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Equity Investment Styles

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Abstract

The aim of this thesis is to investigate the nature of determinants of equity returns as suggested by the CAPM model, in particular, alphas, betas and equity premium and to outline implications for investment managers that statistical and structural analysis of the aforementioned variables may suggest.

The thesis contributes to the existing literature in the following areas. First, it addresses the question of predictive power of historical risk-adjusted portfolio performance measures on determining future equity returns in short and long term horizons. Second, it investigates the stability of beta coefficients and its impact on portfolio risk and seasonality in equity returns. Third, it assesses the question of dividend yield as determinant of portfolio alphas. Finally, it addresses the question of a common factor that may be influencing movements of equity premiums across European markets. All the aforementioned empirical work is the first of this kind, at least to our knowledge, in the UK.

In Chapter One we provide an indirect test of alpha stability. We test if past alphas, information ratio and alpha-to-beta ratio of positive and negative alpha portfolios can be used to determine future portfolio returns. We find that chosen portfolio performance measures do not have any predictive power in the short term investment horizons. However, in the longer term horizons of 24 to 36 months, we document the mean reversion in our portfolio returns and conclude that one can use historical measures of performance to predict returns in the longer run.

In Chapter Two we proceed to investigate if stocks with higher beta (systematic risk) also exhibit higher instability in betas as well, thus causing even greater risk for investors. We also examine the seasonality effect in the UK size-based portfolios and try to relate it to seasonality in betas. Our findings suggest that higher beta stocks do have more time-variant betas. Additionally, we find that equity returns are much higher in December-April than in May-November period but we find no robust evidence that such seasonality in returns is due to seasonality in betas but rather due to investors’ psychology.
In Chapter Three, we assess the relationship between excess returns and dividend yields in the UK market. The econometric analysis reveals U-shaped yield-return relationship in the 1980s and quadratic, bell-shaped, relationship in the 1990s. It seems that such a change in the relationship is driven by the change in the returns pattern of small size stocks in the 1990s. We find no evidence of the tax effect as the explanation of yield-return relationship that we observe.

In Chapter Four we try to identify what may be the common determinant of equity risk premium across European markets. We test for the serial correlation in the stock market returns and the results suggest that serial correlation is not in the level of returns but in the volatility of returns. Hence, if shocks to returns and in turn equity premium are persistent, there can be a scenario of a world-wide shock, which may influence the equity premium across countries in the similar manner driving them in the same direction.

The overall findings of the thesis are indicating instability of CAPM determinants of UK equity returns. If investors are aware of these instabilities, they can adjust their investment strategies accordingly and generate excess returns on their investment.
INTRODUCTION

The notion of different equity investment styles has emerged in the 1970s as members of the investment industry began more actively gathering and analysing data from financial markets and investment managers. Although, at first, style descriptions weren't as well defined as they are today, researchers and analysts noted that clusters of portfolios with similar characteristics have similar performance patterns. In turn, they have started exploring which particular stock characteristics are the key determinants of stock price movements. Such philosophical view about stock price determinants is supported by financial data. To constitute a style, these investment philosophies must be held in common by a group of investors. While the exact implementation of investment style may differ among the investors in the group, the group must agree upon factors that determine stock prices. In this thesis, we will show one view of the determinants of equity returns.

It is commonly accepted in financial theory and practice that equity returns (prices) are determined by a factor model. If one would assume that equity market risk premium is identifiable, than one can regard CAPM model as the most commonly used factor model for determining asset prices. The issue whether this statement is true in practice will be discussed later in this section. The excess returns in the CAPM model are estimated using the following Sharpe-Lintner CAPM equation:

\[
(E(R_i) - R_f) = \alpha_i + \beta_i (E(R_M) - R_f) + \varepsilon_{iM}
\] (1)
According to the theory of CAPM, the excess return on the stock (portfolio), \( (E(R_i) - R_f) \), is proportionate to the market risk premium \( (E(R_M) - R_f) \).

We can define alpha as the intercept in the regression of \( R_i - R_f \) on \( R_M - R_f \) and beta as the slope of that regression. The OLS estimates of \( \beta \) and \( \alpha \) are:

\[
\hat{\beta} = \hat{\rho}_{i,M} \frac{\sigma_i}{\sigma_M} \quad \text{and} \quad \hat{\alpha} = R_i - R_f - \hat{\beta}(R_M - R_f)
\]

Furthermore, the CAPM is assuming that all investors are facing the same universe of assets, have the same investment horizon and have the same expectations about expected returns, variances and covariances. In that case, all efficient portfolios will be the combinations of the tangent portfolio (risky portfolio) and the risk-free asset. Therefore, the main implication of the CAPM model assumptions is that all investors are investing in the same 'market' portfolio and the risk-free asset. However, such theoretical implication of CAPM is unobtainable in practice. It is very difficult for an individual to invest in the entire market portfolio by buying all the available shares because of the costs involved. However, investing in the market through passive investment management, i.e. mutual funds (index funds) that are designed to track the index closely, is easy and relatively inexpensive. A believer in the CAPM would buy an index fund and invest some money in a risk-free asset such as a Treasury Bill.

Let us now analyse the components of equation (1) that this thesis is going to deal with. In practice, it is usually said that CAPM, expressed through equation (1) is a
product of unstable and unidentifiable. Let us explain this further. In theory, alpha and beta parameters estimated through OLS regression model as in equation (1) are assumed to be constant. However, in practice, there is overwhelming evidence that proves that alphas and betas are actually unstable, i.e. they vary over time. Additionally, since Roll’s criticism of the market portfolio in the CAPM, it is widely accepted that market risk premium, \((R_M - R_f)\), is unidentifiable. This is due to the fact that in practice, the market portfolio is unobservable and it is usually proxied by the equity portfolio or even further by the stock market index. That is why Roll suggests that Arbitrage Pricing Theory is more appropriate factor model to be used when pricing assets. Although many authors suggest that factors in the APT model are difficult to identify, there is an overwhelming evidence regarding some factors that determine equity returns, such as dividend yields, price-to-earnings ratios, book-to-market ratios etc. These factors are considered to be source of alphas in the asset pricing models. In this thesis, we will provide statistical and structural approach to analysing these unstable and unidentifiable factors that determine equity returns in the CAPM. Using equation (1), we can present the structure of this thesis through a simple diagram, as shown below:
Chapter one represents the statistical approach to alphas. In this chapter we provide the indirect test of alpha stability by looking at the return behaviour of positive and negative alpha portfolios. Using weighted least squares methodology, we try to determine whether the current performance of positive (negative) alpha portfolios will persist in the future time periods and how long will the persistence of performance be before a mean reversion in portfolio returns occurs. We will examine if investors can make profitable investment strategy based on past alphas and portfolio performance measures that take into account both unsystematic and systematic risk measures.

Chapter two is a statistical approach to betas. In this chapter we will assess the issue of time varying betas and the seasonality effect in UK stock returns. Since large number of empirical evidence suggests that style return differentials are compensation for the risk measured by beta, using Kalman filter methodology, we investigate in this chapter whether investors investing in high beta portfolios incur even greater risk from the dynamics of betas. Additionally, we try to determine the presence of the seasonality effect in the UK stock returns and investigate if it is related to the seasonality in portfolio betas.

In Chapter three, we will apply a more structural approach to alphas. It is well known that alpha, as a measure of portfolio excess return, is a proxy for some other variables such as dividend yield, size, P/E ratio, market-to-book ratio, seasonals etc. In this chapter we will analyse the dividend yield effect in the UK market. The objective of the chapter is to investigate the yield effect on its own and to provide the explanation for any pattern in the behaviour of different dividend yield portfolios that we find. Classifying stocks into yield-size portfolios and using Sharpe-Lintner CAPM
estimation procedure we try to identify whether size has any impact on the return behaviour of different dividend yield portfolios in the 1980s and 1990s.

In Chapter four we deal with the unobservable part of the CAPM equation and focus on the equity premium in the European markets. We are analysing the Equity premium changes from the point of view of local investor and UK international investor. According to the CAPM, the equity premium depends on the stock market returns and their volatility. There is extensive evidence covering time varying volatility of stock market returns which makes it very difficult to provide forward looking estimates of equity premium. There have been series of attempts to model the evolution of the equity premium over time and to relate changes in the equity premium to variables such as dividend yield and earnings yield, i.e. to style determinants. Using ARCH and GARCH methodology in this chapter, we model the changes in equity premium in 16 European countries and try to answer whether equity premium is driven by the volatility of stock returns in all countries under consideration.

Finally, we will present the summary of the conclusions from all four chapters, which will enable the reader to focus directly on the main findings of this thesis.
CHAPTER ONE

REVISITING ALPHAS – UK EVIDENCE
1. Introduction

1.1. Introduction to alphas

We have established in the introduction to this thesis that we can define alpha as an excess return, which is obtained as an intercept in the regression of \( R_i - R_f \) on \( R_M - R_f \). Then, the OLS estimate of alpha can be formulated as:

\[
\hat{\alpha}_i = R_i - R_f - \hat{\beta}(R_M - R_f)
\] (1.1)

Investors who would like to generate excess returns on their investment will pursue active investment policy, i.e. they will invest in risky portfolios other than the 'market' portfolio. Since the aim of active investment management is to form portfolios that will outperform the market, it can be implied that the source of excess returns of those portfolios is the deviation from the market portfolio. This deviation is reflected in the extra amount of unique risk that the investor is willing to take. The main issue that arises from this discussion is how to construct those portfolios that will produce excess returns, i.e. positive alphas.

1.2. Using Alphas for Portfolio Construction

The main use of alphas is in the portfolio construction procedure. The explicit use of alpha is one of the ways to distinguish between a traditional portfolio manager and a systematic portfolio manager. In particular, traditional managers deal with alphas implicitly. It is possible to infer traditional manager's alphas by looking at the manager's portfolios. In other words one can look at the portfolios, assume a portfolio
construction process, and thus ‘reverse engineer’ the alphas that would lead to that portfolio. Systematic managers, on the other hand, use the same procedure as a reality check.

Dealing with alphas is not easy in the sense that alpha is always a result of an informed but *ad hoc* analysis. However, let us consider any asset q. Asset q will have a non-zero weight in the combined portfolio with another asset if and only if $\alpha_q \neq 0$. If $\alpha_q > 0$, then asset q will have a positive weight in the combined portfolio. If the investor has initial wealth W and $\alpha_q < 0$ then the investor will go short $w_q W$ in security q.

Being in a bull or bear market matters relatively little to the institutional investors. Since their aim is to improve long-term rates of return, what matters the most is whatever can boost those returns up, in other words: alphas. According to Arnott and Bernstein (1997) ‘... a positive alpha boosts portfolio wealth by boosting the prospective real income stream that portfolio generates in the future, while a negative alpha does the opposite’. An attempt to create a portfolio that would generate positive alpha always carries the risk of negative alpha that reduces the future real yields. Therefore, the reason why long-term investors are not really interested in bull and bear markets but in alphas is that real yields rise (fall) with any market decline (rally), whereas alpha has a direct impact on prospective future yields. Furthermore, the authors separate alpha into three categories: those that are the result of asset allocation, security selection and, finally, that stemming form arbitrage, but argue that the lines separating these three categories are indistinct.
1.3. What determines alphas?

It has already been noted that stocks and portfolios with positive alphas outperform the market. The question is why? Empirical tests have shown that alpha is a proxy for some other variables that determine excess returns, such as dividend yield, size, P/E ratio, market-to-book ratio, sales growth, leverage, earnings-to-price ratio etc. If there is a sufficient commonality in manager’s investment philosophies, portfolio characteristics and subsequent returns of those portfolios, the type of investing is labeled a style. The existence of style is confirmed by seeing if consistent patterns of returns follow from the style, both in the form of the performance of indexes of stocks selected using a style characteristics and in the average returns of managers following the style. The evidence for styles can be, for example, found in Fama and French (1992), Keim (1985), Levis (1989), etc. In all these studies, authors classify portfolios along one or more style characteristic (e.g. deciles of dividend yield portfolios, pentiles of size portfolios etc). They find differences in the average cross-sectional returns among style groups. Fama and French (1992), for example, also find that the single-factor CAPM fails to explain any of those differences in cross-sectional returns. If more than one style is used for portfolio classification, risk-return model with two risk premiums seems to be, at least \textit{a priori}, more appropriate to use. In other words, it seems that Arbitrage Pricing Theory would be better to use when explaining the cross-sectional returns. However, some may argue that Fama and French (1992) do not believe much in theory when estimating expected return, but they rely heavily on data in terms of looking at average returns of certain factors as the expected returns on those factors. The problem with using the theory lies in the fact that one may not know what portfolio to use to represent the factor or even what to use as the first factor. The problem with the data on the other hand is that very long
time series of data are needed to estimate factor’s expected return. Also, the problem of mismeasuring the market portfolio tends to give stocks with low betas high alphas. There is numerous empirical evidence related to style classification. However, that issue is beyond the scope of this chapter. We will examine the impact of dividend yield on the alpha of a portfolio in Chapter 3 of this thesis.

1.4. Unconditional vs. Conditional Alphas

Alpha, as the unconditional measure of abnormal performance defined in equation (1.1), was created by Jensen (1969). Unconditional parameter estimates are those that ignore the information about changing economic and stock market conditions. Therefore, there can be an error when using alphas to measure excess portfolio returns. Unconditional alphas are estimated by using different benchmark indices such as appropriate market index as well as style indices in the OLS regression model. When the beta coefficient in the equation (1.1) is equal to 1, the calculation of alpha as the measure of excess return can be simplified to:

$$\alpha_i = R_i - R_m$$

(1.2)

In the same manner as we will show in the chapter 2 that beta estimates can change with market conditions, alphas can be dynamic as well. Christopherson, Ferson and Glassman (1998) show in their study that conditional alphas are better predictors of portfolio returns than unconditional ones. Obtaining a single coefficient $\alpha_p$ from the regression model, as in equation (1) in the introduction to this thesis, suggests that the abnormal returns of a portfolio under consideration were constant over time. However, if, in reality, abnormal returns vary over time, a constant alpha may not be an adequate measure to identify the excess returns of a portfolio. Christopherson,
Ferson and Glassman (1998) introduce time varying alpha in the equation (1), where they allow alpha to be a function of $Z$:

$$\alpha_p(t) = \alpha_0 + A_p^T z_t$$

where $Z$ represents the vector of market information variables, $z_t = Z_t - \bar{Z}$ is a normalised vector of deviations of $Z_t$ from the unconditional mean $\bar{Z}$, the coefficient $\alpha_0$ is interpreted as an 'average alpha' or the alpha when all information variables are at their means in which case $z_t = \bar{Z} - \bar{Z} = 0$ and finally, $A_p$ is the vector whose elements measure the sensitivity of the conditional alpha to the deviations of the $Z_t$ from their means. This equation approximates conditional alpha by a linear function.

Additionally, their model includes a time-varying beta that is a function of $Z$:

$$\beta_p(t) = \beta_0 + B_p^T z_t$$

The modified regression that includes time varying alphas and betas tracks the variation of regression parameters over time and it is formulated as:

$$r_{p+1} = \alpha_0 + A_p^T z_t + \beta_0 + B_p^T z_t + u_{p+1}$$

The practical use of conditional performance evaluation can be explained through the example that follows. The authors assume that equity markets can take two equally likely states: 'bull state' when the returns of equity market will be 20% and a 'bear state' when the expected market return will be -20%, expected return on high quality stock -5% and a beta 0.25. The results conditional on the bull market are implying that beta is 1.0, the expected return is 20% and that alpha is zero. The results conditional on the expected bear market are suggesting beta of 0.25 and alpha also of zero. However, when unconditional alpha measurement approach is applied, it does
not take into account the known information about expected bull or bear market and hence reports an incorrect alpha of 7.5% and beta of 0.625. The data sample on which conditional alphas, as described in the model above, were tested includes 261 institutional equity managers in the period 1980 through 1996. Comparing the CAPM alphas and conditional alphas, authors find that CAPM alphas are insignificant at the 95% confidence interval level, while the conditional alphas are much closer to significance overall, with an average t-statistics of 1.53 across style portfolios. Furthermore, they rank stocks according to conditional alphas into quintile portfolios, from the highest to the lowest. They find that high alpha portfolios outperform low alpha ones by 4%, which is very close to 4.09% as reported by Christopherson, Ferson and Glassman (1998). Additionally, portfolio based on the top quintile of conditional alpha outperform portfolio based on the top quintile of CAPM alphas by 0.82%, while portfolios based on the bottom underperform by −1.72%. The next step was to test if the patterns of performance are consistent or whether they vary over market cycles. In particular, in the period 1986-1990, the difference between the cumulative return performance of the CAPM and conditional alpha estimates was very small, with high CAPM alpha portfolios doing slightly better in the period end of 1987-1989. However, in the 1990s, during the bull market, the top portfolios of conditional alphas constantly outperform the top portfolios of the unconditional alphas. In conclusion, while higher conditional alphas do not guarantee superior returns, they are more likely to successfully forecast alphas than some previously available measures.
1.5. Alpha as a portfolio performance measure

Prior to the late 1960s, one of the central problems in finance and investment has been the evaluation of performance of risky portfolios. The main difficulty in measuring portfolio performance was poor assessment and misunderstanding of risk measurement by many investors. It is generally perceived on the financial markets that investors are risk averse, implying that assets of high risk should yield higher returns than those assets of lower risk. Therefore, the effects of different levels of risk on the returns must be taken into account when evaluating portfolio performance.

Jensen (1968) was the first researcher to define alpha as portfolio performance measure. He argues that portfolio performance can be seen as a) forecasting ability of portfolio manager to predict security prices and b) the ability of the portfolio manager to minimise the diversifiable risk in the portfolio. Jensen suggests the absolute measure of fund manager's ability, i.e. alpha, as derived from equation 1, which measures the performance of the mutual funds against an absolute standard, i.e. market index:

\[ \alpha_{IM} = (E(R_i) - R_f) - \beta_{IM}(E(R_M) - R_f) + \varepsilon_{IM} \]  

(1.3)

According to equation (1.3), alpha is allowance for the manager's forecasting ability obtained simply by not constraining the estimating regression to pass through the origin. In other words, Jensen allowed for a non-zero constant, which is nowadays a common way of measuring abnormal returns of portfolios. Positive alpha implies that a portfolio under consideration has positive incremental return which is due solely to the manager's ability to forecast security prices. Analyzing the performance of 115
US mutual funds over the period 1955 – 1964, Jensen finds that on the average funds could not outperform the simple buy and hold strategy, even when returns gross of management expenses were taken into consideration. Jensen also attributes the returns of funds to the random chance rather than the managers’ forecasting ability.

Carlson (1970) looked at the performance of 1) diversified common stock funds, 2) balanced funds and 3) income funds over the period 1948-1967 on the US market. The author finds that the performance of the fund depends on the type of fund studied, time period under consideration and the proxy selected to represent the ‘market’. In particular, by using the Sharpe portfolio performance measure\(^1\) Carlson compares the performance of 3 (above named) types of mutual funds over 11 overlapping decades with the S&P 500, NYSE Composite Index and Dow Jones Industrial Average. He finds that outperformance of the fund against market index depends on the time period under consideration and type of market index used in the analysis. Among other things, the author partially replicates Jensen’s (1968) analysis. In particular, whereas Jensen (1968) subtracted the risk-free rate for each year from the annual return, Carlson applies the mean riskless rate for the total sample period. The rest of the Jensen’s analysis was replicated. Contrary to Jensen, the results of this study suggest that 82 mutual funds analysed netted about 0.6% more per year than their level of systematic risk is implying they should earn. Additionally, 59% of the funds produced better returns than the buy and hold strategy. Carlson also brings up the issue of consistency of the performance of mutual funds and he finds that past

\[^1\] Sharpe performance measure or reward-to-volatility ratio is calculated as:

\[ S = \frac{R_p - R_f}{\sigma_p} \]

where \( R_p \) is the mean return of the portfolio, \( R_f \) is the risk free rate and the \( \sigma_p \) is the standard deviation of the portfolio.
performance results for common stock mutual funds show no consistent predictive value.

Grinblatt and Titman (1994), using 279 funds in the period December 1974 to December 1984, try to provide corrections and improvements on Jensen's alpha, as an approach for evaluating portfolio performance, which has been a subject of the great deal of controversy for the following reasons:

- Benchmark efficiency, which refers to the fact that performance evaluation is sensitive to the choice of the benchmark portfolio, as already noted in the Carlson (1970) study. As noted by in the main criticisms of the CAPM model, there is no observable market portfolio with which to compute beta, implying that Jensen's measure of performance has no relation with the true performance. Grinblatt and Titman use four benchmarks in their analysis and three portfolio performance measures, which will be outlined below. Correlation matrices that examine to which extent the benchmarks matter, suggest that the performance of individual funds depends on the choice of the benchmark. Particularly, negative performance of funds is observed when equally weighted index and factor based indices are used and almost-zero performance with the benchmark formed from securities characteristics.

- Timing ability, which refers to the statistical bias in Jensen's alpha technique, which arises whenever the portfolio under evaluation successfully times the market. In order to correct for this bias, Grinblatt and Titman (1989) proposed the Positive Period Weighting Measure that is not subject to the timing-related bias and Treynor-Mazuy Measure. The definition and derivation of these measures is beyond the scope of this chapter. They find that Jensen's measure and Positive
Period Weighting Measure are virtually identical for majority of the funds regardless of the benchmark, but for those funds that the two measures give different results, the reason may be that those funds do successfully time the market.

- Statistical power, which is related to noisiness of the returns that prevents detecting abnormal performance even when it exists. In particular, authors argue that 2% annual alpha will not usually be statistically significant although, for a fund manager who has $1bn to invest, it represents over $20mn excess return per year. Tests to examine the determinants of mutual funds performance show that the performance is positively related to turnover but not to size or the expenses of mutual funds.

Out of these three criticisms of alpha, the most attention has been given to timing ability of alpha and the examples of those criticisms are given in the following section.

1.5.1. Timing ability of alpha

The problems related to timing ability of Jensen’s alpha can be illustrated by the following graph:
If we assume that the portfolio manager is constrained to invest in either high or low beta portfolio and if the benchmark portfolio is considered to be efficient, the steeper and the flatter line denoting high and low beta portfolio respectively will both pass through the origin of the graph. Additionally, if we assume that an informed investor (i.e. portfolio manager) can receive one of the two signals:

1) Signal that the benchmark will have high return, then he will invest in a high beta portfolio and be in point A or

2) Signal that the benchmark will have low return, then he will invest in a low beta portfolio and be in point B on the graph

However, an uninformed investor would estimate the risk of the investment strategy as the slope of the line connecting points A and B. The slope of that line is steeper than the slope of either low or high beta portfolio implying that such a portfolio has higher risk. The intercept of the line connecting A and B and the y-axis is negative,
implying negative excess returns (alphas) and inferior performance of an informed investor. The empirical studies outlined below show examples that demonstrate that the Jensen's alpha can assign negative performance to a market timer because it is based on an upwardly biased estimate of systematic risk for a market timing investment strategy.

In the paper on optimal utilisation of market forecasts and the evaluation of investment performance, Jensen (1972) (among other things, which are beyond the scope of this chapter), provides the structure for the analysis of measurement problems introduced into the portfolio performance evaluation by market forecasting activities by a portfolio manager. If the manager can forecast future market returns, the simple time series regression will not allow us to separate the excess returns (alphas) that are due to stock selection ability of investment manager from excess returns that are due to his ability to forecast the market. Additionally, Jensen argues that bias problems arise only when the portfolio manager can actually forecast the market.

Admati and Ross (1985) are investigating the problem of measuring investment performance when superior performance is identified with superior information, i.e. if traders possess diverse pieces of private information, superior performance on the basis of better information is a natural process. The authors identify that the key problem to performance evaluation is that the observer has different information set than that of an informed manager or trader, causing the traditional measures of investment performance, such as alphas, to be inappropriate to use. The model used in the paper is a rational expectations equilibrium CAPM, which is characterised by
having many risky assets and a large number of traders who possess diverse pieces of
private information. In other words, in contrast to the traditional CAPM that assumes
homogeneous investors, this model assumes heterogeneous beliefs and asymmetric
information. The model employed is a two-period model, where the trading takes
place only once in the first period and the consumption also takes place only once in
the second period. The findings suggest that an informed trader reacts to the private
information, which is generally correlated with ex-post observable information and
with benchmarks created on the basis of coarser information. Therefore, the returns on
the managed portfolio may seem to an outside observer as: a) mean-variance
inefficient or b) plot below his/her securities market line. The authors recognise that
the model they are using has drawbacks in terms of being a two-period, rather than a
multi-period model.

Dybvig and Ross (1985) analyse the deviations from securities market line (SML)
caused by superior performance that is based on superior information. Roll argues that
the only reason for deviation from the SML is the misspecification of the market
portfolio. Hence, superior performance based on superior information, as suggested in
Admati and Ross (1985), has to be ruled out a priori. In a paper that was assessing
similar issue, Myers and Rice (1979) find that differential information does not
disrupt the validity of the SML. They base their conclusion on two results: a) the first
result that is interpreting SML analysis as it correctly measures the performance of
market participants whose information is security specific and b) the second result
that is interpreted as being valid more generally. Dybvig and Ross criticise that
second result in a sense that they claim it is not general and they use a 'market timing'
example to prove it. According to the authors, if the reference portfolio is considered
to be efficient, a manager who is plotting above the SML has superior information in the context of completely uninformed investor, but one cannot tell whether the information has been used correctly. Also, the model used in the paper suggests that an investor plotting below the SML also has differential information, but this feature would go away in a model in which it is possible to pay transaction costs. On the other hand if the observer has chosen a misspecified index than abnormal returns are simply reflecting the inefficiency, not superior information. In this paper, the manager plots above the observer's SML in the case when there is a risk-free asset and the manager knows nothing about the returns of the uninformed observer's portfolio. However, this is not enough to justify why there is an extensive use of SML analysis in performance evaluation. Therefore, there should be a new performance measurement technique, which should correctly identify superior, ordinary and inferior performance, it should be immune to gaming (meaning that it should work correctly even when the manager being evaluated understands the measure that is being used) and the measure should be sufficiently powerful to be useful in practice. This last requirement seems to be the most important one, but at the same time, the most difficult to implement in practice. This is due to the fact that there is a large amount of noise in common stock returns that makes it impossible to measure significant superior or inferior performance over time periods short enough to be useful.

### 1.6. Persistence of Positive Alphas

Once investors have established how to measure the performance of portfolios, the question arises: how persistent the performance of the portfolio can be? In other words, the question that needs to be answered is referring to the possibility of creating
a trading strategy based on positive alpha portfolios and consistently outperforming the market. The efficient market hypothesis implies that past performance is no guide to future performance after adjusting for risk or other pricing factors. Let us revise first the available empirical evidence related to this issue.

Carhart (1997) is using a CAPM, 3-factor and 4-factor models to estimate alphas of 1892 diversified equity funds over the period 1962 to 1993. The study suggests that mutual fund investors should a) avoid funds with persistently poor performance, b) funds with large returns last year have higher than average returns in the next year but not the years after and c) expense ratios, transaction costs and load fees are negatively related to the performance of the mutual funds. Therefore the evidence that managers can deliver consistently positive alphas is not very strong. The question that stems from this conclusion is: can one apply similar conclusions to equity portfolios?

Christopherson, Ferson and Glassman (1998) study the persistence of the performance of 185 US pension funds in the period 1979-1990. They assume that pension funds have time-varying conditional betas, investment style factor exposures and time-varying conditional alphas. The paper finds evidence in support of the persistence of the performance of pension funds. Specifically, managers that were generating low conditional alphas in the past tend to be abnormally low return managers in the future. No such a strong conclusion has been provided for the managers having positive alphas in the past. The authors raise the question of survivorship of the poor performing investment managers. The authors also argue that the conditional measures offer more information to investors about future performance than the unconditional ones.
Kahn and Rudd (1995) try to establish if ‘last year’s winners are repeating’. To assess the performance of a mutual fund authors use selection or style adjusted returns and information ratios. Style adjusted returns are measuring the fund’s performance against the ‘style’ benchmark (value, growth, small, large index for example). Information ratio is defined as follows:

\[ P = \frac{\alpha_i}{\sigma_i} \]

and it represents the excess return of the fund (alpha) per unit of fund’s specific risk (\(\sigma_i\)).

The persistence of performance between periods 1 and 2 is measured by regressing period 2 performance against period 1 performance:

\[ \text{Performance}(2) = a + b\text{Performance}(1) + \epsilon \]

Where ‘performance’ can be cumulative total returns, cumulative style adjusted returns or information ratios. If coefficient b is positive, it is considered that period 1 performance contains information for predicting period 2 performance and hence, the evidence of persistence exists. The persistence of performance was not found among 300 equity funds in the early 1990s. This implies that investors, unless they have another basis for choosing winners, should not base their investment decision on the past performance of funds. In conclusion, only with statistically significant past performance, investors should choose active managers, otherwise, stick to the passive.
For the time period 1993-1997, Gupta, Prajogi and Stubbs (1999) find that the top quartile managers added value (generated positive alphas) in all asset classes, namely US fixed income, US small cap, US large cap, International fixed income, International equity and Emerging markets equity. However, as far as persistence of performance is concerned, 1993-1997 period was decomposed in two periods of three and two years and only top quartile managers managing US large and US small cap portfolios were not able to repeat their performance from the first period. The authors argue that the information ratio is the strongest predictor of persistence of manager performance because it has the strongest correlation with the number of quarters in which managers have outperformed. They also analyse the relationship between tracking error and alphas (and persistence of alphas) because they believe that it would help investors to manage risk (as measured by tracking error) across their portfolios. They find that for US and international fixed income, US small cap and international equity, the information ratio is maximised in the 2%-4% tracking error range. For US large cap equity, the ratio is maximised at 1%-2% of tracking error, while for emerging market equity managers, information ratio is maximised at low or high end of the tracking error range of 6%-12%.

Goetzmann and Ibbotson (1994) in their study assess the performance of 728 mutual funds over the period 1976 through 1988. There is a potential survivorship bias in the data since some of the mutual funds that were closed down during the sample period are excluded from the analysis. The authors analyse three types of fund returns:

1. Two-year return intervals, to identify the impact of the long-run performance. Particularly, two-year returns for the period 1976-1977 were used to predict performance for the subsequent two years, 1978-1979. The raw returns and alpha
risk adjustment analysis suggests that winner funds (those that have returns above the median) are repeating. The results are significant at 5% confidence level in three out of five periods and combined results are highly significant.

2. One-year returns, in order to investigate potential survivorship bias. The funds were split into high and low variability funds because it is argued that high variability funds could have more survivorship bias (i.e. they are less likely to survive). Also, if the most volatile funds survive, they are likely to have the best performance. Hence, it appears that we will have repeat winners, since some winner/losers and some loser/losers will not survive. The analysis of the data supports the 'repeat winners' hypothesis as in the previous case and indicates existence of the survivorship bias.

3. Monthly returns, in order to maximise the number of independent time periods so as to be able to identify style factors or common variables other than the market that influence cross-sectional dependence in funds' returns. The results indicate that there is a long-term and short-term performance difference among funds and support 'repeat winners' hypothesis as well. Authors suggest that some of that difference may be due to risk factors not corrected by alphas, timing strategies, fees or it may be consistent with differences in fund management skills.

Ferguson (1986) imposes some criticism upon different performance measures, alpha in particular. Ferguson states that the betas of securities in the CAPM depend on the choice of the market portfolio proxy and that alphas in turn, depend upon betas. He furthermore argues that using alphas as performance measurement is as bad as using returns, particularly because one concentrates more on the values of alphas rather than standard deviations of alphas. Hence, he has introduced the information or appraisal
ratio (as defined earlier in this section), that takes into account the standard deviation of alpha. Ferguson introduces another measure of portfolio attractiveness that refers to the significance level of the portfolio’s alpha. Simply, Ferguson treated the appraisal ratio as ‘t’ value in statistics, computed its cumulative probability and scaled it by 100. No empirical support of use of this performance measure was given in this paper. Ferguson concludes that performance measurement is waste of time because nobody will ever know how to measure investment performance and nobody would want to measure investment performance even if they did know how.

1.7. Motivation

As the above reviewed literature is suggesting, alpha is very widely used as a measure of investment performance and as an ultimate long-term objective of the investment manager. Most of the studies outlined in sections 1.1. to 1.6. are analysing the performance and persistence of performance of mutual funds, pension funds or already existent equity (stock market) indices. There are several issues that we would like to address in this chapter:

a) Find evidence of mispricing. In particular, we will use securities’ alphas as an indicator of underpricing or overpricing of securities on the LSE.

b) Test the size of alphas. In particular, we will examine if the alphas are large enough to provide reasonable profits for investors.

c) Test the persistence of alphas, i.e. test for how long will investors buying stocks based on historical alpha values or portfolio performance measures be able to outperform the market.
d) Finally, once analysed issues in a), b) and c), we will give an answer if alpha-based strategies do work. Specifically, we will answer is if it is possible to develop a trading strategy in which investors could invest in portfolios of stocks generating positive alphas in one time period and be sure that those portfolios will provide them with superior returns in the future.
2. Data

The data for this chapter is gathered from the London Share Price Database (LSPD) monthly returns file. The criteria employed in selecting the firms in the sample in a particular year are:

a) the firm has 5 years of available data used to estimate alphas or excess returns before it was included in the sample and

b) firm’s returns were available on LSPD monthly returns file.

In LSPD, there is always a missing return in the first month of trading or the first month the company data has been collected. The period covered in the analysis is from January 1980 through to December 1996. Data prior to 1980 is used to estimate alphas or excess returns of individual companies.

Since the purpose of this chapter is to examine the persistence of performance of positive/negative alpha portfolios and the predictive nature of alphas, we have used our sample of companies to create positive and negative alpha portfolios. The procedure used to form positive and negative alpha portfolios is as follows:

1. Jensen alphas for each stock are estimated using a model as in (1.3). In particular, we have used 60 months prior to January 1980 to estimate alpha of each stock in that month and the procedure was repeated until the end of the sample period. The excess return over the risk free rate of individual stocks and the market was calculated using UK 1 month Treasury bill. As a proxy for the market return we have used the returns of the FTSE All Share Index.

2. Once the alphas of all stocks in each month were estimated, a criteria for inclusion of the stock in a portfolio was created. Specifically, all the stocks that in a
particular month had positive or negative alphas at 10% significance level or less were included in our portfolios. 10% level of significance was chosen to enable us to include more stocks in our analysis and to create as diversified portfolios as possible. Due to a very small number of negative alpha stocks in January and February 1980, we have started forming our portfolios in March 1980.

3. Both positive and negative alpha portfolios are equally weighted and rebalanced on a monthly basis. Portfolio returns are weighted averages of the returns of individual securities in the portfolio.

Hence, one can conclude that past alphas are used as a criteria for forming positive and negative alpha portfolios.
3. Methodology

3.1. Model for measuring persistence of performance

The approach to measuring persistence in this chapter is based on a regression of future excess returns on a measure of past performance or alpha. In particular, the model applied takes the form of the equation (1.4), as in Christopherson, Ferson and Glassman (1998):

\[ R_{p(t,t+\tau)} = \delta_{0,t} + \delta_{1,t} \alpha_{p,t} + u_{p(t,t+\tau)} \quad (1.4) \]

where \( R_{p(t,t+\tau)} \) is the compounded excess return from month \( t \) to month \( t + \tau \) for alpha portfolio \( p \), measured over UK 1 month Treasury bill. Symbol \( \tau \) is denoting the return horizon of 1, 3, 6, 12, 18, 24 and 36 months. \( \alpha_{p,t} \) is alpha measure of the past abnormal performance, estimated using data series upto month \( t \). \( u_{p(t,t+\tau)} \) is the regression error and \( \delta_{1,t} \) is a slope coefficient to be estimated in the regression. If the slope coefficient is statistically different than zero, it means that alpha can be used to predict the future return, i.e. that there is persistence in performance of portfolio \( p \). Equation (1.4) is estimated by generalised least squares (GLS) using a weighted least squares (WLS) approach.

3.2. Generalised Least Squares (GLS) model

If we were to estimate \( \delta_{1,t} \) by using usual OLS estimation procedure, it would not make use of the ‘information’ contained in the unequal variability of the dependent variable, i.e. the compounded excess returns in our case. OLS method assigns equal weight or importance to each observation. In the case of GLS, the heteroskedasticity
is explicitly taken into account and the method is therefore able to produce estimates that are best linear unbiased estimators (BLUE). There are some consequences that can bring bias to the results in the case of using OLS model in the presence of heteroskedasticity. In particular, if one would assume heteroskedasticity and apply OLS model, the F-test and t-test of such a model are likely to give inaccurate results. The inaccuracy would be in that variance of $\hat{\delta}_{t,t^*}$ is too large and that $\hat{\delta}_{t,t^*}$ is statistically insignificant due to the fact that t-value is smaller than what is appropriate. However, if the correct confidence intervals were used, the coefficient may be statistically significant. The procedure which will enable us to obtain the correct confidence intervals in the presence of heteroskedasticity is GLS. On the other hand, a more likely case when applying the OLS model is the one in which heteroskedasticity is ignored. In this case one would obtain OLS standard errors that are too large (for intercept) or too small (for slope coefficient) in relation to those obtained by those OLS allowing for heteroskedasticity. In any case, if there is heteroskedasticity, one should use GLS, although it is not always easy to apply in practice. Finally, as Kandel and Stambaugh (1995) suggest, GLS is superior to OLS in cross-sectional stock return regressions.

### 3.2.1. Derivation of GLS

Let us start from a two-variable model:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \quad (1.5)$$

which for ease of algebraic manipulation we can re-write as:

$$Y_i = \beta_1 X_{0i} + \beta_2 X_i + u_i \quad (1.6)$$

where $X_{0i}=1$ for each $i$
There are several ways in which one can transform variables from equation (1.6) to eliminate heteroskedasticity. We will present one we believe is the most appropriate in our analysis, i.e. derivation of weighted least squares.

If one assumes that heteroskedastic variances $\sigma_i^2$ are known, (1.6) can be divided through by $\sigma_i$ to obtain:

$$\frac{Y_i}{\sigma_i} = \beta_1 \frac{X_{oi}}{\sigma_i} + \beta_2 \frac{X_1}{\sigma_i} + \frac{u_i}{\sigma_i}$$  \hspace{1cm} (1.7)

which can be re-written as:

$$Y_i^* = \beta_1^* X_{oi}^* + \beta_2^* X_1^* + u_i^*$$  \hspace{1cm} (1.8)

to obtain transformed variables. The star sign in parameters $\beta_1^*$ and $\beta_2^*$ is used to distinguish them from the OLS parameters $\beta_1$ and $\beta_2$. The purpose of transforming the original OLS model from (1.5) can be seen through one feature of the transformed error term $u_i^*$:

$$\text{var}(u_i^*) = E(u_i^*)^2 = E\left(\frac{u_i}{\sigma_i}\right)^2$$

Since $\sigma_i^2$ is known, we have:

$$\text{var}(u_i^*) = \frac{1}{\sigma_i^2} E(u_i^2)$$

Also, since $E(u_i^2) = \sigma_i^2$, we have:

$$\text{var}(u_i^*) = \frac{1}{\sigma_i^2} (\sigma_i^2) = 1$$

Therefore, through the transformation, we have obtained that the variance of the transformed error term is now constant or homoskedastic. If the OLS procedure is applied to the transformed model, which still satisfies the assumptions of the classical
OLS model, we would obtain BLUE estimates. Hence, a procedure in which OLS is applied to transformed variables that satisfy the standard least squares assumptions is called generalised least squares or GLS. In order to obtain the GLS estimators, we must follow several steps. First, we write the equation (1.7) in sample regression form to obtain:

\[
\frac{Y_i}{\sigma_i} = \hat{\beta}_1 \frac{X_{oi}}{\sigma_i} + \hat{\beta}_2 \frac{X_i}{\sigma_i} + \frac{\hat{u}_i}{\sigma_i}
\]

or

\[
Y_i^* = \hat{\beta}_1^* X_{oi}^* + \hat{\beta}_2^* X_i^* + \hat{u}_i^*
\]

The second step is to minimise the sum of squared residuals:

\[
\sum \hat{u}_i^2 = \sum (Y_i^* - \hat{\beta}_1^* X_{oi}^* - \hat{\beta}_2^* X_i^*)^2
\]

or

\[
\sum \left( \frac{\hat{u}_i}{\sigma_i} \right)^2 = \sum \left( \frac{Y_i}{\sigma_i} - \hat{\beta}_1 \left( \frac{X_{oi}}{\sigma_i} \right) - \hat{\beta}_2 \left( \frac{X_i}{\sigma_i} \right) \right)^2
\]

If we regard \( \frac{1}{\sigma_i^2} \) as weight, \( w_i \), assigned to least squares estimators, i.e. if we assume that the weight is inversely proportional to the variance of \( u_i \) or \( Y_i \) conditional upon the given \( X_i \). Therefore, it follows that \( \text{var}(u_i \mid X_i) = \text{var}(Y_i \mid X_i) = \sigma_i^2 \). Since we have established at the beginning of this section that \( X_{oi} = 1 \) for each \( i \), then

\[
\frac{X_i}{\sigma_i^2} = \frac{1}{\sigma_i^2} = w_i \text{ and equation (1.9) can be written as:}
\]

\[
\sum w_i \hat{u}_i^2 = \sum w_i (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2
\]

Observations that are coming from the population with larger \( \sigma_i \) will get relatively smaller weight and those with smaller \( \sigma_i \) will get proportionally larger weight in minimising the sum of residual squares.
Differentiating (1.10) with respect to $\hat{\beta}_1^*$ and $\hat{\beta}_2^*$, we obtain:

$$\frac{\partial \sum w_i \hat{u}_i^2}{\partial \beta_1^*} = 2 \sum w_i (Y_i - \hat{\beta}_1^* - \hat{\beta}_2^* X_i)(-1)$$

and

$$\frac{\partial \sum w_i \hat{u}_i^2}{\partial \beta_2^*} = 2 \sum w_i (Y_i - \hat{\beta}_1^* - \hat{\beta}_2^* X_i)(-X_i)$$

Setting the two expressions above equal to zero, we obtain the following two equations:

$$\sum w_i Y_i = \beta_1^* \sum w_i + \beta_2^* \sum w_i X_i$$

$$\sum w_i X_i Y_i = \beta_1^* \sum w_i X_i + \beta_2^* \sum w_i X_i^2$$

Solving these two equations simultaneously, we obtain:

$$\hat{\beta}_1^* = \bar{Y}^* - \hat{\beta}_2^* \bar{X}^*$$

and

$$\hat{\beta}_2^* = \frac{(\sum w_i)(\sum w_i X_i Y_i) - (\sum w_i X_i)(\sum w_i Y_i)}{(\sum w_i)(\sum w_i X_i^2) - (\sum w_i X_i)^2}$$

Note that:

$$\bar{Y}^* = \frac{(\sum w_i Y_i)}{\sum w_i}$$

and

$$\bar{X}^* = \frac{(\sum w_i X_i)}{\sum w_i}$$

When $w_i = w$, a constant for all i, the weighted means in the equation above coincide with the unweighted (or equally weighted) means from the OLS. Since (1.10) minimises a weighted sum of residual squares it is more appropriate to refer to it as a weighted least squares (WLS). WLS is just a special case of the more general estimation technique, GLS. In the context of heteroskedasticity, one can treat WLS and GLS interchangeably.
Using WLS (GLS) in our chapter has one particular advantage. When in the model as in (1.4) weight of \( \frac{1}{\sigma_i^2} \) is assigned to \( \alpha_{pi} \), we obtain essentially an appraisal or information ratio that is used in assessing portfolio performance and it reduces the cross-sectional differences related to variance. Additional advantage is related to heteroskedasticity in data, which was already explained earlier in this section. The drawbacks of the GLS model related to the transformations of the variables are as follows:

a) When there are more than two variables in the model, it would be a problem to decide a priori which of the \( X \) variables should be chosen for transforming the data. Since we are applying two-variable model, we do not face this problem.

b) It is considered that log transformation such as:

\[
\ln Y_i = \beta_1 + \beta_2 \ln X_i + u_i
\]

very often reduces heteroskedasticity when compared with the regression from (1.5). However, although beneficial, such a transformation is not plausible if some of the values of \( X \) or \( Y \) are zero or negative. Hence, we have not decided to use log transformation but rather weighted one.

c) Problem of spurious correlation, i.e. the correlation between the ratios of variables may exist although the original variables are not correlated or random. That is why we will not use the ratio of variables in our transformations.

d) When \( \sigma_i^2 \) is not directly known, all testing procedures used would be valid in large samples. Therefore, one should be cautious in interpreting the results based on the transformations done in small or finite samples.
4. Analysis of the results

4.1. Characteristics of portfolios

If a simple trading strategy of investing in positive or negative past alpha portfolios is employed, we obtain two portfolios with characteristics as in Table 1.

Table 1: Some characteristics of positive and negative alpha portfolios, March 1980-December 1996

<table>
<thead>
<tr>
<th>Portfolio Type</th>
<th>Mean Return</th>
<th>Mean Alphas</th>
<th>Standard deviation</th>
<th>Unconditional Alphas</th>
<th>Unconditional Betas</th>
<th>Min/Max Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Alpha Portfolio</td>
<td>0.45</td>
<td>2.31</td>
<td>4.52</td>
<td>0.05</td>
<td>0.78</td>
<td>-25.51/+12.38</td>
</tr>
<tr>
<td>Negative Alpha Portfolio</td>
<td>0.21</td>
<td>-3.24</td>
<td>7.19</td>
<td>-0.31</td>
<td>0.91</td>
<td>-25.35/+19.58</td>
</tr>
</tbody>
</table>

Mean return represents the average monthly return of the equally weighted positive or negative alpha portfolio, expressed as a percentage. As expected positive alpha portfolio has higher mean return than the negative alpha portfolio by 0.24% per month or 2.88% per year. Christopherson, Ferson and Glassman (1998) report the difference between mean returns of high alpha and low alpha quintile of only 0.2% per year. The difference between these two findings may stem from the fact that Christopherson et al. use quintile portfolios and hence do not differentiate portfolio alpha characteristics as much as we do. In other words, they do not report whether their lowest alpha portfolio includes stocks with negative past alphas or both low positive and negative alpha stocks. Also, Christopherson et al. are not using past performance of individual stocks but fund managers, in particular, pension funds. They state that their database...
almost certainly has survivorship bias since it contains only surviving managers and selection bias as the managers enter database after they attract the attention. Our sample based on the individual stocks avoids both of the aforementioned biases.

Mean alphas represent the average portfolio alphas that are calculated as a weighted average of alphas of individual securities in a portfolio. It is expected, due to the method of portfolio construction, that the mean historical alphas of positive alpha portfolio are positive and of negative alpha portfolio to be negative.

Unconditional Jensen alphas for positive (negative) alpha portfolio are estimated using rolling regressions of positive (negative) alpha portfolio returns over the risk free rate on the market returns in excess of the risk free rate. It was found that positive alpha portfolio generates small positive average alphas over the time period analysed (0.05% per month or 0.6% per year) and negative alpha portfolio generates negative alpha (-0.31% per month or -3.72% per year). Therefore, the average alphas obtained in this way, especially for the positive alpha portfolio do not appear to be large enough to suggest a trading strategy of buying stocks with positive historical alphas and being able to outperform the market. However, the spread between the positive and negative portfolio alpha is 4.23% per year which suggests that one may create a long/short equity portfolio by going long in positive alpha stocks and short in negative alpha stocks and only moderately benefit from the spread in alphas. It should be mentioned here that although the estimated unconditional alpha of the positive (negative) alpha portfolio is expected to be positive (negative), it is not necessarily always the case. Our analysis shows that positive (negative) alpha portfolio generates negative (positive) alphas in 59 (61) out of 144 months estimated but they are
significantly negative (positive) only 5 (9) out of those 59 (61) months. However, when positive (negative) alpha portfolio generated positive (negative) unconditional alpha, it was significant in most of the cases. Therefore, our analysis suggests that alphas of both portfolios are not stable over time. Also, as mentioned above, the statistical significance of these alpha coefficients in table 1 is somewhat in doubt, since in the large number of cases unconditional alphas estimated were not significant. This is consistent with the findings of Christopherson et. al., who say that in their study that 'alphas of future returns are not statistically different than zero'. However if were to recalculate average excess return measured by alpha of positive and negative alpha portfolio taking into account significant alphas and assigning value of zero to all insignificant ones, our findings change dramatically: average annual alpha for the positive alpha portfolio becomes 4.29% and for the negative alpha portfolio it becomes -13.06%, creating a positive/negative alpha spread of 17.35%!

Let us now analyse the risk characteristics of our portfolios. Negative alpha portfolio has larger average monthly standard deviation and beta, in other words higher total risk and systematic risk than the positive alpha portfolio. This finding is consistent with systematic bias in the unconditional CAPM, suggested in a study by Fama and MacBeth (1973), whose results show an inverse relationship between return (as measured by alpha) and risk (as measured by beta and standard deviation).

The last column in Table 1 shows the minimum and maximum return of each portfolio under consideration. If there is a persistence in performance of high and low alpha portfolios to be identified, then a positive alpha portfolio will generate higher future returns than the negative alpha portfolio. In table 1, where return horizon is one
month, we can observe that positive alpha portfolio has higher future mean returns but alphas of both positive and negative alpha portfolio are in many cases statistically equal to zero, making their excess returns in essence zero. In conclusion, the results from Table 1 seem to provide little reliable information about predictive power of past unconditional alphas on the future abnormal portfolio performance. Let us now observe whether our conclusions change when we apply econometrics test to assess predictive power of alphas over one month as well as over longer time horizons. In other words, let us see if the performance of positive/negative alpha portfolio can persist in the future.

4.2. Evidence of Performance Persistence

In the introduction to this thesis we outline that according to commonly accepted asset pricing model, CAPM, risk adjusted stock (portfolio) returns are determined by alpha, beta and the market risk premium:

$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + u_i$ (1.11)

In the above equation, alpha represents the excess return or reward that an investor receives for bearing unsystematic risk whereas beta represents the measure of systematic risk. In general, rational investors take into the account both risk and return variables when making investment decisions, and hence they are interested in excess returns per unit of risk. We also know that the total risk of a security (portfolio) is consisted of both unsystematic and systematic risk. Therefore, dividing the equation (1.11) with unsystematic risk (standard deviation of residuals from 1.11) and systematic risk (portfolio beta) we obtain:
Equations (1.12a) and (1.12b) are suggesting two portfolio performance measures, based on different types of risk:

a) \( \frac{\alpha_i}{\sigma_i} \), which represents the information or appraisal ratio which can easily be obtained through WLS procedure as it was outlined in section 3.2.1 and

b) \( \frac{\alpha_i}{\beta_i} \), alpha per unit of systematic risk, which may be regarded as a modified version of the Treynor ratio

In the following two sections we will examine whether future returns of positive and negative alpha portfolios can be predicted on the basis of past portfolio performance measures as defined in a) and b) above.

In essence, we have created a 'winner' and a 'loser' portfolio based on the historical performance of individual stocks, using the Jensen alpha procedure explained in section 2. The winner/loser portfolio concept was first introduced by DeBondt and Thaler (1985) but the criteria for including stocks into portfolios in their study was different. In particular they have used past cumulative abnormal returns to distinguish between winners and losers. DeBondt and Thaler find that loser portfolios start

\[ \frac{R_i - R_f}{\sigma_i} = \frac{\alpha_i}{\sigma_i} + \frac{\beta_i}{\sigma_i} (R_m - R_f) + \frac{u_i}{\sigma_i} \]  

\[ \frac{R_i - R_f}{\beta_i} = \frac{\alpha_i}{\beta_i} + \frac{\beta_i}{\beta_i} (R_m - R_f) + \frac{u_i}{\beta_i} \]
outperforming the market three years after portfolio formation while winner stocks underperform the market after the same time interval. In other words, they detect the reversal in performance. Let us see now how that compares with our findings.

The issue that we specifically want to address in this section is whether past estimated portfolio performance of the winner (positive historical alpha) portfolio or the loser (negative historical alpha) portfolio can indicate the investment strategy one should pursue in order to persistently outperform the market. More importantly, we will examine how long the strategy should be pursued for.

4.2.1. Information ratio as an indicator of persistence of performance

As we have outlined above in the section 3.1., we are measuring persistence of performance of positive and negative alpha portfolio by using predictive regression from equation (1.4). It is considered to be predictive regression because estimates of alpha are based on past data only. We have tried to establish predictive power of alpha by using past alphas to predict returns of 1, 3, 6, 12, 18, 24 and 36 months horizons. By using the future 1, 3, 6, 12, 18, 24 and 36 months compounded returns as the dependent variable in equation (1.4) we would be able to test the extent to which past alphas determine future returns. Compounded returns are calculated using geometric progression as:

\[(1 + r_{t+\tau}) = (1 + r_1)(1 + r_2) \ldots (1 + r_{\tau}),\]

\[\tau = 1,3,6,12,18,24,36\]
As suggested by Christopherson, Ferson and Glassman (1998), the alternative approach would be to use alpha of the future subperiod as the dependent variable but that would result in correlation of alphas due to the fact that factors causing bias in positive (negative) alphas such as size or yield effect are correlated over time. That correlation can create spurious evidence of performance persistence.

However, before we proceed with weighted least squares estimation of equation (1.4) in which in effect we use appraisal (information) ratio as an independent variable, let us first estimate the coefficients of (1.4) with the OLS procedure that may give us some insight in the predictive power of past alphas for positive and negative alpha portfolio. Specifically, in this estimation procedure independent variable is past alpha, termed as unconditional alpha in Table 1, and the dependent one future return. We are testing the null hypothesis that the $\delta_{t, r}^{\tau}$ coefficient is zero against the alternative hypothesis that it is different from zero. When we use in the estimation procedure return horizon longer than one month ($\tau > 1$), we have autocorrelation in residuals due to overlapping data and hence, biased t-statistics of coefficients estimated. In order to account for this autocorrelation, we use Newey-West adjusted variance-covariance matrix with $\tau - 1$ moving average terms. In particular, Table 2 shows the OLS, Newey-West adjusted alpha coefficients ($\delta_{t, r}^{\tau} \gamma$) for both positive and negative alpha portfolios. Time series used to estimate $\delta_{t, r}^{\tau}$ is From March 1985 through to December 1996 - $\tau$. 
Table 2: Past Alpha As a Measure of Persistence of Future Portfolio Returns.

OLS estimation

<table>
<thead>
<tr>
<th>Return Horizon</th>
<th>Positive Alpha Portfolio</th>
<th>Negative Alpha Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta_{t,t}$</td>
<td>t-ratios</td>
</tr>
<tr>
<td></td>
<td>(Newey-West Adjusted)</td>
<td></td>
</tr>
<tr>
<td>1 month</td>
<td>-0.26</td>
<td>-0.27</td>
</tr>
<tr>
<td>3 months</td>
<td>-1.47</td>
<td>-0.43</td>
</tr>
<tr>
<td>6 months</td>
<td>-4.38</td>
<td>-0.72</td>
</tr>
<tr>
<td>12 months</td>
<td>-14.88</td>
<td>-1.71**</td>
</tr>
<tr>
<td>18 months</td>
<td>-26.79</td>
<td>-2.31*</td>
</tr>
<tr>
<td>24 months</td>
<td>-36.29</td>
<td>-2.95*</td>
</tr>
<tr>
<td>36 months</td>
<td>-62.88</td>
<td>-5.60*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 5% level or less and ** indicates statistical significance at 10%

Table 2 suggests that unconditional alphas of positive alpha (winner) portfolios provide little evidence of predictive ability over the short term horizons. In particular, as the future return horizon is increased, evidence of negative relationship between alphas of positive alpha portfolios and future returns appears. Furthermore, the longer the return horizon, the stronger the relationship between the two variables is. In other words, the longer the return horizon is, the more likely it is that if positive alpha portfolios were generating positive excess returns in the past, they will generate lower returns in the future and vice versa. Such, or any other pattern can not be determined in the relationship between negative alpha portfolio returns and their alphas.
However, we have established in section 3 that it would be more appropriate to apply weighted least squares procedure for estimation of equation (1.4), where the weights used would be the reciprocal value of the standard deviation of residuals from the time series model that was used to estimate alphas. As explained in section 3.2.1., in effect we use information ratio as an independent variable in this procedure. Table 3 shows the results generated using WLS approach to estimate (1.4).

**Table 3: Past Alpha (Information Ratio) As a Measure of Persistence of Future Portfolio Returns, WLS estimation**

<table>
<thead>
<tr>
<th>Return Horizon</th>
<th>Positive Alpha Portfolio</th>
<th>Negative Alpha Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta_{t,t,r}$</td>
<td>$t$-ratios</td>
</tr>
<tr>
<td></td>
<td>(Newey-West Adjusted)</td>
<td></td>
</tr>
<tr>
<td>1 month</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>3 months</td>
<td>2.38</td>
<td>0.54</td>
</tr>
<tr>
<td>6 months</td>
<td>1.77</td>
<td>0.20</td>
</tr>
<tr>
<td>12 months</td>
<td>-7.89</td>
<td>-0.69</td>
</tr>
<tr>
<td>18 months</td>
<td>-17.30</td>
<td>-1.32</td>
</tr>
<tr>
<td>24 months</td>
<td>-24.61</td>
<td>-1.96*</td>
</tr>
<tr>
<td>36 months</td>
<td>-51.46</td>
<td>-4.03*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 5% level or less and ** indicates statistical significance at 10%
The findings from table 3 are very similar to the ones in table 2 obtained through an OLS procedure. Predictive ability of information ratio (alpha) in the short-term horizons is non-existent for both investors investing in positive and negative alpha portfolios. It appears that information ratio (alpha) has predictive ability for positive alpha portfolios after two years. In particular, positive past information ratios (alphas) indicate negative future returns and vice versa. For negative alpha portfolios we also find significant negative relationship between past information ratios (alphas) and future returns when the investment horizon is 3 years, i.e. if a loser portfolio generated relatively low prior information ratios (alphas), they will tend to have relatively high future returns and vice versa. However, the results for the negative alpha portfolio are not as robust as the findings corresponding to positive alpha portfolio. We can also see for both positive and negative alpha portfolios that the magnitude of coefficients is larger for longer time horizons. Christopherson, Ferson and Glassman (1998) perform similar analysis on US pension funds data and use different measures of alpha as determinants of past performance. Similarly to our results, they find that unconditional alphas can be used for assessing persistence of performance for negative alpha funds in the longer time horizons, i.e. horizons longer than 18 months.

Most of the prior evidence is suggesting that persistence is associated mainly with poor performing mutual fund managers. However, in our analysis of stock portfolios, we find the evidence of the reversal of performance of positive and negative alpha portfolios as the investment horizon is increased. Such returns behaviour of positive and negative alpha portfolios appears to be related to mean reversion return behaviour. There is a substantial body of literature which shows that prices have a tendency to revert to their mean over three to five year period (Fama and French.
(1988b) and Poterba and Summers (1988) for example). Poterba and Summers (1988) suggest that stock returns show positive correlation over short periods and negative correlation over longer intervals. They have used data sets for various countries to prove this, among others they have used FT Actuaries Share Price index in the period 1939-1986 to represent the UK market. They find that the UK market displays mean reversion at longer time horizons and positive serial correlation among returns at horizons of less than 12 months. Although in our study we observe positive (however, not significant) relationship between past alphas and compounded returns in shorter time horizons for positive alpha portfolio, we do not find such a relationship for negative alpha portfolios. Therefore, in our data sample, we can conclude that in the short time horizons past alphas are not very good indicators of future performance. However, as the returns horizon is extended to two or three years, we observe that past alphas have predictive power. Such observed predictive power is stronger for positive alpha portfolio rather than for the negative one. Also we find that predictive power of past alphas is improving with an increase in investment horizon. Additionally, Carhart (1997) finds that one should avoid mutual funds with persistently poor performance, whereas funds with higher returns last year will have higher than expected returns in the next year but not in years after that. These findings again indicate mean reversion of returns of best performing funds.

To conclude this section, we have established that good (poor) past performance based on the information ratio performance measure is not necessarily an indicator of a good (poor) future performance in the short and in the long run. We find that alpha based strategies in the short run are not very successful since no significant relationship between past alphas and future returns was identified. However, alpha
based portfolio picking strategies do work over the longer run investment horizons of minimum two to three years, suggesting a mean reversion pattern in portfolio returns.

4.2.2. Alpha/Beta ratio as an indicator of persistence of performance

In this section we examine the predictive power of alpha/beta ratio portfolio performance measure for estimating future returns. We apply the same procedure as in the previous section except that we transform least squared estimates by weighting individual observations with 1/beta. Results are presented in the Table 4 below and are similar to the results obtained when we used Information ratio in the procedure.

Table 4: Past Alpha/Beta Ratio As a Measure of Persistence of Future Portfolio

<table>
<thead>
<tr>
<th>Returns, WLS estimation, weight used: 1/beta</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Return</th>
<th>Positive Alpha Portfolio</th>
<th>Negative Alpha Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>δ_{i,t}, t-ratios</td>
<td>R²</td>
</tr>
<tr>
<td>Horison</td>
<td>(Newey-West Adjusted)</td>
<td></td>
</tr>
<tr>
<td>1 month</td>
<td>0.07</td>
<td>0.071</td>
</tr>
<tr>
<td>3 months</td>
<td>-0.60</td>
<td>-0.15</td>
</tr>
<tr>
<td>6 months</td>
<td>-2.97</td>
<td>-0.29</td>
</tr>
<tr>
<td>12 months</td>
<td>-13.90</td>
<td>-1.48</td>
</tr>
<tr>
<td>18 months</td>
<td>-24.51</td>
<td>-2.16*</td>
</tr>
<tr>
<td>24 months</td>
<td>-32.01</td>
<td>-2.70*</td>
</tr>
<tr>
<td>36 months</td>
<td>-59.11</td>
<td>-5.21*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 5% level or less and ** indicates statistical significance at 10%
One can see that after 18 months, we observe negative and significant $\delta_{t,t+1}$ coefficient for positive alpha portfolio. In other words, our findings suggest that in the short run, in investment horizons shorter than 18 months, alpha/beta ratio does not have a predictive power for positive alpha portfolio returns. However, when the investment horizon is extended to 18 months or over, we observe significant reversal in performance of the positive alpha portfolios. Similar to findings in table 2 and 3, we cannot identify significant relationship between past alpha/beta ratio and future returns of negative alpha portfolios.

The results in this chapter could be improved by using conditional alpha estimates, where alpha is allowed to vary over time. This is a topic that we can expand on in the future research.
5. Conclusions

This chapter provides the analysis of the performance and persistence of performance of positive alpha and negative alpha portfolios. We find that if we take into account only the months over which the excess return of positive alpha and negative alpha portfolio was statistically significant and assign value of zero alpha to all other months, the return spread between the two portfolios is 17.35%. This is representing the return one could have generated if the long/short investment strategy was applied over our sample period. Using UK data for individual stocks we have created positive and negative alpha portfolios on the basis of past unconditional alphas. We test whether past information about alphas, information ratio and alpha/beta ratio has predictive power for determining future returns of positive and negative alpha portfolios. Based on the information contained in past alphas, we find that although in short time period horizons (1 month) positive alpha portfolios seem to maintain their performance (however not significantly) it does not appear to be so in the horizons longer than 24 months. Applying weighted least squares methodology to regress future returns on past appraisal ratios of positive and negative alpha portfolios, we find that there is evidence of reversal of performance for both portfolios. In particular, the returns of winner (positive alpha) portfolios exhibit mean reversion after 24 months and the returns of the loser (negative alpha) portfolio revert their performance after 36 months. This is similar to the mean reversion findings in Fama and French (1988b) and Poterba and Summers (1988) who suggest that mean reversion in common stock returns occurs over three to five year period. Therefore, past information ratios (alphas) are good indicators of the future performance in longer time period horizons, whereas over short-term horizons there seems to be no reliable relationship between past information ratios (alphas) and future returns. Furthermore
if we transform our data to introduce alpha/beta ratio as a portfolio performance measure, we find similar results as before: alpha/beta ratio has predictive power for positive alpha portfolio returns in investment horizons of 18 months or longer. There seems to be no significant relationship between future returns of negative alpha portfolio and past alpha/beta ratio. It is possible that models based on conditional, i.e. time-varying alphas will give more powerful signals of performance persistence or behaviour of our portfolios in the future in general. Further research is needed to address this issue.
CHAPTER TWO

SEASONALITY EFFECT AND TIME VARYING BETA IN THE UK
1. Introduction

Traditional Sharpe-Lintner-Black Capital Asset Pricing Model (CAPM) suggests that the cross-section of expected returns can be explained by the measure of systematic risk, beta (β). For a long time, this form of CAPM has implied the way in which both academics and practitioners have perceived risk and the relationship between the risk and expected return. The relationship tested is presented as:

\[ E(R_p) = R_f + \beta_p (E(R_m) - R_f) \] (2.1)

where \( E(R_p) \) is the expected return on a risky portfolio \( p \), \( R_f \) is the risk-free rate, \( \beta_p \) is the beta coefficient of the risky portfolio \( p \) representing the covariance between the portfolio's return with the market return divided by the variance of the market, and \( E(R_m) \) is the expected return on the market.

Since we have outlined in the introduction to this thesis that CAPM model is based on numerous unrealistic assumptions and unobservable market portfolio, we should point out that CAPM, like any other model, is just an approximation of the real world and it is unreasonable to expect that it will be 100% accurate in pricing assets with respect to their systematic risk.

The model is underlined by the assumption of the positive risk-return trade-off: the expected return on the market must be greater than the risk-free return, i.e. the term \( (E(R_m) - R_f) \) must be positive. This implies that the expected return on any risky portfolio should be positively related to beta coefficient. Many researchers have tested
the relationship between betas and portfolio returns. However, tests were performed on the basis of realised returns, not the expected returns, as originally implied by the CAPM.

Although the model’s assumptions can be criticised, CAPM is deriving an important relationship between the expected return of a security (or a portfolio of securities) and its’ risk. These facts about CAPM enhance the importance of a security’s (portfolio’s) beta that measures its sensitivity to the future market movements. A beta can be regarded as the slope of the market model, which is formulated as:

\[ R_i = \alpha_{iM} + \beta_{iM}R_M + \epsilon_{iM} \]  

(2.2)

If the slope term (\( \beta_{iM} \)) is positive, bearing in mind that the expected value of error term (\( \epsilon_{iM} \)) is zero, the above equation indicates that the higher the market return, the higher the return on the security is likely to be. If the regression line estimated with the market model was constant over time, i.e. it was not changing from period to period, than the beta of a security (portfolio) could be estimated by examining the historical relationship between the returns on the market (which is usually proxied by the market index) and the returns on a security (portfolio). Such a procedure, known as OLS regression, gives us historical beta. This method of beta estimation is widely used in the financial research, starting from Fama and MacBeth (1973) onwards. However, we will see in the sections that follow that there is numerous empirical evidence suggesting that constant betas obtained by using historical data with Fama – MacBeth methodology are inaccurate.
1.1. Evidence on Time Varying Betas

There is considerable number of studies that question whether the assumption about time invariant betas in the market model is valid in the real world. Majority of the evidence that beta stability assumption is invalid stems from the US market. In particular, the evidence about beta instability dates from 1970s. For example, Francis and Fabozzi (1978) use the random coefficient model in which they allow beta coefficient from period to period \( (b_{it}) \) to vary randomly around the mean beta \( (B_i) \). The smaller the divergence of period to period beta from the mean beta (i.e. the smaller the variance), the more accurate the traditional single index model is in assuming beta stability over time. Therefore, the authors try to examine whether the variance \( \text{var} (b_{it} - B_i) = \sigma_i^2 \) is significantly different from zero. In order to do this they have obtained an estimate for the variance of the estimate \( \text{var} (b_{it} - B_i) = \hat{\sigma}_i^2 \). Authors use the sample of 700 stocks with continuous monthly data from December 1965 through December 1971. 99% of that sample had OLS beta coefficient significant at the 5% level. After estimating the variance \( \hat{\sigma}_i^2 \) they find that only 382 out of 700 stocks had the positive estimate of the variance, out of which for 103 stocks variance was significant at 10% level and for 57 stocks at the 5% level. This provides some evidence for beta as the random coefficient rather than stationary. By using the coefficient of randomisation \( \frac{\hat{\sigma}_i}{B_i} \) they test the magnitude of beta’s randomness.

The results show that the beta largely fluctuated around its mean. From the 318 stocks that had negative variance around mean beta, by choosing 20 stocks and using restricted GLS model, authors find that small proportion of this sample (three stocks) has random beta coefficients at the 10% level of significance. However, the overall results are indicating that the beta, at least for some stocks, is a random coefficient
and that it may not be the only variable that should be used for explaining company (portfolio) returns.

In Francis and Fabozzi (1979), the authors find not only that the passage of time influences the change in Single Index model statistics but also the macroeconomic factors. The paper shows that beta coefficient drawn from a sample of 694 NYSE stocks, for period December 1965 through to December 1971, has tendency to change at peaks and troughs of the business cycle. Francis and Fabozzi (1979) divide the sample into two subperiods: recession years and expansion years. They have also formed six arbitrarily partitioned subsamples (without regard to business cycle) which they will use as control groups for comparison with the results from recession-expansion division. They perform the Market model regression in which they introduce two dummy variables: one for alpha coefficient from the regression (it takes value of 1 if recession period and zero otherwise) and one for beta coefficient (it takes the value of 1 if recession period and zero otherwise). Such a model allows for the shifts in both alphas and betas between two different business cycles. The coefficients on dummy variables for alphas and betas are showing the shift at the 5% level of significance. The explanation offered in the paper suggests that the shifts are caused by the changes in US economy that have caused company's risk-return characteristics to change. Also the authors point out that betas are less stable than alphas. As for the arbitrary subsamples, the alphas and betas do not shift as frequently as when we had expansion-recession partition. The study therefore supports the hypothesis that the Single index model is affected by market conditions, implying that the intertemporal instability of betas may partially be due to changes in the business cycle.
Beta instability evidence was also provided by Sunder (1980). The author presents the unbiased and consistent estimates of the market risk variance of common stocks and their respective portfolios. The period analysed is including monthly data on NYSE stocks between January 1926 and December 1975. To test the stationarity of beta, Sunder is using the first-order autoregressive process with a serial correlation very close to 1:

$$\beta_t = \rho\beta_{t-1} + (1-\rho)\beta + \upsilon_t$$

In the above process $E(\upsilon_t) = 0, E(\upsilon_t^2) = \sigma_\upsilon^2, E(\upsilon_t\upsilon_s), t \neq s$. $\rho$ is the first order serial correlation coefficient and $\sigma_\upsilon^2$ is the variance of the disturbance term in the market risk process. The variance of beta is calculated as $\sigma_\upsilon^2 / (1-\rho^2)$ and, if $\rho = 0$, it is equal to $\sigma_\upsilon^2$, if $\rho \to 1$, it is approaching infinity. Sunders refers to $\sigma_\upsilon^2$ as a step variance of the systematic risk process. Sunders analyses the nonstationarity of the market risk of portfolios because from fund manager's point of view the nonstationarity of systematic risk of individual stocks is not of such a great interest. As far as individual stocks are concerned, over the 50-years period examined, 88% of the stocks support the hypothesis of nonstationarity of market risk. Similar results were found for the two 25 year subperiods of the sample. When the diversification is introduced, i.e. grouping the stocks into portfolios, the greatest degree of nonstationarity was found for the period 1938-1944. The same was true for the individual stocks, implying that the periods of high instability of beta for portfolios are the high uncertainty periods for individual equity as well.

Bos and Newbold (1984) also provided evidence for beta instability in the US market. They allow beta to follow first order autoregressive process, as in Francis and Fabozzi
(1978), that includes the random coefficient model, as in Sunder (1980), in which systematic risk is the white noise. Traditionally, in the market model, one would assume that beta is constant through time, but Bos and Newbold assume that beta follows the first order autoregressive process of the form:

\[ \beta_t - \bar{\beta} = \phi(\beta_{t-1} - \bar{\beta}) + a_t \]

where \( \bar{\beta} \) is the mean beta and \( a_t \) is white noise independent of the process \( e_{it} \). In the special case when \( \phi = 0 \), the above autoregressive process becomes the random coefficients model. The data used in the study is covering the monthly data of 464 NYSE companies in the period January 1970 to December 1979. The authors apply Lagrange multiplier test of the null hypothesis stating that the variance of white noise is equal to zero, while the alternative hypothesis is that the variance is positive, with the parameter \( \phi = 0 \). Null hypothesis is rejected for the majority of the series. Furthermore, the null hypothesis of a fixed slope parameter (beta) in the market model has been tested against the alternative that this parameter is stochastic, i.e. following the first order autoregressive process. The results are suggesting stochastic rather than fixed systematic risk. Additionally, Bos and Newbold are testing the null hypothesis stating that beta is purely random against the hypothesis that it is autocorrelated, obeying first-order process, for those stocks that have stochastic beta parameter from the previous test. The findings are suggesting that there is not sufficient evidence against the beta as a random coefficient.

In traditional asset pricing models beta is the only variable needed for explaining returns. However, there is extensive evidence that relates company's returns to other variables such as size, dividend yield, P/E ratio, etc. (for some examples and references refer to chapter 1). Fama and French (1992) are discarding the central
relationship of the CAPM model, that average stock returns are related to market beta. Moreover they find that size, book to market ratio (BE/ME) and earnings to price (E/P) ratio are much better variables to use in explaining the cross-section of average stock returns. Using a monthly data for the period 1963-1990, they find a strong negative relationship between size and returns, positive (0.5) and strong (t-statistic of 5.71) relationship between ln(BE/ME) and returns and a U-shaped relationship between E/P ratio and returns. When authors make portfolios on size alone, they find a positive relationship between average return and beta, implying that the CAPM model is correct in pricing the risk. However, when portfolios are formed on the basis of betas, they do not support the traditional CAPM model. Even when authors rank portfolios first on the basis of size (10 deciles) and than on basis of betas (10 deciles within each size decile, making a 100 portfolios altogether), it is shown that variation in betas that is related to variation in size is positively related to market returns. However, variation in betas that is unrelated to variation in size, gives little variation in average returns, i.e. the beta-return relationship is flat, even when beta is the only variable in the regression. This work by Fama and French (1992) opened a discussion about beta instability once again.

Kothari, Shanken and Sloan (1995), estimated the cross-section of expected stock returns and found a positive relationship between beta and annual portfolio returns, the result directly opposing the Fama and French (1992) results. Please note that Fama and French (1992) used monthly portfolio returns for beta estimation. These contradictory results are indicating that the significance of the results may be sensitive to the way the beta is estimated on the first place. Some of the reasons why one should (could) use annual returns are: CAPM does not say not to use them, there is a
seasonal component in monthly stock returns (January seasonal, Keim (1983)) and betas might be biased due to trading frictions or non-synchronous trading in short-interval returns. Kothari, Shanken and Sloan (1995) are analysing all NYSE and AMEX stocks 1927-1990 period and post 1940 period for comparative purposes with Fama and French (1992). They find that the relationship between book to market ratio and return is weaker than that found by Fama and French (1992).

Bhardwaj and Brooks (1993) applied a ‘bull-bear’ beta approach. In particular, they were examining whether the risk of stocks differs between bull and bear markets. The authors examined the returns on 20 size-ranked portfolios of the NYSE (from 1926 to 1988) and AMEX (from 1963 to 1988). Compared to the median market return over the entire sample period, return of each month is classified as either bull or bear, hence the subsets of bull and bear markets were formed. To test the variability of betas through time, the varying risk model of the following form was applied:

\[ R_t = a_{bull} + (a_{bear} - a_{bull})D_1 + b_{bull}R_{mt} + (b_{bear} - b_{bull})R_{mt}D_1 + e_t \]

The dummy variable \( D_1 \) is equal to one for bear months and zero for bull months. The statistical significance of \( (a_{bear} - a_{bull}) \) and \( (b_{bear} - b_{bull}) \) is determining whether average abnormal returns and systematic risks of a given portfolio are different between bear and bull market months. The results of this study are suggesting the statistically significant differences in systematic risks and abnormal returns on firm size based portfolios. Therefore, this results are suggesting that single index model may not be the most appropriate one for explaining security returns, since it is assuming constant risk. In this study in particular, it has been found that in the bear months, small size stocks were underperforming larger ones, whereas in the bull months they were outperforming. However, the total risk of small stocks in bull
market was much greater than the total risk of larger stocks. This is showing the relationship between business cycles and return premium. The authors find that the superior performance of larger stocks is even greater in months other than January (note that small stock returns are largest in January). Overall, in this study, the negative excess return of small stocks is found.

Similar to Bhardwaj and Brooks (1993), Pettengill, Sundaram and Mathur (1995) examined the relationship between betas and portfolio returns in periods when excess market returns are negative (downmarket returns) and positive (upmarket returns). The authors criticise the Sharpe-Lintner-Black model because it is based on expected rather than actual returns. They state that the previous tests of the relationship between betas and returns must be modified. They believe that the assumption of the CAPM that suggests that the market return will always be greater than the risk-free rate is wrong and that investors should perceive the probability that market return in some instances will be lower than the risk-free rate. This suggests that the relationship between the betas and realised returns is actually different from the relationship between betas and expected returns. The authors have developed methodology that considers the positive relationship between betas and returns during periods of positive excess returns on the market \((E(R_m) - R_f > 0)\) and the negative relationship between betas and returns during periods of negative excess returns on the market \((E(R_m) - R_f < 0)\). Pettengill, Sundaram and Mathur study the monthly returns of the US stocks in the 1936-1990 period, creating 11 subsamples of 15 years each. They form 20 beta ranked portfolios and test the systematic beta-return relationship by using the following regression model:

\[
R_{it} = \hat{\gamma}_{0i} + \hat{\gamma}_{1i} \delta \beta_{it} + \hat{\gamma}_{2i} (1 - \delta) \beta_{it} + \varepsilon_{it}
\]
where $\delta$ is equal to 1 if market excess returns are positive and zero otherwise, $\hat{\gamma}_{1t}$ is estimated in periods with positive market excess returns, so the sign of that coefficient should be positive and $\hat{\gamma}_{2t}$ is estimated in negative market excess return periods, resulting in the negative sign expected on that coefficient, implying that the relationship between betas and returns in upmarket periods should be positive and in the downmarket periods it should be negative. The results are confirming the expectations: $\hat{\gamma}_{1t}$ coefficient has a positive (0.0336) and significant ($t=12.61$) value at 1% level, and $\hat{\gamma}_{2t}$ coefficient has the negative (-0.0337) and also significant ($t=-13.82$) value at 1% level. These results suggest that in the upmarket periods, high beta stocks are outperforming low beta stocks and in the downmarket period low beta stocks are performing better than high beta ones. The authors also find the support for the hypothesis of a positive relation trade-off between beta and average portfolio return.

One of the studies from the UK, Fletcher (1997), also examines the relationship between betas and returns in up market months and down market months and the role of size in the UK stock returns. He examines the period from January 1975 through to December 1994, using the UK stock returns from LSPD. Following Fama and French (1992), they form 100 size-beta portfolios. Without distinction between up markets and down markets, in a majority of the size deciles, the low beta portfolio had higher return than the high beta ones. The size-return relationship is not a monotonic, but rather a U-shaped one. Monthly cross-sectional regressions of portfolio returns, beta and size, are suggesting the negative insignificant relationship between beta and return, consistent with Fama and French (1992) and Jagannathan and Wang (1996).
One of the reasons for this findings is put forward by Pettengill et al (1995), who argued that the flat slope is the result of using realised rather than actual returns in the tests, in which case the relationship between beta and return would be conditional on the market index return. When distinguishing between the up market and down market months, they find positive and significant beta/return relationship in period when market excess return is positive and negative and significant beta/return relationship when the market excess returns are negative. They find that the risk premiums are higher in down market months than in up market months (contradicting the positive risk-return trade-off), and find no reason for that in January effect or October 1987 crash.

Using the dual beta model of Bhardwaj and Brooks (1993), Howthon and Peterson (1998) examine the cross-section of realised stock returns. They use dual-beta asset pricing model, meaning that risk changes according to the period (bull or bear) to explain whether beta alone provides the explanation of cross-sectional returns. The data covers monthly returns of all non-financial NYSE and AMEX firms in the period 1977-1993. They use Fama and French (1992) methodology to construct 100 portfolios classified in deciles based on size first and then again in deciles based on betas. They classify the month as bull (bear) if the return on the market is higher (lower) than the median market return. For each portfolio, Howthon and Peterson are estimating both bull-market and bear-market betas. In the cross-sectional regressions, they use the following independent variables: beta assigned to security i in month t estimated either from the dual or constant risk model, the market value of equity, the book value of equity, the earnings-price ratio if earnings are positive and zero otherwise and earnings price dummy which has the value of 1 if earnings are negative.
and zero otherwise. Each firm in the cross sectional regression has three betas for each portfolio: constant beta, bull-market beta and bear-market beta. The findings are suggesting that a simple regression model that separates months into bull-market and bear-market months is allowing beta to become an important determinant of cross-sectional returns. The average bull-month beta (1.252) is significantly larger than the bear-month one (0.996). Book-to-market ratio is only significant in bear-market periods. Market value has negative relationship with the returns in all January months and February through December bear market months but not February through December in bull market months. The results are consistent Kothari, Shanken and Sloan (1995), implying that the beta is sensitive to the time periods over which it is estimated and the way it has been estimated.

Evans (1994) has developed an empirical model for intertemporal asset pricing that allows for both time-varying risk premia and time-varying betas. He examines the monthly value-weighted portfolio returns of NYSE common stocks between January 1964 and December 1990 for five portfolios of common stocks (1st, 3rd, 5th and 8th decile portfolios obtained by ranking stocks according to market price) two portfolios of Treasury bills and portfolio of long-term Treasury bonds. Evans tests the CAPM model with time-varying betas. A wide selection of information variables (such as lagged realised return on one-month Treasury bill, the spread between the yield on one-month and six-month Treasury bills, the spread between AAA and BAA rated bonds etc.), were used to estimate risk premium. The results are suggesting that betas in smaller deciles vary over the business cycles. The mean change in betas in expansion periods is 0.048 per annum and in recession periods it is −0.143. On the other hand, corporate bond betas are moving counter-cyclically, having a mean
change in the first decile beta of $-0.031$ per year during booms and $0.049$ per year during recessions. Additionally, Evans finds that variation in betas has a little to do with predicting the variability in returns, but, instead, he suggests that the primary source of predictability lies in the variations of the risk premium.

Jagannathan and Wang (1996) criticise the empirical studies of CAPM that assume that betas are intertemporally constant and that the return on the value-weighted portfolio of all stocks is a proxy for the return on aggregate wealth. They assume that stock prices and market risk premium vary over business cycle, therefore, they choose a forecasting variable (yield spread between BAA and AAA rated bonds) that can predict change in business cycle as a variable to forecast market risk premium as well. They state in their analysis that the proxy for the market return should include not only the stock index return but also the return on human capital. Automatically, by relaxing the assumptions of CAPM, Jagannathan and Wang are applying the conditional CAPM. The authors use the same data set as Fama and French (1992), monthly NYSE and AMEX data in the period 1963-1990, forming 100 portfolios, 10 size deciles and within each size decile they form 10 beta deciles. They find that when allowing beta to vary over time, the size effect and the statistical rejections of the parameters in the model become much weaker. When they include human capital variable in the return on aggregate wealth measure, they find that such conditional CAPM has insignificant pricing errors and size effect has no additional explanatory power. Furthermore, authors point out that one should be cautious when interpreting the results that support conditional CAPM for several reasons: approach applied to model betas is simple, events are occurring at monthly or yearly frequencies and, afterall, the CAPM is just a model that approximates the reality.
Kim (1993) investigates the nonstationarity of betas across the firm size and beta magnitude of stocks. He argues that nonstationarity models, used for example by Francis and Fabozzi (1978), Bos and Newbold (1984) and Sunder (1980) allow the beta to change at every time period, i.e. too frequently. In turn, Kim uses the constant beta coefficient model, which assumes stationarity of betas over a certain time period (e.g. 5 years period has been widely used in empirical studies). Kim states that stock returns are affected by the relevant information on the market, and that the parameters related to stock returns will change at certain time points, referred to as 'change points' in this paper. Kim detects the change points and investigates whether there are distinctive characteristics in the beta estimate and behaviour at change points across beta magnitude and firm size portfolios. The data covers the NYSE and AMEX stocks in the period 1926-1990. Using annual rebalancing, Kim forms 10 size-based portfolios and within each size portfolio he constructs 10 beta-based portfolios. He computes the length of the stationary period of beta and beta for each stock in the portfolio and in turn combines those values to obtain the length of stationary interval and beta of the portfolio. The findings are suggesting that high beta firms have shorter stationary intervals than low beta firms, which implies that high beta firms are more unstable. However, the relationship between firm size and stationary interval is not monotonic. The medium size firms have longer stationary interval than the small and the large firms. Additionally, Kim found that the average length of the stationary interval is 54.19 months (about 5 years) and that for the smaller firms shorter interval should be used (approximately 3 years). The frequency of changing points is the same across different months. However, the frequency of changing points has the positive relation to the market returns but the negative relation with the three-months
Treasury bill. The correlation coefficient between the changing points and the market returns is 0.257 with t-statistics of 2.11, whereas the correlation coefficient between the changing points and the 3-month T-bill is -0.358 with t-statistics of -3.04. In conclusion, when the market returns are high the systematic risk changes more frequently and vice versa. Also, greater fluctuation in betas is found for smaller, low-beta and high-beta companies, rather than for larger and medium size companies.

Gregory-Allen, Impson and Karafiath (1994) investigate the beta stability of portfolios vs. individual securities. They find no evidence that portfolio betas are more stable than individual security betas. The authors criticise the methods, which estimate beta coefficient on the basis of historical returns and favour the first order auto regressive random coefficients model (RCM) for generating betas:

\[ \beta_{jt} = \rho \beta_{j(t-1)} + \eta_{jt} \]

However, any advantage of a RCM may be lost due to accumulated estimation errors resulting from the complexity of the model. Alternative for stock return generating process would be to estimate beta with OLS and select the appropriate estimator for the covariance matrix of parameter estimates. If a linear regression model is fitted with the returns generated by RCM, the error terms from the least squared regression will be heteroskedastic and serially correlated. The daily stock returns of US individual stocks and portfolios were obtained for the period of 1050 days, forming subperiods of 500, 250, 200, 125 and 100 observations. Using OLS variance estimator, portfolio betas appear to be less stable than individual securities betas. Applying Newey-West estimator, with 500 observations in each subperiod, they reject the hypothesis that there is no change in beta for 5.97% of individual securities and 4.08% of portfolios. They conclude that portfolio betas are no more or less stable than
securities betas, but neither category is showing stability over time even for intervals as short as 100 days.

Alexander and Chervany (1980), among other findings, outline the possible reasons for difference in opinion about the stability of portfolio betas. Firstly, they examine whether betas in the extreme pentiles are more stable than the ones in the interior pentiles. The data sample in this study is including monthly returns of 160 common stocks listed on the NYSE in the period 1950-1967. Using the mean absolute deviation (MAD) measure of changes in betas, the authors find that betas in the extreme pentiles are less stable than those in the interior ones. Furthermore, they examine the optimal size of the beta estimation interval. The formula for calculating beta (sum of all observations of the ratio of covariance between the market and security and the market variance) is suggesting that increase in the number of observations will reduce the estimation error for beta as well as for the estimated variance of beta. This is implying that the larger estimation intervals are giving more precise beta estimates. However, during longer time intervals (which will increase number of observations for beta estimation), structural changes in betas are possible, making it necessary to determine the optimal time period for beta estimation. Alexander and Chervany find that the optimal period for beta estimation is both four years and six years. Some researchers are also pointing out the nine years estimation period as optimal one. Alexander and Chervany re-examine this issue whether the beta estimation is related to number of securities in the portfolio as well as the issue that beta is becoming more stable with greater diversification of stocks into portfolios. Their results are suggesting that, regardless of the way the portfolios are formed, intertemporal stability of portfolio betas is increasing as the number of securities in
the portfolio rises. They find that portfolios of 10 and more securities have
intertemporally stable betas when the stability is measured by mean absolute
deviation.

Opposing the results suggested in some of the previous papers, such as Francis and
Fabozzi (1978), Kolb and Rodriguez (1990) find that beta distribution is nearly
stationary in the short run as well as in the long run. They use monthly returns from
1926 to 1985 and divide this period into twelve five-year periods. Betas of individual
firms are estimated using market model for all five-year periods. They use Kruskal-
Wallis test to test if betas are identically distributed in all periods. The test rejects the
hypothesis of stationarity over all twelve periods with 1% confidence level.
Furthermore, they test whether betas are stationary from one five-year period to
another. They test the hypothesis that beta is identically distributed in two periods by
using Mann-Whitney test. In 42 out of 66 tests the hypothesis cannot be rejected.
Although there are departures from strict stationarity, they could occur by chance, so
the overall results indicate that betas tend to be stationary between any two periods.
Additionally, they test stationarity between continuous estimation periods and they
cannot find the significant departure from stationarity. Therefore, the suggestion they
make is stating that betas can be regarded as stationary for most practical purposes.

Models used in the above papers are trying to test the dynamics of beta and criticise
the constant beta model. If one can capture the dynamics of beta risk, they could be
sure of outperforming the constant beta model. However, Ghysels (1998) argues that
if time-varying beta is misspecified, one can make large pricing errors, maybe even
larger than with the traditional beta model. In order to test the misspecification of
betas, the author is testing for the structural shifts in the parameters of the conditional CAPM and APT. He uses the data set of monthly returns from January 1927 through January 1988. It is found that the conditional CAPM and APT do not capture very well the time-varying beta and therefore, misprice risk. By calculating the in-sample root mean square error (RMSE) of the conditional models and comparing it with the RMSE of the constant beta CAPM model for 10 size based portfolios and 12 industry based portfolios, it has been shown that the unconditional CAPM outperforms all the other conditional models. The reason for this may be that betas vary over time very slowly and that the conditional CAPM and APT models tend to overstate the volatility of beta over time. Similar results were obtained for both size-based and industry-based portfolios. Ghysels suggests that the non-linear APT is stable in comparison to conditional CAPM and APT models and it has a great potential for further development.

All these studies are providing evidence, more than clearly, that beta coefficient is a subject of intertemporal instability. The above studies are outlining the results of various tests regarding instability of betas, length of estimation periods for betas, size of the portfolio and beta estimation etc. In the next section we will review the studies and commonly used methodologies dealing with the estimation of time-varying beta coefficient, i.e. the issue of modelling beta.

1.1.1. Modelling Time Varying Beta Coefficient

Abell and Krueger (1989) examine the influence of macroeconomic variables on beta coefficient by allowing it to vary with a set of chosen economic characteristics. Starting from the single index market model, authors are allowing the beta coefficient
of the model to vary linearly with a number of factors and call that new model variable beta model (VBM). Single index models and VBMs were estimated for 17 US industry portfolios between January 1980 and December 1986. The coefficients from this period are then used to predict betas from January 1987 to September 1987. Authors use 10 most important macroeconomic variables, such as: budget deficit, consumer price index, M1 money supply, unemployment rate etc. The VBM procedure that was performed on the basis of stepwise regression and 10% F-test. In seven industries only one macroeconomic factor was significant, in four cases more than one factor was significant but in six industries none of the descriptors were significant, so those six industries were left out from the further analysis. In the eleven remaining industries, only US dollar exchange rate and commercial paper rates were found to be insignificantly related to beta during the period under observation. The results suggest that the R-squared of the VBMs is always better than that of the Single index model, implying that VBM can be of use in explaining portfolio returns. As far as the influence of macroeconomic factors on beta is concerned, it has been found that higher interest rate, larger trade deficits, higher oil prices and higher inflation rate had negative influence on beta. In other words, portfolio returns increased when the market was advancing under these conditions, but at the diminishing rate. Moreover, the utility of VBM was tested by using individual portfolio models (specified in the period January 1980 – December 1986) to forecast betas for the future period (January 1987 – September 1987). In order to forecast one period ahead, one had to make assumptions about the subsequent values of macroeconomic variables. In that respect, two different sets of assumptions were made: (1) the value of the macro variable in the forecasting period (January 1987 – September 1987) will be equal to the average value of that variable over the period January-December 1986; and (2) the
average value of the macro variable over the period July-December 1986 will continue into the forecast period. Using single index model, a beta for each industry was estimated for the forecast period and compared with the VBM beta forecasts as well as with the historical beta from single index model. It was shown that variable beta models are more accurate in predicting future betas and the direction in beta's change from the historical to the future period, than naïve models based on the historical beta.

Black, Fraser and Power (1992) estimate random walk betas for 30 authorised UK unit trusts from 1980 through 1989 by using the Kalman filter approach. Authors regard the examination of the performance of UK unit trusts as an indirect test of the efficient market hypothesis, because they are supposed to have superior investment opportunities and, in turn, outperform the market. It is considered that the time-varying risk model would be appropriate to apply in the analysis because the period 1980-1989 was characterised by changes in economic relationships, such as Financial Services Act (1988) and changes regarding capital gains tax. Black, Fraser and Power try to overcome the problem of constant risk factor (beta) in traditional Sharpe-Lintner CAPM model by obtaining maximum likelihood estimates of the parameters of interest, called 'hyper parameters' by using Kalman filter technique. This will allow them to obtain the time varying elements in betas. The empirical findings of this paper are suggesting that, for the first three autocorrelation series, there does not appear to be any significant autocorrelation between the series. Also the autocorrelation is negative in majority of the trust's cases, implying that there is a mean reversion in returns. The relative variance between error terms $\sigma^2_i/\sigma^2_0$ (the hyper-parameter) is showing that only eight out of 30 unit trusts maintain the constant
level of risk. However, it is statistically significant in three cases only. The OLS estimates of market risk and abnormal returns are produced. ARCH tests suggest that there is autoregressive residual heteroskedasticity in around 50% of the trusts. Therefore, the estimates would be biased and tests of significance will be invalidated. The ARCH results are suggesting that one should allow the risk-return relationship of the trusts to vary over time. OLS is showing that there is large number of unit trusts earning abnormal returns, thus contradicting the efficient market hypothesis. The time varying estimation procedure is showing that 21 out of 30 unit trusts are earning abnormal returns. However, one should bear in mind that the data used in this paper are mid-market prices and once the transaction costs are taken into account, the abnormal returns are diminishing. Additionally, the findings may be relevant only for the time period studied. And, finally, regarding the sample size in this study, the authors suggest that more comprehensive analysis will be possible if one would have larger sample of unit trusts.

Wells (1994) is also using Kalman filter technique to estimate time varying betas for the small sample (10) of Swedish stocks. He applies Kalman filter in the random walk model (RW), random coefficients model (RCF) and the mean reverting model (MRV). Wells uses the Akaike information criterion (AIC) to test which model best fits the data (the best model will have the lowest AIC). In this paper, the majority of the cases are best modelled with the random coefficient model, which is stating that there exists a ‘true’ or ‘steady state’ beta around which the estimated coefficient varies randomly. It is possible for the ‘true’ beta to change so the correct model will have beta varying around a step function. The author concludes that each stock must be studied by itself if one wants to understand the factors behind the estimates.
Statistical tests evaluate the performance of the model on the basis of historical data. Since the financial data is used in the paper, more relevant test would be whether an investor could make money by following the forecasts generated by the models under consideration. The model that has the lowest forecasting error will prove to be the superior one. To provide the forecasts, the last estimated state variable in the model was taken and assumed to be constant over the following 24 months. Then, the OLS coefficients were estimated using the last 60 months of the estimation period. Kalman filter outperforms OLS estimates in three out of six cases presented in the paper. The random coefficient model has the lowest mean absolute deviation in three out of six cases. In a more extensive study by Wells, the random walk model gives forecasts with the lowest mean absolute deviation in the case of 24 out of 57 stocks, which is followed by the random coefficients model (the lowest prediction error: 11/57 cases). Although the general conclusion of Wells’ paper is that betas are truly non-stationary, one should take this study with caution because it has been based on a very limited sample.

One of the more recent studies by Faff and Brooks (1998) is examining the time varying betas for Australian industry portfolios. They introduce a variable beta model where beta is the function of some unspecified, hypothetical, variables that explain the time variation in beta risk. The sample period covers monthly compounded returns between January 1974 and December 1992. The methodology of Abell and Krueger (1989) in which betas are modelled in terms of different macroeconomic variables is applied. Three subperiods are used in the paper that might influenced the change in betas: deregulation of Australian market (pre deregulation regime from January 1974 to November 1983), introduction of tax imputation system (pre-imputation regime
from December 1983 to June 1987) and the period after imputation (post imputation regime from July 1987 to December 1992). Faff and Brooks incorporated these regimes in variable beta model by using two dummy variables. The first one has the value of one in the pre-imputation regime (zero otherwise) and the second dummy has the value of one in the post computation period (zero otherwise). Such variable beta model is substituted in market model to form 'regime dependent market model'. They assume that beta is a function of two variables: return on the market and volatility of the risk free rate. When they include this assumption in the market model, it becomes the quadratic market model. Putting the two forms of market model from this paper together, one obtains the 'regime dependent quadratic market model'. Additionally, the dependence of betas on the volatility of the interest rates is included in the model. Such a time-varying beta model was run and it has been found that the Other Metals industry has a constant beta over the three regimes, the market return variation in betas is only significant in the post-imputation regime (because of its positive sign it is implying that beta rises with the market) and volatility term is significant in pre-deregulation (negative sign, indicating that as the volatility rises, the beta increases) and post-imputation regime (positive sign, indicating that as the volatility rises, beta rises as well). Taking the example of a Food and Household Goods industry that has significant market driven variation in betas and risk-free rate volatility driven variation in betas during the pre-deregulation period, Faff and Brooks plot the evolution of the time-varying beta against the beta produced by the Kalman filter estimation. They find that there is a low correlation between the betas and the tendency for greater volatility in time varying betas. Testing for beta stability, they find that stability hypothesis can be rejected across majority of industries. Authors also suggest that adding complexity to the original market model is successful in
modelling time varying betas, because the rejection of stability hypothesis is diminishing across industries, as the model is becoming more complex. Finally, the univariate and multivariate tests were used but they do not provide enough evidence that the time-varying CAPM is appropriate under all circumstances, therefore it should be used with caution.

The paper by Brooks, Faff and McKenzie (1998) compares three techniques used for estimating the time-variant betas. The first model used in this paper to estimate the time-variant betas is the multivariate generalised ARCH model or M-GARCH model. The second is the model proposed by Schwert and Seguin (1990) which suggests the time varying beta market model approach. In particular, the model involves estimating the conditional beta of an industry return series as:

\[ \beta_{it} = b_1 + \frac{b_2}{h_{Mt}} \]

where \( h_{Mt} \) is the conditional variance of the market index (obtained from GARCH model fitted to this return series) and \( b_1 \) and \( b_2 \) are coefficients obtained through the following regression:

\[ R_{it} = a_0 + b_1 R_{Mt} + b_2 r_{Mt} + \varepsilon_{it} \quad (2.3) \]

where \( R_{it} \) is the return on industry \( i \), \( R_{Mt} \) is the market return, \( r_{Mt} = R_{Mt}/h_{Mt} \) and \( \varepsilon_{it} \) is the error term.

The third technique used is Kalman filter approach. In order to establish the relative advantage of one model over another, authors propose the methodology in which in-sample return is forecasted using the market model as follows:

\[ \hat{R}_{it} = \alpha_i + \beta_{it} R_{Mt} \]
where $\beta_t$ is provided by each of the three techniques in the paper and $R_m$ is the return on the market index.

The conditional intercept coefficient series ($\gamma_t$) is generated for the Kalman filter approach and for GARCH and Schwert and Seguin approach it is estimated as the mean industry return minus the mean conditional beta times the mean of the market index returns. Once forecast $\hat{R}_t$ is generated, using each of the conditional series, the accuracy of the forecast may be obtained by using mean absolute forecast error or, as an alternative, mean square forecast error.

The data contains prices of 24 industries from Australian Stock Exchange in the period January 1974 through March 1996. To have the starting point for comparison, the standard market model was estimated for each of the Australian industry portfolios and the results are showing that all betas are significantly different from zero and, in majority of cases, different from unity. The GARCH (1,1) model was fit into each of the 24 industries data and the market indices data. The ARCH and GARCH terms are generally significant, sum to be less than unity and satisfy the positivity assumptions (a and b coefficients $\geq 0$ and c coefficient $>0$ will guarantee the positive definiteness of conditional covariance). Only three industries exhibited negative ARCH parameters and they were excluded from the further analysis. The mean correlation coefficient between each industry return and the market return was 0.0764 indicating a high and significant relationship. Using the conditional variances from the 21 GARCH models and the model estimated for the market index, authors estimate beta coefficients. The first moment beta has the mean value similar to the point estimate beta in each industry. In the second moment estimates, GARCH
conditional series has a high degree of variability with some sectors. The estimation of conditional beta with the second, Schwert and Seguin (SS), approach, requires the conditional variance from the market index, obtained through the GARCH model fitted to the market index. Therefore, the series \( r_{Mt} = R_{Mt}/h_{Mt} \) can be constructed and the regression specified by equation (2.3) was estimated. The R-squared from regression for different industries varies from 0.79 to 0.32. However, the authors have found that the inclusion of the \( b_2r_{Mt} \) term in the regression has added little explanatory power to the simple market model. Using the coefficients from the regression, the series of SS beta was generated. The first moment SS betas have similar values as point beta estimates from the market model and the GARCH betas. The second moment shows that there are differences between SS betas and GARCH betas: range of SS betas is less in each instance. Finally, the Kalman filter approach is involving a restricted sample (January 1976 – March 1996) because in the initial stages of estimation, the approach is generating very large outliers, so the first two years of observations are excluded in order to avoid bias. The first moment parameters of risk are similar to the previous two cases, but in the second moment, Kalman approach generates range of observations that are less than those generated by GARCH. Comparison of the three approaches is telling us that conditional beta is similar in all three cases. However, the range of observed values in betas is largest with GARCH approach and variations in betas are smallest with SS approach. As far as the accuracy of the models is concerned, the in and out of sample forecasts of the industry returns are used and MAE and MSE forecast errors were calculated. All three models are showing better accuracy out of sample rather than in sample. In both, in-sample and out-of-sample forecasts, The Kalman filter approach had superior performance.
Schwert and Seguin (1990) examine the heteroskedasticity in stock returns. In particular, they investigate the relation between the aggregate stock return volatility and the variance of monthly returns to disaggregated portfolios of stocks and they also examine the effect of portfolio heteroskedasticity. To model heteroskedasticity in monthly stock returns, they use estimates of aggregate volatility from daily stock returns to the S&P composite portfolio from 1928 to 1986.

Gonzales-Rivera (1996) is testing conditional CAPM versus the conditional Residual Risk Model. The author designs the bivariate GARCH-in-mean system where she pairs each individual stock return with the market portfolio return. The CAPM is tested with individual stocks and the approach applied avoids the need to form portfolios to correct the measurement errors in betas. The analysis is based on the weekly returns of the American computer industry companies (89 companies, out of which only 3 are active during the whole sample period) from July 7, 1962 to December 29, 1987. As a proxy for the market portfolio, she uses NYSE value-weighted index. She finds that univariate GARCH behaviour is associated only with large firms, implying that the large proportion of firms do not show any (G)ARCH effects. Therefore, it was concluded that the multivariate GARCH model would be much more plausible to use because the volatility of the returns of individual companies are driven by a set of factors. There was no support for the one-dynamic factor model. Also, empirical evidence on the residual risk shows that the variance of the market returns is better predictor of expected returns than the covariance between the individual security returns and the market return. In general, residual risk model is preferred to conditional CAPM and the one-dynamic factor models. Finally, the bivariate model provides an estimate of traditional measure of risk, beta based on the
conditional covariance between the stock and the market and the conditional variance of the market. Constructed series of the betas are showing that in the computer industry mean beta is well above one, implying that it is a high-risk industry. Additionally, the author compares the market risk among securities by using the stochastic dominance criteria. It is found that the leader of the industry, IBM, is the least risky company.

1.2. Time varying beta and seasonality in stock returns

As it will be outlined in the latter sections in this chapter, time varying betas are used to explain the seasonal behaviour of stock market returns. In other words, it is believed that small stocks in particular have higher returns in January than in any other month and the main reason for that is that they also have higher beta in January than in any other month. Let us present some evidence that verifies that the seasonality effect exists in stock market returns.

Rozeff and Kiney (1976), found that for the US for the period 1904-74, the average monthly return was about 0.5%, while the January return was 3.5%. Keim (1983) reports that there is January seasonal in the size effect, i.e. that, in essence, the January seasonal is a small size phenomenon because the small firms earned half of their abnormal returns in January. As an extension to that, Keim (1985) finds that yields are related to size and that, in turn, there is a January seasonal in the yield-return relationship. On the other hand, Lakonishok and Smidt (1988) have assessed the returns of the Dow Jones Industrial Average (DJIA) for 1897-1986 period. Their results are suggesting no January effect. The reason for this finding may be that the DJIA is predominantly consisted of larger companies. In conclusion, in the US, stock
returns of small companies particularly in January have higher returns than in any
other month of the year. An alternative explanation put forward for the January
seasonal, apart from the small size effect, is related to tax. Specifically, the US tax
year ends on the December 31, so investors are assumed to sell poorly performing
shares late in the year to realise capital losses for tax-saving purpose. At the beginning
of the New Year, in January, investors are assumed to start buying back the shares,
pushing the prices up and, in turn, creating higher January returns.

In the UK and Australia, however, Gultekin and Gultekin (1983) find not only
January, but also April and July effect. Their study explored the January effect in 16
countries in the period 1959 to 1979 and has found it in 15. One should note that the
highest return in the UK was found in January. The puzzling thing is that UK tax year
starts on April 6 and the performance of the small companies index HGSCI (Hoare
Govett Smaller Companies Index) relative to FTSE All Share Index is the poorest in
January and April. That means that according to this study, one can use tax
hypothesis, but not the small size phenomenon at the same time, as an explanation for
the April effect. The question is, what explains the July and January effects in the UK
than?

As noted by De Bondt and Thaler (1985), the overreaction effect, which represents
the tendency for extreme performance stocks over various periods to reverse their
performance in the following period, occurs mainly in January.

Finally, as reported by Rogalski and Tinic (1986), risk as well as return behaves
differently in January. In particular, they are questioning whether excess returns of
small stocks in January are statistical artifice resulting from the incorrect measurement of risks in January. Studies such as Banz (1981), Keim (1983) and Reinganum (1981) applied models that assume that systematic risk and equilibrium required rates of return remain constant throughout the calendar months. However, the asset pricing theory doesn’t require risks or risk premiums to be stable over time. If at the beginning of the year small stocks have higher risks for any reason, due to the risk-return trade-off, they should also have higher returns during that period. Hence, Rogalski and Tinic assess the stationarity of commonly accepted risk measures over calendar months. They examine the total, systematic and non-systematic risk of different size firms. The data used are the daily returns of NYSE and AMEX stocks over the 1963-1982 period. They have confirmed the larger average returns in January for the index portfolio (equally weighted index of NYSE plus AMEX stocks) and for the first 15 out of 20 ascending size portfolios. Using variance as the common measure of the total risk, authors find that first five size portfolios have larger variance in January than in any other month, whereas the variance of the index in January is the third largest variance. Using market model, Rogalski and Tinic estimate betas and residual risk for each portfolio. Again, betas for the first five size portfolios are much larger in January than in any other month. The January beta of the smallest firms is found to be two times larger than the January beta of the smallest firms. Similar results are found for the residual variance or the unsystematic risk across portfolio groups. These results provide a possible explanation for the January-size effect and also raise the question of the seasonality in the equity risk premium.
1.3. Motivation

It has been proven, starting from Francis and Fabozzi (1978), that beta of individual stocks and portfolios is not constant as assumed by the CAPM model, but it follows a time-varying pattern. Pervious studies have shown three different approaches for modelling time varying beta: variations of ARCH and GARCH techniques, Schwert and Seguin (1990) approach and Kalman filter. In most of the studies, time-varying betas across different industry portfolios has been analysed, such as in the study of Brooks, Faff and McKenzie (1998). Other studies, such as Alexander and Chervany (1980) examine whether betas of individual stocks have greater time-variability than betas of portfolios. Additionally, size is often used as criteria for grouping stocks into portfolios. However, forming portfolios on the basis of size only is creating a bias regarding variation of portfolio betas due to size. Therefore, large number of studies, in order to see differences in beta variations across size portfolios is using the classification first on the basis of betas and then on the basis of size.

The question that hasn’t been tackled before is regarding the issue of whether particular companies that are more risky, in systematic risk sense, than some other companies on the market are facing even greater risk stemming from the behaviour of the risk measure itself. In particular, the results of this chapter should provide us with the answer of whether companies that have higher beta coefficients also observe greater time variability in betas, and thus incur even greater risk for an investor. Therefore, the UK stocks in this chapter are going to be classified into portfolios according to their beta coefficients, estimated using historical data and the OLS regression. Then, the time-based stability of betas will be examined across different beta portfolios, using the Kalman filter methodology. As far as other UK studies are
concerned, Black, Fraiser and Power (1992) and Hall et. al.. (1989) were looking at time-varying parameters but in an entirely different context from the one in this chapter. In particular, Black, Fraiser and Power (1992) analyse performance of UK unit trusts when allowing market risks to vary and Hall et al. (1989) estimates a multivariate ARCH process for the aggregate indices of four sectors in the UK: they provide time-varying covariances for four sectors against the market index and a time-varying variance for the index.

There are several reasons why we are grouping securities in portfolios according to OLS betas despite the evidence that betas are time-varying. Firstly, investors in the real world would use information on beta available from public data sources such as Bloomberg, Reuters, Datastream, etc. Such betas are estimates representing the slope of the OLS regression of returns of the stock on the returns of the market, proxied by FTSE All Share Index. Secondly, studies such as Gregory-Allen, Impson and Karafiath (1994) investigate whether individual securities betas are more or less stable than portfolio betas and find no evidence that individual securities betas are more volatile than portfolio betas. Also, Alexander and Chervany (1980) suggest that the optimal period for individual security beta estimation is four to six years (we are using five years of monthly data in our study). Finally, from our analysis of portfolio betas that will be presented later in this chapter, Kalman beta estimates and OLS beta estimates are not very different for most of the portfolios, especially those with larger betas.

Furthermore, as the question of seasonality in betas is raised from Rogalski and Tinic (1986) study, we will also examine that issue in detail. Although January effect was
the one found in unconditional betas in the US market, that doesn’t necessarily mean that one would find the same in the UK.
2. Data

Similarly to chapter 1, the first set of data is gathered from the London Share Price Database (LSPD) monthly returns file. The criteria employed in selecting the firms in the sample in a particular year are:

c) the firm has 5 years of available data before it was included in the sample and
d) firm’s returns were available on LSPD monthly returns file.

In LSPD, there is always a missing return in the first month of trading or the first month the company data has been collected. The list of all actively traded companies and dead companies (delisted, subjects to takeovers, mergers, bankruptcy, etc.) in the period January 1980 through to December 1996 was obtained from the Datastream file. The data prior to 1980 was used as well to estimate beta coefficients. The number of firms included in the sample that meet the above requirements varies from around 940 in January 1980 to around 1108 in December 1996. The largest number of companies in the sample was at the beginning of 1980, more than 1500. There is no survivorship bias in the sample. The only restriction for the sample is that it does not include investment trust companies due to their specific characteristics.

Monthly returns given by LSPD are calculated as follows:

\[ R_t = \log_e \left( \frac{(P_t + d_t)}{P_{t-1}} \right) \]

Where \( R_t \) is log return in month \( t \), \( P_t \) is the last traded price in month \( t \), \( P_{t-1} \) the last traded price in \( t-1 \) and \( d_t \) is the dividend declared during month \( t \).

The returns calculated in this way are more accurate because they include both components of returns: income (dividend) and capital gains. Therefore, for the market
returns data, I have used prices and dividend yields for the FTSE All market index from Datastream for the period January 1980 through to December 1996 and calculated returns on the index using the above formula. Data for 60 months prior to January 1980 was also used to estimate beta coefficients of individual companies.

2.1. Formation of Portfolios

The purpose of the chapter is to examine the time-stability of the betas across different portfolios of companies. In order for a security to be included in the sample in month $t$, it has to fulfill a condition of having 60 months of continuous monthly returns prior to month $t$, which is used to estimate beta coefficients. Beta coefficients are estimated by using the OLS regression of stock returns on the market returns that are proxied by FTSE All Share index. Once the betas of individual securities have been calculated, we have divided securities into beta portfolios. In particular, the full sample of securities in month $t$ was sorted into ten decile portfolios according to their OLS betas in that particular month. Portfolios were grouped in the ascending order, where portfolio 1 contains all stocks with smallest betas and portfolio 10 has highest beta stocks.

All portfolios are equally weighted and rebalanced on the monthly basis. Equal weighting is usually done to eliminate the size effect from the portfolio.

Portfolio returns for each month are calculated as average returns of individual stocks in that particular portfolio. We have also calculated the average dividend yield and market values of each portfolio. Finally, the portfolio beta is the average of betas of each security in the portfolio over the sample period. Beta coefficient for each
particular security 'x' for month t is estimated using the OLS regression model if the
security 'x' had continuously available monthly returns five years (60 months) prior
to month t, as explained previously.
3. Methodology for modelling betas

3.1. Kalman Filter

Various methods, such as Kalman filter and ARCH/GARCH models are used in practice to model beta instability. The question is, which method will be more appropriate to use? The findings of several studies suggest that Kalman filter seems more accurate method to use. For example, Brooks, Faff and McKenzie (1998) findings suggest that Kalman filter approach had superior performance over GARCH and Schwert and Seguin (1990) approach, i.e. it was proved to be the most accurate measure of risk out of the three measures examined. The method used for comparison of the beta-modelling techniques is the mean squared error (MSE) and the mean absolute error (MAE) of the forecasted industry returns. For both in-sample and out-of-sample forecasts, Kalman filter approach has shown superior forecasting performance, exhibiting smaller MAE and MSE than ARCH/GARCH or Schwert and Seguin methods. Also, in their study of UK unit trusts, Black, Fraiser and Power (1992) use the Kalman filter and suggest that only 25% of unit trusts maintain the constant level of risk during 1980s. Taking into account these findings, the methodology for time varying beta modelling that will be applied in this chapter will be the Kalman filter.

The Kalman filter procedure refers to an estimation method commonly used to estimate "state-space" models. State-space models originated in the engineering literature and were first in the economic (financial) literature in the late 1960s. This class of models consists of two parts: the transition equation, which describes the evolution of a set of state variables, and the measurement equation, which describes
how the data actually observed is generated from the state variables. The Kalman filter estimation method is an updating method that bases the regression estimates for each time period on last period's estimates plus the data for the current period. There is a considerable computational advantage and possibly some insights (investigating structural changes in the parameters) to be gained by calculating the estimates sequentially as new observations become available. Assuming a single dependent variable for simplicity the Kalman filter model (state-space form) can be written in the following way:

\[ Y_t = \alpha_t + \beta_t X_t + \varepsilon_t \]
\[ \varepsilon_t \sim N(0, \sigma^2 H) \]  
\[ \alpha_t = \lambda_{11} \alpha_{t-1} + \lambda_{12} \beta_{t-1} + u_t \]  
\[ \beta_t = \lambda_{21} \alpha_{t-1} + \lambda_{22} \beta_{t-1} + \eta_t \]

Equation (2.4), which in general is known as a simple market equation, in the technical literature terminology, it becomes the measurement or observation equation. Equation (2.4) can be estimated by OLS regression, but in that case one would assume constant \( \alpha_t \) and \( \beta_t \) parameters. In practice, there is evidence that those parameters are not stable over time. Therefore, to allow the parameters of market model to vary over time, equations (2.5) and (2.6) are introduced and they are known as the transition equations. The \( \alpha_t \) and \( \beta_t \) are called state variables and their initial values are assumed to be known with the following properties:

3 Each time our set of regression parameters is updated by incorporating a new observation (n+1) the "recursive" or "on-line" estimator is
\[ \hat{\beta}_{n+1} = \hat{\beta}_n + \frac{(X_n'X_n)^{-1}X_{n+1}'(Y_{n+1} - X_{n+1}\hat{\beta}_n)}{1 + X_n'X_n^{-1}X_{n+1}'X_{n+1}} \] 

The formula shows that the updated estimator is equal to the previous estimator plus an adjustment factor, which is proportional to the prediction error (innovation).
\[ \beta_0 \sim N(\beta_0, \sigma^2 P_0) \quad \text{and} \quad \alpha_0 \sim N(\alpha_0, \sigma^2 Z_0) \]

In addition, the initial values of \( \lambda \)'s are also assumed to be known and \( u_t \) and \( \eta_t \) are jointly distributed random errors with known variances and covariances:

\[ u_t \sim N(0, \sigma^2 \mathbf{V}) \quad \text{and} \quad \eta_t \sim N(0, \sigma^2 \mathbf{Q}) \]

The fact that prior information for the variance of the measurement error term, the variances and covariances of the transition error terms, the transition parameters and the initial values of the regression coefficients is needed, could be an obstacle which, however, can be handled by assuming that \( H = V = Q = \lambda_y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \). The matrix of coefficients in the transition equation, also called the transition matrix, is assumed to be diagonal and its elements are not time varying. By setting the transition matrix and all other unknown matrices equal to the identity matrix in which diagonal elements are equal to one, as shown above, we obtain a model in which coefficients from the measurement equation are allowed to vary over time as random walks. The vector of prior coefficients \( \alpha_0 \) and \( \beta_0 \) for the measurement equation will be calculated by default from a regression in the initial \( m \) observations of the sample, where \( m \) is the number of coefficients in \( \alpha \) and \( \beta \).

The above Kalman filter equations can be written more compact in a single equation form as follows:
\[ Y_t = \lambda^n \alpha_0 + \sum_{i=1}^{m} \lambda^{m-1}(\beta_{i-1} + u_i) + \left[ \lambda^n \beta_0 + \sum_{i=1}^{m} \lambda^{m-1}(\alpha_{i-1} + \eta_i) \right] X_i + \varepsilon_i \]  

(2.7)
4. Econometric Results

4.1 Summary Statistics

Descriptive statistics on average dividend yields, average market values, average OLS beta coefficients, average returns and standard deviations on returns are given in Table 1.

Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Portfolio number</th>
<th>Dividend Yield</th>
<th>Market Value</th>
<th>Beta Coefficient</th>
<th>Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1</td>
<td>3.61</td>
<td>18.55</td>
<td>-0.036</td>
<td>1.21</td>
<td>3.59</td>
</tr>
<tr>
<td>p2</td>
<td>4.30</td>
<td>38.60</td>
<td>0.267</td>
<td>1.06</td>
<td>4.02</td>
</tr>
<tr>
<td>p3</td>
<td>4.49</td>
<td>69.61</td>
<td>0.419</td>
<td>1.29</td>
<td>4.43</td>
</tr>
<tr>
<td>p4</td>
<td>4.64</td>
<td>332.37</td>
<td>0.541</td>
<td>1.07</td>
<td>4.62</td>
</tr>
<tr>
<td>p5</td>
<td>4.74</td>
<td>369.11</td>
<td>0.653</td>
<td>1.02</td>
<td>4.87</td>
</tr>
<tr>
<td>p6</td>
<td>4.88</td>
<td>439.63</td>
<td>0.758</td>
<td>1.15</td>
<td>5.29</td>
</tr>
<tr>
<td>p7</td>
<td>4.82</td>
<td>452.49</td>
<td>0.857</td>
<td>1.02</td>
<td>5.25</td>
</tr>
<tr>
<td>p8</td>
<td>4.71</td>
<td>547.90</td>
<td>0.962</td>
<td>1.02</td>
<td>5.41</td>
</tr>
<tr>
<td>p9</td>
<td>4.70</td>
<td>447.55</td>
<td>1.099</td>
<td>0.98</td>
<td>5.86</td>
</tr>
<tr>
<td>p10</td>
<td>4.19</td>
<td>233.08</td>
<td>1.405</td>
<td>0.57</td>
<td>6.60</td>
</tr>
</tbody>
</table>

The lowest beta portfolio provided the highest mean monthly returns over the time period under observation (1.21% for portfolio 1 and 1.30% for portfolio 3) whereas the highest beta portfolios provided smallest monthly mean returns for the investors (0.97% for portfolio 9 and 0.57% for portfolio 10). These findings are directly contradicting the CAPM theory that suggests that relationship between beta of the
portfolio and expected return of the portfolio should be positive, because of the expected positive market risk premium. Additionally, the greatest standard deviation in returns was found in portfolio 10 (highest beta portfolio) which exhibited the standard deviation of 6.6% per month and the lowest mean return. The lowest variation in returns was found in the lowest beta portfolio (3.6%) that has one of the highest mean returns. This is again inconsistent with the expectations of positive risk-return tradeoff.

As noted earlier, the betas of individual stocks in the sample are gained by using the OLS regression model that assumes the stability of parameters over time. Hence, the measure of the systematic risk of the portfolio, i.e. portfolio beta is calculated as the weighted average of betas of individual securities in the portfolio:

\[
\beta_p = \sum_{i=1}^{n} w_i \beta_i
\]  

(2.8)

where \( \beta_p \) is portfolio beta \((p = 1 \text{ to } 10)\)

\( w_i \) is the weight assigned to each stock in the portfolio, in this case equal weighting was used

\( \beta_i \) is the beta of individual securities in the portfolio \( p \)

Since the portfolio beta is calculated using time invariant parameters and the evidence provided by the literature suggests that beta parameters are unstable over time, it would be appropriate to assess the time varying beta risk by using the Kalman filter methodology as outlined in the section 3 of this chapter.
4.2 Kalman Conditional Beta Estimates

The issue that we want to tackle in this chapter is whether investors that invest in risky portfolios, which are consisted of stocks that have higher level of systematic risk, are facing even greater risk than originally assumed because of the time-varying characteristic of the measure of risk itself – beta. In particular, we want to examine whether high beta portfolios observe also greater instability over time. In order to test time-variability of betas we use Kalman filter model, which takes into account beta instability. The coefficients and t-statistics for the final state vector estimated through Kalman filter together with the log likelihood function are presented in the Table 2 and compared with OLS estimates.

Table 2: Kalman filter estimates

<table>
<thead>
<tr>
<th>Portfolio Number</th>
<th>Intercept</th>
<th>Conditional Beta</th>
<th>t-statistics (Kalman)</th>
<th>Variance of transition equation</th>
<th>Log of Likelihood Function</th>
<th>OLS beta</th>
<th>t-statistics (OLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1</td>
<td>9.80E-04</td>
<td>0.18</td>
<td>1.53</td>
<td>1.61</td>
<td>432.44</td>
<td>0.42</td>
<td>10.40</td>
</tr>
<tr>
<td>p2</td>
<td>-6.70E-04</td>
<td>0.38</td>
<td>3.14</td>
<td>0.73</td>
<td>425</td>
<td>0.55</td>
<td>13.37</td>
</tr>
<tr>
<td>p3</td>
<td>-0.011</td>
<td>0.56</td>
<td>4.63</td>
<td>2.01</td>
<td>423.58</td>
<td>0.67</td>
<td>16.82</td>
</tr>
<tr>
<td>p4</td>
<td>-0.019</td>
<td>0.64</td>
<td>5.19</td>
<td>0.31</td>
<td>422.2</td>
<td>0.73</td>
<td>18.72</td>
</tr>
<tr>
<td>p5</td>
<td>-0.016</td>
<td>0.73</td>
<td>6.3</td>
<td>1.28</td>
<td>433.77</td>
<td>0.81</td>
<td>21.50</td>
</tr>
<tr>
<td>p6</td>
<td>-0.024</td>
<td>0.84</td>
<td>6.72</td>
<td>0.50</td>
<td>419.94</td>
<td>0.88</td>
<td>21.94</td>
</tr>
<tr>
<td>p7</td>
<td>-0.556</td>
<td>0.87</td>
<td>8.09</td>
<td>1.67</td>
<td>448.85</td>
<td>0.92</td>
<td>26.79</td>
</tr>
<tr>
<td>p8</td>
<td>-0.9</td>
<td>0.94</td>
<td>8.41</td>
<td>1.79</td>
<td>441.93</td>
<td>0.96</td>
<td>28.57</td>
</tr>
<tr>
<td>p9</td>
<td>-0.649</td>
<td>1.03</td>
<td>8.65</td>
<td>0.26</td>
<td>429.18</td>
<td>1.04</td>
<td>27.88</td>
</tr>
<tr>
<td>p10</td>
<td>-2.10E-03</td>
<td>1.13</td>
<td>7.26</td>
<td>0.40</td>
<td>375.173</td>
<td>1.12</td>
<td>23.29</td>
</tr>
</tbody>
</table>

4 All variances of transition equations reported in Table 2 are statistically insignificant, hence the standard errors and respective t-statistics are not reported in the table. However, Harvey (1989) argues that reporting the t-statistics for variances of transition equations is meaningless since its standard errors follow nonstandard distributions.
The parameterisation of risk of Kalman filter and OLS estimation procedure is similar, especially for the larger beta portfolios. The estimates are produced on the basis of 204 observations for each portfolio. The t-statistics of the final state vector from the Kalman filter estimation is the smallest and insignificant at any commonly accepted level of significance for the lowest beta portfolio (1.53) and it increases as the beta of the portfolio increases (8.65 and 7.26 for portfolios 9 and 10 respectively). It means that investors investing in high beta portfolios are not only undertaking higher level of systematic risk but also that that risk is even more significant. As far as the variability of betas over time is concerned, we can see that all the variances of transition equations are different than zero and betas appear to be time-varying but their statistical significance is difficult to determine since standard errors follow non-standard distributions and are thus meaningless (Harvey, 1989).

The correlation coefficient between OLS beta estimate and Kalman conditional beta across all beta portfolios is very large: 0.9988. These findings are very similar to findings of Brooks, Faff and McKenzie (1998) who find the correlation between the OLS beta and Kalman beta across different industry portfolios in their study to be 0.983. However, the OLS beta coefficient is assumed to be constant over time. Therefore, it can show us the level of systematic risk that investors are facing when investing in particular beta portfolio in this study. On the other hand, it doesn’t pick up on the dynamics of beta, hence one should rely more on the Kalman filter estimates as to be more realistic. Let us observe the behaviour of dynamic beta coefficients presented in Figure 1 and Figure 2 below, that show the evolution of state vector beta and smooth vector beta respectively.
Additionally, we provide the average value of the smoothed vector beta, together with the standard deviation and the confidence band, which are presented in Table 3 below.

**Figure 1:**

Evolving state vectors, Kalman filter estimation

**Figure 2:**

Smoothed state vectors, Kalman filter estimation
Additionally, we provide the average value of the smoothed vector beta, together with the standard deviation and the confidence band, which are presented in Table 3 below.

Table 3: Smooth vector beta

<table>
<thead>
<tr>
<th>Portfolio number</th>
<th>Beta Smooth vector beta</th>
<th>Average beta</th>
<th>Standard deviation</th>
<th>Confidence band</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.32</td>
<td>0.10</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.45</td>
<td>0.04</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.59</td>
<td>0.07</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.65</td>
<td>0.04</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.73</td>
<td>0.07</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.82</td>
<td>0.06</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.87</td>
<td>0.07</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.92</td>
<td>0.06</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.99</td>
<td>0.06</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.07</td>
<td>0.09</td>
<td>0.012</td>
<td></td>
</tr>
</tbody>
</table>

If the beta was not time varying in a particular we should expect the standard deviation of the smoothed vector beta over the observed period to be zero. This is clearly not the case. Standard deviation in betas ranges between 4% and 10% for smoothed vector beta. The smooth vector beta standard deviation is the greatest in the smallest and the largest beta portfolio, however since we have shown that the smallest beta portfolio time varying beta is not significant we can say that the investor investing in the largest beta portfolio will incur additional risk stemming from more dynamic beta over time than in any other portfolio.
Additionally, the vast majority of empirical findings indicate that small stocks yield higher risk-adjusted returns than larger ones and the majority of these excess returns occur in the month of January. In some of the studies on seasonality, such as Rogalski and Tinic (1986), the seasonal behaviour of stock returns is the result of higher betas of stocks in January or it has been argued that the January effect is a result of small size effect. Therefore, in the sections to follow, we have classified our stocks into 10 size portfolios and examined the seasonality effect in returns and time variability of monthly betas over the period under observation.

4.3. Seasonality of Returns in Size Portfolios

Similarly to previous beta classification, in this section portfolios 1 to 10 represent different size firms and are ranked in ascending order from the portfolio containing smallest size stocks to the portfolio containing largest size stocks. Let us see some descriptive statistics for size portfolios. The results are given in the Table 4:
We can see that portfolio 10 has distinctively larger size than portfolios 1-9. Usually, as small-cap stocks we classify those that belong to the bottom decile or the lowest 10% of the market. Referring to the study of Reinganum (1992), some institutional investors may consider as small stock even those whose size is in the range of $500 million (around £350 million at the time of writing) to $1 billion (approximately £700 million at the time of writing). If this is applied to our portfolios, it corresponds to market values of portfolios 1-9, which makes only portfolio 10 to be distinctively large portfolio. The table indicates that the average return of the smallest and the largest size portfolios are greater than returns for portfolios 2-9.

Table 4: Average market value and return of 10 size portfolios, 1980-1996

<table>
<thead>
<tr>
<th>Size portfolios</th>
<th>Market value</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1</td>
<td>1.79</td>
<td>18.58</td>
</tr>
<tr>
<td>p2</td>
<td>4.31</td>
<td>11.27</td>
</tr>
<tr>
<td>p3</td>
<td>7.80</td>
<td>11.05</td>
</tr>
<tr>
<td>p4</td>
<td>13.04</td>
<td>11.25</td>
</tr>
<tr>
<td>p5</td>
<td>21.64</td>
<td>11.32</td>
</tr>
<tr>
<td>p6</td>
<td>36.82</td>
<td>13.48</td>
</tr>
<tr>
<td>p7</td>
<td>65.24</td>
<td>15.92</td>
</tr>
<tr>
<td>p8</td>
<td>129</td>
<td>16.77</td>
</tr>
<tr>
<td>p9</td>
<td>333.07</td>
<td>17.42</td>
</tr>
<tr>
<td>p10</td>
<td>2329.3</td>
<td>19.21</td>
</tr>
</tbody>
</table>
Let us analyse the seasonality in size portfolio returns. The average annualised\(^5\) return on 10 portfolios and FTSE All Share index are estimated for each month over the period 1980 through 1996. The results are presented in Table 5.

**Table 5: Average annualised percentage returns on size portfolios month by month, 1980-1996**

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1</td>
<td>47.92</td>
<td>38.33</td>
<td>14.49</td>
<td>34.63</td>
<td>19.87</td>
<td>13.76</td>
<td>-0.17</td>
<td>12.71</td>
<td>4.47</td>
<td>-1.87</td>
<td>3.45</td>
<td>35.40</td>
</tr>
<tr>
<td>p2</td>
<td>42.22</td>
<td>27.44</td>
<td>5.22</td>
<td>35.04</td>
<td>19.71</td>
<td>4.64</td>
<td>5.16</td>
<td>1.65</td>
<td>-10.03</td>
<td>-24.51</td>
<td>-0.23</td>
<td>28.94</td>
</tr>
<tr>
<td>p3</td>
<td>45.84</td>
<td>29.13</td>
<td>10.24</td>
<td>32.35</td>
<td>14.81</td>
<td>8.10</td>
<td>-0.70</td>
<td>2.35</td>
<td>-13.79</td>
<td>-23.15</td>
<td>-4.39</td>
<td>31.78</td>
</tr>
<tr>
<td>p4</td>
<td>49.80</td>
<td>23.15</td>
<td>23.33</td>
<td>34.87</td>
<td>19.11</td>
<td>1.50</td>
<td>-5.54</td>
<td>0.22</td>
<td>-15.43</td>
<td>-26.23</td>
<td>-5.12</td>
<td>35.30</td>
</tr>
<tr>
<td>p5</td>
<td>48.52</td>
<td>34.53</td>
<td>11.60</td>
<td>29.63</td>
<td>10.11</td>
<td>3.47</td>
<td>-1.37</td>
<td>1.74</td>
<td>-16.98</td>
<td>-20.21</td>
<td>-4.99</td>
<td>39.79</td>
</tr>
<tr>
<td>p7</td>
<td>50.27</td>
<td>38.92</td>
<td>20.78</td>
<td>30.09</td>
<td>9.31</td>
<td>3.07</td>
<td>-1.01</td>
<td>5.89</td>
<td>-8.35</td>
<td>-15.53</td>
<td>0.54</td>
<td>57.13</td>
</tr>
<tr>
<td>p8</td>
<td>56.54</td>
<td>32.01</td>
<td>23.79</td>
<td>33.41</td>
<td>3.29</td>
<td>-0.48</td>
<td>6.63</td>
<td>9.38</td>
<td>-10.81</td>
<td>-14.32</td>
<td>1.85</td>
<td>59.98</td>
</tr>
<tr>
<td>p9</td>
<td>51.10</td>
<td>27.58</td>
<td>17.02</td>
<td>34.04</td>
<td>-2.85</td>
<td>3.27</td>
<td>9.74</td>
<td>9.36</td>
<td>-18.24</td>
<td>-7.13</td>
<td>9.37</td>
<td>75.78</td>
</tr>
</tbody>
</table>

Analysing the returns in table 5, one can observe that the first six size portfolios have the highest return in January and the last four (or the largest four) portfolios have the highest return in December! The December effect was identified also by Lakonishok and Smidt (1988) who analysed half-monthly returns in the 90-year period on the US market. They find that high returns in December were the result of trading from pre-Christmas trading day through to New Years trading day. On the other hand, Levis

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\(^5\) Average annualised percentage return for each month is calculated as average monthly percentage return multiplied by 12:
(1989) and Corhay et. al. (1988) have suggested that small firm effect is largest in May. Although in our sample we cannot overall identify the size effect as reported previously in the literature (that smaller stocks will give investors higher returns) we can see from table 5 that this is the case in May. So, we can modify the finding of Levis to state that the small firm effect exists in May but is not necessarily in other months of the calendar year.

Let us examine now the statistical significance of these results. We have tested the existence of seasonality using the parametric method. Parametric methods involve testing the existence of seasonality and the difference in month-to-month mean returns using the following regression model:

\[ \bar{R}_{pt} = a_{1p} + a_{2p}D_{2t} + a_{3p}D_{3t} + \ldots + a_{12p}D_{12t} + e_{pt} \]  

(2.9)

Where \( \bar{R}_{pt} \) is monthly stock return for portfolio \( p \) (\( p = 1 \) to 10) in month \( t \) and \( D_{it} \) are the dummy variables indicating the month of the calendar year. \( D_{it} \) is taking value of 1 for month \( i \) and zero otherwise. \( a_{1p} \) measures the mean return of the month of the year (in our case January) for each portfolio \( p \) against which we want to measure the returns of the other months in the year. Hence, \( a_{1p} \) through \( a_{12p} \) measure the differences between the mean return in January and the returns for the remaining eleven months for each portfolio. The results of these tests are presented in table 6.
Table 6: Coefficients on dummy variables for size based portfolios and market index; Base month January

<table>
<thead>
<tr>
<th>Size portfolios</th>
<th>p1</th>
<th>p2</th>
<th>p3</th>
<th>p4</th>
<th>p5</th>
<th>p6</th>
<th>p7</th>
<th>p8</th>
<th>p9</th>
<th>P10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept: Jan</td>
<td>0.040*</td>
<td>0.035*</td>
<td>0.038*</td>
<td>0.041*</td>
<td>0.040*</td>
<td>0.043*</td>
<td>0.042*</td>
<td>0.047*</td>
<td>0.043*</td>
<td>0.036*</td>
</tr>
<tr>
<td>Feb</td>
<td>-0.008</td>
<td>-0.012</td>
<td>-0.014</td>
<td>-0.022</td>
<td>-0.012</td>
<td>-0.012</td>
<td>-0.010</td>
<td>-0.020</td>
<td>-0.020</td>
<td>-0.020</td>
</tr>
<tr>
<td>Mar</td>
<td>-0.028</td>
<td>-0.031</td>
<td>-0.030</td>
<td>-0.022</td>
<td>-0.031</td>
<td>-0.025</td>
<td>-0.025</td>
<td>-0.027</td>
<td>-0.028</td>
<td>-0.024</td>
</tr>
<tr>
<td>Apr</td>
<td>-0.011</td>
<td>-0.006</td>
<td>-0.011</td>
<td>-0.012</td>
<td>-0.016</td>
<td>-0.020</td>
<td>-0.017</td>
<td>-0.019</td>
<td>-0.014</td>
<td>-0.005</td>
</tr>
<tr>
<td>May</td>
<td>-0.023</td>
<td>-0.019</td>
<td>-0.026</td>
<td>-0.026</td>
<td>-0.032</td>
<td>-0.032</td>
<td>-0.034</td>
<td>-0.044*</td>
<td>-0.045</td>
<td>-0.039</td>
</tr>
<tr>
<td>Jun</td>
<td>-0.029</td>
<td>-0.031</td>
<td>-0.031</td>
<td>-0.040*</td>
<td>-0.038</td>
<td>-0.042*</td>
<td>-0.039</td>
<td>-0.047*</td>
<td>-0.040</td>
<td>-0.027</td>
</tr>
<tr>
<td>Jul</td>
<td>-0.040*</td>
<td>-0.031</td>
<td>-0.039*</td>
<td>-0.046*</td>
<td>-0.042*</td>
<td>-0.048*</td>
<td>-0.043*</td>
<td>-0.042</td>
<td>-0.035</td>
<td>-0.024</td>
</tr>
<tr>
<td>Aug</td>
<td>-0.029</td>
<td>-0.034</td>
<td>-0.036*</td>
<td>-0.041*</td>
<td>-0.039*</td>
<td>-0.041*</td>
<td>-0.037</td>
<td>-0.039</td>
<td>-0.035</td>
<td>-0.021</td>
</tr>
<tr>
<td>Sep</td>
<td>-0.036*</td>
<td>-0.043*</td>
<td>-0.050*</td>
<td>-0.054*</td>
<td>-0.055*</td>
<td>-0.051*</td>
<td>-0.049*</td>
<td>-0.056*</td>
<td>-0.058*</td>
<td>-0.048</td>
</tr>
<tr>
<td>Oct</td>
<td>-0.041*</td>
<td>-0.056*</td>
<td>-0.057*</td>
<td>-0.063*</td>
<td>-0.057*</td>
<td>-0.056*</td>
<td>-0.055*</td>
<td>-0.059*</td>
<td>-0.048*</td>
<td>-0.038</td>
</tr>
<tr>
<td>Nov</td>
<td>-0.037*</td>
<td>-0.035</td>
<td>-0.042*</td>
<td>-0.046*</td>
<td>-0.045*</td>
<td>-0.045*</td>
<td>-0.041*</td>
<td>-0.046*</td>
<td>-0.035</td>
<td>-0.019</td>
</tr>
<tr>
<td>Dec</td>
<td>-0.010</td>
<td>-0.011</td>
<td>-0.012</td>
<td>-0.012</td>
<td>-0.007</td>
<td>-0.005</td>
<td>0.006</td>
<td>0.003</td>
<td>0.021</td>
<td>0.029</td>
</tr>
<tr>
<td>F-test</td>
<td>3.25*</td>
<td>1.56</td>
<td>2.8*</td>
<td>2.19*</td>
<td>2.81*</td>
<td>3.74*</td>
<td>1.73</td>
<td>1.67</td>
<td>1.61</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Note: * is indicating significance at 5% confidence level and ** is indicating significance at 10% level.

We have already established that January returns are higher than in any other month across all beta portfolios and the market index. The coefficients on dummy variables are indicating that the mean returns in December and February to April (May, or June, depending on the portfolio number) are not statistically different from January return for all portfolios except the distinctively larger portfolio 10. Portfolio 10, as the only very large stock portfolio in our sample seems to have statistically equal returns across all calendar months. Therefore, in general, we observe that returns in the first half of the calendar year in the UK are generally much better than in the second half,
at least for smaller stocks. Therefore, we cannot single out a clear-cut January effect as found by Gultekin and Gultekin (1983) or April effect as in Levis (1989). As noted in Lofthouse (1996), the traditional advice of ‘sell in May and go away’ seems to be a profitable strategy here, at least for the smaller size portfolios. As far as market index is concerned, it can be seen that mean returns for all months are lower than the January return but significantly lower in some of the months in the second half of the year, namely September, October and November. The last row of Table 6 is reporting the results of a test of the hypothesis that mean returns were equal across all 12 months. Since we observe the differences in returns in the first and second half of the year (as suggested by Lofthouse), the F-test hypothesis is rejected for five, mainly smaller portfolios under observation. Therefore, we now test our original observation, if the returns in the December – April period are different from the returns in May – November period by using the following regression:

$$\tilde{R}_{pt} = a_p + a_p D_{pt} + \tilde{\varepsilon}_{pt}$$  \hspace{1cm} (2.10)

where $D_{pt}$ takes value of zero for months December through April and 1 otherwise.

The results are presented in the table below:

**Table 7: Differences in December-April and May – November returns for size portfolios**

<table>
<thead>
<tr>
<th>Size portfolios</th>
<th>p1</th>
<th>p2</th>
<th>p3</th>
<th>p4</th>
<th>p5</th>
<th>p6</th>
<th>p7</th>
<th>p8</th>
<th>p9</th>
<th>p10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec-Apr</td>
<td>0.028*</td>
<td>0.023*</td>
<td>0.025*</td>
<td>0.028*</td>
<td>0.027*</td>
<td>0.030*</td>
<td>0.033*</td>
<td>0.034*</td>
<td>0.034*</td>
<td>0.032*</td>
</tr>
<tr>
<td>May-Nov</td>
<td>-0.022*</td>
<td>-0.024*</td>
<td>-0.027*</td>
<td>-0.031*</td>
<td>-0.031*</td>
<td>-0.032*</td>
<td>-0.034*</td>
<td>-0.035*</td>
<td>-0.034*</td>
<td>-0.027*</td>
</tr>
<tr>
<td>F-test</td>
<td>9.60*</td>
<td>9.53*</td>
<td>13.01*</td>
<td>17.43*</td>
<td>14.80*</td>
<td>15.74*</td>
<td>15.36*</td>
<td>14.62*</td>
<td>11.39*</td>
<td>6.74*</td>
</tr>
</tbody>
</table>

Note: * is indicating significance at 5% confidence level and ** is indicating significance at 10% level.
It can be seen from the coefficients that May-November returns are significantly lower than December-April returns at 5% level of confidence across all size portfolios. The F-test hypothesis can be rejected also for all portfolios confirming that the strategy 'sell in May and go away' would be a profitable one in the UK.

4.4. Time varying beta in 10 Size Portfolios

As previously noted, Rogalski and Tinic (1986) suggest that the reason for returns seasonality is seasonality in the beta coefficient of particular size portfolio. Since we have shown that we do not have a clear cut seasonality effect in our portfolio, we will examine the time-variability of betas (similarly to what we have estimated for beta portfolios) and try to see if it may be the cause for the variability in stock returns of small size portfolios. Kalman filter beta estimates are in Table 8.
Table 8: Kalman filter estimates for 10 size portfolios

<table>
<thead>
<tr>
<th>Portfolio Number</th>
<th>Intercept</th>
<th>Conditional Beta</th>
<th>t-statistics (Kalman)</th>
<th>Variance of transition equation⁶</th>
<th>Log of Likelihood Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1</td>
<td>0.0057</td>
<td>0.39</td>
<td>2.22</td>
<td>0.08</td>
<td>350.89</td>
</tr>
<tr>
<td>p2</td>
<td>-0.0084</td>
<td>0.51</td>
<td>2.72</td>
<td>0.38</td>
<td>337.70</td>
</tr>
<tr>
<td>p3</td>
<td>-0.021</td>
<td>0.57</td>
<td>3.33</td>
<td>0.38</td>
<td>354.67</td>
</tr>
<tr>
<td>p4</td>
<td>-0.026</td>
<td>0.64</td>
<td>3.60</td>
<td>0.25</td>
<td>347.91</td>
</tr>
<tr>
<td>p5</td>
<td>-0.011</td>
<td>0.73</td>
<td>3.81</td>
<td>0.05</td>
<td>331.46</td>
</tr>
<tr>
<td>p6</td>
<td>-0.011</td>
<td>0.79</td>
<td>3.92</td>
<td>0.06</td>
<td>321.20</td>
</tr>
<tr>
<td>p7</td>
<td>-0.0075</td>
<td>0.80</td>
<td>3.61</td>
<td>0.05</td>
<td>303.82</td>
</tr>
<tr>
<td>p8</td>
<td>-0.009</td>
<td>0.97</td>
<td>4.36</td>
<td>0.04</td>
<td>301.26</td>
</tr>
<tr>
<td>p9</td>
<td>-0.0067</td>
<td>1.07</td>
<td>4.29</td>
<td>0.18</td>
<td>279.15</td>
</tr>
<tr>
<td>p10</td>
<td>0.0030</td>
<td>1.05</td>
<td>4.07</td>
<td>0.04</td>
<td>271.26</td>
</tr>
</tbody>
</table>

Table 8 indicates that estimated final state vector betas are increasing with the size of the portfolio and that their significance is increasing as well. Additionally, variability of betas over time as indicated by variance of transition equations is much smaller for these portfolios sorted according to size than when portfolios are sorted by betas, as in table 2. This is consistent some of the prior evidence that states that variability of betas in size portfolios is constrained due to size factor. The average smoothed state vector beta, its standard deviation and confidence bands are presented in Table 9 below:

⁶ Standard errors are not reported due to Harvey (1989), who argues that reporting the t-statistics for variances of transition equations is meaningless since its standard errors follow nonstandard distributions.
### Table 9: Smooth vector beta, size portfolios

<table>
<thead>
<tr>
<th>Size</th>
<th>Portfolio number</th>
<th>Smooth vector beta</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average beta</td>
<td>Standard deviation</td>
<td>Confidence band</td>
</tr>
<tr>
<td>1</td>
<td>0.46</td>
<td>0.07</td>
<td>0.014</td>
</tr>
<tr>
<td>2</td>
<td>0.55</td>
<td>0.06</td>
<td>0.008</td>
</tr>
<tr>
<td>3</td>
<td>0.58</td>
<td>0.07</td>
<td>0.014</td>
</tr>
<tr>
<td>4</td>
<td>0.65</td>
<td>0.08</td>
<td>0.011</td>
</tr>
<tr>
<td>5</td>
<td>0.70</td>
<td>0.07</td>
<td>0.016</td>
</tr>
<tr>
<td>6</td>
<td>0.74</td>
<td>0.12</td>
<td>0.014</td>
</tr>
<tr>
<td>7</td>
<td>0.77</td>
<td>0.06</td>
<td>0.015</td>
</tr>
<tr>
<td>8</td>
<td>0.92</td>
<td>0.06</td>
<td>0.012</td>
</tr>
<tr>
<td>9</td>
<td>1.02</td>
<td>0.08</td>
<td>0.013</td>
</tr>
<tr>
<td>10</td>
<td>1.03</td>
<td>0.03</td>
<td>0.017</td>
</tr>
</tbody>
</table>

If the betas of size portfolios were not time varying, the standard deviations of the smoothed state vector from the above table would be equal to zero. We can see that standard deviations for smoothed betas for smaller portfolios 1-9 are much higher than the standard deviation of the betas for distinctively larger portfolio 10. We may use this as an indicator to explain lower variability of portfolio 10 returns observed in table 6 above.

When we apply the same estimation procedure as in (2.10) to test if evolving state vectors from Kalman filter estimation can be used to explain the exact pattern of
returns that we have found in this chapter, i.e. if betas in December to April period are higher than betas in May to November period for 10 size portfolios. We find that betas in May to November period are lower for most of the 10 portfolios but the results are not significant at any generally acceptable level of significance and hence we are not presenting them here.

Since our results are ruling out beta as the explanatory variable for the observed returns pattern in the UK, let us consider some alternative explanations. One alternative explanation put forward for the seasonality effect is tax-loss-selling hypothesis, which will cause seasonality effect in the first month of the calendar year, i.e. April in the UK. Since in our data sample we cannot single out the April effect as the predominant one in size portfolios and also we find that January returns are higher than April returns in all instances, we are convinced that tax-loss-selling explanation of the seasonality does not play the major role in our sample. Hence, the only alternative explanation that could be suggested at this point is stemming from investor's behaviour that is influencing the volume of trading in the first half of the year in the UK. For example, at the turn of the year (December- January), a boost in volume can be liquidity based (New Year salary bonuses) or it can be related to the corporate information releases. On the other hand, towards the end of financial year in the UK (March-April), volume increase may be because investors want to realise their capital losses.
5. Conclusions

Prior empirical evidence suggests that unconditional beta estimates from the OLS regression model are not stable over time and therefore exclude important information for investor. The chapter is examining the time varying component of the systematic risk measure – beta. We have divided our sample of stocks into 10 beta portfolios in order to test whether stocks with higher beta also exhibit higher instability. The conditional betas for 10 portfolios were generated by using Kalman filter approach. Although the parameterisation of risk between OLS and Kalman beta estimate is similar, Kalman conditional beta estimates are suggesting that for portfolio that have greater level of systematic risk, significance of that risk is also higher. When analysing the standard deviations of evolving smoothed state vector beta, we find that betas are time variant across all portfolios (standard deviation is always different than zero) but the largest volatility in betas is present in the smallest and the largest beta portfolio. Since we have established that betas are dynamic, we have proceeded to test if the dynamics of betas is influencing seasonality in stock returns. Since prior studies suggest that smaller stocks have observed higher returns in January than in any other month of the year due to higher betas, we test seasonality effect on 10 size portfolios. We have found that in our sample of stocks, for size based portfolios, there is a pattern in stock returns that shows seasonality in the following manner: returns are higher in the December-April period than in May-November period. Automatically, we rule out tax-loss selling hypothesis as the explanation of the observed stock returns behaviour. We also find that betas for small stock portfolios are more dynamic than beta of the large stock portfolio. However, although our findings suggest that betas in the period May-November are lower than betas in the first half of the year for most of
the portfolios, the results are not statistically significant. Therefore, since betas can also be ruled out as the explanation of the seasonality, possible explanation of seasonal behaviour of stock returns in the UK that one can suggest could be the investors' behaviour, which is increasing the trading volume at the turn of the calendar year and at the turn of the financial year.
CHAPTER THREE

THE DIVIDEND YIELD EFFECT: EVIDENCE FROM THE UK
1. Introduction

1.1. Definition of the Yield Effect

In chapter one, we have outlined several variables that can be used for determining alphas, for example: size, dividend yield, book-to-market ratio, price-earnings ratio, share price etc. The study by Levis (1989), that will be discussed in greater detail later in this chapter, suggests that the variable that has the strongest relationship with excess returns in the UK market is the dividend yield. Hence, in this chapter we will investigate the dividend yield effect in the UK market. The term ‘dividend yield effect’ is regarded as the phenomenon in the stock market behaviour in which case the securities paying high dividend yield should provide investors with higher returns. Whether one can actually make profits by buying the market on high yields and selling when yields are low is a subject of the debate in financial economics for a long time. Many authors consider the yield effect as the market anomaly, practically meaning that investors can make profits by systematically investing in high yield stocks. This contradicts with the market efficiency hypothesis, which states that, in the long run, investors cannot outperform the market. Therefore, if a reasonable explanation were offered to justify where the yield effect stems from, it would no longer be considered as the anomaly. Therefore, many researchers have tried to find the possible reasons for dividend yield effect and to provide support for efficient market hypothesis. Some of the explanations offered are as follows:

- Tax effect. This is one of the most commonly accepted explanations for the yield effect in the US. It was explained by Litzenberger and Ramaswamy (1979). Particularly, they have shown that the positive relationship between yields and
returns is a consequence of different tax rates investors pay for dividends and capital gains. In many countries dividends (income) is taxed at a higher rate than capital gains. Therefore, an investor would purchase higher yielding stocks only if he is offered a compensation for the tax he has to pay, i.e. if those stocks pay higher returns. On the other hand, even if tax rates were equalised, capital gains tax is paid only when the gain is realised, which means that it can be postponed whereas that is not the case with income tax. Litzenberger and Ramaswamy derive the after-tax version of the CAPM (where $R-r_f = \gamma_0 + \gamma_1 \beta + \gamma_2 (d - r_f) + \varepsilon$) and observe the period from 1936 to 1977. 1936 was the first year when dividends became taxable. They compare the results from the cross-sectional regression based on the after tax model and based on before tax CAPM (where $R-r_f = \gamma_0 + \gamma_1 \beta + \varepsilon$). Authors use OLS, GLS and Maximum Likelihood estimation (MLE). The coefficient on the excess dividend yield variable is positive, less than unity and highly significant under all estimation procedures. The magnitude of the coefficient suggests that for the dollar of taxable return investors require around 23, 24 cents of additional before-tax return. They argue that $\gamma_2$ changes over time because it represents the weighted average of individual's marginal tax rates. MLE estimates of $\gamma_2$ are showing that there is no upward trend in that variable over time. Authors also test whether the yield effect reverses itself in non-ex-dividend months. However, results indicate that dividend effect on before-tax returns is positive in both ex-dividend months and non-ex-dividend months. Therefore, in the US, in order for the tax effect to be used as the sole explanation of the yield effect, we should observe positive linear relationship between yields and returns. However, in the UK, we observe somewhat different tax rules related to dividend payments than in US. The American taxation system explained in
Litzenberger and Ramaswamy (1979) study is known as the classical system. Under a classical system, all profits are subject to taxation twice: once in the form of corporation tax and second time in the form of either income tax (if profits are distributed as dividends) or capital gains tax (if profits are retained by the company and the value of shareholders' equity rises). Since rates of income tax are higher than the rates of capital gains tax, dividend income was 'penalised' under the classical tax system. In the UK, since 1973, classical system was replaced with the 'imputation' tax system. This system was meant to reduce the 'tax penalty' on dividend income by 'imputing' part of the company's tax liability to shareholders. In particular, shareholders in the UK receive a 'gross dividend' of £100(1-t)(1-c), where t is corporation tax rate and c is the rate of imputation, currently being a basic rate of 20%. Such a 'gross dividend' is than liable for the personal income tax. Therefore, the shareholder actually receives £100(1-t)(1-c)/(1-m), where m is personal income tax rate. Under this system, the shareholder who pays income tax at the basic rate will pay the same tax regardless of whether company pays dividend or not (due to the fact that c = m). Different groups of shareholders have different preferences regarding dividend payments. As an example, pension funds are dividend tax-exempt, hence they would prefer dividend income. For insurance companies it is known that c = m, however they are still paying capital gains tax. Although there are advantages regarding capital gains tax in the sense that there is a tax-exempt amount (which is not very relevant for institutional investors) and that its payment can always be deferred, insurance companies are favouring dividends. Dividend income of Investment trusts and Unit trusts is taxed at the basic rate, implying that c = m, while any capital gains are tax exempt. Hence, investment and unit trusts can be regarded as tax neutral
investors and their tax preferences are determined by the preferences of individuals investing in a particular trust. As far as individual investors are concerned, the 1988 Finance Act equalised the tax rate paid on personal income and capital gains. One would expect that in such case individuals would unconditionally favour dividends. However, the tax-exemption amount from capital gains tax, currently £6900, as well as the fact that capital gains tax is paid only on realised gains, is eliminating the payment of capital gains tax for large number of private shareholders. Hence, those investors who are not paying capital gains tax and for whom m > c will clearly favour capital gains, not dividends. However, private investors in the UK market are minority. In the 1990s, their proportion in the overall number of investors fell to less than 20%. Therefore, since the majority of investors in the UK are institutions who prefer dividend payments, one can conclude that British tax system is treating dividend income favourably in relation to capital gains. This has been shown in the studies by Poterba and Summers (1984), Ashton (1991) Chui et al. (1992), Bond et al. (1995). Particularly, Poterba and Summers (1984) find that, regardless of the class of the investor, marginal dividend tax rate is negative (-0.0277) compared to the capital gains marginal tax rate (0.1343). Hence, it can be concluded that the tax-based hypothesis in the UK is suggesting the reverse yield-return relationship than the one observed in the US: investors who are paid lower dividends will require higher returns. Specifically, if the tax-based hypothesis is to be used as the plausible explanation for the yield effect in the UK, we should find a negative relationship between dividend yields and risk-adjusted returns: stocks paying high yields should earn low risk-adjusted returns and vice versa. Studies of the yield effect in the UK market by Levis (1989) and Morgan and Thomas (1998) are
showing that such a negative yield/return relationship does not exist implying that tax-based hypothesis does not explain the relationship between dividend yields and risk-adjusted returns.

- The fact that contradicts the tax effect, as defined by Litzenberger and Ramaswamy, is that the evidence exists that states that zero dividend-paying stocks also offer high returns. Elton et al. (1983) provides evidence for that. The authors demonstrate that dividend yield has a large and statistically significant impact on the return above and beyond that explained by zero beta CAPM. The data they use is monthly data on prices, dividends and returns for NYSE stocks for the period January 1927-December 1976. Since they are interested in relationship between CAPM beta and dividend yield, they calculated the estimates of those two variables. In estimating CAPM, beta is calculated based on five years of historical data, using CRSP value weighted index. On the basis of betas stocks were ranked into 20 portfolios and beta for each portfolio was then calculated using rolling regressions for the each overlapping five years period. The authors finally form a sample of beta estimates for 20 portfolios for 40 years periods. Also, return on each portfolio is calculated. Stocks were also grouped according to dividend yield into 20 groups (in a decreasing manner of dividend, where group 20 was zero-dividend group). In order to forecast dividend yield in year, say, 11, they have used the actual yield in year 10 as a forecast. Following this, they are trying to relate excess return to dividend yield of the portfolio. Authors believe that zero-yield stocks behave differently, so the they assess the yield-return relationship by including a Dummy variable in the analysis, taking value of 1 for the portfolio of stocks which paid zero dividends and zero for all other portfolios.
Cross-sectional regressions of average excess returns of 20 portfolios on dividend yield over the 40 years period are showing small but positive and significant relationship between the two variables. They believe that yield effect is small due to zero yield group that has higher return than any other yield group. Therefore, the regression that includes Dummy variable shows the results that suggest that yield coefficient is now large (0.794) and statistically significant at the 1% level. Elton et al. argue that behaviour of zero-yield stocks is due to the small size effect and low price effect. When the same regressions as before were run excluding the stocks of value less than $5, the yield coefficient has improved from 0.794 to 0.865 and is significant at 1% level. In conclusion, Elton et al. find the persistent positive relationship between yield and excess returns for dividend paying stocks. However, highest excess returns of the zero-yield group are clearly showing that tax-hypothesis cannot be applied and that the explanation for such a behaviour lies in the small size, low price stocks.

- Information effect. Miller and Scholes (1982), stating that the yield effect is associated with the information bias hidden in dividend, provide the evidence for this explanation of the yield effect. For example, if a company pays high dividend it will give a signal to the market that it is performing well and, as the result of good news, its share price will rise providing an investor with the higher return. The problem with this explanation lies in the fact that if the company is paying low dividend it is regarded to be a bad news for the stock market. However, low dividend doesn’t have to be associated with financial distress of the company, it can easily mean that the company is retaining more profits in order to finance future investments or projects, which is actually the good news. In particular,
Miller and Scholes (1982), start with the re-examination of the tax effect as a plausible explanation of the yield effect by distinguishing between the short run and the long run dividend yield measures. They find that for the short-run dividend yield measure used in this study, the yield effect is the result of the dividend announcements. The difference between this and previous studies is that in this case authors use individual stocks instead of portfolios in the period 1940-1978. They estimate the after-tax CAPM model and they interpret the dividend yield coefficient that is positive and less than 0.6 as 'tax differential'. For different short-term definitions of dividend yield that they use they have obtained positive and significant yield coefficients and within the plausible range for tax effect. They find that the yield-return relationship is sensitive to the definition of yield. However, they question whether this is the tax effect or the information effect because yield effect appeared to reflect the degree to which different short-run yield measures introduce unwanted information effects. After correcting those measures for information effect, there is no significant yield-return relationship that can be considered as the classical tax effect.

- Dividend yield effect is associated with other market anomalies such as small size effect or January seasonal, as reported by Keim (1985). The evidence has been provided that high dividend paying stocks are, in most cases, stocks that have small market capitalisation. Therefore the reasons for yield effect would be the reasons for small size effect such as: higher risk, low price, neglected stocks etc as noted before by Elton et al. However, this explanation seems to be like a vicious circle: one anomaly is explained by another one, that one by the third one and so on.
• Investors' poor assessment of growth prospects of the company. As an example, a company paying high dividend yield is usually a small company with the low price and is not expected to do very well on the market. However, growth prospects of small companies are very high, they can surprise everyone and their price can rise, providing an investor with the higher return. This explanation doesn't seem plausible if investors assess the growth prospect of a company correctly.

Apart from these explanations of the yield effect, it is important to mention the dividend neutrality hypothesis that is developed by Black and Scholes (1974). It contrasts the tax-effect hypothesis and it is used to explain entirely different pattern of yield-return relationship. In particular, Black and Scholes find no reliable link between portfolios monthly stock return and its long-run dividend yield, i.e. they find that the relationship can be represented by a relatively flat line. They state that in the equilibrium, firms will adjust their dividend policies so that the aggregate supply of dividend meets the aggregate demand for dividends from investors that value dividends as capital gains.

1.2. Empirical Evidence on the Yield Effect

The evidence related to the yield effect has been mixed. Keim (1985) and Blume (1980) report that zero-yield stocks and highest yield stocks are realising higher returns than stocks that belong to the lower dividend yield categories, forming a U-shaped relationship between yields and returns. In particular, the data Keim uses to examine the relationship are all firms that are listed on NYSE and that have returns
for the last 60 months prior to the date it was included in the sample. Each month firms leave and enter the sample, hence, he starts analysis with 429 firms in January 1931 and ends it with 1289 in December 1978. Keim divided the sample of securities into six equally weighted portfolios of increasing dividend yield, starting from zero-dividend firms group. He interprets the dividend yield in month t as the sum of dividends paid in the previous 12 months divided by the stock price in the month t-13.

He reports mean returns for each dividend yield portfolio, average dividends yields and average market values. His findings state that zero dividend stocks have highest returns, whereas returns for dividend paying stocks increase as the yield increases. He also applies one-period Sharpe-Lintner CAPM to estimate excess returns of each portfolio and analyse relationship between risk-adjusted returns and dividend yields. If there is a yield effect, according to CAPM, the estimates of excess returns of portfolios will be systematically related to dividend yields. He rejects the hypothesis that excess returns across portfolios are jointly equal to zero and the hypothesis that average returns are equal across portfolios. Moreover, Keim examines the relationship between yield and size. He formed five size portfolios within the original six dividend yield portfolios, resulting in 30 categories overall. The findings are showing that the smallest firms are concentrated in the zero-dividend group and the highest yield group. The implication of this finding is that the high average returns of zero and highest yield firms may not be due to dividend yield effect but the size effect. In order to examine the interaction between the seasonality (January) effect and the yield effect, he tests the hypothesis that the average returns are equal across portfolios within a month. The hypothesis in the month of January has to be rejected, however for all the other months it cannot be rejected. It means that in months other than
January, returns are smaller and yield effect can be neglected and that relationship between yield and returns is concentrated in January. After testing whether the yield-return relationship should be attributed to differences in tax rates for dividends and capital gains, Keim finds that the yield coefficient in January is too large to be interpreted as the 'marginal tax bracket'. Similar analysis is performed to assess yield-size relationship. Size coefficient is larger in January than in other months and dividend yield coefficient remains significant even when controlling for the size. However, January coefficient is still too large to be interpreted as tax brackets associated with after tax pricing models.

The same yield-return relationship as reported by Keim we can find in the study of Blume (1980). In order to assess the relationship between dividend yield and return, Blume calculates dividend yield as the ratio of dividends paid over 12-month period to the beginning of period price adjusted for general market movements. Data used in the paper are quarterly returns of all NYSE stocks form 1936 to 1976. Quarterly returns are used due to the fact that in US dividends are paid quarterly, so Blume believes that the tax effect might differ in periods when stock went ex-dividend and when it didn't. Each quarter, stocks were sorted into five portfolios of equal size according to their beta coefficient estimated with Fama and MacBeth (1973) methodology. Each of the beta portfolios was then subdivided into five dividend yield groups. The process results in 25 portfolios for each quarter. The same process was repeated for each quarter through 1976. Overall, 164 quarterly cross-sectional regressions were run. Blume assumes that by grouping the stocks according to beta first and then by dividend yield might cause bias against finding the dividend yield effect because within one beta group there will be low variability of dividend yields.
Therefore, he completes the grouping the other way around (first forming five dividend yield groups and than within each of them five beta groups). Regression results were as follows: for portfolios grouped first on beta and then on dividend yield, average coefficient on dividend yield for the whole period was 0.5232 with t-statistics of 2.07. The significance of dividend yield variable is changing over time. For example in two decades 1937-46 and 1957-66 the average coefficient was positive but not significant at any level of significance commonly used. On the other hand, from 1947-56 the dividend yield coefficient was high (0.8743) and highly significant (t-value of 4.27). The same results to the greater or lesser extent were obtained for portfolios grouped first on dividend yield and then on beta. To draw the line between dividend paying and zero dividend stocks, Blume includes Dummy variable in the analysis (1 for zero paying stocks and zero otherwise) and he finds that the dividend yield coefficient becomes even more significant. This shows some evidence of the non-linear relationship between return and dividend yield. Over the period studied, one can observe that the zero dividend paying stock returns are higher than returns of dividend paying groups. Blume’s explanations of the yield effect are including the tax-based hypothesis. However, in a tax system where dividends are taxed at the greater rate than capital gains (US in 1970s), the expected before tax returns should increase with the increase in dividend yield. Since in this paper portfolio returns follow a U-shaped pattern with the increase in dividend yield, tax-based explanation is not the adequate one. Another explanation that the author offers is that market did not manage to anticipate the greater relative growth of dividends for high yield stocks compared to low yield ones.
More recent study by Christie (1990) is reporting that zero-yield stocks are actually realising negative abnormal returns, significantly lower than dividend paying stocks. He argues that this evidence related to zero-yield stocks contradicts previous research due to the differences in sample period they use. In particular, Keim and Blume sample period is including the years prior to the Second World War, whereas Christie analyses stock returns during the period 1945 to 1986. The author classified a firm as a zero dividend if it paid no cash dividends since listing or if it announced it won’t pay the next dividend (non-dividend paying, dividend-initiating and dividend-omitting firms). All other firms are regarded to be dividend paying and are placed in yield quartiles. The author uses the size based risk-adjustment returns model. Specifically, in each month (t), firms are sorted into size deciles based on market value in previous month (t-1) and within each size decile firms are divided into yield quartiles based on yield in the previous month (t-1). Yield groups include both zero-dividend and dividend-paying firms. Expected return of firm i represents the average monthly return of all firms belonging to the same size decile excluded from firm i’s yield category. The main advantage of this size-based model is that the firms are included in the sample in the first month of listing by using their market value at the end of the month to assign them to appropriate size decile. On the contrary, authors like Keim (1983,1985) and Blume (1980) required a security to be listed at least 60 months before it can be included in the sample. The second advantage of this method is that it enables us to control for the size effect. This paper considers size as a proxy for risk. Christie shows that majority of zero-dividend firms belong to the smallest size deciles. The empirical results for the yield-return relationship are surprising in that zero-dividend firms have an average loss of -0.41% per month in comparison with the dividend paying firms of the same size. That pattern can be observed in every
month except January. Additionally, in the small size decile, zero dividend firms are outperforming dividend paying ones by 4.59%. At the smaller scale but at 10% significance level, the same can be concluded for size deciles one through six. These results are very much in contrast with the findings of Keim (1983) who reports that zero-dividend firms have average adjusted monthly return of 0.27%. Christie states that returns of zero-yield stocks observed by Keim and Blume are driven by low price stocks from 1930s. Blume (1980) has also noted in his study that superiority of returns of zero-yield stocks to dividend paying stocks stems from the 1936-1946 decade. Observing the relationship between returns of positive dividend yield paying firms and their yields in this study, one can conclude that there is a positive relationship. Introducing the zero-dividend group in the analysis, the U-shaped yield/return relationship cannot be identified. On the contrary, the behavior of zero dividend firms is following the pattern of other yield categories. The U-shaped relationship is persistent only in January. However, although Christie (1990) recognises that in dividend paying portfolios returns are increasing with the increase in dividend yields, he doesn’t explore this issue but focuses on the differences between zero-dividend and dividend paying stocks of the similar size. Christie further examines whether differences in returns are the consistent with the tax effect. These results suggest that the negative excess returns of zero dividend firms cannot be a result of the tax effect. However, additional explanation that focuses on market expectations of cash dividends is given.

Naranjo et al. (1998) are looking at the sample period from July 1963 through December 1994. They define the dividend yield as $4D/P_{t-1}$, where $D$ is the last declared quarterly dividend before the end of month $t-1$. The criteria authors apply to
include a stock into a portfolio that pays dividends are: in the prior twelve months, the company has to have either four ex-dividend dates or four dividend declaration dates and that the company has no other special dividends declared, that may not recur. The criteria for stocks to be included in the zero-yield portfolio if it had no dividends in the prior year and that it was listed on the CRSP NYSE tape for at least a year. Securities are grouped into one zero-yield portfolio and ten dividend-paying portfolios. They find that zero-yield portfolio returns are outperforming the first four lowest yield-paying portfolios. On the other hand, their evidence shows that in dividend-paying portfolios returns are increasing with the yield, but up to the certain point: the two highest yield portfolios are realising approximately 2.5% lower annual return than the portfolio number eight that has the highest return of all. They also classify stocks into 20 portfolios: first into 5 according to the dividend yield and than within each group according to the market values. Within each size portfolio there is a positive relationship between yield and return as well. The smallest companies are concentrated in the smallest and the largest yield group. Risk-adjusted returns are estimated by applying multifactor asset pricing models, where the factors are: market portfolio, difference between return of portfolios of small and large stocks and the difference between returns of portfolios with high book-to-market and low book-to-market ratio stocks. They perform OLS regressions in each of the eleven yield portfolios, using portfolio return as the dependant variables and the above factors as independent ones. The results of abnormal returns found suggest an existence of the yield effect. To test the hypothesis that the abnormal returns of eleven portfolios are jointly equal to zero, they use F-test that takes the value of 4.57 with a p-value below 0.0001, so the hypothesis is easily rejected. The same test was repeated for 10 dividend-paying portfolios to show that the previous results are not influenced by the
zero-dividend firms. Furthermore, the F-test was completed within each of the 20 portfolios, i.e. within each of the size quartiles. The finding is that the null hypothesis that abnormal returns across dividend yield groups are jointly equal to zero is rejected in all size quartiles except the largest one. This finding is inconsistent with the tax effect. For all twenty dividend yield and size sorted portfolios the dividend yield coefficient is large and significant, but too high to be associated with the tax effect. The authors test whether the yield effect is related to the poor performance of stocks after initial public offering (IPO) or secondary offering by using the basic multifactor regression for eleven portfolios and twenty portfolios that do not include companies that made an issue. The yield effect is still significant. Additional tests done were to see the significance of the earnings-to-price (E/P), book-to-market (B/M) and cashflow-to-price (CF/P) effects and five years sales-to-growth effect on the yield effect. They find that the yield effect is not a proxy for any of the above mentioned effects. Another attempt to use tax hypothesis as an explanation is made by including the implied tax rate (ratio of one-year prime grade municipal yield to the one-year T-bill yield) in the analysis. They find no evidence of the tax effect. Therefore, there are two clear conclusions in this paper: yield effect does exist and it is not related to the ‘tax penalty’ on dividends.

Apart from positive relationship between yield and returns, a pattern indicating negative yield-return relationship can be found. In particular, that pattern shows that portfolio returns are decreasing with an increase in dividend yields. Such negative relationship between yields and returns occurs when investors that prefer cash dividend willingly accept lower before-tax returns on the high yield stocks. This hypothesis used in explaining the negative yield-return relationship is called dividend
preference hypothesis. Christie and Huang (1994) have identified this negative pattern in yield-return relationship in the years that have been singled out of their overall sample. The sample period in their study covers the years from 1946 through to 1994. They use size-based risk adjusted returns as in Christie (1990). Christie and Huang (1994) introduce a new approach in evaluating whether the yield effect is attributable to the tax differential between dividends and capital gains. They introduce analysis where they observe annual pattern of the yield-return relationship year by year across 20 dividend yield portfolios. Specifically, negative yield/return relationship is observed in years 1981, 1982 and 1984. They find that in the years when the tax differential between income and capital gains was the largest (period 1946-1971), the tax effect should have been the most obvious. However, only years 1962, 1968 and 1970 produce results consistent with the tax hypothesis. The overall conclusion is that in the entire sample from 1946-1985, majority of the evidence is consistent with the tax neutrality hypothesis. Christie and Huang are the first authors that introduce the analysis of the yield-return relationship after equalisation of income and capital gains taxes in the US by the 1986 Tax Reform Act (TRA). For post-TRA evidence authors use 1986-1990 years. In 1986, the evidence suggests an upward sloping yield/return relationship, while in years 1987 through 1989 we observe relatively flat yield /return patterns supporting the tax neutrality hypothesis, but there is a slight decline in returns after portfolio 16. In 1990, there is a relatively flat pattern through portfolio 15, but the highest yield portfolios now observe significantly positive returns. According to this paper, the tax based explanation of the yield effect seems implausible because the flattest yield return relationship is in the years of the greatest tax differential and the strongest evidence for the tax-based hypothesis emerges in the year when the TRA
was introduced. Therefore, yield-return relationship is of little or no guidance for investors seeking higher returns.

In addition to the studies that observe the relationship between stock returns and dividend yields, it is important to note that one of the earliest studies, Black and Scholes (1974), suggests that there is no link between portfolio’s monthly return and its long run dividend yield. The data they use is a monthly data on dividends, prices and returns for any common stock listed on NYSE in the period January 1926 - March 1966. Then, they group the securities into five portfolios on the basis of dividend yield and they divide each yield portfolio into five portfolios according to securities’ beta coefficient, ending up with 25 portfolios in total. The authors have supposed that dividend yield is related to return of stocks and that relationship is linear, so they have estimated the cross-sectional regression of the form:

\[ E(R_t) = \gamma_0 + [E(R_m) - \gamma_0] \beta + \gamma_1 (\delta - \delta_m) / \delta_m \]

If \( \gamma_1 \) is significantly different from zero, than dividend policy matters and if it is insignificantly different from zero dividend policy doesn’t matter. Results for the entire period and for six subperiods are showing that the estimate of \( \gamma_1 \) is statistically indifferent from zero. This is implying that expected returns on high yield securities are not statistically different from expected returns on the low yield securities. In other words, dividend yield does not provide investors with sufficient information that will enable investor to make an investment decision. Therefore, they argue that instead of investing in a portfolio with higher yields and having a badly diversified portfolio, it is better to create a highly diversified portfolio.
Pesaran and Timmermann (1995) examine the predictability of US stock returns in the period 1960-1992, using the following variables: dividend yield, earnings-to-price ratio, 1 month Treasury Bill, 12 months Treasury Bond, year-on-year rate of inflation, year-on-year rate of change in industrial output and year-on-year growth rate in the narrow money stock. All the macroeconomic indicators that were used were measured 12 months moving averages to reduce the impact of historical data revisions on the results. The authors use S&P 500 index as a dependent variable and estimate 512 models at each point in time since each model represents a certain combination of the independent variables used. They show that the recursive predictions of returns based on different model selection criteria have similar patterns and show high degree of volatility in 1980s which coincides with the period of high interest rate volatility in the US. Additionally, it can be observed that in the periods when volatility of US market increased, the predictability of returns increased as well (with the exception of market crash episode in October 1987). Furthermore, the authors claim that if the independent variable is included in the model on a continuous basis, such variable is an important factor for predicting stock returns. The only variable to be included in the model throughout the entire sample period is one month lagged value of the 1 month T-bill. From 1970 onwards, the variable that was included in the model in most of the periods is the dividend yield. Monetary growth and industrial production are included in the models more or less continuously after mid to late 1960s. Finally, the inclusion of inflation rate and 12 month T-bond rate in the model was dependent on shocks and 'regime switches'. Therefore, these findings confirm that the predictability of stock returns depends on the business cycles and hence it would be useful to use forecasting procedures that allow for regime changes. Authors find that their forecasting model is more beneficial in the periods of higher volatility in the US.
market. The findings from this paper can be beneficial for expanding the model used in this chapter and include some macroeconomic variables to assess their impact in the predictability of stock returns.

All the research outlined above has been conducted by using the data from the US market, in particular NYSE. Let us now observe the evidence from the UK market.

The evidence from the UK market regarding the yield effect is not as extensive as in the US. In the UK, Levis (1989) investigated the stock market anomalies in general, such as size effect, small P/E ratio, etc. He recognised the strong presence of the yield effect: as portfolio yields are increasing returns are increasing as well, but he offers no explanations for such anomalous behaviour of the stocks. The data used in this paper is from the London Share Price Database (LSPD) monthly returns file and source file. Source file provides the data required to estimate market value, PE multiples, dividend yields and share prices while the monthly returns file contains monthly rates of return (including dividends and capital gains). At the end of each year firms are ranked separately in ascending order according to market value, dividend yield, PE ratio and share price. It is interesting to note that by ranking stocks into dividend yield portfolios, Levis didn’t analyse the zero dividend yield group separately. Instead, he forms one group of smallest yield stocks which is including the zero-yield ones. Portfolio returns are calculated for 12 months commencing the following April by using equal weights for the periods April 1956-March 1985. In order to control the interaction between four effects, Levis constructed combined portfolios. He used both within groups only and within groups plus randomisation methods. According to the first one, all firms are first ranked by the chosen criterion and quintiles are formed.
Then within each quintile firms are ranked on the second variable and quintiles are formed within the existing quintile. Finally, 25 portfolios were formed for each combination of two attributes. This was repeated 25 times in order to use each of the four attributes as primary variable first and the other three for secondary grouping. Overall, there were 300 portfolios with all possible combinations. According to the second method, 25 portfolios generated as above are combined to form randomised portfolios. For example, market size portfolios are constructed by randomising separately with respect to dividend yield, PE and share price. This method results in 60 randomised portfolios. Levis used two main models: one is a limiting stage of simple CAPM and the second is the CAPM. According to the second model, firstly, the beta coefficients are estimated using a 60-month base period. Secondly, base period beta estimates are used to obtain abnormal returns for subsequent holdout period (holdout period runs for 12 months starting from April 1961). Abnormal returns are based on simple CAPM model. The results show that there is a positive relationship between dividend yield and returns. The difference between returns of the two extreme yield portfolios is 10% per annum. It has been found that the size effect is not the most important anomaly on the LSE, in comparison with the dividend yield or the P/E effect. The author also analyses the interaction between the four effects. The results suggest that the size and the P/E effect are independent. The yield effect is persistent across all market size portfolios, however it appears that the size effect is dependent on the yield effect, i.e. it is firmer within the highest yield quintile. The smaller size portfolios have disproportionately large number of high dividend paying firms. Comparison between dividend yield and P/E effect shows that both effects are at work independently. It appears from this study that the strongest relationship that Levis identifies in the UK market is the yield-return relationship.
In the more recent study of dividend yield effect in the UK, Morgan and Thomas (1998) examine the yield-return relationship over the period 1975 to 1993 using monthly total return data from LSPD. The paper replicates Keim (1985) methodology. In this study, for the first time in the UK, zero-yield stocks are forming a distinct group. The authors find what they call 'a clear inverse relationship' between dividend yields and returns. In other words, the yield-return relationship they find is a U-shaped relationship as found by Keim (1985) and Blume (1980) in the US. Morgan and Thomas consider these results based on the UK data as an indirect test of the tax-based hypothesis. Since there is no evidence of the negative yield-return relationship, the authors reject the tax-based hypothesis as the explanation of the yield effect. However, the F-statistic, used to test the null hypothesis that average returns are equal across yield portfolios, cannot be rejected. The t-statistic of the null hypothesis that the mean of the highest yield portfolio equals the mean of the lowest yield portfolio is small (1.56) but shows some difference among portfolios on the individual basis. After applying the Sharpe-Lintner CAPM model to estimate the excess returns of each yield portfolio, results suggest that although there is an element of non-linearity in the yield-return relationship, only excess returns of the two highest and the smallest yield-paying portfolios are significant at the 5% level. Investigating the size effect and its impact on yield effect, the non-linear relationship between yield and size is found. Smallest firms are concentrated in the zero-yield and the highest-yield group which are also providing the highest returns. Research regarding the seasonality of stock return patterns for the UK is different than for the US due to the fact that, apart from the month of January as in US, seasonality effect in UK can be observed in April and September as well. April is the month of the end of the UK tax year and September
effect is unexplained. The hypothesis that returns for each particular portfolio are equal for each month is rejected, emphasising the existence of seasonality among UK stock returns. Non-linearity of return behaviour is best observed during January, March and April. However, after controlling for the influence of zero-yield stocks, there is little evidence of seasonality in the yield effect. Rather, the findings suggest positive influence of non-seasonal dividend yields on risk-adjusted returns (yield coefficient of 0.07 or 0.06 when size is included in the analysis). A unique nature of zero-dividend stocks is offering excess returns on that portfolio in January and April but negative returns in September. This enhances the role of zero-yield stocks in explaining the seasonality of stock returns. Additional findings suggest that in January and April there is a positive size coefficient, while the overall (non-seasonal) size coefficient is negative. This evidence confirms the ambiguity of the size effect in the UK, first imposed by the Levis (1989) study. Furthermore, Morgan and Thomas examine the clientele effect. Clienteles are investing in companies whose dividend policies are such that suit the tax position of that particular client group. Since the demand from different tax clientele groups is varying, the firms are adjusting their dividend policies to meet those various demand levels. This implies that pre-tax returns should be equal across yield portfolios, which is not the case in this paper. Therefore, clientele effect cannot be used to explain the strong yield-return relationship found. Finally, the authors are suggesting that dividend signaling by managers and slow price reaction to those signals by investors are resulting in the positive excess returns of relatively high yielding stocks. Although this is consistent with findings in Morgan and Thomas study, no direct tests have been completed to confirm it.
Similarly to their study on predictability of US stock returns, Pesaran and Timmermann (2000) assess how the model reviewed earlier in this section in Pesaran and Timmermann (1995) can be extended, generalised and applied to the UK market. The variables used as predictors of stock returns are dividend yield on FTSE All Share index, three month Treasury Bill (3M T-Bill) rate, the difference between 3M T-Bill rate in time period t and t-1, the rate of change of retail prices, the change in the yield on 2.5% government consol, January dummy, the rate of change of the money supply, the rate of change in the spot price of oil. As a dependent variable, the authors use FTSE All Share Index returns for the period 1965-1993. The difference between this model and the previous version is that dividend yield, 3M T-Bill and the rate of inflation are always included in the forecasting model and the rest of the variables are included according to the importance in the relevant period. It is reported in the study that out of eight variables included in the regression, lagged dividend yield and change in the oil prices is significant at 1% level. Additionally, the paper shows that it is possible for investors to select a forecasting model recursively, use forecast from the model and improve the risk-return tradeoff offered by the market portfolio proxied by FTSE All Share Index even when ‘real time’ search for forecasting model and transaction costs were taken into account. The authors consider strategy of switching portfolios according to which investors can get in and out of the market. Following that, it is suggested that investors who were out of the market in the period 1973-1975 managed to avoid negative returns but also missed on large rises in returns in January and February 1975. Finally, to explain the predictability of stock returns, authors try to relate variations in expected returns to the changes in risk premia on one hand or consider it as a market inefficiency on the other. This paper can serve as a basis for the expansion of the research topic that will be tackled in this chapter.
1.3. Motivation

The problem that will be tackled in this chapter can be formulated as follows: Is the dividend yield a good predictor of stock returns in the UK and is there a trading strategy that will enable an investor in the UK market to generate profits?

We use the dividend yield variable as a starting point for predicting equity returns for the following reasons: a) Levis (1989) identifies stronger relationship between returns and dividend yields in the UK market than any other variable; b) Morgan and Thomas (1998) find a U-shaped yield return relationship; c) our experimental research indicates the change the yield/return relationship from U-shaped one to a bell-shaped one. We do not limit out analysis to the dividend yield variable, but we try to relate returns to the quadratic yield, market value (size) of the portfolio and systematic risk (beta) of the portfolio. Following Pesaran and Timmermann (1995, 2000), we recognise that there is a scope for the expansion of this study and inclusion of other variables (such as inflation, interest rates etc.) in the model for predicting returns, which will be addressed in the future research.

The objective of the chapter would be to investigate the yield effect on its own and to provide the explanation for any pattern in the behaviour of different dividend yield portfolios that we might find. Emphasis will be put on the size effect, its ambiguity in the UK and interaction with the yield effect. We will search for the possible explanations of the particular pattern in the yield-return relationship to be found. Additionally, the data sample that will be used is more up to date than the one used in the previous studies, covering the period 1980 through to 1996. We believe that this is of significance due to the fact that firstly, this period will cover the recession and
post-recession period in the UK, which may have influenced the return behaviour of some stocks and secondly, there is evidence in the 1990s that small stocks have started behaving differently, earning lower returns than large stocks, which may have some influence on the pattern of the yield-return relationship itself. These issues need to be considered in the empirical findings that follow in this chapter.
2. Data

There are two sets of data used in this chapter. The first set of data is gathered from the London Share Price Database (LSPD) monthly returns file. The criteria employed in selecting the firms in the sample in a particular year are:

e) the firm has dividend yield and market value data available in the year prior to the year of its inclusion in the sample

f) the firm has 5 years of available data before it was included in the sample and

g) firm’s returns were available on LSPD monthly returns file.

In LSPD, there is always a missing return in the first month of trading or the first month the company data has been collected. The list of all actively traded companies and dead companies (delisted, subjects to takeovers, mergers, bankruptcy, etc.) in the period January 1980 through to December 1996 was obtained from the Datastream file. The data prior to 1980 was used as well to estimate beta coefficients. The number of firms included in the sample that meet the above requirements varies from around 940 in 1980 to around 1100 in 1996. The largest number of companies in the sample was at the beginning of 1980, more than 1400. There is no survivorship bias in the sample. The only restriction for the sample is that it does not include investment trust companies. The second set of data represents the dividend yields and market capitalisation of the sample firms. Due to the fact that we are going to use annual portfolio rebalancing, we have collected the annual data for the former variables from the Datastream for the period 1979 through 1995. In order to match companies’ returns from one datasource with the corresponding dividend yields and market values from another datasource, we have used individual company identification (SEDOL) numbers in order to avoid any confusion arising from the possibility of companies...
being listed under different names in LSPD and Datastream. This was done by using a macro in Excel, but for some companies SEDOL numbers have changed (maybe they bought another company or something similar happened) hence, the matching had to be done manually.

There are two main reasons why we have used two different sources of data:

1. The LSPD files provide only monthly data, whereas we needed the annual observations for dividend yield and market capitalisation variables.

2. The returns that could be calculated by using share prices from Datastream are not as accurate as returns ready available on LSPD. This point will be explained below.

Monthly returns given by LSPD are calculated as follows:

\[ R_t = \log \left( \frac{(P_t + d_t)}{P_{t-1}} \right) \]

Where \( R_t \) is log return in month t. \( P_t \) is the last traded price in month t, \( P_{t-1} \) the last traded price in t-1 and \( d_t \) is the dividend declared during month t.

The returns calculated in this way are more accurate because they include both components of returns: income (dividend) and capital gains. Otherwise, if returns were calculated by using Datastream data, we would either have to include the dividend in the formula manually or calculate returns just as a log difference in share prices.

Dividend yield from Datastream is expressed as the ratio of the dividend paid during the twelve months period of the calendar year to the market price of ordinary share at
the end of that period. Black and Scholes (1974) used this measure of dividend yield. Some authors consider that this formula is overstating the dividend yield due to the fact that some companies that had historically low dividend yield might perform badly at the end of the year and their price might drop causing the dividend yield to rise. Therefore, they suggest the alternative measure of dividend yield where the price in the denominator would be \( P_{t-13} \). The reason for this is that if a company performs unsatisfactory at the beginning of the year and its price falls, it will have time to adjust its dividend during the year and maintain the usual payout ratio. However, Keim (1985) shows that both measures are accurate and they give the same qualitative and quantitative results. Additionally, Blume (1980) concludes that the accuracy of the dividend yield formula to be used is purely an empirical question, but that would be a subject of a different debate.

Correspondingly, market capitalisation of the firm represents the market price at the end of the calendar year multiplied by the number of shares outstanding.

**2.1. Formation of Portfolios**

In order to analyse the relationship between dividend yields of London Stock Exchange firms and their returns, the following procedure has been employed. Securities that were included in the sample in year \( t \) have to have 5 years (60 months) of continuous monthly data prior to year \( t \), which is used to estimate beta coefficients. Then, the full sample of securities in year \( t \) was sorted into ten groups according to their dividend yield from year \( t-1 \). The first group represents the zero-dividend firms, whereas all remaining securities are grouped in nine portfolios of the same size (give or take a security). Assigning all zero paying stocks into the same group, results in a
portfolio that has much more constituents than any of the remaining, dividend paying, portfolios. This is in contrast with the procedure used in some previous studies, such as Black and Scholes (1974) in US and Levis (1989) in the UK. According to Elton et al. (1983), this is the right way of grouping securities, because if we were forming portfolio deciles, we might end up with several zero dividend yield portfolios in particular years and remainder would be the dividend paying ones. Since we want to create portfolios whose distinguishable characteristic is the dividend yield, i.e. portfolios with wide range of dividend yields, we accept the argument of Elton et. al. (1983).

All portfolios are equally weighted. Additionally, all portfolios are rebalanced annually. If a company was temporally suspended from trading, and its LSPD returns would be missing, it was removed from the sample during that period.

I have calculated the risk and return characteristics of portfolios alongside their average dividend yields and average market values and presented them in the table format. Values in the tables presented throughout the chapter are results of the following procedures:

1. Returns are calculated as the annualised value of the average monthly returns. The formula employed is:

   \[ R_{pa} = 12 \times (\text{average } R_{pm}) \]

   Where \( R_{pa} \) is annual and \( R_{pm} \) is monthly return of portfolio \( p \).
Note that the data that we originally have from LSPD is the monthly returns. That explains the use of average monthly returns of any portfolio p. A monthly return of portfolio p is computed by combining monthly returns of securities in that portfolio with equal weights. Furthermore, average monthly return \( R_{pm} \) is the arithmetic average of monthly returns of that portfolio p for the period January 1980 through December 1996.

2. Average dividend yield is expressed as a percentage of a share price. The values reported are the average portfolio p yields over the period under observation.

3. Average market values are in millions of pounds and represent the average market capitalisation of stocks in any particular portfolio p over the observed period.

4. Average beta of the portfolio is the average of betas of each security in the portfolio over the sample period. Beta coefficient for each particular security ‘x’ for year t is estimated if the security ‘x’ had continuously available monthly returns five years (60 months) prior to year t. Security returns in year t were regressed on the monthly returns of the market, proxied by FTSE All Share Index. The estimated coefficient from such regression for period t-5 through t-1 will be the anticipated beta coefficient of the security ‘x’ for the year t. The same estimation procedure was repeated for every security in the sample in every year of the sample period. This estimation procedure originated from Fama and McBeth (1973) and was used in the number of studies such as Blume (1980), Elton et. al. (1983), Levis (1989) etc.
3. Summary Statistics

3.1. Yield effect in the UK

The question that should be answered in the following section is: Is there a yield effect on the UK stock market in the period under observation?

Table 1 reports the risk/return/DY/MV values for nine dividend yield portfolios containing the same number of stocks for the period 1980 through 1996:

Table 1: Summary statistics 1980-1996 for 10 yield portfolios

<table>
<thead>
<tr>
<th>Portfolio number</th>
<th>Dividend yield (%)</th>
<th>Anannualised return (%)</th>
<th>Market values (£mn)</th>
<th>Beta of the portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>p0-zero</td>
<td>0</td>
<td>7.02</td>
<td>24.88</td>
<td>0.70</td>
</tr>
<tr>
<td>p1-lowest</td>
<td>1.38</td>
<td>9.84</td>
<td>184.95</td>
<td>0.59</td>
</tr>
<tr>
<td>p2</td>
<td>2.63</td>
<td>11.16</td>
<td>276.83</td>
<td>0.63</td>
</tr>
<tr>
<td>p3</td>
<td>3.44</td>
<td>11.97</td>
<td>364.02</td>
<td>0.67</td>
</tr>
<tr>
<td>p4</td>
<td>4.19</td>
<td>13.08</td>
<td>380.22</td>
<td>0.69</td>
</tr>
<tr>
<td>p5</td>
<td>4.95</td>
<td>13.73</td>
<td>406.47</td>
<td>0.70</td>
</tr>
<tr>
<td>p6</td>
<td>5.74</td>
<td>13.81</td>
<td>446.91</td>
<td>0.71</td>
</tr>
<tr>
<td>p7</td>
<td>6.66</td>
<td>12.17</td>
<td>386.27</td>
<td>0.71</td>
</tr>
<tr>
<td>p8</td>
<td>7.84</td>
<td>14.06</td>
<td>251.50</td>
<td>0.69</td>
</tr>
<tr>
<td>p9-highest</td>
<td>10.94</td>
<td>9.81</td>
<td>168.20</td>
<td>0.69</td>
</tr>
</tbody>
</table>
The results in the above table show that a positive relationship exists between portfolio returns and corresponding dividend yields up to a certain point. We can observe that the zero-yield portfolio is the worst performer, i.e. that it gives average annual return of only 7.02%. For dividend paying portfolios, it can be concluded that as the dividend yield increases the portfolio returns follow the same pattern, but only up to the last (highest) yield portfolio where the decrease in the return of around 4.5% per annum can be noted. In other words, the relationship between dividend yield and the return can be observed as the bell shaped curve. One can say that the return of portfolio 7 is slightly dropping, thus breaking the pattern of increasing returns. However, that decrease in the return is only around 1.5% on the annual basis, which can be considered as insignificant. For justification of this argument we refer to Keim (1985): in his findings, he suggests that returns of dividend paying stocks are increasing with the dividend yield, although monthly returns of the first three yield paying portfolios are showing decreasing pattern and returns of the remainder two portfolios are increasing.

Therefore, we can say that annualised returns the portfolios from the above table first increase with an increase in dividend yield and then decrease, forming a sort of bell shaped curve, i.e. forming a quadratic relationship. The relationship can be presented by the figure 1:
The above results are at variance with the findings of some of the earlier studies. For example, although Blume (1980) and Keim (1985) report that there is a non-linear relationship between portfolio returns and their dividend yields, they find that zero paying stocks and portfolios with the highest yield paying stocks outperformed all the other portfolios. Both authors use similar observation periods: Blume uses monthly data from 1936 through to 1976 and Keim also uses monthly data for the period from 1931 to 1978. More importantly, the parallel should be drawn between results of this chapter and Morgan and Thomas (1998) findings. Following Keim (1985) methodology, Morgan and Thomas are analysing yield effect in the 1975-1993 period. Their results suggest negative, U-shaped, yield-return relationship. Graphically, their findings are presented on the figure 2:
The differences in results stem from the difference in the time period observed: we do not include period 1975-1979 and Morgan and Thomas (1998) did not include 1994-1996 period, making the 8 years difference in the sample. There is evidence that in 1990s in the UK the behaviour of the small stocks’ returns has changed. In 1980s and prior to that period it has been known that small stocks were outperforming larger ones. However, in the last years, from the beginning of 1990s, there have been many reports showing that the small stocks are actually underperforming larger ones. Morgan and Thomas report that small stocks in their sample are concentrated in the smallest and highest yield portfolios, particularly in the zero-yield portfolio. Those are exactly the portfolios that are earning highest returns and that are forming the extreme ends of the u-shaped curve presented in figure 2. Although Morgan and Thomas study covers the beginning of 1990s period, it was not enough to capture the change in the return behaviour of small stocks and its effect on the shape of the yield-return curve. Morgan and Thomas do not break the sample period into subperiods so even if
there was a change in the yield-return relationship in the 1990s, it was overwhelmed by the U-shaped relationship between yield and return in the rest of the sample years (1975-1989). Therefore, to observe the change in the yield-return pattern between 1980s and 1990s due to the change in return behaviour of small stocks, we have split our sample into two parts: 1980-1989 and 1990-1996. Referring to table 1, consistent with Morgan and Thomas findings, small stocks in our sample are concentrated in the extreme portfolios – zero yield, smallest yield and the highest yield portfolio. Hence, plotting the yield-return curve for the 1980s, as in figure 3, we find:

**Figure 3:**

![Relationship between dividend yield and returns 1980-1989](image)

Figure 3 shows almost identical relationship between yield and return as figure 2. Specifically, our analysis shows that in the 1980s yield-return relationship was negative, as reported by Morgan and Thomas (1998). The reason why the curves in figures 2 and 3 are not of exactly the same shape is because figure 2 includes period 1975-1979 when the small stocks were performing better which may be driving the
shape of the curve. In our sample, in the 1990s, yield-return relationship followed the pattern that is actually driving the relationship for the whole sample period, as presented in figure 4:

**Figure 4:**

![Relationship between dividend yields and returns: 1990-1996](image)

Figure 4 is showing the quadratic yield-return relationship. It can be noted that small stocks, being in the highest and the lowest yield groups, are not only underperforming larger stocks, but they are earning negative returns. The conclusion can be drawn that the pattern of the relationship between dividend yield and return has been changed due to the change in return characteristics of small stocks.

The inconsistency between these findings and the findings of Keim (1985) and Blume (1980) might stem from the fact that my analysis is based on the different market, UK rather than US, and that I am using more recent time period. For example, Christie (1990) argues that Keim and Blume are observing high positive abnormal returns in the zero dividend paying portfolio due to the sample period: they use 1930s and 1940s
which, according to Christie, influence those results. Hence the returns behaviour of different dividend yield portfolios shown in this chapter is more consistent with the results shown in the more recent US studies, such as Naranjo et al (1998). They observe a positive but somewhat quadratic relationship between stock returns and dividend yields of the dividend paying portfolios, as it is the case in this chapter. In the rest of the UK evidence available, Levis (1989) confirms the existence of the yield effect. His results are finding the 10% difference in annual returns of the highest and the lowest yield portfolios. The results in this chapter are showing the difference of 7% in annualised returns but between the extreme zero-yield portfolio and portfolio 8. The portfolio with the highest yield, portfolio 9, has only slightly less than 3% higher return than portfolio 0. The main reason for this difference is the change in the pattern of return behaviour across yield portfolios in recent years for the reasons outlined earlier. Levis data sample covers period from 1955 up to 1985 only, whereas the sample in this chapter is extended to 1996 covering the years of underperformance of small stocks and recession years.

Let us now observe the market values of portfolios and their impact on dividend yield effect and their relationship with portfolio returns.

3.2. The role of size in the yield effect

Market values from table 1 are showing the following relationships with other variables:

- As the market size of the portfolio increases, the return of a yield portfolio increases as well. Such positive relationship can be observed in the figure 5.
This is an interesting finding due to the fact that Keim (1985) in the US has found exactly the opposite relationship between the market values and the return. However, in the UK, there is a difference between results reported in Morgan and Thomas (1998) study and the findings of this chapter: we find that smallest stocks have smallest returns, whereas Morgan and Thomas (1998) report that smallest stocks are among those with the highest returns.

- As the dividend yield increases market values of portfolios increase as well up to the certain point, where the further increase in yield causes market values of portfolios to decrease. Therefore, this relationship between portfolio size and dividend yields is a positive, but not linear one, forming a bell shaped curve as in figure 6:
Previous research has shown that zero dividend paying portfolios have the smallest market values (Keim 1985), which is consistent with what we observe in this study, but the positive dividend yields and the market values are inversely related. We can see that the largest size companies in our table 1 are mainly concentrated in the medium yield portfolios. Smallest size companies are concentrated in the lowest and the highest yield groups.

Also, remember that it has been noted above that relationship between dividend yields and return is a sort of bell shaped curve as well. Therefore, by comparing the graphs presented above, we can conclude then that the medium yield companies from our study (larger size companies) have the highest returns. Additionally, lowest return companies from table 1 (which are also the smallest size companies) are constituents of the lowest and the highest yield groups, i.e. portfolios one, two and nine. Therefore, a question arises: is the yield - return relationship then actually the market value - return relationship? The table and graphs are giving some indications that it might be
so. However, additional confirmation is needed from the analysis of the econometric results in the later section.

It can be concluded from the above analysis that the dividend yield effect, defined in the previous studies so as that high yield companies are providing investors with higher returns, exists but only up to the point when the market values of the companies start decreasing with the further increase in yield causing the returns to decrease as well.

Why are the smallest companies offering lower returns? One of the explanations that can be put forward is that the large part of the sample period in this chapter covers the recession years in the UK. Our sample of companies shows that during the second subperiod, recession period in the UK, the average returns of all the companies, especially smaller ones are substantially dropping compared to the returns in the first subperiod. In particular, smaller companies in the zero yield and the highest yield portfolio had negative average annualised returns of $-8.82\%$ and $-3.57\%$ respectively. Although such low returns are partly reflecting the recession period, they are also suggesting that some companies, especially smaller ones, have not been able to improve their performance when the recession period has finished. Since the small size companies are the ones with the higher risk, I believe that they were the most affected by the recession and that they haven’t been able to recover properly after it. Some of the small companies offer high dividend yields, probably in order to attract investors. Others, on the other hand, pay zero or very low dividend yield, either because they cannot afford paying more or because they want to attract tax-concerned
investors. That is a way in which small companies try to compensate investors for giving lower returns.

3.3. The tax effect hypothesis and the yield effect

Since one of the explanations of the yield effect, that the main emphasis were put on in the literature, was the tax effect, I will try to examine the possibility that the yield effect is actually in the function of the tax effect. In particular, Litzenberger and Ramaswamy (1979) were one of the first researchers to give an explanation that the yield effect is induced by the disparity in tax rates for dividend yields and capital gains. In the US, rate at which dividends are taxed is higher than rate for capital gains tax. In such a case, an investor who invests in high yielding stocks will ask for higher return in order to be compensated for tax. However, in the UK, since the 1973 tax imputation system was introduced, as we have shown in the introduction, if the tax based hypothesis was the plausible explanation for the yield effect, we should find the negative relationship between yield and risk-adjusted returns. According to Morgan and Thomas (1998), UK data provides an independent test of the tax-based hypothesis. Since the evidence in this chapter implies positive and quadratic yield-return relationship, it automatically rules out the tax effect as the explanation for the yield-return relationship.
4. Econometric Results

4.1. Time series analysis of the risk adjusted returns

Risk-adjusted returns for each of the 10 yield portfolios under observation are estimated by using the Sharpe-Lintner CAPM model:

\[(R_{pt} - R_{n}) = \alpha_p + \beta_p (R_{mt} - R_{n})\]

Where \(R_{pt}\) is the return for portfolio \(p\) (\(p = 0\) to \(9\)) in month \(t\), \(R_{mt}\) is the market return in month \(t\) proxied by using the FTSE All Share Index returns and \(R_{n}\) is a risk free rate in month \(t\). As a risk free rate of return, 1 month T-bill rate was used. \(\alpha_p\) is an estimate of the risk-adjusted excess return on the portfolio \(p\), \(p = 0\) to \(9\).

The data for the FTSE All Share Index monthly prices and 1month T-bill returns is obtained from Datastream. Since our portfolio returns are calculated using LSPD data, in order for the market returns and T-bill returns to be compatible with portfolio returns, we have used the following formulae to calculate needed variables:

\[R_{mt} = \log_e \left( \frac{(P_{mt} + D_{mt})}{P_{mt-1}} \right)\]

Where \(R_{mt}\) is the return on FTSE All Share Index in period \(t\), \(P_{mt}\) and \(P_{mt-1}\) are the index prices in period \(t\) and \(t-1\) and \(D_{mt}\) is the dividend payment on the index in period \(t\) also obtained from Datastream.

Additionally, log-return on a risk free rate is expressed as follows:

\[R_{n} = \log_e (1 + R_{f})\]
Parameters of the model (alpha and beta) are estimated for each portfolio separately for the entire sample period, 1980–1996, using the time-series data for monthly portfolio returns and monthly FTSE All Share Index returns. The Sharpe-Lintner model suggests that the estimates of \( \alpha_p \) will be systematically related to dividend yields if there is a yield effect. The estimates of alpha and beta for each portfolio are presented in the table 2 below:

Table 2: Alpha and Beta estimates for yield portfolios

<table>
<thead>
<tr>
<th>Portfolio Number</th>
<th>Dividend yield</th>
<th>Alpha t-statistics</th>
<th>Beta t-statistics</th>
<th>( R^2 )</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>p0</td>
<td>0</td>
<td>-0.00791</td>
<td>-2.42</td>
<td>0.88</td>
<td>13.73</td>
</tr>
<tr>
<td>p1</td>
<td>1.38</td>
<td>-0.00476</td>
<td>-2.48</td>
<td>0.75</td>
<td>19.86</td>
</tr>
<tr>
<td>p2</td>
<td>2.63</td>
<td>-0.00382</td>
<td>-2.43</td>
<td>0.78</td>
<td>25.12</td>
</tr>
<tr>
<td>p3</td>
<td>3.44</td>
<td>-0.00308</td>
<td>-2.03</td>
<td>0.77</td>
<td>25.67</td>
</tr>
<tr>
<td>p4</td>
<td>4.19</td>
<td>-0.00238</td>
<td>-1.64</td>
<td>0.81</td>
<td>28.19</td>
</tr>
<tr>
<td>p5</td>
<td>4.95</td>
<td>-0.0019</td>
<td>-1.18</td>
<td>0.81</td>
<td>25.76</td>
</tr>
<tr>
<td>p6</td>
<td>5.74</td>
<td>-0.00197</td>
<td>-1.24</td>
<td>0.84</td>
<td>26.64</td>
</tr>
<tr>
<td>p7</td>
<td>6.66</td>
<td>-0.0035</td>
<td>-1.93</td>
<td>0.87</td>
<td>24.24</td>
</tr>
<tr>
<td>p8</td>
<td>7.84</td>
<td>-0.00176</td>
<td>-0.86</td>
<td>0.84</td>
<td>20.76</td>
</tr>
<tr>
<td>p9</td>
<td>10.94</td>
<td>-0.00555</td>
<td>-2.04</td>
<td>0.88</td>
<td>16.42</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level

It can be seen that all portfolios have negative excess returns. Keim (1985) in the US and Morgan and Thomas (1998) in the UK have completed the same analysis for six yield portfolios (one zero-yield and five dividend-paying portfolios of ascending
dividend yield). However, they find negative excess returns in the lowest yield portfolios, and positive excess returns in portfolios with higher yields. All alphas that Keim reports are also insignificant, but when tested if they are jointly equal to zero across portfolios, the hypothesis is rejected. Morgan and Thomas find significant excess returns for lowest and highest yield paying portfolios. Table 2 is indicating that four lowest yield and the highest yield portfolio have significant excess returns at 1% level. The results are implying a non-linear relationship between dividend yields and risk-adjusted returns that can be shown graphically as well. The question that arises is: can the pattern of behaviour of those excess returns explained by dividend yields? The figure 7 is showing relationship between dividend yields and excess returns, measured by alpha:

![Figure 7](image)

This graph is almost identical as the one when risk-unadjusted returns were used. Therefore, one can conclude that even after adjusting for risk, the quadratic relationship between returns and dividend yields remains the same. What is that relationship driven by? Since the summary statistics results have suggested that there is a relationship between the yield and market values, in the next section we test the
impact of size on the yield – return relationship. In particular, section 4.1.1. examines whether any particular size group influences the non-linear relationship between dividend yields and portfolio returns.

4.1.1. The role of size in the yield effect: using the risk adjusted returns

In order to test if the quadratic yield-return relationship is actually a function of the size effect, we have formed subgroups of portfolios with respect to market values. In particular, within each of the 10 yield portfolios constituent securities are ranked according to their annual market values. Then, within each yield portfolio three new portfolios of equal size (give or take a security) were formed: first representing the smallest companies, second representing medium size companies and the third group representing the companies with the highest market values within a particular yield portfolio. Following such a procedure of classifying securities we finally obtain 30 portfolios in total (3 market values portfolios within each of the 10 yield groups).

The same Sharpe-Lintner model is used to estimate excess return parameters for small medium and large companies within each yield group. As before, we use the time series analysis to estimate alpha and beta parameters of the 30 portfolios. In particular, we regress monthly risk-adjusted returns of each portfolio against market risk-adjusted returns for the period January 1980 through December 1996.

Estimates of the excess returns and betas for small, medium and large companies across different yield groups are shown in tables 3, 4 and 5. Figures 8, 9 and 10 are showing the relationship between excess returns (alphas) and dividend yields of small, medium and large companies respectively.
### Table 3: Alphas and betas of small size companies

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha</th>
<th>t-statistics</th>
<th>Beta</th>
<th>t-statistics</th>
<th>( R^2 )</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>-0.00109</td>
<td>-0.25</td>
<td>0.85</td>
<td>10.01</td>
<td>0.33</td>
<td>100.11*</td>
</tr>
<tr>
<td>P1</td>
<td>1.29</td>
<td>-0.00193</td>
<td>-0.67</td>
<td>0.58</td>
<td>10.17</td>
<td>0.34</td>
<td>103.44*</td>
</tr>
<tr>
<td>P2</td>
<td>2.62</td>
<td>-0.00351</td>
<td>-1.37</td>
<td>0.60</td>
<td>11.84</td>
<td>0.41</td>
<td>140.16*</td>
</tr>
<tr>
<td>P3</td>
<td>3.42</td>
<td>-0.00092</td>
<td>-0.39</td>
<td>0.55</td>
<td>11.95</td>
<td>0.41</td>
<td>142.83*</td>
</tr>
<tr>
<td>P4</td>
<td>4.19</td>
<td>-0.00015</td>
<td>-0.06</td>
<td>0.63</td>
<td>13.04</td>
<td>0.46</td>
<td>170.08*</td>
</tr>
<tr>
<td>P5</td>
<td>4.94</td>
<td>-0.0011</td>
<td>-0.45</td>
<td>0.61</td>
<td>12.70</td>
<td>0.44</td>
<td>161.25*</td>
</tr>
<tr>
<td>P6</td>
<td>5.70</td>
<td>0.000313</td>
<td>0.13</td>
<td>0.62</td>
<td>13.33</td>
<td>0.47</td>
<td>177.60*</td>
</tr>
<tr>
<td>P7</td>
<td>6.66</td>
<td>-0.00367</td>
<td>-1.34</td>
<td>0.68</td>
<td>12.72</td>
<td>0.44</td>
<td>161.75*</td>
</tr>
<tr>
<td>P8</td>
<td>7.86</td>
<td>0.002319</td>
<td>0.88</td>
<td>0.64</td>
<td>12.30</td>
<td>0.43</td>
<td>151.23*</td>
</tr>
<tr>
<td>P9</td>
<td>11.20</td>
<td>-0.00086</td>
<td>-0.29</td>
<td>0.67</td>
<td>11.51</td>
<td>0.40</td>
<td>132.52*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level

### Table 4: Alphas and betas of medium size companies

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha</th>
<th>t-statistics</th>
<th>Beta</th>
<th>t-statistics</th>
<th>( R^2 )</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>-0.01369</td>
<td>-3.32</td>
<td>0.87</td>
<td>10.69</td>
<td>0.36</td>
<td>114.20*</td>
</tr>
<tr>
<td>P1</td>
<td>1.40</td>
<td>-0.0051</td>
<td>-2.20</td>
<td>0.73</td>
<td>15.86</td>
<td>0.55</td>
<td>251.45*</td>
</tr>
<tr>
<td>P2</td>
<td>2.64</td>
<td>-0.00329</td>
<td>-1.79</td>
<td>0.74</td>
<td>20.48</td>
<td>0.67</td>
<td>419.64*</td>
</tr>
<tr>
<td>P3</td>
<td>3.44</td>
<td>-0.00491</td>
<td>-2.49</td>
<td>0.73</td>
<td>18.82</td>
<td>0.64</td>
<td>354.36*</td>
</tr>
<tr>
<td>P4</td>
<td>4.19</td>
<td>-0.00388</td>
<td>-2.00</td>
<td>0.78</td>
<td>20.34</td>
<td>0.67</td>
<td>413.61*</td>
</tr>
<tr>
<td>P5</td>
<td>4.96</td>
<td>-0.00248</td>
<td>-1.18</td>
<td>0.81</td>
<td>19.51</td>
<td>0.65</td>
<td>380.60*</td>
</tr>
<tr>
<td>P6</td>
<td>5.73</td>
<td>-0.00412</td>
<td>-1.91</td>
<td>0.85</td>
<td>20.00</td>
<td>0.66</td>
<td>400.06*</td>
</tr>
<tr>
<td>P7</td>
<td>6.66</td>
<td>-0.00441</td>
<td>-1.89</td>
<td>0.86</td>
<td>18.78</td>
<td>0.64</td>
<td>352.58*</td>
</tr>
<tr>
<td>P8</td>
<td>7.82</td>
<td>-0.00513</td>
<td>-1.99</td>
<td>0.87</td>
<td>17.07</td>
<td>0.59</td>
<td>291.42*</td>
</tr>
<tr>
<td>P9</td>
<td>10.92</td>
<td>-0.00885</td>
<td>-2.72</td>
<td>0.93</td>
<td>14.49</td>
<td>0.51</td>
<td>210.01*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level
### Table 5: Alphas and betas of large size companies

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha</th>
<th>t-statistics</th>
<th>Beta</th>
<th>t-statistics</th>
<th>$R^2$</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>-0.00884</td>
<td>-2.72</td>
<td>0.94</td>
<td>14.68</td>
<td>0.52</td>
<td>215.62*</td>
</tr>
<tr>
<td>P1</td>
<td>1.46</td>
<td>-0.00719</td>
<td>-4.44</td>
<td>0.95</td>
<td>29.77</td>
<td>0.81</td>
<td>886.19*</td>
</tr>
<tr>
<td>P2</td>
<td>2.63</td>
<td>-0.00466</td>
<td>-3.43</td>
<td>0.99</td>
<td>36.86</td>
<td>0.87</td>
<td>1358.8*</td>
</tr>
<tr>
<td>P3</td>
<td>3.45</td>
<td>-0.00336</td>
<td>-2.83</td>
<td>1.01</td>
<td>43.28</td>
<td>0.90</td>
<td>1873.3*</td>
</tr>
<tr>
<td>P4</td>
<td>4.19</td>
<td>-0.0031</td>
<td>-2.85</td>
<td>1.01</td>
<td>47.37</td>
<td>0.92</td>
<td>2243.6*</td>
</tr>
<tr>
<td>P5</td>
<td>4.95</td>
<td>-0.00211</td>
<td>-1.67</td>
<td>1.03</td>
<td>41.29</td>
<td>0.89</td>
<td>1705.3*</td>
</tr>
<tr>
<td>P6</td>
<td>5.69</td>
<td>-0.00209</td>
<td>-1.62</td>
<td>1.04</td>
<td>41.13</td>
<td>0.89</td>
<td>1692.0*</td>
</tr>
<tr>
<td>P7</td>
<td>6.65</td>
<td>-0.00245</td>
<td>-1.55</td>
<td>1.05</td>
<td>33.65</td>
<td>0.85</td>
<td>1132.1*</td>
</tr>
<tr>
<td>P8</td>
<td>7.82</td>
<td>-0.00246</td>
<td>-1.30</td>
<td>1.00</td>
<td>26.91</td>
<td>0.78</td>
<td>723.95*</td>
</tr>
<tr>
<td>P9</td>
<td>10.71</td>
<td>-0.00686</td>
<td>-2.29</td>
<td>1.03</td>
<td>17.47</td>
<td>0.60</td>
<td>305.38*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level

### Figure 8:

**Relationship between alphas and dividend yields for small companies**

![Graph showing relationship between alphas and dividend yields for small companies](image)

### Figure 9:

**Relationship between alphas and dividend yields for medium companies**

![Graph showing relationship between alphas and dividend yields for medium companies](image)
Figure 10:

We can see from tables 3, 4, and 5 and corresponding figures that even when we separate portfolios in terms of the size, there are indications that group of small companies is behaving differently in comparison to the other two groups. It was noted by Morgan and Thomas (1998) that excess returns are showing significant variation over time. As we have outlined before, we have reason to believe that small stocks in our sample are behaving differently in the 1980s and 1990s. In order to examine whether there is a change in the pattern of relationship between excess returns and dividend yields of small companies, as well as large and medium ones, we have completed the same time series analysis as before for the two subperiods: 1980-1989 and 1990-1996. The results presented in tables 6, 7, 8, 9, 10 and 11 and corresponding figures 11, 12, 13, 14, 15 and 16 are implying the following conclusions:

1. The group of small stocks drives the U-shaped relationship between dividend yields and stock returns prior to 1990s. Particularly, in the 80s, as on figure 11, we can identify the U-shaped, inverse, yield-return relationship of small stocks, while figures 13 and 15 are indicating that medium and large companies earn excess
returns that are positively, rather than negatively related to their dividend yields. Additionally, small stocks are the only group of companies in 1980s that is earning positive excess returns, outperforming medium and large companies. Therefore, since the small stocks are particularly concentrated in the zero-yield group as well as in the highest yield one, both in this study and Morgan and Thomas (1998) study, it can be concluded that they are the stocks driving higher returns in those yield groups, thus forming a U-shaped yield-return relationship.

2. The group of small stocks is showing the change in the returns behaviour in the 1990s. The inverse yield-return relationship of small size companies in 1980s has turned positive in 1990s (figure 12). Small stocks in the 1990s are not showing the perfect positive, quadratic yield-return relationship that can be found in the medium and large size groups (figures 14 and 16). One of the reasons for that may be the following: since the small stocks are usually new to the market and financially unstable, some of them, who are in the most distress, may have done worse than the others during the recession period in the 1990s. However, it is clear that their positive excess returns in the 80s have turned negative in the 90s across all yield portfolios. Also, the pattern of the yield-return relationship has changed, from a U-shaped on figure 11 to a positive, quadratic-like one on figure 12.

Therefore, a conclusion may be made from the time series analysis of risk-adjusted returns of small medium and large stocks that the change in the returns behaviour of small companies in the 1990s has influenced the yield-return relationship to change from U-shaped in the 80s to a bell-shaped one in the 90s.
Table 6: Alphas and betas of small companies: 1980-1989

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha t-statistics</th>
<th>Beta t-statistics</th>
<th>R²</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>0.00641</td>
<td>1.17</td>
<td>0.86</td>
<td>8.88</td>
</tr>
<tr>
<td>P1</td>
<td>1.34</td>
<td>0.00209</td>
<td>0.52</td>
<td>0.66</td>
<td>9.22</td>
</tr>
<tr>
<td>P2</td>
<td>2.73</td>
<td>0.00079</td>
<td>0.24</td>
<td>0.64</td>
<td>10.72</td>
</tr>
<tr>
<td>P3</td>
<td>3.59</td>
<td>0.00225</td>
<td>0.81</td>
<td>0.55</td>
<td>11.16</td>
</tr>
<tr>
<td>P4</td>
<td>4.42</td>
<td>0.00223</td>
<td>0.71</td>
<td>0.63</td>
<td>11.39</td>
</tr>
<tr>
<td>P5</td>
<td>5.25</td>
<td>0.00039</td>
<td>0.13</td>
<td>0.62</td>
<td>11.22</td>
</tr>
<tr>
<td>P6</td>
<td>6.13</td>
<td>0.00349</td>
<td>1.14</td>
<td>0.62</td>
<td>11.26</td>
</tr>
<tr>
<td>P7</td>
<td>7.11</td>
<td>0.00189</td>
<td>0.58</td>
<td>0.68</td>
<td>11.80</td>
</tr>
<tr>
<td>P8</td>
<td>8.30</td>
<td>0.00489</td>
<td>1.53</td>
<td>0.63</td>
<td>11.17</td>
</tr>
<tr>
<td>P9</td>
<td>11.20</td>
<td>0.00521</td>
<td>1.51</td>
<td>0.64</td>
<td>10.55</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level

Table 7: Alphas and betas of small companies: 1990-1996

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha t-statistics</th>
<th>Beta t-statistics</th>
<th>R²</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>-0.01176</td>
<td>-1.72</td>
<td>0.78</td>
<td>4.72</td>
</tr>
<tr>
<td>P1</td>
<td>1.21</td>
<td>-0.00797</td>
<td>-2.16</td>
<td>0.34</td>
<td>3.77</td>
</tr>
<tr>
<td>P2</td>
<td>2.47</td>
<td>-0.00974</td>
<td>-2.51</td>
<td>0.48</td>
<td>5.10</td>
</tr>
<tr>
<td>P3</td>
<td>3.18</td>
<td>-0.00539</td>
<td>-1.32</td>
<td>0.55</td>
<td>5.54</td>
</tr>
<tr>
<td>P4</td>
<td>3.86</td>
<td>-0.00355</td>
<td>-0.91</td>
<td>0.60</td>
<td>6.36</td>
</tr>
<tr>
<td>P5</td>
<td>4.49</td>
<td>-0.00325</td>
<td>-0.83</td>
<td>0.57</td>
<td>6.07</td>
</tr>
<tr>
<td>P6</td>
<td>5.08</td>
<td>-0.00418</td>
<td>-1.14</td>
<td>0.62</td>
<td>6.98</td>
</tr>
<tr>
<td>P7</td>
<td>6.01</td>
<td>-0.01156</td>
<td>-2.51</td>
<td>0.66</td>
<td>5.92</td>
</tr>
<tr>
<td>P8</td>
<td>7.24</td>
<td>-0.0013</td>
<td>-0.29</td>
<td>0.65</td>
<td>5.93</td>
</tr>
<tr>
<td>P9</td>
<td>11.19</td>
<td>-0.00934</td>
<td>-1.82</td>
<td>0.71</td>
<td>5.75</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level
Figure 11

Relationship between excess returns and dividend yields of small companies: 1980-1989

![Graph showing the relationship between excess returns and dividend yields for small companies from 1980 to 1989.](image)

Figure 12

Relationship between excess returns and dividend yields of small companies: 1990-1996

![Graph showing the relationship between excess returns and dividend yields for small companies from 1990 to 1996.](image)

Table 8: Alphas and betas of medium companies: 1980-1989

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha t-statistics</th>
<th>Beta t-statistics</th>
<th>R²</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>-0.00867 -1.50</td>
<td>0.91</td>
<td>8.88</td>
<td>0.40</td>
</tr>
<tr>
<td>P1</td>
<td>1.42</td>
<td>-0.00326 -1.07</td>
<td>0.75</td>
<td>13.77</td>
<td>0.62</td>
</tr>
<tr>
<td>P2</td>
<td>2.74</td>
<td>-0.00455 -1.96</td>
<td>0.75</td>
<td>18.40</td>
<td>0.74</td>
</tr>
<tr>
<td>P3</td>
<td>3.63</td>
<td>-0.00389 -1.65</td>
<td>0.72</td>
<td>17.26</td>
<td>0.72</td>
</tr>
<tr>
<td>P4</td>
<td>4.44</td>
<td>-0.00247 -1.06</td>
<td>0.76</td>
<td>18.31</td>
<td>0.74</td>
</tr>
<tr>
<td>P5</td>
<td>5.29</td>
<td>-1.8E-05 -0.01</td>
<td>0.81</td>
<td>18.60</td>
<td>0.75</td>
</tr>
<tr>
<td>P6</td>
<td>6.16</td>
<td>-0.00047 -0.20</td>
<td>0.80</td>
<td>19.22</td>
<td>0.76</td>
</tr>
<tr>
<td>P7</td>
<td>7.13</td>
<td>-0.00108 -0.43</td>
<td>0.82</td>
<td>18.68</td>
<td>0.75</td>
</tr>
<tr>
<td>P8</td>
<td>8.33</td>
<td>-0.00185 -0.62</td>
<td>0.80</td>
<td>15.21</td>
<td>0.66</td>
</tr>
<tr>
<td>P9</td>
<td>10.75</td>
<td>-0.00307 -1.03</td>
<td>0.77</td>
<td>14.67</td>
<td>0.65</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level
Table 9: Alphas and betas of medium companies: 1990-1996

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha (t-statistics)</th>
<th>Beta (t-statistics)</th>
<th>R²</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>-0.02096 (-3.73)</td>
<td>0.74 (5.40)</td>
<td>0.26</td>
<td>29.22*</td>
</tr>
<tr>
<td>P1</td>
<td>1.38</td>
<td>-0.0078 (-2.18)</td>
<td>0.66 (7.67)</td>
<td>0.42</td>
<td>58.77*</td>
</tr>
<tr>
<td>P2</td>
<td>2.498</td>
<td>-0.00156 (-0.51)</td>
<td>0.72 (9.79)</td>
<td>0.54</td>
<td>95.82*</td>
</tr>
<tr>
<td>P3</td>
<td>3.17</td>
<td>-0.00632 (-1.84)</td>
<td>0.75 (9.08)</td>
<td>0.50</td>
<td>82.45*</td>
</tr>
<tr>
<td>P4</td>
<td>3.85</td>
<td>-0.0058 (-1.74)</td>
<td>0.81 (10.10)</td>
<td>0.55</td>
<td>101.97*</td>
</tr>
<tr>
<td>P5</td>
<td>4.49</td>
<td>-0.00598 (-1.62)</td>
<td>0.78 (8.78)</td>
<td>0.48</td>
<td>77.07*</td>
</tr>
<tr>
<td>P6</td>
<td>5.11</td>
<td>-0.00907 (-2.30)</td>
<td>0.96 (10.13)</td>
<td>0.56</td>
<td>102.63*</td>
</tr>
<tr>
<td>P7</td>
<td>5.99</td>
<td>-0.00896 (-2.05)</td>
<td>0.96 (9.05)</td>
<td>0.50</td>
<td>81.83*</td>
</tr>
<tr>
<td>P8</td>
<td>7.09</td>
<td>-0.00951 (-2.11)</td>
<td>1.02 (9.37)</td>
<td>0.52</td>
<td>87.88*</td>
</tr>
<tr>
<td>P9</td>
<td>11.17</td>
<td>-0.01639 (-2.60)</td>
<td>1.32 (8.67)</td>
<td>0.48</td>
<td>75.20*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level

Figure 13: Relationship between dividend yields and excess returns of medium companies: 1980-1989

Figure 14: Relationship between excess returns and dividend yields of medium companies: 1990-1996
### Table 10: Alphas and betas of large companies 1980-1989

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha</th>
<th>t-statistics</th>
<th>Beta</th>
<th>t-statistics</th>
<th>$R^2$</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>-0.00329</td>
<td>-0.82</td>
<td>0.96</td>
<td>13.45</td>
<td>0.60</td>
<td>180.84*</td>
</tr>
<tr>
<td>P1</td>
<td>1.54</td>
<td>-0.00662</td>
<td>-3.25</td>
<td>0.96</td>
<td>26.48</td>
<td>0.86</td>
<td>701.41*</td>
</tr>
<tr>
<td>P2</td>
<td>2.74</td>
<td>-0.00527</td>
<td>-3.02</td>
<td>0.96</td>
<td>31.09</td>
<td>0.89</td>
<td>966.43*</td>
</tr>
<tr>
<td>P3</td>
<td>3.64</td>
<td>-0.00465</td>
<td>-2.92</td>
<td>1.02</td>
<td>36.23</td>
<td>0.92</td>
<td>1312.8*</td>
</tr>
<tr>
<td>P4</td>
<td>4.42</td>
<td>-0.00554</td>
<td>-0.27</td>
<td>0.81</td>
<td>23.03</td>
<td>0.94</td>
<td>1748.8*</td>
</tr>
<tr>
<td>P5</td>
<td>5.27</td>
<td>-0.00193</td>
<td>-1.18</td>
<td>1.00</td>
<td>34.60</td>
<td>0.91</td>
<td>1197.3*</td>
</tr>
<tr>
<td>P6</td>
<td>6.12</td>
<td>-0.00113</td>
<td>-0.79</td>
<td>1.00</td>
<td>34.26</td>
<td>0.91</td>
<td>1173.8*</td>
</tr>
<tr>
<td>P7</td>
<td>7.10</td>
<td>0.001697</td>
<td>0.93</td>
<td>0.99</td>
<td>30.36</td>
<td>0.89</td>
<td>921.75*</td>
</tr>
<tr>
<td>P8</td>
<td>8.30</td>
<td>-0.00039</td>
<td>-0.21</td>
<td>0.92</td>
<td>28.02</td>
<td>0.87</td>
<td>785.08*</td>
</tr>
<tr>
<td>P9</td>
<td>10.76</td>
<td>-0.00185</td>
<td>-0.75</td>
<td>0.90</td>
<td>20.60</td>
<td>0.78</td>
<td>424.21*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level

### Table 11: Alphas and betas of large companies: 1990-1996

<table>
<thead>
<tr>
<th>Portfolio no.</th>
<th>Dividend yield</th>
<th>Alpha</th>
<th>t-statistics</th>
<th>Beta</th>
<th>t-statistics</th>
<th>$R^2$</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0</td>
<td>-0.01677</td>
<td>-3.14</td>
<td>0.87</td>
<td>6.74</td>
<td>0.36</td>
<td>45.39*</td>
</tr>
<tr>
<td>P1</td>
<td>1.35</td>
<td>-0.00802</td>
<td>-2.99</td>
<td>0.94</td>
<td>14.44</td>
<td>0.72</td>
<td>208.65*</td>
</tr>
<tr>
<td>P2</td>
<td>2.47</td>
<td>-0.0037</td>
<td>-1.72</td>
<td>1.06</td>
<td>20.29</td>
<td>0.83</td>
<td>411.64*</td>
</tr>
<tr>
<td>P3</td>
<td>3.18</td>
<td>-0.00158</td>
<td>-0.88</td>
<td>1.00</td>
<td>23.12</td>
<td>0.87</td>
<td>534.61*</td>
</tr>
<tr>
<td>P4</td>
<td>3.87</td>
<td>-0.00349</td>
<td>-1.95</td>
<td>1.11</td>
<td>25.77</td>
<td>0.89</td>
<td>664.33*</td>
</tr>
<tr>
<td>P5</td>
<td>4.48</td>
<td>-0.00225</td>
<td>-1.14</td>
<td>1.09</td>
<td>22.83</td>
<td>0.86</td>
<td>521.03*</td>
</tr>
<tr>
<td>P6</td>
<td>5.07</td>
<td>-0.00303</td>
<td>-1.52</td>
<td>1.15</td>
<td>23.89</td>
<td>0.87</td>
<td>570.60*</td>
</tr>
<tr>
<td>P7</td>
<td>6.02</td>
<td>-0.00806</td>
<td>-3.09</td>
<td>1.20</td>
<td>18.94</td>
<td>0.81</td>
<td>358.69*</td>
</tr>
<tr>
<td>P8</td>
<td>7.14</td>
<td>-0.00506</td>
<td>-1.41</td>
<td>1.20</td>
<td>13.85</td>
<td>0.70</td>
<td>191.88*</td>
</tr>
<tr>
<td>P9</td>
<td>10.65</td>
<td>-0.01339</td>
<td>-2.20</td>
<td>1.37</td>
<td>9.29</td>
<td>0.51</td>
<td>86.31*</td>
</tr>
</tbody>
</table>

* indicates statistical significance at 1% level
4.2. Time series and cross-sectional analysis of the yield effect

The empirical work on the capital asset pricing model, such as the one of Fama and MacBeth (1973), is mainly focused on a cross-sectional relationship between common stock returns and their corresponding beta coefficients. A simple extension of this type of relationship that includes anticipate dividend yields will be employed to the test the impact of dividend yields on stock returns:

\[ r_{pt} = \alpha + \gamma d_{pt} + \theta \beta_{pt} + \epsilon_{pt} \]  

(1.1)

Where \( r_{pt} \) is the total realised return of portfolio \( p \) in the period \( t \)

\( d_{pt} \) is the relevant dividend yield of portfolio \( p \) anticipated in period \( t \) and

\( \beta_{pt} \) is the relevant beta coefficient of portfolio \( p \) in period \( t \) calculated as the weighted average of betas of securities in portfolio \( p \) where equal weights are assigned to all securities: \( \beta_{pt} = \sum w_i \beta_i \)

\( \epsilon_{pt} \) is the error term and
\( \gamma \) and \( \theta \) are coefficients on the above parameters.

The null hypothesis to be tested is that the returns are unrelated to dividend yields and can be formulated as:

\[ H_0 : \gamma = 0 \]

In order to apply this model we have constructed 30 dividend-paying portfolios of equal size and one non-dividend-paying portfolio. Results of the pooled regression given by equation (1.1) for the whole sample period 1980-1996 are given in the table 12:

**Table 12: Parameter estimates of dividend yield and beta variable**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameter Estimates</th>
<th>t - Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>0.019</td>
<td>8.78</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-7.5E-05</td>
<td>-0.41</td>
</tr>
<tr>
<td>( \theta )</td>
<td>-0.013</td>
<td>-5.10</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.048</td>
<td></td>
</tr>
</tbody>
</table>

The dividend yield parameter is statistically insignificant at a usual 95% confidence interval, implying that there is no linear relationship between dividend yields and returns, i.e. that there is no yield effect. \( R^2 \) is also very small 0.047, showing again that linear dividend yield variable on its own does not explain movements in the stock returns, but F-test is significant, taking the value of 13.19. According to the CAPM, one would expect the parameter estimate on beta coefficient to be positive. This is not the case in tables 12 and 13 in this section and tables 14 and 16 in the following
section. The possible explanations for the negative parameter estimate of the beta coefficient will be given in section 4.3.

The summary statistics suggest that the relationship between return and dividend yield may be a non-linear one. In particular, the results in table 1 are suggesting a quadratic yield/return relationship. Therefore, to account for this observation, the quadratic dividend yield term is added in the original model in equation (1.1), to obtain

$$r_{pt} = a + \gamma d_{pt} + \delta d_{pt}^2 + \theta \beta_{pt} + \epsilon_{pt}$$  \hspace{1cm} (1.2)

Table 13 shows the parameter estimates of the pooled regression from equation (1.2) for the whole sample period:

**Table 13: Parameter estimates of dividend yield, quadratic dividend yield and beta variable**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameter Estimates</th>
<th>t – Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.014</td>
<td>5.81</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.002</td>
<td>4.23</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.00012</td>
<td>-4.84</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-0.013</td>
<td>-5.16</td>
</tr>
<tr>
<td>R Squared</td>
<td>0.084</td>
<td></td>
</tr>
</tbody>
</table>

The econometric results in the table above are confirming the positive quadratic relationship between yield and return suggested earlier on in the results of summary statistics. In particular, both dividend yield coefficient and quadratic yield coefficient, although taking small values of 0.002 and -0.0001 respectively, have significant values of the t-test with the right sign. The R-squared has also improved from 0.05 to
0.08 when the quadratic term is taken into account. The F-test is also significant (16.99).

4.2.1. The role of size in the yield portfolio returns behaviour

It was established in the summary statistics in section 3, that there is a relationship between the market values and dividend yields, and hence the relationship between the market values and stock returns. Let us now see how significant market value is in explaining the changes in stock returns and whether its inclusion in a model will change the econometric results previously observed. If we add the market value variable in the equation (1.2) we obtain the following model:

\[ r_{pt} = a + \gamma d_{pt} + \delta d_{pt}^2 + \mu m_{pt} + \theta \beta_{pt} + \varepsilon_{pt} \]  

(1.3)

The parameter estimates from equation (1.3) are in table 14:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Estimates</th>
<th>t – Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.013</td>
<td>5.53</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.0019</td>
<td>4.46</td>
</tr>
<tr>
<td>( \delta )</td>
<td>-0.00013</td>
<td>-5.18</td>
</tr>
<tr>
<td>( \theta )</td>
<td>-0.001</td>
<td>-3.57</td>
</tr>
<tr>
<td>( \mu )</td>
<td>-4.8E-06</td>
<td>-2.42</td>
</tr>
<tr>
<td>R Squared</td>
<td>0.099</td>
<td></td>
</tr>
</tbody>
</table>

Table 14: Parameter estimates of dividend yield, quadratic dividend yield, beta and market value variable
Although the dividend yield parameters are remaining significant at 99% confidence interval, the model is giving very small negative value for the market value parameter $\mu$, however it is significant only at the 1% level of confidence. This implies the negative relationship between the stock returns and market values. We have found indications in the section 3 that the impact of market values on stock returns may have changed over the time period under consideration in this chapter. Let us then divide the sample period in the two subperiods: 1980s, i.e. 1980-1989 period, which should capture the superior performance of small stocks during that period and 1990s, i.e. 1990-1996 period, where the change in the size-return relationship is expected to occur. We are also interested in assessing the impact of that change on the yield coefficient between the first and second subperiod.

Cross-sectional regressions defined by equation (1.3) were run for the two subperiods and the results are presented in tables 15 and 16.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Estimates</th>
<th>t – Statistics</th>
<th>Parameter Estimates</th>
<th>t – Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.007</td>
<td>3.73</td>
<td>$a$</td>
<td>0.012</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.002</td>
<td>4.73</td>
<td>$\gamma$</td>
<td>0.002</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.00012</td>
<td>-4.55</td>
<td>$\delta$</td>
<td>-9.9E-05</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.008</td>
<td>3.44</td>
<td>$\theta$</td>
<td>-0.021</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-1.3E-050</td>
<td>-3.18</td>
<td>$\mu$</td>
<td>4.95E-06</td>
</tr>
<tr>
<td>R Squared</td>
<td>0.10</td>
<td></td>
<td>R Squared</td>
<td>0.057</td>
</tr>
</tbody>
</table>
There are three major findings that can be drawn from tables 15 and 16:

Firstly, regarding the size coefficient, market value parameter estimate from table 15 (1980s) has a very small negative value, but significant at the 1% level. It implies the negative, i.e. inverse relationship between market values and returns in 1980s. For an investor, it means that during that period investing in smaller companies would provide higher returns than the investment in larger ones. On the other hand, in the 1990s, our findings show that the market value coefficient has turned positive. The coefficient is significant at the 10% level, implying that in the 1990s, the larger the size of the company, the larger its returns would be. Directly, this finding is confirming our results from the summary statistics in section 3, where we have first suggested the change in the size-return relationship due to recent underperformance of small companies.

Secondly, regarding the yield effect, it is interesting that the dividend yield coefficient hasn’t changed at all. The same value of 0.002 remains in both subperiods, significant at 1% level in 1980s and 5 % level in 1990s. This indicates that although the shape of the yield-return relationship has changed in 1990s, the significance of dividend yields as an explanatory variable for stock returns behaviour hasn’t changed at all.

These results in this section are consistent with the results in the time series analysis: market values are related to the stock returns but they do not change the strength of the yield-return relationship.
The third set of findings will be explained in the following section 4.3.

4.3. The analysis of the estimate of beta coefficients

CAPM is suggesting a positive relationship between security (portfolio) returns and their systematic risk that is measured by beta. However, in some instances in this chapter, we are observing the negative relationship between the two variables. In particular, negative beta parameter estimates are obtained when applying regression models (1.1), (1.2), and (1.3) on the whole sample period and model (1.3) on the second subperiod of the sample, 1990-1996, as in tables 12, 13, 14 and 16. However, we also find that in the analysis of the first part of the sample period, 1980-1989, we obtain the positive beta parameters as in table 15, which is consistent with the CAPM theory. Specifically, these results are driving us to the third main finding in this chapter: in the 1990s, beta is not the correct measure of the systematic risk anymore.

One of the possible reasons for having positive coefficients on betas in the first subperiod and then negative estimates in the second subperiod may lie in the methodology used to estimate betas of individual securities and, in turn, different portfolios particular securities belong to. Specifically, we use historic data to estimate beta for the future period. It might have happened that in the period 1980-1989 betas were much more stable than in the later years and, hence, hence estimated historic betas were good predictors of the betas in the future period. If the betas became more unstable and volatile in the second half of the sample period, the historic data was not of much use for predicting future betas. Therefore, this mean-reverting pattern of betas in the second subperiod is driving the sign of its parameter estimates for the whole sample period to be negative, as presented in tables 12, 13 and 14, thus contradicting the CAPM.
5. Conclusions

The chapter is showing that there is a significant yield effect on the UK market in the period 1980 to 1996. The yield – return relationship observed is a quadratic one, forming a sort of a bell shaped curve. In particular, zero yield portfolios have the smallest returns, even on the risk-adjusted basis. Furthermore, as the yields are increasing the portfolio returns are increasing as well, up to the point when yield reaches the highest level – that is when returns start dropping again. We have used the time series and cross-sectional analysis to examine and determine the source of this yield-return relationship. We find no evidence that the tax-based hypothesis can be used as a plausible explanation of the yield-return relationship in the period under observation due to the fact that imputation tax system in the UK implies that the yield-return relationship should be negative if the tax hypothesis applies. Our findings are the following:

1. The yield coefficient remains constant throughout the whole sample period.
2. Small size companies are experiencing the change in the returns behaviour in the 1990s, causing the yield-return relationship to change from inverse to a positive one. The market value coefficient in the cross-sectional regression analysis has changed from the negative one in the 80s, implying that small stocks earn higher returns than larger ones, to the positive coefficient in the 90s, implying that small stocks started underperforming larger ones.
3. Beta is an adequate measure of systematic risk in the 80s, but it doesn't continue to be so in the 90s. Finding the possible reasons for this, apart the measurement error in betas, requires further research.
CHAPTER FOUR

EQUITY PREMIUM: THE EUROPEAN OUTLOOK
1. Introduction

1.1. Definition of the equity premium

Equity premium is the difference between the return on common stock and the return on government securities. To calculate equity premium, usually two types of government securities are used. The first popular choice is short-term Treasury bill with maturity of 1 month or 3 months, because it is the closest thing to the risk-free asset. The second group of securities used are the long-term Treasury bonds. It is commonly accepted that 20-year treasury bonds are used when calculating equity premium.

The equity premium may be defined in historical (ex-post) sense and a forward-looking (ex-ante) sense. The ex-post premium is calculated as the difference between the historical average return on common stocks and the average return on treasury bills/bonds. The calculation of the ex-ante premium is not that straightforward and it will be discussed in the later sections.

1.2. Methods of calculation of the equity premium

The most direct way to assess the equity premium is by use of the Sharpe ratio that measures equity premium per unit of total risk, measured by standard deviation. The Sharpe ratio for the market portfolio would be:

\[ S = \frac{E(r_j) - r_F}{\sigma_j} \]

If one knows the Sharpe ratio and the standard deviation of a security’s returns, the equity premium can easily be estimated.
Furthermore, one can calculate equity premium through the CAPM, if he/she knows the value of the risk free rate, security’s beta and the level of the market risk premium:

\[ E(r_j) - r_f = \beta_j (E(r_m) - r_f) \]

The CAPM and most well known asset pricing models give the risk premium of individual assets in terms of the market risk premium but they do not allow assessment of the market risk premium itself. In order to assess the market risk premium as well as the risk premium on any security, we can use a basic model that relates risk to future consumption (investing means foregoing consumption today in order to have opportunity to consume more tomorrow). Hence, in order to measure the risk of investing in equities, we should assess the impact of that investment on the riskiness of future consumption. In other words, the determinant of risk premium is the correlation of asset returns with consumption.

The above stated relationship between returns and consumption can be stated in one equation that is known as the consumption-based CAPM:

\[ E(r_j - r_F) = \gamma \text{Cov}(\Delta C, r_j) = \gamma \sigma_{\Delta C} \sigma_{r_j} \text{corr}(\Delta C, r_j) \]

where \( \Delta C \) is the percentage change in aggregate per capita consumption over the observation period and \( \gamma \) is the level of risk aversion.
The intuition behind this equation is straightforward: risk premium is higher if the level of risk aversion is higher and if the covariance between asset returns and changes in consumption is higher. Note that consumption-based CAPM does not include expected return on the market as an explanatory variable. Therefore, it can be applied to any asset including the market portfolio.

We can use the consumption based CAPM to calculate the theoretical Sharpe ratio:

$$S = \frac{E(r_j) - r_F}{\sigma_{\Delta C}} = \gamma \sigma_{\Delta C} corr(\Delta C, r_j)$$

Observed Sharpe ratio on the US stock market is 0.5. Cornell (1999) states that theoretical Sharpe ratio derived from consumption CAPM is 0.004, which implies that there is a difference between the historically estimated equity premium and the theoretical equity premium suggested by the consumption-based CAPM. We will explore this issue in section 1.3.4.

In this chapter we are aiming to review the evidence related to the historical estimates of the equity premium and to examine the factors that equity premium depends on. Particular emphasis in the empirical part of the chapter will be placed on examining the relationship between volatility of the equity returns and equity premium and modelling of the equity premium.
1.3. Empirical Evidence

1.3.1. Early estimates

One of the earliest studies regarding the equity premium was by Mehra and Prescott (1985). The period under consideration in this study is 1889 – 1978. Authors take the average short-term real interest rate for the period and the average real return on the S&P 500 Composite index to calculate the equity premium. The real interest rate and the real return are adjusted for inflation, therefore the authors find that the real short-term interest rate in the period analysed was 0.8%, the real return on the S&P 500 index was 6.98% and in turn, the equity premium was 6.18%. The authors use Sharpe ratio to estimate equity premium in this study.

Considering these results, authors are raising the question of the 'equity premium puzzle'. It refers to the question if the high value of equity premium should be related to the low value of the average real risk-free rate rather than the high value of the average real equity return.

Possible problem with this study is the fact that after tax returns are under the consideration and they vary across income classes and over time period observed. The study covers 90 years period in which tax rates in the earlier years were quite low. In the latter years, regardless of the tax rate each income group has to pay, the real interest rates were low and the equity premium was high. They conclude that the historical return on stocks has been too high in comparison to the return on the risk free rate to be explained by the commonly used economic models of risk and return.
(consumption based capital asset pricing model), without assuming unreasonably high degree of risk aversion of investors.

Furthermore, Ibbotson and Singuefild (1976) analysed the real returns on equities and risk free assets over the period 1926-1974. During the entire period, US Treasury bills had compounded annual return of 2.2%, which was approximately equal to the rate of inflation. Therefore, real interest rate was close to 0.0%. Over the same period, the real return on equity, based on the S&P 500 Composite index, was greater than 7%, resulting in the equity premium that is exceeding 7%. Since the findings are suggesting high equity premium and low (zero) risk free rate, the same question as before can be asked: do we have equity premium puzzle or the real rate puzzle?

### 1.3.2. More Recent Estimates of Equity Premium

Siegel (1999) examines the estimates of the equity premium derived from historical in the period from 1802 through 1998. Some of the findings of this paper are presented in the tables 1 and 2 below:

**Table 1: Compound annual real returns (%) U.S. data, 1802-1998**

<table>
<thead>
<tr>
<th></th>
<th>Stocks</th>
<th>Bonds</th>
<th>Bills</th>
<th>Gold</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1802-1998</td>
<td>7.0</td>
<td>3.5</td>
<td>2.9</td>
<td>-0.1</td>
<td>1.3</td>
</tr>
<tr>
<td>1802-1870</td>
<td>7.0</td>
<td>4.8</td>
<td>5.1</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>1871-1925</td>
<td>6.6</td>
<td>3.7</td>
<td>3.2</td>
<td>-0.8</td>
<td>0.6</td>
</tr>
<tr>
<td>1926-1998</td>
<td>7.4</td>
<td>2.2</td>
<td>0.7</td>
<td>0.2</td>
<td>3.1</td>
</tr>
<tr>
<td>1946-1998</td>
<td>7.8</td>
<td>1.3</td>
<td>0.6</td>
<td>-0.7</td>
<td>4.2</td>
</tr>
</tbody>
</table>
For the entire period under observation, 1802-1998, the real compounded annualised returns on short-term bonds was 2.9% while the long-term bonds gave real return of 3.5%.

**Table 2: Equity premiums (%) U.S. data, 1802-1998**

<table>
<thead>
<tr>
<th>Period</th>
<th>Equity premium with Bonds</th>
<th>Equity premium with Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Geometric</td>
<td>Arithmetic</td>
</tr>
<tr>
<td>1802-1998</td>
<td>3.5</td>
<td>4.7</td>
</tr>
<tr>
<td>1802-1870</td>
<td>2.2</td>
<td>3.2</td>
</tr>
<tr>
<td>1871-1925</td>
<td>2.9</td>
<td>4.0</td>
</tr>
<tr>
<td>1926-1998</td>
<td>5.2</td>
<td>6.7</td>
</tr>
<tr>
<td>1946-1998</td>
<td>6.5</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Note that the author distinguishes between the arithmetic and the geometric average equity premium. In the CAPM the equity premiums are derived from the arithmetic not geometric returns. It can be seen from table 2 that the manner in which average return is calculated makes the difference in the final result. The question is: which average would be more appropriate to use? If one wants to estimate year-by-year equity premium, the arithmetic average would be a better choice. Conversely, if the estimates of average equity premium over the entire examination period are needed, the geometric average is a better choice. Therefore, the author suggests that it is useful to obtain both arithmetic and geometric equity premium averages.
1.3.3. Some explanations of the real rate puzzle

a) *Real rate puzzle is related only to Mehra and Prescott (1985) sample period*

Siegel (1992) constructs a continuous interest rate series for both UK and US from 1800 to 1990, extending the period analysed by Mehra and Prescott (1985). The reason why researchers haven't analysed interest rates and equity premium prior to 1889 is the lack of data for the risk free asset in the US. However, fixed income market in the UK was highly developed and it has been used to construct a hypothetical short-term risk free rate for the US market prior to 1889. The author distinguishes between real rates and equity premium within Mehra and Prescott (M/P) period and outside that period. The findings are as follows: a) the real return for the US risk free asset is 0.87% during the M/P period but 5.19% outside the period, b) the real return on the UK risk free securities averaged to 0.75% during M/P period and 4.84% outside that period, c) the real rate of return on equity outside M/P period was 7.75%, which is almost identical to the real equity return of 7.79% in M/P period and d) the level of equity premium changes within and outside M/P period but still remains above 1% level.

There are several explanations for small real return on the risk free asset during M/P period that have been put forward, and they are merely related to the impact of the inflation rate on the real return on risk-free asset.

b) *Unanticipated inflation*

One of the reasons for the low real interest rates may be the acceleration of the unanticipated inflation after the World War II and in the 1970s. When expected inflation is used to calculate real rates, the rates that we obtain are called expected or...
ex-ante real rates. Very often past inflation rates are used as proxies for the expected inflation, hence, they are used to calculate realised or ex-post real interest rates. The problem is that the expected real rates may be different from the realised ones. The unanticipated inflation undoubtedly had a negative effect on the realised real returns from long-term bonds.

Additionally, the nature of the inflation has changed during those years because of the collapse of the gold standard. In particular, in 1931, the UK suspended the conversion of sterling into gold and the US followed it in 1933. The final collapse of the golden standard occurred in 1971 when the US forbid foreign central banks to convert dollars into gold. During the gold standard the currency had to be backed by gold and the rate at which gold could be accumulated was controlling the money supply. If the money supply was controlled, so was the inflation. The fact that the golden standard has diminished has incurred high and consistent inflation. The average inflation rate between 1802 and 1932 in the US was 0.003% whereas between 1934 and 1997 it averaged 4.1%. Nowadays, inflation is determined by political forces, which is adding the risk to investing in the fixed income securities. Consequently, this inflationary risk alters both ex-ante and ex-post equity risk premium, especially if it is measured using long-term bonds. Ex-ante risk premium will be reduced because the unpredictable inflation will add the risk to long term bonds, increase their return and in turn reduce the spread between long term bonds and equities. Ex-post risk premium will increase because of the possible increase in inflation during the periods of unexpected inflation that will decrease returns on bonds. Simply, if inflation rises unexpectedly over a longer time period, historical returns on bonds will be low or negative during that period.
1.3.4. Historical vs. theoretical equity premium

As noted in the section 1.2., there is a difference between historically estimated Sharpe ratio (0.5) and the one theoretically implied by the consumption based CAPM (0.004). We have also established that using the historical estimate of the Sharpe ratio Mehra and Prescott (1985) find equity premium of 6%. Theoretical Sharpe ratio implies equity premium of 0.08% if one assumes 20% standard deviation of the market (Cornell (1999)). There are several arguments put forward to explain this discrepancy. Let us examine them in turn.

a) The empirical data are wrong

The problem lies with the empirical estimates of the Sharpe ratio. The historical data overstate the true risk premium. The reasons for this may be survival bias or pure luck. However, it has been argued that the survival bias cannot explain 10 times greater historically observed risk premium from the one implied by the consumption CAPM.

b) High risk aversion

Investors may be much more risk averse than the economists generally believe. However, the coefficient of risk aversion (\(\gamma\)) that has to be assumed in consumption based CAPM in order to reduce the discrepancy between the theoretical and empirical risk premium, has to be so large that it is inconsistent with common sense and everyday behaviour.
c) Nonstandard utility functions

The consumption based CAPM is built on the assumption that each period utility depends only on the amount of consumption. This idea is an oversimplification. Some authors suggest that alterations might bridge the gap between theoretical and empirical equity premium. Only a few of those alterations offer the possibility to explain the gap. One of the alterations offered is that the utility of consumption depends on the investor's standard of living. Cochrane (1997) develops a model in which the risk aversion of investor depends on how far current consumption is from the accustomed standard of living. If consumption drops and approaches the habitual level, investors become more risk averse because they are less willing to accept further declines in consumption and hence, they require higher equity premium. The problem with this model is that the consumption growth rate is highly unpredictable but the model assumes the opposite. Overall, the use of nonstandard utility function does not adequately explain the gap between theoretical and empirically observed equity risk premium.

d) Autocorrelation in returns

If there is autocorrelation in returns, the usual measures of risk such as standard deviation will not be adequate for assessing long term risk. Siegel (1992) tried to examine this issue. He found the negative autocorrelation between stock returns (bad years are likely to be followed by a good one). This decreases the risk of holding stock over the long run. On the other hand, returns on bonds are positively correlated. Therefore, for the longer holding periods, the risk of holding stocks relative to bonds is falling. This implies that the risk premium should be lower than what the analysis
of the short term data predicts. Hence, this is not an adequate explanation for our discrepancies.

e) Time varying expected returns

If there is a nonstationarity in stock returns, two types of risk have to be distinguished: 1) uncertainty regarding the return on equity this period and 2) uncertainty regarding the change of expected returns as time passes. Investors will require compensation for bearing the risk of changes in expected returns so it is possible that variation in expected returns can explain the equity risk premium. Returns must change in a way that makes stocks less attractive to investors and increases return (equity premium) they require. However, stocks can act as a hedge against future changes in expected returns. In particular, when future expected returns rise, current returns drop and vice versa. This 'natural hedge' makes stocks less risky than they are from the point of view of two-period models that look only at history of current returns. Adding variation in expected returns leads to the prediction of lower, not higher, equity risk premium.

f) Heterogeneous Investors

Models of equity risk premium have relied on the assumption of homogeneous investors, meaning that investors are identical in their beliefs regarding expected returns, variances and covariances between equities and their utility functions. When heterogeneous investors assumption is introduced, one can observe unique risks associated with individual investors. However, unique risk is diversifiable and it is not correlated with the aggregate consumption that determines systematic risk and
therefore the risk premium. The problem with these models is that they are very complex.

None of these theories and models outlined in this section fully explains the difference between the theoretical and empirical estimates of the equity premium on their own. Maybe the combination of the above models can give some possible explanations of the equity premium puzzle. However, this does not mean that such an explanation would be the right one, only the possible one.

1.3.5. The problem with historical data as estimates of expected return

According to Cornell (1999), Ibbotson and Sinquefield made long term predictions in 1976 and again in 1982 of real returns for stocks, bonds, Treasury bills and inflation on the basis of their own analysis of the historical data. The forecast periods were 1976-2000 and 1982-2001. Since the actual data for most of their forecast period is available at present, Siegel (1999) has compared the Ibbotson-Sinquefield forecast estimates with the actual data as in table 3 below.

Table 3: Long-term forecasts of real returns – compound annual rates of return

<table>
<thead>
<tr>
<th>Forecast Period</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Bills</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976-2000</td>
<td>Forecast</td>
<td>6.3(23.5)</td>
<td>1.5(8.0)</td>
<td>0.4(4.6)</td>
</tr>
<tr>
<td></td>
<td>Actual</td>
<td>11.0</td>
<td>5.3</td>
<td>2.1</td>
</tr>
<tr>
<td>1982-2001</td>
<td>Forecast</td>
<td>7.6(21.9)</td>
<td>1.8(8.3)</td>
<td>0.0(4.4)</td>
</tr>
<tr>
<td></td>
<td>Actual</td>
<td>14.6</td>
<td>9.9</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Note: the value in parenthesis is the standard deviation of the annual returns.
There is one common characteristic of the forecasts for financial assets in the table 3: they all underestimate the actual return on each asset. Particularly, most serious underestimation is regarding fixed-income securities, where they forecast the real return on bills to be zero, when their realised return value is 2.9%. Similar can be observed for bonds. One can also notice that long-term inflation rate was overestimated, especially in the 1982 forecast, taking the value of 12.8%. Such a value was obtained by deducting low historical real return of bonds from their high nominal rate.

Although there is a large forecast error in Ibbtson-Sinquefield forecasts, they are considered as a benchmark for the risk and return estimates, which is used by both academics and professionals. Their forecasts show that not even fifty years of historical returns data is enough to accurately forecast future returns. The accuracy with which one can measure the historical returns and in turn, risk premium depends on the following:

1. **Statistical Properties of standard deviation**

There is always a degree of variability (standard deviation) of observations from which the average historical risk premium is calculated. Table 3 shows that stock returns exhibit the highest annualised standard deviation of over 21% in both forecast periods. Similarly, Cornell (1999) found that the standard deviation for annual stock returns was 20% and that the standard deviation of the risk premium is 21%. This finding leads to conclude that most of the variation in the equity risk premium is due to variability of stock returns rather than bond or bill returns. Such a high standard
deviation is suggesting that historical returns are so variable that they cannot be taken as a guide to make estimates of the future risk premium. Rather than measuring standard deviation of individual returns, one can measure the standard deviation of the mean return. The property of the standard deviation of the mean is that it declines at a rate, which is approximately equal to the square root of the number of observations. Hence, the larger the number of observations, i.e. the longer the sample period, the smaller the standard deviation of the mean. The direct implication of this would be that the accuracy of forecast based on historical estimates will depend on the choice of the sample period. In general, the shorter the sample period, the greater the variability of the estimates.

2. Stationarity

When we use historical data to estimate the risk premium, we make an assumption that the past data are stationary. However, changes in the stock market and the economy as a whole throughout the years suggest that the historical data used to forecast equity returns and in turn, risk premium may be nonstationary. Nonstationarity of stock returns (risk premium) can be tested by using the volatility of those returns because volatility can be estimated with high degree of accuracy. The estimation of the volatility of returns depends on the frequency of the data. Hence, we can measure volatility of returns as frequently as we want, even at five minutes intervals. Since we are able to measure volatility of stock returns over short time intervals, we can introduce tests of nonstationarity that measure the statistical significance of changes in volatility between two time intervals. If we accept that the economic theory is correct and that higher volatility (risk) implies higher return (risk premium), than time-changing volatility implies nonstationary risk premium.
Additionally, to produce reasonable forward-looking estimates of equity premium, the nature of the nonstationarity should be specified. In order to complete this, we should either specify the variables on which the variation of premium depends or develop a model of how the premium varies over time.

1.4. Modelling the changes in the risk premium

1.4.1. Models based on the variability of returns

One of the first attempts to explain the variations in the risk premium is based on the risk-return relationship itself. One of the widely used models that relates return of an asset (portfolio) to the risk is the CAPM model. There is considerable empirical evidence regarding the relationship between stock returns and variability of those returns which we will outline in this section.

Merton (1980) states that the model, which determines the expected return on the market, depends on the inputs available for that model. If the investors are risk averse, as it is the case in the CAPM, the expected return on the market must be greater than the risk free rate. However, this is not always the case. Merton recognises that the historical mean return on the market may be lower than the current interest rates and that it depends on the inflation rate, hence, the market risk premium may take negative values. Therefore, he specifies the model that accounts for the negative market risk premium. He derives a linear relationship between the equity risk premium and the variance of equity returns. In particular, he states that equity
premium \( (\lambda) \) depends on the level of risk aversion \( (\theta) \) and the variance of the equity returns \( (\sigma^2) \):

\[
\lambda_t = \theta \sigma^2_t
\]

Additionally, his findings are documenting that the market returns vary over time and hence, the model used is adjusted for heteroskedasticity. The data used in the model is US market returns (NYSE Index of all stocks) and interest rate data (The US Treasury Bill Index) for the period 1926-1978.

This study of Breen, Glosten and Jagannathan (1989) is considering the fact that one-month interest rate is useful in forecasting the sign and the variance of the risk premium because it has been shown in the financial literature that there is a significant negative relationship between nominal excess returns on stocks and nominal interest rates. The data used is the US value weighted and equally weighted index NYSE stocks and one-month Treasury bill for the period April 1954 through December 1986. Authors find that treasury bills can forecast the changes in stock index risk premium when the index is value weighted NYSE portfolio. The value of the forecast for an investor who assesses the performance of the forecasting model was 2% (annualised) of the value of the assets managed. As far as equally weighted index is concerned, the model didn’t prove to be statistically beneficial. The study also identifies the presence of the heteroskedasticity in the data.

It is generally accepted in the finance theory that higher risk will imply higher return. However, Golsten, Jagannathan and Runkle (1993) state that the relationship between risk and return across time is somewhat different. There may be times when a larger risk premium may not be required for bearing larger risk for several reasons:
Time periods of higher risk may coincide with time periods in which investors are able to bear particular types of risk.

In the risky periods, investors may wish to save more. Specifically, if all the assets carry risk at certain point in time and there is no asset that can be considered as a risk-free one, the price of the risky asset may rise causing the risk premium to fall.

Therefore, both positive as well as negative covariance between time-dependent expected return and time-dependent variance should be equally accepted in the theory.

This study utilises modified GARCH-M framework to model the stochastic volatility of stock returns. The model allows for 1) seasonal patterns in volatility of returns 2) positive and negative innovations to returns have different impact on the conditional variance (e.g. the fluctuations in stock prices are caused by fluctuations in expected future cashflows; if the riskiness of future cashflows does not change proportionally when investors revise their expectations, then, unanticipated changes in returns will be inversely related to unanticipated changes in future volatility) and 3) nominal interest rates are used as a predictor for a conditional variance. The data used was monthly excess continuously compounded returns on the CRSP value-weighted index for the period April 1951 through December 1989. The findings suggest the negative relationship between conditional expected monthly return and the conditional variance. However, without modifying the GARCH process, the relationship between conditional returns and volatilities is positive but insignificant. It is clear that the specification of the model may considerably change the final result.

Nelson (1991) states that GARCH models used in modelling the relationship between conditional variance and asset risk premia have some drawbacks when applied in
asset pricing due to: a) there is a negative correlation between current returns and future returns volatility but GARCH models are ruling this out by the assumption, b) GARCH models impose parameter restrictions that are often violated by estimated coefficients and that may restrict the dynamics of the conditional variance process and c) it is difficult to interpret in GARCH models whether shocks to conditional variance 'persist' or not. Therefore, using ARCH and GARCH models to model the relationship between equity premium and conditional variance has a serious problem unless a model is introduced that will overcome the problems outlined above. Such an attempt has been made by Nelson who has developed new form of ARCH model to estimate the risk premium on the CRSP value weighted index in the period 1962 through 1987.

Poterba and Summers (1986) examine the influence of the stock market volatility on the level of stock prices and evaluates the changing risk premium hypothesis. Similarly to Merton (1980), authors define the relationship between equity premium and the variance of returns as:

$$\lambda_t = \theta \sigma_t^2$$

In order to be able to analyse the impact of stock volatility changes on stock prices, one must specify the evolution of the equity variance term. The empirical work of Potreba and Summers suggests that in the postwar period, monthly variance of the equity returns follow an autoregressive order one process, AR(1):

$$\sigma_t^2 = \rho_0 + \rho_1 \sigma_{t-1}^2 + \varepsilon_t$$

Automatically, it follows that the monthly values of equity premium follow an AR(1) process:

$$\lambda_t = \theta \rho_0 + \rho_1 \lambda_{t-1} + \theta \varepsilon_t$$
The mean value of monthly equity premium is then:
\[
\bar{\lambda} = \frac{\theta \rho_0}{1 - \rho_1}
\]
and \(\lambda_t - \bar{\lambda}\) evolves according to:
\[
\lambda_t - \bar{\lambda} = \theta \rho_0 + \rho_1 (\lambda_{t-1} - \bar{\lambda}) + \theta \epsilon_t
\]

The above formula suggests that the effect of changes in volatility on the level of share prices is sensitive to the level of serial correlation in monthly volatility, \(\rho_1\).

The period examined in this study was 1928-1984. The volatility estimates are computed from daily returns on the S&P 500 Composite Index. The findings are showing positive serial correlation in the data for both postwar and the whole sample period. Persistence of the volatility is greater for the full sample period (monthly AR(1) coefficient is 0.73) than for the postwar period (monthly AR(1) coefficient is 0.57). The analysis confirms that the null hypothesis of nonstationarity can be rejected and that volatility of stock returns is stationary at very high confidence levels. This is contradicting some of the earlier studies, such as Breen, Glosten and Jagannathan (1989) for example. The results of the paper show that even when volatility is doubled, the level of share prices would be reduced by 23% at most (probably even less). This leads us to believe that fluctuation in volatility, and the changes in equity premium that such a fluctuation causes, can not explain a large proportion of the variation in the stock market's level.

Analysing the evidence in this section, we can conclude that authors disagree on how the changing variability is related to the risk premium: some find positive relationship
between the two, some negative and some no significant relationship whatsoever. Therefore, we can say that if there is any kind of relationship between returns and the variability of the returns, it is probably a very weak one. This implies that variability of returns may not be a good variable for modelling possible changes in the risk premium and that different models should be introduced to complete the modelling task more successfully.

1.4.2. Models based on dividend yield

One of the variables commonly considered to be correlated with future risk premium is the dividend yield. One of the direct methods to estimate the risk premium, free of nonstationarity, is based on the fundamental equity valuation model, i.e. dividend discount model. If we assume that dividends grow at a constant rate we can apply the Gordon’s constant growth model of equity valuation in which:

\[ P_0 = \frac{E(D_1)}{E(r) - E(g)} \]

Since we can forecast future dividends, we can solve the above equation for the expected return:

\[ E(r) = E(g) + \frac{E(D_1)}{P_0} \]

Deducting the current yield on Treasury bills or bonds from the expected return obtained from the formula above yields a forward-looking estimate of the equity risk premium.

This study by Blanchard (1993) extended the basic discounted cashflow model (DCF) for estimating equity premium outlined above. The extended model accounts for
variations in the interest rates and dividend yields. Blanchard estimated the future path of stocks and bond returns and examined the forward looking risk premium estimates. The results of this study show that, as of 1992, the equity risk premium has been following a decreasing trend since 1950s and that its value today is around 2-3%. If Blanchard model was applied on data after 1992 (1992-1997), one would find that predictions of the equity premium are even lower – around 2%. Cornell (1999) predictions of equity premium based on the DCF model are 5.77% over bills and 4.53% over bonds which is much higher than the predictions obtained through Blanchard’s extended DCF model.

Fama and French (1988a) have provided evidence that dividend yields have significant power to predict the ex-post risk premium, especially over the longer time horizons. Using the monthly return on S&P 500 Index, its dividend yield and the return on one-month Treasury bills for the period 1948-1997, Cornell (1999) assesses how powerful dividend yield is in predicting ex-post equity premium. The findings are summarised in table 4 below.

Table 4: Regressions of the ex-post risk premium on dividend yield, 1948-1997

<table>
<thead>
<tr>
<th>Horison</th>
<th>$\alpha$</th>
<th>$t(\alpha)$</th>
<th>$\delta$</th>
<th>$t(\delta)$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>-0.086</td>
<td>1.02</td>
<td>4.45</td>
<td>2.22</td>
<td>0.093</td>
</tr>
<tr>
<td>2 years</td>
<td>-0.218</td>
<td>-1.79</td>
<td>10.01</td>
<td>3.47</td>
<td>0.204</td>
</tr>
<tr>
<td>5 years</td>
<td>-0.79</td>
<td>-3.75</td>
<td>30.59</td>
<td>6.22</td>
<td>0.468</td>
</tr>
</tbody>
</table>

At a 1 year time horison the dividend yield explains 10% of the variation in ex-post equity premium, whereas at a 5 year horison, the explanatory power of the dividend
yield rises to almost 50%. Specifically, high dividend yields of the 1950s predicted the market boom in the 1960s and high dividend yield in the late 1970s preceded the boom of the 1990s. The dividend yield regressions provide some evidence that the risk premium is nonstationary, but the variations are not very large (1% annual change in the risk premium) so this nonstationarity may be considered as a minor issue. This is due to the following: a) the goal is to estimate long-run future risk premium b) in the long run, the evidence shows that dividend yield tends to oscillate around its mean value and hence, c) long run estimates of the risk premium are not affected by nonstationarity in yields, because the future like the past will be characterised by intervals of low and high dividends. This implies that average yield in the future will be near the average yield in the past, so the average risk premium in the future will also be near its past average. Please note that such argument applies only when the variable used to explain nonstationarity in the risk premium doesn't undergo a permanent change.

1.4.3. Models based on Earnings Yield

Earnings yield is nothing else but the inverse of the price-earnings ratio, hence it is defined as earnings per share divided by the share price. The economic theory suggests that there is a long run relationship between the real return on equity and earnings yield. Earnings can be either paid out as dividends or retained for future investment. Either way, they provide additional value for shareholders. If we denote earnings as $X$, in the similar manner as in the previous section, the expected return from the DCF model will be:

$$E(r) = E(g) + \frac{E(X_1)}{P_0}$$
Hence, by deducting the return on Treasury bills or bonds from the expected return one would obtain the forward-looking estimate of the equity risk premium.

Siegel (1998) found that over the 1926-1997 period, compounded real return on equity was 7.7% and earnings yield was 7.2%. This relationship holds even for longer sample periods. In 1998, in the US, the earnings yield on the S&P 500 Index fell below 4%. If Siegel’s (1998) findings hold, that means that real return on equity should be about 4% over the long run, i.e. the estimates of the risk premium should be reduced by approximately 3.7%.

Sorensen and Arnott (1988) suggest that valuation models based on earnings yield and dividend yield can be used as the effective tool in asset allocation, however, they are subject to errors as any other models. In this study in particular, as an improvement for the valuation models, authors offer the enhancement of the earnings yield approach for predicting equity premium by including in the approach a non-quantitative information. Factors that authors list as non-quantifiable are: currency revaluation, favourable earnings surprises, attitudes of foreign investors and real earnings base.

One of the studies that relates both dividend yield and earnings yield to the equity premium is by Reichenstein and Rich (1993). The paper starts from the thought of the rational market school, which states that dividend yield and earnings-to-price ratio tend to move with the unobservable market risk premium and when the market risk premium is large, the future stock returns will be large. The authors propose a model of the estimate of market risk premium (RP) that is based on the Value Line forecasts
of dividends and capital gains. Starting from the zero-growth dividend discount model, RP model is designed to move with the unobservable market risk premium:

\[
\frac{D_t}{P} = \frac{E_t}{P} = R_f + \text{MRP}
\]

According to this equation, dividend yield and earnings yield should be able to predict long run stock returns. They introduce the third variable that should be able to predict stock market returns, the market risk premium or RP:

\[\text{RP} = (\text{VLYLD} + \text{CapGains}) - R_f\]

Where VLYLD is Value Line’s median dividend yield year ahead and CapGains is median annual capital gain.

The data used for the variables are quarterly values from, 1968:1 through 1989:4. Also, they use S&P 500 stock returns and returns on Treasury bill as the risk free rate. Out of the three variables, RP shows consistent ability to predict long-horizon S&P returns, i.e. it explains around 30% of variation in six-quarter S&P returns. It is superior to earnings yield and dividend yield because a) it reflects capital gains as well as expected dividends and b) it relies on Value Line forecasts rather than historical values. Additionally, authors prove that RP is able to predict long-run stock returns in both in-sample and out-of-sample tests. The reason why the RP model is successful in predicting stock returns supported by the findings of this paper is that RP varies with the expected market risk premium, i.e. it is large when the premium is large and vice versa. Finally, authors comment that RP model can be used in tactical asset allocation and market timing strategies and it is useful as the model of expected risk premium and required rate of return on the market.
1.5. Motivation

There have been series of attempts in the past 1) to model the evolution of the equity premium over time and 2) to relate changes in the equity premium to variables such as dividend yield and earnings. In this chapter we will apply approach of modelling the evolution of equity premium over time to 16 European countries. The rationale behind this choice lies in the fact that there is very little, if any research regarding equity premium completed on the European markets. All of the studies outlined in the previous section are concerned with the evolution of the equity premium in the US market, based on the stock market index itself or sectors within that index. We expand the study by including more European markets in the analysis rather than just the UK due to the integration and globalisation of those markets to assess if a Europe-wide shock on returns, and in turn equity premium, will drive those markets in the same direction. In particular, we assess the changes in equity premium from the point of view of the local investor on a particular market and UK international investor. There is a scope in this context for assessing the European equity premium in the framework of international CAPM. This will be the subject of some future research. The question that needs answering is whether the changes in the equity premium are driven by the same factors across different markets or if there are, at the end of the day, some national economic factors that influence those changes.

Let us now explain the rationale behind the methodology we intend to apply. It has been established in the previous section that the equity premium is related to the volatility of the stock market. It is well known that traditional econometric models assume a constant one-period forecast variance. However, we have also provided extensive evidence from the existing research papers referred to in this chapter and
chapter 2 that stock market volatility is time-variant and autoregressive. Financial forecasters find that their ability to predict future movements of returns varies from one period to another. Therefore, it is important to separate predictable (mean) movements in equity returns from unpredictable (residual) movements. One can use ARCH model to make the variance of the residuals predictable. In other words, in the ARCH model the underlying forecast variance is predicted by past forecast errors. According to Engle (1995), the drawback of the ARCH model appears to be the fact that it is more of a moving average rather than an autoregression specification since the conditional variance is a moving average of squared residuals. Financial data, in particular in our case market volatility is autoregressive, so we introduce in our estimation Generalised ARCH (GARCH) model, whose parameters contribute to both autoregressive and moving average structure. One of the first finance applications of the models was in French, Schwert and Stambaugh (1987). They have used daily data on the S&P 500 index (16,000 observations) to estimate how volatility shocks influence equity premium. In the similar manner we will use ARCH and GARCH models to obtain efficient parameters of our model which should help us determine the relationship between equity premium and stock market volatility in different countries under observation. Therefore, our arguments for using this particular estimation procedure are purely of a statistical nature.

7 The derivation and specifications of ARCH model will be outlined in the Methodology section.
8 The derivation and specifications of GARCH model will be outlined in the Methodology section.
2. Data

The data used for estimation of the equity premium are the monthly returns of the price indices for the 16 European countries and the short-term interest rates for those countries are used as a proxy for the risk free rate. All indices used are price indices and they are expressed in both local currency and pounds for the purpose of our analysis. In particular, the data used is outlined in the following table:

<table>
<thead>
<tr>
<th>Country</th>
<th>Index</th>
<th>Risk-free rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>Total Market Index – Belgium</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Denmark</td>
<td>Total Market Index – Denmark</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Finland</td>
<td>Total Market Index – Finland</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Austria</td>
<td>Total Market Index – Austria</td>
<td>VIBOR 3 months offered rate</td>
</tr>
<tr>
<td>Germany</td>
<td>Total Market Index – Germany</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Greece</td>
<td>Total Market Index – Greece</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Ireland</td>
<td>Total Market Index – Ireland</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Italy</td>
<td>Total Market Index – Italy</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Netherlands</td>
<td>Total Market Index – Netherlands</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Norway</td>
<td>Total Market Index – Norway</td>
<td>Interbank 3 months middle rate</td>
</tr>
<tr>
<td>Portugal</td>
<td>Total Market Index – Portugal</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Sweden</td>
<td>Total Market Index – Sweden</td>
<td>Interbank 3 months middle rate</td>
</tr>
<tr>
<td>Spain</td>
<td>Total Market Index – Spain</td>
<td>Interbank 3 months middle rate</td>
</tr>
<tr>
<td>UK</td>
<td>Total Market Index – UK</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Total Market Index – Switzerland</td>
<td>Interbank 3 months offered rate</td>
</tr>
<tr>
<td>France</td>
<td>Total Market Index – France</td>
<td>Interbank 3 months offered rate</td>
</tr>
</tbody>
</table>

Data is obtained through the Datastream International. Datastream provides the following definition of the total market index data: 'Total market calculations do not
include all companies in a market. Instead, the most important companies by market value are chosen, the precise number of constituents varies from market to market, according to the size of market capitalisation and changes to reflect current market conditions.' We have used the indices provided by Datastream rather than the actual stock market indices because: a) for some of the actual indices the prices were not available for a time period long enough to complete reasonable analysis and b) the dividend yield variable, which is used in the return calculations, was not available for most of the actual indices. The period under observation is different for different indices, depending on the time when the data has become available for them. However, the period of analysis for all indices and risk-free rates ends in April 2000. Therefore, the period under consideration will be outlined for each index separately in the necessary sections. We have completed two types of analysis and hence, two sets of results will be presented throughout the chapter:

- The first one will consider the point of view of the local investor, where we have used the data expressed in the local currency and
- The second one will consider the point of view of the UK (international) investor in which case we have completed the analysis using the data transformed into pounds.

In order to transform the local currency into pounds we have applied the following procedure:
• We have obtained the monthly exchange rate data from the Datastream, in particular, the exchange rate of local currency ($S_t$) to 1 UK pound for the period under consideration.

• We have transformed the price of the market index of each country into pounds using the following formula in each period $t$:

$$\frac{\text{Value of the index in local currency in } t}{S_t/\text{£1}} = \text{Value of the index in pounds in } t$$

• Once the data was transformed, we have proceeded with the estimation procedure, as described in sections 3 and 4.
3. Methodology

3.1. Conditional Volatility Approaches

The original tool for analysing volatility forecasts is the autoregressive conditional heteroscedasticity (ARCH) class of models originally developed by Engle (1982) and then further extended by Bollerslev (1986) as the well known generalised ARCH (GARCH) specification. ARCH model is a way to parameterise the time varying conditional variances observed in any stochastic variable. In order to understand the intuition lying behind these processes, it is important to clarify the differences between conditional and unconditional moments. Let \( R_t \) be the value of an economic variable at time \( t \), and assume that it follows an AR(1) process:

\[
R_t = \gamma R_{t-1} + u_t
\]  

(4.14)

with \( E(u_t) = 0, E(u_t, u_k) = 0 \) \( \forall t \neq k \), and \( E(u_t^2) = \sigma^2 \).

Then at time \( t \), the conditional mean of \( R_t \) is:

\[
E(R_t | \phi_{t-1}) = E_{t-1}(R_t) = \gamma R_{t-1}
\]  

(4.15)

where \( E(\cdot | \phi_{t-1}) \) denotes the conditional mean depending on the information set in period \( t-1 \). In simple terms, the conditional mean is the expected value of a random variable, which is a function of other random variables. The conditional variance depends on past periods information and volatility and is given by:

\[
\text{var}(R_t | \phi_{t-1}) = \text{var}(R_t - E_{t-1}(R_t))^2 = E_{t-1}(u_t^2) = h_t
\]  

(4.16)

The unconditional mean and variance can be defined respectively as:

\[
E(R_t) = 0 \quad \text{Var}(R_t) = \frac{\sigma^2}{1-\gamma^2}
\]  

(4.17)

According to Engle & Bollerslev (1986), "...the success of time series models is attributable to the use of the conditional mean for forecasting rather then the
unconditional mean. Similar gains are available for variances from using more sophisticated models of the conditional variances. The ARCH model is a generalisation of this, in that the conditional variance is also made a function of the past. To make this operational, Engle specifies the conditional variance of the following model as:

\[ R_t = \gamma' X_t + u_t, \quad u_t \sim N(0, h_t) \]  

(4.18)

\[ h_t = E_{t-1}(u_t^2) = a_0 + a_1 u_{t-1}^2 \]

Although the above equation is very specific in terms of the parameters employed, in more general terms the specification of the model can be formulated as follows:

\[ R_t | \phi_{t-1} \sim N(\gamma'X_t, h_t) \]

(4.19)

\[ h_t = f(e_{t-1}, \ldots, e_{t-q}, z) \]

where \( h_t \) is a function of \( q \) past innovations plus any other stochastic exogenous variables \( z \). A specific parameterisation of the ARCH(\( q \)) model with \( q \) time lags is:

\[ h_t = a_0 + \sum_{i=1}^{q} a_i u_{t-i}^2 \]

(4.20)

The ARCH models can be estimated using iterative, non-linear maximum likelihood methods. When conditional heteroscedasticity is present, but not correctly modelled, the parameters from an OLS regression will be unbiased. However, the non-linear ML estimator will produce greater efficiency gains [Engle (1982)]. A natural generalisation is to allow past conditional variances to enter the above equation. This alternative lag structure has been proposed by Bollerslev (1986) as the generalised autoregressive conditional heteroscedasticity model GARCH(\( p, q \)), formulated as:

\[ h_t = a_0 + \sum_{i=1}^{q} a_i u_{t-i}^2 + \sum_{j=1}^{p} b_j h_{t-j} = a_0 + A(L) u_t^2 + B(L) h_t \]

(4.21)

where \( p \geq 0, \ q > 0, \ a_0 > 0, \ a_i \geq 0 \ i = 1, \ldots, q, \ b_j \geq 0 \ j = 1, \ldots, p \)
In the GARCH framework, volatility of returns is essentially parameterised as a function of past volatility and new information arrival. An important element under this approach is the heteroscedastic nature of the residuals in the conditional mean equation. The econometric details of the GARCH model are later provided in the next section.

3.2. Modelling Time Varying Volatility

3.2.1. GARCH models

An alternative class of models is the GARCH models originally developed by Bollerslev (1986). GARCH processes represent the nature of volatility and measure the impact of last period's forecast error and volatility in determining current volatility. The appealing feature for the use of these models is that they are able to capture the time series properties of a mixing variable, based on the hypothesis that financial prices are generated by a mixture of distribution, in which the stochastic mixing variable is the rate of information arrival. In addition, empirical studies such as Engle (1982) indicate that percentage changes while serially uncorrelated are not independent. In particular, large or small changes in financial markets tend to be followed by larger or smaller changes, in either direction, a characteristic of financial data usually called volatility clustering. This suggests that usual measures of volatility are temporally dependent (heteroscedastic). Consequently, a meaningful comparison of volatility across different time periods can only be made if the analysis controls for this time dependence. Without this control, researchers can not be certain that observed differences are not simply an incidental result of the temporal dependence. This persistence and any propensity of changes of like magnitude to
cluster in time is captured by the GARCH models. The econometric estimation of the proposed model is presented in the following section.

Nonetheless, like any other econometric process these models do suffer from certain drawbacks. In general, conditional volatility models impose an ARMA structure on conditional variance, allowing volatility shocks to persist over time. That is, conditional volatility applications in returns may exhibit an IGARCH representation \((\hat{a}_1 + \hat{b}_1 \approx 1)\), implying that volatility is affected by current shocks infinitely.

3.2.2. Estimation of the GARCH Model

Because the variance of \( u_t \) depends upon the unobservable past values of \( u_t \), the \( h \) function, and hence the likelihood function, has to be generated recursively. The average log likelihood function for a sample of \( T \) observations, excluding a constant term, is:

\[
L_{\overline{t}} = T^{-1} \sum_{t=1}^{T} l_{\ell_t}
\]

(4.22)

Where the log likelihood term for each entry \( t \) takes the form:

\[
l_{\ell_t} = -\frac{1}{2} [\ln h_t + (R_t - X_t' \gamma) h_t^{-1}]\]

(4.23)

Differentiating with respect to the variance parameter \( w' = (a_0, a_1, ..., a_q, b_1, ..., b_p, \delta) \), we obtain:

\[
\frac{\partial l_{\ell_t}}{\partial w} = \frac{1}{2} h_t^{-1} \frac{\partial h_t}{\partial w} \left( \frac{u_t^2}{h_t} - 1 \right)
\]

(4.24)

\[
\frac{\partial^2 l_{\ell_t}}{\partial w \partial w'} = \left( \frac{u_t^2}{h_t} - 1 \right) \frac{\partial}{\partial w'} \left( \frac{1}{2} h_t^{-1} \frac{\partial h_t}{\partial w} \right) - \frac{1}{2} h_t^{-2} \frac{\partial h_t}{\partial w} \frac{\partial h_t}{\partial w'} \frac{u_t^2}{h_t}
\]

(4.25)
where

\[
\frac{\partial h_t}{\partial w} = v_t + \sum_{i=1}^{q} b_i \frac{\partial h_{t-i}}{\partial w} \tag{4.26}
\]

\[
h_t = w v' \quad v' = (1, L^1 u_i^2, \ldots, L^q u_i^2, L^1 h_i, \ldots, L^p h_i, g)
\]

Going back to Engle (1982) it is obvious the inclusion of the recursive part in equation (4.26). Since it is necessary for the recursive estimation to have pre-sample values \((t \leq 0)\) for \(h_t\) and \(u_t\), it is simple to obtain \(T^{-1} \sum_{i=1}^{T} u_i^2\).

The differentiation with respect to the mean parameters yields:

\[
\frac{\partial \ell_t}{\partial \gamma} = u_t X_t h_t^{-1} + \frac{1}{2} h_t \frac{\partial h_t}{\partial \gamma} \left( \frac{u_t^2}{h_t} - 1 \right) \tag{4.27}
\]

\[
\frac{\partial^2 \ell_t}{\partial \gamma \partial \gamma'} = -h_t^{-1} X_t X_t' - \frac{1}{2} h_t^{-2} \frac{\partial h_t}{\partial \gamma} \frac{\partial h_t}{\partial \gamma'} \left( \frac{u_t^2}{h_t} \right) - 2h_t^{-2} u_t X_t \frac{\partial h_t}{\partial \gamma} + \left( \frac{u_t^2}{h_t} - 1 \right) \frac{\partial}{\partial \gamma'} \left( \frac{1}{2} h_t^{-1} \frac{\partial h_t}{\partial \gamma} \right) \tag{4.28}
\]

where

\[
\frac{\partial h_t}{\partial \gamma} = -2 \sum_{i=1}^{q} a_i X_{t-i} u_{t-i} + \sum_{j=1}^{p} b_j \frac{\partial h_{t-j}}{\partial b} \tag{4.29}
\]

Again the single difference with the simple ARCH(q) regression model is the inclusion of the recursive part in equation (4.29). An iterative procedure will be used to obtain maximum likelihood estimates, and second order efficiency. Let \(s^i\) denote the parameter estimates after the \(i\) iteration. Successive values of these parameters are estimated as follows:

\[
s^{i+1}_t = s^i + c_i \left( \sum_{i=1}^{T} \frac{\partial \ell_t}{\partial s} \frac{\partial \ell_t}{\partial s^i} \right)^{-1} \sum_{i=1}^{T} \frac{\partial \ell_t}{\partial s} \tag{4.30}
\]

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The $c_i$ is a variable step length chosen to maximise the likelihood function and the \( \partial \ell_i / \partial s \) is evaluated at $s^i$. A detailed analysis of the aforementioned algorithm method is beyond the scope of this research.
4. Analysis of the Results

4.1. Estimates of the historical equity premium

Tables 6a and 6b below represent the estimates of the average returns on stocks, short-term interest rates and equity premium in the local currency and in pounds respectively, in each of the 16 countries under examination. The estimation periods for each country are different and they depend on the interest rate or index data availability. Hence, period of equity premium estimation will be outlined for each country separately. The compound equity index returns are calculated as:

\[ R_t = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right) \]

The compound return on the risk free rate is calculated as:

\[ R_{ft} = \ln(1 + R_{ft}) \]

And, finally, the equity premium is calculated as the difference between equity returns and returns on the risk free rate:

\[ EP = R_t - R_{ft} \]

These equity premiums are derived using arithmetic return averages, as generally accepted in the capital asset pricing model.
Table 6a: Compound annual returns and equity premium (local currency)

Values in the parenthesis are annual standard deviations in percentages

<table>
<thead>
<tr>
<th>Country</th>
<th>Estimation period</th>
<th>Stock returns</th>
<th>Interest rates</th>
<th>Equity premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>Oct 89-Apr 00</td>
<td>8.96 (55.23)</td>
<td>5.90 (2.62)</td>
<td>3.06 (55.65)</td>
</tr>
<tr>
<td>Denmark</td>
<td>Jun 88-Apr 00</td>
<td>16.31 (57.66)</td>
<td>6.98 (3.02)</td>
<td>9.33 (58.06)</td>
</tr>
<tr>
<td>Finland</td>
<td>Feb 87-Apr 00</td>
<td>24.04 (98.53)</td>
<td>7.43 (4.05)</td>
<td>16.61 (99.86)</td>
</tr>
<tr>
<td>Austria</td>
<td>Jun 91-Apr 00</td>
<td>0.84 (59.30)</td>
<td>5.03 (2.19)</td>
<td>-4.18 (59.64)</td>
</tr>
<tr>
<td>Germany</td>
<td>Jan 86-Apr 00</td>
<td>11.22 (66.72)</td>
<td>5.34 (2.15)</td>
<td>5.87 (66.91)</td>
</tr>
<tr>
<td>Greece</td>
<td>Apr 94-Apr 00</td>
<td>29.69 (104.17)</td>
<td>13.67 (6.84)</td>
<td>16.03 (106.65)</td>
</tr>
<tr>
<td>Ireland</td>
<td>Jan 84-Apr 00</td>
<td>19.05 (76.96)</td>
<td>8.32 (3.26)</td>
<td>10.73 (76.94)</td>
</tr>
<tr>
<td>Italy</td>
<td>Apr 88-Apr 00</td>
<td>12.44 (81.46)</td>
<td>9.03 (3.23)</td>
<td>3.41 (82.00)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>Jan 79-Apr 00</td>
<td>17.85 (55.20)</td>
<td>6.31 (2.49)</td>
<td>11.54 (55.58)</td>
</tr>
<tr>
<td>Norway</td>
<td>Jan 86-Apr 00</td>
<td>13.78 (86.97)</td>
<td>9.01 (3.76)</td>
<td>4.77 (87.25)</td>
</tr>
<tr>
<td>Portugal</td>
<td>Feb 93-Apr 00</td>
<td>20.87 (69.88)</td>
<td>7.24 (3.40)</td>
<td>13.63 (70.03)</td>
</tr>
<tr>
<td>Sweden</td>
<td>Mar 91-Apr 00</td>
<td>23.30 (79.54)</td>
<td>7.03 (3.07)</td>
<td>16.27 (80.26)</td>
</tr>
<tr>
<td>Spain</td>
<td>Dec 91-Apr 00</td>
<td>20.66 (70.08)</td>
<td>7.38 (3.28)</td>
<td>13.28 (70.38)</td>
</tr>
<tr>
<td>UK</td>
<td>Jan 75-Apr 00</td>
<td>20.05 (68.42)</td>
<td>9.66 (2.99)</td>
<td>10.39 (68.45)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Aug 88-Apr 00</td>
<td>11.64 (55.37)</td>
<td>4.25 (2.57)</td>
<td>7.39 (55.78)</td>
</tr>
<tr>
<td>France</td>
<td>Oct 87-Apr 00</td>
<td>15.59 (69.64)</td>
<td>6.59 (2.72)</td>
<td>9.00 (70.04)</td>
</tr>
</tbody>
</table>

We can see the variety of values of equity premium across different countries under the observation periods. One can argue that the difference in equity premiums among countries stems from the difference in currency or the difference in time periods used in the estimation procedure. It is easy to examine the impact of the currency fluctuations on the level of equity premium. Specifically, we are presenting the estimates of the equity premium for 16 European countries expressed in sterling (£) in the table 6b.
Table 6b: Compound annual returns and equity premium (£)

Values in the parenthesis are annual standard deviations in percentages

<table>
<thead>
<tr>
<th>Country</th>
<th>Estimation period</th>
<th>Stock returns</th>
<th>Interest rates</th>
<th>Equity premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>Oct 89-Apr 00</td>
<td>8.13 (57.83)</td>
<td>5.90 (2.62)</td>
<td>2.23 (57.84)</td>
</tr>
<tr>
<td>Denmark</td>
<td>Jun 88-Apr 00</td>
<td>15.83 (57.05)</td>
<td>6.98 (3.02)</td>
<td>8.85 (57.10)</td>
</tr>
<tr>
<td>Finland</td>
<td>Feb 87-Apr 00</td>
<td>21.52 (100.04)</td>
<td>7.43 (4.05)</td>
<td>14.09 (101.28)</td>
</tr>
<tr>
<td>Austria</td>
<td>Jun 91-Apr 00</td>
<td>-0.64 (62.37)</td>
<td>5.03 (2.19)</td>
<td>-7.34 (62.57)</td>
</tr>
<tr>
<td>Germany</td>
<td>Jan 86-Apr 00</td>
<td>11.57 (67.94)</td>
<td>5.34 (2.15)</td>
<td>6.23 (67.96)</td>
</tr>
<tr>
<td>Greece</td>
<td>Apr 94-Apr 00</td>
<td>22.20 (108.28)</td>
<td>13.67 (6.84)</td>
<td>8.53 (110.48)</td>
</tr>
<tr>
<td>Ireland</td>
<td>Jan 84-Apr 00</td>
<td>18.69 (78.22)</td>
<td>8.32 (3.26)</td>
<td>10.37 (78.02)</td>
</tr>
<tr>
<td>Italy</td>
<td>Apr 88-Apr 00</td>
<td>9.46 (88.52)</td>
<td>9.03 (3.23)</td>
<td>0.43 (88.87)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>Jan 79-Apr 00</td>
<td>18.14 (57.31)</td>
<td>6.31 (2.49)</td>
<td>11.83 (57.56)</td>
</tr>
<tr>
<td>Norway</td>
<td>Jan 86-Apr 00</td>
<td>12.07 (91.28)</td>
<td>9.01 (3.76)</td>
<td>3.07 (91.45)</td>
</tr>
<tr>
<td>Portugal</td>
<td>Feb 93-Apr 00</td>
<td>14.48 (72.51)</td>
<td>7.24 (3.40)</td>
<td>7.25 (72.68)</td>
</tr>
<tr>
<td>Sweden</td>
<td>Mar 91-Apr 00</td>
<td>20.49 (79.20)</td>
<td>7.03 (3.07)</td>
<td>13.46 (79.73)</td>
</tr>
<tr>
<td>Spain</td>
<td>Dec 91-Apr 00</td>
<td>15.35 (74.47)</td>
<td>7.38 (3.28)</td>
<td>7.97 (74.67)</td>
</tr>
<tr>
<td>UK</td>
<td>Jan 75-Apr 00</td>
<td>20.05 (68.42)</td>
<td>9.66 (2.99)</td>
<td>10.39 (68.45)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Aug 88-Apr 00</td>
<td>11.54 (59.60)</td>
<td>4.25 (2.57)</td>
<td>11.29 (59.84)</td>
</tr>
<tr>
<td>France</td>
<td>Oct 87-Apr 00</td>
<td>14.63 (70.29)</td>
<td>6.59 (2.72)</td>
<td>8.03 (70.42)</td>
</tr>
</tbody>
</table>

The results in the table 6b suggest that even after adjusting for different currencies, the variations in equity premium across countries still persist.

Let us examine now what is the impact of the length of the estimation period on the level of equity premium in different countries. If we take as an example from tables 6a and 6b Sweden and Austria, we can see that although the estimation period for equity premium is approximately the same, we obtain two of the most extreme values of equity premium (Austria is the only country in our analysis with the negative...
equity premium, while Sweden is one of the countries with the highest equity premiums).

Furthermore, let us discuss the standard deviations of annualised returns, interest rates and equity premiums given in parenthesis of tables 6a and 6b. It is clear that the volatility of the equity premium depends on the volatility of stock returns rather than volatility of interest rates in all countries in the analysis. This is consistent with the findings of Cornell (1999) who reports that volatility of equity premium in the US market depends on the volatility of US annual stock market returns and is around 20% in the period 1926-1997. Therefore, we are doomed to think that the difference in the level of equity premium in different countries depends more on the difference between the average returns on the equity market in the given time period, rather than the difference in the level of interest rates between two countries.

However, one conclusion can be drawn from that additional analysis. In particular, the level of interest rates in the European countries has fallen in the 1990s and the level of the stock market has risen. The interest rates are falling within the range of 2.09% and 6.31% across Europe, if we exclude Greek interest rate of 13.67% as an outlier, whereas the stock market volatility is somewhat more in tact. In addition, the level of the stock market returns is very high in 1990s in all countries with an exception of Austria. Hence, the high level of the equity premium in 1990s is due to increasing equity returns and decreasing risk free rates.
4.2. Econometric results

Therefore, the returns and risk-free rates used in the analysis that follows are compounded returns and they are calculated using formulae as described in the section 4.1.

For the purposes of having initial point of comparison, the OLS model of the form as described in the equation (4.31) below was estimated:

\[ R_{it} = \alpha + \beta R_{it-1} \]  

(4.31)

where \( R_{it} \) and \( R_{it-1} \) are the return and the lagged return on the market index \( i \) in time period \( t \) and \( t-1 \) respectively and \( \alpha \) and \( \beta \) are coefficients.

Originally, we have run the model of a slightly different form:

\[ R_{it} = \alpha + \beta R_{it-1} + \gamma D_{Y_{it-1}} \]  

(4.32)

where \( R_{it} \) and \( R_{it-1} \) are the return and the lagged return on the market index \( i \) in time period \( t \) and \( t-1 \) respectively and \( D_{Y_{it-1}} \) is the dividend yield on the market index \( i \) in time period \( t-1 \). \( \alpha, \beta \) and \( \gamma \) are coefficients.

The model of this form was originally chosen because it is believed that a) changes in equity premium are driven by nonstationarity in equity returns and b) returns on equity (and, in turn, the equity premium) in time period \( t \) depends on the level of dividend yield in time period \( t-1 \). However, the results when applying this model are showing that dividend yield coefficient in all countries (except UK) was not significant, so we have dropped it out of the model and continued our estimation of equation 4.31.
Additionally, for the reasons we have given in section 1.5 of this chapter, we have tested the presence of the ARCH effect in the model, i.e. we have examined whether the conditional variance of the above model is changing as a function of time.

4.2.1. Serial correlation in returns and impact on the equity premium: The case of the European Domestic investor vs. UK International investor

The results of the OLS model and the ARCH test are presented in summary form in Tables 7a and 7b.
Table 7a: The OLS estimates of the $R_t = \alpha + \beta R_{t-1}$ model and the results of the ARCH test for 16 European countries (local currency)

<table>
<thead>
<tr>
<th>Country</th>
<th>OLS estimates</th>
<th>ARCH test</th>
<th>ML estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\chi^2$</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.008*</td>
<td>0.11*</td>
<td>13.16*</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.010*</td>
<td>0.15*</td>
<td>7.90*</td>
</tr>
<tr>
<td>Finland</td>
<td>0.015*</td>
<td>0.24*</td>
<td>No ARCH</td>
</tr>
<tr>
<td>Austria</td>
<td>0.004</td>
<td>0.25*</td>
<td>42.18*</td>
</tr>
<tr>
<td>Germany</td>
<td>0.008*</td>
<td>0.054</td>
<td>13.13*</td>
</tr>
<tr>
<td>Greece</td>
<td>0.018**</td>
<td>0.17**</td>
<td>7.03*</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.011*</td>
<td>0.17*</td>
<td>7.71**</td>
</tr>
<tr>
<td>Italy</td>
<td>0.011*</td>
<td>0.079</td>
<td>21.56*</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.011*</td>
<td>0.087</td>
<td>5.28*</td>
</tr>
<tr>
<td>Norway</td>
<td>0.009*</td>
<td>0.12*</td>
<td>3.56**</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.007</td>
<td>0.20*</td>
<td>15.65*</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.015*</td>
<td>0.17*</td>
<td>2.70**</td>
</tr>
<tr>
<td>Spain</td>
<td>0.012*</td>
<td>0.12</td>
<td>No ARCH</td>
</tr>
<tr>
<td>UK</td>
<td>0.012*</td>
<td>0.093**</td>
<td>4.82*</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.007*</td>
<td>0.15*</td>
<td>6.27*</td>
</tr>
<tr>
<td>France</td>
<td>0.011*</td>
<td>0.08</td>
<td>22.08*</td>
</tr>
</tbody>
</table>

Note: * is denoting 5% significance or less, and ** is denoting 10% significance

Looking at the significance of lagged returns coefficients from the Table 7a, we can see that the lagged returns explain quite well the dependent variable (returns on the stock market index) in most of the countries under observation. Particularly, the lagged returns coefficient ($\beta$) from the simple OLS estimation is found to be significant in most of the countries with the exception of Spain, Italy, Netherlands,
Germany and France when returns are expressed in the local currency. This implies that prices do not follow a random walk in all other countries apart from the aforementioned ones, i.e. that the markets are not efficient in most of the European countries. Therefore, unconditional, long-run parameter estimates suggest that serial correlation of equity returns is important in most of the countries.

Nevertheless, most of the asset holders are interested in the forecasts of the rate of return and its variance over the holding period, so the unconditional estimates would be unimportant if you plan to buy the asset at time $t$ and sell it at time $t+1$. Hence, we have to see if our conclusions change when we obtain conditional estimates.

As noted earlier, the OLS estimates from table 7a and 7b may be biased due to the fact that time series financial data in the model as in (4.31) may have unequal variance, i.e. periods of unusually high volatility may be followed by periods of relative tranquillity. We have used the plot of Cumulative Sum of Squares of Recursive Residuals (CUSUMSQ) as a test of heteroskedasticity in the data. CUSUMSQ is a test of structural stability and it was rejected for most of the countries under analysis, indicating that data has unequal variance. Since the test is presented in the graphical way, to save the space, results are not presented in this chapter. Additionally, for a few countries, the results of the test were somewhat inconclusive at the 5% significance level. Therefore, we can say that our model in (4.31) is misspecified and we have to model the error term.

Hence, we have extended our analysis with the ARCH test in order to examine the presence of heteroskedasticity in the data in a more formal way. Using this method, it
is possible to simultaneously model the mean and the variance of the series. The results of the \( \chi^2 \) and F-test in the table 7a test suggest that there are significant ARCH effect in all but two countries. Hence, in the GARCH estimation that follows, Spain and Finland will be excluded. When we re-estimate \( \alpha \) and \( \beta \) coefficients from equation (4.31) with ML Cochrane-Orcutt estimation procedure that takes into account the presence of the ARCH effects in the data, the significance of the lagged return variable coefficient changes. The Cochrane-Orcutt estimation is chosen because the estimated standard errors computed under this procedure are valid even if the regression equation contains lagged values of the dependent variable, as it is in our case. In particular, the results from the last two columns of table 7a suggest that the lagged return is of importance for determining current returns only in Denmark, Portugal and Sweden. In effect, our results imply that serial correlation in all but above named countries is not actually in the level of returns but in the volatility of returns.

Let us see now if the results change if we take into account UK international investor who is considering all the returns in the home currency – sterling, as in the table 7b.
Table 7b: The OLS estimates of the $R_u = \alpha + \beta R_{u-1}$ model and the results of the ARCH test for 16 European countries (pound sterling)

<table>
<thead>
<tr>
<th>Country</th>
<th>OLS estimates</th>
<th>ARCH test</th>
<th>ML estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\chi^2$</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.010*</td>
<td>0.06</td>
<td>8.49**</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.011*</td>
<td>0.089</td>
<td>14.35*</td>
</tr>
<tr>
<td>Finland</td>
<td>0.015*</td>
<td>0.19*</td>
<td>No ARCH</td>
</tr>
<tr>
<td>Austria</td>
<td>0.007*</td>
<td>0.16*</td>
<td>25.40*</td>
</tr>
<tr>
<td>Germany</td>
<td>0.011*</td>
<td>0.02</td>
<td>7.97*</td>
</tr>
<tr>
<td>Greece</td>
<td>0.014</td>
<td>0.09</td>
<td>14.88*</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.011*</td>
<td>0.13*</td>
<td>No ARCH</td>
</tr>
<tr>
<td>Italy</td>
<td>0.009*</td>
<td>0.039</td>
<td>3.21**</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.014*</td>
<td>0.02</td>
<td>6.87*</td>
</tr>
<tr>
<td>Norway</td>
<td>0.009**</td>
<td>0.089</td>
<td>3.91*</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.007</td>
<td>0.053</td>
<td>7.29*</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.015*</td>
<td>0.099</td>
<td>7.30*</td>
</tr>
<tr>
<td>Spain</td>
<td>0.010*</td>
<td>0.040</td>
<td>No ARCH</td>
</tr>
<tr>
<td>UK</td>
<td>0.012*</td>
<td>0.093**</td>
<td>4.82*</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.010*</td>
<td>0.11*</td>
<td>3.72**</td>
</tr>
<tr>
<td>France</td>
<td>0.011*</td>
<td>0.096**</td>
<td>10.34*</td>
</tr>
</tbody>
</table>

Note: * is denoting 5% significance or less, and ** is denoting 10% significance

The table 7b shows the results of the same procedures as explained before for table 7a, that is the OLS estimates, the ARCH test and the ML estimates when all stock market returns are converted into home currency of the UK international investor. The lagged return coefficient ($\beta$) is insignificant in most of the countries, the ARCH effects are present in all countries but Finland, Spain and Ireland and the ML
estimation suggests that inefficient markets in Europe appear to be Austria, Denmark, Greece and Sweden. Again, serial correlation in the level of returns persists only in the aforementioned countries, while in all the remaining countries in the analysis we find evidence of serial correlation in the volatility of returns.

To extend our analysis and to estimate GARCH-based time-varying parameters of our model, which would contribute to both moving average and autoregressive structure of volatility, it is necessary to fit GARCH models to the returns data for each of the 14 countries remaining in the local currency analysis and 13 countries in the sterling analysis. The details of these GARCH models are presented in Table 8a and 8b. Parameters estimated are obtained from the estimation of the equation (4.21), which in case of GARCH (1,1) takes the following form:

\[ h_t = a_0 + a_1 u_{t-1}^2 + b_1 h_{t-1}^2 \]  

(4.33)

We have succeeded in modelling equity returns with GARCH (1,1) model in most of the cases. However, due to non-convergence of GARCH (1,1) model for some of the countries in tables 8a and 8b, we have fitted GARCH (0,1) model.
Table 8a: GARCH (1,1) and GARCH (0,1) model estimation for 14 European countries (local currency)

<table>
<thead>
<tr>
<th>Country</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$a_1 + b_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>0.003</td>
<td>0.069*</td>
<td>-0.383*</td>
<td>-0.314</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.00038*</td>
<td>0.093*</td>
<td>0.758*</td>
<td>0.851</td>
</tr>
<tr>
<td>Austria</td>
<td>0.00093*</td>
<td>1.14*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>0.00044*</td>
<td>0.10*</td>
<td>0.88*</td>
<td>0.98</td>
</tr>
<tr>
<td>Greece</td>
<td>0.011*</td>
<td>-0.023</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.00057*</td>
<td>0.15*</td>
<td>0.73*</td>
<td>0.88</td>
</tr>
<tr>
<td>Italy</td>
<td>0.0011*</td>
<td>0.12*</td>
<td>0.68*</td>
<td>0.80</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.0019*</td>
<td>0.087</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Norway</td>
<td>0.0097*</td>
<td>0.025</td>
<td>-0.98*</td>
<td>-0.955</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.00055*</td>
<td>0.24**</td>
<td>0.58*</td>
<td>0.82</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.0017*</td>
<td>0.13</td>
<td>0.49*</td>
<td>0.63</td>
</tr>
<tr>
<td>UK</td>
<td>0.00036*</td>
<td>0.18*</td>
<td>0.73*</td>
<td>0.91</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.0017*</td>
<td>0.47*</td>
<td>-0.16*</td>
<td>0.31</td>
</tr>
<tr>
<td>France</td>
<td>0.00085*</td>
<td>0.12*</td>
<td>0.65*</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Note: * is denoting 5% significance or less, and ** is denoting 10% significance
Table 8b: GARCH (1,1) and GARCH (0,1) model estimation for 13 European countries (pound sterling)

<table>
<thead>
<tr>
<th>Country</th>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(b_1)</th>
<th>(a_1 + b_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>0.0004*</td>
<td>0.039</td>
<td>0.82*</td>
<td>0.859</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.0002*</td>
<td>0.086*</td>
<td>0.85*</td>
<td>0.936</td>
</tr>
<tr>
<td>Austria</td>
<td>0.0003*</td>
<td>0.23*</td>
<td>0.70*</td>
<td>0.93</td>
</tr>
<tr>
<td>Germany</td>
<td>0.0003*</td>
<td>0.10*</td>
<td>0.79*</td>
<td>0.89</td>
</tr>
<tr>
<td>Greece</td>
<td>0.005*</td>
<td>0.11*</td>
<td>0.83*</td>
<td>0.94</td>
</tr>
<tr>
<td>Italy</td>
<td>0.002**</td>
<td>0.14**</td>
<td>0.44**</td>
<td>0.58</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.0003*</td>
<td>0.071**</td>
<td>0.79*</td>
<td>0.861</td>
</tr>
<tr>
<td>Norway</td>
<td>0.0055*</td>
<td>0.044</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.0006**</td>
<td>0.16**</td>
<td>0.64*</td>
<td>0.80</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.0026*</td>
<td>0.15**</td>
<td>0.29</td>
<td>0.44</td>
</tr>
<tr>
<td>UK</td>
<td>0.00036*</td>
<td>0.18*</td>
<td>0.73*</td>
<td>0.91</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.0021*</td>
<td>0.15*</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>France</td>
<td>0.00083*</td>
<td>0.14*</td>
<td>0.67*</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Note: * is denoting 5% significance or less, and ** is denoting 10% significance

The estimates of the GARCH (1,1) model in tables 8a and 8b are statistically significant in most of the cases and indicate that the variance of monthly returns is highly autocorrelated. To compare the persistence implied by the GARCH model with the ARCH model it is useful to consider the sum of \(a_1\) and \(b_1\) parameters, which must be less than 1.00 for the volatility process to be stationary. Examining these results one can see that the majority of the GARCH models are robust in that the ARCH and GARCH parameters in both tables 8a and 8b are generally statistically significant, sum to less than one and satisfy the positivity assumption outlined in section 3.1.1. The only countries that do not meet the positivity assumption in table 8a are Belgium,
Greece, Switzerland and Norway. When the returns are converted into pounds, ARCH and GARCH parameters for all of the countries under analysis satisfy the positivity assumption and sum to less than one. The estimates $a_1$ of GARCH (0,1) model in both tables indicate that the relationship between recent squared errors and the estimate of volatility is not that strong in the countries where GARCH (1,1) model could not be estimated. For those countries, we can suggest that some other (e.g. macroeconomic) factors should be specified to be determined in the model which should determine better stock market returns and in turn equity premium.

Additionally, it should be mentioned that during the experimentation stage we have attempted to estimate alternative specifications of the GARCH family of models. In particular, we have examined if there is GARCH effect in mean and E-GARCH in our estimated model. However, the results were suggesting that there was no GARCH effect in mean or exponential GARCH effect in any of the countries under examination, so the results are not reported in this chapter.

To conclude, in this chapter we find that equity premium is related to the stock market volatility in most of the European countries. ARCH and GARCH model estimates suggest that stock market volatility is serially correlated and that shocks to stock market returns, and in turn equity premium are persistent. Therefore, there can be a scenario where there will be a world-wide shock in the equity markets that may affect the equity premium across countries in the same way, driving the equity premium in the same direction.
5. Conclusions

In this chapter, we analyse the impact of stock market volatility on equity premium in 16 European countries. We consider our results from the point of view of the local investor in each country and from the point of view of the UK international investor. Descriptive statistics results suggest that standard deviation of the equity premium is almost the same as the standard deviation of stock returns rather than standard deviation of interest rates in all countries in the analysis. We base our main findings of this chapter on the serial correlation in returns model estimated through OLS, ARCH and GARCH procedure. OLS results suggest there is serial correlation in the level of returns in most of the countries. However, since we find that our data is heteroskedastic, we identify the ARCH effect in our model and re-estimate it using Maximum Likelihood Cochrane-Orcutt procedure. The results from this procedure show that serial correlation is not in the level of returns but in the volatility of returns in both cases of the local and UK international investor. Additionally, we find that the GARCH models are robust in majority of the countries in that the ARCH and GARCH parameters are generally statistically significant, sum to less than one and satisfy the positivity assumption (where applicable). These results from ARCH and GARCH models imply that volatility in majority of European markets is serially correlated and that shocks to equity market returns, and in turn equity premium, are persistent. We identify several countries, namely Greece and Netherlands (in the local investor analysis) and Norway (in the pound sterling analysis), where our model parameters are not significant implying that there might be factors different than stock market volatility that is driving equity premium in those countries.
SUMMARY OF CONCLUSIONS

This Ph.D. thesis is investigating equity investment styles that can be derived from the determinants of equity returns as defined in the CAPM model. In particular, we have looked at statistical analysis of alphas and betas, structural analysis of alphas using dividend yield and finally, we have analysed equity premium in the European markets. In this section we present the summary of our findings.

In chapter one we analyse the performance and persistence of performance of positive alpha and negative alpha portfolios. Positive and negative alpha portfolios were based on past unconditional alpha of individual stocks in the UK market in the period January 1980-December 1996. We have tested if and how past information about alphas, information ratio or alpha/beta ratio determines future returns. The main findings suggest that the predictive ability of past alphas in the short-term horizons is non-existent for both investors investing in positive and negative alpha portfolios. However, it appears that past information ratio has predictive ability for positive alpha portfolios after two years. In particular, finding an inverse relationship between positive past alphas and returns indicates that positive past alpha portfolios generate negative future returns. For negative alpha portfolios we also find significant negative relationship between past alphas and future returns when the investment horizon is 3 years, i.e. portfolios with relatively low prior alphas will tend to have relatively high future returns. Similar mean reversion in stock portfolios is noted in Fama and French (1988b). Similar results are found when persistence of performance is measured using alpha/beta ratio. As noted before, this analysis used unconditional alpha estimates. However, using conditional alphas will give more powerful signals of performance.
persistence or behaviour of our portfolios in the future in general. This could be a subject for future research.

In chapter two, we examine the time varying beta coefficients and seasonality effect in the UK. One of the main empirical problems in testing the CAPM is that it is assumed that the unconditional alpha and beta estimates from the OLS regression model are constant over time. However, in the reality alphas and betas are dynamic, hence unconditional alphas and betas exclude important information for investor. We have divided our sample of stocks into 10 beta portfolios in order to test whether stocks with higher beta also exhibit higher instability in betas as well, causing even greater risk for investors. Using the simple form of the Kalman filter methodology according to which alphas and betas evolve as random variables, we find that for portfolios that have greater level of systematic risk, the risk is also more significant. Furthermore, we have identified the presence of the seasonality effect in the UK size stock portfolio returns, which differs from the traditional January or April effect. In particular, we find that size sorted portfolio returns are higher in December-April period than in May- November period. We confirm the dynamics of beta for size sorted portfolios as well: betas of smaller portfolios are more time varying than betas of distinctively large portfolios. Furthermore, we find that returns pattern observed in the UK is not a consequence of the tax-loss selling hypothesis or beta seasonality. In particular, betas across all size portfolios are lower in the period May-November, but the results are not statistically significant. The only explanation that we might offer for higher stock returns in December-April period is related to investors' behaviour that is influencing trading volume during those months. In particular, at the turn of the year (December- January), a boost in trading volume can be liquidity based (New
Year salary bonuses) or related to the corporate information releases, whereas towards the end of financial year in the UK (March-April), volume increase may be because investors want to realise their capital losses.

In chapter three, we examined the presence of the yield effect in the UK. We find that in the period from January 1980 through to December 1996, yield-return relationship is a quadratic one, forming a bell shaped curve. This is contradicting some of the US and UK yield effect evidence which covers earlier time periods and suggests U-shaped, negative, yield-return relationship. We believe that the source of this discrepancy lies in the change of the return behaviour of the small-cap companies in the 1990s. It was noted that the lowest and the largest yield portfolios are mainly composed from the small-cap stocks. By splitting the sample into two subperiods, 1980s and 1990s, we have investigated whether return pattern of small size stocks has influenced the shape of the yield-return relationship in the UK. We find that yield coefficient remains constant throughout the whole sample period. The market value coefficient in the cross-sectional regression analysis has changed from the negative one in the 80s, implying that small stocks earn higher returns than larger ones, to the positive coefficient in the 90s, implying that small stocks started underperforming larger ones. We believe that this has caused the yield-return relationship to change from the negative, U-shaped one in the 1980s into positive, bell shaped one in the 1990s. We have ruled out the tax hypothesis as the explanation of the yield effect since the UK imputation tax system suggests that the yield-return relationship should be linear and negative, which is clearly not what we have found. Additionally, we note that beta has negative relationship with returns in the 1990s, implying that it no
longer represents the adequate measure of portfolio risk. However, this should be the subject of a different research.

In chapter four we analyse the equity premium in 16 European countries. We consider our results from the point of view of the local investor in each country and from the point of view of the UK international investor. From the descriptive statistics that shows average annualised stock market returns, risk free rates, equity premiums and their respective standard deviations, we find that the standard deviation of equity premium depends on the standard deviation of stock returns rather than standard deviation of interest rates in all countries in the analysis. Hence, we believe that modelling equity returns will in turn be equivalent to modelling equity premium in those markets. Using a simple serial correlation model, we test if the stock market returns in the current time period (t) depend on the stock market returns from the previous time period (t-1). OLS model estimates suggest that long term, unconditional estimates indicate that serial correlation of stock market returns does have an impact on the long run equity premium. After the ARCH effect in the model is identified and the model is re-estimated using Maximum Likelihood Cochrane-Orcutt procedure, we find that serial correlation coefficient is insignificant in most of the countries and that ARCH model is significant. In particular, these results suggest that there is serial correlation in the volatility of returns rather than returns themselves. Once the GARCH modelling is applied, we find that the majority of the GARCH models are robust in that the ARCH and GARCH parameters are generally statistically significant, sum to less than one and satisfy the positivity assumption (where applicable). These results confirm serial correlation in volatility of returns in most of the countries and imply that the shocks to returns, and in turn equity
premium, are persistent. If there is a scenario according to which there will be a world-wide shock, it will influence the equity premium across countries in the similar manner driving equity premiums in different markets in the same direction.

The above findings have implications for both a) investment strategies and b) asset pricing models and their empirical modelling. Let us examine each set of implications in turn.

As far as investment strategies are concerned, findings from chapter one suggest that investor going long in the positive alpha portfolio should liquidate it after 24 months. There is no such a clear-cut strategy for investors investing in negative alpha portfolio since the results are not that robust even after 36 months. A profitable strategy in such a case would be for investors to create long/short portfolio where they will buy positive alpha portfolio and short-sell negative alpha portfolio, thus earning two alphas plus the interest (earned on the amount the short portfolio was sold for net of the expenses for the prime broker, transaction costs etc.). Furthermore, we find that betas are more significant for higher beta portfolio owner and also time-varying across all beta portfolios. Additionally, as far as seasonality in stock returns is concerned, regardless of the market value (size) of the portfolio, investors should be out of the equity market in the period May-November, since it has been shown that returns in that period are systematically lower over the years. Higher returns could be generated in December-April period, reasons being predominantly related to the behaviour of investors and psychological effects. Findings from Chapter three suggest that in the 1990s profitable strategy for investors would be to invest in medium yield and larger size companies since those would be able to generate the highest returns.
Finally, a European local investor and a UK investor investing across Europe will have his/her returns/equity premium driven in the same direction in most of the markets. Hence, in anticipation of a Europe-wide shock that would have negative effect on the returns, international investor should get out of the market and liquidate his/her international portfolio.

The implications for asset pricing models are straight forward. The fact that we can identify mispriced securities/portfolios that are able to generate positive/negative alphas on the UK market automatically suggests that CAPM as a model for correctly pricing securities and portfolios doesn't work and that UK market is not efficient. Therefore, this thesis provides a test of validity of CAPM model. Additionally, the thesis provides evidence that neither alphas (including variables that determine alphas such as dividend yield) nor betas or equity premium are stable over time. This implies that one should not use traditional least squared estimation of parameters from the CAPM equation since the method assumes constant estimates but rather use modelling techniques to estimate time-varying parameters. Additionally, the use of multi-period asset pricing models may be a way to overcome these problems as well.
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