SYSTEMS EVOLUTION:

The Conceptual Framework And A Formal Model

BY
SHI-JI GAO

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"... the spread, both in width and depth, of the multifarious branches of knowledge during the last hundred odd years has confronted us with a queer dilemma. We feel clearly that we are only now beginning to acquire reliable material for welding together the sum total of all that is known into a whole; but , on the other hand, it has become next to impossible for a single mind fully to command more than a small specialized portion of it.

"I can see no other escape from this dilemma (lest our true aim be lost for ever) than that some of us should venture to embark on a synthesis of facts and theories, albeit with second-hand and incomplete knowledge of some of them -and at the risk of making fools of ourselves.
"So much for my apology."

Erwin Schrodinger, 1944.

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DECLARATION

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ABSTRACT

This research addresses to some of the fundamental problems in systems science. The aim of this study is to:

(1) provide a general conceptual framework for systems evolution;
(2) develop a formal model for evolving systems based on dynamical systems theory;
(3) analyse the evolving behaviour of various systems by using the formal model so far developed.

First of all, it is argued that a system, which can be recognized by an observer as a system, is characterised by some emergent properties at a certain level of discourse. These properties are the results of the interactions between the system's components but not reducible to the individual or summative properties of those components. Any system is such an emergent and organized whole, and this whole can be defined and described as an emergent attractor. To maintain the wholeness in a changing environment, an open system may undergo radical changes both in its structure and function. The process of change is what is called of systems evolution.

On reviewing the existing theories of self-organization, such as "Theory of Dissipative Structure", "Synergetics", "Hypercycle", "Cellular Automata", "Random Boolean Network" et al., a general conceptual framework for systems evolution has been outlined and it is based on the concept of emergent attractor for open systems. The emphasis is placed on the structural aspect of the process of change.

Modern mathematical dynamical systems theory, with the study of nonlinear dynamics as its core, can provide

(a) the concept of "attractor" to describe a system as an organized whole;
(b) simple geometrical models of complex behaviour;
(c) a complete taxonomy of attractors and bifurcation patterns;
(d) a mathematical rationale for the explanations of evolutionary processes.

Based on this belief, a formal model of evolving systems has been developed by using the language of mathematical dynamical systems theory (DST). Attractors and emergent attractors are formally defined. It is argued that the state of any systems can be described by one of the four fundamental types of attractors (i.e. point attractor, periodic attractor, quasiperiodic attractor, chaotic attractor) at a certain level. The evolving behaviour of open systems can be analyzed by looking at the loss of structural stability in the systems. For a full analysis of systems evolution, the emphasis is put on the nonlinear inner dynamics which governs evolving systems.

In trying to apply this conceptual framework and formal model, the evolving behaviour of various systems at different levels have been discussed. Among them are Benard cells in hydrodynamics, Brusselator in chemical systems, replicator systems in biology (hypercycle), predator-prey-food systems in ecology, and artificial neural networks. The complex dynamical behaviour of these systems, like the existence of various types of attractors and the occurrences of bifurcation when the environment changes, have been discussed. In most of the examples, the results in previous studies are cited directly and they are only re-interpreted by using the conceptual framework and the formal model developed in this research. In the study of artificial neural networks, a simple cellular automata network with only three neurons has been constructed and the activation dynamics has been analysed according to the formal model. Different attractors representing different dynamical behaviour of this network have been identified (point, periodic, quasiperiodic, and chaotic attractor). Similar discussions have been applied to a coupled Wilson-Cowan net.

It is believed that the study of systems evolution is one of those attempts to bring systems science out of its primitive stage in which it ought not to be.

Systems science is still in a primitive stage, even more primitive than it ought to be.

Peter Checkland, 1991

Chapter 1 Introduction

1.1 Focus of Study

Not everybody agrees that systems science is a legitimate field of scientific inquiry. For some people who usually are associated with the so called systems community, however, the legitimacy of systems science has been justified [Klir, 1991; Rodriguez-Delgado, 1992]. The objects of study for systems science are systems and associated problems. It is argued that systems science has developed within a movement starting early this century which is usually referred to as systems movement [Checkland, 1981]. The development has led to the current situation: professions of the so called systems scientists have been created, educational institutions devoted to systems science established, conferences regarding systems science held regularly, systems related academic journals published. All these seem to justify that systems science has been legitimated.

The affirmative view regarding the legitimacy of systems science is accepted in this research: systems science is a legitimate field of scientific research and its objects of study are systems. It studies the concept, properties and taxonomy of systems; it explores principles and mechanism concerning the structure, function and evolution of systems; it develops methods and methodologies for the understanding of systems; it applies ideas, techniques and methodologies about systems to solve problems arising from sciences, engineering and human activities. The scope of systems science include systems philosophy, systems theory, systems methodology and techniques, and systems practice (for the structure of systems science, chapter 2 gives a summary in a diagram). The focus of this study is about concepts, principles and models of systems evolution and it belongs to the category of systems theory.

However, systems science is still in a primitive stage, even "more primitive than it ought to be" [Checkland, 1991]. Compared with classic sciences like physics, chemistry, and astronomy etc, systems science still lacks rigourously defined concepts;

fundamental laws and principles, well established methods which can rival classic sciences in generating testable hypothesis and produce useful predictions [Flood, 1990; Checkland, 1991]. One notable example of the unmatured systems science is the lack of a rigourously defined and universally accepted concept of "system". For different people, even the same people at different time, the concept of system means different things.

This research does not intend to solve all these problems in systems science. It mainly concerns systems evolution at the theoretical level and hence can be categorised as the fundamental study in systems science.

1.2 Objectives of Study

Systems evolution is not a totally new concept. The idea that systems can evolve can be traced back, at least , to one of the early holistic thinker C. J. Smuts [Smuts, 1926]. Of course, discussions about the evolution of specific systems, i.e. biological systems evolution, have been widely known since Darwin [Darwin, 1859]. Another important work related to systems evolution is Bergson's "Creative Evolution" [Bergson, 1911], it advocates a general argument about evolution at the philosophical level. Before Bergson, Spencer discussed this problem in his "First Principles" [Spencer, 1971 (edited version)].

This study does not aim to continue the argument originated by those work mentioned above. It aims to study the fundamental concepts, mechanism, principles and models of systems evolution within the domain of systems science. In general, a system is an entity which is regarded as an organized whole. It will change its internal structure to maintain its entity as a whole in a changing environment. This is defined as systems evolution. Certain conditions must be met if a system is to evolve: it must be open, nonlinear, non-equilibrium, with microscopic fluctuations within the system and external perturbations from its environment. This topic has been touched by several important strands of thinking in modern systems research, like the theory of dissipative structures, synergetics et. al.

In this study, the objective is to establish a conceptual framework and a formal model for systems evolution. It starts with the careful examination of several definitions of systems. It is accepted that a recognized system is an organized whole. Starting from this, a dynamical model is employed to described a dynamical system so that the system can be defined and described by an attractor which is the result of the interactions of the system's parts. A system is always an emergent whole. To maintain the organized whole, the system will evolve in a changing environment. This is what systems evolution is all about. Systems evolution defined as such is different from the concept of evolution defined by Spencer et. al. Systems evolution does not necessarily mean that systems evolve progressively, i.e., evolve from lower stage to higher stage, because "progressiveness" is very difficult to justify, especially from the functional point of view. A system that can survive in environment E_1 is not necessarily superior than a system that survives in environment E_2. However, the grand tendence of the evolution of the universe is towards the increase of complexity.

It is believed that there are some general principles, mechanisms and patterns underlying all processes of systems evolution. Several schools of thought about self-organization have made progresses in discovering patterns and establishing principles about systems evolution, like "order through fluctuations", "slaving principle" etc. It will be desirable to construct a general conceptual framework of systems evolution based on the concept of emergent attractor and it should embrace all the known principles and patterns.

It is also the aim of this study to construct a formal model of systems evolution by using mathematical dynamical systems theory (DST). The recent development in mathematical dynamical systems theory has been noticed by its progress in nonlinear dynamics and its pervasions in such diverse field as physics, chemistry, ecology, and economics etc. [Stewart, 1989; Thompson, 1986]. It provides a means to study the complex dynamical behaviour of nonlinear systems. For the study of systems evolution, DST can provide a dynamical model which is often used in systems science. It can also provide a formal definition and classification of attractors which correspond to different state of systems. The techniques of bifurcation analysis in DST are also very useful for exploring patterns and stages during systems evolution. Above all, the concept of structural stability seems to be the essential concept related to systems evolution. For the above accounts, this study sets out to build a formal model for systems evolution by using the language of DST.

Another aim of this study is to apply the conceptual framework and formal model so far developed to analyse the evolution process of systems in various fields or at different space-time scales. Systems to be considered include some well known examples in hydrodynamics, laser, biology, ecology and neural networks.

To summarise, the objective is to explore the concepts, principles, models and examples of systems evolution. It includes the construction of a conceptual framework, the developing of a formal model and the discussion of some applications.

1.3 Scope and Limitations of Study

The study of systems is often confronted with the problem of choosing between a structural description or a functional description of systems [Kampis, 1987]. A structural description method is adopted in this study to avoid the pitfall of the controversies around the functional description of biological evolution. The discussion is centred in exploring the structural change of dynamical systems.

Usually, any discussion of systems evolution can not avoid using concepts like "entropy", "order" and the relations between them. This is touched briefly in this study only to describe systems evolution explanatorily: an open system can change from a less ordered state to a more ordered one by absorbing negative entropy from its environment. Study of the thermodynamics of systems evolution is another line of research worth pursuing [Weber et. al, 1988; Swenson, 1989a], but it is not the main concern of this study.

The structural description of systems evolution is based on the belief that, in principle, the state of any dynamical system can be described by relations between its internal variables [Thom, 1975]. However, this method has been attacked and rejected by some people like Berlinski [Berlinski, 1976]. Although Belinski's argument that early dynamical models do not describe the complexity, discontinuity and nonlinearity of complex systems has been resolved by the progress in mathematical DST, some of his critical comments are still appropriate for the study of systems evolution. In practice, not every dynamical system can be described by a group of dynamical equations as stated in chapter 4. This is especially true for some social-economic systems which are usually difficult to describe mathematically. Even if dynamical equations are given for some systems by related classical sciences, they may not be always in the standard form stated in the formal model. Mathematical DST itself needs development: it is not always possible to find solutions for a particular system, especially analytical solutions.

Apart from the limitations of the formal model arising from describing systems and from DST, there are serval restrictions set by the development of systems science as a whole. Although every effort has been made to ensure that, in this study, the consistence of the meanings of some of the important concepts like systems, emergent properties, attractors, order, evolution etc is maintained, there still lacks a consensus on universally accepted fundamental concepts. They may still mean different things for different people, not only in references cited, but in some other current discussion on this topic as well.

In trying to apply the conceptual framework and formal model so far developed, some examples in various classical areas of science have been cited directly. For those examples, they are only re-interpreted in this thesis and hardly any new results are reported. Two original examples in neural networks are studied according to the formal model and some novel results are achieved. However, the aim of this study is to explore one of the fundamental properties of systems, i.e. systems evolution, the two original models are chosen and analysed only to show that neural systems can evolve when environment changes and the framework and formal model developed in this research can be applied to their analysis. For reasons thus mentioned, they are not fully analysed as some other much studied models such as the Lorenz model. The "big pictures" of the evolution process of these two systems have not been obtained.

It will be a very bold and unjustifiable claim that this study has established a general theory of systems evolution, but it is true that this study tries to contribute to systems science at the fundamental level by addressing to one of the fundamental problems concerning the properties of systems, i.e systems evolution. It is an attempt, among many others, to get systems science out of its present primitive stage. No matter how far this study has covered toward that goal, the study of systems evolution is one of the most important area of research to bring systems science to its maturation

1.4 Outline of Study

After a brief introduction to basic concepts of systems, systems description, systems science, evolution and systems evolution etc. in chapter 2, the scope of systems science is mentioned. The problem of systems evolution is viewed within the framework of this new dimension of our modern sciences. The conflict between the pessimistic view of the universe implied in the second law of thermodynamics and the optimistic one provided by Darwin's theory of evolution is believed to be solvable when we take an open systems' point of view. The state of an open dynamical system is jointly decided by the inner dynamics of the system and the interactions between the system and its environment. Evolutionary behaviour of systems is the necessary result of the joint action of both the inner dynamics and environmental impacts when certain conditions are satisfied. It is argued that systems evolution is one of the general properties of open dynamical systems.

In chapter 3, various schools of thoughts about self-organization have been reviewed which have provided with the general framework for discussion of systems evolution. Brussels school's work on non-equilibrium thermodynamics, Eigen's work on hypercycle, Varela et. al's work on autopoiesis, Haken's work on synegetics, Wolfram etc.'s work on cellular automata, they all look at the same problem, i.e., the evolutionary behaviour of open systems, but from different points of view. Although these work are originated in different fields, like the theory dissipative structures in non-equilibrium thermodynamics, autopoiesis and hypercycle in biology, synergetics and self-organized criticality in physics, and cellular automata in mathematics and computer simulation, they all have reached the same conclusion that open systems can exhibit complex evolutionary behaviour. Different aspects of the process have been stressed in different schools, they all have contributed to lay the foundation for talking about the evolutionary process from the systems point of view. By synthesizing these strands of thought, a more general conceptual framework about systems evolution is established based on the concepts of attractor, emergence and organized whole.

The developing of a formal model of evolving system is reported in chapter 4. With the mathematical preparations introduced Appendix I, the effort is put to construct a formal model of evolving systems by using the language of dynamical systems theory. An open system can be modelled by a group of dynamical equations with parameters representing the constraints of the environment. The invariant sets implied in these dynamical equations decide the macroscopically stable state of the system, i.e, attractors that prescribe the macroscopic behaviour of the system. An evolutionary process is characterized by the loss of structural stability of the dynamical system and described by various bifurcation patterns through which one type of attractor is replaced by another. It is shown that general patterns, principles, and fundamental mechanism can be manifested in this formal model.

In chapter 5, several examples of the application of the general framework and formal model are reported. Some well studied examples like Benard cells, Brusselator etc are re-interpreted by using the proposed model. Structural aspects of systems evolution are stressed

Chapter 6 is devoted to the discussion of the complex behaviour of (artificial) neural networks as adapting and evolving systems. A simple 3-neuron cellular network is shown to exhibit evolution behaviour: it changes from one type of attractor to another while the network as an organized whole is maintained. Same analysis has been done for a coupled Wilson-Cowan nets.

The last chapter concludes this thesis by reviewing the progress that has been made in this study towards the understanding of the complex behaviour of open systems. Further problems worth studying are mentioned. It is argued that systems research is a new kind of human endeavour which fits our modern times. In a constantly changing world, to understand the transformation of the world is very important The study of systems evolution can help us understand the evolving universe of which we are only a part.

Appendix I is about the introduction of modern mathematical dynamical systems theory. The last twenty years has seen the development of dynamical systems theory which was mainly inspired and kindled by the discovery of "chaos" within deterministic dynamical equations. It has now become explicit that nonlinear dynamical systems can exhibit extremely complicated behaviour and this is consistent with the belief that complexity is the intrinsic property of open systems (they usually are nonlinear, non-equilibrium). Basic concepts, principles and theorems in mathematical dynamical systems theory are introduced and some recent advances of the study of nonlinear dynamics are reviewed.

Chapter 2 Systems Research and Systems Evolution

2.1 Systems and systems science

2.1.1 The concept of systems

The term "system" has different meanings under different circumstances for different people. It is often loosely defined as a group or combination of interrelated, interdependent, or interacting elements forming a collective entity or organic whole. Two points are quite essential in any definitions of systems: the first is that there are components which are interrelated or interacting, the other is that a system is a unity, an organized whole. In a formal way, a system **S** is defined as:

$$\mathbf{S} = (\mathbf{E}, \mathbf{R})$$

where **E** represents a group of elements, **R** the relations between them.

A more easily understood verbal definition of system can be given as this:

> *A system is an assembly of components, connected together in an organized way. The components are affected by being in the system and the behaviour of the system is changed if they leave it. This organized assembly does something and has been identified as a particular interest.* ["Systems Behaviour", Open university, pp18]

In this definition, the dialectic relationship between components and the whole is stressed.

What is not explicitly expressed in this definition is that the whole usually possesses some properties which do not come from the properties of individual elements, or the summative properties of all the elements. These properties, not reducible to the parts or the sum of parts, are called emergent properties. Thus a system is such an organized and emergent whole. It becomes conceivable that "*the whole is more than just the sum of the parts*" only when these emergent properties are taken into account.

Stressing the emergent properties of systems, Swenson gives a lengthy definition of emergence, or how emergent properties have emerged:

Emergence

The spontaneous transformation of a set of components (generalized 'atomisms' or 'particles') from an incoherent state, where the space-time correlation between them is confined to mean free path and mean relaxation collision times, to a coherent state exhibiting novel, global, dynamical space-time behaviour, viz., space-time correlations, between atomisms many orders of magnitude greater than mean free path-relaxation times, inaccessible to, not locatable in, and not reducible to the individual or summative behaviour of the separate atomisms; the spontaneous creation of a new set of macroscopic constraints that reduce accessible microstates from some initial set M_n to some much smaller subset M_s to yield a new irreducible level of dynamical space-time behaviour. By the transformation M_n to M_s emergence is always a progressive asymmetrical time-dependent transformation of matter away from equilibrium. 'Spontaneous" means 'goes by itself' without exogenous (outside) 'maker', e.g., since Newtonian machines are explicitly specified and constructed from without, they are not emergent. [Swenson, 1989a].

This definition stresses not only the irreducibility of the emergent properties to properties of parts, it also emphasises the spontaneity of the process in which a system is assembled from separate parts. According to this definition, Newtonian machines are explicitly excluded from having emergent properties. This restricts the meaning and applicability of the term "emergence" to describe mechanical systems. A car is composed of many parts, and as a whole, it can serve as a transportation tool but not any the component parts have this property. It would be more appropriate to call Swenson's definition of emergence as the definition of 'self-organization' process for natural systems. Emergent properties of a system can be defined as what are resulted from the interactions of the parts, but "inaccessible to, not locatable in, and not reducible to the individual or summative behaviour of the separate atomisms".

Arthur Koestler expressed his view on the definition and meaning of the term of systems in a book "*The Ghost in the Machine*" [Koestler, 1967]. Considering the property of a system as a whole, he proposed to define a system as a "holon" which means a whole of elements functioning as an element in a larger whole. In that case, a system is a whole, coming into existence through the organizing of elements and serving as an element in a bigger holon (system). It has not become a universally accepted definition of systems or a popular word in systems community. Recently, Checkland argues that the confusion arising from expressing systems ideas in systems literature should mainly be blamed for the use of the word "system": it means both for parts of the real world perceived to be complex whole (like 'the education system') and for the abstract notion of a whole (a model) [Checkland, 1988, 1991]. He advocates the use of "holon" for the abstract notion of a whole.

The term "system" discussed so far is apparently about the abstract notion of a whole. It is used in this study to mean an organized whole about some real-world entity perceived by human beings. It needs to be stressed here that

(1) a system is an emergent whole, the whole results from the interactions of many interdependent elements, or parts, or subsystems;
(2) a system is always a model of a certain perceived entity.

The second part is stressed by Ashby, Checkland and many others. It reflects the perspectivism in talking about systems: systems do not exist in the real-world and they are only models of certain entities perceived by certain observers. Gaines defines a system as "what is distinguished as a system" [Gaines, 1979] and this definition, sounds tautological, reflects the essential feature of the term "system". The properties discovered in a model, however, are believed to represent the properties of the real-world entity modelled.

Rosen analyses the term "system" from another standing point. He argues that

> *"the word 'system' is never used by itself; it is generally accompanied by an adjective or other modifier: physical system; biological system; social system, economic system; axiom system; religious system; and even "general" system. This usage suggests that, when confronted by a system of any kind, certain of its properties are subsumed under the objective, and other properties are subsumed under the "system,' while still others may dependent essentially on both. The objective describes what is special or particular; i.e., it refers to the specific "thinghood" of the system; the "system" describes those properties which are independent of the specific "thinghood."* [Rosen, 1986]

Parallel to the thinghood, he coined a word "systemhood" to describe those system related properties. He goes further to argue that systems theory is the study of systemhood related properties.

Another term is proposed in this study to describe a special kind of systems whose state changes over time, i.e. dynamical systems. The term suggested is called "attractor". Originally it is a mathematical term defined for a special invariant set for a flow on a manifold, it has been extended to describe the state of general time-dependent systems [Thom, 1975; Ruelle, 1989; Swenson, 1989a]. An attractor is a time-independent (time-asymptotical) state that attracts initial conditions from some region around it. To use attractors to represent a dynamical system stems from the use of dynamical models to describe the state of a system. The formal definition of an attractor will be given in chapter 4, but the following properties of an attractor can be mentioned briefly here: it is invariant, i.e. time-independent; it reflects certain emergent properties of a dynamical process modelled, hence represents a whole; it describes a global process resulted from some local interactions.

To summarise, a system is recognized as an organized whole and it is an emergent entity through the interactions of its components.

2.1.2 Systems science

Once it is clear what a system is about, the domain of study for systems science can be defined. The definition given in a recently published textbook of systems science by Klir is appropriate

> *Systems science is a science whose domain of inquiry consists of those properties of systems and associated problems that emanate from the general notion of systemhood* [Klir, 1991].

Systems science, if it exists, has grown from an intellectual movement which started in 40's this century and is usually referred as "systems movement" [Checkland, 1981]. There are many strands of thought each with a different background that have formed this movement. One common and essential characteristic of all these ideas, methods, models etc. which appeared in this movement is that it is accepted that some properties of a conceived entity can not be reduced to its composing elements, all the elements and their relationships should be considered simultaneously and the entity can only be understood as a whole as a result of these relations. A prototype of such an entity is the organic unit in a biological system. The striking feature of this systems movement is that problems related to "systemhood" have been studied and stressed.

As to the characteristics of systems science, Klir argues that it is essentially different from classical sciences, such as physics, chemistry, biology and astronomy et.al. Those classical sciences study problems related to thinghood, i.e., problems associated with specific physical properties. In contrast, systems science addresses problems which arise from across a wide range of domains and are independent of their physical traits. Classical sciences study entities, systems science studies relations; classical sciences study thinghood while systems science is concerned about systemhood. He argues that systems science and classical sciences are orthogonal to each other and they together form a two-dimensional sciences which characterises the present informational society [Klir, 1985b]. It is stressed in this study that systems science is about systemhood and it is principally different from classical sciences.

As to problems that systems science is trying to study, two of them are very important: isomorphism between systems and the relation between parts and whole.

Isomorphism, or isomorphic relation among systems, was the central problem for general systems theory -- one of the most important schools of thought in systems movement -- in early days [von Bertalanffy, 1968]. It is about the equivalence relations among all systems of interest: if two systems are proved isomorphic, then they share the same properties, i.e, understanding one means that the other is also understood. In the terminologies developed so far, isomorphic systems have the same systemhood related properties. Isomorphic relations are proved to exist in many cases, for example, a pendulum as a system is isomorphic to an electrical circuit with certain structure and their behaviours can be described by the same type of dynamical relations. However, if isomorphic relations are extended to embrace too broad a spectrum of systems, as they usually are, they will become either meaningless or useless. This is one of the reasons that general systems has been sharply criticized [Berlinski, 1976].

The most important relation between parts and whole is about emergence: how the emergent properties of the whole have emerged from interactions of the parts. Due to the emergence in all kinds of systems, systems are recognized as systems and these systems are organized in a hierarchical way: systems at certain level are emergent entities from subsystems at one level below which are emergent entities of sub-subsystems a further level below. The problem of emergence in dynamical systems will be discussed in the following chapters by resorting to a dynamical model.

Another relation between parts and whole is called "self-similarity" or "self-isomorphic" in structure. It is argued, notablely in Miller's "Living Systems Theory", that certain properties of the whole are isomorphic to that of parts as a sub-whole [Miller, 1978]. Miller has discovered that at different levels of living systems, i.e., cells, organs, organisms, groups, organizations, societies, super-nations, the same types of subsystems can be identified, such as reproducer, boundary, distributor, encoder, and decoder etc., totally 19 subsystems. Another isomorphic relation between the whole and parts has also been discovered recently: the self-similarity in spatial-temporal behaviour between parts and whole. In fractals, the shape of a whole is identical to that of the part, as discovered by Mandelbrot [Mandelbrot, 1983]. In chaotic systems, chaotic attractors are discovered having Cantor-set-like self-similar structures [Zeeman, 1988; Thompson et. al, 1986]. However, the isomorphic relation in structure between parts and whole is not as universal as the emergence problem in systems.

To my own opinion, the scope of systems science is very broad and it can be structured as having the following epistemological levels: systems philosophy, systems theories, systems methodologies, and systems practices:

Systems philosophy is about systems thinking and general discourse based on ideas about systems. It concerns about problems like holism versus reductionism, synthesis and analysis etc.and it is the overview of systems thinking and standing point.

Systems theories are about ideas, conceptual frameworks, general principles and models concerned about systems. They include theory of general systems (GST, Cybernetics, Living Systems Theory etc), theory of systems evolution (Theory of dissipative structures, Synergetics, Hypercycles etc.), and concepts of systems. They discuss the following problems.

Elementary concepts: system, structure and function, subsystems and supersystems, information and entropy, systems description etc.

Properties of systems: emergence, wholeness, stability, adaptability, hierarchy, equifinality, instantiality etc..

Typology of systems or taxonomy of systems.: according to what criteria what kind of systems can be grouped together, or forming a hierarchical structure.

Evolution of systems: how systems change: patterns of changes, mechanism underlying changes, general principles etc.

Systems methodologies: methods and techniques, Operational Research (like Mathematical Programming, Networks and Flows etc.), Game Theory, Decision Theory, Systems Analysis, Systems Engineering approach, Soft Systems Methodology et. al.

Systems practices: applications in various fields, like engineering systems management, ecological systems analysis, man-machines systems et. al..

The structure of systems science can be illustrated in figure 2.1.1.

In this study, special interest is paied to the dynamical behaviour of open systems: how a system changes its structure in a changing environments, what is the mechanism for that change, what the general principles for all those changes. These are problems to be discussed in the following chapters. This study is hence about the evolution of systems and it belongs to the category of systems theory.

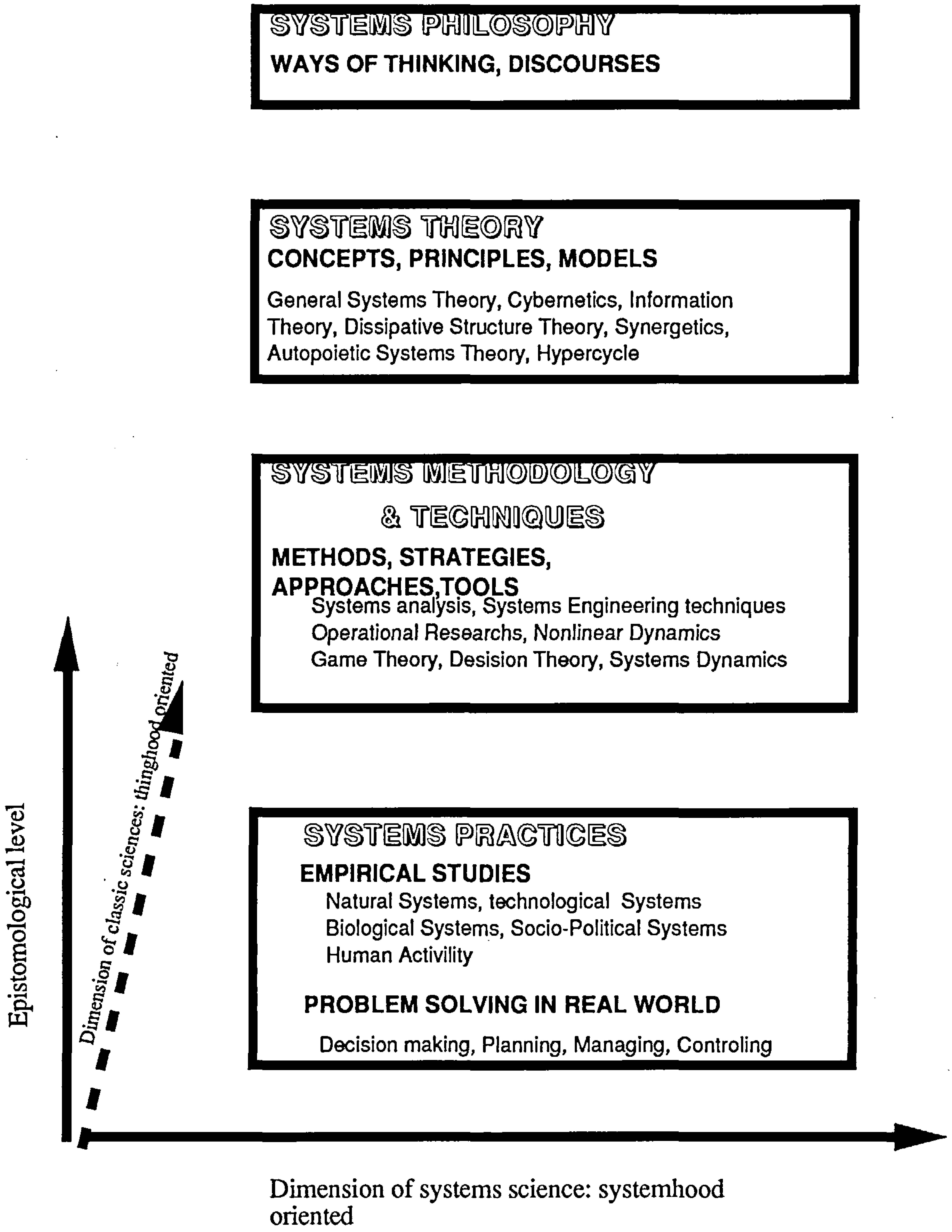


Figure 2.1.1 The structure of systems science.

2.2 Systems science and the science of complexity

2.2.1 Organized complexity

Systems science is sometimes described as the science of complexity [Klir, 1985a, 1985b; et. al, 1988]. Although there have been discussions centered around complexity for many years, there is still no rigourous definition about what "complexity" actually is. It can only be understood intuitively. The concept of complexity is generally related to "a large number of components" and "complex relationships" of systems. The multi-facets of complexity have been revealed by Klir [Klir, 1985a], and other concepts like "hierarchy", "emergent property" are closely related to the concept of complexity. What is more important of complexity for the discourse of systems science is the concept of "organized complexity" proposed by Weaver in an important paper "Science and Complexity" [Weaver, 1948].

According to Weaver, the complexity of a system depends both on the number of composing components and the randomness involved. The degree of the complexity of the system is decided by the number and degree of interrelations of these components, and the degree of randoms involved. He identified two types of complexity: disorganized complexity and organized complexity. Sciences, with different domains of study, address to the problems of simplicity and complexity. Weaver classifies three categories of study according to the degrees of complexity involved and this classification is illustrated in figure 2.2.1 as follows:

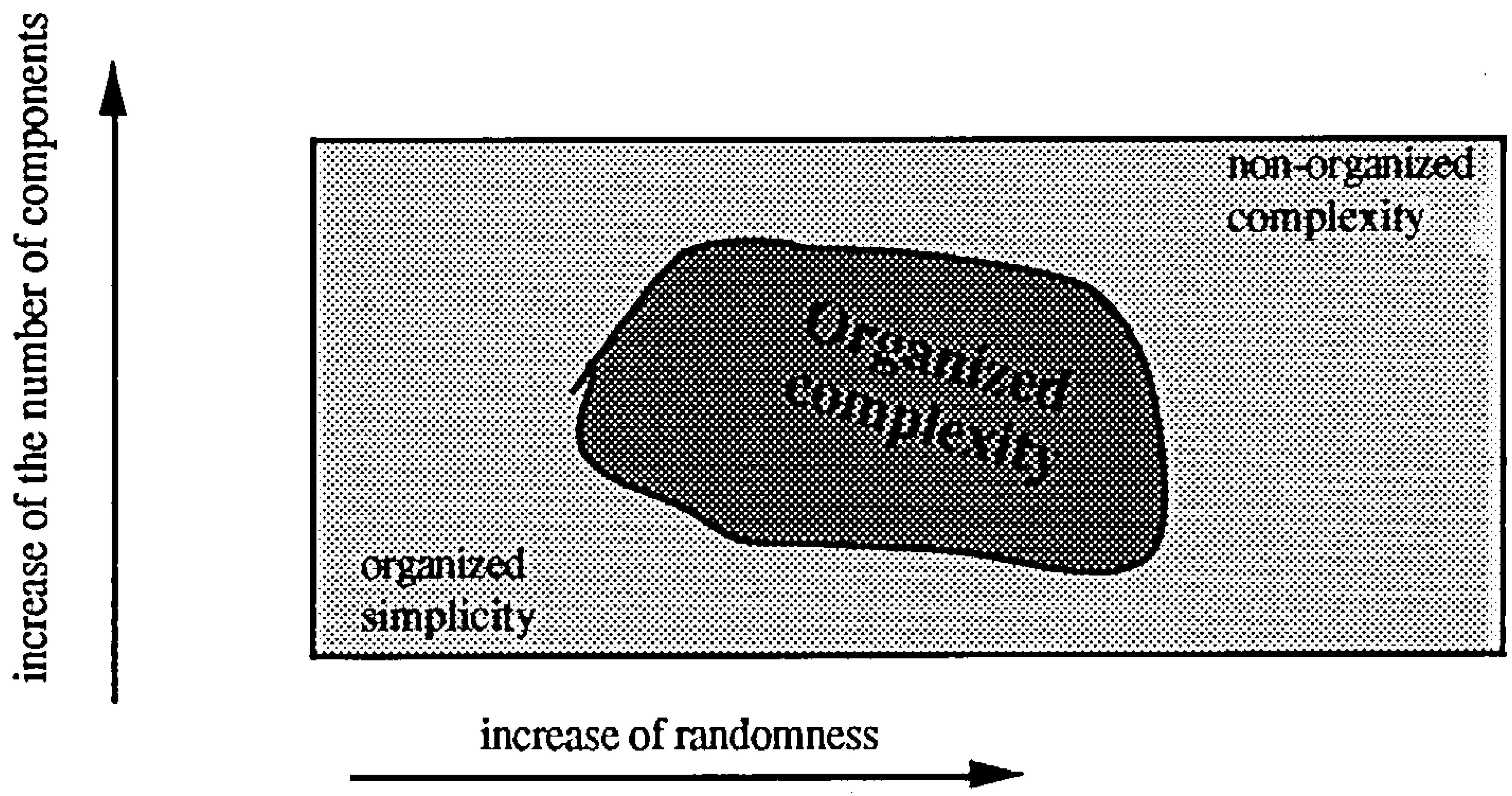


Figure 2.2.1 Different types of complexity

Physics mainly deals with systems possessing organized simplicity where physical laws, found or to be found, are believed to exist to govern the movement of systems, like Newton's kinetic laws, gravitational law etc.. The non-organized complexity, possessed by systems with very large number of components which are in random state, can be tackled by probabilities and statistics. The example is the ideal gas where the inner dynamics of the system is expressed by the law of molecular thermal movement. Although the movement of every molecule is governed by kinetic laws, it is meaningless and also impossible to describe the movement of individual components in practice. However, the collective behaviour of the system is relatively simple: the system as a whole can be described in a collective way, like the distribution of energy, speed etc.. Between these two extremes of systems lies a large amount of systems which have a fairly large number of components and there are strong interconnections among them. In this case, there is another type of complexity which arises from the interrelations between a large number of components and it is defined as the "organized complexity". Examples of organized complexity conceived by Weaver include biological systems, social systems etc. This problem has not been touched effectively by classic sciences, at least not until recently, and, as to be shown, the problem of organized complexity is what systems science sets to attack.

Remember what has been said about systems and systems science: systems are organized wholes of parts. What is important is how those parts interact to each other so that a whole can emerge to exhibit some novel properties. When the number of components become fairly large and the interdependent relations between them very complex, organized complexity has to be addressed to. As a matter of fact, it will be shown in the forthcoming chapters that even a system with only few components but complex relations can exhibit complex behaviour. This belongs also to the problem of organized complexity, but unforeseen by Weaver.

Dealing with systems with organized complexity can be traced back to the early stage of the development of systems science. From the theoretical aspects, general systems theory (GST) has set out to find a new way of looking at the problem of biological systems which could not be described properly within the domain of classic sciences [Bertalanffy, 1968]. To study a biological system, usually many variables must be considered simultaneously: the complex behaviour of a system arises from the interconnections between its forming components and all of them have to be taken into account. This was one of the main concerns when Weaver introduced the concept of "organized complexity". In Bertalanffy's GST, he meant to establish a general theory which, in principle, can deal with the problem of organized complexity arising from all systems. Among those properties of general systems, "equafinality" is a concept which illustrates the characteristics of organized complexity [Bertalanffy, 1968].

Systems engineering is regarded as another important contribution to the development of systems science. According to Checkland [Checkland, 1981], systems engineering belongs to the "hard way" of systems thinking in systems science. It concerns about the principles, methods and techniques applied in organizing large engineering projects from the systems point of view: problems of technology, finance, manpower and their management are intertwined with each other and all of them must be considered together simultaneously. This is another example of organized complexity. The techniques and skills of the management of one large scale engineering problem are transferable to others and here lies the general principles and techniques of systems engineering. The techniques of operational research belong to those mathematical techniques which form the hard core of systems engineering.

In the mean time, there is a strand of thought which is called "soft system methodology" (SSM). The essence of it is that human factors are involved in conceiving, modelling, analysing and designing systems. On account of the “soft” aspect of systems science, another factor of the complexity of systems is touched: the subjectiveness in describing systems. As argued by Flood, the concept of complexity has its subjective meaning [Flood et. al, 1988]. The complexity of a system is always conceived by human beings and therefore, apart from the number of components of a system and the interrelations between components, the viewer's standing point must be taken into account when talking about the problem of systems complexity. This is of vital importance when systems thinking is applied to analyse social-economic systems. However, the subjectiveness of complexity is not the main concern in this study.

Systems science is about the organized complexity of systems and hence the organized complexity is sometimes called systems complexity. In discussions about properties of a system, it is imperative to look at the complexity the system exhibits. Growth, adaptability, evolution are behaviours resulted from the organized complexity of biological systems. Large spatial span, long time duration, large financial and human involvement are the characteristics of organized complexity of large engineering systems. The growth, equilibrium-seeking, recession are phenomena observed in modern economic systems, which intrinsically involve natural and human source, industries, agriculture, education and many other sectors, and political system which strongly influences the economic behaviour. They are complex not only in the spatial-temporal scale that goes beyond the scope of classic sciences: they are complex in the sense that they can not be understood by merely resorting to the properties of their components. The macroscopic behaviour of a national economy do not merely depend on the behaviour of one or a handful of commercial firms or companies, it is the result of the how all different sectors, -- agricultural, industrial, comericial etc. --, are connected to each other and also how they are affected by the international economic environment. These complex behaviours are the reflection of the systems' organized complexity and can only be analyzed by adapting a systems point of view.

When the components and interactions of components are changing in the passage of time, the problem of complexity becomes more complex: dynamical complexity of systems is hence entering discussion.

In Benard hydrodynamics experiment, when a thin layer of viscous liquid is heated uniformly from below, with its upper surface exposed to a cooler air, organized spatial-temporal patterns, i.e, the hexagonal cells, can emerge from the previously homogeneous state if the temperature gradient imposed exceeds a critical point [Haken, 1983a; Swenson, 1989a].

In Belousov-Zhabotinsky reaction, the reactants are pumped in and products are flowed out constantly. When certain critical state is reached, chemical waves can appear suddenly and may be sustained by constant inflow and out-flow [Nicolis et. al, 1977, 1989].

In the osmosis experiment, certain inanimate chemical reactants are put into certain chemical liquid. In the time span of minutes to hours, amazingly some complex patterns like trees, mushrooms, vegetables, and bearded goat et. al can be observed to grow up and they are reminiscent of the complex biological forms which have been found in the natural world [Klir et. al., 1988].

In biology, self-aggregation phenomenon has been discovered in the insect population. In an experiment mentioned by Prigogine, larvae of a coleoptera are initially distributed at random on two sheets of glass. When an artificial nucleus is introduced in a peripheral region of the system, a cluster appears and the density around the imposed centre increases. When the initial density of the population is high, the system will choose a centre itself and the population grows at that point [Prigogine et. al 1984].

In the social economic field, it has been demonstrated that urbanization happens in a way similar to the self-aggregation process in insect population. Commercial centres appear from a homogeneous area because of some random factors and they then start to attract people to immigrate there and further develop to large centres [Allen, 1986].

In the computer experiment of cellular automata, the random initial conditions can lead to organized spatial-temporal patterns by following simple deterministic rule. Further more, artificial lives can be created by setting up simple initial conditions and simple rules and this is believed to provide some new insight into the emergence of complex systems in the universe [Wolfram, 1984; Langton, 1986, 1989].

2.2.2 Systems science and the science of complexity

Although systems science deals with the problem of organized complexity, it does not do so exclusively. Since early 70's, the complex dynamical behaviour of systems in a wide range have been noticed and addressed by some classical sciences, like physics, chemistry, biology etc.. Examples include those cited in last section, e.g. Benard Cells, Belousov-Zhabotinsky reactions etc., they were originally discovered, incidentally, by physicists, chemists and biologists who were puzzled when fist seeing them. Especially when the development in nonlinear dynamical systems studies have revealed that many systems can exhibit complex dynamical behaviour as such, the problem of complexity has been noticed and studied in almost all the classical areas of science. This study of organized complexity is called the "science of complexity".

Institutions bearing the title like "complex systems", "nonlinear systems" etc. have appeared rapidly in recent years. One well-known example is the Santa Fe Institute in USA where physicists, chemists, biologists, computerists etc. are all involved in the study of organized complexity. Different from systems scientists whose main concern is about the complexity of abstract systems arising from the interactions of components, or systems complexity in general, scientists like those in Santa Fe are looking into some specific systems and study their dynamical complexity arising from the nonlinear relations between systems components. Problems under consideration include: nonlinear dynamical behaviour of physical systems (fluid, chemical waves, catalytic networks in gene dynamics etc.), emergent computation of certain model systems (cellular automata, neural network, replicator systems etc.), computer simulation exploring the mechanism underlying natural evolution (artificial life). The core of the "science of complexity" is about the nonlinear dynamics of systems which is sometimes referred to as the study of chaos [Stein, 1989; Jen, 1990; Langton, 1989; Langton et. al, 1992].

Systems science is overlapping with the science of complexity, but they are different. Systems science may discuss systems complexity from a general point of view, like emergent properties etc., the science of complexity concentrates on specific systems. Systems science is systemhood orientated while the science of complexity is more thinghood inclined. The best line of research seems to look at complex systems from both classical sciences and systems science point of view: through systems science, the general properties of complex systems can be applied in the study and some knowledge about other systems may be transferable, like dynamical models; through classical science, the problems associated with thinghood can be addressed by resorting to the specific physical form of systems. For a specific system, its organized complexity essentially depends on both systemhood and thinghood.

To study how a system's components are related to each other so that the system can exhibit rich complexities and to know what the state a system is in at a particular time is regarded as the study of the "being" of the system: its structure, organization, state etc. What is of special interest in this study is the dynamical behaviour of open systems, especially the process when an open system changes from one level of complexity to another or how a system in an disorganized state becomes, with the impact of its environment, organized. In other words, the main concern is about the "becoming" of systems in this study. The following section will trace the history of the study of becoming of various systems.

2.3 Evolution and Thermodynamic Equilibrium: two extremes

2.3.1 Reversible and irreversible process: the role of time

It has been argued that one of the striking characteristics of Newtonian mechanism is that time plays no constructive role in all the processes happening in our machine-like world. Time is merely a parameter which has no direction [Prigogine et. al, 1984]. In the mechanical model of the universe, every system starts at certain initial conditions, follows certain trajectory defined by some universal laws, like the Gravitation Law, and goes on and on for ever. To know the future is just the same as to know the history: you just need to follow the trajectory prescribed by the equations along the time rather than to retrieve the trajectory by change the sign of time. The fate of the universe is defined for ever and what can be done is to try to find those universal laws and write the dynamical equations. The process of changes that happen in the universe is reversible in the sense that to change the time from t to $-t$ in the equations does not change the form of equations, i.e., laws that govern the behaviour of the system remain the same. This is the picture of the world from the Newtonian mechanical point of view.

Thermodynamics studies the absorbing and dissipation of energy of systems. For the first time, time is assigned a direction along which the energy flows one way. It is common sense that when two iron bars, with one "hotter" than the other, are put together, they will eventually change to, through heat conduction, a state at which they are at the same temperature. No one has ever witnessed the reverse process in the nature, i.e, the heat flows from one bar to another, which were at the same temperature at the beginning, and leads to the rise of temperature in one and decrease in the other without any constraints imposed from outside (say, deliberately introducing a heat gradient). The natural process of heat transfer is irreversible. Thermodynamics depicts a picture of the universe which is irreversible in the sense that "useful" energy is constantly digested and the universe is moving to the "heat death" as the second law says.

Irreversible process is also observed in the biological world, in galaxices etc.. Even before Darwin's theory of evolution brought about human's attention to the evolutionary process observed in the biological world, some people had argued that Newtonian mechanism was inadequate in describing the organisation of living matters [Prigogine et. al, 1984]. The processes of change in Newtonian world is reversible but the process of the growth of plants as well as the development of human organs are characterised by irreversible complexification. Those challenges to the doctrine of Newtonian mechanical view of the world have been noticed and stressed since Darwin's work on biological evolution. With the picture of a mechanical world on the one hand and an dynamical, complex world rich of changing behaviour and innovation on the other hand, there is a split imagine of the universe perceived by human beings.

Systems science has been trying to portray a different picture of the world which can, quite possibly, provide some insights to bring to the end the confusion about order and disorder, reversibility and irreversibility.

2.3.2 The second Law of thermodynamics

Although the second law of thermodynamics is regarded by some physicists like Eddington as holding the supreme position among the laws of nature, it is too pessimistic a conclusion from the human's point of view. The claim that entropy always increases in every closed system implies that the universe is doomed to head for a 'horrific' state, i.e., the state of thermodynamic equilibrium: maximum disorder, the complete destruction of any structure and organization, and hence the system is both spatially and temporally homogeneous. It is extremely disappointing for people who, encouraged by the triumph of scientific rationality brought about by the classical sciences since Newton, have placed the human on the top of the nature. They believe in the unlimited power of human rationality inspired by classical sciences and tend to use it to create an ideal world with order, efficiency and justice. It is especially true for the twenty century people who are indulged in the success of the "industrial civilization".

Thermodynamics is the macroscopic description of natural processes. Instead of describing systems by specifying the kinetic states of its components, it concentrates on the tendency of the change of state at a macroscopic level. It is the process not the state that is stressed. The Second Law, as formulated by Clausius, was originally about the energy consumption of closed thermodynamic systems but it is generally interpreted as that free energy is spent and leveled out in any natural process. Entropy change is the quantity which characterises this process. As argued above, time is assigned a direction in thermodynamics along which free energy is always consumed and dissipated and the thermodynamic process is irreversible. This thermodynamic view has challenged the view of a static, reversible world represented by Newtonian paradigm. However, the tendency of evolution to disorder stated by the second Law is obviously contradictory to our observation that order and structure are growing in the biological world. This brings to the conflict with Darwin's evolutionary picture of the biological systems.

2.3.3 The growing of order in the biological world

Prigogine has argued that our scientific heritage includes two basic questions to which till now no answer was provided [Prigogine et. al, 1984]. One is the obvious contradiction between the static view of classical dynamics and the dynamical paradigm of thermodynamics, i.e., the direction of time [Coveny etc., 1990]. The other one is the relation between order and disorder. The famous law of increase of entropy describes the world as evolving from order to disorder while, biological or social evolution shows us the complex emerging from the simple. It has been a dichotomy facing philosophers and natural scientists for a long time.

Unlike the tendency to go to disorder and stable thermodynamic equilibrium as claimed by the second law of thermodynamics, evolution, diversification, and instability are found common in the biological world. Darwin has been undoubtedly credited as the founder of the theory of evolution. This evolutionary paradigm has been strengthened and extended by some great discoveries in this century. The discovery of genes leads to the so-called Neo-Darwinism, Stanley L Miller's experiment on "primordial soup" which is believed to have exited on the surface of the earth hundreds of millions of years ago provides new clues about how life might have come into existence in this planet [Hogan, 1991]. Eigen's Hypercycle adds one new link to the evolutionary chain of the biological world, and the concept of Big Bang and the theory of an expanding universe complete the evolutionary continuum of the universe [Eigen etc., 1979; Hawking, 1988].

In recent years, phenomena of the emergence and growing of order have been observed in diverse fields ranging from simple physical system to complex human societies and a general awareness has been raised about the becoming of the universe. This has entered the research scope of human enquiring and it is believed that systems science holds a unique position to study the general patterns, fundamental principles and the basic mechanism underlying the evolutionary behaviour exhibited by various systems.

2.4 From Being to Becoming: the shift of emphasis in systems science

2.4.1 Seeking a dynamical equilibrium

As argued in Gao et al.[Gao et al, 1990], in the early stage of the development of systems science, the main interest was about the static organization and equilibrium state of self-stabilizing systems. This can be illustrated by analysing the development of one of the most important theoretical contributions to the systems science: cybernetics.

Since its inception, cybernetics, was more or less identified as a science of self-regulating and equilibrating systems. The three basic concepts in early cybernetics are "feedback", "stability", and "state". It has been successfully argued that the mechanism underlying those "equilibrium seeking" systems, whether they be natural or artificial, biological or inanimate, is characterised as having a "negative feedback" loop (see figure 2.4.1). A quick look at the classic monograph on cybernetics, i.e. Wiener's "*Cybernetics: information and communication in animal and machine*" [Wiener, 1948], or preferably, Ashby's "*Design for a brain*" and "*An Introduction to Cybernetics*", will reveal this view [Ashby, 1952, 1956]. Thermostats, physiological regulation of body temperature, automatic steering devices, economic and political processes etc. were studied under a general mathematical model of this negative feedback loop.

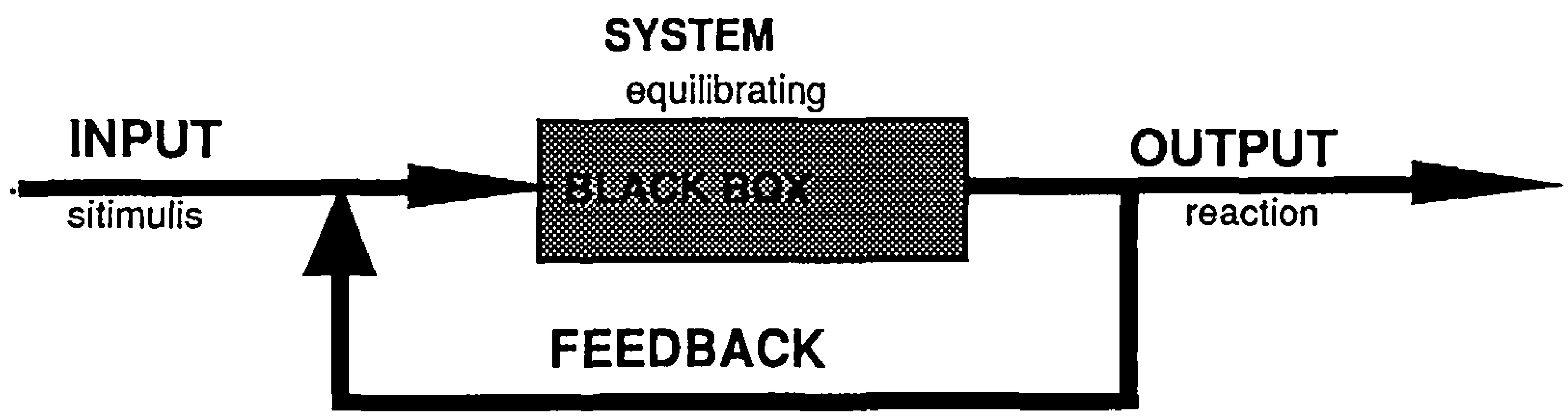


Figure 2.4.1 Equilibrium-seeking System

Homeostasis is a concept originally introduced for living organisms which describes the phenomena that a stable state of the organism is maintained by some organic regulating mechanism in such a way that they occur in an opposite direction to what a corresponding external change would cause according to physical law. It is extended to phenomena of seeking equilibrium in any systems and serves as a synonym of "seeking for equilibrium". A system which reveals a purposeful, goal-seeking behaviour is usually described and analysed under this general framework. Related mathematical models have been developed to describe these phenomena. For example, a dynamical system can be modelled by coupled differential equations. The asymptotic behaviour of the solutions of these equations is employed to describe the system's behaviour of seeking a steady equilibrium state. Application of this framework can be found in physiology, biology, sociology, economics, the design of various serviomachines etc. [Buckley, 1968].

However, less attention has been paid to the emergence and development of new ordered states in systems, either in early cybernetics or in the domain of systems science, until recently. The study of "becoming" occurring at all levels of the universe has greatly enriched our knowledge about the mechanism underlying these evolving behaviours.

2.4.2 From homeostasis to emergent attractor

Derivation-counteracting feedback has been served as the foundation of a self-regulating model which has been widely used in explaining, describing and designing huge ranges of systems seeking dynamic equilibrium. However, there are systems which change their structures and functions significantly over time. These phenomena include the outbreak of war between countries, the evolution of organisms, the rise of cultures of various types; in short, all processes of mutual causal relationships that amplify an insignificant or accidental initial kick, build up deviation and diverge from the initial condition. It is not fair to say that the phenomena of the breakdown of a system's structure and the appearance of a new one by deviation-amplifying feedback has totally escaped the cyberneticians' sight. Maruyama's paper in 1963 "The second cybernetics: deviation-amplifying mutual causal process" was the first, at least to the authors' knowledge, to discuss these phenomena [in Maruyama, 1968]. Examples cited in that paper include international conflict, the coming of a new town from a homogeneous area, the appearance of new type of culture in human history and morphogenesis in biology. These ideas have been shared by later thinkers in their discussions about the general evolution patterns except the term "initial kick" was replaced by other names like "fluctuation" [Nicolis et. al, 1977], "ignorance" [Allen, 1989a]. More detailed discussion about this evolutionary behaviours in open systems appeared in later 70's after the appearing and development of several important schools of thought on self-organization, i.e., "Order Through Fluctuations" [Nicolis et. al, 1977; Prigogine, 1980; Prigogine et. al, 1984], "Synergetics" [Haken, 1983a, 1983b, 1988], "Hypercycles" [Eigen et. al, 1979], "Autopoiesis" [Maturana and Varela, 1980; Zeleny, 1980], and "Theory of General Evolution" [Swenson, 1989a, 1989b].

The statement that every system tends to move to its thermodynamic equilibrium with maximum entropy and non functionary structure is obviously contradictory to the observed evolving world where order grows over time. Early cybernetics was unable to answer this question while it can analyse and describe the mechanism by which a system maintains its ordered structure and converges to a predetermined goal. This was noticed by some physicists like Schrodinger and systems thinkers like von Bertalanffy [Schrodinger, 1944; von Bertalanffy, 1968]. Distinction between closed and open systems was made that an open system can possibly evolve to higher ordered state on the expense of the environment's negentropy while a closed one is doomed to its maximum disordered state as indicted by the second law of thermodynamics. This was the first programme trying to fill this gap between the decrease of order in the universe predicted by thermodynamics laws and the increase of order observed in the biological world by adopting systems point of view but these ideas had not been fully developed in Bertalanffy's GST due to the lack of rigourous conceptual framework, intensive empirical study and powerful mathematical techniques. His main concern there was mainly about a system's structure, function, dynamic interaction between system's components, and equafinality behaviour of systems in general etc. In later 70's, studies on the spontaneous occurrence of coherent functioning structure from previously incoherent sets of components and the maintenance of the new ordered whole at a non-equilibrium state shed new lights on the evolutionary process observed at all levels in our universe [Nicolis et. al, 1977].

Independently developed in different fields, these schools of thought are all based on the principle that in a system which is open to the exchange of matter-energy-information (or matergon-information) with its environment, order can increase by importing negentropy from outside. In Prigogine's "theory of dissipative structures", for instance, the entropy change of an open system dS is split up into to two parts, i.e., entropy increase dSi, due to the irreversible process within the system, and the second part, dSe , the entropy import from its environment . Although in a system far from its thermodynamic equilibrium, $dS_i > 0$ always holds, when the negentropy excess certain level, say $dS_e < dS_i$, the total entropy of the system decreases, i.e.,
$dS = dS_i + dS_e < 0$.

The crucial role of microscopic fluctuations, or "initial kick" in Maruyama's term, within a system has been recognised and explored in depth in these theories mentioned above, especially in "Order Through Fluctuations". Chemically, it might be gradient in the kinetics of reactions, biologically genetic drift, economically the appearance of new products, socially new ideas of creative individuals. These fluctuations are constantly testing the stability of the system. Below some critical point, they are absorbed by the system through its multiple feedback and feedforward networks. But in a nonlinear system, these fluctuations might be amplified by its complex inherent dynamics which are characterised by nonlinear interactions between components (including feedback or forward networks). When some of the fluctuations are amplified, the old structure of the system may collapse and new ordered state may emerge spontaneously (without a recognisable external factor deliberately designing the new structure). This phenomena, i.e. self-organization, is observed at all levels of the universe and is regarded as the fundamental property of all evolutionary process.

2.5 Systems evolution (progressive change: unavoidable)

2.5.1 Evolution in the Darwinian tradition

Although Charles Darwin is regarded as the founding father of The Theory of Evolution, he himself, as Gould argues, had never used the term of "evolution" to imply any superiority of the new species over their ancestors. Darwin's theory of evolution goes nothing beyond "descent with modification" [Gould, 1975]. All species merely adjust themselves to fit the changing environment (*if an amoeba is as well adapted to its environment as we are to ours, who is to say that we are higher creatures?* -- Gould, 1975, pp36.). It was Herbert Spencer who should be credited with the one who advocated of the popular vision of evolution as "progressive complexification". In his *First Principle,* he defines evolution as follows:

> *Evolution is an integration of matter and concomitant dissipation of motion; during which the matter passes from an indefinite, incoherent homogeneity to a definite coherent heterogeneity* [Quoted from Gould, 1975, pp36].

Darwin's theory is about evolution of biological species, Spencer extended this concept to describe the process of evolution in the universe, from the inanimate world to human society. If it is admitted that the current planet, including human species, has evolved from a primordial earth before the appearance of any biological molecules, it must be accepted that the general tendency of evolution is towards a progressive complexification although in some special cases, like in certain biological species, evolution does not necessarily always lead to the increase of complexity in systems. Some other people like Henri Bergson and J.C Smuts also share with Spencer the view of universal evolution towards complexification [Bergson, 1911 (1975); Smuts, 1926]. The avoidance of defining evolution as progressive complexition is mainly caused by the concern that this kind of interpretation of evolution is highly controversial especially when is applied to the domain of human society which has inspired the Social Darwinism. Later in this study the concept of evolution will be discussed mainly from the structural point of view and it will be found that it is closer to the original meaning of Darwin's rather than that of Spencer, but systems evolution is defined to happen not just in biological systems, but in all systems at various level, such as physical, chemical, biological social systems etc..

The old theme about the evolution of biological species, in public's eyes, is implied in the famous doctrine "the survival of the fittest". In brief, the mechanism of evolution can be summarised as:

$$\textbf{Random small variations} \xrightarrow{\text{Natural selection}} \textbf{Evolution of species}$$

(emergence of new species)

It is implied in Darwin's original writing that the reason for evolution of biological species is to adapt to a changing environment, so there is no guarantee of a general improvement in the structure and function of the biological species. Evolution has no direction: only those which fit to the changing environment are selected and the nature is a "*Blind Watch Maker*" [Dawkins, 1986]

Three points need to be mentioned here about Darwin's theory of evolution which are related to our study of systems evolution.

First, the source of evolution is from the random small variations in species (gene drifts). This can be comprehended from the open systems point of view: evolution is originated in the system's inner microscopic fluctuations (random variations). Second, natural selection is to what kind of species is to survive in a changing environment. The emergence of new species and the selection by the environment can be understood from a broader sense about the relationship of systems and environment: the structure of systems must be compatible with constraints set by environment. The last point is about the time duration of evolution. It has been discovered that evolution is a gradual process full of "punctuated equilibrium" points, i.e., the time required for a species to evolve from one state to another is significantly short compared with the time it stayed in a stable state [Gould, 1975]. It implies that evolution is a "catastrophic event" and there is a break of the continuity of species. The mathematics to be used in this study, or catastrophe theory and dynamical systems theory in specific, will be able to reflect the abrupt change during evolution.

2.5.2. Systems evolution

In early days, cybernetics, or systems science in general, was mainly concerned with the static functional analysis of systems which permits understanding of self-stablizing mechanism underlying various systems, and exploring the relation between systems' structure and function. Due to the development of sciences resulting from continuing scientific explorations, it has now reached the stage of understanding the forces, forms, and main stages and phases of universal evolution. The philosophic climate is also created for the study of systems evolution. Even within the domain of systems science, evolutionary thinking has emerged at all epistemological levels.

Systems science consists of systems philosophy, systems theory, systems methodology and techniques and systems practices (see figure 2.1.1). Development at all four levels have contributed to the formation of a new evolutionary paradigm.

At the philosophical level, a system evolving as a whole was already noticed by some philosophers at late last century or early this century, like Spencer, Bergson, Smuts [Spencer 1971; Bergson 1911; Smuts 1926]. In the 20th century, physicist Schrodinger argued, in his small book *"What is life"*, that biological systems can evolve to higher ordered state because they are open to their environment [Schrodinger, 1944]. The pioneers of this century's systems movement like, Bertalanffy stood firmly on the ground that entropy can decrease in open systems. Latter thinkers like Prigogine et. al have contributed further to the evolutionary thinking of open systems [Jantsch, 1980; Prigogine et. al, 1984].

At the theoretical level, concepts, principles, and models of evolutionary systems can be found in various systems theories, especially in later contributions to the study of self-organization, or, “becoming” phenomena in the universe, i.e., the Theory of Dissipative Structures, Synergetics, Hypercycles, Autopoiesis, Cellular Automata, and Artificial Life et. al [Wolfram, 1984; Langton, 1986; 1989]. They have provided us with some conceptual frameworks and concrete plans for the study of systems evolution.

At the methodology level, methods, strategies, approaches, and tools are available for the study of the forces, forms, and stages of evolutionary processes in the universe. It needs to be stressed about the role that the techniques provided by mathematical dynamical systems theory (DST) can play in the effort to understand the mechanism of systems evolution. This is one of the main concerns of this research and it will be discussed later on. Other techniques include Cellular Automata and the general research under the title "Artificial Life" which tries to study the evolutionary behaviour of simple systems governed by some deterministic rules. It is argued that computer simulations of those discrete dynamical systems will help to reveal the evolutionary myth of systems in general.

At the practical level, great attempts have been made in various field to study various evolutionary processes, like in biological systems, ecological systems, socio-economical systems and human societies [Allen, 1986, 1989a, 1989b; Schneider, 1988 et. al]. The evolutionary behaviour of various systems including neural networks will be discussed. This is regarded as the first step for us to understand the evolving universe at different spatial-temporal scales by using the new evolutionary paradigm that has emerged over the last 10 years.

The time has arrived to establish a new chapter in systems science, the chapter of systems evolution, by working at all levels, from philosophical level down to practical level. This marks the shift of paradigm from self-stabilising systems to evolving systems. The study of systems' nonlinear dynamics by modern DST is of special importance for this effect because it provides us with robust concepts and solid methods for some of the evolutionary processes as well as a dialectic attitude and a complementary strategy.

Viewing evolution as a special form of time discourse of open systems where the qualitative change of spatial-temporal behaviour can be observed, it can be defined that: <u>systems evolution is a process during which either a new system emerge by association of formerly unconnected elements (subsystems), or a system changes its structure qualitatively to maintain as an organized whole; it happens in a changing environment.</u>

Several remarks should be made about this definition. It is apparent that this two parts definition reflects two different types of evolution. The first one is about the coming into existence of a system at a new level: unconnected elements are organized to form an emergent at a new spatial-time level. It is demonstrated by examples in physical systems, chemical systems, like Benard cells, chemical waves etc. This kind of evolution is usually called "order out of chaos" or "order out of disorder". This kind of evolution can be defined as <u>systems genesis</u>. Systems genesis happens across levels in the elementary particles -atoms -molecules -biological molecules -cells -organs -organisms-species - groups -society -supernations hierarchy.

The second part of the definition refers to a intra-level evolution: a system evolves in a changing environment simply to stay as an organized whole. No additional parts are added to the system and what has changed is only the system's structure and organization.

The whole process of systems evolution includes both systems genesis and intra-level evolution, and only through this process, various forms of systems have appeared and evolved in this planet. However, evolution of biological systems, or living systems in general (like what is defined by Miller in [Miller, 1978]), has its own unique characteristics: reproduction through inheritance [Yates, 1988]. Evolution happens for species, not for individual systems. Two levels of discussion are involved: random small variations happen at the genotype level and natural selection works at the phenotype level. Favourable traits are inherited through reproduction and the evolution of biological species, or living systems in general, is hence an accumulating process [Dawkins, 1986]. The general discussion about systems evolution may still be applicable in studying biological evolution, but it is not sufficient. Like the application of systems theory in analysing any specific systems, the unique characteristics, which are usually thinghood related, must be considered. It is not the aim of this study to provide a formula which can be used to solve all problems related to systems evolution by direct applying it, and there may never be such a formula. What that is intended to achieve in this study is that, through careful examination of some related models, methods, and examples, certain general principles, mechanisms, ideas, and models may be discovered or established to shine light on problems related to systems evolution, say, conditions, patterns, processes, and trends. The goals to be achieved include a general conceptual framework and a formal model about systems evolution. The aspect of structural change during the process of evolution is of particular interest to us and it will be tackled by using dynamical systems theory.

Chapter 3 Self-Organization: the way that systems evolve

3.1 General principles of self-organizing systems

Long before the rising of a general discussion of "systems evolution", the concept of "self-organization" or "self-organizing systems" had received great attention during the early sixties. A series of conferences devoted to "self-organizing systems" and the proceedings published later recorded, as it is generally agreed, the main development and achievement of those discussions in early days [Yovits et. al (eds), 1960; von Foerster et al.(eds), 1962], although it has been argued that the study can stem back to as early as 40's [Dalenoort, 1989]. The study of self-organizing systems or self-organization went silent for a while and has been rekindled almost two decades later by the study of the spontaneous emergence of coherent spatial-temporal structure in non-equilibrium thermodynamics and other fields.

Despite its importance and usefulness, the concept of "self-organization" or "self-organizing systems" is difficult to define precisely. It depends upon where you draw the boundary and how you conceive the system and its environment. As a matter of fact, von Foerster has even argued in a well-known paper, i.e. "On self-organizing system and their environments", that there are no such things as "self-organizing systems" if you draw the boundary just to encompass the kernel of the system and consider the system as closed or isolated. It has been realized that there von Foerster over emphasised the "self" in "self-organizing". Self-organizing systems do exit only if we take an open systems point of view, as von Foerster himself admitted, in a conditionally stated sentence:

> *"... I propose to continue the use of the term 'self-organizing system', while being aware of the fact that this term become meaningless, unless the system is in close contact with an environment, which posses available energy and order, and with which our system is in a state of perpetual interaction, such that it somehow manages to 'live' on the expenses of this environment."* [von Foerster, 1960, pp6]

This open systems point of view is now regarded as the starting point for any discussion of self-organization and systems evolution and it has been generally accepted that any open systems which have "rich" interactions with the environment can exhibit self-organizing behaviour, provided certain other conditions, such as nonequilibrium, nonlinearity, microscopic fluctuations etc. are met. Bearing in mind that the concept of "open system" has been used in von Bertalanffy's writing in 40's [von Bertalanffy, 1968], it seems strange that the term of "open system" was not used explicitly in those days even in the discussing of self-organizing systems (for example, in von Forester's writing).

It is usually conceived vaguely that the process of "self-organization" or a "self-organizing system" refers either to the spontaneous appearance of order or organization where there was previous none (order out of chaos, as often said) or the automatic replacement of an organization at a lower ordered state by one at a higher ordered state (order out of order). The characteristics of this process is that it goes "by itself" in the sense that there is no recognizable external "agents" responsible for deliberately designing the emerging structure or organization. According to this definition, a self-organizing system is in contrast with systems like a machine, a modern commercial firm etc. which are designed purposefully by human beings, the outside designers, to serve certain goals. To decide whether there is a process of self-organization occurring in an open system, there are two criteria which depend on whether a structural or a functional description in discussion is adopted. For a structural description, the process of "self-organization" can be detected by the appearance of some structural regularities, like the emergence of hexagonal cells in Benard experiment. For a functional description, it can be assessed by examining some aspect of the performance of the system, like the successful adaptation to the changing environment by a biological system. Be it structural or functional, it is natural to describe the process of self-organization as a process in which, in the terms of thermodynamics, an open system's entropy decreases, because concepts like entropy, negentropy, order and "disorder" etc. are connected with each other.

As we mentioned above, self-organization can only occur in open systems. Suppose that there is an open system O_S with E as its environment. The system is open to its environment and there is constant change of matter/energy/information between the system and its environment, as illustrated in figure 3.1.1.

From the thermodynamics point of view, the state of the system can be characterised by its entropy level S and the direction of change of the system is specified by the sign of its entropy change rate dS/dt. The entropy change dS consists two parts, one is dS_i which comes from the irreversible process occurring within the system and the other one, dS_e from the interaction between the system and its environment. Usually, we can denote this as:

$$dS = dS_i + dS_e \qquad (3.1.1)$$

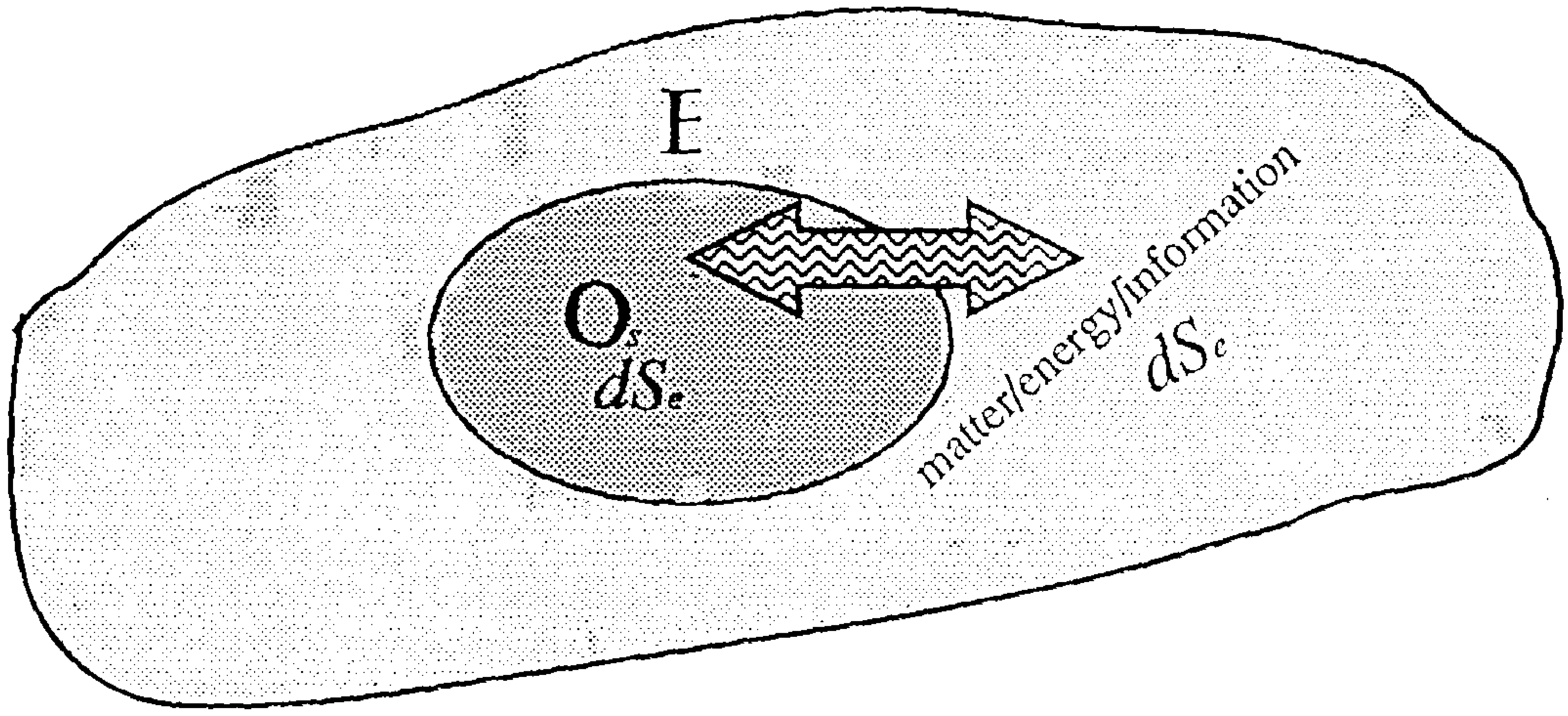


Figure 3.1.1 Open system and its environment

As generally accepted, self-organization happens when $dS < 0$. According to thermodynamics, $dS_i > 0$ always holds when the system is not at its thermodynamic equilibrium. Therefore the condition for self-organization becomes:

$$dS_e < - dS_i < 0, \quad |dS_e| > |dS_i| \tag{3.1.2}$$

That means that when the rate of neg-entropy contribution from the environment exceeds the rate of internal entropy production, the system self-organizes itself.

This criterion for the detection of self-organization has also served as the starting point for discussions of systems evolution. A similar discussion based on the term of "order" rather than "disorder" can be found depending on various forms of the term "order" defined. Among them von Foerster's concept of order is well known.

Based on Shannon definition of "redundancy", von Foerster defines the measure of order of an system as:

$$R = 1 - H/H_m \tag{3.1.3}$$

where H is the entropy level of the system at the state considered, H_m the possible maximum entropy. According to this definition, when the system is at an absolute ordered state, its measure of disorder i.e., entropy $H=0$, that implies $R=1$. When the system is at an absolute disordered state, $H=H_m$, therefore $R = 0$. This is consistent with our intuition about the proper measure of order of a system. For discussions of self-organization, it would be useful to know what this formula can tell.

$$dR = -(H_m\, dH - H\, dH_m) / (H_m)^2 \tag{3.1.4}$$

It is easy to show that :

(i) when H_m = constant., $dR > 0$ if and only if $dH < 0$

(ii) when H =constant, $dR > 0$ if and only if $dH_m > 0$

(iii) when $H, H_m \neq$ constant, $dR > 0$ if and only if $H\, dH_m - H_m\, dH > 0$

These results can be regarded as the general criteria for the process of self-organization. Apparently, result (i) agrees with the above discussion and our intuition about order and disorder. The other results are not so obvious [von Foerster, 1960].

This discussion is based on terms of thermodynamics, it can serve as a very good explanation about the process of self-organization. Because it is highly controversial to extend the concept of entropy beyond the domain of thermodynamics, none of these various forms of definitions of "order" and "disorder" has been widely accepted and similar discussion can only serve as a suggestive explanation. For a detailed discussion about self-organization, or systems evolution in general, based on concepts of thermodynamics, Weber et. al give a very good review and some renewed arguments [Weber, et. al. 1988].

In early days, self-organizing systems were usually related to natural systems like biological systems, learning behaviour of neural networks etc. which are essentially different from those man-made systems like machines. When Ashby talked about self-organizing systems, he was mainly concerned with those complex systems which seek equilibrium state by itself, like a country's economy goes back to normal state (equilibrium state) from a war. He argued that when a system goes to one of the equilibria, it is moving from a large number of possible states to a smaller fraction, or in other words, the system is changing from a more probable state to a less probable one [Ashby, 1962]. In that case entropy of the system decreases and order increases. Going to an equilibrium state is a kind of self-organization in Ashby's term that the system goes from a "bad" organization to a "better" one without the purposeful design of some outside factors. This kind of self-organization is sometimes described as the "goal seeking" behaviour of systems. The prototype of this kind of self-organizing systems is the growth of crystals and the stabilization of national economy. However, what is of particular interest to us is a special kind of self-organizing systems which either develop its own organization from none, or move from a lower ordered state to a higher ordered one and all these happen spontaneously. The state of the system before and after this change is qualitatively different and there is a discontinuity in the system's behaviour over the period of time during the process of self-organization. Although there is not a single external factor which can be identified as responsible for this process, it happens only when there is close interactions between the system and its environment. The system self-organizes itself by absorbing the neg-entropy from its environment in the sense that it dissipates matter/energy/information from its environment. This kind of self-organization is defined as "dissipative self-organization". Examples range from physical systems, such as the often quoted Benard Hydrodynamics experiment [Swenson, 1989a, b], through chemical systems like the Belousov-Zhabotinsky (BZ) chemical wave [Haken, 1983a, 1983b, Skinner et al, 1989, 1991] , to biological systems like the hypercycles [Eigen et. al, 1979]. Compared with the "goal-seeking" type of self-organization as mentioned above, the process of dissipative self-organization is more complex. These systems can exhibit certain novelties and creativities in the sense that the emergent macrostates have new irreversible behavioural / dynamical regimes at new spatial-temporal scales which are inaccessible to and unobserved in separate components. This process increases the degree of complexity in open systems and therefore, in most cases, the notion of "dissipative self-organization" is used as a synonym of systems evolution. Dissipative self-organization, or self-organization in short, is the actual process taking place during systems evolution.

There are different schools of thought about self-organization which set to develop the general principles for the understanding of the spontaneous emergence of dynamical structure at a new spatial-temporal level and to lie the foundations for a general argument of "becoming", i.e. systems evolution, observed at all scales in the universe. The following sections will review critically some of the most important schools and attention is brought especially to the mathematical treatments which are believed to be able to be unified by the mathematical dynamical system theory. One important field not discussed here is self-organization observed in neural systems which will be discussed latter as an original examples of the application of the new evolutionary paradigm.

3.2 Order out of Chaos: self-organization in non-equilibrium systems

Theory of dissipative structures was originated by Ilya Prigogine and his colleagues in Brussels. Primarily in studying the non-equilibrium thermodynamics, it has been discovered that order can emerge in open systems in a far-from-equilibrium state through the exchange of matter/energy /information with its environment. This has been extended and developed to provide a foundation for the description and analysis of the spontaneous transformation of systems from an incoherent state to a coherent state and their further evolution. It has been regarded as the most important school of thought of self-organization. Its impact has gone far beyond the study of non-equilibrium systems on the science community: its philosophic implications has promoted the new thinking about a science of "becoming" in contrast to the science of "being". [Prigogine, 1980; Prigogine et. al, 1984].

The conceptual framework of the theory of dissipative structure is built on notions such as open system, non-equilibrium, fluctuations, nonlinearity, chance and necessity etc. The main conclusion is that "Nonequilibrium is the source of order" and that "order comes through fluctuations". A brief summary of these ideas can be given as following.

Openness The open systems point of view has been explicitly stated and stressed in the theory of dissipative structures. As discussed above, for a systems open to its environment, its overall entropy level is decided by both the inner entropy production arising from the internal frictions of the system and the net entropy input decided by the matter/energy/entropy influx from the environment. When the neg-entropy influx surpasses certain critical point where the internal entropy production is more than compensated, the state of the system may undergo a radical change of its state and new order may come into existence. This has been stressed in the theory of dissipative structures and adapted as the starting for the study of self-organization and systems evolution.

Far-from-equilibrium In the theory of dissipative structures, three different types of thermodynamic state of systems must be distinguished, i.e. equilibrium, linear non-equilibrium (near equilibrium) and nonlinear non-equilibrium (far-from-equilibrium). When the system is at its thermodynamic equilibrium, it has reached its maximum disorder in the general sense and its entropy production d_iS vanishes. At such a state there is not functional structure existing, no direction of time, no "free energy" and the system is in a homogeneous state (or there is no system at all!). The entropy level is characterised by:

$$d_iS = d_eS = 0 \qquad (3.2.1)$$

The total entropy = maximum

This is the ultimate state of any closed systems including our universe, as stated in the second law of thermodynamics.

When the system is in the region of a linear equilibrium --- that is, in the range of validity of Onsager' "reciprocity relation" [Nicolis et. al, 1977] ---, it evolves toward a stationary state and that state is necessarily a non-equilibrium state at which dissipation of energy still exists. It has been proved that this process is governed by the principle of "minimum entropy production" and this non-equilibrium state is characterised by a minimum entropy production $P = d_iS = constant > 0$ which is compatible with the constraints imposed upon the system by its environment [Prigogine et.al, 1984]. For at the thermodynamic equilibrium $P = d_iS = 0$, the state restricted by

$$P = d_iS = constant > 0 \qquad (3.2.2)$$

is hence called a "close-to-equilibrium" state or near-equilibrium state.

There is nothing special at the close-to-equilibrium state which is maintained as a dynamical equilibrium, i.e. the inner entropy production is just compensated by the negative entropy imported from its environment and all quantities that describe the state of a system, like temperature, concentrations, and spatial-temporal structure, become time-independent. This system is in a stable state and its behaviour is predictable in the sense that it will move to the final state determined by the imposed boundary conditions.

Phenomena like the above mentioned Benard convection experiment, BZ reaction etc. are quite different and these systems can exhibit some novel behaviour such as the emergence of new spatial-temporal structure when the environment changes. Such new structures can always be identified in these processes of change and the negative entropy flux to the system from the environment is no longer a linear function of the forces imposed on the system. In this case, the system is within its nonlinear non-equilibrium region or at a far-from-equilibrium state. Nonlinear thermodynamics studies systems at their far-from-equilibrium state and sets out to analyse the process of self-organization through which novel structure and behaviour can be observed which is totally different from what has been understood about close-to-equilibrium systems. "Nonequilibrium is the source of order", as Prigogine has claimed, and this has opened a whole new world where the increasing complexity of this universe is no longer so mysterious once we start to look at systems from this nonlinear nonequilibrium point of view. [Prigogine et. al, 1984, pp287].

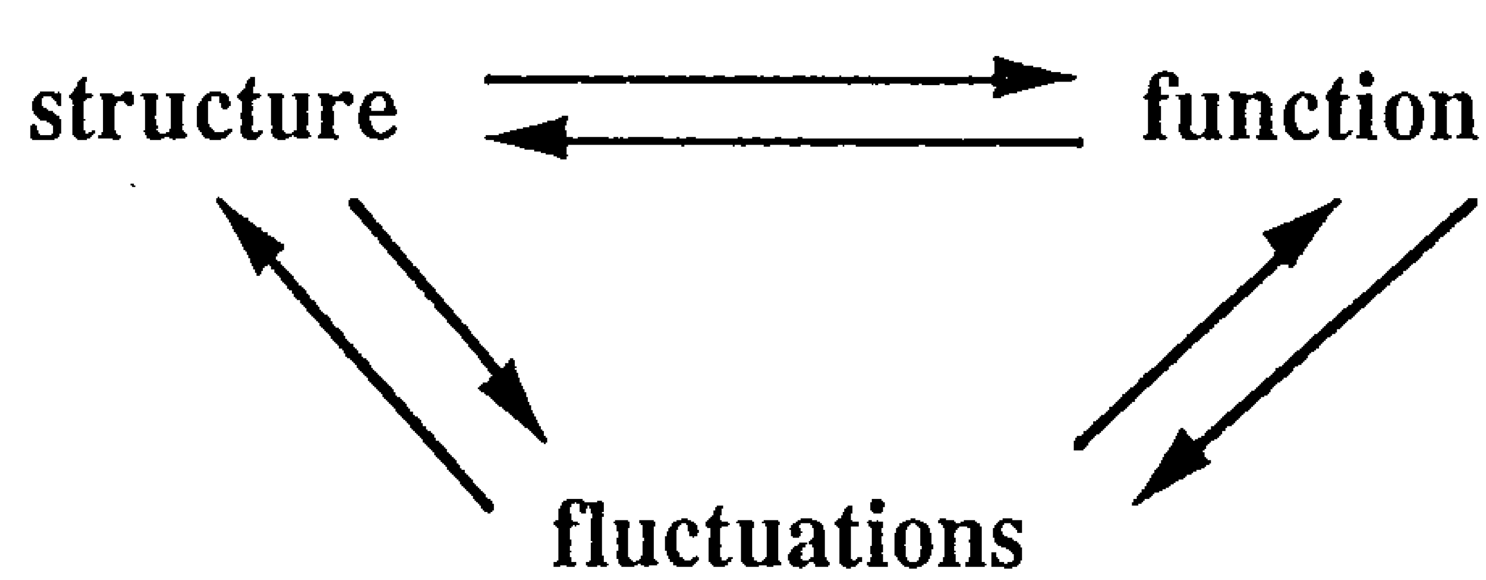
The principle of minimum entropy production is regarded to govern the behaviour of systems in a close-to-equilibrium state, it no longer holds for far-from-equilibrium systems. A system at a far-from-equilibrium state does not tend to equilibrium as it does in the close-to-quilibrium situation. On the contrary, it produces entropy at its maximum rate through the irreversible process occurring within the system and, with the negentropy influx from its environment, tries to stay in a far-from-equilibrium state. That is:

$$dP/dt > 0 \dashrightarrow P_{max} \qquad (3.2.3)$$

This has been defined as the "principle of maximum entropy production" by Swenson and it has been successfully argued that it is this very "principle of maximum entropy production" that governs the behaviour of nonlinear nonequilibrium systems and gives rise to new dynamical spatial-temporal structure in those systems. That is to say that the principle of maximum entropy production is the universal physical law underlying all the self-organization processes [Swenson, 1989a, 1989b].

Fluctuations Openness and far-from-eqilibrium are the necessary conditions for a system to be able to evolve from lower ordered state to higher ordered state, but they are not sufficient. Among the others, one of the most important factors which is emphasized in the theory of dissipative structure is that the system must possess sufficient fluctuations at the microscopic level.

Any system is composed of many interconnected components. The behaviour of the system depends on the interactions of its components, i.e. it depends on how those components are organized rather than what the behaviour of every individual component. For a system at a stable state, its structure and behaviour can be described at the macroscopic level and the movement of the composing components can be neglected. However, at the microscopic level, those components, or subsystems, are in constant motion and constant change. These movements and changes are restricted by the global behaviour of the system through those inter-connections between components. The components or subsystems are also subjected to their own laws at the microscopic level which are different from the laws the system as a whole should watch. Microscopic fluctuations never stop and they constantly test the stability of the system as an organized whole. Usually they are absorbed or damped by the system when the system is macroscopically stable. For a nonlinear, nonequilibrium, and open system, these fluctuations can, coupled with perturbations from the environment, be amplified to destroy the stability of the system and this will usually give rise to new state to the system. This is called "order through fluctuations" in the theory of dissipative structures and it can be illustrated as:



These microscopic fluctuations provide a system with different potential choices for its future state. Depending on which fluctuation is amplified, the system moves to one of the various potential states blueprinted by or prescribed by the various fluctuations. Physically, fluctuations can be thermal movement of molecules, biologically the gene drifts, socially the free will and new ideas of individuals. Each of these fluctuations correspond a potential macroscopic state of the system when it is amplified.

Nonlinearity Essentially, a system capable of self-organizing must have many interconnected components. The complex interdependence is manifested as multiple feedback and feedforward loops. They reflect the nonlinearity of a system's inner dynamics. Only in a nonlinear system as such, a small change arising from internal fluctuations can be amplified through the nonlinear mechanism at a critical point to give birth of a new state to the system . From the mathematical point of view, the behaviour of a linear dynamical system is trivial: its behaviour is decided by some simple invariant sets like node, centre, source etc. For nonlinear dynamical systems, there is a totally different picture: a two dimensional dynamical system can rest on a periodic state and a three dimensional system may even stay at a chaotic state. Only nonlinear systems can have rich bifurcation behaviours and this makes it possible for new spatial-temporal structure emerge from microscopic fluctuations. This is also stressed in the theory of dissipative structures [Nicolis et. al, 1977].

Necessity and chance When the stability of the system is destroyed at a critical point by the amplification of one of those restless fluctuations, the system undergoes a radical change of its structure and behaviour and this very point is called a "critical point" or "bifurcation point". As argued by Prigogine, the microscopic fluctuations introduce the randomness and hence introduce the "arrow of time".

> "*Only when a system behaviour in a sufficiently random way may the difference between past and future, and therefore irreversibility, enter into its description*" [Prigogine et.al, 1984, p16].

Because the system faces different choices initiated by different fluctuations at the bifurcation point, the emergence of new structure is decided not by laws which the system followed previously but rather by some random factors.

At the bifurcation point, the law of large numbers breaks down and the process is dominated by random factors introduced by the microscopic fluctuations [Nicolis et.al, 1977; Prigogine et.al, 1984]. The system has many possible directions of evolution to follow. Which one the system will finally choose is unpredictable. The decision made at this critical point is hence called an "event". After this event, the system enters into a new ordered state and re-assumes its macroscopic stability. The impact of microscopic fluctuations no longer changes the system's macroscopic behaviour until next critical point is achieved. Under the constraints of its environment, the system is stable and its new state is maintained by the constant exchange of matter/energy/ information with the outside world. The macroscopic behaviour of the system follows some laws at the new stage. The microscopic fluctuations have never ceased and always exist to test the system's stability within new macroscopic constraints. When the next critical point is reached, these fluctuations will compete to each other again so that one of them can be chosen, once more, by some random factors or the "mysterious unknown force", to be amplified to dominate the system's behaviour and gives rise to a new order state.

Therefore "chance" and "necessity" both are essential in the description of the process of self-organization of nonlinear non-equilibrium. This can be illustrated by the following bifurcation diagram (figure 3.2.1).

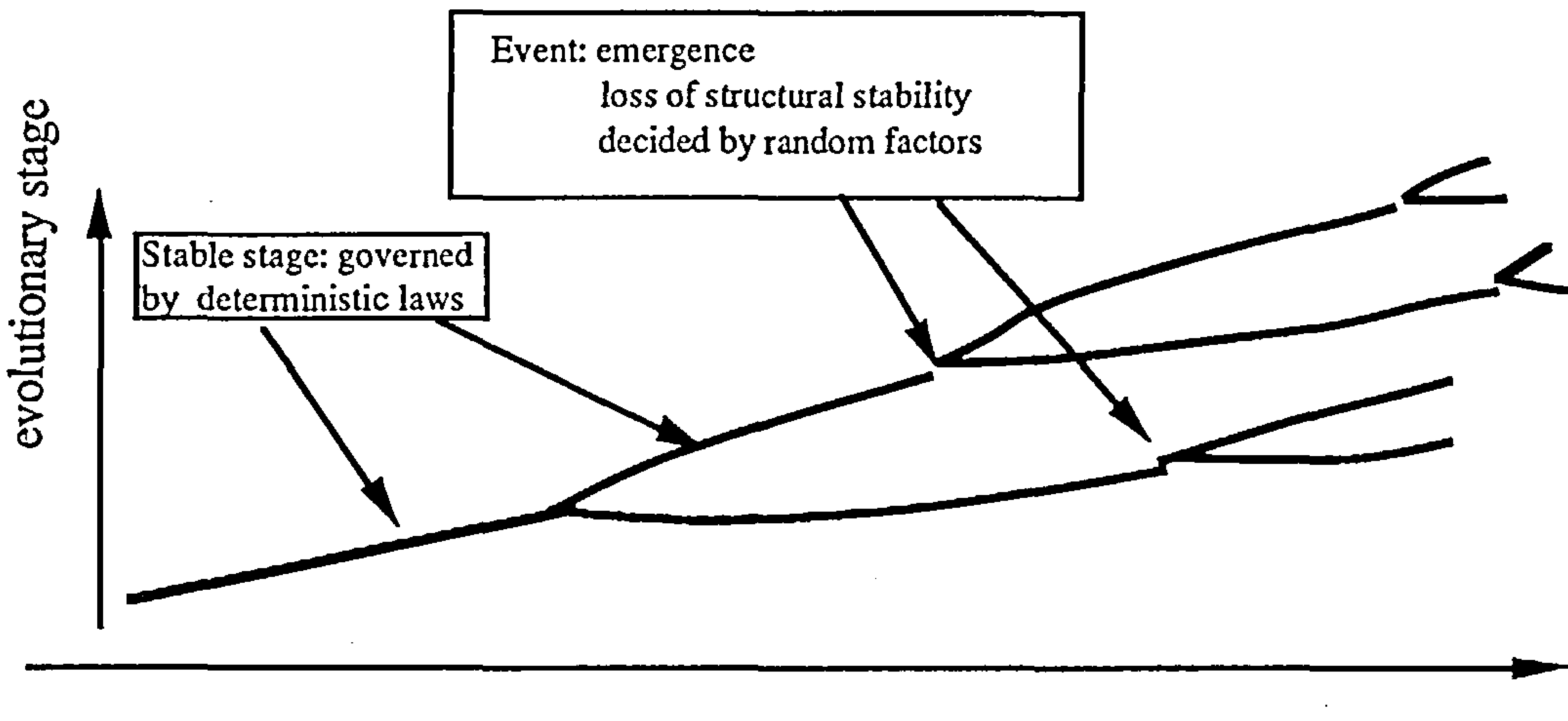


Figure 3.2.1 Chance and necessity: history versus events

The evolution of a far-from-equilibrium system is full of events recorded by critical points. Between any two events the system follows certain deterministic laws which decide the systems' behaviour at the macroscopic level and affect the fluctuations at the microscopic level. During the periods between two events, systems are stable and their macroscopic behaviour can be described and predicted, but, as Prigogine said,

> "*we can never determine when the next bifurcation will arise*"[Prigogine and Stenger, 1984, xxvi].

3.3 Hypercycle: emergence of dynamical functional structure in biological systems

Although Charles Darwin's "The Origin of Species" was published more than one hundred years ago, we still have not had a complete picture about exactly where and how life has come into existence on this planet in the first place. The progress in science and technology over the past century has provided us many assumptions, theories, evidences about possible ways in which the biological world began. The Big

Bang hypothesis sets out to explain the origin of the universe which hosts the solar system and our planet [Hawking, 1988]. Stanley Miller's electrical charge experiment provides evidences that the biological systems might have been formed in the early times of the primordial earth through some elementary physical and chemical reactions under certain special conditions like sustained higher temperature, higher pressure and electric sparks etc.[Miller et.al, 1974]. Various theories about the appearance of pre-biologic molecules have also been proposed and investigated recently [Horgan, 1991]. To study how the biological species evolve from those basic biological molecules, Eigen and Schuster have postulated that there may have been a number of big biological molecules supporting each other by functional interactions and hence had formed some auto-catalytic cycles called "hypercycle" [Eigen et. al, 1979; Eigen et al, 1980]. The theory of hypercycle tends to explain the process of natural selection and mechanism of self-organization in pre-biotic systems at the macroscopic pre-biotic molecules level. For it stresses the complex feedback mechanism, i.e. the auto-catalytic and cross-catalytic cycles -- the manifestation of necessary nonlinearity during self-organization -- it has contributed to the understanding of the general principles and mechanism underlying the process of self-organization in all open systems. The rigourous mathematical treatment in the theory of hypercycle provides a useful guide-line for dealing with self-organization in biological systems and this can be restated latter by using the language of dynamical systems theory.

Catalytic reaction is very fundamental in chemical reactions, especially in organic reactions [Nicolis et.al, 1977; Eigen et. al, 1979; Farmer et.al, 1986]. As it has been known to us for a long time, different enzymes serve as very important catalysts in various basic chemical reactions in living cells of biological systems. The catalytic process can be viewed as a cyclic process of the catalyst through which the catalyst forms different intermediates with the reactants and finally gives the product and the catalyst comes back to its original state and remains individually unchanged. This can be illustrated as follows (figure 3.3.1):

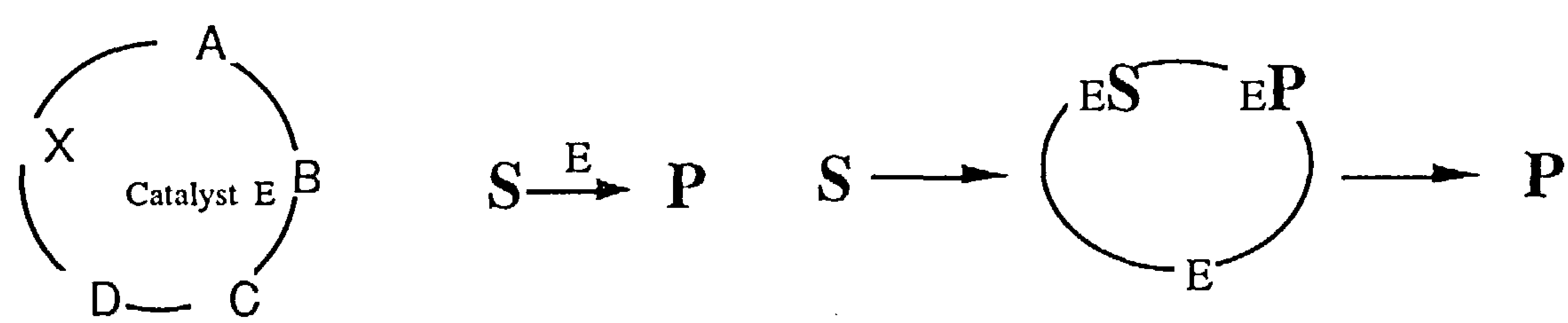


Figure 3.3.1 The catalytic process

If the catalyst itself is also both the reactant and the product, like in the following reaction:

$$X \xrightarrow{I} I \qquad (3.3.1)$$

it is called, following Eigen and Schuster, an autocatalytic cycle [Eigen et. al, 1979]. It is believed that the self-replication of double-stranded DNA is an autocatalytic process. If there is a reaction cycle in which, at least one, but possibly all of the intermediate products are catalysts, it is called a catalytic cycle which represents a higher level of organization in the hierarchy of catalytic scheme.

> "*A catalytic hypercycle is a system which connects autocatalytic or self-replicative units through a cyclic linkage*" [Eigen and Schuster, 1979].

The hierarchy of cyclic reaction network is illustrated in the figure 3.3.1. Eigen and Schuster have argued that hypercycle is the unique reaction cycle found in the selection and evolution of RNA and DNA molecules [Eigen et. al, 1979, Eigen et. al, 1980].

The main assumption in the theory of hypercycle is that self-replicative macromolecules, like RNA and DNA, in a suitable environment can exhibit a behaviour which can be represented by the concept of quasi-species. "*A Quasi-species is defined as a given distribution of macromolecular species with closely interrelated sequences, dominated by one or several (degenerate) master copies*" [Eigen et. al 1979, Eigen et. al, 1980]. These quasi-species are capable of self-reproduction withe the help of some information carriers which are believed to be RNA molecules.

It is also supposed that

> "*... systems of matter, in order to be eligible for selective self-organization, have to inherit physical properties which allow for metabolism, i.e. the turnover of energy rich reactants to energy-deficient products, and for ('noisy') self-reproduction. These prerequisites are indispensable. Under suitable external conditions, they also prove to be sufficient for selection and evolutive behaviour*" [Eigen and Schuster, 1979].

What is Hypercycle? It is

> "*the analogue of Darwinian systems at the next higher level of organization*"

It has been argued that natural selection and evolution, which are the consequences of self-reproduction, operate in the case of molecules as they do in the case of cells or species.

Equipped with these notions and the mathematical techniques of fixed point analysis, Eigen and Schuster set out to analyse the selection and evolution of quasi-species formed by macromolecules.

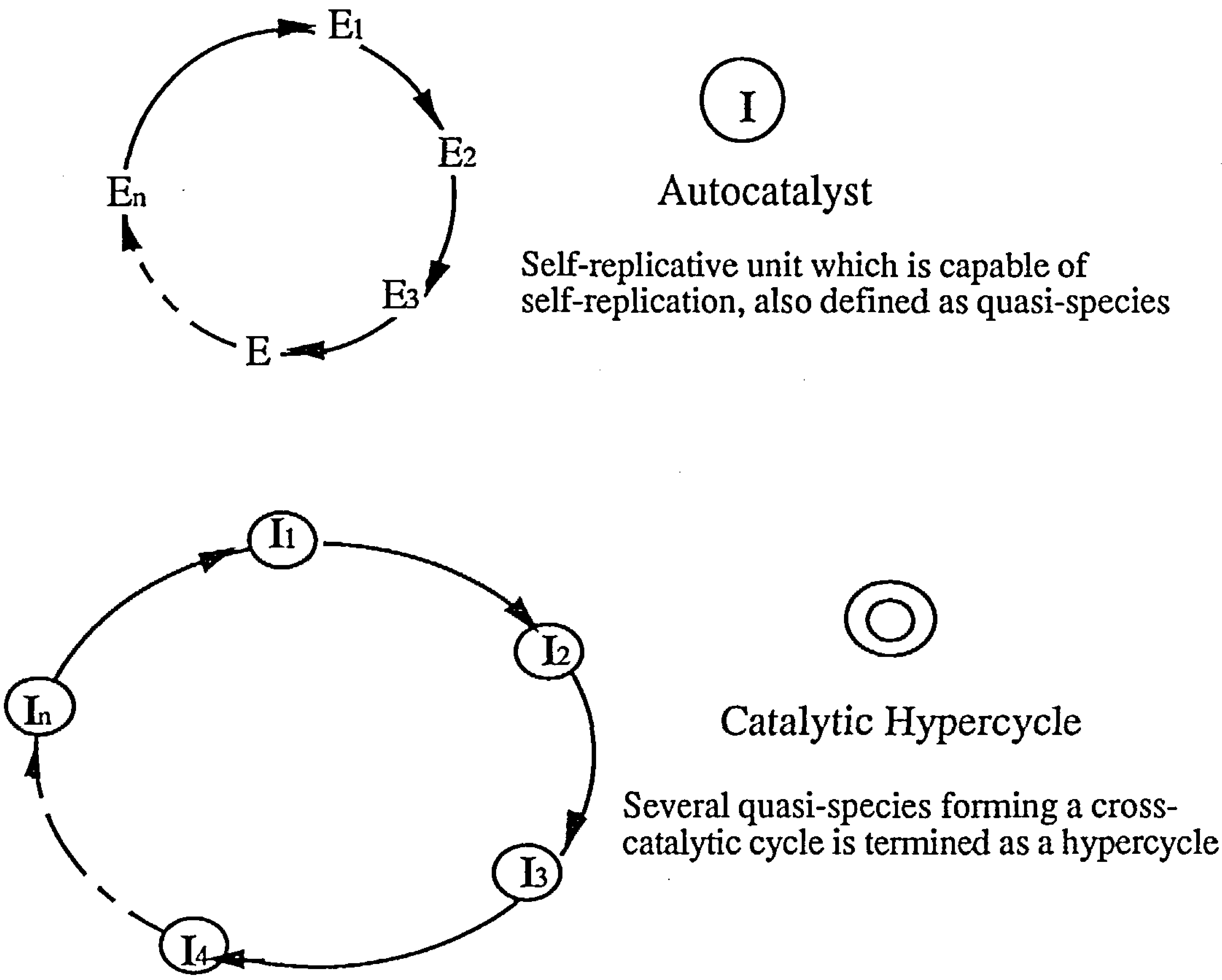


Figure 3.3.2 The hierarchy of hypercycles

The simplest system, according to the above mentioned necessary prerequisites, can be described by a system of differential equations of the following form:

$$dX/dt = (A - \Lambda - \Phi)(X) \qquad (3.3.1)$$

where $X = (x_1, x_2, ..., x_n)$ is the vector of population variables; A reflects the positive contribution (amplification) to X, Λ the negative one (decomposition), while Φ refers to the out flux of X from the system. A quantity called the intrinsic selective value is defined as the result of the metabolism and the decomposition of the molecular species, i.e. $W = A - \Lambda$.

The equation is essentially nonlinear (with **W, A,** Λ in respect to **X**) and mathematical techniques employed here are from dynamical systems theory (fixed points analysis in particular), although not explicitly stated. It has been proved that only hypercycle organizations can fulfil the requirements for a selection of the best functionally linked assemble and it's evolutive optimization, i.e. the information stored in each single replicative unit or reproductive cycle must be maintained and these must establish a cooperation which includes all functionally integrated species [Eigen and Schuster, 1979, Eigen et. al, 1980, Schuster, 1989]. Realistic hypercycle has been analysed in studying the origin of the genetic code and the translation machinery.

The formation of hypercycles is claimed to be the general principle governing natural self-organization at the pre-biotic level. It stresses the complex feedback mechanism in biological systems and this also implies that the non-equilibrium open system capable of self-organization must be essentially involving nonlinear interactions between its components. The nonlinearity is represented by some catalytic networks manifested in the form of nonlinear feedback loops. This is the necessary condition for self-organization in all systems and it has also been stressed, although maybe in different ways, in some other schools of thought about self-organization like the theory of dissipative structures and synergetics. The following theory, i.e. theory of autopoiesis is also devoted primarily to the study of self-organization in biological systems but can be extended to contribute to systems evolution in some important aspects.

3.4 Autopoiesis: self-reproduction and biological organization

Another term relevant to self-organization phenomena often encountered in the systems literature is "autopoiesis". It stems from "auto" (which means "self") and "poiesis" (which means "production") and literally means "self-production". Although coined originally by H. Maturana, F. Varela, and R Uribe to explain the particular characteristics of living systems, it has been extended, sometimes in a rather naive manner, to molecules, organism, nervous systems language, communication, social behaviour and human societies [Varela et. al 1974; Zeleny, 1980; Mingers, 1989]. More accurately, it is employed as a useful metaphor to describe a wide range of systems which can exhibit such autonomous behaviour like self-production and self-organizing. For the theory of autopoiesis aims originally to emphasize the defining phenomenological aspect of living systems, i.e. self-(re)production, rather than to look for general principles underlying the process of self-organization found in various systems, it seems that it can not serve as a general theory about self-organizing systems, or evolving systems. However, it still contributes much to the discussion of self-organization in the sense that it provides some useful metaphors as well as some heuristic ideas. The model of autopoietic systems is essentially mechanical and it resembles, to some extent, to the model of cellular automata which we will explain later in this chapter.

For the discussion of autopoiesis, the distinction between organization and structure of systems needs to be particularly emphasised. A system, or using the original terminology in autopoiesis, a unity, is a whole distinguished from the background (environment) by the observer. The organization of the system is the relation between its components and the necessary properties of the components which characterize the system and define and maintain it as a unity distinguished from its environment. Structure, on the other hand, is the actual temporal and spatial relations between components. It is also regarded that the organization defines a system in general as belonging to a particular type of class while structure describes the actual components and the actual relation of a particular real example of any such systems [Varela, 1986; Mingers, 1989]. Generally, structure and organization are interchangeable in most current systems literature especially when only mechanical systems are concerned where both of them refer to the actual relations between components of a system. It is right in most cases when physical systems are analysed in that way, but it is necessary to distinguish structure and organization when we are dealing with biological systems. For example, the organization of a cell is maintained invariant while its structure may be changed during the life time of that cell, e.g. the concentration of chemicals in cytoplasms may increase or decrease, the shape of the cell wall may change etc.. However, when the structure of a system is changed qualitatively, its organization will change and the identity of the system is no longer invariant. In later discussion, the concept of "structural stability" will be used to describe the situation when the system's organization is maintained while its structure is changed.

Given the clear difference between structure and organization of a system, it shall be noticed that one defining characteristics of biological systems is that during the process of autopoiesis, the organization of components and the component-producing process is maintained invariant through the interactions of components and the influx of matter/energy/ information from the environment. It is also demonstrated that life is a kind of emergent unity which can arise from the interactions of some biological blocks.

As self-production is viewed as the defining and unique properties of biological systems, autopoiesis is coined for this special characteristics of biological systems in the first place. A system having an autopoiesis feature is called an "autopoietic system". An explicit definition of an autopoietic system is as following:

> *"An autopoietic system is:*
> *A unity realized through a closed organization of production process such that (a) the same organization of process is generated through the interactions of their own products (components) and (b) a topological boundary emerges as a results of the same constitutive process* [Zeleny, 1980].

In this definition, the first part specifies the process of self-production while the second part stresses that the process is realized in a unity which produces its own boundary. It characterizes living systems, especially biological systems as contrast to nonliving systems. One striking result of the theory of autopoietic systems is that "all living (biological) systems are autopoietic (self-(re)production)" [Maturana and Varela, 1987].

The typical and best known autopoietic system is the biological cell. It is actually the prototype of autopoietic system which inspired Maturana et. al in the first place [Varela et.al, 1974; Zeleny, 1980].

For a better understanding of what an autopoietic system is, we should look at the six criteria of autopoiesis outlined in the first published article on autopoiesis, i.e. "Autopoiesis: the organization of living systems, its characterization and as a model". The criteria read as:

> *(1) Determine, through interactions, if the unity has an identifiable boundary. If can be, go to step (2);*
> *(2) Determine if there are constitutive elements of the unity that are components of the unity. If these components can be described, go to (3);*
> *(3) Determine if the unity is mechanistic system, that is, the component properties are capable of satisfying certain relations that determine in the unity the interactions and transformations of these components. If this is the case, go to (4);*
> *(4) Determine if the components that constitute the boundaries of the unity constitute these boundaries through preferential neighbourhood relations and interactions between themselves, as determined by their properties in the space of their interactions. If this holds, proceed to (5);*
> *(5) Determine if the components of the boundaries of the unity are produced by the interactions of the components of the unity, either by transformation of previous produced elements, or by transformation and/or coupling of non-components that enter the unity through its boundaries. If yes, go to (6);*
> *(6) If all the other components of the unity are also produced by the interactions of its components as in (5), and if those which are not produced by the interactions of other components participate as necessary permanent constitutive components in the production of other components, you have an autopoietic unity (system) in the space in which its components exist. If this is not the case and there are components of the unity which do*

not participate in the production of other components, you do not have an autopoietic unity. [Verela, et. al 1974].

As can be seen from these six criteria, the concept of autopoiesis or autopoietic system has a very strict meaning as it is deliberately defined for biological systems in particular. The centre of it is self-(re)production. Systems which produce something other than themselves are called **allopoietic**, and systems designed by human with a purpose are named **heteropoietic**. In this way systems are classified into these three different groups .

By looking at these criteria defining an autopoietic system and bearing in mind the bold statement "all biological systems are autopoietic systems", it is not difficult to notice that the following properties are implied in the theory of autopoietic systems.

First of all, living systems are autonomous. The autonomy is maintained by the process of autopoiesis. There is interaction between the autopoietic system and its environment, but the organization of the systems is not decided by the environment but by its internal dynamics.

Secondly, the organization and behaviour of living systems are decided by the relations of components and the interactions of neighbour elements. Thus living systems are essentially **mechanistic** rather than **teleological**. In next section in this chapter, models of cellular automata will be discussed where the structure and behaviour of cellular automata are also decided by deterministic local rules which restrict the state of a site at certain step by referring to states of its immediate neighbours. The computer simulations of autopoietic systems artificial lives are based on the cellular automata model and this provide us considerable courage that we might be able to study the properties of living systems by guided computer simulations.

Thirdly, an autopoietic system arises spontaneously from the interactions of interdependent elements which are connected to each other to form a network of production. The properties of an autopoietic system is independent of the properties of individual elements, like production and reproduction, therefore autopoiesis is an emergent property and an autopoietic system is an emergent whole.

Fourthly, living systems are environment independent in the sense that the organization of an autopoietic system is decided by the inner dynamics of the system, i.e the interactions between the constitutive components rather than the environment or other external designers. Certainly the environment is the source of material and energy, but it has little to do with how the process of autopoiesis should go on and what kind of organization the unity should possess. The process of autopoiesis goes by itself and in this sense this process is viewed as a self-organization process and the autopoietic system a self-organizing system.

An autopoietic systems is regarded to be organizational closed because the product of the organization is the organization itself. The self-production process depends only on the interactions of the immediate neighbourhood involved and it produces, through its own metabolism, the very components, network interactions and boundary which realizes it as an autonomous system. It is the internal dynamics of the autopoietic unity rather than the environment that decides the self-production process hence the organization of the system. The environment, apart from providing the necessary material source and energy for the autopoiesis, also provide some noises and perturbations to the autopoietic system, but in most cases the system can maintain its autonomy and remain to be an autopoietic unity. However, when the noises and perturbations are amplified and coupled with the system's inner instability to shake the structural stability , i.e the organization of the unity, the system will undergo radical change and the identity of the autopoietic unity will be destroyed. Environment can "trigger" mutations within the system, but it can not decide the change of the system. That is to say that the new state of the system is decided by the system's inner dynamics rather than designed by the environment and the change of environment can only help activate the intrinsic mechanism. This is how Maturana and Varela view the relation between the system and its environment and the way of "natural selection" [Maturana and Varela, 1987, pp 101-117]. This view of change is consistent with our view on self-organization and systems evolution observed in a wide range of systems.

It is argued by some people that systems like linguistics, human societies and organizations also exhibit the same characteristics as those of autopoietic systems, namely, the autonomy, the persistence and maintenance of identity despite the change of structure and components, some even suggested that those human organizations like a commercial firm and various social institutions are autopoietic systems [Zeleny, 1980; Ulrich et. al, 1984; Geyer et. al, 1986]. It is arguable because these systems might not meet the six criteria of autopoiesis which are claimed to be the necessary and sufficient universal definition for life or "living systems", but it seems that there is no harm to use the concept of "autopoietic system" as a useful metaphor in discussion about those human institutions. It has also been shown that not only biological ("organic") systems are autopoietic, but also many other non-organic systems, like physical-chemical patterns, inorganic precipitation in osmotic growth, can exhibit the process of autopoiesis [Zeleny, 1980, 1989; Klir et. al, 1988]. As mentioned before, the conclusion is that all biological (living) systems are autopoietic (self-producing) but not all autopoietic systems are necessarily living in the sense of traditional organic biology.

3.5 Synergetics: cooperative behaviour in self-organizing systems

The spontaneous formation of spatial-temporal structures out of less structured or even non-structured states have been observed in such diverse systems like physical systems, chemical systems, biological systems and even human society. The most well known example in physics is the behaviour of laser system. Energy is pumped in a system consisting of active materials and mirrors. At the low level energy input, atoms emit light in an uncorrelated fashion, like the domestic light. When the energy input reaches certain threshold, the essentially random radiation of atoms is replaced suddenly by a completely coherent radiation and the amazingly coherent laser beam can be observed. At that threshold, the system undergoes a radical change and organizes itself by ordering and arranging atoms to behave in a cooperative way. In trying to understand the mechanism underlying this self-organization process, and by consulting to problems in other fields like hydrodynamics, chemistry, biology and ecology, the German physicist Herman Haken has established a new theory called "synergetics" [Haken, 1983a, 1983b].

The aim of synergetics is to deal with systems which are composed of many subsystems (or components) and open to their environment. It tries to establish some general principles according to which systems acquire macroscopically ordered dynamical behaviour through the process of self-organization and these principles should be universal in the sense that they hold all the time irrespective of the nature of the systems. It has been argued that the aim of synergetics has been fulfilled by the discovery that the spontaneous occurrence of order is the result of competition and cooperation between subsystems and this process is governed by the "slaving principle" which says that certain "order parameters" decide the systems' behaviour at the critical point where the stability of the system is lost.

For an open system, its structure and behaviour is essentially decided by the interactions between subsystems.The external perturbations, through the exchange of matter/energy/information, will not affect the system's behaviour significantly when the system is within a stable region. At some critical points, the external conditions, coupled with the internal fluctuations, can amplify the small changes occurring within the system and cause the system to lose its stability. Qualitative change in the system's structure occurs and this gives rise to the system certain new spatial-temporal structure. The appearance of the new ordered state is not merely the result of environment "selection", it is the joint product of the environmental perturbations and the system's inner dynamics. Within the system, there are large amounts of different fluctuations caused by the movement of subsystems at the microscopic level. These fluctuations, potentially, decide the system's behaviour at the macroscopic level when amplified. What kind of new structure the system will possess depends on which of those microscopic fluctuations is chosen finally to be amplified. The different potential states represented by different microscopic fluctuations are called different "modes" of the system. Haken proposes that such different kind of modes compete with each other during self-organization of the system. Eventually, one or a few of modes win over and this decides the system's new structure at the macroscopic level. The appearance of such collective modes defines the order and structure of the overall system at the macroscopic level. The quantities describing these collective modes are called "order parameters" and they are usually the **unstable modes** at a critical point. The order parameters can be physical variables, like the amplitude of waves, or abstract variables like radical ideas in a system consisting a group of human beings [Haken, 1983a].

Once the order parameters have been established, they prescribe the action of the other subsystems, or using the terminology used in Haken's synergetics, order parameters slave other subsystems at the microscopic level. The slaving principle states that "**long-living systems slave short-living systems**" [Haken, 1983a, 1983b].

The notion of order parameters and slaving principle can be expressed mathematically by using the language of dynamical systems theory.

Generally, the behaviour of a non-equilibrium open system can be described by a dynamical equation reflecting the system's inner dynamics, the parameter reflecting the impact of environment and a term describing the systems internal fluctuations.

$$\frac{\partial}{\partial t}x(\sigma,t) = f[\, x(\sigma,t), \Delta, t\,] + \delta[\, x(\sigma,t), t] \qquad 3.5.1$$

where

x(t): the state variables (vector) of the system;

σ: the parameter vector reflecting the impact of environment;

Δ: the spatial gradient imposed on the system;

f: a nonlinear function (vector) describing the inner dynamics of the system;

$\delta(x(\sigma,t), t)$: the stochastic force representing the internal fluctuations.

In the analysis based on dynamical systems theory, we usually neglect the stochastic term in equation 3.5.1 and this will not affect the result of the analysis significantly.

As argued in chapter 2, a system that can be observed as a system is usually at its stable state, or periodic state or an aperiodic state (chaotic state), that is to say it is attracted to a stable attractor. Its state can be specified by a structural stable solution of equation 3.5.1, $x_0(\sigma, t)$. When the environment changes, the system usually remains close to it and the state of the system can be denoted as:

$$x(\sigma, t) = x_0(\sigma, t) + w(\sigma, t)$$

At the critical point, the behaviour of the system needs detailed analysis and the equation can be changed to:

$$\frac{\partial}{\partial t} w(\sigma,t) = \boldsymbol{L}[\ \Delta, \sigma, t\ ]\ w(\sigma,t) + \boldsymbol{N}[\ x(\sigma,t), \Delta, \sigma, t] \qquad 3.5.2$$

Operator $\boldsymbol{L}$ and $\boldsymbol{N}$ represent the linear and nonlinear parts of the dynamics respectively.

When the parameter reaches a certain critical point σ_c, the stable state $x_0(\sigma_c, t)$ becomes unstable. The analysis of the onset of instability is performed by applying the well-established linear analysis techniques.

Denote $\lambda_j(\sigma_c)$ ($j = 1, 2, \ldots, n$) the eigenvalues of the linear operator $\boldsymbol{L}$ and Φ_j the eigenvector respecting to λ_j.

$$\lambda_j(\sigma)\ \Phi_j = \boldsymbol{L}\ \Phi_j \qquad 3.5.3$$

and at the critical point one or more engenvalues fulfil the relation

$$\mathrm{Re}\ \lambda_j(\sigma_c) = 0.$$

In the neighbourhood of the critical point, it is possible to divide the normal mode into two parts according to their different linear relaxation times:

Stable Modes $\lambda_s(\sigma_c)$: $\mathrm{Re}\lambda_s(\sigma) << 0$
Unstable Modes $\lambda_u(\sigma_c)$: $\mathrm{Re}\lambda_u(\sigma_c) \approx 0$

At the critical point, the nonlinear character of the system plays a decisive role in describing the system's unstable behaviour and the state vector is hence expressed as the combination of those linear eigenvectors:

$$x(t) = x_0(t) + \Sigma_u \xi_u(t)\Phi_u + \Sigma_s \xi_s(t)\Phi_s \qquad 3.5.4$$

According to equation 3.5.1 (without the stochastic term), one obtains a set of differential equations for the amplitudes $\xi_i(t)$

$$d_t\xi_u(t) = \lambda_u\xi_u(t) + P_u[\xi_u(t), \xi_s(t), \sigma] \qquad 3.5.5$$
$$d_s\xi_s(t) = \lambda_s\xi_s(t) + P_s[\xi_u(t), \xi_s(t), \sigma] \qquad 3.5.6$$

Since the amplitude of the stable mode has small relaxation time compared with the time scale of the variation of the amplitude of the unstable modes, they are able to follow immediately the slow motion of the amplitudes $\xi_u(t)$. This leads to the existence of an invariant manifold defined by:

$$\xi_s(t) = \xi_s(\xi_u(t)) \qquad 3.5.7$$

It has been proved that, through some iteration procedure and the elimination of the amplitudes of the stable modes, the generalized Ginzburg-Landau equation can be obtained:

$$d_t\xi_u(t) = \lambda_u(\sigma)\xi_u(t) + P_u[\xi_u(t), \xi_s(\xi_u(t)), \sigma] \qquad 3.5.8$$

The state vector is obtained from equation 3.5.4 and 3.5.7 as:

$$x(t) = x_0(t) + \Sigma_u\xi_u(t)\Phi_u + \Sigma_s\xi_s(\xi_u(t)\Phi_s \qquad 3.5.9$$

The whole process through which we obtain the state vector is called the "adiabatic elimination" and it states mathematically the slaving principle. We see that the temporal behaviour of the system close to the unstable point is entirely determined by the dynamics of the amplitudes of the unstable modes. The amplitudes $\xi_u(t)$ are therefore defined as order parameters.

The mathematical treatment is based obviously on the dynamical systems theory, it can be found out later on that there is a general treatment in dynamical systems theory of the unstable behaviour of a system at the critical point and the "centre manifold theory" is the generalization of this adiabatic elimination process. More bifurcation patterns can be found for the complex temporal behaviour of dynamical systems.

As in the theory of dissipative structures, synergetics also stresses the importance of the internal fluctuations in the self-organization process of systems. Another point emphasised in synergetics is that there is generally an identifiable symmetry-breaking process in this process and only through which new spatial-temporal structure can come into existence [Haken, 1983a, 1983b; Jantsch, 1980]

Applications of Synergetics are various. It has been successfully applied in the study of self-organization process in laser, hydrodynamics, chemical waves and also in pattern organization etc.[Haken, 1983a; 1988]. In the study of systems evolution, synergetics provides some important notions such as order parameters and useful techniques.

3.6 Cellular Automata: prototype of discrete dynamical systems

Opposite to the study of self-organization phenomena based on continuous differential equations as discussed in the "theory of dissipative structures", "synergetics" and "hypercycle", cellular automata serve as another class of mathematical models for evolving systems which involve discrete coordinates and variables as well as discrete time steps. Essentially, cellular automata consist of a huge number of components which are locally connected to each other through relatively simple deterministic rules. Recent study of cellular automata based on extensive computer simulation has revealed that cellular automata can exhibit very complicated spatial and temporal behaviour analogous to those found with continuous dynamical systems like cyclic and chaotic behaviour. The other advantage in studying cellular automata is that its behaviour can be analysed by using modern computing techniques: extensive computer simulations. As a matter of fact, the study of cellular automata at present time mainly depends on the technology of computer simulation.

The simplest cellular automata to bear in mind is a line of sites with each site having either value 1 or 0 with initial values assigned to each site, renew them in discrete step and the new value of each site is decided by values of its nearest neighbours on previous step. This is a typical 1-dimensional cellular automata. When such operations are carried on, each site has a sequence of values. Such a system of a line of sites, with each site having assigned values at different time step according to a determinist rule, is a discrete dynamical system. By arranging the line according to the time sequence, we get something like the phase configurations for the continuous dynamical systems and it is called the configuration of that cellular automata. The configuration of a cellular automata reveals its time evolution behaviour. It has been proved that even such a simple cellular automata can exhibit rich evolutionary behaviour: organized spatial pattern can emerge spontaneously in some cellular automata starting from random initial states and following a simple deterministic rule. Therefore, cellular automata is regarded as a potentially useful model for the study of the evolutionary behaviour of systems composed of large number of components and governed by deterministic rules.

Generally, cellular automata can be 1-dimensional (sites arranged on a line). 2-dimensional (sites arranged on a surface as two dimensional lattice) or three dimensional (for high dimensional cellular automata, it is difficult to visualize its spatial pattern). The site may take any finite set of possible values and the renewing rules depend not only on the values of a site's neighbours at the previous time step, but also from other preceding steps. All such cellular automata must have five fundamental defining characteristics:

(1) They consist of a discrete lattice of sites;
(2) They evolve in discrete time steps;
(3) Each site takes on a finite set of possible values;
(4) The value of each site evolves according to the same deterministic rules;
(5) The rules for the evolution of a site depend only on a local neighbourhood of sites around it.[Wolfram 1984].

With these characteristics, it has been argued that cellular automata provide rather general models for homogeneous systems with local interactions to evolve over discrete time steps. It has been illustrated that even in one dimensional cellular automata, they can exhibit complex behaviour, i.e, the system can evolve to homogeneous state, or heterogeneous state like steady state, cyclic state, even chaotic state [Wolfram, 1984].

Cellular automata can be formally defined and analysed. However, the analysis of the spatial and temporal behaviour of cellular automata is mainly based on computer simulations.

Suppose there is an 1-dimensional cellular automaton consisting of N sites and each site can take one of the k different states. Denote $a_i(t)$ the value of the site i at time step t and it is decided by the values of its $2r+1$ nearest neighbourhood at time step $t-1$. Therefore this cellular automaton have totally k^N different states. The deterministic rule which governs the time evolution of this cellular automaton F can be described as follows (equation 3.6.1):

$$a_i(t) = F[a_{i-r}^{(t-1)}, a_{i-r+1}^{(t-1)}, \ldots, a_i^{(t-1)}, \ldots, a_{i+r}^{(t-1)}], \quad i=1,\ldots, N$$

(3.6.1)

or:

$$a_i(t) = f\left[\sum_{j=-r}^{j=r} \alpha_j a_{i+j}^{(t-1)}\right], \quad i=1, \ldots, N \tag{3.6.2}$$

where α_j ($j=-r, \ldots, r$) are integer constants representing weights through which values of a_{i+j} ($j=-r, \ldots, r$) at time step $t-1$ affect a_i's value at time step t.

If $\alpha_j = 1$ for all j ($j=-r, \ldots, r$) and the "null" condition

$$F[0, 0, \ldots, 0] = 0 \quad \text{or} \quad f(0) = 0 \tag{3.6.3}$$

and the symmetrical condition

$$F[a_{i-r}^{(t)}, \ldots, a_i^{(t)}, \ldots, a_{i+r}^{(t)}] = F[a_{i+r}^{(t)}, \ldots, a_i^{(t)}, \ldots, a_{i-r}^{(t)}] \tag{3.6.4}$$

hold for all i, these rules are called "legal" rules. Among the $k^{k(2r+)}$ possible cellular automata rules of the above form (i.e., satisfying (3.6.3) and (3.6.4)), $k^{\frac{k(k+1)}{2}-1}$ are legal rules.

Functions in (3.6.2) can be specified by a numerical "code":

$$C_f = \sum_{n=0}^{(2r+1)(k-1)} k^n f[n]$$

which can be used to indicate a deterministic rule.

For the convience in study, especially in computer simulation, usually only those so called "totalistic rules" are considered. A totalistic rule depends only on the sum of the input values in the followinf form:

$$a_i(t) = f\left(\sum_{j=-r}^{j=r} a_{i+j}^{(t-1)}\right) \tag{3.6.5}$$

As an example, let $k = 2$ and $r = 2$, then there are 32 totalistic legal rules of the one dimensional cellular automaton. The numerical codes for these 32 rules is 0, 2, 4, 6, ..., 60, 62. They are specified like this:

Rule	5	4	3	2	1	0
24	0	1	1	0	0	0
56	1	1	0	1	1	0

where (111110) $\rightarrow$ 5, (11110), (11101), (11011), (10111), (01111) $\rightarrow$ 4 etc.

Starting from a disordered (random) initial condition, spatial patterns of these cellular automata appear to fall into five qualitative classes:

(1) homogeneous state, like state specified by codes 16, 60 ;

(2) simple stable or periodic state, like codes 24, 56 ;

(3) chaotic state, like codes 18, 46 ;

(4) state with some organized complexity like code 20, 52.

(for graphic representations of the trajectories of these automata, see Wolfram 1984 [Wolfram, 1984].

They can be defined as different "attractors" towards which cellular automata will finally evolve irreversibly. The emergence of these organized patterns in cellular automata is the same kind of self-organization phenomena found in many other different systems and it characterises the general behaviour of cellular automata no matter what their dimensions and rules are. It is apparently similar to the taxonomy of attractors found in continuous dynamical systems of which detailed analysis will be given in the following chapters, and for this very reason, cellular automata can be regarded as a very useful tool for the discussion of systems evolution

Recent study has revealed that what is more interesting and more important is the transient behaviour of cellular automata during the evolution process. Before entering into these organized patterns, cellular usually go through a transient period and the transient behaviour could be very complex. It has also been discovered that the space of the rules of cellular automata can be divided into different regions that each of which represents qualitatively different patterns, i.e. attractors, to which cellular automata will ultimately set on.

In a paper by Langton [Langton, 1989], each rule of a class of cellular automata, which is specified by k and r, can be assigned a measurement (usually a number) and hence the rule space of cellular automata can be parameterized by this measurement. It has been discovered that corresponding to different values of the parameter, cellular automata possess different attractors. The rule space is partitioned into different regions of attractors and the transformation from one region to another is a structural change which can be called "bifurcation" by using the terminology of dynamical systems theory. A parameterized rule space is illustrated in the following picture figure 3.6.5.

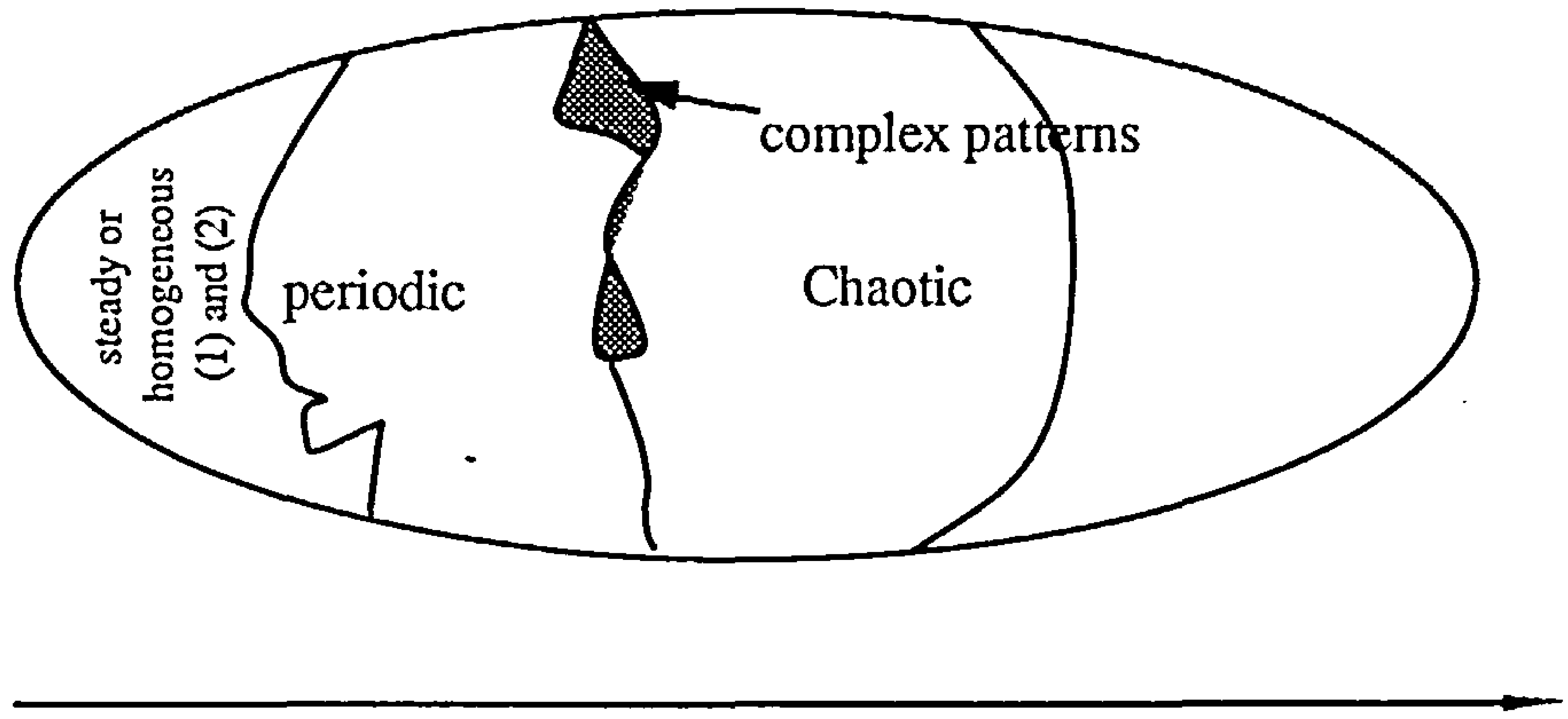


Figure 3.6.1 The picture of the rule space of cellular automata

A diagram of bifurcation on the phase space is found to resemble the fold catastrophe in continuous dynamical systems.

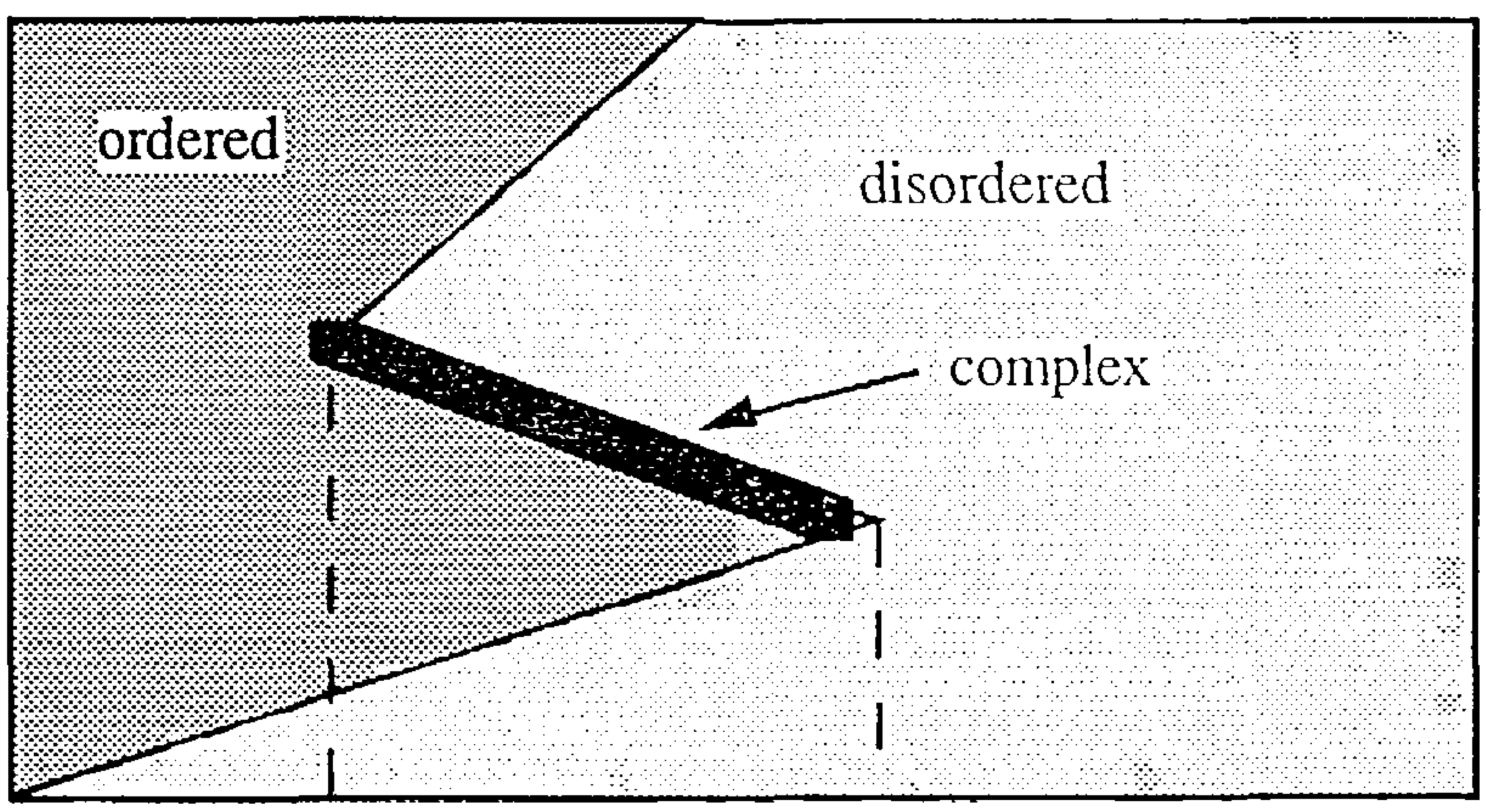


Figure 3.6.2 The picture of The 2D phase-diagram for cellular automata.

The application of cellular automata can be found in various fields. The simultaneous renewal of all its values of a large number of cells makes it potentially useful for parallel computing [Wolfram, 1984; Langton, 1989]. For the behaviour of cellular automata is specified by local rules where the state of a cell is affected by some immediate neighbours discretely, it is regarded as a good model for biological systems which are believed to grow according locally decided rules [Kauffman, 1984, 1989]. This reminds us immediately the theory of autopoietic systems which is believed to be essential for the study of living systems. Actually, one early model of autopoietic system built by Zeleny in 1977 is a cellular automata model [Zeleny, 1977]. Most recently, there emerges a new research field called "Artificial Life" which tends to study the properties of living systems by resorting to some artificially designed systems which are "alive" like osmotic system and some living creatures created on computers [Klir, et.al 1989; Langton, 1989, 1992]. In the study of artificial life, cellular automata plays the central role because most computer simulations of artificial creatures are created by cellular automata.

Apart from these applications, cellular automata has also been employed to study the phenomena of "self-organized criticality" which was initiated by P Bak and his collaborators. The notion of self-organized criticality says that in nature, many systems usually composed of a huge number of components and submitted to some external excitation, naturally evolve into a steady state which is critical in the sense that it displays spatial and temporal lang rang correlations like fractals [Bak et.al, 1988, 1991]. The example that one can visualize is a sandpile that can evolve, as the new sand is constantly dropped on it, towards a critical slope on which avalanches of all sizes can be produced by a single excitation. Threshold cellular automata has been proposed to model this situation [Bak et. al, 1988; Hermann, 1989].

The study of artificial life and self-organized criticality are of great importance for the study of systems evolution. In both cases, cellular automata serves as a very important tool.

3.7 Connectionist Models about complex systems

To some extent, the hypercycle model and the cellular automata model belong to the same type of model of systems: the number of components is usually large (it is larger in Cellular Automata), the components are locally connected to each other in the explicit form of networks, the system emerged is an organized whole which is resulted from the connections of components. This type of model systems is usually called "connectionist model" [Forrest, 1990]. One special type of connectionist models has become very popular fairly recently and that is what usually called the "(artificial) neural network". Due to the emergent computational power of neural networks, it has been widely used in various fields as information-processing systems.Neural network as an adapting and evolving system will be discussed in chapter 6.

For our study of systems evolution, it can be shown that the connectionist model can help us understand how a system emerges from the connected components and how it can evolve in a changing environment. Kauffman has studied another type of connectionist model called "Random Boolean Network" [Kauffman, 1989] and showed that it has some interesting properties which, to us, can be applied for the discussion of systems evolution.

An autonomous (N, K) random boolean network is comprised of N elements which can be described as binary variables (i.e., the value of each variable can only be 0 (off) or 1 (on)). Each element is randomly connected with / or affected by K out of N elements through a boolean function. Each element updates its state at the same moments and the change of state is regulated by boolean functions assigned by the randomly connected elements. Kauffman has shown that

(1) The network will ultimately settle to some stable states called "attractors", be they steady or cyclic. These attractors describe the network as an organized whole that can be recognized as a system.

(2) The number of state cycle attractors is far smaller than the number of total elements. For example a (N, K=2) network can only have $\sqrt{N}$ attractors.

The results have shown that things tend to organize themselves to form stable and structured wholes on conditions that there are rich interactions among them. Result (2) above demonstrates that the forming of organized entity is not governed by the probablistic law which may make this an "improbable event". It supports the idea that life on this planet may have come from inanimate materials by defying the attack which says that according probablistic analysis the emergence of life forms is an event with a infinitely small probability [Kauffman, 1989].

3.8 Systems Evolution: a general property of systems

3.8.1 Towards organized whole: self-organization

The study of Random Boolean Networks has shown that a group of elements, if interacting to each other, tend to get organized by themselves. The organized whole is represented by only a few possible "attractors" prescribed by interactions between elements. The spontaneous formation of an organized whole by associating of a group of formerly unstructured components is what we have discussed in this chapter: self-organization. Through self-organization, new entities cam emerge as organized wholes of parts. Systems are generated from without. It has been argued by some philosophers, especially someone like Spencer and Smuts that things do tend to become organized into a new whole (system) [Spencer, 1971; Smuts, 1926]. When a new kind of whole emerges for the first time, its parts are regarded as the sole source of its nature, structure and stability. The maintaining of the emerged new whole depends on the parts and their cooperations. When the environment in which the whole is situated changes, the interactions between parts may change so that the whole is maintained as an organized entity. The system may evolve and its organization changes qualitatively. All these changes are just for the sole purpose that systems remain as systems. Smuts has called this tendency of systems to become self-organized the "holism" and argued that systems and evolution are closely connected to each other [Smuts, 1926].

The idea that systems may tend to self-organize themselves and order can spontaneously emerge is not exclusively expressed in modern systems thinking. Ancient oriental philosophers achieved the same result intuitively and expressed it thousands of years ago. Capra has once compared the basic ideas of oriental philosophies (sometimes referred as Eastern mysticism) , such as Taoism, Hinduism, and Zen, with the underlying concepts of modern physics and argued that there are fascinating parallels between them [Capra, 1975]. We can find the same parallels between the ideas of self-organization in modern systems research and in the view of a unitary universe in oriental philosophies, especially the view of spontaneous orders emphasized by the ancient Chinese philosophies. Chinese ancient philosophies, especially the most influential Taoist philosophy, stressed this unified view towards human and nature and surprisingly we can find the idea of self-organization expressed in an apparently similar way in the classics of Taoist philosophy *Tao Te Ching* [Yu, 1990].

Taoism has been one of the most important strands of philosophical thought in Chinese culture and its influence is even further reaching than that of Confucianism [Fung, 1958; Yu, 1990]. Taoists view the world as a whole spontaneously created by the movement of Tao (the way, the order), an all embracing first principle and an expression of the intrinsic nature of everything. Tao manifests itself as *Ying* and *Yang* and through their interactions the universe has come into being. All systems in the universe, by using the terminology of modern systems science, owe their existence to the attributes of Tao. What Tao accomplishes was not done purposefully, but is simple spontaneously so. The Taoists' view stressed "non-action" of the humans and proposed to let the intrinsic nature of everything, e.g. Tao, to do its work. In politics, it has advocated that through non-action, everything can be done. In both the nature and society, it believes in the spontaneous orders arising through the movement of Tao. Everything has its own nature and that nature is Tao. Lao Tzu said:

> *Man follows the laws of the earth; Earth follows the laws of heaven; Heaven follows the laws of Tao; Tao follows the laws of its intrinsic nature (the spontaneous).* [*Tao Te Ching*, Chapter 25, Yu, 1990].

Any harmony or order in society could only be the result of the movement of Tao. It is not difficult to discover that the idea of spontaneity perceived by Taoist philosophers is similar to, but not the same as, the idea of self-organization in modern systems science and we may point that the Taoism had over stressed the "self" a little. The Tao, the intrinsic nature of the world, can be expressed, partially, by the inner dynamics of the system (this inner dynamics is expressed in Taoism as the dialectic relations of *Ying* and *Yang*.) .

In short, in the ancient Chinese world view, largely influenced by the Taoism dialectics, there was a harmonious cooperation between all beings and this kind of cooperation arose, not from the orders of a superior authority external to themselves, but from the fact that they were all parts in a hierarchy of wholes forming a cosmic pattern, and what they obeyed were the internal dictates of their own natures-- that nature is Tao [Fung, 1958].

If the eastern wisdom is not so scientific in describing the spontaneity of the emergence of order, modern science and philosophy, at least systems science, support and agree with this view. Makridakis proposes a "second law of systems" which says that, systems tend to increase in their internal order in contrast to the second law of thermodynamics which says that order tends to generally decrease in the universe [Makridakis, 1977]. Systems may and tend to evolve, within suitable environment.

3.8.2 Systems evolution

It has been shown that things tend to get self-organized and systems may evolve. How this will happen? The conditions, and some laws and principles can be established as follows.

In general, an open system can be regarded as a stable attracting centre (attractor) within a field of constantly changing matter flow, energy flow, and information flow [Swenson, 1989a]. It continuously attracts matter, energy, and information to form and then sustain a dynamically functional whole in a state far away from thermodynamic equilibrium. The microscopic fluctuations within the system and the perturbations from the environment are also constantly testing the stability of the system. Under certain conditions, the system maintains its stability (structural stability) by staying at a dynamically equilibrium state through its multiple feedback and feedforward loops as analysed in previous paradigm. When the following conditions are fulfilled the system may move to another new ordered state with properties which are irreducible to its previous state, and the whole as a new attractor is qualitatively different from the old one:

(1) it is open to its environment
(2) it is far from thermodynamics equilibrium
(3) it is governed by nonlinear dynamics
(4) there are microscopic fluctuations

(5) the environment is changing

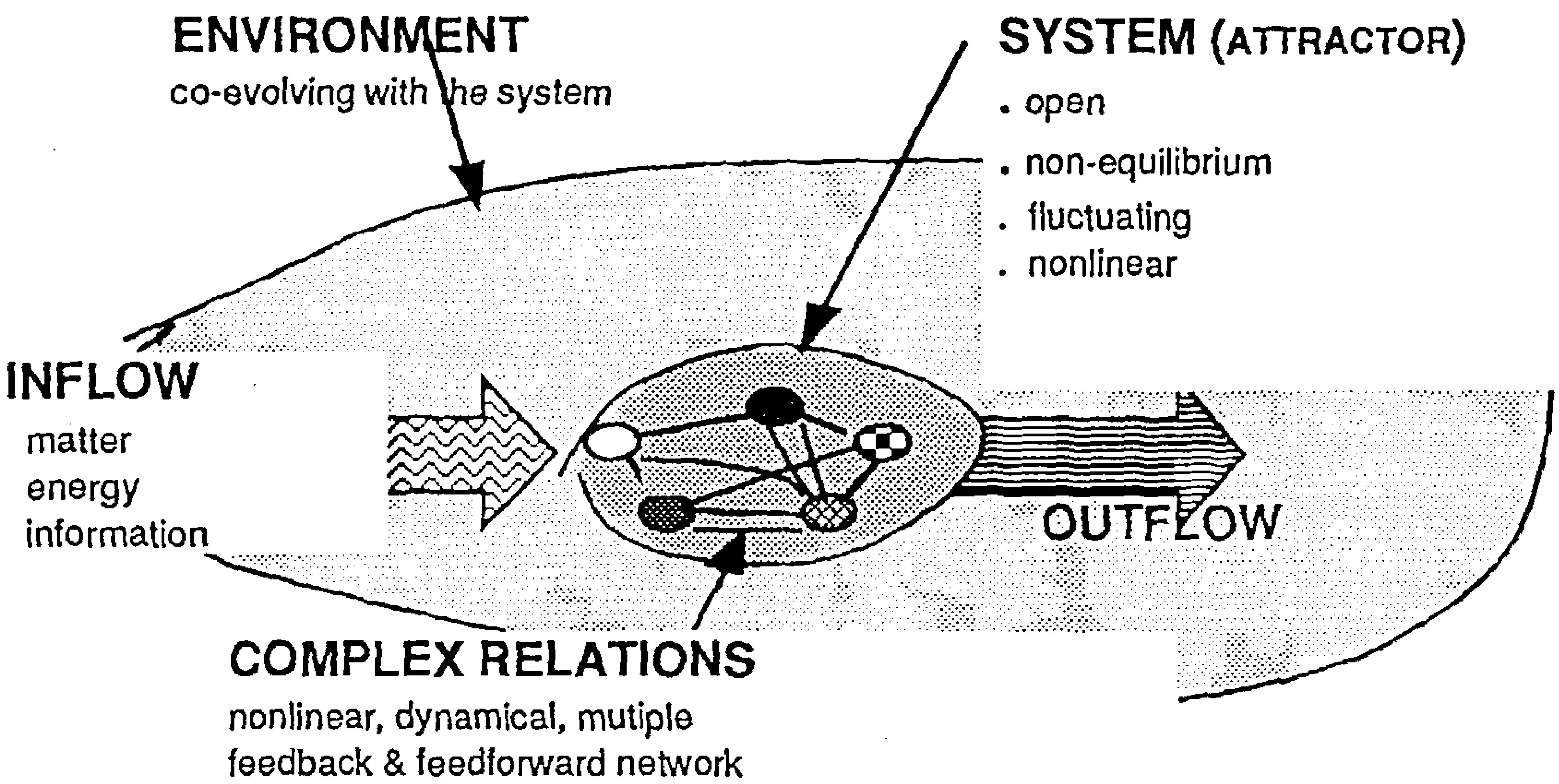


Figure 3.8.1 Open system within a changing environment

It needs to be emphasised that a dialectic attitude must be adopted in any discussions on systems evolution. On the one hand, the effect of the environment, i.e., "natural selection" emphasised by Darwin's evolutionary theory, can trigger the fluctuation-amplifying process in an open system; on the other hand, the system must possess complex inner dynamics and has various microscopic fluctuations. Both aspects, inner and outer, play important roles in the system's evolution and they jointly decide the process and stage of evolution.

As to the laws that govern the evolutionary process, Swenson has established, deductively from the second law of thermodynamics, a law called "the law of maximum entropy". It says that a system which fulfils the above conditions will spontaneously evolve to a new state so that the rate of entropy production is at the maximum possible value. [Swenson, 1989a].

It should be noticed that this thermodynamics law of systems evolution only shows explanatorily to which way a system will evolve (to maximize its entropy production rate), it does not tell us at which state the system will be (because entropy is a global variable). The actual process and result of evolution can only become known through analysing the interrelations of the parts, i.e., studying the internal dynamics.

Through the kinetic description, i.e. dynamical equations, together with the overall constraints imposed by the environment, we can understand the possible patterns of systems evolution. The next chapter is devoted to this problem by building a formal model.

Chapter 4 A formal model for evolving systems

4.1 Systems Science and Formal Systems Theory

4.1.1 Systems, models, Formal Systems Theory and General Systems Study

It is accepted in this study that a system is a model of an object [Ashby, 1956; Klir, 1972]. Checkland goes further to stress the subjectiveness in perceiving and describing systems, especially for systems where human factors are involved. He argues that the system recognized by an observer is decided by his/her Weltanschauung, i.e., the point of view of the world [Checkland, 1981]. Here a system is regarded as a model of an object which chooses to describe some of the many properties of the object. The object is described by a group of variables at a certain space-time resolution level. A system corresponds to an object and the properties of the system, or the model, can always be regarded as the properties of the the object under consideration and hence the system (model) and the modelled object are the same.

Among all the contributions to the development of systems science, general systems theory is one of the most important theoretical traits. In its early stage, general systems theory was regarded as a collection of concepts, ideas, and models about systems. It has been considered as a formal theory, a methodology, a way of thinking, a methodology by different people [Klir, 1972]. Despite the variety of views on general systems theory, there is a common philosophical foundation on which the general systems theory stands and that is "isomorphism". It has long been the belief among those general systems thinkers that through an isomorphic process, knowledge of one specific system can be applied in the analysis of other systems with different physical forms and these knowledge are regarded as the properties of all systems of certain class (certain type of systems in general). These systems properties, like equifinality, wholeness, stability, adaptivity et. al., are transferable in such disparate systems as physical systems, biological systems, ecological systems and human systems etc.. In some other place, isomorphism, or the isomorphic relation is manifested as "metaphor" or "systems analogy" [Flood et. al, 1990].

There have been various expressions of general systems and among them are formal systems theories. According to their forms and logic-mathematical foundations, these various formal theories of general systems can be classified as "axiomatic " systems theory and "dynamical system theory".

4.1.2 Axiomatic Systems Theory

In the category of axiomatic theory there are those theories where the rigourous definitions of systems and the derivation of its implication are based firmly on modern mathematics and modern logic. The definition of systems starts as a formal logic or mathematical expression, with no specific properties assigned to it and is virtually applicable for any systems. Over the past 30 years, three major formal systems theory have been developed, namely: Mesarovic and Takahara's general system [Mesarovic et. al, 1972, 1988], Wymore's wattled system theory [Wymore, 1971], and Klir's systems theory [Klir, 1969; 1985a].

A) Mesarovic's General Systems Theory

Mesarovic and his co-workers have been studying the formal theory of general systems since 60's by using the language of set theory, and latter, the category theory. The definition of a system is an axiomatic statement which is formulated in the most general terms. Additional axioms are then added to formalize pragmatically significant special classes of systems. The study started by giving the definition of system as this:

Definition 4.1.1 A general system S is a relation on non-empty sets:

$$S \subseteq \prod \{ V_i : i \in I \}$$

where $\prod$ denotes the Cartesian product of the sets V_i , which are called the set of element and I is an index set.

V_i can be the set of elements of any systems and the relations among these elements can be of any forms. This definition is regarded as being applicable to any systems. Based on this definition, a framework of general systems has been constructed. Mesarovic etc. have set out to study the properties of general systems like the input-output relation, the hierarchical structure, optimization strategy and decision making etc. [Mesarovic et. al, 1975, 1988].

Recently, other formal theory developed upon Mesarovic etc.'s original idea have been put forward. Among them is Lin and Ma's general systems theory which seems to many people is just the general logic deductions [Lin et.al, 1991].

B) Klir's systems theory and Reconstruction Analysis

Klir's work on general systems theory started in late 60's and his formal theory is based on the general circuit systems theory [Klir, 1969, 1972, 1985a]. Over the past 20 years, he has developed a complete framework of general systems theory and general systems methodology. Compared with other theories of general systems which are usually of a deductive nature, Klir's approach is essentially a inductive one. Rather than defining the concept of a system axiomatically, Klir starts by defining system traits. These traits are simplified which are independent of a specific nature of variables involved. Upon those traits, a sequences of systems like structural system, generative system, meta system, and meta-meta-system can be defined. A specific analysis techniques called reconstructability analysis has been developed which is unique for this theory but has some significant implications in systems philosophy as well as in systems methodology. Compared with Mesarovic's general systems theory, Klir's theory is also the most well known one in the systems community [Klir, 1985a].

4.1.3 Dynamical models of general systems

An alternative to the axiomatic theory of general systems is the dynamical models of systems, as put forward in the first time by L. von Bertalanffy [von Bertalanffy, 1968; Klir, 1972]. The focus of such dynamical models is the dynamical relation among the components of the system considered. Essentially, there are two kinds of dynamical models of general systems, according to the way they describe the dynamical process of the system. One of them is to use the internal variables of the system to express explicitly the dynamical relations of the system's components. The other is to treat the system as a black box and the dynamical structure of the system is reflected in the relationships between the input and output of the system.

The first method assumes that the inner dynamics of a system is known to us, although it is not always the case, and we can understand the dynamical behaviour of the system by studying the equations describing the dynamics. Two types of equations are employed: differential equations and discrete automata.

L.von Bertalanffy first used ordinary differential equations to describe the dynamical behaviour of general systems. It has been claimed that such a dynamical model is very useful for the studying of some properties of general systems, like the equifinality, equilibrium, stability etc.. Actually, the applicability of the same type of dynamical model in various distinct systems, like the physical, chemical, biological and even socio-economic or some others, is part of the claims that a general systems theory is possible and which can describe the properties of general systems [von Bertalanffy, 1968].

The use of such continuous models to describe the dynamical behaviour of various models has been shared by many other people like, Rapoport [Rapoport, 1984], Thom [Thom, 1974], Abraham [Abraham, 1988], Prigogine [Nicolis et. al, 1977, Prigogine, 1980], Haken [Haken, 1983a, 1983b] and many others. The state space description method developed in control systems theory can be viewed as the further development of these dynamical models in a particular domain of systems [Zadeh, 1962] . By adding to the model some concrete contents, they can be applied in various engineering systems, as well as some other systems like economic systems.

Another kind of dynamical model is the discrete automata which were put forward to study the Turing machine [von Neumann, 1966] as well as biological systems [Kauffman, 1989]. The most recent development of such method is the model of cellular automata [Wolfram, 1984]. As showed in 2.6 last chapter, models of cellular automata method assume that the system to be modelled is composed of a large number of simple components, each component is connected to its nearing neighbours and the state of each component is renewed discretely according to some simple local rules. It is believed that such model is extremely useful in the studying of a wide range of systems and it can also be employed as a general model for analysing the complex dynamical behaviour of general systems.

The black box model was once one of the most important models in early systems movement, although not necessarily directly related to the general systems theory. It was first put forward by Wiener in 40's and later generalized as a model for a wide range of systems by Ashby [Wiener, 1948; Ashby, 1956]. By this model, the system is a black box and to us the inter structure of the system, namely how the components of the system are connected to each other, is unknown. Given an input to the system, certain output can be obtained. Only through the analysis of the input-output relation can we "guess" the internal relations. The black box model has also been used as an important metaphor in systems science to refer to those systems whose internal structures are difficult or even impossible to know, like the human brain.

This model is still one of the most important models in control systems theory. The essential step in using this model is to find out the input output relation through the standard techniques called "identification". Various application of this model can be easily found in engineering systems, physiological systems and other systems [D'Azzo et. al, 1987; Finkelstein et.al, 1985].

With the recent development in mathematical dynamical systems theory, we can rethink about these dynamical models by considering its applications in the analysis of the complex dynamical behaviour of open systems, although we should not go so far as to claim that this is the sign of the revitalising of GST.

The above mentioned formal theories of general systems, together with their recent development, have been pursuing the goal set by early systems thinkers, namely to study general properties and laws of systems, to form principles applicable to all types of systems. Some may argue that a theory which tends to include everything is virtually about nothing. From this point of view, a general systems theory of such purpose is impossible. However, those various efforts devoted to the development of general systems have not been a total failure. They have been successful at least to the extent that they provide some common terminologies for the discussion of various systems, characterise some properties of general systems, although they have not, and probably never will, found a universal formula or theory for every system.

Holding the view point that the various theories of systems are only different models which each reflects some of the many aspects of reality and they are not exclusive and can be integrated in their applications, we should be able to argue more objectively that such theory of general systems still needs to have a place in modern systems research. This relates closely to perspectivism which becomes the philosophical tendency of the systems science. To keep the claim as modest as possible, it would be justifiable to say that the following sections are devoted to the outlining of a formal model of a special type of systems, i.e dynamical systems, and through which the evolutionary behaviour of these systems can be explored. The following two chapters will provide us with some applications to demonstrate the applicability of this formal model.

4.2 A Formal Model Based On Mathematical Dynamical Systems Theory (DST)

4.2.1 A Dynamical System Model

This study is about the complex dynamical behaviour of systems in general. The ontological statement can be appropriately expressed by quoting the ancient Greek philosopher Herarclitus' famous saying: "Everything flows and Nothing Stays". It has already been argued that a system capable of evolution must be open to its environment, at a non-equilibrium state, governed by nonlinear inner dynamics, possessing sufficient microscopic fluctuations and subjected to a changing environment. In this chapter, a formal model to describe such kind of systems and their dynamical behaviour is to be constructed.

In the most general sense, a system is situated in a larger environment. The discourse is always over the axis of time. Thus a system is always in certain space-time domain. Denote **B** as the the background of a system in focus (we do not specify what is the particular content of this set), **T** the time axis, $\mathbf{T} = \{t: -\infty < t < +\infty\}$ (or **T** can be $\mathbf{R}^+ = \{0 \le t < +\infty\}$, or $\mathbf{Z} = \{..., -2,-1,0,1,2,...\}$; or $\mathbf{Z}^+ = \{0,1,2,...\}$).

Let **E** be the set of the elements (variables) related to a perceived object, $\mathbf{E} \subset \mathbf{B}$, **R** be the set of some relations on **E**,

$$\mathrm{R} \subseteq \bigcup_{n=1}^{\infty} \mathrm{E}^n \tag{4.2.1}$$

and hence the system can be defined as $\mathrm{S} = (\mathbf{E}, \mathbf{R})$. $\mathbf{Q}=\mathbf{B}-\mathbf{E}$ is called the ***environment*** of system S.

If

$$\mathrm{C} = \{\mathbf{f}: \mathbf{f} \subset \mathrm{E} \times \mathrm{Q}\} \neq \emptyset,$$

i.e., there are couplings between **E** and **Q** we call the system is an ***open*** system and $\mathrm{S} = (\mathrm{E}, \mathrm{R}, \mathrm{C})$.

If the system changes over time **T**, i.e. the system's elements and the relations among them change when time goes, this can be defined as

$$\mathbf{R} \subset \{\mathbf{f} \mid \mathrm{f}: \mathrm{E} \times \mathrm{T} \to \mathrm{E}\} \tag{4.2.2}$$

Such a system is called a (generalized) ***dynamical system.*** The system is hence described as:

$$\mathbf{S} = (\mathrm{E}, \mathrm{R}, \mathrm{T}) \tag{4.2.3}$$

When the system is both dynamical and open, it can be represented as

$$\mathbf{S} = (\mathrm{E}, \mathrm{R}, \mathrm{C}, \mathrm{T}) \tag{4.2.4}$$

Explanatory definitions are given to both a system and a dynamical system. The lack of any concrete contents in the definition makes it only a heuristic tool for understanding the concepts of systems, environment etc, but it can be applied in various situations.

To make these definitions more precise by using rigourous mathematical terminologies, meanings of these sets should be restricted as well as the range of systems to be defined.

Usually the system is described by a group of variables, represented by a vector **X** which takes its value in a set **M** which is defined as a manifold which is usually a smooth metric space like $\mathbf{R}^n$. The relation **R** is specified as a function **F**,

$$\mathbf{F}: \mathbf{M} \times \mathbf{T} \rightarrow \mathbf{M} \tag{4.2.5}$$

The system is hence represented as

$$\mathbf{S} = (\mathbf{X}, \mathbf{F}).$$

From the strict mathematical sense, only when the following conditions are satisfied can we call the system S a dynamical system (as can be seen in the appendix 1):

F: $\mathbf{M} \times \mathbf{T} \rightarrow \mathbf{M}$ is a continuous map (C^r map), if F satisfies:

(1) F (x, 0) = x, for any $x \in$ M;

(2) F(x, s+t) = F(F(x, t), s), for any s, $t \in \mathbf{T}$, $x \in \mathbf{M}$

then F is called a C^0 flow (C^r flow) or a C^0 dynamical system (C^r dynamical system).

This is a rather restrictive condition. An equivalent definition can be made by using the terms of differential equations and that makes the definition less formidable because many systems encountered in various fields are usually described by differential equations (see appendix 1).

Again, by taking into account the relation between the system and its environment, we can define a system open if there are coupling relations between them, i.e., the system is affected by and also affecting its environment. Usually, the dynamical behaviour of the environment is not so obvious as those of the system in focus, or in other words, the environment changes very slowly compared with the system in discussion. In this case, the impact of the environment appears no more than a parameter in the equation describing the behaviour of the system.

A frequently encountered situation is that the dynamical behaviour of an open system can be described by the inner dynamics of the system, i.e. the dynamical relations between the systems components. It is written in the form of differential equations, either partial or ordinary. For a physical system, the dynamical process often unfolds in both space and time. Therefore, the spatial distribution as well as the unfolding along the time axis of the system must be considered. As it has been already known from the previous chapters that there are microscopic fluctuations of composing components within the system at the microscopic level. These fluctuations be denoted by a function δ. However, any description of the behaviour of a system as a whole is at the macroscopic level and in this description, fluctuations at the microscopic have been averaged out. Macroscopic description of a system does not include microscopic fluctuations.

More generally, the spatial-temporal behaviour of a dynamical system can be described as:

$$\frac{\partial X}{\partial t} = f\,(X, \theta, \nabla, \sigma, t) \tag{4.2.6}$$

where:

f describes the system's inner dynamics;

X the state variabe;

$\theta = (a, b, c)$ the spatial coordinates;

$\nabla = \left(\frac{\partial}{\partial a}, \frac{\partial}{\partial a}, \frac{\partial}{\partial a}\right)$ the spatial gradient;

σ the influence from environment.

In principle, this complex form of dynamical equations can be simplified to a simpler one: partial differential equation can be reduced to some ordinary differential equations so that the whole body of knowledge of mathematical dynamical systems theory can be employed to analyse the complex behaviour of the system. In the following section, some important terms like attractor, structural stability and bifurcation etc. will be defined and all definitions are based on the standard autonomous equation which describes a differentiable dynamical system:

$$\frac{dX}{dt} = f\,(X) \tag{4.2.7}$$

where f is a vector field on manifold M which defines the inner dynamics of the system, and X is the state variable and $X \in M$.

In analysing systems evolution, the impact of environment to a system must be taken into account and hence parameters must appear in the dynamical equation explicitly. The discussion will be centred around the complex behaviour of the system caused by the change of environment, namely by varying the parameters. That makes the mathematical analysis extremely difficult if several parameters must be considered simultaneously.

4.2.3 DST: The unified language for evolving systems

In Chapter 3, several important schools of thought about systems evolution, like the theory of dissipative structure, hypercycle, synergetics etc.are mentioned. A brief revisit to these theories will reveal that they not only share some terminologies in the study of the conditions, stages, processes, and general principles underlying various evolving systems, they also use the same mathematical tools to reach their goals [see also, Gao et.al, 1991b].

A) The Theory of Dissipative Structures

The Brussells school's work on dissipative structures is still one of the most influential in the study of systems evolution, not to say it is the most well known one. In analysing the evolutionary process of various systems, special dynamical equations, such as partial differential equations of the reaction-diffusion type, master equation, are employed to model the evolving systems, and many results and techniques of dynamical systems theory have been used [Nicolis et. al, 1977, Prigogine, 1980].

B) Synergetics

In Synergetics, an open system is away from thermal equilibrium and is composed of many different competing and cooperating subsystems. It can undergo self-organization (evolution) in the sense that new ordered state at macroscopic level can emerge spontaneously. To understand the specific process of evolution of a particular system, the key step, according to synergetics, is to detect the order parameters at the critical point. Given dynamical equations describing the system, the mathematical techniques employed to find the order parameters is the "adiabatic approximation". The use of DST in synergetics is plentiful (even more so than in dissipative structure theory). It is even stated that the slaving principle contains some deep theorems of DST as its special cases, such as the centre manifold theorem, the slow manifold theorem, and adiabatic elimination procedure [Haken 1983b, pp316].

C) Hypercycle

The theory of hypercycle studies the self-organization process through which some primitive biological systems can emerge from some special organic molecules. This investigation is based on impressive mathematical treatment. The simplest system capable of selective self-organization can be defined by a dynamical system in such a general form:

$$dX/dt = (A - \Lambda - \Phi)(X) \qquad (4.2.8)$$

where $X = (x_1, x_2, .., x_n)$ is the vector of population variables; A reflects the positive contribution (amplification) to X, Λ the negative one (decomposition), while Φ refers to the out flux of X from the system. The mathematics techniques employed there belong to dynamical systems theory (fixed points analysis in particular), although not explicitly stated.

D) Cellular Automata

Cellular automata is a kind of discrete dynamical system, it is capable of studying the emergence of organized structure from dis-organized structure, the defining characteristic of systems evolution. Although it depends heavily on computer simulations, its behaviour can also be described by using discrete dynamical systems equations. Therefore, it would be possible to use the techniques developed in dynamical systems theory to study cellular automata.

E) Connectionist models

Connectionist models, like Random Boolean Network, Neural networks, explicitly address the problem of how a group of elements are connected to form an organized whole and how the global behaviour of a system emerges from the local behaviour of elements. The dynamical behaviour of a system composed many elements connected as such is modelled by a group of differential equations. All the techniques developed in mathematical systems theory are employed in understanding the adapting and evolving behaviour of these systems.

In all those above mentioned schools of thought, considerable weight has been put into the analysis of the system's nonlinear dynamics in the complete discussion of the evolutionary behaviour of various system. The study of dynamical systems theory consists of the study of structural stability, bifurcation and global behaviour of flows and that is why the modern mathematical dynamical systems theory has been employed. The recent development in mathematical dynamical systems theory has greatly enriched our understanding of the complex behaviours of systems governed by nonlinear dynamics. In principle, any systems evolving over time can be modelled by such dynamical equations. The long run behaviour of the system is prescribed by various attractors which the dynamical equations possess. With an extended introduction to dynamical systems theory (DST) given in Appendix 1, it can be argued that the following features make DST a suitable means for us to study systems evolution.

1. It provides the notion of "attractor" which can be used to describe the state of any systems qualitatively. As it will be demonstrated in the following section, any dynamical system, which is recognized as a system, can be described by an attractor which can be defined either verbally or formally. Attractors describe the rough patterns of systems. We can also use the notion of attractor or emergent attractor to describe and explain the striking property of systems: that "a system is more the sum of its parts" is due to the existing of an emergent attractor that gives the system its identity as an organized whole.

2. DST provides a concise and delicate geometrical model for the analysis of the complex dynamical behaviour of systems. The history to use dynamical model in explaining the behaviour of various systems has been very long, but only recent development in DST (both analytical and computational) enable its use more practical and accessible. Although data can be fed in to get some quantitative results, the method of DST is essentially qualitative rather than quantitative. This makes it more suitable,

compared with other mathematical techniques, for the study of systems evolution because evolution is also a qualitative property of systems.

3. DST provides a complete taxonomy for emergent attractors: there are four fundamental types of emergent attractors, i.e. point attractor, periodic attractor, quasiperiodic attractor and chaotic attractor, that can describe the state of any system at any stage.

4. DST provides a classification of bifurcation patterns that can help us to explain the evolution route of systems. As argued in chapter 2, systems evolution include systems genesis and intra-level evolution. Intra-level evolution means that the number of the components of a system is unchanged, but the structure of the system has changed qualitatively so that it can maintain its integrity in a new environment as an new whole. Those bifurcation patterns studied by DST can be used to describe adequately this intra-level evolution behaviour.

5. DST provides not only a mathematical rationales for us to understand evolution process of systems, but also a way to understand the behaviour of systems in general. Klir once attributed the emergence of systems science to three factors: a) the development of systems ideas in various discipline; b) the development of mathematical theory and techniques; c) the invention and development in computation technology. When talking about the use of mathematics, he mainly emphasised the development of OR techniques for early systems practice in systems analysis and systems engineering [Klir et. al, 1989]. He has also pointed that recent development in DST is very important for the emerging "self-organization paradigm".

This chapter will define and explain the concept of attractors of systems and set to analyse how it can be used to describe the spatial-temporal evolution of dissipative systems. As mentioned in previous sections, DST also brings with the solid mathematical techniques for the study of evolutionary process of particular systems and formal model to be fully developed will be employed to study some particular evolving systems in the following chapters.

4.3 Attractors and the State of Dynamical Systems

4.3.1 The concept of attractors: a historical account

Any system that can be regarded a system is at stable states in a certain sense, otherwise we will be unable to observe it or to describe it. Here stable state means that the system has certain structure which makes it possible for our observers to view the system as an organized whole. Thom has proposed to call such a stable state an attractor:

> *"Every object, or physical form, can be represented as an attractor C of a dynamical system on a space M of internal variables"*. [Thom, 1975, pp320]

Thom considers that any natural process can be described as a dynamical system (M, X) which is defined by a vector field X on the manifold. An attractor F of the system is a closed invariant set under X. He gives the definition of an attractor as such an invariant set satisfying the following conditions:

> *1. There exists an open neighbourhood U of F, called the basin of the attractor F, such that every trajectory starting from a point of U has F as its ω-limit set.*
>
> *2. Every trajectory whose α-limit set contains a point of F is contained in F.*
>
> *3. F is indecomposable, that is, almost every trajectory of X in F is dense in F.* [Thom, 1975, pp39]

According to this definition, an attractor is an attracting set to which every "near-by" trajectories are attracted to. For an observer, an attractor corresponds to a state of the system to which the system will ultimately settle down to even it is perturbed by some noise and driven to the near-by area. A system is always facing both the internal noise as well the external perturbations, therefore we have the concept of a structurally stable attractor.

> *" an attractor F of a field X is called structurally stable, if for every field X_1 sufficiently close (in the C^1 -topology) to X, there are an attractor F_1 and a homeomorphism h of a neighbourhood of F onto a neighbourhood of F_1, throwing trajectories of X onto trajectories X_1.* [Thom, 1975, pp39]

Although it has not been proved that any given field X on M has attractors, still less structurally stable attractors, many dynamical equations encountered, which describe some real-world systems on a R^n space, have attractors of various types.

Thom considers only one type of attractors, i.e. point attractor, in his discussion of catastrophe theory. We will find out latter that there are several other types of attractors which represents different states of complex dynamical systems.

An attractor, as a time-independent state of systems, can also be described by less rigourous mathematical terms as which found in Swenson [Swenson, 1989a, b]:

" The time-independent (time-asymptotic) states, or limit sets, that attract initial conditions from region around them, 'basins of attraction', during a time dependent processes (evolutionary behaviour) as $t \rightarrow \infty$. All real world macroscopic change is irreversible and hence governed by attractors, viz., the instability of entropy-producing processes. In particular, the second law of thermodynamics specifies a maximum entropy attractor, S_{max}, the macrostate with the maximum number of accessible microstates, for all macroscopic changes as $t \rightarrow \infty$. In this sense, all macroscopic change is (i) progressive (goes irreversibly towards an attractor), and (ii) goal-driven (the attractor is the goal). The attractor drives the evolutionary behaviour by virtue of the instability of all states within the basin of attraction but off the attractor." [Swenson, 1989a].

The time-dependent behaviour of a dynamical system, or a dynamical process, is prescribed by time-independent attractors. The dynamical system discussed might be a closed system whose dynamical behaviour is governed by the second law of thermodynamics. When the system is an open system, and driven far away from its thermodynamic equilibrium through the strong interaction between the system and its environment, its dynamical behaviour tends to become very complicated. However, the long term behaviour of both open systems and closed systems can be described by the concept of attractor. As stressed by Swenson, it is very important to bear in mind that there are two types of fundamentally different attractors in the real world physical systems. One is the maximum entropy attractor which is the ultimate attractor to which the whole universe is heading to according to the second law of thermodynamics. Such maximum attractor is what is usually called the "thermodynamical equilibrium" where the system is at its maximum entropy state and all functional structure of the system has vanished (and there is no system at all). Another type of attractors are called "emergent attractors" which represent the emergent behaviour of systems. That means that strikingly new and innovative structures emerge from the system's old structure, through some self-organization process, and such structure is represented by some sort of emergent attractors. Because systems in thermodynamical equilibrium are rare to observe in real world, all systems we can recognize as systems are non-equilibrium systems and hence are only described by emergent attractors. For that reason, the concept of an attractor is usually used to mean an emergent attractor. We can also define a system as an emergent attractor which represents an entity recognized as an organized whole. The adequacy to use attractors to describe the states and behaviours of general dynamical systems can be justified by the following properties of attractors.

1) Attractors describe the "asymptotic" or long-term recurrent behaviour of a dynamical system after transients reflecting initial conditions have died away. They are most of what a system does when observed over a long time.

2) Attractors represent the integrated behaviour of a dynamical system to focus upon and describe the global state of a system through the local connections of

variables. This can be demonstrated by looking into the definition given by Thom or to be given in 4.3.2. Where, the locality of the connections has been passed to the globality of the state of a system as a whole through the passage of time, exactly as the passing from the locality of the differential equations to the globality of its solution as argued by Zeeman [Zeeman, 1986]. Therefore, attractors describe systems as emergent wholes and hence can be employed to define systems.

3) Attractors describe a progressive process: systems move irreversibly towards attractors in the long run.

4) Attractors describe the goal-driven behaviour of dynamical systems and attractors are the goals. No matter where a system starts from, or where it is perturbed to off the attractor, the system will ultimately settle down to an attractor.

5) Attractors exhibit the spontaneous order of dynamical systems and imply a self-organization process. Through the interactions of components, a dynamical system will go to one attractor by itself without requiring any outside work. This self-organization process gives the system some degree of order which is an inherent property of that system.

4.3.2 Definition of Attractors

As seen from the above section, a dynamical system can be modelled by a differential equation as :

$$\frac{dX}{dt} = \mathbf{f}\,(X) \qquad (4.3.1)$$

where $\mathbf{f}$ is a vector field on manifold $\mathbf{M}$ which defines the inner dynamics of the system, and $\mathbf{X}$ is the state variable and $\mathbf{X} \in \mathbf{M}$. The concept of attractor can be defined in more rigourous mathematical terms. An attractor A is a subset on the manifold $\mathbf{M}$ (generally, $\mathbf{M}$ is the state space of a system).

Definition 4.3.1 (**Attractor**). Let A be a subset of manifold $\mathbf{M}$, U is a neighbourhood of A, $A \subset U \subset M$. Suppose that $\cap_{t \geq 0} \mathbf{f}^t U = A$, then U is contracted by $\mathbf{f}$. If A satisfies the following conditions, A is called an attractor of the system S and U is a foundamental set of A:

(1) Attracting: for every open set $V \supset A$, $\mathbf{f}^t U \subset V$ for all sufficiently large t.

(2) Invariant: $\mathbf{f}^t A = A$ for all $t \in \mathbf{R}$.

(3) Indecomposibe: if there is another such A' satisfying (1) and (2), A'=A.

Where the condition (1) can also be expressed in another way:

(1)' For every V, $U \supset V \supset A$, any trajectory starting from a point in V, its ω-limit set is included in A .

In this definition, U is called a fundamental set of attractor A [Ruelle, 1989]. The basin of attraction of an attractor A is defined as a set :

$B=\{x \mid x\in M$, x is attracted to A, i.e., A contains the ω-limit set of trajectory $f^t(x)\}$.

There are many other definitions for attractor. In almost all definitions, it requires that an attractor must have:

> "*... a fundamental set of neighbourhoods, each of which is forward invariant under the flow generated by (the vector field) X*" [Guckenheimer, 1976]

In a paper by Milnor [Milnor, 1985], a number of such definitions have been reviewed and compared. For reasons of mathematical elegance, he proposed an alternative definition which is based on asymptotic behaviour of the system for almost every choice of initial point. For our discussion of evolving systems, under the notions and concepts introduced so far, the above definition is sufficient for us to carry on the argument. Its meaning can be comprehended by referring to Swenson's explanatory definition quoted above.

According to the structure of A, there are the following four different types of attractors.

Definition 4.3.2 (1) (Point attractor) A is an attractor of a dynamical system S. If there is only one point in A, i.e., $A = \{x_0\}$, then A is called a point attractor.

A point attractor has the following properties:

(i) All its (Lyapunov) characteristic components have negative sign.

(ii) Its fractal dimension is 0.

Definition 4.3.2 (2) **(Periodic attractor)** A is an attractor of a dynamical system S. If A is a closed orbit, i.e, there exists T>0, such

for every $x\in A \Rightarrow x(t+T) \in A$ for all t.

then A is called a periodic attractor with period T.

A periodic attractor has the following properties:

(i) All its (Lyapunov) characteristic components have non–positive sign, i.e., either 0, or a negative real number.

(ii) Its dimension is a positive integer ($\dim A \leq \dim M$).

Definition 4.3.2 (3) *(Quasiperiodic attractor)* A is an attractor of a dynamical system S. If A is composed of several periodic orbits at different directions, i.e., $A = (A_1, A_2, \ldots A_n)$

where A_i (i=1, .., n) are periodic orbits with period T_i (i=1, .., n) satisfying:

(i) $\dim A = \sum_{i=1}^{n} \dim A_i$.

(ii) There is at least one irrational ratio $\frac{T_i}{T_j}$ (i, j =1, .., n, i≠j)

then A is called a quasiperiodic attractor.

A quasiperiodic attractor has the following properties:

(i) All its (Lyapunov) characteristic components have non–positive sign, i.e., either 0, or a negative real number.

(ii) Its dimension is a positive integer ($\dim A \leq \dim M$).

A Quasi-periodic attractor is sometimes called a toroidal attractor, or a pseudo-periodic attractor.

Definition 4.3.2 (4) **(Chaotic attractor)** A is an attractor of a dynamical system S. If and only if it has the following properties:

(i) At least one of its (Lyapunov) characteristic exponent has a positive sign.

(ii) The system is sensitive to the initial conditions in the neighbourhood of this attractor.

then A is called a chaotic attractor.

Its dimention is a positive non–integer real numbe, i.e. it is a fractal.

or

Definition 4.3.2.(4') **(Chaotic attractor)** A is an attractor of system S. If it is neither a point attractor, nor a periodic attractor, nor a quasi-periodic attractor, it is called a chaotic attractor

In some literatures, there are only three distinct attractors: point attractor, periodic attractor, and aperiodic attractor. The aperiodic attractor is the name for both quasiperiodic attractor and chaotic attractor because both of them describe the irregular, non-periodic motion of dynamical systems [Ruelle, 1989; Swenson, 1989b]. Here we take the two as different because they are essentially different in the following aspects:

(1) A quasiperiodic attractor describes the quasiperiodic behaviour of a system which is characterised by several periodic motion at different direction with several

fundamental frequencies (T_i/T_j is irrational for some (i, j)). A quasiperiodic attractor is a k-dimensional torus with k the number of fundamental frequencies. However, a chaotic attractor does not have any of such periodic motion in any directions and it describes the near random behaviour of a deterministic system.

(2) The dimension of a quasiperiodic attractor is a positive integer (the dimension of a torus on which this quasiperiodic attractor is found) while the dimension of a chaotic attractor is a positive non-integer real number, i.e., the chaotic attractor has a fractal dimension.

(3) When a system rests at a chaotic attractor, it usually exhibits some sensitive dependence on the initial condition while for a quasiperiodic attractor, the system is resistant to the small purterbation at the initial condition.

Fundamentally, a chaotic attractor is totally different from a quasiperiodic attractor and the other two types of attractors. This point can be justified by discussing the peculiar properties of chaotic attractors in the following section.

4.3.3 State of dynamical systems

The various types of attractors describe the various asymptotic state of a system. In the phase space, they are invariant sets that attract trajectories starting from the nearing region around them. For a system whose state and behaviour can be observed, they represent different types of state that prescribe the long run behaviour of the system.

A) Point attractor and stable state

It is quite familiar for us that a point attractor represents the asymptotic stable state of dynamical systems. It can be the steady state of a perturbed pendulum in the air: without sustained driving force, the resistance of the air will drive it to that steady state finally, no matter how big the amplitude was when first perturbed. It might be the certain height of a water tank with the same amount of water flowing in and flowing out. It can be the concentrations of several chemical reactants in a container where the reaction is maintained with the new reactants added in and the products moved out in a constant speed. It may well be the glucose level in the blood of a normal human body. It can also be the temperature of a room regulated by air conditioning. In some mechanical or physical systems, it is the dynamical equilibrium state to which the system will move to even if it is slightly perturbed. In biological systems it is the well known "Homeostasis" to which the self-regulating mechanism, feedback mechanism in general, will drive the system toward.

Take the often referred Benard hydrodynamic system as an example [see example in chapter 3 and references [Haken, 1983a; Swenson, 1989a], the point attractor represents the stable state of the system when the temperature is not very high. The system can be observed as in a steady state and the heat is conducted by the liquid molecules from the heated bottom to the top.

In the chemical system of the Brusselator [Nicolis et.al, 1977; 1989], the chemical system is at a equilibrium when chemical reactants are pumped in the container at certain speed: this is another example of point attractor which we will go into details in the following chapter.

B) Periodic attractor and oscillating behaviour

Oscillating behaviour has been found in many systems, ranging from electric circuits, chemical waves, biological rhythms, ecological systems and economic long waves etc. In the phase space of such dynamical systems, this kind of oscillating behaviour is represented as a closed orbit, which is usually called a limit cycle. Periodic attractors are employed to represent such stable oscillating behaviour: all near-by trajectories are attracted to such a limit cycle.

First of all, it should be made very clear that not all closed stable orbits are periodic attractors. For there are essentially two different types of dynamical systems: conservative systems and dissipative systems, and the concept of attractors are especially designed for dissipative systems. The former are called Hamiltonian systems and their behaviour on the phase space is described to obey the Liouville theorem: the volume of the trajectories is constant [Prigogine, 1980]. There is no contracting behaviour in conservative systems on the phase space. Closed orbits can be found in Hamiltonian systems but they are not periodic attractors because they are not attracting any trajectories anyway.

In dissipative systems, energy is dissipated constantly. Through the interaction between the system and its environment, manifested as the exchange of energy/matter/information, the system will settle down to a state at which the dissipation of energy by the system is counter-balanced and maintained by such exchange. Apart from the state mentioned above, the dynamical equilibrium state characterised by a point attractor, the system might be in a stable oscillating state: when the system perturbed, slightly, i.e not exceeding the basin of attraction, the system will go back to that oscillating state.

In electric circuits, the oscillating behaviour is very common, but the first well study system was due to Van der Pol through the well known Van der Pol equation:

$$\frac{d^2x}{dt^2} + \alpha\,(x^2-1)\,\frac{dx}{dt} + \omega_0^2 x = A\sin(\omega t) \tag{4.3.2}$$

which describes the self-excited relaxation oscillations for $\alpha >> 0$ and $A=0$.

For various α, ω_o, A and ω, the system will settle down to a stable oscillating state with various periods and amplitudes [Hirsch et.al, 1974, Thompson et. al, 1986].

In non-equilibrium thermodynamics, the Brusselator is known to have such a periodic attractor [Nicolis et. al, 1977; Tomita, 1986]. With A, B as initial reactants, and D, E as final products, whose concentrations are imagined to be imposed as constants throughout by appropriate adding reactants and removing products, the hypothetical reaction is as follows:

$$\begin{aligned} A &\rightarrow X \\ B+X &\rightarrow Y+D \\ 2X+Y &\rightarrow 3X \\ X &\rightarrow E \end{aligned}$$

The dynamical model for this chemical reaction can be written by the following coupled nonlinear rate equation (with the same letters representing the concentrations of those chemicals):

$$\begin{aligned} \frac{dX}{dt} &= A-(B+1)X+X^2Y \\ \frac{dY}{dt} &= BX-X^2Y \end{aligned} \tag{4.3.3}$$

It has been proved, mathematically and experimentally, that stable periodic oscillation exists in this chemical system for certain values of B, i.e., the concentrations of the intermediates X and Y change periodically. This state is represented as a periodic attractor, according to our conceptual framework. The Brusselator is an example in later discussion about bifurcation and the onset of chaos.

Periodic attractors can also be employed to characterise the states of many other systems, like predator-prey-food chain in ecological system [Toro et.al ; 1988]; neural oscillation [Wilson et. al, 1972 etc]; economic long wave [Stermann, 1988, 1989], the cyclic behaviour of international trade system [Zhang, 1989].

C) Quasiperiodic attractor and toroidal motion

Quasiperiodic motion is a kind of oscillation found in many nonlinear dynamical systems. It is irregular, compared with periodic oscillation and, hence is sometimes called as an "aperiodic oscillation". Such an oscillation can be described as this: it is composed of several periodic oscillations along several different directions and their periods are called the fundamental frequencies. Some of the ratio of the fundamental frequencies are irrational numbers. The projections of the orbit along those directions to certain hyperplanes are closed orbits, but the overall motion is not strictly periodic: the trajectory will never pass the same point twice although it can come very near to that point, as near as you like. The trajectory covers the whole surface of a torus. For these reasons, such a motion is called a quasiperiodic oscillation, or a toroidal oscillation. The concept of quasiperiodic attractor characterises the quasiperiodic oscillation, one of the observed complex behaviour of nonlinear dissipative systems.

Quasiperiodic attractors have been found in electric circuits [Parker et.al, 1989; Haken, 1983b]; chemical systems [Tomita, 1986] and many other systems. We will see later that quasiperiodic attractors are also playing an important part in the discussion of the onset of chaos: a periodic attractor can bifurcate to a quasiperiodic attractor and further to a chaotic attractor.

D) chaotic attractor and chaos

Chaos has been a popular word since early 80's and even more so in recent years after the publication of a book called "*Chaos*" by Gleick in 1987 [Gleick, 1987]. It refers to the complicated dynamical behaviour of nonlinear systems: although the system is deterministic and often very simple, like the logistic map, it exhibits a highly irregular behaviour, just like a random motion. The random-like behaviour, called "deterministic chaos", found in simple deterministic systems has excited not only mathematicians, physicists, and applied dynamicists, but also biologists, economists, sociologists and many others, and some scientists go further to argue that there is a entirely new branch of science emerging based on the discovery of chaos (of course, this has caused controversies, hot debates can be found, say, in *The Mathematical Intelligencers*, 1989, Vol, 11, No.3 among some leading mathematicians, and *Behaviour and Brain science*, 1987 No.10, between behavioural scientists). In this research, we will adopt a modest claim and a cautious attitude towards the use of the concept of chaos and chaotic attractor: it is used no more than a concept to describe a nonlinear dynamical system whose behaviour is very complicated (for the discover of chaos, there is a lot of review articles to be referred to, say Ott etc.[Ott, 1981; Eckmann et. al, 1985; Gleick, 1987]).

Chaotic attractors describe the state of a system at which its behaviour is random look-like. It is also called a strange attractor, for it reveals the randomness of a essentially simple deterministic process. Chaotic attractor, or chaos in a dynamical system, has the following properties which make it essentially different from other attractors (see also [Ruelle, 1989; Gleick, 1986; Stewart, 1989]).

1) randomness.
Although the system is deterministic, the time behaviour is so irregular that it resemble the random motion rather than a deterministic process.

2) sensitive dependence on the initial condition
Two trajectories starting very close will eventually diverge and the minor difference in the initial conditions will be amplified to a significant scale.

3) fractal dimension
The trajectory in the phase space will cover densely a region. The dimension of such region has a fractal dimension, i.e., a positive, non-integer number.

4) structural stability
All chaotic attractors, at least all the known chaotic attractors, are structurally stable, despite is irregularity and the sensitive dependence of each individual trajectory on the initial conditions. (Zeeman conjectures that there might be some chaotic repellors, see [Zeeman 1988a]).

5) Order within chaos
Although the behaviour of the dynamical system at a chaotic attractor is so irregular, but the chaotic attractor itself has some intricate structure, like the Cantor-like set and Smale's horseshoe structure found in some chaotic attractors. Chaos has imbeded structures and chaotic attractors are structually stable.

Chaotic attractor has been found in various systems. In principle, any nonlinear dissipative dynamical system with dimension no less than 3 can exhibit chaos. It must be stressed that the concept of "chaos" in our discussion is quite different from what it means in the title of a famous book "*Order out of Chaos*"[Prigogine et.al, 1984]. In that book, chaos means the homogeneous, structureless, or simply disordered state of a system, but chaos or chaotic attractor in mathematics and in our discussion is employed to describe a structurally stable state of systems which implies some fine mathematical structures such as Cantor set. It is very misleading to take these two concepts as the same (largely due to the popularity of these two books, i.e. "*Order out of Chaos*" and "*Chaos*").

4.3.4 problems about attractors

Four distinct attractors have been defined and found in nonlinear dissipative systems which describe four distinct types of asymptotic stable state. Any dynamical system, no matter where it starts in the phase space, will move and settle down to one of these attractors. It is usually supposed that the transient time that a system takes to change from one steady state to another is very short compared to the time scale of the history of the system. It is apparent that this mathematical classification of attractors satisfies the following criteria and the concept of attractors can be used to study the evolutionary behaviour of dissipative dynamical systems.

1) Experimentally identifiable, in spite of the error in observation.

2) Well-founded for the discussion of evolutionary process described by differential equations.

3) Tractable in computer simulation.

[Abraham, 1988]

Mathematically, it is not always possible to define and detect all the attractors analytically from differential equations, ordinary or partial. In most cases when dealing with a system in a chaotic state, it has to rely on computer simulations and numerical analysis, like the construction of phase portraits, the calculation of Lyapunov exponents, fractal dimensions, and spectrum analysis. It is hoped that further progress in mathematical dynamical systems theory and computer simulation techniques will help us to identify attractors more easily.

It has not been proved that every mathematical dynamical system has attractors, but for dynamical systems arising from practical problems, there are always attractors which correspond to the observed state of the systems. For many such systems there is usually more than one type of attractor co-existing and they correspond to different type of the potential state of the system in focus.

4.4 Structural stability and bifurcation

4.4.1 Multiple Attractors

Attractors prescribe and describe the long run behaviour of dissipative systems. In lower dimensional systems (dimension ≤ 3), these attractors can be illustrated on the phase space which can show clearly the relations between the variables, i.e., how one variable changes in accordance with others. Such graph is known in dynamical systems theory as phase portrait which consists limit sets (attractors, repellors, separatrices), basin of attraction and some typical trajectories.

In contrast to the concept of attractors, there are other limit sets for the dynamical systems: repellors and saddle-like limit sets which separate the basins of various attractors. By changing the direction of time in definition 4.3.1, i.e., replacing $t \rightarrow +\infty$ by $t \rightarrow -\infty$, the limit set we get is called a repellor. A repellor repels the trajectories starring from any points from its neighbourhood.

For repellors represent the unstable state of dynamical systems, they are the un-observable states of real-world systems. For example, there is a dynamical system which can be described by a potential function V possessing two local minima and one maximum point (see the following illustration figure 4.4.1).

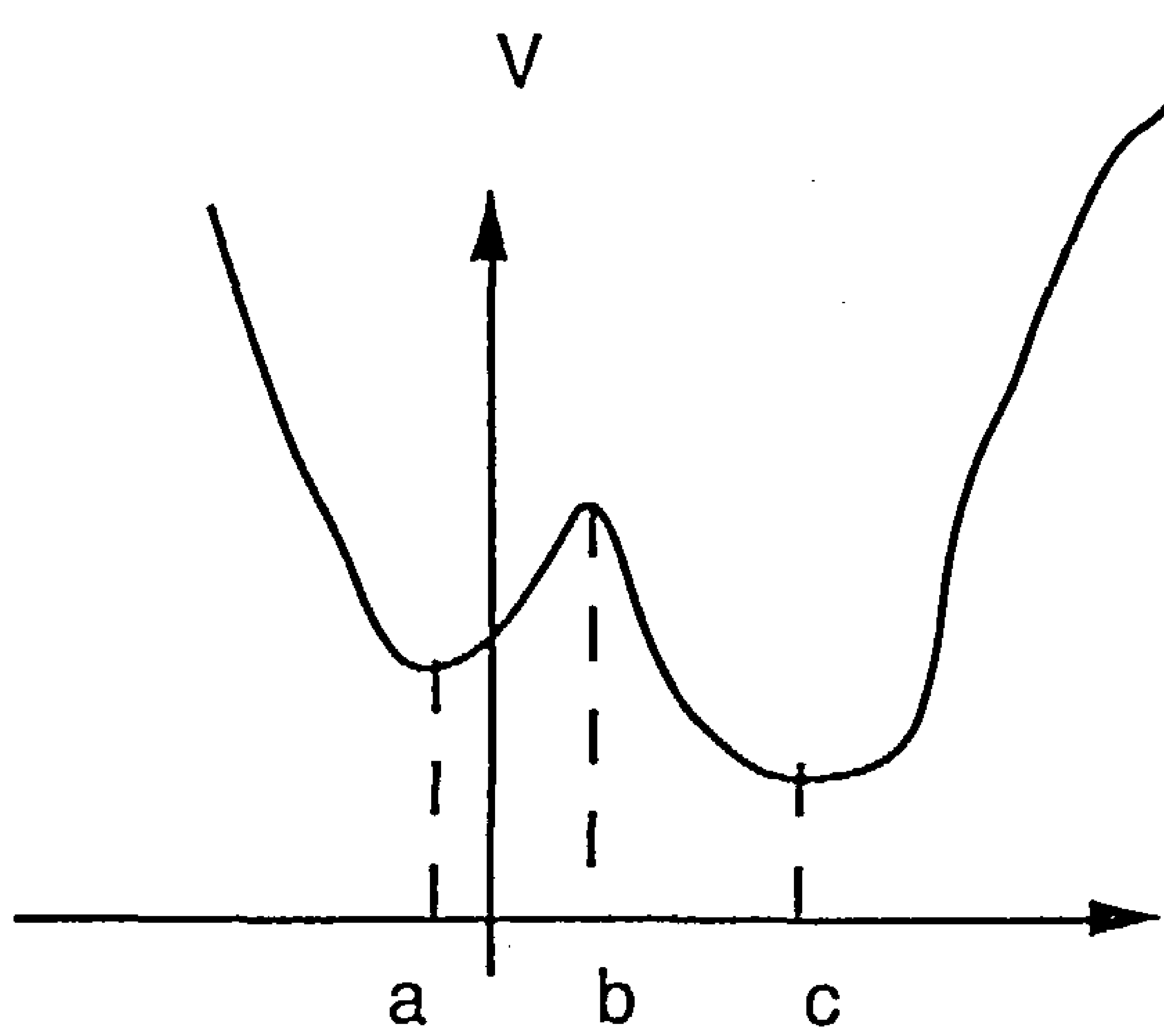


Figure 4.4.1 Attractors and repellors

Points ***a*** and ***c*** are two local minima and they represent the possible state the system can settle down to: they are two attractors. Point ***b*** is a local maximum point representing a state with a high potential. A system can not be observed at state ***b*** : the microscopic fluctuations constantly drive the system away from state ***b*** and once the system is driven away from ***b***, it will fall along the slope to one of the nearing potential valley, i.e to ***a*** or ***c***. By using our terminology here, ***b*** is a repellor and it repels the system from its neighbourhood around ***b*** to certain attractors, ***a*** or ***c***. According to Maxwell convention, the system is always driven to a lower potential state.

It has been proved that there exist point repellors and periodic repellors, quasiperiodic repellors, but no chaotic repellors has been reported existing so far. Zeeman has conjectured that there might exist some chaotic repellors and he has even suggested that the two mathematically proved chaotic attractors, i.e, Smale's horseshoe and the solenoid, can be embedded as chaotic repellors [Zeeman, 1988a]. Because repellors can not be observed, any discussion of repellors must be based on the theoretical analysis of the dynamical equations describing the system.

The point repellor is usually called an unstable fixed point, or a "source". A periodic repellor is an unstable periodic orbit, or an unstable limit cycle. They can be illustrated as the following (figure 4.4.2, 4.4.3).

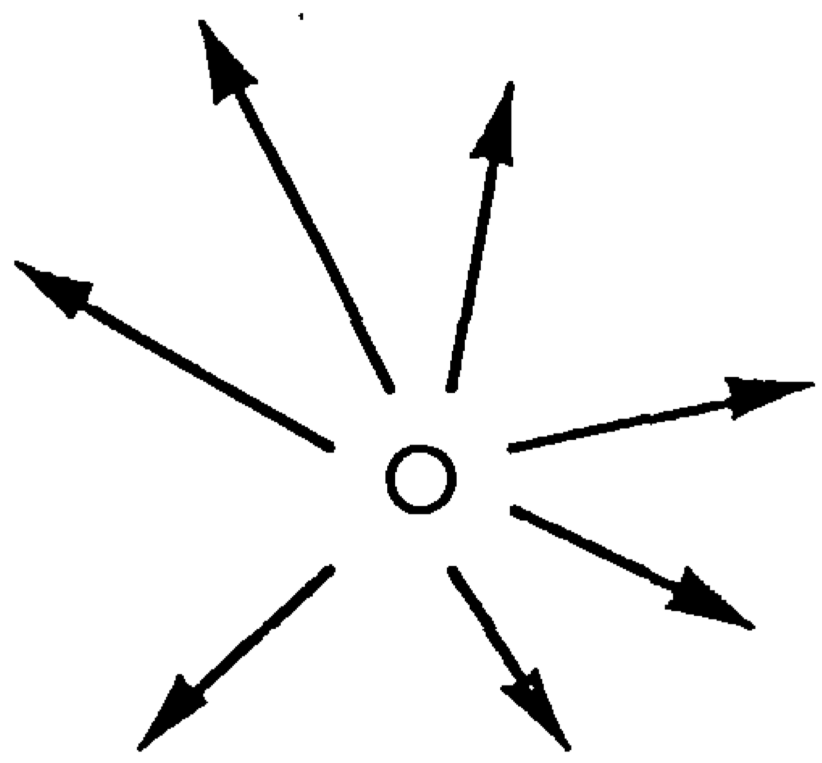


Figure 4.4.2 Point repellor

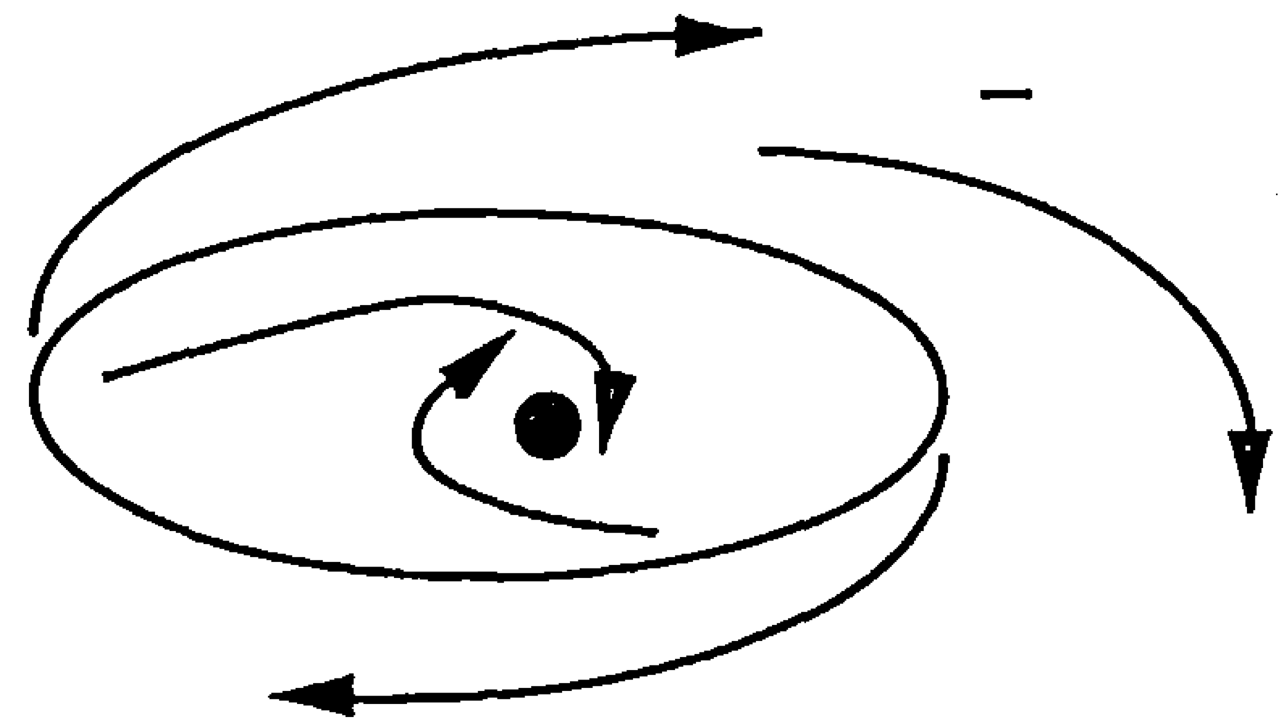


Figure 4.4.3 Periodic repellor

Saddle-like limit sets lie in between attractors and repellors: they are attracting nearby trajectories along certain directions and repelling some others. This can be typically illustrated by the saddle point and a saddle-like limit cycle (figure 4.4.4, 4.4.5).

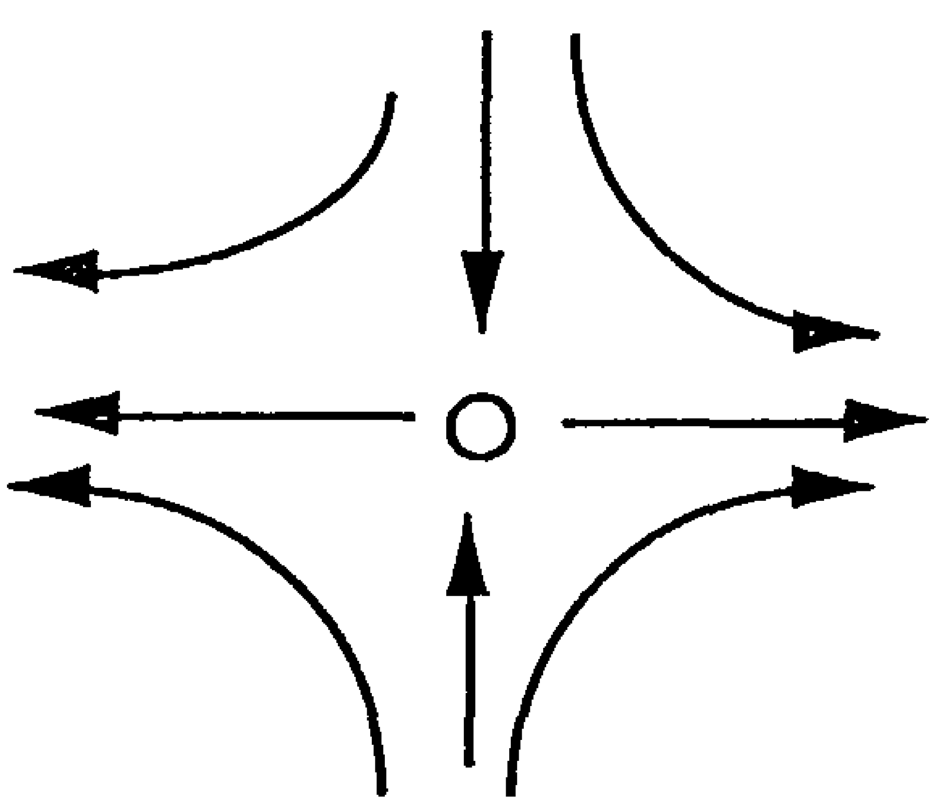


Figure 4.4.4 Saddle point

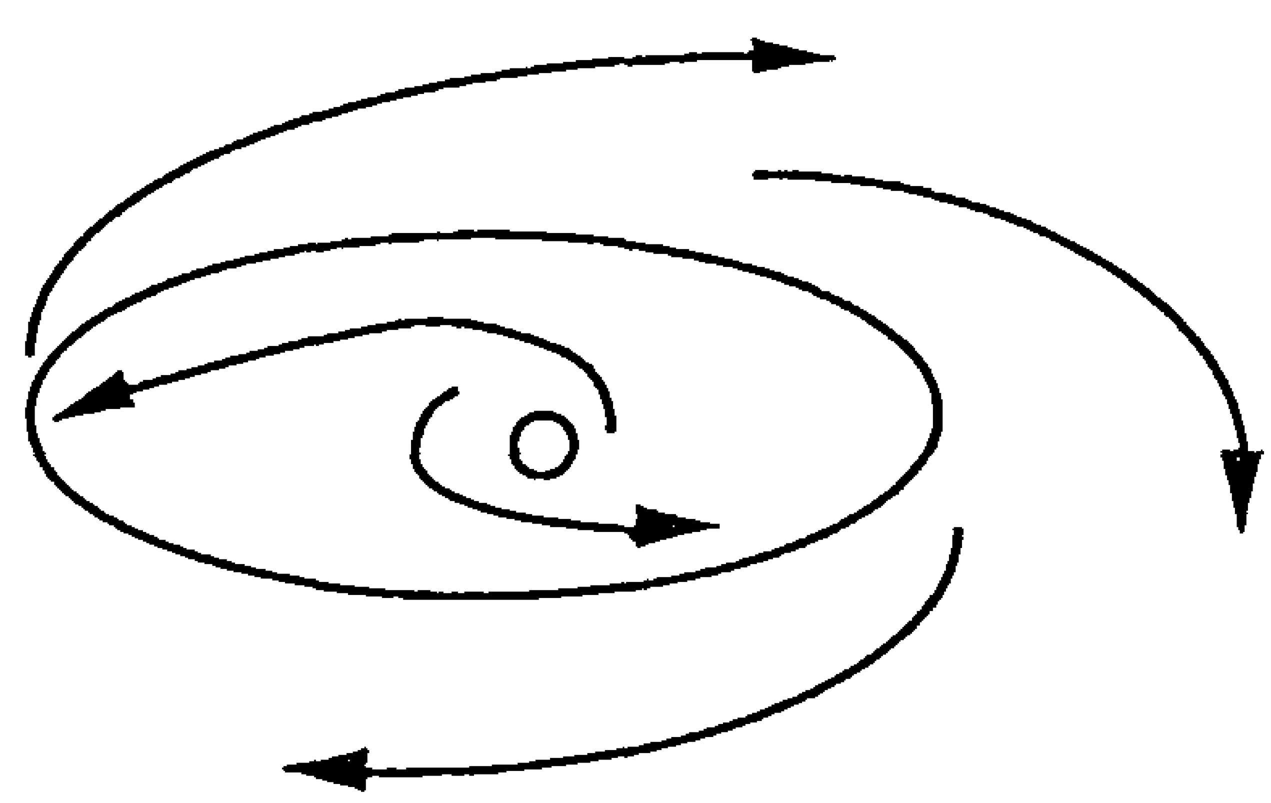


Figure 4.4.5 Saddle-like limit cycle

As we have already known that the basin of attraction of an attractor is the set of all those points such that any trajectories starting from them are attracted by the attractor. If there are more than one attractor existing, each attractor has its own basin of attraction and lying between are sets called **separatrices**.

The separatrices are the boundaries of basins of attraction but they do not belong to any basins. By nature, they can never be observed for they are not stable [Abraham, 1988]. They are usually the saddle-like limit sets but they are restricted to some forms: in one-dimensional phase space, the separatrices are saddle-points; in two-dimensional phase space, they are some sorts of closed curves which might be composed of saddle points or saddle-like closed orbits; in three dimensional space, they are some saddle-like closed surfaces. The attractors, separatrices and repellors, together with basins of attraction form the phase portrait. The phase portrait of a two-dimensional system can be illustrated by the following figure (4.4.6) (only point attractors are presented and the separatrices are saddle-like limit cycles).

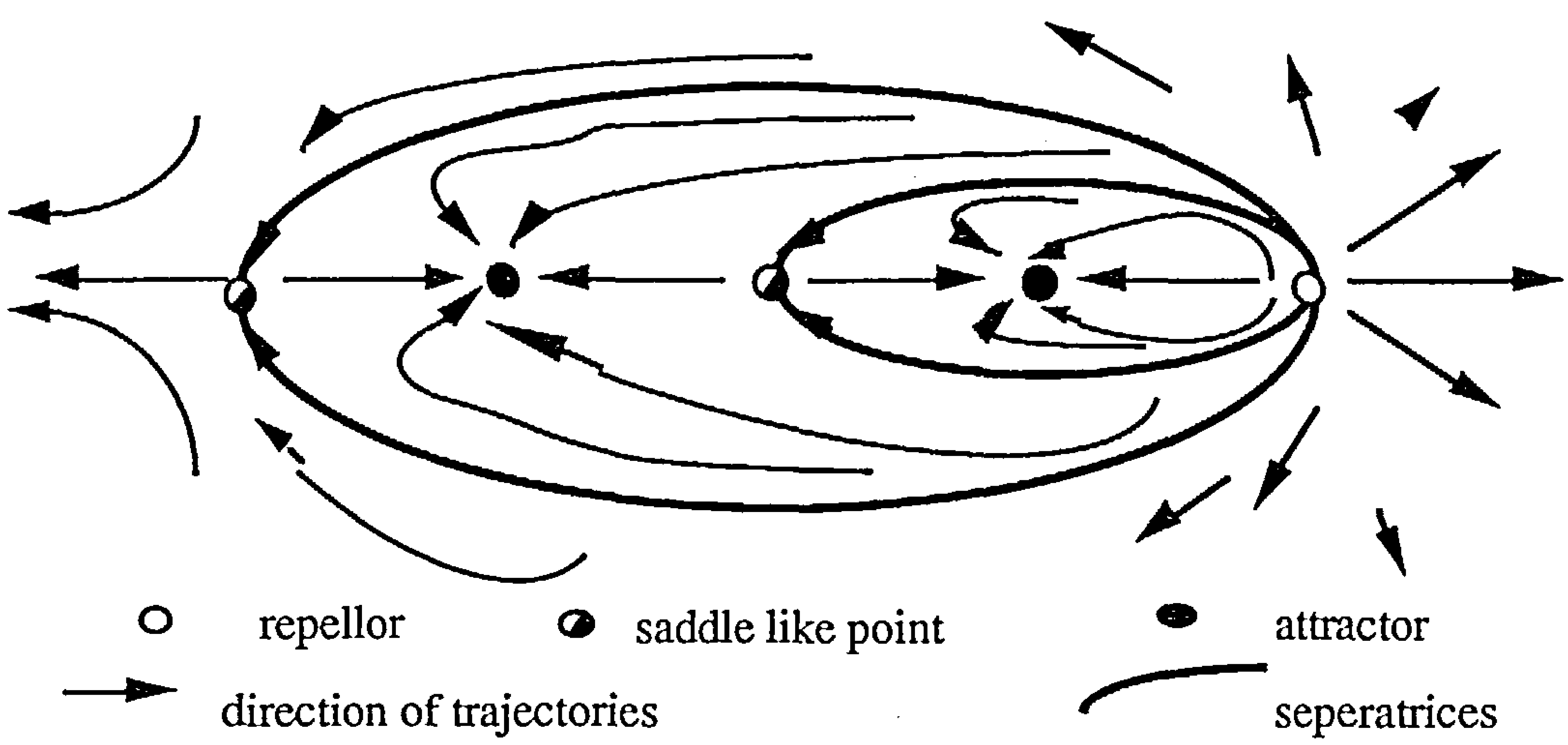


Figure 4.4.6 Phase Portrait with multiple attractors and repellors

Because the separatrices are some sets with (Lebesgue) measure zero in certain measure space (usually the measure space has the same dimension as that of the phase space), they can not be observed (A set with the measure zero is composed of those points representing some events that are "almost impossible"). "Almost" all the initial points of trajectories are situated in certain basins of attraction and hence attracted to certain attractors. The observed state of a dynamical system, that makes it to be recognized as a system, depends on which basin of attractor the system was initially in. For a system with more than one attractor, it can be observed at different state corresponding to different attractors. The attractors are decided by the nonlinear dynamics, the initial condition has nothing to do with how the components of the system is connected to each other dynamically but depends on some "historical reasons" or some random events. Initial conditions and the system's nonlinear inner dynamics together decide the state of the system as observed. If the system starts from different initial points in different basin in the phase space, it may settle down to different attractors ultimately. Therefore we can observe the system at different state or we can even perceive different systems (described by the same dynamical equations)! The change of initial conditions may cause the system to change from one state to another while the phase portrait of the system at that stage is unchanged. However, this is different from the situation when the change of the system's state is caused by the permanent, irreversible change of the environment. In this occasion, the system is driven from one attractor to another because the whole phase space is changed! This is what will be discussed in the following section.

4.4.2 Structural stability, organization and entity of systems

Recall the concept of structural stability given in section 4.3.1. It is about the preservation of trajectories of a dynamical system in the presence of certain "small" perturbations: if its trajectories are kept topologically invariant after being perturbed, the system is called "structurally stable".

In our discussion, a dynamical system is open, nonlinear and at a non-equilibrium state. It is supposed that the system can be described by an autonomous equation with a parameter vector, as in the form given in section 4.3, and here environmental impact is represented in a special form as the change of the parameter. The dynamical behaviour of a system is decided by the complex interrelations between the components, i.e the complex nonlinear inner dynamics of the system. As having been demonstrated, the long run behaviour of such a dynamical system is decided and described by attractors, repellors, and separatrices. To specify the system's state, we need also to specify the system's initial state.

An open system is affected by its environment. Systems evolve only to survive in a changing environment . Therefore, the state of an open dynamical system is no longer independent of the parameter μ. Hence we can have the following notions and definitions.

Denote

N^{μ}: the number of the attractors corresponding to the parameter μ;

T^{μ}: a set describing the type of these attractors, i.e, point attractor, periodic attractor, quasiperiodic attractor, and chaotic attractor, like T^{μ} = (point attractor, periodic attractor).

P^{μ}: the set of the separatrices describing the relative position of these attractors.

The relative position of the attractors describes how these attractors are distributed in the phase space. Between basins of any two attractors lies separatrices. The actual position of each individual attractor is not of crucial importance.

A triplet $\Omega^{\mu} = (N^{\mu} , T^{\mu} , P^{\mu})$ is obtained and will be used to describe the qualitative properties of the system, like structural stability, bifurcation and evolution.

The long term behaviour of a dynamical system is completely decided by its limit sets, and among them, attractors and separatrices alone prescribe the state of the system. The triplet Ω^{μ} includes all the attractors, and implicitly specifies the positions of separatrices and hence provides us with all the information about the possible states of the system which can be realized once the initial conditions are specified. The set of repellors has the Lebesgue measurement 0, and hence is negligible in considering the state of the system.

The separatrices cannot be ignored in specifying and describing the system's behaviour. As having been argued before, the state of a system is described by the attractor it settles down to. If a system has more than one attractor, the system can be observed in any of them. To specify which state the system will be at, we need to know not only the different choices of attractors, but also the initial conditions where the system has started. If the system starts from points in different basins of attraction, it will move to different attractors and hence can be observed at different macroscopic states. The separatrices separate different basins of attraction and hence play an important role in deciding the system's state.

The dynamics decides the set of attractors, that is to say it decides all possible states potentially possessed by a system. The initial conditions, i.e., where the system starts, help the system to realize one of these possibilities. For any system, it can only be observed at one stable state at one time (for the transient is very short compared with the life time of the system, we do not take into account the transient state of a system), this actualization makes the system to make a choice among its various potential states.

Here, the history and chance are intertwined. When the environment is relatively stable, the system must be at an attractor. Like in the phase portrait, moving along the trajectory by reversing the time, theoratically, the history of the system will become known to us: the system must have started from certain initial point in an attracting basin and was then attracted to the attractor in that basin through a dynamical process. When the environment changes, possible states of the system might change accordingly, and therefore attractors, as well as the basins of attraction, might be changed (as will be discussed soon). The state, i.e., the attractor, before this change, becomes the starting point in the new phase portrait which is partitioned to different basins of attraction by the new separatrices. For the upcoming bifurcation event, the system happens to be at a certain position at certain time because it was at that point as a result of previous history. From that point, the system will move to a state described by a new attractor, which may be at a different position in the previous phase portrait that now has been changed. When the system's microstate at the critical point is analysed, it will show that chance has entered the course through the microscopic fluctuations: the microscopic fluctuations have been existing all the time persistently testing the stability of the system. Only at the critical point, the system undergoes a radical change in its dynamics, specified by the change of the parameter, some random factors come to play a crucial role. This will be discussed in the following sections.

4.4.3 Structural stability and bifurcation

Definition 4.4.1 **(structural stability)** For a system S, its possible state is decided by Ω^{μ}. In a changing environment, if Ω^{μ} is kept unchanged for all the μ in the neighbourhood of μ_o, the system is called structurally stable at μ_o, otherwise the system loses its structural stability at that point.

It is apparent from the definition that a system is structurally stable means that all the types, numbers and relative position of the attractors potentially possessed by the system have not been changed.

Definition 4.4.2 **(bifurcation)** For a system S, if its structural stability is lost at the point μ_o, it is called that a bifurcation occurs at that point.

Definition 4.4.3 **(evolution)** For a system S, if it loses its structural stability in a changing environment, this process of change is called the evolution of the system. That is to say, during the evolution of system S, a bifurcation must have happened at certain critical point μ_o

Definition 4.4.4. **(increase of intra-level complexity)** For the various attractors, a partial order < is defined as

point attractor < periodic attractor < quasiperiodic attractor < chaotic attractor.

This partial order defines a direction for the increase of the intra-level complexity of evolving systems.

Evolutionary events only happen when the system loses its structural stability, i.e only when the bifurcation occurs. This is caused by the change of the environment. Systems evolution is defined in this way so that it can be distinguished from the process when the system changes its state from an attractor to another by changing its initial state from one basin of attraction to another. The change of initial condition can only happen when there is an outside designer deliberately doing that. We will concentrate on the process during which a system evolves under the influence of the change of environment. There is not any explicitly identifiable outside "organizers" responsible for this evolutionary change: systems evolution is essentially a self-organization process.

The essence of systems evolution is the qualitative change of systems structure, i.e., the loss of structural stability. When a system is structurally stable, the organization of the system, which is the necessary relations defining the system as an organized whole as such, is unchanged. Organization reflects the qualitative aspects of a system's structure while the structure of a system is the actual relations between components: structure is the "snap shot" of a system's organization at certain time. The definition of structure and organization is very important in dealing with living systems, like autopoietic systems [Varela, 1986].

It can be seen from these definitions that evolution does not necessarily mean the increase of complexity, even intra-level complexity. The sole motivation, if there ever is, of systems evolution, is for systems to survive as systems, as having been mentioned repeatedly. Systems may evolve within certain levels in the direction along which the intra-level complexity increases, but it is equally true that it may evolve to decrease the intra-level complexity, say from a quasiperiodic attractor to a periodic attractor. This agrees with Darwin's opinion that biological evolution often produced "degeneration" in design -- anatomical simplification in parasites, for example [Gould, 1975]. The following section mentions several routes through which systems may evolve, but systems evolution may equally happen in the opposite direction.

4.4.4 Bifurcation patterns and the route of becoming

The process of systems evolution may be manifested in different forms for different systems, but there are some patterns underlying those various systems which are content-free. Be it a physical system, or a biological system or a neural system, the self-organizing behaviour can be described by several prototypes which are *independent of the specific material characteristics of those systems. In our model, this* is termed and described as various bifurcation patterns to be specified below.

As stressed before, the standard dynamical equation for the evolving systems in discussion is an autonomous equation with parameter:

$$\frac{dx}{dt} = f(x,\mu) \tag{4.4.1}$$

(In practice, the original equation is usually more complicated than this. However, it can be deduced, in principle, to this form either by mathematical manipulation or simplification, or by both. Any further development of mathematical techniques will certainly improve our ability to deal with these problems of manipulation and simplification).

Based on this model, the following bifurcation patterns can be identified which are believed to describe the evolutionary processes in various systems, no matter what they are and at which spatial-temporal scale these processes are observed.

a) a point attractor to point attractors

If a system is at a steady state, i.e. at a dynamical equilibrium state, its behaviour is described by a point attractor, or we can simply say that the system is at an point attractor. This point attractor might be one of several attractors possessed by the system at that stage. When the environment changes, i.e., the parameter in the dynamical model changes, the system can be driven to another attractor. The old attractor which the system was previously at might split to two attractors and one of them will attract the system to settle down to. This process is called a bifurcation from one point attractor to two point attractors.

In the phase portrait, this process can be illustrated as follows (Figure 4.4.7):

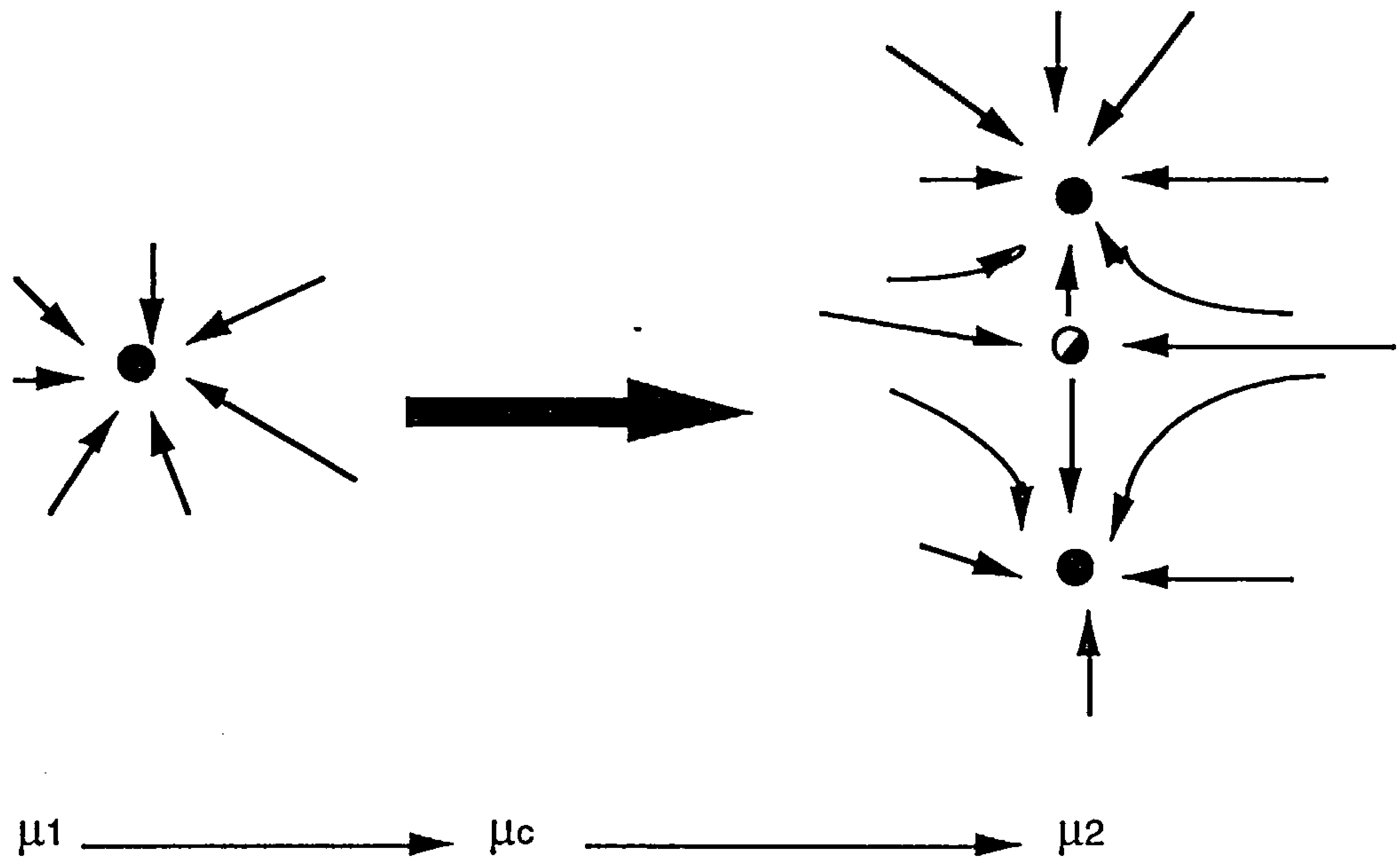


Figure 4.4.7 Bifurcation pattern: from a point attractor to point attractors

As discussed in Appendix 1, this kind of bifurcation is all what the elementary catastrophe theory is about [Thom, 1974; Zeeman, 1982]. The well known example is a gradient dynamical system described by the potential equation:

$$V(x) = x^4 + ax^2 + bx \qquad (4.4.2)$$

When $\Delta = 8a^3 + 27\, b^2 > 0$, the system has only one point attractor;

When $\Delta = 8a^3 + 27\, b^2 < 0$, the system has two point attractors representing two states with different potential levels.

When Δ changes from positive to negative, a bifurcation occurs through which one point attractor splits to two. This elementary catastrophe model can be applied to the analysis of the evolutionary behaviour of a wide range of systems. In a recent paper, Zeeman has applied it to investigate the dynamics of biological evolution [Zeeman, 1986].

Another form of the dynamical equation which is more general than the potential form, is:

$$\frac{dx}{dt} = f(x,\mu)$$
$$= x^3 - \mu x \qquad (4.4.3)$$

When the parameter μ changes from negative to positive, a similar bifurcation can be observed and it is called a pitchfork bifurcation.

c1) periodic attractor to periodic attractors

Like the bifurcation pattern of a point attractor to point attractors, a periodic attractor can bifurcate to two periodic attractors when a single real eigenvalue of the Jacobian matrix changes from 0 to positive [Guckenheimer et. al, 1983, Haken 1983b]. This is illustrated as (figure 4.4.9):

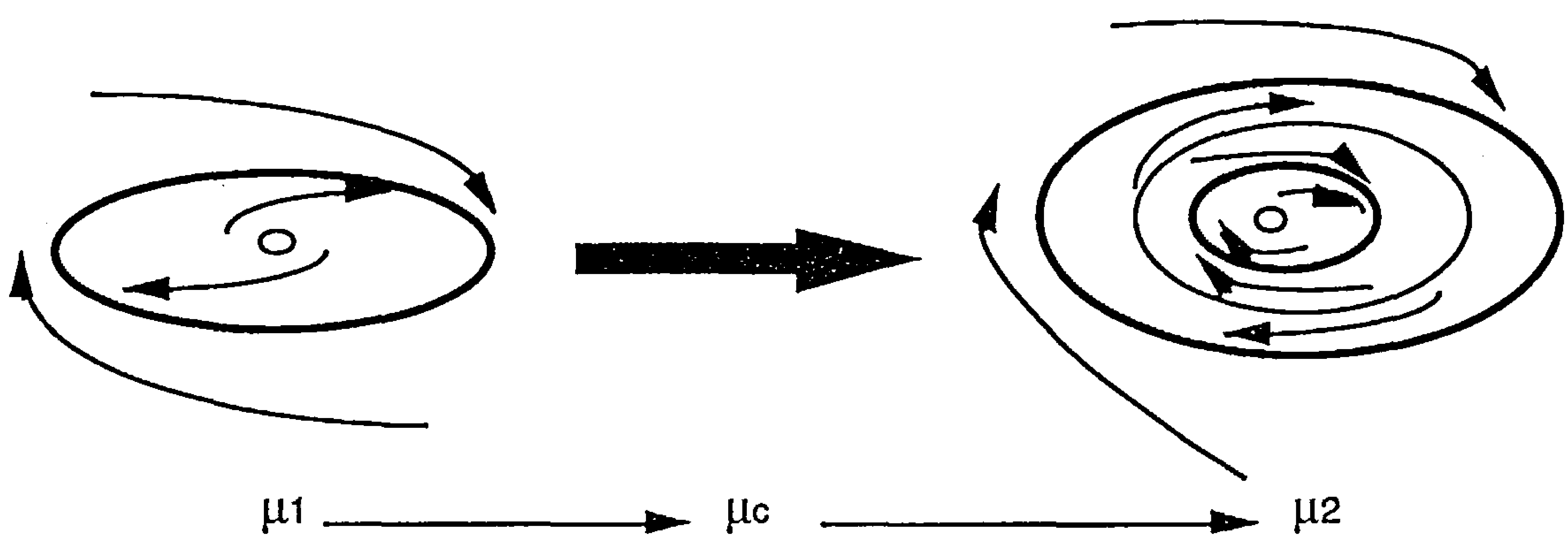


Figure 4.4.9 Bifurcation pattern: from a periodic attractor to periodic attractors

The new periodic attractors describe a new level of state which is different from the previous one: either it is at a new energy level, or at a level of different entropy or in other word at a new level of order. The system, again, has to choose between the two newly emerged periodic attractors and this could be decided by the combination of history, i.e., its previous position in the phase portrait, and chance, represented by the microscopic fluctuations within the system.

c2) a lower dimensional periodic attractor to a higher dimensional attractor

This happens when one or more variables change from constant to time varying as the parameter changes. An old periodic attractor with lower dimension hence gives birth to a periodic attractor with higher dimension. In this case the system is at a state more complicated than the previous one (figure 4.4.10).

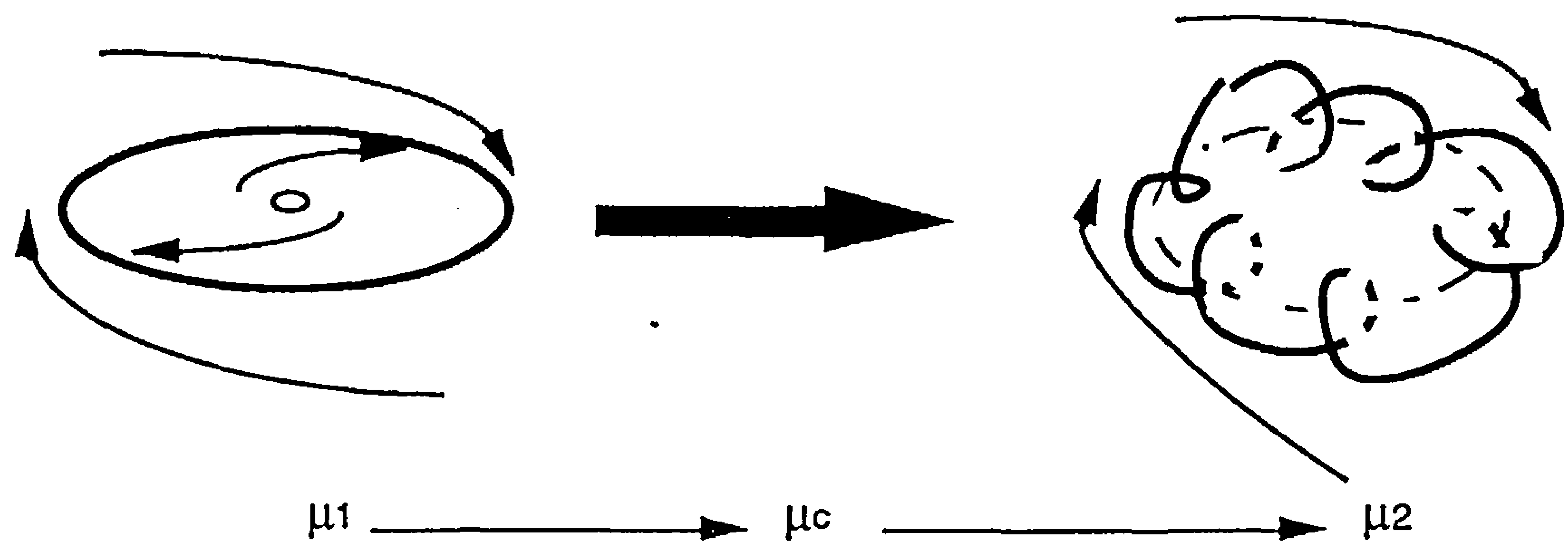


Figure 4.4.10 Bifurcation pattern: from a lower dimensional periodic attractor to a higher dimensional periodic attractor

c3) a periodic attractor to a periodic attractor with doubled period

A periodic attractor with period T can change to a new periodic attractor with period $2T$: the system will take as long as twice the time to return to the same state. This is what we call a period doubling bifurcation (figure 4.4.11).

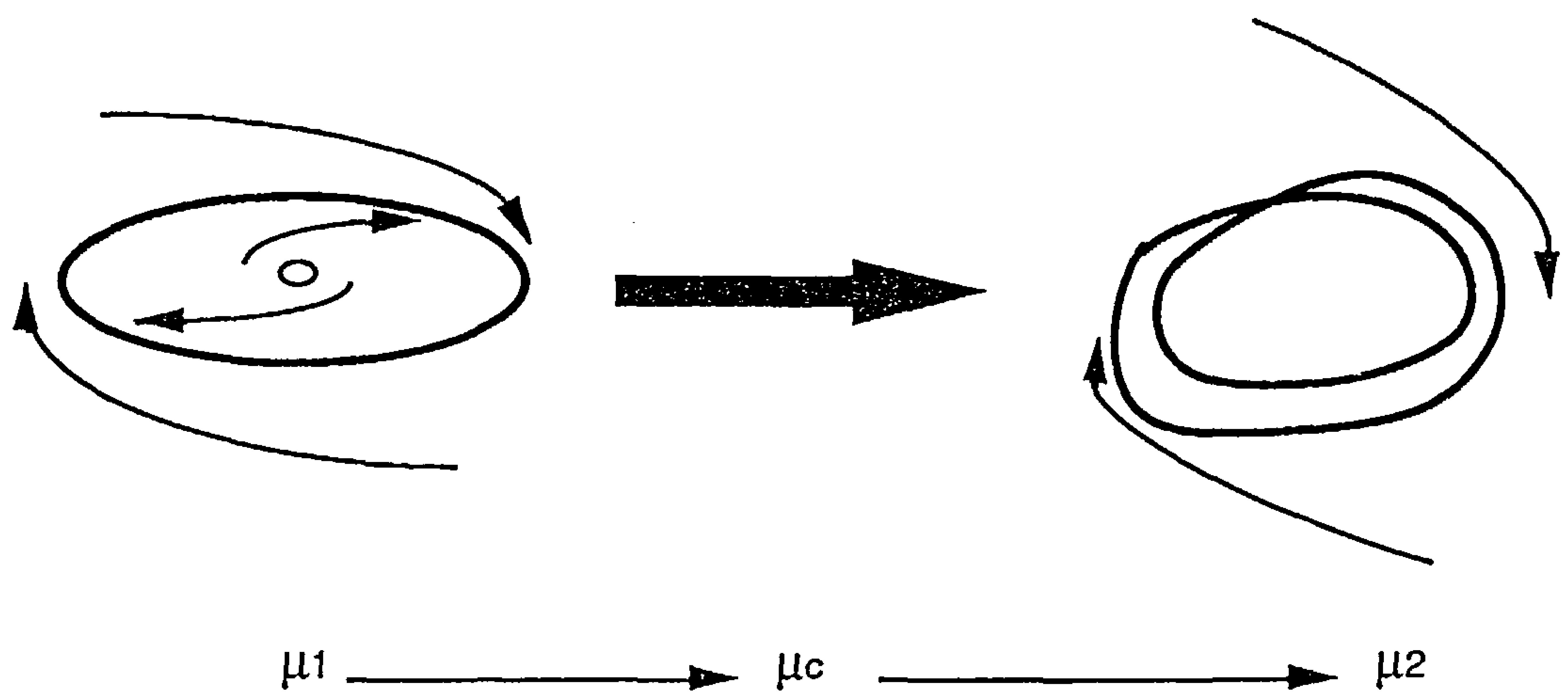


Figure 4.4.11 Bifurcation pattern: period-doubling

The eigenvalues analysis cannot tell us much about this periodic doubling bifurcation. Although it is a much talked topic in nonlinear dynamics, little is yet known to us from rigourous mathematical analysis. Periodic doubling bifurcation was first brought to our attention through the study of one dimensional iteration map, which can generate a two dimensional flow through suspension, and it is closely connected to the emergence of chaotic attractor [Feigenbaum, 1983; Eckmann, 1981]. In later discussion about other bifurcations we shall discuss this very important process through which the chaotic attractor can arise.

d) a periodic attractor to a quasiperiodic attractor;

When several of the total variables oscillate along one direction with certain period, others may change periodically along other directions. These oscillations are called the subharmonic oscillations. A periodic attractor can be a combination of different subharmonic oscillation on condition that all ratios of these different oscillation are rational numbers. When the parameter changes, some of these rational ratios may change to a irrational number and the dynamical behaviour of the system hence may change dramatically: a period attractor is replaced by a quasiperiodic attractor. This is another bifurcation pattern often encountered in electronic circuits and other fields (figure 4.4.12).

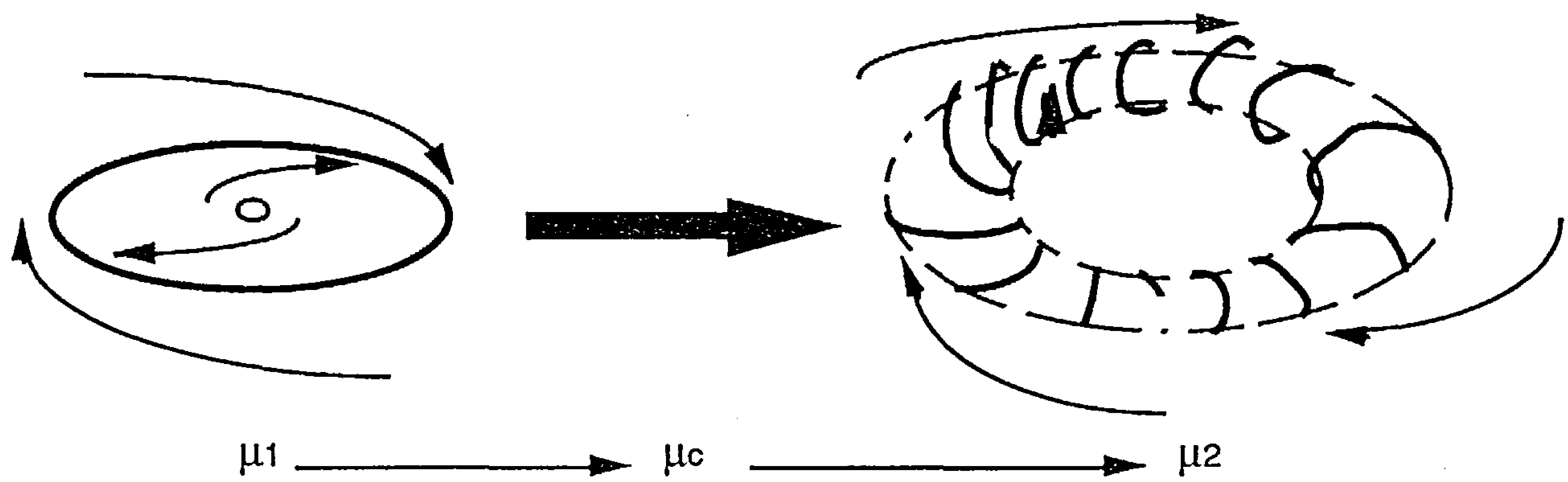


Figure 4.4.12 Bifurcation pattern: from a periodic attractor to a quasi-periodic attractor

e) a periodic attractor to a chaotic attractor;

Compared with the above mentioned bifurcation patterns, the bifurcation resulting in the emergence of a chaotic attractor is a complex process rather than a single step event. It is usually composed of consecutive bifurcation events and this leads to a chaotic attractor. One of these complex consecutive bifurcations is the periodic-doubling process through which a periodic attractor will give birth to a chaotic attractor (figure 4.4.13).

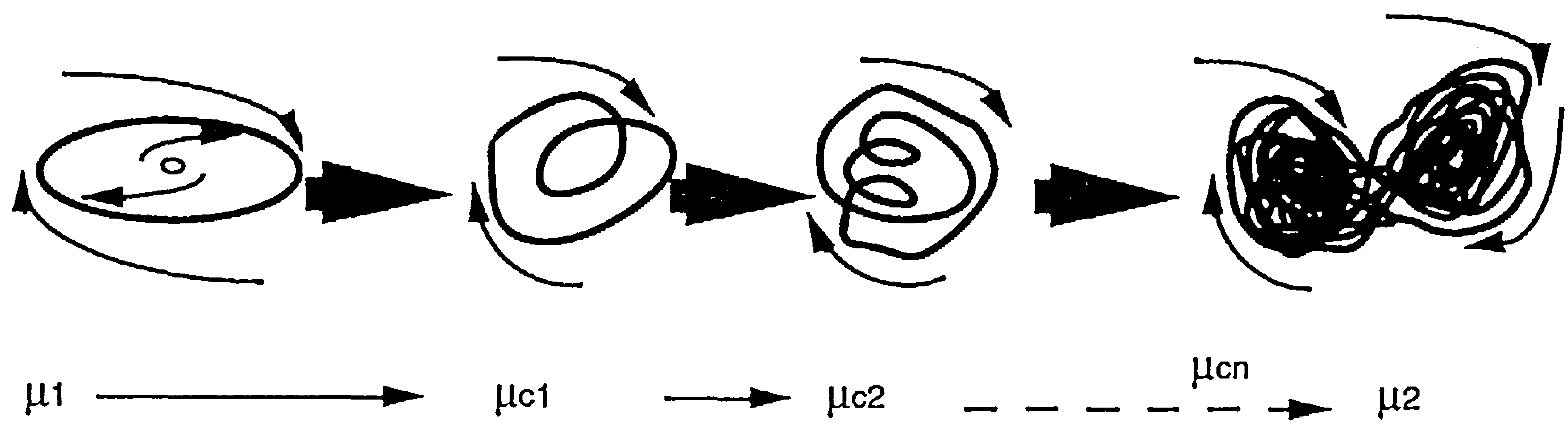


Figure 4.4.13 Bifurcation pattern: period-doubling leading to a chaotic attractor

The study of period-doubling processes is responsible for kindling the early research for the universality in studying chaos. The pioneering work in this field is Feigenbaum's discovery of a universal constant associated with the onset of chaos through periodic doubling in one dimensional iteration map, or one dimensional discrete dynamical system, and this constant is known as the Feigenbaum constant. It is defined as :

$$\delta = \lim_{n\to\infty} \delta_n = \lim_{n\to\infty} \frac{\mu_{n+1}-\mu_n}{\mu_{n+2}-\mu_n} = 4.6692016...$$

where μ_n is the critical value of the parameter at which the period of a system's periodic attractor is doubled the nth time [Feigenbaum, 1983]. This constant δ describes how fast the bifurcation happens. The accumulating point λ_∞ is the critical point at which a periodic attractor appears. It is a universal constant and it is independent of the particulars of any dynamical systems: no matter what the system is, if it has the qualitative properties that enables it to undergo the process of period-doubling, the quantitative properties of this bifurcation process is determined, by this constant.

It is believed that this is a universal constant governing any bifurcation process leading to chaos. Experimental studies have supported this result and some theoretical work has also been done to prove that claim [Lanford, 1982]. As the study of chaotic attractors still lacks general laws and rigourous theoretical results, this constant, although first imported from experimental study, is one of a few known properties about chaos.

f) a quasiperiodic attractor to a chaotic attractor;

Period doubling bifurcation is one of the often encountered and well known bifurcation patterns which leads to chaos. Chaos can also come into existence through another bifurcation event, i.e bifurcation from a quasiperiodic attractor to a chaotic attractor (figure 4.4.14).

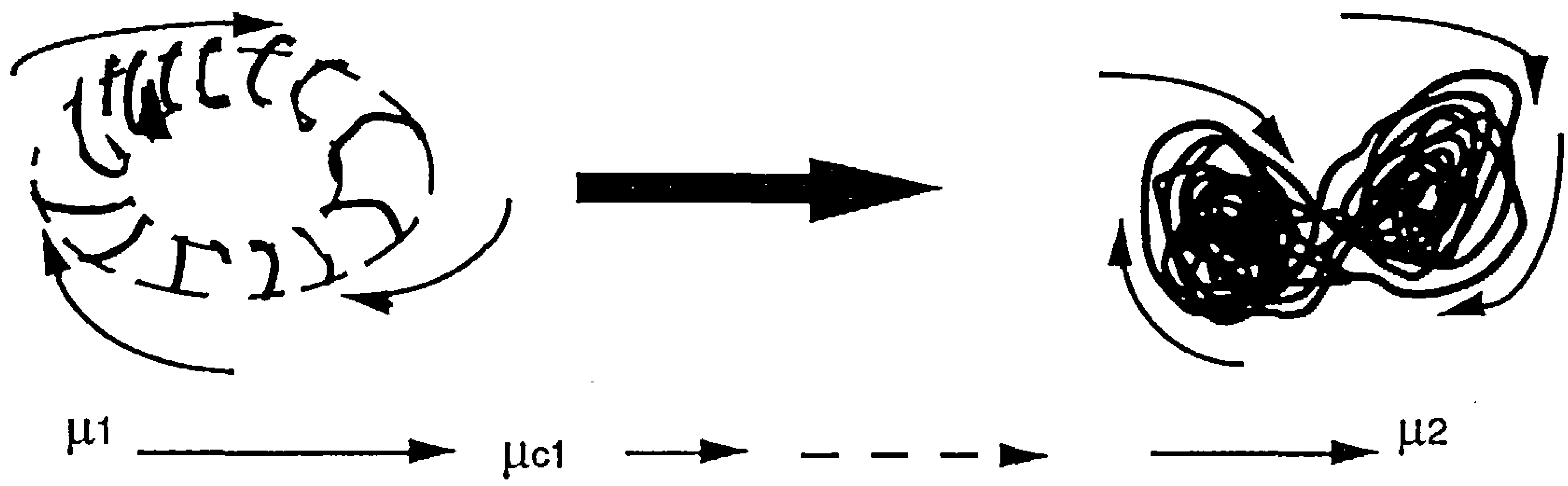


Figure 4.4.14 Bifurcation pattern: from a quasi-periodic attractor to a chaotic attractor

A quasiperiodic attractor is characterised by periods of subharmonic oscillations known as the fundamental frequencies. When the parameter changes, the number of fundamental frequencies increase and the dynamical behaviour of the system becomes more and more complicated. In the phase space, a toroidal orbit on a torus surface with lower dimension is replaced by a surface with higher dimension and this process continues as the parameter changes and ultimately leads to a chaotic attractor. This bifurcation is similar to but different from the period doubling bifurcation, and it is an alternative way that chaotic attractor can emerge [Eckmann et. al, 1985].

In fluid dynamics, these two bifurcation patterns, i.e., period-doubling and quasiperiodic bifurcation leading to chaotic attractor, represent two different routes through which the turbulence can appear. The route of period doubling leading to chaos was proposed by Ruelle and Taken as contrast to the route of increasing the number of fundamental frequencies in quasiperiodic attractor to create turbulence known as Hopf-Landau route [Eckmann, 1981].

According to our formal model, the state of a dynamical system can be represented by various attractors and its complex forms of behaviour are characterised by these complex attractors. When the environment changes, the change of state and behaviour of the system can be described as a process that one type of attractor is changed to another. This process is termed as systems evolution. The definition is consistent with our understanding that a system evolves to maintain itself as a system in a changing environment. Evolution does not always mean the increase of complexity. The evolution routes showed are more appropriate to describe the intra-level evolution than inter-level evolution, the later is often related to systems genesis. However, these patterns can also be employed to describe the possible evolution routes in the inter-level case but those attractors are found at different levels of description.

4.5 Emergent attractors and evolving behaviour

4.5.1 Emergent attractors, meta-attractors and hierarchical structure

Systems evolution always happens at certain a space-time level and the change is about the state of the system at the macroscopic level. It has been widely accepted and, hence taken for granted, that any system is composed of some interacting subsystems at a lower level of spatial-temporal scale, and at the same time the system itself is a composing element, or a subsystem of a "supersystem" at a higher level of scale. The state of the system is the result of the interactions between its subsystems, and at the same time it is constrained by the larger system to which it is only a component. Bearing in mind this point, any discussions about systems evolution must be at certain spatial-temporal scale: we talk about a system as an identifiable organized whole which is the element of "selection" in further evolution of a larger system.

Viewing that a system is composed of interacting subsystems at microscopic level, an attractor is the global result of local interactions between subsystems. The properties of the attractor are not the simple summation of the properties of its subsystems, instead they are the characteristics of the interrelations of those subsystems. This explains why "the whole is more than just the sum of its parts". The properties of a attractor at a macroscopic level is usually known as the emergent property of systems. The attractor which is employed to describe the state of the system is hence called an "emergent attractor".

Compared with the state of its subsystems, emergent attractors explain why a system has some novel, global, dynamical space-time behaviour which is inaccessible to, not locatable in, and not reducible to the individual or summative behaviour of the separate subsystems. These four different emergent attractors, i.e., point attractor, periodic attractor, quasiperiodic attractor and chaotic attractor, are employed to describe the emergent behaviour of the systems compared with its subsystems: each of them characterise how a group of subsystems forms an organized whole to be observed at a macroscopic level to interact with the environment through the matter /energy/ information exchange and how the organized whole subsumes the state and behaviour of the individual subsystems. Emergent attractors best serve to describe the duality of parts and whole.

According to the supersystem -system -subsystem hierarchy, the emergent attractor at certain level is the result of several interacting attractors at a lower level which describe the emergent behaviour of those subsystems which in turn are composed of sub-components at a further lower level. At the mean time, the attractor, representing the system as an organized whole, is interacting with some other attractors at the same level and this will leads to an emergent attractor which describes a supersystem as an organized entity at a higher level. This is a no-ending process along either directions (some others will argue that for the physical systems, the upper limit is the biggest attractor, the universe, and the lower limit is the attractor representing the elementary particles at the most elementary level!).

It is obvious that the hierarchy of systems can be described by the hierarchy of attractors. Each attractor at certain level describes the emergent properties of a system which is composed of several subattractors in a lower level. This attractor is an composing element for attractors at a higher level in the hierarchy. The best example of this hierarchical structure is the biological world which consists of physical particles, chemical molecules, biological molecules, biological cells, tissues, organs, biological entities (animals, plants), biological niches, ecological systems etc. At each level, the state of a system can be represented by any of the four distinct attractors. This can be illustrated as (figure 4.5.1):

↑
supra-supra-attractors
↑
supra-attractor
↑
attractor
↑
sub-attractor
↑
sub-sub-attractor

Figure 4.5.1 The hierarchical structure of emergent attractors

4.5.2 Determinism and chance and the varieties of systems

It is assumed that each complex system, along with the observed state which is its actual existence at certain moment and in a given environment, also has its potential states that determine what the given system might have become under all different possible conditions and what it can never become at all. All these possible states, actual or potential, are specified by the attractors the system may possess. One can only observe an actualised materialization of the system, an actualized attractor; its other attractors can only be described by theory, and they can be judged only subject to its possible actualization or its influence on the actualized attractor. Generally, this is referred as the multiple-stable states of systems. These possible multiple-states may not be realized by one system through systems evolution, but other similar systems with similar inner dynamics at slightly different conditions may have reached other potential states. This explains why some systems have a particular relations with others and why this world is essentially pluralistic. A man-made mechanical system may have multiple-stable states, where "multiple-stable state" has special technical meanings: it can reach all of them through human manipulation, but not by self-organization.

This dual reality of complex system is a consequence of its nonlinear inner dynamics. It is only nonlinear systems that can have a certain set of attractors and repellors. Only one such attractor at any moment of time is actually realized, the rest of them that are alternatives with respect to the former state and they exist only potentially.

Following our previous discussion about the change of a system's state, i.e., the jump from one attractor to another, caused by the change of initial condition and by the change of environment, we can explain what actually happens in the process of systems evolution and how determinism and chance come to jointly determine the actual state of a system at certain moment.

For a certain environment, the observed state of a system, which is open, nonlinear, non-equilibrium and with persisting microscopic fluctuations, is described by one of several potentially possible attractors which are described by Ω^{μ} and the phase portrait. When the environment changes, the system is affected and its state might be changed quantitatively, i.e. the actual position of the attractor in the space might be shifted sightly but the phase portrait remains unchanged. In this case, the system has not lost its structural stability and is structurally stable. It may also be changeed qualitatively: the change of environment causes the change of the phase portrait so that the system has lost its structural stability and has had to jump from the old attractor to a new attractor. Again, this new attractor is one of the several potential attractors possessed by the system under the constraint of the new environment. Deterministic factors, like the system's inner dynamics which reflects the interactions of the system's components, and the coupling between the system and its environment, decide the emergence of all possible attractors and their distributions. At the critical point when the system loses its structural stability, microscopic fluctuations of the systems components become so strong, due to the nonlinearity manifested as positive feedback, that they lead to the destruction of the old attractor and hence the organization of the system. Those potentially possible attractors represent all the possible new states which might be reached by the system and they correspond to all the possible results when those microscopic fluctuations are amplified to dominate the system's behaviour. For a system at a stable state, all these fluctuations are constrained by the system's organization as·a whole and the macroscopic state reflects the compromise of the interactions between the system's components. In a stable state, the impact of these microscopic fluctuations has been averaged out. At this critical point, the average description is no longer valid and the fluctuations are amplified to dominate the system's behaviour. Allen has argued that the biological evolution is not an optimization process, but a process of "learning through ignorance" by which the fluctuations are subject to the selection of the environment which will decide, together the system's inner dynamics, which microscopic fluctuation will be chosen to prescribe the emerging new attractor [Allen, 1989].

At the critical point, minor influences on the system are sufficient to initiate a leap from an initial attractor to a new attractor which is one of those possible attractors decided by the inner dynamics, microscopic fluctuations, and environment impact as we have argued before. The new attractor represents a more differentiated order and organization on the occasions when the system evolves to survive in a more complicated environment. In this circumstances, it is absolutely impossible to predict at the bifurcation point to which attractor the system will settle down and in which direction the system's development will continue. In this situation, the complex behaviour of the system that functions under the conditions of time irreversibility, become non-predeterminitive -- there are not much rules to make it possible to determine precisely or with some degree of probability the system's next state by its inner state and the numerous external influences.

In our model, all possible attractors and their distributions are predictable, but they are deprived of some of the details about the system's actual behaviour: attractors describe the rough patterns of the system. As for example, in the Benard Cell experiment, we can predict from the dynamical model that the system will change from a point attractor to an periodic attractor when the parameter changes to certain critical point. That implies that the system will change from a homogeneous state, at which the heat is transferred from the heated bottom to the cooled top through conduction, to the highly organized patterns, the hexagonal patterns, at which the heat being transferred by convection. However we can never specify in which direction the liquid will be rolling: towards inside or outside.

To understand the details of the evolutionary process, one needs to gather as much information about the particulars of that system as possible, but the above outlined qualitative model can always help us to know how the system will possibly change when the environment changes.

In the following bifurcation diagram (figure 4.5.2), the system is governed by deterministic laws between two bifurcation points. At the bifurcation points, the event is decided by pure chance, although we may have known all the possible states.

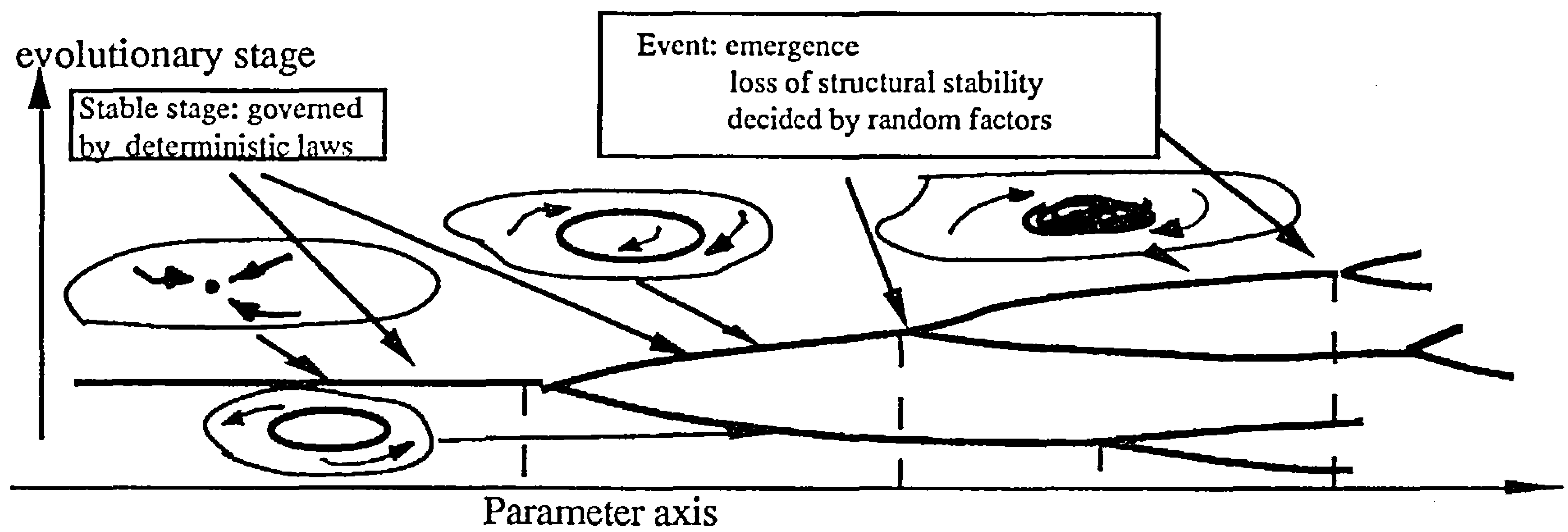


Figure 4.5.2 Big Picture of bifurcation cascade

For a particular system, its observed state is only the actualization of one of many potential attractors the system possesses at that stage. Due to the irreversibility of time, it has not got the chance to revisit all those possible attractors. However, there might be many other systems which have similar structures and organizations and were previously at similar environment when the environment started to change. Because the unpredictable, and uncontrolable factor of chances have played crucial roles in the evolution of these systems, some of them have to settle down to one of the attractors, some others to another. That makes that similar systems evolve to different systems after the bifurcation. For any particular system, its all possible new states arepredetermined, but for all those systems in the similar situation, it looks as though "*God does play dices*". Various systems bifurcate at the same point to different attractors and they will move along different bifurcation branches and arrive at different bifurcation points the next time. Those systems at different branches in the bifurcation have different potential attractors at different bifurcation points, but for systems which move along the same bifurcation branch, they face the same possibilities for their future state at the same time: again they will bifurcate at that point under the influences of chances: **God is playing dice, again!**

The above discussion of systems evolution, especially that about conditions, processes, the relation between chance and necessity etc., is suitable for any evolving systems at any evolution stage. However, the routes of evolution, defined in parallel with the bifurcation patterns found in DST, are more appropriate for describing the intra-level evolution than inter-level evolution. During intra-level evolution, the number of components of the system is the same and the inner dynamics of the system can be described by the same family of dynamical equations. Systems evolution can be adequately described by the bifurcation analysis as described in DST. In the situation of inter-level evolution, new components are added to or taken from the system. The dynamical equations which described the previous behaviour of that system no longer hold and new equations must be found. This inter-level evolution is characterised by systems genesis, the genesis of system at a new space-time level. Therefore, the evolution continuum of systems can be illustrated by the following diagram (Figure 4.5.3).

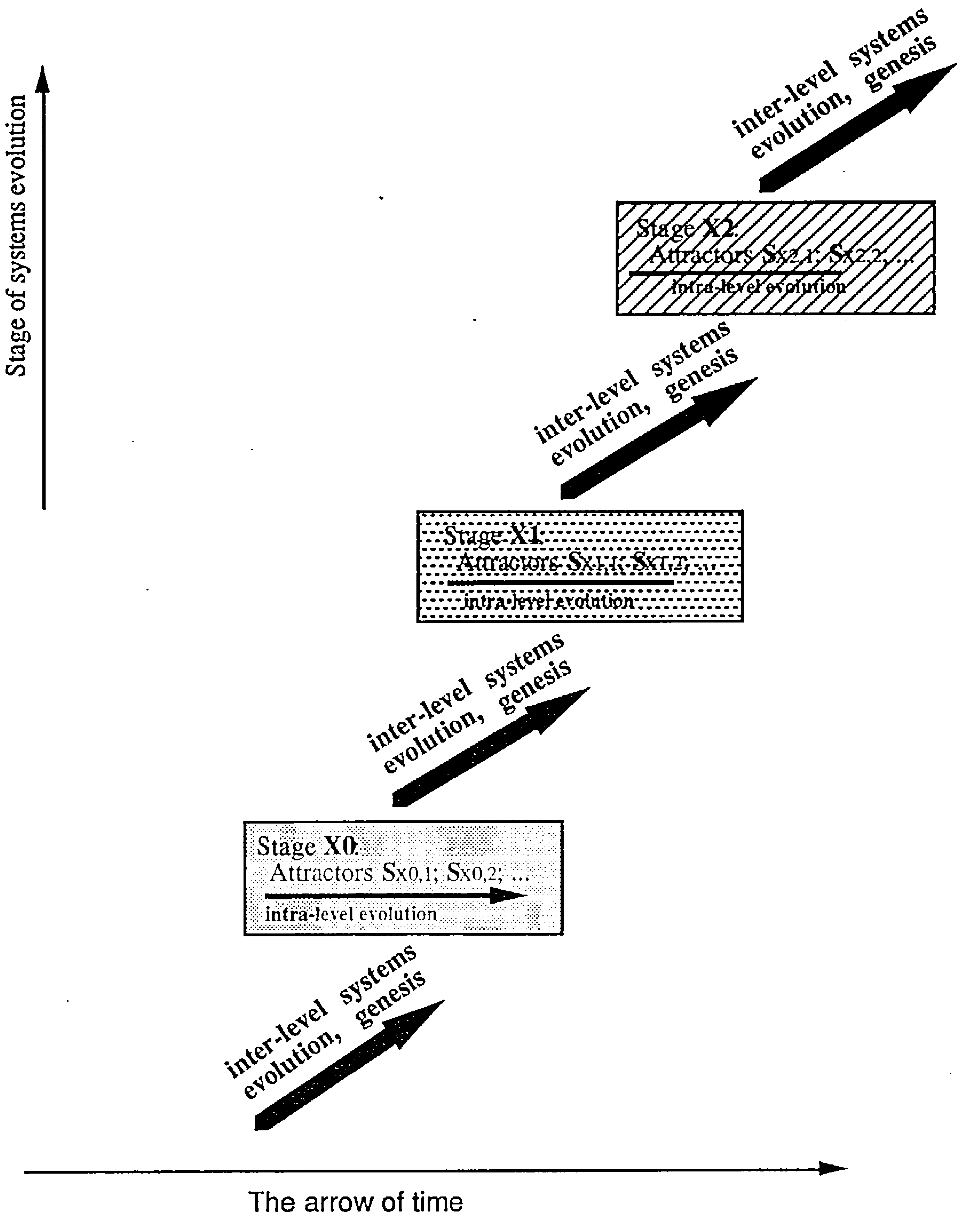


Figure 4.5.3 The global picture of systems evolution

Everything Flows and Nothing Stays. Herarclitus

Chapter 5 Evolving Systems I: Old Examples Revisited

5.1 Introduction

The universe, as we see it, is organized in a hierarchical way, or to be more accurate, it can be perceived as being organized in a hierarchical way. At the fundamental level, there are various "fundamental" particles like protons, neutrons, electrons etc..(there are still arguments about whether they are the most fundamental ones or they are formed by some even smaller particles). From these particles, atoms are formed. Atoms are the basic units which are kept invariant during chemical reactions. Molecules are composed of atoms, either in the form of chemical compounds (different types of atoms held together by chemical forces) or formed by the same type of atoms, like oxygen molecules (O_2). The aggregation of some molecules, called organic molecules, mainly composed of **C**, **N**, **H** and **O**, can form a special type of molecules called organic molecules. Biological macromolecules are formed by organic molecules and they can be further "assembled" to form further complicated biological entities which are usually expressed the form of life. These biological entities, or we can call them biological systems, range from the relatively simple forms, like amoebae and some other forms of bacterium, to the most complicated forms like our human beings. A group of many biological systems, like animals, together with the environment can form a larger system: a biological niche composed of some animals, plants, and geographic territories; a sociological community consisting a group of people, social-political structure, and man-made products like houses, power stations. Such a system in a very large scale are called either a ecological system, or a sociological systems. The planet earth we are residing on is such a big ecological system, a "global village". Look beyond our earth, we can distinguish the solar system composed of nine big planets and among them earth is one with moderate size. Further to that, we have identified the galaxy, super-galaxy, and finally, the universe as a grand whole: it is composed of every physical entities which we, human beings, think or feel that they are existing as objective entities.

We are living in such a hierarchically organized world and to understand it, of which we are only a small but an important part, is one of our dreams. The ancient eastern mysticism regarded the universe as a dynamic whole: there is no past and future, no human and non-human, no material and spirit and there is only the eternal dynamic unit. This eternal whole is called Tao by Taoism, or similar names in Zen and Buddhism. The ancient Greeks shared the same attitude towards the universe. There is a famous saying by Herarclitus, an ancient Greek contemporary of Lao Tzu in China, that says "Everything flows and nothing stays". It implies that, for a long time, we human beings have realized that we are living in a world that everything changes with the passage of time. To view how the world changes differs for different people at different times. Typically, the ancient eastern mystic thought that the world was changing in a cyclic fashion: the same event always comes back over and over again while the eternal whole is maintained. Modern science, originated in the West only three centuries ago, can help us to look at this problem from a "scientific" point of view. Two voices echo two radically different view deduced from this scientific rationale. Biologists, especially evolutionary biologists, have showed how the biological species have changed from some primitive forms to the present complex ones. There is a progressive point of view implied in that theory. On the other hand, thermodynamics has claimed that the universe is moving to its doomed destiny: thermal death with the entropy at its maximum. Both claims, the progressive changes and the thermodynamics' pessimistic point of view, have been supported by solid empirical evidences and scientific logics and it seems that both of them are right in general. How can these two theories be conciliated, which are obviously contradicting to each other, to explain the realities that we as human beings can feel and face everyday?

In the first three chapters, a brief introduction has been given to current scientific research devoted to the explaining of the changing world. It has been argued throughout that the study of systems evolution can helps us to pursue this goal. A recognized entity is described as a system and its behaviour can be analysed by looking into how its components are held together through interactions among them to form a whole. A conceptual framework has been outlined to describe the conditions and the general course of such evolution process. Chapter 4 has put forward a formal model of evolving systems based on mathematical dynamical systems theory. The basic ideas is that an open, nonlinear, non-equilibrium system with microscopic fluctuations can change its structure and behaviour when its environment changes. The state of a dynamical system can be described by one of the four types of fundamental attractors and the change of a system is hence described by the process that the system jumps abruptly from one attractor to another triggered by the environmental change. Such evolution processes happen at all levels in the hierarchical structure of the universe. An attractor at certain level is an emergent attractor in the sense that it is the result of the interactions between sub-attractors at one level below in this hierarchy. It has been mentioned in previous chapters that evolutionary processes, in terms of systems evolution, have been identified in various systems at different levels, ranging from physical systems like a hydrodynamic systems, through biological systems, like a Hypercycle, to the socio-economic systems. In this chapter these systems will be analysed according to the conceptual framework and the formal model so far developed.

Some well known examples in hydrodynamic systems, chemical systems, biological systems, ecological systems, sociological systems and cellular automata will be used to show that the dynamical behaviour of these systems can be described in a unified form and analysed by using the formal model based on dynamical systems theory. Some results are quoted directly from those particular fields and are just re-interprated here by using the terminologies, conceptual framework and formal model developed in this study. In the next chapter about neural networks, a neural network, natural or artificial, is treated as an adaptive and evolving system. Some novel results in studying a simple cellular neural network (CNN) and the coupled Wilson-Cowan nets are reported and implications for the study of neural networks in general are explored. Further studies are suggested.

The following sections will be arranged in this way: at first, a brief description is given to the system in focus as it is usually described, the characteristics of the system and its environment will be analysed: openness, non-equilibrium state, nonlinear relations between systems' components, microscopic fluctuations within the system, the interaction between systems and their environment and the possible changes of the environment. Followed by such descriptions, the dynamical behaviour of systems are described by the formal model developed in chapter 4. Such formal descriptions will be specified according to the particular inner dynamics of particular systems. Based on the detailed descriptions of systems' inner dynamics, results in those particular fields are quoted and theorems and techniques in nonlinear dynamics analysis are employed to describe the state of the systems at different evolving stages. The big picture of the evolving cascade of the systems will be drawn, if appreciate, to show the global behaviour of systems under different conditions.

5.2 Evolving Physical systems

Discoveries that many very simple physical systems can exhibit amazingly complex spatial and/or temporal behaviour have been the driving force and the focus of attentions of the currently blooming study of the science of complexity [Stein, 1989; 1991; Jen, 1990]. These systems are usually open, nonlinear, non-equilibrium and situated in a changing environment and their dynamical behaviour can be discussed within the framework of systems evolution. Some systems, like a laser system and some other electronic devices, are set up purposefully by human beings, but our interest here is not the devices themselves, but the behaviour of the systems. For such man-made systems, mainly mechanical or electric/ electronic devices, the evolution of temporal behaviour is of interest to us. For other physical systems, like Benard hydrodynamic system, the evolving behaviour involves both spatial and temporal aspects. Mathematical dynamical systems theory (DST) based on flows and ordinary differential equations (ODE) is more appreciate in analysing the temporal behaviour, but the spatio-temporal behaviour together can still be discussed within the scheme of systems evolution.

Among various evolving physical systems, two examples are chosen for analysis from the systems evolution point view. One is the often-quoted, much studied but still not yet fully understood Rayleigh-Benard hydrodynamic system [Haken, 1983a]. The other is a laser system.

5.2.1 Benard Cells

The set-up of Rayleigh-Benard convection experiment is very simple: a thin (several millimetre thick) layer of viscous liquid is placed in a flatted pan (more than 10 centimetres wide) and the upper side and lower side are contacted with cooler and hotter source respectively. When the system is heated from blow while the above is kept exploded to cooler source, the dynamical behaviour of the system will change as the temperature varies and a variety of spatio-temporal behaviour can be observed [Haken 1983a; Berge et. al, 1984]. The system can be illustrated as follows (figure 5.2.1):

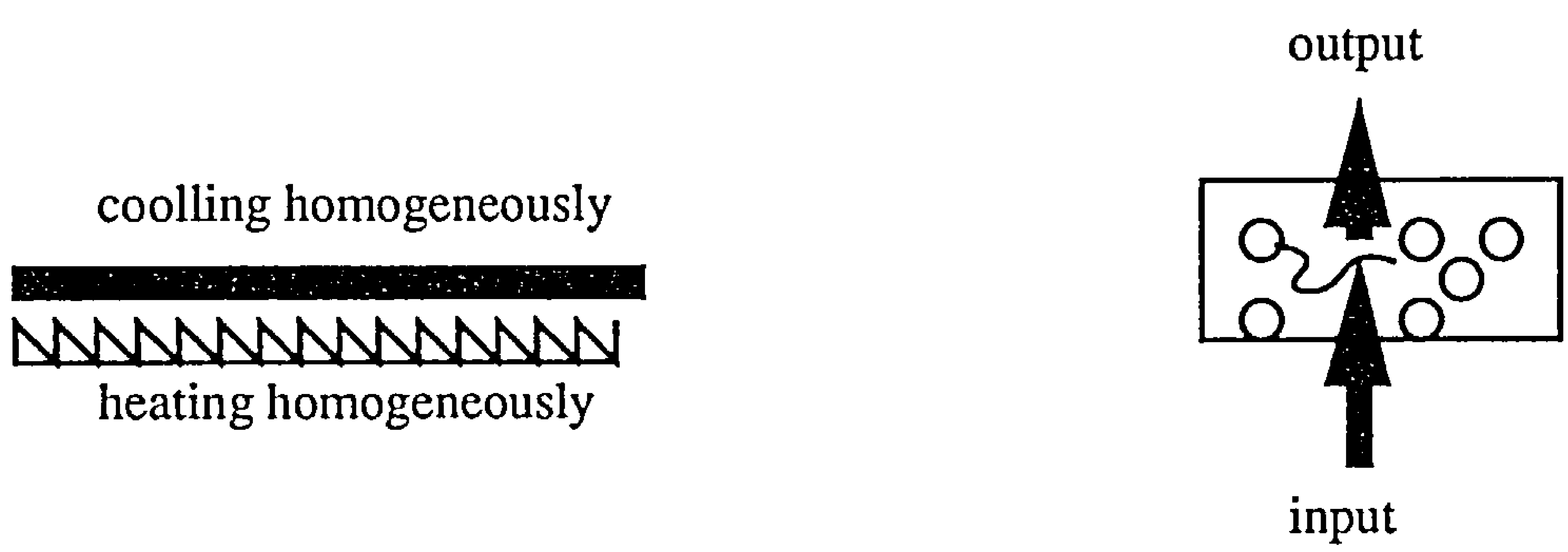


Figure 5.2.1 Benard cells experiment

<u>Openness</u>: Energy is imported and exported through heating and cooling;

Non-equilibrium: The system is apparently far from thermodynamic equilibrium due to the temperature gradient within the system and energy exchange caused by the gradient;

Nonlinearity: The interaction between the liquid molecules is governed by complex hydrodynamic rules and this has not been fully revealed.

Fluctuations: Liquid molecules oscillate all the time and there are also environmental noise like the inhomogeneous heating, the inhomogeneity of the liquid and the oscillation of the experimental devices.

Environment: Although everything, except the layer of liquid itself, can be counted as the environment of the system, the direct affect of the environment comes from the heating and cooling source.The change of environment is represented as the change of the temperature gradient imposed on the system.

According to the very complex mechanism in fluid dynamics, the inner dynamics of this system can be described as:

$$\begin{aligned} &\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = g\varepsilon\Delta T\delta_{i3} - \frac{1}{\rho}\frac{\partial P}{\partial x_i} + \upsilon\nabla^2 u_i \\ &\frac{\partial T}{\partial t} + u_j\frac{\partial T}{\partial x_j} = \kappa\nabla^2 T \\ &\frac{\partial u_i}{\partial x_i} = 0 \end{aligned} \tag{5.2.1}$$

Where:

x_i: ith spatial coordinate; u_i: ith velocity field component; g: gravitational constant;

ε: coefficient of thermal expansion; $\Delta T/H$: imposed temperature gradient; ρ:fluid density;

T: fluid temperature field; P: fluid pressure field; υ: viscosity;

κ: coefficient of thermal conduction; t: time.

The dynamical behaviour of the system is classified as follows:

(1) At low temperature level, heat is transported by conduction. Macroscopically the system is in an homogeneous state;

(2) When the temperature gradient exceeds certain critical point, orderly roll patterns appear (the well known Rayleigh-Benard cells) and heat is transported by convection;

(3) Further increasing the temperature gradient, the fluid motion becomes chaotic [Gilmore, 1981; Haken 1983a, Berge et. al, 1984].

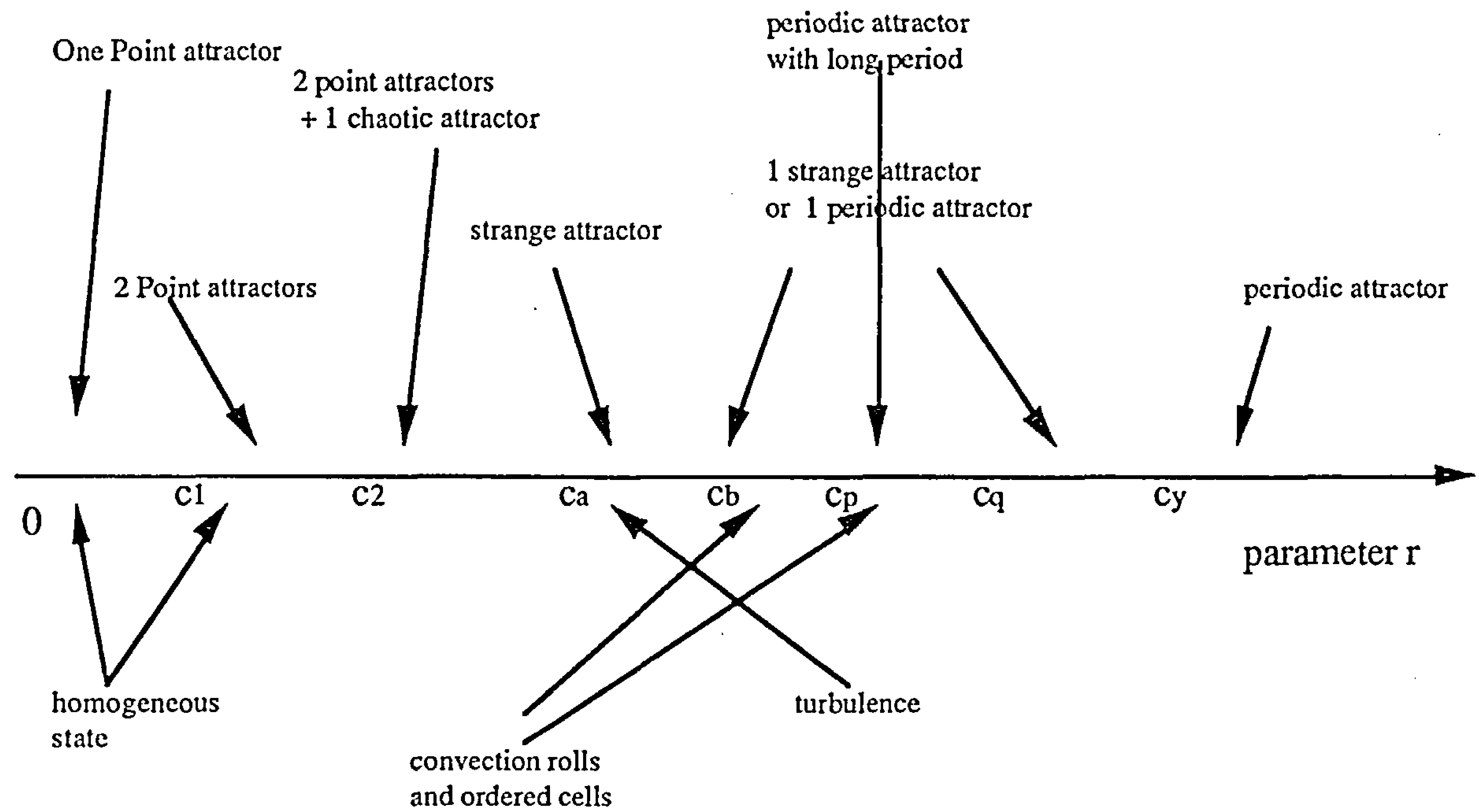
The transformation from (1) to (2) is a genesis phenomenon: heterogeneous structure emerge by association of formerly unconnected molecules, or order out of chaos, as successfully argued by Swenson [Swenson, 1989a]. The appearance of the chaotic motion pattern from ordered cells is also a emergence (order out of order). Despite the physical laws underlying this process, we can look into the structural aspects by using the programme outlined above. According to this perspective, the evolving behaviour of this system can be described as intra-level evolution: the number of components is unchanged (nothing added and nothing subtracted). The system merely change its organization, i.e., how molecules are connected, to meet the constraints imposed by the changing environment.

The dynamical equation derived directly from fluid dynamics is a group of partial differential equations. It is more complex than the typical systems studied in dynamical systems theory (usually about ordinary differential equations). However, under suitable physical assumptions and approximations (which include scaling, the introduction of dimensionless variables, imposing boundary conditions) and further mathematical process of algebraic manipulations and truncations, the equations are simplified to the well known Lorenz equation with x_1, x_2, x_3 representing some collective variables and σ, r, b parameters reflecting the condition of the experiment:

$$\frac{d}{dt}x_1 = -\sigma(x_1 - x_2)$$
$$\frac{d}{dt}x_2 = -x_1(x_3 - r) - x_2$$
$$\frac{d}{dt}x_3 = x_1 x_2 - bx_3 \tag{5.2.2}$$

(It is undoubtedly an over simplified version. However, it is a version that can possibly be handled by DST. For detailed discussion of the process of simplification, see [Gilmore, 1981, pp562-565; Berge et. al, 1984, Appendix D]. It has been proved that this simplified model can not faithfully describe all the dynamical behaviour of the hydrodynamic system, but it can serve to describe the essentially qualitative behaviour of the system and, most importantly, it is analytically as well as numerically easy to handle. The reason we still use this simplified model here is that we are more interested in the qualitative than the quantitative behaviour and the Lorenz model can reflect these qualitative changes of the system's structure and behaviour.).

Using the proposed study scheme, the characteristics of the system are defined by the attractors of the system. The system possesses point attractors, periodic attractors and a chaotic attractor at different values of parameters. This reflects the different dynamical behaviour exhibited by the system at different experimental conditions, viz., steady state at low temperature gradient, honeycombs at moderate temperature gradient, and chaotic motion at high temperature gradient. Point attractor, periodic attractor, and chaotic attractor are the rough patterns which correspond to the qualitatively different behaviour in categorise (1), (2), (3) respectively. Emergence is observed when the system loses its structural stability and bifurcation occurs (figure 5.2.2).



*: *The map between different attractors and different state of the system is not acurate when only one papameter is discussed (for both the variables and parameters are collective variables in the simplified model.)*

Figure 5.2.2 The bifurcation diagram for Benard hydrodynamic system (After Berge et. al, 1984).

5.2.2 Laser systems

Laser systems have long been used as prototypes of self-organizing systems in Synergetics by Haken [Haken, 1983a, 1983b]. The coherent emission is actually the result of the cooperative effects of a huge amount of laser active atoms and it is an emergent behaviour at the macroscopic level.

The ordinary set-up of a laser system consists of a set of atoms embedded in a solid state matrix with mirrors at the two ends. Energy can be pumped into the system and atoms are animated to emit light like a ordinary lamp. When the energy input exceeds a certain threshold, coherent laser can be observed as the output of the system: the laser system is in a new, highly ordered state oat the macroscopic level. According to our terminology, the state of the system is hence characterised by an emergent attractor. The coherent emission can be stable, periodic, or even chaotic under different environments. The system is at non-equilibrium state: atoms are excited to stay in a state at which they emit light. That means the system is at a energy level higher than the thermal equilibrium state at which the Boltzmann distribution function applies.

The system is open because energy is pumped in and light is emitted.

The system is nonlinear because the emission of light is governed by some nonlinear law (Maxwell equations) and there is feedback light and energy from the mirrors.

Fluctuations stem from both the free oscillations of the field in the cavity and the oscillation of the reflecting mirrors.

To understand the behaviour of the laser system, we have to find its inner dynamics, i.e., how the system's components interact to each other. This dynamics has been revealed by the Maxwell equations and Schrodinger equations [Haken 1983a]. In the simplest case, when Maxwell equations and Schrodinger equations are coupled and only the first mode which goes unstable is preserved, we have the following form:

$$
\begin{aligned}
\frac{dE}{dt} &= -kE + gP \\
\frac{dP}{dt} &= -\gamma_{\perp}P + gE\Delta \\
\frac{d\Delta}{dt} &= -\gamma_{//}(\Delta - \Delta_0) - 4gPE^{\cdot}
\end{aligned}
\tag{5.2.3}
$$

where

E: the mode amplitude;

P : a collective variable describing the atomic polarization;

Δ : a collective variable describing the population inversion;

$k,\ \gamma_{\perp},\ \gamma_{//}$: the loss rates for field, polarization and population respectively;

g : a coupling constant;

Δ_0 : population inversion according pump mechanism.

Such a laser system can exhibit different dynamical behaviour under different experiment conditions, i.e. in different environment. Roughly, the following classification can be obtained [see Arecchi et. al , 1982, 1987; Arecchi, 1988]

(1) $\gamma_{\perp} \approx k >> \gamma_{//}$

There is only one type of attractors, i.e. point attractors, can be identified in this equations and that corresponds to the coherence emission in the laser system. The techniques employed is the adiabatic elimination procedure and this method has been also stressed in synergetics by Haken who has extended this idea to the general "slaving principle" [Haken, 1983a,b].

(2) $\gamma_{\perp} >> k \approx \gamma_{//}$

There are two types of attractors, i.e. point attractors and periodic attractors, can be identified in this equations and they correspond to the coherence emission and periodic emission in the laser system respectively.

(3) $\gamma_{\perp} \approx k \approx \gamma_{//}$

The dynamical behaviour of this system is described by a full 3-dimensional equations and all sorts of attractors are feasible. Actually, it has been proved, experimentally, that at least three type of attractors, i.e. point attractor, periodic attractor and chaotic attractor, have been identified [Arecchi et. al, 1982]. The appearance of chaotic attractors is known in laser physics as "coherence collapse" and the intrinsic chaos, due to the nonlinearity of the system, is blamed for these "bad" behaviour of laser systems [Dente et. al, 1988].

Arechhi has reported the experimental confirmation for the evolution of laser systems [Arecchi et.al, 1982; Arecchi, 1988]. In some cases, the environment is changing periodically and that leads to the complex behaviour of the system: the system may evolve to chaotic attractors. In the analysis, this kind of stimulus is represented by a periodic input (the parameter is a function of time).

For a specific laser system, the values of coefficients and the range of the values of parameters can be specified so that the "big picture" of bifurcations can therefore be constructed. Like in the previous example and the one to be discused in the following section, the evolutionary behaviour of the system is made very clear: the bifurcation diagram will be able to indicate explicitly how the system will evolve from one state to another when parameters change from region to another.

5.3 Evolving Chemical systems: Brusselator and Oregonator

The decisive breakthrough in the study of self-organizing systems occurred in later 60's and early 70's in non-equilibrium thermodynamics with the theory and empirical confirmation of the so-called dissipative structure in chemical reaction systems that has revitalised the discussion of self-organization and initiated the study of systems evolution in general. It has brought with it a general conceptual framework and new ordering principles underlying these processes (see chapter 3 for detailed discussions). The theoretical studies include the entropical analysis of thermodynamic processes in near and far from equilibrium systems: in a near equilibrium system, the evolution of a system is governed by the minimum entropy production principle while the maximum entropy production principle applies to systems at a far from equilibrium state [Prigogine etc, 1972; Swenson, 1989a]. Empirical evidences that order can emerge spontaneously in far-from-equilibrium systems came from the elaborated chemical reaction system, the Brusselator, named after the inventors' institution. Later on, a similar chemical system was discovered and named in the same way as the Oregonator.

Detailed report about the experimental set-up of these two chemical systems can be consulted with relevant references [Nicolis et.al, 1977; 1989]. The analysis of the evolving behaviour, spatial-temporal rather than functional, can be conducted as follows according to the programme outlined above (chapter 4).

The Brusselator is a chemical system composed of reactants A and B, and products D and E. The reactants are constantly added in and products removed out from this system and in that case the system is an open system. There are intermediate products, X and Y, and the whole process of the chemical reaction is auto-catalytic and cross-catalytic. The nonlinear interaction of these elements can be schematically illustrated as follows (figure 5.3.1):

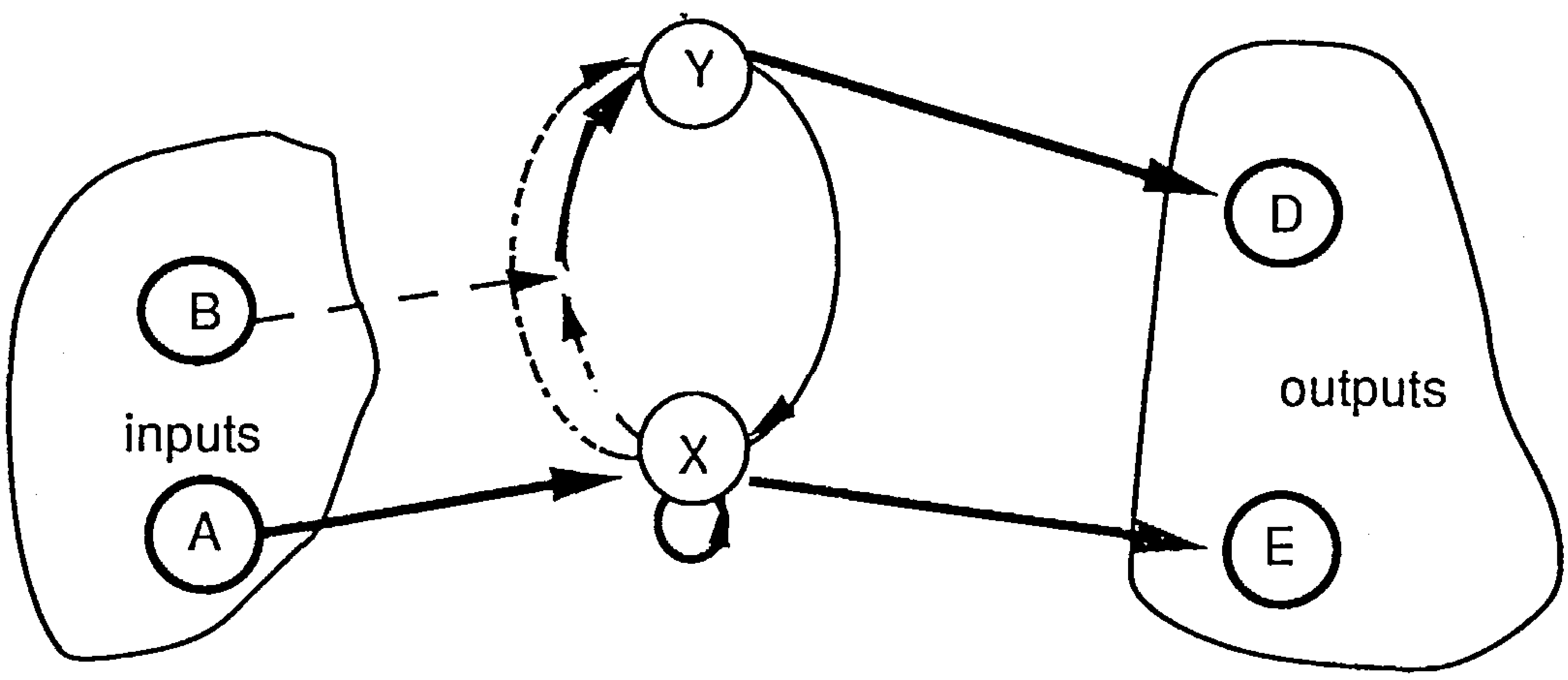


Figure 5.3.1 The schematic illustration of the Brusselator

The Brusselator fulfils the conditions for evolving systems:

It is **open** with input A, B and output D and E, and it also changes energy with the environment. It is apparently at a **far-from-equilibrium** state with the molecular oscillations as **microscopic fluctuations**. The inner dynamics is **nonlinear** and it is manifested as the auto-catalytic and cross-catalytic loops during the reaction process. The nonlinear relations between different elements of the system is clearly indicated in the above diagram.

As we have stated in Chapter 4, the reaction is illustrated as follows:

$$\begin{aligned} A &\rightarrow X \\ B + X &\rightarrow Y + D \\ 2X + Y &\rightarrow 3X \\ X &\rightarrow E \end{aligned}$$

The dynamical model for this chemical reaction can be described by the following coupled nonlinear rate equation (with the same letters representing the concentrations of those chemicals):

$$\begin{aligned} \frac{dX}{dt} &= A - (B+1)X + X^2Y \\ \frac{dY}{dt} &= BX - X^2Y \end{aligned} \qquad (5.3.1)$$

This is the standard form for dynamical systems. The mathematical analysis for this equation is relatively easy because of the simplicity of this equation. It is not difficult to find that the system possesses a point attractor within the region {A, B: A>0, b>0, $B<1+A^2$}. When the environment changes, the system can evolve to a new ordered structure. It has been proved that with A as a constant, when $B<1+A^2$, the system will enter a state of regular oscillation through a Hopf bifurcation process [Hassard et. al, 1980]. It is to say that a periodic attractor may emerge when the system passes certain critical point [Prigogine, 1980].

Brusselator provides an example for the emergence of ordered state in an open non-equilibrium system. This can be theoretically proved and experimentally confirmed. Further work has been done to extend this example to a non-autonomous cases. Tomita has studied the dynamical behaviour of the Brusselator with the appearance of external stimulus [Tomita, 1986]. When the system is driven by periodical external forces, the system can exist even more complicated dynamical behaviour, such as the chaos.

It is assumed that the input reactant A is changing periodically. In the case of a sine function, the concentration of A is changed to $A + a\ Sin\omega t$. Then the dynamical equation is:

$$\frac{dX}{dt} = (A + a\sin(\omega t)) - (B+1)X + X^2Y$$
$$\frac{dY}{dt} = BX - X^2Y \tag{5.3.2}$$

where A and B are remained as constant and a and ω are adjustable parameters. Numerical analysis about this equation has revealed that there exist various attractors for different choices of parameters: point attractors, periodic attractors, quasi-periodic attractors and chaotic attractors. When the environment changes, i.e., when the system is moving from one region to another in the a-ω parameter space, the system can evolve from one ordered state to another. The Bifurcation diagram is illustrated as follows (5.3.2) [Tomita, 1986].

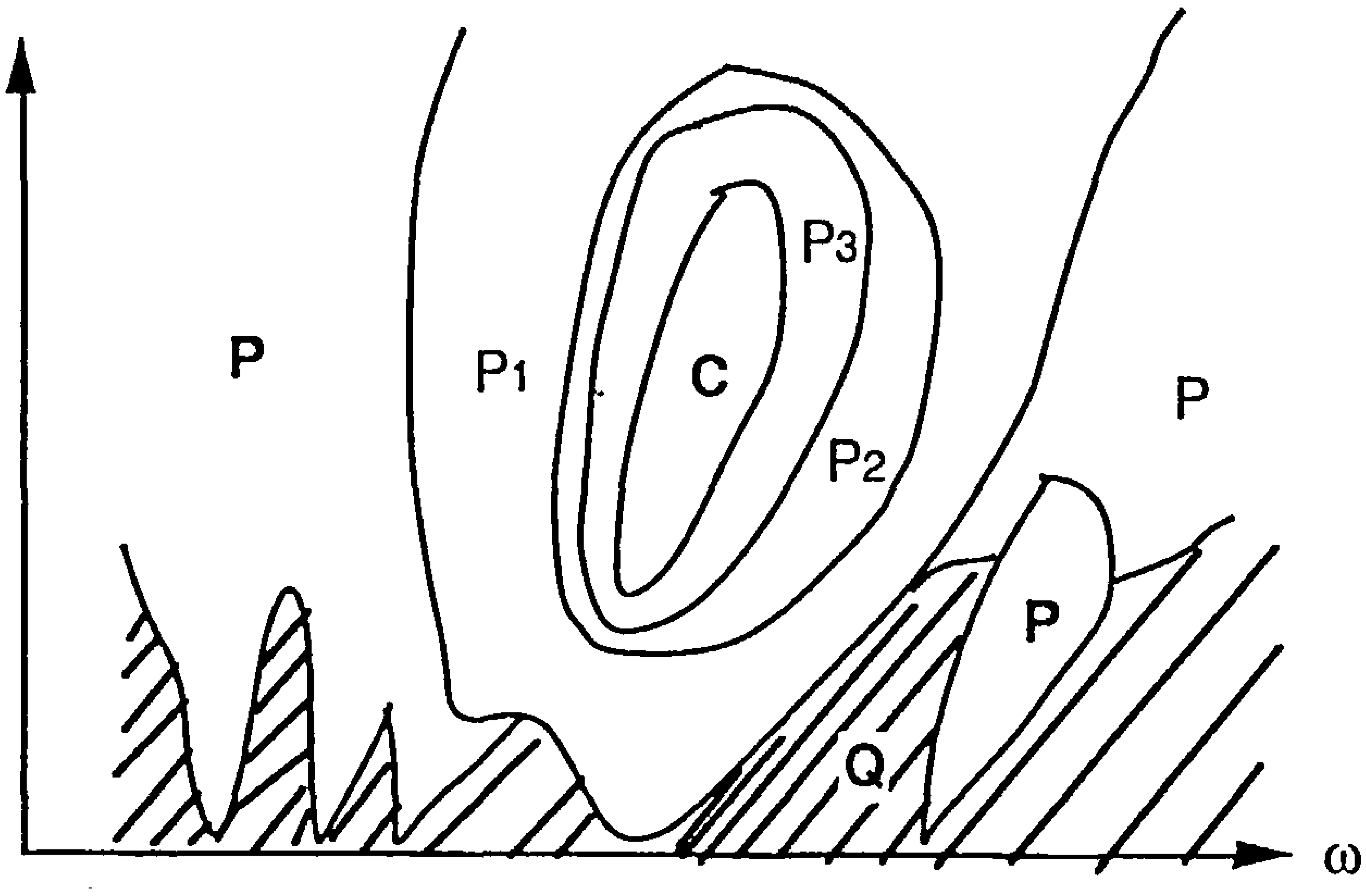


Figure 5.3.2 Parameter Space for periodically-Driven Brusselator

P: periodic attractors (with different periods);

Pn: periodic attractors with periodic 2^n.

Q: quasi-periodic attractors;

C: chaotic attractors

The different regions in the parameter space represent different environment conditions under which the system has different states. When the environment changes, i.e., when the parameters change from one region to the other the system will evolve from one state to the other at which its structure is compatible with the constraints imposed by the new environment. It is apparent that the description is purely structural rather than functional although we could argue that the structure changes in such a way that the system with the new structure is functioning more efficiently in the new environment.

Similar to the Brusselator, there is another example in nonequilibrium thermodynamics which has been employed to demonstrate that order can spontaneously emerge in a nonlinear open system under suitable conditions and this chemical system is usually called Oregonator. In this chemical system, the reaction mechanism can be illustrated as similar to the Brusselator [Nicolis et.al, 1977].

$$A + Y \xrightarrow{k_1} X$$

$$X + Y \xrightarrow{k_2} P$$

$$B + X \xrightarrow{k_{34}} 2X + Z$$

$$2X \xrightarrow{k_5} Q$$

$$Z \xrightarrow{k_6} fY$$

There, A and B are reactants and P and Q reactants. The rate equations which describe the reaction mechanism can be written as:.

$$\frac{dX}{dt} = k_1AY - k_2XY + k_{34}BX - 2k_5X^2$$
$$\frac{dY}{dt} = -k_1AY - k_2XY + fk_6Z$$
$$\frac{dZ}{dt} = k_{34}BX - k_6Z \qquad (5.3.3)$$

This is also of the standard form for describing a dynamical system. Mathematical analysis about this equations, by the aggregation of the parameters, have revealed that point attractors and periodic attractors exist for appreciate parameters. These attractors have been proved to represent the actual state of this chemical systems by empirical confirmations [Nicolis et. al, 1977]. Certainly, we can re-interprete these results by using the formal model of this study, just as we have done for the Brusselator. Further studies can also be carried out by introducing time-varying inputs, like the periodic input of one of the reactants for example. It is highly possible that we may discover more complicated dynamical behaviour of this chemical system, at least theoretically if not empirically.

In the literatures about the study of self-organization, another important example of self-organizing chemical system is the Belousov-Zhabotinsky (BZ) reaction [Nicolis et. al, 1977; Haken, 1983a, Jantsch 1980 et. al]. Like the Brusselator and the Oregonator, it is a chemical systems exhibiting complex spatio-temporal patterns under different environment constraints, it evolves over time. If it is said that the Brusselator is more a theoretical elaboration of self-organizing system, and the Oregonator a deliberately designed experimental system, then the Belousov-Zhabotinsky reaction is regarded as a real discovery by chemists which has urged the scientist to seek a proper explanations [Coveney et. al, 1991]. The BZ reaction and many variants have been used as novel examples to show that ordered spatio-temproal patterns can spontaneously come into existence in chemical systems. Many different ways of explanation have been offered for these reactions. Many theoretical analysis and experiments have been proposed. Recent development can be find in various sources [Special Issue of Physica D, 1991; Skinner et. al, 1991]. We can certainly use BZ reaction as a genuine example for the evolving systems found in nature. The general conceptual framework and the formal model can be employed to describe and study the evolving behaviour of this system.

Another equally important example for self-organization phenomena in chemical systems is the morphogenesis process in biological systems [Turing, 1952]. Turing has tried to use the well-known physical laws to explain how nature has created patterns by itself alone. In doing this, Turing has suggested that the morphogenic processes may be understood by describing the biological cells in terms of their chemical concentrations which are coupled to each other through a diffusing process through the cellular membranes and he showed that chemicals can vary their concentrations to form spatial patterns if several substances with different diffusion rates react with each other. He modelled the simplified diffusion-reaction process with some simple dynamical equations and proved that there exists point attractors and periodic attractors in the equations and they explain the different dynamical behaviour of the system and the appearing of different forms [Turing, 1952]. Further mathematical analysis about Turing model has been offered by Smale in 1974 [Smale, 1974]. In this case, our formal model for evolving systems can be applied directly to describe the evolving process of this process although more detailed mathematical analysis about the more complicated model needs to be carried out before we can strive to find the full explanation of the morphogenic processes. It is worth pointing out that another line of research has also been advocated by other people like Thom whose study scheme is also based on mathematical dynamical systems theory. Actually, the development of the elementary catastrophe theory was intended for the study of biological morphogenesis and further development have also been reported [Thom, 1975, 1986].

5.4 Biological systems: hypercycle and quasi-species

The evolution of biological systems is far more complex than the evolution of physical systems. Compared with the relatively primitive, physically constrained behaviour of some systems like the Benard hydrodynamic systems, biological systems are especially remarkable for their capacity of self-repairing and self-maintaining to restore local stability, and above all, the capacity of reproduction. Some people objects to use of the term "self-organization" to describe biological evolution [Landauer, in Yates, 1988] while some like Kauffman holds a strong view to support the adequacy of the use of self-organization in biological systems evolution. He argues that there is a missed order mechanism underlying evolution and the missing element is spontaneous self-organization. He said that "Darwin did not know self-organization" and evolution is a combination of natural selection and self-organization, interacting in ways that are both profound and not well understood [Kauffman, 1989]. Examples, although not based on empirical data or laboratory experiments, but mainly on mathematics and computers, are plenty to support this view point. Among them is the origin of life itself which have happened as simple molecules organizing themselves to form a kind of primitive metabolism.

One of this model of origin of life is the elementary hypercycle model proposed by Eigen and Schuster [Eigen et. al, 1979]. We have had a brief introduction about hypercycles in chapter 3 and mentioned that it is important not only for the study of the origin of life, --- because it has suggested how the elementary metabolism which is essential for life has come into existence through some basic chemical reactions by using some macromolecules provided by the primordial earth, but also because that the theory of hypercycles has put forward some important concepts and ideas for the study of systems evolution in general. The mathematical treatment is very impressive for mathematical dynamical systems theory has been used successfully to demonstrate how those subsystems have to cooperate to form an organized whole --- a hypercycle --- so that they all can survive to serve as components within the emerged new entity. The formation and evolution process of these hypercycles, or autocatalytic reaction networks in brief, can be re-formulated and reinterpreted by the evolution paradigm as mentioned above.

The following reactor for evolution experiment illustrates the conditions for the emergence of hypercycle networks (figure 5.4.1) [Schuster et. al, 1988].

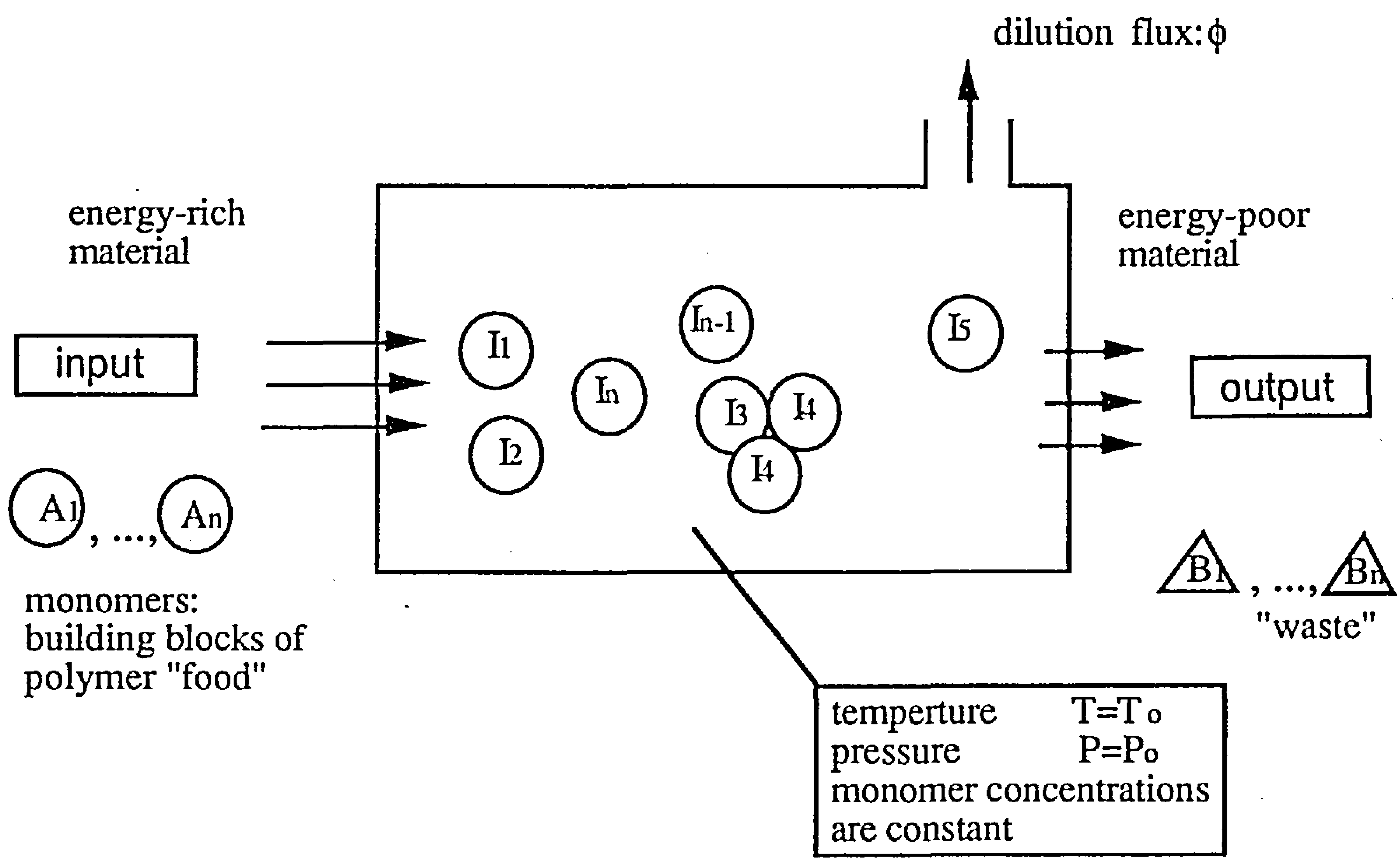


Figure 5.4.1 The dialysis reactor for evolution experiments [following Schuster and Sigmund, 1988].

The dilution flux ϕ is introduced in order to maintain that the sum of the concentrations is constant and it can be controlled by ϕ.

A simple catalysed replication and mutation network consists typically (1) some macroscopic molecules called templates, denoted as I_j, which are to be replicated, (2) catalyst denoted I_i which are templates as well; (3) substrates, materials needed for replication, denoted as (S) and it is assumed that their concentration do not change during the course of replication; (4) the product of the reactions are different species of autocatalytic reaction networks, denoted as I_k.
The basic reaction can be illustrated as:

$$(A) + I_i + I_j \xrightarrow{Q_{kj}A_{ji}} I_k + I_i + I_j; \quad i, j,k = 1, 2, .. n$$

where Q_{kj} and A_{ji} are called mutation rate matrix and replication rate matrix respectively [Eigen et. al, 1979; Boerlijst et al., 1991, Schnabl et al., 1991].

The reaction system satisfies the following conditions:

Open and nonequilibrium: The reaction system is supplied with necessary materials: it is guarantied that the concentrations of substrates is kept constant throughout the reaction. Accordingly, the system is maintained at a far from equilibrium state through these matter/energy/information influx and reactions happening within the system.

Fluctuations: The molecules are in constant motions through the oscillations at a more fundamental levels. Particular units within the macroscopic molecules are also oscillating around their averaged positions and this is the essential factor for the appearances of errors during the replications. By using the biological terms, we can call this the genetic drifts.

Nonlinearity: The reaction is autocatalytic and cross-catalytic and hence the nonlinearity is manifested as these complex feedback and feedforward loops. It becomes more obvious when we start to look at the dynamical equations which describe the reaction process.

With x_i, i=1, 2, .., n representing the concentrations of I_i, the dynamics of this reaction can be described as:

$$\frac{dx_i}{dt} = r_i x_i + k_i x_i x_{i-1} - \phi x_i$$

$$\sum_{i=1}^{n} x_i = c$$

$$r_i = b_i - d_i \tag{4.4.1}$$

where b_i, d_i representing the constant in the self-replication process

$$I_i \xrightarrow{b_i} 2I_i$$

and decay process

$$I_i \xrightarrow{d_i}$$

respectively.

Analysis about the dynamical equations has showed that with $N \leq 4$, there are only point attractors representing the steady stable state of the replication system. When $N \geq 5$, there may exist periodic attractors: the system is evolving to a regular oscillation state. In both the stable steady state and oscillating state, the constituents show cooperative behaviour because their concentrations are controlled by the dynamics of the system as a whole where no population variable vanish. In this way, through the competition and cooperations between different units, new entity manifesting as networks of these interacting units emerge as attractors at a higher level. These attractors characterise the emergence of the elementary hypercycles in the processes of self-reproduction in pre-biotic systems. The existence of these emergent attractors has provided clear evidences for the necessity of hypercycle coupling between those reacting units. Based on this analysis, Eigen and Schuster argued that only the catalytic hypercycles can fulfil the following conditions for the integration of informations that makes the evolution to biological systems from the pre-biotic systems possible:

1) Selection stability of each component due to the favourable competition with error copies;
2) Cooperative behaviour of the components to integrate into the new functional unit
3) Favourable competition of the functional units3 with other less efficient systems.[Eigen et. al, 1979].

Although the evolving behaviour of the pre-biotic system can be described and analysed from the structural point of view, the discussion of its implication in the real biological terms is inevitably interpreted in the functional terms. While leaving the further functional discussions to biologists or biochemists, we can still continue our argument about the emergence of new orders through the self-organization process in pre-biotic systems in terms of structural analysis.

The hypercycle model was originally proposed to demonstrate that the Darwinian evolution scheme (mutation + selection ---> evolution) applies even to the molecules level in the prebiotic and early biological evolutions. This model has been generalized to a model called "replicator system" which is included in a even more general model called "reaction-mutation system" [Schuster et. al, 1983; 1988; Schuster, 1989]. The scheme uses the dynamical systems theory to reveal the complex dynamical behaviour of the systems in discussion and our formal model and conceptual framework can be applied adequately in the discussion. According to Schuster and Sigmund, the general reaction-mutation mechanism can be described as:

$$A + I_i + I_j \xrightarrow{k_j^{(i)}} I_k + I_i + I_j; \quad i, j=1, 2, .. N$$

where I_k is an error copy of I_i under the catalysis of I_j.

The general kinetic equation is:

$$\frac{dx_k}{dt} = x_k\left(\sum_{j=1}^{N} f_{kj}^{(k)} x_j - \frac{1}{c_0}\phi\right) + \sum_{j=1}^{N}\sum_{i\neq k}^{N} f_{kj}^{(k)} x_j x_i\,; \quad k=1, 2, ..., N$$

$$\phi = \sum_{\kappa}^{N} x_k f_k \tag{5.4.2}$$

with the system in a similar condition as considered in the elementary hypercycle model. A special case of this replication and mutation system is the replicator system with the meaning of "replicator system" given by Schuster etc. The complex relation between the constituents of such a system is described as:

$$\frac{dx_k}{dt} = x_k\left(f_k(x_1, .., x_N) - \frac{1}{c_0}\phi\right); \quad k=1, 2, ..., N$$

$$\phi = \sum_{\kappa}^{N} x_k f_k \tag{5.4.3}$$

where $c_o > 0$, f_k representing the interactions term for unit I_k and the term $f=\Sigma x_k f_k$ ensures that the set of state variables $(x_1, ..., x_N)$ with $x_k > 0$ and $x_1 + ... + x_k = c_o$ is invariant.

The analysis of the dynamical behaviour of the general replication-mutation systems is exceedingly complex for even moderate number N (say N=20). However, analytic as well as numerical analysis have shown that in some relatively simple cases like $N \leq 10$ and f_k being some simple functions (constant, linear, or nonlinear with quadric forms etc.), different types of attractors can be identified in the replicator system model. The case of elemental hypercycle has been analysed by Eigen etc as we have mentioned above. Some analysis, mainly numerical analysis, has demonstrated that in the general cases, all the four types of elementary attractors can be found to exist in the replication-mutation systems and some recent results are particularly about the identification of chaotic attractors in the system [Schnable et. al, 1991]. The results have been summarised by Schuster [Schuster et. al, 1988]:

1) For hypercycle equation, $f_{kj}= f_k\, \delta_{j,k-1}$, there is only point attractor for $n \leq 4$, stable quasi-species can be formed. For $n \geq 5$, the intra-

level evolution is manifested as the change from point attractors to periodic attractors.

2) For general equations as above, there are point attractors, periodic attractors, and also chaotic attractors. The routes of intra-level evolution of this system could be any of those studied in chapter 4, except those related to quasi-periodic attractors.

However, these cases are not exclusive, further work need to be carried out so that a complete evolution diagram based on the partition of parameter space can be found. No doubt that further study to unfold the rich dynamical behaviour of the system will help us to understand the evolution process in pre-biotic and early biological systems.

5.5 Ecological systems

Like neural network model, ecological systems is also believed to provide an appropriate platform for systems thinking and systems theory [Schneider, 1988]. In nature, biological species interact with each other through complex energy-consumption chains to form a whole called biosphere on the earth. Ecology studies the relations between plants versus nature (absorption of sunlight, intake of water etc.), plants versus plants, animal versus plants and, animal versus animals. The behaviour of an ecological system is decided by these interactions among the various biological species. The study of ecological systems by using systems ideas, methodologies and techniques becomes more and more important since for such a complex system, with very large spatial scale and time scale, we have to adopt a global thinking and to consider the complex relations between different parts. The idea to consider ecological systems as evolving systems is implied in the study of population dynamics, the co-evolution of different species and the evolution of ecosystems as a whole [May, 1980; Jantsch, 1980; Stenseth, 1986].

The study of ecological systems has also enriched our understanding of the behaviour of systems in general. In the study of systems evolution, substantial contributions have been made by systems ecologists like Howard T Odum and James J. Kay [Odum, 1989; Kay, 1989]. Odum has studied the relation between energy transformation and the hierarchical structure of biological systems. Kay has proposed that ecosystems will organize themselves to maximize the degradation of the available work in incoming energy and that ecosystem will evolve and adapt to maximize the potential for the ecosystem and its component systems to survive. Nature evolves because it abhors a (energy) gradient [Schneider, 1988; Schneider et. al, 1989]. All these statements or claims are consistent with our discussion about systems evolution and agree with the Law of Maximum entropy production [Swenson, 1989a].

The study of the population dynamics of an ecological system consisting of different species has, in recently years, employed mathematical DST to demonstrate that different state of the ecosystem can arise as a result of the complex ways the species interact with each other in a changing environment [May, 1980]. To show that our study scheme can be applied in the study of ecological systems, we use the example of a system of predator-prey and changing resources and a system with three species forming a predator-prey-food chain. It can be shown that such a system can evolve to different ordered structures under different conditions.

The relationship of predator and prey can be illustrated as following by using the influence diagram of systems dynamics (figure 5.5.1):

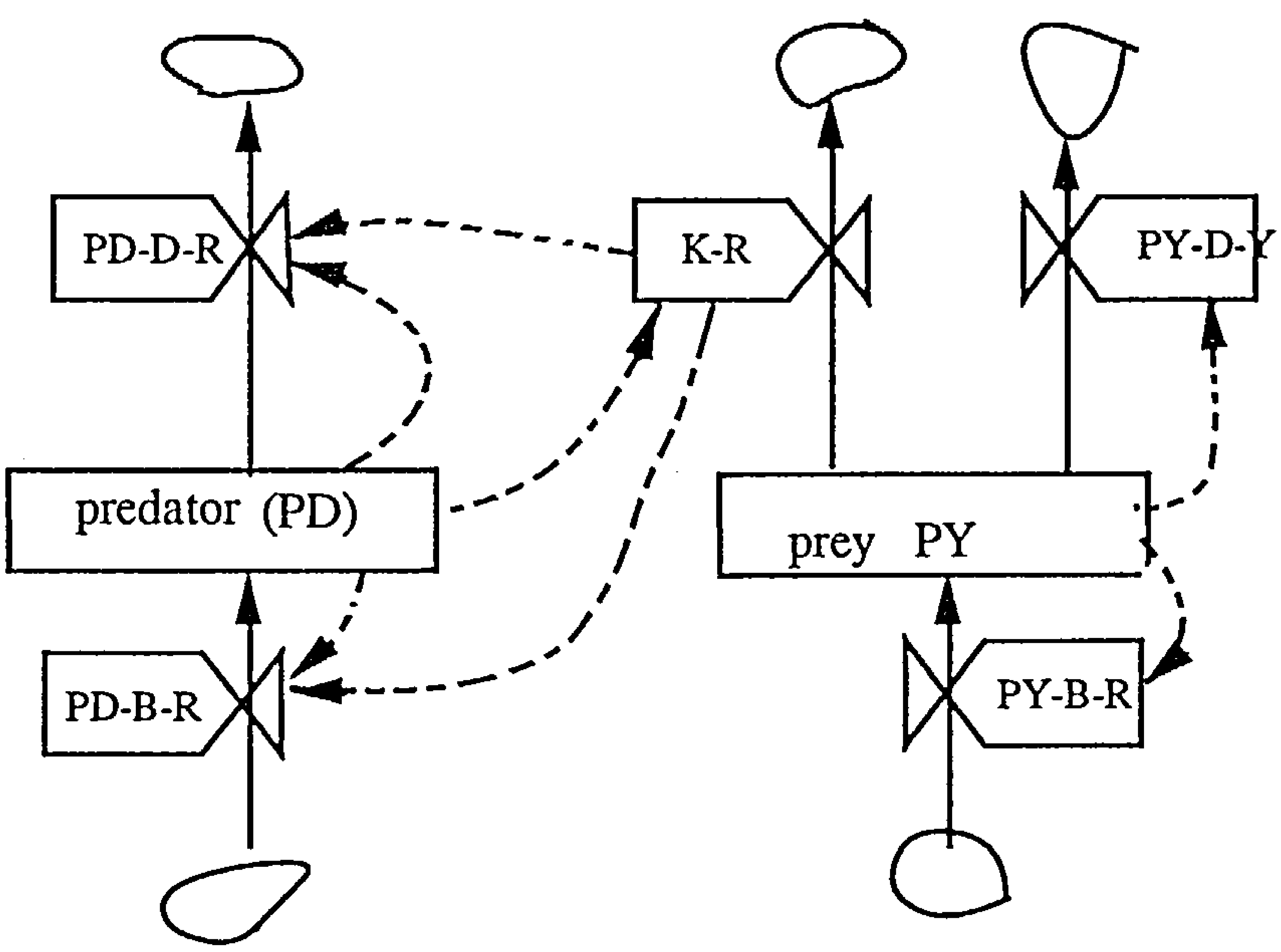


Figure 5.5.1 interactions between predator and prey

PD-D-R: death rate of predator; PY-D-R: death rate of prey;

PD-b-R: birth rate of predator; PD-B-R: birth rate of prey;

K-R: the kill rate of predator;

The interaction between predator and prey can be described by the general Lotka-Voterra equations with limited natural resources N (with x, y the number of predator and prey respectively):

$$\frac{dx}{dt} = x(\ af(y) - R_{PD})$$
$$\frac{dy}{dt} = y(N - g(x) - D_{PY}) \tag{5.5.1}$$

This system has been studied with different choice of functions *f* and *g* (linear, polynomial with different orders et. al). A lot of work has been done to analyse the evolutionary behaviour of ecosystems described by this equations with carious special forms of *f* and *g* [Gardini, 1989; Hofbauer, 1981]. It has been shown that, under different conditions, there exist point attractors and periodic attractors in this system. The change from point a point attractor can be analysed by using Hopf bifurcation theory. If the resource is not constant but changes with time, it can be proved that, at least theoretically, apart from point and periodic attractors, chaotic attractors can also be identified. [Toro and Aracil, 1988].

With the case of three species forming a predator-prey-food chain, the interactions between the different species can be shown as:

$$\frac{dx}{dt} = x(a_{PD}\,f(y) - D_{PD})$$
$$\frac{dy}{dt} = y(a_{PY}g(z) - b\,f(y) - D_{PD})$$
$$\frac{dz}{dt} = z(N - a_{Food}\,h(y) - D_{Food}) \tag{5.5.2}$$

The analysis of the model can be found in [Freedman and Waltman, 1985; Toro and Aracil, 1988].

To construct the bifurcation diagram reflecting the evolution of ecological systems, Toro and Aracil have studied two examples in detail [Toro and Aracil, 1988]. These results can certainly be re-interpreted by using the general framework of systems evolution, as having been done in previous examples. However, these analysis is more meaningful for theoretical deduction and /or in numerical simulation than for understanding real ecological systems. To observe the evolution of such an ecological system is not as easy as to observe the evolution of a chemical system like the BZ reaction. It usually involves a time period up to many years and the data available is also very very scarce. Till now, the detection of chaotic attractors in ecological system is mainly a theoretical work to study dynamical models. However, we can still consider a ecological system composed of several interacting species as an open system. The structural change of the system can be described within the conceptual framework proposed.

5.6 Socio-economic systems: some speculations

The idea to view social systems as evolving systems is not new and it can be traced back to the early debate about social Darwinism [Pines, 1987; Anderson et. al, 1988]. Although social Darwinism has been rejected for social and moral implications, it is still debatable how the social systems evolve. However, to consider the social system as a self-organizing system was not only touched by the eastern philosophies thousands of years ago, as mentioned in chapter 3, it has also been discussed recently from the systems point of view [Jantsch, 1980, 1981; Laszlo, 1989, 1991]. According to the conceptual framework developed in this research, we can have the following analysis or speculations about social systems.

Consider any group of people, say a nation, as a social system, then it is situated in such an environment which is the planet earth with all the creatures on it. The component in such a system is each individual and the relations between them include political, economic, social, and cultural. To consider the social behaviour and economic behaviour of such a system, it is open in the sense that it changes matter /energy / information through its social, political and economic relations with other nations or countries. The microscopic fluctuations are manifested as the free will, creativity, and socio-economic efforts of individuals. The inner dynamics is described as the various complex relations between individuals or groups of individuals in the the various aspects of economic, social, political and cultural domains (sociologists, economists, and philosophers many have different views about the driving forces of the change of society). For the world as a total is changing, any nation as a social system is affected by such a changing environment.

The state of a social system can be roughly described as attractors because the transient time when a nation is in a process of social transformation is short compared with its history. It is very difficult to specify what kind of state is respecting to what kind of attractor, but we can loosely differentiate various qualitatively different states. To say that a social system is in a stable state, or is described as an attractor, it means that the social, economic and political organization are stable and there is no radical change of the social and economic structure, although there might be economic growth, government change et. al. It is also assumed that the time when a nation is in war, either with other nations or in civil war, is short. During war time, the social order and economic structure are destroyed, but we tend not to define it as an "chaotic attractor" rather regard it as a short transient. The tendency in social systems is that the integrity of a society is to be maintained, not destroyed.

About the emergence of social order, ancient Chinese philosophers had perceived that it can only appear spontaneously [Fung, 1958; Yu, 1990; see also chapter 3]. The doctrine of Taoists' philosophy "Through non-action, everything can be done" can be explained as that if we let the system to change according to its own inner dynamics, ordered structure can emerge automatically Unfortunately, the old sages in ancient China saw only a socially and economically closed system maintained at a lower ordered state with low economic efficiency and simple social interactions. To maintain a society in such a low ordered "point attractor" was the ultimate goals for early rulers.

F. von Hayek has also praised the spontaneously generated social orders [Zeleny, 1985]. He argues that social orders, like laws and social conventions, are neither invented by any person nor imposed by any political organization, they have appeared spontaneously through the passage of time. This is the explicit expression of the idea of self-organization in the contest of social systems.

Current situations in many countries are examples of the evolution of open systems: societies evolve to new order state through the economic, political, social and cultural contact with outside world and the driving force is to maintain the societies as organized integrities in the competitive and cooperative environment. However, the new ordered state of any society can not be designed according to some model countries, but only be the result of the complex interplays between the economic factors, political structures, social conventions, cultural histories of nations and external influences. The emerging social orders are transcendental to any individuals or groups of individuals, but the efforts of every individual are not doomed to be neglected. We have witnessed the impacts of individuals in many situations of social evolution over the past few years. Those are examples of how positive attitude of each individual towards social evolution can influence the change of societal systems and the analysis of these examples can be supported by the analysis of systems evolution [Laszlo, 1987, 1991]..

Chapter 6 Evolving systems II: Neural Networks

6.1 Introduction

Neural networks model has been used as a model to study the function of human brain for a long time [See the review by Anderson, 1988, 1990]. The past ten years has witnessed the blooming of the study of neural networks and artificial neural networks mainly for two purposes: as a model to understand how the human brain works, and as a computational system for information processing. For us here, the neural networks model can serve as a prototype for the discussion of evolving systems and our main concern is about the complex dynamical behaviour of neural networks rather than their capacity for information processing.

The reason to chose neural networks as a field to apply the developed conceptual framework and formal model is very obvious:

Firstly, the neural networks model implies the so-called connectionist philosophy as discussed in chapter 3: the behaviour of the system results from the connections of its components and it is emergent. What decides the emergent behaviour is not the properties of individual components, but how they are connected together. This makes it very ideal for the discussion of properties of systems in general: in systems science, the main concern is about emergent behaviour, interaction between subsystems, and the relations between properties of the system as a whole and properties of its components. The strength of neural networks lies on the emergent property of the whole -- emergent computation power in particular -- which is not reducible to each individual neuron [Forrest, 1990].

Secondly, the dynamical behaviour of a neural network, natural or artificial, can be described by dynamical equations. No matter whether it concerns only about the activation dynamics, or the weight dynamics or the mixture of both, mathematical DST can always be employed explicitly and state of a neural network can be analysed by using techniques provided by DST.

Thirdly, neural networks are able to self-organize themselves to some stable states by adjusting the connections between neurons [Kohonen, 1989]. The framework developed in this study about systems evolution can be employed to study the behaviour of neural networks. The general framework and formal model also suggest that neural networks can be treated as open, nonlinear systems with a parameter vector representing the change of environment; therefore neural networks are capable of evolution and they are also, in principle, capable of evolving to more complicated states like specified by periodic attractors and chaotic attractors.

Finally, the results of theoretical analysis can be tested against the empirical analysis or experimental study but here experiments are carried out on computers. Numerical simulations have extended some work about the neural networks, like the simulation of Hodgkin-Huxely model [Degn et. al, 1987]. Other results, like the existences of periodic attractors and chaotic attractors in neural networks, have been confirmed by using electronic circuits [Kepler et. al, 1990].

It is not unnatural to regard neural networks as open and nonlinear systems capable of adaptation and evolution as discussed above. The dynamics of a neural network can be classified into four different types:

(1) the activation dynamics: the state of the neurons can be described by state variables while weights and inputs are regarded as parameters;
(2) weight dynamics: the strength of connection changes while the topology of the network remains the same;
(3) the mix dynamics of both (1) and (2): the weights are adapted while running the activation dynamics;
(4) the dynamics of the network topology: the way which neurons are connected is changing over time.

All the four cases deal with the intra-level evolution of neural networks because the number of elements is not changed, and the dynamical behaviour of the system can be described by the same family of dynamical equations (probably except case 4). The routes of evolution may be similar to those mentioned and discussed in chapter 4.

Although DST will not be able to handle all those cases, the conceptual framework and formal model developed in this study are applicable for analysing neural networks as evolving systems: the long term behaviour of a neural network is described by various attractors and the network adapts to and evolves to a new state when there are some changes in either the input to neurons or the connection strengths. The emergent computational property of most of the neural networks in discussion nowadays is characterised only by one type of attractor: point attractors (symmetric connection -----> point attractors only). However, it has long been realized that other more complex attractors exit: periodic attractors and chaotic attractors have been discovered when the activation dynamics is considered [Clark, 1989; Kepler etc, 1990; Wang, 1991]. Chaotic attractor has also been identified when the weight dynamics is discussed [van der Mass et. al, 1990].

From this point of view, any neural networks violating the symmetrical connection condition is potentially able to exhibit very complex dynamical behaviour no matter how simple its architecture is. It has already been known that chaotic attractors can be identified in neural networks with a number of neurons as small as 2 and as large as around 30 [Clark, 1989]. The following section will report two simple neural nets which display various attractors at different stages, and it is proposed that further work can be carried out to analyse the full bifurcation behaviour corresponding to the change of parameters in a wide range .

6.2 A 3-neuron cellular network

Cellular Neural Network (CNN) is a class of artificial neural networks (ANN) proposed by Chua and Yang in 1988 [Chua and Yang, 1988]. It is made of a massive aggregate of simple and regularly spaced circuits, called cells, just like the cellular automata. It is a neural network because it is made of, like any other artificial neural networks, a large-scale nonlinear analog circuit which processes signals in real time. CNN, with certain constraints in structure, like other ANN, such as symmetry, has been used as a class of information-processing systems (pattern recognition, image processing, voice analysis etc.). Here it has been used to demonstrate that even in a very simple CNN, four distinct types of attractors can be identified. According to the general framework of systems evolution , cellular neural networks, or neural networks in general (physiological or artificial), can be viewed as evolving systems which exhibit adaptive, evolutionary behaviour in a changing environment.

Architecture and dynamics of CNN

In practice, the cells in a CNN are those common electronic linear/ nonlinear circuit elements, like capacitors, resistors, linear/ nonlinear controlled sources and independent sources. Like cellular automata, each cell is connected only to its neighbours. A two dimensional CNN with 5×5 cells can be illustrated as follows (figure 6.2.1).

(Theoretically, a CNN can have may dimensions, but usually only 2 or 3 dimensions are considered).

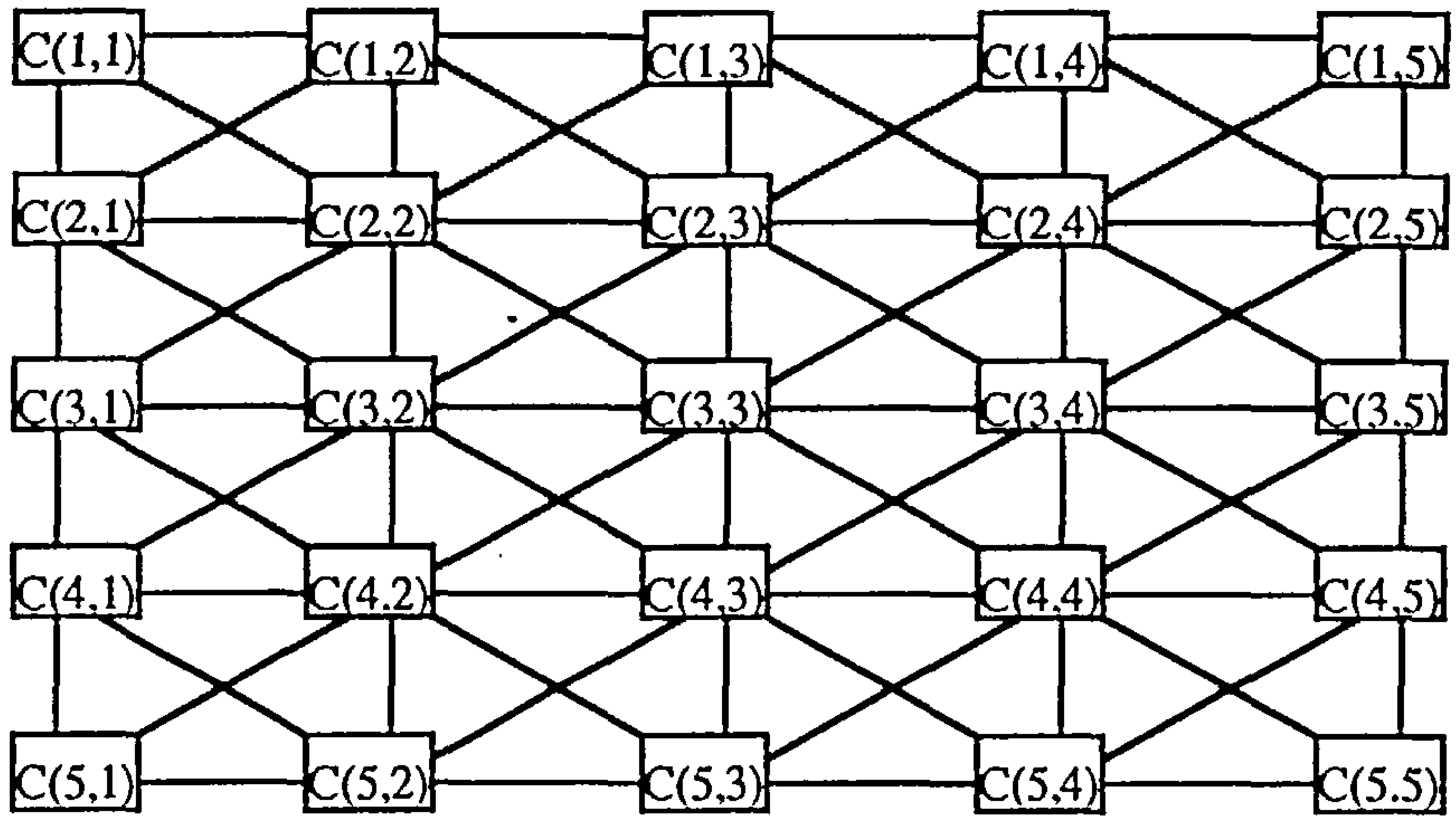


Figure 6.2.1 Architecture of Cellular Neural Networks

For a CNN is composed of electronic circuits, the dynamics of the CNN can be described by laws of physics, namely Kirchhoff's Laws (KCL, KVL). According to these laws, an $m \times n$ CNN, the dynamics of each cell (i,j) can be described as:

$$C\frac{dx_{ij}}{dt} = -\frac{1}{R}x_{ij}(t) + \sum_{C(i,j)\in N(i,j)} a(i,j;\, k,l)\; y_{kl}(t) + \sum_{C(i,j)\in N(i,j)} b(i,j;\, k,l)\; u_{kl} + u_{ij}$$

$$1 \le i \le m,\ 1 \le j \le n; \qquad (6.2.1)$$

where

N(i, j) is the set of all the neighbouring cells connected with C(i,j);

output equation: $y_{ij} = f(x_{ij})$ can be any sigmoind functions;

u_{ij} is the input for cell C(i,j), $1 \le i \le m$, $1 \le j \le n$; they can be either constants or time varing functions;

Constraint conditions must be satisfied:

$$|x_{ij}(0)| \le 1, \quad 1 \le i \le m,\ 1 \le j \le n;$$

$$|\ u_{ij}\ | \le 1, \quad 1 \le i \le m,\ 1 \le j \le n;$$

Parameter assumptions:

$$a(i,j;\, k,l) = a(k,l;\, i,j) \qquad 1 \le i,k \le m,\ 1 \le j,l \le n;$$

$$C > 0,\ R > 0.$$

These equations can be normalized as:

$$\frac{dx_c}{dt} + x_c(t) = a_{cd}y_d + b_{cd}u_d + i_c \qquad (6.2.2)$$

where the first term in the right hand of this equation is the summative of the effect of the neighbouring cells (or neurons) after synapsis, the second part is the summative of the effect of the neighbouring cells through their inputs, the third the input to cell c.

Output function:

$y_c = f(x_c)$ (= a sigmoid function);

Constraint conditions:

$|x_c(0)| \le 1$, $|u| \le 1$.

Some properties, like the dynamic range and stability, have been discussed in the original paper by Chua and Yang, and what is most useful in information processing is a special class of CNN which has symmetrical connections. However, what is of interest to us here is another class of CNNs which are simple (with only few neurons), asymmetrical, but exhibit very complicated dynamical behaviour.

A simple CNN exhibiting complex dynamical behaviour

Consider a very simple case: there are three neurons connected in a simple way: the inhibitory effect from neuron *i* to *j* is as strong as the excitatory effect from neuron j to i. Each cell has an input.

This CNN can be illustrated as:

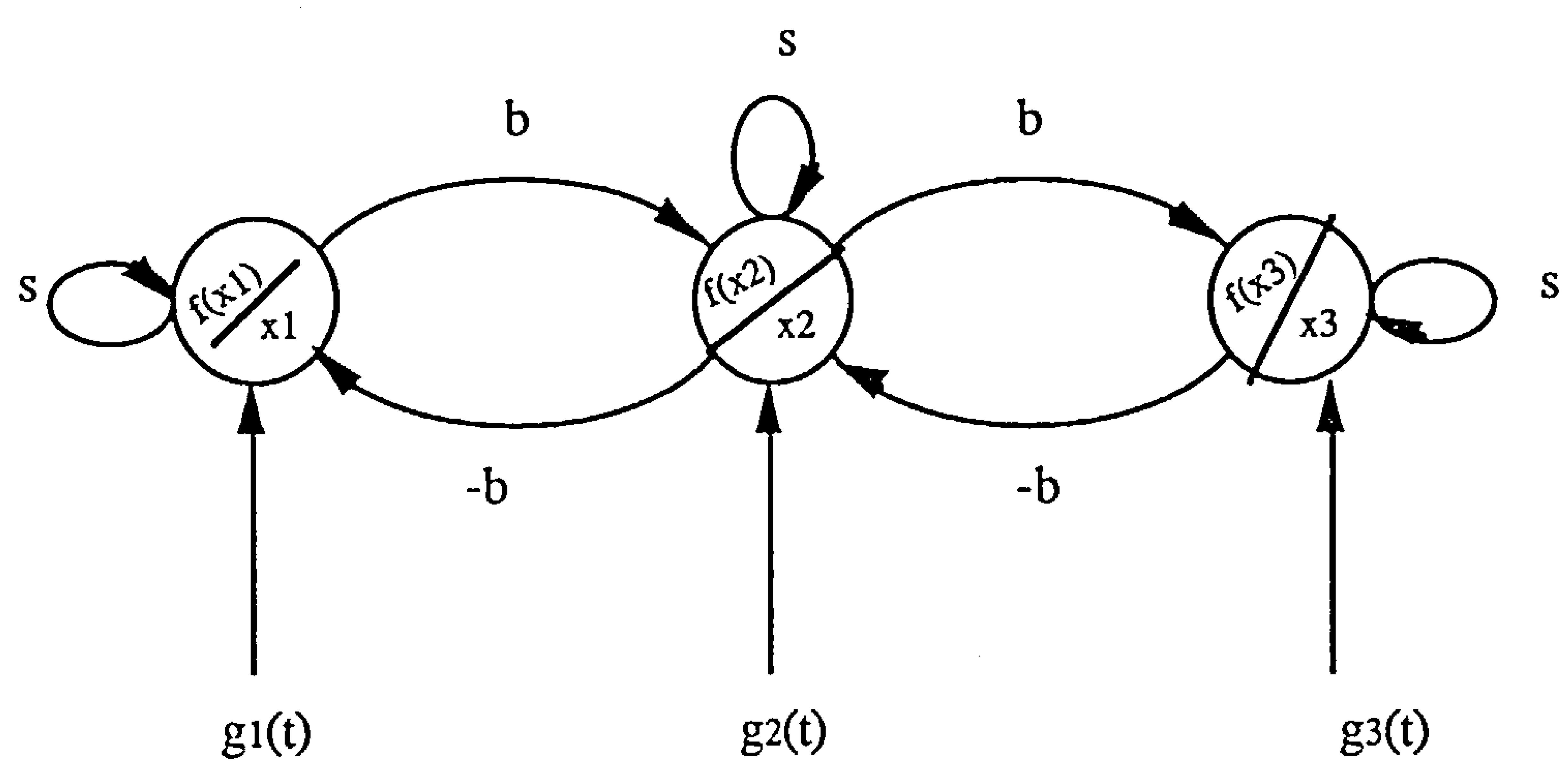


Figure 6.2.2 The architecture of a 3-neuron cellular network

The *activation dynamics* of this network can be described as:

$$\frac{dx_1}{dt} + x_1 = s\,f(x_1) - b\,f(x_2) \qquad\qquad + g_1(t)$$

$$\frac{dx_2}{dt} + x_2 = b\,f(x_1) + s\,f(x_2) - b\,f(x_3) + g_2(t)$$

$$\frac{dx_3}{dt} + x_3 = \qquad\qquad b\,f(x_2) + s\,f(x_3) + g_3(t)$$

(6.2.3)

The dynamics of this system has both the output feedback and input control mechanism. The output feedback loops depend on the interactive parameter *s, b*. The feedback loops and the firing function *f*, a sigmoid type function, reflect the intrinsic nonlinearity of the system. The control mechanism relies on the input signals which might be constant as well as time varying. We tend to regard that these inputs are signs of the strong interaction between the system and its environment.

To analyse the dynamical behaviour, there are numerous choices:

a) The choice of the output function:

The output function can take various forms as long as it is a sigmoid type function. Typical choices include:

$$f(x) = \frac{1}{2}\left(|x+1| - |x-1|\right)$$

$$f(x) = \tanh(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

And the first type is chosen in this study.

b) The choice of input functions:

There are even more choices of the input functions, but they can be usually taken as constants or some bounded periodic functions like *Sin(x)* or *Cos(X)* or the combined forms. With constant inputs, the system is autonomous and DST can be applied directly. However, it is very common to introduce periodic stimulus, so that more complicated attractors such as chaotic attractors can be identified. This can be found in the forced Brusselator model (chapter 4.3), forced nonlinear oscillators in electric circuits, if the constant inputs do not lead to expected complicated behaviours [Tomita, 1986; Thompson et. al, 1986]. However, even these time-varying inputs may seem very artificial, the existence of various attractors reflects the inherent complexity and order of the system in focus. The system changes its organization in response to the change of environment. Periodic inputs are considered in this study to trigger many complex evolutionary behaviour of the system.

c) The choice of weights s, b.

The coefficients s and b represent the strength of intra- and inter- neuron connections. They are the main concern in current study of artificial neural networks as information processing systems, because the change of the weights between neurons may allow a network to classify the input information accordingly. Different connection strength may give rise to the network as a whole different emergent behaviour: this is what exactly this study is meant to deal with.

For the dynamical equations are very much like what have been studied in many other fields: the forced pendulums, the periodically driven van de Pol equation, or forced Duffing equation. It is expected to find various attractors for various choices of output functions, or input signals, or structural parameters. Rich bifurcation diagrams can be obtained if attractors can be detected respecting to all choices of parameters.

As a matter of fact, four distinct attractors, namely, point attractor, periodic attractor, quasiperiodic attractor, and chaotic attractor, have been identified in this simple case:

Case 1 Point attractor

Compared with attractors like periodic attractors and chaotic attractors, point attractors are trivial cases: the system is in a stable state. For neural networks, it means that the system will rest at that point attractor no matter where it starts. For that reason, the system has some sort of memories about the structure at the attractor and it is this capability of memorizing that makes neural networks extremely appropriate for the task of information processing [Hopfield, 1982; 1984].

In this 3-neuron cellular network, many simulations have revealed that without any input signals, there exist a point attractor for choices of $s=b$. Zou and Nossek have proved that in the general cases, when there is no input signals and $b< (s-1)/2$, there is at least one point attractor [Zou and Nossek, 1991a].

Case 2. periodic attractor

The implication of neural networks at periodic attractors for information processing has not been made very clear, but many of such networks in oscillating state have been identified [Wilson et. al, 1972]. In our study, periodic attractors have been identified. Figure 6.2.3 is the phase portrait of a periodic attractor identified for this 3-neuron network, with

$$g_i(t) = 0,\ i=1,2,3,\ b>s-1$$

(with different coefficients b, s, the amplitude of the cycle is different).

Case 3. period-3 attractor

A period-3 attractor has been identified in this model by introducing a periodic signal. Figure 6.2.4 shows the phase portrait of a periodic attractor identified for this 3-neuron network, with

$$g_1(t)=\sin(2\pi t),\ g_2(t)=g_3(t)=0;$$
$$s=3.5,\ b=2.7$$

Case 4. quasi-periodic attractor

By introducing two periodic signals with different frequencies, a quasi-periodic attractor has been identified. Figure 6.2.5 shows the phase portrait of a periodic attractor identified for this 3-neuron network, with

$$g_1(t)=0,\ g_2(t)=\sin(2\pi t), g_3(t)=\cos(\sqrt{2}\pi t)$$
$$s=3.5,\ b=2.7$$

Case 5. chaotic attractor

Through further "manipulation" with the input signals and parameters, a chaotic attractor is found for this system (however, the influence from neuron x_3 to x_2 is cut). The choice of parameter is

$$s=2,\ b=1.5$$

input signals

$$g_1(t)=4.1\sin(4\pi t),\ g_2(t)=g_3(t)=0$$

The phase-portrait of this chaotic attractor is illustrated in figure 6.2.6.
(see also [Zou and Nossek, 1991b].

What is missing in our study about chaotic attractor includes the calculation of Lyapunov components and fractal dimension, the construction of Poincare section and spectrum analysis. These means, including the construction of phase portrait and the sensitivity analysis, are all important for the identification of chaos, as argued in the extended appendix I. However, we suppose it is not necessary to do all these test to justify that one attractor is actually a chaotic attractor, although it is surely very useful. What has been done here about the chaotic attractor of the 3-neuron cellular neural network is not more than what were done for the discovering of Lorenz attractor by Lorenz in 1963 : phase portrait and sensitivity analysis [Lorenz, 1963]. Of course, the other tests can be carried to reveal various other aspect of this chaotic attractor: what is its fractal dimension, what does the Poincare section look like, what is the biggest Lyapunov exponents etc..

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Figure 6.2.3 periodic-attr

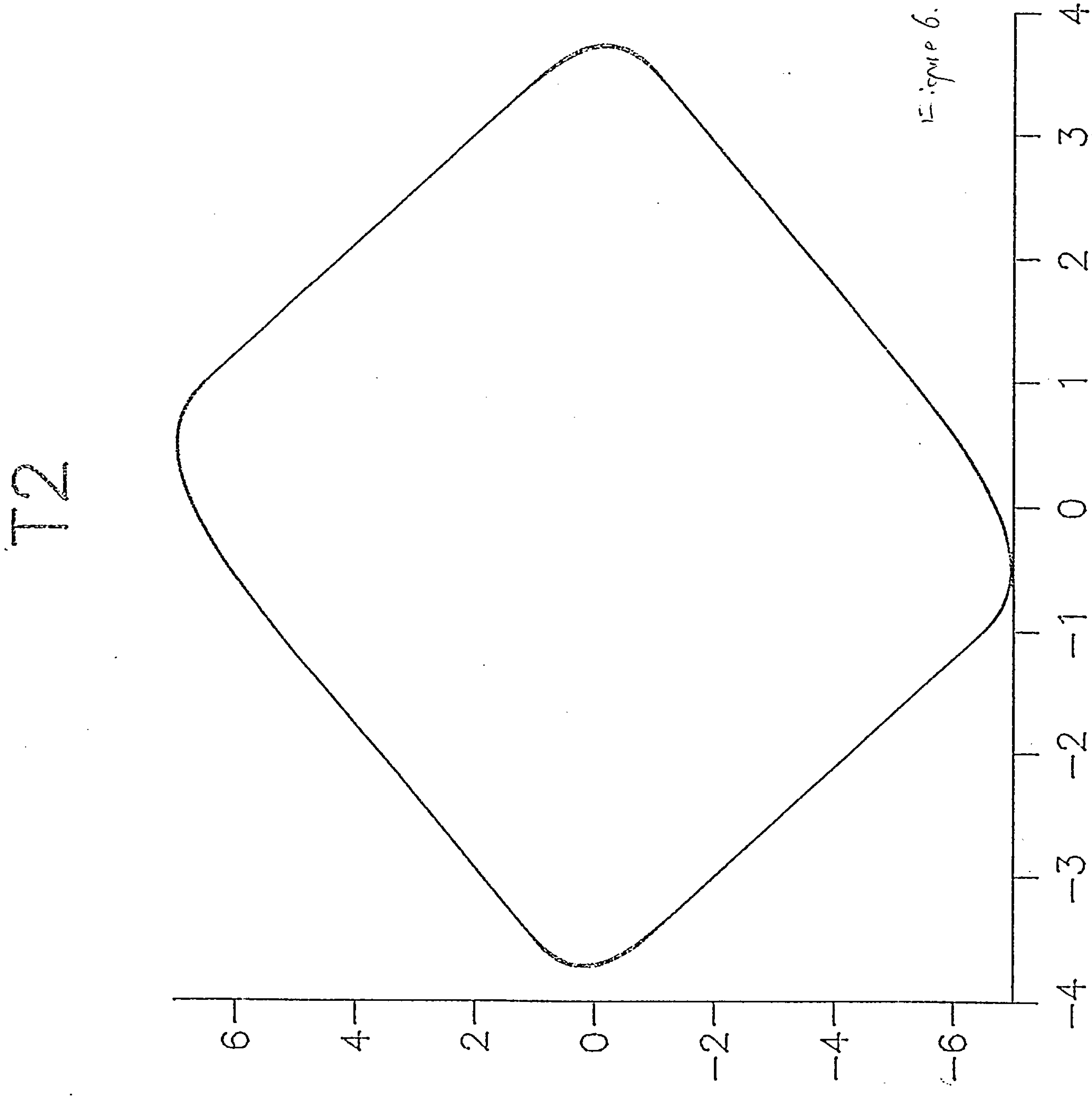


Figure 6.2.3 A Periodic attractor of the 3-neuron CNN

Figure 6.2.c Perioc

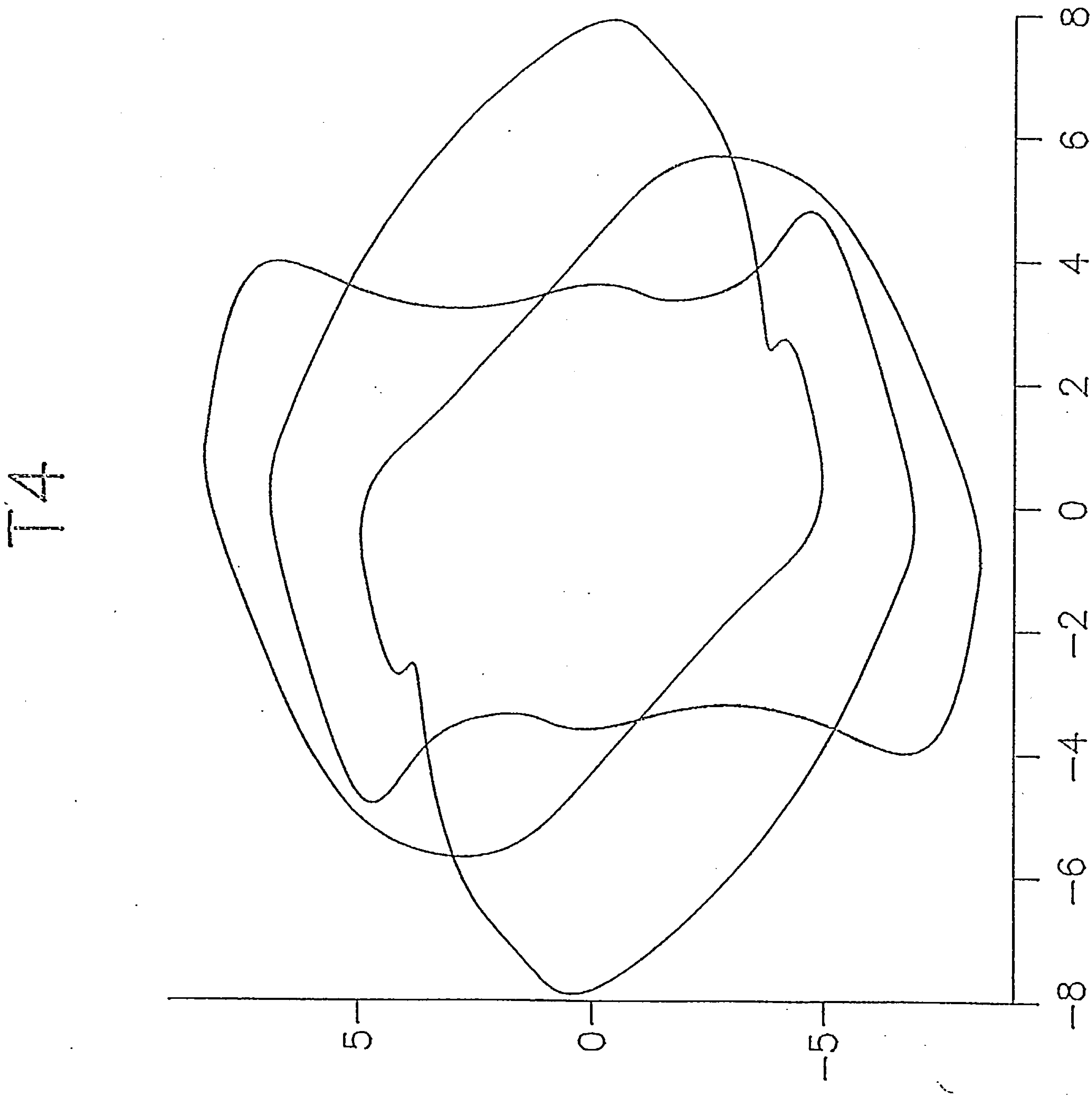


Figure 6.2.4 A Periodic-3 attractor of the 3-neuron CNN

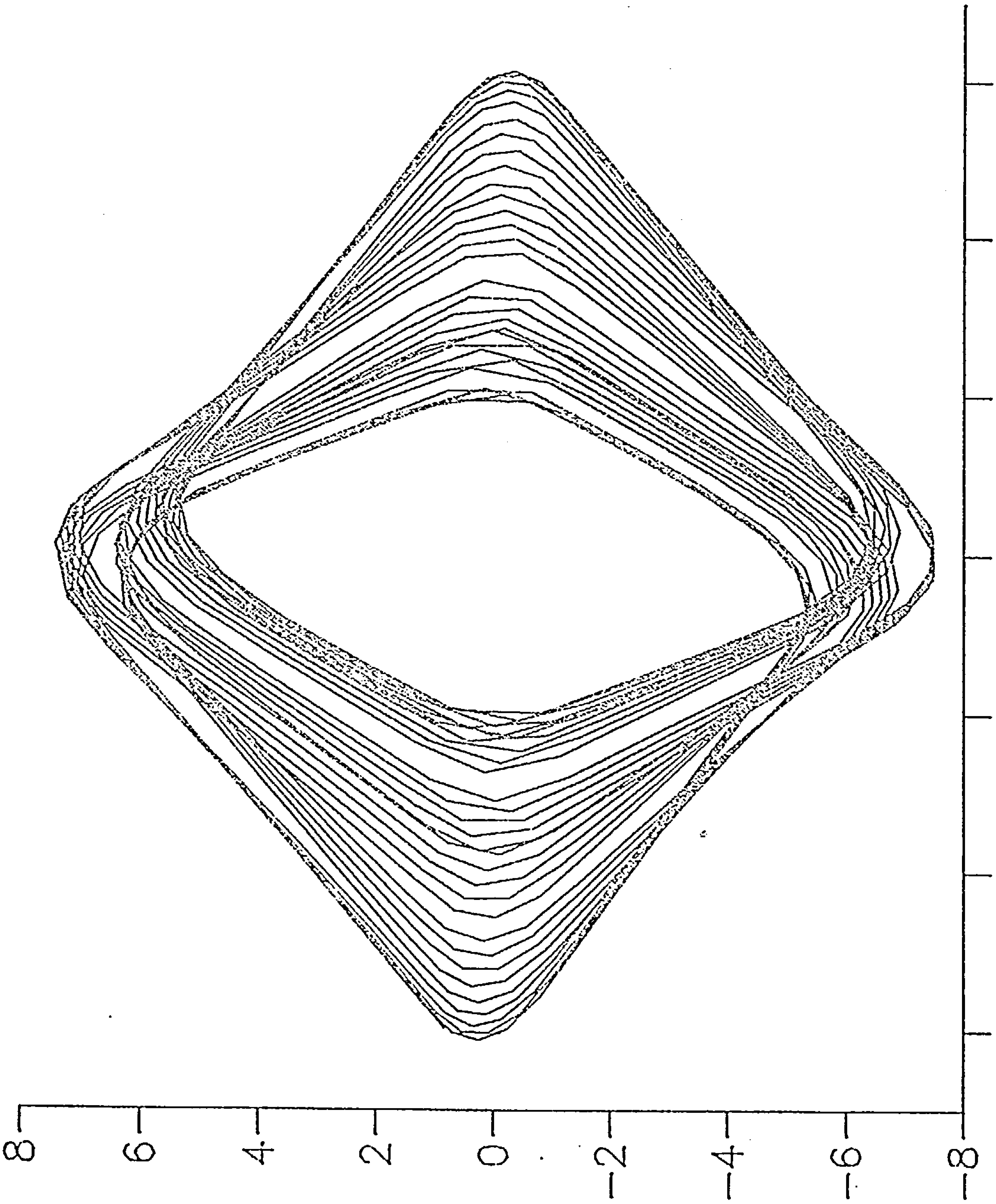


Figure 6.2.5 A quasi-periodic attractor of the 3-neuron CNN

figure 6.2.6

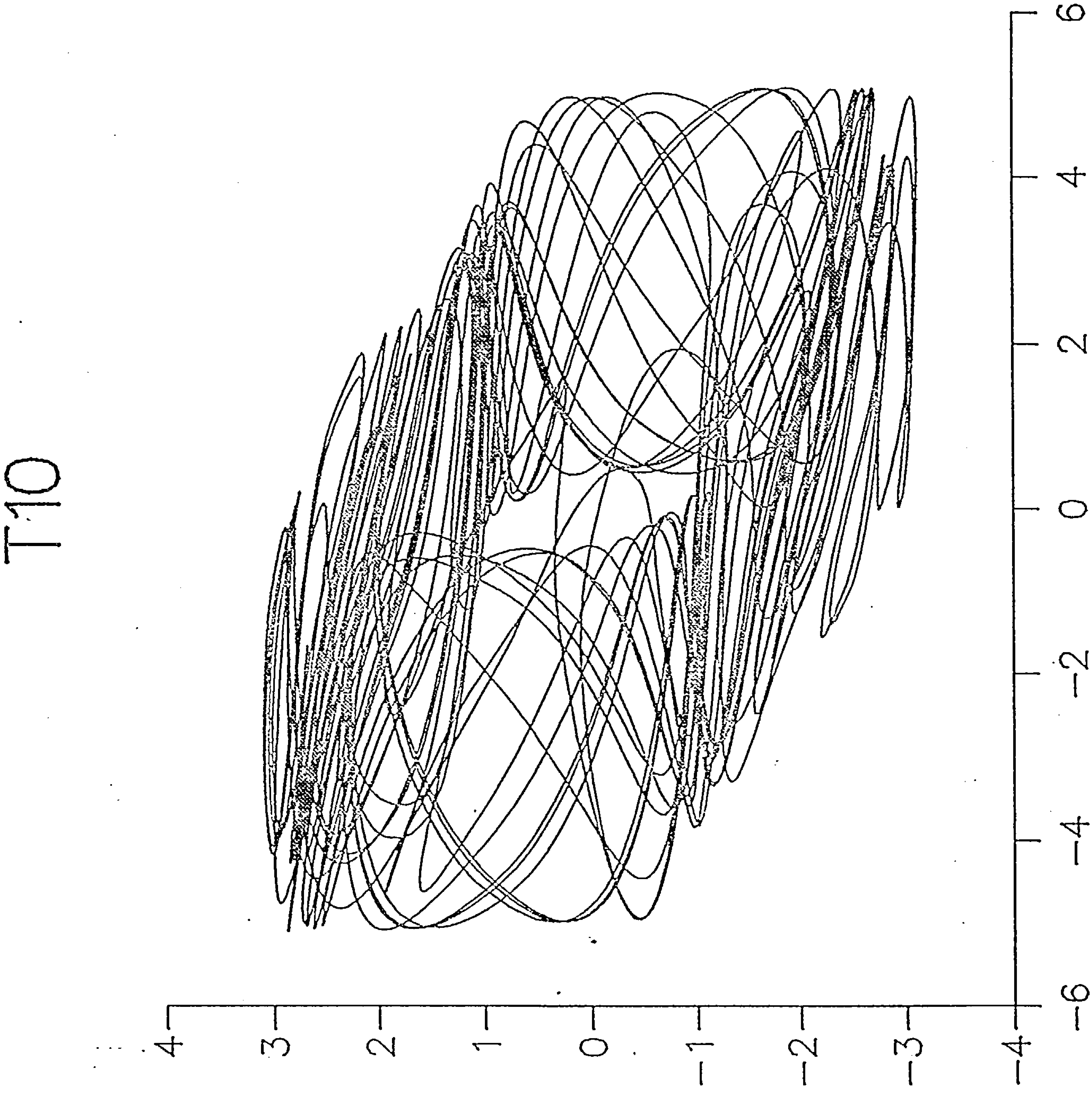


Figure 6.2.6 A chaotic attractor of the 3-neuron CNN

The same kind methods are employed in the following section to study the complex behaviour of a coupled Wilson-Cowan Nets.

6.3 Coupled Wilson-Cowan Nets

The Wilson-Cowan model, proposed in 1972, concerns a pair of neurons among them one is excitatory and one is inhibitory [Wilson et. al 1972]. The connection of them can be illustrated in Fig 6.3.1.

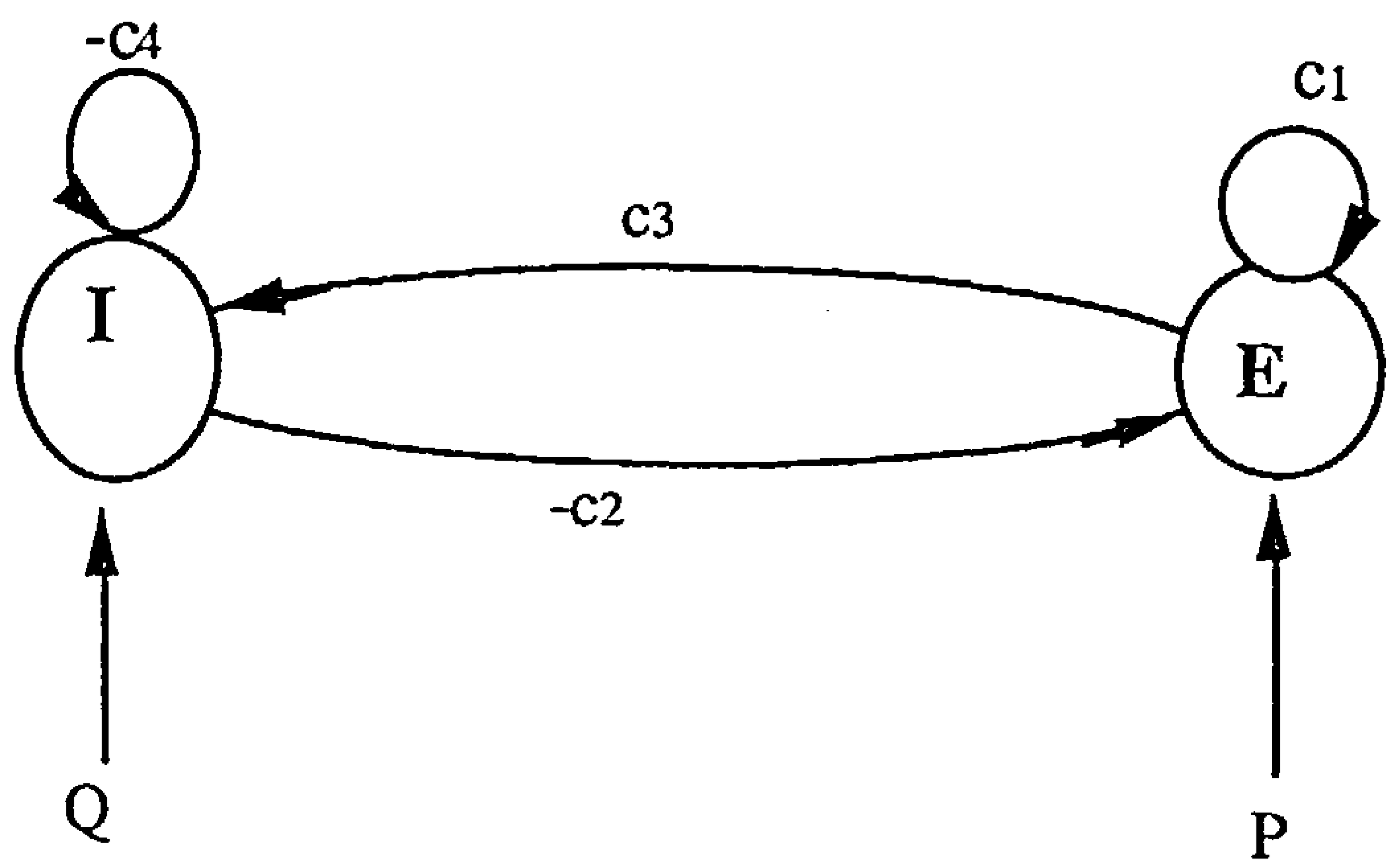


Figure 6.3.1 A Wilson-Cowan Net

The excitatory neuron, represented by **E**, and inhibitory neuron, represented by **I**, are connected in such a way that the inhibitory neuron always inhibit the neural activities: not only itself, but other neurons connected with it. The excitatory neuron excite all neurons connected with it, as well as itself. It is believed that all nervous process of any complexity are dependent upon the interaction of excitatory and inhibitory cells [Wilson and Cowan, 1972].

The activation dynamics of this network can be described as

$$\tau_e\frac{dE}{dt} = -E + (k_e - r_eE)S_e(c_1E - c_2I + P)$$
$$\tau_i\frac{dI}{dt} = -I + (k_i - r_iI)\, S_i(c_3E - c_4I + Q) \tag{6.3.1}$$

where

E: the firing activity of the excitatory neuron at time t;

I: the firing activity of the inhibitory neuron at time t;

c_1, c_2, c_3, c_4 : connection strengths, all are positive constants in this model;

Si, Se: sigmoid-shaped firing functions;

P, Q: input to the excitatory.and inhibitory neuron respectively, in this study, they are constants, not time varying;

τ_e, τ_i, k_e, k_i, r_e, r_i : constants.

With $S(x) = 1/(1+ \exp(-a(x-\theta)) - 1/(a+\exp(a\theta))$, with a and θ depending on whether it is an excitatory or an inhibitory neuron. It has been proved, through phase plane analysis and numerical analysis rather than finding analytical solution, that point attractors and periodic attractors exist corresponding to different constant stimuli [Wilson and Cowan, 1972]. With two pairs of such neurons being connected as in figure 6.3.2,

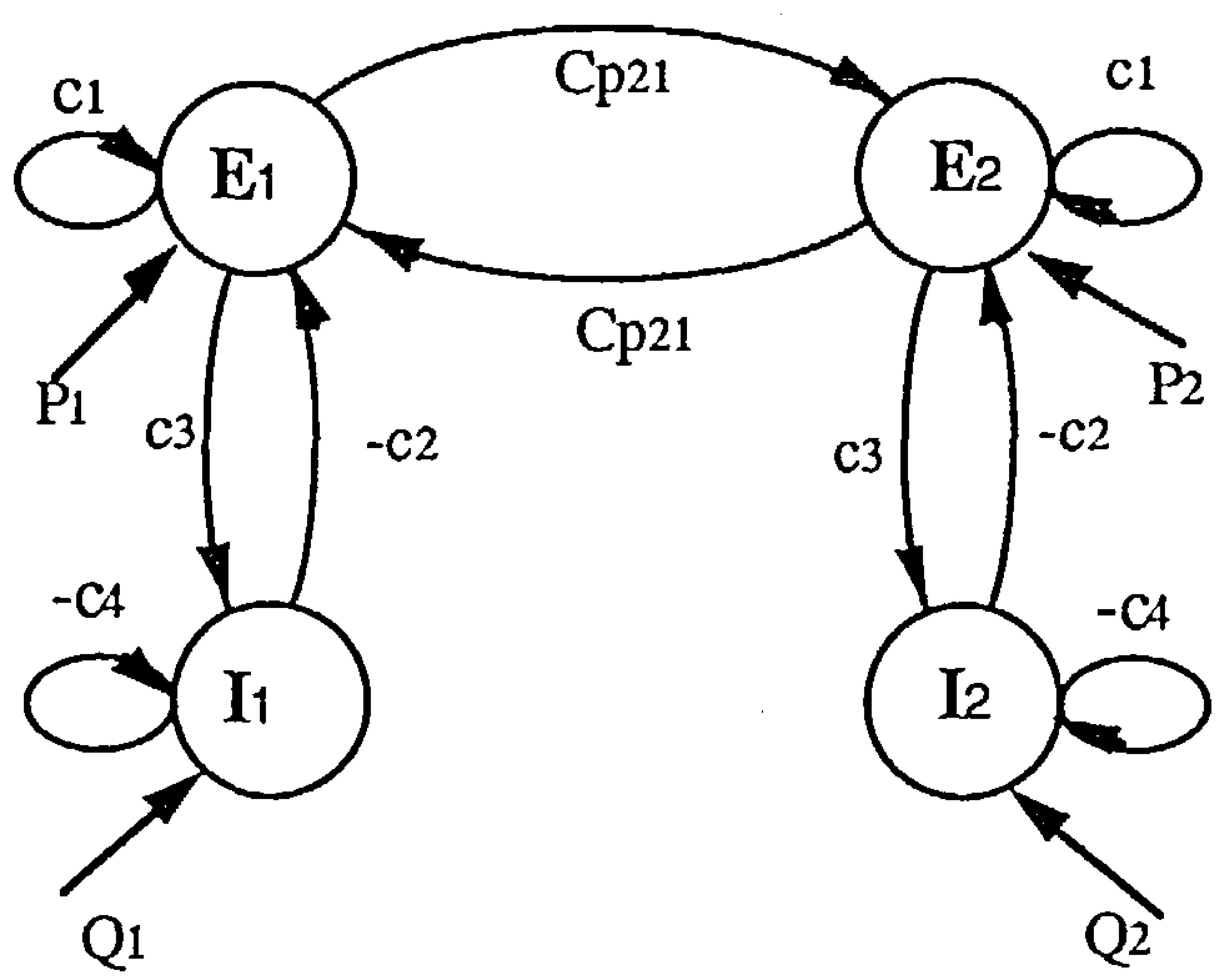


Figure 6.3.2 Coupled Wilson-Cowan Nets

more interesting behaviour is expected to be observed.The dynamical equation is written as:

$$\tau_e\frac{dE_1}{dt} = -E_1 + (k_e - r_eE_1)S_e(c_1[E_1+Cp_{21}\,E_2]-c_2I_1+P_1)$$

$$\tau_i\frac{dI_1}{dt} = -I_1 + (k_i - r_iI_1)\,S_i(c_3E_1-c_4I_1+Q_1)$$

$$\tau_e\frac{dE_2}{dt} = -E_2 + (k_e - r_eE_2)S_e(c_1[E_2+Cp_{12}\,E_1]-c_2I_2+P_2)$$

$$\tau_i\frac{dI_2}{dt} = -I_2 + (k_i - r_iI_2)\,S_i(c_3E_2-c_4I_2+Q_2) \qquad (6.3.2)$$

where Cp_{12}, Cp_{21} , are coupling strengths between two excitatory neurons (when $Cp_{21} = Cp_{12}$, it is a symmetrically coupled).

Our study has shown that two types of complicated attractors can be found in this system, i.e., periodic and quasi-periodic attractors. (The choices of some parameters follow Wilson-Cowan paper as: c_1= 16, c_2=12, c_3=15, c_4=3, a_e=1.3, a_i=2; θ_e= 4, θ_i= 3.7, r_e= r_i =1; some other parameters may be chosen in the further studies)

<u>Case 1: point attractor</u>

Again, point attractors are trivial cases in this model. (say, with coupling Cp12=Cp21= 0.05, and c_1= 13, c_2=4, c_3=20, c_4=2, a_e=1.2, a_i=5; θ_e= 2.7, θ_i= 3.7, r_e= r_i =1).

<u>Case 2: periodic attractor</u>

When inputs are chosen as P1=1.3, P2=0.5, Q1=Q2=0; coupling strength CP12=Cp21= 0.05, a periodic attractor can be identified as shown in figure 6.3.3;

<u>Case 3: quasi-periodic attractor</u>

When input to E2 has been changed to P2=1.5 while others remain the same as in case 2, a quasi-periodic attractor has been identified, as the phase portrait showing in figure 6.3.4.

Further discussion about the complex behaviour of this model can be carried by paying attention to chaotic attractors. Periodic input can also be considered.

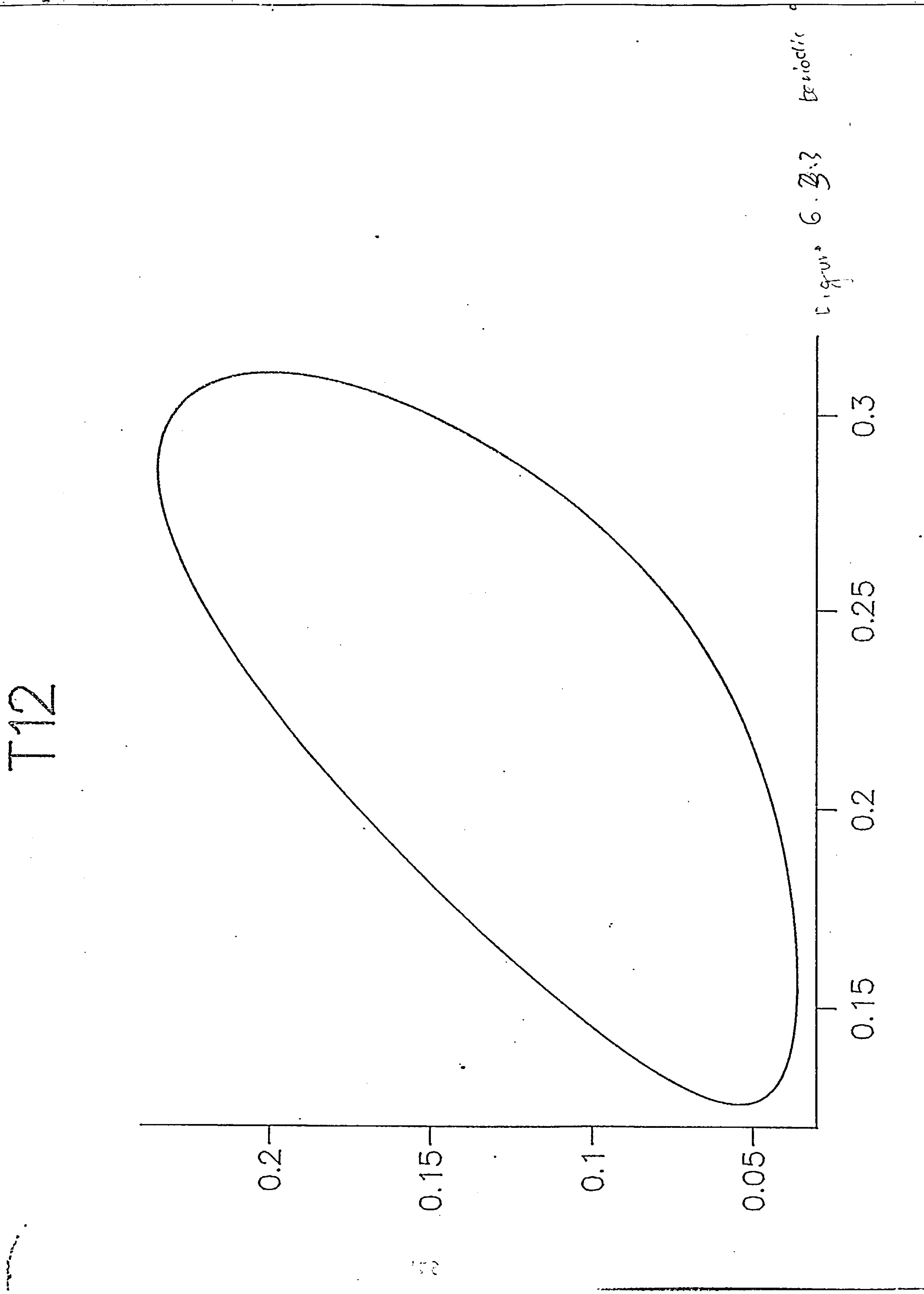


Figure 6.3.3 A periodic attractor of a coupled Wilson-Cowan net

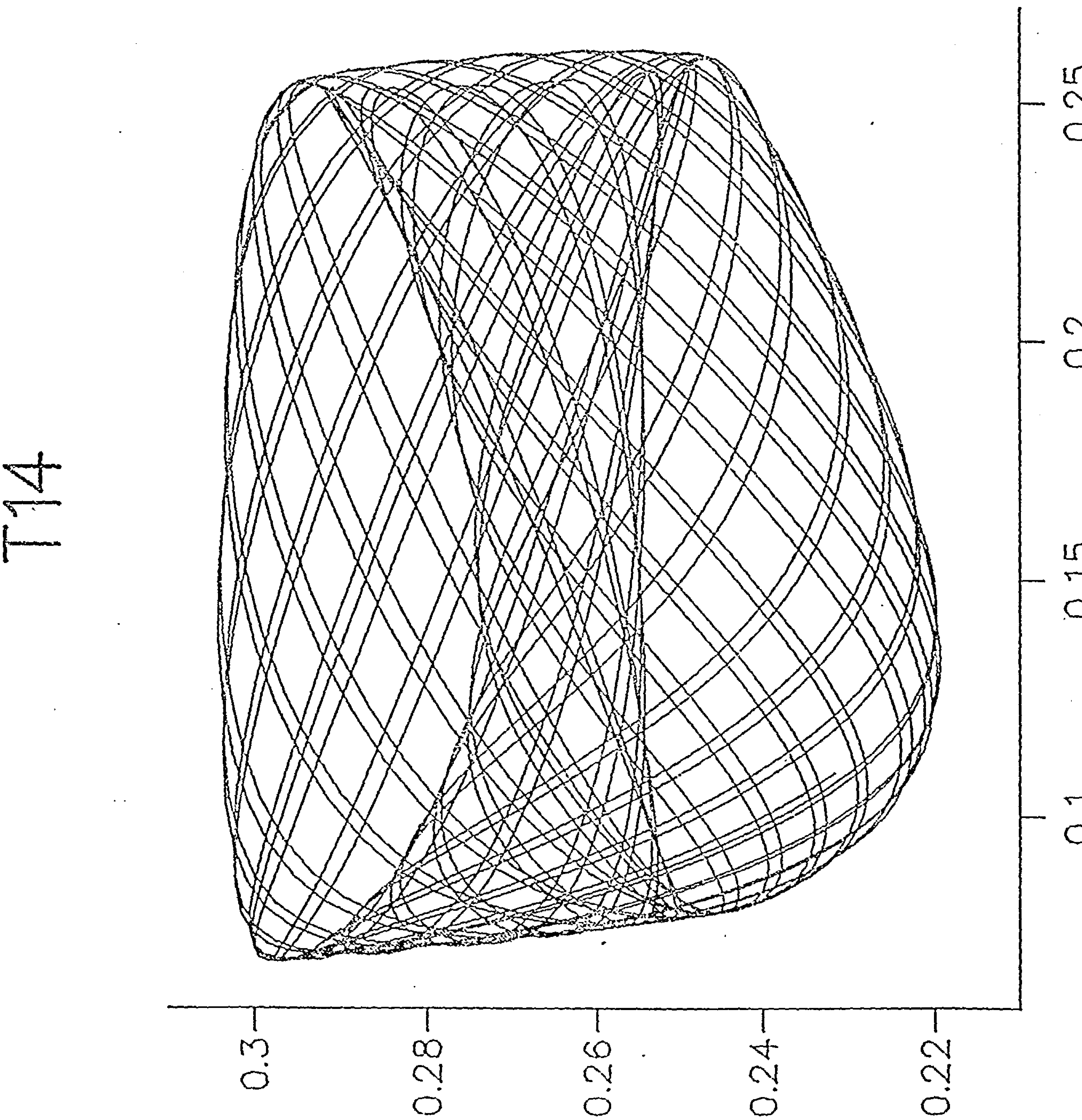


Figure 6.3.4 A quasi-periodic attractor of a coupled Wilson-Cowan net

6.4 Neural networks as adapting and evolving systems

As having been demonstrated, neural networks, if not symmetrically connected, may possess different types of attractors. The dynamical behaviour of neural networks can be analyzed by adapting the conceptual framework and formal model developed in this study: as open, nonlinear, non-equilibrium systems, neural networks can evolve from one state to another, with the change of influences from environment. The focus is about the nonlinear dynamics of those systems: it reflects the inherent complexity and order of evolving neural systems. Most current study about neural networks are centred around the emergent computation power of symmetrically connected systems. The discovery of complicated dynamical behaviour of neural networks demands serious attention [Skarda and Freeman, 1987]

Large-scale neural networks can be constructed by connecting many blocks of small networks of which each is characterised by one of those complex emergent attractors. In the general case, DST is useful but may not be enough for analysing the behaviour of neural networks with a large number of neurons.

In this study about evolution of neural networks, the conceptual framework and formal model developed so far have been applied. The detection of the state of a neural network at a certain time relies on the computer simulation, i.e., finding out attractors by comparative numerical simulations. In principle, all the techniques and skills in studying dynamical systems as mentioned in appendix 1, such as the Hopf bifurcation analysis, Lyapunov exponents, Poincare section etc., can be employed in the analysis, especially in detecting chaotic attractors. However, the construction of phase portrait by comparative numerical simulation is also a reliable means. In this study, computations were tried in both PCs and Sun work stations. To construct the phase portraits of the systems, the 4th order Runge-Kutta algorithm has been employed. In both the CNN and Wilson-Cowan models, to obtain a phase portrait about the system starting from a particular point usually takes around 20 minutes on PCs (386) with 30 000 iterations. It only takes about 10-20 seconds on a Sun Sparc-2 station. To obtain a complete"big picture of the bifurcation cascade") needs thousands times of such simulations. To get some analytic solutions certainly helps to reveal the evolutionary behaviour of a system over a wide range of parameters, but it is not always possible to have analytic solutions. To assist numerical simulations, intuitive knowledge about possible bifurcation patterns is of great help.

To calculate Lyapounov components or to construct the Poincare may require even bigger memories and faster speed (as mentioned in many references about those examples cited in chapter 5). To carry out intensive computation, the efficiency of algorithms should also be considered.

Chapter 7 Conclusions and Further Studies

7.1 Conclusions

To summarise, the following results have been achived in this study;

1. A conceptual framework for systems evolution

Among many other systems concepts, the concepts of emergence and emergent properties have been stressed and they are regarded as the most fundamental concepts in systems science [Checkland, 1981; 1989]. It is argued that the defining characteristics of a system comes from its emergent properties -- it is the emergent properties that give the identity of wholeness to a system. It is the emphasis on the wholeness that makes systems science different from classical sciences.

The entity of an organized whole is described by an attractor. Attractors and emergent attractors have been defined verbally and formally. By employing the concept of emergent attractors, the complex dynamical behaviour of nonlinear dynamical systems can be explained

The ideas of self-organization and evolution of open systems have been synthesized and developed to the idea of systems evolution. It is accepted that systems evolution is defined as the process by which a system changes its structure qualitatively to maintain as organized entity in a changing environment. A structural description of the systems behaviour has been adopted: the evolution is described as spatial and temporal change of the system's structure. On reviewing all the important schools of thought of self-organization and self-organizing systems, general conditions for systems evolution to occur are proposed. This general conceptual framework is firmly based on the idea of open systems.

2 A formal model for evolving systems based on mathematical dynamical systems theory

The argument about systems evolution is based on the structural description of systems. Hence the mathematical dynamical systems theory has been employed to advance our discussion. Recent development in mathematical DST has provided us with convenient and powerful tools for describing the dynamical behaviour of open systems.

In this study, a formal model has been outlined to describe the behaviour of dynamical systems. The concept of attractor has been formally defined and it is argued that there are four types of fundamental attractors, i.e., point attractor, periodic attractor, quasi-periodic attractor and chaotic attractor, that can describe the state of any dynamical systems at any level. Another key concept, i.e., structural stability, is also defined based on the concept of attractors. Evolution is therefore defined when an open system loses its structural stability. The process of evolution is described in a similar way as a bifurcation in dynamical systems theory. By using the results of mathematical DST, several possible evolution patterns are proposed which depend on how the system losses its structural stability.

3. The application of the conceptual framework and the formal model to the description of the evolution process of various systems.

This is one important part of this study, although not the main part. Examples from various disciplines have been used to demonstrate the applicability of the conceptual framework and the formal model. Among them are Benard hydrodynamics experiment and laser systems in physical systems, Brusselator and Oregonator in chemical systems, elemental hypercycles in biological systems, predator-prey-food chain in ecological systems, cellular neural networks in artificial neural networks, and some speculations about social economic systems. In most cases, we take the advantages of using the reported results in those different areas and only re-interpreted them by using the conceptual framework and the formal model. In the study of artificial neural networks, we have constructed a novel example based on the concept of cellular network and applied our framework in studying the evolution process of the elaborated system. Another novel example is the coupled Wilson-Cowan nets. Only the activation dynamics of a 3-neuron cellular network and Wilson-Cowan nets have been studied although other examples are mentioned and commented upon. Various attractors have been identified under different conditions and the evolution process of the system in focus has been discussed. Results of numerical analysis are given. In detecting chaotic attractors in the proposed example, it has mainly relied on numerical simulations and this reflects the constraints of the mathematical techniques available to us. Further studies along this line of research are suggested.

The whole study scheme stresses the implication of the dynamical systems theory in the study of the dynamical behaviour of open systems. In many cases, the effort to understand the evolution behaviour of a system is severely restricted by the means of mathematical techniques available to us at this stage. Fortunately, modern computation technology has provided us a useful alternative for analysing the qualitative behaviour of the system. It is worth mentioning that in the case of a chaotic attractor being detected, the credibility of numerical analysis is not always recognised and accepted.

The construction of a general conceptual framework for systems evolution based on the idea of attractors aims to provide a way of understanding the dynamical universe. The concept of attractor is believed to be very useful for the discussion of systems ideas and hence can help us to resolve the confusions so often encountered when expressing systems ideas.

The formal model is not claimed to be universally applicable. However, it is very useful when we are dealing with natural systems where the dynamics of the system is well understood. As a matter of fact, the current study of the so called "the science of complexity" relies heavily on the mathematical DST and from that point of view, the formal model, which itself is based on the mathematical DST, is applicable in most of these cases [for references about the study of the science of complexity, see Stein 1989, Jen 1991, etc.]. There are reasons, through those examples in chapter 5 and 6, to believe that the conceptual framework and formal model developed in this study can be employed in many cases to analyse the complex dynamical behaviour of many open systems. It is admitted that in most cases, at least in our study so far, the strength of this study schemes lies in its descriptive power rather than its predictive power. It can only show which direction systems evolution will follow along , but not the exact state that a system will evolve to. It is not a genuine surprise because evolution is, essentially, unpredictable.

There is another limit for this formal model: it can, at best, only be employed to study the evolution behaviour of the system for a period of time where the dynamics of the system can be expressed by the dynamical equations. In this study, this evolution process is defined as "intra-level evolution".

When the system arrives at another evolution state, the dynamics governing the behaviour of the system changes and the dynamical equation which describes the dynamics is no longer held. Events break laws suitable for the system at a particular stage. Whenever there is a evolutionary event, the old laws are violated and new laws must be found that can adequately describe the behaviour of the system at the next stage of evolution. This involves systems genesis, and the law that directs systems genesis is called "the law of maximum entropy production".

A global picture of the evolution process can not be obtained by only looking the evolution at one level which is governed by a particular law. The "big picture" of the evolutionary cascade in chapter 4 is patched by looking the laws at different stages of the evolution continuum.

7.2 Further Studies

7.2.1 Systems Science: a new domain of research

The laws of physics are believed to be the most fundamental laws that govern the universe. It has been proved that there are only four types of fundamental forces, i.e., gravitational forces, electromagnetic force, nuclear strong force and nuclear weak force, existing in the universe. The materials that make up the universe are also limited in types, with not more that 120 different atoms. How could these four fundamental forces, acting on such a limited set of materials governed by some limited number of physical laws, give rise to such a huge amount of forms and organizations in this universe? Can physics tell us everything about this process?

Physics, both the Newtonian classic physics and relativistic physics, is based on the reductionist philosophy. In searching for the laws of the nature, things are broken into pieces and properties of a system are reduced to its elements. Reductionism stresses the entity rather than relation, state rather than process, and universality rather than variety. [Toffler, 1984 (in Prigogine et.al, 1984]. This reductionists' point of view had been already facing serious challenges when it came to address the phenomena of life long time ago [Jantsch on the historical account of Vitalism and the holistic thinking, 1980]. It has long been realized, or disputed, that life can not be explained by the laws of physics alone. The study of biology can not be reduced to the study of physics. At most, the laws governing the biological system are compatible with laws of physics [Yates, 1988].

In the long journey struggling to find a proper theory for biological systems, vitalism was proposed: biological systems are teleological [reviewed by Bertalanffy, 1968]. Holism was outlined: life is holistic and it is incomprehensible by looking at those parts that form a biological system [Smuts, 1926]. Then came the open systems point of view of Bertalanffy and Schrodinger: for the former, biological systems are regarded as open systems which are purposeful. The goal-seeking behaviour of these systems can be explained by a general term which is applied to any open systems, i.e., the term of equifinality [Bertalanffy, 1968]. For the later, the phenomena of life can be explained as entropic processes of open systems: the growth and development process of biological systems can be described as a general process of which a system changes to a higher ordered state when the negentropy from the environment exceeds certain critical point. The process of life is characterised by both "order out of disorder" and "order out of order" [Schrodinger, 1944]. Through the study of self-organization, the effort to search for the laws that can explain the myth of life continues. Parallel to the striving for the understanding of biological systems, efforts have also been devoted to the discovering of fundamental laws and principles that can bridge the gap between the laws of physics and the laws for the life science. It has been widely regarded that the contribution from the study of dissipative structure is a significant step towards that goal. Among others, include also the syntheses of thermodynamics and Darwinism [Weber, 1988, Wicken, 1987], the thought implied in the Random Boolean networks of Kauffman [Kauffman, "Darwin did not know self-organization"]. However, we are still not able to say how far we have to go before we can achieve a better understanding of biological systems based on the basic laws of physics.

The striving towards a better understanding of biological systems has contributed a lot in a unique way to the development of a new way of thinking: the emerging of systems thinking and systems science in the twenty century [Smuts, 1926; Bertalanffy, 1968]. The uniqueness of this contribution has come from the unique characteristics of the phenomenon of life. To understand that, an open systems point view must be adopted. It is not surprising that, as generally agreed, one of the most prominent funding father of systems theory or systems science in general, Ludvig von Bertalanffy, came as a biologist. Among many contributions concerning the basic concepts, principles, and mechanisms discovered or perceived by him about systems in general, the open systems point of view is certainly an important one [Bertalanffy, 1968].

In accordance with the systems ideas developed in biology, systems ideas have been developed independently, or coordinately, in many fields, such as economics, mathematics [Rappport, 1984; Wiener, 1948], psychology [Miller, 1978], neurophysiology [Ashby, 1952], and some other field like systems engineering, systems analysis , operational research [Churchman 1979, Ackoff, 1973], philosophy etc. [Buckley, 1968].

Although numerous papers and books have been published in the field of systems science, a lot of research and teaching institutions have been established for the study of systems science, a handful of academic societies have been formed for systems science, national or international conferences, annually, bi-anually or trianuanlly, were held under the titles concerning systems and systems science, a question as simple as "what is a system" or "what is systems science" is still a difficult one for any systems experts to answer. It is not surprising that many scientists, especially those working in the traditional research fields like mathematics, physics and chemistry et. al, are still very sceptical about systems and systems science.

As Checkland pointed out recently, systems science is still in a primitive stage, even more primitive than it ought be [Checkland, 1991]. To some extent, it is more adequate to say that there is a field or research called systems research rather than a science called systems science because, as a science, systems science is still lack of the fundamental concepts, laws, principles etc that can clearly define the domain, method and progresses in this field. Although the word "system" is more popular than any other words nowadays, it has been argued that this has actually destroyed the rich meaning of this concept [Checkland, 1991]. Some other people like Klir has argued that systems science is a new dimension of science which differs from the classic sciences in the way that it concerns the relations of the components of an entity which is perceived as an organized whole [Klir, 1985b].

As shown in the previous chapters, it has been advocated to use ideas of emergence and emergent attractors to explain that a system is an organized whole which is composed of some interconnected components and the wholeness is the result of those components. A system is one that is recognized as a system and it is characterised by some emergent properties.It is argued that any systems which can be recognized as a system may be described and characterised by an attractor. Emergent attractors are actually used to describe the state of any systems except a system at its thermodynamic equilibrium. By using the term of emergent attractor, the property of emergence has been stressed.

Someone may say that there is no such a science called systems science, but only many research fields related to the study of the organized entities as a result of relations between many parts. It is right to some extent. However, if we regard the study of various systems related problems from a system's point of view as an emerging scientific research field and admit that it is still to be yet maturated, we can rightly call this new domain, at least loosely, systems science. The common ground for those studies can be adequately summarised and titled as "systems thinking" which mainly concerns the properties of systems. Systems thinking is in contrast to the mechanistic thinking implied in those classic science represented by physics: it is essentially holistic. Therefore, it is also argued that the emergence of systems science represents the emergence of a paradigm: systems thinking is in contrast to the reductionists' mechanistic thinking. Systems thinking and mechanistic thinking is complementary to each other and they are both very important for the full understanding of the universe.

The need for systems thinking came not only from the need for a better explanation of biological systems as mentioned above, but also from the need to deal with complexity facing our human beings: large-scale engineering project, integrated development of economy and society, complex environment problems involved ecological systems as well as human society etc. Systems science is a science for dealing with the complexity [Flood et. al, 1988].

Systems science is yet to be well established. For the further development of systems science, work that needs to be done includes not only the fundamental studies about the basic concepts, general theories, universal laws, and underlying principles, but good practice in applying these ideas to tackle complexity facing us in the real world [Checkland, 1991; Flood, 1990]. The current study scheme suggested in this research is an attempt to understand the evolution process of open dynamical systems by looking at the structural aspects of various systems. Systems evolution is a general property of open systems

7.2.2 Systems Evolution: a property of systems

Open systems are usually situated in environments which are themselves changing. Due to the structural coupling between a system and its environment, the change of environment may trigger the change of the system so that the system can still be maintained as an organized whole. Phenomenologically, it seems that the change of the system is caused by the environment or started from outside. As a matter of fact, there is always microscopic fluctuations within the system which are constantly trying to break the system's stability. The environment can only serve to amplify the internal noise so that, through the complex interconnections of the system's components, the microscopic fluctuations are strong enough to break the system's stability and give rise to the system a new structure. This new structure must be compatible with the structural constraint from the environment due to the coupling relations between them. From this point of view, it is fair to say that environment acts as the selection force for the change of the system.This process is what we call systems evolution in this research.

The dialectic relations between the impact of the environment and a system's inner dynamics must be stressed. Although systems evolution is essentially internally driven, as says by "order through fluctuation", not externally imposed, the importance of the "triggering" of the evolution by the change of the environment should not be neglected. This can be better explained by the formal model proposed in our study.

Systems evolution happens at every level within the hierarchical structure of the universe, but the biological evolution is essentially not equivalent to the concept of systems evolution in the general sense. Biological evolution is different from the evolution of other natural systems, like the formation of chemical waves, in a radical way: the evolution of biological systems is based on self-reproduction. The evolution is characterised by one unique property of biological systems: "good" traits are inheritable. In the terms of systems evolution, biological evolution include "order out of order".

However, the study of self-organization and systems evolution for general systems has revealed that the force of self-organization is still working in biological evolution. Kauffman has once commented about the current study of biological evolution by saying the Darwin did not know self-organization. He argues that "self-organization" is the missing order principle in explaining biological evolution and to reveal this is the contribution from the study of systems evolution.

From a general systems point of view, every open system, under suitable conditions as mentioned in chapter 3, 4, can evolve to a new ordered state so that it can still be maintained as a system in a changing environment. The entity of any system is described as an attractor. Through the notion of emergent attractors, it can be explained why systems science is different from classic science by resorting to the emergent properties of the system. Also through the notion of emergent attractor, it can be established that systems evolution is the general property of open systems. It is believed that the study of systems evolution helps to understand systems and other systems ideas at a fundamental level.

Systems evolution is more a general view towards the understanding the dynamical behaviour of open systems in general than a concrete theory to study any particular systems. To analyse the complex behaviour of any system relies essentially on the understanding of the dynamics of that particular system in a particular environment while the study scheme in our research can only provide a general guideline to how to look at the problem from a structural point of view. It also relies heavily on the mathematical and computational means available to fully reveal the complex behaviour of nonlinear systems by detecting various attractors at various situations.

7.2.3 New Hope From Dynamical Systems Theory

It has been admitted that the whole research programme about systems evolution is based on mathematical dynamical systems theory which is very useful for studying structure and structural change. The strength of mathematical DST, especially with its recent development in nonlinear dynamics analysis, for the study of systems evolution seems to lie on the following facts:

The bifurcation process, through which one type of attractor is replaced by another, can serve as a prototype of emergent phenomena found in any evolutionary process. As argued in this study, for understanding self-organizing systems, or evolving systems in general, Dynamical Systems Theory (DST) seems to provide with:

1) the notion of "attractor" which can be used to describe the state of any systems qualitatively
2) simple geometric models for systems' complex behaviour;
3) a complete taxonomy of attractors possessed by dynamical systems;
4). DST provides a classification of bifurcation patterns that can help us to explain the evolution route of systems.
5) a mathematical rationale for the complex systems to evolve along a particular evolutionary path among different choices (see also Abraham, 1988).

The hope aroused by the development of DST is not just about those as mentioned above, it also provides us insights, techniques to analyse the complex behaviour of various systems, natural and artificial, and help us to understand the complexity implied in this universe. The later is emphasised in current trend of the science of complexity which has been rigourously advocated by some scientists and research institutions [see reference about Santa Fe Institute study of science of complexity 1989-1992, but there the main concern is about the complex behaviour of natural or man made physical systems which can be dealt with by using DST and the focus is about the discovery of chaotic attractors]. Systems evolution is one of the complex behaviour exhibited by many open systems. Systems science is also a science of complexity, but it is not limited in the complexity of physical systems, but encompasses a broad spectrum of systems including human activities [Flood et.al, 1989; Klir, 1991]. To develop a well-established systems science will help us understand the complex world, both the nature and our human beings.

7.2.4 Connectionist model and Neural Networks.

In chapter 6, it has been demonstrated that neural networks can be treated as adapting and evolving systems, and the study scheme established in this study can be applied to analyse their complex behaviour. There are many problems which need to be considered in future studies, both in systems evolution and in understanding neural networks.

The first is more technical. In chapter 6, various types of attractors have been identified in both the CNN model and the Wilson-Cowan nets, but the complete bifurcation diagrams have yet to be constructed. This requires intensive numerical simulations which are both time consuming and resource consuming (high speed computers and large storage capacity are essential). However, this work is worth doing. The constructions of the complete "evolution cascade" will enrich our understanding not just those two networks , but other networks in general as well.

The second is more philosophical. The myth of the computation power of neural networks lies in that it is the emergent behaviour of the networks: it is formed by connecting many neurons together, but the global behaviour of the network as a whole is irreducible to its components. How did it happen? Although the "emergent property" is the right word, but it can not tell much about how actually emergent properties emerge. Can we find a way to connect components, not necessarily neurons, to give a system the desirable "emergent properties"? Will the connectionist model help to find more efficient way of "systems design", so that some machines as well as human organizations with good "emergence" can be obtained? It is hoped that systems science and classical sciences together can provide the right answer.

Appendix 1. Dynamics: the geometry of behaviour

A1.1 Dynamical systems and flow on manifold

A1.1.1 Flows on manifold

In chapter 3, a critical review is given to various schools of thought of self-organization and it is noticed that except similarities between their conceptual foundations, they also use the same mathematical tools in analysis, i.e, mathematical dynamical systems theory. It is no longer a coincidence if having borne in mind that those theories about self-organization are dealing with the dynamical spatial-temporal behaviour of open systems and, dynamical systems theory, in its early or modern forms, has been served as the mathematics of time in sciences ever since Newton. Hirsh, in a paper in 1984, gives an excellent review about systems, dynamics, origin of science and the use of dynamical systems theory (or dynamical reasoning) as the mathematics of time in sciences [Hirsch, 1984]. He argues that to determine the dynamics for various systems, which are the rules for determining the states of systems which correspond at a given future time to a given present state, is a central problem of science, and "*once the dynamic is given, it is the task of mathematical dynamical systems theory to investigate the patterns of how states change in the long run*". The development in dynamical systems theory over the past twenty years makes it possible for us to understand the complex long run behaviour of those systems of which their dynamics are known to us while the finding of the dynamics remains the main problem in sciences.

A dynamical system is one that its state changes in time, and a mathematical dynamical system consists of the space of states of the system together with a rule called "dynamics" which determines how the state changes in time. Primarily, there are two different types of dynamical systems in mathematics according the time steps involved. If the time changes continuously, these systems are called "continuous dynamical systems". When the time takes discrete time step, they are "discrete dynamical systems". Discrete dynamical systems are not merely systems obtained by discreterizing the "continuous dynamical systems" and they have particular properties in their own and must be studied separately in some cases. Definitions of both continuous and discrete dynamical systems will be given in this section.

For the discussion of continuous dynamical systems, the state space is usually a generalized Euclidean space with certain structure, i.e a differential manifold, and the dynamical system is defined as a flow on such a manifold. The definition of a C^r dynamical system can be given as the following:

Definition A1.1.1 **(Continuous dynamical systems)** Let X be a topological space (C^r differential manifold), $\varphi : \mathbf{R} \times X \rightarrow X$ is a continuous map (C^r map). If φ satisfies:

(1) $\varphi(0, x) = x$, for any $x \in X$;

(2) $\varphi(s+t, x) = \varphi(s, \varphi(t, x))$, for any $s, t \in \mathbf{R}, x \in X$

then φ is called a C^0 flow (C^r flow) or a C^0 dynamical system (C^r dynamical system).

It has been proved that any C^r vector field on a compact C^r differential manifold can generate a C^r flow [Guckenheimer et. al, 1983].

A discrete dynamical system consists a sequence of homeomorphism or diffeomorphism and it can be defined as:

Definition A1.1.2 **(Discrete dynamical systems)** Let X be a topological space (C^r differential manifold) and **Z** the set of integers $\phi : \mathbf{Z} \times X \rightarrow X$ is a homeomorphism (C^r diffeomorphism), if ϕ satisfies:

(1) $\phi^0(x) = id(x)$, for any $x \in X$;

(2) $\phi^{k+l}(x) = \phi^k \circ \phi^l(x)$, for any $k, l \in \mathbf{Z}, x \in X$,

then ϕ is called a discrete dynamical system (a C^r discrete dynamical system) which consists of the sequence of homeomorphism or diffeomorphism.

The relationship between the flow (continuous dynamical system) and discrete dynamical system can be described as following:

For any continuous dynamical system φ, a discrete dynamical system ϕ can be generated by discrete sampling for a period of τ:

$$\ldots\ldots \quad \phi^{-2} = \varphi^{-2\tau}, \ \phi^{-1} = \varphi^{-\tau}, \ \phi^0 = id, \ \phi^1 = \varphi^{\tau}, \ \phi^2 = \varphi^{2\tau}, \ \ldots.$$

On the other hand, for a given discrete dynamical system, it may not be necessarily embeded into a continuous dynamical system [Smale, 1967]. This demonstrates that discrete dynamical system contains some properties which can not be studied by continuous dynamical systems [Aulin, 1987]. However, it has been proved that any discrete dynamical systems generated by a sequence of diffeomorphism can be connected with a flow through appropriate "suspension" [Smale, 1967; Cuckenheimer et. al, 1983].

A1.1.2 Differential equations and dynamical systems

In practice, a dynamical system is usually closely connected with a differential equation. One reason for that is that mathematical dynamical systems theory was originally a branch of ordinary differential equations---it is widely accepted that it was stemed from Poincare's qualitative study of differential equations. What is more important is that differential equations have long served as a means to study the dynamical process of natural systems, the trajectories of the movement of plants in astronomy, for example. The importance of differential equations in the history of science has been echoed by many people and among them, Lie said:

> "*Amonth all mathematical disciplines the theory of differential equations is the most important... It furnishes the explanation of all those elemental manifestation of nature which involves time.*" [Sophus Lie as quoted by Hirsh [1984]

As it can be seen in following sections, mathematical dynamical systems theory, growing up from the study of differential equations, and latter with the influence of differential topology, study mainly the qualitative behaviour of dynamical systems while differential equations concerns more about quantitative aspects dynamical systems. It is not only an important and active field of study itself, it is also a very important means for studying differential equations. As far as the study of the qualitative properties of a dynamical system is concerned, autonomous differential equation and mathematical dynamical systems theory are about the same thing. Formally, or mathematically, a dynamical system (flow) on a C^r differential manifold corresponds to a differential equation (vector field) on the manifold and the other way around. This can be easily shown as follows.

Suppose φ_t: $X \rightarrow X$ is a C^r dynamical system on X, and $x \in X$. Define

$$x(t) = \varphi_t(x) = \varphi(t, x)$$

and a vector field f: $X \rightarrow X$ as :

$$f(x) = \frac{d}{dt} \varphi_t(x) \big|_{t=0}$$

then we have the following differential equation:

$$dx/dt = f(x)$$

with $x(t) = \varphi_t(x)$ being its solutions curve which satisfies the initial condition

$$t=0 \rightarrow x(0) = \varphi(0, x) = x_0$$

On the other hand, given an autonomous differential equation

$$dx/dt = f(x)$$

on a manifold **X** and suppose that the conditions for solutions' existence, uniqueness, and continuity are met. Denote x(t) the solution of the equation with the initial condition:

$$t=0 \rightarrow x(0) = x_0 \in \mathbf{X}.$$

For any $t \in \mathbf{R}$, $x \in \mathbf{X}$, define $\varphi_t(x) = \varphi(t, x) = x(t) = x$ then it is easy to prove that:

(1) $\varphi_t(x) = \varphi(t, x)$ is continuous and differentiable on **X**, and

(2) $\varphi_t(x)$ satisfies

(a) $\varphi_0(x) = x$

(b) $\varphi_{s+t}(x) = \varphi_s(\varphi_t(x)) = \varphi_s \circ \varphi_t(x)$

then we have φ being a C^0 flow (C^r flow) or a C^0 dynamical system (C^rdynamical system) on X.

In the case of nonautonomous differential equations, they can always be transformed into autonomous equations with an additional dimension and hence its solution can generate a flow on a manifold. For partial differential equations, there is not a general procedure and universal method to yield a dynamical system (in the mathematical sense). However, some types of partial differential equations as often encountered in studying natural systems, like "reaction-diffusion" equations, their solutions satisfy sufficiently strong theorems of existence, uniqueness and continuity and hence can generate a dynamical system.

In all the following discussions, autonomous differential equations and flows on a manifold are regarded as the same thing. They are all models about the same dynamical process in focus. Qualitative properties of those dynamical systems can be studied by adapting either the point of view of differential equation or flow on manifold. Usually, limit sets, Poincare section etc. are defined in terms of a flow φ_t on a manifold and the structural stability, bifurcation etc. of a vector field $f = d/dt\ (\varphi_t)$ (which defines a differential equation) on the same manifold.

A1.2 Problems about dynamical systems: stability and structural stability

A1.2.1 Invariant sets

A dynamical system in the real world can, in principle, be modelled by either a flow on a suitable manifold (state space) or described by a group of differential equations. The dynamical behaviour of the flow, long run behaviour of individual orbit, its stability etc. can be represented by some special sets of points in the state space. This leads to concepts like invariant sets, stability, structural stability etc. studied by dynamical systems theory .

For a flow φ_t: $X \rightarrow X$, $t \in \mathbf{R}$, the map $I(x)$: $X \rightarrow X, t \rightarrow x(t)$ is the trajectory of $x \in X$. Its image is called the orbit:

$$O(x) = \{\, x(t) \mid t \in \mathbf{R} \}$$

Definition A1.2.1 **(Invariant set:)** A set $\Lambda \subseteq X$ is said to be invariant under the flow φ if $\varphi_t\,(x) \in \Lambda$ for each $x \in \Lambda$ and all $t \in \mathbf{R}$, i.e. $\varphi_t\,(\Lambda) \subseteq \Lambda$.

Clearly, the orbit O(x) of any point x is an example of an invariant set.

Definition A1.2.2 **(Wandering set and non-wandering set)**

(a) Wandering set $\Upsilon(\varphi)$:

$$\Upsilon(\varphi) = \{\, x \mid \exists\, W \ni x, \forall\, t \in R \Rightarrow \varphi_t(W) \cap W = \varnothing \,\}$$

(b) Non–wandering set $\Omega(\varphi)$:

$$\Omega(\varphi) = \{\, x \mid \forall\, W \ni x, \exists \mid t \mid > t_0 > 0 \Rightarrow \varphi_t(W) \cap W \neq \varnothing \,\}$$

Both wandering set $\Upsilon(\varphi)$ and non-wandering set $\Omega(\varphi)$ are invariant sets, but $\Upsilon(\varphi)$ is an open subset of X while $\Omega(\varphi)$ is closed. Any point in $\Upsilon(\varphi)$ or $\Omega(\varphi)$ is called wandering point or non-wandering point respectively.

Wandering set and non-wandering set characterise the global structure of the flow on manifold, i.e they figure out those special points on manifold which either be carried away or stay where they are along the flow. We are also interested in the direction of movement of individual orbit, i.e the long run behaviour of the trajectory of x, hence we define the limit set of the orbit of x.

Definition A1.2.3 **(Limit sets)**

(a) α–limit set $L_\alpha(x)$:

$$L_\alpha(x) = \left\{\, y \mid y \in X, \exists\, \{t_i\}_{i=1}^{\infty} \subset R, \lim_{i \to \infty} t_i = -\infty \Rightarrow \lim_{i \to \infty} \varphi_{t_i}(x) = y \,\right\}$$

(b) ω–limit set $L_\omega(x)$:

$$L_\alpha(x) = \left\{\, y \mid y \in X, \exists\, \{t_i\}_{i=1}^{\infty} \subset R, \lim_{i \to \infty} t_i = \infty \Rightarrow \lim_{i \to \infty} \varphi_{t_i}(x) = y \,\right\}$$

It is easy to see that both α- and ω- limit sets are invariant under flow φ, and they belong to the non-wandering set:

$$L_\alpha(x) \subseteq \Omega(\varphi), \qquad L_\omega(x) \subseteq \Omega(\varphi) \quad \text{for any } x \in X.$$

These limit sets are very important in the study of the qualitative behaviour of dynamical systems and we go further to describe some of their subsets which are often encountered in the study of vector fields or differential equations.

Definition A1.2.4 **(Stationary point or equilibrium)** If there is a point $x_0 \in X$ which satisfies:

$$\varphi_t(x_0) = x_0 \text{ for all } t \in \mathbf{R}.$$

then x_0 is called a stationary point or equilibrium of the flow.

In the case of a stationary point, $x_0 = L_\alpha(x_0) = L_\omega(x_0)$. Denote all E the set of all the equilibria of the flow and the E is a closed subset of X.

Definition A1.2.5 **(Periodic point and periodic orbit)** If there is a point $x_0 \in X$ and there exists a T>0 such

$$\text{that } \varphi_{t+T}(x_0) = x_0 \text{ for all } t \in R$$

then x_0 is called a periodic point for and the minimum of T is called the period of x_0. In this case, $O(x_0)$ is called a periodic orbit (or cycle) when it is not a stationary point.

Obviously, we can see that $O(x_0) = L_\alpha(x_0) = L_\omega(x_0)$. Both the set of equilibria and the periodic orbit are limit sets and what is more interesting is the way they are approached by the trajectories of those points which is in the neighbourhood of them.

Definition A1.2.6 **(Stability of equilibria)** Denote $x_0 \in X$ is an equilibrium of flow φ, then

(a) x_0 is stable if for every neighbourhood V of x_0 in X, i.e, $x_0 \in V \in X$, there is a neighbourhood $V_1 \quad V$ such that for every $x \in V_1$, $\varphi_t(x) \quad V$ for all $t \in \mathbf{R}$.

(b) x_0 is asymptotically stable if x_0 is stable and V_1 can be chosen so that for every $x \in V_1$, $\varphi_t(x) \to x_0$. as $t \to \infty$.

When an equilibrium is asymptotically stable, we call it a fixed point or a point attractor. It attracts some neighbour of itself. Generally, an attractor needs not to be a point, it can be a cycle or even a torus on a surface. Attractor of a dynamical system can be defined in the general terms.

Definition A1.2.7 **(Attractor)** Let A be a sub-set of X, $U \supset A$ is a neighbourhood of A with $A \supseteq \cap_{t>0} \varphi_t(U)$. A is called an attractor of the system S, if and only if it satisfies the following properties:

(1) Attractivity: for every V with $V \supset A$, we have $V \supseteq \varphi_t(U)$ for all sufficiently large t;

(2) Invariance: $\varphi_t(A) = A$, for all t;

(3) Indecomposibility: if there exist another such set A' satisfying (1) and (2) and $A \supseteq A'$, then $A' = A$.

These attractors are subsets of the limit sets of a flow and they represent some important types of invariant sets. The attractivity implies that they are some local centres on the manifold and to which the trajectories of some neighbourhood points are approaching. They are locally stable and are actually corresponding to the state of a real dynamical systems which are the only meaningful and observable ones in real world. We can differentiate different attractors which represent different state of dynamical systems, i.e point attractor representing stable equilibrium, periodic attractor representing periodic oscillations and chaotic attractor representing chaotic behaviour. They can be mathematically defined and described. Much efforts have been devoted to the finding of the set of non-wandering points, among them, attractors are more interesting than others.

Example

Denote $\varphi = e^{At}$ is a flow on $X=R^2$, and it defines a vector field F=A and hence generates a differential equation

$$d/dt\,(X) = A\,X$$

where $A = ((a, c)^T, (b, d)^T)$

The non-wandering points of the flow, which change when a, b, c, d change, can be characterised as the following (figure A1.21):

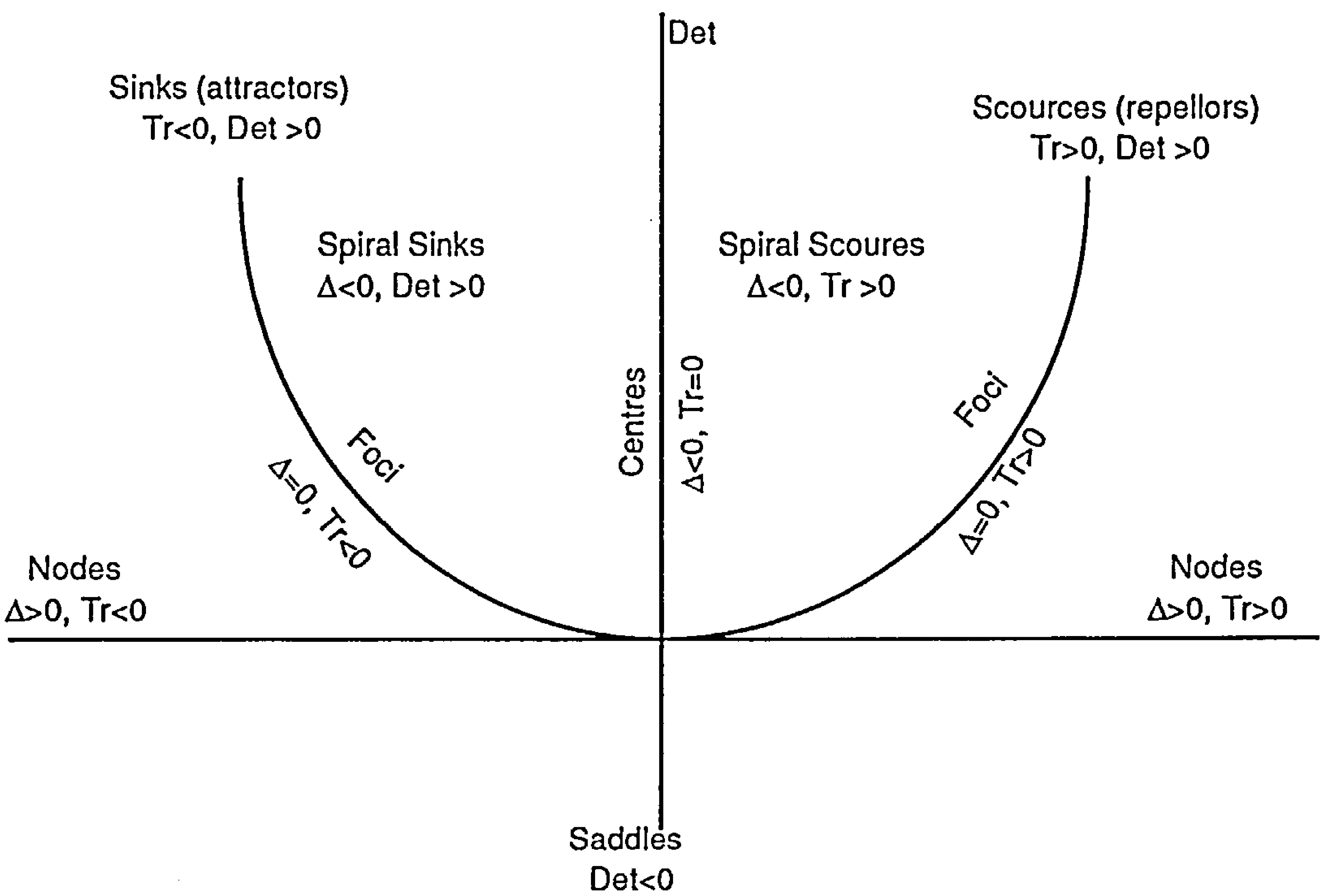


Figure A1.2.1 The taxonomy of limit sets of the 2-dimensional linear dynamical system ($Tr = Tr\,A = a+d$; $Det = Det\,A = ad-bc$; $\Delta = (Tr)^2 - 4\,Det$)

There is no other limit set like periodic orbit or aperiodic orbit in linear dynamical systems, but even though, when the dimensions of the dynamical systems become higher, the taxonomy of the non-wandering points is more difficult. This is nothing comparing with the difficult in finding the limit sets for nonlinear dynamical systems and many work remains to be done.

A famous global theorem concerning the existence of a period orbit of dynamical systems on a 2-dimensional manifold and the taxonomy of non-wandering points of a 2-dimensional nonlinear flow.

Theorem (*Poincare-Bendixson Theorem* [Hirsh and Smale, 1974])
A non-empty, compact limit set of a flow on the plane, which contains no fixed point, is a closed orbit.

There is no general results concerning the location of periodic orbit in higher dimensional systems.

A1.2.2 stability and structural stability

Those invariant sets, especially attractors, are important for understanding the long run behaviour of a flow on manifold. There is another concept which is in the centre of the study of dynamical systems, i.e, the structural stability of a flow or vector field. For convenience, we use vector $F=d/dt\ (\varphi_t)$ rather than flow φ_t in our discussion about the structural stability.

Definition A1.2.7 Equivalent and topologically equivalent: Two C^r vector fields, F, G are said to be C^k equivalent ($r \geq k$) if there exists a C^k diffeomorphism h which takes orbits O(F) of F to orbits O(G) of G. That is to say, for any x and t_1, there is a t_2 such that : $h(F_{t_1}(x)) = G_{t_2}(h(x))$. When k=0, we say that F and G are topologically equivalence,

Definition A1.2.8 Structural stability: A C^r vector field F on a manifold X is structural stable if for any sufficient small C^k perturbations $G = F + \delta$ ($\delta \in C^k(X)$), F and G are topologically equivalent.

(or, there exists a neighbourhood N of F in $C^k(X)$, $N \in C^k(X)$, such that for every $G \in N$, F and G are topologically equivalent.)

Structural stability is a much talked topic and is still one of the most important problem in mathematical dynamical systems theory. The discussion centres on the criteria for deciding whether a flow, or a vector, is structurally stable and whether structurally stable systems are dense in the space of dynamical systems. It has been proved that the set structurally stable systems on 2-dimensional compact manifold is open and dense [Peixoto, 1962]. It is not true for dynamical systems on manifold with dimension ≥ 3. [Smale, 1967]. For 2-dimensional dynamical system, a general result has been proved by Peixoto.

Theorem [Pexoto, 1962].

Let φ is the flow on 2-dimensional manifold X, φ (or its vector field F) is structural stable if and only if it satisfies that:

(i) all equilibrium are hyperbolic;

(ii) all closed orbit are hyperbolic;

(iii) there are no orbits connecting saddle points.

(where a limit set is hyperbolic means that the eigenvalues of the vector field at that point (set) is not equal to 0).

Although the concept of structural stability is one of the key concept in our formal model of evolving systems, mathematically, it is not perfect for discussion of the stability of dynamical system: structurally stable systems are not dense for all finite dimensional compact manifold. One can trace back to the definition of equivalence relation on which the concept of structural stability is based. New definitions of equivalence relation lead to new types of of stable dynamical systems or stable vector fields. The ultimate goal is to have the definition restrictive enough to permit classification of the stable ones, but , at the same, to have the stable vector fields *generic*, that is, the intersection of a countable sequence of open dense sets. Much of the research in finite dimensional abstract dynamical systems in the last twenty years has been devoted to this general problems [Smale, 1967; Peixoto, 1973]. One of the advances made recently is by Zeeman which defines an ε-equivalence hence ε-stability as an alternative to structural stability. It is argued that the the new definition has a number of advantages over structural stability, one of them is that (ε-)stable systems are dense for on any finite dimensional compact manifold. For the definition of ε-equivalence is based on the Fokker-Plank equation, it is particularly aimed at dissipative nonlinear systems, including those with chaotic attractors [Zeeman, 1988b].

A1.2.3 Vector fields with parameters

The property of structural stability is the characteristics of individual dynamical system and it reflects how robust the system is under the external perturbations. It is of vital importance especially when we use a vector field on a manifold to describe a dynamical process. If the vector field is structurally stable, then the model can faithfully, to certain extent, describe the behaviour of the modelled system, or otherwise, it may be totally wrong to use it as a model. The importance of the structural stability of the dynamical model, or a vector field, which is employed to model a dynamical system, is emphasized by Arcil [Arcil, 1986].

From the practical point of view, a dynamical system is usually exposed to a changing environment (as as discussed in chapter 2 and 3). In the model, the impact of the environment to the behaviour of the system is usually represented by a parameter or a parameter vector μ. As these parameters are varied, changes may occur in the qualitative structure of the system for certain parameter values. This change is called a bifurcation and it is closely related with the loss of the structural stability of the system.

With the appearance of a parameter vector, the dynamical system takes form like:

$$d/dt\,(x) = F_{\mu}(x)\,, \quad (x, t) \in X \times R, \mu \in V \ \ (V \text{ is a subset of } R^m)$$

and the flow is then represented as

$$\varphi_{\mu}(x, t) = \varphi_{\mu, t}(x)$$

When there is a parameter vector appearing in the vector field, a family of vector fields or flows are obtained. Invariant sets, like equilibria, periodic orbit etc. can be defined, but it has to take into account parameter λ and those invariant sets might not be the same for all $\mu \in V$. This leads to the concept of bifurcation.

Definition A1.2.8 **(Bifurcation)** F_{μ} is a family of vector fields on a n-dimensional manifold X, and $\mu \in V$ (V is a subset of R^m) is a parameter vector. When the field is not structurally stable at certain $\lambda_0 \in V$, we say that the vector fields (or dynamical system) bifurcate at λ_0 and λ_0 is called a bifurcation point.

The definition of bifurcation is based on the concept of "structural stability", structurally stability of a dynamical system is usually analysed by detecting bifurcation points. Various methods are proposed to find the bifurcation point of vector fields and it is known that there more numeric methods then analytical methods in bifurcation analysis.

As an example, considering dynamical systems described by the following one-parameter families of vector fields on $X = V = R$.

(a) $F_\mu(x) = \mu - x^2$

(b) $F_\mu(x) = \mu x - x^2$

(c) $F_\mu(x) = -(1+\mu)\, x^2$.

The set of equilibrium can be easily obtained analytically by setting $F_\mu(x) = 0$. The phase portrait of these systems and their bifurcation diagram can be illustrated as the following (figure A1.2.2):

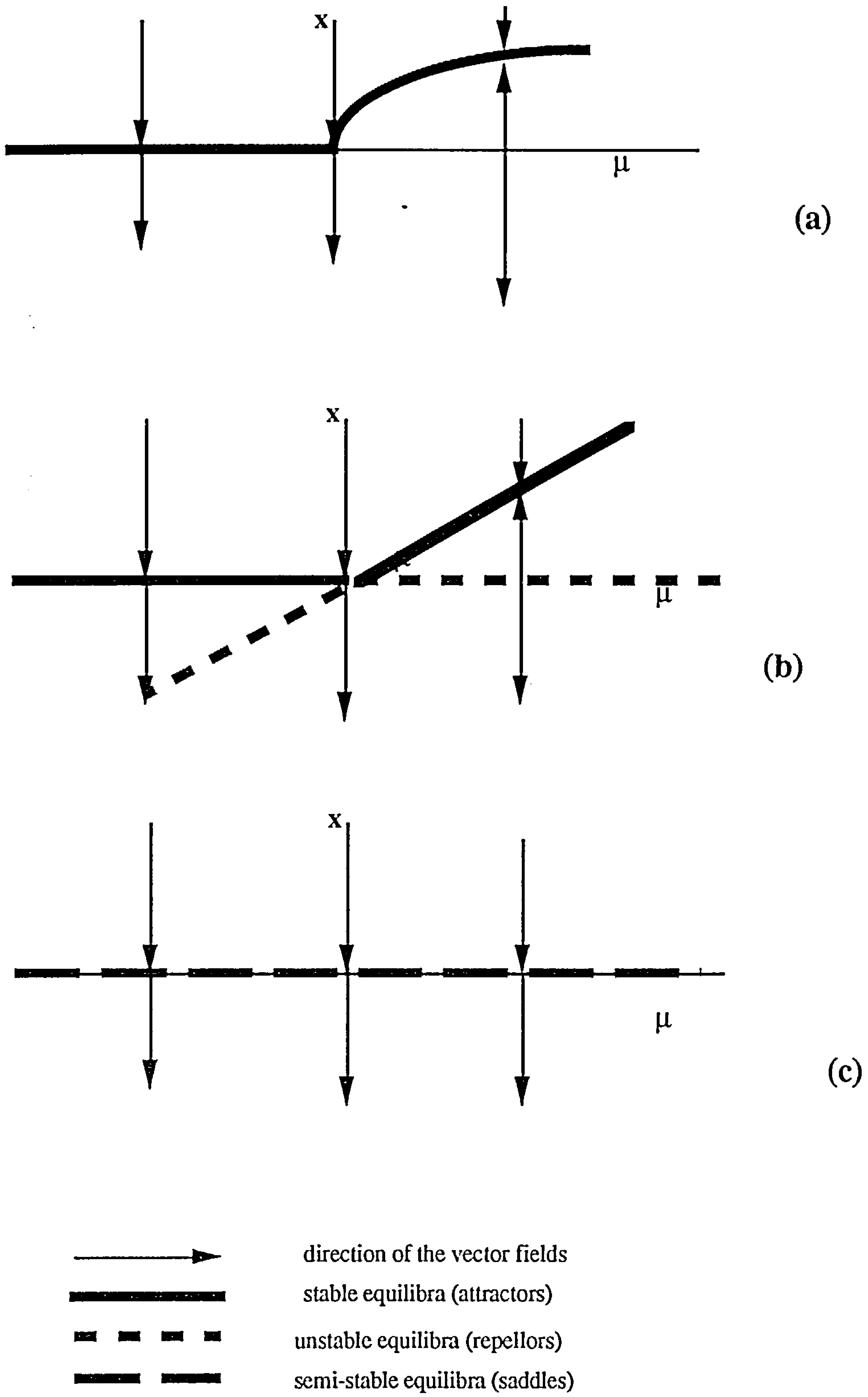


Figure A1.2.2 Bifurcation diagram for vector fields (a), (b), (c).

This is the simplest case about the local bifurcation of dynamical systems. When the system has a higher dimension, the bifurcation diagram can be very complicated. Apart from some special case, there is no analytic solutions for a general bifurcation problem, especially when there are complex attractors (repellors) appearing through bifurcation, like bifurcation to periodic attractors and chaotic attractors. In that case, it has to largely depend on the numerical techniques to find out bifurcation points and new attractors. This numerical method is even more than "convenient" and "important" when there is a bifurcation to a chaotic attractor. It is the an important method that can helps to identify the appearance of a chaotic attractor, because there is no analytical representation of a chaotic attractor. Various local bifurcation patterns will be discussed in section A1.4

A1.3 Invariant Manifold

A1.3.1 Invariant manifold

For the equilibrium is the only limit point of a linear dynamical system, the global behaviour of such a linear system depends on the properties of its equilibrium which in turn are decided by the matrix describing this system, like the example of a linear flow on the plain [see the above section]. In a high dimensional system, to decide the stability of the equilibrium becomes more difficult, but, in principle, it can be done analytically. There is a whole body of knowledge about linear systems which provides numerical as well as analytical techniques for the stability analysis of the system's equilibrium [D'Azzo et. al, 1987]. It is the foundation of linear control system analysis and design and it relies on matrix analysis.

A nonlinear dynamical system can exhibit more complex behaviour than a linear one can do. It usually has a very complicated non-wandering set which, especially with its subset called "attractors", prescribes the long ran behaviour. The phase space, or the state manifold, is divided into different regions, which we call the basins of attraction, and in each region the system is attracted to an attractor. Depending on where the system starts, the system's long run behaviour is determined by the type of attractor in that basin. On the boundary of a basin, which is called a "separatrix" and is usually a saddle-like limit point (sets), the behaviour of the system is decided not by a attractor, but rather than by whether it is on the stable manifold or the unstable manifold (see the following definition). Therefore, if we want to give a global picture of the behaviour of a system on the whole state manifold, we must look into different local attractors in different attracting basins separated by some usually complicated separatrices. This is a sharp contrast with the a linear dynamical system where its behaviour is decided globally by its equilibrium point. When there are external parameters, the behaviour of a nonlinear system becomes even more difficult because the change of parameter can lead to bifurcations. It is known from the above section that bifurcation means the loss of structural stability and the behaviour of the system can change qualitatively.

In dynamical systems theory, geometrical methods are employed which discuss the global structure of systems [Palis and Melo, 1982]. However, there are few global results in dynamical system theory which can be applied in the study of nonlinear dynamical systems arising in practical fields of research . A more practical method, which has been adopted by almost all the applied mathematicians in application, is based on a procedure of linearization. It is employed to study the local property of a system by linearizing a dynamical system in the neighbourhood of a limit point, or a limit set (in most cases, it is a fixed point) and then use the result of linear systems analysis. The following theorems guarantee that the procedure of linearization is valid in studying the local property of the system.

For a non–linear flow φ_t on X, there is a corresponding vector field f which gives the usual differential equation $\dot{x} = f(x), x \in X$. An equilibrium (or fixed point) x^* of the flow is given by $f(x^*) = 0$. Then the linearization of $\dot{x} = f(x)$ at x^* is a linear dynamical system:

$$\dot{y} = Df(x^*)\, y$$

where

$$DF(x^*) = \left[\frac{\partial f_i}{\partial x_j} \right]_{i,j=1}^{n} \Big|_{x=x^*}$$ is the Jacobian at the fixed point

and $y = (y_1, \ldots, y_n)^T$ are local coordinates at x^* on X.

Definition A1.3.1 **(Hyperbolic point)**

An equilibrium x* of a flow φ_t (field *f*) is said to be hyperbolic if no eigenvalues of D*f* (x*) has zero part.

With a hyperbolic equilibrium x*, the nonlinear dynamical system can be simplified at that point locally to a linear flow.

Theorem (*Hartman- Grobman* [Guckenheimer et. al, 1983])

Let x* be a hyperbolic fixed point of x' = f(x) with flow φ_t on X. Then there is a neighbourhood N of x* on which φ is equilvalent to the linear flow exp(Df(x*)t)x.

With this theorem the local behaviour of a nonlinear dynamical system can be studied by looking into an equivalent linear flow whose behaviour is quite well known [Hirsch and Smale, 1974]. The following theorem says more about the system's local behaviour.

Theorem (*Invariant manifold*)

Let φ_t be a flow on X with a hyperbolic fixed point x^*. Then on a sufficiently small nerghbourhood N of x^*, $x^* \in N \subset X$, there exist local stable and unstable manifolds:

$$W^s_{loc}(x^*) = \{ x \in X \mid \varphi_t(x) \to x^* \text{ as } x \to +\infty \}$$

$$W^u_{loc}(x^*) = \{ x \in X \mid \varphi_t(x) \to x^* \text{ as } x \to -\infty \}$$

of the same dimension of E^s and E^u for the linear vector fleid $DF(x^*)$ and tangent to them at x^*.

These theorems enable us to study the behaviour of nonlinear dynamical systems locally by resorting to its local equivalence of linear flows which is relatively easy to handle (there are well established techniques for linear flows and the whole body of knowledge depends heavily on the matrix analysis). By patching together all those pictures about the system's local behaviour, the global behaviour is clear.

A1.3.2 Centre manifold theorem

In the hyperbolic case, the local behaviour of a nonlinear dynamical system can be studied by a linear one, as we discussed above. In the non hyperbolic case, the problem is getting harder. By extending the idea of invariant manifold theorem to the local behaviour at non-hyperbolic fixed points of nonlinear flows, a centre manifold theorem is obtained. Without loss of generality, assume that the fixed point being dealt with is at the origin.

Theorem (*Centre Manifold theorem*)

The origin is the fixed point of a C^r flow on X and f is the corresponding vector field. Let $A = Df(0)$ and divide the spectrum of A into three parts, σ_s, σ_c, σ_u with:

$$\sigma_s = \{ \lambda \mid \mathrm{Re}\lambda < 0\}$$

$$\sigma_c = \{ \lambda \mid \mathrm{Re}\lambda = 0\}$$

$$\sigma_u = \{ \lambda \mid \mathrm{Re}\lambda > 0\}$$

Let the generalized eigenspaces of σ_s, σ_c, and σ_u be E^s, E^c, and E^u respectively. Then there exist C^r stable and unstable local manifolds W^s_{loc} and W^u_{loc} tangent to E^s and E^u at 0 and a C^{r-1} centre (local) manifold W^c_{loc} tangent to E^c at 0. The manifolds W^s_{loc}, W^c_{loc}, and W^u_{loc} are all invariant for the flow φ_t. The stable and undtable local manifolds are unique, but W^c_{loc} need not be.

In general, the centre manifold theorem isolates the complicated asymptotic behaviour by locating an invariant manifold, centre manifold, tangent to the subspace spanned by the generalized eigen space of eigenvalues on the imaginary axis. It is very important and useful in situations where bifurcation occurs. Actually, the centre manifold provides a means for systematically reducing the dimension of the state spaces which need to be considered when analysing bifurcations of a given type. It underlies the well developed bifurcation theory, i.e, Hopf bifurcation theory. [Marsden and McCracken, 1976] and Carr [Carr, 1981].

Apart form its technical usefulness, it is also implied that the behaviour of a dynamical system can only rely on a few factors at a critical point. This has been used as a rigourous support for Haken's "slaving principle" in synergetics [Haken, 1983a; 1983b].

The invariant manifold theorem, especially the centre manifold theorem provides us with an important tool tool for studying the complex behaviour of nonlinear dynamical systems, particularly in the appearance of parameters. The analytical as well as numerical analysis of a nonlinear dynamical system through linearization procedure is rested on a solid mathematical theory. Although there are many cases that can not be tackled by this method, for example, in the case of chaotic behaviour, it at least enables us to solve part of the big problem.

A1.4 Limit sets: definitions and descriptions

A1.4.1 description and classification of limit sets

A. equilibrium or fixed point

As stated in Hartman-Grobman theorem, the local behaviour of a nonlinear dynamical system at a hyperbolic point can be studied by its linear equivalence. In the case of 2-dimensional dynamical systems, typically systems on the plain R^2, the global behaviour of a linear dynamical systems is very clear and so is the local behaviour of any plain dynamical systems at a fixed point. Referring to the 2-dimensional linear dynamical system discussed in section A1.2, the behaviour of the system is decided by the property of its fixed points. When the fixed point is hyperbolic, it belongs to one of the three distinct types: point attractor (asymptotic stable), point repellor (unstable), saddle point (attracting-while-repelling) depending on the signs of the eigenvalue of the system given by the matrix A. In the degenerate case, it is a centre. They can be illustrated in the following figure.

In higher dimensional manifold, it is difficult to visualize these three different kinds of hyperbolic fixed point, but an attractor can be connected with the attracting behaviour of that fixed point, a repellor with the repelling behaviour, and the saddle with the attracting-and-repelling behaviour of the system. For any nonlinear dynamical systems on a manifold with any dimensions, they exhibit the same attracting, or repelling, or attracting-and-repelling local behaviour in the neighbour of equilibria depending on whether the fixed points are attractors, or repellors or saddle points. To decide the type of a fixed point, what needs to be done is to find the signs of the eigenvalues of its Jacobian at that equilibrium point. This is described in the following theorem.

Theorem [Hirsch, 1984] *Let $x_0 \in X$ be an fixed point of a vector field f on X, σ) C the spectrum of the linear operator $Df(x_0)$ on the tangent space X_{x_0}, then*

(a) If Re $\lambda < 0$ for all $\lambda \in \sigma$, then x_0 is a point attractor;

(b) If Re $\lambda > 0$ for all $\lambda \in \sigma$, then x_0 is a point attractor;

(c) If Re $\lambda < 0$ for some $\lambda \in \sigma$, and Re $\lambda < 0$ for others, then x_0 is a saddle point..

It is obvious from Hartman-Grobman theorem and the classification of fixed point of discussed in figure A1.4.1.

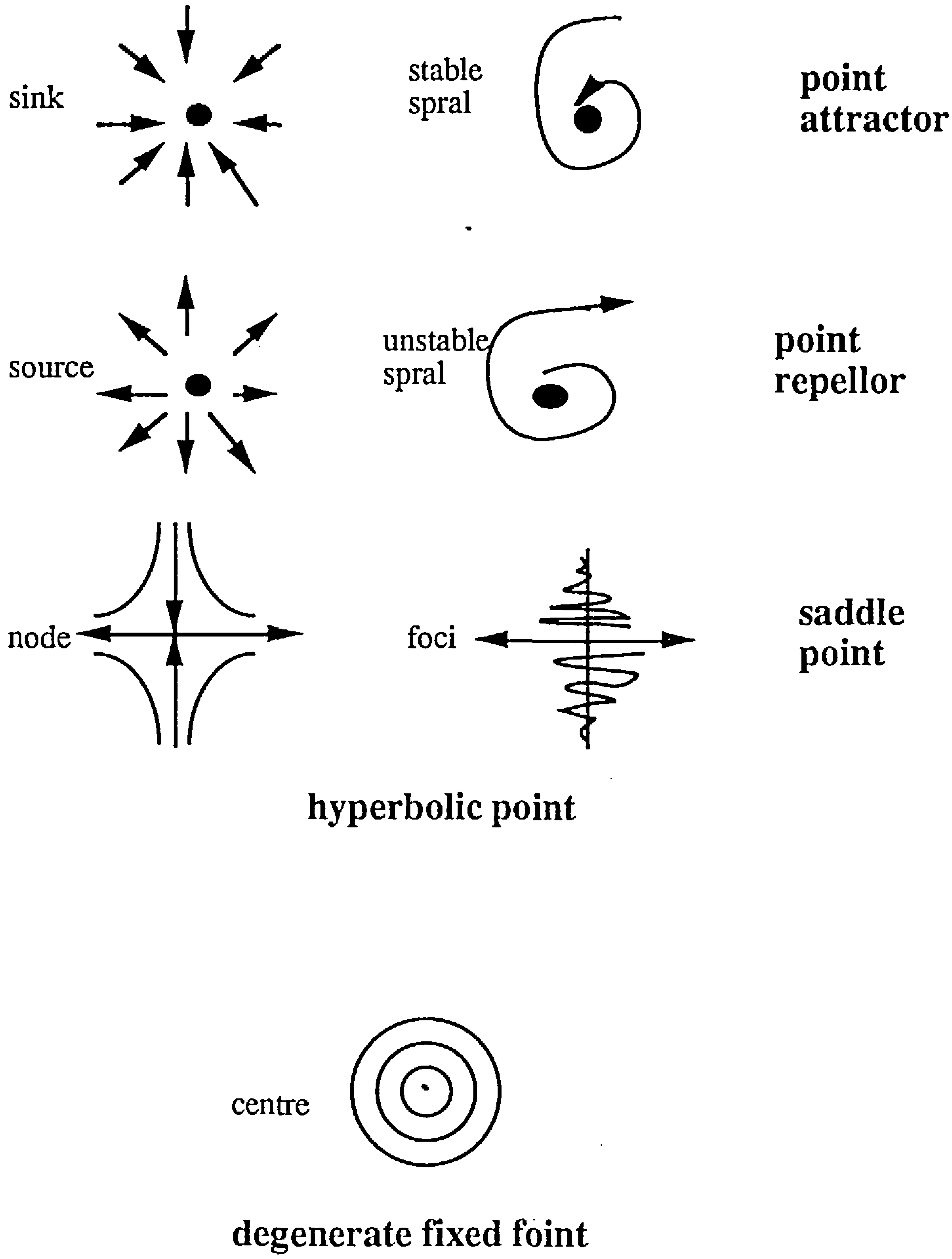


Figure A1.4.1 Hyperbolic and degenerate points

B *Limit cycles*

For a nonlinear dynamical system on a manifold M with dimension≥2, the above equilibria do not exclusively describe its local behaviour. It is often encountered that the system may have a complex nonwandering set which consists a periodic orbit or even a chaotic orbit. They represent the complex asymptotic behaviour in a sufficient small neighbourhood of those limit sets. In practice, the cyclic movement, or oscillation of a system is observed. In dynamical systems theory, such kind of oscillation behaviour is defined as a periodic orbit, we have already given the definition in mathematical terms in section A1.2. Similar to the classification of fixed points, we have three distinct periodic orbits, i.e., periodic attractor, periodic repellor, and saddle-like periodic orbit (in manifold with dimension ≥ 3). They can be illustrated as:

Figure A1.4.2 Periodic orbits

When it is a straightforward procedure to find a fixed point and to decide its type (subject to the difficult of solving algebra equations either analytically or numerically), it is a difficult task to find the periodic orbit. It is by equally difficult to find out whether a known periodic orbit is attracting, repelling, attracting-while-repelling. As a matter of fact, a great deal of efforts has been devoted to the analysis of the existence, stability, and type of periodic orbit of a dynamical system. The most successful case is when the dynamical system is described by a vector field which is a polynomial of the state variables. For plain dynamical systems (systems on R^2), some special techniques have been developed to detect the periodic orbits [Guckenheimer et. al, 1983].

There is also an elegant theorem concerning the existence of a periodic orbit in dynamical system on any manifold and it is called Poincare-Bendixson theorem [see section A1.3]. Theoretically, it can be used in any situation. However, there is no similar theorem for dynamical systems with dimensions higher than 3. There is a delicate method for finding a periodic orbit and it is based on concepts like "Poincare Section" and first return map and we will discuss it in the following section.

Hopf bifurcation theory has also been widely used in locating periodic orbits. It provides analytical method for finding the periodic orbit as well as a index for its stability and it can be applied in a wide range of systems as long as there is a Hopf bifurcation. [Marsden et. al, 1976].

C quasi-periodic orbits

A typical quasi-periodic orbit is one on the torus surface T^2. It is the combination (Cartesion product) of two periodic orbits with different periods and the ratio of these two periods is an irrational number. The behaviour of such a quasiperiodic orbit is a bit more peculiar than a periodic attractor: it moves on a surface of a torus and it never re-passes what it has passed. It is wandering on the surface and it covers every point of that surface.

A quasi-periodic orbit exhibits some regularities because its behaviour can be decomposed to two periodic behaviour. It also exhibit some singularities because it is not exactly periodic: although it may come very close to its previous trajectory it never passes exactly a point it passed before. For these reasons it is called a quasi-periodic orbit.

A quasi-periodic orbit, especially the one on T^2 which is usually called "a torus", is a favourite for mathematicians working on abstract dynamical systems theory where it is easy to construct and its properties are very interesting [Smale, 1967]. When a dynamical system is represented as a diffeomorphism, a torus is an elegant example for demonstrating generic properties like structural stability, transversality etc.. Haken has reported that such quasi-periodic orbits do exist in practical systems (see, [Haken, 1983b], et. al.).

For some people, quasi-periodic orbits are grouped together with chaotic orbits as "aperiodic orbits" (or "aperiodic attractors"). It is, however, in this study that quasiperiodic attractors and chaotic attractors are treated as two different types of attractors.

torus attractor
(asymptotic stable torus)

torus repellor
(unstable torus)

Figure A1.4.3 Quasi-periodic orbits

Poincare section is one kind of the techniques which can be used to identify quasi-periodic attractors. Another one is the Lyapunov spectrum which we will introduce in the following section.

D chaos and chaotic attractors

Chaos, or chaotic attractor, is a very popular topic nowadays. It refers to the phenomena that dynamical behaviour of a deterministic system is in any practical sense unpredictable and its quantitative behaviour has an extreme sensitivity to initial conditions. There is a very good introductory book on chaos by Gleick, although it is not so mathematical [Gleick, 1987]. Another interesting introductory paper is by Shaw [Shaw, 1981]. For a more mathematical discussion about chaos, see Guckenheimer and Holmes [Guckenheimer et. al, 1983].

The best way to get a feeling about what a chaotic attractor looks like is still to look at examples. There are several "classical" examples of chaotic attractors, i.e. Lorenz attractor, Rossler attractor, in flows and Smale's Horseshoe, Henon attractor for diffeomorphism [Shaw, 1981; Cvitanovic, 1989; Holden, 1986]. In a less restrict mathematical form, a chaotic attractor A can be defined as following:

Definition A1.4.1 **(chaotic attractor)**

A is an attractor of a dynamical system, we call A a chaotic attractor if and only if the asymptotic behaviour of the system in the neighbourhood of A depends sensitively on the initial conditions.

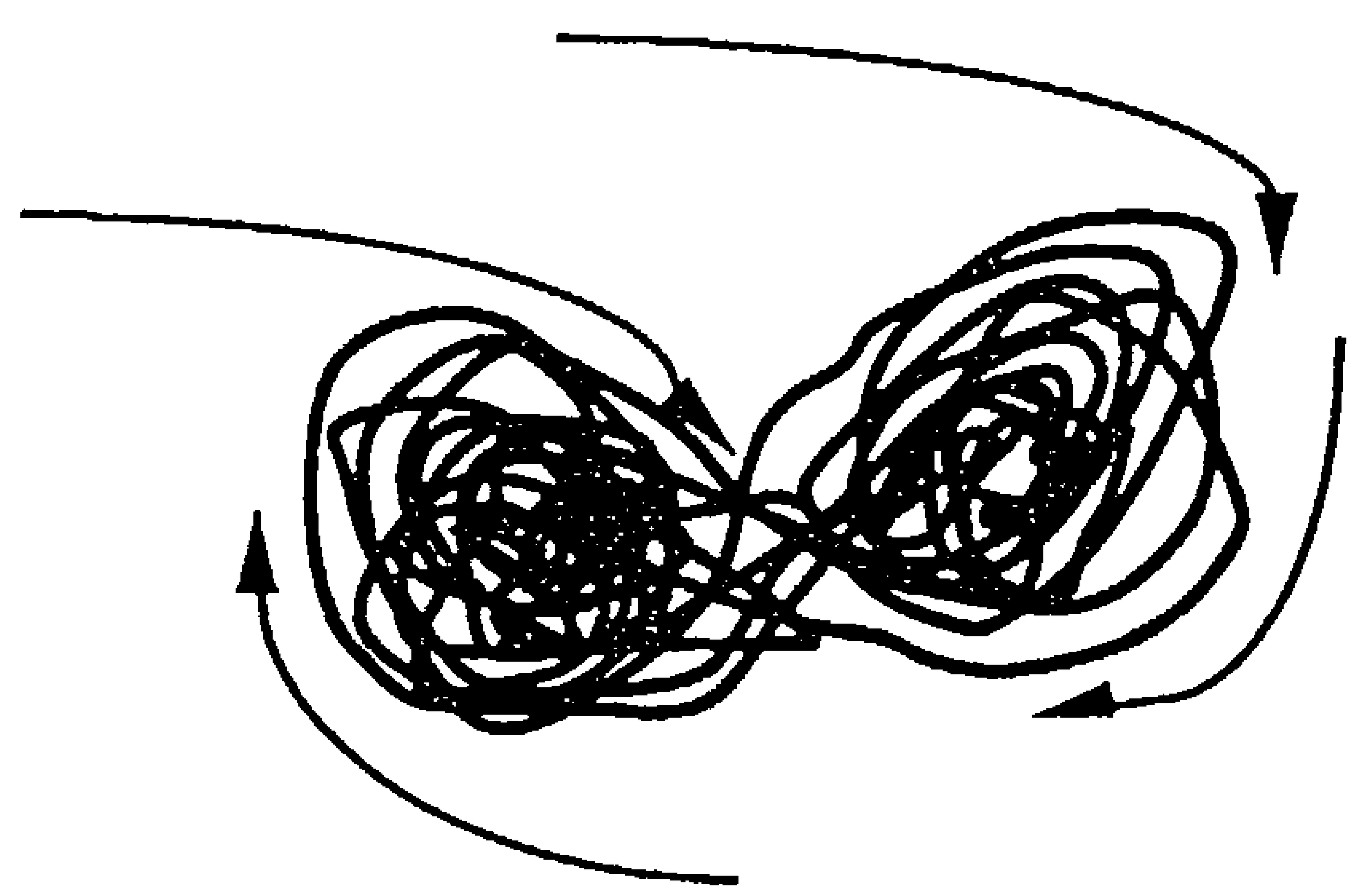


figure A1.4.4 A chaotic attractor

To find out that whether an attractor is chaotic or not, there is almost non rigourous analytical techniques except in a very special case where a Smale Horseshoe can be found [Zeeman, 1988a]. In most cases, we have to rely on numerical analysis which can provide us some information about the attractor. These numerical-analysis-dependent methods include construction of Poincare section, calculation of Lyapunov exponent, and calculation of fractal dimension and power spectra (Fourier spectrum) [Berge et. al, 1984; Eckmann and Ruelle, 1985]. They will be discussed in the following section.

A1.4.2 Description of limit sets

To detect and describe the limit set of a nonlinear dynamical system becomes very difficult when the limit set consists some complicated orbits like periodic orbit or chaotic orbit. When Many methods stressing numerical analysis, like using Lyapunov exponent, calculating fractal dimension (for chaotic orbit), have received attention over the past ten years. Here we give a brief introduction of four of the most used methods finding and describing periodic, quasi-periodic, and chaotic orbit.

A. *Poincare section*

The principle underlying this method is fairly simple: to construct a diffeomorphism from a flow and analyse the behaviour of this diffeomorphism. This can be defined in rigourous terms.

Definition A1.4.2 **(Cross section)**

Let φ be a flow on M with vector field F and suppose that Σ is a submanifold with $\dim \Sigma = \dim M - 1$ satisfying:

(1) every orbit of φ meets Σ for arbitrarily large $t>0$ and $t<0$;

(2) if $x \in \Sigma$ then $F(x)$ is not tangent to Σ.

then Σ is said to be a cross section of the flow φ.

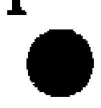
Let $y \in \Sigma$ and there are many $t>0$ and $t<0$ satisfying $\varphi_t(y) \in \Sigma$. Denote $\tau(y)$ be the least positive time for which $\varphi_{t(y)}(y) \in \Sigma$, then we can define a Poincare map as follows.

Definition A1.4.3 **(first return map)**

The first return map, or Poincare Map, $P: \Sigma \to \Sigma$ is defined to be:

$$P(y) = \varphi_{t(y)}(y), \qquad y \in \Sigma$$

By construction P is a diffeomorphism and it can be used to study the properties of flows in a manifold one higher dimension. As a matter of fact, an point attractor of P in Σ corresponds to a periodic attractor in the phase portrait of φ on M. When the Poincare section has a complex structure, the underlying dynamical system has a more complicated behaviour [Berge et. al, 1984]. Actually, this method is also used for finding out other complex limit set of dynamical systems. This can be illustrated as follows (figure 1.4.5):



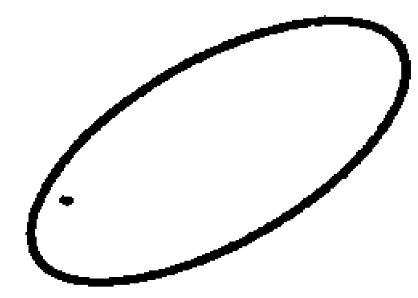


Figure A1.4.5 Poincare map

This method is also elegant in theory, but there are difficulties in practice. One of the difficulties is that not every flow has a cross section hence this Poincare section method is not always valid [Guckenheimer et. al, 1983]. Another problem is that since the definition of the Poincare map relies on the knowledge of the flow, Poincare map can not de computed unless general solutions of the corresponding differential equations have been obtained. It is a severe restriction for using this method. However, there are other techniques developed to extend this method by marring it with other methods. They include:

(1) the use of perturbation and averaging methods to approximate the Poincare map [Guckenheimer et. al, 1983]
(2) numerical analysis [Eckmann and Ruelle, 1985].

While the first one relies on mathematically proved perturbation techniques, the second one is based on not-so-rigorous numerical method which generating Poincare section through data processing. Both of them have been used in nonlinear dynamics analysis of complex physical and engineering systems [Thompson and Stewart, 1986; et. al].

B: power spectra (Fourier spectrum)

In abstract dynamical systems theory, the Poincare section method enjoys the beauty of theoretical elegance when it is employed to deal with lower-dimensional systems, but it is not convenient to use in situations where the explicit solution of a differential equation is not known. It is even worse when only a time series is given. Fortunately, there are some methods developed to cope with these problems and one of them is the study of Fourier spectrum or "power spectra".

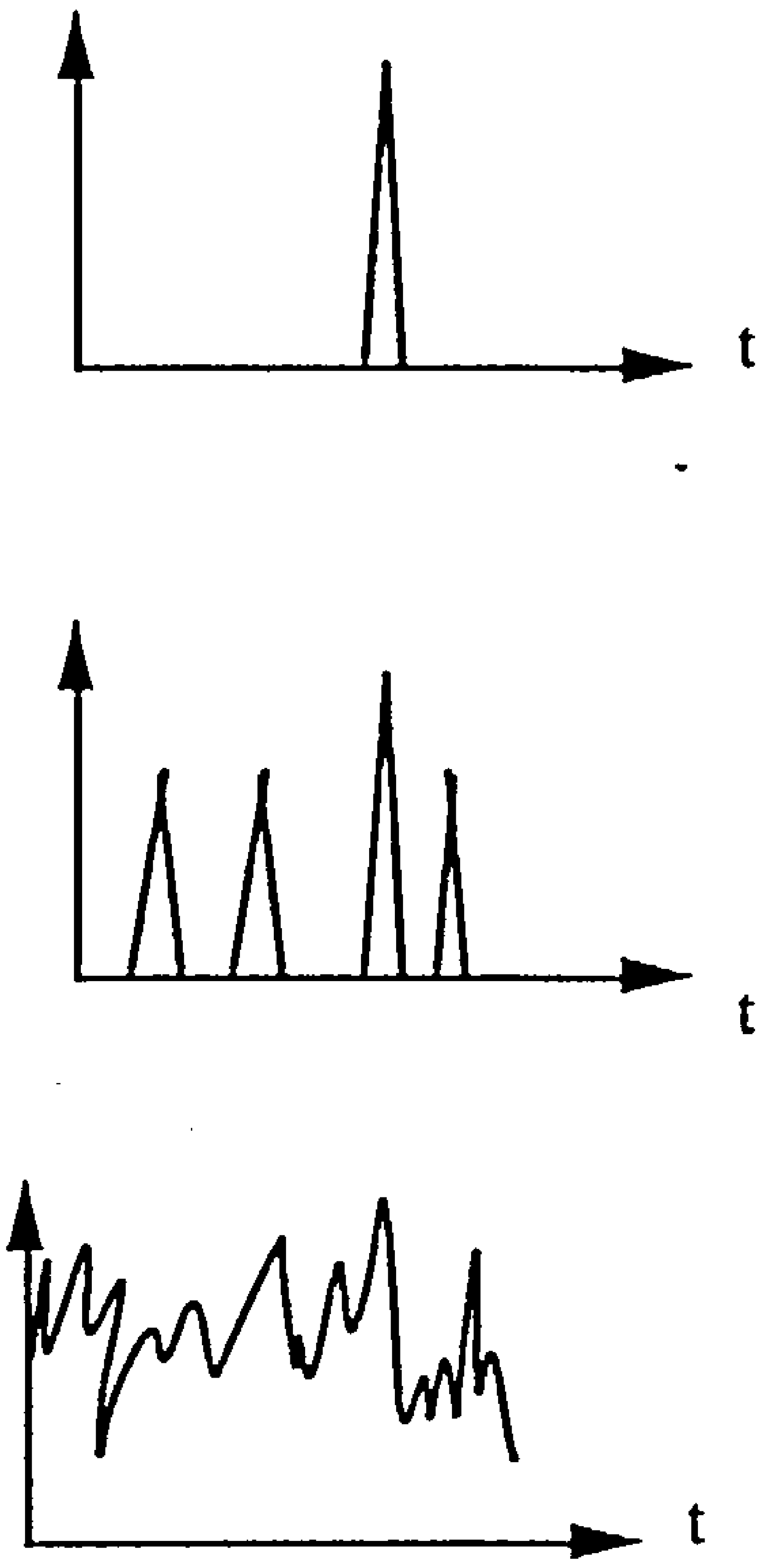
Fourier spectrum method has long been used to extract useful information from a experimentally provided time series of dynamical systems in various fields and it is very successful in some cases where regular (periodic) oscillation and irregular (aperiodic) oscillation must be differentiated [Berge, 1984]. When the oscillation is a superposition of oscillations which differ in amplitude, period, ratio of harmonics etc., Fourier spectrum can provide the necessary information to describe the oscillation. Here in our discussion, this method is employed to analyse the dynamical behaviour of a system from its time series often generated experimentally.

The Fourier transform in continuous and discrete form are quite familiar to us [See any text book]. Through these transform, a time series can be transformed into another one called "Fourier spectrum" which is believed to convey information for the detection of the system's behaviour.

It has been proved that different kinds of Fourier spectra correspond to different oscillation behaviour of a dynamical system corresponding to a time series studied [Berge et. al, 1984; Eckmann & Ruelle, 1985]. The results can be summarised as:

(1) If there is only one fundamental frequency and a few harmonics in the Fourier spectrum, the oscillation is periodic and hence the system has a periodic orbit there (if the orbit is an attractor as well, the system has a periodic attractor).
(2) If the spectrum has several peaks representing different fundamental frequencies, the oscillation is quasiperiodic and hence the system has a quasi-periodic periodic.
(3) If the spectrum is continuous, it indicts that the oscillation is chaotic and hence the system posses a chaotic orbit.

These can be illustrated in figure A1.4.6.



Fourier spectrum for periodic orbit:
one peak (or several peaks with
one foundemental frequences)

Fourier spectrum for quasi periodic orbit:
several peaks
representing fundemental frequences

Fourier spectrum for chaotic orbit:
continuous spectrum

figure A1.4.6 Fourier spectrum for different oscillating orbits.

While Fourier spectrum method is powerful in detecting complex limit sets like periodic and chaotic orbit, it can not be used to detect fixed points (although it is relatively easy to find fixed points). Many work has been done in Fourier spectrum analysis and many numerical algorithms have been developed to implement it in computers. In general, it is very good for detecting oscillating orbits and it is one of the often used method to find out a chaotic orbit. However, it provides no information about how such orbits look like on phase plain which are very important in studying a chaotic orbit.

C Lyapunov Exponent

Another well developed and often used method for determining the limit sets of a dynamical system is Lyapunov exponent method. It is based on the so called "ergodic theory" of dynamical systems . The basic idea is that there is a topologically invariant quantity called Lyapunov exponent (or characteristic exponent) which reflects the expanding or contracting property of a trajectory on phase plane [Eckmann and Ruelle, 1985].

The Lyapunov exponent, or a Lyapunov number is the average exponential rate of divergence or convergence of near by orbits in phase [Shaw, 1981]. It is usually employed to characterize the divergence of trajectories on an attractor of a dynamical system. When the exponent is negative, the trajectory is contracting hence the associated limit set is attracting. Any system contains at least one positive Lyapunov exponent is defined to be chaotic and the magnitude of the exponent reflects the time scale on which the dynamical behaviour of a system becomes unpredictable.

For a dynamical system on manifold X described as:

$$x' = F(x)$$

we are concerned here the expanding and contracting property of a trajectory starting from the initial condition x.

Denote

$$T_x = \left(\frac{\partial F_i}{\partial x_j}\right)_{i,j=1,..n} \quad \text{and} \quad T_x^t = \left(\frac{\partial F_i^t}{\partial x_j}\right)_{i,j=1,..n}$$

and it has been proved that the following limits exist:

$$\lim_{t\to\infty}\left((T_x^t)^* \, T_x^t\right)^{\frac{1}{2t}} = \Lambda_x$$

$$\lim_{t\to\infty} \frac{1}{t} \ln \|T_x^t u\| = \lambda^{(i)}, \quad \text{if } u \in E_x^{(i)} \setminus E_x^{(i-1)}$$

where $\lambda^{(1)} > \lambda^{(2)} > ... > \lambda^{(n)}$ are the logarithms of the eigenvalues of Λ_x, $E_x^{(i)}$ is the subspace of R^n corresponding to the eigenvalues $\leq e^{\lambda^{(i)}}$ of Λ_x, and $\| \cdot \|$ is the norm (not necessarily Euclidean These $\lambda^{(i)}$ are called the Lyapunov exponent of the dynamical system.

These Lyapunov exponents are related to the expanding or contracting nature of different directions in phase space. When the exponent is negative, the trajectory is contracting along that direction. When it is positive, it is expanding. A attractor is a limit set to which any trajectory around it is contracting in all directions. In such a way, the various type of limit sets of a dynamical system is connected to Lyapunov exponents. In three dynamical systems, this relation can be illustrated as figure A1.4.7

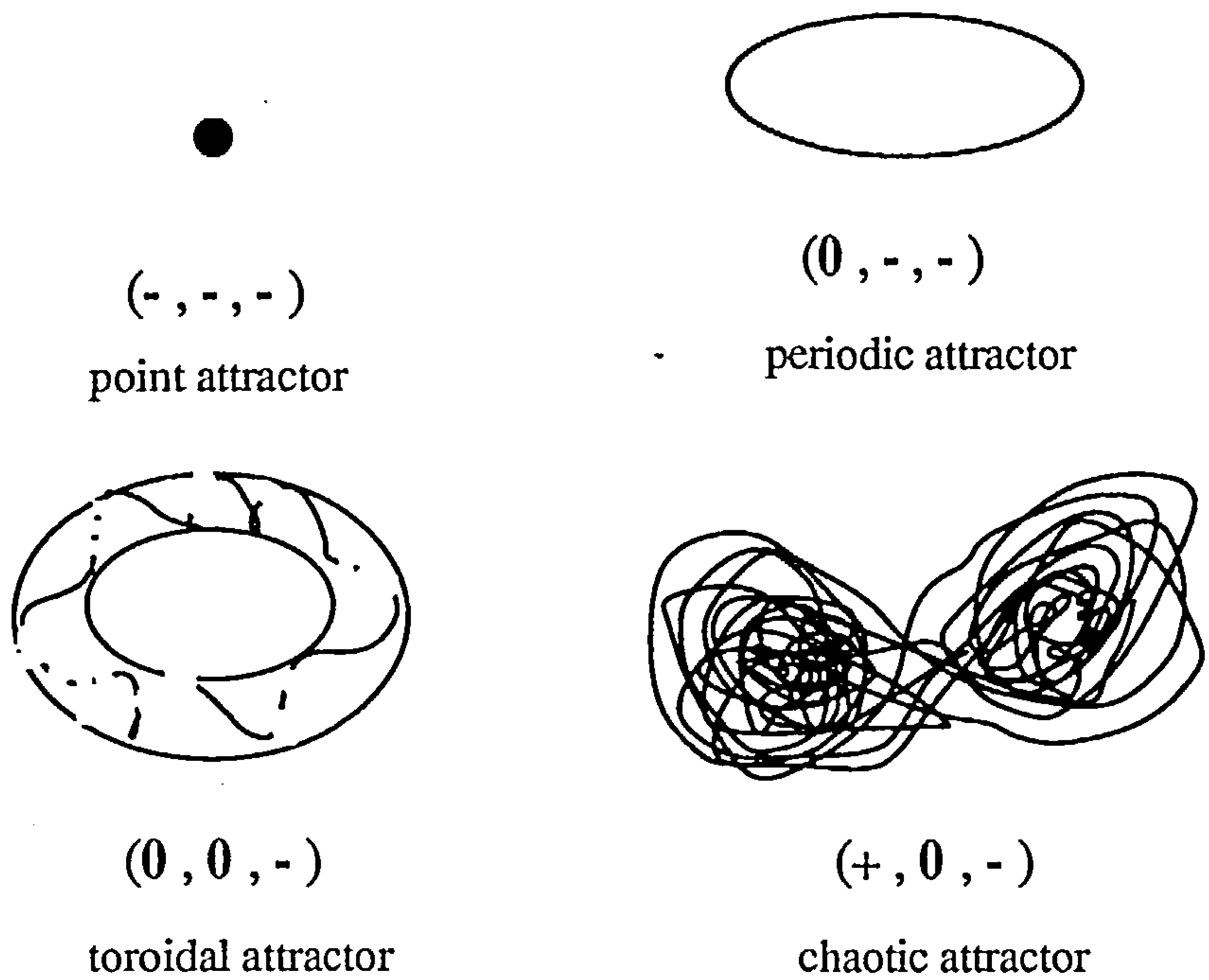


Figure A1.4.7 The Lyapunov exponents and various attractors.

Compared with the spectrum method, Lyapunov exponent method provides the information of a trajectory in the phase space. Again, we need intensive computation to find out the exponents in the case when the time series of a dynamical system is presented. To determine the Lyapunov exponent from a time series known experimentally is one of the active research fields related to nonlinear dynamics [Wolf et. al, 1985]. For the means to detect and describe chaotic attractor is so scarce, to calculate the Lyapunov exponent is the most used method. With the appearance of a positive exponent, we know for sure that the system is in a chaotic state and a chaotic attractor is hence found.

D Fractal Dimension

The study of chaos is connected with a new field of research called “Fractal Geometry” although the latter is in a wider sense regarded as a new geometry for complex forms or dynamics. The concept of fractal dimension and “fractal geometry” were introduced by a French mathematician Mandelbrot for the study of complex form originally found in complex iteration maps like the Julia map [Mandelbrot, 1983]. It is based on the concept of "Hausdoff dimension" and has then been generalized to measure various complex forms, notably forms with the property of self-similarity.

The fractal dimension of a set in a metric space is related to the concept of "capacity dimension" and "Hausdoff dimension". They can be defined as follows.

Let A be a subset in a compact Banach space X and the natural metric to use is the one defined by the norm. Suppose A can be covered by the minimum number of N(r, A) of open balls of radius r. Then we define:

$$\dim_K(A) = \lim_{r\to 0} \sup \frac{\ln N(r, A)}{\ln(1/r)}$$

as the capacity of A. It is also called the ***capacity dimension*** of A.

If we denote by σ a covering of A by a family of sets σ_K with diameter $d_K = \operatorname{diam} d_k \le r > 0$, we write

$$m_r^\alpha(A) = \inf_\alpha \sum_K (d_K)^\alpha$$

We call $m^\alpha(A) = \lim_{r\to 0} m_r^\alpha(A)$ the ***Hausdoff measure of A in dimension*** α, and

$$\dim_H A = \sup\{\alpha : m^\alpha(A) > 0\}$$

is defined the ***Hausdoff dimension*** of A.

It is easy to see that for every compact set A, the following inequality holds:

$$\dim_H A \le \dim_K A$$

When $\dim_H A = \dim_K A$, we call it a ***fractal dimension*** of set A, i.e.

$$\dim_F A = \dim_H A = \dim_K A$$

In general, the fractal dimension of a set is in agreement with the usually Euclidean dimension, i.e, a straight line has dimension 1, the plain has dimension 2. However, they are not always the same and the fractal dimension of a set is not necessarily an integer (this is why it is called "fractal dimension"). For a set with complex structure, like the Cantor set, the fractal dimension is ln2/ln3 [Farmer et. al 1983] while the Euclidean dimension is 0. Actually, the full strength of the fractal geometry lies in the fact that the property of being non-integer makes it in a particular position for describing of chaotic attractor of nonlinear dynamical systems [Eckmann &Ruelle, 1985; Stein, 1989].

Fractal dimension is topologically invariant. It is used as a quantity for the description of the limit set of a dynamical system. It has been proved that qualitatively different limit sets have different fractal dimensions and the they can be illustrated in table A1.4.1

Table A1.4.1 Fractal dimensions for different limit sets of 3-dimensional systems

limit set	fractal dimension
fixed point	0
periodic orbit	1
quasiperiodic orbit	2
chaotic orbit (Lorenz attractor)	2<d<3 (2.07)

To calculate the fractal dimension of a limit set is the way to detect the type of that limit set and it is of vital importance for the detection and description of chaotic attractor. Apart from fractal dimension, there are many other dimensions defined for the description of chaotic attractor, like the information dimension, Lyapunov dimension at. al.. A great deal of effort has been devoted to this subject and the calculation of fractal dimension form part of the core of nonlinear dynamics [Mandelbrot, 1983; Eckmann et. al, 1985].

So far in this section, we have given the definitions and descriptions of different limit sets possibly possessed by nonlinear dynamical systems. They describe the asymptotic properties of the trajectories of a system and prescribe the system's long run behaviour independent of the initial conditions.

A1.5 Local bifurcation

A1.5.1 Local bifurcation

The complex behaviour of a dynamical system is locally decided by its limit sets, in particular by four distinct attractors: point attractor, periodic attractors, quasiperiodic attractors and chaotic attractors. However, many systems with practical interests are affected by some external parameters. When these parameters change, the limit set or attractors will change and hence the long run behaviour of the system. It is known from section A1.3 that this is called a bifurcation.

Those limit sets are defined locally, so are various bifurcations. To discuss the various local bifurcation patterns often encountered in the study of nonlinear dynamics, assume the the dynamical system is specified by a field vector f on manifold X and described as:

$$dx/dt = f(x, \mu) \qquad (x, \mu) \in X \times C ,$$

X is a n-dimensional manifold and μ is a parameter vector on a r-dimensional manifold C.

When the parameter vector μ is in a space with dimension ≥ 2, it is usually very difficult to analyse the bifurcation except in a very special case which will be discussed in the following section. In our discussion here, assume that C =R, i.e., there is only one parameter.

Further suppose that discussion is restricted to to fixed point bifurcation, i.e the bifurcation happens on the set of fixed points:

$$L = \{(x_0, \mu_0) : f(x_0, \mu_0) = 0\}$$

It is already known that a fixed point bifurcation happens at (x_0, μ_0) when the Jacobian of f, $J_x f(x_0, \mu_0)$ at that point has zero or pure imaginary eigenvalues. When there is a simple zero eigenvalues, the bifurcation is called a elementary bifurcation. When there is a pair of pure imaginary eigenvalues, it is called a Hopf bifurcation.

A1.5.2 Elementary Bifurcations

Let us first consider the case of a simple zero eigenvalue with corresponding right eigenvector φ and left eigenvector ψ, we assume that

$$f(x_0, \mu_0) = 0, J_x f(x_0, \mu_0)\varphi = 0, \quad ||\varphi|| = 1. \qquad (*)$$

there are following local bifurcation patterns at the bifurcation point (x_0, μ_0). [Langford, 1981; 1983].

A Saddle-node bifurcation

Assume that in addition to the condition (*), we have:

$$a \equiv \psi J_\mu f(x_0, \mu_0) \neq 0, \quad b \equiv \psi J_{xx} f(x_0, \mu_0)\, \varphi\varphi \neq 0$$

Then in the neighbourhood of (x_0, μ_0) there exists a unique branch of solutions of the equation for the dynamical system discussed. The solution is of the form (x(r), μ(r)) for small r in R, given by:

$$x(r) = x_0 + r\varphi + 0(r^2) , \mu(r) = \mu_0 + r^2(-b/(2a)) + 0(r^3)$$

This is called a saddle-node bifurcation and the bifurcation diagram is showed in Figure A1.5.1 (a).

B Transcritical bifurcation

Suppose that the system has a trivial solution $(x(\mu), \mu)$ for all $\mu \in R$. Without loss of generality, we denote $x = x_0 \equiv 0$. Assume that in addition to the condition (*), we have:

$$c \equiv \psi J_{x\mu} f(x_0, \mu_0)\varphi \neq 0, \quad b \equiv \psi J_{xx} f(x_0, \mu_0)\varphi\varphi \neq 0$$

Then in the neighbourhood of (x_0, μ_0) there exists a unique smooth branch of nontrivial solutions $(x(r), \mu(r))$ of the form :

$$x(r) = r\varphi + 0(r^2), \ \mu(r) = \mu_0 + r^2(-b/(2c)) + 0(r^2)$$

This is called a transcritical bifurcation and the bifurcation diagram is showed in Figure A1.5.1 (b).

C Pitchfork Bifurcation

If the vector field f satisfies the symmetry condition:

$$Sf(x, \mu) = f(Sx, \mu) \qquad \text{and } S\mu = -\mu$$

where S is the linear operator on X (the frequently encountered case is that f is an odd function.). Together with the condition (*), we assume that :

$$c \equiv \psi J_{x\mu} f(x_0, \mu_0)\varphi \neq 0, \quad d \equiv \psi J_{xxx} f(x_0, \mu_0)\varphi\varphi\varphi \neq 0$$

Then in the neighbourhood of (x_0, μ_0) there exists a unique smooth branch of nontrivial solutions $(x(r), \mu(r))$ of the form :

$$x(r) = r\varphi + 0(r^2), \ \mu(r) = \mu_0 + r^2(-d/(6c)) + 0(r^4)$$

This is called a transcritical bifurcation and the bifurcation diagram is showed in Figure A1.5.1 (c).

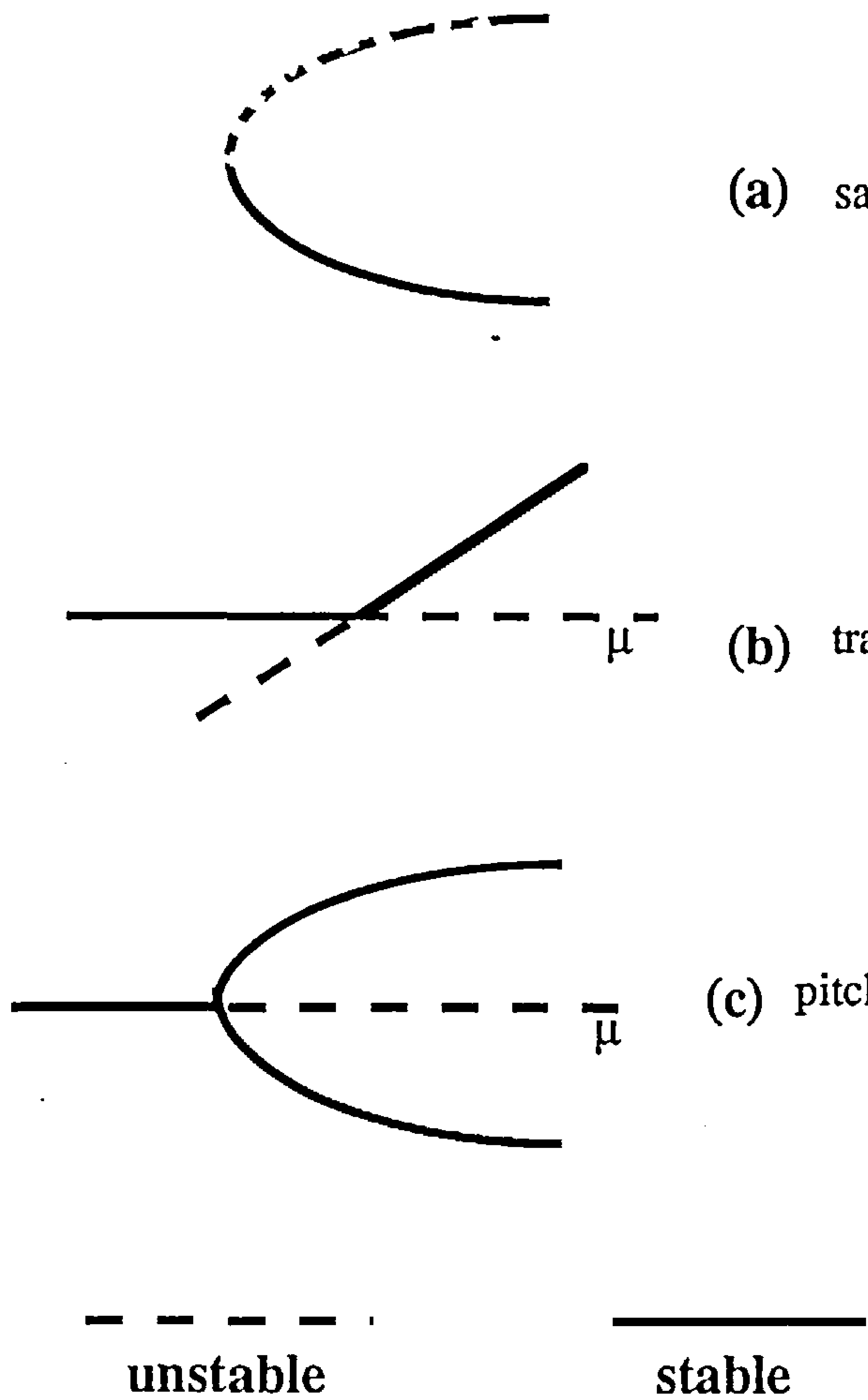


Figure A1.5.1 three elementary bifurcation patterns

The stability of the new branch of solutions after these different kinds of elementary bifurcation can be decided by some standard techniques of eigenvalues analysis of matrix, like the Lyapunov-Schemit method [Langford, 1983]. These bifurcation patterns can be described in the following normal forms:

saddle-node $\mu \pm x^2 = 0$

transcritical $\mu x \pm x^2 = 0$

pitchfork $\mu x \pm x^3 = 0$

A1.5.3 Hopf bifurcation

When there is a pair of pure imaginary eigenvalues, a qualitatively different bifurcation pattern can be observed which is called Hopf bifurcation. It is about the bifurcation through which a fixed point gives birth to a periodic orbit [Marden and McCracken, 1976; Hassard et. al, 1981].

Hopf bifurcation is the most studied and most well known local bifurcation. It happens in a dynamical system with one parameter when a pair of imaginary eigenvalues goes across the imaginary axis. This bifurcation is believed to be responsible for the appearance of oscillation behaviour in a wide range systems, like in a chemical system (Brusselator), mechanical system, hydrodynamics, biological system , ecological system economic system [Hassard et. al, 1981; Zhang, 1989; Guckenheimer et. al, 1983].

There different vision about the Hopf-bifurcation theorem and here we give theorem in a recipe-like form [Hassard et al, 1981]:

Theorem: Hopf Bifurcation (A Recipe–Summary [Hassard et. al, 1981])

Suppose a differential equation described by a vector field f with a single parameter υ:

$$\frac{dx}{dt} = f(x, \mu)$$

the following criteria can be used to find a Hopf bifurcation an find out the stability, period of the

1. *Find the equilibriun (or equilibria)* $x_*(\mu)$ *of the vector field by setting:* $f(x_*, \mu) = 0$, *and th the eigenvalues of the Jacobian matrix:*

$$J(\mu)\,/_{\xi=\xi_*} = \left\{ \frac{\partial f^i}{\partial x_j}(x_*(\mu); \mu) \qquad (i,j = 1, \ldots, n) \right\},$$

and order them according to $Re\lambda_1 \geq Re\lambda_2 \geq, \ldots \geq Re\lambda_v$.

2. *Find the critical value* μ_χ *such that* $Re\lambda_1(\mu_\chi) = 0$. *If*

(a) λ_1 *and* λ_2 *are a congugate pair* $(\lambda_1(\mu) = \overline{\lambda_2}(\mu))$ *for* μ *in an open interval includin*

(b) $Re\lambda_1^3(\mu_\chi) \neq 0$,

(c) $Im\lambda_1(\mu_\chi) \neq 0$, *and*

(b) $Re\lambda_\varphi(\mu_\chi) < 0 \quad (j = 3, \ldots, n)$,

then there is a periodic orbit appearing in the neighbourhood of μ_χ*.and we say that there hopf bifurcation.*

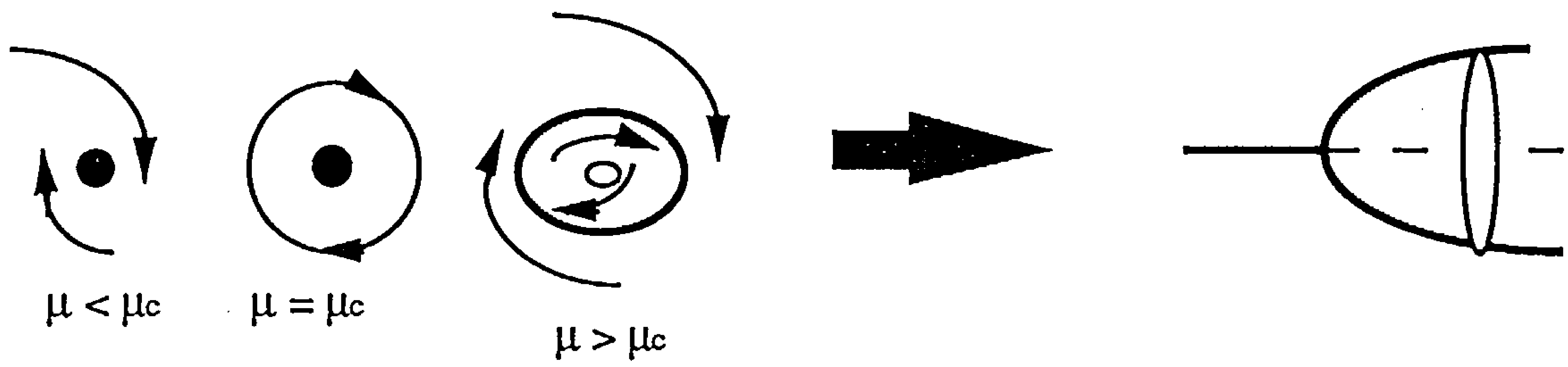
3 *There are two different kind of bifurcations:*

(a) *if* $x_*(\mu_\chi)$ *is an attractor, then, there is a periodic attractor for* $\mu \in (\mu_\chi, \mu')$ *for some this is called a supercritical hopf bifurcation.*

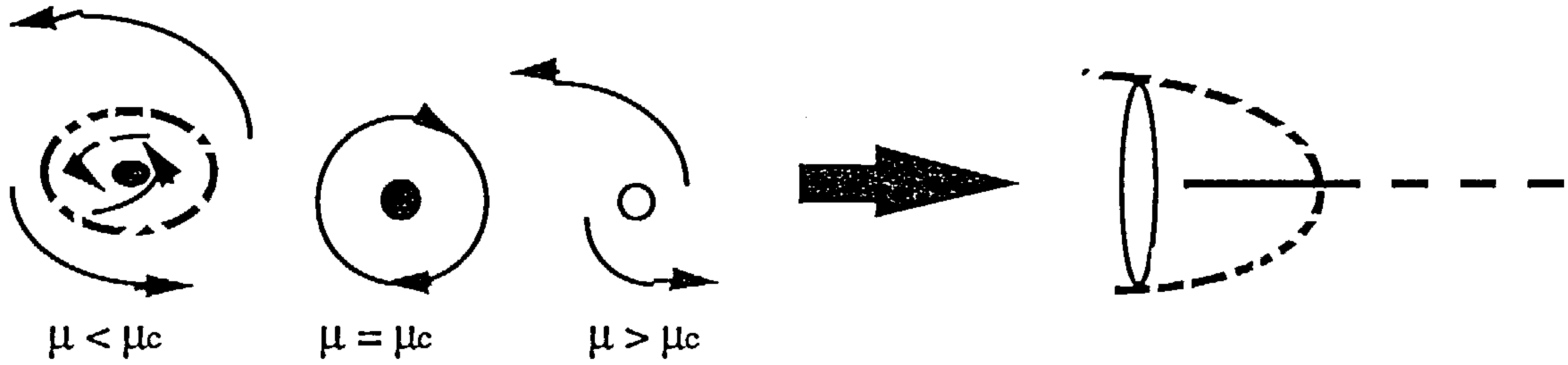
(b) *if if* $x_*(\mu_\chi)$ *is a centre with maginal stability, then, there is a periodic repellor for* $\mu \in (\mu'', \mu_\chi)$ *for some* $\mu'' < \mu_\chi$ *and this is called asubcritical hopf bifurcation.*

(There are formulas for the periodic orbit and the period and index for the stability of the periodic orbit, see [Hassard et. al, 1981])

The supercritical and subcritical Hopf bifurcation can be illustrated in figure A1.5.2



supercritical Hopf Bifurcation



subcritical Hopf Bifurcation

Figure A1.5.2 Hopf Bifurcation

The Hopf bifurcation is called a subtle bifurcation because the amplitude of the periodic orbit is a continuous function of the parameter. There are other bifurcation patterns where the amplitude of the appearing periodic orbits not continuous over the parameter and it is called an catastrophic bifurcation [Abraham, 1985; Thompson & Stewart, 1986]. Such catastrophic bifurcations, like "explosive" bifurcation [Zeeman, 1982] and "dangerous" bifurcation [Abraham, 1985] have been observed in a wide range systems, especially in engineering systems [Stewart et. al, 1986].

A1.5.4 Other bifurcation patterns

The above mentioned bifurcation patterns, include the Hopf bifurcation are the fundamental local bifurcation patterns in nonlinear dynamical systems. For a moderate nonlinear system with one or two parameters, there are many other bifurcations which are more complicated [Langford, 1983; Haken, 1983b; Abraham, 1985; Stewart et. al, 1986]. Except in some very special case, there is usually no analytical solutions for these bifurcations. Especially when there is the appearance or disappearance of a complex attractor, like a quasiperiodic attractor or a chaotic attractor, it is usually not possible to resort to analytical techniques for the bifurcation analysis, not mention the analytical forms of the new branch of solutions.

One well-known path of bifurcation leading to the appearance of a chaotic attractor is through "periodic doubling cascade" [Feigenbaum, 1983; Thompson and Stewart, 1986]. During this process, a periodic attractor with period 1 bifurcates so that to give birth to a periodic attractor with period 2. The new periodic attractor undergoes a further bifurcation, when the parameter changes further, to a periodic attractor with period 4. The process goes on and on, and usually a periodic attractor with a period 2^n bifurcates to a periodic attractor with a period 2^{n+1}. When the system reaches a accumulating point, there appears a chaotic attractor. This "period-doubling" process has been observed in various situation, and in the case of a map, it has been proved that there is a universal constant, called Feigenbaum constant, governing this process [Feigenbaum, 1983].

A1.6 Catastrophe Theory

A1.6.1 dynamical systems with a potential function

As mentioned in the above section, except in some special cases like Hopf bifurcation, there is no universally applicable analytical solutions to the bifurcation problems. Bifurcation analysis becomes more difficult in the following case:

(1) In a system with dimension ≥ 3, there might be many different attractors and hence there are many possibilities that the bifurcation can occur. When a bifurcation leading to the appearance of periodic attractors or chaotic attractor occurs, it is usually impossible to have an analytical solution.

2) When there is more than one parameter in the system, bifurcation becomes even more difficult and we have to take account of the change of all of the parameters of which the change of each of them can lead to complex bifurcation patterns.

Of course, when the dimension of a system is high and also the parameter vector is multiple dimensional, things almost become unmanageable. In that case, we have to mostly depend on numerical simulation [Thompson and Stewart, 1986]. Every system has to be treated individually and the bifurcation analysis usually means tentative computer simulations on large and fast computers.

In a very special case when the dynamical system is a gradient one, there is a well developed theory to analyse the bifurcation from point a attractor to point attractors when several parameters change simultaneously. This is called the elementary catastrophe theory, or catastrophe theory in general.

Catastrophe theory was originally introduced in early 70's as "*a new mathematical method for describing the evolution of forms in nature*" [Zeeman, 1977], and Thom goes further to claim that it "*... has to be considered as a theory of general morphology*" [Thom, 1975]. From the dynamical systems theory point of view, catastrophe theory studies the qualitative behaviour of dynamical systems with several external parameters but where only point attractors are considered. For elementary catastrophe theory, it deals with a special kind of dynamical systems, i.e. gradient systems which can be described by a "potential function" V of state x and parameter μ, i.e, $V(x, \mu)$. Here the states x lie in some Euclidean space and μ as a variable in a lower dimensional Euclidean space (dimension $\leq$ 4). The qualitative behaviour of such a dynamical system is described by its equilibria set and it changes when the parameter vector changes. Elementary catastrophe theory has classified that when dimension of $\mu \leq 4$ those f have seven local canonical forms which describe the qualitative change of the system depending on the change of those parameters [Thom, 1975; Zeeman, 1977].

In Thom's seminar book, catastrophe theory is based on the idea of dynamical systems theory and the theoretical analysis is centered about structural stability, generic property, transversality etc. while elementary catastrophe theory is treated by others from the point of view of singularity theory [Poston et. al, 1978; Smale, 1978]. In Thom's book, catastrophe theory " *.... is not a mathematical theory, but a whole body of ideas*" and its philosophical and methodological implications go far beyond a new mathematical tool [Thom, 1975; Zeeman, 1977]. However, here we discuss catastrophe theory, to be accurate, elementary catastrophe theory, within the framework of dynamical systems theory and it serves as a body of mathematical knowledge which deals with the bifurcation behaviour of a special kind of dynamical systems concerning only their equilibria, or fixed points.

Denote a dynamical system by a vector field f and it is described by:

$$dx/dt = f(x, \mu) \qquad (x, \mu) \in X \times C ,$$

X is a n-dimensional manifold and μ is a parameter vector on a r-dimensional manifold C.

It is gradient if and only if there is a function V: $X \times C \rightarrow R$ such that

$$f(x, \mu) = - \mathit{grad}\, V(x, \mu) \qquad (\mu \text{ is regarded as constant})$$

where *grad* V(x, μ) is the gradient of V(x, μ), called the potential function, which is defined as:

$$\mathit{grad}\ V(x, \mu) = \left(\frac{\partial}{\partial x_1} V(x, \mu), \ \ldots, \ \frac{\partial}{\partial x_n} V(x, \mu) \right)^T$$

This is possible only a very strict condition is met, i.e.:

$$\frac{\partial f_i}{\partial x_j} = \frac{\partial f_j}{\partial x_i}, \quad i, j = 1, \ldots, n$$

where f_k is the k th element of f. This is a severe limit to the application of elementary catastrophe theory in bifurcation analysis of dynamical systems.

Apart from the elementary catastrophe theory which deals with the bifurcation of equilibria of a gradient system, Smale has proved a theorem concerned about the structural stability of gradient systems.

Theorem [Smale, 1961 (see Smale, 1967)]

Gradient systems for which all fixed points are hyperbolic and all intersections of stable and unstable manifold transversal, are structurally stable.

This is one of few theorems about structural stability of dynamical systems with any dimensions.

A1.6.2 Elementary catastrophe

Under this circumstance, the behaviour of a gradient dynamical system can be analysed by looking a family of functions:

$$V: X \times C \rightarrow R$$

where: X is the state space, dim X = n;

C is the parameter space, or control space, dim C = r and it is called the *codimension* of V. In elementary catastrophe theory, $r \leq 4$.

The *catastrophe manifold* M is a subset of X x C defined by:

$$M = \{ (x, \mu) \mid D\, V(x, \mu) = 0 \}$$

The *catastrophe map* χ is the restriction to M of the natural projection

$$\pi: \quad X \times C \rightarrow C$$

for which: $\pi(x, \mu) = \mu$

The *singularity set* S is the set of singular points in M of χ , at which, χ is singular, i.e:

$$S = \{(x, \mu) \mid (x, \mu) \in M, \text{rank}(D\chi) < r\}$$

The bifurcation set B is the image $\chi(S)$ in C:

$$B = \{ \mu \mid \mu \in \chi(S) \; C \}$$

The bifurcation set B is the set on which the number and nature of the equilibria of the system change. It lies in the parameter space, i.e control space C and when the parameter changes so that it crosses the boundary of the bifurcation set, the system loses its structural stability. The state or behaviour of the system changes qualitatively, hence a bifurcation occurs. The usually case is that the system jumps from one state to a qualitatively different one and a bifurcation from one point attractor to two point attractors (plus an unstable equilibrium) can be observed. This can be illustrated by the following *fold catastrophe and cusp catastrophe*:

(1) Fold catastrophe

The potential function of a gradient dynamical system can be reduced to the canonical form as:

$$V_a = V(x, a) = \frac{1}{3} x^3 + ax, \qquad x \in R, a \in R$$

The catastrophe manifold:

$$M = \left\{ (x, a) \mid \frac{d}{dx} V_a(x) = 0 \right\}$$
$$= \{ (x, a) \mid x^2 + a = 0\}$$

The bifurcation set in the control space is:

$$B = \left\{ a \mid (x, a) \in M \text{ and } \frac{d^2}{dx^2} V_a(x) = 0 \right\}$$
$$= \{ 0\}$$

It is easy to show that when the parameter a changes from positive to negative, the system will change from a state with no minimum potential to a state of minimum potential, i.e a point attractor. The bifurcation diagram can be illustrated as follows:

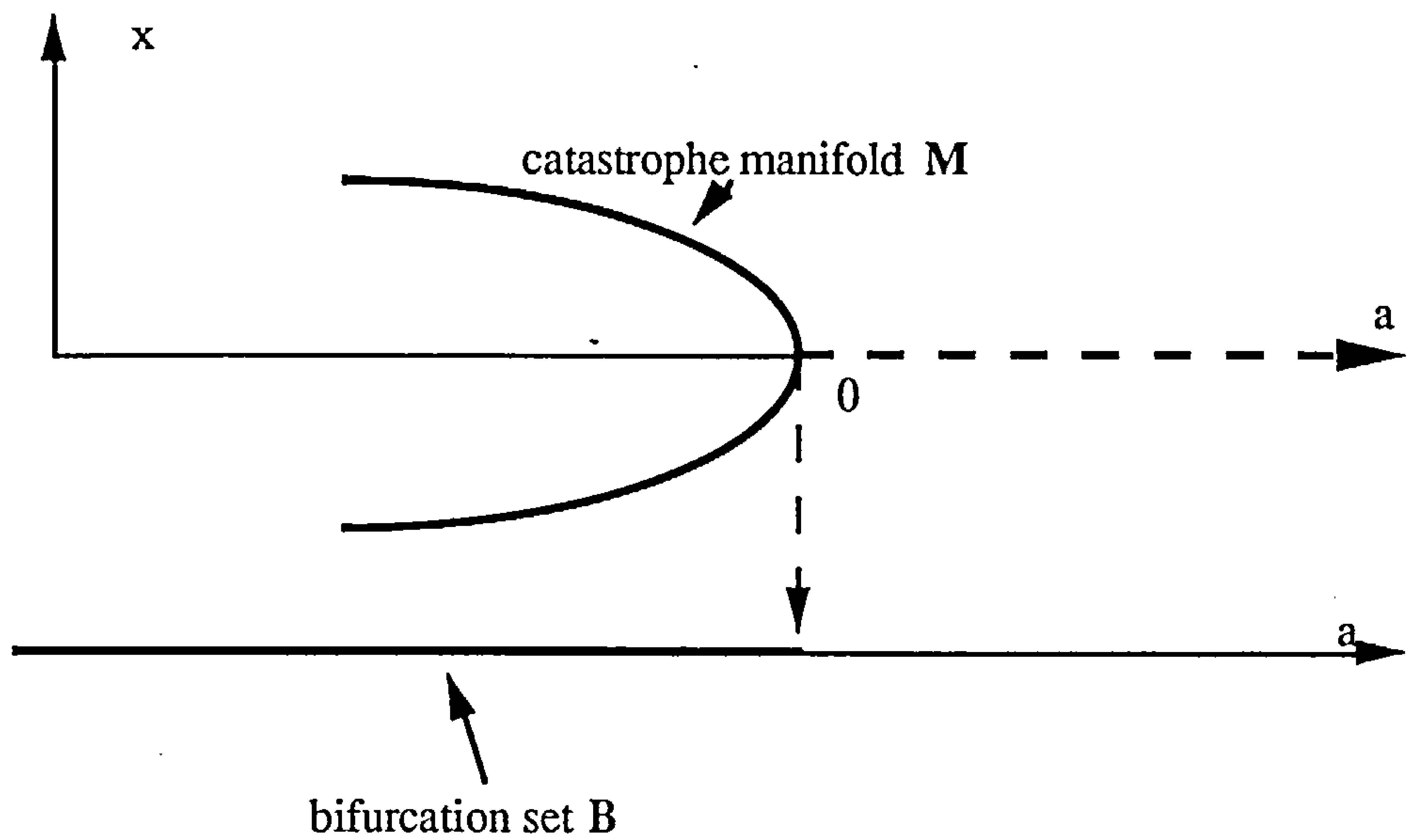


Figure A1.6.1 Illustration of fold catastrophe

This can be compared with the fold bifurcation discussed in section A1.3. It a quite trivial case in elementary catastrophe theory. Others are more complicated.

(2) Cusp catastrophe

The potential function is a fourth order polynomial of state variable x with two control parameters (a, b). It can be written in the canonical form, through the standard procedure called "universal unfolding", as follows:

$$V_{ab} = V(x, a, b) = \frac{1}{4}x^4 + \frac{1}{2}ax^2 + bx, \qquad x \in R,\ (a, b) \in R^2$$

We say that function V has corank 1 and codimension 2. According the outlined procedure, we can detect the catastrophe manifold M, bifurcation set B etc as follows.

Catastrophe manifold:

$$M = \left\{ (x, a, b) \mid \frac{d}{dx} V_{ab}(x) = 0 \right\}$$

$$= \{ (x, a, b) \mid x^3 + ax + b = 0 \}$$

The singular set of M is:

$$S = \left\{ (x, a, b) \mid (x, a, b) \in M \text{ and } \frac{d^2}{dx^2} V_{ab}(x) = 0 \right\}$$

$$= \{ (x, a, b) \mid x^3 + ax + b = 0 \text{ and } 3x^2 + a = 0 \}$$

The *bifurcation set* is:

$$B = \chi(S) = \pi|_M (S) = \{ (a, b) \mid (x, a, b) \in S \}$$

$$= \{ (a, b) \mid 4a^3 + 27b^2 = 0 \}$$

Comparing with the fold catastrophe, the cusp catastrophe exhibits many original properties which lies the full strength and originality of elementary catastrophe theory. This is made more clear by looking at the catastrophe surface.

1) Modality and inaccessibility

Look at the catastrophe manifold (or catastrophe surface) which represents the equilibrium of th system. There is a folding part corresponding certain values of parameters a and b. There the potential function has more than one local minimum point which means that the physical system has many possible distinct states. Some of them, like those points in the middle surface of the folding region, are inaccessible by the system (which are corresponding to maximum potential points).

(2) Catastrophe: sudden jump

When the parameters change across the boundary of the bifurcation set in the control space, the system can change abruptly from one equilibrium state to another: either jumps from a point in the lower space to one in the upper space or vice versa. It can be observed in physical systems, like the Zeeman catastrophe machine, or social systems, like the outbreak of war, that tiny change in the forces or motivations lead to abrupt change in behaviour [Zeeman, 1977]. This is where the name "catastrophe theory" comes from.

(3) Divergence

It is easy to see that starting from two near points near the origin the system can move towards different part of the catastrophe surface which implies that the system stay in two distinct states because of the small difference in the initial condition. It can explain that why a small, negligible factor can lead to a totally different state even the environment is almost the same (that means the parameter changes the same way). However, divergence here is different from the effect of random factors we are going to discuses latter.

(4) Hysteresis

The behaviour of the system becomes very interesting when the parameters change to cross the bifurcation set. We can see from figure A1.4.2 the following phenomena: when parameter a increases to cross B, the system moves in the lower surface and jumps suddenly to the upper surface at a bifurcation point, but the system will not jump from the upper surface to the lower one at the same point. The whole process is not strictly reversible.

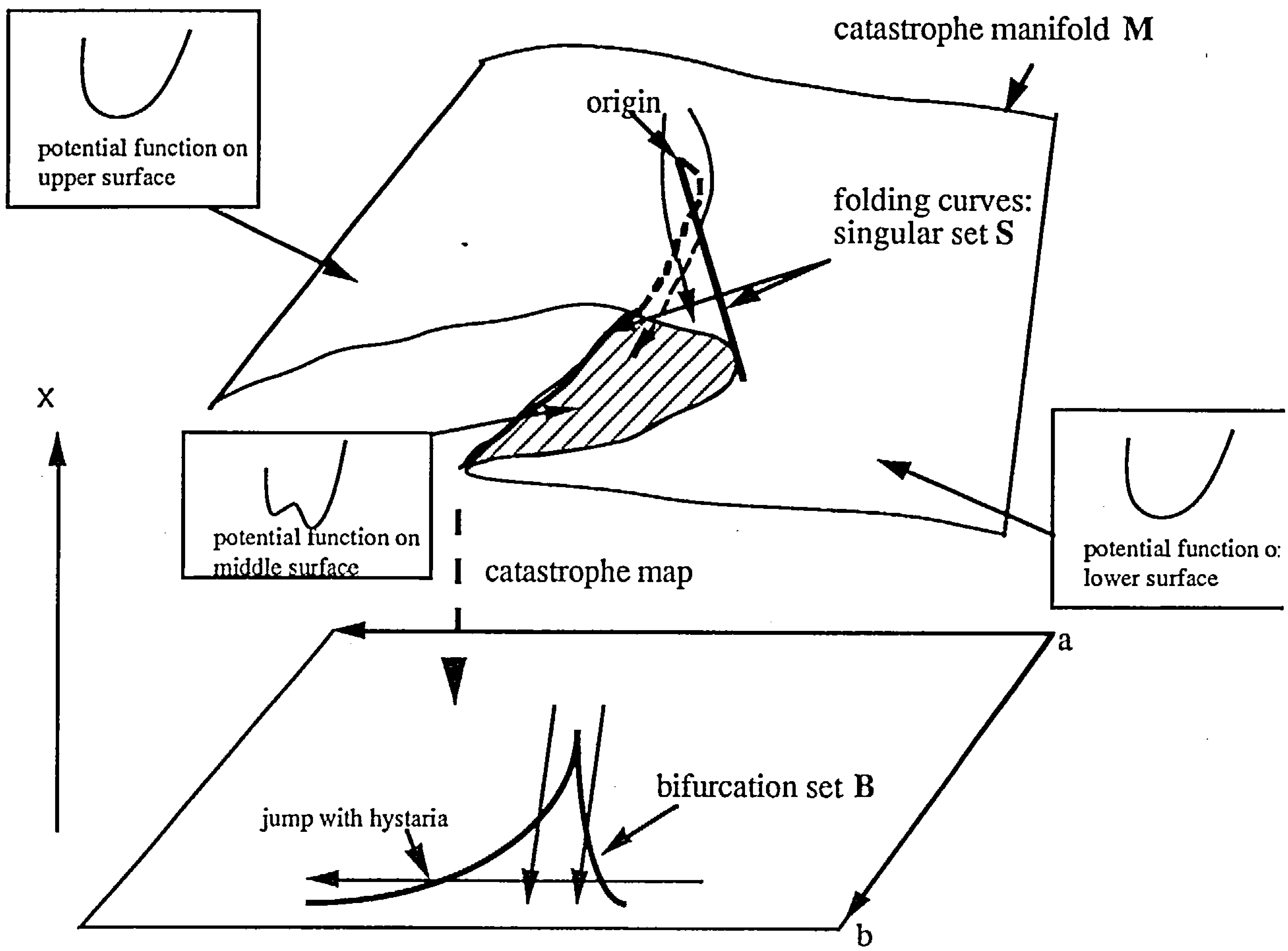


Figure A1.6.2 Illustration of the cusp catastrophe

Those are the properties of gradient dynamical systems revealed by the elementary catastrophe theory. In those more complicated elementary catastrophes, like swallowtail, butterfly et. al, these phenomena can also be observed and described.

It has been proved in elementary catastrophe theory that there are only seven canonical catastrophes for all those gradient dynamical systems with not more than A1 parameters and these forms can be summarized as:

Table A1.6.1 Seven Elementary Catastrophes

Catastrophe	number of parameters	number of state variables	Universal Unfolding
fold	1	1	$\frac{1}{3}x^3 - a_1x$
cusp	2	1	$\frac{1}{4}x^4 - \frac{1}{2}a_1x^2 - a_2x$
swallowtail	3	1	$\frac{1}{5}x^5 - \frac{1}{3}a_1x^3 - \frac{1}{2}a_2x^2 - a_3x$
butterfly	4	1	$\frac{1}{6}x^6 - \frac{1}{4}a_1x^4 - \frac{1}{3}a_2x^3 - \frac{1}{2}a_3x^2 - a_4x$
hyperbolic	3	2	$x_1^3 + x_2^3 + a_1x_1x_2 + a_2x_1 + a_3x_2$
ellipic	3	2	$x_1^3 - x_1x_2^2 + a_1(x_1^2 + x_2^2) + a_2x_1 + a_3x_2$
parabolic	4	2	$x_1^2x_2 + x_2^4 + a_1x_1^2 + a_2x_2^2 + a_3x_1 + a_4x_2$

Among all the seven elementary catastrophes, cusp catastrophe is the most studied, well understood, and most important one. It has a solid physical system as its prototype, e.g the Zeeman catastrophe machine [Poston and Stewart, 1978]. It exhibits all those characteristics of a gradient dynamical system that a catastrophe model can reveal. This cusp catastrophe model has been used in a wide range of systems including behaviour systems, social systems [Zeeman, 1977; Poston and Stewart, 1978]. There has been many controversies about the applicability of catastrophe theory in behaviour and social sciences and further doubts have been aroused about the originality of catastrophe theory in general [Smale, 1978]. However, the application of catastrophe theory in the physical systems, engineering systems in particular, is well justified [Poston and Stewart, 1978; Gilmore, 1981]. Here in our discussion of dynamical systems, we use it as a body of mathematical knowledge which can be used to analyse the bifurcation of gradient systems their behaviour are affected by more than one parameter. The idea of catastrophe theory as it is presented in Thom's original book "*Structural stability and Morphogenesis*" goes beyond that: it provides a general framework for studying any dynamical process by using dynamical systems theory. This reflects, latter on, in Abraham's "morphodynamics" and the whole vision of modern mathematical dynamical systems. This is the philosophical, theoretical and methodological foundation of this study. The concepts of "structural stability", "attractor" is generalized to described the state and the dynamical process of systems.

A1.7 Global bifurcation

For nonlinear dynamics with external parameters, there are various forms of local bifurcation patterns, as can be seen from the above sections. The global behaviour of such a system depends not only on where the system starts (the initial conditions), but also which region the parameter is in. When the parameter changes over a wide range of values, it is possible that various local bifurcations can be observed. To paste all these local bifurcation diagrams together, we can have a picture of the global bifurcation patterns over the whole range of the parameter [Abraham &Shaw, 1985; 1988].

With such a global bifurcation picture, the time evolution of the structure of the system becomes clear and it is known that what the state of the system might be depends on value of the parameters in the parameter space. To get such a global bifurcation picture, it needs to work out all the local bifurcation patterns and it is by no means a easy job. As mentioned above, there are still many work to be done before all the local bifurcation patterns are known to us. Among all these problems, bifurcations leading to quasiperiodic attractors and chaotic attractors need more attention. For the lack of analytical techniques in those situations, we have largely rely on the intensive and tentative computer simulations. Various numerical methods and many new algorithms have been developed to detect the limit sets. The above mentioned methods, i.e., Poincare section, Fourier spectra, Lyapunov exponent and fractal dimension have been used widely in achieving this goal. The power of these methods have been demonstrated by some researchers, like Thompson and Stewart who use computational techniques as a main means for the study of nonlinear dynamics [Thompson et. al, 1986; Thompson, 1989; Thompson et al, 1990].

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