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Playing Outside: Excursions from the Tonality in Jazz Improvisation

Volume III of III:
Chapters 4 & 5, Appendices

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Chapter 4

Steve Coleman: “Symmetry” and “Sum” Systems in “Cross-Fade” (1991)

Overview

In examining “Cross-Fade”, we will see that Steve Coleman, like John Coltrane and Robert Irving III, is a composer who uses a carefully devised (and seemingly unique) set of strategies to play outside the general key of the piece. In contrast with most other musicians, however, he has spoken and written in some detail (in interview and via his website) about these strategies; his “Sum” and “Symmetry” systems. We shall see that these systems involve the simultaneous enlargement of jazz theory in a modernist, quasi-atonal manner, alongside the careful choice of relevant material from within the jazz tradition that complies with these specific theories.

Steve Coleman is an alto saxophonist, and a great early influence upon him (this due to the interest of his father) was Charlie Parker. Steve Coleman wrote out and practised many of Parker’s solos.¹ More recently, he has become renowned for a more pluralistic approach, leading a multiplicity of different groups which act as vehicles for his compositions. These groups often perform and record several projects alongside each other, each functioning as a focus for an area of compositional and improvisational interest. He calls the umbrella organisation for these groups “M-BASE”, which was formed in 1982.

¹ Hrebeniak (1991) p. 19

“The BASE stands for Basic Array of Structured Extemporisation. We wanted it to grow and be big and that’s why we called it Macro, thus M-BASE.”²

Steve Coleman is typical of many contemporary jazz musicians in that he often promotes the African-American oral tradition over theory per se. For example, he has often made clear that his use of irregular time signatures is the natural result of melodies composed in a “non-western” manner, and not the result of a deliberate attempt to create knowingly strange metric templates.³ However, as we shall see, he has also taken great efforts to develop his own complex theoretical models that are often self-consciously mathematical in structure. As an inheritor of the freedoms fought for jazz (and society) in the 1960s, he clearly wishes to build upon (and thus be a part of the development of) the jazz tradition whilst maintaining links with its earliest forms and sources. Further, he does not wish to simply replicate jazz styles of the past and, as such, be seen to act in a reactionary (i.e. what Giddins has called a “Neo-classical”⁴) way. Thus, at the core of his approach is a desire to be flexible, to be open to new, but clearly structured, approaches:

“For us [M-Base] this means expressing our experiences through music that uses improvisation and structure as two of its main ingredients. There is no limitation on the kind of structures or the type of improvisation, or the style of the music...One of the main ideas in M-Base is growth through creativity. As we learn through our experiences then the music will change and grow to reflect that. The idea is not to develop some musical style and then play it forever.”⁵

² Alleyne (1990) p. 19

³ See, for example, Coleman (1995a) p. 1

⁴ This debate was well established at the time of the release of this CD: Giddins (1985) pp. xi-xv

⁵ Coleman (1995a) p. 1

At times this pluralistic flexibility extends into a relativist philosophical approach:

“...the concept of which style is better than any other style has no place here.

Since the goal is the expression of culture and philosophy, there is no “better”.”⁶

Similarly, Steve Coleman’s stated desire to develop new musical concepts continually is

“not about leaving anyone behind.”⁷

“In every interview that I’ve ever done and when I talked to anyone I made a point to tell them that I was not the leader of M-Base and that there was no leader. This made no difference to western thinking journalist who insisted that there was a leader, and normally it was written that I started (or was the leader of) M-Base.”⁸

This collectivist nature of M-Base is discussed at length by Iyer (1996), who sees M-Base as an extension of the tradition of such groups as the Chicago AACM, Sun Ra (and his Arkestras) and the Black Artist’s Group in St. Louis. Iyer, as a “newer member of the [M-Base] collective,” confirms that M-Base represents an on-going development of creative methods.⁹ Steve Coleman has said that

“I am only the catalyst and portal through which the energy that is holding this particular incarnation of creative relationships together is working. But other

⁶ Coleman (1995a) p. 1

⁷ DuMars (1995) p. 3

⁸ Iyer (1995) p. 3

⁹ Iyer (1996) p. 7

individuals respond to these vibrations by opening themselves to these creative energies and this is what makes it a collective on this plane of existence."¹⁰

Thus, he strives for equality in representation for styles, methods within the composition process and, further, for the members of the collectives. The journeys undertaken (composing, rehearsing, playing and improvising) are seen as plural and open to debate and interaction. Steve Coleman often uses language in the way in which Coltrane did in the 1960s. He has said that his goal is

"...to be on the path of creative expression...In other words, to be on the path is in itself success".¹¹

Further, Steve Coleman is interested in, and uses, computers in his work. Santoro describes a music computer group meeting in New York in 1988 at which Steve Coleman and Joe Ravo demonstrated their improvising computer system:

"Terms fly around the room: Pythagoras' Golden Section, Symmetry, the Fibonacci sequence, machine language, MIDI and SMPTE."¹²

His philosophy and interests are at the heart of the two composition systems mentioned above that Steve Coleman uses and has publicly described: the "Symmetry" and "Sum" systems. We will see that these theories are universal in nature, based on symmetrical structures at the heart of the 12-note system. This ordered source for his music parallels

¹⁰ Iyer (1996) p. 15

¹¹ Iyer (1995) p. 1

(a) his desire for rationality and equality of representation at a stylistic and social level, (b) by extension, his self-enforced reduction in his status as leader and (c) his interest in and use of mathematics and modern music computer technology.

“Symmetry” system: Alleyne (1990) and Hrebeniak (1991)

When I first heard Steve Coleman’s music, I immediately wondered what strategies he employs to structure his improvisations, which seem to spend as much time being “out” as “in”, although most seem firmly based around various key centres. My interest in Steve Coleman’s music was shared by (tenor saxophonist) Phil Veacock, who kindly showed me two articles based upon interviews that Coleman gave to Alleyne (1990) and Hrebeniak (1991). These interviews were published about the time of the release of the CD “Black Science”, which includes the track “Cross-Fade”.¹³ In these two articles Steve Coleman describes in fairly general terms his approach to composing and performing during this period. However, he also describes a system, which he calls “Symmetry”, thus:

“Back in Chicago I used to practise outside, trying to develop a bigger sound. I had a big afro at that time and I used to wear this shit on my hair, this afro-sheen type stuff. They had these little yellow bees in Chicago, whenever you wore something sweet like perfume they would come and start fucking with you. At first it was just an irritant, but after a while I noticed that these bees had a funny way of moving in terms of their flight patterns. I’m talking out of order here, but I was an artist before I was a musician...I noticed that these bees flew in funny patterns. When I look at birds flying, they fly straight, swooping up and down.

¹³ Santoro (1994) p. 294

These bees were flying in a wiggly, hanging pattern. They were always moving, but it was in a very small area, sort of like gnats round a light. This motion intrigued me and I started thinking how come people don't play like that. People play in a very 'bird' type fashion, even Charlie Parker played like that. So I started thinking that if you could play ideas that had more to do with bee motion, rather than bird motion, then obviously you would sound different. It's a rhythmic thing and a melodic thing. When I got to New York, one day I woke up, maybe I had a dream, and the germ of the idea of how you could play like that hit me. I call it *symmetry*, which to this day some people think is bullshit."¹⁴

"I've been working on Symmetry since 1978. There are a lot of chord structures considered symmetrical. For example, with the C augmented triad E is the focus, being equidistant from the G[#] and the C. You have to know you're intervals well to deal with these things. When playing I could pick a phrase and work in intervals, in equal sections around it. The focus for me is not the key centre - it's almost like a gravity centre, or things orbiting the sun in the solar system. It really has to do with figuring out different ways of contracting and expanding away from centre-points in terms of composition, improvisation or rhythm. I just throw out the things that don't sound good. Now Symmetry is balanced out by the Golden Section which is lopsided. The two complement each other perfectly. Golden Section proportions sound pleasing to people's ears and exist in almost all world folk music - I've checked it out. The symmetrical stuff is more abstract and angular sounding. When I make a non-symmetrical move in one direction, if I follow the strict rule that I made up, I have to make an equally non-symmetrical move in the opposite direction to balance the forces."¹⁵

¹³ "Black Science" (1991) Novus PD83119. Indeed, Hrebeniak's interview/article (1991) is entitled "Black Scientist".

¹⁴ Alleyne (1990) pp. 18-19

¹⁵ Hrebeniak (1991) p. 20

Note the similes employed by Steve Coleman here: the “wiggly, hanging pattern” of the bees, the “gnats round a light” and “things orbiting the sun in the solar system”. These represent extensions into the third dimension beyond the two-dimensional flight of a bird (e.g. *the “Bird”*, Charlie Parker) flying “...straight, swooping up and down”. Notice also the use of the word “phrase” in the fourth sentence: this confused me for some time. We will see below that, in “Cross-Fade”, Steve Coleman generally picks *single notes* (i.e. not phrases) and works in equal intervals around these. Thus, we must read the word “note” where Steve Coleman uses the word “phrase” here. However, his use of the “Symmetry” system does indeed generate a melodic pattern, which is more usually called a phrase.

“Sum” system: from “The Music of Steve Coleman” website (1997)

Steve Coleman first described his “Sum” system on the internet in a relatively brief article which outlined its importance to his composition “Cross-Fade”, thus:

“Melodically and harmonically the concept of “Cross-Fade” deals with tonal centers in terms of space (as opposed to the standard tonality which deals in key centers). In other words, what is important here is the position of tones in space or distance. So when I speak of improvising with regard to a “sum 11 tonal center” I am speaking of a tonality that has an axis (or spatial center) of sum 11. Sum 11 means that the tones B-C (also F-G flat) are the spatial tonal centers of this section of the composition. For the improviser this means improvising with this spatial tonality in mind. To go into some examples of improvising in a “sum 11” tonality is beyond the scope of this presentation and any [sic] would take many chapters. I do plan on releasing a small book on the subject later but I can say that one necessity would be learning to think spacially with the mind as well as using your

ears. For those musicians who just don't want to deal with this I've also written a more traditional tonality (F sharp minor) which can be substituted but that is not what I had in mind when I created the song, it is only a worst case substitute. If you use the sum concept you will be closer to the melodic material in the composition (and the concept in general).

The reason for the term "sum" comes from the concept of adding note numbers. If tones C through B chromatically are represented by the numbers 0 through 11 respectively, then it is possible to "add" tones together and come up with sums. The sums represent the axis (or center point) between the two tones being added together. For a "sum 8" axis, any two tones that add up to the number 8 would be considered a sum 8 interval. For example D sharp and F (3+5) would add up to 8. The center (or axis) of D sharp and F is E and E (which is also sum 8 or 4+4, an axis always implies at least four tones, in this case the E unison represents two tones but if thought of from another perspective B flat and B flat is also the axis, i.e. $10+10=20$ minus $12 = 8$), the same goes for C sharp and G (also sum 8). So the axis of the "sum 11" interval would be B and C (Since we are dealing with 12 tones the whole system is based on 12, so you can continually subtract the number 12 from any sum that is 12 or greater until the sum is below 12).

To keep your balance rhythmically also requires developing a sense of spatial placement. This is not unlike being able to feel where the bridge is in say changes based on the song "I Got Rhythm" which many players have learned to do (through repetition, doing and hearing). The same concepts can be applied here but it is necessary to hear and play the rhythms many times to learn this sense of balance. Start by learning the drum chant."¹⁶

Where Steve Coleman uses the word “tones” here we should read “notes”. although his choice of this term creates a resonance with the phrase “12-tone” (music) and suggests connections with such atonal theory. Further, it has struck me that there is a similarity between the “Sum” system and the theories of Perle (1977), both in terms of structure and nomenclature (although I do not know whether Steve Coleman has been directly influenced by this text; much conventional atonal theory describes similar systems. of course. For example, note the use of the term “sum” in the following diagram presented by Rahn (1980. p. 50) for the mapping T_6I (the equivalent of Steve Coleman’s “Sum 6”):

x	0	1	2	3	4	5	6	7	8	9	10	11
$T_6I(x)$	6	5	4	3	2	1	0	11	10	9	8	7
sums	6	6	6	6	6	6	6	6	6	6	6	6

Table 4-1 Rahn’s diagram for mapping T_6I (1980. p. 50)

“Symmetrical Movement Concepts” website (2000)

In contrast with the sources examined so far, this website is a relatively lengthy, thorough and carefully written description of both the “Symmetry” and “Sum” systems. Further, it demonstrates how these two systems are interrelated. The two sections of specific interest to this thesis are “Melodic Material Generated By Symmetrically Derived Laws of Motion” (which I will call the “Symmetry” system for short) and “Harmonic Material Generated In Symmetrical Space” (which I will call the “Sum” system for short). These are reproduced in Appendix 6. and I direct the reader here unless they are already familiar with this website.¹⁷

¹⁶ Coleman. Steve (1997) p. 1

Here there seem to be even closer similarities between Steve Coleman's and Perle's theories. Perle's musical language developed from a self-confessed "misunderstanding" of Schoenberg's twelve-tone system.¹⁸ His "modal" system negates the redundancy of the atonal nature of the chromatic set, using "symmetrical chords and progressions"¹⁹ at the centre of his work. He defines "symmetrical progressions" as "the simultaneous unfolding of inversionally related pitches", and "symmetrical chords" as "pitch-class collections that may be analyzed into inversionally complementary sub-collections".²⁰ Again, I am not sure of any direct influence of Perle's work upon Steve Coleman, but point out these similarities for interest's sake.

Coleman presents several theoretical examples in this text ("Examples 3, 4 and 5"), the first of which he describes in some detail. I present here my analysis of these three examples. This, I suggest, will not only allow us to consider Steve Coleman's techniques within his own, considered, theoretical context, but will also lead the reader gradually into the relative complexities of "Cross-Fade" itself. Thus, I do not propose this analysis as a replacement for that provided by Steve Coleman. However, as we shall see, there are some symmetrical structures that he does not describe (or even "cue" with a circled axis) directly in these examples. I have recreated the diagrams and notation from the website as closely as possible (the brackets and annotation are mine).²¹

¹⁷ I include the whole of these texts in this thesis on the grounds that they may disappear from the internet in the future.

¹⁸ Perle (1977) p. ix. I do not wish to suggest by this that Steve Coleman does not understand something. Rather I feel that Perle's personal approach to what might be viewed as a mathematically closed set (i.e. the material of serialism) parallels the flexibility a jazz musician might require when designing such a musical system.

¹⁹ Perle (1977) pp. 10-15

²⁰ Perle (1977) p. 10

²¹ There is one small cosmetic difference: the boxes around notes and pairs of notes on Examples 3, 4 and 5 are circular/ovoid in the original scores.

Example 3

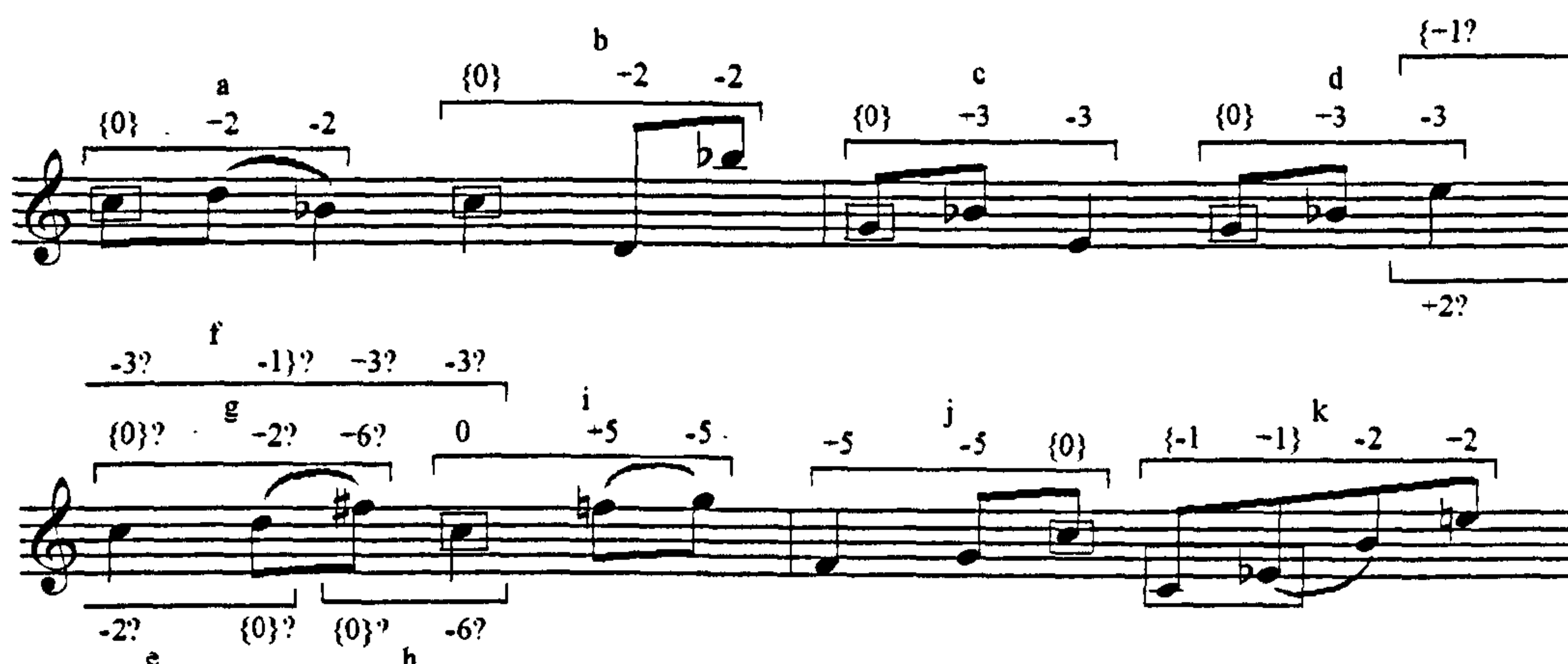


Fig. 4-1 "Example 3" from Steve Coleman's "Symmetrical Movement Concepts" website

The bracket marked *a* opens this example with a typical, quintessential pattern: a pivot note of C followed by a D (a tone higher) and a Bb (a tone lower). The C, then, is the "axis", and acts as a mental focus for the other two notes. I have annotated the note C with the sign {0}. The curly brackets here mean that C is the (singular) axis. The notes D and Bb are, respectively, two semitones above (annotated +2) and below (-2) the axis. Note that this set of {C, D, Bb} might be placed in many conventional jazz contexts, e.g. within a C Dorian or C Mixolydian tonality. The melodic structures in this example show a gradual move away from such a strict adherence to such conventional jazz formulae – note the B natural at *k*, for example. In practice, we might not expect such a gradual move away from a tonality. This is because although each of the pivot notes belongs to an underlying tonality, strict adherence to interval patterns defined by the "Symmetry" system is likely to generate notes from outside that tonality. However, here Steve Coleman carefully gradually introduces dissonant material to the student. Pattern *b* is an example that shows that these cells are not obligatorily local by octave: although in "Cross-Fade" Steve Coleman usually operates within a given octave and avoids large intervals in his melodic forms. Pattern *c* is similar to *a*, although here the

interval concerned a 3 semitones, or a minor 3rd. Pattern *d* is similar to *b* in that it shows that a given set of notes can be performed across octave boundaries, here creating a purely ascending melody. Pattern *g* shows what at first glance seems to be an error. That is, the D that follows the C is then answered by an F#. However. Let us recall Steve Coleman's statement that

“One of the exceptions to this rule is when the interval that you play is one of the Symmetrical Intervals in spiral number one, those intervals being a major second, major third, tritone, minor sixth, minor seventh, octave etc. Then you don't have to make the equal movement in the opposite direction.”²²

In retrospect, the opposite outcome of this rule seems to be what Steve Coleman meant by the following statement, made earlier to Hrebeniak:

“When I make a non-symmetrical move in one direction, if I follow the strict rule that I made up, I have to make an equally non-symmetrical move in the opposite direction to balance the forces.”²³

However, here I suggest the analyses shown by brackets *e* and *f*. Pattern *e* has the note D as an axis the C and E respectively (this kind of retrograde melodic construction, with the axis at the end of a phrase is soon to be shown by Steve Coleman himself at *j*).

Pattern *f* is a more complex suggestion, the first to use a paired “axis” of D and E. This suggests an (unheard) “true” axis of Eb. Thus D and E are -1 and +1 of this true axis, Eb. The curly brackets that have surrounded notes that represent a single axis up to this

²² Coleman, Steve (2000)

²³ Hrebeniak (1991) p. 20

point are divided amongst the (heard) notes that form (or represent an unheard) single axis. Thus E is annotated “{+1}”, and D is annotated “{-1}”. This follows Steve Coleman’s description that

“...all axis [sic] are two tones as in the initial axis of spirals one and two, and they are all either a unison or a minor second as in examples 1 and 2.”²⁴

Of course, these phrases *e* and *f* could easily be an over-enthusiastic description on my part. I may be finding a pattern where nonesuch was imagined by the composer. This is a risk with all musical analysis, of course. A study of any melody would reveal a certain number of melodic symmetries of this kind. However, having examined “Cross-Fade”, I feel that Steve Coleman is so fluent with this system that it seems possible that he has included some in the composition, and then omitted to provide the reader with sufficient “axis” circles (squares in my notation). However, if we look closely at the annotations for *e* and *f*, we can see that the paired axis of *f* is split up by another note (marked in **bold**):

<i>e</i> :	+2	-2	{0}		
<i>f</i> :	{+1	-3	-1}	+3	-3

Table 4-2 Annotation of *e* and *f* from Example 3

So, although *f* is a phrase that can be deemed to be made of notes that form a symmetrical set, the performance of this would be considerably more complex than the single axis pattern *e*. Thus, on balance, I feel that *e* is a more likely correct analysis. It is a simpler pattern amongst other simple patterns and it would seem easier to conceive of

²⁴ Coleman, Steve (2000)

patterns (especially at speed) where the note(s) that form the axis are adjacent (whereas in *f* they are not). Indeed, this too is borne out by my analysis of "Cross-Fade". Further, let us note that the +6 annotation given to the F# in *g* might as well be -6, bearing in mind the octave equivalence rule suggested by patterns *b* and *d*. I have decided to use both forms of annotation: +6 where the melodic movement is up a tritone, and -6 where the melodic movement is down a tritone. Pattern *h* is a further possibility, based upon the idea that *e* has completed with the note D at beat 2 and that the following F# and C have a deliberate asymmetry (being a tritone apart). However, given the half-bar length of all of the other examples, let us assume that this C belongs to pattern *i*. At pattern *i*, we find an example of a set of notes that have become representative to Steve Coleman of the whole system: he uses them extensively in his description of these examples, emphasizing that this set *must* be completed:

"But if you play one of the Non-Symmetrical Intervals in spiral number two, for example, axis C to the tone F (an interval of a perfect fourth), then you must play a G after that, according to these laws of movement (actually it would be more accurate to say that after playing the tone F you must then 'complete' the symmetrical motion)."

Indeed, as we shall see, this {C, F, G} set occurs in all three of these examples, and, in other keys (= {0, 5, 7}), is a feature of the Intro Solo of "Cross-Fade". Pattern *j*, as mentioned above, shows a retrograde melodic phrase (or, to use Steve Coleman's phrase, "...you can do the reverse and move towards an axis."). That is, the musician has prepared the entire structure {C, F, G} in advance in their head, and reveals it in reverse order. Although this is not a very complex undertaking, perhaps we should not be surprised to find some sets of notes used more commonly than others in this way. Pattern *k* sees Steve Coleman give the example of a paired axis. Here C and Eb represent

the (unheard) axis notes of Db and D (sum 3 on Steve Coleman's table). In my analysis, I would name this axis pair Db and D as {0 and 0}. Thus the (heard) notes C and Eb are annotated {-1 and +1} respectively. In pattern *k*, the B and E that follow are (theoretically) chromatically adjacent to the already heard C and Eb. Where a similar axis pair occur in "Cross-Fade", we will see a similar, local, approach to completion of symmetrical phrases. That is, where the axis pair are separated by an interval of a minor or Major 3rd, the other (often following) notes in the phrase are usually no more distant, being either a semitone, a tone, or a minor 3rd away from the central axis.

Example 4

The image shows two staves of musical notation. The first staff contains patterns labeled a, b, c, d, e, and f. Pattern a has notes with intervals {-1, -1}, {-2, -2}, and {0}. Pattern b has notes with intervals {-1, +1}, {-5, -5}, and {+3, -3}. Pattern c has notes with intervals {-2, +2}. Pattern d has notes with intervals {-1, -1} and {0}. Pattern e has notes with intervals {-6, -1} and {0}. Pattern f has notes with intervals {-3, -3}. The second staff contains patterns labeled g, h, i, and j. Pattern g has notes with intervals {0, 0}, {-6, -6}, and {+5, -5}. Pattern h has notes with intervals {+1, -1}. Pattern i has notes with intervals {+1, -1} and {-4, -4}. Pattern j has notes with intervals {+2, -2}, {-4, -4}, and {-4, -4}.

Fig. 4-2 "Example 4" from Steve Coleman's "Symmetrical Movement Concepts" website

Now we will examine Example 4. Pattern *a* is a transposed version of *k* in Example 3. Note the common note C in these two patterns. Further, pattern *b* is identical to *i* in Example 3. This simple beginning is contrasted by the next structure, namely that the four-note pattern *c* reaches across patterns *b* and *d*. Pattern *c* has an obvious (unheard) axis of F#, although it is noteworthy that this is the equivalent of the axis C, which is the starting point for phrases *b* and *d*. Pattern *e* further asserts this relationship between these two (equivalent) axes, but continues with an asymmetric move to a B at beat 4. Patterns *f*

and *g* share a common paired axis, and, indeed, might well be read as a single (six-note) pattern with a central paired axis of {D, Eb}. The quaver rest at beat 3 of bar 3 not only provides a space in the melody, it also notifies a stop/start point for symmetrical patterns in this example. This characteristic is shared with the only other rest to be found within these three examples: at bar 5 of Example 5). Despite this small population, we shall find this to be an important structural characteristic of "Cross-Fade". Patterns *h* and *i* have a common (unheard) axis of Ab. Further, note that the (similarly unheard) axis of *j* is D, the tritone "partner" of the Ab axis of *h* and *i*.

Example 5

Example 5 is a musical score on a single staff, featuring a series of notes with various intervallic annotations. The annotations are organized into groups labeled with letters and symbols, indicating specific intervals or sets of intervals between notes.

Annotations and Groups:

- a:** {0} +4 -4
- b:** {+1 -1} -2 +2 +3 -3 {0}
- c:** {0} +4 -4
- d:** {-1 +1} +2 -2
- e:** -3 +3 {0 0}
- f:** {+1 -1} -2
- g:** {+1 -1} -2
- h:** +2 {0} +5 -5
- i:** {0} +4 -4
- j:** +1 -1 {0}
- k:** {0} +1 -1
- l:** {0} +6
- m:** {+1 -1} +2 -2
- n:** +5 {+1 -1} -5
- o:** -5 +5 {0 0}
- p:** +3 -3 {-2 +2}
- q:** {0} +6
- r:** {0} -6
- s:** -1 +1
- t:** {0} {0} +6
- u:** {0} +1 -1 -4
- v:** {+1 -1} +2 -2
- w:** +4 {-1 +1}
- x:** +2 -2 {0 0}
- y:** {0 0} +1 -1
- z:** -1 {0} +1
- aa:** {+1 -1} +4 -4
- bb:** {0} -6
- cc:** {0} +6
- dd:** {0} -3 +3
- ee:** {0} +3 -3
- ff:** {0} -5 +5

Fig. 4-3 "Example 5" from Steve Coleman's "Symmetrical Movement Concepts" website

Example 5 (Fig. 4-3) opens with pattern *a*, a structure without a circled axis. This is the first case of an Augmented triad found in these examples. After pattern *b* (axis circled), a further (axis uncircled) Augmented triad is seen (*c*). That these are (a) relatively rare events and (b) uncircled is curious in that the Augmented triad is the symmetrical example given by Steve Coleman to Hrebeniak (p. 20). However, we shall see another (the axis circled, that time) at *i*. A diminished triad follows as pattern *d*, and, let us note, the pattern that leads out of this, *e*, is identical to pattern *a* in example 4. Further overlapping systems follow: *f* and *g*, although the latter does not have its axis circled by Steve Coleman. Pattern *h* is the {C, F, G} set – we have now seen this in all three examples, mostly in “natural” order. Pattern *i* is, as noted above, a further Augmented triad with a circled axis. This is followed by a pair of local chromatic melodic phrases, each the retrograde inversion of the other: *j* and *k*. Pattern *l* is an (legally complete) asymmetric move from G to Db (tritone). Bar 6 begins with the aforementioned rest, and, interestingly, a repeated F note (marked *). This, naturally, gives additional weight to the start of this particular phrase. Here, there seems to be no symmetrical partner for this repeated F. However, we shall see repeated notes used in “Cross-Fade” which spread out a symmetrical melodic pattern over a series of bars (the notes A and G in bars 3-5). A sequence of overlapping patterns then follows (*m*, *n*, *o* and *p*), some provided with circled axes, some not. Patterns *q* and *r* are overlapping, and asymmetrical (a rather complex description for three notes, two of which are the same...). This is the only such example that I have found. Pattern *t* (a further tritone) acts as a link between *s* and *u*, which are both further examples of local chromatic melodic phrases. However, unlike *j* and *k*, these are merely retrograde (i.e. not retrograde *and* inverted). There now follows a frenzy of overlapping and interrelated symmetrical material, and Steve Coleman has provided a correspondingly large set of circled axes. Understandably, perhaps, he does not do so for the chromatically adjacent set of *z*, the tritones of *aa* and *bb*, and, relatedly, the diminished triad patterns of *dd* and *ee*; all of these patterns, although correct, are

redundant. Pattern *cc*, however, is beyond the range of *aa*, and is a tidy (Symmetrical Interval) movement to pattern *ff*.

Summary of Examples 3, 4 and 5

These three examples show an increasing degree of Chromatic Saturation. Example 3 has nine pitches (no Db, Ab or A). Examples 4 and 5 cover the entire chromatic scale. Further, in Example 5 we find 11 of the 12 notes of the chromatic scale in the first two bars and, in the last fourteen notes, we find all 12 of the chromatic scale (F# and B are repeated). Further, we can find several elements of atonal music: retrograde and inverted material. These structures are easily generated with the "Symmetry" system. Octave equivalence is also an important element of this theory (seen, for example, in Ex.3, *b* and *d*). On the other hand, we can see that this fully chromatic music is mostly derived from excursions away from a conventional tonality. That is, nearly all of the single axes (i.e. those annotated with {0}) in the first two of these three examples are natural (a notable exception being the use of Eb at *k* in Example 3 and *g* in Example 4). However, just as we see an increase in Chromatic Saturation across the three examples, we can also see that sharp and flat notes are increasingly used as axis material in Example 5. The use of a home key as a launch pad for "Symmetry" system excursions is one that we shall see a great deal of in our analysis of Steve Coleman's solos in "Cross-Fade". Further, all of these examples show a good deal of repetition of melodic patterns in specific keys, even matching note orders and pitch contours. Whilst this is perhaps a useful characteristic of a teaching text, we will find that Steve Coleman is especially fond of using motifs in "Cross-Fade", these seemingly derived from such exercises.

FULL SCORE

CROSS-FADE

STEVE COLEMAN

$\text{♩} = \text{ca. } 145$ Sax solo (fast rate of change)

Alto Sax 1

Alto Sax 2

Piano

Gtr

Bass

Drums

open improvisation (slow rate of change)

Alt 1

Alt 2

Pno

Gtr

Bass

Dr

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Fig. 4-4 "Cross-Fade" (Steve Coleman's score)

The image displays a musical score for a piece titled "Cross-Fade" by Steve Coleman. The score is organized into two systems, each containing six staves for different instruments: Alt 1, Alt 2, Pno (Piano), Gtr (Guitar), Bass, and Dr (Drums). The notation is in standard musical notation, with notes, rests, and other musical symbols. The first system includes a double bar line and a repeat sign. The second system includes the word "fine" written below the first five staves. The score is written in a single system, with the two systems of staves separated by a double bar line and a repeat sign.

Alt 1

Alt 2

Pno

Gtr

Bass

Dr

fine

fine

fine

fine

fine

fine

Fig. 4-4 "Cross-Fade" (Steve Coleman's score)

[E♭ m] (or num 5 4cm1 correct)*

Alt 1

Alt 2

Pno (or num 11 [F♯ m] total correct*)

Gtr

Bass

Dr

==

Alt 1

Alt 2

Pno

Gtr

Bass

Dr

Fig. 4-4 "Cross-Fade" (Steve Coleman's score)

Cross-Fade

from "Black Science" (1991)
Novus PD83119

Steve Coleman

Intro Solo F# Blues:

$\bullet = c.145$

Alto Saxophone

Drums

snare

kick

+1

2

A. Sax.

"solo (fast rate of change)"

Bass

"open improvisation (slow rate of change)"

Dr.

cowbell

hi-hat

a

b

c

$\{0\} +1$

$-1 \{0\}$

$+1 \{0\} -1$

$\{+1 -1\} -4$

$+4$

$-1 \{0\} +1$

ss

-3

3

$* C^{o7}$

Fig. 4-5 "Cross-Fade" (notation of recording)

A. Sax.

Bass

Dr.

3

+1 -1 {0} -1 {0}

d

+3 {+1 -1} -3 {0, 2, 5, 7} b e

uu

-1 {0} +1

{-1 +1} +3 {0, 2, 5, 7} b tt



A. Sax.

Bass

Dr.

4

{0} f {0} +2 +3 -2 {0} -2 -3 -2

g h

3 *

vv

-1 {0} +1

F# Major Blues: I bIII III

F# Blues: bVII #IV V bVII I IV bIII

Fig. 4-5 "Cross-Fade" (notation of recording)

5

A. Sax.

Bass

Dr.

$\{0, 2, 5, 7\} b$ $\{0, 2, 5, 7\} b$

F# Blues F# m.p....

$\{-1 +1\} -4 +4 +2$ $\{+0 -0\} -2$ $\{0\} +5$

F#m⁷ i

k -5 m +5 {0}

ww D:II III I V



6

A. Sax.

Bass

Dr.

$\{0, 2, 5, 7\} b$ $\{0, 2, 5, 7\} b$

...F# m.p. F# Blues

$\{-1 +1\} -3 \{+1\}$ $\{-1 +1\} \{0\} +1$

-5 -3 {-1} +1} +3

$\{0\} -5_0 +5$ $\{0, 2, 5, 7\} b$ -2 {0} +2

$\{+1 -1\} +4 -4 -1\} \{+1\}$ q r

F#m⁷ p

-1 xx {0} +1

Fig. 4-5 "Cross-Fade" (notation of recording)

7

A. Sax.

Bass

Dr.

$-3 \{0\}$ E^{b0} $+3 \{0\}$

$F\# m.p.$

v $-2 \{0\} +2$

u $\{0\} +5 -5$

$F\# Blues$ $+1 \{0\}$ x

$-3 \{-1 +1\} +3$

$\{0, 2, 5, 7\} a$

w

3

3

8

A. Sax.

Bass

Dr.

$+1$ $+1$ -1 $\{0\}$ $+3$ -1

y $\{0, 2, 5, 7\} b$ $\{+1 -3$ $-1\}$ $\{0, 2, 5, 7\} b$

z $\{-1 +1\} -3 +3$

-5 $+5$ $\{0\}$ $+5$ $\{0\}$ -5

-2 $\{0\}$ $\{0\}$ yy $+2$

$*$

Fig. 4-5 "Cross-Fade" (notation of recording)

9

A. Sax.

Bass

Dr.

aa {0, 2, 5, 7} b

+3 {+1} -1} -3

+4 {+1 -1} -1} -4 -1}

F[#]m⁷ bb

{0} +1 cc -1

+2 dd {0}



10

A. Sax.

Bass

Dr.

ee -3 {-2} * +2} +3

-3 {0} -4 -3 {0} * +3 +4

-1 {0} hh

* {0} +1 gg

-2

Fig. 4-5 "Cross-Fade" (notation of recording)

11

A. Sax.

Bass

Dr.

{0} +1

jj {0} -2 +1 {0} +1 -1

kk

{0} +5 +6!

ii



12

A. Sax.

Bass

Dr.

(F# Blues)

mm F# minor pentatonic

+5 -5 -2 {0} -2 +5 +2

nn

{0, 2, 5, 7} b

-3 {-1 +1} +3

{0} oo +6

*

-5 +5 {0}

+3 +1 -3 {-1}

{0, 2, 5, 7} b

ll

Fig. 4-5 "Cross-Fade" (notation of recording)

13 F# minor: I & bIII qq {0}-2 {0}+2

A. Sax. {0} pp +5 -5 *

Bass 3 -2 {0}+2 zz

Dr.

Fig. 4-5 "Cross-Fade" (notation of recording)

Head 0'46"; 1'19"; 1'45"; 2'15"; 3'19" (to fine)

The musical score is written for six instruments: A. Sax. 1, A. Sax. 2, Pno, El. gtr, Bass, and Dr. The time signature is 9/4. The key signature has one sharp (F#). The score includes several annotations and markings:

- A. Sax. 1 and 2:** Both staves start with a measure marked '15'. A bracket labeled 'A' spans measures 4 and 5 in both parts.
- Pno:** The piano part includes the annotation "[F#m] (or sum 11 tonal center)" above measures 3, 5, and 6.
- El. gtr:** The electric guitar part has a bracket labeled "j" above measures 2, 3, and 4, and another bracket labeled "k" above measures 5 and 6. The annotation "(tacit 1st time)" is placed above measure 2.
- Bass:** The bass part includes several annotations: "{0}" above measure 2, "a" above measure 3, "-1 +1" above measures 4 and 5, "{0}" above measure 6, "+1 b" above measure 7, and "-1" above measure 8.
- Dr.:** The drum part features a series of eighth notes with 'x' marks below measures 2 through 7, indicating specific drum hits.

Fig. 4-5 "Cross-Fade" (notation of recording)

The musical score is arranged in six staves, each with a label to its left:

- A. Sax. 1**: Treble clef. Starts at measure 16. Contains two boxed sections labeled **B** and **C**. Section B contains two measures of music. Section C contains two measures of music.
- A. Sax. 2**: Treble clef. Contains two boxed sections corresponding to the ones in A. Sax. 1.
- Pno**: Bass clef. Contains a series of notes and rests across the measures.
- El. gtr**: Treble clef. Contains a series of notes and rests. Above the staff, there is a bracket labeled **l** spanning the first four measures, and another bracket labeled **m** spanning the last two measures.
- Bass**: Bass clef. Contains a series of notes and rests. Above the staff, there are several markings: **0}**, **+1**, **c**, **-1**, **{0}**, **{0}**, **d**, and **-2!**.
- Dr.**: Drum staff. Contains a series of notes and rests, with 'x' marks indicating specific drum hits.

Fig. 4-5 "Cross-Fade" (notation of recording)

The musical score consists of six staves, each with a label to its left:

- A. Sax. 1**: Treble clef. Measure 17 is marked with a 'D' above a bracket. A 'fine' marking is placed below the staff at the end of the first measure.
- A. Sax. 2**: Treble clef. A 'fine' marking is placed below the staff at the end of the first measure.
- Pno**: Bass clef. A 'fine' marking is placed below the staff at the end of the first measure.
- El. gtr**: Treble clef. Notes are marked with 'n' and 'o' above them. A 'fine' marking is placed below the staff at the end of the first measure.
- Bass**: Bass clef. Notes are marked with '+1', '{0}', 'e', '-1', '{0}', 'f', '+1', '-1', '+1', 'g', and '{0}' above them. A 'fine' marking is placed below the staff at the end of the first measure.
- Dr.**: Drum staff with a double bar line. A 'fine' marking is placed below the staff at the end of the first measure.

Fig. 4-5 "Cross-Fade" (notation of recording)

18

A. Sax. 1

A. Sax. 2

Pno

El. gtr

Bass

Dr.

E F F¹ G

p

-1 +1 {0}^h -1

+2 +1 {0} -1 -2

i

Fig. 4-5 "Cross-Fade" (notation of recording)

1'01" 1st Solo

G Whole Tone F# Blues

19 "[Ebm] (or sum 5 tonal center)"

A. Sax.

Pno

Bass

(Part for Solos)

"[F#m] (or sum 11 tonal center)"

Diagram showing musical notation for A. Sax., Pno, and Bass. The A. Sax. staff includes a 1'01" 1st Solo section. The Pno staff shows a sequence of chords. The Bass staff shows a sequence of notes and rests. The diagram includes various musical notations such as G Whole Tone, F# Blues, and specific tonal centers like [Ebm] and [F#m].

20

A. Sax.

Pno

Bass

G minor pentatonic F# Blues F# m.p. F# m.p.

Diagram showing musical notation for A. Sax., Pno, and Bass. The A. Sax. staff includes a 20 section. The Pno staff shows a sequence of chords. The Bass staff shows a sequence of notes and rests. The diagram includes various musical notations such as G minor pentatonic, F# Blues, F# m.p., and Cm7.

Fig. 4-5 "Cross-Fade" (notation of recording)

21

A. Sax.

F# Dorian

F# m.p.

G m.p.

$\{0, 2, 5, 7\} b$ $\{0, 2, 5, 7\} c$ $\{0, 2, 5, 7\} c$ $\{0, 2, 5, 7\} d$

3 3 3 *

$\{-1+1\}-3+3-3\{+1-1\}+3$ $\{-1-3+1\}+3\{-1-3+1\}+3\{-1+1\}$

o p q s

Pno

Bass



22

A. Sax.

F# Dorian

(Cm7)

C Dorian

C Blues

$\{0, 2, 5, 7\} e$

6

$-3+3\{+1-1\}+3+3\{+1-3$ $\{+1-1\}$ $\{0-2\{0-2\{0-2\{00\}+2$ $-4\{-1+1\}$

0}-1 G#m7 u v w x y z

Pno

Bass

Fig. 4-5 "Cross-Fade" (notation of recording)

Musical score for "Cross-Fade" (notation of recording). The score is written for five instruments: A. Sax., A. Sax. 2, Pno, El. gtr, and Bass. The Dr. (Drum) part is also shown at the bottom. The notation includes various musical symbols such as notes, rests, and accidentals. Above the A. Sax. staff, there are annotations: "aa", "-1 {0} +1 +1", and "(Head)". A bracket labeled "23" spans the first four measures of the A. Sax. staff. A bracket labeled "+4" spans the first four measures of the A. Sax. 2 staff. The Dr. staff shows a sequence of notes and rests, with "etc." at the end.

Fig. 4-5 "Cross-Fade" (notation of recording)

Solo 2'00"

35 F# minor pentatonic:

El. gtr



El. gtr



El. gtr



El. gtr

Fig. 4-5 "Cross-Fade" (notation of recording)

F# Blues

2nd Solo 2'44"

A. Sax. 47

Bass

(Head Arrangement)

A. Sax. 48

Bass

A. Sax. 49

Bass

F# Dorian

B W.T. F# m.p.

t +1 -1

v G#07

{+1-1} -4 +4{+1}

3 3

+1 {0} -2! {0} -6

-2 {0} +2 s u

r

h C07 {-1+1} -4+4-4

G m.p. +5-3{0}+3-5{0}+3

i j E09

k Bbm

m F# Aux. Dim.

Cm7

(G) (F) -1+1{00}-1 +3{0}-3{0}+6-1{0}

l n o q

{0}+4 p

a -2 {0} +2 {0} +2

c {0} +2

e -5 +5 {0} -5 g +5 {0}

f {0}

-1 +1 {0} -3 {0} +3 (min 3rd)

cf. m.20 b d

Fig. 4-5 "Cross-Fade" (notation of recording)

50

A. Sax.

Bass

F# m.p. G m.p. F# Dorian

+4

B W.T. F# m.p. (G) (F)

F# Blues

B W.T. *

F# m.p.

(F)(G)

-2 {0} +2

w

-2 {0 +2 0}

x

-2 {+1 -1} +2

y

+2 {+1 -1}

z

-2 {0} +2 -2 -2 +2 {0} {+1}

{+1 +3 -1} -3 {+1 -1}

bb

+1

aa

{0} -1

==

51

A. Sax.

Bass

(Head 5)

-1} -4 +4

F#m7

cc

(Part for Solos)

etc.

Fig. 4-5 "Cross-Fade" (notation of recording)

“Cross-Fade” – Steve Coleman (Fig. 4-5)

Analysis Using “Symmetrical Movement Concept” Theories

(from “Black Science” (1991) Novus PD83119)

The piece “Cross-Fade” is the ninth track on Steve Coleman and the Five Elements’ album “Black Science”. My interest in this piece in particular was first raised when Steve Coleman published a score and short description of “Cross-Fade” on his website (see Figure 4-4).¹ The short, but accurate (computer-printed) score follows jazz convention and consists mainly of the four-bar Head arrangement, offering blank bars where solos are to be played. This score formed a basis for my own, extended notation of the performance (see Figure 4-5). This notation includes three distinct improvisations by Steve Coleman, which I have labelled Intro Solo, and 1st and 2nd Solos. We will see that Steve Coleman employs both “Symmetry” and “Sum” systems in “Cross-Fade” to a great extent. Although the “Symmetry” system is essentially used for improvisation and the “Sum” system is essentially used for composition in “Cross-Fade”, their inter-relatedness and structure would by no means limit their uses in this way. (Indeed, as we shall see, both the alto and bass parts in “Cross-Fade” show improvised use of motifs derived from the “Sum” system.) Further, much of the analysis that follows uses conventional jazz language, and seeks to find commonly-used chords and scale sets within Steve Coleman’s music. This might be seen as antagonistic to his statement that his music avoids such constructions:

“When I play, I’m not thinking scales. A lot of younger players are so locked into scales that they can’t think of anything else. I’ve often wondered why the two modes, major and minor, dominate out of the countless diatonic structures.

After studying African, Eastern and Bulgarian folk music, I decided that it

wasn't necessary to use just major and minor, and consciously abandoned it.

Now I'm working with cells. There's nothing mystic about that - they're just small musical constructs which I manipulate to get two types of sound motion, stationary and in transit. I've also been looking for geometric ways of doing progression rather than thinking in standard terms.”²

However, as we shall see, Steve Coleman does not avoid common structures from the jazz tradition. Rather, he actively chooses those which can be derived from the “Symmetry” and “Sum” systems. The truth behind the above quotation seems to be that Steve Coleman's strategies *take precedence* over such traditional, scale based systems (as found in bebop and modal jazz), yet are influenced by them. Let us note immediately the rondo-like structure of this piece, alternating between the head and solos:

1. 2/4 bar
2. Introduction (Intro Solo)
3. 2/4 bar
4. Head 1
5. Alto Sax 1st Solo – Steve Coleman
6. Head 2
7. Piano Solo – James Weidman
8. Head 3
9. Guitar Solo – David Gilmore
10. Head 4
11. Drum Solo – Marvin “Smitty” Smith
12. Alto Sax 2nd Solo – Steve Coleman
13. Head 5 (interrupted)

Table 4-3 Rondo-like structure of “Cross-Fade”

¹ Coleman, Steve (1997); copyright 1990 Goemon Publishing Co.

The head acts as an interlude, grounding this otherwise generally abstract piece in a manner rare for a jazz composition. This form also promotes a concept of equality between head and solos. However, this equality does not extend to exposure per musician; Steve Coleman has two conventional solos, and he improvises in the introduction. Further, the rondo-like form is eventually disrupted: the drum solo is not followed by a head, and acts as a kind of introduction to Steve Coleman's 2nd solo. We will examine the head first.

Heads 1 to 4 (bars 15-18)

I suggest that the drum part was composed first. This is a common practice in popular music composition at a computer/sequencer workstation (Steve Coleman is known for his use of computers and software music sequencer programs to compose his music³). This four-bar drum pattern acts as an ostinato through the piece, and is only altered for the drum solo (which is, similarly, based on quaver durations). Further, Steve Coleman suggests that musicians wishing to play the piece should start "...by learning the drum chant."⁴ Also, this drum pattern is typical of composition using a sequencer on a computer. The frequencies of notes played on the four instruments of kick and snare drums, cowbell and hi-hat suggest a desire to promote a general equality of use of the elements of the drum kit, and, further, the kick drum is not used to emphasize the first beat in each bar. This means that the 9/4 time signature is not emphasized, although there are occasions in the recording where Smith emphasizes the first beat of the bar with a cymbal crash. This ordering of the drum part is paralleled by the uniformity of dynamics to be found in the head. Thus, most of the performance is dynamically neutral,

² Hrebeniak (1990) pp. 19-20

³ See, for example, the credit given to "Dr-T" software on "Sine Die" (1988) Pangaea 461159 1.

⁴ Coleman, Steve (1997)

creating a flat musical surface.⁵ This neutrality is further emphasized by an equality of volume ascribed to each musician in the mix. Also, let us note that all of the music in Steve Coleman’s score of the head is “quantised” to a quaver level.⁶ The actual performance on the CD of this piece accurately reflects the rigid nature of these rhythms as they may have sounded on the computer-based version.

Use of the “Sum” system in the Head

If we group the note sets for each instrument, we find:

Alto 1:	D	E \flat	E				A	B \flat	B
Alto 2:	C	D \flat	D	E \flat	E	F	F \sharp	A	B \flat B
Piano:							G		
Guitar:				E	F		A \flat	A	
Bass:		D	E \flat	E	F	F \sharp	G		

Table 4-4 Note sets by instrument in “Cross-Fade”

We can see that although many are subsets of other sets, all 12 notes of the chromatic scale are present in Table 4-4. However, further examination reveals the following symmetries in these data:

⁵ This limited dynamic range may have as much to do with the use of audio compressors in the studio as a desire to de-emphasise the first beat of the bar. Smith’s occasional cymbal crashes on beat 1 seem calmed in this way.

⁶ “Quantisation” is generally described as the act of snapping notes to a virtual rhythmic grid in a computer sequencer. However, here I am referring to the use of such software in the preparation of a score by fixing the durations of notes in each part. In Steve Coleman’s score every part has the quaver value as its lowest common denominator.

Given:

C	Db	D	Eb	E	F	F#	G	Ab	A	Bb	B
0	1	2	3	4	5	6	7	8	9	10	11

We may see the following mirror symmetries:

Alto 1:			D	Eb	E				A	Bb	B
			2	3	<u>4</u>				<u>9</u>	10	11

Alto 2:	C	Db	D	Eb	E	F	F#		A	Bb	B
	0	1	2	3	4	5	<u>6</u>		<u>9</u>	10	11

Piano:							G				
							<u>7</u>				

Guitar:				E	F			Ab	A		
				4	<u>5</u>			<u>8</u>	9		

Bass:		D	Eb	E	F	F#	G				
		2	3	<u>4</u>	<u>5</u>	6	7				

Table 4-5 Symmetrical structure of note sets by instrument in “Cross-Fade”

Here, the underlined notes represent the symmetrical axes (or axis, in the piano part) of their note sets. We can perform a little mathematics on these note sets to show that each instrument can be assigned a value from the “Sum” system, thus:

Alto 1:	D	E \flat	E		A	B \flat	B
	2	3	<u>4</u>		<u>9</u>	10	11

= Sum 1: {D, B} 2 + 11 = 13; 13 - 12 = 1
 {E \flat , B \flat } 3 + 10 = 13; 13 - 12 = 1
 {E, A} 4 + 9 = 13; 13 - 12 = 1

Alto 2:	C	D \flat	D	E \flat	E	F	F \sharp		A	B \flat	B
	0	1	2	3	4	5	<u>6</u>		<u>9</u>	10	11

= Sum 3: {C, E \flat } 0 + 3 = 3
 {D \flat , D} 1 + 2 = 3
 {E, B} 4 + 11 = 15; 15 - 12 = 3
 {F, B \flat } 5 + 10 = 15; 15 - 12 = 3
 {F \sharp , A} 6 + 9 = 15; 15 - 12 = 3

Piano:									G
									<u>7</u>

= Sum 2: {G, G} 7 + 7 = 14; 14 - 12 = 2

Guitar:			E	F			A \flat	A
			4	<u>5</u>			<u>8</u>	9

= Sum 1: {E, A} 4 + 9 = 13; 13 - 12 = 1
 {F, A \flat } 5 + 8 = 13; 13 - 12 = 1

Bass:	D	E \flat	E	F	F \sharp	G
	2	3	<u>4</u>	<u>5</u>	6	7

= Sum 9: {D, G} 2 + 7 = 9
 {E \flat , F \sharp } 3 + 6 = 9
 {E, F} 4 + 5 = 9

Table 4-6 Calculation of “Sum” system values for instruments in “Cross-Fade”

Noting that each axis/paired axis in Table 4-6 has an equivalent possibility a tritone away, we can construct the following table:

	Sum:	Axes:
Alto 1:	Sum 1	{C, Db} / {F#, G}
Alto 2:	Sum 3	{Db, D} / {G, Ab}
Piano:	Sum 2	{Db} / {G}
Guitar:	Sum 1	{C, Db} / {F#, G}
Bass:	Sum 9	{E, F} / {Bb, B}

Table 4-7 Sum values and Axes for instruments in “Cross-Fade”

Thus it seems possible that the piece was composed using subsets of sum pairs where the value of Sum *x* is different (although not unique) for each instrument. However, there seems to be no higher level of organisation; the table above reveals that two instruments share “Sum 1” as their set resource, and I can find no reason for the particular list of sums found here. Further, the parts variously have axis material in or out of their limited sets. Also, there appears to be no structure relating to chromatic saturation across these notes sets; although, uniquely, if we sum the set used for Alto 2 with the (possible) axis of {G, Ab}, we achieve the chromatic scale.

If we examine the axis (/ -es) for each instrument, we find that the range of notes used is quite limited, ranging from E to Ab (here the axes are marked in **bold**):

Alto 1:	D	Eb	<u>E</u>	<u>F#</u>	<u>G</u>	<u>A</u>	Bb	B				
Alto 2:	C	Db	D	Eb	E	F	<u>F#</u>	<u>G</u>	<u>Ab</u>	<u>A</u>	Bb	B
Piano:							<u>G</u>					
Guitar:					E	<u>F</u>	<u>F#</u>	<u>G</u>	<u>Ab</u>	<u>A</u>		
Bass:		D	Eb	<u>E</u>	<u>F</u>	F#	G					
Axis notes:				E	F	F#	G	Ab				
Frequency:				1	1	2	4	1				

Table 4-8 Showing range of axis material in instrument note sets

Here we see no even spread of axis material across the chromatic scale. However, there may be an extension of the “Symmetry” system to be found here. That is, the limits E and Ab might be seen as satellites, equidistant from the home key of F#:

-2	{0}	+2
E	F#	Ab

Table 4-9 E and Ab equidistant from F#

Finally, before moving on, let us note that a G occurs in every part as a member of a limited set and/or an axis member, and, indeed, is the most common axis member, upsetting any possible balance suggested by Table 4-9.

Bass

The first pitched instrument composed for in this piece would seem to be the bass. This would see Steve Coleman continue the conventional order of popular music composition with a computer, mentioned above.

The opening motif of this bass part in bar 15 (marked *a*) starts with a sustained F# (annotated as an axis: {0}), followed by the two notes immediately chromatically below and above this F#: i.e. F and G (annotated -1 and +1). Here, then, we have a simple, pure, even iconic expression of the “Symmetry” system described above, although the durations of the notes concerned reflect an emphasis upon the tonic. There are many sets of brackets with annotated numbers, chords and scales in the notation, describing some simple and some relatively complex constructions. In every case the data represent a

pattern that I have found in the performance that relates to Steve Coleman's stated theories.

Pattern *b* is a phrase similarly built with symmetrical material, but here we see a paired axis of {E, F}. Pattern *c* has its axis (Eb) at the end of the phrase. Note that the note D in phrase *c* (marked *) is the only occurrence of such a note in the bass line. It is crucial, however, to my theory of the use of the "Sum" system in the head (see below). Pattern *d* is the only example of a (legal) Symmetrical Interval in the bass line. (This and following Symmetrical Intervals will be annotated just as this one: i.e. by means of an exclamation mark.) At the start of bar 17 we hear again the three notes F#, F and G (marked *e*) that formed the opening motif of *a*. However, here they appear in a new order (i.e. G, F#, F), and this time the phrase starts on beat 1 of the bar. For this set to occur at the beginning of bars 1 and 3 of this 4-bar head emphasizes its importance to Coleman, although a listener unfamiliar with the piece (and out of step with the 9/4 time signature) might very well not hear this structure. Patterns *f* and *g* continue the use of symmetrical material in a straightforward, non-overlapping way. Next, the ordering of the {F, F# G} set found at the start of bar 17 (marked *e*), is repeated at the centre of bar 18 (marked *h*), but here can be seen to operate as part of a larger symmetrical structure (marked *i*). Note that pattern *i* is the only part of the bass line which has annotation of +2 or -2. However, this characteristic is perhaps offset by the descending chromatic nature of this phrase. With regard to the use of the {F, F#, G} set in the bass line let us note that the first appearance of this cell (*a* at bar 15) emphasises the tonic note of the piece by starting with a sustained F#. However, the latter occurrences of this cell (*e* and *h* at bars 17 and 18) emphasize the symmetrical nature of this set (and, thus, de-emphasize the tonic) by ordering the notes with F# at the centre.

The bass part, then, consistently follows the "Symmetry" system. However, this system cannot be said to have been extended to the other parts, when seen individually.

Guitar

The Guitar part, although also based upon a limited set, has none of the “Symmetry” system characteristics that we have seen pervade the Bass part. However, there are several other symmetrical structures that help to construct this part.

Let us note that the pattern *j*, at the very start of the Head, is immediately repeated as *l*. Pattern *m* is a further repeat of *j*, but is aborted by the end of the 2nd bar of the Head (i.e. halfway through the Head). Further material is derived from the overlap of patterns *j* and *l*. That is, pattern *k*, which is used as *n*. Pattern *o* is a subset of *p*, which in itself is a retrograde version of the first pattern, *j*. Further, in the 2nd and later Heads, the F# played by the guitar on beat 1 of bar 15 is matched by an F# on the last beat of the head, further extending the range of *j* and *p* (annotated with dotted brackets).

Thus, in the guitar part, we find translational (i.e. simple repeats) and retrograde symmetries. This structure is unique to the guitar part in the Head.

Piano

The piano part is the only part that uses a “Sum” system value which is an even number, and therefore has a single axis. The simplicity of the piano part may be a reflection of its relationship with other parts at a “Sum” system level. We will examine this and other relationships now.

Relationships between Parts in the Head

Alto 1 and 2 have similar rhythmic structure, as do the Bass and Guitar parts, although this is subverted on occasion. Thus, these pairs of parts may be the result of the composition of one part, followed by the editing of a copy of this initial part (this is simple to achieve with most music sequencing software). Although, as we have seen, each part is constructed from a limited set of notes, it is only the bass part that shows use of the “Symmetry” system in the composition of phrases. We might postulate, then whether there are relationships *between* parts in the head which relate to the “Sum” or “Symmetry” systems. Returning to the “Sum” values for each part, we should perhaps note the following symmetry:

{C, Db}	{Db}	{Db, D}
Alto 1	Piano	Alto 2

Table 4-10 Symmetry of “Sum” values of Alto 1, Piano and Alto 2 parts

That is, the axis of the piano part acts a kind of meta-axis to the Alto 1 and 2 parts. However, this analysis ignores the fact that the guitar part shares the {C, Db} axis with Alto 1, and has no obvious “partner” under this system. Further, this relationship is not continued through all of the parts. However, if we sum the axes for *all* of the parts we find a (not surprisingly) symmetrical scale: C, Db, D, E, F, F#, G, Ab, Bb, B. This scale is interesting in that it might be seen as the sum of G minor pentatonic and Ab minor pentatonic (or Bb Pentatonic and B Pentatonic). Let us recall that (a) the minor pentatonic scale is an important set in this piece, (b) that G and Ab appear as the paired axis of Alto 2, and (c) that Bb and B appear as the paired axis of the bass part. However, although the axes have these relationships, there are not strict note pair relationships between actual parts: for example. Alto 1 and Alto 2:

Alto 1: Alto 2:

C	
Db	
D	Eb Db Bb A B D C
Eb	B C Db
E	D C B
F	
F#	
G	
Ab	
A	F F#
Bb	A E
B	F A

Table 4-11 Simultaneous note events of Alto 1 and Alto 2 parts

Nor are there strict note pair relationships between the Guitar and Bass parts:

Guitar: Bass:

C	
Db	
D	
Eb	
E	E D Eb G F#
F	F# E F G
G	
Ab	F E F#
A	F# G Eb F E
Bb	
B	

Table 4-12 Simultaneous note events of Guitar and Bass parts

Further, the frequency of occurrence of each note for each instrument shows no regular pattern with regard to the “Sum” value for that instrument. If we sum the values for each instrument’s notes across the entire melody and then subtract the relevant multiple of 12 we find the following remainders:

Alto 1	123	$123/1 = 123$, remainder 0
Alto 2	131	$131/3 = 43$, remainder 2
Piano	91	$91/2 = 45$, remainder 1
Guitar	191	$191/1 = 191$, remainder 0
Bass	125	$125/9 = 13$, remainder 8

Table 4-13 Showing that “Sum” values do not divide neatly into instrument note frequencies

This inconsistency suggests that Steve Coleman has simply composed a satisfactory melody from each relevant set for each instrument, but has not matched up pairs (etc.) of notes in terms of their values within the “Sum” system. The “Sum” system, thus, has supplied pairs of notes to a limited set, and then that limited set has been used to freely compose a melody. There seems little point, then, in searching for a more complex pattern of note choice that relates to what we might term a “serial” use of the limited set. However, it does seem that there is a level of pitch organisation to be found when we look at several parts simultaneously. This may or may not be a deliberate act on Steve Coleman’s part and may simply be a consequence of his use of the “Sum” system to create sets of notes for each part. For example, let us examine the four notes shared between Alto 1 and Alto 2, grouped in Box A (beat 6 of bar 15). These form the following symmetrical set:

Box A:

-1	{0	0}	+1
B	C	Db	D

Table 4-14 Annotation of Box A

Boxes B, C, D and E reveal similar sets:

Box B:

-1	{0	0}	+1
C	Db	D	Eb

Box C:

-2	-1	{0	0}	+1	+2
B	C	Db	D	Eb	E

Box D:

-3	{-2	+2}	+3
E	F	Bb	B

Box E:

-3	{-1	+1}	+3
A	B	D	E

Table 4-15 Annotation of Boxes B, C, D and E

Note that all of these boxes contain notes that occur simultaneously. Box F is a similar case, but here we must all for notes overlapping in time:

Box F:

-4	{-1	+1}	+4
B	D	E	G

Table 4-16 Annotation of Box F

Further, the symmetry found in Box F is maintained as the two Alto parts move up a semitone (creating a new box, marked F¹):

Box F ¹ :			
-3	{0	0}	+3
C	Eb	E	G

Table 4-17 Annotation of Box F¹

On the last beat of the head (beat 9, bar 18) each instrument plays a note of a crotchet value (this is even true of the drum part). Of the notes in Box G, the Bass, Guitar and Alto parts can be seen to form a symmetrical structure:

Box G:			
-3	{0	0}	+3
Eb	F#	F#	A

Table 4-18 Annotation of Box G

This set is noteworthy for several reasons. Firstly, the axis of the set is the home key of F#. Secondly, two instruments play this note. There are no other such unisons in the head (except Head 5; see later). Thirdly, these notes occur by pitch in the simple order of an Eb diminished triad in root position: Eb, F#, A. That is, the Eb is the lowest sounding pitch, the two F#s are in the middle, and the A is played high by Alto 1. Fourthly, this is the only such chordal event in the head, and it is striking that it falls on the last beat. The piano part, by contrast, continues obstinately with its octave G.

There is a further detail that has caught my eye with regard to this Eb dim triad. That is, it is only on this last beat of the Head that Alto 2 plays an F#. As noted above, this note is essential for Alto 2 to have a limited set of notes that has a paired axis. By contrast, in the bass part (the only other part that can claim F# as a member of its limited set) the note F# occurs 6 times, and is annotated no less than 4 times as an axis in my analysis.

These note frequencies suggest to me that Steve Coleman may have composed this last event in the head early on in the writing process.

“[F#m] (or sum 11 center)” and “[Ebm] (or sum 5 center)”

Above the piano part at the start of the head in Steve Coleman’s score is the instruction “[F#m] (or sum 11 center)”. Further, at the start of the solo section are the instructions “[Ebm] (or sum 5 center)” and “[F#m] (or sum 11 center)”, respectively. It seems to me that the phrase “[F#m]” means “a conventional jazz (/blues) F# minor tonality” (i.e. an F# minor pentatonic and/or an F# Blues scale⁷). At first glance it might seem as if these two sum values (5 and 11) refer to different symmetrical strategies. However, I suggest that they are simply transpositions of each other for the relevant instruments. That is, the axis of Sum 5 is {D, Eb} (this being for instruments in Eb), which is a minor 3rd below the axis of Sum 11, {F, F#} (this being for instruments in C). Here is a simple table that shows this relationship:

C	Db	D	Eb	E	F	F#	G	Ab	A	Bb	B
0	1	2	3	4	5	6	7	8	9	10	11
(2 + 3 = 5)				(5 + 6 = 11)							

Table 4-19 Relationship of sum5 and sum11 to tonal centres of Eb and F#

Thus, the “[Ebm] (or sum 5 center)” instruction is for the Alto part(s), and the “[F#m] (or sum 11 center)” instruction is for the other (C concert) parts of piano, guitar and bass. However, as we have already noted, the sets of notes used for the piano, guitar and bass parts in the head are based in Sum 2, 1 and 9 respectively (i.e. not Sum 11) and, thus, this instruction, we might postulate, is only meant to refer to the solos. In which

⁷ Let’s recall that F#m is the “second best” approach.

case, we might expect the solos to either (a) show relationships of notes and/or scales that are based upon the Sum 11 spiral (i.e. {B, C}; {Db, Bb}; {D, A}; {Eb, Ab}; {E, G}; {F, F#}), or (b) relate to Steve Coleman's "Sum" table (shown at the end of Appendix 6). In fact, as we shall see later, this latter possibility is the actual case.

Further, let us note that this theory is based around the theoretical home key of C. That is, C = 0. Although "Cross-Fade" is in F# minor, F# does not = 0. Rather, F# *always* = 6. Thus, these "Sum" instructions are independent of the key of the piece.

Head 5

Head 5 (see Fig. 4-6) is shorter than the previous versions; it stops abruptly at the start of its third bar (bar 53), as noted by Steve Coleman in his original score (I have included this *fine* at beat 2 of bar 17). This is the end of the tune. In fact, in a rare instance on a jazz CD, the following track ("Black Phonemics (Reprise)") starts immediately, apparently interrupting "Cross-Fade". In fact, this 5th Head seems to have been curtailed because of a mismatch between the bass and the upper parts. In this regard let us now note that the bass part for the solos is different to that for the Head (we will examine this more closely below). Let us recall the structure of the piece, and note which bass part is being played where (i.e. the bass part for the head, or for the solos):

3'19" (to fine)

51 Head 5

The musical score consists of two systems, each with six staves. The instruments are labeled on the left: A. Sax. 1, A. Sax. 2, Pno, El. gtr, Bass, and Dr. The time signature is 9/4. The key signature has one sharp (F#). Measure 51 is marked with a box labeled 'Head 5'. Measure 52 is marked with a box labeled '52'. The score includes various musical notations such as notes, rests, and accidentals.

A. Sax. 1

A. Sax. 2

Pno

El. gtr

Bass

Dr.

A. Sax. 1

A. Sax. 2

Pno

El. gtr

Bass

Dr.

Fig. 4-6 Head 5 of "Cross-Fade"

53

A. Sax. 1

A. Sax. 2

Pno

El. gtr

Bass

Dr.

fine

fine

fine

fine

fine

fine

Fig. 4-6 Head 5 of "Cross-Fade"

Bass part played:

1. 2/4 bar	Tacet
2. Introduction (Intro Solo)	Intro
3. 2/4 bar	Tacet
4. Head	Head
5. Alto Sax 1 st Solo	Solo
6. Head 2	Head
7. Piano Solo	Solo
8. Head 3	Head
9. Guitar Solo	Solo
10. Head 4	Head
11. Drum Solo	Solo
12. Alto Sax 2 nd Solo	Head
13. Head 5 (interrupted)	Solo

Table 4-20 Structure of “Cross-Fade”

This shows that there is no head between the Drum Solo (11.) and the Alto 2nd Solo (12.). However, the bassist, perhaps understandably, continues to oscillate between Head and Solo bass parts. Whilst this seems to produce acceptable results for Steve Coleman’s 2nd solo, at Head 5 the differences are more challenging. This is because of the appearance of unisons previously absent in the head arrangement (these are marked with boxes on the score). The F in the bass part at the end of bar 51 forms a unison with the guitar part. This is then almost immediately followed by a further unison between the bass part and Alto 1 at beat 2 of bar 52. These two unisons, unique to Head 5, further suggest to me that this difference is not premeditated, and that it is due to the confusion described above. Further, let us note that the final F# and G in this bass part, combined with the Alto 1 and 2 parts, form an F#7 > G7 cadence in the last few beats of the piece. Further, this cadence is supported by the F#7+9 chord formed by these parts at the end of the previous bar:

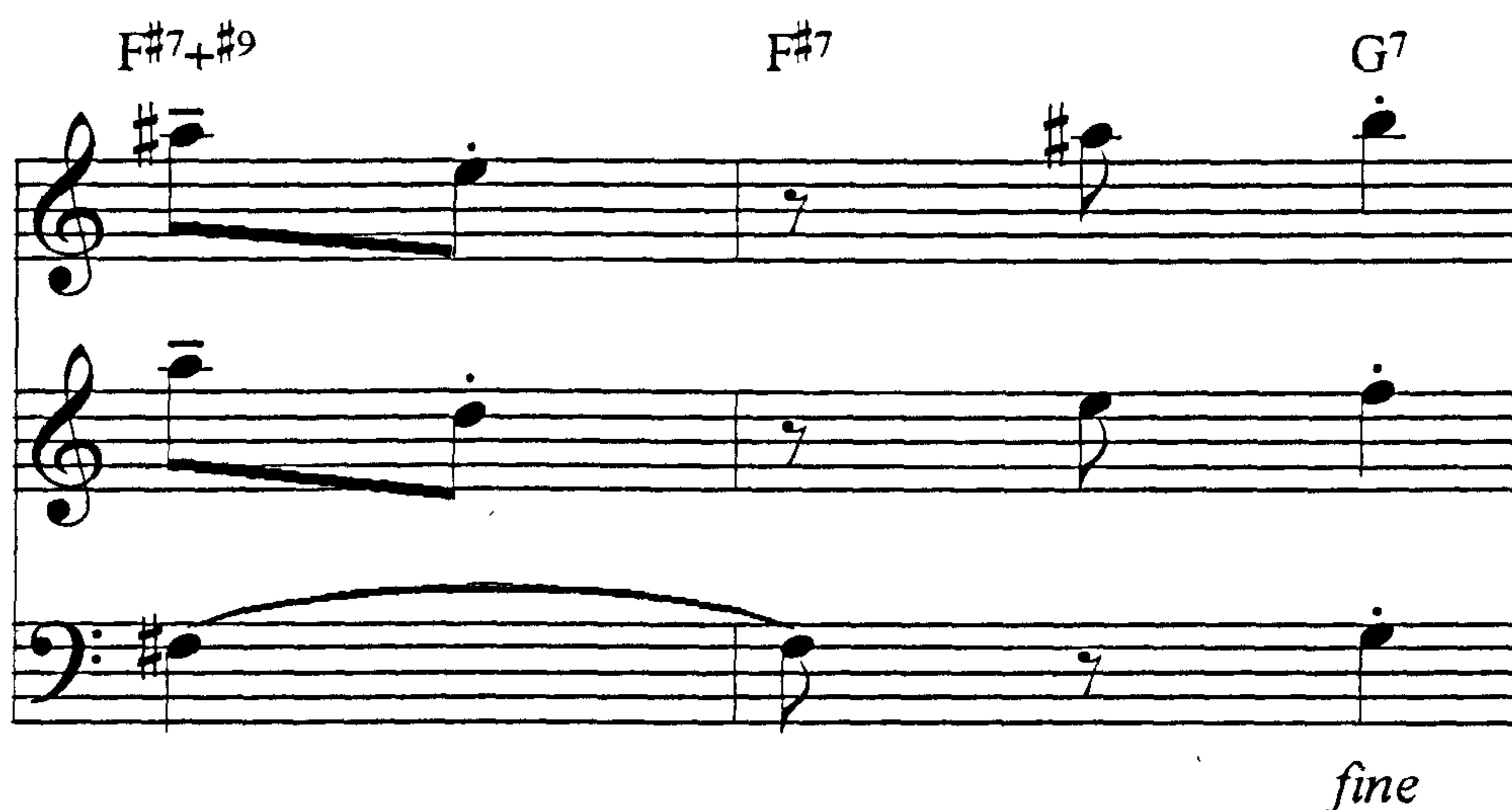


Fig 4-7 Final Two Bars of "Cross-Fade": Alto 1, Alto2 and Bass Parts

Thus, although it seems possible that the performance stopped in this last head, it was used perhaps because, in general, it represented the best take of the session. With this in mind, it seems possible that the *fine* marking in Steve Coleman's score may have been added after the event (note that he places the *fine* marking on an F# note in the bass part, whereas this instrument actually plays a G as its final note). The choice of this particular point as the position of the audio edit is intriguing in that this cadence includes not only a tonic chord of F#, but also the bII (the chord of G7).

Analysis of the solo material within "Cross-Fade" shows extensive use of the "Symmetry" system, most notably by Steve Coleman, but also by his fellow musicians. The details of this analysis are presented in this thesis within Appendix 7, and I direct the interested reader there. Summaries of this analysis follows in the main text.

I will now present a summary of the symmetrical motifs present throughout this piece.

Symmetrical Motifs

Having examined “Cross-Fade” in a chronological manner, we will now collect together and examine the various types of symmetrical structures used in this piece, these being found most especially within Steve Coleman’s three solos. Although these structures are not thematic in the usual sense, their frequency and natures suggest that they are premeditated patterns. Thus, we will refer to these as “symmetrical motifs”.¹ These are as follows:

- {0, x, y} Sets (where F# = 0)
- Chromatic Scale Subsets
- dim7 Chords (Fdim7 and F#dim7)
- {0, 2, 5, 7} Sets
- F#m7, Cm7 and G#m7 Chords
- {0, 5, 7} Set (where F# does not = 0)
- F# minor pentatonic and G minor pentatonic Scales
- Whole Tone Scale Subsets
- F# Dorian and C Dorian
- Eb9 Chord
- F# Auxiliary diminished Scale Subset

This list represents the chronological appearance of these symmetrical motifs within “Cross-Fade”. We will examine these motifs one at a time.

¹ Steve Coleman seems to refer to these structures as “cells” in Hrebeniak (1991) p. 20.

- **$\{0, x, y\}$ Sets (where $F\# = 0$)**

This is the core structure at the heart of the “Symmetry” system. In each case, the musician performs a pattern built upon a set of (a) the tonic note $F\#$ and (b) two other notes, equally distant from that tonic. As suggested above, this seems to be the actual expression of Steve Coleman’s description that he might “...pick a phrase and work in intervals, in equal sections around it”.² Let us note that there are 6 possible such $\{0, x, y\}$ sets: $\{0, 1, 11\}$, $\{0, 2, 10\}$, $\{0, 3, 9\}$, $\{0, 4, 8\}$, $\{0, 5, 7\}$ and $\{0, 6, 6\}$. With $F\#$ as 0, these are $\{F\#, G, F\}$, $\{F\#, Ab, E\}$, $\{F\#, A, Eb\}$, $\{F\#, Bb, D\}$, $\{F\#, B, Db\}$ and $\{F\#, C, C\}$. But which of these sets occur in Steve Coleman’s solos in “Cross-Fade”? Let us examine them individually:

$$\{0, 1, 11\} = \{F\#, G, F\}$$

This set is revealed initially in the bass motif at bar 15, then adapted in bars 17 and 18. Steve Coleman also uses this set as a climax at the end of his 1st solo at bar 23. It is at the heart of the C Blues scale, used in bar 23. There is an extended version of this at bars 10-11 (pattern *hh*) where $F\#$ Blues may be seen as the $\{0\}$ axis, with interpolated by the notes F and G. Similarly, at bar 50, the notes F and G are used within $F\#$ minor material (mostly $F\#$ minor pentatonic).

$$\{0, 2, 10\} = \{F\#, Ab, E\}$$

This set appears at bar 13 as the climax of Steve Coleman’s Intro Solo.

$$\{0, 3, 9\} = \{F\#, A, Eb\}$$

This set can be found at the opening of bar 7. This occurrence is noteworthy for its sympathetic rhythmic structure which further emphasises the iconic nature of the set. This set is also used as basis of the final chord in the head (beat 9 of bar 18).

$$\{0, 4, 8\} = \{F\#, Bb, D\}$$

Despite this Augmented triad pattern being that chosen by Steve Coleman to explain his “Symmetry” system to Hrebeniak³, I have been unable to find an occurrence of this set in “Cross-Fade”. However, note pattern *p* at bar 48. This is a legally incomplete Symmetrical Interval pattern using the notes Bb and F#.

$$\{0, 5, 7\} = \{F\#, B, Db\}$$

Despite the high occurrence of $\{0, 5, 7\}$ sets throughout this piece (see below), there is no such where $0 = F\#$.

$$\{0, 6, 6\} = \{F\#, C, C\}$$

This structure only appears as the last two notes in the Alto 2 part in the head as C. F#. None of Steve Coleman’s solos show this pattern; although most of the other possible tritones occur once in his solos: {Db, G} at *ii* in bar 11; {Eb, A} at *m* in bar 20; {E, Bb}

² Hrebeniak (1991) p. 20

³ Hrebeniak (1991) p. 20

at o in bar 48; and $\{D, Ab\}$ at u in bar 49. Further, I have found three occurrences of the tritone $\{E, Bb\}$ in the piano solo (a , d and j).

Summary: $\{0, x, y\}$ Sets (where $F\# = 0$)

Let us note that $\{0, 1, 11\}$ occurs in more parts of the music than any other of these sets. Further, it and $\{0, 2, 10\}$ appear at the climaxes of two of Steve Coleman's solos. Further, the only other $\{0, x, y\}$ set that occurs is $\{0, 3, 9\}$, this being found within a solo, and within the head. Thus, these three $\{0, x, y\}$ sets are the only ones that occur in "Cross-Fade". Further, by frequency of use of these sets seems to reflect a hierarchy of importance. That is, the more fundamental a set to Steve Coleman, then the smaller the intervals between $\{0 \text{ and } x\}$ and $\{0 \text{ and } y\}$. That is, where f is the frequency of occurrence of a given set:

$$f\{F\#, F, G\} > f\{F\#, E, Ab\} > f\{F\#, Eb, A\}$$

Further, the contour of all of these examples suggests a common derivation. That is, in most of the cases noted here the lower of the two non-axis notes (e.g. $\{11\}$ from $\{0, 1, 11\}$) occurs first. For example, at t in bar 7, the (lower) $D\#$ precedes the (higher) A . This structure can also be found in the other occurrence of this $\dim 7$ triad at the end of the head (beat 9 of bar 18). The notable exceptions to this rule are the two bass motif variations at bars 17 and 18.

- **Chromatic Scale Subsets**

These simple ascending/descending patterns always occur within “Cross-Fade” when Steve Coleman is playing quickly, and are derived from adjacent subsets of the chromatic scale:

<i>b</i> at bar 2:	{A, Bb, B}
<i>s</i> at bar 21:	{Ab, A, Bb, B}
<i>aa</i> at bar 23:	{F#, F, G}
<i>l</i> at bar 48:	{Bb, B, C, Db}
<i>q</i> at bar 48-49:	{A, Bb, B}
<i>aa</i> at bar 50:	{B, C, Db}

Table 4-21 Chromatic scale subsets

Pattern *b* seems to be a conventional melodic chromatic movement designed to move from one pattern (*a*) to the next (*c*). Pattern *s*, following as it does a rest, seems to exist more for its own sake. Despite this difference, I suggest that both of these patterns are an expression of the idea that *any* (adjacent) subset of the chromatic scale is qualified to occur in a “Symmetry” derived piece. This postulation is supported by the fact that the axes of these sets, which are {Bb} and {A, Bb}, are non-conventional (i.e. not derived from F# Blues). Pattern *aa*, on the other hand, is clearly simply a part of the C Blues pattern started at the end of bar 22. Pattern *l* in bar 48 and *aa* in bar 50 are also conventional: they have axes of {B, C} and {C}, respectively (i.e. these axes are both to be found in F# Blues). Pattern *q* (at bar 48-49) appears to function like pattern *b* (bar 2), that is, it links together two other symmetrical sets. Thus, it is with regard to three of these six examples that we might argue that Steve Coleman is using the chromatic scale as a source scale from which to randomly select adjacent subsets (in the knowledge that

they will, in themselves, be symmetrical). However, let us note that these three examples consist of almost identical material, and only cover a small local subset of the chromatic scale: {Ab, A, Bb, B}. Further, let us note that all of these contain either F, F#, B or C.

- **dim7 Chords (Fdim7 and F#dim7)**

There are three occurrences of dim7 arpeggios in Steve Coleman's solos. These are Cdim7 (bar 2 and 48) and G#dim7 (bar49). We should recognise straight away that these annotations are not exclusively correct because all dim7 chords have four possible letter names. The Cdim7 (pattern *c*) at bar 2 has been annotated as such because of the sustained C that ends the arpeggio. However, Steve Coleman could equally well be thinking of this chord as an Eb, F# or Adim7. The second dim7 arpeggio (pattern *h*) at bar 48 is the same note set as pattern *c* in bar 2, and, thus, it makes sense to call it Cdim7 (although the note A is repeated). The third and final dim7 arpeggio occurs as *v* in bar 49, where the notes that start and end the rapid downwards phrase are both G#s; hence the name G#dim7. Thus, I have named these three arpeggios "Cdim7" and "G#dim7". However, we should note that these two arpeggios might equally be named F#dim7 and Fdim7, thus complying with the "Sum 11" instruction for the solos; recall this has an axis of F–F#. These two arpeggios show symmetry about the axes of F and F#:

Fdim7				
B	D	F	Ab	B
-6	-3	{0}	+3	+6

F#dim7				
C	Eb	F#	A	C
-6	-3	{0}	+3	+6

Table 4-22 Structures of Fdim7 and F#dim7

Further evidence which suggests that this is the rationale for the inclusion of these sets in this way is (a) the fact that the only other possible dim7 set (Dbdim7) is absent from the piece, (b) the proximity of *h* and *v* in bars 48 and 49, and (c) the fact that F#dim7 is the most frequently occurring of these arpeggios.

• {0, 2, 5, 7} Sets

There are five {0, 2, 5, 7} sets used by Steve Coleman in this solo (marked *a* to *e*):

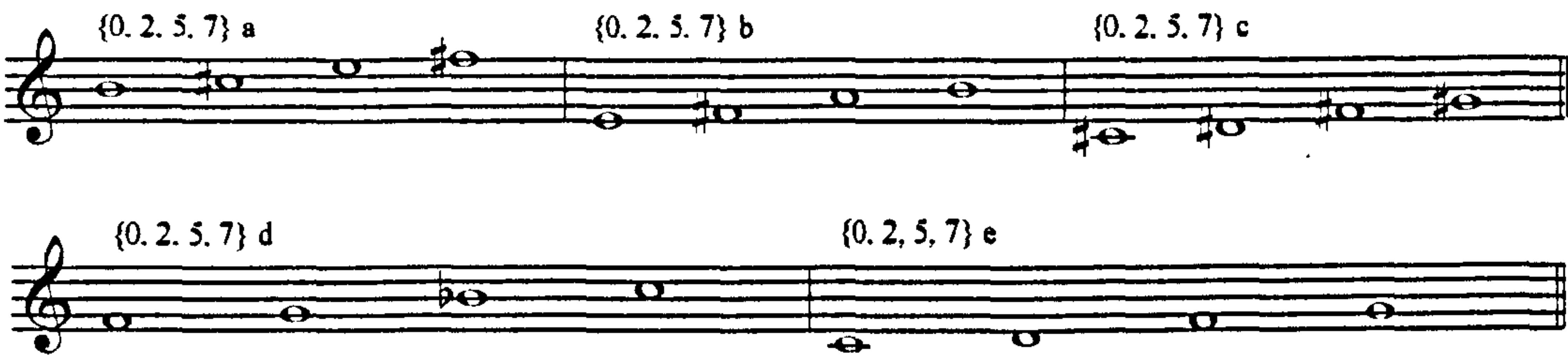


Fig. 4-8 {0, 2, 5, 7} Sets: *a*, *b*, *c*, *d* and *e*

Let us note that *a* and *b* sum to F# minor pentatonic. Also, sets *d* and *e* sum to G minor pentatonic. There are no other pairs of the {0, 2, 5, 7} cell which can sum to the minor

pentatonic scale. This suggests premeditation on behalf of Steve Coleman in the use of and transposition of this particular cell. Further, sets *b* and *c* sum to F# Dorian (there is actually a further possible {0, 2, 5, 7} set generated from F# Dorian, which is {F#, G#, B, C#}, but Steve Coleman does not use this set in the piece). Summed together, these five sets cover the chromatic scale, which might be expressed thus: F# Dorian + G minor pentatonic = chromatic scale (actually four of these sets will do it, thus: $b + c + d + e$). Thus, we might say that *a*, *b*, and *c* are the “in” sets, and that *d* and *e* are the “out” sets. Further, noting the inclusion of *c*, we might suggest that Coleman is happier to extend “in” material into the F# Dorian scale, whilst leaving “out” material firmly in G minor pentatonic. This, of course still maintains the characteristic of high polarity between the “in” and “out” material (i.e. G minor pentatonic is the complement of F# Dorian). This would not occur with smaller or larger subsets of these scales. All of the occurrences of the {0, 2, 5, 7} sets have their members grouped tightly together, usually with notes of identical duration, and within a limited interval range. In fact, they nearly all extend to cover a range of a Perfect 5th, except, that is, for *n* (bars 20-21), which has a range of a minor 7th. Let us examine in more detail these sets, focusing upon where they appear, and how they start:

{0, 2, 5, 7}x: start note: in/out:

Intro Solo

<i>b</i>	B	in
<i>b</i>	B	in
<i>b</i>	E	in
<i>b</i>	B	in
<i>b</i>	E	in
<i>a</i>	B	in
<i>b</i>	B	in
<i>b</i>	F#*	in
<i>b</i>	B	in
<i>b</i>	B	in
<i>b</i>	E	in

1st Solo

<i>a</i>	B	in
<i>e</i>	C	out
<i>d</i>	F	out
<i>a</i>	F#	in
<i>b</i>	F#*	in
<i>c</i>	C#	in
<i>c</i>	D#*	in
<i>d</i>	G*	out
<i>e</i>	G	out

2nd Solo

<i>none</i>	—	—
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Table 4-23 Occurrences and start notes of {0, 2, 5, 7} sets

{0, 2, 5, 7} sets in the Intro Solo

Note that the Intro Solo is characterised by an almost exclusive use of type *b*. Indeed, the only occurrence of type *a* is within the rapidly ascending F# minor pentatonic list in bar 7, suggesting that the analysis of pattern *v* as such a {0, 2, 5, 7}*a* set is unnecessarily detailed. Further, note that the start note is usually B or E, and that these two notes are the last and first in the normal list of this set: {E, F#, A, B}. There is a single case where the start note is internal to such a normal list (i.e. not first or last): the F# (marked * in the table above). This high frequency use of the {0, 2, 5, 7}*b* set suggests that it is seen as a more resolved subset of F# minor pentatonic than {0, 2, 5, 7}*a*. This is confirmed by Fig. 4-14, which shows the importance of the note A to this piece.

{0, 2, 5, 7} sets in the 1st Solo

By contrast, the 1st Solo is characterised by use of the entire range of {0, 2, 5, 7} sets, *a* to *e*. Further, there is not a single identical start note by set type (see Table 4-43). Also, there is an increased use of start notes which are internal to the set lists (marked with *s in Table 4-43). Since we saw {0, 2, 5, 7}*b* function as the most resolved subset of F# minor pentatonic in the Intro Solo, we might expect its transpositional equivalent {0, 2, 5, 7}*d* to be the most resolved subset of the “out” scale of G minor pentatonic. However, Steve Coleman gives equal time and space to both *d* and *e* set types in the 1st Solo.

{0, 2, 5, 7} sets in the 2nd Solo

The absence of any such sets in the 2nd Solo further emphasises their deliberate use in the previous solos, offsetting the possibility (especially in the case of types *a* and *b*) that

they are merely randomly generated subsets of core scalar material. (We might note that there is an incomplete {0, 2, 5, 7} *d* set at the start of bar 50.)

Finally, let us note that these sets are usually performed at the start of each of the solos (a) with their axis notes performed adjacently within the melodic phrase and (b) with a simple upward or downward contour. However, as each solo develops, the sets are played with their axes separated and, relatedly, more complex melodic contours are employed.

- **F#m7, Cm7 and G#m7 Chords**

There are eight occurrences of m7 arpeggios in Steve Coleman's solos: F#m7 (bars 5, 6, 9 and 50-51); Cm7 (bars 20, 22-23 and 48); and G#m7 (bar 22). Most of the F#m7 chords have complex contours. By contrast, all of the Cm7 chords have simple, upward/downward contours. The higher frequency of occurrence of the F#m7 chord, its many appearances in the Intro Solo, its more advanced contours and its use as the final material in the 2nd Solo all suggest that this is seen as the simpler chord. (This, of course, is not surprising, since it is derived from the core scale of F# minor pentatonic: see also "Key structure re the Alto Sax", below). However, the chords of F#m7 and Cm7 are closely related with regard to the "Symmetry" system. As noted above, these two chords share an axis: the note F (a hidden axis, in the case of F#m7):

F#m7						
B	C#	E	F	F#	A	B
-6	-4	-1	{0}	+1	+4	+6

Cm7				
C	Eb	F	G	Bb
-5	-2	{0}	+2	+5

Table 4-24 Showing that F#m7 and Cm7 share the axis F

(This axis of F could equally be B, with the Cm7 chord having the hidden axis.) The similar frequency of occurrences of the F#m7 and Cm7 arpeggios reflects this interrelated nature. Further, let us recall the special relationship of F#dim7 and Cdim7, examined above. However, the G#m7 chord occurs once only, and does not relate to the “Sum 11” F–F# axis in the same way. In fact, this suggests that the appearance of this G#m7 chord is not deliberate in the way that the appearances of the F#m7 and Cm7 chords are. As suggested above, it may more likely be a result of the use of the F# Dorian scale in compensation for the “out” section *r* in bar 21.

- **{0, 5, 7} Set (where F# does not = 0)**

We have already examined this piece for occurrences of the {0, 5, 7} set where F# = 0, and found none. However, here we will allow any transposition of the {0, 5, 7} set. Such a {0, 5, 7} set is, of course, the result of a choice of any three members of a {0, 2, 5, 7} set, and thus, we might take the view that we have already examined this piece for this set in the {0, 2, 5, 7} section above. However, let us recall the ubiquitous {C, F, G} set in “Examples 3, 4 and 5” from Steve Coleman’s website (examined above), and the importance given to this set in his analysis. In fact, in the Intro Solo of “Cross-Fade”,

we find one such pattern occurring frequently. That is, at *m*, *n*, *u*, *y*, *ll* and *mm* we can find the identical pitches E, A and B strictly listed in either an ascending or a descending order. These three notes are to be found within the {0, 2, 5, 7}b set, also ubiquitous within the Intro Solo. The high frequency of this set, and, further, the general absence of this set in his later two solos suggests that this is a distinct motif for Steve Coleman (although there is a single further example in the 2nd Solo: at *e*, bar 47).

- **F# minor pentatonic and G minor pentatonic Scales**

Of course, F# minor pentatonic is the core scale of this piece, but there are also a couple of instances where it seems to be used as a motif in itself. Firstly, there is the rapid ascending statement starting at beat 5 of bar 7. Secondly, pattern *mm* at bar 12 sees a deliberate attempt to cover the scale, this phrase noteworthy for its ascending and descending sections. With regard to G minor pentatonic, there are two rapidly ascending examples of this scale: *h* at bar 20; and *i* at bar 48. Also note the short (three note) excursion into G minor pentatonic near the start of bar 50.

Let us note that, like the Fdim7 and F#dim7 chords examined above, these particular two scales may be seen to have a relationship with the “Sum 11” axis of F–F#, thus:

F# minor pentatonic

B	C#	E	<i>F</i>	F#	A	B
-6	-4	-1	{0}	+1	+4	+6

G minor pentatonic

C	D	F	<i>F#</i>	G	Bb	C
-6	-4	-1	{0}	+1	+4	+6

Table 4-25 Showing the hidden F and F# axes of F# and G minor pentatonic

• **Whole Tone Scale Subsets**

In the light of what I have suggested re the chromatic scale subsets (examined above), we might expect Steve Coleman to use the Whole Tone scale in a similar way. However, here the evidence is much less clear. There are four examples of Whole Tone scale subsets in Steve Coleman's solos. There are two types, each occurring twice: {G, A, B, Db} (bars 19 and 50) and {B, Db, Eb} (bars 49 and 50). Both of these sets are in fact derived from axis material from F# minor pentatonic, and, thus show none of the axis independence from F# Blues that we found in the Chromatic Scale Subsets, for example *b* and *s*, above. Further, let us note that both of these sets contain only a single note outside of F# Blues (G and Eb, respectively), and that this is a characteristic of much other "Symmetry"-derived melodic material that we have seen so far in the piece. Thus, I do not believe that Steve Coleman uses the Whole Tone (unlike the chromatic) scale as a source scale for (adjacent) sets in this piece.

• **F# Dorian and C Dorian**

Although F# minor pentatonic (and its close relation, F# Blues) may be regarded as the core scalar material of “Cross-Fade”, there are a few instances when Steve Coleman uses F# Dorian, notably as compensation for “out” material. This scale is symmetrical about its own axis:

F# Dorian

C#	D#	E	F#	G#	A	B
-5	-3	-2	{0}	+2	+3	+5

Table 4-26 Structure of F# Dorian

Note however, that this scale usually occurs in subset form, only appearing once as a relatively adjacent set: that is, in the first 5 beats of bar 21. There is only one occurrence of the C Dorian set: at bar 22. However, let us note that this 7-note scale is (a) completely listed within 9 notes and (b) continued for a further 9 notes (of 5 pitches); these characteristics suggest that its use here is quite deliberate. We have already noted that C Dorian has a (hidden) axis of F#:

C Dorian

C	D	E \flat	F	F#	G	A	B \flat	C
-6	-4	-3	-1	{0}	+1	+3	+4	+6

Table 4-27 C Dorian with hidden F# axis

However, the Cm7 subset that opens this section has an F axis:

Cm7 Subset		
Eb	<i>F</i>	G
-2	{0}	+2

Table 4-28 Cm7 subset with F axis

Thus, the axis shifts from F to F# between these two otherwise strongly related structures. The rest at beat 4 of bar 22 neatly separates these two structures. Note that the C Dorian set is complete even if we start looking for it after the Cm7 subset (i.e. from beat 5 of bar 22).

- **Eb9 Chord**

It was noted above that the Eb9 arpeggio, although symmetrical in itself, does not have an axis of either F or F#, but, rather, E (= Bb).

Bb	Db	Eb	<i>E</i>	F	G	Bb
-6	-3	-1	{0}	+1	+3	+6

Table 4-29 Eb9 chord, hidden E axis

Use of either an E9 or F9 arpeggio (with axes of F and F#, respectively) would have maintained the “Sum 11” axis. This (concert) Eb9 chord is, for Alto saxophone, C9 (axis Db). I wonder if the choice of this particular transposition represents Steve Coleman remembering this structure in the key that he may have first calculated it (recall that C = 0 in his descriptions of the “Symmetry” system, generally speaking), rather than the key(s) that relate to this piece.

- **F# Auxiliary diminished Scale Subset**

As noted above, we hear 7 notes of this 8-note scale (no Db). Note that the performance of this scale grows outwards from the Cm7 arpeggio, which has, in itself, a symmetrical relationship with the F# Auxiliary diminished scale:

F# Auxiliary diminished									
F#	G	A	Bb	<i>B</i>	C	(Db)	Eb	E	
-6	-5	-3	-1	{0}	+1	(+3)	+5	+6	
[Cm7]:	G		Bb		C		Eb		

Table 4-30 Symmetrical relationship of Cm7 and F# Auxiliary diminished

Although its appearance at bar 48 is the solitary example, there are several reasons that lead me to believe that this represents (like C Dorian) a conscious use of the F# Auxiliary diminished scale by Steve Coleman. Firstly, we can see an almost complete performance of this scale. Secondly, there is an unusual adjacent pair of Symmetrical Interval patterns (*o* and *p*) within the structure that may (in this light) actually be randomly generated. Thirdly, as mentioned above, the scale subset grows outwards from the core Cm7 arpeggio; both share the (hidden) axis B. (This maintained axis of B through Cm7 and F# Auxiliary diminished is on contrast with the Cm7 subset through C Dorian pattern examined above, where the axis shifts from F to F#.) Note that A, E and F# quickly follow the Cm7 arpeggio, almost as if Steve Coleman is “filling in” the missing notes. Indeed, if we permit the B at the start of bar 49 to be seen as either (a) a repeat of the (hidden) axis, or (b) an error (instead of C), then this scale could be seen to continue through beats 1 and 2 of bar 49. Thus, perhaps the missing Db is to be found as the axis of *r* in bar 49! That Steve Coleman attempts to use all of the notes from this scale is reflective of its structure; it would not be “correct” just to perform a short adjacent subset, as we saw with the chromatic scale. Further, note that both F#m7 and (if

we include the (hidden) B axis) F# minor pentatonic are subsets of this scale (this can be seen in Table 4-50 above).

Symmetrical Material with “Sum 11” Axes

Here is a summary list of the material used by Steve Coleman that has (included or hidden) “Sum 11” axes of either F (=B) or F# (=C):

Axis of F (=B):

Fdim7

F#m7

Cm7 (and subset)

F# minor pentatonic

G Whole Tone

F# Auxiliary diminished Scale Subset

Axis of F# (=C):

F#dim7

G minor pentatonic

G Whole Tone

C Dorian

As implied in some of the above paragraphs, I suggest that it is in this material that we find a realisation of the “[xm] (or sum y center)” instruction. That is, because this piece is based on “Sum 11”, Steve Coleman chooses material based upon the axes of F and F#. This structure is defined in his “Spirals Number One & Two” Table (reproduced as

Table 4-27, above). I have searched “Cross-Fade” carefully for other ways in which this “Sum 11” instruction may have been interpreted, but I have found none. Further, there are no melodic constructions to be found in Steve Coleman’s solos that do not fall into this analysis (beyond those many phrases that are built using the basic “Symmetry” strategy). Clearly, there are a considerable number of symmetrical structures that have either an F or F# axis ($64 \times 2 = 128$; see Appendix 7). However, Steve Coleman has chosen sets that are already part of the conventional jazz language. Thus, it seems that from the very beginning of the 1st Solo we can see a new level of use of the “Symmetry” system, beyond the basic “Symmetry” strategy first heard in the Introduction. That is, one where precalculated material with an axis derived from the “Sum 11” axis pair is inserted into the improvisation. Thus, the phrase “[xm] (or sum y center)” is an instruction to include within the solo these particular sets that (a) are commonly used in jazz and (b) have the required symmetry. Further, let us note the relatively high speed with which many of these sets are performed; notably, the Cdim7 and F# minor pentatonic sections in the Intro Solo, the G minor pentatonic, Cm7 and C Dorian sections in the 1st Solo, and the F# Auxiliary diminished in the 2nd Solo. This high degree of fluency further suggests premeditation. Thus, I suggest that all of these sets are precalculated symmetrical motifs.

Motif Contour in Performance

There is a further level of detail with regard to motifs that also suggests that they are premeditated. That is, the specific contours chosen in the execution of the set concerned, which usually vary from one performance to the next. Generally speaking, the first time that a motif appears as a complete list in Steve Coleman’s playing, it appears either in ascending or descending form. This is true of:

- Cdim7 (bar 2)
- {0, 2, 5, 7}b (bar 3)
- Ebdim (bar 7)
- F# minor pentatonic (bar 7)
- {0, 2, 5, 7}a (bar 7)
- G Whole Tone Subset (bar 19)
- {0, 2, 5, 7}e (bar 20)
- G minor pentatonic (bar 20)
- Cm7 (bar 20)
- C Blues (bars 22-23)
- G#dim7 (bar 49)

However, nearly all of these are performed later with a more complex contour. Thus, we might argue, these motifs are fundamental material that require stating in a pure manner, but then are open for (indeed *require* in this improvisational context) alteration and interpretation. That is, increasing complexity of contour is used here as a variation technique. Another example, not in the list above, is the F#m7 arpeggio at the end of the 2nd Alto Solo. Although this is not the first occurrence of this chord, perhaps its special position here enforces its simplicity of contour. With this in mind, let us note that the previous F#m7 (bar 5), and, further, the Eb9 (bar 48) arpeggios *almost* follow the rule; both contours are changed at the end from an otherwise upward or downward direction. On the other hand, this upward/downward restriction is never placed upon the F# minor pentatonic (and Blues) scale, suggesting it is the *resource* for the piece, not a *motif* as such (the closest this scale gets to being performed straight up or down is the mostly descending list that fills bar 9). However, the performance of the {B, C, C#} motif found at *a* (bar 2) and *x* (bars 7-8) in the Intro Solo shows a different approach. Here, the motif

seems to have been deemed too straightforward to deserve a performance as a simple ascending or descending list. Perhaps this deliberate complication is due to the fact that the set is wholly derived from the F# Blues scale and, as such, may be seen as insufficiently complex to warrant a simple (upward or downward) performance. Further, the (slightly more complex) zigzag contour of a melody such as C, B, C# would, perhaps, sound too similar to the bass motif F#, F, G, found throughout the head.

Subset Relationships between Symmetrical Motifs:

Let us also note that many of the symmetrical motifs are subsets of each other:

F# Dorian (*includes amongst its subsets*):

F# minor pentatonic

F#m7

{0, 2, 5, 7} a, b, c

{E, A, B}

F# Blues:

F# minor pentatonic

F#m7

{0, 2, 5, 7} b, c

{E, A, B}

{B, C, C#}

F# minor pentatonic:

F#m7

{0, 2, 5, 7} b, c

{E, A, B}

G minor pentatonic:

{0, 2, 5, 7} d, e

{0, 2, 5, 7}b:

{E, A, B}

C Blues:

Cm7

Cm7 subset {Eb, G}

{0, 1, 11} = {F, F#, G}

C Dorian:

Cm7

Cm7 subset {Eb, G}

F#dim7:

{0, 3, 9} = {F#, A, Eb}

F# Auxiliary diminished:

F#dim7

{0, 3, 9} = {F#, A, Eb}

Cm7

Cm7 subset {Eb, G}

These subset relationships might be seen as providing an advantage in the provision of solo material using the “Symmetry” system. That is, (a) the interrelation of these sets makes recall simple and (b) this subset list shows a variety of “start” notes, and thus provides a wide range of melodic possibilities. Further, we have seen several examples of patterns in Steve Coleman’s solos that, whilst in themselves do not show adherence to the “Symmetry” system, are other subsets of some of the commonly-used (and premeditated) sets listed here.

A Comparison of Steve Coleman's Solos

There are important differences between the Intro Solo on one hand and the 1st and 2nd Solos on the other. Firstly, let us note the large difference in note density (the number of notes per bar) between (a) the Intro Solo and (b) the 1st and 2nd Solos:

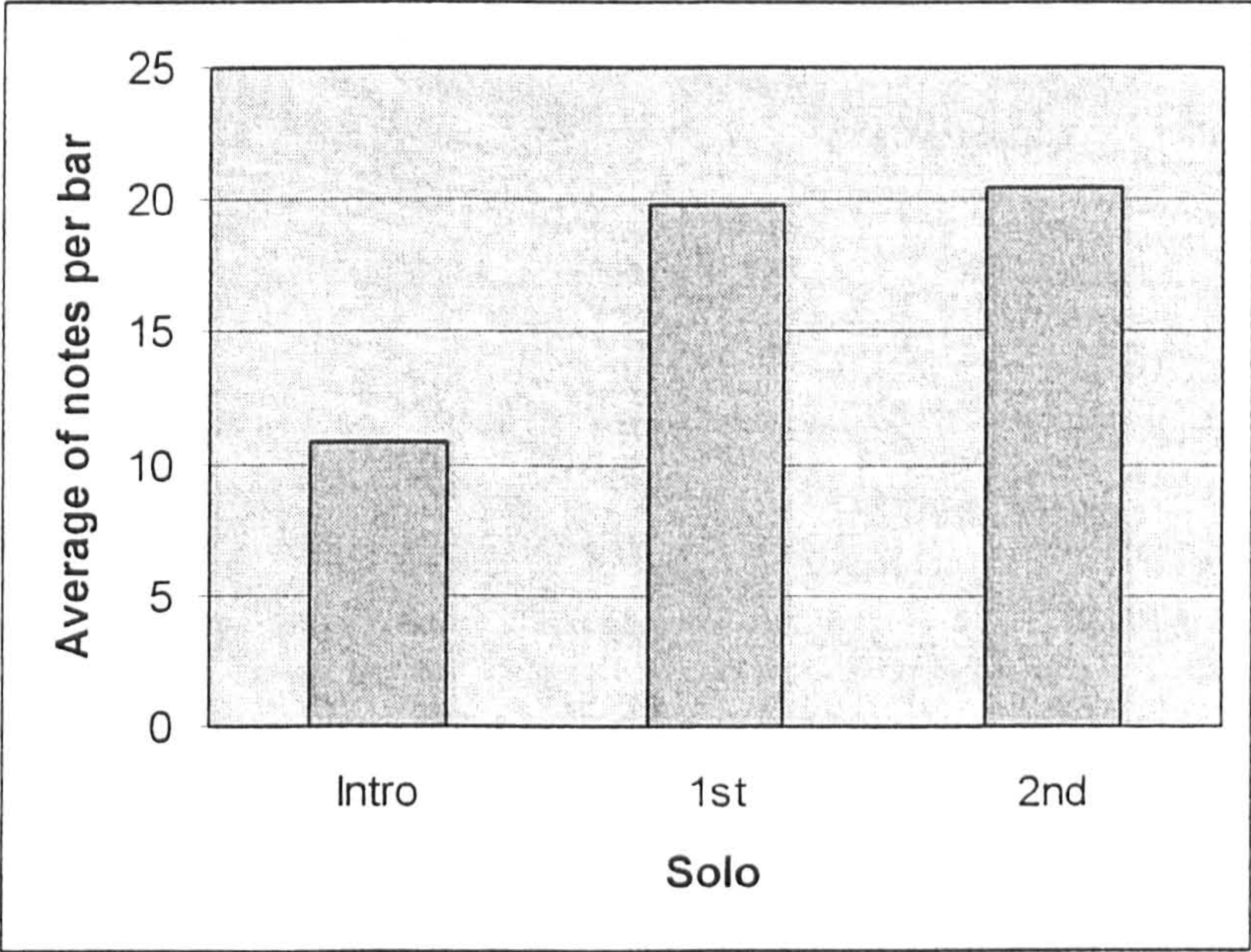


Fig. 4-9 Graph Showing Note Density of Intro, 1st and 2nd Solos

The 1st and 2nd Solos have a very similar density, with a slight increase at the 2nd Solo (19.8 and 20.5, respectively), but both are nearly twice as dense as the Intro Solo. Secondly, there is an increase in speed of change from strategy to strategy. Thirdly, there is an increasing use of the precalculated Symmetrical Motifs (defined above), and a corresponding decrease in use of the more conventional “Symmetry” patterns (i.e. a decrease in the direct melodic use of the core scale F# minor pentatonic/Blues). Fourthly, there is an increase in the speed of chromatic saturation. These four changes result in a shift away from the home key of F# minor and an increase in the use of material from the complement set (F Dorian), as shown in the following graph:

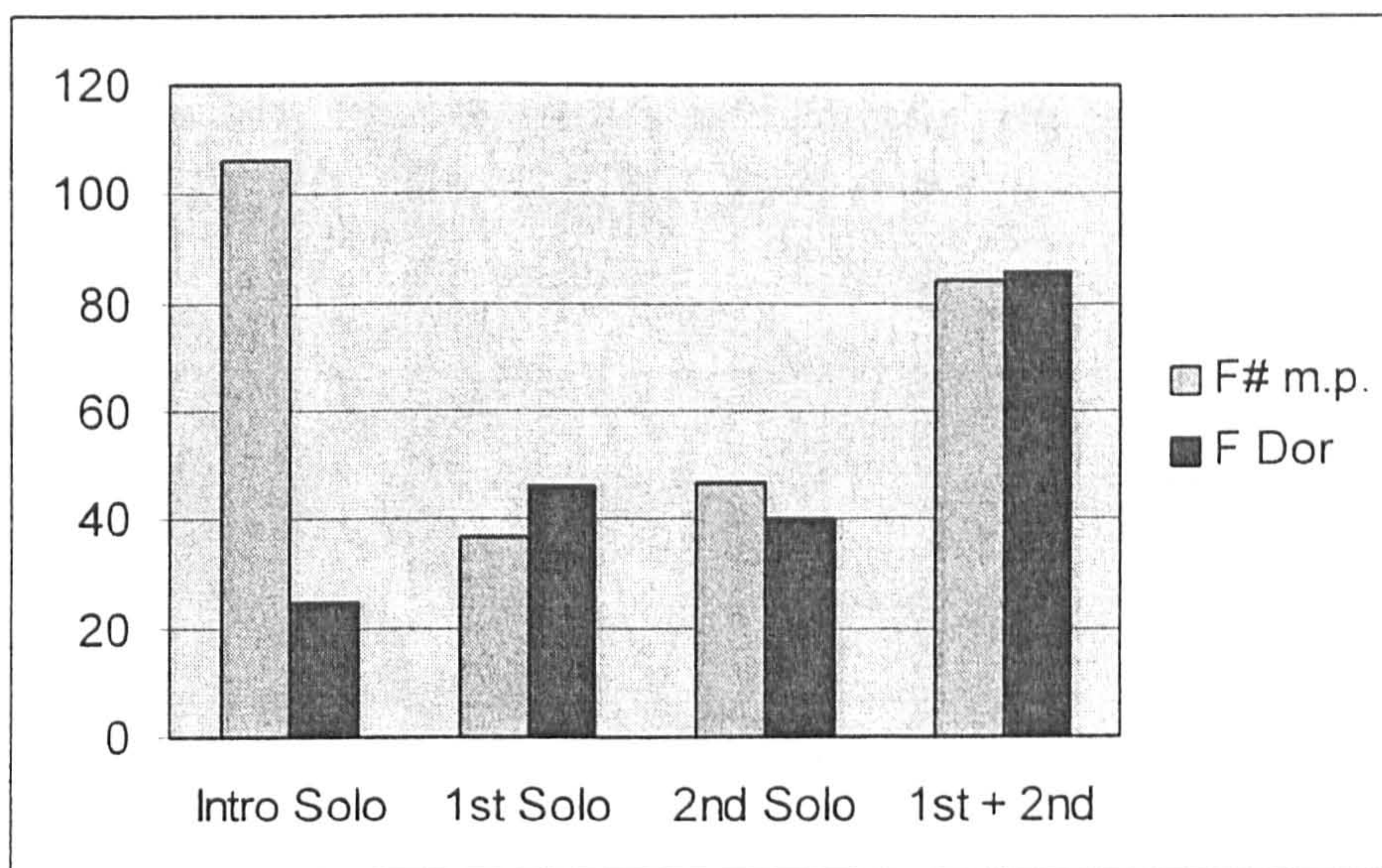


Fig. 4-10 Graph Showing Use of F# minor pentatonic and F Dorian (Complement Set) in Solos

The two rightmost bars in Fig. 4-12 reveal that the 1st and 2nd Solos have, on average, an approximate equality of notes from F# minor pentatonic and F Dorian.

Emphasis on G rather than F

Given what we have seen with regard to the design of the “Symmetry” system, we might expect to find a global balance of F and G (notes and chord/scale material) within this piece. That is, if we asked Charlie Chance (see Chapter 2) to randomly improvise using the core “Symmetry” strategy, we would expect to find an equal number of notes from the keys of F and G minor pentatonic or Blues scales. Let us see if this is so. If we sum the occurrence of each pitch class for all three solos, we can draw the following graph:

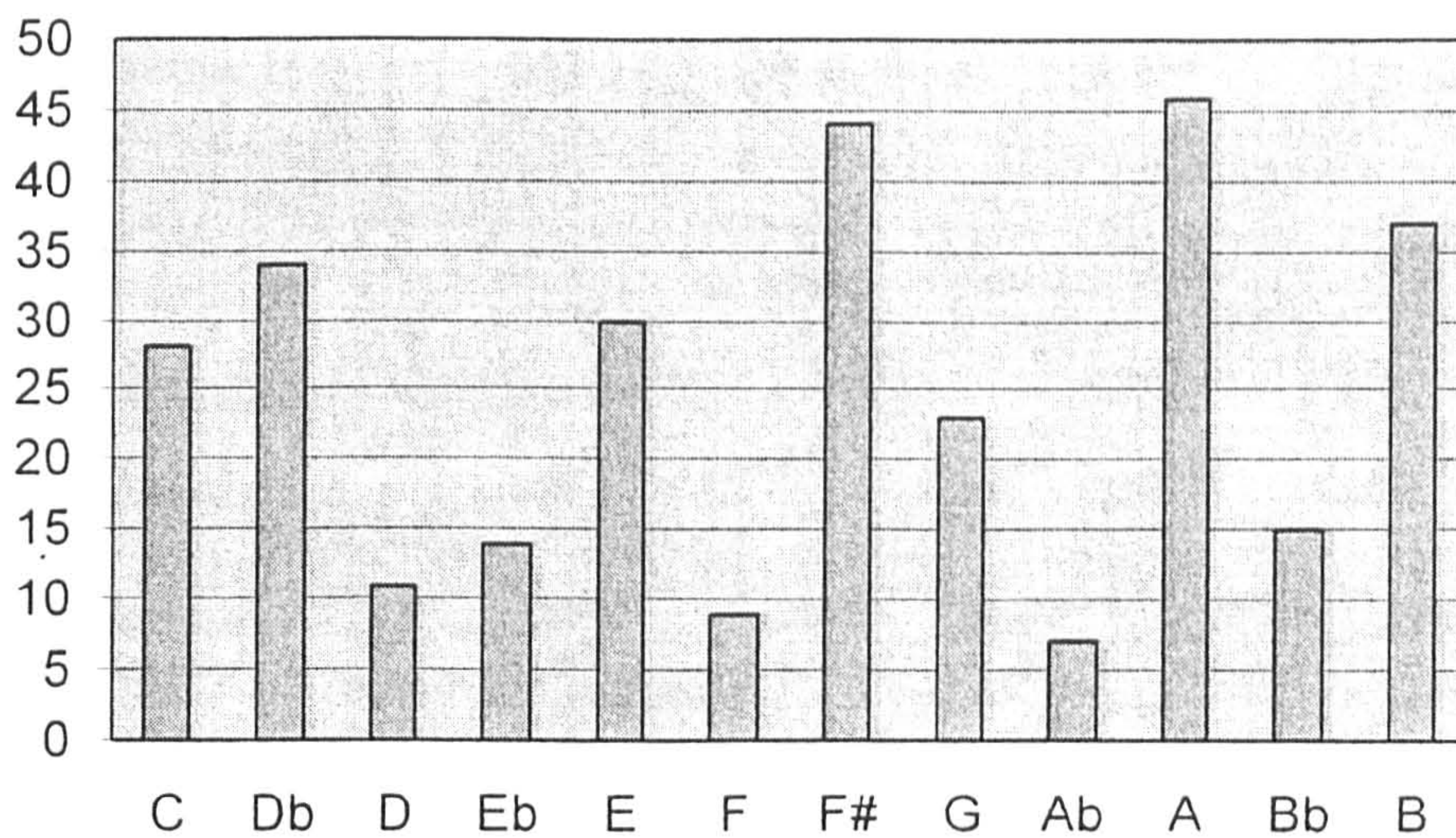


Fig. 4-11 Graph Showing Pitch Class Frequency in Solos

If we reorder the x axis of this graph to collect the F# minor pentatonic set to the left and its complement (F Dorian) to the right, we get the following graph:

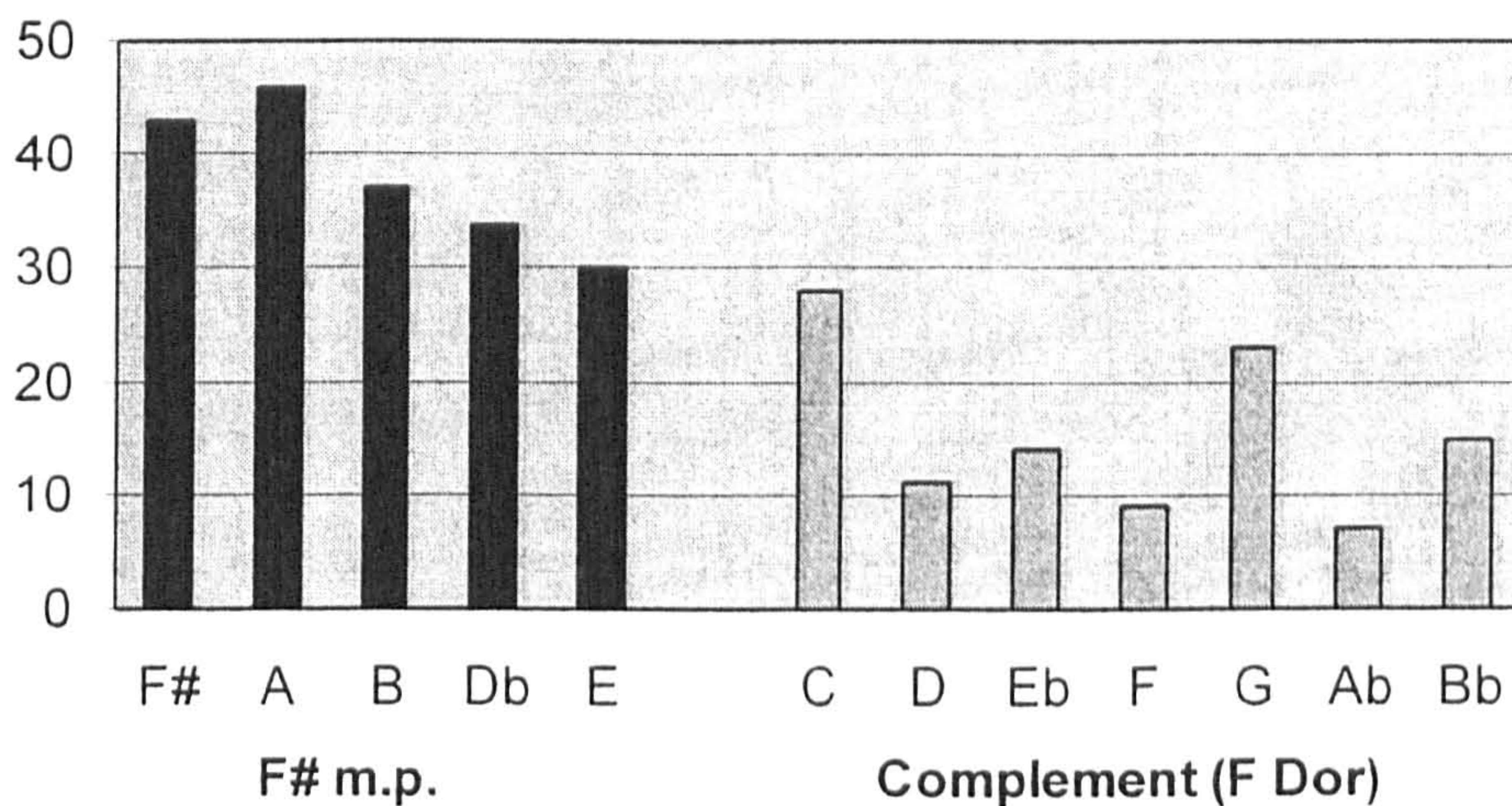


Fig. 4-12 Graph Showing Pitch Class Frequency in Solos Ordered By F# minor pentatonic and F Dorian (Complement Set)

Given the relatively high use of F# minor pentatonic in the Intro Solo, it is not surprising to see this scale as the dominant group in this graph. Note also that C, the note that converts F# minor pentatonic into F# Blues, is the next most frequently occurring note. Note especially that the note G occurs more than twice as frequently as the note F. This means that there is not a global balance of occurrences of the notes F and G as predicted above. Further, if we count the number of notes that belong to F#, F and G (NB order) minor pentatonic in the Intro Solo, 1st Solo and 2nd Solo, we can construct the following graph:

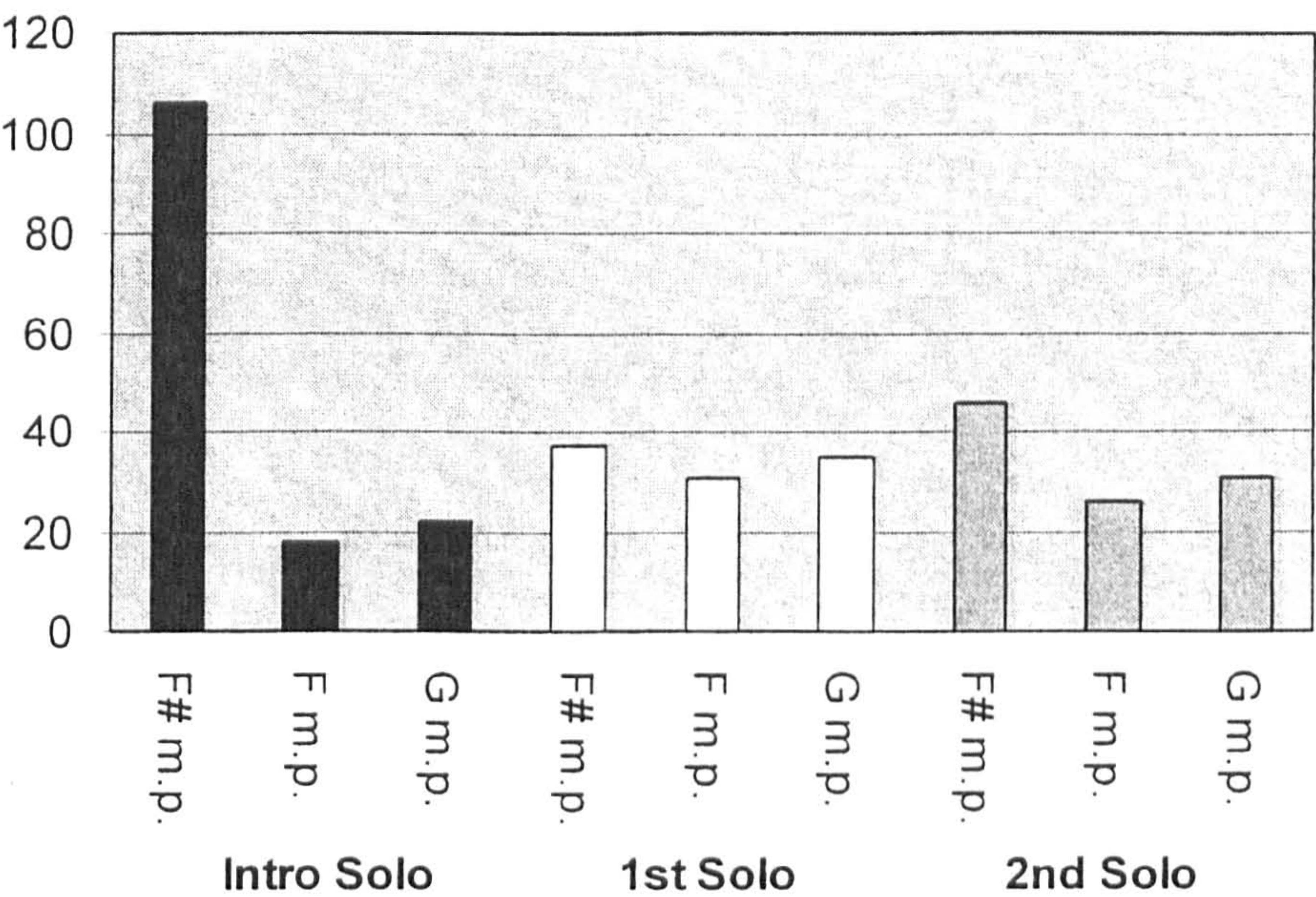


Fig. 4-13 Graph Showing Pitch Class Frequency in each Solo, grouped By F#, F and G minor pentatonic

This shows two key trends. Firstly, whilst in the Intro Solo, the home key of F# minor predominates, in the later two solos, the three keys are more equally represented. Secondly, the key of G minor pentatonic is better represented than the key of F in all three solos. This structure is, naturally, also apparent if we sum these data by scale for these three solos:

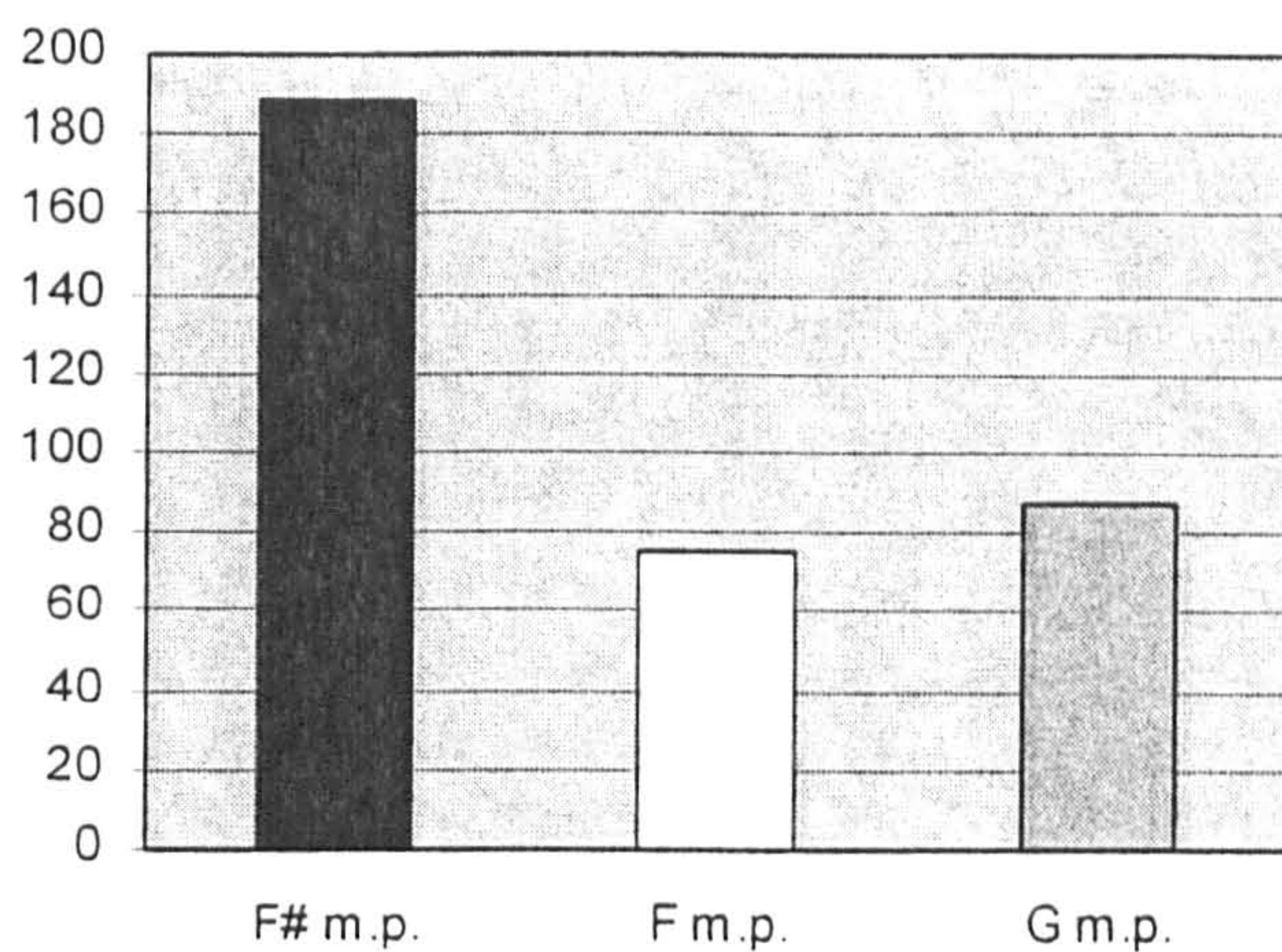


Fig. 4-14 Graph Showing Pitch Class Frequency for all Solos, grouped By F#, F and G minor pentatonic

Further to this general statistical evidence, there is to be found a large number of examples (mostly within Steve Coleman's solos) where G may be seen to dominate F:

Intro Solo

- Steve Coleman's solo in the Introduction contains many interpolated Gs, but only a single F (and this F is immediately chromatically resolved to an F#, reducing any sense of independence).

Head

- The piano part in the head consists entirely of an octave G
- G is the most common axis member of the limited sets used for each part in the

Head

- The 5th (interrupted) head ends on an F#7 > G7 cadence.

1st and 2nd Solos

- The only non-scale tone generated by the use of the (repeated) Whole Tone pattern {G, A, B Db} is G (bars 19 and 50)
- All Whole Tone subsets (such as they are) belong to the set that we might equally call “G Whole Tone” or “F Whole Tone”. However, when using these sets, Steve Coleman never includes the note F in his melodies
- There are numerous occurrences of G minor pentatonic in these solos but none of F minor pentatonic (bars 20, 21, 48 and 50)
- It is the note G in {0, 2, 5, 7} *d* set in bar 21 that is repeated
- F# and G are most frequent start notes for the densest collection of {0, 2, 5, 7} cells (marked *a* to *e*), found in bars 20-22
- There is a *missing* F in the rapidly ascending (and otherwise complete) C Blues in bar 22
- At bar 23 there is a repeated G in the -1 0 +1 pattern: also, this is the highest note in the solo

- In Steve Coleman's 1st solo, G naturals appear by themselves, interpolated into the F# Blues material. By contrast, F naturals generally only appear *adjacently* to a G natural, and then only as a member of one of the {0, 2, 5, 7} d or e sets. There is only one exception to this: the F near the start of bar 23. Here, however, this F is not truly solitary in that it is part of the -1 0 +1 pattern. This suggests that G notes are independent in a way that F notes are not. The 2nd Solo has more independent F notes, but there are still more Gs.
- The two phrases in bar 50 end on G and start on F - however, the G is sustained more than three times as long; also, this is the highest note in the solo
- The scale of F# Auxiliary diminished contains F# and G, but no F (bar 48).

Piano Solo

- The piano solo is noteworthy for its interpolation of G (rather than F) notes.

Caveats

There are, however, a few caveats that I wish to mention:

- I have interpreted the {0, 2, 5, 7} d and e sets as subsets of G minor pentatonic. However, this set could be seen as a subset of, say, F Dorian. Thus my approach might be said to prioritise (G) minor pentatonic "out" material over (F) Dorian (although this is the 7-note complement of F# minor pentatonic). However, let us

recall that $\{0, 2, 5, 7\}_d + \{0, 2, 5, 7\}_e = G$ minor pentatonic, and, whereas $\{0, 2, 5, 7\}_b + \{0, 2, 5, 7\}_c = F^\#$ Dorian, there is no pair of $\{0, 2, 5, 7\}$ sets used by Steve Coleman that sum to F Dorian (these are in fact $\{C, D, F, G\}$ (= set type e) and $\{Eb, F, Ab, Bb\}$ (this latter set is not used))

- Beat 7 of bar 49 contains an adjacent Bb, B and C. Further, bar 25 contains a phrase based upon Bb and B. These might suggest possible uses of the F Blues scale as a resource. However, as I have shown, I do not believe this to be the true scalar source of these notes
- There is a general equality of appearance of F and G in $\{0, 2, 5, 7\}_d$ and e sets; also in a few larger scalar patterns (e.g. (a) the Eb9 chord at bar 48 and (b) at the third and fourth notes of bar 50)
- Attention is drawn to the note F within *j* in the 2nd Alto Solo (bar 48) by its formation of a Bbm arpeggio that follows the pattern *k*
- Many of the symmetrical motifs used in the 1st and 2nd solos (described above) contain both F and G notes in theory (e.g. C minor pentatonic, C Blues, Eb9).

On balance, then, we can see that in this performance Steve Coleman prefers the bII to the VII, both as a note and the key of larger chord/scale constructions, despite the fact that the “Symmetry” and “Sum” systems might give equal weight to this conjugate pair. “Cross-Fade”, then, has more in common than we might have guessed with those pieces examined in Chapter 1 that use the bII transposition in order to derive “out” material. I suggest that Steve Coleman’s preference for G over F is for two reasons:

Firstly, it “sounds right”; i.e.. subjectively to Steve Coleman. Let us recall that he uses the “Symmetry” system with the caveat that he will “...throw out the things that don’t sound good”¹. That is, material generated by the “Symmetry” system that tends towards the bII and away from the VII of the home key sounds more correctly “out” to Steve Coleman. Let us recall that Steve Coleman was firstly a bop stylist, heavily influenced by Charlie Parker, and that bop is dominated by bII > I cadential structures. Further, let us note that whilst G minor pentatonic has little in common with the home key of F# minor pentatonic, F minor pentatonic may be seen to contain two notes from F# Dorian (G# and D#). Thus, G minor pentatonic may be made to sound more “out” than F minor pentatonic.

Secondly, by emphasising G, and avoiding F, Steve Coleman may be compensating for the importance given to F in the “Sum 11” instruction; i.e. the F–F# axis. Thus, I postulate that, in “Cross-Fade”, F has a theoretical weight on one side of a central F#, and G has an actual, employed weight on the other:

-1	{0}	+1
F	F#	<i>G</i>
Theoretical (Sum 11)		<i>Sounded</i>
Ignored		Emphasised

Table 4-31 Model of F, F# and G use in “Cross-Fade”

This analysis ignores the possibility that the axis of {F, F#} from the “Sum 11” instruction is to be read as, say {F Dorian, F# minor pentatonic}. This would allow all “out” material to belong to the key of F, and all “in” material to belong to the key of F#. However, although this structure is mathematically reasonable, it is not explored in this piece, and thus, I suggest, it is not a correct analysis. Consider the following points:

¹ Hrebeniak (1991) p. 20

- As seen in the graphs above, the note F occurs relatively infrequently in this piece (further, its minor 3rd, Ab, is the *least* frequent note in this piece)
- F Dorian would provide two further {0, 2, 5, 7} sets, beyond those that sum to G minor pentatonic: {Bb, C, Eb, F} and {Eb, F, Ab, Bb}. Neither of these occur in this piece
- The special characteristic of F# minor pentatonic and F Dorian is that they sum to the chromatic scale, yet nowhere in this piece is this specifically explored.

“Drop Kick”

Let us now briefly examine another piece by Steve Coleman, “Drop Kick” (title track of “Drop Kick” (1993) Novus 01241 63144 2), since it seems that this piece, like “Cross-Fade”, also has an F–F# axis. Vijay Iyer has written about “Drop Kick”:

“The pitch content of his lines plays with the ambiguity of the static “tonality” established by the other instruments. The sound may be called F# minor, though the most heavily accented bass note is F (or E#), as is the most frequent guitar note. The simultaneous presence of these two “tonalities” provides much material for the improviser...Coleman’s initial phrase sound roughly like F# minor, but gradually the pitch organization is explored in less direct ways.”²

In “Drop Kick”, the parts enter one at a time, starting with the bass:



Fig. 4-15 "Drop Kick" – Bass Guitar

This clearly shows (a) the F# minor tonality (expressed by a perfect list of F# minor pentatonic and an F#m triad arpeggio), and (b) the surrounding F notes (marked with *s). The guitar enters next, mostly playing Fs (as noted by Iyer). The other notes (Ab and A) may be seen to function in both keys of F and F# in a jazz manner: they are the II and bIII of F# (= minor), and the bIII and III of F (=Blues):



Fig. 4-16 "Drop Kick" – Electric Guitar

Next, the piano performs a pair of inversionally equivalent F# Dorian clusters and the Alto Saxophones enter shortly afterwards, playing a pair of minor 3rd intervals with symmetrical contour:

² Iyer (1996) p. 11



Fig. 4-17 "Drop Kick" – Piano



Fig. 4-18 "Drop Kick" – Alto Saxophones

Thus, we can see that in "Drop Kick" (released in 1993, three years after "Cross-Fade") we can easily find material in the head that relates to the F–F# axis. This is in contrast with "Cross-Fade", where the head does not relate to the F–F# axis. Further, there seems to be less use of G material in Steve Coleman's solos in "Drop Kick" than in "Cross-Fade", and the F–F# axis seems to be the true source for these improvisations.³ We will now examine a possible reason for Steve Coleman's preference for this particular axis in "Cross-Fade" and "Drop Kick".

³ This statement should be qualified. I have not notated the solos in "Drop Kick", merely auditioned them carefully.

Key structure re the Alto Saxophone

Let us now consider the scales and keys used by Steve Coleman in “Cross-Fade” and “Drop Kick”, focusing upon the minor pentatonic, and examining their Black/White structure of the alto sax scales as they would appear at a keyboard.

concert (C):	alto sax (Eb):	Black/White:
F# minor pentatonic	Eb minor pentatonic	B-B-B-B-B
F minor pentatonic	D minor pentatonic	W-W-W-W-W
G minor pentatonic	E minor pentatonic	W-W-W-W-W
C minor pentatonic	A minor pentatonic	W-W-W-W-W

Table 4-32 Black/White structure of minor pentatonic scales for C and Eb instruments

Note that the three “out” keys (F, G and C) transpose as D, E and A for the alto. These three keys form the complete list of white note minor pentatonic scales at the keyboard (i.e. no flats or sharps). Similarly, the “in” key of Eb is the only black note minor pentatonic scale. If we extend the minor pentatonics into Blues scales, although we now find mixes of Black and White at the keyboard, the polarised relationship (between {D, E, A} and {Eb}) is maintained for this set of keys:

concert (C):	alto sax (Eb):	Black/White:
F# Blues scale	Eb Blues scale	B-B-B-W-B-B
F Blues scale	D Blues scale	W-W-W-B-W-W
G Blues scale	E Blues scale	W-W-W-B-W-W
C Blues scale	A Blues scale	W-W-W-B-W-W

Table 4-33 Black/White structure of blues scales for C and Eb instruments

Aside from being visually clear at the keyboard (the input device for Steve Coleman's computer sequencer, presumably), the keys of D, E and A minor pentatonic are probably the most easy to play on the saxophone. This is because these three minor pentatonics use no side keys, and for the Blues scale, the characteristic #IV (= bV) is, in each case, a side key. Thus the keys of D, E and A minor pentatonic have a remarkably similar spatial structure. Further, let us note that, by being "natural" scales, the minor pentatonics of D, E and A promote a natural fluency, offsetting the fact that they are being used as "out" keys against the home tonality (for Alto) of Ebm. That this is a strategy used in both "Cross-Fade" and "Drop Kick" suggests that Steve Coleman is deliberately choosing a spatially difficult key as the "in" material and spatially easy keys for the "out" material. With this in mind, let us note that it is in the 1st and 2nd Solos of "Cross-Fade" that Steve Coleman is playing fastest, rapidly moving from motif to motif, and where he plays as many notes from (the five note) F# minor pentatonic as from its (seven note) complement, F Dorian. Further, the most rapidly performed sections of these two (1st and 2nd) solos are both "out" patterns. These are the C Blues sextuplet at bar 22 and the Eb9 quintuplet at bar 48.

Chapter 5

Conclusions

All of the various strategies and systems examined in this thesis are the result of conscious departures from an underlying tonality. Further, their design means that they can all be “turned on like a tap” by a musician improvising at speed.¹ Also, all of them are manufactured extensions of the tonal and rhythmic language of jazz as well as employing methodologies sourced in the improvisational tradition.

In my Introduction I defined the use of terms such as “outside” with regard to this thesis, and examined texts from the literature (both pedagogical and historical/musicological) that are concerned with this area of research. Further, I proposed a chronology of the evolution of “playing outside”.

I examined 20 excerpts from jazz solos in Chapter 1, all of which display at least one section where a musician plays “outside”, according to the definition provided by the Introduction. Whilst recognising that the performances under examination were diverse, I have described three common strategies of “playing outside”: motivic, scalar and spatial. These strategies were found to be surprisingly distinctive in the excerpts under analysis. Further, sets of these three strategies were sometimes found within solos, and I have extrapolated a hierarchical relationship of “outness” between these strategies.

In the excerpts that I have examined, motivic sections were generally found to consist of relatively small (2- and 3-note) sets. This characteristic was also seen to be true of the

“A Love Supreme” motif, studied in Chapter 2. By contrast, scalar and spatial sections generally consisted of larger note sets. The motivic and scalar strategies tended to use the bII transposition (just as may be seen in Russell’s (1953) theoretical example), whereas spatial playing, dependent upon the physical structure of the instrument concerned, showed a variety of relationships with the underlying tonality of the solo. Transposition to the bII also saw many musicians change mode; the frequency and diversity of this was a real surprise to me. (Russell (1953) does not examine these latter common strategies in any detail.)

Perhaps unsurprisingly, spatial playing showed greatest diversity in outcome by instrument. Spatial strategies at the keyboard were seen to employ the Black/White structure of the keys. Moving across strings on a guitar’s neck and the use/non-use of side keys on the saxophone were seen to be the physical actions used in spatial playing on those instruments. However, I was surprised to see that so many of the examples of spatial playing are (a) fairly extended and (b) highly integrated (at least at a rhythmic level) within the body of the solo concerned.

Choice of “out” scale was also found to be important; the avoidance of the Blues scale was seen to be a common strategy in order to reduce redundancy of notes between “in” and “out” sections, and thus increase the “polarity” of such sections.

With regard to placement, examination of these 20 excerpts showed “playing outside” to be highly organised within conventional cadential structures. For example, excursions occurred nearly always either (a) within the *last* bar(s), (b) at the penultimate bar or (c) within the *first* bar(s) of a 2-, 4-, 8-, 12-, 16-, 32- (etc.) bar long section.

¹ Dr. Gerry Farrell, my supervisor, used this evocative phrase in a personal conversation.

“Compensation” of “out” material with unambiguous statements of “in” material was also seen to be a common strategy. The use of “ $\{x, x-1, x+1\}$ sets” was also found in many of the excerpts, providing a local chromatic framework for the benefit of both listener and performer. A resonance of these $\{x, x-1, x+1\}$ sets was found in the “Symmetry” system of Steve Coleman in Chapter 4.

Finally, examination of live and studio versions of the music concerned suggested that “playing outside” is a more comfortable activity on the stage than in a recording studio.

In examining John Coltrane’s motivic strategy in “Acknowledgement” from “A Love Supreme” in Chapter 2 I have shown the range of skills and premeditation that went into the construction of a section of this famous jazz performance. My analysis has shown that the characteristics of the chosen motif under transposition allow that motif to function as a musical metaphor for Coltrane’s personal struggle (for example, the characteristics of “independence” and “integration”). I have also shown that the motif is not suited to rapid chromatic saturation, and have argued that Coltrane was not attempting such a task with the motif in “Acknowledgement”. Further, I have examined Coltrane’s use of identical sets in previous pieces and his recognition of their use in world musics.

In assessing the level of premeditation at bars 137-172 of “Acknowledgement” I have shown that the list of transpositions and resultant effects such as chromatic saturation are unlikely to be randomly generated. Rather, I have argued that Coltrane has created a premeditated, *quasi*-random journey through all twelve keys. I have also demonstrated, using various methods, that an important strategy of Coltrane’s in performing bars 137-172 was the avoidance of redundancy. That is, Coltrane continually surprises the listener with the direction, interval and order of transpositions of the motif.

Further, I suggest two possible sources for the list of transpositions at bars 137-172: (a) a construction of transpositions designed to cover the chromatic set and (b) the Mother and Grandmother chords to be found in Slonimsky's "Thesaurus" (1947). Whilst both theories seem reasonable, the latter possibility seems the most likely to me, and thus my thesis builds upon and supports the work of Demsey (1991). In this light, Slonimsky's book increasingly appears to be Coltrane's musical "bible".

Importantly, although the Mother and/or Grandmother chords may have been the original source, we find only parts of them represented in the choice of transpositions in bars 137-172. In fact, as I have argued, Coltrane may be seen to be improvising the list of transpositions, interpreting a memory (perhaps a written version?) of the list of transpositions provided by the Mother and/or Grandmother chords. This is a direct corollary of the construction of a strictly tonal solo using chord/scale relationships and other jazz conventions for, say, a jazz standard. Quite astonishingly, Coltrane simultaneously applies the hierarchy of transpositions (which I have analysed using "out factor" values) whilst employing this strategy.

Finally, I have examined three pieces by Eric Dolphy which encode their numeric titles within the music, and, in the light of that analysis, suggest possible relationships between his composition "245" and Coltrane's "A Love Supreme".

Chapter 3 examined the special set of "outside" strategies used by Robert Irving III on Miles Davis' albums "Decoy" and "You're Under Arrest". I have named these strategies "Pedal Chains" and "Quartal Chains", and have showed that their characteristics have precedents in earlier compositions by Irving for Davis. Both types of Chain strategies may be described as being highly spatial in nature in that Irving usually allows the Black/White structure of the keyboard to dictate the list of acceptable chord sets and transpositions of those sets. There is a clear link between some of the Pedal Chain visual

structures and those used by Jimmy Smith in “The Sermon” (examined in Chapter 1), further asserting the role of the keyboard in their design.

In the previous chapter, I showed Coltrane’s flexibility in his application of the Mother/Grandmother lists. Similarly, Irving’s systems represent a core language that may be adjusted to fit tonal and metrical circumstances. However, in calculating a dissonance hierarchy for the Pedal and Quartal Chains, I have been able to show that Irving generally seeks to maximise the contrast between adjacent chain elements against the underlying tonality, even at the cost of supplanting the Black/White visual strategy. Further, this characteristic is true for both Pedal and Quartal Chains. This approach was made transparent after analysis of Irving’s actions in the two pieces in E minor (“Katia” and “You’re Under Arrest”). In these recordings he variously uses (a) a small subset of the common Pedal Chains, (b) chords unique to that piece and (c) a small subset of the common Quartal Chains.

Further, Irving sometimes adapts his chains to provide compositional contrast. For example, he uses a set of chords which demonstrate a relatively small difference in dissonance values in the head of “That’s Right” in order to provide contrast between the head and solo sections.

Chapter 4 considered the structure and application of Steve Coleman’s “Sum” and “Symmetry” systems, predominantly with regard to his composition “Cross-Fade”. I have shown that these systems are natural extensions of his musical philosophy, and how he has (rarely for a jazz musician) gradually revealed their structure through interviews and his expanding website. I have shown that the head of “Cross-Fade” is constructed using material derived from both the “Sum” and “Symmetry” systems. However, the head is not a slave to these techniques, and shows evidence of being

otherwise freely composed. Further, I have analysed Head 5 separately, suggesting a reason for the abrupt edited ending of the track.

I was not surprised to find that the theoretical examples of Steve Coleman's melodic technique (self-) published on the internet were pedagogical (i.e. neat, progressive and sometimes densely packed with structure). However, this (along with the freely composed quality of parts of the head) in no way prepared me for the extreme fluency and diverse application of the "Symmetry" system in his solos for "Cross-Fade". Clearly, this fluency, and the use of a wide range of premeditated structures, nearly all based around axes of the underlying tonality of F# minor pentatonic/Blues ("Symmetrical Motifs"), suggest a great deal of personal practise and commitment on behalf of Steve Coleman.

Further, I have shown that many of the premeditated structures may be seen as subsets of each other and that this offers recall and melodic advantages. I have also suggested that the main reason for the apparent emphasis upon the bII over the VII through the piece is based in a compensation for the F-F# axis structure.

My analysis also shows that Steve Coleman has successfully communicated some of his theories to his sidemen. David Weidman (piano) is seen to be familiar with the strategies employed by his leader, and includes a few symmetrically-derived structures of his own.

Finally, I have examined the related symmetrical structures of "Drop Kick", and have shown that for both this piece and "Cross-Fade" Steve Coleman carefully chooses the key in order to promote instrumental fluency on the alto sax.

Whereas Coltrane's "journey" in "Acknowledgement" is clearly motivic, Irving's chain strategies are spatial. However, examination of Steve Coleman's use of his "Sum" and

“Symmetry” strategies in “Cross-Fade” revealed traces of all three of the “outside” strategies defined in Chapter 1: i.e. motivic, scalar *and* spatial. Steve Coleman’s use of $\{0, x, y\}$ sets might be seen as motivic in that a given interval is repeated (although inverted); I have described a similar case of an interval being used as a motif in George Benson’s “Fly By Night” (Chapter 1). On the other hand, the resulting sums of all x and y notes form *scalar* transpositions of the core scale of F# minor pentatonic through the whole piece (e.g. F minor pentatonic and G minor pentatonic). Also, as noted above, Steve Coleman’s choice of key for “Cross-Fade” creates a black/white polarity at a keyboard, and this might be seen as a spatial approach.

John Coltrane, Robert Irving III and Steve Coleman have designed their strategies and systems to be fully chromatic, and they all exploit this potential in their performances. In “Acknowledgement”, Coltrane choses a simple motif and transposes it through all twelve keys. Robert Irving III moves though chains of chromatically related chords. Steve Coleman designs and employs systems that leap outwards from a core minor pentatonic scale, and thus cover the chromatic scale. Clearly, unlike John Coltrane, Steve Coleman is not transposing his core set in a random or quasi-random manner. However, it is clear that Steve Coleman achieves chromatic saturation in “Cross-Fade” generally as fast as does Coltrane in bars 137-172 of “Acknowledgement”, and that his use of symmetrical structures is greatly responsible for this. Robert Irving III also covers the chromatic scale by associating adjacent transpositions of chains.

Further, all three use relatively strict theoretical models, which are then played within the improvisational process. Coltrane performs in a quasi-random fashion, with the model of the Mother and/or Grandmother chord in mind. Robert Irving III reacts to the melody line of soloists, the bass line, and other rhythmic events in the music. Steve Coleman constantly reacts to the results of his own (sometimes premeditated) material generated by the “Symmetry” system in his improvisations.

Future research

I can see many opportunities for future research that would extend the work presented here. With regard to Chapter 1, I would like to find and analyse more excerpts in order to test further (and possibly extend?) the subdivisions of motivic, scalar and spatial playing. As I write up this thesis I am only able to note that I have discovered several more excerpts, but have as yet neither notated nor analysed them. I feel that there must be many more interesting examples to investigate. Some of the musicians examined in this thesis are represented by more than a single excerpt and I feel that there is also probably more to examine and say concerning their individual approaches to playing “outside”.

As noted above, my ideas about Coltrane’s use of Slonimsky’s “Thesaurus” build and extend the work of Demsey (1991), and I now wonder if there is more of Coltrane’s music that may have its source in that book. Further, I wonder whether the “out factor” table of motif transposition has application to (freer) improvisations by Coltrane that followed “A Love Supreme”. With regard to Robert Irving III, there are several later albums which I know that he has made, and I wonder to what extent he has employed his chain strategies in these recordings. Similarly, I feel certain that there are many compositions and improvisations of Steve Coleman that would reveal use of his “Sum” and “Symmetry” systems, given the fluency and sophistication of his composing and improvising on “Cross-Fade”.

Also, as mentioned in the Introduction, I feel that there are opportunities to define quintessential “out” scales, motifs and strategies, and that these might be helpful with regard to teaching student jazz musicians. Such strategies and material could be found using computer searches similar to those variously employed in this thesis. Indeed, this

particular area of study is currently the most interesting to me; I have already begun to undertake this work, and I hope to publish the results of these searches elsewhere.

Appendix 1: Commonly-used Scales

One of the assumptions made by this thesis is that we may make a division in jazz music between “commonly-used” scales and other (rarer) scales. Here is a list of the scales that this thesis sees as being “commonly-used” in jazz. This has been compiled from Coker et al. (1970), Haerle (1980) and Levine (1995). (Abbreviations used in this thesis are shown in brackets.) An example of each scale is shown in the key of C.

Major

Ionian (Ion); the “Major” scale	C D E F G A B
Dorian (Dor)	C D Eb F G A Bb
Phrygian (Phr)	C Db Eb F G Ab Bb
Lydian (Lyd)	C D E F# G A B
Mixolydian (Mix)	C D E F G A Bb
Aeolian (Aeo)	C D Eb F G Ab Bb
Locrian (Loc)	C Db Eb F Gb Ab Bb

Jazz Melodic

Jazz Melodic (Jazz Mel); the Melodic minor scale	C D Eb F G A B
Dorian,b2 (Dor,b2)	C Db Eb F G A Bb
Lydian Augmented (Lyd Aug)	C D E F# G# A B
Lydian,b7 (Lyd,b7)	C D E F# G A Bb
Mixolydian,b6 (Mix,b6)	C D E F G Ab Bb
Locrian,#2 (Loc,#2)	C D Eb F Gb Ab Bb
Super Locrian (Sup Loc); sometimes called “altered” or “diminished whole-tone”	C Db Eb Fb Gb Ab Bb

Harmonic minor

Harmonic minor scale (Harm. min.)	C D Eb F G Ab B
2 nd Mode	C Db Eb F Gb A Bb
3 rd Mode	C D E F G# A B
4 th Mode	C D Eb F# G A Bb
5 th Mode	C Db E F G Ab Bb
6 th Mode	C D# E F# G A Bb
7 th Mode	C Db Eb Fb Gb Ab Bbb

Pentatonic

Major Pentatonic	C D E G A
Minor pentatonic	C Eb F G Bb

The term “Pentatonic” implies the Major version. The minor mode is always referred to as the “Minor Pentatonic”.

Blues

Blues (= minor Blues)	C Eb F F# G Bb
Major Blues	C D Eb E G A

Thus, where I have written “Blues” in the text, I am referring to the minor mode. This scale has the potential to have 6 modes, but composers and theorists generally prefer to express an F Blues played over a C bass as “F Blues/C” or “F Blues/C Bass”, rather than as “the 5th mode of F Blues”.

Whole Tone

Whole Tone (W.T.)	C D E F# Ab Bb
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Diminished

Diminished (ascending Whole tone, Semitone pattern) (Dim.)	C D Eb F Gb Ab A B
Auxiliary Diminished (ascending Semitone, Whole tone pattern) (Aux. Dim.)	C Db Eb E F# G A Bb

Appendix 2: The 2048 Scales

This table provides not only the list of 2048 scales, but also the 2048 Lists of Transpositions (LOTs).

#:	scale:			
1	C			
2	C	Db		
3	C	D		
4	C	Eb		
5	C	E		
6	C	F		
7	C	F#		
8	C	G		
9	C	Ab		
10	C	A		
11	C	Bb		
12	C	B		
13	C	Db	D	
14	C	Db	Eb	
15	C	Db	E	
16	C	Db	F	
17	C	Db	F#	
18	C	Db	G	
19	C	Db	Ab	
20	C	Db	A	
21	C	Db	Bb	
22	C	Db	B	
23	C	D	Eb	
24	C	D	E	
25	C	D	F	
26	C	D	F#	
27	C	D	G	
28	C	D	Ab	
29	C	D	A	
30	C	D	Bb	
31	C	D	B	
32	C	Eb	E	
33	C	Eb	F	
34	C	Eb	F#	
35	C	Eb	G	
36	C	Eb	Ab	
37	C	Eb	A	
38	C	Eb	Bb	
39	C	Eb	B	
40	C	E	F	
41	C	E	F#	
42	C	E	G	
43	C	E	Ab	
44	C	E	A	
45	C	E	Bb	

46	C	E	B
47	C	F	F#
48	C	F	G
49	C	F	Ab
50	C	F	A
51	C	F	Bb
52	C	F	B
53	C	F#	G
54	C	F#	Ab
55	C	F#	A
56	C	F#	Bb
57	C	F#	B
58	C	G	Ab
59	C	G	A
60	C	G	Bb
61	C	G	B
62	C	Ab	A
63	C	Ab	Bb
64	C	Ab	B
65	C	A	Bb
66	C	A	B
67	C	Bb	B
68	C	Db	D Eb
69	C	Db	D E
70	C	Db	D F
71	C	Db	D F#
72	C	Db	D G
73	C	Db	D Ab
74	C	Db	D A
75	C	Db	D Bb
76	C	Db	D B
77	C	Db	Eb E
78	C	Db	Eb F
79	C	Db	Eb F#
80	C	Db	Eb G
81	C	Db	Eb Ab
82	C	Db	Eb A
83	C	Db	Eb Bb
84	C	Db	Eb B
85	C	Db	E F
86	C	Db	E F#
87	C	Db	E G
88	C	Db	E Ab
89	C	Db	E A
90	C	Db	E Bb
91	C	Db	E B
92	C	Db	F F#
93	C	Db	F G
94	C	Db	F Ab
95	C	Db	F A
96	C	Db	F Bb
97	C	Db	F B
98	C	Db	F# G
99	C	Db	F# Ab
100	C	Db	F# A
101	C	Db	F# Bb
102	C	Db	F# B
103	C	Db	G Ab
104	C	Db	G A

105	C	Db	G	Bb
106	C	Db	G	B
107	C	Db	Ab	A
108	C	Db	Ab	Bb
109	C	Db	Ab	B
110	C	Db	A	Bb
111	C	Db	A	B
112	C	Db	Bb	B
113	C	D	Eb	E
114	C	D	Eb	F
115	C	D	Eb	F#
116	C	D	Eb	G
117	C	D	Eb	Ab
118	C	D	Eb	A
119	C	D	Eb	Bb
120	C	D	Eb	B
121	C	D	E	F
122	C	D	E	F#
123	C	D	E	G
124	C	D	E	Ab
125	C	D	E	A
126	C	D	E	Bb
127	C	D	E	B
128	C	D	F	F#
129	C	D	F	G
130	C	D	F	Ab
131	C	D	F	A
132	C	D	F	Bb
133	C	D	F	B
134	C	D	F#	G
135	C	D	F#	Ab
136	C	D	F#	A
137	C	D	F#	Bb
138	C	D	F#	B
139	C	D	G	Ab
140	C	D	G	A
141	C	D	G	Bb
142	C	D	G	B
143	C	D	Ab	A
144	C	D	Ab	Bb
145	C	D	Ab	B
146	C	D	A	Bb
147	C	D	A	B
148	C	D	Bb	B
149	C	Eb	E	F
150	C	Eb	E	F#
151	C	Eb	E	G
152	C	Eb	E	Ab
153	C	Eb	E	A
154	C	Eb	E	Bb
155	C	Eb	E	B
156	C	Eb	F	F#
157	C	Eb	F	G
158	C	Eb	F	Ab
159	C	Eb	F	A
160	C	Eb	F	Bb
161	C	Eb	F	B
162	C	Eb	F#	G
163	C	Eb	F#	Ab

164	C	Eb	F#	A
165	C	Eb	F#	Bb
166	C	Eb	F#	B
167	C	Eb	G	Ab
168	C	Eb	G	A
169	C	Eb	G	Bb
170	C	Eb	G	B
171	C	Eb	Ab	A
172	C	Eb	Ab	Bb
173	C	Eb	Ab	B
174	C	Eb	A	Bb
175	C	Eb	A	B
176	C	Eb	Bb	B
177	C	E	F	F#
178	C	E	F	G
179	C	E	F	Ab
180	C	E	F	A
181	C	E	F	Bb
182	C	E	F	B
183	C	E	F#	G
184	C	E	F#	Ab
185	C	E	F#	A
186	C	E	F#	Bb
187	C	E	F#	B
188	C	E	G	Ab
189	C	E	G	A
190	C	E	G	Bb
191	C	E	G	B
192	C	E	Ab	A
193	C	E	Ab	Bb
194	C	E	Ab	B
195	C	E	A	Bb
196	C	E	A	B
197	C	E	Bb	B
198	C	F	F#	G
199	C	F	F#	Ab
200	C	F	F#	A
201	C	F	F#	Bb
202	C	F	F#	B
203	C	F	G	Ab
204	C	F	G	A
205	C	F	G	Bb
206	C	F	G	B
207	C	F	Ab	A
208	C	F	Ab	Bb
209	C	F	Ab	B
210	C	F	A	Bb
211	C	F	A	B
212	C	F	Bb	B
213	C	F#	G	Ab
214	C	F#	G	A
215	C	F#	G	Bb
216	C	F#	G	B
217	C	F#	Ab	A
218	C	F#	Ab	Bb
219	C	F#	Ab	B
220	C	F#	A	Bb
221	C	F#	A	B
222	C	F#	Bb	B

223	C	G	Ab	A
224	C	G	Ab	Bb
225	C	G	Ab	B
226	C	G	A	Bb
227	C	G	A	B
228	C	G	Bb	B
229	C	Ab	A	Bb
230	C	Ab	A	B
231	C	Ab	Bb	B
232	C	A	Bb	B
233	C	Db	D	Eb E
234	C	Db	D	Eb F
235	C	Db	D	Eb F#
236	C	Db	D	Eb G
237	C	Db	D	Eb Ab
238	C	Db	D	Eb A
239	C	Db	D	Eb Bb
240	C	Db	D	Eb B
241	C	Db	D	E F
242	C	Db	D	E F#
243	C	Db	D	E G
244	C	Db	D	E Ab
245	C	Db	D	E A
246	C	Db	D	E Bb
247	C	Db	D	E B
248	C	Db	D	F F#
249	C	Db	D	F G
250	C	Db	D	F Ab
251	C	Db	D	F A
252	C	Db	D	F Bb
253	C	Db	D	F B
254	C	Db	D	F# G
255	C	Db	D	F# Ab
256	C	Db	D	F# A
257	C	Db	D	F# Bb
258	C	Db	D	F# B
259	C	Db	D	G Ab
260	C	Db	D	G A
261	C	Db	D	G Bb
262	C	Db	D	G B
263	C	Db	D	Ab A
264	C	Db	D	Ab Bb
265	C	Db	D	Ab B
266	C	Db	D	A Bb
267	C	Db	D	A B
268	C	Db	D	Bb B
269	C	Db	Eb	E F
270	C	Db	Eb	E F#
271	C	Db	Eb	E G
272	C	Db	Eb	E Ab
273	C	Db	Eb	E A
274	C	Db	Eb	E Bb
275	C	Db	Eb	E B
276	C	Db	Eb	F F#
277	C	Db	Eb	F G
278	C	Db	Eb	F Ab
279	C	Db	Eb	F A
280	C	Db	Eb	F Bb
281	C	Db	Eb	F B

282	C	Db	Eb	F#	G
283	C	Db	Eb	F#	Ab
284	C	Db	Eb	F#	A
285	C	Db	Eb	F#	Bb
286	C	Db	Eb	F#	B
287	C	Db	Eb	G	Ab
288	C	Db	Eb	G	A
289	C	Db	Eb	G	Bb
290	C	Db	Eb	G	B
291	C	Db	Eb	Ab	A
292	C	Db	Eb	Ab	Bb
293	C	Db	Eb	Ab	B
294	C	Db	Eb	A	Bb
295	C	Db	Eb	A	B
296	C	Db	Eb	Bb	B
297	C	Db	E	F	F#
298	C	Db	E	F	G
299	C	Db	E	F	Ab
300	C	Db	E	F	A
301	C	Db	E	F	Bb
302	C	Db	E	F	B
303	C	Db	E	F#	G
304	C	Db	E	F#	Ab
305	C	Db	E	F#	A
306	C	Db	E	F#	Bb
307	C	Db	E	F#	B
308	C	Db	E	G	Ab
309	C	Db	E	G	A
310	C	Db	E	G	Bb
311	C	Db	E	G	B
312	C	Db	E	Ab	A
313	C	Db	E	Ab	Bb
314	C	Db	E	Ab	B
315	C	Db	E	A	Bb
316	C	Db	E	A	B
317	C	Db	E	Bb	B
318	C	Db	F	F#	G
319	C	Db	F	F#	Ab
320	C	Db	F	F#	A
321	C	Db	F	F#	Bb
322	C	Db	F	F#	B
323	C	Db	F	G	Ab
324	C	Db	F	G	A
325	C	Db	F	G	Bb
326	C	Db	F	G	B
327	C	Db	F	Ab	A
328	C	Db	F	Ab	Bb
329	C	Db	F	Ab	B
330	C	Db	F	A	Bb
331	C	Db	F	A	B
332	C	Db	F	Bb	B
333	C	Db	F#	G	Ab
334	C	Db	F#	G	A
335	C	Db	F#	G	Bb
336	C	Db	F#	G	B
337	C	Db	F#	Ab	A
338	C	Db	F#	Ab	Bb
339	C	Db	F#	Ab	B
340	C	Db	F#	A	Bb

341	C	Db	F#	A	B
342	C	Db	F#	Bb	B
343	C	Db	G	Ab	A
344	C	Db	G	Ab	Bb
345	C	Db	G	Ab	B
346	C	Db	G	A	Bb
347	C	Db	G	A	B
348	C	Db	G	Bb	B
349	C	Db	Ab	A	Bb
350	C	Db	Ab	A	B
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352	C	Db	A	Bb	B
353	C	D	Eb	E	F
354	C	D	Eb	E	F#
355	C	D	Eb	E	G
356	C	D	Eb	E	Ab
357	C	D	Eb	E	A
358	C	D	Eb	E	Bb
359	C	D	Eb	E	B
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361	C	D	Eb	F	G
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365	C	D	Eb	F	B
366	C	D	Eb	F#	G
367	C	D	Eb	F#	Ab
368	C	D	Eb	F#	A
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389	C	D	E	F#	A
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391	C	D	E	F#	B
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393	C	D	E	G	A
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395	C	D	E	G	B
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397	C	D	E	Ab	Bb
398	C	D	E	Ab	B
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400	C	D	E	A	B
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415	C	D	F	A	B
416	C	D	F	Bb	B
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419	C	D	F#	G	Bb
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433	C	D	Ab	A	Bb
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436	C	D	A	Bb	B
437	C	Eb	E	F	F#
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456	C	Eb	E	A	B
457	C	Eb	E	Bb	B
458	C	Eb	F	F#	G

459	C	Eb	F	F#	Ab
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461	C	Eb	F	F#	Bb
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492	C	Eb	A	Bb	B
493	C	E	F	F#	G
494	C	E	F	F#	Ab
495	C	E	F	F#	A
496	C	E	F	F#	Bb
497	C	E	F	F#	B
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513	C	E	F#	Ab	Bb
514	C	E	F#	Ab	B
515	C	E	F#	A	Bb
516	C	E	F#	A	B
517	C	E	F#	Bb	B

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520	C	E	G	Ab	B
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523	C	E	G	Bb	B
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528	C	F	F#	G	Ab
529	C	F	F#	G	A
530	C	F	F#	G	Bb
531	C	F	F#	G	B
532	C	F	F#	Ab	A
533	C	F	F#	Ab	Bb
534	C	F	F#	Ab	B
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548	C	F#	G	Ab	A
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555	C	F#	Ab	A	B
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557	C	F#	A	Bb	B
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559	C	G	Ab	A	B
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561	C	G	A	Bb	B
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563	C	Db	D	Eb	E F
564	C	Db	D	Eb	E F#
565	C	Db	D	Eb	E G
566	C	Db	D	Eb	E Ab
567	C	Db	D	Eb	E A
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569	C	Db	D	Eb	E B
570	C	Db	D	Eb	F F#
571	C	Db	D	Eb	F G
572	C	Db	D	Eb	F Ab
573	C	Db	D	Eb	F A
574	C	Db	D	Eb	F Bb
575	C	Db	D	Eb	F B
576	C	Db	D	Eb	F# G

577	C	Db	D	Eb	F#	Ab
578	C	Db	D	Eb	F#	A
579	C	Db	D	Eb	F#	Bb
580	C	Db	D	Eb	F#	B
581	C	Db	D	Eb	G	Ab
582	C	Db	D	Eb	G	A
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584	C	Db	D	Eb	G	B
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587	C	Db	D	Eb	Ab	B
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589	C	Db	D	Eb	A	B
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591	C	Db	D	E	F	F#
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594	C	Db	D	E	F	A
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596	C	Db	D	E	F	B
597	C	Db	D	E	F#	G
598	C	Db	D	E	F#	Ab
599	C	Db	D	E	F#	A
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601	C	Db	D	E	F#	B
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604	C	Db	D	E	G	Bb
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607	C	Db	D	E	Ab	Bb
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611	C	Db	D	E	Bb	B
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613	C	Db	D	F	F#	Ab
614	C	Db	D	F	F#	A
615	C	Db	D	F	F#	Bb
616	C	Db	D	F	F#	B
617	C	Db	D	F	G	Ab
618	C	Db	D	F	G	A
619	C	Db	D	F	G	Bb
620	C	Db	D	F	G	B
621	C	Db	D	F	Ab	A
622	C	Db	D	F	Ab	Bb
623	C	Db	D	F	Ab	B
624	C	Db	D	F	A	Bb
625	C	Db	D	F	A	B
626	C	Db	D	F	Bb	B
627	C	Db	D	F#	G	Ab
628	C	Db	D	F#	G	A
629	C	Db	D	F#	G	Bb
630	C	Db	D	F#	G	B
631	C	Db	D	F#	Ab	A
632	C	Db	D	F#	Ab	Bb
633	C	Db	D	F#	Ab	B
634	C	Db	D	F#	A	Bb
635	C	Db	D	F#	A	B

636	C	Db	D	F#	Bb	B
637	C	Db	D	G	Ab	A
638	C	Db	D	G	Ab	Bb
639	C	Db	D	G	Ab	B
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643	C	Db	D	Ab	A	Bb
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646	C	Db	D	A	Bb	B
647	C	Db	Eb	E	F	F#
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652	C	Db	Eb	E	F	B
653	C	Db	Eb	E	F#	G
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656	C	Db	Eb	E	F#	Bb
657	C	Db	Eb	E	F#	B
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682	C	Db	Eb	F	Bb	B
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691	C	Db	Eb	F#	A	B
692	C	Db	Eb	F#	Bb	B
693	C	Db	Eb	G	Ab	A
694	C	Db	Eb	G	Ab	Bb

695	C	Db	Eb	G	Ab	B
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702	C	Db	Eb	A	Bb	B
703	C	Db	E	F	F#	G
704	C	Db	E	F	F#	Ab
705	C	Db	E	F	F#	A
706	C	Db	E	F	F#	Bb
707	C	Db	E	F	F#	B
708	C	Db	E	F	G	Ab
709	C	Db	E	F	G	A
710	C	Db	E	F	G	Bb
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716	C	Db	E	F	A	B
717	C	Db	E	F	Bb	B
718	C	Db	E	F#	G	Ab
719	C	Db	E	F#	G	A
720	C	Db	E	F#	G	Bb
721	C	Db	E	F#	G	B
722	C	Db	E	F#	Ab	A
723	C	Db	E	F#	Ab	Bb
724	C	Db	E	F#	Ab	B
725	C	Db	E	F#	A	Bb
726	C	Db	E	F#	A	B
727	C	Db	E	F#	Bb	B
728	C	Db	E	G	Ab	A
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733	C	Db	E	G	Bb	B
734	C	Db	E	Ab	A	Bb
735	C	Db	E	Ab	A	B
736	C	Db	E	Ab	Bb	B
737	C	Db	E	A	Bb	B
738	C	Db	F	F#	G	Ab
739	C	Db	F	F#	G	A
740	C	Db	F	F#	G	Bb
741	C	Db	F	F#	G	B
742	C	Db	F	F#	Ab	A
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744	C	Db	F	F#	Ab	B
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746	C	Db	F	F#	A	B
747	C	Db	F	F#	Bb	B
748	C	Db	F	G	Ab	A
749	C	Db	F	G	Ab	Bb
750	C	Db	F	G	Ab	B
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752	C	Db	F	G	A	B
753	C	Db	F	G	Bb	B

754	C	Db	F	Ab	A	Bb
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756	C	Db	F	Ab	Bb	B
757	C	Db	F	A	Bb	B
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759	C	Db	F#	G	Ab	Bb
760	C	Db	F#	G	Ab	B
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762	C	Db	F#	G	A	B
763	C	Db	F#	G	Bb	B
764	C	Db	F#	Ab	A	Bb
765	C	Db	F#	Ab	A	B
766	C	Db	F#	Ab	Bb	B
767	C	Db	F#	A	Bb	B
768	C	Db	G	Ab	A	Bb
769	C	Db	G	Ab	A	B
770	C	Db	G	Ab	Bb	B
771	C	Db	G	A	Bb	B
772	C	Db	Ab	A	Bb	B
773	C	D	Eb	E	F	F#
774	C	D	Eb	E	F	G
775	C	D	Eb	E	F	Ab
776	C	D	Eb	E	F	A
777	C	D	Eb	E	F	Bb
778	C	D	Eb	E	F	B
779	C	D	Eb	E	F#	G
780	C	D	Eb	E	F#	Ab
781	C	D	Eb	E	F#	A
782	C	D	Eb	E	F#	Bb
783	C	D	Eb	E	F#	B
784	C	D	Eb	E	G	Ab
785	C	D	Eb	E	G	A
786	C	D	Eb	E	G	Bb
787	C	D	Eb	E	G	B
788	C	D	Eb	E	Ab	A
789	C	D	Eb	E	Ab	Bb
790	C	D	Eb	E	Ab	B
791	C	D	Eb	E	A	Bb
792	C	D	Eb	E	A	B
793	C	D	Eb	E	Bb	B
794	C	D	Eb	F	F#	G
795	C	D	Eb	F	F#	Ab
796	C	D	Eb	F	F#	A
797	C	D	Eb	F	F#	Bb
798	C	D	Eb	F	F#	B
799	C	D	Eb	F	G	Ab
800	C	D	Eb	F	G	A
801	C	D	Eb	F	G	Bb
802	C	D	Eb	F	G	B
803	C	D	Eb	F	Ab	A
804	C	D	Eb	F	Ab	Bb
805	C	D	Eb	F	Ab	B
806	C	D	Eb	F	A	Bb
807	C	D	Eb	F	A	B
808	C	D	Eb	F	Bb	B
809	C	D	Eb	F#	G	Ab
810	C	D	Eb	F#	G	A
811	C	D	Eb	F#	G	Bb
812	C	D	Eb	F#	G	B

813	C	D	Eb	F#	Ab	A
814	C	D	Eb	F#	Ab	Bb
815	C	D	Eb	F#	Ab	B
816	C	D	Eb	F#	A	Bb
817	C	D	Eb	F#	A	B
818	C	D	Eb	F#	Bb	B
819	C	D	Eb	G	Ab	A
820	C	D	Eb	G	Ab	Bb
821	C	D	Eb	G	Ab	B
822	C	D	Eb	G	A	Bb
823	C	D	Eb	G	A	B
824	C	D	Eb	G	Bb	B
825	C	D	Eb	Ab	A	Bb
826	C	D	Eb	Ab	A	B
827	C	D	Eb	Ab	Bb	B
828	C	D	Eb	A	Bb	B
829	C	D	E	F	F#	G
830	C	D	E	F	F#	Ab
831	C	D	E	F	F#	A
832	C	D	E	F	F#	Bb
833	C	D	E	F	F#	B
834	C	D	E	F	G	Ab
835	C	D	E	F	G	A
836	C	D	E	F	G	Bb
837	C	D	E	F	G	B
838	C	D	E	F	Ab	A
839	C	D	E	F	Ab	Bb
840	C	D	E	F	Ab	B
841	C	D	E	F	A	Bb
842	C	D	E	F	A	B
843	C	D	E	F	Bb	B
844	C	D	E	F#	G	Ab
845	C	D	E	F#	G	A
846	C	D	E	F#	G	Bb
847	C	D	E	F#	G	B
848	C	D	E	F#	Ab	A
849	C	D	E	F#	Ab	Bb
850	C	D	E	F#	Ab	B
851	C	D	E	F#	A	Bb
852	C	D	E	F#	A	B
853	C	D	E	F#	Bb	B
854	C	D	E	G	Ab	A
855	C	D	E	G	Ab	Bb
856	C	D	E	G	Ab	B
857	C	D	E	G	A	Bb
858	C	D	E	G	A	B
859	C	D	E	G	Bb	B
860	C	D	E	Ab	A	Bb
861	C	D	E	Ab	A	B
862	C	D	E	Ab	Bb	B
863	C	D	E	A	Bb	B
864	C	D	F	F#	G	Ab
865	C	D	F	F#	G	A
866	C	D	F	F#	G	Bb
867	C	D	F	F#	G	B
868	C	D	F	F#	Ab	A
869	C	D	F	F#	Ab	Bb
870	C	D	F	F#	Ab	B
871	C	D	F	F#	A	Bb

872	C	D	F	F#	A	B
873	C	D	F	F#	Bb	B
874	C	D	F	G	Ab	A
875	C	D	F	G	Ab	Bb
876	C	D	F	G	Ab	B
877	C	D	F	G	A	Bb
878	C	D	F	G	A	B
879	C	D	F	G	Bb	B
880	C	D	F	Ab	A	Bb
881	C	D	F	Ab	A	B
882	C	D	F	Ab	Bb	B
883	C	D	F	A	Bb	B
884	C	D	F#	G	Ab	A
885	C	D	F#	G	Ab	Bb
886	C	D	F#	G	Ab	B
887	C	D	F#	G	A	Bb
888	C	D	F#	G	A	B
889	C	D	F#	G	Bb	B
890	C	D	F#	Ab	A	Bb
891	C	D	F#	Ab	A	B
892	C	D	F#	Ab	Bb	B
893	C	D	F#	A	Bb	B
894	C	D	G	Ab	A	Bb
895	C	D	G	Ab	A	B
896	C	D	G	Ab	Bb	B
897	C	D	G	A	Bb	B
898	C	D	Ab	A	Bb	B
899	C	Eb	E	F	F#	G
900	C	Eb	E	F	F#	Ab
901	C	Eb	E	F	F#	A
902	C	Eb	E	F	F#	Bb
903	C	Eb	E	F	F#	B
904	C	Eb	E	F	G	Ab
905	C	Eb	E	F	G	A
906	C	Eb	E	F	G	Bb
907	C	Eb	E	F	G	B
908	C	Eb	E	F	Ab	A
909	C	Eb	E	F	Ab	Bb
910	C	Eb	E	F	Ab	B
911	C	Eb	E	F	A	Bb
912	C	Eb	E	F	A	B
913	C	Eb	E	F	Bb	B
914	C	Eb	E	F#	G	Ab
915	C	Eb	E	F#	G	A
916	C	Eb	E	F#	G	Bb
917	C	Eb	E	F#	G	B
918	C	Eb	E	F#	Ab	A
919	C	Eb	E	F#	Ab	Bb
920	C	Eb	E	F#	Ab	B
921	C	Eb	E	F#	A	Bb
922	C	Eb	E	F#	A	B
923	C	Eb	E	F#	Bb	B
924	C	Eb	E	G	Ab	A
925	C	Eb	E	G	Ab	Bb
926	C	Eb	E	G	Ab	B
927	C	Eb	E	G	A	Bb
928	C	Eb	E	G	A	B
929	C	Eb	E	G	Bb	B
930	C	Eb	E	Ab	A	Bb

931	C	Eb	E	Ab	A	B
932	C	Eb	E	Ab	Bb	B
933	C	Eb	E	A	Bb	B
934	C	Eb	F	F#	G	Ab
935	C	Eb	F	F#	G	A
936	C	Eb	F	F#	G	Bb
937	C	Eb	F	F#	G	B
938	C	Eb	F	F#	Ab	A
939	C	Eb	F	F#	Ab	Bb
940	C	Eb	F	F#	Ab	B
941	C	Eb	F	F#	A	Bb
942	C	Eb	F	F#	A	B
943	C	Eb	F	F#	Bb	B
944	C	Eb	F	G	Ab	A
945	C	Eb	F	G	Ab	Bb
946	C	Eb	F	G	Ab	B
947	C	Eb	F	G	A	Bb
948	C	Eb	F	G	A	B
949	C	Eb	F	G	Bb	B
950	C	Eb	F	Ab	A	Bb
951	C	Eb	F	Ab	A	B
952	C	Eb	F	Ab	Bb	B
953	C	Eb	F	A	Bb	B
954	C	Eb	F#	G	Ab	A
955	C	Eb	F#	G	Ab	Bb
956	C	Eb	F#	G	Ab	B
957	C	Eb	F#	G	A	Bb
958	C	Eb	F#	G	A	B
959	C	Eb	F#	G	Bb	B
960	C	Eb	F#	Ab	A	Bb
961	C	Eb	F#	Ab	A	B
962	C	Eb	F#	Ab	Bb	B
963	C	Eb	F#	A	Bb	B
964	C	Eb	G	Ab	A	Bb
965	C	Eb	G	Ab	A	B
966	C	Eb	G	Ab	Bb	B
967	C	Eb	G	A	Bb	B
968	C	Eb	Ab	A	Bb	B
969	C	E	F	F#	G	Ab
970	C	E	F	F#	G	A
971	C	E	F	F#	G	Bb
972	C	E	F	F#	G	B
973	C	E	F	F#	Ab	A
974	C	E	F	F#	Ab	Bb
975	C	E	F	F#	Ab	B
976	C	E	F	F#	A	Bb
977	C	E	F	F#	A	B
978	C	E	F	F#	Bb	B
979	C	E	F	G	Ab	A
980	C	E	F	G	Ab	Bb
981	C	E	F	G	Ab	B
982	C	E	F	G	A	Bb
983	C	E	F	G	A	B
984	C	E	F	G	Bb	B
985	C	E	F	Ab	A	Bb
986	C	E	F	Ab	A	B
987	C	E	F	Ab	Bb	B
988	C	E	F	A	Bb	B
989	C	E	F#	G	Ab	A

990	C	E	F#	G	Ab	Bb
991	C	E	F#	G	Ab	B
992	C	E	F#	G	A	Bb
993	C	E	F#	G	A	B
994	C	E	F#	G	Bb	B
995	C	E	F#	Ab	A	Bb
996	C	E	F#	Ab	A	B
997	C	E	F#	Ab	Bb	B
998	C	E	F#	A	Bb	B
999	C	E	G	Ab	A	Bb
1000	C	E	G	Ab	A	B
1001	C	E	G	Ab	Bb	B
1002	C	E	G	A	Bb	B
1003	C	E	Ab	A	Bb	B
1004	C	F	F#	G	Ab	A
1005	C	F	F#	G	Ab	Bb
1006	C	F	F#	G	Ab	B
1007	C	F	F#	G	A	Bb
1008	C	F	F#	G	A	B
1009	C	F	F#	G	Bb	B
1010	C	F	F#	Ab	A	Bb
1011	C	F	F#	Ab	A	B
1012	C	F	F#	Ab	Bb	B
1013	C	F	F#	A	Bb	B
1014	C	F	G	Ab	A	Bb
1015	C	F	G	Ab	A	B
1016	C	F	G	Ab	Bb	B
1017	C	F	G	A	Bb	B
1018	C	F	Ab	A	Bb	B
1019	C	F#	G	Ab	A	Bb
1020	C	F#	G	Ab	A	B
1021	C	F#	G	Ab	Bb	B
1022	C	F#	G	A	Bb	B
1023	C	F#	Ab	A	Bb	B
1024	C	G	Ab	A	Bb	B
1025	C	Db	D	Eb	E	F F#
1026	C	Db	D	Eb	E	F G
1027	C	Db	D	Eb	E	F Ab
1028	C	Db	D	Eb	E	F A
1029	C	Db	D	Eb	E	F Bb
1030	C	Db	D	Eb	E	F B
1031	C	Db	D	Eb	E	F# G
1032	C	Db	D	Eb	E	F# Ab
1033	C	Db	D	Eb	E	F# A
1034	C	Db	D	Eb	E	F# Bb
1035	C	Db	D	Eb	E	F# B
1036	C	Db	D	Eb	E	G Ab
1037	C	Db	D	Eb	E	G A
1038	C	Db	D	Eb	E	G Bb
1039	C	Db	D	Eb	E	G B
1040	C	Db	D	Eb	E	Ab A
1041	C	Db	D	Eb	E	Ab Bb
1042	C	Db	D	Eb	E	Ab B
1043	C	Db	D	Eb	E	A Bb
1044	C	Db	D	Eb	E	A B
1045	C	Db	D	Eb	E	Bb B
1046	C	Db	D	Eb	F	F# G
1047	C	Db	D	Eb	F	F# Ab
1048	C	Db	D	Eb	F	F# A

1049	C	Db	D	Eb	F	F#	Bb
1050	C	Db	D	Eb	F	F#	B
1051	C	Db	D	Eb	F	G	Ab
1052	C	Db	D	Eb	F	G	A
1053	C	Db	D	Eb	F	G	Bb
1054	C	Db	D	Eb	F	G	B
1055	C	Db	D	Eb	F	Ab	A
1056	C	Db	D	Eb	F	Ab	Bb
1057	C	Db	D	Eb	F	Ab	B
1058	C	Db	D	Eb	F	A	Bb
1059	C	Db	D	Eb	F	A	B
1060	C	Db	D	Eb	F	Bb	B
1061	C	Db	D	Eb	F#	G	Ab
1062	C	Db	D	Eb	F#	G	A
1063	C	Db	D	Eb	F#	G	Bb
1064	C	Db	D	Eb	F#	G	B
1065	C	Db	D	Eb	F#	Ab	A
1066	C	Db	D	Eb	F#	Ab	Bb
1067	C	Db	D	Eb	F#	Ab	B
1068	C	Db	D	Eb	F#	A	Bb
1069	C	Db	D	Eb	F#	A	B
1070	C	Db	D	Eb	F#	Bb	B
1071	C	Db	D	Eb	G	Ab	A
1072	C	Db	D	Eb	G	Ab	Bb
1073	C	Db	D	Eb	G	Ab	B
1074	C	Db	D	Eb	G	A	Bb
1075	C	Db	D	Eb	G	A	B
1076	C	Db	D	Eb	G	Bb	B
1077	C	Db	D	Eb	Ab	A	Bb
1078	C	Db	D	Eb	Ab	A	B
1079	C	Db	D	Eb	Ab	Bb	B
1080	C	Db	D	Eb	A	Bb	B
1081	C	Db	D	E	F	F#	G
1082	C	Db	D	E	F	F#	Ab
1083	C	Db	D	E	F	F#	A
1084	C	Db	D	E	F	F#	Bb
1085	C	Db	D	E	F	F#	B
1086	C	Db	D	E	F	G	Ab
1087	C	Db	D	E	F	G	A
1088	C	Db	D	E	F	G	Bb
1089	C	Db	D	E	F	G	B
1090	C	Db	D	E	F	Ab	A
1091	C	Db	D	E	F	Ab	Bb
1092	C	Db	D	E	F	Ab	B
1093	C	Db	D	E	F	A	Bb
1094	C	Db	D	E	F	A	B
1095	C	Db	D	E	F	Bb	B
1096	C	Db	D	E	F#	G	Ab
1097	C	Db	D	E	F#	G	A
1098	C	Db	D	E	F#	G	Bb
1099	C	Db	D	E	F#	G	B
1100	C	Db	D	E	F#	Ab	A
1101	C	Db	D	E	F#	Ab	Bb
1102	C	Db	D	E	F#	Ab	B
1103	C	Db	D	E	F#	A	Bb
1104	C	Db	D	E	F#	A	B
1105	C	Db	D	E	F#	Bb	B
1106	C	Db	D	E	G	Ab	A
1107	C	Db	D	E	G	Ab	Bb

1108	C	Db	D	E	G	Ab	B
1109	C	Db	D	E	G	A	Bb
1110	C	Db	D	E	G	A	B
1111	C	Db	D	E	G	Bb	B
1112	C	Db	D	E	Ab	A	Bb
1113	C	Db	D	E	Ab	A	B
1114	C	Db	D	E	Ab	Bb	B
1115	C	Db	D	E	A	Bb	B
1116	C	Db	D	F	F#	G	Ab
1117	C	Db	D	F	F#	G	A
1118	C	Db	D	F	F#	G	Bb
1119	C	Db	D	F	F#	G	B
1120	C	Db	D	F	F#	Ab	A
1121	C	Db	D	F	F#	Ab	Bb
1122	C	Db	D	F	F#	Ab	B
1123	C	Db	D	F	F#	A	Bb
1124	C	Db	D	F	F#	A	B
1125	C	Db	D	F	F#	Bb	B
1126	C	Db	D	F	G	Ab	A
1127	C	Db	D	F	G	Ab	Bb
1128	C	Db	D	F	G	Ab	B
1129	C	Db	D	F	G	A	Bb
1130	C	Db	D	F	G	A	B
1131	C	Db	D	F	G	Bb	B
1132	C	Db	D	F	Ab	A	Bb
1133	C	Db	D	F	Ab	A	B
1134	C	Db	D	F	Ab	Bb	B
1135	C	Db	D	F	A	Bb	B
1136	C	Db	D	F#	G	Ab	A
1137	C	Db	D	F#	G	Ab	Bb
1138	C	Db	D	F#	G	Ab	B
1139	C	Db	D	F#	G	A	Bb
1140	C	Db	D	F#	G	A	B
1141	C	Db	D	F#	G	Bb	B
1142	C	Db	D	F#	Ab	A	Bb
1143	C	Db	D	F#	Ab	A	B
1144	C	Db	D	F#	Ab	Bb	B
1145	C	Db	D	F#	A	Bb	B
1146	C	Db	D	G	Ab	A	Bb
1147	C	Db	D	G	Ab	A	B
1148	C	Db	D	G	Ab	Bb	B
1149	C	Db	D	G	A	Bb	B
1150	C	Db	D	Ab	A	Bb	B
1151	C	Db	Eb	E	F	F#	G
1152	C	Db	Eb	E	F	F#	Ab
1153	C	Db	Eb	E	F	F#	A
1154	C	Db	Eb	E	F	F#	Bb
1155	C	Db	Eb	E	F	F#	B
1156	C	Db	Eb	E	F	G	Ab
1157	C	Db	Eb	E	F	G	A
1158	C	Db	Eb	E	F	G	Bb
1159	C	Db	Eb	E	F	G	B
1160	C	Db	Eb	E	F	Ab	A
1161	C	Db	Eb	E	F	Ab	Bb
1162	C	Db	Eb	E	F	Ab	B
1163	C	Db	Eb	E	F	A	Bb
1164	C	Db	Eb	E	F	A	B
1165	C	Db	Eb	E	F	Bb	B
1166	C	Db	Eb	E	F#	G	Ab

1167	C	Db	Eb	E	F#	G	A
1168	C	Db	Eb	E	F#	G	Bb
1169	C	Db	Eb	E	F#	G	B
1170	C	Db	Eb	E	F#	Ab	A
1171	C	Db	Eb	E	F#	Ab	Bb
1172	C	Db	Eb	E	F#	Ab	B
1173	C	Db	Eb	E	F#	A	Bb
1174	C	Db	Eb	E	F#	A	B
1175	C	Db	Eb	E	F#	Bb	B
1176	C	Db	Eb	E	G	Ab	A
1177	C	Db	Eb	E	G	Ab	Bb
1178	C	Db	Eb	E	G	Ab	B
1179	C	Db	Eb	E	G	A	Bb
1180	C	Db	Eb	E	G	A	B
1181	C	Db	Eb	E	G	Bb	B
1182	C	Db	Eb	E	Ab	A	Bb
1183	C	Db	Eb	E	Ab	A	B
1184	C	Db	Eb	E	Ab	Bb	B
1185	C	Db	Eb	E	A	Bb	B
1186	C	Db	Eb	F	F#	G	Ab
1187	C	Db	Eb	F	F#	G	A
1188	C	Db	Eb	F	F#	G	Bb
1189	C	Db	Eb	F	F#	G	B
1190	C	Db	Eb	F	F#	Ab	A
1191	C	Db	Eb	F	F#	Ab	Bb
1192	C	Db	Eb	F	F#	Ab	B
1193	C	Db	Eb	F	F#	A	Bb
1194	C	Db	Eb	F	F#	A	B
1195	C	Db	Eb	F	F#	Bb	B
1196	C	Db	Eb	F	G	Ab	A
1197	C	Db	Eb	F	G	Ab	Bb
1198	C	Db	Eb	F	G	Ab	B
1199	C	Db	Eb	F	G	A	Bb
1200	C	Db	Eb	F	G	A	B
1201	C	Db	Eb	F	G	Bb	B
1202	C	Db	Eb	F	Ab	A	Bb
1203	C	Db	Eb	F	Ab	A	B
1204	C	Db	Eb	F	Ab	Bb	B
1205	C	Db	Eb	F	A	Bb	B
1206	C	Db	Eb	F#	G	Ab	A
1207	C	Db	Eb	F#	G	Ab	Bb
1208	C	Db	Eb	F#	G	Ab	B
1209	C	Db	Eb	F#	G	A	Bb
1210	C	Db	Eb	F#	G	A	B
1211	C	Db	Eb	F#	G	Bb	B
1212	C	Db	Eb	F#	Ab	A	Bb
1213	C	Db	Eb	F#	Ab	A	B
1214	C	Db	Eb	F#	Ab	Bb	B
1215	C	Db	Eb	F#	A	Bb	B
1216	C	Db	Eb	G	Ab	A	Bb
1217	C	Db	Eb	G	Ab	A	B
1218	C	Db	Eb	G	Ab	Bb	B
1219	C	Db	Eb	G	A	Bb	B
1220	C	Db	Eb	Ab	A	Bb	B
1221	C	Db	E	F	F#	G	Ab
1222	C	Db	E	F	F#	G	A
1223	C	Db	E	F	F#	G	Bb
1224	C	Db	E	F	F#	G	B
1225	C	Db	E	F	F#	Ab	A

1226	C	Db	E	F	F#	Ab	Bb
1227	C	Db	E	F	F#	Ab	B
1228	C	Db	E	F	F#	A	Bb
1229	C	Db	E	F	F#	A	B
1230	C	Db	E	F	F#	Bb	B
1231	C	Db	E	F	G	Ab	A
1232	C	Db	E	F	G	Ab	Bb
1233	C	Db	E	F	G	Ab	B
1234	C	Db	E	F	G	A	Bb
1235	C	Db	E	F	G	A	B
1236	C	Db	E	F	G	Bb	B
1237	C	Db	E	F	Ab	A	Bb
1238	C	Db	E	F	Ab	A	B
1239	C	Db	E	F	Ab	Bb	B
1240	C	Db	E	F	A	Bb	B
1241	C	Db	E	F#	G	Ab	A
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1243	C	Db	E	F#	G	Ab	B
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1245	C	Db	E	F#	G	A	B
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1247	C	Db	E	F#	Ab	A	Bb
1248	C	Db	E	F#	Ab	A	B
1249	C	Db	E	F#	Ab	Bb	B
1250	C	Db	E	F#	A	Bb	B
1251	C	Db	E	G	Ab	A	Bb
1252	C	Db	E	G	Ab	A	B
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1254	C	Db	E	G	A	Bb	B
1255	C	Db	E	Ab	A	Bb	B
1256	C	Db	F	F#	G	Ab	A
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1260	C	Db	F	F#	G	A	B
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1262	C	Db	F	F#	Ab	A	Bb
1263	C	Db	F	F#	Ab	A	B
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1265	C	Db	F	F#	A	Bb	B
1266	C	Db	F	G	Ab	A	Bb
1267	C	Db	F	G	Ab	A	B
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1276	C	Db	G	Ab	A	Bb	B
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1279	C	D	Eb	E	F	F#	A
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1281	C	D	Eb	E	F	F#	B
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1283	C	D	Eb	E	F	G	A
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1296	C	D	Eb	E	F#	Ab	A
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1298	C	D	Eb	E	F#	Ab	B
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1342	C	D	Eb	G	Ab	A	Bb
1343	C	D	Eb	G	Ab	A	B

1344	C	D	Eb	G	Ab	Bb	B
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1346	C	D	Eb	Ab	A	Bb	B
1347	C	D	E	F	F#	G	Ab
1348	C	D	E	F	F#	G	A
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1401	C	D	F#	Ab	A	Bb	B
1402	C	D	G	Ab	A	Bb	B

1403	C	Eb	E	F	F#	G	Ab
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1456	C	Eb	F#	G	A	Bb	B
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1458	C	Eb	G	Ab	A	Bb	B
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1461	C	E	F	F#	G	Ab	B

1462	C	E	F	F#	G	A	Bb
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1466	C	E	F	F#	Ab	A	B
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1468	C	E	F	F#	A	Bb	B
1469	C	E	F	G	Ab	A	Bb
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1474	C	E	F#	G	Ab	A	Bb
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1477	C	E	F#	G	A	Bb	B
1478	C	E	F#	Ab	A	Bb	B
1479	C	E	G	Ab	A	Bb	B
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1481	C	F	F#	G	Ab	A	B
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1483	C	F	F#	G	A	Bb	B
1484	C	F	F#	Ab	A	Bb	B
1485	C	F	G	Ab	A	Bb	B
1486	C	F#	G	Ab	A	Bb	B
1487	C	Db	D	Eb	E	F	F# G
1488	C	Db	D	Eb	E	F	F# Ab
1489	C	Db	D	Eb	E	F	F# A
1490	C	Db	D	Eb	E	F	F# Bb
1491	C	Db	D	Eb	E	F	F# B
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1493	C	Db	D	Eb	E	F	G A
1494	C	Db	D	Eb	E	F	G Bb
1495	C	Db	D	Eb	E	F	G B
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1498	C	Db	D	Eb	E	F	Ab B
1499	C	Db	D	Eb	E	F	A Bb
1500	C	Db	D	Eb	E	F	A B
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1502	C	Db	D	Eb	E	F#	G Ab
1503	C	Db	D	Eb	E	F#	G A
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1506	C	Db	D	Eb	E	F#	Ab A
1507	C	Db	D	Eb	E	F#	Ab Bb
1508	C	Db	D	Eb	E	F#	Ab B
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1510	C	Db	D	Eb	E	F#	A B
1511	C	Db	D	Eb	E	F#	Bb B
1512	C	Db	D	Eb	E	G	Ab A
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1514	C	Db	D	Eb	E	G	Ab B
1515	C	Db	D	Eb	E	G	A Bb
1516	C	Db	D	Eb	E	G	A B
1517	C	Db	D	Eb	E	G	Bb B
1518	C	Db	D	Eb	E	Ab	A Bb
1519	C	Db	D	Eb	E	Ab	A B
1520	C	Db	D	Eb	E	Ab	Bb B

1521	C	Db	D	Eb	E	A	Bb	B
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1523	C	Db	D	Eb	F	F#	G	A
1524	C	Db	D	Eb	F	F#	G	Bb
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1614	C	Db	Eb	E	F	F#	G	A
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1616	C	Db	Eb	E	F	F#	G	B
1617	C	Db	Eb	E	F	F#	Ab	A
1618	C	Db	Eb	E	F	F#	Ab	Bb
1619	C	Db	Eb	E	F	F#	Ab	B
1620	C	Db	Eb	E	F	F#	A	Bb
1621	C	Db	Eb	E	F	F#	A	B
1622	C	Db	Eb	E	F	F#	Bb	B
1623	C	Db	Eb	E	F	G	Ab	A
1624	C	Db	Eb	E	F	G	Ab	Bb
1625	C	Db	Eb	E	F	G	Ab	B
1626	C	Db	Eb	E	F	G	A	Bb
1627	C	Db	Eb	E	F	G	A	B
1628	C	Db	Eb	E	F	G	Bb	B
1629	C	Db	Eb	E	F	Ab	A	Bb
1630	C	Db	Eb	E	F	Ab	A	B
1631	C	Db	Eb	E	F	Ab	Bb	B
1632	C	Db	Eb	E	F	A	Bb	B
1633	C	Db	Eb	E	F#	G	Ab	A
1634	C	Db	Eb	E	F#	G	Ab	Bb
1635	C	Db	Eb	E	F#	G	Ab	B
1636	C	Db	Eb	E	F#	G	A	Bb
1637	C	Db	Eb	E	F#	G	A	B
1638	C	Db	Eb	E	F#	G	Bb	B

1639	C	Db	Eb	E	F#	Ab	A	Bb
1640	C	Db	Eb	E	F#	Ab	A	B
1641	C	Db	Eb	E	F#	Ab	Bb	B
1642	C	Db	Eb	E	F#	A	Bb	B
1643	C	Db	Eb	E	G	Ab	A	Bb
1644	C	Db	Eb	E	G	Ab	A	B
1645	C	Db	Eb	E	G	Ab	Bb	B
1646	C	Db	Eb	E	G	A	Bb	B
1647	C	Db	Eb	E	Ab	A	Bb	B
1648	C	Db	Eb	F	F#	G	Ab	A
1649	C	Db	Eb	F	F#	G	Ab	Bb
1650	C	Db	Eb	F	F#	G	Ab	B
1651	C	Db	Eb	F	F#	G	A	Bb
1652	C	Db	Eb	F	F#	G	A	B
1653	C	Db	Eb	F	F#	G	Bb	B
1654	C	Db	Eb	F	F#	Ab	A	Bb
1655	C	Db	Eb	F	F#	Ab	A	B
1656	C	Db	Eb	F	F#	Ab	Bb	B
1657	C	Db	Eb	F	F#	A	Bb	B
1658	C	Db	Eb	F	G	Ab	A	Bb
1659	C	Db	Eb	F	G	Ab	A	B
1660	C	Db	Eb	F	G	Ab	Bb	B
1661	C	Db	Eb	F	G	A	Bb	B
1662	C	Db	Eb	F	Ab	A	Bb	B
1663	C	Db	Eb	F#	G	Ab	A	Bb
1664	C	Db	Eb	F#	G	Ab	A	B
1665	C	Db	Eb	F#	G	Ab	Bb	B
1666	C	Db	Eb	F#	G	A	Bb	B
1667	C	Db	Eb	F#	Ab	A	Bb	B
1668	C	Db	Eb	G	Ab	A	Bb	B
1669	C	Db	E	F	F#	G	Ab	A
1670	C	Db	E	F	F#	G	Ab	Bb
1671	C	Db	E	F	F#	G	Ab	B
1672	C	Db	E	F	F#	G	A	Bb
1673	C	Db	E	F	F#	G	A	B
1674	C	Db	E	F	F#	G	Bb	B
1675	C	Db	E	F	F#	Ab	A	Bb
1676	C	Db	E	F	F#	Ab	A	B
1677	C	Db	E	F	F#	Ab	Bb	B
1678	C	Db	E	F	F#	A	Bb	B
1679	C	Db	E	F	G	Ab	A	Bb
1680	C	Db	E	F	G	Ab	A	B
1681	C	Db	E	F	G	Ab	Bb	B
1682	C	Db	E	F	G	A	Bb	B
1683	C	Db	E	F	Ab	A	Bb	B
1684	C	Db	E	F#	G	Ab	A	Bb
1685	C	Db	E	F#	G	Ab	A	B
1686	C	Db	E	F#	G	Ab	Bb	B
1687	C	Db	E	F#	G	A	Bb	B
1688	C	Db	E	F#	Ab	A	Bb	B
1689	C	Db	E	G	Ab	A	Bb	B
1690	C	Db	F	F#	G	Ab	A	Bb
1691	C	Db	F	F#	G	Ab	A	B
1692	C	Db	F	F#	G	Ab	Bb	B
1693	C	Db	F	F#	G	A	Bb	B
1694	C	Db	F	F#	Ab	A	Bb	B
1695	C	Db	F	G	Ab	A	Bb	B
1696	C	Db	F#	G	Ab	A	Bb	B
1697	C	D	Eb	E	F	F#	G	Ab

1698	C	D	Eb	E	F	F#	G	A
1699	C	D	Eb	E	F	F#	G	Bb
1700	C	D	Eb	E	F	F#	G	B
1701	C	D	Eb	E	F	F#	Ab	A
1702	C	D	Eb	E	F	F#	Ab	Bb
1703	C	D	Eb	E	F	F#	Ab	B
1704	C	D	Eb	E	F	F#	A	Bb
1705	C	D	Eb	E	F	F#	A	B
1706	C	D	Eb	E	F	F#	Bb	B
1707	C	D	Eb	E	F	G	Ab	A
1708	C	D	Eb	E	F	G	Ab	Bb
1709	C	D	Eb	E	F	G	Ab	B
1710	C	D	Eb	E	F	G	A	Bb
1711	C	D	Eb	E	F	G	A	B
1712	C	D	Eb	E	F	G	Bb	B
1713	C	D	Eb	E	F	Ab	A	Bb
1714	C	D	Eb	E	F	Ab	A	B
1715	C	D	Eb	E	F	Ab	Bb	B
1716	C	D	Eb	E	F	A	Bb	B
1717	C	D	Eb	E	F#	G	Ab	A
1718	C	D	Eb	E	F#	G	Ab	Bb
1719	C	D	Eb	E	F#	G	Ab	B
1720	C	D	Eb	E	F#	G	A	Bb
1721	C	D	Eb	E	F#	G	A	B
1722	C	D	Eb	E	F#	G	Bb	B
1723	C	D	Eb	E	F#	Ab	A	Bb
1724	C	D	Eb	E	F#	Ab	A	B
1725	C	D	Eb	E	F#	Ab	Bb	B
1726	C	D	Eb	E	F#	A	Bb	B
1727	C	D	Eb	E	G	Ab	A	Bb
1728	C	D	Eb	E	G	Ab	A	B
1729	C	D	Eb	E	G	Ab	Bb	B
1730	C	D	Eb	E	G	A	Bb	B
1731	C	D	Eb	E	Ab	A	Bb	B
1732	C	D	Eb	F	F#	G	Ab	A
1733	C	D	Eb	F	F#	G	Ab	Bb
1734	C	D	Eb	F	F#	G	Ab	B
1735	C	D	Eb	F	F#	G	A	Bb
1736	C	D	Eb	F	F#	G	A	B
1737	C	D	Eb	F	F#	G	Bb	B
1738	C	D	Eb	F	F#	Ab	A	Bb
1739	C	D	Eb	F	F#	Ab	A	B
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1741	C	D	Eb	F	F#	A	Bb	B
1742	C	D	Eb	F	G	Ab	A	Bb
1743	C	D	Eb	F	G	Ab	A	B
1744	C	D	Eb	F	G	Ab	Bb	B
1745	C	D	Eb	F	G	A	Bb	B
1746	C	D	Eb	F	Ab	A	Bb	B
1747	C	D	Eb	F#	G	Ab	A	Bb
1748	C	D	Eb	F#	G	Ab	A	B
1749	C	D	Eb	F#	G	Ab	Bb	B
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1751	C	D	Eb	F#	Ab	A	Bb	B
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1753	C	D	E	F	F#	G	Ab	A
1754	C	D	E	F	F#	G	Ab	Bb
1755	C	D	E	F	F#	G	Ab	B
1756	C	D	E	F	F#	G	A	Bb

1757	C	D	E	F	F#	G	A	B
1758	C	D	E	F	F#	G	Bb	B
1759	C	D	E	F	F#	Ab	A	Bb
1760	C	D	E	F	F#	Ab	A	B
1761	C	D	E	F	F#	Ab	Bb	B
1762	C	D	E	F	F#	A	Bb	B
1763	C	D	E	F	G	Ab	A	Bb
1764	C	D	E	F	G	Ab	A	B
1765	C	D	E	F	G	Ab	Bb	B
1766	C	D	E	F	G	A	Bb	B
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1769	C	D	E	F#	G	Ab	A	B
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1777	C	D	F	F#	G	A	Bb	B
1778	C	D	F	F#	Ab	A	Bb	B
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1780	C	D	F#	G	Ab	A	Bb	B
1781	C	Eb	E	F	F#	G	Ab	A
1782	C	Eb	E	F	F#	G	Ab	Bb
1783	C	Eb	E	F	F#	G	Ab	B
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1785	C	Eb	E	F	F#	G	A	B
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1787	C	Eb	E	F	F#	Ab	A	Bb
1788	C	Eb	E	F	F#	Ab	A	B
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1793	C	Eb	E	F	G	Ab	Bb	B
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1795	C	Eb	E	F	Ab	A	Bb	B
1796	C	Eb	E	F#	G	Ab	A	Bb
1797	C	Eb	E	F#	G	Ab	A	B
1798	C	Eb	E	F#	G	Ab	Bb	B
1799	C	Eb	E	F#	G	A	Bb	B
1800	C	Eb	E	F#	Ab	A	Bb	B
1801	C	Eb	E	G	Ab	A	Bb	B
1802	C	Eb	F	F#	G	Ab	A	Bb
1803	C	Eb	F	F#	G	Ab	A	B
1804	C	Eb	F	F#	G	Ab	Bb	B
1805	C	Eb	F	F#	G	A	Bb	B
1806	C	Eb	F	F#	Ab	A	Bb	B
1807	C	Eb	F	G	Ab	A	Bb	B
1808	C	Eb	F#	G	Ab	A	Bb	B
1809	C	E	F	F#	G	Ab	A	Bb
1810	C	E	F	F#	G	Ab	A	B
1811	C	E	F	F#	G	Ab	Bb	B
1812	C	E	F	F#	G	A	Bb	B
1813	C	E	F	F#	Ab	A	Bb	B
1814	C	E	F	G	Ab	A	Bb	B
1815	C	E	F#	G	Ab	A	Bb	B

1816	C	F	F#	G	Ab	A	Bb	B
1817	C	Db	D	Eb	E	F	F#	G
1818	C	Db	D	Eb	E	F	F#	G
1819	C	Db	D	Eb	E	F	F#	G
1820	C	Db	D	Eb	E	F	F#	G
1821	C	Db	D	Eb	E	F	F#	Ab
1822	C	Db	D	Eb	E	F	F#	Ab
1823	C	Db	D	Eb	E	F	F#	Ab
1824	C	Db	D	Eb	E	F	F#	A
1825	C	Db	D	Eb	E	F	F#	A
1826	C	Db	D	Eb	E	F	F#	Bb
1827	C	Db	D	Eb	E	F	G	Ab
1828	C	Db	D	Eb	E	F	G	Ab
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1830	C	Db	D	Eb	E	F	G	A
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1832	C	Db	D	Eb	E	F	G	Bb
1833	C	Db	D	Eb	E	F	Ab	A
1834	C	Db	D	Eb	E	F	Ab	A
1835	C	Db	D	Eb	E	F	Ab	Bb
1836	C	Db	D	Eb	E	F	A	Bb
1837	C	Db	D	Eb	E	F#	G	Ab
1838	C	Db	D	Eb	E	F#	G	Ab
1839	C	Db	D	Eb	E	F#	G	Ab
1840	C	Db	D	Eb	E	F#	G	A
1841	C	Db	D	Eb	E	F#	G	A
1842	C	Db	D	Eb	E	F#	G	Bb
1843	C	Db	D	Eb	E	F#	Ab	A
1844	C	Db	D	Eb	E	F#	Ab	A
1845	C	Db	D	Eb	E	F#	Ab	Bb
1846	C	Db	D	Eb	E	F#	A	Bb
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1850	C	Db	D	Eb	E	G	A	Bb
1851	C	Db	D	Eb	E	Ab	A	Bb
1852	C	Db	D	Eb	F	F#	G	Ab
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1854	C	Db	D	Eb	F	F#	G	Ab
1855	C	Db	D	Eb	F	F#	G	A
1856	C	Db	D	Eb	F	F#	G	A
1857	C	Db	D	Eb	F	F#	G	Bb
1858	C	Db	D	Eb	F	F#	Ab	A
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1860	C	Db	D	Eb	F	F#	Ab	Bb
1861	C	Db	D	Eb	F	F#	A	Bb
1862	C	Db	D	Eb	F	G	Ab	A
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1865	C	Db	D	Eb	F	G	A	Bb
1866	C	Db	D	Eb	F	Ab	A	Bb
1867	C	Db	D	Eb	F#	G	Ab	A
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1869	C	Db	D	Eb	F#	G	Ab	Bb
1870	C	Db	D	Eb	F#	G	A	Bb
1871	C	Db	D	Eb	F#	Ab	A	Bb
1872	C	Db	D	Eb	G	Ab	A	Bb
1873	C	Db	D	E	F	F#	G	Ab
1874	C	Db	D	E	F	F#	G	Ab

1875	C	Db	D	E	F	F#	G	Ab	B
1876	C	Db	D	E	F	F#	G	A	Bb
1877	C	Db	D	E	F	F#	G	A	B
1878	C	Db	D	E	F	F#	G	Bb	B
1879	C	Db	D	E	F	F#	Ab	A	Bb
1880	C	Db	D	E	F	F#	Ab	A	B
1881	C	Db	D	E	F	F#	Ab	Bb	B
1882	C	Db	D	E	F	F#	A	Bb	B
1883	C	Db	D	E	F	G	Ab	A	Bb
1884	C	Db	D	E	F	G	Ab	A	B
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1886	C	Db	D	E	F	G	A	Bb	B
1887	C	Db	D	E	F	Ab	A	Bb	B
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1897	C	Db	D	F	F#	G	A	Bb	B
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1899	C	Db	D	F	G	Ab	A	Bb	B
1900	C	Db	D	F#	G	Ab	A	Bb	B
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1902	C	Db	Eb	E	F	F#	G	Ab	Bb
1903	C	Db	Eb	E	F	F#	G	Ab	B
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1905	C	Db	Eb	E	F	F#	G	A	B
1906	C	Db	Eb	E	F	F#	G	Bb	B
1907	C	Db	Eb	E	F	F#	Ab	A	Bb
1908	C	Db	Eb	E	F	F#	Ab	A	B
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1910	C	Db	Eb	E	F	F#	A	Bb	B
1911	C	Db	Eb	E	F	G	Ab	A	Bb
1912	C	Db	Eb	E	F	G	Ab	A	B
1913	C	Db	Eb	E	F	G	Ab	Bb	B
1914	C	Db	Eb	E	F	G	A	Bb	B
1915	C	Db	Eb	E	F	Ab	A	Bb	B
1916	C	Db	Eb	E	F#	G	Ab	A	Bb
1917	C	Db	Eb	E	F#	G	Ab	A	B
1918	C	Db	Eb	E	F#	G	Ab	Bb	B
1919	C	Db	Eb	E	F#	G	A	Bb	B
1920	C	Db	Eb	E	F#	Ab	A	Bb	B
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1923	C	Db	Eb	F	F#	G	Ab	A	B
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1927	C	Db	Eb	F	G	Ab	A	Bb	B
1928	C	Db	Eb	F#	G	Ab	A	Bb	B
1929	C	Db	E	F	F#	G	Ab	A	Bb
1930	C	Db	E	F	F#	G	Ab	A	B
1931	C	Db	E	F	F#	G	Ab	Bb	B
1932	C	Db	E	F	F#	G	A	Bb	B
1933	C	Db	E	F	F#	Ab	A	Bb	B

1934	C	Db	E	F	G	Ab	A	Bb	B	
1935	C	Db	E	F#	G	Ab	A	Bb	B	
1936	C	Db	F	F#	G	Ab	A	Bb	B	
1937	C	D	Eb	E	F	F#	G	Ab	A	
1938	C	D	Eb	E	F	F#	G	Ab	Bb	
1939	C	D	Eb	E	F	F#	G	Ab	B	
1940	C	D	Eb	E	F	F#	G	A	Bb	
1941	C	D	Eb	E	F	F#	G	A	B	
1942	C	D	Eb	E	F	F#	G	Bb	B	
1943	C	D	Eb	E	F	F#	Ab	A	Bb	
1944	C	D	Eb	E	F	F#	Ab	A	B	
1945	C	D	Eb	E	F	F#	Ab	Bb	B	
1946	C	D	Eb	E	F	F#	A	Bb	B	
1947	C	D	Eb	E	F	G	Ab	A	Bb	
1948	C	D	Eb	E	F	G	Ab	A	B	
1949	C	D	Eb	E	F	G	Ab	Bb	B	
1950	C	D	Eb	E	F	G	A	Bb	B	
1951	C	D	Eb	E	F	Ab	A	Bb	B	
1952	C	D	Eb	E	F#	G	Ab	A	Bb	
1953	C	D	Eb	E	F#	G	Ab	A	B	
1954	C	D	Eb	E	F#	G	Ab	Bb	B	
1955	C	D	Eb	E	F#	G	A	Bb	B	
1956	C	D	Eb	E	F#	Ab	A	Bb	B	
1957	C	D	Eb	E	G	Ab	A	Bb	B	
1958	C	D	Eb	F	F#	G	Ab	A	Bb	
1959	C	D	Eb	F	F#	G	Ab	A	B	
1960	C	D	Eb	F	F#	G	Ab	Bb	B	
1961	C	D	Eb	F	F#	G	A	Bb	B	
1962	C	D	Eb	F	F#	Ab	A	Bb	B	
1963	C	D	Eb	F	G	Ab	A	Bb	B	
1964	C	D	Eb	F#	G	Ab	A	Bb	B	
1965	C	D	E	F	F#	G	Ab	A	Bb	
1966	C	D	E	F	F#	G	Ab	A	B	
1967	C	D	E	F	F#	G	Ab	Bb	B	
1968	C	D	E	F	F#	G	A	Bb	B	
1969	C	D	E	F	F#	Ab	A	Bb	B	
1970	C	D	E	F	G	Ab	A	Bb	B	
1971	C	D	E	F#	G	Ab	A	Bb	B	
1972	C	D	F	F#	G	Ab	A	Bb	B	
1973	C	Eb	E	F	F#	G	Ab	A	Bb	
1974	C	Eb	E	F	F#	G	Ab	A	B	
1975	C	Eb	E	F	F#	G	Ab	Bb	B	
1976	C	Eb	E	F	F#	G	A	Bb	B	
1977	C	Eb	E	F	F#	Ab	A	Bb	B	
1978	C	Eb	E	F	G	Ab	A	Bb	B	
1979	C	Eb	E	F#	G	Ab	A	Bb	B	
1980	C	Eb	F	F#	G	Ab	A	Bb	B	
1981	C	E	F	F#	G	Ab	A	Bb	B	
1982	C	Db	D	Eb	E	F	F#	G	Ab	A
1983	C	Db	D	Eb	E	F	F#	G	Ab	Bb
1984	C	Db	D	Eb	E	F	F#	G	Ab	B
1985	C	Db	D	Eb	E	F	F#	G	A	Bb
1986	C	Db	D	Eb	E	F	F#	G	A	B
1987	C	Db	D	Eb	E	F	F#	G	Bb	B
1988	C	Db	D	Eb	E	F	F#	Ab	A	Bb
1989	C	Db	D	Eb	E	F	F#	Ab	A	B
1990	C	Db	D	Eb	E	F	F#	Ab	Bb	B
1991	C	Db	D	Eb	E	F	F#	A	Bb	B
1992	C	Db	D	Eb	E	F	G	Ab	A	Bb

1993	C	Db	D	Eb	E	F	G	Ab	A	B
1994	C	Db	D	Eb	E	F	G	Ab	Bb	B
1995	C	Db	D	Eb	E	F	G	A	Bb	B
1996	C	Db	D	Eb	E	F	Ab	A	Bb	B
1997	C	Db	D	Eb	E	F#	G	Ab	A	Bb
1998	C	Db	D	Eb	E	F#	G	Ab	A	B
1999	C	Db	D	Eb	E	F#	G	Ab	Bb	B
2000	C	Db	D	Eb	E	F#	G	A	Bb	B
2001	C	Db	D	Eb	E	F#	Ab	A	Bb	B
2002	C	Db	D	Eb	E	G	Ab	A	Bb	B
2003	C	Db	D	Eb	F	F#	G	Ab	A	Bb
2004	C	Db	D	Eb	F	F#	G	Ab	A	B
2005	C	Db	D	Eb	F	F#	G	Ab	Bb	B
2006	C	Db	D	Eb	F	F#	G	A	Bb	B
2007	C	Db	D	Eb	F	F#	Ab	A	Bb	B
2008	C	Db	D	Eb	F	G	Ab	A	Bb	B
2009	C	Db	D	Eb	F#	G	Ab	A	Bb	B
2010	C	Db	D	E	F	F#	G	Ab	A	Bb
2011	C	Db	D	E	F	F#	G	Ab	A	B
2012	C	Db	D	E	F	F#	G	Ab	Bb	B
2013	C	Db	D	E	F	F#	G	A	Bb	B
2014	C	Db	D	E	F	F#	Ab	A	Bb	B
2015	C	Db	D	E	F	G	Ab	A	Bb	B
2016	C	Db	D	E	F#	G	Ab	A	Bb	B
2017	C	Db	D	F	F#	G	Ab	A	Bb	B
2018	C	Db	Eb	E	F	F#	G	Ab	A	Bb
2019	C	Db	Eb	E	F	F#	G	Ab	A	B
2020	C	Db	Eb	E	F	F#	G	Ab	Bb	B
2021	C	Db	Eb	E	F	F#	G	A	Bb	B
2022	C	Db	Eb	E	F	F#	Ab	A	Bb	B
2023	C	Db	Eb	E	F	G	Ab	A	Bb	B
2024	C	Db	Eb	E	F#	G	Ab	A	Bb	B
2025	C	Db	Eb	F	F#	G	Ab	A	Bb	B
2026	C	Db	E	F	F#	G	Ab	A	Bb	B
2027	C	D	Eb	E	F	F#	G	Ab	A	Bb
2028	C	D	Eb	E	F	F#	G	Ab	A	B
2029	C	D	Eb	E	F	F#	G	Ab	Bb	B
2030	C	D	Eb	E	F	F#	G	A	Bb	B
2031	C	D	Eb	E	F	F#	Ab	A	Bb	B
2032	C	D	Eb	E	F	G	Ab	A	Bb	B
2033	C	D	Eb	E	F#	G	Ab	A	Bb	B
2034	C	D	Eb	F	F#	G	Ab	A	Bb	B
2035	C	D	E	F	F#	G	Ab	A	Bb	B
2036	C	Eb	E	F	F#	G	Ab	A	Bb	B
2037	C	Db	D	Eb	E	F	F#	G	Ab	A Bb
2038	C	Db	D	Eb	E	F	F#	G	Ab	A B
2039	C	Db	D	Eb	E	F	F#	G	Ab	Bb B
2040	C	Db	D	Eb	E	F	F#	G	A	Bb B
2041	C	Db	D	Eb	E	F	F#	Ab	A	Bb B
2042	C	Db	D	Eb	E	F	G	Ab	A	Bb B
2043	C	Db	D	Eb	E	F#	G	Ab	A	Bb B
2044	C	Db	D	Eb	F	F#	G	Ab	A	Bb B
2045	C	Db	D	E	F	F#	G	Ab	A	Bb B
2046	C	Db	Eb	E	F	F#	G	Ab	A	Bb B
2047	C	D	Eb	E	F	F#	G	Ab	A	Bb B
2048	C	Db	D	Eb	E	F	F#	G	Ab	A Bb B

Appendix 3: Probability Calculations

re Vol. I, p. 79

Probability (a)

This probability (P) is calculated by dividing one by 12 factorial (written $12!$), thus

$$P = \frac{1}{12!}$$

$$= \frac{1}{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 0.000000002...$$

= roughly 1 in 500 million.

Probability (b)

I found this probability relatively harder to calculate, and discovered that such a task relies upon the use of a set of nested equations. In order to calculate the chances that all 12 keys are covered from a random 26 selections, I started with simpler material. Flipping a coin twice has four possible outcomes (H for heads, T for tails): HH, HT, TH or TT. (Note that I did not actually flip any coins; this probability work is theoretical, not experimental – but see below). Of these results, two interest us since they cover all possible “sides” of the object (i.e. just as Coltrane covers the

chromatic set with his transpositions in bars 137-172): HT and TH. Thus, of the four possible outcomes, two may be called “good”. Therefore, the probability of such a “good” outcome may be seen to be

$$P = \frac{2}{4}$$

$$= 0.5$$

I wrote out lists of the permutations of results for three, four and five tosses of a coin and discovered the following probabilities. If we allow three tosses of a coin in which to achieve both a head and a tail result, then the permutations are

HHH

HHT

HTH

THH

HTT

THT

TTH

TTT

And thus the probability may be seen to be

$$P = \frac{6}{8}$$

$$= 0.75$$

Likewise, for four tosses,

$$P = \frac{14}{16}$$

$$= 0.875$$

For five tosses,

$$P = \frac{30}{32}$$

$$= 0.9375$$

It might be imagined that for all cases we might extend what seems to be happening here, i.e.

$$P = \frac{x}{x+2}$$

Or

$$P = \frac{x}{x+y}$$

Where y = the number of “sides” of a given object. However, as we shall see, things are not so simple.

I then undertook to examine a theoretical object that has three “sides”. For this, list analysis revealed that, for three tosses

$$P = 0.222...$$

For four tosses $P = 0.444...$, and for 5 tosses $P = 0.617$. I then extended these data until it was sufficient to enable the discovery of the underlying formulae that would enable me to make the larger calculation re bars 137-172.

Here are the results tabulated, with numbers in **bold** show the number of sides to the given object generating the result, and numbers in *italics* show the number of tosses:

	P =
2, 2:	0.5
2, 3:	0.75
2, 4:	0.875
3, 3:	0.222...
3, 4:	0.444...
3, 5:	0.617283951
4, 4:	0.09375 (this is the chance of being randomly dealt all four suits with playing cards)
4, 5:	0.234375
4, 6:	0.380859375

Analysis revealed the following structure where the number of sides = number of tosses:

For 2, 2:

$$P = \frac{2!}{2^2}$$

Similarly, for 3, 3:

$$P = \frac{3!}{3^3}$$

And so on.

Further, each probability calculation was seen to provide numerical data for each set of calculations for higher numbers of tosses and sides:

2, 2:

$$P = \frac{2!}{2^2} = \frac{2}{4} = 0.5$$

2, 3: (employs the 2 from the 2/4 above)

$$P = \frac{2(2+1)}{2^3} = \frac{6}{8} = 0.75$$

2, 4: (employs the 6 from the 6/8 above)

$$P = \frac{2(6+1)}{2^4} = \frac{14}{16} = 0.875$$

3, 3:

$$P = \frac{3!}{3^3} = \frac{6}{27} = 0.222...$$

3, 4: (employs one 6 from 3, 3 and another 6 from 2, 3 above)

$$P = \frac{3(6+6)}{3^4} = \frac{36}{81} = 0.444...$$

3, 5: (employs the 36 from 3, 4 and the 14 from 2, 4 above)

$$P = \frac{3(36+14)}{3^5} = \frac{150}{243} = 0.617283951...$$

4, 4:

$$P = \frac{4!}{4^4} = \frac{24}{256} = 0.09375$$

4, 5: (employs the 24 from 4, 4 and the 36 from 3, 4 above)

$$P = \frac{4(24+36)}{4^5} = \frac{240}{1024} = 0.234375$$

4, 6: (employs the 240 from 4, 5 and the 150 from 3, 5 above)

$$P = \frac{4(240+150)}{4^6} = \frac{1560}{4096} = 0.380859375$$

And so on...

I built an extended string of these formulae in Microsoft Excel on my PC and from this have been able to calculate that for the case of randomly selecting 26 keys and then checking to see if all twelve members of the chromatic set have been covered:

i.e. for 12, 26:

$$P = 0.215471706$$

= roughly 1 in 5.

Probability (c)

A similar approach was taken with the calculation of this probability. Lists of outcomes for objects with x “faces” were analysed after $x+1$, $x+2$, etc. “goes”, ignoring results which contained adjacent repeats. (The word “go” is used to mean a random selection, e.g. the toss of a coin. Each “go” delivers a single result.) A further set of nesting formulae was derived, and extended in Microsoft Excel until the model reached a 12, 26 level (as above). From this I discovered that the number of key lists that (a) cover all 12 keys and (b) do not have adjacent repeats is

$$= 3.843329761995471 \times 10^{26}$$

(This number was refined beyond the detail allowed by Excel by using my PC’s calculator). The number of permutations of key lists from 26 “goes”

$$= 12^{26}$$

Therefore, the probability of covering all 12 keys and not getting any adjacent repeats in 26 “goes”

$$= \frac{3.8433... \times 10^{26}}{12^{26}} = 0.033573114$$

= roughly 1 in 30.

p. 80

A subtle variation of Probability (c) was used here. In this case I needed to allow for the deliberate disallowing of adjacent repeats. Thus, only one of the other 11 may follow each key, creating a new denominator:

$$= 12(11^{25})$$

Therefore, the probability of covering all 12 keys disallowing repeats in 26 goes

$$= \frac{3.8433... \times 10^{26}}{12(11^{25})} = 0.295600825$$

= roughly 1 in 3.

p. 88

These probability data assess the likelihood that a scale of a given length could be randomly selected from any of the 2048 scales. As shown in my 1997 paper, the 2048 scales have the following frequency of distribution (x = scale length, f = frequency):

x	f
1	1
2	11
3	55
4	165
5	330
6	462
7	462
8	330
9	165
10	55
11	11
12	1

Thus, the probability of randomly generating any of the sets of LOTs that sum to the chromatic scale may be calculated. For a set of such LOTs, S , each member being of a set of all scales of a given length of population f , then

$$P = S \frac{1}{f}$$

For example, there are 5 x five-note LOTs that sum Scale #33 (the “A Love Supreme” set) to the chromatic scale. There are 330 five-note scales (or LOTs). Thus the probability of randomly generating such a LOT in five “goes” is

$$P = 5 \frac{1}{330} = 0.02...$$

p. 90

These probabilities proved impossible for me to calculate theoretically, and so I generated these data experimentally with programs that I wrote in Visual Basic running within Microsoft Excel. These programs acted as random key generators. I suggest an accuracy range of circa +/-5% based upon analysis of the data as they were collected. I ran the programs through sufficient iterations (and with offset start points to compensate any commonalties within the host software) until I was confident whether or not random generation of LOT #248 was likely.

p. 94

The statement that the average number of keys of the motif required to achieve chromatic saturation (under random saturation) is 8.24 (2 d.p.) was also calculated experimentally using a program running in Visual Basic within Microsoft Excel over an extended period. A results averaging system applied to several runs of the program was employed to refine the detail of the result to 6 decimal places beyond the 2 provided for in the text.

Appendix 4: Slowest and Fastest Scales at Chromatic Saturation under Random Transposition

Using my computer as a random transposition generator, I have calculated the following tables of data, which show which of the 2048 scales are the slowest and fastest at achieving chromatic saturation under random transposition.

no. of notes	#/2048	scale	notes	Forte: Prime Form	Interval Vector
1	1	C		-	-
2	5	C E	Major 3 rd interval	-	-
3 (a)	14	C Db Eb	"ALS" set minor and Major triads	3-2	[111000]
3 (b)	15	C Db E		3-3	[101100]
3 (c)	16	C Db F		3-4	[100110]
3 (d)	25	C D F		3-7	[011010]
3 (e)	35	C Eb G		3-11	[001110]
4 (a)	80	C Db Eb G	13 th chord	4-Z29	[111111]
4 (b)	86	C Db E F#	7#9 chord	4-Z15	[111111]
5 (a)	250	C Db D F Ab		5-Z38	[212221]
5 (b)	298	C Db E F G		5-Z18	[212221]
6 (a)	599	C Db D E F# A		6-Z46	[233331]
6 (b)	648	C Db Eb E F G		6-Z10	[333321]
7 (a)	1037	C Db D Eb E G A		7-Z12	[444342]
7 (b)	1047	C Db D Eb F F# Ab		7-Z36	[444342]
7 (c)	1055	C Db D Eb F Ab A		7-Z18	[434442]
7 (d)	1086	C Db D E F G Ab		7-Z38	[434442]
8 (a)	1506	C Db D Eb E F# Ab A		8-Z29	[555553]
8 (b)	1523	C Db D Eb F F# G A		8-Z15	[555553]
9 (a)	1818	C Db D Eb E F F# G A		9-2	[777663]
9 (b)	1821	C Db D Eb E F F# Ab A		9-3	[767763]
9 (c)	1827	C Db D Eb E F G Ab A		9-4	[766773]
9 (d)	1828	C Db D Eb E F G Ab Bb		9-7	[677673]
9 (e)	1855	C Db D Eb F F# G A Bb		9-11	[667773]
10 (a)	1982	C Db D Eb E F F# G Ab A	i.e. all 10 note scales <i>except</i> chromatic scale minus tritone	-	-
10 (b)	1983	C Db D Eb E F F# G Ab Bb		-	-
10 (c)	1985	C Db D Eb E F F# G A Bb		-	-
10 (d)	1988	C Db D Eb E F F# Ab A Bb		-	-
10 (e)	1992	C Db D Eb E F G Ab A Bb		-	-
11	2037	C Db D Eb E F F# G Ab A Bb	Thus, all 11 note scales	-	-
12	2048	C Db D Eb E F F# G Ab A Bb B	Chromatic scale	-	-

Table 1: Scales that are SLOWEST at achieving Chromatic Saturation under Random Transposition

no. of notes	#/2048	scale	notes	Forte: Prime Form	Interval Vector
1	1	C		-	-
2	7	C F#	Tritone	-	-
3	43	C E Ab	Augmented triad	3-12	[000300]
4	164	C Eb F# A	dim7 chord	4-28	[004002]
5	284	C Db Eb F# A	dim7 (+1)	5-31	[114112]
6	849	C D E F# Ab Bb	WholeTone Scale	6-35	[060603]
7	1101	C Db D E F# Ab Bb	WholeTone (+1)	7-33	[262623]
8	1636	C Db Eb E F# G A Bb	Auxiliary Diminished	8-28	[448444]
9	1879	C Db D E F F# Ab A Bb	3 x Augmented triad	9-12	[666963]
10	1997	C Db D Eb E F# G Ab A Bb	Chromatic scale minus tritone	-	-
11	2037	C Db D Eb E F F# G Ab A Bb	Thus, all 11 note scales	-	-
12	2048	C Db D Eb E F F# G Ab A Bb B	Chromatic scale	-	-

Table 2: Scales that are FASTEST at achieving Chromatic Saturation under Random Transposition

The scales given in both of these tables are Prime Forms, thus they also represent (a) their own inversions and (b) all possible modes. Thus, for example, 3(a) C Db Eb in the Slowest Table above represents:

#/2048	scale
14	C Db Eb
21	C Db Bb
23	C D Eb
31	C D B
65	C A Bb
66	C A B

Thus, for each of the tables, there are a number of Prime Forms, and a related number of actual scales (i.e. from the 2048 possible):

	Number of Prime Forms	Number of Scales (of the 2048)
SLOWEST	31	326
FASTEST	12	44

From this simple table it can be seen that the Fastest scales are considerably rarer than their Slowest counterparts. One reason for this is the inherent symmetry found in the Fastest scales, and the equivalent asymmetry in the Slowest scales.

The data from Forte (1973) is given to show that there is a strong correlation between the structure of the Interval Vector of a given scale and its relative speed at Chromatic Saturation under random transposition. Generally speaking, the smoother the Interval Vector (e.g. 4-Z29 [111111]), then the *slower* the scale will achieve Chromatic Saturation. By contrast, the more contoured the Interval Vector (e.g. 4-28 [004002]), then the *faster* the scale will achieve Chromatic Saturation.

Further, generally speaking, a scale of a given length in both tables will have its complement as the result in the corresponding section of the table. That is, for example, in Table 2 we find that 3-12 (C E Ab) and its complement 9-12 (C Db D E F F# Ab A Bb) both appear. However, whilst this pattern holds true for most of the data in these tables, note the results for 2-note and 10-note; 5-note and 7-note scales in Table 1. Further, 5-31 and 7-33 in Table 2 are clearly not complements of each other.

Further, let us note that although all scales of 6 notes and above in the Fastest table achieve Chromatic Saturation by a single transposition to the bII, this is only true of Prime Forms 9(d), 9(e), 10(b), etc. of the scales from the Slowest table.

Commonly-used Scales

We have seen that the 7-note scale which is fastest at achieving Chromatic Saturation under random Transposition is #1101 C Db D E F# Ab Bb (= Whole Tone (+1) 7-33

[262623]). Immediately after this in terms of speed are the Major scale (and its modes) and the (tritone-related) chromatic list C Db D Eb E F F#.

The hierarchy of commonly-used 7-notes scales is as follows (these represent the scale named and their modes):

FASTEST

Major

Jazz melodic

Harmonic minor

SLOWEST

Note also the absence of the so-called Augmented (and Auxiliary Augmented) scales from Table 2 (FASTEST). We might see this scale as being formed by the joining of two chromatically spaced Augmented triads. Despite their being mentioned (and even encouraged) in various theory teaching texts (for example Coker et al. (1970) pp. 134-138; Haerle (1980) pp. 34-36), they are rare in jazz playing. Although “symmetrical” in structure (repeating minor 2nd and minor 3rd interval patterns), they are not as fast as the Whole Tone scale, with its larger order of symmetry. All of the other commonly called “symmetrical” scales mentioned by these authors – and commonly used by jazz musicians – occur in Table 2 above (Whole Tone, Diminished and Auxiliary Diminished, Chromatic).

Appendix 5: A Small Omission in Slonimsky (1947)

On pages 177-178 of his “Thesaurus of Scales and Musical Patterns”, Slonimsky gives his “Division of Twelve Tones into Four Mutually Exclusive Triads”. As we noted above, Demsey (1991) attempts to show that Coltrane used the {Cm, Dm, E, F#} triad set in the construction of the melody of “Miles’ Mode”, a postulation with which I have disagreed. However, in examining this possibility, I have discovered an omission in Slonimsky’s book. The “Division of Twelve Tones into Four Mutually Exclusive Triads” section consists of 21 bars of four triads per system, with each set of four triads covering (without repetition) the notes of the chromatic scale (we might note that the set chosen by Demsey is in fact the very first):

“Two Major and Two Minor Triads”

1. Cm Dm E F#
2. Cm D E Bbm
3. C D G#m Bbm
4. C F#m G#m Bb
5. C Dm F# G#m
6. Cm E F#m Bb

“Two Augmented, One Minor, One Minor Triad”

7. C Dm Eb+ F#
8. Cm Db+ D+ E
9. C+ Db+ Eb Bm
10. C Db+ D+ G#m

“Augmented, Major, Minor, Diminished Triads”

- 11. Eb F#m Bo C+
- 12. C Db+ Ebm G#o
- 13. Cm D+ Fo A
- 14. Co Db+ E Gm

“Two Diminished, One Major, One Minor Triads”

- 15. Co Fo Gm A
- 16. Co Dm E Go
- 17. Cm D Fo Bbo
- 18. C Ebo Abo Bbm

“Four Augmented Triads”

- 19. C+ Db+ D+ Eb+
- 20. C+ D+ Eb+ F+
- 21. C+ Eb+ F#+ A+

These sets of chords (progressions) tend to be grouped in modal sets, and thus these can be reduced to a core progression (i.e. by choosing the most packed to the left of the modes). Similarly, the results for progressions 19 to 21, due to their reliance upon augmented triads alone, are actually all inversionally equivalent, and thus redundant.

Progressions 1 to 6 can be subdivided into two modally-equivalent groups: (a) progressions 1 to 4 and (b) 5 to 6, thus:

(a):

1. Cm Dm E F#
2. Cm D E Bbm
3. C D G#m Bbm
4. C F#m G#m Bb

(b):

5. C Dm F# G#m
6. Cm E F#m Bb

Thus, by reduction, we can see that Slonimsky gives 6 distinct possible progressions, even though he groups them by title as a set of five. Searching for these data independently, I have found a seventh type (my search was undertaken with my PC and Microsoft Excel - presumably Slonimsky only had paper and pencil at his disposal). This seventh type is constructed from three diminished and one augmented triad, thus:

“Three Diminished, One Augmented Triad”

Co Db+ Eo Abo

Therefore, there are actually 7 “types” of progression. Let’s order them by conventional triad ordering; i.e. first major, then minor, then augmented, then diminished. In this list, the new type appears as number 6:

Type x:

1	C	Dm	F#	Abm
2	Cm	Db+	D+	E
3	Cm	Dm	E	F#
4	C+	Db+	D+	Eb+
5	Co	Db+	E	Gm
6	Co	Db+	Eo	Abo
7	Co	Dm	E	Go

Similar results may be obtained if we include the sus4 chord as a kind of “honorary” common triad (as permitted by some theorists¹; Slonimsky does not include this sus4 triad), thus:

Type x (sus4):

1	C	Dm	Ebsus4	F#sus4
2	Cm	Db	D+	Esus4
3	Cm	Do	Esus4	F#
4	Co	Db	Esus4	Gm
5	Co	Do	Esus4	Go
6	Csus4	D+	Esus4	Absus4
7	Csus4	Ebsus4	F#sus4	Asus4

Interestingly, again there are seven core progressions.

¹ See, for example, Haerle (1980), p. 7

Appendix 6: Steve Coleman's "Symmetrical Movement Concept"

Texts

Found at: <http://cnmat.cnmat.berkeley.edu/~mbase/Symmetry_Movement.html>

Notes: I have recreated the diagrams and notation from the website as closely as possible; however, I wish to point out that the boxes around notes and pairs of notes on Examples 3, 4 and 5 are ovoid in the original scores.

BALANCE

We live in a world of immense beauty. There are a multitude of forms with countless variations on simple themes. I want to speak here about balance and make some comments about how balance can be achieved musically. There are countless ways that architectural balance can be musically achieved from the micro to macro level. Since attention to detail has always been an important factor for me, and these things are not usually discussed, I would like to initiate some dialog on this subject. The most obvious kinds of balance that come to mind are the various forms of symmetry (i.e. bilateral, etc.) that can be applied musically, using intuitive and logical methods. Symmetry is a fact of nature and one of the oldest fascinations of humanity. Some of the more obvious ways in which symmetrical musical balance could be realized are through melody, rhythm, tonality, form, harmony and instrumentation. As well as the structural considerations of symmetrical musical forms I will also discuss these structures from a dynamic point of view, i.e. as they progress through time.

MELODIC MATERIAL GENERATED BY SYMMETRICALLY DERIVED LAWS OF MOTION

-by Steve Coleman-

This theory was originally a melodic theory. I named it Symmetry because the motion of the melodies involves an expansion and contraction of tones around an axis tone or axis tones (i.e. around a center point). The expansion and contraction involved is almost always equal on both sides of the axis, hence the term Symmetry. This is basically a melodic system that obeys it's own laws of motion. It moves according to the gravity of this motion more than anything else but it can also be adapted to deal with the gravity of other types of tonality such as cells or the traditional dominant-tonic harmonic system. When I first started dealing with symmetry I only dealt with the laws of motion produced by the system without any regard for other types of tonality. This is how I would suggest others to learn the system to get a feel for thinking in these terms. Also a complete knowledge of intervals and their relationships (always thinking in terms of semi-tones) would be extremely helpful.

I began by writing symmetrical exercises for myself. Then I practiced these exercises to get my fingers and ears used to moving and hearing these ideas. It was only after doing this that I practiced improvising within these structures playing at first in an open manner (not based on any outside structure such as a song). Later I adapted these improvisations to structures and forms. I did this by slowly integrating the ideas with the more traditional improvisational style I was already playing. My goal was not to play in a totally symmetrical style (as this would be as boring as playing all major scales) but to integrate the style and give myself more options when I improvise.

The basic system involves what I call two spirals. They are tones that move out equally in half steps from an axis (which is always at least two tones).

If the axis of the first spiral is the two tone C-C unison (one octave above middle C, it could be in any octave) then from that unison C, you move out (spiral out) each tone in a different direction in half steps, i.e. C-C, then B on the bottom and C sharp on the top: B flat on bottom and D on top; A on bottom and D sharp on top; A flat on bottom and E on top; G on bottom and F on top; G flat on bottom and F sharp on top (at this point you are at the beginning of the spiral again, or the symmetrical mirror image of the spiral): F on bottom and G on top; E on bottom and G sharp on top; E flat on bottom, A on top; D on bottom and A sharp on top; D flat on bottom and B on top; C on bottom and C on top (this is your starting point one octave above and one octave below your original tones). You're thinking two tones at a time and they're spiraling out together. This I call spiral number one.

As you spiral out from C-C (the axis) and you think of the interval between the tones then C to C is a unison. The next tones in the spiral are B and C sharp, the interval between B and C sharp is a major second. Next in the spiral is B flat on bottom and D on top, that's a major third. Then A on bottom and D sharp on top, that's an augmented fourth or a tritone. Continuing, A flat on bottom and E on top, that's an augmented fifth (could also be thought of as a minor sixth). Then G on bottom and F on top is a minor seventh. The next tones are G flat on bottom and F sharp on top, these tones are an octave apart and are really the same as the beginning of the spiral. All symmetry has two axis and in this system they are always a tritone apart from each other, more on this later. As you keep spiraling out until you reach the two C's two octaves apart the important thing about the spiral is not the tones themselves, but the intervals between each of the tones as you spiral out each half step. It is these resulting intervals that are formed in spiral number one (Unison, Maj 2nd, Maj 3rd, Tritone, min 6th and min 7th) that I call Symmetrical Intervals (see example 1). This is important to remember as it forms the foundation for the laws of melodic motion. Note that beginning with the tones

intervals formed in spiral number one, as a matter of fact they are all the intervals missing from spiral number one. I call the intervals in spiral number two (min 2nd. min 3rd, Perfect 4th. Perfect 5th, Maj 6th and Maj 7th) Non-Symmetrical Intervals (see example 2).



EXAMPLE 2 [Spiral # 2]

The basis of the laws of movement are as follows. Thinking monophonically if you have an initial tone which you mentally consider to be the axis, when you move in one direction from this axis then generally you must move the same distance in the opposite direction from that same axis. For example, you play a C, then the next tone you play is a D above the C . which is a major second away from C, then the following tone you must play is the tone a major second below C, which would be B flat. In other words, for the same distance that you moved above C you must play the tone that is that same distance below C (see example 3 - measure 1 - beats 1 and 2, in these examples the axis are circled). Actually it doesn't matter whether the D is above or below the C as long as you remember which direction you moved to get to the first tone, so that you move in the opposite direction (the same distance) to reach the second tone (see example 3 - measure 1 - beats 3 and 4). Obviously, you must know your intervals very well to think like this quickly in an improvisational context. For another example if you played G and consider the G as an axis and then play B flat as your next tone, then the following tone you play must be E. With G being the axis, B flat is a minor third above G and E is a minor third below G (see example 3 - measure 2 - beats 1 and 2). Now you do not have

to play the E a minor third below G, you could play the E or the B flat in any octave. But you need to be *thinking* in terms of G being the axis and the other tones 'surrounding' the axis (see example 3 - measure 2 - beats 3 and 4).

One of the exceptions to this rule is when the interval that you play is one of the Symmetrical Intervals in spiral number one, those intervals being a major second, major third, tritone, minor sixth, minor seventh, octave etc. Then you don't have to make the equal movement in the opposite direction. You can chose to, but you don't have to. For example if you play the tone C as the axis, and the you play the tone D, you don't necessarily have to play B flat after that. You could pick any tone at that point. But if you play one of the Non-Symmetrical Intervals in spiral number two, for example, axis C to the tone F (an interval of a perfect fourth), then you must play a G after that. according to these laws of movement (actually it would be more accurate to say that after playing the tone F you must then 'complete' the symmetrical motion, more on this below). In this example the tone G can be in any octave, but you need to play a G because F is a perfect fourth above C and G is the perfect fourth below C (see example 3 - measure 3).



EXAMPLE 3

There are many variations to the above laws of movement that are still considered symmetrical movements according to this theory. For example, instead of moving away from an axis you can do the reverse and move towards an axis. You can play the tone F,

then the tone G, and then play the axis tone C (see example 3 - measure 4 - beats 1 and 2). Or you could think of the axis as two tones further apart than a unison or a minor 2nd (actually, to be technically correct, even when it seems like there is only a one tone center all axis are two tones as in the initial axis of spirals one and two, and they are all either a unison or a minor second as in examples 1 and 2). So you could initially play C and E flat. Now, logically, you might think that the next tone you have to play is A, because if C is the axis and if the E flat is a minor third above C then you would think you have to play an A because A is a minor third below the axis tone C. But you can play C and E flat and then play B and E natural! This is because you can think of the C and E flat together as an axis. Then you can expand out a half step on either side of this axis and play B and E natural (the true axis in this situation are the tones D flat/D or G/A flat). This movement would still be within the rules because mentally you're using C and E flat together as an axis (see example 3 - measure 4 - beats 3 and 4). So there are many variations depending on what you mentally think of as the axis. Some sample symmetrical movements are listed in example 4, try and follow the logic of the movements. Example 5 is a symmetrical melody which connects different movement logics in one idea, demonstrating how the movement laws can flow together. The circles will give you some hints on where to look for the axis. This is a more complex example, notice the 'nested' axial movements ('nested' meaning some tones of one axis overlap and share tones with adjacent axis). It is possible to generate shapes that contain an entire chain of nested axial movements, however the resulting melodies would be extremely jagged and not necessarily sound musical, unless of course that is the desired effect.



EXAMPLE 4



EXAMPLE 5

All of the examples above are written in atonal space. The analysis of the axial progression can be thought of in any number of different ways. In other words it is possible to analyze these same passages differently and still be well within the given laws. Notice that the above examples require thinking in small cells of ideas, at least initially. However there is a linear gravity involved in the thinking which requires that the improviser become fluent in thinking in two directions simultaneously. It should be clear from Example 5 that it helps to be able to think at least two or three tones

backward and forward in time. This is a different skill than the normal way of thinking as retention of individual tones, as well as phrases, needs to be practiced. For instance, in Example 5 at the end of measure 2, the tones F and E are the axis, not only of the tones immediately following (A and C) but also of the preceding tones (Db and Ab)! Overall this produces a sort of accordion effect in time. Not that this will be heard by the average person, especially given the speed of execution, but it will be felt and it does have an effect.

After this what has to be obtained is fluency in progressing from one idea to the next in seamless motion, building up to higher levels of complexity in the communication of ideas. This is similar to the progression from words, to sentences, to grammar, and finally to communication of conceptual ideas in linguistic expressions. Also, with some imagination, the same ideas could be merged with any other logic. It is not my goal here to simply write down ideas for others. I simply want to demonstrate that there are many possibilities to be explored, definitely more than have already been explored. I have been working with the ideas above for at least 22 years now and I still have not found any end in sight!

HARMONIC MATERIAL GENERATED IN SYMMETRICAL SPACE

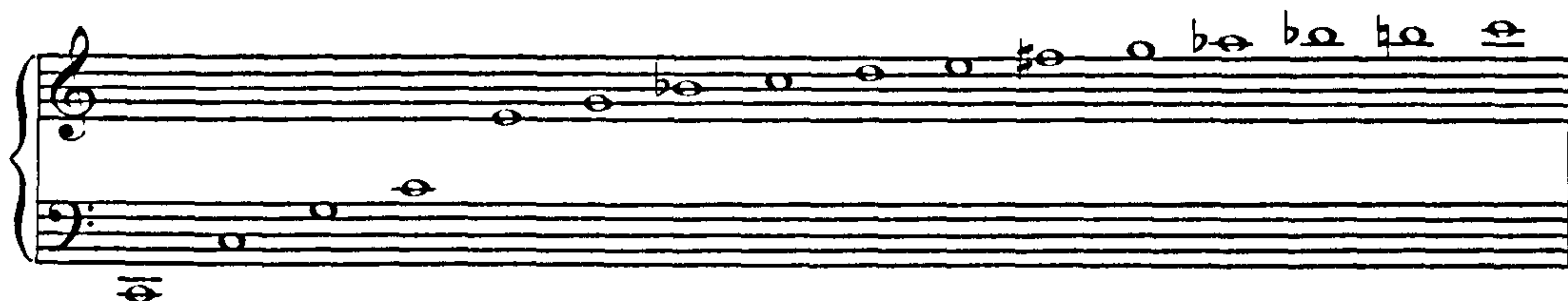
Over the years I have been exploring several ideas which could be expansions of the symmetrical laws of motion mentioned above. Most of these ideas are based on the various concepts of 'gravity' and what can be generally called 'binding' and 'unbinding' (i.e. different types of laws of attraction). The melodic concept discussed above and other related harmonic concepts all deal with tonal centers in terms of spatial geometry, as opposed to the standard tonality which deals in tonal key centers in terms of tonics. These different approaches can be looked at as different types of 'gravity'. Here we could

borrow two terms coined by music theorist Ernst Levy, calling the concept of gravity that results in the traditional tonic-based tonality **Telluric Gravity** or **Telluric Adaptation** and the concept of gravity that is at the basis of centers of 'geometric space' **Absolute Conception**. In Telluric Adaptation our perception of gravity is based on laws of attraction that are influenced by our sense of 'up' and 'down'. Thus we tend to look at the harmonic series only from the 'bottom-up' perspective, with the 'fundamental' on the bottom. This is a 'terrestrial' mode of thinking influenced by the fact that we live on Earth and tend to localize our concept of space according to our everyday situation. In Absolute Conception what is important is the position of the tones in space and their distance. Here the harmonic series is seen as 'spiraling' out from a 'generator' (as opposed to a tonic or fundamental) so as to produce both an 'Overtone' and 'Undertone' series (see example 6-a)! Absolute Conception is based on a 'universal' mode of thinking that results when you look at the Earth, other planets, satellites and stars from the point of view of how they relate to each other in space. So the difference is the way the gravity operates from a 'terrestrial' or 'telluric' perspective (on Earth we tend to think of the gravitation pull in one direction, 'down') and how gravity operates from a 'universal' or 'absolute' perspective (in space we tend to think of objects orbiting around a gravity source or being pulled towards the source from a multidirectional perspective). In the absolute conception partials are thought of as 'orbiting' around a generator tone producing both overtone and undertone energy. These two different concepts of gravity, telluric and absolute, will be explored in more detail below.

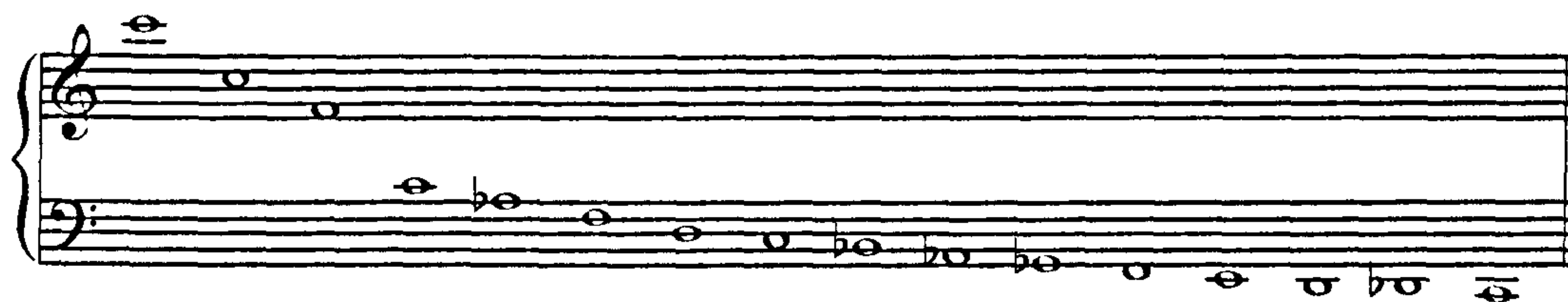
EXAMPLE 6-a

Absolute conception of the Harmonic Series based on the generator tone C

Overtone Series



Undertone Series



I just want to say a few things about the information series above. The tones shown are the closest equal temperament equivalents to the actual tones that are in the series. In some books the 11th partial is listed as F natural instead of F sharp. However I believe that this is technically wrong as the ratio 11:8 is closer to F# by a very small amount (if C is generator). There are 100 cents to an equal temperament half step (for example between F and F# there are 100 cents). The ratio 11:8 (which is 11:1 sounded in the same octave as the generator) is 551 cents above the generator. An equal temperament perfect 4th is 500 cents above the generator and an equal temperament augmented 4th is 600 cents above the generator. Since 550 cents would be an exact quarter tone between a perfect 4th and an augmented 4th then 11:8 is closer to F# as the distance between 11:8 and F natural is 51 cents and the distance between 11:8 and F# is only 49 cents. The tone is actually closer to F# and books that list F natural as being the 11th partial in a

harmonic series with C as generator are not correct. Of course this means that the corresponding tone, the 11th partial in the undertone, series is Gb. However since most books do not deal with the undertone series we don't have to worry about that.

Also the 13th partial is listed as being A natural instead of A flat. Again this is technically wrong as the ratio 13:8 is definitely closer to Ab (again if C is generator). The ratio 13:8 is 841 cents above the generator (13:8 is 13:1 octave reduced). An equal temperament minor 6th is 800 cents above the generator and an equal temperament major 6th is 900 cents above the generator. So the ratio 13:8 is closer to being Ab because the distance between 13:8 and Ab is 41 cents and the distance between 13:8 and A natural is 59 cents. The tone is closer to Ab and books that list A natural as being the 13th partial in a harmonic series with C as generator are not correct. This also means that the corresponding tone, the 13th partial, in the undertone series is E natural.

In terms of the nomenclature that can be used to express the actual tones which act as axis (melodic) or generators (harmonic) in the Absolute Conception I propose using **Sum Notation** as the main terminology. So when I speak of improvising with regard to a 'sum 11 tonal center' I am speaking of an Absolute tonality that has an axis (or spatial center) of sum 11. Sum 11 means that the tones B-C (also F-G flat) are the spatial tonal centers of this section of the composition. For the improviser this means improvising with this spatial tonality in mind.

One necessary skill required for this mode of thinking would be to learn how to hear spatially with the mind as well as with the ears (actually it is all in the mind). In other words learning to construct mental images of the 'geometric space' and to be able to 'hear' inside of that space.

The reason for using the term 'sum' comes from the concept of adding note numbers. If tones C through B chromatically are represented by the numbers 0 through 11 respectively, then it is possible to 'add' tones together to arrive at their sums. The sums represent the axis (or center point) between the two tones being added together. For a 'sum 8' axis, any two tones that add up to the number 8 would be considered a sum 8 interval. For example D sharp and F (3+5) would add up to 8. The center (or axis) of D sharp and F is E and E (which is also sum 8 or 4+4, an axis always implies at least four tones. in this case the E-E unison represents two tones but if thought of from another perspective B flat and B flat is also the axis, i.e. 10+10=20 minus 12 = 8). The same goes for C sharp and G (also sum 8). So the axis of a sum 11 interval would be B and C (i.e. 11 + 0 = 11. Since we are dealing with 12 tones the entire tonal system then . for the purposed of octave reduction, you can continually subtract the number 12 from any sum that is 12 or greater until the sum is below 12). Notice that all of the even sums are the result of any interval in spiral number 1 above and all of the odd sums are the result of intervals from spiral number two. Example 6-b is a table that is a summary of the relationships between spirals, axis and sums:

EXAMPLE 6-b

SPIRALS NUMBER ONE & TWO

AXIS TONES	SUM #	AXIS TONES	SUM #
C-C	SUM 0	C-Db	SUM 1
C#-C#	SUM 2	C#-D	SUM 3
D-D	SUM 4	D-Eb	SUM 5
Eb-Eb	SUM 6	Eb-E	SUM 7
E-E	SUM 8	E-F	SUM 9
F-F	SUM 10	F-F#	SUM 11
F#-F#	SUM 0	F#-G	SUM 1
G-G	SUM 2	G-Ab	SUM 3
Ab-Ab	SUM 4	Ab-A	SUM 5
A-A	SUM 6	A-Bb	SUM 7
Bb-Bb	SUM 8	Bb-B	SUM 9
B-B	SUM 10	B-C	SUM 11

This may initially be a little confusing but many things that are unfamiliar are confusing at first. With a little work it can be as natural as any other internalized system.

Appendix 7: Analysis of Solo Material in “Cross-Fade”

Introduction (bars 1–14)

Note the use of the instructions “fast rate of change” in the alto part and “slow rate of change” in the bass part. If we assume that this refers to rhythmic density, then we find that this marking is only obeyed in bar 2; beyond this point, the Alto and Bass parts have a similar frequency of notes. This section is based upon F# Blues, although, as we shall see, Steve Coleman uses the “Symmetry” system extensively in his improvisations. By contrast, the Bass part uses little (if any) of the “Symmetry” system to generate material. Thus, the “rate of change” instructions may (like the “fine” mark in the Head) have been added to the score by way of explanation of this difference *after* the recording had been made. I will describe this Introduction one instrument at a time: first Alto, and then Bass.

Alto in Introduction (bars 1–14)

The opening phrase played by Steve Coleman is quite complex. The section marked *a* consists of a {B. C. Db} set with C as the axis. Let us note that this is not performed as a basic -1, 0, +1 list, but, rather, Steve Coleman freely selects from this set to create a (symmetrical) +1 {0} +1 -1 {0} -1 pattern. This shows fluency with such a basic pattern beyond what we have seen in the Examples from Steve Coleman’s website (analysed above): however, this parallels (by tritone) the use of the {F. F# G} set used by the bass in the head. Further, the section marked *b*, although simply a chromatic section leading to the Cdim7 (marked *c*) can also be seen as a product of the “Symmetry” system. That is, the A# acts as an axis for the notes chromatically above and below it. Whilst this may

seem an enthusiastic interpretation, if we allow this as a possibility then the entire first phrase may be seen to be generated by means of the "Symmetry" system. The Cdim7 downwards arpeggio (marked *c*) completes this phrase, and contains the only note so far not derived from the scale of F# Blues: namely, the D# at beat 6 (marked *). This arpeggio exhibits strong symmetrical structure and is performed with such fluency (at a semiquaver level) that it seems to be premeditated material. Further, this is the first of several appearances of dim7 arpeggios in Steve Coleman's solos. I have annotated this Cdim7 arpeggio with a (heard) paired axis of {F#, A}, derived from a theoretical axis of {G, Ab} (or, a tritone away {Db, D}). However, let us note that a dim7 pattern may be seen to have four such axes. Here, then, I am assuming that the first notes have an importance re the analysis of the section. Let us note that, whilst phrase *d* is a variation of *a*, there is little thematic variation after this. Indeed, Steve Coleman seems to use this relatively conventional melodic relationship just to "get the solo going", so to speak: examination of later events has revealed no other use of such structural formulae. The pattern marked *e* is a common phrase formed by a downward set of notes from F# minor pentatonic (or Blues scale). However, let us note that it (a) is symmetrical, (b) shares the same axes as the Cdim7 arpeggio at *c*, above (i.e. F# and A), and, like *c*, it completes a legato melodic phrase. Thus, like *c* above, it too seems premeditated to some extent. Further, note that I have annotated this set as a "{0, 2, 5, 7}b" set. We will see these sets used a great deal as a motif in Steve Coleman's solos as the piece progresses, and I shall explain and examine the different types ({0, 2, 5, 7}a to e) later in this chapter. The phrase marked *f* is based around an A axis, and extends from F# (-3) to C (+3), thus:

F#	G	A	B	C
-3	-1	{0}	+1	+3

Table App.7-1 Structure of pattern *f*

Note especially the sections marked *g* and *h*, which further reveal this symmetry: these are phrases that, respectively, ascend and descend from the A axis to the upper and lower limits of the set. Further, the G note immediately before section *h*. is, after all, a slurred note, and, thus, we might read *g* and *h* as being directly adjacent to each other. The result of this use of the “Symmetry” system is that *f* contains many Gs (marked with *s): a note outside F# Blues. This note G is similar to the D# found in *c*, above, in that it is a solitary “out” note in a pattern otherwise from within F# Blues. There is also a level of melodic structure to this pattern that focuses upon the A and G notes, at the expense of, say the Bs (equidistant from the A as the G). Here is that part of the notation, with brackets to show the simple theme created by the repetition of the note A, which is then transposed down a tone to G (the G in brackets in bar 4 is a grace note).



Fig App.7-1 Bars 3 to 5 of Alto Intro Solo

From the following score (Fig. App.7-2), which compares the two phrases by placing one above the other, it can be seen that the rhythm is not identical, but it is certainly sufficiently close to create a simple theme and variation idea in the listener's mind. This also gives even greater importance in the listener's mind to the "out" note, G.

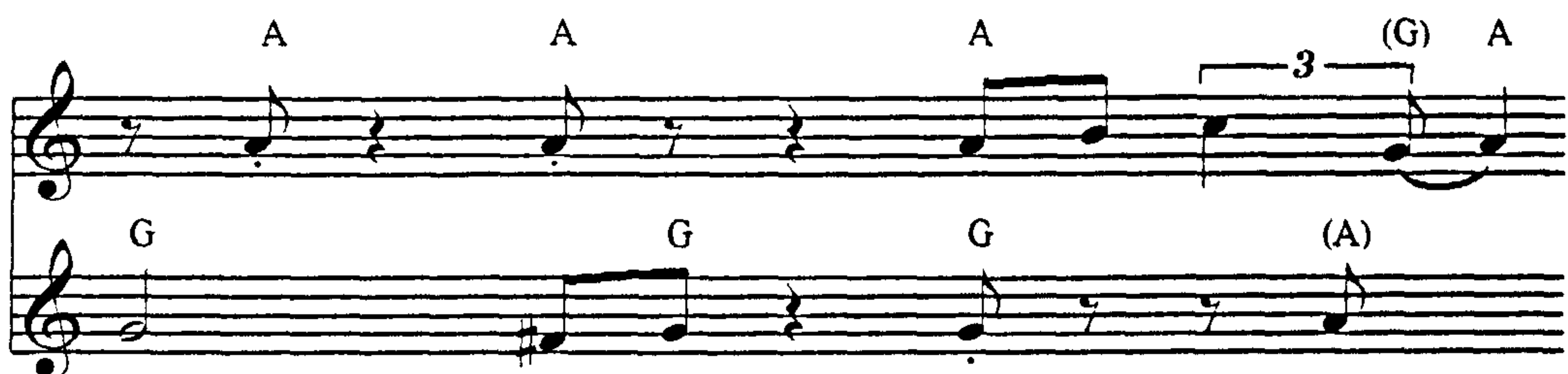


Fig App.7-2 Comparison of Two Phrases from bars 3 to 5 of Alto Intro Solo

Pattern *i* is an F#m7 arpeggio which transforms (overlaps with) pattern *j* which generates the non-tonal note of D (marked *). This D note then has a further role in pattern *k*, which has a {B, C} axis, this pair being rhythmically separated from the surrounding music. Pattern *l* shows the core symmetrical set here. However, let us note that in *l* none of the {+1 and -1} annotations are directly adjacent to each other. Further, let us note that this entire melodic phrase may be subdivided into several {0, 2, 5, 7}_b sets: three within *l*, and a fourth at *q*. These two characteristics suggest to that the use of {0, 2, 5, 7} sets here is motivic, or riff based, and that Steve Coleman is using their inherent structure to continue in a "Symmetry" system way without calculating new sets "on the fly". Note the sets of *m*, *n* and *o*. These all form sus4 triads in simple ascending or descending order. Whilst they are a natural consequence of the performance of {0, 2, 5, 7} sets in non-simple order. let us recall the iconic importance of the {C, F, G} set in the examples from Steve Coleman's website, described above. The repeat of the notes E and F# in the section marked *p* seems to be a confirmation that these two notes are the paired axis of this set.

However, as noted above, the second occurrence of this pair (at beat 4) also form part of the {0, 2, 5, 7} set, annotated *q*. Note that at *i*, which was a previous use of an F#m7 arpeggio, the paired axis was {A, C#}. Pattern *r* is a possible analysis of the following few notes, but I suggest that the B and C# found here really belong to the pattern annotated *s*. This analysis leads me to the conclusion that Steve Coleman is using the F# minor pentatonic scale as a core list here, with a special emphasis upon {0, 2, 5, 7} sets derived from this minor pentatonic scale. Let us note at this point that the minor pentatonic is in itself derivable from the "Symmetry" system, with an axis of B, thus:

-5	-2	{0}	+2	+5
F#	A	B	C#	E

Table App.7-2 Structure of F# minor pentatonic (axis of B)

However, the Blues scale has no such global symmetry (although Steve Coleman is clearly interested in the {B, C, C#} set, since it is located centrally in the F# Blues scale). Now, let us note the use of the note C to form F# Blues sets at beat 6 of bar 5 and beat 8 of bar 6. We can see that C occurs at the start and very near the end of this section based on F# minor pentatonic and {0, 2, 5, 7} sets. Thus, the note C acts as a kind of boundary, or punctuation to the melodic material. We will see more of this in Steve Coleman's 1st Solo.

Bar 7 opens with an extremely clear, even iconic, use of the "Symmetry" system. An F# axis generates an Ebdim triad pattern (marked *t*). Note the clarity given to this pattern by the repeat of (a) the axis tone and (b) the rhythmic structure. Further emphasis is drawn to this pattern by its starting with the (alien) D# note (marked *). This contrasts sharply with pattern *c*, where the note D# seems to occur more directly as a consequence of the

use of the “Symmetry” system. Further, let us recall the Ebdim triad found at beat 9 of bar 18 (annotated as Box G). In both cases Eb is the low note, A the high note, and there are two F#s. The speed of the F# minor pentatonic section that follows, like pattern *c* described above, increases the likelihood of premeditation. Pattern *u* reveals another sus4 arpeggio, and, at *v*, we can see another simple +2 {0} -2 annotation (as at *r*, above). Further, pattern *w* is a {0, 2, 5, 7} a set in ascending order. However, we might expect this kind of detail in such an ascending F# minor pentatonic list. This entire section seems to act as (a rare) tonic scale compensation for the “outness” of the Ebdim arpeggio heard just before, this sense emphasised by the F# at the end and top of this phrase. The semiquaver rest at the centre of *w* parallels the crotchet rest found in *t*. Pattern *x* continues Steve Coleman’s use of the {B, C, C#} set in a free way; here we can see neither the order -1 {0} +1, nor its retrograde, +1 {0} -1. Patterns *y*, *z* and *aa* reveal not only use of the {0, 2, 5, 7} b set, but also, in the cases of *y* and *z*, place emphasis upon the melodic forms of this set that reveal the notes E, A and B as a {0, 5, 7} set (annotated as -5 +5 {0}, etc.). The sustained A note suggests the start of pattern *bb*, another F#m7 arpeggio. Here the axis seems to be {E, F#}, as at *p*. At *cc* and *dd* we see an overlapping of the {A, B, C#} and {B, C, C#} symmetrical sets. In retrospect, this has been previously seen at *r* and *s* in bar 6. Pattern *ee* shows a chromatic extension of the F#dim {F#, A, C} set thus:

-4	-3	{0}	+3	+4
F	F#	A	C	C#

Table App.7-3 Structure of pattern *ee*

However, this analysis requires us to ignore the B note (marked *). On the other hand, annotation *ff* provides an explanation for the B – but here we must ignore the A note (similarly marked *)! Thus, here we see no straightforward single structure derived from

the "Symmetry" system. In this light, the F# at the end of bar 10 might be seen as a conventional tonic resolution to an irregular melody. However, the presence of the simple *gg* set suggests a further analysis. Let us note that all of bars 10 and 11 consists of notes from F# Blues *except for* an F (at beat 5 bar 10) and a G (at beat 5 bar 11). I think that these two notes act as a symmetrical pair around the "axis" not of a single note F#, but, rather, around *the entire tonality* of F# Blues (annotated *hh*). Note the ascending list of the F# Blues scale which omits only the note E. Just like the compensatory ascending F# minor pentatonic list in bar 7, the phrase in bar 10 also ends on the tonic note. The idea that the entire F# Blues scale is acting as the axis is further suggested by the fact that the F is below all of the other notes in this section and the G is above all of the other notes in this section. Further, the lack of simple explanation provided by *ee* and *ff* is matched by the (illegal) move to G in *ii* (marked +6!). Let us recall that the set of {F, F#, G} is the motif central to the bass line in the head. Here, I suggest, Steve Coleman is using this motif in a prolongational manner, this explanation exonerating the "illegal" move. If we allow this analysis, then the F# at the end of bar 10 takes on a special significance as a kind of pivot note. The phrase *ee* that precedes it is all below this F#, and the music in bar 11 is all above it. Further, these two sections are very similar both in duration and note density. Also, the F in bar 10 is a minor 9th below, and the G in bar 11 can be found exactly the same interval above.

The emphasis put upon the F# Blues scale (and a corresponding de-emphasis of the "Symmetry" system per se) continues beyond the G note at beat 5 of bar 11. Although *jj* and *kk* are both legal constructions under the "Symmetry" system, they show less integrity than those already seen in this Introduction. Further, the first four notes of bar 12 are a conventional F# Blues melody and have no "Symmetry" system structure. Pattern // shows a possible {0, 2, 5, 7} b set, although this one feels less distinct than those found

previously. The contour of *mm* separates it from the surrounding notes, the A that precedes it acting as an anacrusis. The pattern *nn* seems to perform the role of catalyst, and then Steve Coleman uses a C# to complete the F# minor pentatonic list, inserting a repeated E to create rhythmic balance and reduce the interval from the A down to the C#. Note the two similar $-5 +5 \{0\}$ annotations shown in this bar: both use the notes B, A and E in descending order. Pattern *oo* sees Steve Coleman use a (legal) Symmetrical Interval structure: there is no symmetrical partner for this event (unlike the “illegal” pattern *ii* above). Rather, the simple I and bIII of F# minor are heard at the start of bar 13. This is the first section of the Intro Solo that evades a simple “Symmetry” system analysis. In retrospect, I suggest that this is because Steve Coleman is “treading water”, concentrating instead upon the preparation of the line with which he completes his solo. The annotation *pp* shows a rare example of the tonic F# with the equidistant pair of B and C#, forming a $\{0, 5, 7\}$ where 0 = the tonic note. However, its rhythmic structure makes this a less than certain analysis. More likely is pattern *qq*, where we see a further use of the $\{B, C, C\# \}$ set. The rests that surround it further pronounce its internal structure, yet it is a retrograde version of *kk*, where, I have argued, Steve Coleman is thinking more of F# Blues as his material, rather than the “Symmetry” system per se. At *rr* we see a pattern reminiscent of that found at *t* earlier. Here, as at *t*, F# is the axis (and, thus, the final motif in this Intro Solo ends with material based around the tonic axis). However, the other notes are a tone above and below, not the minor 3rd found at *t*. Put another way, whereas *t* is a $\{0, 3, 9\}$ set, *rr* is a $\{0, 2, 10\}$ set. Further, *rr* ends on the note G#, outside the tonic scale of F# Blues. Although this note G# is the II (=9th) of the key, and the preceding E functions as a conventional bVII, it really sits out in the recording. Looking back from this final, iconic pattern *rr*, we might say that bar 12 and the start of bar 13 are relatively disorganised. Although they show use of the “Symmetry” system, they are less refined than much of what has been heard previously. I suggest that the (a) the unusual event at *oo*, (b) the

simplicity of approach taken at the start of bar 12 and (c) the relative lack of refinement is precisely due to Steve Coleman's desire to place this final "Symmetry" system flourish at precisely the end of the bar.

An important point regarding the choice of axes in this Intro Solo is that, in nearly every case, they are sourced in F# Blues, the "home" scale of the Intro Solo. In these cases, then, the "outside" material is wholly derived from the "inside" material. Further, there is extensive use of symmetrical structures inherent to the Blues scale: for example, the emphasis upon the {B, C, C#} set at the centre of the F# Blues scale. This approach is maintained throughout the piece. Thus, Steve Coleman is not using his "Symmetry" system to generate strictly atonal material. Rather, the melodies, although often using notes outside of the key of F# Blues, are essentially based within that tonality.

Finally, I will make two other general points. Firstly, let us note that we have used (a) conventional jazz scale and chord nomenclature, (b) atonal analysis techniques and (c) "Symmetry" analysis techniques in order to understand the complexity of constructions used by Steve Coleman. Indeed, as we saw in the final bar of this solo, the use of the "Symmetry" system means that Steve Coleman ends his solo on a note outside the home scale of F# Blues. This recalls to mind a (more general) statement made by Hrebeniak:

"Like his namesake Ornette, Steve Coleman has investigated the reductive suppositions of European harmony and formulated a system embracing the anti-cadence."¹

¹ Hrebeniak (1991) p. 19

Secondly, there is no attempt to resolve “out” motifs generated by the “Symmetry” system beyond the single example of compensation found in bar 7. Thirdly, we might note that, by the end of this Intro Solo, Steve Coleman has played 11 of the 12 notes of the chromatic scale. Thus, his use of the “Symmetry” system has enabled a (almost) fully chromatic improvisation. We shall see that this develops further in the 1st and 2nd Alto Solos.

Bass in Introduction (bars 1–14)

The pattern *ss* in bar 2 is a simple chromatically ascending line, resolving to the scale tone E at the end of the bar. However, let us note the structural equivalence of this set to that of the Alto at the start of the piece: Alto {B, C, Db}; Bass {D, Eb, E}. The similarity of these sets suggests that they may be thematic. That is, they are both simple {0, 1, 2} sets which have been derived from the “Symmetry” system and are perhaps used here to act as a kind of launch pad for both instruments’ improvisations. Further, let us note that it is these sets that (a) the Alto repeats shortly after (*d* in bar 3), and, (b) the bass uses 4 bars later (bar 6; also, note the following F# and A are identical). At *tt* the bass plays an ascending {0, 2, 5, 7}_b set. However, let us note that this, like pattern *uu*, is just part of an ascending F# Blues scale that fills most of bar 3. Further, the axis of the bass set is Eb, which is not in F# Blues. Thus, although the bass is using sets that generally obey “Symmetry” system rules, their context and axis detail suggests that this is more by luck or poor imitation than careful design. On the other hand, Steve Coleman’s use of the same {0, 2, 5, 7} set at *e* in the same bar is more distinct: it has a rhythmic integrity and a contour that separates it from its surroundings. Perhaps Steve Coleman’s use of this set at *e* in a *descending* order is a conscious symmetrical “answer” to the *ascending* bass part

of *tt*. The bass continues its exploration of the Blues tonality with both Major and (minor) Blues scale material in bar 2, with the pattern marked *v* being the simple result of an A# grace note between the scale tones of A and B. However, bar 3 shows two intriguing patterns. Firstly, the use of the note G at the start of the bar (marked with *s) and the D Major rapid arpeggio at beats 5-6. I suggest that these two events are responses to material heard in the Alto part. The Gs are a response to the Gs played by Steve Coleman in bar 4: let us note especially the melodic role of the final note in the Alto part in this bar. With regard to the D major arpeggio in the bass, note that at *j* in the Alto we hear notes III, II and I from the key of D. Thus, the "out" material found in the bass part may be seen to be derived from a listening to the Alto part. Pattern *xx*, as noted above, is constructed from the same note set heard four bars earlier at *ss*. However, here it occupies less than half of the time (i.e. three and a half beats at *xx* versus eight beats at *ss*), and the opening D note is reduced to a quaver in duration. At *yy* (in bar 8) there seems at first glance to be a melodic pattern in the bass that may have been derived from the "Symmetry" system. Let us note that its source is F# minor (C# = {0}), and that it creates a note outside this source: the D# (marked *). Further, this four-note pattern is separated from the surrounding melodic material by rests. Further, the symmetrical nature of *yy* is emphasised by repetition of C# at the octave. However, it seems equally possible that this pattern is derived from a combination of (a) the ubiquitous F# Blues scale (used from the start of the bar until the low C# at beat 8), and (b) the D, D#, E motif found at *ss* and *xx*. In this regard, note the sustained E that follows at beat 2 of bar 10, and the additional D# in bar 9 (marked *), which is followed by a further E. Further, pattern *zz* in bar 13 also seems to be the result of using F# Blues (actually F# minor pentatonic, of course), rather than a deliberate use of the "Symmetry" system to construct this melody.

Thus, the bass part in this Introduction, whilst providing exciting rhythmic counterpoint to Steve Coleman's solo, generally shows much less use of the "Symmetry" system, preferring instead the F# Blues as a straightforward resource. Indeed, although there are several sections that have seemed, on initial inspection, to be deliberately constructed by means of the "Symmetry" system, closer examination of their context and detail has revealed that this is not likely.

1st Sax Solo (bars 19-23) – Steve Coleman

Steve Coleman starts this solo with what at first glance appear to be (merely) two conventional ascending G Whole Tone patterns, the latter of which transforms into an F# Blues melody. Note that both ascending patterns commence with a G (both marked *), thus suggesting that Steve Coleman may be thinking of G as the tonic/name of this scale. However, analysis with regard to the "Symmetry" system reveals a different rationale for this section, along with further detail. Patterns *c* and *d* are non-overlapping structures with similar axis material ($\{A, B\}$ and $\{A\}$, respectively). Thus, the Whole Tone subset used here is generated by the "Symmetry" system, just as we saw in the Intro Solo. Pattern *e* is a subset of *f*, the first of several $\{0, 2, 5, 7\}$ sets to be found in this solo. However, none of the other such sets have a similar $\{0, +5, -5\}$ structure as their first three notes. Further, note that the F# Blues section (the last 5 notes of bar 19) begins as an F# minor pentatonic pattern, and it is only the final C natural (marked *) which characterises this as a Blues scale pattern. In fact, this C note seems to stimulate the choice of the following material, and thus, in my annotation, is bracketed together with it (i.e. the bracket annotated *g* which extends into bar 20). This is the first of several occurrences of relatively lengthy rests in this solo which divide two phrases that have a

shared axis. It seems that in these cases the rest may signify “thinking time” for Steve Coleman whilst he calculates a possible conclusion of a phrase. This is unlike what we found in the Intro Solo, where sets are more usually clustered together, without rests (comparison between this section and *s* at bar 6 shows up this distinction nicely), and demonstrates a subtle shift in philosophy of delivery of these sets. In fact, pattern *g* in bar 20 is only a part of a rapidly executed (and complete) G minor pentatonic scale, which in itself has two structures of note: a distinct {0, 2, 5, 7}d set (marked *h*), and the symmetrical list marked *i*. In a sense, then, this argument means that the F# Blues pattern found in bar 19 has been retrospectively altered to become merely F# minor pentatonic: the C in bar 19 now “belongs” to the G minor pentatonic in bar 20. On the other hand, we clearly hear an F# Blues melody in bar 19; here, then, Steve Coleman at once satisfies a conventional jazz language and his own version of it. Further, let us recall that we have found other musicians using the Blues scale for the home key alongside the (slightly simpler) minor pentatonic for “out” material at bII (see the section entitled “Scalar” in the Conclusions of Chapter 1). Also, let us note that whilst *i* may be seen to have an axis from within F# Blues, *g* and *h* have axis material which is completely outside of this scale. We will return to this important change below. For now, we will move onwards, noting the thematic relationship between the two phrases to be found in bar 19 and 20. Both start with “out” material based on G, and close with an F# Blues section. These two F# Blues sections are closely related. However, such variation plays no real part in the following solo: a similar strategy was employed in the Introduction Solo (see analysis of phrases *a* and *d*, above). Steve Coleman resolves the “out” section in G minor pentatonic with an F# minor pentatonic pattern (actually a further {0, 2, 5, 7}a set, marked *j*), which, similar to the music in the previous bar, ends with a C (at beat 6, marked *), suggesting F# Blues (and revealing the further pattern *k*). However, in this case the phrase then leads onwards, out of the home key of F#, with the C note acting as a pivot note.

Here, at *l*, we find a five-note construction, with B as the axis and a downward Cm7 arpeggio as the other material. Let us note that the only other five-note pattern was the F# minor pentatonic set at *mm* in the Intro Solo (bar 12). This, considered alongside the contour of the Cm7 arpeggio, suggests premeditation of this pattern. The notes A and B at the end of bar 20 indicate a return to the home key of F# minor pentatonic, and, although an incomplete “Symmetry” structure in themselves, can be seen as stimulus for the following material. Here, just as at *g*, a rest breaks up a symmetrically coherent (and otherwise inexplicable) set of notes. Further, just as at *g*, the music after the rest is a {0, 2, 5, 7} set. Pattern *m* is a rare, but legal, Symmetrical Interval found between *l* and *n*. Bar 21 consists almost exclusively of a collection of what appear to be prefabricated {0, 2, 5, 7} sets. Let us note that these show development of contour in that *n* and *o* have their paired axis appearing as adjacent notes, whereas *p* and *r* have their paired axis separated by another note from the relevant {0, 2, 5, 7} set. Pattern *q* shows that, further, *p* and *r* are symmetrically related. Note that the first five notes of this bar are a complete F# minor pentatonic list, and, as we noted above, this scale can be derived from the “Symmetry” system, thus:

-5	-2	{0}	+2	+5
F#	A	B	C#	E

Table App.7-4 Structure of F# minor pentatonic (axis of B) (copy of Table App.7-2)

However, this F# minor pentatonic scale is only maintained for 6 notes in this bar: the use of prefabricated {0, 2, 5, 7} sets takes precedence over this core scale key, although they clearly relate to it. The D#s and G#s found in *o* and *p* complicate F# minor pentatonic into F# Dorian. Note that the choice of contour of set {0, 2, 5, 7}c places these notes from F# Dorian at the end of the set. The “out” set of {0, 2, 5, 7}d found in *r* shows

similarity of contour with *p*. and, further, similarity of pitch, contour and duration with *h*. This relationship with the last heard performance of G minor pentatonic and, also, the fact that this {0. 2. 5. 7}d set contains two G notes (marked with *s) suggest that Steve Coleman is here thinking of this pattern as a G minor pentatonic subset.

The next section is open to several possible analyses, but I suggest the following. Pattern *s* is a chromatic section leading to *t*, a G#m7 arpeggio. This arpeggio has an axis of {B. D#}. This D# is clearly outside F# Blues, and I suggest that this riff has more to do with a desire to compensate the "out" section of *r* with a return to the home key of F# minor than with strict adherence to the "Symmetry" system and its related, prefabricated structures. With this in mind, we might note that the notes found in the F# Dorian bracket form a fairly regular B13 arpeggio, and recall that B7 is the V to the (m)II of F#m7 (this progression being common in bebop and modal jazz). Pattern *u* is especially interesting in that the first two intervals (A to G: G to D# (=Eb)) are both Symmetrical Intervals, and thus require no further notes. However, Steve Coleman seems to choose to resolve this particular set. His solution is to resolve the first part of *u* with *v* (the crucial note being the F, marked *), again allowing the rest to act as no barrier to pattern construction.

Pattern *v* is in itself a symmetrical phrase: {0. 2. 5, 7}e. This, like the {0, 2. 5. 7}d heard just before, is also a (in fact the only other possible) subset of G minor pentatonic. Further, let us note that the only other previous appearance of set {0, 2, 5, 7}e was at *g* in bar 20, where it simply ascended (whereas at *v* it simply descends).

Taking a wider view of bar 22, I have found other structures that may be seen to relate to the "Symmetry" system. We have already noted that the opening section is based in F# Dorian (due to the G#m7 arpeggio at *t*). This is balanced by the use of C Dorian in the latter part of the bar. This C Dorian section has further levels of detail. Firstly, the D# (=

Eb) at beat 3, and the G that precedes it form a subset of Cm7. Secondly, pattern *v* is a {0, 2, 5, 7}d set. Thirdly, *w* and *x*, although more difficult to find, do create a seamless use of the "Symmetry" system up to the more conventional pattern *y*. However, I suggest that here Steve Coleman is "treading water", so to speak, and planning the timing and material for the end of his solo (I have described a similar strategy at bar 13 of the Intro Solo, above). Further, the repeated, low G, A pattern in *x* creates a sense of tension and acts as contrast to the ascending material that follows. In the last beat of bar 22, C Dorian is replaced by C Blues as the scale in use, and Steve Coleman finishes his solo with a flourish of a diminished triad (*y*), a Cm7 arpeggio (*z*) and, in bar 23, the quintessential {F, F#, G} set (*aa*). Let us note that this latter set is the key motif in the bass part of the Head which is just starting. Further, just as at the end of the Intro Solo, this solo ends with a pattern using F# as the axis, which, in itself, ends with a note outside of the home key of F# Blues. Note the repeated G at the end of the solo (these marked with *s). That Steve Coleman uses the C Blues scale for this section further suggests that the final {F, F#, G} section is premeditated: C Blues is the only conventional jazz scale that includes these three notes. Further, we might note that C Blues is a sum of two sets already used by Steve Coleman, thus: Cm7 + {F, F#, G} = C Blues.

Before moving on, let us examine the Cm7 arpeggio at the cusp of bar 22 and 23 in more detail. Unlike the Cm7 arpeggio at bar 20, which has an axis note B immediately preceding it, here we have no such heard axis. However, we can still find an axis from F# Blues, if we allow it to be described as a *hidden* axis (here the hidden axis is shown in *italics*):

Cm7				
G	Bb	B	C	Eb
-4	-1	{0}	+1	+4

Table App.7-5 Cm7, hidden B axis

However, equally, the true hidden axis of this arpeggio may in fact be the note a tritone away from B, that is, F:

Cm7				
C	Eb	F	G	Bb
-5	-2	{0}	+2	+5

Table App.7-6 Cm7, F axis

Let us now recall that the instruction for this 1st Solo is “[Ebm] (or sum 5 center)” (which, as we have noted already, transposes to “[F#m] (or sum 11 center)” in concert pitch). If we examine Steve Coleman’s “Sum” table (Example 6-b: see end of Appendix 6), we see that the boxes relating to Sum 11 contain the entries “F–F#” and “B–C”:

EXAMPLE 6-b

SPIRALS NUMBER ONE & TWO

AXIS TONES	SUM #	AXIS TONES	SUM #
C-C	SUM 0	C-Db	SUM 1
C#-C#	SUM 2	C#-D	SUM 3
D-D	SUM 4	D-Eb	SUM 5
Eb-Eb	SUM 6	Eb-E	SUM 7
E-E	SUM 8	E-F	SUM 9
F-F	SUM 10	F-F#	SUM 11
F#-F#	SUM 0	F#-G	SUM 1
G-G	SUM 2	G-Ab	SUM 3
Ab-Ab	SUM 4	Ab-A	SUM 5
A-A	SUM 6	A-Bb	SUM 7
Bb-Bb	SUM 8	Bb-B	SUM 9
B-B	SUM 10	B-C	SUM 11

Table App.7-7 Steve Coleman's "Spirals Number One & Two" Table

Thus, we might expect to find a realisation of this instruction in the solo(s), but not necessarily in the head. Steve Coleman is recognising the interchangeability of these two axis pairs by including two entries for each of "Sum x". Let us now recall that both the G Whole Tone subset (bar 19) and the G minor pentatonic scale (bar 20) can be seen to have axes derived from within F# Blues, thus:

G Whole Tone subset 1 (*c*, bar 19)

G	A	B	C#
-3	{-1	+1}	+3

G Whole Tone subset 2 (*d*, bar 19)

G	A	B
-2	{0}	+2

G minor pentatonic (*e*, bar 20)

G	Bb	C	D	F
-5	-2	{0}	+2	+5

Table App.7-8 Scales with axes from F# Blues

However, similar *hidden* axis material can be found for their complete lists, thus:

G Whole Tone

Db	Eb	F	F#	G	A	B
-5	-3	-1	{0}	+1	+3	+5

G minor pentatonic

C	D	F	F#	G	Bb	C
-6	-4	-1	{0}	+1	+4	+6

Table App.7-9 Scales with hidden F# axis

As we can see, these hidden axes are the same, being the tonic note F# (this means they could also have the axis note C). Further, the C Dorian scale used in bar 22 can be seen to have the same possible hidden axis:

C Dorian

C	D	E _b	F	F _±	G	A	B _b	C
-6	-4	-3	-1	{0}	+1	+3	+4	+6

Table App.7-10 C Dorian with hidden F# axis

Also, the strange Cm7 subset in bar 22 might be explained, like Cm7 itself (see above), by a hidden F axis note:

Cm7 Subset

E _b	F	G
-2	{0}	+2

Table App.7-11 Cm7 subset with F axis

Finally, the F# minor pentatonic scale and the F#m7 chord, both used extensively in this piece, may be seen to have an axis of B (as noted above) or (as shown here) F:

F# minor pentatonic

B	C#	E	F	F#	A	B
-6	-4	-1	{0}	+1	+4	+6

Table App.7-12 F# minor pentatonic with hidden F axis

F#m7

C#	E	F	F#	A
-4	-1	{0}	+1	+4

Table App. 7-13 F#m7 with hidden F axis

Before moving on, we should note that all of the above structures are from within the jazz tradition and are commonly described as elements of jazz theory. Further, they all have axes of either F or F# (or notes from within F# Blues).

Bass in 1st Sax Solo (bars 19-23)

The bass has a special part with which to accompany the solos. I have notated it only once since it is identical in each case.

Pattern *a* in bar 19 is a variation of pattern *a* in bar 15 (in the head). The use of the note E (forming a Symmetrical Interval with the preceding F#) is more complex, being (as allowed by Steve Coleman's rules) answered by a (non-symmetrical) G. Let us recall that it was a similar F# to E melody that ends the first two bars of the head (beats 7 to 9 of bar 16). The following three notes (F#, F and E) show real difference in contour from the head, although they maintain the rhythm written in the head arrangement. However, by the second bar, the rhythm in the Head is subverted. This second phrase, *b*, whilst showing a distinct axis of {E, F}, is less transparent than those we found in the head: the repeated E in bar 20 complicates the symmetry of the phrase. Annotation of the remainder of this bass line has proved more complex than annotation of the bass line of the head: here more numerous interpretations of intent seem possible. Like the solo that it accompanies, this bass part may have been constructed with symmetrical structures spanning rests (a characteristic that I did not find in the head). For these reasons, I have not annotated this bass part beyond *b*. However, this bass part for the solos clearly has similarities with, and some differences from the head. For example, the limited set of notes used for the head {D, Eb, E, F, F#, G} is maintained here. Further, the note D

occurs once only, and in a similar position: in the middle of the second bar of each section. However, the ordering of the notes into melodic phrases in the bass part for the solos owes a lot less to the "Symmetry" system than we found in the bass part for the head.

As we noted earlier, the bassist plays the bass part for the head under Steve Coleman's 2nd Alto Solo, and plays the bass part for the solos under Head 5. This seems to be why it may have been aborted before the end, forcing the edit into the next track as an emergency measure.

Piano Solo (bars 27-31) – David Weidman

This solo is essentially based in F# minor pentatonic, with the addition of three notes: D# (=Eb), G and A# (= Bb) (marked with *s). Note that these three additional notes often occur (a) at the start and end of phrases, (b) at the top and bottom of melodic contours, and (c) on the beat. These three characteristics emphasise this "out" method. An initial analysis suggests that Weidman is using F# minor pentatonic as the "in" scale, and that the (relatively frequently occurring) notes of G and A# (= Bb) are used to suggest the "out" key of G minor. Recall that we have just seen Steve Coleman's use of G minor pentatonic in his 1st Solo as "out" material. However, this piano solo does not use the "out" material as groups of notes separate from the "in" material, as does Steve Coleman. A G is never adjacent to an A#, for example. Further, let us note that in bar 30 each of the A#s seem to resolve to an A natural.

However, this initial analysis ignores the note Eb. Let us note that the three additional notes of Eb G and Bb form the Eb triad, and that this might be seen as (a) a handy mnemonic, if not derived from the "Symmetry" system per se or (b) a subset of another "out" set. In this regard, let us note that Eb, G and Bb are common to C Dorian (to be found in Steve Coleman's 1st Solo), Cm7 (to be found in Steve Coleman's 1st and 2nd Solo) and Eb9 (to be found in Steve Coleman's 2nd Solo – but see below).

Beyond this note set, we can find several structures that may have been formed with regard to the "Symmetry" system as used by Steve Coleman. At *a* we find a Symmetrical Interval (tritone), immediately followed by *b*, a palindromic melodic phrase. Further, the first note of *c* could be seen as acting (retrospectively) as the axis for the tritone *a*, thus:

E	G	A#
-3	{0}	+3

Table App.7-14 Structure of *a* and *c*

Further, this pattern *c* includes *d*, which is an octave transposition of *a*, suggesting that it is a premeditated motif, and that G is the known axis. Patterns *e* shows a further rationale for the inclusion of the note G: as a member of a C# diminished triad. Pattern *f* shows not only a repeated (and thus emphasised) axis note (B), but also a symmetrical contour to the entire phrase. Pattern *g* shows possible use of the system, although the notes that precede it do not, and, thus, the pattern that I have annotated may exist simply by chance. By contrast, however, *h* and *i* show deliberate, and, indeed, relatively complex symmetrical structures. Pattern *h* has a (hidden) axis of B, and is split in time by *i*, which shares this axis, yet sounds it. In a sense, *i*, consisting as it does of F# minor pentatonic material, acts as a temporal "in" axis around which the two "out" halves of *h* orbit. It is possible, of

course, to analyse *h* and *i* as a single pattern with the single axis of B. but the contour and rhythm of this section encourages me to suggest that they were performed as separate sets. Pattern *j* is a repeat of the tritone found at *a* and *d*. but conceived at yet another octave. By octave, *a* might be seen as the axis to *d* (an octave above) and *j* (an octave below), thus:

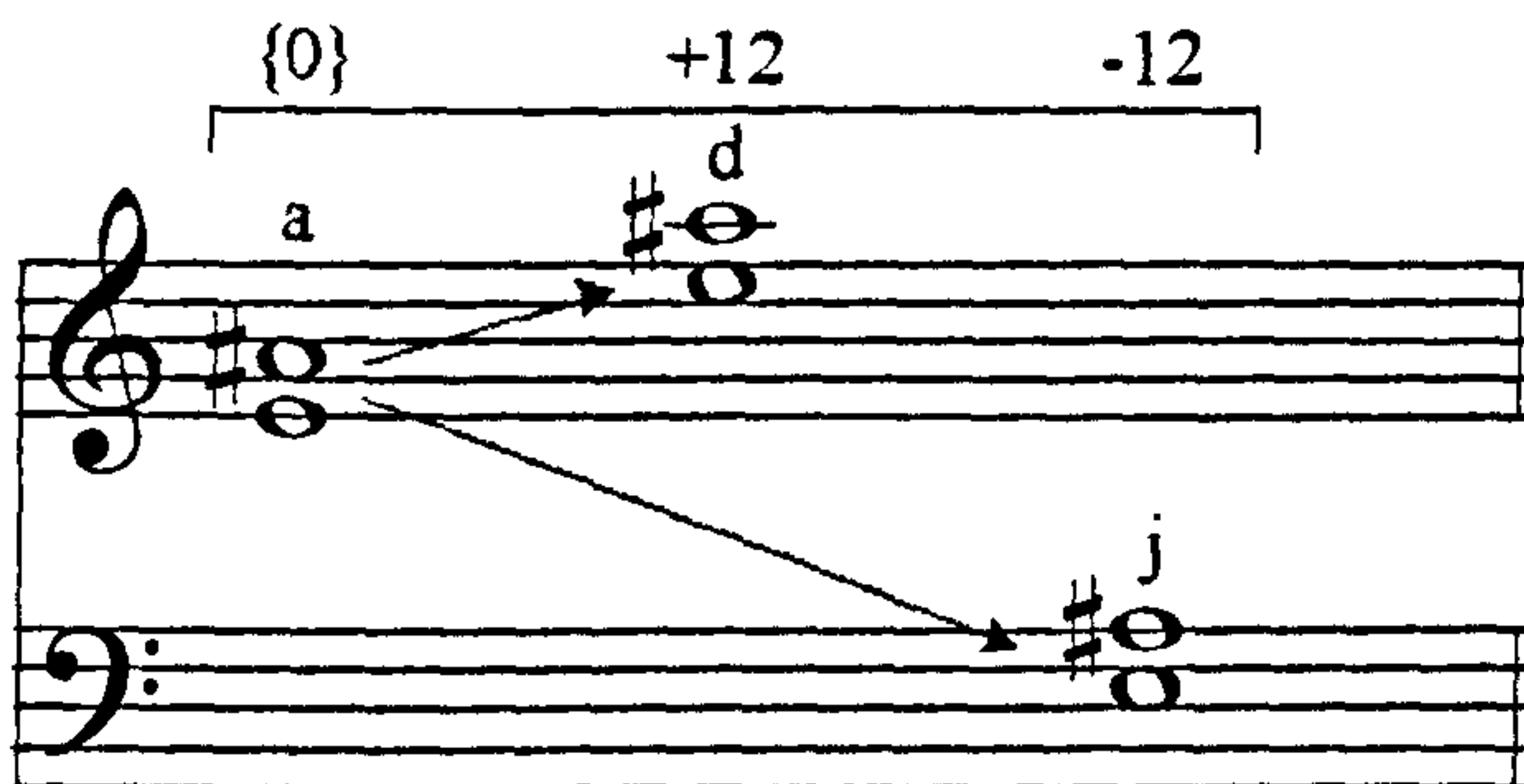


Fig. App.7-3 {E. A#} Tritones at *a*, *d* and *j*

It is clear, then, that Weidman not only uses Steve Coleman's "Symmetry" system. but also extends this to include some symmetrical methods of his own in this solo.

Electric Guitar Solo (bars 35-38) – David Gilmore

This guitar solo is similar to the piano solo examined above in that F# minor pentatonic is the core scale. However, symmetrical structures are rarer. Pattern *a*, for example. being constructed solely of notes from within F# minor pentatonic. may just exist by coincidence. Further, on the fourth quaver of bar 36 there is a solitary G note (marked *) that does not, in itself, seem to be part of a melodic phrase derived from the "Symmetry" system. This is also true of a further G note, placed on the second quaver of bar 37 (also marked *). However. these may be functioning similarly to those Gs and Bbs found in the

Piano Solo, examined above. That is, they are individually interpolated within an improvisation based in F# minor pentatonic. Pattern *b* is a localised {0. 2. 5. 7}b set. However, members of this set extend a note before and several notes after pattern *b*, and thus, just as at patterns *d*, *f* and a similar {0. 2. 5. 7}a set in bar 37. the appearance of this “Symmetry” system derived material may be merely coincidence: the notes that precede it do not form symmetrical structures. Note however that the (identical) repeat of pattern *a* as *c* at the end of bar 36 is similarly separated from the surrounding notes (a rest before *a*, a sustained F# before *b*). Pattern *e*, however, is a retrograde statement of *d*, suggesting that Gilmore may be including such symmetry in his repertoire. The minor pentatonic list that is *g*, of course, shows symmetry: as does the {0. 2. 5. 7}a set that overlaps it at *h*. Let us also note, however, that *g'* may be seen to be the retrograde inversion of *g*².

When we arrive at *i*, however, we suddenly hear new material: an F and Eb in the last beat of the solo (marked with *s). We might note that this forms a Symmetrical Interval, but this seems an unlikely explanation for these notes in this solo. I suggest that these two notes relate to the other note which we have heard outside of F# minor pentatonic, namely G, in the following way:

Eb	F	G
-2	{0}	+2

Table App.7-15 Structure of *i*

Further, similar to sets found in Steve Coleman’s 1st Solo, this symmetrical pattern may be seen to have a hidden axis from within C minor pentatonic, thus:

F	G	B	Eb	F
-6	-4	{0}	+4	+6

Table App.7-16 C minor pentatonic, hidden B axis

Thus, this use of a limited “out” set parallels Weidman’s use of the {Eb, G, Bb} set in the piano solo. And again, like Weidman’s “out” set, the notes Eb, F and G can be found in Eb9 (but see below) and C Dorian. Although this solo has less use of symmetrical systems found in the piano solo, we can say that Gilmore, like Weidman, has developed his own symmetrical structures, most noticeably in the use of retrograde and retrograde inversion structures. Further, both Gilmore and Weidman use F# minor pentatonic as their core scale, whereas Steve Coleman uses F# Blues.

2nd Sax Solo (bars 47- 51) – Steve Coleman

This solo has none of the thematic structure shown at the start of the Introduction Solo and 1st Solo. Here Steve Coleman plays five distinct, rapidly executed phrases of complex structure, separated by rests of at least a crotchet in duration. Each of these phrases shows extensive use of the “Symmetry” system.

Steve Coleman opens this solo with an assertive F# Blues phrase, designed to reveal many of the symmetrical structures within this scale. After a single, tonic F#, the simple *a* overlaps *b*: notice how this latter pattern, by placing the axis as the last note, disrupts what might have otherwise been a more conventional upward list of the F# Blues scale. Pattern *c* is a Symmetrical Interval structure, and is followed by *d*, a rather specialised F# diminished triad pattern (specialised in that this is a rare use of the Blues scale – here, of

course, it follows the “Symmetry” system). Pattern *e* is a motif that we have seen used frequently in the Intro Solo. This pattern is then transposed down a minor 3rd and performed as *g*. Note that this forms a further structure, annotated *f*. The rhythm of this section further encourages the sense that this is a premeditated device. The minor 3rd interval of this transposition may then inspire the last few notes of this first phrase at the start of bar 48: a Cdim7 arpeggio, performed relatively quickly. This speed of performance is maintained, and, for the first time, melodic material becomes dislocated rhythmically from the rhythm section.

The second phrase opens with *i*, an ascending G minor pentatonic list, which, as we noted above, has a hidden axis of F#. Note that, like the ascending G minor pentatonic scale in bar 20, the highest note is the F. This is followed by an ascending Eb7 arpeggio, which is extended by the following F into an Eb9 chord, annotated *j*. This chord of Eb9 is interesting on several counts. Firstly, its *sounded* axis (Bb) is clearly not derived from F# Blues. However, if we allow a *hidden* axis from within F# Blues, then we can find a structure, using the tritone E as the axis:

Bb	Db	Eb	<i>E</i>	F	G	Bb
-6	-3	-1	{0}	+1	+3	+6

Table App.7-17 Eb9 chord, hidden E axis

Thus, unlike the Cm7 (and subset) and C Dorian structures found in the 1st Solo (i.e. those other structures that have no sounded axis from F# Blues), this Eb9 chord does not have an axis of either F or F# (= B or C). Use of either an E9 or F9 arpeggio (with axes of F and F#, respectively) would have maintained continuity with previous material.

Thus, here Steve Coleman seems to be extending the remit of the “Symmetry” and “Sum”

systems beyond what we have already seen. Alternatively, it seems possible that he has recalled the wrong (premeditated) arpeggio. If Steve Coleman is allowing Eb9 to appear here because it has a hidden axis that is neither F or F# (although E, of course, is within of F# Blues), then this would “open the floodgates”, so to speak, from a mathematical perspective. That is, just about any symmetrical pattern *in any key* would be permissible under this sort of a rule. This seems unlikely, given the specific and limited nature of the sets that fill the rest of his solos. Finally, with regard to this Eb9 chord, notice how the internal -3, {0}, +3 structure, annotated *k*, is varied to become the Bbm triad that follows. This, in itself, adds emphasis to the F note, compensating its being placed an octave below where we might expect, that is had the Eb9 arpeggio been continued in its upwards direction. The latter two notes of this Bbm triad, Bb and Db, act as the catalyst for a new pattern, *l*. Pattern *l* generates a final B, which, in turn, acts as the catalyst for a downward Cm7 arpeggio, *m*, similar to that heard at bar 20. In this case, then, unlike the previous Cm7 arpeggio at bar 22, we *hear* the axis of B. The last two notes of this Cm7 chord, Eb and C, form a diminished triad pattern, *n*, and this is then followed by the Symmetrical Interval of a tritone at *o*. Pattern *q*, a simple chromatic set of three notes gives way to *r*, also three notes, but this time with intervals a tone apart (and thus a subset of a B (etc.) Whole Tone scale), the axis being C#. The Symmetrical Interval of a Major 2nd follows as *s*, which, with the (as yet unbalanced) C# that follows, resolves the phrase back into the home key of F# minor. This is the end of the second phrase in this solo.

Before continuing with the third phrase, let us note that the last two and a half beats of bar 48 may be annotated as a subset of the F# Auxiliary diminished scale, a so-called “symmetrical” scale.² That 7 of the 8 notes in this scale are present (there is no Db)

² See, for example, Haerle (1980), pp. 34-36

suggests its use may be a knowing extension of the (similarly common) Cm7 arpeggio. This scale, like the Cm7 chord, has a possible (hidden) axis of B, thus:

F# Auxiliary diminished								
F#	G	A	Bb	B	C	(Db)	Eb	E
-6	-5	-3	-1	{0}	+1	(+3)	+5	+6

Table App.7-18 F# Auxiliary diminished. hidden B axis

(That we don't hear a Db (bracketed in Table App.7-18 above to denote its non-appearance in the music) means that these 10 notes may not be annotated as a complete "Symmetry" pattern.) Note that we hear a B immediately preceding this scale subset. Of course, despite this heard B note, the axis of this scale could equally be D, Ab or, importantly, F. Thus, like many structures found in the 1st Solo, this scale may have been derived from the "Sum 11" approach (i.e. generating F-F# (= B-C) axes), as described above. This relatively "out" scale then gives way to a relatively "in" F# Dorian tonality at the start of bar 49 (recall that we heard Steve Coleman also specifically using F# Dorian as resolving material in his 1st Solo, at the start of bar 22).

Returning now to the start of the third phrase we can see that pattern *t* explains and balances the C# at the end of the second phrase. It becomes part of a simple chromatic symmetrical set { {0}, +1, -1 }, with, as we have seen in some patterns in the 1st Solo, a rest between two of its elements. The Symmetrical Interval *u* follows which leads to *v*, a G#dim7 arpeggio. Just as previous dim7 arpeggios have been notable for their rapid performance after notes of quaver values (or accurate multiples thereof), this G#dim7 arpeggio is similarly separated from its surroundings by being performed at the level of quaver triplets, having been preceded by rapid semiquaver patterns. This marks the end of

the third phrase, but this G#dim7 arpeggio also extends across the rest that follows, forming a contour of this extended arpeggio not dissimilar to that found at *h*, i.e. the Cdim7 arpeggio at the start of bar 48. Note how this dim7 arpeggio, like the others in this piece (at bar 2 and 48), has a descending contour and is placed at the end of a larger melodic phrase.

The fourth phrase, starting in bar 50, initially jumps between F# minor pentatonic and G minor pentatonic. Here there is no use of the "Symmetry" system per se. What we can say, however, is that these are subsets of material already explored and known to be symmetrical. Patterns *w* and *x* are, intriguingly, identical to *r* and *s* heard very recently at the start of bar 49. However, here the final note of the phrase is a G, creating a further symmetrical pattern, *y*.

The fifth and final phrase in this solo commences with pattern *z*, which is then linked by *aa* to *bb*. Pattern *bb*, aside from having a paired axis of {A, B} itself, is melodically subdivided into two equal subsets, these having a {B} and {A} axis respectively, which, further, are transposed retrograde versions of each other. This complex structure only generates a single note outside of F# minor pentatonic, however: the G at beat 9 (marked *). Further, although it might be tempting to read pattern *bb* as a B Whole Tone scale subset, we should note that this pattern consists of the identical set found at bar 19 {G, A, B, Db}, which is, similarly, not derived from this (symmetrical) scale. Steve Coleman then completes the phrase with a downward arpeggio of an F#m7 chord. Although we have already heard two such arpeggios already (in the Intro Solo at bars 5 and 9), we will now note this chord has a B (= F) axis:

F#	A	B	C#	E
-5	-2	{0}	+2	+5

Table App.7-19 F#m7. B axis

This characteristic is also true of the Cm7 arpeggios heard in the 1st and 2nd Alto Solos. and, thus, these two tritone m7 chords may, in this context, be seen as theoretically related (and premeditated) sets. This fifth phrase is similar to those phrases that end the Intro Solo and the 1st Solo, in that all three consist of a range of complex “Symmetry” derived structures. Interestingly, these three final phrases all show a generally upward or downward contour. This characteristic may aid their performance, and, further, may be seen to give a more global melodic structure to a collection of “Symmetry” derived sets.

Finally, let us examine bar 50 in a little more detail. Firstly, note that Bar 50 has general ascending/descending symmetry, about a central point of beat 5 (which is a crotchet rest). This creates a contrast with the ends of the Intro Solo and 1st Solo, which end with (carefully prepared) ascending final phrases. Secondly, note that the ascending element commences in F# minor pentatonic, and that the descending element (in bar 51) ends in the same scale (this has a similarity with use of F# Blues as the {0} axis in pattern *hh*, at bars 10-11). Thirdly, the bracketed G and F notes which surround the rest at beat 5 might be seen to have a hidden axis of F# (much like the other “out” material examined above). thus:

F	F#	G
-1	{0}	+1

Table App.7-20 Notes F and G with hidden F# axis

This, of course, is the set at the heart of the bass part in the head. The idea that these notes are being used by Steve Coleman in such an iconic, thematic manner is encouraged by the fact that, although the G and F notes may be rationalised using "Symmetry" analysis, they are noticeably outside the sections of F# Dorian and F# Blues that immediately precede and follow them. In a sense, then, the crotchet rest takes the place of the F# axis. Further, this structure may be seen as a more complex variation of the (F) and (G) notes at beats 1 and 2 of bar 50. The F# in the bass part at beat 5 of bar 50 seems to be a coincidence, because, as described above, this is not the usual solo bass part, but, rather, the bass part for the head arrangement. However, there is still the possibility that Steve Coleman may have prepared this phrase, in the knowledge that in the bass part that we might expect to be played during the solos there is *also* an F# played on beat 5 (see beat 5, bar 22).

Appendix 8: The 64 Scales with Palindromic Interval Construction

This set of 64 scales is first described in my 1997 paper (pp. 218-220). There I show that each of these scales has a special characteristic with regard to common notes under transposition. However, it was not listed in full. I include it here to show the full range of such symmetrical material available to the improviser. The numbers in the first column refer to the 2048 scales (see Appendix 2).

1	C						
7	C	F#					
22	C	Db	B				
30	C	D	Bb				
37	C	Eb	A				
43	C	E	Ab				
48	C	F	G				
102	C	Db	F#	B			
137	C	D	F#	Bb			
164	C	Eb	F#	A			
184	C	E	F#	Ab			
198	C	F	F#	G			
268	C	Db	D	Bb	B		
295	C	Db	Eb	A	B		
314	C	Db	E	Ab	B		
326	C	Db	F	G	B		
378	C	D	Eb	A	Bb		
397	C	D	E	Ab	Bb		
409	C	D	F	G	Bb		
452	C	Eb	E	Ab	A		
464	C	Eb	F	G	A		
498	C	E	F	G	Ab		
636	C	Db	D	F#	Bb	B	
691	C	Db	Eb	F#	A	B	
724	C	Db	E	F#	Ab	B	
741	C	Db	F	F#	G	B	
816	C	D	Eb	F#	A	Bb	
849	C	D	E	F#	Ab	Bb	
866	C	D	F	F#	G	Bb	
918	C	Eb	E	F#	Ab	A	
935	C	Eb	F	F#	G	A	
969	C	E	F	F#	G	Ab	
1080	C	Db	D	Eb	A	Bb	B
1114	C	Db	D	E	Ab	Bb	B
1131	C	Db	D	F	G	Bb	B
1183	C	Db	Eb	E	Ab	A	B
1200	C	Db	Eb	F	G	A	B
1233	C	Db	E	F	G	Ab	B
1308	C	D	Eb	E	Ab	A	Bb
1325	C	D	Eb	F	G	A	Bb
1358	C	D	E	F	G	Ab	Bb

1413	C	Eb	E	F	G	Ab	A	
1551	C	Db	D	Eb	F#	A	Bb	B
1585	C	Db	D	E	F#	Ab	Bb	B
1597	C	Db	D	F	F#	G	Bb	B
1640	C	Db	Eb	E	F#	Ab	A	B
1652	C	Db	Eb	F	F#	G	A	B
1671	C	Db	E	F	F#	G	Ab	B
1723	C	D	Eb	E	F#	Ab	A	Bb
1735	C	D	Eb	F	F#	G	A	Bb
1754	C	D	E	F	F#	G	Ab	Bb
1781	C	Eb	E	F	F#	G	Ab	A
1851	C	Db	D	Eb	E	Ab	A	Bb B
1865	C	Db	D	Eb	F	G	A	Bb B
1885	C	Db	D	E	F	G	Ab	Bb B
1912	C	Db	Eb	E	F	G	Ab	A B
1947	C	D	Eb	E	F	G	Ab	A Bb
2001	C	Db	D	Eb	E	F#	Ab	A Bb B
2006	C	Db	D	Eb	F	F#	G	A Bb B
2012	C	Db	D	E	F	F#	G	Ab Bb B
2019	C	Db	Eb	E	F	F#	G	Ab A B
2027	C	D	Eb	E	F	F#	G	Ab A Bb
2042	C	Db	D	Eb	E	F	G	Ab A Bb B
2048	C	Db	D	Eb	E	F	F#	G Ab A Bb B

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