The Applicability of Statistical Techniques to Credit Portfolios with Specific Reference to The Use of Risk Theory in Banking

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Contents

Chapter 3

Theory and Practice of Credit Risk Treatment in Bank Loan Pricing

Chapter 4

Dataset Used for Empirical Analysis

List of Tables

Glossary of Terms

CAR	:	Capital Adequacy Regulation
CART	:	Marginal Capital At Risk (for a specific loan)
CAD	:	EC Capital Adequacy Directive (EC 93/96)
ERORC	:	Excess Return on Regulatory Capital
GLIM	:	Generalised Linear Interactive Modelling
IID	:	Independent Identically Distributed
LIBOR	:	London Interbank Offered Rate
NSSBF		National Survey of Small Business Finance (USA)
RAC	:	Risk Adjusted Capital
ROE	:	Return On Equity
ROC	:	Return On Capital
RAROC	:	Risk Adjusted Return On Capital
RORAC	:	Return On Risk Adjusted Capital
RARORAC	:	Risk Adjusted Return On Risk Adjusted Capital
RBC	:	Risk Based Capital
VAR	:	Value At Risk

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Abstract

This thesis examines the use of statistical techniques in credit portfolio management, with emphasis on actuarial and risk theoretic pricing and reserving measures. The bank corporate loan portfolio is envisaged as an insurance collective, with margins charged for credit risk forming premium income, provisions made forming claims outgo, and variation over time in provisioning and profitability producing a need for reserves.

The research leads to the computation of portfolio specific measures of risk, and suggests that a value-at-risk (VAR) based reserve computation has several advantages over current regulatory reserving methodology in bank capital adequacy regulation (CAR) with respect to non-residential private sector loan portfolios. These latter, current CAR practices are invariably used by banks to compute the respective capital adequacy backing required on loans. A loan pricing model is developed that allocates capital required by reference to observed provisioning rates across a total of 64 differing combinations of rating factors. This represents a statistically rigorous return on risk adjusted capital (RORAC) approach to loan pricing.

The suggested approach is illustrated by reference to a particular portfolio of loans. The reserving and pricing measures computed are portfolio specific, but the methodology developed and tested on the specific portfolio (dataset) of loans has a wider, more general applicability. A credit market comprising portfolios which are both more and less risky than the original particular portfolio is hypothesised, and existing regulation is compared to VAR regulation in the context of the hypothesised credit market. Fewer insolvencies are observed using the VAR framework than under existing regulation, and problem portfolios are identified earlier than under existing regulation.

For the particular portfolio of loans, existing algorithm-based loan pricing is compared with the proposed loan pricing model. Significant differences are observed in loan pricing by reference to gearing and collateral, and the elimination of observed inefficiencies in pricing is recommended. Although the proposed model has some limitations, it is argued to be an improvement on existing regulatory and banking practice.

Introduction

This thesis develops portfolio choice models of asset allocation for commercial bank corporate lending. It does not specify a complete model of the banking process. Statistical and specifically actuarial techniques are applied to produce both global value at risk and individual loan pricing procedures, and these are proposed as an alternative to present practice in these areas.

A wealth of microeconomic banking literature exists, chiefly dating from the mid-1960s; the early literature attempts to model the bank as a complete financial intermediary. Subsequent literature focuses on individual component operations. A significant breakthrough in terms of credit pricing was Black and Schole's development in 1973 of option pricing formulae, based on stochastic calculus, which enabled the valuation of a wide range of contingent claims. Merton's development of Black and Schole's original approach led in 1974 to a closed form mathematical evaluation of credit risk. Subsequent development has led to a rich literature on deposit insurance.

Independently of the banking literature, insurance theorists had already developed a collective theory of risk, building portfolio models of collectives of independent risks from consideration of the mathematics of individual risks. The insurance literature uses conventional calculus in constructing its building blocks, and although the two literatures have obvious parallels, their development has been separate. Thus, for example, option pricing formulae were in common use in insurance applications from the mid 1950s, although their introduction into the finance literature did not occur until the early 1970s.

The application of insurance pricing and solvency techniques to the evaluation of credit risk is a central focus of this thesis. If that part of loan margin dedicated to credit risk coverage is envisaged as an insurance premium, then this part of a bank's operations may be considered as being directly equivalent to those of an insurance company. The premium income is the sum of credit risk margins payable by borrowers; the claim outgo is the sum of losses incurred by the bank on individual loans; and the underlying reserve is that sum of money which will ensure the continued solvency of the credit risk process at some acceptable probability.

This approach allows the application of insurance theory directly to bank lending, and provides fresh insights into capital adequacy backing for loans and individual loan pricing.

These insights are illustrated relative to present practice by reference to an actual loan portfolio, comprising approximately 20% of the United Kingdom corporate bank lending market. Where substantial differences exist between the results of the insurance based approach and current practice, these differences are examined.

Banks have a medium to long term involvement in lending. The objective in using insurance pricing methodology to price credit risk is to price correctly default risk in the medium term, and not for a specific time interval. The overall premium sufficiency of the model is of paramount importance. The responsiveness of the model to current credit conditions is dealt with by the application of linear control theory, with the speed of response being dictated by the extent to which reserves can absorb systemic shocks. Reserves and loan pricing are interconnected in many ways, so that one cannot be considered independent of the other. The approach adopted in this thesis is to quantify required reserves from observation of past loan loss experience, and to price loans so as to remunerate adequately the quantified reserves.

Chapter I

Aims, Methodology and Structure

Introduction

The purpose of this chapter is to set out the main research questions to be addressed, to describe clearly the methodology to be applied in addressing the research questions, and to set out a guidemap illustrating the progression of the research.

1.1 Aims

The main research question is the following: can statistical, and specifically actuarial techniques of risk measurement, be used to improve the present measurement and treatment of credit risk in bank corporate lending portfolios?

This immediately devolves into two related sub-questions, as credit risk is priced both with respect to its portfolio impact and with respect to the individual loan generating a specific credit risk. The portfolio sub-question becomes: can actuarial techniques be used to develop a value at risk (VAR) model of capital adequacy, with loan pricing structured to reflect this value at risk? The individual loan sub-question becomes: can actuarial techniques be used to improve present practice in loan pricing?

In addressing the above sub-questions it is necessary to review relevant literature, to consider the present treatment of credit risk, to develop alternative capital adequacy backing and loan pricing models, to compare the results of these models in the context of an actual loan portfolio, to consider the differences observed, and to simulate the future operation of such models. These requirements are dealt with in succeeding chapters.

The application of methodology from outside of mainstream banking literature is a key to the development of alternative models in this thesis. In this sense, the thesis develops a kind of inter-disciplinary (banking, insurance, finance and statistical) model framework. This methodology is now discussed.

1.2 Methodology

The proposed methodology is to use the collective theory of risk to value credit risk as if the bank were operating as a credit risk insurance company, independent of its loan granting activities. As will be seen in the relevant literature, Merton and Bodie (1993) describe the fundamental identity:

Loan Guarantee = Default Free Loan - Risky Loan (1.1) which specifies that the price of a loan guarantee is simply the price of a default free loan less the price of an identical loan with credit risk attached. A bank could invest entirely in default free assets, in which case no loan guarantees would be issued. Banks choose to make risky loans in the belief that income in the form of their charges for loan guarantees will in aggregate compensate them for the inevitable outgo in the form of losses on defaulting loans.

If the issue of loan guarantees is separated from the physical transfer of money to the borrower, then the former, the "unbundled lending" as described by Asay and Albertson (1990 p16), has a very small balance sheet (when compared to the total lending balance sheet) composed primarily of reserves and credit enhancements. Assets which previously appeared as loans on the bank's balance sheet now appear as "securities", with lending retaining only the credit risk on loans originated and sold. The sale of loans may be merely an intercompany securitisation.

The default free loan referred to in (1.1) is generally taken to be a domestic government security of similar term and payment type (fixed or floating interest rate) to the risky loan being considered. If liquidity risk is charged to depositors and serviced out of an existing portfolio of readily realisable assets, then the cost of a loan guarantee may be regarded as the price of credit risk.

This leads directly to the conclusion that the cost of a loan guarantee, given that loans are certified at face value, must be the value of the yield margin over comparable government securities charged for the loan by the bank. Note that liability composition or cost is not discussed. The only interest is in the returns available to the bank for its issuance of loan guarantees, and not the overall profitability of the bank. Of course, the return on equity (ROE) on the "unbundled lending" balance sheet described above may be calculated.

1.2.1 The Collective Theory of Risk

A brief introduction to the collective theory of risk is given here-under, since it is integral to the methodology of this thesis. This introduction is paraphrased from Daykin Pentikainen and Pesonen (1994, pp1-63). Specific applications will be discussed in fuller detail later as and when they are introduced.

Early risk theory was mainly concerned with life assurance and was based on individual risks, with the behaviour of a portfolio of such risks being deduced as the sum of individual outcomes (Bohlmann (1909)). The studies of Lundberg (1909, 1919) and Cramer (1926, 1930, 1954) became known as the collective theory of risk: the occurrence of claims was dealt with collectively, without reference to individual risks.

Using the collective theory, in terms of probability calculus, we construct a doubly stochastic aggregate claim amount model, where both the number of claims and the size of each claim are stochastic variables. In the specific applications within this thesis, a claim may be considered to have occurred when a loan moves into default - defined as more than 90 days overdue with no rescheduling agreement in place, and the claim may be considered to be of positive amount when final and specific provision of some monetary amount greater than zero has been made.

Consider the accumulated number $k(t)$ of defaults occurring during a time period 0 to t as a function of (t). $k(t)$ is a stochastic process, postulated to satisfy the following three conditions:

1. The number of defaults occurring in any two disjoint time intervals are independent.
2. No more than one default may arise from the same event.

3. The probability that a default occurs at a given fixed time point is equal to zero.

If this is the case, then the number of defaults occurring within a given fixed time period is Poisson distributed (proof given in Daykin Pentikainen and Pesonen (1994 Appendix A.)), with probabilities corresponding to different k values given by:

$$P_K = P_K(n) = e^{-n} \frac{n^K}{\angle K} \qquad \text{where } n = \mathrm{E}(\mathrm{K}) \tag{1.2}$$

Examining each of these conditions in turn:

1. The condition of independence seems to be immediately violated in practice, as loan performance is known to co-vary through time (see, for example Bennett (1984), Chirinko and Guill (1991), Sinkey (1992)).

   If loan default probability can be characterised as being independent over short time periods, but subject to similar changes in background risk intensity over longer time periods, then a generalised form of the Poisson distribution is applicable.

   In this case, observed covariance over time is a function of the time period over which loan defaults are assumed independent, and the volatility of changes in background risk intensity. In general the longer the former and the lower the latter, the closer the portfolio is to satisfying the original Poisson condition; the shorter the former and the higher the latter, the larger the positive covariance, and the closer the portfolio is to the generalised Poisson model. Changes in risk intensity are often handled by introducing an auxiliary variable, generating what is commonly known as a mixed Poisson distribution.

2. It is possible for a loan default in one company to produce a domino effect, causing a chain of subsequent defaults linked to the first. In banking, this effect is referred to as "associated risk", and loans are generally grouped to exclude its effects. If all default costs arising from a single event are treated as a single default cost, then condition 2 will not be violated. In practice, unrecognised associated risk may cause condition 2 to be approximately rather than exactly satisfied.

3. It is a characteristic feature of defaults that they occur randomly in such a way that their exact times of occurrence are unpredictable, thus automatically satisfying condition 3.

Poisson variables have the useful properties of being additive with respect to both size and time, of having moments which are a function of only one parameter, of having successive values which may be recursively calculated, and of asymptotically approaching the normal distribution as n increases.

It has been mentioned previously that the standard Poisson law premises may not be valid because of external background factors, such as economic conditions, the credit cycle, etc. The default process can in fact be composed of trends, cycles, short-term seasonal oscillations and pure random fluctuation.

When the variation of default risk intensity is random, the stochastic variation can be interpreted as random changes of the Poisson parameter from its expected long run average n. The changes may be described by a multiplicative factor q such that $E(q)=1$. If $q>1$, default risk intensity is higher than expected and *vice versa*. If the value of q is fixed in any time period and equal to some value (q), then the first condition is again satisfied and the conditional default number variable k is a mixed Poisson variable. In this case:

$$P_k = E\left(P_k(n.q)\right) = \int_0^\infty e^{-Nq} \frac{(nq)}{\angle k}\, dH(q) \tag{1.3}$$

where H(q) is the distribution function of the mixing variable (q).

Specifically, where H(q) is defined as gamma (h,h), that is gamma with mean value as expected equal to 1, then:

$$H(q) = \frac{1}{\Gamma(h)} \int_0^{hq} e^{-z} z^{h-1} dz \tag{1.4}$$

with variable h open to choice by fitting. As h $\rightarrow \infty$ we reapproach the simple Poisson distribution, and as h $\rightarrow$ 0 the distribution flattens as the mixing variable implies a very large range for (q). Where this form of mixing distribution is used, the resulting point probabilties are given by the distribution:

$$P_k = \binom{h+k-1}{k} \times p^h \times (1 - p)^k \tag{1.5}$$

where $p = \dfrac{h}{n+h}$, also known as the Polya distribution.

Note that the mixing distribution and its moments may be described in terms of its two parameters n and h.

Similar auxiliary variables may be defined for differential risk within a portfolio; in such applications they are described as structure, rather than mixing variables.

The default process is now generalised to include consideration of default cost amounts. Let k denote the number of defaults from a portfolio of loans in a certain time period. The aggregate default cost X during that time period is:

$$X = \sum_{i=1}^{k} Z_i \tag{1.6}$$

where Z_i is the cost of the *i*th default during the period. If there are no defaults, then k=0. Variables of the form (1.4) are known as random sums, since the number k of the summands is a random number as well as the individual value of the summands.

The distribution of total portfolio default cost may be constructed as:

$$F(x) = PROB(X \leq x) = \sum_{k=0}^{\infty} P_k \times PROB\left\{\sum_{i=1}^{k} Z_i \leq x\right\} \tag{1.7}$$

For this result to hold, it is necessary for the default cost variables to be independent of the random number of defaults k. When this distribution has been constructed, ruin probabilities may be specified and required reserves calculated.

1.2.2 Pricing of Individual Insurance Risk

In order to avoid adverse selection, the general insurance approach to risk classification is to sort individual risks into subsections which are as homogeneous as possible with regard to rating factors likely to influence the probability of occurrence of the insured event. The pure risk premium charged represents the observed historic cost per unit adjusted for inflation and known risk effects, for risks subject to an identical combination of rating factors. This is analogous to, but more statistically rigorous than the banking practice of using classification and regression trees: for example as in Frydman Altman and Kao (1985), or Marais, Pattel and Wolfson (1984).

The differences between banking and insurance practice in this area will be discussed as they arise.

1.2.3 Research Methodology

The research methodology used in this thesis is summarised as follows:

1. To apply the collective theory of risk to an existing corporate loan portfolio.
2. To deduce required capital adequacy using a VAR approach.
3. To develop a multiplicative, multi-factor loan pricing model, designed to compensate VAR capital adequacy as previously calculated.
4. To compare the proposed loan pricing model results to current loan pricing within the portfolio.
5. To compare the proposed VAR capital adequacy requirement with present practice, using a simulation based approach.

In order to make comparison relevant, in the case of the existing corporate loan portfolio, total VAR is constrained to be equal to existing regulatory capital requirements, and total VAR return is constrained to equal historic average return on regulatory capital requirements. This has implications for both VAR and loan pricing measures.

The research is broadened to include hypothesised portfolios varying with respect to risk, profitability and size, and the implications for VAR and loan pricing are discussed. The applicability and limitations of the research are then discussed.

1.3 Structure

The present chapter is a statement of research aims, a brief description of the underlying methodology, and a presentation of the structure of this thesis.

Chapter 2 consists of a sectioned literature review, with individual reviews of the following relevant areas of the literature:

1. Modelling the banking firm
2. Information, credit rationing and collateral
3. Portfolio allocation
4. Option pricing deposit insurance, and regulatory capital
5. General insurance
6. Regulation

Each of these subsections of the overall literature review will be indexed as to its relevance to subsequent chapters of the thesis.

Chapter 3 consists of a review of present practice with respect to bank credit pricing, including a discussion of its operational significance, its strengths and weaknesses and how the proposed approach might be used to address identified weaknesses. The thesis as a whole aims to improve on present practice so that this chapter is required to establish the baseline on which improvements are to be generated and compared.

Chapter 4 is a discussion of trends in bank credit risk, with specific reference to the dataset to be used in empirical comparisons. It sets out the dataset employed, carries out exploratory data analysis and comments on the applicability and generalisability of results to be derived using the dataset as a test bed. Chapter 4 also contains a clear specification of the uses of the dataset in developing the framework proposed in this thesis.

Chapter 5 establishes that the building blocks of the process, i.e. individual risks, are identical in mathematical terms, subject to common assumptions. The purpose of this exercise is to demonstrate that the difference between current practice and that proposed in this thesis is a difference in architecture rather than raw materials. The use of insurance methodology enables the elimination of a known bias in option pricing, and permits the numerical computation of stochastic collateral/stochastic firm value solutions, which had not been possible under previous formulations.

Chapter 6 develops the value at risk (VAR) portfolio model, demonstrates that riskier banks will have larger VAR, and that actuarial soundness in deposit insurance schemes would be achieved using a combination of VAR and level insurance premiums. The extent of moral hazard and adverse selection may be reduced by comparison with existing capital adequacy schemes, and may be further reduced by the application of linear control theory to smooth surplus capital and loan pricing.

Chapter 7, equipped with VAR calculated in the previous chapter, allocates return on VAR by proportion to observed default losses across a range of rating factor cells. A transition matrix is used to reflect prior probabilities of cell movement, in order to charge correctly a two period loan for its average credit risk throughout its term. Proposed charges for credit risk using the above approach are compared with charges currently being levied in the dataset introduced in Chapter 4. Where significant differences exist these are discussed.

Chapter 8 simulates the operation of a number of credit portfolios under 2 approaches: the first representing current practice and the second representing the approach proposed in this thesis. With an equivalent starting reserve, and subject to similar feedback loops, the improvement associated with the proposed methodology should be represented by fewer observed insolvencies, and a tighter stochastic bundle, than is experienced in present practice.

Chapter 9 summarises the thesis, identifies conclusions which may be drawn from the research contained therein, and points to limitations of this research.

Chapter 2

Literature Review

Introduction

The purpose of this chapter is to chart the chronological development and current state of research effort in those areas which have a bearing on the research developed in this thesis. Table 2.0 below sets out the subsections into which the literature review is divided, with the final column in the table providing an index of subsequent chapters to which the specific subsection is directly relevant. In essence, the current chapter outlines the "broad chart" from within which the subsequent "routes" taken by the thesis will be traced.

Table 2.0

Breakdown of Literature Review

Section Number	Topic	Relevant Chapters
1	Modelling the banking firm	1, 3, 5, 6
2	Information, credit rationing and collateral	3, 4, 5, 7
3	Portfolio allocation	3, 6, 7, 9
4	Option pricing and deposit insurance, and regulatory capital	5, 6, 8
5	General insurance	1, 5, 6, 7
6	Regulation	4, 6, 9

Thus, for example, the information based-literature is of relevance to thesis chapters 3, 4, 5 and 7; the option pricing, deposit insurance and regulatory capital literature to thesis chapters 5, 6, and 8 etc.

Each finance section of the literature review is prefaced by a chronological table giving year, author and main research area. The purpose of these tables is to order the progression of research in the relevant area.

2.1 Modelling the Banking Firm

Table 2.1 summarises the chronological progression of the literature modelling the banking firm.

Table 2.1

Modelling the Banking Firm Literature

(Chronological progression)

Year	Author	Brief Description / comment
1888	Edgeworth	Foundation of mathematics of banking
1963	Schull	Monopoly, multiple product, price discrimination
1970	Brucker	Microeconomic approach
1971	Klein	Monopoly model describing size and structure
1971	start of	Portfolio allocation literature
1973	start of	Option pricing and deposit insurance literature
1977	start of	Information literature
1980	Fama	Banking in general equilibrium in finance theory
1980	Baltensperger	A survey of alternative approaches with emphasis on partial models
1980	Sealey	Complete model with risk aversion, resource costs, and deposit rate setting
1981	start of	Credit rationing and collateral literature
1982	Langhor	A defence of Klein's work against Baltensperger's criticisms
1983	O'Hara	A dynamic model of the banking firm
1984	Diamond	Financial intermediation and delegated monitoring
1984	Santomero	Survey on models to date
1986	Devinney	Credit rationing in a complete theory of the banking firm
1987	Lefebvre	Financial intermediation viability from deposit contract viewpoint
1987	James	Empirical evidence on banks as delegated monitors
1987	Bryan	Process model of banking
1989	Bernanke and Gertler	A neo-classical economic model of bank behaviour
1990	Asay and Albertson	Process model of banking
1994	Rajan	Fluctuations in bank credit policies
1995[b]	Berger	Capital and earnings in relationship banking

The basic mathematics of banking were set out by Edgeworth (1888). Microeconomic research into the banking firm in the modern era began with Schull (1963). Schull viewed a bank as transforming a pool of deposit funds into credit products in a manner designed to maximise profit: banks extend credit in any product until its marginal revenues equate to the marginal cost of general credit extension. The marginal cost of general credit extension in equilibrium was seen by Schull to be equal to the average revenue in the short-term government securities market.

Brucker (1970) extended Schull's analysis and used an analysis of the competitiveness of loan classes in a bank's asset mix as a proxy for monopoly power enjoyed by banks in various loan markets. Brucker (1970, p6) makes the first specific reference in the literature to the riskiness of loan portfolio when he refers to "The ratio of net loan loss to total loan "used" as an *ex post* proxy measure for the level of risk within a state economic area".

Klein (1971) produced the first complete microeconomic model of the banking firm. He stated that the expected return on loans must be less than the contracted rate of interest since the latter represents the maximum the bank can receive. Generally:

$$E_L < r \quad if \quad \sigma_L > 0 \tag{2.1}$$

where r is the contracted rate, E_L is the expected loan return and σ_L is the standard deviation of the probability distribution of loan payments. All borrowers were assumed to be identical and non-interest terms of loans were assumed exogenous, leading to his conclusion that default risk was exogenous to the bank. He did point out that such a conclusion was unlikely, but he wanted to abstract from problems connected with loan risk appraisal in the development of his model.

Fama (1980) regarded banks as issuing deposits and using the proceeds to purchase securities, a loan being merely another form of (closed-market) security. Banks operate to fashion portfolios of such securities as will appeal to their depositors, receiving in return a competitively determined fee for their services. The willingness of banks to provide access to the accounting system of exchange in the face of reserve requirements is what differentiates them from other financial intermediaries. Reserve requirements are viewed as a tax on depositors leading banks to attract funds for access to the accounting system of exchange at lower interest rates than those payable on outside investments.

Baltensperger (1980) presented a survey and discussion of banking literature up to that time. Many of the deficiencies of early models had been addressed by then and his paper represented the then contemporary theory. He postulated (1980, p1)......"the main economic functions of financial firms are those of consolidating and transforming risks, and of serving as dealers or 'brokers' in the credit markets (the basis of which is the existence of transaction and information costs in these markets)". Uncertainty, informational problems and adjustment costs had now moved centre stage. This paper is the first to acknowledge explicitly information and administration costs and to associate costly portfolio adjustments with liquidity risk. The paper consolidates the work of many academics, chief among them Orr and Mellon (1961), Poole (1968), Frost (1971) and Koskela (1976), in treating the banks reserve and liquidity management decision as involving inventory optimisation under stochastic demand.

Treating the rate of interest charged on loans, r, as net of information and transaction costs, a declining demand function for loans is postulated. Allying this demand function to Schull's original model produces marginal lending until the risk adjusted return on lending equals that obtained on Government securities. Government securities are regarded as effectively a kind of dumping ground for excess liabilities.

The chief criticism of complete models of the banking firm in Baltensperger's view is their reliance on monopoly power (i.e. banks operating as price makers in markets), and the model tendency to collapse under competitive forces. A model based on resource costs, on the other hand, can function regardless of market behaviour.

Baltensperger advocates the consideration of costs involved in different credit contracts, default risks and overall insolvency risk, and the consideration of similar costs on the liability side. He points to the conclusion by Pyle (1971) that intermediation can only occur if there is a positive dependence between net rates of interest on loans and deposits. He notes that the probability for intermediation to be profitable increases with the yield difference between loans and deposits - allowing for resource costs- and the degree of positive correlation between them.

He concludes by specifying a model based on expected profit maximisation and price taking in all markets, and uses the model to determine jointly asset structure, liability structure and firm scale. Langhor (1982), in a reply to Baltensperger, points to the fact that liability recomposition by a bank leads to asset adjustment if necessary, occurring in its Government securities portfolio rather than its loan portfolio. Thus, in his view Klein's (1971) original assertion that the estimated density function of deposit outflow is independent from the size of the loan portfolio is in fact correct, to the extent that the bank has not exhausted its portfolio of Government securities.

For the purpose of this thesis a fully specified complete bank model is not necessary, as we are concentrating solely on the credit management process. However these models provide important insights and contributions to the latter. From this point on, models will be summarised algebraically, as the more current literature is surveyed.

Sealey (1980) in an important contribution used the following assumptions

1). Deposit rate setting by bank (to fix liability price)
2). Random deposit supply
3). Stochastic loan rates
4). Perfectly competitive loan markets
5). Single period planning horizon
6). Liquidity costs

to specify a deposit supply function

$$D^s = D(R_D U); \quad \left(\frac{dD}{dR_D}\right) \succ 0; \quad \left(\frac{dD}{dU}\right) \succ 0 \tag{2.2}$$

where D^s is the quantity supplied; R_D is the interest rate; U is not known *ex ante*, but has a known subjective probability distribution, and reflects random supply; and R_L is the random return on loans which has a known subjective density and range $0 \leq R_L \geq \infty$.

The simple balance sheet composition is: $L = D^P + Z$

where L = Loan, Z = a composite variable measuring the difference between money market borrowing and lending and D^P = quantity of deposits purchased.

Decisions as to the quantity of loans and interest rate on deposits are made prior to the start of the 1-period horizon. At the start of the horizon period the quantity of deposits and return on loans become known. *Ex post* liquidity costs are imposed equal to $l(Z)$.

The objective is assumed to be the maximisation of utility of profit, defined as:

$$\max_{L,R_D} E\left\{\max_{D^P,z} \left[U\left(R_L L - R_D D^P - l(Z) - C_D\left(D^P\right) - C_L(L)\right)\right]\right\} \tag{2.3}$$

$C_D\left(D^P\right)$ and $C_L(L)$ are the resource cost of servicing deposits and loans.

E is an expectations operator and *U* is the utility function assumed by the present author to have $U' \succ 0,\ U'' \prec 0$ i.e. the bank is risk averse.

The first order condition specifically applying to loans is:

$$\frac{\sigma}{\sigma L} E\left[U\right] = E\left[U'\left[R_L - P - C'_L\ (L)\right]\right] = 0 \tag{2.4}$$

P being the rate of adjustment of *Z* to *ex post* liquidity.

The last equation may be rearranged as:

$$E\left[R_L\right] = C'_L\ (L) + P\ -\ \frac{Cov\left(U',\ R_L\right)}{E\left[U'\right]} \tag{2.5}$$

Where $Cov\left(U',\ R_L\right)$ is the covariance between the marginal utility of profits and the marginal revenue on loans.

The last term is the risk premium due to risk aversion by the bank and it is an increasing function of loan extensions. Under the model the utility of profit prevents the bank from equating marginal revenue with marginal cost on the lending process as long as covariance is negative. If profit is an increasing function of loan interest rates then this will be the case. Further, under the above conditions the risk of profit is increasing in total loans.

If the random loan rate is purely the result of loan default, then it is reasonable to assume positive correlation between the loan rate and deposit level. Essentially Sealey (1980) is proposing that risk averse management will stop short of equating marginal revenues to marginal costs, its stopping point being dictated by the size of negative covariance between utility of profit and loan rate.

O'Hara (1983) presents a dynamic model of the banking firm. She discriminates between management and shareholders, and characterises management as defining the risk averse stance of banks. This is a direct result of agency problems, causing management to depart from the simple profit maximisation which may be sought by shareholders. Management is viewed as maximising its utility subject to regulatory and shareholder constraints. A further appeal is given to the model by its endogenous determination of the composition of the banks asset portfolio.

The manager's problem is defined as:

$$\max E\left[\sum_{0}^{\infty} \alpha^{t} U(X_{t})\right] \tag{2.6}$$

Where U is the management's (risk averse) utility function and α is a discount rate applied to future time periods. A direct result of this problem is that the manager, because of utility of positive value in future time periods, increasing in value as time progresses, will not have any incentive to drive a solvent bank into insolvency. Management is assumed to be prevented by regulation or otherwise from asset stripping the bank.

The first major conclusion is that if the return on loans increases, management will increase the bank's holding of loans. In setting up the partial derivative, O'Hara (1983, p130) refers to $\bar{\mathrm{i}}$, as being either "the stated interest minus expected loan losses or simply the level of interest rates". The model is flawed in that an increase in $\bar{\mathrm{i}}$ caused by a reduction in expected loan losses will have a very different effect on customer behaviour than one caused by a move in general interest rates. O'Hara's model is insufficiently dynamic in that it fails to describe customer response to bank actions.

The second major conclusion is that as interest rates paid on deposits increase, the manager will reduce bank holdings of risky debt, responding to management's own decreasing absolute risk aversion, due to the fact that loan riskiness is an increasing function of interest rates.

The final major conclusion is that changes in regulation will not necessarily produce predictable results in bank behaviour. O'Hara notes that the increased collateral paradox noted by Stiglitz and Weiss (1981) may also influence banks to choose riskier portfolios when faced with higher capital requirements. This paradox has led to a rich banking regulation literature.

O'Hara's basic construct is postulated as a real world (or more inductive) approach. However, she appears to underestimate the impact of customer behaviour on management's freedom of action.

Diamond (1984), in a major contribution to the literature develops a theory of financial intermediation that fits in many ways the present author's purpose. This is the most recent "complete" theory in the literature and is deserving of detailed discussion. Diamond focuses on minimum cost production of information useful for solving incentive problems. Cost, information and incentives had previously been discussed, but this paper draws the strands together in a powerful and significant way.

The information production task is delegated to the intermediary, giving rise to incentive problems. In analysing the determinants of delegation costs Diamond develops a model in which a financial intermediary has a net cost advantage over borrowing and lending direct, and answers the basic question as to why banks exist. Diversification proves the key.

Diamond's model envisages N risk neutral entrepreneurs, each endowed with the technology for an indivisible investment project with stochastic returns. These entrepreneurs have zero wealth, but their project output has expected value greater than *R*, the competitive interest rate within the economy. Individual project returns are denominated $\tilde{y}$, $0 \leq \tilde{y} < \infty$. Everyone agrees on the distribution of $\tilde{y}$, but the actual realisation of $\tilde{y}$ is observed only by the entrepreneur. The entrepreneur must choose an incentive contract which depends only on observable (by the intermediary) variables and makes the lender anticipate a competitive expected dividend. Positive penalties are assumed to attach to bankruptcy, e.g. loss of reputation, management time, etc.

The optimal contract maximises the risk-neutral entrepreneurs expected return, given a minimum expected return to lenders of *R*. By specifying a non-pecuniary penalty function dependent on *Z* (assuming project returns normally distributed and *Z* as the standard normal variable), the optimal contract for the entrepreneur is shown to be a debt contract with face value *H* where *H* is the smallest value providing an expected return of *r* to the lenders. Positive probability of non pecuniary penalty means that the optimal contract is costly. As long as this cost is less than the cost of the lender observing $\tilde{\mathbf{y}}$, the optimal contract will be preferred.

If *K > 0* is the cost of observing $\tilde{y}$, then possible situations are no monitoring, each lender monitoring separately, and delegated monitoring by all lenders to one or more, acting as agents. The last pays when

$$K + D \leq \min\left[E_{\tilde{y}}\left[\phi^*(\tilde{y})\right](m.k)\right] \qquad (2.7)$$

Where there are M Lenders, D is costs of delegation, $E_{\tilde{y}}$ is the expectation operator, and $\phi^*(.)$ is the non pecuniary penalty probability density function.

A financial intermediary having zero wealth is envisaged as acting as delegated monitor for its depositors. The intermediary will receive payment $g_i(.)$ in respect of the i_{th} of N entrepreneurs monitored $g_i(y_i) \leq y_i$. Total payments are:

$$G_N = \sum_{i=1}^{N} g_i(Y_i) \tag{2.8}$$

The intermediary must pay away $R.\ N.$ For the intermediary to remain in business $P(G_N \geq N.R) = 1$. Since the entrepreneur can pay at most Y_i, then

$$P(G_N \geq N.R) \leq P\left(\sum_{i=1}^{N} \tilde{y}_i \geq N.R\right) \tag{2.9}$$

Any entrepreneur with $P(\tilde{y}_i \geq R) = 1$ will finance directly.

The intermediary will only contract with entrepreneurs whose monitoring costs exceed the intermediaries physical and delegation costs.

The model thus far provides evidence that delegated monitoring can justify the existence of financial intermediaries in certain specific situations. When the benefits of diversification are superimposed, the prevalence of intermediaries as providers of funds, particularly to small and medium sized entrepreneurs, is clearly understandable. The diversification benefits about to be discussed are central to this thesis.

For a financial intermediary to be viable, the depositors must receive *r* per unit deposited, the intermediary must receive an expected return after all costs of at least zero, and each entrepreneur must retain an expected return at least as high as that obtainable contracting directly with depositors.

With independent and identically distributed projects, the per entrepreneur cost D_n is a monotonically decreasing function of N as the probability of average returns per project being in the penalty zone decreases. In reality projects may be neither independent nor identically distributed, but to the extent that they are not perfectly correlated, diversification benefits may still accrue to the intermediary. If projects are independent, then as their number grows without bound the cost of monitoring converges to its physical cost K.

Leverage in any intermediary will be high, as the net worth of the intermediary will be:

$$N(R + D_N + K) - N\left(R + \frac{D_N}{2}\right) = N\left(\frac{D_N}{2} + K\right) \tag{2.10}$$

As $D_n \to 0$ the net worth of the intermediary will be simply $N.K.$

Diamond (1984) extends the model to exclude exogenous observable correlative factors such as economic variables, assuming these to be hedged in futures markets. The residual uncorrelated risk is assumed independent. The statistical procedures in Chapters 6 and 7 will test for correlation of risk, thereby indirectly testing Diamond's conclusions.

Assuming risk averse agents implies a constant payment to lenders. Assuming risk averse depositors also implies a constant payment to them by the intermediary. The intermediary must inevitably charge its depositors a lower risk premium than the entrepreneur. This can only occur if the intermediary has a less severe trade off between risk sharing and incentives than the depositor (i.e. a less concave utility function), and can thus be delegated the monitoring task.

Essentially this is an indirect application of a utility inequality, which directly parallels its application in general insurance. Financial intermediaries, specifically banks, may be considered as default risk insurers. This can again be observed empirically, by applying general insurance principles to the valuation of credit portfolios.

At the individual bank level, the intermediary consists of a single agent, and diversification is sought by adding risks. This will only be justified when adding independent risk reduces per entrepreneur risk aversion, a situation which seems to correspond to reality .

Decreasing absolute risk aversion is characterised by a similar type of utility function to that chosen by O'Hara (1983) i.e.

$$U'(.) > 0, \quad U'' < 0 \tag{2.11}$$

With the additional conditions that

$$U'''(.) \geq 0, \quad U''''(.) \geq 0 \tag{2.12}$$

with one inequality strict.

These results are less testable than those previously commented on, but do seem to confirm more closely to real world conditions.

A direct implication of delegated monitoring is that intermediary assets are illiquid. Interestingly Diamond (1984 p 410) mentions <u>both</u> commercial banks and insurance companies as being the most obvious applications of his model. Thus he draws a direct parallel between the activities of both types of intermediary, suggesting that similar methodologies are applicable to each.

In an appendix, Diamond (1984 pp 411-414) proves that if the agent has an exponential utility function, adding independent identically distributed risk will not reduce the per project risk premium in the single agent case, and thus a high order polynomial utility function is required for this form of diversification to add value.

Santomero (1984) provides a complete literature survey. He refers to the information literature (then) as still in its formative stages, and identifies the flow of information as being a key element in the market for financial assets. Specifically relating to portfolio choice models of asset allocation (one of several which he surveys) Santomero (p 587) views this type of model as the result of the following type of maximisation process:

$$\max_{A_i D_j} \pi = \sum_{i} R_{Ai} A_i - \sum_{j} R_{Dj} D_j \tag{2.13}$$

where $\frac{dR_{Ai}}{dA_i} \prec 0 \quad \forall_i$

$$\frac{dR_{Dj}}{dD_j} \succ 0 \quad \forall_j \neq m$$

$$\frac{\mathrm{dR_m}}{\mathrm{dD_m}} = 0$$

These first order conditions result in:

$$\left(\frac{dR_{Ai}}{d_{Ai}}\right) A_i + R_{Ai} = R_{Dm} = \left(\frac{dR_{Dj}}{dD_j}\right) + R_{Dj} \tag{2.14}$$

that is, one market is perfectly competitive. Klein (1971), for example, uses the government security market. Separation exists between asset allocation and deposit structure. Variation in demand for loans of one type does not affect other loan decisions, as all loan rates balance at a marginal rate dictated by the perfectly competitive market.

A minor adjustment to this model would have:

$$\frac{dR_{Di}}{dD_j} \succ \frac{dR_m}{dD_m} \succ 0 \qquad \forall_j \neq m \tag{2.15}$$

In this case, excess demand for funds from the competitive market produces some upward move in rates, but this move would be less than that caused by similar demand in the less competitive market of the bank's own deposit base.

This formulation produces a dynamic equilibrium, whereby the bank will resource from the less competitive market until its cost of funds exceeds that in the more competitive market. Thereafter the bank will resource from the more competitive market, until marginal costs equate to marginal revenues.

If the government securities market is identified as the more competitive market, then the return on government securities must be bounded on the upside by the return on low risk loans (to prevent "round tripping", creditworthy customers arbitraging against the bank by borrowing to invest profitably in government securities) and on the downside by the return on deposits (to prevent existing government securities holders from accessing the beneficial aspects of the deposit contract at zero or negative cost).

In this modified model, returns generated by the bank would be by means of a variable spread defined as follows:

$$\max_{A_i D_j D_m} \pi = \sum_i R_{Ai} A_i + R_m D_m - \sum_i R_{Di} Di \tag{2.16}$$

Where the bank has a government security portfolio. The actual returns may be subdivided into a) those generated by asset allocation to assets other than government securities, and b) those generated by deposits being sourced at all-inclusive costs below the prevailing government security yields. As Santomero (1984) observes, the factoring of resource costs into the above models does not alter their underlying logic.

This interpretation underlies this thesis, which concentrates on the first part of the subdivision, that is, a). Consideration of this type of model leads inevitably to a process view of the bank: its consideration as the sum of several businesses. The fact that loan margins are in practice quoted by reference to some benchmark government yield would imply that the variability of spread largely depends on the second part of the subdivision (i.e. b)), as loan rates on floating rate advances would immediately adjust to prevailing government security yields. However, a major source of variability in asset allocation-driven returns remains to be considered. This source is in fact the deviation of actual loan losses from those expected, and is the main research focus of this thesis.

Much of the research after 1984 has focused on information properties of the banking firm.

Lefebvre (1986) makes the point that random consumption patterns of individuals gives rise to early liquidation, and that this itself may be sufficient to raise individual return variance sufficiently above bank return variance to justify the bank deposit contract, even if there exists no difference between the portfolio alternatives available to the bank or investor. However, economies of scale in evaluation, plus the existence of financial intermediaries as sorters of projects, will give a better selection of projects than blind diversification on the part of the investor. Lefebvre advances the latter two arguments as giving banks a higher expected return than individuals, but does not comment on any variance impact. His arguments may provide other reasons for the existence of financial intermediaries, centring on the individual's consumption function and economies of scale in the selection process. This may be seen as strengthening Diamond's diversification case, but from the deposit contract viewpoint. Thus, even if the benefits of asset diversification are disputed, consumption diversification may help the overall diversification case.

James (1987) finds empirical evidence that borrowers pay the cost of reserve requirements on certificates of deposit *(CDs).* He concludes that banks provide some special service with their lending activity which is not available from other borrowers. The implication of James' findings are that banks may be able to charge a greater than economically justifiable interest rate to borrowers because of the existence of information effects in bank borrowing. This would imply that market equilibrium should lie at some point above the yield on prevailing government securities in a credit process model.

Bryan (1987) advocates a business process view of the banking firm, with the firm providing deposit, lending, investment and fee based advice services. He models each of these services as a separate business process, and his lending service model is closely analogous to the basic credit process modelled in this thesis.

Bernanke and Gertler (1989) develop a neo-classical business cycle model which may equip bank credit models with an interactive capacity. Traditional bank models have the deficiency that the bank is viewed as operating in a perfectly competitive loan market. However, the dynamics of the loan market may also be affected by customer decisions, which may in aggregate contract or expand the total loan market independent of banker's actions. In this respect a simple illustration is appropriate. Suppose an entrepreneur perceives an improvement in business conditions, which he interprets as reducing his default probability. The bank through which he finances remains unaware of the improvement, and continues to charge him based on his historic default probability.

Given that in a perfect market for loans the bank will now be overcharging the entrepreneur, the entrepreneur may decide to repay part of his loan. The bank may respond by lowering its interest rate to a level corresponding to the new loan level, but continue to overcharge by not reflecting the original improvement in creditworthiness. The entrepreneur continues to repay the loan, with equilibrium only being restored when the entrepreneur has either fully repaid the loan or has exhausted other sources of capital. The key point is that instability in volumes of loans can result from customer as well as banker action. Therefore, not only can credit rationing arise from customer action, but the total size of loan market is itself stochastic, and may lead to equilibrium substantially prior to the declining demand curve equating to returns on Government securities. This effect could, of course, also be produced by decreasing absolute risk aversion. Empirical evidence suggests the former, which means that a fully developed microeconomic model of the credit process needs to allow for customer, as well as bank action. The causation argument presented by Bernanke and Gertler is disputable (in fact causation can run the other way), but their paper does add a new dimension to the analysis of bank credit.

Asay and Albertson (1990) develop Byan's (1987) approach, and advocate the notional securitisation of all loans, leaving the lending business to be modelled as the sum of reserves plus credit enhancements. Their model of the bank's lending business is identical to that used in this thesis, although this thesis proposes a differing form of analysis for both reserves and credit enhancements.

Rajan (1994) develops an interactive model of bank managers with short term horizons influenced by both other managers and demand conditions. The model hypothesises the acceptance, in certain conditions, of negative net present value projects for credit extension by such managers. Development of the model leads to a theory of low frequency business cycles driven by bank credit policies. Rajan points to *the fact that credit policies change over* time as evidence that loans are a product of more than pure financial calculations. In the short term, bank management may attempt to manipulate current earnings in an effort to boost its reputation. In the long term, such efforts are doomed to failure, but this short term model does at least behaviourally help to explain observed credit cycles. It may be viewed at an extension of Bernanke and Gertler's (1989) neo-classical model.

Berger (1995)[b] investigates the relationship between capital and earnings in banking. He finds a positive relationship through the 1980's in the US, with evidence that this relationship has changed during the early 1990's. He infers that banks may have "overshot" their optimal capital ratios, as reduced real risk lowered optimal ratios while regulatory changes and higher earnings raised actual capital ratios. This conclusion implies that optimal capital ratios are a product of fluctuating real risks borne by banks. If this is the case, then value at risk *(VAR)* models should enable banks to target these ratios more accurately than blanket capital adequacy requirements.

2.2 Information, Credit Rationing and Collateral

Table 2.2 below summarises the chronological progression of information, credit rationing and collateral literature.

Table 2.2

Information, Credit Rationing and Collateral Literature

(Chronological Progression)

Year	Author	Brief description / comment
1977	Leland and Pyle	Information deficits and bank structure
1980	Campbell and Kracaw	Signalling hypothesis and information production
1981	Stiglitz and Weiss	Loan price as information source
1982	Thakor	Commitments and bank risk
1985	Bester	Screening versus rationing
1985	Chan and Kanatas	Asymmetric valuation and collateral
1986	Chan, Thakor and Greenbaum	Information acquisition by banks
1986	Devinney	Credit rationing in a theory of the banking firm
1987[a]	Besanko and Thakor	Asymmetric information model of credit market
1987[b]	Besanko and Thakor	Collateral and rationing: sorting equilibria
1989	Greenbaum, Kanatas and Venezia	Informationally advantaged lender policies
1989	Diamond	Reputation acquisition
1989	James and Weir	Relationship information
1990	Sharpe	Informational capture by banks
1990	Berger and Udell	Collateral and loan quality
1992	Berger and Udell	Evidence of credit rationing
1992	Holden and Subrahmanyam	Information and imperfect competition
1992	Rajan	Informed banks bargaining power
1992	Calem and Rizzo	Hospital lending as evidence of banks informational role
1994	Petersen and Rajan	Relationship lending
1995	Berger and Udell	Relationship lending in small firms
1995	Antzoulatos	Credit rationing and rational behaviour

Leland and Pyle (1977) discussed information asymmetries in the context of financial intermediation, observing that borrowers will always be better informed than lenders, resulting in a lender requiring to charge for his information deficit. Economies of scale in information production encourage the emergence of financial intermediaries.

Campbell and Kracaw (1980) developed a signalling hypothesis, which stated that the upper bound for the costs of financial intermediation given information asymmetries could not exceed the costs of a direct credit quality signal by the borrower to the capital markets. Borrowers would issue debt directly if it were cheaper for them to do so. As the costs of a direct signal of credit quality would be lower for larger companies, financial intermediaries concentrate their efforts in the area where credit quality signals are most expensive to generate, i.e. in lending to small and medium sized companies.

Stiglitz and Weiss (1981) point to the possibility of multiple equilibria in markets where banks use the interest rate payable as a screening device. As interest rates increase, borrower creditworthiness may decrease, with the possibility of lower overall returns from high interest rate lending. Further the level of interest rate payable may itself have a direct bearing on the riskiness of projects undertaken by borrowers. The authors use a simple model to present many, often counter-intuitive conclusions. Specifically, they identify default probability as acting to reduce expected returns in a similar way to bankruptcy costs in the Modigliani Miller corporate capital structure model.

If interest rates are set by the bank, then only projects above a certain risk threshold will be considered by entrepreneurs. This risk threshold is increasing in interest rates. The expected return to the bank on a loan is a decreasing function of its riskiness. The adverse selection effect outweighs rising interest rates due to the change in mix of applicants. Market equilibrium can feature either a single interest rate at or below market-clearing or several interest rates with excess demand for credit at each one. Collateral requirements may have an adverse selection effect in encouraging less risk-averse borrowers. Banks will seldom seek to steal competitor customers, because they will only attract the least profitable. In equilibrium, each bank may have excess loanable funds, but no bank will lower its interest rate. The large number of conclusions drawn are a direct result of the simple observation that the expected quality of a commodity is a function of its price. This paper is of particular appeal due to its large number of empirically testable conclusions, which may be incorporated into credit process models.

Thakor (1982) is of direct interest in that he presents a valuation model for variable rate loan commitments. A less than fully drawn loan facility may be regarded as a commitment in this sense. The unsurprising conclusion is that commitments increase in value in an uncertain interest rate environment. This paper relates to fee based commitments, but a minor model alteration can be made in order to adjust the model to UK conditions. The paper points to the partial takedown phenomenon as being deserving of study and indeed this will form part of our later empirical research in this thesis.

Bester (1985) argues that where information asymmetries exist, banks may eliminate credit rationing (which results from pooling different risks at one offer level) by simultaneously setting both the applicable rate of interest and required collateral. At any given interest rate level, borrowers will be sorted by collateral and *vice versa*. An assumption necessary for this argument to hold is that higher collateral enforces selection of less risky projects. Investors with low bankruptcy probability are shown to be more willing to accept higher collateral requirements at any given interest rate, and will thus be sorted into acceptable risks from the banks viewpoint. Investors with high bankruptcy probability will be sorted into higher interest rate contracts with lower collateral requirements. Providing all applicants for credit are within acceptable boundaries with respect to interest rates payable and collateral levels, sorting will ensure that credit rationing does not occur in equilibrium. Bester's argument relies heavily on the role of collateral in lending agreements.

Chan and Kanatas (1985) examine the role of collateral in terms of marginal lending decisions. The existence of high collateral levels may encourage lenders to back projects with relatively high default probabilities; the collateral will make good their potential losses. Thus collateral does not act as a sorting mechanism in the monotone sense suggested by Bester (1985). The reader is referred to Devinney's (1986) conclusions (section 2.1) with regard to collateral, which agree with those of Chan and Kanatas.

Chan, Thakor and Greenbaum (1986) view the bank as possessing an information processing technology which gives it a cost advantage over other lenders in screening loan applicants. They submit that the durability of information has reduced in recent times because of inter-temporal fluctuations in borrower credit ratings. Also they argue that a narrowing of interest rate spreads has increased banks ruin probability and has reduced screening expenditure which has led to poorer quality loans becoming more common.

Specifically, they postulate that screening is a matter of choice for an individual bank, denoted by a real value scalar α and with feasible set $A=(0,\overline{\alpha})$, and cost strictly increasing, strictly convex $V(\alpha)$. This screening expenditure is financed by bank equity and equilibrium is reached when expenditure on screening is exactly offset by expected savings in default costs.

Economies of scale may exist through cross sectional re-usability as well as temporal re-usability of information.

Chan, Thakor and Greenbaum ascribe the disimprovement in bank asset quality to the reduction in screening expenditures, and the reduction in output from those expenditures. However, there is no empirical evidence of screening expenditures being reduced; in fact, given the increasing default costs being experienced, it is equally likely that screening expenditures have in fact increased. A much more likely explanation for the undoubted disimprovement in asset quality is in fact the improvement in information technology, enabling borrowers to signal their true creditworthiness to the market directly, and in a more cost effective fashion than heretofore. This has led to the self selection of larger corporates away from bank borrowing as a means of finance, and evidence of this selection would be a relative lack of quoted companies in the loan portfolio under study. Thus, increased competition may be directly responsible for better risks leaving bank portfolios and thus disimproving average asset quality.

Devinney (1986) has examined the economic role of collateral. He defines collateral as "that amount which the bank receives, with certainty, at no cost when a borrower defaults". Thus, his discussion is limited to fixed collateral. Many forms of collateral are not fixed e.g. shares or property pledged as security, so that Devinney's definition is restrictive. He regards fixed collateral as moving the bank's income distribution to the right.

Devinney also assumes that no bad borrower will accept a collateralisation requirement of more than 100%. (Most mortgagees might disagree!) Essentially Devinney argues that substitution would enable a potential borrower to avoid borrowing by liquidating his/her collateral. In a real world sense this does not appear to be the case. Devinney also refers to collateral in escrow, so perhaps his definition assumes a borrower redepositing against a loan.

Devinney also argues that collateralisation cannot be used as a signal since non-defaulters will exit the market before defaulters. Thus while collateral has a direct impact on risk, it cannot directly signal default probability. Therefore, it acts only to reduce bank losses on default, given that default has occurred. Only if collateralisation costs poor credits more or if asset levels are positively correlated with ability to repay will collateralisation be a positive quality signal.

Besanko and Thakor (1987)[a] study a credit market with risk neutral agents and *ex ante* uninformed lenders. They sort by loan quantity, interest rate, and collateral. They conclude that credit will not be rationed, low risk borrowers pay higher interest rates than high risk borrowers, low risk borrowers obtain higher quantities of credit than they would if they disclosed full information, and, finally, low risk borrowers pledge higher collateral amounts than high risk borrowers. Their conclusions derive from the ability of the lender to sort borrowers into as many risk classes as there are types of borrower through a suggested four-dimensional sorting process, relevant dimensions being credit granting probability, loan quality, loan interest rate and required collateral. This sorting process is closely analogous to the loan pricing model developed in our subsequent chapter 7, although the sorting criteria differ. Besanko and Thakor (1987)[b] consider two types of loan market, monopolistic and competitive. In monopolistic markets they find that collateral will not be used unless it makes the loan riskless, and that increasing interest rates may cause high risk borrowers to exit the market prior to low risk borrowers. This second conclusion runs counter to Stiglitz and Weiss (1981), and is a result of sorting allowing *ex post* good borrowers to be observed as opposed to pooling and credit rationing, which does not permit such observation.

In competitive loan markets, sorting occurs through the inverse relationship between interest rates and collateral, permitting the identification of risk classes. Credit rationing may occur even where deposit supply is elastic and collateral is available. As collateral increases, the probability of the occurrence of credit rationing tends toward zero.

The above strand of the literature closely parallels this thesis. However, the interest rate is not used as a direct sorting mechanism herein. By using financial gearing, loan utilisation and collateral as sorting variables, risk groups are identified which are hypothesised to be homogenous with respect to default probability. The historic cost as a proportion of funds lent in these risk groups is then used to provide a pricing mechanism. Similar sorting techniques are used, but interest rates are derived, as opposed to the above models that take interest rates as exogeneously determined. In this thesis interest rates are a conditional sorting device, conditioned on loan utilisation, with low utilisation rate attracting lower interest rate charges.

Greenbaum, Kanatas, and Venezia (1989) examine a loan market in which an incumbent lender is better informed than other potential lenders. They find that the incumbent lender's interest rate, given client search costs and heterogeneous potential offers of credit, will exceed the incumbent lenders cost of funds, and will be higher than the average of competing potential offers. Potential lenders are prepared to make "loss leading" credit offers lower than their cost of funds. The expected remaining duration of a lender-client relationship is decreasing in the length of the existing relationship. Thus, clients are more likely to leave the longer their relationship with their existing bank.

James and Weir (1990) examine the benefits of a borrowing relationship in terms of underpricing of stock offers. They find that the existence of an established bank borrowing relationship reduces investor uncertainty and thus the underpricing of initial public offerings (IPO's) in the borrowers stock.

Sharpe (1990) analyses the "informational capture" of a bank's existing client base, and proposes implicit contracts whereby future period interest rates are implicitly contracted at the time the initial rate is set, subject to satisfactory borrower performance. This makes possible loss leading by banks, backed by reputation. "Informationally captured" clients yield their bank a profit in future time periods, but will not in aggregate yield a new bank profit at similar interest rates. This may explain the "stickiness" of credit customers, and applying Bernanke and Gertler's (1989) logic could explain the downward spiral associated with intermediary insolvency in the depression of the 1930s. Once again, the stickiness of credit customers is empirically testable. Banks although *ex ante* competitive in the credit market, may be *ex post* monopolists due to their ability to offer "informationally captured" clients interest rate terms below markets to the extent that their being poached is economically sub optimal for an *ex ante* competitor.

Berger and Udell (1990) examine collateral as an indicator of loan quality. They distinguish between the sorting by private information paradigm which is prevalent in the literature and the sorting by observed risk paradigm evident in conventional practice. Essentially they argue that previous models only allowed sorting by reference to variables set by the lender, while denying the lender any ability to sort *a priori*. They point out that observable risky borrowers may be required to pledge more collateral, rather than being offered an interest rate/collateral trade off as postulated by earlier sorting models.

Using an empirical portfolio of 1,000,000 loans made by 460 US banks, over the years 1977-1988, they establish that collateral is most often associated with riskier borrowers, riskier loans and riskier banks. Thus, riskier than average firms borrow on a secured basis, the average secured loan tends to be riskier than the average unsecured loan, and banks which make a higher fraction of unsecured loans tend to have riskier portfolios.

Berger and Udell's findings suggest that the value of recourse to collateral less than offsets the sorting by observed risk effect associating higher risk borrowers with higher collateral. These findings will be discussed in the context of the development of loan pricing models in Chapter 7.

Calem and Rizzo (1992) examine lending to hospitals perceived to be financially weak. They find a strong positive relationship between bank loans (as a share of borrowing) and profitability. This they present as evidence of hidden factors enhancing the bank borrowing hospital's financial standing. This study may be taken as further empirical evidence on the ability of banks to sort risk, and on the unique nature of bank lending agreements in terms of their existence as an information source.

Rajan (1992) discusses the fact that although the information benefits of bank financing are well covered in the literature, the costs associated with the bank's power to influence firms profits are not well understood. He examines how firm's choice of source and priority of debt attempts to limit these costs. This may lead firms into multiple sourcing project finance, and setting differing priorities for differing source payments. The effect of this may be to weaken the information benefits while simultaneously reducing the costs of relationship banking. Over time this may contribute to a disimprovement in loan quality.

Berger and Udell (1992) use the same portfolio as in their previous paper to examine empirically the significance of credit rationing. They would expect that if credit rationing were significant, the proportion of loans issued under commitment should rise in tight credit market conditions. They find no evidence of this, and conclude that credit rationing is not a significant macroeconomic phenomenon.

Moore (1993) revisits the ground covered by Bernanke and Gertler (1989) and states that a borrower will seek to improve his credit status by retaining earnings to the extent that his/her resulting credit savings exceed his/her opportunity loss in retaining earnings in the first place.

Petersen and Rajan (1994) use a small business administration U.S. database to show that single banking relationships increase the availability of finance over time but do not necessarily reduce its price. As in Rajan (1992) they find that any attempts to use multiple sources of credit both increases price and reduces availability rather than price.

Berger and Udell (1995) use the same data-source as Petersen and Rajan (1994) to solve the asymmetric information problems associated with small firm finance. With specific reference to letters of credit (L/C's) they find that borrowers with longer relationships pay less, and are less likely to pledge collateral. These results are consistent with relationship banking generating valuable information about borrower quality.

Berger and Udell comment on transaction-driven rather than relationship-driven loans, and identify L/C's as an example of the latter. This may explain the different results derived by them to those of Petersen and Rajan (1994), given their specific focus. A composite result would suggest that L/C's in relationship banking reduce in price over time, but this effect may be masked by other loan transactions which increase availability of finance over time, though not necessarily at a reduced price.

These last results imply that a durable single relationship may be valuable, and appear to contradict Greenbaum, Kanatas and Vanezia's (1989) model and Sharpe's (1990) informational capture model which predict increased interest rates over time. The debate seems likely to continue, and the eventual resolution may lie in the sorting paradigms first mentioned by Berger and Udell (1990). If loan sorting is by difference to observed risk, then this risk should reduce over time as lenders reduce information asymmetries, thus producing cheaper loans for given collateralisation levels. If, alternatively, loan sorting is primarily by private information, then information asymmetries will persist, and loans will not cheapen. With reference to the present thesis, the proposed sorting mechanism is firmly by reference to observed risk, so that the models therein belong (in a taxonomic sense) to the observed risk side of the debate, although empirical reduction in interest rates by duration of relationship would only be produced by the introduction of a new risk class, which requires statistical justification.

As the quality of data available improves, more risk should become observable, and less should remain in private hands for given risk quanta. It seems reasonable to suppose that improvements in information technology and risk classification will produce an improvement in data quality, leading to observable risk growing and private information risk decreasing over time. Thus the sorting by observed risk paradigm is likely to dominate.

2.3 Portfolio Allocation

Table 2.3 below summarises the chronological progression of portfolio allocation literature.

Table 2.3

Portfolio Allocation Literature

(Chronological Progression)

Year	Author	Brief description / comment
1952	Markowitz	Portfolio selection, mean variance, efficiency
1971	Pyle	Modern portfolio theory, hedging, quantity setting
1974	Hart and Jaffee	Portfolio theory, quantity setting, random deposit rates
1979	Kane and Buser	Diversification by government security holdings
1981	Brealey, Hodges and Selby	Simulation of loan portfolio risk
1984	Bennett	Diversification in global loan portfolios
1991	Gelles	Risk measurement for banks
1991	Avery and Berger	Loan commitment risk
1992	Sinkey	Management of loan portfolios
1994	Glantz	Management of loan portfolios

Following Markowitz's (1952) path-breaking development of a mean variance framework for asset portfolios, Pyle (1971) used such a framework to determine necessary conditions for financial intermediation. He used a 3-asset model featuring a riskless asset in addition to loans and deposits. His results imply that covariance of lending and deposit rates is extremely important, and a positive loan premium and /or a negative deposit premium must exist for the firm to make risky loans and to intermediate deposits. In this thesis we concentrate only on the positive loan premium. As previously discussed, a modification of the competitive asset market model should produce a variable loan premium which will have a minimum value of zero, as a value less than zero would permit loan arbitrage.

The specification for the optimal solution in these models is critically dependent on the returns to assets and/or liabilities and in most cases these are assumed exogeneously given. This does not apparently give the bank any role in determining asset returns through its lending policies. The banks own objective function must play a part in establishing the feasible portfolio set, and return maximisation may not be a first best solution. Hart & Jaffee (1974) first postulated that the Markowitz/Tobin/Litner model of an efficient portfolio could directly be applied to financial intermediaries. The intermediary is assumed to maximise a utility function defined over the mean and standard deviation of returns. The authors recognised a shortcoming in their own approach in respect of quantity setting by banks as opposed to stochastic quantities, and confined their considerations to portfolios of traded securities. They showed that if the intermediary has a Neumann-Morgenstern utility function and if the terminal value of portfolio assets are multi-normally distributed, then a unique efficient portfolio solution exists.

Hart and Jaffee (1974) develop a separation theorem under some restrictive assumptions, which may work in an environment in which no risk-free asset exists i.e. a fully liability-funded bank. Arguably all banks which maintain portfolios of government securities are already fully liability-funded if government securities are viewed as a risk free asset.

Kane and Buser (1979) theorised that financial intermediaries are specialised producers of diversification services. They examined bank holdings by number of U.S. treasury securities as evidence of diversification, and concluded that stockholders were prepared to pay considerable diversification costs to increase confidence that actual portfolio variance was at or below an acceptable level. Marginal diversification benefits were found to be a decreasing function of size. Their work pointed to information risk as being a possible cost element mitigating against "home made" diversification, and paved the way for the delegated monitoring/agency theory of banking which prevails today (Diamond (1984) *et al*). They again concentrated only on marketable securities, and did not discuss diversification of credit portfolios.

Bennet (1984) suggests the direct application of portfolio theory to bank loan portfolios, with banks pricing aggressively to secure business acting as a relative hedge to other portfolio components. According to his reasoning, banks should focus on the absolute variability of returns on their entire loan portfolio. However, apart from referring to hypothetical simulations by Brealey *et al* (1981) and constructing a simple illustrative example, Bennet provides only a recipe for what banks might do. The effectiveness of diversification remains to be tested. The present author intends *inter alia* to assess directly how diversified an actual loan portfolio is.

Gelles (1991) proposes a simple risk measure $\sigma_R / \overline{R}$, the coefficient of variation of profit, being the standard deviation of profit divided by the average realisation of the random variable profit. This will be an appropriate measure for any risk averse bank, and the author contends that it is relevant regardless of the nature of the risky asset distribution. He maintains that any exogenous action on the part of regulators which increases the probability of negative bank profit is to be avoided. The measure is simple, observable and appears to make practical sense. However, the author's assumption that if R is normally distributed then the measure is monotonically positively related to the probability that bank profit (and thus end-period wealth) will be less than zero deserves examination if only because no evidence is given that R is in fact normally distributed, and because not all risky options are linear transformations of a given distribution.

Chirinko and Guill (1991) analyse industry loan losses in the United States; they link these losses to macroeconomic and industry variables in order to derive a loan loss equation. They downgrade financial markets alone as unlikely to prove useful in assessing depository institution risks. It is argued that existing capital standards, decided by broad asset group allocation, focus on the wrong target. Diversification ignoring covariance can lead to a substantial mis-statement of risk exposure.

The authors focus on portfolio characteristics allied to cash flow characteristics for specific industries and macroeconomic variables in order to produce risk measures of mean, standard deviation and to locate area to the right of some critical level (representing excessive loan losses) for the loan losses of a particular bank. Assuming loan losses normally distributed the mean and standard deviation characterise the distribution. This may be an heroic assumption.

The authors conclude that much is lost by ignoring industry-specific information when analysing loan portfolio risk. They also conclude that covariation of portfolio risk is of great importance, specifically in terms of the critical region of the loan loss distribution, which is likely to be of interest to regulators.

This study closely parallels the main body of this present thesis. However industry-wide loan losses are not available to the present author, so that emerging information is confined to that provided by the specific portfolio used to develop and test the proposed methodology. Further Chirinko and Gull (1991) are concerned more with areas of concern to regulators, rather than the efficient portfolio required for ongoing management. However, both as an indication of an alternative methodology and as a reference point for loan loss distribution analysis, their study has merit. The extent to which the loan loss distribution may be skewed could have important implications for just those probabilities with which the authors are most concerned. The study is therefore incomplete, but will be referred to in Chapter 6.

Avery and Berger (1991) examine bank commitments, or agreements to lend in some future period at a price agreed now. They conclude that commitments increase a bank's risk exposure, but may or may not have a disproportionate effect depending on the sorting of borrowers to whom commitments are to be given versus the moral hazard and adverse selection activities of the borrowers themselves. This paper is of interest in the area of facility utilisation, which will prove to be a risk determinant later in the current study.

Sinkey (1992) identifies four major drawbacks in closing the gap between research and practice in loan classification, which is vital in mean variance analysis of asset portfolios. These are:

1. The inability to quantify the relationship aspect of the lending process.
2. The reluctance of lenders to share information with researchers.
3. The lack of data on rejected borrowers.
4. The backward looking nature of classification studies.

He suggests that: "Timely and accurate pro forma information on the decision inputs required by lenders combined with similar forecasts of competitive and economic conditions would be ideal for the prevention, identification and resolution of borrower problems at commercial banks" (Sinkey (1992 p 514)).

Recent work on bank relationships by duration and the availability of some databases are beginning to make inroads into the first two drawbacks, but the last two are more problematic. In terms of competitive loan pricing the last drawback is particularly significant. For the purpose of this thesis, the focus is less on highly responsive loan pricing than on the solvency of banking institutions given some control mechanism allowing banks to respond to default experience within their own portfolios. Sinkey describes a highly competitive and responsive loan market, which would imply a rapid adjustment of loan pricing to current economic developments. This is one end of a loan pricing spectrum, the other end of which is permanently fixed loan margins.

The problem is to produce a loan pricing framework which is sufficiently responsive to current conditions, while not so responsive as to jeopardise the solvency of the lender. This topic will be further discussed in Chapter 6.

Glantz (1994) refers to an expected default frequency model (EDF) which will be seen to be one of the key building blocks of the insurance approach to pricing credit risk. His model depends on the existence of publicly traded stock, from whose performance the value of debt may be inferred. As most bank loans are made to firms whose stock is unquoted, the EDF model proposed by Glantz is unsuitable for valuation of most bank loans. This thesis relies on the past performance of similar loans to estimate asset volatility. Thus asset volatility is inferred directly from examination of relevant portfolio risk groups, given that no market value is readily available for the computation of such volatility in the case of single borrowers. This marks a key difference between the two approaches to loan pricing.

2.4 Option Pricing and Deposit Insurance

Table 2.4 summarises the chronological progression of option pricing and deposit insurance literature.

Table 2.4

Option Pricing and Deposit Insurance Literature

(Chronological Progression)

Year	Author	Model Description / comment
1973	Black and Scholes	Option pricing of corporate liabilities
1974	Merton	Option pricing of corporate default risk
1977	Merton	Option pricing of deposit insurance
1978	Sharpe	Variable capital fixed premium deposit insurance
1981	Buser, Chen and Kane	Optimal capital under existing regulation
1983	Horvitz	Variable premium deposit insurance feasibility
1986	Brickley and James	Stock price response to deposit insurance
1986	Goodman and Santomero	Variable premium deposit insurance
1989	Acharya and Dreyfus	Pricing deposit insurance with variable reorganisations
1989	Furlong and Keeley	Regulatory influences on risk taking
1990	Gorton and Santomero	Quoted debt response to bank risk
1990	Goudreau and King	Recent banking profitability
1990	Keeley	Charter value as a risk determinant
1991	Genotte and Pyle	Regulation and bank risk
1991	Avery and Berger	Risk based capital
1991	Bradley, Wambeke and Whidbee	Risk weights and risk based capital
1991	Flannery	Error in regulatory measurement systems
1992	Chan, Greenbaum and Thakor	A fair deposit insurance system
1993	Allen and Saunders	Deposit insurance reform
1993	Merton and Bodie	Deposit insurance reform
1993	Benston and Kaufman	Deposit insurance reform

Black and Scholes (1973) made a significant breakthrough in the mathematical evaluation of corporate liabilities by enabling the valuation of equity as a call option on firm value. Merton (1974) developed Black and Scholes work, using the isomorphic relationship between levered equity and a call option on firm value, to develop a valuation model for corporate debt and thence to deduce a risk structure of interest rates dependent on the volatility of firm value and the ratio of indebtedness to firm value. Merton acknowledges a bias in his work, since his use of stochastic calculus did not permit a unique solution, without his use of a discount function at the risk-free (as opposed to the risky) rate of interest.

Merton (1977) applied option pricing theory to the problem of deposit insurance and completed the basic product analytics. However, Merton did not specify the process whereby volatility of either bank or firm value might be computed. Merton provides a blueprint for what would be done if these volatilities were available. This present thesis provides a framework for the computation of these volatilities.

Merton (1978) investigates who pays for deposit insurance. He concludes that in a competitive banking industry, the equilibrium return on deposits will be reduced by the auditing costs of the intermediary, with the actual put option price of deposit insurance being paid by the shareholders.

Sharpe (1978) examines fixed premium deposit insurance, and concludes that an equitable insurance system must allow capital requirements to vary between banks of varying risk. Sharpe's main argument is closely analogous to that in this thesis. We do not attempt to price deposit insurance, but our methodology equates ruin probability between banks of varying risk using a VAR approach. Our methodology directly implies variable capital requirements coupled with deposit insurance premiums which would vary little between banks.

Buser, Chen and Kane ((1986) point out that deposit insurance and regulation are closely linked, and that regulatory authorities may charge an implicit insurance premium by their activities. They argue that contingent regulatory interference is an implicit premium, the size of premium increasing as the probability of such interference increases. Optimal bank capital is that which minimises the cost per dollar deposited of the sum of implicit and explicit premiums. Banks operate close to the interference threshold (as is evidenced by US banks frequently shifting on and off the FDIC's problem list in large numbers) in response to their capital positions weakening or strengthening due to unanticipated cyclical swings in economic activity.

Horvitz (1983) argues against variable deposit insurance premiums on the basis that risk intensity is likely to be inherently far less stable than in conventional insurances. The effect of this would be to have banks agreeing to insure deposits when they perceived such insurance to be to their advantage, and not otherwise. Deposit insurance premiums would increase as banks could least afford to meet their cost. The overall effect would be a deficit for the insurance fund, due to adverse selection by banks, and substantial reliance on deposit insurance in times of economic difficulty. The maintenance of a solvent, relatively stable fund would be extremely difficult. Horvitz concludes that capital standards, rather than deposit insurance premiums, should vary. Although not considering the issues in the same light as Horvitz, the methodology proposed in this thesis would avoid the difficulties mentioned by him.

Brickley and James (1986) investigate whether access to the deposit insurance system has a discernible effect on the stock prices of financial intermediaries during periods of financial stress. Specifically, by reference to savings and loan institutions in the United States, they find that stock returns reflect investor expectations concerning the reaction of insuring agents to changes in risk (in terms of insurance availability). Changes in portfolio values need not be reflected fully in changes in the equity value of the institution. This has implications for studies such as those by Marcus and Shaked (1984), which utilise estimates of the variance of equity returns in an option pricing approach to the cost of deposit insurance. The use of this methodology may understate the true price of the insurance, which is portfolio rather than equity-based.

Goodman and Santomero (1986) examine variable rate deposit insurance in a wider social context. They argue that such a scheme has costs in the real (as opposed to the financial sector), by increasing the cost of funds to the real sector, and thus increasing the probability of social costs associated with the bankruptcy of borrowing firms. In addition to being actuarially fair, the social costs referred to must also be taken into account. The authors do not recommend a scheme of insurance, but confine their conclusions to the above. An implication which may be drawn is that with positive social costs and the difficulties already referred to by Horvitz (1983), variable rate deposit insurance is problematic in costing and implementation.

Acharya and Dreyfus (1989) point to difficulties in pricing insurance when the claim costs of such insurance is directly influenced by the actions of the insurers themselves. Optimal policies pursued by regulators for both bank closures and deposit insurance premiums may be derived as functions of rate of flow of bank deposits, the interest rate on such deposits, the risk free interest rate, and audit/administration costs. The insurer must simultaneously determine his/her optimal closure policy and respective level of premium. The optimal closure level is found to be at an asset to deposit level of at least 1: that is, some excess of assets over deposits. The authors point out that these conclusions may be bank-specific in that they ignore possible "knock-on" effects of bank closure.

Furlong and Keeley (1989) examine the response by banks to increases in required regulatory capital; they conclude that a value-maximising bank will have diminishing incentives to increase asset risk as capital increases. Such a bank will raise such increased capital, rather than reducing instead its balance sheet size. The analysis requires that regulators do not relax efforts to limit asset risk and size. In this environment, more stringent capital requirements will reduce the risk exposure of the deposit insurance system.

Gorton and Santomero (1990) examine quoted bank subordinated debt to establish whether financial markets enforce a risk discipline on banks by reducing the value of such debt as the issuing bank increases its asset risk. They do not find evidence of such discipline, implying that regulators should not rely on financial market signals.

Both Keeley (1990) and Goudreau and King (1990) examine banking profitability, the former over a longer timescale. Keeley analyses the US banking problems of the 1980s as being due to moral hazard associated with fixed rate deposit insurance availability as bank charter values fell, through increased competition. He concludes that in an environment where bank charters have reduced in value, thereby removing a barrier to unfettered moral hazard, the then current deposit insurance system needed reform.

Genotte and Pyle (1991) analyse the effect of deposit guarantees on bank's, loan portfolios. They view bank loans as market securities. It is concluded by them that deposit guarantees lead to inefficient investment and that capital requirements may lead banks to increase asset risk. The key difference between their approach and that of Furlong and Keeley (1990) is their consideration of bank loans as market securities, which will respond in value to capital requirements. Loan portfolio payoffs are characterised by a present value V and a risk index σ. Banks seek to maximise the net present value of loan portfolios $J(V, \sigma)$. Deposit insurance allows the bank to fund such portfolios at the riskless rate. The net present value of the bank is the value of the subsidy represented by deposit insurance and the net present value referred to above. The subsidy is strictly increasing in value as insured deposits increase. Banks would buy traded assets without limit in the absence of capital requirements. However, traded assets require inputs in the form of information about relative risk.

The effect of an increase in capital requirements may be to reduce scale, but to cause the bank to effect this reduction by reducing assets in increasing order of risk in order to maintain or increase the subsidy value. If assets already held have a risk elasticity greater than the risk reduction effect of increasing capital, the perverse effect may be that an increase in capital requirements may increase the probability of bankruptcy. This approach requires the existence of risky securities in sufficient quantity to enable the required elasticity to exist.

Avery and Berger (1991) examine the now current capital adequacy requirement referred to as risk-based capital (RBC). They conclude that RBC is an improvement, and is stricter than previous requirements for large banks and for the system as a whole. Banks representing more than 25% of all bank assets would have failed these standards in 1989. They observe that the ideal dataset for the analysis of the effectiveness of RBC would include information on the performance of individual loans, which was not available at the time. They conclude that a risk-based deposit insurance scheme using similar weights to RBC (plus some rewards for higher capital) would "likely" be an improvement on flat rate deposit insurance. They point out the risk of using data from a past regulatory environment to assess the effectiveness of RBC, and mention that RBC penalises large banks because of their relatively higher proportion of off balance sheet activities. RBC is an improvement, but could itself be improved. This thesis proposes a VAR computation of capital requirements which is argued to represent a further improvement on RBC.

Bradley, Wambeke and Whidbee (1991) calculate the required RBC to ensure actuarial equity in the deposit insurance scheme over the 1985-88 period in the United States. They conclude that RBC would have been insufficient, and would need to have been set at a 20% standard rather than 8%. They also point out that losses may have been inflated over their chosen period by resolution problems in savings and loans, so that 20% may represent an overestimate of required capital on standard assets. They further point out that interbank deposits and residential mortgages may be relatively safer than 20% and 50% of standard weightings, respectively.

Flannery (1991) examines the real world difficulties of establishing bank asset values, and points out that errors in measurement can cause mispricing of deposit insurance. In such an environment, Flannery shows that the impact of these errors on private sector allocations can be minimised with a combination of risk-related capital standards, and risk-related deposit insurance premia. Key to this conclusion is that measured asset risk provides a differing yardstick as it varies, implying that both leverage and insurance premia should vary as asset risk varies. Flannery observes that moral hazard could still exist between measures of asset risk. This could be countered by a retrospective adjustment by regulators.

As error in asset risk measurement is reduced, the scope for "game playing" is reduced. However asset risk will vary in any event due to underlying economic conditions, so that variable leverage and premia would obtain even if measurement error were eliminated. This thesis attempts to minimise measurement error by specifying a VAR model unique to each bank, and specifies insolvency probabilities of sufficiently small order as to concentrate the adjustment to changing asset risk on the leverage component. Thus, what is proposed herein is the reduction of measurement error, the risk adjustment process to take place through varying capital, and a small residual theoretical deposit insurance premium relative to preset measurement and standards.

Chan, Greenbaum and Thakor (1992) examine whether deposit insurance can be fairly priced in a deregulated environment. Where private information and moral hazard exist, the former can be overcome by offering each bank a menu of choice whereby capital is inversely related to the required deposit insurance premium. However, the moral hazard can only be overcome by the introduction of subsidies, implying that fair deposit insurance is not possible. These subsidies may be generated out of social cost savings (Goodman and Santomero (1986)) so that fair deposit insurance may still be possible for society as a whole, even though Chan, Greenbaum and Thakor indicate that it is unlikely that a private sector insurer could provide this service. The reduction of moral hazard associated with VAR models (where the responsiveness of VAR to increased observed risk may be optimised) may reduce the extent of required subsidy. If moral hazard were to tend toward zero, then deposit insurance could be fairly priced with a variable deductible.

Allen and Saunders (1993) point out that the value of subsidy in deposit insurance may be overstated by simply valuing the deposit insurance contract as a put option. In fact the option is a callable put if the insurer can act to close the bank prior to shareholders. This call provision has value, represented by savings generated by early official closure. This value must be subtracted from the simple put value in estimating the extent of the subsidy. If the regulator rewards lower-risk banks by exercising forbearance (delaying exercise of the call), then beyond some asset risk threshold deposit insurance may be of diminishing value to shareholders.

Merton and Bodie (1993), Flannery (1993) and Benston and Kaufman (1993) discuss functional reform of the deposit insurance system in the light of the arrival of RBC. The main focus of this thesis is not, in fact, on deposit insurance, although the author recognises that any VAR model must have implications for deposit insurance as capital levels and deposit insurance are substantially interrelated. Given these kinds of linkage, and bearing in mind that deposit insurance is not a central issue in this thesis, the preceding review is confined solely to the evolution of the literature, rather than its present practical application.

2.5 Insurance

A chronological development of the insurance literature is not furnished, because most of the mathematical functions associated with insurance were completed by less well-known Scandinavian mathematicians like Bohlmann (1909), Lundberg (1909),(1919) in the early part of this century. Whereas a generalised theory of financial intermediation was required and sought in order to examine the reasons for the existence and activities of banks, no such theory was necessary to describe the benefits of risk pooling arrangements embodied in insurance intermediaries. The latter focus was essentially on statistical properties themselves in a particular context or application.

More recent literature begins with Ammeter (1953) who investigated the pricing of risk pooling between insurance companies. Cramer (1954) refined the earlier models by establishing a mathematical framework for the existence and operation of single period insurers, assuming inter-temporal stationarity of risk.

Subsequently the literature diverges into several branches:

1. *Simulation models of insurance companies in operation.* A basic model was first completed by Sugars (1973). This model did not contain any feedback, and limited itself to the consideration of stochastic bundles on limited numbers of observations of random variation in the claims process. Subsequently a detailed blueprint for insurance company simulation was developed by Beard, Pentikainen, and Pesonen (1984). Their model contained extensive interaction, developed by Daykin and Hey (1989) and Daykin Pentikainen and Pesonen (1994).

2. *Insurance Pricing:* Wilson (1977) first modelled insurance product pricing in a market where both moral hazard and adverse selection existed. Biger and Kahane (1978) and Blazenko (1986) concentrated on the design of insurance products which would minimise these twin problems. Hsaio, Changseob and Taylor (1990), Bond and Crocker (1991) and Brockman and Wright (1992) concentrated on the endogenous categorisation of risks, providing a perspective on the relative magnitude of risks by reference to past experience of similar risks. This enabled statistically rigorous procedures to be adopted in pricing insurance risk, assuming broad inter-temporal stationarity in risk intensity.

3. *Implicit contracts:* This literature concentrates on the implicit promise of an insurer to review a short-term insurance contract at some price similar to that being presently charged, provided there is no material change in the insured risk. Lof (1983) investigates the problem from a linear control perspective, with product price and reserves each partly absorbing risks associated with the implicit contract. Rubenstein and Yaari (1983) examine the emergence of moral hazard over a multi-period insurance contract, which may not be present in single period insurances. Rosen (1985) examines implicit contracts in general, and concludes that a first period contract must be priced to reflect the implied option granted for all

future periods, thus leading to higher prices for such contracts than for genuine single period contracts.

Cooper and Hayes (1987) continue the development of the literature by examining how the moral hazard problem may be overcome by allowing the insured to participate in the benefits of his/her own good housekeeping. A no-claims discount would be a specific example of such participation. As part of the loan pricing model developed in this thesis, a similar methodology is advocated, whereby clients of banks would be rewarded with a lower interest rate payable for lower loan utilisation levels. This represents a form of lower risk discount encouraging the borrower to reduce his/her exposure to risk of default, and to participate in such savings as result.

Underwriting cycles: Cummins and Harrington (1987) and Taylor (1988) have investigated underwriting cycles, the former with respect to regulation and the latter with respect to pricing and solvency. They both conclude that underwriting cycles exist, and therefore such non stationarity of risk intensity and/or insurance pricing must be allowed for in solvency calculations.

Claim Payment: It may be many years before claims are finally settled, and an increasing amount of practical work has gone into estimating the claim "tail" in respect of payments to be made in the future on claims which have already occurred. Renshaw (1989) uses generalised linear interactive modelling to extrapolate such future claim costs from those incurred in the past, adjusted for ensuing inflation and volume of business. Verrall (1989) develops a state space representation of the chain ladder technique of reserving for such claims. Verrall (1993) points out that these techniques may be used even where negative incremental claims exist (as would be the case if provisions were "written back", or if claim reserves were found to be surplus to requirements). Verrall's approach is to transform the mean of the distribution being used to ensure that all claims are positive in the transformed state, and then to calculate reserves reversing the transformation after relevant calculations.

Provisioning in banking is similar to claims reserving in insurance, with full provision in respect of default losses being completed approximately 15 months after default notification. Thus reserving techniques similar to those used in insurance models are necessary to price loans accurately.

An excellent compendium of insurance literature is contained in the series by Kluwer Academic Publishers (1985, 1986, 1989[a], 1989[b], 1991 and 1992).

2.6 Regulation

A very extensive and rich regulation literature (much of it US based) exists, dealing with reasons for and effectiveness of banking regulation. As this thesis does not propose to enter directly the regulatory arena, and as regulation insofar as it is interrelated with deposit insurance has been discussed in Section 2.4, we confine our review to current regulations embodied in the following:

EC 86/635	On the preparation of consolidated accounts
EC 89/643	On solvency ratios
EC 89/646	On European co-ordination of laws regulations and administrative provisions
EC 92/30	On supervision
EC 93/6	On capital adequacy

The Basle (1988) risk-based capital adequacy standards were incorporated into European Community (EC) law in EC 89/643. These standards apply to commercial bank "banking books", including unquoted loans to be held to maturity.

Capital must be held in proportion to the weighted risk as determined by the application of the above standards to a bank's holdings of earning assets. Such capital requires remuneration (as it forms the "risk capital" of the banking entity) and gives rise to a need to earn some target rate of return on capital. This target rate of return is an integral part of loan pricing calculation.

Subsequently EC 93/6 drew a distinction between assets held for resale ("traded assets") and assets which were to be held to maturity. In respect of the former, it set out a building block VAR framework for calculation of capital required to back bank trading books.

Essentially, this thesis argues the merits of applying a VAR-type approach to both traded and non traded assets, and proposes a method of computation in the case of non traded assets, that imputes VAR from observable portfolio-specific inputs to a standard model.

Current regulation with regard to risk-based capital (EC 89/643) is compared to that implied by a VAR model of credit risk, analogous in operation to EC 93/6 which relates to traded assets. Discussion is confined to differing implications for capital adequacy and insolvency risk, and does not attempt to address regulation in terms of its own specific literature and policy arena. This thesis is concerned with capital adequacy regulation only to the extent that it requires that loan pricing has to recover *inter alia* the minimum required rate of return on the bank equity "allocated" (on risk adjusted criteria) to a loan. In short, regulation and the respective regulatory capital are essentially a datum to this thesis.

2.7 Conclusion

The relevant banking and insurance literatures have has been reviewed selectively under 6 main headings. These have set the proposed research in context by reference to the evolving theory of financial intermediation (which discusses the existence and methods of operation of banks); the information asymmetries that characterise the activities of financial intermediaries (with the use of credit rationing and collateral as means to minimise the effect of such asymmetries); the portfolio allocation activities of financial intermediaries (this thesis sets out a framework for mean variance efficiency and resource allocation using a VAR methodology); option pricing and deposit insurance as a means of quantifying the risk of loan portfolios (this thesis equates option and insurance pricing methodologies, and develops a variable capital fixed insolvency probability model to address the twin issues of deposit insurance and capital requirement); insurance pricing with specific reference to multi-period insurances; and finally regulation.

Some additional references are cited in the context of specific chapters of the thesis. These are included in the full bibliography at the back of the thesis.

Chapter 3

Theory and Practice of Credit Risk Treatment in Bank Loan Pricing

Introduction

The previous chapter selectively surveyed the main literature from which important strands of the present thesis develop. This chapter focuses more specifically on credit risk treatment in bank loan pricing, the central concern of this study. The purpose of this chapter is to discuss the nature and operational significance of credit and other loan-related risks. A simplified bank loan-pricing function is presented and discussed in the context of present theory and practice. The strengths and limitations of present pricing practice are explored. The alternative bank-specific approach in this thesis is introduced, and the key variables integral to the proposed approach are identified.

3.1 A Simple Bank Loan Pricing Function

3.1.1 Discussion of Loan Risks

The primary risk to which a lender is exposed is that the borrower will not repay the loan either wholly and/or according to the agreed repayment schedule; this risk is known generally as credit or default risk. Other risks associated with bank lending are interest rate risk, (the risk that market interest rates may differ from those implicit in the interest rate payable), and liquidity risk (the risk that a bank may not have the resources to meet legitimate liquidity demands).

Sinkey (1992) points out that banks acquire reputational capital as monitors of credit risk. While this reputational capital is valuable, liquidity risk is much reduced, as runs on banks, known as liquidity crises, emerge only when markets devalue a bank's reputational capital. Recent US and other banking experiences suggest that the underlying reasons for the devaluation of banking reputational capital can usually be traced to excessive credit risk, manifested as heavy loan losses e.g. Continental Illinois (1982) and Bank of New England (1991). For a bank which does not take on excessive credit risk, money management techniques are generally sufficient to absorb short-term liquidity fluctuations, with government security portfolios available as a backstop, before unquoted asset portfolios require realisation. Thus, liquidity risk is not material to loan pricing (although it may be material to a bank's ability to extend a loan).

With respect to interest rate risk, loans may be priced at fixed or floating interest rates. Given that most bank liabilities are short-term floating interest rate sensitive, interest rate risk may exist when interest rate resets on loans occur less frequently than resets on liabilities, so that an interest reset gap emerges. In practice, for example, fixed interest rates result as a combination of floating interest rate loans with floating/fixed interest rate swaps. The interest rate risk associated with fixed interest loans is priced into the swap contract, which is separate from the loan.

Loans may be priced by reference to the bank's own cost of funds, by reference to a benchmark rate of interest (e.g. prime rate or base rate), or by reference to the interest rate prevailing on a floating interest rate government obligation of similar duration to the loan being granted. With respect to the first of these methods Sinkey (1992, p518) observes:

> "If on a risk adjusted basis bond returns exceed loan returns, and if a bank is not under regulatory pressure for not serving the lending needs of its community, then, if the interests of bank shareholders are to be served, the bank should be investing in bonds".

Asay and Albertson (1990 p14) note that when a bank chooses to hold a loan

> "...it is choosing to engage in money management, holding a security with a yield commensurate with asset backed securities in the capital markets, as well as choosing to be in the loan origination business....it may make perfect sense for the bank to sell its implicit asset backed securities and replace them with higher yielding securities of equivalent risk even if the banks funding costs are lower than the yield on the asset backed securities created and sold". The "sale" may be notional.

From a theoretical portfolio allocation viewpoint, basing loan pricing on the bank's cost of funds is only strictly justified if a direct link can be established between depositors prepared to accept lower than market interest rates in the expectation of a future subsidised loan. Given separated competition in both liability and asset markets, such a link does not appear plausible. Thus, for a profit-maximising bank, asset choice should in theory be independent of liability choice; asset pricing, therefore, should be independent of liability pricing.

With respect to the second method, the crucial question is, how representative is the benchmark rate? If the benchmark rate is closely correlated with short-term default free assets in the market in question it and represents a competitive part of the portfolio choice open to the bank, then pricing by reference to the rate is an option. Disintermediation in financial markets has tended to make reference rates such as the US prime rate, the UK base rate, or the German Lombard rate, for example, less representative of prevailing asset market interest rates, thus making such benchmark rates differ from the true rates available for asset allocation purposes.

With reference to the last choice, prevailing government security rates represent those applicable to a security free of default risk, and subject to identical interest rate risk to that of the comparable asset which a loan represents. If day-to-day liquidity risk is accepted as a money management function charged to depositors, and that liquidity risk compelling (forced) loan sales or realisations is of very low order, then the difference between the interest rate payable on a loan, net of administration costs, and that payable on a government security of similar reset and duration, represents the charge payable to the bank in respect of default risk. Thus the third choice is appropriate for this research, which concentrates on default risk and is only concerned with developing a more market-orientated (opportunity cost) system of practical bank loan pricing. It also fits in with modern CAPM and other pricing methods which are built around opportunistic, externally focused pricing.

3.1.2 Development of a Basic Loan Pricing Model

In equilibrium, Sinkey (1992 pp 517-525) proposes the following generalisable loan pricing model:

$$(1+r^*)(1-d)=(1+r) \tag{3.1}$$

where: r* = interest rate on risky loan

d = expected loss per unit lent

r = risk free interest rate

The bank must generate sufficient interest income on its loans to compensate itself for expected default costs and resulting earnings uncertainty and to produce an identical return to that payable on a risk free asset.

Re arranging, the respective default risk premium is

$$(r*-r)=\left(1+r^*\right)d \tag{3.2}$$

This general type of approach is further developed in Chapter 5 of the current thesis. Critical to the solution of this equation is the value of *d,* the expected loss per unit lent. In practice this is a function of the well-known 'Cs' of credit (see next paragraph). The model should be forward looking, as it relates to future expected values.

Sinkey (1992 p 522) extends the model to express *d* in terms of its functional components which he identifies as "information quality, character, level and stability of cash flow, real net worth and guarantees". He further argues that each of the above is a function of a "state of the economy" variable. This last variable leads to covariation of risk within loan portfolios necessitating the introduction of a further term, *p*, representing the markup or markdown for portfolio risk.

Equation 3.2 is thus transformed to

$$r^{*} = \frac{(1+r)}{1-d-p} - 1 \qquad (3.3)$$

Equation 3.3 decomposes to a respective risk premium:

$$r^{*} - r = d + p + (d + p)\, r^{*} \qquad (3.4)$$

requiring investors to be compensated for "the time value of money (r), default risk (d), portfolio risk (p), and interaction effects $|d+p|\,\mathrm{r}^{*}$. As "p is difficult to compute", these latter equations 3.3 and 3.4 are "more interesting for their theoretical insights than for direct and practical empirical applications." Nevertheless, they do embody the essence of practical loan-pricing in banks. Sinkey's model is the distillation of widespread practical experiences and theory.

The approach adopted in this thesis has many close parallels with Sinkey's model. The "state of the economy" variable corresponds to risk intensity; the "level and stability of cash flow" to utilisation; the "real net worth" to gearing; and "guarantees" to security. All of the above are 'rating factors' within the model proposed in Chapter 7.

Typical credit analysis focuses on *(1-d)*, the probability of full repayment. Given that such probabilities in most banking applications will be close to unity, apparent accuracy is relatively easily obtained. However, given the highly leveraged nature of lending, small percentage movements in (*1-d*) occasioned by large percentage movements in *d*, can have serious operational implications for underlying financial measures. Table 3.1 illustrates this point:

Table 3.1

Lending performance of 27 large U.S. banks (1986-89)

	Return on Equity %	Loan spread over securitised rates %	Provision as % loans	Operating Exposures as % Revenues
Average of 27	13.5	2.56	0.84	47
Average of top 5	23.9	3.34	0.90	43
Average of bottom 5	2.0	2.19	1.23	54

Source: Asay and Albertson (1990)

Asay and Albertson examined 27 large U.S. banks over the period 1986-89. These banks were of similar size and operated in a similar economic environment. The average spread achieved was 1.72% for all banks, with 2.44% for the top 5 and 0.96% for the bottom 5. As measured by provisions, the top banks made marginally riskier than average loans, with the bottom banks making substantially riskier loans. While operational efficiencies were greater for the top banks, the key determinant of their ability to produce substantially higher return on equity was their ability to secure higher loan spreads on loans of approximately average quality.

While this table is subject to many caveats *vis a vis* accounting policies, heterogeneity of asset portfolios etc, it does illustrate that there is wide disparity of achievement in the required default risk premium.

3.1.3 Modern Portfolio Loan Pricing Models

In a development of Sinkey's (1992) model, Wyman (1992) proposes two possible variations applicable to individual categories of lending across a portfolio. Wyman states that either the capital backing required or the return required on capital is proportionate to the expected default cost per unit lent in the category. In a further development, Coopers and Lybrand (1995) maintain that both the size of capital backing and the return required on this capital should be proportionate to risk (this fact had been recognised by Wyman (1992), but he proposed two differing approaches rather than a single integrated approach). The integrated approach is to be favoured as it allows explicitly for both expected and unexpected risk. It is also reflective of current basic thinking and practice (at least by the more sophisticated managements) in this area.

The first approach produces a return on risk adjusted capital (RORAC); the second a risk adjusted return on capital (RAROC); and the last a risk adjusted return on risk adjusted capital (RARORAC). These may be represented algebraically as follows:

RORAC $$\sum_{i=1}^{N} C_i^*\left[r^* - r\right] = \sum_{i=1}^{N} V_i\left[d_i + p_i + (d_i + p_i)r^*\right] \quad (3.5)$$

RAROC $$\sum_{i=1}^{N} kV_i\left[r_i^* - r\right] = \sum_{i=1}^{N} V_i\left[d_i + p_i + (d_i + p_i)r_i^*\right] \quad (3.6)$$

RARORAC $$\sum_{i=1}^{N} C_i^*\left[r_i^* - r\right] = \sum_{i=1}^{N} V_i\left[d_i + p_i + (d_i + p_i)r_i^*\right] \quad (3.7)$$

where

C_i^* is the required capital backing for business type i

r^* is the required return on lending defined for the entire portfolio

V_i is the volume of lending in business type i

d_i is the expected default cost per unit lent in business type i

p_i is the portfolio effect per unit lent in business type i

k is a constant

r_i^* is the required return on capital defined for business type i

r is the risk free rate of interest

and there are N business types in the portfolio

In this context, VAR (value at risk) is defined as the maximum amount of capital backing which may be lost due to default, at some appropriate probability threshold, either over the entire portfolio, or over portfolio subsections. The calculation of VAR directly implies that a risk adjusted capital (RAC) approach is appropriate, due to its expositional simplicity with regard to capital requirements. While it is theoretically possible to compute a risk-adjusted return on resulting RAC, this has not been attempted in this thesis for the following reasons:

1. The portfolio has not been observed for long enough for meaningful variance estimates to have emerged for specific lending categories.

2. The small size of certain lending categories (as few as 6 loans exposed to risk of default) does not justify the additional precision required.

Thus, the RORAC approach has been adopted, with uniform return on risk adjusted capital. This thesis proposes the use of the collective theory of risk to evaluate VAR in respect of the entire portfolio, and the decomposition of the portfolio into business areas, permitting loan pricing to incorporate both VAR for the relevant area, and the portfolio effect due to covariance with the remainder of the portfolio.

3.2 Current Loan Risk Assessment

Historically, the credit granting decision was judgmental, with experienced loan officers acting in an underwriting capacity. Wyman (1991), Maniktala (1991) and Ferrari (1992), for example, refer respectively to "one or two individuals" using "their judgement", to the "industry consistently failing to adequately differentiate for risk in its pricing", and to "traditional rule of thumb methods of loan pricing ineffective and unreliable".

Altman (1968) and Beaver (1968) analysed the predictive ability of financial ratios in terms of forecasting bankruptcy probability, the former using multiple discriminant analysis (MDA). Their approach was to examine samples of financially healthy firms compared with failed firms of similar size and industry.

It may be noted that MDA concentrates only on the probability of default rather than its expected cost. Nevertheless, some form of MDA or credit scoring is used by most banks, if only as a guide to credit granting decisions. Dependent variables may be both non financial and subjective (Argenti (1976)).

Mensah (1984) observed that many of the financial ratios used as a predictive variables are themselves highly variable through time, thus rendering their interpretation difficult at differing points in the economic cycle.

Altman *et al*(1981) suggest that a suitable application of a failure prediction model could lie in the area of loan grading, where the lending decision had already been taken. By implication, the model could afford to be less than precise when allocating by risk within a portfolio, rather than the more substantial precision required to reduce the costs associated with rejecting good and accepting bad borrowers.

Most credit scoring models used internally by banks to grade corporate loans for risk purposes rely on similar techniques to MDA. Specific ratios are given weights, the actual value of these variables are then multiplied by the relevant weights, and the result is a ranking score. If this score is set at an index level of, say, 60 in respect of a barely acceptable loan applicant, and 0 in respect of a risk free borrower, then what results is an ordinal ranking of credit scores, corresponding to what the operator of such a system believes is the relative risk of loans.

Typically in such a credit scoring system, there are 10 risk grades, with grades 7-10 comprising loans which have disimproved in quality since the loan granting decision (Foss (1992), Wyman(1992)). This structure is typical of that encountered in major banking organisations with which the present author has had employment experience (National Westminister Bank, Citibank, Citicorp), and given its frequent reference in standard works (Sinkey (1992), Glantz (1994)) on loan pricing, it is taken as part of industry standard practice.

The weaknesses in such credit scoring/failure prediction models are as follows:

a) When one refers to the original studies in this area, the requirement of paired samples (to eliminate size and industry effects) meant that sample sizes were extremely small e.g. Altman (1968) sample size 33, Deakin (1972) sample size 32, Dambolena (1980) sample size 23. The number and extent of statistical inferences which may be drawn from such small databases must be open to question.

b) Because of the nature of their construction, an implicit assumption exists that relevant variables are linearly related throughout their ranges. This is implausible, and must mean that loans are being classified heterogeneously under such models, with respect to default risk.

c) What results from the application of these models is an ordinal ranking of credit risk. Overall credit risk is not quantified, nor are the relative magnitudes of risk grades. Users of such models state that the historic experience of similar grade loans may be used in pricing (Charlton (1991),Wyman (1992)), but if the loans have been incorrectly classified in the first place, then the incorrect historic loss experience may be applied to the pricing of specific loans.

d) As already mentioned, relevant variables may change over time, so that coefficients and cut-off points may only be valid for short time periods. Further, the long duration of original studies (e.g. Altman (1968) used data over 20 years, Mensah (1984) over 9 years) meant that such studies reflected moving averages of relevant variables, rather than absolute values.

Their strengths are:

a) they are relatively straightforward to apply;

b) they distinguish the economic cost of risk, as opposed to the regulatory cost of risk, which is presently level for all private sector non-residential mortgage, non interbank credits (Basle Committee (1988)).

The model proposed in this thesis addresses the above weaknesses in order:

a) the dataset used to develop and test the proposed model comprises a large loan portfolio;
b) the relevant variables need not be linearly related;
c) loan pricing, together with required mathematical reserves result directly from application of the model;
d) relevant variables are permitted to vary through time by means of changes in risk intensity.

3.3 Regulatory Versus Economic Capital Adequacy

3.3.1 Regulatory Capital for "Other" Private Sector Assets

The current regulatory environment (Basle Committee (1988)) requires minimum capital adequacy ratios: that is regulatory defined (tier 1 and 2) capital, of 0% in respect of government assets, 1.6% in respect of interbank assets, 4.0% in respect of residential mortgages, and 8.0% in respect of other private sector assets - defined as all private sector lending other than that covered by previous reference. These standards are referred to as risk-based capital (RBC) relative to previous uniform standards, and apply only to commercial banks 'banking books' (comprising assets generally to be held to maturity).

In relation to other private sector assets, RBC standards fail to discriminate by relative risk within the regulatory risk class categories, thus permitting banks to select assets of varying risk but still subject to uniform regulatory capital requirements (Foss (1992)). By similar reasoning, Maniktala (1991) points out that regulators do not reward banks for engaging in less risky business within a given sector.

If loan pricing were to remunerate regulatory capital, then the computation of required loan margin would produce a uniform solution. If, for example, banks target net ROE was 15%, the capital to asset ratio was 8%, the banks tax rate was 30%, and yield on short-term government securities was 7%, then this solution would be:

$$\frac{0.15 \times 0.08}{0.7} - 0.08 \times 0.07 = 0.0115 \tag{3.8}$$

or 1.15% for all other private sector lending, prior to adjustment for overhead, and provisioning cost. This size of margin could discourage more creditworthy potential borrowers, who could raise capital more cheaply from other sources.

Regulatory capital, then, may be quite a restricted version of 'real' or 'economic' capital banking needed. The latter is more specific to a particular loan sub-section or individual loan (especially where substantial heterogeneity of risk exists within the respective loan sub-section).

3.3.2 Economic Capital for "Other" Private Sector Assets

Wyman (1992) discusses expected risk which he defines as "the average loss on a particular category of loan over a credit cycle, taking the form of a risk charge in basis points assigned to all loans within that category". Unexpected risk is represented by the volatility of likely losses in any one year relative to the expected loss; he correspondingly found that this volatility is directly proportional to the expected loss. Given that capital is required to absorb such volatility, he points out that capital should be differentially allocated to each category of risk.

Allocation of capital in proportion to actual ('real') observed or expected default cost across a portfolio produces "economic capital, and its remuneration directly from relevant portfolio subsections, produces an economic" cost of capital levied proportionate to risk (Wyman 1992, p 21).

Sinkey (1992) refers to the market value of equity as "cutting the veils", which surround either the accounting or regulatory definitions of capital. Economic capital is the marked-to-market value available to provide capital backing for a bank's overall activities. When such capital is allocated in proportion to risk, we have sectoral economic capital.

3.3.3 Specific Bank Economic Versus Regulatory Capital

For given insolvency probability, economic capital (as defined by Wyman (1992)) may be greater or less than required regulatory capital. In the case of a bank making high risk loans for which it is failing to charge an adequate margin, it is likely that for an acceptable risk of insolvency, regulatory capital would be inadequate. The chief failing in RBC is that it is insufficiently responsive to risk, and may facilitate adverse selection in banking. The alternative idea of regulating economic capital is intuitively appealing for the following reasons:

1) the scope for adverse selection by banks would be considerably reduced; a bank increasing the risk of its assets within any risk category, then, would be required to dedicate more capital;
2) banks which are currently relatively risky would be required to hold more capital than relatively safe banks;
3) banks would be rewarded in terms of lower capital requirements for directing lending toward safer projects;
4) loan pricing would be expected to be more efficient than at present as specialised lending intermediaries evolved.

There are two major problems with regard to the use of economic capital for regulatory purposes:

1) The statistically accurate measurement of such capital for individual banks (Rajan (1994)).
2) The accurate reporting (identification and measurement) of credit quality (Berger King and O'Brien (1991)).

These issues will be discussed in subsequent sections.

3.4 A VAR Model for Bank Loan Portfolios?

3.4.1 Preliminary Discussion

Jackson, Maude and Perraudin (1995) discuss the present trend towards increasing reliance by bank regulators on capital requirements covering particular risks. Such capital requirements limit the amount of risk that can be taken, while distorting the behaviour of regulated institutions less than direct controls such as restrictions on competition or asset choice. Merton and Bodie (1993) argue that asset choice must be tightly constrained if the underlying institution offers deposits which are effectively government guaranteed.

The Basle Committee on Banking Supervision (1995) in a consultative document (extending its earlier 1988 scheme for international bank capital adequacy) has proposed two approaches for the calculation of capital requirements for securities trading books: a building block approach which concentrates on individual categories of traded assets, and combines these as independent risks, and a model based approach which takes a "whole book", simulation based approach to aggregate trading risks. The building block approach is exemplified in the Capital Adequacy Directive (1993/6/EEC), which deals with specific market risks, and aggregates resulting capital requirements into a total trading book capital-adequacy requirement. Jackson, Maude and Perraudin (1995) explain that the building block approach may be inherently flawed, because if its constraints bind, banks may well select portfolios strictly inside the "whole book" mean variance efficient frontier, which may have higher variance than portfolios which would otherwise have been chosen.

The VAR approach adopted in this thesis imputes a value at risk for the loan portfolio, as if loans were traded assets. When equipped with this value, VAR may be calculated using identical methodology to that applicable to traded asset portfolios, though over a longer time period. Portfolio experience provides a revaluation mechanism directly analogous to "mark to market" in the case of traded assets.

The model-based approach takes as its starting point VAR analysis. Within VAR analysis, the techniques may be parametric or non-parametric. The parametric approach imposes distributional assumptions on the individual asset returns, the most commonly used of which is that returns are stationary, joint normal and independent over time. These assumptions are difficult to justify. The non-parametric approach involves simulation using a long run of historical data, and does not impose distributional assumptions. For market data, Jackson Maude and Perraudin (1995) conclude that a non-parametric model fits the observed risk distribution better than a parametric model.

This work on traded assets VAR indicates that if a VAR model is to be constructed for untraded credit instruments, then a non-parametric, portfolio based approach avoids many of the difficulties associated with parametric VAR or building block approaches.

3.4.2 Requirements for VAR Models of Credit Risk

The following conditions would require to be satisfied by any viable VAR model of credit risk:

1. VAR as calculated to be directly proportionate to observed risk (Wyman (1992));
2. VAR to be responsive to movements in observed risk, but not so responsive as to cause capital requirements to fluctuate excessively (Wyman (1992));

3. a standard framework of calculation to be adopted (Jackson Maude & Perraudin (1995));
4. methods of calculation to be "whole book" (i.e. taking all exposures combined) and non parametric (Jackson Maude & Perraudin (1995));
5. results seen to be a justifiable improvement on present practice.

It is not proposed to enter the private information risk/observed risk paradigm debate (Berger and Udell (1990)), except to comment that under a VAR framework, private information should reduce as a proportion of total risk relative to its present level, because of the conditional requirement that banks report their total provisions to their regulators, as opposed to their total credit outstanding, thus enabling regulators to observe defaults. Banks reporting detailed specific provisions would rapidly provide a database for regulatory use in assessing the "reasonableness" of specific provisions. Theoretically, as the available information on observed risk improved, it would be possible to subdivide the portfolio into its individual constituent loans and to price these individually. However, such a detailed analysis would produce extremely complex pricing models. Viewed pragmatically, what is required is a model capable of subdividing the portfolio into homogeneous sub groups, while retaining ease of application. Brockman and Wright (1992) describe this problem as a "trade off between accuracy and simplicity." This issue is discussed further in Chapter 7.

Observed risk should increase as a proportion of total risk due both to the conditional requirement noted above, and because methods of calculation of observed risk should improve as observed risk measurement becomes the focus of regulators attention. Thus, adverse selection will be no larger a potential problem than under present regulation, and may reduce as the direct implications of risky lending feed through in resulting increased capital required.

Three immediately observable inputs into the calculation of observed risk are the mean profitability of lending over time, the mean provisioning rate over time, and the volatility of the provisioning rate. Both Rajan (1994) and Berger King and O'Brien (1991) have referred to the ability of banks to "massage" earnings and provisions on a short term basis. The mean profit and provisioning rates should be chosen over a time period sufficient to encompass one complete business cycle, thereby ensuring that such techniques are difficult to employ. The volatility of the mean provisioning rate may be understated due to the massaging activities referred to earlier, so that a conservative approach may be required in VAR treatment, requiring higher VAR than that indicated by a purely statistical observed volatility approach.

The responsiveness of VAR to movement in observed risk is a key feature of any model. If the model is too responsive, a strongly pro-credit cycle lending policy results. An under-responsive model, on the other hand, simply freezes banks' relative risk, without supplying any incentive toward risk reduction. The adverse selection problem continues much as before, with each bank simply having a different starting point in terms of differential observed risk.

Conditions three and four, taken together, imply that measures of risk relating to a bank's entire credit portfolio be input into a uniform regulatory stochastic model, with a requirement being that not more than ε% of insolvencies be observed in X thousand iterations (ε and x to be agreed).

The final condition requires that change should not be made for 'changes sake'. Tangible improvements which might be observable are a reduction in required capital for given ε% insolvencies, indicating improved capital efficiency; reduced numbers of insolvencies for given capital, indicating improved capital allocation; or an arguable reduction in adverse selection due to the responsiveness of risk measures to idiosyncratic bank action.

If a VAR approach is to be viable, the questions are : what risk measures, unique to a particular bank, are to be input, and what uniform stochastic model is to be used?

3.5 Variables for a VAR Model of Credit Risk

The relevant observable variables available in respect of a specific bank are:

1. The profitability of lending: Clearly a bank engaging in profitable lending regardless of provisioning is of less regulatory concern than one which is lending unprofitably.
2. The mean provisioning rate over time: by definition, riskier loans entail higher provisions.
3. The volatility of provisioning rates over time: if provisioning rates vary widely between accounting periods, less reliance may be placed on mean profitability or provisioning rate, and correspondingly greater reliance on reserves.

These variables permit the calculation of the distribution function of default cost over time, using the kind of methodology outlined in Chapter 1.

In respect of a uniform stochastic model to be furnished by the regulators for use by the banks, relevant inputs in addition to those above are:

1. Taxation assumptions: Since banks fund capital adequacy partially out of retained earnings, after-tax profitability is the relevant measure.

2. Dividend assumptions: Since bank shareholders expect dividends, and given that the imputation corporate taxation environment favours

distributions to gross shareholders, some distribution assumption is necessary.

3. Inflation assumptions: Since inflation causes the nominal money stock to rise, and thereby puts pressure on a bank's ability to finance its stock of reserves, some future inflation assumption is necessary.

4. Short-term interest rate assumptions: Since reserves have earning power, this variable is directly linked to the inflation assumption.

5. The insolvency probability ε%.

6. The timescale over which the model simulates.

Relevant output (assuming VAR-related reserve requirements) is the reserve requirement, in respect of an individual bank.

Key variables are the bank specific inputs, the choice of ε%, the relevant timescale, and the resultant reserves. For short-term simulations (e.g. 1 year), profitability is simply stated net of taxation and dividends, and interest rates and inflation are ignored. This greatly simplifies the use of such a model, with only minor associated approximation error, and provides an estimate of minimum permissible VAR.

The equation used for such short-term solvency calculations is deterministic in form, is a direct application of the collective theory of risk, and is as follows (Daykin, Pentikainen and Pesonen ((1994) p159):

$$\mu_r = Y_\varepsilon P\sqrt{\tfrac{r_2}{N} + \sigma_q^2} \; - \; \lambda P + R_Y$$

$$\text{where: } R_y = P\frac{\left(Y_\varepsilon^2 - 1\right)\left({}^{r_3}/_{N^2} + 3r_2\,{}^{\sigma_q^2}/_N + Y_q\sigma_q^3\right)}{6\left({}^{r_2}/_N + \sigma_q^2\right)} \tag{3.9}$$

μ_r	are the specific required reserves
Y_ε	is the unit normal variate corresponding to ε%
r_2, r_3	are risk indices obtained by dividing the relevant moment of the cost of one default about the origin by mean default cost to the relevant power.
σ_q^2	is the variance of the mixing variable.
P	is the mean expected default cost for all defaults i.e. *m.n* where m is the mean expected cost of one default and *n* is the number of defaults expected.
R_Y	is a term which corrects for the skewness of the distribution
Y_q	is the skewness of the mixing variable

With the exception of the choice of ε, this equations inputs are bank-specific. For common ε, reserves are strictly decreasing in λ and strictly increasing in n (the number of defaults), m (the average cost per default), r_2 and r_3 (the variability of default cost), and R_Y (the skewness of overall default cost).

Thus required reserves are reducing in profitability, increasing in both risk and volatility of risk, and are proportionate to observed risk adjusted for volatility and skewness. Such short-term reserves are likely to prove highly responsive to short term movements in default rates, so that a relevant timescale must be chosen for a full simulation exercise. Note that the equation explicitly allows for non-stationarity (through the existence of the mixing variable *q*) of the

Poisson parameter n, and that covariance is built into the equation by the assumption that the structure variable q produces movements in risk intensity which are 100% correlated between changes in risk intensity. When risk intensity is moving rapidly, the portfolio covariance increases; when risk intensity is stable or moving slowly, portfolio covariance decreases.

A model form has been established which is proportionate to risk, responsive to risk, is the result of a uniform equation, and is fitted using non parametric procedures. An important research aim of this thesis is, of course, to establish whether this methodology produces a sufficient improvement in present practice to justify a (proposed) change in that practice.

Conclusion

Loan related risks have been discussed, focusing on the treatment of credit risk in loan pricing models. Existing regulation with regard to credit portfolios has been outlined, and a clear distinction drawn between regulatory and economic capital adequacy. The shortcomings of existing pricing and regulation have been identified, and the possibility of using a bank-specific VAR methodology to address these shortcomings has been discussed. Some necessary preconditions for a successful VAR approach have been set out, together with a risk theoretic equation which, *a priori*, appears to satisfy these preconditions. Consideration is now directed towards an empirical dataset within which present practice may be compared to that implied by the proposed VAR model, with respect to loan pricing and capital adequacy.

Chapter 4

Dataset Used for Empirical Analysis

Introduction

It is proposed to compare the methodology of this thesis with present practice in the context of an existing bank credit portfolio. To set the portfolio in context, trends in credit conditions are discussed over the time period 1980/94. The bank and portfolio comprising the dataset are profiled, and some exploratory data analysis is effected. The limitations and strengths of the dataset are discussed. Finally the specific deployment of the dataset within this study is clearly specified.

4.1 Trends in Bank Provisioning: The Credit Cycle

The following table illustrates five UK major banks' provisions excluding LDC provisions, as a percentage of loans outstanding since 1980.

Table 4.1

Annual Provisions as a Percentage of Loans Outstanding

Year	Bank					5 Banks
	Bank of Scotland	Barclays	Lloyds	National Westminister	Royal Bank of Scotland	Overall Unweighted Average
1980	0.54	0.50	0.48	0.54	0.47	0.51
1981	0.59	0.39	0.42	0.14	0.28	0.36
1982	0.82	0.73	0.83	0.58	0.50	0.69
1983	0.84	0.70	0.76	0.62	0.56	0.70
1984	0.85	0.94	0.80	0.67	0.46	0.74
1985	0.85	0.87	0.82	0.69	0.47	0.74
1986	0.96	0.76	0.61	0.71	0.87	0.78
1987	0.33	0.53	0.37	0.35	0.67	0.54
1988	0.55	0.41	0.45	0.40	0.46	0.45
1986	0.70	0.46	0.14	0..63	0.48	0.68
1990	1.18	1.23	1.21	1.24	0.97	1.17
1991	1.32	0.55	3.19	2.06	1.60	1.94
1992	1.45	2.09	2.69	1.52	1.84	1.92
1993	1.20	1.55	1.53	1.11	095	1.27
1994	0.77	0.52	0.77	0.54	0.51	0.62
Overall Average	0.86	0.88	1.07	0.79	0.74	0.87

Source: Smith New Court United Kingdom Clearing Bank Research (1995)

The average provisioning rate on total loans outstanding over 15 years for our sample of 5 banks was 87 basis points, with the highest provisioning bank at 107 basis points and the lowest at 74 basis points (23% higher and 15% lower than average). In short, no two banks are typically the same in this key area. These data confirm that it is entirely possible (indeed typical) within one regulatory system to have reporting banks up to 20% more or less risky (if one measures risk by provisioning rate) than average over a long period.

There is some evidence of cyclical behaviour, with provisioning rates reaching low points in 1981, 1987/88 and 1994, relative to immediately preceding experience. On the evidence of the above table (which does represent a significant part of the UK banking industry), one might posit that the duration of the bank credit cycle over 1980 to 1994 was approximately 6 years, from trough to trough.

There may also be some evidence from the same data that the amplitude of the credit cycle is increasing. The average provisioning rate for the sample peaked in 1986 at 78 basis points. The next peak, in 1991/2, was at approximately 193 basis points, around 150% higher than the 1986 peak.

Measuring risk by provisioning, then, one can suggest that lending activity became apparently more risky in the early 1990s in the United Kingdom than at any point in the 1980s. Whether the United Kingdom domestic recession of the early 1990s had unique features which make this increase in amplitude a temporary phenomenon, the present researcher cannot state. However, it is a fact that UK banks (compared with most other European banks) did suffer particularly badly from domestic loan losses during the recession of the late 1980s and into the early 1990s.

UK banks lending, then, apparently became a substantially riskier activity in the early 1990s compared with earlier periods. This has implications for loan pricing and for solvency, and these are examined in subsequent chapters.

4.2 The Bank and Dataset Employed

4.2.1 Bank Profile

The bank used in the current study is one of the UK "Big 4". It has a large market price to book asset value per share ratio, (its shares trade at a substantial premium to asset value), reflecting *inter alia* management ability, and holds substantially in excess of the 8% minimum capital requirement under existing regulation (Basle 1988). The bank's current capital adequacy ratio is 11%. The sample bank, then, is inferred to have large reputational capital (from its market to book ratio) (Sinkey 1992). At present capitalisation it has apparent substantial positive charter value and low default risk (Keeley 1990). For this particular bank insolvency risk appears to be substantially less important than the efficient use of its capital.

In the practical, contemporary management of its loan portfolio, the bank is interested in the pricing of its loan guarantees (Merton and Bodie 1993), the mean, variance and skewness of its provisions (Daykin, Pentikainen and Pesonen (1994)), and, the future development of its loan portfolio performance (Daykin and Hey 1989). During recent years, the bank's management has attached high priority to and devoted considerable resources to these key areas. Management clearly recognises the (differential) portfolio effects of different configurations of its loan portfolio and their respective policy implications. Credit risk appraisal is apparently sophisticated by bank practice standards and risk-based loan pricing (see Chapter 3) is being developed.

4.2.2 The Dataset Employed

The dataset comprises an average of 59,000 loans, and is the UK corporate loan book (comprising loans greater than £20,000), of the above bank. Overall performance is available in respect of each of the years 1990/93 inclusive, while a detailed breakdown is available for the 21-month period July 1993 to March 1995.

Loan pricing throughout has been conducted by the bank using a credit scoring model, with an algorithm reflecting Argenti[1] scores (Argenti (1976)), tangible net worth, gearing and other variables; this algorithm remains confidential to the bank. The model allocates loans to 10 risk grades, and produces target pricing ranging between 100 and 650 basis points above the floating London Interbank Offered Rate (LIBOR) for the relevant reset period. LIBOR is taken as a good proxy for returns available on corresponding government securities. The bank states that it has a consistent 20% share of the relevant market, and that the dataset is representative of that market.

4.3 Details of Dataset

4.3.1 Description of Dataset

In respect of the calendar years 1990/4 inclusive, the following information is available on the specific loan portfolio used in this research:

1. Portfolio size at beginning and end of the calendar year.
2. Weighted (by loan amount) average margin income per annum.
3. Provisions per annum.

[1] Argenti (1976) assessed management quality by survey, and incorporated his results into a management quality scoring system.

In respect of the period July 1993/March 1995 the following data are also available:

1. Records of individual loans, in detail.
2. Records of specific defaults by loan, in detail.
3. Records of specific provisions by loan, including defaults outstanding as at July 1993.

The reason that detailed records are not available prior to mid-1993, is that the bank in question did not maintain a statistical database in this form prior to that time. The detailed loan records themselves contain all financial information relating to the customers' own accounts, including the respective Argenti scores, industry codes, collateral levels, industry status, risk grade, whether under report (early watch), whether accounts are up to date, target loan margin, existing loan margin, existing loan margin and facility size

In addition, the detailed default records provide date of default in respect of those loans in default. The specific provision records detail provisions to date in respect of unresolved defaults, with total provisions in respect of resolved defaults.

The statistics available from July 1993 onward permit the researcher to engage in detailed loan pricing and provisioning analysis over a 21 month period; they also permit the parameterisation of a simulation model for longer time periods.

4.3.2 A Necessary Assumption

One of the major reasons for using the insurance approach for analysing cost in terms of frequency and severity is the belief that providing the nature of the insured risk does not change and providing that claim data are immunised against changes in monetary values, the distribution by amount of the cost of one claim should remain constant or change very slowly through time (Daykin Pentikainen and Pesonen (1994 p57)). Thus, changes in risk intensity manifest themselves through changes in default frequency, causing total amount payable to fluctuate. Applying this assumption enables one to use the entire database, by fitting implied default frequencies to observed total default costs. It may be noted that this assumption is rendered invalid if there has been a fundamental change in lending type or business risk within the portfolio over the 5 year period. Effectively, it is assumed that the type and business mix have remained proportionate to that observed by the researcher in detail over the 21 month period July 1993 to March 1995 for the five years 1990/94 inclusive, and that fluctuations in provisions have been caused by changes in default frequency only.

4.4 Exploratory Data Analysis

4.4.1 Summarised Portfolio Experience

The following table 4.2 shows loan portfolio performance over the period 1990/94. Administrative expenses have been assumed at 0.50% of outstanding balances throughout; the researcher does not have detailed calculations of such expenses, since they form part of the banks overhead, and they are not specifically allocated to this portfolio. The level of 0.50% is chosen for the following reasons:

1. Other studies (i.e. Asay and Albertson 1990), have identified overall administrative expenses in corporate lending in the range 0.75%-1.00%.
2. This portfolio is confined to larger loans, and therefore would be expected to have lower than average overhead, due to economies of scale. For this reason, administrative expenses lower than those in earlier studies have been assumed.

Table 4.2

Portfolio Experience 1990/94 (calendar years)

Year	1990	1991	1992	1993	1994	Overall Average
Average Portfolio Size £BN	11.4	11.5	11.0	10.5	11.2	11.1
Average Loan Margin %	2.20	2.30	2.35	2.40	2.28	2.31
Average Provision %	1.10	2.06	1.31	0.48	0.25	1.04
Margin Net of Provision %	1.10	0.24	1.04	1.92	2.03	1.27
Administrative Costs %	0.50	0.50	0.50	0.50	0.50	0.50
Net Margin	0.60	(0.26)	0.54	1.42	1.53	0.77
Implied Excess Return on Regulatory Capital %	7.5	(3.2)	6.8	17.8	19.1	9.6

Source: Empirical

The portfolio has averaged £11.1BN in outstandings. The average loan margin has been 231 basis points, ranging between 220 basis points in 1990 and 240 basis points in 1993. The average provision has been volatile around a mean of 104 basis points, ranging from 206 basis points in 1991 to 25 basis points in 1994. Net margin has been correspondingly volatile. Implied excess return on regulatory capital (ERORC) has been obtained by dividing net margin % by the 8% minimum capital requirement.

The last row illustrates how difficult a target short term return on capital is, and may illustrate that such targeting may only be reasonable in the medium term.

4.4.2 Utilisation

Using the 21 month (July 1993-March 1995) detailed dataset, the following table sets out average utilisation by risk grade.

Table 4.3

Average utilisation by risk grade

Risk Grade	Average Utilisation %
1=Best	8.42
2	15.72
3	19.60
4	28.16
5	43.63
6	59.55
7	72.06
8	78.79
9	85.42
10=Worst	90.76
Average	36.01

Source: Empirical

Note: Average utilisation defined as average observed drawdown.

Table 4.3 illustrates a bias in utilisation toward higher risk loans. It suggests that utilisation itself may be a significant risk factor, and that portfolio studies should be conducted more by reference to amount utilised rather than by facility size if correct allocation of cost to risk is to be achieved.

4.4.3 Turnover

Again, using the detailed database over the same 21 month period, the following table 4.4 sets out recent turnover statistics.

Table 4.4

Portfolio Turnover Statistics (number of loans)

6 month period to	Jan 1994	July 1994	Jan 1995
Average number of accounts	58627	55122	62403
Number of leavers	11229 (19.1%)	19320 (35.0%)	16286 (26.1%)
Number of Joiners	11342 (19.3%)	15724 (28.6%)	23589 (37.89%)

Source: Empirical

Table 4.4 above, admittedly over a short time period, suggests average annual withdrawal rates ('leavers) of 53.5%, and annual entry rates ('joiners') of 57.2%, implying very high client mobility. Either the portfolio comprises a stable core of clients, with an extremely mobile non core element, or all clients are equally mobile. If the latter is the case, then work on the durability of banking relationships and on the benefits accruing to borrower's in a long term relationship (Peterson and Rajan, 1994; Berger and Udell, 1995) may only apply to a tiny minority of loans, and may not have a significant impact on loan pricing. For example, if all clients are equally mobile, then only 2% of relationships last 5 years, and only 0.05% of relationships last 10 years.

Even if these calculations are only partially correct, it seems that only a small fraction of borrowers succeed in maintaining a long term banking relationship. Pricing by duration of relationship, then, may only affect a small proportion of loans.

4.5 Relevance of Dataset

4.5.1 Applicability of Dataset

The dataset using the necessary assumption discussed earlier (section 4.3.2), is representative of large UK bank corporate lending in the years 1990/94 inclusive. It comprises approximately 2.5% of the dataset size explored by Petersen and Rajan (1994) and Berger and Udell (1995) in the case of the United States NSSBF Data. Therefore, while the data are of sufficient volume to permit the exploration of basic loan pricing and solvency relationships, it cannot be used for more detailed research necessitating richer data, such as researching the rate at which information benefits accrue to the borrower in a long term relationship. The dataset used in this research, however, does represent a portfolio of real banking relationships, and permits inferences to be made about risk factors in pricing and about risk in a capital adequacy context.

4.5.2 Generalisability of Dataset

The statistics calculated from the portfolio are specific to the portfolio itself, so that loan pricing and insolvency risk calculations may not be wholly applicable to other bank lending portfolios. Nevertheless, given the similarity of provisioning patterns evident when one compares the 1990/94 data in Table 4.1 with Table 4.2, results calculated using this dataset are likely to be indicative of the general order of risk within UK corporate banking. Although the size and timing of provisions vary between portfolios and banks, default risk appears to be subject to similar influences over time. Thus, while calculated risk for a specific portfolio may not be generally applicable, the principles underlying the calculations are, and given the evident correlation (calculated at 0.88) between provisions within the dataset and UK bank loan loss provisioning generally, the dataset may be interpreted as broadly indicative of prevailing default risk conditions within the UK corporate bank lending market over the 1990/94 time period.

4.5.3 Strengths of the Dataset

The dataset provides an extremely detailed breakdown of each individual loan within the portfolio, both for loans which remain good and those which have defaulted. This enables the consideration of all plausible risk factors and combinations thereof in using the building block insurance approach to the production of a default frequency model. With respect to default cost, the dataset provides 3,887 observed resolved defaults, of which 910 gave rise to a provisioning need. This is equivalent to 911 individual data points mapping the distribution of the cost of one default. In loan pricing applications, this wealth of data points allows one to consider the portfolio subdivided by collateralisation and size of loan, retaining sufficient data points in each case to fit empirically the resulting distribution. The dataset thus permits the pricing of loans by reference to a significant number of risk factors, and allows the researcher to calculate with substantial accuracy required reserves for the portfolio in question over the relevant time period.

4.5.4 Limitations of the Dataset

The methodology used in this thesis is to develop and calculate portfolio-specific loan pricing and capital adequacy models. While the calculations in respect of this portfolio are accurate, it must be borne in mind that the detailed results apply only to this portfolio and its respective period in time.

One must be extremely careful (despite the potential generalisability of much of this research in the context of big UK bank's loan provisioning policies) not to draw unwarranted detailed and specific inferences about other banks or portfolios from the results. In this context it as well to emphasise that this dataset is to be used primarily for comparing methodologies, rather than in parameterising general models.

4.5.5 The Dataset in Perspective

It may be as well to re-emphasise that this dataset is effectively and first and foremost a simulation 'testbed' for the system proposed in this thesis. The essence of the research approach is to test and compare the proposed system to current practice. Given the selected dataset and respective bank involved, and for the reasons argued in this chapter, we assume (at best) that these data are broadly representative of bank corporate lending practice. At the very least (and worst) they represent a 'snapshot' of practical loan management for a major bank player during the specific period spanned by the selected dataset. Even in this kind of 'worst case scenario' (from a research methodology view point) the proposals still have significant potential merit since they may suggest the need to explore further more general improvements over current practice.

4.6 Uses of Dataset

The described dataset, then, provides a real credit environment which may be used as a testbed for hypotheses. In Chapters 6,7, and 8 of this thesis, the dataset is used for three distinct purposes. These are summarised below.

4.6.1 General Solvency Model Application (Chapter 6)

The statistics calculated from the dataset are input into equation 3.4, enabling the calculation of relevant mathematical reserves associated with an assumed insolvency probability. From Section 4.1 it is observed that portfolios may be more or less risky than the dataset itself. Two additional portfolios are hypothesised, 20% more risky (by number of defaults) and 20% less risky respectively, and used to demonstrate that for given reserves, insolvency probability varies monotonically, though not linearly, with risk. Holding insolvency probability constant, reserves are shown to vary monotonically but not linearly, with respect to risk.

Loan pricing is then allowed to vary, by assuming margins 20% greater and 20% less, respectively, than those in the dataset. The responsiveness of reserves to loan pricing is shown to be linear and monotonically decreasing. A smaller portfolio is also examined, permitting greater random variation. Finally, linear control of pricing and reserves is discussed using the dataset to illustrate differing scenarios.

4.6.2 Loan Pricing Application (Chapter 7)

The detailed testing and analysis of significant risk factors are carried out using the individual loans which make up the dataset. On construction of a default frequency model, loans are allocated to risk cells, each cell representing a specific combination of risk factors. Expected costs per default are allocated to each cell, using the dataset's own default experience. Finally, an economic loan pricing model is developed by assigning required return on capital to cells, weighted by observed default cost experience by cell. The loan pricing model thus obtained is directly compared to the bank's existing pricing, and differences between the two are examined.

4.6.3 Simulation Applications (Chapter 8)

A regulatory environment is hypothesised, containing a universe of banks, ranging from 20% more risky to 20% less risky than the specific dataset used. Using historic dataset parameters, together with future taxation, dividend, inflation, interest rate and expense assumptions, the behaviour of our group of banks is observed under the following two scenarios:

1. existing regulation;
2. a regulatory system where capital adequacy relates to bank specific risk measures.

The second scenario, using equivalent assumptions, should effectively sort the universe of banks, so that required reserves vary, but insolvency probability is relatively constant; thus permitting the application of a level premium deposit insurance scheme. The result of this sorting should be to produce fewer insolvencies (as a proportion of simulations run) than under the first scenario.

It may be noted that the dataset used in this application is assumed to represent the centre of a universe of portfolios with respect to risk. The dataset is used to illustrate the relative superiority of the second form (i.e. bank specific regulatory capital), which is derived by applying a standard model to bank-specific risk measures. This application of the dataset simply assumes that the dataset is representative of a banking environment, which of course, may and does vary substantially as simulations progress. As emphasised earlier (section 4.5.5) a principle is being illustrated, using real numbers, and no particular significance (other than its use as a 'realistic' or 'practical-based' simulation testbed) should attach to the choice of dataset.

4.7 Conclusion

This chapter discussed UK banking provisions, the dataset used by the researcher, and its proposed uses. Before proceeding to empirical study, there is a need to establish that the application of insurance methodology to loan pricing produces identical results, given common assumptions, to those produced by conventional option pricing models so that the results are directly comparable. This is demonstrated in the next chapter.

Chapter 5

Unbiased Corporate Loan Pricing In The Presence of Risky Collateral

Introduction

The pricing of corporate debt, where the issuer is subject to default risk, is investigated. The research extends the work of Merton (1974), who defined a risk structure of interest rates using the isomorphic relationship between the levered equity of a firm and the value of a call option on that firms equity.

In Section I Merton's approach is summarised. In Section II Merton's valuation formula is replicated, valuing debt as a deductible, using insurance-based mathematics and conventional calculus. In Section III a distribution-free approach is derived, which solves directly for risk interest rates, eliminating an acknowledged bias in Merton's work. *Section IV deals with* collateral in both fixed and variable forms. The first form may be dealt with easily, and involves a simple adjustment to either Section II or Section III methodologies. The second form does not permit a closed form solution using either methodology, but is accurately handled using insurance mathematics, which is free of distributional assumptions.

In fact most collateral is variable. Therefore, the approach using insurance methodology to define a risk structure of interest rates, is advocated over Merton's original work, as being inherently more flexible and permitting more accurate solutions.

5.1 Option Based Loan Pricing

This section concentrates on the development of a theory subsequently referred to as the "risk structure of interest rates", as opposed to the traditional "term structure of interest rates", used in pricing bonds where there exists a significant probability of default. An attempt is made to account for unanticipated changes in a firm's default probability as opposed to unanticipated changes in interest rates.

Merton (1974) proposes an operational formula for the risk structure of interest rates. He uses the isomorphic price relationship between the levered equity of the firm and a call option on the value of equity and incorporates weak perfect market assumptions in addition to the following:

(a) trading in assets takes place continuously in time;
(b) the risk structure of interest rates is flat and known with certainty;
(c) the force of interest equals r per unit time;
(d) the dynamics for the value of the firm, V, through time is described by a diffusion type stochastic process with the stochastic differential equation.

$$dV = (\alpha V - c)dt + \sigma V dz \tag{5.1}$$

where α is the instantaneous expected rate of return on the firm per unit time, *c* is the total payouts by the firm per unit time σ^2 is the instantaneous variance of the return on the firm per unit time, *dz* is a standard Gauss Wiener process. Assumption (d) requires that price movements are continuous and that the (unanticipated) returns on the securities be serially independent consistent with the efficient markets hypothesis.

Merton assumes that there exists a security whose market value at any point in time can be written as a function of the value of the firm and time. By Ito's Lemma, the instantaneous returns on the security and firm value are shown to be perfectly correlated. A three security portfolio comprising the security, the firm and riskless debt may be constructed with a value of zero in aggregate investment terms at any point in time, thus giving rise to a non-stochastic (zero) return in order to avoid arbitrage profits.

If *Y* = *F[v,t]* is the value of the debt security, then from Merton's assumptions the following holds:

$$0 = \tfrac{1}{2}\sigma^2 V^2 F_{vv} + (rV - C)F_v - rF + F_t + C_y \tag{5.2}$$

Equation (2) is a parabolic partial differential equation for F[2], which must be satisfied by any security whose value can be written as a function of the value of the firm and time.

In order to distinguish debt from equity, a complete specification must include two boundary conditions and an initial condition. All parameters are available at the outset except for variance, which may be estimated.

[2] Note that F, the value of debt, depends on the interest rate, the variance of firm value, the payout policy of the firm, and the expected collateral payout, in addition to firm value and time. F does not depend on the expected rate of return on the firm, the risk preferences of investors or, any other assets available to investors.

Consider a Firm with:

(a) a single homogeneous class of debt

(b) equity

Suppose the debt provisions specify that:

(i) the firm promises to pay a total of *B* to debt-holders on a specified calendar date *T*,

(ii) if this payment is not made, debt-holders immediately take over the company,

(iii) the firm may not engage in share or debt issue or share repurchase.

If there are no payouts, then *F*, the value of the debt issued equals:

$$\tfrac{1}{2} \quad \sigma^2 V^2 F_{vv} + rVF_v - rF - F_{\tau\tau} \tag{5.3}$$

where:

$\tau = T - t$ represents the length of time to debt maturity

In order to value debt, Merton requires an initial condition and two boundary conditions to be defined:

The initial condition is:

$$F[V,0] = MIN[V,B] \qquad \text{(5.4 a)}$$

The first boundary condition is:

$$F(0,T) = f(0,t) = 0 \qquad \text{(5.4 b)}$$

Debt F, and Equity f, can only assume non negative values.

The following regulatory condition may substitute for the second boundary condition:

$$\frac{F(V,\tau)}{V} \leq 1 \tag{5.4 c}$$

Equations (5.4a) through (5.4c) allow one to solve equation (5.3) directly for the value of debt. Note also that the value of equity is simply the value of a call option on the value of the firm, with exercise price B. From this fact, and noting also that $F = V - f$, the value of the debt is as follows:

$$F[v,\tau] = B e^{-r\tau}\left\{ \phi\left[h_2\left(d,\ \sigma^2\ \tau\right)\right] + \tfrac{1}{d}\ \phi\left[h_1\left(d,\sigma^2\tau\right)\right]\right\} \tag{5.5}$$

where:

$$d = \frac{Be^{-r\tau}}{V},$$

$$h_1\left(d,\sigma^2\tau\right) = -\frac{\left[\tfrac{1}{2}\ \ \sigma^2\tau - \log(d)\right]}{\sigma\sqrt{\tau}},$$

$$h_2\left(d,\sigma^2 t\right) = \frac{-\left[\tfrac{1}{2}\ \ \sigma^2\tau + \log(d)\right]}{\sigma\sqrt{\tau}}.$$

Restating (5.5) in terms of yield, instead of prices, results in equation (5.6):

$$\mathrm{R}(\tau) - \mathrm{r} = -\tfrac{1}{\tau}\log\ \left\{\phi\left[\mathrm{h}_2\left(\mathrm{d},\sigma^2\mathrm{t}\right)\right] + \tfrac{1}{\mathrm{d}}\ \phi\left[\mathrm{h}_1\left(\mathrm{d},\sigma^2\tau\right)\right]\right\} \tag{5.6}$$

where:

$$e^{-R(\tau)\tau} = F[V,\tau] / B$$

and $R(\tau)$-r is a loan margin. In Merton's framework[3], the risk structure of interest rates depends only on the variance of firm value and the ratio of the present value (at rate r) of the promised payment to the value of the firm d.

The work of both Merton (1974) and Black and Scholes (1973) have been subject to empirical tests with some success. Empirical results support the hypothesis that the distribution of future firm value is lognormal.

5.2 Reinsurance Based Loan Pricing

Consider a bank which owns the entire single homogeneous class of debt with provisions as specified in Section I. To ease comparison with Merton's work, firm value is assumed to be lognormally distributed with time dependent parameters, such that:

$$Log\,(V) \sim N\left(r\tau + \frac{1}{2}\sigma^2\tau,\ \sigma^2\tau\right) \tag{5.7}$$

The process about to be described may be envisaged, from the shareholder viewpoint as a negative claim, standard excess of loss contract. From the bank's point of view its claim on firm assets is to an extent a mirror image of the shareholder claim (i.e. F=V-f when the firm remains solvent), but varies as firm value falls below its claim (i.e. min(V, B) when the firm defaults). Thus, two distinct scenarios are considered, when developing a valuation for debt in

[3] Merton proceeds substantially further to analyse the term premium, bankruptcy costs, perpetual and callable bonds. However, this further analysis is not of concern at present.

this section: one where the firm remains solvent, the other where the firm defaults. In Section IV, when collateral is considered, additional outcomes / scenarios are included in the valuation of debt. A standard excess of loss contract may be described as follows[4]: Ammeter (1953).

$$E(y) = \int_B^\infty (V - B) f(v) dv \tag{5.8}$$

where *E(y)* is the expected (negative) claim, *V is current firm* value, *B* is the bond payment at time *T*, f(v) is the lognormal *P.D.F* $(r\tau, \sigma^2\tau)$, *and cash flows will* be discounted to the present, using a rate of interest *r* per unit time.

From the shareholder viewpoint, evaluation of the integral by parts yields equation (5.9):

$$\int_B^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{\left\{-(\log V - r\tau)^2/(2\sigma^2\tau)\right\}} dv = \int_{B^*}^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{\left\{-(w - r\tau)^2/2\sigma^2\tau\right\}} e^w dw \tag{5.9}$$

where:

$$W = \log V, \quad dW = \left(\frac{1}{V}\right) dV, \; dV = e^W dW$$

Completing the square in *W*, Currie (1993) yields the following:

$$\int_{B^*}^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{\left\{-\left(w - \left(r\tau + \sigma^2\tau\right)\right)^2 \Big/ \left(2\sigma^2\tau\right)\right\}} dw \, e^{r\tau + \sigma^2\tau/2} = E(v) \times PROB\left(W_1 \succ B^*\right) \tag{5.10}$$

[4] This begins the process of evaluation of the first expression in Merton's formulation, i.e. when the firm remains solvent, using reinsurance based mathematics and conventional calculus, in the valuation of debt.

where $W_1 \sim N\left(r\tau + \sigma^2\tau,\ \sigma^2\tau\right)$

In all such cases, the payment to the bank will be limited to *B* so that the present value of *B* may be substituted for *E(V)* in order to present the bank's outcome.

Thus for the bank we have:

$$Be^{-r\tau} \times PROB\left(W_1 \succ B^{\bullet}\right) \tag{5.11}$$

For non negative values, the natural logarithm of firm value is a monotonic increasing function of firm value, allowing us to manipulate the probability statement algebraically.

Examining the statement, PROB ($W_1 > B^*$),in isolation (allowing for growth in V), at exercise, the present value of the firm is:

$$PROB\left(LogV + r\tau + \frac{1}{2}\sigma^{2\tau} \succ LogB \right) = PROB\left(Ve^{r\tau + \frac{1}{2}\sigma^{2}\tau} \succ B \right) \tag{5.12}$$

$$= PROB\left(Ve^{\frac{1}{2}\ \sigma^2\tau} \succ Be^{-r\tau}\right),$$

$$= PROB\left(e^{\frac{1}{2}\sigma^2\tau} \succ \frac{Be^{-r\tau}}{V} \right),$$

$$= PROB\left(\tfrac{1}{2}\ \ \sigma^2\tau \succ \log\ \frac{Be^{-r\tau}}{V} \right),$$

$$= PROB\left(\tfrac{1}{2}\ \ \sigma^2\tau \succ \log\ (d)\right),\ \text{ as defined by Merton,}$$

$= PROB\left(½ \quad \sigma^2\tau - \log(d) \succ 0\right),$ [5] $N\ (\sigma^2\tau,\ \sigma^2\tau)$

after discounting at *r*

$$= \phi\left(\frac{½ \quad \sigma^2\tau - \log(d) - \sigma^2\tau}{\sigma\sqrt{\tau}}\right),$$

$$= \phi\left(-\left[½ \quad \sigma^2\tau + \log(d)\right] / \sigma\sqrt{\tau}\right).$$

Thus, the bank's outcome in this eventuality (i.e. where the firm remains solvent) is:

$$Be^{-r\tau}\ \phi\ \left[h_2\left(d, \sigma^2\tau\right)\right] \tag{5.13}$$

corresponding exactly to the first term in Merton's expression for the value of debt (see equation (5.5)).

To evaluate the second eventuality,-that is, where the firm defaults,-it must be remembered that if firm value falls below *B* at expiry, the bank only receives the residual value of the firm. From the shareholder viewpoint, the second part of the integral is (suspending the concept of limited liability for shareholders, the bank always gets paid, and a residual positive claim exists momentarily):

$$Be^{-r\tau}\int_o^{Be^{-r\tau}} f(v)\,dv = Be^{-r\tau} \quad x \quad PROB\left(ve^{½ \quad \sigma^2\tau} \prec Be^{-r\tau}\right) =$$

$$Be^{-r\tau} \times PROB\left(Log(V) + \frac{1}{2}\sigma^2\tau \prec Log\left(Be^{-r\tau}\right)\right) \tag{5.14}$$

where: $\text{Log}(\text{V}) \sim N\left(0, \sigma^2\tau\right)$, after discounting at rate *r*.

[5] This represents a probability density function , normally distributed, with parameters $\sigma^2\ \tau$, $\sigma^2\ \tau$.

That is, the residual positive claim is equal to the discounted value of the debt multiplied by the probability that the log of the discounted expected value of the firm, at maturity, is less than the log of the discounted value of the debt.

Substituting the bank outcome, the following is derived:

$$V * PROB\left(\log(\mathrm{v}) + \tfrac{1}{2}\sigma^2\tau \prec \log\left(Be^{-r\tau}\right)\right) \tag{5.15}$$

Again, examining the probability statement in isolation, and valuing at present, but allowing for growth in *V* in excess of the valuation rate *r*, we have:

$$PROB\left(\, Log\, V + \tfrac{1}{2} \quad \sigma^2\tau < \log\, \left(Be^{-r\tau}\right)\,\right)$$

$$= PROB\,\left(Ve^{\frac{1}{2} \quad \sigma^2\tau} < Be^{-r\tau}\right)$$

$$= PROB\left(e^{\frac{1}{2} \quad \sigma^2\tau} < \frac{Be^{-r\tau}}{V}\right)$$

$$= PROB\left(\tfrac{1}{2} \quad \sigma^2\tau < \log\,(d)\right) \qquad N\left(0, \sigma^2\tau\right) \,.$$

$$= \phi\,\left(-\left[\left(\tfrac{1}{2} \quad \sigma^2\tau \;-\; \log\,(d)\right)\right] / \,\sigma\sqrt{t}\right) \tag{5.16}$$

Thus the bank's outcome in the second eventuality (i.e. where the firm defaults) is:

$$V\phi\left[h_1\,\left(d, \sigma^2\,\tau\,\right)\right] \tag{5.17}$$

which corresponds exactly to Merton's (1974) second term, as defined in equation (5.5), given that there are no other possible outcomes by our application of reinsurance mathematics, i.e. the firm remains solvent or defaults. Thus, equations (5.13) and (5.17) replicate Merton's valuation formula for corporate debt, as defined in equation (5.5), through the use of insurance mathematics, in contrast to grounding this value in option theory.

The use of stochastic calculus by Black and Scholes may have been driven by a desire to specify *absolutely* the process by which firm values change, but any empirical observation that a firm's value through time follows a lognormal distribution would have permitted the straightforward application of conventional calculus.

It may be necessary for continuous trading to take place in order to establish instantaneous variance, but in truth, reinsured assets such as firms lent to by banks are rarely traded, the vast majority of such firms being unquoted.

This application of insurance mathematics sets out to demonstrate its equivalence to option pricing methodology under similar assumptions, and shows that the bank credit process may be described, at least in part, as an insurance implementation. There may be substantial opportunities for the application of actuarial and insurance techniques, not only in bank lending, but in many other areas of finance. The value of the firm using reinsurance methodology may be assumed to have a wide variety of distributional forms, permitting closed form solutions in many cases. Conventional calculus provides substantially more flexibility than stochastic calculus, is easier to apply, and does not suffer any loss of accuracy under similar assumptions.

5.3 An Alternative and More Accurate Calculation of the Pricing of Corporate Debt.

Although the equivalence of pricing methods using Merton's assumptions in a reinsurance setting has been demonstrated, the biased upwards estimate of initial gearing, resulting from Merton's procedure of discounting risky debt at the risk-free rate of interest, has yet to be addressed. Effectively, Merton's assumption produces a uniform gearing slope throughout the term of a loan regardless of the level of $R(\tau)$. This produces substantial bias with increasing $R(\tau)$, by systematically overstating gearing in the early part of a loan's term. An alternative calculation solves for $R(\tau)$ directly, and thus avoids the need for Merton's assumption.

Consider an arbitrarily large portfolio of N independent identically distributed firms, each with an obligation to pay 1 at time T if each firm is valued identically, then the total portfolio of debt has value at present of:

$$\sum_{i=1}^{N} e^{-\tau R(\tau)} \tag{5.18}$$

where $R(\tau)$ is the appropriate force of interest for the valuation of the risky debt.

In order to satisfy non-arbitrage conditions, the portfolio must return a risk-free force of interest $r(\tau)$ to an investor; once the arbitrarily large portfolio size has resulted in the disappearance of random variance, the outcome of any investment in such a portfolio is predefined.

The portfolio in fact returns the risk free rate IFF:

$$e^{\tau r(\tau)} = e^{\tau \mathrm{R}(\tau)} \, \mathrm{PROB}\big(\mathrm{V}(\tau) \succ 1\big) - \int_0^1 (1-x) f(x,\tau)\, dx \tag{5.19}$$

The profits resulting from successful payers for loans of term τ (first term in right hand side expression) are assumed to offset exactly the shortfall on those who fail to make the payment (second term). The first term of the right hand side shows the appropriate discount rate applied to risky debt multiplied by the probability that the value of the firm, at exercise, remains solvent (i.e. >1), while the second term of the right hand side represents the shortfall in payment multiplied by the probability density function appropriate, the resulting expected value being integrated over the range of possible shortfall values. Rearranging (5.19) results in the following:

$$\frac{e^{\tau r(\tau)} + COST(\tau)}{1 - PROB\big(V(\tau) \prec 1\big)} = e^{\tau \mathrm{R}(\tau)}$$

$$\tau \, R(\tau) = LN\left[\frac{e^{\tau r(\tau)} + COST(\tau)}{1 - PROB\big(V(\tau) \prec 1\big)}\right]$$

$$R(\tau) = \tfrac{1}{\tau} LN\left[\frac{COST\ (\tau) + e^{\tau r(\tau)}}{1 - (PROB\ V\ (\tau) < 1)}\right] \tag{5.20}$$

where COST (τ) is the expected cost of defaults per unit lent per period τ (i.e. the second right hand term in equation (19))and (PROB V (τ) < 1) is the probability of default at end period τ, i.e. the appropriate force of interest is simply that which recoups the shortfall from those who fail to make full payment from those who succeed. Clearly, the higher the proportion of successful firms the lower the shortfall and the closer the force of interest to the risk free force.

Interest rate margins resulting from this computation are directly compared to those computed using Merton's methodology in Tables 5.1(a) through 5.1(c). Clearly the bias in Merton's method is substantial over the range of gearing levels considered appropriate for banks.

Accordingly, the method outlined in this section is used to price loans, ignoring collateral, while collateral is taken into account in the next section. This method assumes that the contract is always expressed as a loan, and does not consider conversion to equity in the event of a default. Merton on the other hand, assumes conversion of debt into equity by means of a lender taking over the firm once its residual value falls below zero. Thus, for highly geared firms (outside the normal range of banking activities) this $R(\tau)$ exceeds Merton's, because debt/equity conversion is not assumed, however a certain proportion of successful firms continuing to subsidise those who fail is assumed; The issue of debt by negative net worth firms is in any event unlikely, and therefore we propose the use of this method throughout. This method has the advantage of providing a valuation interest rate directly, which is applicable to the nominal value of outstanding indebtedness.

The choice of probability density function used for the estimation of future firm value is entirely open to choice, although for direct comparison with Merton's estimates, the lognormal distribution is used herein.

Table 5.1

Pricing Discrepancy resulting from the Options Approach as opposed to the Reinsurance based Approach

Firm value is always assumed to be lognormally distributed, and the risk free rate of interest is taken as 6% throughout. In Panel A the volatility of firm value is 25% and loan term is 1 year. Firm value volatility is increased to 40% in Panel B, with the loan term remaining constant at 1 year. Panel C keeps the volatility of firm value at 25% as in Panel A, though increases the loan term to a period of 3 years. An [a] corresponds to banker's gearing of 300%, while [b] indicates all debt, no equity, prior to discounting debt, and [c] refers to a level of debt outstanding twice firm value, also prior to discounting debt.

	Loan Margin %		Bias	
B/V	Option Approach	Reinsurance Approach	Absolute	%
		Panel A		
0.10	0.000	0.000	-	-
0.20	0.012	0.007	0.005	71
0.30	0.169	0.115	0.054	47
0.40	0.769	0.577	0.192	33
0.50	2.079	1.703	0.376	22
0.60	4.202	3.765	0.437	12
0.70	7.094	6.987	-0.107	2
0.75[a]	8.793	9.091	-0.298	-3
0.80	10.638	11.606	-0.968	-8
1.00[b]	19.120	25.358	-6.238	-25
1.20	28.639	45.609	-16.970	-37
1.50	43.399	87.063	-43.664	-50
2.00[c]	66.654	174.312	-107.658	-62
		Panel B		
0.10	0.004	0.002	0.003	+142
0.20	0.180	0.095	0.085	+90
0.30	0.980	0.596	0.384	+64
0.40	2.670	1.819	0.850	+47
0.50	5.230	3.949	1.280	+32
0.60	8.510	7.077	1.432	+20
0.70	12.330	11.247	1.084	+10
0.75[a]	14.400	13.728	0.668	+5
0.80	16.540	16.471	0.068	-
1.00[b]	25.620	29.990	-4.637	-15
1.20	35.050	42.607	-12.184	-26
1.50	49.010	78.555	-29.542	-38
2.00[c]	70.560	138.893	-68.330	-49

Table 5.1 Continued

	Loan Margin %		Bias	
B/V	Option Approach	Reinsurance Approach	Absolute	%
		Panel C		
0.10	0.056	0.028	0.028	+102
0.20	0.469	0.290	0.178	+61
0.30	1.281	0.905	0.376	+42
0.40	2.375	1.850	0.525	+28
0.50	3.644	3.077	0.568	+18
0.60	5.018	4.540	0.478	+11
0.70	6.446	6.201	0.245	+4
0.75a	7.171	7.095	0.077	-1
0.80	7.900	8.026	-0.127	-2
1.00b	10.804	12.060	-1.256	-10
1.20	13.637	16.458	-2.821	-17
1.50	17.678	23.451	-5.773	-25
2.00c	23.793	35.446	-11.653	-33

Source: Calculated values in line with assumptions

As a result of the uniform gearing slope assumption discussed above, substantial over and under-pricing discrepancies arise with changing gearing rates when following Merton's approach. The reinsurance based approach takes account of the effect of gearing alterations when pricing debt. As can be clearly seen from Tables 5.1 (Panel A through Panel C), overpricing occurs in the earlier stages of gearing, with the interest rate differential rising up to the 0.6 gearing mark and falling thereafter, with underpricing dominating from the 0.8 gearing mark. Overpricing is substantially greater in the higher volatility categories with a maximum interest rate differential of 143 basis points, arising in the 40% firm volatility, 1 year time frame category. Longer time frames appear to diminish substantially the pricing discrepancy arising from the uniform gearing slope assumption.

5.4 Loan Pricing with Collateral

Consider the existence of collateral *independent* of the firm. The bank is assumed to make no action while firm value exceed indebtedness, but to reserve the right to foreclose should firm value fall below indebtedness, and to maintain that right should the sum of firm value and collateral fall below indebtedness.

Two specific types of collateral are considered: The first is fixed: for example, the capital amount of a risk free investment, pledged against the loan. The second, and more common form, is independent of the firm and the loan and its value is stochastic over time. The first type involves a relatively simple adjustment to both Merton's and the reinsurance formula, but can have a dramatic effect in reducing the loan margin. The second involves a convolution, and does not permit a closed form solution. However, numeric techniques do permit solutions, and their comparison with examples without taking collateral into account again indicates reduced loan margins.

5.4.1 Fixed Collateral

This would be the case where a capital sum, but not the interest thereon was pledged as collateral. If the collateral value exceeds indebtedness, then the bank is absolutely certain of its repayment. The mathematics are trivial, and using either Merton's or the reinsurance pricing technique produces no loan margin, i.e. the bank's investment is free of credit risk.

Where collateral *(S)* is less than indebtedness, a relatively simple adjustment in both cases produces a revised loan margin. In the Merton case, simply alter the definition of *d* to:

$$\frac{(B-S)e^{-r\tau}}{V} \tag{5.21}$$

i.e. substitute

$$(B-S)e^{-r\tau} \quad \text{for} \quad Be^{-r\tau} \tag{5.22}$$

The mathematics proceed as before. Fixed collateral in this case simply lowers the gearing ratio, or the proportion of firm value represented by debt payment commitments. This is equivalent to lowering the exercise level of the option from *B* to *B - S*, and will produce a corresponding reduction in delta. The relative levels of *V, B*, and *S* will produce differing reductions in loan margins (see Table 5.2).

In the reinsurance case, the bank's contract is modified by the inclusion of a deductible *S*. The starting integral is modified to:

$$\int_{B-S}^{\infty}\left(V-(B-S)\right)f(v)\,dv \tag{5.23}$$

and its integration by parts, with substitution of the bank outcome proceeds as before, to produce identical results to Merton, as modified herein.

TABLE 5.2

Loan Margins per annum, expressed as a percentage

Merton's definition of gearing, which is the ratio of bond to firm value (including bond) is used. This differs from the Bankers definition of gearing i.e. bond/equity. As expected *the required loan margin* is reduced with increased collateral, with greater effect as the debt ratio (B/V) is raised. Loan margins are calculated on the basis of a one year loan term, with firm volatility of 40%, a risk free rate of 6% and fixed collateral.

B/V	Collateral 0%	Collateral 25%	Collateral 50%	Collateral 75%
20%	0.095	0.030	0.002	0.000
40%	1.818	0.596	0.095	0.002
60%	7.070	3.004	0.596	0.032
80%	16.453	7.077	1.819	0.095
100%	29.995	13.728	3.949	0.346

Source: Calculated values in line with assumptions

* All debt, no equity

It is clear from Table 5.2 that the addition of collateral (in this case fixed) substantially reduces the required loan margin, with the greatest *absolute* basis reduction occurring upon initial introduction of collateral, and with the significance of the reduction rising with higher debt levels (i.e. a 1600 basis point fall occurs at a debt level of 100% upon inclusion of 25% fixed collateral).

5.4.2 Stochastic Collateral

Stochastic collateral occurs when the collateral pledged is not risk-free, but is independent of firm value. There is no straightforward method of modifying Merton's approach to allow for a second stochastic potential source of repayment to the bank. Because the bank may only consider collateral in the event of firm value falling below B, the first term of Merton's expression remains valid. The second term becomes a conditional probability multiplied by an option for which there is no closed form solution. Much of the appeal of Merton's formula is lost, as it can at best be represented by:

$$F[V,\tau t] = Be^{-r\tau}\left[\phi\left[h_2\left(d,\sigma^2\tau\right)\right]\right] + \phi\left[h_1\left(d,\sigma^2\tau\right)\right]$$

$$\left\{ Be^{-r\tau} \int_B^{\infty} F(k-B)\, f(k)\, dk + Ke^{-r\tau} \int_0^B F(k-B)\, f(k)\, dk \right\} \tag{5.24}$$

where $K = V + S$ and $F(k)$ is the joint density of the sum $V+S$. This joint cumulative function of $V+S$ is derived by the convolution:

$$\int_0^{\infty} F_v(K-S)\, f(s)\, ds \quad \text{or} \quad \int_0^{\infty} F_s\,(K-V)\, f(v)\, dv \tag{5.25}$$

depending on which of V and S is selected. As $f(v)$ is presumably already available, it is likely that the second form of convolution would be chosen. The distribution function of S is required. Note that even the adaptation of Merton's formula only applies where V has the lognormal density function. A return to conventional calculus is appropriate, as it allows a valuation expression to be developed free of distributional assumptions.

Adopting reinsurance methodology to equation (5.23), the valuation formula becomes:

$$Be^{-r\tau}\left[PROB\ \mathrm{V} \succ Be^{-r\tau}\right] + \left[\mathrm{PROB\ V} \prec Be^{-r\tau}\right] \times$$
$$\left[Be^{-r\tau}\left[PROB(K) \succ Be^{-r\tau}\right] + K\left[PROB(K) \prec Be^{-r\tau}\right]\right] \tag{5.26}$$

Any valid distributional form may be chosen for V and S, leading to a similar convolution calculation of the distribution of K. Equation (5.26) defines the value of risky debt as: the discounted value of debt by the probability of remaining solvent, plus the probability of default multiplied by both the discounted value of debt by the probability that firm value and collateral exceed the present value of debt obligations, and the joint firm and collateral value by the probability of a shortfall arising between this value and the present value of the debt obligations. Effectively, the existence of collateral broadens the range of outcomes in which the bank can expect full payment. The extent of this broadening is a function of the size of collateral pledged, and its probability density function of future values.

Table 5.3

Loan Margins, % per annum for differing firm and collateral volatility levels

Firm value and collateral value are both assumed lognormally distributed. Collateral is assumed to be independent of the firm. Loan margins are based on a one year loan term and a risk free rate of 6% throughout. Panel A calculates these loan margins on the basis of firm volatility and collateral volatility of 25%, while Panel B revises these margins on the basis of a higher firm volatility and collateral volatility of 40%. In Panel C firm volatility is maintained at 40% while collateral volatility has risen to 60%.

B/V	Collateral 0%	Collateral 25%	Collateral 50%	Collateral 75%	Collateral 100%
Panel A					
.20	0.007	0.000	0.000	0.000	0.000
.40	0.577	0.102	0.022	0.006	0.002
.60	3.765	0.840	0.209	0.061	0.020
.80	11.606	2.864	0.769	0.238	0.082
1.00	25.358	6.608	1.829	0.587	0.210
Panel B					
20%	0.095	0.019	0.006	0.002	0.000
40%	1.818	0.482	0.163	0.065	0.029
60%	7.070	2.090	0.748	0.313	0.146
80%	16.453	5.111	1.870	0.800	0.381
100%	29.955	9.567	3.510	1.516	0.731
Panel C					
20%	0.095	0.021	0.008	0.003	0.002
40%	1.818	0.502	0.200	0.096	0.051
60%	7.070	2.135	0.886	0.437	0.240
80%	16.453	5.148	2.162	1.082	0.602
100%	29.955	9.515	3.982	2.000	1.122

Source: Empirical

It is clear from Table 5.3 (Panel A through Panel C) that stochastic collateral, as with fixed, can significantly reduce the required loan margin, hence the price of debt, with the greatest impact occurring upon initial introduction of collateral. Thus, the required loan margin continually reduces, as the level of collateral increases, but at a decreasing rate. Obviously the larger the *S* the smaller the loan margin.

The relationship between *risky interest rates* and the *volatility* in underlying collateral is not so obvious. In general, the more volatile the collateral the higher the loan margins demanded, with the differential between the margins based on a low and high volatility level, increasing with increased collateral. In certain circumstances, the asymptotic right hand tail of the lognormal distribution of future collateral values can reduce the risk interest rate chargeable as volatility of collateral rises. This would clearly occur only when the right hand tail of the lognormal distribution was operative i.e. when:

(a) collateral has relatively small values,
(b) the ratio B/V is at high values,
(c) volatility in underlying collateral is at high absolute values,

Study of the ratio $S/_{V\sigma_s^2}\tau$ indicates that beyond a threshold of 20% in this ratio, loan margins will increase monotonically with $\sigma_s^2\ \tau$.

These considerations lead directly to the following propositions:

1: the existence of positive value independent collateral *(S)* reduces the riskiness of individual loans (Table 5.1),

2: risk is monotonic decreasing in *S*, and limits at 0, for given *V*, *B*, and constant volatility in *S* (Tables 5.1 & 5.2),

3: for values of the ratio $S/_{V\sigma_s^2}\tau$ in excess of 20% risk is monotonic increasing in the volatility of *S*, for given *S, V,* and *B* (Table 5.3).

The development of a full model is unnecessary, because propositions (1)-(3) are evident from the preceding discussion. For an illustration of stochastic collateral loan margins, see Table 5.3; for a discussion of the partial derivatives of $R(\tau)$, see section 5.6.

With respect to the stochastic nature of the collateral, comparing Table 5.2 with Table 5.3 (Panel B), it is evident that for low levels of collateral volatility, stochastic collateral gives rise to lower required margins than for its fixed counterpart (given the level of debt), whereas for high levels of collateral, fixed collateral yields cheaper loans than stochastic since the riskiness of the stochastic collateral outweighs the potential benefits. For minute levels of collateral, the more volatile the collateral the better, since there is a higher chance that a very small amount of collateral can yield a significant amount.

5.5 Applications of Stochastic Collateral

As Devinney (1986) has pointed out, if collateral is in fact fixed and risk free, and accounts for a significant proportion of loan finance, then the self-financing option would require greater study.

Collateral value in general is volatile. When recession occurs, it can have two distinct effects on collateral value. If recession is accompanied by inflation (which, apart from the most recent recession in the UK (1989/91) it has been), collateral values are often enhanced to the point that loan losses in many cases are made good, and overall loan losses are manageable. As in the 1989-1991 UK recession, if inflation does not stand alongside recession, asset prices fall in many instances, leading to a shortfall in collateral arising as the risk intensity of the default function increases. Thus, a rise in risk intensity is sometimes accompanied by a rise in loan loss severity, producing large loan losses.

In fact, a double stochastic process is at work. The amount at risk in the case of a loan with stochastic collateral is a function both of firm value and collateral value. To confine consideration to firm value would be remiss. It should be possible to examine collateral by type, and to group type of collateral in ascending order of risk.

5.6 Note on the Partial Derivatives of R(τ)

In simple terms, and free of distributional assumptions:

$$R(\tau) = \tfrac{1}{\tau}\ LN\left(\frac{COST\ (\tau) + e^{\tau r(\tau)}}{1-(PROB\ \ V(\tau)<1}\right) \tag{5.27}$$

with the derivative $\dfrac{dR(\tau)}{d^{*}}$ equal to the following:

$$\frac{COST(\tau)^{1^{*}}\ (1 - PROB\ V(\tau) < 1)\ +\ (COST\ (\tau) + e^{\tau r(\tau)})\ \ (PROB\ V(\tau)<1)^{1^{*}}}{(1 - PROB\ V\ (\tau)<1)^{\,2}} \tag{5.28}$$

where:

* represents the relevant variable,

1* represents the partial derivative with respect to that variable,

R(τ) is the risk interest rate actually charged,

τ is the term of the loan,

Cost (τ)is the expected cost function of defaults per unit lent for term τ,

r(τ) is the risk free rate interest rate, and

PROB V (τ) is the probability density function of default appropriate for period τ.

And because LN is monotonically increasing in positive R(τ) for τ constant, its partial derivatives will behave in similar manner to those of R(τ). Initially, both τ and r(τ) are assumed constant, meaning that the partial derivative with respect to other variables will affect the COST (τ) term and the PROB V (τ) term only.

Considering each of these variables in turn, with respect to their likely effect on the probability of default and the expected cost of default, hence on the appropriate risk interest rate to be charged, the following results:

a). V or firm value. As firm value increases, the probability of default must not increase and will normally reduce. Similarly for greater firm value, expected cost must reduce, so that the partial derivative must reduce, leading to the conclusion that R(τ) is decreasing in V.

b). B or size of indebtedness. As B increases, so must the probability of default and the expected cost of default. The net effect is to increase the value of R(τ), and thus we can conclude that R(t) is increasing in B.

c). S or size of collateral. As S increases, the probability of default and its expected cost reduce. The effect is to decrease the value of R(τ) and we can conclude that R(τ) is decreasing in S.

d). σ_V^2 or volatility of firm value. As the volatility of firm value increases, so does the probability of default, for all realistic combinations of V and B, and so also does the expected cost of default. The effect is to increase the value of R(τ), and it can be concluded that R(τ) is increasing in σ_V^2 (in the range of values likely to be taken by V and B).

e). σ_s^2 or volatility of collateral value. As the volatility of collateral value increases, the probability of default may either increase or decrease, depending on the size of collateral relative to firm value. For extremely small values of ${}^{SB}\!/_{V}$, the impact of σ_s^2 is likely to be trivial in any event, while for large values of ${}^{SB}\!/_{V}$, the effect would be to increase the probability of default. A problem arises since this function is <u>not</u> well behaved through its range for all combinations of S, B and V.

This problem is analogous to the following example. Suppose a bank has lent £100 against £5 collateral. Paradoxically, the £5 collateral held in oil exploration shares (high σ_s^2) has a greater cost-reducing effect than if the £5 were held in cash, in which case the £5 reduction in expected cost would be trivial in the context of the loan. Therefore, for small collateral values, high σ_s^2 can be preferable to low σ_s^2 values, from an R(τ) perspective. Similar problems could theoretically arise in the case of σ_v^2 but only where firm value was extremely small in proportion to indebtedness.

Allowing τ, and r(τ) to vary, produces slightly more complex partial derivatives, but the following is readily observable.

With respect to τ, as τ increases, the ratio $\frac{Be^{-R(\tau)\tau}}{V}$ on average is lowered, thus reducing the *average* probability of default through the loan term. Similarly the expected cost is expected to reduce. The longer discount period has a similar effect to a continuous reduction in the ratio of ${}^{B}\!/_{V}$ throughout the term. R(τ) is therefore a decreasing function of term.

With respect to r(τ), increases in the risk free rate of interest will be matched almost pari passu by increases in R(τ). The secondary effect of lower average gearing with higher r(τ) will be of a substantially lower order. Therefore R(τ) will be an increasing function of r(τ).

With respect to $[R(\tau) - r(\tau)]$, the risk margin, the rate of increase in margin with respect to $r(\tau)$ will be a decreasing function due to the second order effect referred to above. Thus $[R(\tau) - r(\tau)]$ is a decreasing function of $r(\tau)$.

Conclusion

Re-insurance based credit pricing, using an expected value of loss approach, has been shown to produce identical results to option-based credit pricing, under identical assumptions. The expected value approach enables a numeric solution to the double stochastic firm value/collateral problem, which is not feasible using the option-based approach. An existing and acknowledged bias in the option-based approach has been eliminated, and the re-insurance based approach permits a choice of distributional form.

This chapter has shown that individual loans may be evaluated usefully as insurance contracts. Thus the collective theory of risk may be applied directly to portfolio evaluation and individual loan pricing.

Chapter 6

Modelling Bank Lending as an Insured Risk Process

Introduction

In the previous chapter it has been demonstrated that the building blocks of the insurance process, under similar assumptions, produce identical results to the option pricing approach in the case of individual loans. In this chapter, the performance of a collective of such building blocks is examined, using portfolio historic performance to parameterise endogenously the default cost function, unique to a particular portfolio. Having estimated the appropriate default cost function, the reserve requirement for a bank operating such a portfolio can now be specified. In this respect, the additivity of the Poisson parameter is a powerful feature, allowing one to decompose portfolio performance over time and homogeneous portfolio subsections. The current capital adequacy requirement is discussed in the context of the above mathematical reserve requirement, and methods of control of reserves and pricing are suggested.

Major differences between current practice within the UK bank dataset used in this study and the modelling approach developed in this and succeeding chapters are set out below:

Calculation	**Current Practice**	**Proposed Model**
Capital adequacy	Basle (1988)	Portfolio specific VAR using standard formula
Loan pricing	MDA-type Algorithm	Multi-factor rating
ROE	10% per annum (historic experience)	10% per annum on VAR
RORAC	Not used	Directly calculated

6.1 Background Development

Santomero (1984) states "If one accepts" the "view that bank liabilities are essentially 100% insured, then the entire issue of bank capital and risk taking should be recast in terms of a discussion of insurance pricing". He continues (1984 p 604):

> "Literature on optimal bank capital is a bit vague and very model specific".

If the current, minimum capital-adequacy requirement of 8% (capital to risk weighted assets) appropriate to lending of the type included within our dataset is taken as indicative of required reserves, then this reserve level appears to be amply sufficient (based on our dataset bank's empirical experience) to ensure the *de facto* 100% insurance of bank liabilities, even without deposit insurance, since the probability of insolvency (i.e. liability excess) is extremely small for the chosen portfolio, and also for riskier portfolios. It is possible to hypothesise the existence of portfolios which would have significant insolvency probability at a reserve level of 8%, but average provisions for these portfolios would be at 3-5 times the level observed in the portfolio under examination.

The 8% capital adequacy requirement is a permanent minimum excess required for asset portfolios. Regulators may have insisted on this level for the following reasons:

1. To ensure that an asset excess existed, or to reduce substantially the probability of a deficit, even where loan portfolios were compulsorily liquidated.

2. To allow for any systematic overvaluation of assets which might be a feature of banks under regulatory pressure.

3. To control the rate of credit creation by means of bank lending. In this case, the imposition of capital controls would slow the rate at which banks could finance credit creation internally.

4. The required asset excess may form a pool, which can be drawn on by asset recomposition (effectively, selling loans and buying government securities). To the extent that asset recomposition may take time to effect, the regulators may have been taking a medium term view of bank insolvency probability, rather than the short term view implied by the regularity of required reporting.

Points 1) to 3) are not considered here, as the relevant information is not available which would enable us to comment on average realisation proceeds in bank insolvencies; nor are the economic consequences of an 8% capital requirement in terms of credit creation within this remit.

With respect to 4), the likely medium-term reserve requirement is examined, and is concluded to be larger than the short term requirement, and may provide an insight into the choice of the 8% level.

The model introduced in Chapter 1 is now developed.

6.2 Development of Model

6.2.1 Underlying Logic

In the preceding chapter, with respect to individual loans, it was argued that the valuation process consisted of evaluating an integral representing the continuous distribution of future firm value multiplied by the cost resulting from the particular firm value. Such a product integral only takes positive values when the cost resulting is positive, so that the range of the integral covers only those values for which indebtedness exceeds firm value.

Algebraically

$$\int_0^B (V - B) f(V) dV \tag{6.1}$$

represents the integral of the product of the probability that firm value (*V*) is below indebtedness (*B*), and the firm value corresponding to that probability *(f(v))*.

For a risk collective, this integral is summed, giving:

$$\sum_{i=0}^{N} \int_0^B (V - B) f(V) dV \tag{6.2}$$

for an N loan portfolio.

Recognising that, in a risk collective, numbers of loans defaulting, as well as numbers of loans exposed to risk of default, are available, the insurance approach is to approximate the probability of default for an individual loan by dividing the observed number of defaults by the number of loans exposed to risk of default. For a portfolio, multiplying through by the total number of loans produces simply the observed number of defaults.

Thus for an individual loan:

$$\text{Probability of default} = \frac{\text{Number of Observed defaults}}{\text{Number of loans exposed to risk of default}}$$

for homogeneous loans.

For a risk collective, the observed defaults are the result of only one sample path followed by the portfolio. They form a maximum likelihood estimator, for the mean of a Poisson or Polya distribution of all possible sample paths. In the Poisson case, the maximum likelihood estimator for the mean number of defaults is simply the number of observed defaults. A probability density function is thus built:

$$\sum_{k=0}^{N} {}_{n}P_{k}$$

where ${}_{n}P_{K}$ is the probability of observing exactly k defaults where the mean expected number of defaults equals N.

This approximates:

$$\sum_{i=0}^{N} \int_{0}^{B} (V - B) \qquad (6.3)$$

the first part of (6.2).

With respect to cost, the insurance assumption is that the distribution of cost of one default is independent and identically distributed. This is equivalent to stating that the knowledge that *K* defaults have occurred does not *yield any* information other than the mean expected cost of *K* defaults i.e. one default, *a priori*, does not differ from another with respect to expected cost or other moments.

Thus, total default cost is conditional on the number of defaults observed, and arises as follows:

$$F(X) = PROB(x \leq X) = \sum_{K=1}^{N} {}_nP_K \times PROB\left(\sum_{i=1}^{K} z_i \leq x\right) \qquad (6.4)$$

$$= \sum_{K=1}^{N} {}_nP_K . S^{K*}(X) \qquad (6.5)$$

F(X) is the cumulative total default cost, evaluated at X.

In respect of an N loan portfolio, where $\sum_{i=1}^{K} Z_i$ is the distribution of cost of exactly *K* defaults, and $S^{K*}(X)$ is the K_{th} convolution of the common default cost function S for 1 default.

Note that the approximation to (6.2) represented by (6.5) is dependent on the accuracy of the two underlying statistical assumptions with respect to a) observed default numbers, and b) default cost in respect of 1 default having a common distribution.

Permitting default cost numbers to vary through time (i.e. the Polya process), introduces a further variable *h*, which is a measure of the volatility of the underlying Poisson parameter, and is again estimated from portfolio observation.

Thus, the collective risk approach approximates the summation of the integral (6.1), parameterising the approximation by using the method of moments, the relevant moments being calculated directly from the underlying portfolio.

6.2.2 Criticism of the Above Approach

6.2.2.1 Distributional Assumptions

It has been suggested that the Binomial distribution may be appropriate to default frequency as either a loan defaults or it does not. Firstly, it is possible for a loan to default more than once, so that a loan may recover and subsequently default a number of times. Secondly, the rate of default per quarter must be a low order (maximum of 2 - 3%), in order for the bank to survive (although high margin loans could theoretically default at higher rates), so that the loss of accuracy in using the Poisson approximation to the Binomial distribution is not substantial. Thirdly, the Poisson distribution, being additive with respect to both size and time, does provide substantial mathematical tractability.

The use of the Poisson process as an approximation to what is essentially a part Binomial process may lead to a maximum specification error set out in Table 6.1 (Taylor 1986).

Table 6.1

Maximum Specification Error: Poisson V's Binomial

	Binomial	Poisson	Error*	
			(1)	(2)
Mean	N_p	n		
Variance	N_{pq}	n	+0.4%	+2.2%
Coefficient of Skewness	(q-p)/ $\sqrt{Npq}$	1/ $\sqrt{n}$	+0.4%	+2.3%

Source: Empirical

* Error:

1. For default frequency 0.4% per quarter (current experience)
2. For default frequency 2.2% per quarter (maximum historic experience).

In both cases, the Poisson approximation produces an overestimate of the last two parameters, the overestimate being broadly proportional to default frequency. The Poisson approximation is deemed to offer sufficient flexibility to compensate for possible low order specification error (Feller, 1950).

Variance in excess of mean leads to estimation error both in the underlying Poisson parameter and in the structure variable chosen for fitting. Simultaneous fitting is used in order that the mixing distribution fits the first three moments of the underlying distribution.

6.2.2.2 Methodology

Cummins (1986, p 285) has suggested

> "In spite of its mathematical sophistication, actuarial ruin theory has been of limited practical applicability. The principal problems are:
>
> (1) The mathematical intractability of most of the results.
>
> (2) The nearly universal tendency to ignore investment risk.
>
> (3) The failure to recognise that insurance companies operate in a market economy where insurance premiums and asset prices are determined by the interaction of supply and demand".

Derrig (1989, p 303/304) observes that "Increasingly complex techniques were developed, mostly in Europe, under the rubic of "Ruin Theory" which sometimes seemed to show that surprisingly low levels of surplus were needed to keep the probability that liabilities would exceed assets ("ruin") at low levels".

Even though the above refer to insurance companies, they must be directly addressed in any real world model of bank lending as an insured credit process. In order:

(1) The mathematical tractability or otherwise of the results is not of major import providing the model accurately reflects portfolio development. Parsimony in modelling is greatly to be desired, and general risk theoretic results are assumed easily understandable.

(2) Given a building block presentation investment risk is subject to a separate reserve requirement (93/6/EEC), and is thus considered separate to credit risk.

(3) Cummins (1986) and Derrig (1989): both authors refer to the stationarity assumption implicit in risk theoretic insurance models. Cummins suggests indirectly a cyclical model, whereby premiums and asset prices vary through time, while Derrig refers to "surprisingly low levels of surplus".

Any model which purports to represent the credit process accurately must allow for non-stationary parameters, reflecting the cyclical tendencies referred to by Cummins (1986), Horwiz (1983). In our proposed model, default probabilities will only be assumed constant within quarter, and may vary substantially between quarters. With respect to Cummin's suggestion that premium rates vary through time, making loan pricing levels a random variable conflicts with one of the main aims of this thesis, which is to establish a consistent medium-*term loan pricing structure.*

In a control sense, the effects of allowing premiums to vary are examined, but a particular strategy is not recommended. We shall model current loan pricing against stochastic default rates, and impute required reserves. Thus, modelling the adequacy of loan pricing, as opposed to .modelling solvency in the conventional sense, is proposed.

Having addressed Cummins and Derrig's criticisms, a risk theoretic model is proposed that is mathematically tractable and which will impute a risk free return to reserves. The model, referred to as the General Solvency Model, is intended to be a stationary reserve model.

With regard to the General Solvency Model, extensive estimation of the pattern of past defaults was necessary, due to the paucity of pre-May 1993 data within our dataset. Therefore, the model results should be interpreted as an indication of required reserves for this branch of the credit process, rather than an absolute reserve requirement. It does at least partially address the criticisms in the previous section in recognising that credit conditions are not stationary with regard to risk.

6.3 The General Solvency Model

6.3.1 Choice of Default Frequency for Estimation

The underlying construct that loans are independent with respect to default risk over short time periods, but 100% correlated with respect to changes in risk intensity between time periods, yields an approximate test as to the time period for which to assume independence. The longer the time period chosen, the less correlated is loan performance and *vice versa*. Over the 21 month period for which detailed statistics are available from the dataset used by the researcher, the average loan correlation within the portfolio is estimated to have equalled 97% (100 samples of size 1,000 were taken at random, and the average correlation calculated).

This would imply that fitting the correlation estimate as the thesis model, only 3% (100% - 97%) of total variance should be represented by the Poisson parameter. Table 6.2 illustrates the effect of time period choice on variance. It may be noted that this implies the portfolio is dominated by changes in risk intensity.

Table 6.2

Percentage of Variance Represented by Poisson Parameter

Time Period	1 month	3 months	6 months	1 year
Poisson Parameter / Total Variance	4.0%	3.0%	2.0%	0.4%

Source: Empirical

Based on Table 6.2, the 3 month period would appear most representative of portfolio performance.

Other practical reasons for the choice of a 3 month period are:

1. The variability of monthly default rates does not in general have an impact on reserves of sufficient size or duration to render its study worthwhile.

2. The continuation of trend (if any) reflected in quarterly data is judged to correspond more closely to perceived reaction time in terms of loan pricing, and thus little benefit would be gained by studying very short term variability in provisioning.

3. A quarterly analysis corresponds to the time period chosen for financial analysis of the insurance business (Cummins (1986), Cummins and Harrington (1989)).

6.3.2 Estimation of Quarterly Default Frequencies

In respect of the time period 1.1.90 to 30.6.93, crude annual default rates exist for the portfolio. A moving average process was applied to calculate implied quarterly default frequencies, required as an input to the thesis model. Independent verification for these implied frequencies was obtained by requiring the residual defaults outstanding implied by these frequencies coupled with a default resolution table to reconcile to defaults outstanding as at July 1993.

These implied quarterly default frequencies are assumed accurate subject to the composition of the underlying portfolio not having substantially altered over the last 5 years, and the pattern of default resolution also not having markedly changed (for reconciliation purposes only).

Table 6.2 sets out these estimates, the actual observed data, and the variance calculation for the estimation of the Poisson parameter n and the incomplete gamma variable h.

Table 6.3

Estimated Historic, Actual Current Default Frequencies and Implied Parameter Variance

Quarter	Frequency	Contribution to Variance Sum
1 90-13	740	497
2 90-12	850	7691
3 90-11	970	43139
4 90-10	1100	114041
1 91-9	1210	200435
2 91-8	1320	311029
3 91-7	1440	459277
4 91-6	1340	333737
1 92-5	1240	228192
2 92-4	1150	150311
3 92-3	1050	82771
4 92-2	840	6037
1 93-1	640	14957
2 93 0	430	110423
3 93 1	353	167526
4 93 2	278	234546
1 94 3	277	235516
2 94 4	245	267599
3 94 5	152	372466
4 94 6	219	295175
1 95 7	164	357963
	AVE: 762.3	**TOTAL: 3993333**

Source: Empirical

Notes: Table 6.3

a) $i)\ \hat{n} = 762.3$

$ii)$ h derived from $\sigma_n^2 = 199667 = \hat{n} + \frac{(\hat{n})^2}{h} \Rightarrow h = 2.9$

$iii)$ σ_q^2, y_q derived from $\sigma_q = \frac{1}{\sqrt{h}}$, $y_q = \frac{2}{\sqrt{h}} \Rightarrow \sigma_q^2 = 0.342$

$y_q = 1.170$

Methodology from Daykin, Pentikainen and Pesonen (1994 pp 45-54)

b) A h value of 2.9 implies wide dispersion of the underlying parameter, i.e. highly variable default rates through time.

c) A γ_q of 1.17 implies that the distribution of the default rate parameter is highly positively skewed.

6.3.3 Cost Per Default

Using in the General Solvency Model the approach suggested by Daykin, Pentikainen and Pesonen (1994), the first three moments of the cost per default may be estimated direct from empirical data. Underlying assumptions are that cost of default is crystallised at the instant of default, and that after indexing for inflation the cost per default remains relatively stable over time, implying a slow changing or constant mix of underlying loans.

Table 6.4

Moments and Risk Indices of the Distribution of Cost of One Default

(1)	MEAN COST	= m =	£2.9966×10^{4}
(2)	$\sum_{0}^{\infty}(COST)^2$	= a_2 =	£5.59×10^{10}
(3)	$\sum_{0}^{\infty}(COST)^3$	= a_3 =	2.9316×10^{17}
(4)	$\dfrac{\sum(COST)^2}{(MEAN\ \text{COST})^2}$	$=\dfrac{(2)}{(1)^2}=r_2$	63.36
(5)	$\dfrac{\sum(COST)^2}{(MEAN\ \text{COST})^2}$	$=\dfrac{(3)}{(1)^3}=r_3$	10920.8

Source: Empirical

Notes to Table 6.4:

a) Based on 3887 resolved defaults of which 910 resulted in provisions being raised.

b) This is a “risky” distribution, high risk indices (r_2, r_3) being occasioned by several large (£5-£15 million) individual provisions.

6.4 Estimation of F(x), for the Dataset

One could proceed to the recursive estimation of F(x), the cumulative distribution of total default. This estimation procedure could involve up to

5×10^9 individual arithmetic operations, and therefore a reasonable approximation is sought. Daykin, Pentikainen and Pesonen (1994 p 129) suggest the approximation formula:

$$F(X) \approx N\left(-\frac{3}{\gamma_x} + \sqrt{\frac{9}{\gamma_x^2} + 1 + \frac{6}{\gamma_x}\left(\frac{x-\mu_x}{\sigma_x}\right)}\right) \qquad (6.6)$$

which is valid where the coefficient of skewness does not exceed 1.2. Note that the co-efficient of skewness γ_q at 1.17 is close to the upper feasible limit for this approximation, so that relative error may be large. However, note also that small proportions of the total function in reserve estimation are being dealt with, so that even though relative error may be large, absolute error in relation to size of reserves or required loan pricing is likely to remain small.

This approximation formula (6.6) allows one to specify the upper tail of the distribution function F(x) as a transformed quadratic standard normal distribution, and permits the application of equation 3.7 directly to the calculation of reserves. The following assumptions underly the given results.

a) Insolvency probability is set at two differing levels, approximately one in 40, (γ_ε =2.0) and one in 800 ((γ_ε =3.09, allowing for relative error of 25% in our approximation of F(x)).

b) Interest earnings are ignored. Alternatively, an equivalent assumption would be that a dividend equal to the risk free rate of interest earned on reserves is paid each year.

c) The portfolio earns returns of 0.77% of loans outstanding (historic experience).

d) Administrative expenses total 0.50% of loans outstanding.

Table 6.5 below illustrates required reserves as a percentage of loans outstanding for given ruin probabilities.

Table 6.5
Required Reserves and Ruin Probabilities for Dataset/Portfolio Reserves % Loans Outstanding

γ_ε	Time Period	
	1 Year	**5 Years**
2.0	0.86	4.10
3.09	2.22	10.78
2.75	1.68	8.00
6.63	8.00	N.R.*

Source: Empirical

*Not relevant

Notes: Table 6.5

a) Ruin is defined as loss of more than stated reserves.

b) Required reserves increase with time.

In terms of interpretation, the portfolio requires excess reserves of 0.86% of loans outstanding in order to avoid regulatory constraints on its asset choice at one in 40 probability on a one year view. At one in 800 probability it would require excess reserves of 2.22%, again on a one year view. This would imply a total reserve of 8.86% or 10.22% respectively.

If rapid recomposition of assets were possible, the minimum capital adequacy requirement of 8% would imply an infinitesimal probability of insolvency in any one year (γ_ε =6.63 corresponds to approximately $1\text{x}10^{-7}$). On a 5 year view, excess reserves of 4.10% of loans outstanding (1 in 40) and 10.78% (1 in 800), respectively, would be required to avoid regulatory constraints. This would imply a total reserve of 12.10% or 18.78%, respectively. The insight provided by the 5 year view is that regulators may expect banks which become asset constrained to remain so, and wish to provide themselves with resources sufficient for a medium term workout, financed by asset recomposition.

In this context, the minimum capital adequacy requirement of 8% would imply $\gamma_\varepsilon = 2.75$ corresponding to approximately 3.0×10^{-3} ruin probability (on the artificial assumption of no asset recomposition). Asset recomposition would commence when the ratio dipped below 8% and would continue until regulatory capital was exhausted; the bank at that stage possessing an asset portfolio comprising only government securities.

In practice, observation of current capital adequacy ratios of UK clearing banks would imply adherence to the first interpretation above, so that sufficient asset excess is held by these banks to ensure that regulatory constraints on asset choice have less than a 1% probability of occurrence. Quite small excess reserves thus ensure unconstrained asset choice for these banks, at the margin and in the short term.

6.4.1 Comparative Static Analysis

Given that the 8% minimum capital adequacy requirement is uniform, it is interesting to explore the variability of mathematical reserves in response to changing provisions, loan margins and size. 20% greater and lesser provisions, 20% higher and lower average loan prices, and a bank 10% of the size of this portfolio are hypothesised, and the effect of these on reserves are examined.

This is equivalent to the assumption that there exist, within the uniform regulatory environment, banks which are 20% more and less risky (see Table 3.1), 20% more and less expensive in terms of loan pricing and 90% smaller, and examine the effect of these on reserves.

Table 6.6 below, shows reserves required as a percentage of loans outstanding for specified ruin probabilities.

Table 6.6

Reserves Required: Unchanged Policies at Given γ_ε

γ_ε	(a)	(b)	(c)	(d)	(e)	(f)
			Bank			
			(i) 1 year time period. Reserves % loans outstanding			
2.0	1.39	0.33	0.86	0.36	0.30	1.34
3.09	3.03	1.44	2.22	1.72	2.73	3.08
			(ii) 5 year time period. Reserves % loans outstanding			
2.0	6.74	1.78	4.10	1.88	6.32	4.36
3.09	14.7	6.83	10.78	8.56	13.00	11.16

Source: Empirical

Key: (a) and (b) 20% higher and lower provisions respectively.
(c) The actual observed portfolio.
(d) and (e) 20% higher and lower loan prices respectively.
(f) 10% of portfolio size, permitting greater random variation.

Clearly, mathematical reserves are influenced both by provisions and loan pricing, with provisions having a non-linear impact on reserves and loan pricing a linear impact. Profitability appears as a direct reduction in required reserves in equation (3.4). A small size effect is also apparent.

Table 6.7 below illustrates the γ_ε levels associated with Table 6.6 banks, in this case assuming a uniform capital adequacy requirement of 8%.

Table 6.7

γ_ε Values Associated With 8% Uniform Capital Adequacy

	Bank					
Time Period	**(a)**	**(b)**	**(c)**	**(d)**	**(e)**	**(f)**
1 Year	5.84	7.67	6.63	6.87	6.39	5.53
5 Years	2.28	3.42	2.75	3.19	2.32	2.68

Source: Empirical

Evaluating the unit normal variate in the 5 year case, produces insolvency "probabilities" of 1.13%, 0.169%, 0.298%, 0.071%, 1.02%, and 0.321%, respectively. These insolvency "probabilities" differ substantially, and indicate the potential difficulties associated with uniform capital adequacy requirements. These "probabilities" considerably overstate the true ruin probability, as risk would have been sharply reduced by asset recomposition prior to the 5 year horizon. Given timely asset recomposition, true ruin probability would remain infinitesimal, and quite close to 5 times the 1 year unit normal variate probability values.

6.4.2 Toward a VAR System

The preceding calculation (simulations) have been on an artificial basis, since they do not take into account the risk reduction effects of asset recomposition. The shareholders would have lost their entire investment in the bank some time before the asset recomposition process was complete, as their claim on bank assets would be subordinate to that of debt holders. Clearly, the major risk faced by shareholders is asset recomposition risk, associated with the loss of part of their invested capital, rather than insolvency risk associated with the loss of all of their invested capital.

The feasibility of total asset recomposition into a portfolio of government securities must be open to question, as the bank would have ceased to be a bank at that stage and would have transformed itself into a mutual fund (Fama (1980)). Evidence from the United States (Allen and Saunders (1993)) would suggest that regulators would foreclose prior to completion of the process.

Asset recomposition risk may be defined as the risk that the bank's asset choice becomes constrained by capital adequacy. Under present regulation it is a real risk for safer banks, whose true mathematical capital requirement is likely to be lower than the 8% minimum {Banks (b) and (d) Table 6.6}. Bank (b) in particular may have decided to make low risk loans, and may have accepted returns on those loans commensurate with the lower risk on its portfolio. It could thus find itself in a position where it was unable to remunerate its regulatory minimum capital at a competitive rate. Of course, if information asymmetries did not exist, its shareholders would accept lower and safer expected returns. However, if profit is the only observable output, investors may not recognise bank (b)'s relatively safer strategy. Banks (a) and (e) might also find it difficult to remunerate their regulatory capital, but for entirely understandable reasons. Bank (a) is relatively risky, while Bank (e) is relatively less profitable.

What is required is a system capable of discrimination between Banks (a) to (f) inclusive, with respect to the specific nature of their credit portfolios. Required reserves should be a function of a portfolio-specific VAR calculation, sensitive to provisions and profitability.

6.4.3 Investor Choice and CAPM Inputs

The existing regulatory system does not recognise any difference between Banks (a) to (f). The following table sets out the CAPM (capital asset pricing model) inputs generated by each of these banks, and presents the opportunity set available to investors.

Table 6.8
Set of Available Investments

Bank	Expected Return to Equity %	Standard Deviation of Equity Investment %
(a)	12.0	16.25
(b)	19.0	10.9
(c)	15.5	13.6
(d)	23.8	13.6
(e)	7.2	13.6
(f)	15.5	17.3

Source: Empirical

Assumptions:

1. Each bank has capital adequacy of 10% (6% equity, 4% debt).
2. Debt yields 5% in excess of risk free rate.
3. Expected excess return = Average of last 5 years excess return.
4. Risk free rate = 6% per annum
5. Standard deviation calculated as
   {0.5 (1 year reserves (γ_ε = 2.0) + Profit)}/ .06

CAPM would value these banks, from a shareholder viewpoint as follows: (d)$\geq$(b)>(c)>(f)>(a)>(e).

If CAPM can so easily sort these hypothetical banks by value, why not the regulator? The regulator may have a differing opportunity set to the investor, but should still be capable of ranking these banks by relative insolvency risk.

6.5 A specific VAR Computation

From Section 3.4.2, a VAR formula is required which is directly proportionate to risk, which responds to movements in risk, has a standard calculation framework, calculates for the "whole book", and is a justifiable improvement on present practice.

From Section 6.4, the formula to incorporate provisioning and profitability measures is required. Taxation has thus far been ignored, and simplifying assumptions have been made about dividends. Taxation and dividends both reduce the extent to which reserves may be financed out of internal resources. Taxation effects are included in the computation, and a generous dividend constraint is imposed. To ease comparison, and also to assuage regulators, the formula is constrained in order to produce an identical capital adequacy requirement to that operative at present in the case of the original dataset.

The proposed computation formula is as below:

$$\text{VAR Capital \%} = \left[2 \times \sum_{1}^{5} \text{Provisions \%} - (1-t)\sum_{1}^{5} \text{Margins \%}\right] \times \text{Loans Currently Outstanding} \tag{6.7}$$

Subject to a minimum value of 2% (to avoid infinitesimal or negative values) where:

i) VAR CAPITAL % is the computed value at risk capital requirement, expressed as a percentage of loans currently outstanding,

ii) $\sum_{1}^{5}$ provisions % is simply the sum of the past 5 years provisions expressed as a percentage of loans then outstanding,

iii) $\sum_{1}^{5}$ margins % is the sum of the past 5 years margins, expressed as a percentage of loans then outstanding and,

iv) t is the operative corporate profit taxation rate, currently assumed to equal 38%.

The margin calculation would take the form

$$\text{Margin \%} = \frac{\text{Interest Income - Administration Expenses - Provisions}}{\textit{Average Total Amount of Loans Outstanding}}$$

The provision calculation would take the form[6] :

$$\text{Provision \%} = \frac{\text{Provisions}}{\text{Average Total Amount of Loans Outstanding}}$$

This computation produces results as outlined in the following table.

[6] Both these formulae relate to the year in question.

Table 6.9

VAR Reserves and 5 Year Probability Measures

Bank	(a)	(b)	(c)	(d)	(e)	(f)
VAR Reserves %	10.82	5.28	8.00	6.45	9.55	8.00
Insolvency γ_ε	2.75	2.91	2.75	2.92	2.63	2.68
Probability Measure %	0.298	0.181	0.298	0.175	0.427	0.368

Source: Empirical

The effect of the VAR computation has been to reduce substantially the observed variation in the probability measure. This measure does not reflect real insolvency probability, but is nevertheless an index of relative risk. Variation in this measure has been reduced by 98% from its pre VAR spread (Section 6.4.2).

In declaring dividend payments, if banks were limited in their declarations by directly attributable streams of income as follows:

$$0.005 \times (1-t)\left[\sum_{1}^{5} \frac{\text{margins \%}}{\text{VAR capital \%}} + \sum_{1}^{5} \frac{\text{average interbank rate \% per annum}}{100}\right] \tag{6.8}$$

this has the effect of reducing the observed variation in the probability measure by constraining loss making banks, while allowing profitable banks to distribute a high proportion of their net earnings.

The reasoning behind this dividend formula is that reserves earn both margin as computed plus an opportunity cost equal to the inter-bank rate. At least theoretically, some small proportion of reporting banks could distribute more than their annualised net return, thus reducing their capital base by excess distribution. This would only apply in the case of banks with little or no provisioning requirement from year to year.

The following table illustrates the probability measures resulting from this dividend restriction.

Table 6.10
5 Year Probability Measures for VAR Computation with Restricted Dividends

Bank	(a)	(b)	(c)	(d)	(e)	(f)
Insolvency γ_ε	2.45	2.53	2.42	2.50	2.41	2.36
Probability Measure %	0.71	0.59	0.78	0.62	0.80	0.91

Source: Empirical

The probability measure has increased, because permissible dividends are on average, larger than those assumed in the earlier computations. However the variation in this measure has reduced by a further 65%.

One is now very close to a "level playing field" in terms of probability measures, the variation in these measures being only 0.7% of its level as calculated using conventional capital adequacy. Thus a VAR capital-adequacy-fixed premium deposit insurance scheme seems feasible. Whether this proposed approach actually represents a viable improvement on present practice awaits a full simulation approach and comparative evaluation 'tests', developed in Chapter 8.

It is now appropriate to turn our attention to the responsiveness of the VAR calculation to changes in current credit market conditions.

6.6 Premium Control

Many authors (e.g. Rubenstein and Yaari (1983), Rosen (1985) Cooper and Hayes (1987)) have discussed implicit contracts and multiperiod insurances. Essentially, the acceptance of a customer by an insurance company, or bank, carries with it the implicit contract that the customer will be acceptable in future time periods, subject only to no material adverse change in customer circumstances. Thus, for example, insurance companies offer no claims bonus systems to clients with superior claim records (Rubenstein and Yaari (1993)) thus countering the moral hazard problem. Similarly banks will informationally capture clients (Sharpe (1990)) and will make continuing reputable client offers of credit which will render their departure to a competitor bank unlikely.

If, however, the underlying risk intensity is unstable, and banks do not propose to lose money, then some form of control must be a feature of the system. Banks will require relatively stable pricing to avoid alienating their client base. Thus, banks must take long term views of client credit worthiness, and must be prepared to honour the implicit future contract at a price not markedly different to that on a current contract. Banks responsiveness to changes in credit market conditions is circumscribed by the desire to offer clients a stable credit environment. This obviously requires the bank to operate as a kind of "shock absorber" and indeed is one of the main reasons for regulatory and actual reserves.

Löf (1983) has described the situation prevailing for many banks. Using his notation, if G_N is the accumulated surplus, then

> "It should be so big that the risk that annual claims cannot be met is sufficiently small. The premium should not vary too much in time. G_N should not grow too large and it is good if it does not vary too wildly. In

particular G_N ought to be stable (equitable) i.e. if X_N (annual claims) does not have an increasing trend and bounded fluctuations, the same ought to be true about G_N".

Faced with unstable risk intensity, banks require large reserves. However they remain open to the simultaneous criticism of being lemming-like (all lending too cheaply when risk intensity rises) or of profiteering (when risk intensity falls and is unmatched by a reduction in charges).

Neither Ruin Theory, which requires large and increasing G_N to avoid ultimate insolvency, nor Credibility Theory (which ultimately leads to large and increasing G_N), satisfactorily address the above problem.

The requirement of control is to smooth premiums and reserves simultaneously, and Löf's proposed method is an application of linear control theory.

Firstly, assume the system is stable. Shareholders require <u>excess</u> returns of 10% per annum to compensate them for risk. Administration charges amount to 0.25% on loans. Then the relationship between G_N, G_{N-1} and P (on the assumption that "Pure Premium" is paid away in default cost) is as follows:

$$G_N = .9091\, G_{N-1} + P(1+\lambda) - P \tag{6.9}$$

For stable reserves we have:

$$0909 + P\,(1 + \lambda) - P = 0 \tag{6.10}$$

G = Reserves

P = Pure Premium

λ = Safety Loading

Assume P = 1, G = 8P

A minimum safety loading of 73% plus administration costs is required in an absolutely stable system. This is very similar to the current average situation prevailing in the dataset, although the system is unstable.

Löf introduced coefficients of variation for both premium and reserves, directly analogous to Gelles risk measure of profit. These are defined as:

$$V_p = \frac{\sigma_p}{P}\left(\frac{V}{X}\right) \tag{6.11}$$

$$V_g = \frac{\sigma_g}{G}\left(\frac{V}{X}\right) \tag{6.12}$$

$$V_r = \frac{\sigma_r}{R}(Gelles) \tag{6.13}$$

and permit random variation of claim, V (in the Poisson case $\frac{V}{X}=1$) Löf imputes a reserve target, and in the specific case where $\frac{\sigma_G}{G} \leq \frac{1}{8}$ the direct implication is that the coefficient of variation for reserves may not exceed $1/8$.

Extending the model to banking, we see that $\frac{V}{X}$ is substantially larger than 1 (variance substantially exceeds mean). Making the assumption that $\left(\frac{V}{X}\right)=2$, then the coefficient of variation of reserves is reduced to $^1/_{16}$, directly implying that premiums must be responsive to changes in risk intensity.

In our dataset, regulatory reserves are set at approximately 8 times pure premium. Gelles observes that average loan loss provisions in the US in years 1985-89 amounted to 0.982%. Current capital adequacy standards on this basis, would amount to 8 times pure premium.

This directly implies that a permitted coefficient of variation of $^{1}/_{16}$, in reserves allows reserves to fluctuate quite widely in terms of premium. The excess reserves carried by banks may allow them to smooth loan charges, and to absorb the resultant reserve volatility.

Because of the inherent volatility of default cost, there must be a shock absorber within the system. If reserves are targeted, risk (to the bank) is held to a low order, and price shocks are immediately absorbed into loan charges. Alternatively, if loan charges are targeted for stability, extremely volatile profits and somewhat volatile reserves result.

An ideal solution would spread risk so that some volatility was apparent in all measures, but not as much as would be the case if each measure bore the entire risk on its own.

The regulator faces a reserving problem in deciding the appropriate time period over which shocks will be absorbed by reserves. The suggested VAR approach in Section 6.5 assumes that 20% of any shock is absorbed each year, for 5 years after the shock. A shorter time period over which to sum provisions and margins implies greater variability in reserves.

The choice of a 5 year time period was the result of observation of provision and profitability changes in the early 1990's. The capacity of banks to absorb changes greater than approximately 80 basis points per annum in capital adequacy requirements is limited. Thus, any greater permitted change would require them to shrink their balance sheets, or to seek additional capital. It is assumed that the regulator would not seek such developments and therefore would choose a time period which permitted banks to absorb such shocks internally. Hence the choice of a 5 year period.

The optimisation problem facing the banks is how much volatility to absorb on its own account, and how much to pass to the customer. Clearly the absolute boundary on its own absorption is set by the market. A solution to the problem is achievable, but only when boundaries have been specified. This represents a major strategic hurdle for any bank, but the solution in the case of an individual bank is beyond the scope of this thesis.

An illustrative example is presented below. Premium control is one aspect of a fuller simulation process discussed in Chapter 8.

Using the dataset to illustrate:

G = £920 million (i.e. 8% of £11.5 Bn)

$\overline{X}$ = £120 million (i.e. 1.04% of £11.5Bn)

$\overline{P}$ = £208 million

i.e. {(2.31%-0.50%) (margin income - expenses) x £11.5Bn}

$\overline{R}$ = £88 million

V = 3

V_p = .05 b). Unspecified elsewhere

V_G = .0625 a). Unspecified elsewhere

V_r = Unspecified

V_x = 0.59 calculated from dataset

Scenario:

a). Maximum permitted reserves move 19.3% per annum (y_ε = .001).

b). Maximum permitted charges move 15.5% per annum.

c). 50% of variance absorbed by reserves, balance through charges.

d). All variance absorbed by reserves.

The following Table 6.11 sets out the implications for the *Gelles* measure for each of the above scenarios.

Table 6.11

Co-efficient of Variation of Profit Under Differing Assumptions

	a).	b).	c).	d).
$\frac{\sigma_r}{\overline{R}}$	0.65	0.69	0.42	0.80

Source: Empirical

Obviously, the simple illustration could be significantly enhanced if it was equipped with the strategic decision parameters appropriate to an individual bank. None of the above scenarios eliminate the probability of losses occurring. Scenario c) is the safest (probability of loss 0.9%), and d) the riskiest (probability of loss 10.5%) However c) implies an ability to raise charges by 50%, which may not be feasible.

Scenario d) would be appropriate if charges were to remain permanently fixed. By observation, b) would be the scenario closest to the present operation of the portfolio, implying a probability of loss of 7.5% in each year of operation.

Conclusion

The solvency model first discussed in Chapter 1 has been parameterised by values calculated from the dataset. Using the spread of performances observed in Table 4.1, the existence of a number of banks differing from the bank represented by the dataset has been hypothesised. A probability measure of insolvency risk on unchanged policies for these banks has been developed and it is shown that this measure differs significantly between banks under current regulation.

An alternative VAR model for capital adequacy is proposed, using bank specific risk measures. The application of the model is simplified so that only readily available figures are used in its computation. It has been shown that this VAR model produces relatively uniform insolvency probability measures, permitting the possible introduction of fixed premium deposit insurance. Finally, aspects of control have been discussed, with reserves, loan pricing and profit acting as shock absorbers within the system.

The next logical step is to develop a detailed individual loan pricing model which is then compared to current pricing within the dataset in order to establish whether existing credit pricing is efficient or may be improved. The following chapter proceeds to this next step.

CHAPTER 7

Development of an Individual Loan Pricing Model

Introduction

In the preceding Chapter, concentration was focused on overall portfolio measures, e.g. default frequency, cost per default, current loan provisioning, overall margin income, profitability and reserves. While of interest in their own right, these measures do not permit the bank to price individual loans, other than very crudely. This chapter seeks to develop a model to enable loans to be priced by reference to several categorical variables (Altman (1969) Altman et al (1981)).

7.1 Structure of Model

The model structure which follows was chosen for the following reasons:

1. All rating factors may be shown to be theoretically justified (Merton (1974), Devinney (1986)).

2. Empirical research has shown that these factors have a practical risk effect.

3. The number of factor levels has been chosen as a direct result of tests of the data.

This preliminary presentation of structure is further developed below:

Defaults are grouped by gearing (2 levels), utilisation (8 levels) security (2 levels) and size (2 levels, for severity only), giving a total of 64 (2 x 8 x 2 x 2) cells within which an individual loan may be categorised. The underlying assumption is that the four chosen variables are of substantial value in subdividing (or segmenting) the riskiness of loans. Techniques to be used in developing the model are identical to those used in statistical insurance work [Brockman & Wright (1992)] (See Chapter 6 for justification of use).

The basic insurance relationship, valid for cells and for the overall portfolio is:

Pure Premium = Claim Frequency x Claim Severity

This form of relationship is valid for solvency calculations, as in Chapter 6 where the entire historic experience is being used, and also for estimates of overall current profitability where 21 month past data are aggregated.

7.2 Assumptions and Methodology Underlying the Loan Pricing Model

7.2.1 Assumptions:

1. In any future time period, loans are independent. It is not possible, *a priori*, to determine which loans are about to default, and default on any loan does not confer any information about other loans.

2. The portfolio composition remains stable over time. This assumption applies to industry grouping etc., but allows the gearing, utilisation and size to vary over time, as these factors are inbuilt.

3. Changes in risk intensity apply uniformly across the portfolio

7.2.2 Methodology

For the portfolio as a whole, and for each specific subsection, the provisioning rate in future time periods is modelled as a weighted average of provisions made within the time period, the weights being historic default frequencies, and the provisions being the average provision in respect of the time period for a single defaulting loan.

7.3 Practical Difficulties

The assumptions and methodology proposed are identical to those used by Brockman & Wright (1992), and are subject to their shortcomings. In addition:

1. The assumption that loans are independent with respect to their single period risk of default depends very much on identifying associated risk successfully. If this is not done correctly, then the independence assumption is open to question.

2. The paucity of observations of actual default costs (spread over 64 cells) means that in practice security is assumed constant over broad ranges of loans differing only with respect to size of loan and size of security %. Examination of the portfolio indicates that this assumption is reasonable.

3. Assumptions about portfolio composition and risk intensity are open to accusations of oversimplification, and unless true, will be a further source of pricing error.

4. A very small tail of large cost defaults exists, substantially increasing the variance of cost amount.

Addressing the difficulties in order:

1. Loans are currently grouped to exclude known associated risk. The fact that risk covaries across the portfolio is known, but is assumed to be dealt with by the common risk intensity forecast. Loans are assumed 100% correlated with respect to changes in risk intensity, and 100% correlated by cell, but totally uncorrelated within cell. Overall estimates of covariance or correlation may be produced by summing total variance, and allocating the variance to the three headings just discussed. A correlation estimate will be given by the ratio of the sum of variance by risk intensity and by cell, to the total sum of variance. Thus, even though the assumption of independence appears less than credible, it is only applicable to the cells within which loans are categorised, and not to the process as a whole.

A minor difficulty with respect to correlation is the now standard assumption of correlation by industry group, leading to a regulatory requirement for portfolios of loans diversified among industries. Detailed examination of the portfolio has produced no evidence that such diversification works, as all loans seem broadly correlated, with industry grouping being irrelevant, at least in the short term. Hence industry group is not a rating factor in the model. In this respect this thesis differs from Dale (1993), who did find evidence that provisions vary by industry over the period 1976 to 1991. There is some evidence in Dale's (1993) illustrations that covariance increased markedly towards the end of his period of observation. Industry differences in provisioning may have been obscured by the fact that all industries entered the recession of 1989 to 1992 simultaneously, and the resulting high covariance masked underlying industry differences over the years 1990 to 1994 inclusive. Even if this were the case, relying on diversification by industry was apparently ineffective in the latter period.

2. Examination of provisioning rates across cells provided little evidence that these rates differ, other than by size of loan and in the case of secured loans. The difference between provisioning rates on secured and unsecured loans will be taken into account.

3. Once again, examination of the portfolio indicates relative stability, with migration by risk group being of a relatively low order and almost cancelling over the period under review.

   There remains a risk, however, that the nature of the banks business might change in future periods in a way which renders the above assumption invalid.

4. The existence of a few large defaults may mean that the loan pricing model is appropriate up to a certain loan level, with case-by-case estimation being appropriate above a certain size of loan. Loan prices were produced for the whole portfolio, and for the portfolio excluding loans above £5 million. Only overall pricing for the whole portfolio is illustrated below, as pricing is similar whether or not such loans are included. The impact on risk and resultant required reserves has been referred to in Chapter 6.

If the truncated portfolio is considered, then the relevant subsection produces lower required reserves, and greater statistical accuracy in provision forecasting, but a residual portfolio of large loans remains to be dealt with on a case-by-case basis.

7.4 Choice of Risk Factors

Under credible conditions, Merton has shown that the price of loan guarantees is a function of volatility of firm value and the ratio of debt to firm value including debt (Merton 1974, Chapter 5). In the vast majority of cases, both firm value and debt are not traded - in fact non traded loans may be a necessary condition for banking intermediation (Diamond (1984), Sharpe (1990) *et al*).

The volatility of firm value is not available to us. However, a proxy measure of this volatility-and more important, the loan portfolio's sensitivity to volatility-has been derived by observing a measure of risk intensity, (Chirinko & Guill, 1991, Chapter 6).

The allocation of risk intensity among loans concentrates on measures of the ratio of debt to firm value.

a). Gearing: This is essentially given by an accounting calculation. As given in the data, it is the ratio of debt to equity, corresponding to the bankers definition. This definition is modified to the following:

$$\frac{\text{Debt}}{\text{Equity} + \text{Debt}}$$

prior to being incorporated as a rating factor. Thus, the rating factor used corresponds to the Chapter 5 definition of gearing. As individual loan files are only updated on average once each year, current gearing figures will on average be at least 6 months out of date. However, observation indicates that gearing changes only slowly, so that gearing may be interpreted as a measure of broad default risk relative location.

The reason for modification of the definition of gearing is that certain firm values are comprised of debt only, equity having disappeared effectively. Using the bankers definition of gearing produces a debt equity ratio of infinity, rendering valuation using gearing as a risk factor impossible in these cases.

b). Utilisation: Avery and Berger (1991) have analysed commitments in detail. As pointed out in Chapter 2, the direct analogy to commitments in US banking is undrawn facilities in UK Banking. Utilisation is used as a measure of financial stress because there is an observed correspondence within the portfolio between utilisation rates approaching 100% and default.

c). Security: As pointed out in Chapter 5, security has the direct effect of lowering potential default losses, given that default has occurred. The reason for its inclusion in a frequency model is that in many cases, firms with negative net equity (bankers definition) are kept out of a default situation by the existence of independent security or resources which enable the firm to continue trading. Thus, security in this particular case is assumed to have an effect on default frequency.

d). Size of loan: considered only by reference to severity.

The choice of rating factors is one among many possible choices. If they correctly subdivide by "true" risk, then a bank using a model of the above type will find it possible to select potentially profitable business from the universe of banks not using such models, or at worst, to avoid being selected against. The rating factors have been theoretically justified in earlier chapters; their empirical justification will appear as a difference in loan prices between cells, and ultimately in improved performance, reflecting the banks choice of new business from the lower risk/higher profit cells, relative to current portfolio structure.

7.5 Number of Levels Per Risk Factor

The starting point for the number of levels is the portfolio itself. Repeated queries of the database were necessary to produce the chosen number of levels. Historic portfolio experience must differ by level for choice of levels to have any validity. Secondly, the increasing/decreasing risk associated with the chosen factor must have a theoretical and practical justification. Finally the rate of increase/decrease of risk with respect to the factor under consideration and the overall risk associated with the factor indicate the relative sensitivity of the factor to risk.

With respect to the factors chosen, all factors and levels produced differing observed historic loan pricing. All factors are theoretically justifiable. With regard to gearing, Table 5.1 in Chapter 5 illustrates its effect on loan pricing. Fitting gearing independently as a one factor model produced low empirical default costs for low levels of gearing, with an exponential increase. On the basis of gearing alone, 6 levels would have been appropriate (see Table 7.1)

Table 7.1

Parameter for Gearing Levels

(Multiplicative Poisson)

Gearing Level %	**Gearing Factor**	**Parameter -4.646**
<50	1	0.000 (Aliased)
50 ≤ G < 100	2	0.7493
100 ≤ G < 150	3	1.146
150 ≤ G < 200	4	1.054
200 ≤ G < 500	5	1.267
500 ≤ G	6	1.503
Scaled deviance	986.42	Residual Degrees of Freedom 191
Single Factor Gearing	630.17	Residual Degrees of Freedom 186

Source: Empirical

However, when utilisation was fitted in conjunction with gearing, all gearing levels other than the first became statistically indistinguishable. This may mean that utilisation itself has very substantial explanatory power *vis a vis* increased gearing. Intuitively, we would expect high levels of gearing to be associated with high levels of utilisation (assuming the bank was prepared to lend to some prudential limit) and *vice versa*. This appears to be the case when empirical evidence is considered (see Table 7.2).

Table 7.2

Parameters for Gearing and Utilisation

(Multiplicative Poisson)

Gearing Level %	Gearing Factor	Parameter -5.210	S.E. 0.1542
G < 50	1	0.000	(Aliased)
50 ≤ G < 100	2	0.518	0.1290
100 ≤ G 150	3	0.730	0.1315
150 ≤ G 200	4	0.594	0.1475
200 ≤ G < 500	5	0.718	0.1227
500 ≤ G	6	0.615	0.1162
Utilisation Level %			
V ≤ 25	1	0.000	(Aliased)
25 < V ≤ 35	2	0.364	0.1901
35 < V ≤ 45	3	0.542	0.1859
45 < V ≤ 60	4	0.822	0.1638
60 < V ≤ 75	5	1.191	0.1590
75 < V ≤ 83	6	1.354	0.1619
83 < V ≤ 90	7	1.666	0.1646
90 < V	8	2.022	0.1667

Source: Empirical

Note: Table 7.2:

Smooth progression of utilisation parameters, with irregular and approximately constant gearing parameters as follows:

Scaled deviance	986.42	Residual degrees of freedom	191
Single factor utilisation	354.06	Residual degrees of freedom	184
Two factor utilisation and gearing	310.24	Residual degrees of freedom	179

The final choice of 2 levels for this factor was influenced by its relatively insignificant effect on pricing for low levels, its exponential increase mid-range (accounted for in the factor model by utilisation levels) and its theoretical maximum.

Utilisation as a measure of financial stress proved highly responsive. The greatest number of levels (8) was chosen for this factor, as empirical observation indicated substantially differing experience by utilisation.

Security (as intimated in Tables 5.1 - 5.3, Chapter 5) has a substantial effect, in many ways inverse to gearing. The choice of only 2 levels for this factor is as much a response to the way in which loans are granted as to its risk significance. In fact, loans are generally either unsecured (by independent security), or fully secured, with few partly-secured loans in existence. The choice of 2 levels was dictated by the clear distinction between unsecured and fully secured loans. Security, by investigation, has proved stochastic, as the range of outturns for realised security as opposed to its book value has proved highly variable. For the purposes of claim frequency analysis, security is taken as an accounting entry. Its variability will be addressed as part of the default cost amount analysis, to follow.

The full table of rating factors and corresponding levels is set out in Table 7.3 below:

Table 7.3

Multiplicative Poisson: Rating Factors

Rating Factor	Gearing %	Utilisation %	Security %	Size Severity Only
Level 1	$0 < G \le 50$	$U \le 25\%$	$0 \le S < 100$	< £100,000
2	$50 < G \le \infty$	$25\% < U \le 35\%$	$100 \le S$	≥ £100,000
3		$35\% < U \le 45\%$		
4		$45\% < U \le 60\%$		
5		$60\% < U \le 75\%$		
6		$75\% < U \le 83\%$		
7		$83\% < U \le 90\%$		
8		$90\% < U$		
9				
10				

Source: Empirical

Unequal intervals are a function of observation; intervals are chosen by reference to observed risk.

7.6 Choice of Default Frequency Model

In Chapter 6 using Daykin, Pentikainen & Pesonen's (1994) underlying structure, defaults were hypothesised to follow a Poisson process. Brockman & Wright (1992) comment on Johnson & Hey's (1989) specification of a Poisson process with regard to claim frequency, followed in that case by a constant variance claim frequency model. Brockman & Wright themselves propose a Poisson error structure, whereby the variance of claim number is equal to the cell mean. They further point out that variance may be allowed to exceed mean by means of a simple model adjustment, which allows mean claim frequency forecasts to remain unaltered, but increases overall variance.

They do warn that the "fit" of the model in terms of the explanatory factors, must be good in order to permit the variance constraint to be eased.

With respect to the default frequency model to be used, it is proposed to follow Brockman and Wright's (1992) blueprint (i.e. to assume that defaults are Poisson distributed): firstly constraining variance to equal cell mean, and secondly permitting variance to exceed mean. Standardised residuals are produced for the first assumption only, as results were found to be acceptable.

The software used is the GLIM package, and a multiplicative model is proposed in order to investigate interaction of factors. Where variance equals mean, a pure Poisson process is being proposed; where variance is not so constrained, a distribution of negative binomial type results.

There exists a key difference between the motor insurance claim frequency model of the above authors and the default frequency model proposed by the present author: A motor vehicle can have more than one accident, whereas a single loan may only default once (under normal circumstances)!

Firstly, it is in fact possible for a loan to default more than once. Loans may enter and leave the default state through recovery in the firms finances and subsequently re-enter the default state (and thus be counted as a separate default) should the recovery prove temporary. Secondly, the fact that a large portfolio is involved means that the effect of approximating a part Binomial (for single defaults), part Poisson (for more than 1 default) distribution using a Poisson assumption is not likely to be substantial, given that residual heterogeneity within cells may already have invalidated the strict I.I.D. assumption, and that we know the default frequency to be bounded by operational factors at quite low levels. What is faced is a trade off "between accuracy and simplicity" (Brockman & Wright 1992) (see also Chapter 6), and the simplicity of the Poisson assumption is chosen at the expense of a small - and conservative loss of accuracy. For a full discussion of the Poisson approximation to the Binomial for large N, see McCullagh and Nelder (1989, Chapter 6 p103 - 107 and Appendix B).

The data used, as mentioned earlier, is the default experience arising from a portfolio of approximately 59,000 loans over a 21 month period. Parameter estimates, significance levels and residuals are shown in Tables 7.4, 7.5 and 7.6.

Table 7.4

Parameter Estimates: Default Frequency Model

Parameter	Estimate	Standard Error	Parameter Name
1	-5.124	0.155	1
	0	aliased	UTIL(1)
2	0.3966	0.1901	UTIL(2)
3	0.5921	0.1858	UTIL(3)
4	0.8851	0.1636	UTIL(4)
5	1.262	0.1583	UTIL(5)
6	1.426	0.1599	UTIL(6)
7	1.716	0.1601	UTIL(7)
8	2.066	0.1626	UTIL(8)
	0	aliased	GEAR(1)
9	0.5966	0.1091	GEAR(2)
	0	aliased	SEC(1)
10	-0.3019	0.05003	SEC(2)

Source: Empirical

Table 7.5

Significance Levels: Analysis of Deviance

Model	Degrees of Freedom D	Scaled Deviance Q	F Statistic
No Factors	31	738.70	
Utilisation	24	106.34	3.797
Gearing	23	68.25	8.597
Security	22	30.75	12.638

Source: Empirical

Notes: Table 7.5

1). F statistic = $$\frac{(Q_i - Q_{i-1})/(D_i - D_{i-1})}{Q_i / D_i}$$

2). All factors significant at 99% level

3). No product factors significant

Table 7.6

Standardised Residuals: Default Frequency Model

Cell	G-U-S	Observed	Fitted	Standardised Residual	Exposed to Risk
1	1-1-1	0.005738	0.005953	-0.147	2,788
2	1-1-2	0.006401	0.004402	1.13	1,406
3	1-2-1	0.00432	0.008851	-1.638	1,157
4	1-2-2	0.006844	0.006544	0.11	877
5	1-3-1	0.011923	0.010762	0.29	671
6	1-3-2	0.004809	0.007958	-0.882	624
7	1-4-1	0.015367	0.014426	0.219	781
8	1-4-2	0.011686	0.010667	0.289	856
9	1-5-1	0.014980	0.021034	-1.023	601
10	1-5-2	0.014178	0.015554	-0.293	705
11	1-6-1	0.034217	0.024777	0.648	117
12	1-6-2	0.023690	0.018321	0.515	169
13	1-7-1	0	0.033107	-0.444	6
14	1-7-2	0	0.024481	-0.390	6
15	1-8-1	0.254372	0.046984	5.366	31
16	1-8-2	0	0.034742	-1.108	35
17	2-1-1	0.010546	0.010810	-0.108	1,802
18	2-1-2	0.005934	0.007993	-0.669	843
19	2-2-1	0.016234	0.016071	0.060	2,218
20	2-2-2	0.015702	0.011884	1.218	1,210
21	2-3-1	0.019733	0.019542	0.065	2,280
22	2-3-2	0.014986	0.01445	0.167	1,401
23	2-4-1	0.026737	0.026196	0.233	4,862
24	2-4-2	0.018092	0.019370	-0.529	3,316
25	2-5-1	0.039714	0.038196	0.595	5,867
26	2-5-2	0.027324	0.028243	-0.392	4,259
27	2-6-1	0.043914	0.044991	-0.367	5,215
28	2-6-2	0.034233	0.033268	0.323	3,739
29	2-7-1	0.057426	0.060119	-0.714	4,232
30	2-7-2	0.048656	0.044454	1.054	2,795
31	2-8-1	0.087869	0.085318	0.413	2,231
32	2-8-2	0.055211	0.063087	-1.171	1,395

Note: Residuals well behaved, with the exception of one outlier associated with very small exposed to risk.

7.7 Conditional Default Cost Model

Brockman & Wright (1992) advocate a multiplicative gamma error structure model for fitting to motor claim severity data, with coefficient of variation assumed constant. For a variety of reasons, an overall model is impractical for loan portfolio analysis. Firstly, it cannot be claimed in respect of loans, that a similar severity distribution applies regardless of size. Secondly the existence of security on some loans and not on others must have repercussions on the severity of default cost. In the light of information under loan size and security, the global allocation of default cost across the portfolio appears sub optimal; a better overall fit should be attained by fitting piecewise.

The default cost data made available is, not surprisingly, more detailed than simple frequency data. It is ultimately proposed to produce loan margins on an annualised basis; it is insurance industry practice to index historic default costs to today's date, and to produce an annual average default cost in today's prices in order to immunise the portfolio against already observed changes in monetary values and to produce a pricing structure appropriate to current default cost.

The combination of frequency and default cost models will give us a pricing structure, but one which is not yet applicable to loans granted presently.

The piecewise fitting referred to above, involves fitting 4 separate distributions (2 for each security level x 2 for each loan amount) to 16 cells (2 for each gearing level x 8 for each utilisation level) in each case.

The data available comprise 3,533 defaults, of which 829 gave rise to provisioning need.

Tables 7.7, 7.8 and 7.9 give a breakdown of the default data by security and loan amount.

Table 7.7

Breakdown of Default Frequencies by Security and Size of Loan

Security Factor Level	Loan Size Factor Level	Number of Observations				
		Zero Cost		Positive Cost		Total
1	1	998	(70.0%)	427	(30.0%)	1425
1	2	481	(60.9%)	309	(39.1%)	790
2	1	786	(94.4%)	47	(5.6%)	833
2	2	439	(90.5%)	46	(9.5%)	485
Total		**2704**	**(76.5%)**	**829**	**(23.5%)**	**3533**

Source: Empirical

Note: Security increases the proportion of zero cost defaults

Table 7.8

Conditional and Unconditional Mean and Variance of Default Cost Amount

Security Factor Level	Loan Size Factor Level	Conditional (on positive cost)		Unconditional	
		£ Mean	Variance	£ Mean	Variance
1	1	34725	$554.59.10^{6}$	10405	419.13×10^{6}
1	2	254055	$349.80.10^{9}$	99371	151.97×10^{9}
2	1	25812	$507.68.10^{6}$	1456	63.58×10^{6}
2	2	453423	$186.90.10^{10}$	43005	191.46×10^{9}
Total		**139205**	$\mathbf{248.78.10^{9}}$	**32664**	$\mathbf{61.80.10^{9}}$

Source: Empirical

Note: Security reduces unconditional expected cost, but its stochastic nature is revealed, when we consider the conditional large loan mean cost.

Table 7.9

Higher Moments of Default Cost Amount

Security Factor Level	Loan Size Factor Level	Conditional (on positive Cost)			Unconditional		
		a_3 £	Skewness	Kurtosis	a_3 £	Skewness	Kurtosis
1	1	$1.1718.10^{14}$	1.3527	5.860	$3.5113.10^{13}$	2.4369	9.9980
1	2	$1.8357.10^{8}$	7.5089	73.4076	$4.6932.10^{17}$	11.3424	169.3275
2	1	$8.4634.10^{13}$	2.5317	11.9770	$4.7753.10^{12}$	8.8653	112.0520
2	2	$1.2515.10^{19}$	3.8881	18.0992	$1.1870.10^{18}$	13.8736	210.6419
Total		**$1.3787.10^{18}$**	**10.2531**	**127.4226**	**$3.2351.10^{17}$**	**20.6603**	**518.8445**

Source: Empirical

Notes: Table 7.9

1) Based on defined, indexed default costs. Excludes 354 defaults included in Table 4.6, of which 81 gave rise to provisioning cost.

2) Extremely high skewness and kurtosis point to a distribution asymptotic to both X and Y axis, and give rise to great difficulty in fitting distributions, particularly to the important "body" of the distribution.

3). Best fit provided by the logistic distribution, but insufficient accuracy obtained.

As may be observed from the moments of the default cost distribution in Tables 7.8 and 7.9, the combination of relatively low mean with high 2nd, 3rd and 4th moments, and the fact that the distribution is effectively asymptotic to both X and Y axes, mean that the loss of accuracy entailed in fitting a smooth curve to the empirical distribution is unjustified. Using the "Best Fit" software

package, no acceptable fit was found among 22 distributions tested, and thus it is proposed to fit the distribution empirically.
The above findings are now applied to price loans currently being granted.

7.8 The Transition Matrix

The observation of default cost per cell and the basing of loan prices on cell experience does not allow for the migration of loans between cells. The correct loan pricing formula is a probability weighted average of all cells, so that a loan is appropriately charged throughout for its underlying probability of improvement or disimprovement in quality. A transition matrix is necessary to calculate the current state probabilities of cell movement. This transition matrix is unlikely to be stable over time, but deviations from its implied levels will provide a valuable insight into changing portfolio conditions.

Based on most recent experience, the appropriate current transition matrix is set out in Table 7.10. This matrix is effectively the probability that a loan retains its current risk rating, or moves to another risk rating group within a 12-month period. (As loans are reviewed annually, the underlying assumption is that appropriate charges on a 2 year loan may only be adjusted once, at the mid point of the loan term). If monitored, changes in this matrix over time can provide information as to how portfolio risk is changing over time. An increase in the probability that a loan moves to a higher risk grade, *ceteris paribus*, would indicate that either the current pace of improvement in credit conditions was slowing, or that credit conditions were beginning to disimprove. The corollary is obviously also the case.

Table 7.10 is appropriate only to loans currently being granted if there is no change in the rate of improvement in credit conditions currently operative (see Table 6.1, Chapter 6, for overall portfolio default frequency by quarter). The high turnover noted in Table 4.4, Chapter 4, effectively means that any initial selection effect is not likely to be large.

All loans will be adjusted by the relevant transition matrix entries prior to final pricing.

Table 7.10

Current probabilities of Risk Grade Movement within 12 months.

	1	2	3	4	5	6	7	8
1	61.88%	14.92%	7.15%	7.68%	5.19%	2.11%	0.82%	0.25%
2	18.21%	42.05%	11.54%	13.44%	8.68%	3.96%	1.68%	0.44%
3	8.67%	13.10%	37.92%	19.96%	11.69%	0.53%	2.63%	0.78%
4	4.52%	7.73%	10.29%	46.64%	18.28%	8.12%	3.16%	1.26%
5	2.33%	3.64%	5.42%	17.65%	47.21%	15.11%	6.58%	2.07%
6	1.31%	2.20%	2.64%	8.79%	21.08%	45.42%	14.30%	4.26%
7	0.84%	1.17%	1.73%	5.31%	11.25%	16.34%	52.00%	11.37%
8	0.45%	1.15%	1.54%	3.33%	6.78%	10.81%	19.83%	56.11%

Source: Empirical

7.9 Loan Pricing

The construction of loan pricing, follows directly from the preceding discussion. Provisioning rates allied to default frequencies produce an expected cost per cell, which must be at least equalled by cell income, so that overall portfolio income prior to administrative expenses and return to capital exactly equals expected cost, i.e. a "Pure Premium" calculation.

Tables 7.11-7.12 set out estimates of loan pricing on a "Pure Premium" basis, applicable to current loans. Note that no margin is taken to compensate bank shareholders for risk, and that no allowance is made for costs.

Table 7.11

Loans < £100,000: Pure Risk Premium

	Sec 1, Gear 1	Sec 1, Gear 2	Sec 2, Gear 1	Sec 2, Gear 2
1	1.22%	2.19%	0.11%	0.22%
2	1.15%	2.08%	0.11%	0.22%
3	1.08%	1.90%	0.11%	0.22%
4	1.08%	1.90%	0.11%	0.18%
5	1.15%	2.04%	0.11%	0.22%
6	1.18%	2.26%	0.11%	0.22%
7	1.26%	2.69%	0.14%	0.25%
8	1.86%	3.26%	0.18%	0.32%

Source: Empirical

Table 7.12

Loans >= £100,000: Pure Risk Premium

	Sec 1, Gear 1	Sec 1, Gear 2	Sec 2, Gear 1	Sec 2, Gear 2
1	0.32%	1.00%	0.29%	0.72%
2	0.39%	1.15%	0.25%	0.68%
3	0.47%	1.15%	0.25%	0.54%
4	0.57%	1.29%	0.25%	0.54%
5	0.90%	1.61%	0.36%	0.61%
6	1.40%	2.19%	0.36%	0.75%
7	3.23%	2.62%	0.64%	1.11%
8	2.55%	3.05%	0.86%	1.36%

Source: Empirical

For illustration purposes only, if, for example total excess capital of 8% of loans outstanding were to require a return of 10% per annum to compensate for risk, and if administration costs were assumed to amount to 0.50% per annum for all loans, the annual margins payable would be as set out in Tables 7.13 and 7.14 (excess return to capital levied proportional to observed default cost).

Table 7.13

Loans < £100,000: Expense Loaded Premium

	Sec 1, Gear 1	Sec 1, Gear 2	Sec 2, Gear 1	Sec 2, Gear 2
1	2.54%	4.19%	0.70%	0.86%
2	2.44%	4.02%	0.69%	0.85%
3	2.31%	3.82%	0.68%	0.83%
4	2.31%	3.80%	0.68%	0.82%
5	2.45%	4.06%	0.69%	0.84%
6	2.51%	4.32%	0.70%	0.86%
7	2.61%	5.02%	0.73%	0.94%
8	3.65%	5.99%	0.81%	1.04%

Source: Empirical

Table 7.14

Loans >= £100,000: Expense Loaded Premium

	Sec 1, Gear 1	Sec 1, Gear 2	Sec 2, Gear 1	Sec 2, Gear 2
1	1.01%	2.20%	0.97%	1.71%
2	1.16%	2.42%	0.93%	1.64%
3	1.26%	2.43%	0.91%	1.42%
4	1.45%	2.68%	0.92%	1.42%
5	2.01%	3.22%	1.10%	1.57%
6	2.85%	4.21%	1.12%	1.77%
7	5.99%	4.95%	1.59%	2.36%
8	4.81%	5.73%	1.95%	2.80%

Source: Empirical

7.10 Using The Results: Comparison with Existing Pricing

Tables 7.15 and 7.16 set out the difference between the banks current charging structure, and that produced by the loan pricing model, with reasonable allowance for return to equity and administration costs. The proposed pricing model has been constrained to produce approximately existing revenues, so that direct comparison is possible, to establish whether systematic relative overpricing/underpricing of specific cells exists.

If existing pricing is representative of the market, then scope may exist for concentrating marketing effort on areas offering attractive risk/reward ratios, and de-emphasising areas which are relatively less attractive.

Table 7.15

Loans < £100,000: Current Margin Less Proposed Margin

Utilisation	Sec 1, Gear 1	Sec 1, Gear 2	Sec 2, Gear 1	Sec 2, Gear 2
1	0.40	-1.21	2.15	1.92
2	0.39	-0.70	2.10	2.30
3	0.62	-0.37	2.27	2.47
4	0.69	-0.22	2.33	2.51
5	0.70	-0.37	2.50	2.65
6	0.30	-0.52	2.58	2.67
7	0.11	-0.92	3.21	2.76
8	-0.16	-1.77	1.99	2.92

Source: Empirical

Table 7.16

Loans >= £100,000: Current Margin Less Proposed Margin

Utilisation	Sec 1, Gear 1	Sec 1, Gear 2	Sec 2, Gear 1	Sec 2, Gear 2
1	-0.56	-1.01	0.69	0.00
2	-0.43	-0.98	0.62	0.40
3	-0.37	-0.91	0.74	0.46
4	-0.13	-0.87	0.67	0.53
5	-0.50	-1.16	1.09	0.57
6	-1.50	-1.68	0.87	0.57
7	-3.99	-2.38	2.41	0.33
8	-2.07	-2.98	0.28	0.03

Source: Empirical

Broadly speaking, smaller loans appear substantially more profitable than larger loans. Two major differences between the algorithm used by the bank at present, and the model presented in this chapter are:

1. Collateral

   Secured loans are priced by the bank substantially higher than their cost. All of columns 3 and 4 in each of Tables 7.15 and 7.16 are positive, indicating that excess returns are being generated in secured lending: all but 1 of columns 1 and 2 in each of these tables are negative, indicating that unsecured lending is failing to generate the returns implicit in our model.

   The pattern is consistent, and suggests that some proportion of charges be moved from secured to unsecured lending. In practical terms, charges for secured loans should be reduced, and those for unsecured loans increased, until column entries in Tables 7.15 and 7.16 are balanced closer to zero; in this case columns 1 and 2 taken together, should approximately equal columns 3 and 4 taken together.

2. Gearing

   There is evidence that the risk reduction effect of low gearing is insufficiently taken into account in current pricing. Highly geared lending, whether secured or unsecured (i.e. columns 2 and 4) produces lower excess returns than low geared lending (i.e. columns 1 and 3). This would imply that charges should be increased for the former and reduced for the latter, similar to the procedure followed for collateral.

The algorithm presently used by the bank seems otherwise effective, with relative stability in patterns of overcharging/undercharging relative to cost, when each column is analysed. One exception to this effectiveness is the pattern in columns 1 and 2 of Table 7.16 where undercharging increases exponentially with utilisation. For higher utilisation levels, this may be a function of the bank's maximum margin of 650 basis points.

However, while it may be argued that the algorithm merely requires adjustment, and therefore that the loan pricing model does not represent a significant improvement on present practice, it must be pointed out that a loan pricing model of the type proposed herein was necessary to identify anomalies in the algorithm, and its continued application would be necessary to ensure that the revised algorithm continued to represent underlying experience.

7.11 Comparison of Portfolio Mean Income and Standard Deviation

The proposed scale of charges produces higher mean income. There are several reasons for this.

The proposed scale of charges assumes weighted average gross margin income of 2.34% in order to remunerate regulatory capital at 10% per annum over the entire 5 year period. Current charges, on the other hand, have recently reduced, reflecting improved provisioning trends, and produce weighted average gross margin income of 2.20%. This would produce charges on average 6.4% higher under the proposed scale than those currently being operated.

The choice of 10% per annum for remuneration of regulatory capital is dictated by the fact that this is the actual rate of return achieved by the dataset over the last 5 years. This rate is chosen to enable direct comparison of models.

The size of cross subsidisation within the existing portfolio is such that deficits on loans for which such deficits exist, more than absorb surplus profits on loans which are charged a higher than proposed margin. This has the effect of reducing current mean income by 8.0% relative to its level were the proposed scale of charges to operate.

At present, the proposed scale of charges would produce mean income after provisioning of £65.05 million per quarter, versus £56.22 million per quarter at present. The reader is cautioned that these figures represent specific results for the portfolio in question, and no general inferences may be drawn from them. To the extent that cross subsidisation exists within bank portfolios, mean income may be increased by the elimination of subsidies, and the continuation of charging at least the observed economic cost in all risk groups.

For the specific bank in question, effort might be directed towards increasing the pricing of unsecured lending, with any reduction in volumes resulting from such increases giving rise to resources which might then be directed toward secured lending. Clearly, the current cross subsidisation is significant, and might allow the above effort to continue for some time, rather than as an "at the margin" exercise.

With regard to portfolio standard deviation, the proposed scale of charges produces a standard deviation of profit of £2.0 million at present, versus £1.6 million for the market responsive charges operative at present. If we constrain the proposed scale of charges to equate to existing charges by eliminating 1. above, then the associated standard deviation of profit falls to £1.88 million, and mean income after provisioning to £61.16 million. It may be inferred that the latter mean standard deviation combination would be preferable to that currently operative.

In practice, what would emerge would be a hybrid portfolio, retaining or slightly reducing excess profits where these are currently a feature, and reducing or eliminating opportunity losses. The size of excess profits and opportunity losses are estimated to be significant, so that the current portfolio would not be efficient in a Markowitz framework. The actual portfolio resulting from the effort referred to above cannot be estimated, as the elasticity of credit demand for each risk group is unknown. Equally, it would be unrealistic to assume perfect credit markets, because we know that the existing portfolio represents 20% of its total market.

It should be cautioned again that the above observations apply to a specific portfolio over a relatively short time period. Such effects as have been observed may or may not be a feature of other credit portfolios. If subsidisation is present, so that some loans are charged less than opportunity cost, then the income characteristic of any portfolio may be improved. In this specific portfolio, subsidisation has been identified and its elimination would enhance profitability.

Conclusion

A factor model for loan pricing has been derived, based on expected cost. The results of this model have been applied to impute a scale of operative charges. These charges have been compared to those currently operative; reducing or eliminating differences could enhance portfolio profitability, and might improve portfolio allocation in favour of those areas producing revenue in excess of expected cost. A fundamental assumption is that expected cost remains in line with historic experience of actual cost.

Simulation of the operation of a number of credit portfolios over longer time periods is now pursued.

Chapter 8

Simulation of the Credit Process

Introduction

A framework for a VAR computation of reserves has been developed in previous chapters. This computation involves only observable measures, is developed by reference to a risk *theoretic equation*, and has been constrained to produce equal regulatory reserves to those currently operative in the case of the dataset. The range of banks reporting to a regulatory authority has been hypothesised to contain 6 in total, with banks with provisions 20% higher and lower than this dataset, banks with profitability 20% higher and lower than this dataset, and finally a bank 10% of the size of this dataset. The operation of such a range of credit portfolios under both the present and proposed regulatory requirements will now be simulated. With equivalent assumptions, the superiority of the proposed VAR regulatory requirement should manifest itself in fewer observed insolvencies.

8.1 Discussion of Models and Statement of Common Features

The volatility of underlying loan losses requires any simulation to possess significant feedback, in order to constrain outcomes into a reasonable stochastic bundle. One significant area of difference in the credit process compared with simulation models in insurance is the relatively simple nature of investments undertaken by banks, so that reduced variability of investment returns may partly compensate for the volatility of default losses.

Daykin and Hey's (1989) paper is taken as the starting point due to its expositional simplicity. Modifications are required and are discussed while proceeding. Viewing the credit portfolio in isolation from the bank's other activities we have:

Inflow: Loan Margin Income
Short Term Interest Income on Reserves

Outflow Loan Provisions
Administrative Expenses
Taxation
Dividends

Thus a simple model, with only primary variable input, would be:

$$A(t) = A(t-1) + R(t)\big(A(t-1)\big) + L(t) - P(t) - E(t) - T(t) - D(t) \qquad (8.1)$$

Where:

A(t) is the amount of assets at end year t.

R(t) is the average short term interest rate in year t.

L(t) is the loan margin income in year t.

P(t) is the provision requirement in year t.

E(t) is the administration expense in year t.

T(t) is the taxation charge in respect of year t - 1.

D(t) is the dividend payable in respect of year t-1.

Reserves are assumed to be held in short term deposits, and taxation and dividends are assumed paid after a 1 year lag. Loan provisions are a slow moving average, with an average 2 years run off. Thus, on average current loan provisions are assumed to relate to loans made 2 years ago. In an inflationary environment this can make a significant difference to the simulation. Most of the flow variables would be expected to bear some relationship to inflation over time, so that a model for future inflation is necessary. Daykin and Hey's model is:

$$LN(1+i(t))-U_q=\alpha_q\left[LN(1+i(t-1))-U_q\right]+\sigma_q Z_q(t) \qquad (8.2)$$

Where:

$i(t)$ is the rate of inflation.

U_q is the average inflation rate.

α_q is the serial correlation of last years inflation rate with this years.

σ_q is the amplitude of random "white noise".

Z_q (t) is a sequence of independent identically distributed unit normal variables.

Note: This corrects Daykin and Hey's misstatement of the underlying equation (J.I.A. 116 1989).

Values suggested by Wilkie for the above on the basis of 60 years data are: U_q = .05, α_q = 0.6 and σ_q = .05. These have been criticised by Daykin and Hey as producing too many negative inflation observations. For more recent experience they suggest U_q = .07 α_q = 0.6 and σ_q = .03. The author suggests (with 5 low inflation years added to the experience!): U_q = .05, α_q = 0.6 and σ_q = .03.

The average short-term interest rate in any year is assumed to be normally distributed around a mean 2% above current inflation, with standard deviation of 1.5%, making negative interest rates possible, but extremely unlikely.

Loan margin income growth is assumed to be normally distributed around a mean of current inflation plus 2.5%, with standard deviation of 5% per annum. Administration expenses are assumed to be normally distributed about inflation with standard deviation of 2% per annum.

It is assumed that negative taxation is possible, and that taxation is at an annual rate of 38%. The negative taxation possibility assumes that the overall bank does not operate at a loss over prolonged periods, and thus either tax can immediately be reclaimed by offset against other currently profitable areas, or tax losses can be offset against taxable income, after a short time lag. Taxation may be expressed as follows:

$$T(t) = .38\left[L(t-1) - P(t-1) - E(t-1) + R(t-1)A(t-2)\right] \tag{8.3}$$

assuming interest income on reserves subject to tax.

Dividends are assumed payable in arrears, based on (6.8). The basic simulation equation thus becomes:

$$\begin{aligned} A(t) = A(t-1) + R(t)A(t-1) + L(t) - P(t) - D(t-1) - E(t) \\ -.38\left[L(t-1) - P(t-1) - E(t-1) + R(t-1)A(t-2)\right] \end{aligned} \tag{8.4}$$

8.2 Differences in Model Operation

In the previous section, the common features of the proposed simulation model were set out. The differences with respect to current and proposed VAR regulation are only with respect to starting reserve values and definition of insolvency. In other words, exactly identical operating conditions are assumed to apply, regardless of the regulatory framework.

In the case of the existing regulatory framework, **(Model 1),** the starting points for relevant values are as given below:

Reserves :	11% of loan volumes outstanding
Loan margins :	Bank dependent
Provisions :	Bank dependent
Expenses :	0.5% of loan volumes outstanding
Short term interest rate :	6.0%
Dividend :	Bank dependent

Insolvency in this case is defined as reserves at any point in time falling below 8% of loan volumes outstanding.

In the case of the proposed VAR regulatory framework (**Model 2**), the starting points for relevant values are again as given below:

Reserves :

$$1.375 \times \left[2\sum_{1}^{5} \text{Provisions }\% \; - \; (1-t)\sum_{1}^{5} \text{Margins }\% \right] \times \text{Loans currently outstanding}$$

Loan margins :	Bank dependent
Provisions :	Bank dependent
Expenses :	0.5% of loan volumes outstanding
Short term interest rate :	6.0%
Dividend :	Bank dependent

Insolvency in this case is defined as reserves at any point in time falling below:

$$\left[2\sum_{1}^{5}\text{Provisions }\% \;-\; (1-t)\sum_{1}^{5}\text{Margins }\%\right]\times \text{Loans currently outstanding} \qquad (8.5)$$

One now proceeds to set out the simulation results for the chosen range of banks under both models.

8.3 Presentation and Discussion of Results

Table 8.1

Presentation of Results From Simulation of the Credit Process

Bank (a) refers to 20% higher provisions, Bank (b) refers to 20% lower provisions, Bank (c) refers to the original dataset, and Bank (d) refers to 20% higher margins, while Bank (e) refers to 20% lower margins.

Bank	Simulation	Model	Number of insolvencies observed after				Percentage insolvencies observed after			
			5yrs	10yrs	15yrs	20yrs	5yrs	10yrs	15yrs	20yrs
Overall	2500	1	-	10	100	323	-	0.4	4.0	12.9
		2	1	31	86	122	-	1.2	3.4	4.9
(a)	500	1	-	2	17	68	-	0.4	3.4	13.6
		2	-	5	8	14	-	1.0	1.6	2.8
(b)	500	1	-	4	50	119	-	0.8	10.0	23.8
		2	-	7	33	47	-	1.4	6.6	9.4
(c)	500	1	-	1	8	28	-	0.2	1.6	5.6
		2	-	4	13	20	-	0.8	2.6	4.0
(d)	500	1	-	1	7	28	-	0.2	1.4	4.6
		2	1	11	21	28	0.2	2.2	4.1	5.6
(e)	500	1	-	2	18	85	-	0.4	3.6	17.0
		2	-	4	11	13	-	0.8	2.2	2.6

Source: Empirical

A total of 2500 simulations were run, comprising 500 for each of Banks (a) to (e) inclusive. Bank (f) was not included as random variation caused by small size does not lend itself to simulation. Overall, substantially fewer insolvencies were observed to occur in total (122 under the VAR approach as opposed to 323 under the present 8% regulation). The overall 20-year observed insolvency rate was 4.9% for the VAR methodology, versus 12.9% for present regulation. There is a bias present in the simulation which may involve banks paying dividends under the simulation payment formula which causes them to become insolvent under existing regulation, but not under the VAR approach. This bias is estimated at 40% from consideration of the outturn for the Bank (c) simulation, where identical starting capital and operating conditions produce a correspondingly greater percentage insolvencies. Even allowing for this bias, it does appear that the VAR approach produces fewer insolvencies overall than present regulation.

In the earlier years, however, the VAR approach produces higher numbers of insolvencies compared with present regulation. This may be explained by the observation that banks which are under financial strain, either because of high provisions, low margins or a combination of both, are likely to find themselves facing a VAR solvency requirement greater than 8% of loans outstanding as is the case under present regulation. Thus, the need for corrective action may be signalled sooner under the VAR methodology than under present regulation. However, a rising capital requirement as credit conditions disimprove may encourage pro-cyclical behaviour by regulated banks, causing lending to contract as provisions rise and *vice versa*. The extent to which this might occur would be a function of the number of operationally constrained banks within a regulatory system at any point in time and could only be quantified by a regulator.

Examining the individual bank simulations in turn results in the following:

1. Bank (a) (20% higher provisioning rates than standard)

   Under the VAR (Model 2) approach, Bank (a) is equipped with higher initial capital than under the Model 1 approach. Notwithstanding this higher capital, the VAR requirement is more onerous than 8%, producing 5 as opposed to 2 insolvencies after 10 years. Subsequently, lack of profitability causes more Model 1 banks to fail as time progresses, so that after 20 years 13.6% of Model 1 simulations are observed as insolvencies versus 2.8% of Model 2 simulations.

2. Bank (b) (20% lower provisioning rates than standard)

   Intuitively, one would expect fewer failures than the number observed, because under both models, Bank (b) is more profitable than Bank (a). However, this increased profitability permits the payment of substantially higher dividends than under the previous scenario. High dividend payments associated with a VAR requirement lower than 8% precipitates a large number of Model 1 insolvencies, while the over distribution of excess profits earned also increases Model 2 insolvencies.

   It should be pointed out that the dividend formula presented in Chapter 6 is intended to represent a maximum distribution, rather than an automatic entitlement. Prudent management in Bank (b) would restrict dividend payouts while solvency was under threat. A minor adjustment to the maximum permissible dividend could sharply reduce observed insolvencies for Bank (b), but if applied generally, would see no insolvencies in other bank types. As the purpose of these simulations is to observe and compare insolvency experience in all bank types, this result has seen been allowed to stand, even though most unlikely to occur in practice.

3. Bank (c) (Parameters as in original dataset)

Here the evidence of the bias referred to in earlier discussion is seen. Insolvency rates are some 40% higher for Model 1 than for Model 2, notwithstanding identical starting capital and operating assumptions. This is caused by the relationship of dividends to VAR capital, and again may lead Model 1 banks to overdistribute inadvertently. Once again, changing the distribution rules by model type invalidates the observation of identical sample paths, so that this bias cannot be eliminated.

4. Bank (d) (20% higher margins than standard)

As excess earnings may be paid away as dividends, approximately the same number of insolvencies as under the previous scenario is evident. This is the only scenario in which Model 2 does not outperform Model 1. The explanation for this is that lower initial capital under Model 2 is not recouped from retained earnings, because these earnings are fully distributed. Starting with capital of 9.0% of loans outstanding, versus 11.0% for Model 1, Model 2 banks remain at higher risk of insolvency throughout. However, total insolvencies in banks of type (d) are of lower than average proportion, and dividend policy is available to reduce insolvency risk.

5. Bank (e) (20% lower margins than standard)

This shows the widest discrepancy in observed insolvencies of any bank type (85 versus 13). Once again, lack of profitability is to blame, compensated in the case of Model 2, by higher initial capital.

8.4 Interpretation of Findings

The VAR methodology suggested in this thesis has been observed to produce fewer overall insolvencies than present regulation, on a 20 year time horizon. It may be argued, thus, that it represents an improvement on present regulation.

If, however, the simulation were to have run for 10 years only, the VAR methodology would have been observed to produce 3 times as many insolvencies as present regulation, and the opposite argument might hold sway.

The argument in favour of VAR is more subtle than when the simulation clock stops; after all, regulation does not affect banking operations in this specific model, so that one cannot simply claim victory for VAR after 20 years as opposed to defeat after 10 years.

Viewed overall and in a practical setting, the VAR methodology does appear to identify weak banks earlier than present regulation. The differential initial capital associated with VAR permits a larger number of banks to survive the simulation process. From a regulatory standpoint, fewer weak banks identified earlier than otherwise has two main advantages; the ability to concentrate remedial regulatory effort on fewer banks at any point in time, and the knowledge that relatively high residual capital will be available within banks which require their attention due to high provisions, lack of profits or both. The main disadvantages are the possibility of pro-cyclical action by operationally constrained banks, and the over-optimistic reporting of provisions and earnings by the industry, thus reducing its calculated VAR capital.

The former disadvantage has been discussed. The latter has been addressed by the requirement that provisions and margins are calculated over five years. The scope for management of reported earnings (Rajan (1994)) is substantially reduced the longer the timescale over which such management is required. The choice of five years is not accidental; it corresponds to the peak-to-trough duration of the banking cycle (Table 4.1).

From the individual bank viewpoint, Banks (a) to (e) inclusive have much less variable insolvency probability under VAR than under current regulation (Table 8.1). This may mean that a level premium deposit insurance scheme becomes more feasible. Further, VAR allows banks to specialise without requiring substantial regulatory capital which cannot be compensated without increasing risk if such specialisation is at the lower end of the risk return spectrum of private sector banking. Banks are free to price loans in the knowledge of a closer correspondence between economic and regulatory capital than exists at present. Efficiencies might be generated, and informational asymmetries reduced.

Banks are not passive victims in the regulatory process. If the advantages of VAR outweigh its disadvantages in terms of computation, reporting, and reduction of scope for adverse selection, then banks can canvass the regulators for such regulation. The regulator similarly cannot impose regulation by fiat. Thus, VAR is likely only to emerge provided both the banking industry and its regulators are in favour. This section has outlined the advantages and disadvantages from the perspective of each. These advantages and disadvantages are summarised below:

Advantages

Regulator	: Fewer problem banks identified earlier.
	: Concentration of remedial effort.
	: Higher asset excess in problem banks than under current regulations.
Regulated Banks	: Reduction in adverse selection.
	: No capital 'dead-weight'.
	: Ability to specialise.
	: Direct link to economic capital.

Disadvantages

Regulator	: Greater sophistication required.
	: Risk of pro cycle behaviour by regulated banks.
	: Resulting possible increase in systemic risk.
Regulated Banks	: Expense of configuring systems.
	: Greater exposure to 'shocks', i.e. sudden changes in VAR adequacy.

The conclusion of this thesis is now developed in the following chapter.

Chapter 9

Summary, Conclusions, and Limitations

9.1 Summary

The research area addressed in this thesis has been the application of statistical, specifically actuarial, techniques in credit portfolio management. A large credit portfolio has been used to illustrate the development and application of statistical techniques to produce VAR and loan pricing models. A major portion of the work was concerned with the non stationarity of underlying risk, and the subdivision of the portfolio into homogeneous cells with respect to factors already observed to influence risk. This has allowed mathematical reserves to be calculated, permitted their approximation using observable portfolio measures and has been employed to develop both VAR and loan pricing models.

In summary, the focus of the thesis has been in the following areas:

- A statement of the research aims, objectives and methodology.
- An examination of relevant banking and statistical literature.
- A discussion of current industry practice.
- An introduction to the dataset used to illustrate proposals concerning VAR methods.
- A demonstration of the equivalence of actuarial techniques in credit pricing to option based credit pricing, together with some suggested improvements in the latter.
- The parameterisation of a solvency model, based on the dataset, and its development into a value at risk (VAR) model.
- The specification of a loan pricing model, its comparison to existing pricing within the dataset, and the identification of anomalies in existing pricing.
- The simulation of future portfolio states, for both VAR and existing regulatory environments.

The main findings are now summarised in more detail

9.1.1 Value At Risk versus Risk Based Capital

In Chapter 3, risk based capital was shown to be a misnomer in respect of "other" private sector credit. The requirements of a VAR model were identified and discussed in the context of a clear distinction between industry and regulatory definitions of capital.

9.1.2 Strengths and Limitations of Dataset

In Chapter 4, a dataset was introduced which was subsequently used to illustrate numerically the differing models analysed and compared within the study. To the extent that this dataset is broadly representative of its industry (UK bank corporate lending over the years 1990 to 1994), models proposed in this study are potentially generalisable. The latter is reinforced since the dataset was used to develop and test principles (rather than to forecast or test specific hypotheses) using some broadly realistic data. 'Hypothetical' data could also have been used, but the researcher chose to obtain 'real' data for the 'simulation testbed' developed in the study.

9.1.3 Option Pricing of Credit versus Insurance Pricing

In Chapter 5, the equivalence of the two approaches under common assumptions is demonstrated. The expected value approach of the latter enabled a known bias to be eliminated. The existence of a second stochastic process relating to independent security has been explicitly allowed for in loan pricing in a suggested extension of existing theory. This equivalence allows insurance mathematics to be applied directly in the evaluation of bank credit portfolios.

9.1.4 Empirically Parameterised Models

Using historic portfolio experience, a solvency model relevant to long term strategy was constructed. An interesting result is that required reserves compute to a similar order to the operational ratios apparently used in the banking system, so that the actuarial risk theoretic approach developed validates rather than alters existing reserve levels.

The reserves computed were expressed in terms of observable portfolio measures, and subsequently used to illustrate an alternative VAR model of solvency.

9.1.5 Loan Pricing Models

Substantial differences between default costs have been shown to exist in subsections of the portfolio when it is subdivided by factors known to affect risk. The comparison of the loan pricing model to existing charging structures indicated that portfolio composition and pricing are sub-optimal, and that overall returns could be enhanced by the elimination of cross subsidisation.

The bank which provided the dataset has expressed interest in pursuing the research methodology contained herein, specifically with respect to loan pricing, thereby providing some evidence supporting its practical application.

9.1.6 Simulation Results

The simulation of a proposed VAR solvency model compared with existing capital adequacy standards identified substantial differences in operation. Some key potential advantages to both regulators and regulated banks were identified with respect to the proposed model. Disadvantages exist also, and both parties must be satisfied prior to the introduction of a VAR approach that advantages solidly outweigh properly addressed disadvantages.

This thesis cannot conclude that VAR regulation is to be preferred in all circumstances, but it does offer many apparent advantages not possessed by the current industry framework.

9.2 Limitations of the Research

While the thesis has provided an extensive illustration of the applicability of statistical techniques, it should be stressed that the illustration has been by reference to a specific portfolio, although the techniques themselves are well grounded in the literature. One must be cautionary that specific model values cannot be assumed generally applicable; each portfolio must be evaluated individually. Nevertheless it must be re-emphasised that a specific loan portfolio dataset was chosen for the following two reasons:

- The data were made available to the researcher.

- Using the dataset is preferable if the proposals to this thesis are to have practical merit, rather than 'inventing' a simulation testbed (which would itself be an acceptable research approach if these data were not available). As a result, it is the *principles* of the proposed methodology which form the major contribution of this research.

No matter how apparently precise statistical information may be, it cannot be used as a substitute for sound judgement. This thesis advocates VAR and loan pricing models as tools rather than solutions, and does not argue that it has found the answer to all credit problems.

VAR capital gives the manager an estimate of capital requiring remuneration for risk; it does not equip him with the means of its remuneration. Similarly loan pricing models may be priced to remunerate capital at risk regardless of which type of loan is added at the margin; strict adherence may mean that few loans are added to the portfolio.

However, these models can and do provide an insight into where capital is at risk within an organisation, and into how it might be remunerated at least cost for this assumption of risk.

9.3 Areas for Further Research

There remain many important areas where the techniques of this thesis might be extended. Particularly pertinent examples are:

- The investigation of other credit portfolios to produce industry representative models.
- A comparison of actual versus expected portfolio performance over an extended period of time, permitting the application of a RARORAC loan pricing approach, and the further refinement of control techniques.
- Portfolio optimisation by reference to asset swapping/sharing.
- Testing these proposals and gaining feedback from practicing loan officers in banks.

In peroration, this thesis has established the relevance of statistical techniques to the management of solvency and pricing, and may equip bank management with an effective strategic tool. This contribution has been made in a challenging and significant area of research and there are other areas of exploration as defined briefly above.

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APPENDIX A

Historic Income, Default Cost and Gross Profitability of Selected Bank Dataset

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provisioning Cost	Net Margin Income	Net Margin Income Per Pound Lent
1-1-1	1	Q3 93	1,310	13,101,963	85,234	0	85,234	0.6505%
1-1-1	1	Q4 93	1,468	14,571,568	94,354	0	94,354	0.6475%
1-1-1	1	Q1 94	1,535	15,043,652	96,572	-13,249	109,821	0.7300%
1-1-1	1	Q2 94	1,168	11,358,481	77,527	0	77,527	0.6826%
1-1-1	1	Q3 94	1,090	10,342,219	77,207		77,207	0.7465%
1-1-1	1	Q4 94	1,332	12,638,421	90,710		90,710	0.7177%
1-1-1	1	Q1 95	1,467	13,483,663	98,007		98,007	0.7269%
1-1-1	2	Q3 93	539	8,220,045	53,953	-8	53,961	0.6565%
1-1-1	2	Q4 93	557	8,411,580	54,657	0	54,657	0.6498%
1-1-1	2	Q1 94	550	8,148,659	52,931		52,931	0.6496%
1-1-1	2	Q2 94	490	7,186,906	48,427		48,427	0.6738%
1-1-1	2	Q3 94	493	7,172,861	50,841		50,841	0.7088%
1-1-1	2	Q4 94	574	8,175,723	57,054		57,054	0.6978%
1-1-1	2	Q1 95	590	8,476,697	60,399		60,399	0.7125%
1-1-1	3	Q3 93	304	6,860,599	47,532	0	47,532	0.6928%
1-1-1	3	Q4 93	336	7,683,825	51,143		51,143	0.6656%
1-1-1	3	Q1 94	346	7,882,050	52,601		52,601	0.6674%
1-1-1	3	Q2 94	294	6,776,105	46,847		46,847	0.6914%
1-1-1	3	Q3 94	297	6,816,572	49,956		49,956	0.7329%
1-1-1	3	Q4 94	335	7,645,389	55,341		55,341	0.7238%
1-1-1	3	Q1 95	343	7,716,484	56,114		56,114	0.7272%
1-1-1	4	Q3 93	352	11,260,905	77,468	0	77,468	0.6879%
1-1-1	4	Q4 93	400	12,774,468	90,490	-852	91,342	0.7150%
1-1-1	4	Q1 94	420	13,190,524	96,754	0	96,754	0.7335%
1-1-1	4	Q2 94	375	11,533,569	86,566		86,566	0.7506%
1-1-1	4	Q3 94	361	11,313,879	86,511		86,511	0.7646%
1-1-1	4	Q4 94	425	13,193,059	100,169		100,169	0.7593%
1-1-1	4	Q1 95	454	13,784,598	102,869		102,869	0.7463%
1-1-1	5	Q3 93	323	12,087,705	83,127		83,127	0.6877%
1-1-1	5	Q4 93	356	13,159,837	90,994	0	90,994	0.6914%
1-1-1	5	Q1 94	377	13,995,733	98,616		98,616	0.7046%
1-1-1	5	Q2 94	329	12,542,801	90,657		90,657	0.7228%
1-1-1	5	Q3 94	328	11,952,394	89,731	-1,737	91,468	0.7653%
1-1-1	5	Q4 94	351	12,356,462	95,797	0	95,797	0.7753%
1-1-1	5	Q1 95	357	12,210,802	94,546		94,546	0.7743%
1-1-1	6	Q3 93	85	3,419,486	24,411		24,411	0.7139%

1-1-1	6	Q4 93	86	3,314,958	24,311	0	24,311	0.7334%
1-1-1	6	Q1 94	82	3,262,431	22,816		22,816	0.6993%
1-1-1	6	Q2 94	70	2,706,412	18,972		18,972	0.7010%
1-1-1	6	Q3 94	60	2,265,924	15,763		15,763	0.6957%
1-1-1	6	Q4 94	49	1,911,211	12,855		12,855	0.6726%
1-1-1	6	Q1 95	42	1,736,658	12,080		12,080	0.6956%
1-1-1	7	Q3 93	5	279,751	2,261		2,261	0.8082%
1-1-1	7	Q4 93	5	320,325	2,515		2,515	0.7850%
1-1-1	7	Q1 94	3	185,931	1,235		1,235	0.6641%
1-1-1	7	Q2 94	3	232,058	1,557		1,557	0.6710%
1-1-1	7	Q3 94	5	263,804	1,912		1,912	0.7248%
1-1-1	7	Q4 94	4	208,281	1,565		1,565	0.7514%
1-1-1	7	Q1 95	4	202,586	1,476		1,476	0.7286%
1-1-1	8	Q3 93	41	1,879,186	17,797	92	17,706	0.9422%
1-1-1	8	Q4 93	37	1,558,652	15,297	-662	15,959	1.0239%
1-1-1	8	Q1 94	34	1,440,059	13,783	0	13,783	0.9571%
1-1-1	8	Q2 94	15	607,509	5,467		5,467	0.8999%
1-1-1	8	Q3 94	14	527,598	4,813		4,813	0.9122%
1-1-1	8	Q4 94	18	625,980	6,300		6,300	1.0064%
1-1-1	8	Q1 95	16	543,951	5,593		5,593	1.0283%

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provision-ing Cost	Net Margin Income	Net Margin Income Per Pound Lent
1-1-2	1	Q3 93	513	5,099,377	39,329	0	39,329	0.7713%
1-1-2	1	Q4 93	558	5,560,600	43,377	812	42,565	0.7655%
1-1-2	1	Q1 94	598	5,969,370	46,636		46,636	0.7813%
1-1-2	1	Q2 94	928	9,180,484	66,972		66,972	0.7295%
1-1-2	1	Q3 94	1,217	11,840,889	90,972		90,972	0.7683%
1-1-2	1	Q4 94	1,504	14,175,168	105,515	0	105,515	0.7444%
1-1-2	1	Q1 95	1,712	15,751,295	117,738	-216	117,954	0.7489%
1-1-2	2	Q3 93	924	13,693,222	106,555	-765	107,320	0.7837%
1-1-2	2	Q4 93	1,019	14,856,371	117,528	517	117,011	0.7876%
1-1-2	2	Q1 94	1,057	15,446,605	121,578	-2,464	124,042	0.8030%
1-1-2	2	Q2 94	1,079	15,767,139	121,663	-34,624	156,287	0.9912%
1-1-2	2	Q3 94	1,135	16,406,457	134,224	4,513	129,711	0.7906%
1-1-2	2	Q4 94	1,331	18,908,695	156,163	0	156,163	0.8259%
1-1-2	2	Q1 95	1,404	19,979,172	165,918	0	165,918	0.8305%
1-1-2	3	Q3 93	972	22,931,492	186,086	0	186,086	0.8115%
1-1-2	3	Q4 93	1,069	24,686,872	199,260	-706	199,966	0.8100%
1-1-2	3	Q1 94	1,101	24,997,518	201,467	668	200,799	0.8033%
1-1-2	3	Q2 94	1,083	24,299,216	198,445	-137	198,582	0.8172%
1-1-2	3	Q3 94	1,090	24,045,328	204 006	-1	204,007	0.8484%
1-1-2	3	Q4 94	1,180	26,280,360	224,841	10,136	214,705	0.8170%
1-1-2	3	Q1 95	1,192	26,784,174	231,813	-1,601	233,414	0.8715%
1-1-2	4	Q3 93	2,215	69,829,521	589,388	1,476	587,912	0.8419%
1-1-2	4	Q4 93	2,413	75,722,788	637,298	-76,222	713,520	0.9423%
1-1-2	4	Q1 94	2,386	74,393,830	620,933	5,960	614,973	0.8266%
1-1-2	4	Q2 94	2,134	66,828,456	555,339	4,934	550,405	0.8236%
1-1-2	4	Q3 94	2,139	66,464,136	582,926	-4,829	587,755	0.8843%
1-1-2	4	Q4 94	2,353	73,074,889	650,455	35	650,420	0.8901%
1-1-2	4	Q1 95	2,336	72,825,462	648,650	-60,425	709,075	0.9737%
1-1-2	5	Q3 93	2,899	112,647,074	978,418	266,817	711,601	0.6317%
1-1-2	5	Q4 93	3,099	119,390,449	1,025,669	24,134	1,001,535	0.8389%
1-1-2	5	Q1 94	3,143	120,564,787	1,015,991	43,229	972,762	0.8068%
1-1-2	5	Q2 94	2,849	110,602,536	929,806	21,640	908,166	0.8211%
1-1-2	5	Q3 94	2,824	108,949,358	974,334	-11,519	985,853	0.9049%
1-1-2	5	Q4 94	3,072	116,629,695	1,066,551	-28,129	1,094,680	0.9386%
1-1-2	5	Q1 95	2,958	111,464,494	1,019,669	439	1,019,230	0.9144%

1-1-2	6	Q3 93	2,883	119,141,117	1,074,992	26,652	1,048,340	0.8799%
1-1-2	6	Q4 93	3,057	125,483,318	1,117,270	-124,128	1,241,398	0.9893%
1-1-2	6	Q1 94	3,054	125,016,881	1,110,584	-64,017	1,174,601	0.9396%
1-1-2	6	Q2 94	2,668	109,466,548	980,817	16,035	964,782	0.8813%
1-1-2	6	Q3 94	2,729	111,119,339	1,076,419	-26,250	1,102,669	0.9923%
1-1-2	6	Q4 94	3,074	123,576,516	1,217,975	49,678	1,168,297	0.9454%
1-1-2	6	Q1 95	3,141	125,420,423	1,194,531	-104,902	1,299,433	1 0361%
1-1-2	7	Q3 93	2,639	112,503,265	1,023,721	-24,734	1,048,455	0.9319%
1-1-2	7	Q4 93	2,808	118,455,046	1,078,168	-26,707	1,104,875	0.9327%
1-1-2	7	Q1 94	2,762	116,635,315	1,065,065	-45,686	1,110,751	0.9523%
1-1-2	7	Q2 94	2,402	102,029,318	951,569	-55,058	1,006,627	0.9866%
1-1-2	7	Q3 94	2,613	110,860,293	1,125,952	-34,560	1,160,512	1 0468%
1-1-2	7	Q4 94	3,019	126,254,994	1,288,977	8,995	1,279,982	1.0138%
1-1-2	7	Q1 95	3,097	127,334,406	1,303,417	10,116	1,293,301	1.0157%
1-1-2	8	Q3 93	1,686	74,225,797	708,080	109,008	599,072	0.8071%
1-1-2	8	Q4 93	1,585	69,270,150	659,347	88,519	570,828	0.8241%
1-1-2	8	Q1 94	1,465	63,812,448	604,227	-1,025	605,252	0.9485%
1-1-2	8	Q2 94	1,123	48,344,606	468,116	29,198	438,918	0.9079%
1-1-2	8	Q3 94	1,339	58,036,569	619,651	-731	620,382	1.0689%
1-1-2	8	Q4 94	1,613	69,224,020	716,225	-40,703	756,928	1.0934%
1-1-2	8	Q1 95	1,687	71,330,447	747,833	-145,769	893,602	1.2528%

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provision-ing Cost	Net Margin Income	Net Margin Income Per Pound Lent
1-2-1	1	Q3 93	624	6,654,406	41,797	0	41,797	0.6281%
1-2-1	1	Q4 93	701	7,269,392	44,832		44,832	0.6167%
1-2-1	1	Q1 94	738	7,506,560	46,265	0	46,265	0.6163%
1-2-1	1	Q2 94	633	6,233,136	41,093		41,093	0.6593%
1-2-1	1	Q3 94	660	6,361,266	45,761		45,761	0.7194%
1-2-1	1	Q4 94	871	8,250,422	57,485		57,485	0.6968%
1-2-1	1	Q1 95	1,058	9,796,576	69,722		69,722	0.7117%
1-2-1	2	Q3 93	335	5,207,910	34,251		34,251	0.6577%
1-2-1	2	Q4 93	361	5,539,260	36,357		36,357	0.6564%
1-2-1	2	Q1 94	372	5,749,990	37,768		37,768	0.6568%
1-2-1	2	Q2 94	354	5,480,103	36,381		36,381	0.6639%
1-2-1	2	Q3 94	384	5,770,305	39,928		39,928	0.6920%
1-2-1	2	Q4 94	497	7,334,594	50,644		50,644	0.6905%
1-2-1	2	Q1 95	616	8,866,963	61,434		61,434	0.6928%
1-2-1	3	Q3 93	183	4,349,693	29,296	0	29,296	0.6735%
1-2-1	3	Q4 93	218	5,216,112	34,710	0	34,710	0 6654%
1-2-1	3	Q1 94	226	5,475,129	35,160		35,160	0.6422%
1-2-1	3	Q2 94	220	5,446,810	35,956		35,956	0.6601%
1-2-1	3	Q3 94	244	6,038,150	43,318		43,318	0.7174%
1-2-1	3	Q4 94	326	7,776,673	56,545	-841	57,386	0.7379%
1-2-1	3	Q1 95	375	8,569,162	63,354	0	63,354	0 7393%
1-2-1	4	Q3 93	291	9,659,308	67,003		67,003	0.6937%
1-2-1	4	Q4 93	324	10,770,809	76,158	0	76,158	0.7071%
1-2-1	4	Q1 94	328	10,981,417	78,587		78,587	0.7156%
1-2-1	4	Q2 94	319	10,537,332	74,618		74,618	0.7081%
1-2-1	4	Q3 94	359	11,733,352	88,825		88,825	0.7570%
1-2-1	4	Q4 94	460	15,044,843	113,970		113,970	0.7575%
1-2-1	4	Q1 95	548	17,546,527	133,168		133,168	0.7589%
1-2-1	5	Q3 93	289	12,159,404	85,803	971	84 832	0.6977%
1-2-1	5	Q4 93	327	13,704,611	96,305	0	96,305	0.7027%
1-2-1	5	Q1 94	332	13,675,066	95,214		95,214	0.6963%
1-2-1	5	Q2 94	298	12,081,530	85,676		85,676	0.7091%
1-2-1	5	Q3 94	315	12,267,943	94,771		94,771	0.7725%
1-2-1	5	Q4 94	375	14,229,464	110,984		110,984	0.7800%
1-2-1	5	Q1 95	426	15,472,500	120,524	0	120,524	0.7790%

1-2-1	6	Q3 93	87	4,010,805	33,502		33,502	0.8353%
1-2-1	6	Q4 93	91	4,188,739	34,126	0	34,126	0.8147%
1-2-1	6	Q1 94	92	4,111,262	33,050		33,050	0.8039%
1-2-1	6	Q2 94	88	4,040,959	31,896		31,896	0.7893%
1-2-1	6	Q3 94	81	3,739,206	28,158		28,158	0.7530%
1-2-1	6	Q4 94	79	3,573,201	26,856		26,856	0.7516%
1-2-1	6	Q1 95	66	2,754,847	22,024		22,024	0.7995%
1-2-1	7	Q3 93	6	230,207	2,185		2,185	0.9492%
1-2-1	7	Q4 93	5	203,300	1,748		1,748	0.8598%
1-2-1	7	Q1 94	6	235,190	1,829		1,829	0.7775%
1-2-1	7	Q2 94	3	138,950	784		784	0.5644%
1-2-1	7	Q3 94	3	150,909	1,252		1,252	0.8296%
1-2-1	7	Q4 94	5	229,353	1,975		1,975	0.8613%
1-2-1	7	Q1 95	4	222,692	2,001		2,001	0.8987%
1-2-1	8	Q3 93	28	1,375,468	12,909		12,909	0.9385%
1-2-1	8	Q4 93	23	1,075,203	9,843		9,843	0.9154%
1-2-1	8	Q1 94	22	949,350	8,262		8,262	0.8703%
1-2-1	8	Q2 94	18	810,487	6,672		6,672	0.8232%
1-2-1	8	Q3 94	18	702,782	5,808		5,808	0.8264%
1-2-1	8	Q4 94	22	844,795	6,763		6,763	0.8005%
1-2-1	8	Q1 95	26	1,097,092	8,036		8,036	0.7325%

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provision-ing Cost	Net Margin Income	Net Margin Income Per Pound Lent
1-2-2	1	Q3 93	218	2,296,326	17,623		17,623	0.7675%
1-2-2	1	Q4 93	265	2,729,842	20,773		20,773	0.7610%
1-2-2	1	Q1 94	277	2,826,627	21,801		21,801	0.7713%
1-2-2	1	Q2 94	431	4,359,022	30,371	0	30,371	0.6967%
1-2-2	1	Q3 94	604	5,995,368	43,566		43,566	0.7267%
1-2-2	1	Q4 94	812	7,795,737	55,896		55,896	0.7170%
1-2-2	1	Q1 95	991	9,199,696	65,563	0	65,563	0.7127%
1-2-2	2	Q3 93	456	7,031,411	51,715	-879	52,594	0.7480%
1-2-2	2	Q4 93	522	7,959,612	57,630	0	57,630	0.7240%
1-2-2	2	Q1 94	572	8,756,915	66,156		66,156	0.7555%
1-2-2	2	Q2 94	615	9,132,298	69,508		69,508	0.7611%
1-2-2	2	Q3 94	694	10,334,782	80,571		80,571	0.7796%
1-2-2	2	Q4 94	862	12,673,451	100,046	-113,372	213,418	1.6840%
1-2-2	2	Q1 95	965	13,817,246	109,354	0	109,354	0.7914%
1-2-2	3	Q3 93	536	13,213,127	104,143		104,143	0.7882%
1-2-2	3	Q4 93	590	14,694,148	114,979		114,979	0.7825%
1-2-2	3	Q1 94	604	14,917,533	115,040		115,040	0.7712%
1-2-2	3	Q2 94	604	14,783,648	115,312		115,312	0.7800%
1-2-2	3	Q3 94	650	15,536,264	127,662	0	127,662	0.8217%
1-2-2	3	Q4 94	808	18,852,541	155,772		155,772	0.8263%
1-2-2	3	Q1 95	947	21,423,886	173,840		173,840	0.8114%
1-2-2	4	Q3 93	1,191	40,434,450	323,685	0	323,685	0.8005%
1-2-2	4	Q4 93	1,290	43,814,310	347,019	2,046	344,973	0.7874%
1-2-2	4	Q1 94	1,324	44,676,594	354,945	0	354,945	0.7945%
1-2-2	4	Q2 94	1,303	43,899,743	346,766	0	346,766	0.7899%
1-2-2	4	Q3 94	1,403	46,469,353	385,662		385,662	0.8299%
1-2-2	4	Q4 94	1,735	56,363,961	471,627	0	471,627	0.8368%
1-2-2	4	Q1 95	1,950	61,786,761	516,991	0	516,991	0.8367%
1-2-2	5	Q3 93	1,584	65,705,389	548,579	-4,500	553,079	0.8418%
1-2-2	5	Q4 93	1,752	72,153,918	601,378	0	601,378	0.8335%
1-2-2	5	Q1 94	1,803	74,514,364	625,980	0	625,980	0.8401%
1-2-2	5	Q2 94	1,679	69,385,932	588,838		588,838	0.8486%
1-2-2	5	Q3 94	1,776	73,034,376	647,964		647,964	0.8872%
1-2-2	5	Q4 94	2,093	83,879,992	747,365	2,000	745,365	0.8886%
1-2-2	5	Q1 95	2,323	90,746,171	796,733	0	796,733	0.8780%

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provisioning Cost	Net Margin Income	Net Margin Income Per Pound Lent
1-2-2	1	Q3 93	218	2,296,326	17,623		17,623	0.7675%
1-2-2	1	Q4 93	265	2,729,842	20,773		20,773	0.7610%
1-2-2	1	Q1 94	277	2,826,627	21,801		21,801	0.7713%
1-2-2	1	Q2 94	431	4,359,022	30,371	0	30,371	0.6967%
1-2-2	1	Q3 94	604	5,995,368	43,566		43,566	0.7267%
1-2-2	1	Q4 94	812	7,795,737	55,896		55,896	0.7170%
1-2-2	1	Q1 95	991	9,199,696	65,563	0	65,563	0.7127%
1-2-2	2	Q3 93	456	7,031,411	51,715	-879	52,594	0.7480%
1-2-2	2	Q4 93	522	7,959,612	57,630	0	57,630	0.7240%
1-2-2	2	Q1 94	572	8,756,915	66,156		66,156	0.7555%
1-2-2	2	Q2 94	615	9,132,298	69,508		69,508	0.7611%
1-2-2	2	Q3 94	694	10,334,782	80,571		80,571	0.7796%
1-2-2	2	Q4 94	862	12,673,451	100,046	-113,372	213,418	1.6840%
1-2-2	2	Q1 95	965	13,817,246	109,354	0	109,354	0.7914%
1-2-2	3	Q3 93	536	13,213,127	104,143		104,143	0.7882%
1-2-2	3	Q4 93	590	14,694,148	114,979		114,979	0.7825%
1-2-2	3	Q1 94	604	14,917,533	115,040		115,040	0.7712%
1-2-2	3	Q2 94	604	14,783,648	115,312		115,312	0.7800%
1-2-2	3	Q3 94	650	15,536,264	127,662	0	127,662	0.8217%
1-2-2	3	Q4 94	808	18,852,541	155,772		155,772	0.8263%
1-2-2	3	Q1 95	947	21,423,886	173,840		173,840	0.8114%
1-2-2	4	Q3 93	1,191	40,434,450	323,685	0	323,685	0.8005%
1-2-2	4	Q4 93	1,290	43,814,310	347 019	2,046	344,973	0.7874%
1-2-2	4	Q1 94	1,324	44,676,594	354,945	0	354,945	0.7945%
1-2-2	4	Q2 94	1,303	43,899,743	346,766	0	346,766	0.7899%
1-2-2	4	Q3 94	1,403	46,469,353	385,662		385,662	0.8299%
1-2-2	4	Q4 94	1,735	56,363,961	471,627	0	471,627	0.8368%
1-2-2	4	Q1 95	1,950	61,786,761	516,991	0	516,991	0.8367%
1-2-2	5	Q3 93	1,584	65,705,389	548,579	-4,500	553,079	0.8418%
1-2-2	5	Q4 93	1,752	72,153,918	601,378	0	601,378	0.8335%
1-2-2	5	Q1 94	1,803	74,514,364	625,980	0	625,980	0.8401%
1-2-2	5	Q2 94	1,679	69,385,932	588,838		588,838	0.8486%
1-2-2	5	Q3 94	1,776	73,034,376	647,964		647,964	0.8872%
1-2-2	5	Q4 94	2,093	83,879,992	747,365	2,000	745,365	0.8886%
1-2-2	5	Q1 95	2,323	90,746,171	796,733	0	796,733	0.8780%

1-2-2	6	Q3 93	1,604	72,221,672	625,715	-9,918	635,633	0.8801%
1-2-2	6	Q4 93	1,740	77,472,894	659,406	-22,596	682,002	0.8803%
1-2-2	6	Q1 94	1,824	80,520,754	681,814	-1,844	683,658	0.8490%
1-2-2	6	Q2 94	1,717	75,732,235	646,434	2,496	643,938	0.8503%
1-2-2	6	Q3 94	1,784	76,745,427	703,984	0	703,984	0.9173%
1-2-2	6	Q4 94	2,158	90,974,137	826,542	4,352	822,190	0.9038%
1-2-2	6	Q1 95	2,413	99,155,362	888,704	0	888,704	0.8963%
1-2-2	7	Q3 93	1,475	66,871,474	591,376	-1,278	592,654	0.8863%
1-2-2	7	Q4 93	1,586	72,023,581	634,686	42,419	592,267	0.8223%
1-2-2	7	Q1 94	1,582	71,644,032	627,242	-14,497	641,739	0.8957%
1-2-2	7	Q2 94	1,411	64,128,445	568,921	-6,102	575,023	0.8967%
1-2-2	7	Q3 94	1,579	70,058,802	679,847	5,983	673,864	0.9619%
1-2-2	7	Q4 94	1,925	83,932,605	796,231	3,913	792,318	0.9440%
1-2-2	7	Q1 95	2,175	92,464,741	873,421		873,421	0.9446%
1-2-2	8	Q3 93	977	45,782,974	427,337	-23,298	450,635	0.9843%
1-2-2	8	Q4 93	925	42,849,914	398,981	26,894	372,087	0.8684%
1-2-2	8	Q1 94	886	41,318,187	382,079	-28,542	410,621	0.9938%
1-2-2	8	Q2 94	713	32,744,767	304,176	3,633	300,543	0.9178%
1-2-2	8	Q3 94	854	39,178,245	397,426	0	397,426	1.0144%
1-2-2	8	Q4 94	1,051	47,343,820	466,008	0	466,008	0.9843%
1-2-2	8	Q1 95	1,184	52,976,800	526,076	0	526,076	0.9930%

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provision-ing Cost	Net Margin Income	Net Margin Income Per Pound Lent
2-1-1	1	Q3 93	1,508	383,356,857	1,091,194		1,091,194	0.2846%
2-1-1	1	Q4 93	1,603	421,026,452	1,120,017		1,120,017	0.2660%
2-1-1	1	Q1 94	1,617	478,510,005	1,248,079	1,721	1,246,358	0.2605%
2-1-1	1	Q2 94	1,316	440,340,035	1,246,161	0	1,246,161	0.2830%
2-1-1	1	Q3 94	1,253	558,784,389	1,564,601	-1,912	1,566,513	0.2803%
2-1-1	1	Q4 94	1,413	604,895,462	1,519,958	0	1,519,958	0.2513%
2-1-1	1	Q1 95	1,417	610,065,552	1,517,726	0	1,517,726	0.2488%
2-1-1	2	Q3 93	631	299,039,066	758,738	11,426	747,312	0.2499%
2-1-1	2	Q4 93	637	297,042,099	755,630	0	755,630	0.2544%
2-1-1	2	Q1 94	640	315,124,386	793,908	0	793,908	0.2519%
2-1-1	2	Q2 94	566	318,991,592	774,907		774,907	0.2429%
2-1-1	2	Q3 94	563	300,031,522	862,104		862,104	0.2873%
2-1-1	2	Q4 94	624	377,961,055	1,004,883		1,004,883	0.2659%
2-1-1	2	Q1 95	623	421,077,665	1,103,183		1,103,183	0.2620%
2-1-1	3	Q3 93	348	268,280,434	755,474	8,976	746,498	0.2783%
2-1-1	3	Q4 93	374	236,687,224	713,630	0	713,630	0.3015%
2-1-1	3	Q1 94	366	240,923,551	665,741		665,741	0.2763%
2-1-1	3	Q2 94	320	217,219,570	611,047		611,047	0.2813%
2-1-1	3	Q3 94	329	260,032,427	755,022		755,022	0.2904%
2-1-1	3	Q4 94	344	266,859,637	656,508		656,508	0.2460%
2-1-1	3	Q1 95	329	256,649,241	618,730		618,730	0.2411%
2-1-1	4	Q3 93	391	259,926,520	874,708		874,708	0.3365%
2-1-1	4	Q4 93	405	269,721,800	865,862		865,862	0.3210%
2-1-1	4	Q1 94	406	326,128,553	953,149		953,149	0.2923%
2-1-1	4	Q2 94	362	328,901,442	978,033		978,033	0.2974%
2-1-1	4	Q3 94	354	339,811,669	1,025,268		1,025,268	0.3017%
2-1-1	4	Q4 94	397	344,473,116	1,106,040		1,106,040	0.3211%
2-1-1	4	Q1 95	355	335,960,995	1,058,842		1,058,842	0.3152%
2-1-1	5	Q3 93	263	224,090,387	665,729	0	665,729	0.2971%
2-1-1	5	Q4 93	279	206,706,032	622,589	0	622,589	0.3012%
2-1-1	5	Q1 94	283	171,483,824	554,276	18,713	535,563	0.3123%
2-1-1	5	Q2 94	258	139,984,617	470,730		470,730	0.3363%
2-1-1	5	Q3 94	229	142,509,371	499,309		499,309	0.3504%
2-1-1	5	Q4 94	239	142,920,902	510,483		510,483	0.3572%
2-1-1	5	Q1 95	211	123,125,994	473,801		473,801	0.3848%

2-1-1	6	Q3 93	55	16,177,163	81,075		81,075	0.5012%
2-1-1	6	Q4 93	58	18,721,817	90,298		90,298	0.4823%
2-1-1	6	Q1 94	57	17,243,979	87,674		87,674	0.5084%
2-1-1	6	Q2 94	51	16,125,879	83,904		83,904	0.5203%
2-1-1	6	Q3 94	40	10,994,081	60,738		60,738	0.5525%
2-1-1	6	Q4 94	33	11,806,216	51,998		51,998	0.4404%
2-1-1	6	Q1 95	26	15,380,918	51,427		51,427	0.3344%
2-1-1	7	Q3 93	2	313,491	3,076		3,076	0.9813%
2-1-1	7	Q4 93	2	351,361	3,455		3,455	0.9833%
2-1-1	7	Q1 94	2	209,848	2,937		2,937	1.3994%
2-1-1	7	Q2 94	2	307,913	3,763		3,763	1.2221%
2-1-1	7	Q3 94	1	96,018	1,560		1,560	1.6250%
2-1-1	7	Q4 94	2	202,793	534		534	0.2633%
2-1-1	7	Q1 95	2	170,787	534		534	0.3126%
2-1-1	8	Q3 93	11	3,399,870	23,644		23,644	0.6954%
2-1-1	8	Q4 93	9	2,661,688	17,156		17,156	0.6445%
2-1-1	8	Q1 94	6	1,315,415	8,592		8,592	0.6532%
2-1-1	8	Q2 94	3	624,809	4,013		4,013	0.6423%
2-1-1	8	Q3 94	3	596,258	4,631		4,631	0.7766%
2-1-1	8	Q4 94	4	702,901	5,754		5,754	0.8186%
2-1-1	8	Q1 95	9	2,976,898	19,757		19,757	0.6637%

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provision-ing Cost	Net Margin Income	Net Margin Income Per Pound Lent
2-1-2	1	Q3 93	504	100,552,650	345,026		345,026	0.3431%
2-1-2	1	Q4 93	545	92,647,785	334,353		334,353	0.3609%
2-1-2	1	Q1 94	552	96,418,521	349,317		349,317	0.3623%
2-1-2	1	Q2 94	832	160,910,213	506,205		506,205	0.3146%
2-1-2	1	Q3 94	1,065	197,355,488	659,234		659,234	0.3340%
2-1-2	1	Q4 94	1,198	235,869,805	723,338		723,338	0.3067%
2-1-2	1	Q1 95	1,244	251,637,847	772,760		772,760	0.3071%
2-1-2	2	Q3 93	952	213,076,864	823,236	-848	824,083	0.3868%
2-1-2	2	Q4 93	1,009	214,548,130	809,645	0	809,645	0.3774%
2-1-2	2	Q1 94	1,064	220,746,991	885,138	0	885,138	0.4010%
2-1-2	2	Q2 94	1,090	240,257,737	954,634		954,634	0.3973%
2-1-2	2	Q3 94	1,121	255,699,104	1,034,804		1,034,804	0.4047%
2-1-2	2	Q4 94	1,226	313,374,241	1,173,468		1,173,468	0.3745%
2-1-2	2	Q1 95	1,171	311,491,613	1,134,960		1,134,960	0.3644%
2-1-2	3	Q3 93	1,071	365,936,154	1,547,936	-1,403	1,549,338	0.4234%
2-1-2	3	Q4 93	1,150	389,174,219	1,593,496	14,112	1,579,384	0.4058%
2-1-2	3	Q1 94	1,195	432,338,687	1,784,300	-15,383	1,799,683	0.4163%
2-1-2	3	Q2 94	1,141	417,330,340	1,696,683	502	1,696,181	0.4064%
2-1-2	3	Q3 94	1,185	491,051,123	2,039,351	0	2,039,351	0.4153%
2-1-2	3	Q4 94	1,307	509,491,069	2,092,255	-16,453	2,108,708	0.4139%
2-1-2	3	Q1 95	1,279	505,219,292	2,008,528	0	2,008,528	0.3976%
2-1-2	4	Q3 93	2,527	998,362,301	4,524,636	-11,073	4,535,709	0.4543%
2-1-2	4	Q4 93	2,607	1,061,793,770	4,900,201	-10,101	4,910,302	0.4625%
2-1-2	4	Q1 94	2,628	1,144,290,568	5,321,418	-60,389	5,381,807	0.4703%
2-1-2	4	Q2 94	2,504	1,188,139,174	5,416,823	36,111	5,380,712	0.4529%
2-1-2	4	Q3 94	2,517	1,195,192,221	5,403,827	-47,858	5,451,685	0.4561%
2-1-2	4	Q4 94	2,643	1,261,081,028	5,577,066	-13,799	5,590,865	0.4433%
2-1-2	4	Q1 95	2,530	1,162,881,109	5,228,865	17,379	5,211,486	0.4482%
2-1-2	5	Q3 93	2,951	1,327,998,424	6,573,047	1,062,254	5,510,794	0.4150%
2-1-2	5	Q4 93	3,044	1,331,102,408	6,783,578	-192,409	6,975,987	0.5241%
2-1-2	5	Q1 94	3,006	1,313,312,236	6,770,588	69,044	6,701,544	0.5103%
2-1-2	5	Q2 94	2,722	1,270,531,506	6,351,166	312,093	6,039,073	0.4753%
2-1-2	5	Q3 94	2,711	1,254,296,046	6,404,860	-199,095	6,603,955	0.5265%
2-1-2	5	Q4 94	2,881	1,317,923,265	6,700,500	-751,841	7,452,341	0.5655%
2-1-2	5	Q1 95	2,794	1,308,688,939	6,726,935	-53,950	6,780,885	0.5181%

2-1-2	6	Q3 93	2,517	930,702,148	5,719,812	276,098	5,443,715	0.5849%
2-1-2	6	Q4 93	2,529	938,919,419	5,600,062	-136,581	5,736,643	0.6110%
2-1-2	6	Q1 94	2,468	910,380,631	5,348,613	74,102	5,274,511	0.5794%
2-1-2	6	Q2 94	2,160	829,054,076	4,816,211	1,771,159	3,045,052	0.3673%
2-1-2	6	Q3 94	2,048	734,551,189	4,424,232	-215,985	4,640,217	0.6317%
2-1-2	6	Q4 94	2,092	700,899,428	4,299,756	165,471	4,134,285	0.5899%
2-1-2	6	Q1 95	1,961	573,724,372	3,661,802	-40,236	3,702,038	0.6453%
2-1-2	7	Q3 93	1,601	528,530,698	3,434,710	2,600,456	834,254	0.1578%
2-1-2	7	Q4 93	1,618	540,457,465	3,478,999	292,542	3,186,457	0.5896%
2-1-2	7	Q1 94	1,550	543,670,966	3,442,696	-219,103	3,661,799	0.6735%
2-1-2	7	Q2 94	1,294	447,149,747	2,875,894	-5,034,834	7,910,728	1.7691%
2-1-2	7	Q3 94	1,362	470,462,963	3,131,049	600,307	2,530,742	0.5379%
2-1-2	7	Q4 94	1,463	496,092,354	3,315,436	430,132	2,885,304	0.5816%
2-1-2	7	Q1 95	1,446	469,630,873	3,092,835	-253,238	3,346,073	0.7125%
2-1-2	8	Q3 93	889	437,597,979	2,710,227	254,685	2,455,542	0.5611%
2-1-2	8	Q4 93	813	421,785,015	2,608,386	321,361	2,287,025	0.5422%
2-1-2	8	Q1 94	749	385,277,410	2,336,906	323,273	2,013,633	0.5226%
2-1-2	8	Q2 94	555	257,436,027	1,660,712	76,975	1,583,737	0.6152%
2-1-2	8	Q3 94	635	296,070,932	1,924,919	374,866	1,550,053	0.5235%
2-1-2	8	Q4 94	738	302,427,015	2,059,518	1,330,336	729,182	0.2411%
2-1-2	8	Q1 95	750	277,619,668	1,957,984	3,520,530	-1,562,546	-0.5628%

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provisioning Cost	Net Margin Income	Net Margin Income Per Pound Lent
2-2-1	1	Q3 93	681	61,850,701	254,441		254,441	0.4114%
2-2-1	1	Q4 93	716	68,627,445	275,769	0	275,769	0.4018%
2-2-1	1	Q1 94	731	79,246,071	308,155		308,155	0.3889%
2-2-1	1	Q2 94	601	70,155,077	282,807		282,807	0.4031%
2-2-1	1	Q3 94	565	60,510,403	265,383		265,383	0.4386%
2-2-1	1	Q4 94	657	68,811,456	284,710		284,710	0.4138%
2-2-1	1	Q1 95	695	82,244,756	339,902		339,902	0.4133%
2-2-1	2	Q3 93	445	108,034,854	360,495	0	360,495	0.3337%
2-2-1	2	Q4 93	480	84,268,237	332,071	0	332,071	0.3941%
2-2-1	2	Q1 94	489	71,107,942	303,326		303,326	0.4266%
2-2-1	2	Q2 94	444	65,901,775	287,158		287,158	0.4357%
2-2-1	2	Q3 94	445	65,589,025	280,716		280,716	0.4280%
2-2-1	2	Q4 94	491	91,156,826	357,630		357,630	0.3923%
2-2-1	2	Q1 95	502	104,256,082	399,124		399,124	0.3828%
2-2-1	3	Q3 93	356	101,537,263	431,925		431,925	0.4254%
2-2-1	3	Q4 93	385	123,735,553	460,785		460,785	0.3724%
2-2-1	3	Q1 94	390	129,256,347	482,688		482,688	0.3734%
2-2-1	3	Q2 94	354	119,695,832	453,546		453,546	0.3789%
2-2-1	3	Q3 94	356	119,437,025	467,255		467,255	0.3912%
2-2-1	3	Q4 94	393	128,653,442	504,206		504,206	0.3919%
2-2-1	3	Q1 95	362	104,181,379	446,344		446,344	0.4284%
2-2-1	4	Q3 93	488	159,708,932	772,938		772,938	0.4840%
2-2-1	4	Q4 93	499	170,780,071	787,475		787,475	0.4611%
2-2-1	4	Q1 94	477	191,094,164	775,775		775,775	0.4060%
2-2-1	4	Q2 94	453	215,785,809	822,630		822,630	0.3812%
2-2-1	4	Q3 94	472	218,060,093	886,418		886,418	0.4065%
2-2-1	4	Q4 94	525	246,468,016	1,007,238		1,007,238	0.4087%
2-2-1	4	Q1 95	507	277,758,625	1,068,886		1,068,886	0.3848%
2-2-1	5	Q3 93	420	174,435,642	777,842		777,842	0.4459%
2-2-1	5	Q4 93	399	151,574,847	678,924	0	678,924	0.4479%
2-2-1	5	Q1 94	394	166,838,837	747,284		747,284	0.4479%
2-2-1	5	Q2 94	367	153,682,870	695,589		695,589	0.4526%
2-2-1	5	Q3 94	337	135,517,815	617,928		617,928	0.4560%
2-2-1	5	Q4 94	352	139,483,158	688,687		688,687	0.4937%
2-2-1	5	Q1 95	335	105,965,460	564,649		564,649	0.5329%

2-2-1	6	Q3 93	108	33,042,162	168,347		168,347	0.5095%
2-2-1	6	Q4 93	104	30,558,702	156,475		156,475	0.5120%
2-2-1	6	Q1 94	98	21,175,863	124,644		124,644	0.5886%
2-2-1	6	Q2 94	84	20,103,620	113,701		113,701	0.5656%
2-2-1	6	Q3 94	70	20,201,537	108,642		108,642	0.5378%
2-2-1	6	Q4 94	63	30,960,481	163,015		163,015	0.5265%
2-2-1	6	Q1 95	43	26,268,362	133,977		133,977	0.5100%
2-2-1	7	Q3 93	3	461,268	2,410		2,410	0.5224%
2-2-1	7	Q4 93	3	437,635	2,287		2,287	0.5226%
2-2-1	7	Q1 94	2	376,987	2,062		2,062	0.5471%
2-2-1	7	Q2 94	1	209,279	1,177		1,177	0.5625%
2-2-1	7	Q3 94	1	209,279	1,177		1,177	0.5625%
2-2-1	7	Q4 94	1	209,279	1,177		1,177	0.5625%
2-2-1	7	Q1 95	0	74,031	740		740	1.0000%
2-2-1	8	Q3 93	22	9,178,105	51,458		51,458	0.5607%
2-2-1	8	Q4 93	17	6,642,724	36,243		36,243	0.5456%
2-2-1	8	Q1 94	12	2,821,426	18,701		18,701	0.6628%
2-2-1	8	Q2 94	6	972,189	7,577		7,577	0.7794%
2-2-1	8	Q3 94	9	1,962,479	10,964		10,964	0.5587%
2-2-1	8	Q4 94	13	3,130,977	17,734		17,734	0.5664%
2-2-1	8	Q1 95	13	2,992,740	17,159		17,159	0.5734%

Facility-Security-Gearing Identifier	Utilisation Level	Quarter	Number of Loans Outstanding	Debit Balances	Gross Margin Income	Provisioning Cost	Net Margin Income	Net Margin Income Per Pound Lent
2-2-2	1	Q3 93	206	12,677,334	74,112	0	74,112	0.5846%
2-2-2	1	Q4 93	213	15,001,613	70,625		70,625	0.4708%
2-2-2	1	Q1 94	225	15,589,846	73,954		73,954	0.4744%
2-2-2	1	Q2 94	353	29,918,715	127,352		127,352	0.4257%
2-2-2	1	Q3 94	465	46,126,449	198,947		198,947	0.4313%
2-2-2	1	Q4 94	524	43,956,580	186,597		186,597	0.4245%
2-2-2	1	Q1 95	553	45,273,324	193,727		193,727	0.4279%
2-2-2	2	Q3 93	490	54,300,787	268,234		268,234	0.4940%
2-2-2	2	Q4 93	521	55,069,227	280,559		280,559	0.5095%
2-2-2	2	Q1 94	512	52,447,718	270,310		270,310	0.5154%
2-2-2	2	Q2 94	541	61,238,737	306,496		306,496	0.5005%
2-2-2	2	Q3 94	560	66,033,215	332,536		332,536	0.5036%
2-2-2	2	Q4 94	629	74,245,993	369,887		369,887	0.4982%
2-2-2	2	Q1 95	616	85,046,170	464,300		464,300	0.5459%
2-2-2	3	Q3 93	673	117,092,231	605,256	0	605,256	0.5169%
2-2-2	3	Q4 93	705	131,483,513	636,019	3,586	632,433	0.4810%
2-2-2	3	Q1 94	716	150,127,340	689,866		689,866	0.4595%
2-2-2	3	Q2 94	708	159,615,028	733,743		733,743	0.4597%
2-2-2	3	Q3 94	734	185,157,378	884,875		884,875	0.4779%
2-2-2	3	Q4 94	825	208,829,626	1,010,989		1,010,989	0.4841%
2-2-2	3	Q1 95	831	289,775,635	1,621,944		1,621,944	0.5597%
2-2-2	4	Q3 93	1,723	440,399,643	2,363,821	0	2,363,821	0.5367%
2-2-2	4	Q4 93	1,814	490,899,036	2,634,234	0	2,634,234	0.5366%
2-2-2	4	Q1 94	1,840	539,796,930	2,800,157		2,800,157	0.5187%
2-2-2	4	Q2 94	1,789	546,279,765	2,838,563		2,838,563	0.5196%
2-2-2	4	Q3 94	1,888	593,248,201	3,126,947		3,126,947	0.5271%
2-2-2	4	Q4 94	2,096	663,498,084	3,459,125		3,459,125	0.5213%
2-2-2	4	Q1 95	2,083	682,712,735	3,452,248	0	3,452,248	0.5057%
2-2-2	5	Q3 93	2,454	848,022,257	4,689,885	539	4,689,347	0.5530%
2-2-2	5	Q4 93	2,492	859,770,199	4,733,406	133,398	4,600,008	0.5350%
2-2-2	5	Q1 94	2,464	835,499,430	4,717,994	-7 536	4,725,530	0.5656%
2-2-2	5	Q2 94	2,297	819,795,507	4,568,448	12	4,568,436	0.5573%
2-2-2	5	Q3 94	2,309	930,248,937	5,021,191	0	5,021,191	0.5398%
2-2-2	5	Q4 94	2,469	942,211,084	5,076,857	0	5,076,857	0.5388%
2-2-2	5	Q1 95	2,373	896,245,409	4,803,026	4,834	4,798,192	0.5354%

2-2-2	6	Q3 93	2,064	755,466,097	4,470,389	-2,720,030	7,190,419	0.9518%
2-2-2	6	Q4 93	2,073	771,405,576	4,274,555	-9,163	4,283,718	0.5553%
2-2-2	6	Q1 94	2,009	704,514,967	4,012,941	-160,832	4,173,773	0.5924%
2-2-2	6	Q2 94	1,768	540,451,523	3,284,658	1,969	3,282,689	0.6074%
2-2-2	6	Q3 94	1,696	469,681,367	3,067,900	0	3,067,900	0.6532%
2-2-2	6	Q4 94	1,766	473,446,010	3,061,247	0	3,061,247	0.6466%
2-2-2	6	Q1 95	1,646	491,580,684	3,047,371	-10,713	3,058,084	0.6221%
2-2-2	7	Q3 93	1,251	408,079,102	2,559,348	15,792	2,543,556	0.6233%
2-2-2	7	Q4 93	1,229	379,974,925	2,430,861	191,604	2,239,257	0.5893%
2-2-2	7	Q1 94	1,184	349,112,394	2,259,856	25	2,259,831	0.6473%
2-2-2	7	Q2 94	997	270,671,314	1,737,183	562	1,736,621	0.6416%
2-2-2	7	Q3 94	1,055	304,605,030	1,919,817	9,830	1,909,987	0.6270%
2-2-2	7	Q4 94	1,130	298,752,256	1,966,831	-31,573	1,998,404	0.6689%
2-2-2	7	Q1 95	1,155	289,859,409	1,941,993	-4,989	1,946,982	0.6717%
2-2-2	8	Q3 93	579	163,361,193	1,143,233	15,828	1,127,405	0.6901%
2-2-2	8	Q4 93	527	148,123,043	1,025,084	83,082	942,002	0.6360%
2-2-2	8	Q1 94	468	121,014,241	879,984	9,521	870,463	0.7193%
2-2-2	8	Q2 94	353	96,783,041	691,500	82,207	609,293	0.6295%
2-2-2	8	Q3 94	398	112,765,723	777,525	16,684	760,841	0.6747%
2-2-2	8	Q4 94	462	134,511,981	911,418	8	911,410	0.6776%
2-2-2	8	Q1 95	461	130,040,359	915,238	32,670	882,568	0.6787%