Guest Workers in the Underground Economy

by

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Abstract

Guest-worker programs have been providing rapidly growing economies with millions of foreign workers over the last couple of decades. With the duration of stay strictly limited by program rules in some host countries and wages paid to guest workers often set at sub-market levels, many migrants choose to overstay and seek unauthorized employment. The model we develop examines how the wage of illegal aliens and the flow of guest workers transiting to undocumented status are affected by the rules of the program, enforcement measures of the host country, and market conditions facing migrants at home and abroad. Lengthening the duration of official work permits is found to decrease the stock of undocumented workers, but it has an ambiguous effect on their wage. An expansion in the allowed inflow of documented guest workers has a negative impact on the wage of undocumented workers and an ambiguous effect on their stock.

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Key Words: Temporary migration, undocumented workers, underground economy

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1 Introduction

Rapid economic growth and demographic factors have combined to create shortages of low-skilled labor in many economies. Germany and other Western European countries addressed this problem in the 1960s and 70s by establishing guest-worker programs. In the Middle East, temporary migration schemes have expanded to the point where foreign guest workers in some of the states on the Arabian Peninsula accounted for 80-90% of the workforce in 2004 (see Kapiszewski, 2006). Over the last two decades, growth in East Asia has also generated significant shortages of low-skilled workers. The response of the authorities in South Korea, Taiwan, Hong Kong, Singapore, Brunei, Japan, and Malaysia was to establish programs for the recruitment of temporary foreign workers (sometimes classified as trainees) from other, relatively poorer Asian economies such as Nepal, Bangladesh, Indonesia, Sri Lanka, Thailand, India, Pakistan, the Philippines, and Viet Nam. Since these programs offer wages to guest workers that are considerably higher than the wages prevailing in the source countries of program participants, there is typically no difficulty in attracting migrants.\(^3\) The problem is making sure they go back home when their work permit expires. In fact large numbers of guest workers remain abroad illegally in order to accumulate additional savings by working in the underground economy.\(^4\) What makes clandestine employment particularly attractive is that in many cases it offers a higher wage and more flexible conditions of employment when

\(^3\)See the Human Development Report (2009).

compared with the official guest-worker programs. Surveys of Thai contract workers as well as of undocumented migrants employed in the more advanced countries of East Asia, indicate that wages of undocumented workers in Japan, South Korea, and Taiwan can exceed the wages of foreign contract workers by 50% or more, depending on the occupation. The other side of the coin is that undocumented workers face strict deportation measures if apprehended by the authorities. Japan, Singapore, South Korea, Taiwan, Malaysia, Saudi Arabia, United Arab Emirates, and other GCC states are well known for their strict enforcement of laws pertaining to residency of foreign nationals. In addition to apprehension and deportation, an illegal alien sometimes faces corporal punishment, a fine and even a jail sentence. Table 1 provides an indication of the role that guest-workers play in these economies and the attitude of the host-country authorities towards illegal immigrants. Column 1 describes the inflows of contract workers in 2010-2011 from a select group of source countries (SGSC), consisting of Nepal, Bangladesh, Indonesia, Sri Lanka, Thailand, India, Pakistan, the Philippines, and Viet Nam. Column 2 gives the total stock of foreign labor from all sources, including migrants with rights to permanent residence, such as Peruvians and Brazilians of Japanese ancestry in Japan, while Column 3 shows the size of the total labor force of the host country.

\textsuperscript{5}See Jones and Pardthaisong (1999), Sobieszczyk (2000), Hahn and Choi (2006), and Park (2008)). When foreign workers are classified as trainees, the difference can be much greater. Ihwan (2005) reports that a trainee in South Korea who transited from a contractual employer to work as a painter in the underground economy was able to increase his earnings by a multiple of 8. The fact that undocumented migrants in East Asian economies can earn more than official guest workers or trainees is in sharp contrast with what is observed in Western advanced countries, where undocumented status typically implies lower earnings. More will be said on this in Section 3 below.

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<table>
<thead>
<tr>
<th>Host States</th>
<th>Inflow of Guest Workers from SGSC (thousands)</th>
<th>Total Stock of Foreign Labor-2010 (thousands)</th>
<th>Total Labor Force-2010 (millions)</th>
<th>Maximum Guest-Worker Stay</th>
<th>Strict Deportation Policies</th>
<th>Fines and Penalties for Overstaying</th>
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<td>Yes</td>
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</table>

Notes: In Column 1, inflows of guest workers from Nepal, Indonesia, and Thailand are for the year 2011, from Bangladesh, Sri Lanka, India, Philippines and Viet Nam are for the year 2010, and for Pakistan for 2008. The figures are for overseas workers whose departure is recorded by the government agencies in the origin country. Coverage of individual departures for employment may be partial or limited to employment under bilateral agreements. For a detailed description of the different sources used to build the statistics on the inflows, see OECD (2012, p.169) and for the stocks of foreign labor in 2010, displayed in Column 2, see OECD (2012, p.168). *Statistics on the stocks of foreign labour for GCC States are not available but immigrants represent around 52% of the populations living in GCC States, with a large variation across countries ranging from 32% in Saudi Arabia to 89% in United Arab Emirates (Thiollet, 2015). Data on the size of the labor force is from the World Bank for the year 2010 and from the CIA Fact Book for Taiwan for the year 2010. For more details on the guest-worker policies, see Asis (2006), Abella (2009), and IOM (2010).

Table 1: The role of guest-workers from South and South-East Asia in the main labor-importing countries of East Asia and the Middle East and the policies of the latter with respect to illegal immigration.

One would expect that there is a strong connection between the guest-worker programs through which the migrants enter the economy and the equilibrium wage and employment of clandestine labor in the underground economy. The main contribution of our paper is to provide a theoretical analysis of these links. More specifically, our purpose is to address some of the key questions that arise in this context: How do the rules of the guest-worker program
affect the propensity of foreign contract workers to overstay and become illegal aliens? What is the role of employer sanctions, worksite inspections, and deportation policies in controlling the stock of undocumented workers? What is the relationship between the conditions facing documented guest workers and the market for undocumented labor? How do wages of clandestine workers respond to the policies in place? These and other related questions are of major importance to a growing number of countries that rely heavily on guest workers to meet shortages in their market for low-skilled labor. The literature on temporary migration is only beginning to address them.

The connection between temporary migration of contract workers and illegal immigration was examined for the first time in the context of a theoretical model developed by Epstein, Hillman and Weiss (1999). They study the problem facing a documented guest worker who has to decide whether or not to run away from his employer and overstay in the host country for an extra year if he receives an offer of undocumented employment. Their analysis is conducted within a framework where the authorities require the employer to post a bond for each imported worker, with the bond forfeited if the migrant does not leave the country when the permit expires. Subsequent works by Schiff (2007, 2011) focus on the links between illegal and guest-worker migration from a macroeconomic perspective, with the goal of analyzing the policies required to attain the optimal proportion of documented to undocumented workers employed in the economy. The article by Djajić and Michael (2013) is in a similar vein, although the focus is on the host-country problem of setting the optimal duration of the
permit issued to guest workers.

Djajić (2013) takes a somewhat different approach by examining the conditions under which foreign contract workers have sufficiently strong incentives to return home once their work permit expires, while Djajić and Vinogradova (2015) examine the question of how long it is optimal for a guest worker to overstay in the host country when the incentives for voluntary return at the end of the contract happen to be inadequate. That analysis is conducted taking the labor-market conditions of the host country as given. The present paper goes a step further to specify the structure of the market for undocumented labor and determine endogenously the equilibrium stock of illegal aliens and their wage rate. An important feature of the model is that it relates these key endogenous variables to the characteristics of the guest-worker program and a wide range of immigration policy instruments of the host country. Our positive approach therefore stands in contrast with the existing literature, which focuses on defining the optimal policies while neglecting the complex links between the official temporary migration programs and the clandestine labor market.

In terms of its approach, our work is also closely related to the recent contribution by Camacho, Mariani, and Pensierosos (2013). They study how fiscal and migration policies affect both illegal migration and the size of the informal economy. A distinctive feature, however, is that they do not model the interactions between a guest-worker program and the clandestine labor market, which are at the center of our analysis. They focus instead on the role of fiscal policy as a factor influencing whether firms choose to operate in the formal sector.
or informally, in which case they can tap the market for undocumented workers. One of the key findings of Camacho, Mariani, and Pensierosos (2013) is that illegal immigration and the level of informal activity depends non-monotonically on the tax rate imposed on the firms.

Thus the focus of our paper is on the market for low-skilled undocumented foreign labor in an economy with a guest-worker program of the type used to bring contract workers to the advanced and emerging economies of East Asia over the last two decades. Section 2 defines the problem facing an individual program participant and examines the conditions under which it pays to overstay and seek clandestine employment. Both the rules of the guest-worker program and a wide range of immigration policies and enforcement measures influence the behavior of foreign workers. Relevant policies in the East Asia setting include the quota on the number of guest-workers admitted each year, the wage they are offered under the terms of the program, the duration of their work permit, the proportion of their official wage withheld to guarantee contract completion and return to the source country, deportation measures and fines imposed on those who overstay, and the penalties imposed on employers of undocumented aliens. Section 3 considers the problem facing employers of undocumented foreign workers and derives the demand schedule for clandestine labor. Section 4 examines the implications of changes in policy instruments on the market wage of clandestine labor and the stock and flow of undocumented workers. Finally, Section 5 concludes the paper with a summary of the model’s main policy implications.
2 Return or Overstay?

Let us assume that under an exogenously given quota, the host country admits each year \( G \) low-skilled workers from a source country on a temporary basis. The work permits are valid for \( \tau \) years and non-renewable.\(^6\) They offer migrant workers the opportunity to earn the wage \( \bar{W} \), which is greater than the wage, \( W \), that these same workers can earn in their country of origin. We take both \( \bar{W} \) and \( W \) as given. Participants are assumed to have a time horizon of \( T \) years. The undiscounted lifetime earnings of a guest worker who obeys the rules of the program and returns home after serving for \( \tau \) years as a contract worker in the host country are thus given by

\[
Y = \bar{W}\tau + W(T - \tau).
\]

Instead of returning, as required by program rules, a guest worker may choose to overstay. This outcome was frequently observed in South Korea in the 1990s, when more than half of the foreign participants in their trainee program ended up working clandestinely in the underground economy. Out of a total of 110,250 trainees admitted into the program, 63,515 transited to the underground economy as of December 2001 (see Hahn and Choi, 2006). In 2006, about 7% of the 330,000 foreign workers in Taiwan were reported missing and presumably working without documentation (see Abella, 2009).

\(^6\)The maximum duration of stay for a low-skilled migrant (trainee) is, for example, three years in Japan, four years in Singapore and Cyprus, five years in Israel, and two 3-year stays (with the first stay followed by a mandatory return to the country of origin), for a maximum of six years, in South Korea and Taiwan (see Asis, 2006, Abella, 2009, and IOM, 2010).
The motive for overstaying is to accumulate more savings by working clandestinely at the wage \( \tilde{W} \), which we take to be endogenously determined by the supply and demand for undocumented labor in the host country. Let us assume that the failure to return home when the work permit expires implies the loss of withheld wages and it exposes the migrant to strict deportation measures and a fine, \( \Phi \). We denote by \( \beta \) the proportion of a guest worker’s wage withheld by the employer, as fixed by the rules of the guest-worker program.\(^7\) Withheld earnings are returned to the worker only at the end of the contract period, just at the moment of departure from the host country. Thus a worker who chooses to overstay, forfeits the withheld wages and has an expected undiscounted lifetime income of

\[
Y^n = \tilde{W} (1 - \beta) \tau + \tilde{W} \psi - \Phi + W (T - \tau - \psi),
\]

where \( \psi \) is the expected duration of a worker’s clandestine employment in the underground economy before being apprehended, fined, and deported. A more vigorous deportation policy...

\(^7\)The practice of withholding wages of guest workers can be traced back to the Bracero program in the U.S.A. (1942-1964). The program required employers to withhold 10 percent of a bracero’s earnings, payable to the worker upon his return to Mexico (Ruhs, 2002). In East Asia, foreign workers are often obliged by their employers to participate in forced-savings programs, where a portion of the worker’s salary is withheld and deposited into a savings account to which the worker does not have access until his term of work is complete. In Taiwan, for example, forced savings of up to 30 percent of a worker’s salary is legally permitted. This mechanism provides the employer with what is sometimes referred to as “guarantee money” or “runaway insurance” (Verite, 2011). Similarly, Japanese employers often require from recruiting agents to have foreign trainees sign a contract agreeing that a part of their wages will be withheld as a “security deposit” until their departure from Japan. An example of such a contract reads: “In addition to the 25,000 Yen in cash paid monthly to Party B (the trainee), the remaining wage will be deposited at a Japanese bank under Party B’s name and the passbook will be managed by party B’s assigned company, to be returned to Party B before returning to their home country” (CLB, 2011, p.36). The practice is current in other Asian labor-importing countries, even where it is not authorized. In Malaysia, NGOs have reported that the police often fail or refuse to investigate complaints relating to the withholding of wages (United States Department of State, 2011). Amnesty International (2009) offers a more detailed documentation of evidence on cases of salary withholding in South Korea. Withholding of a guest worker’s wage is also a common practice in the Middle East. For the cases of Qatar and the other GCC states, see Garner et al (2013).
in the host country implies a smaller value of $\psi$. According to the Japanese Ministry of Justice, 32,661 individuals from 99 countries went through deportation proceedings in 2009 (Williams (2010)). This amounts to roughly 1/3 of the estimated stock of 100,000 undocumented aliens living in Japan. Those who overstay can therefore expect to work in the underground economy for only about 3 years, on average, before being apprehended and deported. In Malaysia, where the estimated stock of illegal aliens from Indonesia is reported to be roughly 450,000, the number of Indonesians deported every month is around 10,000. This suggests that an undocumented Indonesian migrant in Malaysia can expect to work in the underground economy on average for approximately 4 years before being deported (see Vinogradova, 2011). The expected duration of an undocumented stay is very similar in other East Asian economies with strict deportation policies, such as Singapore, South Korea, and Taiwan.

We assume that workers who take part in the guest-worker program are averse to violating the laws of the host country. If they remain in the host country as illegal aliens they will be subject to arrest and deportation. This imposes a psychological cost of becoming an undocumented worker. Let us suppose that individuals are heterogeneous in terms of what they perceive to be the non-pecuniary cost of transiting to undocumented status. This cost is assumed to have a monetary equivalent of $\rho$ that is distributed identically and independently across generations of guest workers according to the density function $f(\rho)$ and distribution $F(\rho)$ over $[\underline{\rho}, \overline{\rho}]$. In such an environment, a risk-neutral guest worker prefers to overstay rather than return home when the work permit expires only if this increases the expected
lifetime income net of the cost, \( \rho \). Thus the condition for overstaying can be written as \( \rho < (\bar{W} - W) \psi - \bar{W} \beta \tau - \Phi \), and the proportion of workers overstaying is equal to:

\[
(3) \quad F((\bar{W} - W) \psi - \bar{W} \beta \tau - \Phi)
\]

With the immigration quota allowing only \( G \) workers to be admitted into the host country per unit of time and granted work permits of the duration \( \tau \), the steady-state stock of guest workers is \( \tau G \). If a fraction \( F((\bar{W} - W) \psi - \bar{W} \beta \tau - \Phi) \) of the flow due to return to the source country decides to overstay until apprehended and deported, this implies that at each point in time \( F((\bar{W} - W) \psi - \bar{W} \beta \tau - \Phi) G \) guest workers transit to the underground economy. Thus the undocumented labor inflow to the underground economy depends on host-country policies, as captured by the parameters \( \psi, \beta, \tau, \bar{W}, \Phi \) and \( G \), on the wage in the source country, \( W \), the market wage for clandestine labor in the underground economy, \( \bar{W} \), and on the distribution, \( F(\rho) \) of guest workers’ preferences for avoiding undocumented status. To simplify the analysis, we assume in what follows that \( \bar{W}(1 - \beta) > W \), so that a guest worker who intends to overstay does not have an incentive to run away from his contractual employer before time \( \tau \) in order to take up employment in the underground economy. The assumption that an undocumented alien can expect to work undetected in the underground economy for just \( \psi \) units of time, implies that a premature departure from his legal employer before the end of the contract entails spending correspondingly more time in the source country earning the wage \( W \) rather than \( \bar{W}(1 - \beta) \). As can be seen in eq. (2), running away from the legal employer (i.e.,
voluntarily reducing τ) lowers the expected lifetime earnings when \( \bar{W}(1 - \beta) > W \).

### 3 Demand for Undocumented Labor

Not all sectors and firms in the host country are authorized to employ foreign workers. Guest-worker programs have been specifically developed in South Korea, Taiwan, Singapore, Japan, and other economies in the region, to alleviate shortages of low-skilled labor in specific industries, such as manufacturing, cleaning and other services, agriculture, construction, and fish processing (see Park, 2012). For simplicity we will refer to these activities as belonging to Sector \( E \), the one eligible to hire guest workers. Employers in other industries (or Sector \( I \), hereafter) are ineligible in the sense of being obliged to look for workers in the local labor market, although to some extent they may be able to conceal employment of undocumented aliens.\(^8\) Such employment has the advantage that illegal aliens can be paid less than the native workers. In addition, it is possible to evade payroll taxes as well as a number of obligations that employers have in relation to documented labor. They don’t have to provide undocumented workers with health insurance, vacation pay, sick days, etc. (see, e.g., Djajić (1997) and Sobieszczyk (2000, p.402)). The downside is that employers of illegal aliens face penalties if their infractions are uncovered by the authorities.

\(^8\)In Japan, for example, undocumented immigrants learn by word of mouth or SMS messages which enterprises and labor contractors are willing to risk fines by disregarding workers’ legal status. One striking feature of East Asian economies such as Japan, Singapore, South Korea and Taiwan, is the very small size of their informal sector (Farrell, 2006).
America and Western Europe, it is typically assumed that illegal aliens face a wage penalty that reduces their earnings in relation to those of the natives and legal immigrants. The environment facing foreign workers is quite different in East Asia, where the interests of the employers have played a prominent role in the design of their guest-worker programs. The programs are intended not only to alleviate shortages in the labor market, but also to generate large rents for the firms that hire guest workers. Wages of trainees and guest workers are therefore set at levels considerably lower than those paid to native workers with the same qualifications. This results in a three tier wage structure, where the native workers receive higher wages than the illegal aliens (reflecting their legal status and the internal enforcement measures that deter employers from hiring undocumented foreign workers), while illegal aliens can earn higher wages than do documented guest workers. What sustains this relationship among the three wage levels is the large gap between the wage earned by natives and that earned by documented guest workers. This creates incentives for both the employers and overstaying guest workers to participate in the market for undocumented labor at a wage rate between the two extremes.

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10 From a theoretical perspective, there is nothing in our model that precludes the possibility of \( W \) being lower than \( \bar{W} \), depending on the policy environment which governs the supply and demand for undocumented labor. We focus on the case of \( W > \bar{W} \) as it corresponds to the market conditions facing foreign workers and trainees in economies such as Japan and South Korea. In Japan, for example, Sobieszczyk (2000) notes that a Thai documented migrant employed as an industrial worker in 1997 reported earning $1,171 per month plus room, while unauthorized migrants in the same country and the same occupation earned in 1996 between $2,076 and $4,193 per month plus room. She finds that earnings of undocumented migrants interviewed for her study are on average 103 percent higher than those of documented contract workers (Sobieszczyk, 2000, p.401). See Ihlwan (2005) for the case of South Korea.
It is important to bear in mind that employers in the $E$ sector have privileged access to low-cost documented guest workers. This reduces their incentive, when compared with employers in the $I$ sector, to hire illegal aliens. Moreover, hiring undocumented workers may jeopardize their eligibility to participate in the guest-worker program, which puts them under relatively greater legal, economic, and moral pressure to abstain from hiring undocumented labor. In what follows, we shall therefore assume that only firms in the $I$ sector have an incentive to engage in hiring undocumented workers.

Let us assume that Sector $I$ has $J$ identical firms whose owners may find it attractive to hire illegal aliens. Each firm has a fixed amount of capital, $\bar{K}$, producing output, $Q$, according to a CRS production function with low-skilled labor as the only variable factor.\footnote{Later in the paper, we discuss the case where firms in Sector I are heterogeneous and consider the possibility of allowing both $\bar{K}$ and $J$ to vary in the long run.} The number of native workers employed by the firm is denoted by $L$ and the number of undocumented workers by $U$.

\begin{equation}
Q = Q(\bar{K}, L + U).
\end{equation}

As indicated in eq. (4), the two types of workers are assumed to be perfect substitutes in production, although they enjoy different legal status.\footnote{The assumption that illegal immigrants and native workers are prefect substitutes in the low-skilled sectors where they are employed is standard in the theoretical literature on illegal immigration (see, e.g., Ethier, 1986, Bond and Chen, 1987, Djajić, 1987, 1997, 1999, Bandyopadhyay and Bandyopadhyay, 1998, Gaytan-Fregoso and Lahiri, 2000, Bandyopadhyay, 2006, and Woodland and Yoshida, 2006). The few empirical studies that examine the degree of substitutability between documented and undocumented labor, focusing on the US economy, suggest that the degree of substitutability is quite high (see Grossman, 1984 and Bean, Lowell and Taylor, 1988).} This has important implications...
with respect to their compensation. Let us assume that a firm found to employ undocumented workers must pay a penalty $\phi$ for each such worker detected on its premises.\textsuperscript{13} The probability, $\pi$, of a firm being caught with undocumented workers depends, of course, on the visibility of such employment to outsiders, including its competitors, clients, and the authorities. As all firms in Sector $I$ are of the same size, it is most realistic to assume that this visibility increases at an increasing rate with the number of undocumented workers hired. We can then write $\pi = \pi(U)$, with $\pi'(U) > 0$ and $\pi''(U) > 0$. The profit function of each of the $J$ firms is thus given by:

\begin{equation}
(5) \quad \Pi = Q(\bar{K}, L + U) - W^*L - \bar{W}U - \pi(U)U\phi,
\end{equation}

where $W^*$ is the market wage that a firm in sector $I$ faces when hiring native workers. We shall assume that $W^* = g(G\tau)$, with $g'(G\tau) < 0$. That is, $W^*$ depends on the stock of documented guest workers, $G\tau$, employed in Sector $E$ and the degree of mobility of native workers between Sectors $E$ and $I$, as reflected in the slope of the $g(.)$ function. A greater absolute value of the slope signifies a higher degree of intersectoral mobility of native workers. Thus an expansion of the guest worker program that admits a larger stock of foreign labor into Sector $E$, lowers the cost of hiring native workers in Sector $I$. This is based on the presumption that there is likely to be at least some degree of mobility of native workers between Sectors $E$ and $I$.\textsuperscript{14}

\textsuperscript{13}For earlier theoretical studies that model employer sanctions in a similar way, see Ethier (1986), Djajić (1997), Yoshida (2000) and Woodland and Yoshida (2006).
\textsuperscript{14}If we were to consider the extreme case in which there is no mobility of natives between $E$ and $I$, we would
Profit maximization by firms in Sector $I$ implies that each of them will hire workers up to the point where the marginal productivity of both types of labor is equal to its respective marginal cost, i.e. $Q_2 = W^*$ for native workers and $Q_2 = \tilde{W} + \pi(U)\phi + U\phi\pi'(U)$ for undocumented workers. It follows that:

$$W^* - \tilde{W} = \pi(U)\phi + U\phi\pi'(U) = \pi(U)\phi(1 + \eta), \quad (6)$$

where $\eta \equiv U\pi'(U)/\pi(U) > 0$ is the elasticity of $\pi(\cdot)$ with respect to $U$. We can then express this relationship between the demand for undocumented labor by each of the $J$ firms and the market wage of clandestine workers, $\tilde{W}$, as a function of the model’s parameters, including $G, \tau,$ and $\phi$, and the internal-enforcement intensity, which determines the position and shape of $\pi(U)$.

$$\tilde{W} = g(G\tau) - (1 + \eta)\pi(U)\phi \quad (7)$$

Note that $\partial\tilde{W}/\partial U = -\phi[\eta\pi(U)/U](2 + \eta\pi'(U)) < 0$, where $\eta\pi'(U) \equiv \pi''(U)/\pi'(U) > 0$ is the elasticity of $\pi'(U)$ with respect to $U$. Thus the demand-side relationship between $\tilde{W}$ and $U$, as given by eq. (7), can be depicted by the negatively sloped $dd$ schedule in Figure 1. In the next section, we join the supply and demand sides of the market for undocumented labor to determine $\tilde{W}$ and $U$.

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set $g'(G\tau) = 0$ in our comparative statics exercises that follow. The qualitative results turn out to be the same as those that pertain to the case of what we refer to be a low degree of intersectoral mobility.
4 Equilibrium in the Clandestine Labor market

Assuming that the market for undocumented labor clears at all times, the stock of illegal aliens, $N$, must be equal to the demand by the $J$ firms in Sector $I$ (i.e., $N = JU$). The evolution of the stock is governed by the dynamics of entry and exit of undocumented workers into and out of the underground economy. With respect to the dynamics of exit, we assume that apprehensions of illegal aliens can take place either on the premises of the employer in the context of worksite inspections (in which case the worker is deported and the employer fined) or outside of the workplace (in which case only the worker is deported), thanks to random as well as targeted identity checks or tipoffs received by the enforcement authorities.\(^{15}\) The total number of apprehensions (and deportations) per unit of time is thus given by $\pi(U) + \lambda N$, where $\lambda$ is the probability that an undocumented alien is apprehended during leisure time outside of the workplace, which we take to be an exogenous policy variable, and $\pi(U)$ is the probability of detection and apprehension at the workplace. Having established earlier that the steady-state flow of guest workers transiting into the underground economy is given by $F((\bar{W} - W)\psi - \bar{W} \beta \tau - \Phi)G$ and noting that $\psi = \frac{1}{\pi(U) + \lambda}$, we conclude that $N$ evolves according to the following differential equation: $\frac{dN}{dt} = F(\frac{\bar{W} - W}{\pi(U) + \lambda} - \bar{W} \beta \tau - \Phi)G - [\pi(U) + \lambda]N$. It can be readily shown that this equation is stable. Focussing only on the stationary equilibrium where $dN/dt = 0$, and noting that $N = JU$, we have

\(^{15}\)We assume that unless an illegal alien is detected at a worksite, the employer can deny having any links with that worker.
(8) \[ F\left( \frac{\tilde{W} - W}{\pi(U) + \lambda} - \tilde{W} \beta \tau - \Phi \right) G - [\pi(U) + \lambda] JU = 0. \]

Eqs. (7) and (8) enable us to solve for the equilibrium level of \( \tilde{W} \) and \( U \), as functions of the model’s parameters. We are particularly interested in exploring the links between the structure of the guest-worker program and the equilibrium in the clandestine labor market, as characterized by the following variables: The stock of undocumented labor and the equilibrium wage paid to illegal aliens. Also of interest in the present context is the question of how enforcement measures interact with program rules to shape the behavior of migrants and firms that hire undocumented workers.

### 4.1 A Larger Guest-Worker Program

We consider first the effects of an expansion of the guest worker program, as measured by the allowed inflow of guest workers, \( G \), holding the duration \( \tau \) of each worker’s contract constant. Equation (7) shows that for any given \( U \), the wage that firms are willing to pay undocumented workers falls following an increase in \( G \). A larger inflow of guest workers creates more slack in the labor market of Sector \( E \), the one eligible to hire guest workers. This puts downward pressure on the wages of natives who we assumed are at least to some extent mobile between Sectors \( E \) and \( I \). Some native workers will thus move into Sector \( I \), reducing the sector’s demand for undocumented foreign labor. This exerts negative pressure on \( \tilde{W} \) in the sense that the \( dd \) schedule in Figure 1 shifts down by the amount \( \tau g'(\tau G) dG \). On the supply side, the positively sloped \( ss \) schedule depicts the relationship between \( \tilde{W} \) and \( U \) corresponding to
eq. (8). A reduction in $\widetilde{W}$ lowers the proportion of guest workers willing to transit to the underground economy (which is reflected in a movement down along the $ss$ schedule), while an increase in $G$ enlarges the pool of migrants who might be tempted to do so (shifting the $ss$ schedule to the right). As shown in the Appendix, the latter effect dominates, causing the equilibrium stock of undocumented workers to increase if

$$\text{(9)} \quad F(A) > \frac{\tau g'(G\tau)}{\pi(U) + \lambda} f(A)G,$$

where $A \equiv \frac{W - \bar{W}}{\pi(U) + \lambda} - \bar{W}/\beta - \Phi > 0$ is a guest worker’s expected monetary payoff from transiting to undocumented status rather than returning to his country of origin at time $\tau$. Thus if condition (9) is satisfied, the downward shift of the $ss$ schedule exceeds that of $dd$, as shown in Figure 1, resulting in $dU/dG > 0$ and $\widetilde{W}$ falling by more than $\tau g'(\tau G)dG$. Alternatively, if the deterrent effect of a lower $\widetilde{W}$ dominates the direct scale effect of an increase in $G$ on the number of guest workers transiting to the underground economy, $F(A) < \frac{\tau g'(G\tau)}{\pi(U) + \lambda} f(A)G$ and $dU/dG < 0$. In this case $\widetilde{W}$ drops by less than $\tau g'(\tau G)dG$. In host countries where mobility of native workers between the eligible and ineligible sectors is relatively low, (i.e., $|g'(G\tau)|$ is small), we would expect the direct scale effect to dominate and hence the overall effect on $U$ to be positive. These results are summarized in Proposition 1.

**Proposition 1:**

An increase in the flow of guest workers, $G$, has an ambiguous effect on the stock of undocumented workers employed in the underground economy and a negative effect on their
wage. If the degree of mobility of native workers between sectors is sufficiently low, the stock of undocumented workers in the underground economy increases following an expansion of the guest-worker scheme.

4.2 Increase in Contract Duration

Consider next the effect of an increase in $\tau$, the duration of time that guest workers are legally allowed (and obliged) to work for their contractual employer in Sector $E$. For a given $G$, a longer $\tau$ increases once again the stock of guest workers. Assuming that native workers are mobile to some extent between Sectors $E$ and $I$, this puts downward pressure on the demand for undocumented labor in the underground economy. The $dd$ schedule therefore shifts down and to the left in Figure 2. On the supply side of the market for undocumented labor, for a given guest-worker salary, $\bar{W}$, and salary-withholding rate, $\beta$, an increase in $\tau$ implies that a larger amount of foreign earnings is forfeited by a guest worker, should he decide to transit to the underground economy rather than return to his country of origin. This deters overstayers, shifting the $ss$ schedule up and to the left. In consequence, the stock of undocumented workers unambiguously falls, while the wage of undocumented workers may either rise or fall, depending on whether the leftward shift of $ss$ is larger or smaller than that of $dd$. As shown in the Appendix, for a sufficiently low degree of intersectional mobility of native workers, the $ss$ locus shifts more than $dd$ does, resulting in an increase in $\bar{W}$.\(^\text{16}\) This is the case depicted

\(^{16}\)For $\bar{W}$ to increase with $\tau$, the necessary and sufficient condition is that \(\frac{\partial \bar{W}}{\partial \tau} = \frac{\partial G}{\partial U} (f(A)G \frac{\pi'(U)(\bar{W} - \bar{W}}{\pi(U) + \lambda} - \pi'(U)JU - (\pi(U) + \lambda)J) + \frac{1}{\pi(U)}[W \beta f(A)G \phi \eta \pi(U)/U](2 + \eta \phi U) > 0.\) To the
in Figure 2. We can thus establish the following Proposition:

**Proposition 2:**

*An increase in the duration, \( \tau \), of the contract offered to guest workers, decreases the stock of undocumented workers and has an ambiguous effect on their wage. If the degree of mobility of native workers between sectors is sufficiently low, the wage of undocumented workers increases following an increase in the duration of the contract.*

### 4.3 Role of Employer Sanctions

Consider next the role of policies aimed at discouraging employers in Sector \( E \) from hiring undocumented labor. We examine two measures: The magnitude of the penalty, \( \phi \), paid by a firm for each undocumented worker detected on its premises and the probability, \( \pi(U) \), of detecting and apprehending undocumented labor at the workplace. Both instruments serve to shift the demand curve for undocumented labor to the left.\(^{17}\) An increase in the penalty extreme, if there is no mobility of native workers between sectors (i.e. \( g'(G\tau) \) equals zero), the effect is unambiguous and positive.

\(^{17}\)For the sake of simplicity, we do not consider here the fiscal implications of collecting bigger fines or funding tougher enforcement measures. However, as pointed out by an anonymous referee, to the extent that enforcement spending is financed through distortionary taxes, the budgetary impact of any set of measures could have secondary repercussions on behavior, particularly in relation to underground economic activity. In the extreme case of economies on the Araban Peninsula, this is not likely to be the case as government expenditures are covered by resource rents rather than distortionary taxes. Considering other contexts where the authorities rely on distortionary taxation to cover public sector spending, it can be argued that the fiscal impact of tougher immigration control measures is unlikely to be very significant. This is because immigration control budgets typically represent only a tiny fraction of total government spending. If we take the U.S.A. as an example, where data on enforcement spending are readily available, 2012 immigration control expenditures (including border enforcement, worksite enforcement, detention and removal of noncitizens, visa controls and travel screening, etc.) amounted to $17.9 billion (Meissner et al, 2013) out of total expenditures at the federal, state and local levels of $6.1 trillion. A doubling of enforcement spending would therefore require an increase in taxes of less than 1/3 of one percent to keep the budget deficit from increasing. Thus if we were to generalize our model and introduce explicitly public sector finances, this would not have a notable impact on the comparative
\( \phi \), however, has no impact on the supply side. It shifts only the \( dd \) schedule down and to the left, resulting in an unambiguous decline in both \( U \) and \( \tilde{W} \).

An exogenous increase in the probability of detecting and apprehending undocumented workers on the premises of a firm, due to more frequent worksite inspections, for example, results in an upward shift of the function \( \pi(U) \). This obviously diminishes the attractiveness of hiring undocumented labor, shifting the \( dd \) locus to the left. It also reduces the incentive of a guest worker to transit to undocumented status, as it lowers the expected duration of the employment phase in the underground economy and hence the expected payoff enjoyed by an overstayer. Moreover, an upward shift of \( \pi(U) \) increases the deportation rate, which has a negative impact on the stock of undocumented labor. Both effects on the supply side operate in the same direction to displace the \( ss \) schedule up and to the left. Thus a tightening of the worksite inspection regime shifts both the \( dd \) and \( ss \) schedules to the left. This reduces the stock of illegal aliens, while having an ambiguous effect on \( \tilde{W} \). These results are summarized in Proposition 3, with the related algebra provided in the Appendix.

**Proposition 3:**

An increase in the penalty, \( \phi \), paid by the firms for hiring undocumented workers or an exogenous increase in the probability, \( \pi(U) \), of detecting and apprehending an undocumented alien at the workplace, decreases the stock of undocumented workers. While an increase in \( \phi \) lowers their wage, an exogenous increase in \( \pi(U) \) affects it ambiguously.
4.4 Other Policy Instruments

The effects of policy instruments that only affect the supply of undocumented workers are much simpler to analyze, as they only shift the ss locus while leaving dd unaffected. As may be seen in eq. (8) an increase in any of the following parameters: $\Phi$, $\bar{W}$, $\beta$, and $\lambda$, decreases the flow of guest workers transiting to the underground economy. An increase in either the fine, $\Phi$, paid by apprehended undocumented workers for violating the conditions of their visa or in the amount of earnings withheld by their contractual employers, $\beta \bar{W}$, decreases the monetary payoff enjoyed by an overstayer and hence the flow of guest workers transiting to the underground economy. An increase in the probability of apprehension outside of the working place, $\lambda$, has the additional effect of helping lower the stock of undocumented workers by increasing the outflow of illegal aliens back to their country of origin. All these measures, therefore, shift the ss schedule to the left, contributing to a reduction in the stock of illegal aliens and an increase in the equilibrium wage of undocumented workers. We thus have Proposition 4.

**Proposition 4:**

An increase in the fine ($\Phi$) paid by undocumented workers, in the official wage ($\bar{W}$) paid to guest workers, in the share ($\beta$) of a guest worker’s earnings withheld by the contractual employer pending contract completion, or in the probability ($\lambda$) of being caught outside the workplace, decreases the number of undocumented workers in the underground economy and increases their wage.
These results show that most of the repressive instruments (increased apprehensions and deportations of undocumented workers and more severe penalties for overstaying) have the expected effect. They lower the number of illegal aliens in the economy and raise the wage paid to undocumented labor. It is interesting to note, however, that an increase in the frequency of worksite inspections has a very different effect on the equilibrium wage of clandestine labor when compared with an intensification of controls outside the workplace, as captured in our model by an increase in $\lambda$. Stricter controls outside of the workplace increase the wage of undocumented workers, while an intensification of the controls at the workplace has an ambiguous effect on the wage. This is because the latter policy reduces both the supply and the demand for undocumented labor, while the former reduces only the supply. Also note the asymmetry between the effects of fines imposed on the employers and those imposed on the undocumented aliens. Larger fines, $\phi$, imposed on the employers reduce only the demand for undocumented labor, causing the equilibrium wage to fall, while larger fines, $\Phi$, imposed on the undocumented workers have a negative effect only on the supply side, resulting in a higher wage.

Our comparative statics results are summarized in Table 2 below.
Policy Variable | Impact on the Stock of Undocumented Workers | Impact on the Wage of Undocumented Workers
---|---|---
$G$: quota on the inflow of guest workers | Ambiguous* | -
$\tau$: work permit duration | - | Ambiguous*
$\phi$: firm’s penalty | - | -
$\pi(U)$: probability of apprehension at work | - | Ambiguous
$\Phi$: fine paid by undocumented worker | - | +
$W$: wage of a guest worker | - | +
$\beta$: proportion of the wage withheld | - | +
$\lambda$: probability of apprehension outside work | - | +

Notes: *indicates a positive effect if the degree of intersectoral mobility of native workers is sufficiently low.

Table 2: The impact of policy changes on the stock of undocumented workers and their wage.

### 4.5 Labor-Market Conditions in Sector E and in the Source Country

An increase in the source-country wage makes overstaying less attractive. This causes the $ss$ locus to shift to the left. As the $dd$ schedule is unaffected, $\tilde{W}$ increases and $U$ falls. One can also easily show that a tightening of labor-market conditions in the host country, which increases the wage of native workers in the sense of an exogenous upward shift of the function $g(G\tau)$, causes the $dd$ schedule to shift to the right, while leaving $ss$ unaffected. As a result, both $\tilde{W}$ and $U$ tend to increase. These results are summarized in Proposition 5:

**Proposition 5:**

*Tighter labor-market conditions in the destination country result in a larger number of undocumented workers in the underground economy and an increase in their wage. An improvement in the labor-market conditions in the source country lowers the number of undocumented workers.*
workers in the host country and causes their wage to rise.

4.6 Heterogeneous Firms

We have made a number of simplifying assumptions to facilitate the exposition. One of these assumptions is that all firms hiring illegal aliens are identical. It is important to note that if firms in Sector I are not identical, this does not change the qualitative findings of our paper. If firms in Sector I are heterogeneous in terms of their capital stock, for example, or ability to avoid detection of wrongdoing – with each firm $i$ having an idiosyncratic $\pi_i(U_i)$ function – any policy measure that makes hiring undocumented workers less attractive, still results in a leftward shift of the $dd$ schedule. Thus policy measures that reduce the demand for undocumented labor in our basic model would do so as well in an extension with heterogeneous firms, except that the leftward shift of the $dd$ schedule would not only reflect changes in the demand for undocumented labor at the level of each firm that hires illegal aliens (the intensive margin) but also at the extensive margin, as fewer firms may be willing to hire undocumented workers.

5 Conclusions

While a guest-worker program tends to reduce shortages of labor in the host country and diminish the incentive for employers to hire undocumented aliens, it can also contribute to an expansion in the supply of undocumented labor if workers choose to overstay after the
expiration of their work permits. This paper examines the links between a guest-worker program and the supply and demand for clandestine labor in the underground economy. Our main focus is on the question of how the program rules and the enforcement measures of the immigration authorities influence the behavior of illegal immigrants and their employers to determine the wage and the stock of undocumented workers.

The principal findings of the paper may be summarized as follows. An increase in the flow of guest workers admitted into the economy lowers the wage of undocumented workers, but it has an ambiguous effect on the stock of illegal aliens. If the degree of intersectoral mobility of native workers is sufficiently low, an increase in the inflow of guest workers generates a larger stock of undocumented labor. By contrast, allowing each of the guest workers to remain longer in the host country, decreases the stock of undocumented labor and has an ambiguous effect on their wage, which is positive if the degree of intersectoral mobility of native workers is sufficiently low.

These results have important policy implications. Noting that the stock of documented guest workers is simply the product of the allowed inflow and the duration of each worker’s authorized stay, our findings suggest that countries requiring an increase in the stock of documented guest workers can achieve this objective with a more favorable outcome in terms of illegal-immigration control, by increasing the duration of each guest worker’s stay, rather than by increasing the allowed inflow.

A bigger penalty imposed on firms found to be employing undocumented workers or an
exogenous increase in the probability of detecting and apprehending undocumented aliens at
the workplace (due to more frequent worksite inspections), both tend to lower the economy’s
stock of undocumented labor. The effects of the two policies on the clandestine wage rate,
however, are different. While the former measure lowers it, the latter has an ambiguous effect.
Repressive policies aimed at illegal aliens, such as increased identity checks outside of the
workplace and tougher deportation measures and fines for overstaying, all have the expected
effect of lowering the stock of undocumented labor in the economy and raising the equilibrium
wage received by illegal aliens.

Since we consider the capital stock and the number of firms operating in the underground
economy to be given, our analysis pertains mainly to the short and intermediate run. A
long-run analysis of the clandestine labor market would need to consider the possibility of the
number of firms and the capital stock of each firm contracting or expanding in each sector
in response to variations in the profitability of their operations due to changing conditions
on the sector’s labor market. In the long run, the prices of goods and services produced by
the two sectors would also have to be treated as endogenous. An earlier paper by Djajić
(1997), focussing on the short- and long-run effects of illegal immigration in the context of a
model with perfect international capital mobility and intersectoral mobility of native workers,
which is only partial in the short run, provides an indication of how our economy would react
to policy changes when everything is allowed to adjust.18 Insights provided by that earlier

18Note that the Djajić (1997) model is quite different from the one developed in the present study. It is
designed to examines the implications of a once-and-for-all entry of illegal aliens into a three-sector economy
study, especially in relation to adjustments in the capital stock and the resulting changes in the demand for labor, suggest that an expansion of the guest-worker program (i.e., an increase in $G$) would result in an increase in the number of firms employing illegal aliens in the long run, with the wage paid to undocumented workers falling by less than it does in the short run. By contrast, repressive enforcement measures targeting illegal aliens that were examined in Section 4.4 have an adverse effect on the profitability of firms in Sector $I$ operating with the aid of undocumented labor. This encourages exit and a contraction of existing firms over time, lowering the demand for such labor and contributing to a reduction in the wage of undocumented workers. Overstaying is thereby discouraged and the stock of illegal aliens in the underground economy will tend to decline over time. In consequence, the quantitative impact of these policies on the wage of undocumented workers can be expected to be smaller in the long run than it is in the short run, while the negative impact on the stock of undocumented labor should be stronger. We can thus think of the findings presented in our paper as being particularly relevant in the short to medium run, although we would expect the qualitative results to remain largely intact in the long-run.

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that employs skilled and unskilled labor, along with capital, to produce intermediate and final goods. There is no guest-worker program in that economy and hence no possibility of documented workers transiting to the underground economy.
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Appendix

Equations (7) and (8) can be rewritten as:

\[ F(\frac{\tilde{W} - W}{\pi(U) + \lambda}) - \tilde{W} \beta - \Phi G - (\pi(U) + \lambda) J U = 0 \equiv H(U, \tilde{W}) \]

\[ g(G\tau) - (1 + \eta)\pi(U)\phi - \tilde{W} = 0 \equiv M(U, \tilde{W}) \]

where \( U \) and \( \tilde{W} \) are endogenously determined and all other variables \( G, \tau, \phi, \pi, \lambda, \Phi, \beta, W, \bar{W} \) are determined exogenously by policy measures.

Let’s study now the static comparative with respect to \( Z \), which represents any parameter of the following set \( \{G, \tau, \phi, \pi, \lambda, \Phi, \beta, W, \bar{W}\} \).

Let’s denote

\[ A = \frac{\tilde{W} - W}{\pi(U) + \lambda} - \tilde{W} \beta - \Phi \]

and \( J = \begin{bmatrix} \frac{\partial H}{U} & \frac{\partial H}{\partial \bar{W}} \\ \frac{\partial M}{U} & \frac{\partial M}{\partial \bar{W}} \end{bmatrix} \) we can easily check that \( |J| > 0 \)

\[ |J| = f(A)G\frac{\pi'(U)(\bar{W} - W)}{(\pi(U) + \lambda)^2} + \pi'(U)JU + (\pi(U) + \lambda)J \\
+ \frac{f(A)G}{\pi(U) + \lambda} [\phi(\pi'(U)/U)(2 + \eta\pi'(U))] \]

After writing successively:

\[ J_u = \begin{bmatrix} -\frac{\partial H}{U} & \frac{\partial H}{\partial \bar{W}} \\ -\frac{\partial M}{U} & \frac{\partial M}{\partial \bar{W}} \end{bmatrix} \] and \( J_{\bar{W}} = \begin{bmatrix} \frac{\partial H}{U} & -\frac{\partial H}{\partial Z} \\ \frac{\partial M}{U} & -\frac{\partial M}{\partial Z} \end{bmatrix} \)

we can use Cramer’s rule and study \( (\partial U/\partial Z) = \frac{|J_u|}{|J|} \) and \( (\partial \bar{W}/\partial Z) = \frac{|J_{\bar{W}}|}{|J|} \).

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Since $\frac{\partial H}{\partial \tilde{W}} = f(A)G$ and $\frac{\partial M}{\partial \tilde{W}} = -1$ we find

$$J_u = \begin{bmatrix} -\frac{\partial H}{\partial Z} & \frac{f(A)G}{\pi(U) + \lambda} \\ -\frac{\partial M}{\partial Z} & -1 \end{bmatrix}$$

and we obtain easily:

$$\frac{\partial U}{\partial Z} = \frac{\partial H}{\partial Z} + f(A)G\pi(U) + \lambda \frac{\partial M}{\partial Z}$$

Studying:

$$\frac{\partial H}{U} = -f(A)G\frac{\pi'(U)(\tilde{W} - W)}{(\pi(U) + \lambda)^2} - \pi'(U)JU - (\pi(U) + \lambda)J$$

entails that: $\frac{\partial H}{U} < 0$.

Moreover $\frac{\partial M}{U} = -\phi(\eta\pi(U)/U)(2 + \eta\pi'U)$ entails that: $\frac{\partial M}{U} < 0$

**Comparative statics with respect to $G$**

$$\frac{\partial U}{\partial G} = \frac{\partial H/\partial G + \tau\pi'(U)}{|J|}$$

Using $\frac{\partial H}{\partial G} = F(A)$ and $\frac{\partial M}{\partial G} = \tau\pi'(G\tau)$ we show that the sign of $\frac{\partial U}{\partial G}$ is a priori ambiguous. It is positive if and only if the following condition is satisfied:

$$F(A) > -\frac{\tau\pi'(G\tau)}{\pi(U) + \lambda}f(A)G$$

We now turn to studying $\left(\frac{\partial \tilde{W}}{\partial G}\right) = \frac{|J_{\tilde{W}}|}{|J|} \text{ with } J_{\tilde{W}} = \begin{bmatrix} \frac{\partial H}{U} & -\frac{\partial H}{\partial G} \\ \frac{\partial M}{U} & -\frac{\partial M}{\partial G} \end{bmatrix}$

Using $\frac{\partial M}{U} < 0; -\frac{\partial M}{\partial G} = -\tau\pi'(G\tau) > 0; \frac{\partial H}{U} < 0$ and $\frac{\partial H}{\partial G} = F(A)$, we find that $\frac{\partial \tilde{W}}{\partial G}$ is negative as can be shown easily since

$$\frac{\partial \tilde{W}}{\partial G} = \frac{-F(A)}{|J|}[\phi(\eta\pi(U)/U)(2 + \eta\pi'U)] + \frac{\tau\pi'(G\tau)}{|J|}[f(A)G\frac{\pi'(U)(\tilde{W} - W)}{(\pi(U) + \lambda)^2} + \pi'(U)JU + (\pi(U) + \lambda)J]$$
Comparative statics with respect to \( \tau \)

Using \( \frac{\partial U}{\partial \tau} = \frac{\partial H/\partial \tau + \frac{L(A)G}{\mu(U)+\lambda} \partial M/\partial \tau}{|J|} \) with \( \partial H/\partial \tau = -\bar{W}\beta f(A)G < 0 \) and \( \partial M/\partial \tau = Gg'(G\tau) \) entails:

\[
\frac{\partial U}{\partial \tau} = \frac{Gg'(G\tau)}{\pi(U) + \lambda} - \bar{W}\beta \frac{f(A)G}{|J|} \\
\frac{\partial U}{\partial \tau} < 0
\]

Using \( \partial H/U < 0; \partial M/\partial \tau = -Gg'(G\tau); \partial H/\partial \tau = -\bar{W}\beta f(A)G \) and \( \partial M/U < 0 \) we find that \( \partial \bar{W}/\partial \tau = \frac{|J_{\bar{W}}|}{|J|} \) with \( J_{\bar{W}} = \begin{bmatrix} \partial H/U & -\partial H/\partial \tau \\ \partial M/U & -\partial M/\partial \tau \end{bmatrix} \) is ambiguous in general as can be shown easily since

\[
\partial \bar{W}/\partial \tau = \frac{Gg'(G\tau)}{|J|} \left\{ f(A)G \frac{\pi'(U)(\bar{W} - W)}{(\pi(U) + \lambda)^2} + \pi'(U)JU + (\pi(U) + \lambda)J \right\} \\
+ \frac{\bar{W}\beta f(A)G}{|J|} [\phi(\eta\pi(U)/U)(2 + \eta\pi(U))]
\]

Comparative statics with respect to \( \phi \)

\[
\frac{\partial U}{\partial \phi} = \frac{\partial H/\partial \phi + \frac{L(A)G}{\mu(U)+\lambda} \partial M/\partial \phi}{|J|} \\
\partial H/\partial \phi = 0 \\
\partial M/\partial \phi = -(1 + \eta)\pi(U) < 0.
\]

Therefore, we find easily that

\[
\frac{\partial U}{\partial \phi} = -f(A)G \frac{(1 + \eta)\pi(U)}{|J|(\pi(U) + \lambda)}
\]

Therefore \( \frac{\partial U}{\partial \phi} < 0. \)

Let’s consider now \( J_{\bar{W}} = \begin{bmatrix} \partial H/U & -\partial H/\partial \phi \\ \partial M/U & -\partial M/\partial \phi \end{bmatrix} \)
\[-\partial M/\partial \phi = (1 + \eta)\pi(U) > 0 ; \partial H/U < 0 \] and \(\partial H/\partial \phi = 0\) entail that \(\partial \tilde{W}/\partial \phi < 0\) as can be shown easily since:

\[
\partial \tilde{W}/\partial \phi = -\frac{(1 + \eta)\pi(U)}{|J|} \left[ f(A)G\pi'(U)(\tilde{W} - W) + \pi'(U)JU + (\pi(U) + \lambda)J \right]
\]

**Comparative statics with respect to \(\pi(U)\)**

To simplify the notations, let’s assume that \(\pi(U) = \pi + \rho(U)\) and that the effect of the policy is a (constant) shift in the parameter \(\pi\).

\[
J_u = \begin{bmatrix}
-\partial H/\partial \pi & \partial H/\partial \tilde{W} \\
-\partial M/\partial \pi & \partial M/\partial \tilde{W}
\end{bmatrix}
\]

with \(\partial H/\partial \pi < 0\), which is also true more generally for any exogenous increase in \(\pi(U)\), noted \(\partial \pi(U)\), following more effective detection or apprehension of undocumented workers at the workplace, since we find:

\[
\partial H/\partial \pi(U) = -\frac{(\tilde{W} - W)}{\pi(U) + \lambda) \phi} f(A)G - JU < 0.
\]

Moreover, \(\partial M/\partial \pi(U) = -(1 + \eta)\phi < 0\).

We can now sign easily \(\partial U/\partial \pi(U) = \frac{\partial H/\partial \pi(U) + \frac{f(A)G}{\pi(U) + \lambda} \partial M/\partial \pi(U)}{|J|}\) and find that \(\partial U/\partial \pi(U) < 0\).

Let’s consider now \(J_W = \begin{bmatrix}
\partial H/\partial U & -\partial H/\partial \pi(U) \\
\partial M/\partial U & -\partial M/\partial \pi(U)
\end{bmatrix}\)

Since \(\partial M/U < 0 ; \partial H/\partial U < 0 ; \partial H/\partial \pi(U) < 0, \partial M/\partial \pi(U) = -(1 + \eta)\phi < 0\) we find that \(\partial \tilde{W}/\partial \pi(U)\) is ambiguous in general as can be shown easily by studying the following:

\[
\partial \tilde{W}/\partial \pi(U) = \frac{(1 + \eta)}{|J|} \phi \left\{ -f(A)G\pi'(U)(\tilde{W} - W) + \pi'(U)JU - (\pi(U) + \lambda)J \right\}
\]

\[+ \frac{1}{|J|} \left\{ -\frac{W - W}{(\pi(U) + \lambda)^2} f(A)G - JU \right\} \{-\phi(\eta \pi(U)/U) (2 + \eta \pi U)\} \]}
Comparative statics with respect to $Z = \lambda, \Phi, \beta, W$, or $\tilde{W}$

$$J_u = \begin{bmatrix} -\partial H / \partial Z & \partial H / \partial \tilde{W} \\ -\partial M / \partial Z & \partial M / \partial \tilde{W} \end{bmatrix}$$

Using $\partial H / \partial Z < 0$ and $\partial M / \partial Z = 0$ yields $\partial U / \partial Z = \frac{\partial H / \partial Z}{|J|}$, which shows that $\partial U / \partial Z < 0$.

In particular $\partial U / \partial \lambda = \frac{\tilde{W} - W}{(\pi(U) + \lambda)^2} f(A)G + JU |J|^{-1}$

$$J_{\tilde{W}} = \begin{bmatrix} \partial H / U & -\partial H / \partial Z \\ \partial M / U & -\partial M / \partial Z \end{bmatrix}$$

$\partial M / \partial Z = 0$; $\partial H / U < 0$; $-\partial H / \partial Z > 0$; and $\partial M / U < 0$ yield $\partial \tilde{W} / \partial Z > 0$.

In particular

$$\partial \tilde{W} / \partial \lambda = \frac{\tilde{W} - W}{(\pi(U) + \lambda)^2} f(A)G + JU \ast [\phi(\eta \pi(U) / U)(2 + \eta' U)] |J|^{-1}$$

$$\partial \tilde{W} / \partial \Phi = \frac{f(A)G \ast [\phi(\eta \pi(U) / U)(2 + \eta' U)]}{|J|}$$
FIGURES
Notes: An increase in the inflow of guest workers (G) shifts the demand schedule for undocumented labor (dd) to the left and the supply schedule (ss) to the right, with the subscript 1 signifying the new equilibrium values of the endogenous variables.
Figure 2: Effects of an increase in the duration of a guest worker’s contract on the stock of clandestine workers, $U$, and their wages, $\bar{W}$.

Notes: An increase in the contract duration of guest workers shifts both the demand schedule for undocumented labor ($dd$) and the supply schedule ($ss$) to the left, with the subscript 1 signifying the new equilibrium values of the endogenous variables.