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Testing the Uniqueness of the Open Bosonic String Field Theory Vacuum

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Abstract: The operators K_n are generators of reparameterization symmetries of Witten's cubic open string field theory. One pertinent question is whether they can be utilised to generate deformations of the tachyon vacuum and thereby violate its uniqueness. We use level truncation to show that these transformations on the vacuum are in fact pure gauge transformations to a very high accuracy, thus giving new evidence for the uniqueness of the perturbatively stable vacuum. Equivalently, this result implies the vanishing of some discrete cohomology classes of the BRST operator in the stable vacuum.

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Contents

1. Introduction

In the last two years, there has been a huge amount of work done to understand tachyon condensation by using Witten's cubic open bosonic string field theory [1]. The fate of a space-filling D25-brane in the open bosonic string theory is described by Sen's three conjectures ([2], [3]). The first proposes that the difference in energy of the tachyon between the perturbative vacuum and the perturbatively stable vacuum exactly cancels the tension of the D-brane. The second asserts that after the tachyon condenses, all open string degrees of freedom disappear, leaving us with the closed string vacuum. The last conjecture states that non-trivial field configurations correspond to lower-dimensional D-branes.

The first and third conjectures have been shown to be true to a very high level of accuracy ([4] - [21]); they have also been proven analytically in Boundary String Field Theory ([22] - [28]). The second conjecture however, is by now the most puzzling. Roughly, it can be regarded at three different levels of stringency. A weak statement is that all perturbative conventional open string excitations disappear from the perturbatively stable vacuum. There has been several works testing this statement from various approaches: one could show that some flat directions are removed, as was done in [35, 36], or that the kinetic terms of the string field fluctuations are absent as in [37], or by the usage of toy models in field theory ([29], [30], [31], [32], [33], [34]) as well as the boundary state formalism ([38, 40]).

A slightly stronger statement is that not only the conventional perturbative open string excitations disappear, but more precisely the full cohomology of the new BRST operator around the tachyon vacuum vanishes. As usual, the cohomology could include discrete states in addition to conventional excitations. In [41, 42], Rastelli, Sen and Zwiebach have proposed that after a field redefinition, the new BRST operator may be taken to be simply c_0, or more generally a linear combination of operators of the form $(c_n + (-)^n c_{-n})$. The cohomology of such operators is manifestly trivial, and thus these authors are proposing this more stringent form of the second conjecture. Using this simple BRST operator on the vacuum, they were able to find solutions corresponding to the D-25 brane and lower dimensional D-branes.

Finally, a third level of understanding the second conjecture is that the perturbatively stable vacuum should correspond precisely to the closed string vacuum. A possible interpretation of this statement is that we should be able to isolate closed string excitations. Indeed, it is well-known that closed string perturbative amplitudes can be in principle isolated from cubic open string field theory diagrams. Thus closed string physics is there, though in a rather unmanageable form. It may be that closed string states appear more manifestly around the tachyon vacuum. If this is the case, perhaps one could obtain a description which differs from the explicit one provided by closed string field theory [43]. For recent discussions of closed strings in the tachyon vacuum see [44, 38].

A full understanding of Sen's conjectures, especially the second, would probably require the knowledge of the analytic solution for the perturbatively stable vacuum in cubic Open String Field Theory (OSFT), which is not yet known. However, we can still make progress by using various methods; in particular we will use the level truncation scheme to show that certain deformations from the perturbatively stable vacuum belong to the trivial cohomology of the BRST operator Q_{Ψ_0} governing the spectrum of the string field theory around the tachyon vacuum. This provides evidence for the second level of the second conjecture, viz., the disappearance of discrete excitations.

Our idea is the following. It is well known that the cubic OSFT has a *reparameterization symmetry* generated by operators ([45], [46], [47], [48]):

$$K_n := L_n - (-)^n L_{-n}.$$

Hence if Ψ is a solution of the equation of motion, i.e., $Q_B\Psi + \Psi \star \Psi = 0$, so is $e^{\epsilon K_n}\Psi$; this follows immediately from properties (2.2-2.3) of K_n which we will list in the next section. In other words, we can generate new solutions by acting $e^{\epsilon K_n}$ on a known solution, and in particular, the perturbatively stable vacuum Ψ_0 of OSFT (which we will always assume to lie in the Feynman-Siegel gauge).

A problem subsequently arises. From the physics point of view, we expect the tachyon vacuum solution to be unique, i.e., there should be no moduli space of the tachyon vacuum solution. On the other hand we seem to be able to deform Ψ_0 by $e^{\epsilon K_n}$ with arbitrary parameters ϵ and n.

In order that this seeming paradox may be consistent with physical intuition, there are two possibilities. Firstly it may be that $K_n\Psi_0 = 0$ for all n, which would imply that $e^{\epsilon K_n}\Psi_0 = \Psi_0$ and that no new tachyon vacuum solutions are generated. At face value, this possibility is very unlikely to be true because the action of K_n takes a solution in the Siegel gauge out of it, and a miraculous cancellation would be needed. In fact, we have verified that the K_n's do not annihilate the tachyon condensate. This leaves us with another choice, i.e., though $\epsilon K_n\Psi_0$ may not vanish, it could be a *pure gauge transformation* for any n and ϵ.

The purpose of this note is to show that it is indeed the case that $K_n\Psi_0$ is a pure gauge transformation. Our result can be summarized as follows. First by using a recursive relation obtained from the algebra of the K_n's, we show that it is enough to demonstrate that if the action of K_1 and K_2 on the tachyon vacuum Ψ_0 are pure gauge transformations, so too are K_n for all n. Then we use the level truncation scheme to calculate $K_1\Psi_0$ and $K_2\Psi_0$ up to levels 5 and

4 respectively. We then show that they are indeed pure gauge transformations to an excellent accuracy of 1.5% for K_1 (resp. 1.6% for K_2).

The statement that $K_n\Psi_0$ is a pure gauge transformation for any n is equivalent to the assertion that the discrete zero momentum state $K_n\Psi_0$ is Q_{Ψ_0} exact. That is, these discrete BRST-closed states are actually BRST-trivial. In a very nice recent work, Ellwood and Taylor [50] have addressed the triviality of the cohomology classes associated to continuous non-zero momentum deformations of the tachyon vacuum. More precisely, they discuss the scalar excitations at even levels and show that if they are Q_{Ψ_0} closed, they are Q_{Ψ_0} exact also to very high accuracy, thus giving the first convincing evidence for the disappearance of (a subset) of the conventional open string excitations. Our results, by focusing on discrete cohomology, complement their work. Therefore, our works jointly support, from different view-points, the triviality of the cohomology and hence the validity of Sen's second conjecture.

The outline of the paper is as follows. In Section 2 we review the key properties of the K_n operators and show that it suffices to consider only $K_{1,2}$. Level truncation was subsequently applied in Section 3 for $K_2\Psi_0$ up to level 4, and in Section 4 for $K_1\Psi_0$ up to level 5 while most of the details of the involved computations are left to the Appendix. Finally we end with concluding remarks and open questions in Section 5.

2. The K_n Symmetry of Cubic String Field Theory

It is a well known fact that the subalgebra[2] of the Virasoro algebra generated by the following operators

$$K_n = L_n - (-)^n L_{-n}, \tag{2.1}$$

is a symmetry of Witten's Cubic String Field Theory ([45, 41]). Because $K_{-n} = (-1)^{n+1}K_n$ we need only consider the cases of $n \geq 1$. These operators have the following properties:

$$[K_n, Q_B] = 0 \tag{2.2}$$

$$K_n(A \star B) = (K_nA) \star B + A \star (K_nB) \tag{2.3}$$

$$\langle K_nA, B\rangle = -\langle A, K_nB\rangle\,, \tag{2.4}$$

[2]It is in fact the maximal subalgebra that leaves the mid-point of the string invariant.

where A and B are arbitrary string fields, and Q_B is the conventional BRST operator. Incidentally, comparing (2.3) and (2.4) with similar properties for Q_B, we notice that there is no sign factor $(-1)^A$ here because K_n is a ghost number zero Grassman even operator.

Using (2.3) it is easy to show that $e^{K_n}(A\star B) = (e^{K_n}A)\star(e^{K_n}B)$. Therefore if $Q_B\Psi+\Psi\star\Psi=0$, so too is $Q_B(e^{K_n}\Psi)+(e^{K_n}\Psi)\star(e^{K_n}\Psi)=0$, where we have used (2.2). In other words, using the symmetry generators K_n, we can obtain new solutions of the equation of motion by acting on a known solution. As we have argued in the introduction, this poses a question about the uniqueness of the tachyon vacuum. On the one hand, from the physics point of view, we expect that the tachyon vacuum should be unique. On the other hand, we can seemingly generate new solutions by acting e^{K_n} on the vacuum. For these two ideas to be consistent, we must propose that *the action of K_n on the tachyon vacuum Ψ_0 should be a pure gauge transformation*, i.e.,

$$K_n\Psi_0 \stackrel{?}{=} \delta\Psi_0 \equiv Q_{\Psi_0}\Lambda = Q_B\Lambda + \Psi_0\star\Lambda - \Lambda\star\Psi_0. \tag{2.5}$$

It is the checking of the conjecture (2.5) with which this present paper is concerned. We remark in passing that there seems to be the possibility that $K_n|\Psi_0\rangle = 0$. However this is highly unlikely because though Ψ_0 is in the Feynman-Siegel gauge, the K_n action does not preserve this gauge. Indeed we have verified at low levels that this triviality does not seem to be the case so that we need to return to address (2.5).

First we check the consistency of the conjecture. Because we have $Q_{\Psi_0}Q_{\Psi_0}=0$ on the right hand side of (2.5) due to nilpotency, so too must we get zero when we act Q_{Ψ_0} on the left. This is indeed so:

$$\begin{aligned}
Q_{\Psi_0}K_n\Psi_0 &= Q_B(K_n\Psi_0) + \Psi_0\star(K_n\Psi_0) + (K_n\Psi_0)\star\Psi_0\\
&= K_n(Q_B\Psi_0) + K_n(\Psi_0\star\Psi_0)\\
&= K_n\{Q_B\Psi_0 + \Psi_0\star\Psi_0\}\\
&= 0,
\end{aligned}$$

where in the second step we have used $[K_n, Q_B]=0$ (2.2) and in the last step, the equation of motion (the expression in the braces) of Ψ_0. Notice that this check requires no usage of any special properties of the tachyon vacuum, so for any solution of the equation of motion $Q_B\Psi+\Psi\star\Psi=0$, we always have $K_n\Psi$ being Q_Ψ closed. Our conjecture is the statement that when $\Psi=\Psi_0$ is the tachyon vacuum, $K_n\Psi_0$ is not only closed, but also exact, whence BRST-cohomology trivial. To show this is true is our work.

Naively it seems to be difficult to check that all the K_n actions are mere pure gauge transformations because there are an infinite number of them. However, we can show that it suffices to check for K_1 and K_2, then by iteration $n \geq 3$ follows. This can be done in two steps. Firstly we recall that the K_n's form an algebra:

$$[K_n, K_m] = (n-m)K_{n+m} - (-1)^m (n+m) K_{n-m}. \tag{2.6}$$

Secondly we can show that if for some n and m,

$$K_n \Psi_0 = Q_{\Psi_0} \Lambda_n, \qquad K_m \Psi_0 = Q_{\Psi_0} \Lambda_m,$$

then

$$[K_n, K_m]\Psi_0 = Q_B \tilde{\Lambda}_{n,m} + \Psi_0 \star \tilde{\Lambda}_{n,m} - \tilde{\Lambda}_{n,m} \star \Psi_0 = Q_{\Psi_0} \tilde{\Lambda}_{n,m}, \tag{2.7}$$

and hence pure gauge, where

$$\tilde{\Lambda}_{n,m} = K_n \Lambda_m - K_m \Lambda_n + \Lambda_n \star \Lambda_m - \Lambda_m \star \Lambda_n. \tag{2.8}$$

Combining (2.6), (2.7) and (2.8), we see instantly that if the conjecture is true for K_1 and K_2, then by iteration, we would have the result for all $K_{n \geq 3}$.

3. The Exactness of $K_2 \Psi_0$

In this section, we check that $K_2 \Psi_0$ is a pure gauge transformation, which would imply that $K_2 \Psi_0$ is BRST-exact. First we do the calculation at level two, which is very simple. We use this example to demonstrate our method, then we go further to level four. For the details, the reader is referred to the Appendix.

Before proceeding, let us make some general remarks which is explained further in the Appendix. The tachyon solution Ψ_0 of [7] has only even level components. So if the gauge parameter Λ is in an even (resp. odd) level, $\Psi_0 \star \Lambda - \Lambda \star \Psi_0$ will contain only even (resp. odd) levels as well; this is shown in (A.1). Furthermore, since Q_B does not change the level and K_2 increases or decreases the level by two, to see whether K_2 on Ψ_0 is a pure gauge, we can restrict the gauge parameters to be in even levels only. Likewise, for K_1, because it increases or decreases the level by one, $K_1 \Psi_0$ must have only odd levels. Therefore, in this case we can restrict all gauge parameters to be in odd levels only. In particular we will focus on levels $2, 4$ for K_2 and $3, 5$ for K_1.

3.1 Fitting at Level 2

Up to level two, there are four components for the string field:

$$|\Psi\rangle = \eta_{0,1}\,|\Omega\rangle + \eta_{2,1} b_{-1} c_{-1}\,|\Omega\rangle + \eta_{2,2} b_{-2} c_0\,|\Omega\rangle + \eta_{2,3} L^m_{-2}\,|\Omega\rangle\,, \tag{3.1}$$

where the η's are numerical coefficients and L^m_{-n} are matter Virasoro operators. Furthermore, $|\Omega\rangle = c_1\,|0\rangle$ and $|0\rangle$ is the $SL(2,\mathbf{R})$ invariant vacuum[3]. For simplicity, we denote the basis of the fields as a row vector with four components so that

$$(\eta_{0,1},\ \eta_{2,1},\ \eta_{2,2},\ \eta_{2,3}) := \eta_{0,1}\,|\Omega\rangle + \eta_{2,1} b_{-1} c_{-1}\,|\Omega\rangle + \eta_{2,2} b_{-2} c_0\,|\Omega\rangle + \eta_{2,3} L^m_{-2}\,|\Omega\rangle\,.$$

To this convention of notation of fields we shall adhere.

The numerical values for these coefficients have been computed to great precision in the Feynman-Siegel gauge[7]. At level $(2,6)$ (here we use their convention that (L,I) refers to truncating fields up to level L and interactions up to level I; also we shall use their normalization), the vacuum field (3.1) is

$$(\eta_{0,1},\ \eta_{2,1},\ \eta_{2,2},\ \eta_{2,3}) = (0.39765,\ -0.13897,\ 0,\ 0.040893). \tag{3.2}$$

Up to level two, for the gauge parameter $|\Lambda\rangle$ of ghost number 0, there is only one numerical parameter $\mu_{2,1}$:

$$|\Lambda\rangle = \mu_{2,1} b_{-2}\,|\Omega\rangle\,, \tag{3.3}$$

and the gauge transformation of (3.1) up to level two is already given in [49] as

$$\begin{aligned}
\delta\eta_{0,1} &= \mu_{2,1}\left(-\frac{16}{9}\eta_{0,1} - \frac{464}{243}\eta_{2,1} + \frac{128}{81}\eta_{2,2} + \frac{1040}{243}\eta_{2,3}\right)\\
\delta\eta_{2,1} &= \mu_{2,1}\left(-3 - \frac{176}{243}\eta_{0,1} - \frac{11248}{6561}\eta_{2,1} - \frac{6016}{6561}\eta_{2,2} + \frac{11440}{6561}\eta_{2,3}\right)\\
\delta\eta_{2,2} &= \mu_{2,1}\left(-1 - \frac{224}{81}\eta_{0,1} + \frac{992}{6561}\eta_{2,1} + \frac{1792}{729}\eta_{2,2} + \frac{14560}{2187}\eta_{2,3}\right)\\
\delta\eta_{2,3} &= \mu_{2,1}\left(1 + \frac{80}{243}\eta_{0,1} + \frac{2320}{6561}\eta_{2,1} - \frac{640}{2187}\eta_{2,2} - \frac{9296}{6561}\eta_{2,3}\right),
\end{aligned} \tag{3.4}$$

which we have confirmed term by term.

On the other hand, we remind the reader that

$$K_2 := L_2 - L_{-2} = L^m_2 + L^g_2 - L^m_{-2} - L^g_{-2},$$

[3]Our notation is different from that in [7]. We use here, for the matter part, the universal basis instead of the oscillator basis.

where $L^g_m := \sum_{n=-\infty}^{\infty}(2m-n) : b_n c_{m-n} : -\delta_{m,0}$ is the ghost Virasoro operator with : : being the creation-annihilation normal ordering. Recalling (3.1), we have

$$K_2 |\Psi\rangle = (3\eta_{2,1} + 4\eta_{2,2} + 13\eta_{2,3}) |\Omega\rangle + 3\eta_{0,1} b_{-1} c_{-1} |\Omega\rangle + 2\eta_{0,1} b_{-2} c_0 |\Omega\rangle - \eta_{0,1} L^m_{-2} |\Omega\rangle . \tag{3.5}$$

We are now ready to check our proposal (2.5) up to level 2 accuracy, i.e., can one tune the parameter $\mu_{2,1}$, so that

$$K_2 |\Psi_0\rangle = \delta |\Psi_0\rangle \tag{3.6}$$

would hold?

The left hand side of (3.6) is obtained by substituting the numerical results of (3.2) into (3.5):

$$K_2 |\Psi_0\rangle = (0.11469,\ 1.1930,\ 0.79531,\ -0.39765).$$

The right hand side of (3.6) is obtained via substitution of (3.2) into (3.4):

$$\delta |\Psi_0\rangle = \mu_{2,1}(-0.26656,\ -2.9785,\ -1.8485,\ 1.0238).$$

Now we have 2 (Euclidean) vectors $(K_2 |\Psi_0\rangle)_i$ and $(\delta |\Psi_0\rangle)_i$ of equal length which we wish to be as close as possible if (2.5) were to hold. We subsequently choose the parameter $\mu_{2,1}$ by performing a least-squares fit on these two vectors by minimizing the Euclidean distance between the two.

$$|K_2 |\Psi_0\rangle - \delta |\Psi_0\rangle | := \left(\sum_i \left[(K_2 |\Psi_0\rangle)_i - (\delta |\Psi_0\rangle)_i \right]^2 \right)^{\frac{1}{2}} .$$

To this procedure we shall refer as "best fit." At the present level we arrive at

$$\mu_{2,1} = -0.40732.$$

Putting this value into $\delta |\Psi_0\rangle$ we get $\delta |\Psi_0\rangle = (0.10857,\ 1.2132,\ 0.75290,\ -0.41702)$ and whence

$$K_2 |\Psi_0\rangle - \delta |\Psi_0\rangle = (0.0061153,\ -0.020207,\ 0.042406,\ 0.019368).$$

A good estimator for our results is the normalized quantity,

$$\epsilon := \frac{|K_2 |\Psi_0\rangle - \delta |\Psi_0\rangle |}{|K_2 |\Psi_0\rangle |},$$

which we wish to be as close to 0 as possible. Using the above values, we have $\epsilon = 0.034294$. Therefore we conclude that up to level 2, our conjecture is accurate to 3.4%.

3.2 Fitting at Level 4

And thus we continue and to higher levels we shall go. Now we keep the string field solution up to level four and compare the two sides of $K_2 |\Psi_0\rangle$ and $Q_{\Psi_0}\Lambda$ also up to level 4.

As we mentioned before, we can restrict the gauge parameters to be of even levels as well, thus we can write Λ as:

$$\begin{aligned} |\Lambda\rangle &= \mu_{2,1}b_{-2}|\Omega\rangle + \mu_{4,1}b_{-4}|\Omega\rangle + \mu_{4,2}b_{-2}b_{-1}c_{-1}|\Omega\rangle \\ &\quad + \mu_{4,3}b_{-3}b_{-1}c_0|\Omega\rangle + \mu_{4,4}b_{-1}L^m_{-3}|\Omega\rangle + \mu_{4,5}b_{-2}L^m_{-2}|\Omega\rangle\,, \end{aligned} \tag{3.7}$$

which has six numerical parameters.

Due to the overwhelming length of the gauge transformation and K_2 action on Ψ_0 to this level, we leave their presentation to the Appendix. Again in accordance with our convention, we can write the field into a vector with 14 components in the order

$$(\eta_{0,1}, \eta_{2,i}, \eta_{4,j}) \qquad (i = 1,2,3; j = 1,2,..,10).$$

In this notation, the tachyon vacuum at level $(4,12)$ is given by

$$\begin{aligned} |\Psi_0\rangle = (&0.40072,\ -0.15029,\ 0,\ 0.041595,\ 0.041073,\ 0.024192,\ 0.013691, \\ &0,\ 0,\ -0.0037419,\ 0,\ 0.0050132,\ 0,\ -0.00043064) \end{aligned} \tag{3.8}$$

We need now check (3.6) to level 4. The K_2 action on the left hand side is given by

$$\begin{aligned} K_2|\Psi_0\rangle = (&0.089868,\ 1.2947,\ 0.75306,\ -0.42277,\ 0.75143,\ 0,\ -0.15029, \\ &0,\ -0.30057,\ 0,\ 0,\ 0.27507,\ 0.083189,\ -0.041595) \end{aligned} \tag{3.9}$$

and $\delta|\Psi_0\rangle$ on the right hand side is a numerical function of the 6 μ parameters obtainable by substitution of (3.8) into the appropriate expressions in the Appendix.

Again we minimize $|K_2|\Psi_0\rangle - \delta|\Psi_0\rangle|$ and find the parameters as

$$\mu_{2,1} = -0.54013, \qquad \mu_{4,1} = 0.18957, \qquad \mu_{4,2} = -0.37946,$$

$$\mu_{4,3} = -0.37645, \qquad \mu_{4,4} = -0.12019, \qquad \mu_{4,5} = -0.022464$$

Subsequently, we obtain

$$\epsilon = \frac{|K_2|\Psi_0\rangle - \delta|\Psi_0\rangle|}{|K_2|\Psi_0\rangle|} = 0.016078.$$

In conclusion then, the accuracy increases from 3.4% at level 2 to 1.6% at level 4.

4. The Exactness of $K_1\Psi_0$

Having checked the validity of our conjecture (2.5) for K_2 to within 1.6%, in this section we check if the K_1 action is a pure gauge transformation. As we have mentioned in the beginning of the last section, we can restrict the gauge parameters to odd levels only. Naïvely the first nontrivial test is to expand $|\Lambda\rangle$ to only level 1 which has 1 free parameter. However, because to level 1 $K_1\Psi_0$ has only 1 component, we would be lead to the trivial fitting of 1 parameter to 1 constraint. Therefore we must start with level 3, by which we mean that we expand Λ to level 3 and Ψ_0 to level 2 and thus $K_1\Psi_0$ to level 3.

4.1 Fitting at Level 3

Up to level 3, we have four free parameters $\mu_{1,1}$ and $\mu_{3,i}, i = 1, 2, 3$ in the gauge parameter:

$$|\Lambda\rangle = \mu_{1,1}b_{-1}|\Omega\rangle + \mu_{3,1}b_{-3}|\Omega\rangle + \mu_{3,2}b_{-2}b_{-1}c_0|\Omega\rangle + \mu_{3,3}b_{-1}L^m_{-2}|\Omega\rangle$$

Once again the data of Ψ_0 to level 2 was given in (3.2). The K_1 action and gauge transformation are subsequently presented in the Appendix. Since $K_1\Psi_0$ is at level 3, we have 6 fields in the basis and a general field may be represented as $(\eta_{1,1}, \eta_{3,i})$ with $(i = 1, .., 5)$. Upon substitution of the numerical values in (3.2), we have, to level 3,

$$K_1|\Psi_0\rangle = (-0.25868,\ -0.41692,\ 0,\ 0,\ 0.040893, -0.040893).$$

We perform the same procedure as in the previous section and minimize $|K_1|\Psi_0\rangle - \delta|\Psi_0\rangle|$ to obtain the least-square fitting parameters:

$$\mu_{1,1} = 0.88605, \quad \mu_{3,,1} = -0.15821, \quad \mu_{3,,2} = 0.42491, \quad \mu_{3,,3} = 0.23200.$$

Consequently, the measure of our fit is given by

$$\epsilon = \frac{|K_1|\Psi_0\rangle - \delta|\Psi_0\rangle|}{|K_1|\Psi_0\rangle|} = 0.036030$$

Thus accuracy is achieved to within 3.6%, not so bad for this level.

4.2 Fitting at Level 5

To achieve greater accuracy, let us keep the string field up to level 5 and check (2.5). Its two sides $K_1|\Psi_0\rangle$ and $Q_{\Psi_0}\Lambda$ are both up to level 5, which in our notation is a vector of length 22, with 16 components at level 5 in addition to those in the previous subsection (indeed as remarked before, we need not include the even levels):

$$(\eta_{1,1},\ \eta_{3,i},\ \eta_{5,j}) \qquad (i = 1, ..., 5; j = 1, ..16).$$

In the same vein, we can restrict the gauge parameters to odd levels only:

$$\begin{aligned}
|\Lambda\rangle &= \mu_{1,1} b_{-1} |\Omega\rangle \\
&+ \mu_{3,1} b_{-3} |\Omega\rangle + \mu_{3,2} b_{-2} b_{-1} c_0 |\Omega\rangle + \mu_{3,3} b_{-1} L^m_{-2} |\Omega\rangle \\
&+ \mu_{5,1} b_{-5} |\Omega\rangle + \mu_{5,2} b_{-2} b_{-1} c_{-2} |\Omega\rangle + \mu_{5,3} b_{-3} b_{-1} c_{-1} |\Omega\rangle \\
&+ \mu_{5,4} b_{-3} b_{-2} c_0 |\Omega\rangle + \mu_{5,5} b_{-4} b_{-1} c_0 |\Omega\rangle + \mu_{5,6} b_{-1} L^m_{-4} |\Omega\rangle \\
&+ \mu_{5,7} b_{-2} L^m_{-3} |\Omega\rangle + \mu_{5,8} b_{-3} L^m_{-2} |\Omega\rangle + \mu_{5,9} b_{-2} b_{-1} c_0 L^m_{-2} |\Omega\rangle \\
&+ \mu_{5,10} b_{-1} L^m_{-2} L^m_{-2} |\Omega\rangle ,
\end{aligned}$$

which has 14 parameters μ.

Once again, the gauge transformation and K_1 action on Ψ_0 can be found in the Appendix. And thus equipped, the left hand side of (3.6) gives

$$\begin{aligned}
K_1 |\Psi_0\rangle = (&-0.25043,\ -0.33721,\ 0.054765,\ -0.013691,\ 0.021593, \\
&-0.046608,\ 0.20537,\ 0.096767,\ 0.065265,\ 0.027382,\ 0, \\
&0.024192,\ 0.013691,\ -0.011656,\ 0.0037419,\ 0.0050132, \\
&0,\ 0.015040,\ 0,\ 0,\ -0.00086128,\ 0.00043064).
\end{aligned}$$

Finally we minimize $|K_1 |\Psi_0\rangle - \delta |\Psi_0\rangle|$ and find the best-fit gauge parameters as:

$$\begin{aligned}
&\mu_{1,1} = 0.96221, && \mu_{3,1} = -0.16665, && \mu_{3,2} = 0.42762, \\
&\mu_{3,3} = 0.19259, && \mu_{5,1} = -0.027057, && \mu_{5,2} = -1.2515, \\
&\mu_{5,3} = 0.31370, && \mu_{5,4} = 1.0733, && \mu_{5,5} = -0.30612, \\
&\mu_{5,6} = -0.091788, && \mu_{5,7} = 0.21383, && \mu_{5,8} = -0.30555, \\
&\mu_{5,9} = 0.19208, && \mu_{5,10} = 0.050724,
\end{aligned}$$

with an error estimate of:

$$\epsilon = \frac{|K_1 |\Psi_0\rangle - \delta |\Psi_0\rangle|}{|K_1 |\Psi_0\rangle|} = 0.015128.$$

So the accuracy increases from 3.6% at level 3 to 1.5% at level 5.

5. Concluding Remarks and Open questions

Sen's second conjecture remains to be fully understood. A strong version of the conjecture states that the entire spectrum of the open string should disappear from the perturbatively stable vacuum Ψ_0 and hence the cohomology of Q_{Ψ_0} should be trivial. A reparameterization symmetry generated by K_n in bosonic OSFT seems to be able to deform the tachyon vacuum whereby violating its uniqueness. In this paper we have given a strong evidence in favor of the second conjecture by explicitly showing that $K_n\Psi_0$ is merely a pure gauge transformation and thus gives no new moduli to the tachyon vacuum. Using a level truncation scheme, we have demonstrated that $K_{1,2}$ are pure gauge up to level 5 (resp. level 4) to within an excellent accuracy of 1.5% (resp. 1.6%), and that all other K_n are so by iteration.

Many open questions are of immediate interest for investigation; we list a few here.

- An immediate check one could perform, as a test to the validity of the level truncation procedure, is to see to what accuracy is $K_n\Psi_0$ closed, i.e., though $Q_{\Psi_0}K_n\Psi_0$ should be identically zero, level truncation spoils this and it would be interesting to check the numerics.

- As we mentioned before, we can generate new solutions by acting $e^{\epsilon K_n}$ on a known solution. We can apply this method to, for example, lump solutions ([9]-[14]) and see what will happen. Indeed as is with Ψ_0, it is unlikely that K_n will annihilate the lump solution for all n, so we probably will obtain deformations of lumps. The question is then to see if these new solutions are gauge equivalent to known lump solutions or if they do generate inequivalent new physical states. If the answer is the latter, we would generate a part of the moduli space to which the lumps belong. One particularly interesting example would be the solution generated by $e^{\epsilon K_1}$. Because K_1 changes the level by one unit, by acting on the lumps we may obtain new solutions which correspond to marginal deformations.

- In this paper and in [50] only part of the cohomology of Q_{Ψ_0} is proven to be trivial. It will be very interesting to see if the entire cohomology is trivial. In other words, if we have an arbitrary deformation $\delta\Psi_0$ around the tachyon vacuum Ψ_0 which is closed $Q_{\Psi_0}\delta\Psi_0 = 0$, it must be exact, i.e., there exists a gauge parameter Λ such that $Q_{\Psi_0}\Lambda = \delta\Psi_0$. One particular set of interesting deformations is those without momentum dependence because they are related to the possible moduli space of translationally invariant solutions. When the solution is unique, from a physical point of view, we should expect those deformations to be in the trivial cohomology. Proving the triviality of zero-momentum cohomology should be readily tractable by level truncation.

- It is known that at the perturbative vacuum, K_n is a good symmetry of the theory. Indeed, $[K_n, Q_B] = 0$. However, in the tachyon vacuum Ψ_0 we have

$$[K_n, Q_{\Psi_0}]A = (K_n\Psi_0) \star A - (-)^A A \star (K_n\Psi_0) \equiv [K_n\Psi_0, A],$$

  which is not zero in general. This is in accord with [41, 42], where the candidate BRST operators of the tachyon vacuum do not generally commute with the K_n operators[4]. There may be a gauge in which the tachyon vacuum $\tilde{\Psi}_0$ satisfies $K_n\tilde{\Psi}_0 = 0$ for all n, but we think this is unlikely. However, a subalgebra of K_n might be a symmetry of the tachyon vacuum. Any conclusions on these questions would have implications for the SFT around the tachyon vacuum.

Note added

After the first version of this preprint was released, H. Hata sent us a formal proof that the $K_n\Psi_0$ are pure gauge. We thank him for pointing this out to us and, with his permission, we reproduce his proof here: The proof uses the following three points:

(1) The K_n can be expressed as an anticommutator: $K_n = \{Q_B, B_n\}$, with $B_n = b_n - (-1)^n b_{-n}$.

(2) The B_n obey a Leibnitz rule for the star-product: $B_n(A\star C) = (B_nA)\star C + (-1)^A A\star(B_nC)$.

(3) The equation of motion: $Q_B\Psi_0 + \Psi_0 \star \Psi_0 = 0$.

Using the above, we can express $K_n\Psi_0$ in the following way:

$$\begin{aligned} K_n\Psi_0 &= \{Q_B, B_n\}\Psi_0 \\ &= Q_B(B_n\Psi_0) + B_n(Q_B\Psi_0) \\ &= Q_B(B_n\Psi_0) - B_n(\Psi_0 \star \Psi_0) \\ &= Q_B(B_n\Psi_0) + \Psi_0 \star (B_n\Psi_0) - (B_n\Psi_0) \star \Psi_0, \end{aligned}$$

showing that (2.5) holds, by taking $\Lambda = B_n\Psi_0$.

[4] We thank B. Zwiebach for a discussion of this point.

The work presented in this paper therefore reduces to a new check of the consistency of the level truncation method. The above proof also immediately answers our open question concerning deformations of lumps. Indeed, it can be seen from the proof, that for $K_n\Psi$ to be pure gauge, Ψ only needs to be a solution of the equation of motion. The proof thus applies to a lump solution as well as to the vacuum.

Checking to what accuracy is $K_n\Psi_0$ closed, namely to see how well the property $Q_{\Psi_0}^2 = 0$ holds in the level truncation, would still be a good check of the level truncation. And of course, studying other parts of the cohomology, as well as looking for a subalgebra of the K_n leaving the vacuum invariant, are still important open questions.

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A. Appendix

In this Appendix we shall tabulate the details used in our calculations. In subsections A.1 and A.2 we present the basis of the fields for ghost numbers 0 and 1, In A.3, we present the action of K_1 and K_2 on the string field theory vacuum to level 4. Finally in subsections A.4 and A.5 we present the gauge transformations of the vacuum to level 5.

A.1 The Basis of Ghost Number 1 Fields

As Ψ_0 is ghost number 1, we here tabulate the basis of the ghost number 1 fields up to level 5, consisting of a total of 14 in even levels and 22 in odd levels. The numerical parameters $\eta_{\ell,i}$ denote the expansion coefficient of the field Ψ at the i-th field at level ℓ. For the vacuum these parameters have been computed to great precision in [7]; we use their results at level $(4, 12)$.

Level	Field	Coefficient	vev at level (4,12)
0	$\lvert\Omega\rangle = c_1 \lvert 0\rangle$	$\eta_{0,1}$	0.40072
1	$b_{-1}c_0 \lvert\Omega\rangle$	$\eta_{1,1}$	0
2 (3 fields)	$b_{-1}c_{-1} \lvert\Omega\rangle$	$\eta_{2,1}$	-0.15029
	$b_{-2}c_0 \lvert\Omega\rangle$	$\eta_{2,2}$	0
	$L^m_{-2} \lvert\Omega\rangle$	$\eta_{2,3}$	0.041595
3 (5 fields)	$b_{-1}c_{-2} \lvert\Omega\rangle$	$\eta_{3,1}$	0
	$b_{-2}c_{-1} \lvert\Omega\rangle$	$\eta_{3,2}$	0
	$b_{-3}c_0 \lvert\Omega\rangle$	$\eta_{3,3}$	0
	$L^m_{-3} \lvert\Omega\rangle$	$\eta_{3,4}$	0
	$b_{-1}c_0L^m_{-2} \lvert\Omega\rangle$	$\eta_{3,5}$	0
4 (10 fields)	$b_{-1}c_{-3} \lvert\Omega\rangle$	$\eta_{4,1}$	0.041073
	$b_{-2}c_{-2} \lvert\Omega\rangle$	$\eta_{4,2}$	0.024192
	$b_{-3}c_{-1} \lvert\Omega\rangle$	$\eta_{4,3}$	0.013691
	$b_{-4}c_0 \lvert\Omega\rangle$	$\eta_{4,4}$	0
	$b_{-2}b_{-1}c_{-1}c_0 \lvert\Omega\rangle$	$\eta_{4,5}$	0
	$L^m_{-4} \lvert\Omega\rangle$	$\eta_{4,6}$	-0.0037419
	$b_{-1}c_0L^m_{-3} \lvert\Omega\rangle$	$\eta_{4,7}$	0
	$b_{-1}c_{-1}L^m_{-2} \lvert\Omega\rangle$	$\eta_{4,8}$	0.0050132
	$b_{-2}c_0L^m_{-2} \lvert\Omega\rangle$	$\eta_{4,9}$	0
	$L^m_{-2}L^m_{-2} \lvert\Omega\rangle$	$\eta_{4,10}$	-0.00043064

Level	Field	Coefficient	vev at level (4,12)
5 (16 fields)	$b_{-1}c_{-4}\|\Omega\rangle$	$\eta_{5,1}$	0
	$b_{-2}c_{-3}\|\Omega\rangle$	$\eta_{5,2}$	0
	$b_{-3}c_{-2}\|\Omega\rangle$	$\eta_{5,3}$	0
	$b_{-4}c_{-1}\|\Omega\rangle$	$\eta_{5,4}$	0
	$b_{-5}c_{0}\|\Omega\rangle$	$\eta_{5,5}$	0
	$b_{-2}b_{-1}c_{-2}c_{0}\|\Omega\rangle$	$\eta_{5,6}$	0
	$b_{-3}b_{-1}c_{-1}c_{0}\|\Omega\rangle$	$\eta_{5,7}$	0
	$L^{m}_{-5}\|\Omega\rangle$	$\eta_{5,8}$	0
	$b_{-1}c_{0}L^{m}_{-4}\|\Omega\rangle$	$\eta_{5,9}$	0
	$b_{-1}c_{-1}L^{m}_{-3}\|\Omega\rangle$	$\eta_{5,10}$	0
	$b_{-2}c_{0}L^{m}_{-3}\|\Omega\rangle$	$\eta_{5,11}$	0
	$b_{-1}c_{-2}L^{m}_{-2}\|\Omega\rangle$	$\eta_{5,12}$	0
	$b_{-2}c_{-1}L^{m}_{-2}\|\Omega\rangle$	$\eta_{5,13}$	0
	$b_{-3}c_{0}L^{m}_{-2}\|\Omega\rangle$	$\eta_{5,14}$	0
	$L^{m}_{-3}L^{m}_{-2}\|\Omega\rangle$	$\eta_{5,15}$	0
	$b_{-1}c_{0}L^{m}_{-2}L^{m}_{-2}\|\Omega\rangle$	$\eta_{5,16}$	0

A.2 The Basis of Ghost Number 0 Fields

The gauge transformation parameter $|\Lambda\rangle$ is of ghost number 0, thus we here present the basis for ghost number 0 fields. Analogous to the previous subsection, we use $\mu_{\ell,i}$ for $\ell = 1,..,5$, and i indexing within each level to denote the coefficient of the expansion of $|\Lambda\rangle$ into the basis. A least-squares fit was then performed in order to minimize the difference between the K action on the vacuum and the gauge transformation therefrom. Below, the columns Fit n refer to the solution of the parameters μ at the best-fit at level n.

Level	Field	Coefficient	Fit 2	Fit 3	Fit 4	Fit 5
1	$b_{-1}\|\Omega\rangle$	$\mu_{1,1}$		0.886		0.962
2	$b_{-2}\|\Omega\rangle$	$\mu_{2,1}$	-0.407		-0.540	
3	$b_{-3}\|\Omega\rangle$	$\mu_{3,1}$		-0.158		-0.167
	$b_{-2}b_{-1}c_0\|\Omega\rangle$	$\mu_{3,2}$		0.425		0.428
	$b_{-1}L^m_{-2}\|\Omega\rangle$	$\mu_{3,3}$		0.232		0.193
4	$b_{-4}\|\Omega\rangle$	$\mu_{4,1}$			0.190	
	$b_{-2}b_{-1}c_{-1}\|\Omega\rangle$	$\mu_{4,2}$			-0.379	
	$b_{-3}b_{-1}c_0\|\Omega\rangle$	$\mu_{4,3}$			-0.376	
	$b_{-1}L^m_{-3}\|\Omega\rangle$	$\mu_{4,4}$			-0.120	
	$b_{-2}L^m_{-2}\|\Omega\rangle$	$\mu_{4,5}$			-0.0225	
5	$b_{-5}\|\Omega\rangle$	$\mu_{5,1}$				-0.0271
	$b_{-2}b_{-1}c_{-2}\|\Omega\rangle$	$\mu_{5,2}$				-1.25
	$b_{-3}b_{-1}c_{-1}\|\Omega\rangle$	$\mu_{5,3}$				0.314
	$b_{-3}b_{-2}c_0\|\Omega\rangle$	$\mu_{5,4}$				1.07
	$b_{-4}b_{-1}c_0\|\Omega\rangle$	$\mu_{5,5}$				-0.306
	$b_{-1}L^m_{-4}\|\Omega\rangle$	$\mu_{5,6}$				-0.0918
	$b_{-2}L^m_{-3}\|\Omega\rangle$	$\mu_{5,7}$				0.214
	$b_{-3}L^m_{-2}\|\Omega\rangle$	$\mu_{5,8}$				-0.306
	$b_{-2}b_{-1}c_0L^m_{-2}\|\Omega\rangle$	$\mu_{5,9}$				0.192
	$b_{-1}L^m_{-2}L^m_{-2}\|\Omega\rangle$	$\mu_{5,10}$				0.0507

A.3 K_1 and K_2 Actions on $|\Psi_0\rangle$

We act K_1 and K_2 on the vacuum Ψ_0 (only the action on nonzero components of Ψ_0 is kept):

$$
\begin{aligned}
K_1\Psi_0 = & [(-\eta_{0,1}-\eta_{2,1})b_{-1}c_0|\Omega\rangle] + [(3\eta_{2,1}+\eta_{4,1}+3\eta_{4,2})b_{-1}c_{-2}|\Omega\rangle \\
& + (4\eta_{4,3})b_{-2}c_{-1}|\Omega\rangle + (-\eta_{4,3})b_{-3}c_0|\Omega\rangle \\
& + (\eta_{2,3}+5\eta_{4,6}+3\eta_{4,10})L^m_{-3}|\Omega\rangle + (-\eta_{2,3}-\eta_{4,8})b_{-1}c_0L^m_{-2}|\Omega\rangle] \\
& + [(5\eta_{4,1})b_{-1}c_{-4}|\Omega\rangle + (4\eta_{4,2})b_{-2}c_{-3}|\Omega\rangle + (\eta_{4,2}+3\eta_{4,3})b_{-3}c_{-2}|\Omega\rangle \\
& + (2\eta_{4,3})b_{-4}c_{-1}|\Omega\rangle + (\eta_{4,2})b_{-2}b_{-1}c_{-2}c_0|\Omega\rangle + (\eta_{4,3})b_{-3}b_{-1}c_{-1}c_0|\Omega\rangle \\
& + (3\eta_{4,6}+\eta_{4,10})L^m_{-5}|\Omega\rangle - (\eta_{4,6})b_{-1}c_0L^m_{-4}|\Omega\rangle + (\eta_{4,8})b_{-1}c_{-1}L^m_{-3}|\Omega\rangle
\end{aligned}
$$

$$+ (3\eta_{4,8})b_{-1}c_{-2}L^m_{-2}\,|\Omega\rangle + (2\eta_{4,10})L^m_{-3}L^m_{-2}\,|\Omega\rangle - (\eta_{4,10})b_{-1}c_0L^m_{-2}L^m_{-2}\,|\Omega\rangle]$$

$$\begin{aligned} K_2\Psi_0 &= [(3\eta_{2,1} + 13\eta_{2,3})\,|\Omega\rangle] \\ &+ [(3\eta_{0,1} - \eta_{4,1} + 5\eta_{4,3} + 13\eta_{4,8})b_{-1}c_{-1}\,|\Omega\rangle + (2\eta_{0,1} - 2\eta_{4,2})b_{-2}c_0\,|\Omega\rangle \\ &+ (-\eta_{0,1} + 6\eta_{4,6} + 3\eta_{4,8} + 34\eta_{4,10})L^m_{-2}\,|\Omega\rangle] + [(-5\eta_{2,1})b_{-1}c_{-3}\,|\Omega\rangle \\ &+ \eta_{2,1}b_{-3}c_{-1}\,|\Omega\rangle + (2\eta_{2,1})b_{-2}b_{-1}c_{-1}c_0\,|\Omega\rangle + (-\eta_{2,1} + 3\eta_{2,3})b_{-1}c_{-1}L^m_{-2}\,|\Omega\rangle \\ &+ (2\eta_{2,3})b_{-2}c_0L^m_{-2}\,|\Omega\rangle - \eta_{2,3}L^m_{-2}L^m_{-2}\,|\Omega\rangle] \end{aligned}$$

A.4 Gauge Transformation of the Even Level String Field

Let us present the heuristics of the computation required in the gauge transformation $\delta\Psi := Q_B\Lambda + \Psi\star\Lambda - \Lambda\star\Psi$. The only non-trivial part is the computation of the $\star$-product. Since we are working under a level-truncation scheme, to compute $B\star C$ for string fields B and C, it suffices to find, level-by-level, the coefficients of the expansion of the star-product into the basis of each level, i.e.,

$$B\star C = \sum_{\ell,i} x_{\ell,i}\psi_{\ell,i},$$

with $\psi_{\ell,i}$ the i-th field basis at level ℓ and $x_{\ell,i}$ the coefficients we wish to determine. Defining the orthonormal basis $\tilde{\psi}_{\ell,i}$, so that

$$\langle\tilde{\psi}_{\ell,i}, \psi_{\ell',i'}\rangle = \delta_{\ell\ell'}\delta_{ii'},$$

where $\langle\cdot,\cdot\rangle$ is the BPZ inner product, we arrive at $x_{\ell,i} = \langle\tilde{\psi}_{\ell,i}, B\star C\rangle$, which simplifies by the definition $\langle A, B\star C\rangle := \langle A, B, C\rangle$, to

$$x_{\ell,i} = \langle\tilde{\psi}_{\ell,i}, B, C\rangle.$$

For an example, let us determine the coefficient x in

$$|\Omega\rangle \star b_{-2}\,|\Omega\rangle = x\,|\Omega\rangle + \ldots$$

The orthogonal state to $|\Omega\rangle$ is $c_0\,|\Omega\rangle$, therefore $x = \langle c_0\,|\Omega\rangle, |\Omega\rangle, b_{-2}\,|\Omega\rangle\rangle = -\frac{8}{9}$ in a normalization where $\langle|\Omega\rangle, |\Omega\rangle, |\Omega\rangle\rangle = 3$ in accordance with [7]. The computation of the 3-correlator we leave the reader to a vast literature [6, 7, 46, 48]. As another example, let us compute

$$b_{-1}c_{-1}\,|\Omega\rangle \star b_{-2}\,|\Omega\rangle = xb_{-2}c_0\,|\Omega\rangle + \ldots.$$

The orthogonal state to $b_{-2}c_0 \left|\Omega\right>$ is $c_{-2}\left|\Omega\right>$, whence $x = \langle c_{-2}\left|\Omega\right>, b_{-1}c_{-1}\left|\Omega\right>, b_{-2}\left|\Omega\right>\rangle = \frac{496}{6561}$.

We point out further that a simplification is at hand due to the relation:

$$\langle A, B, C\rangle = (-1)^{1+g(A)g(B)+\ell(A)+\ell(B)+\ell(C)}\langle A, C, B\rangle, \tag{A.1}$$

where $g(X)$ and $\ell(X)$ are the ghost number and level of X respectively (we take $g(|\Omega\rangle) = 1$).

This simplification (A.1) is crucial to the observations in the second paragraph at the beginning of Section 3. We need to compute $\Phi\star\Lambda - \Lambda\star\Phi$, so we expand it into the basis A and the coefficients are

$$\langle A, \Phi, \Lambda\rangle - \langle A, \Lambda, \Phi\rangle = \langle A, \Phi, \Lambda\rangle(1 + (-)^{g(A)g(\Phi)+\ell(A)+\ell(\Phi)+\ell(\Lambda)})$$

In our case, we have always that $g(A) = 2$, so we must have

$$\ell(A) + \ell(\Phi) + \ell(\Lambda) = \text{even};$$

otherwise the coefficient would be zero.

For example, when $|\Phi\rangle = |\Omega\rangle$ and $|\Lambda\rangle = b_{-2}\left|\Omega\right>$, only even levels of A have non zero coefficients, while when $|\Phi\rangle = |\Omega\rangle$ and $|\Lambda\rangle = b_{-1}\left|\Omega\right>$, only the odd levels of A have non zero coefficients. Of such a simplification we have taken great advantage in the computations of Sections 3 and 4.

We present below the gauge transformation on a string field. Here we consider the case that the string field has only even levels, so for the gauge transformation of even levels we have only even level gauge parameters while for the gauge transformation of odd levels we have only odd level gauge parameters. We divide the gauge transformation into two parts. The first part ($\delta^{(1)}\eta_{\ell,i}$) is $Q_B\Lambda$, which is exact at every level. The second part ($\delta\eta_{\ell,i}$) is $\Psi_0\star\Lambda - \Lambda\star\Psi_0$; it is approximate in the level truncation.

$Q_B\Lambda$ **part:**

$$\begin{aligned}
\delta^{(1)}\eta_{2,1} &= -3\mu_{2,1} \\
\delta^{(1)}\eta_{2,2} &= -\mu_{2,1} \\
\delta^{(1)}\eta_{2,3} &= 1\mu_{2,1} \\
\delta^{(1)}\eta_{4,1} &= -7\mu_{4,1} + 5\mu_{4,2} + 6\mu_{4,3} - 52\mu_{4,4} \\
\delta^{(1)}\eta_{4,2} &= -6\mu_{4,1} - 3\mu_{4,2} - 13\mu_{4,5} \\
\delta^{(1)}\eta_{4,3} &= -5\mu_{4,1} - 1\mu_{4,2} - 2\mu_{4,3} \\
\delta^{(1)}\eta_{4,4} &= -3\mu_{4,1} - 2\mu_{4,3} \\
\delta^{(1)}\eta_{4,5} &= -3\mu_{4,2} + 4\mu_{4,3}
\end{aligned}$$

$$
\begin{aligned}
\delta^{(1)}\eta_{4,6} &= \mu_{4,1} + 2\mu_{4,4} \\
\delta^{(1)}\eta_{4,7} &= \mu_{4,3} - 3\mu_{4,4} \\
\delta^{(1)}\eta_{4,8} &= \mu_{4,2} - 4\mu_{4,4} - 3\mu_{4,5} \\
\delta^{(1)}\eta_{4,9} &= -3\mu_{4,5} \\
\delta^{(1)}\eta_{4,10} &= \mu_{4,5}
\end{aligned}
$$

for even level and

$$
\begin{aligned}
\delta^{(1)}\eta_{3,1} &= -5\mu_{3,1} + 4\mu_{3,2} - 13\mu_{3,3} \\
\delta^{(1)}\eta_{3,2} &= -4\mu_{3,1} - 2\mu_{3,2} \\
\delta^{(1)}\eta_{3,3} &= -2\mu_{3,1} - \mu_{3,2} \\
\delta^{(1)}\eta_{3,4} &= \mu_{3,1} + \mu_{3,3} \\
\delta^{(1)}\eta_{3,5} &= \mu_{3,2} - 2\mu_{3,3} \\
\delta^{(1)}\eta_{4,1} &= -9\mu_{5,1} + 6\mu_{5,2} + 7\mu_{5,3} + 8\mu_{5,5} - 130\mu_{5,6} - 78\mu_{5,10} \\
\delta^{(1)}\eta_{4,2} &= -8\mu_{5,1} - 4\mu_{5,2} + 6\mu_{5,4} - 52\mu_{5,6} \\
\delta^{(1)}\eta_{4,3} &= -7\mu_{5,1} - \mu_{5,2} - 3\mu_{5,3} - 4\mu_{5,4} - 13\mu_{5,8} \\
\delta^{(1)}\eta_{4,4} &= -6\mu_{5,1} - 2\mu_{5,3} - 2\mu_{5,5} \\
\delta^{(1)}\eta_{4,5} &= -4\mu_{5,1} - \mu_{5,4} - 3\mu_{5,5} \\
\delta^{(1)}\eta_{4,6} &= -4\mu_{5,2} - 5\mu_{5,4} + 6\mu_{5,5} + 13\mu_{5,9} \\
\delta^{(1)}\eta_{4,7} &= -4\mu_{5,3} + 3\mu_{5,4} + 5\mu_{5,5} \\
\delta^{(1)}\eta_{4,8} &= \mu_{5,1} + 3\mu_{5,6} + \mu_{5,7} + \mu_{5,10} \\
\delta^{(1)}\eta_{4,9} &= \mu_{5,5} - 4\mu_{5,6} \\
\delta^{(1)}\eta_{4,10} &= \mu_{5,3} - 5\mu_{5,6} - 3\mu_{5,7} - 3\mu_{5,10} \\
\delta^{(1)}\eta_{4,11} &= \mu_{5,4} - 4\mu_{5,7} - \mu_{5,9} \\
\delta^{(1)}\eta_{4,12} &= \mu_{5,2} - 6\mu_{5,6} - 5\mu_{5,8} + 4\mu_{5,9} - 34\mu_{5,10} \\
\delta^{(1)}\eta_{4,13} &= -4\mu_{5,7} - 4\mu_{5,8} - 2\mu_{5,9} \\
\delta^{(1)}\eta_{4,14} &= -\mu_{5,4} - 4\mu_{5,8} - \mu_{5,9} \\
\delta^{(1)}\eta_{4,15} &= \mu_{5,7} + \mu_{5,8} + 2\mu_{5,10} \\
\delta^{(1)}\eta_{4,16} &= \mu_{5,9} - 4\mu_{5,10}
\end{aligned}
$$

for odd level (only nonzero contributions are listed).

$\Psi_0 \star \Lambda - \Lambda \star \Psi_0$ **part:**

Here we show only $\delta\eta_{0,1}$ and $\delta\eta_{1,1}$. For the complete results up to levels 4 and 5 for all η's, due to the enormity of the expressions, the reader is referred to the web-page http://pierre.mit.edu/˜yhe/gaugetransf.dvi.

$$\begin{aligned}
\delta\eta_{0,1} = &\frac{-16\eta_{0,1}\mu_{2,1}}{9} - \frac{464\eta_{2,1}\mu_{2,1}}{243} + \frac{128\eta_{2,2}\mu_{2,1}}{81} + \frac{1040\eta_{2,3}\mu_{2,1}}{243} \\
&-\frac{8576\eta_{4,1}\mu_{2,1}}{6561} + \frac{496\eta_{4,2}\mu_{2,1}}{729} + \frac{7040\eta_{4,3}\mu_{2,1}}{6561} - \frac{2816\eta_{4,4}\mu_{2,1}}{2187} \\
&+\frac{6016\eta_{4,5}\mu_{2,1}}{6561} - \frac{2080\eta_{4,6}\mu_{2,1}}{243} + \frac{30160\eta_{4,8}\mu_{2,1}}{6561} - \frac{8320\eta_{4,9}\mu_{2,1}}{2187} \\
&-\frac{112736\eta_{4,10}\mu_{2,1}}{6561} + \frac{352\eta_{0,1}\mu_{4,1}}{243} + \frac{6112\eta_{2,1}\mu_{4,1}}{6561} - \frac{2816\eta_{2,2}\mu_{4,1}}{2187} \\
&-\frac{22880\eta_{2,3}\mu_{4,1}}{6561} - \frac{290560\eta_{4,1}\mu_{4,1}}{177147} + \frac{32864\eta_{4,2}\mu_{4,1}}{177147} - \frac{9472\eta_{4,3}\mu_{4,1}}{19683} \\
&+\frac{61952\eta_{4,4}\mu_{4,1}}{59049} - \frac{7424\eta_{4,5}\mu_{4,1}}{19683} + \frac{45760\eta_{4,6}\mu_{4,1}}{6561} - \frac{397280\eta_{4,8}\mu_{4,1}}{177147} \\
&+\frac{183040\eta_{4,9}\mu_{4,1}}{59049} + \frac{2480192\eta_{4,10}\mu_{4,1}}{177147} + \frac{176\eta_{0,1}\mu_{4,2}}{243} + \frac{11248\eta_{2,1}\mu_{4,2}}{6561} \\
&+\frac{6016\eta_{2,2}\mu_{4,2}}{6561} - \frac{11440\eta_{2,3}\mu_{4,2}}{6561} + \frac{17536\eta_{4,1}\mu_{4,2}}{19683} + \frac{217136\eta_{4,2}\mu_{4,2}}{177147} \\
&+\frac{14720\eta_{4,3}\mu_{4,2}}{177147} - \frac{7424\eta_{4,4}\mu_{4,2}}{19683} - \frac{80512\eta_{4,5}\mu_{4,2}}{177147} + \frac{22880\eta_{4,6}\mu_{4,2}}{6561} \\
&-\frac{731120\eta_{4,8}\mu_{4,2}}{177147} - \frac{391040\eta_{4,9}\mu_{4,2}}{177147} + \frac{1240096\eta_{4,10}\mu_{4,2}}{177147} + \frac{8192\eta_{2,1}\mu_{4,3}}{6561} \\
&+\frac{303104\eta_{4,1}\mu_{4,3}}{177147} + \frac{131072\eta_{4,2}\mu_{4,3}}{177147} - \frac{139264\eta_{4,3}\mu_{4,3}}{177147} - \frac{532480\eta_{4,8}\mu_{4,3}}{177147} \\
&+\frac{212992\eta_{2,3}\mu_{4,4}}{6561} + \frac{2129920\eta_{4,6}\mu_{4,4}}{19683} - \frac{13631488\eta_{4,7}\mu_{4,4}}{177147} - \frac{4046848\eta_{4,8}\mu_{4,4}}{177147} \\
&-\frac{1703936\eta_{4,9}\mu_{4,4}}{177147} - \frac{20873216\eta_{4,10}\mu_{4,4}}{177147} + \frac{1040\eta_{0,1}\mu_{4,5}}{243} + \frac{30160\eta_{2,1}\mu_{4,5}}{6561} \\
&-\frac{8320\eta_{2,2}\mu_{4,5}}{2187} - \frac{120848\eta_{2,3}\mu_{4,5}}{6561} + \frac{557440\eta_{4,1}\mu_{4,5}}{177147} - \frac{32240\eta_{4,2}\mu_{4,5}}{19683} \\
&-\frac{457600\eta_{4,3}\mu_{4,5}}{177147} + \frac{183040\eta_{4,4}\mu_{4,5}}{59049} - \frac{391040\eta_{4,5}\mu_{4,5}}{177147} + \frac{3117920\eta_{4,6}\mu_{4,5}}{177147} \\
&-\frac{1703936\eta_{4,7}\mu_{4,5}}{177147} - \frac{3504592\eta_{4,8}\mu_{4,5}}{177147} + \frac{966784\eta_{4,9}\mu_{4,5}}{59049} + \frac{5034016\eta_{4,10}\mu_{4,5}}{59049}
\end{aligned}$$

$$\delta\eta_{1,1} = \frac{-16\eta_{0,1}\mu_{1,1}}{9} + \frac{16\eta_{2,1}\mu_{1,1}}{81} + \frac{896\eta_{2,2}\mu_{1,1}}{243} + \frac{1040\eta_{2,3}\mu_{1,1}}{243}$$

$$
\begin{aligned}
&-\frac{640\eta_{4,1}\mu_{1,1}}{2187}+\frac{5488\eta_{4,2}\mu_{1,1}}{6561}-\frac{640\eta_{4,3}\mu_{1,1}}{729}-\frac{15616\eta_{4,4}\mu_{1,1}}{6561}\\
&-\frac{7808\eta_{4,5}\mu_{1,1}}{6561}-\frac{2080\eta_{4,6}\mu_{1,1}}{243}-\frac{1040\eta_{4,8}\mu_{1,1}}{2187}-\frac{58240\eta_{4,9}\mu_{1,1}}{6561}\\
&-\frac{112736\eta_{4,10}\mu_{1,1}}{6561}-\frac{80\eta_{0,1}\mu_{3,1}}{81}+\frac{80\eta_{2,1}\mu_{3,1}}{729}-\frac{7040\eta_{2,2}\mu_{3,1}}{6561}\\
&+\frac{5200\eta_{2,3}\mu_{3,1}}{2187}-\frac{159872\eta_{4,1}\mu_{3,1}}{177147}+\frac{7600\eta_{4,2}\mu_{3,1}}{6561}-\frac{3200\eta_{4,3}\mu_{3,1}}{6561}\\
&+\frac{9472\eta_{4,4}\mu_{3,1}}{19683}+\frac{108160\eta_{4,5}\mu_{3,1}}{177147}-\frac{10400\eta_{4,6}\mu_{3,1}}{2187}-\frac{5200\eta_{4,8}\mu_{3,1}}{19683}\\
&+\frac{457600\eta_{4,9}\mu_{3,1}}{177147}-\frac{563680\eta_{4,10}\mu_{3,1}}{59049}-\frac{256\eta_{0,1}\mu_{3,2}}{243}-\frac{9472\eta_{2,1}\mu_{3,2}}{6561}\\
&+\frac{8192\eta_{2,2}\mu_{3,2}}{6561}+\frac{16640\eta_{2,3}\mu_{3,2}}{6561}-\frac{114688\eta_{4,1}\mu_{3,2}}{177147}-\frac{6400\eta_{4,2}\mu_{3,2}}{177147}\\
&+\frac{81920\eta_{4,3}\mu_{3,2}}{177147}-\frac{114688\eta_{4,4}\mu_{3,2}}{177147}+\frac{303104\eta_{4,5}\mu_{3,2}}{177147}-\frac{33280\eta_{4,6}\mu_{3,2}}{6561}\\
&+\frac{615680\eta_{4,8}\mu_{3,2}}{177147}-\frac{532480\eta_{4,9}\mu_{3,2}}{177147}-\frac{1803776\eta_{4,10}\mu_{3,2}}{177147}+\frac{1040\eta_{0,1}\mu_{3,3}}{243}\\
&-\frac{1040\eta_{2,1}\mu_{3,3}}{2187}-\frac{58240\eta_{2,2}\mu_{3,3}}{6561}-\frac{120848\eta_{2,3}\mu_{3,3}}{6561}+\frac{41600\eta_{4,1}\mu_{3,3}}{59049}\\
&-\frac{356720\eta_{4,2}\mu_{3,3}}{177147}+\frac{41600\eta_{4,3}\mu_{3,3}}{19683}+\frac{1015040\eta_{4,4}\mu_{3,3}}{177147}+\frac{507520\eta_{4,5}\mu_{3,3}}{177147}\\
&+\frac{3117920\eta_{4,6}\mu_{3,3}}{177147}-\frac{1703936\eta_{4,7}\mu_{3,3}}{177147}+\frac{120848\eta_{4,8}\mu_{3,3}}{59049}+\frac{6767488\eta_{4,9}\mu_{3,3}}{177147}\\
&+\frac{5034016\eta_{4,10}\mu_{3,3}}{59049}+\frac{5680\eta_{0,1}\mu_{5,1}}{6561}-\frac{5680\eta_{2,1}\mu_{5,1}}{59049}+\frac{152960\eta_{2,2}\mu_{5,1}}{177147}\\
&-\frac{369200\eta_{2,3}\mu_{5,1}}{177147}+\frac{26240\eta_{4,1}\mu_{5,1}}{4782969}-\frac{717680\eta_{4,2}\mu_{5,1}}{1594323}+\frac{227200\eta_{4,3}\mu_{5,1}}{531441}\\
&-\frac{1804544\eta_{4,4}\mu_{5,1}}{4782969}-\frac{803200\eta_{4,5}\mu_{5,1}}{1594323}+\frac{738400\eta_{4,6}\mu_{5,1}}{177147}+\frac{369200\eta_{4,8}\mu_{5,1}}{1594323}\\
&-\frac{9942400\eta_{4,9}\mu_{5,1}}{4782969}+\frac{40021280\eta_{4,10}\mu_{5,1}}{4782969}+\frac{304\eta_{0,1}\mu_{5,2}}{2187}-\frac{105136\eta_{2,1}\mu_{5,2}}{177147}\\
&-\frac{103040\eta_{2,2}\mu_{5,2}}{59049}-\frac{19760\eta_{2,3}\mu_{5,2}}{59049}-\frac{730240\eta_{4,1}\mu_{5,2}}{4782969}-\frac{1423184\eta_{4,2}\mu_{5,2}}{1594323}\\
&-\frac{352640\eta_{4,3}\mu_{5,2}}{531441}+\frac{91904\eta_{4,4}\mu_{5,2}}{1594323}+\frac{7936640\eta_{4,5}\mu_{5,2}}{4782969}+\frac{39520\eta_{4,6}\mu_{5,2}}{59049}\\
&+\frac{6833840\eta_{4,8}\mu_{5,2}}{4782969}+\frac{6697600\eta_{4,9}\mu_{5,2}}{1594323}+\frac{2141984\eta_{4,10}\mu_{5,2}}{1594323}+\frac{80\eta_{0,1}\mu_{5,3}}{81}\\
&-\frac{80\eta_{2,1}\mu_{5,3}}{729}+\frac{26240\eta_{2,2}\mu_{5,3}}{177147}-\frac{5200\eta_{2,3}\mu_{5,3}}{2187}+\frac{390272\eta_{4,1}\mu_{5,3}}{1594323}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1519120\eta_{4,2}\mu_{5,3}}{1594323}+\frac{3200\eta_{4,3}\mu_{5,3}}{6561}+\frac{84736\eta_{4,4}\mu_{5,3}}{1594323}+\frac{18560\eta_{4,5}\mu_{5,3}}{59049}\\
&+\frac{10400\eta_{4,6}\mu_{5,3}}{2187}+\frac{5200\eta_{4,8}\mu_{5,3}}{19683}-\frac{1705600\eta_{4,9}\mu_{5,3}}{4782969}+\frac{563680\eta_{4,10}\mu_{5,3}}{59049}\\
&+\frac{1280\eta_{0,1}\mu_{5,4}}{2187}+\frac{47360\eta_{2,1}\mu_{5,4}}{59049}+\frac{40960\eta_{2,2}\mu_{5,4}}{177147}-\frac{83200\eta_{2,3}\mu_{5,4}}{59049}\\
&+\frac{9060352\eta_{4,1}\mu_{5,4}}{4782969}-\frac{98560\eta_{4,2}\mu_{5,4}}{531441}-\frac{409600\eta_{4,3}\mu_{5,4}}{1594323}+\frac{212992\eta_{4,4}\mu_{5,4}}{4782969}\\
&+\frac{1515520\eta_{4,5}\mu_{5,4}}{4782969}+\frac{166400\eta_{4,6}\mu_{5,4}}{59049}-\frac{3078400\eta_{4,8}\mu_{5,4}}{1594323}-\frac{2662400\eta_{4,9}\mu_{5,4}}{4782969}\\
&+\frac{9018880\eta_{4,10}\mu_{5,4}}{1594323}+\frac{5632\eta_{0,1}\mu_{5,5}}{6561}+\frac{15872\eta_{2,1}\mu_{5,5}}{19683}-\frac{212992\eta_{2,2}\mu_{5,5}}{177147}\\
&-\frac{366080\eta_{2,3}\mu_{5,5}}{177147}-\frac{163840\eta_{4,1}\mu_{5,5}}{531441}-\frac{1235456\eta_{4,2}\mu_{5,5}}{4782969}-\frac{229376\eta_{4,3}\mu_{5,5}}{1594323}\\
&+\frac{360448\eta_{4,4}\mu_{5,5}}{531441}-\frac{409600\eta_{4,5}\mu_{5,5}}{531441}+\frac{732160\eta_{4,6}\mu_{5,5}}{177147}-\frac{1031680\eta_{4,8}\mu_{5,5}}{531441}\\
&+\frac{13844480\eta_{4,9}\mu_{5,5}}{4782969}+\frac{39683072\eta_{4,10}\mu_{5,5}}{4782969}-\frac{2080\eta_{0,1}\mu_{5,6}}{243}+\frac{2080\eta_{2,1}\mu_{5,6}}{2187}\\
&+\frac{116480\eta_{2,2}\mu_{5,6}}{6561}+\frac{3117920\eta_{2,3}\mu_{5,6}}{177147}-\frac{83200\eta_{4,1}\mu_{5,6}}{59049}+\frac{713440\eta_{4,2}\mu_{5,6}}{177147}\\
&-\frac{83200\eta_{4,3}\mu_{5,6}}{19683}-\frac{2030080\eta_{4,4}\mu_{5,6}}{177147}-\frac{1015040\eta_{4,5}\mu_{5,6}}{177147}-\frac{226516160\eta_{4,6}\mu_{5,6}}{1594323}\\
&-\frac{17039360\eta_{4,7}\mu_{5,6}}{531441}-\frac{3117920\eta_{4,8}\mu_{5,6}}{1594323}-\frac{174603520\eta_{4,9}\mu_{5,6}}{4782969}-\frac{604222528\eta_{4,10}\mu_{5,6}}{4782969}\\
&+\frac{4259840\eta_{2,3}\mu_{5,7}}{177147}+\frac{42598400\eta_{4,6}\mu_{5,7}}{531441}-\frac{54525952\eta_{4,7}\mu_{5,7}}{4782969}-\frac{80936960\eta_{4,8}\mu_{5,7}}{4782969}\\
&-\frac{34078720\eta_{4,9}\mu_{5,7}}{4782969}-\frac{417464320\eta_{4,10}\mu_{5,7}}{4782969}+\frac{5200\eta_{0,1}\mu_{5,8}}{2187}-\frac{5200\eta_{2,1}\mu_{5,8}}{19683}\\
&+\frac{457600\eta_{2,2}\mu_{5,8}}{177147}-\frac{604240\eta_{2,3}\mu_{5,8}}{59049}+\frac{10391680\eta_{4,1}\mu_{5,8}}{4782969}-\frac{494000\eta_{4,2}\mu_{5,8}}{177147}\\
&+\frac{208000\eta_{4,3}\mu_{5,8}}{177147}-\frac{615680\eta_{4,4}\mu_{5,8}}{531441}-\frac{7030400\eta_{4,5}\mu_{5,8}}{4782969}+\frac{15589600\eta_{4,6}\mu_{5,8}}{1594323}\\
&+\frac{28966912\eta_{4,7}\mu_{5,8}}{4782969}+\frac{604240\eta_{4,8}\mu_{5,8}}{531441}-\frac{53173120\eta_{4,9}\mu_{5,8}}{4782969}+\frac{25170080\eta_{4,10}\mu_{5,8}}{531441}\\
&+\frac{16640\eta_{0,1}\mu_{5,9}}{6561}+\frac{615680\eta_{2,1}\mu_{5,9}}{177147}-\frac{532480\eta_{2,2}\mu_{5,9}}{177147}-\frac{1933568\eta_{2,3}\mu_{5,9}}{177147}\\
&+\frac{7454720\eta_{4,1}\mu_{5,9}}{4782969}+\frac{416000\eta_{4,2}\mu_{5,9}}{4782969}-\frac{5324800\eta_{4,3}\mu_{5,9}}{4782969}+\frac{7454720\eta_{4,4}\mu_{5,9}}{4782969}\\
&-\frac{19701760\eta_{4,5}\mu_{5,9}}{4782969}+\frac{49886720\eta_{4,6}\mu_{5,9}}{4782969}-\frac{109051904\eta_{4,7}\mu_{5,9}}{4782969}-\frac{71542016\eta_{4,8}\mu_{5,9}}{4782969}
\end{aligned}
$$

$$+\frac{61874176\eta_{4,9}\mu_{5,9}}{4782969}+\frac{80544256\eta_{4,10}\mu_{5,9}}{1594323}-\frac{112736\eta_{0,1}\mu_{5,10}}{6561}+\frac{112736\eta_{2,1}\mu_{5,10}}{59049}$$
$$+\frac{6313216\eta_{2,2}\mu_{5,10}}{177147}+\frac{5034016\eta_{2,3}\mu_{5,10}}{59049}-\frac{4509440\eta_{4,1}\mu_{5,10}}{1594323}+\frac{38668448\eta_{4,2}\mu_{5,10}}{4782969}$$
$$-\frac{4509440\eta_{4,3}\mu_{5,10}}{531441}-\frac{110030336\eta_{4,4}\mu_{5,10}}{4782969}-\frac{55015168\eta_{4,5}\mu_{5,10}}{4782969}-\frac{604222528\eta_{4,6}\mu_{5,10}}{4782969}$$
$$+\frac{166985728\eta_{4,7}\mu_{5,10}}{4782969}-\frac{5034016\eta_{4,8}\mu_{5,10}}{531441}-\frac{281904896\eta_{4,9}\mu_{5,10}}{1594323}-\frac{279502912\eta_{4,10}\mu_{5,10}}{531441}$$

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