"Modelling Energy Markets and Pricing Energy Derivatives"

Thesis by

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ABSTRACT

The main objective of this thesis is to provide an empirical assessment of the popular methodologies for modelling the underlying spot price dynamics in energy markets. After a brief introduction in the alternative forms of derivation that may be used for speculative and risk management purposes in energy markets, we assess the performance of the standard Black's framework in modelling energy prices.

For the first time in the literature we use a powerful and realistic data set which covers oil, gas and electricity markets and tests the appropriateness of the Geometric Brownian Motion process to explain the observed dynamics of the spot prices in these markets. We also provide spreadsheet based computer algorithms to price popular energy derivatives based on the Geometric Brownian Motion specifications.

In Chapter-3 we try to accommodate observed stylised facts in the spot price behaviour, namely mean reversion and jumps. For the first time in the literature we test a jump diffusion model, and a mean reversion jump diffusion model against our broad data set and compare the findings to the Black's Geometric Brownian Motion specifications.

In Chapter-4 we use a forward curve approach as an alternative-modelling framework to the spot price models. Based upon an almost proprietary data set of historical forward curves, we determine the number of independent factors that are needed to model the forward curve's dynamic evolution.

After carrying out principal component analysis on historical forward data a three-factor-model emerges as the most appropriate for energy markets in general. The first factor being the volatility (level effect), the second the smile and the third sesonality.

Finally in Chapter-5 of the thesis we compare the ability of spot models (Jump Diffusion and Mean Reversion Jump Diffusion Model) and forward curve based models to price WTI options. The results show that the Jump Diffusion Model is the best model as the option prices given are very accurate in comparison with the other models and closest to the market observed options prices.

CHAPTER 1

INTRODUCTION TO ENERGY DERIVATIVES

1.1 Introduction

Virtually every entity in today's world is exposed to economic risks. Some of these risks are unavoidable, some can be avoided with good management. Some risks are, in one way or another, insurable, others are not. One particular form of risk is exposure to commodity price fluctuations. The Gulf War for instance, greatly affected crude oil prices. The market forward prices of crude oil contained information on how long it would take the production side to respond to the sudden imbalance between supply and demand. Spot and short-term forward prices spiked, while longer-term future contracts remained relatively stable. In this case the mean reversion as exhibited in forward prices was tied to how quickly the production side could bring the system back in balance[1]. A similar high price volatility can be found in the Electricity market. For example the summer heat wave of 1995 caused electricity prices to jump to a multiple of their average price levels[2]. However, temperatures spiked only for several days and prices reverted to equilibrium. In this case the mean reversion had to do with the dissipation of an event.

Energy markets around the world are under going rapid deregulation, leading to more competition, increased volatility in energy prices and exposing participants to potentially much greater risks. Deregulation impacts both consumers and producers and has lead to a heightened awareness of the need for risk management and the use for derivatives controlling exposure to energy prices.

The process of managing risks generally has three phases: identifying the risks: quantifying the risks: and then managing the risks. The balance of this thesis explores the specifics of modelling energy prices and pricing energy derivatives. Therefore, understanding both the quantitative methodologies and the fundamentals of a market place is very important.

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[1] "Crude Oil Hedging Benchmarking Price Protection Strategies" Edward N. Krapels and Michael Pratt, Energy Security Analysis, Inc. Risk Books Energy & Power Special Reports.

[2] "Electricity Trading and Hedging" Edward N. Krapels, Energy Security Analysis, Inc. Risk Books, Risk Executive Report.

1.2 Futures Options and over-the –counter Instruments

1.2.1Forward contracts

A forward contract is an agreement between a buyer and seller to purchase or sell a specified amount of a commodity on a fixed future date at a pre-determined price - e.g., 1 million barrels of Saudi Arabia Light crude oil will be delivered 6 months from now, at a price of US$ 18 per barrel. Forward contracts have the advantage that future prices are locked in, which permits the determination of fixed purchasing prices - e.g., a public transport company buys a large part of its projected fuel needs on a forward basis, and is thus able to guarantee consumers the transport prices remain unchanged over this period. Also, they strengthen the links between a specific seller and buyer. Moreover, no cash changes hands until the contract is finally settled - so there are no cash-flow problems linked to the use of forwards, contrary to the situation with futures contracts (as we will see later).

Nevertheless, standard forward contracts suffer from a lack of flexibility (it is difficult to get out of a transaction), and moreover, one is exposed to a counterpart risk (several exporters were left with large losses when a few large trading companies went bankrupt in recent years). Traders are also exposed to a risk of default on the side of exporters - in effect, it is rather common that exporters decide to default on a fixed-price forward contract when, after the contract's signature, prices increase. Because of this default risk, traders often pay a relatively low price on forward contracts.

Forward contracts can be standardised and traded on an organised exchange - this is the case for a number of commodities in India, China and Indonesia. Tradable forward contracts offer more flexibility than non-tradable forward contracts, because the initial buyer or seller has the ability to transfer his contract to someone else. Also, they provide a means to discover future prices. On the other hand, they still allow less flexibility than futures contacts, because they assume deliver of the commodity.

Summing up, forward contracts can be a good way to secure a market outlet, at a fixed price. However, they are most suited to entities with a good reputation - others will receive comparatively low prices, because of the "risk premium" that traders will build into the price they offer. Also, forward contracts do not offer much flexibility.

1.2.2Futures contracts

Futures contracts are very similar to standardised forward contracts but they give rise to, primarily, financial transactions, not to physical transactions, as is the case of forward contracts. Use of the futures market will result in profits or losses that, ideally, will compensate the profits or losses made on a physical transaction[3]. This is best explained with an example.

Suppose that an exporter has an inventory. When he is able to sell this inventory at the current price of US$ 17/bbl., he will break even (that is, have a profit that is equal zero). When prices decline, he makes a loss, when they increase, he will make a profit. As the exporter cannot be sure of future price developments, he has an entirely speculative position.

What this exporter can do to secure an acceptable profit is to sell futures contracts for an amount similar to the amount he has in stock. At the moment he sells the crude he has in stock, he simultaneously closes out his futures position by buying the same number of contracts as he had initially sold. He has initially sold futures contracts for the then current crude oil price; when prices decline, he will have a profit on these contracts, when they increase, he makes a loss. The profit/loss profile of the futures market operation is thus exactly opposite to that of the physical market transaction. This, in turn, implies that if prices go down, the value of the inventory declines, but this is compensated by profits in the futures market. The exporter has thus hedged his inventory, that is, protected it against adverse price movements.

Also, there is a major constraint on the use of futures. The futures exchanges are rather secure places in which to trade, but to ensure market security, market users have to pay an initial deposit when they buy or sell a futures contract, and then have to pay margin calls when futures market prices move against them. In this way, the exchange reduces the risk of default of its users; the deposit provides sufficient collateral for the potential losses which may ensue during any one day when prices move adversely; and if prices move adversely, users have to make up the loss by paying a margin calls before the market opens again the next day (if they do not do so, their position is liquidated). For users, this may pose a cash-flow problem: deposits as

[3] International Petroleum Exchange: "An Introductory guide to oil futures on the International Petroleum Exchange"

well as margin calls have to be paid in convertible currencies; and margin calls in general have to be paid within 24 hours.

In the above example, futures were used to protect the value of an inventory. In the same way, futures can serve to guarantee the price of a crop which has not been harvested yet, the price of metals that are likely to be produced in a year's time, or the costs of importing fuels for the coming two-year period. Basically, companies can buy or sell futures contracts at the moment that they find prices attractive, and thus more or less fix the future "net prices" that they will receive or pay for the commodities that they expect to buy or sell.

Futures can also be used for other purposes. One of the simplest ways to use these instruments is the smoothing of the fluctuations in prices for small-scale producers or consumers. For example, small exporters of commodities may only produce enough to make two or three shipments of their product in a year, or may be forced by logistical factors (lack of storage facilities, seasonal transport problems) to sell their products during a limited period of time. They risk to sell at moments that prices are exceptionally or seasonally low. In order to ensure that the average price received reflects the year's average, rather than merely the average of the shipment months, the exporter can sell futures contracts during the non-shipment months, and buy these back during the shipment months.

The role of intermediaries, such as the producers of cocoa butter manufacturers, oil refiners or oilseeds crushers, also lends itself well to the use of futures and options. The profitability of the processing of raw materials is extremely sensitive to small variations in the price of the actual commodity being processed. Unless processors can pass unanticipated raw material price changes directly on to their customers, they have little choice but to turn to risk-management tools to hedge this price risk. For example, both for crude oil and for certain oil products futures contracts are traded. The differential between the two, for a certain month, represents a "refining premium", at times, this premium can be quite low on the spot market, while the forward differential between the prices of crude oil and oil products has become so wide that refineries can "lock in" the profits of refining: this hedging operation can provide full insurance against adverse movements in the refining margin, by selling oil products futures at the same time as a purchase of crude oil futures.

In the forgoing paragraph, futures were used to "simulate" a processing operation. In a parallel manner, futures can be used to "simulate" physical trade decisions. Say that an exporter sees world market prices, which he finds very attractive, but he happens to have no oil which he can sell on the world market. To replicate a physical sale, he can sell futures contracts, and liquidate this position later when he has obtained local materials and sold them. Say that he has a client who is willing to offer an interesting price for forward delivery, but he does not own the product yet - by buying futures contracts he can replicate the actual purchase of physicals, and if prices increase, he will have to pay a higher price to the oil producers, but will be compensated by profits on his futures market operation.

Summing up, futures contracts can be used:

- to avoid the effects of fluctuations in prices for producers who, because of their limited production volume or seasonal factors, are not able to spread out their sales over the year; or for consumers, who because of their limited size cannot spread out their purchases;
- to protect the value of inventories, or partly finance the cost of storage;
- to secure a processing margin;
- to "lock in" future prices at an attractive level; and
- to improve marketing policies.

The main disadvantages of using futures contracts are that:

- they freeze up working capital;
- although they may provide protection against unfavourable price changes, they do not permit profiting from favourable ones.

1.2.3 Option contracts

The purchase of option contracts gives the right, but not the obligation, to buy or sell a futures contract at a given price. When prices move favourably, this right will not be exercised, and therefore, the purchase of options provides protection against unfavourable price movements, while permitting to profit from favourable ones.

For an option, one has to pay a premium - this is the maximum loss from the option purchase. However, when prices develop negatively, an options buyer will make a profit which is more or less commensurate with the extent of the price decline (again, this is somewhat oversimplified).

These option profits will offset the losses on a physical inventory. The result is that the total losses have been limited (to a level that the company finds affordable), while the possibility to profit from the price improvements still exists. In many countries, this form of price protection may be politically more acceptable than the use of futures.

The other advantage of using options is that only the sellers of options have to pay margins. Hence producers in countries with non-convertible currencies who are contemplating a hedging strategy can, by buying options (in their case, put options - this will be discussed later), avoid the possible problems caused by the need for foreign exchange to meet margin deposits or maintenance margins. Similarly, consumers can hedge without having to pay margin calls by buying options - in this case, call options - which give the right to buy commodities at a given price.

Options may be a better hedging vehicle than futures in the case of an uncertain supply - e.g. in the case of an oil company that cannot be sure of the quantity it will be able to ship. They are often used to protect prices in deals with not fully reliable partners. If a fixed price deal with a seller has been concluded, and this position is covered with a futures contract, one may get stuck with a loss-making uncovered futures contract if the physical leg of the transaction disappears. When options are used, traders' losses are limited to the upfront premium they paid[4].

The sale of options allows the generation of some profits, but at a high risk, at least if those selling ("writing") are not properly protected by, for example, physical inventory. Producers currently make virtually no use of the potential benefits of selling options to create value from surplus stocks or production flexibility. The benefits of option sales are long run: in the short run the producer may do worse than if he had not sold the option. For example, in the case of the sale of a call option, if the market declines, the buyer of the call will not exercise the option, and the producer will keep the option premium, thus lowering the cost of the inventory. But if the price rises, the option is exercised. However, the producer still receives the original strike price plus the premium: his losses on the options are only compensated if he can dispose of physical commodities, through the high prices he will receive for his physical goods. In the long run, the producer will make a not insignificant profit. However, the necessary logistical skills to manage this type of operations are still

[4] New York Mercantile Exchange (1989): Nynex Energy Options-Strategies at a glance.

largely absent in developing countries, and under present conditions, most would be best advised to refrain from selling options.

Summing up, the purchase of options may serve the same purposes as the purchase or sale of futures contracts, with the main difference being that options leave open the possibility of profiting from favourable price movements, and that they cause less cash-flow problems. This may make them, in the short term, a more acceptable tool of managing risk and of improving commercialisation strategies than futures. The sale of options gives advanced users the possibility to create extra value from the inventories.

1.2.4 Commodity swaps

Commodity swaps are a very recent market innovation, dating from the late 1980s. To put it very simply, swaps can be used to guarantee income streams. In the case of commodity sales, a company's expected income stream during a certain period is equal to the amount of commodities it expects to sell, multiplied by expected prices over this period. In actual physical sales, prices may turn out to be higher or lower, and consequently, the company's income stream will be higher or lower than expected. If the company enters into a swap, it will receive a compensatory financial payment if prices are indeed lower, but will have to give up its unexpected benefits if prices turn out to be higher than expected: thus, it has a more or less guaranteed "net price".

A swap is thus a purely financial instrument. Normally, a producer (or consumer) enters into a swap with a bank or a large trading company. The advantages of swaps over futures are that they are available for periods much longer than 1 year; that they can be tailor-made to cover the needs of a certain company, in terms of commodities and risks covered; and that their requirements in terms of margin payments are much less stringent than on futures markets[5].

Swaps are often attractive to lenders or investors, as they provide security for the cash-flow of the company to whom they are lending, and thus the ability of the company to repay a loan or pay a dividend is improved by providing this type of long-

[5] Intercapital Commodity Swaps, Petroleum Intelligence Weekly Special Report (1990): The complete guide to oil price swaps.

term custom-designed hedge. Therefore, much of the current use of swaps is in the context of project finance.

Summing up, swaps can be used:

- to lock in the price of commodities for a long period of time (e.g., locking in oil import or export prices for a three to five year period poses few problems);
- to secure the income stream of operations or new investments; and
- to help attract more capital at more favourable conditions.

1.2.5 Commodity bonds and loans

Commodity bonds and loans are bonds and loans with a repayment (of the principal and/or interest rate) linked to commodity prices. This link can be in two major forms:

- **The loan or bond type**: the principal and/or interest payment on this loan or bond is repaid with the financial equivalent of a fixed amount of a commodity. For example, bonds of US$ 400 each are written. The yearly interest payments are equal to the price of 0.1 ounce of gold, and after 4 years, a sum equal to the value of 1 ounce of gold is reimbursed. If during the last year gold prices have increased to an average of US$ 500/ounce, the investor obtains US$ 50 in interest payments, and his bond is redeemed at US$ 500; but if gold prices have declined to US$ 300, he only receives US$ 30, respectively US$300. This type of commodity bonds or loans serve to protect a company's long-term risk exposure.
- **The option type**: the investor, or lender, gets the choice to have at maturity of the loan/bond, reimbursement of a fixed amount, or reimbursement in the monetary equivalent of a fixed amount of a commodity. For instance, in the case of a US$ 400 bond, the investor gets the choice to redeem the bond at US$ 400, or to be paid the price of 1 ounce of gold. This type of commodity bonds or loans are often used to obtain finance more easily and at a lower cost: the commodity option is thrown in as a sweetener.

Commodity bonds and loans are usually linked to investment projects (they have become very important to finance gold mining projects), or to debt rescheduling (they have played a role in the rescheduling of the official debts of Mexico, Venezuela, Uruguay and Nigeria). Their commodity coverage is somewhat limited: most commodity bonds and loans issued so far have been linked to gold, silver and oil: some are issued on aluminium, copper, nickel, palm oil, and coffee and cocoa.

Summing up, commodity bonds and loans can serve several goals:

- They can provide access to financial markets, which may not otherwise be available. As a result bonds may lead to the improved creditworthiness of the company or country concerned;
- In a high-inflation economy, commodity-linked bonds (in particular gold-linked bonds) can provide sufficient anti-inflation guarantee to bond buyers, thus allowing the issuing government (or company) to pay a somewhat lower net interest on the bond;
- Cash-strapped producing companies or countries can expect to pay lower interest rates as a result of issuing commodity bonds;
- Commodity bonds with long term commodity warrants attached can smooth the earnings of enterprises in commodity producing countries. In practice, the bonds can be designed to link the interest or principal payments of the bond to revenues rather than prices of the export commodity.
- They can be used as risk management tools, not only by commodity producers, but also by commodity consumers. In this respect, the commodity warrants that are often attached to bonds can be used by commodity consumers to hedge against commodity price increases above a certain pre-determined strike price[6].

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[6] J.P. Morgan & Co. (eds), Commodity -Linked Finance, Euromoney Books (London 1992).

1.3 Hedging with Futures (A basic description)

When a market participant uses futures market instruments, it is either in order to speculate or to hedge. Only hedge strategies will be explored here, or in other words: operations made to provide protection.

The basis of hedging with futures is the establishment of a position in the futures market that is equal and opposite to a position in the underlying oil product. In short, the principle of hedging is that the loss in one market should be offset by an "equivalent" gain in the other market. With such a strategy one locks in a price to be paid or received for future deliveries.

Two basic hedging strategies can be summarised here. Firstly, hedges can be undertaken in order to offset price risk that has arisen in a physical contract; this is known as "offsetting hedge" (the fundamental principle is to maintain a balanced book - each physical transaction must be balanced by an opposite futures operation). Secondly, hedges can serve to lock-in an attractive price level, or in other words by securitising profits on anticipated business. This last strategy may seem to be a speculative one but is the opposite: locking-in a good price removes the speculative part from a transaction by fixing the sales price at a level above known costs (in the case of a seller) or fixing a purchasing price at a lower level than allowed for in the fuel costing[7].

Different market participants can use futures to hedge their position. A producer can sell forward in a futures market in order to hedge sales of physical to clients, whether these sales are based on a long-term contract or spot. Exporters, merchants, manufacturers, and on the other side importers and customers may also benefit from futures hedging when they have to buy, sell or both.

An example on hedging with futures by an oil producer:

A producer of crude oil has to deliver 200,000 bbl. of crude in August. He considers actual market prices attractive and expects prices to drop considerably between now (May) and August; he wants to lock in his sales price at today's levels. Because of the anticipated price fall, prices for futures contracts in October are at a discount to the current market price: October contracts are sold for US$ 14.5/bbl., while the prevailing price is US$ 15.25/bbl. To lock in this price, he sells October crude oil contracts. If the producer is correct, and prices in August are indeed lower than at

[7] Phibro Energy (1990): Long Dated Oil Derivatives, mimeo

present, the gain on the futures contacts will compensate for the eventual losses on the physical market. In the event of an unexpected price rise however, the gains on the physical market will be somewhat lower than without the futures market position. Suppose the producer is right and the spot price on August 11th is US$ 14.00, while October contracts are traded against US$ 13.25.

Date	**Cash market**	**Futures market**
May 8	Spot price of US$ 15.25/bbl.	Sells October crude oil futures at US$ 14.50
August 11	Sells 200,000 bbl. of crude oil at US$ 14.00/bbl.	Buys October crude oil futures at US$ 13.25
Result	US$ 2,800,000	Gain of US$ 1.25 * 200,000 bbl. = US$ 250,000

The result on the cash market is 200,000 * US$ 14 = 2,800,800. The futures market activities have resulted in a gain of US$ 1.25 * 250,000. The total result is US$ 2,800,000 + US$ 250,000 = US$ 3,050,000, which means a price per barrel of US$ 15.25. This is equal to the May spot price and US$ 1.25 over the price that would be realised without the futures market operations.

An example on hedging with futures by an oil consumer

Suppose autumn has been exceptionally warm, and gas oil prices in November are US$ 140/tonne. A road transport company, having continuous need for gas oil, has to keep the prices for his customers unchanged over a period of 6 months. The transporter wishes to lock in this attractive price, which is US$ 8 below his price to break-even. Prices may well rise during these 6 months and to hedge this risk, the transporter buys June futures contracts on the IPE at US$ 144.00.

Date	Cash market	Futures market
November 21	Spot price of US$ 140/tonne.	Buys May Gas Oil futures at US$ 144.00
May 10		Sells May Gas Oil futures at US$ 146.00
	Gas Oil bought cash at the average price of US$ 142/tonne.	
Result		Gain of US$ 2.00/tonne

In May, the average price paid for the gas oil inputs bought on the cash market turns out to be higher than the November price. On average, US$ 142/tonne has been paid. The futures market position is closed out at the price of US$ 146.00, meaning a profit on the futures contracts of US$ 2.00/tonne. The overall result is that the transport company has an effective buying price of US$ 140, which is the attractive cash price in November that he wanted to lock in.

Example on hedging with futures by an oil refiner

Imagine an oil refiner who wants to secure his market share because of increased competition. He enters into a forward contract for delivery of 1 million gallons of oil products in three months time. To meet this obligation, he will need an amount of 1,427,850 gallons of crude oil. His risk, therefore, is that between now and January 1st, the price of crude oil will increase.

Since he has already agreed to a forward price at which to sell the oil products, a price increase could easily cause him to lose money in the forward sale. To protect himself against a risk of price increase, he will buy the crude oil in the futures market.

Date	Cash market	Futures market
November 1	Sells for Febr. shipment oil products equivalent to 33,995 bbl. crude oil (equal to 1,427,850 gallons) at US$ 16/bbl.	Buys 34 Febr. crude oil futures at US$ 15.70
January 1	Buys 33,995 barrels of crude oil at US$ 17/bbl.	Sells 34 Febr. crude oil futures at US$ 16.70
Result	Loss of US$ 1 * 33,995 bbl. = US$ 33,995	Gain of US$ 1 * 34,000 bbl. = US$ 34,000

Notice that the number of barrels needed for the cash market deliver did not exactly correspond to the number of barrels bought on the futures market. Since the crude oil futures size is 42,000 gallons per contract, the calculation for the number of contracts bough is:

Cash position / contract size = # of contracts

The calculation is: 1,427,850 gallons/42,000 = 33.9964

Rather than leaving any position of the cash position open to price risk, the higher number of contracts should be purchased. In this case, the hedger purchased 34 contracts instead of 33.

On November 1, the refiner sold the oil products forward for US$ 16/bbl. On January 1, he had to buy crude oil to fill his order for US$ 17/bbl. This is a loss of US$ 1 per bbl. Since his cash market position was for 33,996.4 bbl., the total loss on the cash market was US$ 33.995.

Luckily, he had an off-setting position on the futures market. On November 1, he bought crude oil futures at the price of US$ 15.70/bbl. On January 1, he was able to sell these contracts at the price of US$ 16.70/bbl. Since the futures market position was for 34,000 barrels, the total gain on the futures market was US$ 1 * 34,000 = US$ 34,000.

Combining the loss on the cash position with the gain on the futures position leaves the refiner with a small gain of US$ 5. Had the position remained unhedged, his loss would have been US$ 33,995.

1.4 Hedging with Options (A basic description)

Using futures to cover price risks can provide price protection, but has one important disadvantage: while strongly reducing the likelihood of losses, the possibility to benefit from price improvements is also lost. Options do not have this disadvantage:

By buying an option, protection can be obtained against unfavourable price movements, while the possibility to profit from favourable ones remains.

This is the basic reason for the use of options for hedging purposes.

To determine what option might be useful to protect against price risks, firstly the risks have to be identified: are price rises or on the contrary price declines the risk? Then, to protect against the effects of a price change, an option can be bought giving profits when prices move in the direction that the buyer wants to protect against. Losses on the physical goods will then be compensated by profits on the options, just like is the case of futures contracts.

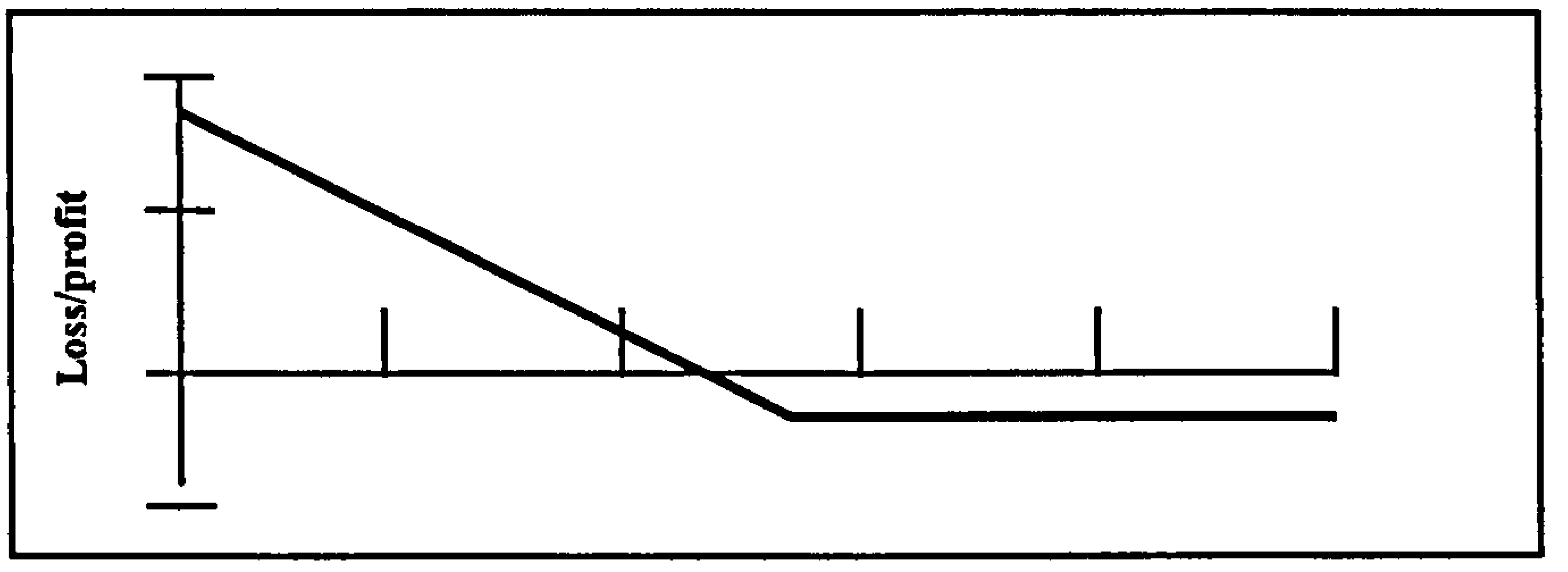
In the case of futures contracts this kind of price protection is paid for by giving up the possibility of profiting from improved prices. In the case of options, a fixed price has to be paid: the premium[8].

For example, if money would be lost when prices decline (such as is the case for a producer who is to sell his production, or for a trader who has unsold commodities in stock), an option that gives a profit when prices decline should be bought. This is called a put option. Mathematically the payoff for a put option can be written as follows:

$$\text{Max}\,(0, K-S) \qquad (1.1)$$

[8] J.P. Morgan Securities Inc., Derivatives Research (15 March,10 May, 6 July 1995)

Graphically, this looks as follows:



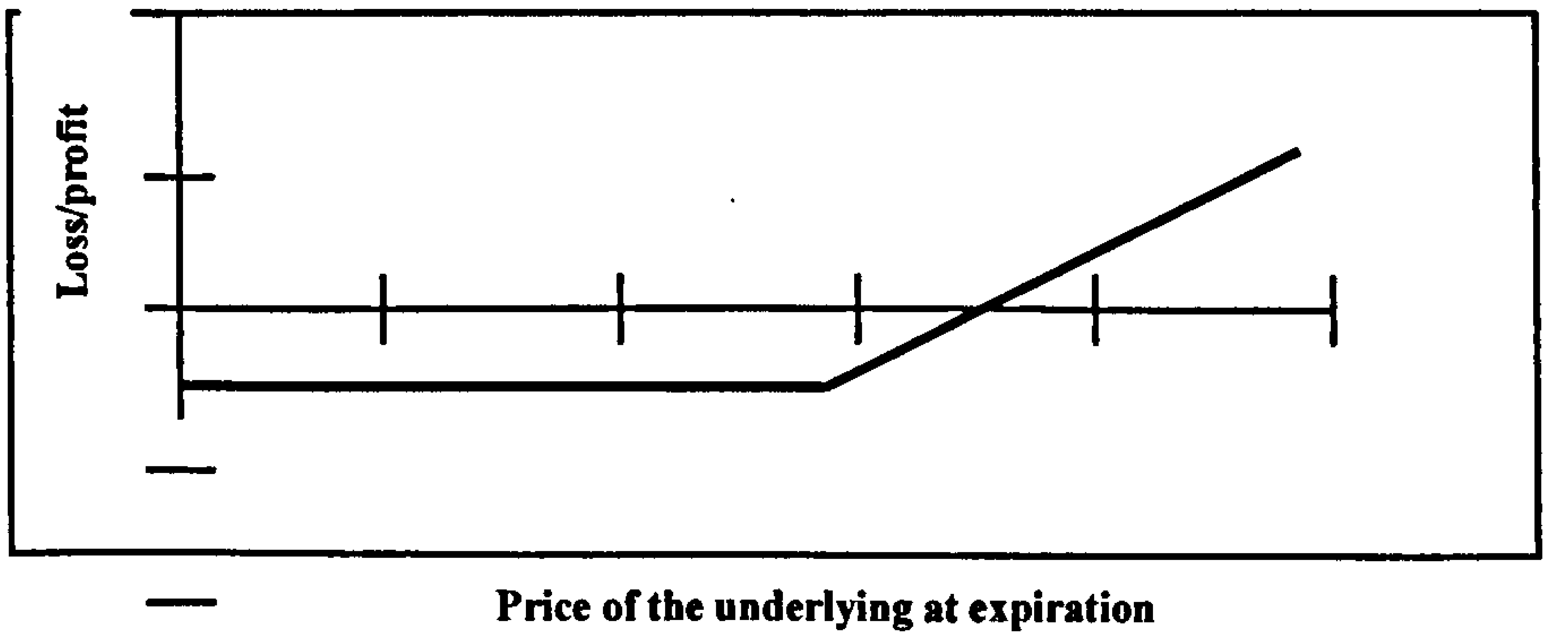
Price of the underlying at expiration

As can be seen in the chart of the put option, declining prices cause losses on the physical transactions, but buying a put option gives a profit.

If a price increase would involve a loss of money (such as is the case for a consumer who still has to buy the commodities he needs, or for a trader who has sold commodities for a fixed price, while he does not have these commodities in stock), an option can be bought which gives a profit when prices increase. Such an option is called a call option. The payoff to a call option can be described mathematically as follows:

$$\text{Max}\,(0, S\text{-}K) \qquad (1.2)$$

Graphically, this looks as follows:



Price of the underlying at expiration

When prices increase, losses are made on the physical position, but buying a **call option** will then give profits. The result is: protection against price increases, but with still the possibility to profit from price declines.

The option is like an insurance: it provides protection against price declines (put option) or price increases (call option), and simultaneously the possibility to profit from reverse price movements. How this exactly works will be explained in the following paragraphs, what is important here is to understand how options can be used for covering price risks. It should be underlined that in particular the insurance function of options will be discussed in this and the following paragraphs: selling options is the reserve of highly advanced players because of the enormous financial risks involved. So, foremost option buying strategies will be looked at.

Options are rather complicated instruments, and before turning to a number of options applications, some definitions and theory on their technical details will be given.

Generally speaking, an option is a contract granting its buyer a right, but not the obligation to buy or sell a defined quantity of the underlying product (for example a futures contract) at a pre-fixed price, and to do so during a period agreed beforehand or upon the contract's expiry. The option buyer, also called its holder, can choose to let the option expire or to exercise it. The writer or seller of an option has the obligation to fulfil the contract when the buyer decides to exercise.

At the time of transaction, a price is set in terms of the option contract, specifying a price at which the underlying, for example a futures contract, may be bought or sold. This price is called the strike price, or the exercise price. The option buyer may take (call) or make (put) delivery of the contract against this price. *When* the holder of an option can exercise his right depends on the option type. American-style options may be exercised at any time between the date of purchase and the date of expiration. European-style options may only be exercised at the date of expiration.

When buying an option, a price has to be paid to obtain the rights laid down in the contract. This is called the premium. It is, like other prices, determined by market forces of supply and demand. As are other market-determined prices, premiums are subject to fluctuations.

The option premium is composed of two elements: the **intrinsic value** and the **time value.** The intrinsic value is the difference between the price of the underlying futures contract and the strike price. It is either positive or zero and indicates the value of the option at any time. For a call option, the intrinsic value is the price of the underlying futures contract minus the strike price :the intrinsic value of a put option is equal to the strike price less the futures price. So, a US$ 17 crude oil option will have a positive intrinsic value for call options with a strike price below US$ 17 and for put options that have a strike price of above US$ 17. When the futures price rises compared to the strike price of a call option, its premium will probably increase because of increased intrinsic value. It becomes more likely that exercise of the call will be profitable. The premium of a put decreases when the price of the underlying future rises, since the intrinsic value diminishes: it will become less and less interesting to exercise the rights conveyed under the option contract. Following the same reasoning, for a futures price decrease, the premium for a call option with a certain strike price becomes less since the chance of its exercise diminishes, and the price for a put option goes up.

The amount by which an option's premium exceeds the option's value is called its time value or extrinsic value, the second component of the premium. The life-time of an option is one of the factors that determines the time value. Suppose the other factors - the price of the underlying futures, the strike price, volatility and short-term interest rates - remain the same. Then, the time value decreases when maturity approaches. The time value is highest when the strike price equals the price of the underlying asset. When these two prices diverge, the time value decreases. This is because, the chances of exercise of the option, as well as the losses of an options seller, will be higher. The time value is, so to say, a time-related flexibility value of an option.

Suppose a trader has bought a call on a July heating oil futures contract, strike price US$ 0.54/gallon. At the same time, the July heating oil futures are traded at US$ 0.51. Its intrinsic value is zero, but still a premium has to be paid for this option. This is because through time, the price on the futures market for the July contract may well rise. In other words: the option still has time value, equal to the premium. So, even if during the life of a call option, the strike price is higher than the quoted price for the underlying futures contract, the call has time value, equalling its premium. Closer to maturity, the time value gets lower to become zero at expiration. Then, the option maintains only its intrinsic value.

Imagine that all factors determining the time value remain stable and only the volatility of the price of the futures contract underlying the option increases. Bearing in mind that the option serves as an insurance against price changes of the futures contract, it is obvious that the price that has to be paid to obtain this protection will rise. When the futures price is more volatile, there is a growing chance that in its life the option may become worthwhile to exercise. This induces sellers to ask a higher premium, for the risks they run get higher.

Let us take the example of a trader who has bought a call option on a September sweet crude oil futures contract for US$ 0.58/bbl. Its strike price is US$ 20.30/bbl. If the September sweet crude oil futures moves above the strike price, the buyer will exercise his right to a long futures position at a price that is lower than the market price for the futures contract. For instance, when the September contract is traded at US$ 21.00/bbl., exercise of a long call option will give a profit of US$0.70/bbl. (not taking into account the premium paid). When the strike price is lower than the real market price for the underlying futures contract, a call option is said to be in-the-money. Imagine the same scenario for the September sweet crude oil put option, strike price US$ 20.30/bbl. When the market price rises to US$21.00/bbl., the holder of the put option will not exercise his option because of the loss involved. A put option on a futures contract that has an actual market price above the strike price laid down in the option, is said to be out-of-the-money. Suppose the September futures price falls to US$ 19.50/bbl.: then it becomes worthwhile for the buyer of a put option to exercise because the option is now in-the-money. For a holder of a call, the exercise of the option would result in a loss, or in other words, the option is out-of-the-money. If the strike price equals the market price of the underlying, calls as well as puts are at-the-money.

Regardless of whether the buyer of the option decides to exercise his right, the premium, once paid to the seller of an option, remains in the hands of the seller. The maximum amount the options buyer can lose on his option position, is the premium paid for the option. His profit potential is virtually unlimited. A seller, on the other hand, can only have profits limited to the premium size: his loss can be unlimited, since prices may move to unforeseen low or high levels. If the price of the underlying asset drops below the strike price minus the premium paid, the buyer of a put will exercise his right and the seller has to take delivery against payment of the, relatively high, strike price.

If a call buyer decides to exercise his option when the strike price is below the price of the underlying, the seller is obliged to deliver the underlying asset. If he does not possess it, he will have to buy the asset on the market and suffer a loss equal to the difference between the market price and the strike price (less the premium which he has collected), which can be enormous when supply is tight. The sale of a call without the previous purchase of the underlying asset is said to be "naked". It is a position meant to collect the premium, a purely speculative and very risky position.

An option trader who has a net selling position, in other words he has sold more option contracts than he bought, is said to be short in options. If a market participant has bought more contracts than he sold, he has a long position. For instance, an option trader who has sold 20 options, and bought 6 options, is short 14 options. And for example, somebody who is long calls has bought the right to buy a certain asset at the strike price: if prices go up, he will profit by exercising his call and reselling the underlying on the market at a higher price. The same is true for another options position, namely a short put: in that case, a price increase will prevent the buyer from exercising and the options seller collects the premium. Following the same reasoning, a price decline is beneficial to participants having a long put position, for they can sell the underlying at the strike price and will be able to procure the underlying asset on the market at a lower price. And short call investors will also benefit from a price drop: the buyer of a call will not exercise his right and the premium paid remains at the seller's account.

The buyer of an option can choose to let the contract expire, to exercise it, or to close his position before expiration. An options seller has only two choices: to wait for the option to expire, or to close his position before the expiration date is reached. When closing out an options position, attention should be paid not only to the expiration date, the offsetting position, and of course the underlying asset, but also to the strike price of the relevant option. This means that a long call position can be closed out by selling calls with the same expiration date and the same exercise price; similarly, when having the same expiration date and equal exercise prices, a short call can be closed by a long call transaction, a long put can be offset by the sale of a put, and a short put position can be closed by the purchase of put options.

For example, it is now May and an October crude oil put option strike price US$ 17.00 is traded at 55 cents/bbl. In mid July, the option expires. The futures contract is traded at US$ 16.00/bbl. The spot price is US$ 15.75/bbl. The option's premium is now US$ 1.10/bbl. Let us first consider the situation if, in the first half of July, the trader would have exercised his option. Then he sells crude oil futures under the option against the strike price of US$ 17.00, first buying futures at the current price of US$ 16.00. His futures profit is US$ 1.00/bbl. The trader sells his crude oil on the spot market at US$ 15.75/bbl. The net selling price the trader obtains for his crude is the sale on the spot market plus his futures market result, less the cost the option, which is US$ 15.75 + US$ 1.00 - US$ 0.55 = US$ 16.20/bbl. If, on the other hand, the trader decides to close out his options position, his transactions are the following: in May, he buys a put option, October maturity and strike price US$ 17.00, at US$ 0.55/bbl., to close out this options position, the trader sells a US$ 17.00 put at a premium of US$ 1.10/bbl. His option profit is the difference between the premium paid in May and the premium earned in July, namely US$ 0.45/bbl. He sells his crude on the spot market at US$ 15.75/bbl., which makes his overall result US$ 16.20/bbl.

To conclude this paragraph, some examples about long put and call positions are posed underneath.

Example 1: The manager of a trading company has an unsold stock of heating oil. The market forecast is that heating oil prices will rise. Therefore, by buying put options he protects himself against the risk of declining prices, while maintaining the possibility to profit from price rises.

Example 2: Suppose an oil producer has explored a new well and has just started drilling. The oil well proceeds will be sold in two months. By buying put options he protects himself against the risk of declining prices, while maintaining the possibility to profit from price rises.

1.4.1 Option contracts traded on an organised market:

The most liquid crude oil and petroleum products futures markets, are nowadays complemented by option markets. Considering the petroleum sector, the main options traded on the exchanges are[9]:

- Options on the IPE Brent Crude Oil futures:
- Options on the NYMEX Sweet Crude Oil futures:
- Options on the IPE Gas Oil futures:
- Options on the NYMEX Heating Oil futures: and
- Options on the NYMEX Gasoline futures.

Options on IPE Brent Crude Oil futures

Underlying Instrument:	1 IPE Brent Crude Oil futures contract of 1,000 barrels
Maturities:	As of futures contracts
Strike prices:	Multiples of US$ 0.50 per barrel
Last trading day:	Three days before the cessation of trading of the underlying futures contract
Expiration:	One hour after cessation of trading at the last trading day, 20.15 h.
Tick Size:	US$ 0.01 per barrel

Brent Crude Oil options contracts traded on the IPE reached a volume of 531,742 in 1993. This represents 531,742,000 barrels of Brent, which is over 2 times 1993 Brent crude oil production.

The contract maturities are standardised. Note that the option's maturities correspond to the futures delivery months. This enables traders to implement arbitrage strategies between the two instruments.

Strike price increments vary according to the underlying futures price. If the underlying contract is traded at 20 for example, the strike prices 19, 19.5, 20, 20.5, 21 etc. represent the out-of-the-money, at-the-money and in-the-money prices. There is a minimum of seven strike prices quoted.

The expiration date is the last day on which the option may be exercised, which, for the Brent crude contract, is the last trading day of the underlying futures contract. This is possible because this contract is cash-settled; for contracts which provide for physical deliver, the last trading day of options is generally the last day before the futures contract enters into its delivery phase that is, in general one month before final contract expiry. This is to reduce the likelihood that options are used to squeeze the market.

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[9] Chicago Board of Trade (1989): Options on agricultural futures (3rd ed.), International Petroleum Exchange: Gas Oil Traded Options.

At the close of business on the last day of trading all options over US$ 0.25 in-the-money, will be automatically exercised into the underlying Brent futures at the strike price of the option. Declaration instructions have to be given to the clearing house at latest one hour after close of business.

The minimum price fluctuation of the option, its tick size, is US$ 0.01 per barrel. i.e. US$ 10 per contract.

Options on NYMEX Sweet Crude Oil futures

Underlying Instrument:	1 NYMEX Sweet Crude Oil futures contract of 1,000 barrels (42,000 gallons)
Maturities:	Twelve consecutive months plus three long-dated options at 18, 24, and 36 months out.
Strike prices:	Multiples of US$ 0.50 per barrel for the first nine strike prices: the increments are US$ 1 for the next three strike prices and US$ 5 for the nearest higher or below the nearest lower existing strike price.
Last trading day:	The Friday immediately preceding the expiration of the underling futures contract as long as there are three days left to the futures expiration. Otherwise, the option expires the second Friday prior to the futures expiration.
Expiration:	5.30 pm on the last trading day, or 45 minutes after underlying futures settlement price is posted, is the last possible exercise time.
Tick Size:	US$ 0.01 per barrel.

The NYMEX Sweet Crude Oil option is traded from 9.45 am to 3.10 pm (New York time). These are the same trading hours as for the underlying futures contract. As for the underlying futures, out-of-hours trading is possible via the NYMEX Access trading system.

The quoted strike prices are those of five puts and five calls above and below the strike price that is closest to the previous day's close of the underlying futures contract.

Options on IPE Gas Oil futures

Underlying Instrument:	1 IPE Gas Oil futures contract of 100 tonnes of gas oil
Maturities:	Nine consecutive months, including the current month
Strike prices:	Multiples of US$ 5 per tonne
Last trading day:	The 5th business day prior to cessation of trading in the underlying gas oil futures contract
Expiration:	12.00 hours, as for the underlying gas oil futures
Tick Size:	US$ 0.05 per tonne

In 1993, the volume of IPE Gas Oil options was only 136,859 contracts.
The holder of an IPE Gas Oil options contract has to right to buy or sell the underlying IPE Gas Oil futures contract on any date up to the expiry date. Trading months are similar to those of the futures contract, namely nine months in advance including the current month. In this respect the IPE Gas Oil options and futures differ from the Brent contracts, under which trading in the maturity month is not allowed. For example, the January 1995 Gas Oil option contract is traded up to Thursday 5th of January.

Options on NYMEX Heating Oil and Gasoline futures

Underlying Instrument:	1 NYMEX Heating Oil futures contract: respectively Gasoline futures contract of 1,000 barrels
Maturities:	Twelve consecutive months
Strike prices:	Multiples of US$ 0.02 per gallon, except for the first three months listed at US$ 0.05 above and below the at-the-money strike price
Last trading day:	The Friday immediately preceding the expiration of the underlying futures contract as long as there are three trading days left in the futures contract. Otherwise trading stops the second Friday of the month prior to the delivery month of the underlying futures contracts
Expiration:	5.30 pm on the last trading day
Tick Size:	US$ 0.0001 per gallon (US$ 4.20 per option contract)

Options on NYMEX Heating Oil and on NYMEX Gasoline futures, represent a futures contract of 1,000 barrels each. Trading hours are the same for the underlying futures contracts as for the options, namely 9.50 am to 3.10 pm (New York time). Both options can be traded after the open outcry market is closed via the electronic trading system Access.

Prices are quoted in dollars and cents per gallon. Strike prices after the first three delivery months are listed only as even numbers, for example US$ 0.4600, 0.4800 and 0.5000 etc.

1.5 Oil hedging with swaps

Types of swaps offered on the over-the-counter market are[10]:

a. Straightforward (or "plain vanilla") swaps;

b. Collar swaps;

c. Participation swaps;

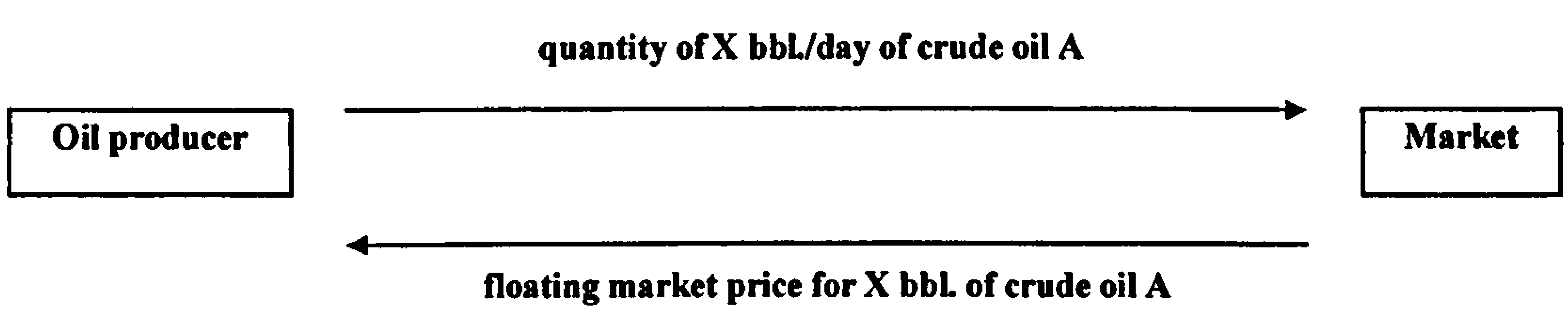
d. Specialised swaps.

1.5.1 Straightforward swaps

The straightforward swap is an exchange of a single fixed price and a single floating price without the possibility for any of the parties involved to take advantage of positive market swings.

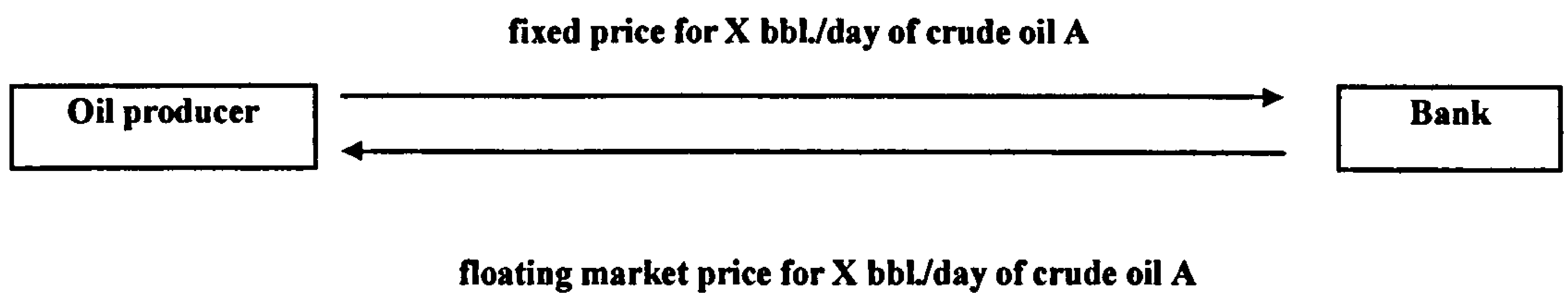
Example of a straightforward swap between an oil producer and a bank:

An oil producer sells his production on the market, in quantities of X barrels per day. His revenue is the market price for the product at the moment of sale.



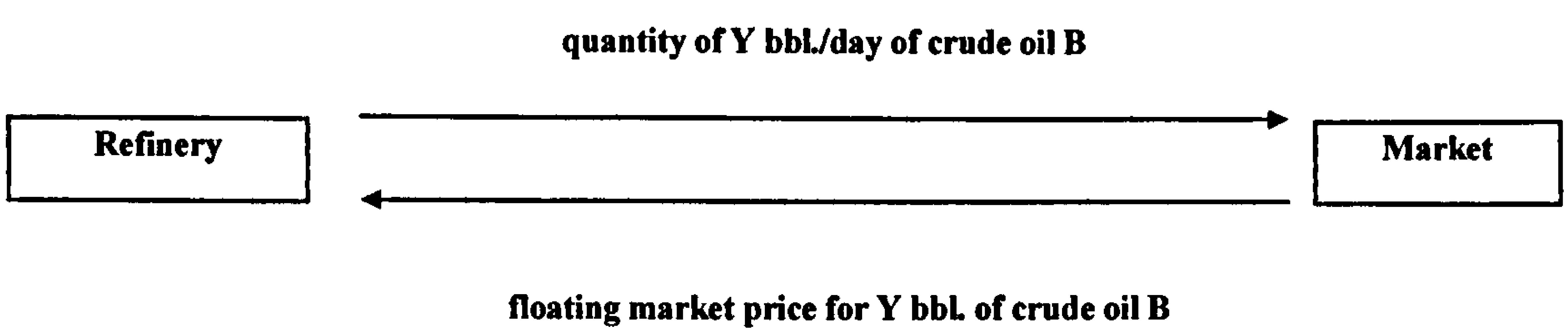
The producer has plans to expand his drilling capacity, and wants to fix his revenues to be sure that he can repay the loan that he needs for the investment. He finds a bank willing to guarantee the fixed price, and as a result to carry the price risk involved.

[10] Enron, (1995): Managing Energy Price Risks, Risk Publications, London.

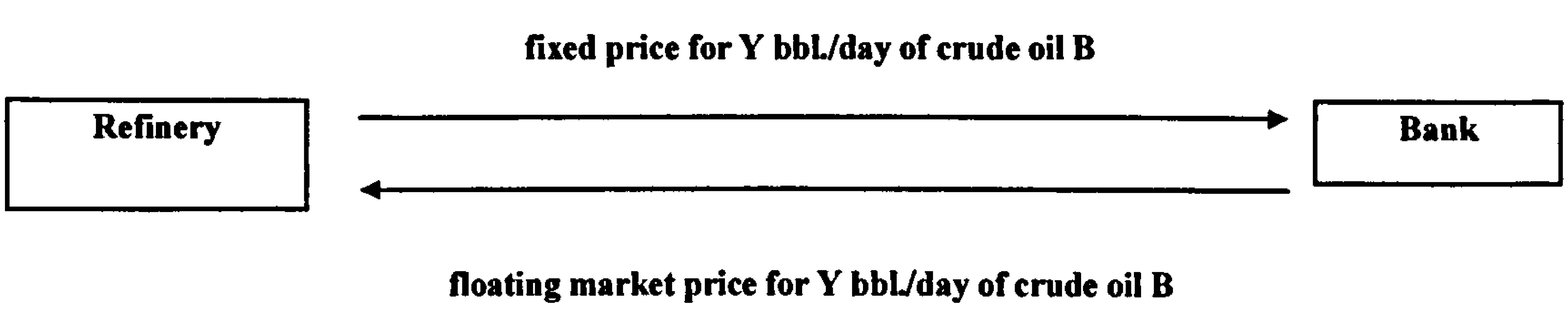


The oil producer receives a floating market price for his crude oil sales, which he pays to the bank in exchange for the fixed price laid down in the swaps agreement. The oil producer no longer is exposed to the price risk of his daily X barrels of crude oil (A) sales. The bank, on the other hand, has to pay the fixed price, while receiving the floating market price, thus being expose to the price risks involved.

Consider the opposite case, an oil consumer, for instance a refinery, who wishes to have fixed price obligations for his input purchases. Normally, he buys the quantity of Y barrels of crude oil daily on the market against the prevailing spot price



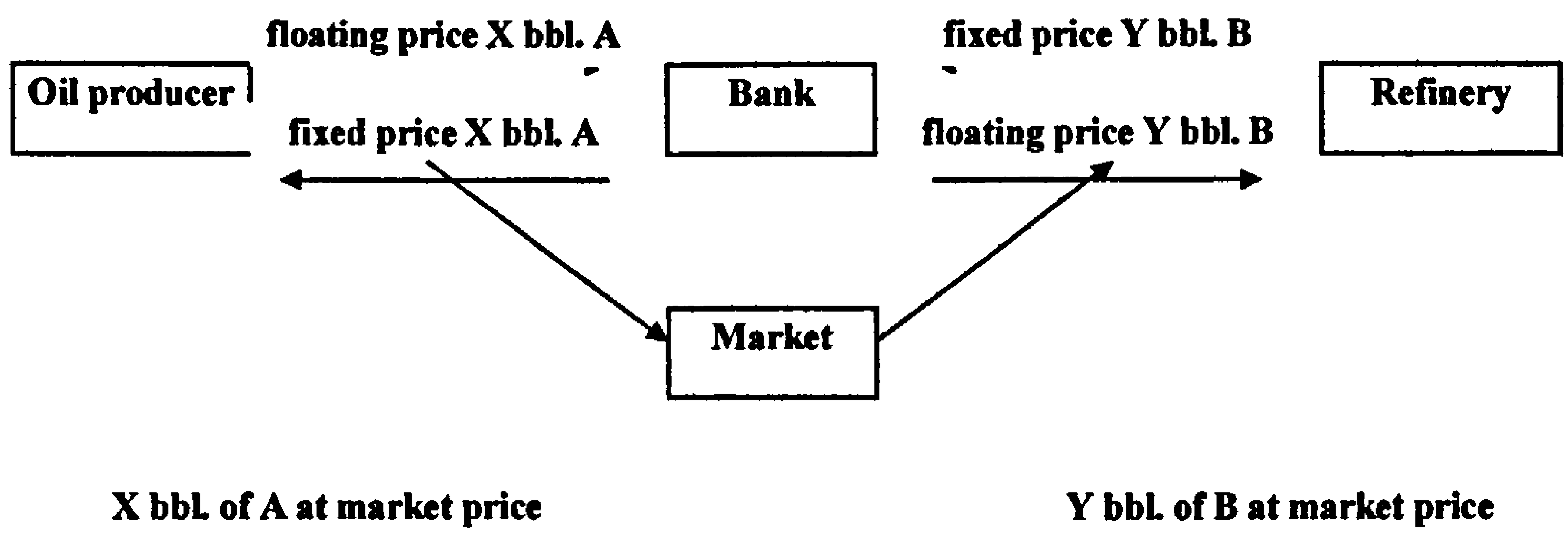
The refiner approaches a bank to agree on a consumer swap arrangement:



The refinery pays net a fixed price for his crude oil (b) purchases. The price risks are carried by the bank, which guarantees the market price in exchange for a floating price.

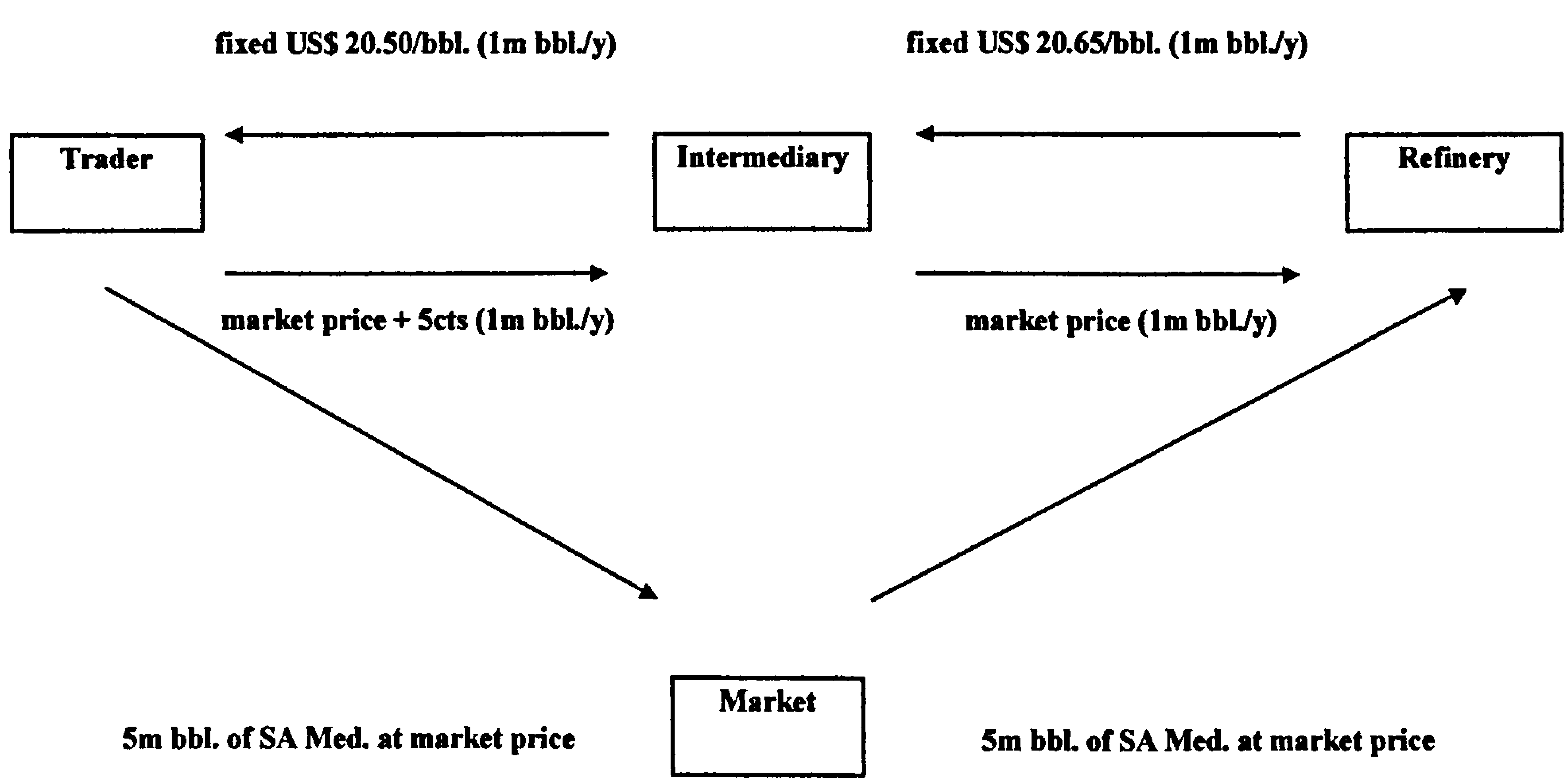
A bank normally does not want to be exposed to these kind of price risks for the often long duration of the swap. So, the bank offsets the risks on the futures and options market, when product similar contracts are available. If this is not the case, it can look for a third party that wants to take the opposite side in a swap.

In the earlier examples, the bank can offset the price risks of the producer swap by arranging a swap with a party that wants to buy the product against a fixed price, in this case: the refinery. In the reverse, he can lay off the price risks of a consumer swap by coupling it to a swap with a producer.



As follows from this figure, there are four factors that determine if the swap, constructed in this way, is possible or not: A, B, X and Y. If the physical products (A) and (B) crude oil are not similar, it is very likely that prices have other characteristics. Then exchanging the cash flows resulting from the sales and purchase of the products (A) and (B), may not be interesting at all to the bank. Furthermore, if the quantities are not the same, the bank still has a price risk exposure; this problem can be solved by finding yet another consumer/producer. Often, the bank has to warehouse part of the price risks, until another participant is found.

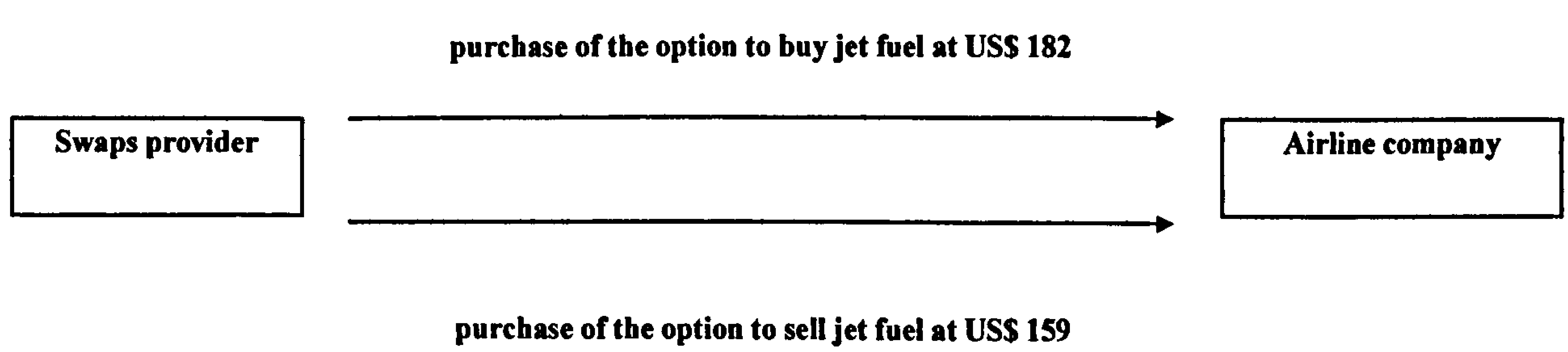
Example:



In 1989, an oil trader and a refiner agree a swap with an intermediary for 5,000,000 barrels of Saudi Arabia Medium crude oil over a period of five years with annual payments, starting this same year. The oil trader sells the Saudi Arabia crude on the physical market, receiving the floating market price. He pays the intermediary the floating market price plus 5 cents/bbl. for the 1,000,000 barrels sold. In return he receives a fixed price of US$ 20.50/bbl. for 1 million barrels per year. The refiner buys 1 million barrels per year on the physical market, paying the floating market price. He receives the floating price from the intermediary, paying in return a fixed price of US$ 20.65/bbl. for the annual purchase of 1 million barrels.
The intermediary can charge a commission for connecting the two participants, but he can also formulate the prices paid and received in such a way that it is levied indirectly.

1.5.2 Collar swap (also called min-max)

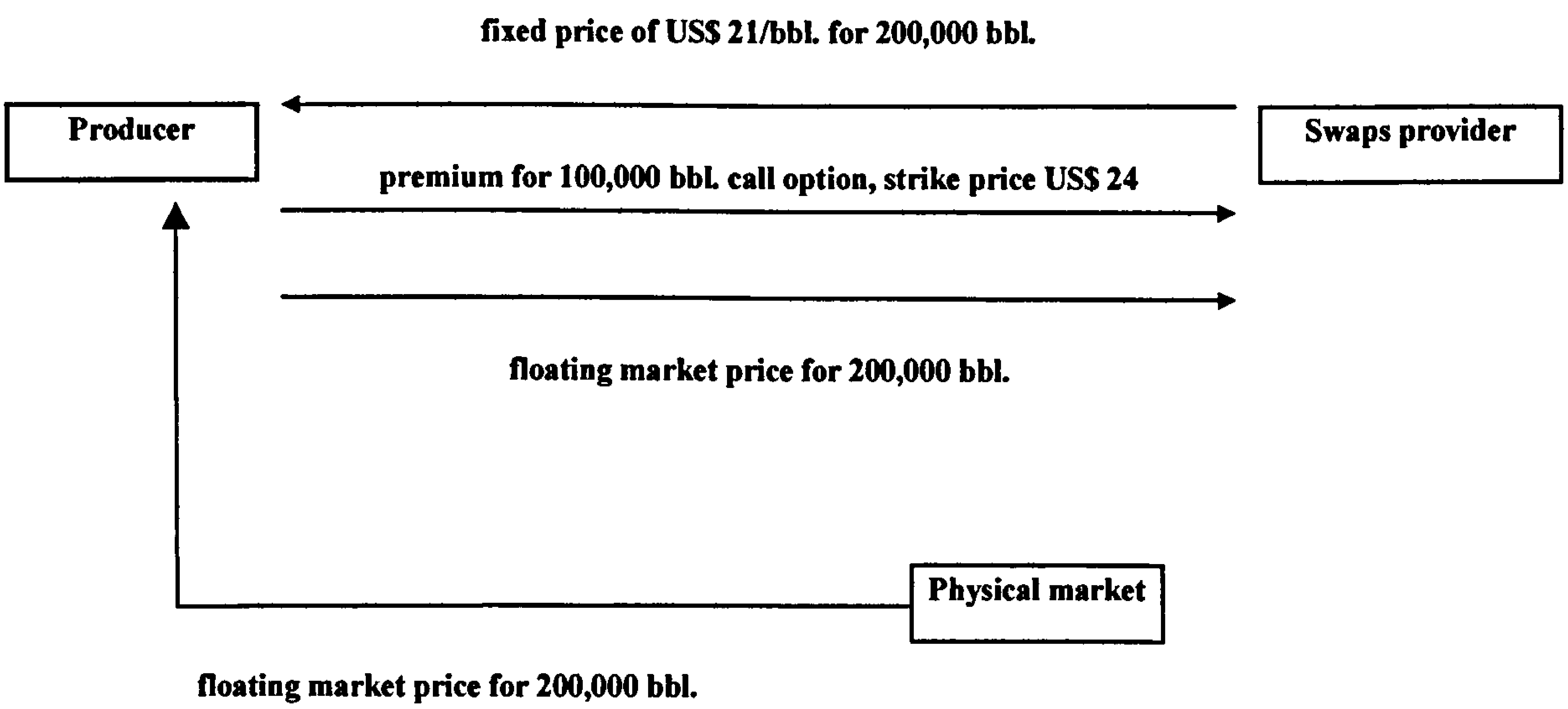
Imagine a airline company budgets for a maximum jet kerosene price of US$ 182 per tonne. He believes, on the other hand, that prices might dip as low as US$ 159 per tonne. What he wants is to pay not more than US$ 182 for the jet fuel, with the latitude to profit from lower prices down to US$ 159. The collar swap enables the flight company to realise this goal. By locking in a floor price (US$ 159) and a ceiling (US$ 182), the airline company is allowed to sacrifice less of the benefit from favourable price movements than would be the case under a straightforward swap transaction.



The collar swap locks in a minimum price and a maximum price. In other words: the swap locks in a price range unlike the straightforward swap under which a single price is guaranteed.

1.5.3 Participation swap

The participation swap is similar to the straightforward swap, with the difference that under this swap the buyer or seller does not have to sacrifice all of his potential gains. Instead, the swaps participant has the option to keep a certain share of the possible windfalls, should the price level rise above a certain price level. Let us take the example of an oil producer who has arranged a swap with a swaps provider.



As you can see in the foregoing flow-chart, the producer has negotiated an option to pay the floating market price for 200,000 bbl. of crude oil, but with the possibility to benefit from a price rise above US$24/bbl. In return for this possibility, he has to pay the swaps provider a premium. Note that the participation swap still guarantees a fixed price (in this example the fixed price laid down in the contract is US$ 21/bbl.).

1.5.4 Specialised swap

Specialised swaps embrace many different sophisticated swap variants. In general, these swaps are of a short-term nature. Particularly for the oil products for which no organised futures market exists, the specialised swap product is a popular instrument to deal with different (price) risks. Jet fuel swaps and refinery margin swaps are examples of this instrument which demands a high degree of expertise and management capacities.

The provider of a specialist swap is often deeply involved in the oil market itself, like a large oil company is. By having a natural long or short position in oil, he is very well qualified to develop a flexible strategy to offset the risks involved.

A growing sophistication of swaps can be seen from the fact that these have become increasingly tailor-made. For example, "floating for floating" or "basis" swaps have become rather popular. In these specialised swaps, a certain price relation is fixed: e.g. between sweet and sour oil; between crude oil and heating oil (a "crack spread" swap); between WTI and Brent oil or between New York and Singapore prices (to hedge locational price developments); or between July and November prices (to hedge seasonality). Dozens of different basis swaps are traded. These basis swaps are very useful to reduce the basis risks of using the existing futures or forward markets. They are used by a wide range of parties, with the international oil and trading companies in a dominant position.

There is a very active market in WTI, Brent, Dubai and Tapis swaps for maturities of less than three years, and also some swap activity exists in several other crude's, on the basis of Platts Oilgram prices. In the United States, natural gas swaps are traded on a large scale. The spread market WTI/heating oil is also very active; in general, these swaps trade for less than six months. As concerns product swaps, trade in heating oil swaps is considerable in several regions for period of up to 18 months; there is also an active market in Northwest Europe and Singapore gas oil. Over-the-counter options as well have a high turnover in most of these markets. It should be noted that the periods covered by these contracts largely coincide with the maturity of futures contracts. Swaps are preferred over futures because they are felt to correspond better to the needs of users. For example, in the natural gas swap market in the United States, price settlement is based on a price reporting service rather than on

NYMEX futures prices which are felt to show a weak correlation with cash prices because the NYMEX contract expires before spot delivery prices are determined.

Summary

In this chapter we have introduced energy derivatives, describing some simple structures and some basic risk management techniques. In the following chapter we look at the applicability of Geometric Brownian Motion for modelling energy price processes and price energy derivatives based upon this (underlying) dynamics.

CHAPTER 2
OPTION VALUATION

2.1 Introduction

The aim of this chapter is to test the Black's model approach for modelling energy prices and to develop computer algorithms for pricing the main derivative products used in energy markets. We discuss closed form solutions lattices and Monte Carlo Simulation methods.

•

2.2 Detailed Option Model Implementation

Model implementation should not be confused with option model derivation, although with some implementation techniques it is hard to separate the two. Model derivation in its basic form is the derivation of the differential equation for the option price. How we get from this from this differential equation to the option price is what I refer to as the model implementation.

There are a number of implementation techniques. We will concentrate on the most common ones: the closed-form solutions (as exemplified by the famous Black-Scholes and Black equations), approximations to the closed-form solutions, and the tree building methodologies.

The closed-form implementation methodology involves the solving of the differential equation for the option price to obtain an equation that defines the option price as a function of the market variables and modelling parameters.

Arguably, closed-form solutions for option prices provide the ideal implementation methodology. Since they provide us with a simple equation, which can be easily programmed and implemented on the trading floor. Such equations are easy to use and quick to give us the option value as well as the risk calculations when we need them.

Unfortunately, the closed-form solutions are typically extremely hard to arrive at. The more complicated the market place is, the more complicated is the differential equation for the option price. The more complicated the terms of settlement of the option, the more difficult it becomes to satisfy the boundary condition of the option in solving the differential equation.

In the end, in order to arrive at closed form solutions, we usually need to make many simplifying assumptions about both the market variables and the option settlements character. The end result of these simplifications is that while providing us with a practical and easy to use option-pricing methodology, the closed form solution may not reflect the reality of the market behaviour. Examples of such simplification include assuming that the volatilities are constant over time when they are not, assuming that the underlying market price is lognormal when could be mean reverting, and treating the option settlement as a discrete price when it is actually an average of discrete prices.

It is such simplifications that force us to calculate corrections to the closed-form option price implementation.

Two famous closed-form option-pricing models are the Black-Scholes model and the Black model. Both assume that the option settlement prices are lognormal and have constant volatilities. Next, the derivation of these models is briefly discussed.

2.3 The Black-Scholes Model

Part A

In this chapter we will look at a number of different methodologies that have been developed for pricing options. These approaches have been developed under the Black Scholes Model (BSM) assumptions of no arbitrage, risk neutral valuation, a non-dividend-paying asset, constant interest rates and constant volatility. In a risk neutral world, all assets earn the riskless rate of interest, thus the actual expected return on the asset does not appear in the Black Scholes formula. From the perspective of this chapter, however, the most important assumption in the BSM model is the mathematical description of how the asset prices evolve through time. This is the well-known Geometric Brownian Motion (GBM) assumption where proportional changes in the asset price, denoted by S, are assumed to have constant instantaneous drift, μ, and volatility, σ. The mathematical description of this property is given by the following stochastic differential equation:

$$dS = \mu S dt + \sigma S dz \qquad (2.1)$$

Here dS represents the increment in the asset price process during a small interval of time dt, and dz is the underlying uncertainty driving the model and represents an increment in a Weiner process during dt. The risk-neutral assumption implies that the drift can be replaced by the riskless rate of interest (i.e. $\mu = r$). Any process describing the stochastic behaviour of the asset price will lead to a characterisation of the distribution of future asset values and the assumption in equation (2.1) implies that future asset prices are lognormally distributed. Equation (2.1) can be written in terms of the natural logarithm of the spot price, $x = \ln S$, as follows:

$$dx = (r - \frac{1}{2}\sigma^2)dt + \sigma dz \qquad (2.2)$$

These extensions are straightforward to incorporate into Monte Carlo simulations but lead to the loss of the analytical tractability of the BSM model.
The BSM model described by the equation (2.1) can be descretised as follows:

$$\Delta x_i = (r - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{1i} \qquad (2.3)$$

We are going to test the BSM (equation 2.3) in the following markets:

Brent IPE from January 1995 to 29 December 2000(Figure-2.1), WTI from January 1995 to 29 December 2000 (Figure-2.2), Natural Gas from January 1995 to 30 October 2000 (Figure-2.3), California-Oregon Border On Peak (Figure-2.4) & Off Peak (COB) (Figure-2.5) electricity spot price from December 1996 to 30 December 2000, Mid-Columbia On Peak (Figure-2.6) & Off Peak (Figure-2.7) (MC) over-the-counter electricity spot price from April 1996 to December 2000 and the Southwest Power Pool On Peak (Figure-2.8) & Off Peak (Figure-2.9) (SPP) electricity spot price from April 1996 to December 2000.

Figure-2.1

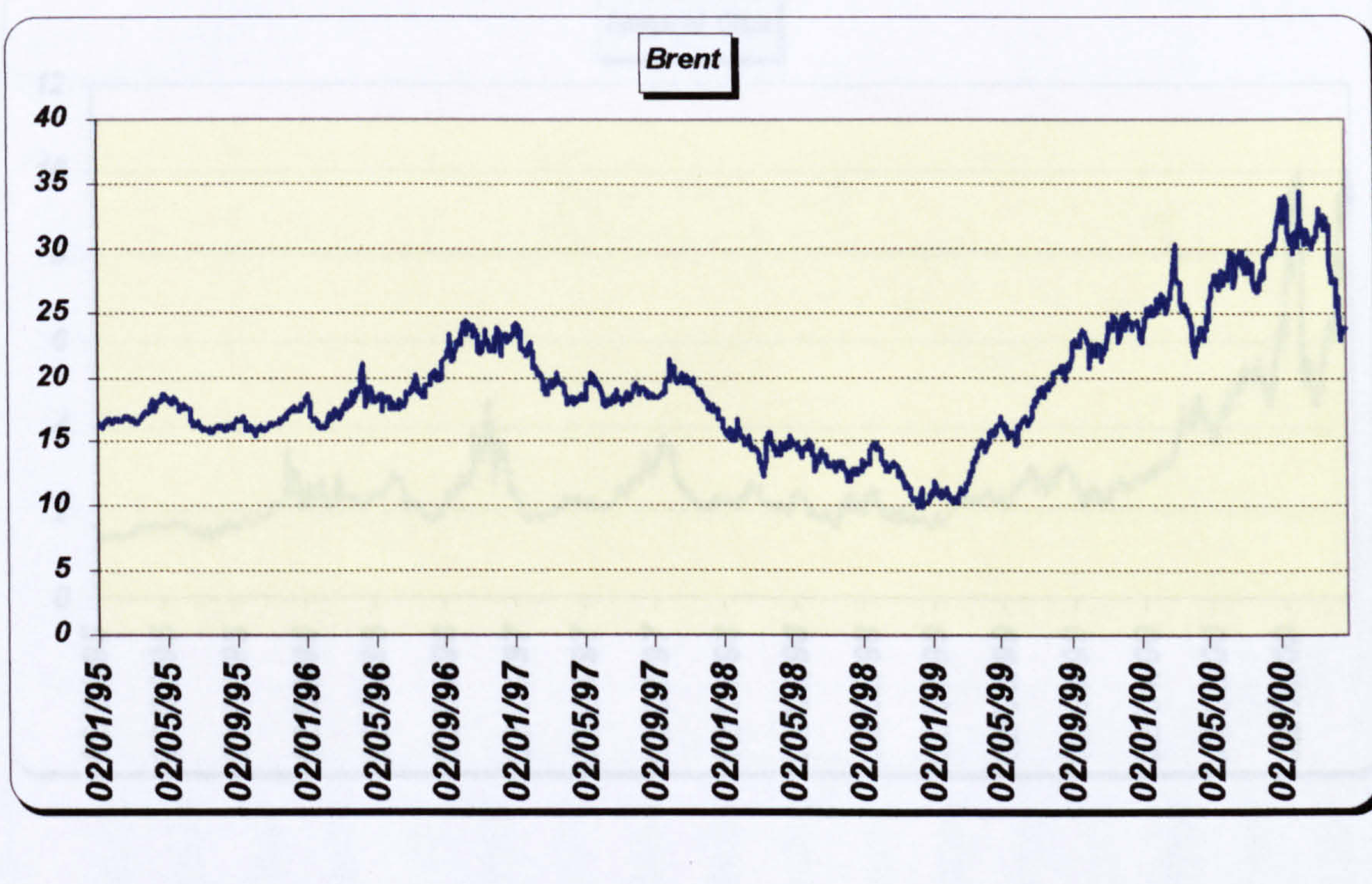


Figure-2.2

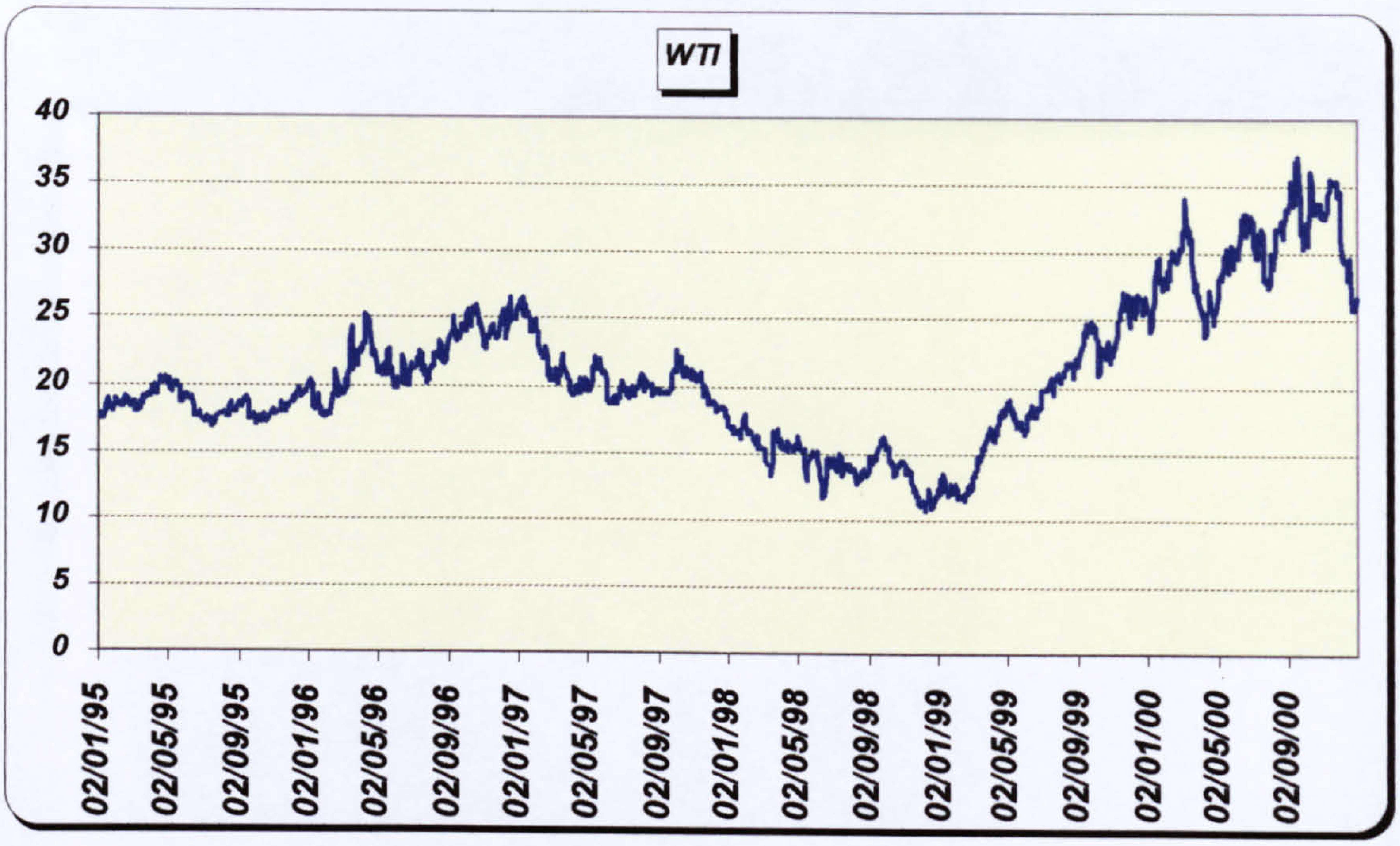


Figure-2.3

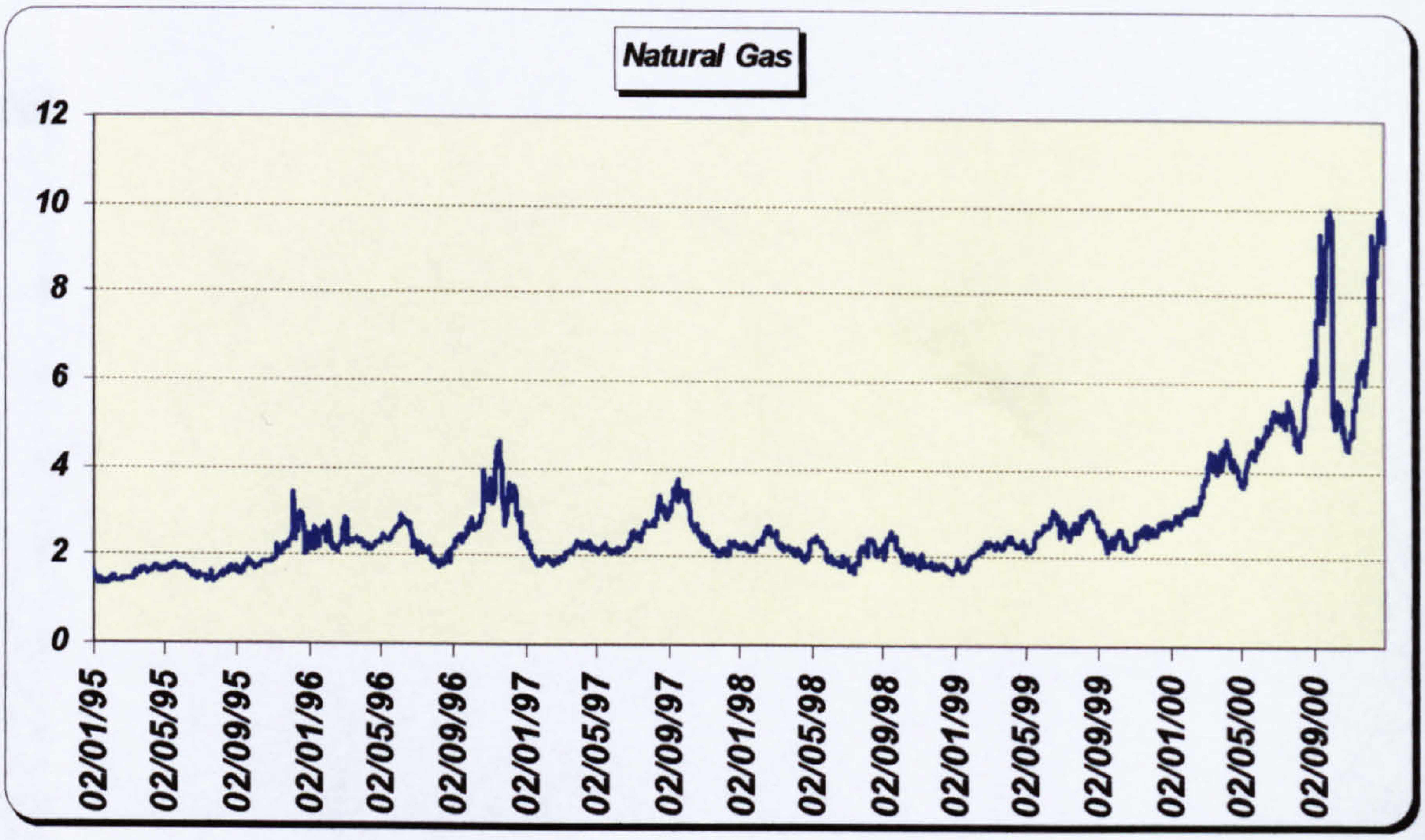


Figure-2.4

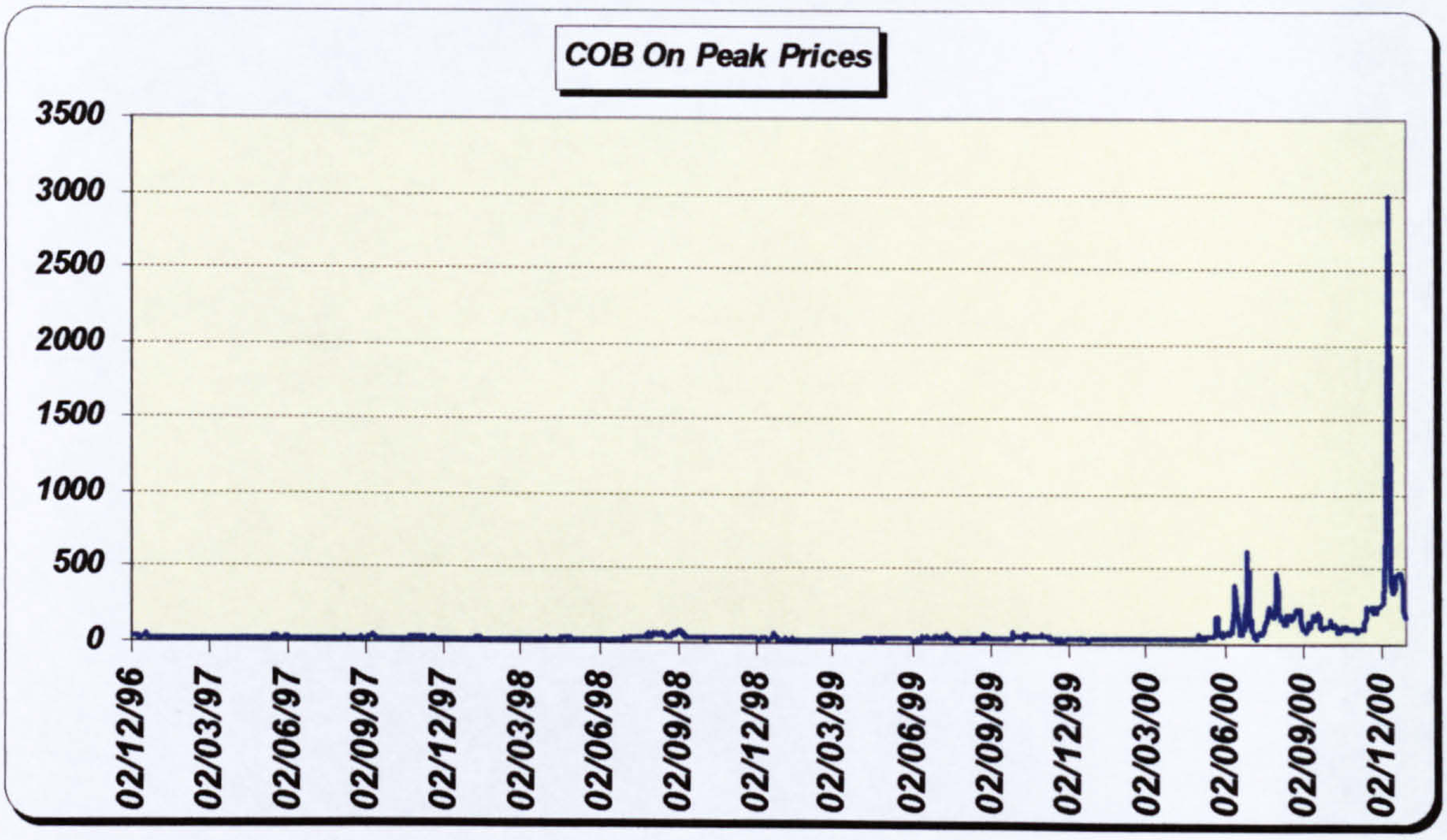
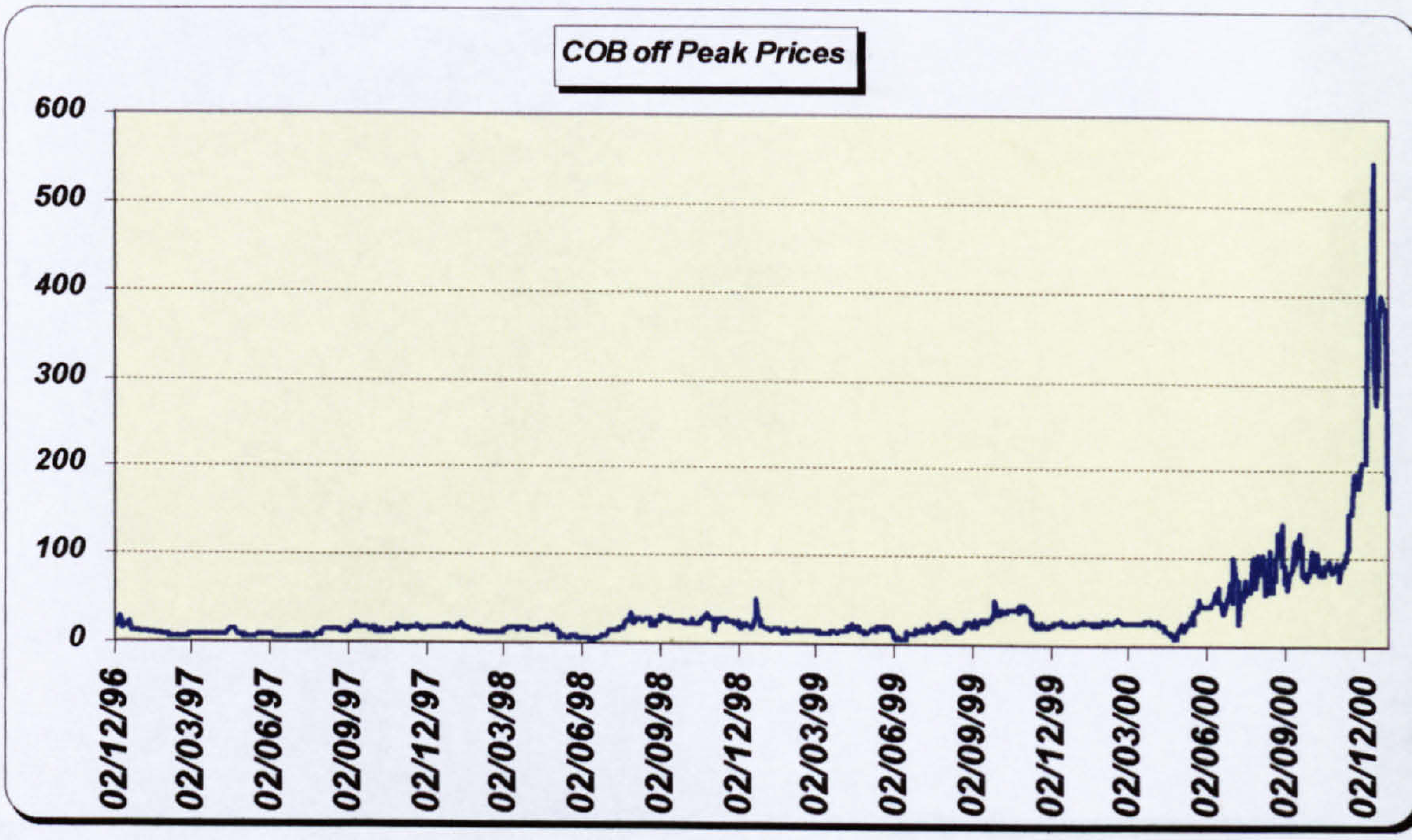


Figure-2.5



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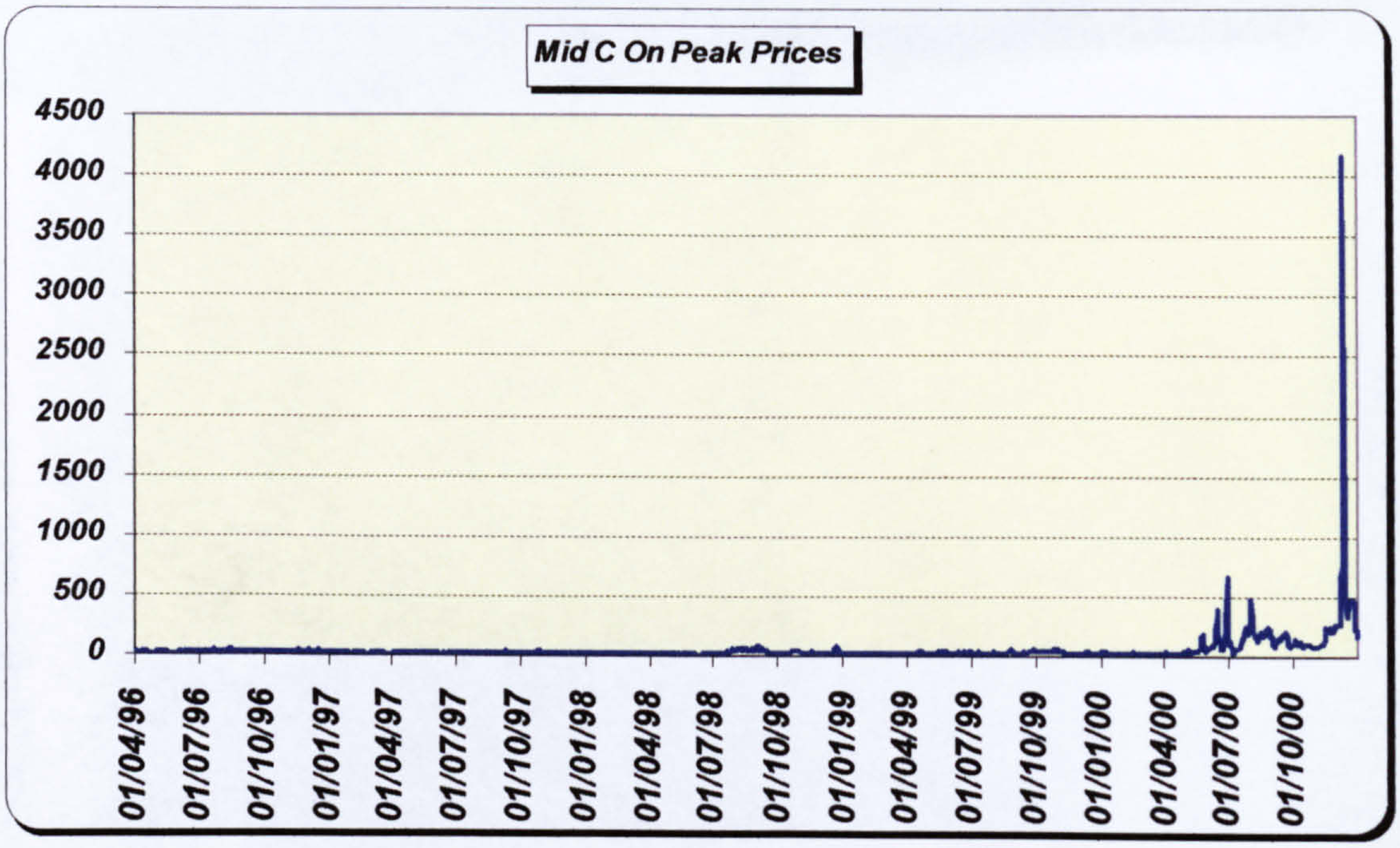


Figure-2.7

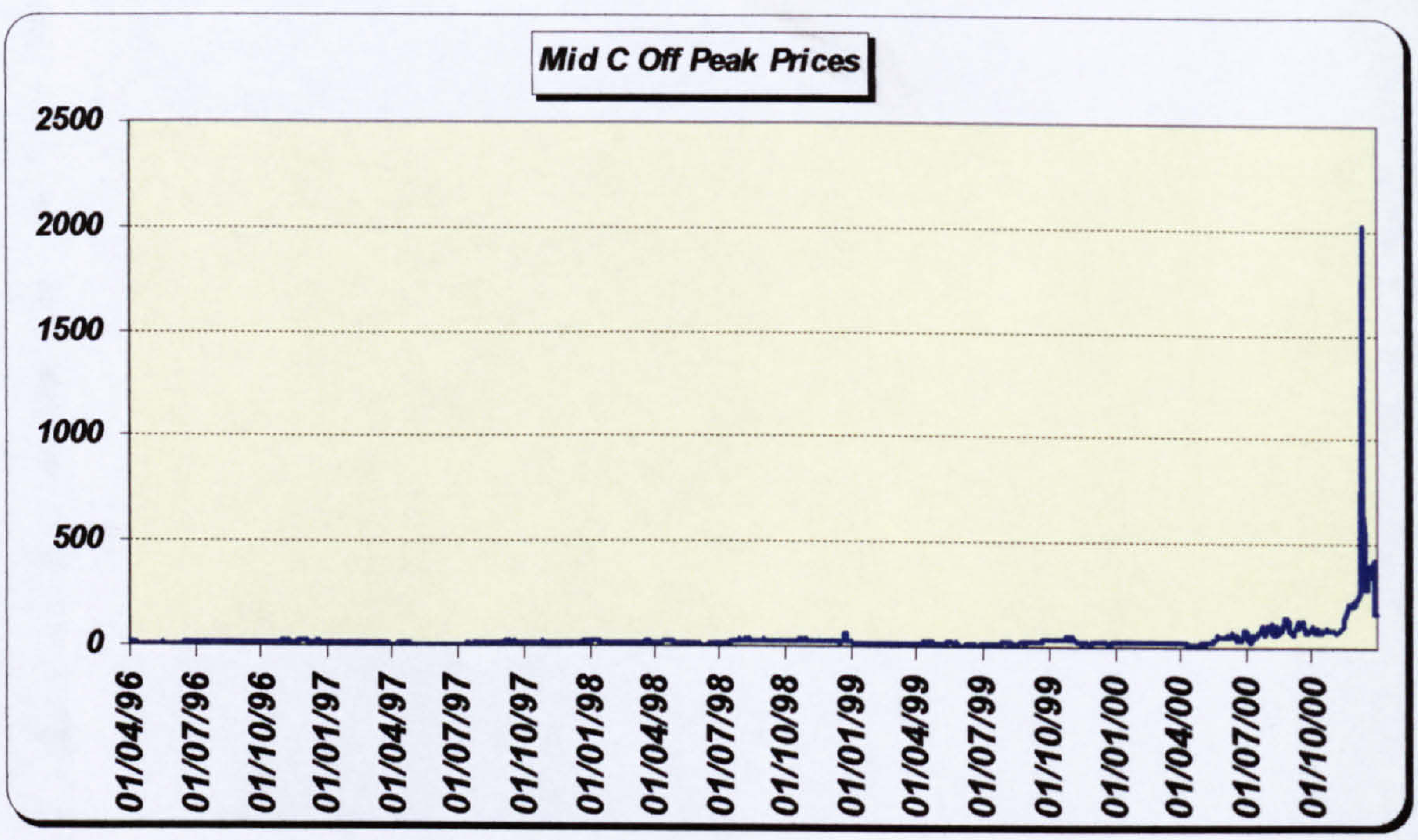


Figure-2.8

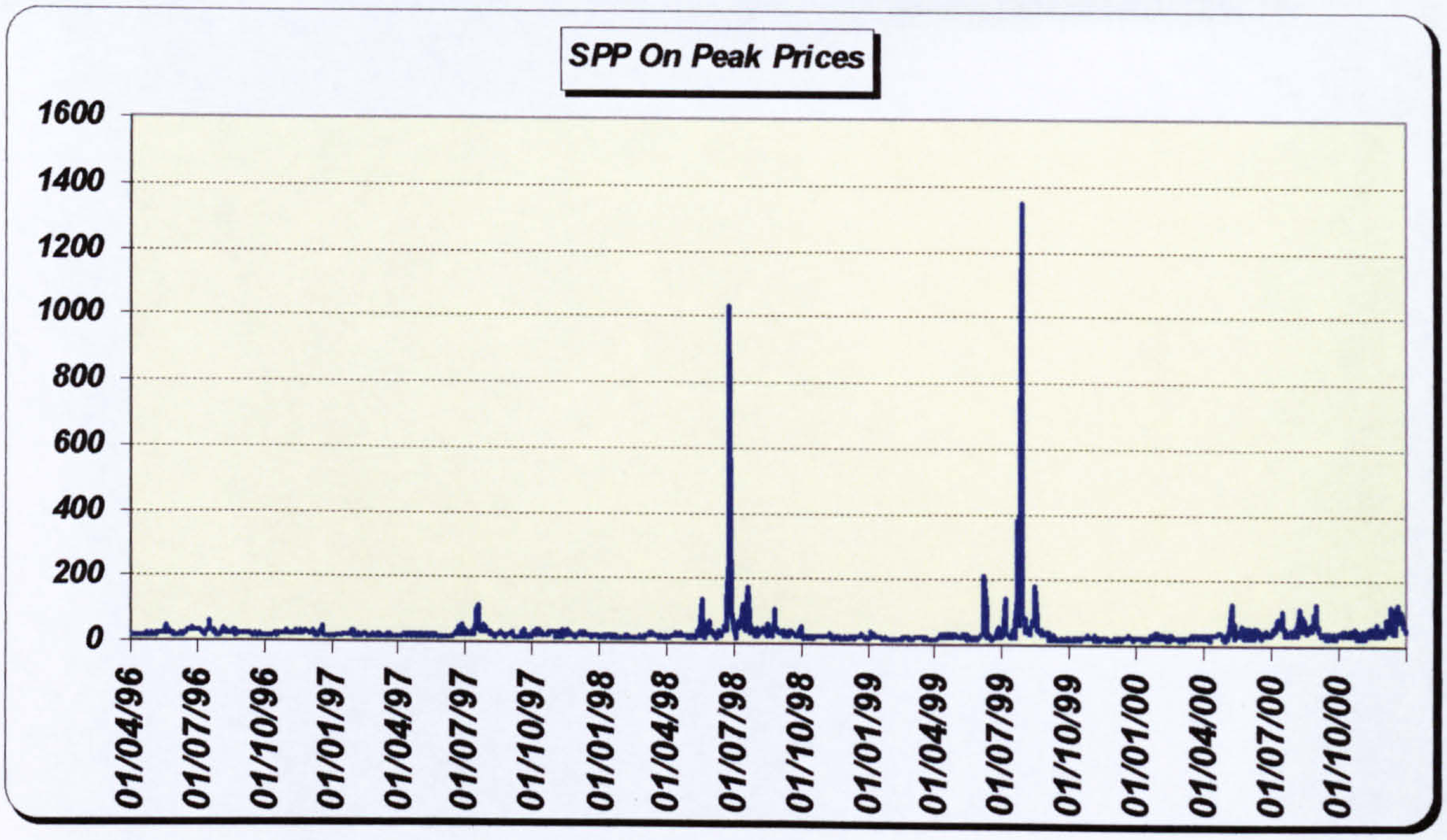
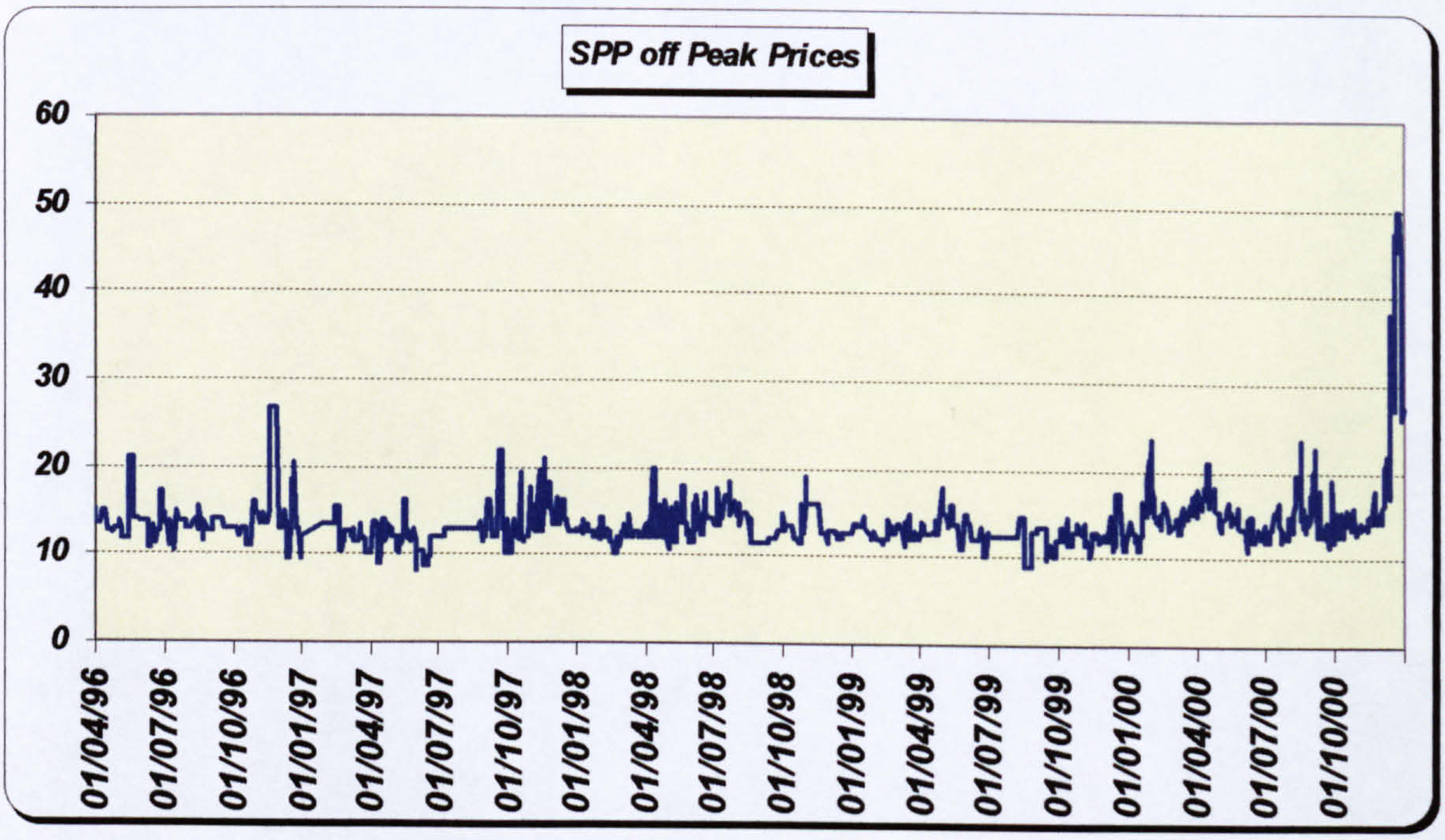


Figure-2.9



We are using Monte Carlo Simulation in order to implement the BSM (equation 2.3). Appendix-2.1 shows the computer algorithm for the Black-Scholes GBM model. The computer language we are using to implement the computer algorithm for the BSM is Visual Basic. Monte Carlo Simulation is going to be discussed in more detail later on in this chapter. We apply ten thousands simulations for each one of the sixty-six consecutive historical chosen randomly from the data available observations in order get more accurate results. Sixty-six data points corresponds to three months worth of observations which we feel is a large enough sample which we can draw conclusions about the spot price behaviour based on the BSM.
We assume that the interest rate is 5% and the volatility σ can be estimated from the historical price returns. The process of estimation of the volatility can be broken into several steps that can be easily carried out in a spreadsheet.

Step 1. Calculate logarithmic price returns.

This can be accomplished by forming the price ratios S_t / S_{t-1} and taking the natural logarithms of these ratios. Price returns are typically calculated as $r = (S_t / S_{t-1}) - 1$. The logic of the approach described above is that for relatively small x, $\ln(1+x) \cong x$. Taking the natural log of S_t / S_{t-1} is equivalent to taking the natural log of $1+r$, and this in turn is roughly equal to r.
The use of natural log returns has also some other additional advantages. If one wants to calculate a log return over a longer time period, say from t to $t+n$, corresponding to the ratio S_{t+n} / S_t, one can convert into $(S_{t+n} / S_{t+n-1})(S_{t+n-1} / S_{t+n-2})........(S_{t+1} / S_t)$. Given that a log of a product is equal to the sum of the logs, one can easily show that a log return over the longer time period can be calculated as the sum of log returns for the sub periods.

Step 2. Calculate the standard deviation of the logarithmic price returns.

Step 3. Annualise the standard deviation by multiplying it by the correct factor.

As a first approximation the annualisation factor depends on the price data frequency. In the case that the data is monthly, the factor is $\sqrt{12}$, for weekly data is $\sqrt{52}$. For the daily data available for each calendar year one has to use $\sqrt{365}$. If the information is available for trading days only, the standard number is $\sqrt{252}$. For example, if the price of a barrel of oil at the close on Monday is $16.00, and at the close on Tuesday is $16.20, then close to close price return is ln (16.20)-ln (16.00)=0.0124.
On an annualised basis ($\sqrt{252}$ *0.0124=0.1968=19.68%). The realised volatility between these two days is 19.68%.
Table-2.1 shows the annualised volatility σ.

Table-2.1

Market	Annualised Volatility σ
WTI crude oil	36.2018%
IPE Brent	30.8532%
Natural Gas	66.4777%
COB On Peak	290.2306%
Mid-C On Peak	282.3249%
SPP On Peak	434.0962%
COB Off Peak	202.5722%
Mid-C Off Peak	221.5380%
SPP Off Peak	160.9265%

WTI

Figure-2.10

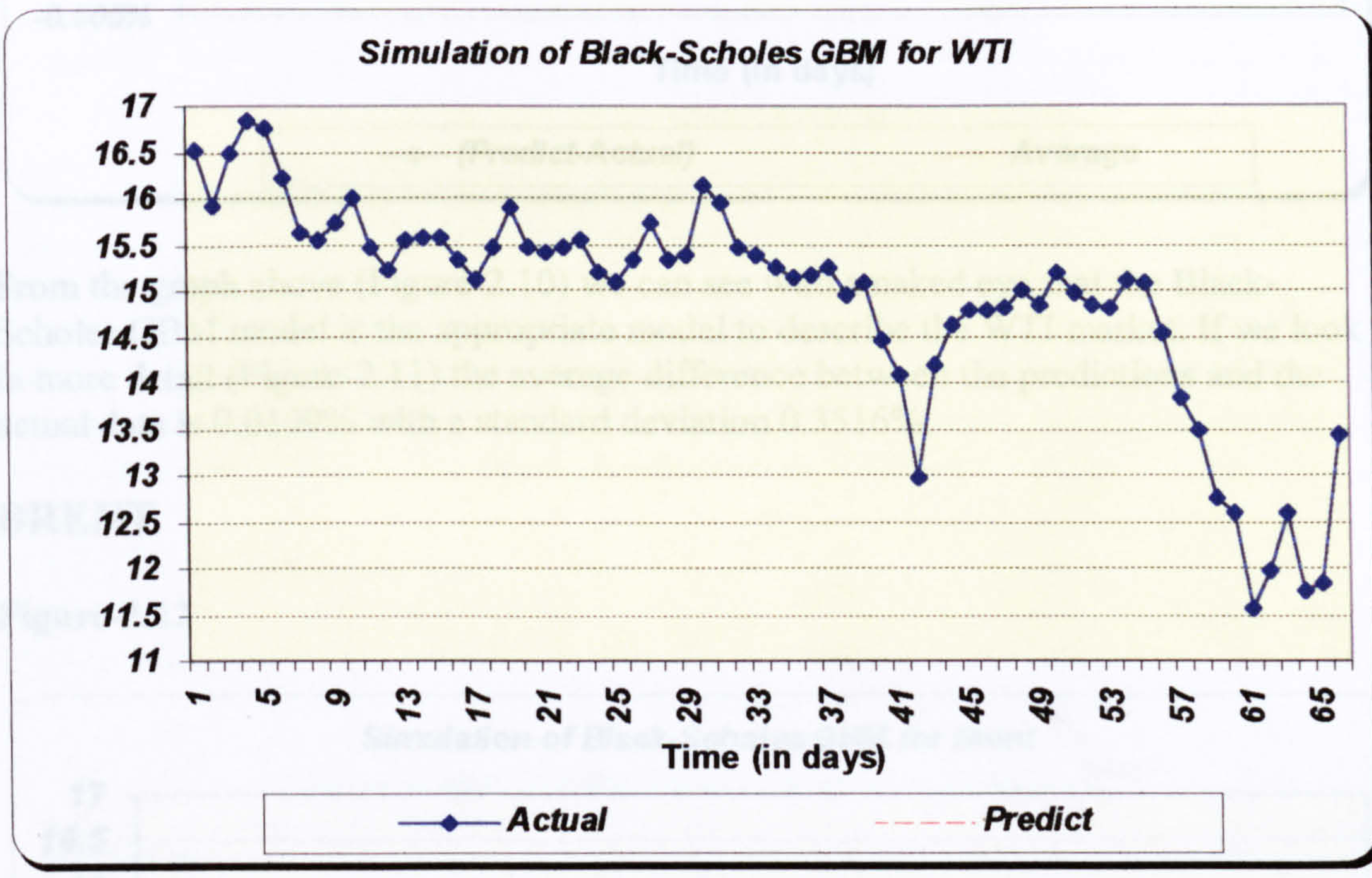
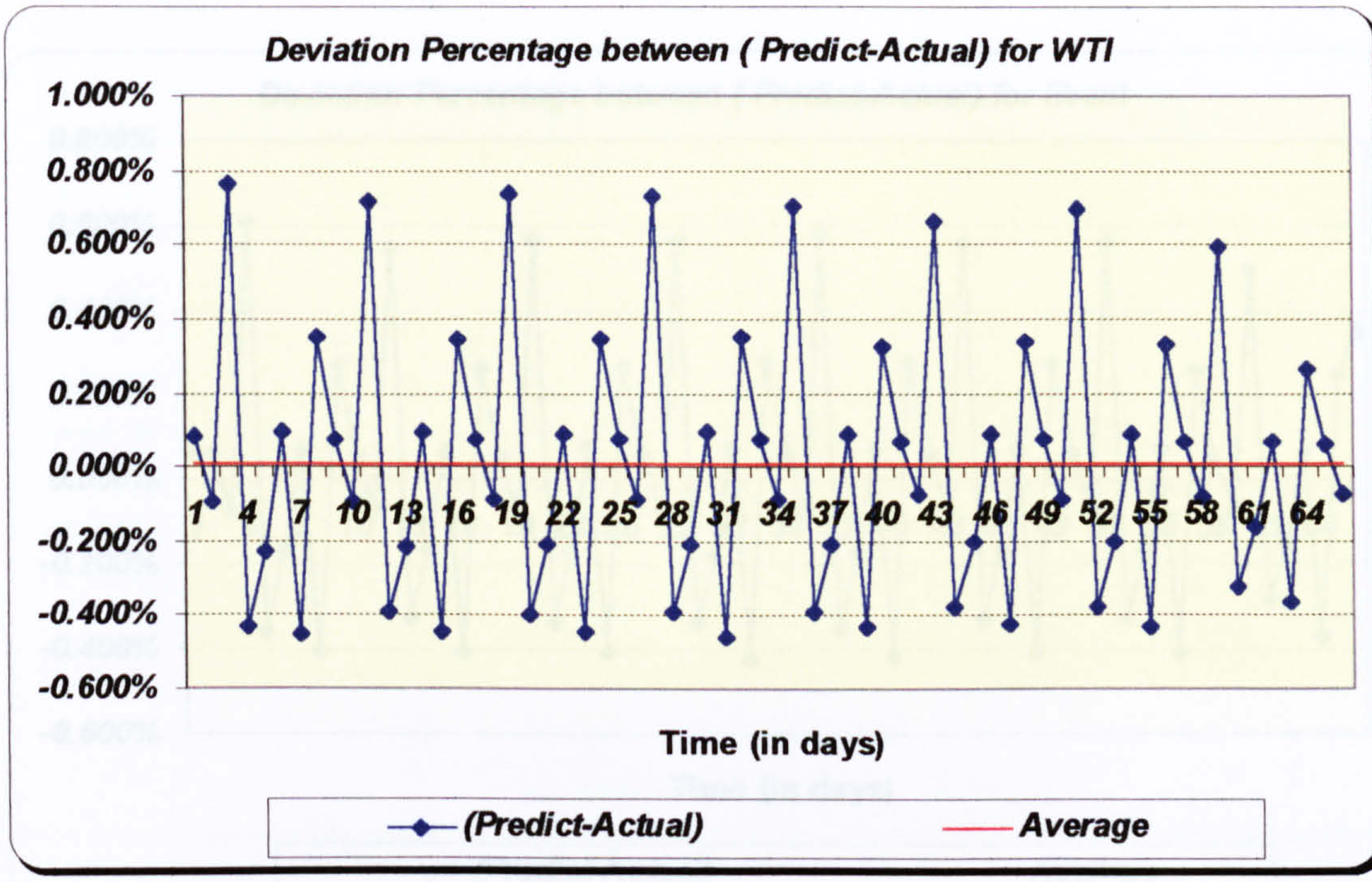


Figure-2.11



From the graph above (Figure-2.10) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the WTI market. If we look in more detail (Figure-2.11) the average difference between the predictions and the actual data is 0.0109% with a standard deviation 0.3516%.

BRENT

Figure-2.12

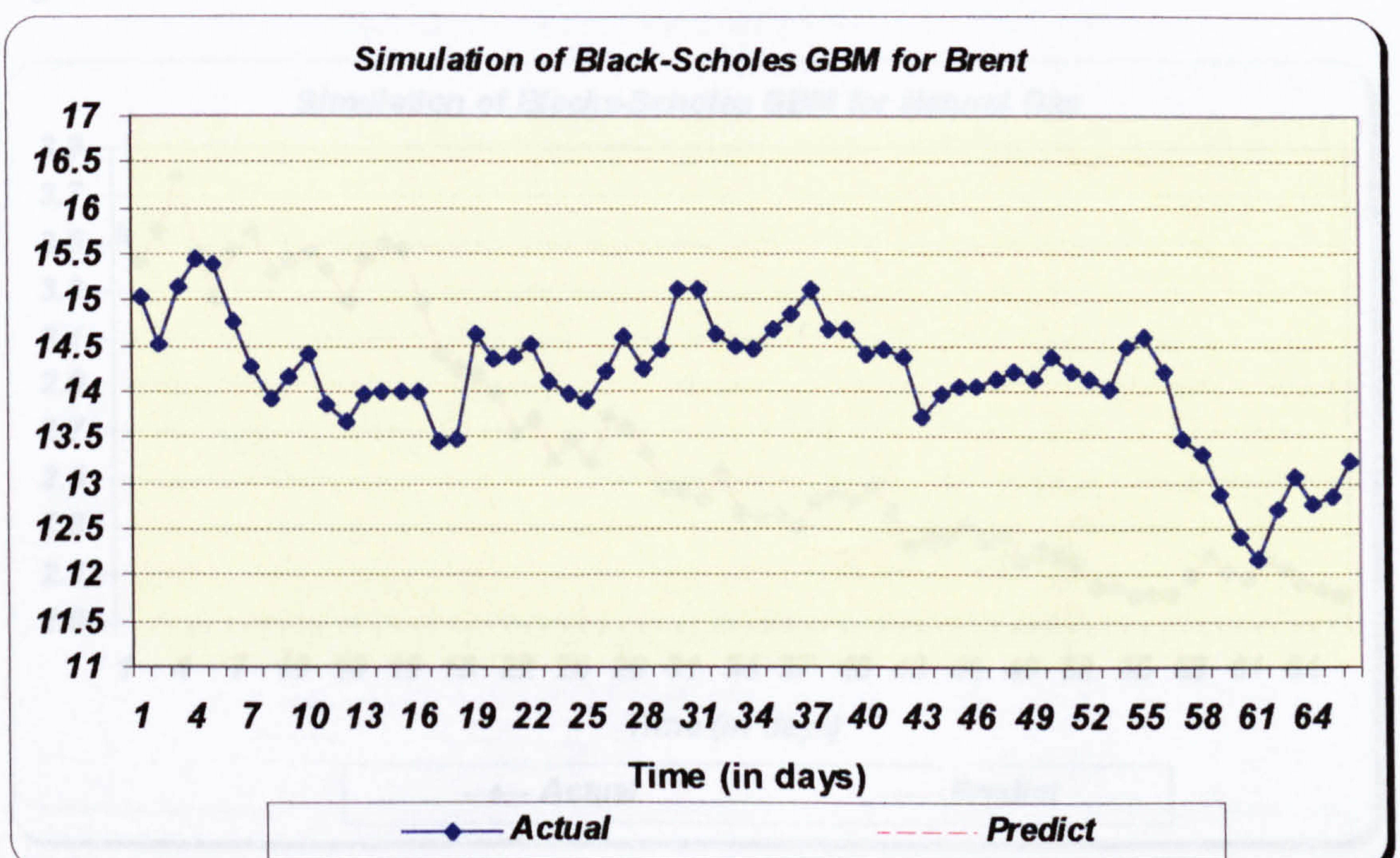
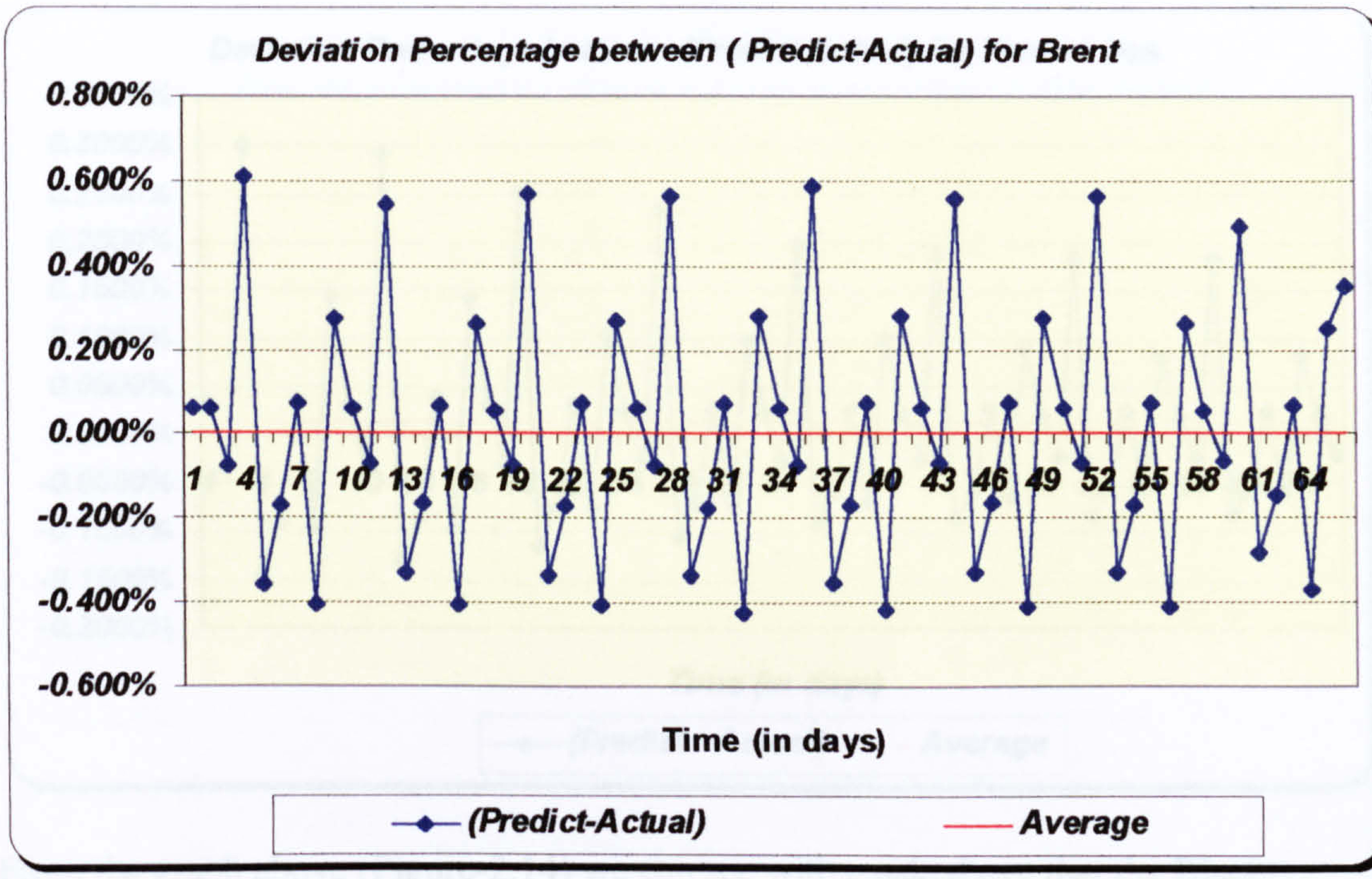


Figure-2.13



From the graph above (Figure-2.12) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the Brent market. If we look in more detail (Figure-2.13) the average difference between the predictions and the actual data is 0.0051% with a standard deviation 0.2976%.

NATURAL GAS

Figure-2.14

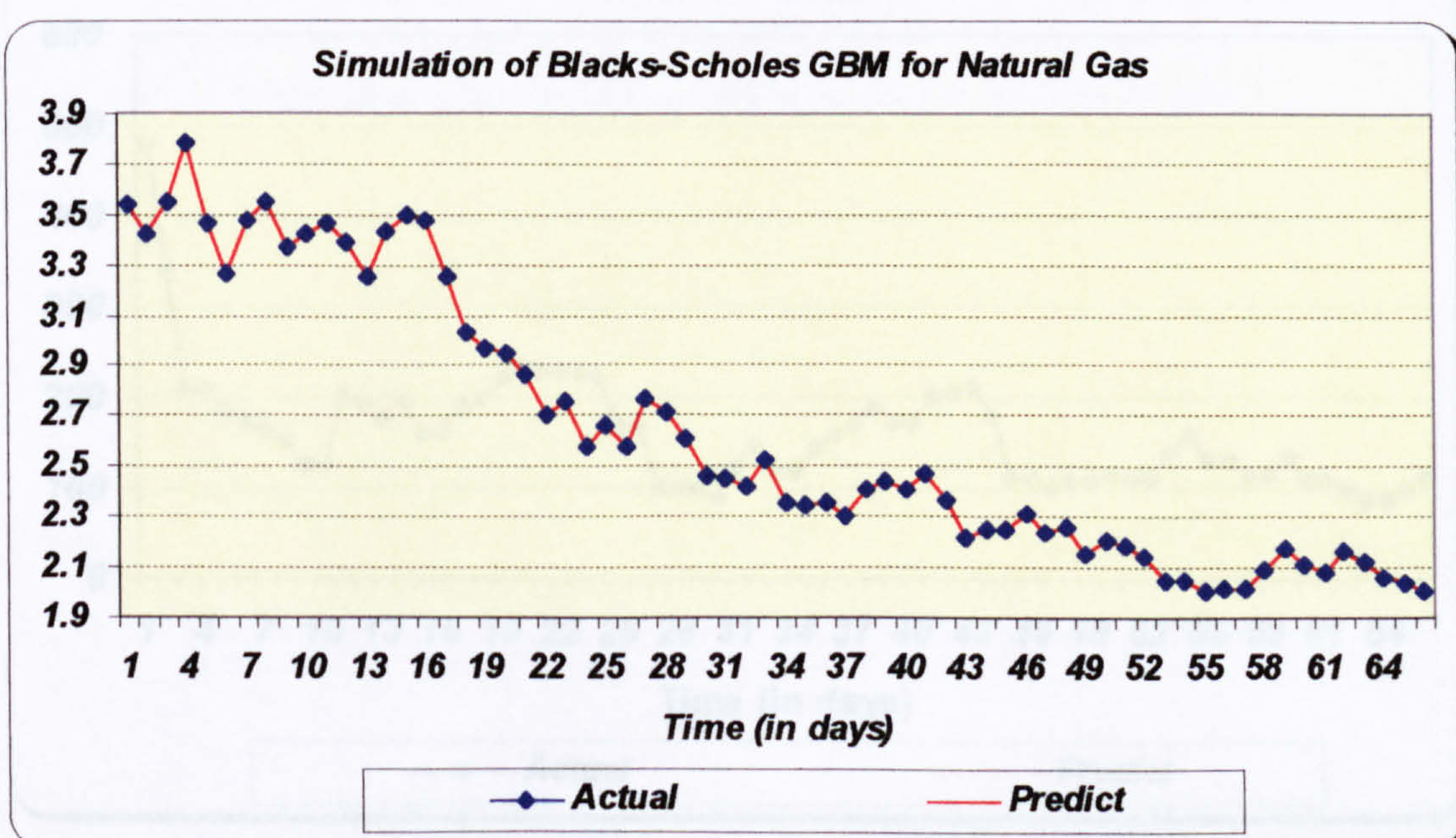
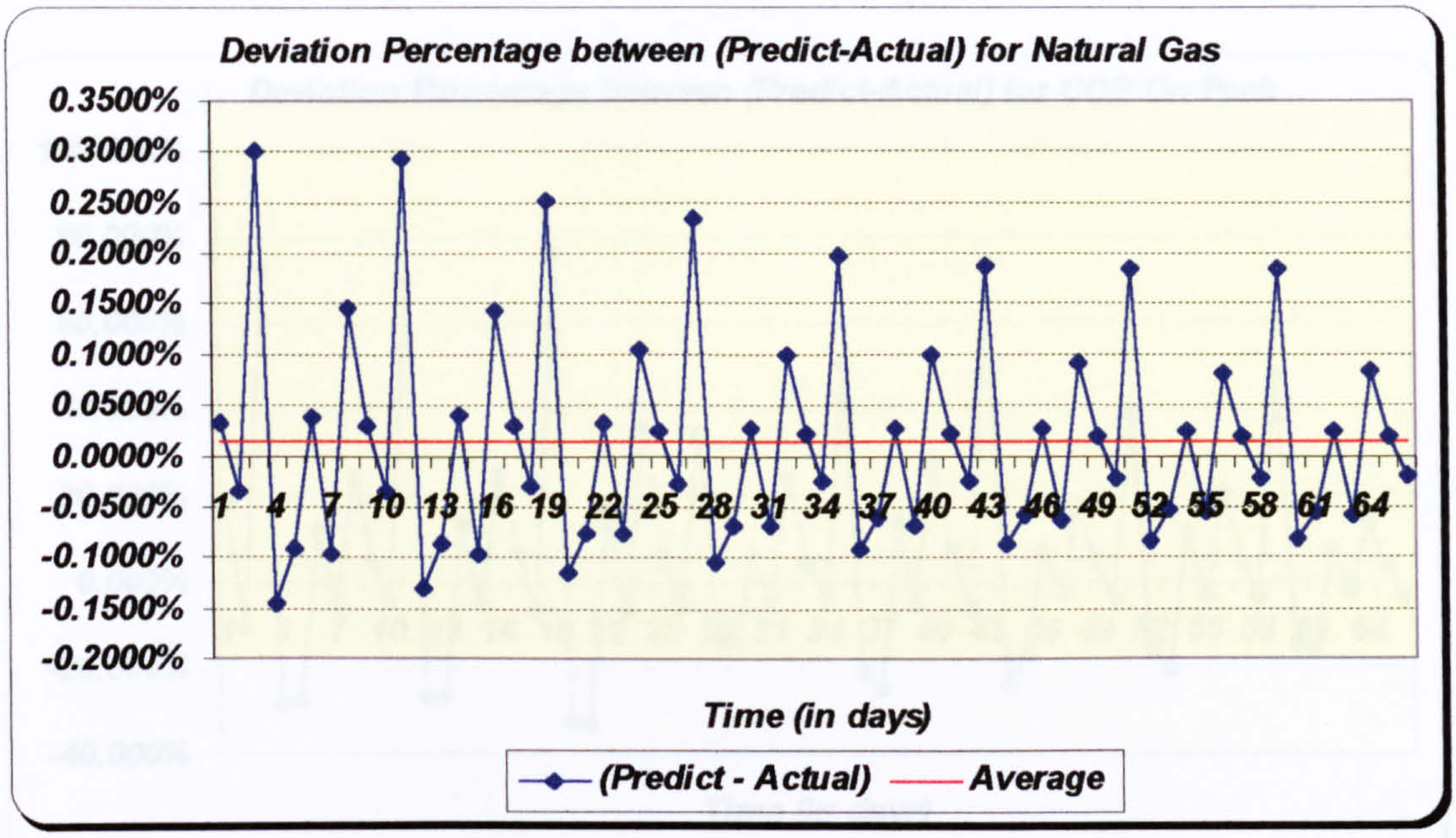


Figure-2.15



From the graph above (Figure-2.14) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the Natural Gas market. If we look in more detail (Figure-2.15) the average difference between the predictions and the actual data is 0.0141% with a standard deviation 0.1048%.

COB On Peak Prices

Figure-2.16

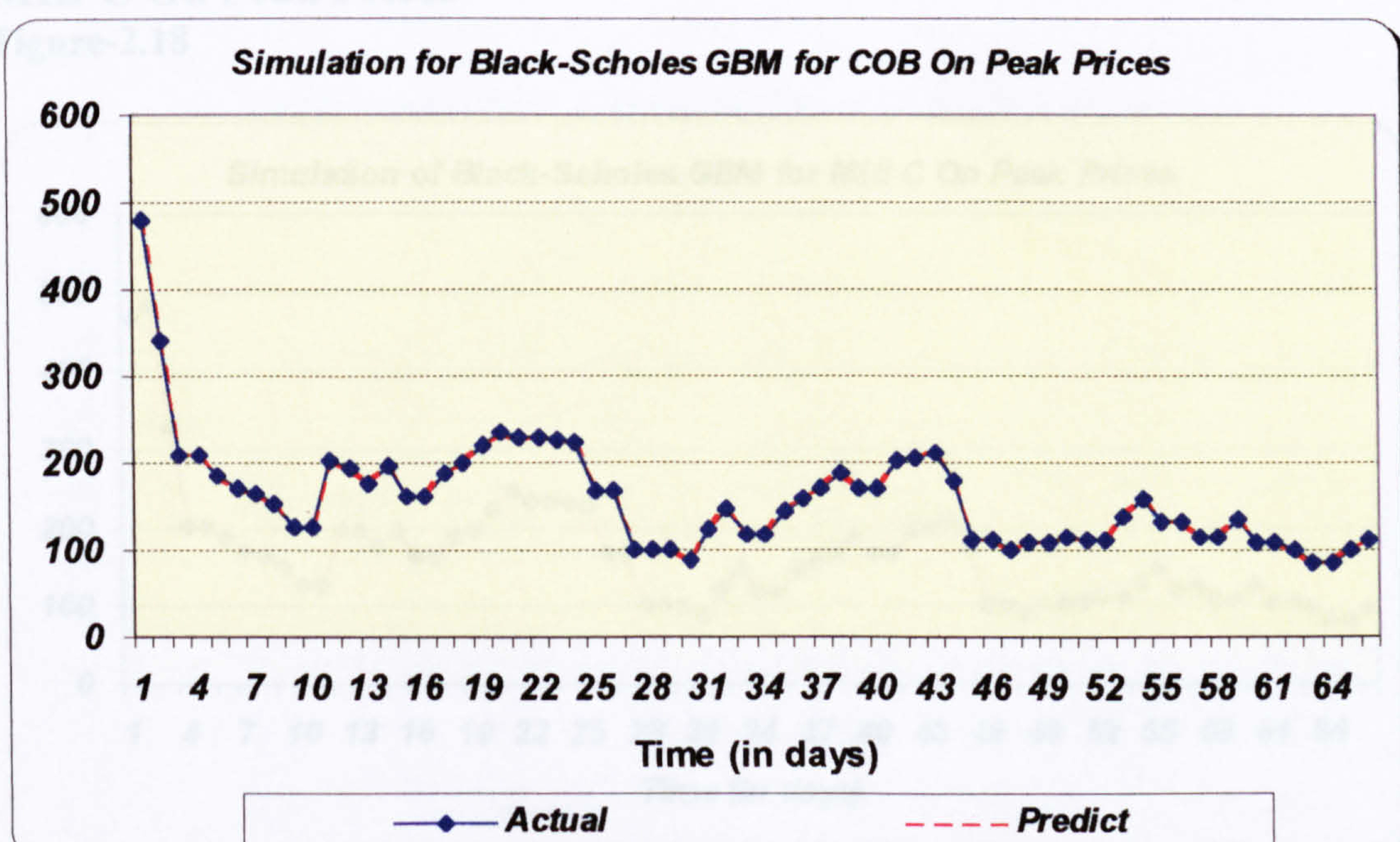
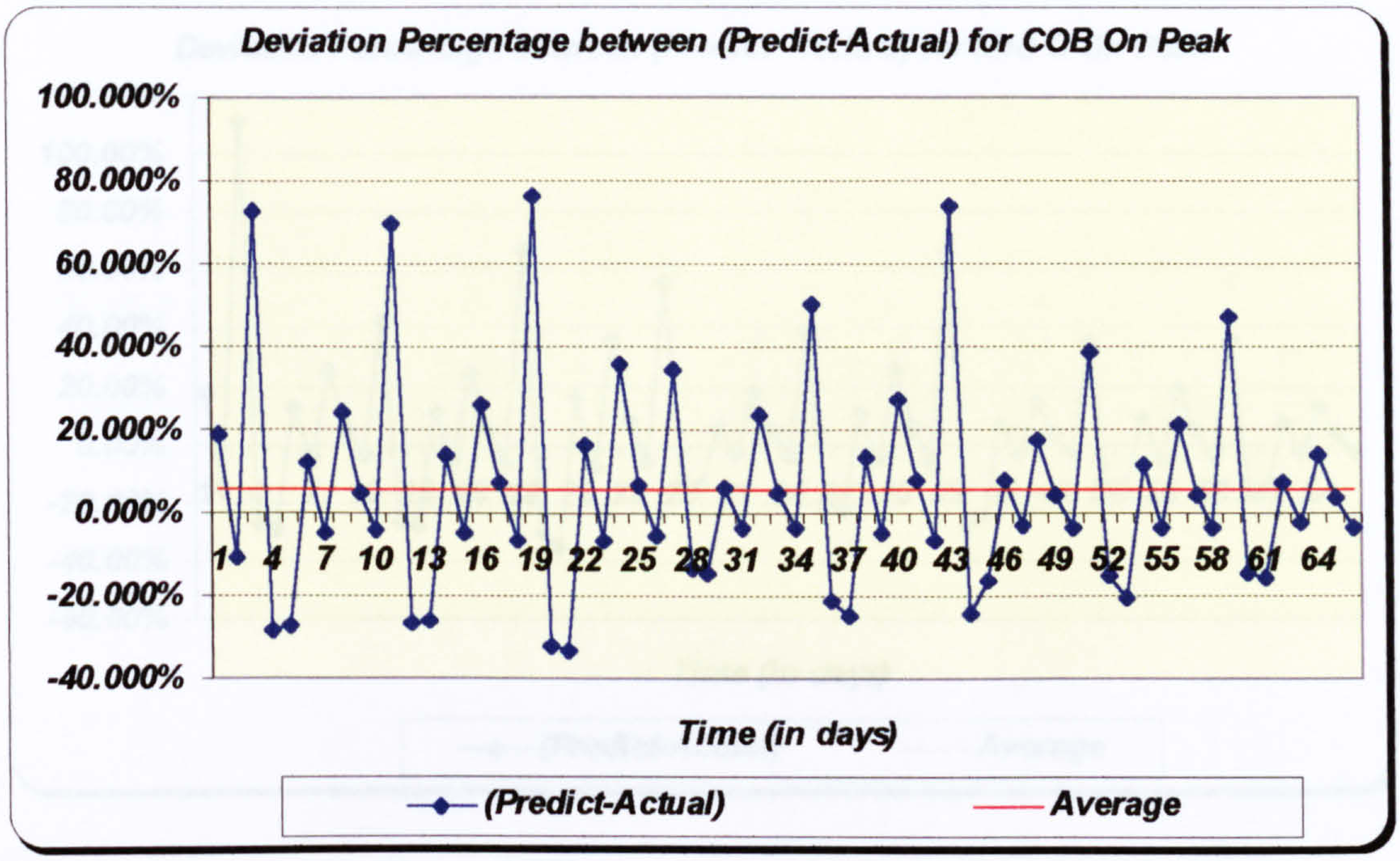


Figure-2.17



From the graph above (Figure-2.16) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the COB On Peak electricity market. If we look in more detail (Figure-2.17) the average difference between the predictions and the actual data is 5.545% with a standard deviation 25.554%.

MID C On Peak Prices

Figure-2.18

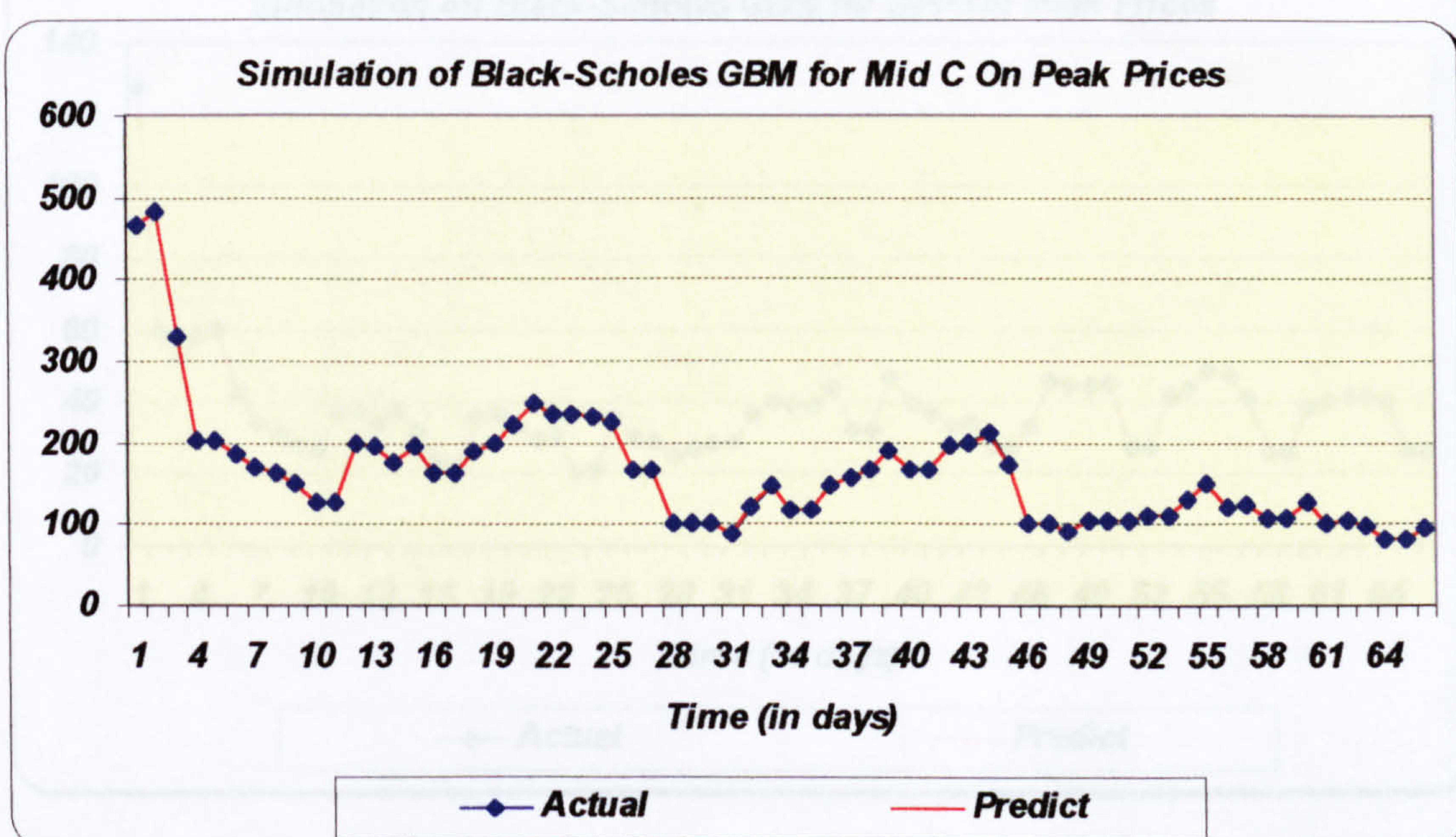
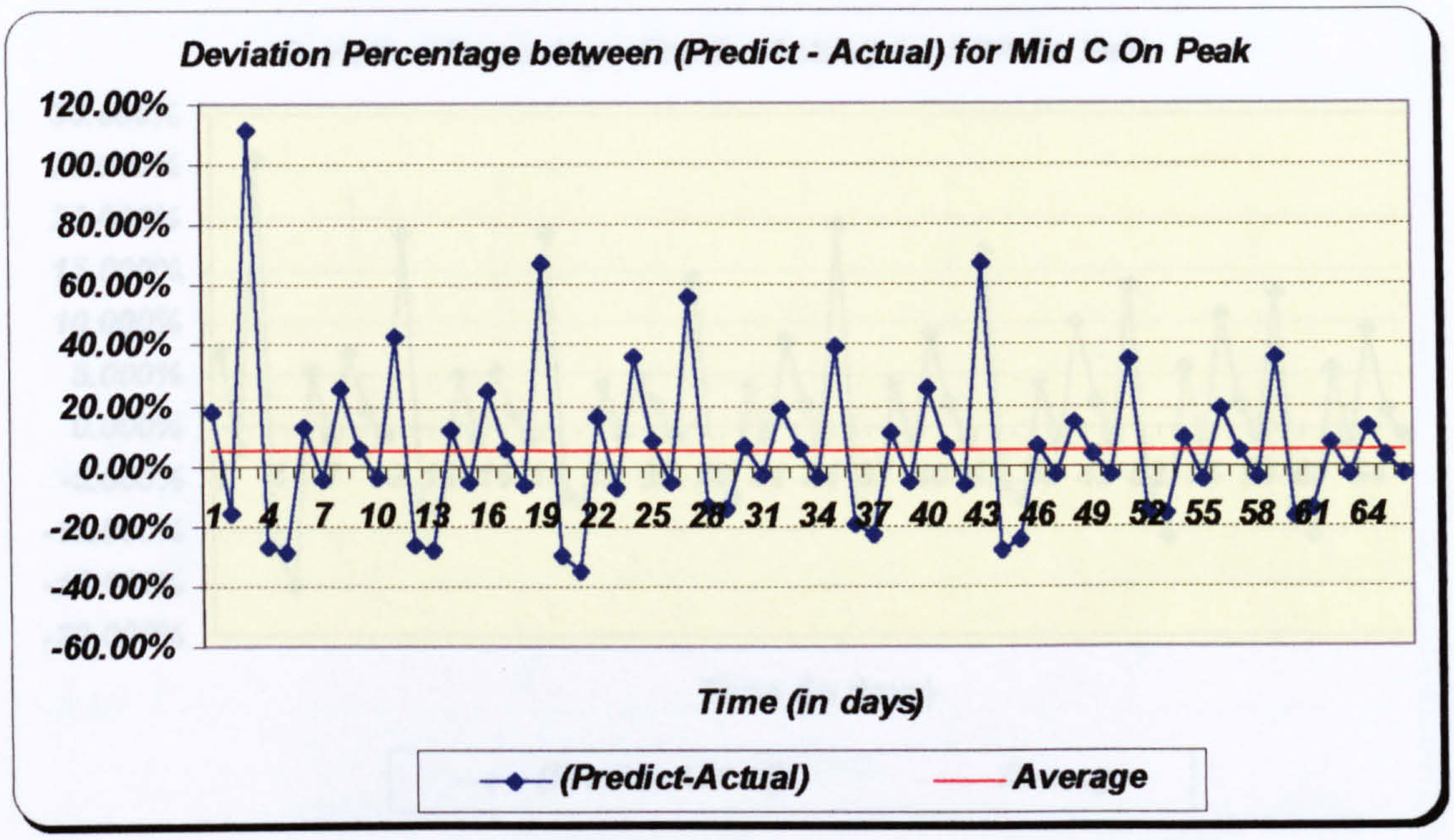


Figure-2.19



From the graph above (Figure-2.18) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the Mid C On Peak electricity market. If we look in more detail (Figure-2.19) the average difference between the predictions and the actual data is 5.05% with a standard deviation 26.03%.

SPP On Peak Prices

Figure-2.20

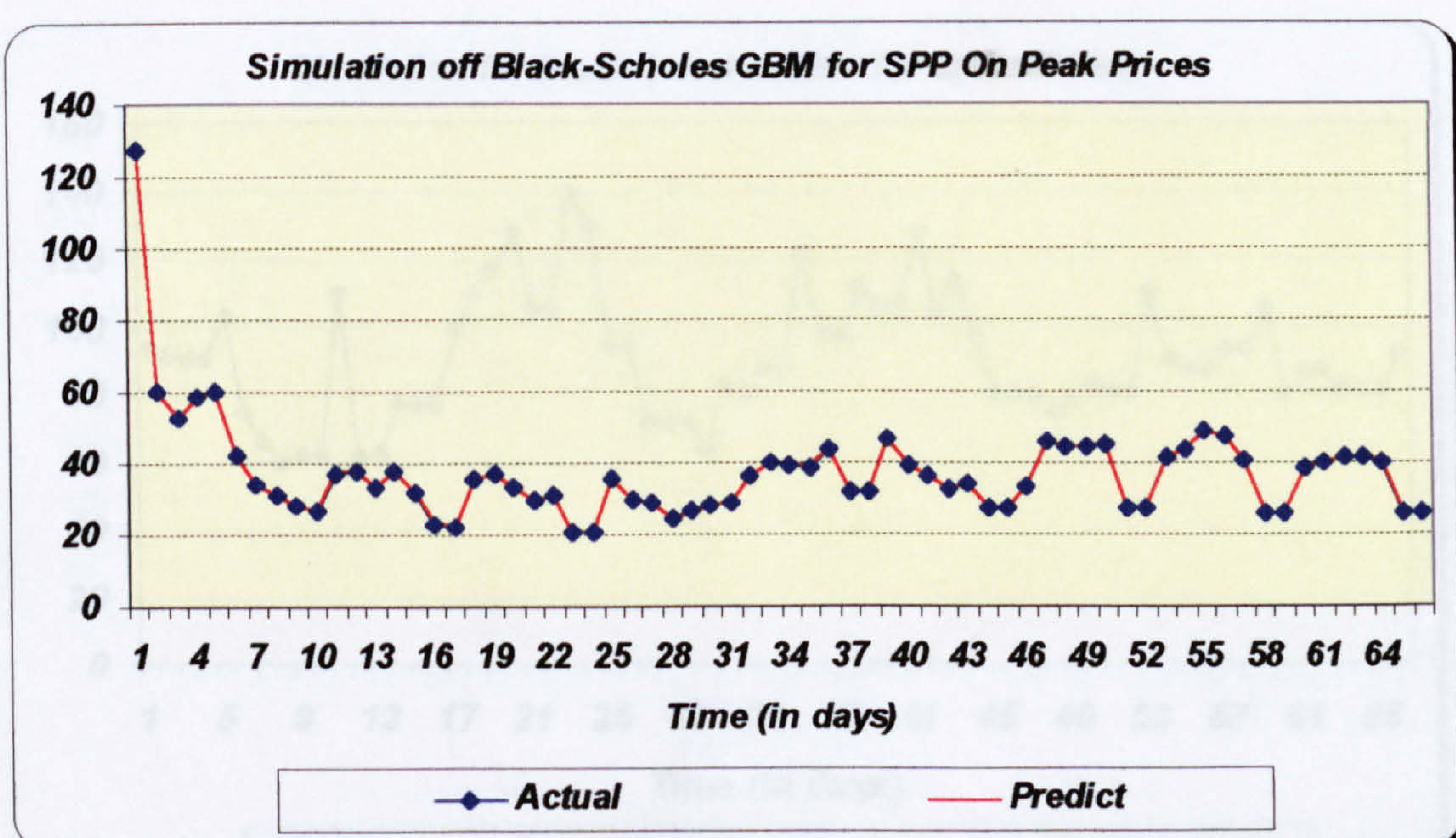
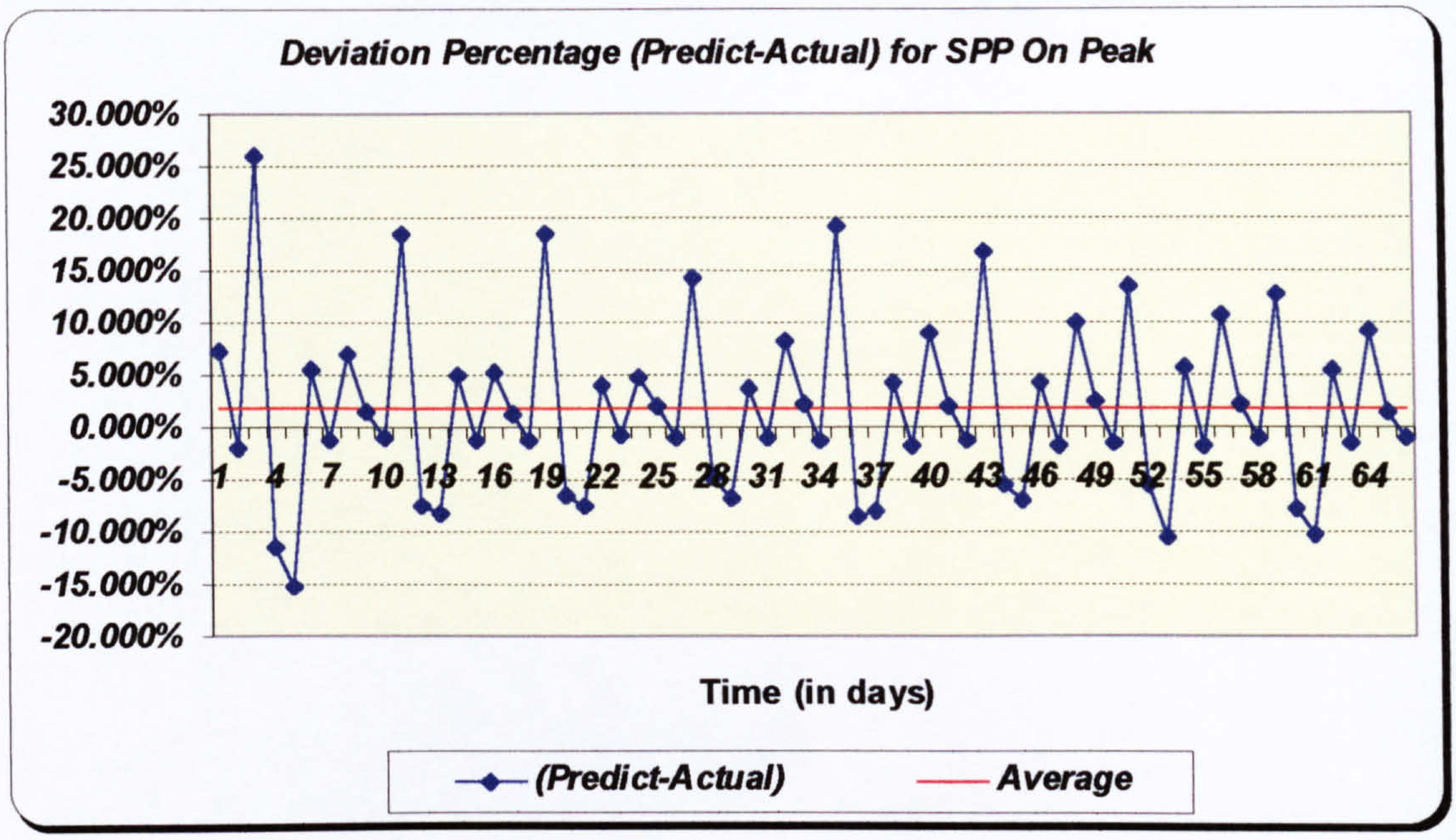


Figure-2.21



From the graph above (Figure-2.20) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the SPP On Peak electricity market. If we look in more detail (Figure-2.21) the average difference between the predictions and the actual data is 1.694% with a standard deviation 8.2316%.

COB Off Peak Prices

Figure-2.22

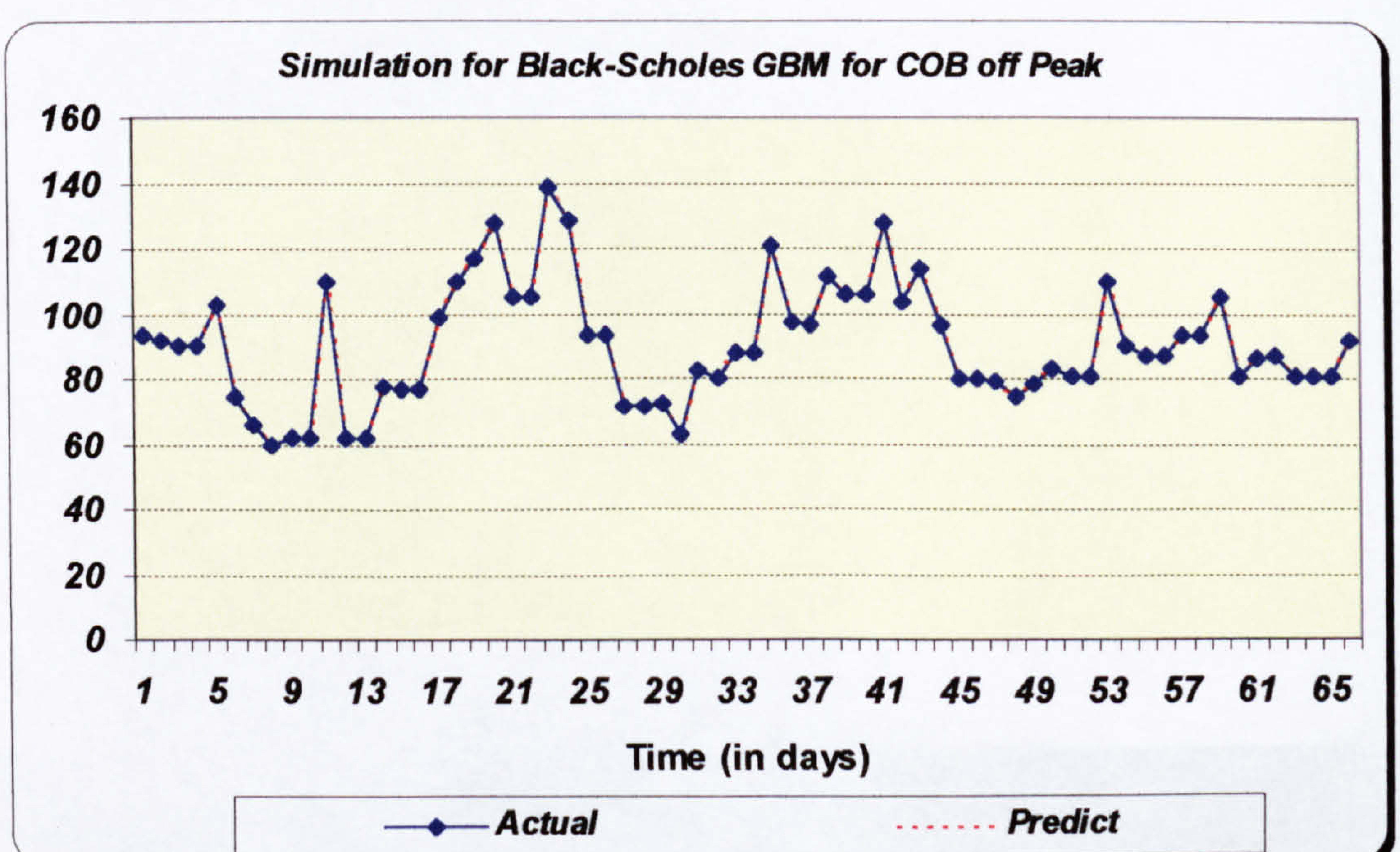
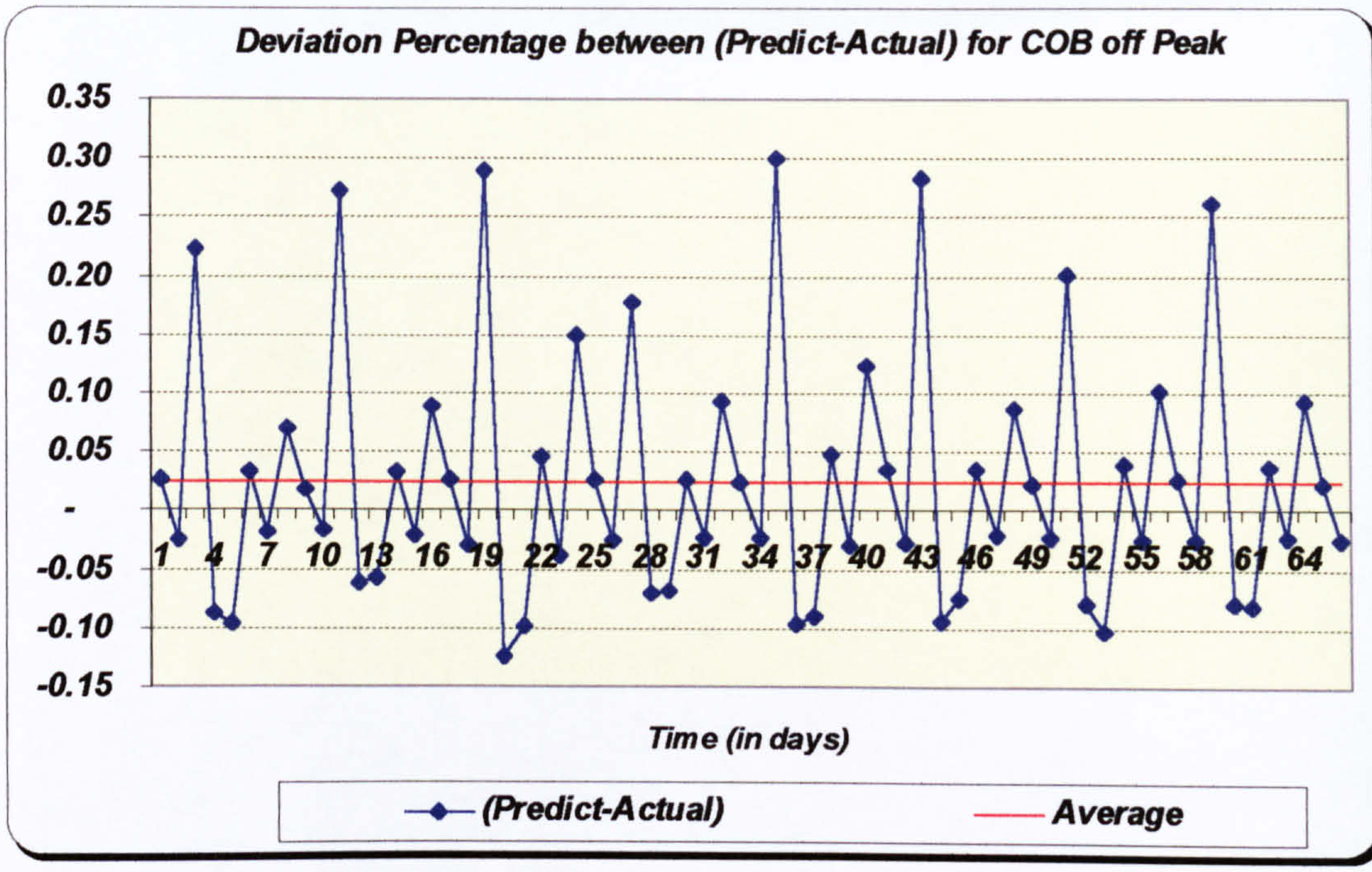


Figure-2.23



From the graph above (Figure-2.22) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the COB Off Peak electricity market. If we look in more detail (Figure-2.23) the average difference between the predictions and the actual data is0.024with a standard deviation 0.105.

MID C Off Peak Prices

Figure-2.24

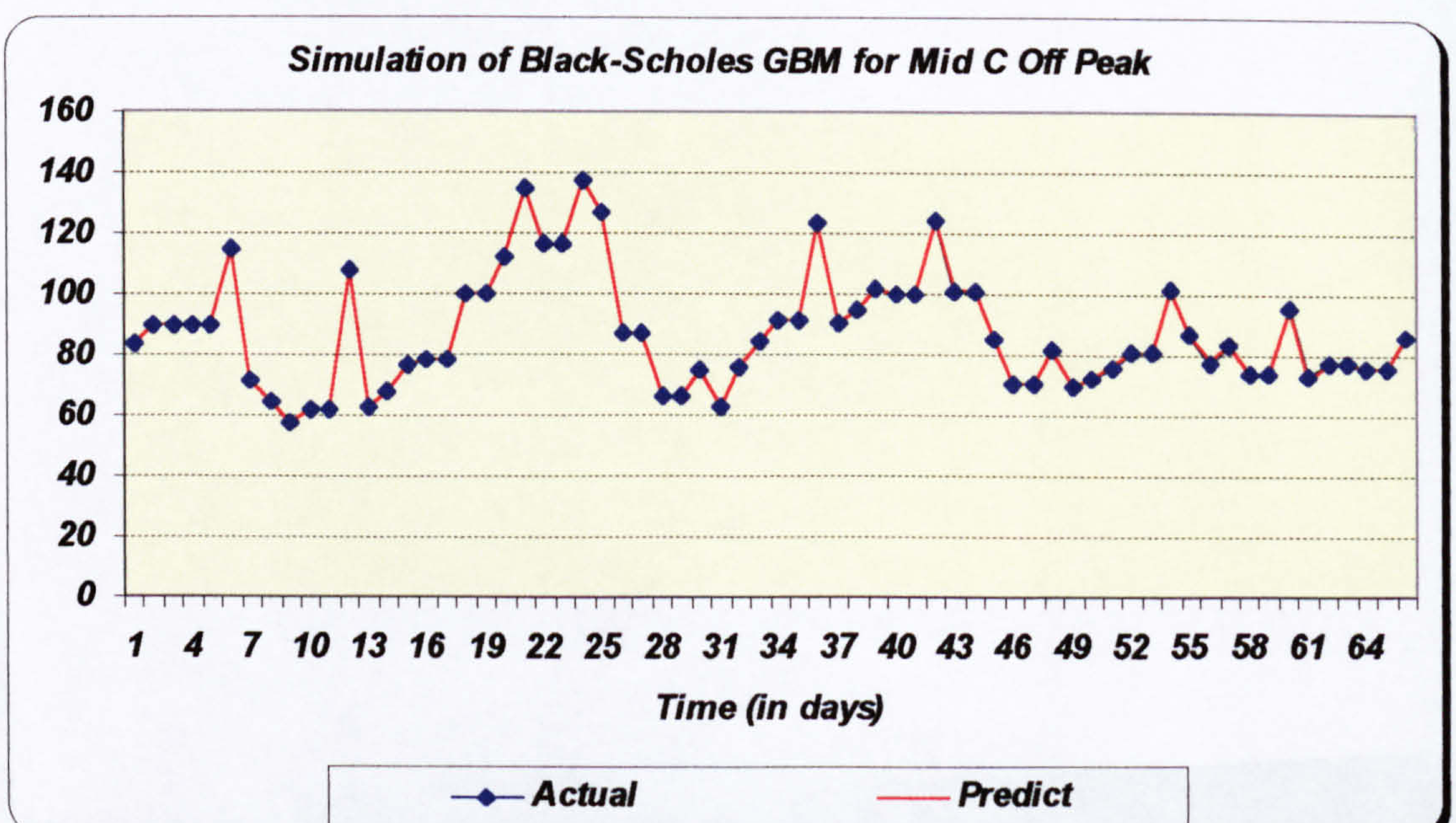
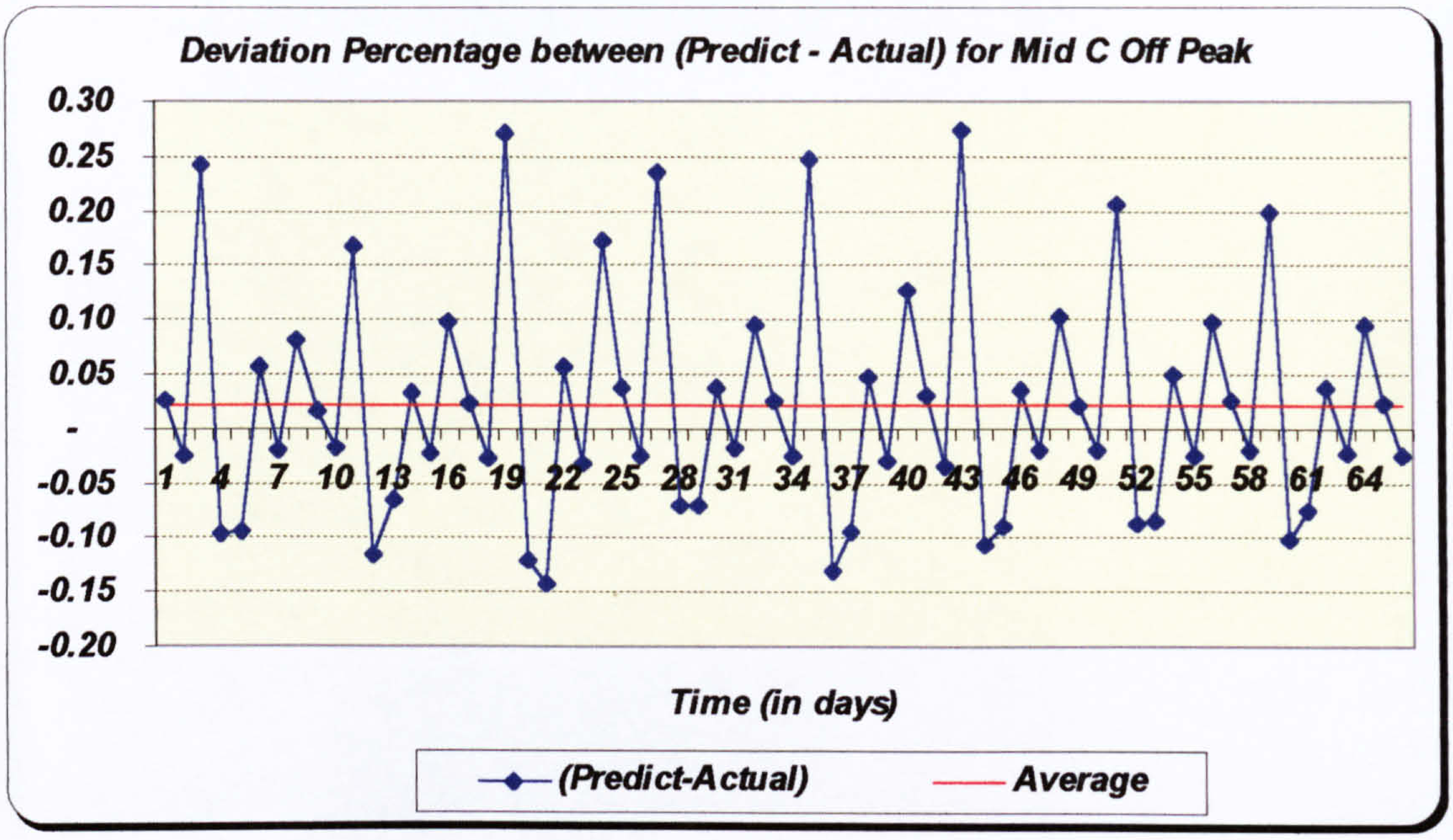


Figure-2.25



From the graph above (Figure-2.24) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the Mid C Off Peak electricity market. If we look in more detail (Figure-2.25) the average difference between the predictions and the actual data is0.0204with a standard deviation 0.1033.

SPP Off Peak Prices

Figure-2.26

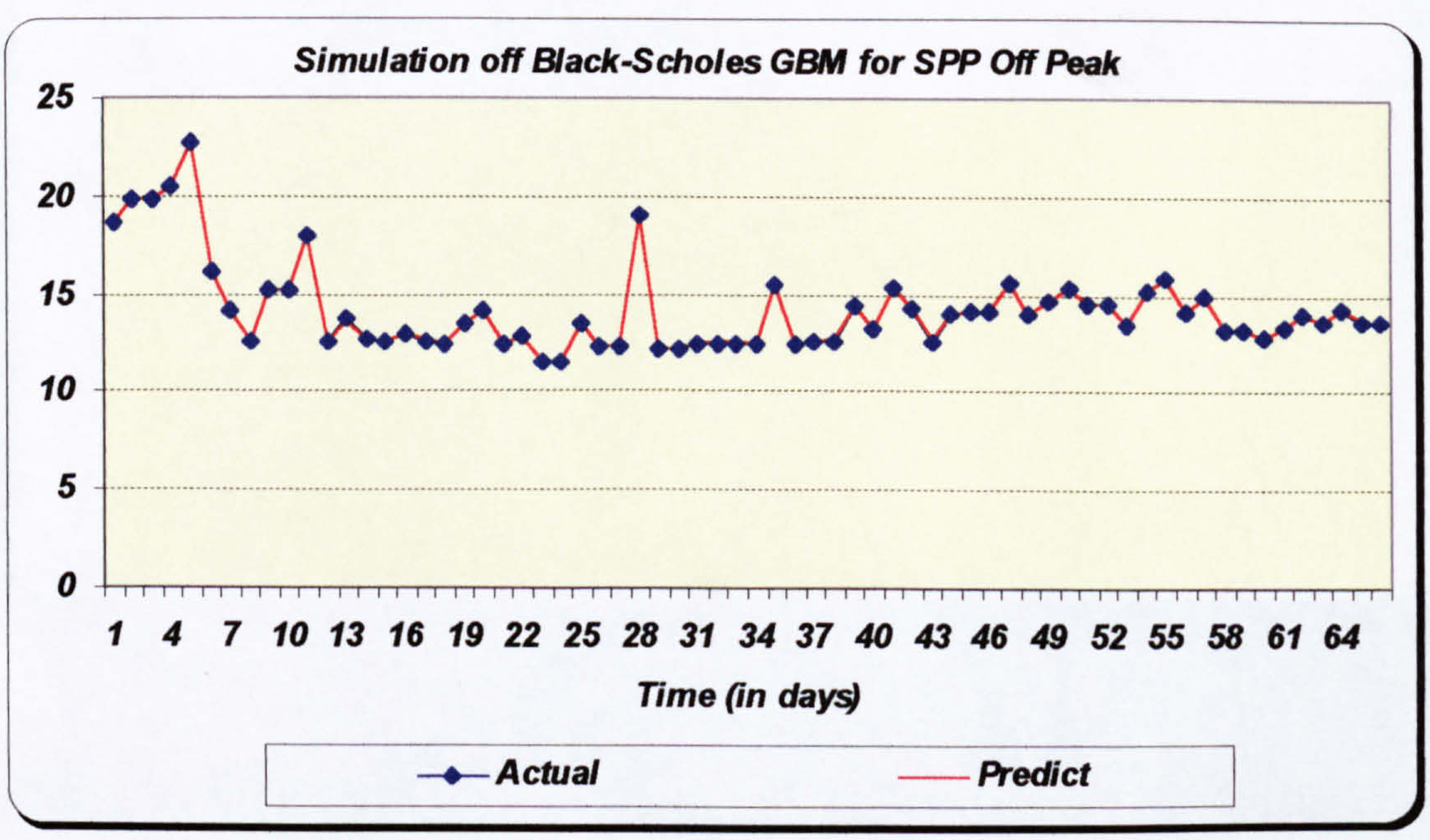
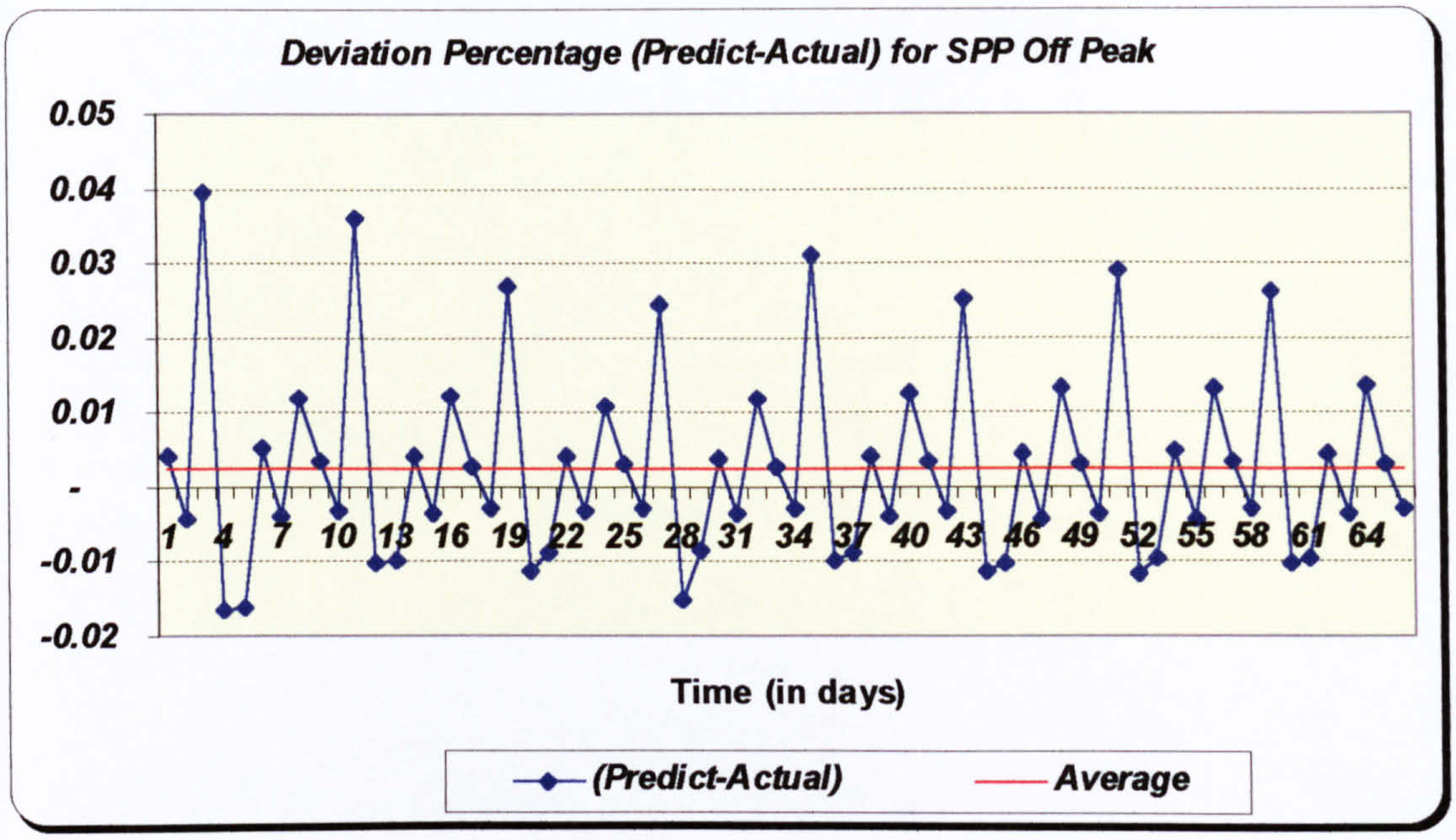


Figure-2.27



From the graph above (Figure-2.26) we can see with a naked eye that the Black-Scholes GBM model is the appropriate model to describe the SPP Off Peak electricity market. If we look in more detail (Figure-2.27) the average difference between the predictions and the actual data is0.0025with a standard deviation 0.0128.
Based on the results above, we conclude that the Black-Scholes GBM approach is the appropriate for modelling the oil, natural gas and electricity market for the periods we tested the model. Because of that we can develop computer algorithms for pricing the main derivatives products used in the energy markets based on the Black-Scholes environment.

PART-B

The Black-Scholes closed form solution for option prices is probably the most famous option pricing methodology. It is so easy to use so that it can be implemented on the trading floor by the traders themselves.

In this simple world, we make the assumption that an option position can be perfectly hedged with the spot price and we can use the bank's services to borrow and lend money at a risk-free interest rate. This leads us to derive the differential equation for the option price:

$$\frac{\vartheta C}{\vartheta t}+\frac{1}{2}\frac{\vartheta^2 C}{\vartheta S^2}\sigma^2 S^2+rS\frac{\vartheta C}{\vartheta S}-rC=0 \qquad (2.4)$$

where: C = call option price

S = spot price

K = strike price

r = discount (risk-free) rate

σ = spot price volatility

Solving this differential equation and imposing the boundary constraint that the option price must equal the option parity value at expiration, we obtain the closed-form solution for the option price:

$$C_{BS} = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{2.5}$$

$$d_1 = \frac{\ln(S/K) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{2.6}$$

$$d_2 = \frac{\ln(S/K) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{2.7}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{2.8}$$

$$N(x) = \int_{-\infty}^{x} \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy \tag{2.9}$$

where: T = time of option expiration

t = time of option evaluation

N (.) = cumulative normal distribution function

This is the famous Black-Scholes option solution. It is a function of the current spot price, the spot price volatility, the risk-free rate, option's strike price, and the time to option's expiration. The value of a European put can be calculated in a manner similar to a European call. The closed-form solution is the following:

$$P_{BS} = Ke^{-r(T-t)}N(-d_2) - SN(-d_1) \tag{2.10}$$

The computer algorithm of the Black Scholes Model is shown in the Appendix-2.2 of this chapter.

2.4 The Black Model

If, instead, the option settles not on the spot price at the time of the option's expiration, but rather a forward price, we end up using the forward to hedge the option and not the spot price. The forward price on a lognormal spot price, as defined above, is given by:

$$F = Se^{r(T-t)} \tag{2.11}$$

and the change in the forward price over time dt is then given by:

$$dF = (\mu - r)Fdt \tag{2.12}$$

Since the forward price contract is an arrangement that carries no cost of financing, our hedge to the option price requires no borrowing of money from the bank. This changes the option differential equation in (2.4) to:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\frac{\partial^2 C}{\partial F^2}\sigma^2 F^2 - rC = 0 \tag{2.13}$$

Solving this differential equation for the option price results in the closed-form solution in terms of the forward price rather than the spot price. This is the also famous Black option-pricing model:

$$C_B = Fe^{-r(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{2.14}$$

$$d_1 = \frac{\ln(F/K) + (\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{2.15}$$

$$d_2 = \frac{\ln(F/K) + (-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{2.16}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{2.17}$$

$$N(x) = \int_{-\infty}^{x} \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}}dy \tag{2.18}$$

where: T = time of expiration and forward price expiration.

t = time of option valuation.

N (.) = cumulative normal distribution function. (2.19)

The, price of the corresponding put option is given by:

$$P_B = Fe^{-r(T-t)}N(-d_1) - Ke^{-r(T-t)}N(-d_2) \tag{2.20}$$

The computer algorithm of the Black's Model is shown in the Appendix-2.3 of this chapter.

2.5 Approximations to Closed Form Solutions

Making simplifying assumptions in the derivation of closed-form solutions, such us that the option settlement prices are lognormal with a constant volatility-i.e. flat volatility term structure-and that the settlement price is defined by a single factor model, when in reality this may not be the case, leads us to come up with approximation and/or correction techniques that allows us to continue using the closed form solutions in complex markets.
One approximation technique is as simple as adjusting the volatility inputs fed into the closed form solution, to properly reflect the way markets act. Or they may be as complicated as calculating the higher order correction terms to the closed form solution to capture the market option prices. Either way, we still end up with an equation for calculating the option prices, and as such it remains relatively easy to program and use on the trading desk.
Such corrections to Black-Scholes or Black option pricing equations allow us to price fairly easily all kinds of European-style options, including Asian options on averages of prices, whose settlement price may be path-dependent.
The potential problem of making adjustments to closed-form solutions is that we have to know when it is appropriate to do so, and when the corrections simply do not capture all these is to capture. Thus such methodology always need to be used with caution and with an understanding (by the traders) of what its boundaries are.
The model corrections attempt to allow for correct pricing. However, even if we achieve this, we are still left with potentially risk calculations. This is probably the greatest drawback of this methodology. The tree building methodology, the binomial model offers a way to get this right as well.

2.6 Binomial Option Pricing

The binomial method is certainly the most widely used numerical method to price to price American options on stocks, futures and currencies. The method was published by Cox, Ross and Rubinstein (1979) and Rendleman and Barter (1979).They introduced how to construct a recombining binomial tree that discretize the geometric Brownian motion. At the limit, a binomial tree (with a very large number of time steps) is equivalent to the continuous-time Black-Scholes formula when pricing European options. More interesting, the binomial model handles the pricing of American options, where no closed-form solution exists, as well as several exotic options. In a sense, in the binomial tree approach the life of the option is subdivided into a number of time intervals. In each interval, the price of the underlying can move into a small number of states. For example, in the binomial tree method, the price in an interval can go up with a probability, say p, and go down with a probability (1-p). The magnitudes and probabilities of the price shifts are determined from the stochastic process assumed for the price of the underlying by requiring that the distribution of the tree prices has the correct mean and variance at each time step. The binomial model described in this section is the well-known Cox-Ross-Rubinstein binomial tree. Including the cost of carry term b, the model can be used to price European and American Options on stocks ($b = r$), stocks and stock indexes paying a continuous dividend yield q (b = r-q), futures (b = 0), and currency options with foreign interest rate r_f ($b = r - r_f$). The asset price of each node is set equal to:

$$S u^i d^{j-i}, \quad i = 0, 1, \ldots\ldots\ldots\ldots, j \qquad (2.21)$$

Where the up and down jump size that the asset price can take at each time step Δt apart is given by

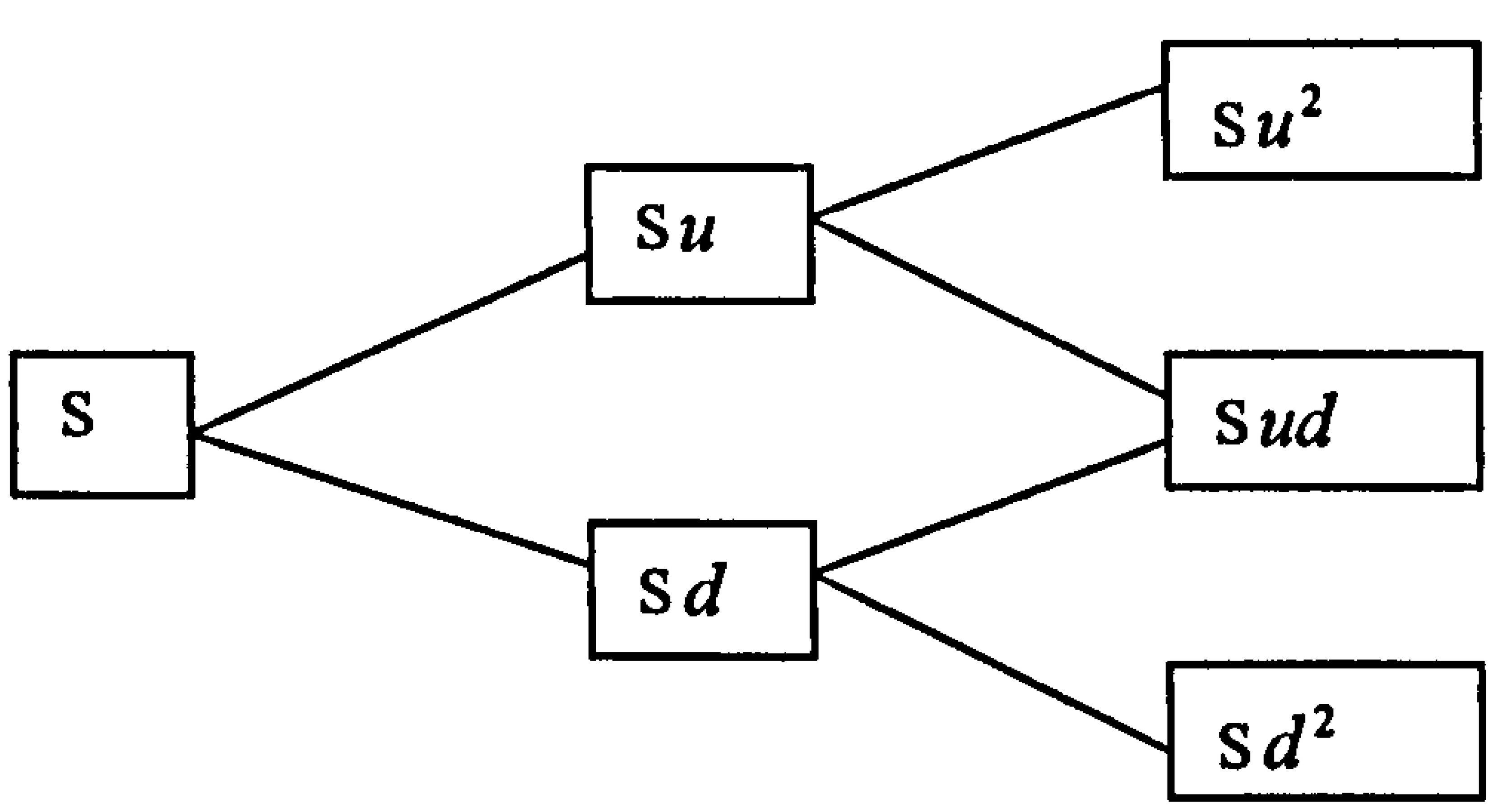
$$u = e^{\sigma\sqrt{\Delta t}} \quad (2.22), \quad d = e^{-\sigma\sqrt{\Delta t}} \quad (2.23)$$

where $\Delta t = T/n$ is the size of each time step, and n is the number of time steps. The probability of the stock price increasing at the next step is the following:

$$p = \frac{e^{b\sqrt{\Delta t}} - d}{u - d} \qquad (2.24)$$

The probability of going down must be 1-p and the probability of going either up or down equals unity. The up and down jump size and the up and down probability are chosen to match the first two moments of the stock price distribution (mean and variance). This ensures that the binomial tree is the discretization of the geometric Brownian motion.
The value of the American put option can now easily be found by standard back ward induction.

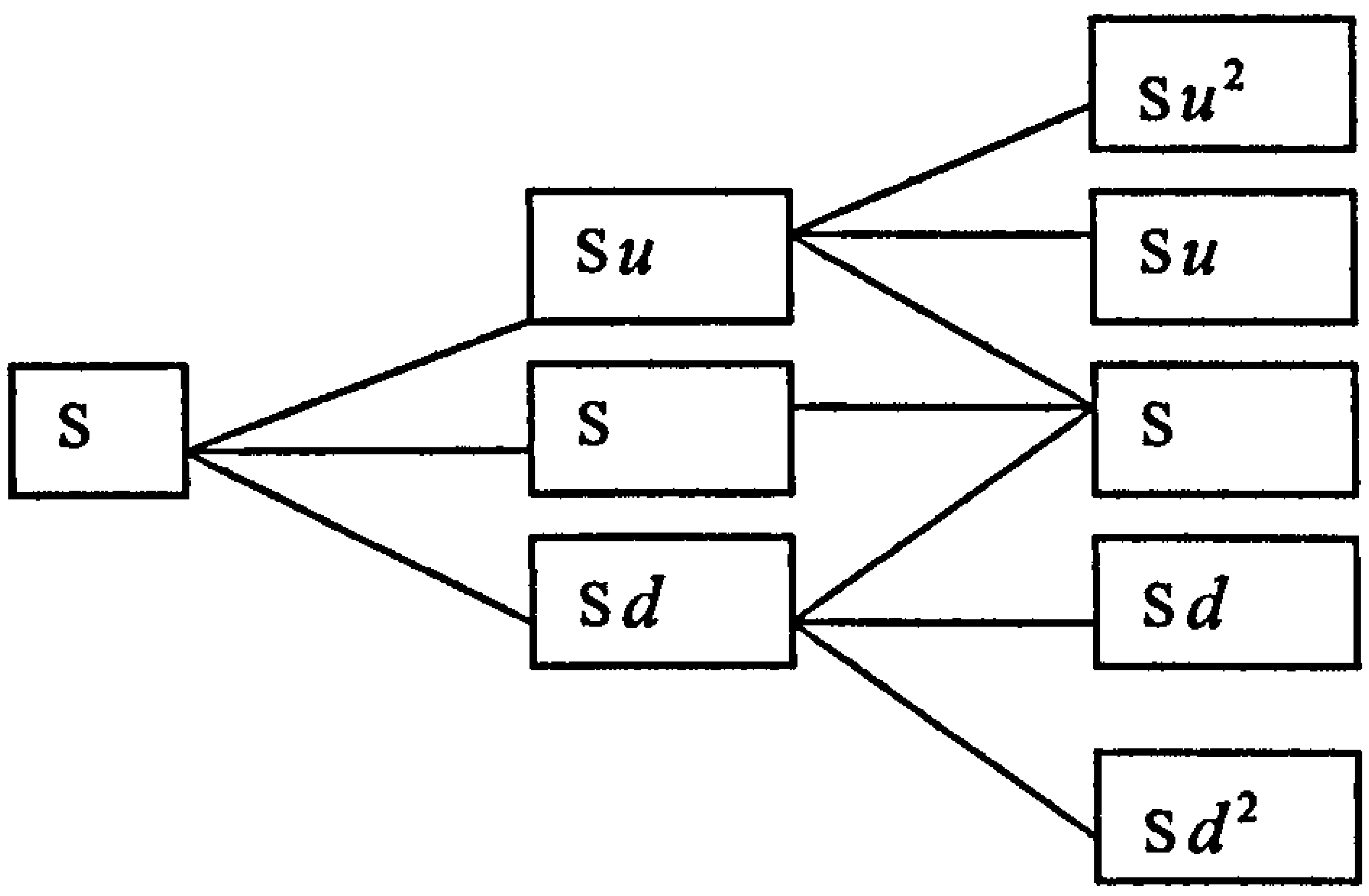
$$P_{j,i} = \max\{X - S\, u^i d^{j-i}, e^{-r\Delta t}\,[p\, P_{j+1,i+1} + (1-p)\, P_{j+1,i}]\} \qquad (2.25)$$



The computer algorithm of the Binomial Model is shown in the Appendix-2.4 of this chapter.

2.7 Trinomial Option Pricing

Trinomial trees in option pricing, introduced by Boyle (1986), are similar to binomial trees, only now instead of the up-and-down move, we have one more degree of freedom: the sideways move. Trinomial trees can be used to price both European and American options on a underlying asset.



Because the asset price can move in three directions from a given node, compared with only two in a binomial tree, the number of time steps can be reduced to attain the same accuracy as in the binomial tree. This makes trinomial trees more efficient than binomial trees.
There are several ways to choose jump size and move the probabilities in a trinomial tree that give the same result when the number of time steps is large. To discretize a geometric Brownian motion, the jump sizes and probabilities must match the first two moments of the distribution (the mean and the variance). One possibility is to build a trinomial tree where the asset price at each node can go up, stay at the same level or go down. In that case, the up-and –down jump sizes are:

$$u = e^{\sigma\sqrt{2\Delta t}} \quad (2.26), \quad d = e^{-\sigma\sqrt{2\Delta t}} \quad (2.27)$$

and the probability of going up and down respectively are:

$$p_u = \left(\frac{e^{b\Delta t/2} - e^{-\sigma\sqrt{\Delta t/2}}}{e^{\sigma\sqrt{\Delta t/2}} - e^{-\sigma\sqrt{\Delta t/2}}}\right)^2 \tag{2.28}$$

$$p_d = \left(\frac{e^{\sigma\sqrt{\Delta t/2}} - e^{b\Delta t/2}}{e^{\sigma\sqrt{\Delta t/2}} - e^{-\sigma\sqrt{\Delta t/2}}}\right)^2 \tag{2.29}$$

The probabilities must sum to unity. Thus the probability of staying at the same asset price level is:

$$p_m = 1 - p_u - p_d \tag{2.30}$$

T is the time to maturity in years, b is the cost of carry, $\Delta t = T/n$ is the size of each time step, and n is the number of time steps.
The computer algorithm of the Trinomial Option Model is shown in the Appendix-2.5 of this chapter.

2.8 Spread Option Pricing

The payoff from a European call spread option on two futures contracts is max (F_1- $F_2 - X$,0). The payoff from a put option is similarly max ($X - F_1 + F_2$,0). A European spread option on forwards or futures contracts can be valued by using the standard Black (1976) model by performing the following transformation:

$$c = \max(F_1 - F_2 - X, 0) = \max(\frac{F_1}{F_2 + X} - 1, 0)(F_2 + X) \qquad (2.31)$$

$$p = \max(X - F_1 + F_2, 0) = \max(1 - \frac{F_1}{(F_2 + X)}, 0)(F_2 + X) \qquad (2.32)$$

The value of a call or put is:

$$c = (F_2 + X)\{e^{-rT}[FN(d_1) - N(d_2)]\} \qquad (2.33)$$

$$p = (F_2 + X)\{e^{-rT}[N(-d_2) - FN(-d_1)]\}, \qquad (2.34)$$

where $d_1 = \frac{\ln(F) + (\sigma^2/2)T}{\sigma\sqrt{T}}$, (2.35) $d_2 = d_1 - \sigma\sqrt{T}$, $F = \frac{F_1}{F_2 + X}$, (2.36)

and the volatility of $\frac{F_1}{F_2 + X}$ (2.34) can be approximated by the following equation:

$$\sigma^2 = \sigma_1^2 + [\sigma_2 \frac{F_2}{(F_2 + X)}]^2 - 2\rho\sigma_1\sigma_2 \frac{F_2}{(F_2 + X)}$$

where:

F_1 = Price on futures contract 1.
F_2 = Price on futures contract 2.
X = Strike price.
T = Time to expiration of the options in years.
r = Risk free interest rate in years.
σ_1 = Volatility of futures 1.
σ_2 = Volatility of futures 2.
ρ = Correlation between the two futures contracts.

The computer algorithm of the Spread Option Model is shown in the Appendix-2.6 of this chapter.

2.9 Asian Option Pricing

Asian options are the options whose final payoff is based in some way on the average level of an energy price (spot, forward or future) during some or all of the life of the option. Of the range of exotics options, Asian options in the equity and commodity markets are priced and risk managed as being almost vanilla in that they are now one of the best understood options and are very popular as a means to hedge exposure. There are two basic styles of Asian option; average price options and average strike options. The payoffs to the four main standard Asian options are summarised in the following table-2.2 where K is the strike price for the fixed strike options.

Table-2.2 Payoffs of the standard Asian Options

Name	Payoff
Average Price Call Option:	$\max(0, \frac{1}{m}\sum_{k=1}^{m} S_k - K)$
Average Price Put Option:	$\max(0, K - \frac{1}{m}\sum_{k=1}^{m} S_k)$
Average Strike Call Option:	$\max(0, S_T - \frac{1}{m}\sum_{k=1}^{m} S_k)$
Average Strike Put Option:	$\max(0, \frac{1}{m}\sum_{k=1}^{m} S_k - S_T)$

In general the main use of Asian Options is hedging an exposure to the average price over a period of time. For example, large buyers of electricity often require hedge their average fuel cost as the price they charge to the customers are based on the average purchase prices. Asian options also fit the risk profile of energy producers who need to meet budget targets on average prices.

2.9.1 Geometric Average-Rate Options

If the underlying asset is assumed to be lognormally distributed, the geometric average ($(x_1 .... x_n)^{1/n}$) of the asset will itself be lognormally distributed. As originally shown by Kemma and Vorst(1990) the geometric option can be priced as a standard option by changing the volatility and cost-of-carry term:

$$c = Se^{(b_A - r)T} N(d_1) - Xe^{-rT} N(d_2) \tag{2.38}$$

$$p = Xe^{-rT} N(-d_2) - Se^{(b_A - r)T} N(-d_1) \tag{2.39}$$

where

$$d_1 = \frac{\ln(S/X) + (b_A + \sigma_A{}^2/2)T}{\sigma_A\sqrt{T}}, \quad (2.40) \quad d_2 = d_1 - \sigma_A\sqrt{T}, (2.41)$$

and the adjusted volatility is equal to

$$\sigma_A = \frac{\sigma}{\sqrt{3}} \tag{2.42}$$

The adjusted cost of carry is set to

$$b_A = \frac{1}{2}(b - \frac{\sigma^2}{6}) \tag{2.43}$$

The computer algorithm of the Geometric Average Rate Option Model is shown in the Appendix-2.7 of this chapter.

2.9.2 Arithmetic Average-Rate Options

It is not possible to find a closed form solution for the valuation of options on an arithmetic average $(\frac{x_1 + x_2 + ...+ x_n}{n})$. The main reason for this is that when the asset is assumed to be lognormally distributed, the arithmetic average will not itself have a lognormal distribution. Arithmetic average-rate options can be priced by analytical approximations, as presented below or with Monte Carlo simulations which we are going to present later on this chapter.

The Turnbull and Wakeman Approximation.

The formula below is based on the work of Turnbull and Wakeman (1991). The approximation adjusts the mean and variance so that they are consistent with the exact moments of the arithmetic average. The adjusted mean, b_A and the variance, σ_A^2, are then used as an input in the Black-Scholes formula:

$$c \approx Se^{(b_A - r)T_2} N(d_1) - Xe^{-rT_2} N(d_2) \tag{2.44}$$

$$p \approx Xe^{-rT_2} N(d_2) - Se^{(b_A - r)T_2} N(d_1) \tag{2.45}$$

$$d_1 = \frac{\ln(S/X) + (b_A + \sigma_A^2/2)T_2}{\sigma_A\sqrt{T_2}}, (2.46) \quad d_2 = d_1 - \sigma_A\sqrt{T_2} \quad (2.47)$$

where T_2 is the remaining time to maturity. In addition,

$$\sigma_A = \sqrt{\frac{\ln(M_2)}{T} - 2b_A} \tag{2.48}$$

$$b_A = \frac{\ln(M_1)}{T} \tag{2.49}$$

The exact first and second moments of the arithmetic average are:

$$M_1 = \frac{e^{bT} - e^{b\tau}}{b(T-\tau)} \tag{2.50}$$

$$M_2 = \frac{2e^{(2b+\sigma^2)T}}{(b+\sigma^2)(2b+\sigma^2)(T-\tau)^2} + \frac{2e^{(2b+\sigma^2)\tau}}{b(T-\tau)^2}[\frac{1}{2b+\sigma^2} - \frac{e^{b(T-\tau)}}{b+\sigma^2}] \tag{2.51}$$

where T is the original time to maturity, and τ is the time to the beginning of the average period, the strike price must be replaced by X_S, and the option value must be multiplied by $\frac{T_2}{T}$, where

$$X_S = \frac{T}{T_2}X - \frac{T_1}{T_2}S_A, \tag{2.52}$$

where S_A is the average asset price during the realised or observed time period $T_1 (T_1 = T - T_2)$. The formula doesn't work for trivial cost of carry ($b = 0$).
The computer algorithm of the Arithmetic Average Rate Option Model is shown in the Appendix-2.8 of this chapter.

2.9.3 Levy's Approximation

Another alternative is the Levy (1992) Asian option approximation:

$$c_{Asian} \approx S_E N(d_1) - X^* e^{-rT_2} N(d_2), \tag{2.53}$$

$$S_E = \frac{S}{Tb}(e^{(b-r)T_2} - e^{-rT_2}) \tag{2.54}$$

$$d_1 = \frac{1}{\sqrt{V}}\left[\frac{\ln(D)}{2} - \ln(X^*)\right], d_2 = d_1 - \sqrt{V} \tag{2.55}$$

$$X^* = X - \frac{T-T_2}{T}S_{A,}\ (4.54),\ V = \ln(D) - 2[rT_2 + \ln(S_E)]\ \ (4.55),\ D = \frac{M}{T^2} \tag{2.56}$$

$$M = \frac{2S^2}{b+\sigma^2}\left[\frac{e^{(2b+\sigma^2)T_2} - 1}{2b+\sigma^2} - \frac{e^{bT_2} - 1}{b}\right] \tag{2.58}$$

The Asian put value can be found using put call parity:

$$p_{Asian} = c_{Asian} - S_E + X^* e^{-rT_2} \tag{2.59}$$

where :

S_A = Arithmetic average of the known asset price fixings.
S = Asset Price

X = strike price of option.
r = Risk free interest rate
b = cost of carry rate
T_2 = Remaining Time to maturity
T = Original time to maturity
σ = Volatility of natural logarithms of return of the underlying asset.

The formula does not allow for $b = 0$.
The computer algorithm of the Levy's Approximation Option Model is shown in the Appendix-2.9 of this chapter.

2.9.4 Curran's Approximation

Curran (1992) has developed an approximation method for pricing Asian options based on the geometric conditioning approach. Curran claims that this method is more accurate than other closed form approximations presented earlier.

$$c \approx e^{-rT}\left[\frac{1}{n}\sum_{i=1}^{n} e^{\mu_i+\sigma_i^2/2} N\left(\frac{\mu-\ln(\hat{X})}{\sigma_x}+\frac{\sigma_{xi}}{\sigma_x}\right) - XN\left(\frac{\mu-\ln(\hat{X})}{\sigma_x}\right)\right] \tag{2.60}$$

where

S= Initial asset price.
X= Strike price of option.
r = Risk free rate.
b = Cost of carry.
T = Time to expiration in years.
t_1 = Time to first averaging point.
Δt = Time between averaging points.
n = Number of averaging points.
σ = Volatility of the asset.
$N(x)$ = The cumulative normal distribution function.

$$\mu_i = \ln(S) + (b - \sigma^2/2)t_i \tag{2.61}$$

$$\sigma_i = \sqrt{\sigma^2[t_1 + (i-1)\Delta t]} \tag{2.62}$$

$$\sigma_{xi} = \sigma^2\{t_1 + \Delta t[(i-1) - i(i-1)/(2n)]\} \tag{2.63}$$

$$\mu = \ln(S) + (b - \sigma^2/2)[t_1 + (n-1)\Delta t/2] \tag{2.64}$$

$$\sigma_x = \sqrt{\sigma^2[t_1 + \Delta t(n-1)(2n-1)/6n]} \tag{2.65}$$

and

$$\hat{X} = 2X - \frac{1}{n}\sum_{i=1}^{n} \exp\left\{\mu_i + \frac{\sigma_{xi[\,]}[\ln(X) - \mu]}{\sigma_x^2} + \frac{\sigma_i^2 - \sigma_{xi}^2 / \sigma_x{}^2}{2}\right\}. \tag{2.66}$$

2.10 COMPOUND OPTIONS

Compound option is an option that allows the holder to buy or sell another option for a fixed price. Compound options occur in energy markets in the form of captions and floptions as well as calls or puts on simple calls or puts. A caption is an option on a cap and a floption is an option on the floor.
Compound options offer a method of locking in commodity price protection at an initial cost, which is lower than that of the purchase of a call or put. These options are also useful for locking in the cost of price protection is contingent on some future event. The compound options lose value as they come closer to expiration, with the at-the-money and the out-of-the money option values approaching to zero for very short tenors.
A model for pricing compound options was first published by Geske (1977). It was later extended and discussed by Geske (1979) in a Black-Scholes world.

Call on Call.
The payoff is: $\max[c_{BS}(S, X_1, T_2) - X_2, 0]$, where X_1 is the strike price of the underlying option, X_2 is the strike price of the option on the option, and $c_{BS}(S, X_1, T_2)$ is the Black-Scholes formula with strike X_1 and the time to maturity T_2.

$$c_{call} = Se^{(b-r)T_2} M(z_1, y_1; \rho) - X_1 e^{-rT_2} M(z_2, y_2; \rho) - X_2 e^{-rt} N(y_2), \tag{2.67}$$

where

$y_1 = \dfrac{\ln(S/I) + (b + \sigma^2/2)t_1}{\sigma\sqrt{T_2}}$, (2.68) $y_2 = y_1 - \sigma\sqrt{t_1}$ (2.69)

$z_1 = \dfrac{\ln(S/X_1) + (b + \sigma^2/2)T_2}{\sigma\sqrt{T_2}}$, (2.70) $z_2 = z_1 - \sigma\sqrt{T_2}$ (2.71)

$\rho = \sqrt{\dfrac{t_1}{T_2}}$ (2.72)

where S is the price of the underlying at time t_1, I is the underlying price which makes the underlying option equal to X_2 at time t_1, T_2 is the time to maturity on the underlying option, t_1 is the time to maturity on the option on the option, σ is the annualised volatility, r is the risk-free rate, ρ is the correlation coefficient and $M(z, y, \rho)$ is the bivariate cumulative normal distribution function.

Put on Call
Payoff: $\max\left[X_2 - c_{BS}(S, X_1, T_2\ );0\right]$

$$p_{call} = X_1 e^{-rT_2} M(z_2, -y_2; \rho) - Se^{(b-r)T_2} M(z_1, y_1; -\rho) + X_2 e^{-rt} N(-y_2), \qquad (2.73)$$

where the value of I is found by solving the following equation:

$$c_{BS}(I, X_1, T_2 - t_1) = X_2{}^1 \qquad (2.74)$$

Call on put
Payoff:$\max\left[p_{BS}(S, X_1, T_2) - X_2;0\right]$

$$c_{put} = X_1 e^{-rT_2} M(-z_2, -y_2; \rho) - Se^{(b-r)T_2} M(-z_1, -y_1; \rho) - X_2 e^{-rt_1} N(-y_2) \qquad (2.75)$$

Put on Put
Payoff: $\max\left[X_2 - p_{BS}(S, X_1, T_2);0\right]$

$$p_{put} = Se^{(b-r)T_2} M(-z_1, y_1; -\rho) - X_1 e^{-rT_2} M(-z_2, y_2; -\rho) + X_2 e^{-rt_1} N(y_2), \qquad (2.76)$$

where the value of I is found by solving the following equation

$$p_{BS} = (I, X_1, T_2 - t_1) = X_2 \qquad (2.77)$$

The computer algorithm of Compound Option Model is shown in the Appendix-2.10 of this chapter.

2.11 LOOKBACK OPTIONS

With lookback options the payout is a function of the highest or lowest price at which the underlying asset trades over some period during the life of the option. There are two basic styles of lookback option; fixed strike and floating strike options.

2.11.1 Fixed Strike Lookback Options

In a fixed strike lookback call, the strike is fixed in advance, and at expiry the option pays out the maximum of the difference between the highest observed price, $S_{\max}$, in the option lifetime and the strike X, 0. Similarly, a put at expiry pays out the maximum of the difference between the fixed price X and the minimum observed price, $S_{\min}$, and 0. The lookback option is much more expensive than the corresponding European option. Fixed strike lookback options can be priced using the Conze and Viswanathan (1991) formula.

[1] We are using the Newton-Raphson algorithm in order to calculate the value of I. (see Appendix-2.10).

Fixed strike lookback call

$$c = Se^{(b-r)T}N(d_1) - Xe^{-rT}N(d_2) + Se^{-rT}\frac{\sigma^2}{2b}\left[-\left(\frac{S}{X}\right)^{\frac{-2b}{\sigma^2}} N\left(d_1 - \frac{2b}{\sigma}\sqrt{T}\right) + e^{bT}N(d_1)\right],$$

(2.78)
where

$$d_1 = \frac{\ln(S/X) + (b + \sigma^2/2)T}{\sigma\sqrt{T}}, \text{ (2.79)} \quad d_2 = d_1 - \sigma\sqrt{T}, \text{ (2.80)}$$

when $X \le S_{max}$

$$c = e^{-rT}(S_{max} - X) + Se^{(b-r)T}N(e_1) - S_{max}e^{-rT}N(e_2) + Se^{-rT}\frac{\sigma^2}{2b}\left[-\left(\frac{S}{S_{max}}\right)^{\frac{-2b}{\sigma^2}} N\left(e_1 - \frac{2b}{\sigma}\sqrt{T}\right) + e^{bT}N(e$$

(2.81)
where

$$e_1 = \frac{\ln(S/S_{max}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}}, \text{ (2.82) } e_2 = e_1 - \sigma\sqrt{T} \text{ (2.83)}$$

Fixed strike lookback put

$$p = Xe^{-rT}N(-d_2) - Se^{(b-r)T}N(-d_1) + Se^{-rT}\frac{\sigma^2}{2b}\left[\left(\frac{S}{X}\right)^{\frac{-2b}{\sigma^2}} N\left(-d_1 + \frac{2b}{\sigma}\sqrt{T}\right) - e^{bT}N(-d_1)\right]$$

(2.84)
and when $X \ge S_{min}$

$$p = e^{-rT}(X - S_{min}) - Se^{(b-r)T}N(-f_1) + S_{min}e^{-rT}N(-f_2) +$$

$$Se^{-rT}\frac{\sigma^2}{2b}\left[\left(\frac{S}{S_{min}}\right)^{\frac{-2b}{\sigma^2}} N\left(-f_1 + \frac{2b}{\sigma}\sqrt{T}\right) - e^{bT}N(-f_1)\right] \quad \text{(2.85)}$$

where

$$f_1 = \frac{\ln(S/S_{min}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}}, \text{ (2.86)} \quad f_2 = f_1 - \sigma\sqrt{T} \text{ (2.87)}$$

The computer algorithm of the Fixed Strike Lookback Option Model is shown in the Appendix-2.11 of this chapter.

2.11.2 Floating Strike Lookback Options

A floating strike lookback call gives the holder of the option to buy the underlying at the lowest at the lowest price observed, $S_{\min}$, in the life of the option. Similarly, a floating strike lookback put gives the option holder the right to sell the underlying at the highest price observed, $S_{\max}$, in the option's lifetime. The payoff from a standard floating strike lookback option is:

$$c(S, S_{\min}, T) = \max(S - S_{\min}; 0) = S - S_{\min} \tag{2.88}$$

and for the put is

$$p(S, S_{\max}, T) = \max(S_{\max} - S; 0) - S_{\max} - S \tag{2.89}$$

Floating strike lookback options were originally introduced by Goldman, Sosin and Gatto. (1979).

Floating strike lookback call

$$c = Se^{(b-r)T} N(a_1) - S_{\min} e^{-rT} N(a_2) + Se^{-rT} \frac{\sigma^2}{2b}$$
$$\left[\left(\frac{S}{S_{\min}} \right)^{\frac{-2b}{\sigma}} N\left(-a_1 + \frac{2b}{\sigma}\sqrt{T} \right) - e^{-bT} N(-a_1) \right] \tag{2.90}$$

where

$$a_1 = \frac{\ln(S/S_{\min}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (2.91) \qquad a_2 = a_1 - \sigma\sqrt{T} \quad (2.92)$$

Floating strike lookback put

$$p = S_{\max} e^{-rT} N(-b_2) - Se^{(b-r)T} N(-b_1) + Se^{-rT} \frac{\sigma^2}{2b}$$
$$\left[-\left(\frac{S}{S_{\max}} \right)^{\frac{-2b}{\sigma^2}} N\left(b_1 - \frac{2b}{\sigma}\sqrt{T} \right) + e^{bT} N(b_1) \right] \tag{2.93}$$

where

$$b_1 = \frac{\ln(S/S_{\max}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (2.99) \qquad b_2 = b_1 - \sigma\sqrt{T}. \quad (2.94)$$

The computer algorithm of the Floating Strike Lookback Option Model is shown in the Appendix-2.12 of this chapter.

2.12 BARRIER OPTIONS

In recent years one of the most significant growth areas in new options types has been in the area of barrier options. Barrier options are standard options that either cease to exist (Knock-out) or come in to existence (Knock-in) if the underlying price (usually a spot energy price or the price of the forward contract) crosses a predetermined level – the barrier (which we denote by *H)*. Barrier options were originally introduced because they are cheaper than otherwise identical standard options due to the fact that the option can cease to exist or never come to the existence. There four basic types of barrier options- the barrier can be above the current underlying price (up-barriers) or below (down-barriers) and the option can cease to exist (knock-out) or come in to existence (Knock in) when the barrier level is crossed. Each of these four types can be either a call or a put, which leads to eight different barrier options. A standard variation of the options just described are options which pay a predetermined cash rebate if an out option disappears or an in option never appears.

A typical example of a barrier option is the up-and-out put purchased by an energy producer to hedge its natural long position. An up-and-out put may be an attractive alternative to the vanilla put option as it is less expensive and provides the same price protection if prices move down from the current levels. However, if prices move up, the increase in the underlying commodity's price reduces the need for downside price protection at the original strike. If the price moves up sufficiently to cross the barrier and extinguish the option, the owner may consider re-entering a hedge by buying another put at a higher strike price.

Merton (1973) and Reiner and Rubinstein (1991a), (see also Rich 1994) have developed formulas for pricing standard barrier options:

$$A = \phi S e^{(b-r)T} N(\phi x_1) - \phi X e^{-rT} N(\varphi x_1 - \varphi\sigma\sqrt{T}) \tag{2.95}$$

$$B = \varphi S e^{(b-r)T} N(\varphi x_2) - \varphi X e^{-rt} N(\varphi x_2 - \phi\sigma\sqrt{T}) \tag{2.96}$$

$$C = \varphi S e^{(b-r)T} (H/S)^{2(\mu+1)} N(\eta y_1) - \varphi X e^{-rT} (H/S)^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{T}) \tag{2.97}$$

$$D = \phi S e^{(b-r)T} (H/S)^{2(\mu+1)} N(\eta y_2) - \phi X e^{-rt} (H/S)^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T}) \tag{2.98}$$

$$E = K e^{-rT} [N(\eta x_2 - \eta\sigma\sqrt{T}) - (H/S)^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T})] \tag{2.99}$$

$$F = K[(H/S)^{\mu+\lambda} N(\eta z) + (H/S)^{\mu-\lambda} N(\eta z - 2\eta\lambda\sigma\sqrt{T})], \tag{2.100}$$

where

$$x_1 = \frac{\ln(S/X)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}, \quad (2.101) \quad x_2 = \frac{\ln(S/H)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \quad (2.102)$$

$$y_1 = \frac{\ln(H^2/SX)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \quad (2.103), \quad y_2 = \frac{\ln(H/S)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \quad (2.104)$$

$$z = \frac{\ln(H/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \quad (2.105), \quad \mu = \frac{b-\sigma^2/2}{\sigma^2} \quad (2.106), \quad \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}} \quad (2.107).$$

2.12.1 "In" Barriers

In options are paid for today but first come into existence if the asset price S hits the barrier H before expiration. It is possible to include a prespecified cash rebate K, which is paid out at option expiration if the option has not knocked in during its lifetime.

Down-and-in-call $S>H$

Payoff: max $(S\text{-}X,\ 0)$ if $S \le H$.

$c_{di(X>H)} = C + E \qquad \eta = 1, \phi = 1$

$c_{di(X<H)} = A - B + D + E \qquad \eta = 1, \phi = 1$

Up-and-in-call $S<H$

Payoff: max $(S\text{-}X,\ 0)$ if $S \ge H$.

$c_{ui(X>H)} = A + E \qquad \eta = -1, \phi = 1$

$c_{ui(X<H)} = B - C + D + E \qquad \eta = -1, \phi = 1$

Down-and-in-put $S>H$

Payoff: max $(X\text{-}S,\ 0)$ if $S \le H$.

$p_{di(X>H)} = B - C + D + E \qquad \eta = 1, \phi = -1$

$p_{di(X<H)} = A + E \qquad \eta = 1, \phi = -1$

Up-and-in-put

Payoff: max $(X\text{-}S,\ 0)$ if $S \ge H$.

$p_{ui(X>H)} = A - B + D + E \qquad \eta = -1, \phi = -1$

$p_{ui(X<H)} = C + E \qquad \eta = -1, \phi = -1$

where η, φ are binary variables with range $\{1, -1\}$.

2.12.2 "Out" Barriers

Out options are similar to standard options except that the option is knocked out or becomes worthless if the asset price S hits the barrier before expiration. It is possible to include a prespesified cash rebate K which is paid out if the option is knocked out before expiration.

Down-and-out-call $S>H$

Payoff: max $(S\text{-}X,\ 0)$ if $S>H$.

$c_{do(X>H)} = A - C + F \qquad \eta = 1, \phi = 1$

$c_{do(X<H)} = B - D + F \qquad \eta = 1, \phi = 1$

Up-and-out-call $S<H$

Payoff: max $(S\text{-}X,\ 0)$ if $S<H$.

$c_{uo(X>H)} = F \qquad \eta = -1, \phi = 1$

$c_{uo(X<H)} = A - B + C - D + F \qquad \eta = -1, \phi = 1$

Down-and-out-put $S>H$.

Payoff: max $(X\text{-}S,\ 0)$ if $S>H$

$p_{do(X>H)} = A - B + C - D + F \qquad \eta = 1, \phi = -1$

$P_{do(X<H)} = F \qquad \eta = 1, \phi = -1$

Up-and-out-put $S<H$

Payoff: max $(X\text{-}S,\ 0)$ if $S<H$.

$p_{uo(X>H)} = B - D + F \qquad \eta = -1, \phi = -1$

$p_{uo(X<H)} = A - C + F \qquad \eta = -1, \phi = -1$

where η, φ are binary variables with range $\{1, -1\}$.

The computer algorithm of the Barrier Option Model is shown in the Appendix-2.13 of this chapter.

2.13 BINARY (DIGITAL) OPTIONS

Binary options, also known as digital options, have discontinuous payoffs and they aren't widely used in the energy market. The main distinction between European options and binary options is simply this:

- The payout of a European option is related to the difference between the underlying and the strike price.
- The payout of a binary option is determined by whether or not the underlying is above the strike price. The amount paid out is independent of the difference.

Digital (or binary) options typically pay either a constant value or zero depending on whether the payoff condition is satisfied or not.

Examples of such options are provided by cash-or-nothing and asset-or-nothing options.

2.13.1 CASH-OR-NOTHING OPTIONS

To illustrate: A European call option stuck at $100 will pay $5 if the underlying ends at $105, $10 if it ends at $110, and $20 if it ends $120.A binary call option sruck at $100 will payout $1 if the underlying ends at $101 and will payout the same $1 even if the underlying ends up at $110,$120, or $150.This means that if the strike is $100, the binary option has a payout of $1 if the underlying is priced at $100.001.On the other hand if the underlying price is $99.999, the binary option pays zero.

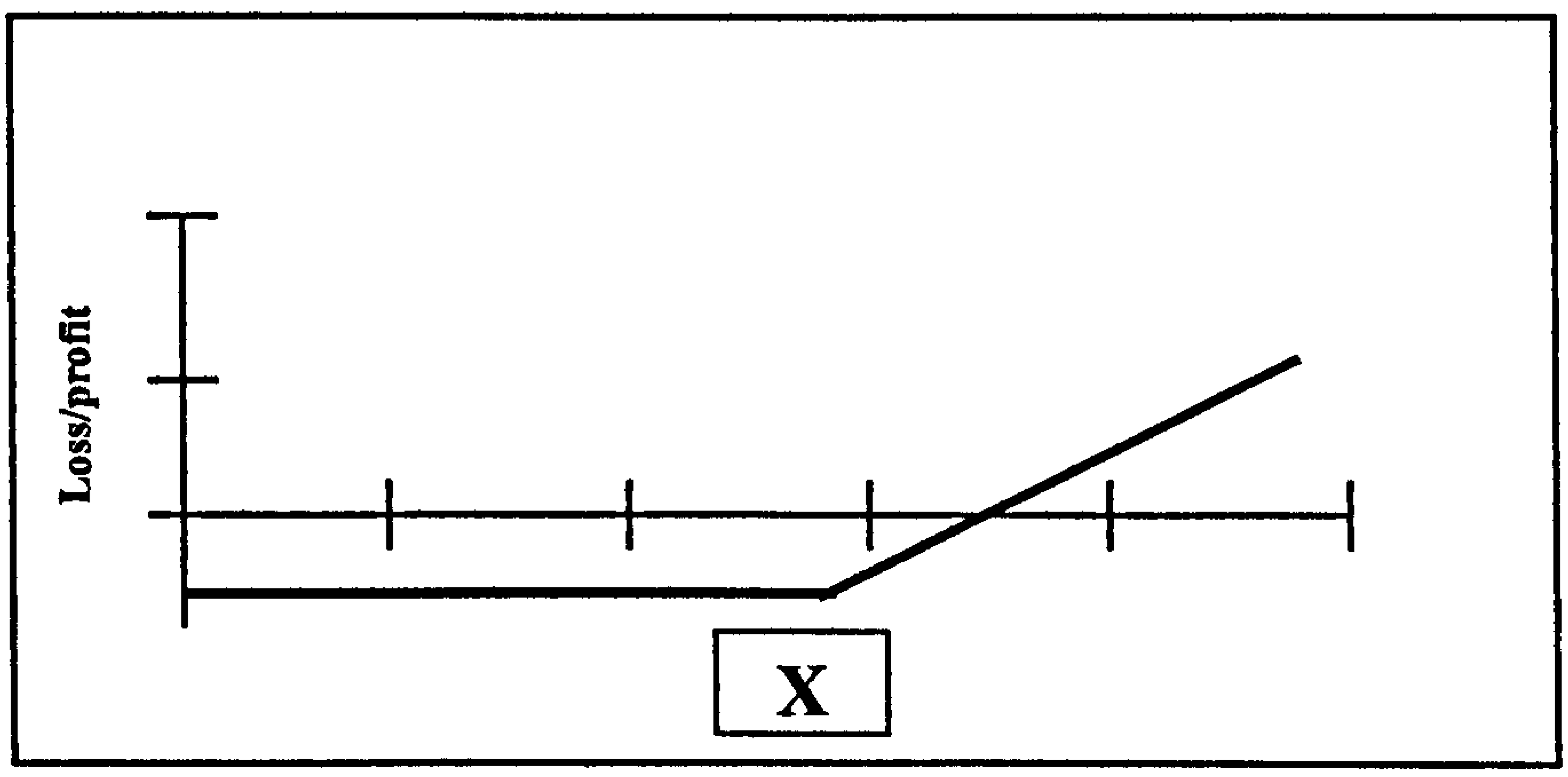


Figure-2.28 The payout of a standard European option. Its payout function is max $(S_t - X, 0)$.

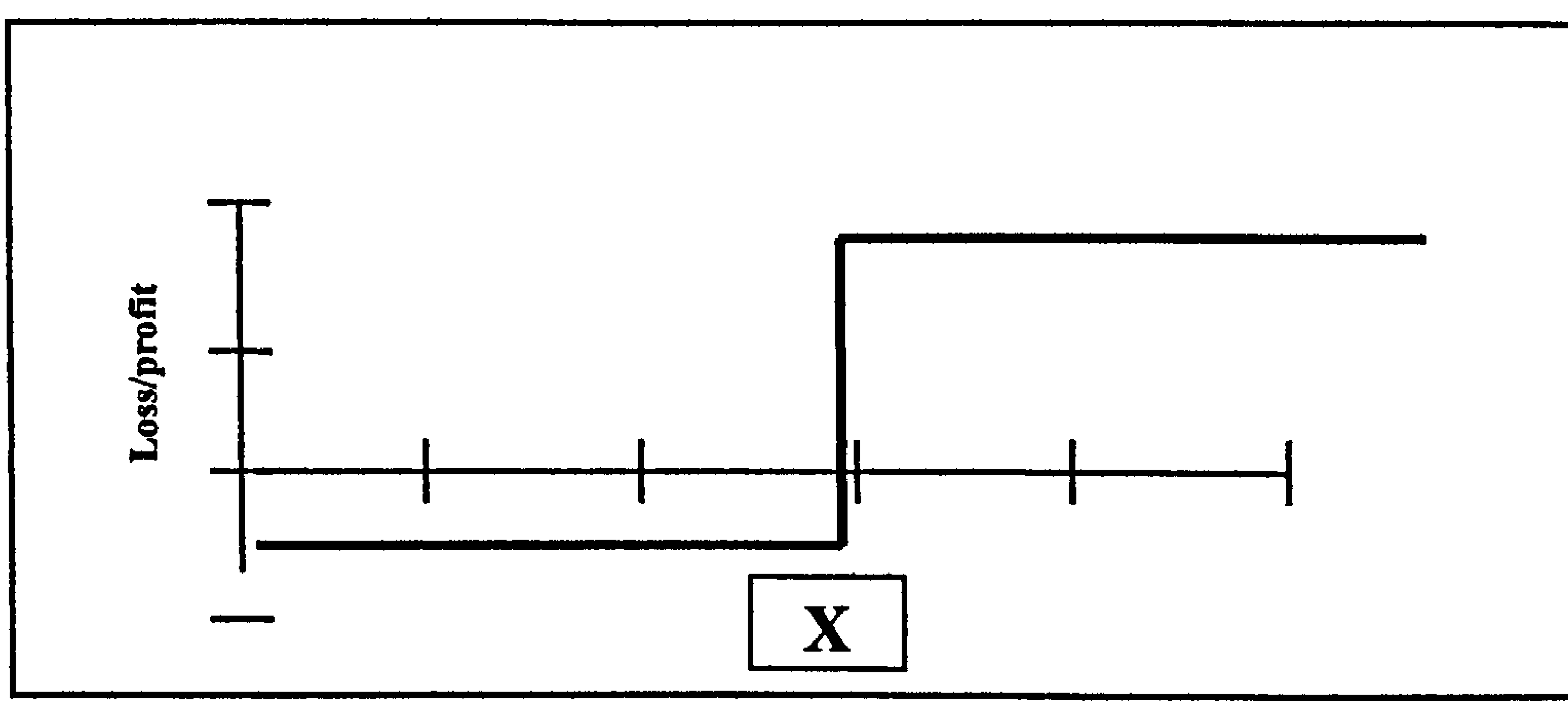


Figure-2.29 The payout of a binary option. It pays out a $1 if $S_t > X$; otherwise it pays out 0.

As mentioned above, a cash-or-nothing call option pays out $1 if the underlying, S, is above the strike, X. The same payout is given regardless of the difference between S and X. A binary put option pays $1 if the underlying is below the strike price.

The valuation of a cash-or-nothing option is extremely simple, it pays out a cash amount if the option is in-the-money. The payoff from a call is 0 if $S \leq X$ and K if $S > X$. The payoff from a put option is 0 if $S \geq X$ and K if $S < X$. Valuation of cash-or-nothing options can be made using the formula described by Reiner and Rubinstein (1991b):

$$C = Ke^{-rt} N(d\ ) \tag{2.108}$$

$$P = Ke^{-rt} N(-d\ ) \tag{2.109}$$

Here C is the price of the binary call option, P is the price of the binary put option, *N()* is the cumulative standard normal distribution function, *r* is the risk free interest rate and *t* is the time to expiration. This is nothing but the last part of the Black's formula, where:

$$d\ = \frac{\ln(S/X) + (-\sigma^2/2)T}{\sigma\sqrt{T}} \tag{2.110}$$

where

S = Spot price of the underlying

X = Strike price

σ = Volatility

Therefore, the price of the binary option is given by the area under the curve.

What is the probability of the underlying ending in the money, discounted to today's rate? If we price a binary option struck at \$100,the current spot price is \$100, the price of the option is \$0.48.This can be regarded as the probability that the underlying will end up above \$100 (roughly 50%) discounted by one year. The option has virtually no time decay.

Pricing the same option with only two weeks to expiration give us a price of \$0.50. Of course, the probability that the underlying will end up \$100 is still roughly 50%. With two weeks to expiration, we have just changed the present value somewhat.

The computer algorithm of the Cash-or-nothing Option Model is shown in the Appendix-2.14 of this chapter.

2.13.2 ASSET-OR-NOTHING OPTIONS

The asset-or-nothing call option pays 0 if $S \le X$ and S if $S > X$. Similarly, a put option pays 0 if $S \ge X$ and S if $S < X$. The option can valued using the formula described by Reiner and Rubinstein (1991b):

$$C = Se^{-rT} N(d) \tag{2.111}$$

$$P = Se^{-rT} N(-d) \tag{2.112}$$

Here C is the price of the binary call option, P is the price of the binary put option, $N()$ is the cumulative standard normal distribution function, r is the risk free interest rate and t is the time to expiration. This is nothing but the first part of the Black's formula, where:

$$d = \frac{\ln(S/X) + (\sigma^2/2)T}{\sigma\sqrt{T}} \tag{2.113}$$

The computer algorithm of the Asset-or-nothing Option Model is shown in the Appendix-2.15 of this chapter.

2.14 MONTE CARLO SIMULATION

Monte Carlo simulation is another numerical method that is often useful when no closed form solution is available. Monte Carlo simulating in option pricing, originally introduced by Boyle (1977), can be used to value most types of European options (i.e. arithmetic average-rate options where only closed form solutions are available). The Monte Carlo simulation, we will here limit ourselves to processes where the natural logarithm of the underlying asset follows geometric Brownian motion. That is, the process governing the asset price S is given by

$$S + dS = S\exp\left[(\mu - \frac{1}{2}\sigma^2)\delta t + \sigma dz\right] \tag{2.114}$$

where the dz is a wiener process with a standard deviation one and mean zero. To simulate the process, we split it up at discrete intervals, Δt apart.

$$S + \Delta S = S\exp\left[(\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\varepsilon_t\sqrt{\Delta t}\right], \tag{2.115}$$

where ΔS is a discrete change in S in the chosen time interval Δt, and ε_t is a random drawing from a standard normal distribution. Most computer languages have built in functions that draw randomly from a standard normal distribution. If the computer languages only have a function to draw randomly a number Z between zero and one, this can be easily be transformed into a random number from a standard normal distribution ε by using the relationship

$$\varepsilon = \sum_{i=1}^{12} Z_i - 6 \qquad (2.116)$$

The main disadvantage of Monte Carlo simulation is that it is computer intensive. Thousand of simulations are typically necessary to price an option with satisfying accuracy.

The computer algorithm of the European Option Model with Monte Carlo Simulation is shown in the Appendix-2.16 of this chapter.

2.14.1 Two Assets

Monte Carlo simulation can easily be extended to options on two underlying assets.

$$S_1 + \Delta S_1 = S_1 \exp\left[(\mu_1 - \frac{1}{2}\sigma_1^2)\Delta t + \sigma_1 a_{1,t}\sqrt{\Delta t}\right] \qquad (2.117)$$

$$S_2 + \Delta S_2 = S_2 \exp\left[(\mu_2 - \frac{1}{2}\sigma_2^2)\Delta t + \sigma_2 a_{2,t}\sqrt{\Delta t}\right] \qquad (2.118)$$

Correlation between the two assets is allowed by setting

$$a_{1,t} = \varepsilon_{1,t} \qquad (2.119)$$

$$a_{2,t} = \rho\varepsilon_{1,t} + \varepsilon_{2,t}\sqrt{1-\rho^2}, \qquad (2.120)$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are two independently random numbers from a standard normal distribution. The two asset Monte Carlo simulation is useful in the pricing of European Asian spread options. These are options whose payoff depends on the difference between the arithmetic average of the two assets at expiration.
The computer algorithm of the Two-Asset Spread Option Model with Monte Carlo Simulation is shown in the Appendix-2.17 of this chapter.

2.15 Summary

As we mentioned at the beginning the aim of this chapter is to test the Black's model approach for modelling energy prices and to develop computer algorithms for pricing the main derivative products used in energy markets.
Based on the results on Part A, we conclude that the Black-Scholes GBM approach is the appropriate for modelling the oil, natural gas and electricity market for the periods we tested the model. Consequently, we developed computer algorithms for pricing the main derivatives products used in the energy markets based on the Black-Scholes environment and described some numerical procedures often implemented by practitioners to evaluate derivative prices.

Appendix-2.1

Black Scholes Computer Algorithm with Monte Carlo Simulation

```
Public Sub BSModel()

Dim p, r, f, sig, k, y, T, M, n, Z, PutCall, nodays
Dim diff, drift, dt, lnp, sump, test, test2, rand1, rand2, rand3, jump, CT, sum_CT
Dim i, j, pcf, b As Integer

nodays = (Range("F").End(xlDown).Row) - 1

For b = 1 To nodays

p = Cells(1 + b, 1).Value
Z = Cells(1 + b, 2).Value
PutCall = Cells(1 + b, 3).Value
r = Cells(1 + b, 4).Value
sig = Cells(1 + b, 5).Value
T = Cells(1 + b, 6).Value
M = Cells(1 + b, 7).Value
n = Cells(1 + b, 8).Value



dt = T / n

drift = (r - 0.5 * sig ^ 2) * dt

diff = sig * Sqr(dt)

sump = 0
CT = 0
sum_CT = 0

For j = 1 To M

lnp = Log(p)

   For i = 1 To n

   test = Rnd

         If test = 0 Then
         test = test + 0.0000001
         End If

         rand1 = Application.NormSInv(test)

   lnp = lnp + drift + diff * rand1
```

```
  Next i

  If LCase(PutCall) = "call" Then

  pcf = 1

  End If

  If LCase(PutCall) = "put" Then

  pcf = -1

  End If

  sump = sump + Exp(lnp)
  CT = Application.Max(pcf * (Exp(lnp) - Z), 0)
  sum_CT = sum_CT + CT

Next j


Cells(1 + b, 9).Value = Exp(-r * T) * (sump / M)
Cells(1 + b, 10).Value = Exp(-r * T) * (sum_CT / M)

Next b

End Sub
```

Appendix-2.2

2.2.1 Black Scholes Computer Algorithm

The Black Scholes function returns the call price if the (call-put) flag is set equal to "c" or the put price when set equal to "p". In the computer code $v = \sigma$.

```
Function BlackScholes (CallPutFlag As String, S As Double, X _
        As Double, T As Double, r As Double, v As Double) As Double


   Dim d1 As Double, d2 As Double


   d1 = (Log(S / X) + (r + v ^ 2 / 2) * T) / (v * Sqr(T))
   d2 = d1 - v * Sqr(T)
   If CallPutFlag = "c" Then
      BlackScholes = S * CND(d1) - X * Exp(-r * T) * CND(d2)
   ElseIf CallPutFlag = "p" Then
```

```
    BlackScholes = X * Exp(-r * T) * CND(-d2) - S * CND(-d1)
  End If
End Function.
```

Where **CND** is the cumulative normal distribution function N(x). The following approximation of the cumulative normal distribution function N(x) produces values to within four decimal place accuracy.

$$N(x) = \int_{-\infty}^{x} \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-y^2}{2}} dy$$

$$N(x) = \{1 - n(x)(a_1 k + a_2 k^2 + a_3 k^3) \quad \text{when } x \geq 0 \text{ or}$$

$$N(x) = \{ \ 1\text{-}N(\text{-}x) \quad \text{when } x < 0,$$

Where

$$K = \frac{1}{1 + 0.33267x}$$

$$a_1 = 0.4361836$$

$$a_2 = -0.1201676$$

$$a_3 = 0.9372980$$

$$n(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

The next approximation produces values of N(x) to within six decimal places of the true value.

$$N(x) = \{1 - n(x)(a_1 k + a_2 k^2 + a_3 k^3 + a_4 k^4 + a_5 k_5) \quad \text{when } x \geq 0$$

or

$$N(x) = \{ \ 1\text{-}N(\text{-}x) \quad \text{when } x < 0,$$

$$K = \frac{1}{1 + 0.2316419x}$$

$$a_1 = 0.319381530$$

$$a_2 = -0.356563782$$

a_3 =1.781477937

a_4 = -1.821255978

a_5 = 1.330274429

2.2.2 Cumulative Normal Distribution Function Computer Algorithm

The cumulative normal distribution function *CND* returns values of *N* to within six decimal places accuracy.

```
Function CND(X As Double) As Double

   Dim L As Double, K As Double
   Const a1 = 0.31938153:  Const a2 = -0.356563782: Const a3 = 1.781477937:
   Const a4 = -1.821255978:  Const a5 = 1.330274429

   L = Abs(X)
   K = 1 / (1 + 0.2316419 * L)
   CND = 1 - 1 / Sqr(2 * Pi) * Exp(-L ^ 2 / 2) * (a1 * K + a2 * K ^ 2 + a3 * K ^ 3 + a4
* K ^ 4 + a5 * K ^ 5)

   If X < 0 Then
      CND = 1 - CND
   End If
End Function
```

Appendix-2.3

Black's Computer Algorithm.

```
Public Function Black76(CallPutFlag As String, F As Double, X _
            As Double, T As Double, r As Double, v As Double) As Double

   Dim d1 As Double, d2 As Double

   d1 = (Log(F / X) + (v ^ 2 / 2) * T) / (v * Sqr(T))
   d2 = d1 - v * Sqr(T)
   If CallPutFlag = "c" Then
```

```
    Black76 = Exp(-r * T) * (F * CND(d1) - X * CND(d2))
  ElseIf CallPutFlag = "p" Then
    Black76 = Exp(-r * T) * (X * CND(-d2) - F * CND(-d1))
  End If
End Function.
```

Appendix-2.4

Binomial Option Model Computer Algorithm

Computer Algorithm
The computer code returns the value of a European or American call, or put option. Setting the *AmeEurFlag=a* gives American Option values, *AmeEurFlag=e* gives European values. In the computer code $v = \sigma$ and $dt = \Delta t$

```
 Function CRRBinomial(AmeEurFlag As String, CallPutFlag As String, S As Double,
X As Double, T As Double, _
        r As Double, b As Double, v As Double, n As Integer) As Double

  Dim OptionValue() As Double
  Dim u As Double, d As Double, p As Double
  Dim dt As Double, Df As Double
  Dim i As Integer, j As Integer, z As Integer

  ReDim OptionValue(n + 1)

  If CallPutFlag = "c" Then
    z = 1
    ElseIf CallPutFlag = "p" Then
    z = -1
  End If

  dt = T / n
  u = Exp(v * Sqr(dt))
  d = 1 / u
  p = (Exp(b * dt) - d) / (u - d)
  Df = Exp(-r * dt)

  For i = 0 To n
    OptionValue(i) = Max(0, z * (S * u ^ i * d ^ (n - i) - X))
  Next

  For j = n - 1 To 0 Step -1:
    For i = 0 To j
      If AmeEurFlag = "e" Then
        OptionValue (i) = (p * OptionValue(i + 1) + (1 - p) * OptionValue(i)) * Df
      ElseIf AmeEurFlag = "a" Then
        OptionValue (i) = Max((z * (S * u ^ i * d ^ (Abs(i - j)) - X)), _
```

```
            (p * OptionValue(i + 1) + (1 - p) * OptionValue(i)) * Df)
        End If
      Next
    Next
    CRRBinomial = OptionValue (0)
End Function.
```

Appendix-2.5

Trinomial Option Model Computer Algorithm

Computer Algorithm
In the computer code, $v = \sigma$ and $dt = \Delta t$. The computer code starts with the statement "Option Base 0".This statement forces the arrays to start counting from 0.

Trinomial tree

```
Public Function TrinomialTree(AmeEurFlag As String, CallPutFlag As String, S As
Double, X As Double, T As Double, _
        r As Double, b As Double, v As Double, n As Integer) As Double


   Dim OptionValue() As Double
   Dim dt As Double, u As Double, d As Double
   Dim pu As Double, pd As Double, pm As Double
   Dim i As Integer, j As Integer, z As Integer
   Dim Df As Double

   ReDim OptionValue(n * 2 + 1)

   If CallPutFlag = "c" Then
      z = 1
      ElseIf CallPutFlag = "p" Then
      z = -1
   End If

   dt = T / n
   u = Exp(v * Sqr(2 * dt))
   d = Exp(-v * Sqr(2 * dt))
   pu = ((Exp(b * dt / 2) - Exp(-v * Sqr(dt / 2))) / (Exp(v * Sqr(dt / 2)) - Exp(-v *
Sqr(dt / 2)))) ^ 2

   pd = ((Exp(v * Sqr(dt / 2)) - Exp(b * dt / 2)) / (Exp(v * Sqr(dt / 2)) - Exp(-v *
Sqr(dt / 2)))) ^ 2
   pm = 1 - pu - pd
   Df = Exp(-r * dt)

   For i = 0 To (2 * n)
      OptionValue(i) = Max(0, z * (S * u ^ Max(i - n, 0) * d ^ Max(n * 2 - n - i, 0) -
X))
   Next
```

```
  For j = n - 1 To 0 Step -1
    For i = 0 To (j * 2)
      If AmeEurFlag = "e" Then
        OptionValue(i) = (pu * OptionValue(i + 2) + pm * OptionValue(i + 1) + pd
* OptionValue(i)) * Df
      ElseIf AmeEurFlag = "a" Then
        OptionValue(i) = Max((z * (S * u ^ Max(i - j, 0) * d ^ Max(j * 2 - j - i, 0) -
X)), _
        (pu * OptionValue(i + 2) + pm * OptionValue(i + 1) + pd * OptionValue(i))
* Df)
      End If
    Next
  Next
  TrinomialTree = OptionValue (0)
End Function.
```

Appendix-2.6

Spread Option Model Computer Algorithm

Computer Algorithm

```
Function SpreadApproximation(CallPutFlag As String, f1 As Double, f2 As Double,
X As Double, T As Double, _
        r As Double, v1 As Double, v2 As Double, rho As Double) As Double

  Dim v As Double, F As Double
  Dim d1 As Double, d2 As Double

  v = Sqr(v1 ^ 2 + (v2 * f2 / (f2 + X)) ^ 2 - 2 * rho * v1 * v2 * f2 / (f2 + X))
  F = f1 / (f2 + X)

  SpreadApproximation = GBlackScholes(CallPutFlag, F, 1, T, r, 0, v) * (f2 + X)
End Function.
```

Appendix-2.7

Geometric Average Rate Option Model Computer Algorithm

Computer Algorithm

```
Public Function GeometricAverageRateOption (CallPutFlag As String, S As Double,
SA As Double, X As Double, _
        T As Double, T2 As Double, r As Double, b As Double, v As Double) As
Double


  Dim t1 As Double 'Observed or realized time period
  Dim bA As Double, vA As Double

  bA = 1 / 2 * (b - v ^ 2 / 6)
```

```
  vA = v / Sqr(3)

  t1 = T - T2

  If t1 > 0 Then
     X = (t1 + T2) / T2 * X - t1 / T2 * SA
     GeometricAverageRateOption = GBlackScholes (CallPutFlag, S, X, T2, r, bA,
vA) * T2 / (t1 + T2)
  ElseIf t1 = 0 Then
     GeometricAverageRateOption = GBlackScholes (CallPutFlag, S, X, T, r, bA,
vA)
  End If

End Function
```

Appendix-2.8

Arithmetic Average Rate Option Model Computer Algorithm

Computer Algorithm.

```
Public Function TurnbullWakemanAsian (CallPutFlag As String, S As Double, SA
As Double, X As Double, _
        T As Double, T2 As Double, tau As Double, r As Double, b As Double, v As
Double) As Double

  Dim m1 As Double, m2 As Double, t1 As Double
  Dim bA As Double, vA As Double

  m1 = (Exp(b * T) - Exp(b * tau)) / (b * (T - tau))
  m2 = 2 * Exp((2 * b + v ^ 2) * T) / ((b + v ^ 2) * (2 * b + v ^ 2) * (T - tau) ^ 2) _
  + 2 * Exp((2 * b + v ^ 2) * tau) / (b * (T - tau) ^ 2) * (1 / (2 * b + v ^ 2) - Exp(b *
(T - tau)) / (b + v ^ 2))

  bA = Log(m1) / T
  vA = Sqr(Log(m2) / T - 2 * bA)
  t1 = T - T2

  If t1 > 0 Then
     X = T / T2 * X - t1 / T2 * SA
     TurnbullWakemanAsian = GBlackScholes(CallPutFlag, S, X, T2, r, bA, vA) *
T2 / T
  Else
     TurnbullWakemanAsian = GBlackScholes(CallPutFlag, S, X, T2, r, bA, vA)
  End If
End Function.
```

Appendix-2.9

Levy's Approximation Option Model Computer Algorithm

Computer Algorithm

```
Public Function LevyAsian(CallPutFlag As String, S As Double, SA As Double, X
As Double, _
            T As Double, T2 As Double, r As Double, b As Double, v As Double) As
Double

   Dim SE As Double
   Dim m As Double, d As Double
   Dim Sv As Double, XStar As Double
   Dim d1 As Double, d2 As Double

   SE = S / (T * b) * (Exp((b - r) * T2) - Exp(-r * T2))
   m = 2 * S ^ 2 / (b + v ^ 2) * ((Exp((2 * b + v ^ 2) * T2) - 1) / (2 * b + v ^ 2) -
(Exp(b * T2) - 1) / b)
   d = m / (T ^ 2)
   Sv = Log(d) - 2 * (r * T2 + Log(SE))
   XStar = X - (T - T2) / T * SA
   d1 = 1 / Sqr(Sv) * (Log(d) / 2 - Log(XStar))
   d2 = d1 - Sqr(Sv)

   If CallPutFlag = "c" Then
      LevyAsian = SE * CND(d1) - XStar * Exp(-r * T2) * CND(d2)
   ElseIf CallPutFlag = "p" Then
      LevyAsian = (SE * CND(d1) - XStar * Exp(-r * T2) * CND(d2)) - SE + XStar *
Exp(-r * T2)
   End If
End Function.
```

Appendix-2.10

Compound Option Model Computer Algorithm

Computer Algorithm

Options on options

```
Public Function OptionsOnOptions(TypeFlag As String, S As Double, X1 As Double,
X2 As Double, t1 As Double, _
            T2 As Double, r As Double, b As Double, v As Double) As Double

   Dim y1 As Double, y2 As Double, z1 As Double, z2 As Double
   Dim I As Double, rho As Double, CallPutFlag As String

   If TypeFlag = "cc" Or TypeFlag = "pc" Then
      CallPutFlag = "c"
   Else
      CallPutFlag = "p"
   End If
```

```
  I = CriticalValueOptionsOnOptions(CallPutFlag, X1, X2, T2 - t1, r, b, v)

  rho = Sqr(t1 / T2)
  y1 = (Log(S / I) + (b + v ^ 2 / 2) * t1) / (v * Sqr(t1))
  y2 = y1 - v * Sqr(t1)
  z1 = (Log(S / X1) + (b + v ^ 2 / 2) * T2) / (v * Sqr(T2))
  z2 = z1 - v * Sqr(T2)

  If TypeFlag = "cc" Then
    OptionsOnOptions = S * Exp((b - r) * T2) * CBND(z1, y1, rho) - X1 * Exp(-r *
T2) * CBND(z2, y2, rho) - X2 * Exp(-r * t1) * CND(y2)
  ElseIf TypeFlag = "pc" Then
    OptionsOnOptions = X1 * Exp(-r * T2) * CBND(z2, -y2, -rho) - S * Exp((b - r)
* T2) * CBND(z1, -y1, -rho) + X2 * Exp(-r * t1) * CND(-y2)
  ElseIf TypeFlag = "cp" Then
    OptionsOnOptions = X1 * Exp(-r * T2) * CBND(-z2, -y2, rho) - S * Exp((b - r)
* T2) * CBND(-z1, -y1, rho) - X2 * Exp(-r * t1) * CND(-y2)
  ElseIf TypeFlag = "pp" Then
    OptionsOnOptions = S * Exp((b - r) * T2) * CBND(-z1, y1, -rho) - X1 * Exp(-r
* T2) * CBND(-z2, y2, -rho) + Exp(-r * t1) * X2 * CND(y2)
  End If
End Function.

'// Calculation of critical price options on options (I)
Private Function CriticalValueOptionsOnOptions(CallPutFlag As String, X1 As
Double, X2 As Double, T As Double, _
        r As Double, b As Double, v As Double) As Double

  Dim Si As Double, ci As Double, di As Double, epsilon As Double

  Si = X1
  ci = GBlackScholes(CallPutFlag, Si, X1, T, r, b, v)
  di = GDelta(CallPutFlag, Si, X1, T, r, b, v)
  epsilon = 0.000001
  '// Newton-Raphson algorithm
  While Abs(ci - X2) > epsilon
    Si = Si - (ci - X2) / di
    ci = GBlackScholes(CallPutFlag, Si, X1, T, r, b, v)
    di = GDelta(CallPutFlag, Si, X1, T, r, b, v)
  Wend
  CriticalValueOptionsOnOptions = Si
End Function.
```

The ***Newton-Raphson method*** is an efficient way to find the value of I. I is the underlying price which makes the underlying option equal to X_2 at time t_1. The method seldom spends more than two to three searches before it converges to the value of I. Let

$$X_1 = X_1 - \frac{(c_i - X_2)}{d_i}$$

until $|c_i - X_2| \geq \varepsilon$, at which point c_i is the price of the option (call or put) with strike X_1, ε is the desired degree of accuracy (in our case 0.000001), d_i is the delta of the option with strike X_1, X_1 is the strike price of the underlying option and X_2 is the strike price of the option on the option.

Where *CND* is the cumulative normal distribution function and CBND is the cumulative bivariate normal distribution function described below.
The standardised cumulative normal function returns the probability that one random variable is less than a and that a second random variable is less than b when the correlation between the two variables is ρ:

$$M(a,b;\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{q} \int_{-\infty}^{b} \exp\left[-\frac{x^2 - 2\rho xy + y}{2(1-\rho^2)} \right] dx \quad dy$$

Drezner (1978) has developed a method for approximating the cumulative bivariate normal distribution function. This approximation produces values of $M(a,b;\rho)$ to within six decimal places accuracy.

$$\varphi(a,b;\rho) = \frac{\sqrt{1-\rho^2}}{\pi} \sum_{i=1}^{5} \sum_{j=1}^{5} x_i x_j f(y_i, y_j),$$

where

$$f(y_i, y_j) = \exp[a_1(2y_i - a_1) + b_1(2y_i - b_1) + 2\rho(y_i - a_1)(y_i - b_1)]$$

$$a_1 = \frac{a}{\sqrt{2(1-\rho^2)}}, \quad b_1 = \frac{b}{\sqrt{2(1-\rho^2)}}$$

$x_1 = 0.24840615$

$x_2 = 0.39233107$

$x_3 = 0.21141819$

$x_4 = 0.033246670$

$x_5 = 0.00082485334$

$y_1 = 0.10024215$

$y_2 = 0.48281397$

$y_3 = 1.0609498$

$y_4 = 1.7797294$

$y_5 = 2.6697604$

If the of a, b, and ρ is non positive, compute the cumulative bivariate normal probability using the following rules:

1.If $a \leq 0, b \leq 0$, and $\rho \leq 0$, then

$$M(a,b;\rho) = \varphi(a,b;\rho)$$

2.If $a \leq 0, b \geq 0$, and $\rho \geq 0$, then

$$M(a,b\rho) = N(a) - \varphi(a,-b;-\rho)$$

3.If $a \geq 0, b \leq 0,$ and $\rho \geq 0,$ then

$$M(a,b;\rho) = N(b) - \varphi(-a,b;-\rho)$$

4.If $a \geq 0, b \geq 0,$ and $\rho \leq 0$, then

$$M(a,b;\rho) = N(a) + N(b) - 1 + \varphi(-a,-b;\rho)$$

In these circumstances where the product a,b and ρ is positive compute the cumulative bivariate normal function as:

$$M(a,b;\rho) = M(a,0;\rho_1) + M(b,0;\ p_2) - \delta,$$

where $M(a,0;\rho_1)$ and $M(b,0;\rho_2)$ are computed from the rules where the product of a,b and ρ is negative, and

$$\rho_1 = \frac{(\rho a - b)Sign(a)}{\sqrt{a^2 - 2\rho ab + b^2}}, \qquad \rho_2 = \frac{(\rho b - a)Sign(b)}{\sqrt{a^2 - 2\rho ab + b^2}}$$

$$\delta = \frac{1 - Sign(a) * Sign(b)}{4}, \qquad Sign(x) = \begin{cases} +1 \text{ when } & x \geq 0 \\ \text{-1 when } & x < 0 \end{cases}$$

Computer Algorithm
The $CBND(a, b, \rho)$ function returns the standardised bivariate normal probability that the first variable is less than a and the second variable is less than b where ρ is the correlation between the two variables.

The cumulative bivariate normal distribution function
Public Function CBND(a As Double, b As Double, rho As Double) As Double

```
    Dim X As Variant, y As Variant
    Dim rho1 As Double, rho2 As Double, delta As Double
    Dim a1 As Double, b1 As Double, Sum As Double
    Dim I As Integer, j As Integer

    X = Array(0.24840615, 0.39233107, 0.21141819, 0.03324666, 0.00082485334)
    y = Array(0.10024215, 0.48281397, 1.0609498, 1.7797294, 2.6697604)
    a1 = a / Sqr(2 * (1 - rho ^ 2))
    b1 = b / Sqr(2 * (1 - rho ^ 2))

    If a <= 0 And b <= 0 And rho <= 0 Then
        Sum = 0
        For I = 1 To 5
            For j = 1 To 5
```

```
            Sum = Sum + X(I) * X(j) * Exp(a1 * (2 * y(I) - a1) _
            + b1 * (2 * y(j) - b1) + 2 * rho * (y(I) - a1) * (y(j) - b1))
         Next
      Next
      CBND = Sqr(1 - rho ^ 2) / Pi * Sum
   ElseIf a <= 0 And b >= 0 And rho >= 0 Then
      CBND = CND(a) - CBND(a, -b, -rho)
   ElseIf a >= 0 And b <= 0 And rho >= 0 Then
      CBND = CND(b) - CBND(-a, b, -rho)
   ElseIf a >= 0 And b >= 0 And rho <= 0 Then
      CBND = CND(a) + CND(b) - 1 + CBND(-a, -b, rho)
   ElseIf a * b * rho > 0 Then
      rho1 = (rho * a - b) * Sgn(a) / Sqr(a ^ 2 - 2 * rho * a * b + b ^ 2)
      rho2 = (rho * b - a) * Sgn(b) / Sqr(a ^ 2 - 2 * rho * a * b + b ^ 2)
      delta = (1 - Sgn(a) * Sgn(b)) / 4
      CBND = CBND(a, 0, rho1) + CBND(b, 0, rho2) - delta
   End If
End Function.
```

Appendix-2.11

Fixed Strike Lookback Option Model Computer Algorithm

Computer Algorithm.

```
// Fixed strike lookback options
Public Function FixedStrikeLookback(CallPutFlag As String, S As Double, SMin As
Double, SMax As Double, X As Double, _
            T As Double, r As Double, b As Double, v As Double) As Double

   Dim d1 As Double, d2 As Double
   Dim e1 As Double, e2 As Double, m As Double

   If CallPutFlag = "c" Then
      m = SMax
   ElseIf CallPutFlag = "p" Then
      m = SMin
   End If

   d1 = (Log(S / X) + (b + v ^ 2 / 2) * T) / (v * Sqr(T))
   d2 = d1 - v * Sqr(T)
   e1 = (Log(S / m) + (b + v ^ 2 / 2) * T) / (v * Sqr(T))
   e2 = e1 - v * Sqr(T)

   If CallPutFlag = "c" And X > m Then
      FixedStrikeLookback = S * Exp((b - r) * T) * CND(d1) - X * Exp(-r * T) *
CND(d2) _
```

```
      + S * Exp(-r * T) * v ^ 2 / (2 * b) * (-(S / X) ^ (-2 * b / v ^ 2) * CND(d1 - 2 * b /
v * Sqr(T)) + Exp(b * T) * CND(d1))
   ElseIf CallPutFlag = "c" And X <= m Then
      FixedStrikeLookback = Exp(-r * T) * (m - X) + S * Exp((b - r) * T) * CND(e1) -
Exp(-r * T) * m * CND(e2) _
      + S * Exp(-r * T) * v ^ 2 / (2 * b) * (-(S / m) ^ (-2 * b / v ^ 2) * CND(e1 - 2 * b /
v * Sqr(T)) + Exp(b * T) * CND(e1))
   ElseIf CallPutFlag = "p" And X < m Then
      FixedStrikeLookback = -S * Exp((b - r) * T) * CND(-d1) + X * Exp(-r * T) *
CND(-d1 + v * Sqr(T)) _
      + S * Exp(-r * T) * v ^ 2 / (2 * b) * ((S / X) ^ (-2 * b / v ^ 2) * CND(-d1 + 2 * b /
v * Sqr(T)) - Exp(b * T) * CND(-d1))
   ElseIf CallPutFlag = "p" And X >= m Then
      FixedStrikeLookback = Exp(-r * T) * (X - m) - S * Exp((b - r) * T) * CND(-e1)
+ Exp(-r * T) * m * CND(-e1 + v * Sqr(T)) _
      + Exp(-r * T) * v ^ 2 / (2 * b) * S * ((S / m) ^ (-2 * b / v ^ 2) * CND(-e1 + 2 * b /
v * Sqr(T)) - Exp(b * T) * CND(-e1))
   End If
End Function.
```

Appendix-2.12

Floating Strike Lookback Option Model Computer Algorithm

Computer Algorithm.

Floating strike lookback options

```
Function FloatingStrikeLookback(CallPutFlag As String, S As Double, SMin As
Double, SMax As Double, T As Double, _
        r As Double, b As Double, v As Double) As Double

   Dim a1 As Double, a2 As Double, m As Double

   If CallPutFlag = "c" Then
      m = SMin
   ElseIf CallPutFlag = "p" Then
      m = SMax
   End If

   a1 = (Log(S / m) + (b + v ^ 2 / 2) * T) / (v * Sqr(T))
   a2 = a1 - v * Sqr(T)
```

```
  If CallPutFlag = "c" Then
    FloatingStrikeLookback = S * Exp((b - r) * T) * CND(a1) - m * Exp(-r * T) *
CND(a2) + _
    Exp(-r * T) * v ^ 2 / (2 * b) * S * ((S / m) ^ (-2 * b / v ^ 2) * CND(-a1 + 2 * b / v
* Sqr(T)) - Exp(b * T) * CND(-a1))
  ElseIf CallPutFlag = "p" Then
    FloatingStrikeLookback = m * Exp(-r * T) * CND(-a2) - S * Exp((b - r) * T) *
CND(-a1) + _
    Exp(-r * T) * v ^ 2 / (2 * b) * S * (-(S / m) ^ (-2 * b / v ^ 2) * CND(a1 - 2 * b / v
* Sqr(T)) + Exp(b * T) * CND(a1))
  End If
End Function.
```

Appendix-2.13

Barrier Option Model Computer Algorithm

Computer Algorithm

```
Function StandardBarrier(TypeFlag As String, S As Double, X As Double, H As
Double, K As Double, T As Double, _
        r As Double, b As Double, v As Double)

   Dim mu As Double
   Dim lambda As Double
   Dim X1 As Double, X2 As Double
   Dim y1 As Double, y2 As Double
   Dim Z As Double

   Dim eta As Integer   'Binary variable that can take the value of 1 or -1
   Dim phi As Integer   'Binary variable that can take the value of 1 or -1

   Dim f1 As Double   'Equal to formula "A" in the thesis
   Dim f2 As Double   'Equal to formula "B" in the thesis
   Dim f3 As Double   'Equal to formula "C" in the thesis
   Dim f4 As Double   'Equal to formula "D" in the thesis
   Dim f5 As Double   'Equal to formula "E" in the thesis
   Dim f6 As Double   'Equal to formula "F" in the thesis

   mu = (b - v ^ 2 / 2) / v ^ 2
   lambda = Sqr(mu ^ 2 + 2 * r / v ^ 2)
   X1 = Log(S / X) / (v * Sqr(T)) + (1 + mu) * v * Sqr(T)
   X2 = Log(S / H) / (v * Sqr(T)) + (1 + mu) * v * Sqr(T)
   y1 = Log(H ^ 2 / (S * X)) / (v * Sqr(T)) + (1 + mu) * v * Sqr(T)
```

```
y2 = Log(H / S) / (v * Sqr(T)) + (1 + mu) * v * Sqr(T)
Z = Log(H / S) / (v * Sqr(T)) + lambda * v * Sqr(T)

If TypeFlag = "cdi" Or TypeFlag = "cdo" Then
   eta = 1
   phi = 1
ElseIf TypeFlag = "cui" Or TypeFlag = "cuo" Then
   eta = -1
   phi = 1
ElseIf TypeFlag = "pdi" Or TypeFlag = "pdo" Then
   eta = 1
   phi = -1
ElseIf TypeFlag = "pui" Or TypeFlag = "puo" Then
   eta = -1
   phi = -1
End If

 f1 = phi * S * Exp((b - r) * T) * CND(phi * X1) - phi * X * Exp(-r * T) * CND(phi
* X1 - phi * v * Sqr(T))
 f2 = phi * S * Exp((b - r) * T) * CND(phi * X2) - phi * X * Exp(-r * T) * CND(phi
* X2 - phi * v * Sqr(T))
 f3 = phi * S * Exp((b - r) * T) * (H / S) ^ (2 * (mu + 1)) * CND(eta * y1) - phi * X
* Exp(-r * T) * (H / S) ^ (2 * mu) * CND(eta * y1 - eta * v * Sqr(T))
 f4 = phi * S * Exp((b - r) * T) * (H / S) ^ (2 * (mu + 1)) * CND(eta * y2) - phi * X
* Exp(-r * T) * (H / S) ^ (2 * mu) * CND(eta * y2 - eta * v * Sqr(T))
 f5 = K * Exp(-r * T) * (CND(eta * X2 - eta * v * Sqr(T)) - (H / S) ^ (2 * mu) *
CND(eta * y2 - eta * v * Sqr(T)))
 f6 = K * ((H / S) ^ (mu + lambda) * CND(eta * Z) + (H / S) ^ (mu - lambda) *
CND(eta * Z - 2 * eta * lambda * v * Sqr(T)))


 If X > H Then
    Select Case TypeFlag
       Case Is = "cdi"
          StandardBarrier = f3 + f5
       Case Is = "cui"
          StandardBarrier = f1 + f5
       Case Is = "pdi"
          StandardBarrier = f2 - f3 + f4 + f5
       Case Is = "pui"
          StandardBarrier = f1 - f2 + f4 + f5
       Case Is = "cdo"
          StandardBarrier = f1 - f3 + f6
       Case Is = "cuo"
          StandardBarrier = f6
       Case Is = "pdo"
          StandardBarrier = f1 - f2 + f3 - f4 + f6
       Case Is = "puo"
          StandardBarrier = f2 - f4 + f6
    End Select
```

```
  ElseIf X < H Then
    Select Case TypeFlag
      Case Is = "cdi"
        StandardBarrier = f1 - f2 + f4 + f5
      Case Is = "cui"
        StandardBarrier = f2 - f3 + f4 + f5
      Case Is = "pdi"
        StandardBarrier = f1 + f5
      Case Is = "pui"
        StandardBarrier = f3 + f5
      Case Is = "cdo"
        StandardBarrier = f2 + f6 - f4
      Case Is = "cuo"
        StandardBarrier = f1 - f2 + f3 - f4 + f6
      Case Is = "pdo"
        StandardBarrier = f6
      Case Is = "puo"
        StandardBarrier = f1 - f3 + f6
    End Select
  End If
End Function
```

Appendix-2.14

Binary (Cash-or-nothing) Option Model Computer Algorithm

```
'// Cash-or-nothing options
Public Function CashOrNothing(CallPutFlag As String, S As Double, X As Double,
K As Double, T As Double, r As Double, v As Double) As Double

  Dim d As Double

  d = (Log(S / X) + ( - v ^ 2 / 2) * T) / (v * Sqr(T))

  If CallPutFlag = "c" Then
    CashOrNothing = K * Exp(-r * T) * CND(d)
  ElseIf CallPutFlag = "p" Then
    CashOrNothing = K * Exp(-r * T) * CND(-d)
  End If
End Function
```

Appendix-2.15

Binary (Asset-or-nothing) Option Model Computer Algorithm

```
'// Asset-or-nothing options
Public Function AssetOrNothing(CallPutFlag As String, S As Double, X As Double,
T As Double, r As Double, v As Double) As Double

   Dim d As Double

   d = (Log(S / X) + ( v ^ 2 / 2) * T) / (v * Sqr(T))

   If CallPutFlag = "c" Then
      AssetOrNothing = S * Exp((b - r) * T) * CND(d)
   ElseIf CallPutFlag = "p" Then
      AssetOrNothing = S * Exp((b - r) * T) * CND(-d)
   End If
End Function
```

Appendix-2.16

European Option Model Computer Algorithm with Monte Carlo Simulation

Computer Algorithm for European Options (calls and put options)

```
// Monte Carlo plain vanilla European option
Public Function MonteCarloStandardOption(CallPutFlag As String, S As Double, X
As Double, T As Double, _
            r As Double, b As Double, v As Double, nSteps As Integer, nSimulations
As Integer) As Double

   Dim dt As Double, St As Double
   Dim Sum As Double, Drift As Double, vSqrdt As Double
   Dim i As Integer, j As Integer, z As Integer

   dt = T / nSteps
   Drift = (b - v ^ 2 / 2) * dt
   vSqrdt = v * Sqr(dt)

   If CallPutFlag = "c" Then
      z = 1
   ElseIf CallPutFlag = "p" Then
      z = -1
   End If

   For i = 1 To nSimulations
      St = S
      For j = 1 To nSteps
         St = St * Exp(Drift + vSqrdt * Application.NormInv(Rnd(), 0, 1))
      Next
```

```
    Sum = Sum + Max(z * (St - X), 0)
  Next

  MonteCarloStandardOption = Exp(-r * T) * (Sum / nSimulations)

End Function.
```

Appendix-2.17

Two Asset Asian Option Model Computer Algorithm with Monte Carlo Simulation

Computer Algorithm for two asset Asian spread options.

```
Monte Carlo two asset Asian spread option
Public Function MonteCarloAsianSpreadOption(CallPutFlag As String, S1 As
Double, S2 As Double, _
        X As Double, T As Double, r As Double, b1 As Double, b2 As Double, v1
As Double, v2 As Double, rho As Double, _
        nSteps As Integer, nSimulations As Integer) As Double

  Dim dt As Double, St1 As Double, St2 As Double
  Dim i As Integer, j As Integer, z As Integer
  Dim Sum As Double, Drift1 As Double, Drift2 As Double
  Dim v1Sqrdt As Double, v2Sqrdt As Double
  Dim Epsilon1 As Double, Epsilon2 As Double, Average1 As Double, Average2 As
Double

  If CallPutFlag = "c" Then
    z = 1
  ElseIf CallPutFlag = "p" Then
    z = -1
  End If

  dt = T / nSteps
  Drift1 = (b1 - v1 ^ 2 / 2) * dt
  Drift2 = (b2 - v2 ^ 2 / 2) * dt
  v1Sqrdt = v1 * Sqr(dt)
  v2Sqrdt = v2 * Sqr(dt)

  For i = 1 To nSimulations
    Average1 = 0
    Average2 = 0
    St1 = S1
    St2 = S2
    For j = 1 To nSteps
      Epsilon1 = Application.NormInv(Rnd(), 0, 1)
      Epsilon2 = rho * Epsilon1 + Application.NormInv(Rnd(), 0, 1) * Sqr(1 - rho ^
2)
      St1 = St1 * Exp(Drift1 + v1Sqrdt * Epsilon1)
```

```
      St2 = St2 * Exp(Drift2 + v2Sqrdt * Epsilon2)
      Average1 = Average1 + St1
      Average2 = Average2 + St2
    Next
    Average1 = Average1 / nSteps
    Average2 = Average2 / nSteps
    Sum = Sum + Max(z * (Average1 - Average2 - X), 0)
  Next

  MonteCarloAsianSpreadOption = Exp(-r * T) * (Sum / nSimulations)

End Function.
```

CHAPTER 3

SPOT PRICE BEHAVIOUR

3.1 Introduction

Historically the majority of work on modelling energy and commodity prices has been focused on stochastic processes for the spot price and other key variables, such us the convenience yield and interest rates (examples include; Schwartz (1997), Gibson & Schwartz (1990), Miltersen & Schwartz (1998), Hilliard & Rays (1998)).
Gibson & Schwartz (1990) presented a different approach to the valuation of commodity derivatives. They develop a two-factor pricing model where the first factor is the spot price of the commodity, and the second factor is the instantaneous convenience yield. They analyse its performance in valuing short as well long-term oil contracts. Their main empirical results show that the model performs well in valuing short-term contracts such as futures. The computed theoretical present values of one to ten years ahead deliverable oil barrel seem to be low and hence suggest that the risk premium for long term oil investments is high. Furthermore the two-factor model t is able to explain the intrinsic difference in price volatility between spot and future contracts as well as its decreasing maturity pattern observed among the latter. Finally they show that although they apply the model to financial securities whose payoff structure is linear in the spot price of crude oil, it can easily be extended to any more complex payoff structure characterising the option features of real and financial oil claims.
Schwartz (1997) compares three models of the stochastic behaviour of commodity prices that take into account mean reversion, in terms of their ability to price existing futures contracts and their implication with respect to the valuation of other financial and real assets. The first model is a simple one-factor model in which the logarithm of the spot price of the commodity is assumed to follow a mean reverting process of the Ornstein-Uhlenbeck type. The second model is a variation of the two factors Gibson & Schwartz (1990) model. The second factor in this model is the convenience yield of the commodity and it is assumed to follow a mean reverting process. Finally, he extends Gibson & Schwartz (1990) model by introducing a third stochastic factor, the instantaneous interest rate that is also assumed to follow a mean reverting process as in Vasicek (1977). One of the main difficulties in the empirical implementation of commodity price models is that frequently the factors or state variables are not directly observable. In many cases the spot price of a commodity is so uncertain that the corresponding futures contract closest to maturity is used as a proxy for the spot price. The instantaneous convenience yield is even more difficult to estimate.
Hilliard & Rays (1998) extend this three-factor model by introducing jumps in the spot price of the commodity and by using the term structure of the interest rates to eliminate the market price of interest rate risk in their fundamental pricing equation. However, they leave the market price of convenience yield risk as a parameter (to be determined in equilibrium) in their pricing formulae's.
Miltersen & Schwartz (1998) develop a model that generalises and combine the two approaches by using all the information in the initial term structure of both interest rates and commodity futures prices. In addition assuming normality of continuously compounded forward interest rates and convenience yields and log-normality of the spot price of the underlying commodity, they obtain closed-form solutions for the

pricing of options on futures prices, which are in the spirit of Black and Scholes (1973) and Merton (1973). Also in the development of the model, they distinguish between forward and future convenience yields, a distinction that has not been recognised in the existing literature at the time. It is empirically stylised fact that the most commodity price processes are mean reverting (see Section 3.3). Standard no-arbitrage arguments completely determine the drift of the price processes under an equivalent martingale measure leaving no room for explicit modelling of mean reversion via the drift of the spot commodity price. However, the spot convenience yield process enters the drift of the spot commodity price under an equivalent martingale measure in such a way that a positive correlation between the spot commodity price and the spot convenience yield will have a mean reversion effect on the spot commodity price even under an equivalent martingale measure. Clearly, this has an impact on the option prices. The option-pricing model that Miltersen & Schwartz (1998) developed took this phenomenon into account.

All the models described above, are no arbitrage based models, which they have one fundamental disadvantage; the factors or state variables of these models are not directly observable (predominantly as the convenience yield). So the empirical implementation of these models is very difficult.

For this reason in this chapter we extend the Black-Scholes model, introduced and discussed in detail in chapter 2 to take into account empirical results and the observed in the spot price process of energy markets. For the first time in the literature we test the mean reversion, jump diffusion and mean reversion jump diffusion model using high quality data across the different energy markets, and we compare the relative performance of these models compare to the Black's approach. The price processes we discuss in detail are the following:

- Mean Reverting behaviour of the energy spot price (the tendency of spot prices to move back towards their long-term level).
- Jump Diffusion behaviour.
- Combined Mean Reverting and Jump behaviour.

The application of Monte Carlo Simulation on these processes is also described. The processes mentioned above, are the extensions to the Black-Scholes Geometric Brownian Motion model by assuming constant parameters. These parameters (volatility, mean reversion rate, jump parameters, long term level) will be estimated from historical data with robust estimation methods, which are presented in this chapter. We incorporate these parameters into the Monte Carlo Simulations of these processes and compare them to see which one of these processes is the appropriate to describe the oil market the natural gas, and the electricity market.

3.2` Mean Reversion

The mean-reversion behaviour has been considered to be one of the most important features of commodities. Basic microeconomics theory tells that, in the long run, the price of a commodity ought to be tied to its long-run marginal production cost or, "*in case of a cartelized commodity like oil, the long-run profit-maximising price sought by cartel managers*" (Laughton&Jacoby,1995,p.188).
In other words, although oil prices have short-term oscillations, they tend to revert back to a "normal" long-term equilibrium level.
Production cost varies largely across the countries, mainly due to the geologic features, and most of the lower cost countries belong (or are influenced) by the OPEC cartel.

Hence, even with a growing non-OPEC production, the OPEC role remains very important in the production game of the petroleum industry. The large oil prices rises in February-April 1999 are mainly due to the articulation power of OPEC (and its eventual allies) in reducing the production.

There is strong empirical evidence that oil prices are mean reverting, see for example Gibson & Schwartz (1990), Brennan (1991), Cortazar & Schwartz (1994), Schwartz (1997)[1].

Pindyck & Rubinfeld (1991, chapter 15) using a Dickey-Fuller unit root test, rejected the random walk hypothesis for a very long time series (more than 100 years). But they point out that the oil price reversion to a long-run equilibrium level is likely to be slow.

Other important mean-reverting evidence comes from the futures market, as pointed out by Baker et al (1998, pp.124-127). This is summarised as follows:
First, the term structure of futures prices is decreasing (toward the "normal" long-run level, in backwardation) if the spot prices are "high", and is increasing (in contango) if prices are "low".
Second, if the prices are random walk, the volatility in the futures prices should equal the volatility of the spot price, but the data show that spot prices are much more volatile than futures prices. In both cases, the mean-reverting model is much more consistent with the futures prices data than the random walk model. In addition, the econometric tests from futures term structure performed by Bessembinder et al (1995, p.373-374) also reveals strong mean-reversion for oil prices and agricultural commodities (but weak reversion for precious metals and financial assets).

---

[1] See also "The Stochastic Behaviour of Commodity Prices"and the econometric tests of Dragana Pilipovic (Energy Risk 1998, table 4-9, p.78, WTI petroleum).

Schwartz (1997) presented a classic model for mean reverting spot price behaviour in energy markets which is represented by the following equation:

$$dS = \alpha(\mu - \ln S)Sdt + \sigma Sdz \qquad (3.4)$$

In this model σ is the volatility of the stock and the spot price mean reverts to the long-term level $\overline{S} = e^{\mu}$ at a speed given by the mean reversion rate, α which is taken to be strictly positive. If the spot price is above the long-term level $\overline{S}$, then the drift of the spot price will be negative and the price will turn to revert towards the long-term level. Similarly, if the spot price is below the long-term level then the drift will be positive and the price will tend to move back towards $\overline{S}$. At any point in time, the spot price will not necessarily move back towards the long-term level as the random change in the spot price may be of the opposite sign of the component.

3.4 Estimation of the Mean Reversion Rate

The mean reversion rate of the spot energy price can be estimated relatively simply and robustly via linear regression.[2] We consider here the simple mean reverting process for the natural logarithm form of the energy spot price in equation (3.4), $x = lnS$;

$$dx = a\ (\mu\ - x)\ dt + \sigma dz. \qquad (3.5)$$

This can be discretised as follows:

$$\Delta x_t\ = \alpha_0 + \alpha_1 x_t\ + \sigma\varepsilon_t,\ \varepsilon_t \sim N(0,1) \qquad (3.6)$$

where $\alpha_0 = a\ \mu\ \Delta t$ and $\alpha_1\ = -a\ \Delta t$.This implies that observations of the spot price through time can be considered $\alpha_1\ = -a\ \Delta t$ as observations of the linear relationship between Δx_t and x_t in the presence of noise (represented by $\sigma\varepsilon_t$).Therefore, if we regress observations of Δx_t against x_t we can obtain estimates of $\alpha_0 = a\ \mu\ \Delta t$ and $\alpha_1\ = -a\ \Delta t$ as the estimates of the intercept and slope of the linear relationship. Since we know the time interval between observations Δt we can obtain estimates of a and μ.

The market dailydata we are going to use in order to calculate the mean reversion rate are for the years between 1995 and 2000 for the WTI crude oil, Brent crude oil, and natural gas prices.

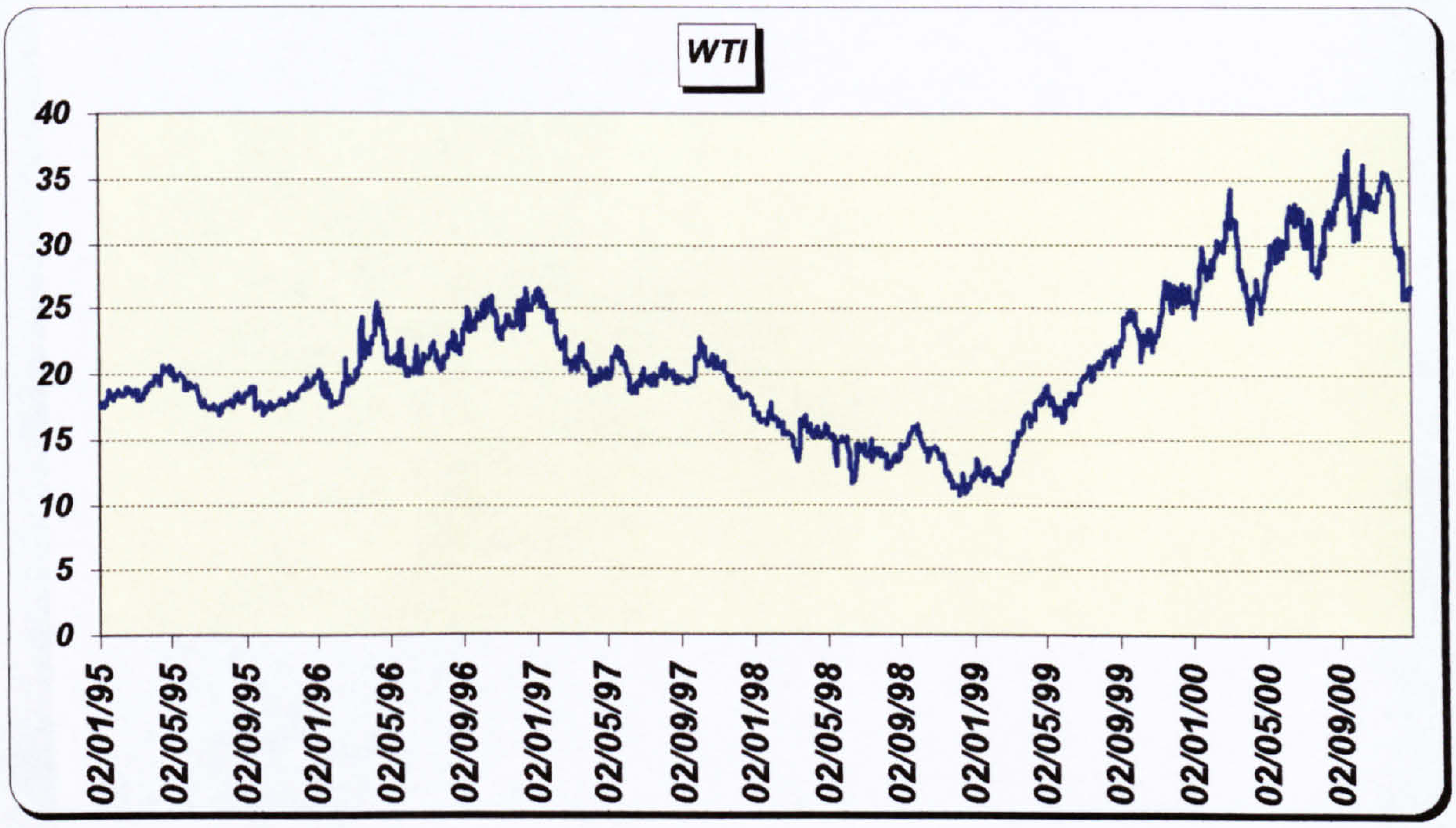
Electricity market data were available for relatively shorter periods. Since the electricity markets are just recently in the process of deregulation, we have to keep in mind that any parameters estimated from this data set may not be necessarily the parameters will be seeing in the future. The deregulation of the electricity markets is

[2] EPRM November 2000 "Making the most of Mean Reversion" by Les Clewlow, Chris Strickland & Vince Kaminski). See also Energy Derivatives Pricing & Risk Management 2000by Les Clewlow & Chris Strickland page29

bound to cause changes in the way the prices act. Accordance with Schwartz (1997) we use futures to approximate the spot prices.
The markets to be analysed are:

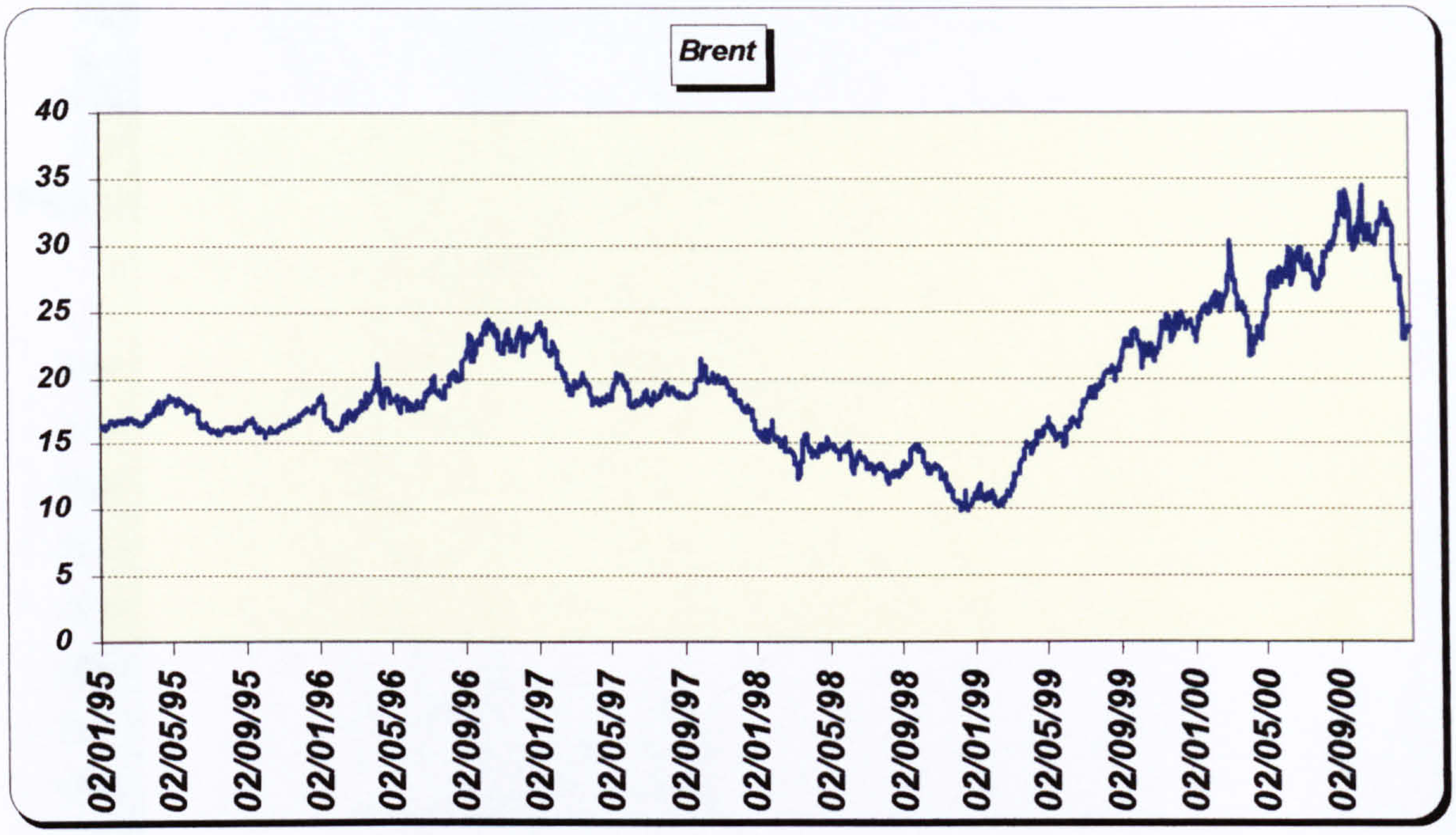
- West Texas Intermediate (WTI) crude oil first nearby future from the New York Mercantile Exchange (NYMEX). Figure-3.1 plots the time series for the first nearby WTI future from January 1995 to 29 December 2000.

Figure-3.1



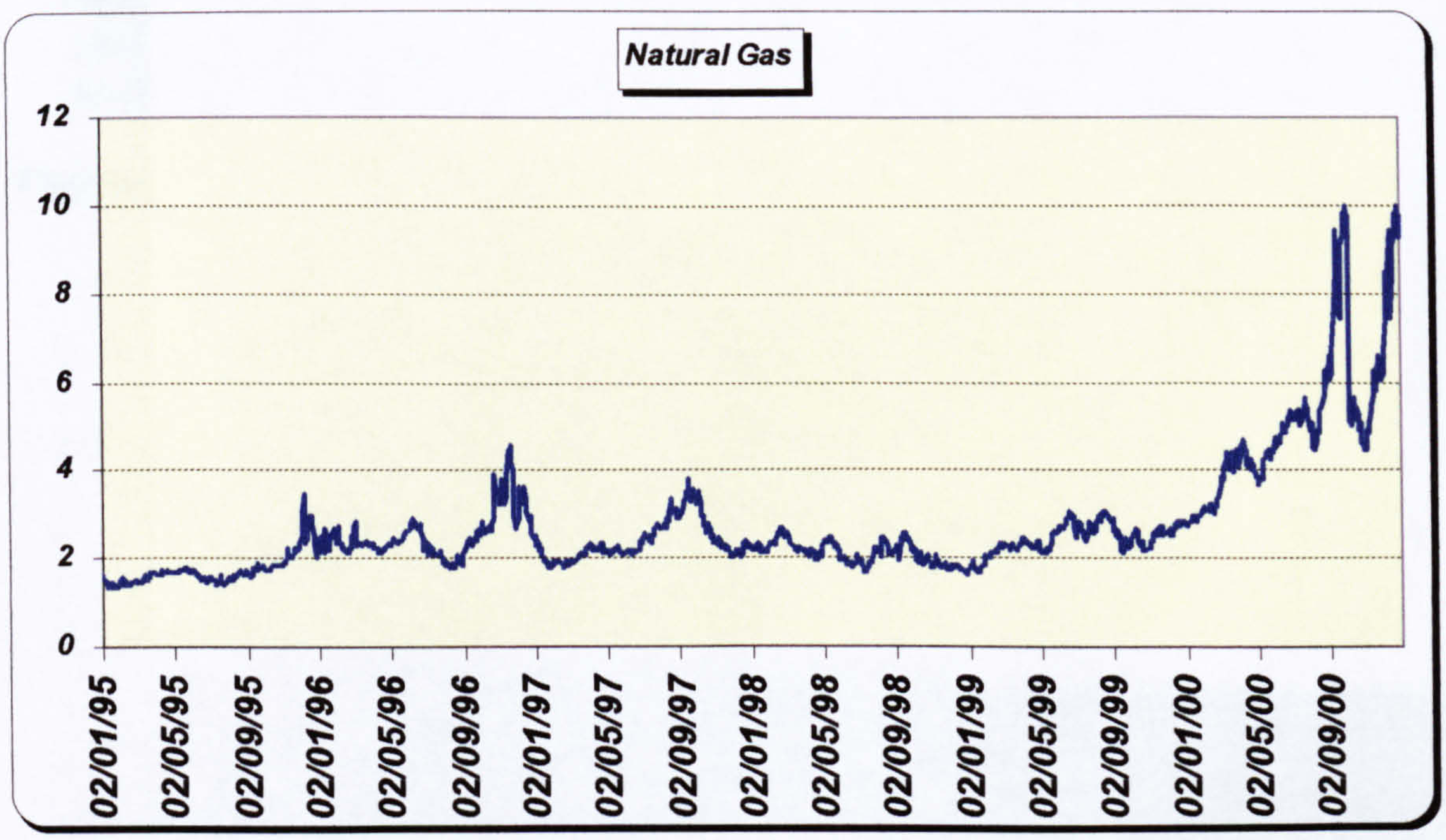
- Brent crude oil first nearby from IPE. Figure-3.2 plots the time series for the first nearby Brent IPE future from January 1995 to 29 December 2000

Figure-3.2



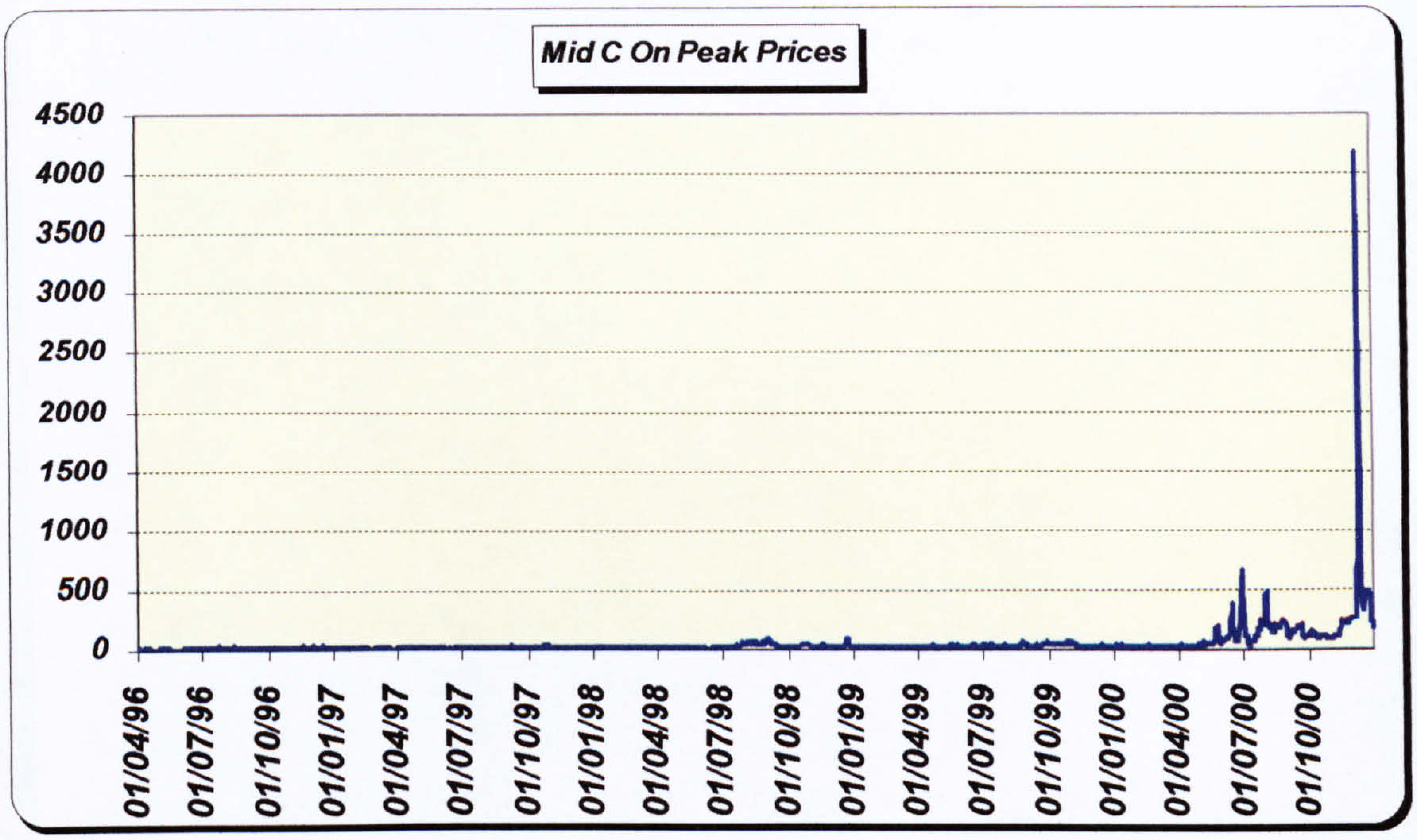
- Natural gas (NG) futures from NYMEX. Under the NYMEX futures contract natural gas is delivered over the next calendar month past contract expiration at the Henry Hub delivery node near the Gulf of Mexico. Figure-3.3 plots the time series for the first nearby NG future contract between January 1995 to 30 October 2000.These futures are based on delivery of natural gas over the whole contract month.

Figure-3.3



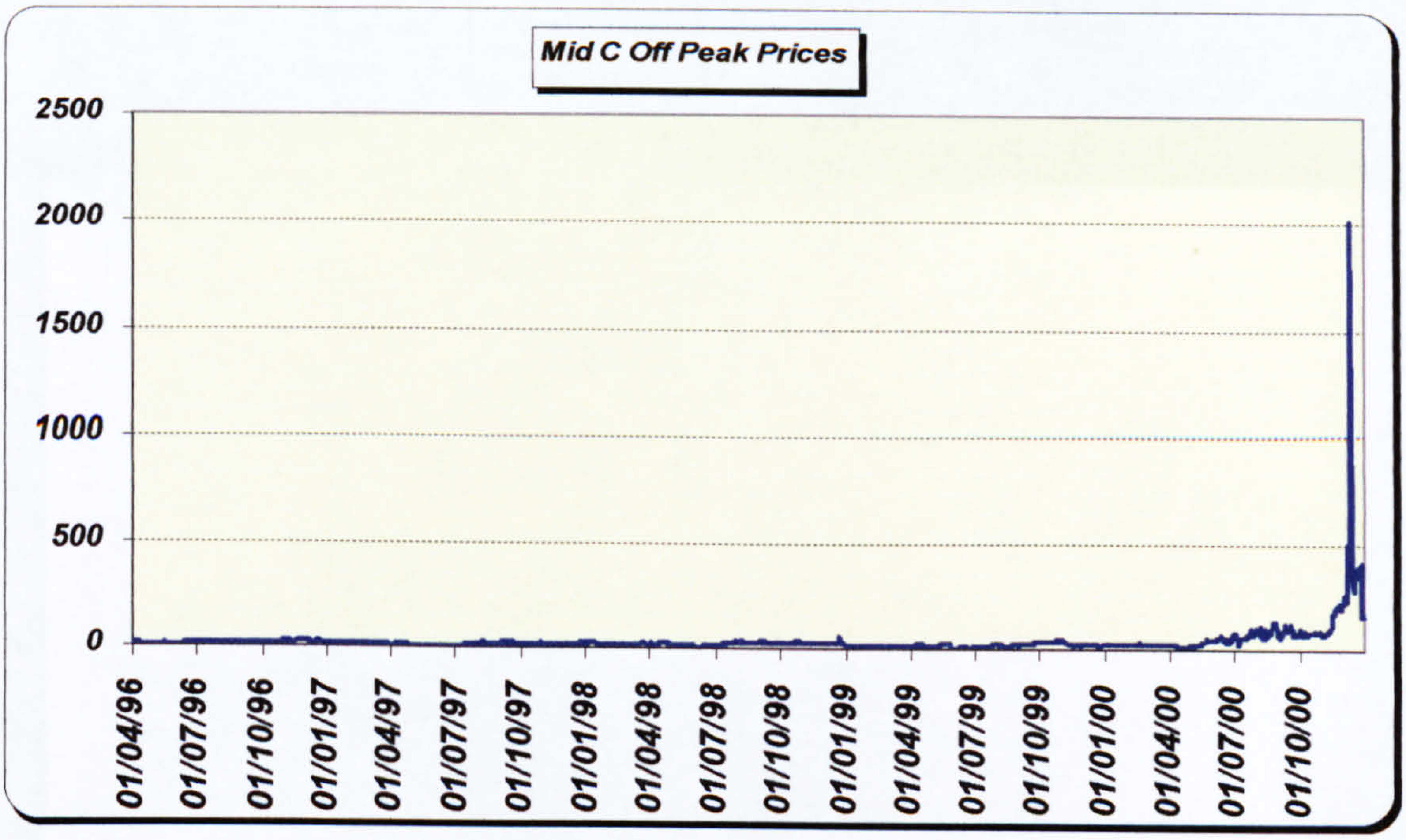
- The Mid-Columbia (MC) On Peak over-the-counter electricity spot price. We are using the Dow Jones daily price index (closing prices) during April 1996 to December 2000.Figure-3.4 plots the time series for the MC's on-peak spot prices for that period.

Figure-3.4



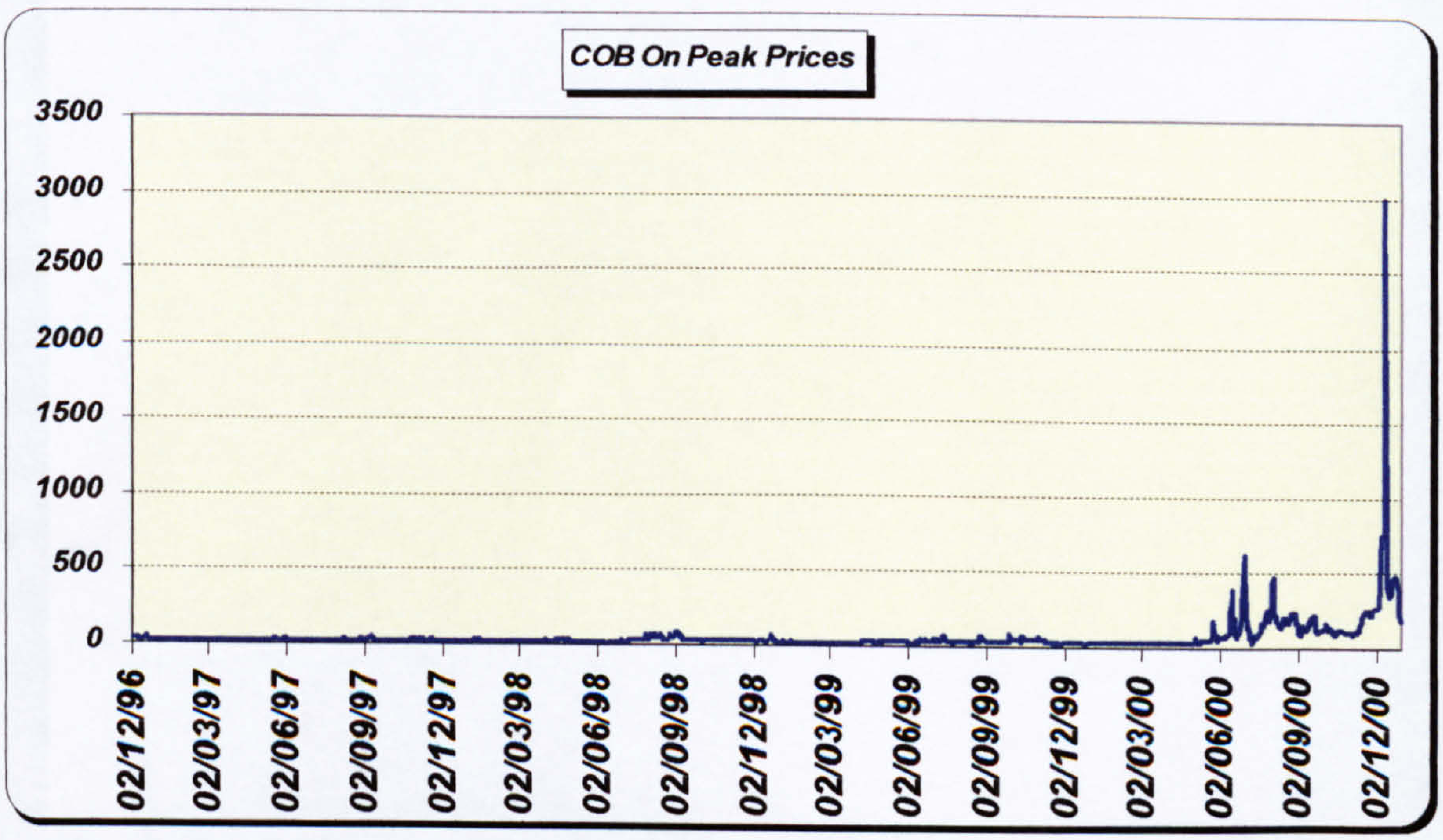
- The Mid-Columbia (MC) Off Peak over-the-counter electricity spot price. We are using the Dow Jones daily price index (closing prices) during April 1996 to December 2000.Figure-3.5 plots the time series for the MC's off-peak spot prices for that period.

Figure-3.5



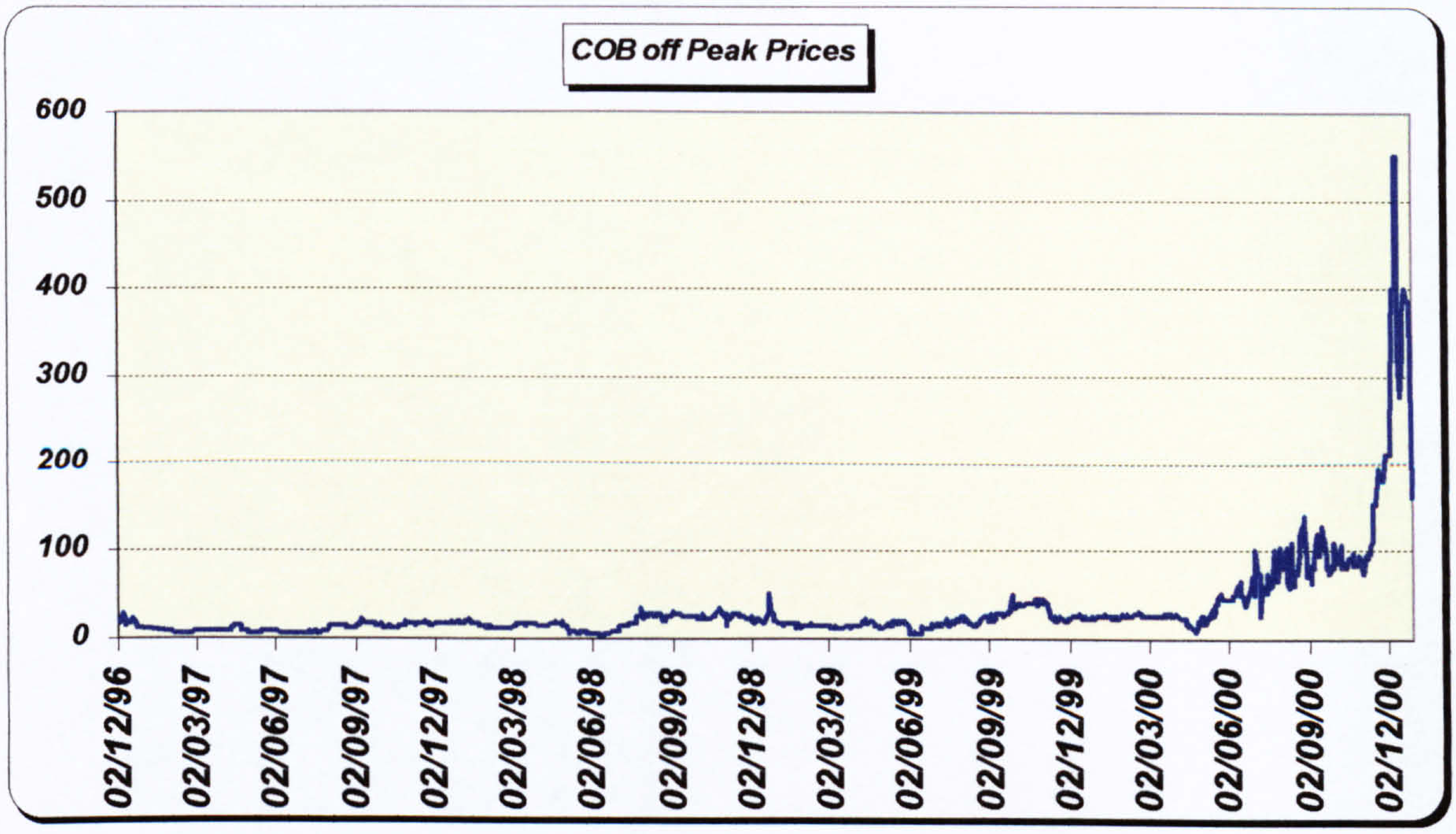
- California-Oregon Border (COB) on-peak spot prices from the Dow Jones daily price index (closing prices). Figure-3.6 plots the time series for COB's on peak spot prices from December 1996 to 30 December 2000.

Figure-3.6



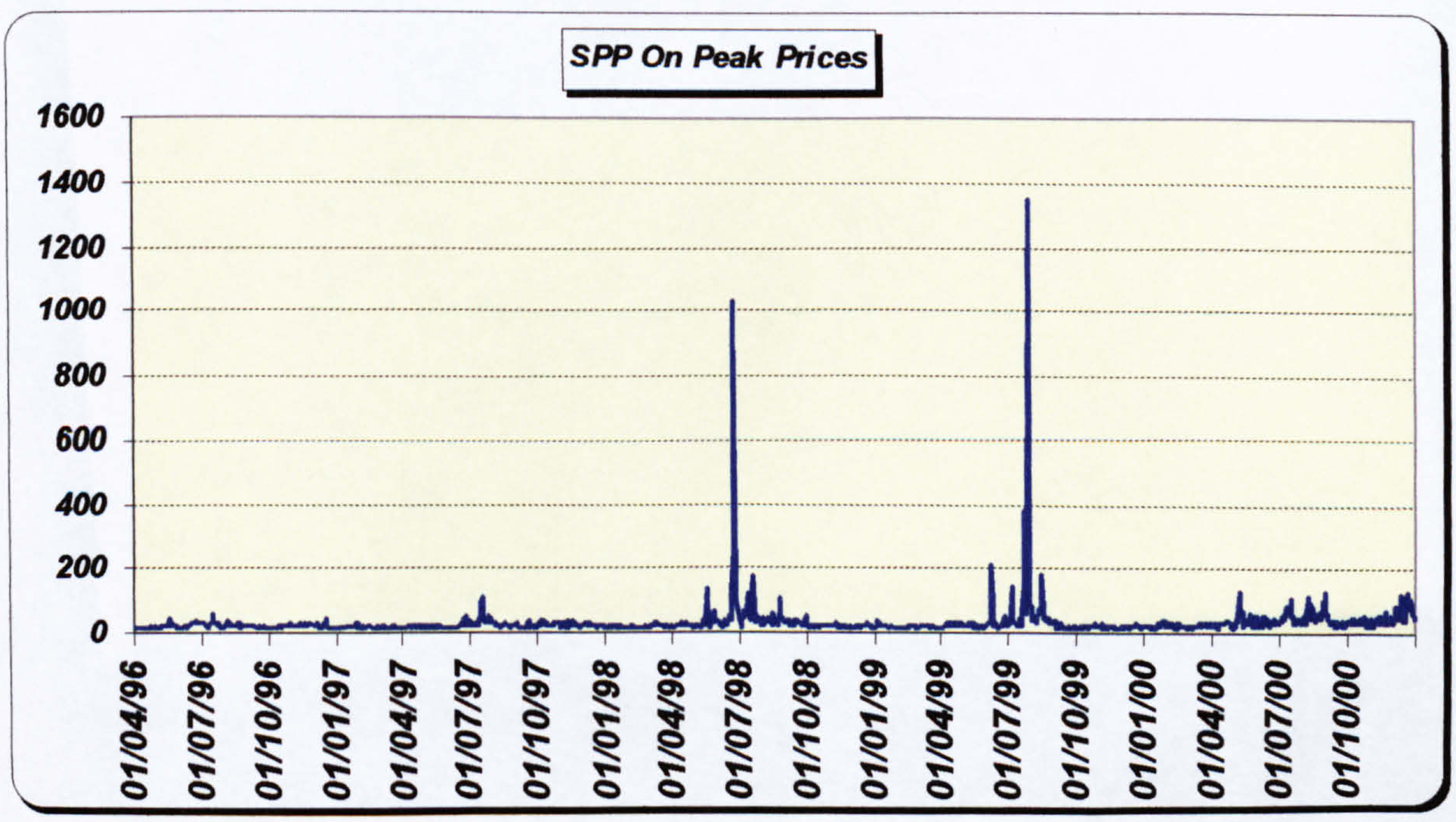
- California-Oregon Border (COB) off-peak spot prices from the Dow Jones daily price index (closing prices). Figure-3.7 plots the time series for COB's off peak spot prices from December 1996 to 30 December 2000.

Figure-3.7



- The Southwest Power Pool (SPP) on peak prices. We will be using the Dow Jones daily price index (closing prices) from April 1996 to December 2000.Figure-3.8 a plots the time series for the SPP's on-peak spot prices for that period.

Figure-3.8



- The Southwest Power Pool (SPP) off peak prices. We will be using the Dow Jones daily price index (closing prices) from April 1996 to December 2000.Figure-3.9 a plots the time series for the SPP's off-peak spot prices for that period.

Figure-3.9

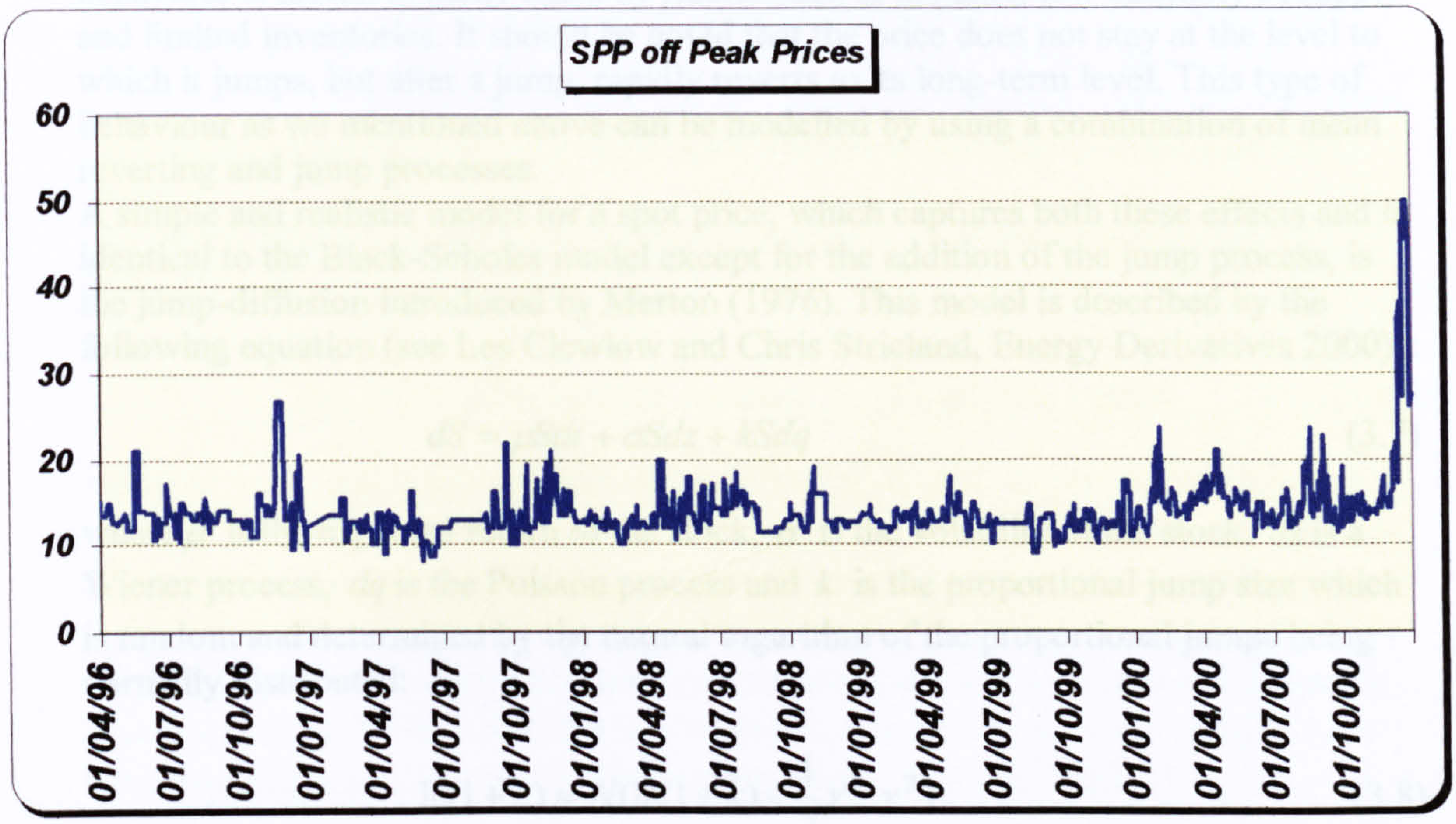


Table-3.1 shows the mean reversion analysis for a range of spot energy prices. The estimates from the linear regression are α_1 and α_o with standard errors in parentheses.

Table-3.1

Market	α_1	α	α_o
WTI crude oil	-0.0035 (0.00221)	5.5822	0.0109(0.00668)
IPE Brent	-0.0026 (0.00898)	4.0816	0.0078(0.00556)
Natural Gas	-0.0023 (0.00279)	3.7469	0.0033(0.00277)
COB On Peak	-0.0242 (0.06348)	31.0599	0.0853 (0.02253)
Mid-C On Peak	-0.0211 (0.00552)	31.0999	0.0709 (0.01868)
SPP On Peak	-0.1527 (0.01388)	226.477	0.5004(0.04575)
COB Off Peak	-0.0092 (0.00426)	11.8485	0.0293(0.01326)
Mid-C Off Peak	-0.01036(0.00405)	15.2513	0.0314(0.01202)
SPP Off Peak	-0.1304 (0.01305)	193.3899	0.3400(0.03410)

Mean reversion rates for most energy prices, with the obvious exception of the electricity are relatively low (see Table-1). The On Peak Electricity Prices have higher mean reversion rate than the Off Peak. The mean reversion process generally produces a readjustment that is less abrupt.

3.5 Mean Reversion Jump Diffusion

Energy prices often exhibit sudden, unexpected and discontinuous changes. Jump behaviour is driven in many cases by fluctuations in demand, low elasticity of supply and limited inventories. It should be noted that the price does not stay at the level to which it jumps, but after a jump, rapidly reverts to its long-term level. This type of behaviour as we mentioned above can be modelled by using a combination of mean reverting and jump processes.

A simple and realistic model for a spot price, which captures both these effects and is identical to the Black-Scholes model except for the addition of the jump process, is the jump-diffusion introduced by Merton (1976). This model is described by the following equation (see Les Clewlow and Chris Stricland, Energy Derivatives 2000)[3]:

$$dS = \mu S dt + \sigma S dz + kS dq \qquad (3.7)$$

where μ is the expected return of the stock, σ is the volatility of the stock, dz is a Wiener process, dq is the Poisson process and k is the proportional jump size which is random and determined by the natural logarithm of the proportional jumps being normally distributed:

$$\ln(1+k) \approx N(\ln(1+\bar{k}) - \frac{1}{2}\gamma^2, \gamma^2) \qquad (3.8)$$

where $\bar{k}$ is the mean jump size and γ is the standard deviation of the proportional jump size which we call the jump volatility. The jump process (dq) is a discrete time process-jumps do not occur continuously but at specific instants of time. Therefore, for typical jump frequencies, most of the time dq =0 and only takes the value 1 when a randomly timed jump occurs. When no jump is occurring the spot price behaviour is identical to the Geometric Brownian Motion process is only different when a jump occurs. If the jump frequency becoming very small, so that the chances of a jump occurring are close to zero, then we would get a Geometric Brownian Motion process. Similarly, if the jump volatility were very small, so that even if jumps were very frequent their size would be insignificant, then this would result in the spot price behaviour following a Geometric Brownian Motion process. Under that assumption the jump diffusion model, which is represented by equation (3.7), can be written in terms of the natural logarithm of the spot price, $x = \ln S$, as follows:

$$dx = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dz + k dq \qquad (3.9)$$

The proportional jump returns in equation (3.7) are normally distributed and therefore symmetrical. That is, the number of positive and negative jumps and the range of

[3] See also Les Clewlow, Chris Stricland &Vince Kaminski Making the most of Mean Reversion Energy and Power Risk Management (EPRM), Risk Waters Group 5(8); November 2000b,Les Clewlow, Chris Stricland &Vince Kaminski Spot Simulation Processing (EPRM), Risk Waters Group 5 (9); December 2000c

sizes of the proportional jumps will be equal on average. In reality the distribution of jump return sizes of energy spot prices is positively skewed. A simple way to incorporate this property into equation (3.7) is to have the proportional jumps drawn from a normal distribution, but with different jump volatilities for the positive and negative jumps.
Another simple alternative would be to have the proportional jumps drawn from a negatively shifted lognormal distribution, this would give a lower limit on the negative jump returns. These extensions are straightforward to incorporate into Monte Carlo simulations but lead to the loss of the analytical tractability of the Merton's model.
The jump-diffusion model described by the equation (3.7) can be discretised as follows:

$$\Delta x_i = (r - \phi \bar{k} - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{1i} + (\bar{k} + \gamma\varepsilon_{2i})(u_i < \phi\Delta t) \tag{3.10}$$

where $\mu = (r - \phi\bar{k})$ under the risk neutral process[4], ε_1 and ε_2 are independent standard normal random variables and u is a uniform (0,1) random sample. The term $(u_i < \phi\Delta t)$ is taken to be one if the condition is true and zero otherwise. This generates jumps randomly at the correct average frequency. When a jump occurs, its size is the mean jump size plus a normally distributed random amount with standard deviation γ determined by ε_2. In order that the frequency of jumps is correctly simulated, the time step Δt must be relative to the jump frequency such that $\phi\Delta t << 1$.
Given the previous evidence that energy markets are mean reverting hence it seems appropriate to combine mean reversion and jumps into the same model.
This model is called Mean Reversion Jump Diffusion and can be represented by the following equation:

$$dS = a(\mu - \ln S)Sdt + \sigma Sdz + kSdq \tag{3.11}$$

where S is the spot price, α is the mean reversion rate, μ is the long term average value of *lnS* in the absence of jumps, *dz* is Wiener process, *k* is the mean jump size, γ is the standard deviation of the proportional jump (jump volatility), ϕ is the average number of jumps per year and *dq* is the Poisson process.
Applying Ito's Lemma for $x = \ln S$, we get:

$$dx = (a(\mu - x) - \frac{1}{2}\sigma^2)dt + \sigma dz + kdq \tag{3.12}$$

The equation (3.12) can be discretised in logarithmic form as follows:

$$\Delta x_i = (\alpha(\mu - x_i) - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{1i} + (\bar{k} + \gamma\varepsilon_{2i})(u_i < \varphi\Delta t) \tag{3.13}$$

[4] This assumption is very important because it turns out that we cannot apply risk-neutral valuation to situations where the size of the jump is systematic. (see John C Hull "Options Futures and other Derivatives" pg. 498-499)

Once the jump parameters $\phi, \bar{k}, \sigma, \gamma$ have been estimated from the historical data and settle $\Delta t = 1/252$ then we can test which one of these models is most appropriate to capture the behaviour of the spot price for the oil market, natural gas and electricity market.

3.6 Estimation of Jump Process Parameters

The estimation of jump parameters for energy prices is complicated by the fact that the jumps can only be observed as part of a time series of the normal non-jump behaviour of the price. Typically, we will not have any information on the exact time the jump occurs.
If we assume that jumps are relatively infrequent and not too large then we can get an estimate of the diffusion volatility in the usual way by calculating the sample standard deviation of returns. Based on this estimate of the volatility we can then look for actual returns that were larger than we would expect (for a chosen probability=3 times the sample standard deviation) in the absence of jumps and identify these extreme returns as jumps. Given that we have identified some returns as jumps we should recalculate the estimate of the diffusion volatility by recalculating the sample standard deviation of returns with the jumps returns excluded. This will give us a lower estimate of the diffusion volatility. So, using this new estimate of the diffusion volatility, we can look for more returns, which exceed the chosen limit. This approach is called a Recursive Filter. (See Les Clewlow and Chris Stricland, Energy Derivatives 2000, Les Clewlow and Chris Stricland et al. 2000b)[5]. Also the parameters we mentioned above ($\phi, \bar{k}, \sigma, \gamma$) can be identified using the maximum likelihood method (Ball and Torous, 1983; Lien and Strom, 1999; Les Clewlow and Chris Stricland 2000) but empirical analysis suggests that the Recursive Filter does pick out the lower frequency higher volatility jump components instead of the higher frequency lower volatility jumps that are better estimated using the maximum likelihood method.

3.6.1 Recursive Filter Estimation of the Jump-Diffusion Parameters

We are going to apply this method to estimate the Jump-Diffusion parameters for: Brent IPE from January 1995 to 29 December 2000,WTI from January 1995 to 29 December 2000, Natural Gas from January 1995 to 30 October 2000, California-Oregon Border On Peak & Off Peak (COB) electricity spot price from December 1996 to 30 December 2000, Mid-Columbia On Peak & Off Peak (MC) over-the-counter electricity spot price from April 1996 to December 2000 and the Southwest Power Pool On Peak & Off Peak (SPP) electricity spot price from April 1996 to December 2000. Figures- (3.10-3.18) shows us the price returns of the data we mentioned above.

[5] See also Les Clewlow, Chris Stricland &Vince Kaminski Jumping the Gaps Energy and Power Risk Management (EPRM), Risk Waters Group 5(10); January 2001a.

Figure-3.10

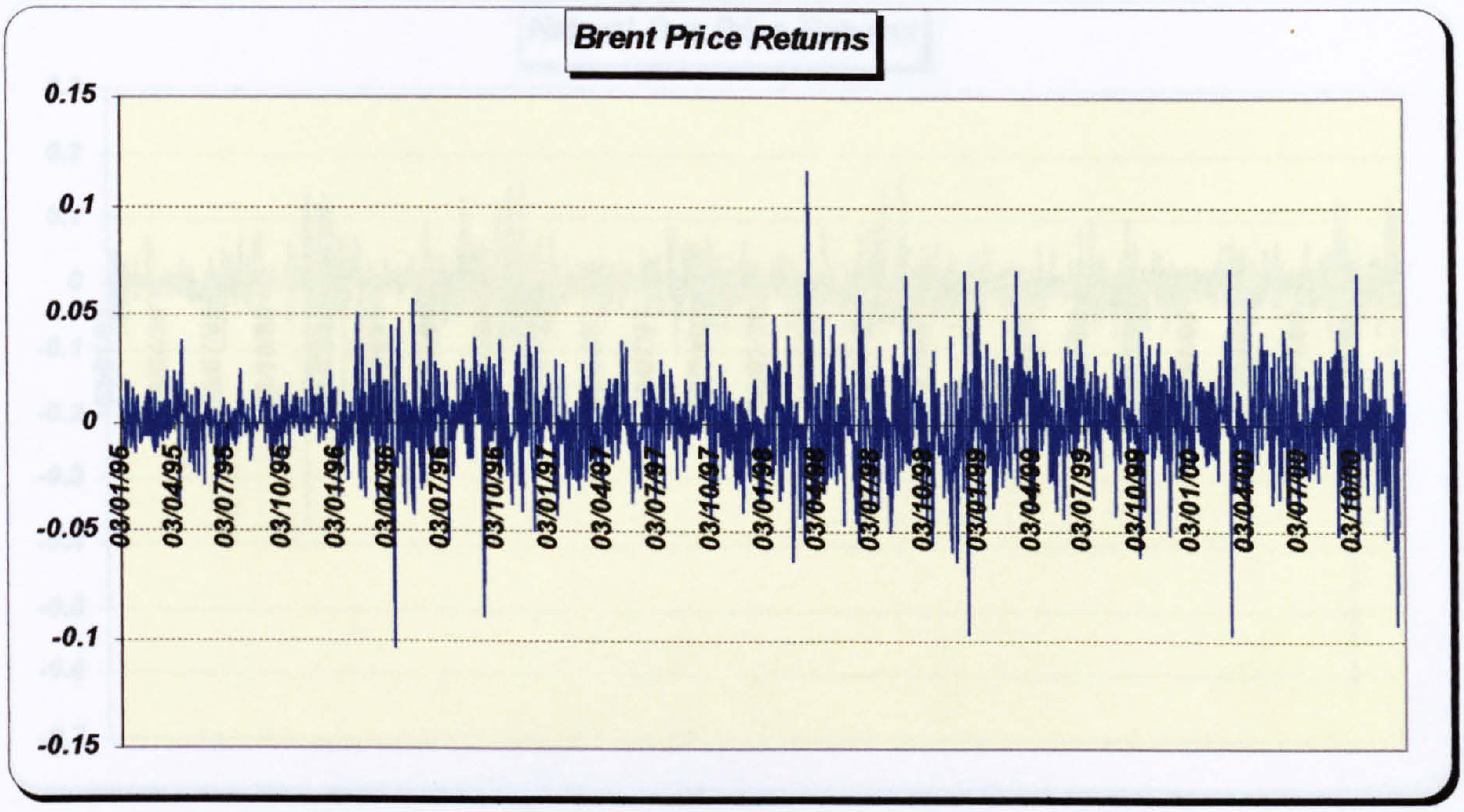


Figure-3.11

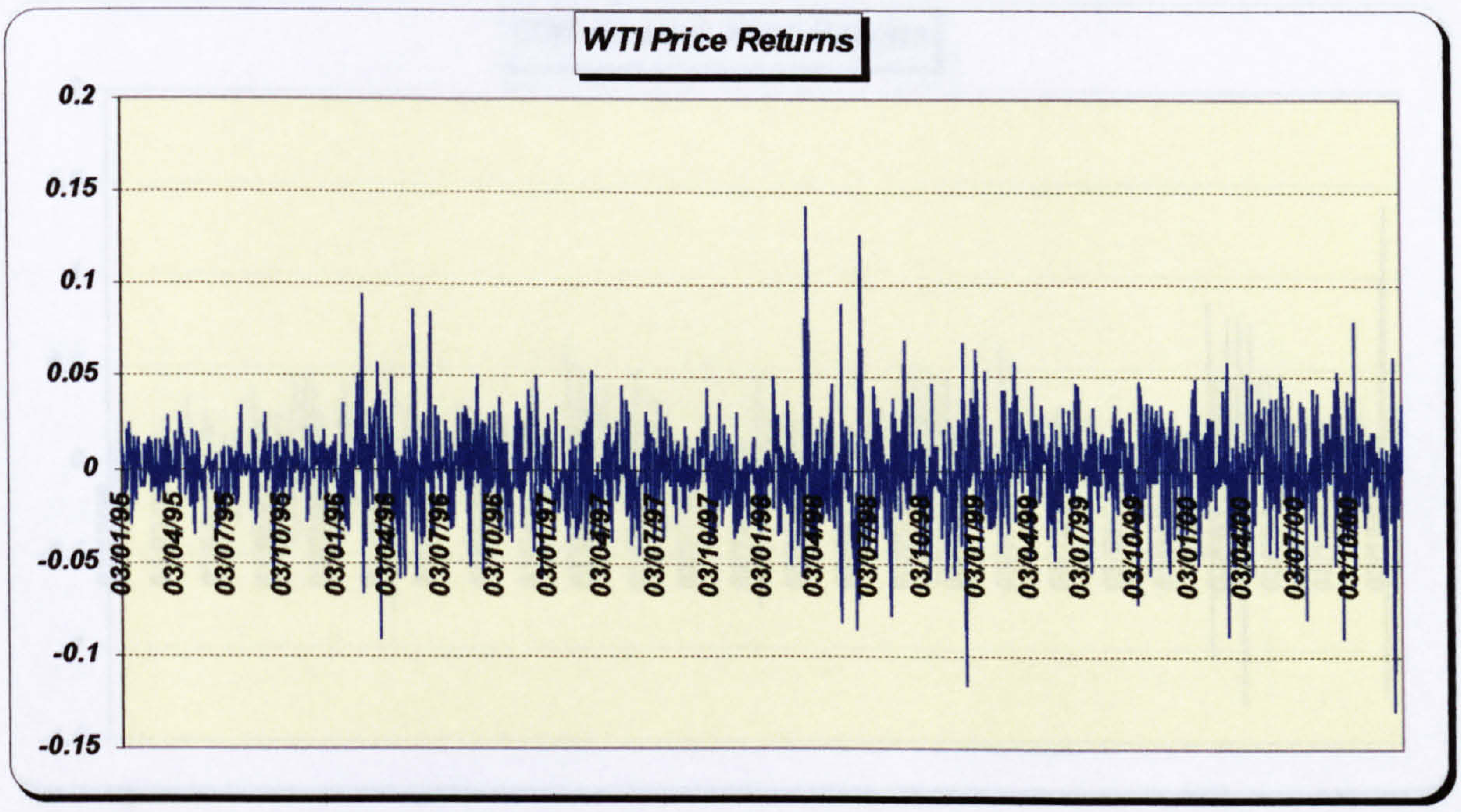


Figure-3.12

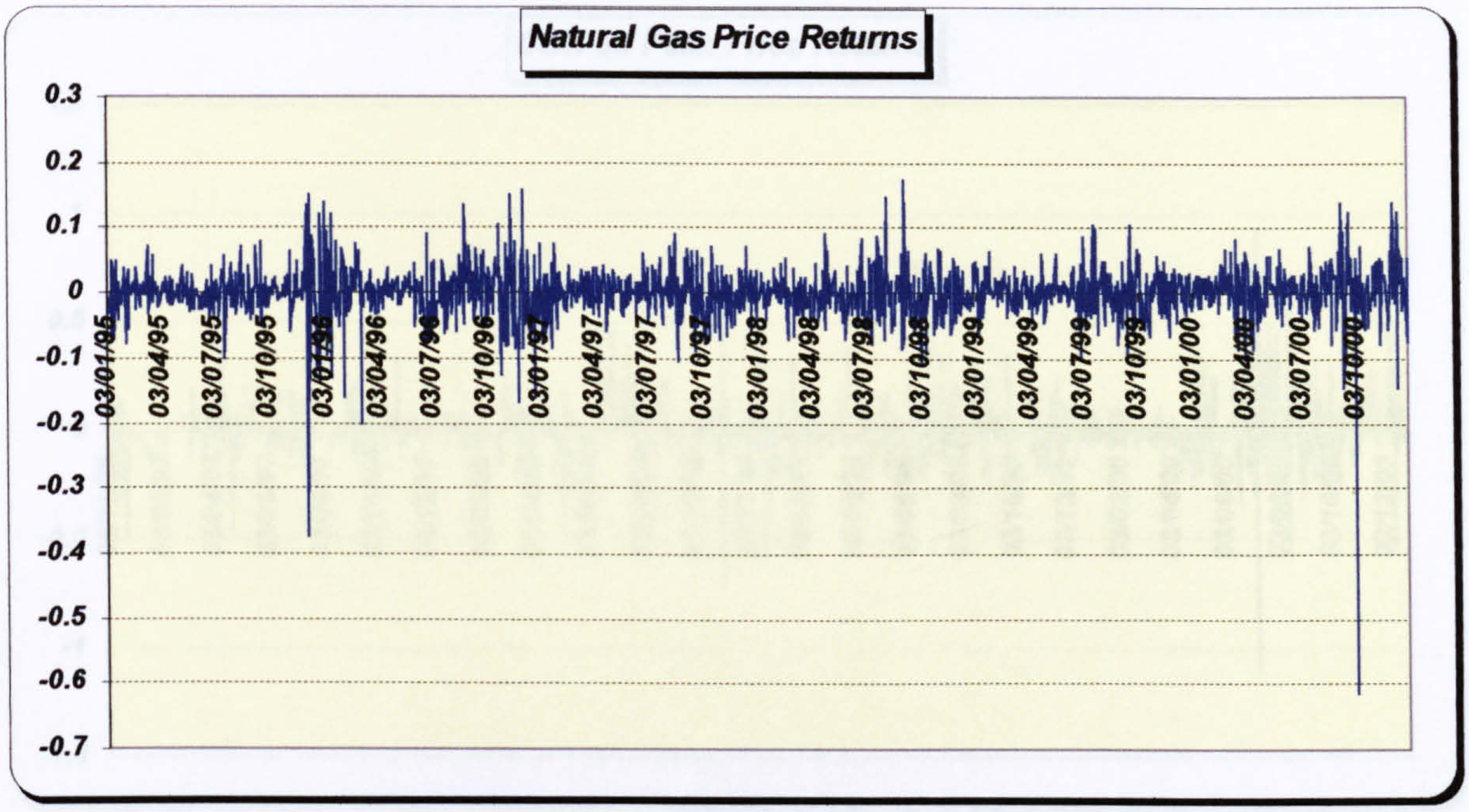


Figure3.13

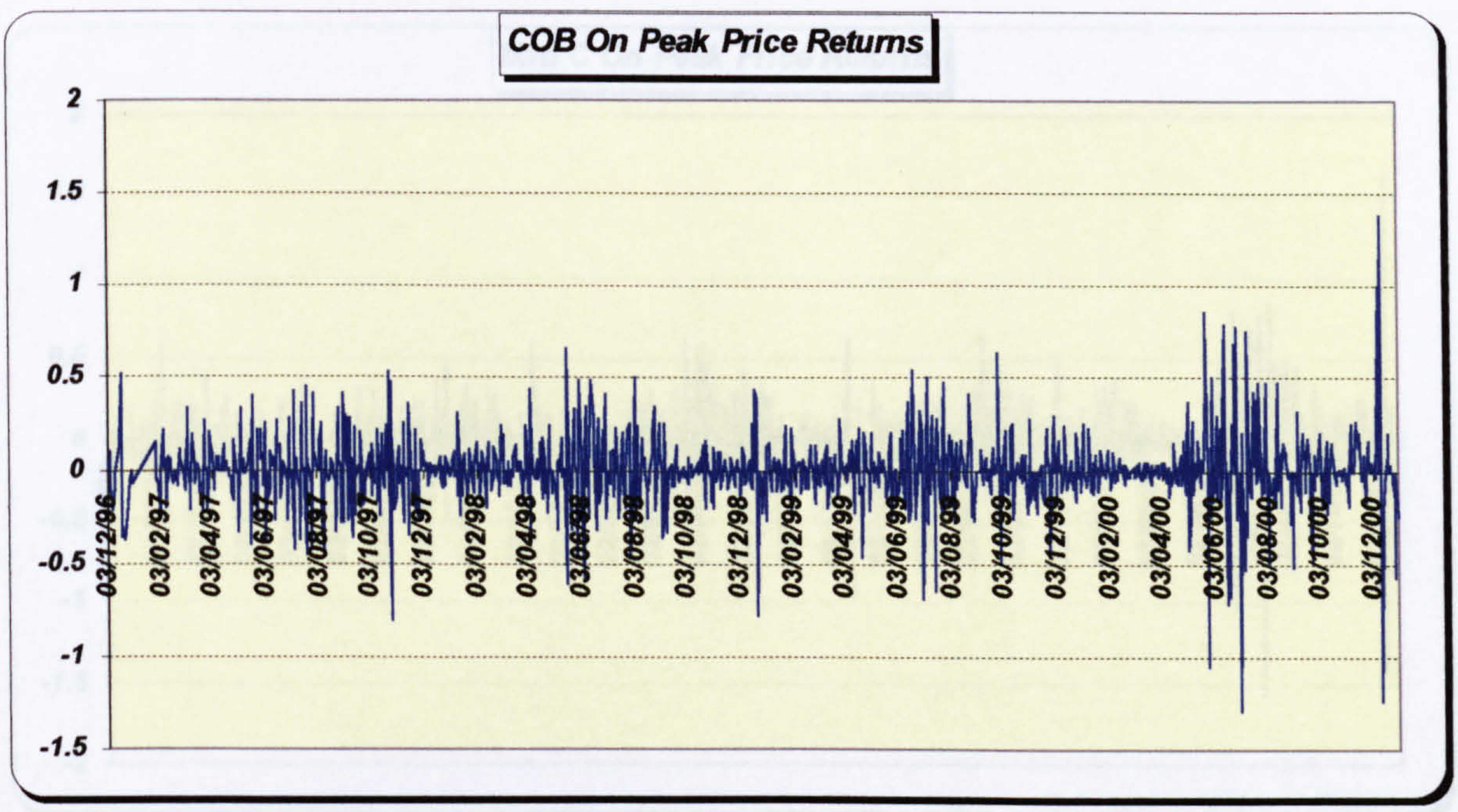


Figure-3.14

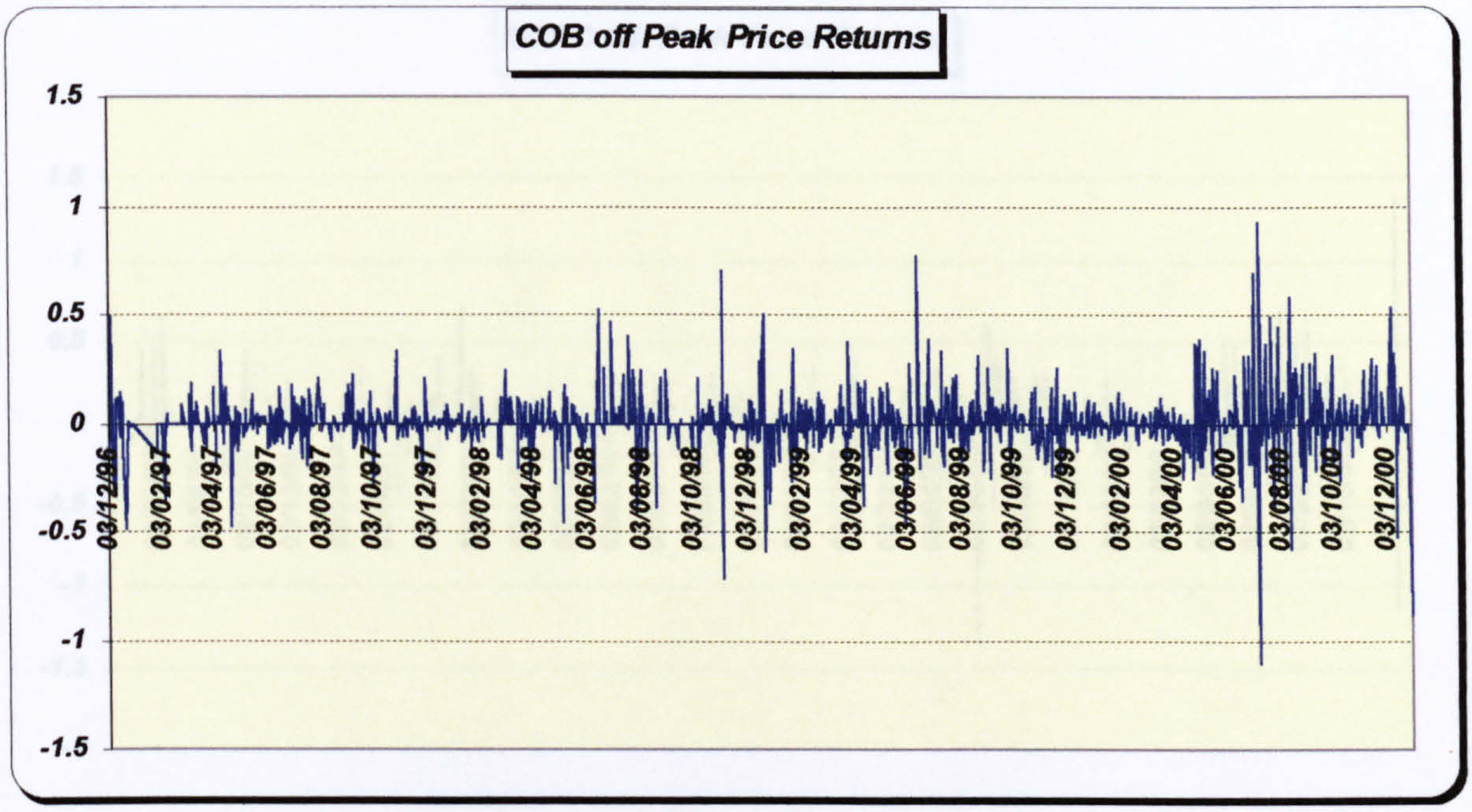


Figure-3.15

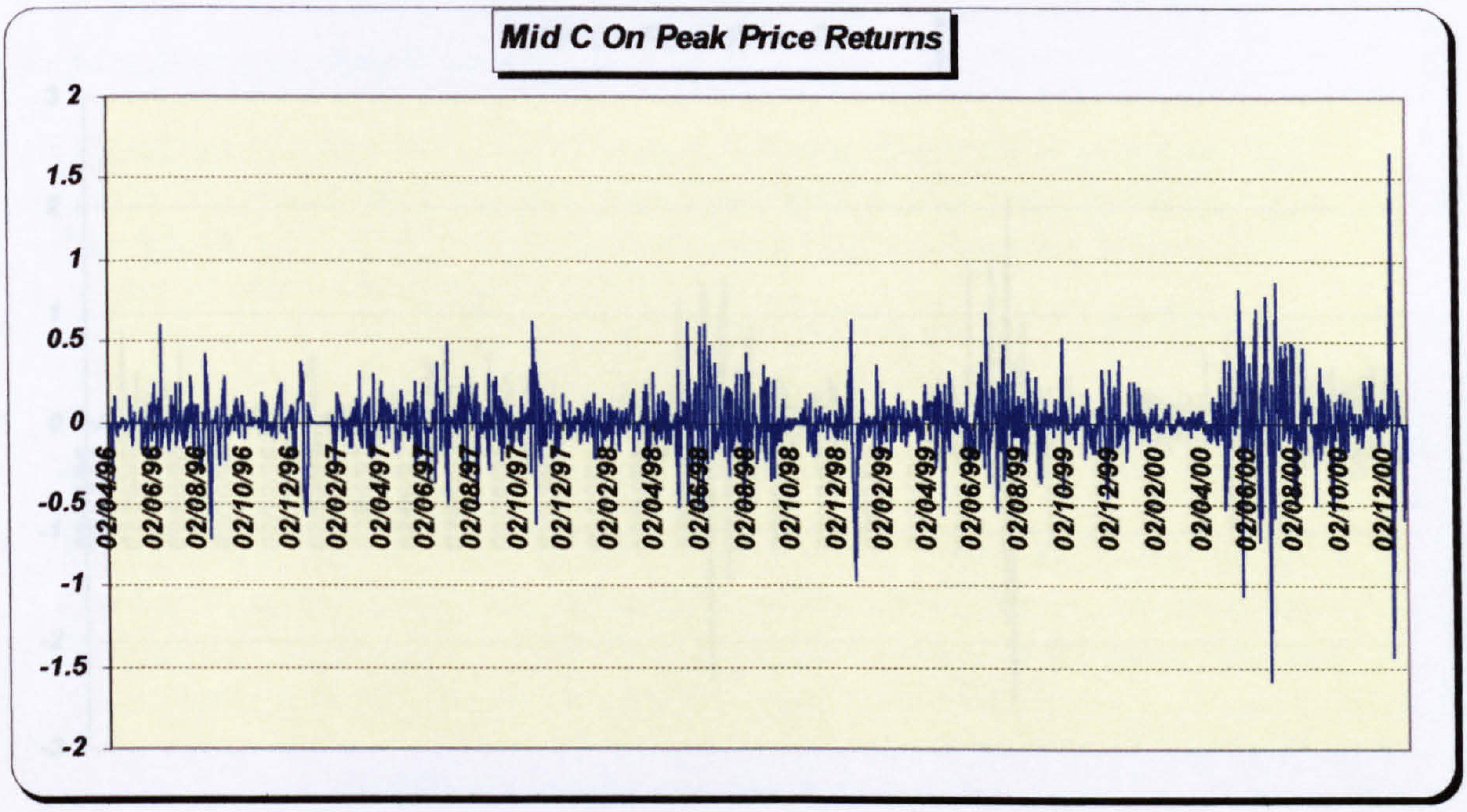


Figure-3.16

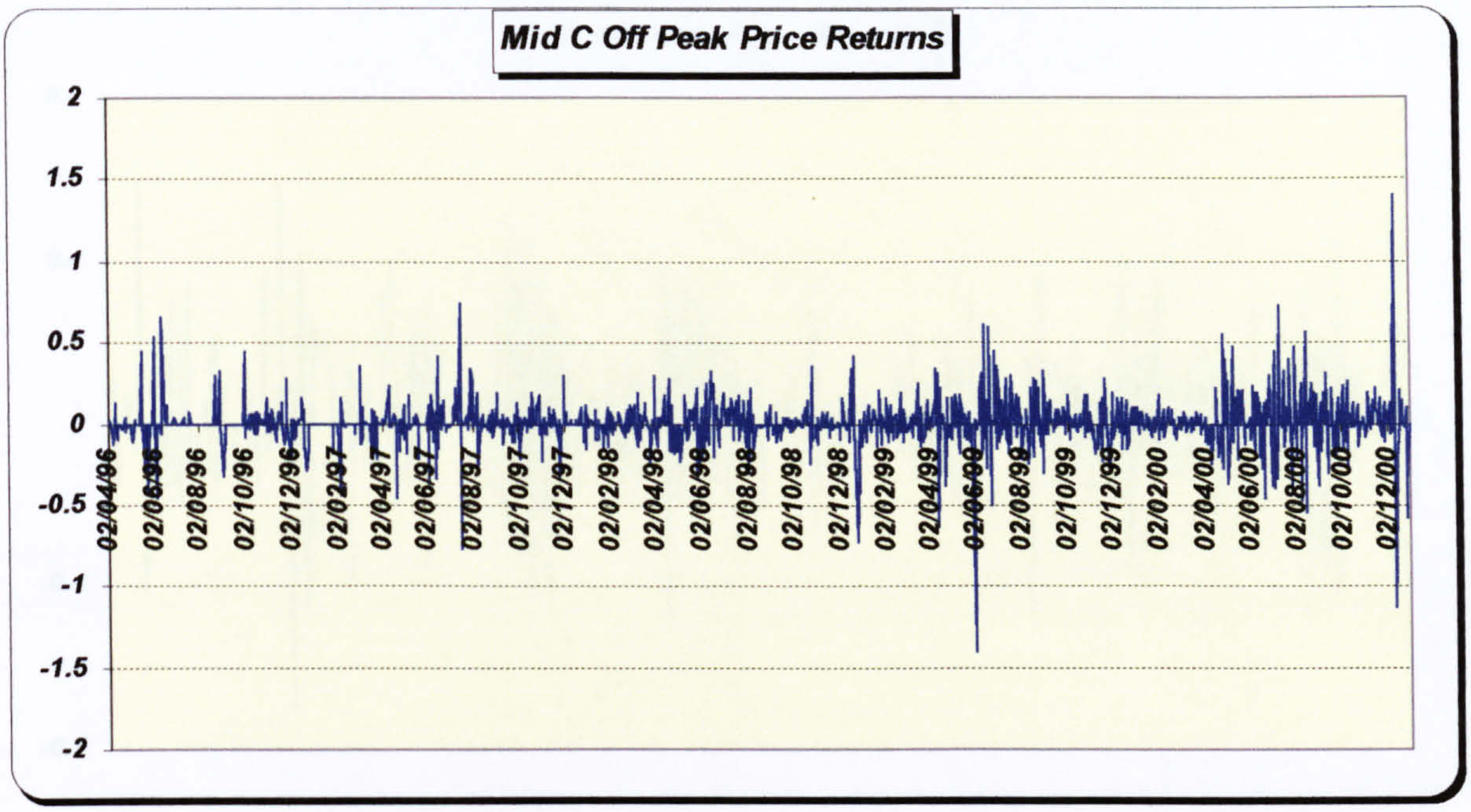


Figure-3.17

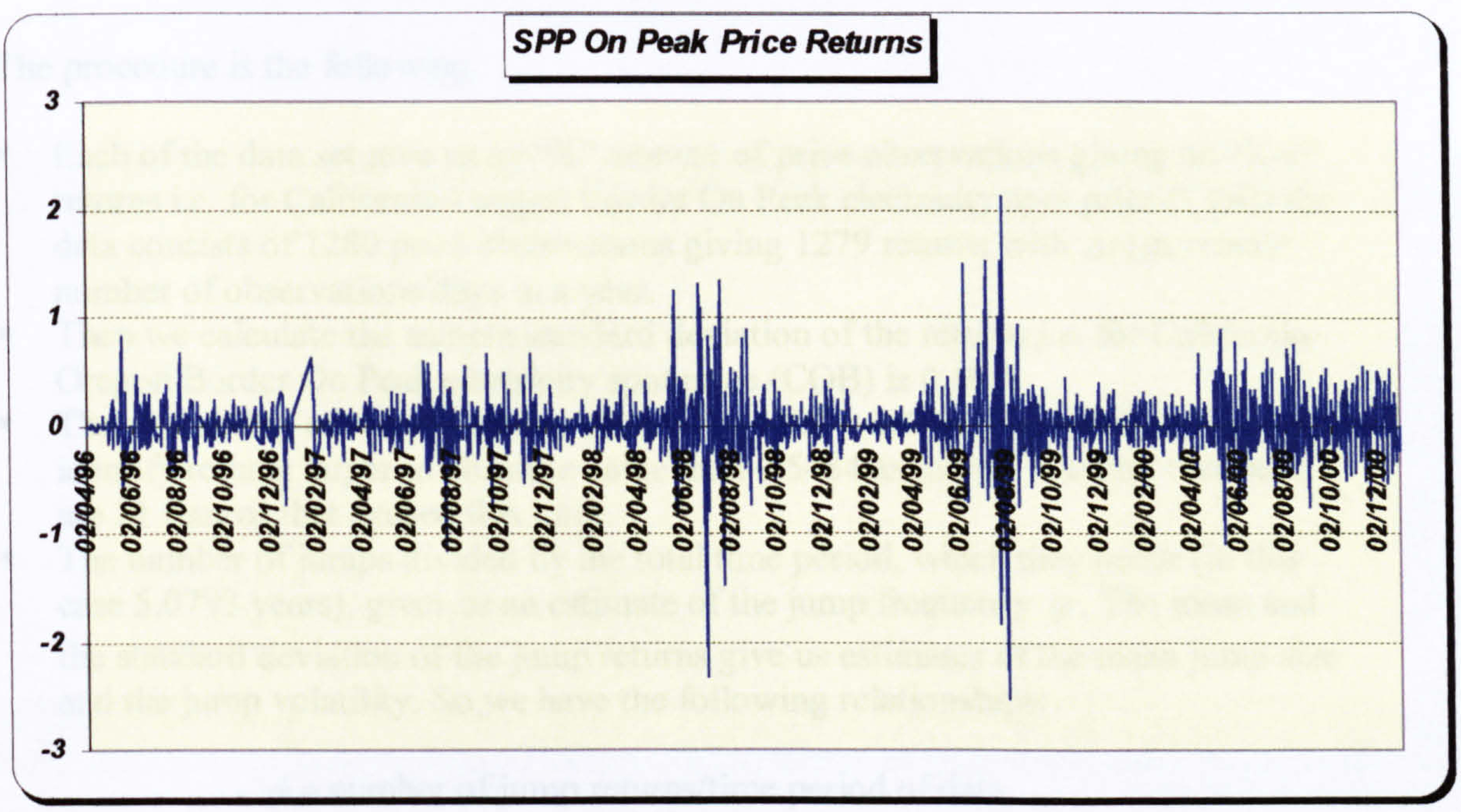
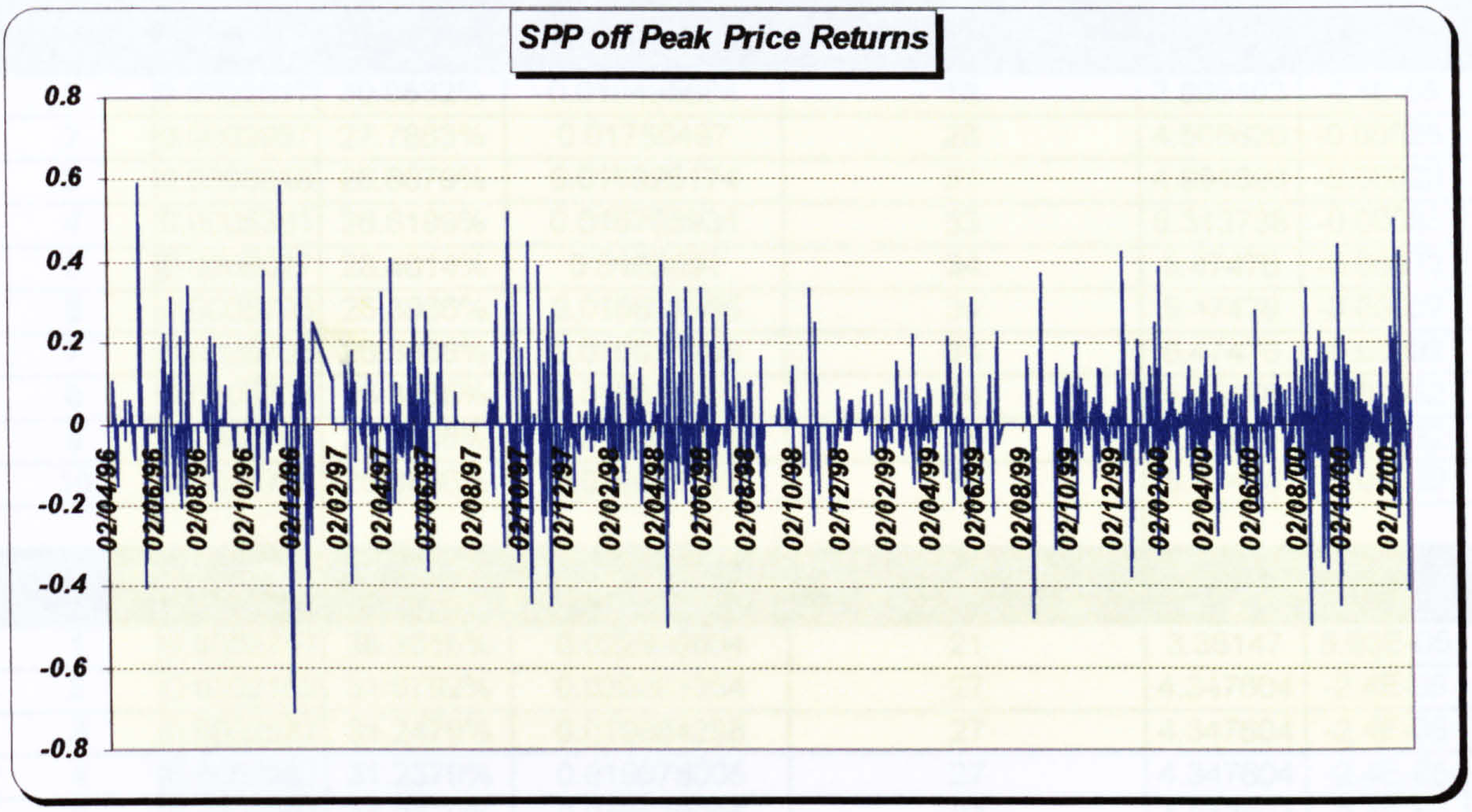


Figure-3.18



The procedure is the following:

- Each of the data set give us an "X" amount of price observations giving an "X-1" returns i.e. for California-Oregon Border On Peak electricity spot price (COB) the data consists of 1280 price observations giving 1279 returns with Δt (in years)= number of observations/days in a year.
- Then we calculate the sample standard deviation of the returns.i.e. for California-Oregon Border On Peak electricity spot price (COB) is 0.1828
- The probability of returns greater than 3 X 0.1828 = 0.5484, therefore we begin to identify returns larger in absolute value than 0.5484 as jumps. We find that there are 21 returns that exceed this limit.
- The number of jumps divided by the total time period, which they occur (in this case 5.0793 years), gives us an estimate of the jump frequency φ. The mean and the standard deviation of the jump returns give us estimates of the mean jump size and the jump volatility. So we have the following relationships:

$$\phi = \text{number of jump returns/time period of data}$$

$$\bar{k} = \text{average jump of returns}$$

$$\gamma = \text{standard deviation of jump returns}$$

Table-3.2 give us the results of repeating this process for Brent, WTI, Natural Gas, California-Oregon Border (COB) On Peak & Off Peak, Mid-Columbia (MC) On Peak & Off Peak, and the Southwest Power Pool (SPP) On Peak & Off Peak prices.

Table-3.2

Interations	Mean	Volatility	Standard Deviation	BRENT Jumps	ϕ	κ	γ
1	0.0002517	30.8532%	0.019435684	18	2.898403	-4.4E-05	0.008444
2	0.0002957	27.7883%	0.01750497	28	4.508626	-0.00025	0.009541
3	0.0005048	26.8679%	0.016925174	31	4.991693	-0.00029	0.009801
4	0.0005381	26.6199%	0.016768931	33	5.313738	-0.00035	0.009961
5	0.0006027	26.4614%	0.0166691	34	5.47476	-0.00032	0.010042
6	0.0005706	26.3866%	0.016621965	34	5.47476	-0.00032	0.010042
7	0.0005706	26.3866%	0.016621965	34	5.47476	-0.00032	0.010042
8	0.0005706	26.3866%	0.016621965	34	5.47476	-0.00032	0.010042
9	0.0005706	26.3866%	0.016621965	34	5.47476	-0.00032	0.010042
10	0.0005706	26.3866%	0.016621965	34	5.47476	-0.00032	0.010042

Interations	Mean	Volatility	Standard Deviation	WTI Jumps	ϕ	κ	γ
1	0.0002747	36.2018%	0.022805004	21	3.38147	5.93E-05	0.010809
2	0.0002152	31.8782%	0.020081354	27	4.347604	-2.4E-05	0.011514
3	0.0002987	31.2479%	0.019684298	27	4.347604	-2.4E-05	0.011514
4	0.0002987	31.2379%	0.019678005	27	4.347604	-2.4E-05	0.011511
5	0.0002987	31.2379%	0.019678005	27	4.347604	-2.4E-05	0.011511
6	0.0002987	31.2379%	0.019678005	27	4.347604	-2.4E-05	0.011511
7	0.0002987	31.2379%	0.019678005	27	4.347604	-2.4E-05	0.011511
8	0.0002987	31.2379%	0.019678005	27	4.347604	-2.4E-05	0.011511
9	0.0002987	31.2379%	0.019678005	27	4.347604	-2.4E-05	0.011511
10	0.0002987	31.2379%	0.019678005	27	4.347604	-2.4E-05	0.011511

Interations	Mean	Volatility	Standard Deviation	Natural Gas Jumps	ϕ	κ	γ
1	0.0011526	66.4777%	0.041877022	23	3.710627	-0.00074	0.025085
2	0.0018931	53.1646%	0.033490572	40	6.453265	-0.00049	0.027632
3	0.0016429	49.9115%	0.031441293	44	7.098592	-0.00049	0.028065
4	0.0016446	49.2820%	0.03104477	45	7.259923	-0.00055	0.028155
5	0.0017046	49.1328%	0.030950724	45	7.259923	-0.00055	0.028155
6	0.0017046	49.1328%	0.030950724	45	7.259923	-0.00055	0.028155
7	0.0017046	49.1328%	0.030950724	45	7.259923	-0.00055	0.028155
8	0.0017046	49.1328%	0.030950724	45	7.259923	-0.00055	0.028155
9	0.0017046	49.1328%	0.030950724	45	7.259923	-0.00055	0.028155
10	0.0017046	49.1328%	0.030950724	45	7.259923	-0.00055	0.028155

Interations	Mean	Volatility	Standard Deviation	COB On Peak Jumps	ϕ	κ	γ
1	0.0014053	290.2306%	0.18282812	21	4.134375	-0.00163	0.109162
2	0.0030341	232.7657%	0.146628616	47	9.253125	-3.7E-05	0.129765
3	0.0014412	204.4489%	0.128790721	65	12.79688	0.001869	0.138891
4	-0.000463	188.6508%	0.118838856	77	15.15938	0.001289	0.143428
5	0.0001149	179.8011%	0.113264068	82	16.14375	0.001025	0.145034
6	0.0003792	176.5272%	0.11120166	87	17.12813	0.001287	0.146543
7	0.0001168	173.3548%	0.10920327	90	17.71875	0.001544	0.147408
8	-0.000139	171.4920%	0.108029817	91	17.91563	0.001798	0.147686
9	-0.000394	170.8813%	0.107645126	92	18.1125	0.00205	0.147958
10	-0.000646	170.2769%	0.107264376	94	18.50625	0.002051	0.148507

11	-0.000646	169.0702%	0.106504223	95	18.70313	0.0018	0.14878
12	-0.000396	168.4731%	0.106128068	95	18.70313	0.0018	0.14878

Interations	Mean	Volatility	Standard Deviation	COB Off Peak Jumps	ϕ	κ	γ
1	0.0016557	202.5722%	0.127608495	28	5.5125	0.000625	0.087313
2	0.0010298	147.7408%	0.093067985	61	12.00938	0.002339	0.102174
3	-0.000682	121.3251%	0.07642761	90	17.71875	0.00249	0.108992
4	-0.000833	105.2644%	0.066310327	116	22.8375	0.001175	0.113094
5	0.0004793	93.6752%	0.059009838	134	26.38125	0.000569	0.115252
6	0.0010852	86.7959%	0.054676252	155	30.51563	0.000946	0.117261
7	0.0007088	79.7269%	0.050223229	164	32.2875	0.00032	0.118004
8	0.0013347	76.9063%	0.048446424	173	34.05938	0.000432	0.118657
9	0.0012222	74.3356%	0.04682704	181	35.63438	0.001103	0.119191
10	0.0005512	72.1516%	0.045451225	187	36.81563	0.001316	0.119566
11	0.0003383	70.5656%	0.044452159	196	38.5875	0.001631	0.12009
12	2.3E-05	68.2710%	0.04300671	206	40.55625	0.001227	0.120653
13	0.0004277	65.7406%	0.041412685	215	42.32813	0.000735	0.121125
14	0.0009198	63.5231%	0.040015821	225	44.29688	0.001119	0.1216
15	0.0005358	61.1913%	0.038546889	237	46.65938	0.001296	0.122132
16	0.0003585	58.4548%	0.03682307	248	48.825	0.001208	0.122581
17	0.0004467	56.0415%	0.035302798	258	50.79375	0.001369	0.122956
18	0.000285	53.9237%	0.033968726	266	52.36875	0.00137	0.123231
19	0.0002847	52.3135%	0.032954387	276	54.3375	0.00153	0.123549
20	0.0001242	50.3816%	0.031737407	292	57.4875	0.001684	0.124019
21	-2.95E-05	47.3694%	0.02983992	303	59.65313	0.001748	0.12431
22	-9.35E-05	45.4004%	0.028599556	314	61.81875	0.001678	0.124574
23	-2.39E-05	43.5481%	0.027432704	323	63.59063	0.001878	0.124773
24	-0.000224	42.0609%	0.026495906	330	64.96875	0.001815	0.124915
25	-0.00016	40.9914%	0.025822171	337	66.34688	0.001755	0.125051
26	-0.0001	39.9445%	0.025162677	346	68.11875	0.001694	0.125215
27	-3.99E-05	38.6300%	0.024334641	363	71.46563	0.001982	0.125502
28	-0.000327	36.1669%	0.022783032	382	75.20625	0.002039	0.125792
29	-0.000385	33.5260%	0.021119404	408	80.325	0.001941	0.126141
30	-0.000286	30.0631%	0.018937965	438	86.23125	0.001651	0.12648
31	3.65E-06	26.2719%	0.016549756	475	93.51563	0.002023	0.126797
32	-0.000369	22.0154%	0.013868397	511	100.6031	0.001809	0.127033
33	-0.000155	18.3199%	0.011540436	561	110.4469	0.001901	0.127255
34	-0.000247	13.8795%	0.00874329	622	122.4563	0.001668	0.127432
35	-1.36E-05	9.0211%	0.005682756	683	134.4656	0.001576	0.12752
36	7.822E-05	5.0655%	0.003190979	749	147.4594	0.001647	0.127554
37	6.939E-06	1.7592%	0.001108211	788	155.1375	0.001659	0.127558
38	-5.02E-06	0.4257%	0.000268145	810	159.4688	0.001655	0.127559
39	-5.71E-07	0.0502%	3.16347E-05	813	160.0594	0.001654	0.127559
40	0	0.0000%	0	813	160.0594	0.001654	0.127559

Interations	Mean	Volatility	Standard Deviation	Mid C On Peak Jumps	ϕ	κ	γ
1	0.0017419	282.3249%	0.177847995	29	4.964674	-0.00077	0.114799
2	0.0025058	215.6084%	0.135820494	55	9.415761	-0.00037	0.131473
3	0.0021103	190.1182%	0.119763222	75	12.83967	0.001144	0.138639
4	0.0005971	176.7854%	0.111364332	85	14.55163	0.000214	0.141532
5	0.0015269	170.8086%	0.107599326	91	15.5788	0.000661	0.143092

6	0.0010796	167.5075%	0.105519836	96	16.43478	0.000882	0.144297
7	0.0008584	164.8829%	0.103866482	101	17.29076	0.000668	0.145458
8	0.0010727	162.2915%	0.10223406	105	17.97554	0.000247	0.146346
9	0.0014935	160.2613%	0.100955156	106	18.14674	0.000454	0.146561
10	0.0012866	159.7693%	0.100645194	107	18.31793	0.00066	0.146773
11	0.0010808	159.2806%	0.100337374	108	18.48913	0.000865	0.146982
12	0.0008759	158.7939%	0.10003074	108	18.48913	0.000865	0.146982
13	0.0008759	158.7939%	0.10003074	108	18.48913	0.000865	0.146982

Interations	Mean	Volatility	Standard Deviation	Mid C Off Peak Jumps	ϕ	κ	γ
1	0.0021831	221.5380%	0.139555813	33	5.649457	-0.00046	0.09798
2	0.0026395	157.7361%	0.099364404	69	11.8125	-9.5E-06	0.112204
3	0.0021911	131.7307%	0.082982538	95	16.26359	0.000727	0.117984
4	0.0014549	118.3042%	0.07452462	112	19.17391	0.001241	0.120648
5	0.0009401	111.2229%	0.070063868	129	22.08424	0.001379	0.12292
6	0.0008023	104.7651%	0.065995781	140	23.96739	0.001233	0.124183
7	0.000949	100.9449%	0.063589306	150	25.67935	0.00097	0.125204
8	0.0012117	97.7140%	0.061554019	159	27.22011	0.001356	0.126062
9	0.0008255	94.8912%	0.059775867	167	28.58967	0.001358	0.126771
10	0.0008236	92.4826%	0.058258591	173	29.61685	0.001357	0.127275
11	0.000825	90.7189%	0.057147513	179	30.64402	0.001591	0.127749
12	0.0005904	89.0187%	0.056076497	184	31.5	0.001939	0.128128
13	0.0002423	87.6244%	0.055198156	187	32.01359	0.002053	0.128346
14	0.0001291	86.8092%	0.054684665	189	32.35598	0.002277	0.128487
15	-9.52E-05	86.2699%	0.054344935	191	32.69837	0.002277	0.128628
16	-9.55E-05	85.7371%	0.054009326	193	33.04076	0.002277	0.128767
17	-9.57E-05	85.2091%	0.053676668	195	33.38315	0.002497	0.128901
18	-0.000315	84.6802%	0.053343538	197	33.72554	0.002498	0.129037
19	-0.000316	84.1566%	0.053013655	198	33.89674	0.002606	0.129102
20	-0.000425	83.8944%	0.052848473	200	34.23913	0.002822	0.12923
21	0.0001986	83.3749%	0.052521267	202	34.58152	0.002821	0.129361
22	-0.00064	82.8610%	0.052197544	204	34.92391	0.003035	0.129486
23	-0.000853	82.3453%	0.051872643	206	35.2663	0.003035	0.129614
24	-0.000853	81.8370%	0.051552472	208	35.6087	0.002824	0.129745
25	-0.000642	81.3351%	0.051236311	210	35.95109	0.003033	0.129864
26	-0.000851	80.8300%	0.050918121	213	36.46467	0.00272	0.130055
27	-0.000539	80.0873%	0.050450232	218	37.32065	0.003029	0.130349
28	-0.000847	78.8385%	0.049663586	225	38.51902	0.002928	0.13076
29	-0.000747	77.1174%	0.048579425	230	39.375	0.002629	0.131046
30	-0.000447	75.9175%	0.047823548	232	39.71739	0.002628	0.131155
31	-0.000446	75.4447%	0.047525693	232	39.71739	0.002628	0.131155

Interations	Mean	Volatility	Standard Deviation	SPP On Peak Jumps	ϕ	κ	γ
1	0.0005694	434.0962%	0.273454874	23	3.908294	0.00401	0.174084
2	-0.003438	334.6644%	0.210818786	53	9.006069	0.002999	0.201796
3	-0.002428	292.8891%	0.184502767	66	11.2151	0.003414	0.209297
4	-0.002843	279.1936%	0.175875453	74	12.57451	0.004143	0.212931
5	-0.003573	271.9959%	0.171341305	80	13.59407	0.004138	0.215497
6	-0.003569	266.8550%	0.168102826	88	14.95347	0.004142	0.218708
7	-0.003573	260.1895%	0.163903968	92	15.63318	0.004134	0.220244
8	-0.003565	256.9027%	0.161833465	93	15.8031	0.004464	0.220605

9	-0.003895	256.0959%	0.161325225	93	15.8031	0.004464	0.220605
10	-0.003895	256.0959%	0.161325225	93	15.8031	0.004464	0.220605

Interations	Mean	Volatility	Standard Deviation	SPP off Peak Jumps	ϕ	κ	γ
1	0.0003978	160.9265%	0.101374182	43	7.306811	0.001566	0.070759
2	-0.001168	115.1235%	0.072521003	85	14.4437	0.001278	0.082745
3	-0.000879	92.8452%	0.058486972	122	20.73095	0.001998	0.088415
4	-0.001599	78.4878%	0.049442664	157	26.67835	0.00123	0.091829
5	-0.000833	67.8761%	0.042757895	198	33.64531	0.000966	0.094657
6	-0.000569	57.2777%	0.036081589	234	39.76264	0.000796	0.096446
7	-0.000399	49.1980%	0.03099184	276	46.89953	0.00093	0.097929
8	-0.000532	41.1424%	0.025917249	327	55.56575	0.00038	0.099175
9	1.78E-05	32.8115%	0.020669271	380	64.57181	0.000118	0.100059
10	0.000279	25.1578%	0.015847956	443	75.27714	0.000346	0.100699
11	5.134E-05	13.3884%	0.00843389	558	94.81861	0.000424	0.101273
12	-2.69E-05	5.8241%	0.003668826	612	103.9946	0.000382	0.101332
13	1.585E-05	1.9667%	0.001238919	644	109.4322	0.000395	0.10134
14	2.396E-06	0.3897%	0.000245474	658	111.8112	0.000398	0.10134
15	-4.47E-07	0.0273%	1.72145E-05	659	111.9811	0.000398	0.10134
16	0	0.0000%	0	659	111.9811	0.000398	0.10134

As we can see from the Table-3.2 above for Brent and Natural Gas we needed to do six interations to estimate the jump diffusion parameters, for WTI we needed to do four interations, for COB On Peak and Mid C On Peak we needed to do twelve interations for SPP On Peak Prices we needed to do nine interations, for COB Off Peak we needed to do forty interations, for Mid C Off Peak we needed to do thirty one interations and for SPP Off Peak we needed to do sixteen interations. That indicates that in the electricity market we can see large shocks, which don't last for a long period of time but can happen frequently.

Since the jump parameters $\phi, \bar{k}, \sigma, \gamma$ and $\Delta t = 1/252$ have been estimated from the historical data we can insert them to the equations (3.3), (3.10) and (3.13) and see which model is the most appropriate to capture the spot price behaviour, the Jump Diffusion or the Mean Reversion Jump Diffusion. For the Mean Reversion Jump Diffusion model we have to estimate the mean reversion rate and the long-term average without the jumps. For the Black-Scholes Geometric Brownian Motion Model the volatility we are using is the volatility with the jumps, which has been calculated in the first interation on the Table 3.2 above.
Since the jumps have been estimated we have to exclude them from the historical data and estimate the Mean Reversion Rate and the long-term average. In order to calculate the mean reversion rate we apply the same method that we described at the beginning of this chapter and the long-term average is the average price of the historical data. Table-3.3 shows the mean reversion analysis for a range of spot energy prices without the jumps. The estimates from the linear regression are α_1 and α_o with standard errors in parentheses.Table-3.4 shows the long-term average without the jumps. Also we assume that the interest rate is 5%. We are using Monte Carlo Simulation (see Espen Gaarder Haug, Option Pricing Formulas 1998 page 139-

142)[6] in order to implement these models (see Appendix-3.1 for the jump diffusion model and Appendix-3.2 for the mean reversion jump diffusion model). The computer language we are using to implement the computer algorithms for jump diffusion model and the mean reversion jump diffusion is Visual Basic. The main drawback of Monte Carlo Simulation is that is computer intensive. We apply ten thousands simulations for each one of the sixty-six consecutive historical chosen randomly from the data available observations in order get more accurate results. Sixty-six data points corresponds to three months worth of observations which we feel is a large enough sample which we can draw conclusions about the spot price behaviour.

Table-3.3

Market	α_1	α	α_o
WTI crude oil	-0.0034 (0.00221)	5.3599	0.0107(0.00668)
IPE Brent	-0.0023 (0.00184)	3.6615	0.0072(0.00541)
Natural Gas	-0.0023 (0.00289)	3.5804	0.0033(0.00285)
COB On Peak	-0.0268 (0.00697)	31.8601	0.0935(0.02445)
Mid-C On Peak	-0.0255 (-0.02555)	34.8508	0.0845(0.02104)
SPP On Peak	-0.1872 (0.01567)	260.2723	0.6044(0.05097)
COB Off Peak	-0.0051 (0.00866)	2.4048	0.0215(0.02653)
Mid-C Off Peak	-0.0086 (0.00424)	10.7524	0.0266(0.01234)
SPP Off Peak	-0.1845 (0.02076)	152.2418	0.4759(0.05361)

Table-3.4

Market	long-term average
WTI crude oil	20.86
IPE Brent	19.16
Natural Gas	2.70
COB On Peak	46.23
Mid-C On Peak	39.92
SPP On Peak	28.73
COB Off Peak	33.86
Mid-C Off Peak	25.85
SPP Off Peak	13.38

[6] see also Les Clewlow and Chris Strickland, Implementing Derivatives Models 1998 page (82-87)

BRENT

Figure-3.19

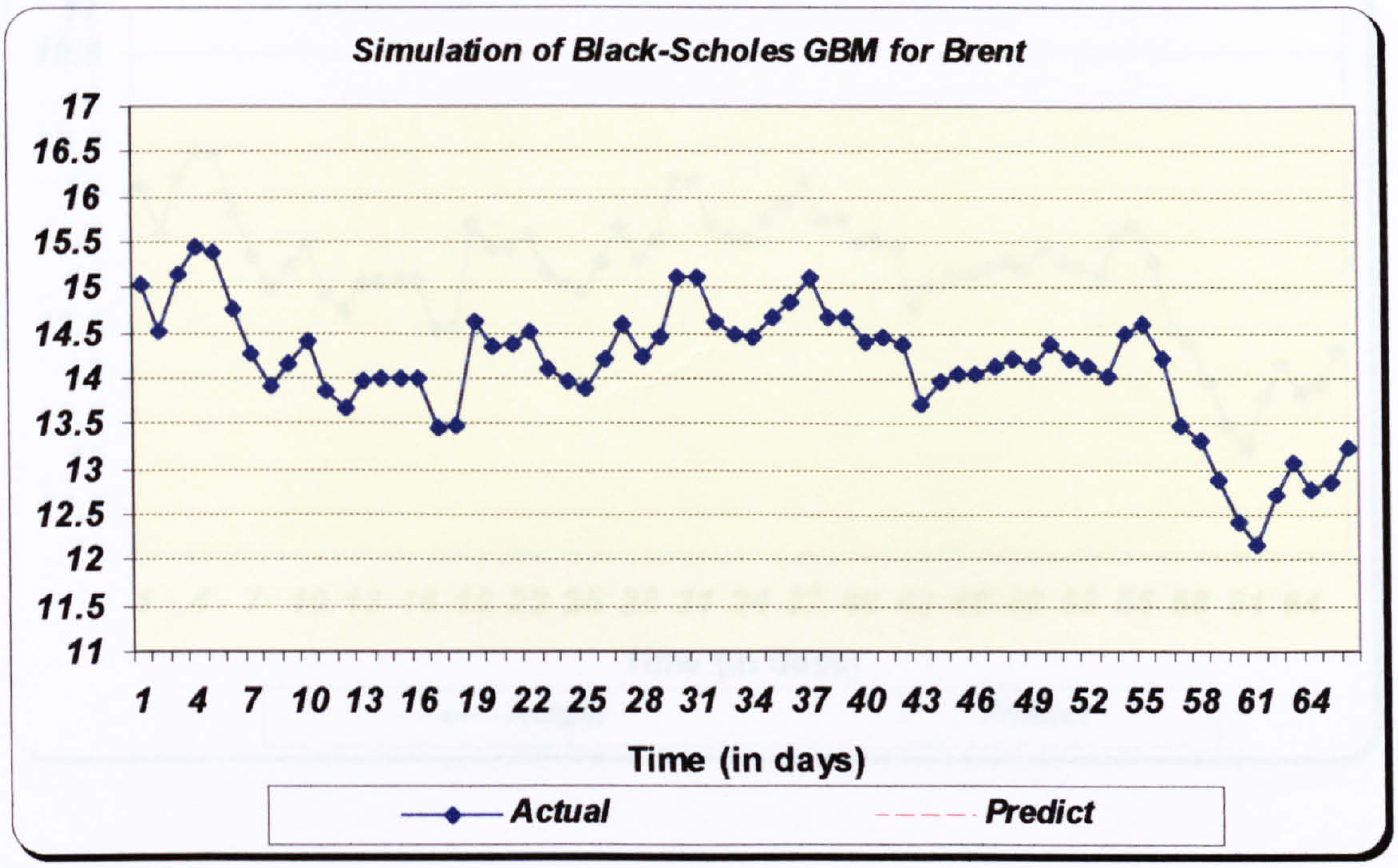


Figure-3.20

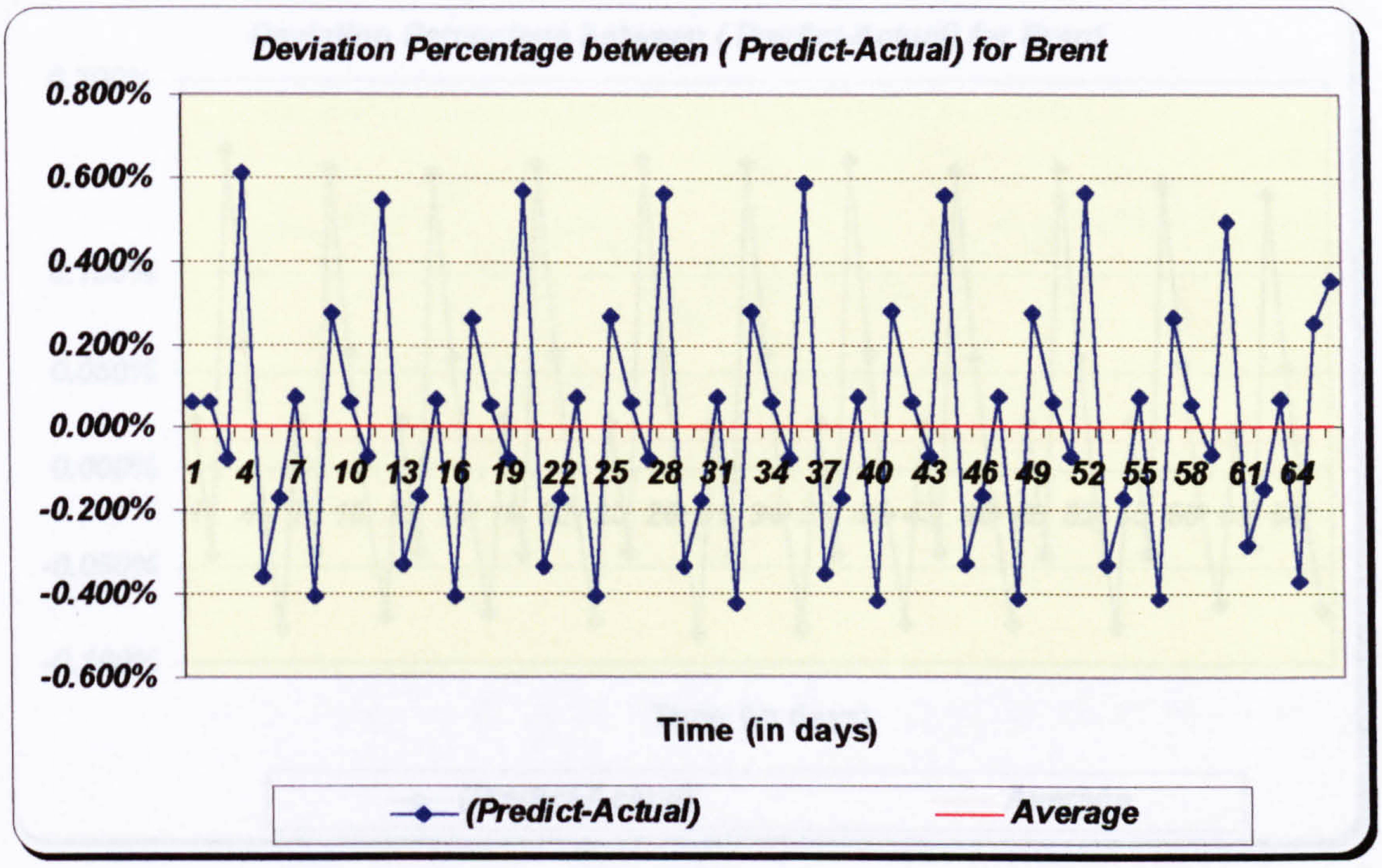


Figure-3.21

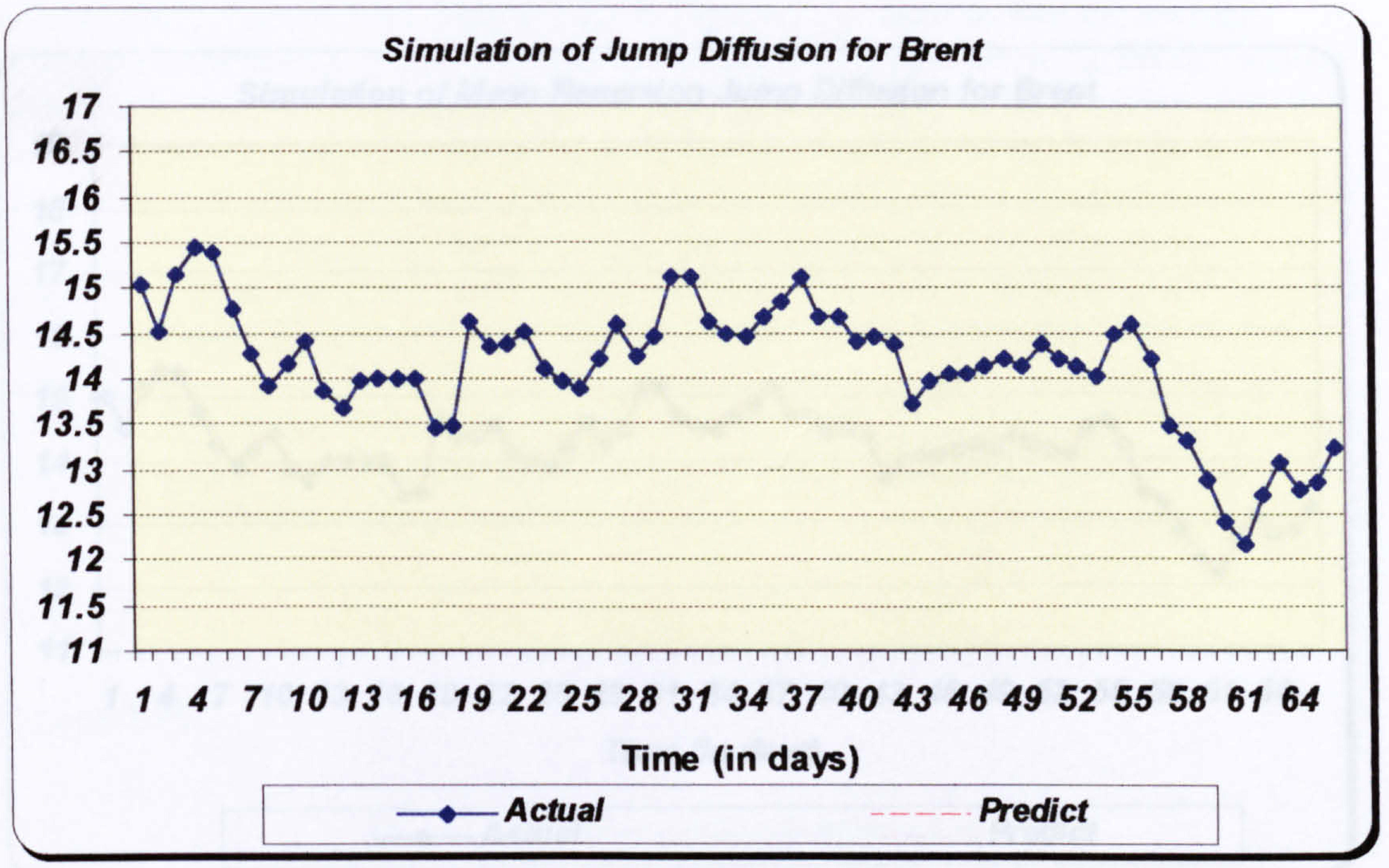


Figure-3.22

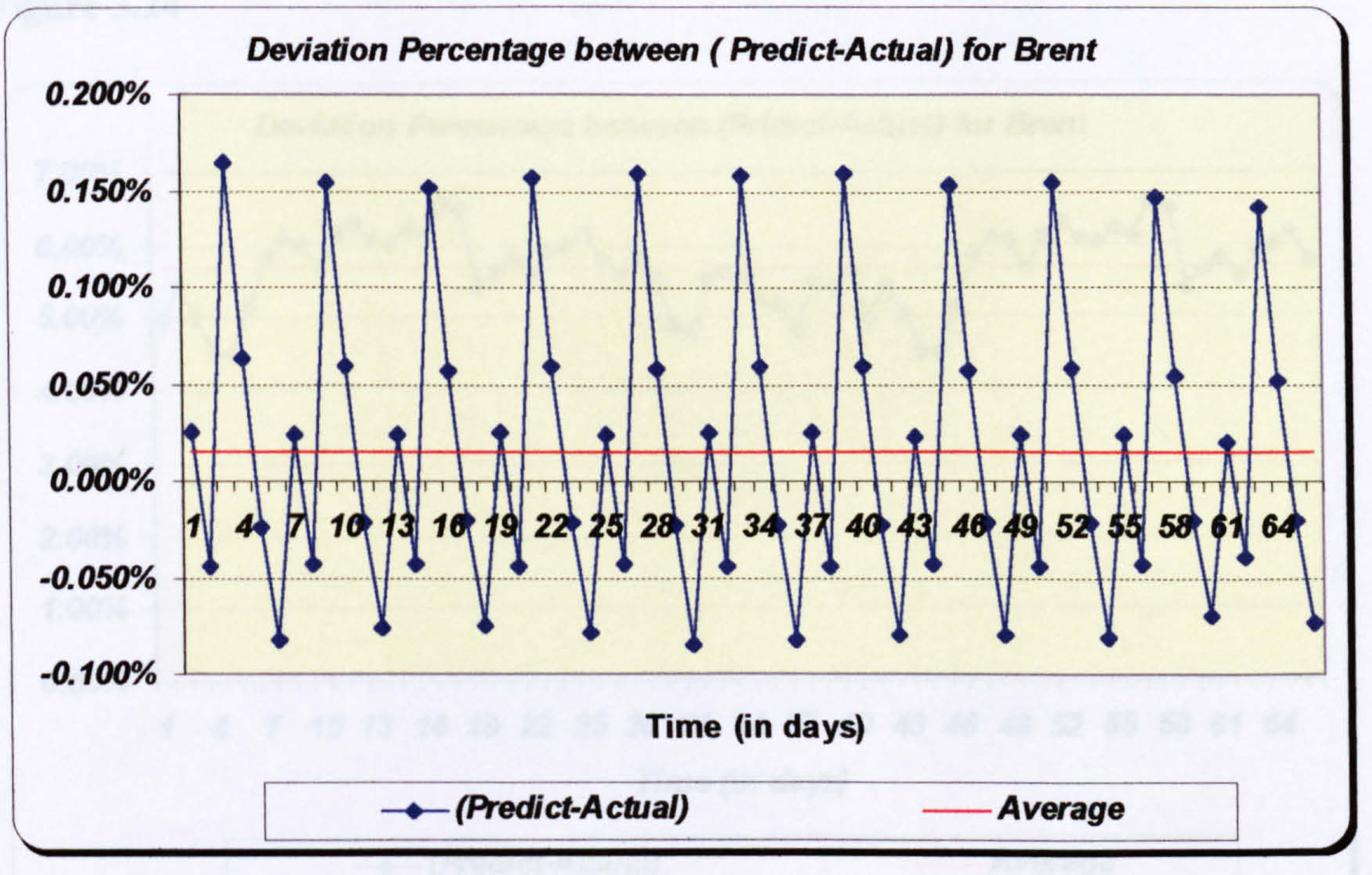


Figure-3.23

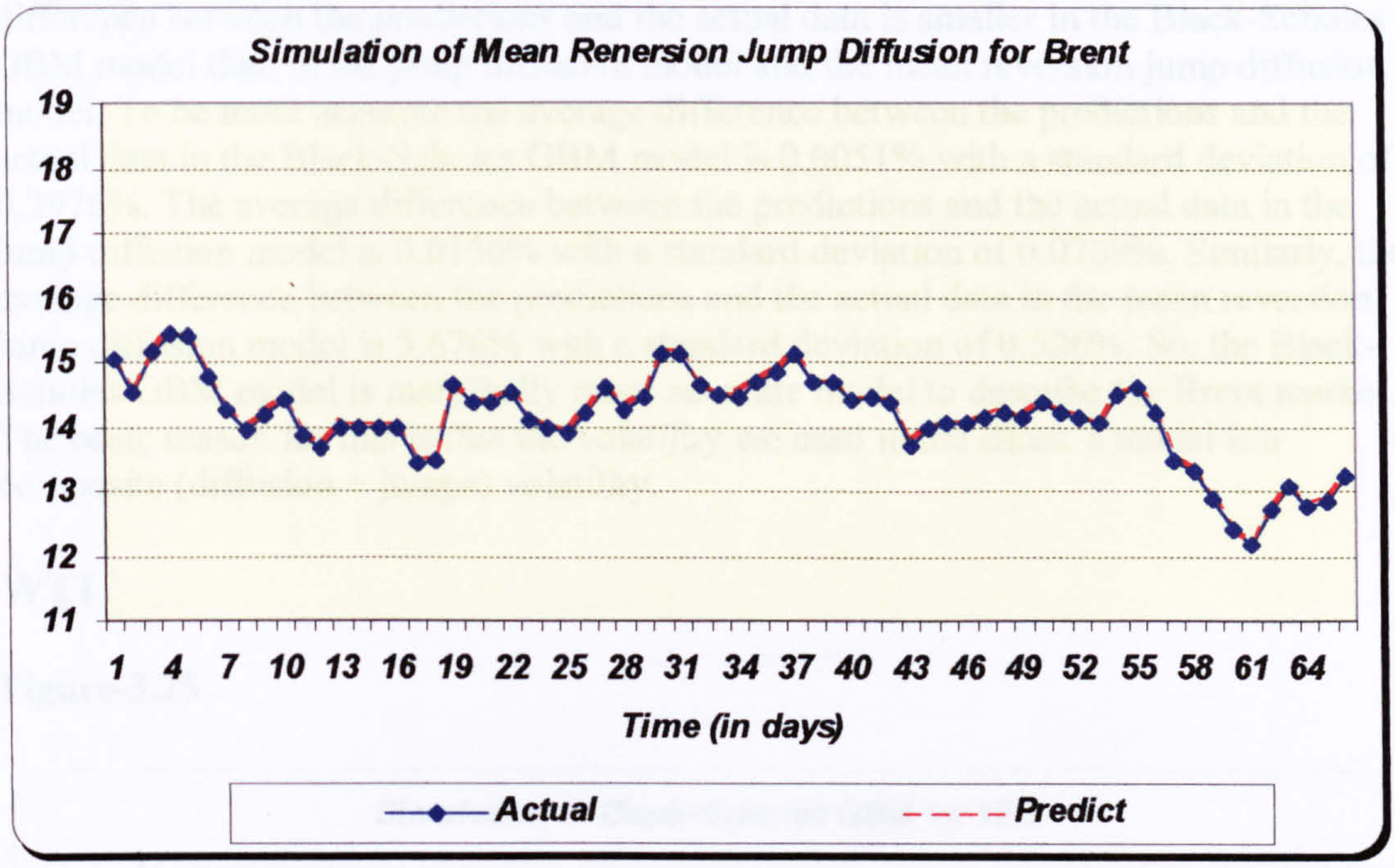
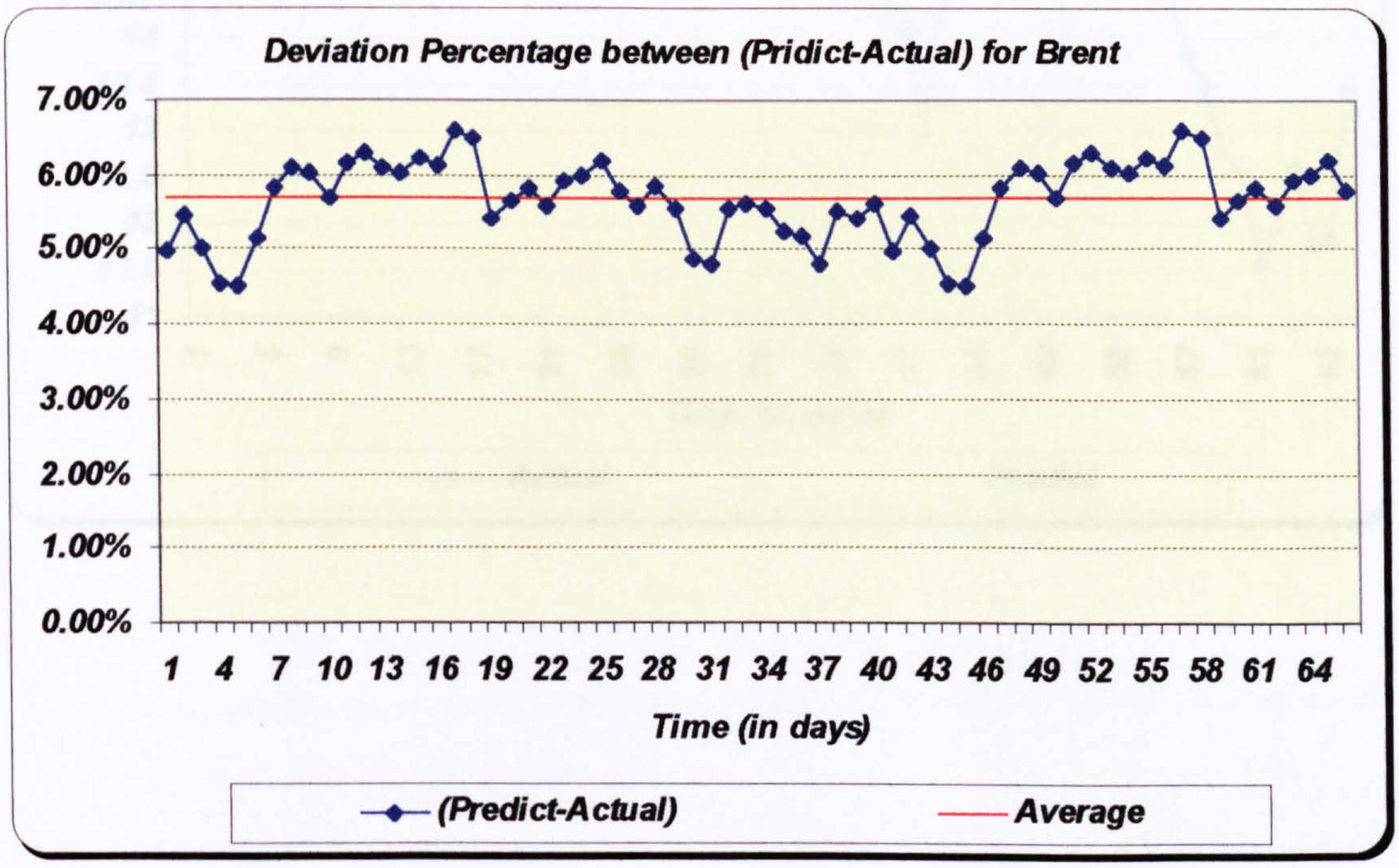


Figure-3.24



From the graphs above (Figure-3.19, Figure-3.21& Figure-3.23) we can see with a naked eye that all three models, the Black-Scholes GBM, the jump diffusion and the mean reversion diffusion are appropriate to describe the Brent market. But if we look in more detail, Figure-3.20, Figure-3.22 & Figure-3.24 show us that the average difference between the predictions and the actual data is smaller in the Black-Scholes GBM model than in the jump diffusion model and the mean reversion jump diffusion model. To be more accurate the average difference between the predictions and the actual data in the Black-Scholes GBM model is 0.0051% with a standard deviation of 0.2976%. The average difference between the predictions and the actual data in the jump diffusion model is 0.0156% with a standard deviation of 0.0769%. Similarly, the average difference between the predictions and the actual data in the mean reversion jump diffusion model is 5.676% with a standard deviation of 0.526%. So, the Black-Scholes GBM model is marginally more accurate model to describe the Brent market. The basic reason for this is that the volatility we used in the Black's model is a composite (diffusion + jumps) volatility.

WTI

Figure-3.25

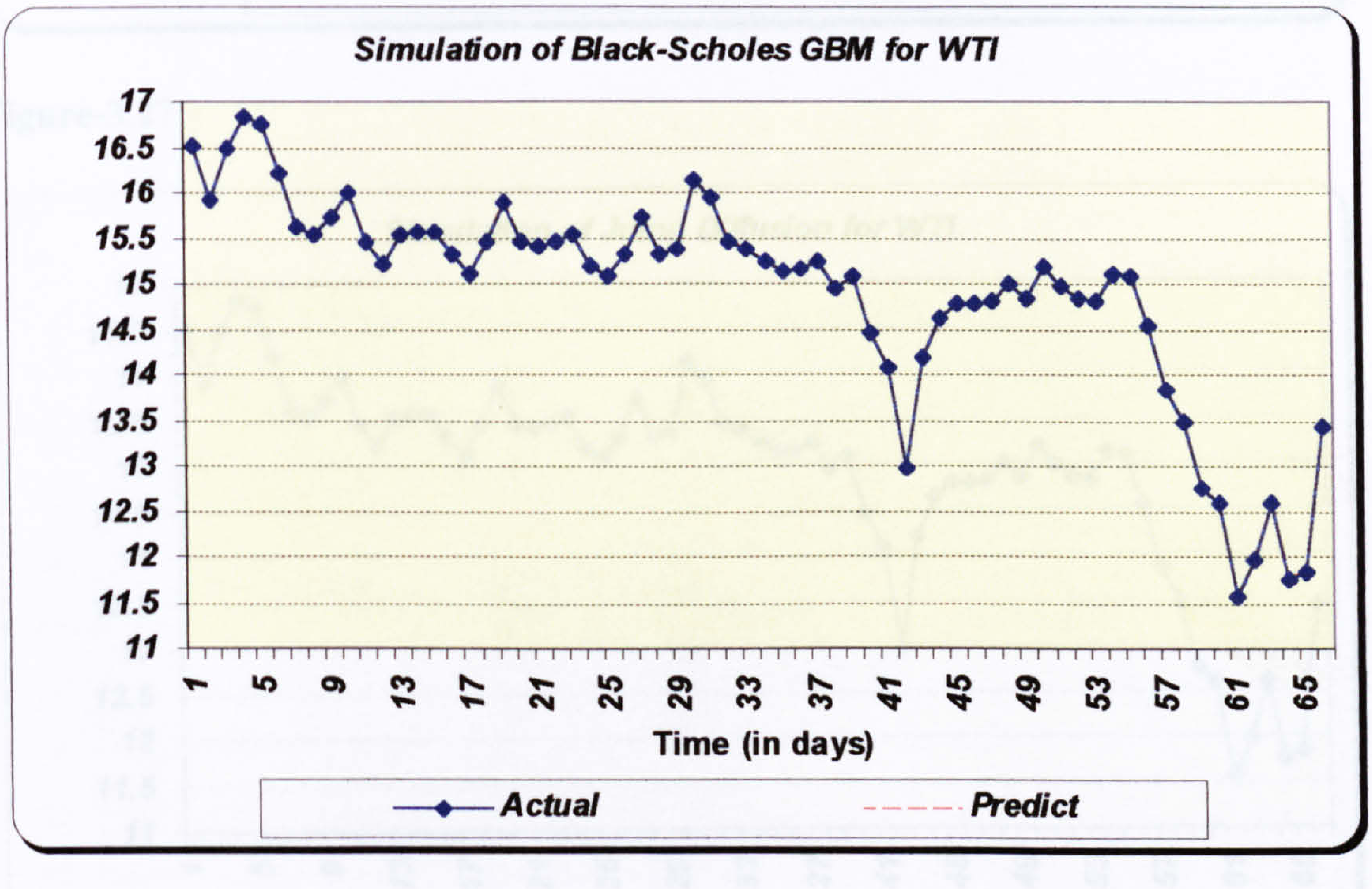


Figure-3.26

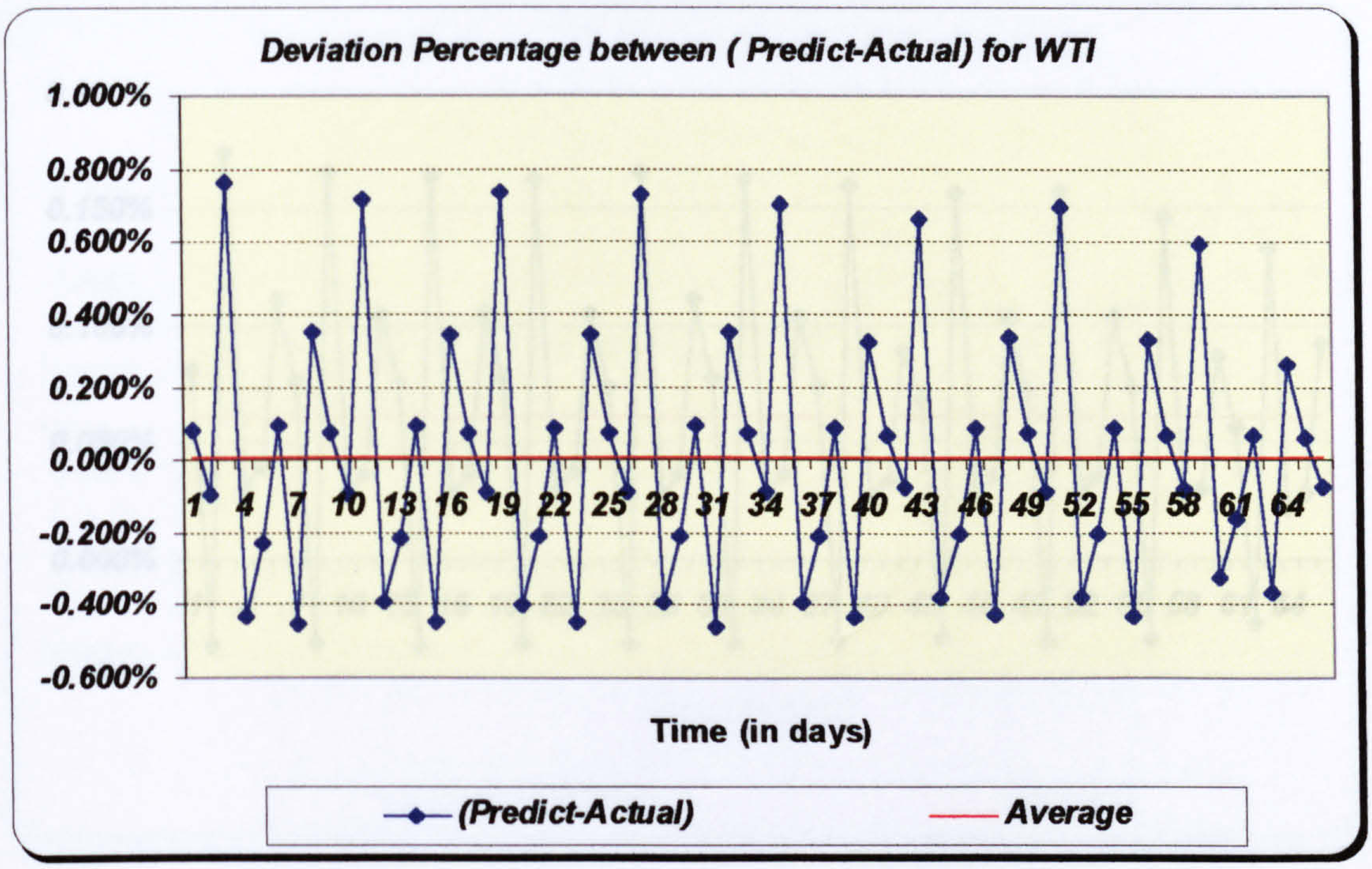


Figure-3.27

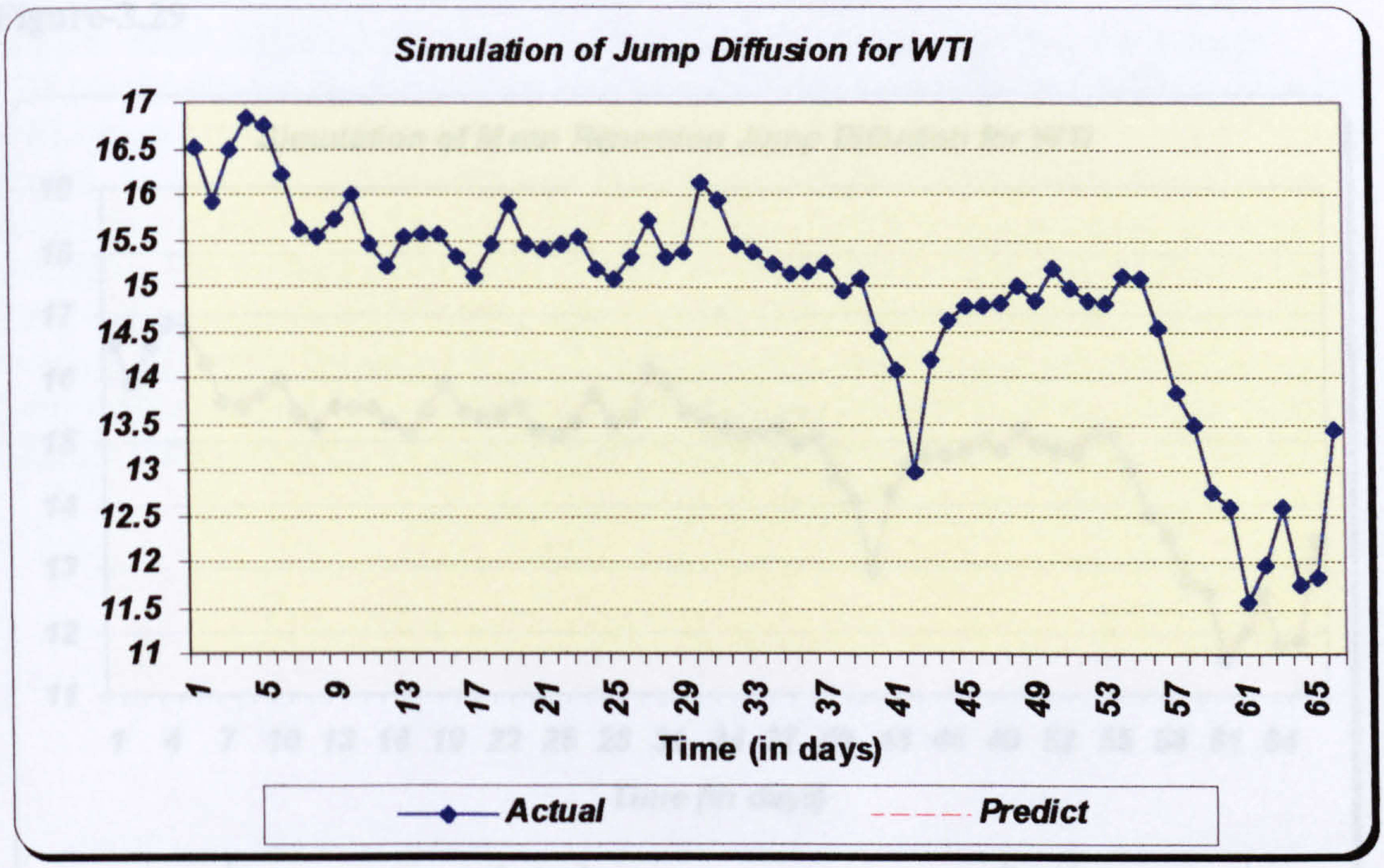


Figure-3.28

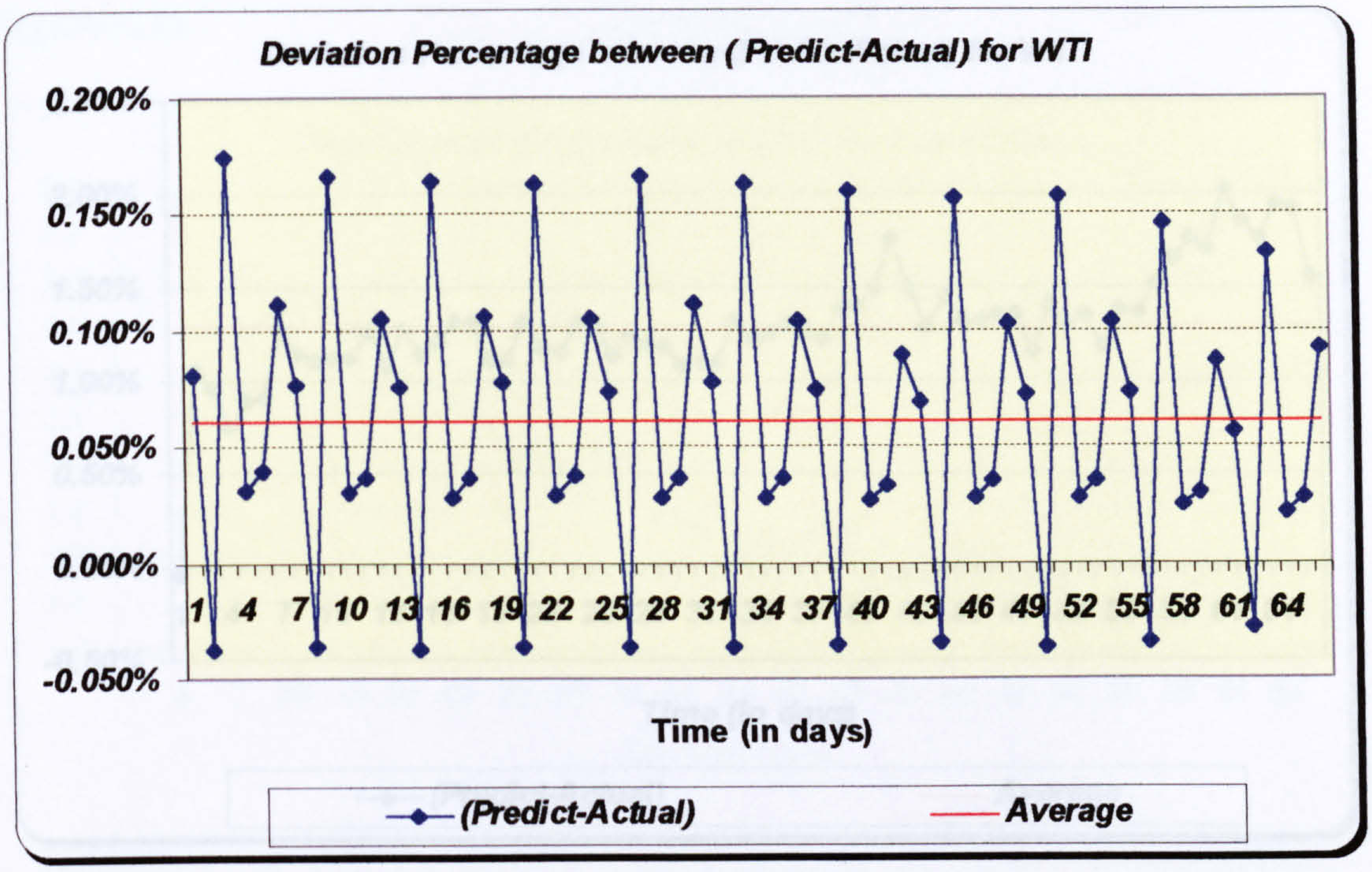


Figure-3.29

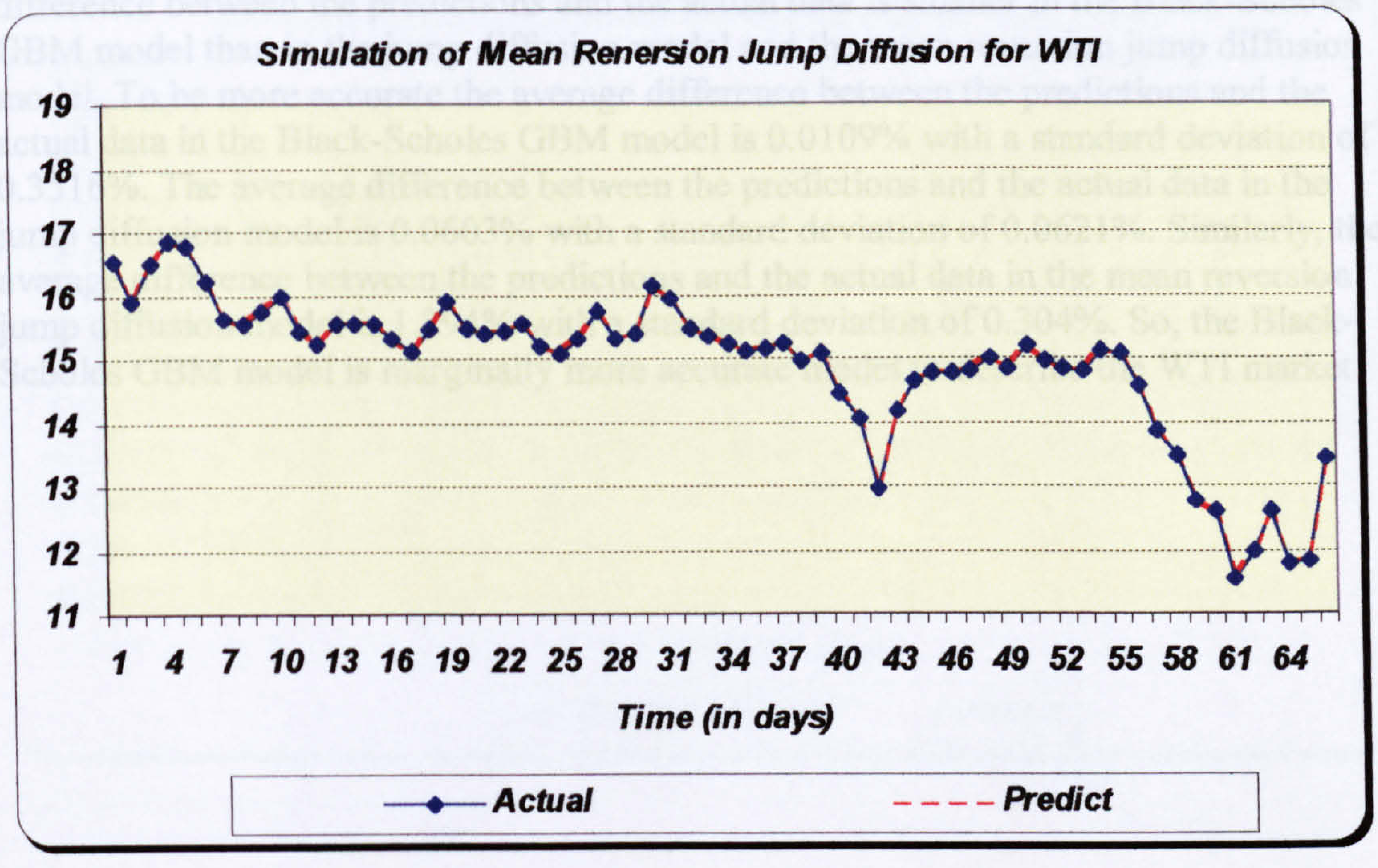
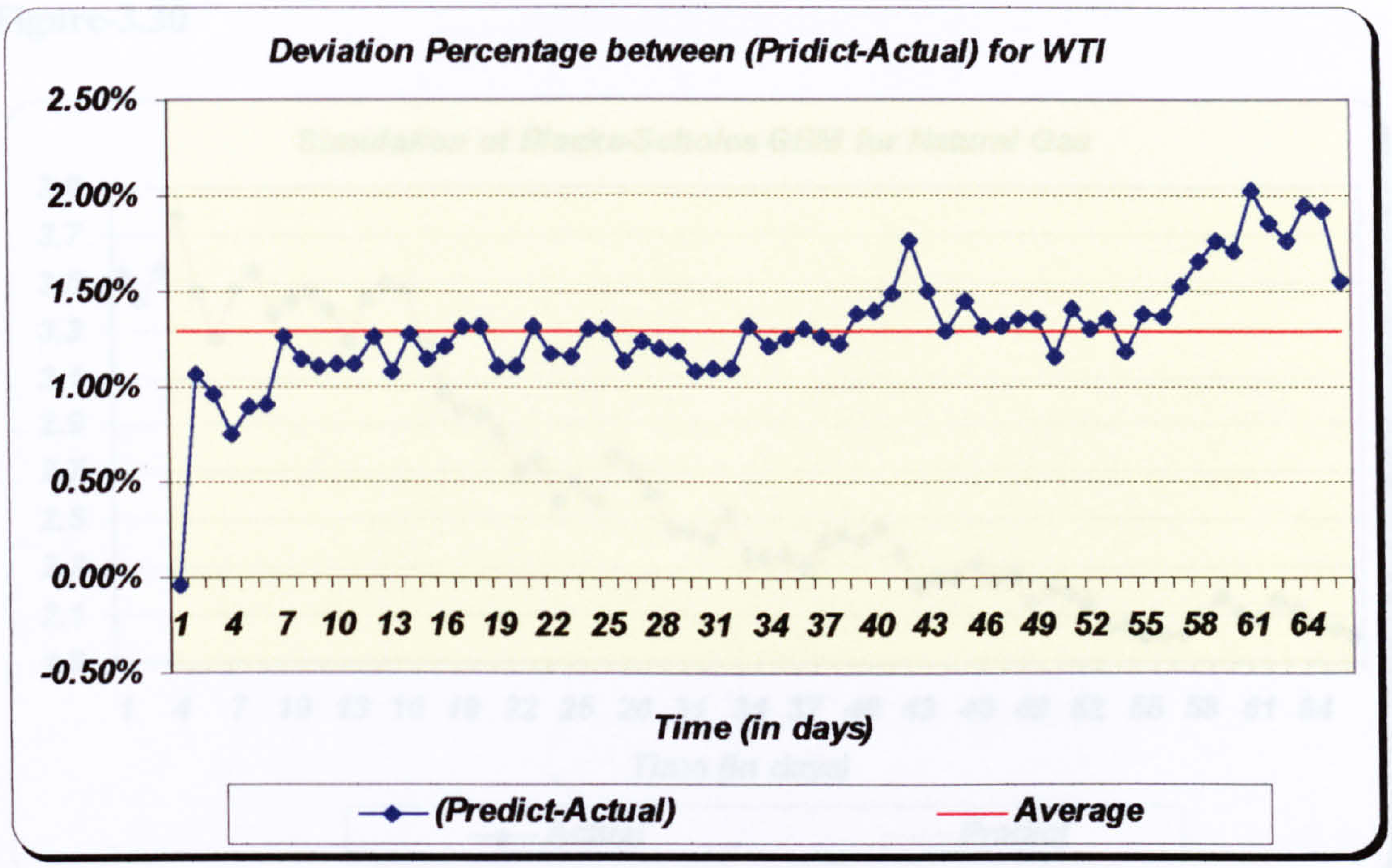


Figure-3.30



From the graphs above (Figure-3.25, Figure-3.27 & Figure-3.29) we can see with a naked eye that all three models, the Black-Scholes GBM, the jump diffusion and the mean reversion diffusion are appropriate to describe the WTI market. But if we look in more detail, Figure-3.26, Figure-3.28 & Figure-3.30 show us that the average difference between the predictions and the actual data is smaller in the Black-Scholes GBM model than in the jump diffusion model and the mean reversion jump diffusion model. To be more accurate the average difference between the predictions and the actual data in the Black-Scholes GBM model is 0.0109% with a standard deviation of 0.3516%. The average difference between the predictions and the actual data in the jump diffusion model is 0.0603% with a standard deviation of 0.0621%. Similarly, the average difference between the predictions and the actual data in the mean reversion jump diffusion model is 1.294% with a standard deviation of 0.304%. So, the Black-Scholes GBM model is marginally more accurate model to describe the WTI market.

NATURAL-GAS

Figure-3.30

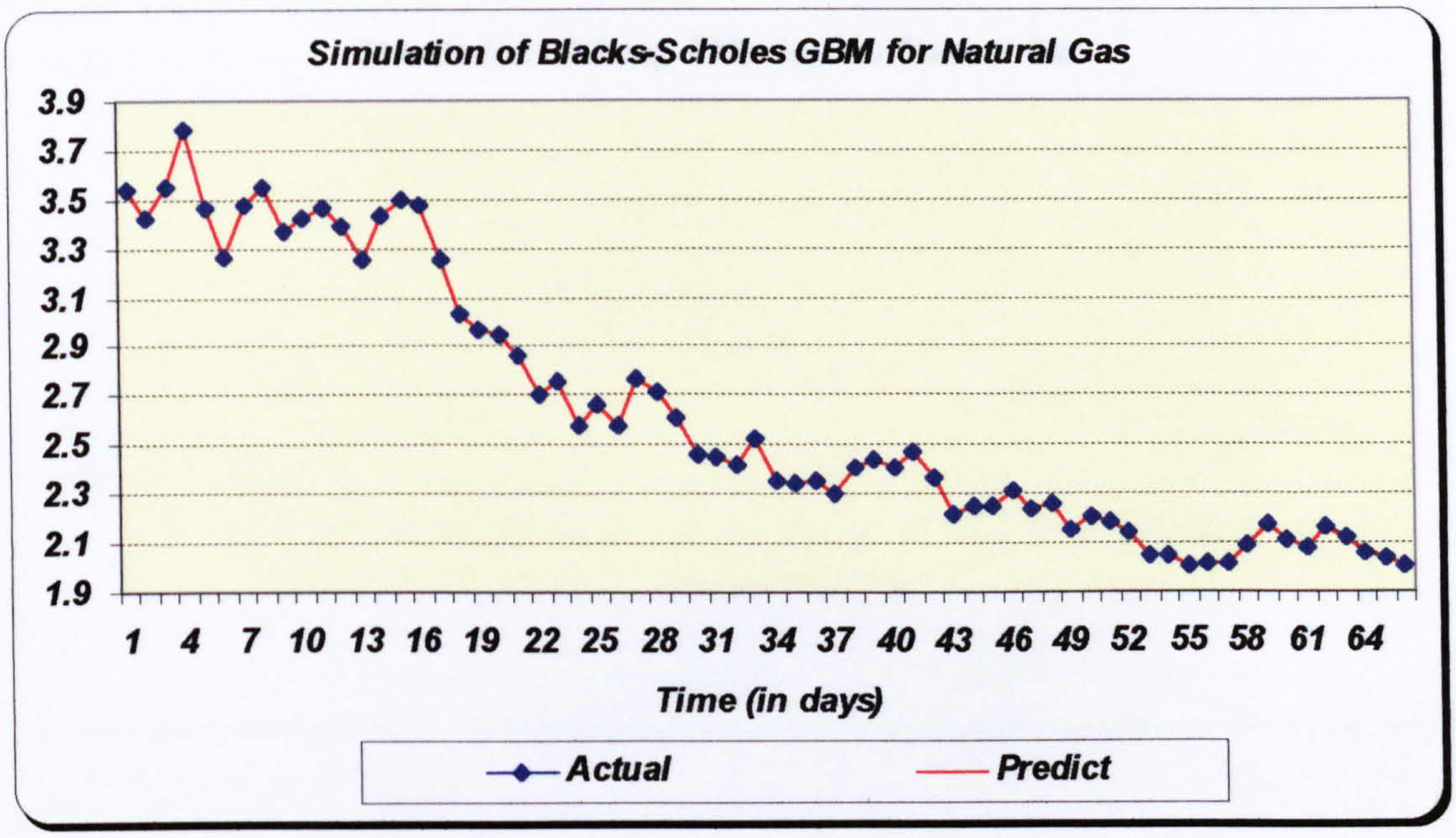


Figure-3.31

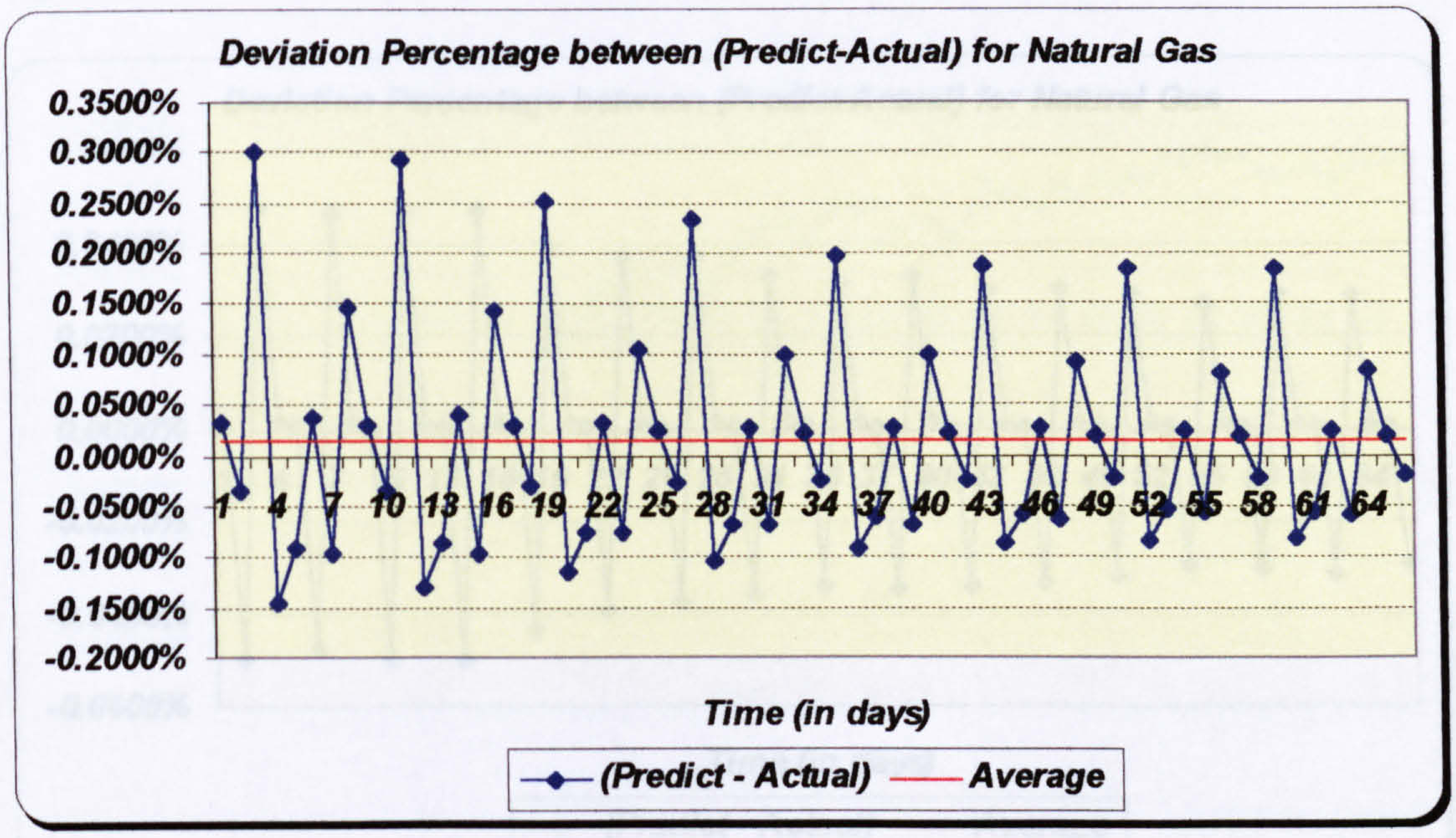


Figure-3.32

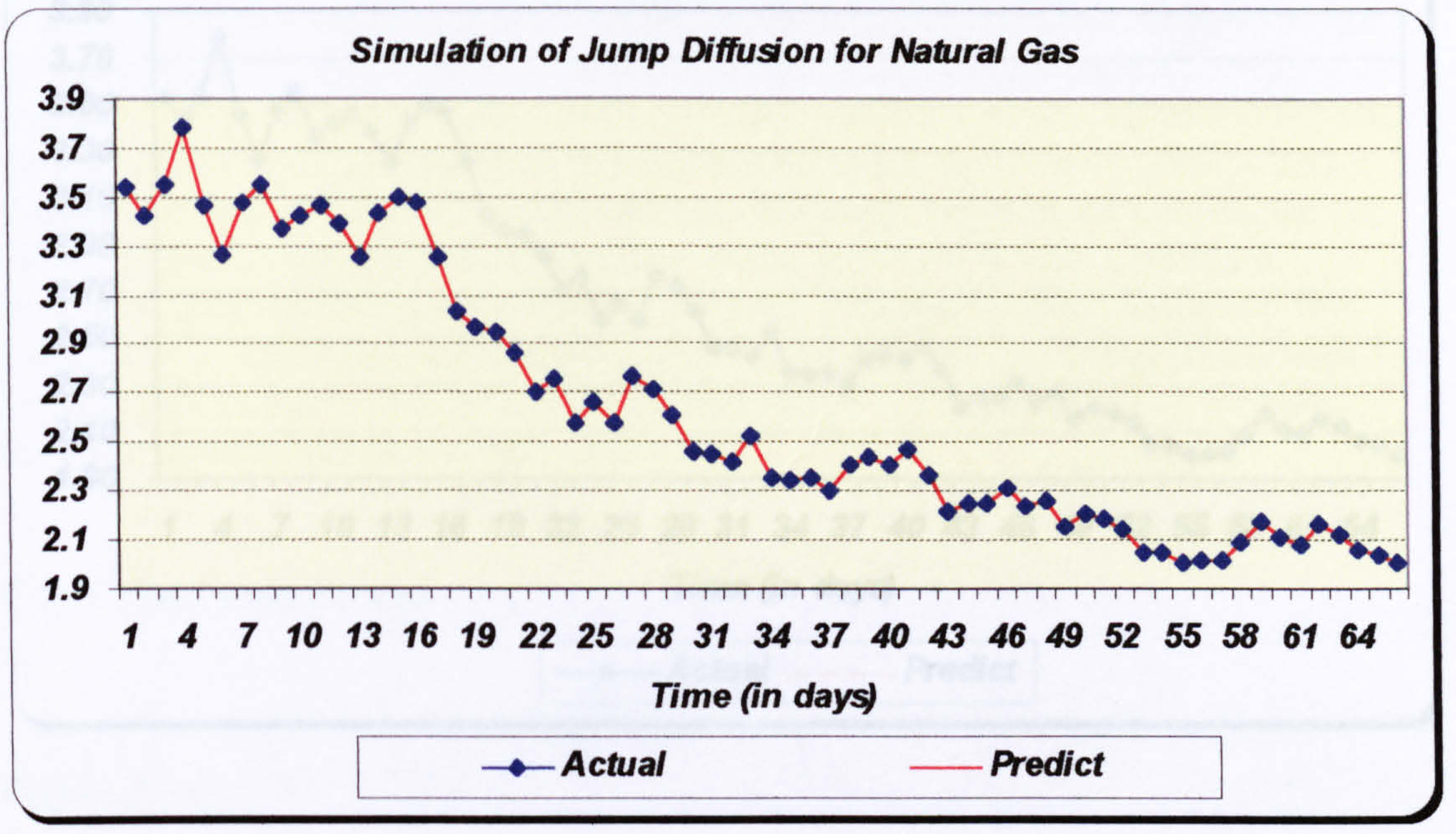


Figure-3.33

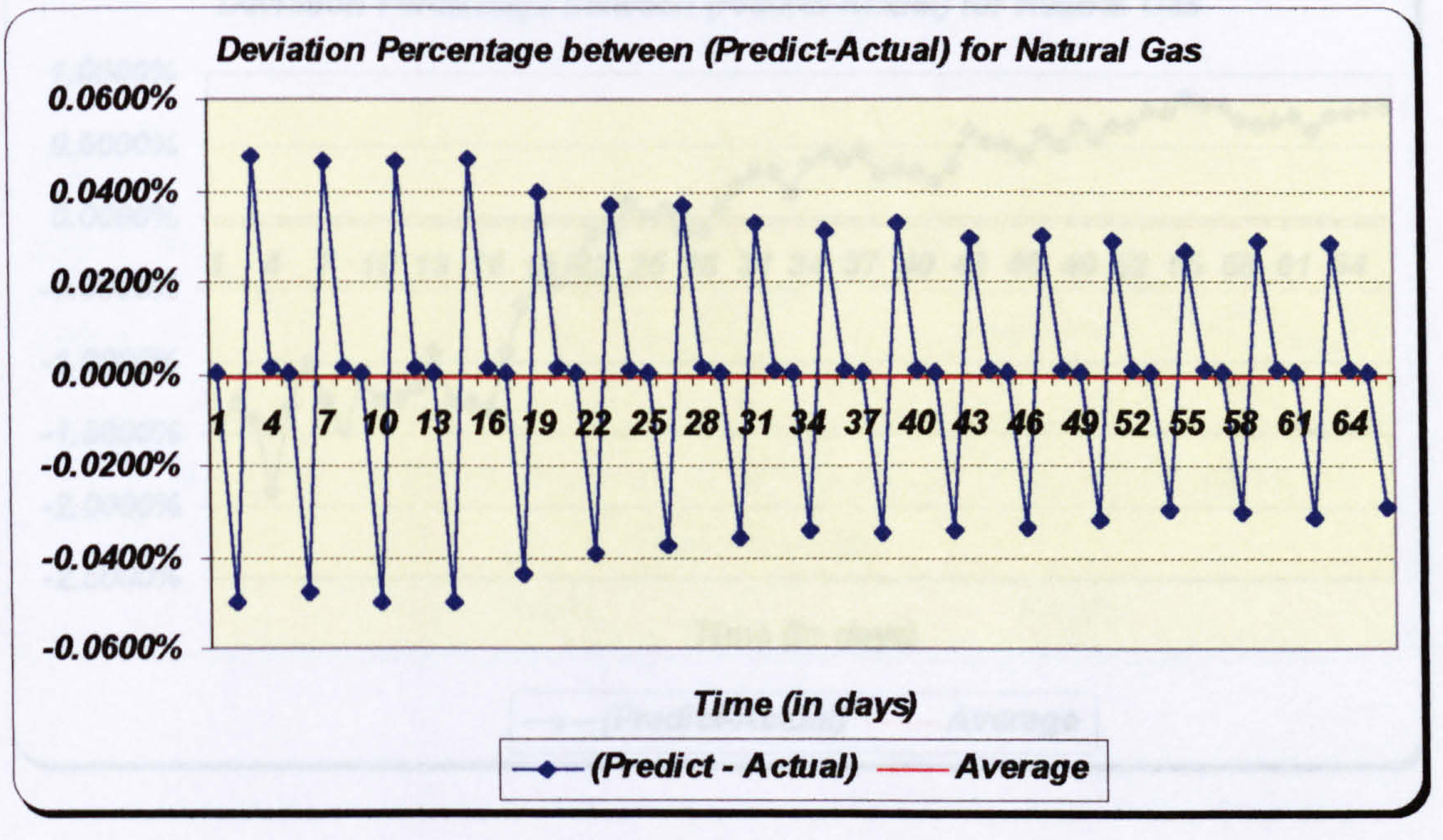


Figure-3.34

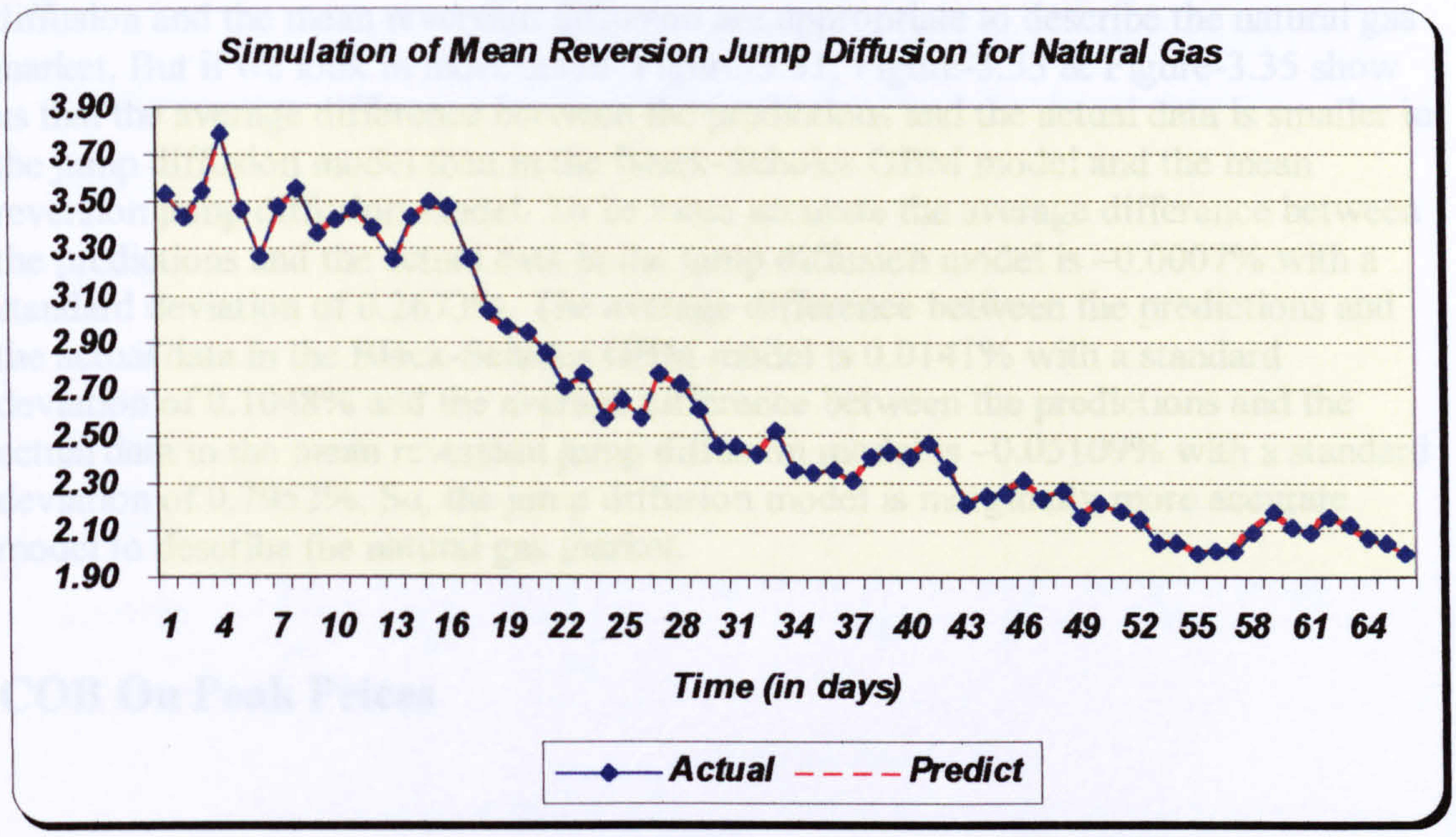
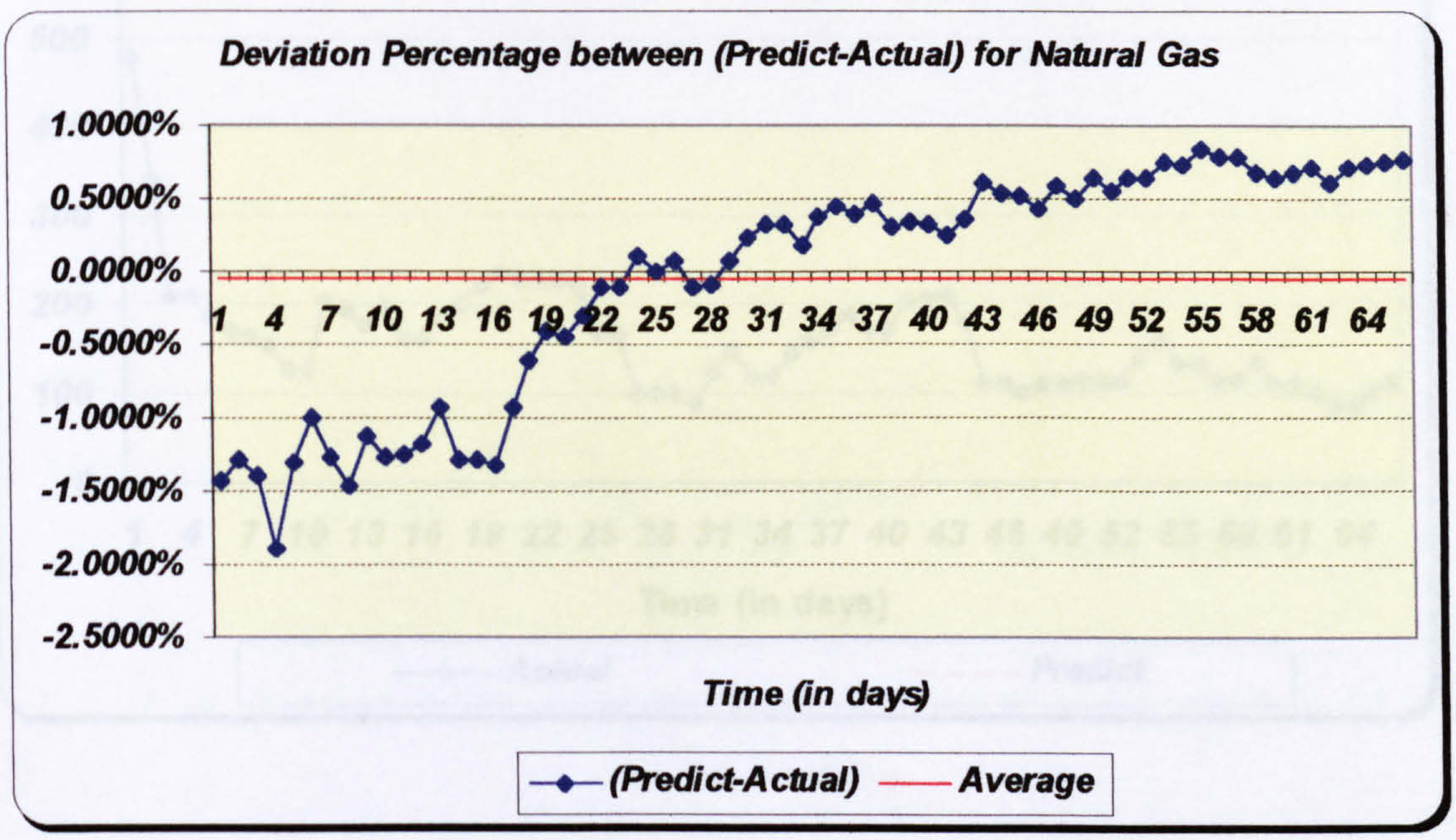


Figure-3.35



Similarly from the graphs above (Figure-3.30, Figure-3.32 & Figure-3.34) we can see with a naked eye that all three models, the Black-Scholes GBM model, the jump diffusion and the mean reversion diffusion are appropriate to describe the natural gas market. But if we look in more detail, Figure-3.31, Figure-3.33 & Figure-3.35 show us that the average difference between the predictions and the actual data is smaller in the jump diffusion model than in the Black-Scholes GBM model and the mean reversion jump diffusion model. To be more accurate the average difference between the predictions and the actual data in the jump diffusion model is –0.0007% with a standard deviation of 0.2673%. The average difference between the predictions and the actual data in the Black-Scholes GBM model is 0.0141% with a standard deviation of 0.1048% and the average difference between the predictions and the actual data in the mean reversion jump diffusion model is –0.05109% with a standard deviation of 0.7952%. So, the jump diffusion model is marginally more accurate model to describe the natural gas market.

COB On Peak Prices

Figure-3.36

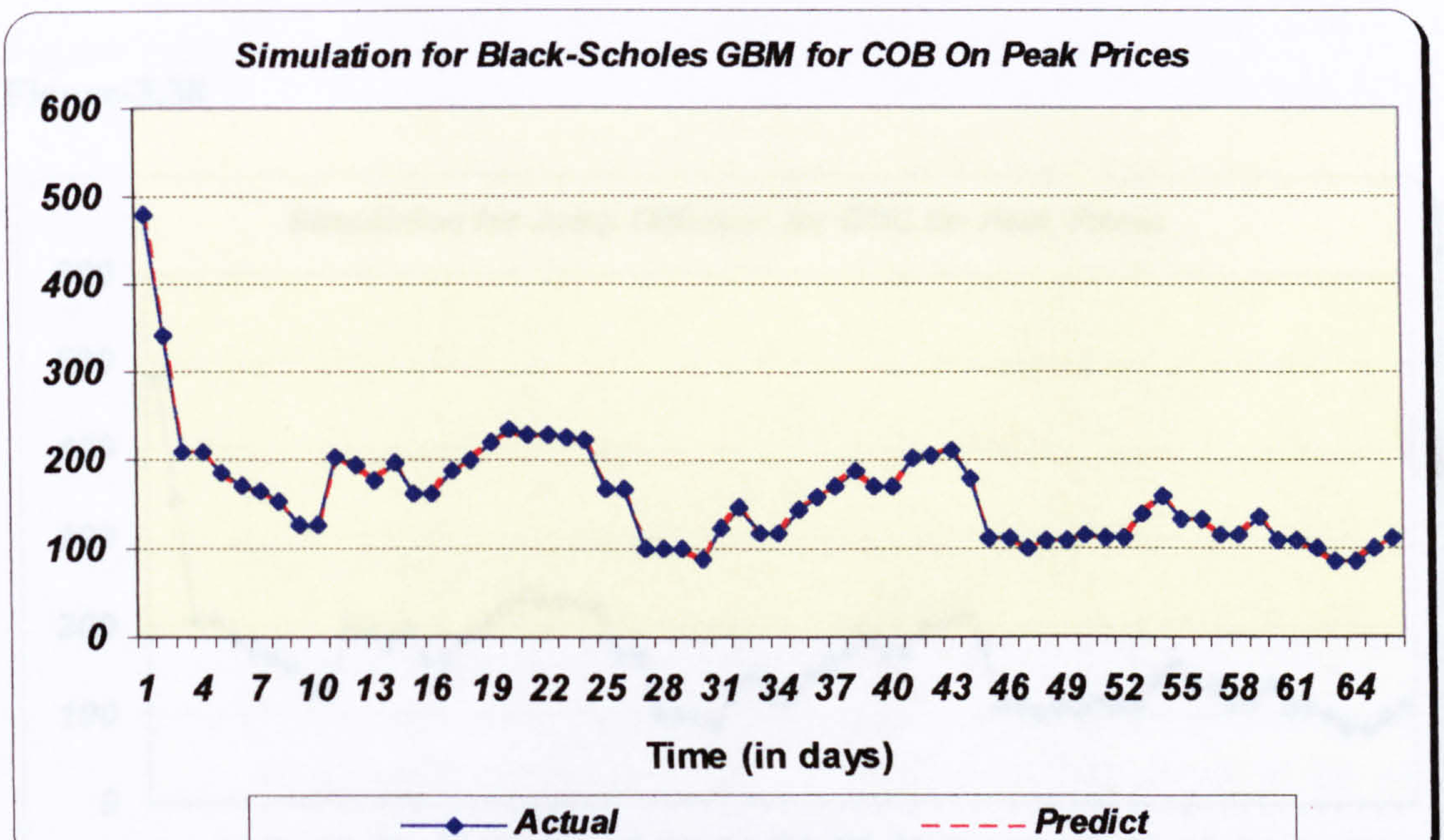


Figure-3.37

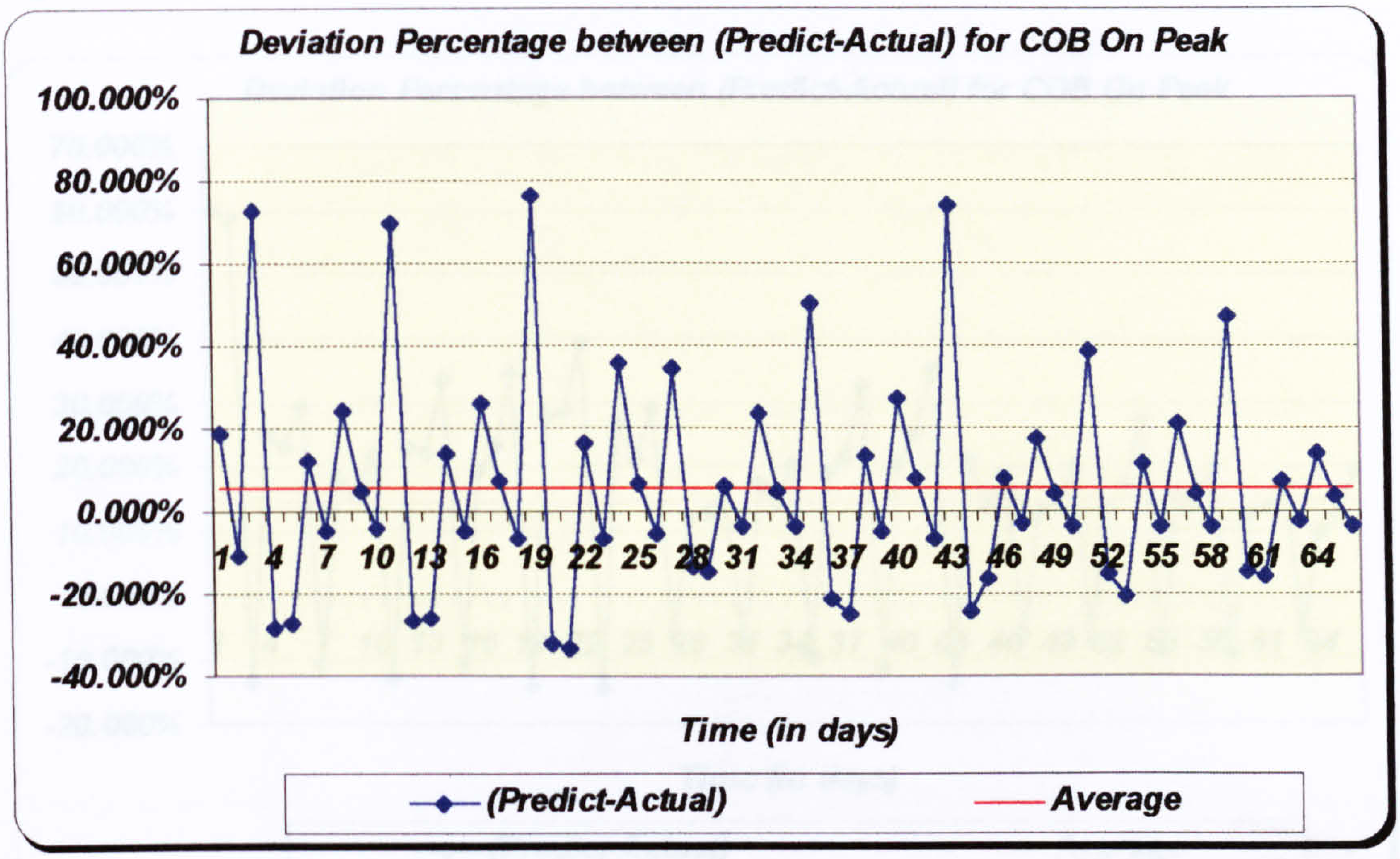


Figure-3.38

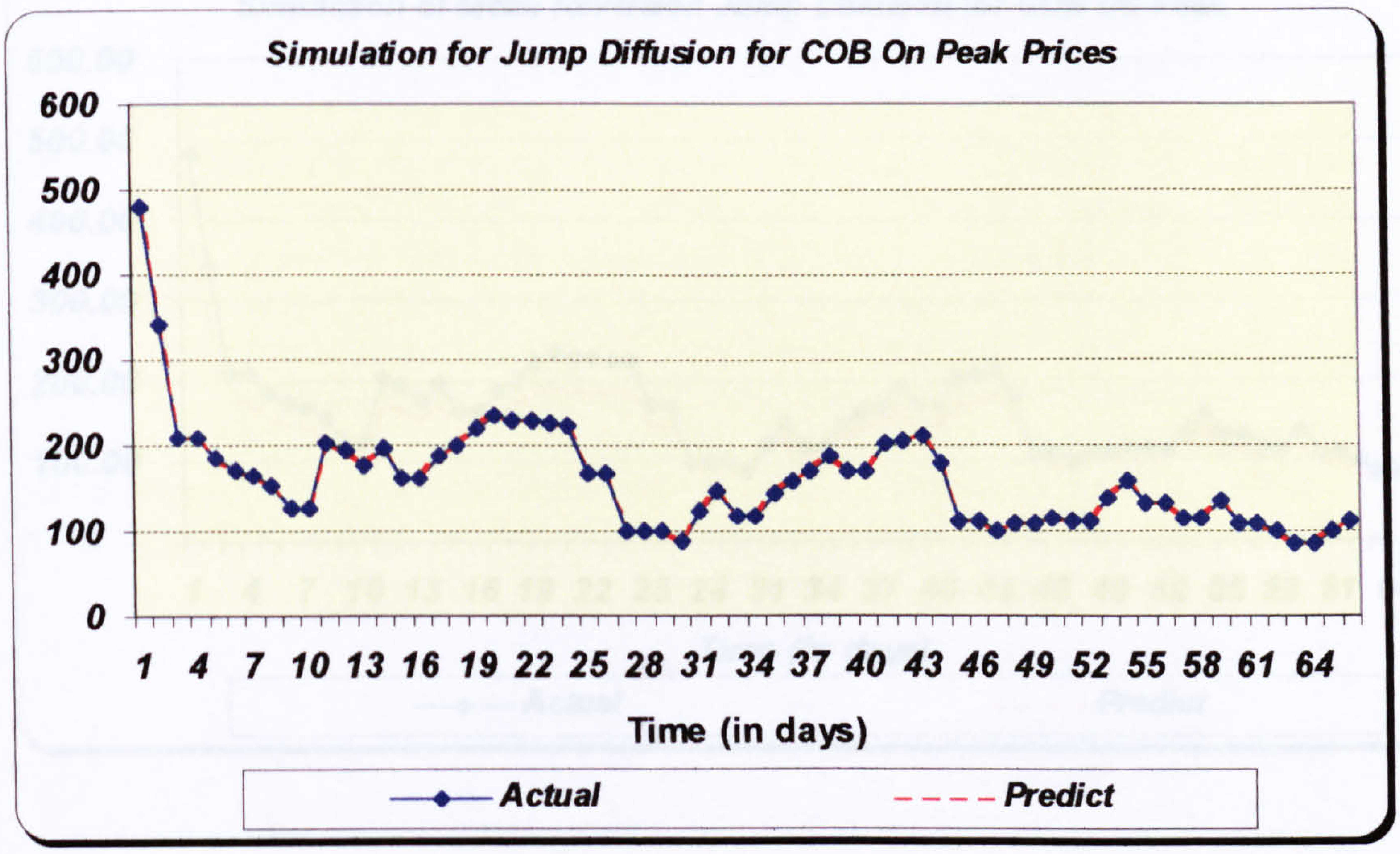


Figure-3.39

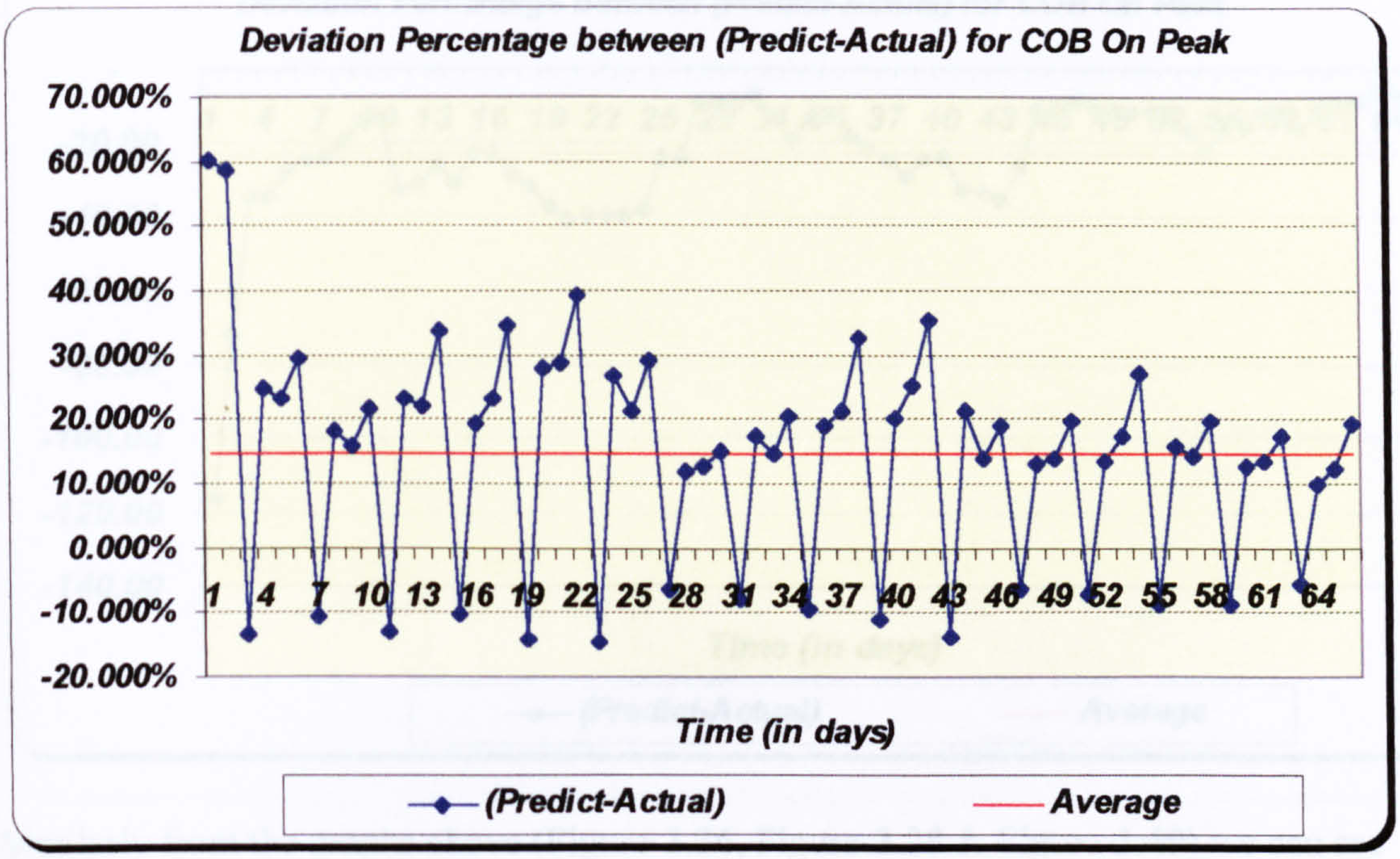


Figure-3.40

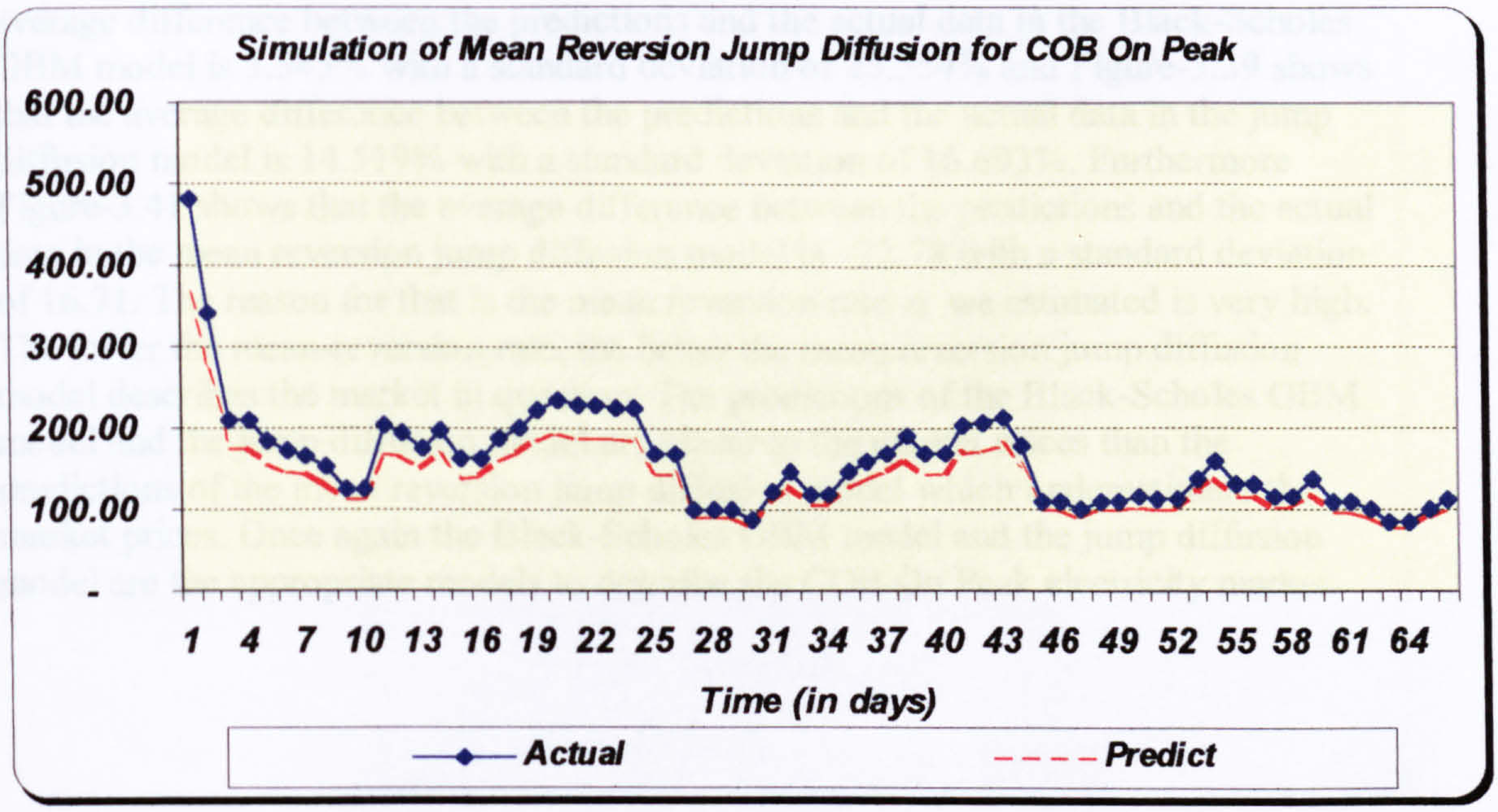
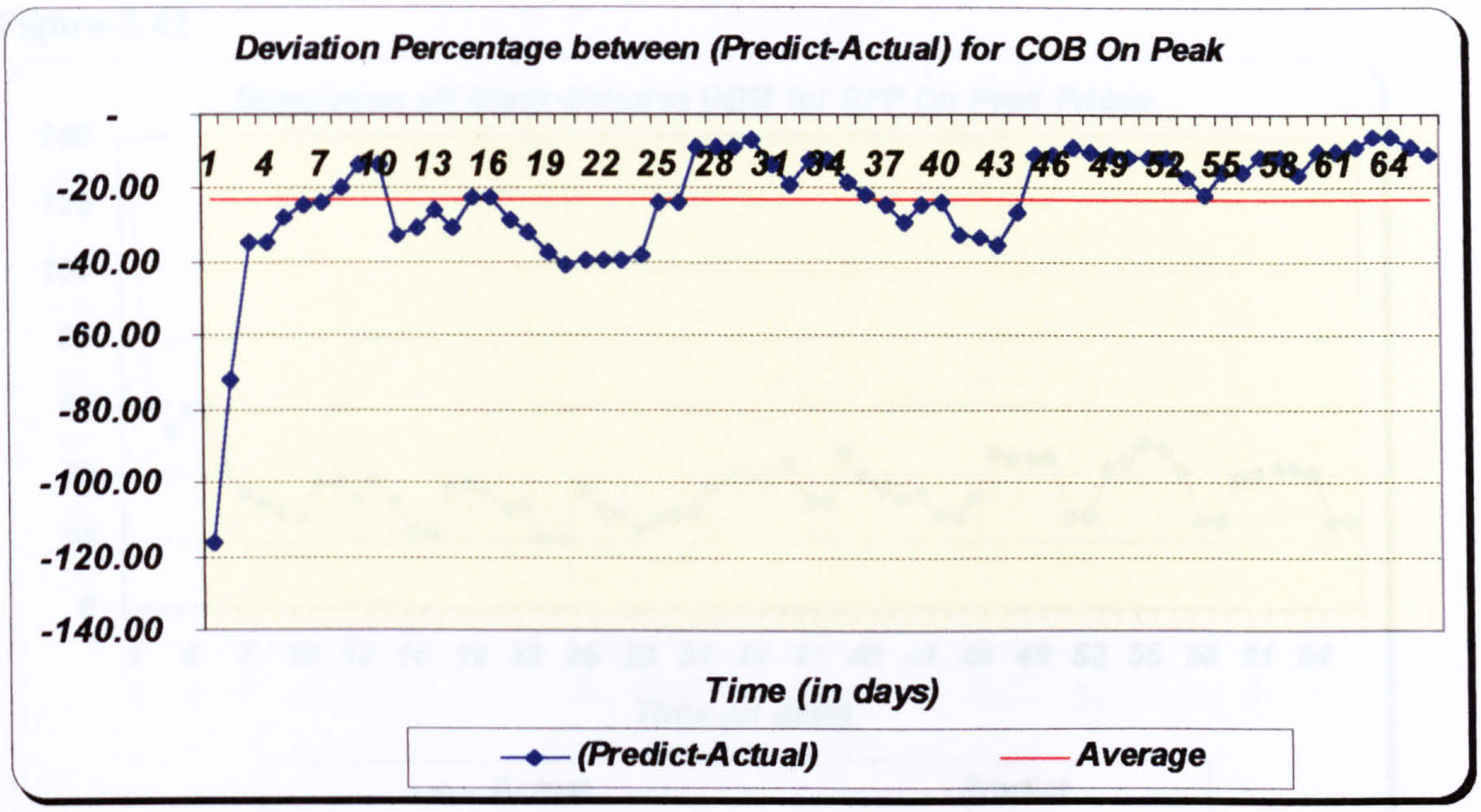


Figure-3.41



Similarly from the graphs above (Figure-3.36, Figure-3.38 & Figure-3.40) we can see with a naked eye that the Black-Scholes GBM model and the jump diffusion model is the appropriate model to describe the COB On Peak electricity market and not the mean reversion jump diffusion. If we look in more detail, Figure-3.37shows that the average difference between the predictions and the actual data in the Black-Scholes GBM model is 5.545% with a standard deviation of 25.554% and Figure-3.39 shows that the average difference between the predictions and the actual data in the jump diffusion model is 14.519% with a standard deviation of 16.693%. Furthermore Figure-3.41 shows that the average difference between the predictions and the actual data in the mean reversion jump diffusion model is –22.78 with a standard deviation of 16.71. The reason for that is the mean reversion rate α we estimated is very high. The lower the mean-reversion rate, the better the mean reversion jump diffusion model describes the market in question. The predictions of the Black-Scholes GBM model and the jump diffusion model are closer to the market prices than the predictions of the mean reversion jump diffusion model which underestimate the market prices. Once again the Black-Scholes GBM model and the jump diffusion model are the appropriate models to describe the COB On Peak electricity market.

SPP On Peak Prices

Figure-3.42

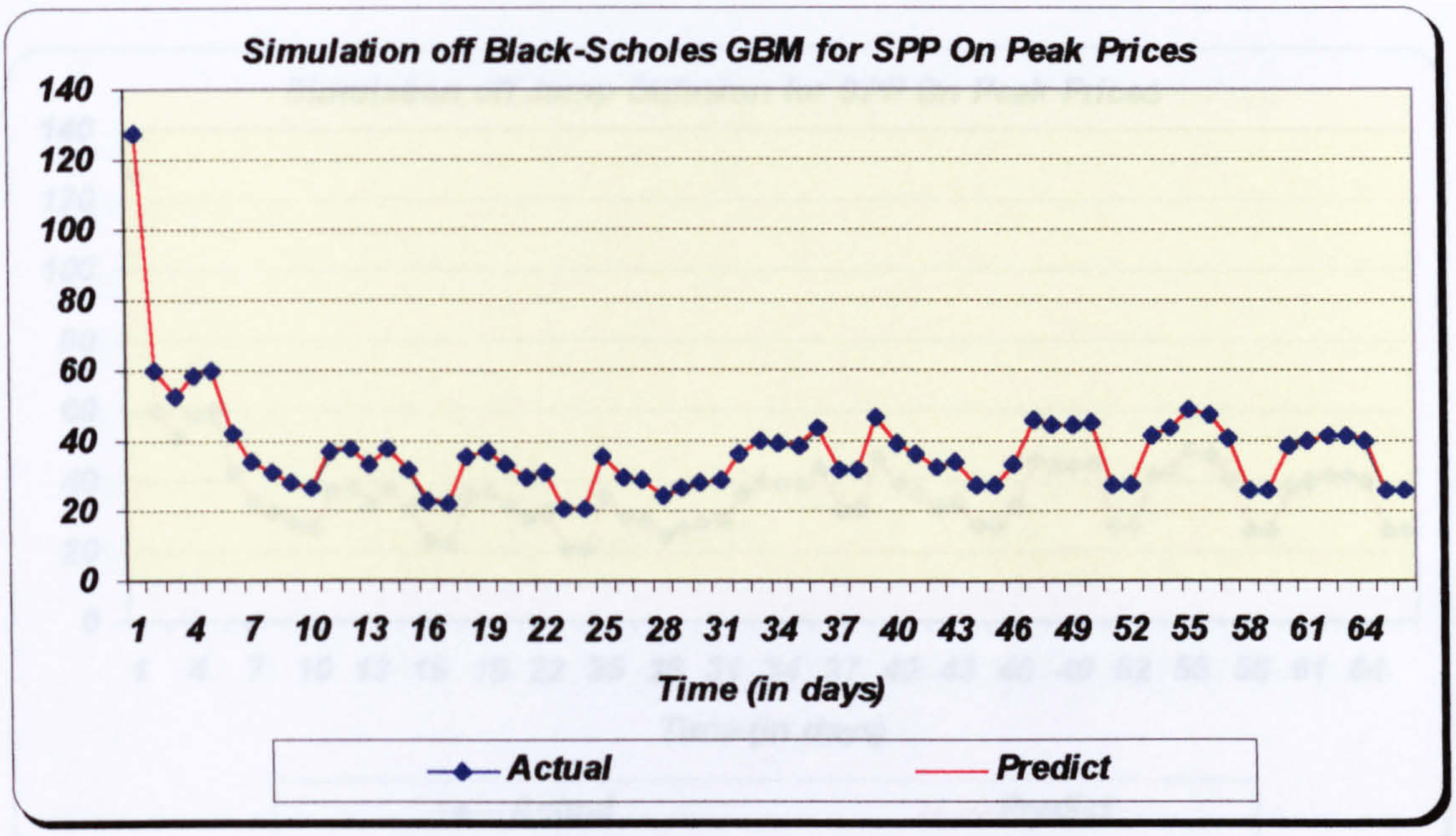


Figure-3.43

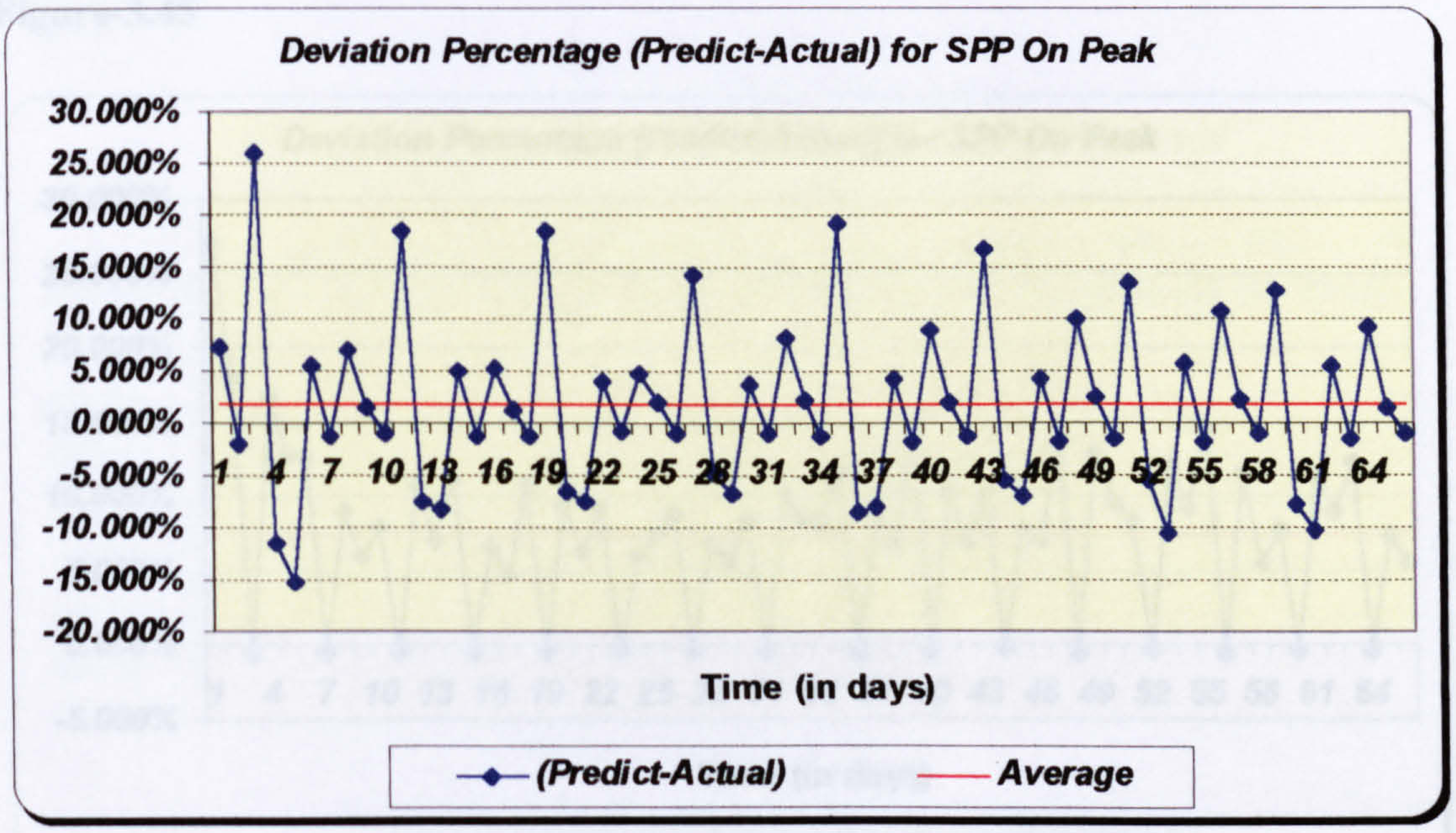


Figure-3.44

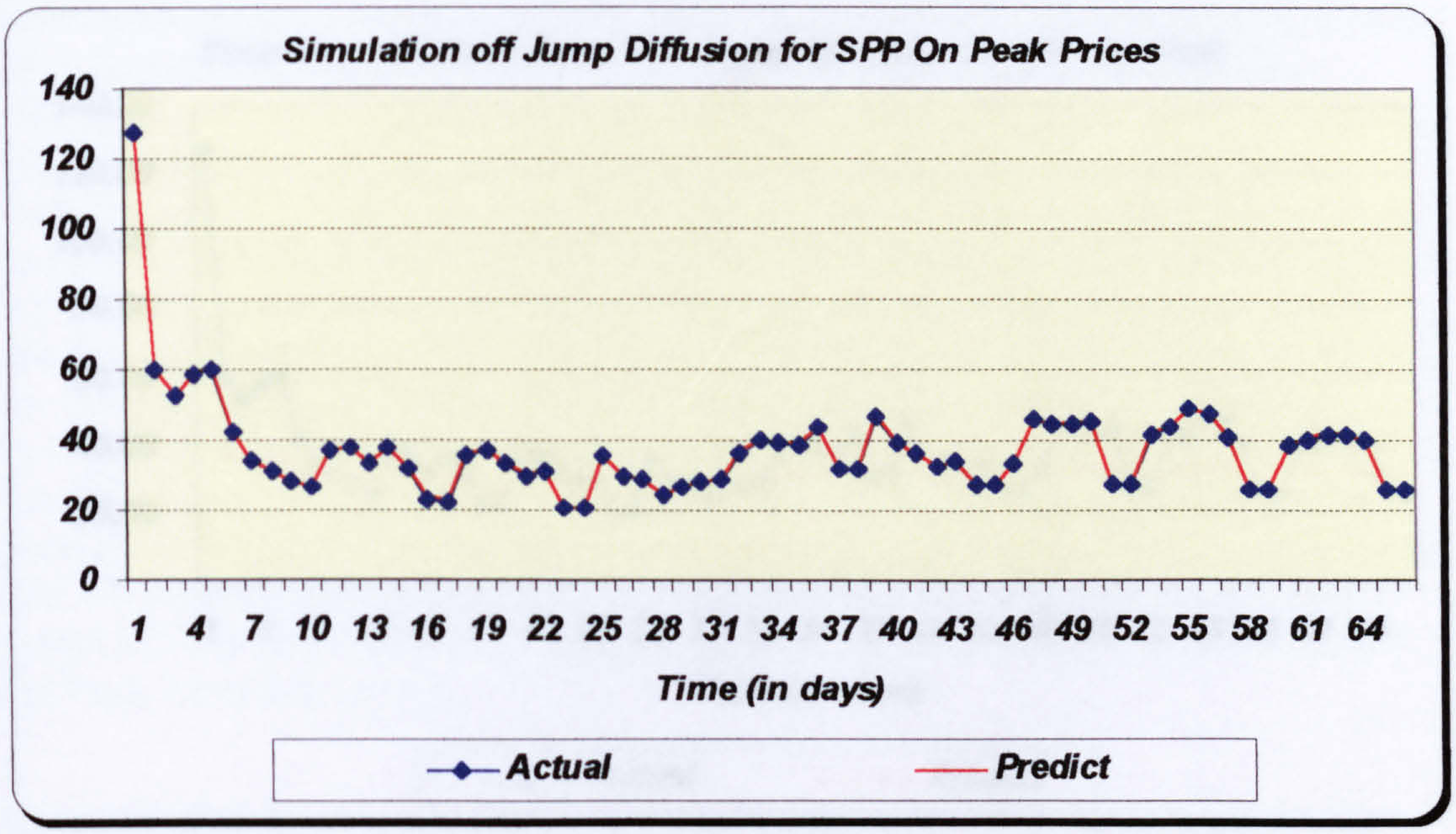


Figure-3.45

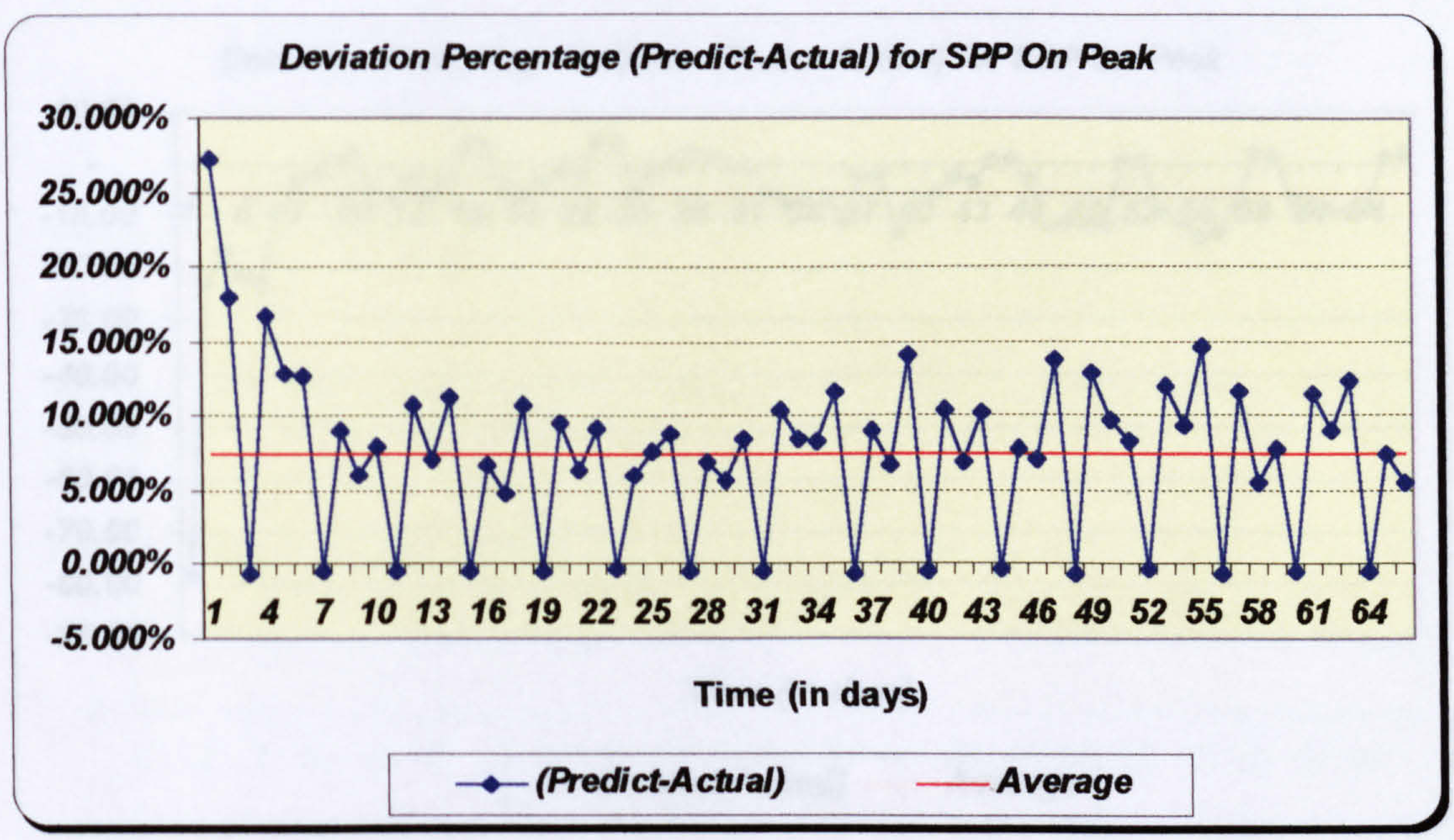


Figure-3.46

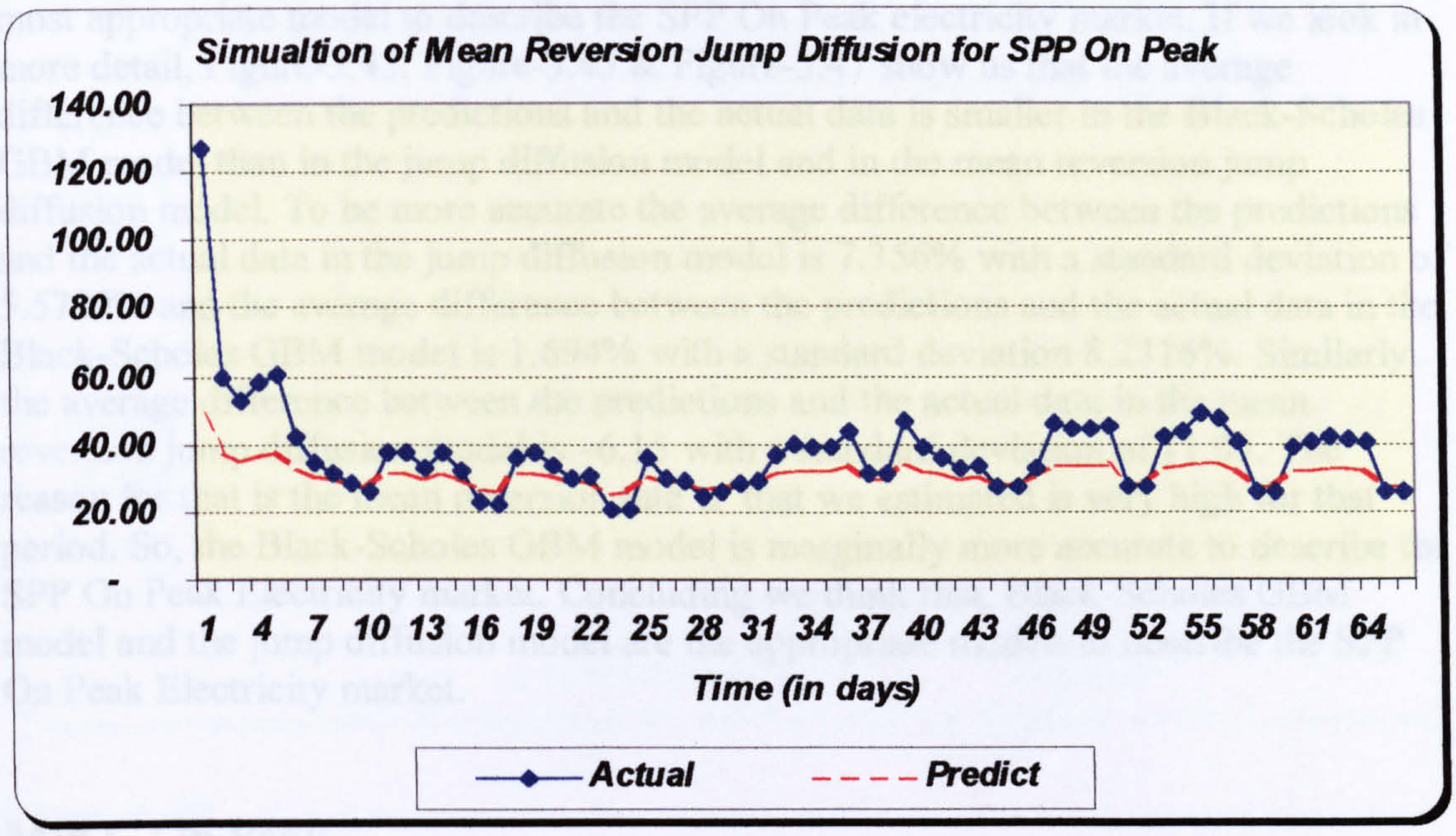
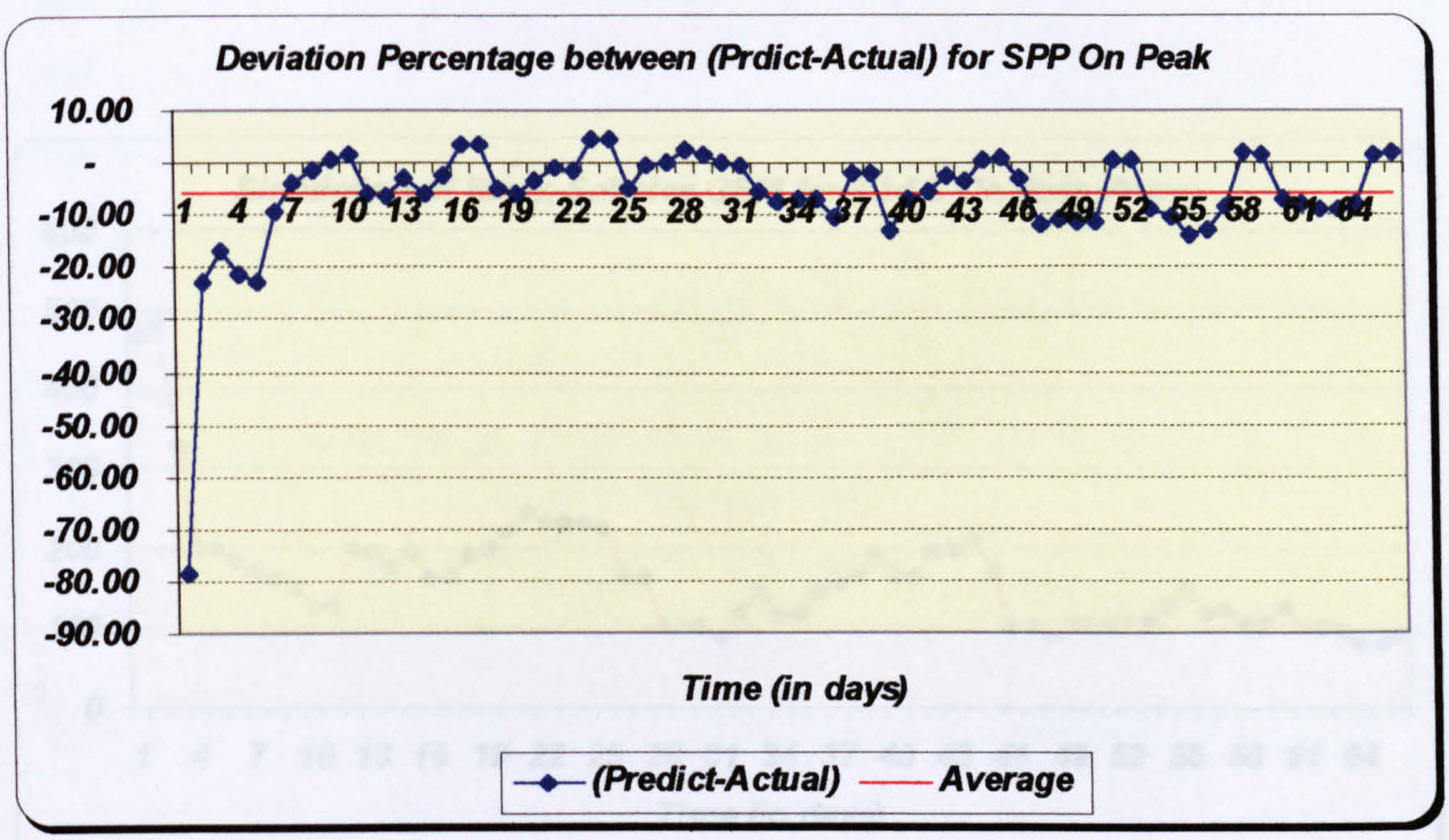


Figure-3.47



Furthermore from the graphs above (Figure-3.42, Figure-3.44 & Figure-3.46) we can see with a naked eye that the Black-Scholes GBM and the jump diffusion model is the most appropriate model to describe the SPP On Peak electricity market. If we look in more detail, Figure-3.43, Figure-3.45 & Figure-3.47 show us that the average difference between the predictions and the actual data is smaller in the Black-Scholes GBM model than in the jump diffusion model and in the mean reversion jump diffusion model. To be more accurate the average difference between the predictions and the actual data in the jump diffusion model is 7.356% with a standard deviation of 5.5729% and the average difference between the predictions and the actual data in the Black-Scholes GBM model is 1.694% with a standard deviation 8.2316%. Similarly the average difference between the predictions and the actual data in the mean reversion jump diffusion model is –6.15 with a standard deviation of 11.05. The reason for that is the mean reversion rate α that we estimated is very high for that period. So, the Black-Scholes GBM model is marginally more accurate to describe the SPP On Peak Electricity market. Concluding we think that, Black-Scholes GBM model and the jump diffusion model are the appropriate models to describe the SPP On Peak Electricity market.

Mid C On Peak

Figure-3.48

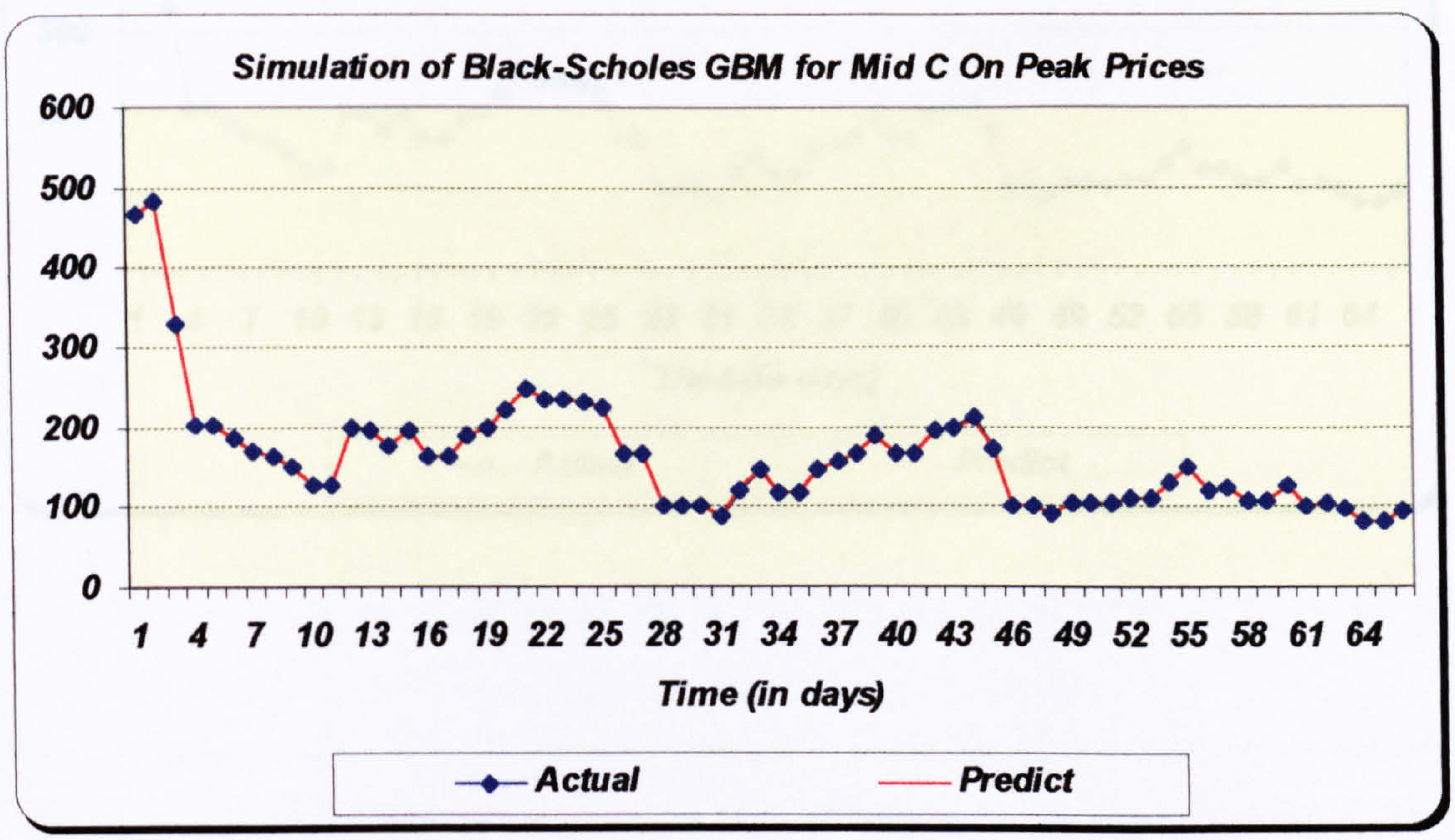


Figure-3.49

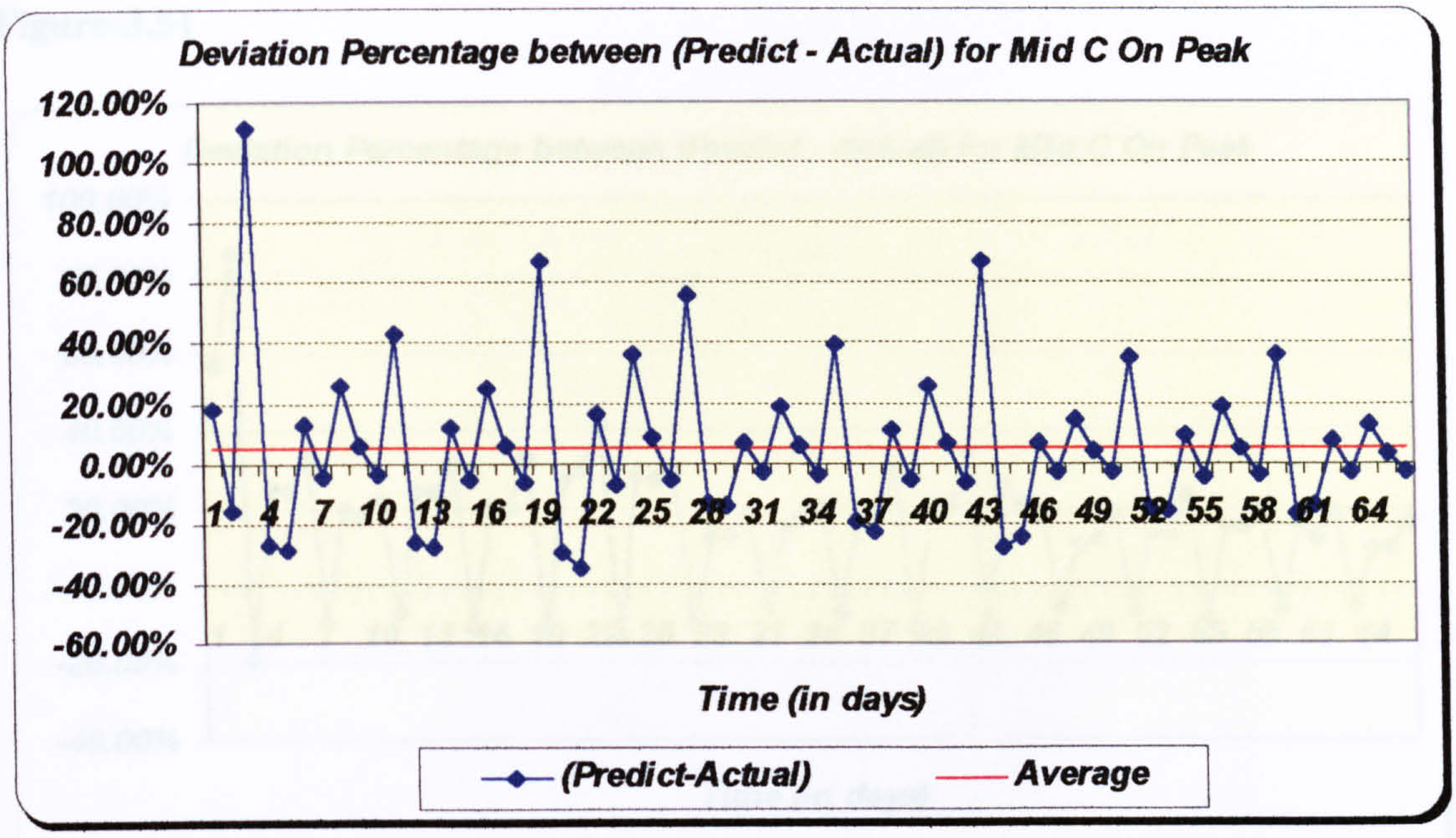


Figure-3.50

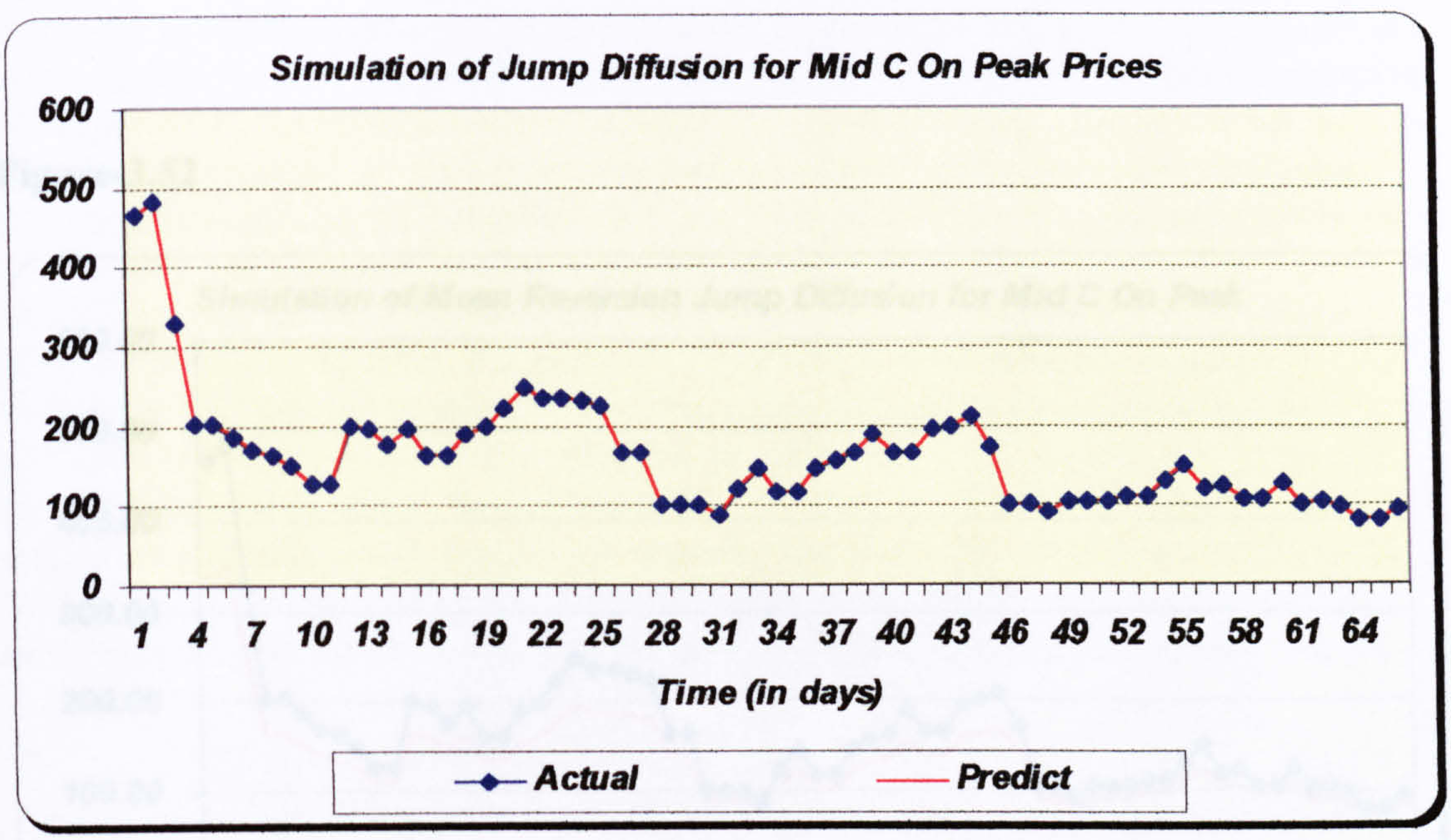


Figure-3.51

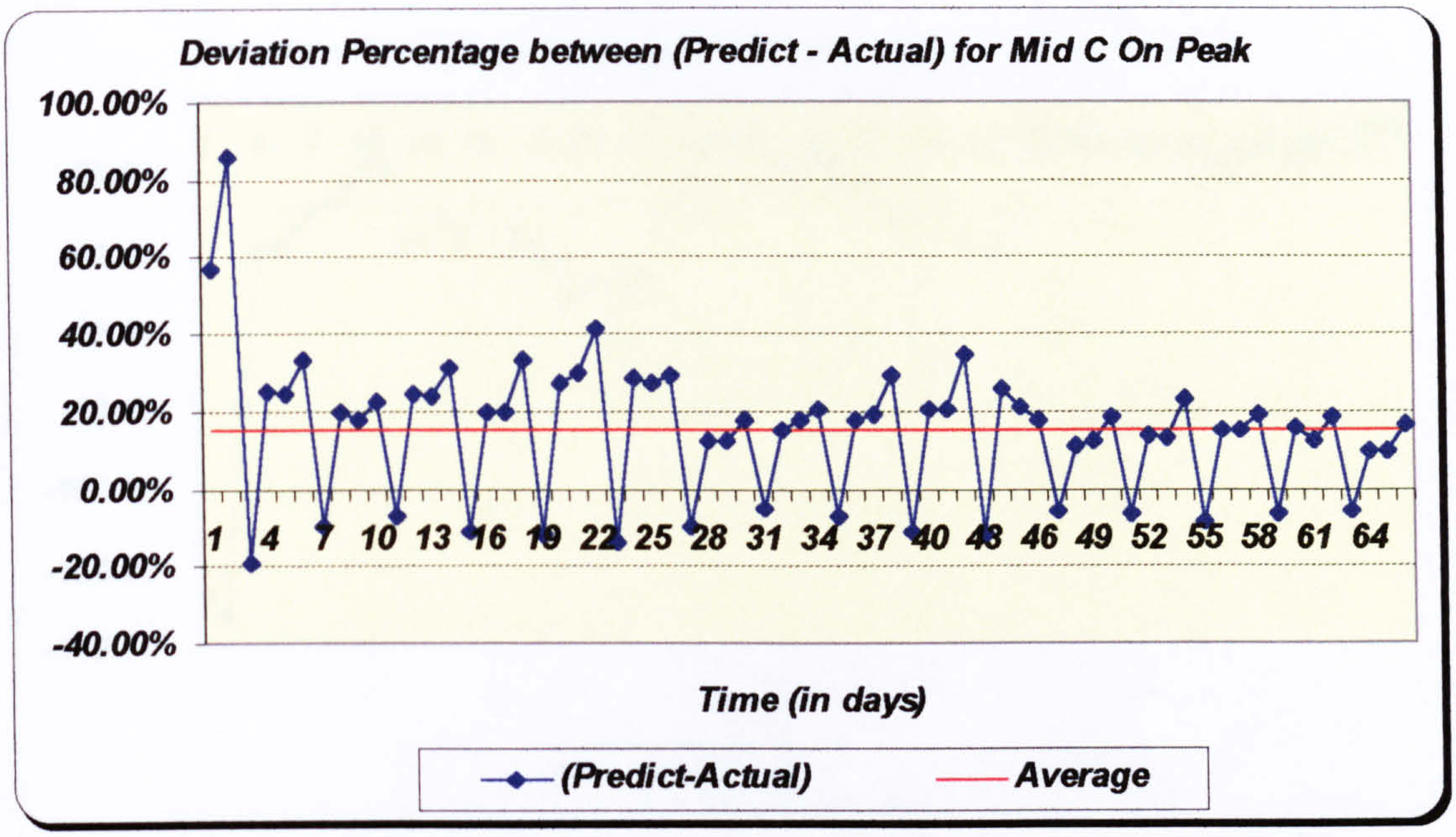


Figure-3.52

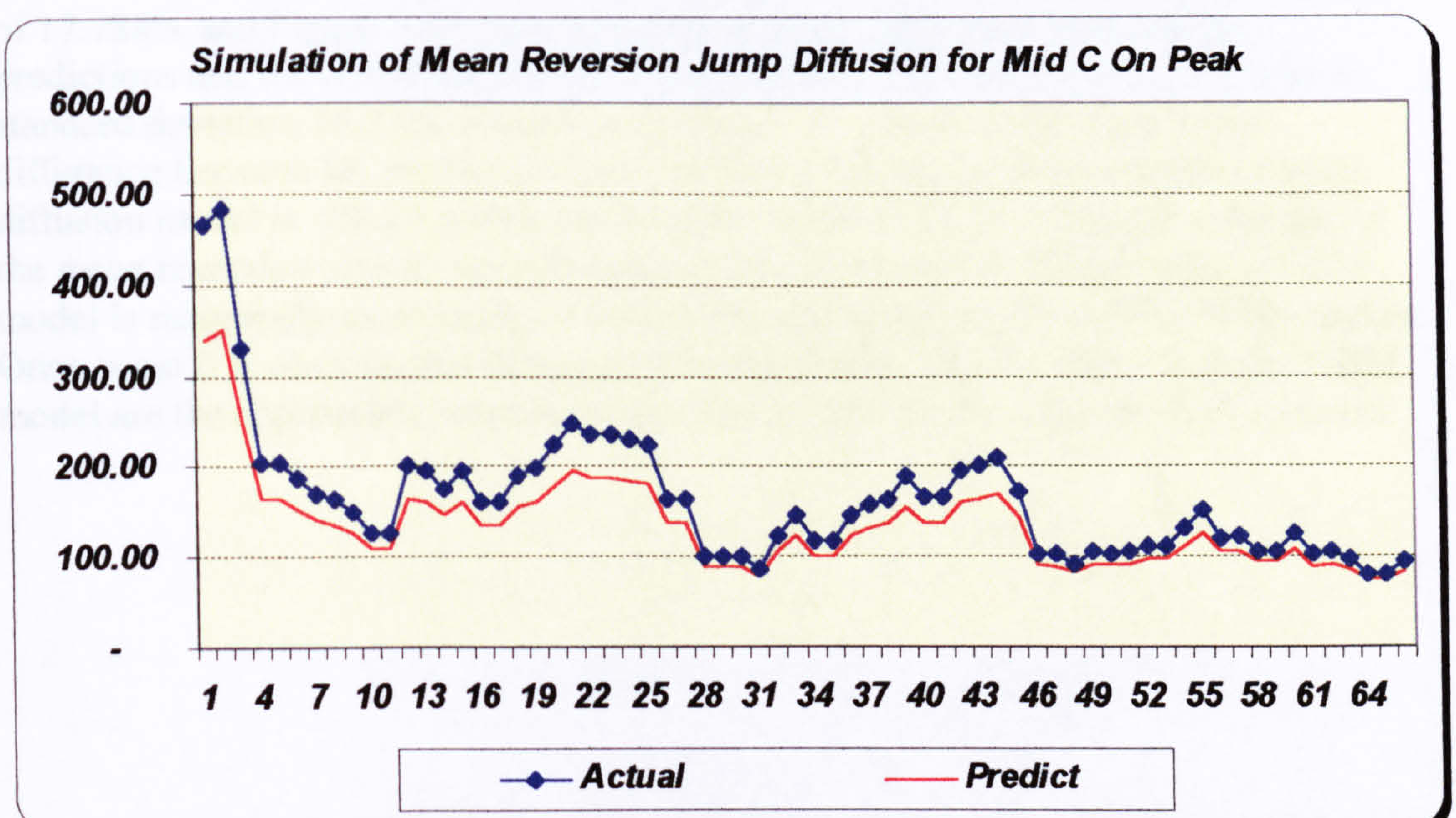
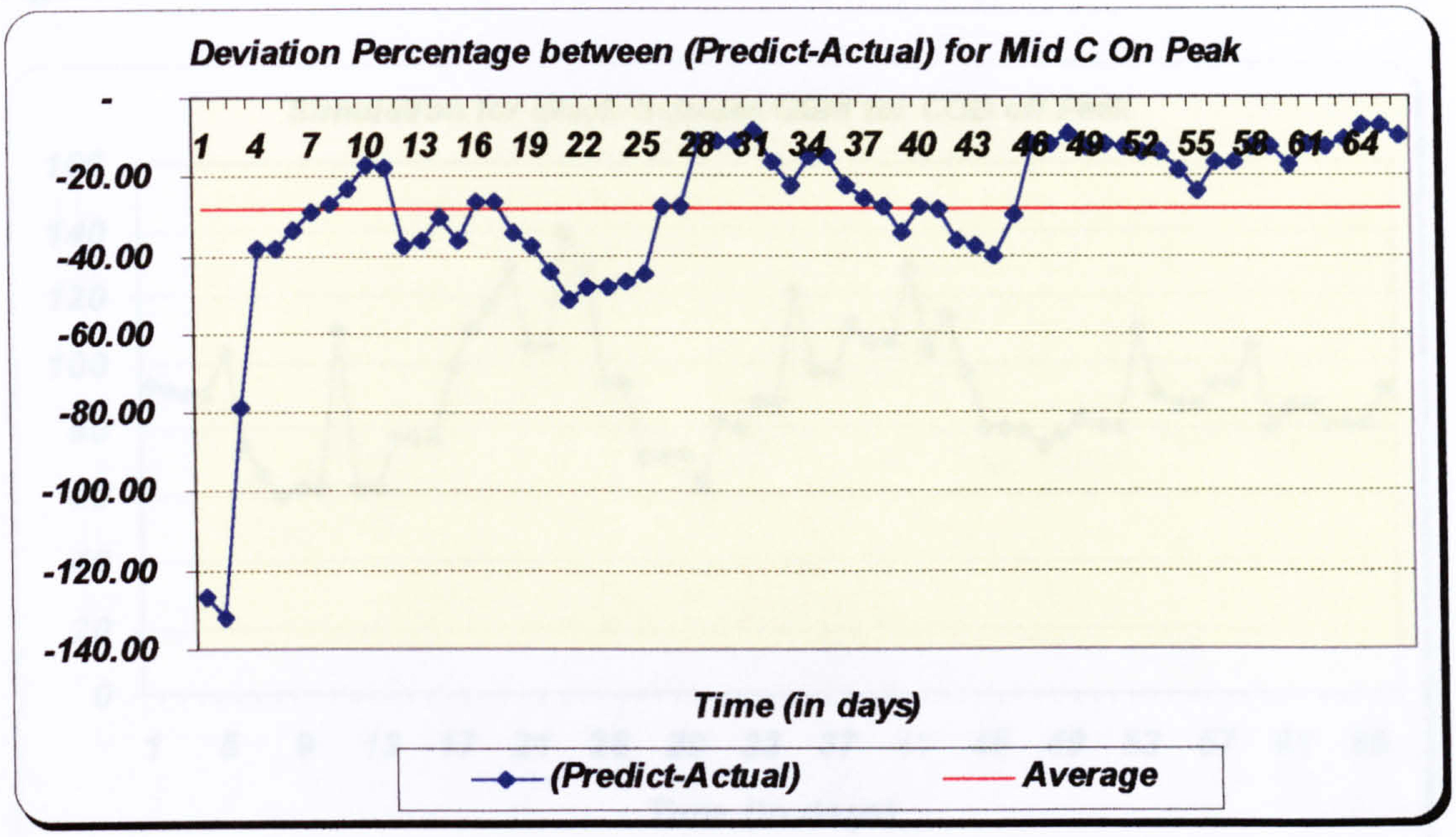


Figure-3.53



Similarly from the graphs above (Figure-3.48, Figure-3.50 & Figure-3.52) we can see with a naked eye that the Black-Scholes GBM model and the jump diffusion model is the appropriate model to describe the Mid C On Peak electricity market. If we look in more detail, Figure-3.51 shows that the average difference between the predictions and the actual data in the jump diffusion model is 15.244% with a standard deviation of 17.788% and Figure-3.49 shows that the average difference between the predictions and the actual data in the Black-Scholes GBM model is 5.05% with a standard deviation 26.03%. Furthermore Figure-3.53 shows that the average difference between the predictions and the actual data in the mean reversion jump diffusion model is –28.22 with a standard deviation of 22.67. The reason for that is the mean reversion rate α we estimated is very high. So, the Black-Scholes GBM model is marginally more accurate to describe the Mid C On Peak Electricity market. Once again it is obvious that the jump diffusion model and the Black-Scholes GBM model are the appropriate models to describe the Mid C On Peak electricity market.

COB Off Peak Prices

Figure-3.54

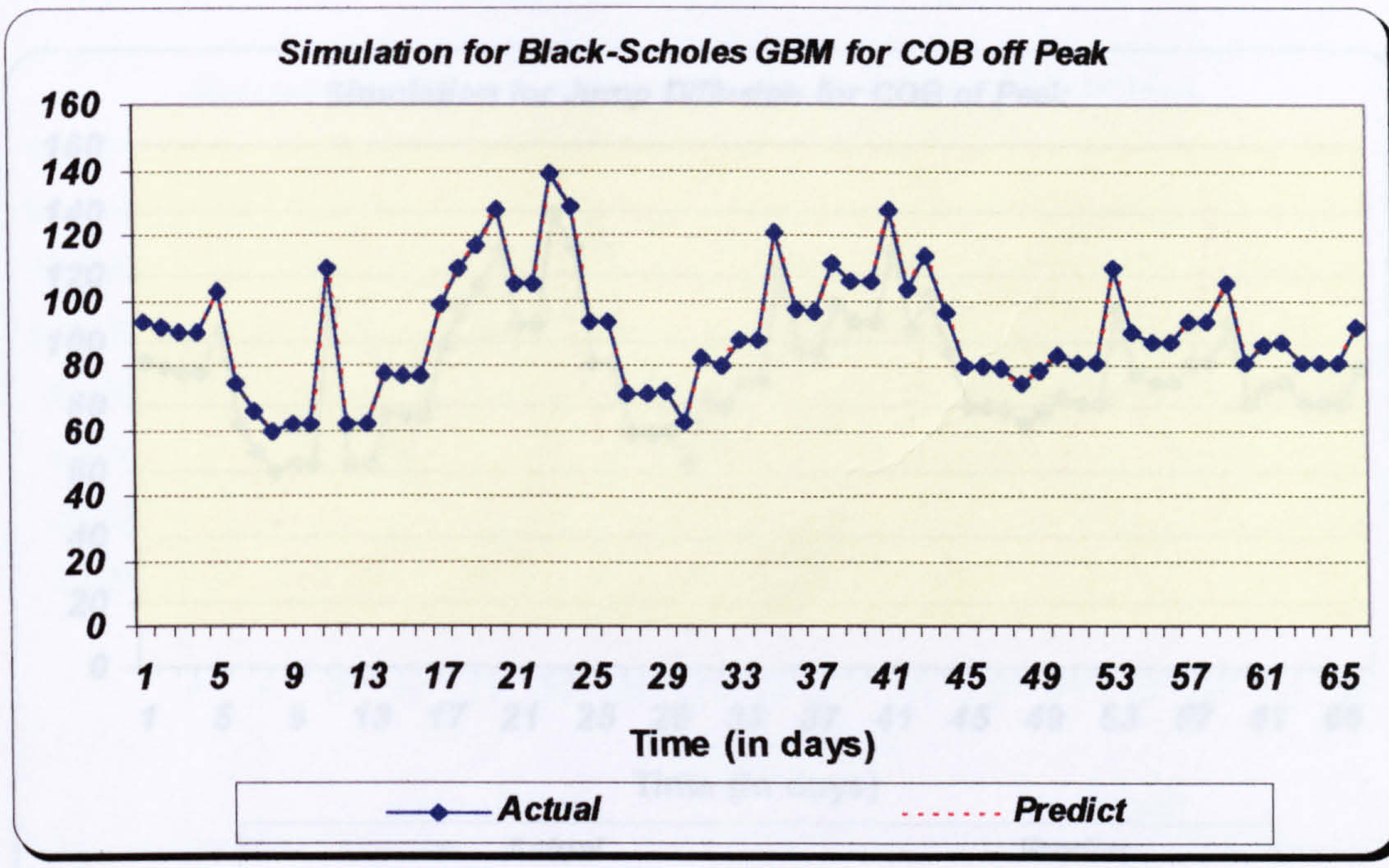


Figure-3.55

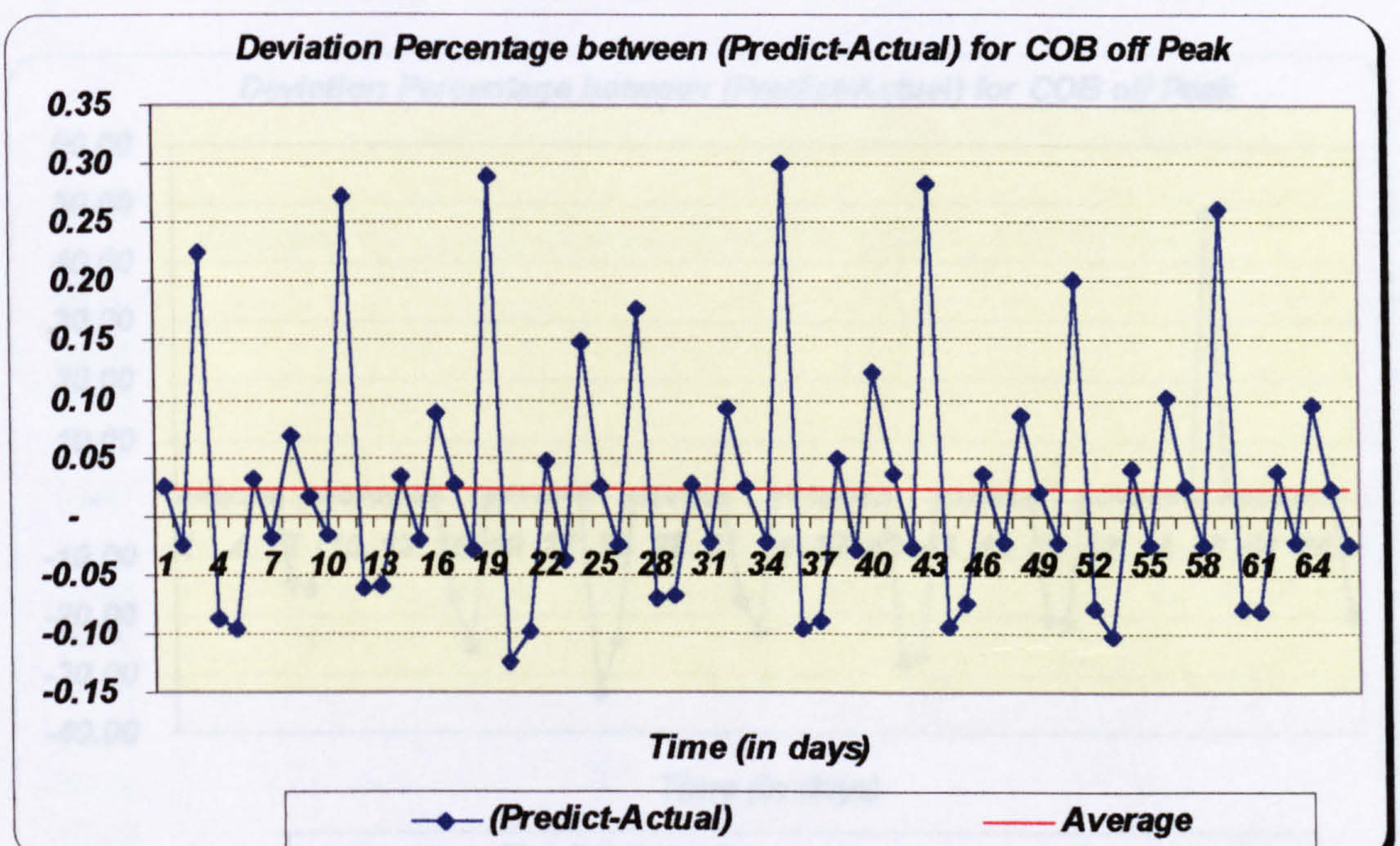


Figure-3.56

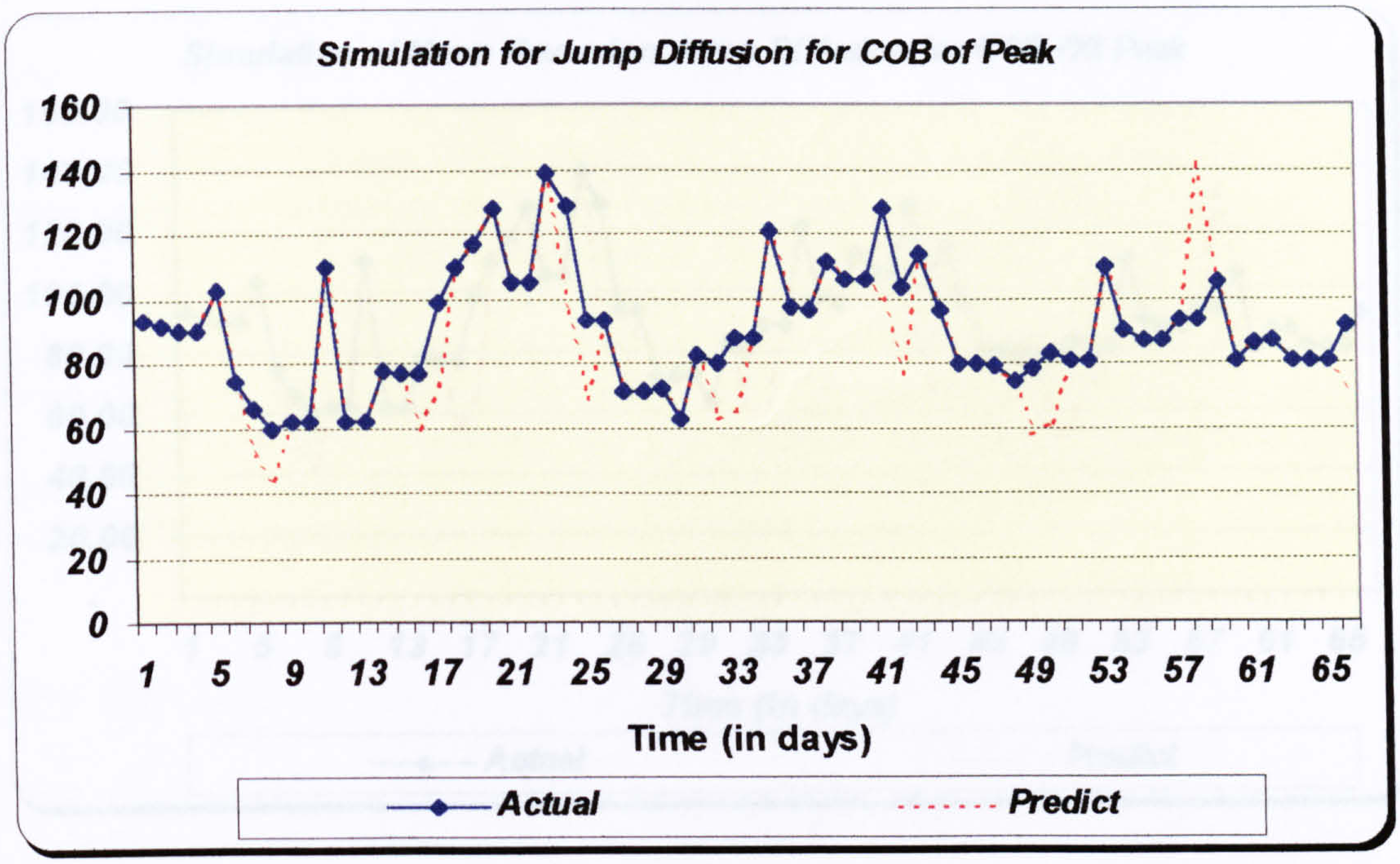


Figure-3.57

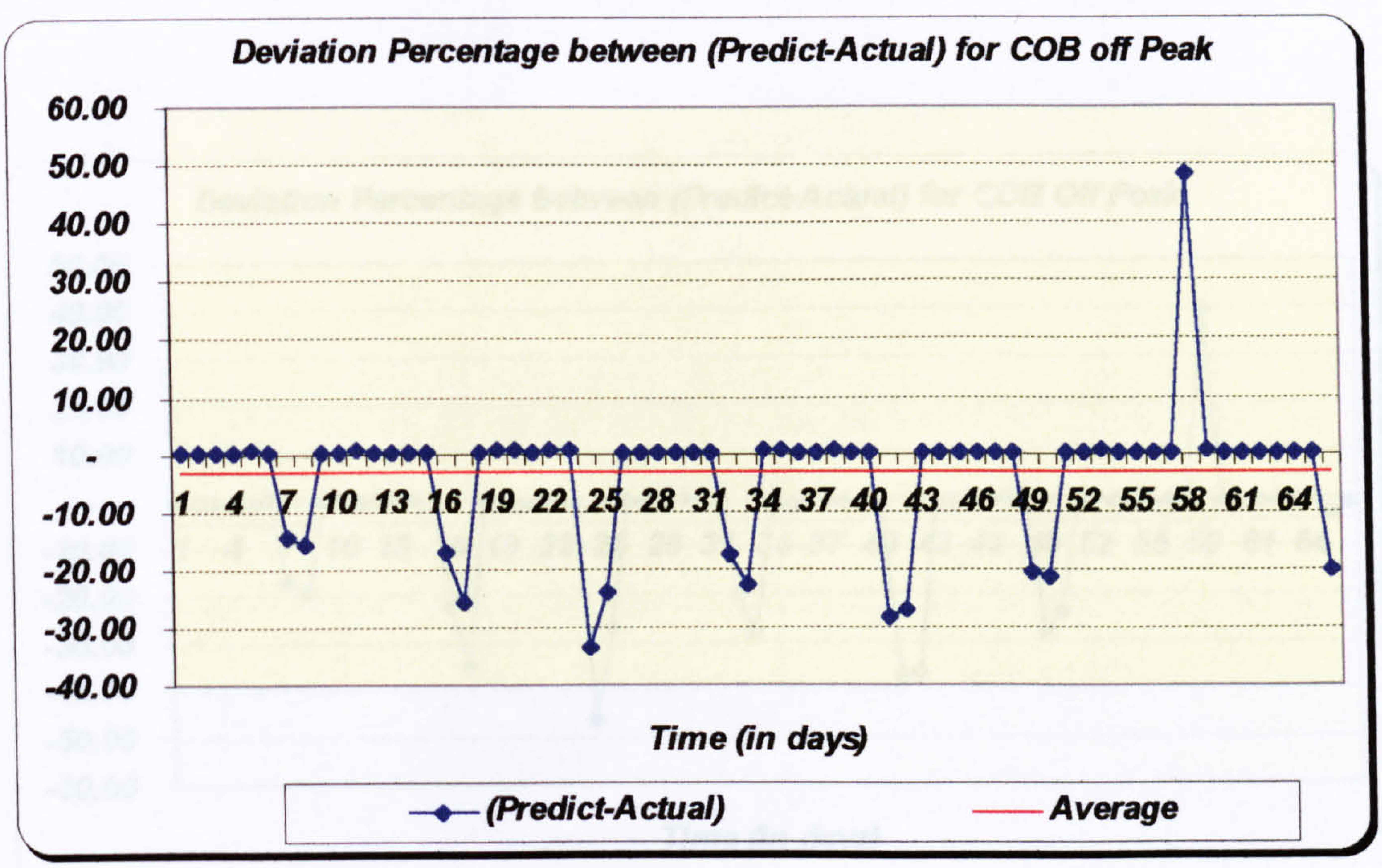


Figure-3.58

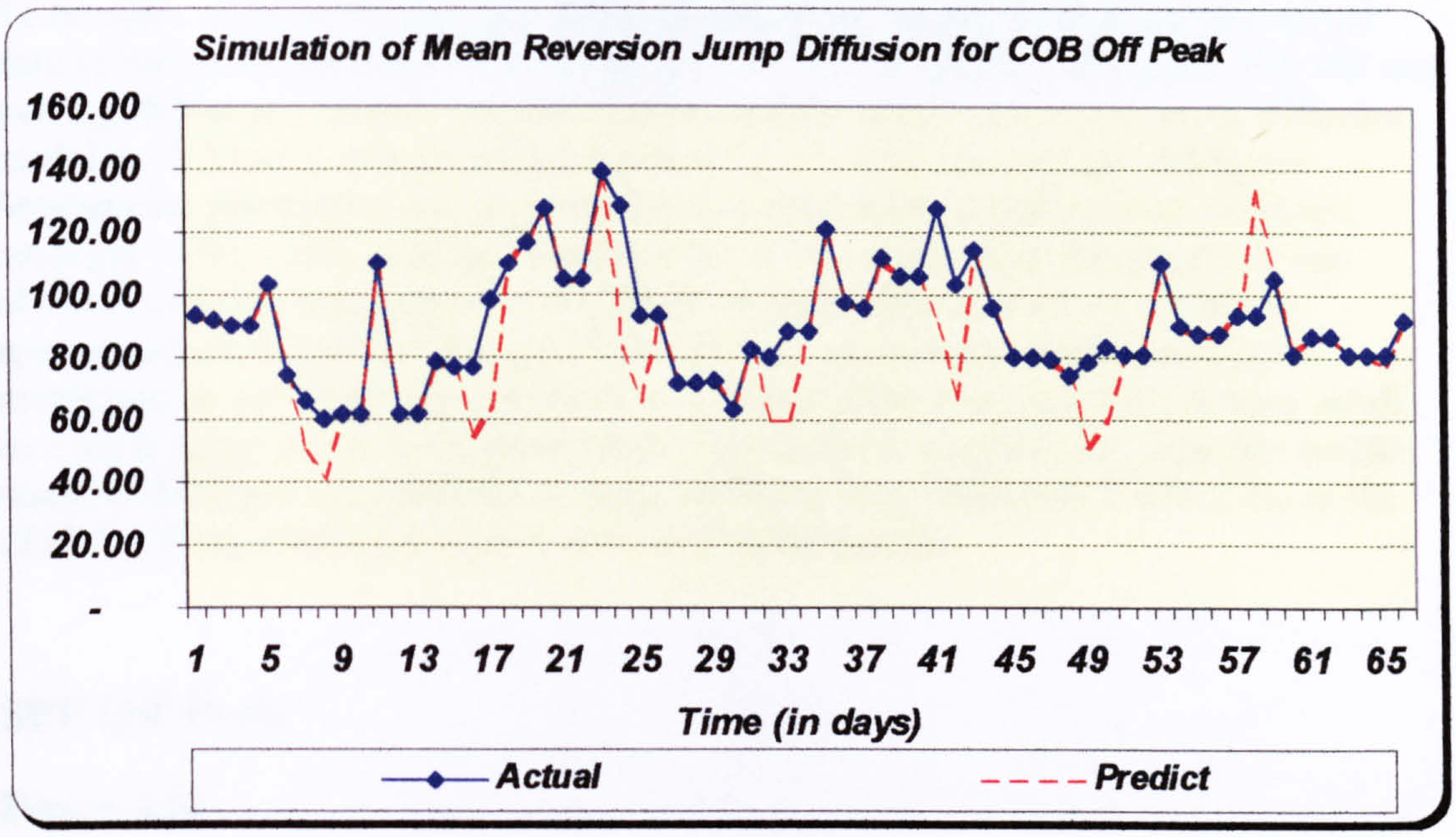
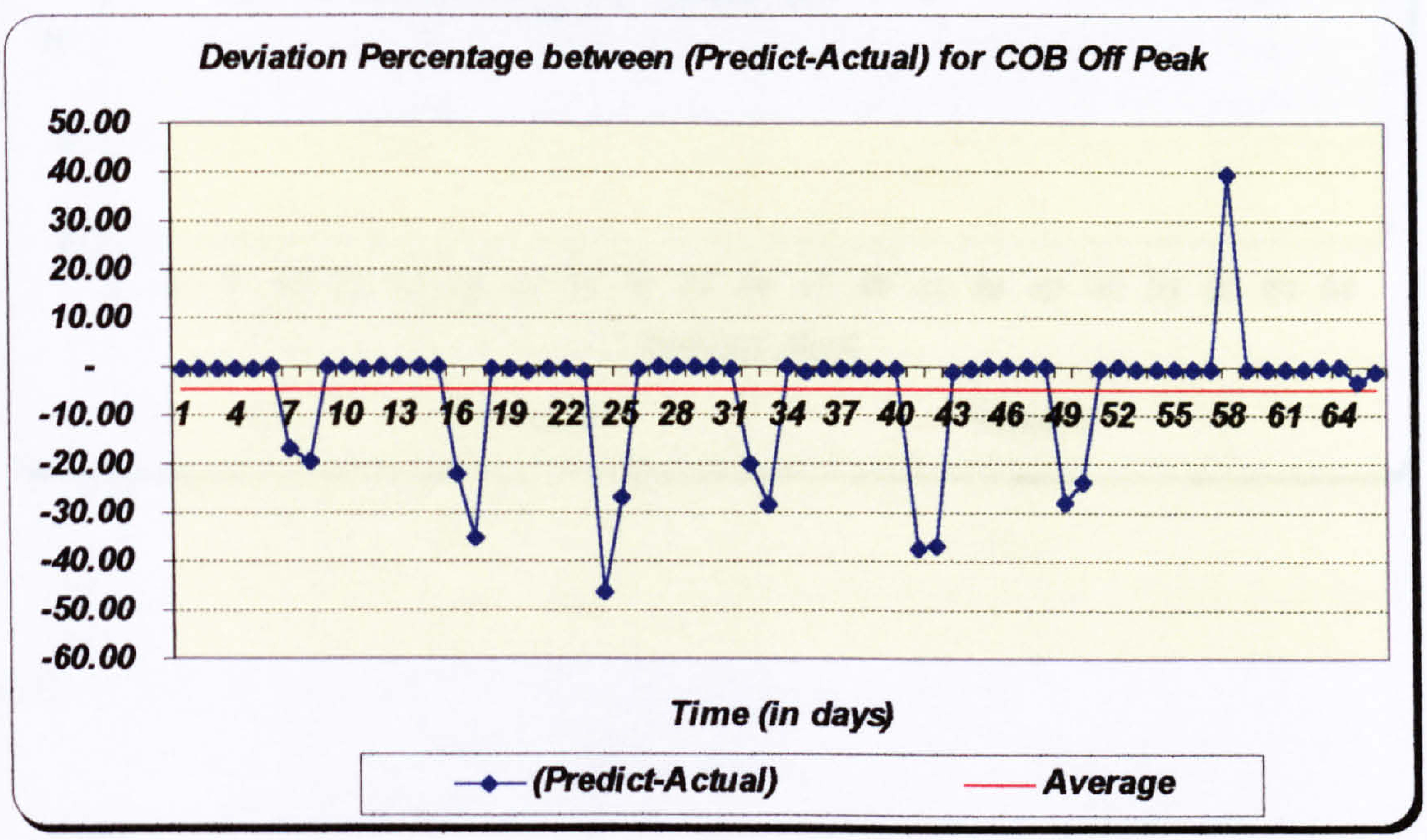


Figure-3.59



Similarly from the graphs above (Figure-3.54, Figure-3.56 & Figure-3.58) we can see with a naked eye that from the three the models the Black-Scholes GBM, is appropriate to describe the COB Off Peak electricity market. Even if we look in more detail, Figure-3.55, Figure-3.57& Figure-3.59 show us that the average difference between the predictions and the actual data is smaller in the Black-Scholes GBM model than in the jump diffusion model and the mean reversion jump diffusion model. To be more accurate the average difference between the predictions and the actual data in the Black-Scholes GBM model is 0.024 with a standard deviation of 0.105 and average difference between the predictions and the actual data in the jump diffusion model is –3.32 with a standard deviation of 11.33. Also the average difference between the predictions and the actual data in the mean reversion jump diffusion model is –4.93 with a standard deviation of 12.74. Apart from the results, it was obvious from the beginning that the Black-Scholes GBM model would be the appropriate model to describe the COB Off Peak electricity market because we mentioned at the beginning (see Section 3.5) that if the jump volatility is very small, so even if jumps are very frequent their size would be insignificant, then this would result a GBM process. Here the jump volatility is very small (see Table-3.2), so the COB Off Peak electricity market follows a GBM process.

SPP Off Peak

Figure-3.60

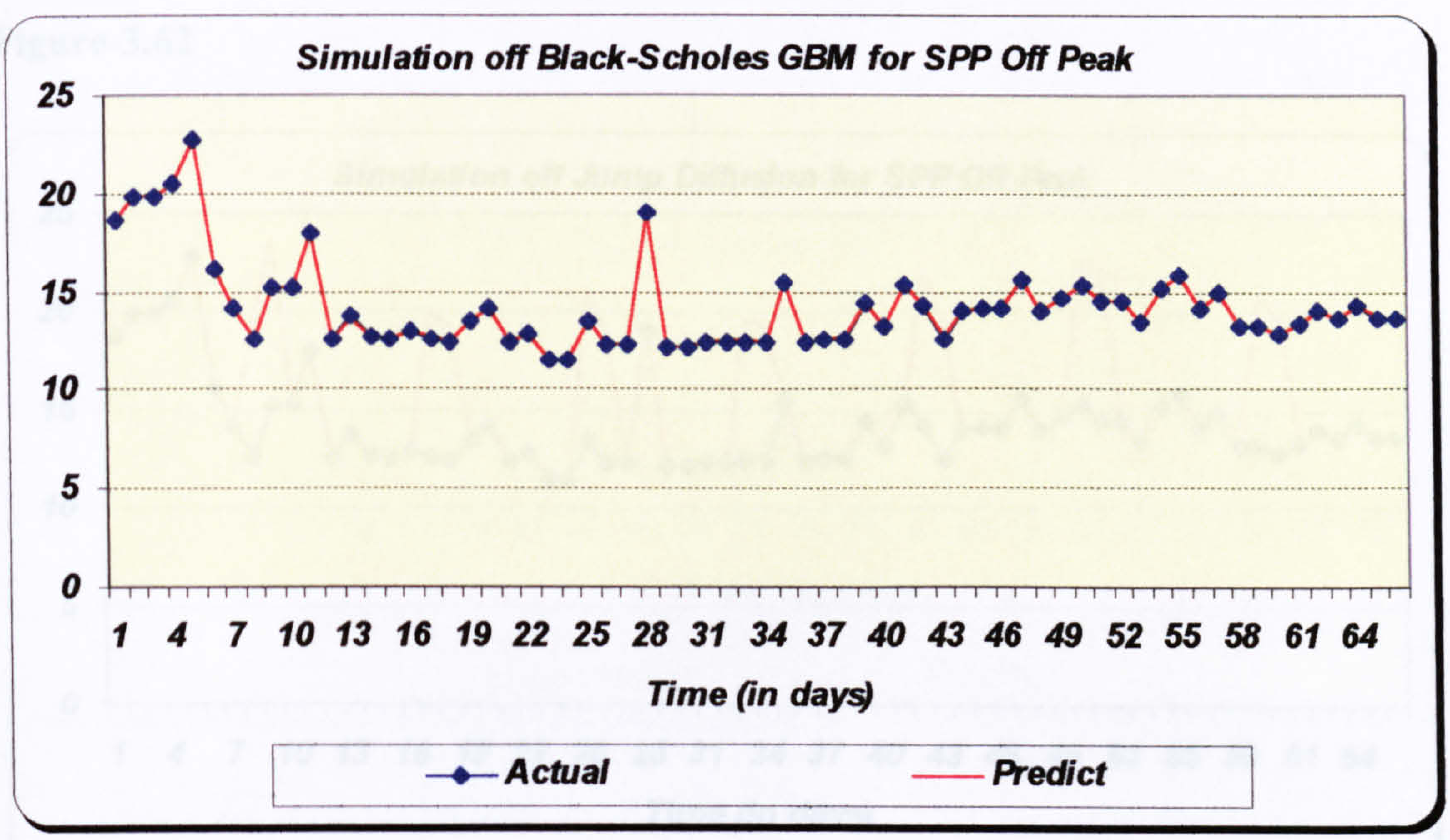


Figure-3.61

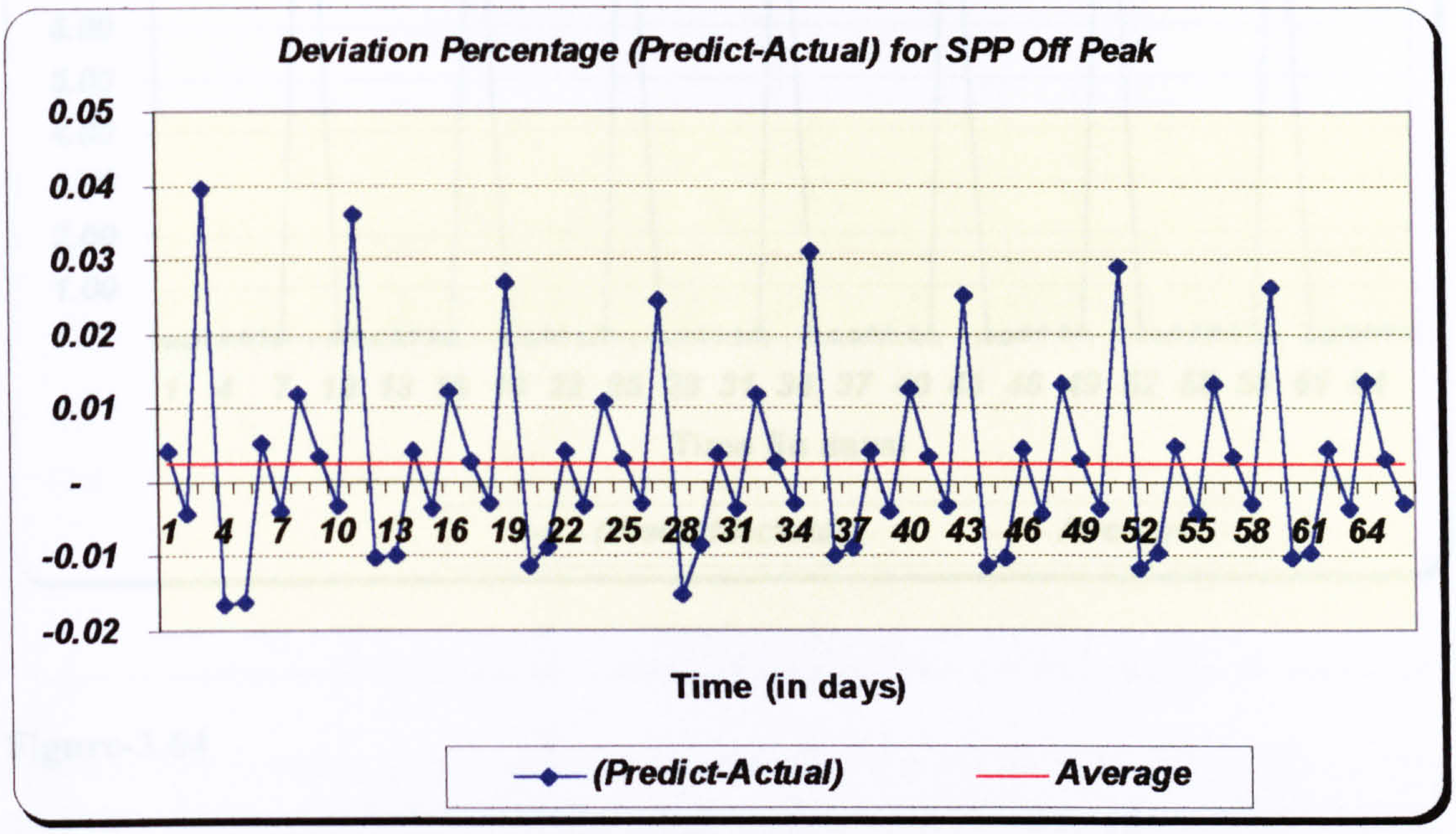


Figure-3.62

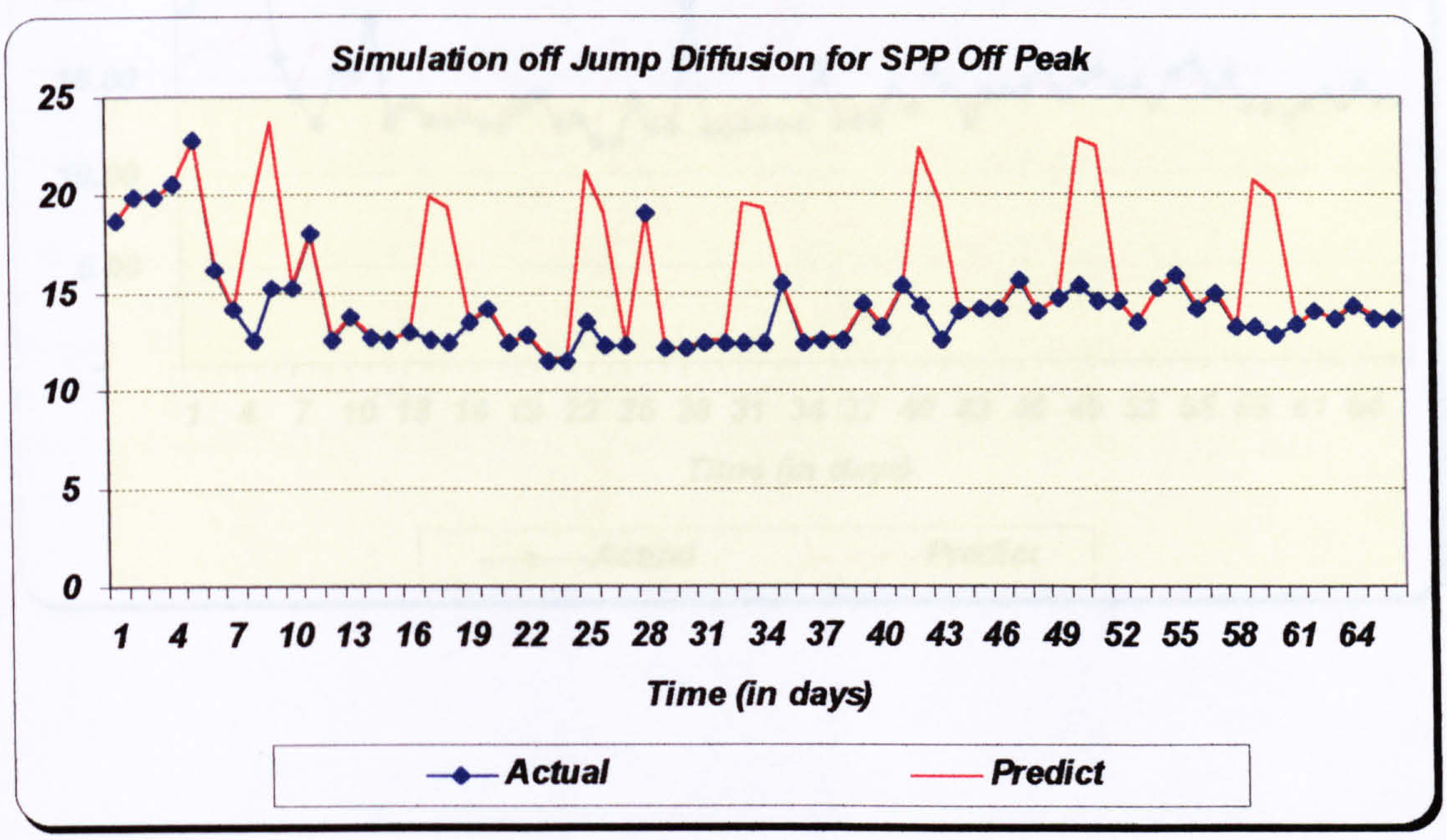


Figure-3.63

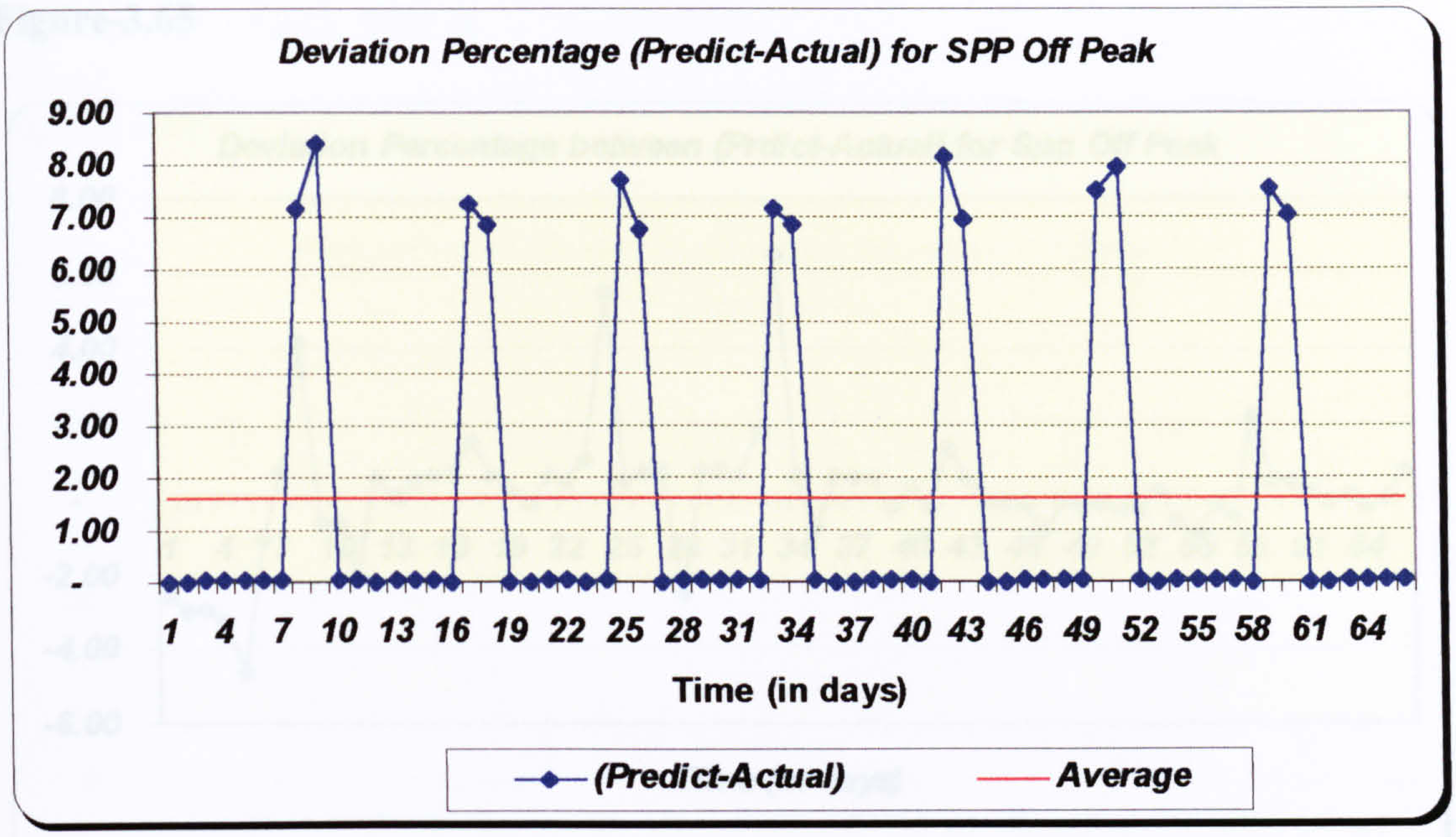

Deviation Percentage (Predict-Actual) for SPP Off Peak
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
-
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64
Time (in days)
(Predict-Actual)
Average

Figure-3.64

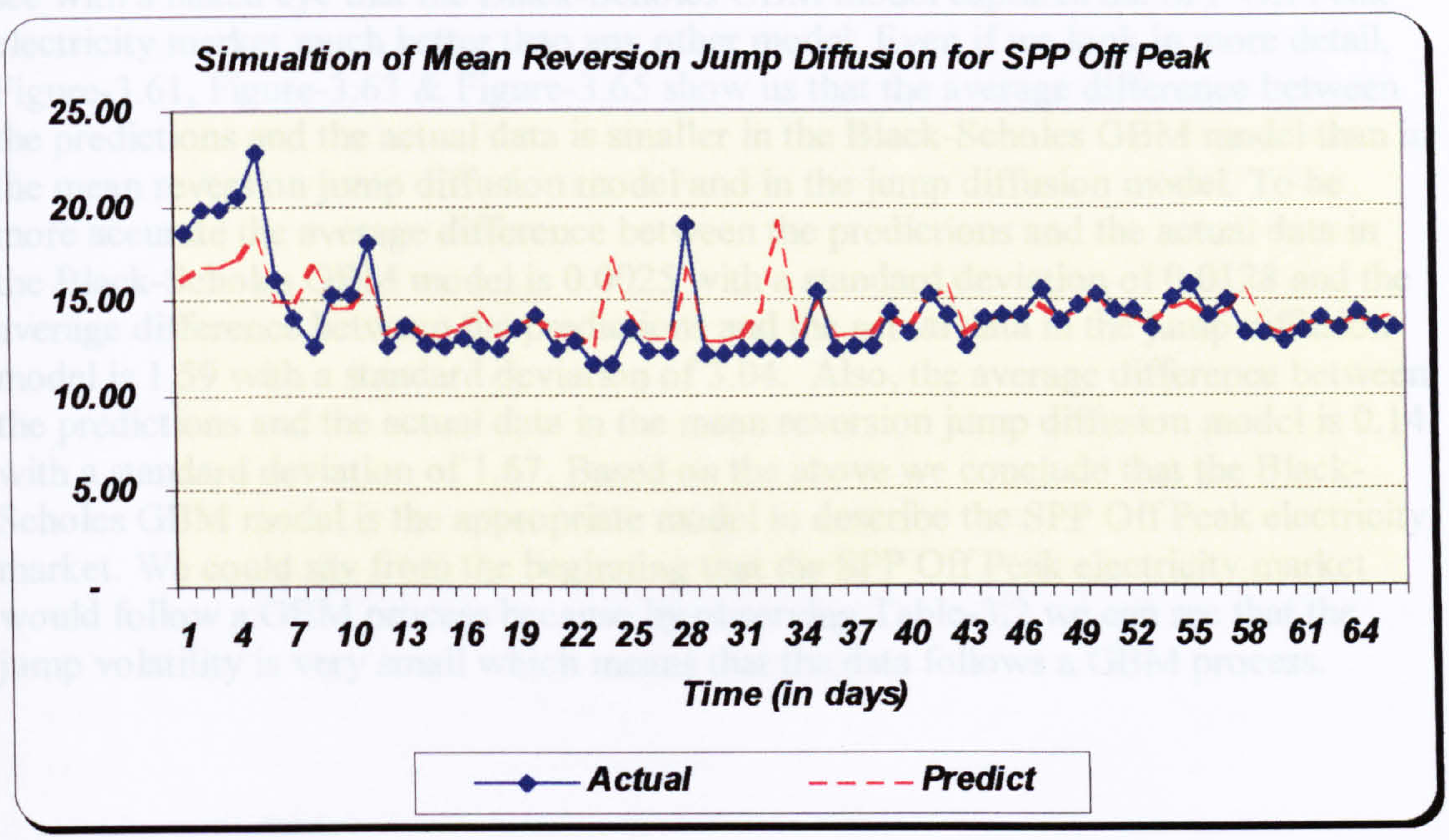
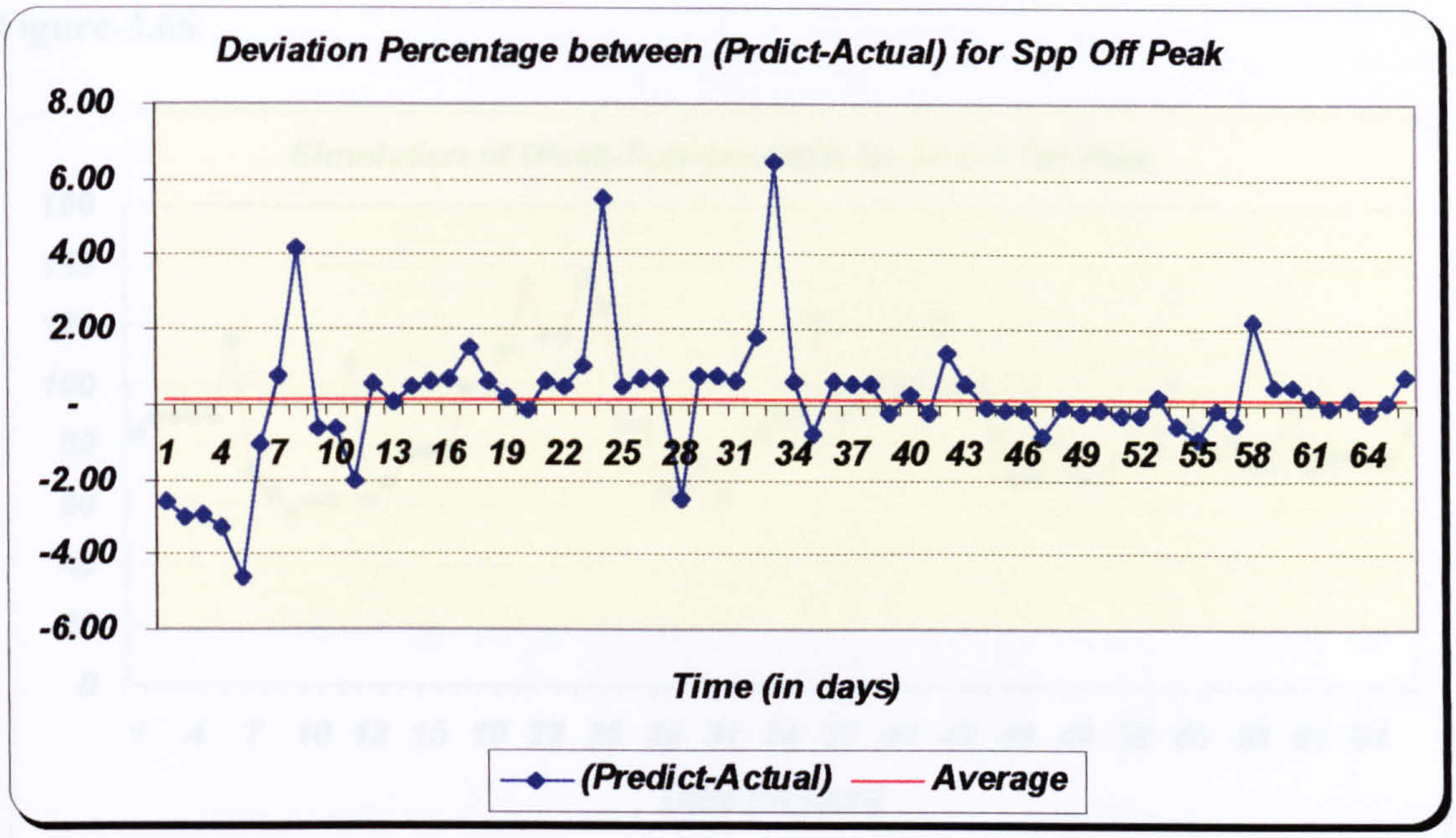

Simualtion of Mean Reversion Jump Diffusion for SPP Off Peak
25.00
20.00
15.00
10.00
5.00
-
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64
Time (in days)
Actual
Predict

Figure-3.65



Furthermore from the graphs above (Figure-3.60, Figure-3.62 & Figure-3.64) we can see with a naked eye that the Black-Scholes GBM model captures the SPP Off Peak electricity market much better than any other model. Even if we look in more detail, Figure-3.61, Figure-3.63 & Figure-3.65 show us that the average difference between the predictions and the actual data is smaller in the Black-Scholes GBM model than in the mean reversion jump diffusion model and in the jump diffusion model. To be more accurate the average difference between the predictions and the actual data in the Black-Scholes GBM model is 0.0025 with a standard deviation of 0.0128 and the average difference between the predictions and the actual data in the jump diffusion model is 1.59 with a standard deviation of 3.04. Also, the average difference between the predictions and the actual data in the mean reversion jump diffusion model is 0.14 with a standard deviation of 1.67. Based on the above we conclude that the Black-Scholes GBM model is the appropriate model to describe the SPP Off Peak electricity market. We could say from the beginning that the SPP Off Peak electricity market would follow a GBM process because by observing Table-3.2 we can see that the jump volatility is very small which means that the data follows a GBM process.

MID-C Off Peak Prices

Figure-3.66

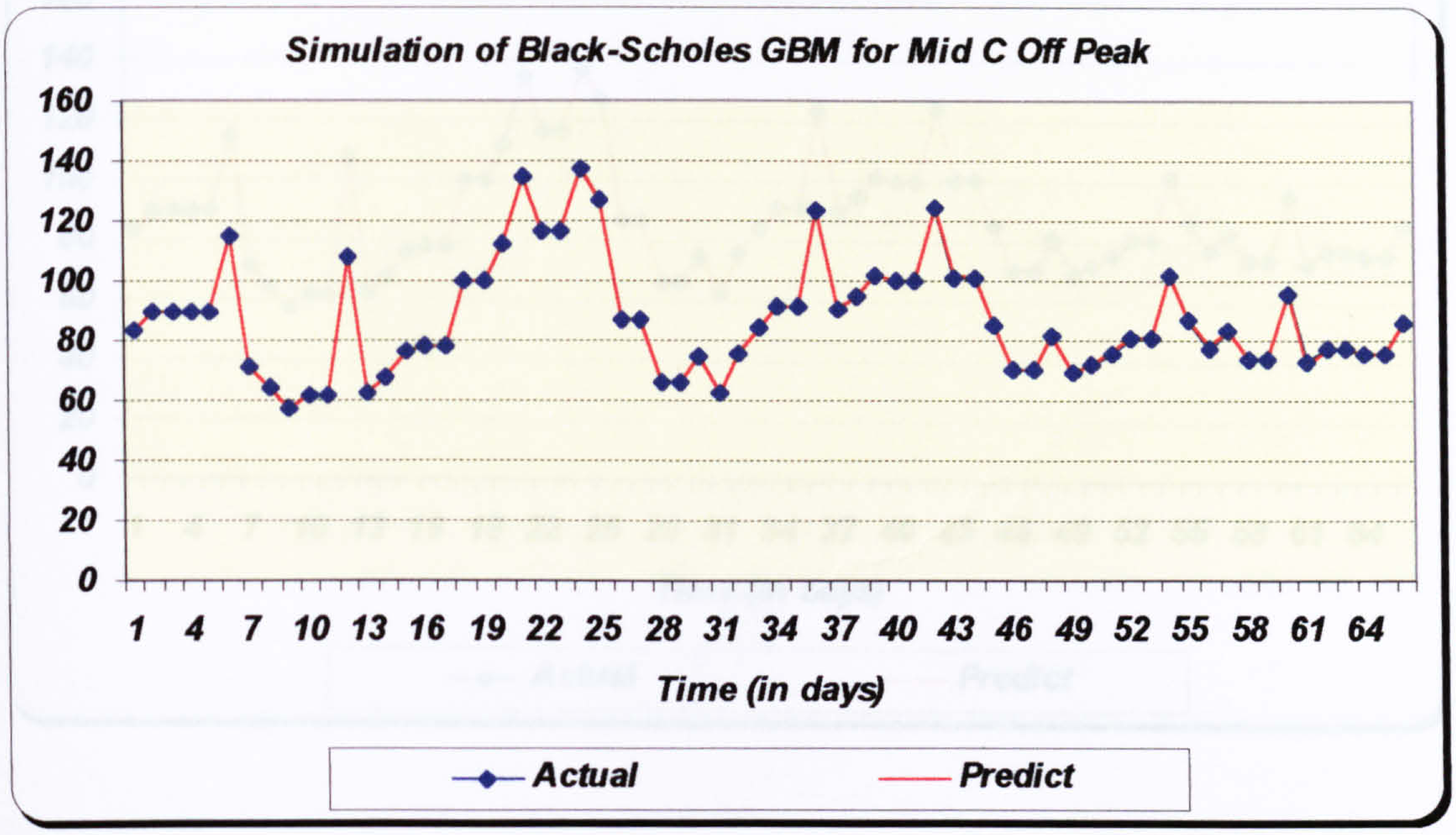


Figure-3.67

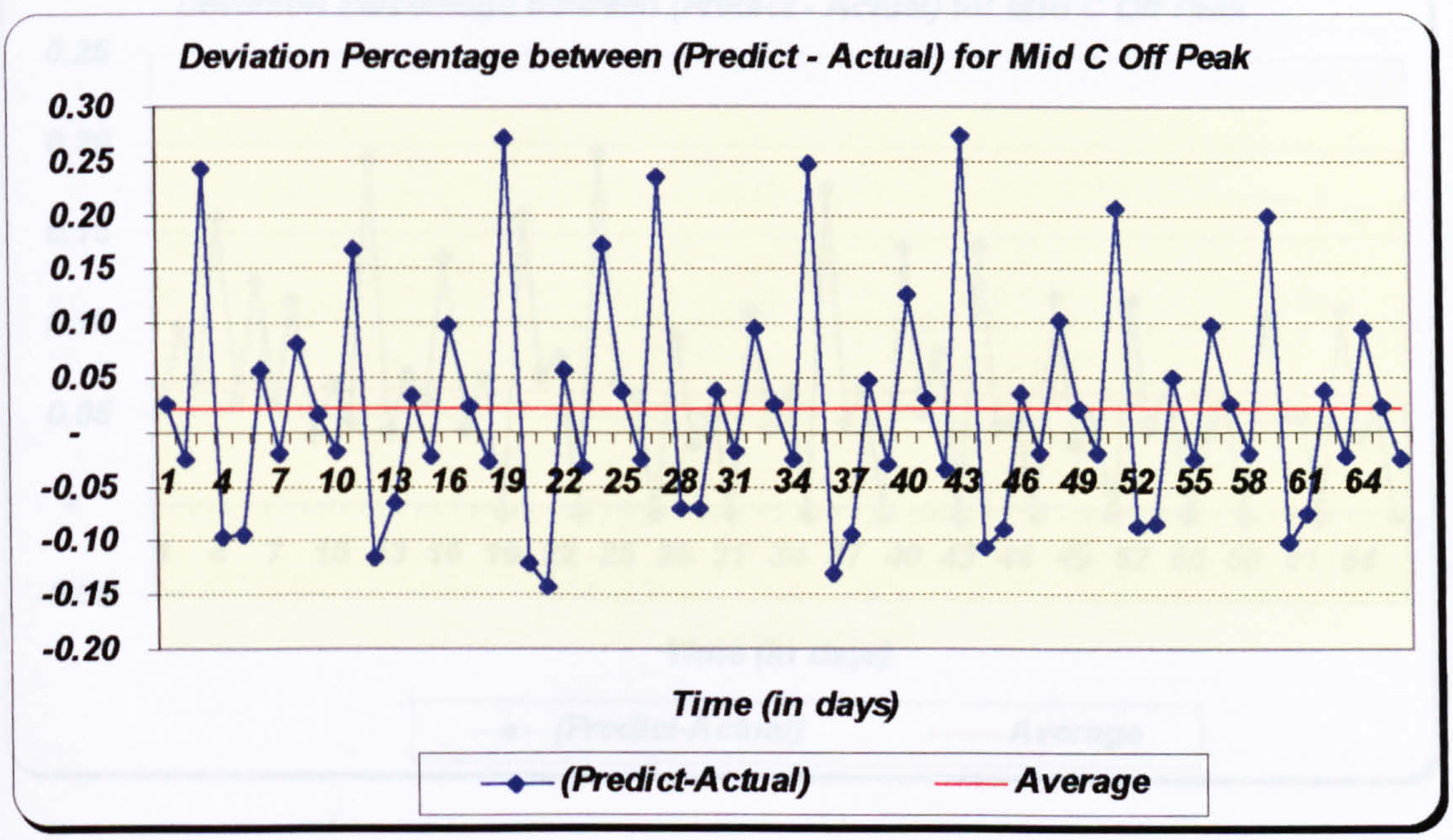


Figure-3.69

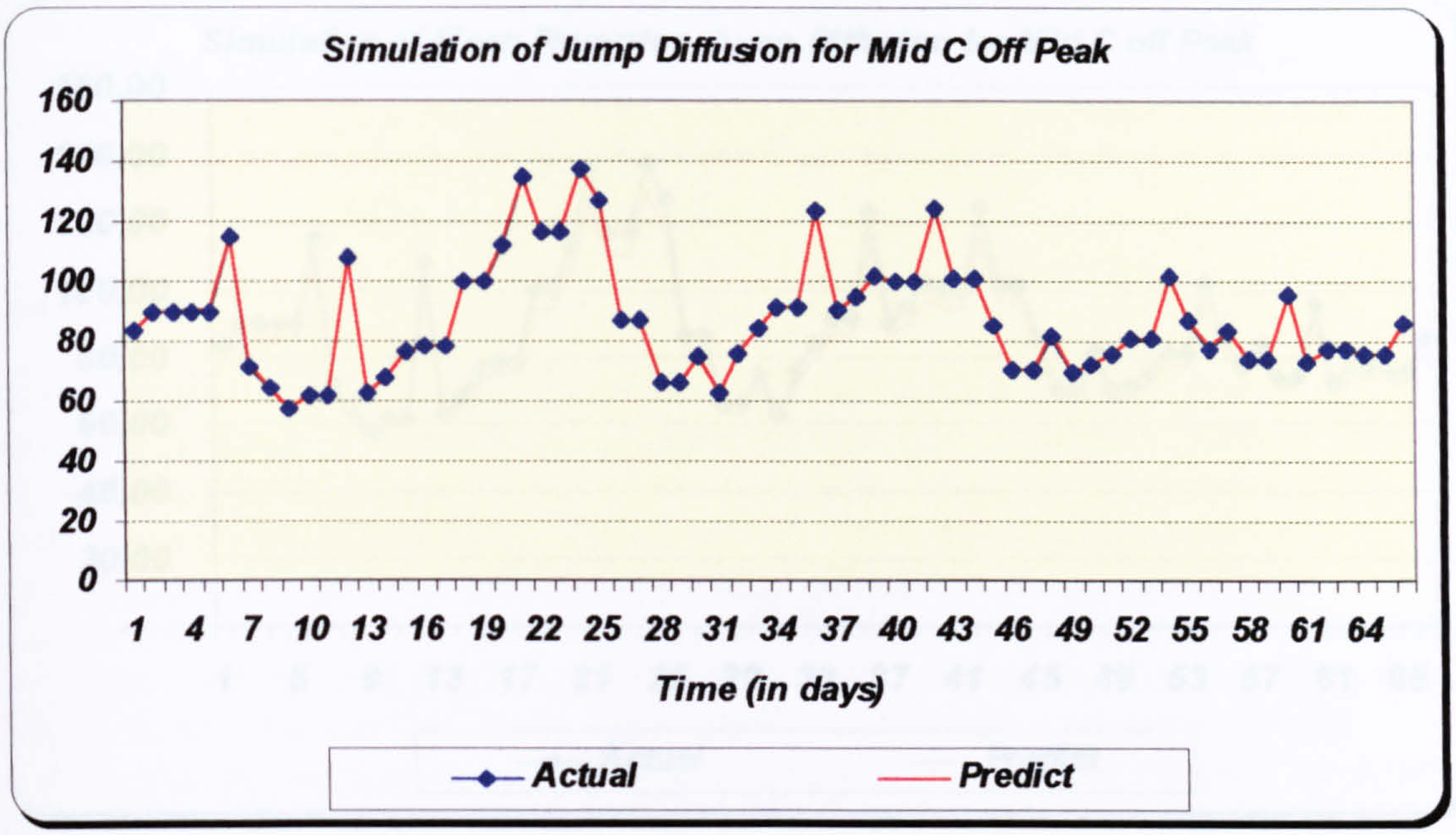


Figure-3.70

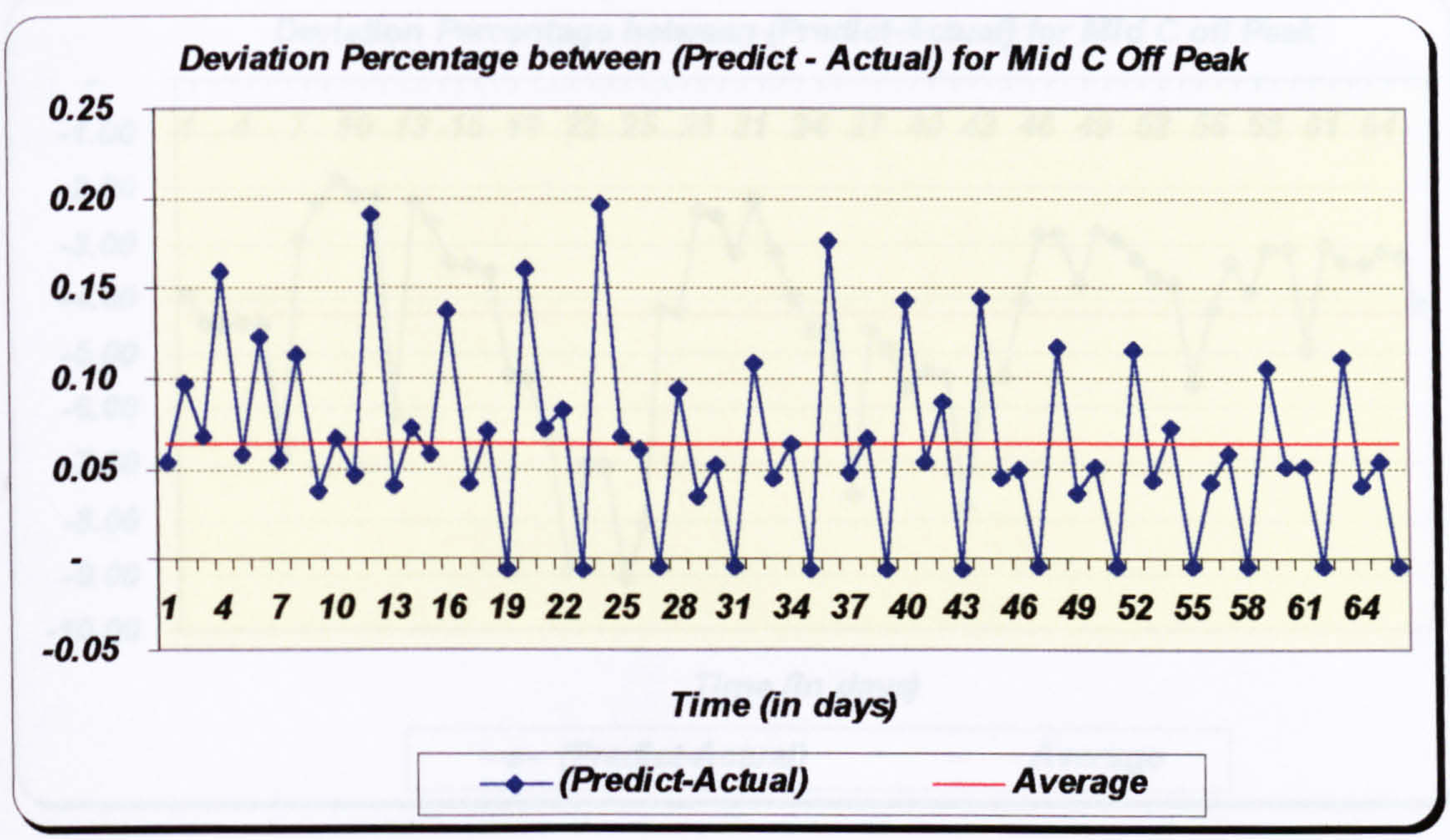


Figure-3.71

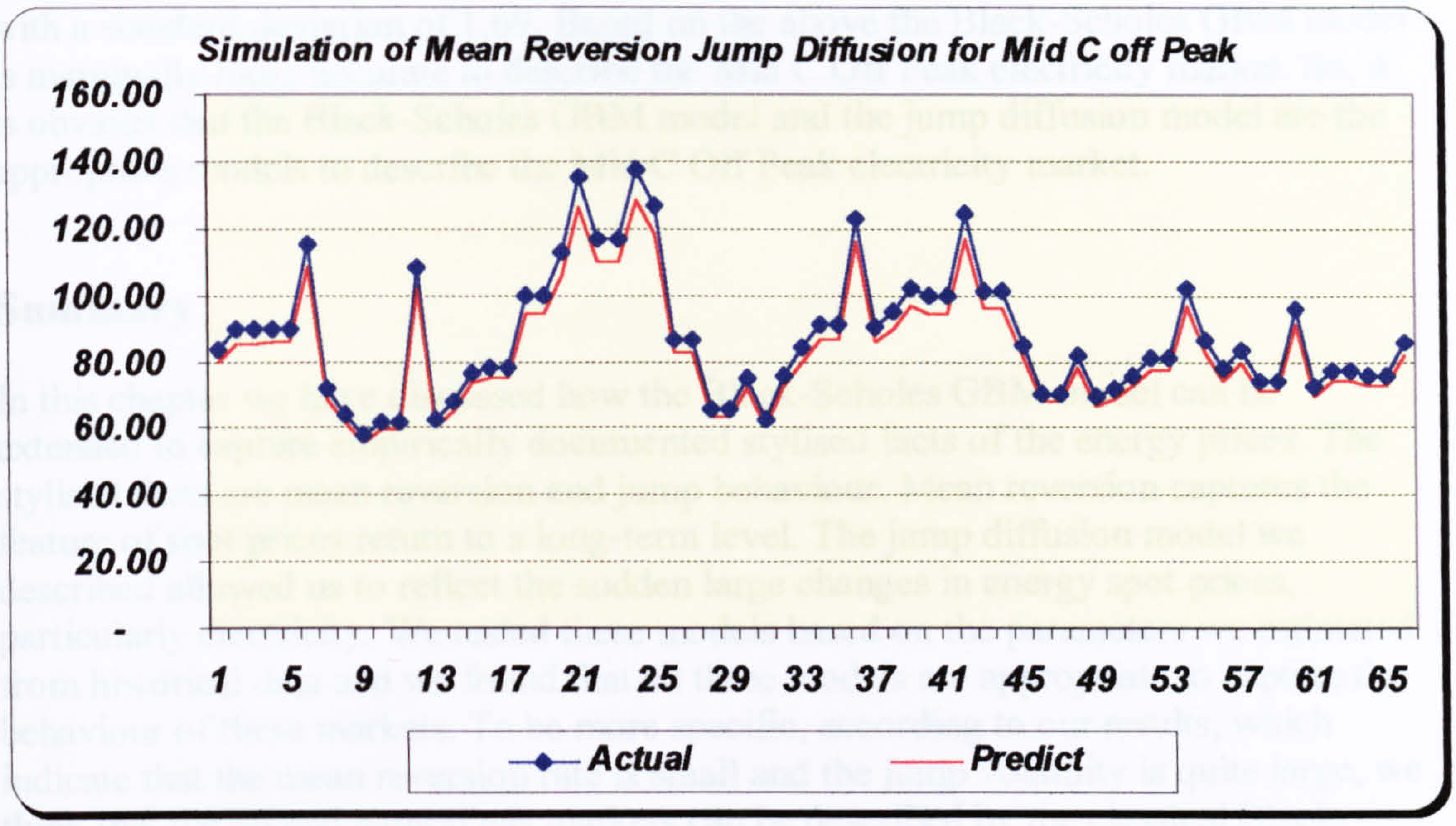
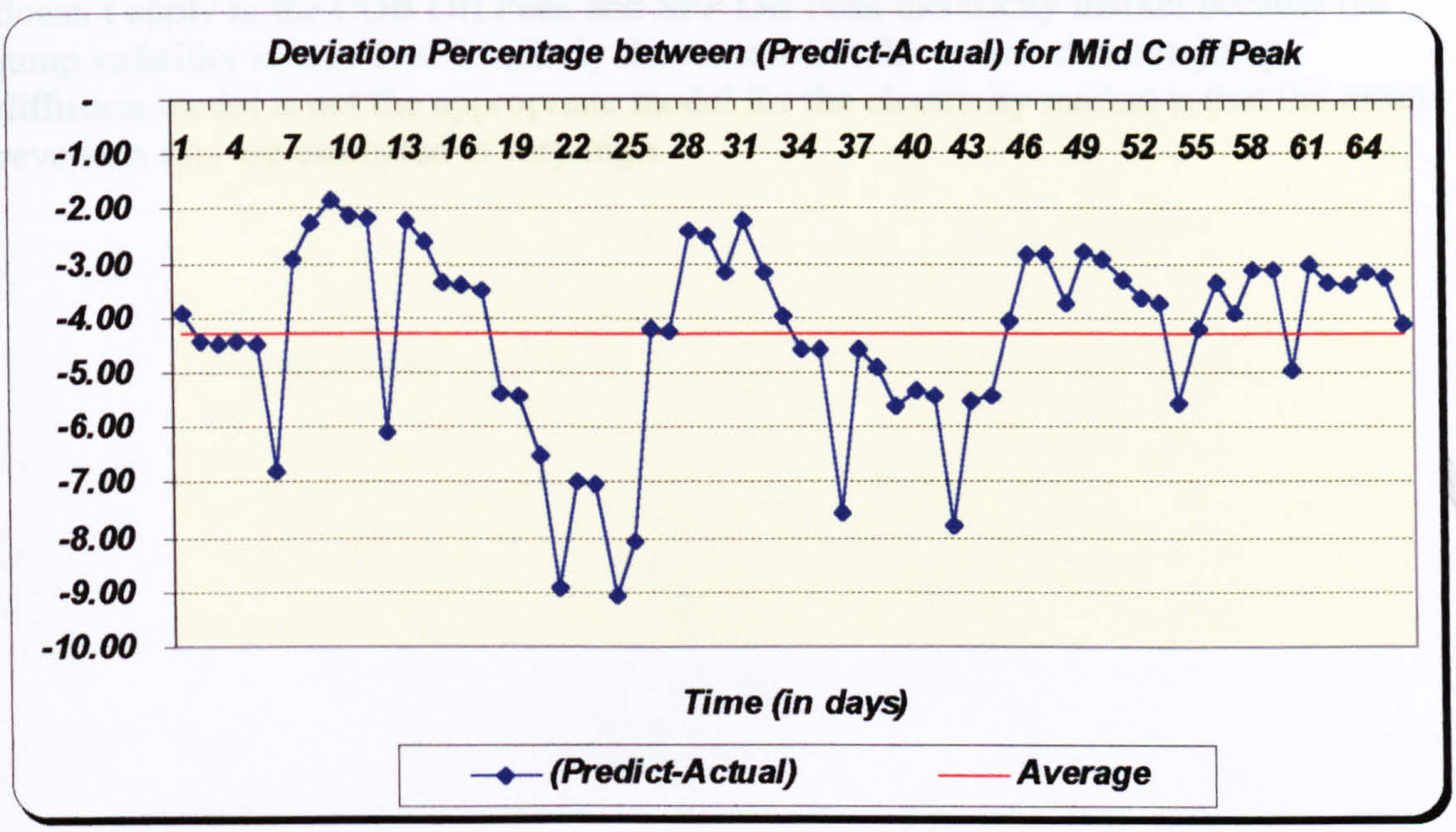


Figure-3.72



Similarly from the graphs above (Figure-3.66, Figure-3.68 & Figure-3.70) we can see with a naked eye that the Black-Scholes GBM model and the jump diffusion model is the appropriate model to describe the Mid C Off Peak electricity market. If we look in more detail, Figure-3.67 shows that the average difference between the predictions and the actual data in the Black-Scholes GBM model is 0.0204 with a standard deviation of 0.1033,Figure-3.69 shows that the average difference between the predictions and the actual data in the jump diffusion model is 0.0643 with a standard deviation of 0.0514 and Figure-3.71 shows that the average difference between the predictions and the actual data in the mean reversion jump diffusion model is –4.30 with a standard deviation of 1.69. Based on the above the Black-Scholes GBM model is marginally more accurate to describe the Mid C Off Peak electricity market. So, it is obvious that the Black-Scholes GBM model and the jump diffusion model are the appropriate models to describe the Mid C Off Peak electricity market.

Summary

In this chapter we have discussed how the Black-Scholes GBM model can be extended to capture empirically documented stylised facts of the energy prices. The stylised facts are mean reversion and jump behaviour. Mean reversion captures the feature of spot prices return to a long-term level. The jump diffusion model we described allowed us to reflect the sudden large changes in energy spot prices, particularly electricity. We tested these models based on the parameters we estimated from historical data and we found that all three models are appropriate to capture the behaviour of these markets. To be more specific, according to our results, which indicate that the mean reversion rate is small and the jump volatility is quite large, we think that the oil and natural gas markets can be described by the classical Black-Scholes GBM as well as by the jump diffusion and the mean reversion jump diffusion models. In the electricity market we have noticed that the jump behaviour and the Black-Scholes is the dominant empirical characteristic. The jump diffusion model doesn't apply in the COB Off Peak and SPP Off Peak electricity market because the jump volatility is very low. Similarly the reason that the mean reversion jump diffusion model is not the appropriate model for the electricity market is that the mean reversion rate we estimated is very high.

Appendix-3.1

The computer algorithm for the jump-diffusion model (equation-3.10) is the following:

```
Public Sub JumpDiffusionOptionModel()

Dim p, r, f, sig, k, y, T, M, n, Z, PutCall, nodays
Dim diff, drift, dt, lnp, sump, test, test2, rand1, rand2, rand3, jump, CT, sum_CT
Dim i, j, pcf, b As Integer

nodays = (Range("F").End(xlDown).Row) - 1

For b = 1 To nodays

p = Cells(1 + b, 1).Value
Z = Cells(1 + b, 2).Value
PutCall = Cells(1 + b, 3).Value
r = Cells(1 + b, 4).Value
sig = Cells(1 + b, 5).Value
f = Cells(1 + b, 6).Value
k = Cells(1 + b, 7).Value
y = Cells(1 + b, 8).Value
T = Cells(1 + b, 9).Value
M = Cells(1 + b, 10).Value
n = Cells(1 + b, 11).Value

dt = T / n

drift = (r - f * k - 0.5 * sig ^ 2) * dt

diff = sig * Sqr(dt)

sump = 0
CT = 0
sum_CT = 0

For j = 1 To M

lnp = Log(p)

   For i = 1 To n

   test = Rnd

         If test = 0 Then
         test = test + 0.0000001
         End If
```

```
        rand1 = Application.NormSInv(test)

    lnp = lnp + drift + diff * rand1


        rand2 = Rnd

        If rand2 < (f * dt) Then

            test2 = Rnd

            If test2 = 0 Then
            test2 = test2 + 0.0000001
            End If

        rand3 = Application.NormSInv(test2)

        jump = k + (y * rand3)

        lnp = lnp + jump

            End If


    Next i

    If LCase(PutCall) = "call" Then

    pcf = 1

    End If

    If LCase(PutCall) = "put" Then

    pcf = -1

    End If

    sump = sump + Exp(lnp)
    CT = Application.Max(pcf * (Exp(lnp) - Z), 0)
    sum_CT = sum_CT + CT

Next j


Cells(1 + b, 12).Value = Exp(-r * T) * (sump / M)
Cells(1 + b, 13).Value = Exp(-r * T) * (sum_CT / M)

Next b
```

```
End Sub
```

Appendix-3.2

The computer algorithm for the mean-reversion-jump-diffusion model (equation-3.13) is the following:

```
Public Sub MeanReversionJumpDiffusionOptionModel()

Dim p, r, f, sig, k, y, T, M, n, a, u, Z, PutCall, nodays, test6
Dim diff, drift, dt, lnp, sump, test, test2, rand1, rand2, rand3, jump, CT, sum_CT
Dim i, j, pcf, b As Integer

nodays = (Range("F").End(xlDown).Row) - 1

For b = 1 To nodays

p = Cells(b + 1, 1).Value
Z = Cells(b + 1, 2).Value
PutCall = Cells(b + 1, 3).Value
a = Cells(b + 1, 4).Value
u = Cells(b + 1, 5).Value
r = Cells(b + 1, 6).Value
sig = Cells(b + 1, 7).Value
f = Cells(b + 1, 8).Value
k = Cells(b + 1, 9).Value
y = Cells(b + 1, 10).Value
T = Cells(b + 1, 11).Value
M = Cells(b + 1, 12).Value
n = Cells(b + 1, 13).Value


dt = T / n

diff = sig * Sqr(dt)

sump = 0
CT = 0
sum_CT = 0

For j = 1 To M

lnp = Log(p)

   For i = 1 To n

drift = (a * (u - lnp) - 0.5 * sig ^ 2) * dt
```

```
test = Rnd

    If test = 0 Then
    test = test + 0.0000001
    End If

    rand1 = Application.NormSInv(test)

lnp = lnp + drift + diff * rand1



    rand2 = Rnd

    If rand2 < (f * dt) Then


       test2 = Rnd

       If test2 = 0 Then

       test2 = test2 + 0.0000001
       End If

     rand3 = Application.NormSInv(test2)

     jump = k + (y * rand3)

     lnp = lnp + jump

       End If



'Cells(1 + i + j, 17 + (8 * (b - 1))).Value = lnp
'Cells(1 + i + j, 18 + (8 * (b - 1))).Value = drift
'Cells(1 + i + j, 19 + (8 * (b - 1))).Value = diff
'Cells(1 + i + j, 20 + (8 * (b - 1))).Value = rand1
'Cells(1 + i + j, 21 + (8 * (b - 1))).Value = k
'Cells(1 + i + j, 22 + (8 * (b - 1))).Value = y
'Cells(1 + i + j, 23 + (8 * (b - 1))).Value = rand3
'Cells(1 + i + j, 24 + (8 * (b - 1))).Value = Exp(lnp)
Next i

If LCase(PutCall) = "call" Then

pcf = 1

End If
```

```
   If LCase(PutCall) = "put" Then

   pcf = -1

   End If


   sump = sump + Exp(lnp)
   CT = Application.Max(pcf * (Exp(lnp) - Z), 0)
   sum_CT = sum_CT + CT
'Cells(1 + j, 17 + b).Value = Exp(lnp)
Next j



Cells(1 + b, 14).Value = Exp(-r * T) * (sump / M)
Cells(1 + b, 15).Value = Exp(-r * T) * (sum_CT / M)

Next b

End Sub
```

CHAPTER 4
PRICING OIL DERIVATIVES BASED UPON THE FORWARD CURVE APPROACH

4.1 Introduction

For derivative pricing, many industry participants require the forward curve to be an input into the derivative price model, rather than an output from it, as is the case with the constant parameter versions of the spot price models in chapter 3. In this chapter we show how a multi-factor forward model can be developed in stages and we then apply it to the oil market. We show how the multi-factor model can be calibrated to market observable data, and derive formulae for pricing of standard energy derivatives. We also discuss how the model can be adapted to handle seasonal volatilities and volatility smiles.

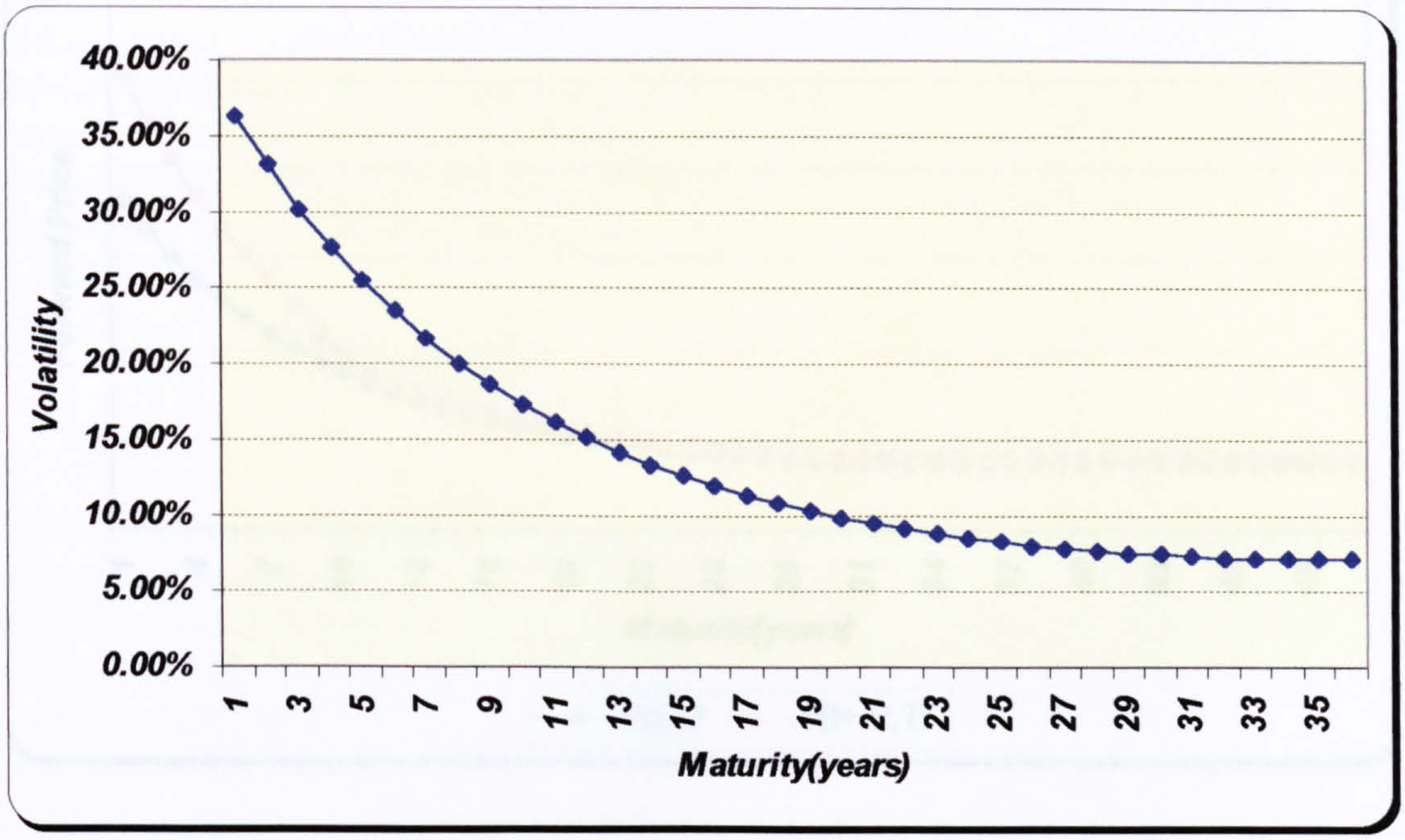
4.2 A Simple Model for the Forward Curve

A simple single factor model of the forward curve can be represented by the following stochastic differential equation:

$$\frac{dF(t,T)}{F(t,T)} = \sigma e^{-\alpha(T-t)} dz(t) \tag{4.1}$$

The inputs to the model are: the observed forward curve $F(t,T)$ which denotes the forward price at time t for maturity date T, and $\sigma e^{-\alpha(T-t)}$ which is the single factor or volatility function associated with a source of risk $dz(t)$. Notice that equation (4.1) has no drift term because futures and forwards contracts have zero initial investment, their expected return in a risk-neutral world must be zero, implying that the process describing their evolution has zero drift. The volatility function of equation (4.1) has a very simple negative exponential form, which we illustrate in Figure-4.1.

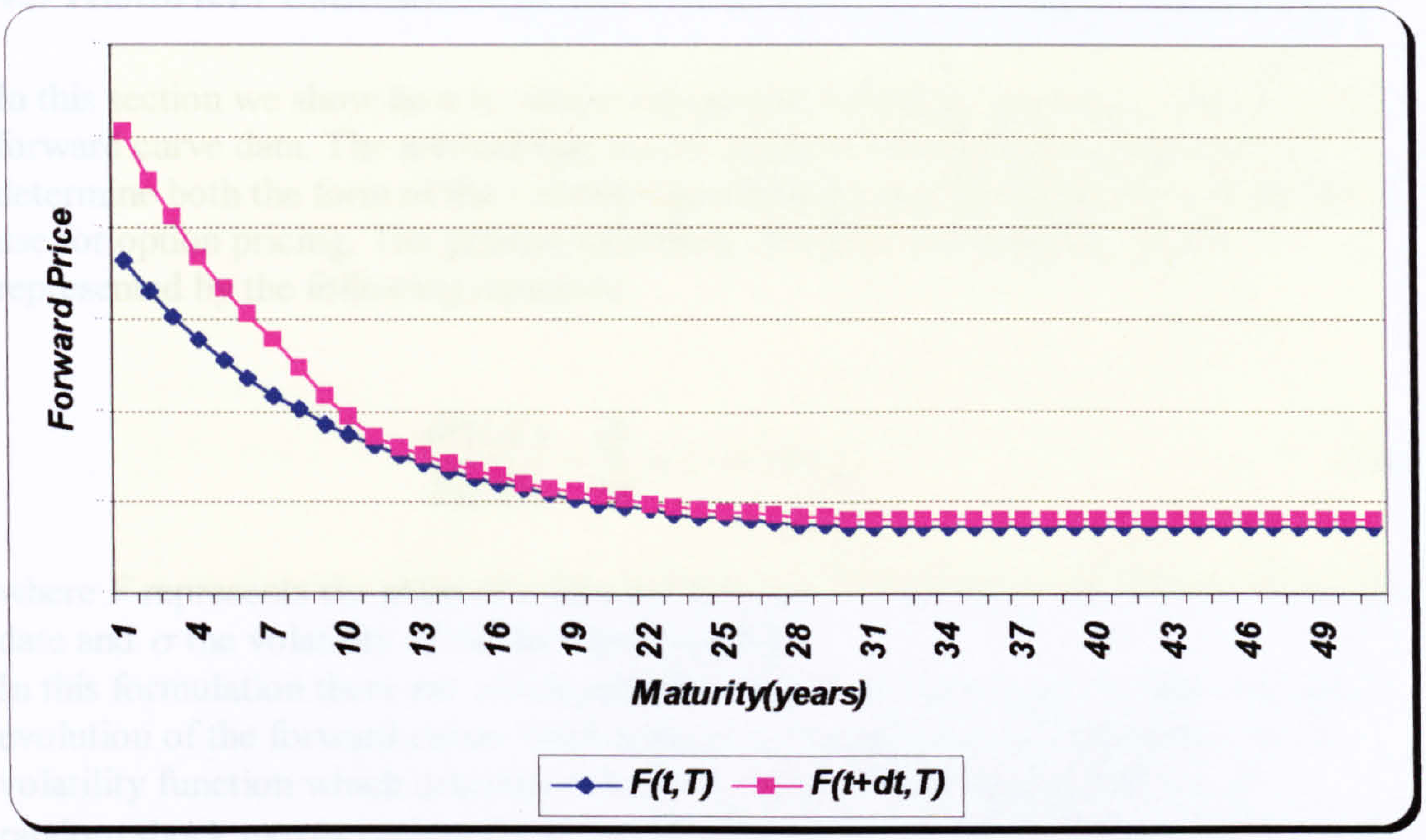
Figure-4.1



For this volatility function short dated forward returns are more volatile than long dated forwards-information occurring in the market today has little effect on, say, the five year forward price but can have a significant effect on the one month forward price. The parameter values used for the Figure-4.1 are $\alpha = 1$ and $\sigma = 0.40$. Here σ represents the overall volatility of the forward curve whilst α tells us how fast the forward volatility curve attenuates with increasing maturity. With an α of 100% we see that the one month forward has a volatility of about 37%, decreasing to roughly 2% for the three year forward.

Figure-4.2 shows a graphical representation of the evolution of the forward curve described by equation (4.1) with a volatility function of this type. The lower curve denoted by $F(t,T)$, is the original observed forward curve, whilst the upper curve, $F(t+dt,T)$, represents the curve after a small time step dt where there has been a positive shock to the system ($(dz(t) > 0)$. In this case the whole forward curve shifts up, with each point a multiple of $\sigma e^{-\alpha(T-t)}$ of the shock.

Figure-4.2



The volatility function is not restricted to have the parameterised form of equation (4.1). Instead we can allow the function to be very general:

$$\frac{dF(t,T)}{F(t,T)} = \sigma(t,T)dz(t) \tag{4.2}$$

where $\sigma(t,T)$ would be read as the time t volatility of the T maturity forward price return. We can determine from market data what the form of $\sigma(t,T)$ should be.
In order to determine empirically the form of the volatility function(s) we can look at the historical evolution of the market forward data. One method that can be used to determine the set of common factors that drive the dynamics of the forward curve is principal component analysis (PCA) or eigenvector decomposition of the covariance matrix. This procedure can be utilised to simultaneously identify the number of important factors and estimate the volatility functions. We show in detail how empirically to estimate the volatility functions in section 4.3.
Principal component analysis can also give us insights into how many factors we need to realistically model the forward curve.
If we need more than one factor to describe the forward curve we can amend the model described by equation (4.2) by adding additional sources of risk and volatility functions. For example, a three-factor model is given by:

$$\frac{dF(t,T)}{F(t,T)} = \sigma_1(t,T)dz_1(t) + \sigma_2(t,T)dz_2(t) + \sigma_3(t,T)dz_3(t) \tag{4.3}$$

where $\sigma_1(t,T), \sigma_2(t,T)$ and $\sigma_3(t,T)$ are the volatility functions and $dz_1(t), dz_2(t)$ and $dz_3(t)$ representing the independent sources of uncertainty (Brownian motions).

4.3 Historical Estimation of the Forward Curve Volatility Functions

In this section we show how to obtain the general volatility functions from historical forward curve data. The method that we are going to describe allows the user to determine both the form of the volatility functions as well as the number of factors to use for option pricing. The general multifactor forward curve model can be represented by the following equation:

$$\frac{dF(t,T)}{F(t,T)} = \sum_{i=1}^{n} \sigma_i(t,T) dz_i(t) \tag{4.4}$$

where F represents the price of a forward contract, T represents the contract maturity date and σ the volatility of the forward contract.
In this formulation there are n independent sources of uncertainty, which drive the evolution of the forward curve. Each source of uncertainty has associated with it a volatility function which determines by how much, and in which direction that random shock moves each point of the forward curve. The $\sigma_i(t,T)$ are, therefore, the n volatility functions associated with the independent sources of risk $dz_i(t)$.In practice we would usually set $n = 1,2$ and 3.After an application of Ito's Lemma the equation (4.4) can be represented in logarithmic form, as:

$$d\ln F(t,T) = -\frac{1}{2}\sum_{i=1}^{n} \sigma_i(t,T)^2 dt + \sum_{i=1}^{n} \sigma_i(t,T) dz_i(t) \tag{4.5}$$

This can be discretized for small time changes $\Delta(t)$ as:

$$\Delta \ln F(t,t+\tau_j) = -\frac{1}{2}\sum_{i=1}^{n} \sigma_i(t,t+\tau_j)^2 \Delta t + \sum_{i=1}^{n} \sigma_i(t,t+\tau_j) \Delta z_i \tag{4.6}$$

Equation (4.6) implies that changes in the natural logarithms of the forward prices with relative maturities $\tau_j, j = 1,.........m$ are jointly normally distributed. We can compute the sample covariance matrix of these forward prices in the standard way:

$$\sigma_{i,j} = \frac{1}{N}\sum_{k=1}^{N} (x_{ik} - \bar{x}_i)(x_{jk} - \bar{x}_j) \tag{4.7}$$

where there are N samples ($k = 1,........, N$) of x_{ik} and x_{jk} which are defined as:

$$x_{ik} = \ln(F(t_k, t_k + \tau_i)) - \ln(F(t_k - \Delta t, t_k - \Delta t + \tau_i)) \tag{4.8}$$

and where $\bar{x}_i, \bar{x}_j$ are the sample means. The time interval Δt is chosen to be daily. The discretized volatility functions, $\sigma_i(t, t + \tau_j)$; $i = 1,......, n$ and $j = 1,........,m$, are recovered by eigenvector decomposition of the covariance matrix.

The decomposition yields the set of independent factors, which drive the evolution of the variables underlying the covariance matrix. It decomposes the covariance matrix, which we denote by Σ, into n eigenvectors υ_{ji} and associated eigenvalues λ_i such that

$$\Sigma = \Gamma\Lambda\Gamma^T \tag{4.9}$$

where

$$\Gamma = \begin{vmatrix} \upsilon_{11}\upsilon_{12}\ldots\ldots\upsilon_{1n} \\ \upsilon_{21}\upsilon_{22}\ldots\ldots\upsilon_{2n} \\ \ldots\ldots\ldots\ldots\ldots \\ \upsilon_{n1}\upsilon_{n2}\ldots\ldots\upsilon_{nn} \end{vmatrix}$$

and

$$\Lambda = \begin{vmatrix} \lambda_1 0\ldots\ldots\ldots 0 \\ 0\lambda_2\ldots\ldots\ldots 0 \\ \ldots\ldots\ldots\ldots\ldots \\ 00\ldots\ldots\ldots\lambda_n \end{vmatrix}$$

and where the superscript T here denotes transpose. The columns of Γ are the eigenvectors. The eigenvalues represent the variances of the independent factors, which drive the forward points in proportions determined by the eigenvectors. The volatility functions are then obtained as $\sigma_i(t, t+\tau_j) = \upsilon_{ji}\sqrt{\lambda_i}\sqrt{252}$. (4.10)

We use the principal component analysis methodology above to determine the volatility functions from historical forward curve data for:

- Brent IPE from January 1999 to 16th February 2001, (Figure-4.3)
- WTI from January 1999 to 16th of February 2001, (Figure-4.4)
- Natural Gas from January 1999 to 16th of February 2001, (Figure-4.5)
- IPE Gasoil form January 1999 to 16th February 2001, (Figure-4.6)
- NYMEX Heating Oil from January 1999 to 16th of February 2001 (Figure-4.7)
- NYMEX Unleaded from January 1999 to 16th February 2001 (Figure-4.8)

The data that we are using are the settlement prices for the first nine months because they are the most liquid and they are the most reliable.

The first step is to construct a time series of forward price returns according to the equation (4.8). Next we compute the sample covariance matrix by applying equation (4.7). The following tables show the covariance matrices for: WTI crude oil futures, Brent crude oil futures, IPE Gasoil futures, Heating Oil futures, Unleaded futures and Natural Gas futures.

Table-4.1 Covariance Matrix For WTI Crude Oil Futures

	F(1m)	F(2m)	F(3m)	F(4m)	F(5m)	F(6m)	F(7m)	F(8m)	F(9m)
F(1m)	0.00061	0.00052	0.00047	0.00043	0.00040	0.00038	0.00036	0.00035	0.00033
F(2m)	0.00052	0.00048	0.00044	0.00041	0.00038	0.00036	0.00035	0.00033	0.00032
F(3m)	0.00047	0.00044	0.00041	0.00039	0.00037	0.00035	0.00033	0.00032	0.00031
F(4m)	0.00043	0.00041	0.00039	0.00037	0.00035	0.00033	0.00032	0.00030	0.00029
F(5m)	0.00040	0.00038	0.00037	0.00035	0.00033	0.00032	0.00031	0.00029	0.00028
F(6m)	0.00038	0.00036	0.00035	0.00033	0.00032	0.00031	0.00029	0.00028	0.00028
F(7m)	0.00036	0.00035	0.00033	0.00032	0.00031	0.00029	0.00029	0.00027	0.00027
F(8m)	0.00035	0.00033	0.00032	0.00030	0.00029	0.00028	0.00027	0.00027	0.00026
F(9m)	0.00033	0.00032	0.00031	0.00029	0.00028	0.00028	0.00027	0.00026	0.00026

Table-4.2 Covariance Matrix For Brent Crude Oil Futures

	F(1m)	F(2m)	F(3m)	F(4m)	F(5m)	F(6m)	F(7m)	F(8m)	F(9m)
F(1m)	0.0006	0.00052	0.00046	0.00043	0.00040	0.00037	0.00036	0.00035	0.00034
F(2m)	0.0005	0.00049	0.00044	0.00041	0.00039	0.00035	0.00035	0.00034	0.00033
F(3m)	0.00046	0.00044	0.00041	0.00039	0.00037	0.00033	0.00033	0.00032	0.00031
F(4m)	0.00043	0.00041	0.00039	0.00037	0.00035	0.00032	0.00032	0.00031	0.00030
F(5m)	0.00040	0.00039	0.00037	0.00035	0.00033	0.00031	0.00031	0.00030	0.00029
F(6m)	0.00037	0.00035	0.00033	0.00032	0.00031	0.00029	0.00029	0.00028	0.00028
F(7m)	0.00036	0.00035	0.00033	0.00032	0.00031	0.00029	0.00029	0.00029	0.00028
F(8m)	0.00035	0.00034	0.00032	0.00031	0.00030	0.00028	0.00029	0.00029	0.00028
F(9m)	0.00034	0.00033	0.00031	0.00030	0.00029	0.00028	0.00028	0.00028	0.00029

Table-4.3 Covariance Matrix For IPE Gasoil Futures

	F(1m)	F(2m)	F(3m)	F(4m)	F(5m)	F(6m)	F(7m)	F(8m)	F(9m)
F(1m)	0.00050	0.00046	0.00042	0.00037	0.00034	0.00032	0.00029	0.00028	0.00026
F(2m)	0.00046	0.00045	0.00041	0.00037	0.00034	0.00032	0.00029	0.00028	0.00026
F(3m)	0.00042	0.00041	0.00039	0.00036	0.00033	0.00031	0.00029	0.00027	0.00025
F(4m)	0.00037	0.00037	0.00036	0.00034	0.00031	0.00029	0.00027	0.00026	0.00024
F(5m)	0.00034	0.00034	0.00033	0.00031	0.00030	0.00029	0.00027	0.00025	0.00023
F(6m)	0.00032	0.00032	0.00031	0.00029	0.00029	0.00028	0.00026	0.00025	0.00023
F(7m)	0.00029	0.00029	0.00029	0.00027	0.00027	0.00026	0.00025	0.00024	0.00023
F(8m)	0.00028	0.00028	0.00027	0.00026	0.00025	0.00025	0.00024	0.00024	0.00022
F(9m)	0.00026	0.00026	0.00025	0.00024	0.00023	0.00023	0.00023	0.00022	0.00023

Table-4.4 Covariance Matrix For Heating Oil Futures

	F(1m)	F(2m)	F(3m)	F(4m)	F(5m)	F(6m)	F(7m)	F(8m)	F(9m)
F(1m)	0.00081	0.00061	0.00052	0.00046	0.00042	0.00038	0.00036	0.00034	0.00033
F(2m)	0.00061	0.00055	0.00050	0.00046	0.00043	0.00039	0.00037	0.00035	0.00034
F(3m)	0.00052	0.00050	0.00048	0.00045	0.00042	0.00038	0.00036	0.00033	0.00032
F(4m)	0.00046	0.00046	0.00045	0.00044	0.00041	0.00038	0.00035	0.00033	0.00031
F(5m)	0.00042	0.00043	0.00042	0.00041	0.00040	0.00037	0.00035	0.00033	0.00031
F(6m)	0.00038	0.00039	0.00038	0.00038	0.00037	0.00036	0.00034	0.00032	0.00031
F(7m)	0.00036	0.00037	0.00036	0.00035	0.00035	0.00034	0.00033	0.00032	0.00031
F(8m)	0.00034	0.00035	0.00033	0.00033	0.00033	0.00032	0.00032	0.00032	0.00030
F(9m)	0.00033	0.00034	0.00032	0.00031	0.00031	0.00031	0.00031	0.00030	0.00030

Table-4.5 Covariance Matrix For Unleaded Futures

	F(1m)	F(2m)	F(3m)	F(4m)	F(5m)	F(6m)	F(7m)	F(8m)	F(9m)
F(1m)	0.00064	0.00054	0.00046	0.00040	0.00036	0.00034	0.00031	0.00030	0.00031
F(2m)	0.00054	0.00052	0.00045	0.00040	0.00036	0.00034	0.00031	0.00029	0.00029
F(3m)	0.00046	0.00045	0.00043	0.00038	0.00034	0.00032	0.00029	0.00028	0.00027
F(4m)	0.00040	0.00040	0.00038	0.00037	0.00032	0.00030	0.00028	0.00027	0.00025
F(5m)	0.00036	0.00036	0.00034	0.00032	0.00033	0.00029	0.00027	0.00027	0.00025
F(6m)	0.00034	0.00034	0.00032	0.00030	0.00029	0.00030	0.00027	0.00027	0.00025
F(7m)	0.00031	0.00031	0.00029	0.00028	0.00027	0.00027	0.00027	0.00026	0.00024
F(8m)	0.00030	0.00029	0.00028	0.00027	0.00027	0.00027	0.00026	0.00028	0.00025
F(9m)	0.00031	0.00029	0.00027	0.00025	0.00025	0.00025	0.00024	0.00025	0.00027

Table-4.6 Covariance Matrix For Natural Gas Futures

	F(1m)	F(2m)	F(3m)	F(4m)	F(5m)	F(6m)	F(7m)	F(8m)	F(9m)
F(1m)	0.000950	0.000820	0.000685	0.000608	0.000563	0.000511	0.000470	0.000443	0.000409
F(2m)	0.000820	0.000770	0.000682	0.000628	0.000556	0.000488	0.000444	0.000415	0.000386
F(3m)	0.000685	0.000682	0.000663	0.000640	0.000540	0.000449	0.000398	0.000371	0.000352
F(4m)	0.000608	0.000628	0.000640	0.000660	0.000555	0.000452	0.000385	0.000349	0.000331
F(5m)	0.000563	0.000556	0.000540	0.000555	0.000520	0.000450	0.000385	0.000335	0.000305
F(6m)	0.000511	0.000488	0.000449	0.000452	0.000450	0.000423	0.000380	0.000332	0.000288
F(7m)	0.000470	0.000444	0.000398	0.000385	0.000385	0.000380	0.000380	0.000346	0.000297
F(8m)	0.000443	0.000415	0.000371	0.000349	0.000335	0.000332	0.000346	0.000355	0.000322
F(9m)	0.000409	0.000386	0.000352	0.000331	0.000305	0.000288	0.000297	0.000322	0.000329

The following tables summarise the results of the eigenvectors decomposition and the eigenvalues λ. [1]

[1] We import the covariance matrices in Mathematica (software package) which will compute the eigenvalues and the eigenvectors

WTI

Table-4.7 Eigenvalues (λ) for WTI Crude Oil Futures

λ	0.003205	0.00012	1.8E-05	8.05E-06	2.14E-06	8.60E-07	5.42E-07	4.39E-07	-3.50E-07

The Table-4.7 shows us that the first three eigenvalues are significant. Adding these we get 0.003357 and then dividing each λ with 0.003357 we can see which eigenvalue is the most important. Figure-4.8 plots the eigenvalues for the WTI crude oil futures. The figure shows that the first eigenvalue is the most important, explaining 95.47 % of the total variation in the evolution of the future curve. Together the first two factors explain 99.12% of the total variation, with the first three factors explaining 99.63%. The other factors as we mentioned above are not significant so a three-factor model is sufficient to explain the evolution of the WTI data over that period.

By applying the formula (4.10) we obtain the resulting volatility functions. The columns of Table-4.9 indicate the results of this calculation. The volatility functions illustrated in Figure-4.9 are the first three columns of Table-4.9. We apply the same method for the IPE Gasoil, Heating Oil, Unleaded and IPE Brent.

Table-4.8 Eigenvectors for WTI Crude Oil Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	0.4122	0.3818	0.3591	0.3380	0.3200	0.3075	0.2957	0.2836	0.2745
F(2m)	0.3818	0.3076	0.0034	0.1050	0.1645	0.2301	0.2651	0.3039	0.3410
F(3m)	0.3591	0.3768	0.4616	- 0.3013	- 0.1269	0.0165	0.1312	0.3384	0.4257
F(4m)	0.3379	0.7800	0.3769	- 0.1197	0.0003	- 0.0357	- 0.0117	0.0034	0.0245
F(5m)	0.3214	0.0190	0.5100	- 0.3907	0.0000	- 0.0046	0.1457	0.3042	0.3962
F(6m)	0.3076	0.0510	0.1660	0.0841	0.0002	- 0.0056	0.0024	0.0398	- 0.0205
F(7m)	0.2957	0.0135	0.3147	- 0.0079	- 0.0000	0.0017	0.0020	- 0.7285	0.6452
F(8m)	0.2836	0.0541	0.3571	- 0.6943	- 0.0000	0.0018	0.4276	0.1922	- 0.3472
F(9m)	0.2756	0.0380	0.0268	0.5133	0.0001	- 0.0062	- 0.0817	- 0.1212	0.1119

Figure-4.8

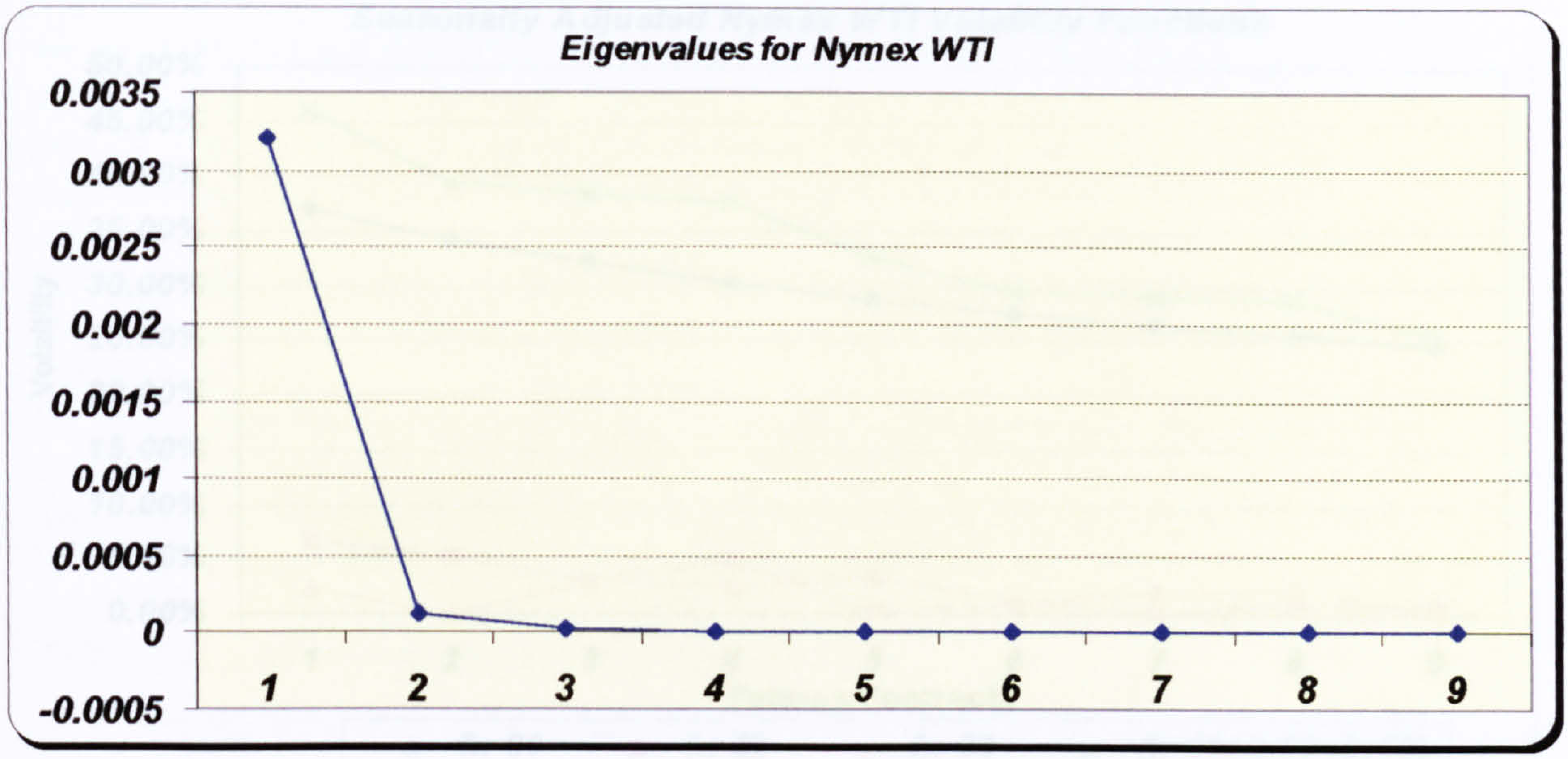
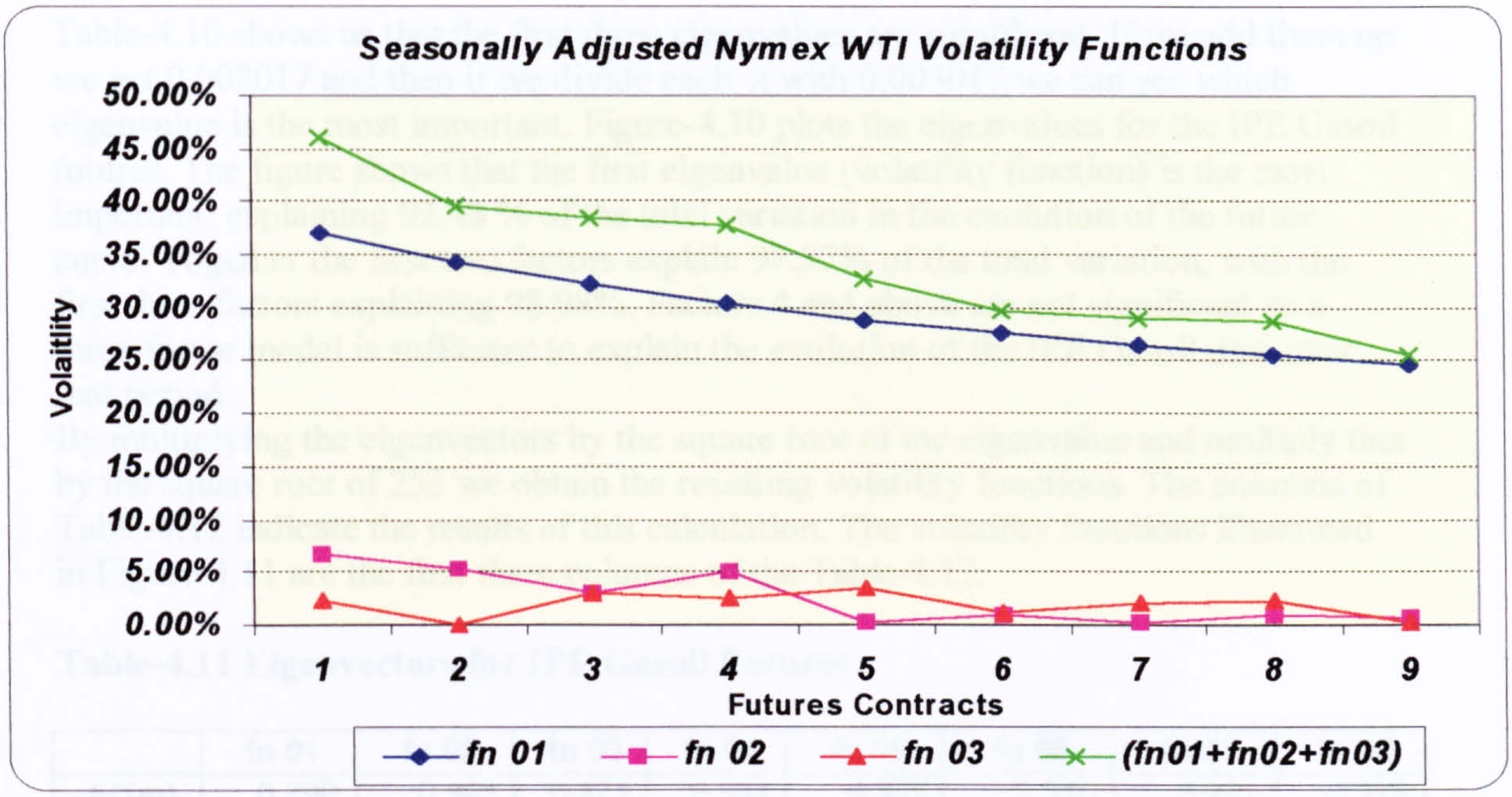


Table-4.9 Volatility Functions for WTI Crude Oil Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	37.05%	6.71%	2.41%	1.52%	0.74%	0.45%	0.35%	0.30%	0.26%
F(2m)	34.31%	5.41%	0.02%	0.47%	0.38%	0.34%	0.31%	0.32%	0.32%
F(3m)	32.28%	3.00%	3.09%	-1.36%	-0.29%	0.02%	0.15%	0.36%	0.40%
F(4m)	30.37%	5.00%	2.52%	-0.54%	0.00%	-0.05%	-0.01%	0.00%	0.02%
F(5m)	28.88%	0.33%	3.42%	-1.76%	0.00%	-0.01%	0.17%	0.32%	0.37%
F(6m)	27.64%	0.90%	1.11%	0.38%	0.00%	-0.01%	0.00%	0.04%	-0.02%
F(7m)	26.58%	0.24%	2.11%	-0.04%	0.00%	0.00%	0.00%	-0.77%	0.61%
F(8m)	25.49%	0.95%	2.39%	-3.13%	0.00%	0.00%	0.50%	0.20%	-0.33%
F(9m)	24.77%	0.67%	0.18%	2.31%	0.00%	-0.01%	-0.10%	-0.13%	0.11%

Figure-4.9



In Figure-4.9 above:

- **fn01** constitutes the **volatility function** (or level effect). In Figure 4.9 the volatility declines with maturity because the short maturity forward prices are more volatile than longer maturity forward prices.
- **fn02** indicates the volatility **smile**. We have drawn this conclusion from the shape of the curve and the practical knowledge of the market. In the front traded months, there are many options with different strikes from which a "smile" can be implied. The further into the future you look, due to less liquidity and more uncertainty, this effect is harder to discern, hence in the diagram we see fn02 tending to zero. In order to capture the smile/skew in the option pricing procedure the only variable we have to calibrate is the volatility. In the Black's model we have to make an adjustment in the volatility in order to capture the smile/skew of the market.[2]
- **fn03** indicates the **seasonality** in the volatility. The shape of this curve when added to the fn01 volatility function creates the "sign-wave" shape of volatiltiy seen in the market. Again, this effect is less "pronounced" the further in the future we look due to less liquidity in the market.
- **(fn01+fn02+fn03)** represents the final volatility curve with the smile and the seasonality effects included.[3]

GASOIL

Table-4.10 Eigenvalues (λ) for IPE Gasoil Futures

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[2] For more details regarding the smile see Chapter 5.

[3] The data has been tested for the period January –1999 to February 2000 and we reached to similar results to those above.

λ	0.0027901	0.0001537	4.3E-05	1.29E-05	7.93E-06	3.8E-06	2.68E-06	1.5E-06	1.83E-06

Table-4.10 shows us that the first three eigenvalues are significant. If we add them up we get 0.003017 and then if we divide each λ with 0.003017 we can see which eigenvalue is the most important. Figure-4.10 plots the eigenvalues for the IPE Gasoil futures. The figure shows that the first eigenvalue (volatility function) is the most important, explaining 92.48 % of the total variation in the evolution of the future curve. Together the first two factors explain 97.57% of the total variation, with the first three factors explaining 98.98%. Factors 4 and above are not significant so a three-factor model is sufficient to explain the evolution of the IPE Gasoil data over that period.
By multiplying the eigenvectors by the square root of the eigenvalue and multiply that by the square root of 252 we obtain the resulting volatility functions. The columns of Table-4.12 indicate the results of this calculation. The volatility functions illustrated in Figure-4.11 are the first three columns of the Table-4.12.

Table-4.11 Eigenvectors for IPE Gasoil Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08
F(1m)	0.398	0.391	0.371	0.344	0.325	0.310	0.291	0.278
F(2m)	0.577	0.363	0.171	0.003	0.141	0.258	0.338	0.371
F(3m)	0.491	0.004	0.289	- 0.437	- 0.360	- 0.152	0.073	0.248
F(4m)	0.244	0.670	0.053	- 0.190	- 0.288	- 0.456	- 0.069	0.096
F(5m)	0.036	0.019	0.510	- 0.391	0.000	- 0.005	0.146	0.304
F(6m)	0.021	0.051	0.166	0.084	0.000	- 0.006	0.002	0.040
F(7m)	0.015	0.014	0.315	- 0.008	- 0.000	0.002	0.002	- 0.729
F(8m)	0.009	0.054	0.357	- 0.694	- 0.000	0.002	0.428	0.192
F(9m)	0.000	0.038	0.027	0.513	0.000	- 0.006	- 0.082	- 0.121

Figure-4.10

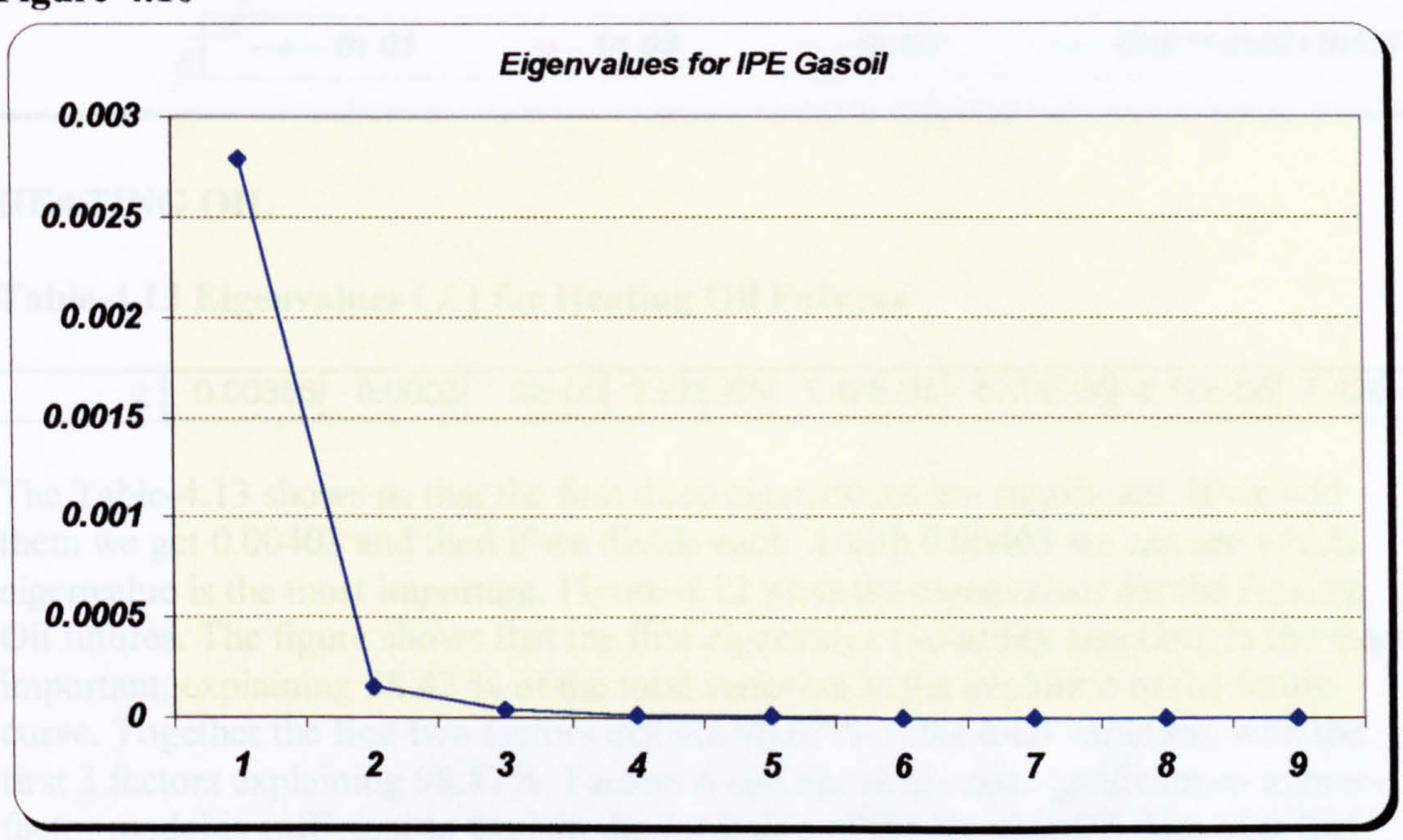
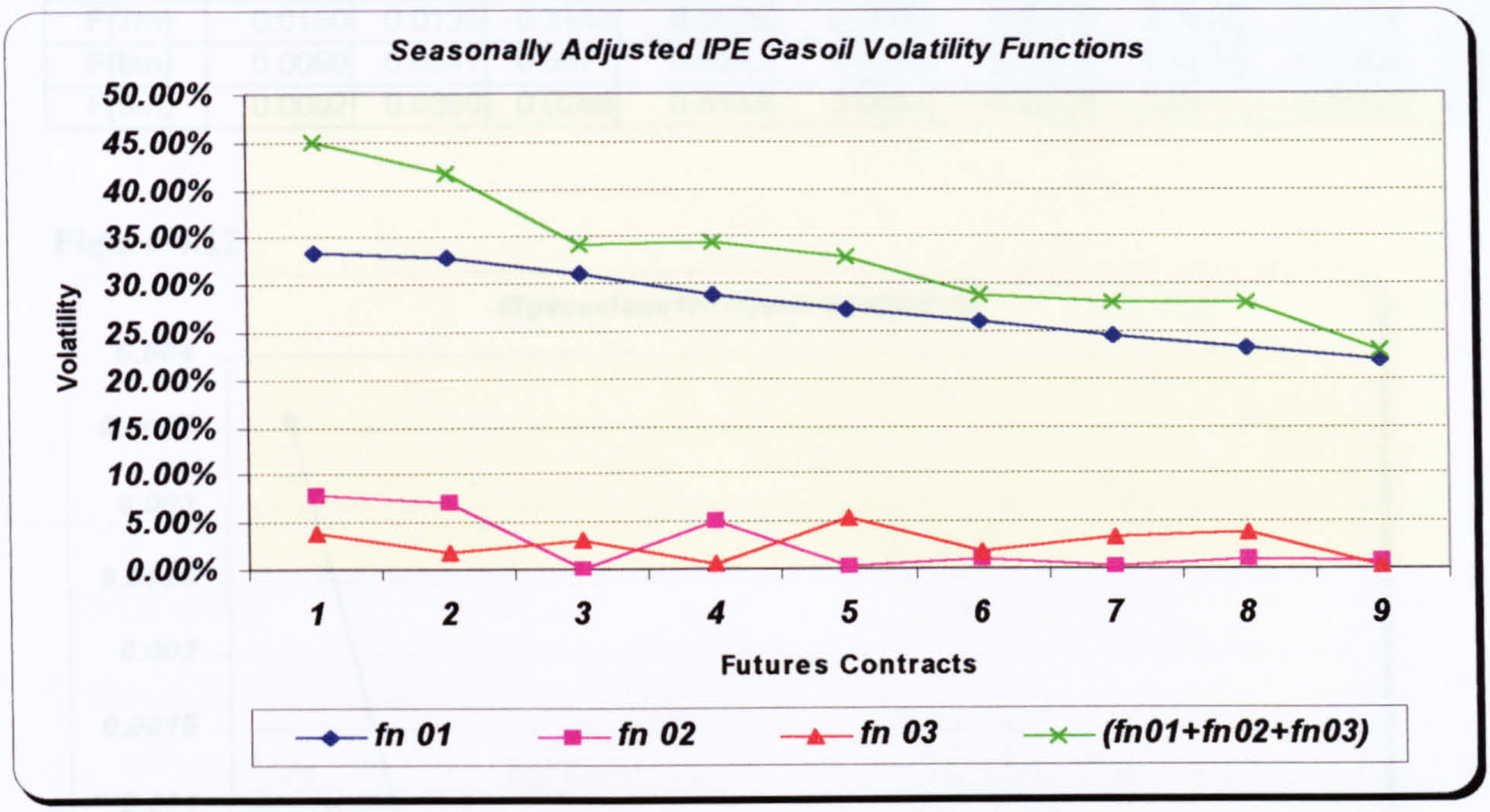


Table-4.12 Volatility Functions for IPE Gasoil Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	33.39%	7.70%	3.85%	1.96%	1.45%	0.96%	0.76%	0.54%	0.56%
F(2m)	32.82%	7.15%	1.77%	0.02%	0.63%	0.80%	0.88%	0.72%	0.88%
F(3m)	31.14%	0.07%	2.99%	-2.49%	-1.61%	-0.47%	0.19%	0.48%	1.10%
F(4m)	28.81%	5.00%	0.54%	-1.08%	-1.29%	-1.41%	-0.18%	0.19%	0.81%
F(5m)	27.28%	0.37%	5.28%	-2.23%	0.00%	-0.01%	0.38%	0.59%	0.85%
F(6m)	25.99%	1.00%	1.72%	0.48%	0.00%	-0.02%	0.01%	0.08%	-0.04%
F(7m)	24.41%	0.27%	3.26%	-0.05%	0.00%	0.01%	0.01%	-1.41%	1.39%
F(8m)	23.28%	1.06%	3.70%	-3.96%	0.00%	0.01%	1.11%	0.37%	-0.75%
F(9m)	21.92%	0.75%	0.28%	2.93%	0.00%	-0.02%	-0.21%	-0.24%	0.24%

Figure-4.11



HEATING OIL

Table-4.13 Eigenvalues (λ) for Heating Oil Futures

λ	0.00356	0.0003	9E-05	2.27E-05	1.42E-05	4.17E-06	3.17E-06	7.42E-07	5.71E-07

The Table-4.13 shows us that the first three eigenvalues are significant. If we add them we get 0.00403 and then if we divide each λ with 0.00403 we can see which eigenvalue is the most important. Figure-4.12 plots the eigenvalues for the Heating Oil futures. The figure shows that the first eigenvalue (volatility function) is the most important, explaining 88.42 % of the total variation in the evolution of the future curve. Together the first two factors explain 96.67% of the total variation, with the first 3 factors explaining 98.87%. Factors 4 and above are not significant so a three-factor model is sufficient to explain the evolution of the Heating Oil data over that period.

By multiplying the eigenvectors by the square root of the eigenvalue and multiplying that by the square root of 252 we obtain the resulting volatility functions. The columns of Table-4.15 indicate the results of this calculation. The volatility functions illustrated in Figure-4.13 are the first three columns of Table-4.15.

Table-4.14 Eigenvectors for Heating Oil Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	0.4130	0.3845	0.3608	0.3432	0.3262	0.3066	0.2904	0.2779	0.2677
F(2m)	0.7636	0.2255	0.0456	-0.1000	-0.2002	-0.2557	-0.2873	-0.3048	-0.2703
F(3m)	0.3824	0.2320	0.4162	0.4117	0.2322	-0.0170	-0.2273	-0.3785	-0.4562
F(4m)	0.2445	0.6700	0.0525	-0.1897	-0.2884	-0.4562	-0.0685	0.0959	0.3759
F(5m)	0.0360	0.0190	0.5100	-0.3907	0.0000	-0.0046	0.1457	0.3042	0.3962
F(6m)	0.0210	0.0510	-0.1660	0.0841	0.0002	-0.0056	0.0024	0.0398	-0.0205
F(7m)	0.0150	0.0135	-0.3147	-0.0079	0.0000	0.0017	0.0020	-0.7285	0.6452
F(8m)	0.0090	0.0541	0.3571	-0.6943	0.0000	0.0018	0.4276	0.1922	-0.3472
F(9m)	0.0002	0.0380	0.0268	0.5133	0.0001	-0.0062	-0.0817	-0.1212	0.1119

Figure-4.12

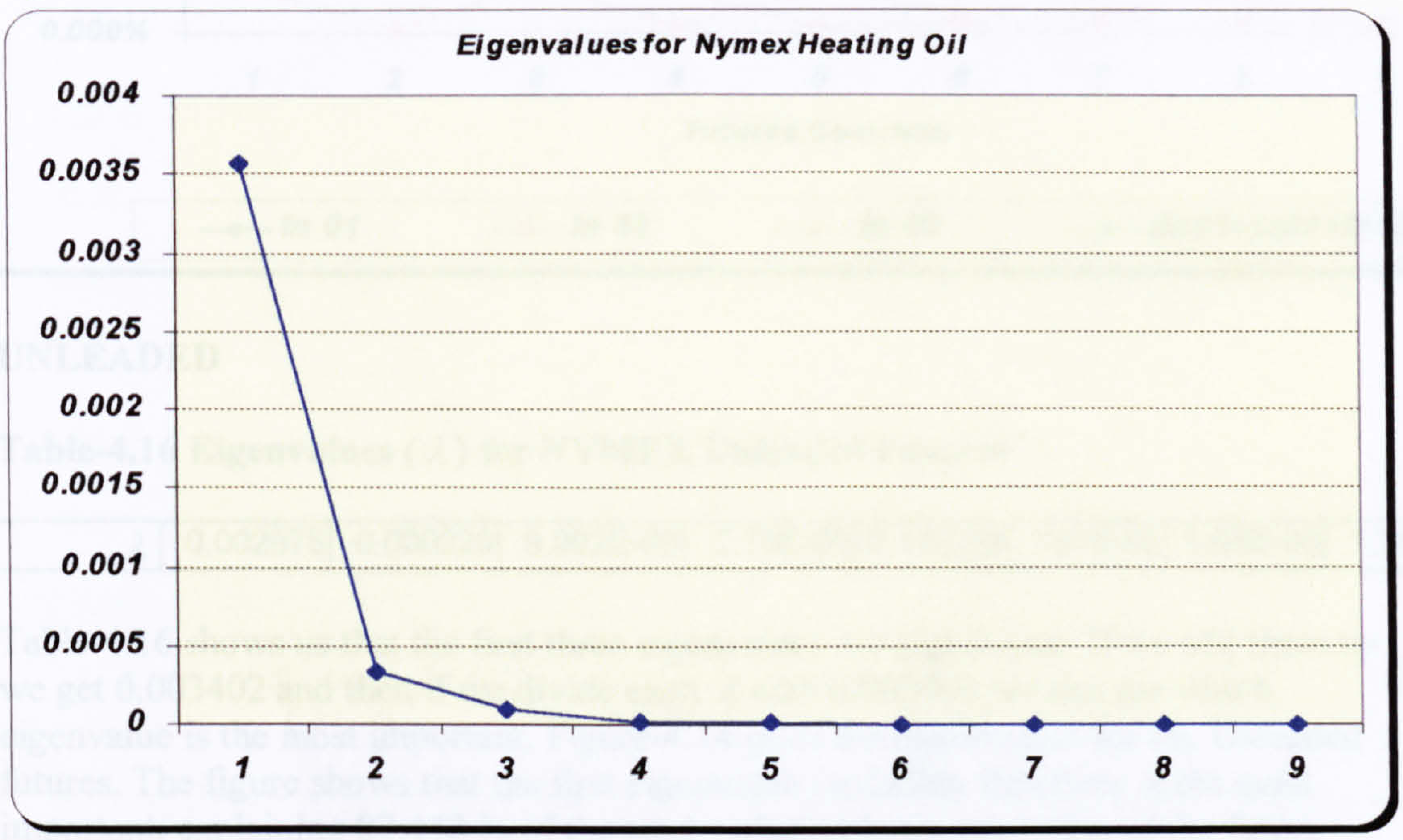
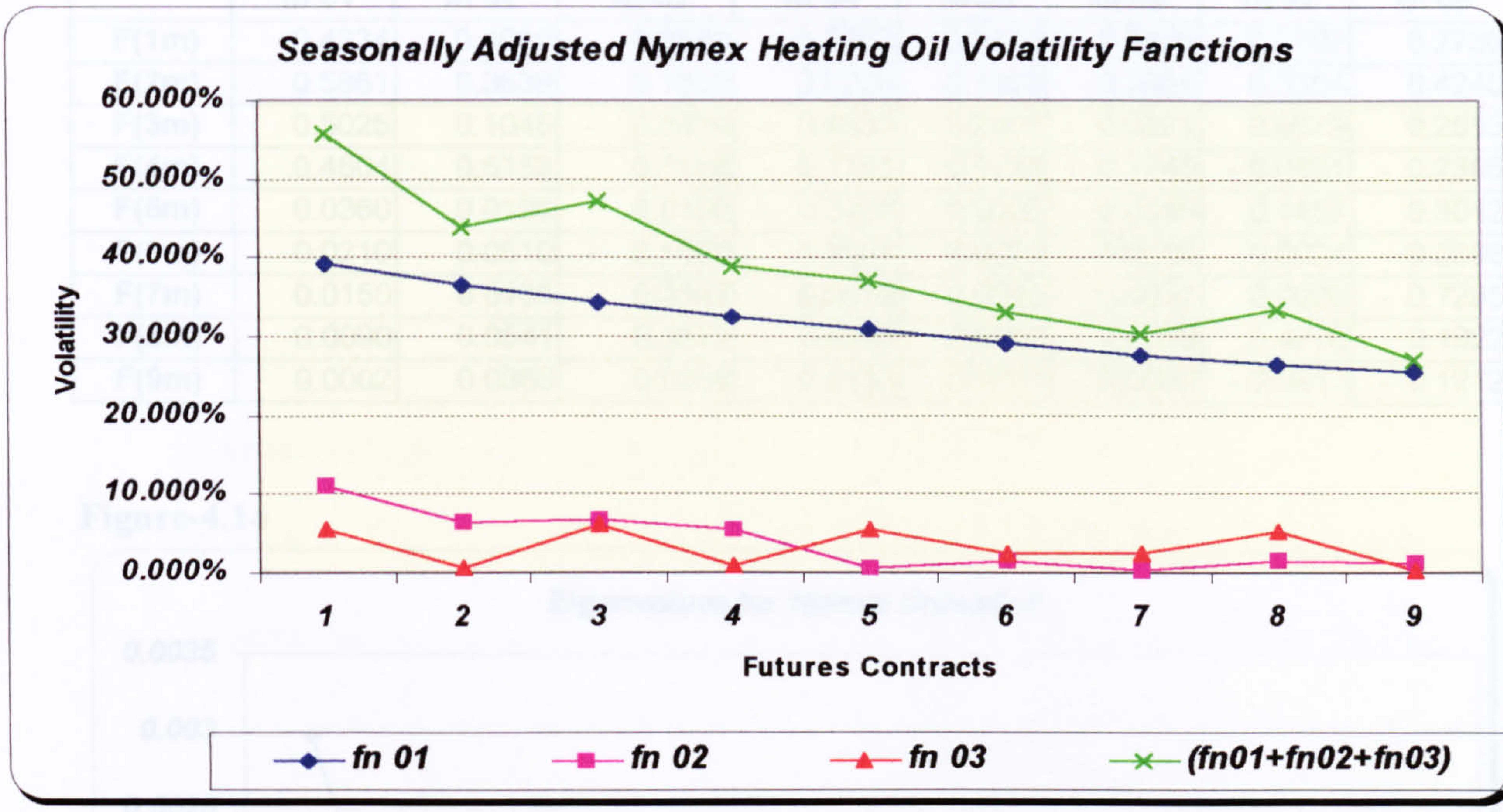


Table-4.15 Volatility Functions for Heating Oil Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	39.120%	11.129%	5.385%	2.598%	1.951%	0.994%	0.821%	0.380%	0.321%
F(2m)	36.419%	6.528%	0.681%	-0.757%	-1.197%	-0.829%	-0.812%	-0.417%	-0.324%
F(3m)	34.178%	6.716%	6.213%	3.116%	1.389%	-0.055%	-0.642%	-0.517%	-0.547%
F(4m)	32.506%	5.500%	0.784%	-1.436%	-1.725%	-1.479%	-0.194%	0.131%	0.451%
F(5m)	30.900%	0.550%	5.500%	-2.957%	0.000%	-0.015%	0.412%	0.416%	0.475%

F(6m)	29.040%	1.476%	2.478%	0.637%	0.001%	-0.018%	0.007%	0.054%	-0.025%
F(7m)	27.506%	0.391%	2.500%	-0.060%	0.000%	0.005%	0.006%	-0.996%	0.774%
F(8m)	26.327%	1.565%	5.330%	-5.255%	0.000%	0.006%	1.209%	0.263%	-0.416%
F(9m)	25.361%	1.099%	0.400%	3.885%	0.001%	-0.020%	-0.231%	-0.166%	0.134%

Figure-4.13



UNLEADED

Table-4.16 Eigenvalues (λ) for NYMEX Unleaded Futures

λ	0.002975	0.000229	9.092E-05	2.79E-05	2.14E-05	1.97E-05	1.66E-05	1.28E-05	8.73E-06

Table-4.16 shows us that the first three eigenvalues are significant. If we add them up we get 0.003402 and then if we divide each λ with 0.003402 we can see which eigenvalue is the most important. Figure-4.14 plots the eigenvalues for the Unleaded futures. The figure shows that the first eigenvalue (volatility function) is the most important, explaining 87.448 % of the total variation in the evolution of the future curve. Together the first two factors explain 94.179% of the total variation, with the first 3 factors explaining 96.85%. The other factors as we mentioned above are not significant so a three-factor model is sufficient to explain the evolution of the Unleaded data over that period.
By multiplying the eigenvectors by the square root of the eigenvalue and multiplying that by the square root of 252 we obtain the resulting volatility functions. The columns of Table-4.18 indicate the results of this calculation. The volatility functions illustrated in Figure-4.15 are the first three columns of the Table-4.18.

Table-4.17 Eigenvectors for NYMEX Unleaded Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	0.4234	0.4010	0.3684	0.3352	0.3143	0.3006	0.2792	0.2736	0.2644
F(2m)	0.5861	0.3639	- 0.1352	0.0239	0.1928	0.2664	0.3364	0.4240	0.3226
F(3m)	0.5025	0.1048	- 0.3518	- 0.4533	- 0.2907	- 0.0621	0.0679	0.2555	0.4991
F(4m)	0.4504	0.6153	0.1168	- 0.1761	- 0.1723	- 0.1746	- 0.0631	- 0.2395	0.5064
F(5m)	0.0360	0.0190	0.5100	- 0.3907	0.0000	- 0.0046	0.1457	0.3042	0.3962
F(6m)	0.0210	0.0510	- 0.1660	0.0841	0.0002	- 0.0056	0.0024	0.0398	- 0.0205
F(7m)	0.0150	0.0135	- 0.3147	- 0.0079	- 0.0000	0.0017	0.0020	- 0.7285	0.6452
F(8m)	0.0090	0.0541	0.3571	- 0.6943	- 0.0000	0.0018	0.4276	0.1922	- 0.3472
F(9m)	0.0002	0.0380	0.0268	0.5133	0.0001	- 0.0062	- 0.0817	- 0.1212	0.1119

Figure-4.14

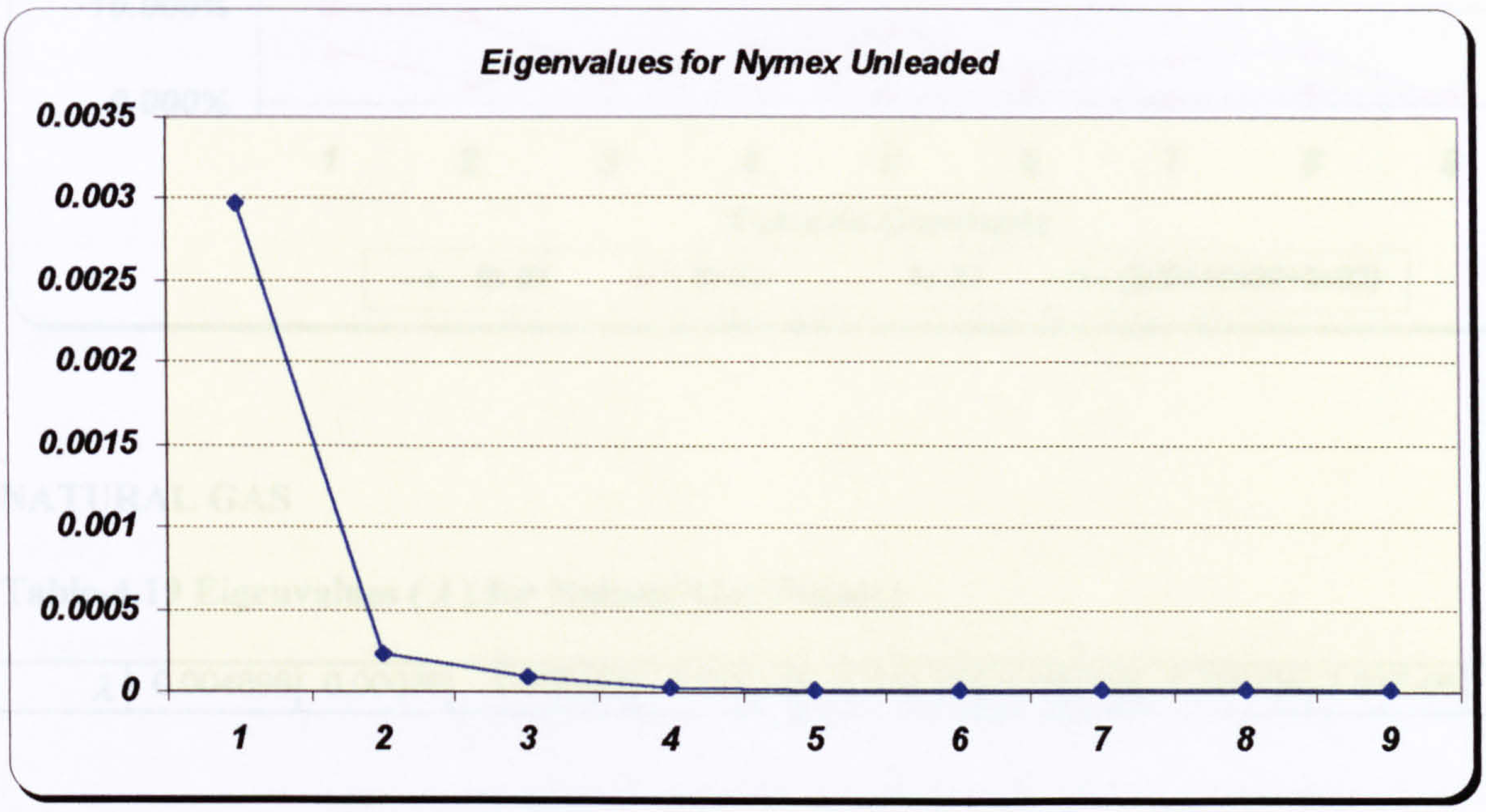
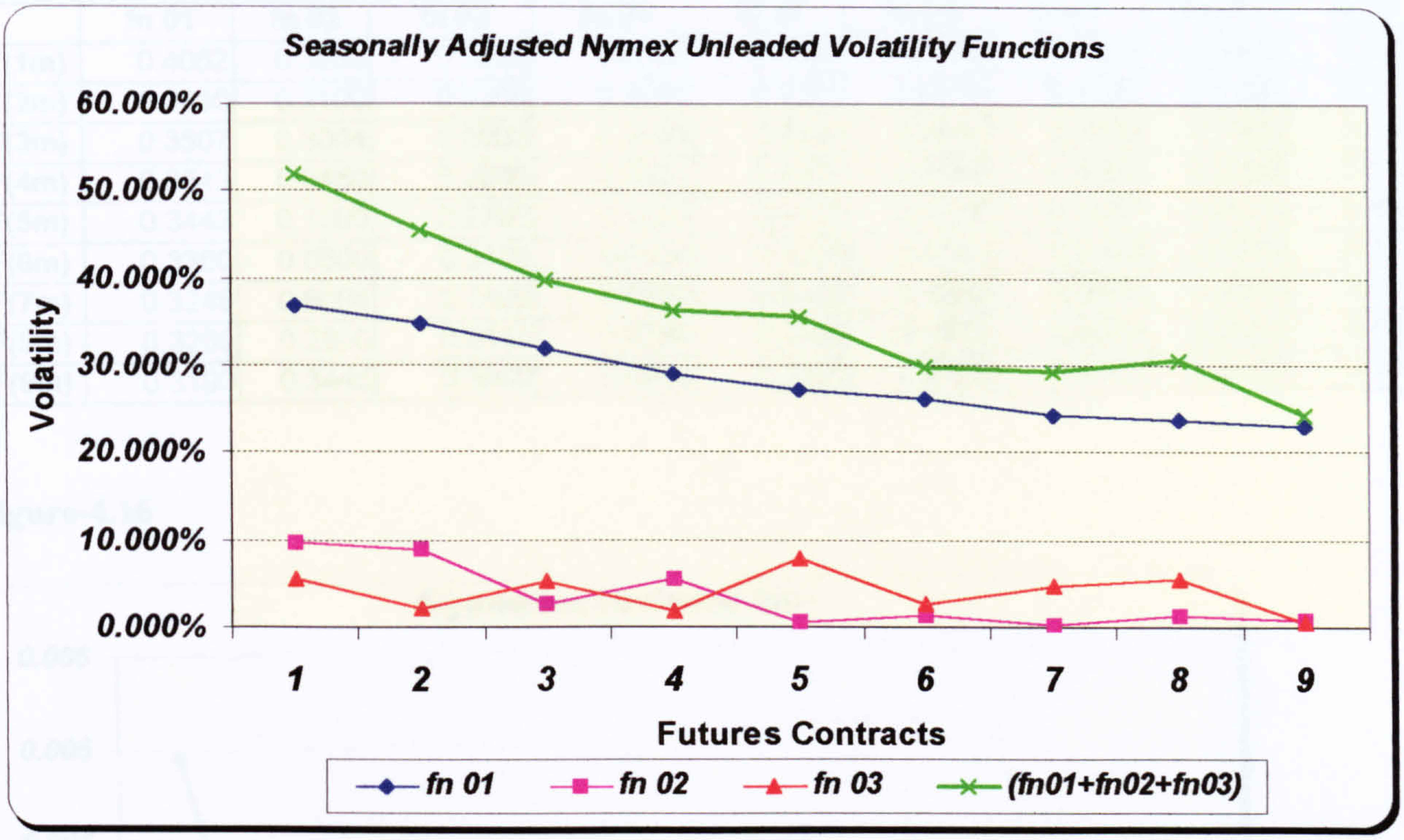


Table-4.18 Volatility Functions for NYMEX Unleaded Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	36.664%	9.634%	5.576%	2.810%	2.309%	2.119%	1.803%	1.554%	1.240%
F(2m)	34.724%	8.742%	2.047%	0.200%	1.417%	1.878%	2.173%	2.408%	1.513%
F(3m)	31.897%	2.516%	5.325%	3.800%	-2.136%	-0.438%	0.439%	1.451%	2.340%
F(4m)	29.020%	5.500%	1.768%	1.476%	-1.266%	-1.231%	-0.408%	-1.360%	2.374%

F(5m)	27.211%	0.456%	7.720%	3.275%	0.000%	-0.032%	0.941%	1.727%	1.858%
F(6m)	26.029%	1.225%	2.513%	0.705%	0.001%	-0.039%	0.016%	0.226%	-0.096%
F(7m)	24.173%	0.324%	4.763%	0.067%	0.000%	0.012%	0.013%	-4.137%	3.026%
F(8m)	23.689%	1.299%	5.405%	5.820%	0.000%	0.013%	2.762%	1.092%	-1.628%
F(9m)	22.890%	0.912%	0.406%	4.303%	0.001%	-0.044%	-0.528%	-0.688%	0.525%

Figure-4.15



NATURAL GAS

Table-4.19 Eigenvalues (λ) for Natural Gas Futures

λ	0.004896	0.00036	1.17E-04	1.17E-05	1.17E-05	3.18E-05	1.32E-05	7.33E-06	4.80E-06

Table-4.19 shows us that the first three eigenvalues are significant. If we add them up we get 0.005677 and then if we divide each λ with 0.005677 we can see which eigenvalue is the most important. Figure-4.16 plots the eigenvalues for the Natural Gas futures. The figure shows that the first eigenvalue (volatility function) is the most important, explaining 86.42 % of the total variation in the evolution of the future curve. Together the first two factors explain 92.77% of the total variation, with the first 3 factors explaining 94.84%. The other factors as we mentioned above are not significant so a three-factor model is sufficient to explain the evolution of the Natural Gas data over that period.
By multiplying the eigenvectors by the square root of the eigenvalue and multiplying that by the square root of 252 we obtain the resulting volatility functions. The

columns of Table-4.21 indicate the results of this calculation. The volatility functions illustrated in Figure-4.17 are the first three columns of the Table-4.21.

Table-4.20 Eigenvectors for Natural Gas Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	0.4062	0.3800	- 0.5000	- 0.3300	- 0.3060	- 0.2700	- 0.2500	- 0.2300	- 0.2234
F(2m)	0.3830	0.2100	- 0.3200	- 0.3600	- 0.1800	0.0170	0.1700	0.3100	0.3916
F(3m)	0.3507	0.3004	- 0.0600	- 0.3553	- 0.3300	- 0.2640	- 0.2200	- 0.1600	- 0.1055
F(4m)	0.3543	0.0450	- 0.2000	- 0.3321	- 0.0207	0.2569	0.4600	0.3800	0.0126
F(5m)	0.3443	0.1300	- 0.2700	- 0.1200	0.3100	0.4330	0.1490	- 0.2900	- 0.4500
F(6m)	0.3300	0.0500	- 0.3100	- 0.0524	0.4200	0.1742	- 0.2900	- 0.2010	0.1700
F(7m)	0.3245	0.6004	0.1400	- 0.4600	0.0400	0.2060	0.0600	- 0.2800	- 0.0110
F(8m)	0.3200	0.2800	0.2115	0.0780	- 0.2200	- 0.1877	0.6000	- 0.2800	- 0.1800
F(9m)	0.3100	0.3445	0.5900	- 0.4900	0.3400	- 0.0950	- 0.1200	- 0.3400	0.1800

Figure-4.16

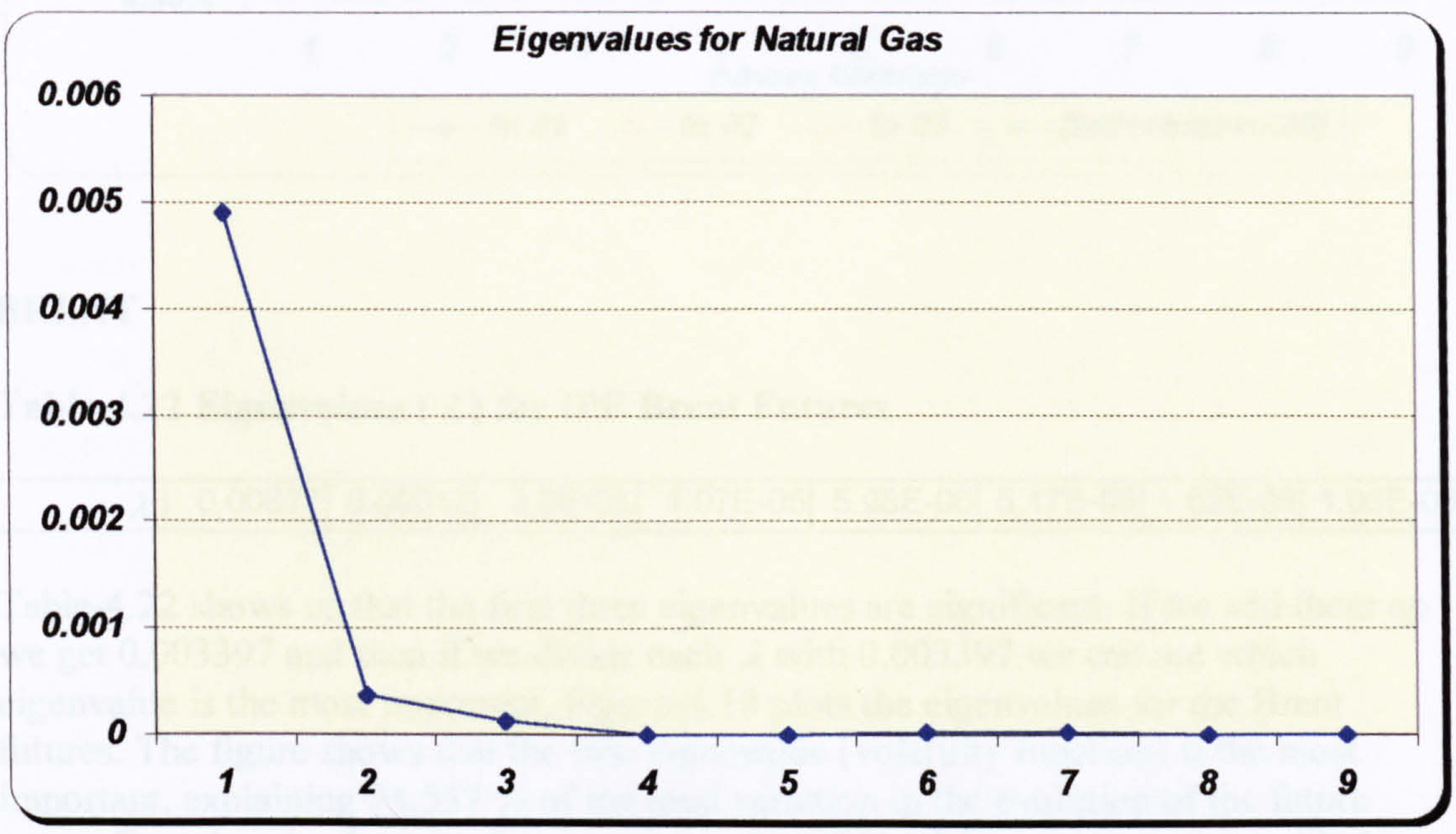
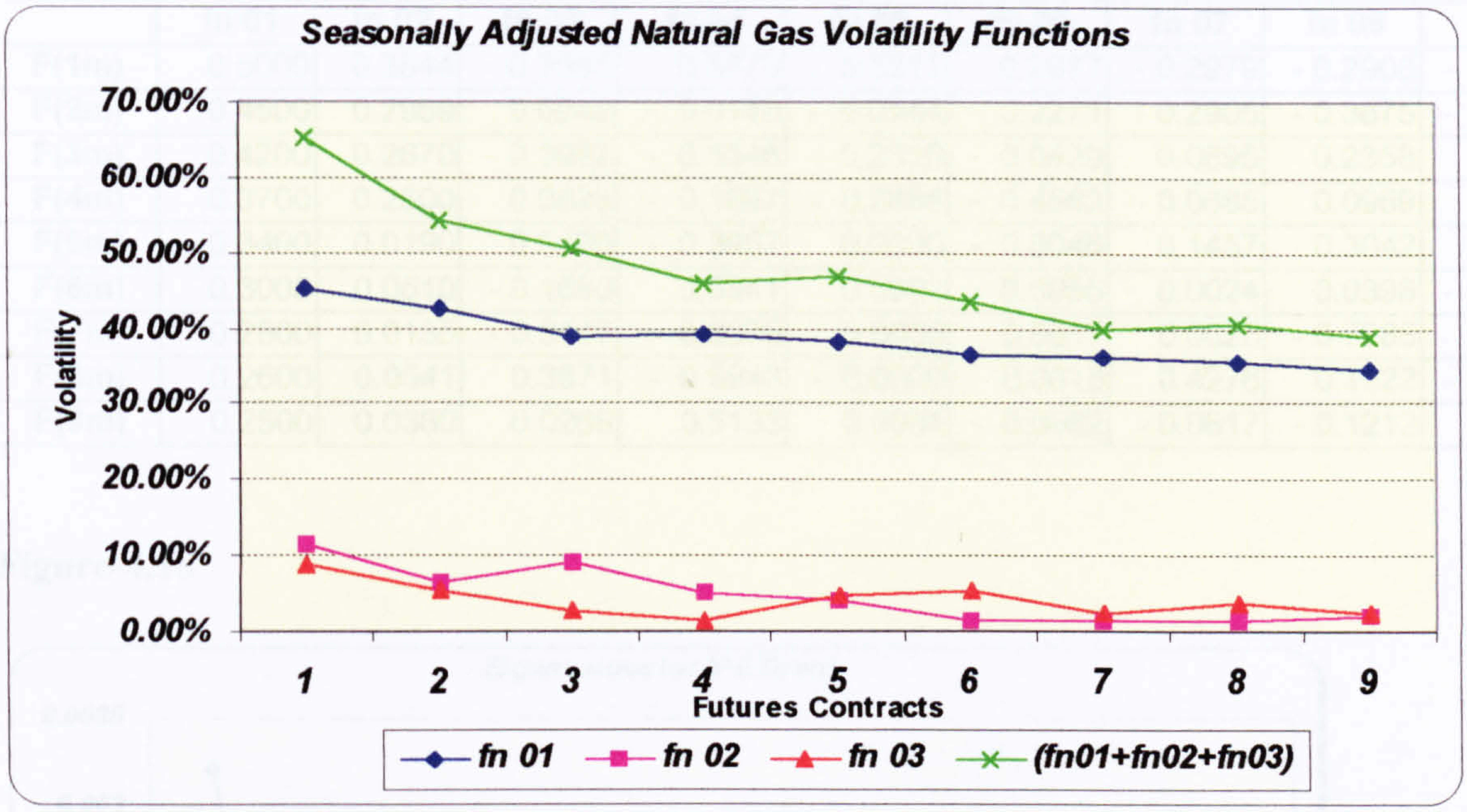


Table-4.21 Volatility Functions for Natural Gas Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	45.12%	11.44%	8.60%	5.68%	-5.26%	-2.42%	-1.44%	-0.99%	-0.78%
F(2m)	42.54%	6.32%	5.51%	6.19%	-3.10%	0.15%	0.98%	1.33%	1.36%
F(3m)	38.95%	9.05%	2.60%	6.11%	-5.68%	-2.36%	-1.27%	-0.69%	-0.37%
F(4m)	39.35%	5.00%	1.50%	5.71%	-0.36%	2.30%	2.66%	1.63%	0.04%
F(5m)	38.24%	3.92%	4.65%	2.06%	5.33%	3.88%	0.86%	-1.25%	-1.56%

F(6m)	36.65%	1.51%	5.33%	0.90%	7.23%	1.56%	-1.67%	-0.86%	0.59%
F(7m)	36.04%	1.25%	2.41%	7.91%	0.69%	1.85%	0.35%	-1.20%	-0.04%
F(8m)	35.54%	1.25%	3.64%	1.34%	-3.79%	-1.68%	3.46%	-1.20%	-0.63%
F(9m)	34.43%	2.00%	2.50%	8.43%	5.85%	-0.85%	-0.69%	-1.46%	0.63%

Figure-4.17



BRENT

Table-4.22 Eigenvalues (λ) for IPE Brent Futures

λ	0.00321	0.00012	3.6E-05	1.07E-05	5.98E-06	5.17E-06	1.82E-06	1.06E-06	6.44E-07

Table-4.22 shows us that the first three eigenvalues are significant. If we add them up we get 0.003397 and then if we divide each λ with 0.003397 we can see which eigenvalue is the most important. Figure-4.18 plots the eigenvalues for the Brent futures. The figure shows that the first eigenvalue (volatility function) is the most important, explaining 94.557 % of the total variation in the evolution of the future curve. Together the first two factors explain 98.198% of the total variation, with the first 3 factors explaining 99.25%. Factors and above are not significant so a three-factor model is sufficient to explain the evolution of the Brent data over that period. By multiplying the eigenvectors by the square root of the eigenvalue and multiplying that by the square root of 252 we obtain the resulting volatility functions. The columns of Table-4.24 indicate the results of this calculation. The volatility functions illustrated in Figure-4.19 are the first two columns of the Table-4.24.

Table-4.23 Eigenvectors for IPE Brent Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	0.5000	0.3844	- 0.3565	- 0.3373	- 0.3211	- 0.2977	- 0.2979	- 0.2900	- 0.2842
F(2m)	0.4500	0.2969	0.0942	- 0.0146	- 0.0944	- 0.2271	- 0.2905	- 0.3675	- 0.4286
F(3m)	0.4200	0.2670	- 0.3982	- 0.3346	- 0.2616	- 0.0430	0.0895	0.2358	0.4425
F(4m)	0.3700	0.2500	0.0525	- 0.1897	- 0.2884	- 0.4562	- 0.0685	0.0959	0.3759
F(5m)	0.3400	0.0190	0.5100	- 0.3907	0.0000	- 0.0046	0.1457	0.3042	0.3962
F(6m)	0.3000	0.0510	- 0.1660	0.0841	0.0002	- 0.0056	0.0024	0.0398	- 0.0205
F(7m)	0.2800	0.0135	- 0.3147	- 0.0079	- 0.0000	0.0017	0.0020	- 0.7285	0.6452
F(8m)	0.2600	0.0541	0.3571	- 0.6943	- 0.0000	0.0018	0.4276	0.1922	- 0.3472
F(9m)	0.2500	0.0380	0.0268	0.5133	0.0001	- 0.0062	- 0.0817	- 0.1212	0.1119

Figure-4.18

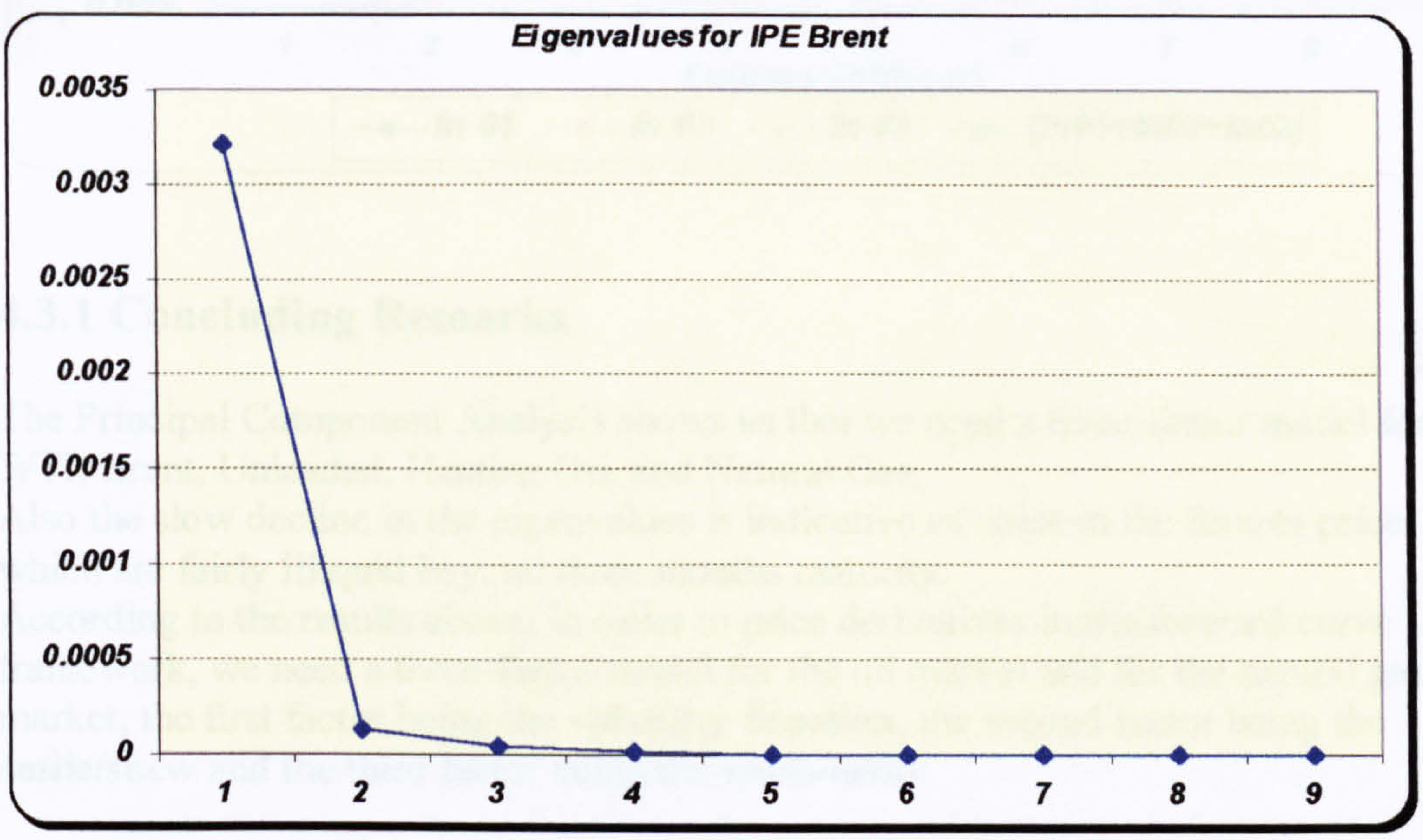
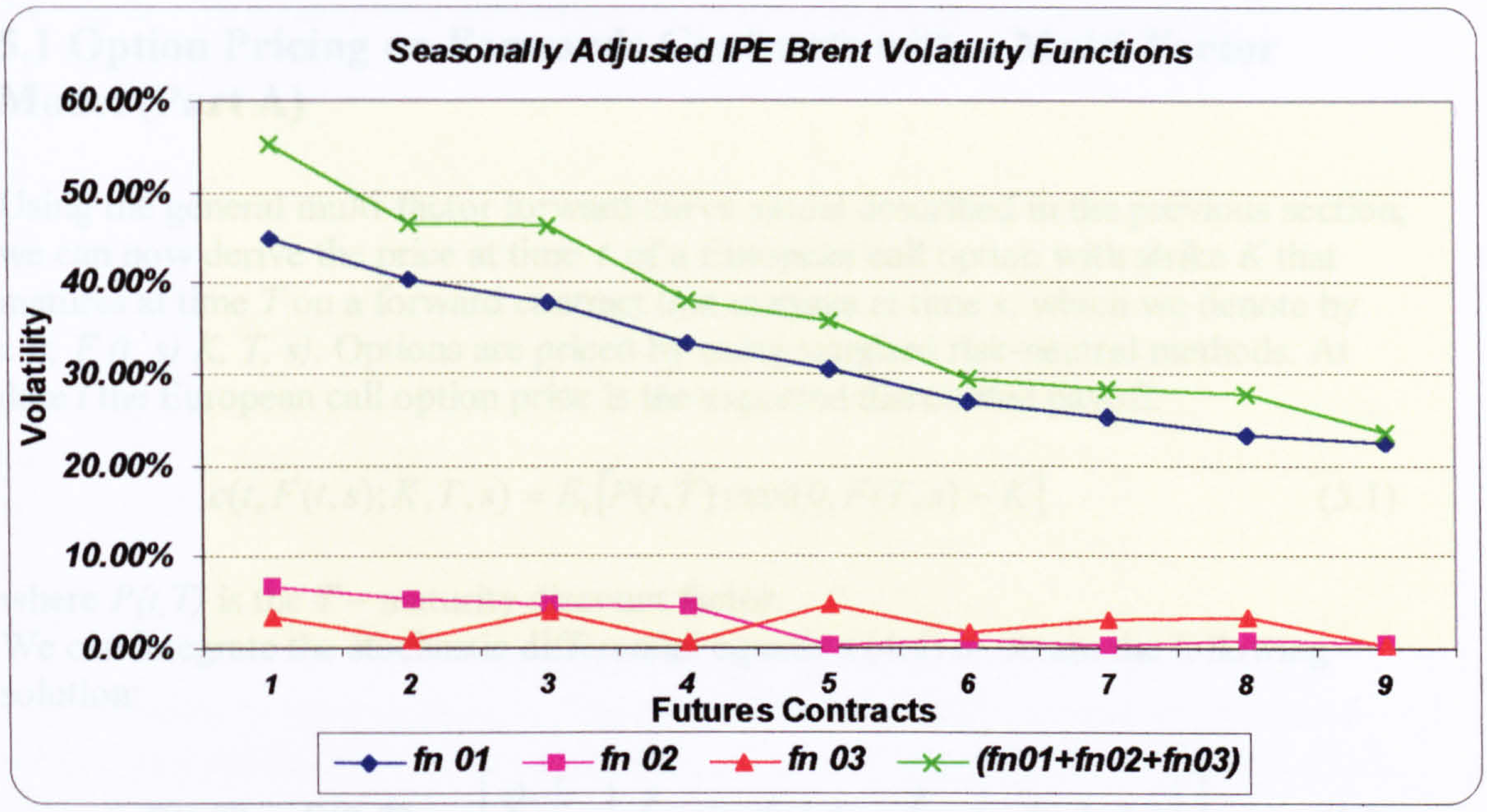


Table-4.24 Volatility Functions for IPE Brent Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	44.98%	6.79%	3.39%	-1.75%	-1.25%	-1.08%	-0.64%	-0.47%	-0.36%
F(2m)	40.48%	5.24%	0.90%	-0.08%	-0.37%	-0.82%	-0.62%	-0.60%	-0.55%
F(3m)	37.79%	4.71%	3.79%	-1.73%	-1.02%	-0.16%	0.19%	0.38%	0.56%
F(4m)	33.29%	4.41%	0.50%	-0.98%	-1.12%	-1.65%	-0.15%	0.16%	0.48%
F(5m)	30.59%	0.34%	4.85%	-2.02%	0.00%	-0.02%	0.31%	0.50%	0.50%
F(6m)	26.99%	0.90%	1.58%	0.44%	0.00%	-0.02%	0.01%	0.06%	-0.03%

F(7m)	25.19%	0.24%	2.99%	-0.04%	0.00%	0.01%	0.00%	-1.19%	0.82%
F(8m)	23.39%	0.95%	3.40%	-3.60%	0.00%	0.01%	0.92%	0.31%	-0.44%
F(9m)	22.49%	0.67%	0.25%	2.66%	0.00%	-0.02%	-0.17%	-0.20%	0.14%

Figure-4.19



4.3.1 Concluding Remarks

The Principal Component Analysis shows us that we need a three-factor model for WTI, Brent, Unleaded, Heating Oil, and Natural Gas.
Also the slow decline in the eigenvalues is indicative of noise in the futures prices which are fairly illiquid beyond three months maturity.
According to the results above, in order to price derivatives in the forward curve framework, we need a three-factor model for the oil market and for the natural gas market, the first factor being the **volatility function**, the second factor being the **smile/skew** and the third factor being the **seasonality**.

CHAPTER 5
PRICING WTI OPTIONS ON FORWARD CONTRACTS BASED UPON THE MULTI FACTOR MODEL

5.1 Option Pricing on Forwards Contracts with a Multi-Factor Model (Part A)

Using the general multi-factor forward curve model described in the previous section, we can now derive the price at time t of a European call option with strike K that matures at time T on a forward contract that matures at time s, which we denote by c *(t, F (t, s) K, T, s)*. Options are priced by using standard risk-neutral methods. At date t the European call option price is the expected discounted payoff:

$$c(t,F(t,s);K,T,s) = E_t\left[P(t,T)\max(0,F(T,s)-K\right] \tag{5.1}$$

where $P(t,T)$ is the $T-$ maturity discount factor.
We can integrate the stochastic differential equation (4.4) to obtain the following solution:

$$F(t,T) = F(0,T)\exp\left[\sum_{i=1}^{n}\left\{-\frac{1}{2}\int_0^t \sigma_i(u,T)^2 du + \int_0^t \sigma_i(u,T)dz_i(u)\right\}\right] \tag{5.2}$$

This equation expresses the forward curve at time t in terms of it's initially observed state (time 0) and integrals of the volatility functions.
From equation (5.2) the natural logarithms of the forward prices at time T are normally distributed:

$$LnF(T,s) \approx N\left(\ln F(t,s) - \frac{1}{2}\sum_{i=1}^{n}\left\{\int_t^T \sigma_i(u,s)^2 du\right\}\sum_{i=1}^{n}\left\{\int_t^T \sigma_i(u,s)^2 du\right\}\right) \tag{5.3}$$

Using this result it is straightforward to show that the equation (5.1) is given by:

$$c(t,F(t,s);K,T,s) = P(t,T)[F(t,s)N(h) - KN(h-\sqrt{\omega})] \tag{5.4}$$

where

$$h = \frac{\ln(F(t,s)/K) + \frac{1}{2}\omega}{\sqrt{\omega}} \tag{5.5}$$

ω is the integral of the futures return variance over the life of the option.
The corresponding pricing formula for a standard put option, $p(t,F(t,s);K,T,s)$ can be easily obtained by put-call parity:

$$p(t,F(t,s);K,T,s) = P(t,T)[KN(-h+\sqrt{\omega}) - F(t,T)N(-h)] \tag{5.6}$$

European call and put options are then given by equation (5.4) and (5.6) respectively.
If ω is replaced by:

$$\omega = \sigma^2 (T-t) \tag{5.7}$$

then we have the **Black's (1976) Model**[1].
As was shown in Chapter 2 the Black's model is the standard industry tool for pricing the wide range of options found in the energy market.
If ω is replaced by:

$$\omega = \sum_{i=1}^{n} \left\{ \int^{T} \sigma_i (u,s)^2 du \right\} \tag{5.8}$$

then we have the **Multi-Factor Model.**

Clewlow and Strickland (1999a) show that for the single-factor restriction European calls and puts futures option prices are calculated via the equations (5.4) and (5.6) with ω given by:

$$\omega = \int^{T} \sigma(u,s)^2 du = \int^{T} \sigma^2 e^{-2\alpha(s-u)} du = \frac{\sigma^2}{2\alpha}\left(e^{-2\alpha(s-T)} - e^{-2\alpha(s-t)}\right) . \tag{5.9}$$

But according to our results in section 4.3 of Chapter 4 in the oil market we need a three-factor model. We can derive that the model applying equation (5.8) with $n = 3$.
The equation takes the following form:

$$\omega = \sum_{i=1}^{n=3} \left\{ \int^{T} \sigma_i (u,s)^2 du \right\} =$$

$$\frac{\sigma_1^2}{2a_1}(e^{-2a_1(s-T)} - e^{2a_1(s-t)_1}) + \frac{\sigma_2^2}{2a_2}(e^{-2a_2(s-T)} - e^{2a_2(s-t)_1}) + \frac{\sigma_3^2}{2a_3}(e^{-2a_3(s-T)} - e^{2a_3(s-t)_1}) \tag{5.10}$$

Here σ_1 represents the (fn01+fn02+fn03) volatility function or in other words the implied volatility, σ_2 the fn02 volatility function and σ_3 the fn03 volatility function whilst α_1, α_2 & α_3 tells us how fast the fn01 volatility curve, fn02 & fn03 attenuates with increasing maturity. We can calculate the α_i form the following equation:

$$\mathrm{F(1m,2m...9m)} = \mathrm{e}^{-\alpha(T-t)} \tag{5.11}$$

i.e. the α for the WTI for the fn01 volatility function (see Table-4.9) is:

$$37.05\% = e^{-\alpha(0.9643)} \Rightarrow \alpha = -\frac{\ln(37.05\%)}{1.052} \Rightarrow \alpha = 1.029.$$

T is the expiry of the ninth contract and t is the expiry of the first contract in this case.

---

[1] see Chapter 2 (Option Valuation)

As we can observe from Figure-4.9, Figure-4.11, Figure-4.13, Figure-4.15, Figure-4.17 and Figure-4.19 the volatility function fn01, fn02 has a simple negative exponential form whereas fn03 doesn't. Hence the third part of the equation (5.10) $\frac{1}{2a_3}(e^{-2a_3(s-T)} - e^{2a_3(s-t)_1})$ doesn't apply as the third factor fn03 is not found to be exponentially decreasing. We make the assumption that we have to replace it with the average of the prices of the fn03 (AVG fn03) which were calculated in the previous section of this chapter. Hence equation (5.10) takes the following form:

$$\omega = \sum_{i=1}^{n=3} \left\{ \int^{T} \sigma_i(u,s)^2 du \right\} =$$

$$\frac{\sigma_1^2}{2a_1}(e^{-2a_1(s-T)} - e^{2a_1(s-t)_1}) + \frac{\sigma_2^2}{2a_2}(e^{-2a_2(s-T)} - e^{2a_2(s-t)_1}) + \sigma_3^2 AVGfn03 \qquad (5.12)$$

The computer algorithms of the Black's model and the Three-Factor model above shown in Appendix-5.1 and Appendix-5.2 of this chapter.

5.2 Empirical Results

Arguably one of the most important issues in the use of any model for pricing and hedging derivatives is the calibration of the model to market data. Calibration is the process of choosing model parameters so that the prices returned by the model coincide with the observed market prices. Calibrating an energy model is analogous to choosing, or implying, a volatility parameter for the Black's model, when valuing say stock or index options, that equates the model price with market price. However in the Black's model the single variable that carries all the information is the volatility.
We have documented the need for extending the Black and Scholes model to allow for jumps and mean reversion rate.
Therefore, we are going to price options based on the historical parameters we calculated i.e. mean reversion rate, the first three factors from the PCA (Principal Component Analysis), and number of jumps, for the oil market.
Our previous PCA analysis in Chapter-4 shows us that for the oil market we need a three-factor model for option pricing. In order to make sure that the three-factor model is the ideal model we have to check it with market data and compare it with the Black's model, and see which one is closer to the market data. The market observed data are WTI options prices for the first nine months (Aug-02, Sep02, ...Apr03.) and are taken on the 31st of May 2002.
Our previous PCA analysis was based on data between January 1999 to 16th February 2001. As the option prices were taken from the market on the 31st of May 2002, we repeat the PCA analysis using more recent data from 1st of May 2001 to 31st of May 2002. We think 1 year worth of data will give an excellent idea about the recent market behaviour. We follow the same procedure that we described in Section 4.3 of Chapter-4. The first step is to construct a time series of forward price returns according to equation (4.9). Next we compute the sample covariance matrix by applying equation (4.8). The following table (Table 5.1) shows the covariance matrix for WTI crude oil futures.

Table-5.1 Covariance Matrix For WTI Crude Oil Futures

	F(1m)	F(2m)	F(3m)	F(4m)	F(5m)	F(6m)	F(7m)	F(8m)	F(9m)
F(1m)	0.000806	0.000732	0.000669	0.00063	0.000597	0.000568	0.00054	0.000515	0.000493
F(2m)	0.000732	0.00069	0.000632	0.000596	0.000566	0.000539	0.000513	0.00049	0.000469
F(3m)	0.000669	0.000632	0.000588	0.000554	0.000525	0.0005	0.000477	0.000456	0.000438
F(4m)	0.00063	0.000596	0.000554	0.000525	0.000499	0.000476	0.000455	0.000436	0.000418
F(5m)	0.000597	0.000566	0.000525	0.000499	0.000476	0.000456	0.000436	0.000418	0.000401
F(6m)	0.000568	0.000539	0.0005	0.000476	0.000456	0.000436	0.000418	0.000401	0.000386
F(7m)	0.00054	0.000513	0.000477	0.000455	0.000436	0.000418	0.000401	0.000385	0.000371
F(8m)	0.000515	0.00049	0.000456	0.000436	0.000418	0.000401	0.000385	0.000371	0.000357
F(9m)	0.000493	0.000469	0.000438	0.000418	0.000401	0.000386	0.000371	0.000357	0.000345

Table-5.2 Eigenvalues (λ) for WTI Crude Oil Futures

λ	0.004533	5.8E-05	1.3E-05	4.08E-06	1.76E-06	8.69E-07	-8.25E-07	3.16E-07	-2.01E-08

Table-5.2 shows us that the first two eigenvalues are significant. Adding these we get 0.00461 and then dividing each λ with 0.00461 we can see which eigenvalue is the most important. Figure-5.1 plots the eigenvalues for the WTI crude oil futures. The figure shows that the first eigenvalue is the most important, explaining 98.319 % of the total variation in the evolution of the future curve. Together the first two factors explain 99.579% of the total variation. The other factors as we mentioned above are not significant so a two-factor model is sufficient to explain the evolution of the WTI data over that period.
By applying the formula (4.10) we obtain the resulting volatility functions. The columns of Table-5.3 indicate the results of this calculation. The volatility functions illustrated in Figure-5.2 are the first two columns of Table-5.4.

Table-5.3 Eigenvectors for WTI Crude Oil Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	0.4106	0.7119	- 0.5400	- 0.1421	- 0.0256	- 0.0174	- 0.0174	0.0209	- 0.0139
F(2m)	0.3863	0.2519	0.4245	0.7595	- 0.0848	0.0743	0.0094	- 0.0408	0.0821
F(3m)	0.3571	0.0703	0.5248	- 0.4545	- 0.3637	- 0.0453	- 0.2248	0.0834	- 0.4399
F(4m)	0.3384	- 0.0055	0.2455	- 0.4025	0.2278	0.3583	0.3512	0.1002	0.5884
F(5m)	0.3228	- 0.1524	0.0043	- 0.0680	0.3134	- 0.4652	- 0.3643	- 0.6080	0.2187
F(6m)	0.3087	- 0.2153	- 0.0773	0.0332	0.4959	- 0.1966	0.5118	0.1137	- 0.5401
F(7m)	0.2950	- 0.2904	- 0.1570	0.1323	0.0633	- 0.2065	- 0.4288	0.7199	0.1936
F(8m)	0.2826	- 0.3323	- 0.2718	0.1010	0.0244	0.7206	- 0.3027	- 0.2395	- 0.2380
F(9m)	0.2713	- 0.3825	- 0.2932	0.0277	- 0.6776	- 0.2161	0.3827	- 0.1465	0.1396

Figure- 5.1

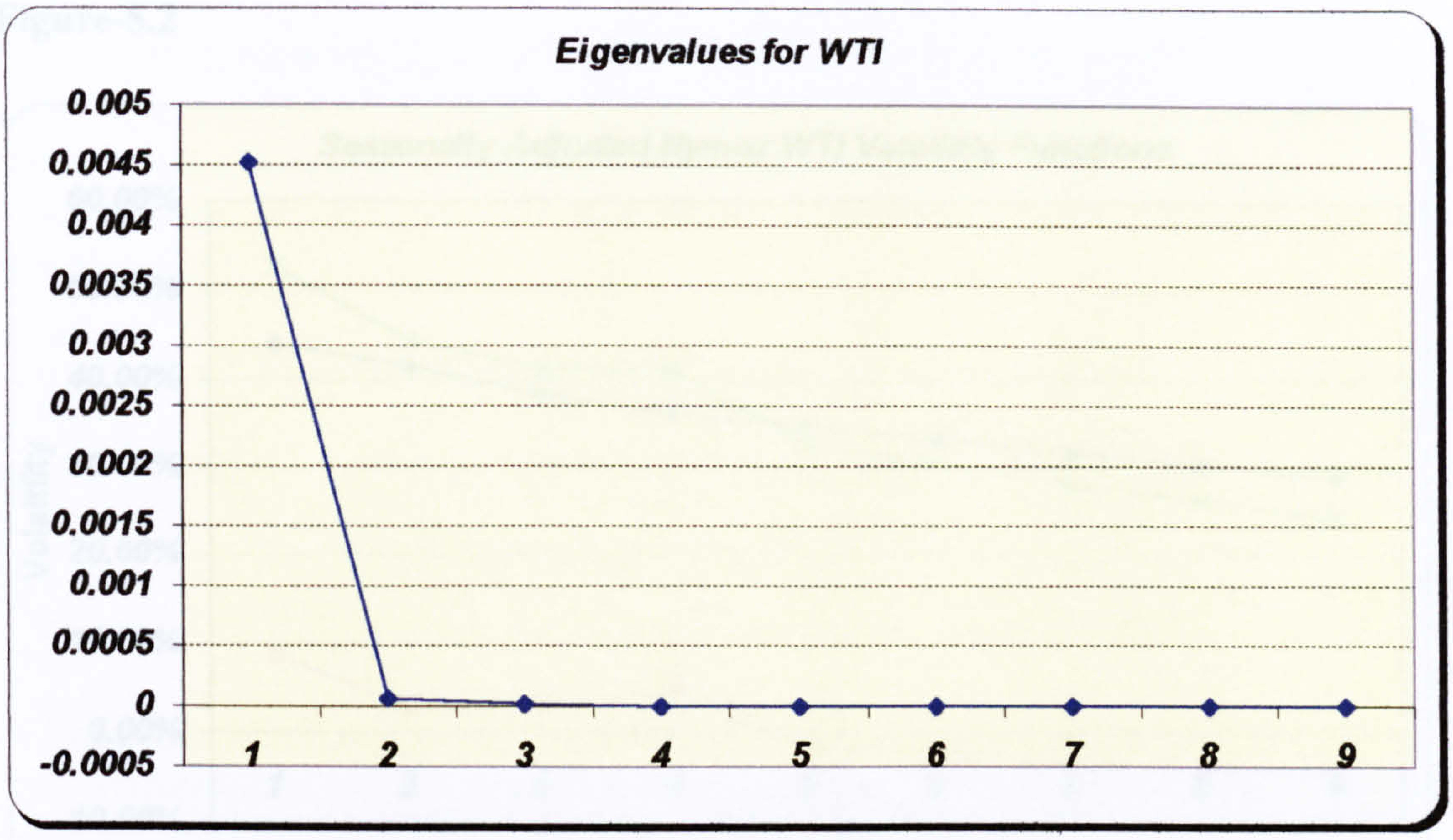
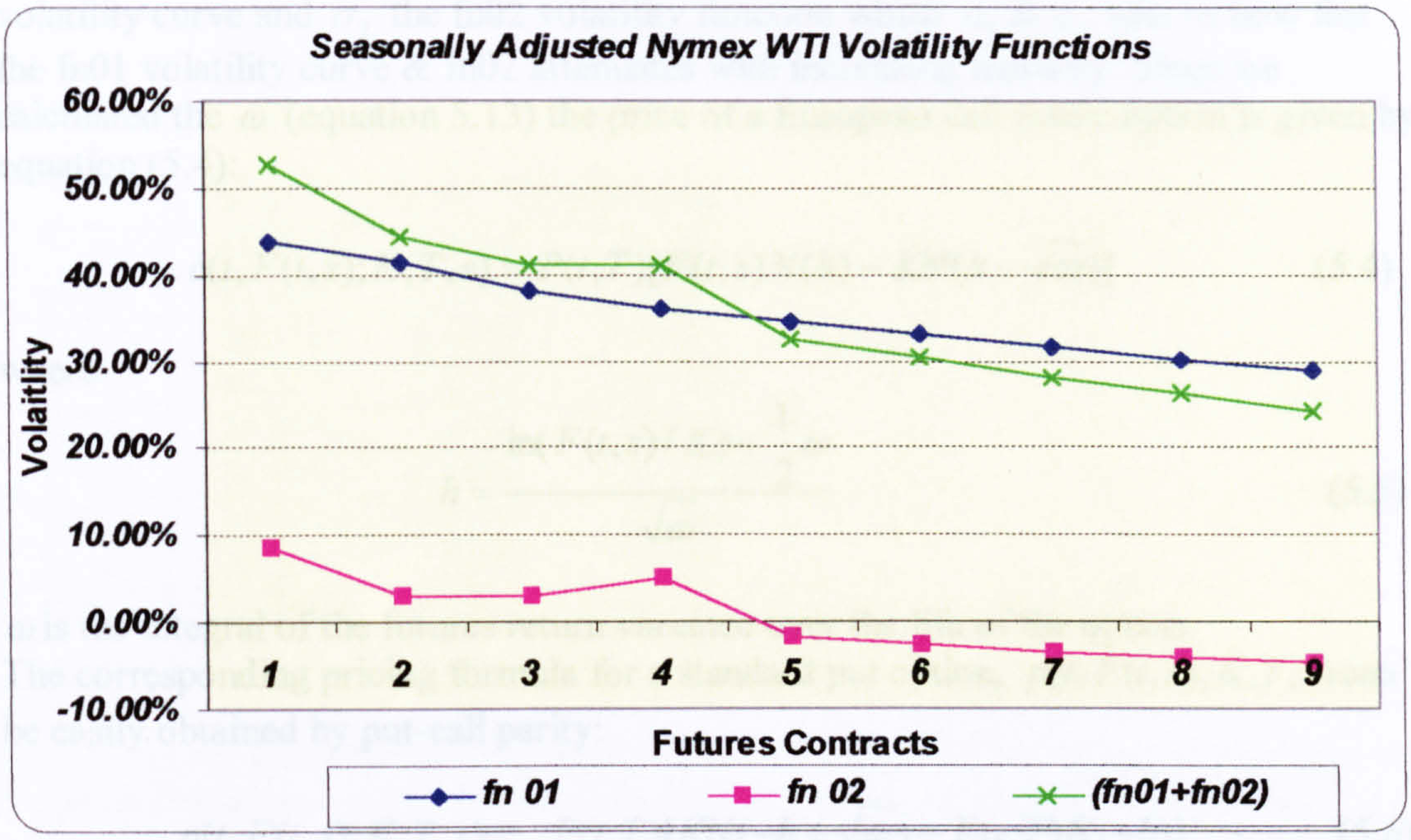


Table-5.4 Volatility Functions for WTI Futures

	fn 01	fn 02	fn 03	fn 04	fn 05	fn 06	fn 07	fn 08	fn 09
F(1m)	43.88%	8.61%	-3.12%	-0.46%	-0.05%	-0.03%	#NUM!	0.02%	#NUM!
F(2m)	41.29%	3.05%	2.45%	2.44%	-0.18%	0.11%	#NUM!	-0.04%	#NUM!
F(3m)	38.16%	3.00%	3.03%	-1.46%	-0.76%	-0.07%	#NUM!	0.07%	#NUM!
F(4m)	36.17%	5.00%	1.42%	-1.29%	0.48%	0.53%	#NUM!	0.09%	#NUM!
F(5m)	34.50%	-1.84%	0.02%	-0.22%	0.66%	-0.69%	#NUM!	-0.54%	#NUM!
F(6m)	32.99%	-2.60%	-0.45%	0.11%	1.04%	-0.29%	#NUM!	0.10%	#NUM!
F(7m)	31.53%	-3.51%	-0.91%	0.42%	0.13%	-0.31%	#NUM!	0.64%	#NUM!
F(8m)	30.20%	-4.02%	-1.57%	0.32%	0.05%	1.07%	#NUM!	-0.21%	#NUM!
F(9m)	28.99%	-4.63%	-1.69%	0.09%	-1.43%	-0.32%	#NUM!	-0.13%	#NUM!

Figure-5.2



In Figure-5.2 above (as explained earlier in Chapter-4):

- **fn01** constitutes the volatility function.
- **fn02** indicates the volatility smile in the front traded months, there are many options with different strikes from which "smile" can be implied. The further in the future you look, due to less liquidity and more uncertainty, this effect is harder to discern, hence in the diagram we see fn02 tending lower than zero.
- **(fn01+fn02)** represents the final volatility curve with the smile effects included.

The PCA shows us that for the oil market between 1st May 2001 to 31st May 2002 we need a two-factor model for option pricing. As we can see, different chronological periods give different results. We have shown in Section 4.3 of Chapter-4, based on the PCA analysis, that for the oil market we need a three-factor model between 1st January 1999 to 16th February 2001 for pricing the options, but between 1st May 2001 to 31st May 2002 we need a two-factor model. This is because the market is changing continuously, i.e. fundamentals, liquidity, events, OPEC, 11 September, etc. Furthermore, it is widely known that PCA analysis is sensitive to the specific period used.
In order to make sure that the two-factor model is legitimate we have to check it with market data and compare it with the Black's model, and see which is closer to the market data. We can derive the two-factor model by applying equation (5.8) with n=2. The equation takes the following form:

$$\omega = \sum_{i=1}^{n=2} \left\{ \int^{T} \sigma_i(u,s)^2 du \right\} = \frac{\sigma_1^2}{2a_1}(e^{-2a_1(s-T)} - e^{2a_1(s-t)_1}) + \frac{\sigma_2^2}{2a_2}(e^{-2a_2(s-T)} - e^{2a_2(s-t)_1}) \quad (5.13)$$

Here σ_1 represents the (fn01+fn02) volatility function (implied volatility), the final volatility curve and σ_2 the fn02 volatility function whilst α_1 & α_2 tells us how fast the fn01 volatility curve & fn02 attenuates with increasing maturity. Since we calculated the ω (equation 5.13) the price of a European call future option is given by equation (5.4):

$$c(t,F(t,s);K,T,s) = P(t,T)[F(t,s)N(h) - KN(h-\sqrt{\omega})] \quad (5.4)$$

where

$$h = \frac{\ln(F(t,s)/K) + \frac{1}{2}\omega}{\sqrt{\omega}} \quad (5.5)$$

ω is the integral of the futures return variance over the life of the option.
The corresponding pricing formula for a standard put option, $p(t,F(t,s);K,T,s)$ can be easily obtained by put-call parity:

$$p(t,F(t,s);K,T,s) = P(t,T)[KN(-h+\sqrt{\omega}) - F(t,T)N(-h)] \quad (5.6)$$

The market observed data are WTI options prices for the first nine months (Aug 02, Sep 02, ..., Apr 03) and are taken on the 31st May 2002.
Our first step is to price the at-the-money options with the Black's model in order to find the at-the-money volatility (Table-5.9). Secondly we are going to price the at-the-money options with the at-the-money volatility with the two-factor model and compare the results with the Black's model. Also we are going to price the out-of-the-money options with the Black's model using the at-the-money volatility in order to find the smile/skew. In order to capture the smile/skew in the Black's model the only variable we have to calibrate is the volatility. Table-5.11 shows the market call/put option prices and the comparison of all the prices across the two models. In the Black's model we made an adjustment in the volatility in order to capture the smile/skew of the market. In the two-factor model, the factors that we are using to calculate the option prices have been calculated from the PCA analysis. From the PCA analysis we concluded that **(fn01+fn02)** represents the final volatility curve with the smile effects included, so by finding the at-the-money volatility from the option prices we know the final volatility curve. In order to use this implied volatility curve to price these options with the two-factor model, we have to break it down in two factors fn01 & fn02, which added together give us the implied volatility curve. Hence, we have the following equation:

$$\textit{Implied Volatility} = fn01 + fn02 \quad (5.14)$$

Our question is: how can we calculate the fn01 & fn02? We can calculate the percentage fn02/Implied Volatility, fn01/ Implied Volatility from PCA analysis and then apply these percentages to the implied volatility calculated from the Black's model.

Table-5.4 shows us the volatility functions for WTI from the PCA analysis. The first two factors, fn01 and fn02, are significant. If we add them up, we have the final volatility curve.

Table-5.5 shows the final volatility curve and the first two factors fn01 & fn02 from the PCA analysis.

Table-5.5

	fn 01	fn 02	(fn01+fn02)
F(1m)	43.88%	8.61%	52.49%
F(2m)	41.29%	3.05%	44.33%
F(3m)	38.16%	3.00%	41.16%
F(4m)	36.17%	5.00%	41.17%
F(5m)	34.50%	-1.84%	32.65%
F(6m)	32.99%	-2.60%	30.39%
F(7m)	31.53%	-3.51%	28.01%
F(8m)	30.20%	-4.02%	26.18%
F(9m)	28.99%	-4.63%	24.37%

As we can see from Table-5.5 and the Figure-5.2 the volatility function fn01, fn02 has a simple negative exponential form but fn02 has negative numbers. The highest negative number is –4.63% so we make the following modification: We add that number with the opposite sign 4.63, to the fn02 in order to give us positive numbers. Table-5.6 shows us the final volatility curve and the first two factors fn01 & fn02.

Table-5.6

	fn 01	fn 02	(fn01+fn02)
F(1m)	43.88%	13.24%	57.12%
F(2m)	41.29%	7.68%	48.96%
F(3m)	38.16%	7.63%	45.79%
F(4m)	36.17%	9.63%	45.80%
F(5m)	34.50%	2.79%	37.28%
F(6m)	32.99%	2.03%	35.02%
F(7m)	31.53%	1.12%	32.64%
F(8m)	30.20%	0.61%	30.81%
F(9m)	28.99%	0.00%	29.00%

Our next step is to find the percentage of fn01 & fn02 to the final volatility curve from the PCA analysis. Table-5.7 shows us the percentage of fn01 & fn02 to the final volatility curve.

Table-5.7

	% of fn 01 to the Final Volatility Curve	% of fn 02 to the Final Volatility Curve
F(1m)	76.817%	23.183%
F(2m)	84.320%	15.680%
F(3m)	83.339%	16.661%
F(4m)	78.972%	21.028%
F(5m)	92.526%	7.474%
F(6m)	94.216%	5.784%
F(7m)	96.579%	3.421%
F(8m)	98.020%	1.980%
F(9m)	99.990%	0.010%

Table-5.8 shows the implied volatility from the option prices. If we multiply the implied volatility with the numbers from the Table-5.7 we can calculate the implied factors fn01 & fn02. Table-5.9 shows us the implied factors fn01 & fn02.

Table-5.8

	ATM VOL
Aug-02	52.50%
Sep-02	40.00%
Oct-02	36.00%
Nov-02	38.00%
Dec-02	32.50%
Jan-03	30.50%
Feb-03	30.00%
Mar-03	29.00%
Apr-03	28.15%

Table-5.9

	ATM VOL	Implied fn01	Implied fn02
Aug-02	52.50%	40.329%	12.171%
Sep-02	40.00%	33.728%	6.272%
Oct-02	36.00%	30.002%	5.998%
Nov-02	38.00%	30.009%	7.991%
Dec-02	32.50%	30.071%	2.429%
Jan-03	30.50%	28.736%	1.764%
Feb-03	30.00%	28.974%	1.026%
Mar-03	29.00%	28.426%	0.574%
Apr-03	28.15%	28.147%	0.003%

Based on the factors above (Table-5.9) we can price the WTI options with the two-factor model and compare them with the Black's model.

Table-5.10 shows us the interest rates that we are using to price the options. We took the interest rates from Bloomberg on the 31st of May 2002.
Appendix-5.3 gives the computer algorithm of the Two-Factor model.

Table-5.10

	R
1 month	1.78
2 month	1.768
3 month	1.75
4 month	1.741
5 month	1.74
6 month	1.738
7 month	1.742
8 month	1.748
9 month	1.759

Table-5.11

WTI	SETTLE	STRIKE	CALL	PUT	B.M. CALLS	B.M. PUTS	TWO FACTOR MODEL CALLS	TWO FACTOR MODEL PUTS
Aug-02	24.85	21.50	3.66	0.33	3.611	0.301	3.593	0.256
Aug-02	24.85	22.00	3.26	0.43	3.211	0.404	3.190	0.351
Aug-02	24.85	22.50	2.85	0.54	2.835	0.529	2.811	0.469
Aug-02	24.85	23.00	2.51	0.67	2.483	0.680	2.456	0.613
Aug-02	24.85	23.50	2.18	0.84	2.159	0.857	2.130	0.785
Aug-02	24.85	24.00	1.89	1.04	1.862	1.060	1.831	0.984
Aug-02	24.85	24.50	1.61	1.26	1.594	1.292	1.562	1.213
Aug-02	**24.85**	**25.00**	**1.36**	**1.51**	**1.353**	**1.503**	**1.321**	**1.470**
Aug-02	24.85	25.50	1.14	1.79	1.140	1.835	1.108	1.756
Aug-02	24.85	26.00	0.95	2.09	0.954	2.145	0.922	2.068
Aug-02	24.85	26.50	0.79	2.43	0.791	2.480	0.761	2.405
Aug-02	24.85	27.00	0.64	2.78	0.652	2.836	0.623	2.765
Aug-02	24.85	27.50	0.53	3.16	0.533	3.212	0.507	3.147
Aug-02	24.85	28.00	0.43	3.56	0.433	3.607	0.409	3.547
Aug-02	24.85	28.50	0.35	3.98	0.349	4.018	0.327	3.963
Sep-02	24.79	21.50	3.78	0.52	3.725	0.494	3.656	0.398
Sep-02	24.79	22.00	3.41	0.64	3.355	0.624	3.279	0.515
Sep-02	24.79	22.50	3.04	0.77	3.007	0.774	2.923	0.655
Sep-02	24.79	23.00	2.72	0.94	2.682	0.946	2.591	0.818
Sep-02	24.79	23.50	2.4	1.12	2.380	1.141	2.283	1.005
Sep-02	24.79	24.00	2.12	1.33	2.101	1.359	2.000	1.217
Sep-02	24.79	24.50	1.84	1.55	1.845	1.600	1.742	1.455
Sep-02	**24.79**	**25.00**	**1.62**	**1.83**	**1.613**	**1.821**	**1.509**	**1.717**
Sep-02	24.79	25.50	1.41	2.12	1.403	2.149	1.299	2.002
Sep-02	24.79	26.00	1.22	2.42	1.215	2.456	1.112	2.311
Sep-02	24.79	26.50	1.05	2.75	1.047	2.782	0.947	2.641

Sep-02	24.79	27.00	0.9	3.09	0.898	3.127	0.802	2.991
Sep-02	24.79	27.50	0.76	3.45	0.767	3.489	0.676	3.360
Sep-02	24.79	28.00	0.65	3.83	0.652	3.868	0.567	3.746
Sep-02	24.79	28.50	0.55	0	0.552	4.261	0.473	4.148
Oct-02	24.65	21.00	4.26	0.65	4.150	0.616	4.020	0.430
Oct-02	24.65	21.50	3.89	0.78	3.789	0.752	3.645	0.546
Oct-02	24.65	22.00	3.54	0.92	3.446	0.907	3.289	0.683
Oct-02	24.65	22.50	2.34	1.08	3.124	1.081	2.955	0.840
Oct-02	24.65	23.00	2.88	1.25	2.822	1.274	2.642	1.019
Oct-02	24.65	23.50	2.59	1.45	2.540	1.487	2.352	1.221
Oct-02	24.65	24.00	2.32	1.67	2.278	1.719	2.084	1.445
Oct-02	**24.65**	**24.50**	**2.06**	**1.91**	**2.064**	**1.916**	**1.839**	**1.691**
Oct-02	24.65	25.00	1.82	2.17	1.814	2.241	1.615	1.960
Oct-02	24.65	25.50	1.61	2.46	1.611	2.530	1.413	2.249
Oct-02	24.65	26.00	1.43	2.77	1.426	2.836	1.230	2.558
Oct-02	24.65	26.50	1.26	3.09	1.258	3.159	1.067	2.887
Oct-02	24.65	27.00	1.11	3.45	1.107	3.497	0.922	3.233
Oct-02	24.65	27.50	0.97	3.64	0.971	3.851	0.793	3.597
Oct-02	24.65	28.00	0.85	4.16	0.849	4.218	0.680	3.975
Nov-02	24.51	21.00	4.27	0.81	4.142	0.801	4.072	0.639
Nov-02	24.51	21.50	3.92	0.95	3.798	0.954	3.723	0.779
Nov-02	24.51	22.00	3.59	1.11	3.473	1.125	3.392	0.937
Nov-02	24.51	22.50	-	1.28	3.167	1.313	3.081	1.115
Nov-02	24.51	23.00	2.96	1.47	2.879	1.519	2.789	1.312
Nov-02	24.51	23.50	2.68	1.68	2.609	1.743	2.517	1.529
Nov-02	24.51	24.00	2.42	1.91	2.359	1.984	2.264	1.765
Nov-02	**24.51**	**24.50**	**2.43**	**2.42**	**2.410**	**2.401**	**2.030**	**2.020**
Nov-02	24.51	25.00	1.93	2.42	1.912	2.519	1.815	2.294
Nov-02	24.51	25.50	1.72	2.70	1.715	2.810	1.618	2.586
Nov-02	24.51	26.00	1.54	3.01	1.534	3.117	1.438	2.896
Nov-02	24.51	26.50	1.37	3.34	1.369	3.440	1.275	3.222
Nov-02	24.51	27.00	1.22	3.68	1.219	3.776	1.127	3.563
Nov-02	24.51	27.50	1.07	4.17	1.083	4.125	0.994	3.919
Nov-02	24.51	28.00	0.94	4.38	0.960	4.488	0.874	4.288
Dec-02	24.37	21.00	4.28	0.97	4.112	0.956	3.834	0.557
Dec-02	24.37	21.50	3.95	1.13	3.783	1.121	3.481	0.690
Dec-02	24.37	22.00	3.62	1.29	3.471	1.303	3.148	0.843
Dec-02	24.37	22.50	3.31	1.47	3.177	1.501	2.836	1.017
Dec-02	24.37	23.00	3.03	1.67	2.900	1.716	2.544	1.212
Dec-02	24.37	23.50	2.76	1.89	2.641	1.948	2.274	1.428
Dec-02	24.37	24.00	2.5	2.13	2.400	2.196	2.024	1.664
Dec-02	**24.37**	**24.50**	**2.23**	**2.36**	**2.228**	**2.355**	**1.795**	**1.922**
Dec-02	24.37	25.00	1.99	2.62	1.968	2.739	1.586	2.199
Dec-02	24.37	25.50	1.78	2.9	1.776	3.034	1.396	2.495
Dec-02	24.37	26.00	1.59	3.2	1.600	3.342	1.225	2.810
Dec-02	24.37	26.50	1.42	2.88	1.438	3.664	1.071	3.142
Dec-02	24.37	27.00	1.26	0	1.290	4.000	0.933	3.491
Dec-02	24.37	27.50	1.13	4.21	1.155	4.347	0.810	3.854
Dec-02	24.37	28.00	1	4.57	1.032	4.707	0.701	4.232
Jan-03	24.20	20.50	4.51	0.88	4.357	0.865	4.034	0.458
Jan-03	24.20	21.00	4.16	1.02	4.022	1.019	3.669	0.576
Jan-03	24.2	21.50	3.83	1.18	3.704	1.190	3.323	0.713

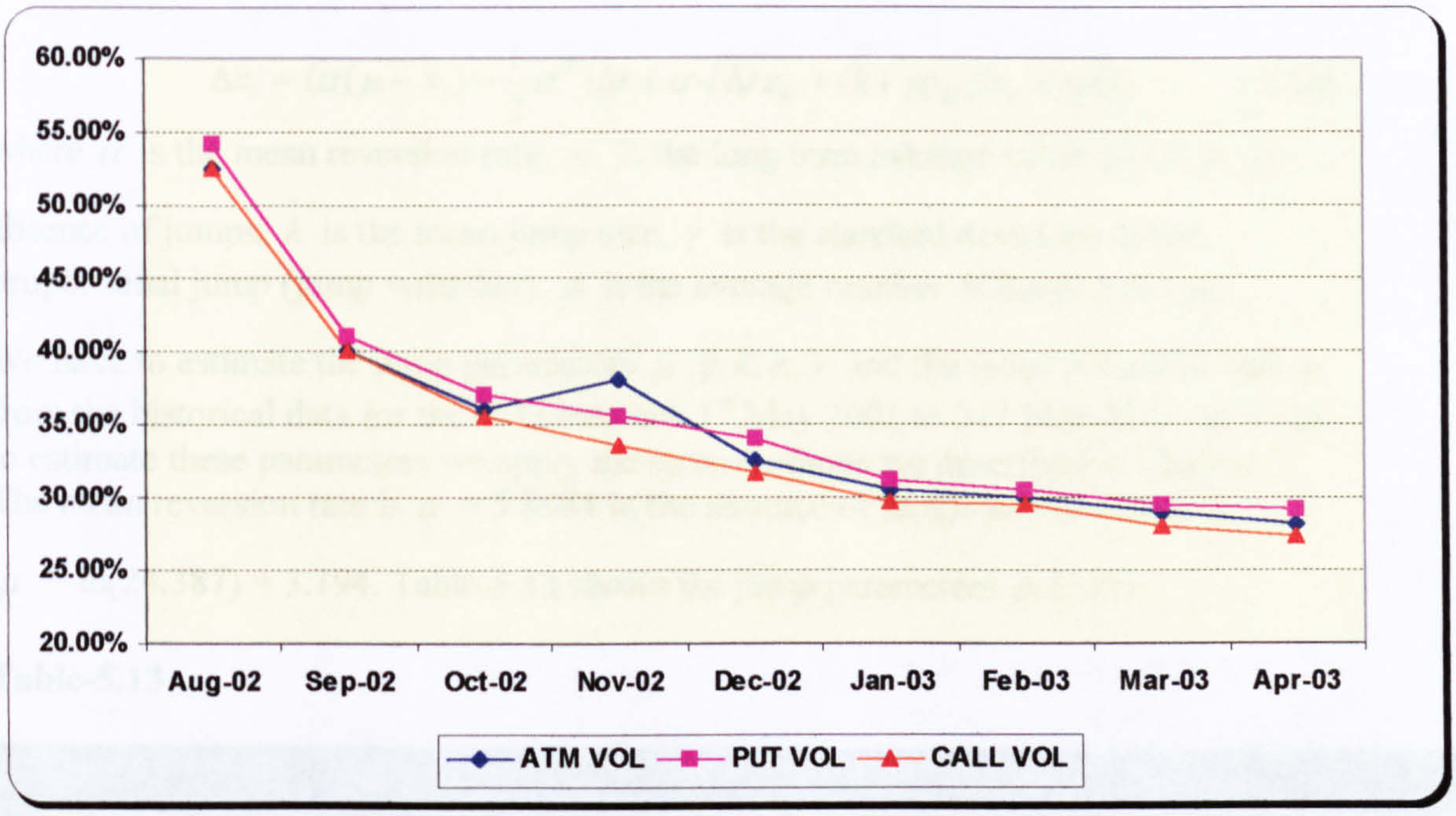
Jan-03	24.2	22.00	2.74	1.36	3.403	1.378	2.997	0.871
Jan-03	24.2	22.50	2.54	0	3.119	1.581	2.692	1.049
Jan-03	24.2	23.00	2.94	1.75	2.852	1.802	2.409	1.249
Jan-03	24.2	23.50	2.67	1.97	2.603	2.038	2.146	1.469
Jan-03	**24.2**	**24.00**	**2.41**	**2.21**	**2.427**	**2.233**	**1.905**	**1.711**
Jan-03	24.2	24.50	2.17	2.47	2.153	2.558	1.684	1.974
Jan-03	24.2	25.00	1.96	2.76	1.953	2.841	1.483	2.256
Jan-03	24.2	25.50	1.76	3.05	1.767	3.139	1.301	2.558
Jan-03	24.2	26.00	1.58	0	1.596	3.450	1.137	2.877
Jan-03	24.2	26.50	1.41	0	1.439	3.775	0.991	3.214
Jan-03	24.2	27.00	1.25	0	1.295	4.112	0.860	3.566
Feb-03	24.06	20.50	4.53	1.04	3.780	0.410	3.603	0.182
Feb-03	24.06	21.00	-	1.20	3.407	0.524	3.199	0.259
Feb-03	24.06	21.50	3.88	1.37	3.054	0.658	2.817	0.357
Feb-03	24.06	22.00	0	1.55	2.723	0.813	2.460	0.480
Feb-03	**24.06**	**24.50**	**2.26**	**2.69**	**1.457**	**1.880**	**1.090**	**1.513**
Feb-03	24.06	25.00	2.05	2.97	1.223	2.208	0.902	1.805
Feb-03	24.06	26.00	1.68	0	0.899	2.841	0.601	2.465
Feb-03	24.06	26.50	1.51	0	0.765	3.184	0.484	2.828
Feb-03	24.06	27.00	1.35	0	0.648	3.544	0.386	3.212
Mar-03	23.92	22.00	3.55	1.66	3.368	1.653	2.795	0.961
Mar-03	23.92	23.00	-	2.08	2.854	2.101	2.238	1.359
Mar-03	**23.92**	**24.00**	**2.48**	**2.56**	**2.489**	**2.565**	**1.763**	**1.840**
Mar-03	23.92	26.00	1.68	3.01	1.670	3.786	1.047	3.034
Mar-03	23.92	26.50	1.52	-	1.519	4.111	0.911	3.376
Mar-03	23.92	27.00	1.37	4.55	1.380	4.448	0.790	3.732

From the market price data in the tables above we can see positive skew to the out-of-the-money puts and negative smile to the out-of-the-money calls. Hence, in order to price an out-of-the-money put with Black's model we need higher volatility in the model and in order to price an out-of-the-money call we need to input lower volatility into the model. Figure-5.3 illustrates the at the money volatility, the out-of-the-money call volatility and the out-of-the-money put volatility. Also we observe that two-factor model performs better for the nearby future to the longer expiration contracts at the beginning, it better captures the skew to the puts in the Aug-02 and Sep-02 future contract and also it captures better the smile to the calls in the same contracts. But on the rest of the contracts the two-factor model performs very poorly, it doesn't capture satisfactorily the smile to the calls and the skew to the puts, which means that the factors that we calculated historically are correct according to history but inappropriate going forward. This is because the market is changing all the time i.e. fundamentals, liquidity, events, OPEC, etc. and these factors have not been taken into account in our option pricing. That is why the professionals calibrate their models according to the observed market data. History is very important because it gives us a very good indication about the market but the future can be radically different. After 11th September the market changed drastically and traders are interested in buying out-of-the-money puts for the protection of their books, which is why puts are so well bid above the at-the-money volatility. They are much less interested in upside calls. If the market becomes bullish again then we will have the opposite scenario (calls being well bid, puts being well offered).

Table-5.12

	ATM VOL	PUT VOL	CALL VOL
Aug-02	52.50%	54.25%	52.50%
Sep-02	40.00%	41.00%	40.00%
Oct-02	36.00%	37.00%	35.50%
Nov-02	38.00%	35.50%	33.50%
Dec-02	32.50%	34.00%	31.75%
Jan-03	30.50%	31.25%	29.75%
Feb-03	30.00%	30.50%	29.50%
Mar-03	29.00%	29.50%	28.00%
Apr-03	28.15%	29.25%	27.35%

Figure-5.3



5.3 Option Pricing with Monte Carlo Simulation (Part B)

In this section of the chapter we are going to price the same options with the Mean Reversion Jump Diffusion Model and the Merton's Jump Diffusion Model and compare them. For these two models there isn't any analytical formulae so, Monte Carlo Simulation (first used by Boyle, 1977)[2] provides a simple and flexible method for valuing the options prices for those two models.
We presented these models on Chapter-3; the Jump Diffusion Model has the following form:

$$\Delta x_i = (r - \phi \bar{k} - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{1i} + (\bar{k} + \gamma\varepsilon_{2i})(u_i < \phi\Delta t) \qquad (5.15)$$

and the Mean Reversion Jump Diffusion has the following form:

$$\Delta x_i = (\alpha(\mu - x_i) - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{1i} + (\bar{k} + \gamma\varepsilon_{2i})(u_i < \varphi\Delta t) \qquad (5.16)$$

where α is the mean reversion rate, μ is the long term average value of *lnS* in the absence of jumps, $\bar{k}$ is the mean jump size, γ is the standard deviation of the proportional jump (jump volatility), ϕ is the average number of jumps per year.

We have to estimate the jump parameters $\mu, \phi, \bar{k}, \sigma, \gamma$ and the mean reversion rate α from the historical data for the WTI between 1st May 2001 to 31st May 2002. In order to estimate these parameters we apply the same methods we described in Chapter 3. The mean reversion rate is α = 5.8684 in the absence of jumps and μ which is μ = ln(24.387) = 3.194. Table-5.13 shows the jump parameters $\phi, \bar{k}, \sigma, \gamma$.

Table-5.13

Interations	Mean	Volatility	Standard Deviation	WTI Jumps	ϕ	κ	γ
1	-0.000474	43.9466%	0.027683732	3	2.661972	-0.00135	0.013417
2	0.000873	38.3627%	0.024166205	5	4.43662	-0.00079	0.015023
3	0.000316	36.8957%	0.023242104	5	4.43662	-0.00079	0.015023
4	0.000316	36.8305%	0.023201011	5	4.43662	-0.00079	0.014997
5	0.000316	36.8305%	0.023201011	5	4.43662	-0.00079	0.014997
6	0.000316	36.8305%	0.023201011	5	4.43662	-0.00079	0.014997
7	0.000316	36.8305%	0.023201011	5	4.43662	-0.00079	0.014997
8	0.000316	36.8305%	0.023201011	5	4.43662	-0.00079	0.014997
9	0.000316	36.8305%	0.023201011	5	4.43662	-0.00079	0.014997
10	0.000316	36.8305%	0.023201011	5	4.43662	-0.00079	0.014997

[2] see Chapter 2 of the Thesis (Option Valuation)

Since the parameters μ ,$\phi, \bar{k}, \sigma, \gamma$ and α have been estimated we apply ten thousands simulations (M = 10000), or in other words ten thousand simulated paths and for each simulated path we compute the pay-off of the call option max (0, $S_T - K$). To obtain the estimate of the call price we simply take the discounted average of the simulated payoffs:

$$C = \exp(-rT)\frac{1}{M}\sum_{j=1}^{M}\max(0, S_T - K) \qquad (5.17)$$

Appendix-5.4 gives the computer algorithm for the Jump Diffusion Model and Appendix-5.5 gives the computer algorithm for the Mean Reversion Jump Diffusion Model.
The option prices are illustrated in Table-5.14.

Table-5.14

WTI					Calls	Puts	Calls	Puts
	SETTLE	STRIKE	CALL	PUT	Merton's J.D.M	Merton's J.D.M	MRJD	MRJD
Aug-02	24.85	21.50	3.66	0.33	3.768	0.264	#VALUE!	#VALUE!
Aug-02	24.85	22.00	3.26	0.43	3.290	0.380	#VALUE!	#VALUE!
Aug-02	24.85	22.50	3.27	0.54	2.933	0.494	#VALUE!	#VALUE!
Aug-02	24.85	23.00	2.51	0.67	2.578	0.651	#VALUE!	#VALUE!
Aug-02	24.85	23.50	2.18	0.84	2.278	0.814	#VALUE!	#VALUE!
Aug-02	24.85	24.00	1.89	1.04	1.955	1.007	#VALUE!	#VALUE!
Aug-02	24.85	24.50	1.61	1.26	1.730	1.245	#VALUE!	#VALUE!
Aug-02	**24.85**	**25.00**	**1.36**	**1.51**	**1.433**	1.500	**#VALUE!**	**#VALUE!**
Aug-02	24.85	25.50	1.14	1.79	1.206	1.800	#VALUE!	#VALUE!
Aug-02	24.85	26.00	0.95	2.09	1.046	2.100	#VALUE!	#VALUE!
Aug-02	24.85	26.50	0.79	2.43	0.887	2.410	#VALUE!	#VALUE!
Aug-02	24.85	27.00	0.64	2.78	0.716	2.746	#VALUE!	#VALUE!
Aug-02	24.85	27.50	0.53	3.16	0.582	3.142	#VALUE!	#VALUE!
Aug-02	24.85	28.00	0.43	3.56	0.350	3.522	#VALUE!	#VALUE!
Aug-02	24.85	28.50	0.35	3.98	0.220	3.887	#VALUE!	#VALUE!
Sep-02	24.79	21.50	3.78	0.52	3.876	0.470	3.442	0.038
Sep-02	24.79	22.00	3.41	0.64	3.572	0.587	3.027	0.074
Sep-02	24.79	22.50	3.04	0.77	3.350	0.680	2.530	0.118
Sep-02	24.79	23.00	2.72	0.94	2.909	0.801	2.156	0.196
Sep-02	24.79	23.50	2.4	1.12	2.525	0.963	1.776	0.313
Sep-02	24.79	24.00	2.12	1.33	2.299	1.103	1.417	0.454
Sep-02	24.79	24.50	1.84	1.55	1.945	1.170	1.105	0.665
Sep-02	**24.79**	**25.00**	**1.62**	**1.83**	**1.790**	**1.455**	**0.854**	**0.903**
Sep-02	24.79	25.50	1.41	2.12	1.582	1.690	0.643	1.162
Sep-02	24.79	26.00	1.22	2.42	1.335	1.881	0.446	1.513
Sep-02	24.79	26.50	1.05	2.75	1.186	2.156	0.321	1.867
Sep-02	24.79	27.00	0.9	3.09	1.033	2.370	0.223	2.276
Sep-02	24.79	27.50	0.76	3.45	0.875	2.628	0.153	2.686
Sep-02	24.79	28.00	0.65	3.83	0.706	2.936	0.108	3.160
Sep-02	24.79	28.50	0.55	0	0.662	3.206	0.068	3.649
Oct-02	24.65	21.00	4.26	0.65	4.542	0.560	4.613	0.182

Oct-02	24.65	21.50	3.89	0.78	4.193	0.642	4.111	0.253
Oct-02	24.65	22.00	3.54	0.92	3.811	0.746	3.790	0.335
Oct-02	24.65	22.50	2.34	1.08	3.389	0.936	3.318	0.450
Oct-02	24.65	23.00	2.88	1.25	3.107	1.114	2.949	0.583
Oct-02	24.65	23.50	2.59	1.45	2.900	1.324	2.710	0.754
Oct-02	24.65	24.00	2.32	1.67	2.570	1.532	2.344	0.899
Oct-02	**24.65**	**24.50**	**2.06**	**1.91**	**2.339**	**1.742**	**2.057**	**1.134**
Oct-02	24.65	25.00	1.82	2.17	2.033	2.032	1.811	1.400
Oct-02	24.65	25.50	1.61	2.46	1.850	2.286	1.570	1.583
Oct-02	24.65	26.00	1.43	2.77	1.720	2.541	1.300	1.875
Oct-02	24.65	26.50	1.26	3.09	1.522	2.864	1.141	2.166
Oct-02	24.65	27.00	1.11	3.45	1.325	3.220	0.936	2.460
Oct-02	24.65	27.50	0.97	3.64	1.111	3.578	0.820	2.845
Oct-02	24.65	28.00	0.85	4.16	0.991	3.925	0.710	3.242
Nov-02	24.51	21.00	4.27	0.81	4.900	0.841	5.404	0.377
Nov-02	24.51	21.50	3.92	0.95	4.495	0.969	4.878	0.459
Nov-02	24.51	22.00	3.59	1.11	4.090	1.109	4.585	0.561
Nov-02	24.51	22.50	-	1.28	3.802	1.307	4.250	0.704
Nov-02	24.51	23.00	2.96	1.47	3.520	1.515	3.850	0.859
Nov-02	24.51	23.50	2.68	1.68	3.218	1.735	3.567	0.989
Nov-02	24.51	24.00	2.42	1.91	3.020	1.955	3.118	1.192
Nov-02	**24.51**	**24.50**	**2.43**	**2.42**	**2.710**	**2.177**	**2.979**	**1.385**
Nov-02	24.51	25.00	1.93	2.42	2.499	2.455	2.750	1.624
Nov-02	24.51	25.50	1.72	2.70	2.299	2.737	2.500	1.865
Nov-02	24.51	26.00	1.54	3.01	2.072	3.080	2.203	2.074
Nov-02	24.51	26.50	1.37	3.34	1.940	3.345	1.870	2.360
Nov-02	24.51	27.00	1.22	3.68	1.714	3.633	1.772	2.651
Nov-02	24.51	27.50	1.07	4.17	1.575	3.982	1.620	2.991
Nov-02	24.51	28.00	0.94	4.38	1.420	4.320	1.520	3.314
Dec-02	24.37	21.00	4.28	0.97	4.730	0.770	6.062	0.290
Dec-02	24.37	21.50	3.95	1.13	4.386	0.864	5.649	0.355
Dec-02	24.37	22.00	3.62	1.29	4.025	1.050	5.289	0.442
Dec-02	24.37	22.50	3.31	1.47	3.715	1.203	4.950	0.544
Dec-02	24.37	23.00	3.03	1.67	3.410	1.412	4.650	0.720
Dec-02	24.37	23.50	2.76	1.89	3.128	1.610	4.213	0.800
Dec-02	24.37	24.00	2.5	2.13	2.901	1.822	3.762	0.949
Dec-02	**24.37**	**24.50**	**2.23**	**2.36**	**2.626**	**2.083**	3.512	1.106
Dec-02	24.37	25.00	1.99	2.62	2.383	2.324	3.200	1.314
Dec-02	24.37	25.50	1.78	2.9	2.180	2.608	2.900	1.520
Dec-02	24.37	26.00	1.59	3.2	1.976	2.900	2.691	1.758
Dec-02	24.37	26.50	1.42	2.88	1.830	3.195	2.382	1.977
Dec-02	24.37	27.00	1.26	0	1.642	3.520	2.167	2.253
Dec-02	24.37	27.50	1.13	4.21	1.458	3.846	1.960	2.535
Dec-02	24.37	28.00	1	4.57	1.250	4.193	1.740	2.780
Jan-03	24.20	20.50	4.51	0.88	5.200	0.698	7.689	0.179
Jan-03	24.20	21.00	4.16	1.02	4.755	0.809	7.325	0.233
Jan-03	24.2	21.50	3.83	1.18	4.384	0.972	6.850	0.296
Jan-03	24.2	22.00	2.74	1.36	4.050	1.135	6.380	0.370
Jan-03	24.2	22.50	2.54	0	3.745	1.280	6.055	0.453
Jan-03	24.2	23.00	2.94	1.75	3.477	1.495	5.603	0.554
Jan-03	24.2	23.50	2.67	1.97	3.243	1.671	5.299	0.654
Jan-03	**24.2**	**24.00**	**2.41**	**2.21**	**2.916**	**1.915**	4.920	0.801

Jan-03	24.2	24.50	2.17	2.47	2.690	2.181	4.514	0.930
Jan-03	24.2	25.00	1.96	2.76	2.455	2.350	4.251	1.067
Jan-03	24.2	25.50	1.76	3.05	2.289	2.704	3.870	1.244
Jan-03	24.2	26.00	1.58	0	2.093	2.948	3.625	1.423
Jan-03	24.2	26.50	1.41	0	1.876	3.258	3.320	1.664
Jan-03	24.2	27.00	1.25	0	1.650	3.600	3.025	1.860
Feb-03	24.06	20.50	4.53	1.04	7.147	1.452	21.584	0.122
Feb-03	24.06	21.00	-	1.20	6.654	1.615	21.239	0.149
Feb-03	24.06	21.50	3.88	1.37	6.474	1.787	20.800	0.181
Feb-03	24.06	22.00	0	1.55	6.300	1.965	20.164	0.223
Feb-03	**24.06**	**24.50**	**2.26**	**2.69**	**4.842**	**3.001**	18.414	0.423
Feb-03	24.06	25.00	2.05	2.97	4.807	3.228	17.820	0.409
Feb-03	24.06	26.00	1.68	0	4.304	3.572	16.884	0.530
Feb-03	24.06	26.50	1.51	0	3.966	3.872	16.366	0.654
Feb-03	24.06	27.00	1.35	0	3.754	4.235	16.193	0.665
Mar-03	23.92	22.00	3.55	1.66	4.260	1.334	9.497	0.232
Mar-03	23.92	23.00	-	2.08	3.650	1.717	8.727	0.355
Mar-03	**23.92**	**24.00**	**2.48**	**2.56**	**3.175**	**2.123**	7.879	0.485
Mar-03	23.92	26.00	1.68	3.01	2.285	3.182	6.780	0.892
Mar-03	23.92	26.50	1.52	-	2.079	4.250	5.916	1.040
Mar-03	23.92	27.00	1.37	4.55	1.867	5.300	5.653	1.191
Apr-03	23.78	20.50	-	1.24	5.140	0.880	13.011	0.880
Apr-03	23.78	22.00	0	1.78	4.258	1.370	11.609	1.370
Apr-03	**23.78**	**24.00**	**2.48**	**2.7**	**3.276**	**2.232**	9.758	2.232
Apr-03	23.78	26.00	1.7	2.21	2.374	3.231	8.172	3.230

As we can see form the Table-5.14 above, the Jump Diffusion Model (JDM) captures very well the call option prices (out and in the money) and the put options prices (in-the-money and especially the out-of-the-money) up to November future contract. Beyond the November contract the JDM captures satisfactorily the call option and put option prices.

The Mean Reversion Jump Diffusion Model (MRJD) behaves very poorly according to the market observed option prices, it doesn't capture the smile/skew of the calls and the puts. Between the two models the JDM is far more superior and extremely accurate compared to the MRJD model.

Finally by comparing Table-5.11 with Table-5.14 we can conclude that the JDM is the appropriate model to price the WTI options. It captures the price of the out-of- and in-the money puts and the price of out-of- and in-the-money calls better than any other model we tested. Based on our analysis it is quite obvious that the best model is the JDM as the option prices given are very accurate in comparison with the other models and closest to the market observed options prices.

5.4 Summary

In this chapter we have developed a general framework for pricing and risk management of energy derivatives on the historical forward curve for energy prices. Also we discussed the estimation of the volatility function of the forward curve from historical data including the seasonality and the volatility smile/skew. We priced options based upon a two-factor model and compared the results with the Black's model. Finally, we used the Jump Diffusion and Mean Reversion Jump Diffusion models for the underlying process for oil prices and priced standard European options. The results show that the JDM is the best model as the option prices given are very accurate in comparison with the other models and closest to the market observed options prices.

Appendix-5.1

Black's (1976) Model Computer Algorithm

```
'// Black (1976) Options on futures/forwards
Public Function Black76(CallPutFlag As String, F As Double, X _
        As Double, T As Double, r As Double, v As Double) As Double

  Dim d1 As Double, d2 As Double

  d1 = (Log(F / X) + (v ^ 2 / 2) * T) / (v * Sqr(T))
  d2 = d1 - v * Sqr(T)
  If CallPutFlag = "c" Then
     Black76 = Exp(-r * T) * (F * CND(d1) - X * CND(d2))
  ElseIf CallPutFlag = "p" Then
     Black76 = Exp(-r * T) * (X * CND(-d2) - F * CND(-d1))
  End If
End Function
```

Appendix-5.2

Three Factor Model Computer Algorithm

```
Function BSOptionValue3(iopt, F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2,
sigma3, alpha3)
' Returns the Black-Scholes Value (iopt=1 for call, -1 for put; q=div yld)

  Dim ert, NDOne, NDTwo

  ert = Exp(-r * topt)
  If F > 0 And K > 0 And topt > 0 And sigma > 0 And sigma2 > 0 And sigma3 > 0
Then
     NDOne = Application.NormSDist(iopt * BSDOne3(F, K, r, topt, tfwd, sigma,
alpha, sigma2, alpha2, sigma3, alpha3))
     NDTwo = Application.NormSDist(iopt * BSDTwo3(F, K, r, topt, tfwd, sigma,
alpha, sigma2, alpha2, sigma3, alpha3))
     BSOptionValue3 = iopt * ert * (F * NDOne - K * NDTwo)
  Else
     BSOptionValue3 = -1
  End If
End Function

Function BSDOne3(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2, sigma3, alpha3)
' Returns the Black-Scholes d1 value

 BSDOne3 = (Log(F / K) + (0.5 * w3(sigma, alpha, sigma2, alpha2, sigma3, alpha3,
topt, tfwd))) / (Sqr(w3(sigma, alpha, sigma2, alpha2, sigma3, alpha3, topt, tfwd)))

 End Function
```

```
Function BSDTwo3(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2, sigma3, alpha3)
' Returns the Black-Scholes d2 value
  BSDTwo3 = BSDOne3(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2, sigma3,
alpha3) - (Sqr(w3(sigma, alpha, sigma2, alpha2, sigma3, alpha3, topt, tfwd)))
End Function

Function w3(sigma, alpha, sigma2, alpha2, sigma3, alpha3, topt, tfwd)
' Returns value of volatility function
w3 = ((sigma ^ 2) / (2 * alpha)) * (Exp(-2 * alpha * (tfwd - topt)) - Exp(-2 * alpha *
tfwd)) + ((sigma2 ^ 2) / (2 * alpha2)) * (Exp(-2 * alpha2 * (tfwd - topt)) - Exp(-2 *
alpha2 * tfwd))
End Function
```

Appendix-5.3

Two-Factor Model Computer Algorithm

```
Function BSOptionValue2(iopt, F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2)
'  Returns the Black-Scholes Value (iopt=1 for call, -1 for put; q=div yld)

  Dim ert, NDOne, NDTwo

  ert = Exp(-r * topt)
  If F > 0 And K > 0 And topt > 0 And sigma > 0 And sigma2 > 0 Then
    NDOne = Application.NormSDist(iopt * BSDOne2(F, K, r, topt, tfwd, sigma,
alpha, sigma2, alpha2))
    NDTwo = Application.NormSDist(iopt * BSDTwo2(F, K, r, topt, tfwd, sigma,
alpha, sigma2, alpha2))
    BSOptionValue2 = iopt * ert * (F * NDOne - K * NDTwo)
  Else
    BSOptionValue2 = -1
  End If
End Function

Function BSDOne2(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2)
'  Returns the Black-Scholes d1 value

 BSDOne2 = (Log(F / K) + (0.5 * w2(sigma, alpha, sigma2, alpha2, topt, tfwd))) /
(Sqr(w2(sigma, alpha, sigma2, alpha2, topt, tfwd)))
'BSDOne = -0.72927
End Function

Function BSDTwo2(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2)
'  Returns the Black-Scholes d2 value
  BSDTwo2 = BSDOne2(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2) -
(Sqr(w2(sigma, alpha, sigma2, alpha2, topt, tfwd)))
End Function

Function w2(sigma, alpha, sigma2, alpha2, topt, tfwd)
' Returns value of volatility function
```

```
w2 = ((sigma ^ 2) / (2 * alpha)) * (Exp(-2 * alpha * (tfwd - topt)) - Exp(-2 * alpha *
tfwd)) + ((sigma2 ^ 2) / (2 * alpha2)) * (Exp(-2 * alpha2 * (tfwd - topt)) - Exp(-2 *
alpha2 * tfwd))
End Function

Function BSOptionValue3(iopt, F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2,
sigma3, alpha3)
' Returns the Black-Scholes Value (iopt=1 for call, -1 for put; q=div yld)

   Dim ert, NDOne, NDTwo

   ert = Exp(-r * topt)
   If F > 0 And K > 0 And topt > 0 And sigma > 0 And sigma2 > 0 And sigma3 > 0
Then
      NDOne = Application.NormSDist(iopt * BSDOne3(F, K, r, topt, tfwd, sigma,
alpha, sigma2, alpha2, sigma3, alpha3))
      NDTwo = Application.NormSDist(iopt * BSDTwo3(F, K, r, topt, tfwd, sigma,
alpha, sigma2, alpha2, sigma3, alpha3))
      BSOptionValue3 = iopt * ert * (F * NDOne - K * NDTwo)
   Else
      BSOptionValue3 = -1
   End If
End Function

Function BSDOne3(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2, sigma3, alpha3)
' Returns the Black-Scholes d1 value

 BSDOne3 = (Log(F / K) + (0.5 * w3(sigma, alpha, sigma2, alpha2, sigma3, alpha3,
topt, tfwd))) / (Sqr(w3(sigma, alpha, sigma2, alpha2, sigma3, alpha3, topt, tfwd)))

End Function

Function BSDTwo3(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2, sigma3, alpha3)
' Returns the Black-Scholes d2 value
   BSDTwo3 = BSDOne3(F, K, r, topt, tfwd, sigma, alpha, sigma2, alpha2, sigma3,
alpha3) - (Sqr(w3(sigma, alpha, sigma2, alpha2, sigma3, alpha3, topt, tfwd)))
End Function

Function w3(sigma, alpha, sigma2, alpha2, sigma3, alpha3, topt, tfwd)
' Returns value of volatility function
w3 = ((sigma ^ 2) / (2 * alpha)) * (Exp(-2 * alpha * (tfwd - topt)) - Exp(-2 * alpha *
tfwd)) + ((sigma2 ^ 2) / (2 * alpha2)) * (Exp(-2 * alpha2 * (tfwd - topt)) - Exp(-2 *
alpha2 * tfwd))
End Function
```

Appendix-5.4

Merton (1976) Jump Diffusion Model computer Algorithm with Monte Carlo Simulation

```
Public Function JDOM(p, Z, PutCall, r, sig, f, k, y, T, M, n)

'Dim p, r, f, sig, k, y, T, M, n, Z, PutCall, nodays
Dim diff, drift, dt, lnp, sump, test, test2, rand1, rand2, rand3, jump, CT, sum_CT
Dim i, j, pcf, b As Integer

'nodays = (Range("F").End(xlDown).Row) - 1

'For b = 1 To 3

'p = Cells(1 + b, 1).Value
'Z = Cells(1 + b, 2).Value
'PutCall = Cells(1 + b, 3).Value
'r = Cells(1 + b, 4).Value
'sig = Cells(1 + b, 5).Value
'f = Cells(1 + b, 6).Value
'k = Cells(1 + b, 7).Value
'y = Cells(1 + b, 8).Value
'T = Cells(1 + b, 9).Value
'M = Cells(1 + b, 10).Value
'n = Cells(1 + b, 11).Value



dt = T / n

drift = (r - f * k - 0.5 * sig ^ 2) * dt

diff = sig * Sqr(dt)

sump = 0
CT = 0
sum_CT = 0

For j = 1 To M

lnp = Log(p)

   For i = 1 To n

   test = Rnd

        If test = 0 Then
        test = test + 0.0000001
```

```
      End If

      rand1 = Application.NormSInv(test)

   lnp = lnp + drift + diff * rand1


      rand2 = Rnd

      If rand2 < (f * dt) Then


         test2 = Rnd

         If test2 = 0 Then
         test2 = test2 + 0.0000001
         End If

      rand3 = Application.NormSInv(test2)

      jump = k + (y * rand3)

      lnp = lnp + jump

         End If




   Next i

   If LCase(PutCall) = "call" Then

   pcf = 1

   End If

   If LCase(PutCall) = "put" Then

   pcf = -1

   End If

   sump = sump + Exp(lnp)
   CT = Application.Max(pcf * (Exp(lnp) - Z), 0)
   sum_CT = sum_CT + CT

Next j
```

```
'Cells(1 + b, 12).Value = Exp(-r * T) * (sump / M)
'Cells(1 + b, 13).Value = Exp(-r * T) * (sum_CT / M)
JDOM = Exp(-r * T) * (sum_CT / M)
'Next b

End Function
```

Appendix-5.5

Mean Reversion Jump Diffusion Model computer Algorithm with Monte Carlo Simulation

```
Public Function MRJDOM(p, Z, PutCall, a, u, r, sig, f, k, y, T, M, n)

'Dim p, r, f, sig, k, y, T, M, n, a, u, Z, PutCall, nodays
Dim diff, drift, dt, lnp, sump, test, test2, rand1, rand2, rand3, jump, CT, sum_CT
Dim i, j, pcf, b As Integer

'nodays = (Range("F").End(xlDown).Row) - 1

'For b = 1 To 1

'p = Cells(2, 1).Value
'Z = Cells(2, 2).Value
'PutCall = Cells(2, 3).Value
'a = Cells(2, 4).Value
'u = Cells(2, 5).Value
'r = Cells(2, 6).Value
'sig = Cells(2, 7).Value
'f = Cells(2, 8).Value
'k = Cells(2, 9).Value
'y = Cells(2, 10).Value
'T = Cells(2, 11).Value
'M = Cells(2, 12).Value
'n = Cells(2, 13).Value


dt = T / n



diff = sig * Sqr(dt)

sump = 0
CT = 0
sum_CT = 0
```

```
For j = 1 To M

lnp = Log(p)

drift = (a * (u - lnp) - 0.5 * sig ^ 2) * dt

   For i = 1 To n

   test = Rnd

         If test = 0 Then
         test = test + 0.0000001
         End If

         rand1 = Application.NormSInv(test)

   lnp = lnp + drift + diff * rand1


         rand2 = Rnd

         If rand2 < (f * dt) Then


            test2 = Rnd

            If test2 = 0 Then
            test2 = test2 + 0.0000001
            End If

         rand3 = Application.NormSInv(test2)

         jump = k + (y * rand3)

         lnp = lnp + jump

            End If



    Next i

    If LCase(PutCall) = "call" Then

    pcf = 1
```

```
    End If

    If LCase(PutCall) = "put" Then

    pcf = -1

    End If


    sump = sump + Exp(lnp)
    CT = Application.Max(pcf * (Exp(lnp) - Z), 0)
    sum_CT = sum_CT + CT

Next j



'Cells(2, 14).Value = Exp(-r * T) * (sump / M)
'Cells(2, 15).Value = Exp(-r * T) * (sum_CT / M)
MRJDOM = Exp(-r * T) * (sum_CT / M)
'Next b
End Function
```

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