CREDIBILITY THEORY AND

EXPERIENCE RATING IN GENERAL INSURANCE

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Abstract

The efficient use of available data to forecast future performance is one of the central concerns of modern business. In general insurance, especially, profit or loss depends upon charging the correct premium for the individual risk underwritten.

When a new contract is signed, the insurer may know very little about the individual characteristics of the risk he has insured, and the premium charged is based upon prior statistics gathered from a large collective of similar, but also somewhat heterogeneous contracts grouped together under the same group rate. However, as the individual contract continues over several years, experience data can be gathered to reflect individual risk characteristics.

A simple but effective approach to estimating an experience rated premium is provided by credibility theory. Credibility theory is concerned with the weight to be given to the small amount of recent individual experience as compared with the mass of the data from the past collective experience.

This research was undertaken to review different views and approaches to experience-rating in line with credibility theory considering fire insurance as an example of application ; also to develop a mathematical formula for experience rating for the pure premium keeping in mind the conditions and the facilities available in the Egyptian market. The formula developed takes into account the variability of the claim amount and the claim frequency. Two classes of non-industrial fire insurance are analysed and tables for the credibility factors involved in the developed formula are constructed.

CHAPTER ONE

INTRODUCTION

1.1 Risk And Insurance

Risk is a universal feature of modern economic life. In everyday life we face a great number of risks. Risk has been defined simply as uncertainty [Willet, 1951] ; but other authors refer to it as uncertainty in regard to loss [Denenberg, 1964]. In insurance, risk is generally referred to as the possibility of financial loss. From that point of view, risk concerns the state of some financial relationship between an insurer and the insured, while uncertainty is simply a state of mind [Pfifer, 1956]. Another definition of risk has been given as "... Risk is an invariable deviation from expectation which may or may not reflect a reduction or disappearance in value ..." [Athrton, 1964]. This sort of risk of an economic nature most of the people wish to avoid, and they try to find ways for handling such risks.

Risks may be classified as speculative risk or as pure risk [Blanchard, 1961] according to their nature. Risk is said to be speculative if there is a possibility of both loss and gain as in gambling or in commercial activity. Risk is said to be pure risk if there is only the possibility of loss as in fire, flood, death .... There are direct causes of pure risks such as fire, flood .... and indirect causes or conditions which lie behind the occurrence of loss which we call hazards.

Hazard increases the likelihood of loss and may make the loss more severe. The basic types of hazard are physical, criminal and those due to social behaviour (other than criminal).

As a matter of fact there is no escape from the presence of risk, but the degree of risk may be reduced. There are ways of meeting risk such as avoidance, prevention, distribution and transfer.

1.2 Different views of insurance and nature of insurance

Insurance is one of the ways of dealing with risk. There is a wide range of different views as to the nature of insurance ; one definition of insurance is that of risk transfer [Head, 1967] which is the promise of payment by the insurer and the premium payment by the insured. Another definition is in terms of techniques. Mehr and Commak (1966), for example, define insurance as .... social device for reducing risk by combining a sufficient number of exposure units to make their individual losses collectively predictable. The predictable loss is thus shared proportionately by all those in the combination. Also, Mehr and Hedges [1963] say a device will be deemed to be insurance if it applies the law of large numbers so that the requirement for future funds to cover loss is predictable with reasonable accuracy, and it provides some definite method for raising these funds by levies against the units covered by the scheme.

In accordance with this view the essential feature of insurance is the manner in which losses are predicted and shared.

A third suggestion is that a complete definition should include a combination of these criteria [Willet, 1951]. As an example Rigel and Miller (1966) define insurance "... Insurance is the transfer of risk with added features 1 - combination of risk 2 - estimation of future losses ...".

Insurance, on the basis of its function as described above, is the creation of the counterpart of risk, which is security. Insurance does not decrease the uncertainty for individuals as to whether or not the event will occur, nor does it alter the probability of occurrence, but it does reduce the financial loss connected with the event. In fact, an insurance contract provides a valuable factor in the freedom from the burden of uncertainty. While it is theoretically possible to insure all possibilities of financial losses (for pure risks), some are not insurable. The main characteristics of insurable risk are :

1. There must be sufficiently large numbers of homogeneous exposure units to make losses reasonably predictable.

2. The loss produced by risk must be capable of financial measurement ; also the type of losses must be relatively difficult to counterfeit.

3. The loss must be accidental.

4. The loss must not be catastrophic.

Insurance may be divided into two major divisions : life and non life insurance. Non life insurance can be subdivided into certain classes on the basis of the perils insured against or the fundamental nature of a particular programme. In general, non life insurance has some common features :

1. The claim size is not known in advance.

2. The premium charge may change from year to year.

3. More than one claim can arise under the same policy.

In our study we are concerned with the problem of estimating the pure premium (expected claim per policy) using fire insurance claim data from the Egyptian market.

1.3 Rating Insurance

Insurance is usually classified as a service and the rate (premium rate) of insurance is the price per unit of insurance (unit of exposure). Like any other price, it is a function of cost of the services and of market conditions. However, in insurance, unlike other services, the transactions are exchanges of money for money, not money for goods or services which directly meet needs.

It is an exchange of money now for money payable in the future, contingent on the occurrence of certain events insured against.

The main purpose of rate making is to determine the price for the service which will bring in sufficient funds for paying future losses, covering the costs of operation and providing a margin for contingencies and profit. The major problem we face in rating is the fact that, the amount of loss which will occur is unknown, which means that the cost of the operation is unknown. The only solution to this problem is the estimation of the future loss using past experience as a basis.

We have to distinguish between rate and premium. The rate is the price per unit of protection ; the premium is the rate multiplied by the number of units of protection purchased. The premium the insured pays is called "gross premium" and is composed of two parts - one part is designed to provide for the payment of losses called the "pure premium" (when expressed as a percentage it is called expected loss ratio) . The second part is called loading and covers the expenses of the operation. In our study we are more concerned with the estimation of the pure premium.

A satisfactory rate should satisfy criteria as to adequacy, reasonableness, equity, stability, flexibility, and provide for incentive for loss prevention. For achieving these criteria, the risk should be classified in such a way that the rate provides for an equitable distribution of cost by being closely adjusted to the inherent hazard of the individual risk. This problem will be discussed in more detail in Chapter 2.

1.4 Claim Definition in Fire Insurance

The insurance device does not prevent loss, but it reduces the uncertainty of loss. Actually the contract of fire insurance presupposes the existence of some physical object which is capable of being destroyed by fire, and upon this loss the insured has the right of indemnity.

Some writers define fire as a "combination in which oxidisation takes place so rapidly that a flame or glow is produced. Furthermore, the fire must be hostile, that is, it must be of such a character that it goes outside its normal confines" [Green, 1964] . If the fire escapes its confine, it becomes hostile and all losses resulting from it are covered by the insurance contract.

Most fire losses are partial but total losses are not uncommon. It is easy to determine that the fire has occurred, but it is difficult to determine how much the insured has lost by fire.

When we come to the point of studying the fire losses (claims) it is important to divide the fire risk into qualitative homogeneous classes according to the factors which increase the possibility of loss. The main factors are :

1. Location
2. Occupation (by type, and operation)
3. Exposures (the chance of loss from outside sources)
4. Construction (i.e. materials)
5. Fire protection

Having grouped the claim data into homogeneous classes of risk, and having determined the levels of risk in each class, the problem is to determine the expected loss per policy for each class using the past experience.

1.5 Experience rating

The term "experience rating" is used to refer to a prescribed method for determining the risk rate depending in whole or in part on the risk's own experience. Risks whose rates have been determined in accordance with such a procedure are said to be experience rated [Paul Dorweller 1934].

The object of experience rating is to determine a more equitable rate for the individual risk based on the evidence presented by its own experience. It is recognised that individual risks within a class are not alike and that there exists inherent differences due to variation in the physical and moral hazard factors affecting the risk experience. Some of these differences are of such a nature that it is difficult to define them and they can not be associated with conditions measurable in advance.

These differences can not be assumed due to chance. Experience rating is considered as the most practical method to recognise the variation produced by such factors.

Experience rating is applicable whenever there is a large variation among the risk categories which make up the classification, and where the risk categories are of such a nature that they may be expected to develop individual risk experience of appreciable evidential value. Most lines of casualty insurance have classifications somewhat nonhomogeneous as a result of the merger of the experience available and the lack of knowledge of the risk elements entering into the composition of hazard.

There are two forms of experience rating plans, in which the risk experience is used to determine the risk rate.

1 - Prospective experience rating

2 - Retrospective experience rating

Prospective experience rating is used to determine rates for a period in the future. Retrospective experience rating is used to determine a final rate to apply to a past period. Both methods present a definite way of recognising variation in the inherent hazard of risks.

The essential operation of experience rating consists of comparing the risk experience and class experience on a common premium and loss basis, assigning to the risk experience a weight depending on the size of the risk experience and to the class experience the complementary weight. The adjusted rate or experience rate may be looked upon as a weighted average of the rate indicated by the risk loss experience, and the manual rate i.e. the rate indicated by the class experience.

1.6 Meaning of credibility

The weight used in the experience rating formula mentioned above is called the credibility factor. Credibility as it was originally introduced into actuarial science, is a measure of credence that the actuary considers should be attached to a particular volume of data for the purpose of rate making.

Thus we may say that the loss experience under a new class of insurance is "too small to be credible" implying that the expereince which will develop in the future may well be very different from that so far collected, and also implying that we have more confidence in our prior knowledge based in other data such as current rates for similar classes.

Also, we may say that the experience is fully credible which implies that the experience is adequate to establish the rate without reference to the previous rate.

In many cases, the volume of data is too small to be fully credible, but large enough to have some credibility.

A scale of credibility can be established which gives a credibility of zero to data too small to be of any use for rate making and one for credibility to data which are fully credible.

It is a fact that credibility is not an inherent property of the data which can be calculated by a simple mathematical rule such as for the mean and standard deviation, but it is the amount of credence which can be given to the volume of the data for the specific purpose for which the data are to be used. *

When we come to consider the revision of any important set of rates for a class of insurance such as fire insurance, we normally have a volume of data which is fully credible for the purpose of determining the overall rate level which is required. However, each of these major classes includes a very large number of subclasses and there will never be sufficient data to provide credible rate revision for each individual rate. Most rating systems contain a pattern of association between various rates. As an example, in

* The credibility weight to be given to a certain volume of data in experience rating formula is not the same weight to be given to the same data if they are used for some other purpose, such as independent rating.

fire insurance the rate may have five types of classification :

1. Location
2. Construction
3. Occupancy
4. Protection
5. Outside exposure.

If there are three subclasses in each classification, there will be (3^5 ≡) 243 subgroups of the data, no one of which could individually be credible. If it is assumed that the change in rates according to the first type of classification (location) raises or lowers all rates in equal proportion, then by grouping all data by location only with a necessary adjustment to reflect differences in distribution by the other classifications, we can establish the location differential with a reasonable degree of credibility.

By regrouping the data in another way, other classification differentials can be determined.

In fact, credibility theory is concerned with establishing measures and standards of full credibility. In practice, the choice of too high a standard for full credibility would considerably delay the response to changes in risk factors, and may lead to overall inadequacy of premium levels. The standard of full credibility is not normally important in itself, but it is important as a means of introducing consistency in the rate making procedure and in establishing a relationship in respect of reliability between different volumes of experience.

1.7 Historical development of credibility theory for experience rating

Credibility theory for experience rating is largely an invention of North American actuaries. It was first introduced by Witney 1918 for experience rating, in particular for workmens' compensation insurance, where some industrial companies with a large number of employees and favourable safety records pressed for the recognition in their insurance premium of their apparently superior claim record.

Since then the theory has had wide application by different workers in different branches of property and casualty insurance. Michelbacher (1918) was the first to apply the theory for workmens' compensation. Wheeler (1930), Perryman (1932), Wittic (1958) , Bailey(1959) and many others have applied the theory to automobile rate making. Hurley (1954 & 1959) applied the theory for fire insurance and multiple peril rating problems.

The problem of full credibility and partial credibility was discussed by Perryman (1937),Kormes (1952),Hurley (1954) and Longely-Cook (1962).

Witney's theoretical structure for experience rating remained the only one available until 1942, when Bailey presented a new theoretical foundation based upon Bayesian probability. Despite the fact that Bayesian probability was unpopular at that time and Bailey's view did not get wide acceptance and although his approach to the problem was different from Whitney, the results are the same and confirm the value of early work.

Mayerson (1964) resurrected Bailey's work by relating it to modern Bayesian statistics, where the basic credibility equation is developed from a principle now known as conjugate prior analysis. The Bayesian approach to the theory soon becomes acceptable.

Bailey's original results were generalised by Jewell (1974) who showed that the basic credibility equation [ZA+(1-Z)E] could be derived for many conjugate priors of the exponential family.

Buhlmann(1967) contributed to the theory by proving that the basic credibility equation was a least square estimate under the conditions explained in Chapter 4.

Currently, credibility theory is viewed as a method for estimating various moments or parameters of the probability distribution of total loss. It does not matter whether one derives estimates of the quantities from the classical approach [Whitney] or from Bayesian principles, because in the most important cases the revised estimates have the familiar form "ZA + (Z-1)E". However, it is important to recognise that ultimately these credibility adjusted values are used to tell the insurer what he should charge in future.

1.8 Purpose and Outline of Study

Determination of the rates of premium to be charged for insurance cover in a given line of insurance is an important aspect of insurance company policy. Prospective experience rating takes into consideration past experience, in lines of credibility theory, as an efficient procedure for rate determination and rate revision.

The research aim in this study is to review the practical aspect of credibility theory for experience rating and to develop a mathematical formula for experience rating for the pure premium considering fire insurance as an example.

The research includes six chapters. The first chapter is an introduction which includes the definition of different terms usually used in insurance, literature and in some places in this study. The aim of this chapter is to indicate the nature of insurance business.

The aim of the second chapter is to discuss the rating problem and the factors to be taken into account for classification in fire insurance with reference to data from the Egyptian market. Classical statistical tests of significance are suggested to test the correctness of the classification.

The classical approach to credibility theory review is discussed in Chapter 3. In Chapter 4 the collective risk theory approach and the Bayesian approach to credibility are discussed and appraised.

In Chapter 5 a mathematical formula for experience rating for the pure premium is developed in the lines of the Buhlmann credibility formula using the principles developed in Chapter 3 and Chapter 4. The experience rating formula has three components :

1 - Average claim outgo ,

2 - Variability of the claim amount,

3 - Variability of the claim frequency.

Each component has a credibility coefficient.

A simple explicit formula is obtained for determining the level of full credibility for each of the three components.

In Chapter 6 there is an empirical study for two classes of non-industrial fire insurance using data from the Egyptian market.
The primary claim amount is obtained using a multisplit plan with an initial amount £E1500 for class I and £E8000 for class II and a discounting factor 0.6. A credibility table is then constructed for each of the credibility factors involved in the experience rating formula.
The table is left in raw form and can be modified in the light of such a specification as the level of self-rating.

CHAPTER TWO

RATE MAKING AND CHARACTERISTIC OF FIRE INSURANCE

2.1 Problem of rate making

An insurance premium is the price for a promise to indemnify the insured against financial losses resulting from the contingency insured against. The premium reflects the underwriter's assessment of the expected claim experience, the necessary expenses and a margin for profit. It is customary to express the premium as a rate per one, per hundred, or per thousand units of exposure. The choice of unit of exposure depends on two considerations :

"First, that it should be conveniently ascertainable, and second, that it should accurately reflect the scope and degree of the obligation which the insurance carrier undertakes" [Michelbacher, 1925].

The selection of unit to measure the extent [Morwbrey, 1921] of the hazard involved in an insurance transaction is not difficult. It is the measurement of the quality of the hazard which gives rise to problems. Even within a line of insurance it is expected that the probability of loss is much greater for some risks than others. It is fair, therefore, to require from some a correspondingly higher rate of premium. The economic function of insurance has been defined as the safe and equitable distribution of the burden of contingent loss. "Equitable" means distributing the loss among the insured in accordance with the inherent hazard or risk distribution of those who enter into the general pool. The mean of the risk distribution when insurance is conducted as a business is the premium which is paid in advance.

Also, market conditions have a great effect. The presence of competition between the insurance companies makes it necessary to match the premium rates as closely as possible to the risk by the sub-division of risks into homogeneous subgroups. There must, however, be a limit since if the process of relating premium to risk were to be carried to its extreme, each policy holder would be called upon to pay his own claims.

As far as may be practical, the risks are divided into classes, without unnecessarily destroying the reliability of the class experience. The policies in each of the classes are treated as of "equal" hazard and each risk would take the average hazard of the class.

The rate making plan for a given line of insurance would therefore provide as a first step a scheme for the appropriate grouping of the risks according to the various degrees of hazard. In order to put this into practice and to successfully perform the task of determining the rates, the following conditions are to be fulfilled :

a. The plan should provide for identification of claim producing possibilities.

b. It should be possible to recognise the expenses of management attributable to the individual risk.

c. A method should exist for measuring the claim producing potentiality.

d. It should be possible to relate the measurement of the claim producing potentiality to the unit of exposure, and hence to assess the variation in premium.

Furthermore, it is not sufficient to assume the existence of a function that will express the condition of each of the elements of risk.

Instead, what is essential for the successful rating plan is that this function assumes a simple expression for practical purposes.

The fact is that there are rapid changes in the economic and social conditions which have enormous influence on the three types of hazard (legal, moral, physical) attached to risk. It would be reasonable for the rating plan to be responsive to changes in these conditions. The extent to which responsiveness is to be built into the rating plan would, however, depend upon the implication of the change for the expected claim experience of the risk on the one hand, and the requirement of the stability of rates on the other hand. The exact nature of the combination of stability and responsiveness would vary from one line of insurance to another.

As multiple classification is an accepted principle for rating and experience rating in all lines of insurance, there are two requirements for successful application of this principle.

1. A suitable classification system.
2. Availability of a sufficient volume of accurate data on exposure to loss and losses realised covering a considerable period of time, and indefinitely known circumstances, which may be expected to show little or no change in time.

In some lines of insurance, none of these requirements are satisfied. On these lines the rate quoted is largely a matter of personal and intuitive judgment.

The types of rating plans are few in number but there are innumerable detailed differences. Common rating plans are :

a. Class rates : class rates are used for large groups of homogeneous risks such as automobile insurance and dwelling fire insurance. Class rates provide an excellent method of sharing losses and expenses among a large number of policy holders, and are also the most reliable method of predicting future losses. On the other hand, they do not recognise individual differences, which might be significant.

b. Individual rates : individual rates are used for risks which do not fit within a class of homogeneous risks, i.e., there are so many individual rating characteristics that a class rate cannot account for the variables in an equitable fashion. A rate which is designated for an individual risk is able to reflect better its actual loss experience, such as a large risk which has enough exposure units to reflect its losses in the premium. Most commercial risks fall within this category, although some smaller commercial risks may be class rated for partial coverage.

The advantages of individual rating plans are obvious. This results in rates which are fairer to large risks, encourage safety programs and other means of loss reduction, because reduced losses are reflected directly in the premium. Of course, there are disadvantages, and one of them is that it is easy to give too much rate reduction.

c. Schedule rates : schedule rates are developed by comparing lists containing the average characteristics of all risks of the same general type. Each risk has its rate adjusted by the degree to which it varies from the standard. If a risk is better in some categories, it gets credit ; if it is worse in others, it gets a debit. The total of the variances determines the extent of modification made for the individual risk as compared with the standard rate.

Schedule rating, rather than class rating, is commonly used with commercial fire risks because frequency of loss is low but the severity of loss is high.

There are two disadvantages of schedule rating : (1) it is dependent upon the judgment of the person making the inspection, and (2) it places emphasis only on physical factors.

d. Experience rating : Experience rating is developed in the same way as schedule rates except that modifications are made for the past loss experience of the policyholder rather than for his physical characteristics. Such a plan requires a fairly large risk and a reasonable loss frequency. This method is used for larger commercial risks for such coverages as liability, burglary, and workman's compensation.

Experience rating plans assume that immediate past loss experience will continue in the future and they are considered a prospective rating system.

e. Retrospective rates : Retrospective rates use the experience of the current period to determine the rate for that period. The insurer collects a deposit premium when the policy is written, and at the end of the period, calculations are made to determine if the policyholder will get a partial return of his deposit, or if he has to pay an additional premium. A maximum premium is stated at the outset.

2.2 The Characteristic of the Fire Hazard

Basic fire policy cover properties against loss or damage by fire, lightning and damage caused by removal of property for protection against insured peril.

Fire hazards may be divided into two broad classes, physical and psychological (moral). Physical hazards are those material factors which may give rise to fire or contribute to its spread. Psychological hazards are the intangible human qualities which may have the same result.

I - Physical hazard is easier to detect than moral hazard because physical characteristics are specific and objective ; many are capable of quite precise measurement. Common physical hazards such as lighting power generating, power transmission, heating, ventilating systems are found in greater or less degree in all sorts of risk. Rates in fire insurance depend to a great extent on the physical characteristics of the risk insured such as the construction elements of the building, occupancy, degree of police or fire protection, exposures from adjacent buildings, climatic and weather conditions, location and facilities for controlling or minimizing loss.

All elements of physical hazard are susceptible to inspection and study. In many cases the insurance company may ask the insured to make some minor changes to reduce or eliminate hazard. The investigation of most large claims shows that the insured did not meet the insurance company's requirements. Table (2-1) shows the distribution of the claims from year 1970 to 1979 according to the cause of the fire "for direct insurance by the company under investigation".

From Table (2-1) it can be seen that 0.712 of the amount of claims is due to physical factors affecting the risk. The most important factor is electrical shortage due to poor electrical connections and is represented by 0.39 of the number of claims (Chart(2-1) shows the percentage of the amount of loss and the number of claims for the years 1970-1979).

Table (2-2) shows the average of claim and loss ratio for each class of business in urban and rural areas. It can be seen that industrial projects have the highest loss ratio and the largest loss average per accident. The projects insured in this class are individually rated and have large amounts of unit of exposure, which make the insured have the power to influence the insurance company to make the rate very low. Also, the class of storage (warehouse) is higher than the rest of the class and this is also due to the large amount of units of exposure involved. At the same time, the loss ratio for this class and the average loss per accident in the urban areas are higher than the loss ratio and the average loss in the rural areas because the average amount exposed to risk in each storage is larger for urban than for rural areas. The average per storage in urban areas is £E684,290 and in rural areas £E246,848. In the other classes the loss ratio for rural areas is slightly larger than that of the urban areas except for the classes of woodwork, food industry and hotels.

II - Psychological (moral) hazard is the most difficult to detect, and yet, may be the most dangerous. In spite of its potential severity, its true nature is seldom understood. Psychological hazard can be divided into two classes : voluntary and involuntary.

Voluntary hazard is synonymous with "moral turpitude" as purposefully causing or increasing fire losses, failure to take measures for extinguishing or preventing the spread of fire, and defrauding the insurance company by the exaggeration of loss. Purposeful fire is the common* type of moral hazard. While the number of accidents from this cause is 1.73% of the total number of the accidents for the period 1970-79, the amount of claims is 11.11% of all claims for

* see Table (2-1)

this period ; this demonstrates the potential severity of this cause. The average claim per accident is also the second highest average per accident of all classes.

Involuntary (moral) hazard comprises those intangible elements in human character and skill which do not reflect on the honesty of the person involved but which, nevertheless, are strong in their effect on fire loss. Common types of this sort of hazard are negligence of ordinary protection of the property against loss, carelessness, incompetent management, and lack of trust. To some extent these qualities of hazard can be detected by evidence which can be observed by the producer or the insurance inspector. The easiest type of moral hazard to observe is poor housekeeping, poor maintenance and irresponsible conduct. From Table (2-1) it is seen that accidents caused by cigarettes represent 37% of the total number of accidents during the period (1970-1979). The Chart (2-2) also shows the high frequency of this cause during this period.

It is difficult to contain the moral hazard in the rating structure and the insurance company should reject the risk if it has a large degree of this element. Ownership (private or public ownership) presents another hazard which has an element of turpitude. Table (2-3) shows that the loss ratio for public ownership is more than the loss ratio for private ownership.

2.3 Fire Insurance Coverage

Fire insurance on building and content of building and industrial manufacturing in Egypt is one of the oldest and most established lines of insurance. Fire insurance coverage constitutes the third largest share of non life insurance premiums. Table (2-4) shows the growth of the non life insurance market and the amount of fire insurance premium in the period 1970-1979.

Commercial fire insurance is more subject to large losses and is more sensitive to changes in the economic conditions of the country. Fire losses tend to increase as the economic condition worsens.

Coverage of fire insurance in the Egyptain market can be divided into two parts :

1. Industrial risks
2. Non-industrial risks

Most industrial risks are individually rated because they are very large and complicated risks. Industrial risks constitute 55% of the premiums paid during the period of the investigation. Non-industrial risks include building construction, building content, retail shops, warehouses, woodwork, metalwork and garages. Table (2-5) shows the distribution of the premiums between the classes for the company under investigation.

The basic fire policy covers property against loss by fire, lightning and damage caused by removal for protection. The added perils of extended coverage are common in commercial fire policies. Perils of extended coverage are as follows : windstorm, hail, riot, vandalism, aircraft, vehicle, smoke, earthquake, sprinkler leaking and water leaking. A fire policy can be extended to cover consequential losses such as loss of profit and third party liability. A small proportion of industrial risk is insured against such risks. Non-industrial risk is not accustomed to extended coverage. It seems to the writer that the insurers underestimate or misunderstand the extended coverage. Not less than 10% of the reported claims were not paid because the cuase of the damage was not covered, though it could have been covered at the outset by the appropriate addition to premium. (Table (2.6) shows the distribution of the unpaid claims in relation to the cause of the accidents .

From Table (2.6) it can be seen that damage by water represents a substantial number of unpaid claims. While the insured has no right to apply for the claim ((his contract has not any clause to indemnify him against the cause of damage)) but he did apply, this means that he did not understand the condition of the contract or was misled.

Always an insurance company tries hard to secure insurance to cover the true value (i.e. the actual value of the property) ; but often, it is difficult to ascertain the actual value, particularly as to equipment and stock, also inflation has a great effect upon this problem. Table (2.7) shows the number of those proved to be under-insured and the average fraction of full cover of insurance for the under-insured in years 1970-1979.

Often with a multiple location policy the insured will have stocks in more than one location in the country, and the insurance company will discover that his insurance cover is much less than the actual value of his total stocks. In some cases, the sum insured is only 15% of the stock and while the claim is the full indemnity for the loss. Most of these cases are discovered only when the event of loss occurs and the insurance company makes its investigation to assess the amount of claim. In these cases the insurance company applies the co-insurance clause (average clause) which indicates that the policy holder will share every loss to the percentage extent that his insurance cover is deficient.

Commercial fire rates are customarily given as annual rates for each £E1000. Some policies are written for more than one year and the insured obtains a reduction if he pays the premium in advance for the whole period covered by the policy. Table 2-8 shows the rate of reduction.

2.4 Factors Considered in classifying and rating

The first factor considered in rating is the location. The location can have an important effect on the desirability of the property from a fire insurance standpoint. Cities and towns are graded according to fire protection classes. The factors which establish the grading of a city are :

i availability of water and water pressure.

ii fire department, personnel, equipment, efficiency.

iii police protection

These factors together measure the anticipated ability to control the fire. Egypt is divided into two regions :

i Region A (urban area)

1 - City of Cairo

2 - City of Alexandria

3 - El Hay El Efringy of Esmalia

4 - El Hay El Efringy of Sues

ii Region B (rural area)

Rest of the country

Table (2-9) shows the amount of premium in each region for each class of insurance.

In addition, four major factors are used in classifying and rating for commercial fire risks. These four factors must be considered by the insurance rating bureau engineers who establish the rates. The four major factors are construction, occupancy, protection and exposure (the COPE factors) (Robert B. Holtom, 1973 p. 384-388).

1 - Construction :-

A basic consideration in rating and underwriting is the construction of the building insured or containing the property insured. Various types of material may be used such as wood, steel, cement. This material may be classified as fire resistive, semi-fire resistive and combustible.

Egyptian rating organisation groups building construction into two groups (Appendix (2.1) presents the rates in force from 1970-1979).

a - Fire-resistive construction which constitutes 98% of the insured property.

b - Buildings constructed of frame wood or a percentage of frame wood. This class is divided into three categories :

  i   wood amounting to 25% of construction material.

  ii wood amounting from 25% to 50% of construction material.

  iii wood amounting from 50% to 100% of construction material.

Other factors may enter into the construction consideration too. One of these is the number of fire divisions in the building and whether the partitions which divide the building into sections are properly restricting the spread of fire. This factor is of vital interest in selecting risks as well as in rating.

2 - Occupancy :-

The use of a building and the type of business within it, constitutes the occupancy. Multiple occupancies are common in commercial buildings, so the underwriter must identify all occupancies. Many considerations enter into the occupancy factor. The type of operation performed is critical, inherent hazards in such operations as spray painting, cooking, furniture refurnishing and the handling of the inflammable liquids and gases. The materials being used are important. Some materials are more inflammable than others. Other materials are more susceptible to damage by smoke or any water which is used to extinguish a fire.

Smoking, cooking, air-conditioning equipment, heating, wood working and many other potential hazards of occupancies also must be considered. Even a vacant building has its hazards, even when those caused by people and operations in other occupancies are not present; the unique hazard of vacancy does exist.

Rate organisation divides equipment or materials involved in industrial hazards into 6 groups as in Appendix (2.2).

3 - Protection (Loss Prevention) :-

Loss prevention factors measure the reduction of risks by the reduction of the chance that a loss will occur or by reducing its severity if it does occur. Protection can be divided into two elements : (1) public protection, which is an element of location referred to earlier , (2) private protection . Private protection refers to the insured's effort to protect his property by, for example, the installation of an automatic sprinkler system and alarms, watchman services, portable extinguishers, chemical systems and other means of preventing, detecting and extinguishing fires. Usually, this private protection results in premium reduction. In Egypt the amount of reduction is determined by the rating organisation engineers on their own individual basis.

4 - Exposure :-

The chance of loss from an outside source is called exposure. The outside source may be another building, a railroad, or many other exterior exposures. An exposure may be presented from other occupancies in the building, so a great deal of information must be secured about such occupancies. In addition, the degree of exposure is important, involving the question of distance between risks, fire division, the openings through stair wells and air ducts, and all other factors which could prevent or contribute to the spread of fire. The degree to which the outside sources expose the risk to loss is important in underwriting and rating.

It would seem that this factor is under-estimated by the insurance inspector in the Egyptian market, because in many cases the cause of the accident is outside exposure, and outside exposure, such as a main electric box near the object insured is rated as a standard risk as also is a window in a shop looking on to a light shaft full of waste, which is easy to notice.

Rating organisation in Egypt divides the country into two regions as already stated. For each class of risk the organisation establishes specific rates for each region. Each class of risk consists of a number of categories (risks) which are assumed to be homogeneous and have the same amount of hazard on average.

At this point the question arises as to whether the classification is correct i.e. whether the categories are correctly grouped, and whether this question can be answered statistically.

Since the rating plans are designed to adjust the price of insurance for each class on the basis of the loss experience of each class, the loss portion of the premiums (pure premium) should approximate to the loss payments. There are two reservations to this assumption :-

1 - The hazard of any risk (category) as measured by the expected loss cost per exposure unit may differ from that of the class because of the crudeness of the classification (grouping).

2 - Due to chance fluctuation alone, the actual loss of any category (risk) may differ from what would be expected even if its hazard were known; in other words, even if the risk were perfectly classified, there would be no reason to expect that the actual losses in any year would be equal to the expected losses.

3 - A third reservation should also be mentioned, namely that the pure premium which is the loss portion of the office rate is itself an estimate of the hazard of the group and the expected loss determined on the basis of this premium seldom reflects the true average hazard. Furthermore, even if it did, errors in the application of the rate would distort the result.

For the present, it will be assumed that the pure premium is a true estimate of the class hazard and that rates are applied correctly.

The only types of variation that will be considered, therefore, will be those due to the crudeness of the rating classification and to chance.

2.5 Common statistical tests of significant differences

Statistical theory is largely concerned with sampling from populations ; and much of the observation of insurance experience is a sampling process (David Haston, 1960). In our study the population will consist of all people who are potential users of a given line of insurance. This population is divided into subpopulations (classes ≡ strata) so that the risk units (categories) in each class are homogeneous. A sample of size N is drawn, where N is the number of policyholders. The loss of each policyholder is observed, and assigned to its class, and these losses make up the sample data, assuming that the forces producing the actual losses are to continue to operate without limit of time.

We want to apply statistical tests to examine the homogeneity of the classes. A statistical hypothesis is made about a parameter or a measure defining the risk class. This hypothesis may state for example that the expected losses per exposure unit of risk (for any category in the class) are equal to those of the class which may be calculated from the pure premium. Chance fluctuation alone will cause the sample values to vary around the hypothecated parameter. In other words differences are expected. The validity of the statistical hypothesis is tested by deciding whether the difference is greater than can be accounted for by chance i.e. is significant. A decision can then be made as to whether the category was or was not properly classified.

But this is always a probability statement. It cannot be stated that the category is definitely not a member of that class. All that can be said is that if the hypothesis is true, the probability of drawing such a deviant is very small. On the other hand, if the difference is not statistically significant, the hypothesis that the risk is properly classified is accepted, but again this statment is only a probability statement. What is meant, is that if the classification is correct, the chance that the actual losses of the risk would differ from those expected on the basis of the class rate by as much as, or more than, is observed, is greater than the critical level of significance.

Usually the level of significance is set at a probability of .01 or .05 but if it is needed to be less conservative, it can be moved up to .1 or .2. It has to be borne in mind also that in rejecting the hypothesis because the observed difference of observation from hypothesis is larger than expected by chance e.g. beyond a chance of .05 may be false. Since a chance of .05 means that the event must occur once in twenty times and the observation may indeed be just this occurrence [Type I error]. Also, in accepting the hypothesis because the difference is small may equally be in error since the difference may, in fact, be a large deviation from a difference true rate (Type II error). The likelihoods of the two types of error are related, one increasing as the other decreases, with a change in the level of significance chosen. It is not possible to eliminate either one without making the other a certainty. (The two types of error are indicated in Table [2-16]).

[2.10]

ACTION	Hypothesis	
	True	Not True
Accept hypothesis	Correct	Type II error
Not accept hypothesis	Type I error	Correct

The only solution to this problem is to effect a compromise. Usually one type of error is more important in practice than the other, and the probability of that type of error is fixed at a desired value, the probability of the other type being held at a minimum by choosing an appropriate test of significance and , if possible , an appropriate sample size. These tests are valuable in that they indicate whether it is reasonable to believe that the risk is properly classified.

Distribution of claims according to the cause of fire

TABLE (2.1)

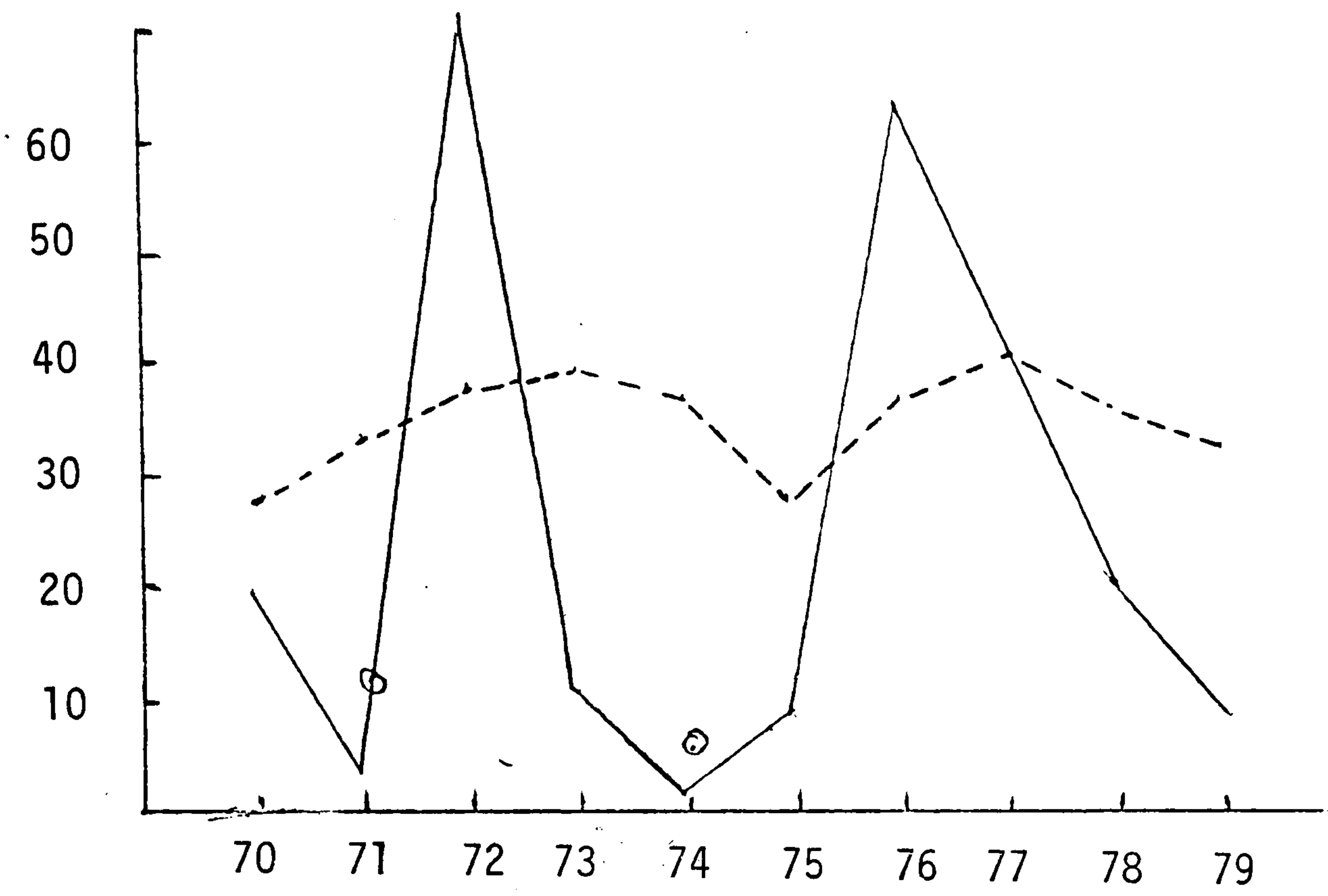
CAUSE OF LOSS	PERCENTAGE		AVERAGE		
	Of all claims	excluding extremes	for each cause	for each cause excluding extremes	frequency of claims
CIGARETTES	14.24	27.02	1618	1818	621
ELECTRICAL SHORTAGE	11.62	22.05	1250	1250	656
CHIMNEY SPARKLE*(1)	13.19	4.86	13895	2740	67
PURPOSE*(2) FIRE	11.11	10.70	27033	15304	29
EXPLOSION	6.06	11.50	8068	8068	53
TECHNICAL FAULT	5.41	10.26	4062	4062	94
SELF INTERACTION	3.39	6.43	5566	5566	43
DIRECT FIRE	3.31	6.29	2750	2750	85
FLAMMABLE LIQUID	0.25	0.47	1266	1266	14
VANDALISM	0.14	0.26	1944	1944	5
DAMAGE BY WATER	0.06	0.11	689	689	6
CHEMICAL INTERACTION *(3)	31.22	0.05	734325	888	3
TOTAL	100.00	100.00	4213	2228	1675

* within the period of the investigation, there were three very extreme accidents

a - one accident in 1978 amounted to 2,201,200 *(3)

b - two accidents in 1975 : one amounted to 750,143 *(1) and the other one resulted in three claims amounting to 397,910 *(2).

Graph (2.1)a

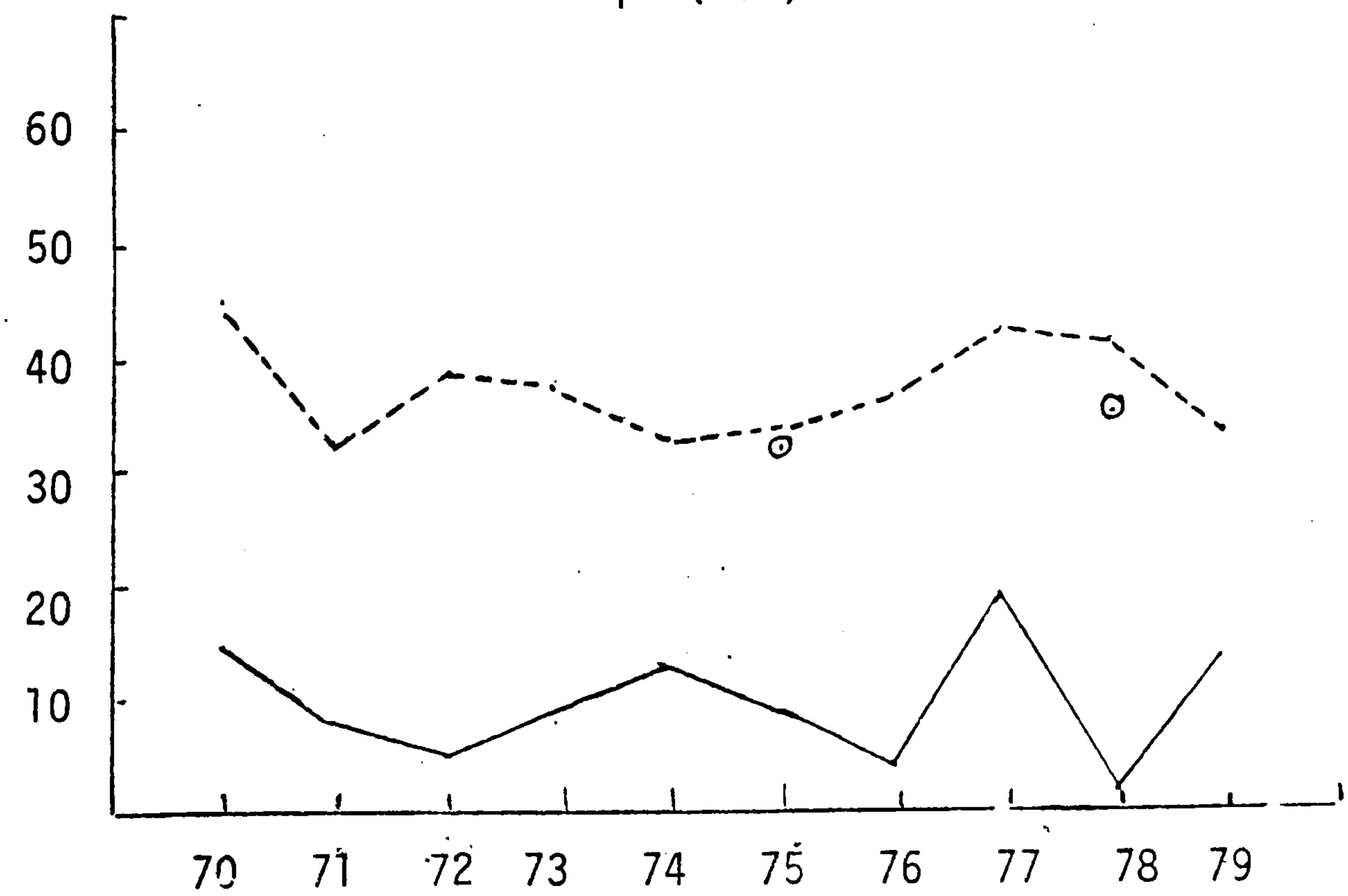


Percentage of number and amount of loss because of cigarettes

---- percentage of number of claims

—— percentage of amount of claims

Graph (2.1)b



Percentage of number and amount of claims because of electrical shortage

Average claim and loss ratio for each class of business

TABLE (2.2)

Class of insurance	Urban area		Rural area		Urban & rural areas	
	Average	Loss Ratio	Average	Loss Ratio	Average	Loss Ratio
Residential building	416	.025339	694	.028279	434	.025625
Hotels	1248	.041569	2949	.261956	1580	.056582
Stores & shops	1705	.133083	1359	.092967	1681	.129875
Wood work	2439	.249415	9819	1.262093	4284	.46164
Metal work	432	.012786	260	.014067	417	.013055
Cinema & theatres	801	.15050	0	0	801	.139049
Garages	6991	.225305	333	.002791	6436	.167736
Warehouses	9972	.455199	5839	.193461	6999	.359745
Rice & flour mills	2585	.309849	3067	.126718	2626	.17366
Food industry	2280	.031197	7949	.271850	4052	.068228
Textile industry	889	.080741	3850	.542477	967	.088689
Other industry	7444	.720407	29930	1.235339	8387	.768337

$$\text{Average} = \frac{\text{amount of claims}}{\text{no. of claims}}$$

$$\text{Loss Ratio} = \frac{\text{amount of claim}}{\text{amount of premium}}$$

Loss ratio for public and private ownership

TABLE (2.3)

YEAR	LOSS RATIO		PUBLIC & PRIVATE
	PUBLIC	PRIVATE	
79	.4666	.0614	.3838
78	.1520	2.4310	.5379
77	.2115	.0830	.1872
76	.1759	.0325	.1527
75	.4240	.8823	.4871
74	.0987	.0753	.0960
73	.1196	.0353	.1109
72	.1337	.0172	.1244
71	.1774	.0657	.1574
70	.0850	.0750	.0838

$$\text{Loss Ratio} = \frac{\text{amount of loss}}{\text{amount of premium}}$$

The total amount of premium paid for the class of non life insurance and the total premium paid for the class of fire insurance

TABLE (2.4)

	NON LIFE* INSURANCE	FIRE INSURANCE*	PERCENTAGE
79	79669	17129	21.5
78	69748	13662	19.6
77	50893	9825	19.3
76	43494	8218	18.8
75	38272	8064	21.1
74	29165	6241	21.4
73	18874	5703	30.0
72	16298	4476	27.5
71	16273	4451	27.3
70	15145	4237	28.0

* amount in E£1000

TABLE (2.5)

Class of insurance	Premium (Public)	Premium (Private)	Total	Percentage of the class
Residential building	1303686	713996	2017682	.0621437
Hotels	367064	470565	837629	.0257986
Stores & shops	1847435	1077251	2924686	.0900787
Woodwork	68488	42877	111365	.0034300
Metalwork	186282	68942	255244	.0078608
Cinema & theatres	15389	19160	34549	.0010641
Garages	626227	294607	920834	.0283613
Warehouses	6090183	1303339	7393522	.2277172
Rice & flour mills	520723	0	520723	.0160380
Food industry	2691161	159179	2850340	.0877892
Textile industry	7635293	198537	7833830	.2412785
Industry	5886992	880635	6767627	.2084399
Total	27238923	5229088	32468011	1.0000000

Distribution of the premium between the classes of insurance

The distribution of unpaid claims according to the cause of the accident

TABLE (2.6)

Year	Damage by Water	Liability	Accident	Vandalism	Electrical* Surge	Other Causes	Total	Total / Total no of claims
79	16	4	3	4	3	4	34	.109
78	12	4	4	5	5	6	36	.105
77	10	5	5	6	5	11	42	.120
76	12	8	3	12	10	20	65	.240
75	14	5	2	-	5	4	30	.136
74	11	3	6	4	10	6	40	.172
73	16	6	12	3	8	3	48	.180
72	13	4	2	5	7	7	38	.151
71	18	5	8	4	10	5	50	.231
70	5	6	5	3	8	9	31	.154

* The insurance contract in Egypt does not indemnify the insured against damage by electrical surge

Average fraction of full cover of insurance for under insured

TABLE (2.7)

Year	Average proportion of full insurance for under insured *1	No. of cases	Number of cases as a percentage of total number of claims
79	.4466	48	16.2
78	.5546	40	13.0
77	.6400	18	6.1
76	.2976	6	2.9
75	.6527	18	9.5
74	.7167	20	10.4
73	.7104	10	4.6
72	.5721	16	7.5
71	.4711	16	9.6
70	.7094	18	10.6

*1 Average insurance for the under insured =
$\sum$ sum insured x fraction of full insurance/$\sum$ sum insured

Rate of reduction for period more than a year

TABLE (2.8)

Period In Years	Reduction per unit
2	.125
3	.1667
4	.1875
5	.2000
6	.208
7	.214
8	.218
9	.222
10	.229

The total premiums paid in each region for each class of insurance

TABLE (2.9)

Class of insurance	Urban area A	Rural area B	Total A+B
Residential building	1821457	196226	2017682
Hotels	780567	57063	837629
Stores & shops	2690772	233911	2924686
Woodwork	88026	23339	111365
Metalwork	236741	18483	255224
Cinema & theatres	31920	2629	34549
Garages	682594	238240	920834
Warehouses	4556553	2836970	7393522
Rice and flour mills	133478	387245	520723
Food industry	2411733	438609	2850340
Textile industry	7698992	134837	7833830
Other industries	6137689	629938	6767627
Total	27270522	5197489	32468011

APPENDIX 2-1

	Rate for £E1000	
	A	B
1 - Building construction		
i constructed of fire resistive material		
a - residential only	0.9	2.0
b - all or part of the building used as retail shop or warehouse	1.0	2.5
ii constructed of frame wood		
a - frame wood constitutes up to 25% of construction material	1.0	2.5
b - frame wood constitutes up to 50% of construction material		
c - frame wood constitutes over 50% of construction material	10.0	10.0
2 - Furniture	1.25	2.75
jewellery and valuable objects if they constitute more than 25% of the sum insured	1.875	4.3125
3 - Hotels		
i for building construction	1.5	3.75
ii for furniture	1.875	4.125
5 - Public places		
i construction	1.0	2.5
ii furniture	1.875	4.3125
7 - Churches & Mosques		
i construction	.72	1.6
ii furniture	1.25	2.75

		Rate for £E1000	
		A	B
8 -	Hospitals & Schools		
i	construction	.9	2.0
ii	furniture	1.25	2.75
9 -	Building under construction	1.125	2.5

Group I

Acetin acid in plastic tanks

Aluminium oxide

Arsenic oxide

Beer

Carbon dioxide bottles

Carbrolactum in iron barrels

Carbon tetrachloride

Copper

Fashion shops

Formica

Freon bottles

Grain

Graphite powder

Iron

Iron oxide

Lead

Magnesium oxide

Nickel oxide

Rice

Shoe shop

Thread (cotton, silk, man-made fibre)

Tin

Wheat grain

Wood paste

Group 2

Animal food

Artwork shops

Bicycle retailer

Bookbinder

Bookshop

Calcium cyanide

Calcium nitrate

Calcium phosphate (super)

Carpet retailer or warehouse

Chocolate

Coffee

Coffee shop

Diaries

Dried dates

Dry cleaners

Electrical equipment retailer

Fragrant odours

Furniture shops and warehouses

Hairdresser

Jewellers shop

Lighting equipment

Medical wholesaler

Optician

Oxygen bottle

Painting

Paper in warehouses

Pastry shop

Peanuts

Pharmacy

Photographer

Plumber

Printers

Public houses

Raw wool

Red or yellow mercuric oxide

Restaurant

Shoe repair

Sports

Stationary

Sugar in warehouses

Sulphur dioxide

Tailor

Tea

Tobacco retail shops

Unlicenced grocers

Watch retailer

Wines

Wool fibre

Group 3

Aluminium bicarbonate

Aluminium sulphate

Ammonium hydroxide

Animal hair

Antimony pentoxide

Baker

Black mercury oxide

Borax

Calcium hydroxide

Calcium oxide

Calcium sulphate

Cement

China

Copper sulphate

Dusting powder

Electrical workshops

Ferrous chloride

Ferrous sulphate

Fertilizer

Fire arms

Flour

Glass

Glue

Hatter

Insecticides (paste or powder)

Ironmonger

Jasmine (paste)

Licenced grocers

Lime water

Methyl chloride

Nicotine sulphate

Nylon

Pastry shop with baking facility

Photo equipment

Plastics

Polyethylene (granules)

Potassium bicarbonate

Potassium bromide

Potassium chloride

Potassium sulphate

Potassium persulphate

Records

Red lead oxide

Silk fibres

Sodium bicarbonate

Sodium bromide

Sodium hypersulphate

Sodium persulphate

Sodium silicate

Sodium sulphide

Starch

Synthetic fibres

Talcum powder

Trichlorethylene

Zinc Powder

Group 4

Ammunition

Amylcetate

Anhydrous acetic acid

Barium

Bitumen (asphalt)

Benzyl acetate

Butter

Camphor

Candles

Cellulose fibres

Cork

Ethyl acetate

Fish oil

Fish fat

Formic acid

Fruit spirit

Fur

Henna

Hexa methylene

Hydrochloric acid

Industrial odours

ISO propyl alcohol 76%

Liquid paint (not cellulose)

Lubricants

Man made silk fibre

Matches

Medical cotton

Methylchloride

Napthalene

Potassium hydroxide

Sheep fat

Shoe polish

Sodium hydroxide

Spirits 70% alcohol

Stearic acid

Sulphuric acid

Tar

Vaseline

Vegetable oil

Wood

Zinc oxide

Group 5

Acetic acid

Alcoholic beverage

Ammonium nitrate

Aniline

Calcium carbide

Chromic acid

Concentrated extract (inflammable)

Ethyl alcohol

Hydrogen in iron cylinders

Insect killer (aerosol)

Liquid paraffin

Man-made wool fibre

Metal varnishings

Nitric acid

Nitro benzene

Oxalic acid

Phosphric matches

Raw rubber

Red phosphorous

Sodium nitrate

Turpentine oil

CHAPTER THREE

EXPERIENCE RATING AND CREDIBILITY

3.1 Introduction

Experience rating is one of the possible methods of trying to reach reasonable* premiums by starting from a hypothetical value (say π_o) and subsequently correcting it by using actual claims experience as it emerges.

Witney (1918) established the basis for experience rating and arrived at an analytical formula which is known as the credibility formula for experience rating.

$$p = p_o + z(p_1 - p_o) \qquad (3.1\text{-}1)$$

where

p_1 = the existing class rate

p_o = the rate indicated by the current experience

z = the allowed percentage of the difference between the existing rate and the rate developed from the recent experience of the risk

p = the final rate for the risk

Actually, the formula (3.1-1) has some important advantages

1 - it presents a functional relationship between the existing rate and the current experience.

* "reasonable premiums, means at least that the mean premium should not be too far from the actual expected value of claims, and also should not show too much random fluctuation. (R. E. Beard et. al. Risk Theory". Cambridge University Press, 1979"

2 - the credibility factor z is a function of the data used in experience rating, which would make the credibility formula respond to the indication of the current experience.

The current experience, and consequently z, will depend on four factors :

1 - Risk-exposure

2 - the degree of hazard

3 - the degree of risk homogeneity within the class

4 - credibility of the existing rate.

It is evident that the larger the number of units of exposure, the larger the credence that can be given to the risk-experience. Also the same relationship holds in the case of the hazard ; the larger the degree of the hazard, the larger the number of the accidents for the same exposure and therefore the more trustworthy the average. The standard deviation can be taken as a measure of dispersion. If the risk is homogeneous within the class, it means that the standard deviation is small, and the risk-experience that departs from the average can be regarded as due to chance rather than to any inherent variability in the degree of hazard. The final factor to be taken into account, is the credibility of the existing rate. It would be wise to give greater weight to the existing rate if it is established upon sufficient experience.

Development in the use of formula (3.1-1) can be divided into two approaches

1 - classical approach

2 - risk theory approach

3.2 Credibility Experience Rating - Classical Approach

It is common practice for an insurance company to adjust their current rate using their past experience. The most popular formula used is

$$\theta = \frac{A-E}{E} \tag{3.2-1}$$

A = Actual losses during the period of experience

E = Expected loss (earned pure premium during the period of experience)

θ = amount of adjustment (modification factor)

Formula (3.2-1) is simple and logical, but it does not account for the number of claims or the number of units of exposure during the period of the experience rating. Multiplying formula (3.2-1) by credibility factor would account for factors which affect the rate adjustment (Perryman, 1938) such as

1 - Number of units of exposure

2 - Number of claims

3 - Amount of actual loss

The basic credibility formula for modification of the rate is

$$\theta = \frac{zA + (1-z)E}{E} \tag{3.2-2}$$

The numerator of (3.2-2) is the weighted mean of the previously expected loss ratio and the observed actual loss ratio. Formula (3.2-2) can be written in the form

$$\theta = (1-z) + \frac{A}{E}z \qquad (3.2\text{-}3)$$

Formula (3.2-3) consists of three terms (Perryman, 1938) :

1 - unity corresponding to no change from existing rate for instance, if $z = 0$.

2 - $-z$ being the credit if the actual loss is zero i.e. $A = 0$.

3 - $+z \frac{A}{E}$ being the change for the actual loss A compared with expected loss.

Also formula (3.2-2) indicates for two extreme values

$z = 0$ which indicates zero credibility value for the current experience.

$z = 1$ which indicates full credibility for the current experience when a large volume of risk experience is available.

The main problem is to determine z. There is no agreed statistical techniques available to determine z. Most of the methods used for determining z are empirical and although they worked in practice, they were hard to justify mathematically (Mayerson, 1964).

3.2-1 Some Properties of z

Witney in his paper (1918) arrives at two expressions as approximations for the value of z, called first and second approximations. The first approximation is the one most known and since applied.

$$z = \frac{p_n}{p_n+k} = \frac{E}{E+k} \qquad (3.2\text{-}4)$$

where k is a constant

Actually k is a function in E as it is from Witney derivation, it is assumed constant for practical applications.

Perryman (1938) says the value k is determined from consideration of the "swing" * it is desired to give to the plan.

Expression (3.2-4) gives z a value between 0 and 1, continually increasing as E increases but never quite reaching unity. In fact, if z is plotted as a function of E, z moves along a branch of the hyperbola which has z = 1 as an asymptote.

From equation (3.2-4), z must satisfy the conditions

1 - z should not be less than zero or greater than unity

$$0 \leqslant z < 1 .$$

2 - z should increase as the size of the risk increases.

3 - as the size of the risk increases the percentage charge for any loss of given size should decrease.

Condition (1) is directly obtained from the relation between z and E. Also condition (2) can be derived from the same relationship, since as E increases, z asymptotically approaches to 1 (point of self rating) For condition (3), if we go back to the third part of equation (3.2-3)

* "Swing" of the plan is defined as the credit of the debit granted on the basis of a given loss record . Paul Darweiler "A Survey of Credibility in Experience Rating" PCAS XXI 1934 (1-25).

we find that the risk charge is $\frac{z}{E}A$, so that if E is increased for any given A (amount of loss), the risk charge will decrease with the increase of E. This condition can be expressed mathematically as

$$\left.\begin{array}{lll} \text{i} & - & 0 < z < 1 \\ \text{ii} & - & \frac{dz}{dE} \geq 0 \quad \text{not negative} \\ \text{iii} & - & \frac{d}{dE}\left(\frac{z}{E}\right) \leq \end{array}\right\} \qquad (3.2\text{-}5)$$

which satisfy equation (3.2-4), where k is greater than zero. Equation (3.2-4) is known as the k-formula.

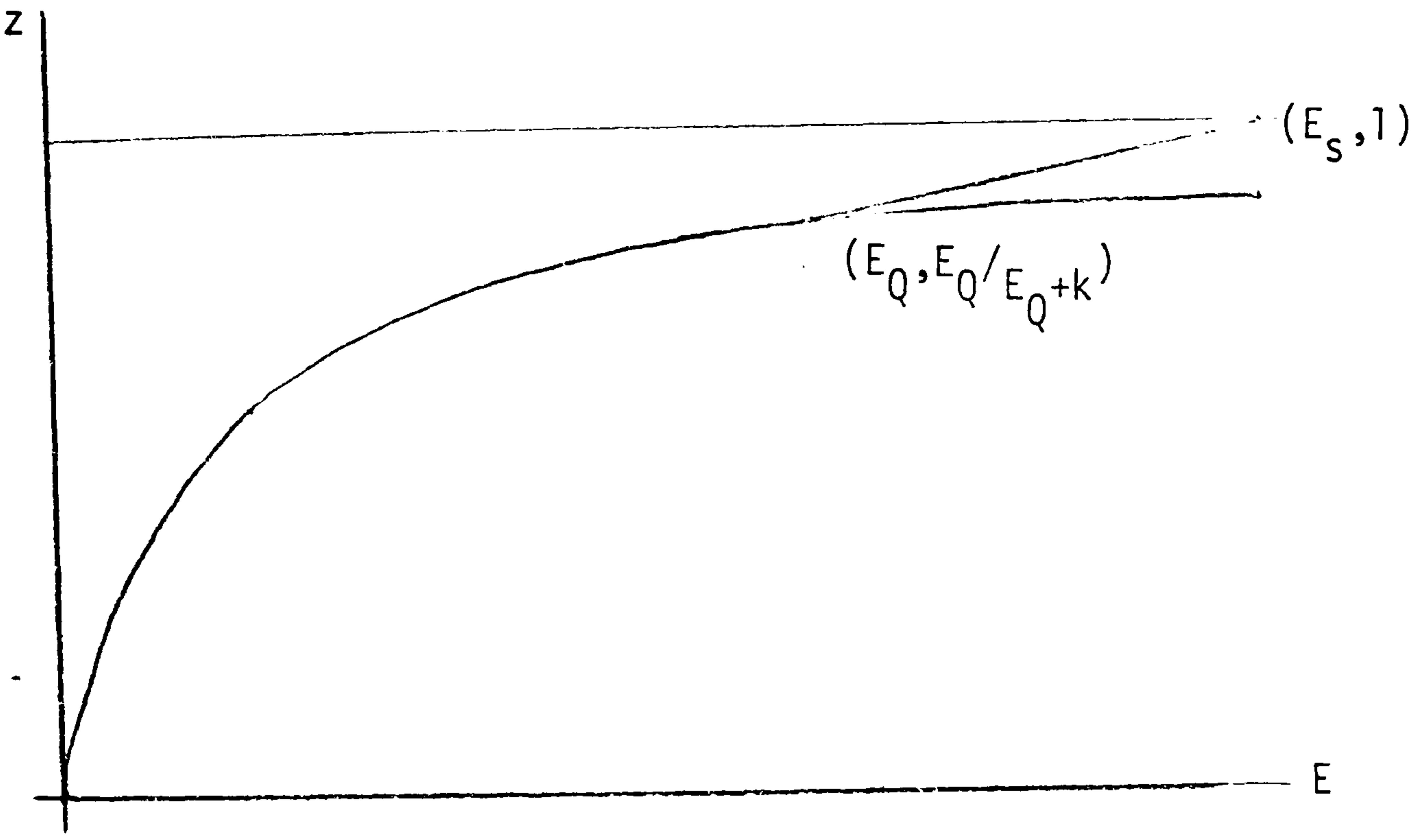
3.2-2 Adjustment for self rating

It has been shown that formula (3.2-4) for z gives values that approach unity as E increases, but never reaches this value. However, for practical reasons it is often desirable that for risks over a certain size the credibility z be exactly unity, i.e. in terms of equation (3.2-2), ignoring the existing rate when a certain size of the risk experience becomes available. When the size of experience reaches this point the risk is said to be self-rated. Considering E as a measure for the risk experience, the E_s is the self-rating point. If the point of self-rating is fixed, equation (3.2-4) would be modified to indicate this condition. There are several approaches to this modification. The obvious one is putting equation (3.2-4) in the form

$$\left.\begin{array}{lll} 1 - z = & \frac{E}{E+k} & \text{if} \quad E < E_s \\ & 1 & \text{if} \quad E \geq E_s \end{array}\right\} \qquad (3.2\text{-}6)$$

which implies a sudden change at the point E_s and this is not desirable.

2 - draw a straight line from an arbitrary point $(E_Q , E_Q / E_Q+k)$ to the self rating point (E_s,L) [Fig. 1].



$$z = \frac{E}{E+k} \qquad \text{if } E < E_Q$$

$$= \frac{k}{(E_Q+k)} \frac{(E-E_Q)}{E_s-E_Q} + \frac{E_Q}{E_Q+k} \qquad E_Q \leq E \leq E_s \qquad (3.2\text{-}7)$$

$$= 1 \qquad E > E_s$$

This also would have discontinuity at E_Q and E_s .

3 - instead of using an arbitrary point E_Q , draw a tangent from the point $(E_s,1)$, touching the curve $z = \frac{E}{E+K}$ at $E = E_Q$.

$$z = \frac{E}{E+k} \qquad E < E_Q$$

$$= 1 - \frac{4k(E_s-E)}{(E_s+k)^2} \qquad E_Q \leq E \leq E_s \qquad (3.2\text{-}8)$$

$$= 1 \qquad E > E_s$$

This expression will have discontinuity at E_s .

4 - It would have been better, while making the change , to have drawn a curve (i.e. second degree parabola) touching the line z = 1 at $E = E_s$ and also touching the curve $z = \frac{E}{E+k}$ at $E = E_Q$.

A simple parabola of the m^{th} degree

$$z = 1 - H(E_s-E)^m$$

where H is constant and m not less than one, will touch z = 1 at

$E = E_s$ and $z = E/E+k$ at $E = E_Q$

$$\text{if} \quad \frac{k}{E_Q+k} = H(E_s-E_Q)^m$$

$$\text{and} \quad \frac{k}{(EQ+k)^2} = mH(E_s-E_Q)^{m-1}$$

from which $m = \frac{E_s-E_Q}{E_Q+k}$ and $H = \frac{km}{(E_s-E_Q)^{m-1}}$ (3.2-9)

Then we can either chose m > 1 and then

$$E_Q = \frac{E_s-mk}{m+1}$$

$$\text{and} \quad H = \frac{k(m+1)^{m+1}}{m^m(E_s+k)^{m+1}} \qquad (3.2\text{-}10)$$

or chose E_Q which must be less than $E_Q = \frac{E_s-k}{2}$, then m and H can be calculated from formula (2.2-6).

If E_Q is taken as zero $\quad m = \frac{E_s}{k}$, $H = E_s - \frac{E_s}{k}$

Then by taking $\frac{E_s}{k} > m > 1$ or $0 \leqslant E_Q < \frac{E_s - k}{2}$

we can obtain an equation of a simple parabola (not usually second degree) which touches the hyperbola $z = \frac{E}{E+k}$ at $E = E_Q$ and the line $z = 1$ at $E = E_s$.

Formula (3.2-1) would be

$$z = \frac{E}{E+k} \qquad 0 \leqslant E \leqslant E_Q$$

$$= 1 - H(E_s - E)^m \qquad Q \leqslant E \leqslant E_s$$

$$= 1 \qquad E > E_s$$

To determine which parabola (or which value of E_Q) other conditions have to be called upon.

Probably for most purposes the second degree parabola obtained by putting m = 2 will be satisfactory and in this case

$$E_Q = \frac{(E_s - 2k)}{3}$$

$$H = 27k/(4(E_s+k)^3$$

and

$$z = 1 - \frac{27k(E_s - E)^2}{4(E_s+k)^3}$$

Also, it will be recognised that cases (1) , (2) and (3) can be obtained by appropriate choices of m and E_Q and case (4) can be obtained by putting m = 1. Note also that if $E_Q = 0$ we use the parabola all the way from 0 to E_s in place of the original hyperbola.

5 - In the previous forms k is constant; this form is suggested.

$$z = \frac{E}{E+k_E} \tag{3.2-12}$$

where k_E is decreasing as E increases. k_E has the following properties

$k_E = k$ for $E \leq Q$

$\frac{d}{dE} k_E = 0$ at E_Q and E_s

$k_E = 0$ at E_s

This form is suggested also by Perryman (1938).

6 - Mark Kormes (1952) suggests another approach to the point of self-rating, where z would begin with a certain size of risk and would reach self-rating (z = 1) for another size, both lower and upper limits being selected by judgment.

$$z = \frac{E+f_k}{E+k} \tag{3.2-13}$$

where f is a function of E which varies from 0 when $E = E_Q$ to 1 when $E = E_s$. The conditions for f are

i) $\frac{df}{dE} = 0$ $E = E_Q$ and $E = E_s$

ii) $\frac{dz}{dE} > 0$ $\Longrightarrow$ $f = 0$ $E < E_Q$

iii) $\frac{d}{dE}\ \frac{z}{E} < 0 \quad \Longrightarrow \quad f = 1 \qquad E \geq E_s$

Condition i) leads to a Bernoullian differential equation

$$\frac{df}{dE} = Af^2 + Bf$$

which has a solution

$$f = \frac{c}{1+e^{a+bE}} \quad .$$

To determine c,a and b, select f_1, f_2 and f_3 at suitable equidistant values of f; then we have the following relation

$$c = \frac{2f_1f_2f_3 - f_2^2(1+f_2)}{f_1f_3 - f_2^2}$$

$$b = \frac{1}{n} \ln \frac{f_1(c-f_2)}{f_2(c-f_1)}$$

$$a = \ln \frac{(c-f_1)}{f_1}$$

where n is the number of units on the E axis between two consecutive values of E corresponding to f_1, f_2 and f_3. Choice of the three values of f will always be determined by practical requirements.

7 - Robert Hurtey (1954) used another method for determining the self rating point

$$z = \frac{E-c}{E-c+k} \qquad (3.2\text{-}14)$$

which has the same original form (equation 3.2-14)

$$z = \frac{E^*}{E^*+k} \qquad E-c = E^*$$

The Hurley procedure is that c is constant determined so that the credibility curve will start at the statistical norm for zero credibility. The point E_Q of 67% credibility in linear interpolation over (c,E_s) would coincide with the corresponding 67% value from the above equation.

Thus

$$0.67 = \frac{E_Q-c}{E_Q-c+k}$$

$$E_Q = c + 0.67(E_s-c)$$

so that $k = 0.33\ (E_s-c)$

z is modified for self-rating at E_s by multiplying by

$$\alpha = \frac{(E_s-c) + 0.33(E_s-c)}{(E_s-c)} = 1.33$$

thus $z = \dfrac{1.33(E-c)}{(E-c)+0.33(E_s-c)}$

3.3 Determination of the level of full credibility

It is known that the expected claim experience is a stochastic process. It depends on the variation in the size of claims and the number of claims during the experience period. Mowbray (1914) defined the dependable (credible) pure premium as

"one for which the probability is high that it does not differ from the absolute (true) pure premium by more than an arbitrary limit which may be selected in view of the other factors referred to".

From this definition, credible premiums would be expressed in probabilistic form within a certain fixed percentage of the true value; the definition in this form involves two parameters :

1 - the probability p of the fully credible premium.

2 - the percentage of deviation allowed from the true premium.

3.3-1 Level of full credibility for the number of claims

The empirical probability of any accident in any class is the claim frequency obtained by dividing the total number of claims by the number of exposures. If q equals the probability of an accident and p = 1-q, and the number of units of exposure is n, the probability of m accidents will have a binomial distribution

$$p_r(m) = \binom{n}{m} q^m (1-q)^{n-m} \qquad (3.3\text{-}1)$$

with mean nq and variance nq(1-q).

The probability that the number of claims in n exposure units will be within ± 100k of nq will be

$$\text{i.e.} \quad p = p_r\{(1-k)nq \leq nq \leq (1+k)nq\} \qquad (3.3\text{-}2)$$

where

$$p = \sum_{r=(1-k)nq}^{r=(1+k)nq} c_r^n \, q^r (1-q)^{n-r}$$

Since the binomial distribution tends to the normal * distribution

* when n is large, the normal distribution can be used to approximate to the binomial probabilities, but q should be moderate; if q is very small or close to 1 the binomial distribution is quite skewed, and it takes a larger n to get a reasonable degree of approximation than when p is moderate.

as $n \to \infty$

$$p = \frac{1}{\sqrt{2\pi npq}} \int_{-knq}^{+knq} e^{-\frac{x^2}{2npq}} dx \tag{3.3-3}$$

Let $t = \frac{x}{\sqrt{npq}}$

$$p = \frac{1}{\sqrt{2\pi}} \int_{0}^{\frac{knq}{\sqrt{npq}}} e^{-\frac{t}{2}} dt \tag{3.3-4}$$

If the criterion for full credibility is to be $p = 1-\alpha$, from the standard normal distribution table we can determine the value of $t = a$ which corresponds to probability $1-\frac{\alpha}{2}$.

$$a_{1-\frac{\alpha}{2}} = \frac{knq}{\sqrt{npq}} \tag{3.3-5}$$

From (3.3-5) we determine the number of units of exposure n_s which are required for full credibility.

$$n_s = \left(a_{1-\frac{\alpha}{2}}\right)^2 \frac{p}{qk^2} \tag{3.3-6}$$

In this formula for n_s it is assumed that :

1 - The probability of hazard is constant throughout the period of observation.

2 - The normal error at the critical region is sufficiently close to the binomial at the critical area (Mowbray 1914).

These conditions in practice are not totally satisfied. The probability q is usually very small so that the frequency distribution is skew and not symmetrical as is assumed. Also, if the period of experience is long it would be doubtful if the probability of hazard is constant. Also, from equation (3.3-6), it can be seen that credible number of units of exposure depend on

i - accident frequency

ii - the value of k% (the allowable departure from the expected value).

iii - the probability integral adopted

If probability integral and accident frequency are constant, the volume of exposure varies inversely as the square root of the allowable variation. With fixed values for the allowable departure and the probability integral, the required number of units of exposure varies inversely with the accident frequency (q) (Wheeler 1930). With a given accident frequency and a fixed value for the allowable departure, n_s varies directly with the probability integral and in a ratio greater than the square.

Also (3.3-6) can be written in the form

$$\left.\begin{aligned} qn_s &= \left(a_{1-\frac{\alpha}{2}}\right)^2 \frac{p}{k^2} \\ m_s &= \left(a_{1-\frac{\alpha}{2}}\right)^2 \frac{p}{k^2} \end{aligned}\right\} \qquad (3.3\text{-}7)$$

m = number of accidents.

where m_s is the number of claims which can be used instead of the number of exposure units as a requirement for full credibility.

In the above discussion it is assumed that the number of claims from i The units of exposure are m_i and m_i is equal to 0 or 1.If m_i takes the values 0,1,2, ... during the experience period (one year).

Let the period of experience be divided into very small intervals of length s such that no more than one accident would happen in any one interval. The probability of any accident in any interval will be $\frac{q}{s}$ (for any one unit of exposure).

From equation (3.3-1), the probability of r accidents at the interval s would be rewritten

$$\binom{n}{r}(1 - \frac{q}{s})^{n-r}\left(\frac{q}{s}\right)^r \qquad (3.3\text{-}8)$$

Let $n \to \infty$ as $\frac{q}{s} \to 0$ and $\lambda = \left(\frac{q}{s}\right)n \Rightarrow \left(\frac{q}{s}\right) = \frac{\lambda}{n}$.

The expression (3.3-8) can be written

$$\frac{n(n-1)....(n-r+1)}{r!}\left(\frac{\lambda}{n}\right)^r\left(1 - \frac{\lambda}{n}\right)^n\left(1 - \frac{\lambda}{n}\right)^{-r}$$

$$= \frac{1(1 - \frac{1}{n})...(1 - \frac{r+1}{n})}{r!}\lambda^r\left(1 - \frac{\lambda}{n}\right)^r\left(1 - \frac{\lambda}{n}\right)^n$$

as $n \to \infty$ this expression tends to

$$\lambda^r \frac{e^{-\lambda}}{r!}$$

which is a poisson distribution, and in this case the expectation of the number of claims will equal the variance of the number of claims and once again the n_s requirement is

$$\left.\begin{aligned} n_s &= \left(a_{1-\frac{\alpha}{2}}\right)^2 \frac{1}{\lambda k^2} \\ \lambda n_s = m_s &= \left(a_{1-\frac{\alpha}{2}}\right)^2 \frac{1}{k^2} \end{aligned}\right\} \qquad (3.3\text{-}7)$$

Dropkin (1959) and Leroy J. Simon (1960) studies indicate that the distribution of claim frequency where there is a possibility of multiple claims per unit of exposure, is not a poisson distribution because their studies indicate that the mean and variance of the frequency distribution are not equal as in the poisson assumption. They suggest the negative binomial or Pearson type III as the claim frequency .

3.3-2 <u>The criteria for full credibility in terms of mean and standard deviation</u>

It would be easier if the criteria for full credibility could be represented in terms of the mean and standard deviation of the frequency distribution of claims.

Let μ and σ be the mean and the standard deviation of a normal random variate x. Then the probability that an observation differs from the mean by less than $\delta = k\mu$ is given by

$$p = \frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{-\delta}^{\delta} e^{-\frac{x^2}{2\sigma^2}} \, dx .$$

Let $t = \frac{x}{\sigma}$

$$p = \frac{2}{\sqrt{2\pi}} \int_{0}^{\frac{ku}{\sigma}} e^{-\frac{t^2}{2}} \, dt$$

From the Table of the standard normal distribution we can determine for a given value of p the value of t as a function of p.

Thus if $\frac{k\mu}{\sigma} \geq f(p) \quad \therefore \quad \frac{\sigma}{\mu} \leq \frac{k}{f(p)}$ (3.3-10)

In rate making or experience rating, the number of observations is large, say n, so if the frequency distribution of the individual observations is not normal, the frequency distribution of the mean usually tends to be normal. If the mean for one observation is μ and standard deviation is σ, the mean of n observations will be μ and the standard deviation $\frac{\sigma}{\sqrt{n}}$. The original problem is the value of n for credible premium i.e. how large n should be so that the probability p that the variation of the mean of the n observations from the true value is less than 100k%

$$\text{i.e.} \quad \frac{\sigma}{\mu\sqrt{n}} \leq \frac{k}{f(p)}$$

$$\text{i.e.} \quad n_s \geq \frac{\sigma^2}{\mu^2} \frac{\{f(p)\}^2}{k^2} \qquad (3.3\text{-}11)$$

Expression (3.3-11) determines the minimum value for n.

3.3-3 The level of full credibility for the pure premium

In the above discussion it has been assumed that with a credible accident frequency the pure premium is credible, but the pure premium is the product of the accident frequency and the average claim. Let

$$A = \sum_{i=1}^{n} X_i$$

A = Total amount of claims

X_i = amount of i^{th} claim for i^{th} unit of exposure

n = number of units of exposure,

where $X_1, X_2, \ldots, X_n$ are independent random variables and they are independent of n.

$$E\,X_i = \mu$$

$$\text{var}\,X_i = \sigma^2$$

Accident frequency $= \lambda$

Accident frequency standard deviation $= \sqrt{\frac{\lambda}{n}}$

The pure premium $\pi = \mu\lambda$

As before we assume that the μ has a normal distribution with average $E(\mu) = \mu \quad \text{var}(\mu) = \frac{\sigma^2}{n\lambda}$ (number of claims $n\lambda$)

$$\therefore \quad E(\pi) = \mu\lambda$$

$$\text{var}\,\pi = \mu^2 \frac{\lambda}{n} + \lambda^2 \frac{\sigma^2}{n\lambda}$$

Thus

$$\frac{\text{variance of } \pi}{(\text{mean of } \pi)^2} = \frac{1}{n\lambda} + \frac{\sigma^2}{n\lambda\mu^2} = \frac{1}{n\lambda}\left\{1 + \frac{\sigma^2}{\mu^2}\right\}$$

using formula (3.3-11)

$$\lambda n_s \geqslant \frac{2\{f(p)\}^2}{k^2}\left(1 + \frac{\sigma^2}{\mu^2}\right) \qquad (3.3\text{-}12)$$

From (3.3-10) it can be seen that the number of claims required for full credibility for the pure premium is increased by $(1 + \frac{\sigma^2}{\mu^2})$ of the number of claims required for credibility of accident frequency.

If the variation of the size of claim is ignored, formula (3.3-12) would be the same as formula (3.3-7).

3.3-4 Credibility of pure premium (independent of distribution assumption)

In the above discussion, it was assumed that the expected number of claims was very large so that the average claim cost is approaching normal distribution and the claim frequency was approximately normally distributed. Also, it was assumed that the pure premium was approximately normally distributed. The approximate normality of the pure premium is the implication of the Central Limit Theorem. It should be noted that the density of the product of two normal random variables, both of which are normal, is not normal. Mayerson (1963) has derived a criterion for full credibility for a pure premium based only on the moments of the distribution of the number of claims and on the moments of the claim amount distribution, without making any assumption about the specific form of either distribution.

For certain p and k, a classification is fully credible if

$$P_r\ \{(1-k)E(A) \leq A \leq (1+k)E(A)\} \geq p$$

which is equivalent to

$$P_r\ \left\{ \frac{-kE(A)}{\sigma A} \leq \frac{A-E(A)}{\sigma A} \leq \frac{kE(A)}{\sigma A} \right\} \geq p$$

The problem is to find the percentile of the distribution $\frac{A-EA}{\sigma A}$. The Cornish-Fisher expansion expresses a percentile a_p of the distribution of $\frac{A-E(A)}{\sigma A}$ as a percentile of the standardised normal distribution α_p plus certain correction terms, which adjust for the departure from normality of the distribution of A. The expansion is

$$a_p = \alpha_p + \frac{\gamma_1}{6}(\alpha_p^2-1) + \left[\frac{\gamma_2}{24}(\alpha_p^3-3\alpha_p) - \frac{\gamma_1^2}{36}(2\alpha_p^3-5\alpha_p)\right]$$

where

$$\gamma_1 = \frac{E[A-E(A)]^3}{\sigma_A^3}$$

$$\gamma_2 = \frac{E[A-E(A)]^4}{\sigma_A^4}$$

where $E(n) = \lambda \quad E[(n-\lambda)^r] = \lambda_r \quad r = 2,3, \ldots.$

$E(X) = \mu \quad E[(X-\mu)^r] = \mu_r \quad r = 2,3, \ldots.$

$E(A) = \lambda\mu$

$$E[(A-E(A))^2] = \mu_2\lambda + \mu^2\lambda_2$$

$$E[(A-E(A))^3] = \mu_3\lambda + 3\mu_2\mu\lambda_2 + \mu^3\lambda_3$$

$$E[(A-E(A))^4] = \mu_4\lambda + 3\mu_2^2(\lambda_2-\lambda+\lambda^2) + 4\mu\mu_3\lambda_2 + 6\mu^2\mu_2(\lambda_3+\lambda\lambda_2) + \mu^4\lambda_4$$

The distribution of A would be skew, with a long tail on the right-hand side. Therefore, for values of p of interest

$$a_{(1-p)/2} > a_{(1+p)/2}$$

Let

$$\frac{kE(A)}{\sigma A} = a_{(1+p)/2}$$

For simplicity, use only two terms of the expansion

$$\frac{kE(A)}{\sigma_A} = \alpha_p + \frac{E[A-E(A)]^3}{6\sigma_A^3} (\alpha_p^2-1)$$

$$\therefore \quad \frac{k\mu\lambda}{\sqrt{\mu_2\lambda+\mu^2\lambda_2}} = \alpha_p + \frac{\mu_3\lambda+3\mu_2\mu\lambda_2+\mu^3\lambda_3}{6(\mu_2\lambda+\mu^2\lambda_2)^{3/2}}$$

$$k\lambda = \alpha_p\sqrt{\lambda} \sqrt{\frac{\lambda_2}{\lambda} + \frac{\mu_2}{\mu^2}} + \frac{\alpha_p-1}{\sigma} \frac{\frac{\lambda_3}{\lambda} + \frac{3\lambda_2}{\lambda}\frac{\mu_2}{\mu^2} + \frac{\mu_3}{\mu^3}}{\frac{\lambda_2}{\lambda} + \frac{\mu_2}{\mu^2}} \qquad (3.3\text{-}14)$$

if terms involving the third moment are ignored, we get the following equation :

$$\lambda = \frac{\alpha_p}{k^2}\left(\frac{\lambda_2}{\lambda} + \frac{\mu_2}{\mu^2}\right) \qquad (3.3\text{-}15)$$

Formula (3.3-10) is a special case of (3.3-13). If we put in (3.3-11) $\lambda_2 = \lambda$ (i.e. the variance and the mean of the frequency distribution of the number of claims are equal), then we get

$$\lambda = \frac{\alpha_p^2}{k^2} (1 + \frac{\sigma^2}{\mu^2})$$

Also, if we take $\lambda = \lambda_2$ and $\mu_2 = 0$, i.e. we do not take account of the variation in the size of claim, we arrive at formula (3.3-9)

$$\lambda = \alpha_p^2 \times \frac{1}{k^2}$$

This method is more appropriate for the computation of the pure premium, since it takes account of both variation in the frequency of the number of claims and the variation in the size of the claim and also there is no need to make an assumption about the underlying probability distribution.

3.4 Credibility multisplit plan

In the work developed earlier in this Chapter, we have been dealing with the case of an experience rating plan with no splits, that is, where all losses are given equal weight. From time to time the insurance company faces a very large payment in one claim, which needs more attention. In an ordinary credibility plan a single loss of, say, 10000 gets as much weight as ten losses of 1000 each; it is both theoretically and practically desirable to give the very large losses much more weight.

In the ordinary split plan, (both actual and expected) losses are divided into "normal" and "excess" i.e. the experience on losses limited to certain amount per claim form the normal part, and the experience on losses in excess of this certain amount forms the "excess" part. The expected losses are divided in the same way (from the available statistics) and the final rate is the sum of the adjusted rates for each of the two parts.

A multisplit plan consists of different ways of dividing the total losses into "normal" and "excess" ; it reduces all losses to normal (for distinction we will call it primary loss) and excess using the following formula :

Let S_0 = an amount regarded as the normal claim amount.

a = a real number less than one used as a discounting factor for effecting a decreasing weightage to the different strata of an individual claim amount.

S_i = individual claim amount (expected or actual).

$$S_{in} = \text{"primary" component of } S_i$$

$$= S_o \sum_{i=o}^{r-1} a^i + R\, a^r$$

$$= S_o \frac{(1-a^r)}{1-a} + R\, a^r$$

where r = the integer part of the real number $\frac{S_i}{S_o}$

$$R = S_i - r\, S_o$$

$$S_{ie} = \text{the excess component of } S_i \; .$$

Note that the maximum discounted value would be $\frac{S_o}{1-a}$.

In this case the modification factor (3.2-2) may be split into two parts :

$$= \frac{E_n}{E} \; \frac{Z_n A_n + (1-Z_n)E_n}{E_n} + \frac{E_e}{E} \; \frac{Z_e A_e + (1-Z_e)E_e}{E_e}$$

$$= \frac{Z_n A_n + (1-Z_n)E_n + Z_e A_e + (1-Z_e)E_e}{E_e}$$

where E_n, A_n, Z_n denote normal expected loss, actual loss and credibility respectively, and E_e, A_e, Z_e denote the actual excess loss, expected excess loss and credibility for the excess loss noting that $E_n + E_e = E$ and $A_n + A_e = A$ where

$$Z_n = \frac{E_n}{E_n + K_n}$$

and $$Z_e = \frac{E_e}{E_e + K_e} \; .$$

By making K_e much larger than K_n we gave much less credibility to the excess losses . Z_n and Z_e can be modified to attain the self rating point in accordance with the principles set out in paragraph (3.2-2).

CHAPTER FOUR

A BAYESIAN APPROACH TO CREDIBILITY AND COLLECTIVE RISK THEORY

4.1 Individual risk theory and collective risk theory

The risk theory approach to insurance has been developed by European actuaries, especially Scandinavians.

The two most general formal models of insurance, collective risk theory and individual risk theory, have both been developed in this field. Risk theory provides a logical context within which current insurance problems may be ordered and analysed. Beard (1975) summarises general insurance problems in this statement -

"... There are two aspects of general insurance in which knowledge of the structure of the element of the risk variation is needed ; first in the rate fixing process and second in the question of financial stability , and this can be generalised respectively into the knowledge of the mean and the variance of the risk involved ..." .

In this study the first aspect of the risk process (rate fixing) will be our concern.

Individual risk theory, sometimes called classical insurance theory uses the individual policy as a basic unit of analysis. A claim probability can be associated with each unit of risk (policy). The claim amount will be limited to the maximum company liability as specified in the policy. Let X_k be a random variable measure for the claim arising from the k^{th} policy. Then $\sum_{k=1}^{N} X_k$ will be the total amount of the claim occurring from all the policies in force. However, according to the central limit theorem, the sum of the random variables will be normally distributed if the number of the policies is large.

Collective risk theory was first intorduced by Filip Lundberg (1909) and developed by many others including Esscher (1932), Ammeter (1948), Cramer (1955). In this approach the individual policy structure is disregarded and instead the portfolio is considered as a whole - the claim process can be described in terms of a sequence of random numbers which have time parameters. For simplicity , formal treatment can be as follows :

(a) operational time : the length of any time interval will be measured by the expected number of claims qt arising during this period, given the number and types of claims in force with the company.

(b) frequency distribution for claims : given the above definition of the operational time, the probability of exactly "n" claims in time interval t is P(n/qt).

(c) distribution of total amount of claims occur within the time t is S(t) where S(t)

$$S(t) = X_1 + X_2 + X_3 + \dots + X_n \tag{4.1-1}$$

S(t) is a random variable with distribution function

$$F(S,qt) = P\ (S/qt,\ S < S) \qquad 4.1\text{-}2)$$

F(S,qt) is the product of the probability that a certain number of claims occur (frequency) and the probability that these claims will be of a certain size (severity).

It is assumed that

1 - X_i in equation (4.1-1) are independent and each has a distribution function P(x) [P(x) is the distribution of a single claim].

2 - The number of claims n is independent of the size of the claim x.

Then the distribution function in the expression (4.1-2) takes the form

$$F(S,qt) = \sum_n P(n/q)\, P^{n*}(x) \qquad (4.1\text{-}3)$$

where $P^{n*}(x)$ is the n^{th} convolution of P(x) with itself which is the conditional probability that if the number of claims is exactly n, the sum S of these n claims is less than S . As has been said before, the distribution of each x is independent of n (number of claims).

The moment of the distribution P(x) will be denoted by

$$C_i = \int_0^\infty x^i \, dP(x) \qquad (4.1\text{-}4)$$

and the moment generating function will assume to exist and will be denoted by

$$M(\theta) = \int_0^\infty e^{\theta x} \, dP(x) \qquad (4.1\text{-}5)$$

The main problem in this approach is to determine the compound function F(S,qt). The mean of F(S,qt) is the pure premium which is our concern. In the next two sections we will attempt to find the mean and variance of $F(\overline{S},qt)$ when t is very large (limiting distribution of F(S,qt)).

4.1-1 The distribution of F(S,qt) when the number of claims have a Poisson distribution :

Let t be the number of units of exposure in force for the entire accounting period (noting that a policy in force for the entire accounting period adds one to t and policies in force for part of the period will add a fraction to t). Let n be the number of losses having a Poisson distribution, and the probability of loss is q, i.e. expected number of losses at the operational time t is qt. Then

$$P(n/t) = P(n/qt) = \frac{(qt)^n e^{-qt}}{n!}$$

from 4.1-3

$$F(S,t/qt) = \sum \frac{(qt)^n e^{-qt}}{n!} p^{n*}(x) \qquad (4.1\text{-}6)$$

The expression (4.1-6) is called a generalised Poisson function or compound Poisson function [Beard et al (1978), Miller and Hakman (1974)] since the moment generating function of two independent random variables is the product of the corresponding moment generating function;it follows that the moment generating function of the n^{th} convolution $P^{*n}(x)$ is $[M(\theta)]^n$

The moment generating function for the compound Poisson process in (4.1-6) is

$$\phi(\theta) = \sum_n \frac{e^{qt}\,[qt]^x}{n!} \int e^{\theta x}\, d\, P^{*n}(x)$$

$$= e^{-qt} \sum_n \frac{[qt]^n\,[M\theta]^n}{n!}$$

$$= e^{-qt}\, e^{qtM(\theta)}$$

$$= e^{qt[M(\theta)-1]}$$

$\therefore \quad E(S) = qt\, C_1 \qquad E(S^2) = [qt\, C_1]^2 + qt\, C_2$

Since $Var(s) = E(S^2) - [E(S)]^2\ qt\, C_2$

Let $\quad y = \dfrac{S-qt\, C_1}{\sqrt{qt\, C_2}}$

when t is very large, y has a normal* distribution $N(0,1)$, hence S has an approximate normal distribution $N(qt\, C_1,\ qt\, C_2)$, and the average claim per policy $\frac{S}{t}$ has a normal distribution $N(q\, C_1,\ q\, C_2/t)$

4.1-2 Distribution of F(S,qt) when the number of claims have a negative binomial distribution :

In many practical problems the mean and the variance of the number of claims distribution are not equal which is the case for the Poisson distribution Longley-Cook (1962), Dropkin (1959), Simon (1950 & 1961) and Hewitt (1960). Show that the negative binomial is more appropriate for the claim frequency than the Poisson distribution.

* because the moment generating function of y tends to $e^{-\theta/2}$ when $t \to \infty$, which is the moment generating function for the standard normal distribution.

In the case of the negative binomial

$$P(n/t,\alpha) = \frac{n+\alpha-1!}{n!\alpha-1!} \left(\frac{\beta}{t+\beta}\right)^{\alpha} \left(\frac{t}{t+\beta}\right)$$

where $\frac{\beta}{t+\beta} = q \qquad (1-q) = \frac{t}{t+\beta}$.

From (4.1-3)

$$\phi(\theta) = \sum_n \frac{n+\alpha-1!}{n!\alpha-1!} \left(\frac{\beta}{t+\alpha}\right)^{\alpha} \left(\frac{t}{t+\alpha}\right)^{n} \int e^{\theta x} \, dP^{*n}(x)$$

$$= \sum_n \frac{n+\alpha-1!}{n!\alpha-1!} \left(\frac{\beta}{t+\alpha}\right)^{\alpha} \left[\frac{tM(\theta)}{t+\beta}\right]$$

$$= \left(\frac{\beta}{t+\beta}\right)^{\alpha} \left[1 - \frac{tM(\theta)}{t+\beta}\right]^{-\alpha}$$

$$= \left[1 - \frac{t(1-M(\theta))}{B}\right]^{-\alpha} \qquad (4.1\text{-}8)$$

$$E(S) = \frac{\alpha t}{\beta} C_1 \qquad E(S^2) = \frac{\alpha t}{\beta} C_2 + \frac{(\alpha+1)\alpha t^2 C_1^2}{\beta^2}$$

$$\mathrm{Var}(S) = \frac{\alpha t}{\beta} C_2 + \frac{\alpha t^2 C_1^2}{\beta^2} = \frac{\alpha t(\beta C_2 + t C_1^2)}{\beta^2}$$

Let $y = \frac{S}{E(S)} = \frac{S\beta}{C_1 \alpha t}$

y is dependent on t

and limit $t \to \infty$ $\displaystyle\int_0^{\infty} e^{\theta y} dy = \frac{1}{(1-\theta/\alpha)^{\alpha}}$

i.e. y has a limited distribution, which is a gamma distribution with parameters α , β where $\alpha = \beta$. S will have an approximate gamma distribution with parameters α and $\frac{\beta}{C_1 t}$.

Hence the average claim per policy $\frac{S}{t}$ will have a gamma distribution with parameters α and $\frac{\beta}{C_1}$.

The objective of credibility theory from the Bayesian viewpoint is to find the posterior distribution of the pure premium i.e. the distribution for average claim per policy.

4.2 Bayesian approach to credibility

The formula (3.1-1) has been accepted because it is logical and reasonable to give the indication of the large volume of data more consideration or weight than the indication of a small volume of data. The problem of determining the normalising constant k has not got a proper statistical solution. The Bayesian approach was first introduced to credibility theory by Bailey (1945 & 1950) and clarified by Mayerson (1964). Bailey shows that equation 3.1-1 is an exact Bayesian forecast under certain assumptions, and the normalizing constant k can be determined statistically in terms of the mean and variance of the posterior distribution.

The restriction we have in the Bayesian approach is that we have to know the distribution of the process which generates the data, and the prior distribution of the population parameters at the outset. The first is called the likelihood distribution.

The question which should be asked is for what prior distribution can we say that the current data are fully credible i.e. for what prior distribution can we say that the posterior distribution is independent of the prior distribution because of the large size of the data ? The choice of the prior distribution should be that one which underlies the previous rate for the class of risk in question. Once the prior distribution has been chosen, the prior distribution and the distribution of the data can be combined together using the Bayes theorem to obtain the posterior distribution.

Since in (4.1-3) it is assumed that numbers of claims and the individual claim amount distribution are independent, then we can proceed to the Bayesian estimation of the parameters of each distribution.

4.2-1 Application of Bayesian theorem

Let θ be the parameter of risk class, have prior distribution $F(\theta)$ and let $\tilde{y}$ be the sample statistic which summarizes all the sample information involving θ .

Then the posterior distribution is the conditional density of θ given the observed value y of the statistic $\tilde{y}$

$$f(\theta/\tilde{y} = y) = \frac{f(\theta,y)}{f(y)} \qquad (4.2\text{-}1)$$

But $f(\theta,y) = f(\theta)f(y/\theta)$

and the marginal distribution

$$f(y) = \int_{-\infty}^{\infty} f(\theta,y)d\theta = \int_{-\infty}^{\infty} f(\theta)f(y/\theta)d\theta$$

hence the posterior density can thus be written

$$f(\theta/y) = \frac{f(\theta)f(y/\theta)}{\int_{-\infty}^{\infty} f(\theta)f(y/\theta)d\theta} \tag{4.2-3}$$

where $f(y/\theta)$ represents the likelihood function.

Let m_θ be the mean of θ, σ_θ^2 the variance of θ.

Let m_y be the mean of y, σ_y^2 the variance of y.

Bailey (1950) suggests $E(\theta/y)$ as estimates of θ. It should be noticed that $E(\theta/y)$ is a function of y only; it may be called the regression function of θ on y

$$E(\theta/y) = \int_{-\infty}^{\infty} \theta f(\theta/y)d\theta = a+by \tag{4.2-4}$$

(Note that $E(\theta/y)$ is not always a linear function).

$$= \int_{-\infty}^{\infty} \frac{\theta f(\theta,y)d\theta}{f(y)} = a+by \tag{4.2-4}$$

$$= \int \theta f(\theta,y)d\theta = (\alpha + by)f(y) \tag{4.2-5}$$

By integrating both sides in respect to y

$$\mu_\theta = a + by \tag{4.2-6}$$

Multiply (4.2-5) by y then integrate on y

$$E(\theta.y) = a\mu_y + bE(y^2)$$

$$\rho_{\theta y}\sigma_\theta\sigma_y + \mu_\theta\mu_y = a\mu_y + b(\sigma_y^2 + \mu_y^2) \tag{4.2-7}$$

From (4.2-6) and (4.2-7)

$$a = \mu_\theta - \rho_{\theta y}\frac{\sigma_\theta}{\sigma_y}\mu_y$$

$$b = \rho_{\theta y} \frac{\sigma_\theta}{\sigma_y}$$

Thus $$E(\theta/y) = (\mu_\theta - \rho_{\theta y} \frac{\sigma_\theta}{\sigma_y} \mu_y) + \rho_{\theta y} \frac{\sigma_\theta}{\sigma_y} y$$

Let $$\left.\begin{aligned} A &= \rho_{\theta y} \frac{\sigma_y}{\sigma_\theta} \\ B &= \mu_y - A\mu_\theta \end{aligned}\right\} \quad \text{coefficients of the regression line } E(y/\theta)$$

$$E(\theta/y) = \mu_\theta - \frac{\rho_{\theta y}^2}{A} (B + A\mu_\theta) + \frac{\rho_{\theta y}^2}{A} y$$

$$= (1 - \rho_{\theta y}^2)\mu_\theta + \rho_{\theta y}^2 \frac{y - B}{A} \qquad (4.2\text{-}8)$$

where A and B are coefficients of the regression line $E(y/\theta)$.

From (4.2-8) it can be seen that the credibility z which is given to the observed data in equation (3.1-1) is the square of the correlation coefficient between θ and y when the data are combined with prior knowledge. It should be noted also that this has the desired property $z(\equiv\rho^2) \leq 1$.

In Chapter 3 we discussed two cases. 1 - when the distribution of the number of claims is binomial. 2 - when the distribution of the number of claims is Poisson.

We will discuss these cases again from the point of view of the Bayesian approach.

4.2-2 The distribution of the number of claims is binomial.

In this case the likelihood distribution (distribution of the observations) is binomial, because of potential difficulties in combining the likelihood function and the prior density function. Bayesian statisticians have developed the concept of conjugate prior distributions which essentially are families of distribution that ease the computational burden when used as prior distributions. This concept adds another restriction to this approach. The form of the likelihood function would restrict the prior distribution to a certain family of distributions.

If the distribution of the number claims F(n/q) is binomial, then it would be helpful if the prior distribution F(q) is assumed to be a Beta distribution.

$$F(n/q,t) = \frac{t!}{n!t-n!} q^n(1-q)^{t-n} \qquad (4.2\text{-}9)$$

$$F(q) = k\, q^r(1-q)^{m-r} \qquad (4.2\text{-}10)$$

$k = \dfrac{1}{\beta(r+1, m-r+1)}$, the posterior density function for q, using equation (4.2-3)

$$f(q/n,t) = \frac{kq^{n+r}(1-q)^{t+m-n-r}}{k\int_0^1 q^{n+r}(1-q)^{t+m-n-r}}$$

$$= \frac{q^{n+r}(1-q)^{t+m-n-r}}{B(m+r+1,\ t+m-n-r+1)} \qquad (4.2\text{-}11)$$

Using equation (4.2-4)

$$E(q/n,t) = \int_0^1 q\ f(q/n,t)dq$$

$$= \frac{B(n+r+2,\ t+m-n-r+1)}{B(n+r+1,\ t+m-n-r+1)}$$

$$= \frac{n+r+1}{t+m+2} \qquad (4.2\text{-}11)$$

which is linear in n, so we may write

$$E(q/n,t) = \frac{t}{t+m+2}\,\frac{n}{t} + \frac{m+2}{t+m+2}\,\frac{r+1}{m+2}$$

$$= z\,\frac{n}{t} + (1-z)\,\frac{r+1}{m+2} \qquad (4.2\text{-}12)$$

where $z = \frac{t}{t+m+2}$ (4.2-13)

Since (q) has

mean $\mu_q = \frac{r+1}{m+2}$

and variance $\sigma_q^2 = \frac{(r+1)(m-r+1)}{(m+2)^2(m+3)} = \frac{\mu_q(1-\mu_q)}{(m+3)}$

Note that, for fixed values of $m, z \to 1$ as $t \to \infty$, so we may write z in terms of the mean and the variance of f(q)

$$z = \frac{t\sigma_q^2}{(t+m+2)\sigma_q^2}$$

$$= \frac{t\sigma_q^2}{(t-1)\sigma_q^2 + (m+2)\sigma_q^2}$$

$$= \frac{t\sigma_q^2}{t\sigma_q^2 + \mu_q(1-\mu_q)-\sigma_q^2}$$

$$= \frac{t}{(t-1)+k} \qquad (4.2\text{-}14)$$

where $k = \frac{\mu_q(1-\mu_q)}{\sigma_q^2}$

Note that equation (4.2-14) corresponds to equation (3.2-1) and we do not need to make any assumption about the normalising constant k.

4.2-3 The distribution of number of claims is Poisson

If the distribution of the number of claims is $f(n/q,t)$ and is a Poisson distribution, it is convenient that the prior distribution $f(q)$ be a gamma distribution

$$f(n/q,t) = \frac{(tq)^n e^{-tq}}{n!} \qquad (4.2\text{-}15)$$

$$f(q) = k\, q^{r-1} e^{-\alpha q} \qquad (4.2\text{-}16)$$

$k = \dfrac{\alpha^r}{\Gamma n}$, the posterior density function for q

$$f(q/n,t) = \frac{q^{r+n-1} e^{-(t+\alpha)q}}{\int_0^\infty q^{r+n-1} e^{(t+\alpha)q}\, dq}$$

$$= \frac{(t+\alpha)^{r+n-1} q^{r+n-1} e^{-(t+\alpha)q}}{\int_0^\infty [(t+\alpha)q]^{r+n-1} e^{-(t+\alpha)q} dq}$$

$$= \frac{(t+\alpha)}{\Gamma r+n} [(t+\alpha)q^{r+n-1} e^{-(t+\alpha)q} \qquad (4.2\text{-}17)$$

hence $$E(q/n,t) = \int_0^\infty q f(q/n,t) dq$$

$$= \frac{1}{\Gamma r+n} \int_0^\infty [(t+\alpha)q]^{r+n} e^{-(t+\alpha)q} dq$$

$$= \frac{1}{t+\alpha} \frac{\Gamma r+n+1}{\Gamma n+r} = \frac{n+r}{t+\alpha}$$

which is linear in n, so we may write

$$E(q/n,t) = \frac{t}{t+\alpha}\,\frac{n}{t} + \frac{\alpha}{t+\alpha}\,\frac{r}{\alpha}$$

$$= z\,\frac{n}{t} + (1-z)\,\frac{r}{\alpha} \qquad (4.2\text{-}18)$$

Hence $z = \dfrac{t}{t+\alpha}$ (4.2-19)

Since $f(q)$ has mean $\mu_q = \dfrac{r}{\alpha}$ and variance $\sigma_q^2 = \dfrac{r}{\alpha^2}$

hence the normalizing factor $\alpha = \dfrac{\mu_q}{\sigma_q}$ (4.2-20)

4.2-4 The distribution of the amount of claim is exponential

Exponential distribution is one of the classical examples of collective risk theory [Hickman (1975), Lundberg (1909)] .

Let $f(x/\delta)$ be the distribution of claim amount. The prior distribution is the same as for Poisson distribution i.e. a gamma distribution.

The density function for individual claims

$$f(x/\delta) = \left(\frac{1}{\delta}\right) e^{-\frac{x}{\delta}} \qquad x > 0 \quad \text{and} \quad \delta > 0 \qquad (4.2\text{-}21)$$

If exactly n_o claims occur such that

$$S_o = x_1 + x_2 + \ldots + x_{n_o}$$

the likelihood function is :

$$f(S/\delta_o, n_o) = \left(\frac{1}{\delta}\right)^{n_o} e^{-S_o/\delta}$$

then the conjugate prior distribution will be

$$f(\delta) = k\left(\frac{1}{\delta}\right)^{n_1} e^{-S_1/\delta}$$

and the posterior distribution for δ is

$$f(\delta/n' = n_0+n_1,\ S' = S_0+S_1) = \frac{\left(\frac{1}{\delta}\right)^{n'} e^{-S'/\delta}}{\int_0^\infty \left(\frac{1}{\delta}\right)^{n'-2} e^{-S'/\delta}\frac{d\delta}{\delta^2}} \qquad (4.2\text{-}22)$$

$$= \frac{S'^{(n'-1)}}{\sqrt{n'-1}} \left(\frac{1}{\delta}\right)^{n'} e^{-S'/\delta} \qquad (4.2\text{-}23)$$

Using equation (4.2-4)

$$E(\delta/n',S') = \frac{S'^{(n-1)}}{\sqrt{n'-1}} \int_0^\alpha \left(\frac{1}{\delta}\right)^{n'-1} e^{-S'/\delta}\ d\delta$$

$$= \frac{S'}{\sqrt{n-1}} \int_0^\infty S'^{n'-2}\left(\frac{1}{\delta}\right)^{n-3} e^{-S'/\delta}\ \frac{d\delta}{\delta^2}$$

$$= S'\ \frac{\sqrt{n'-2}}{\sqrt{n'-1}}$$

$$\frac{S'}{n'-2} \qquad (4.2\text{-}24)$$

which is linear in S' (total amount of claims).

Since $S' = S_0 + S_1$, and $n' = n_0 + n_1$, then we may write

$$F(\delta/n',S') = \frac{S_0}{n_0}\left(\frac{n_0}{n'-2}\right) + \frac{S_1}{n_1-2}\ \frac{n_1-2}{(n'-2)}$$

$$= \frac{S_0}{n_0} z + \frac{S_1}{n_1-2}\ (1-z) \qquad (4.2\text{-}25)$$

where $z = \frac{n_1}{n_1+n_2-2}$

The exponential case for claim size is simple because the distribution has only one parameter. The problem is more difficult if the claim size distribution has more than one parameter. Miller and Hackiman (1974) demonstrate an example for the gamma distribution as the claim amount distribution.

$$f(x/\gamma,\delta) = \left(\frac{\gamma}{\delta}\right)^{\gamma} \frac{x^{\gamma-1}}{\Gamma\gamma} e^{-\frac{\gamma}{\delta}x} \qquad x > 0 \text{ and } \gamma,\delta > 0 \quad (4.2\text{-}26)$$

which has a mean δ .

A conjugate prior for the parameters γ , δ has a density function

$$f(\gamma_1\delta/n_1S_1) = k(\Gamma\gamma)^{-n_1} \left(\frac{\gamma}{\delta}\right)^{n_1\gamma+1} p'^{\gamma-1} e^{-\left(\frac{\gamma}{\delta}\right)S_1}$$

where $p = \prod_{1=1}^{n} X_i$

as has been seen above the posterior distribution would have the same form as the prior distribution. The posterior distribution for (γ,δ) is

$$f(\gamma,\delta/n',S') = k'(\Gamma\delta)^{-n'} \left(\frac{\gamma}{\delta}\right)^{n'\gamma+1} p'^{\gamma-1} e^{-\left(\frac{\gamma}{\delta}\right)S'} \qquad (4.2\text{-}27)$$

The problem is to obtain the marginal distribution for δ only ; that makes it necessary to examine the likelihood contours of equation (4.2-26).

Miller and Hackiman (1975) show that the density function for

δ should be of the form

$$p(\delta/n',S') = c\left(\frac{1}{\delta}\right)^{n'\gamma+1} e^{-\frac{\gamma}{\delta}S'}$$

If the posterior distributions for the number of claims and the amount of claims have been found, the posterior distribution of the pure premium can be found easily by compounding both posterior distribution of the number of claims and the amount of claims.

4.2-5 The distribution of the pure premium

The posterior distribution of the pure premium i.e. the posterior distribution for $E[n]E[S/n=1]/t$ i.e. the expected claim payment on a single unit of risk during the policy period.

For the case where the number of claims has a Poisson distribution and the size of claims is exponential, the posterior distribution for q and δ [probability of claim, expected size of claim] from equation (4.2-17) and (4.2-23)

$$f(q,\delta/n,n',r',S',t) = \frac{(t+\alpha)}{\Gamma(r+n)} [(t+\alpha)q]^{r+n-1} e^{-(t+\alpha)q}$$

$$\frac{S'^{(n'-1)}}{\Gamma(n'-1)} \left(\frac{1}{\delta}\right)^{n'} e^{-S'/\delta} \qquad (4.2\text{-}27)$$

Let $\mu = q\delta$ and $v = \delta$; we want the distribution of μ .

$$\phi(q,\delta) = \psi(u,v)|J|$$

where $|J| = \frac{1}{\delta}$

$$f(u/n,n',t,\alpha,S') = A\, u^{n+r-1} \int \left(\frac{1}{v}\right)^{r+n+n'-1} e^{-\frac{1}{v}[S'+(t+\alpha)\mu]}\, dv$$

$$= \frac{A}{\Gamma r+n+n'} \quad \frac{\mu^{n+r-1}}{[S'+(t+\alpha)\mu]^{n'+n+r-1}} \qquad (4.2\text{-}28)$$

which is a member of the Pearson type VI family of distributions. The importance of that distribution is that it summarises all the information available about the risk parameters that determine the pure premium. Developing the posterior distribution for the pure premium for a two parameter distribution needs more advanced mathematical techniques, numerical integration and graphical display to guide the selection of the prior distribution for the parameters.

Jewell (1974), using measure theory and advanced mathematical technique, has developed the posterior distribution for the simple exponential family of risk models and proves that the credibility mean of such a process is exact Bayesian.

4.3 A Distribution-free credibility model

In the above work we have made some assumptions about the distribution of the claim amount and number of claims. Now using the collective risk model, without assumptions about the distribution of the claim amount and the number of claims , we try to show that the credibility equation (3.2-1) is a least square estimate for the pure premium and to obtain z and k using the Bayesian approach.
This problem was first discussed by Buhlmann(1967) and Taylor (1975).

Assume that the insurance portfolio of a particular line of insurance consists of n classes.

The risk within each class is homogeneous but may differ from class to class. Let

x_i = the loss ratio of the i^{th} class, x_i is a random variate and these variates are mutually independent, and identically distributed.

θ_i = the parameter which characterises the distribution of x_i.

Let $F(x_i/\theta_i)$ be the distribution of x_i and $p(\theta_i)$ is the distribution of θ .

Also let

$$E(x_i/\theta_i) = \mu(\theta_i)$$, and $\hat{\mu}_i$ is the estimate of $\mu(\theta_i)$.

Also let $$\bar{x} = \frac{x_1+x_2+\ldots+x_n}{n}$$

noting that the credibility formula (3.2-1) is a linear approximation of $\mu(\theta_i)$. Following the lines of Bulhmann(1967) we try to show that (3.2-1) is a least square approximation for $\mu(\theta_i)$ for some value of z.

Let $1-z = a$ and $z = b$

$$\hat{\mu}_i = a + b x_i \quad . \tag{4.3-1}$$

Then we want to choose a and b which minimize ϕ , where

$$\phi = \iint [\hat{\mu}_i - \mu(\theta_i)]^2 f(x_i/\theta_i) f(\theta) dx d\theta \tag{4.3-2}$$

i.e. we want the rates of a and b which satisfy the condition

$$\frac{\partial\phi}{\partial a} = \frac{\partial\phi}{\partial b} = 0$$

$$\frac{\partial\phi}{\partial a} = \int_X\int_\theta 2[a + bx_i - \mu(\theta_i)]\ f\ (x_i/\theta)\ f\ (\theta)dx\ d\theta$$

$$= 2\int_\theta [a+b\ \mu(\theta_i) - \mu(\theta_i)]\ f\ (\theta)d\theta$$

But $\int_\theta \mu(\theta_i)\ f\ (\theta)d\theta = E_\theta[\mu(\theta)] = \overline{x}$ (4.3-3)

hence

$$a = (1-b)\overline{x} \qquad (4.3\text{-}4)$$

$$\frac{\partial\phi}{\partial b} = \int\int 2x_i[a+bx_i-\mu(\theta_i)]\ f\ (x_i/\theta_i)\ f\ (\theta)dx\ d\theta \qquad (4.3\text{-}5)$$

Now let

$$x_i = \overline{x} + [\mu(\theta_i)-\overline{x}] + [x_i - \mu(\theta_i)]$$

and the integrant factor

$$a + b\mu(\theta_i) - \mu(\theta_i) = b[x_i-\mu(\theta_i)] + (b-1)[\mu(\theta_i)-\overline{x}] .$$

The two substitutions used in (4.3-5) by Taylor (1975)

$$\frac{\partial\phi}{\partial b} = \int_\theta\int_X [\overline{x} + \{\mu\ (\theta_i)-\overline{x}\} + \{x_i-\mu(\theta_i)\}]$$

$$[b\{x_i-\mu(\theta_i)\} + (b-1)\{\mu(\theta_i)-\overline{x}\}fx_i(\theta_i)\ f\ (\theta)dx\ d\theta$$

Multiplying the two factors of the integrand,and integrating term by term, all the terms will vanish except two ,

$$\frac{\partial\phi}{\partial b} = \int_\theta \int_x [(b-1)\{\mu(\theta_i)-\overline{x}\}^2 + \{x_i-\mu(\theta_i)\}^2] d(x_i/\theta_i) d\theta$$

$$0 = (b-1) \, var_\theta [\mu(\theta)] + b \, E_\theta[var(x_i/\theta)]$$

hence

$$b = \frac{var_\theta[\mu(\theta)]}{var_\theta[\mu(\theta)] + E_\theta[var(x_i(\theta))]} \qquad (4.3\text{-}6)$$

and the credibility relation would be

$$\mu_i^n = (1-z)\overline{x} + z\,x_i \quad \text{and} \quad z = b \qquad (4.3\text{-}7)$$

Since x_i are independent and are identically distributed, then

$$var_\theta[\mu(\theta)] = E[\overline{x} - \mu(\theta)]^2$$

$$= \frac{1}{n} E[x_1 - \mu(\theta)]^2 = \frac{1}{n} E\{\sigma_\theta^2\}$$

$$z = \frac{n}{n + \dfrac{E\{\sigma_\theta^2\}}{var[\mu_\theta]}} \qquad (4.3\text{-}8)$$

where k is the normalising constant,

Let

$v = E[\sigma_\theta^2]$

$w = var[\mu(\theta)]$

$$\text{hence} \quad k = \frac{E\{\sigma_\theta^2\}}{\mathrm{var}\{\mu(\theta)\}} = \frac{v}{w} \tag{4.3-9}$$

The normalising constant k, is the same as that which Whitney (1918) hinted about in his work; using the normal distribution for the class hazard and a mixture of arguments calling on Bayesian and maximum likelihood techniques Whitney finds that

$$k = \frac{p(1-p)}{\varepsilon^2}$$

where p is the indicated class hazard, and ε being the variation of hazard within the class. Also Jewell (1972) arrives at the same form for k. Also, it will be noted that there is a similarity between (4.3-9) and the k factor in (4.2-14) [Beta-Binomial case] and (4.2-20) [Gamma-Poisson case].

z in (4.3-8) satisfies the property $0 \leq z \leq 1$. Since $v, w > 0$ also if $v \to 0$, i.e. the variation within risk class is very small, then $z = 1$, which implies that x_i provides a good estimate for $\mu(\theta_i)$. Also, if $\mathrm{var}[\mu(\theta)]$ is very large, $k \to 0$, and since $\mathrm{var}[\mu(\theta)]$ measures the variation between risk classes, which implies $k = 0$ and x_i is the only estimate for $\mu(\theta_i)$.

Also, if v is very large, then $z \to 0$, which means that there is a very large variation within the risk class, and implies that there is little information about $\mu(\theta_i)$. The same result obtains $\mathrm{var}[\mu(\theta)]$ is very small, i.e. there is little variation between risk classes, and $\bar{x}$ would be a good estimator for $\mu(\theta_i)$.

Buhlmann(1968) has adopted this procedure for experience rating leading to two very useful properties , i.e.

1. sum of correct premium over the portfolio = expected income by the experience rating formula from the portfolio.

2. sum of correct premiums over any part of the portfolio is characterised by claims experience = expected income by experience rating from the same part of the portfolio (which means impossibility of antiselection).

CHAPTER FIVE

CREDIBILITY APPROACH TO RATE MAKING IN GENERAL INSURANCE

5.1 Introduction

In fire insurance as well as other forms of general insurance, it is known that there are many factors affecting individual risk experience (discussed in Chapter two). Almer (1975) summarises the statistical problem of general insurance in the following way :

1 - Insurance risk depends on several dominant parameters
2 - All risk groups are non-homogeneous, composed of unknown homogeneous sub-groups, and the risk in different sub-groups will often vary by 20% to 50% or even more.
3 - The "risk a priori" varies from year to year and generally has a marked trend.
4 - Claims amounts are determined not only by sums insured, but principally by random circumstances in the risk situation preceding actual accident.
5 - The distribution of claim amounts varies with risk group and period. Some statistical results seem to derive from the structure of the risk situation (with or without actual claims experience).

These points are coupled with the problem of paucity of data. For independent rate making, an expereince rating plan is needed to provide a rating plan which is responsive to change over time and insures an equitable treatment to individual risk. Various experience rating plans have these features, which would satisfy the fundamental

In this case, a simple model for premium calculation would exist such that

i. premium income = value of the claim outgo

$$= \int v^t C_t \, dt \qquad (5.1\text{-}1)$$

where v is a discount factor, $C_t \, dt$ the claim outgo in the interval $(t, t+\delta t)$.

ii. premium = rate x number of the units of exposure

or rate = premium / number of units of exposure.

In general insurance, the period of contract is usually short (which is in contrast to normal long term of life insurance) so it is customary to ignore the discount factor v in (5.1-1) ; hence (5.1-1) is reduced to

$$\text{premium income} = \int C_t \, dt = \underset{\sim}{C} \qquad (5.1\text{-}2)$$

In fire insurance, like any other lines of non-life insurance, C, the total claim outgo is determined by the individual circumstances for each case, which is not known at the time of fixing the rate. It has been shown in Chapter 3 and 4 that claim outgo depends on two elements.

1 - The occurrence (or non-occurrence) of the event that causes loss in respect of the insured.

2 - The extent of the loss, which determines the amount of claim paid by the insurer.

Thus, let x be a random variate having the value 1 or 0 according criterion of experience rating. [Robert Bailey 1961].

1 - Each unit of loss (in monetary terms) should contribute to the risk adjusted rate an amount equivalent to the amount of information it provides regarding future losses of the same risk for the same amount of exposure.

2 - The risk premium should not fluctuate widely from year to year.

3 - One unit of loss (in monetary value) should not increase the adjusted rate by more than one unit.

It should be noted that criteria 2 and 3 impose limitations on criteria (1).

Also the plan would be expected to make an explicit use of the available experience of the class as well as of the individual risk.

Rating and experience rating for fire insurance does not depend only on statistical analysis but also on the engineering assessment of the risk.

This chapter will be devoted to the description and development of the mathematical models necessary to estimate the pure risk premium.

5.2 Elements of rate making

The perfect solution for rate making would be where the claim experience is known at the time of rate making with complete certainty -

to whether there is an event of claim or not, respectively and let S be the amount of a claim given that an event of claim has occurred. Then the claim outgo, $C = xS$, is also a random variable having a compound distribution function $F(x,S)$. The pure risk premium ϕ is a function of this compound distribution which assigns a non-negative number to each risk C.

$$\text{Premium} = \phi[F(x,S)] \quad .$$

Actually ϕ relates the claim producing possibility to the variation in the premium (different forms of the function ϕ will be discussed in the next section) . Since S and x are independent random variables, the relationship can be written in the form

$$F(x,S) = B(x) \cdot G(S)$$

where $B(x)$ represents the cumulative probability function of the variable x and $G(S)$ is the cumulative probability function of S given $x = 1$.

From the above discussion, it can be seen that ϕ , $B(x)$ and $G(S)$ constitute the basis for rate making.

5.3 Premium Calculation Principle

If the distribution of the amount of claims $F(x,S)$ is known, the amount of the premium can be calculated according to the following possible principles:

1 - The expected value principle (net premium principle)

$$\phi(C) = (1+\lambda)E(C)$$

2 - The standard deviation principle

$$\phi(C) = E(C) + \alpha\sqrt{\text{var}\, C}$$

3 - The variance principle

$$\phi(C) = E(C) + B\ \text{var}\, C$$

4 - The zero utility principle

The premium $\phi(C)$ is arrived at as a result of the equation

$$E\{\mu[\phi(C) - C]\} = \gamma$$

where $\mu(\omega)$ is the utility function of the company profit; this function should satisfy the following requirements :

a - $\mu(\omega)$ has to be continuous

b - $\mu(\omega)$ has to be non-decreasing

c - $\mu'(\omega)$ has to be non-increasing

This function measures the profit achieved by the company. The constant γ represents the safety loading including the profit desired or expected by the Company.

Gerber 1974 mentions other principles.

5 - The mean value principle

Let $v(\omega)$ be a continuous strictly increasing function, then one defines

$$\phi(C) = v^{-1}E(v(C))$$

where v^{-1} denotes the inverse function of v.

6 - The maximal loss principle

For $p > 0$, $q = 1 - p > 0$ one sets

$$\phi(C) = pE(C) + q \max(C)$$

where max(C) denotes the right end point of the range of C.

The principle of calculating the premium is additive, if the premium assigned to the sum of two independent risks is the sum of the premiums that are assigned to the two risks individually, i.e. the premium in fire insurance is additive if the total premium for two houses insured separately is equal to the premium for the two houses insured as one object.

Principles 1 and 3 fulfil the requirement of additivity. Principle 2 (standard deviation principle) does not satisfy this requirement. Additivity of the premium is to be considered desirable both in regard to practice and theory.

The utility function (and the principles 5 and 6) are very interesting in theory, but in practice may be of very little importance. The fulfilment of the additivity requirement depends on the utility function (note that if the utility function is linear, the resulting premium is equivalent to the expected value principle). In the following sections, more properties of the first three functions will be discussed in more detail.

5.4 Preliminary assumptions

The symbols we will use in the following work will be the common symbols used in most literature concerning this subject.

Let v be the parameter that represents the claim producing possibility (the set of hazards associated with risk) in respect of claim frequency or claim severity. In general, v will be a vector or set of descriptive statements which characterise the risk elements in respect of one of these two aspects of heterogeneity, associated with a specific class.

For convenience, the specific risk can be represented by $R(v_1,v_2)$, where v_1 represents the claim frequency and v_2 represents the claim severity *. $R(v\ ,\cdot)$ would represent specific risk only in respect of claim frequency and $R(\cdot,v_2)$ would represent specific risk only in respect of claim severity.* $R(\cdot,\cdot)$, or simply R, would denote a risk randomly selected from the class.

$p(v_1,v_2)$ = risk premium (or rate for only one unit of exposure for the risk $R(v_1,v_2)$.

p = the class premium for R (collective premium).

Actually true risk premium is unknown and will remain so for a given risk (the only exception is if the risk can be observed over a very long period of time, and the claim experience during this period is stationary, i.e. the risk condition remains unchanged over the course of time). Usually engineering assessment of risk plays a major part in determining the risk premium. At the same time the class premium

* i.e. severity relative to the other risks in the class

can be determined more accurately from the claim statistics.

Let $B(X/v)$ = cumulative probability function for $R(v_1,\cdot)$ for random variate x.

$G(S/v)$ = cumulative probability function for $R(\cdot,v_2)$ for random variate S

and let $g(S/v)$ represent the probability that a loss occurs and a claim presented to the insurer and his liability would not exceed the amount S.

$B(x)$ and $G(S)$ are the cumulative probability function for R, i.e. for a risk taken at random from the class.

$\theta(v,v_2)$ is the structure function of the class with respect to the claim frequency and claim severity. $\theta(v_1,v_2)$ represents the frequency distribution of the group (v_1,v_2) over the class and

$$\theta(v_1,\cdot) = \sum_{v_2} \theta(v_1,v_2)$$

$$\theta(\cdot,v_1) = \sum_{v_1} \theta(v_1,v_2)$$

respectively the structure functions with respect to claim frequency and claim severity. From Chapter 4 it can be seen that

$$B(x) = \frac{\sum B(x/v)\theta(v_1)}{\sum\theta(v,\cdot)}$$

$$G(S) = \frac{G(s/v)\theta(\cdot,v)}{\sum\theta(\cdot,v)}$$

It would be convenient to study the relation between $R(v_1, v_2)$ and R.

$$\text{Let} \quad \pi(v_1) = E[x/v] \quad , \pi = E[x]$$

$$\pi = E[x] = E\{E\{x/v\}\} = E[\pi(v)] \qquad (5.4\text{-}1)$$

where π and $\pi(v)$ are the mean values of the distribution of x, for R and $R(v_1,\cdot)$ respectively.

Also, let $\underline{\mu}(v)$ and $\underline{\mu}$ be the mean values of the conditional distribution of S, for $R(\cdot,v)$ and R respectively , then

$$\underline{\mu} = E[S/x = 1] = E\{E[S/v, x = 1]\}$$

$$\therefore \quad \underline{\mu} = E\{\underline{\mu}(v)\} \qquad (5.4\text{-}2)$$

In general, the higher order central moments are denoted by

$$\underline{\mu}_r(v) = E\{[S-\underline{\mu}(v)]^r/v, x = 1\}$$

$$\underline{\mu}_r = E\{[S-\underline{\mu}]^r/x=1\}$$

$$= E\{([S-\underline{\mu}(r)] + [\mu(r)-\mu])^r/x=1\} \qquad (5.4\text{-}3)$$

In particular, the second central moment (r=2) is of more interest

$$\sigma^2(v) = \underline{\mu}_2(v)$$

$$\sigma^2 = \underline{\mu}_2$$

$$= E\{\sigma^2(v)\} + var\{\underline{\mu}(v)\} \qquad (5.4\text{-}4)$$

The following results will be used in the following work.

$$1 - \quad var(x) = E\{var(x/v_1) + var\{E(x/v_1)\}$$

$$= E\{\pi(v_1)(1-\pi(v_1))\} + var\{\pi(v_1)\}$$

$$= \pi(1-\pi) \qquad (5.4\text{-}5)$$

Since $\pi = E\{\pi(v_1)\}$

and $var[\pi(v)] = E\{\pi^2(v_1)\} + E^2\{\pi(v_1)\}$

$$\left.\begin{aligned} 2 - \quad E\{xS/v_1,v_2\} &= \pi(v_1)\mu(v_2) \\ E\{xS\} &= \pi\mu \end{aligned}\right\} \qquad (5.4\text{-}6)$$

$$\left.\begin{aligned} 3 - \quad var\{xS/v_1,v_2\} &= \pi(v_1)\sigma^2(v_2) + \mu^2(v_2)\pi(v_1)[1-\pi(v_1)] \\ var[x,S] &= \pi\sigma^2 + \mu^2\pi(1-\pi) \end{aligned}\right\} \qquad (5.4\text{-}7)$$

For proof of relations 2 and 3 see Appendix (1).

5.5 Requirement and construction of the premium function ϕ

From the elementary principle of insurance, as far as practical, the risks are divided into classes $R(v_1,v_2)$ differentiating the hazard of various risks. Therefore, all risks with specification (v_1,v_2) may be treated as equal hazards. Hence, each risk $R(v_1,v_2)$ would be charged the average premium of the group (v_1,v_2) . The obvious value for ϕ is

$$\phi[B(x), G(S)] = E[xS/v_1,v_2]$$

and the premium of this risk is

$$p(v_1,v_2) = \pi(v_1)\mu(v_2) \qquad (5.5\text{-}1)$$

From (5.4-6) also

$$p = \pi\mu \tag{5.5-2}$$

The function ϕ should satisfy the following properties :

1 - For a large number of risks, for each group (v_1,v_2) the resulting premium is such that

total premium income = total claim outgo

2 - The risk premiums are unbiased in aggregate

3 - The function ϕ is iterative (the function $\phi(p = \phi(x))$ is iterative if and only if $\phi(x/y)$ is a risk and $\phi(x) = \phi[\phi(x/y)]$ for any pair of risks x and y). This property can be described in our case as applying the function ϕ first within the group (v_1,v_2) then between the different groups ; if the resulting premium is equal to the class premium, then the function ϕ is said to be iterative.

4 - The function ϕ should be additive (mentioned in 5.3)

5 - The function ϕ should have a provision for fluctuation over a short period of time. To ensure sufficient reserve to meet the fluctuation overtime, this provision is affected by the insurance company's policy for retention and re-insurance.

a - Expected value principle

$$\phi[B(x/v_1) , G(S/v_2)] = (1+\lambda)E[xS/v_1,v_2]$$

$$= (1+\lambda)\pi(v_1)\pi(v_2)$$

The expected value principle is often used in life insurance but it is seldom used in property and casualty insurance because of the heterogeneity of the claim experience. This principle is unbiased but not iterative.

b - The standard deviation principle

$$\phi[B(x/v_1),G(S/v_2)] = E[xS/v_1,v_2] + B\sqrt{var[xS/v_1,v_2]}$$

$$= \pi(v_1)\mu(v_2) +$$

$$B[\pi(v_1)\sigma^2(v_2) + \pi(v_1)(1-\pi(v_1)\mu^2(v_2)]^{\frac{1}{2}}$$

This principle makes specific provision for the contingency, and takes into consideration the variability of claim size. The standard deviation principle is most frequently used in property and casualty insurance.

At the same time it is not unbiased, not iterative and not additive.

c - Variance principle

$$\phi[B(x/v_1), G(S/v_2)] = E[xS/v_1,v_2] + B\ var[xS/v_1,v_2] .$$

Hence $p(v_1,v_2) = \pi(v_1)\mu(v_2) + B[\pi(v_1)\sigma^2(v_2) + \pi(v_1)(1-\pi(v_1))\mu^2(v_2)]$

Also the class premium

$$p = \pi\mu + B[\pi\sigma^2 + \pi(1-\pi)\mu^2]$$

The variance principle is unbiased and also iterative. To prove that it is unbiased - since

$$E[p(v_1,v_2)] = \pi\mu + BE\{var[xS/v_1,v_2]\}$$

By using equation (5.4-4) and (5.4-7) and ignoring terms containing $var[\pi(v_1)]$

$$E[p(v_1,v_2)] = \pi\mu + B[\pi\sigma^2+\pi(1-\pi)\mu^2] - B\pi^2\ var[\mu(v_2)] \simeq p$$

Since the last term is of very small order.

To prove it is iterative, apply ϕ function twice, firstly within the groups and secondly between the groups.

$$\phi\{\phi[B(x/v_1)\ ,\ G(x/v_2)]\}$$

$$= E[p(v_1,v_2)] + B\ var[p(v_1,v_2)]$$

$$= E\{\pi(v_1)\mu(v_2)+B[\pi(v_1)\sigma^2(v_2)+\pi(v_1)(1-\pi(v_1))\mu^2(v_2)]\}$$

$$+B\ var\{\pi(v_1)\mu(v_2)+B[\pi(v_1)\sigma^2(v_2)+\pi(v_1)(1-\pi(v_1)\mu^2(v_2)]\}$$

dropping terms containing B^2

$$\simeq E\{\pi(v_1)\mu(v_2)+B[\pi(v_1)\sigma^2(v_2)+\pi(v_1)(1-\pi(v_1))\mu^2(v\ )]\}$$
$$+ B\ var\{\pi(v_1)\mu(v_2)\}$$

$$\simeq \pi\mu + B\{\pi E[\sigma^2(v_2)] + \pi(1-\pi)E[\mu^2(v_2)]\}$$
$$+ B\{\pi^2 var[\mu(v_2)]\}$$

$$\simeq \pi\mu + B\{\pi\sigma^2 \quad +\pi(1-\pi)\mu^2\}$$

Thus ϕ is iterative.

Also, the variance principle takes into account the variability in the distributions of x and S. While the variance principle satisfies all the conditions [from 1 to 5] and has much consideration in favour of it , it is not as popular as the standard deviation principle. It should be noted also that the expected value principle is contained in the variance principle as a special case when B = 0. Buhlmann (1970) shows that in the case of zero utility, if the utility function is quadratic then the variance principle is the first approximation for the premium.

In the following work we shall base our consideration on the variance principle because of favourable theoretical and practical properties.

5.6 Experience rating and credibility premium

In practice, we have observation

$$D_n = \{x_1, x_2, \ldots x_n, S_1, S_2, \ldots, S_n\} \qquad (5.6\text{-}1)$$

about a risk $R(v_1, v_2)$, (the case n = 0 signifies that no experiences are available for the risk) and it is required to estimate the risk premium $p(v_1, v_2)$ for $R(v_1, v_2)$ by taking into consideration the experience D_n . Also we have to consider two facts

1 - the claim frequency is too small i.e. $\pi(v_1)$ is too small .

2 - the claim amount distribution (distribution of S) is usually heavily skew with a long tail on the right-hand side.

These facts have great effect on the efficiency of the estimates of $\pi(v_1), \mu(v_2)$ and $\sigma^2(v_2)$ derived from the actual experience D_n. The efficiency of these estimates would be very low even with moderate size of experience, which consequently affects the reliability of the estimate of $p(v_1,v_2)$ [since this estimate constitutes a component of the premium].

As has been shown, in experience rating a compromise is to give to the estimate from the data weight only to the extent of its credibility, and use the experience available for other groups of the class.

The classical approach to credibility , discussed in Chapter 3 , introduces an arbitrary choice of the formula for experience rating as well as in the formula for the partial credibility. The only criticism of that approach concerns the method of obtaining the normalising factor. At the same time, the Bayesian approach discussed in Chapter 4 has a sound theoretical property, but requires knowledge of the distribution of the claim process which is difficult to obtain.

Buhlmann's credibility formula discussed in (4.5) provided a logical statistical basis for experience rating and found a statistical measure for determining the normalising factor. However, this formula can be adopted directly in such situations where only one of the two elements of the claim, claim frequency and claim severity, is relevant.

In order to avoid having to repeat assumptions, it will be convenient to describe in outline the credibility formula for experience rating before suggesting the modified formula which accounts for both elements of claim process in the following section.

Let y be a random variate, having a cumulative probability function $F(y/v)$ for $R(v)$ and $H(y)$ for R (Note the equivalence between this notation and the notation in Chapter 4).

The variance principle for the risk premium and class premium is given by

$$p(v) = \mu(v) + B\sigma^2(v)$$

and $$p = \mu + B\sigma^2$$

Taking account of the experience, $p(D_n)$ the credibility premium, where

$D_n = \{y_1 y_2, \ldots, y_n\}$

is defined to be

$$p(D_n) = E[p(v)/D_n] + B \operatorname{var}[\mu(v)/D_n]$$

$$= E[\mu(v)/D_n] + BE[\sigma^2(v)/D_n] + B \operatorname{var}[\mu(v)/D_n] \qquad (5.6\text{-}1)$$

This formula can be broken into two parts

a - an approximation part $E[p(v)/D_n]=E[\mu(v)/D_n]+B\,E\,[\sigma^2(v)/D_n]$

b - a fluctuation part $B \operatorname{var}[\mu(v)/D_n]$

The approximation part has the following properties

1 - It approximates the risk premium $p(v)$ with the minimum square error over the entire class (see 4.5).

2 - It is the only approximation whose mean, over all such risks R, which have the same experience as D_n is equal to the mean

risk premium over this part of the whole class (Buhlmann 1961).

3 - It approaches the risk premium p(v) asymptotically for every group (v) (Doop 1949).

4 - The fluctuation part var $[\mu(v)/D_n]$ approaches zero asymptotically.

5 - The risk premium is equal to the class premium when there is no experience for the risk (n = 0) and it converges to the risk premium for a very large claim experience ($n \to \infty$).

The experience rating premium $p(D_n)$ has three components.

These components can easily be determined using Bayesian theory (explained in Chapter 4) if the distribution function F(y/v) and the structure function $\theta(v)$ are known, but this is not so in our case.

Each term of the formula (5.6-1) can be approximated by appropriate credibility formula of the type in (4.5) and by an appropriately chosen "linear" function as follows :

a - approximation to expected value part

$$E[\mu(v)/D_n\} \doteq a + b\bar{y}$$

$$\text{where } \bar{y} = \frac{1}{n} \sum y_i \qquad (5.6\text{-}2)$$

can be approximated by

$$z_{\mu}\bar{y} + (1-z_{\mu})\mu \qquad (5.6\text{-}3)$$

$$\mu = E[\mu(v)]$$

and $z_{\mu} = \dfrac{n}{n+k_{\mu}}$ (5.6-4)

From (4.3-8) we get

$$k_{\mu} = \frac{E[\sigma^2(v)]}{var[\mu(v)]}$$

b - approximation of variance part

$$E\{\sigma^2(v)/D_n] \doteqdot c + d \textstyle\sum^2$$

where $\sum^2 = \dfrac{1}{n-1} \displaystyle\sum_{i=1}^{n} (y_i-\bar{y})^2$ (5.6-5)

can be expressed as

$$z\sigma^2 \textstyle\sum + (1-z_{\sigma^2})E[\sigma^2(v)] \qquad (5.6\text{-}6)$$

where $z_{\sigma^2} = \dfrac{n-1}{(n-1)+2k_{\sigma^2}}$ (5.6-7)

and $k_{\sigma^2} = \dfrac{E[\sigma^4(v)]}{var[\sigma^2(v)]}$ (5.6-8)

c - approximation of fluctuation part

$$var[\mu(v)/D_n(v) \doteqdot E[\mu(v) - (a+b\bar{y})]^2$$

can be expressed as

$$(1-z_{\mu})var[\mu(v)] \qquad (5.6\text{-}9)$$

Then the credibility premium

$$p(D_n) = \{z_\mu \bar{y} + (1-z_\mu)E[\mu(v)] + B\{z_{\sigma^2} \sum^2$$

$$+ (1-z_{\sigma^2})E(\sigma^2(v)] + B(1-z_\mu)var[\mu(v)]$$

using equation (5.4-5) and (5.4-3)

$$p(D_n) = [z_\mu \bar{y} + (1-z_\mu)\mu\} + B\{z_{\sigma^2} \sum^2 + (1-z_{\sigma^2})\sigma^2\}$$

$$+ B(z_{\sigma^2}-z_\mu) \, var[\mu(v)] . \qquad (5.6\text{-}10)$$

The last term is very small and would have little effect if ignored. For determining the size of experience required for self-rating, n_S for expected value, and $n_{S\sigma^2}$ for the variance component of the credibility formula may be given by the inequality

$$p_r\{(1-k)\mu \leq \bar{y} \leq (1+k)\mu\} \geq 1-p \qquad (5.6\text{-}11)$$

$$\text{and} \quad p_r\{(1-k)E[\sigma^2(v)] \leq \sum^2 \leq (1+k)E[\sigma^2(v)]\} \geq 1-p \qquad (5.6\text{-}12)$$

k,p have the same meanings as in (3.3-1) [k represents the percentage deviation on either side of $\bar{y}$ or $\sum^2$, and p is the probability of deviation]. Since the claim distribution is heavily skew, $n_{S\mu}$ and $n_{S\sigma}$ will be large even for moderately stringent [k,p] requirements calling upon the central limit theorem for solving this inequality since

$$var[\bar{y}] = \frac{1}{n} E[\sigma^2(v)] + var[\mu(v)]$$

$$= \frac{1}{n} A \qquad + B \qquad (5.6\text{-}13)$$

where $A = E\sigma^2(v)$ and $B = var[\mu(v)]$

and $var\sum = \frac{1}{n}[E[\mu_4(v)] - \frac{n-3}{n-1}E\{\sigma^4(v)\}$

$+ var[\sigma^2(v)]$

$= \frac{2}{n-1} E[\sigma^4(v)] + var[\sigma^2(v)]$

$$= \frac{1}{n-1} A' + B' \tag{5.6-14}$$

where $A' = 2E[\sigma^4(v)$, $B' = var[\sigma^2(v)]$

then

$$a - n_{S\mu} = \frac{AC}{1-BC} \tag{5.6-15}$$

$$\text{where} \quad C = \left[\frac{ap}{k\mu}\right]^2 \tag{5.6-16}$$

where ap = percentile of the normal distribution with zero mean and unity variance

$$b - n_{S\sigma^2} = 1 + A'C'/(1-B'C')$$

$$C' = \left[\frac{ap}{kE(\sigma^2(v))}\right]^2 \tag{5.6-17}$$

5.7 Modifying formula for credibility premium

The credibility premium (5.6-10) accounts only for one element of claim process . From the principles developed in (5.5) the following formula which can account for the two elements of the claim process (claim frequency and claim severity) can be introduced.

$$p(D_n) = E[p(v_1,v_2)/D_n] + B\,\text{var}\,[\pi(v_1)\mu(v_2)/D_n]$$

$$= E[\pi(v_1)\mu(v_2)/D_n] + B\,E\,[\pi(v_1)\sigma^2(v_2)/D_n]$$

$$+ B[\pi(v_1)(1-\pi(v_1)\mu^2(v_2)/D_n]$$

$$+ B\,\text{var}\,[\pi(v_1)\mu(v_2)/D_n] \qquad (5.7\text{-}1)$$

It should be noticed that, this equation has four components. The first component represents the expected value part. The second and third terms represent the variance part, and the fourth term represents the fluctuation part.

Each of the four terms of equation (5.7-1) will be estimated by a linear function of the data D_n.

a - First Component

We approximate $E[\pi(v_1)\mu(v_2)/D_n]$ by $a+b(\overline{XS})$

where

$$\overline{XS} = \frac{1}{n}\sum_{i=1}^{n} x_i S_i$$

Using the result of (4.3) we can express this part as

$$z_{\pi\mu}\,\overline{XS} + (1-z_{\pi\mu})\pi\mu \qquad (5.7\text{-}2)$$

where

$$z_{\pi\mu} = \frac{n}{(n-1)+k_{\pi\mu}} \qquad (5.7\text{-}3)$$

$$k_{\pi\mu} = \frac{E\{\text{var}[\pi(v_1)\mu(v_2)]\}}{\text{var}\{\mu(v_2)\pi(v_1)\}}$$

$$= \frac{\pi\sigma^2 + \pi(1-\pi)\mu^2}{\pi\, var[\mu(v_2)] + \mu^2 var[\pi(v_1)]} \tag{5.7-4}$$

b - The Second Component

$B\,E\,[\pi(v_1)\sigma^2(v_2)/D_n]$ can be replaced by

$B\,\pi\,E[\sigma^2(v_2)/D_n]$

Let $n' = \sum_{i=1}^{n} x_i$.

Then $E[\sigma^2(v_2)/D_n]$ can be approximated by $c + d\Sigma^2$ where

$$\Sigma^2 = \frac{1}{n'-1} \sum_{i=1}^{n} (S_i - \bar{S})^2 \tag{5.7-5}$$

Then the approximation is

$$z_{\sigma^2} \Sigma^2 + (1 - z_{\sigma^2}) E[\sigma^2(v_2)] \tag{5.7-6}$$

where

$$z_\sigma = \frac{n'-1}{(n'-1) + 2k_{\sigma^2}}$$

and

$$k_{\sigma^2} = \frac{E[\sigma^4(v_2)]}{var[\sigma^2(v_2)]} \tag{5.7-7}$$

c - The Third Component can be similarly approximated by

$\mu^2 B\,E\,[\pi(v_1)(1-\pi(v_1))/D_n]$

Then $E[\pi(v_1)(1-\pi(v_1))/D_n] \doteqdot C_2 + d_2\, \gamma^2$

$$\gamma = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})$$

$$\frac{n}{n-1}\, \bar{X}\, (1-\bar{X}) \tag{5.7-8}$$

Thus the approximation can be expressed as

$$z_{\pi(1-\pi)}\gamma^2 + (1-z_{\pi(1-\pi)})\; E[\pi(v_1)(1-\pi(v_2)) \tag{5.7-9}$$

where

$$z_{\pi(1-\pi)} = \frac{(n-1)}{(n-1)+2k_{\pi(1-\pi)}} \tag{5.7-10}$$

$$k_{\pi(1-\pi)} = \frac{E[(\pi(v_1)(1-\pi(v_1)]^2}{var[\pi(v_1)(1-\pi(v_1))]} \tag{5.7-11}$$

d - The Fourth Component

$var[\pi(v_1)\mu(v_2)/D_n]$ can be written as

$$var[\pi(v_1)\mu(v_2)/D_n] = E\{[\pi(v_1)\mu(v_2) - E(\pi(v_1)\mu(v_2))/D_n]^2\}$$

From the approximation of the first terms

$$E\{[\pi(v_1)\mu(v_2) - (z_{\pi\mu}\, \overline{XS} + (1-z_{\pi\mu})\pi\mu)/D_n]^2\}$$

$$= E\{[\pi(v_1)\mu(v_2) - z_{\pi\mu}\, \overline{XS} - (1-z_{\pi\mu})\pi\mu\}^2$$

according to equation (5.7-3) for $z_{\pi\mu}$

$$var[\pi(v_1)\mu(v_2)/D_n] = (1-z_{\pi\mu})\{\pi\ var[\mu(v_2)]$$

$$+ \mu^2\ var[\pi(v_1)]\} \tag{5.7-12}$$

From (5.7-2) and (5.7-6) and (5.7-9) and (5.7-12), the modifying credibility formula would have the form

$$\{z_{\pi\mu}\ \overline{XS} + (1-z_{\pi\mu})\pi\mu\} + \pi B\{z_{\sigma^2}\sum^2 + (1-z_{\sigma^2})E(\sigma^2(v_2))\}$$

$$+ \mu^2 B\{z_{\pi(1-\pi)}\ \gamma^2 + (1-z_{\pi(1-\pi)})E[\pi(v_1)(1-\pi(v_1))]\}$$

$$+ (1-z_{\pi\mu})B\{\pi^2 var[\mu(v_2)] + \mu^2 var[\pi(v_1)]\} \tag{5.7-13}$$

$$= z_{\pi\mu}\ \overline{XS} + (1-z_{\pi\mu})\pi\mu + \pi B\{z_{\sigma^2}\sum^2 + (1-z_{\sigma^2})\sigma^2\}$$

$$+ \mu^2 B\{z_{\pi(1-\pi)}\gamma^2 + (1-z_{\pi(1-\pi)})\pi(1-\pi)\} + R_0 \tag{5.7-14}$$

where $R_0 = B\{\pi^2(1-z_{\pi\mu}) - \pi(1-z_{\sigma^2})\ var[\mu(v_2)]$

$+ \mu^2(z_{\sigma^2}-z_{\pi\mu})var[\pi(v_1)]$

which can be ignored.

The modifying formula has three components

a - the first component represents the expected value of claim experience, and has credibility coefficient $z_{\pi\mu}$ for the average claim outgo.

b - the second component accounts for the variability of the claim severity S, and has a credibility coefficient z_{σ^2} for the variation of the claim outgo.

c - The third component accounts for the variability in claim frequency x, and has credibility for the variation of the number of claims.

Hence, the suggested formula takes into consideration the two elements of the claim process.

5.8 Level of full credibility for the modifying formula

a - For the first component, the expected value component

since $E[\frac{1}{n} \sum x_i S_i] = \pi\mu$

and $var\{\frac{1}{n} \sum S_i x_i\} = E\{var(\overline{XS}) + var\{E\ \overline{XS}\}$

$$= \frac{1}{n}\{\pi\sigma^2 + \pi(1-\pi)\mu^2\} + \frac{n-1}{n}\{\pi^2 var[\mu(v_2)]$$

$$+ \mu^2\ var[\pi(v_1)\}$$

$$= \frac{1}{n}A \qquad + \frac{n-1}{n}B \tag{5.8-1}$$

where $A = \pi\sigma^2 + \pi(1-\pi)\mu^2$

and $B = \pi\ var[\mu(v_2)] + \mu^2\ var[\pi(v_1)]$

Let a be the percentile of the distribution of

$$T = \frac{\frac{1}{n} \sum x_i S_i - \pi\mu}{\sqrt{var\ \frac{1}{n} \sum x_i S_i}} \tag{5.8-2}$$

The [k,p] requirement

$$P_r\{a_{p_1} \leq T \leq a_{p_2}\} \geq 1-p \qquad (5.8\text{-}3)$$

where $p_2 - p_1 = 1-p$

since for positively skew distribution

$$a_{1-p/2} > | a_{p/2}$$

For the relevant values of p required for full credibility a conservative estimate value of $n_{S\pi\mu}$ may be obtained from

$$a_p = \frac{k\pi\mu}{\sqrt{var[\frac{1}{n}\sum x_i S]}} \qquad (5.8\text{-}4)$$

$$\text{i.e. } n_{S\pi\mu} = \frac{(A-B)C}{(1-BC)} \qquad (5.8\text{-}5)$$

a_p would be obtained in terms of n_p, the corresponding percentile of the normal distribution with mean zero and variance unity, and some correction terms by using the Cornish-Fisher expansion as suggested by Mayerson in (2.3).

The second component has $\sum^2$ as the underlying statistic

$$\sum^2 = \frac{1}{n'-1} \sum_{i=1}^{n'} (S_i - \bar{S})^2 \qquad (5.8\text{-}6)$$

Then

$$E(\sum^2) = E(\sigma^2(v_2))$$

$$\text{and } var(\sum^2) = var[\sigma^2(v_2)] + \frac{2}{n'-1} E[\sigma^4(v_2)]$$

$$= \quad A + \frac{1}{n'-1} B$$

where

$$A = \mathrm{var}[\sigma^2(v_2)]$$

$$B = 2E[\sigma^4(v_2)]$$

and $n_{s\sigma^2}$, the level for full credibility for this component is

$$n_{s\sigma^2} = \frac{1 + BC}{1 - BC}$$

$$\text{where } C = \left[\frac{a_p}{k\,E\,[\sigma^2(v)]}\right]^2$$

The third component γ^2

$$\gamma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i-\bar{x}) = \frac{n}{n-1} \bar{X}\,(1-\bar{X})$$

$$E\gamma^2 = E[\pi(v_1)(1-\pi)]$$

$$\mathrm{var}\ \gamma^2 = \mathrm{var}[\pi(v_1)(1-\pi(v_1)] + \frac{2}{n-1} [\pi(v_1)(1-\pi(v_1))]^2$$

$$= A + \frac{1}{n-1} B$$

where

$$A = \mathrm{var}[\pi(v_1)(1-\pi(v_1)]$$

$$B = 2E[\pi(v_1)(1-\pi(v_1))]$$

The level of full credibility is

$$n_{s\pi}\ (1-\pi) \quad = \quad \frac{1 + AC}{1 - BC}$$

$$C \quad = \quad \left[\frac{a_p}{kE\{\pi(v_1)(1-\pi(v_1)\}} \right]^2$$

Let n be the number of accidents having a probability distribution B(x) and let S be the amount of claims paid in particular accidents having a probability distribution G(S).

Let $\phi_x(t)$ be the probability generating function for the random variable x and $\phi_s(t)$ for s.

$$\phi_x(t) = \sum t^n \, b(x)$$

$$\phi_s(t) = \sum t^n \, g(s)$$

Let n be the number of accidents occurring, and let z

$$z = S_1 + S_2 + \dots + S_n$$

the probability generating function for z is

$$\phi_z = \sum_{z=0}^{\infty} t^z \, \{\text{probability of paying } z\}$$

$$= \sum_x \sum_{S_1} \sum_{S_2} \dots \sum_{S_n} t^{S_1+S_2+\dots S_n} g(S_1)g(S_2)\dots g(S_x)b(x)$$

$$= \sum_x \phi_x \{ \phi_s(t)\} \qquad (1)$$

but the moment generating function $\phi_x(t)$ is related to the probability generating function through the relation

$$\phi_x(t) = m_x(\log(t)) \qquad (2)$$

Thus we can get the moment generating function of z as

$$m_z(t) = \phi_x\{\phi_s(\exp(t))\} \tag{3}$$

to obtain the moment we differentiate 3 with respect to t, at t = 0

$$m_z'(t) = \phi_x'\{\phi_s(\exp(t))\}\phi_s'(\exp(t))\exp(t) \tag{4}$$

$$m_z''(t) = \phi_x''\{\phi_s(\exp(t))\} .\{\phi_s'(\exp(t))\}^2 .\exp(2t)$$

$$+ \phi_{(x)}'\{\phi_s(\exp(t)\} . \phi_s'' (\exp(t)).\exp(2t)$$

$$+ \phi_{(x)}'\{\phi_s(\exp(t)\} . \phi_s' (\exp(t)).\exp(t) \tag{5}$$

Note that at t = 0 exp(t) = 1, also $\phi_x(1) = 1$

$$\text{also} \quad \phi_x'(t)\Big|_{t=1} = E(x)$$

$$\phi_x''(t)\Big|_{t=1} = E(x^2)-E(x)$$

$$= var(x) + E^2(x)-E(x) \tag{6}$$

The same conditions exist for $\phi_s(t)$.

∴ from (3) we get

$$E(z) = \phi_{(x)}'(1).\ \phi_s'(1)$$

$$= E(x)\ E(s) \tag{7}$$

from (5)

$$E(z^2) = \phi_x''(1)\,[\phi_s'(1)]^2 + \phi_x'(1)\phi_s''(1) + \phi_x'(1)\phi_s'(1)$$

$$= [\mathrm{var}\,x + E^2(x) - E(x)]E^2(s) + E(x)[\mathrm{var}(s) + E^2(s) - E(s)]$$

$$+ E(x)E(s)$$

$$= \mathrm{var}(x)E^2(s) + \mathrm{var}(s)E(x) + E^2(s)E^2(x) \qquad (8)$$

Therefore

$$\mathrm{var}(z) = \mathrm{var}(x)E^2(s) + \mathrm{var}(s)E(x) \qquad (9)$$

since

$$\mathrm{var}(z) = E(z^2) - E^2(z).$$

CHAPTER SIX

EXPERIENCE RATING , AN EMPIRICAL STUDY

6.1 Introduction

Credibility theory approach to experience rating has been widely applied by casualty actuaries in the United States for a long time. In practice, for rate revision, the theory was applied to a heterogeneous collection of risks in the portfolio. While this credibility approach can handle a heterogeneous collection of risks, this approach in fire insurance is criticised by Hurley (1954) because it does not differentiate between classes. It would be logical and desirable to divide the total portfolio of lines of insurance into broad classes which would have several advantages.

1 - The credibility tables would be more appropriate to the inherent hazard of the class.

2 - The handling of the data for rating or rate revision would become easier by dividing the data into groups.

3 - The treatment of the risk would be easily understood by the management who are not equipped with actuarial and statistical knowledge.

In chapter two we discussed different factors affecting the claim experience; according to these factors it would be convenient to first divide the data according to the protection group [protected ≡ area A, unprotected ≡ area B].

Secondly, since many factors affecting the risk are relevant to the type of occupancy, the risk is subgrouped according to the occupancy.

Other factors such as construction, private protection and special features not relevant to occupancy would be considered.

It is the practice in fire insurance to divide the sum insured covering a property into two parts :

1 - sum insured for the construction cover

2 - sum insured for the content cover

and the rate would be different for each part. From the data collected from the non-industrial fire branch, it is found that the property is either covered for content or for the construction.

If the number of occupancies covered are very large, broad grouping according to Appendix 2.2 would be helpful for handling the data.

If one fire division has more than one occupancy, it would be charged the higher rate for the more hazardous part and accordingly will belong to the category of the hazardous occupancy.

6.2 Source and Choice of Data

In Egypt, the Association of Insurance has the responsibility of fixing the insurance rate for different lines of insurance, and by the Law the insurance companies must follow their rates. There are no detailed statistical data to ensure that the rates have sound scientific justification. Each company working in the market is represented at the Association of Insurance. The decision for any change concerning the rate (increasing or decreasing, moving some category of risk from one class to another class) are taken by these representatives, relying upon their experience and wise judgement. The information they have is restricted to the amount of premiums collected and the total claim payment for a calendar year grouped in lines of the reinsurance treaty (the Association has the responsibility of fixing the level of retention for the insurance cover).

By the end of each year, each of the insurance companies in the market do some statistics in lines of the reinsurance cover [group the risks according to the lines of reinsurance cover] and determine the loss ratio relating the premium earned in the calendar year to the amount of claim paid during the year [not according to the date of accident occurrence].

It was proposed at the outset of the study to investigate the loss ratio for different groups of non-industrial fire insurance in the Egyptian market on the lines of credibility theory. Because of the difficulty of obtaining data, particularly detailed under-writing statistics for the different groups of this branch, the study is limited to the development of the experience rating formula for the pure premium and obtaining the credibility tables for the minimum number of contracts and claims required for that purpose. Also, for the same reason mentioned, the study is limited to one company of the three direct companies working in the market . Its share of fire insurance business in the market is 33 % (according to the statistics of the Egyptian insurance organisation during the period of investigation).

The data is collected for a period of 10 years (1970-1979). During the period of investigation the rate did not change. The data collected consists of underwriting data and claim data.

6.2-1 Claim data

When a claim is presented to the company, a file would be opened and given a reference number in serial. At the same time the claim would be registered in the reporting book. Any action concerning the claim would be in the claim file [i.e. the history of the claim at the claim file]. When the claim was settled, the amount of settlement is registered at the reporting book and the claim file is closed and archived. All information obtained about claims is gathered from the original claim file. This information can be classified into two groups :

I - Underwriting Information

1 - date of policy in force.
2 - date of termination of the contract.
3 - the sum insured.
4 - amount of premium.
5 - location (we give code 1 for Area A and code 2 for Area B).
6 - occupancy (rate group and category). We give a code number for each rate group and another code for each category within the rate group.
7 - construction. We give codes for the different types of construction.
8 - public or private owned (we give code 1 for private ownership and code 2 for public ownership).

II Claim Information

1 - Date when the accident occurred, date when the accident was reported.

2 - Date of claim settlement.

3 - Total amount of claim consisting of three parts

i - Amount of claim paid = C

ii - Expense directly related to settlement of the claim = E

iii - The recovery from the accident = R

Total amount of claim = C + E - R

4 - Adequacy of the insurance

5 - Cause of the accident (given a code for each cause of accident)

6 - Any outside exposure contributing to the loss (given a code for each outside exposure cause)

Table (6.1) was used for recording this claim information.

6.2-2 Underwriting information

The only sources found for underwriting information are

1 The underwriting department

2 Central fire reinsurance department

The underwriting department does not keep information about the

Form used for gathering of claim data

Table (6.1)

UNDERWRITING INFORMATION										CLAIM INFORMATION							
No of contract	In Force		Location	Construction	Occupancy		Owner -ship	Sum Insured	Premium	Date of Accident	Date of Settle-ment	Claim amount			adequancy of insurance	cause of accident	outside exposure
	from	to			group	sub group						C	E	R			

policy for more than three years and it was found difficult to get the information required in the limited time available. The reinsurance department does keep the underwriting information for a longer period.

Any policy underwritten, renewed, or any action concerning the underwriting is reported to the reinsurance department. The department has four books :

1. New business
2. Renewal
3. Cancellation
4. Alteration

It has been found that the only way to get the required information is through these books. The information obtained was :

1. Date the policy came into force.
2. Date the policy terminated.
3. Sum insured.
4. Premium
5. Location
6. Occupation (rate group)

For many contracts there is difficulty in getting the right information for the sum insured and for the amount of the premium at the same time. The only information we can rely upon is the number of contracts for each group of business which we used in the model developed in chapter (5). In the class of warehouses we face a different problem. The contract may cover more than one warehouse in different locations with a fixed amount for the total cover and maximum amount of cover per warehouse (the contract is subject to monthly reports for the average value of commodities in the warehouse during each month). Neither the number of warehouses nor their locations are in the contract. This situation would suggest that it is necessary to rate the contract according to the individual characteristics.

The total number of contracts during the period of the investigation amounts to 72521 contracts , from this total (66085) belong to the two classes of business studied,1 - construction and content of residential buildings (class I). 2 - small businesses and retail shops (class II).

The number of claims settled or in the process of settlement was 149 for class I and 293 for class II.

6.2-3 Claim reporting

The fire insurance policy requires the insured to give immediate written notice to the insurer within 15 days of the accident at most. The insured should send to the company proof of loss including the time and the origin of loss, the interest of the insured and all other parties in the property , the actual cash value of the items damaged by the fire, the assessment of loss and all other insurance contracts, if any in operation. It was observed that almost all claims were reported within the first and second day from the accident (for the industrial branch the situation is different, especially for small losses). Only two cases were observed which were presented after a long period from the accident occurrence.

6.2-4 Claim settlement

As soon as a claim is reported to the company, the property is inspected by a trained employee from the claims department who assesses the amount of damage and the actual value of the property.

If the amount of the claim is large, the company uses a specialised adjustor for this purpose.

Accidents or damages reported are not claims if the hazard causing the loss is not covered by the contract. This class of reported claims amounted to 10 % of the total number of claims reported.

Most claims were settled near to the claim department's assessment of the amount of the claim. Delay in claim settlement does not have a great effect on the amount of claim [Table (6.2) shows the distribution of the number of claims settled over time during the period of investigation]. The common reasons for delay of claim settlement are

1 - The insured does not agree with the company assessment of the damage.

2 - The property is greatly under-insured and the insured is not satisfied with the company assessment of the actual value of the property.

In most cases, it is found that the settlement is within 10 % of the company's assessment of the loss. From Table (6.2) it can be seen that 66.6 % of claims are settled within 6 months, 23.4 % are settled within a year, and 10 % are settled in a period of more than a year, which corresponds to the percentage of the

Distribution of number of claims settled over time

Table 6.2

Period	Trading shops and small businesses		Construction and content		Total	
	Number	Percentage	Number	Percentage	Number	Percentage
0- 1	29	9.9	26	17.8	55	12.5
1- 2	41	14.0	33	22.6	74	16.9
2- 3	34	11.6	13	8.9	47	10.7
3- 4	37	12.6	22	15.1	59	13.5
4- 5	18	6.1	15	10.3	33	7.5
5- 6	17	5.9	7	4.8	24	5.5
6- 9	46	15.7	13	8.9	59	13.4
9-12	36	12.3	8	5.5	44	10.0
12-18	23	7.8	3	2.1	26	15.9
18 >	12	4.1	6	4.0	18	4.1
	293		146		439	

number of claims of under-insured property during the period.

6.2-5 Treatment of the data for the economic effect

For analysis of data collected over a long period of time (more than one year) it would be appropriate to treat the data to eliminate the economic effect. Economic effect on claim experience can be summarised in

1 - Inflation effect

2 - Change in risk factor

The sum insured and the amount of claim are subject to an inflation effect.

This effect can be eliminated by using a suitable index. For the amount of claim, we used different index numbers for different groups of risk.

For building construction and buildings under construction we used the index number for the construction material. For the content of residential buildings, hotels and public places we used a household index.

For small businesses and retail shops we used the wholesale index number [Table (6.3) shows the different index used and Table (5.4) shows the weight used as a multiplier for each year]. Treatment of the claim data for changes in risk factors is ignored since the rate did not change during the period of investigation.

Price Index

Table 6.3

Year	Building Material	Household	Wholesale Price Index
70	100.0	100	100.0
71	104.5	104	100.4
72	104.5	104	101.2
73	105.3	109	108.1
74	113.0	128	123.6
75	129.1	138	132.9
76	138.9	142	143.3
77	149.1	156	156.7
78	163.7	185	179.7
79	187.8	190	197.0
80	205.8	209	229.0

Source : Egyptian Agency for public mobilisation and statistics - The annual statistical yearbook (1970-1979)

Weight multiplier for each year

Table 6.4

Year	Building Construction	Household	Wholesale Price
70	2.058	2.09	2.296
71	1.969	2.01	2.281
72	1.769	2.01	2.263
73	1.954	1.917	2.118
74	1.821	1.633	1.853
75	1.594	1.514	1.723
76	1.482	1.472	1.598
77	1.380	1.340	1.461
78	1.257	1.130	1.274
79	1.096	1.1	1.162
80	1.000	1.00	1.00

6.3 Preliminary statistical study for experience rating

The distribution of the amount paid, by frequency and by amount for the two classes studied are given in Table (6.5) and Table (6.6).

The important feature that emerges from Table (6.5) and (6.6) is that the distribution underlying the data is highly skew.

Many probability functions were tried to find a good fit to the observed distribution of the claim amounts.

Finally, the following function was suggested for class I

$$f(s) = \frac{1}{B(a,b)} x^{a-1} (1-x)^{b-1}$$

where a = ·34 and b =·4 .

The area under the distribution function is approximated by the following formula :

$$F(x < X) = \frac{1}{B(\alpha,b)} \int_0^X x^{\alpha-1}(1-x)^{b-1}$$

$$= \frac{1}{B(\alpha,b)} \int_0^X \sum_{i=0}^{i=b-1} \binom{b-1}{0} x^{\alpha+i-1}$$

$$= \frac{1}{B(a,b)} \sum_{i=0}^{b-1} \binom{b-i}{0} \frac{X^{\alpha+i}}{\alpha+i}$$

Table (6.7) presents the observed and expected frequencies. An X^2 test is used to examine the goodness of the fit

$$\chi^2 = \sum \frac{(E-0)^2}{E} = 11.6$$

with 7 degrees of freedom ; the test confirms that the fit is not very bad.

For class II, many distribution functions were tried, but we could not find any one to fit the data because of the irregularity of the frequency.

For class I - An important feature emerges from Table (6.5) - that the number of claims exceeding the amount £E1500 is 11% of the total number of claims for this class, but these account for over 60% of the total amount paid. The largest 8 claims account for about 40% of the total amount paid.

At the same time 89% of the amount of claims are less than or equal to £E1500. It is decided to compute the primary claim amount by using the multisplit formula

$$S_p = S_o\left(\frac{1-a^r}{1-a}\right) + Ra^r \qquad (6.3\text{-}1)$$

where $S_o = 1500$

$r = \frac{S}{S_o}$ (S is the observed amount of claim)

$R = S - rS_o$

$a = 0.6$

For class II - An important feature emerges from Table (6.6) - that 10.85% of the total number of claims exceed £E8000, but account for about 60% of the total amount of claims. The largest 8 claims

Observed frequency distribution for class I

Table 6.5

Interval	Frequency of no. of claims		Total amount of claim paid	
	No. of claims	Percentage	Total amount	Percentage
0- 100	56	38.4	2630.9	3.25
100- 200	25	17.1	3587.7	4.43
200- 300	13	8.9	3278.9	4.05
300- 600	18	12.3	6926.4	8.55
600-1000	10	6.8	6318.7	7.79
1000-1500	8	5.5	9300.2	11.48
1500-2500	8	5.5	16783.5	20.71
2500 >	8	5.5	3220.10	39.74
	146	100.00	81027.4	100.00

Mean = 519.863

Variance = 611051.016

Observed frequency distribution of claims for class II

Table 6.6

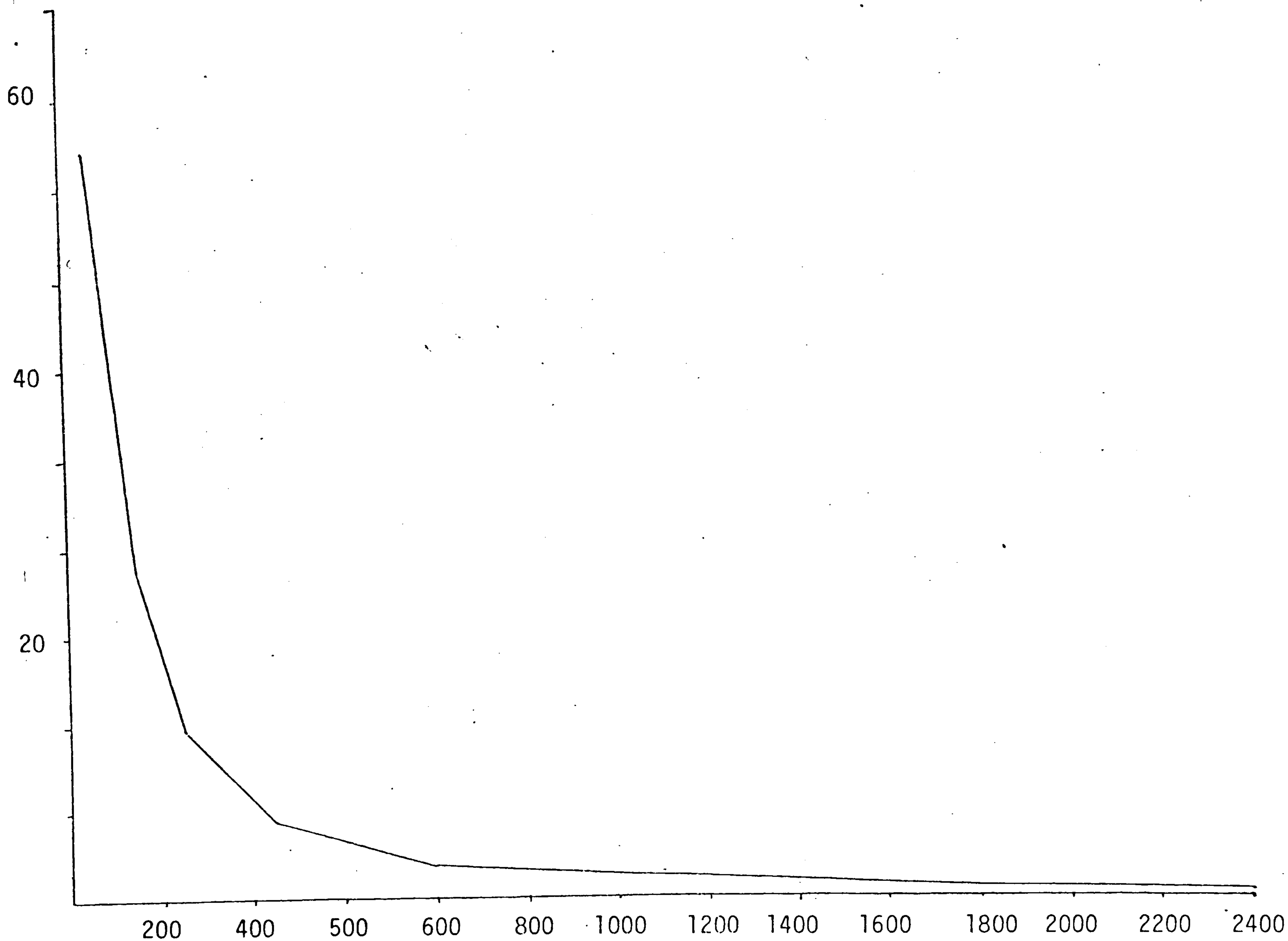
Interval	Number of claims		Amount of claims	
	Frequency	Percentage	Total amount of claims	Percentage
1- 200	79	26.96	6539.2	.81
200- 400	36	12.28	10728.8	1.33
400- 600	29	9.90	14375.5	1.78
600- 1200	25	8.53	23167.1	2.87
1200- 1800	26	8.87	27086.4	4.59
1800- 2400	20	5.83	41745.7	5.17
2400- 3000	13	4.44	34119.9	4.22
3000- 4000	13	4.44	45943.4	5.64
4000- 5000	10	3.41	44784.7	5.55
5000- 6000	5	1.71	25630.4	3.17
6000- 8000	6	2.05	40506.3	5.02
8000-10000	10	3.41	85918.1	10.64
1000-150000	6	2.05	72479.7	8.98
15000-20000	7	2.39	119456.2	14.79
20000 >	8	2.73	205091.0	25.39
Total	293	1.00	807572.4	100.00

mean = 2796.2491

standard deviation = 5128.582

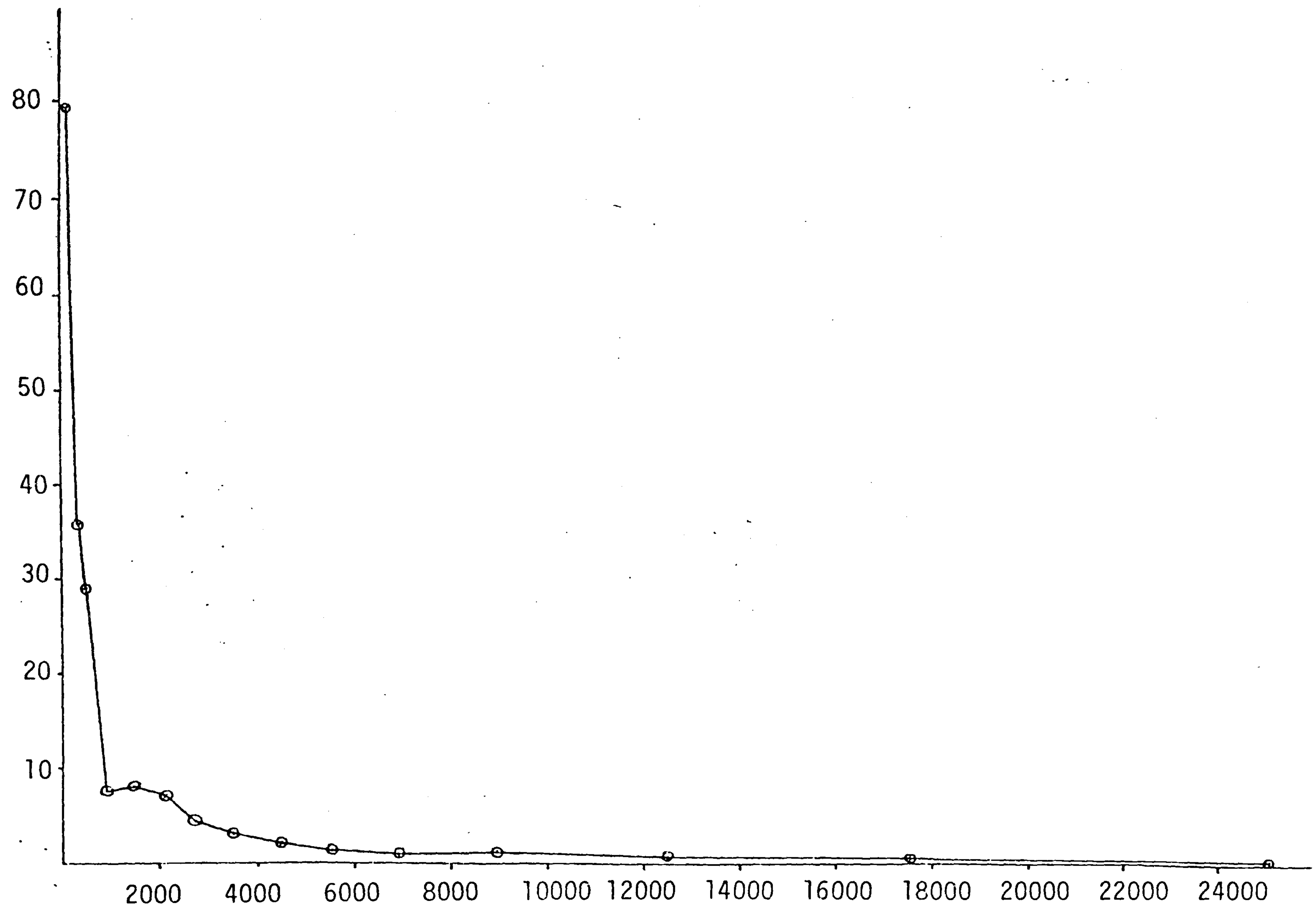
Frequency Curve for Class I

Graph (1)



Frequency Curve for Class II

Graph (2)



Observed and Expected frequency for the class
of building construction and content

Table 6.7

	Probability		Frequency	
	Observed	Expected	Observed	Expected
0- 100	38.4	39.0	56	56.94
100- 200	17.1	9.9	25	14.45
200- 300	8.9	6.7	13	9.78
300- 600	12.3	13.8	18	20.15
600- 800	6.8	9.1	10	13.29
1000-1500	5.5	7.6	8	11.09
1500-2500	5.5	8.1	8	11.83
2500 >	5.5	5.8	8	8.47
			146	146

The mean and variance for groups of Class I

Table 6.8

group*	No. of contract	No. of claims	$\pi(v_1)$ Mean of X	$\pi(v_1)[1-\pi(v_1)]$ Variance of X	$\mu(v_2)$ Mean of S_p	$\sigma(v_2)$ Variance S_p
group (1)	14623	38	0.0026	0.00259	573.228	586789.336
group (2)	9214	48	0.0052	0.00518	198.506	131722.484
group (3)	2820	10	0.0035	0.00353	935.756	722812.532
group (4)	9161	25	0.0027	0.00272	432.535	295946.727
group (5)	1628	25	0.0154	0.01512	660.263	920378.516
	37446	146	0.0039	0.00388	465.675	453814.578

group (1) building construction
group (2) content of offices and residential flats
group (3) building under construction
group (4) public places
group (5) hotels

The mean and variance for groups of Class II

Table 6.9

group*	No. of contract	No. of claims	$\pi(v_1)$ Mean of X	$\pi(v_1)[1-\pi(v_1)]$ Variance of X	$\mu(v_2)$ Mean of S_p	$\sigma(v_2)$ Variance
group (1)	6090	44	.0072	.9928	2136.011	13906582.250
group (2)	12636	142	.0112	.9888	1394.518	11359272.563
group (3)	7204	56	.0078	.9922	4165.603	121904533.000
group (4)	1412	26	.0184	.9816	4941.223	28677650.750
group (5)	1297	25	.0193	.9807	4832.534	35190485.500
Total	28639	293	.0102	.9898	2643.534	33047581.250

according to the rate groups in Appendix (2.2) account for over 25% of the total amount paid.

The multisplit formula (6.3-1) is used to compute the primary for class II with

$$S_o = 8000$$

$$a = 0.6$$

The number of contracts in force during the period of investigation, number of claims and the mean and variance of the two variables of the pure premiums x and S, for each group within the two classes (S represents the primary) are presented in Tables (6.8) and (6.9).

6.3.1 Statistical test for homogeneity of the claim experience over the groups of each class :

Three statistical tests suggested for this purpose are

I Test of the homogeneity of claim occurence

II Test of homogeneity of the variation of the amount of claim

III Test of homogeneity of average amount of claim

I - Contingency analysis for claim frequency

For each class, we test the hypothesis $\pi_{ij}(v_1) = \pi_{kj}(v_1)$ by testing the independence of the following contingency table [Everitt 1977].

group	no. of contract with claim	no. of contract without claim	total
(1)	38	14585	14623
(2)	48	9166	9214
(3)	10	2810	2820
(4)	25	9136	9161
(5)	25	1603	1628
Total	146	37300	37446

group	no. of contract with claim	no. of contract without claim	total
(1)	44	6046	6090
(2)	142	12494	12636
(3)	56	7148	72024
(4)	26	1386	1412
(5)	25	1272	1297
Total	293	28346	28639

The test statistic is

$$X^2 = \sum_{i=1}^{5} \sum_{j=1}^{2} \frac{(n_{ij}-E_{ij})^2}{E_{ij}}$$

where $i = 1,2,3,4,5$ for the group

$j = 1,2$ with claim, without claim

$n_{ij} =$ the number of the observation in row i in Column 5

$E_{i1} = P_{i.} \times n_{i.}$ $\quad P_{i.} = \frac{\text{number of claims}}{\text{total number of contracts}}$

$E_{i2} = P_{.j} \times n_{.j}$ $\quad P_{.j} = \frac{\text{number of contracts without claim}}{\text{total number of contracts}}$

$P_{i.} + P_{.j} = 1$

For class I $\chi = 74.227$, with 4 degrees of freedom; the test is significant at all levels.

For class II $\chi^2 = 31.0613$, with 4 degrees of freedom; the test is significant at all levels.

II Bartlett test for the homogeneity of the variance

The hypothesis tested is $\sigma_i^2(v_2) = \sigma_j^2(v_2)$. This test is suggested by [Bryant 1960] . For simplicity let $\sigma_i(v_2) = \sigma_i$, the test statistic is

$$\chi^2 = \frac{\left[\sum_{i=1}^{5} (n_i-1)\text{lin}\,\bar{\sigma} - \sum_{i=1}^{5} (n_i-1)\sigma_i^2\right]}{1 + \frac{1}{3(k-1)}\left[\sum_{i=1}^{5} \frac{1}{n_i-1} - \frac{1}{\sum_{i=1}^{n} (n_i-1)}\right]}$$

where k = 5

n_i = the number of claims in group i

σ_i = the variance of claims in group i

$$\bar{\sigma} = \frac{\sum (n_i \ \sigma_i^2)}{\sum n_i-1}$$

For class I $\chi^2 = 39.17$, with 4 degrees of freedom; the test was significant at all levels .

For class II $\chi^2 = 251.43$, with 4 degrees of freedom; the test was significant at all levels.

III Non exact test for the homogeneity of mean when the variances are not equal

This test is suggested by Snedecor (1958) . The hypothesis tested is $\mu_i(v_2) = \mu_j(v_2)$ and $\sigma_i \neq \sigma_j$, for simplicity.

Let $\mu_i(v_2) = \mu_i$. The test statistic is

$$F(\nu_1,\nu_2) = \sum_{i=1}^{5} \frac{\dfrac{c_i(\mu_i-\mu_c)^2}{k-1}}{1 + \dfrac{2(k-2)}{k^2-1} \sum \left[\dfrac{1-c_i/\sum c_i)^2}{n_i-1}\right]}$$

where k = number of groups

$$c_i = \frac{n_i}{\sigma_i^2}$$

$$c = \sum_{i=1}^{5} \frac{c_i\mu_i}{\sum_{i=1}^{5} c_i}$$

$$\nu_1 = k-1$$

$$\nu_2 = \frac{1}{\dfrac{3}{(k^2-1)} \left[\sum \dfrac{(1-c_i/\sum_{i=1}^{5} c_i)^2}{n_i-1}\right]}$$

For class I $F(\nu_1,\nu_2) = 4$ with $\nu_1 = 4$ and $\nu_2 = 40.65$.

The test is significant at all levels.

For class II $F(\nu_1,\nu_2) = 5.547$ with $\nu_1 = 4$ and $\nu_2 = 69.44$

The test is significant at all levels.

6.4 Experience Rating

In Chapter 5, the following credibility formula was derived.

$$[\pi_{\pi\mu} \overline{XS} + (1 - z_{\pi\mu})\pi\mu]$$

$$+ \pi B[z_{\sigma^2} \sum^2 + (1 - z_{\sigma^2})\sigma^2]$$

$$+ \mu^2 B[z_{\pi(1-\pi)}\gamma^2 + (1 - z_{\pi(1-\pi)})\pi(1-\pi)]$$

with

$$z_{\pi\mu} = \frac{n}{(n-1)+k_{\pi\mu}}$$

$$k_{\pi\mu} = \frac{\pi\sigma^2 + \mu^2\pi(1-\pi)}{\pi \mathrm{var}[\mu(v_2)] + \mu^2 \mathrm{var}[\pi(v_1)]}$$

Where n is the number of contracts ,

$$z_{\sigma^2} = \frac{(n'-1)}{(n'-1)+2k_{\sigma^2}}$$

$$k_{\sigma^2} = \frac{E[\sigma^4(v_2)]}{\mathrm{var}[\sigma^2(v_2)]}$$

Where n' is the number of claims,

$$z_{\pi(1-\pi)} = \frac{(n-1)}{(n-1)+2k_{\pi(1-\pi)}}$$

$$k_{\pi(1-\pi)} = \frac{E[\pi(v_1)(1-\pi(v_1)]^2}{\mathrm{var}[\pi(v_1)(1-\pi(v_1)]}$$

Table (6.4) and Table (6.5) are used for estimating the various parameters of this formula. The parameters for the three k formula are calculated and presented in Table (6.6).

Using the results in Table (6.6), the formula for the credibility coefficients for class I are

$$Z_{\pi\mu} = \frac{n}{n+12.75918}$$

$$Z_{\sigma^2} = \frac{n'-1}{n'+5.834}$$

$$Z_{\pi(1-\pi)} = \frac{n-1}{n+3.5804}$$

For class II they are

$$Z_{\pi\mu} = \frac{n}{n+17.833}$$

$$Z_{\sigma^2} = \frac{n'}{n'+2.1158}$$

$$Z_{\pi(1-\pi)} = \frac{n}{n+20.7677}$$

In Appendix (6-1) detailed tables are given for the three credibility coefficients and n(n') for each class. The tables can be used to know the minimum n(n') for given levels of credibility, and also to determine the credibility coefficient for a given n(n'). These tables are left in crude form and can be modified in the light of such specifications as the level of self-rating.

Table (6.11) and (6.12) give a summary of the relation between (z,n) for classes I and II respectively, and graphs (6.3) and (6.4) present the relationship between the credibility coefficients and n(n').

The value of the parameters of k formulas

Table 6.10

parameter	Class I	Class II
μ	465.675	2643.534
π	0.0039	0.0102
$var[\mu(v_2)]$	$48.285\ 10^{3}$	$21.14706\ 10^{5}$
$var[\pi(v_1)]$	$0.71342\ 10^{-5}$	$0.108953\ 10^{-4}$
$var[\sigma^2(v_2)]$	$85.20759\ 10^{9}$	$19.65023\ 10^{14}$
$E[\sigma^4(v_2)]$	$29.11526\ 10^{10}$	$20.57166\ 10^{14}$
$var[\pi(v_1)(1-\pi(v_1))]$	$0.68958\ 10^{-5}$	$0.10352\ 10^{-4}$
$E[\pi(v_1)(1-\pi(v_1))]^2$	$0.21924\ 10^{-4}$	$0.11267\ 10^{-3}$
$k_{\pi\mu}$	13.75918	18.832
k_{σ^2}	3.41701	1.55579
$k_{\pi(1-\pi)}$	3.17932	10.88388

Relation between n(n') and z for Class I

Table 6.11

z	n for $z_{\pi\mu}$	n' for z_{σ^2}	n for $z_{\pi(1-\pi)}$
0.1	1	1	1
0.2	3	2	2
0.3	5	3	3
0.4	8	5	5
0.5	12	7	7
0.6	18	11	10
0.7	29	16	15
0.8	50	28	26
0.9	112	62	58
0.99	1240	677	630
0.999	12525	6832	6357

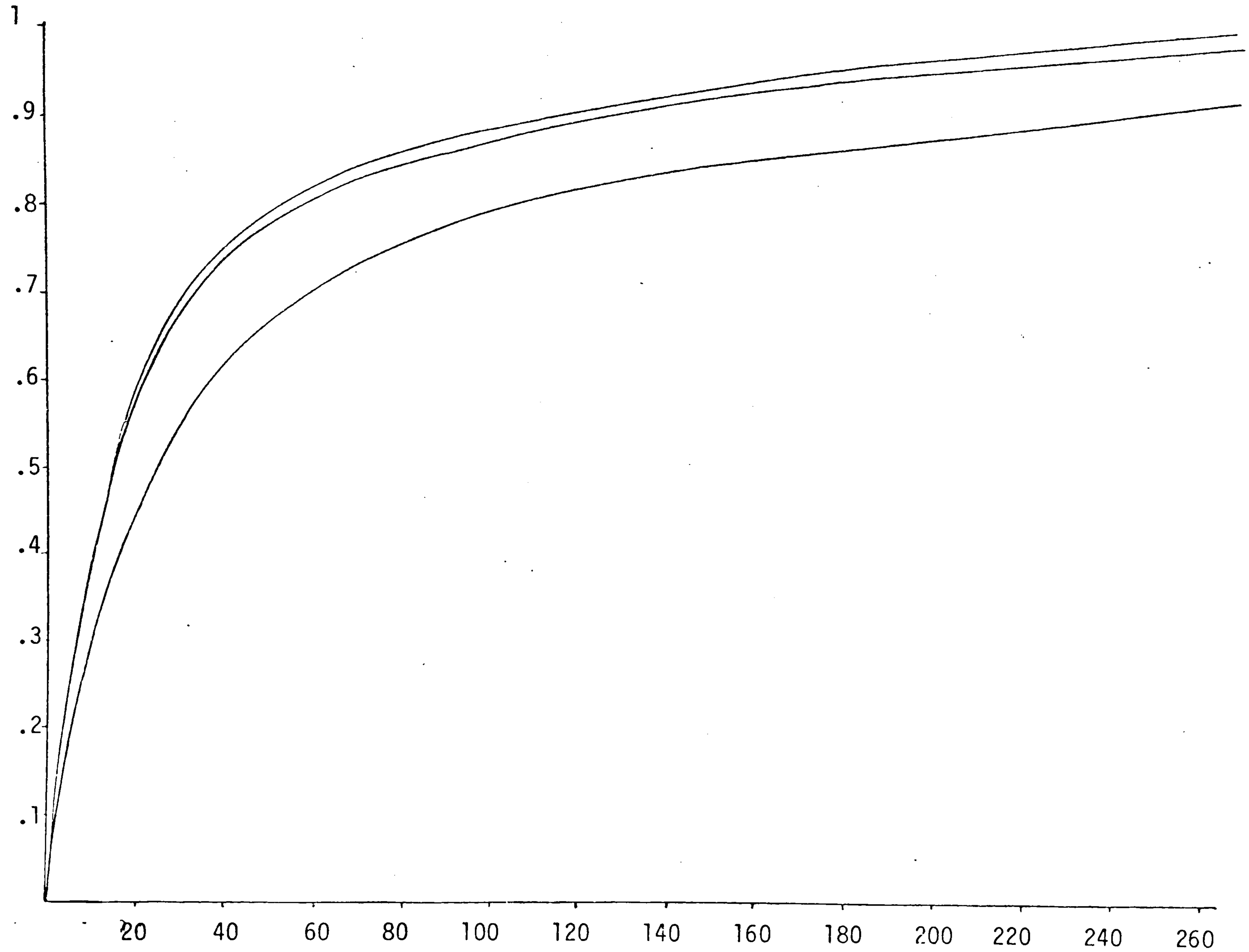
Relation between n(n') and z for Class II

Table 6.12

z	n for $z_{\pi\mu}$	n' for z_{σ^2}	n for $z_{\pi(1-\pi)}$
0.1	1	1	1
0.2	4	1	6
0.3	7	2	10
0.4	11	3	15
0.5	17	4	22
0.6	25	5	33
0.7	40	8	51
0.8	68	13	88
0.9	155	29	196
0.99	1706	309	2156
0.999	17228	3111	21760

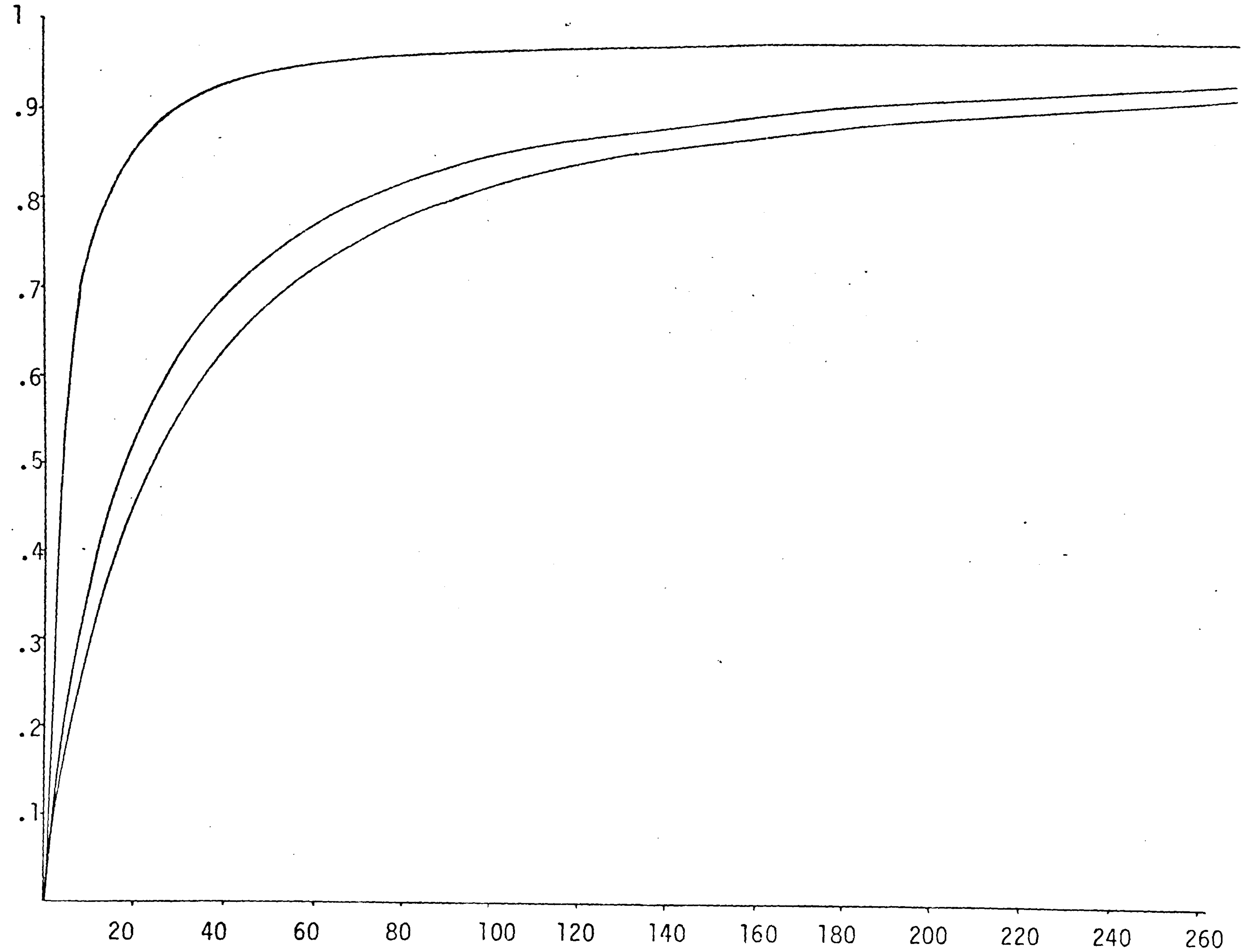
Credibility curves for Class I

Graph 6.3



Credibility curves for Class II

Graph 6.4



APPENDIX (6-1)

TABLE (1)

$$z_{\pi\mu} = \frac{n}{n + 17.833}$$

Class I

$z_{\pi\mu}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.	0	0	0	0	0	0	0	0	0	0
0.01	0	0	0	0	0	0	0	0	0	0
0.02	0	0	0	0	0	0	0	0	0	0
0.03	0	0	0	0	0	0	0	0	0	0
0.04	0	0	0	0	0	0	0	0	0	0
0.05	0	0	0	0	0	0	0	0	0	0
0.06	0	0	0	0	0	0	0	0	0	0
0.07	0	0	0	0	1	1	1	1	1	1
0.08	1	1	1	1	1	1	1	1	1	1
0.09	1	1	1	1	1	1	1	1	1	1
0.10	1	1	1	1	1	1	1	1	1	1
0.11	1	1	1	1	1	1	1	1	1	1
0.12	1	1	1	1	1	1	1	1	1	1
0.13	1	1	1	1	1	1	1	1	2	2
0.14	2	2	2	2	2	2	2	2	2	2
0.15	2	2	2	2	2	2	2	2	2	2
0.16	2	2	2	2	2	2	2	2	2	2
0.17	2	2	2	2	2	2	2	2	2	2
0.18	2	2	2	2	2	2	2	2	2	2
0.19	2	2	2	2	3	3	3	3	3	3
0.20	3	3	3	3	3	3	3	3	3	3

1

TABLE (1) continued

$z_{\pi\mu}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.21	3	3	3	3	3	3	3	3	3	3
0.22	3	3	3	3	3	3	3	3	3	3
0.23	3	3	3	3	3	3	3	3	3	3
0.24	3	3	4	4	4	4	4	4	4	4
0.25	4	4	4	4	4	4	4	4	4	4
0.26	4	4	4	4	4	4	4	4	4	4
0.27	4	4	4	4	4	4	4	4	4	4
0.28	4	4	4	4	4	4	5	5	5	5
0.29	5	5	5	5	5	5	5	5	5	5
0.30	5	5	5	5	5	5	5	5	5	5
0.31	5	5	5	5	5	5	5	5	5	5
0.32	5	5	5	5	6	6	6	6	6	6
0.33	6	6	6	6	6	6	6	6	6	6
0.34	6	6	6	6	6	6	6	6	6	6
0.35	6	6	6	6	6	6	6	6	6	7
0.36	7	7	7	7	7	7	7	7	7	7
0.37	7	7	7	7	7	7	7	7	7	7
0.38	7	7	7	7	7	7	7	7	7	7
0.39	8	8	8	8	8	8	8	8	8	8
0.40	8	8	8	8	8	8	8	8	8	8
0.41	8	8	8	8	8	8	8	8	8	9
0.42	9	9	9	9	9	9	9	9	9	9
0.43	9	9	9	9	9	9	9	9	9	9
0.44	9	9	9	9	10	10	10	10	10	10
0.45	10	10	10	10	10	10	10	10	10	10
0.46	10	10	10	10	10	10	10	10	11	11
0.47	11	11	11	11	11	11	11	11	11	11
0.48	11	11	11	11	11	11	11	11	11	11
0.49	12	12	12	12	12	12	12	12	12	12
0.50	12	12	12	12	12	12	12	12	12	12

TABLE (1) continued

$z_{\pi\mu}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	13	13	13	13	13	13	13	13	13	13
0.52	13	13	13	13	13	13	13	13	14	14
0.53	14	14	14	14	14	14	14	14	14	14
0.54	14	14	14	14	14	15	15	15	15	15
0.55	15	15	15	15	15	15	15	15	15	15
0.56	15	16	16	16	16	16	16	16	16	16
0.57	16	16	16	16	16	16	17	17	17	17
0.58	17	17	17	17	17	17	17	17	17	17
0.59	18	18	18	18	18	18	18	18	18	18
0.60	18	18	18	19	19	19	19	19	19	19
0.61	19	19	19	19	19	20	20	20	20	20
0.62	20	20	20	20	20	20	20	21	21	21
0.63	21	21	21	21	21	21	21	21	22	22
0.64	22	22	22	22	22	22	22	22	23	23
0.65	23	23	23	23	23	23	23	24	24	24
0.66	24	24	24	24	24	24	24	25	25	25
0.67	25	25	25	25	25	26	26	26	26	26
0.68	26	26	26	26	27	27	27	27	27	27
0.69	27	28	28	28	28	28	28	28	28	29
0.70	29	29	29	29	29	29	30	30	30	30
0.71	30	30	30	31	31	31	31	31	31	32
0.72	32	32	32	32	32	33	33	33	33	33
0.73	33	34	34	34	34	34	34	35	35	35
0.74	35	35	36	36	36	36	36	36	37	37
0.75	37	37	37	38	38	38	38	39	39	39
0.76	39	39	40	40	40	40	41	41	41	41
0.77	41	42	42	42	42	43	43	43	43	44
0.78	44	44	44	45	45	45	46	46	46	46
0.79	47	47	47	48	48	48	48	49	49	49
0.80	50	50	50	51	51	51	52	52	52	53

TABLE (1) continued

$z_{\Pi u}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.81	53	53	54	54	54	55	55	55	56	56
0.82	57	57	57	58	58	59	59	59	60	60
0.83	61	61	62	62	62	63	63	64	64	65
0.84	65	66	66	67	67	68	68	69	69	70
0.85	71	71	72	72	73	73	74	75	75	76
0.86	76	77	78	78	79	80	80	81	82	83
0.87	83	84	85	86	86	87	88	89	90	91
0.88	91	92	93	94	95	96	97	98	99	100
0.89	101	102	103	104	105	106	107	109	110	111
0.90	112	114	115	116	117	119	120	122	123	125
0.91	126	128	129	131	133	134	136	138	140	142
0.92	144	146	148	150	152	154	156	159	161	163
0.93	166	169	171	174	177	180	183	186	189	192
0.94	196	199	203	207	211	215	219	223	228	233
0.95	238	243	248	254	259	265	272	278	285	293
0.96	300	308	317	326	335	345	356	367	379	391
0.97	405	419	434	451	469	488	509	532	557	584
0.98	614	646	683	724	770	822	882	951	1031	1126
0.99	1240	1379	1553	1777	2076	2493	3120	4165	6254	12525

1

TABLE (2)

$$z_\sigma^2 = \frac{n'-1}{n'+5.3534}$$ Class I

z_σ^2	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.	1	1	1	1	1	1	1	1	1	1
0.01	1	1	1	1	1	1	1	1	1	1
0.02	1	1	1	1	1	1	1	1	1	1
0.03	1	1	1	1	1	1	1	1	1	1
0.04	1	1	1	1	1	1	1	1	1	1
0.05	1	1	1	1	1	1	1	1	1	1
0.06	1	1	1	1	1	1	1	1	1	1
0.07	1	1	1	1	1	1	1	1	1	1
0.08	1	1	1	1	1	1	1	1	1	1
0.09	1	1	1	1	1	1	1	1	1	1
0.10	1	1	1	1	1	1	1	1	1	1
0.11	1	1	1	1	1	1	1	1	1	1
0.12	1	1	1	1	1	1	1	1	2	2
0.13	2	2	2	2	2	2	2	2	2	2
0.14	2	2	2	2	2	2	2	2	2	2
0.15	2	2	2	2	2	2	2	2	2	2
0.16	2	2	2	2	2	2	2	2	2	2
0.17	2	2	2	2	2	2	2	2	2	2
0.18	2	2	2	2	2	2	2	2	2	2
0.19	2	2	2	2	2	2	2	2	2	2
0.20	2	2	2	2	2	2	2	2	2	2

TABLE (2) continued

Z_{σ}^2	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.21	2	2	2	2	2	2	2	2	2	2
0.22	2	2	2	2	2	2	2	3	3	3
0.23	3	3	3	3	3	3	3	3	3	3
0.24	3	3	3	3	3	3	3	3	3	3
0.25	3	3	3	3	3	3	3	3	3	3
0.26	3	3	3	3	3	3	3	3	3	3
0.27	3	3	3	3	3	3	3	3	3	3
0.28	3	3	3	3	3	3	3	3	3	3
0.29	3	3	3	3	3	3	3	3	3	3
0.30	3	3	3	3	3	3	4	4	4	4
0.31	4	4	4	4	4	4	4	4	4	4
0.32	4	4	4	4	4	4	4	4	4	4
0.33	4	4	4	4	4	4	4	4	4	4
0.34	4	4	4	4	4	4	4	4	4	4
0.35	4	4	4	4	4	4	4	4	4	4
0.36	4	4	4	4	4	4	4	4	4	4
0.37	5	5	5	5	5	5	5	5	5	5
0.38	5	5	5	5	5	5	5	5	5	5
0.39	5	5	5	5	5	5	5	5	5	5
0.40	5	5	5	5	5	5	5	5	5	5
0.41	5	5	5	5	5	5	5	5	5	5
0.42	5	5	5	6	6	6	6	6	6	6
0.43	6	6	6	6	6	6	6	6	6	6
0.44	6	6	6	6	6	6	6	6	6	6
0.45	6	6	6	6	6	6	6	6	6	6
0.46	6	6	6	6	6	6	6	6	7	7
0.47	7	7	7	7	7	7	7	7	7	7
0.48	7	7	7	7	7	7	7	7	7	7
0.49	7	7	7	7	7	7	7	7	7	7
0.50	7	7	7	7	7	7	8	8	8	8

TABLE (2) continued

z_σ^2	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	8	8	8	8	8	8	8	8	8	8
0.52	8	8	8	8	8	8	8	8	8	8
0.53	8	8	8	8	8	8	8	8	8	8
0.54	9	9	9	9	9	9	9	9	9	9
0.55	9	9	9	9	9	9	9	9	9	9
0.56	9	9	9	9	9	9	9	9	9	10
0.57	10	10	10	10	10	10	10	10	10	10
0.58	10	10	10	10	10	10	10	10	10	10
0.59	10	10	10	10	10	11	11	11	11	11
0.60	11	11	11	11	11	11	11	11	11	11
0.61	11	11	11	11	11	11	11	12	12	12
0.62	12	12	12	12	12	12	12	12	12	12
0.63	12	12	12	12	12	12	12	12	13	13
0.64	13	13	13	13	13	13	13	13	13	13
0.65	13	13	13	13	13	13	14	14	14	14
0.66	14	14	14	14	14	14	14	14	14	14
0.67	14	14	15	15	15	15	15	15	15	15
0.68	15	15	15	15	15	15	15	15	16	16
0.69	16	16	16	16	16	16	16	16	16	16
0.70	16	17	17	17	17	17	17	17	17	17
0.71	17	17	17	17	18	18	18	18	18	18
0.72	18	18	18	18	18	19	19	19	19	19
0.73	19	19	19	19	19	19	20	20	20	20
0.74	20	20	20	20	20	20	21	21	21	21
0.75	21	21	21	21	21	22	22	22	22	22
0.76	22	22	22	23	23	23	23	23	23	23
0.77	23	24	24	24	24	24	24	24	24	25
0.78	25	25	25	25	25	25	26	26	26	26
0.79	26	26	27	27	27	27	27	27	27	28
0.80	28	28	28	28	29	29	29	29	29	29

TABLE (2) continued

z_{σ}^2	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.81	30	30	30	30	30	31	31	31	31	31
0.82	32	32	32	32	32	33	33	33	33	34
0.83	34	34	34	35	35	35	35	36	36	36
0.84	36	37	37	37	37	38	38	38	39	39
0.85	39	40	40	40	40	41	41	41	42	42
0.86	42	43	43	44	44	44	45	45	45	46
0.87	46	47	47	47	48	48	49	49	50	50
0.88	51	51	52	52	53	53	54	54	55	55
0.89	56	56	57	58	58	59	59	60	61	61
0.90	62	63	63	64	65	66	66	67	68	69
0.91	70	70	71	72	73	74	75	76	77	78
0.92	79	80	81	82	84	85	86	87	89	90
0.93	91	93	94	96	97	99	100	102	104	106
0.94	108	109	111	114	116	118	120	123	125	128
0.95	130	133	136	139	142	146	149	153	156	160
0.96	165	169	174	178	184	189	195	201	207	214
0.97	221	229	238	247	257	267	278	291	304	319
0.98	335	353	373	396	421	449	482	519	563	615
0.99	677	753	848	970	1133	1361	1702	2272	3412	6832

TABLE (3)

$$z_{\pi(1-\pi)} = \frac{n-1}{n+3.5804} \qquad \text{Class I}$$

$z_{\pi(1-\pi)}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.	1	1	1	1	1	1	1	1	1	1
0.01	1	1	1	1	1	1	1	1	1	1
0.02	1	1	1	1	1	1	1	1	1	1
0.03	1	1	1	1	1	1	1	1	1	1
0.04	1	1	1	1	1	1	1	1	1	1
0.05	1	1	1	1	1	1	1	1	1	1
0.06	1	1	1	1	1	1	1	1	1	1
0.07	1	1	1	1	1	1	1	1	1	1
0.08	1	1	1	1	1	1	1	1	1	1
0.09	1	1	1	1	1	1	1	1	1	1
0.10	1	1	1	1	1	1	1	1	1	1
0.11	1	1	1	1	1	1	1	1	1	1
0.12	1	1	1	1	1	1	1	1	1	1
0.13	1	1	1	1	1	1	2	2	2	2
0.14	2	2	2	2	2	2	2	2	2	2
0.15	2	2	2	2	2	2	2	2	2	2
0.16	2	2	2	2	2	2	2	2	2	2
0.17	2	2	2	2	2	2	2	2	2	2
0.18	2	2	2	2	2	2	2	2	2	2
0.19	2	2	2	2	2	2	2	2	2	2
0.20	2	2	2	2	2	2	2	2	2	2

1

TABLE (3) continued

$z_{\pi(1-\pi)}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.21	2	2	2	2	2	2	2	2	2	2
0.22	2	2	2	2	2	2	2	2	2	2
0.23	2	2	2	2	2	2	2	2	2	2
0.24	3	3	3	3	3	3	3	3	3	3
0.25	3	3	3	3	3	3	3	3	3	3
0.26	3	3	3	3	3	3	3	3	3	3
0.27	3	3	3	3	3	3	3	3	3	3
0.28	3	3	3	3	3	3	3	3	3	3
0.29	3	3	3	3	3	3	3	3	3	3
0.30	3	3	3	3	3	3	3	3	3	3
0.31	3	3	3	3	3	3	3	3	3	3
0.32	3	4	4	4	4	4	4	4	4	4
0.33	4	4	4	4	4	4	4	4	4	4
0.34	4	4	4	4	4	4	4	4	4	4
0.35	4	4	4	4	4	4	4	4	4	4
0.36	4	4	4	4	4	4	4	4	4	4
0.37	4	4	4	4	4	4	4	4	4	4
0.38	4	4	4	4	4	4	4	5	5	5
0.39	5	5	5	5	5	5	5	5	5	5
0.40	5	5	5	5	5	5	5	5	5	5
0.41	5	5	5	5	5	5	5	5	5	5
0.42	5	5	5	5	5	5	5	5	5	5
0.43	5	5	5	5	5	5	5	5	5	5
0.44	5	6	6	6	6	6	6	6	6	6
0.45	6	6	6	6	6	6	6	6	6	6
0.46	6	6	6	6	6	6	6	6	6	6
0.47	6	6	6	6	6	6	6	6	6	6
0.48	6	6	6	6	6	6	7	7	7	7
0.49	7	7	7	7	7	7	7	7	7	7
0.50	7	7	7	7	7	7	7	7	7	7

TABLE (3) continued

$Z_{\pi(1-\pi)}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	7	7	7	7	7	7	7	7	7	7
0.52	7	7	7	7	7	8	8	8	8	8
0.53	8	8	8	8	8	8	8	8	8	8
0.54	8	8	8	8	8	8	8	8	8	8
0.55	8	8	8	8	8	8	8	8	9	9
0.56	9	9	9	9	9	9	9	9	9	9
0.57	9	9	9	9	9	9	9	9	9	9
0.58	9	9	9	9	9	9	10	10	10	10
0.59	10	10	10	10	10	10	10	10	10	10
0.60	10	10	10	10	10	10	10	10	10	10
0.61	10	10	11	11	11	11	11	11	11	11
0.62	11	11	11	11	11	11	11	11	11	11
0.63	11	11	11	11	12	12	12	12	12	12
0.64	12	12	12	12	12	12	12	12	12	12
0.65	12	12	12	12	13	13	13	13	13	13
0.66	13	13	13	13	13	13	13	13	13	13
0.67	13	13	14	14	14	14	14	14	14	14
0.68	14	14	14	14	14	14	14	14	15	15
0.69	15	15	15	15	15	15	15	15	15	15
0.70	15	15	15	16	16	16	16	16	16	16
0.71	16	16	16	16	16	16	17	17	17	17
0.72	17	17	17	17	17	17	17	17	18	18
0.73	18	18	18	18	18	18	18	18	18	19
0.74	19	19	19	19	19	19	19	19	19	19
0.75	20	20	20	20	20	20	20	20	20	21
0.76	21	21	21	21	21	21	21	21	22	22
0.77	22	22	22	22	22	22	23	23	23	23
0.78	23	23	23	23	24	24	24	24	24	24
0.79	24	25	25	25	25	25	25	25	26	26
0.80	26	26	26	26	27	27	27	27	27	27

TABLE (3) continued

$Z_{\pi(1-\pi)}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.81	28	28	28	28	28	29	29	29	29	29
0.82	29	30	30	30	30	30	31	31	31	31
0.83	32	32	32	32	32	33	33	33	33	34
0.84	34	34	34	35	35	35	35	36	36	36
0.85	37	37	37	37	38	38	38	39	39	39
0.86	40	40	40	41	41	41	42	42	42	43
0.87	43	43	44	44	45	45	45	46	46	47
0.88	47	48	48	48	49	49	50	50	51	51
0.89	52	52	53	54	54	55	55	56	56	57
0.90	58	58	59	60	60	61	62	63	63	64
0.91	65	66	66	67	68	69	70	71	72	73
0.92	74	75	76	77	78	79	80	81	82	84
0.93	85	86	88	89	90	92	93	95	97	98
0.94	100	102	104	106	108	110	112	114	116	119
0.95	121	124	127	129	132	135	139	142	146	149
0.96	153	157	161	166	171	176	181	187	193	199
0.97	206	213	221	230	239	248	259	271	283	297
0.98	312	329	347	368	392	418	448	483	524	572
0.99	630	701	789	903	1054	1266	1584	2114	3174	6357

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TABLE (4)

$z_{\pi\mu} = \frac{n}{n+17.833}$ Class II

$z_{\pi\mu}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.	0	0	0	0	0	0	0	0	0	0
0.01	0	0	0	0	0	0	0	0	0	0
0.02	0	0	0	0	0	0	0	0	0	0
0.03	0	0	0	0	0	0	0	0	0	0
0.04	0	0	0	0	0	0	0	0	0	0
0.05	0	0	0	0	0	1	1	1	1	1
0.06	1	1	1	1	1	1	1	1	1	1
0.07	1	1	1	1	1	1	1	1	1	1
0.08	1	1	1	1	1	1	1	1	1	1
0.09	1	1	1	1	1	1	1	1	1	1
0.10	1	1	1	1	2	2	2	2	2	2
0.11	2	2	2	2	2	2	2	2	2	2
0.12	2	2	2	2	2	2	2	2	2	2
0.13	2	2	2	2	2	2	2	2	2	2
0.14	2	2	2	2	2	2	2	2	2	3
0.15	3	3	3	3	3	3	3	3	3	3
0.16	3	3	3	3	3	3	3	3	3	3
0.17	3	3	3	3	3	3	3	3	3	3
0.18	3	3	3	3	3	3	3	3	3	4
0.19	4	4	4	4	4	4	4	4	4	4
0.20	4	4	4	4	4	4	4	4	4	4

TABLE (4) continued

$z_{\pi\mu}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.21	4	4	4	4	4	4	4	4	4	4
0.22	4	4	4	4	4	5	5	5	5	5
0.23	5	5	5	5	5	5	5	5	5	5
0.24	5	5	5	5	5	5	5	5	5	5
0.25	5	5	5	5	5	5	5	5	5	6
0.26	6	6	6	6	6	6	6	6	6	6
0.27	6	6	6	6	6	6	6	6	6	6
0.28	6	6	6	6	6	6	6	6	6	7
0.29	7	7	7	7	7	7	7	7	7	7
0.30	7	7	7	7	7	7	7	7	7	7
0.31	7	7	7	7	7	7	7	7	8	8
0.32	8	8	8	8	8	8	8	8	8	8
0.33	8	8	8	8	8	8	8	8	8	8
0.34	8	8	8	8	9	9	9	9	9	9
0.35	9	9	9	9	9	9	9	9	9	9
0.36	9	9	9	9	9	9	9	9	10	10
0.37	10	10	10	10	10	10	10	10	10	10
0.38	10	10	10	10	10	10	10	10	10	10
0.39	11	11	11	11	11	11	11	11	11	11
0.40	11	11	11	11	11	11	11	11	11	11
0.41	11	12	12	12	12	12	12	12	12	12
0.42	12	12	12	12	12	12	12	12	12	12
0.43	13	13	13	13	13	13	13	13	13	13
0.44	13	13	13	13	13	13	13	13	13	14
0.45	14	14	14	14	14	14	14	14	14	14
0.46	14	14	14	14	14	14	15	15	15	15
0.47	15	15	15	15	15	15	15	15	15	15
0.48	15	15	16	16	16	16	16	16	16	16
0.49	16	16	16	16	16	16	16	17	17	17
0.50	17	17	17	17	17	17	17	17	17	17

TABLE (4) continued

$z_{\pi\mu}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	17	18	18	18	18	18	18	18	18	18
0.52	18	18	18	18	18	19	19	19	19	19
0.53	19	19	19	19	19	19	19	19	20	20
0.54	20	20	20	20	20	20	20	20	20	20
0.55	21	21	21	21	21	21	21	21	21	21
0.56	21	22	22	22	22	22	22	22	22	22
0.57	22	22	23	23	23	23	23	23	23	23
0.58	23	23	23	24	24	24	24	24	24	24
0.59	24	24	25	25	25	25	25	25	25	25
0.60	25	25	26	26	26	26	26	26	26	26
0.61	26	27	27	27	27	27	27	27	27	28
0.62	28	28	28	28	28	28	28	28	29	29
0.63	29	29	29	29	29	29	30	30	30	30
0.64	30	30	30	31	31	31	31	31	31	31
0.65	32	32	32	32	32	32	32	33	33	33
0.66	33	33	33	33	34	34	34	34	34	34
0.67	34	35	35	35	35	35	35	36	36	36
0.68	36	36	36	37	37	37	37	37	38	38
0.69	38	38	38	38	39	39	39	39	39	40
0.70	40	40	40	40	40	41	41	41	41	41
0.71	42	42	42	42	43	43	43	43	43	44
0.72	44	44	44	44	45	45	45	45	46	46
0.73	46	46	47	47	47	47	48	48	48	48
0.74	49	49	49	49	50	50	50	50	51	51
0.75	51	51	52	52	52	53	53	53	53	54
0.76	54	54	55	55	55	56	56	56	57	57
0.77	57	58	58	58	59	59	59	60	60	60
0.78	61	61	61	62	62	62	63	63	64	64
0.79	64	65	65	66	66	66	67	67	68	68
0.80	68	69	69	70	70	71	71	72	72	72

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TABLE (4) continued

$z_{\pi\mu}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.81	73	73	74	74	75	75	76	76	77	77
0.82	78	79	79	80	80	81	81	82	82	83
0.83	84	84	85	85	86	87	87	88	89	89
0.84	90	91	91	92	93	93	94	95	96	96
0.85	97	98	99	100	100	101	102	103	104	104
0.86	105	106	107	108	109	110	111	112	113	114
0.87	115	116	117	118	119	120	121	122	124	125
0.88	126	127	128	130	131	132	133	135	136	138
0.89	139	140	142	143	145	146	148	150	151	153
0.90	155	156	158	160	162	164	166	168	170	172
0.91	174	176	178	180	183	185	187	190	192	195
0.92	198	200	203	206	209	212	215	218	222	225
0.93	228	232	236	240	243	247	252	256	260	265
0.94	270	274	279	285	290	296	301	307	314	320
0.95	327	334	341	349	357	365	374	383	393	403
0.96	413	424	436	448	461	475	489	505	521	538
0.97	557	577	598	621	645	672	700	732	766	803
0.98	844	889	940	996	1059	1131	1213	1308	1419	1549
0.99	1706	1897	2137	2445	2855	3430	4292	5728	8602	17228

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TABLE (5)

$z_\sigma^2 = \frac{n'}{n'+2.1158}$ Class II

z_σ^2	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.	1	1	1	1	1	1	1	1	1	1
0.01	1	1	1	1	1	1	1	1	1	1
0.02	1	1	1	1	1	1	1	1	1	1
0.03	1	1	1	1	1	1	1	1	1	1
0.04	1	1	1	1	1	1	1	1	1	1
0.05	1	1	1	1	1	1	1	1	1	1
0.06	1	1	1	1	1	1	1	1	1	1
0.07	1	1	1	1	1	1	1	1	1	1
0.08	1	1	1	1	1	1	1	1	1	1
0.09	1	1	1	1	1	1	1	1	1	1
0.10	1	1	1	1	1	1	1	1	1	1
0.11	1	1	1	1	1	1	1	1	1	1
0.12	1	1	1	1	1	1	1	1	1	1
0.13	1	1	1	1	1	1	1	1	1	1
0.14	1	1	1	1	1	1	1	1	1	1
0.15	1	1	1	1	1	1	1	1	1	1
0.16	1	1	1	1	1	1	1	1	1	1
0.17	1	1	1	1	1	1	1	1	1	1
0.18	1	1	1	1	1	1	1	1	1	1
0.19	1	1	1	1	1	1	1	1	1	1
0.20	1	1	1	1	1	1	1	1	1	1

1

TABLE (5) continued

z_σ^2	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.21	1	1	1	1	1	1	1	1	1	1
0.22	1	1	1	1	1	1	1	1	1	1
0.23	1	1	1	1	1	1	1	1	1	1
0.24	1	1	1	1	2	2	2	2	2	2
0.25	2	2	2	2	2	2	2	2	2	2
0.26	2	2	2	2	2	2	2	2	2	2
0.27	2	2	2	2	2	2	2	2	2	2
0.28	2	2	2	2	2	2	2	2	2	2
0.29	2	2	2	2	2	2	2	2	2	2
0.30	2	2	2	2	2	2	2	2	2	2
0.31	2	2	2	2	2	2	2	2	2	2
0.32	2	2	2	2	2	2	2	2	2	2
0.33	2	2	2	2	2	2	2	2	2	2
0.34	2	2	2	2	2	2	2	2	2	2
0.35	2	2	2	2	2	2	2	2	2	2
0.36	2	2	2	2	2	2	2	2	2	2
0.37	2	2	2	2	2	2	2	2	2	2
0.38	2	2	2	2	2	2	2	2	2	2
0.39	2	2	3	3	3	3	3	3	3	3
0.40	3	3	3	3	3	3	3	3	3	3
0.41	3	3	3	3	3	3	3	3	3	3
0.42	3	3	3	3	3	3	3	3	3	3
0.43	3	3	3	3	3	3	3	3	3	3
0.44	3	3	3	3	3	3	3	3	3	3
0.45	3	3	3	3	3	3	3	3	3	3
0.46	3	3	3	3	3	3	3	3	3	3
0.47	3	3	3	3	3	3	3	3	3	3
0.48	3	3	3	3	3	3	3	3	3	3
0.49	3	4	4	4	4	4	4	4	4	4
0.50	4	4	4	4	4	4	4	4	4	4

TABLE (5) continued

z_σ^2	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	4	4	4	4	4	4	4	4	4	4
0.52	4	4	4	4	4	4	4	4	4	4
0.53	4	4	4	4	4	4	4	4	4	4
0.54	4	4	4	4	4	4	4	4	4	4
0.55	4	4	4	4	4	4	4	4	4	4
0.56	4	4	4	5	5	5	5	5	5	5
0.57	5	5	5	5	5	5	5	5	5	5
0.58	5	5	5	5	5	5	5	5	5	5
0.59	5	5	5	5	5	5	5	5	5	5
0.60	5	5	5	5	5	5	5	5	5	5
0.61	5	5	5	5	5	5	5	6	6	6
0.62	6	6	6	6	6	6	6	6	6	6
0.63	6	6	6	6	6	6	6	6	6	6
0.64	6	6	6	6	6	6	6	6	6	6
0.65	6	6	6	6	6	6	6	6	6	7
0.66	7	7	7	7	7	7	7	7	7	7
0.67	7	7	7	7	7	7	7	7	7	7
0.68	7	7	7	7	7	7	7	7	7	7
0.69	7	7	7	8	8	8	8	8	8	8
0.70	8	8	8	8	8	8	8	8	8	8
0.71	8	8	8	8	8	8	8	8	8	8
0.72	9	9	9	9	9	9	9	9	9	9
0.73	9	9	9	9	9	9	9	9	9	9
0.74	9	9	9	9	10	10	10	10	10	10
0.75	10	10	10	10	10	10	10	10	10	10
0.76	10	10	10	11	11	11	11	11	11	11
0.77	11	11	11	11	11	11	11	11	11	11
0.78	12	12	12	12	12	12	12	12	12	12
0.79	12	12	12	12	12	13	13	13	13	13
0.80	13	13	13	13	13	13	13	14	14	14

TABLE (5) continued

Z_{σ}^2	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.81	14	14	14	14	14	14	14	14	14	15
0.82	15	15	15	15	15	15	15	15	15	16
0.83	16	16	16	16	16	16	16	16	17	17
0.84	17	17	17	17	17	17	18	18	18	18
0.85	18	18	18	19	19	19	19	19	19	19
0.86	20	20	20	20	20	20	21	21	21	21
0.87	21	22	22	22	22	22	22	23	23	23
0.88	23	24	24	24	24	24	25	25	25	25
0.89	26	26	26	26	27	27	27	28	28	28
0.90	29	29	29	29	30	30	30	31	31	32
0.91	32	32	33	33	34	34	34	35	35	36
0.92	36	37	37	38	38	39	39	40	41	41
0.93	42	42	43	44	45	45	46	47	48	48
0.94	49	50	51	52	53	54	55	56	57	58
0.95	60	61	62	64	65	67	68	70	71	73
0.96	75	77	79	81	84	86	89	92	95	98
0.97	101	105	109	113	117	122	127	133	139	146
0.98	153	161	170	180	192	205	220	237	257	280
0.99	309	343	386	442	516	620	775	1035	1554	3111

1

TABLE (6)

$$z_{\pi(1-\pi)} = \frac{n}{20.7677} \qquad \text{Class II}$$

$z_{\pi(1-\pi)}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.	1	1	1	1	1	1	1	1	1	1
0.01	1	1	1	1	1	1	1	1	1	1
0.02	1	1	1	1	1	1	1	1	1	1
0.03	1	1	1	1	1	1	1	1	1	1
0.04	1	1	1	1	2	2	2	2	2	2
0.05	2	2	2	2	2	2	2	2	2	2
0.06	2	2	2	2	2	2	2	2	2	2
0.07	2	2	2	2	2	2	2	2	2	2
0.08	2	2	2	2	2	3	3	3	3	3
0.09	3	3	3	3	3	3	3	3	3	3
0.10	3	3	3	3	3	3	3	3	3	3
0.11	3	3	3	3	3	3	3	3	3	3
0.12	3	3	4	4	4	4	4	4	4	4
0.13	4	4	4	4	4	4	4	4	4	4
0.14	4	4	4	4	4	4	4	4	4	4
0.15	4	4	4	4	4	4	5	5	5	5
0.16	5	5	5	5	5	5	5	5	5	5
0.17	5	5	5	5	5	5	5	5	5	5
0.18	5	5	5	5	5	5	5	6	6	6
0.19	6	6	6	6	6	6	6	6	6	6
0.20	6	6	6	6	6	6	6	6	6	6

TABLE (6) continued

$z_{\pi}(1-\pi)$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.21	6	6	6	6	6	6	6	7	7	7
0.22	7	7	7	7	7	7	7	7	7	7
0.23	7	7	7	7	7	7	7	7	7	7
0.24	7	7	7	7	8	8	8	8	8	8
0.25	8	8	8	8	8	8	8	8	8	8
0.26	8	8	8	8	8	8	8	8	8	9
0.27	9	9	9	9	9	9	9	9	9	9
0.28	9	9	9	9	9	9	9	9	9	9
0.29	9	9	9	10	10	10	10	10	10	10
0.30	10	10	10	10	10	10	10	10	10	10
0.31	10	10	10	10	10	11	11	11	11	11
0.32	11	11	11	11	11	11	11	11	11	11
0.33	11	11	11	11	11	11	12	12	12	12
0.34	12	12	12	12	12	12	12	12	12	12
0.35	12	12	12	12	12	12	13	13	13	13
0.36	13	13	13	13	13	13	13	13	13	13
0.37	13	13	13	13	14	14	14	14	14	14
0.38	14	14	14	14	14	14	14	14	14	14
0.39	14	14	15	15	15	15	15	15	15	15
0.40	15	15	15	15	15	15	15	15	16	16
0.41	16	16	16	16	16	16	16	16	16	16
0.42	16	16	16	16	17	17	17	17	17	17
0.43	17	17	17	17	17	17	17	17	17	18
0.44	18	18	18	18	18	18	18	18	18	18
0.45	18	18	18	19	19	19	19	19	19	19
0.46	19	19	19	19	19	19	19	20	20	20
0.47	20	20	20	20	20	20	20	20	20	21
0.48	21	21	21	21	21	21	21	21	21	21
0.49	21	21	22	22	22	22	22	22	22	22
0.50	22	22	22	23	23	23	23	23	23	23

1

TABLE (6) continued

$z_{\pi(1-\pi)}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	23	23	23	23	24	24	24	24	24	24
0.52	24	24	24	24	24	25	25	25	25	25
0.53	25	25	25	25	25	26	26	26	26	26
0.54	26	26	26	26	26	27	27	27	27	27
0.55	27	27	27	27	28	28	28	28	28	28
0.56	28	28	28	29	29	29	29	29	29	29
0.57	29	29	30	30	30	30	30	30	30	30
0.58	31	31	31	31	31	31	31	31	32	32
0.59	32	32	32	32	32	32	33	33	33	33
0.60	33	33	33	34	34	34	34	34	34	34
0.61	35	35	35	35	35	35	35	36	36	36
0.62	36	36	36	36	37	37	37	37	37	37
0.63	38	38	38	38	38	38	39	39	39	39
0.64	39	39	40	40	40	40	40	40	41	41
0.65	41	41	41	41	42	42	42	42	42	43
0.66	43	43	43	43	44	44	44	44	44	44
0.67	45	45	45	45	46	46	46	46	46	47
0.68	47	47	47	47	48	48	48	48	49	49
0.69	49	49	49	50	50	50	50	51	51	51
0.70	51	52	52	52	52	53	53	53	53	54
0.71	54	54	54	55	55	55	55	56	56	56
0.72	56	57	57	57	58	58	58	58	59	59
0.73	59	60	60	60	61	61	61	61	62	62
0.74	62	63	63	63	64	64	64	65	65	65
0.75	66	66	67	67	67	68	68	68	69	69
0.76	69	70	70	71	71	71	72	72	73	73
0.77	73	74	74	75	75	75	76	76	77	77
0.78	78	78	79	79	80	80	80	81	81	82
0.79	82	83	83	84	84	85	85	86	86	87
0.80	88	88	89	89	90	90	91	92	92	93

1

TABLE (6) continued

$Z_{\pi(1-\pi)}$	0.	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.81	93	94	95	95	96	96	97	98	98	99
0.82	100	100	101	102	102	103	104	105	105	106
0.83	107	108	108	109	110	111	111	112	113	114
0.84	115	116	117	117	118	119	120	121	122	123
0.85	124	125	126	127	128	129	130	131	132	133
0.86	134	135	136	138	139	140	141	142	144	145
0.87	146	147	149	150	151	153	154	156	157	159
0.88	160	162	163	165	166	168	170	171	173	175
0.89	177	178	180	182	184	186	188	190	192	194
0.90	196	199	201	203	205	208	210	213	215	218
0.91	221	223	226	229	232	235	238	241	244	247
0.92	251	254	258	261	265	269	273	277	281	285
0.93	290	294	299	304	309	314	319	324	330	336
0.94	342	348	354	361	367	375	382	389	397	406
0.95	414	423	432	442	452	462	473	485	497	510
0.96	523	537	552	567	583	601	619	638	659	681
0.97	704	729	756	785	816	849	886	925	968	1015
0.98	1067	1124	1188	1259	1339	1430	1534	1653	1793	1958
0.99	2156	2398	2700	3089	3607	4333	5422	7236	10866	21760

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Conclusion

Nationalisation of insurance companies in Egypt has had a great effect in the insurance industry by eliminating the competition between companies and unifying the rates. After 1974 new economic laws have been in force which allow private investment to operate in the insurance market. This new situation would require the regulating body to lay down new rules governing the conduct of insurance business especially for rate fixing and rate revision. In addition, it requires the insurance company to develop methods of rate making which are not only consistent and sound in operation, but also can be presented to the insurance supervisory authority as producing rates both fair to the insurance company and to the insured public.

Experience rating is a sound and effective tool for rating and rate revision. It is a fair method for class rate modification because it is based on proven methods for measuring the effect of factors which influence the risk experience and can not be defined (such as moral hazard).

The present work provides a specific method for obtaining the risk component of the rate which is as accurate as possible for a particular risk. The method is backed by theory, which would eliminate personal bias, and thus ensure consistent rates. The parameters underlying the mathematical model are computed from the statistics of the collective, due weight being given to the risk experience. The final rate can be computed by reference to the expense of management and the reinsurance policy of the company. An analytical study of the expense of management and the designing of an information system for generating the required data on a scientific basis would be areas for further research.

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CAS = CASUALTY ACTUARIAL SOCIETY

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