similarly, if Q is a transformation that brings (A,C) to the corresponding Popov form (A_0,C_0), then $B_0=Q^-B$ is uniquely defined and (A_0,B_0,C_0) is an output based canonical form. The Popov canonical forms (A_c,B_c,C_c), (A_0,B_0,C_0) are related to the realization of transfer function based on canonical right, left MFDs, that is those which are in a "echelon type form" [Kailath, 1]. Alternative canonical forms, based on the ideas of balancing the controllability and observability Grammians have been defined [Ober & McF., 1] such forms are more robust model parameter uncertainties and play a key role in model reduction. The canonical forms and invariants under co-ordinate transformations are important in system parametrisation, identification and model reduction.

(II) Invariants and Canonical forms under co-ordinate transformations and feedback

The transformations $\mathcal{H}_B^r$, $\mathcal{H}_B^l$, $\mathcal{H}_k$ contain as subgroups the $\mathcal{H}_c^{ls}$, $\mathcal{H}_c^{os}$, $\mathcal{H}_c$ thus, a number of the co-ordinate transformations invariants are not preserved under the more general groups, which are considered now.

(a) Coordinate transformations and state feedback on S(AB)

Under the action of $\mathcal{H}_B^r$ group (input, state co-ordinate transformations and state feedback) on S(A,B) systems, we obtain an equivalence class of systems $E_b(A,B)$ referred to as the Brunovski orbit of S(A,B). If $\mathcal{I}_c=\{\mu_i, i\in p\}$ are the set of controllability indices, or equivalently cmi of P(s) and $\mathcal{D}_{ID} = \{(s-\lambda i)^{\tau_i}, \lambda\in\mathbb{C}, i=1.....,k\}$ is the set of fed of $P_c(s)$ (defining the structure of input decoupling zeros) then we may summarise the properties of $\mathcal{E}_B(A,B)$ as follows [Brun., 1], [Kalman, 2], [Karc. & MacB., 1]:

Result (4.25): For the Brunovski orbit $\mathcal{E}_B(A,B)$ the following hold true:

(i) The sets $\mathcal{I}_c$, $\mathcal{D}_{ID}$ are complete and independent invariants of $\mathcal{E}_B(A,B)$

(ii) There is a uniquely defined canonical form, the generalised Brunovsky form, $S(A_B,B_B)$, which in pencil form is described by

$$P_c^B(S) = \left[sI - A_{B,} - B_B\right] = \left[\begin{array}{c|c|c} sI - A_c & 0 & -B_c \\ - & - & - \\ 0 & sI - A_{ID} & 0 \end{array}\right] \tag{4.21}$$

where $A_{ID} \stackrel{\Delta}{=}$ diag $\{J_{\tau_i}(\lambda_i), i=1....k\}$, $J_{\tau_i}(\lambda_i)$ is the Jordan block associated with $(s-\lambda i)^{\tau_i}$, $A_c \stackrel{\Delta}{=}$ diag $\{H_j: J=\mu_1,....,\mu_p\}$, H_j is the jxj standard nilpotent matrix and $B_c \stackrel{\Delta}{=}$ bl-diag $\{w_j, j=\mu_1, ..., \mu_p\}$, $w_j= [0,..0,1]^t\in R^J$.

□

$S(A_c,B_c)$ is the controllable subsystems and if S(A,B) is controllable, then $sI-A_{ID}$ is not present in (4.23). Controllability indices and the structure and values of decoupling zeros are the only invariants under $\mathcal{H}^r_B$.

Remark (4.11): Controllability indices are essential for identification and study of control theory problems such as: assignment of Jordan forms by state feedback [Rosen., 2], structure and parametrisation of controllability subspaces [Wonham, 1] etc. It seems, that the most relevant for our present work is the value of the controllability index μ.

□

(b) Co-ordinate transformation and output injection on S(A,C)

The results in the previous section have their duals for the Brunovski orbit $\mathcal{E}_B(A,C)$, obtained from S(A,C) under $\mathcal{H}^1_B$. The essence of the duality is that defined by transposition. The set of observability indices $\mathcal{I}_o$ and set of fed of $P_o(s)$, $\mathcal{D}_{OD}$, defining the structure of output decoupling zeros are complete invariants and the corresponding canonical form, is obtained from (A_B, B_B) by transposition [Karc. & MacB., 1].

(c) Co-ordinate transformations, state feedback and output injection on S(A,B,C,D): Kronecker invariants and canonical form

For the S(A,B,C,D) state space model with transfer function G(s), (n:states, l:inputs, m:outputs) the action of the Kronecker group $\mathcal{H}_K$ on S produces an equivalence class $\mathcal{E}_k(A,B,C,D)$. The natural tool to represent S(A,B,C,D) is the system matrix pencil P(s) ((n+m)x(n+p))

$$P(s) = \begin{bmatrix} sI - A & -B \\ -C & -D \end{bmatrix} \tag{4.22}$$

We assume $\rho(P(s))=r$, $\rho(G(s))=\rho$ (Over **R**(s)) and $\rho([C,D]) = m$, $\rho([B^t,D^t])=p$. The pencil P(s) is characterised by Kronecker invariants [Gant., 1], which are defined below.

Definition (4.13): For the system S(A,B,C,D), described by P(s) we define:

(i) $\mathcal{D}_z \underline{\underline{\Delta}} \{(s\text{-}z_i)^{\tau_i}$, ie $\underset{\sim}{\tau}\}$ the set of fed, which define the finite zero structure of S(A,B,C,D); the number $n_f = \sum_{i=1}^{\pi} \tau_i$ is called the finite zero order of the system.

(ii) $\mathcal{D}_\infty \underline{\underline{\Delta}} \{s^{qi}: 1=q_1=...=q_\delta<q_{\delta+1} \le ... \le q_\sigma\}$ the set of i-ed, which define the infinite zero structure of S(A,B,C,D); i-ed of the $\hat{s}$ type are called linear infinite zero divisors (lizd) and those of the $\hat{s}^q$ q>1, are called nonlinear infinite zero divisors (n-l.zd). The number $n_\infty = \sum_{i=1}^{\delta} (q_i\text{-}1)$ is defined as the infinite zero order of the system.

(iii) $\mathcal{J}_r = \{\varepsilon_i: 0<\varepsilon_1\leq...\leq\varepsilon\nu\}$, $\mathcal{J}_l = \{n_i:0<\eta_1\leq...\leq\eta_t\}$ are the sets of cmi, rmi respectively of P(s) and they are called the right-, left- indices of the system. The numbers $n_r \underline{\Delta} \sum_{i=1}^{p} \in_i$, $n_l \underline{\Delta} \sum_{i=1}^{t} \eta_i$ are called the right, left-order respectively of the system.

□

Remark (4.12): The finite and infinite zero structure is characterised in physical terms by the frequency transmission problem. The right, left indices are associated with the blocking of families of signals, which are not necessarily of the simple exponential type [Karc. & Kouv., 1].

□

The importance of the $\mathcal{D}_z$, $\mathcal{D}_\infty$, $\mathcal{J}_r$, $\mathcal{J}_l$ sets defined on S(A,B,C,D) is described below [Morse, 1], [Thorp, 1], [Karc. & MacB., 1]

Result (4.26): For the Kronecker orbit $\mathcal{E}_k$(A,B,C,D) the following hold true:

(i) The set $\{\mathcal{D}_z;\mathcal{D}_\infty;\mathcal{J}_r;\ \mathcal{J}_l\}$ defined on S(A,B,C,D) is a complete and independent invariant.

(ii) There is a uniquely defined canonical form, the Kronecker canonical form $S(A_k,B_k,C_k,D_k)$, which is pencil form is described by:

$$P_K(s)=\begin{bmatrix} sI-A_K & -B_K \\ -C_K & -D_K \end{bmatrix}=\left[\begin{array}{cccc:ccc} sI-A_\varepsilon & 0 & 0 & 0 & -B_\varepsilon & 0 & 0 \\ 0 & sI-A_\eta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sI-A_\infty & 0 & 0 & -B_\infty & 0 \\ 0 & 0 & 0 & sI-A_f & 0 & 0 & 0 \\ \hdashline 0 & -C_\eta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_\infty & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I_\delta \end{array}\right] \quad (4.23)$$

where: A_ε = diag$\{A_j:j=\varepsilon_1,...,\varepsilon_\nu\}$, η_r x η_r, A_η = diag$\{A_j:j=\eta_1,...,\eta_t\}$, $\eta_l x \eta_l$; A_∞ = diag $\{A_j:j=f_i,...,f_{\sigma-\delta}, f_i=q_{i-1}, i=\delta+1,...,\sigma\}$, nxn_∞ ; A_f = diag $\{J_{ti}(z_i): i\in\pi\}$, η_f x η_f; where $J_{ti}(z_i)$ are Jordan blocks characterising $(s-z_i)^{r_i}$, $A_j \underline{\Delta} H_j$, is the jxj standard nilpotent matrix and $C_\eta \overset{\Delta}{=}$ bl.diag $[v^t_j: j= \eta_1, ...,\eta_t\}$, π x η_l ; $C_\infty \underline{\Delta}$ bl.diag $\{v^t_j: j=f_1,...,f_{\sigma-\delta}\}$, $(\sigma-\delta)$ x η_∞ ;

$B_\infty \overset{\Delta}{=}$ bl.diag $\{w_j: j=f_i,...,f_{\sigma-\delta}\}$, η_∞ x$(\sigma-\delta)$; $B_\varepsilon \overset{\Delta}{=}$ bl.diag $\{w_j:j=\varepsilon_1,...,\varepsilon_v\}$, η_rxv ; where $v^t_j=[1,0,...0]$, 1 x j and $w_j = [0,...,0,1]^t$ is a jx1 vector.

(iii) If r,ρ are the ranks of P(s), G(s) respectively, then the following relationships hold true amongst the numbers of the invariants.

(a) $r = n+\rho$, $v= l-\rho$, $t = m-\rho$, $n=n_f + n_\infty + n_r + n_l$

(b) $\sigma = \rho$ and $\delta = \rho(D)$

(c) There are zero cmi, zero rmi, iff $[B^t,D^t]^t$, [C,D] are rank deficient respectively

(iv) If $G_k(s)$ is the Smith form at s=∞ of G(s) [Vard. et al, 1], then the transfer function matrix of $S(A_k, B_k, C_k, D_k)$ is:

$$G_k(s) = C_k(sI-A_k)^{-1} B_k+D_k = \left[\begin{array}{c|c} M^*_\infty(s) & 0_{\sigma,\rho} \\ \hline 0_{\iota,\sigma} & 0_{\iota,\rho} \end{array}\right], \; M^*_\infty(s) = \text{diag}\,[s^{1-q}_1 ...., s^{1-q}_\mu\} \qquad (4.24)$$

□

The above summary of results demonstrates the structure of state-space models under the most general types of transformations that may be applied to them. The importance of the result is that it establishes the numbers, and relationships between different invariants, which enter into the solvability condition of many control problems.

Remark (4.13): The number of divisors at infinity of P(s) is equal to the rank of G(s). There exists a number of linear divisors at infinity equal to the rank of D; for strictly proper systems, all divisors at infinity are nonlinear, i.e. $q_l \geq 2$. The orders of infinite zeros are defined by $f_i = q_i - 1$, when q_i are the degrees of nonlinear divisors at infinity. The f_i define the generic asymptotic root locus pattern and terminal Nyquist phases. If $\rho(D) = \rho(G(s))$, then G(s) has no infinite zeros, or equivalently all q_i's are equal to 1. For strictly proper, square systems with ρ=m=p, all orders of infinite zeros of G(s), f_i, are equal to 1, iff $\rho(CB) = m = p$; higher order of infinite zeros emerge when $\rho(CB) < m = p$.

□

Remark (4.14): The Kronecker form $S(A_k,B_k,C_k,D_k,)$ is maximally uncontrollable and unobservable and the dimension of the minimal system is defined by the infinite zero order.

State feedback and output injection are equivalent to post-, pre-multiplication of transfer function by $\mathbf{R_{pr}}(s)$ unimodular matrices, the special element of $\mathcal{H}_k$ that reduce S to its Kronecker form, is equivalent to a pair of $\mathbf{R_{pr}}(s)$ unimodular matrices which reduce G(s) to its Smith form at $s=\infty$ of G(s).

□

Remark (4.15): For right regular systems ($\rho = p$), $n_r = 0$ (no right indices) and for left regular systems ($\rho = m$), $n_\ell = 0$ (no left indices). For left-right regular systems ($\rho = m = p$) (square nondegenerate systems), $n_r = n_\ell = 0$ and $n_f + n_\infty = n$, which shows that total number of finite and infinite zeros is equal to the dimension of the state space. For such systems, the total number of finite zeros satisfies the conditions:

(i) $D \neq 0$: $n_f \leq n$ and $n_f = n$, iff $\rho(D) = m = p$

(ii) $D = 0$: $n_f \leq n-m = n-p$ and equality holds, iff $\rho(CB) = m = p$

For strictly proper square systems, the number $n - m = n - p$ defines an upper bound on the total number of finite zeros.

□

The right and left indices are related to problems such as squaring down, model matching etc. Their relationships to transfer function invariants will be discussed later.

Remark (4.16): The finite zeros of P(s) and Z(s) (zero pencil) are the same. If q_i, $i \in \underset{\sim}{\tau}$ are the degrees of divisors at $s=\infty$, with $q_i \geq 3$ of P(s), then the degrees of restricted zero divisors of Z(s) are q_i-2, $i \in \underset{\sim}{\pi}$.

□

4.4.4. Transfer function invariants

With a transfer function matrix $G(s) \in \mathbf{R}(s)^{mxp}$ we may always associate the $\mathcal{K}$-coprime MFDs, $G = A^{-1} B_1 = B_2 A_2^{-1}$, where $\mathcal{K}$ is $\mathbf{R}[s]$, $\mathbf{R_{pr}}(s)$, or $\mathbf{R_p}(s)$ and with them we associate the left-, right-MFD matrices.

$$T_l = [A_1, B_1] \in \mathcal{K}^{m\times(m+p)}, \quad T_r \stackrel{\Delta}{=} \left[\frac{B_2}{A_2}\right] \in \mathcal{K}^{(m+p)\times p} \tag{4.25}$$

On G(s), T_ℓ, T_r we may apply different types of transformations which are based on the ring $\mathcal{K}$ which is used to describe fractionally a rational function. The $\mathcal{K}$-unimodular matrices define these transformations and the basic tools are those defined by the Smith, Smith-McMillan forms, as well as those of Hermite, Hermite-McMillan forms. The results are summarised next and their significance for the structure of linear systems is also discussed. Throughout this section it is assumed that $\rho(G) = r \leq \min(m, \ell)$ and that $\mathcal{K}$ is any Euclidean ring such that **R** (s) may be expressed as the field of fractions of $\mathcal{K}$. For control theory applications $\mathcal{K}$ is **R** [s], $\mathbf{R}_{pr}(s)$, $\mathbf{R}_p(s)$, or $\mathbf{R}_o(s)$ (rational functions which have no poles at s = 0).

(a) Smith McMillan forms over $\mathcal{K}$

If L, R are $\mathcal{K}$ unimodular matrices ($L \in U(m, \mathcal{K})$, $R \in U(\ell, \mathcal{K})$), then the natural equivalence $\mathcal{E}_k$ is defined by pre-, post-multiplication of G by L,R and: LGR is the general element of the orbit (equivalence class) $\mathcal{E}_k$ (G). If $G \in \mathcal{K}^{mxp}$, a canonical form and invariants is defined by the Smith-McMillan form over $\mathcal{K}$ [Kailath, 1], [Vard. & Karc., 1], [Vard. et al, 1].

Result (27): The orbit $\mathcal{E}_k$ (G) is characterised by a canonical form M^K_G, the Smith McMillan form over $\mathcal{K}$, where:

$$M^{\mathcal{K}}_G = \begin{bmatrix} M^{*\mathcal{K}}_G & | & O \\ - & | & - \\ O & | & O \end{bmatrix} \begin{matrix} \uparrow \\ r \\ \downarrow \\ \uparrow \\ m-r \\ \downarrow \end{matrix} \qquad M^{*K}_G = \text{diag}\{\varepsilon_i / \psi_i, i \in r\} \tag{4.26}$$

$\leftarrow r \rightarrow \leftarrow p-r \rightarrow$

where $(\varepsilon_i, \psi_i,)$ are $\mathcal{K}$ coprime, they are uniquely defined (modulo $\mathcal{K}$ units) and satisfy the divisibility properties: $\varepsilon_1 | \varepsilon_2 | \ldots\ldots | \varepsilon_r$, $\psi_r | \psi_{r-1} | \ldots\ldots | \psi_1$. The ε_i, ψ_i, $i \in r$, are the elementary $\mathcal{K}$-zero-pole, -pole-functions of G and together with r define & complete and independent set of invariants under $\mathcal{E}_k$ equivalence.

□

Remark (4.17): The Smith-McMillan form over the different rings $\mathcal{K}$ reveals the following information about the system:

(i) For $\mathcal{K} = \mathbf{R}$ [s] indicates the zero, pole structure of G over $\mathbb{C}$. The polynomials ε_i, ψ_i define the finite zeros, poles of G and ∂ ($\prod\psi_i$) the finite McMillan degree δ_M of G. This canonical form does not reveal any information about the structure of G at $s = \infty$.

(ii) For $\mathcal{K} = R_p(s)$, $\mathcal{P} = \Omega U\{\infty\}$ indicates the zero, pole structure of G over $\mathcal{P}$. The proper and Ω-stable functions ε_i,ψ_i define the zeros, poles of G in the region $\mathcal{P}$ and thus characterise infinite zero, pole structure, as well as Ω-nonminimum phase, Ω-instability structure of G. This canonical form does not reveal anything about the structure of G in the region Ω^c (the complement of Ω with respect to $\mathbb{C}$).

(iii) For $\mathcal{K} = \mathbf{R}_{pr}(s)$ indicates the zero, pole structure of G at infinity only, but nothing about the structure of G over $\mathbb{C}$. The ε_i, ψ_i are then proper rational functions of the type $(1/s)^q$, $q \geq 0$ indicating the orders of infinite zeros, poles of G.

(iv) For $\mathcal{K} = \mathbf{R}_0(s)$ indicates the zero, pole structure of G at s=0 only, but nothing about the structure of G over **C**-{0}, or s=∞. The ε_i, ψ_i are polynomials of the type s^P, $p \geq 0$, indicating zero, pole type of G at s=0.

□

Smith-McMillan forms reveal the basic pole zero structure over different subsets of $\mathbf{C} \cup \{\infty\}$; the standard tool for analysis is the form over **R**[s], whereas that over $\mathbf{R}_p(s)$ is essential for studies of stabilisation in the generalised Ω-sense. The Smith-McMillan forms over $\mathbf{R}_{pr}(s)$, or $\mathbf{R}_0(s)$ are local, since the reveal the structure at $\{\infty\}$, or $\{0\}$ respectively; the first is important for characterisation of properness and infinity zero structure, whereas the second is essential for the study of steady-state-tracking disturbance rejections.

(b) Smith forms over K

If $G \varepsilon \mathcal{K}^{mxp}$, then the Smith-McMillan form is reduced to the $\mathcal{K}$ -Smith form, which is defined as in (4.29b) with the only difference that all the ψ_i's are 1; that is the G has no poles over $\mathcal{K}$, but only possibly zeros. Smith forms are essential tools for $\mathcal{K}$-coprimeness tests and thus they are involved in the characterisation of irreducible $\mathcal{K}$-MFDs, as well as solvability of matrix equations over $\mathcal{K}$.

(c) Rational vector spaces and transfer function matrix invariants

Under $\mathcal{E}_k$ type of equivalence the column, row spaces of a transfer function change. A richer set of invariants, which is directly related to pre-, post-compensation of transfer functions, is defined under left-, or right $\mathcal{K}$-unimodular equivalence. If $G \in \mathbf{R}(s)^{mxp}$, $L \in U(m,\mathcal{K})$, $R \in U(\ell,\mathcal{K})$, where $U(\ell,\mathcal{K})$ denotes the ℓ linear group over $\mathcal{K}$, then G and $G' = GR$ are $\mathcal{K}$-right equivalent, and is denoted by $GE^{r}_{\mathcal{K}}G'$, and G and $G'' = LG$ are $\mathcal{K}$-left equivalent, and is denoted by $GE^{\ell}_{\mathcal{K}}G''$; the corresponding equivalence classes, orbits are denoted by $E^{r}_{K}(G)$, $E^{\ell}_{K}(G)$.

Definition (4.14): Let $G \in \mathbf{R}(s)^{mxp}$, $\rho(G) = r \leq \min(m,p)$ and let $G = A^{-1}_1 B_1 = B_2 A_2^{-1}$ be $\mathcal{K}$-coprime left, right MFDs ($\mathcal{K}=\mathbf{R}[s]$, $\mathbf{R}_p(s)$, $\mathbf{R}_{pr}(s)$). With the given G we define:

(i) $\mathcal{X}_{c,G} \triangleq \text{col.sp}_{r(s)}\{G\}$, $\mathcal{X}_{r,G} \triangleq \text{row.sp}_{lR(s)}\{G\}$ as the R(s)-column-, row-vector space of G respectively and $\mathcal{N}_{r,G} \triangleq \mathcal{N}_r\{G\}$, $\mathcal{N}_{\ell,G} = \mathcal{N}_{\ell}\{G\}$ as the R(s)-right-, left-null space of G correspondingly.

(ii) $\mathcal{Y}_{l,G} \triangleq \text{row.sp}_{R(s)}\{T_l\}$, $\mathcal{Y}_{r,G} \triangleq \text{colsp}_{lR(s)}\{T_r\}$ as the R(s)-composite-left-, right-space of G respectively, where T_l, T_r are defined by (4.25).

(iii) $\mathcal{M}^{k}_{c,G} \triangleq \text{col.sp}_k\{B_2\}$, $\mathcal{M}^{k}_{r,G} \triangleq \text{rowsp}_k\{B_1\}$ as the $\mathcal{K}$ column-, row- module of G respectively and $\mathcal{T}^{k}_{\ell,G} \triangleq \text{rowsp}_k\{T_l\}$, $\mathcal{T}^{k}_{\ell,G} \triangleq \text{colsp}_k\{T_r\}$ as the $\mathcal{K}$ -composite-left-, right-module of G correspondingly.

(iv) $\mathcal{M}^{K}_{c,G}$ is the set of all $\underline{x} \in \mathcal{V}^m$ vectors which are in $\mathcal{X}_{c,G}$ and $\mathcal{M}^{K}_{r,G}$ is the set of all $\underline{y} \in \mathcal{K}^p$ vectors such that $\underline{y}^t \in \mathcal{X}_{r,G}$.

□

For any rational transfer function matrix the following general invariants may be established [Kailath, 1], [Vard. & Karc., 1], [Rosen., 1]:

Result (4.28): For all rings $\mathcal{K}$: $\mathbf{R}[s]$, $\mathbf{R}_p(s)$, $\mathbf{R}_{pr}(s)$ the following properties hold true:

i) $\mathcal{X}_{c,G}$, $\mathcal{N}_{\ell,G}$ are invariants of $\mathcal{E}'_k(G)$ and $\mathcal{X}_{r,G}$, $\mathcal{N}_{r,G}$ are invariants of $\mathcal{E}_k(G)$; these properties also hold true for $\mathcal{K}=\mathbf{R}(s)$.

ii) For all $\mathcal{K}$-MFDs, not necessarily coprime, $\mathcal{Y}_{\ell,G}$ is invariant for the left MFDs and $\mathcal{Y}_{r,G}$ is invariant for the right MFDs.

iii) $\mathcal{M}^{k}_{C,G}$ is invariant of $\mathcal{E}^{r}_{K}(G)$ and $\mathcal{M}^{K}_{r,G}$ is invariant of $\mathcal{E}'_{K}(G)$.

iv) $\mathcal{T}^{K}_{l,G}$, $\mathcal{T}^{k}_{r,G}$, are complete invariants for all left, right K-coprime MFDs respectively.

v) $\mathcal{M}^{*K}_{c,G}$, $\mathcal{M}^{*K}_{r,G}$ are maximal K-modules which have the following properties:

(a) If $\rho(G) = p$, G may be factorised as: $G = B_r Z_r D_r^{-1}$, $B_r e \mathcal{K}^{mxp}$, Z_r, $D_r E \mathcal{K}^{pxp}$, where $(B_r Z_r, D_r)$ is $\mathcal{K}$-right coprime, B_l is $\mathcal{K}$-right-irreducible and $col.sp_K(B_r)=M^{k}_{c,G}{}^{*}$; furthermore, $M^{k}_{c,G}$ is invariant for any GQ, $Q \in U(p,\mathbf{R}(s))$

(b) If $\rho(G) = m$, G may be factorised as: $G = D_l^{-1} Z_l B_l$, $B \in \mathcal{K}^{mxp}$, Z_l, $D_l \in \mathcal{K}^{mxm}$, where $(D\ell, Z\ell\ B\ell)$ is $\mathcal{K}$-left coprime, B_l is $\mathcal{K}$-left-irreducible and row $sp_K(B\ell) = M^{k}_{c,G}{}^{*}$ furthermore, $M^{k}_{c,G}{}^{*}$ is invariant for any PG, $P \in U(m,\mathbf{R}(s))$.

□

The above summary of results clearly indicates that the theory of transfer function invariants is related to the theory of invariants of rational vector spaces and $\mathcal{K}$-modules contained in them [Forney, 1], [Kailath, 1], [Vard. & Karc., 2]. This theory is quite rich and becomes rather concrete, in terms of the theory of minimal bases [Forney, 1], [Vard. & Karc., 2] or equivalent by using tools from exterior algebra [Karc. & Gian., 1]. For the case $\mathcal{K}$=R[s], some useful interpretations of the above mathematical result are as follows:

Remark (4.18):

(i) Pre-, post-compensation of G by a square full rank rational compensator leaves invariant the rational vector spaces $\mathcal{X}_{r,G}$, $\mathcal{X}_{c,G}$ respectively; thus, $\mathcal{X}_{r,G}$, $\mathcal{X}_{c,G}$ are not spaces characterising a single transfer function, but a family of transfer functions.

(ii) The rational vector spaces $\mathcal{Y}_{l,G}$, $\mathcal{Y}_{r,G}$ characterise all left-, right- MFDs of G and thus are common to all state space models that have a common transfer function; these spaces are "personal" spaces of G. The modules $\mathcal{T}_{l,G}$, $\mathcal{T}_{r,G}$ (defined for $\mathcal{K}$ = R[s] characterise all left-, right- R[s] coprime MFDs and thus they are invariants of all minimal realisations of G.

(iii) The modules $\mathcal{M}^{k}_{c,G}{}^{*}$, $\mathcal{M}^{k}_{c,G}{}^{*}$ for $\mathcal{K} = R_{pr}(s)$, R(s) define invariants under post-, pre-multiplication respectively by proper, proper and stable square rational transfer functions.

□

For a rational vector space $\mathcal{X}$, $\mathcal{X} \in \mathbf{R}^{n}(s)$, with $\dim\mathcal{X} = \rho$, the theory of invariants, based on the polynomial interpretation, has three alternative directions.

(i) Minimal degree **R**[s] – bases

(ii) R[s] - Hermite forms

(iii) Plücker Matrices

and they are defined below for the general $\mathcal{X}$ and then specialised to the rational vector spaces associated with a transfer function G.

Definition (4.15): Let $X(s) \in \mathbf{R}[s]^{nxp}$ be a polynomial basis matrix of $\mathcal{X}$, i.e. $\rho(X)=p$, $col.sp_{lR(s)}\{X\}=\mathcal{X}$, and let $X(s) = [...,x_i(s)...]$, $x_i(s) \in \mathbf{R}^n[s]$, $\partial[x_i(s)] \underline{\Delta} \delta_i$, $i \in \underset{\sim}{p}$

(i) [Forney, 1] $\mathcal{X}(s)$ will be called an R[s]-minimal basis (R[s]-MB) if it is right irreducible (no finite zeros) and column reduced (full rank high column coefficient matrix). It is called an ordered -**R**[s]-MB if $\delta_i \leq \delta_{i+1}$, $\forall i \in \underset{\sim}{\rho}$. The set $\mathcal{J}_x \underline{\Delta} \{\delta_i, iep:\delta_i \leq \delta_{i+1}\}$ is called the Forney dynamical indices (FDI) of X(s) and $\delta_F = \sum_{i=1}^{p} \delta_i$ the Forney dynamical order (FDO) of X(s).

(ii) [Karc. & Gian., 1] The polynomial multivector $g(x) \underline{\Delta} x_1(s) \wedge .... \wedge \underline{x}_p(s) = C_p(X) \in \mathbf{R}[s]^v$, $v = \binom{n}{p}$, is defined as an R[s]-Grassmann representative (**R**[s]-GR) of $\mathcal{X}$. If X(s) is right irreducible, then g(X) is called a canonical **R**[s]-GR. If g(X) is canonical and $\partial[g(X)] \underline{\Delta} \delta$, then it may be expressed in terms of: P_δ the Plücker matrix of $\mathcal{X}$ as: $g(X) = P_\delta \underline{e}_\delta(s)$, $\underline{e}_\delta(s) = [1,s,...,s^\delta]^t$, $P_\delta \in \mathbf{R}^{vx(\delta+1)}$.

□

Result (4.29) [Forney, 1]: For any rational vector space $\mathcal{X}$, the following hold true:

(i) All **R**[s]-MBs of $\mathcal{X}$ define the same **R**[s]-module M*, which is a maximal Noetherian module.

(ii) All ordered -**R**[s]-MB have the same set $\mathcal{J}_x$ of FDIs and thus $\mathcal{J}_x$ and δ_F are invariants of $\mathcal{X}$.

(iii) There exists a uniquely defined **R**[s]-MB, the echelon type basis, the elements of which uniquely characterise $\mathcal{X}$.

(iv) If $X(s) \in R(s)^{nxp}$ is any rational basis of $\mathcal{X}$, then it may be factorised as:

$$X(s) = N(s)\, Z(s)\, D(s)^{-1} \tag{4.27}$$

where N(s) is an **R**[s]-MB, Z(s), D(s) are pxp polynomial matrices defining the finite zeros, poles respectively of X(s).

□

For a transfer function matrix G, $G \in \mathbf{R}_{pr}(s)^{mxp}$, with $\rho(G) = \min(m,p)$, we have the rational vector spaces $\mathcal{X}_{c,G}$, $\mathcal{X}_{r,G}$, $\mathcal{N}_{r,G}$, $\mathcal{N}_{l,G}$, as well as $\mathcal{Y}_{l,G}$ and $\mathcal{Y}_{r,G}$. The Forney dynamic indices and Forney order of these subspaces will be denoted by $\mathcal{I}(\mathcal{A})$, $\delta_F(\mathcal{A})$, where $\mathcal{A}$ is the corresponding space. Some additional properties are given below [Kailath, 1]

Result (4.30): For the family of rational vector spaces associated with the transfer function matrix G, $G \in R_{Pr}(s)^{mxp}$ $\rho(G) = \min(m, p)$, we have the following properties:

(i) $\mathcal{I}(\mathcal{Y}_{\ell,G})$ defines the observability indices and $\mathcal{I}(\mathcal{Y}_{rG})$ the controllability indices of any realization of G; furthermore, $\delta_F(\mathcal{Y}_{r,G}) = \delta_F(\mathcal{Y}_{\ell,G}) = \delta_M(G)$, the McMillan degree of G.

(ii) If m>p, then $\mathcal{N}_{r,G} = O$, $\delta_F(\mathcal{X}_{c,G}) = \delta_F(\mathcal{N}_{\ell,G})$, and $\mathcal{I}(\mathcal{X}_{r,G}) = \{0, ..,0\}$, that is $\mathcal{I}_\ell$ is an **R**[s] - MB of $\mathcal{X}_{r,G}$.

(iii) If m<p, then $\mathcal{N}_{\ell,G} = O$, $\delta_F(\mathcal{X}_{r,G}) = \delta_F(\mathcal{N}_{r,G})$ and $\mathcal{I}(\mathcal{X}_{c,G}) = \{0,...,0\}$, that is $\mathcal{I}_m$ is an **R**[s] - MB of $\mathcal{X}_{c,G}$.

(iv) If m= p, $\mathcal{N}_{\ell,G} = O$, $\mathcal{N}_{r,G} = O$, $\mathcal{I}(\mathcal{X}_{c,G}) = \{0,...,0\}$, $\mathcal{I}(\mathcal{X}_{r,G}) = \{0,..,0\}$, that is $\mathcal{I}_m$ is an R[s]-MB of $\mathcal{X}_{c,G}$ and $\mathcal{X}_{r,G}$.

□

The nontrivial set $\mathcal{I}(\mathcal{X}_{c,G})$, when m≥p, or $\mathcal{I}(\mathcal{X}_{r,G})$, when m≤p, will be referred to as external dynamical indices (EDI) of G and are invariants under square full rank post-, pre-compensation. These indices are important in the study of compensation, as well as squaring down of systems. An alternative, complete set of transfer function invariants, which is useful in the study of DAP problems is defined below [Karc. & Gian., 2]:

Result (4.19): For any rational vector space $\mathcal{X}$ the following properties hold true:

(i) If $g(X_1)$, $g(X_2)$ are any two **R**[s] -GRs, of $\mathcal{X}$, then $g(X_1) = g(X_2)c$,where $c \in \mathbf{R}(s)$.

(ii) A canonical **R**[s]-GR is coprime (has no zeros) and uniquely characterises $\mathfrak{X}$ (modules $c \in \mathbf{R}$, $c \neq 0$); furthermore, $\delta[g(X)] = \delta_F(X)$.

(iii) A Plücker matrix is a complete invariant (modules $c \in \mathbf{R}$, $c \neq 0$) of $\mathfrak{X}$.

□

A canonical -**R**[s]-GR, or equivalently a Plücker matrix is a complete invariant of $\mathfrak{X}$ and in this sense is equivalent to the echelon type minimal basis of $\mathfrak{X}$. For a given G, the corresponding Plücker matrices, are defined below [Karc. & Gian., 3]:

Remark (4.20):

(i) If $m>p$, $P_c(G)$ is the $\binom{m}{p}$ x $(\delta_{F,c} + 1)$ Plücker matrix of $\mathfrak{X}_{c,G}$, where $\delta_{F,r} = \underline{\Delta}\delta_F(\mathfrak{X}_{c,G})$; the Plücker matrix of $\mathfrak{X}_{r,G}$ is $P_r(G) = 1$.

(ii) If $m<p$, $P_r(G)$ is the $(\delta_{F,r} + 1)$ x $\binom{p}{m}$ is the Plücker matrix of $\mathfrak{X}_{r,G}$, where $\delta_{F,r}\underline{\Delta}\delta_F$ $(\mathfrak{X}_{r,G})$, the Plücker matrix of $\mathfrak{X}_{c,G}$ is $P_c(G) = 1$.

(iii) If $m=p$, $P_r(G) = 1$, $P_c(G) = 1$, are the Plücker matrices of $\mathfrak{X}_{r,G}$, $\mathfrak{X}_{c,G}$.

(iv) $P(T_\ell)$, $P(T_r)$ are the $n+1$ x $\binom{m+p}{m}$, $\binom{m+p}{p}$, x $(n+1)$ Plücker matrices of $\mathcal{Y}_{\ell,G}$, $\mathcal{Y}_{r,g}$ respectively, where $n = \delta_m(G)$.

□

Plücker matrices associated with the basic matrix pencils may also be defined, as it has been shown in the previous section. The matrices $P_c(G)$, $P_r(G)$ are essential in the study of zero assignment problems by "squaring down", whereas $P(T_\ell)$, $P(T_r)$ are crucial in the study of pole assignment by constant, or dynamic output feedback.

(d) Hermite, Hermite-McMillan forms and invariants

With a transfer function matrix we associate rational vector spaces, as well as $\mathcal{K}$-modules. The notions of $\mathcal{K}$-right-, -left-equivalence defined before is intimately related to compensation theory under special types of compensators; thus, if $\mathcal{K}$-$\mathbf{R}_{pr}(s)$, or $\mathbf{R}_p(s)$, then the corresponding equivalence classes are systems obtained under proper, proper and stable

pre-, or post compensation. The theory of right-, -left- equivalence produces types of invariants based on the modules contained in a rational vector space. We distinguish:

(i) Transfer functions (matrices) with elements from a given ring $\mathcal{K}$.

(ii) Transfer functions (matrices) with elements rational functions, i.e. fractions of the elements of $\mathcal{K}$.

The first case is related to the theory of Hermite forms, whereas the second to the case of Hermite-McMillan forms. In the following, by $\mathcal{K}$ we mean either of the cases $\mathbf{R}(s)$, $\mathbf{R}_{pr}(s)$, $\mathbf{R}_{p}(s)$.

(i) **Hermite forms:** We consider matrices $G \in \mathcal{K}^{mxp}$, assume $\rho(G) = m$ $(m \leq p)$ and consider the case of $\mathcal{K}$-left equivalence, $\mathcal{E}^{l}_{k}$. The case of $\mathcal{K}$-right equivalence, as well as the case where $\rho(G) < \min(m,p)$ may be found in the references [Marcus & Minc. 1].

Result (4.31): For a matrix G with the above properties there exists $L \in U(m,\mathcal{K})$ such that

$$LG = \mathcal{H}^{l,K}_{G} = \begin{bmatrix} 0 & \cdots & 0 & x & \cdots & x & \cdots & x & \cdots & x \\ 0 & \cdots & 0 & 0 & \cdots & & \cdots & x & \cdots & x \\ \vdots & & \vdots & \vdots & & & & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & x & \cdots & x \end{bmatrix} \begin{matrix} \leftarrow 1 \\ \leftarrow 2 \\ \vdots \\ \leftarrow m \end{matrix} \qquad (4.28)$$

$\uparrow n_1$ $\uparrow n_2$ $\uparrow n_m$

where $\mathcal{H}^{l,K}_{G}$ is called the $\mathcal{K}$-Hermite row form of G and its elements associated with the p_i rows i m and γ_j columns, j p satisfy the conditions:

(a) $\forall\ i \in \underset{\sim}{m}$, the ρ_i row has a leading nonzero $\mathcal{K}$-monic element h_{ini} (leading entry) such that $1 \leq n_1 < n_2 < \ldots < n_m \leq n$

(b) $\forall\ i \in \underset{\sim}{m}$, then

(i) If $h_{ini} = 1$, $h_{jni} = 0$, $\forall\ j<i$.

(ii) If $h_{ini} \neq 1$, $\partial\,[h_{jni}] < \partial\,[h_{ini}]$, $\forall\ j<i$ s.t. $h_{jni} \neq 0$.

(c) $\forall \gamma_j$ row s.t. $j<n_1$ is zero.

(d) $\forall \gamma_j$ s.t. $n_i < j < n_{i+1}$ with $i \in \underset{\sim}{m-1}$, then the last m-i entries of γ_j are zero.

□

The Hermite forms $\mathcal{H}_G^{l,k}$ is unique (module diagonal scaling by units) and its elements define a complete set of invariants of $E'_k(G)$.

A similar result may be stated for $\mathcal{K}$-right equivalence and the corresponding $\underline{\mathcal{K}}$-Hermite column form of G is denoted by $\mathcal{H}^{r,K}{}_G$. The set of indices $\mathcal{J}= n_i \; i \in \underset{\sim}{m}\}$ are defined as Hermite indices (row or column) and they are also invariants.

(ii) **Hermite McMillan forms:** For a general rational matrix $G \in \mathbf{R}(s)^{mxp}$ of full rank we may define canonical forms under $\mathcal{K}$-left, right-equivalence as follows: Let every element of G be expressed as coprime fraction of elements of $\mathcal{K}$ and let d be the least common multiple of the denominator of the elements of G. Then we may write:

$G = \frac{1}{d} N, \; N \in \mathcal{K}^{mxl}$. If $\mathcal{H}_N^{l,k}$ is the $\mathcal{K}$-row-Hermite form of N we may write:

$\mathcal{H}_N^{l,k} = L\,N, \; L \in U(m, \mathcal{K})$ and thus

$$\mathcal{H}_G^{l,k} = \frac{1}{d}\mathcal{H}_N^{l,k} = L\,G \tag{4.29}$$

is defined as the $\mathcal{K}$-row-Hermite-McMillan form of G, where in $\mathcal{H}_G^{l,k}$ all possible numerator-denominator cancellations are assumed to have been carried out.

Result (4.32): The $\mathcal{K}$-row-Hermite-Macmillan form of G, $\mathcal{H}_G^{l,k}$ is a complete invariant of $\mathcal{E}_k^l(G)$.

□

Note that the structure of $\mathcal{H}_G^{l,k}$ is similar to that of $\mathcal{K}$-row-Hermite form, i.e. "upper staircase", but its elements are rational functions. The corresponding structure and result for the K-row-Hermite-Macmillan form is similar. Some of the invariants of transfer functions under, left, -right K-equivalence are summarised below [Vard. & Karc., 2]

Result (4.21): Let $G \in \mathbf{R_{pr}}(s)^{mxp}$ and assume that $\rho(G) = p$ $(m \geq p)$.

(i) The set of Forney dynamical indices $\mathcal{J}(\mathcal{X}_{c,G})$ and Forney order δ_F of $\mathcal{X}_{c,G}$ are invariants of $\mathcal{E}^r_k(G)$ for all $\mathcal{K}$ of the type $\mathbf{R}[s]$, $\mathbf{R_{pr}}(s)$, $\mathbf{R_p}(s)$.

(ii) The set of finite zeros and poles of G, together with their corresponding multiplicities are invariants of $\mathcal{E}^r_K(G)$ for $\mathcal{K}=\mathbf{R}[s]$; the infinite zeros, poles are not necessarily invariant under this equivalence.

(iii) The set of infinite zeros and poles of G, together with their corresponding multiplicities are invariants of $\mathcal{E}^r_K(G)$ for $\mathcal{K}=\mathbf{R_{pr}}(s)$; the finite zeros, poles are not necessarily invariant under this equivalence.

(iv) The set of zeros and poles of G, together with their corresponding multiplicities, in $\mathbf{P}=\Omega U\{\infty\}$ are invariants of $\mathcal{E}^r_K(G)$ for $\mathcal{K}=\mathbf{R_p}(s)$; the poles and zeros of G in Ω^c (the complement of Ω with respect to C) are not necessarily invariant under this equivalence

(v) If δ_m, δ_F, $z\infty$, Z_f are the McMillan degree, Forney order of $\mathcal{X}_{c,G}$, total numbers of infinite, finite zeros of G, then $\delta_M = z\infty + z_f + \delta_F$.

□

The last relationship indicates that under all types of compensation which preserve δ_F, the difference between McMillan degree and total number of zeros remains constant. Since for square systems $\delta_F = 0$, this also indicates that for square systems, the McMillan degree is equal to the total number of zeros. The Forney order plays a crucial role under squaring down [Karc. & Gian., 1], since it indicates the total number of newly created zeros.

4.5. CONCLUSIONS: EVALUATION AND EMERGING ISSUES

The aim of this chapter was to provide a unifying detailed review of the fundamental linear system properties, property indicators and invariants, which emerge as tools for control design and have the potential to develop also as important instruments for intervention in Global Instrumentation (GI) and Early Process Design (EPD) in general. The review of concepts and results did not aspire to cover everything and a number of criteria and indicators for which testing is not easy with the current computational means have been neglected. The emphasis so far has been on the structural aspects, as these are expressed in terms of invariants. The role of graph structural dimension has not been examined and there

is a need for study of the related properties in a setup where the interconnection graph explicitly appears in the model. However, this area is beyond the scope of the thesis. The major systems and control type issues that emerge from the requirements of the overall integrated design structural framework are briefly summarised below:

(i) **Relationships between Invariants and Property Indicators:** The relationship between structural aspects such as invariants and property indicators are not well understood for the majority of them. Thus, which are the "good" and which are the "bad" values of invariants, as well as the way structure imposes limitations on the compensation of property indicators is an open area of research for control theory and design. As new results are produced they will have to be integrated in the overall framework. There is rich control theory literature, but the field is by no means closed.

(ii) **Model parameters and Invariant Structure:** The functional relationships between system model parameters (the design tools in EPD) and invariants are not always simple and explicit which imposes severe difficulties in developing systematic procedures for assigning values for all invariants, and thus shaping appropriately the property indicators. There is a number of properties evaluated by criteria simply determined by model parameters; however invariants have in general more complex links to model parameters. The general issues of specifying the functional relationships for implicit invariants and indicators is still open in Control Theory.

(iii) **Graph Structure, Invariants and Indicators:** The general theory as developed so far, does not take into account the fact that most systems have an underlying graph structure which imposes constraints on the types of the system invariant that may exist, their possible values and thus in turn the nature of design indicators. The existing theory on generic system properties (examined in a subsequent chapter) assumes the system with no special structure, whereas graph theory has mainly focused on assessing properties, rather than the induced types and values of invariants stemming from a given graph. Once more, this area is still in its early stages of development as a topic of control theory.

(iv) **System Structure and Control Design:** The system structure, as expressed in terms of the graph and the structural invariants has been the backbone of the synthesis methodologies, but they have not been systematically used in the development of design methodologies. The overall view of the relationship between the model parameters, graph structure, invariants and performance indicators is illustrated in

Figure (4.4) and this provides the background to a global design methodology that exploits the system structure. The integration of structural analysis to design, is long overdue and the areas where some urgent steps are needed are in the development of structural diagnostics for selection of type of the control scheme, nature of the dynamics, centralised versus decentralised, as well as development of design methodologies based on the structural approaches. Certain aspects of this structural framework are considered in following chapters.

(v) **Problem Dimensionality:** Analysis and design methodologies of the control area are well developed for reasonable dimension systems, but problems start to arise when we deal with large composite systems. The difficulties are not only due to computations and related problems, but also arise in the conceptual design level. Issues of partitioning the design problem, sequencing of the design of partitioned problems and design of decentralisation control schemes are not well developed; especially the first two are based on experience, and heuristics. Graph theory makes a useful contribution in problems such as evaluation of some properties on large systems, problem decomposition and selection of decentralisation, but it has to combine the method to lead sharper results. Furthermore, graph methodology relies on the state space formulation, which imposes a number of limitations. Problem decomposition and sequencing of design are much more complex problems and the requirements of global process operations have a very significant impact on the adopted solutions.

(vi) **Sensitivity and Robustness Issues:** The structural framework assumes models, which are well defined, and relies on the evaluation of the types and values of invariants. Issues of sensitivity of the type-values of invariants under model uncertainty are crucial for the framework to develop as an effective diagnostics and design tool. Although there exist studies on the generic values and type of invariants, the sensitivity and robustness issues, especially under structured uncertainty, have not being properly considered. Developments in this area also depend on developments with the area (ii) above.

(vii) **Variable Complexity Modelling:** In the framework of EPD we may have to deal with models of variable conceptual and dynamic complexity. Development of a theory for assessing the evolution of system properties and types/values of structural characteristics, in a nesting of progressively more complex models, is an open area. This is crucial for the development of diagnostics for EPD. The area is linked to

robustness studies, but it has a much richer content since we move within different types of families of system models.

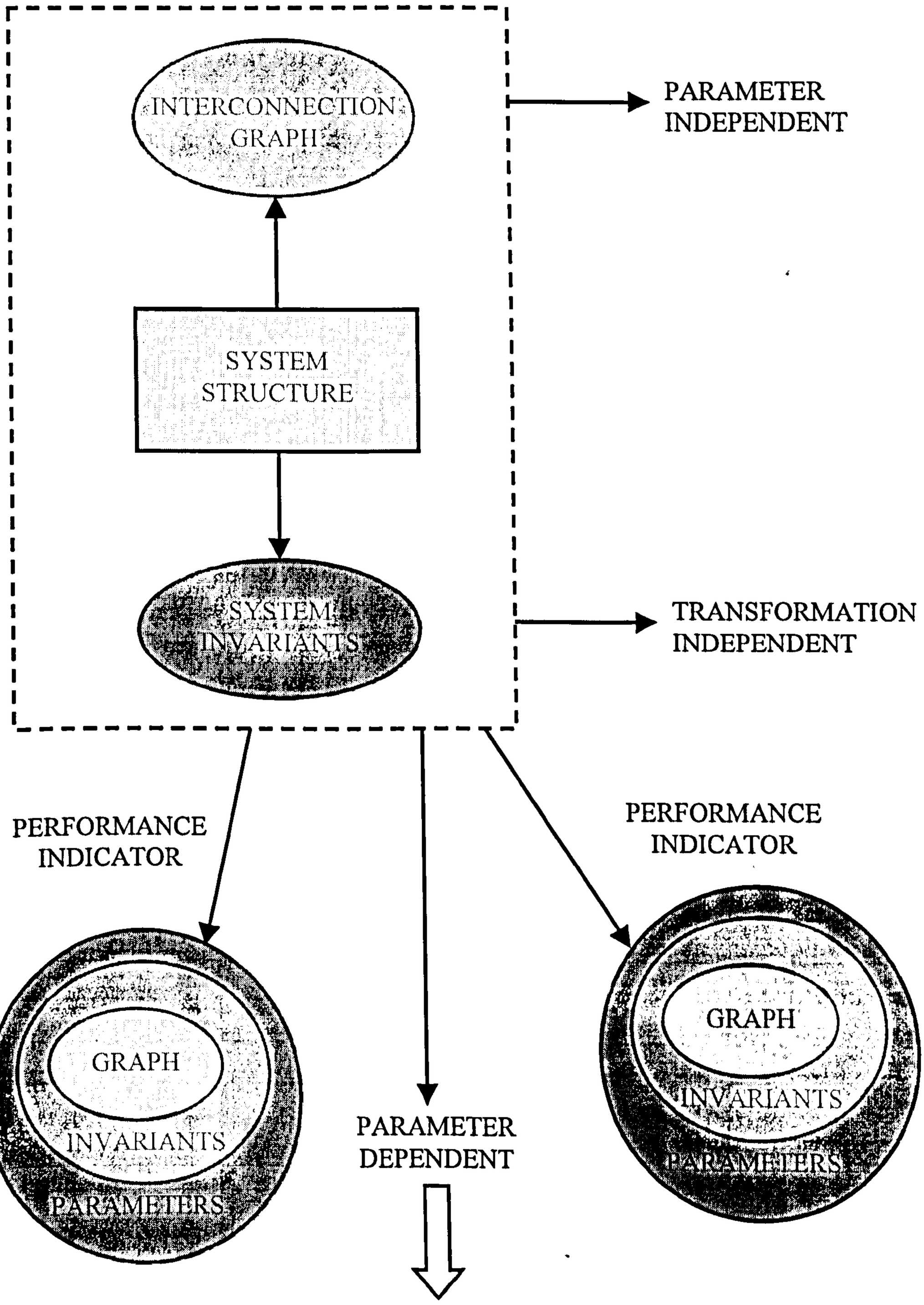

Figure (4.4): A Structural Genetic View of System Model Behaviour

(viii) **Model Interface Issues:** The need for integrating into control design requirements coming from the operational layers, as well as the implementation of designs through computer schemes introduce new challenges for control theory and design which have to do with mixed models, mixed signals and alternative time scale operations. The area of hybrid systems emerges as a dynamic area for handling interface problems in the global operational mode.

(ix) **Generalised Synthesis in Integrated Design:** Development of methodologies for Global Instrumentation and possible tools that may assist in process synthesis is a major challenge. The formation of the interconnection graph for composite systems may be addressed as a complex design problem, where apart from particular area practices, the formation of system properties may be taken as an objective. Similarly, proceeding from diagnostics to systematic design methodologies using the new control theory and design tools, advocated above, is also a major challenge. The overall philosophy in the above two areas is the control of the structure evolution mechanism through those design stages, aiming at assignment of desirable features, where possible, or at least avoidance of undesirable features. An overall philosophy for the challenging problems of integrated or global system design that exploits the structural methodology considered here is summarised in Figure (4.5). In this figure, the generation of alternatives and their structural evaluation leads successively in the selection of the "best" solution amongst a number of possible alternatives and by developing a genetic selection approach.

(x) **Computational Issues:** The area of computations is central for the development of CAD that can accompany the above developments. The algebraic and geometric methodologies of the structural framework have special requirements, which have not being addressed properly before. Computing algebraic invariants and transformation of geometric conditions into testable criteria is a major challenge. Numerical issues related to algebraic computations on inaccurate engineering models, the optimal merging of symbolic and numerical computations and issues of computational complexity, especially for graph type computations are essential. Current CAD packages deal mainly with property, design indicators and rarely use elaborate computations for assessing system structural characteristics. The development of the different aspects of algebraic computations is crucial for the overall development of the topic.

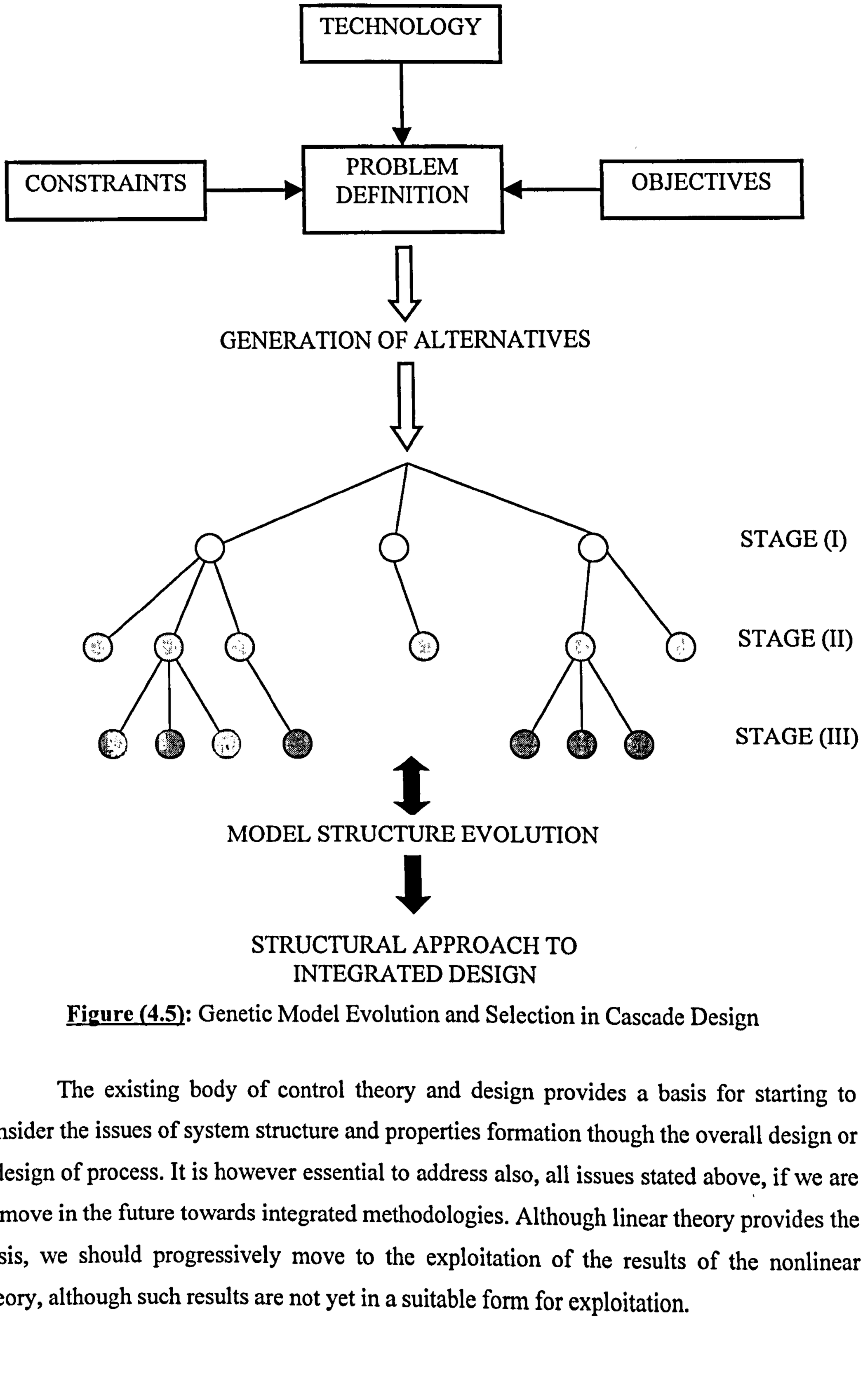

Figure (4.5): Genetic Model Evolution and Selection in Cascade Design

The existing body of control theory and design provides a basis for starting to consider the issues of system structure and properties formation though the overall design or redesign of process. It is however essential to address also, all issues stated above, if we are to move in the future towards integrated methodologies. Although linear theory provides the basis, we should progressively move to the exploitation of the results of the nonlinear theory, although such results are not yet in a suitable form for exploitation.

Chapter 5

MODEL ORIENTATION ISSUES IN EARLY DESIGN: A STATE SPACE APPROACH

Chapter 5

MODEL ORIENTATION ISSUES IN EARLY DESIGN: A STATE SPACE APPROACH

5.1. INTRODUCTION

An integral part of the overall modelling problem for systems is the definition of process variables and their subsequent classification into control variables (inputs), command variables (outputs) and other internal variables. Heuristics linked to the specific domain of applications, or methodologies such as graph analysis, Lagrangian methodology etc. may be used in specific cases for handling issues of nonredundancy in representations and classification of variables. A natural system description that makes no distinction as far as the role of process variables and their dependence, or independence is for the linear case the matrix pencil model (first order differential descriptions), or the general polynomial, or autoregressive model. In this chapter, the focus is on implicit, or matrix pencil models, which characterise the behaviour of all unclassified process variables into inputs, outputs and internal variables. This is referred to as Model Orientation Problem (MOP) and its solutions are systems of the standard state space type, classified as General Singular, Singular and Regular systems. Investigating the conditions under which the MOP is solvable, as well as the characterisation of the structural properties of these solutions and the classification of the emerging solutions are the main topics considered here.

Amongst the solutions to MOP are regular type linear systems of the S(A,B,C,D) type; these are not always suitable for control design since they may be characterised by input, output

structure redundancy and they may be degenerate. Defining subsystems of S(A,B,C,D) by reducing the number of inputs, outputs such that the reduced systems S′(A,B′,C′,D′) are well conditioned, as far as nondegeneracy, input, output regularity and minimality (controllability and observability), is a problem referred to as Well Conditioning Problem [Karc. & Vaf., 1] (WCP). This problem is considered in a subsequent chapter in the case where the inputs, outputs are physical variables and thus input, output reduction implies selection of subsets of such variables. This corresponds to selection of submatrices of the original transfer function, which have certain input, output cardinality and desirable properties.

The problems considered here are integral parts of "early design" of processes [Rijn., 1], [Karc., 1] and are considered in the context of linear systems using results based on the algebraic structure of systems and in particular the Kronecker structure of matrix pencils [Gant., 1]. The emphasis here is on the characterisation of the desirable properties and the definition of a framework within which the design problems are addressed.

5.2. STATEMENT OF THE PROBLEM

Physical modelling based on description of basic elements and use of interconnection topology may be used for large families of systems. If all important variables are included and there is no effort to guarantee their minimality, and classify them into inputs, internal variables, the emerging descriptions are referred to as implicit [Apl. 1], [Lewis, 1] and in the case of first order linear differential descriptions they correspond to the matrix pencil or generalised autonomous description [Karc. & Hayt., 1]:

$$S(F,G): F\,p\underline{\xi} = G\,\underline{\xi},\quad F,G \in R^{\alpha\times\upsilon} \tag{5.1}$$

where p is the differentiation, or shift operator and $\underline{\xi}$ is the vector of all problem variables. The natural operator associated with such descriptions is the matrix pencil $sF-G$ and thus, the study of such descriptions relies on the structure of $sF-G$. For control, as well as handling issues of creating composite structures, it is important to classify the variables in $\underline{\xi}$ into internal

variables, or states $\underline{x}$, assignable, or control variables $\underline{u}$, and measurement, or dependent variables $\underline{y}$. This is expressed in terms of the transformation:

$$\underline{\xi} = Q\,\underline{\tilde{\xi}}, \text{ where } \underline{\tilde{\xi}} = \left[\underline{x}', \underline{u}', \underline{y}'\right]' \text{ and } Q \in \Re^{\upsilon \times \upsilon},\ |Q| \neq 0 \tag{5.2}$$

Q will be called an orientation transformation (OT) and if the original variables in $\underline{\xi}$ are physical and it is desired to preserve them, then Q has to be of the permutation type and will be called physical OT. For first order linear descriptions the most general form of oriented models is the general singular (GS) description:

$$S(E,A,B,C,D): E\,p\,\underline{x} = A\,\underline{x} + B\,\underline{u},\ \underline{y} = C\,\underline{x} + D\,\underline{u} \tag{5.3}$$

where $E, A \in \Re^{\sigma \times n}$, $B \in \Re^{\sigma \times p}$, $C \in \Re^{m \times n}$, $D \in \Re^{m \times p}$, where $\tau = m + \sigma$, $\upsilon = n + p + m$, and in general $\sigma \geq n$. In the case where $\sigma = n$, S will be called singular and if $\sigma = n$ and $|E| \neq 0$, then the description will be called regular and it is equivalent to the standard state space description:

$$S(A,B,C,D): p\,\underline{x} = A\,\underline{x} + B\,\underline{u},\ \underline{y} = C\,\underline{x} + D\,\underline{u} \tag{5.4}$$

Defining an OT Q (general or physical) such that S(F,G) is reduced to S(E,A,B,C,D) or S(A,B,C,D) forms is termed model orientation problem (MOP) and it is considered here. Part of this study is to determine the conditions under which S(F,G) may be reduced to the GS, singular or regular descriptions. In this paper the general case of OT is considered, rather than the more restricted case of physical OT.

Regular descriptions S(A,B,C,D) defined as solutions of MOP may not have good properties as far as control design; in fact, they may have degenerate transfer functions and be characterised by input, output structure degeneracy. Defining subsystems of S(A,B,C,D) by reduction of the input, output structure such that the reduced system S(A,B′,C′,D′) has desirable properties is referred to as input-output structure reduction problem (I-ORP) and

includes problems such as the squaring down [Karc. & Gian. 1]. When the resulting model has physical input, output variables and it is desired to preserve them, then the I-ORP takes the special form, where only a α set of existing inputs and a β set of existing outputs is used, which leads to an $S_{\alpha,\beta} = S(A, B_\alpha, C_\beta, D_{\alpha,\beta})$ subsystem with corresponding transfer function $H_{\alpha,\beta}(s)$. The objective here is to select the α , β sets such that the resulting $S_{\alpha,\beta}$, $H_{\alpha,\beta}(s)$ is well structured as far as certain properties, which may include input, output regularity, nondegeneracy, minimality etc. Such a problem will be referred to, in short, as well conditioning by input-output reduction (WCP) and is considered in a subsequent chapter. Note that in a transfer function matrix setup, WCP is equivalent to defining submatrices of $H(s)$ by eliminating certain columns and rows and which have desirable properties. An integral part of this problem is the parameterisation of the maximal input, output cardinality solutions [Karc. & Vaf., 1]. The problem of Model Orientation is considered as an initial step in the development of oriented early models and is considered first.

5.3. THE MODEL ORIENTATION PROBLEM: CHARACTERISATION OF SOLUTIONS

Consider the matrix pencil description S(F,G) of Equation (5.1) with an associated matrix pencil $sF-G$ of $\tau \times \upsilon$ dimensions. For such a pencil the general theory of Strict Equivalence suggests that the corresponding equivalence class is characterised by a set of invariants known as Kronecker invariants. We assume sF-G to be general and thus have the following Kronecker invariants [Gant., 1]:

$$D_f = \{(s-\lambda_i)^{\tau_i}, i \in \underset{\sim}{\theta} : \sum \tau_i = n_f\}$$

$$D_\infty = \{\hat{s}^{q_1} : q_1 \geq \ldots \geq q_\mu \geq 1,\ q_{\mu+1} = \ldots = q_{\mu+\delta} = 1, \sum_{i=1}^{\mu} q_i = n_\infty\} \qquad (5.5)$$

$$I_r = \{\eta_i : \eta_1 \geq \ldots \geq \eta_l \geq 1, \eta_{l+1} = \ldots = \eta_{l+g} = 0, \sum_{i=1}^{l} \eta_i = n_r\}$$

$$I_c = \{\varepsilon_i : \varepsilon_1 \geq \ldots \geq \varepsilon_v \geq 1, \varepsilon_{v+1} = \ldots = \varepsilon_{v+h} = 0, \sum_{i=1}^{v} \varepsilon_i = n_c\}$$

where D_f, D_∞ denote the set of finite (fed), infinite elementary divisors (ied), I_r is the set of row minimal indices (rmi) and I_c the set of column minimal indices (cmi). Note that δ is the number of linear ied, g is the number of zero rmi and h the number of zero cmi. The above set of invariants completely characterises the Kronecker canonical form $sF_\kappa - G_\kappa$ of $sF-G$, which is defined under strict equivalence. That is there exists a pair $(R,Q): R \in \Re^{\tau\times\tau}, |R| \neq 0, Q \in \Re^{\upsilon\times\upsilon}, |Q| \neq 0$ such that:

$$sF_\kappa - G_\kappa = R(sF-G)Q = block-diag\{0_{g,h}; ..sH_q - I_q ..; ..sI - J_\tau(\lambda)..; ..; L_\varepsilon(s); ..; ..; L_\eta(s)..\} \qquad (5.6)$$

where $sH_q - I_q$ characterises $\hat{s}^q$, with H_q the $q\times q$ nillpotent matrix, $J_\tau(\lambda)$ is the Jordan form associated with $(s-\lambda)^\tau$, $L_\varepsilon(s) = s[I_\varepsilon, \underline{0}] - [\underline{0}, I_\varepsilon]$ characterises $\varepsilon > 0$, and $L_\eta(s) = s[I_\eta, \underline{0}]' - [\underline{0}, I_\eta]'$ corresponds to $\eta > 0$.

The problem considered here is the characterisation of the types of oriented models which may be derived from S(F,G), as well as the development of the methodology that can be used for finding such solutions. It is first noted that:

Remark (5.1): If $R \in \Re^{\tau\times\tau}, |R| \neq 0$, then the space of solutions (smooth and distributions) of S(F,G) and S(RF,RG) are the same.

□

The above suggests that left transformations do not affect the solution space and thus may be used to simplify the original description S(F,G).

Proposition (5.1): If $sF-G$ has g zero rmi, then there exists $R \in \Re^{\tau\times\tau}, |R| \neq 0$ such that:

$$R[F,G] = \left[\begin{array}{c} F', G' \\ \hline 0_g \end{array}\right] \qquad (5.7)$$

and the solutions of S(F,G) and S(F′,G′) are identical.

Proof: The presence of g zero rmi implies that there exist g independent vectors $\underline{v}_i$ such that $\underline{v}_i'[F,G]=\underline{0}'$, clearly $N_l[F,G]$ has dimensions exactly g and thus there is R that reduces [F, G] to the form (5.7).

□

An S(F,G) system with zero rmi will be called reducible; otherwise, it will be called irreducible. Proposition (5.1) implies that we can always assume it to be an irreducible form and this is assumed in the following. The existence of solutions to MOP is considered next.

Lemma (5.1) [Karc. 6]: Consider the irreducible system S(F,G) with Kronecker invariants as described in (5.5). There always exist a strict equivalence pair (R,Q) such that:

$$R(sF-G)Q=sF^*-G^* \tag{5.8a}$$

where: $sF^*-G^*=$

$$\left[\begin{array}{cc|cc|c|c|c} sI-A_f & & & & 0 & & \\ & sH_\infty-I & & & & & \\ \hline & & sI-A_c & & -B_c & & \\ & & & sI-A_r & 0 & 0_h & \\ \hline 0 & & & C_r & 0 & & \\ \hline & & 0 & & & & I_\delta \end{array}\right] = \tag{5.8b}$$

$$=\left[\begin{array}{c|c|c} sE'-A' & -B' & 0 \\ \hline 0 & 0 & I_\delta \end{array}\right] \tag{5.8c}$$

where A_f is $n_f\times n_f$ and it is characterised by D_f, $sH_\infty-I$ is $n_\infty\times n_\infty$ and $sH_\infty-I=block-diag\{sH_{q_i}-I, q_i>1\}$, A_c is $n_c\times n_c$ and corresponds to all $\varepsilon_i>0$, B_c is $n_c\times\nu$ and full rank, A_r is $n_r\times n_r$ and corresponds to all $\eta_j>0$ and C_r is $l\times n_r$ and has full rank.

□

The above Lemma is now used to establish the following main result:

Theorem (5.1): Consider the irreducible system S(F,G):

$$S(F,G):\ F\,p\underline{\xi} = G\,\underline{\xi},\quad F,G \in \Re^{r\times v} \tag{5.9a}$$

where the associated pencil $sF-G$ is assumed to have a general structure as described in (5.5). There always exists an $R \in \Re^{r\times r}, |R| \neq 0$, and a transformation $Q \in \Re^{v\times v}$, $|Q| \neq 0$, such that:

$$\underline{\xi} = Q\underline{\xi}' = Q\begin{bmatrix}\underline{x}'\\ \underline{u}'\\ \underline{y}'\end{bmatrix},\quad R(pF-G)Q = pF'-G' \tag{5.9b}$$

which reduces S(F,G) to the equivalent oriented description:

$$S(F',G'):\ (pF'-G')\underline{\xi}' = \begin{bmatrix} pE'-A' & -B' & 0\\ -C' & -D' & I_\delta\end{bmatrix}\begin{bmatrix}\underline{x}'\\ \underline{u}'\\ \underline{y}'\end{bmatrix} = 0 \tag{5.9c}$$

where $pE'-A'$ is a nonsquare of dimensions $(n'+l)\times n', n' = n_f + n_\infty + n_r + n_c$, B' is $(n'+l)\times(v+h)$, C' is $\delta\times n'$ and D' is $\delta\times(v+h)$.

Proof: By Lemma (5.1) the form (5.8c) is established and thus exists (R, Q') such that:

$$R(pF-G)Q'\begin{bmatrix}\underline{x}'\\ \underline{u}'\\ \underline{0}\end{bmatrix} = \left[\begin{array}{c|c|c} pE'-A' & -B' & 0\\ \hline 0 & 0 & I_\delta\end{array}\right]\begin{bmatrix}\underline{x}'\\ \underline{u}'\\ \underline{0}\end{bmatrix} = \underline{0}$$

which also leads to: $R(pF-G)Q'\begin{bmatrix}\underline{x}'\\ \underline{u}'\\ \underline{0}\end{bmatrix} = \left[\begin{array}{c|c|c} pE'-A' & -B' & 0\\ \hline 0 & 0 & I_\delta\end{array}\right]\begin{bmatrix}\underline{x}'\\ \underline{u}'\\ \underline{0}\end{bmatrix} = \underline{0}$

or $\begin{bmatrix} pE'-A' & -B' & 0 \\ -C' & -D' & I_\delta \end{bmatrix} \begin{bmatrix} \underline{x}' \\ \underline{u}' \\ C'\underline{x}'+D'\underline{u}' \end{bmatrix} = 0$ and by setting $\underline{y}' = C'\underline{x}' + D'\underline{u}'$ the result is established. The dimensionality of matrices follows from Lemma (5.1).

□

The above result establishes the existence of a general singular system as the solution to MOP. Furthermore, the construction of such transformation is intimately linked to derivation of Kronecker canonical forms, which is behind the construction of the form $sF^* - G^*$ of (5.8b). Theorem (5.1) together with Lemma (5.1) establish a relationship between the Kronecker structure of $pF-G$ and the nature of solutions of MOP and this is described by the following corollaries. Any solution of MOP corresponds to the derivation of a system with inputs, outputs and thus provides a realisation of the original implicit (pencil form); such solutions will be referred to as **oriented realisations** of S(F,G).

Corollary (5.1): Any irreducible system S(F,G) has an oriented realisation $S(E',A',B',C',D')$ which has the following properties:

(i) For any oriented realisation we have that:

- **(a)** The number of linear ied δ defines the number of outputs implied by the implicit description and characterise linear relations amongst the implicit variables.
- **(b)** The total number of cmi $\nu+h$ defines the number of inputs, that is the number of variables that can be arbitrarily assigned amongst the implicit variables.

(ii) $S(E',A',B',C',D')$ is general singular, if and only if the set of Kronecker invariants contains nonzero rmi.

(iii) $S(E',A',B',C',D')$ is singular, if and only if the set of Kronecker invariants has no nonzero rmi and contains nonlinear ied.

(iv) $S(E',A',B',C',D')$ is regular, if and only if the set of Kronecker invariants has no nonzero rmi and no nonlinear ied.

Proof:

(i) By inspection of (5.8b), it follows that E', A' are nonsquare if and only if $C_r \neq 0$, which however holds true if and only if the pencil has nonzero rmi. Given that C_r follows by permutation of the elements of the Kronecker form, the block C_r cannot be eliminated by either column, or row transformations.

(ii) From part (i) the pencil must not have nonzero rmi and thus the resulting $sE' - A'$ is square. Note that E' is then singular if and only if there is the $sH_\infty - I$ block, which is part of the Kronecker form and exists if and only if the original pencil has nonlinear ied.

(iii) From parts (i) and (ii) the result follows immediately.

□

The presence of nonzero rmi in the pencil $pF - G$ implies that oriented realisations are of the nonsquare, or general singular type. Regarding the original description S(F,G) this has some additional implications on redundancy of the representation. We first note that for general singular representations the dynamic part is described by the pencil $[pE' - A', -B']$, where $pE' - A'$ is nonsquare. A standard representation of this pencil (defined in a nonunique manner by column permutations) is the pencil $[pE'' - A'', -B'']$, where $[pE'' - A'']$ is square. Clearly, standard representations may be extended to the S(F′,G′) description of (9c), (by using similar column permutations and partitioning). A standard representation will be called normal, if $pE'' - A''$ is a regular pencil ($|pE'' - A''| \neq 0$). The existence of standard and normal realisations is established by the following result.

Proposition (5.1): Consider a general irreducible system S(F, G). The following properties hold true:

(i) Every general singular oriented realisation leads to standard realisations $S(E'', A'', B'', C'', D'')$ by permutation of the components of the implicit vector, that is:

$$S(F'',G''):\begin{bmatrix} pE''-A'' & -B'' & 0 \\ -C'' & -D'' & I_\delta \end{bmatrix}\begin{bmatrix} \underline{x''} \\ \underline{u''} \\ \underline{y''} \end{bmatrix}=0 \tag{5.10}$$

(ii) If $pF-G$ has nonzero rmi, then any oriented realisation S(F′,G′) of S(F,G) has every standard representation S(F′′,G′′) with $pE''-A''$ singular.

(iii) There exist normal realisations of order n'', if and only if pF-G has nonzero rmi.

Proof: The oriented realisation is general singular if only if $sF-G$, or $sF'-G'$ has nonzero rmi. The existence of a nonzero rmi η implies that there exists $\underline{v}(s)$ such that: $\underline{v}(s)'(sF'-G')=0$, with $\partial[\underline{v}(s)]=\eta>0$.

The above implies $[\underline{v}_1(s)',\underline{v}_2(s)']\begin{bmatrix} pE'-A', & -B', & 0 \\ -C', & -D' & I_\delta \end{bmatrix}=0$ and thus $\underline{v}_2'(s)=\underline{0}'$ and hence $\underline{v}_1'(s)[pE'-A',-B']=\underline{0}'$ and $[pE'-A',-B']$ has also a rmi η. Given that a normal representation is obtained from the previous pencil with column permutations the result follows.

□

The above property clearly suggests that general singular representations cannot be used for working out transfer functions. This is due to the fact that there is some redundancy in the components of $\underline{x'}$ vector and this does not allow the derivation of transfer functions. Removing the redundancy is essential for obtaining transfer functions and is considered below:

Corollary (5.2): Let S(F′,G′) be a general singular oriented realisation of S(F,B). The following properties hold true:

(i) There always exist n_r independent linear relations amongst the coordinates of the original vector $\underline{\xi}$.

(ii) The space of solutions of S(F′,G′) is given by the set of n_r linear relations and the solutions of a reduced system defined below:

$$S(\tilde{F},\tilde{G}):\begin{bmatrix} p\tilde{E}-\tilde{A} & -\tilde{B} & 0 \\ -\tilde{C} & -D' & I_s \end{bmatrix}\begin{bmatrix} \underline{\tilde{x}} \\ \underline{u'} \\ \underline{y'} \end{bmatrix}=0 \tag{5.11}$$

where $\underline{\tilde{x}}$ is a vector of dimension $n_f+n_\infty+n_c$, $\underline{u'}$, $\underline{y'}$ are as before and with the associated pencil $p\tilde{F}-\tilde{G}$ having the same Kronecker invariants with $pF-G$ except the set of rmi.

(iii) The realisation $S(\tilde{E},\tilde{A},\tilde{B},\tilde{C},\tilde{D})$ is normal.

Proof: Starting from (5.9c), where $[pE'-A',-B']$ has the form described by (5.8b), we may partition $\underline{x}'=[\underline{\tilde{x}}',\underline{\hat{x}}']$ where $\underline{\hat{x}}'$ corresponds to $pI-A_r$ and $\underline{\tilde{x}}'$ is the part associated with A_f, H_∞, A_c matrices. Then, (9c) is equivalent to the following equations:

$$\left[\begin{array}{c|c|c|cc|c} pI-A_f & & & 0 & 0 & \\ \hline & pH_\infty-I & & 0 & 0 & 0 \\ \hline & & pI-A_c & -B_c & 0 & \\ \hline & -\tilde{C} & & -D' & & I_s \end{array}\right]\begin{bmatrix} \underline{\tilde{x}} \\ \underline{u'} \\ \underline{y'} \end{bmatrix}=0 \tag{5.12}$$

$$\begin{bmatrix} pI-A_r \\ \hline C_r \\ \hline \tilde{\tilde{C}}' \end{bmatrix}\underline{\hat{x}}=0 \tag{5.13}$$

Note that (5.13) is obtained by row permutation of blocks associated with nonzero rmi and thus (5.13) is equivalent to solution of l sets of equations of the type:

$$\begin{bmatrix} p & 0 & \dots & 0 \\ -1 & p & & 0 \\ 0 & -1 & \ddots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & p \\ 0 & 0 & \dots & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = 0 \tag{5.14}$$

which has a unique solution $v_1 = v_2 = \dots = v_n = 0$. Thus, (a.2) has a solution $\underline{\hat{x}} = 0$, and (5.12) describes the dynamic part of the system equations. It is clear from its construction that the pencil defining (5.12), ie.

$$p\tilde{F} - \tilde{G} = \left[\begin{array}{ccc:cc} pI - A_f & 0 & 0 & 0 & 0 \\ 0 & pH_\infty - I & 0 & 0 & 0 \\ 0 & 0 & pI - A_c & -B_c & 0 \end{array}\right] = \left[p\tilde{E} - \tilde{A}, -\tilde{B}\right] \tag{5.15}$$

has the same Kronecker invariants with $pF - G$, except the linear ied (corresponding to I_δ) and the nonzero rmi. Furthermore, the corresponding realisation given by (5.12) is singular, and not general singular. The n_r dimensional zero vector $\underline{\hat{x}} = 0$ implies the existence of n_r linear and independent relations amongst the coordinates of ξ. In fact, if Q is the orientation transform, then

$$\underline{\xi} = Q \begin{bmatrix} \underline{\tilde{x}} \\ \underline{0}' \\ \underline{u}' \\ \underline{y} \end{bmatrix} \rightarrow \begin{bmatrix} \underline{\tilde{x}} \\ \underline{0}' \\ \underline{u}' \\ \underline{y} \end{bmatrix} = \hat{Q}\xi = \begin{bmatrix} \hat{Q}_1 \\ \hat{Q}_2 \\ \hat{Q}_3 \\ \hat{Q}_4 \end{bmatrix} \underline{\xi} \tag{5.16}$$

from which $\hat{Q}_2 \underline{\xi} = 0$. This implies that $\underline{\hat{x}} = 0$ is equivalent to n_r relationships implied from $\hat{Q}_2 \underline{\xi} = 0$.

□

The proof of the above result is constructive and indicates how the set of n_r linear relations is derived from the orientation transformation, as well as a suggested procedure to construct any reduced systems $S(\tilde{F},\tilde{G})$ that expresses the dynamic solutions. The singular system $S(\tilde{E},\tilde{A},\tilde{B})$ defined by $[p\tilde{E}-\tilde{A},-\tilde{B}]$ will be called a reduced realisation of S(F,G) and its properties are described below.

Corollary (5.3): For any $S(\tilde{E},\tilde{A},\tilde{B})$ reduced realisation of S(F,G), the following hold true:

(i) The pencil $[p\tilde{E}-\tilde{A},-\tilde{B}]$ has as Kronecker invariants the set of fed, nonlinear ied and cmi of S(F,G).

(ii) The number of inputs is given by the number of cmi $\nu+h$ of S(F,G). Furthermore, h expresses the order of redundancy of the input structure i.e. number of dependent inputs.

(iii) The system S(E,A;B) is controllable, if and only if S(F,G) has no fed and ied. Furthermore, the system is regular if and only S(F,G) has no ied.

Proof: Part (i) follows from the proof of Corollary (5.2). The number of inputs is defined by the number of arbitrarily assignable variables in the original pencil form. Only the blocks associated with nonzero cmi introduce independent arbitrarily assignable functions as indicated below for an ε cmi block.

$$\begin{bmatrix} p & -1 & 0 & \dots & 0 & 0 \\ 0 & p & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & p & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_\varepsilon \\ u_\varepsilon \end{bmatrix} = 0 \tag{5.17}$$

which is equivalent to the standard controllable representation:

$$[pI_\varepsilon - H_\varepsilon]\begin{bmatrix} x_1 \\ \vdots \\ x_\varepsilon \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u_\varepsilon \tag{5.18}$$

and this completes the existence of v independent and dynamically significant inputs. The existence of h zero cmi indicates arbitrarily assignable functions, which however are not affecting the dynamic behaviour and are linked to the zero column block in B' of (5.8b), or dependency of columns of $\tilde{B}$ with h right nullity.

The controllability, uncontrollability properties readily follow from the standard results for singular systems.

□

The analysis here provides a solution to MOT for general autonomous description S(F,G), a characterisation of the type of resulting oriented realisations and a procedure to construct them based on Kronecker form transformations. So far, no constraint has been imposed on the orientation transformation and the derivation of the different types of oriented realisation is based on the use of Kronecker canonical form, that is the reduction of the system to the Kronecker form and then use of permutation transformations. The proof of the results is constructive and thus algorithms may be developed which follow the steps of the proofs.

In the case where the variables in the implicit vector are physical variables, Q is constrained and some special transformations are required which retain the physical nature of independent variables in the implicit vector $\underline{\xi}$. The general Kronecker theory cannot be used in this case and there is a need to use transformations of strict equivalence on the left and permutations on the right. This is an important area, which is still open (as far as theory of matrix pencils).

5.4. CONCLUSIONS

The problem of model orientation (MOP) that is linked to early modelling of processes has been considered within the setup of state space descriptions. The general case of strict

equivalence transformations is considered here. The derivation of the different forms of oriented realisations is based on the Kronecker canonical form reduction and the proof of the results is of constructive nature, which permits the development of algorithmic procedures. For implicit models with physical variables, preservation of the physical nature of variables implies that the orientation transformation has to be of the permutation type and this requires some restricted form of strict equivalence of matrix pencils, where we have a general transformation of strict equivalence on the left and permutations on the right. The study of such equivalence is essential before we consider the physical variables version of MOP. This is an open problem for future research.

It is worth pointing out that the solutions of MOP considered here are not necessarily system models with good features and properties for control design. In fact, such models may be degenerate, have redundancy in the input, output structure and may have large input, output dimensions, which are not suitable for control design. Working out smaller models with good control characteristics is an important problem and it is considered in the following chapter. This is referred to as well conditioning by input – output reduction.

In this chapter MOP has been considered within a state space setup. However, the first progenitor model may be of the autoregressive (general polynomial matrix type). This case requires tools from the general polynomial matrix theory (rather than Kronecker form) and it is an issue for further research.

Chapter 6

WELL CONDITIONING OF EARLY PROCESS MODELS

Chapter 6

WELL CONDITIONING OF EARLY PROCESS MODELS

6.1. INTRODUCTION

The derivation of models that can be used for early design stages studies of processes requires the use of the process flowsheet (system interconnection graph), the availability of simple models describing the fundamental dynamics of subprocesses and the selection of control (input) and measurement (output) variables. Before commencing the investigation of the properties of the resulting model it is useful to include all possible inputs and outputs; at a later stage we can then determine the effective subsets of inputs, outputs using different controllability, operability criteria. Such models corresponding to all possible inputs and all possible outputs will be referred to as progenitor models [Karc., 1]. Progenitor models are derived on the basis that possible inputs, outputs are selected using heuristics, physical arguments and thus the resulting transfer function may be of large dimensions and possibly not well behaved. The essential feature of such models is that the input, output variables are physical variables, on which specifications may be imposed, and that this transfer function contains as parts all possible (smaller dimension) transfer functions that may be used in actual design. Transfer functions corresponding to subsets of the potential input and output sets are referred to as effective models and are submatrices of the progenitor transfer function. Different families of effective models may be defined by fixing the cardinality of the input, output effective sets or by requiring that the input, output sets contain certain fixed physical variable sets. Characterising such families of models, in terms of a range of important properties, is an important part of the process controllability studies.

This chapter deals with a specific problem within the general area of selecting effective models, when criteria, such as the nondegeneracy of the effective transfer function, the nonredundancy of the instrumentation schemes (independence of selected sensors and actuators) and the controllability and observability of the resulting system are used. Nondegeneracy is a fundamental property for the effective model, since it is linked to the output function controllability [Rosen., 1], and thus to the solvability of a number of control problems. Conditions for the characterisation of system degeneracy and redundancy of the input, output structure of the system are derived in terms of the state space parameters; these conditions also indicate the criteria required to guarantee nondegeneracy and input, output scheme nonredundancy. For the cases of proper and strictly proper progenitor models simple and quite broad sufficient conditions of the rank type are given, which guarantee nondegeneracy and nonredundancy. The characterisation of the controllability and observability properties is performed here using the McMillan degree and the associated properties of Hankel matrices [Ants. & Mich., 1]. Such approach is faster and more suitable for selecting effective models. The selection of maximal dimension effective models, which have all of the previous properties, is then tackled by deploying a procedure that defines the "most orthogonal basis" [Mitr. & Karc., 1] for a given set of vectors, without transforming the data of the set. The approach suggested here leads to a parameterisation of all maximal dimension effective models, which are nondegenerate and input, output nonredundant. The elements of this set may then be used for the selection of models having additional desirable properties, such as avoiding high order infinite zeros. Amongst the additional properties that may be considered are those of avoiding nonminimum phase properties of the resulting models, as well as more general criteria expressing overall control for control design and known as "process controllability" [Morari, 1]. The work here is considered as a first stage in the process of selection of "good" early stage design models.

This chapter is structured as follows: In Section 2, the problem is introduced as part of the early systems design and the objectives of the work are described. In Section 3, we deal with the problem of Input, Output Redundancy and establish their links to system degeneracy. In Section 4, a type of degeneracy is examined, which is not linked to input, output redundancy, but it is a property of the internal model structure of the system. Sufficient conditions for avoiding this type of degeneracy, also, guarantee the absence of infinite zeros for the resulting model. In Section 5, we deal with the characterisation of the family of controllable and observable effective models based on the characterisation of McMillan degree of Hankel matrices. In Section 6, the results of the previous sections are

used to parameterise and select maximal cardinality well-conditioned models (as far as degeneracy, input, output redundancy, controllability and observability). Finally, in Section 7, the conclusions of this chapter are presented.

6.2. STATEMENT OF THE PROBLEM

The development of models, which may be used for evaluation of alternatives is an integral part of the Early Process Design of process plants [Rijn. 1]. Such models are usually developed for the entire plant, are based on the selected process flowsheet (interconnection graph) and involve the use of simple models of the subprocesses) As such, they have a large dimension and their final structure is determined when the control structure is decided.

The selection of control structures is a topic that has attracted a lot of interest within the process control area ([Mor. & Steph., 1], [Gov. & Pow., 1], [Georg. & Fl., 1], [Skog. & Postl., 1] [Rijn., 1], [Karc., 10] and references there in). This problem involves a number of key subproblems [Karc., 10], which are:

(i) The classification of process variables into potential inputs, outputs and referred to as Model Orientation Problem (MOP).

(ii) Specification of effective sets of inputs, outputs on an oriented model and referred to as Model Projection Problem (MPP).

(iii) Deciding on the way we couple effective inputs and outputs for control design purposes and referred to as Input – Output Coupling Problem (I-O.C.P.). Most of the attention so far has been focused on I-O.C.P., when heuristics and diagnostic indicators have been used.

For the first two problems, less attention has been given, especially from the Control Theory viewpoint; with the exception of the work in [Mor. & Steph., 1], [Geo. & Floud., 1], [Karc., 2], [Karc. & Gian., 1] on some specific problems. In this chapter we are concerned with the selection of the effective sets of inputs and outputs of a system, in order to satisfy certain criteria for the resulting transfer function, such as the system nondegeneracy, the nonredundancy of the input, output scheme and controllability, observability of the resulting model. Such problems belong to the MPP family.

We assume, that a linearised model of the system is given, for which the classification of system variables (implicit variables) into systems and outputs has been

already decided. At the early stages of design it is desirable to include as inputs, all possible variables that can be used as variables to be controlled and measured; these inputs, outputs are referred to as potential sets. The model that corresponds to the potential inputs, outputs provides the basis for deriving all subsequent models based on effective input, output sets and it is thus referred to as the progenitor model. The characteristic of the progenitor model is that all inputs and outputs are physical variables that can be acted upon and measured. Given that the classification of internal variables into inputs, outputs has been done mainly with physical, process based criteria, a progenitor model may not be well-behaving. That is the transfer function may be degenerate and there is redundancy in the input, output schemes and a number of other fundamental properties may not have good values (i.e. condition number etc.). Note that a progenitor model represents all our knowledge about the system at a given stage of early design and the McMillan degree of the progenitor transfer function represents the natural order n of the system.

System models, which are degenerate, are not good for subsequent design since they do not satisfy the basic condition of the output function controllability. It is thus desirable to select subsets of the potential inputs and outputs (by elimination of some elements of the potential sets), such that the resulting transfer function is "well-conditioned" in some sense. Amongst the basic criteria we can use are the properties of nondegeneracy, controllability and observability of the system model and nonredundancy of the input and output scheme. Any submodel that satisfies the above three properties and has maximal cardinality for the input and output set will be called a normal progenitor model; clearly, a system may have more than one such models. The problem we consider also here is the parameterisation and systematic construction (by avoiding listing and testing of all possible submodels) of the family of normal progenitor models.

Assume that the progenitor model is described by the minimal state space equations:

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + B\underline{u}\ ,\ A \in \Re^{n\times n}\ ,\ B \in \Re^{n\times r} \\ \underline{y} &= C\underline{x} + D\underline{u}\ ,\ C \in \Re^{q\times n}\ ,\ D \in \Re^{q\times r} \end{aligned} \tag{6.1}$$

with a corresponding transfer function $H(s) = C(sI - A)^{-1}B + D \in \Re^{q\times r}(s)$ and let $\rho = \text{rank}_{\Re(s)}\{H(s)\}$ be the normal rank of H(s). Clearly $\rho \le \min(q, r)$ and whenever strict inequality holds, then the system is called degenerate; when equality holds the system is called nondegenerate. The significance of ρ is described below [Rosen., 1].

Remark (6.1): ρ defines the maximal number of output variables that may be controlled independently (output function controllability criterion). Furthermore, ρ defines the minimal number of independent inputs required to control ρ outputs.

□

Definition (6.1): For the system $S(A,B,C,D)$ for which $r,q \le n$, we define the numbers:

$$\tau_r \stackrel{\Delta}{=} \text{rank}\left\{\begin{bmatrix} B \\ D \end{bmatrix}\right\} \le r, \quad \tau_\ell \stackrel{\Delta}{=} \text{rank}\left\{[C, D]\right\} \le q \tag{6.2}$$

If $\tau_r < r, (\tau_\ell < q)$ the system will be said to have input (output) redundancy; otherwise, i.e. if $\tau_r = r\,(\tau_\ell = q)$, then it will be said to be regular.

□

Regularity of the model is clearly equivalent to nonredundancy of both sensor and actuator schemes and it is desirable property, which however may not hold on a progenitor model. The problem we consider here is described below:

PROBLEM: Given the progenitor model described by H(s) or with S(A,B,C,D), define:

(i) A maximal cardinality subset of the potential input and output sets such as that the resulting transfer function is nondegenerate, has the maximal possible normal rank and it is also regular.

(ii) Amongst the solutions of (i), determine whether there exist solutions, which have McMillan degree equal to that of H(s).

(iii) Parameterise all solutions with the properties described above.

□

The solution of problem (i) will be referred to as well-conditioning of Progenitor models and part (ii) describes the property that the resulting model is both controllable and observable. Note that controllability and observability are notions defined on S′(A,B′,C′,D′) where A corresponds to the minimal realisation of H(s). The latter problem will be referred to as normal conditioning of Progenitor Models. The existence of such solutions, as well as the parameterisation of them (when such solutions exist) will be examined here. Within the same classes of problems we may also consider more relaxed cases such as stabilisability and detectability [Kailath, 1] and more detailed model properties such as absence of nonminimum phase properties, avoidance of high order infinite zeros etc. More general

properties referred to as "process controllability" [Morari, 1] may be used for subsequent evaluations. The overall problem under consideration is the study of properties of the submatrices of the rational transfer function matrix H(s) obtained by elimination of certain sets of columns, rows. Of special interest is the definition of those submatrices H'(s), which preserve certain properties of H(s), but avoid certain undesirable properties. The study of well-conditioning is considered first.

6.3. INPUT, OUTPUT REDUNDANCY AND SYSTEM DEGENERACY

The notion of redundancy of the input, output map of the progenitor model is linked to some type of redundancy of the resulting model and it is the topic of this section. This form of degeneracy will be referred to simply as simple, to distinguish it from an alternative form of degeneracy characterised by properties of the internal mechanism and referred to as strong. The latter is examined in the following section.

The unifying thing between redundancy and degeneracy is that they both relate to properties of kernels of transfer function, or matrix pencil models. The state space description S(A,B,C,D,) may be represented in the s-domain as:

$$\begin{bmatrix} sI-A & -B \\ -C & -D \end{bmatrix}\begin{bmatrix} \underline{x}(s) \\ \underline{y}(s) \end{bmatrix}=\begin{bmatrix} \underline{x}_0 \\ -\underline{y}(s) \end{bmatrix},\ P(s)=\begin{bmatrix} sI-A & -B \\ -C & -D \end{bmatrix} \tag{6.3}$$

where $P(s) \in \Re^{(n+q)\times(n+r)}(s)$ is the Rosenbrock System Matrix pencil [Rosen., 1].

Definition (6.2): For the system described by S(A,B,C,D), we shall denote by $\mathcal{Z}_r \overset{\Delta}{=} \mathcal{N}_r\{P(s)\}, \mathcal{Z}_\ell \overset{\Delta}{=} \mathcal{N}_\ell\{P(s)\}$ the right, left null spaces of P(s). Then,

(i) A pair of polynomial vectors $\underline{x}(s)\in\Re^n[s], \underline{u}[(s]\in\Re^r(s)$ will be said to be a right pair and the composite vector $\underline{\zeta}(s)=[\underline{x}(s)^t, \underline{u}(s)^t]$ a right vector, if

$$P(s)\,\underline{\zeta}(s)=\underline{0} \tag{6.4}$$

(ii) A pair of polynomial vectors $\underline{y}(s)\in\Re^n[s], \underline{v}(s)\in\Re^q[s]$ will be said to be a left pair and the composite vector $\underline{\xi}(s)^t=[\underline{y}(s)^t, \underline{v}(s)^t]$ a left vector, if

$$\underline{\xi}(s)^t P(s) = \underline{0} \tag{6.5}$$

□

For a right (left) pair $\underline{\zeta}(s)$ we define by $\partial[\underline{\zeta}(s)]$ its degree. An interesting property of the degree is described below [War. & Eck., 1]:

Remark (6.2): For any right pair $(\underline{x}(s), \underline{u}(s))$, left pair $(\underline{y}(s), \underline{v}(s))$ we have that:

$$\partial[\underline{u}(s)] = \partial[\underline{x}(s)] + 1, \quad \partial[\underline{v}(s)] = \partial[\underline{y}(s)] + 1 \tag{6.6}$$

□

Furthermore, all right pairs $(\underline{x}(s), \underline{u}(s))$ with $\partial[\underline{\zeta}(s)] = 0$, we have $\underline{x}(s) = 0$ and $\underline{u}(s) = \underline{u} \in \Re^r$. Similarly, for all left pairs $(\underline{y}(s), \underline{v}(s))$ with $\partial[\underline{\xi}(s)] = 0$, we have $\underline{y}(s) = 0$ and $\underline{v}(s) = \underline{v} \in \Re^q$. The above leads to the following interpretation of the significance of right, left constant vectors [Gant., 1]:

Proposition (6.1): For the system S(A,B,C,D) the following holds true:

(a) There exists a right constant vector $\underline{\zeta} = [\underline{0}^t, \underline{u}^t]^t \neq \underline{0}$ if and only if

$$\begin{bmatrix} B \\ D \end{bmatrix} \underline{u} = 0, \underline{u} \neq 0 \quad \Leftrightarrow \quad \text{rank}\left\{\begin{bmatrix} B \\ D \end{bmatrix}\right\} < r \tag{6.7}$$

(b) There exists a left constant vector $\underline{\xi}^t = [\underline{0}^t, \underline{v}^t]^t \neq \underline{0}^t$, if and only if

$$\underline{v}^t [C, D] = 0^t, \underline{v}^t \neq 0 \quad \Leftrightarrow \quad \text{rank}\{[C, D]\} < q \tag{6.8}$$

□

The above readily follows from the definition and clearly establishes the presence of input, or output redundancy as equivalent to the existence of constant, right, or left vectors correspondingly. In the following we shall denote by:

$$\eta = \dim \mathcal{N}_r\{P(s)\}, \quad \theta = \dim \mathcal{N}_\ell\{P(s)\} \tag{6.9}$$

The following result establishes some interesting properties of η, θ numbers.

Proposition (6.2): For the system S(A,B,C,D), let $\tau = \text{rank}_{\Re(s)}\{P(s)\}$ and $\rho = \text{rank}_{\Re(s)}\{H(s)\}$. Then the following properties hold true:

(i) $\tau = n + \rho$, where n is the number of states.

(ii) $$\eta \overset{\Delta}{=} \dim \mathcal{N}_r\{P(s)\} = \dim \mathcal{N}_r\{H(s)\} = r - \rho \tag{6.10}$$

$$\theta \overset{\Delta}{=} \dim \mathcal{N}_\ell\{P(s)\} = \dim \mathcal{N}_\ell\{H(s)\} = q - \rho \tag{6.11}$$

Proof:

(i) Note that:

$$P'(s)=\begin{bmatrix} I_n & 0 \\ C(sI-A)^{-1} & I_q \end{bmatrix}\begin{bmatrix} sI_n - A & -B \\ -C & -D \end{bmatrix}\begin{bmatrix} I_n & (sI-A)^{-1}B \\ 0 & I_r \end{bmatrix}=\begin{bmatrix} sI_n - A & 0 \\ 0 & -C(sI_n - A)^{-1}B+D \end{bmatrix} \tag{6.12}$$

Thus $P'(s)$ and $P(s)$ are equivalent and, thus, $\tau = \text{rank}_{\mathfrak{R}(s)}\{P(s)\} = \text{rank}_{\mathfrak{R}(s)}\{P'(s)\} =$

$= n + \text{rank}_{\mathfrak{R}(s)}\{H(s)\} = n + \rho$

(ii) From the above we have: $\eta = r + n - \tau = r + n - (n+\rho) = r - \rho = \dim \mathcal{N}_r\{H(s)\}$,

$\theta = q + n - \tau = q + n - (n+\rho) = q - \rho = \dim \mathcal{N}_l\{H(s)$

□

A direct consequence of the above lemma is:

Proposition (6.3): The system is degenerate, if and only if:

$$\tau = \text{rank}_{\mathfrak{R}(s)}\{P(s)\} < \min(n+r, n+q) \tag{6.13}$$

Proof:

(i) If $q \geq r$ and the system is not input regular, then $\mathcal{N}_r\{P(s)\} \neq \{0\}$ and thus $\tau < n+r$ which implies degeneracy. The $q \leq r$ case follows similarly.

(ii) From part (i) it follows that $\tau < n+r$, $\tau < n+q$ and thus $\tau < \min(n+r, n+q)$ and this implies degeneracy.

(iii) Consider the case $q \geq r$ and $\tau_l < r$, then $-\tau_l > -r$ and $q - \tau_l > q - r$. This condition implies that there exists a set of $q - \tau_l$ linearly independent vectors $\{\underline{v}_i, i = 1, \ldots, q - \tau_\ell\}$ such that:

$$\underline{v}_i^t[C\ D] = \underline{0}^t \tag{6.14}$$

The above is equivalent to:

$$\left[\underline{0}^t, \underline{v}_i^t\right]\begin{bmatrix} sI - A & -B \\ -C & -D \end{bmatrix} = \underline{0} \tag{6.15}$$

and thus also to:

$$\left[\underline{0}^t, \underline{v}_i^t\right]\begin{bmatrix} sI - A & -B \\ -C & -D \end{bmatrix}\begin{bmatrix} I_n & (sI - A)^{-1} B \\ 0 & I_r \end{bmatrix} = \underline{0} \tag{6.16}$$

or,

$$\left[\underline{0}^t, \underline{v}_i^t\right]\begin{bmatrix} sI_n - A & 0 \\ -C & -H(s) \end{bmatrix} = \underline{0} \tag{6.17}$$

or,

$$\underline{v}_i^t\, H(s) = \underline{0}^t \tag{6.18}$$

Since there are $q - \tau_1$ constant independent vectors in $\mathcal{N}_\ell\ \{H(s)\}$, it follows:

$$\dim \mathcal{N}_\ell\{H(s)\} = q - \rho \geq q - \tau_1 > q - r \tag{6.19}$$

and thus:

$$\rho < r \text{ and } \rho < q, \text{ i.e. } \rho < \min(r, q) \tag{6.20}$$

□

That is, we can use either P(s) or H(s) for characterisation of the property. Furthermore, degeneracy implies that both null spaces of P(s) or H(s) are nontrivial and degeneracy is equivalent to that possibly only one of the two null spaces is nontrivial ($\neq \{0\}$).

Remark (6.3): The property of degeneracy is linked to the loss of output (input) function controllability [Rosen., 1], [Ants. & Mich., 1] since the existence of a right inverse of H(s) is necessary and sufficient condition for output function controllability. Thus, $\mathcal{N}_r\{P(s)\} = 0$, or $\mathcal{N}_r\{H(s)\} = 0$ are conditions for output function controllability of the corresponding model.

□

Some relationships between degeneracy and input, output loss of regularity are described below:

Proposition (6.4): For the system S(A,B,C,D) the following properties hold true:

(i) If $q \geq r$ ($q \leq r$) and the system is not input (output) regular, then it is degenerate.

(ii) If a system is not input and not output regular, then it is degenerate.

(iii) Let $\tau_\ell = \text{rank}[C, D]$, $\tau_r = \text{rank}\begin{bmatrix} B^t & D^t \end{bmatrix}$. Then,

(a) If $q \geq r$ and $\tau_\ell < r$, the system is degenerate.

(b) If $q \leq r$ and $\tau_r < q$, then the system is degenerate.

□

For the pencil P(s), the right, left null spaces $\mathcal{N}_r\{P(s)\}$, $\mathcal{N}_\ell\{P(s)\}$ are characterised by a set of column, row minimal indices (cmi, rmi) [Gant., 1], which also here may be referred to as right, left indices of P(s) [Forney, 1]. Such sets are denoted by $I_P{}^c = \{\varepsilon_i : i = 1, \ldots\ldots \eta = n - \rho\}$, $I_P{}^r = \{\mu_j : j = 1, \ldots\ldots \theta = q - \rho\}$ and may have t_r zero cmi and t_ℓ zero rmi; in fact,

$$t_r = r - \text{rank}\left\{\left[B^t, D^t\right]^t\right\} = r - \tau_r \leq r - \rho \qquad (6.21a)$$

$$t_\ell = q - \text{rank}\{[C, D]\} = q - \tau_\ell \leq q - \rho \qquad (6.22b)$$

The numbers t_r, t_ℓ which characterise 0 – cmi, 0 – rmi respectively, express the order of input, output redundancy and will be referred to as input-, output – redundancy index correspondingly. The use of t_r, t_ℓ indices provides some additional insight on redundancy and leads to the following remarks.

Remark (6.4): The numbers $\tau_r = \text{rank}\{[B^t, D^t]\}$ and $\tau_\ell = \text{rank}\{[C, D]\}$ provide bounds for $\rho = \text{rank}_{\mathcal{R}(s)}\{H(s)\}$ and in particular

$$\rho \leq \min(\tau_r, \tau_\ell) \qquad (6.23)$$

The case of $\rho = \min(\tau_r, \tau_\ell)$ implies:

(a) If $\tau_r = \min(\tau_r, \tau_\ell)$, then all indices in $I_P{}^c$ are zero, or the set is empty; in particular, if $r > \tau_r$, then all cmi are zero and if $r = \tau_r$, then $I_P{}^c$ is empty and the system is nondegenerate.

(b) If $\tau_\ell = \min(\tau_r, \tau_\ell)$, then all indices in I_P^r are zero, or the set is empty; in particular, if $q > \tau_\ell$ then all indices in I_P^r are zero and if $q = \tau_\ell$, then I_P^r is empty and the system is nondegenerate.

(c) If $\rho = \tau_r = \tau_\ell$ and at least one of r, q is equal to ρ, then clearly we have nondegeneracy and redundancy for the index that is greater than ρ. If r, q > ρ, then we have both degeneracy and input, output degeneracy.

□

The case where $t_r = r - \rho$ ($t_\ell = q - \rho$) is referred to as total input – (output-) irregularity. When at least one such condition holds true, this implies that degeneracy of the transfer function may be removed by eliminating redundancy in the corresponding part of the instrumentation map. The results in this section show that there is link between input, output redundancy and system degeneracy. The type of system degeneracy inferred from the input, output redundancy will be called simple. Another type of degeneracy that may exist even under input and output regularity is considered next; this is linked to properties of the internal mechanism and shall be referred to as strong degeneracy.

6.4. STRONG SYSTEM DEGENERACY

In the previous section, issues of degeneracy and input, output redundancy were examined, which are linked to zero values of cmi, rmi. Here we will consider the case of nonzero indices. We shall denote by $\mathcal{Z}_r \stackrel{\Delta}{=} \mathcal{N}_r\{P(s)\}, \mathcal{Z}_\ell \stackrel{\Delta}{=} \mathcal{N}_l\{P(s)\}$ and $\tau_\ell = \text{rank}[C, D]$, $\tau_r = \text{rank}[B^t, D^t]$. The study of strong degeneracy is an issue that is linked in a certain way to the characterisation of infinite zeros. Such links will become explicit. The sets of indices I_p^c, I_p^r associated with $\mathcal{Z}_r$, $\mathcal{Z}_\ell$ respectively may contain nonzero indices and this is characterised by the following result.

Proposition (6.5): For any system S(A, B, C, D) with r inputs, q outputs, transfer function H(s) and $\rho = \text{rank}_{\mathfrak{R}(s)}\{H(s)\}$ the following properties hold true:

(a) The numbers ρ, τ_r, τ_l, r, q satisfy the conditions:

$$\rho \le \tau_r \le r \text{ and } \rho \le \tau_\ell \le q \tag{6.24}$$

(b) The system has $\tau_r - \rho$ nonzero cmi, if and only if

$$\rho < \tau_r \leq r \tag{6.25}$$

and all such indices are nonzero, if $\tau_r = r$

(c) The system has $\tau_\ell - \rho$ nonzero rmi, if and only if

$$\rho < \tau_\ell \leq q \tag{6.26}$$

and all such indices are nonzero, if $\tau_\ell = q$.

Proof:

(a) By (6.21 a), (6.21 b) and the fact that $\tau_r \leq r$ and $\tau_l \leq q$, part (a) readily follows.

(b) The number of nonzero cmi is $n - t_r = \tau_r - \rho$ and such indices exist only when $n - t_r = \tau_r - \rho > 0$. In the case where $\tau_r = r$ then clearly $t_r = 0$. Part (c) follows along similar lines.

□

In the following, we consider the case where $q \geq r$ and we shall assume that (6.25) holds true, i.e. we have at least one nonzero cmi. This implies that there exists a right pair $\underline{x}(s), \underline{u}(s)$, where:

$$\underline{x}(s) = \underline{x}_0 + s\,\underline{x}_1 + \ldots\ldots + s^{k-1}\,\underline{x}_{k-1} \tag{6.27}$$

$$\underline{u}(s) = \underline{u}_0 + s\,\underline{u}_1 + \ldots\ldots + s^{k-1}\,\underline{u}_{k-1} \tag{6.28}$$

such that:

$$\begin{aligned} (sI - A)\,\underline{x}(s) &= B\,\underline{u}(s) \\ C\,\underline{x}(s) + D\,\underline{u}(s) &= 0 \end{aligned} \tag{6.29}$$

The above lead to the following result:

Proposition (6.6): The system S(A,B,C,D) with $q \geq r$ and $\rho < r$ has a right index with value k, at most if and only if there exists a set of vectors $\{\underline{u}_0, \underline{u}_1, \ldots, \underline{u}_k, u_k \neq 0\}$ such that the following conditions are satisfied:

$$\begin{bmatrix} A^kB & A^{k-1}B & A^{k-2}B & \dots & A^2B & AB & B \\ CA^{k-1}B & CA^{k-2}B & CA^{k-3}B & \dots & CAB & CB & D \\ CA^{k-2}B & CA^{k-3}B & CA^{k-4}B & \dots & CB & D & 0 \\ \vdots & \vdots & \vdots & & & \vdots & \\ CAB & CB & D & \dots & 0 & 0 & 0 \\ CB & D & 0 & \dots & 0 & 0 & 0 \\ D & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{u}_k \\ \underline{u}_{k-1} \\ \underline{u}_{k-2} \\ \vdots \\ \underline{u}_2 \\ \underline{u}_1 \\ \underline{u}_0 \end{bmatrix} = \underline{0} \tag{6.30}$$

Proof: Substituting the expressions of $\underline{x}(s), \underline{u}(s)$ from (6.27), (6.28) into (6.29), we have:

$$(sI - A)(\underline{x}_0 + s\,\underline{x}_1 + \dots + s^{k-1}\,\underline{x}_{k-1}) = B(\underline{u}_0 + s\,\underline{u}_1 + \dots + s^k\,\underline{u}_k)$$

$$C(\underline{x}_0 + s\,x_1 + \dots + s^{k-1}\,\underline{x}_{k-1}) + D(\underline{u}_0 + s\,u_1 + \dots + s^1\,\underline{u}_k) = 0$$

By equating coefficients of equal powers, it follows that:

$$\begin{aligned} &\underline{x}_{k-1} = B\,\underline{u}_k \\ &\underline{x}_{k-2} = A\,B\,\underline{u}_k + B\,\underline{u}_{k-1} \\ &\vdots \end{aligned} \tag{6.31}$$

$$\underline{x}_0 = A^{k-1}\,B\,\underline{u}_k + A^{k-2}\,B\,\underline{u}_{k-1} + \dots + A\,B\,\underline{u}_2 + B\,\underline{u}_1 \tag{6.32}$$

$$0 = A^k\,B\,\underline{u}_k + A^{k-1}\,B\,\underline{u}_{k-1} + \dots + A^2\,B\,\underline{u}_2 + A\,B\,\underline{u}_1 + B\,\underline{u}_0$$

$$\begin{aligned} &C\,\underline{x}_0 + D\,\underline{u}_0 = 0 = C\,A^{k-1}\,B\,\underline{u}_k + C\,A^{k-2}\,B\,\underline{u}_{k-1} + \dots + C\,A\,B\,\underline{u}_2 + C\,B\,\underline{u}_1 + D\,\underline{u}_0 \\ &C\,\underline{x}_1 + D\,\underline{u}_1 = 0 = C\,A^{k-2}\,B\,\underline{u}_k + C\,A^{k-3}\,B\,\underline{u}_{k-2} + \dots + C\,A\,B\,\underline{u}_3 + C\,B\,\underline{u}_2 + D\,\underline{u}_1 \\ &\vdots \\ &C\,\underline{x}_{k-1} + D\,\underline{u}_{k-1} = 0 = C\,B\,\underline{u}_k + D\,\underline{u}_{k-1} \\ &D\,\underline{u}_k = 0 \end{aligned} \tag{6.33}$$

By combining the above the result follows.

□

The above condition may now be used to derive conditions for non-degeneracy of transfer functions and thus also procedures for redesign of the system to guarantee non-degeneracy. For the given system, we define the following set of matrices:

$$M_0 = \begin{bmatrix} B \\ D \end{bmatrix}, M_1 = \left[\begin{array}{cc} AB & B \\ \hline CB & D \\ D & 0 \end{array}\right], M_2 = \left[\begin{array}{ccc} A^2B & AB & B \\ \hline CAB & CB & D \\ CB & D & 0 \\ 0 & 0 & 0 \end{array}\right], \dots,$$

$$M_k = \left[\begin{array}{ccccc} A^kB & A^{k-1}B & \dots & AB & B \\ \hline CA^{k-1}B & CA^{k-2}B & \dots & CB & D \\ CA^{k-2}B & CA^{k-3}B & \dots & D & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ CAB & CB & \dots & 0 & 0 \\ CB & D & \dots & 0 & 0 \\ D & 0 & \dots & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc:c} A^kB & \dots & AB & B \\ \hline & & N_k & \end{array}\right] \tag{6.34}$$

In terms of the above matrices, we may state some tests for nondegeneracy as shown below. We first note:

Lemma (6.1): If $q \geq r$, then the maximal possible value of right index of P(s) is:

(i) If $D \neq 0$ and rank $(D) = \delta$, then $\varepsilon_{max} = n - q + 2\delta - 1$.

(ii) If $D = 0$, then $\varepsilon_{max} = n - q - 1$.

Proof:

(i) If $D \neq 0$ and rank $(D) = \delta$, there exists a pair of transformations $Q \in \Re^{q \times q}, R \in \Re^{r \times r}$, $|Q|, |R| \neq 0$ such that:

$$\begin{bmatrix} I_n & 0 \\ 0 & Q \end{bmatrix} \begin{bmatrix} sI - A & -B \\ -C & -D \end{bmatrix} \begin{bmatrix} I_n & 0 \\ 0 & R \end{bmatrix} = \begin{bmatrix} sI - A & -BR \\ -QC & -QDR \end{bmatrix} = Q'P(s)R' = P'(s)$$

where:

$$QDR = \begin{bmatrix} I_\delta & 0 \\ 0 & 0 \end{bmatrix} = D', QC = C', BR = B'$$

By partitioning C', B' according to the partitioning of D', we have:

$$P'(s) = \left[\begin{array}{cc:c} sI - A & -B'_\delta & B'_{r-\delta} \\ -C'_\delta & -I_\delta & 0 \\ \hline -C'_{q-\delta} & 0 & 0 \end{array}\right] = \left[\begin{array}{cc:c} s\tilde{E} - \tilde{A} & & -\tilde{B} \\ \hline & -\tilde{C} & 0 \end{array}\right] \tag{6.35}$$

The zero structure of $P'(s)$ [Karc. & Kouv., 1] is defined by the zero pencil $\tilde{Z}(s) = s\tilde{N}\tilde{E}\tilde{M} - \tilde{N}\tilde{A}\tilde{M}$, where $\tilde{N}$ is a $(n - r + 2\mu)$ x $(n + \delta)$ left annihilator of $\tilde{B}$ and $\tilde{M}$ is a

$(n+\delta)$ x $(n-q+2\delta)$ right annihilator of $\tilde{C}$. Clearly, $\tilde{Z}(s)$ has dimension $(n-r+2\mu)$ x $(n-q+2\delta)$ and $n-r+2\mu \geq n-q+2\delta$. For such a pencil the maximal possible value of a right index is when $\varepsilon_{max} + 1 = n - q + 2\delta$, i.e. smallest of the two dimensions; this follows by inspection of the possible structure of the Kronecker form of $\tilde{Z}(s)$ [Gant., 1]. Part (ii) follows from Part (i) for $\delta = 0$.

□

Theorem (6.1): For the system S(A,B,C,D) with $q \geq r$, the following properties hold true:

(i) If D has full rank, then the system has no right indices of any value and it is thus non-degenerate.

(ii) If $D \neq 0$ and rank $(D) = \delta < r$, then the system is non-degenerate, if and only if the matrix M_σ is full rank, where $\sigma = n - q + 2\delta - 1$.

Proof:

(i) From Proposition (6.6), it follows that if rank (D) = r, then from equation (6.30) we have that $D\,\underline{u}_k = 0$. Clearly, this implies $\underline{u}_k = 0$ and this in turn (from (6.30)) yields $D\,\underline{u}_{k-1} = 0$; again we have $\underline{u}_{k-1} = 0$ and by obvious induction, $\underline{u}_k = 0$ for all $k = 0, 1, 2,....$ It is now clear that since there is no u(s) and thus no x(s) satisfying (6.31), the system is non-degenerate.

(ii) By condition (6.34) if there is a right index $\varepsilon < \tau$ then M_ε has a right kernel and from the structure of M_k for $\forall k \geq \varepsilon$ we shall also have $\mathcal{N}_r\{M_n\} \neq \{0\}$. Since τ is the maximal possible value of a right index, if $\mathcal{N}_r\{M_\tau\} = \{0\}$, then also for $\forall k \geq \tau$ $\mathcal{N}_r\{M_k\} = \{0\}$, since otherwise we are led to a contradiction (existence of a right index greater than τ). This completes the proof.

□

The above results for the case of strictly proper systems have the following form.

First, define the matrices:

$$\tilde{M}_0 = [B],\ \tilde{M}_1 = \left[\begin{array}{c|c} AB & B \\ \hline CB & 0 \end{array}\right],\ \tilde{M}_2 = \left[\begin{array}{cc|c} A^2B & AB & B \\ \hline CAB & CB & 0 \\ CB & 0 & 0 \end{array}\right],\ldots$$

$$\ldots, \quad \tilde{M}_k = \left[\begin{array}{ccccc|c} A^kB & \ldots & A^2B & AB & B \\ \hline CA^{k-1}B & \ldots & CAB & CB & 0 \\ CA^{k-2}B & \ldots & CB & 0 & 0 \\ \vdots & & \vdots & \vdots & \vdots \\ CAB & \ldots & 0 & 0 & 0 \\ CB & \ldots & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccccc|c} A^kB & \ldots\ldots & A^2B & AB & B \\ \hline & & \tilde{N}_k & & 0 \end{array}\right] \tag{6.36}$$

Theorem (6.1) leads to the following corollary:

Corollary (6.1): For the system S(A,B,C) with $q \geq r$, the following properties hold true:

(i) If CB is full rank, then the system has no right indices and the system is non-degenerate.

(ii) The system with CB rank deficient is non-degenerate, if and only if the matrix $\tilde{M}_{\tau'}$ is full rank, where $\tau' = n - q - 1$.

Proof:

(i) Clearly, we have that there exists a 0-right index if the matrix $[B^t, 0]^t$ or equivalently B looses rank. However, if rank (CB) = r, then it is necessary that rank (B) = r, because, otherwise $\exists\, \underline{v} : \underline{v} \neq 0$ and $B\underline{v} = 0 \rightarrow CB\underline{v} = 0$ and this leads to a contradiction. Thus, there is no 0-right index. Following similar arguments to those in the proof of the Theorem, it follows that there is no other index of any value k.

(ii) Part (ii) follows along similar lines.

□

The results in this section provide criteria for a type of degeneracy, and thus loss of output function controllability, which depends on the models, inner structure and will be referred to as strong degeneracy. The distinction between the simple and strong type is the nature of associated indices, that is zero and non-zero respectively. Note that the characterisation of this type of degeneracy is based on the right nullity properties of matrices $M_k, \tilde{M}_k$, which have as integral parts the matrices $N_k, \tilde{N}_k$ introduced by the partitioning of $M_k, \tilde{M}_k$ as indicated by (6.31), (6.36). These matrices are of the Toeplitz type and their right nullity properties are linked to the characterisation of state space infinite zeros of the system [Karc. & Hayt., 1]. The state space characterisation of infinite zeros of a

system S(A,B,C,D) (based on the notion of infinite elementary divisors of the associated system matrix P(s)) leads to a result that shows the links between strong degeneracy and infinite zeros.

Let us denote the following sequence of matrices for the system S(A,B,C,D):

$$Q_0 = [D], Q_1 = \begin{bmatrix} D & 0 \\ CB & D \end{bmatrix}, \ldots\ldots, Q_k = \begin{bmatrix} D & 0 & 0\ldots & 0 & 0 \\ CB & D & 0 & 0 & 0 \\ CAB & CB & D & 0 & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ CA^{k-1}B & CA^{k-1}B & \ldots & CB & D \end{bmatrix} \qquad (6.37)$$

and for the strictly proper case the sequence:

$$\tilde{Q}_1 = [CB], \tilde{Q}_2 = \begin{bmatrix} CB & 0 \\ CAB & CB \end{bmatrix}, \ldots\ldots, \tilde{Q}_k = \begin{bmatrix} CB & 0 & \ldots & 0 \\ CAB & CB & 0 & 0 \\ \vdots & & \ddots & \vdots \\ CA^{k-1}B & CA^{k-2}B & \ldots & CB \end{bmatrix} \qquad (6.38)$$

If we denote by $\gamma_i \stackrel{\Delta}{=} \eta_r(Q_i), \tilde{\gamma}_i \stackrel{\Delta}{=} \eta_r(\tilde{Q}_i)$ the right nullities of the above matrices, then we have the following characterisation of infinite zeros:

Theorem (6.2): Assume that S(A,B,C,D) is nondegenerate and let H(s) be the corresponding transfer function. Then,

(i) The sequence $J_\infty = \{\gamma_0, \gamma_1, \ldots, \gamma_k, \ldots\}$ is a Piecewise Arithmetic Progression [Vard. & Karc., 1], that satisfies the relationship:

$$2\gamma_i \geq \gamma_{i-1} + \gamma_{i+1} \qquad i = 0,1,\ldots \quad \gamma_{-1} = 0 \qquad (6.39a)$$

and the singular points, defined by those i for which:

$$\delta_i = 2\gamma_i - \gamma_{i-1} - \gamma_{i+1} > 0, \qquad i = 0,1,2,\ldots. \qquad (6.39b)$$

characterise the degrees of infinite elementary divisors (ied) of P(s) and δ_i denotes their corresponding multiplicity.

(ii) If $\{\hat{s}^{q_i},\ i=1,2,...\mu,\}$ is the set of ied of P(s), then the orders of infinite zeros of H(s) are defined by the nontrivial elements $(\neq 0)$ of the set $\left\{\hat{q}_i : \hat{q}_i = q_i - 1,\ \ \forall i \in \underset{\sim}{\mu}\right\}$.

Proof: For nondegenerate systems, the systems matrix pencil P(s) is right regular and thus:

$$P(s) = \begin{bmatrix} sI - A & -B \\ -C & -D \end{bmatrix} = s\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A & B \\ C & D \end{bmatrix} = sF - G \tag{6.40}$$

The infinite elementary divisors (ied) are characterised by the properties of the right nullities of the following sequence of Toeplitz matrices defined on the pair (F,G) [Vard. & Karc., 1], [Karc. & Hayt., 1]:

$$T_\infty^1 = F,\ T_\infty^2 = \begin{bmatrix} F & 0 \\ G & F \end{bmatrix},\ldots\ldots,\ T_\infty^k = \begin{bmatrix} F & 0 & \ldots & 0 & 0 \\ G & F & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & F & 0 \\ 0 & 0 & \ldots & G & F \end{bmatrix} \tag{6.41}$$

If we denote by $\eta_k \overset{\Delta}{=} \eta_r\left(T_\infty^k\right)$ the right nullities of the T_∞^k matrices and by $\gamma_i \overset{\Delta}{=} \eta_r(Q_i)$, then we have the following relationships: For k=1, $\eta_r\left(T_\infty^1\right) = r = \eta_1$; For k=2 we have:

$$T_\infty^2 = \begin{bmatrix} F & 0 \\ G & F \end{bmatrix} = \left[\begin{array}{cc:cc} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline A & B & I & 0 \\ C & D & 0 & 0 \end{array}\right] \xrightarrow{\text{equivalent}} \left[\begin{array}{ccc:c} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & D & 0 \\ \hdashline 0 & 0 & 0 & 0 \end{array}\right] = \tilde{T}_\infty^2 \tag{6.42}$$

and thus:

$$\eta_r\left(T_\infty^2\right) = \eta_r\left(\tilde{T}_\infty^2\right) = \eta_r(Q_o) + r = \eta_r(D) + r = \eta_2 = \gamma_o + r \tag{6.43}$$

For the general T_∞^k, by using elementary column and row operations it is readily shown that for k>2 we have that T_∞^k may be reduced to the following equivalent matrix:

$$\tilde{T}_{\infty}^{k} = \left[\begin{array}{ccc|ccc|c} I & & & & & & \\ & \ddots & & & 0 & & \\ & & I & & & & \\ \hline & & & D & 0 & \cdots\ 0 & 0 \\ & 0 & & CB & D & \cdots\ 0 & \\ & & & \vdots & \vdots & \ddots\ \vdots & \\ & & & CA^{k-3}B & CA^{k-4}B & \cdots\ D & \\ \hline & & & 0 & & & 0 \end{array}\right] = \left[\begin{array}{ccc|c} I & & & \\ & \ddots & 0 & 0 \\ & & I & \\ \hline & 0 & Q_{k-2} & \\ \hline & 0 & & 0 \end{array}\right] \qquad (6.44)$$

and thus:

$$\eta_r\left(T_{\infty}^{k}\right) = \eta_r\left(\tilde{T}_{\infty}^{k}\right) = \eta_r\left(Q_{k-2}\right) + r = \eta_2 = \gamma_{k-2} + r \qquad (6.45)$$

□

Clearly, [Vard. & Karc., 1], $\{\eta_k\}$ is a piecewise arithmetic progression and thus also the $\{\gamma_i\}$ sequence. The singular points of the $\{\eta_k\}$, or $\{\gamma_i\}$ sequences define the degrees of ied of P(s). The relationship between degrees of ied and orders of infinite zeros of H(s) (Part (ii)) is a known result established in [Karc. & Kalog., 1].

Remark (6.5): The sufficient conditions for avoiding strong degeneracy, i.e. D full rank (proper systems), CB full rank (strictly proper systems) imply that the sequence J_{∞} is {0} for the proper case or $J_{\infty} = \{rk,\ k = 1,2,....\}$, i.e. arithmetic progression for the strictly proper case. In either case the transfer function H(s) has no infinite zeros (in the algebraic sense).

□

The above suggests that the sufficient conditions for avoiding nondegeneracy i.e. D, or CB full rank, have the additional property that they force the corresponding transfer function not to have infinite zeros. Such systems have the advantage that they can be controlled in a relatively simple way.

6.5. NORMAL CONDITIONING OF PROGENITOR MODELS

Given the progenitor model described by the transfer function matrix $H(s) \in \Re^{q \times r}(s)$ and with a McMillan degree n, there is always a minimal realisation S(A,B,C,D). It is this model which represents our entire knowledge for the system. By

deleting a subset α of inputs and a subset β of the outputs we obtain a resulting system $S(A,B_\alpha,C_\beta,D_{\alpha,\beta})$; this model may be well conditioned (nondegenerate and input, output regular), but it may not necessarily be controllable and observable. Clearly, the standard tests for controllability and observability on all possible system representations $S(A,B_\alpha,C_\beta,D_{\alpha,\beta})$ may be used, but the procedure is rather cumbersome. Here we shall use alternative tests based on the McMillan degree, which may combine with the conditions for nondegeneracy in a more natural way. We note first the following standard results from linear systems [Ants. & Mich., 1], [Kailath, 1].

Lemma (6.2): Let H(s) be transfer function and S(A,B,C,D) be a realisation of H(s). S(A,B,C,D) is a minimal realisation of H(s), if the McMillan degree of H(s) is: $\delta_M(H) = \partial\{|sI - A|\}$.

□

Using the above result we note the following:

Proposition (6.6): Let H(s) be a transfer function, S(A,B,C,D) the corresponding minimal system and $H_{\alpha,\beta}(s)$ be the submatrix defined from H(s) by eliminating the α set of inputs and β set of outputs. If $S(A,B_\alpha,C_\beta,D_{\alpha,\beta})$ is the resulting system, then it is minimal if and only if $\delta_M(H) = \delta_M(H_{\alpha,\beta})$.

Proof: The subsystem $S(A,B_\alpha,C_\beta,D_{\alpha,\beta})$ has dimension of its state space equal to $\delta_M(H)$. If the corresponding transfer function $H_{\alpha,\beta}(s)$ has $\delta_M(H_{\alpha,\beta}) < \delta_M(H)$, then clearly it is not minimal. If $\delta_M(H_{\alpha,\beta}) = \delta_M(H)$, then Lemma (6.2) is established.

□

The result follows directly from Lemma (6.2) and the construction of $H_{\alpha,\beta}(s)$, or $S(A,B_\alpha,C_\beta,D_{\alpha,\beta})$. We now consider the state space characterisation of the McMillan degree, which is established as shown below. Let us consider the Laurent series expression of H(s) [Ants. & Mich., 1], i.e.

$$H(s) = H_o + \hat{H}(s) = H_o + H_1 \cdot s^{-1} + H_2 \cdot s^{-2} + H_3 \cdot s^{-3} + ... \qquad (6.46)$$

where $\hat{H}(s)$ is the strictly proper part and the $q \times r$ real matrices H_o, H_1,... are the Markov parameters where:

$$H_o = D, \quad H_i = CA^{i-1}B, \quad i = 1,2,..... \tag{6.47}$$

The Hankel matrix $M_H(i,j)$ of order (i,j) corresponding to the Markov parameter sequence $H_1, H_2, \ldots$ is defined as the $iq \times jr$ matrix given by:

$$M_H(i,j) \stackrel{\Delta}{=} \begin{bmatrix} H_1 & H_2 & ..... & H_j \\ H_2 & H_3 & ..... & H_{j+1} \\ \vdots & \vdots & & \vdots \\ H_i & H_{i+1} & ..... & H_{i+j-1} \end{bmatrix} \tag{6.48}$$

Lemma (6.3) [Ants. & Mich., 1]: The McMillan degree of the transfer function H(s) is the rank of $M_H(v,v)$, where v is the degree of the least common denominator of the entries of H(s).

□

By computing the least common multiple (lcm) of the entries of H(s), say $d_H(s)$, then $v = \partial\{d_H(s)\}$. Using the Markov parameters {CB,CAB,....) we may define the matrix:

$$M_H(v.v) \stackrel{\Delta}{=} M_H^v = \begin{bmatrix} CB & CAB & ..... & CA^{v-1}B \\ CAB & CA^2B & ..... & CA^vB \\ \vdots & \vdots & & \vdots \\ CA^{v-1}B & CA^vB & ..... & CA^{2v-1}B \end{bmatrix} \tag{6.49}$$

Clearly, $\text{rank}\{M_H^v\} = \delta_M(H)$ and a searching procedure for the submatrices $H_{\alpha,\beta}(s)$ with the same McMillan degree with H(s) can be defined as indicated below:

Definition (6.3): Let $\{CB, CAB, ..., CA^kB, ...\}$ be the Markov parameters associated with the H(s) progenitor model, $\alpha = (i_1, ..., i_\sigma)$ be a set of indices characterising inputs of the {1,2,…,r} set and $\beta = (j_1, ..., j_\tau)$ be a set of indices characterising outputs of the {1,2,…,q} set. We shall denote by $C_\beta A^k B_\alpha$ the submatrix of CA^kB obtained by eliminating the α set of columns and β set of rows of CA^kB. We define as the $M_{H_{\alpha\beta}}^v$ Hankel submatrix of M_H^v the matrix:

$$M^{v}_{H_{\alpha\beta}} = \begin{bmatrix} C_\beta B_\alpha & C_\beta A B_\alpha & \cdots\cdots & C_\beta A^{v-1} B_\alpha \\ C_\beta A B_\alpha & C_\beta A^2 B_\alpha & \cdots\cdots & C_\beta A^{v} B_\alpha \\ \vdots & \vdots & & \vdots \\ C_\beta A^{v-1} B_\alpha & C_\beta A^{v} B_\alpha & \cdots\cdots & C_\beta A^{2v-1} B_\alpha \end{bmatrix} \tag{6.50}$$

□

Using the matrices $M^{v}_{H_{\alpha\beta}}$ we may now state the following result.

Corollary (6.2): Let S(A,B,C,D) be a minimal realisation of H(s) and $S(A,B_\alpha,C_\beta,D_{\alpha,\beta})$ the subsystem obtained by deleting the α set of inputs and β set of outputs. The subsystem $S(A,B_\alpha,C_\beta,D_{\alpha,\beta})$ is both controllable and observable, if and only if

$$\text{rank}\left(M^{v}_{H}\right) = \text{rank}\left(M^{v}_{H_{\alpha\beta}}\right) \tag{6.51}$$

□

The above result readily follows from Proposition (6.6) and Lemma (6.3). This result may be used to formulate the basis for a searching method for controllable and observable subsystems of H(s).

Remark (6.6): For strictly proper transfer functions H(s), a search for maximal rank $M^{v}_{H_{\alpha\beta}}$ submatrices of M^{v}_{H} which is based on a full rank $C_\beta B_\alpha$, guarantees nondegeneracy, no infinite zeros and minimality (controllability and observability) of the resulting subsystem.

□

6.6. WELL CONDITIONING OF TRANSFER FUNCTIONS: SELECTION PROCEDURES AND PARAMETERISATIONS

The results in the previous sections provide criteria for selecting subsystems of H(s), or P(s) which satisfy the input, output regularity requirements and the conditions for non-degeneracy. Although, input, output redundancy may imply degeneracy, input, output regularity does not guarantee non-degeneracy. Guaranteeing non-degeneracy may be achieved by using the sufficient conditions based on the D, CB matrices, or testing selections using the full rank tests based on $M_\tau, \tilde{M}_\tau$ matrices. Note that conditions based on $M_\tau, \tilde{M}_\tau$ are not easy to use for making initial selections, which are made using input,

output regularity as a selection criterion. Two different strategies for model selection can be made:

(I) **Direct Method**: Selection based on sufficient conditions.

(II) **Indirect Method**: Selection based on input, output regularity and search for nondegeneracy.

6.6.1. DIRECT METHOD FOR WELL-CONDITIONING

We assume that $q \geq r$ and that the S(A,B,C,D) model is degenerate. If the system is proper, $D \neq 0$, then degeneracy implies that D is rank deficient and if the system is strictly proper, then necessarily CB has to be rank deficient.

Remark (6.7): If the system S(A,B,C,D) with $q \geq r$ is degenerate, a redesign procedure leading to $\tilde{S}(A,\tilde{B},\tilde{C},\tilde{D})$ with $\tilde{D}$ full rank guarantees the creation of a system which is non-degenerate and has full rank input and output structure.

□

Remark (6.8): If the system S(A,B,C) with $q \geq r$ is degenerate, a redesign procedure leading to $\tilde{S}(A,\tilde{B},\tilde{C})$ with $\tilde{C}\tilde{B}$ full rank guarantees the creation of a system which is non-degenerate and has full rank input and output structure.

□

The meaning of redesign of D, or CB is that we aim to define a maximal subset of the columns of D, or CB that guarantee the maximal full rank property. This procedure is clearly sufficient, but not necessary and leads to a system of smaller dimensions, as far as input, output structure is concerned. Note that we would like to achieve this selection without transforming the matrices D, CB, since it is desirable to keep the physical variables involved in the original model. This leads to the following definition. The redesign problem clearly, becomes trivial, if general input, output coordinate transformations are used. The problem under study here is important only when we want to retain the original set of physical variables. In the following we shall use the definition:

Definition (6.4): Let $T=[\underline{t}_1,\underline{t}_2,...,\underline{t}_r]\in\Re^{q\times r}, q\geq r$ with $\text{rank}(T)=\rho<\min(q,r)$. Any ρ-subset of the set $\{\underline{t}_i, i\in\underline{r}\}$ of columns that is linearly independent is said to form a natural basis for the space colsp {T}. If the set $\{\underline{t}_i, i\in\underline{r}\}$ is normalised $\{\|\underline{t}_i\|=1, i\in\underline{r}\}$, every natural basis has a measure of orthogonality σ and thus every natural basis $\{\underline{t}_{i_1},...,\underline{t}_{i_\rho}\}$ may be referred to as a σ-natural basis. The natural basis with the highest degree of orthogonality will be called a proper basis of colsp {T}.

□

The selection of a proper basis for a set of vectors has been previously addressed in algebraic computations [Mitr. & Karc., 1] as a problem of selection of "best uncorrupted base" and an algorithm for achieving this has been introduced. In the above definition an important ingredient is the notion of orthogonality of the set. This may be introduced using the notion of the Grammian [Gant., 1] or condition numbers. The former is used here.

Definition (6.5) [Gant., 1]: Let $\underline{x}_1,\underline{x}_2,...,\underline{x}_m$ be vectors $\in\Re^n$. The matrix defined by:

$$G=\begin{bmatrix}(\underline{x}_1,\underline{x}_1) & (\underline{x}_1,\underline{x}_2) & \cdots & (\underline{x}_1,\underline{x}_m)\\ (\underline{x}_2,\underline{x}_1) & (\underline{x}_2,\underline{x}_2) & \cdots & (\underline{x}_2,\underline{x}_m)\\ \vdots & \vdots & & \vdots\\ (\underline{x}_m,\underline{x}_1) & (\underline{x}_m,\underline{x}_{21}) & \cdots & (\underline{x}_m,\underline{x}_m)\end{bmatrix} \tag{6.52}$$

where (,) denotes inner product, is called the Gram matrix of the vectors $\underline{x}_1,\underline{x}_2,...,\underline{x}_m$ and the determinant $G_m=G(\underline{x}_1,\underline{x}_2,...,\underline{x}_m)=|G|$ is called their Grammian.

□

Note [Gant., 1] that the vectors $\underline{x}_1,\underline{x}_2,...,\underline{x}_m$ are linearly independent, if and only if their Grammian is nonzero; in general, we have that $|G|\geq 0$ and the following property holds true (Hadamard's inequality):

$$G(\underline{x}_1,\underline{x}_2,...,\underline{x}_m)\leq G(x_1)\cdot G(x_2)\cdot....\cdot G(x_m) \tag{6.53}$$

Note that $G(\underline{x}_i)=\|\underline{x}_i\|_2$ and if the vectors are of unit length (i.e. $\|\underline{x}_i\|^2=1, i=1,2,...,m)$, then

$$0\leq G(\underline{x}_1,\underline{x}_2,...,\underline{x}_m)\leq 1 \tag{6.54}$$

Remark (6.9): An alternative test for closeness to normality of a normalised selected set with a basis matrix A, can be based on the condition number of the corresponding matrix. In fact, the deviation from unity of the $\|\cdot\|_2$ condition number is a measure of proximity to orthogonality. This number measures the elongation of the hyperlipsoid associated with A i.e. $\{A\underline{x} : \|\underline{x}\|_2 = 1\}$.

□

We will use the Grammian as the criterion for selection of natural bases with degree of orthogonality greater than a given number $\underline{\sigma}\,(0 < \sigma \leq 1)$. A procedure for such selection will be described later on and will be referred to as natural basis selection. The set of all natural bases with orthogonality σ : $\underline{\sigma} \leq \sigma \leq 1$ will be called the $\{\underline{\sigma}\}$- set of natural bases. We may now summarise the selection procedure as follows:

DIRECT METHOD FOR WELL-CONDITIONING

Let $T = [\underline{t}_1, \underline{t}_2, ..., \underline{t}_r] \in \Re^{q \times r}, q \geq r$ be a matrix that may represent D, or CB, ρ = rank {T} and assume all its columns to be normalised (i.e. $\|\underline{t}_i\| = 1$). The selection of the well-conditioned model involves:

STEP (1): Select an acceptable order of orthogonality $\underline{\sigma}$. and using the natural basis selection we define $\{\underline{\sigma}\}$- set of matrices $\{T_a : T_a = [\underline{t}_{i_1}, \underline{t}_{i_2}, ..., \underline{t}_{i_\rho}] \in \Re^{i \times \rho}$ such that the corresponding set has orthogonality degree $\sigma \geq \underline{\sigma}$.

STEP (2): For every set of indices $a = (i_1, i_2, ..., i_\rho)$ associated with the $\{\underline{\sigma}\}$- set of matrices $\{T_a\}$, define the subsystems $\{H, \underline{\sigma}\} = \{H_a : a = (i_1, i_2, ..., i_\rho)\}$, having as inputs those corresponding to the set $a = (i_1, i_2, ..., i_\rho)$ of indices defined before. This procedure leads to a set of systems $\langle \zeta, \underline{\sigma} \rangle = \{H_a(s), \underline{\sigma}\}$ for which D_a, or CB_a is a matrix with orthogonality order at least $\underline{\sigma}$.

□

The above procedure produces submodels, which are always non-degenerate and are input, output regular. However, it may lead to systems with unnecessarily small numbers of inputs (outputs), if rank of D, CB are small. The second approach aims at avoiding such problems.

6.6.2. INDIRECT METHOD FOR WELL-CONDITIONING

The second approach is based on the selection and parameterisation of all subsets of inputs and outputs for which input and output regularity is guaranteed and then testing for non-degeneracy using the tests derived before. Once we rely on the selection of natural bases for selecting the suitable input, output sets of variables. For the progenitor model S(A,B,C,D) we denote by:

$$F=\begin{bmatrix}B\\D\end{bmatrix}=[\underline{f}_1,\underline{f}_2,\ldots,\underline{f}_r]\in\Re^{(q+n)\times r},\ G=[C,D]=\begin{bmatrix}\underline{g}_1^{\,t}\\\vdots\\\underline{g}_q^{\,t}\end{bmatrix}\in\Re^{q\times(n+r)} \tag{6.55}$$

and let rank (F) = $\tau_r \le r$, rank (G) = $\tau_\ell \le q$ and rank $_{\Re(s)}\{H(s)\}=\rho$. Without loss of generality we may also assume that the columns of F and the rows of G are normalised.

Definition (6.5): For the matrices F, G we shall denote by:

$$\begin{aligned}\{F\}&\stackrel{\Delta}{=}\left\{F_\beta : F_\beta=[\underline{f}_{j_1},\ldots,\underline{f}_{j_{\tau_r}}]\in\Re^{(q+n)\times\tau_r},\ \beta=(j_1,\ldots,j_{\tau_r})\right\}\\ \{G\}&\stackrel{\Delta}{=}\left\{G_\gamma : G_\gamma=\left[\underline{g}_{l_1},\ldots,\underline{g}_{l_{\tau_l}}\right]^t\in\Re^{\tau_l\times(n+r)},\ \gamma=(l_1,\ldots,l_{\tau_\ell})\right\}\end{aligned} \tag{6.56}$$

the set of all submatrices of F, G which correspond to the natural bases of F, G respectively. The subsets of {F}, {G}, which have a degree of orthogonality greater or equal to some value $\{\underline{\sigma}\}$, will be denoted by $\{F\}_{\underline{\sigma}}, \{G\}_{\underline{\sigma}}$ correspondingly. The set of sequences defined by:

$$\Omega_F\stackrel{\Delta}{=}\left\{\forall\beta:\beta=(j_1,\ldots,j_{\tau_r})\right\},\quad \Omega_G\stackrel{\Delta}{=}\{\forall\gamma:\gamma=(l_1,\ldots,l_{\tau_\ell})\} \tag{6.57}$$

characterising the natural bases of F, G will be referred to as the characteristics of F, G respectively. For every $\beta\in\Omega_F$ and $\gamma\in\Omega_G$ we shall denote by $S_{\beta,\gamma}(A,B,C,D)$ the subsystem of S(A,B,C,D) corresponding to the β set of inputs and γ set of outputs.

□

Remark (6.11): For proper systems S(A,B,C,D), D ≠ 0, the subsystem $S_{\beta,\gamma}(A,B,C,D)$ that corresponds to some $\beta\in\Omega_F$ and $\gamma\in\Omega_G$ is not necessarily input and output regular. This

implies that the process of selecting sets $\beta \in \Omega_F$ and $\gamma \in \Omega_G$ to guarantee input and output regularity are not always independent. In fact, although we can always make the system input regular with cardinality τ_r, or output regular with cardinality τ_ℓ, achieving both may not be possible.

□

The above indicates that progenitor models may be classified as shown below:

Definition (6.6): Given a system S(A,B,C,D) we say that:

(i) It is input-output independent, if any selection of the maximal τ_r number of independent inputs does not affect the selection of the maximal number τ_ℓ of independent outputs and vice versa; otherwise, it is called input-output dependent.

(ii) It is called input-output regularisable, if for at least a $\beta \in \Omega_F$ there is a $\gamma \in \Omega_G$ such that the subsystem $S_{\beta,\gamma}(A,B,C,D)$ is input, output regular; otherwise, it is called input-output non-regularisable.

□

The above notions are important in the construction of well conditioned systems are examined below:

Proposition (6.8): The system S(A,B,C,D) is input-output independent if the following conditions holds true:

$$\text{rank}[C,D] = \text{rank}[C] \tag{6.58}$$

$$\text{rank}[B^t, D^t] = \text{rank}[B^t] \tag{6.59}$$

Proof: If rank [C, D] = rank [C], then any selection $\beta \in \Omega_F$ produces some D_β submatrix and rank [C, D_β] = rank [C, D]. Thus, any choice of $\gamma \in \Omega_G$ based on the properties of rank of C leads to system $S_{\beta,\gamma}$ that is input, output regular.

□

Remark (6.11): A strictly proper system S(A,B,C,D) is always an input-output independent system. Furthermore, any input-output independent system is always input-output regularisable.

□

An input-output dependent system, may, or may not be input-output regularisable. The selection of the maximal number of inputs, outputs in order to guarantee input and output regularity is more complicated and requires a searching method that will be described below. We first note:

Remark (6.12): For any progenitor model S(A,B,C,D) the maximal number of inputs and outputs required to guarantee input and output regularity is τ_r, τ_l respectively. These values can always be achieved for input-output independent systems, but not necessarily for the case of input-output dependent, where they act as upper bounds.

□

The problem of determining the maximal values of cardinality of inputs, outputs, as well as the parameterisation of the corresponding family of systems is considered below in an algorithmic manner. The overall family of such systems will be denoted by $\langle f \rangle$ and every subfamily, with (r', q') input, output cardinality (which is input-output regular) will be denoted by $\langle f \rangle_{r',q'}$. $\langle f \rangle$ will be referred to as the input-output regular family and can always be partitioned as a union of subsets with different indices (r', q').

Searching Algorithm for determining the input-output regular family <f>

Consider the progenitor model S(A,B,C,D) and let $\tau_r = \text{rank}[B^t, D^t] = \tilde{r}, \tau_l = \text{rank}[C, D] = \tilde{q}$ and assume for the sake of simplicity of presentation that $\tilde{r} < \tilde{q}$. Defining $\langle f \rangle$ and the corresponding indices (r', q') involves the following:

CASE (I): Input – Output Independent Systems

For this case the maximal cardinality is $(\tilde{r}, \tilde{q})$ and the family of $\langle f \rangle_{\tilde{r},\tilde{q}}$ systems is constructed as:

Maximal Cardinality Family: Consider the sets of indices $\Omega_F = \{\beta = (j_1, \ldots, j_{\tilde{r}})\}$, $\Omega_G = \{\gamma = (l_1, \ldots, l_{\tilde{q}})\}$. If B_β, C_γ, $D_{\beta,\gamma}$ denote the submatrices corresponding to these indices

then for $\forall \beta \in \Omega_F$ and $\forall \gamma \in \Omega_G$ the subsystem $S(A, B_\beta, C_\gamma, D_{\beta,\gamma})$ is a maximal cardinality $(\tilde{r}, \tilde{q})$ input-output regular subsystem.

CASE (II): Input – Output Dependent Systems

For this case the search involves a number of steps:

STEP (1): For all $\beta \in \Omega_F$ define the submatrices D_β corresponding to the set β of columns, $q_\beta = \text{rank } [C, D_\beta]$, and let:

$$q_1 = \max \{q_\beta, \forall \beta \in \Omega_F\} \tag{6.60}$$

(a) $q_1 = \tilde{q}$: Then the search stops and the maximal number of inputs, outputs that guarantee regularity is $(\tilde{r}, \tilde{q})$ and the system is input-output regularisable. For this case the parameterisation of the family is done as follows:

Maximal Cardinality Family: Let Ω'_F be the subset of sequences of Ω_F for which $q_\beta = \tilde{q}$. For every such $\beta \in \Omega'_F$ we shall denote by $\{\gamma(\beta)\}$ all sequences in Ω_G, which correspond to natural bases of G row space. Thus, we define the set of sequences $\Omega_{F,G} \stackrel{\Delta}{=} \{(\beta, \gamma) \forall \beta \in \Omega'_F \text{ and } \gamma \in \gamma\{\beta\}\}$ and for all $(\beta, \gamma) \in \Omega_{F,G}$ the maximal cardinality $(\tilde{r}, \tilde{q})$ regular family is defined by $S(A, B_\beta, C_\gamma, D_{\beta,\gamma})$.

(b) $q_1 < \tilde{q}$: Then the system is not input, output regularisable and $(\tilde{r}, q_1)$ is a maximal number of inputs solution. The corresponding family of solutions with $(\tilde{r}, q_1 < \tilde{q})$ cardinality is constructed as before.

If a reduced input cardinality and increased output cardinality is desirable, then we proceed to the following step.

STEP (2): For the matrix F, define all sets of $\tilde{r} - 1$ independent vectors of the columns of F (lexicographically ordered), denote this set by $\{F\}_1$ and let the corresponding indices be:

$$\Omega_F^1 \stackrel{\Delta}{=} \{\beta^1 : \beta^1 = (j_1, \ldots, j_{\tilde{r}-1})\} \tag{6.61}$$

For the set Ω_F^1 repeat STEP (1) and this leads to a new solution pair $(\tilde{r}-1, q_2)$ where $q_2 \geq q_1$. The construction of the corresponding family of subsystems follows along the lines described in STEP (1).

The above algorithmic procedure defines the maximal cardinality for input, output regularity, as well as producing a parameterisation of $\langle f \rangle_{\tilde{r},\tilde{q}}$ family, as well as families with orders less than $(\tilde{r}, \tilde{q})$. We can now proceed to the description of the overall methodology for well-conditioning using the Indirect Method.

Indirect Method for Well-conditioning

For the system S(A, B, C, D) we define the maximal cardinality pair $(\tilde{r}, \tilde{q})$ for which input, output regularity is guaranteed and let $\langle f \rangle_{\tilde{r},\tilde{q}}$ be the corresponding family of input, output regular models parameterised by pairs of sequences $(\beta, \gamma) \in \Omega_{F,G}$ with $\beta = (j_1, ..., j_{\tilde{r}}), \gamma = (l_1, ..., l_{\tilde{q}})$. The general element of this family is denoted by $S_{\beta,\gamma} \stackrel{\Delta}{=} S(A, B_\beta, C_\gamma, D_{\beta,\gamma})$. For each $S_{\beta,\gamma}$ we proceed with testing as follows:

STEP (1): If $D_{\beta,\gamma} \neq 0$ and rank $(D_{\beta,\gamma}) = \min(\tilde{r}, \tilde{q})$ or $\tilde{D}_{\beta,\gamma} = 0$ and rank $(C_\gamma B_\beta) = \min(\tilde{r}, \tilde{q})$, then system is degenerate and the search stops.

STEP (2): If $D_{\beta,\gamma} \neq 0$ and rank $(D_{\beta,\gamma}) < \min(\tilde{r}, \tilde{q})$, or $\tilde{D}_{\beta,\gamma} = 0$ and rank $(C_\gamma B_\beta) < \min(\tilde{r}, \tilde{q})$, then test for full rank of the Toeplitz matrix, or respectively Toeplitz matrix $\tilde{M}_{\tau'}$. If M_τ, $\tilde{M}_{\tau'}$ are full rank, then the system is nondegenerate and the search stops. Otherwise, the system is degenerate and we proceed to the testing of another $S_{\beta,\gamma}$ subsystem.

STEP (3): If all elements of $\langle f \rangle_{\tilde{r},\tilde{q}}$ have been tested for degeneracy and there is no element, which is nondegenerate, repeat the analysis of Steps (1), (2) for the smaller order family $\langle f \rangle_{\tilde{r}-1,\tilde{q}}$ etc. The overall procedure always leads to a nondegenerate system.

□

The system of $(\tilde{r}, \tilde{q})$-maximal cardinality subsystems, which are input-output regular and nondegenerate, will be denoted by $\langle f \rangle^{o}_{\tilde{r},\tilde{q}}$ and $\Psi_{F,G}$ will denote the corresponding pairs of (β, γ) sequences.

6.6.3. SELECTION OF NATURAL BASES

The analysis presented so far is based on selection of all possible natural bases and frequently that subset that satisfies certain orthogonality conditions. The construction of such bases is considered here. Let $T = [\underline{t}_1, \underline{t}_2, ..., \underline{t}_r] \in \Re^{q \times r}$, $q \geq r$ with rank (T) = ρ, $\rho <$ min (q, r). The set of all natural bases from the set $\{\underline{t}_1, \underline{t}_2, ..., \underline{t}_r\}$ may be constructed as follows:

Construction of Natural Bases

Let $C_\rho(T) \in \Re^{\binom{q}{q} \times \binom{r}{\rho}}$ denote the ρ – th compound matrix of T [Marcus & Minc, 1] and let $\omega = [i_1, i_2, ..., i_\rho] \in Q_{\rho,r}$ be the sequences characterising the columns of $C_\rho(T)$, i.e.

$$C_\rho(T) = [..., \underline{t}_\omega \wedge, ...], \quad \underline{t}_\omega \wedge = \underline{t}_{i_1} \wedge ... \wedge \underline{t}_{i_\rho} \tag{6.62}$$

where $\underline{t}_\omega \wedge = \underline{t}_{i_1} \wedge ... \wedge \underline{t}_{i_\rho}$ denotes exterior product of the corresponding vectors. If $\Psi_{\rho,r}$ denotes the subset of $Q_{\rho,r}$ that corresponds to nonzero vectors $\underline{t}_\omega \wedge$, then any set $\{\underline{t}_{i_1}, ..., \underline{t}_{i_\rho} : \underline{t}_\omega \wedge \neq 0\}$ is a natural basis. This produces a parameterisation of all such bases in terms of the sequences of $\Psi_{\rho,r}$.

Selection of Natural Bases with Given Orthogonality

The set $\Psi_{\rho,r}$ of sequences of $Q_{\rho,r}$ parameterises all proper bases. However, different bases may have different orthogonality properties. Without loss of generality we may assume that the columns of T are normalised, i.e. $\|\underline{t}_i\| = 1, \forall i \in \underline{r}$. If we use the value of the Grammian as the measure of orthogonality, a classification of the natural bases may be achieved using the following result:

Proposition (6.8): Let $T = [\underline{t}_1, \underline{t}_2, ..., \underline{t}_r] \in \Re^{q \times r}$, $\|\underline{t}_i\| = 1, \forall i \in \underline{r}$, ρ= rank (T) $\leq$ min(r,q) and let:

$$G = G\left(\underline{t}_1,...,\underline{t}_r\right) = T^t\, T \in \Re^{r\times r} \tag{6.63}$$

be the Gram matrix of the vectors $\{\underline{t}_1, \underline{t}_2,..., \underline{t}_r\}$ and let

$$C_\rho\,(G) = C_\rho\,(T^t\, T) \in \Re^{\binom{r}{\rho}\times\binom{r}{\rho}} = \left[c_{ij}\right] \tag{6.64}$$

be the ρ – compound of G. The diagonal elements c_{ii} correspond to all sequences $\omega = \left(i_1,...,i_\rho\right) \in Q_{\rho,r}$ and represent $\left\|\underline{t}_\omega \wedge\right\|$. In particular:

a) $c_{ii} = 0$, if $\underline{t}_\omega \wedge = 0$, i.e. $\{\underline{t}_{i_1},...,\underline{t}_{i_\rho}\}$ dependent.

b) $c_{ii} > 0$, if $\underline{t}_\omega \wedge \neq 0$, i.e. $\{\underline{t}_{i_1},...,\underline{t}_{i_\rho}\}$ is a natural basis.

c) The element with the maximal value c^* corresponds to a sequence $\omega = \left(i_1,...,i_\rho\right) \in \Psi_{\rho,r}$ which characterises the most orthogonal natural basis of T.

□

The above result readily follows from the definition of the Grammian and the interpretation of the Binet – Cauchy Theorem. Clearly by inspection of all the $\binom{r}{\rho}$ diagonal elements of $C_\rho\,(G)$ we can order all natural bases according to degree of orthogonality.

6.7. CONCLUSIONS

The problem of selecting subsystems of a progenitor model S(A,B,C,D), or H(s), which have maximal input and output cardinality, are input-output regular and are nondegenerate has been considered in detail. We have given criteria for the presence of input, output redundancy and system degeneracy, and developed procedures for how we can avoid such properties. The results lead to parameterisation of all subsystems, which are input-output regular and nondegenerate and have maximal cardinality $(\tilde{r}, \tilde{q})$, and leads to the family $\langle f\rangle^{\circ}_{\tilde{r},\tilde{q}}$. Every system in $\langle f\rangle^{\circ}_{\tilde{r},\tilde{q}}$ has $\tilde{r}$-inputs and $\tilde{q}$-outputs and it is parameterised by a set of sequences $(\beta, \gamma) \in \Psi_{F,G}$ defining the subsets of inputs and outputs that have to be considered. Every element $S(A,B_\beta,C_\gamma,D_{\beta,\gamma}) \in \langle f\rangle^{\circ}_{\tilde{r},\tilde{q}}$ does not necessarily have a structure that is desirable, as far as other properties are concerned. In fact, $S_{\beta,\gamma}$ may

be either uncontrollable, and/or unobservable and other properties may not hold true. This family $\langle f \rangle^{\circ}_{\tilde{r},\tilde{q}}$ may then be used as the starting point for additional investigations and conditions based on properties of Hankel matrices are given which also guarantee controllability and observability for the resulting system. An additional advantage of the current procedure is that the sufficient conditions for avoiding strong degeneracy also lead to systems which have no infinite zeros and thus to models with a simple structure. Searching for conditions, which lead to systems with minimum phase characteristics, as well as making the search for minimal subsystems more systematic are problems for future research.

Chapter 7

THE FORMATION OF COMPOSITE SYSTEMS AND THE ROLE OF INPUTS AND OUTPUTS

Chapter 7

THE FORMATION OF COMPOSITE SYSTEMS AND THE ROLE OF INPUTS AND OUTPUTS

7.1. INTRODUCTION

The area of Composite Systems is one of the important aspects of Large Scale Systems Theory. The composite nature of the problem implies that the system is formed as a synthesis of subprocesses according to some interconnection rule, frequently referred to as process flowsheet (layout) and which acts as a progenitor of the underlined graph (derived when we specify the subprocess models). It is the composite nature that makes the overall process model, not an amorphous input-output model, but one reflecting the nature of subprocesses and the process flowsheet. The latter provides the first of the two aspects of the model system structure, the second being those referred to as structural invariants. The study of relationships between subprocesses and overall processes on the level of models and system properties has been an area of interest for many application areas. The way subprocesses enter the composite structure, in terms of interconnecting local variables (subsystem connecting inputs, outputs and effective control inputs and measured outputs) affects drastically the overall properties of the composite system. Most of the previous work [Saeks & DeC., 1], [Vid., 1] deal with the study of properties of composite systems without seeing the interconnection scheme and the selection of local input, output structure as design parameters. A first attempt to link model composition to feedback was made in [Call. & Des., 1] and subsequently developed in [Karc., 9]. Problems connected to the local input, output structure selection are referred to as Model Composition Problems (MCP). A general scheme that addresses interconnection rules and local selection of inputs, outputs

simultaneously and which treats both issues as an equivalent control problem will be presented later in this thesis. The overall emphasis is to address issues of subsystem input, output selection, which are linked to the given interconnection rule.

In this chapter, we will extend recent results on the feedback interpretation of the creation of composite systems [Karc., 10] by developing the notion of completeness of the representation and developing further tests, which may characterise issues of deviation from completeness. The role of selection of subsets of inputs and outputs in structures, which are partially fixed, is considered here by examining some boundary cases corresponding to the total loss of variables at the subsystem level. The general case is shown to be equivalent to a generalised input, output squaring down [Karc. & Gian., 2] for which the exterior algebra framework may be deployed.

7.2. PROBLEM, DEFINITIONS AND BACKGROUND

Composite systems are synthesised by connecting subprocesses (subsystems) according to rules, which are expressing an interconnection topology and, frequently, these rules are expressed by graphs. In recent years, [Cal. & Des., 1], [Karc., 9], [Will., 1], there has been some re-emergence of interest in understanding the evolution of properties of the composite systems. Our approach, here, follows the framework and objectives set up in [Karc., 9] and [Karc., 10] and aims at developing an understanding of the evolution of system properties as a function of:

(i) Properties of the subprocesses.

(ii) Properties of the interconnection graph.

(iii) Local selection of inputs and outputs (subsystem level).

It is a further objective to develop a representation that will allow the understanding of evolution of system properties in the following additional cases:

(a) Evolution from early to late stages.

(b) Variable complexity of modelling of physical streams.

The above two problems are linked to the study of operability in Early – Late Design of chemical processes, where there is interest in evaluating system properties on the

basis of a given stage model and predict such properties on following stage models, when the topology remains the same, but more detailed subprocess models are used. The work, here, is based entirely on linear models of the state-space, transfer function, or MFD type of representation.

We have seen in Chapter 5 that the implicit description of a system (state–space matrix pencil or polynomial – autoregressive) is natural, as far as describing the process itself, with no reference to its environment and makes no classification of implicit variables into inputs, outputs and internal variables. However, when interactions with a certain environment are considered, then an orientation is introduced and it is primarily defined by the nature and properties of the environment. Such an orientation will be referred to as a natural orientation and its main characteristics are introduced in the following diagram:

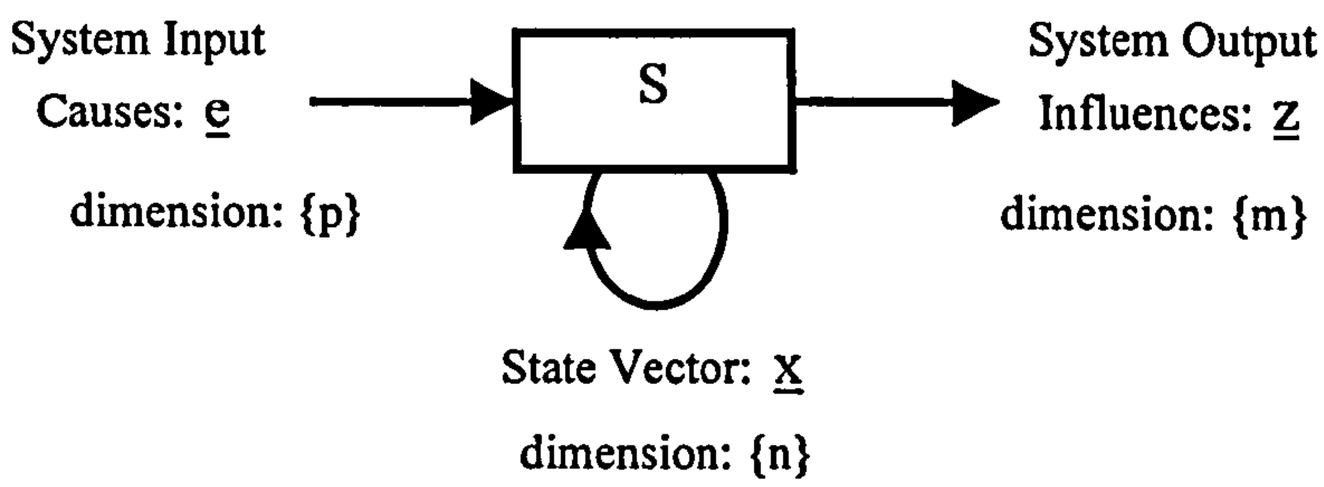

Figure (7.1a): System and its input-output structure

The above figure implies, a certain causality, in the sense that certain internal variables (represented by rates of internal variables) are affected by external causes and internal variables themselves influence events and behaviour outside the system variables and, thus, become causes for other events external to the system. This leads to the following extended definition of system inputs and outputs, which will be used in the subsequent developments.

The vector $\underline{e}$ expresses all variables or combinations of internal variables, which are, influenced by external to the system causes and it will be called input causes vector. The vector $\underline{z}$ expresses all internal variables or combinations of them, which affect or act as causes for events outside the system and it will be called output influences vector.

Note that $\underline{e}$ is assumed to be p-dimensional and includes variables, which are independently assigned (usual inputs) and variables with specific behaviour determined by external influences due to interactions with other systems (other system influences), or of the noise, disturbance type. We may express $\underline{e}$ as:

$$\underline{e} = \omega(\underline{u}, \underline{d}, \underline{f}) \tag{7.1a}$$

where $\underline{u}$ denotes the vector of independently assignable inputs (normal control inputs), $\underline{d}$ is the vector of possible disturbances and $\underline{f}$ is a vector expressing influences from subsystems. For the linear case, we may express (7.1a) as:

$$\underline{e} = L\underline{u} + T\underline{d} + \underline{f} = \underline{v} + \underline{f} \tag{7.1b}$$

where $L \in R^{p \times r}, T \in R^{p \times \sigma}$ and $\underline{f} \in R^{p}$. In the following, we shall restrict ourselves to the case, where there are no disturbances (i.e. $\underline{d} = 0$) and that $\underline{f}$ is generated by different subsystem influences. Similarly, if $\underline{z}$ is the m-dimensional vector of system output influences, then the vectors of measurements $\underline{w}$ and command variables $\underline{y}$ (to be controlled), may be expressed as:

$$\underline{y} = \rho(z) \in R^{q}, \ \underline{w} = \tau(\underline{z}) \in R^{q}, \ \ q, \nu \leq m \tag{7.2a}$$

and for the linear, non-dynamic case, we may express them, as:

$$\underline{y} = K\underline{z}, \ K \in R^{q \times m}, \ \underline{w} = P\underline{z}, \ P \in R^{\nu \times m} \tag{7.2b}$$

The above discussion suggests that the system may be viewed in a more explicit form as:

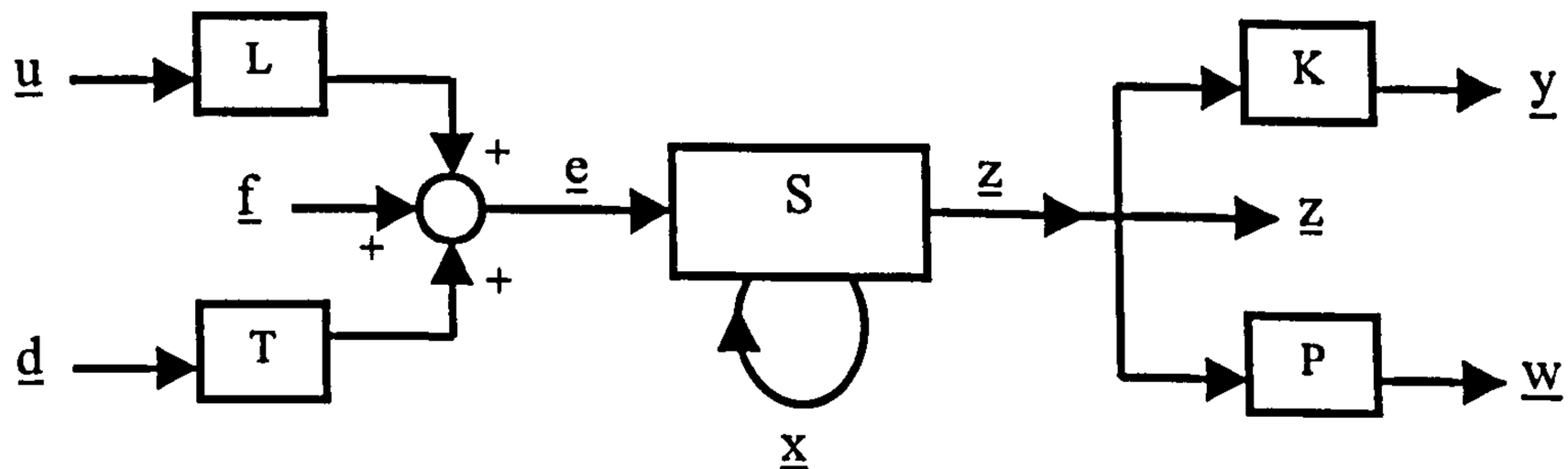

Figure (7.1b): System and its detailed input-output structure

The system defined by the triple $(\underline{e}, \underline{x}, \underline{z})$ is referred to as a progenitor system and will be assumed to be of the regular state-space type and will be denoted by:

$$S(A,B,C,D):\begin{matrix}\dot{\underline{x}} = A\underline{x} + B\underline{e} \\ \underline{z} = C\underline{x} + D\underline{e}\end{matrix} \tag{7.3}$$

Its transfer function matrix $G(s) \in R^{m \times p}(s)$, will also be referred to as progenitor transfer function.

Remark (7.1): The progenitor system S(A,B,C,D) is defined on the basis of general input causes and system output influences and it is not the system that can be used necessarily for control design. It is the physical nature of the process that is manifested by the system, with its natural flows, that defines the properties and structure of the associated models. The definition of such a system implies some ability, knowledge of how the system may be embedded in its environment.

□

Given that the embedding of a system in its environment is defined by physical, operational considerations, the resulting system S(A,B,C,D) or its transfer function may not be well conditioned. In fact, S may be singular, uncontrollable, unobservable, degenerate etc. In the following, we shall assume that the resulting system is of regular state-space type. Clearly, systems and transfer functions from the inputs $\underline{u}$ and command outputs $\underline{y}$, may be defined using the primary model introduced by the progenitor system.

Definition (7.1): Consider a set $\{\Sigma\} = \{S_i(A_i,B_i,C_i,D_i), i = 1,2,...,\mu$ of linear progenitor systems, such that:

$$S_i(A_i,B_i,C_i,D_i):\begin{matrix}\dot{\underline{x}}_i = A_i\underline{x}_i + B_i\underline{e}_i \\ \underline{z}_i = C_i\underline{x}_i + D_i\underline{e}_i\end{matrix} \tag{7.4}$$

with transfer functions $G_i(s) \in R^{m_i \times p_i}(s)$. The system defined by:

$$S_a(A_a,B_a,C_a,D_a):\begin{matrix}\dot{\underline{x}}_a = A_a\underline{x}_a + B_a\underline{e}_a \\ \underline{z}_a = C_a\underline{x}_a + D_a\underline{e}_a\end{matrix} \tag{7.5a}$$

$$\text{where } \underline{x}_a = \left[\underline{x}_1^t,...,\underline{x}_\mu^t\right]^t,\ \underline{e}_a = \left[\underline{e}_1^t,...,\underline{e}_\mu^t\right]^t,\ \underline{z}_a = \left[\underline{z}_1^t,...,\underline{z}_\mu^t\right]^t \tag{7.5b}$$

$$A_a = bl-diag\left\{A_i,\ i\in \underset{\sim}{\mu}\right\},\ B_a = bl-diag\left\{B_i,\ i\in \underset{\sim}{\mu}\right\}$$
$$C_a = bl-diag\left\{C_i,\ i\in \underset{\sim}{\mu}\right\},\ D_a = bl-diag\left\{D_i,\ i\in \underset{\sim}{\mu}\right\} \qquad (7.5c)$$

is called the aggregate of $\{\Sigma\}$ and the transfer function matrix is:

$$G_a(s) = bl-diag\left\{G_i(s),\ i\in \underset{\sim}{\mu}\right\} \qquad (7.5d)$$

Let us denote the space of all values of the aggregate vectors $\underline{e}_a$, $\underline{z}_a$ by $\mathcal{E}_a$, $\mathcal{Z}_a$ respectively and denote by $\underline{\zeta}_a$ the composite vector $\underline{\zeta}_a = \left[\underline{e}_a^{\,t}, \underline{z}_a^{\,t}\right]^t$ and $\tilde{\mathcal{Z}}_a$ the corresponding space. Any function $\mathcal{F}$:

$$\mathcal{F}:\ \tilde{\mathcal{Z}}_a \to \tilde{\mathcal{Z}}_a \qquad (7.6a)$$

defines an action on S_a aggregate system, denoted by *, i.e.

$$S_a * \mathcal{F} = S_c \qquad (7.6b)$$

$\mathcal{F}$ is then called an interconnection rule and S_c is defined as the composite system of S_a induced by $\mathcal{F}$. In the following, we shall consider different composition rules defined on aggregate systems. A large class of important rules are those where the function of composition is defined on a restricted domain and co-domain, i.e.:

$$\bar{\mathcal{F}}:\ \mathcal{Z}_a \to \mathcal{E}_a \qquad (7.7a)$$

and thus the action * on S_a is defined by:

$$S_a * \bar{\mathcal{F}} = S_c \qquad (7.7b)$$

The above composition rule is referred to as feeding interconnection rule and its properties will be considered in some detail here. Before we examine this interesting class of interconnections, we consider some basic interconnection schemes.

7.3. BASIC INTERCONNECTIONS SCHEMES AND TOPOLOGIES

A process is always synthesised by connecting subprocesses (subsystems). The aim, here, is to investigate the links between the structural aspects of the subsystems and the interconnection graph. This problem is of immense importance, especially in the early stages of designing systems by interconnecting subprocesses, since it has important implications on the synthesis of composite structures with desirable control structure characteristics. Some basic assumptions in dealing with composite systems, represented by transfer function matrices or by minimal state-space descriptions are summarised below:

(i) There is no loading effect in any connection of two subsystems; that is, the transfer function of each subsystem remains unchanged after the connection [Chen, 1].

(ii) A system is represented by its transfer function matrix (that is, it is controllable and observable), or more generally, the system is stabilisable and detectable [Wonham, 1]. It will, also, be assumed that the transfer functions are rational and proper.

Note that the assumption that the subsystems are completely characterised by their transfer functions does not imply that the composite systems are completely characterised by their transfer functions. We consider proper systems $S_i(A_i,B_i,C_i,D_i)$, with transfer function matrices $G_i(s) = C_i(sI - A_i)^{-1}B_i + D_i$, $i = 1,2,....$ An interconnected system consisting of a number of subsystems S_i, will be denoted by S_c. The composite system will be called well formed, if all closed-loop transfer functions are well-defined and well posed if all closed-loop transfer functions are well defined and proper [Cal. & Des., 1]. The basic interconnection schemes are shown below:

Cascade Connection: For this scheme $\underline{e}_2 = \underline{z}_1$ and it implies a feeding interconnection rule:

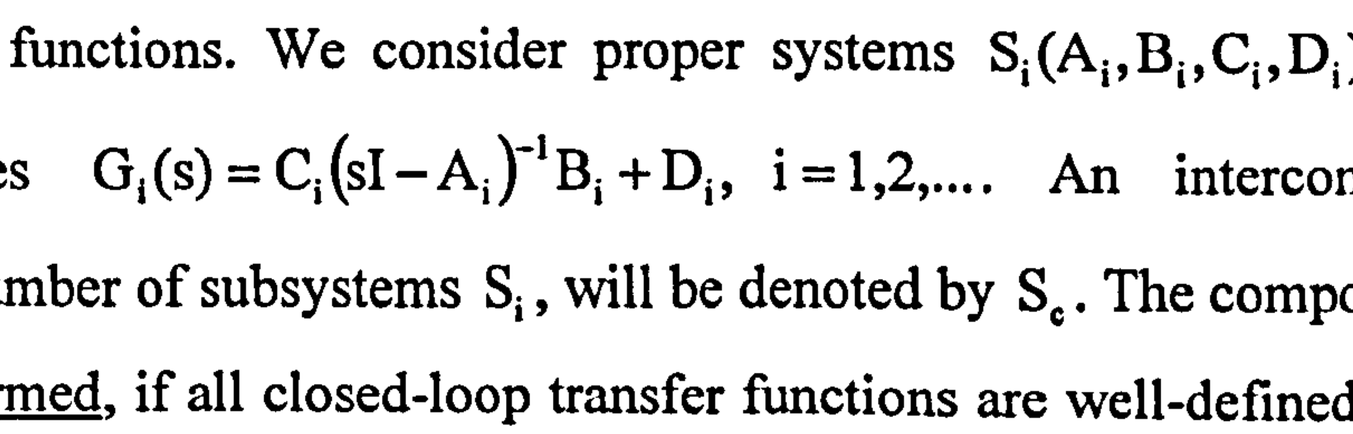

Figure (7.2): Cascade or Tandem Connection

Parallel Connection: For this scheme $\underline{e} = \underline{e}_1 = \underline{e}_2$ and thus it belongs to the general class of interconnection rules and implies dependence on variables.

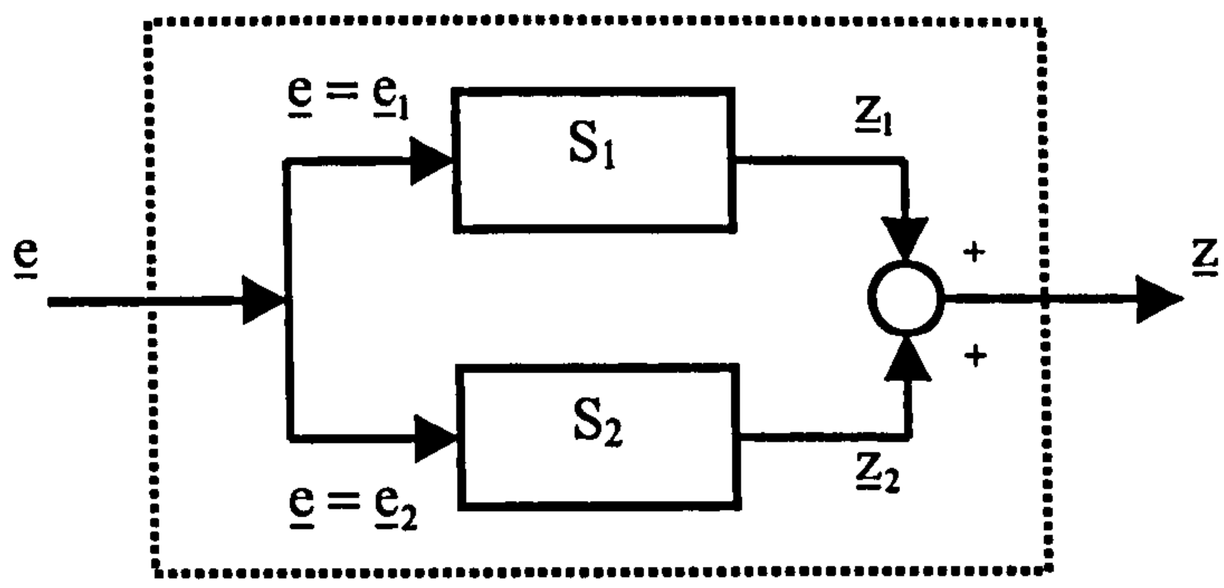

Figure (7.3): Parallel Connection

Feedback Connection: For this scheme $\underline{e}_2 = \underline{z}_1, \underline{u}_1 = \underline{e} - \underline{z}_2$, as it is of the feeding interconnection type. It will be shown that a large class of rules may be characterised in terms of this scheme.

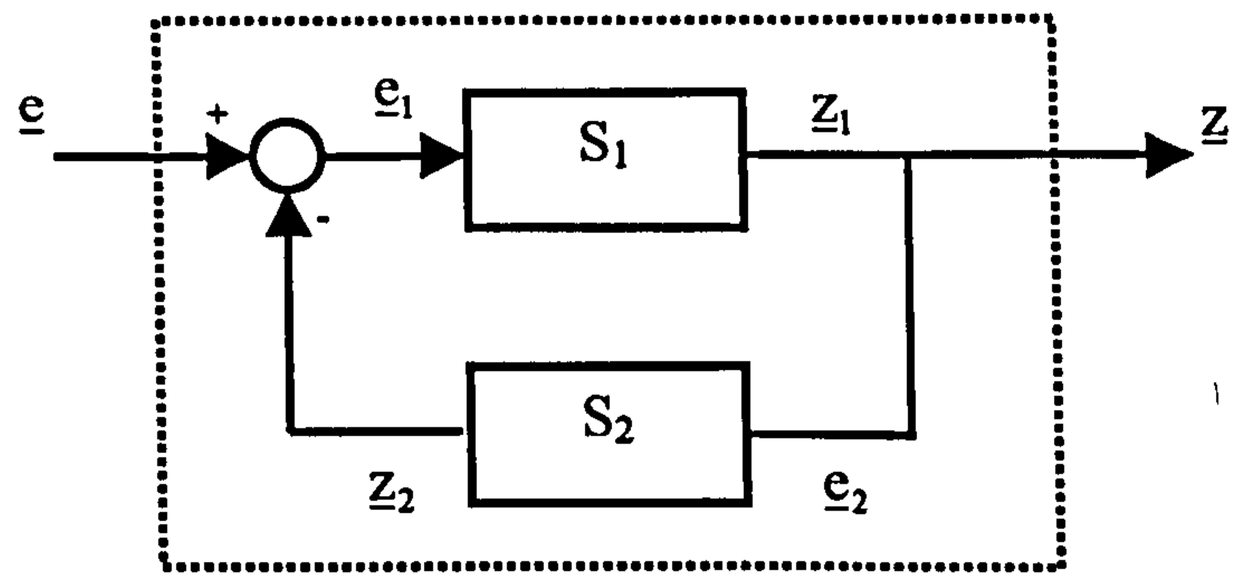

Figure (7.4): Feedback Connection

The composite systems described above are defined by the composite state-space descriptions and whether the composite transfer functions describe these systems depends on the relationships between poles and zeros of the subsystems [Rosen., 2], [Chen, 1] etc. Note that the above connections are well posed under the following conditions:

(a) Cascade connection: Always.

(b) Parallel connection: If $G_1(s) \neq -G_2(s)$.

(c) Feedback connection: If $|I + G_1(\infty)G_2(\infty)| = |I + D_1D_2| \neq 0$.

For two systems S_1, S_2, which are completely characterised by their proper transfer function matrices $G_1(s), G_2(s)$, any composite well posed connection of S_1 and S_2 is completely characterised by its composite transfer matrix $G_{12}(s)$, if and only if [Chen, 1]:

$$\delta_m(G_{12}(s)) = \delta_m(G_1(s)) + \delta_m(G_2(s)) \tag{7.8}$$

For the different types of connections described above, the latter condition for the representation of the composite system by its composite transfer function matrix may become more explicit as conditions for coprimeness of the polynomial matrices defined by the R[s]-irreducible MFDs of $G_1(s)$ and $G_2(s)$ (see [Chen, 1], [Kailath, 1] etc.). For the simple case of single-input, single-output (SISO) systems S_i, which are completely characterised by their proper rational functions $g_i(s), i = 1,2$, we have the following:

(a) The tandem connection of systems S_1 and S_2 is completely characterised by $g_{12}(s) = g_2(s)\, g_1(s)$, if and only if there is no pole-zero cancellation between $g_1(s)$ and $g_2(s)$.

(b) The parallel connection of systems S_1 and S_2 is completely characterised by $g_{12}(s) = g_2(s) + g_1(s)$, if and only if $g_1(s)$ and $g_2(s)$ do not have any pole in common.

(c) The feedback connection of systems S_1 and S_2 is completely characterised by $g_{12}(s) = \left(1 + g_1(s)\, g_2(s)\right)^{-1} g_1(s)$, if and only if there is no pole of $g_2(s)$ cancelled by any zero of $g_1(s)$.

The problem of representation of composite systems by their composite transfer function is always related to controllability and observability of the composite system. The interconnection configurations of Figures (7.2)-(7.4) do not always have these two properties. Controllability and observability of a system always depend on the selection of the inputs and outputs. An enlarged feedback configuration, denoted in Figure (7.5), has always the property of controllability and observability for the composite input vector $\left[\underline{u}_1^t, \underline{u}_2^t\right]^t$ and output vector $\left[\underline{z}_1^t, \underline{z}_2^t\right]^t$ and will be called the complete feedback configuration. Such configuration is used in the discussion of the general control design problem and it is well posed if $\left|I + G_1(\infty) G_2(\infty)\right| \neq 0$. For such a configuration:

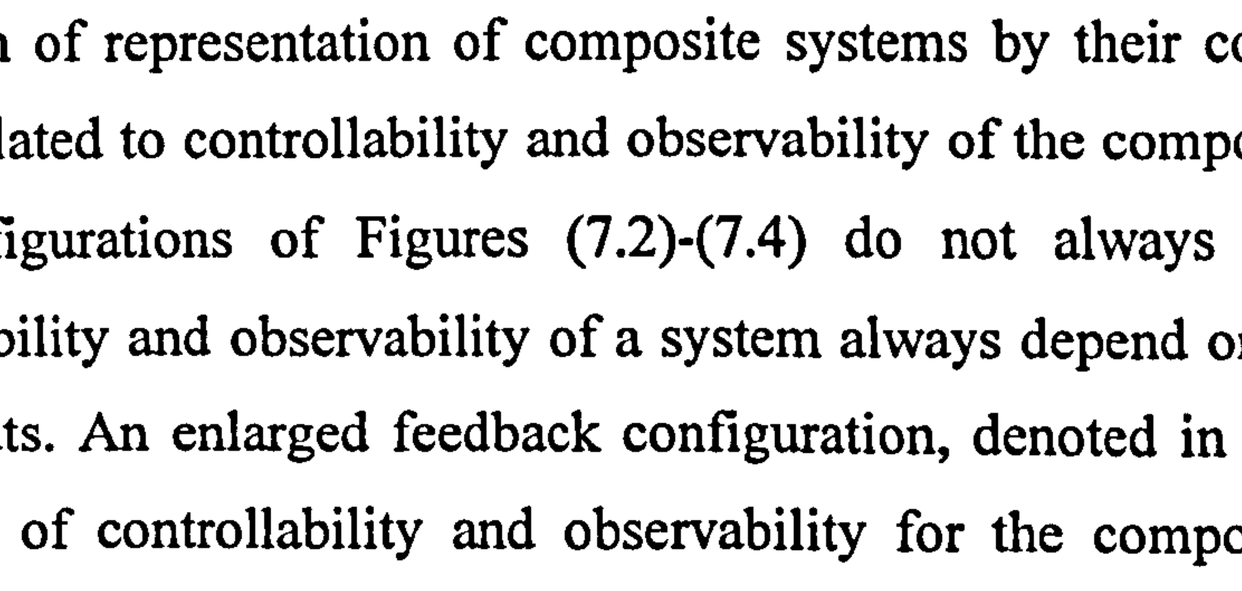

Figure (7.5): Complete Feedback Configuration

$$\begin{bmatrix} \underline{z}_1(s) \\ \underline{z}_2(s) \end{bmatrix} = H(s) \begin{bmatrix} \underline{u}_1(s) \\ \underline{u}_2(s) \end{bmatrix}, \text{ where } H(s) = \begin{bmatrix} I_1 & G_2(s) \\ -G_1(s) & I_1 \end{bmatrix}^{-1} \tag{7.9}$$

and H(s) exists under the well possedness assumption and it is known as error transfer function (other transfer functions may also be defined). If S_{12}^{CF} denotes the composite state-space equations and assume that $G_i(s)$ are complete representations of S_i, then H(s) completely describes the S_{12}^{CF} composite system [Chen, 1], [Vid., 1].

7.4. THE GENERAL CONFIGURATION OF COMPOSITE SYSTEMS

We now consider a general representation for composite systems that has been introduced in [Cal. & Des., 1] and further developed in [Karc., 10]. The aim is to develop this scheme further and explore the role of selection of local inputs, outputs on the overall system properties. The representation, which is described here, is referred to as canonical composite representation (CCR) [Karc., 10] and it is based on a number of general assumptions, which are described next. We consider a family of systems $\{\Sigma\} = \{S_k, k = 1,...,\mu$ with transfer functions for the subsystems $G_k(s), G_k(s) \in R_{pr}^{m_l \times p_l}(s)$. The interconnection rule $\mathcal{F}$, which is considered here, is based on the following assumptions [Karc., 10]:

Local Well Connectedness Assumptions (LWCA): The physical linking of a subsystem S_k to the rest of the subsystems implies that there is a connecting input vector $\underline{e}_k$, having as coordinates all variables connected directly to at least one subsystem output or external variable (manipulated or disturbance) and a connecting output vector $\underline{z}_k$, with coordinates all variables, which feed at least one of the subsystems or measured variables. Considering linear systems, we assume that the transfer functions: $\underline{e}_k \rightarrow \underline{z}_k$ are well defined, that is:

$$\underline{z}_k(s) = G_k(s)\, e_k(s), \; k = 1,2,...,\mu \tag{7.10}$$

and that the subsystems transfer functions $G_k(s)$ are proper. These assumptions are referred to as Local Well Connectedness and $G_k(s)$ is the k-th connecting transfer function.

Furthermore, if $G_k(s)$ represents a minimal system (controllability from $\underline{e}_k$ and observability from $\underline{z}_k$), then the system satisfies the Local Well Formedness Assumption (LWFA). The system represented by the transfer function matrix:

$$G_a(s) = bl - diag\{G_k(s),\ k = 1,2,...\mu\} \tag{7.11}$$

is the system aggregate and it is denoted by $S_a\ (S_k, k \in \underset{\sim}{\mu})$.

Local Well Structured Assumption (LWSA): For every subsystem, we shall denote by $\underline{u}_k$, $\underline{y}_k$ the effective input and output vector, that is the vector of assignable input variables and measured-command variables respectively. We shall assume that $\underline{y}_k$ is a subvector of $\underline{z}_k$, in the sense that

$$\underline{y}_k = K_k \underline{z}_k,\ K_k \in R^{q_k \times m_k},\ q_k \leq m_k \tag{7.12a}$$

and that $\underline{u}_k$ is a reduced vector of $\underline{e}_k$, in the sense that

$$\underline{e}_k = L_k \underline{u}_k + \underline{f}_k = \underline{v}_k + \underline{f}_k,\ L_k \in R^{p_k \times r_k},\ r_k \leq p_k \tag{7.12b}$$

where $\underline{f}_k$ is some vector of dependent variables and the coordinates in $\underline{u}_k$ are independently assignable (control or disturbance) variables. This assumption is referred to as Local Well Structured Assumption.

Global Well Formedness Assumption (GWFA): Consider the aggregate system S_a $(S_k, k \in \mu)$ that satisfies the Local Well Formedness and Local Well Structured Assumptions. The composite system $S_c\ (S_a; F)$ will be called Globally Well Formed, if the interconnection rule $F : \underline{z}_1 \times \underline{z}_1 ... \times \underline{z}_\mu \rightarrow \underline{e}_1 \times ... \times \underline{e}_\mu$ represented by the diagram below:

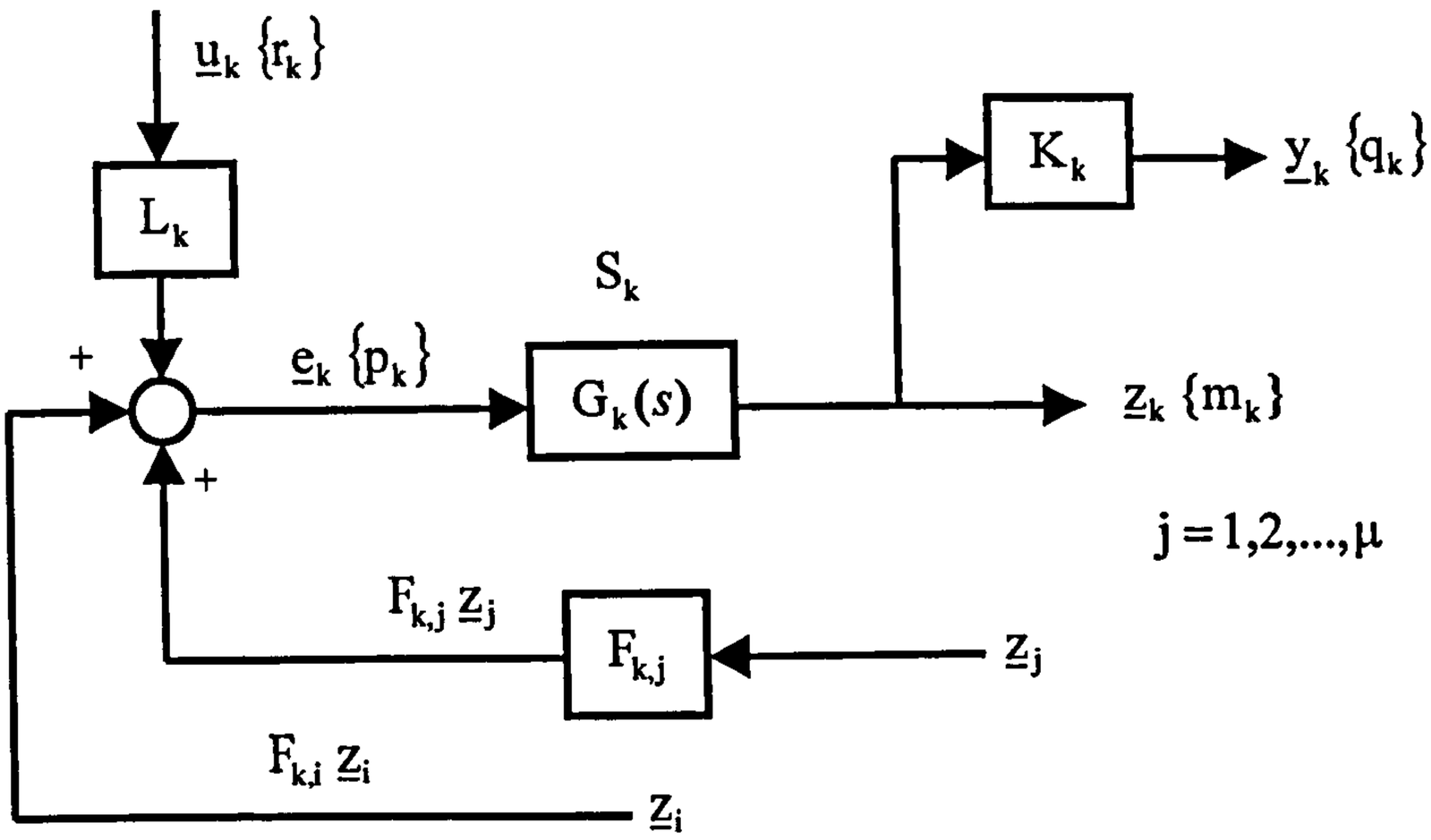

Figure (7.6): Composition Assumptions

satisfies the assumptions:

(i) Its output is the subsystem vector $\underline{z} = \left[\underline{z}_1{}^t,...,\underline{z}_\mu{}^t\right]^t$.

(ii) Its inputs $\underline{e}_k$ are expressed by:

$$\underline{e}_k = \sum_{i=1}^{\mu} F_{k,i}\,\underline{z}_i + \underline{\upsilon}_k \qquad (7.12c)$$

where $F_{k,i}$ are real matrices and $\underline{\upsilon}_k$ are external vectors. We may assume that whatever interactions between the local vectors $\underline{z}_k$ and its inputs are already taken into account and, thus, we may write the condition (7.12c) as:

$$\underline{e}_k = \sum_{i=1,\, i\neq k}^{\mu} F_{k,i}\,\underline{z}_i + \underline{\upsilon}_k \qquad (7.12d)$$

If the above is assumed, then the globally well formed system will be referred to as normal.

(iii) The transfer function from $\underline{\upsilon}_a = \left[\underline{\upsilon}_1{}^t,...,\underline{\upsilon}_\mu{}^t\right]^t \rightarrow \underline{e}_a = \left[\underline{e}_1{}^t,...,\underline{e}_\mu{}^t\right]^t$ is defined and it is proper.

□

If we define $\underline{z}_a = \left[\underline{z}_1{}^t,...,\underline{z}_\mu{}^t\right]^t$, then (7.12c) may be expressed as:

$$\begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \vdots \\ \underline{e}_\mu \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1\mu} \\ F_{21} & F_{22} & \cdots & F_{2\mu} \\ \vdots & \vdots & \ddots & \vdots \\ F_{\mu 1} & F_{\mu 2} & \cdots & F_{\mu\mu} \end{bmatrix} \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \\ \vdots \\ \underline{z}_\mu \end{bmatrix} + \begin{bmatrix} \underline{\upsilon}_1 \\ \underline{\upsilon}_2 \\ \vdots \\ \underline{\upsilon}_\mu \end{bmatrix} \quad \text{or } \underline{e}_a = F\,\underline{z}_a + \underline{\upsilon}_a \tag{7.13a}$$

In the case, where the interconnection is normal, then F has the following structure:

$$F = \begin{bmatrix} 0 & F_{12} & \cdots & \cdots & F_{1\mu} \\ F_{21} & 0 & F_{23} & \cdots & F_{2\mu} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ F_{\mu 1} & F_{\mu 2} & \cdots & \cdots & 0 \end{bmatrix} \tag{7.13b}$$

The matrix F is "structured", in the sense that it has blocks of zeros in certain locations, indicating the topology of the connectivities. F will be called the <u>interconnection matrix</u> of the composite system. Given that

$$\underline{z}_a(s) = G_a(s)\,e_a(s) \text{ and } \underline{e}_a(s) = F\,G_a(s)\,e_a(s) + \underline{\upsilon}_a(s) \tag{7.14a}$$

$$\text{or } \left(I - F\,G_a(s)\right)\underline{e}_a(s) = \underline{\upsilon}_a(s) \tag{7.14b}$$

The third of the assumptions of GWF structure implies that $\left|I - FG_a(s)\right| \neq 0$. Under this assumption, (7.14a) and (7.14b) imply:

$$\underline{e}_a(s) = \left(I - F\,G_a(s)\right)^{-1}\underline{\upsilon}_a(s) \tag{7.15a}$$

$$\text{and } \underline{z}_a(s) = G_a(s)\left(I - F\,G_a(s)\right)^{-1}\underline{\upsilon}_a(s) \tag{7.15b}$$

and, thus, the transfer function of the composite system, from $\underline{\upsilon}_a \rightarrow \underline{z}_a$, is:

$$\hat{H}_c(s) = G_a(s)\left(I - F\,G_a(s)\right)^{-1} \tag{7.16}$$

It is clear that the interconnection topology is represented by the structured matrix F and the composite system is the result of the action of the output feedback F on the aggregate system. Assuming that at each subsystem, we have the presence of a control local vector $\underline{u}_i$ and local measurement $\underline{y}_i$, then

$$\underline{\upsilon}_i = L_i \underline{u}_i, i \in \underset{\sim}{\mu},\ \underline{y}_i = K_i \underline{z}_i, i \in \underset{\sim}{\mu} \tag{7.17a}$$

$$\underline{\upsilon}_a = \left[\underline{\upsilon}_1^t,...,\underline{\upsilon}_\mu^t\right]^t,\ \underline{u}_a = \left[\underline{u}_1^t,...,\underline{u}_\mu^t\right]^t,\ \underline{y}_a = \left[\underline{y}_1^t,...,\underline{y}_\mu^t\right]^t \tag{7.17b}$$

and thus: $\underline{\upsilon}_a = L_a \underline{u}_a,\ L_a = \text{bl}-\text{diag}\{L_1,...,L_\mu\}, \underline{y}_a = K_a \underline{z}_a,\ K_a = \text{bl}-\text{diag}\{K_1,...,K_\mu\}$ (7.17c)

and we have the following representation of the composite structure:

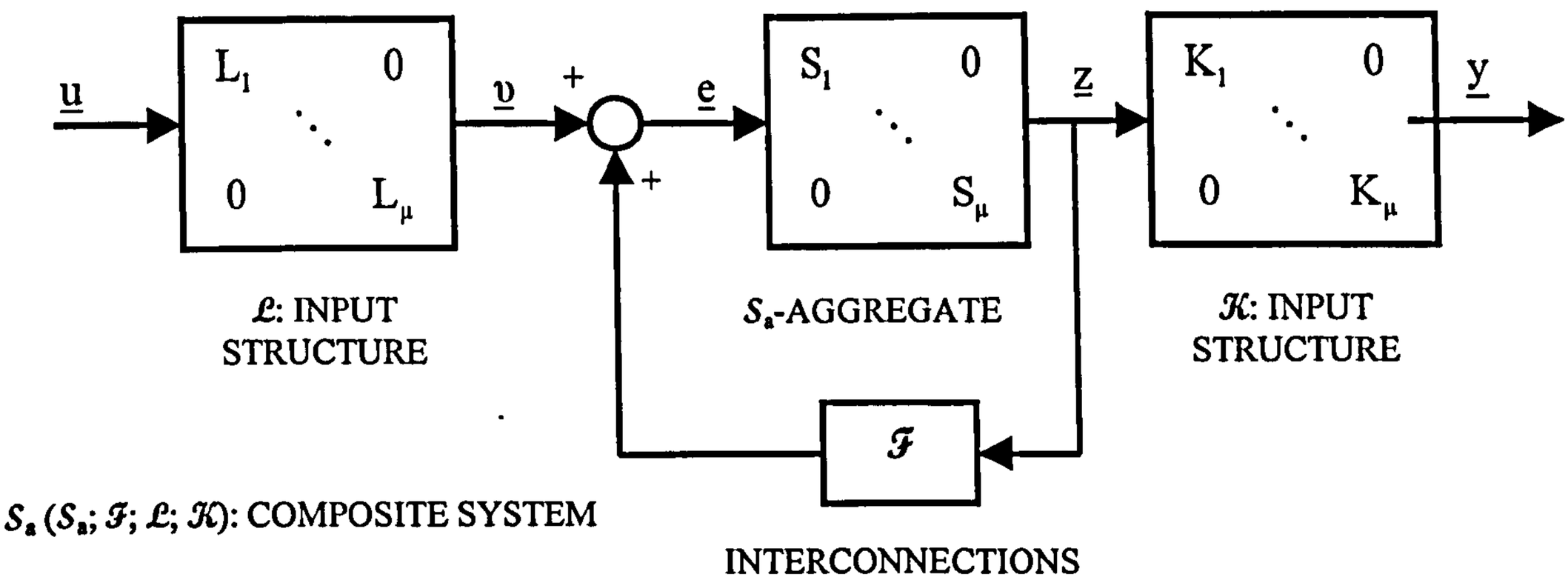

Figure (7.7): Equivalent Feedback Configuration of Composite Systems [Karc., 10]

It is clear that the interconnection graph acts as feedback and the selection of effective inputs, outputs is represented as input, output constant compensators and the composite transfer function from $\underline{u}_a \rightarrow \underline{y}_a$ is given by:

$$H_c(s) = K_a \hat{H}_c(s) L_a = K_a G_a(s)\left(I - F G_a(s)\right)^{-1} L_a \tag{7.18}$$

It is clear, that the above expresses a decentralised Model Projection Problem (MPP) (as defined in Chapter 3), of a special type, with a progenitor transfer function $\hat{H}_c(s)$. An important special case of the above configuration is defined by considering the properties of the vectors $\underline{\upsilon}_k$ in (7.12c), i.e. $\underline{e}_k = \sum_{i=1}^{\mu} F_{k,i} \underline{z}_i + \underline{\upsilon}_k$. Note that the number of independent variables p_k in $\underline{e}_k$ determines the local inputs in the k-th subsystem and this has to do with the physics of the particular problem. The number of independent variables r_k in $\underline{\upsilon}_k$ determines the effective number of local inputs and this leads to the following characterisation.

Completeness Assumption [Karc., 10]: The well formed composite system will be said to be complete, if the following two further conditions hold true:

(i) Every effective subsystem output $\underline{y}_k$, $k=1,2,...,\mu$ satisfies the condition $\underline{y}_k = Q_k \underline{z}_k$, where Q_k is square and invertible, i.e. $Q_k \in R^{m_k \times m_k}$, $|Q_k| \neq 0$.

(ii) Every external subsystem vector $\underline{v}_k$, $k=1,2,...,\mu$ has as many independent coordinates as the dimension of $\underline{e}_k$ input vector, i.e. $\underline{e}_k = R_k \underline{v}_k$ with R_k square and invertible, i.e. $R_k \in R^{p_k \times p_k}$, $|R_k| \neq 0$. Under these conditions the vectors $\underline{v}_k$ and $\underline{y}_k$ will be called full.

□

The completeness assumption implies that to every interconnection input, output there corresponds a same dimension control vector, output vector with the same number of independent variables to those of the interconnecting input, output respectively. As a result of completeness the composite and the aggregate are output feedback and input, output co-ordinate transformation equivalent and, thus, they have the same basic structural characteristics. The composite configuration established here is referred to as Canonical Composite Representation and issues of Global Instrumentation may be discussed within this configuration as system and feedback design problems. Guaranteeing the validity of the above assumptions is both a matter of modelling and selection of input, output schemes. In fact, each assumption is linked to a problem and these problems are considered next.

(i) **Local Well Formedness Problem (LWFP)**: The properties of well connectedness and well formedness at subsystem level are closely linked to the design of individual subprocesses. Given that the way subprocesses are connected is defined by their nature, it seems that deviation from such assumptions may be handled for most cases simply by redesign. There are, however, cases, such as electric networks, where the potential input, output connecting sets are larger than the effective ones. This provides the possibility of addressing issues of design for well formedness based on a combination of MOP and MPP methodologies.

(ii) **Local Well Structuring Problem (LWSP):** The essence of Local Well Structuring is that we have to identify the effective connecting inputs, outputs $\underline{e}_k$, $\underline{z}_k$ respectively (based on the modelling of interconnections), the potential control variables and

outputs $\underline{u}_k$, $\underline{y}_k$ respectively, and we have to guarantee that Equations (7.12a), (7.12b) hold. It may be, frequently, the case that $\underline{y}_k$ is not a subvector of $\underline{z}_k$ and that $\underline{v}_k$ is not related to $\underline{e}_k$. In this case, it is always possible to expand the vectors $\underline{e}_k$, $\underline{z}_k$ to $\underline{e}'_k$, $\underline{z}'_k$ vectors, such that (7.12a), (7.12b) are satisfied. The latter implies rechecking of the LWFA and appropriate modifications of the $G_k(s)$ transfer function. Problems, once more, rely on modelling, as well as on the prediction of requirements, as far as what are the needed inputs, outputs on the final model. When such problems are considered, the input, output vectors $\underline{v}_k$, $\underline{y}_k$ are treated as potential vectors and leave the final design of effective vectors to an MPP.

(iii) **Global Well Formedness Problem (GWFP):** Under the assumption that LWFP, LWSP have been solved, the Global Well Formedness Problem is essentially a modelling problem, since it requires definition of F_{kj} matrices and, thus, F, such that $\{I - FG_a(s)\}$ is non-singular. When there is flexibility in the design of F, the objective may be extended in designing F, such that the progenitor model $\hat{H}_c(s)$ is stable, and that in addition it has a number of other desirable properties. The current framework provides the means for the potential use of feedback control design concepts in the areas of process synthesis.

(iv) **Decentralised Model Projection Problems (DMPP):** The general configuration of Figure (7.7) clearly suggests that the final selection of inputs, outputs is reduced to an MPP, where L and K are block diagonal. This problem is, then, addressed on the progenitor model $\hat{H}_c(s)$ with the different criteria described for MPPs and with the additional requirement that L_a, K_a are block diagonal.

Example (7.1): Consider the composite system shown in Figure (7.8), where the subsystems are represented by the regular state-space models S_i, having as corresponding transfer functions $G_i(s)$, $i = 1,2,3$.

The LWF assumption implies that the transfer functions

$$\underline{z}_i(s) = G_i(s)\, e_i(s), \quad i = 1,2,3 \tag{7.19a}$$

are defined and are proper. The LWF assumption implies that the systems defined as $S_i, i = 1,2,3$ are minimal state-space descriptions and thus they are defined by their $G_i(s)$ transfer functions. The aggregate transfer function is then defined by:

$$\begin{bmatrix} \underline{z}_1(s) \\ \underline{z}_2(s) \\ \underline{z}_3(s) \end{bmatrix} = \begin{bmatrix} G_1(s) & 0 & 0 \\ 0 & G_2(s) & 0 \\ 0 & 0 & G_3(s) \end{bmatrix} \begin{bmatrix} \underline{e}_1(s) \\ \underline{e}_2(s) \\ \underline{e}_3(s) \end{bmatrix} \tag{7.19b}$$

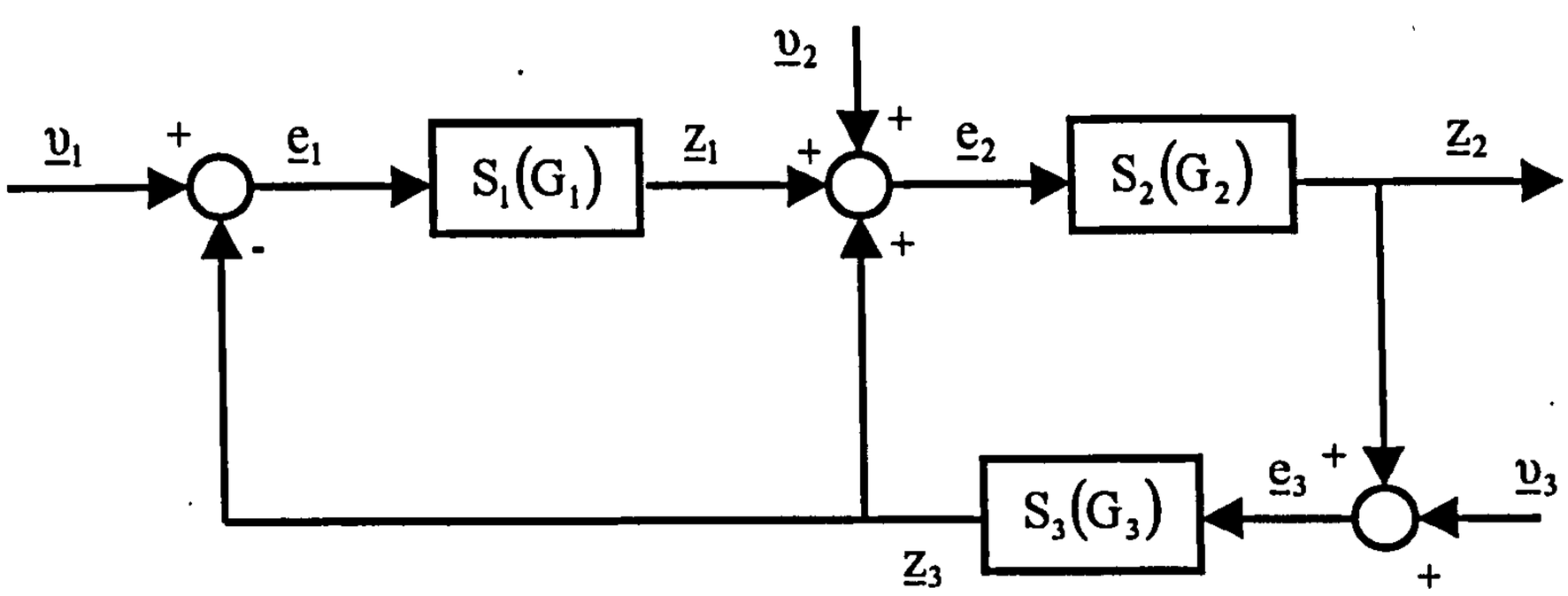

Figure (7.8): Composite System

For the scheme, we have the relationships:

$$\underline{e}_1 = -\underline{z}_3 + \underline{\upsilon}_1, \quad \underline{e}_2 = \underline{z}_1 + \underline{z}_3 + \underline{\upsilon}_2, \quad \underline{e}_3 = \underline{z}_2 + \underline{\upsilon}_3 \tag{7.19c}$$

where $\underline{\upsilon}_1$, $\underline{\upsilon}_2$, $\underline{\upsilon}_3$ are externally generated vectors with dimensions the same as those of $\underline{u}_1$, $\underline{u}_2$, $\underline{u}_3$ respectively. The representation of the interconnection is then defined by:

$$\begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \underline{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -I \\ I & 0 & I \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \\ \underline{z}_3 \end{bmatrix} + \begin{bmatrix} \underline{\upsilon}_1 \\ \underline{\upsilon}_2 \\ \underline{\upsilon}_3 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 & -I \\ I & 0 & I \\ 0 & I & 0 \end{bmatrix} \tag{7.19d}$$

The GWF assumption implies that the matrix

$$I - FG_a(s) = \begin{bmatrix} I & 0 & G_3 \\ -G_1 & I & -G_3 \\ 0 & -G_2 & I \end{bmatrix} \tag{7.19e}$$

is non-singular or R(s), i.e. $|I - FG_a(s)| \neq 0$. Under the latter assumption the composite transfer function exists and

$$\hat{H}_c(s) = G_a(s)\{I - FG_a(s)\}^{-1} \tag{7.19f}$$

The transfer function is proper if $G_i(s), i = 1,2,3$ are proper and $|I - FG(\infty)| \neq 0$. The completeness assumption for this system, means the following:

(i) The measurable outputs are $\underline{y}_1 = \underline{z}_1$, $\underline{y}_2 = \underline{z}_2$, $\underline{y}_3 = \underline{z}_3$, or coordinate transformations of $\underline{y}_1$, $\underline{y}_2$, $\underline{y}_3$ respectively.

(ii) The vectors $\underline{v}_1$, $\underline{v}_2$, $\underline{v}_3$ have independent variables equal to their dimensionality, i.e. all elements in them are independent.

□

7.5. PROPERTIES OF THE GENERAL COMPLETE COMPOSITE SYSTEM

The above representation of composite systems (as a feedback configuration) has important implications for the present work:

(i) It provides a systematic method for representing composite systems (with implications on the transition from process configurations to process transfer functions).

(ii) It allows the formulation of the process synthesis problem (interconnection of subprocesses) as a feedback design problem.

The GWF assumption is equivalent to the existence of a proper rational composite transfer function $\hat{H}_c(s)$. The existence of a rational $\hat{H}_c(s)$ is equivalent to that $|I - FG_a(s)| \neq 0$, whereas the properness is expressed by the following condition [Vid., 1]:

Proposition (7.1): If $G_a(s)$ is strictly proper, then $\hat{H}_c(s)$ is always proper. If $G_a(s)$ is a proper transfer function, then the composite configuration described by $\hat{H}_c(s)$ transfer function is proper if and only if

$$|I - FG_a(\infty)| = |I - G_a(\infty)F| \neq 0 \tag{7.20}$$

A composition rule, which guarantees properness, also guarantees the existence of transfer function $\hat{H}_c(s)$, but not vice versa. Schemes resulting in proper composite transfer functions are referred to as well posed. Under the GWF assumptions, the composite system

is defined and it is equivalent to an output feedback scheme applied on the aggregate system. By assuming completeness, the system may be represented in the simplified form of Figure (7.9), as shown below:

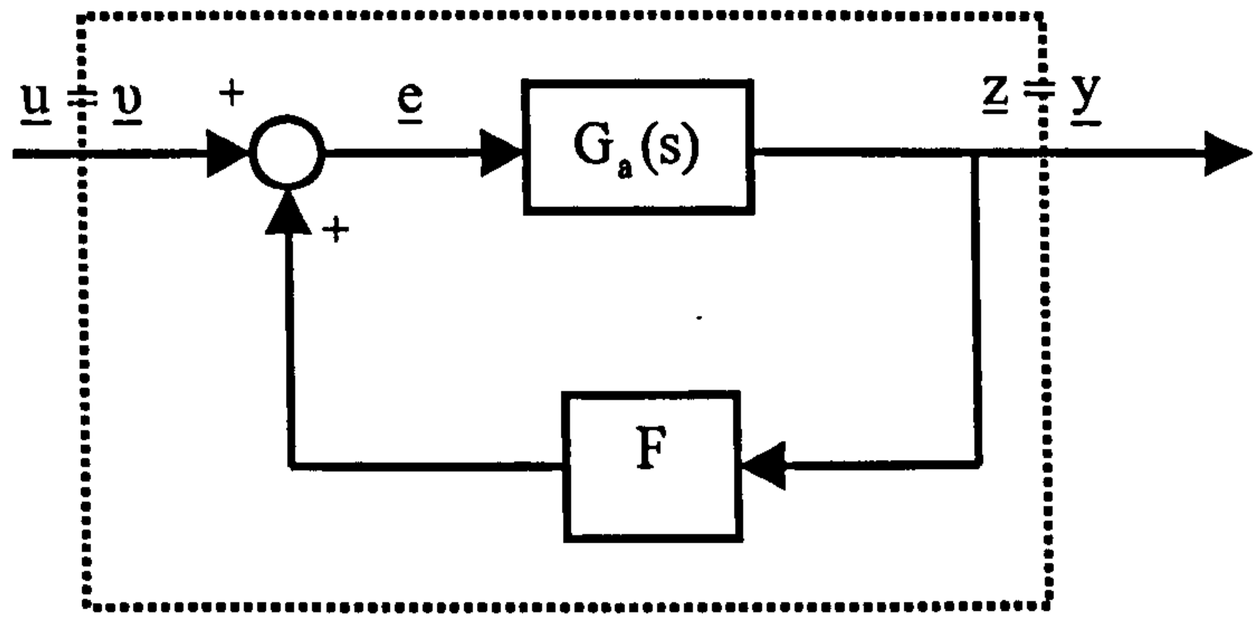

Figure (7.9): Interconnected System S_c under the Completeness Assumption

Lack of completeness implies some decentralised input, output squaring down, as this is described by Equation (7.18) and Figure (7.7). In the following section, firstly, we shall consider properties of the complete representation and then of the non-complete representation. A number of properties relating the aggregate and the complete composite system are considered next.

7.5.1. Well Formedness of Complete Composite Systems

We consider a complete composite system, which is well posed and with subsystems described by state-space equations as:

$$S_i(A_i,B_i,C_i,D_i): \begin{matrix} \dot{\underline{x}}_i = A_i\underline{x}_i + B_i\underline{e}_i \\ \underline{z}_i = C_i\underline{x}_i + D_i\underline{e}_i \end{matrix} \quad i=1,2,...,k \tag{7.21a}$$

where the aggregate vectors are:

$$\underline{e} = \begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \vdots \\ \underline{e}_k \end{bmatrix}, \ \underline{x} = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \vdots \\ \underline{x}_k \end{bmatrix}, \ \underline{z} = \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \\ \vdots \\ \underline{z}_k \end{bmatrix}, \ \underline{v} = \begin{bmatrix} \underline{v}_1 \\ \underline{v}_2 \\ \vdots \\ \underline{v}_k \end{bmatrix} \tag{7.21b}$$

and thus the aggregate system is described by:

$$\underset{=\dot{\underline{x}}}{\begin{bmatrix}\dot{\underline{x}}_1\\ \dot{\underline{x}}_2\\ \vdots\\ \dot{\underline{x}}_k\end{bmatrix}} = \underset{=\bar{A}}{\begin{bmatrix}A_1 & 0 & \cdots & 0\\ 0 & A_2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & 0 & \cdots & A_k\end{bmatrix}} \underset{=\underline{x}}{\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \vdots\\ \underline{x}_k\end{bmatrix}} + \underset{=\bar{B}}{\begin{bmatrix}B_1 & 0 & \cdots & 0\\ 0 & B_2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & 0 & \cdots & B_k\end{bmatrix}} \underset{=\underline{e}}{\begin{bmatrix}\underline{e}_1\\ \underline{e}_2\\ \vdots\\ \underline{e}_k\end{bmatrix}} \tag{7.21c}$$

$$\text{or } \dot{\underline{x}} = \bar{A}\underline{x} + \bar{B}\underline{e} \tag{7.21d}$$

$$\text{where } \underline{e} = \begin{bmatrix}\underline{\upsilon}_1\\ \underline{\upsilon}_2\\ \vdots\\ \underline{\upsilon}_k\end{bmatrix} + F\begin{bmatrix}\underline{z}_1\\ \underline{z}_2\\ \vdots\\ \underline{z}_k\end{bmatrix} \tag{7.21e}$$

where F is a matrix expressing interconnections. Furthermore,

$$\underset{=\underline{z}}{\begin{bmatrix}\underline{z}_1\\ \underline{z}_2\\ \vdots\\ \underline{z}_k\end{bmatrix}} = \underset{=C}{\begin{bmatrix}C_1 & 0 & \cdots & 0\\ 0 & C_2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & 0 & \cdots & C_k\end{bmatrix}} \underset{=\underline{x}}{\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \vdots\\ \underline{x}_k\end{bmatrix}} + \underset{=D}{\begin{bmatrix}D_1 & 0 & \cdots & 0\\ 0 & D_2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & 0 & \cdots & D_k\end{bmatrix}} \underset{=\underline{e}}{\begin{bmatrix}\underline{e}_1\\ \underline{e}_2\\ \vdots\\ \underline{e}_k\end{bmatrix}} \tag{7.21f}$$

$$\text{or } \underline{z} = \bar{C}\underline{x} + \bar{D}\underline{e} \tag{7.22a}$$

Thus, the composite system becomes:

$$\begin{aligned}\dot{\underline{x}} &= \bar{A}\underline{x} + \bar{B}\underline{e}\\ \underline{z} &= \bar{C}\underline{x} + \bar{D}\underline{e}\end{aligned} \tag{7.22b}$$

$$\text{and } \underline{e} = \underline{\upsilon} + F\underline{z} \tag{7.22c}$$

where F is a matrix expressing interconnections. From the above, (i.e. (7.22a) and (7.21c)), we have $\underline{e} = \underline{\upsilon} + F\underline{z} = \underline{\upsilon} + F\left(\bar{C}\underline{x} + \bar{D}\underline{e}\right)$ or

$$\underline{e} = \underline{\upsilon} + F\bar{C}\underline{x} + F\bar{D}\underline{e} \quad \text{or} \quad \left(I - F\bar{D}\right)\underline{e} = \underline{\upsilon} + F\bar{C}\underline{x} \tag{7.22d}$$

Assuming that the feedback configuration is well formed, i.e. $\left|I-F\overline{D}\right| \neq 0$, then we may define:

$$\Delta = \left(I - F\overline{D}\right)^{-1} \tag{7.22e}$$

From (7.22d) and (7.22e), we get: $\underline{e} = \Delta\underline{\upsilon} + \Delta F\overline{C}\underline{x}$ (7.22f)

and thus $\dot{\underline{x}} = \overline{A}\underline{x} + \overline{B}\underline{e} = \overline{A}\underline{x} + \overline{B}\Delta\underline{\upsilon} + \overline{B}\Delta F\overline{C}\underline{x}$ or $\dot{\underline{x}} = \left(\overline{A} + \overline{B}\Delta F\overline{C}\right)\underline{x} + \overline{B}\Delta\underline{\upsilon}$ (7.23a)

and $\underline{z} = \overline{C}\underline{x} + \overline{D}\underline{e} = \overline{C}\underline{x} + \overline{D}\Delta\underline{\upsilon} + \overline{D}\Delta F\overline{C}\underline{x}$ or $\underline{z} = \left(\overline{I} + \overline{B}\Delta F\right)\overline{C}\,\underline{x} + \overline{D}\Delta\underline{\upsilon}$ (7.23b)

We may summarise the above analysis as follows:

Proposition (7.2): The composite system state equations of the well posed complete system are given by:

$$\begin{aligned} \dot{\underline{x}} &= \left(\overline{A} + \overline{B}\Delta F\overline{C}\right)\underline{x} + \overline{B}\Delta\underline{\upsilon} \\ \underline{z} &= \left(\overline{I} + \overline{B}\Delta F\right)\overline{C}\,\underline{x} + \overline{D}\Delta\underline{\upsilon} \end{aligned} \tag{7.24}$$

where $\Delta = \left(I - F\overline{D}\right)^{-1}$, $\left|I - F\overline{D}\right| \neq 0$ and $\overline{A}, \overline{B}, \overline{C}, \overline{D}$ are the state-space parameters describing the aggregate model.

□

Remark (7.2): If every subsystem is strictly proper, then $\overline{D} = 0$ and $\left|I - F\overline{D}\right| = 1$ and, thus, for all composition rules, the configuration is well formed.

□

Remark (7.3): For a generic aggregate system S_a and a generic interconnection rule F, the well formedness assumption is true.

□

7.5.2. The Composite System Pole Polynomial of a Complete Configuration

The composite system pole polynomial of a well-formed system is defined by:

$$\varphi_c(s) = \left| sI - \overline{A} - \overline{B}\Delta F\overline{C} \right| \tag{7.25}$$

and it is clear that $\Delta F = \tilde{F}$ acts as an output feedback rule that, in general, changes the location of the composite system poles from those of the aggregate system. Important issues to be considered are:

(i) The effect of the interconnection rule on generating stable composite system behaviour from stable or unstable aggregate behaviours.

(ii) Characterisation of interconnecting structures that generate composite unstable behaviours from stable aggregate behaviours.

(iii) Characterisation of interconnecting structures that preserve certain parts of the aggregate dynamics.

The first two issues may be considered within the decentralised pole assignment, whereas the last is equivalent to the characterisation of fixed modes under decentralisation. An alternative way of studying these problems is to consider the return difference of the composite configuration of Figure (7.9) defined by:

$$R(s) = I - FG_a(s) \tag{7.26}$$

If every subsystem is represented by a right coprime MFD, i.e. $G_i(s) = N_i(s)\,D_i(s)^{-1}$, $i = 1,2,..,k$, then R(s) may be expressed as:

$$R(s) = \left[\text{diag}\,\{D_i(s)\} - F\,\text{diag}\,\{N_i(s)\}\right]\left[\text{diag}\,\{D_i(s)\}\right]^{-1} = D_c(s)\cdot D_a(s)^{-1} \tag{7.27a}$$

where:
$$\begin{aligned} D_c(s) &= \text{diag}\,\{D_i(s), i = 1,...,k\} - F\,\text{diag}\,\{N_i(s), i = 1,...,k\} \\ D_a(s) &= \text{diag}\,\{D_i(s), i = 1,...,k\} \end{aligned} \tag{7.27b}$$

and thus the composite transfer function H(s), may be expressed as:

$$H_c(s) = \text{diag}\,\{N_i(s), i = 1,...,k\}\,D_c(s)^{-1} \tag{7.27c}$$

Theorem (7.1): If for the subsystems the MFDs $G_i(s) = N_i(s)\, D_i(s)^{-1}$, $i = 1,2,..,k$ are coprime, then the MFD in (7.27c) for the complete composite system H(s) is also coprime. Furthermore,

(i) The composite system pole polynomial is given by

$$\varphi_c(s) = |D_c(s)| = |\,\mathrm{diag}\,\{D_i(s)\} - F\,\mathrm{diag}\,\{N_i(s)\}| \tag{7.28}$$

(ii) The zeros of the composite systems are given as the union of the zeros of the subsystems.

Proof: The MFD in (7.27c) has denominator $D_c(s)$ with degree equal to the McMillan degree of the composite system, which as an output feedback system equivalent to the aggregate must be equal to the sum of McMillan degrees of the subsystems. Thus, since the degree of the denominator is equal to the McMillan degree, the MFD is coprime.

□

The above expression of the composite system pole polynomial is more general than that of (7.25), since it does not rely on the assumption of well posedness of the configuration. The MFD based expression for $\varphi_c(s)$ allows the study of the three classes of problems previously defined. Using the Binet - Cauchy Theorem [Marcus & Minc, 1], we can deploy the exterior algebra framework [Karc. & Gian., 1] for the study of $\varphi_c(s)$, that is

$$\varphi_c(s) = |D_c(s)| = C_p\{[I,F]\}\; C_p\left\{\begin{bmatrix}\mathrm{diag}\,\{D_i(s)\}\\ \hline \mathrm{diag}\,\{N_i(s)\}\end{bmatrix}\right\} \tag{7.29}$$

We may define:

$$\tilde{F} = I + F,\quad T_c^r(s) = \begin{bmatrix}\mathrm{diag}\,\{D_i(s)\}\\ \hline \mathrm{diag}\,\{N_i(s)\}\end{bmatrix},\quad p = \sum_{i=1}^{k} p_i \tag{7.30a}$$

and thus: $\varphi_c(s) = C_p\left(\tilde{F}\right)\, C_p\left(T_c^r(s)\right)$ (7.30b)

The latter expression provides the means for studying the shaping of the composite system pole polynomial using the exterior algebra framework [Karc. & Gian., 1]. In fact, if

$$\underline{\tilde{f}}\wedge = C_p\left(\tilde{F}\right),\ \ \underline{t}_c^r(s)\wedge = C_p\left(T_c^r(s)\right) \rightarrow \varphi_C(s) = \left\langle \underline{\tilde{f}}\wedge, \underline{t}_c^r(s)\wedge \right\rangle \tag{7.30c}$$

where by $\underline{a}\wedge$ we denote the corresponding exterior products [Marcus & Minc, 1]. Of course, the problem here is not the design of F as an arbitrary output feedback, since F is defined by the natural topology of the interconnections. However, Equation (7.30c), may be used for a computation of $\varphi_c(s)$ in a systematic way, but also to study problems of redesign of interconnection by a minor alteration of its structure. This problem is defined below:

Redesign Problem: Given a composite system $S_c(S_a;F)$ that has an undesirable pole polynomial, redesign F by minor alterations to improve the properties of the modified composite system.

□

Alterations can be described in terms of a matrix F', such that the resulting matrix of composition is:

$$\tilde{F} = F + F' \tag{7.31a}$$

This problem may, then, be expressed as a pole assignment by output feedback, since

$$\varphi_c(s) = \left| \text{diag}\left\{D_i(s)\right\} - F\,\text{diag}\left\{N_i(s)\right\} - F'\,\text{diag}\left\{N_i(s)\right\} \right| \tag{7.31b}$$

and by defining $D_c(s) = \text{diag}\left\{D_i(s)\right\} - F\,\text{diag}\left\{N_i(s)\right.$, then

$$\varphi_c(s) = \left| D_c(s) - F'\,\text{diag}\left\{N_i(s)\right\} \right| \tag{7.31c}$$

which is within the pole assignment by output feedback framework. It should be noted, however, that F' is a structured matrix with a topology very close to that of F.

An important problem that is associated with characterising the nature of F composition is the presence of fixed modes between the aggregate and the composite system. The study of fixed modes is established by the following result.

Theorem (7.2): Let $S_c(S_a;F)$ be a composite system described by subsystems with coprime MFDs $G_i(s) = N_i(s)\,D_i(s)^{-1}$, $i = 1,..,k$ and a composition rule F. The fixed modes between the aggregate and the composite system are defined by the zeros of the Smith form of

$$Q(s) = \begin{bmatrix} F\,\mathrm{diag}\{N_i(s)\} \\ \hline \mathrm{diag}\{D_i(s)\} \end{bmatrix} = \begin{bmatrix} F\,N_a(s) \\ \hline D_a(s) \end{bmatrix} \tag{7.32}$$

Proof: If there are fixed modes between the aggregate and the composite system, then the MFD of the return difference R(s) in (7.27a) is not coprime and the fixed modes emerge as zeros of the Smith form of the matrix

$$\begin{bmatrix} \mathrm{diag}\{D_i(s)\} - F\,\mathrm{diag}\{N_i(s)\} \\ \hline \mathrm{diag}\{D_i(s)\} \end{bmatrix} = Q'(s) \tag{7.33a}$$

However, there exists a unimodular matrix, such that:

$$\begin{bmatrix} -I & \mathrm{diag}\{D_i(s)\} \\ 0 & I \end{bmatrix} Q'(s) = \begin{bmatrix} F\,\mathrm{diag}\{N_i(s)\} \\ \hline \mathrm{diag}\{D_i(s)\} \end{bmatrix} \tag{7.33b}$$

and this establishes the necessity. Sufficiency is obvious.

□

The above result provides a simple test for determining the fixed dynamics of the composition process and this is demonstrated by the following example.

Example (7.2): Consider the composite system shown below:

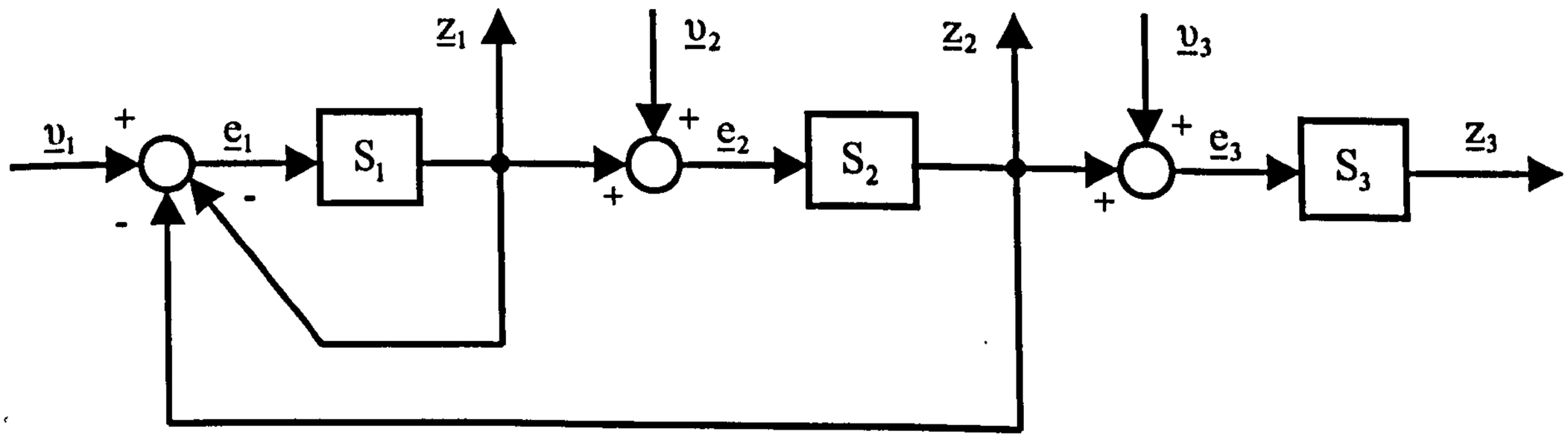

Figure (7.10): Composite System

For the above system, the subsystems are described by: $S_i \rightarrow G_i(s) = N_i(s)\,D_i(s)^{-1}$, $i = 1,..,3$, which are coprime MFDs and the composite rule is:

$$\underline{e} = \begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \underline{e}_3 \end{bmatrix} = \begin{bmatrix} \underline{\upsilon}_1 \\ \underline{\upsilon}_2 \\ \underline{\upsilon}_3 \end{bmatrix} - \begin{bmatrix} I & I & 0 \\ -I & 0 & 0 \\ 0 & -I & 0 \end{bmatrix} \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \\ \underline{z}_3 \end{bmatrix} = \underline{\upsilon} - F\underline{z}$$. The Q(s) matrix defining the fixed modes is:

$$Q(s) = \left[\begin{bmatrix} I & I & 0 \\ -I & 0 & 0 \\ 0 & -I & 0 \end{bmatrix} \begin{bmatrix} N_1(s) & 0 & 0 \\ 0 & N_2(s) & 0 \\ 0 & 0 & N_3(s) \end{bmatrix} \begin{bmatrix} D_1(s) & 0 & 0 \\ 0 & D_2(s) & 0 \\ 0 & 0 & D_3(s) \end{bmatrix}^{-1} \right] = \left[\begin{bmatrix} N_1(s) & N_2(s) & 0 \\ -N_1(s) & 0 & 0 \\ 0 & -N_2(s) & 0 \end{bmatrix} \begin{bmatrix} D_1(s) & 0 & 0 \\ 0 & D_2(s) & 0 \\ 0 & 0 & D_3(s) \end{bmatrix}^{-1} \right]$$

The above is equivalent by left equivalence to

$$\left[\begin{array}{ccc} 0 & 0 & 0 \\ N_1(s) & 0 & 0 \\ D_1(s) & 0 & 0 \\ \hline 0 & N_2(s) & 0 \\ 0 & D_2(s) & 0 \\ \hline 0 & 0 & D_3(s) \end{array}\right] = Q'(s) \quad \text{or} \quad Q'(s) \sim Q''(s) = \left[\begin{array}{ccc} I & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & I & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & D_3(s) \end{array}\right]$$

The equivalence of Q'(s) to Q''(s) follows from the coprimeness (right) of $(N_1(s), D_1(s))$ and $(N_2(s), D_2(s))$. Thus, the poles of S_3 system are fixed between the aggregate and the composite system.

□

7.5.3. Structural Properties of the Complete Composite Configuration

The fact that for complete configurations the interconnection rule acts as an output feedback has a number of implications as far as structural system properties are concerned. We first note:

Remark (7.4): Any constant output feedback law is a special case of a state feedback and a special case of output injection.

□

As a result of the above observation and the structural properties under state feedback and output injection (see Chapter 4), we have the following properties:

Theorem (7.3): Let $S_c(S_a;F)$ be a complete composite system where the subsystems are described by the state space descriptions $S_i(A_i,B_i,C_i,D_i)$, $i=1,2,...,k$. For any interconnection rule F, the following properties hold true:

(i) Controllability properties, as they are expressed by controllability indices and input-decoupling elementary divisors, are the same for the aggregate and the composite system.

(ii) Observability properties, as they are expressed by observability indices and output-decoupling elementary divisors, are the same for the aggregate and the composite system.

(iii) The zero structure, as this is defined by the Kronecker invariants of the system matrix pencil, is the same for the aggregate and the composite system.

□

The above result is a direct consequence of the observation that the interconnection rule for composite complete structures is equivalent to an output feedback. This together with the classical Kronecker Theory ([Morse, 1], [Thorp, 1], [Karc. & MacB., 1]) establishes the result. A direct consequence of the above result is the equivalence of structural properties of complete composite systems to those of the aggregate, as described below:

Corollary (7.1): Let $S_c(S_a;F)$ be a complete composite system where $S_i(A_i,B_i,C_i,D_i)$, $i=1,2,...,k$ are the subsystems. The following properties hold true:

(i) $S_c(S_a;F)$ is controllable (stabilisable), if and only if, all pairs (A_i,B_i), $i=1,2,...,k$ are controllable (stabilisable).

(ii) $S_c(S_a;F)$ is observable (detectable), if and only if, all pairs (A_i,C_i), $i=1,2,...,k$ are observable (detectable).

(iii) $S_c(S_a;F)$ is input-output controllable (functionally controllable), if and only if, all subsystems $S_i(A_i,B_i,C_i,D_i)$, $i=1,2,...,k$ are input- output controllable (functionally controllable).

(iv) $S_c(S_a;F)$ is nondegenerate, if and only if, every subsystem $S_i(A_i,B_i,C_i,D_i)$ is nondegenerate.

□

Corollary (7.2): Let $S_c(S_a;F)$ be a complete composite system where $S_i(A_i,B_i,C_i,D_i)$, $i=1,2,...,k$ are the subsystems. The following properties hold true:

(i) The invariant zeros of $S_c(S_a;F)$, finite and infinite, are defined as the union of the corresponding sets of zeros of all subsystems $S_i(A_i,B_i,C_i,D_i)$, $i=1,2,...,k$.

(ii) The right (left) Kronecker indices of the system matrix of S_c are given as the union of the right (left) Kronecker indices of all subsystems $S_i(A_i,B_i,C_i,D_i)$, $i=1,2,...,k$.

(iii) The controllability (observability) indices and input (output) decoupling elementary divisors of S_c are given as the union of the corresponding controllability (observability) indices and input (output) decoupling elementary divisors of all subsystems $S_i(A_i,B_i,C_i,D_i)$, $i=1,2,...,k$.

□

The above demonstrate that completeness is the idealistic assumption that permits the transferring of the structural properties from the subsystem level to the composite system. Some further implications of the above results are:

Corollary (7.3): For the complete composite system $S_c(S_a;F)$ with subsystems $S_i(A_i,B_i,C_i,D_i)$, we have the properties:

(i) The dimension of the controllable subspace is equal to the sum of the dimensions of the controllable subspaces of the subsystems defined by the (A_i,B_i) pairs.

(ii) The dimension of the unobservable subspace is equal to the sum of the dimensions of the unobservable subspaces of the subsystems defined by the (A_i,C_i) pairs.

(iii) The $S_c(S_a;F)$ state-space representation is minimal, if and only if all $S_i(A_i,B_i,C_i,D_i)$ representations are minimal.

□

The significance of the completeness assumption is that properties of the subsystems are transferred by aggregation to the composite structure irrespective of the nature of F. the implications of deviating from completeness are considered next.

7.6. DEVIATING FROM COMPLETENESS AND SYSTEM PROPERTIES

The essence of the completeness assumption is that the number of independent control variables for every subsystem is equal to the dimension of subsystem input causes (dimension of $\underline{e}_i$ local vector) and the number of independent measurements for the subsystems is equal to the dimension of local output influences (dimension of $\underline{z}_i$ local vector). The dimensionality of $\underline{e}_i$, $\underline{z}_i$ depends on the nature of the specific system and, in general, the dimensionality of independent inputs (control) and outputs (measurements) is less than the ideal dimensions implied by the completeness assumption. Some of the effects of deviating from completeness on the resulting system properties are considered here. A framework is introduced for studying specific cases and a number of results corresponding to certain representative cases expressing deviation from completeness are considered.

Assuming that the complete system is well-formed and that the subsystems are described by $S_i(A_i,B_i,C_i,D_i)$, $i=1,2,...,k$, then the complete composite system has the following description (see (7.24)):

$$\dot{\underline{x}} = \left(\overline{A} + \overline{B}\Delta F\overline{C}\right)\underline{x} + \overline{B}\Delta\underline{\upsilon} \tag{7.34a}$$

$$\underline{z} = \left(\overline{I} + \overline{B}\Delta F\right)\overline{C}\,\underline{x} + \overline{D}\Delta\underline{\upsilon} \tag{7.34b}$$

where $\Delta = \left(I - F\overline{D}\right)^{-1}$, $\left|I - F\overline{D}\right| \neq 0$ and $\overline{A},\overline{B},\overline{C},\overline{D}$ are the aggregate state-space parameters. The corresponding transfer function is expressed as:

$$\underline{z}(s) = \hat{H}_c(s)\,\underline{\upsilon}(s) \tag{7.35}$$

$$\hat{H}_c(s) = G_a(s)\left(I - F\,G_a(s)\right)^{-1} = \left(I - G_a(s)\,F\right)^{-1}G_a(s) \tag{7.36}$$

or in a coprime MFD representation as:

$$\begin{aligned}\hat{H}_c(s) &= \text{diag}\left\{N_i^r(s)\right\}D_c^r(s)^{-1} = N_c^r(s)\,D_c^r(s)^{-1}\\ D_c^r(s) &= \text{diag}\left\{D_i^r(s)\right\} - F\,\text{diag}\left\{N_i^r(s)\right\} = D_a^r(s) - F\,N_a^r(s)\end{aligned} \tag{7.37a}$$

$$\begin{aligned}\hat{H}_c(s) &= D_c^\ell(s)^{-1}\,\text{diag}\left\{N_i^\ell(s)\right\} = D_c^\ell(s)^{-1}N_c^\ell(s)\\ D_c^\ell(s) &= \text{diag}\left\{D_i^\ell(s)\right\} - \text{diag}\left\{N_i^\ell(s)\right\}F = D_a^\ell(s) - N_a^\ell(s)\,F\end{aligned} \tag{7.37b}$$

where $G_i(s) = N_i^r(s) D_i^r(s)^{-1} = D_i^\ell(s)^{-1} N_i^\ell(s)$ are coprime MFDs for the subsystems. From Theorem (7.1), the MFDs in (7.37a), (7.37b) are respectively right, left coprime. The relationship between input causes – control variables and output influences – measurements is described by:

$$\underline{v}_i = L_i \underline{u}_i,\ L_i \in R^{p_i \times r_i},\ p_i \geq r_i,\ i \in \underset{\sim}{k} \tag{7.38a}$$

$$\underline{y}_i = K_i \underline{z}_i,\ K_i \in R^{q_i \times m_i},\ q_i \leq m_i,\ i \in \underset{\sim}{k} \tag{7.38b}$$

and the transfer function between aggregate inputs and aggregate outputs is expressed by:

$$H_c(s) = K_a \hat{H}_c(s) L_a,\ K_a = \text{diag}\{K_i, i \in \underset{\sim}{k}\},\ L_a = \text{diag}\{L_i, i \in \underset{\sim}{k}\} \tag{7.39a}$$

Completeness implies that $H_c(s)$ and $\hat{H}_c(s)$ are equivalent by constant input, output coordinate transformations of the decentralisation type. Loss of completeness implies that at least for a subsystem $p_i > r_i$ or $q_i < m_i$ and, thus, $H_c(s)$ and $\hat{H}_c(s)$ are no longer equivalent in the above sense. Loss of completeness may frequently occur due to the nature of the $\underline{e}_i$, $\underline{z}_i$ variables (not being able to define independent control actions equal to the dimension of $\underline{e}_i$ and not being able to measure as many independent variables as the dimension of $\underline{z}_i$). The complete internal system described by $S_c(S_a;F)$ or $\hat{H}_c(s)$ acts as a "progenitor" model for all systems defined for $\underline{u}_i$, $\underline{y}_i$ sets and the overall problem of shaping properties by selection of (L_i, K_i) local pairs is a form of a generalised squaring down [Karc. & Gian., 2]. In the following, we shall examine some special cases, which correspond to extreme cases, where there is total loss of local control variables or measurement variables. The general case of input-output squaring down is not considered here but may approached within the exterior algebra framework of the squaring down problem [Karc. & Gian., 2].

7.6.1. Controllability Properties under Total Loss of Subsystem Control Inputs

Consider a complete composite system and assume that, for a fixed i subsystem, all the external inputs are not used (i.e. this occurs when interconnection elements are dynamic and no assignable input is available for them). In this case, the corresponding subsystem has as inputs, those coming from the interconnections only and it does not possess the

completeness property. The composite system description with total loss of subsystem inputs is described by Equations (7.34a), (7.34b). Without loss of generality, we may assume that all subsystems are strictly proper, which implies that $\overline{D}=0$ and, thus, $\Delta=I$. Under this assumption, the system is described by:

$$\dot{\underline{x}}=\left(\overline{A}+\overline{B}F\overline{C}\right)\underline{x}+\overline{B}\underline{\upsilon},\ \ \underline{z}=\overline{C}\,\underline{x} \tag{7.40}$$

Assuming that all local inputs are controlled independently with the exception of those of the i-th subsystem for which there are no controls, then from $\underline{\upsilon}$, we get a $\underline{u}$ control vector of the type:

$$\underline{u}_i^{'}=\left[\underline{u}_i^t,\ldots,\underline{u}_{i-1}^t;\underline{0}^t;\underline{u}_{i+1}^t,\ldots,\underline{u}_k^t\right]^t,\ \ \underline{u}_i=\underline{\upsilon}_i \tag{7.41a}$$

and this implies that the first of Equations (7.40), may be reduced to:

$$\dot{\underline{x}}=\left(\overline{A}+\overline{B}F\overline{C}\right)\underline{x}+\tilde{B}_i\,\tilde{\underline{u}}_i,\ \ \tilde{\underline{u}}_i=\left[\underline{u}_i^t,\ldots,\underline{u}_{i-1}^t;\underline{u}_{i+1}^t,\ldots,\underline{u}_k^t\right]^t \tag{7.41b}$$

$$\tilde{B}_i=\begin{bmatrix} B_1 & 0 & \cdots & & \cdots & 0\\ 0 & \ddots & & & & \vdots\\ \vdots & & B_{i-1} & 0 & & \\ & & 0 & 0 & & \\ & & 0 & B_{i+1} & & \vdots\\ \vdots & & & & \ddots & 0\\ 0 & \cdots & & \cdots & 0 & B_k \end{bmatrix} \tag{7.41c}$$

i.e. $\tilde{B}_i$ is obtained from $\overline{B}$ by deleting the i-th column block. We may define the left annihilator of $\tilde{B}_i$, $\tilde{N}_i$ by solving $\tilde{N}_i\,\tilde{B}_i=0$. It is obvious, that

$$\underbrace{\begin{bmatrix} N_1 & 0 & \cdots & & & \cdots & 0\\ 0 & \ddots & & & & & \vdots\\ \vdots & & N_{i-1} & & & & \\ & & & I_{n_i} & & & \\ & & & & N_{i+1} & & \vdots\\ \vdots & & & & & \ddots & 0\\ 0 & \cdots & & & \cdots & 0 & N_k \end{bmatrix}}_{\overset{\Delta}{=}\tilde{N}_i}\underbrace{\begin{bmatrix} B_1 & 0 & \cdots & & \cdots & 0\\ 0 & \ddots & & & & \vdots\\ \vdots & & B_{i-1} & 0 & & \\ & & 0 & 0 & & \\ & & 0 & B_{i+1} & & \vdots\\ \vdots & & & & \ddots & 0\\ 0 & \cdots & & \cdots & 0 & B_k \end{bmatrix}}_{\overset{\Delta}{=}\tilde{B}_i}=0 \tag{7.42a}$$

and, thus, the corresponding input restriction pencil is:

$$R_i(s) = s\tilde{N}_i - \tilde{N}_i\left(\overline{A} + \overline{B}F\overline{C}\right) = s\tilde{N}_i - \tilde{N}_i\overline{A} - \tilde{N}_i\overline{B}F\overline{C} \tag{7.42b}$$

The controllability properties are investigated by examining the above pencil, where:

$$s\tilde{N}_i - \tilde{N}_i\overline{A} = \begin{bmatrix} sN_1 - N_1A_1 & 0 & & & \dots & 0 \\ 0 & \ddots & & & & \vdots \\ \vdots & & sN_{i-1} - N_{i-1}A_{i-1} & & & \\ & & & sI - A_i & & \vdots \\ \vdots & & & & \ddots & 0 \\ 0 & \dots & & & \dots & sN_k - N_kA_k \end{bmatrix} \tag{7.42c}$$

The problem of computing $R_1(s)$ is, thus, reduced to the computation of $\tilde{N}_i\overline{B}F\overline{C}$.

Note that

$$\tilde{N}_i\overline{B} = \begin{bmatrix} & & & 0 & & & \\ & & & \vdots & & & \\ & & & 0 & & & \\ 0 & \dots & 0 & B_i & 0 & \dots & 0 \\ & & & 0 & & & \\ & & & \vdots & & & \\ & & & 0 & & & \end{bmatrix}, \quad F = \begin{bmatrix} F_{11} & F_{12} & \dots & F_{1i} & \dots & F_{1k} \\ \vdots & \vdots & & \vdots & & \vdots \\ F_{i1} & F_{i1} & & F_{ii} & & F_{ik} \\ \vdots & \vdots & & \vdots & & \vdots \\ F_{k1} & F_{k2} & \dots & F_{ki} & \dots & F_{kk} \end{bmatrix} \tag{7.43a}$$

and thus

$$\tilde{N}_i\overline{B}F\overline{C} = \begin{bmatrix} & & 0 & & \\ & & \vdots & & \\ & & 0 & & \\ B_iF_{i1} & \dots & B_iF_{ii} & \dots & B_iF_{ik} \\ & & 0 & & \\ & & \vdots & & \\ & & 0 & & \end{bmatrix} \overline{C} = \begin{bmatrix} & & 0 & & \\ & & \vdots & & \\ & & 0 & & \\ B_iF_{i1}C_1 & \dots & B_iF_{ii}C_i & \dots & B_iF_{ik}C_k \\ & & 0 & & \\ & & \vdots & & \\ & & 0 & & \end{bmatrix} \tag{7.43b}$$

The above, together with (7.42c) lead to the explicit form for the restricted input-state pencil that characterises the controllability properties.

Proposition (7.2): The controllability properties of the incomplete system corresponding to the total loss of the i-th subsystem local inputs are determined by the rank properties of the pencil:

$$R_i(s)=\begin{bmatrix} sN_1-N_1A_1 & 0 & \cdots & & \cdots & 0 \\ 0 & \ddots & & & & \vdots \\ -B_iF_{i1}C_1 & \cdots & -B_iF_{i,i-1}C_{i-1} & sI-A_i-B_iF_{ii}C_i & \cdots & -B_iF_{ik}C_k \\ \vdots & & & & \ddots & 0 \\ 0 & \cdots & & \cdots & 0 & sN_k-N_kA_k \end{bmatrix} \qquad (7.44)$$

□

The above result may be generalised for every i and extended to any combination of indices i, j etc. The expression for $R_i(s)$ may be used for:

(i) Studying the effect of the structure F on the loss of controllability, when total loss of subsystem inputs occur, as well as the location of the formed input decoupling zeros.

(ii) Distinguish the phenomena depending on the parameters of subsystems $S_i(A_i,B_i,C_i)$ and those depending only on the F structure.

It should be pointed out that the loss of external inputs results in a pencil $R_i(s)$, whose Kronecker structure is no longer expressed as a direct sum of the Kronecker structures of the subsystems. The role of matrix F, expressing the interconnections, is crucial in determining the composite system properties.

7.6.2. Observability Properties under Total Loss of Subsystem Outputs

We consider a composite system and we shall examine the case where all the i-th subsystem outputs are not measured. The composite system description is given by (7.34a), (7.34b) and for the strictly proper case by (7.40). For the sake of simplicity, we consider the strictly proper case. Under the assumption that the i-th subsystem influences are not measured, then the resulting output vector is:

$$\underline{y}=\left[\underline{z}_1^t,\ldots,\underline{z}_{i-1}^t;\underline{0}^t;\underline{z}_{i+1}^t,\ldots,\underline{z}_k^t\right]^t,\ \underline{z}_i=\underline{y}_i \qquad (7.45a)$$

and the corresponding reduced vector $\underline{z}_i' = \left[\underline{z}_i^t,\ldots,\underline{z}_{i-1}^t;\underline{z}_{i+1}^t,\ldots,\underline{z}_k^t\right]^t$ has an associated matrix $\tilde{C}_i$ defined by deleting the i-th row block from $\bar{C}$:

$$\tilde{C}_i = \begin{bmatrix} C_1 & 0 & \cdots & & & \cdots & 0 \\ 0 & \ddots & & & & & \vdots \\ \vdots & & C_{i-1} & 0 & 0 & & \\ & & 0 & 0 & C_{i+1} & & \vdots \\ \vdots & & & & & \ddots & 0 \\ 0 & \cdots & & & \cdots & 0 & C_k \end{bmatrix} \tag{7.45b}$$

i.e. the block containing C_i has been deleted. We may define the right annihilator of $\tilde{C}_i$, $\tilde{M}_i$ by solving the equation $\tilde{C}_i\tilde{M}_i = 0$. It is obvious, that

$$\underbrace{\begin{bmatrix} C_1 & 0 & \cdots & & & \cdots & 0 \\ 0 & \ddots & & & & & \vdots \\ \vdots & & C_{i-1} & 0 & 0 & & \\ & & 0 & 0 & C_{i+1} & & \vdots \\ \vdots & & & & & \ddots & 0 \\ 0 & \cdots & & & \cdots & 0 & C_k \end{bmatrix}}_{=\tilde{C}_i} \begin{bmatrix} M_1 & 0 & \cdots & & & \cdots & 0 \\ 0 & \ddots & & & & & \vdots \\ \vdots & & M_{i-1} & & & & \\ & & & I_{m_i} & & & \\ & & & & M_{i+1} & & \vdots \\ \vdots & & & & & \ddots & 0 \\ 0 & \cdots & & & \cdots & 0 & M_k \end{bmatrix} = 0 \tag{7.45c}$$

$$\underbrace{\qquad\qquad}_{=\tilde{M}_i}$$

and thus the corresponding restriction pencil is:

$$T_i(s) = s\tilde{M}_i - \left(\bar{A} + \bar{B}F\bar{C}\right)\tilde{M}_i = s\tilde{M}_i - \bar{A}\tilde{M}_i - \bar{B}F\bar{C}\tilde{M}_i \tag{7.46a}$$

The observability properties are investigated by examining the above pencil, where

$$s\tilde{M}_i - \bar{A}\tilde{M}_i = \begin{bmatrix} sM_1 - A_1M_1 & 0 & \cdots & & \cdots & 0 \\ 0 & & & & & \vdots \\ \vdots & & sM_{i-1} - A_{i-1}M_{i-1} & & & \\ & & & sI - A_i & & \vdots \\ \vdots & & & & & 0 \\ 0 & \cdots & & & \cdots & sM_k - A_kM \end{bmatrix} \tag{7.46b}$$

The problem of computing $T_i(s)$ is, thus, reduced to the computation of $\tilde{N}_i\bar{B}F\bar{C}$.

$$\overline{C}\tilde{M}_I = \begin{bmatrix} & & 0 & & \\ & & \vdots & & \\ & & 0 & & \\ 0 & \dots & C_i & \dots & 0 \\ & & 0 & & \\ & & \vdots & & \\ & & 0 & & \end{bmatrix}, \quad F = \begin{bmatrix} F_{11} & F_{12} & \dots & F_{1i} & \dots & F_{1k} \\ \vdots & \vdots & & \vdots & & \vdots \\ F_{i1} & F_{i1} & & F_{ii} & & F_{ik} \\ \vdots & \vdots & & \vdots & & \vdots \\ F_{k1} & F_{k2} & \dots & F_{ki} & \dots & F_{kk} \end{bmatrix} \tag{7.46c}$$

and thus

$$\overline{B}F\overline{C}\tilde{M}_I = \overline{B}\begin{bmatrix} & & F_{1i}C_i & & \\ & & \vdots & & \\ 0 & \dots & F_{ii}C_i & \dots & 0 \\ & & \vdots & & \\ & & F_{ki}C_i & & \end{bmatrix} = \begin{bmatrix} & & B_1F_{1i}C_i & & \\ & & \vdots & & \\ 0 & \dots & B_iF_{ii}C_i & \dots & 0 \\ & & \vdots & & \\ & & B_kF_{ki}C_i & & \end{bmatrix} \tag{7.46d}$$

From the above, we have the form of the state-output restricted pencil $T_i(s)$ and this leads to the following result.

Proposition (7.3): The observability properties of the incomplete system corresponding to the total loss of the i-th subsystem local outputs are determined by the rank properties of the pencil:

$$T_i(s) = \begin{bmatrix} sM_1 - A_1M_1 & 0 & -B_1F_{1i}C_i & \dots & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & -B_{i-1}F_{i-1,i}C_i & & \\ & & sI - A_i - B_iF_{ii}C_i & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \dots & -B_kF_{ki}C_i & 0 & sM_k - A_kM_k \end{bmatrix} \tag{7.47}$$

The above expression can be generalised for every i and extended to any combination of indices i, j etc. The expression for $T_i(s)$ may be used for:

(iii) Studying the effect of the structure F on the loss of observability, when total loss of subsystem outputs occur, as well as the location of the formed output decoupling zeros.

(iv) Distinguish the phenomena depending on the parameters of subsystems $S_i(A_i, B_i, C_i)$ and those depending only on the F structure.

It should be pointed out that the loss of outputs results in a pencil $T_i(s)$, whose Kronecker structure is no longer expressed as a direct sum of the Kronecker structures of the subsystems. The role of matrix F, expressing the interconnections, is crucial in determining the composite system properties.

7.6.3. The Zero Structure under Total Loss of Subsystem Inputs, Outputs

Consider a complete composite system and assume that all the i-th subsystem external inputs are not used (i.e. this occurs when interconnection elements are dynamic and no assignable input is available for them). In this case, the corresponding subsystem has as inputs, those coming from the interconnections only. The zero pencil of the composite system is described by:

$$
s\tilde{N}_i\overline{M}-\tilde{N}_i\overline{A}\,\overline{M}=\begin{bmatrix} sN_1-N_1A_1 & 0 & \dots & & \dots & 0 \\ 0 & \ddots & & & & \vdots \\ -B_iF_{i1}C_1 & \dots & -B_iF_{i,i-1}C_{i-1} & sI-A_i-B_iF_{ii}C_i & \dots & -B_iF_{ik}C_k \\ \vdots & & & & \ddots & 0 \\ 0 & \dots & & \dots & 0 & sN_k-N_kA_k \end{bmatrix}\times
$$

$$
\times\begin{bmatrix} M_1 & & \dots & 0 \\ & \ddots & & \vdots \\ & M_i & & \\ \vdots & & \ddots & \\ 0 & \dots & & M_k \end{bmatrix}=\begin{bmatrix} sN_1M_1-N_1A_1M_1 & & \dots & 0 \\ & \ddots & & \vdots \\ & sM_i-A_iM_i & & \\ \vdots & & \ddots & \\ 0 & \dots & & sN_kM_k-N_kA_kM_k \end{bmatrix} \tag{7.48}
$$

where $\tilde{N}_i$ has been defined by Equation (7.42a).

Remark (7.5): The above pencil may be generalised for every i and extended to any combination of indices i, j etc.

□

The above matrix may be used to study the location of the zeros when total loss of subsystem inputs occurs. It should be pointed out that loss of external inputs, results in a zero pencil, whose Kronecker structure is again expressed as a direct sum of the Kronecker structures originating from the subsystems. The role of matrix F, expressing the

interconnections, is not crucial in determining the zero properties of the composite system. In connection, with the above statement, we have the following theorem.

Theorem (7.4): The observability structure of the i-th subsystem, for which we have lost all its inputs, as defined by the corresponding state-output pencil, enters into the zero structure of the composite system.

□

By observability structure, we mean the structure of the output decoupling zeros and rmi of the state-output pencil. Next, we investigate the zero properties of the composite system under the loss of subsystem outputs. Consider a complete composite system and assume that all the i-th subsystem outputs are not measured. The zero pencil of the composite system is:

$$
\begin{aligned}
&s\tilde{N}_i\overline{M}-\tilde{N}_i\overline{A}\,\overline{M}= \\
&=\begin{bmatrix} N_1 & & \cdots & 0 \\ & \ddots & & \vdots \\ & & N_i & \\ \vdots & & & \ddots \\ 0 & \cdots & & N_k \end{bmatrix}\begin{bmatrix} sM_1-A_1M_1 & 0 & -B_1F_{1i}C_i & \cdots & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & -B_{i-1}F_{i-1,i}C_i & & \\ & & sI-A_i-B_iF_{ii}C_i & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & -B_kF_{ki}C_i & 0 & sM_k-A_kM_k \end{bmatrix}= \\
&=\begin{bmatrix} sN_1M_1-N_1A_1M_1 & & \cdots & 0 \\ & \ddots & & \vdots \\ & & sN_i-N_iA_i & \\ \vdots & & & \ddots \\ 0 & \cdots & & sN_kM_k-N_kA_kM_k \end{bmatrix}
\end{aligned}
\tag{7.49}
$$

Remark (7.6): The above pencil may be generalised for every i and extended to any combination of indices i, j etc.

□

The above leads to a dual result to that of Theorem (7.4) that corresponds to total loss of subsystem outputs occurs that characterises the resulting zero structure.

Theorem (7.5): The controllability structure of the i-th subsystem, for which we have lost all its outputs, enters into the zero structure of the composite system.

□

By controllability structure, we mean the structure defined by the input decoupling zeros and cmi of the state-output pencil.

Theorem (7.6): Internal dynamics of the i-th subsystem, as defined by the eigenvalues and Segre characteristics of the corresponding subsystem, become part of the zero structure of the composite system under the total loss of input and output corresponding to the i-th subsystem.

□

7.6.4. Example of Total Loss of Subsystem Inputs, Outputs

Consider the complete composite system of Example (7.1), where the subsystems are strictly proper and are represented by $S_i(A_i, B_i, C_i)$, $i = 1,2,3$, as shown below:

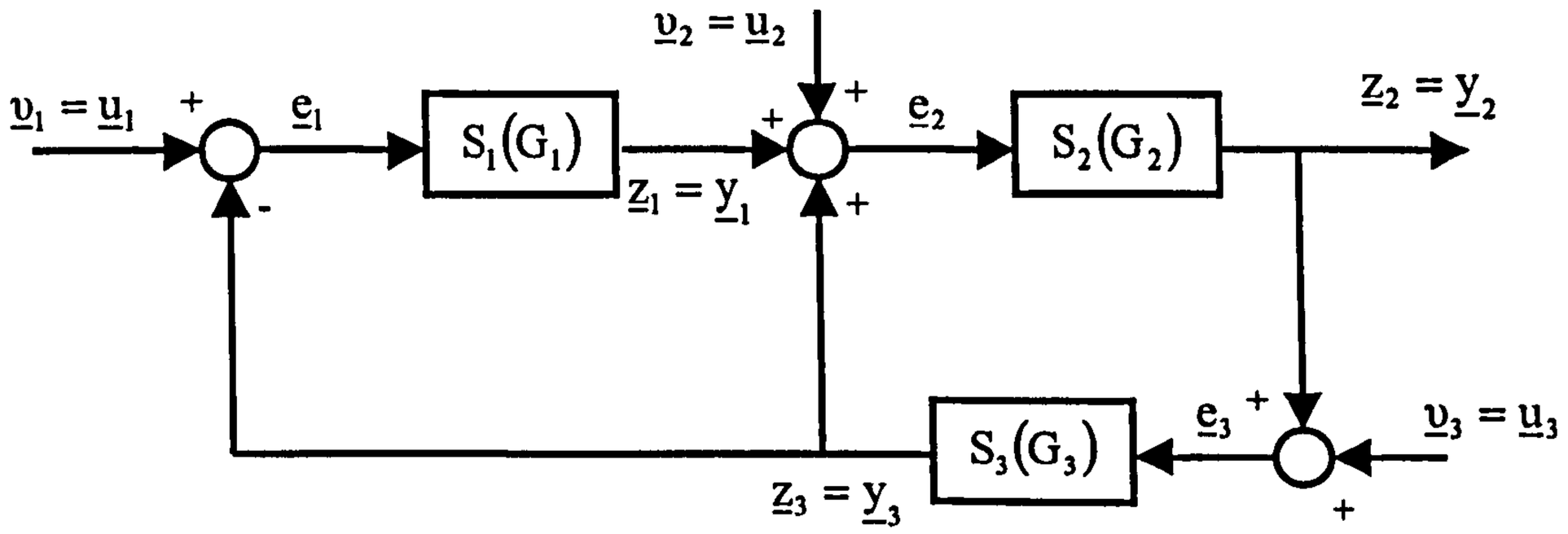

Figure (7.11): Complete Composite System

For this system, the aggregate state equations and the equations of the composite system are described, as shown below:

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \\ \dot{\underline{x}}_3 \end{bmatrix} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix} + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \underline{e}_3 \end{bmatrix} \tag{7.50a}$$

$$\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \end{bmatrix} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix}$$

$$\text{where: } \begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \underline{e}_3 \end{bmatrix} = \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -I \\ I & 0 & I \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \end{bmatrix}, \quad \text{i.e. } \underline{e} = \underline{u} + F\underline{y} \tag{7.50b}$$

and thus the composite state matrix $A_c = \overline{A} + \overline{B}F\overline{C}$ and the composite system equations are:

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \\ \dot{\underline{x}}_3 \end{bmatrix} = \begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix} + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \end{bmatrix}$$
$$\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \end{bmatrix} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix} \qquad (7.50c)$$

We consider, next, the restriction pencils of the composite system. In this example, three different cases are considered. First, the full input - full output case is given. For each subsystem, we associate the input-state restriction pencil, the state-output restriction pencil and the zero pencil, as shown below:

$$\begin{aligned} &S_1(A_1,B_1,C_1) \rightarrow sN_1 - N_1A_1,\ sM_1 - A_1M_1,\ sN_1M_1 - N_1A_1M_1 \\ &S_2(A_2,B_2,C_2) \rightarrow sN_2 - N_2A_2,\ sM_2 - A_2M_2,\ sN_2M_2 - N_2A_2M_2 \\ &S_3(A_3,B_3,C_3) \rightarrow sN_3 - N_3A_3,\ sM_3 - A_3M_3,\ sN_3M_3 - N_3A_3M_3 \end{aligned} \qquad (7.51)$$

where N_i and M_i are the left and right annihilators of B_i and C_i, respectively, for $i = 1,2,3$.

Next, we consider the case where total loss of subsystem input structure has occurred, Assume that $\underline{u}_1 = 0$ (without loss of generality). This leads to the following reduced composite system description:

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \\ \dot{\underline{x}}_3 \end{bmatrix} = \begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix} + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \begin{bmatrix} 0 \\ \underline{u}_2 \\ \underline{u}_3 \end{bmatrix} \qquad (7.52a)$$

$$\text{or } \dot{\underline{x}} = \begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_2 & 0 \\ 0 & B_3 \end{bmatrix} \begin{bmatrix} \underline{u}_2 \\ \underline{u}_3 \end{bmatrix} \quad \text{and} \quad \tilde{N}_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{bmatrix} \qquad (7.52b)$$

To investigate the controllability properties when there is total loss of subsystem input, we may define the input-state restriction pencil, as follows:

$$s\tilde{N}_1 - \tilde{N}_1 A = \begin{bmatrix} I & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{bmatrix} \begin{bmatrix} sI - A_1 & 0 & -B_1C_3 \\ B_2C_1 & sI - A_2 & -B_2C_3 \\ 0 & B_3C_2 & sI - A_3 \end{bmatrix} =$$
$$= \begin{bmatrix} sI - A_1 & 0 & -B_1C_3 \\ 0 & sN_2 - N_2A_2 & 0 \\ 0 & 0 & sN_3 - N_3A_3 \end{bmatrix} \quad (7.52c)$$

Equation (7.52c) shows that the loss of inputs of one of the subsystems results possibly in the presence of finite elementary divisors in the input-state restriction pencil. Therefore, the system may become uncontrollable. However, in general, the latter property depends on diagonal matrices such as the $-B_1C_3$ matrix. To demonstrate that the properties of controllability depend on the interconnection graph, we observe that the pencil $R(s) = s\tilde{N}_1 - \tilde{N}_1 A$ in (7.52d) is strictly equivalent by permutation of blocks to:

$$R'(s) = \left[\begin{array}{c|cc} sN_2 - N_2A_2 & 0 & \\ \hline 0 & sI - A_1 & -B_1C_3 \\ & 0 & sN_3 - N_3A_3 \end{array}\right] = \left[\begin{array}{c|c} sN_2 - N_2A_2 & 0 \\ \hline 0 & \bar{R}(s) \end{array}\right] \quad (7.52d)$$

The above suggests that part of the controllability structure, that is, the one connected with the 2^{nd} subsystem, is part of the controllability structure of the resulting system. The rest of the properties depend on the structure of the reduced pencil $\bar{R}(s)$. In this case, part of the controllability properties of the subsystem are transferred to the composite system

□

Remark (7.7): Total loss of input channels for any of the three subsystems may result in structural uncontrollability. This, however, is a property entirely dependent on the system graph and the location of the deviation from completeness.

□

Consider the case, where only the first and second outputs are measured (without loss of generality). The reduced composite system description is given by:

$$\dot{\underline{x}} = \begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix} \underline{x} + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \underline{u}, \quad \underline{y} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \end{bmatrix} \underline{x} \quad (7.53a)$$

and a right annihilator $\tilde{M}_1$ is given by: $\tilde{M}_1 = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & I \end{bmatrix}$ (7.53b)

To investigate the observability properties, when there is loss of output, we define the state-output restriction pencil as:

$$s\tilde{M}_1 - A\tilde{M}_1 = \begin{bmatrix} sI - A_1 & 0 & -B_1C_3 \\ B_2C_1 & sI - A_2 & -B_2C_3 \\ 0 & -B_3C_2 & sI - A_3 \end{bmatrix} \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} sM_1 - A_1M_1 & 0 & -B_1C_3 \\ 0 & sM_2 - A_2M_2 & -B_2C_3 \\ 0 & 0 & sI - A_3 \end{bmatrix} \quad (7.53c)$$

Equation (7.53c) shows that the loss of output of one of the subsystems may result in the presence of finite elementary divisors in the state-output restriction pencil. Therefore, the system may become unobservable. However, the latter property depends on the B_1C_3, B_2C_3 matrices. Note that the observability properties of the first subsystem (expressed in terms of $sM_1 - A_1M_1$) are transferred to the observability properties of the composite system.

Remark (7.8): Loss of outputs of one of the subsystems may result in system unobservability.

□

There are cases where two or more inputs/outputs are lost, in which case the procedure for solving the problem is more complex. The above example demonstrates that when there is total loss of inputs or outputs, then the interconnection structure plays a crucial role in defining the controllability, observability properties of the resulting system. The characterisation of the resulting structural properties then depends on the properties of the interconnection graph, manifested in the structure of the matrix F. Although, we have considered cases of total loss of inputs, outputs at subsystem loss, the approach may be extended to partial losses, i.e. more generic forms of deviation s from completeness. Such cases may be treated as cases of squaring down [Karc. & Gian., 2] at subsystems inputs and/or outputs, but then analysis becomes much more complicated.

7.7. CONCLUSIONS

The representation of composite systems has been considered and this has led to a general configuration where physical input causes and external physical influences lead to the definition of the notion of completeness for composite configurations. Composition emerges as output feedback on the complete configuration, whereas loss of completeness appears as input, output decentralised squaring down. Some general results, that are linked to the total loss of inputs, outputs at subsystem level, are derived, which indicate how parts of the Kronecker structure are affected by the loss of inputs, outputs. The matrix pencil framework used for studying controllability, observability and zero structure of systems under total loss of inputs, outputs is natural and allows the study of effects of specific interconnection graphs on such properties. The general case of loss of completeness at subsystem level may be studied by the exterior algebra framework of the squaring down problem [Karc. & Gian., 2].

Chapter 8

IDENTIFICATION OF STRUCTURAL CHARACTERISTICS IN EARLY PROCESS MODELLING

Chapter 8

IDENTIFICATION OF STRUCTURAL CHARACTERISTICS IN EARLY PROCESS MODELLING

8.1. INTRODUCTION

The evaluation of early design models is an integral part of the process controllability, observability and operability studies. Such an evaluation aims at predicting characteristics, features of the full model at early stages with simple indicators. The basic methodology, we adopt here, is the evaluation of properties of the model by exploiting the underlining structure. Systems and Control Theory has predicted a lot of results which link structure to system properties and this defines the classical algebraic [Kailath, 1], [Rosen., 1], geometric [Wonham, 1], [Will., 1], algebrogeometric approaches [Brock. & Byr., 1], [Karc. & Gian., 1] etc. Structural approaches are linked to the theory of invariants and to the classification of values and types of such invariants according to the conditioning for the presence or absence of certain system properties. The characteristic of early process models is their simplicity and their endemic uncertainty, as far as the exact values of the parameters are concerned. For models with uncertainty in the parameters, there are a number of important issues to consider which are linked to the evaluation of structural properties. These include issues such as:

(i) Classification of system properties to generic, non-generic for systems with no known interconnection graph topology and parameter uncertainty.

(ii) Identification of structural characteristics on models with certain structure and parameter uncertainty.

In fact, models defined by certain general parameters, such as numbers of inputs, states, outputs but no other specification of parameters may be referred to as generic (with the parameter class). Models, which however have some additional features fixed, such as interconnection graph fixed, dominant dynamics of subsystems fixed etc., but with uncertainty in the remaining parameters, will be referred to as structured. Both generic and structured uncertain models are very important for evaluation of early process models. In this chapter, we are concerned with:

(a) Classification of unstructured and uncertain generic system properties.

(b) Evaluation of two important structural characteristics of structured uncertain transfer function models: the generic values of infinite zeros and the evaluation of the generic value of the McMillan degree.

The latter problem is referred to as "Structural Identification Problem" [Karc. et al, 2]. The results here extend those in [Karc. et al, 2] and provide an implementation of a theoretical algorithm [Vard. et al, 1] for evaluation of the infinite zero structure.

8.2. GENERICITY, SYSTEM PROPERTIES AND GENERIC VALUES OF SYSTEM INVARIANTS

8.2.1. Introduction: The Genericity Assumption

This section reviews the concept of genericity, as this is used in the context of state space models, and examines the generic, nongeneric nature of the fundamental system properties and key structural characteristics, the system invariants. The results of the review provide an important input to the problems of structure assignment, or structure formation avoidance in Global Instrumentation and have to be seen together with those on the generic solvability of families of control problems. This information may provide the basis for a library on model features and characteristics, which in the future should be seen as part of advanced design packages. The results here deal with unstructured models (no assumptions on any underlined graph) and provide useful input to the overall parameterisation of system problems when the genericity property holds true.

Since we are interested in the process design at the early stages, it seems to be appropriate to consider simple models as linear time invariant and finite dimensional. These

may be parameterised by quadruples (A, B, C, D) (modulo coordinate transformations). The study of systems, their classification and finally the characterisation of solvability conditions is based upon them. As it will be presented later, the system properties may be expressed in terms of tests based on the quadruple (A, B, C, D) describing the system. These tests however, when the system is of the large-scale type, are computationally expensive and have numerical problems. It is thus worth developing simple tests (rules of thumb), based on very rough characteristics of the systems that allow the testing of a property (roughly initially) without resorting to the full test. This is where the notion of genericity plays an important role. A system property is generic when it does not occur only in a negligible subset of the set of systems (i.e. the set of all quadruples (A, B, C, D)), or in other words when it occurs on a system taken randomly. Genericity depends on what we consider as set of systems of interest. This may be the set of all systems of p-inputs, m-outputs and n-states, or a subset of it, where A is fixed, or a subset of it, such that (A, B, C, D) has a certain graph structure.

To make the idea of genericity precise, we borrow some terminology from algebraic geometry. Consider polynomials $\varphi(\lambda_1,...,\lambda_n)$ with coefficients in R. A variety $V \subset R^N$ is defined to be the locus of common zeros of a finite number of polynomials $\varphi_1,...,\varphi_k$: $V = \{P \in R^N : \varphi_i(P_1,...,P_N) = 0, i \in k\}$. For example, one can prove [Wonham, 1], [Hirst & Smale, 1], that the set of all (A, B, C, D) of fixed dimensions modulo coordinate state transformations is a variety.

A property Π on V is merely a function $\Pi : V \to \{0,1\}$, where $\Pi(P) = 1$ (or 0) holds (or fails) at P. Let V be a proper variety, we shall say that Π is a generic relative to V, provided $\Pi(P) = 0$ only for points $P \in V' \subset V$ where V' is a proper subvariety of V; and that Π is generic provided such a V' exists. If Π is generic, we sometimes write $\Pi = 1(g)$. As V' is a locus of zeros of polynomials in V, the subset of V such that the property is not true is a negligible set (measure zero).

From this point onwards, when the term state space generic system is used, it will describe the underlining set of systems, which has p-inputs, m-outputs and n-states without assuming any special structure on (A, B, C, D). An alternative form of genericity may be defined on transfer function models. In fact, by fixing the number of inputs and outputs and leaving the elements in the matrix free (SISO transfer functions with arbitrary dynamics), we have the set of unstructured generic transfer functions; such transfer functions are very general models and their McMillan degree may be unbounded. Subsets of this general sets

are those where the McMillan degree is bounded and those where the elements of the transfer function have fixed dominant dynamics (due to modelling assumptions on the subsystems) but otherwise free elements. The first subset is referred to as Bounded McMillan degree generic transfer functions and the second as Dynamically Structured generic transfer functions. Both classes of models of the transfer function type are important for early process studies. The latter will be used for the study of the generic McMillan degree.

8.2.2. Genericity, Invariants and Properties

In this section, some of the most fundamental properties of linear system models will be examined, which are essential in understanding the dynamic behaviour of performance characteristics of control systems and their generic properties will be summarised. A detailed account of the topic may be found in [Karc., 11]. By generic properties or values, we mean the properties – values on the general element of the set under consideration. For the different set of invariants, their properties as far as genericity results may be summarised as shown below [Karc., 11]:

(i) Eigenvalues, Poles and Eigenvectors

Genericity Properties: a) The eigenvectors can have any value; the only restriction is that the product of left and right eigenvectors is identity, b) The poles can have any value in the complex plane. Generically, the poles can get distinct values and the generic Segre characteristic of every pole contains only one element i.e. the geometric multiplicity of every pole is one.

Remark (8.1): Repeated poles may emerge in many physical systems due to the nature of system dynamics and the underlined graph. Such models are topologically or dynamically structured.

□

(ii) Finite Zeros and Zero Directions

Genericity Properties: a) The presence of zeros is generic only on systems which are square. b) For square systems there is nothing we can say on the distribution of zeros on the complex plane and for their associated zero directions; the zeros are generically distinct and their number is n-m, that is all zeros are generically finite. c) Given that zeros are defined on

both the state space and transfer function, the generic values of the two sets are the same (due to that generically the model is both controllable and observable). d) A generic nonsquare system has no zeros and a generic square system has no infinite zeros.

(iii) Infinite zeros

Genericity Properties: If G(s) is a transfer function of a system of p-inputs and m-outputs, then if G(s) is proper generic it has no infinite zeros. If $m = p$ and if G(s) is strictly proper, we have m first order infinite zeros (P(s) m second order infinite zeros), whereas if G(s) is proper we have no infinite zeros.

Remark (8.2): For generic transfer function models, which are affected by dynamic or topological structuring, we may get values for orders of infinite zeros, which are different than the general generic properties above.

□

(iv) Stability of Linear Systems

Genericity Properties: The notion of stability for linear systems depends on root distribution on a certain area of the complex plane. The sets of stable or unstable polynomials are both sets of infinite measure. Thus stability, instability can not be associated with genericity. The equivalence, however, of internal, external stability is a generic property, since controllability and observability are generic properties for a system.

(v) Controllability, Observability, Minimality

Genericity Properties: The controllability and observability properties of a system depend on the rank of a matrix and therefore, they depend polynomially on the system. This together with the fact that we can always find controllable and observable systems proves that these properties are generic. In fact, uncontrollability (existence of input decoupling zeros) and unobservability (existence of output decoupling zeros) are problems equivalent to the existence of a nontrivial greatest common divisor of a set of appropriate polynomials; thus, uncontrollability – unobservability are non-generic properties.

Remark (8.3): For systems, which are structured by an interconnection graph and where inputs, outputs are selected with some physical criteria (not randomly), uncontrollability and unobservability may become a generic property within this family of structured models.

This necessitates the differentiation between the standard controllability, observability notions and those depending on graph, which are referred to as structural [Rijn., 1].

□

8.2.3. Generic Values of Invariants

On a given system, we may apply different types of transformations, some of them corresponding to a change of representation and some others having a compensation or feedback interpretation. The theory of system invariants is important for Control Theory and Design since it describes structural characteristics, which remain unaffected under the transformation and thus are indirectly related to the limits of compensation. Their importance for the design at the early stages is that for certain types of them, it might be possible to assign desirable values by early design decisions or avoid the formation of undesirable ones. This may precondition the presence of desirable properties or exclude the formation of undesirable ones. Here, we examine the generic values that invariants may take when they generically exist. A more detailed account on the topic is in [Karc., 11].

(i) Segre Characteristic and Segre Index

For a given matrix $A \in R^{n \times n}$ and with eigenvalues $\{\lambda_i\}$, the Segre index is the maximal of all cardinalities of the λ_i – Segre characteristic sets (maximum geometric multiplicity of all eigenvalues of A).

Generic Values: As a generic A has distinct eigenvalues the Segre characteristic generically is equal to {1}. The Segre index is generically equal to 1.

Role to Control Problems: The geometric multiplicity plays an important role on the controllability properties of the eigenvalue. In fact, if this multiplicity is greater than the number of inputs (outputs), then the eigenvalue is uncontrollable (unobservable). The importance of the Segre index lies in the fact that if it exceeds the number of inputs p, then the system is uncontrollable.

Remark (8.4): For structured systems by a graph of interconnections or dynamically generic models, the Segre index may be different than the generic value of 1. For such cases, its computation is important for defining the minimal required number of inputs and outputs, which may precondition controllability and observability.

□

(ii) Controllability, Observability Indices

The controllability indices $\{c_1,c_2,\ldots,c_p\}$ is the set of column minimal indices of the pencil $[sI-A,B]$, whereas the observability indices $\{d_1,d_2,\ldots,d_m\}$ is the set of all row minimal indices of the pencil $[sI-A^t,C^t]^t$.

Generic Values: Although, they can take any value that satisfies $n=\sum_{i=1}^{l} c_i=\sum_{i=1}^{m} d_i$, when the system is controllable and observable, the generic values of these indices are [Wonham, 1]:

$$\text{contr. ind.}=\left\{\frac{c,\ldots,c}{p-r_1},\frac{c+1,\ldots,c+1}{r_1}\right\},\ \text{obs. ind.}=\left\{\frac{d,\ldots,d}{m-r_2},\frac{d+1,\ldots,d+1}{r_2}\right\}$$

$$\text{where } c=\left[\frac{n}{p}\right],\ r_1=n-pc,\ d=\left[\frac{n}{m}\right],\ r_2=n-md \text{ and } [\cdot] \text{ denotes integer part.}$$

Role to Control Problems: These indices play an important role to many problems; the more important of these being:

a) Assignment of invariant factors via state feedback where the invariant factors ψ_i must satisfy: $\sum_{i-1}^{r}\deg\psi_{p+1-i}\le\sum_{i=1}^{r}c_i,\ r=1,2,\ldots,p.$

b) Arbitrary pole placement via static output feedback [Lev. & Karc., 1], where there must exist a controllability index such that: $\left[\frac{n}{p}\right]\le c_i\le m$.

(iii) Essential orders

The essential orders q_i of a system are defined to be the least integer $k\in 0,1,\ldots,n-1$ such that $\underline{c}_iA^kB\neq 0$, where $\underline{c}_i$ denote the rows of C.

Generic Values: They can take values for 0 to n-1; however, they are generically equal to 0 as CB has generically nonzero rows.

Role to Control Problems: The essential orders play a role in the decoupling problem where the matrix formed by the $c_i A^{q_i} B$ must be nonsingular.

(iv) Infinite zeros

For the generic values of infinite zeros, we have the properties:

Generic Values: If G(s) is a transfer function of a system of p-inputs and m-outputs with $p \neq m$, then if G(s) is proper generic it has no infinite zeros. If $m = p$ and G(s) is strictly proper, we have m first order infinite zeros, whereas if G(s) is proper, we have no infinite zeros.

Role to Control Problems [Desc. & Dion, 1], [Mal. & Kuc., 1]**:** The infinite zeros play a crucial role to the exact model matching problem and as a result to the (open loop) disturbance rejection problem. As a matter of fact, the equation $T_1 X = T_2$ has a proper solution X (T_1, T_2 matrices are proper), if and only if $[T_1 \quad -T_2]$ and T_1 have the same infinite zero structure. If T_1 is square, then for generic T_1 and T_2, the model matching problem does not have a proper solution. A second problem, where the infinite zeros have a central role is the problem of decoupling. The system (C,A,B) is decouplable if and only if the infinite zero orders of (C,A,B) are respectively equal to the infinite zero orders of the subsystems (c_i,A,B), where c_i is the i-th row of C.

(v) Forney Dynamical Order and Forney Indices of Transfer Functions

The Forney invariants of a rational vector space are the Echelon invariants of a minimal polynomial basis of this space. The Forney dynamical order of a transfer function is the polynomial degree of the minimal basis of the rational vector space spanned by the columns (or rows) of the transfer function. The Forney indices are the indices of the minimal polynomial basis spanning this space.

Generic Values: The Forney index can take any value from zero to n, whereas the indices can take any value $\delta_1, \delta_2, \ldots, \delta_p : \sum_{i=1}^{p} \delta_i \leq n$. However, for a generic proper system the values of the Forney indices are the generic values of controllability indices, whereas, in the strictly proper case, they are the generic values of the controllability indices minus one.

Role to Control Problems: The value of the Forney order and indices play an important role in the zero assignment problem via squaring down. The number of assignable zeros is generically equal to the Forney order and arbitrary zero assignment may be achieved generically if and only if $m(m-p) \geq \delta$.

8.2.4. Summary

The system properties and structural aspects related to invariants have been reviewed for the family of generic systems, for which, we make the assumption that there is no underlined graph structure. The results on generic values of invariants and generic system properties together with those dealing with the generic solvability conditions are prime inputs to the problems of structure assignment or structural features avoidance. The work here has been restricted to fundamental system properties and invariants and they provide the basis of a library on system characteristics and properties, which in the future is envisaged as an indispensable complement to design. The case of unstructured systems (no underlying graph) was examined here, since generic properties on given graph composite systems is an area still in its early stages. The list of invariants considered here are by no means exhausted. Some additional results on the generic rank properties of the Plücker matrices (entering in the solvability of pole - zero assignment problems) are discussed in [Karc., 11].

8.3. GENERIC PROPERTIES OF TRANSFER FUNCTIONS AT INFINITY

8.3.1. Pole – Zero Structure at Infinity and Valuations

The poles and zeros of a system at $s = \infty$ are important in the sense that they provide the information on the system behaviour at $s = \infty$ [Kailath, 1], [Vard. et al, 1]. There are several problems why it is important to keep track of the behaviour at $s = \infty$. Poles at $s = \infty$ characterise nonproper systems (or systems with differentiators), as may arise in constructing inverse systems, while the zeros at ∞ are important, for example, in studying the asymptotic behaviour of multivariable root loci, decoupling etc. For scalar systems with n poles and m zeros, m>n, m of the closed-loop poles will converge to the n-m zeros at infinity under high gain. A similar conclusion can be made for multivariable systems. The definitions given for the poles and the zeros over R[s] do not extend to $s = \infty$, as it can be

shown that the R[s]-unimodular matrices used to transform N(s) into the Smith form, can have both poles and zeros at ∞. So by unimodular transformations, the information at $s = \infty$ will be destroyed. It is important to observe that the pole – zero information of the system is preserved for any finite frequency when under R[s]-unimodular transformations but not always the structure at infinity. If we make the bilinear transformation:

$$s = \frac{a\lambda + b}{c\lambda + d} \tag{8.1}$$

where $c \neq 0$ and $ad - bc \neq 0$, which will merely transform the complex s plane into itself, then this transformation will move the point at $s = \infty$ to the point $\lambda = -\frac{d}{c}$. The Smith-McMillan form for $H(\lambda)$ will accurately reflect the behaviour of $H(\lambda)$ at all points except those at $s = \infty$. In particular, the Smith-McMillan structure at $\lambda = -\frac{d}{c}$ will accurately reflect that of H(s) at $s = \infty$. If the constants a, b, c, d are chosen as $a = d = 0, b = c = 1$, then $s = \frac{1}{\lambda}$ and with the substitution $s = \frac{1}{\lambda}$, the pole-zero information at $s = \infty$ can be studied at $\lambda = 0$.

An alternative way of calculating the pole-zero structure of a system both at finite and infinite frequencies is to use the valuation method [Kailath, 1], [Vard. et al, 1], which characterises the Smith-McMillan form directly. Define for a scalar rational function g(s) the discrete valuation at $s = \infty$ by:

$\upsilon_\infty(g) \equiv$ the ∞ valuation of g(s) = denominator degree - numerator degree (SCALAR CASE)

$\upsilon_\infty^{(i)}(g) \equiv$ the algebraically smallest of valuations at $s = \infty$ of all the $i \times i$ minors of H(s)
(MATRIX CASE)

The Smith-McMillan form at $s = \infty$ is defined by:

$$M_\infty(s) = \text{diag}\{s^{\sigma_1(\infty)}, \ldots, s^{\sigma_r(\infty)}\} \tag{8.2}$$

where:

$$\sigma_1(\infty) = \upsilon_\infty^{(1)}, \sigma_2(\infty) = \upsilon_\infty^{(2)} - \upsilon_\infty^{(1)}, \ldots \tag{8.3}$$

For transfer functions which have fixed number of inputs and outputs, but contain uncertain elements, the computation of the generic infinite structure is an important problem and it is considered next.

8.3.2. Computation of Infinite Zero Structure for Generic Transfer Function Models

The family of unstructured generic models in the frequency domain is defined formally, as shown below:

Definition (8.1): For transfer function models G(s), where G(s) is proper, it is assumed that the number of inputs (p), outputs (m) are fixed, but the $g_{ij}(s)$ elements of G(s) are generic proper rational functions. The family of systems is denoted by $\sum_{pr}(p,m)$.

□

Then for generic systems in the family $\sum_{pr}(p,m)$, we have the following result:

Theorem (8.1): [Karc. & Kouv., 1] For generic systems of the $\sum_{pr}(p,m)$ family, the following properties hold true:

- The generic element of $\sum_{pr}(p,m)$ has no infinite zeros. If the system is strictly proper, then the generic system of $\sum_{pr}(p,m)$ has min{m, p} number of first order infinite zeros.
- If $m \neq p$, the generic system has no finite zeros. If $m = p$, the generic proper system has n finite zeros and the generic strictly proper system has $n-m$ finite zeros, where n is the McMillan degree of G(s).

□

We consider a family of systems described in the frequency domain with a fixed structure. A system H(s) is structured when it has fixed order of numerators and denominators in each placement of H(s). We may illustrate this as:

$$H(s)=\begin{bmatrix} A_1^2A_2 & A_1 & A_3 \\ A_3 & A_2^2 & A_1A_2A_3 \\ 0 & A_4 & A_1^2 \end{bmatrix} \tag{8.4}$$

where $A_i, i=1,2,3,4$ are dynamic models in the frequency domain whose orders are fixed while the coefficients in both the denominator and the numerator may take arbitrary values. We define the valuation at infinity as:

$$\upsilon_\infty(h_{ij})=\upsilon_\infty^{ij}(H)\equiv \text{The element}-\text{wise valuation of } H(s)=\left[u_\infty^{ij}\right]$$

where $\left[u_\infty^{ij}\right]=\upsilon_\infty^{ij}(h_{ij}), i=1,...,m, j=1,...,p$. Note that for scalar rational functions, u_∞^{ij} are expressed as differences between denominator and numerator polynomial degrees and thus are given by the roll-off rate of the Bode magnitude plot. Hence, the roll-off rate defines the structure at infinity for the scalar case. This idea may be generalised to the matrix case under certain genericity assumptions and may thus provide a simple way for finding the generic Smith-McMillan form at infinity of a rational matrix representing an uncertain early model. To show this, let $H(s)\in R^{p\times m}(s)$ and $H_{j_1,j_2,...,j_k}^{i_1,i_2,...,i_k}$ be the k-th order minor composed of rows $i_1,i_2,...,i_k$ and columns $j_1,j_2,...,j_k$ of T(s). Then:

$$H_{j_1,j_2,...,j_k}^{i_1,i_2,...,i_k}=\sum^{k!}(-1)^{\mu}\left(h_{i_1p_1},h_{i_2p_2},...,h_{i_kp_k}\right) \tag{8.5}$$

where $p_1,p_2,...,p_k$ is a permutation of $j_1,j_2,...,j_k$ and the sum is taken over all possible permutations (the exponent μ is the number of transpositions required to go from the natural order $j_1,j_2,...,j_k$ to $p_1,p_2,...,p_k$. Taking valuations of both sides of (7.5) yields:

$$\upsilon_\infty\left\{H_{j_1,j_2,...,j_k}^{i_1,i_2,...,i_k}\right\}\geq \min\left\{\upsilon\left(h_{i_1p_1},h_{i_2p_2},...,h_{i_kp_k}\right)\right\}=\min\left(\sum_{\ell=1}^{k}\upsilon_\infty\left(h_{i_\ell p_\ell}\right)\right) \tag{8.6}$$

For the construction of the Smith-McMillan form at infinity, we require the valuation of all order minors, i.e.

$$\upsilon_\infty^{(k)}=\upsilon_\infty^{k}(H)=\min\left\{\upsilon_\infty\left\{H_{j_1,j_2,...,j_k}^{i_1,i_2,...,i_k}\right\}\right\}\geq \min\left(\sum_{l=1}^{k}\upsilon_\infty\left(h_{i_lp_l}\right)\right) \tag{8.7}$$

where the minima are taken over all indices $1 \le i_1 < \ldots < i_k \le p$, $1 \le j_1 < \ldots < j_k \le m$. Clearly, $\upsilon_\infty(t_{ij})$ are the asymptotic roll-off rates of the individual entries of the rational matrix. Before the $\upsilon_\infty(t_{ij})$ can be used to find the $\upsilon_\infty^{(k)}(H)$, we have to establish the conditions under which equality holds in (8.7). This is given by the following Lemma [Vard. et al, 1]:

Lemma (8.1): Given two rational functions $t_1(s)$ and $t_2(s)$, then if $\upsilon_\infty(t_1(s)) \neq \upsilon_\infty(t_2(s))$

$$\upsilon_\infty(t_1(s)+t_2(s)) = \min\{\upsilon_\infty(t_1(s)), \upsilon_\infty(t_2(s))\} \tag{8.8}$$

Proof: Let $t_i(s) = \frac{n_i(s)}{d_i(s)}, i = 1,2$ where $n_i(s)$ and $d_i(s)$ are polynomials, then

$$t_1(s) + t_2(s) = \frac{n_1(s)\,d_2(s) + n_2(s)\,d_1(s)}{d_1(s)\,d_2(s)} \tag{8.11}$$

since $\upsilon_\infty(t_1(s)) \neq \upsilon_\infty(t_2(s))$, we have that $\deg(n_1(s)) + \deg(d_2(s)) \neq \deg(n_2(s)) + \deg(d_1(s))$, therefore:

$$\upsilon_\infty(t_1(s)+t_2(s)) = \deg(d_1(s)) + \deg(d_2(s)) - \max[\{\deg(n_1(s)) + \deg(d_2(s))\},$$
$$\{\deg(n_2(s)) - \deg(d_1(s))\}] = \min\{\delta(t_1(s)), \delta(t_2(s))\}$$

□

This Lemma shows that if there is only one least valuation product term $\sum_{l=1}^{k} \upsilon_\infty(h_{i_l p_l})$ in (8.7), then equality holds in (8.7). If there are more than one least valuation product term, however, cancellations may occur during the formation of the least valuation minor of a given order k. Almost any small perturbation to the transfer function matrix will remove these valuation-increasing cancellations and decrease the order of some of the infinite zeros and generate finite ones. In the general case, therefore, equality holds in (8.7) even if there is more than one least valuation product term in this expression. This leads to the following result:

Proposition (8.1): Given two generic rational functions f(s), g(s) with valuations $\alpha = \upsilon_\infty(f)$ and $\beta = \upsilon_\infty(g)$ respectively, then

$$\upsilon_\infty(f \cdot g) = \upsilon_\infty(f) + \upsilon_\infty(g) \text{ and } \upsilon_\infty(f \pm g) = \min\{\upsilon_\infty(f), \upsilon_\infty(g)\} \tag{8.12}$$

□

It is thus clear that the computation of the generic Smith-McMillan form at infinity may be simplified by using a representation of the valuations of elements and computations based on integer matrices.

Remark (8.5): A structured transfer function matrix $H(s) \in R_{pr}^{m\times p}(s)$ can be transformed into an integer matrix $\upsilon_\infty(H)$ of the same dimension whose entries are the valuations of the corresponding entries in the structured transfer function matrix.

□

The computation of valuations for generic transfer functions may be considerably reduced by using the following definition:

Definition (8.2): Given an integer matrix $H \in Z^{m\times p}$, then the value of valuation of every element is its valuation order. A path is a sequence of m elements selected from the matrix with no two elements from the same column or from the same row. The length of a path is the number of non-zero elements in the path. The valuation weight of the path is defined as the sum of the orders of the elements in the path. The minimal weight of all the independent paths of length i of the matrix is denoted by $\Gamma_{\upsilon_\infty}^{i}(H)$.

□

The above notions can be defined for the integer matrix $\upsilon_\infty(H)$ as proposed before. We can now establish the following result concerning the valuation of H(s) at infinity:

Proposition (8.2): The $i\times i$-th order valuation of the generic structured H(s) at infinity is equivalent to the minimal weight of length i in matrix $\upsilon_\infty(H)$.

Proof: If the minimal weight of any independent paths with length i is t_i, we have to prove that there exists an $i\times i$ order minor in H(s) such that the valuation of this minor at infinity

is t_i. Construct an $i \times i$ order minor by taking the minor just containing the minimum weight path. Then the order of the minor will be $i \times i$ and by employing Proposition (7.1), the result is established.

□

Example (8.1): Consider the system defined by equation (8.4). If we assume that the components in the transfer function are of the form:

$$A_1 = \frac{a_{11}}{s^2 + b_{11}s + b_{12}},\ A_2 = \frac{a_{21}s^2 + a_{22}s + a_{23}}{b_{21}s^4 + b_{22}s^3 + b_{23}s + b_{24}}, A_3 = \frac{a_{31}s + a_{32}}{b_{31}s + b_{32}},\ A_4 = \frac{a_{41}}{s^2 + b_{41}s + b_{42}}$$

then the element-wise valuation of the matrix H(s) is given as:

$$\upsilon_\infty(H) = \begin{bmatrix} 6 & 2 & 0 \\ 0 & 4 & 4 \\ \infty & 2 & 4 \end{bmatrix}$$

The generic valuation of 1×1 minors of the matrix H(s) at infinity is the minimum along all the entries or $\upsilon_\infty^{(1)}(H) = 0$. The minimal weight paths of the 2×2 minors:

$$\begin{bmatrix} 6 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 4 \\ \infty & 2 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 4 \\ \infty & 4 \end{bmatrix}, \begin{bmatrix} 6 & 2 \\ \infty & 2 \end{bmatrix}, \begin{bmatrix} 6 & 0 \\ \infty & 4 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix}$$

are 2, 0, 4, 2, 6, 4, 8, 10, 2 respectively. So the generic valuation of the 2×2 minors at infinity is 0, i.e. $\upsilon_\infty^{(2)}(H) = 0$. The generic valuation of the 3×3 matrix at infinity is the minimum weight path of $\upsilon_\infty(H)$ which is 2, so $\upsilon_\infty^{(3)}(H) = 2$. Finally, we have: $\upsilon_\infty^{(1)}(H) = 0$, $\upsilon_\infty^{(2)}(H) = 0$, $\upsilon_\infty^{(3)}(H) = 2$ and therefore the rational matrix generically has one zero of order 2 at infinity and no generic poles.

□

Remark (8.6): Assuming that the normal rank of the matrix H(s) is r, because we consider only the proper system, the difference between the total number of zeros and the total number of poles at infinity is given by the minimal weight of all the independent paths with a length r.

□

Remark (8.7): From the above definition and analysis, it is clear that the number of generic poles and zeros at infinity remains unchanged as long as the element-wise valuation of the rational matrix is the same, because then all the minors of all orders will be the same. In fact, this happens when the valuations of the elements A_i are fixed.

□

Remark (8.8): In order to find the minimal weight of all the independent paths, the methods proposed in Section 8.3.2 for finding the maximal weight of all the independent paths can be adapted. The only change needed is to mark the minimum weight path instead of the maximum weight path.

□

8.3.3. The Computation of the Generic Infinite Structure and the Bode Diagrams

Given the generic transfer function $H(s) \in R^{m\times p}(s)$, (note that H(s) may be given in terms of the frequency plots of its elements) the plotting of Bode diagrams of its elements provides a matrix $\upsilon_\infty(H)$ with elements the valuations of the individual elements $h_{ij}(s)$ defined by the roll-off rates. The matrix $\upsilon_\infty(H)$ will be referred to as the asymptotic Bode representation of H(s). The computation of the Smith-McMillan form at infinity of the generic H(s) is based on the following algorithmic procedure, derived from the results in [Vard. et al, 1]:

Algorithm for Generic Structure at Infinity

Given the matrix $\upsilon_\infty(H)$, constructed from the asymptotic roll-off rates of the Bode diagrams $h_{ij}(s)$ of H(s), we compute the Smith-McMillan form at infinity as indicated below. We first denote by $\upsilon^i_\infty(H) \in Z$ the least valuation among the valuations of all minors of H(s) of order j, $j \in \underset{\sim}{\rho}$, where ρ is the normal rank of H(s), where $\upsilon^0_\infty(H) = 0$.

Step (1): Compute for all $i \in \underset{\sim}{\rho}$ the $\upsilon^i_\infty(H)$ generic values of the i-th order valuation as the minimal weight of length i in the matrix $\upsilon_\infty(H)$, $\Gamma^i_{\upsilon_\infty}(H) \overset{\Delta}{=} \upsilon^i_\infty(H)$.

Step (2): Define the generic invariant functions:

$$i_1(s) = s^{\upsilon_\infty^0(H)-\upsilon_\infty^1(H)}, i_2(s) = s^{\upsilon_\infty^1(H)-\upsilon_\infty^2(H)}, \ldots, i_\rho(s) = s^{\upsilon_\infty^{\rho-1}(H)-\upsilon_\infty^\rho(H)}, \upsilon_\infty^0(H) = 0 \tag{8.11}$$

Step (3): The generic Smith-McMillan structure at infinity is then defined by:

$$S_{H(s)}^{\infty} = \left[\begin{array}{c|c} \mathrm{diag}\{i_1(s), i_2(s), \ldots, i_\rho(s)\} & 0 \\ \hline 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} \mathrm{diag}\left\{s^{q_1}, s^{q_2}, \ldots, s^{q_k}; \frac{1}{s^{\bar{q}_{k+1}}}, \ldots, \frac{1}{s^{\bar{q}_\rho}}\right\} & 0 \\ \hline 0 & 0 \end{array}\right] \tag{8.12}$$

where the orders of generic infinite poles (defined for nonproper systems) are given by the set $\{q_1, \ldots, q_k\}$ and the orders of generic infinite zeros are given by the set $\{\bar{q}_{k+1}, \ldots, \bar{q}_\rho\}$, where $q_i > 0$ and $\bar{q}_j > 0$.

□

To illustrate the above procedure consider the following example [Vard. et al, 1]:

Example (8.2): Let H(s) be defined by:

$$H(s) = \begin{bmatrix} \frac{1}{(s+1)^2} & s^3 & \frac{s^2}{s+1} \\ \frac{s+2}{s^2+0.2s+1} & \frac{1}{s^3} & \frac{1}{(s+2)^2} \end{bmatrix}$$

For this transfer function the asymptotic Bode representation is obtained from the plots:

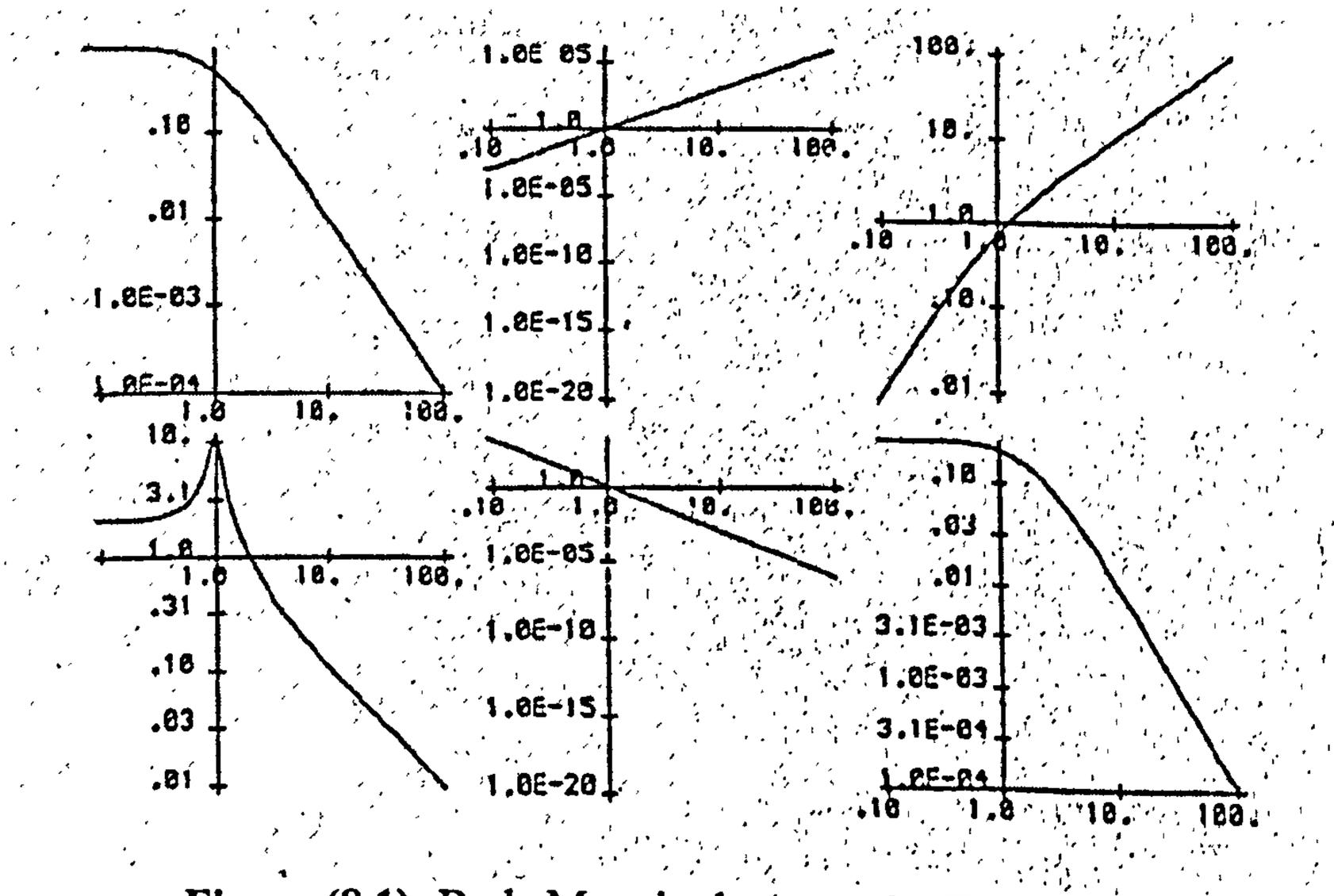

Figure (8.1): Bode Magnitude Array for Example (8.2)

$$\upsilon_\infty(H)=\begin{bmatrix}2 & -3 & -1\\ 1 & 3 & 2\end{bmatrix}$$

Using the algorithm procedure, the generic values of $\upsilon_\infty^i(H)$, are:

$$\upsilon_\infty^1(H)=-3,\ \upsilon_\infty^2(H)=\min\{2+3,1-3,2+2,1-1,2-3,3-1\}=-2$$

and thus:

$$i_1(s)=s^{\upsilon_\infty^0(H)-\upsilon_\infty^1(H)}=s^{0-(-3)}=s^3, i_2(s)=s^{\upsilon_\infty^1(H)-\upsilon_\infty^2(H)}=s^{-3-(-2)}=s^{-1}$$

and the generic Smith-McMillan form at infinity is:

$$S_{H(s)}^\infty=\begin{bmatrix}s^3 & 0 & 0\\ 0 & \frac{1}{s} & 0\end{bmatrix}$$

The advantages of the algorithm for large dimension problems is demonstrated by the following example:

Example (8.3): Consider a 7×5 transfer function matrix H(s) defined by:

$$H(s)=\begin{bmatrix}
s^2+1 & \frac{1}{s^4} & \frac{s^3+2}{s^2+0.5s+2} & \frac{0.2}{1+5s} & \frac{s-3}{s+7}\\
\frac{2s}{(s+3)^2} & (s+1)^2 & s^3 & \frac{5}{s-4} & \frac{2}{2s+7}\\
\frac{s+2}{s^3-2s^2+5s-7} & \frac{4s^2+3}{s+4} & \frac{1}{(2s-3)^2} & \frac{(s-2)^3}{s} & \frac{1}{(s+2)^2}\\
\frac{s+1}{s-2} & s^3-2s+3 & \frac{s^4+6s^2-3}{s-4} & \frac{s+1}{s^3} & s\\
\frac{0.5}{1+4s} & \frac{s-1}{2s^2+s} & \frac{s}{s^5-2s^4+3s^3-4s^2+5s-6} & s^2+1 & s^4\\
\frac{1}{s+1} & \frac{2}{10s+1} & \frac{s-5}{s} & 2s+3 & s^2+3\\
\frac{2}{s^2} & \frac{s}{s+2} & \frac{0.7}{1+3s} & s^2 & \frac{1}{s^3+2s-5}
\end{bmatrix}$$

For the above transfer function matrix, the valuation matrix $\upsilon_\infty(H)$ is:

$$\upsilon_\infty(H) = \begin{bmatrix} -2 & 4 & -1 & 1 & 0 \\ 1 & -2 & -3 & 1 & 1 \\ 2 & -1 & 2 & -2 & 2 \\ 0 & -3 & -3 & 2 & -1 \\ 1 & 1 & 4 & -2 & -4 \\ 1 & 1 & 0 & -1 & -2 \\ 2 & 0 & 1 & -2 & 3 \end{bmatrix}$$

Using the algorithm procedure, the generic values $\upsilon_\infty^i(H)$ (for $i = 1,...,5$) are:

$$\upsilon_\infty^0(H) = 0,\ \upsilon_\infty^1(H) = -4,\ \upsilon_\infty^2(H) = -7,\ \upsilon_\infty^3(H) = -10,\ \upsilon_\infty^4(H) = -12,\ \upsilon_\infty^5(H) = -14$$

and thus:

$$i_1(s) = s^{\upsilon_\infty^0(H)-\upsilon_\infty^1(H)} = s^{0-(-4)} = s^4,\ i_2(s) = s^{\upsilon_\infty^1(H)-\upsilon_\infty^2(H)} = s^{-4-(-7)} = s^3,\ i_3(s) = s^{\upsilon_\infty^2(H)-\upsilon_\infty^3(H)} = s^{-7-(-10)} = s^3,$$

$$i_4(s) = s^{\upsilon_\infty^3(H)-\upsilon_\infty^4(H)} = s^{-10-(-12)} = s^2,\ i_5(s) = s^{\upsilon_\infty^4(H)-\upsilon_\infty^5(H)} = s^{-12-(-14)} = s^2$$

and the generic Smith-McMillan form at infinity is:

$$S_{H(s)}^\infty = \begin{bmatrix} s^4 & 0 & 0 & 0 & 0 \\ 0 & s^3 & 0 & 0 & 0 \\ 0 & 0 & s^3 & 0 & 0 \\ 0 & 0 & 0 & s^2 & 0 \\ 0 & 0 & 0 & 0 & s^2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The algorithm used here is based on finding weights of paths of different order and this is reduced to properties of integer matrices. This topic is similar to that considered in the next chapter and the role of integer matrices will be further explored there. The results given in the following section apply also here with the obvious necessary changes (min instead of max).

8.4. STRUCTURAL IDENTIFICATION OF THE GENERIC McMILLAN DEGREE OF A TRANSFER FUNCTION MATRIX

8.4.1. Introduction

The McMillan degree of a transfer function model is one of the most important structural characteristics and plays an important role in deciding the quality of a system model, as far as allowing solvability of certain control problems. In this section, the problem of identifying the generic McMillan degree of a rational matrix is considered. The transfer function matrices of interest for early process design studies are those referred to as "Structured Transfer Functions" (STF) matrices and have certain elements fixed to zero, some elements being constant and other elements expressing the simple dynamics of the subsystem models used. It is also assumed that although the general rule for interconnecting the subprocesses is known, the exact nature of the underlying interconnection graph (dependent on the knowledge of the actual McMillan degree of subprocesses) is not known. For the family of STF matrices the problem of determining the generic McMillan degree is considered using genericity arguments and an optimisation procedure based on path properties on non-negative integer matrices. The problem considered here is another of the problems of the area of Structural Identification, where the evaluation of structural characteristics on structured models is under investigation with robust computation methods. Once more, the overall approach adopted here is based on properties of non-negative integer matrices and the problem is reduced to an optimal path problem. The proposed solution is of special interest to large-scale early models, where the use of the approaches applicable for small dimension exact models is not appropriate. Note that the solution of the problem considered here is of similar nature to the one previously considered for the infinite structure and relies on properties of integer matrices.

In this section, the generic McMillan degree of a rational matrix is investigated. From the definition, the McMillan degree of a rational matrix can be calculated from the orders of the denominators of the matrix in Smith-McMillan form. So algorithms can be designed to first transform the rational matrix into Smith-McMillan form by using unimodular transformations and then find the sum of the orders of the denominators. This method is impractical in terms of computations to obtain the Smith-McMillan degree. An alternative has been suggested by MacFarlane and Karcanias [MacF. & Karc., 1], that is to obtain the pole polynomial as the least common multiple of the minors of all orders. The

order of this least common multiple gives the McMillan degree. This method may also be used for the computation of the generic form of the Smith-McMillan form, as well as the unstable McMillan degree. This method does not require the transformation of the rational matrix into Smith-McMillan form and computationally is more practical, faster and exploits the parameter uncertainty.

In the early design stages, the exact values of the parameters in the elements of the transfer functions are not known exactly. Yet, it is desirable to have some knowledge on the McMillan degree since it indicates the complexity of the system. Given the structure of the transfer function matrix and the type of the non-zero entries of the matrix, the evaluation of the McMillan degree of such systems will be termed as generic evaluation of the McMillan degree and the McMillan degree will be termed as generic McMillan degree of the given structured uncertain model. The poles and zeros of a system provide important information in the study of the multivariable root-locus design, system properties at infinity etc. When the parameters of the system are not given exactly, we also wish to calculate the generic degree of the poles and zeros at infinity. A robust early estimation of the value of the McMillan degree is essential for conditioning the solvability of many early design problems, such as selection of minimal required number of inputs, outputs irrespective of the values of model parameters.

8.4.2. Generic Structured Transfer Function Matrices

It is worth pointing out that the models at the early design stage provide only some structural information about the system. The structural information includes the fixed poles in the transfer function matrix, the orders of the non-zero entries of the matrix and some repeated patterns due to specific dynamic units, which are modelled with certain known dynamic complexity. But, otherwise, the values of the parameters of the transfer functions are not known exactly. Assume that all the entries of the transfer function are proper rational functions. For a system of dimensions $m \times p$, a structured overall transfer function matrix $H(s)$ is $H(s) \in R_{pr}^{m \times p}[s]$. The notion of structured transfer function matrices of variable complexity is demonstrated by the following examples.

Example (8.4): Consider the system of Figure (8.2), which can be represented by the aggregated model as in Figure (8.3), where the three subsystems are described by:

$$\underline{y}_1 = G_1 \cdot \underline{e}_1, \quad \underline{y}_2 = G_2 \cdot \underline{e}_2, \quad \underline{y}_1 = G_3 \cdot \underline{e}_3 \tag{8.13}$$

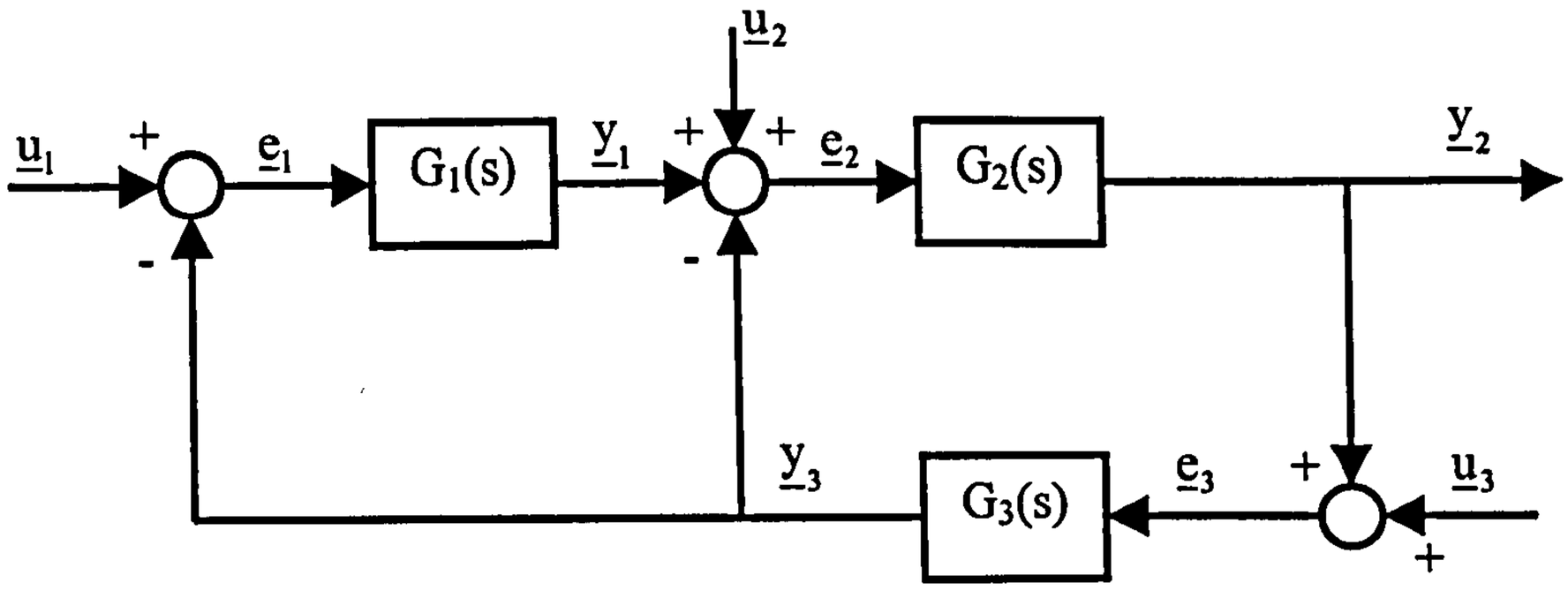

Figure (8.2): A General System

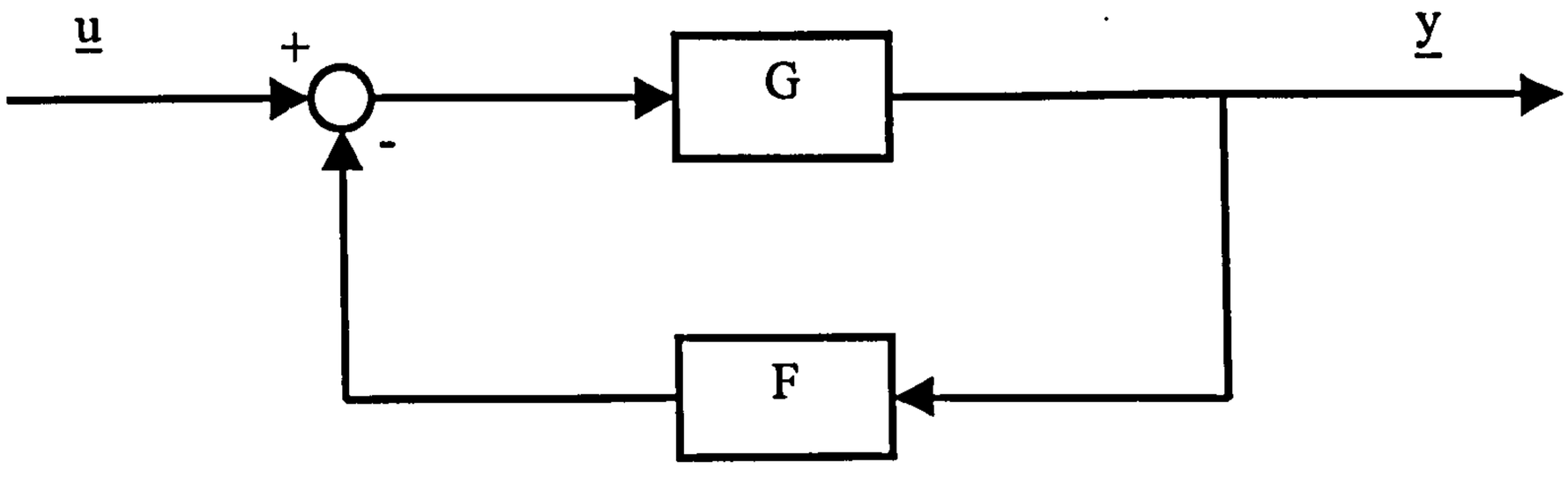

Figure (8.3): Aggregated Model

For the aggregated model, we have:

$$\underline{e} = \underline{u} - F\,\underline{y}, \quad \underline{y} = G(s)\,(I + FG(s))^{-1}\underline{u} \tag{8.14}$$

where $\underline{u} = \left[\underline{u}_1^T, \underline{u}_2^T, \underline{u}_3^T\right]^T$, $\underline{y} = \left[\underline{y}_1^T, \underline{y}_2^T, \underline{y}_3^T\right]^T$, $\underline{e} = \left[\underline{e}_1^T, \underline{e}_2^T, \underline{e}_3^T\right]^T$ and

$$G(s) = \begin{bmatrix} G_1(s) & & 0 \\ & G_2(s) & \\ 0 & & G_3(s) \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 & I \\ -I & 0 & I \\ 0 & -I & 0 \end{bmatrix}$$

The matrix F represents the interconnections, while matrix G(s) represents the dynamics of the aggregated system. Assume that the subsystems are all of dimension 2×2:

$$G_1(s) = \begin{bmatrix} \dfrac{1}{(s+1)(s+5)(s+10)} & \dfrac{6}{(s+1)(s+2)(s+5)(s+10)} \\ \dfrac{1}{(s+2)(s+5)(s+10)} & \dfrac{1}{(s+1)(s+2)(s+5)(s+10)} \end{bmatrix}$$

$$G_2(s)=\begin{bmatrix}\frac{2.5}{s+1} & 0\\ 0 & \frac{0.75}{s+2}\end{bmatrix},\ G_3(s)=\begin{bmatrix}\frac{0.12(s-1)}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)}\\ \frac{6}{(s+1)(s+2)} & \frac{s-2}{(s+1)(s+2)}\end{bmatrix}$$

Following the procedure in the previous section, the structure of the aggregate system can be modelled by the Boolean and steady-state models as:

$$G_s=\begin{bmatrix}1&1&0&0&0&0\\1&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&1\\0&0&0&0&1&1\end{bmatrix},G_o=\begin{bmatrix}0.02&0.06&0&0&0&0\\0.01&0.01&0&0&0&0\\0&0&2.5&0&0&0\\0&0&0&0.375&0&0\\0&0&0&0&-0.06&0\\0&0&0&0&3.0&-1\end{bmatrix}$$

and if we model the system by taking only the dominant pole, then we have:

$$G_1=\begin{bmatrix}\frac{0.02}{s+1}&\frac{0.06}{s+1}&0&0&0&0\\\frac{0.01}{s+2}&\frac{0.01}{s+1}&0&0&0&0\\0&0&\frac{2.5}{s+1}&0&0&0\\0&0&0&\frac{0.375}{s+2}&0&0\\0&0&0&0&\frac{-0.06(s-1)}{s+1}&\frac{s}{s+1}\\0&0&0&0&\frac{3.0}{s+1}&\frac{s-2}{s+2}\end{bmatrix}$$

The above models G_s, G_0, G_1 are approximations of the aggregate transfer function model G(s). The resulting composite system has an overall transfer function defined by:

$$H=G(I+FG)^{-1} \tag{8.15}$$

Note that:

$$(I+FG)=\begin{bmatrix}I&0&G_3\\-G_1&I&G_3\\0&-G_2&I\end{bmatrix}\text{ and }(I+FG)^{-1}=\begin{bmatrix}X_1&X_2&X_3\\X_4&X_5&X_6\\X_7&X_8&X_9\end{bmatrix} \tag{8.16}$$

where:

$$X_1 = I - G_3G_2(G_1G_3G_2 + G_3G_2 + I)^{-1}G_1,\ X_2 = -G_3(G_2G_1G_3 + G_2G_3 + I)^{-1}G_2$$

$$X_3 = -G_3(G_2G_1G_3 + G_2G_3 + I)^{-1},\ X_4 = (G_1G_3G_2 + G_3G_2 + I)^{-1}G_1$$

$$X_5 = G_2^{-1}(G_2G_1G_3 + G_2G_3 + I)^{-1}G_2,\ X_6 = G_2^{-1}\{(G_2G_1G_3 + G_2G_3 + I)^{-1} - I\} \qquad (8.17)$$

$$X_7 = G_2(G_1G_3G_2 + G_3G_2 + I)^{-1}G_1,\ X_8 = (G_2G_1G_3 + G_2G_3 + I)^{-1}G_2$$

$$X_9 = (G_2G_1G_3 + G_2G_3 + I)^{-1}$$

and thus the composite transfer function has the form:

$$H = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix}\begin{bmatrix} X_1 & X_2 & X_3 \\ X_4 & X_5 & X_6 \\ X_7 & X_8 & X_9 \end{bmatrix} = \begin{bmatrix} H_1 & H_2 & H_3 \\ H_4 & H_5 & H_6 \\ H_7 & H_8 & H_9 \end{bmatrix} \qquad (8.18)$$

where $H_i \neq 0, i = 1,2,...,9$. Note that the aggregate dynamics and topology, represented by F, are not easily identified by the values of H_i.

□

A composite system, such as the one represented by the above example, which does not allow identification of interconnections and aggregate dynamics from its transfer function H(s) will be called <u>full</u>. Not all of the composite systems have this property and this is demonstrated by the following example.

<u>Example (8.5)</u>: Consider the aggregate system of Example (8.4), but with an interconnection matrix:

$$F = \begin{bmatrix} I & 0 & -I \\ 0 & I & 0 \\ 0 & I & -I \end{bmatrix} \text{ and } (I+FG) = \begin{bmatrix} I+G_1 & 0 & -G_3 \\ 0 & I+G_2 & 0 \\ 0 & G_2 & I-G_3 \end{bmatrix} \qquad (8.19)$$

and thus

$$(I+FG)^{-1} = \begin{bmatrix} X_1 & X_2 & X_3 \\ X_4 & X_5 & X_6 \\ X_7 & X_8 & X_9 \end{bmatrix} \qquad (8.20a)$$

where the elements of the matrix are defined by:

$$X_1 = (I+G_1)^{-1},\ X_2 = -(I+G_1)^{-1}G_3(I-G_3)^{-1}G_2(I+G_2)^{-1},\ X_3 = (I+G_1)^{-1}G_3(I-G_3)^{-1},$$
$$X_4 = 0,\ X_5 = (I+G_2)^{-1},\ X_6 = 0,\ X_7 = 0,\ X_8 = -(I-G_3)^{-1}G_2(I+G_2)^{-1},\ X_9 = (I-G_3)^{-1} \quad (8.20b)$$

and thus

$$H = G(I+FG)^{-1} = \begin{bmatrix} H_1 & H_2 & H_3 \\ 0 & H_5 & 0 \\ 0 & H_8 & H_9 \end{bmatrix} \quad (8.20c)$$

By comparison of (8.19) and (8.20b), we see that part of the topology of interconnections is preserved in the overall transfer function as fixed blocks of zeros and thus the topology is partly reflected in the structuring of the transfer function. By inspection of (8.20b), it is also clear that part of the aggregate dynamics is also reflected in the composite system, but not in a straightforward manner.

Transfer functions with fixed blocks of zeros, representing part of the interconnection topology, will be referred to as topologically structured transfer functions. The first example demonstrates that not all interconnection topologies are reflected, even partly, on the transfer function.

Example (8.6): Consider the aggregate system of the previous examples with the interconnection topology of the cascade type as shown in Figure (8.4), for which:

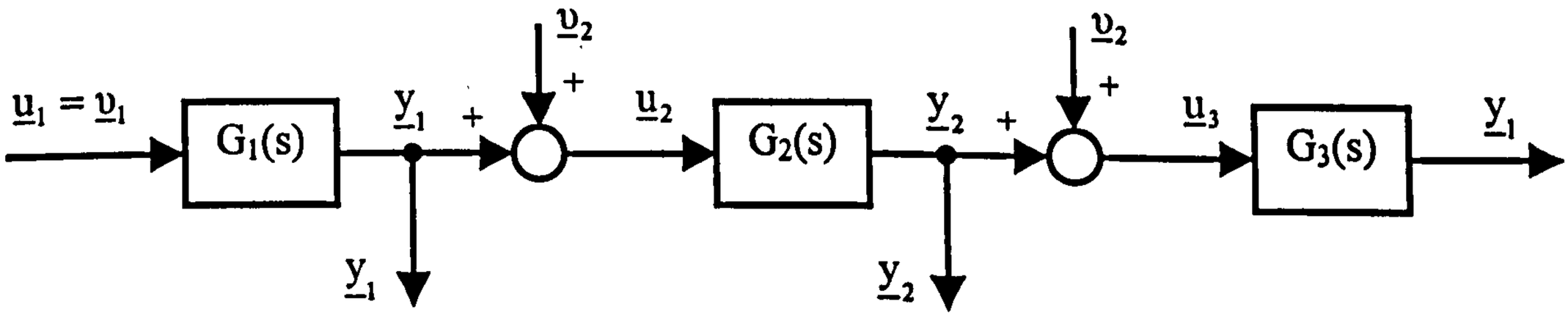

Figure (8.4): Cascade interconnection

$$\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \end{bmatrix} = \begin{bmatrix} G_1\underline{u}_1 \\ G_2\underline{u}_2 \\ G_3\underline{u}_3 \end{bmatrix} = \begin{bmatrix} G_1\underline{v}_1 \\ G_2\underline{v}_2 + \underline{y}_1 \\ G_3\underline{v}_3 + \underline{y}_2 \end{bmatrix} =$$

$$= \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix} \begin{bmatrix} \underline{\upsilon}_1 \\ \underline{\upsilon}_2 \\ \underline{\upsilon}_3 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ -I & 0 & 0 \\ 0 & -I & 0 \end{bmatrix} \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix} \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \end{bmatrix} \tag{8.21}$$

$\overset{\Delta}{=} G$ $\overset{\Delta}{=} F$

and the composite transfer function is defined by:

$$\underline{y} = H\,\underline{\upsilon}, \text{ where } H(s) = G(s)\,(I + FG(s))^{-1}\underline{\upsilon} \tag{8.22}$$

The composite transfer function has the form:

$$(I+FG) = \begin{bmatrix} I & 0 & 0 \\ -G_1 & I & 0 \\ 0 & -G_2 & I \end{bmatrix} \text{ and } H = \begin{bmatrix} G_1 & 0 & 0 \\ G_2G_1 & G_2 & 0 \\ G_3G_2G_1 & G_3G_2 & G_3 \end{bmatrix} \tag{8.23}$$

Note that once more, part of the interconnection structure is preserved, as well as the open loop dynamics, as they are expressed by the poles of the individual transfer functions.

Consider now the modified structure that involves some feedback and is described in Figure (8.5) for which:

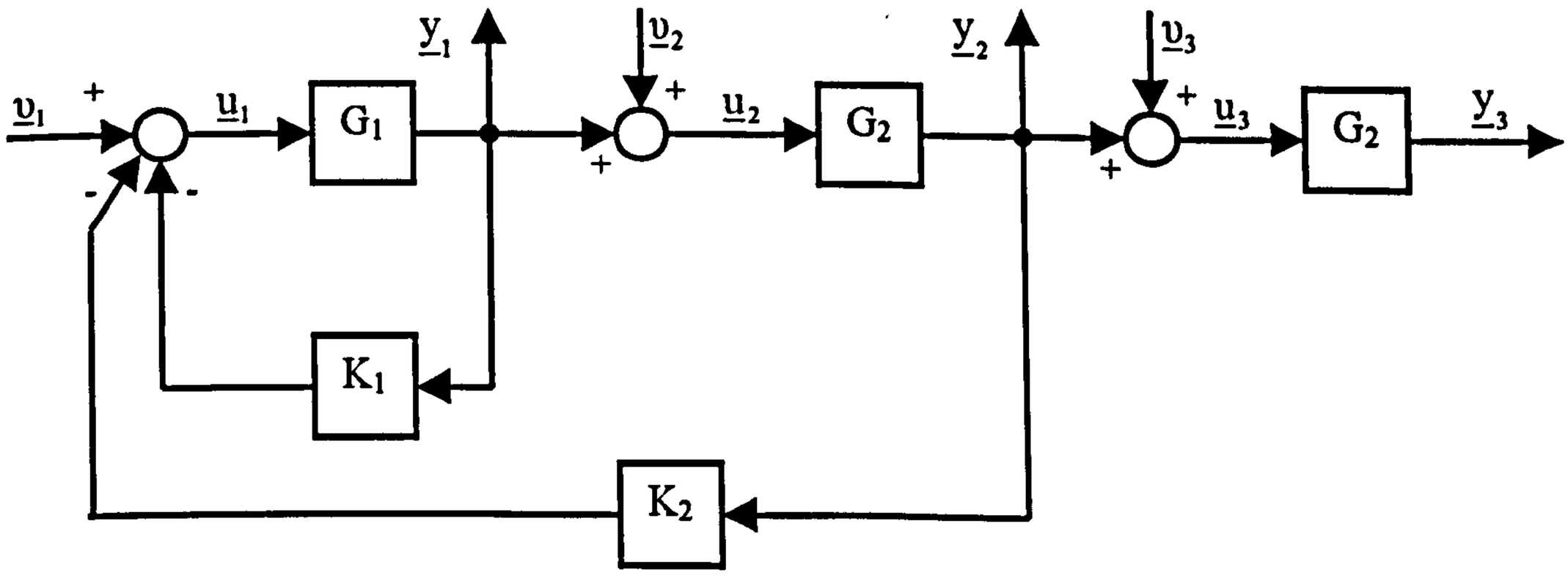

Figure (8.5): Cascade Composition with Partial Feedback

$$\underline{u} = \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \end{bmatrix} = \begin{bmatrix} \underline{\upsilon}_1 \\ \underline{\upsilon}_2 \\ \underline{\upsilon}_3 \end{bmatrix} - \begin{bmatrix} K_1\underline{y}_1 + K_2\underline{y}_2 \\ -\underline{y}_1 \\ -\underline{y}_2 \end{bmatrix} = \begin{bmatrix} \underline{\upsilon}_1 \\ \underline{\upsilon}_2 \\ \underline{\upsilon}_3 \end{bmatrix} - \begin{bmatrix} K_1 & K_2 & 0 \\ -I & 0 & 0 \\ 0 & -I & 0 \end{bmatrix} \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \end{bmatrix}$$

$$\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \end{bmatrix} = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \end{bmatrix} \tag{8.24a}$$

and thus

$$\underline{u} = \underline{v} - F G \underline{u} \longrightarrow \underline{u} = G (I + FG)^{-1} \underline{v} \text{ and } \underline{y} = H \underline{v} = G (I + FG)^{-1} \underline{v} \tag{8.24b}$$

We may now compute $(I+FG)^{-1}$. In fact,

$$(I+FG) = \begin{bmatrix} I+K_1G_1 & K_2G_2 & 0 \\ -G_1 & I & 0 \\ 0 & -G_2 & I \end{bmatrix} \text{ and } (I+FG)^{-1} = \begin{bmatrix} X_1 & X_2 & 0 \\ X_4 & X_5 & 0 \\ X_7 & X_8 & I \end{bmatrix} \tag{8.24c}$$

with

$$\begin{aligned} X_1 &= (I+K_1G_1+K_2G_2G_1)^{-1}, \; X_2 = -(I+K_1G_1+K_2G_2G_1)^{-1}K_2G_2 \\ X_4 &= G_1(I+K_1G_1+K_2G_2G_1)^{-1}, \; X_5 = I - G_1(I+K_1G_1+K_2G_2G_1)^{-1}K_2G_2 \\ X_7 &= G_2G_1(I+K_1G_1+K_2G_2G_1)^{-1}, \; X_8 = G_2 - G_2G_1(I+K_1G_1+K_2G_2G_1)^{-1}K_2G_2 \end{aligned} \tag{8.24d}$$

from which the overall transfer function becomes:

$$H = \begin{bmatrix} G_1X_1 & G_1X_2 & 0 \\ G_2X_4 & G_2X_5 & 0 \\ G_3X_7 & G_3X_8 & G_3 \end{bmatrix} \tag{8.24e}$$

and this indicates once more that part of the fixed zero structure is preserved, as well as the presence of some of the subsystem dynamics.

□

The above examples indicate that the composite systems may be modelled in terms of the subsystem models $\{G_1(s), G_2(s), G_3(s)\}$ and the interconnection structure matrix F. Such models are referred to as Internal Progenitor Models (IPM) [Karc.8,] and frequently

lead to "structured" transfer function matrices for the composite system (when the subsystem models are fixed). A structured transfer function matrix is a transfer function with certain elements fixed to zero, some elements being constant and other elements expressing the simple dynamics of the subsystems. The transfer function of a composite system for which the underlined interconnection matrix F is not known or not explicitly stated, is called an External Progenitor Model (EPM) [Karc.8,] and the only evidence in the interconnection structure is that provided by the structured nature of the overall transfer function. Structured transfer function matrices frequently arise as models in the Early Process Design Stages and some of their basic problems associated with their structural characteristics will be considered here. It will be assumed throughout the following, that the transfer function is given but the underlined matrix F is not known. It will be further assumed that the transfer function matrices, which are considered, have apart from the fixed zeros and the fixed dominant dynamics, elements with uncertain or generic values. Such transfer functions will be referred to as Generic Structured Transfer Function Matrices (GSTF). The examples and the genericity assumptions for the non-fixed elements imply that for such transfer functions, we have the following properties:

(i) Fixed zeros may appear in certain locations.

(ii) Known or unknown origin, and thus class dynamics, may appear in a repeated way throughout the transfer function.

(iii) Generic and non-repeated elements may appear, but with known dynamic order.

The fixed zeros emergence is the result of topology and has been demonstrated by the examples, as well as the appearance of open-loop dynamics in a repeated way in the overall transfer function. From the examples (see nature of (8.20b), (8.24d)), it is also clear that open-loop known transfer functions generate, even under gain uncertainty, families of transfer functions of known dynamic origin. There also exist elements in the overall transfer function, which are generated in a random way (due to parameter uncertainty in interconnections or gain uncertainty in numerator dynamics of the aggregate system). The above properties can be explored in identifying important concepts, such as the McMillan degree of generic structured transfer function matrices and it is considered next.

8.4.3. The Computation of Generic McMillan Degree for the Class of GSTF Matrices

For small dimension GSTF matrices the standard procedures for computing the value of the McMillan degree may be applied. For large dimension problems, however, we need methods, which explore the basic properties of GSTF matrices and thus lead to efficient ways for computing the generic McMillan degree. In this section, we consider the properties of GSTF and show that such problems have a discrete algebraic flavour based on the properties of integer matrices. To motivate the subsequent analysis, we consider an example of a 3×3 structured proper rational transfer function matrix H(s):

$$H(s)=\begin{bmatrix} A_1^2A_2 & A_1 & A_3 \\ A_3 & A_2^2 & A_1A_2A_3 \\ A_1 & A_4 & A_1^2 \end{bmatrix} \tag{8.25}$$

where the elements A_i, i=1,..,4 are repeated patterns representing, for instance, constant terms, the first or second order dynamics:

$$A_1=\frac{c_1}{s+a_1}, A_2=\frac{c_2s+b_3}{s^2+b_1s+b_2},\ldots \tag{8.26}$$

where the a_i, b_i are fixed and the c_i are constants which take generic values. By using partial fraction expansion, we can decompose the transfer function matrix as:

$$\begin{aligned} H(s)&=\begin{bmatrix} A_1^2A_2 & A_1 & A_3 \\ A_3 & A_2^2 & A_1A_2A_3 \\ A_1 & A_4 & A_1^2 \end{bmatrix}= \\ &=\underbrace{\begin{bmatrix} A_1^2 & A_1 & 0 \\ 0 & 0 & A_1 \\ A_1 & 0 & A_1^2 \end{bmatrix}}_{H_1(s)}+\underbrace{\begin{bmatrix} A_2 & 0 & 0 \\ 0 & A_2^2 & A_2 \\ 0 & 0 & 0 \end{bmatrix}}_{H_2(s)}+\underbrace{\begin{bmatrix} 0 & 0 & A_3 \\ A_3 & 0 & A_3 \\ 0 & 0 & 0 \end{bmatrix}}_{H_3(s)}+\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & A_4 & 0 \end{bmatrix}}_{H_{41}(s)} \end{aligned} \tag{8.27}$$

and the matrices $H_i(s)$ will be called simple structured matrices. In general, we define the structured and simple structured matrices as:

Definition (8.3): A simple structured matrix is a structured transfer function matrix where all its elements are of one of the following types: $A_i = \frac{c_i}{s+a_i}$ or $A_2 \frac{c_{i1}s+c_{i0}}{s^2+b_{i1}s+b_{i0}}$, where the poles are fixed but the numerator constants are taking generic values.

□

A structured transfer function matrix can always be decomposed into a set of simple structured transfer function matrices by use of partial fraction expansion method for each of the dynamic terms.

Remark (8.9): If $\{\lambda_1, \lambda_2, ..., \lambda_p\}$ are the fixed pole locations of a structured transfer function, then H(s) may always be expressed as:

$$H(s) = H_1(s) + H_1(s) + ... + H_p(s) \qquad (8.28)$$

where $H_i(s)$ are simple structured transfer functions, corresponding to λ_i fixed pole.

□

For the structured transfer function matrices, we define:

Definition (8.4): The generic McMillan degree of the structured transfer function $H(s) \in R_{pr}^{m\times p}[s]$ is the McMillan degree when the gain parameters of the entries take generic values.

□

Remark (8.10): In the computation of any minor of a generic rational matrix, there is no pole zero cancellation occurring.

□

By the above definition, Remark (8.10) and the definition of McMillan degree based on the minors [MacF. & Karc., 1], we have the result:

Theorem (8.2): The generic McMillan degree of the structured transfer function matrix $H(s) \in R_{pr}^{m\times p}[s]$ is equal to the sum of the generic McMillan degrees of the simple structure matrices $H_1(s), H_2(s), \ldots$. That is, if $\delta_{gm}(H_i)$ denotes the generic McMillan degree of a simple structured transfer function H_i, then

$$\delta_{gm}(H) = \delta_{gm}(H_1) + \delta_{gm}(H_2) + \ldots \qquad (8.29)$$

Proof: Without loss of generality, we assume that $H_i(s)$ and $H_j(s)$ are two simple structured transfer function matrices of H(s), which are associated with the fundamental dynamics $A_i = \frac{1}{s+\lambda_i}$ and $A_j = \frac{1}{s+\lambda_j}$, $\lambda_i \neq \lambda_j$. We prove that if the generic McMillan degrees of the matrices $H_i(s)$, $H_j(s)$ are $\delta_{gm}(H_i)$, $\delta_{gm}(H_j)$, then the contributions of the terms $A_i = \frac{1}{s+\lambda_i}$ and $A_j = \frac{1}{s+\lambda_j}$ towards the generic McMillan degree of the structured transfer function H(s) are exactly $\delta_{gm}(H_i)$ and $\delta_{gm}(H_j)$.

Because the generic McMillan degrees of the matrices $H_i(s)$ and $H_j(s)$ are $\delta_{gm}(H_i)$ and $\delta_{gm}(H_j)$, the least common multiples of all minors of the matrices $H_i(s)$ and $H_j(s)$ are $\frac{\alpha}{(s+\lambda_i)^{\delta_{gm}(H_i)}}$ and $\frac{\beta}{(s+\lambda_j)^{\delta_{gm}(H_j)}}$, respectively. Under the genericity assumption, there will be no pole-zero cancellations among the constituent parts in the determinants of the minors. Therefore, the terms in the least common multiple of all minors of the matrix H(s) due to terms $A_i = \frac{1}{s+\lambda_i}$ and $A_j = \frac{1}{s+\lambda_j}$ are of the forms $\frac{\alpha'}{(s+\lambda_i)^{\delta_{gm}(H_i)}}$ and $\frac{\beta'}{(s+\lambda_j)^{\delta_{gm}(H_j)}}$, respectively, i.e. the contributions towards the generic McMillan degree of the structured transfer function matrix due to the fundamental dynamics $A_i = \frac{1}{s+\lambda_i}$ and $A_j = \frac{1}{s+\lambda_j}$ are $\delta_{gm}(H_i)$ and $\delta_{gm}(H_j)$, respectively. This proves the result.

□

Remark (8.11): The evaluation of the generic McMillan degree of a structured transfer function matrix is reduced to finding the generic McMillan degrees of the simple structured matrices $H_i(s)$ obtained by the partial fraction decomposition.

□

In the following, the methods of computing the generic McMillan degree of the simple structured transfer function matrices are examined. Firstly, we define the concepts of order, path and weight.

Definition (8.5): Given a simple structured matrix $H_i(s) \in R_{pr}^{m\times p}[s]$, $m \le p$, the McMillan order of an entry in the matrix is the pole multiplicity of the i-th fundamental dynamics. A path is a sequence of m elements selected from the matrix with no two elements from the same column or the same row. The length of a path is the number of non-zero elements in the path. The McMillan weight of a path is defined to be the sum of the orders of the elements in the path. The maximal weight of all the independent paths of the matrix is denoted by $\Gamma_M(H_i)$.

□

Remark (8.12): The constant terms of the structured transfer function matrix do not contribute to the McMillan weight. From the definition of the generic McMillan degree, the constant terms do not contribute to the generic McMillan degree. So the constant terms are equivalent to fixed zero elements, as far as the computation of the weight is concerned.

□

Remark (8.13): A path with the maximal weight does not necessarily have to be the longest path, as displayed by the following simple structured matrix:

$$H(s)=\begin{bmatrix} A^4 & A \\ A^2 & 0 \end{bmatrix}$$

For this matrix, there are two paths: $h_{11} \to h_{12}$ with length 1 and weight 4, $h_{12} \to h_{21}$ with length 2 and weight 3.

□

In the following, we study the simple structured matrices $H_i(s)$. Because, only, the non-zero dynamic elements need to be considered, and the non-zero entries represent the same dynamic unit with different orders, for simplicity of notation, we use the orders of the dynamics of the entries only. For example, in (8.27) the matrix $H_1(s)$ is simplified as:

$$H(s)=\begin{bmatrix} A_1^2A_2 & A_1 & A_3 \\ A_3 & A_2^2 & A_1A_2A_3 \\ A_1 & A_4 & A_1^2 \end{bmatrix} \to I_1=\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad (8.30)$$

In general, a map can be defined between a simple structured matrix $H_i(s)$ and an integer matrix I_i, such that the entries of the integer matrix correspond to the orders of the

entries in $H_i(s)$. This representation will be referred to as natural mapping of a simple structured matrix. Note that all definitions relating to order, path, length, weight can be transferred from simple structured matrices to integer matrices of the same dimension. The partial fraction decomposition of (8.27), clearly, suggests that with any generic structured transfer function H(s) with fixed dynamics $\Lambda = \{\lambda_i, \lambda_i \in C\}$, we may always associate the simple structured matrices $\{H_{\lambda_i}(s)\}$ and thus a corresponding set of integer matrices $\{I_{\lambda_i}\}$. Concerning the relationship between the generic McMillan degree and the weight of paths, we have the following result.

Corollary (8.1): The generic McMillan degree of the simple structured matrix $H_i(s)$ is equal to the maximal weight, that is $\delta_{gm}(H_i) = \Gamma(H_i)$.

Proof: For a given simple structured matrix $H_i(s)$ with a maximal weight $\Gamma_M(H_i(s))$, we prove that the order of the least common multiple of all the minors of all orders of the matrix $H_i(s)$ is generically $\Gamma_M(H_i(s))$ and so is the generic McMillan degree. Without loss of generality, we assume $A_i = \frac{1}{s+\lambda_i}$. Firstly, we prove that $\delta_{gm}(H_i) \geq \Gamma_M(H_i)$. Assume that the length of a maximum weight path is l_Γ. From the definition of path, it is clear that there exists a minor of order $l_\Gamma \times l_\Gamma$, which contains this path. The denominator of the determinant of the minor is generically $(s+\lambda_i)^{\Gamma(H_i)}$ because there is no cancellation involved under the genericity assumption, therefore, the order of the least common multiple of all minors is greater or equal to $\Gamma_M(H_i)$. Secondly, we prove that if $\delta_{gm}(H_i) > \Gamma_M(H_i)$, then there exists an independent path whose weight ω^* satisfies $\omega^* > \Gamma(H_i)$. If $\delta_{gm}(H_i) > \Gamma_M(H_i)$, then there must exist at least a minor whose denominator has an order exactly $\delta_{gm}(H_i)$. Since there is no cancellation among the terms of the constituent parts to the minor, there exists a set of elements selected from different rows and columns whose product is in the form $\frac{\alpha}{(s+\lambda_i)^{\delta_{gm}(H_i)}}$. In other words, there exists a path, which has a weight ω^* and $\omega^* = \delta_{gm}(H_i) > \Gamma_M(H_i)$. This contradicts the assumption that $\Gamma_M(H_i)$ is the maximal weight. So we have $\delta_{gm}(H_i) = \Gamma_M(H_i)$.

□

The establishment of the relationship between generic McMillan degree and integer matrices allows the translation of such problems to equivalent searching problems on integer matrices. A number of techniques are considered next, which are based on properties of integer matrices. Some interesting background notation and results are considered first.

8.4.4. Representations, Irreducibility, Weight and Complexity of Natural Matrices

Let $A \in N^{m \times p}$, where N is the set of non-negative integers. Without loss of generality, we may assume $m \geq p$. The matrix A may be expressed as:

$$A = [\underline{a}_1, \underline{a}_2, ..., \underline{a}_p], \ \underline{a}_1 \in N^m, \ i \in \underset{\sim}{p} \tag{8.31}$$

For every column $\underline{a}_i$, we define as its content, the ordered set of distinct values of the numbers in $\underline{a}_i$, as:

$\mathcal{U}(\underline{a}_i) \stackrel{\Delta}{=} \{\delta_{1i} > \delta_{2i} > ... > \delta_{\sigma(i)i} > 0\}$, where δ_{1i} is the weight of the i-th column. Using this notation, we may represent matrix A as shown below:

$$A = \underbrace{A_1 \begin{bmatrix} \delta_{11} & & & 0 \\ & \delta_{12} & & \\ & & \ddots & \\ 0 & & & \delta_{1p} \end{bmatrix}}_{\stackrel{\Delta}{=}\Delta_1} + \underbrace{A_2 \begin{bmatrix} \delta_{21} & & & 0 \\ & \delta_{22} & & \\ & & \ddots & \\ 0 & & & \delta_{2p} \end{bmatrix}}_{\stackrel{\Delta}{=}\Delta_2} + ... + \underbrace{A_\mu \begin{bmatrix} \delta_{\mu 1} & & & 0 \\ & \delta_{\mu 2} & & \\ & & \ddots & \\ 0 & & & \delta_{\mu p} \end{bmatrix}}_{\stackrel{\Delta}{=}\Delta_\mu} \tag{8.32}$$

where $\mu = \max\{\sigma(i), i \in \underset{\sim}{p}\}$, the matrices $A_1, A_2, ..., A_\mu$ are $m \times p$ Boolean matrices (i.e. having only 0, 1 elements) and in the Δ_k matrices δ_{jk} take the k-th value from $\mathcal{U}(\underline{a}_i)$ and if $k > \sigma(j)$, then we set $\delta_{jk} = 0$. We shall refer to the decomposition in (8.32) as the weighted Boolean representation of A; the matrices A_i are referred to as the Boolean coefficients and Δ_i as order matrices.

Equation (8.32), may be also written as:

$$A = [A_1, A_2, \ldots, A_\mu] \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_\mu \end{bmatrix} \quad (8.33)$$
$$\triangleq \langle A \rangle$$

and $\langle A \rangle$ is referred to as a weighted Boolean representation. Every column of $\langle A \rangle$ is Boolean and it characterised by an index δ_{ki}, where δ_{ki} is the i-th value of the Δ_k matrix. It is clear that relation (8.33) establishes an isomorphism between the matrices of $N^{m \times p}$ and the Boolean matrices. We illustrate this description, in terms of an example.

Example (8.7): Consider the matrix:

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 4 & 2 & 6 \\ 1 & 1 & 0 \\ 2 & 3 & 0 \\ 0 & 1 & 5 \end{bmatrix} = [\underline{a}_1, \underline{a}_2, \underline{a}_3]$$

Clearly, $\mathcal{U}(\underline{a}_i) = \{4 > 2 > 1\}, \mathcal{U}(\underline{a}_i) = \{3 > 2 > 1\}, \mathcal{U}(\underline{a}_i) = \{6 > 5 > 3\}$ and we can express A as:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & & 0 \\ & 3 & \\ 0 & & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & & 0 \\ & 2 & \\ 0 & & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 3 \end{bmatrix} = A_1\Delta_1 + A_2\Delta_2 + A_3\Delta_3$$

and the a-Boolean representation is:

$$\langle A \rangle = \left[\begin{array}{ccc:ccc:ccc} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$
$$\begin{array}{ccccccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 4 & 3 & 6 & 2 & 2 & 5 & 1 & 1 & 3 \end{array}$$

An alternative equivalent representation of A, which also defines an isomorphism between natural and Boolean matrices is defined below:

$$A = \underbrace{[\underline{a}_{11}, \underline{a}_{12}, .., \underline{a}_{1\sigma(1)}]}_{\triangleq \tilde{A}_1} = \underbrace{\begin{bmatrix} \delta_{11} \\ \delta_{21} \\ \vdots \\ \delta_{\sigma(1)1} \end{bmatrix}}_{\triangleq \underline{\delta}_1} + \underbrace{[\underline{a}_{21}, \underline{a}_{22}, .., \underline{a}_{2\sigma(2)}]}_{\triangleq \tilde{A}_2} = \underbrace{\begin{bmatrix} \delta_{12} \\ \delta_{22} \\ \vdots \\ \delta_{\sigma(2)2} \end{bmatrix}}_{\triangleq \underline{\delta}_2} + .. + \underbrace{[\underline{a}_{p1}, \underline{a}_{p2}, .., \underline{a}_{p\sigma(p)}]}_{\triangleq \tilde{A}_p} = \underbrace{\begin{bmatrix} \delta_{1p} \\ \delta_{2p} \\ \vdots \\ \delta_{\sigma(p)p} \end{bmatrix}}_{\triangleq \underline{\delta}_p} \quad (8.34)$$

or that

$$\underbrace{A = [\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_p]}_{\triangleq \{A\}} \begin{bmatrix} \underline{\delta}_1 & & & 0 \\ & \underline{\delta}_2 & \ddots & \\ 0 & & & \underline{\delta}_p \end{bmatrix}}_{\triangleq \Delta} \quad (8.35)$$

and $\{A\}$ will be referred to as the b-weighted Boolean representation. Every column of $\{A\}$ is Boolean and it is characterised by a value in $\mathcal{U}(\underline{a}_i)$ and the index i. Clearly, this alternative representation, also, introduces an isomorphism between $N^{m \times p}$ and Boolean matrices and shall be used subsequently in our analysis.

Example (8.7) (cont.): The b-Boolean representation of the matrix of Example (8.7) is:

$$A = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\triangleq \underline{\delta}_1} \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\triangleq \underline{\delta}_2} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{\triangleq \underline{\delta}_3} \begin{bmatrix} 6 \\ 5 \\ 3 \end{bmatrix}$$

and thus the b-Boolean representation is:

$$\{A\} = \left[\begin{array}{ccc|ccc|ccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] = [\tilde{A}_1, \tilde{A}_2, \tilde{A}_3]$$

$$\begin{array}{ccccccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 4 & 2 & 1 & 3 & 2 & 1 & 6 & 5 & 3 \end{array}$$

□

It should be noted that the b-representation is more economical, since the a-representation may contain also some zero columns. In the following, both representations will be used.

A matrix $A \in N^{m\times p}$ is called column irreducible, if the matrix A_1 of the weighted Boolean representation, referred to, also, as high coefficient matrix, has full structural rank; otherwise, the matrix is called column reducible. The set of indices $\{\delta_{11},\delta_{12},...,\delta_{1p}\}$ is referred to as the set of column weight or column degrees and the number $\delta(A)\overset{\Delta}{=}\delta=\sum_{i=1}^{p}\delta_{1i}$ is called the complexity of A. The irreducible, reducible classification of natural matrices has the following implications.

Remark (8.14): From the definition of the weight of the matrix, we have that the complexity of A is always an upper bound for the weight $\gamma(A)=\gamma$, i.e. $\gamma\le\delta$.

□

Theorem (8.3): Let $A \in N^{m\times p}$, $m \ge p$ and assume the representation defined by (8.34). The following properties hold true:

(i) If A is irreducible, then its weight $\gamma(A)=\gamma$ is equal to the complexity $\delta(A)=\delta$, i.e.

$$\gamma=\delta=\sum_{i=1}^{p}\delta_{1i}$$

(ii) If A is reducible, then $\gamma(A)\le\delta$. Furthermore, if $\alpha\overset{\Delta}{=}\{H_k : H_k \in N^{n\times q}, q<p$ denotes the set of all matrices of A made up from subsets of its columns such that H_k is column irreducible, then

$$\sigma=\max\{\delta(H_k),\forall H_k\in A\}\le\gamma(A)<\delta \tag{8.36}$$

Proof:

(i) If A is irreducible, i.e. A_1 has full structural rank, then this implies the existence of a path that contains the maximal column values $\delta_{1i}, i\in\underset{\sim}{p}$ and $\gamma=\sum_{i=1}^{p}\delta_{1i}$.

(ii) If A is reducible, this implies that there exists no path of length p that passes through the values $\{\delta_{1i}, i\in\underset{\sim}{p}\}$. A maximal weight path has, thus, to be found amongst those

based on a subset of $\{\delta_{1i}, i \in \underset{\sim}{p}\}$, say $\delta_{1i_1},...,\delta_{1i_\mu}$, whereas the rest of the path corresponds to elements $\{\delta'_{1j_1},...,\delta_{1j_\nu}\}$, where the values $\delta'_{1j_1} < \delta_{1j_1},...,\delta'_{1j_\nu} < \delta_{1j_\nu}$ and thus $\gamma < \sum_{i=1}^{p} \delta_{1i}$. The rest of the proof follows from the above arguments. In fact, by considering all possible column irreducible submatrices, we define lower bounds for $\gamma(A)$ and thus the maximal of all of them defines also a lower bound for $\gamma(A)$.

□

We shall refer to the number σ, introduced above, as the index of A.

Example (8.8): For the matrix A of Example (8.7), we have:

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ 4 & 3 & 6 \end{matrix}$$

and its rank is clearly 2, i.e. the matrix is reducible and thus $\gamma(A) < 4+3+6 = 13$. The set a for the matrix A is:

$$a = \left\{ \begin{bmatrix} 2 \\ 4 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 0 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 4 & 2 \\ 1 & 1 \\ 2 & 3 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 2 & 6 \\ 1 & 0 \\ 3 & 0 \\ 1 & 5 \end{bmatrix} \right\}$$
$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 4 & 3 & 6 & 4 & 3 & 3 & 6 \end{matrix}$$

and thus the index of the matrix A is defined as: $\sigma = \max\{4, 3, 6, 4+3, 3+6\} = 9$ and thus, the weight satisfies the inequality $9 \leq \gamma(A) < 13$.

□

The search for the exact value of the weight $\gamma(A)$ is now based on the interpretation of the definition of $\gamma(A)$, which may be summarised as follows:

Remark (8.15): Let us assume that a path that gives the weight $\gamma(A)$ is associated with columns $\{i_1, i_2, ..., i_\mu\}$ of the set $\{1,2,...3\}$ and that $\gamma(A) = \gamma_{1i_1} + \gamma_{2i_2} + ... + \gamma_{\mu i_\mu}$. Clearly, $\gamma_{1i_1} \in \mathcal{U}(\underline{a}_{i_1}), ..., \gamma_{1i_\mu} \in \mathcal{U}(\underline{a}_{i_\mu})$ and the Boolean matrix, which is defined as the submatrix of $\{A\}$ that corresponds to indices $(\gamma_{1i_1}, \gamma_{2i_2}, ..., \gamma_{\mu i_\mu})$ has full structural rank. The search for the value of $\gamma(A)$ is thus reduced to finding a submatrix $\hat{A}$ of $\{A\}$, which has the properties:

(i) Only one column is selected from each of the blocks $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_p$ to form $\hat{A}$.

(ii) The resulting matrix $\hat{A}$ has full structural rank.

(iii) The complexity of the matrix $\hat{A}$ is maximal.

□

An alternative interpretation of $\hat{A}$ is given below.

Remark (8.16): For a matrix $A \in N^{m \times p}$, $m \geq p$ with column contents $\mathcal{U}(\underline{a}_i) = \{\delta_{1i} > \delta_{2i} > ... > \delta_{\sigma(i)i} > 0\}$, $i \in \underset{\sim}{p}$, the weight of A is defined as a sum $\gamma(A) = \gamma_{1i_1} + \gamma_{2i_2} + ... + \gamma_{\mu i_\mu}$, $\mu \leq p$, such that each γ_{ki_k} takes value from only one $\mathcal{U}(\underline{a}_i)$. In addition, we have:

(i) The set of Boolean vectors of $\{A\}$ corresponding to γ_{ki_k}, $k \in \underset{\sim}{\mu}$ is structurally independent.

(ii) The sum in $\gamma(A)$ takes a maximal value.

□

Clearly, Remark (8.16) is a restatement of Remark (8.15), but it is in a form that indicates the basics of a new algorithm that leads to the computation of $\gamma(A)$ in an efficient way. This is described in the following section.

8.4.5. A New Algorithm for Determining the Weight of Natural Matrices

Consider the matrix $A \in N^{m \times p}, m \geq p$, assume that its columns are ordered to descending weight and form the table of the column contents, i.e.

col.(1)	col.(2)	.....	col.(p)
δ_{11}	δ_{12}	.....	δ_{1p}
δ_{21}	δ_{22}		δ_{2p}
⋮	⋮		⋮
$\delta_{\sigma(1)1}$	$\delta_{\sigma(2)2}$	.....	$\delta_{\sigma(p)p}$

Table of column contents

where $\delta_{11} \geq \delta_{12} \geq ... \geq \delta_{1p}$. The search for the weight $\gamma(A)$ involves the following steps:

STEP (I): (Preliminary Step)

Define the b-Boolean representation of the matrix A, as shown in (8.36) and test column irreducibility by finding the structural rank of the high coefficient matrix.

(a) If A is column irreducible, then $\gamma(A) = \sum_{i=1}^{p} \delta_{1i} = \delta$ and the search stops.

(b) If A is reducible, then $\gamma(A) < \delta$ and compute the index of A, denoted by $\sigma(A) \stackrel{\Delta}{=} \sigma$ (as described in Theorem (8.3)). The search for $\gamma(A)$ then continues and involves the following major steps:

STEP (II): (Generation of Fundamental Sequences)

The objective of this step is to generate the set of all p-term sequences $(x_1, x_2, ..., x_p)$, such that $x_i \in \mathcal{U}(\underline{a}_i)$, $i \in \underset{\sim}{p}$. This set may be generated from the Table of Column Contents and the objective is to order it according to descending complexity δ of the sequence, where $\delta = \sum_{i=1}^{p} x_i$. A procedure to generate this step involves the following:

(a) The first sequence is $(\delta_{11}, \delta_{12}, ..., \delta_{1p})$.

(b) Consider all sequences $(\delta_{11}, ..., \delta_{1k-1}, y_k, ..., \delta_{1p})$ obtained from $(\delta_{11}, ..., \delta_{1p})$ by keeping p-1 elements fixed and by substituting the k-th element δ_{1k} by $y_k \in \{\delta_{2k}, ..., \delta_{\sigma(k)k}\}$.

(c) Consider all sequences $(\delta_{11}, ..., y_k, ..., y_\ell, ..., \delta_{1p})$, obtained from $(\delta_{11}, ..., \delta_{1p})$, by keeping any p-2 elements fixed and by substituting the δ_{1k} and $\delta_{1\ell}$ elements by $(y_k, y_\ell) \in \{\delta_{2k}, ..., \delta_{\sigma(k)k}\} \times \{\delta_{2\ell}, ..., \delta_{\sigma(\ell)\ell}\}$.

(d) The general step is to keep any $p-\mu$ elements of $(\delta_{11},\delta_{12},...,\delta_{1p})$ fixed at their position and substitute the non-fixed elements $\delta_{1k},\delta_{1\ell},...,\delta_{1v}$ by elements $(y_k,y_\ell,...,y_v)$ obtained from the sequences $(y_k,y_\ell,...,y_v)\in\{\delta_{2k},..,\delta_{\sigma(k)k}\}\times\{\delta_{2\ell},..,\delta_{\sigma(\ell)\ell}\}\times\{\delta_{2v},..,\delta_{\sigma(v)v}\}$.

(e) The above procedure terminates with the definition of the element $(\delta_{21},\delta_{22},...,\delta_{2p})$. If σ is the reduced complexity of A and $\sigma>\delta_{21}+\delta_{22}+...+\delta_{2p}$, then the procedure terminates, since all further combinations lead to smaller values for the weight of the resulting paths. If $\sigma<\delta_{21}+\delta_{22}+...+\delta_{2p}$, we repeat steps (a) to (e), by starting now from the sequence $\delta_{21}+\delta_{22}+...+\delta_{2p}$. The set of sequences generated as above, will be denoted by $\mathcal{Q}(A)$ and will be called the set of <u>fundamental sequences</u> of A.

<u>STEP (III)</u>: (Ordering of Fundamental Sequences)

The set of sequences $\mathcal{Q}(A)$ generated, as above, is now ordered in descending complexity and the resulting set is denoted by $\hat{\mathcal{Q}}(A)$.

<u>STEP (IV)</u>: (Complexity and Index of Fundamental Sequences)

Let {A} be the b-Boolean representation of A, i.e. $\{A\}=\equiv[\tilde{A}_1,\tilde{A}_2,...,\tilde{A}_p]$. Every column of $\tilde{A}_i$ is a Boolean vector, which is "graded", i.e. corresponds to a value of $\mathcal{U}(\underline{a}_i)$ and it is, thus, denoted by $\underline{a}_i(x_i)$, $x_i\in\mathcal{U}(\underline{a}_i)$. For every $q=(x_1,x_2,...,x_p)\in\mathcal{Q}(A)$, there corresponds a p-set of Boolean vectors defining a matrix

$$A(q)\overset{\Delta}{=}\{\underline{a}_1(x_1),...,\underline{a}_p(x_p)\},\ \underline{a}_i(x_i)\in\tilde{A}_i \tag{8.37}$$

where $\delta(q)\overset{\Delta}{=}\sum_{i=1}^{p}x_i$ is the complexity of q, as well as A(q), and $\sigma(A(q))\overset{\Delta}{=}\sigma(q)$ is the index of A(q) evaluated, as indicated by part (ii) of Theorem (8.3). If A(q) is column irreducible, then, $\sigma(q)=\delta(q)$, otherwise (column reducible) $\sigma(q)<\delta(q)$. A sequence for which $\sigma(q)=\delta(q)$ will be called <u>irreducible</u>; otherwise ($\sigma(q)<\delta(q)$), it will be called <u>reducible</u>.

STEP (V): (Searching Algorithm)

Theorem (8.4): Given a matrix $A \in N^{m\times p}$, then from the ordered set of fundamental sequences $\mathcal{Q}(A)$,

$$\mathcal{Q}(A) = \{q_1 \geq q_2 \geq \ldots \geq q_i \geq q_{i+1} \geq \ldots \geq \delta(q_i) \geq \delta(q_{i+1})\} \tag{8.38}$$

the weight $\gamma(A) = \gamma$ is determined by a procedure based on $\delta(q_i)$ and $\sigma(q_i)$ relationships:

(i) Starting from q_1, then:

(a) If $\sigma(q_1)=\delta(q_1)$, i.e. q_1 is irreducible, then procedure terminates and $\gamma(A) = \sigma(q_1) = \delta(q_1) = \delta(A)$.

(b) If $\sigma(q_1)<\delta(q_1)$, i.e. q_1 is reducible, then:

(1) If $\sigma(q_1) \geq \delta(q_2)$, the procedure terminates and $\gamma(A) = \sigma(q_1)$.

(2) If $\sigma(q_1) < \delta(q_2)$, continue the search to q_2. Furthermore, the search is restricted to those q_i, for which $\delta(q_i) > \sigma(q_1)$.

(ii) The general step is carried out for the sequence q_i, for which $\delta(q_i) > \max\{\sigma(q_1),\ldots,\sigma(q_{i-1})\}$. Then,

(a) If $\sigma(q_i)=\delta(q_i)$, i.e. q_i is irreducible, the procedure terminates and $\gamma(A) = \sigma(q_i) = \delta(q_i)$.

(b) If $\sigma(q_i)<\delta(q_i)$, i.e. q_i is reducible, then:

(1) If $\max\{\sigma(q_1),\ldots,\sigma(q_i)\} \geq \delta(q_{i+1})$, the procedure terminates and $\gamma(A) = \max\{\sigma(q_1),\ldots,\sigma(q_i)\}$.

(2) If $\max\{\sigma(q_1),\ldots,\sigma(q_i)\} < \delta(q_{i+1})$, continue the search to q_{i+1}. Furthermore, the search may continue up to that q_j, for which $\max\{\sigma(q_1),\ldots,\sigma(q_i)\} < \delta(q_j)$.

Proof:

(i) Note that irreducibility of q_1 implies that $\gamma(A)=\sigma(q_1)=\delta(q_1)$. If $\sigma(q_1)<\delta(q_1)$, then if $\sigma(q_1)\geq\delta(q_2)$, the procedure stops and there is no need to carry on with the investigation of q_2, since any search based on q_2, q_3 etc, will give rise to $\sigma(q_i)\leq\delta(q_i)\leq\delta(q_2)$ and, thus, $\sigma(q_1)\geq\sigma(q_i),\forall i$. If $\sigma(q_1)<\delta(q_2)$, then the search continues, with a searching horizon, being only those q_i, for which $\delta(q_i)>\sigma(q_1)$.

(ii) The general step is proved along similar lines. Thus, if $\delta(q_i)>\max\{\sigma(q_1),...,\sigma(q_{i-1})\}$, then irreducibility of q_i, i.e. $\sigma(q_i)=\delta(q_i)$, implies that the procedure terminates, since for any q_j, $j=i+1,...$, $\sigma(q_i)=\delta(q_i)\geq\delta(q_j)\geq\sigma(q_j)$. Similarly, the procedure terminates when $\max\{\sigma(q_1),...,\sigma(q_i)\}\geq\delta(q_{i+1})$, since for any $j=i+1,...$, $\max\{\sigma(q_1),...,\sigma(q_i)\}\geq\delta(q_{i+1})\geq\delta(q_j)\geq\sigma(q_j)$ and thus there is no need for further search. The search continues when $\max\{\sigma(q_1),...,\sigma(q_i)\}<\delta(q_{i+1})$; however, the value $\max\{\sigma(q_1),...,\sigma(q_i)\}$ provides an improved prediction for the maximum number of sequences q_i, which has to be considered in order to determine the value of $\gamma(A)$.

□

The above result provides a systematic procedure that leads to the value of $\gamma(A)$ in a very small number of steps. The above procedure uses the minimal possible number of steps since it is based on ordering and exploits fully the property of irreducibility, which is behind the determination of $\gamma(A)$.

Example (8.9): For the matrix A of Example (8.7), we have the following b-Boolean representation of the column ordered matrix A' (re-ordering of columns of A).

$$A'=\begin{bmatrix}3&2&2\\6&4&2\\0&1&1\\0&2&3\\5&0&1\end{bmatrix}\rightarrow\{A'\}=\left[\begin{array}{ccc:ccc:ccc}0&0&1&0&1&0&0&1&0\\1&0&0&1&0&0&0&1&0\\0&0&0&0&0&1&0&0&1\\0&0&0&0&1&0&1&0&0\\0&1&0&0&0&0&0&0&1\end{array}\right]$$

$$\begin{array}{ccc}\uparrow&\uparrow&\uparrow\\6&4&3\end{array}\qquad\begin{array}{ccccccccc}\uparrow&\uparrow&\uparrow&\uparrow&\uparrow&\uparrow&\uparrow&\uparrow&\uparrow\\6&5&3&4&2&1&3&2&1\end{array}$$

The Table of Column Contents is, then:

col.(1)	col.(2)	col.(3)
6	4	3
5	2	2
3	1	1

The application of the searching algorithm is demonstrated below, step by step.

STEP (I): (Preliminary Step)

Note that $\delta(A) = \delta(A') = 6+4+3 = 13$ and by Example (8.7) $\sigma(A') = \sigma(A) = 9$ and thus, $9 \le \gamma(A) < 13$.

STEP (II): (Generation of Fundamental Sequences)

We define the set of fundamental sequences, as detailed below:

(1) $(6,4,3) \rightarrow \delta = 13$

(2) $(6,4,x) \rightarrow (6,4,2) \rightarrow \delta = 12$

$\rightarrow (6,4,1) \rightarrow \delta = 11$

$(6,x,3) \rightarrow (6,2,3) \rightarrow \delta = 11$

$\rightarrow (6,1,3) \rightarrow \delta = 10$

$(x,4,3) \rightarrow (5,4,3) \rightarrow \delta = 12$

$\rightarrow (3,4,3) \rightarrow \delta = 10$

(3) $(6,x,x) \rightarrow (6,2,2) \rightarrow \delta = 10$

$\rightarrow (6,2,1) \rightarrow \delta = 9$

$\rightarrow (6,2,2) \rightarrow \delta = 9$

$\rightarrow (6,1,1) \rightarrow \delta = 8$

$(x,4,x) \rightarrow (5,4,2) \rightarrow \delta = 11$

$\rightarrow (5,4,1) \rightarrow \delta = 10$

$\rightarrow (3,4,2) \rightarrow \delta = 9$

$\rightarrow (3,4,1) \rightarrow \delta = 8$

$(x,x,3) \rightarrow (5,2,3) \rightarrow \delta = 10$

$\rightarrow (5,1,3) \rightarrow \delta = 9$

$\rightarrow (3,2,3) \rightarrow \delta = 8$

$\rightarrow (3,1,3) \rightarrow \delta = 7$

(4) $(5,2,2) \rightarrow \delta = 9$

and there is no need to consider any more sequences, since the resulting complexity is less than 9.

STEP (III) (Ordering of Fundamental Sequences)

We consider, only those sequences for which $\delta \geq 9$ and we order them, as:

(13.1): (6,4,3)

(12.1): (6,4,2)
(12.2): (5,4,3)

(11.1): (6,4,1)
(11.2): (6,2,3)
(11.3): (5,4,2)

(10.1): (6,1,3)
(10.2): (3,4,3)
(10.3): (6,2,2)
(10.4): (5,4,1)
(10.5): (5,2,3)

(9.1): (6,2,1)
(9.2): (6,1,2)
(9.3): (3,4,2)
(9.4): (5,1,3)
(9.5): (5,2,2)

STEPS (III) & (IV): (Indices of Sequences and Search)

For each of the above sequences, we compute the index σ and then use the searching based on Theorem (8.4).

(12.1): (6,4,2) $\rightarrow \delta = 12, \ \sigma = 8$

(12.2): (5,4,3) $\rightarrow \delta = 12, \ \sigma = 12$

Given that (5,4,3) is irreducible, the search terminates here and $\gamma(A) = 12$.

Remark (8.17): The speed, with which $\gamma(A)$ was determined, suggests that for sequences with high complexity, it is worth computing their index immediately, before we proceed to

the computation of the whole set of sequences and order them. This may speed up more the convergence of the algorithm to the true value of $\gamma(A)$.

□

8.5. SUMMARY

In this chapter, some important system properties have been examined, which have been considered on uncertain and structured transfer functions. Genericity arguments have been used to reduce the computational complexity of the exact computations. The work is of special interest to large dimension transfer functions, where the complexity and computational cost of the exact computations is high. The work here, has been focused on the generic infinite zero structure and the generic McMillan degree. Other types of invariants, of significance for the structuring of early process models, such as minimal indices, may be also examined within the same framework, that is computations on integer matrices. These issues, however, are for future research.

The computation of the McMillan degree of a structured transfer function matrix has been considered here, using the properties of column irreducibility of natural (integer) matrices. This algorithm avoids the general searching methods suggested in [Karc. et al, 2] and determines the optimal solutions in a small number of steps. The problem of the generic McMillan degree computation seems to be equivalent to a "maximum matching flow problem" [Gon. & Min., 1] for which alternative solutions exist. The comparison of this new, algebra based, framework to the standard, graph theory based, methods is a subject for future research.

Chapter 9

THE PROBLEM OF CONTROL STRUCTURE SELECTION: TRANSFER FUNCTION METHODS

Chapter 9

THE PROBLEM OF CONTROL STRUCTURE SELECTION: TRANSFER FUNCTION METHODS

9.1. INTRODUCTION

The problem of selection of inputs, outputs and subsequently, of a control structure is an important activity within the overall design of a process and it is characterised by many different aspects. It is an activity of complex character and has not been properly addressed so far as an integration issue within the area of process and control design. The aim of this chapter is to introduce an overall framework on the control structure selection, review the process-based methodologies of the interaction analysis type and provide the basis for the development of relevant software. The software and its evaluation in terms of examples will be presented in the next chapter. As such, this chapter provides a bridge between the traditional interaction measures and the general problem of selecting control schemes.

The overall problem of control structure selection is in the boundaries between Global Instrumentation and Control Design. In fact, it involves issues related to the structuring of the controller and as such, deals with the selection of effective sets of inputs, outputs involved in the control action, deciding on the nature of the coupling of such variables (centralised or decentralised) and determining the type and order of controller dynamics within a given structure. Aspects of the general problem (model orientation and well conditioning of the model) have already considered before. Thus, here we present the overall picture and then specialise to process based methodologies of the interaction type. These methodologies are based on heuristics and simple diagnostics and they are relevant

for problems where the number of inputs and outputs is small. The emphasis here is on frequency domain diagnostics and indicators. Developing software for this mature area of interaction analysis allows the testing of ideas on real life examples and thus, helps to understand the limitations of traditional techniques. The main bulk of the work here is only a part of the overall picture. A framework for an integrated methodology is presented at the end and this provides a roadmap for the area.

9.2. THE GENERAL PROBLEM OF CONTROL STRUCTURE SELECTION

Classical Control Design is a problem defined on a system with well defined input, output structure, specified structure of the controller both in terms of coupling of input-output variables and type and order of dynamics. These types of assumptions are issues that have been decided prior to control design and they are at the boundaries between Global Instrumentation and Control. These problems are referred to in short as Control Structure Selection and it is a composite activity that involves the following specific clusters of problems:

a) Classification of variables, definition of orientation of the model and development of the progenitor model.

b) Definition of effective sets of inputs, outputs for Control Design and Process Monitoring.

c) Structuring of the feedback coupling of the control scheme and designing its decentralisation.

d) Deciding on the type and order of dynamics in the structured controller.

The above ordering is quite natural and in each of these families we have a number of problems that may be addressed at the early or late stages of the design and for systems with large or small scale dimensions. The current effort is based on transfer function type models and the emphasis is on small dimension problems.

The classification of variables is a problem that is not always solved using physical modelling arguments. Very frequently it may lead to progenitor models which are not well defined. The specific issues involved in the selection of a well defined progenitor model and the procedure that can be used to define a well behaved model have been considered in Chapter 5 for the case of state space implicit descriptions. The structuring of an effective

input, output structure has been considered in Chapter 6, where the well conditioning of models to guarantee certain desirable properties has also been discussed. Having decided the required input, output structure of the feedback scheme, the issue that has to be decided is that of the structure of the feedback scheme, i.e. centralised versus decentralised, and if decentralised, then the exact nature of decentralisation; the latter involves the partitioning and pairing, as well as the order of dynamics for the particular channels. The design of decentralisation is a problem that has been only partially addresses and has as integral parts the following subproblems [Karc. et al, 1]:

(c.1) Graph methodologies for structured state space models.

(c.2) Genericity and structural methodologies for selection of decentralisation.

(c.3) Diagnostics based on property indicators.

(c.4) Interaction analysis and diagnostics.

The class of Graph Methodologies is particularly useful for Large Scale Systems with an explicit knowledge of the process interconnection structure. Work in this area is based on state-space models and exploits the underlying structure. Properties such as structural controllability, observability, decomposition of the overall system [Siljak, 1] are important criteria. For generic models with a nonexplicit underlying structure, results on generic solvability of control problems may provide criteria for selection of decentralisation [Lev. & Karc., 1], [Lev. & Karc., 2]. Within the third class, we have a number of methodologies aiming for structures which avoid the emergence of fixed modes and almost fixed modes [Lev. & Karc., 3], [Lev. & Karc., 4] and thus well conditioning. The classes of problems (c.2), (c.3) may be applied for both state space and transfer function descriptions and are relevant for both small and large scale systems.

The interaction analysis and related diagnostics is a well developed body of techniques which has provided so far the bulk of techniques used especially in the process area. The diagnostics are based on heuristics and certain property indicators. The input, output dimensionality of the systems where such techniques are used has to be rather small. The methodology and diagnostics are based on the use of simple models. The dominant current approach is to screen alternatives based on the diagnostics. This mature area is reviewed first and then software is produced to transform it to a design tool. A MATLAB toolbox, that realises the various tests provides the means for testing the methodology on real life systems and thus evaluates them, will be presented in the next chapter. The

development of an approach for integration of the various approaches is given at the end of this chapter

9.3. INTERACTION ANALYSIS: A REVIEW OF METHODOLOGIES

9.3.1. Introduction and Definitions

The design of the structure of a controller, as expressed by the coupling of pairs of subvectors of inputs and outputs has been a problem that has been recognised in the process control area for a number of years as a design problem that requires its own diagnostics and property indicators. For a system represented by an input-output, transfer function model G(s) with p inputs and m outputs, the problem of control structure selection may be expressed in the following way:

Control Structure Selection: For the given system, represented by G(s), Control Structure Selection involves the following:

(i) Generate all possible partitions $\mathcal{P}_I = \{\underline{u}_\lambda\}$ of the input vector and all possible partitions of the output vector $\mathcal{P}_O = \{\underline{y}_\mu\}$ with variable dimensionality of the corresponding vectors $\underline{u}_\lambda, \underline{y}_\mu$.

(ii) Generate all possible compatible pairings from the set of partitions $\mathcal{P}_I, \mathcal{P}_O$, i.e. $\Pi = \{\forall \pi : \pi = \{(\underline{u}_{i_1}, \underline{u}_{j_1}), ..., (\underline{u}_{i_p}, \underline{u}_{j_p})\}\}$. Every compatible pairing Π defines a structured controller $K_\Pi \in \Re^{m \times p}$, or $K_\Pi(s) \in \Re(s)^{m \times p}$, where structuring implies that the controller has elements which are fixed zeros, irrespective of whether the other elements are constant or dynamic.

(iii) Evaluate the performance of every controller K_Π corresponding to compatible pairing π using some appropriate criteria, diagnostic tests and select the best.

The above process may involve clarification of best solutions for every class of pairings, where the class is defined by the dimensionality of the input, output vectors involved; the meaning of the class of controllers is a measure of their complexity and thus solutions may be classified according to their complexity and performance. The above

process classifies the different possible solutions and it has a combinatorial character that makes it rather difficult to use when the dimensionality of the input and output vectors is large. As such the methods considered here are suitable for small dimensionality problems, where generating all possible pairings is not an awesome problem.

The class of structured controllers produced by the above process contains the decentralised solutions. The resulting scheme may be represented as shown below:

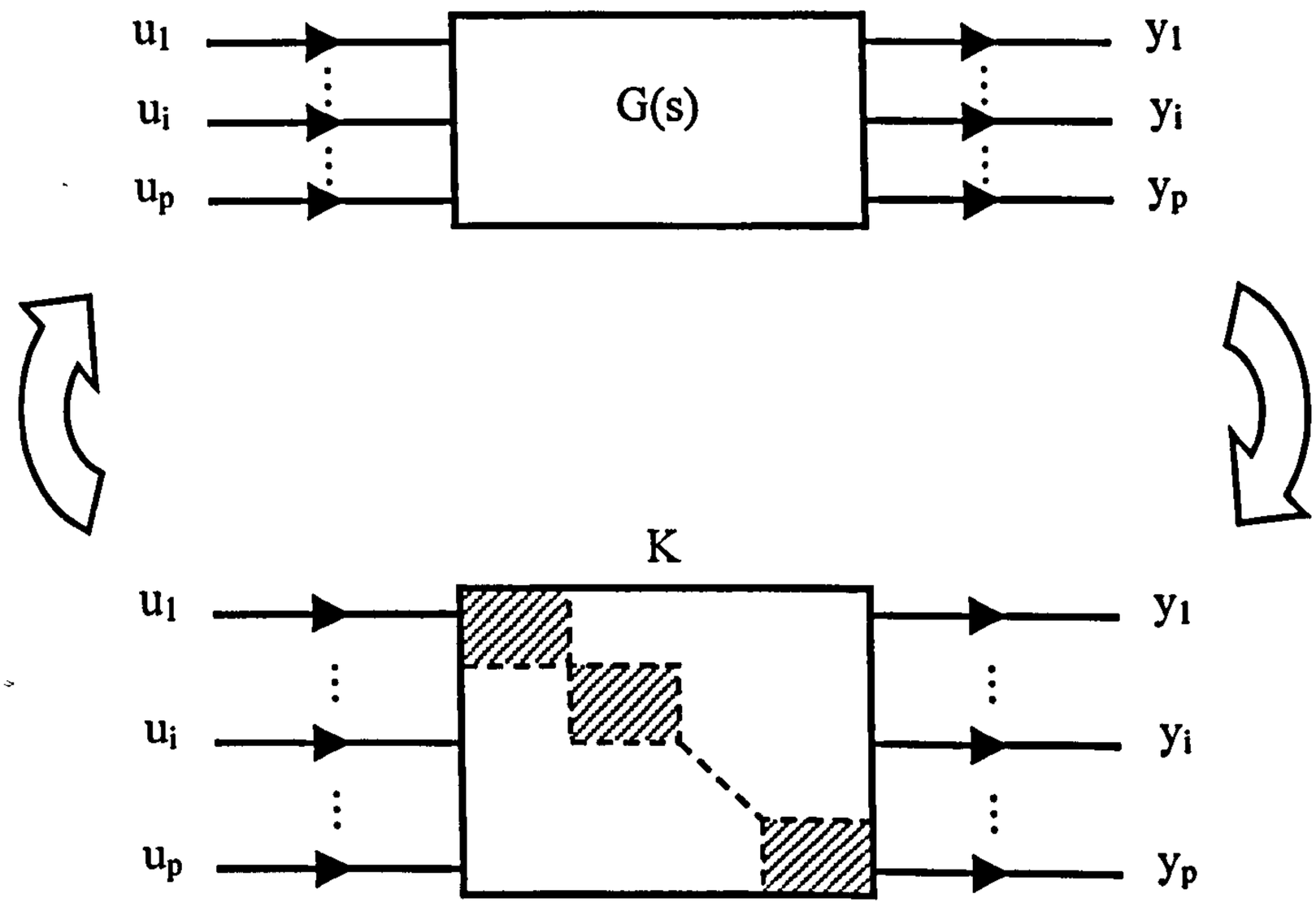

Figure (9.1): Structured Feedback Scheme

The major approaches for control structure selection based on criteria measuring the degree of coupling between the different partitions are referred to as interaction methods and are considered next.

9.3.2. Relative Gain Array (RGA)

The most widely known and used interaction analysis method is the Relative Gain Array (RGA), which was proposed by Bristol [Bris., 1]. This method focuses to a desirable feature of the process, which is to have the effective process gain remain invariant, regardless of the other control loops. It utilises the ratio of the process gain by a given controller with all other loops open over the process gain by a given controller with all other loops closed. Thus, each element λ_{ij} in the matrix Λ is a measure of the relative gain between controlled variable y_i and manipulated variable u_j, and the only information

needed for the estimation of the Relative Gain Array is the steady state gain matrix u_j. We consider the open loop sensitivity of a controller:

$$\left(\frac{\partial y_i}{\partial u_j}\right)_{u_k,k\neq j} \tag{9.1}$$

and the closed loop sensitivity of the controller:

$$\left(\frac{\partial y_i}{\partial u_j}\right)_{y_k,k\neq i} \tag{9.2}$$

where $u_{k,k\neq j}$ indicates all controllers except u_j are held constant. Thus, the elements of the RGA will be expressed as:

$$\lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_{u_k,k\neq j}}{\left(\frac{\partial y_i}{\partial u_j}\right)_{y_k,k\neq i}} \tag{9.3}$$

Bristol recommended two guidelines for the selection of the controller pairing:

1. Pair the controlled and manipulated variables in such a way that the corresponding relative gains are positive and as close to one as possible.
2. Avoid the pairing of controlled and manipulated variables that yield negative relative gains, because such pairings indicate, either an unstable system, or an inverse responding system.

This particular method takes into account only steady-state information of the process, ignoring any dynamic considerations, such as very large time delays or time constants, that are an important factor in the selection of the pairs of controlled and manipulated variables. In conclusion, the RGA provides two important items of information; firstly, it provides a measure of the process interactions, and secondly, it

supplies a recommendation concerning the most effective pairing of controlled and manipulated variables.

9.3.3. Niederlinski Stability Theorem

Useful information regarding the stability of a proposed multi-loop control system can be acquired by a theorem originally developed by Niederlinski [Nied., 1] and later corrected by Grosdidier et al [Grosd. et al, 1] . This theorem is based on three assumptions:

- If $G_{pij}(s)$, denotes the (i,j) element of the process transfer function $G_p(s)$, then each $G_{pij}(s)$ must be proper, rational and stable.
- Each of the n feedback controllers in the multi-loop control system contains integral action.
- Each individual control loop is stable when any of the other n-1 loops are opened.

If the proposed system satisfies the above assumptions then, the closed-loop system is unstable if:

$$\frac{|K|}{\prod_{i=1}^{n} k_{ii}} < 0 \qquad (9.4)$$

where K is the steady-state gain matrix of the system, with corresponding elements k_{ii} and $|K|$ denotes the determinant of K.

This theorem provides a sufficient but not necessary condition for instability. If the inequality is satisfied, then the closed-loop system is unstable. In the case where the inequality does not hold, then the closed-loop system may or may not be stable, depending on the numerical values of the controller settings. McAvoy [McAvoy, 1] reports several examples where apparently reasonable RGA pairings result in unstable closed-loop systems. Thus, it is important to consider the process dynamics and also check to ensure that a proposed pairing does not satisfy the inequality in Equation (9.4).

9.3.4. Dynamic Relative Gain Array (DRGA)

Several researchers, Tung and Edgar [Tung & Edgar, 1], McAvoy [McAvoy, 1], Grosdidier and Morari [Grosd. & Mor., 1] etc. have suggested alternatives and extensions to Bristol's Relative Gain Array, in order to the dynamic features of the process. Several extensions, which include dynamic effects, have been proposed. The dynamic relative gain array proposed by Tung and Edgar [Tung & Edgar, 1] is used more widely.

Assuming that the system is controllable and observable and employing a state-space model of the process, it can be shown that the resulting Dynamic Relative Gain Array will be given by:

	u_1	u_2	$\cdots$	u_m
y_1	$a_{11}(s)$	$a_{12}(s)$	$\cdots$	$a_{1m}(s)$
y_2	$a_{21}(s)$	$a_{22}(s)$	$\cdots$	$a_{2m}(s)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
y_m	$a_{m1}(s)$	$a_{m2}(s)$	$\cdots$	$a_{mm}(s)$

(9.5)

where:

$$\alpha_{ij}(s)=\frac{G_{ij}(s)\Gamma_{ji}}{s},\quad \begin{cases} i=1,2,\cdots,m \\ j=1,2,\cdots,m \end{cases} \tag{9.6}$$

where $G_{ij}(s)$, Γ_{ij} are the elements of the $C(sI-A)^{-1}B$ matrix and the $\left[C(-A)^{-1}B\right]^{-1}$ matrix respectively.

Having presented a frequency domain version of the dynamic RGA, a time domain interpretation can easily be defined. Such a procedure finally leads us to the following result:

	u_1	$\cdots$	u_m
y_1	$\left(\frac{\partial y_1}{\partial u_1}\Big\|_u\right)\Big/\left(\frac{\partial y_1}{\partial u_1}\Big\|_y\right)$	$\cdots$	$\left(\frac{\partial y_1}{\partial u_m}\Big\|_u\right)\Big/\left(\frac{\partial y_1}{\partial u_m}\Big\|_y\right)$
$\vdots$	$\vdots$		$\vdots$
y_m	$\left(\frac{\partial y_m}{\partial u_1}\Big\|_u\right)\Big/\left(\frac{\partial y_m}{\partial u_1}\Big\|_y\right)$	$\cdots$	$\left(\frac{\partial y_m}{\partial u_m}\Big\|_u\right)\Big/\left(\frac{\partial y_m}{\partial u_m}\Big\|_y\right)$

(9.7)

Thus, a very similar matrix, to the Relative Gain Array matrix, is produced, taking into account any dynamic considerations. Tung and Edgar propose that a proper control structure can be selected by finding the dominant terms in the dynamic relative gain array. The elements of the dynamic relative gain array having large absolute values indicate the recommended feedback loops. It is possible that different loop pairings are recommended at low and high frequencies. The use of a multivariable controller could be beneficial in such cases. Because this interpretation is based on gains only, one should analyse the effect of delays separately.

9.3.5. Performance - Relative Gain Matrix (P-RGA)

The notion of the D-RGA has already been presented and its use as a screening tool for alternative control structures has been shown. Even though the RGA as discussed before has many interesting properties, it can not address the multitude of problems that arise in interaction analysis.

The RGA matrix, as already defined, has some interesting algebraic properties ([Grosd. et al., 1]):

(a) It is scaling independent. Mathematically, $\Lambda(D_1GD_2) = \Lambda G$ where D_1 and D_2 are diagonal matrices.

(b) All row and column sums are equal to one.

(c) Any permutation of rows or columns in G results in the same permutation in the RGA.

(d) If G(s) is triangular (and hence also if it is diagonal), $\Lambda(G) = I$.

(e) Relative perturbations in elements of G and in its inverse are related by $d[G^{-1}]_{ji} / [G^{-1}]_{ji} = -\lambda_{ij}\, dg_{ij}/g_{ij}$.

An inadequacy of the RGA is that it, as property (d) outlines, even though interaction may not pose a problem, significant one-way coupling may exist. To overcome this problem, the performance relative gain array (P-RGA) can be introduced [Hovd. & Skog., 1]. The PRGA-matrix is defined as:

$$P(s) = \tilde{G}(s)G(s)^{-1} \tag{9.8}$$

where $\tilde{G}(s)$ is the matrix consisting of only the diagonal elements of G(s), i.e. $\tilde{G} = \text{diag}\{g_{ii}\}$. The matrix P was originally introduced at steady-state by [Grosd., 1] in order to understand the effect of directions under decentralised control. The elements of P are given by:

$$p_{ij} = g_{ij}(s)\left[G^{-1}\right]_{ji} = \frac{g_{ii}(s)}{g_{ji}(s)}\lambda_{ji}(s) \tag{9.9}$$

Note that on one hand the diagonal elements of the RGA and the PRGA are identical, but on the other hand the PRGA does not have all the algebraic properties of the RGA. PRGA must be recomputed whenever G is rearranged, whereas RGA only needs to be rearranged in the same way as G. PRGA is independent of input scaling, that is $P(GD_2) = P(G)$, but it depends on output scaling. This is reasonable since performance is defined in terms of the magnitude of the outputs.

The measures above may be extended to non-square systems by introducing the pseudoinverse. However, the usefulness of the measures, at least for analysing decentralised control, then seems to be limited.

9.3.6. Block Relative Gain (BRG)

Because of its many useful properties, the RGA has gained large applicability in the engineering world. However, at the same time, its original development as a scalar and its presentation in a single array, unnecessarily limited its applicability exclusively to SISO control loops. By formulating and extending the relative gain concept and its properties from a scalar to a matrix, a more powerful synthesis framework is formed, that can address a broader class of control problems, such as the synthesis of decentralised control structures that are not restricted to SISO control loops. This new concept is referred to as Block Relative Gain.

Control system synthesis starts with a given set of measurements, $\underline{y}$, and manipulated variables, $\underline{u}$. The input-output model $\underline{y}(s) = G(s)\,\underline{u}(s)$ is usually assumed to be the one to describe the plant dynamics, with the transfer function matrix G(s) considered to be square. In decentralised plant control, different subsets of outputs are assigned to different subsets of inputs and each such assignment forms a subsystem G_{ii}. In classical

feedback terms this implies that output measurements of an individual subsystem will affect the manipulated inputs of that subsystem only via its own control law. Alternative subsystems and, thus, decentralised control structures can be systematically generated by partitioning G(s) into blocks of different dimensions and also due to alternative ways of assigning inputs and outputs to the blocks, figure (9.2).

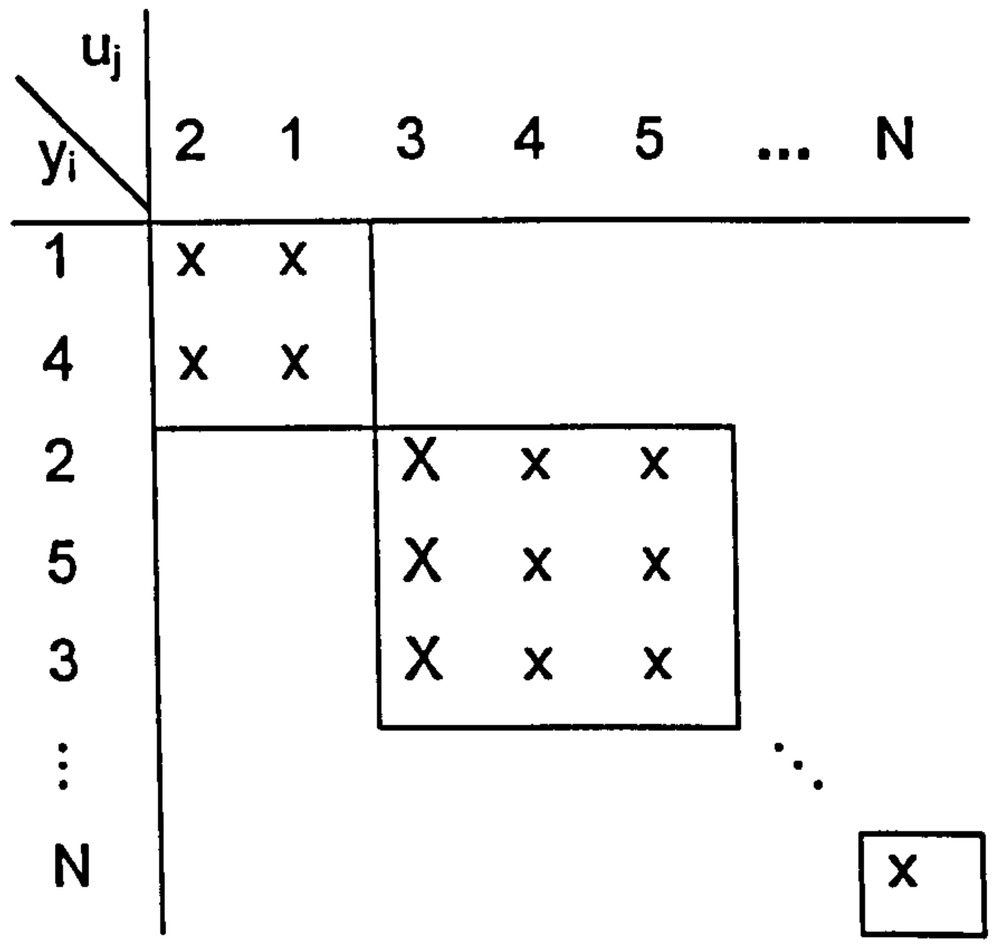

Figure (9.2): Partitioning of G(s) into blocks of different dimensions.

Note that in this type of partitioning, subsystems are viewed as aggregates of control loops and not as groups of process units. Thus, block partitioning of G(s) may not necessarily correspond to a particular process decomposition and the resulting decentralised control system does not have to be compatible with any arrangement of subsystems of process unit operations. However, this does not preclude the possibility of specifying the process decomposition first and then structuring the decentralised control systems within the boundaries of the individual process subsystems. In some cases, this may eliminate the synthesis of undesirable decentralised control structures right from the beginning and reduce the potential combinatorial problems encountered in the block partitioning procedure.

To better understand the concept of BRG, one has to consider a square $(n \times n)$ transfer function matrix G(s), partitioned as follows (the s is dropped for convenience):

$$G = \begin{array}{c} \begin{matrix} n & n-m \end{matrix} \\ \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] \end{array} \begin{array}{c} m \\ n-m \end{array}, \text{ with } \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = G \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{9.10}$$

The plant is to be controlled by a decentralised control structure in which the first m outputs y_1 are interconnected with the first m inputs u_1 and the last n-m outputs y_2 are interconnected with the last n-m inputs u_2. The corresponding feedback configuration is shown in figure 9.3. The controller K and the filter F are given by:

$$K = \begin{matrix} m & n-m \\ \left[\begin{array}{c|c} K_1 & 0 \\ \hline 0 & K_2 \end{array}\right] & \begin{matrix} m \\ n-m \end{matrix} \end{matrix}, \quad F = \begin{matrix} m & n-m \\ \left[\begin{array}{c|c} F_1 & 0 \\ \hline 0 & F_2 \end{array}\right] & \begin{matrix} m \\ n-m \end{matrix} \end{matrix} \tag{9.11}$$

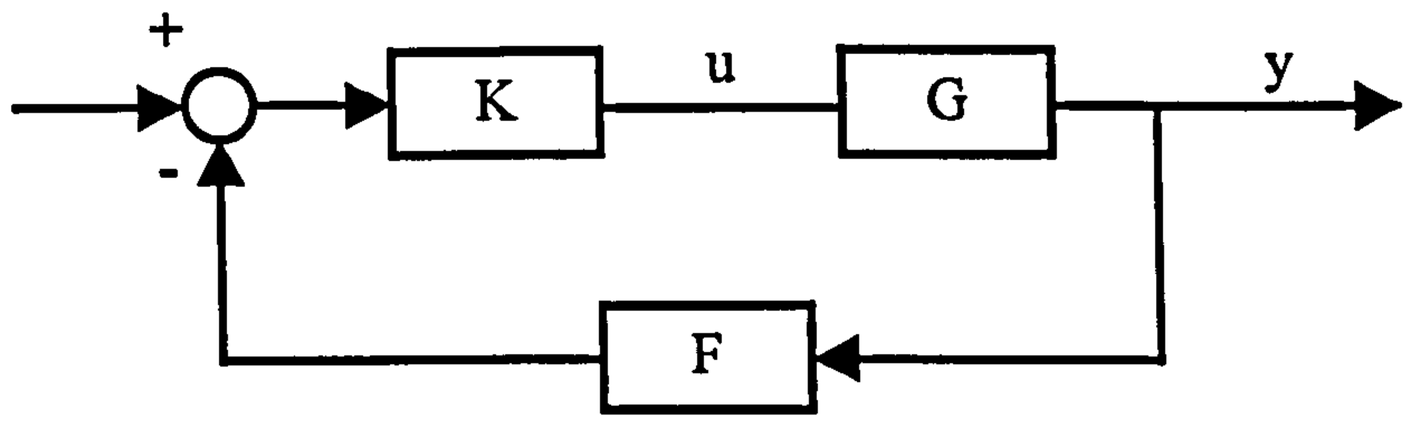

Figure 9.3: Decentralised feedback system

The following relations hold:

$$y = Gu \quad \text{and } u = G^{-1}y \text{ (assuming } G^{-1} \text{ exists)} \tag{9.12}$$

Then, if G_{22} is nonsingular,

$$\left.\frac{\partial y_1}{\partial u_1}\right|_{u_2=0,F=0} = G_{11}, \left.\frac{\partial y_1}{\partial u_1}\right|_{u_2=0,F_1=0,F_2=0} = \left(\left[G^{-1}\right]_1\right)^{-1} = G_{11} - G_{12}G_{22}^{-1}G_{21} \tag{9.13}$$

where $\left[G^{-1}\right]_{11}$ is the first $m \times m$ block of G^{-1}:

$$G^{-1} = \begin{bmatrix} \left[G^{-1}\right]_{11} & \left[G^{-1}\right]_{12} \\ \left[G^{-1}\right]_{21} & \left[G^{-1}\right]_{22} \end{bmatrix} \tag{9.14}$$

According to equation (9.13), G_{11} denotes the block gain between $\underline{y}_1$ and $\underline{u}_1$ when all the loops are open, i.e. $F = 0$. Similarly, $\left(\left[G^{-1}\right]_{11}\right)^{-1}$ is the block gain between $\underline{y}_1$ and $\underline{u}_1$ when the first m loops are open, i.e. $F_1 = 0$ and the last $n-m$ loops are closed, i.e., $F_2 = 0$, and under perfect control, i.e., $\underline{y}_2 = 0$.

The m-dimensional Block Relative Gain (left and right) can then be defined as:

$$BRG_\ell = \left[\left.\frac{\partial y_1}{\partial u_1}\right|_{u_2=0,F=0}\right]\cdot\left[\left.\frac{\partial y_1}{\partial u_1}\right|_{u_2=0,F_1=0,F_2=0}\right]^{-1} = G_{11}\cdot\left[G^{-1}\right]_{11} \tag{9.15}$$

$$BRG_r = \left[\left.\frac{\partial y_1}{\partial u_1}\right|_{u_2=0,F_1=0,F_2=0}\right]\cdot\left[\left.\frac{\partial y_1}{\partial u_1}\right|_{u_2=0,F=0}\right] = \left[G^{-1}\right]_{11}\cdot G_{11} \tag{9.16}$$

Note that in the case of one-dimensional BRG, left and right BRG's become identical (since G_{11} is scalar) and reduce to the classical Bristol's RGA. In the case of n-dimensional BRG, $BRG_\ell = BRG_r = I$. The significance of the BRG in relation to the closed-loop performance can be derived from a study of the following three cases:

<u>**Case 1**</u>: $F_1 = 0, F_2 = 0$ (no feedback)

<u>**Case 2**</u>: $F_1 = 0, F_2 = I$ (feedback of the last $n-m$ outputs to the last $n-m$ inputs).

<u>**Case 3**</u>: $F_1 = 0, F_2 = 0$ (feedback of the first m [last $n-m$] outputs to the first m [last $n-m$] inputs, respectively).

From the results (the extensive calculations can be found in [Manous. et al, 1], the answer to the important question about the significance of the relative gain for the performance of the closed-loop system can be derived. That is that the closed-loop performance of the $m\times m$ block under consideration, when the other $n-m$ outputs are under perfect control, is a continuous function of BRG_ℓ. When $BRG_\ell = I$, which implies $BRG_r = I$, the closed-loop performance of the $m\times m$ block is as if this block was isolated from the rest of the plant and operating under the influence only of its own control law. This makes it clear what kind of information one should expect from BRG and in what sense it can be considered as a measure of interaction.

9.3.7. <u>Dynamic Block Relative Gain (D-BRG)</u>

When defining the block relative gain and deriving its relation to the closed-loop performance, the usual assumption of perfect control for the plant outputs has been made.

This assumption always holds at zero frequency (i.e., at steady state) by the use of integral control action. However, it may not hold for all the frequencies especially when non-minimum phase and/or strictly proper blocks are present. For such cases, the assumption of perfect control over the whole frequency range can be relaxed, [Manous. et al, 1]. If one wished to investigate interactions over the whole frequency range, BRG could be extended to a Dynamic-BRG and become a frequency-dependent interaction measure, that need not be modified, if there are no right half-plane transmission zeros in the complementary subsystem, which is supposed to work under perfect control. On the other hand, when RHP zeros exist, one either evaluates the equations at steady state only or, if interested in all the frequencies, one can use an appropriately modified D-BRG as given in [Manous. et al, 1].

BRG, as it was previously defined, is related to the first m outputs and m inputs of the plant. As a result, it will depend on how the n outputs and n inputs are ordered in G(s). Since the number of all possible rearrangements of n objects is n!, n outputs and n inputs can be ordered in $(n!)\cdot(n!)$ possible combinations. Calculating an m-dimensional BRG for each such combination would result to a total of $(n!)^2$ BRG computations, which would be an enormous task for large n's. In order to resolve this combinatorial problem, certain theorems were presented [Manous. et al, 1]. The results were the following:

- BRG_l (BRG_r) is not affected by the ordering of the last $n-m$ inputs and $n-m$ outputs and the ordering of the first m inputs (outputs). Furthermore, for all G's that contain the same first m inputs and m outputs but different arrangements, the corresponding BRG's turn out to be trivial rearrangements of each other. Consequently, they can be considered equivalent. Thus for an m-dimensional subsystem containing a unique group of inputs and outputs, only one of the equivalent BRG's needs to be examined. This means that for an n-dimensional system, the number of calculations for an n-dimensional BRG drops from $(n!)^2$ to $\binom{n}{m}\cdot\binom{n}{m}$, which is a significant reduction for large systems.
- BRG_l (BRG_r) does not depend on the scaling of the last $n-m$ inputs and outputs and the first m inputs (outputs), but it does depend on the scaling of the first m outputs (inputs). However, if the first m outputs (inputs) are all scaled in the same way, then BRG_l (BRG_r) is not affected at all.

- The diagonal elements of BRG's are well defined i.e. they remain on the diagonal but not necessarily at the same locations when G is trivially rearranged. This implies that, for all the BRGs corresponding to a particular group of m inputs and outputs, the designer needs to examine only m diagonal terms.
- The well-known property of the RGA that elements of each row and each column add to 1, also applies to BRGs. This is a direct consequence of the previous result and also of the fact that the n-dimensional BRG is the identity matrix.

The aim of the DBRG is to provide an acceptable block partitioning of the plant matrix G(s). Such a task is considered to be accomplished if all the BRG's of different dimensions corresponding to the diagonal blocks of different dimensions $G_{ii}(s)$'s, are close to an identity matrix. To quantify this closeness and define the set of viable BRG's, the following procedure is necessary:

Let $B(1,\varepsilon)$ denote a neighbourhood in the complex plane with centre at $(1,0)$ and radius $\varepsilon = \varepsilon(\omega)$. Then we say that a BRG is viable (i.e. close to identity) if its diagonal elements and eigenvalues belong to neighbourhoods $B(1,\varepsilon_1)$ and $B(1,\varepsilon_2)$, respectively, for all frequencies ω. The selection process is the following:

First consider the highest degree of decentralisation – i.e. 1×1 block partitioning of $G-$ that would yield a total of N SISO assignments (or pairings). For this, all the one-dimensional BRGs are first evaluated at $s=0$. Among the viable ones, those which establish a $1-1$ correspondence between the plant's inputs and outputs, are selected. If such alternatives do not exist, then there is no acceptable partitioning using $1-1$ blocks only. In that case, assignment is not complete and one proceeds with two-dimensional BRGs. In case there exists an acceptable $1-1$ block partitioning for $s=0$ but viability and/or acceptance are violated at frequencies other than $\omega=0$, the study of two-dimensional BRGs is again necessary.

The next step in the process is the study of two-dimensional BRGs. We first study only BRG_ℓs. The BRG_ℓs, whose diagonal elements are not close to 1, are screened out first. Among the remaining BRG_ℓs, only those with eigenvalues close to 1 are retained since these are the BRG_ℓs that are close to the identity matrix. Since the eigenvalues of BRG_r and BRG_ℓ are the same, a detailed study of BRG_rs is deemed unnecessary, in case the eigenvalues of BRG_ℓ are close to 1. If this is not the case, the diagonal elements of

BRG_rs should be calculated from the RGA and their closeness to one should be examined as a final screening criterion.

The diagonal terms of all the two-dimensional BRG_ts are the elements of the column vectors that result from every possible addition of two columns of the RGA. Thus if one of these column vectors has $p > 2$ elements within $B(1, \varepsilon_1)$ this implies that there exist only $p!/[2!(p-2)!]$ two-dimensional BRG_ts that should be further considered. Among these BRG_ts, those with eigenvalues outside $B(1, \varepsilon_2)$ are rejected. The remaining BRG_ts are the two-dimensional viable BRG_ts for $s = 0$ and for one of the column vectors discussed above. The screening process is repeated for all the possible column vectors and for all frequencies other than $\omega = 0$ and ultimately gives all viable two-dimensional BRG_ts.

Searching for an acceptable partitioning over the sets of both two- and one-dimensional viable BRG_ts is the next step. If one is found, the procedure concludes; otherwise it continues with the study of BRG_ts of higher dimension, in the same manner, until a solution is achieved. The process is guaranteed to conclude since, in the worst case, it will lead to a centralised full control structure that corresponds to an n-dimensional BRG. It should be mentioned that $\varepsilon_1(\omega)$, $\varepsilon_2(\omega)$ are free parameters through which the designer can affect the screening process and establish what an acceptable degree of interaction is.

Having presented the procedure, one can easily understand the advantages of the DBRG. Different block partitioning of input and output sets leads to alternative decentralised control structures, among which the best are selected by the systematic screening procedure that utilises various important properties of BRG. These properties effectively reduce the combinatorial problems and make the analysis of large-scale systems feasible.

9.3.8. Scaled Gain Matrix (SGM)

This method was first proposed by [Liesl., 1]. The method aims to provide a control system designer useful information on interactions in a form that is easy to interpret. It is based on the scaling of input and output variables. Although a large gain between an input and an output indicates strong interaction, the process gain matrix can not be directly used for interaction analysis, because it depends on the scaling of input and output variables. In this method the input and output variables are rescaled, so that in the new gain matrix,

corresponding to the rescaled variables, the elements are directly comparable with each other.

Consider an $m \times n$ process transfer function matrix G(s). The basic idea behind the method is to scale input and output variables in such a way that the average gain in each row and column of the process model is one at a given frequency. This is achieved using the following iterative procedure:

Step 1 Calculate the gain matrix at the desired frequency ω^*, $|G(j\omega)^*|$ This is the first estimate of the scaled gain matrix Ψ, i.e. for $k = 1$ set

$$\psi_{ij}^k = \left| g_{ij}(j\omega^*) \right| \tag{9.17}$$

Step 2 Scale the rows of Ψ^k in such a way that in each row the average value of the elements is equal to one.

$$\psi_{ij}^{k+1} = \frac{n \cdot \psi_{ij}^k}{\sum_{j=1}^{n} \psi_{ij}^k} \tag{9.18}$$

Step 3 Scale the columns of Ψ^{k+1} in such a way that in each column the average value of the elements is equal to one.

$$\psi_{ij}^{k+2} = \frac{m \cdot \psi_{ij}^{k+1}}{\sum_{j=1}^{m} \psi_{ij}^{k+1}} \tag{9.19}$$

Step 4 Stop if the changes between Ψ^k and Ψ^{k+2} are sufficiently small. Otherwise set $k \leftarrow k+2$ and go to step 2.

The procedure converges towards the scaled gain matrix (SGM) that is unique for each matrix. In the Scaled Gain Matrix, the average value of the elements in each row and column is one. The interpretation of this interaction matrix is simple: values larger than one indicate strong interaction and values smaller than one indicate weak interaction. The largest

elements in Ψ then indicate the inputs and outputs, which should be connected in the feedback controller. The SGM, unlike the RGA, can be used even when the number of inputs and outputs differ. The SGM is also applicable to the analysis of dynamic effects without any special extensions. The SGM is a method for finding possible control structure candidates. Stability and performance constraints imposed by these control structures have to be analysed using alternative measures.

9.3.9. Disturbance Cost and Disturbance Condition Number

The quality assessment of the controllability and resiliency of a process has generated considerable interest. The term resiliency, as used in this text, was introduced by Morari [Morari, 1], and later Perkins [Perk., 1] presented an approach for the simultaneous design of processes and their control systems that addresses plantwide controllability directly. In the context of disturbance rejection the notion of adequate disturbance resiliency means the ability to reject disturbances entering the process quickly enough to meet performance specifications.

The notion of resiliency, as explained above, has lead to the proposition of two methods that are used to measure this property. Assuming a linearised model of the process, the open loop response of the outputs is expressed in terms of the variations of the inputs and disturbances:

$$\underline{y}(s) = \underline{\underline{P}}(s)\,\underline{u}(s) + \underline{\underline{P_d}}(s)\,\underline{d}(s) \tag{9.20}$$

Lewin [Lewin, 1], suggested the norm of the actuator response, $\|\underline{u}(s)\|_2$, as a quantitative measure of the control effort needed in order to reject a given disturbance vector. This norm is called the Disturbance Cost:

$$\|\underline{u}(s)\|_2 = \left\|\underline{\underline{P}}^{-1}(s)\,\underline{\underline{P_d}}(s)\,d(s)\right\|_2 \tag{9.21}$$

Skogestad and Morari [Skog. & Mor., 1], introduced a normalised measure similar to the Disturbance Cost, the Disturbance Condition Number:

$$\kappa_d\{\underline{\underline{P}}\} = \frac{\left\|\underline{\underline{P}}^{-1}(s)\,d'(s)\right\|_2}{\left\|d'(s)\right\|_2}\,\bar{\sigma}[\underline{\underline{P}}\} \tag{9.22}$$

which depends only on the direction of the disturbance $d'(s)$ and not its magnitude and the maximum singular value of $\underline{\underline{P}}$, $\bar{\sigma}\{\underline{\underline{P}}\} = \max_i \lambda_i\{\underline{\underline{P}}^T\underline{\underline{P}}\}$, where λ_i are the eigenvalues of $\underline{\underline{P}}^T\underline{\underline{P}}$. These two methods provide indications for the settling time for the disturbance rejection and the limitations, which depend on actuator constraints.

9.3.10. Closed Loop Disturbance Gain

Another method that provides diagnostics for disturbances that are difficult to reject was proposed by Hovd and Skogestad [Hovd & Skog., 1]. The Closed Loop Disturbance Gain (CLDG) can be defined as:

$$\Delta = \{\delta_{ik}\} = G_{diag}(s)\cdot G^{-1}(s)\cdot G_{dist}(s) \tag{9.23}$$

where $G_{diag}(s)$ is a matrix containing only the diagonal transfer functions of G(s) and $G_{dist}(s)$ is the matrix of the disturbances of transfer functions having a very similar form as the one of equation (9.17).

The components of the CLDG matrix correspond to the influence of any disturbance to every output. The higher the value of any component of the CLDG matrix δ_{jk}, the more difficult is the rejection of the corresponding disturbance k from output j. It is also important to note that the evaluation of the CLDG is scaling dependent, since it depends on the expected magnitude of the disturbances and outputs and it has to be re-evaluated for every possible combination of control pairings.

It is obvious that it is crucial that the variables are scaled properly. In general, the variables should be scaled to be within the interval –1 to 1, that is their desired or expected magnitudes should be normalised to be less than 1. For the case of the CLDG, which depends on scaling of the inputs u and the disturbances z, recommended scaling for inputs is that an input u_j of magnitude 1 should correspond to the allowable input signal and for disturbances that a disturbance z_k of magnitude 1 should correspond to the largest expected disturbance.

9.3.11. Morari Index of Controllability

For this method, it is important to define the notion of the Decentralised Integral Controllability. A plant G (corresponding to a particular pairing) is Decentralised Integral Controllable (DIC) if it is possible to design a diagonal controller for the plant, which has the following properties:

- It has integral action (no offset for tracking)
- It yields stable individual loops
- It is such that the system remains stable when all the loops are closed simultaneously
- It has the property that each loop gain may be reduced independently with a factor $e_i\ (0 \le e_i \le 1)$ without introducing instability

It has to be noted that DIC is a property of the plant and the particular control structure (pairing) chosen.

Another method that investigates whether a possible variable pairing will produce an unwanted dynamic behaviour and therefore a plant which is not Decentralised Integral Controllable is implied, is the Morari Index of Controllability (MIC) [Skog. & Mor., 2]. The Morari Index of Controllability is defined as:

$$\text{MIC} = \text{Re}\{\lambda e\{^{+}((0))\} \tag{9.24}$$

where $G^{+}(0)$ is the plant steady-state matrix with the signs adjusted so that all the diagonal elements have positive signs. Pairing which have negative MIC should be avoided. Another variant of the above method is the one, which eliminates the pairings for which:

$$\text{Re}\{\lambda(E(0))\} < -1 \tag{9.25}$$

where matrix E is used in the interaction measures derived by Grosdidier and Morari [Grosd. & Mor., 1] and is given by:

$$E = (G - G_{diag}) \cdot G_{diag}^{-1} \tag{9.26}$$

where matrices G and G_{diag} are the same as in (9.23). This method is actually equivalent as to eliminate pairings that have eigenvalues of the "Jacobi iteration matrix" greater than one [Mij. et al, 1].

An interesting point regarding the investigation of cases of pairings that produce a plant, which is not Decentralised Integral Controllable as shown in the above two methods is that they can be thought of as special cases of a more general method, which eliminates pairings for which there exists a matrix K (which is diagonal with positive entries), such:

$$\mathrm{Re}\{\lambda(G^{+}(0)\cdot K)\}<0 \qquad (9.27)$$

These two methods are not necessary and sufficient conditions for a plant to be DIC. This means that if a plant fails to satisfy the conditions set by these two methods it is not DIC, but there may be plants that pass the tests, but still turn out not to be DIC.

9.3.12. Singular Value Analysis (SVD)

Singular Value Decomposition (SVD) is a promising analytical technique, used to solve important control problems, such as the evaluation of the robustness of a proposed control scheme and the determination of the optimal multi-loop control configuration. This method can provide a powerful and computationally efficient tool for analysing matrix systems [Fors. et al, 1] and it is the basis for many diagnostics for control system design.

A systematic approach to the synthesis of regulatory process control structure can be formulated. The analysis can be performed over the frequency range that is of practical importance for the particular process, so that both static and dynamic aspects can be considered. An additional important feature of the SVD strategy is its ability to identify modelling aspects, such as model mismatch, which affect the performance of the resulting process control structure. Also, the strategy can show whether or not a structural decoupler will be effective in minimising interactions between loops. A compensator can be designed for the range of frequencies most likely to affect the process. Since we are primarily interested in the control structure, rather than in the actual controller design, the analysis is based on the open loop transfer function. The approach provides insights into important closed loop system properties: stability [Post. et al., 1], sensitivity [Weber & Bros., 1], and invertibility [Mor., 2].

The application of the SVD to the $m \times n$ transfer function matrix G(s) leads to:

$$G(s) = Z(s)\Lambda(s)\Lambda^{T}(s) \tag{9.28}$$

where: $$\Lambda(s) = \begin{bmatrix} \Delta(s) & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} p \\ m-p \end{matrix}, \quad p = \operatorname{rank} G(s) \le \min(m,n) \tag{9.29}$$

$$\begin{matrix} p & n-p \end{matrix}$$

$\Delta(s)$ is a diagonal matrix whose entries are the singular values of $G(s)$. This decomposition implies that $(m-p)$ measurements and $(n-p)$ manipulated variables can be deleted without altering the input-output accessibility and manipulability of the system. However, a preliminary system analysis should include all the input and output variables.

Suppose $\sigma_1(s), \sigma_2(s), \cdots, \sigma_p(s)$ are distinct singular values of $G(s)$, then $Z(s)$ can be partitioned as $Z(s) = [z_1(s) \vdots z_2(s) \vdots \cdots \vdots z_p(s) \vdots z_{p+1}(s), \cdots, z_m(s)]$, and $V(s)$ can be partitioned as $V(s) = [v_1(s) \vdots v_2(s) \vdots \cdots \vdots v_p(s) \vdots v_{p+1}(s), \cdots, v_n(s)]$, where $z_i(s)$ and $v_i(s)$ (for i=1,...,p) are the singular decomposition vectors which correspond to the i-th singular value and $z_j(s)$ and $v_j(s)$ (for j=p+1,...,n) are the remaining decomposition vectors which correspond to the zero singular values. Thus, a singular value decomposition of the matrix $G(s)$ defines an input space spanned by a set of orthonormal basis vectors $\{v_i(s)\}_1^p$ and a gain space defined by the set of singular values $\{\sigma_i(s)\}_1^p$. Furthermore, a one-to-one correspondence is established between these spaces as it is illustrated in Figure (9.4):

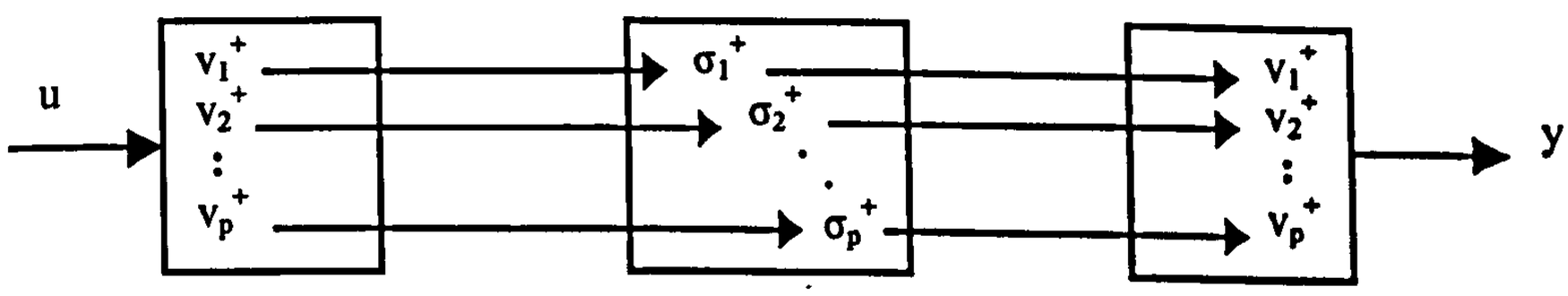

Figure (9.4): Geometric interpretation of the SVD [Lau et al., 1]

It is now possible to interpret the transfer function matrix geometrically. An input vector in the direction of $v_i^T(s)$ propagates through the input space, is scaled by the gain $\sigma_i(s)$, and reappears in the output direction $z_i(s)$. From the above, it is easy to express $G(s)$ in terms of the singular values:

$$y(s) = \left[\sum_{1}^{p} \sigma_i(s) z_i(s) v_i^T(s)\right] u(s) \tag{9.30}$$

By expanding $y(s)$ and $u(s)$ in the standard basis vectors $\{e_k^m\}_{k=1}^m$ and $\{e_j^n\}_{j=1}^n$, where the superscripts refer to the vector dimension, we finally obtain:

$$y_k(s) = \sum_{i=1}^{p} \sigma_i(s) \sum_{j=1}^{n} u_j(s) \langle W_i(s), E_{kj} \rangle \tag{9.31}$$

where $\langle W_i(s), E_{kj} \rangle \equiv \left(e_k^{m^T} z_i(s)\right)\left(v_i^T(s) e_j^n\right)$. The product $\langle W_i(s), E_{kj} \rangle$ may be interpreted geometrically as a measure of the alignment of the singular decomposition vectors $z_i(s)$ and $v_i^T(s)$ to the standard basis vectors in the appropriate space, as is illustrated in figure (9.5):

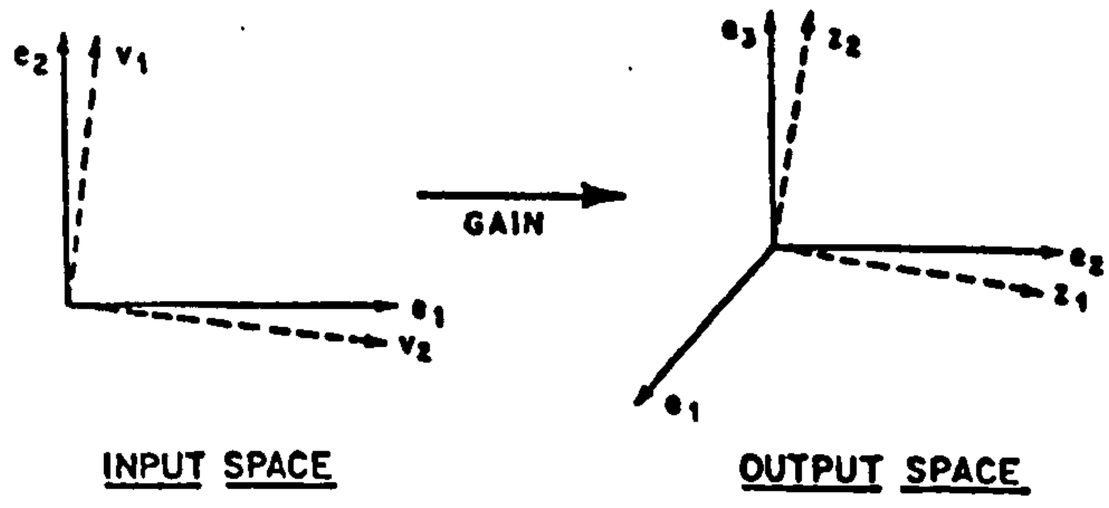

Figure (9.5): Pictorial representation of alignments between singular value vectors and standard basis vectors for a 3-input x 2-input system.

Another important matrix property is the Condition Number:

$$CN = \sigma_1 / \sigma_r \tag{9.32}$$

where σ_1 and σ_r are the largest and smallest nonzero singular value of the steady-state gain matrix. The Condition Number is a positive number that provides a measure of how ill-

conditioned the gain matrix is and additional information on how sensitive is the matrix to variations of its elements. Thus, processes, which have poorly conditioned gain matrices, require excessive control action, by changing the manipulated variables in order to affect the controlled variables.

Singular values of the transfer function matrix may be used to evaluate stability margins for multi-input-multi-output (MIMO) systems in the same manner as the amplitude ratio is used in single-input/single-output (SlSO) systems [Doyle & Stein, 1]. However, no measure of interaction or systematic search procedure is considered. As illustrated in a previous part, Morari [Morari, 2] used the SVD to quantify the control performance attainable in a process and interpreted implementability and sensitivity of the plant, concepts, which quantify the resiliency of the plant, in terms of the norms of the transfer function operator. It is interesting that two problems, at opposite ends of the hierarchy in process design and control structure synthesis, emerge closely related after the appropriate analysis. In addition, the SVD strategy can be used to identify modelling aspects, such as model mismatch, which affect the performance of the resulting control structure.

9.3.13. Structured Singular Value

An approach, which provides a sufficient condition for having a Decentralised Integral Controllable plant, is given in terms of the structured singular value μ of E (equation (9.26)) [Doyle & Stein, 1]. The structured singular value provides a generalisation of the singular value σ and the spectral radius ρ and can be defined by:

$$\mu(M)^{-1} = \min_{\Delta}\{\sigma(\Delta)/\det(I - M\Delta\Delta = 0 \text{ for structured } \Delta\} \qquad (9.33)$$

This measure is used to get necessary and sufficient conditions for robust stability and robust performance. According to the conditions for a plant to be DIC [Grosd. & Mor., 1], the following rule results:

$$\mu(E(0)) < 1 \qquad (9.34)$$

where it should be noted that: $\rho(E) \leq \mu(E) \leq \rho(|E|)$

and we therefore have that the eigenvalues of E(0) should always be greater than –1 (as dictated by equation (9.25)) and their magnitude ρ(E(0)) be preferably less than 1 (as proposed by equations (9.33) and (9.34)). The generalisation of DIC follows since the use of μ(E) allows the individual loops to be different and therefore can be independently designed.

9.3.14. Discussion

From all the mentioned methods and tests that have been developed, which belong to the area of Interaction Analysis and are used to decide the pairings that are to be made in the control structure, it is obvious that necessary conditions for Decentralised Integral Control (DIC) are of particular interest. A violation of any of these tests implies that DIC is not possible and the corresponding pairing may be eliminated. For most process plants the majority of the alternatives may be eliminated using such conditions.

The main reason for problems encountered with decentralised controllers are the interactions caused by the off-diagonal elements in a given plant G. If these elements are "small" then the corresponding interactions are weak and decentralised control is simple. If the interactions are large, then it might happen that the sign of the plant gain between a specific plant input and output changes sign as other loops are closed. Integral control, which is known to depend on knowing the plant gain, is then not possible. All of the conditions that were presented are therefore in terms of avoiding pairings where the plant gain may change sign as other loops are changed. It is stressed here that this discussion is concerned only with tests that use steady state data.

The Relative Gain Array (RGA) has the important advantage that it is very simple to compute and does not have to be recomputed to investigate alternative pairings. This is because a permutation of the rows or columns in the plant G, corresponding to a change in pairings, results in the same permutation in the RGA (Bris., 1). Consequently, one should always start by eliminating pairings according to what is proposed by the evaluation of the RGA. The rest of the conditions and tests can subsequently be used to make the final decision over the control structure of the process. In general, interaction type methodologies rely on testing alternative pairings. For large dimension problems such methodologies are linked to a combinatorial explosion (huge number of tests). Most of these tests have also been modified in order to take into account the dynamic effects of the interaction between the possible pairings of the control structure.

All the methods and tests discussed in this chapter will be implemented in the developed software. The implementation will be discussed in Chapter 10.

9.4. AN INTEGRATED APPROACH TO CONTROL STRUCTURE SELECTION

Interaction analysis is a methodology that is suitable for small dimension problems and thus relevant for issues addressed at the subsystem level. The overall selection of control structure for models, which have been already formed and are well conditioned, is an area involving many more steps and an approach that can be used as described below [Karc. et al, 1]:

Step (1): Use knowledge on the process, geographical location of process units and operational requirements to define a first appraisal of options as far as centralisation versus decentralisation.

This step aims to take into account the particulars of the application area and nature of the problem. This knowledge is indispensable and it is part of the overall problem specification. What is expected at this stage, is the development of the first structuring of the schemes in terms of superblocks, which may require some further structuring subsequently. It is worth mentioning that the requirements of the overall problem decomposition based either on performance optimisation (operational) or subproblem design, have to be taken into account here. This area is dominated by the process dependent specifics, heuristics but there is also a need for work which has to be based on the systematic study of the problem decomposition (operational and design aspects). This area of work may be considered as a part of the control structure selection on a whole plant.

Step (2): Use results on the generic solvability of decentralised control problems to produce a first parameterisation of alternatives.

The study of decentralised control problems has produced some results characterising generic solvability, which leads to parameterisation of possible partitions of input, output channels, which permit solvability of these control problems. These results depend on structural characteristics such as the McMillan degree and numbers of input, outputs, infinite zero structure etc. This topic has been considered in Chapter 7. This analysis is the first of the analytical steps in the evaluation of the alternative schemes.

Step (3): Use of graph analysis methodology and the concept of structural fixed modes for evaluation of alternatives defined by the previous step.

For systems which have an explicit graph structure, a procedure for evaluating alternatives based on the exclusion of structural fixed modes may be used as a first structural methodology that uses the most basic structural aspect, the system graph. It is clear that the results have to exploit deeper structural characteristics based on the graph rather than those of the previous step. In parallel to this activity we have a step that aims at reducing the overall dimensionality of the problem by decomposing the large problem to smaller ones.

Step (4): Develop methodologies for decomposing a large dimension design problem to smaller problems.

The decomposition of the process is not dictated by any computational considerations but is an integral part of the design strategy. Process decomposition reveals the aggregates of unit operations and chemical reactors, which must be centrally controlled. Note that the process decomposition can be directed towards developing the independently controlled groups of units, in terms of regulation or optimisation. Both criteria can be applied to the same process simultaneously and nearly independently. Although this may sound contradictory, process decomposition for regulatory purposes will be feasible within the bounds of the groups established from the process decomposition for optimising control purposes. To split a process into subprocesses, which are optimised separately, one must be able to decompose the overall objective function linearly, and one part of it must be associated with every subsystem. The minimal size of a subsystem is usually dictated by that restriction. For optimisation, the magnitude of the subproblems has to be balanced against the effort to coordinate solutions. In addition, the solution should not be too sensitive to the exact satisfaction of the interconnection constraints. Otherwise, the required co-ordination algorithm needs to be involved as well.

Step (5): Use of interaction analysis diagnostics based on steady state models or simple dynamic models, in order to evaluate the alternatives produced at the previous stage.

Progressing from graph models to decomposed systems using small steady state or simple dynamic models, we may use the large number of diagnostics of the RGA, BRGA type to evaluate further the options specified by the previous step. After this stage we may progress to further evaluation, which is described in the next step.

Step (6): Use of advanced structure selection diagnostics based on linear dynamic models and parameter dependent structural characteristics.

At this stage, we proceed with the evaluation of the available options using linear models and parameter dependent properties such as fixed modes (non structural), almost fixed modes under various dynamic modes, properties of the rank of decentralised Plücker matrices, strong instability and minimum phase phenomena etc. The exterior algebra diagnostics, which include a large number of tests, used in this step are described in [Lev. & Karc, 4]. Within this family the Decentralised Markov parameters [Lev. & Karc., 2] are first used, since the computations involved are relatively simple and then we proceed to the more complex algebra tests. In all these studies we use as a test the avoidance of the formation of undesirable characteristics (fixed, almost fixed modes, loss of rank of Plücker matrices) or preconditioning of properties (full rank of Plücker matrices). In fact, the decentralised Markov parameter test also provides the means to modify the centralised input, output structure in order to guarantee certain properties.

Step (7): Use of diagnostics based on performance indicators to evaluate the alternative decentralisation schemes, which have specified in the previous step, on a full dynamic linear model.

Having exhausted all structural methodologies and tests to reduce the set of options (necessary conditions have been mostly used), we now use computationally intensive methodologies such as singular value analysis, structural singular values, properties of balanced realisations, energy requirements for coupling etc. Such criteria are described in the section on interaction analysis. A state space based methodology exploring measures of system properties for evaluation of alternatives is given in [Nankoo, 1]. The methodology is based on the use of simple models first and then progressively moves to more detailed models and more detailed structural criteria. The current emphasis in the approach is the screening of the bad choices and then the final selection is made according to performance dependent criteria and multiobjective optimisation. A procedure for sorting out various criteria can be based on specifying the structure firstly and then use optimisation for the fine tuning of the parameters.

9.5. CONCLUSIONS

The problem of control structure selection has been considered from the transfer function viewpoint and using the traditional techniques, which have been developed within the process control area and referred to in short as interaction analysis. The development of software, as presented in the next chapter, has enabled the testing of these methods and has revealed the need for extending the framework by integrating it with procedures that allow the selection of inputs, outputs and well conditioned progenitor models, as well as techniques that allow the handling of structured, large scale problems and state space approaches which exploit measures of presence of system properties and formation of structural characteristics.

An attempt has been made to develop an overall approach that allows the consideration of the different dimensions of the problem. This approach is based on exploiting the underlying system structure, going progressively from unstructured model diagnostics, to graph structure based results, to model parameter dependent invariants and finally performance indicators. This structural methodology reflects the overall structural philosophy and it is quite logical for the overall problem. In fact, starting with a large number of options, we first use simple theory and criteria and progressively, by reducing the set of options, we start using more detailed and meaningful criteria, which however are associated with more computationally intensive procedures. What we have provided so far is an overall methodology and in the various steps, new, as well as known results are used. There are many areas, which need development if we are to move to an integrated and substantial structure selection diagnostics framework. Generating the different alternatives in a systematic and not ad hoc manner, sorting out the multiobjective decision problem of alternative criteria and finally moving for evaluation to design are open challenges in the future. So far we have relied on the structural approach, which is quite meaningful at early stages and for sorting out many options. At the later stages, there is a need to develop optimisation methodologies for tuning parameters within a given selected structure. This is also an important area for future research, where tools from the H_∞ optimisation methodology, may combine with the structural approaches to provide powerful hybrid methodologies.

Chapter 10

SOFTWARE DEVELOPMENT AND DESIGN EXAMPLES

Chapter 10

SOFTWARE DEVELOPMENT AND DESIGN EXAMPLES

10.1. INTRODUCTION

In the first part of this chapter, the various methodologies, presented in Chapter 9, are implemented in a control structure selection toolbox. This toolbox can be used to assist the design of a control structure selection or, for the case of an existing structure, to help draw conclusions about the behaviour of the system and provide insights about a possible restructuring of the control scheme. Several examples are used to illustrate the different indicators used.

The second part consists of a numerical example, which illustrates the method, proposed in Chapter 6, for the well conditioning of a degenerate system.

10.2. INTERACTION ANALYSIS TOOLBOX

The toolbox was developed using MATLAB 5.3, Release 11 (MATLAB is a registered trademark of The Math Works Inc.). Part of this program was developed in [Nist., 1] for previous versions of MATLAB. All the developed programs are included in the companion disk of this thesis. By typing in the command prompt of MATLAB "mmenu", the user is presented with the main screen of the program (Figure 10.1), which includes all the possible options available.

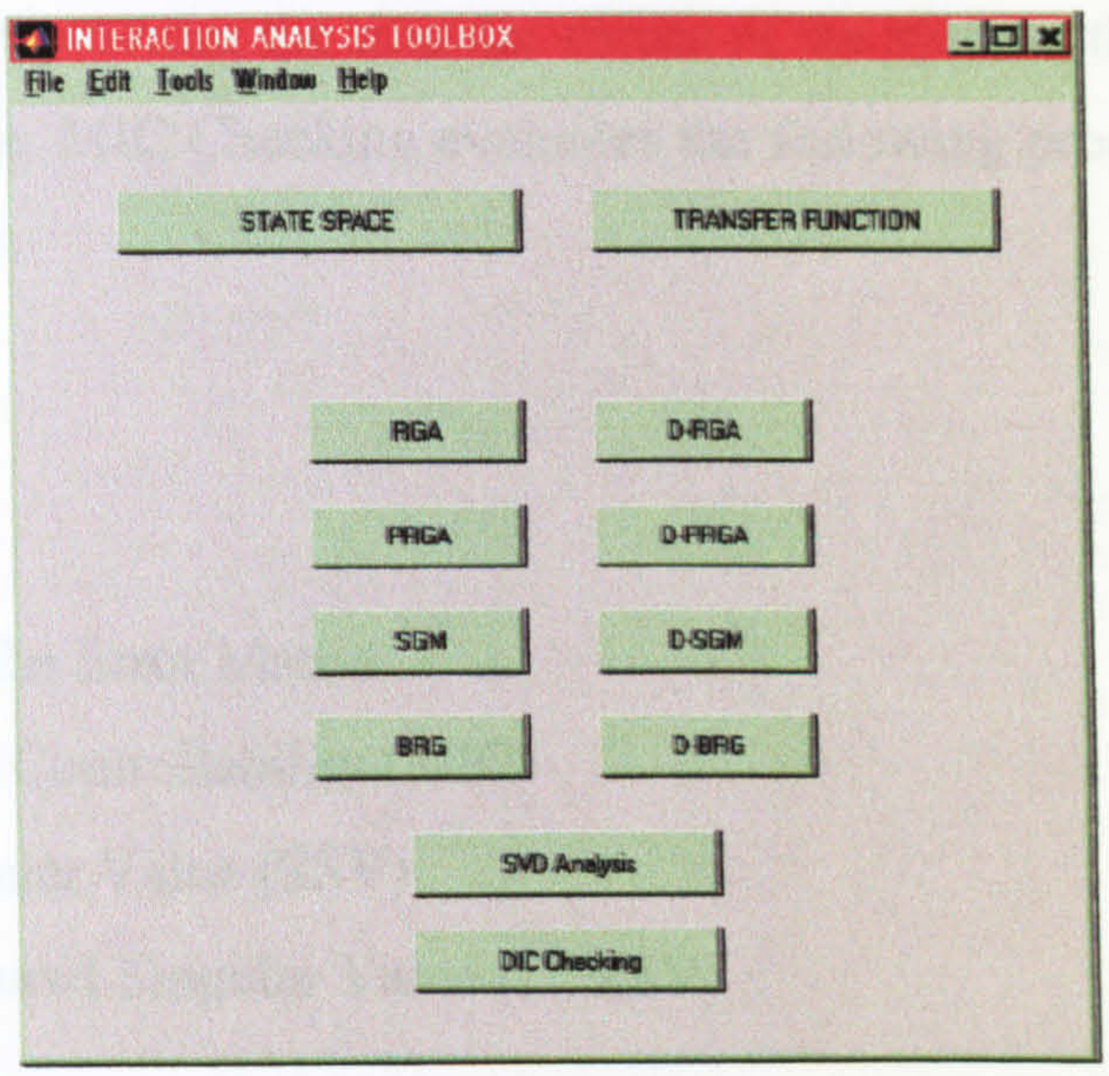

Figure (10.1): Main User Interface

The user needs to insert the system of his choice either in state-space form or as a transfer function, as well as the frequency range. The interaction indicators that are included are the following: RGA, D-RGA, PRGA, D-PRGA , SGM, D-SGM, BRG, D-BRG, SVD, DIC.

Indicators 1, 3, 5 and 7 correspond to static systems (frequency w=0), whereas, 2, 4, 6, 8 to dynamic systems (range of frequency needed). The SVD button (Figure 10.2) and the DIC button (Figure 10.3) provide the user with more choices:

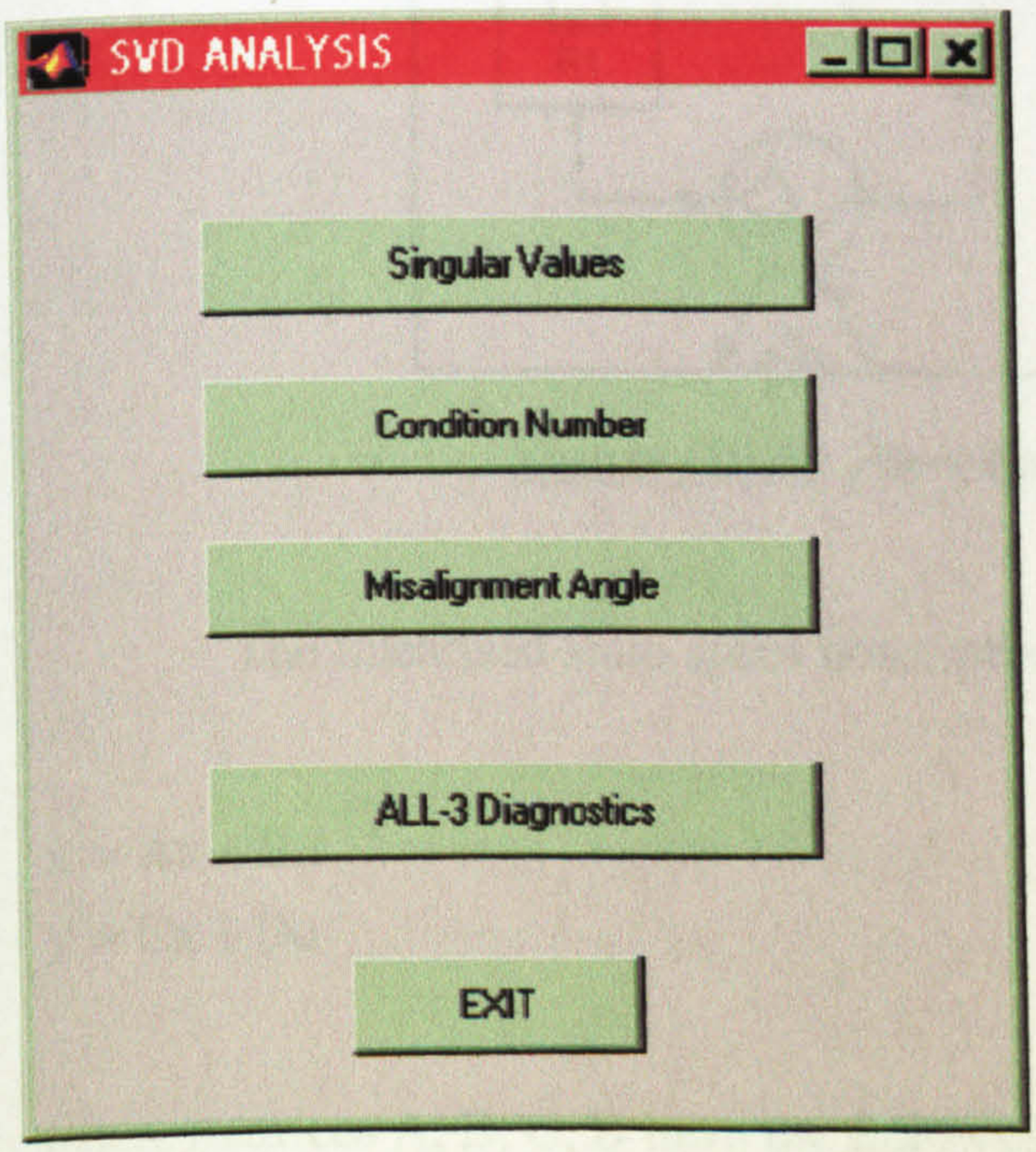

Figure (10.2): SVD Analysis

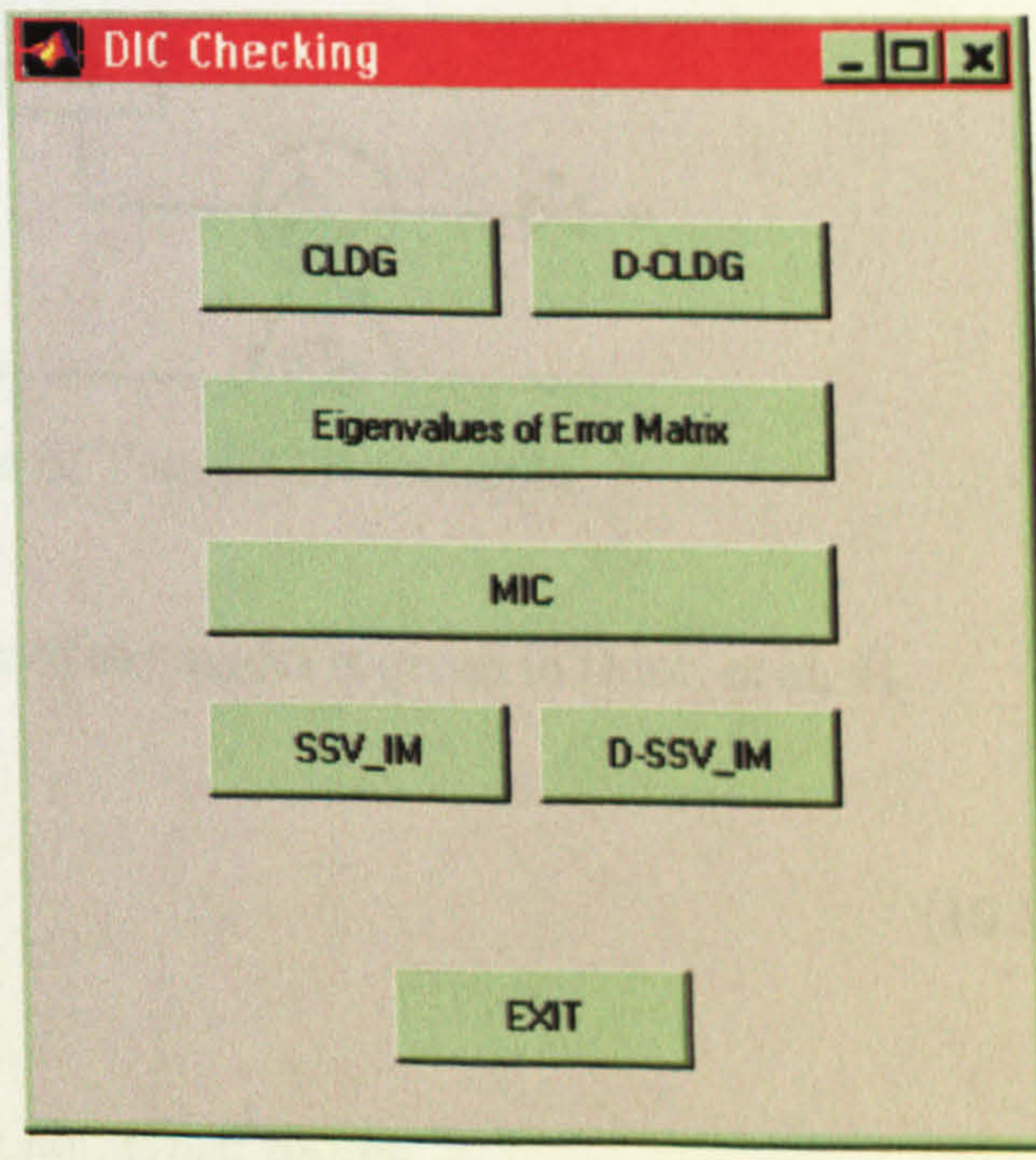

Figure (10.3): DIC Checking

SVD Analysis provides the singular values of the system, the condition number and the misalignment angle. MIC Checking evaluates the following properties, which have been described in Chapter 9:

1. CLDG
2. D-CLDG
3. Eigenvalues of the Error Matrix
4. Morari Index of Controllability (MIC)
5. Structured Singular Value (SSV)
6. Dynamic Structured Singular Value (D_SSV)

10.3. EXAMPLE OF TWO CSTRs

The example used is that of a sequence of two continuous stirred reactor tanks (CSTR) used for a highly exothermic liquid phase reaction. The model is described in more detail in [Pros., 1]. A flowsheet of the process is given in Figure (10.4).

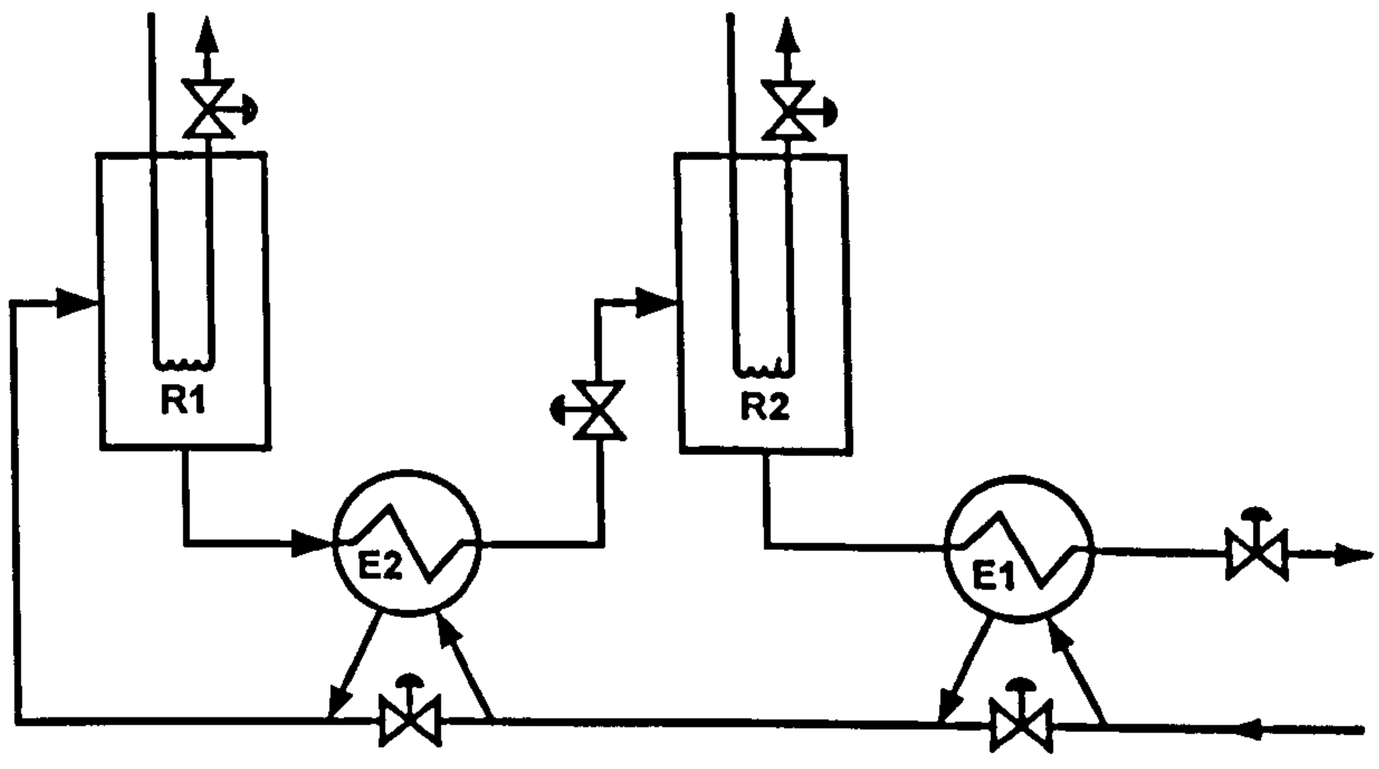

Figure (10.4): Flowsheet of Two CSTR Example

The linearised state-space description of the model is given in [Mor. et al, 1].

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \qquad (10.1)$$

where matrices A, B, C, D have the following numerical forms:

$$A=\begin{bmatrix} -163.98 & 0 & 0 & 0 & 50.345 & 50.345 & 0 & 0 \\ 30.238 & -91.003 & 24.335 & 24.335 & 0 & 0 & 0 & 0 \\ 0.08928 & 0.31251 & 0.83744 & 0 & 0 & 0 & 0.03441 & 0 \\ 17.383 & 24.335 & 68.998 & -117.67 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.63636 & 0.13264 & 0 & 0 & 0.047315 \\ 50.345 & 0 & 0 & 0 & 108.75 & -209.44 & 0 & 0 \\ 0 & 0 & -3986.2 & 0 & 0 & 0 & -60.557 & 0 \\ 0 & 0 & 0 & 0 & -743.85 & 0 & 0 & -24.464 \end{bmatrix}$$

$$B=\begin{bmatrix} 2784 & 0 & 0 & 0 \\ -740.81 & 1110.6 & 0 & 0 \\ -2.1874 & -5.2062 & 0 & 0 \\ -425.86 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1073.5 & 0 \\ 0 & 0 & 0 & -427.73 \end{bmatrix}$$

$$C=\begin{bmatrix} 0.20408 & 0.71430 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad D=\begin{bmatrix} -4.997 & -11.9 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The above model was examined with the software developed and the results are presented next.

10.3.1. RGA

RGA is the most widely used interaction measure. The results for the Two-CSTR model are presented in the following table:

$$RGA = \begin{matrix} 0.0006 & \mathbf{0.9995} & -9.9659 & 1.0412 \\ -0.5333 & 0.0001 & \mathbf{1.5333} & -2.1510 \\ \mathbf{1.6241} & 0.0004 & -0.5332 & -0.0913 \\ -0.0914 & 0.0001 & 0.0001 & \mathbf{1.0913} \end{matrix}$$

According to Niederlinski stability theorem, this system is stable.

The highlighted values are the ones that appear most promising for pairing and so the proposed pairings are: $1 \rightarrow 3,\ 2 \rightarrow 1,\ 3 \rightarrow 2,\ 4 \rightarrow 4$. It is obvious, though, that for inputs 2 and 3 the proposed pairing gives a value not very close to 1 and that indicates strong interactions between the pairs. Also, output 1 gives a very good value, close to 1, for input 4, but this choice has to be discarded because, then, input would have no other possible pairing. However, the application of the Niederlinski stability theorem suggests that the system is stable.

10.3.2. D-RGA

The D-RGA is an extension of the RGA that incorporates the dynamic behaviour of the system. The frequency range is taken to be between 0.001 and 10 rad/sec. The result for the proposed pairings is:

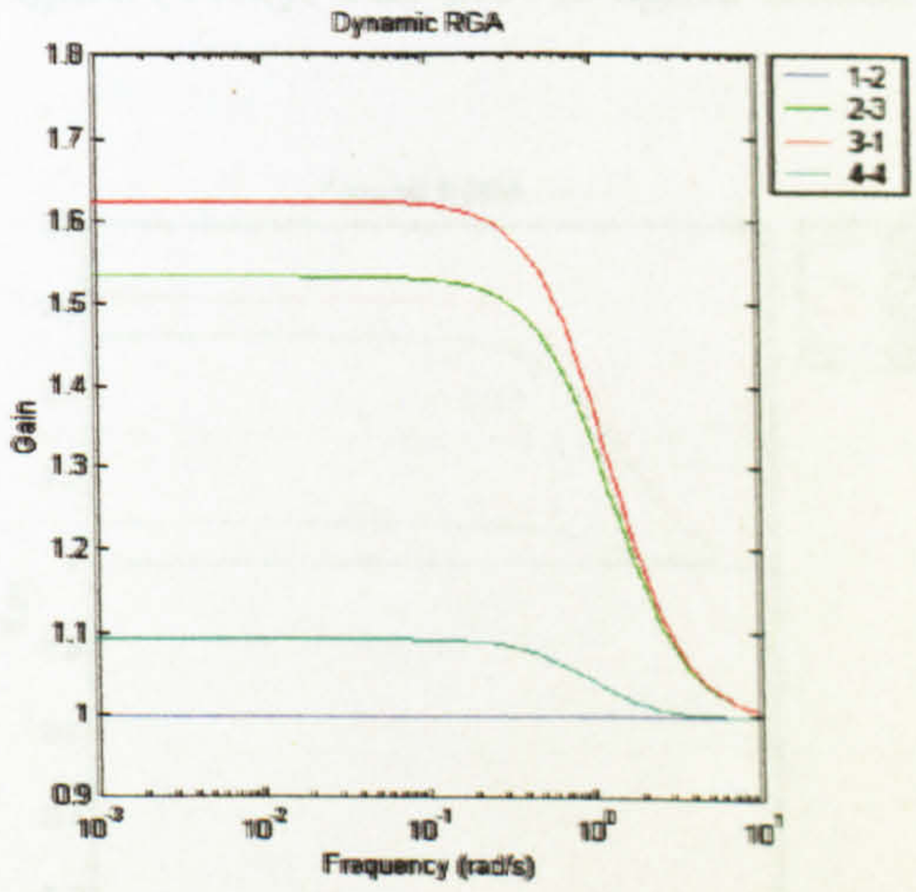

Figure (10.5): D-RGA Results

It should be noted that the user can plot any possible pairing and these four are the ones corresponding to the ones proposed by the RGA. The results, here, show that for inputs 2 and 3 the interactions are rather high for low frequencies, but settle down as the frequency increases.

10.3.3. PRGA

The results for the P-RGA are:

$$P-RGA = \begin{matrix} 0.0006 & \mathbf{0.9990} & -9.5509 & 1.2944 \\ 0.0001 & 0.0001 & \mathbf{1.5342} & -2.1365 \\ \mathbf{1.6197} & 0.0035 & -0.5332 & -0.0916 \\ -0.004 & 0.0001 & -0.0053 & \mathbf{1.1004} \end{matrix}$$

The results are similar to those produced by the RGA and the proposed pairings (highlighted in the above table) remain the same. The advantage of the P-RGA against the RGA is the fact that it is independent of input scaling. This can be very helpful, especially in cases where the transfer function needs to be rearranged, during the design process.

10.3.4. D-PRGA

The dynamical version of the above indicator and for the same frequency range gives the result shown in Figure (10.6). The plot is again similar to the one produced by the D-RGA.

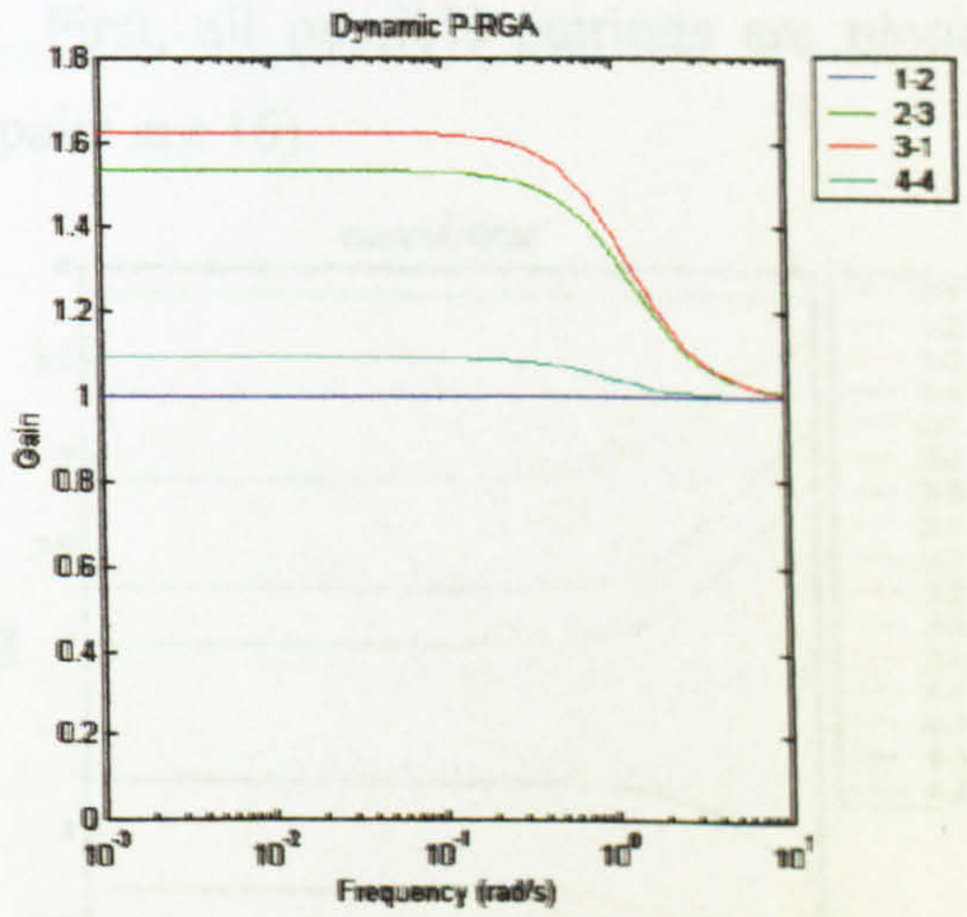

Figure (10.6): D-PRGA Results

10.3.5. SGM

In this method, the inputs and outputs are rescaled, so that the average value of the elements in each row and column is one. Thus, the elements can be directly compared to each other. The result for the given system is:

$$SGM = \begin{matrix} 0.0286 & \mathbf{3.8540} & 0.0064 & 0.0099 \\ 1.2679 & 0.0381 & \mathbf{2.2927} & 0.4350 \\ \mathbf{2.0039} & 0.0799 & 1.2607 & 0.6876 \\ 0.6995 & 0.0279 & 0.4401 & \mathbf{2.8674} \end{matrix}$$

The elements of the SGM matrix provide useful information. Values larger than 1 indicate strong interaction and values smaller than 1 indicate weak interaction. Then, the largest elements indicate which inputs and outputs should be paired. The highlighted elements are the ones that are the largest and correspond to the same pairing scheme proposed according to the RGA. It has to be noted that there exist other significant interactions between other inputs and outputs, i.e. the pairs (1,2) and (3,3).

10.3.6. D-SGM

The D-SGM is an extension of the SGM, as it provides the dynamic behaviour over a given frequency range. First, all possible pairings are plotted (for a 4×4 system, the possible combinations of pairs are 16).

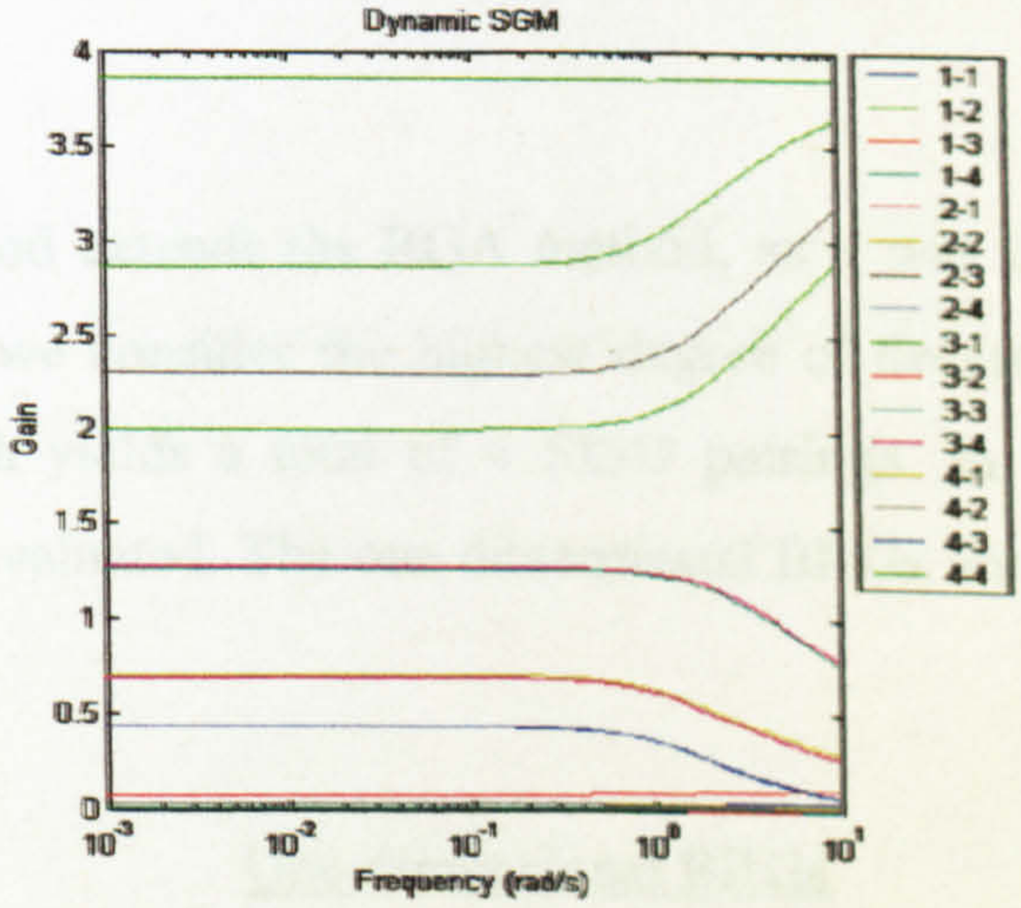

Figure (10.7): D-SGM Results for all Possible Pairings

From this plot, we can deduct useful information about the behaviour of all the pairings. As the frequency becomes higher, the effect of the proposed pairings on the interaction becomes stronger. On the other hand, the pairings, which from the static SGM were identified as possible candidates (pairings (1,2) and (3,3)) appear to be losing their strength and will not interact between them as much. The plot for the proposed pairing is:

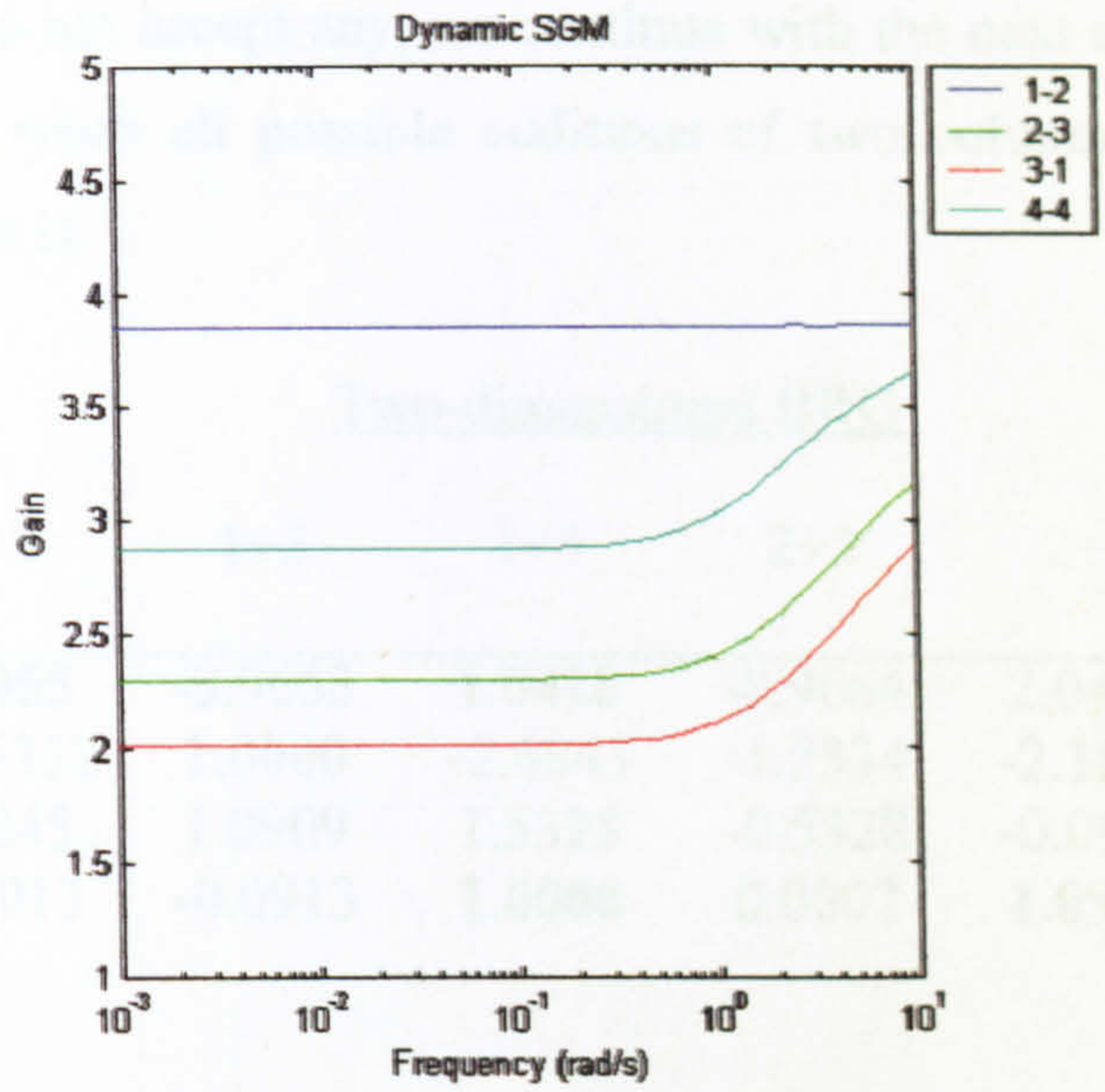

Figure (10.8): D-SGM Results for Proposed Structure

From this graph, it is clearer, that the interactions for pairs (3,2), (1,3) and (4,4) become stronger as the frequency goes up, whereas, the interaction for the pair (2,1) seems rather unaffected in the frequency range.

10.3.7. BRG

The BRG method extends the RGA method, as it was explained in Section 9.3.6 [Man. et al, 1]. Firstly, we consider the highest degree of decentralisation, i.e. 1×1 block partitioning of G, which yields a total of 4 SISO pairings. In this manner, all the one-dimensional BRGs are evaluated. The one dimensional BRGs are shown below in the RGA form:

One-dimensional BRGs

u_j / y_i	1	2	3	4
1	0.0006	**0.9995**	-9.9659	**1.0412**
2	-0.5333	0.0001	**1.5333**	-2.1510
3	**1.6241**	0.0004	-0.5332	-0.0913
4	-0.0914	0.0001	0.0001	**1.0913**

Depending on the desired value for the error step, we can either accept some values or not. In this case, though, we have several that are very close to 1, i.e. value corresponding

to pair (2.1). If we do not accept any, we continue with the next step. The two-dimensional BRGs are available when all possible additions of two columns of the above RGA are performed. The result is:

Two-dimensional BRG

u_j / y_i	1+2	1+3	1+4	2+3	2+4	3+4
1	**1.0055**	-9.9653	**1.0418**	-8.9664	2.0407	-8.9247
2	-0.05332	**1.0000**	-2.6843	-1.5334	-2.1509	-0.6177
3	1.6245	1.0909	1.5328	-0.5328	-0.0909	-0.6245
4	-0.0913	-0.0913	**1.0000**	0.0002	**1.0914**	**1.0914**

The highlighted elements are the most promising. In general, we would have to continue examining the three-dimensional BRGs etc. In our case, we have elements very close to one. We will have to relax the restriction of the error step very much, i.e. 0.8 in order to achieve a 3-1 system, which means that for this value the system can be split into one 3×3 MIMO system and one 1×1 SISO system.

10.3.8. D-BRG

In the example, used in this case, there were elements in the one dimensional BRG that were very close to 1. So the Dynamical version of the BRG method does not provide any additional useful information.

10.3.9. SVD Analysis

The Singular Value Decomposition Analysis contains three different system properties. The singular values of the system, the condition number and the misalignment angle of the system as a function of the frequency, the same frequency range has been used. The results for these three properties are:

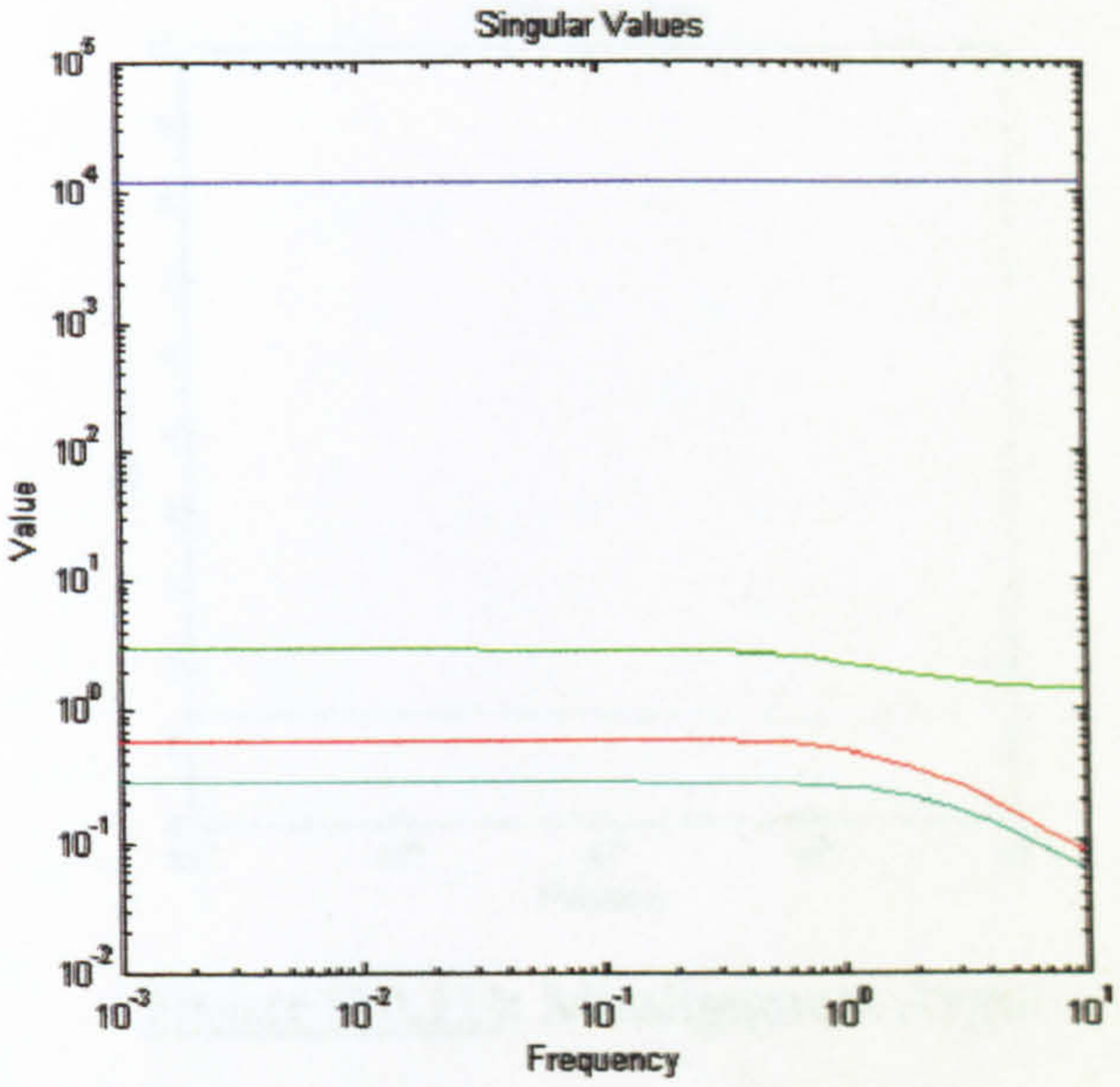

Figure (10.9): Singular Values

The graphical representation of the singular values is very similar to the Bode plot of the system. Therefore, it is expected for them to remain constant in low frequencies and decrease linearly, as the frequency becomes higher.

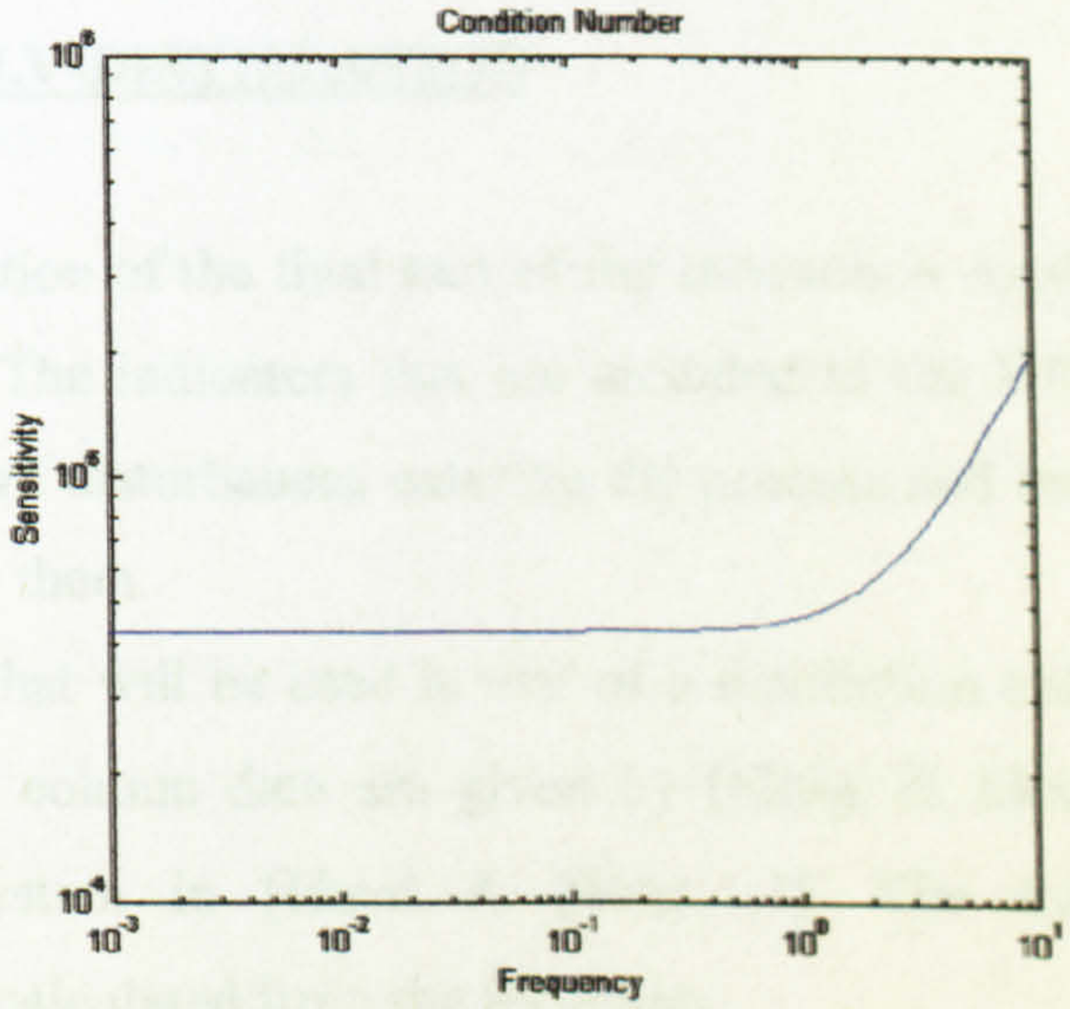

Figure (10.10): Condition Number

From the graph corresponding to the behaviour of the condition number as a function of the frequency, it can be seen that it increases as we move to higher frequencies. Consequently, the sensitivity of the system increases at higher frequencies.

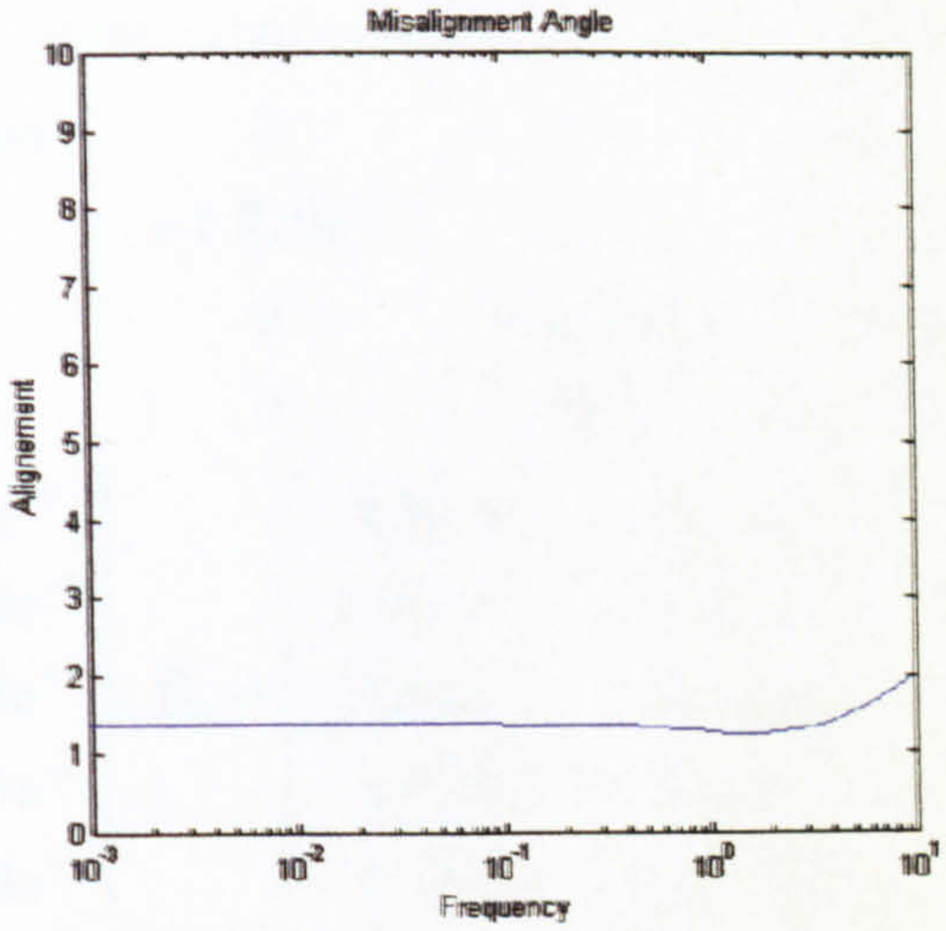

Figure (10.11): Misalignment Angle

From the plot of the misalignment angle, it can be seen that, although, it goes up slowly as the frequency goes higher, it still keeps low values (i.e. values <15°). This means that the system has a natural loop structure and there is no need to introduce any compensators, in order to reduce the interactions within the system.

10.4. EXAMPLE OF LV-DISTILLATION

For the illustration of the final part of the Interaction Analysis Toolbox, a different example will be used. The indicators that are included in the DIC Checking are used for systems that have known disturbances entering the process and correspond to the ability of the system to cope with them.

The example that will be used is that of a distillation column, controlled with the LV-configuration. The column data are given by [Skog. & Mor., 1] and the state-space description of the system in [Hovd & Skog., 1]. The transfer function matrices $G(s)$ and $G_d(s)$ can be calculated from the formulae:

$$G(s) = C(sI - A)^{-1}B + D$$

$$G_d(s) = C(sI - A)^{-1}B_d + D_d$$

where the state-space matrices are:

$$A=\begin{bmatrix} -5.161e^{-3} & 0 & 0 & 0 & 0 \\ 0 & -7.366e^{-2} & 0 & 0 & 0 \\ 0 & 0 & -1.829e^{-1} & 0 & 0 \\ 0 & 0 & 0 & -4.620e^{-1} & 9.895e^{-1} \\ 0 & 0 & 0 & -9.895e^{-1} & -4.620e^{-1} \end{bmatrix}$$

$$B=\begin{bmatrix} -6.296e^{-2} & 6.236e^{-2} \\ 5.481e^{-3} & -1.719e^{-2} \\ 3.041e^{-3} & -1.078e^{-2} \\ -1.856e^{-2} & -1.393e^{-2} \\ -1.229e^{-1} & -5.608e^{-3} \end{bmatrix},\ B_d=\begin{bmatrix} -9.364e^{-3} & -1.333e^{-2} \\ 1.960e^{-2} & 8.018e^{-3} \\ 3.266e^{-3} & -2.116e^{-2} \\ -2.827e^{-2} & 5.319e^{-3} \\ -6.784e^{-3} & 2.719e^{-3} \end{bmatrix}$$

$$C=\begin{bmatrix} -7.223 & -5.170 & 3.836 & -1.633e^{-1} & 1.121 \\ -8.913 & 4.728 & 9.876 & 8.425 & 2.186 \end{bmatrix},\ D=D_d=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For the above system, the DIC Checking part of the software is used and the results are presented next.

10.4.1. CLDG

The steady-state effect of the two disturbances is:

$$G_d(0)=\begin{bmatrix} 1.7660 & 19.6484 \\ 15.6974 & 21.2712 \end{bmatrix}$$

Both disturbances have large magnitudes and seem to be difficult to reject. The pairing that will be examined is that of output 1-disturbance 1 and the steady-state CLDG for the system is:

$$CLDG=\begin{bmatrix} -44.2477 & 0.6730 \\ 65.2039 & 11.3567 \end{bmatrix}$$

From the CLDG, it can be seen that disturbance 2 has a very small effect on output 1 at steady-state. Similarly, the effect of disturbance 2 on output 2 is not that large if compared with effect of disturbance 1 to both outputs. Thus, the designer will have a

problem of rejecting disturbance 1, no matter what the pairing will be, since it affects both outputs very strongly.

10.4.2. D-CLDG

The dynamic behaviour of the CLDG as a function of the frequency can be seen in the following graph:

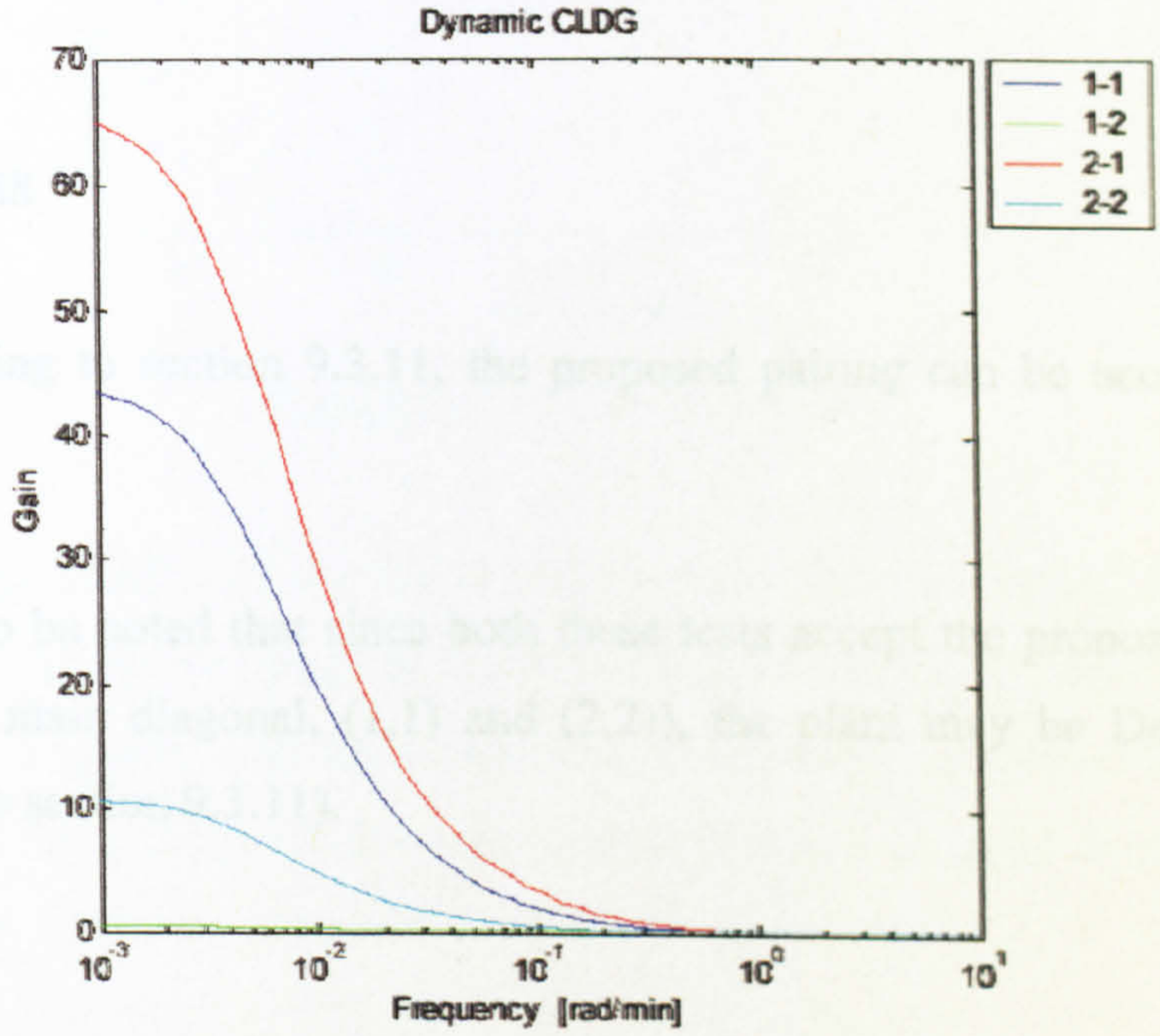

Figure (10.12): D-CLDG of the System

From the graph, it can be seen that the effect of both disturbances on both outputs decreases rapidly for higher frequencies. The problem of rejecting disturbance 1 still remains, since the range of frequencies might not be always available in real life operating.

10.4.3. Eigenvalues of Error Matrix

The program can also evaluate the eigenvalues of the Error Matrix (see section 9.3.11). The result is:

Eigenvalues of Error Matrix = 0.9861
-0.9861

The pairing is accepted since the real part of the eigenvalues of the Error Matrix is less than –1.

10.4.4. MIC

The next test performed on the system, is that of the Morari Index of Controllability. The result returned by the program is:

MIC = 1.3575
195.8628

According to section 9.3.11, the proposed pairing can be accepted since MIC is greater than 1.

It has to be noted that since both these tests accept the proposed pairings (i.e. the pairings on the main diagonal, (1,1) and (2,2)), the plant may be Decentralised Integral Controllable (see section 9.3.11).

10.4.5. SSV

A further indicator for the stability and the performance of the process is the Structured Singular Value. The result for the given plant is:

SSV = 0.9861

This satisfies the conditions for a plant to be Decentralised Integral Controllable [Grosd. & Mor., 1], since the value is less than one. The final indicator used is the Dynamic version of the static SSV.

10.4.6. D-SSV

The Dynamic Structured Singular Value tracks the change of the structured singular value as a function of the frequency. The graph yielded for this method is:

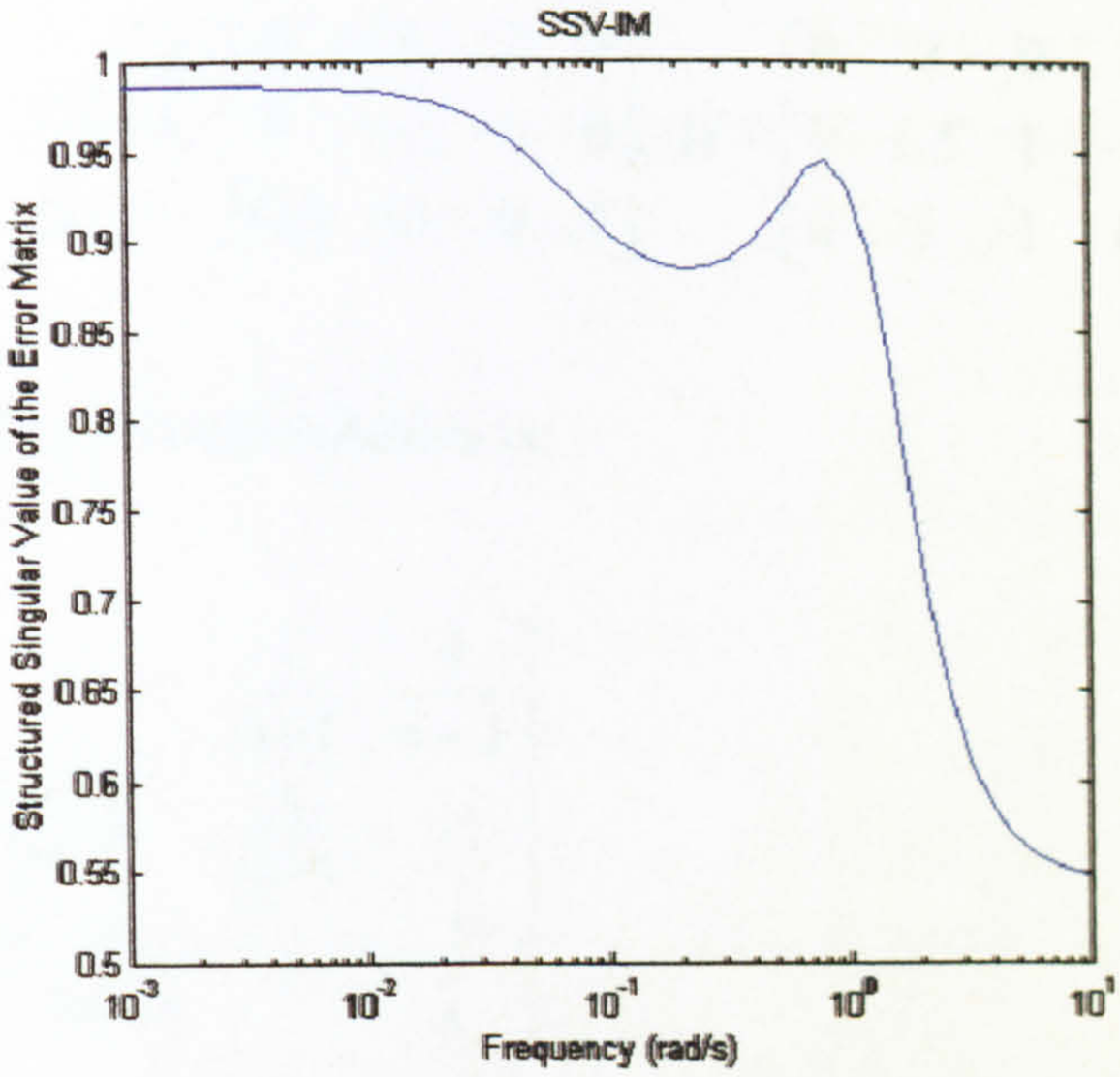

Figure (10.13): D-SSV Results

This graph reinforces the indication obtained from the static version of the SSV. The value of the Structured Singular Value stays less than 1 for the complete frequency range. This result, together with the eigenvalues of the error matrix and the MIC, enables us to conclude that the pairing used leads to decentralised integral controllability and the system will not be affected by model uncertainties.

10.5. Well Conditioning Example

Consider the system described by the state-space matrices:

$$A=\begin{bmatrix} 0 & 1.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C=\begin{bmatrix}1 & 0.5 & 0 & 0 & 0 & 0 & 1 & 2 & 0\\ 0 & 0 & -1 & 0.5 & 0 & 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & -0.5 & 0 & 0 & 1\end{bmatrix},\quad D=\begin{bmatrix}0 & 2 & 0 & 1\\ 0 & 1.5 & 1 & -1\\ 0 & 1 & 1 & 0\end{bmatrix}$$

and the corresponding transfer function is:

$$H(s)=\begin{bmatrix}\frac{s+1}{s^2-3} & 2 & \frac{1}{s+1} & \frac{s}{s-2}\\ -\frac{1}{s} & \frac{3s-1}{2s-1} & \frac{s}{s+1} & -1\\ 0 & \frac{s^2+1}{s^2-2s+3} & 1 & \frac{1}{s}\end{bmatrix}$$

The system has 9 states ($n=9$), 4 inputs ($r=4$) and 3 outputs ($q=3$), by using the same notation as in Chapter 6, where the Well Conditioning algorithm was described. Firstly, we evaluate the rank of the transfer function matrix H(s). This is achieved by giving arbitrary values to the Laplace transform s and evaluating the rank of the resulting numerical matrix. In this case the rank of the H(s) is $\rho=2$. It is clear that $\rho<\min(q,r)$, and the system is degenerate. Since the rank is 2, then 2 is the maximal number of output variables that may be controlled independently. Furthermore, 2 is the minimal number of independent inputs required to control 2 outputs [Rosen., 1]. The next step is to find the numbers:

$$\tau_r \stackrel{\Delta}{=} \text{rank}\left\{\begin{bmatrix}B\\ D\end{bmatrix}\right\}=4,\ \tau_\ell=\text{rank}\left\{[C,D]\right\}=3$$

Since $\tau_r=4=r$, then the system is input regular. Also, $\tau_\ell=3=q$ and the system is output regular. The Rosenbrock System Matrix Pencil has rank: $\tau=n+\rho=9+2=11$. Also, the dimensions of the right and the left null spaces of P(s) are found to be: $\eta=\dim N_r\{P(s)\}=r-\rho=4-2=2$, $\theta=\dim N_\ell\{P(s)\}=q-\rho=3-2=1$. We can recheck the degeneracy of the system, using Condition (6.13): $\tau=\text{rank}_{\mathcal{R}(s)}\{P(s)\}<\min(n+r,n+q)$, which holds true since $\tau=11<\min(9+4,9+3)=12$.

The right and left null spaces of the Rosenbrock System Matrix Pencil P(s) are characterised by a set of column, row indices [Gantm., 1], which also may be referred to as right, left indices of P(s) [Forney, 1]. Such sets are denoted by $I_p^c = \{\varepsilon_i : i = 1,...,\eta = n-\rho\}$, $I_p^r = \{\mu_j : j = 1,...,\theta = q-\rho\}$ and may have t_r zero cmi and t_ℓ zero rmi; in fact, $t_r = r - \text{rank}\left\{\left[B^t, D^t\right]^t\right\} = r - \tau_r = 4-4 = 0$, $t_\ell = q - \text{rank}\left\{\left[C, D\right]\right\} = q - \tau_\ell = 3-3 = 0$.

The numbers t_r and t_ℓ, express the order of input, output redundancy respectively. In our case, as we proved earlier, they are zero since our system is input and output regular. So the system has only nonzero cmi and rmi. So the system was shown to be degenerate and input, output regular. Along the same lines, strong degeneracy can be examined.

10.6. Conclusions

Having gone through all the proposed interaction indicators for the given example, the possibilities of selecting a control structure can be evaluated. The methods provide insights in different aspects of the behaviour of the process, either static or dynamic.

For the first example, the first indication for the possible pairings between inputs and outputs was given by the RGA. The decision over the pairings was, then, reinforced with the dynamic version of the RGA, which showed that in the given frequency range and especially for higher frequencies the pairs chosen remained suitable. The Performance-RGA and its dynamic version (D-PRGA) did not provide any further information but could prove useful if there where scaling matters regarding the inputs and outputs of the transfer function. The SGM introduced some concerns regarding the effect of strong interactions between other inputs and outputs than the ones chosen to be paired. These concerns were relaxed by the results of the D-SGM, which showed that the effect of the other interactions diminishes for higher frequencies and stop presenting problems for the selected control structure. The BRG and D-BRG methods provide a different viewing of the problem. They investigate the alternatives of taking the control structure as a 3-1 system (i.e. one 3×3 MIMO and one 1×1 SISO system) instead of the initial 4×4 MIMO system. Finally, the SVD analysis provided further results that strengthen the decision over the pairings. The condition number shows that for the proposed structure the sensitivity of the system increases for higher frequencies. The misalignment angle retains low values for the desired bandwidth and so there is no need for any compensators to be introduced.

These indicators provide insights for the possible pairings between inputs and outputs. In more complex systems they can not provide conclusive results to assist the decision over the selection of the control structure. In this example, all the indicators were pointing at the same control structure. The designer would be in a very difficult position when getting conflicting results from different indicators since there is not a way of weighting one over another. In such cases, the designer will have to use the indicators that match his specific process using other means, i.e. heuristic rules etc. Similarly, especially when deciding the control structure of a plant after it has been designed, the designer may be restricted by physical aspects of the process, economics etc. and have to discard favourable pairings that are being proposed by the indicators for less favourable ones.

After going through the methodologies for the second example, the proposed pairing seems adequate to reject the disturbances that enter the process. It leads to plant, which is Decentralised Integral Controllable and for the given frequency range can successfully cope with the disturbance rejection. Furthermore, for higher frequencies the other interactions become insignificant.

For complex systems of much higher dimension, the interaction analysis has to become part of a more general philosophy, which according to dimensionality selects the most appropriate methodology. Such an approach has been described in Chapter 6. The availability of alternative tests requires some appropriate means for selecting choices of coupling for problems where conflicting results may arise. This may be achieved by introducing composite indices where the different indicators are weighted and decision is finally reduced to an optimisation problem.

The criteria for the presence of input, output redundancy and system degeneracy have been illustrated here in detail. The importance of the proposed method is that with basic matrix calculations, one can identify the inputs, outputs that can be eliminated from the progenitor model, in order to create a nondegenerate system. The conditions that lead to systems with desired characteristics, as well as making the search for minimal subsystems more systematic are problems for future research.

For large dimension models the algebraic computations involved require special algorithms, which take into account dimensionality of the problem and lead to efficient results. Some of the approaches developed in the structural identification part of this thesis are of relevance here. Developing software in this area also requires use of symbolic computations, appropriately used for certain types of computations.

Chapter 11

CONCLUSIONS

Chapter 11

CONCLUSIONS

This thesis has dealt with the study of a number of system and control theory-type problems that emerge in the effort to develop a systems and control theory framework for the integrated design of the early stages of chemical processes. Central to this attempt is the recognition that many issues referred to as process controllability, operability, process synthesis and control structure selection have not been considered from the system theory viewpoint in a systematic way. The framework, within which the work has developed, has been based on the following important observations:

- Early design of chemical processes starts with simple models for subprocesses and a structuring selection of the process flowsheet, which has to be evaluated with its potential to accept control design solutions and guarantee optimality.
- Evaluation of the quality of such arrangements is based on economic criteria and some design indicators but not assessed by the powerful tools of the control theory.
- Early process models are not necessarily well structured, as far as their potential to accept control solutions is concerned, and they have potential for improvement by modification of the input, output structure and possibly the structuring of the process flowsheet. Such models may be having subdominant dynamics reflecting the nature of subprocesses, are of large dimensions and are characterised by parameter uncertainty.
- Control structure selection has been based on criteria valid for small dimension problems, some of them are heuristics and the problem area lacks a unifying methodology, which can be used at different stages of the life cycle of design and which is relevant for problems with variable dimension.

The underlying philosophy behind the current research is that models evolve through the life cycle of design and that having a methodology to intervene in such evolution is essential. Furthermore, this evolution has two alternative dimensions:

(i) Model structure evolution from process synthesis, selection of inputs, outputs and specification of control structure.

(ii) Early-late design stages evolution of models, which is associated with an evolution of complexity from the early stages to the late stages, when full models are required.

Control theory has been viewed from a different perspective than the traditional. In fact, system structure expressed by invariants and system graph are now viewed as indicators of the quality of the resulting system rather than the tools, which provide the basis for control design. Thus, new control theoretic problems emerge, which are based on the shaping of such indicators, as the result of the early system design stages. Problems of structure evolution and performance indicators shaping are central to this alternative consideration of the key control theoretic notions.

The thesis has made contributions in the development of the system theoretic framework for integrated design by developing results in the following areas:

- Further development of the system aspects of the global instrumentation framework.
- Formulation and development of solutions for the state-space version of the model orientation problem.
- Development of the theory for the well structuring of large dimension early stage models by reduction of the set of potential inputs, outputs and parameterisation of such solutions.
- Development of diagnostics for the McMillan degree and the infinite zero structure for large-scale early models by exploiting model uncertainty and genericity and using properties of integer matrices. The results provide tools for evaluating alternative selections at early stages based on structural criteria.
- The representation of composite structures based on physical interconnection streams and the completeness assumption provides a novel set-up for examining process synthesis. This enables traditional control theoretical tools and methodologies, such as the pole assignment by output feedback theory, to be used in this new set-up. The

representation of lack of completeness as decentralised input, output squaring down links the local selection of inputs, outputs to the general squaring down, for which the exterior algebraic framework [Karc. & Gian., 1] is relevant and can be used.

- Control structure selection has been examined from the traditional process control viewpoint, which is relevant for small dimension problems (inputs, outputs dimensionality) and software has been developed in the form of a small MATLAB toolbox. An integrated framework for the life cycle of control structure selection has been specified but this area requires considerable further work.

The work in this thesis has specified a large number of open issues within the area of integrated design. A few have been considered here but many more remain open. Amongst those, which deserve further investigation are:

(a) The problem of well conditioning of early models requires further work in the areas of:

- Use of additional structural characteristics for the evaluation of early models and, especially, in the area of structured state-space models. This area will involve the use of theoretic tools and graph-based structural properties.
- Development of computational procedures for large-scale models taking advantage of structured, sparse matrix properties.

(b) Model orientation has been considered in a state-space set-up. Extensions are needed for the case of autoregressive models based on polynomial descriptions, as well as establishing explicit parameterisation of the oriented families. Using such families, we can impose additional conditions coming from the physics of the problem.

(c) The structural identification problem has been considered for the McMillan degree and the infinite zero structure. Extension of the work is required in the areas of:

- Structural identification of minimal indices using genericity and the special structure of Toeplitz matrices.
- Use of graph techniques for the development of the optimal solutions of the optimisation problems defined on integer matrices.

(d) The control structure selection is an area that requires extensions to handle issues of flexibility in the design. Furthermore, the current framework requires development using multi-objective optimisation to sort out conflicts between the alternative criteria. Further work is needed to embed interaction analysis indicators within a richer design framework as that described at the end of Chapter 9.

(e) The area of transforming process synthesis to a standard control design is challenging and requires additional work in the following areas:

- Development of the generalised input, output decentralisation squaring down using exterior algebra and algebraic geometry tools.
- Use of graph-based structural properties for evaluation of alternative composition schemes.
- Study of evolution of system properties within the framework of a fixed graph but with variability in the complexity of subsystem models, as we move from the early to the late design stages.

The above provide a sample of research problems, which emerge as extensions of the framework and the results of this thesis. The investigation of such issues, will contribute to a more developed framework, supported by results for the early design of processes.

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