Dynamic Pension Funding Models

by

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AS ORIGINAL

To my little pearls

Sarah, Ahmed and Yasmine

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List of Abbreviation

AAIB	Arab African International Bank
AL	Actuarial Liability
CIPI	Cost-induced performance index
COLA	Cost-of-Living Adjustments
COMPS	Contracted-Out Money Purchase Scheme
COS	Contracted-Out Scheme
COSRS	Contracted-Out Salary Related Scheme
CPI	Consumer Price Index
EISA	Egyptian Insurance Supervisory Authority
ERISA	Employee Retirement Income Security Act
FFA	Fellow of the Faculty of Actuaries
FIA	Fellow of the Institute of Actuaries
GDP	Gross Domestic Product
GN9	Guidance Notes 9
MISA	Ministry of Insurance and Social Affairs
NAIP	National Authority for Insurance and Pensions
NASI	National Authority for Social Insurance
NIB	National Investment Bank
NIC	National Insurance Contributions
NRA	Normal Retirement Age
OPRA	Occupational Pension Regulatory Authority
PAYG	Pay-As-You-Go
RIPI	Ratio-induced performance index
SCR	Standard Contribution Rate
SERPS	State –Earnings- Related Pension Scheme
S2P	State Second Pension
SSIS	State Social Insurance System
UL	Unfunded Liability

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DECLARATION

Abstract

Achieving an adequate income in the old age to maintain the standard level of living after retirement has been a challenge to pension schemes for a long time. In fact, approaching this goal has led to a global pension crisis considering all the economic and demographic changes and the conflicting interests of employers and employees over time. This research aims to deriving different deterministic and stochastic dynamic pension funding models for defined benefit schemes within the mathematical framework of optimal control theory and dynamic programming. The practical implementation of these dynamic models into one of the largest Egyptian defined benefit occupational pension schemes - as a case study – is a tool to examine how they act in the reality, and provide the management of the pension fund with a dynamic plan instead of the static ones that have been used in such a volatile market. Taking into consideration the optimal contribution rate of the mutual interests of both the employer and the employees by including a mixed middle term in the dynamic pension funding models. This represents both the contribution rate risk and the solvency risk and could provide a solution for one of the pension schemes problems.

Chapter Two
Background of Social Security System in Egypt

2.1 Introduction

The development of the Egyptian social security system has been affected by considerable political, social and economic changes since its inception. Certain laws have been issued while many others have been adjusted or even amended for the following reasons:

- extending the coverage to more sectors in the society;
- increasing the benefits for the insured persons;
- regulating or reorganizing the system for achieving more effectiveness and efficiency.

The current Egyptian social security system can be subdivided into:

- State Social Insurance System;
- Contracted-Out Schemes;
- Occupational Pension Schemes or Private Pension Funds as they are called in Egypt;
- Personal insurance policies.

This chapter will be a brief review of the four above mentioned systems, which represent the entire current social protection provisions in Egypt. In some parts, we refer to other countries experience to understand some similarities and differences between the Egyptian system and other developed countries systems such as: UK. Private Pension Funds will be used throughout the thesis to refer to the Egyptian occupational pension schemes, either defined benefit or defined contribution pension schemes.

2.2 State Social Insurance System (SSIS)

The State Social Insurance System is considered the main social security pension system in Egypt. It is a compulsory, funded scheme through two separate funds: one for the government sector and the other for the public and private sectors, while pay-as-you-go (PAYG) for the casual workers who are covered by the law no.112 of 1980, and defined benefit system which according to the present law, extends to all working people.

In the following sub-sections, we will briefly look at the historical background and the modifications of the laws applied with their effect on the benefits and contributions specified in the Egyptian national system, and the different aspects of SSIS.

2.2.1 Historical Background

The historical background of SSIS can be traced through three different stages, each stage represents a period of time where main changes had been made to affect the laws of the social insurance system (El-Sayyad, 1999), they are:

- **First stage: Before 23rd of July 1952 Revolution**

The inception of the Egyptian social insurance system dates back to the mid-nineteenth century and precisely on 26th December, 1854 when the first law was issued to provide pensions for old age, death and invalidity, but that was only for a selected group of government employees. At that time they were not obliged to pay any contributions as these pensions were funded by the Public Treasury.

Due to the increase of Egyptian debts because of the Suez Canal inauguration, a law was issued on 16th October 1870 to enforce payment of 3.5% of the employees' salaries as contributions for the first time. In 1882 and 1884, laws were issued to cover more sectors of the government employees. The contribution rate was raised to 5% as a result of increasing the benefits offered to them.

Both the System and benefits were adjusted by law no. 5 of 1909 to extend the coverage of the social protection provision, and later in 1929, law no.27 was issued to increase the employees' contributions to 7.5%, but thereafter abolished in 1935.

In 1942, the first law covering the employees against work injuries was issued, and was followed by law no.117 of 1950 to cover occupational diseases as well.

- **Second stage: the period following 1952 Revolution**

During the period from1935 to 1952 (the Revolution year), a large sector of government employees and workers were not covered by any pension scheme. Encouraging and providing security for the working people was one of the main objectives of the Revolution, so that a number of laws that dealt with organizing the state pension scheme for the working people in all sectors, such as: laws 1952, 1956 and 1963 for employees in the government sector, and others 1955, 1959, 1961 and 1964 for the public and private sectors, were issued during the period after the revolution to cover the members against old age, disability, death, health insurance, work injuries and unemployment.

- **Third stage: the contemporary stage and recent developments - the period from 1975 till now -**

In the mid-1970's and the beginning of the 80s, a number of basic laws have been issued to cover all the working people inside and outside Egypt:

- Law no.79 of 1975 (General Social Insurance System) for employees in government, public and private sectors.
- Law no.108 of 1976 for employers and the self employed.
- Law no.50 of 1978 to regulate the voluntary social insurance system for Egyptians working abroad.
- Law no.112 of 1980 (Comprehensive Social Insurance System) for all Egyptian working people who are not covered by any of the previous laws.

These laws are considered the four columns of SSIS and regulate the current coverage of social security system in Egypt. There are two authorities responsible for the application of these four laws under the supervision of the Ministry of Insurance and Social Affairs (MISA), they are:

- National Authority for Insurance and Pensions (NAIP).
- National Authority for Social Insurance (NASI).

The NASI is in charge of the General Social Insurance System, i.e. Law no 79 of 1975 by managing two separate funds, the first is: Government Sector Fund which deals with the government employees, the second is: Public and Private Fund for the employees who work for the public and private units.

2.2.2 Coverage

The social security systems around the world aim to cover the whole working people against the potential risks of losing their income. In 2001, the SSIS covered 38.9% of the total population, 92.8% of the active population (working people from age (15 -65)) with total contributors 71.5% and total beneficiaries 28.5%.

The number of insured persons covered by SSIS under the four different laws during the period from 1993 to 2003 is shown in Table 2.1.

Table 2.1

The number of the insured persons under the four main social insurance laws from 1993/94 to 2002/03

(in thousands persons)

Year	Law 79/ 1975	Law 108/1976	Law 50 /1978	Law 112/1980	Total
1993 / 94	8976	1502	56	5355	15889
1994 / 95	8702	1576	68	5537	15883
1995 / 96	9072	1650	20*	5707	16449
1996 / 97	9232	1716	21	5834	16803
1997 / 98	9335	1756	22	5837	16950
1998 / 99	9692	1820	23	5918	17453
1999 / 00	9757	1837	16	5920	17530
2000 / 01	10044	1876	18	5922	17860
2001 / 02	10422	1924	15	5942	18303
2002 / 03	10604	1966	16	5966	18552

Source: MISA, 1998/ 99 and 2002/03.

* This number is an adjusted number starting from 1995/ 96, which represents only the insured persons under this law who are paying contributions regularly.

Table 2.1 indicates that the majority of the contributors during this period were under the coverage of law no. 79 of 1975, the casual workers under law no. 112 of 1980 were in the second rank, then followed by the number of employers and self-employed. Egyptians working abroad constituted the smallest number under the law no. 50 of 1978, this could refer to the voluntary feature of the membership with the possibility that they might get the social insurance coverage they need in the country in which they live. It is also notable that the number of Egyptians working abroad has a decreasing trend compared with other categories. This is due to cancellation of the membership of the insured persons if there is a delay in payment of their contributions for more than 6 months according to the law.

Law no.79 of 1975 not only covers the majority of the Egyptian employees in government, public and private sectors as previously mentioned, but also extends to cover the members of their families or "the dependants". These include: widow, divorcee, spouse, sons, daughters, parents, brothers and sisters under certain requirements.
The following Table 2.2 shows the total number of pensioners and dependants and the amounts of benefits paid to both of them from1993/94 to 2002/03.

Table 2.2
The number of pensioners and their dependants and the amount of benefits paid from 1993/94 to 2002/03

Year	Pensioners (in thousands)	Dependants (in thousands)	Total beneficiaries	Benefits (in million LE)
1993/ 94	1436	4409	5845	5415.1
1994/ 95	1482	4505	5987	6357.3
1995/ 96	1545	4548	6093	7291.4
1996/ 97	1601	4695	6296	8548.6
1997/ 98	1726	4808	6534	9781
1998/ 99	1834	5012	6846	11264
1999/ 00	1952	4954	6906	12848
2000/ 01	2062	5060	7122	14602
2001/ 02	2138	5173	7311	17124
2002/ 03	2199	5183	7382	18967

Source: MISA, 1998/99 and 2002/03.

In 2002/2003, the number of pensioners were 2.199 millions and the number of the dependants were about 5.183 millions. This indicates that 70% of the beneficiaries are "dependants" under this law. During the period from 1993/94 to 2002/03, the average number of the dependants represents 73% of the total beneficiaries, i.e. only 27% of the benefits go to the pensioners who are basically responsible of paying the contributions.

All the four laws of SSIS cover the beneficiaries against the old age, disability and death, however, law no 79 of 1975 provide more benefits like:

- Work injuries insurance.
- Health insurance.
- Unemployment insurance.
- Social patronage insurance for pensioners (which include: establishing homes for elderly people, concessions or reduction in fares for transportation, flight and theatre tickets,...etc).

The old age, disability and death benefits are similar under laws no.79, 108 and 50 of 1975, 1976 and 1978, the coverage under law no 12 of 1980 is different as it is mainly for the casual workers. Thus, a reference to the contributions and the benefits of the old-age, disability and death covered in the General Social Insurance System would, at this point, be of relevance to our study about pension schemes. In the following subsections the contribution rates, benefits and eligibility of old age, disability and death are illustrated.

2.2.3 Contributions

The contribution rate is a fixed percentage of the pensionable salary which is paid by the employer, employee and the government. The pensionable salary that is used for the calculations of contribution rates and benefits is based on:

(i) The basic salary: which an employee receives and that is determined according specific labour list or work contracts.

(ii) The variable salary: that can be defined as: all payments the insured person gets from his original job apart from basic salaries. These include:

- incentives;

- commissions and gratuities;
- allowances;
- bonuses, shares of the firm's profit;
- social increases such as: the high cost of living increases;
- overtime wages.

Since 1992, 100% of the variable salaries have been taken into account in calculating the benefits. The total contribution rate for old-age, disability and death is 26% of the pensionable salary. It is divided as follows:

- 15% paid by the employer;
- 10% paid by the employee;
- 1% paid by Public Treasury.

2.2.4 Benefits

The pension is considered the main income provided by SSIS to the insured persons at normal retirement age (NRA), disability or death. Since the calculation of the contributions is based on both the basic and the variable salary, the pension is also calculated accordingly, i.e. the total pension consists of basic salary pension and the variable salary pension, as follows:

$$TP = BSP + VSP \qquad (2/1)$$

$$BSP = 1/45 \,.\, N.\, ABS \qquad (2/2)$$

$$VSP = 1/45 \,.\, N.\, (1.02)\, AVS \qquad (2/3)$$

where

TP: total pension

BSP: basic salary pension

VSP: variable salary pension

1 / 45: is the accrual rate which is usually 1 / 60 in the UK.

N : no of contribution years.

ABS: average basic salary during the last two years of contribution.

AVS: average variable salary during the contribution period since April 1984, or the contribution period if less.

The variable salary pension was introduced to the system in addition to the basic one due to the increase in salaries, represented in the variable salaries, in response to the rise in the inflation rate in 1984 (Mohamed, 1997). The pensioners were badly affected as the pensions had not been indexed to inflation, so that, the variable salary had been included in the definition of the pensionable salary, and the variable salary pension has been calculated since July 1992 (Maait, 2003).

In addition to the pensions offered, the beneficiaries are entitled to other benefits under certain requirements[1], such as: lump sum, end of service remuneration, additional indemnity, death grant, funeral expenses and marriage grant.

2.2.5 Eligibility

The insured person will be entitled to a pension under the following main requirements:

(a) In the case of retirement

- Reaching the normal retirement age (60) for both men and women.
- A minimum contribution period of 120 months.
- For early retirement, the contribution period must not be less than 240 months, with a possibility of having a full pension from the age of 55 and a reduced pension from the age of 40.

(b) In the case of disability or death

- Termination of service because of (total or partial) disability or death whatever the contribution period of the insured person.
- Death or total disability during one year after retirement provided the insured person is not over sixty and has not been paid the lump sum indemnity, whatever

[1] The requirements of these other benefits are not mentioned in detail in section 2.2.5, as this could be covered in further research which considers studying the SSIS.

the period of contribution (but not less than three consecutive months or six intermittent months).

- Death or total disability after one year of retirement (60 or less) provided that the minimum period of contribution is 120 months and the lump sum has not been paid.

It should be noted that the pension in the last two cases is calculated in the same way as for the retirement pension except that the monthly earnings average is calculated according to the last year of the contribution period, or the whole contribution period if less than one year.

To protect the value of benefits of the insured persons in the case of transfer from a job in one sector to another, the law no 79 of 1975 stated that all the years of contribution must be taken into account as one period when calculating the benefits; each fund would be responsible for the years which the insured person spent working for the sector and should transfer its share in benefits to the other fund.

2.2.6 Taxation

According to the law, contributions of both employer and employee, all investment operations, pensions and all other benefits are exempted from taxes, duties and other fees imposed by the government. Furthermore, the cases held by authorities or insured persons or beneficiaries are exempted from court fees.

Taxation of social security contributions and benefits is different in other countries, for example, in the UK and the US, the employer contributions are tax-deductible and the employees contributions are not in the social security systems and other compulsory plans . Moreover, the taxation of social security benefits is dependent on the benefits received, for example: retirement pensions, widows pensions/allowances are taxable as earnings in the UK. The social security benefits are currently taxable in the US for persons whose income and social security benefits exceed a specified base amount (Watson Wyatt, 2000 and 2004).

2.2.7 Investment

SSIS plays an important role in the economy of the country as the Government relies heavily on its funds, which reach billions of Egyptian Pounds, to finance the social and economic projects of the Government.

Table 2.3 shows the total assets of SSIS and the amounts invested from 30/6/94 to 30/6/2003 - as the fiscal year ends on the 30th of June each year-.

Table 2.3

Total assets of SSIS and amounts of investments from 1993/ 94 to 2002/03

(in million LE)

Year	Total assets	Total investments
1993/ 94	56955.1	53750.6
1994/ 95	67783.3	63687.5
1995/ 96	80348.6	74906.7
1996/ 97	95131.4	87246.5
1997/ 98	111859.6	100352
1998/ 99	130212.7	114018.8
1999/ 00	150217	129951
2000/ 01	171564	147851
2001/ 02	195849	167615
2002/ 03	222287	189776

Source: MISA, 1998/ 99 and 2002/03.

The total assets shown in Table 2.3 represents the total reserves that appears in the government, public and private sectors funds statements, and is held in the scheme to cover its liabilities (including expenditures). Table 2.3 indicates that an average of 89% of these assets is invested in order to guarantee a desired return that helps with the contributions paid by the employers and employees to cover the benefits offered by the schemes.

However, most of the investments (90% or more) are deposited in the National Investment Bank (NIB) in two separate funds: one for the government sector and the other for the public and private sectors. Approximately 5% or less of this amount is invested in other types of investment such as:

- Bonds;
- equities;
- deposits in Banks.

Table 2.4 shows the relative importance of different types of investment in both government, public and private sectors funds which represent SSIS in 2002/03.

Table 2.4
The relative importance of the investments in government, public and private funds in 2002/03

%

Type of Investment	Government sector	Public and private sector
NIB	90.9	93.7
Government bonds	1.0	1.3
Total	91.9	95.0
Equities	0.8	0.9
Deposits	7.1	4.0
Loans*	0.2	0.1
Total	100	100

Source: MISA, 2002/2003.

* Loans: are those guaranteed loans that are given to the members upon their entitlement to pensions.

The investments of both the government, public and private sectors funds show a prudent investment strategy of the funds management. This is clearly seen in Table 2.4 with a percentage from 92-95% of the assets invested in NIB and the government bonds in 2002/03. Deposits had a percentage from 4% to 7% and the equities were around 1% only in both funds. Although this reflects a low risk investment policy, it may not be the optimal strategy when inflation is increasing and the need to achieve extra income to cover any deficit that may arise.

Here, it is worth to mention that the annual rate of return provided by National Investment Bank on the deposits has generally been lower than the market rates. It had actually increased from 6% to13% during the period from 1989 to1992 - according to the Government Actuary's recommendations –. Thus, a higher return on investments had been achieved in order to cover the deficit of the two funds that had arisen in the actuarial valuations reports from 1959 to 1978 (Sabra, 1998). This rate has been reduced to 11% since 1997 according to the decision of the board of directors of NIB no.99 of the year 1997.

Nowadays, the experience of other countries around the world shows that the management of the pension funds tends to adopt different investment strategies and have a portfolio of assets with different risk-return characteristics to respond to the volatility of the economic, demographic and political factors in both domestic and international environments. For example: the pension funds in the UK had 66% of total fund assets invested in domestic and overseas equities in 1989 (Blake, 1992), this percentage increased to an average of 70 % in 2000 (Blake, 2003).

2.2.8 Main problems of State Social Insurance System

Although the Egyptian social insurance system offers a good coverage to the pensioners and a wide range of benefits to their dependants, there are some problems that need to be tackled for a better performance of the main social security scheme, they are:

- Maintaining the real value of pensions, taking into consideration the economic factors such as: inflation rates and salary growth rates (Salem, 2000).
- Keeping the balance between the inputs and outputs, i.e. the income and the expenditures of the system to prevent the occurrence of deficits.
- Evaluating the investment policy of the management of the funds to cope with the economic and demographic changes in the market, and at the same time to learn from the international experience of managing pension funds in other countries,

and the way that they reach their desired portfolio of assets without putting a burden on the promised benefits or levels of contributions.

- The need for a reform in order to increasing the efficiency of the system as it serves all working people in Egypt. Considering the poverty problem in Egypt, the social security system needs to be more efficient and reformed for better allocation of the resources of the scheme and the redistribution of the benefits among the population to serve the poor people in facing their social risks more than the better off (Loewe, 2000).

2.3 Contracted- Out Schemes

Contracted-Out schemes are those schemes which have the opportunity to opt out of SSIS and to offer their members with at least the benefits provided by the State. The establishment and abolishing of such schemes in Egypt with their different aspects will be discussed in the following sub-sections.

2.3.1 Historical background

In late seventies, the Egyptian Government moved towards a capitalist system by adopting the "open door" policy, in which there was a trend towards the privatization of the public units along with encouragement of international investments in Egypt. The idea of providing better benefits to the working people along with achieving a better performance of SSIS was a subject of discussion for the privatization of the social security system. Thus, in 1980 law no.64 was issued to organize the regulations and legislation of contracting- out schemes, the Egyptian Government gave approval to companies to contract- out of SSIS provided that better benefits are offered to their employees, under these two conditions:

(1) The number of employees in the company should not be less than 1000.

(2) The capital of the company should not be less than LE10 million.

Up till now, only seven banks and a marine transportation company have made use of this law. The reason for this limited number is that Law no.230 of 1989 was issued to amend the former law and cease underwriting such schemes. At that time, they have been considered a direct threat to SSIS, this threat was explained by the interest of the state to have an overall control of the management of the pension scheme and the investment of its total funds. On the other hand, the inequality of benefits, offered to the members of these schemes compared with the State's members, would force the Government to raise its benefits offered by SSIS (Maait, 2003).

The experience in the UK shows that the reasons for introducing Contracted-Out schemes were: the desire to introduce an earnings-related system without duplication of the provision; since the occupational pension schemes have already existed, it was also to avoid reducing the funded occupational pension provision by employers; and finally to keep down the social security costs (Daykin, 2001).

Currently in the UK, employees can opt out of the State Second Pension (S2P)and its processor the State Earnings-Related Pension Scheme (SERPS) if they are members of a contracted-out occupational pension scheme or an approved personal pension plan, and in return they pay a reducing rate of National Insurance Contributions (NICs) called the contracting –out rebate (Sullivan, 2004). This is also applied in Japan where there is a reduction in the contributions as a further incentive to contract-out of Employee Pension Insurance (Rein and Turner, 2004).

Although a lot of detailed changes have taken place in the British pension schemes over the years, Contracted-Out Salary Related Schemes (COSRS) and Contracted-Out Money Purchase Schemes (COMPS) are still among the choices of the pension provisions to the working people. All the public sector schemes and about 23.8% of the private sector schemes are contracted-out schemes (Occupational Pension Schemes 2004, 2005). In 2001, there was a report carried out by the Government Actuary, and presented to the UK Parliament on the reductions and rebates for both types of contracted-out schemes covering the period from 6th of April 2002 to 5th of April 2007 (GAD and SSSS, 2001). However, the recent movements towards the money-purchase schemes and the suggestion of abolishing the SERPS and its successor the State Second Pension (S2P) (Ward, 2004), raise a big question about the continuity of these contracted-out schemes.

2.3.2 Characteristics

Egyptian Contracted-Out schemes are registered, supervised and controlled by the Ministry of Insurance and Social Affairs (MISA). This is to ensure the strength of the scheme's financial status and its ability to cover the liabilities to protect the members' rights. In return the schemes are obliged to pay 1% of their funds as a supervision fee to MISA.

Every scheme is an independent entity with a board of directors that regulates all of its activities and determines the types of benefits offered to the members and the eligibility to them. However, they are obliged to offer at least the same benefits offered by the State, and have to amend their benefits to respond to any enhancements of the benefits made by the State.

These schemes are well-established, financially strong and have a good capability to invest their funds more effectively, and this helps them to offer similar types of benefits as the state but with higher values, while the contribution rate of the employees remains equal to the same percentage determined by SSIS, i.e. 10 % of the pensionable salary.

In contracted- out schemes, the contributions and all other resources of the fund except the return of investment are tax exempted as well as all the benefits that paid to the members including: pensions, indemnities, grants and cash payments.

Finally, the insured person can transfer from contracted-out scheme to another according to the rules of each scheme's status as mentioned earlier. The member can also transfer to the State scheme and vice versa according to the rules issued by the Prime Minister.

2.3.3 Investment

In 1999, the total funds of the eight contracted- out schemes were LE 620 million plus US$ 30.2 million held by the Arab African International Bank (AAIB). There was an increase of LE 72.2 million (13%) and US$ 6.3 million (26%) compared with the previous year. It is notable that the funds of these schemes are still limited compared with those held in both SSIS and the Private Pension Funds, this is actually due to the small fixed number of the Contracted-Out schemes.

According to the law, all Contracted-Out Schemes must invest at least 50% of their funds in the National Investment Bank. In 1999, the total amounts invested in NIB reached LE 311.1 million. The invested amounts ranged from 51.9 % to 100 % of the total funds according to the investment strategy of each scheme. It should be noted that the amounts in US Dollars are not invested in NIB, instead they are directed to other types of investment determined by the investment managers of AAIB.

Table 2.5 indicates the asset allocation of the Contracted-out schemes in 1999 which was equal to LE 658.3 million, this amount was a sum of LE 620 million and a converted amount of US$ equals to LE 38.3 million. Apart from the amount invested in NIB – which is 50.2% of the total Egyptian funds held in the schemes – there was more than 50% of the amounts invested in deposits where as 23.6 % were invested in equities and 11.7% in shares, the bonds and loans represented only 9.4% and 3.3% of the investments respectively.

The investment strategies of Contracted-Out schemes showed more diversification of the portfolio of assets held by the schemes compared with the investments of SSIS. This is clearly seen in Table 2.5 as the percentage of the amounts invested in NIB and deposits was approximately 75%, and the amounts invested in equities and shares were 18.7% compared with only 1% in SSIS.

Table 2.5
The asset allocation of the funds of Contracted-Out Schemes in 1999

Investments	Invested amounts (millions LE)	Relative importance %
NIB	311.1	47.3
Total (1)	**311.1**	**47.3**
Deposits and accounts in banks	180.1	27.4
Equities for exchange	82.2	12.5
Shares in companies	40.7	6.2
Government bonds	32.8	4.9
Guaranteed loans	11.4	1.7
Total (2)	**347.2**	**52.7**
Total (1) + (2)	658.3	100

Source: MISA, 1999

2.3.4 Actuarial Valuation

The actuarial valuation must be carried out by a qualified actuary[2] at the time of registration and regularly once every 5 years, unless MISA asks for an earlier valuation in the case of any difficulty facing the financial status of the scheme, or an adjustment is made to the statute of the scheme.

2.4 Private Pension Funds

The need for a supplementary system to the state social insurance systems arises in many countries for many reasons including:

- The increase in benefits costs, due to the changes in the social, demographic and economic factors, which set a challenge for the state social insurance systems to provide the working people with the promised benefits, that secure their standard of living after retirement or in the case of any contingency;
- a new movement for rewarding the faithful employees;

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[2] A qualified actuary according to the Egyptian Law is the one who registered and qualified by EISA upon having the Fellowship of the Institute of Actuaries in the UK or an equivalent degree from the US.

- the increasing needs of the working people to get more benefits than those provided by the state schemes.

Occupational pension schemes started as supplementary voluntary systems in companies and unions long time ago. These types of schemes have developed rapidly, and then spread in many countries around the world. At the present time, Private Pension Funds in Egypt are considered the main complementary pension schemes that provide the insured persons with a coverage above the maximum level offered by SSIS. As these schemes are of a main concern of this thesis, they will be studied in more detail in the following subsections.

2.4.1 Historical background

The development of Private Pension Funds in Egypt could be traced through four stages, they are:

- **First stage: Before 1950**

Friendly societies were the common type of these schemes at that time, they were mainly financed by the contributions of their members to cover them in the case of death. Thereafter, the coverage was expanded to offer benefits in the case of retirement, loss of income and marriage. The contributions were very limited and they ranged from 5p[3] to 25p, accordingly, the benefits were also limited and ranged from LE 3 to LE 50.

- **Second stage: law no. 156 of 1950**

Law no.156 of 1950 was issued to organize the friendly societies after their numbers had increased. It stated that the organizations with a minimum contributions equal to LE300 annually should be registered and supervised by the Insurance Supervisory Authority while the schemes with less amount of contributions should be under the supervision of the Social Affairs Authority.

[3] p = piasters, where LE1 = 100 piasters.

After applying this law, there was not a significant change in the amounts of both the contributions and benefits that had been used before 1950, although employers started to contribute to such schemes. Since then employers' contributions become one of the major sources of funds for these schemes.

- **Third Stage: Law no. 54 of 1975**

By issuing law no. 54 of 1975, a clear definition of Private Pension Funds was stated as well as all the regulations needed to set up, manage, organize and control these schemes. At that time, there were 145 schemes registered at EISA under this law, the number of the total members were 555,000 and the total funds were LE 15 million (Galoul, 1992).
During the period from 1980 to 1983 these schemes were highly spread and their number reached 234 in 1983 with an increase rate of 75% compared with 1980.

- **Fourth stage: period from 1983 to 2000**

On the 6th of July 1983, the Political and Economic Affairs Committee held and decided to cease the registration of these schemes for the reason of setting up a unified complementary system. This unified system would be a second system to the main State scheme, but actually, it could not be as flexible as the private pension funds in determining the benefits and contributions, according to the preference of the employers and employees. Therefore, a debate about this unified system arose concerning the importance of these schemes to the employees, the difficulties that may be faced in managing this unified scheme as well as the differences in the needs of both the employers and members and their ability to pay.

As a result of the complaints of employees and labour unions, on 24th of May 1986 the Committee decided to allow for the registration of new schemes once again. Thus, since 1986 the number of the private pension funds have increased from 234 schemes to 617 in 2003, this indicates a considerable increase in the number of the schemes over 17 years. The total assets held in these schemes reached LE 14.198 million in 2003 and total members was over 3 million (EISA, 2003/2004).

2.4.2 Characteristics

Article 1 in law no.54 of 1975 defines the private pension fund as "Every scheme in any society, union, authority or individuals who are in the same work or in one profession, or have any other social relationship (with no capital), held for offering its members and beneficiaries: compensations, cash payments, and pensions according to its statutes (regulating system) in any of the following cases:

- Marriage of the member, or his offspring, or reaching a certain age, or death of the member or any of his dependants.
- Retirement or loss of income.
- Inability to work due to sickness or accidents.
- Any other purposes approved by EISA, such as: end of service remuneration, funeral expenses, ...etc.

Here, it is worth mentioning that the main reason for using the term of private pension fund in Egypt is that it is considered a wider definition than occupational pension schemes. This can be seen from the legal definition, where it includes all those schemes that can be established under any social relationship other than profession or work such as the friendly societies.

According to the same law, all these schemes should be registered in the Egyptian Insurance Supervisory Authority (EISA) in order to start any of their activities.

Thus, in most countries the general characteristics of occupational pension schemes can be summarized in the following:

- They are usually the main supplementary schemes to the State systems. Hence, they provide an extra coverage to the insured persons against the loss of income due to different contingencies.
- The funds of these schemes are considered an important tool for investments in different projects that achieve some social and economic objectives, so that they help in the growth of the country's economy.

- The possibility to enhance the benefits or reduce the contributions according to the surplus or deficit achieved in the schemes in response to the market volatility.
- An incentive to keep the highly qualified employees and experts and to remunerate them at the end of their service, and a compensation to the early leavers in the case of withdrawal or redundancy (usually conditioned with a certain length of service).
- A tax incentive as the benefits are tax exempted.

The specific characteristics of the Egyptian Private Pension Funds and the differences with SSIS and Contracted-Out Schemes (COS) - bearing in mind that the COS have a very similar system to the SSIS - are highlighted below:

- Private Pension Funds offer different benefits to the members which are usually in the form of a lump sum in addition to those provided by the State and COS, which usually provide pensions to the beneficiaries.
- The majority of the schemes are defined benefit schemes like the SSIS and COS. In other countries like UK, the majority of the occupational pension schemes used to be defined benefit, before they recently have had a strong trend towards the defined contribution schemes.
- They are voluntary schemes for both employees and employers under the supervision of EISA, while the State Social Insurance System is a compulsory system under the supervision of MISA for all working people under coverage.

  In the Contracted-Out Schemes, they are compulsory schemes under the supervision of MISA, as they are representative of the SSIS.

  Although the voluntary feature of setting up a single-employer pension scheme is applied in many countries like: UK, Germany, Switzerland, Finland and Norway, it is mainly for the employers, where they make a commitment as part of the terms of employment to provide different benefits to their employees (Fédération des Experts Comptables Européens, 1995).

- All the financial and legislation aspects of the schemes are stated in the "Regulatory system" or "statutes" (which are similar to the trust deed in UK) whereas all the aspects of SSIS and consequently COS are governed by laws.
- Each scheme is considered a separate legal entity managed by a board of directors representing the members and probably the employer, so that it is responsible for all the decisions made in the scheme including the investment strategy. The Contracted-Out Schemes are also considered separate legal entities under supervision of MISA, their activities are managed by the board of directors which is also responsible to decide about the different types of investments of 50% of the total assets.

  On the other hand, there are two authorities responsible for the activities of SSIS, and most of their funds are invested in NIB as mentioned earlier.

  The occupational pension schemes in the UK and Ireland are separate legal entities (Trust) managed solely by the sponsoring employer (a sole trustee) or with other trustees appointed by the employer and in some cases the employees (Fédération des Experts Comptables Européens, 1995).

2.4.3 Contributions

Unlike the State Social Insurance System, contribution rates in Private Pension Funds are not governed by law, instead they are stated precisely in each statute of each scheme, and are advised by the actuary. They differ from one scheme to another according to:

- The needs of its members.
- The ability of both the employers and the employees to pay.
- The types of benefits required to be provided in the case of the defined benefit schemes.
- The demographic and economic factors relevant to the scheme.

Therefore, both the contribution rates and the pensionable salary are stated clearly in the statute. All the contributions are tax exempted for a maximum amount per member equals to LE1000 in public and private non- state schemes. The contributions that are paid by the employees and the employers can take different forms, the most common of which are:

(a) Employees' contributions

The employees are considered the essential resource of financing the Private Pension Funds, their contributions can be:

- **A fixed percentage of salaries**

This percentage is determined by the actuary and stated in the statute of each scheme as a fixed percentage of the salaries of all members. The advantage of this method is that the contributions are deducted directly and regularly from the salaries and so could be properly collected and invested.

- **A fixed percentage of incentives**

This percentage is directly deducted from the members' incentives or shares of profits, which is a part of the variable salary. The disadvantage of this method is that the amount of contributions as well as the time of payments could be irregular.

- **A fixed percentage of salaries and a fixed percentage of profits or incentives**

The contributions in this case comprise two parts: one is a fixed percentage of the salaries and the second part is deducted from the incentives or the shares of the profits. This method actually has got the advantages of both methods mentioned above, and thus, it is widely used particularly in the profitable large financial units in Egypt.

(b) Employers' contributions

The employers' contributions like the employees' contributions are determined for each scheme by the actuary according to its statute, they can take one of the following forms:

- **An initial sum at the time of setting up the scheme**

The employer may only pay an amount of money at the time of setting up the scheme. The disadvantage of this method is that there is no regular contributions paid to the scheme by the employer. So that, at the time of establishing the scheme, the members who are approaching the retirement age will probably benefit from this rather than the younger or new members.

- **A fixed contribution**

This fixed contribution is paid regularly to the fund in one of the following types:

- A fixed percentage of salaries: which depends on the definition of the pensionable salary.
- A fixed sum per year: which is deducted either from the achieved profits or the incentives of the employees.
- A fixed percentage of salaries and a fixed sum per year.

- **An initial sum and a regular contribution**

According to what is stated in the statutes of each scheme and agreed by the members, this kind of contribution could take the following forms:

- An initial sum and a fixed percentage of salaries: which is the most common type applied in the schemes.
- An initial sum and an annual fixed sum paid regularly in the beginning or at the end of every year. The disadvantage of this method is that the increasing number of members in the scheme will lead to the decrease of the average share paid to each member. Also, this is largely affected by the financial status of the companies and their achievements over the year, i.e this form is more suitable for the large profitable companies which can assure the payment of the annual contribution.

2.4.4 Benefits

Since most of the Private Pension Funds are defined benefit schemes, the benefits should be precisely determined in advance with the eligibility for both members and the dependants (if any) by the management of the fund. In some schemes, the eligibility specified in SSIS may be used as a guide for application, but in any case this should stated clearly in the statute of each scheme. This is followed by specification of the contribution rates that cover the promised benefits by the actuary as mentioned earlier.

Most of these schemes pay the benefits in the form of the lump sum in case of retirement, death and disability which are considered the main coverage offered to the members.
In 1999 there were only 40 schemes offered pension to their members. The UK experience shows that the occupational pension schemes usually offer pensions to their members at the end of their service. Although the lump sum on retirement does not provide longevity insurance and may be invested or spent imprudently (Khorasanee, 2001/2002), it is still the common form of the benefits offered by the Private Pension Funds.

Lastly, it is important to mention that all types of benefits are tax exempted, and the member should have a minimum period of contribution in order to be entitled to the benefits as mentioned earlier. So that, he/she has no right to transfer his accrued rights to another fund, i.e. the member has to leave the current fund and settle his/her accrued rights before starting a new membership in a new fund.

In the following subsections, the major benefits offered by these schemes will be studied in more detail.

(a) Normal Retirement Age (NRA)

For calculating the benefit paid on retirement age which is 60 years for both men and women, the statutes of the schemes must state the following:

- The maximum period previous of setting up the scheme which is allowable for members to include in their membership of the scheme.
- A clear definition of the pensionable salary that will be used in calculating the benefit as previously mentioned .
- The accrual rate in case of offering pensions to the beneficiaries and /or the number of months that the lump sum will depend upon.

Thus, the lump sum on retirement is calculated depending on both the number of months for the years of service prior to setting up the scheme and the number of months of every year of membership after setting up the scheme.

(b) Death

The lump sum is calculated in the same way as the retirement benefit, i.e. the lump sum is calculated upon the number of months defined in the statute of the scheme to be entitled to the benefit, taking into consideration the years of membership until the death occurs.

(c) Disability

Disability must be defined in the statute of each scheme as well as the type of disability which is covered by the scheme. Permanent total disability that leads to the termination of service is usually covered by the majority of the schemes. The lump sum is calculated in the same way as death and retirement taking into consideration the period till the disability occurs. Permanent partial disability is covered in some of the schemes, the lump sum given in this case is calculated as a part of the total disability according to the percentage of disability as clarified in the tables attached to the statute.

(d) Early retirement

In this case, the member is entitled to a remuneration or part of it, only if he/she has got the minimum duration of membership required or he/she reaches a cértain age defined in the statute of each scheme. Otherwise the contributions will be refunded to the members. For example, this could be stated according to the age of the member as follows:

Less than 45	contributions refund
45 – 50	50% of remuneration
50 – 55	75 % of remuneration
55 less than 60	90% of remuneration

Although this helps the companies to keep the qualified employees for longer periods, the recent trend of privatization of public sector may lead to change these percentages in the case of withdrawal or redundancy. In fact, redundancy has affected some of the Private Pension Schemes in Egypt and caused serious problems in their funds. This led to winding-up the schemes in some cases, where they couldn't pay the promised benefits to the members.

(e) Other benefits

Some schemes may offer cash payments in cases of marriage, death of spouse or parents, giving birth and medical care, they are usually equal to two months salaries in each case.

In 2002, the statistics implied that NRA, disability and death benefits got the highest shares with 45%, 22.5% and 18.1% respectively, followed by pension, others[4] and refund of contributions with small shares around 4%, other benefits such as: medical care, death of relative and having children got very small shares as well as the marriage benefit which got the smallest share. This can be seen in Table 2.6 which illustrates the relative importance of the types of benefits offered to the beneficiaries in 2002.

2.4.5 Management of the fund

According to law no 54 of 1975, a board of directors should be elected from the members at the time of setting up the scheme to manage the fund until the first assembly is held. Thereafter, the board of directors is elected for 3 years from the members and the representatives of the company (in the case of employers' contributions).

Each scheme should held the records needed to carry out their operations efficiently, such as: the membership record, the total assets and the investments, the revenues and expenses records including the amounts of the contributions and benefits.

These records are checked and approved by EISA to ascertain the accuracy of the activities carried out by the management of the fund and to secure the members from any theft, fraud, misappropriate of the management, this is similar to the role that the trustees play in the pension funds in the UK. In return, there is a supervision fee equals to 1% of the total contributions paid to EISA.

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[4] Other benefits that are offered in some schemes include: funeral fees, marriage of sons or daughters or any other benefits stated in the scheme and are approved by EISA.

Table 2.6
The relative importance of the types of the benefits offered to the beneficiaries in 2002

Types of Benefits	The benefits (thousands LE)	Relative importance %
NRA	635383	45.0
Death	255205	18.1
Disability	317240	22.5
Pension	63082	4.47
Refund	45998	3.26
Marriage	179	0.013
Children	6423	0.455
Death of relative	824	0.058
Medical care	29529	2.09
Others	57043	4.04
Total	**1410906**	**100**

Source: EISA, 2003/2004

2.4.6 Investment

Investment strategies of Private Pension Schemes play an important role in achieving a satisfactory level of return on assets held by the scheme, which considers another vital resource to the scheme. In fact, Private Pension Funds has got a better chance of investing their assets compared with both SSIS and Contracted – Out Schemes, regarding the restrictions by laws which control their investment strategy and enforce them to invest about 90% and 50% of the funds respectively in National Investment Bank.

The allocation of assets and the restrictions applied to the investments with the return on different types of assets are important to cover in more detail below.

(a) Allocation of assets

The types of assets which the pension schemes normally invest in are:

- Government bonds;

- bank deposits;
- equities;
- commercial bonds;
- properties;
- cash;
- loans (to members or commercial organization);
- overseas investment

The Private Pension Funds invest in all the previous types of assets except overseas investment. However, they adopt the same prudent investment strategy as SSIS and the Contracted-Out Schemes. From Table 2.7 we can see that the investments in Government bonds and bank deposits reached 92.9% in 2002 where as equities represented only 3.4% of the investments. Properties, loans and other investments were around 1% for each of them. Although there are fewer restrictions on asset allocations of the Private Pension Funds compared with SSIS and the Contracted-Out Schemes (mentioned below in sub subsection (b)), the management of the funds prefer to invest their funds in low risk assets.

Table 2.7

The relative importance of the investments of the Private Pension Funds in 2002

Investments	Invested amounts (millions LE)	Relative importance %
Government bonds	6955	63.8
Equities	376	3.4
Bank deposits	3169	29.1
Properties	165	1.5
Loans	125	1.1
Others	111	1.0
Total	10901	100

Source: EISA, 2003/2004

The fact that the stock market is emerging in Egypt, at the present time, can provide a good explanation of why most of the funds are invested in Government bonds and bank deposits rather than equities. Therefore, the development of the stock market could have a big influence in achieving a high return on investment in equities and dealing with the inflation problems. At the same time, the issuing of index-linked bonds by the Government could be a good solution in providing assets that could be used to match the liabilities. Overseas investment of the schemes' funds has not yet permitted by EISA in Egypt, although there is an opinion that a limited share of investment in equities in overseas territories should be allowed (GAD, 1997).

(b) Restrictions on allocation of assets

The Private Pension Funds have the opportunity to choose the types of assets that they would like to invest in, taking into account the percentages determined in Article 14 in law no. 54 of 1975 for securing the members, these percentages are:

- 25% at least in Government bonds.
- A maximum of 15% in bonds, with a maximum of 5% in bonds from one company or 20% of the capital of this company.
- A maximum of 25% in equities with the same condition applied in bonds.
- A maximum of 10% in property.
- A maximum of 25% in loans to the members with no more than 75% of his/her accrued liabilities.
- Bank deposits with a maximum of 10% in one bank.
- 10% in other investments approved by EISA.

These percentages allowed by law provide the management of the funds some flexibility in choosing a diversified portfolio of assets that achieve the desired gains. This emphasizes that the investment strategies of the funds are not directed to high risky assets, as the investments in equities reached 3.4% while the maximum legal percentage is 25%. On the other hand, the investment in Government bonds was about 64% while the minimum legal percentage is 25%.

The restrictions on investment of pension funds are common in many countries, for example: in the UK these restrictions are set out in the Trustee Investment Act 1961, and the self-investment is maximum 5% of the assets of the scheme. In Switzerland: there are restrictions for all financial units to have a maximum of 50% of the funds invested in shares (Fédération des Experts Comptables Européens, 1995).

2.4.7 Actuarial valuation

The actuarial valuation should be made at least once every 5 years by a qualified actuary like the Contracted-Out schemes. However, EISA may ask for another valuation or even an annual one in the case of deficit or any serious problems facing the scheme. The specification of the minimum frequency for actuarial valuation is common, e.g. in UK, an actuarial report is required at least every three years

2.4.8 Main problems of Private Pension Funds

The main problems facing the Egyptian occupational pension schemes are:

- The occurrence of deficit in some of the schemes that could lead to wind-up the scheme and threaten the rights of the members. The number of funds in deficit were in total 37 in 1998 (MISA Report, 1998) classified according to the causes of deficit as follows:
  - Insolvency problems (22 funds), this insolvency mainly arose as a result of the privatization of the public sector and the redundancy of workers, as there are 170 funds in these Public units, the Ministry of Public Employment issued a decision to cover any deficit in the funds caused by the redundancy of workers.
  - Debts in the financial statements of the fund (6 funds), which should be scheduled and paid on the due dates.
  - An actuarial deficit (4 funds) resulting mainly from the difference between the experience and the actuarial assumptions that used in the actuarial valuation.
  - Insufficient resources resulting from facing problems in collecting the contributions from the members or the employers, or not achieving

the expected return on investments (5 funds).

- Theft, fraud and misappropriate management of the funds: although there are restrictions under law on the authorities given to the board of directors, and this is supervised by EISA, there is still a problem dealing with that and resolve the board of directors and appoint another one.
- Funds are not legally registered in EISA, there were a total number of fourteen unregistered schemes in 1998: three universities, eight labor unions and three ministries(MISA Report, 1998). Since they have not been under the supervision of either MISA or EISA, there is no adequate information about them. Thus, it is recommended that these schemes should be registered in EISA whatever its type in order to protect the members' rights and ensure the accuracy of the procedures and their activities.

2.5 Personal insurance policies

Considering the religious and cultural factors in Egypt, the market of life insurance for either the individual policies or the group policies is limited, it only represents about 18% of the total insurance premiums (EISA, 1999/2000), while in EU countries the share of life premiums to total premiums is 43.5% and in some countries like: the UK, it may exceed 60% (European Business, 2002). Other factors that are relevant include: the repressive regulatory environment, the lack of competition and product innovation, as there are only 8 national insurance companies that underwrite life insurance policies, and finally the prevailing mistrust between insurance companies and their clients (Vittas, 1998).

In fact, focusing on the annuity and life insurance market could be one of the solutions to the reform of the social protection programs in Egypt (Loewe, 2000). This can be achieved by tackling the Egyptian market problems through:

- rising the awareness of the Egyptians about the ways that they can deal with the social risks.
- creating a competitive market by increasing the number of both the national and multinational insurance companies. There are currently five multinational

life insurance companies in the Egyptian market. This could be useful for a potential annuity market, where more products and better services are offered, and hence, helps to reduce the mistrust between the insurance companies and the clients.

Summary

This chapter reviewed briefly the whole social security system in Egypt. It explained the State Social Insurance System coverage with the contributions paid and benefits provided in Old –age, death and disability contingencies only. The other risks covered by SSIS are not of the relevance of our research so they are not mentioned at this stage.

Contracted- Out Schemes were briefly studied as they are the only schemes that opt out of the state social insurance system. Also the individual plans represented in the personal insurance policies were mentioned with the problem of the limited annuity market.

Private Pension Funds (or occupational pension schemes), which are the main supplementary system to the State Social Insurance System, were studied in more detail as they are of a main concern in our research. A brief description of the Private Pension Funds and its types were mentioned to understand the background of these schemes. This could be helpful in the application of dynamic pension funding plans on a defined benefit private pension fund later in Chapter 6. Different aspects have been considered such as: the common types of contributions and benefits in these schemes, the actuarial valuation and investment strategies issues were discussed with the problems that face the three main pillars, i.e. the State Social Insurance System, Contracted-Out Schemes and Private Pension Funds.

Chapter Three
Presentation of mathematical model

3.1 Introduction

A key consideration when establishing a pension scheme is how it will be financed. Different types of pension schemes: state pension schemes or occupational pension schemes, group or single schemes, defined benefit or defined contribution schemes, have their own financing strategies which determine the income and outgo of cash flows of the scheme.

In chapter 2, an overall study about the pension schemes was provided to understand how they operate with a particular focus on the Egyptian pension system. In this chapter, we will concentrate on studying the different ways of financing pension schemes. Therefore, the general concepts of the pension funding process will be explained. This is to provide the basic ground for deriving the dynamic pension funding models for a defined benefit pension scheme using the mathematical tool of dynamic programming and control theory.

3.2 Financing pension schemes

Financing pension schemes refers to the way the pension benefits are financed over time to be paid to the beneficiaries in the future, which is normally after the retirement age. This includes the amount and timing of the payment of the contributions to meet the accrued benefits. The methods of financing the pension schemes fall into two main categories: funded schemes and its alternative unfunded schemes or pay-as-you-go (PAYG), both of them will be described in the following subsections.

3.2.1 Funded schemes

The term of pension funding is related to the funded schemes, which are those schemes where the contributions are paid and invested in different types of assets, in order to accumulate in a fund or a reserve over years that will be used to pay the promised benefits in the future. In that sense, the Egyptian General Social Insurance System (i.e. Law no 79 of 1975) is a funded scheme through two funds: one for the government employees and the other for the public and private employees. Also Law no 108 of 1976 for the employers and Law no 50 of 1978 for the Egyptian working abroad are funded schemes. Moreover, the majority of the private pension funds in Egypt and the occupational pension schemes in UK (Blake, 2003) are funded schemes.

In funded schemes, the investment strategy is crucial as it determines the allocation of assets, thus, the amounts accumulated in the fund from investing in these earning assets in the future. It is clear by definition that the defined contribution schemes are funded schemes whereas defined benefit pension schemes could be funded or unfunded schemes. The pension schemes can also be partially funded as previously mentioned in section 1.2.3.

Furthermore, Blake (2003) classified various types of the pension funds in UK as:

- provident and pooled funds according to the type of the fund where the assets are held, either it is an individual account for each member in the former case, or a pooled fund for all members in the latter case;
- internal and external funds: the amount of funds can be invested internally, i.e. in the employer company assets such as: the book reserves in Germany, or externally in other companies;
- insured and self insured funds: the insured funds are the small schemes that usually depend on life insurance companies to arrange their business, while the large schemes undertake the responsibilities of running their own scheme and investing its funds.

According to the characteristics of the Private Pension Schemes in Egypt, we can say that most of the schemes are pooled funds as they are group pension schemes, where all the contributions are pooled and invested as one sum. Further, they are restricted by law to self-investment with a maximum of 10% of total assets as shares in one company.

Also, the majority of the Private Pension Funds are self-insured schemes, where the management of the fund runs the scheme and invest the assets, for example: one of the largest scheme (that we shall study later in chapter 6 as a case study) is responsible for managing 34 small schemes because of its well-known experience. In addition, it manages a group insurance policy for its members instead of the life insurance company. On the other hand, the small schemes also act as self-insured schemes, unless they have group insurance policies for their members which are managed by life insurance companies.

3.2.2 Pay-as-you-go (PAYG)

PAYG or unfunded schemes are the other extreme way of financing the pension schemes, i.e there is no accumulated fund to pay for the pensions in the future, rather, the income of the scheme, which comprises of the collected contributions plus any other revenues, should be used to cover the pensions of the beneficiaries on a yearly basis. Therefore, the investment strategy does not play any role in these schemes. We should note that the overall cost of PAYG schemes is generally lower than the funded schemes. This cost includes: the initial set up costs, the costs of running the scheme and the administrative costs (Blake, 2003).

Although cost can be regarded as an advantage of these schemes, the absence of accumulated invested assets can increase the costs of these schemes in the long term compared with the funded scheme. This has been explained by Khorasanee (1999) as a disadvantage of PAYG schemes. For example: in an unfunded schemes providing salary-related benefits, the rate of pensionable payroll should grow at a faster rate than the investment return that could be achieved in a funded scheme. Also, in the case of termination, there could be no fund available to cover the accrued liabilities.

The state social systems are usually unfunded scheme, for example: in UK, central government civil servants is funded by the Exchequer. In Egypt, Law no 112 of 1980 for casual workers is also considered PAYG scheme.

3.3 Actuarial valuation

The actuarial valuation is an investigation of the current and future income and outgo of the scheme, in other words, it includes an analysis of future income, represented in the contributions paid by the employer and employees and the investment of the assets, and a projection of the expenditures streams, represented in the benefits offered to the current and future beneficiaries, according to specific assumptions.

Pension schemes are obliged by law to carry out the actuarial valuation periodically, where this period can vary from one scheme to another within a maximum limit determined by law, for example: the actuarial valuation should be conducted at least every five years in Egypt, while in UK it is at least every three years. However, it is common for large schemes in both countries to carry out the actuarial valuation annually with the financial statements.

The purpose of the actuarial valuation and the key points for carrying out the actuarial valuation are crucial to the pension funding process, hence, they are illustrated below with focusing on the defined benefit schemes.

3.3.1 Purpose

The purpose of actuarial valuation has been discussed in several researches. From our point of view, the purpose of the actuarial valuation can be seen from a short term and long term perspectives as follows:

- In the short term
    - Examining the financial position of the scheme and ensuring that there is no discontinuance problems facing the scheme.
    - Providing information to the main interested parties, such as: the management of the fund, the employer, the members and the regulatory and supervisory authorities, other financial or accounting parties, or any other entity (or authorised entity as the actuarial valuation reports are not

necessarily published) that might be interested in the scheme and its activities.

- In the long term
  - Projecting the cash flows of the scheme under specific actuarial assumptions - which will be explained in subsection 3.3.2 - to make sure that the resources are sufficient to cover the expenditures of the scheme, hence, the contributions paid by the employers and the employees with the return on investment should be sufficient to pay the current and future benefits within the framework of the trust deeds and the rules of the scheme.
  - Providing recommendations to the management of the fund to make adjustments to the contribution rates or the benefits provided according to the surplus/deficit resulted from the scheme's transactions over time.

3.3.2 Elements of actuarial valuation

The actuarial valuation is carried out by the actuary who is, according to the Actuarial Profession (2000), either a Fellow of the Faculty of Actuaries (FFA) or the Institute of Actuaries (FIA) in UK. The actuary should be well qualified to advice on pension scheme issues, especially about the potential benefit payments, scheme liabilities and how to balance the interests of the various parties involved, and an expert in assessing and communicating the likely financial impact of uncertain future events.

In Egypt, the actuary is appointed by the board of directors according to the Law no 54 of 1975. In UK, the Pensions Act 1995 requires the trustees to appoint a named individual actuary (the Scheme Actuary) for the defined benefit schemes, nevertheless, they can appoint other actuaries as advisors. Hence, the appointed actuary carries out the actuarial valuation regularly as explained earlier, and has to report it to the supervisory authorities (EISA in Egypt or the Pensions Regulator – which took over from OPRA on 6th April 2005 - in UK).

Ultimately, four main tasks are required to enable the actuary to conduct the actuarial valuation, and reach his conclusions about the financial position of the scheme and the recommended contribution rate, they are:

- Preparing the available relevant data of the scheme and its members;
- setting up the actuarial assumptions which will be essential for the pension funding plan;
- choosing the pension funding method in which the standard contribution rate and the actuarial liability are calculated;
- choosing the asset valuation methods that are approved by the management of the fund in accordance with the rules of the scheme.

In order to understand the steps of setting up a pension funding plan, each of these tasks will be discussed in more detail below.

(a) Appropriate data

The preparation of required data for each scheme is an essential process for starting the actuarial valuation. Thus, the data provided by the scheme should be complete, accurate and consistent. Hence, this enables the actuary to reach the accurate output about the recommended contribution rate and the accrued liabilities of the scheme.

In UK, according to paragraph 2.2.5 of GN9 "the actuary must take reasonable steps to satisfy himself or herself that the data provided is of adequate quality for the purpose of the valuation."

The importance of the required data was explained by King at the beginning of last century, he suggested that each member's data is recorded in cards, each card would include: the date of birth of the member, the date of entry into the scheme, the date of exit, the mode of exit and any past contributions to the scheme. This data with the salary scale would be used to construct a service table for the scheme's members

(Ngwira, 2004). At present time, advanced software packages (eg: Applaud-Pension ™ software) are used to help the actuaries in preparing their data.

According to Lee (1986) the following data are needed for the actuarial valuation:

(i) An up-to-date copy of the trust deed and rules of the scheme, further, any significant changes in the scheme's assets or liabilities made by the trustees.

(ii) A copy of the accounts and the balance sheet for each year since the previous valuation. This is considered the financial data of the scheme.

(iii) Details of each employee member, ex-member with preserved pension rights and current pensioner of the scheme and any other person who is entitled to the benefits at the date of the valuation. Age, length of membership, pensionable salary are among the most important data that should be available for each member. This is referred to the membership data of the scheme.

We can add to that, the national economic and demographic statistics published by the government and international bodies about the population of the country and the economic trends.

(b) Actuarial assumptions

The actuarial assumptions refer to the economic and demographic assumptions that are estimated by the actuary to be used in calculating the contribution rate and the actuarial liability of the scheme. The estimation of these assumptions are affected by the economic environment and demographic trends prevalent in each country.

The fact that these assumptions are estimated to be used in projections of cash flows of the pension plans in the future, as one of the purposes of the actuarial valuation, involves uncertainty about the real values that these assumptions might take in the future in the volatile environments. Consequently, the difference between the actual experience and the assumptions affects the income and outgo of the scheme, and may lead to a surplus or deficit in the pension fund.

The estimation of the actuarial assumptions has been considered by Thornton and Wilson (1992) who used the projections to reach the best estimates of these assumptions. However, Fujiki (1994) used the averaging of the past experience to adjust the assumptions. In fact, researchers consider a range of future projections of the economic and demographic trends as a basis for choosing the assumptions. Thus, Frees et al (2004) use different stochastic approaches to project the four economic assumptions for the Social Security fund: inflation rate, investment returns, wage rate and unemployment rate. Further, studying and forecasting of mortality trends as a major demographic assumption has been a subject of series of studies by Haberman and Renshaw (1997, 1999, 2000, 2003 and 2005).

For the purpose of our study, the main economic and demographic assumptions are explained, in turn, below.

(i) Economic assumptions

The economic assumptions are the factors that affect the amounts of income and outgo of the pension scheme, these include:

- *Salary increases*: The increases in salaries could be a general salary escalation and/or a promotional salary increase. The former refers to the average pay increases experienced by working population as a whole. The latter refers to the career advancement of particular individuals due to age or seniority or performance (Maait, 2003). Both types of increase affect the pensionable salary (basic salaries and/or variable salaries in Egypt). Since the increase in salaries earned from year to year by the members affects both the contributions paid and the benefits received from the scheme, the formulae of determining the contribution rate and the actuarial liability should be indexed to such increases (AAA, 2004).

- *Inflation rate*: Barro (1997) defines the inflation rate as "the percentage change in a price index between two periods of time", this price index could be based either on the Gross Domestic Product (GDP) or the Consumer Price Index (CPI) (Blanchard, 1997).

In many schemes especially in UK and US, the benefits are usually adjusted for inflation, by the cost-of-living adjustments (COLA) which is based on the changes of the (CPI). In countries with high inflation, if the pensions are not indexed with inflation, the retirees may receive a markedly inadequate income compared with their earnings pre-retirement. Consequently, the retirees may face severe problems or literally suffer from poverty after few years of retirement. This has been the case in Russia since the early 1990s (Blanchard, 1997), and also in Egypt, where the inflation rate had reached 27% at the end of the eighties before it was brought under control and declined to 5% from 1997-1999 (Maait, 2003).

- *Rate of return on investment*: is one of the important economic factors to the pension schemes. The rates of the investment returns have a vital effect on the income of the scheme. According to The Faculty and Institute of Actuaries (SA4, 2005), two interest rates are required to be used: the first one is the discount rate, which is used in calculating the past service liabilities, it should vary to reflect volatility in the market. The second one is the long term return on assets, which reflects the average rate of future earnings expected on the fund allowing for the reinvestment of future income, this would be expected to be relatively stable from year to year. Thus, the latter interest rate may affect the income to the fund from different investments, hence, influence the financial status of the scheme. The effect of the rates of investment returns on the contribution rates and fund levels in defined benefit schemes are examined by Haberman (1994) and Haberman et al (1997). Regarding the rapid changes in the financial markets, at the present, the rates of investment returns in the actuarial valuation should be carefully estimated in order to avoid highly unexpected gains or losses on the schemes funds.

(ii) Demographic assumptions

The demographic assumptions are the demographic factors that deal with the population of the schemes, thus, these assumptions constitute the multiple decrement model (or the service table) of the scheme. These include:

- *Mortality rate*: this rate affects the number of the beneficiaries (males and females) who are expected to live and receive the benefits, on the other hand, the number of the active members who are expected to die before reaching the retirement age. Mortality rates can vary across the different occupational pension schemes. Significant differences in mortality rates have been detected in UK across the large occupational pension schemes (Gardner, 2005). Besides the difference in wealth of pension members - which explains a part of these differences according to Gardner (2005) - the membership structure of the scheme, i.e. the number of young contributors and older ones, the gender of the active members and pensioners, their marital status are among the factors that affect the mortality rates of the occupational pension schemes.

  At present, studying the future trends in mortality rates is an important issue in the developed countries because of the impact of increasing life expectancy and ageing population. Vaupel et al (2005) explain that mortality at older ages has declined dramatically since 1950 in developed countries, for example: while there were 265 centenarians in England and Wales in 1950, they increased to 6680 in 2000. Progress in economic developments, social improvements and advances in medicine are among the reasons for the increasing trend in life expectancy in the industrialised countries over the last 160 years. The problem of ageing populations creates challenges for all retirement benefit systems in needing to pay the promised benefits for longer periods.

  Although this situation is serious in many developed countries (like UK, US, Japan and western Europe), it is not yet important in Egypt, where there is no ageing population problem, since the proportion of the working people to the elderly is high.

- *Ill-health retirement*: is another factor that can affect the number of the active members of the scheme. The member is usually entitled for the pension if he/she stops working due to illness. For defined benefit pension plans, the types of disability (either partially or fully) and the rules of eligibility to pension, according to the severity of illness, are usually specified in the rules of the scheme. Further, the reduction factors of the ill health retirement that are used to calculate the accrued pension should also be stated in the rules of the scheme or the trust deed. Often, there is no minimum age for ill-health retirement, for example: in Egypt and UK (Booth et al, 2005).

- *Withdrawal or early retirement*: affects the number of active members of the schemes. This contingency usually puts a financial burden on the scheme, particularly if the number of early leavers is high due to political or economic decisions. In Egypt, the percentage of early retirement pensioners to the total pensioners reached 12.43% in 1998 compared with 4.04% in 1979. The analysis of pensioners, in the same year, reveals that the early retirement pensioners are from the public sectors (because of the privatization of many units in this sector in 1990s as mentioned in chapter 2) more than other sectors, and from females more than males (Maait, 2003).

Other factors that might be taken into consideration when studying the demographic characteristics of the population generally - not specifically for defined benefit occupational pension schemes - include:

- *Rate of Fertility*: refers to the average number of children born to a woman during her life time. It is usually applied to the national population, and mainly to determine whether the work force will be sufficient to cover the benefits paid to the retirees. A fertility rate of 2.1 children per woman is required for the population to replace itself (Booth et al, 2005). This means that a higher fertility rates would result in an increasing population, and hence, an improved financial position for the scheme as the number of workers increases.

The total fertility rate (average number of children) was 3.51 in 2000 in Egypt and was expected to be 3.29 in 2005. This is obviously higher than the rates obtained in the developed countries like UK, Germany and Italy, which are equal to 1.70, 1.34 and 1.21 respectively in 2000 (UN, 2000). The expectation that these rates will decline further below the replacement rate, highlights the existence of the pension crisis facing these countries.

- *Immigration rate*: could be among the demographic factors that should be taken into consideration, especially in some countries like: US and Canada, where the immigration rate is high and the economy depends on the young immigrants.

  On the other hand, accept more immigration could provide a solution for the pension crisis in countries that face demographic problems such as: ageing population and low birth rates like UK, Germany and Italy (Blake and Mayhew, 2004). Increasing the number of working people, by allowing a large number of immigrants to the labour market compared with the number of the pensioners, will definitely affect the dependency ratio[1] in the country both in the short term and long term. Accordingly, this affects the number of the new members joining the occupational pension schemes. This rate can be considered in the DB schemes as it affects the number of the new entrants joining the scheme. Colombo (2005) studies and develops the population plan theory, in which the population of the DB scheme grows, because of the new entrants in the scheme (rather than birth), assuming a randomly evolving population.

(c) General Pension Funding Plan

According to Sung (1997), the general pension funding plan is the combination of the primary pension funding methods and supplementary funding methods, based on going-

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[1] $\textit{Dependency ratio} = \dfrac{\textit{the number of people above retirement age}}{\textit{the number of working people}}$

on valuation basis (i.e. the scheme does not have problems of discontinuance) to reach the recommended contribution rate. Thus, the recommended contribution rate consists of two parts: the first part is the standard contribution rate (SCR) or the normal cost calculated using the primary funding methods. The second part is the adjustment to the SCR, which is calculated using the supplementary methods. The primary pension funding methods and the supplementary pension funding methods are explained, in turn, below.

(i) Primary pension funding methods

There is no unique pension funding method that must be used in the pension schemes according to specific circumstances. The pension funding methods can vary from one scheme to another according to the choice of the scheme's actuary, which should comply with the trust deeds, the rules of the scheme and the objectives of the management of the fund.

Therefore, the most common primary pension funding methods are four methods specified by the Institute of Actuaries in order to calculate the actuarial liability (AL) and the standard contribution rate, they are:

- Attained Age method
- Entry Age method
- Projected Unit method
- Current Unit method

According to the actuary's perspective of the main objective of the pension funding plan, i.e. whether it is mainly about the stability of the contribution rate or the security of the accrued benefits of the members, these methods can fall into two categories:

- **Prospective methods:** which target a stable contribution rate (e.g. Attained Age and Entry Age);
- **Accrued benefit methods:** which target the actuarial liability (e.g. Current unit and Projected unit).

Although this classification is commonly used in UK, Haberman (1994) mentioned another categorisation based on the mathematical structure of the fundamental equations, where the funding methods classified into: individual funding methods (e.g. Entry Age, Projected Unit and Current Age); and Aggregate funding methods (e.g. Attained Age and Aggregate).

A brief description of these funding methods, following the former classification, and the basic formulae used to calculate SCR and AL are given below (AEC, 2001), assuming that: the pension scheme is a final salary pension scheme, where the benefits at retirement is related to the member's salary at retirement and accrue by reference to the service in the scheme; and the rate of return is (i) which is used as a discount rate of interest $\left(\frac{1}{1+i}\right)$ in calculating the present values of benefits and earnings.

- **Prospective methods**

Attained Age method versus Entry Age method

Under the Attained Age method, SCR is determined as a stable rate paid by the members over the expected future membership of all members, in order to accumulate (with investment returns) to the value of benefits that are expected to accrue over that future membership period. Hence, the SCR is calculated as follows:

$$SCR = PVF / PVPE \tag{3.1}$$

where:

PVF: the expected present value of all future benefits to present members after the valuation date, by reference to projected earnings.

PVPE: the expected present value of total projected earnings for all members throughout their expected future membership.

On the other hand, the SCR under the Entry Age method is calculated in a similar way to the Attained Age method, except that the contributions and the benefits are equated over the full expected period of the membership, including any past service accrued at the valuation date. This full expected period of membership will be based on a single assumed entry age for all of the members. The assumed entry age may be chosen as one of the actuarial assumptions, or it may be derived from inspection of the actual entry ages of the members, hence the formula for calculating SCR is:

$$SCR = (PVF)_x / (PVPE)_x \tag{3.2}$$

where:

$(PVF)_x$: the expected present value of future benefits for a member joining the scheme at the assumed entry age (x), by reference to projected earnings.

$(PVPE)_x$: the expected present value of projected earnings for the member throughout his/her expected future membership.

The actuarial liability under both methods is similar, although its value will be different according to the value of SCR., thus, the formula for the actuarial liability for both methods is:

$$AL = PVTB - (SCR \,.\, PVPE) \tag{3.3}$$

where:

PVTB: the expected present value of total benefits accrued at the valuation date (from accrued or past service, owed to pensioners and owed to deferred pensioners) based on projected final earnings for members in service.

- **Accrued benefit methods**

Projected Unit Method versus Current Unit Method

Since the target is the actuarial liability in these methods, we are concerned primarily to calculate the AL. Thus, under the Projected Unit method, the actuarial liability is the discounted value of the benefits that have accrued over the past period of membership of the beneficiaries. In determining this value, allowance is made for any future expected inflationary growth of the on-going benefits up to retirement age. Similarly, the SCR is based on one year time period as follows:

$$AL = PVTB$$

$$SCR = PV1 / PV1E \tag{3.4}$$

where:
PV1: the expected present value of all benefits that will accrue in the year following the valuation date, by reference to projected earnings.
PV1E: the expected present value of all members' earnings in that year.

For the Current Unit method, the actuarial liability is determined in the same way but without making an allowance for any inflationary growth (earnings growth). The SCR is calculated based on one year time period and the percentage of earnings increase to determine the amounts needed to cover the liabilities over this year. The formulae of Current Age method for AL and SCR are:

$$AL = PVTBc$$

where:
PVTBc: the expected present value of total benefits accrued at the valuation date, based on current earnings for members in service.

$$SCR = (PV1 + (PVTBc \,.\, e)) / PV1E \qquad (3.5)$$

where:
e: the rate of earning increase over the next year.

Finally, a fifth funding method can be included to the four common funding methods mentioned above, it is called the Aggregate method. Unlike the four funding method mentioned above, the Aggregate method does not define a normal cost or actuarial liability, instead, it is considerably regarded as a type of the adjustments of the contributions (Haberman, 1997), thus RCR is determined each year at time t by this formula:

$$RCR = [\,(PVB_t - F_t) / PVS_t] \,.\, S_t \qquad (3.6)$$

where
PVB_t: is the expected present value of benefits (for all members including pensioners at time t.
PVS_t: the expected present value of future salaries(of active members) at time t.

S_t: is the total salaries of active members at time t.

(ii) Supplementary pension funding methods

The supplementary funding methods are used to adjust the standard contribution rate according to the surplus or deficit emerges as a result of the difference between the experience and the actuarial assumptions, as these assumptions are unlikely to held exactly in real practice. Thus, the recommended contribution rate at time t is:

$$(RCR)_t = (SCR)_t \pm (Adj)_t \tag{3.7}$$

Clearly, the adjustment will have a positive sign if the scheme is in deficit or it is deducted from SCR if the scheme is in surplus.

The most widely used methods by actuaries for adjusting the contributions are: Spread method and amortisation of loss method. In addition, Owadally (1998) has pointed out other ways of adjusting the contributions, such as:

- contribution holidays: which are taken by the employers in the case of large surpluses, particularly if there is a maximum surplus that is allowed to be held in the scheme (as in the UK);
- immediate cash injections: in the case where the surplus or deficit required to be removed immediately rather than gradually;
- ignoring gains and losses if they are within a given corridor: this is the case when the surplus or deficit is small and it is expected that they will be cancelled out if the actuarial assumptions are "on average" correct;
- establishing a reserve, particularly, in the schemes where the book value of assets are used.

In UK, the 1986 Finance Act required that pension-fund surpluses to be reduced to no more than 5% within five years (Blake, 2003). A plan should be presented to the Inland Revenue setting out the action to reduce the overfunding, which could be: employer contribution holiday; employee contribution holiday; increases to benefits or a refund to employer (SA4, 2005). Unlike the British experience, the Egyptian Law of Private

Pension Funds does not determine either a maximum limit of surplus or a spread period, rather, it is known that these funds are not established for the purpose of profit, and hence, the surplus is usually directed to enhance the benefits received by the members.

The two common supplementary pension funding methods used by the actuaries are explained below.

- **Spread method**

The concept of this method is to spread the unfunded liability (UL) which occurs at time t over an agreed period of time "spread period" in the future. This is defined by Dufresne (1986) as "the amount of adjustment of the SCR is equal to the unfunded liability divided by the present value of an annuity for a fixed term" where the unfunded liability is equal to the difference between the actuarial liability and the fund held in the scheme at time t:

$UL_t = AL_t - F_t$

$$Adj_t = \frac{AL_t - F_t}{\ddot{a}_{\overline{m|}}} \qquad (3.8)$$

where

UL_t: the unfunded liability at time t;

Al_t: the actuarial liability at time t;

F_t: the fund at time t.

$\ddot{a}_{\overline{m|}}$: the expected present value of an annuity for a fixed term m, calculated at the rate of discount being used in the valuation.

This method of spreading the whole surplus or deficit at periodic intervals is favoured in UK. Dufresne (1986) has examined the effect of varying the rates of return in a form of white noise process on the contributions and fund levels under the spread method, further, Haberman (1994) has studied this effect, assuming that the rate of return is represented by an autoregressive model for the corresponding force of interest, to consider the optimal range of values of "m", i.e. the spread period. This is followed by Haberman (1997), where the variability of the rate of return on the contribution rates and fund levels in defined benefit schemes with considering the possible choices of the spread period is studied.

- **Amortization of loss method**

This method is widely used in US and Canada, it is based on the actuarial gain and loss experienced at each valuation date, the actuarial loss during this intervaluation period (t-1, t) will be amortized over a fixed term in the future. Dufresne (1986, 1989) has examined how the contributions and fund levels are affected by the variation of rate of return in a form of iid (identically independent distributed) considering amortizing the intervaluation gains or losses over a fixed number of years (e.g 5 or 15). In 1988, he presents a stochastic approach for determining optimal values of these number of years. This can be described mathematically as follows:

$L_t = UL_t -$ *{value of* UL_t *had all actuarial assumptions been realized during the period (t-1, t))*

$$Adj_t = \sum_{s=0}^{m-1} \frac{L_{t-s}}{\ddot{a}_{\overline{m|}}} \qquad (3.9)$$

where

L_t: the actuarial loss during time (t-1, t), in which each L_s is liquidated by m payments of amount $L_s / \ddot{a}_{\overline{m|}}$ made at valuation dates s, s + 1,...., s + m -1.

According to the US Employee Retirement Income Security Act of 1974 (ERISA), the actuarial gains or losses should be amortized over a period from 5 to 15 years. Further, Owadally (1998) has concluded that there is an optimal amortization period range [1, m*] which is larger than the optimal spread period range. Also, the amortization of gains and losses over a fixed term yields greater fund security than when surpluses and deficits are spread over a moving term of equal length. However, the spreading surpluses and deficits may be regarded as more efficient than the amortization method as the latter method involves the feedback of delayed information into the pension funding process. More investigations about the efficient amortization of actuarial gains/losses and optimal funding in pension plans have been carried out by Owadally and Haberman (1999, 2000 and 2004).

(d) Asset valuation methods

The asset valuation is another main part of the scheme actuary's responsibilities. The aim of this valuation is to assess the level of assets held in the scheme, in order to ensure that the future cash flows of the assets can be used to meet the accrued benefits. Hence, the actuary should choose the appropriate method of asset valuation which satisfies the objectives of the management of the fund, among them are: consistency, realism, smoothness and dynamics {Owadally, (1998), Owadally and Haberman (2004)}. Besides, the actuary should be aware of the different characteristics and uses of each asset class in choosing the appropriate method of valuation, i.e the value of assets and their return differ according to the different characteristics mentioned by Blake (2003), which are: degree of liquidity, degree of capital-value certainty, degree of income certainty, inflation risk, default risk and the currency risk, and their uses in a volatile or stable market.

Since the chosen asset valuation method determines the value of the asset at the valuation date that can minimize the fluctuations of the asset values, consequently, the surplus and deficiencies that might arise in the future, the asset valuation affects the adjustments of the contributions due to the gains or losses experienced in the scheme.

In this part, we will briefly mention the different asset valuation methods as a part of the actuarial valuation for defined benefit pension plans, as the investment strategies and asset allocation modeling are not considered in our models at this stage. Thus, the most common asset valuation methods are defined below:

- Market Value method: it values the assets at market prices on the valuation date. This method could achieve the objective of realism, as the assets are valued according to the current prices prevalent in the market, or as an average of the current and past values (a smoothed value). However, it may not provide a good estimate of the value of the assets in the future, especially if a large percentage of the assets held in the portfolio have a degree of capital-value uncertainty. This method is commonly used in UK and US [Owadally and Haberman, (2001), The Society of Actuaries' committee on Retirement Systems Research, (2001)].

- Discounted cash flow method: it discounts the future income of asset to the valuation date. It has been commonly used in the UK until recently but it is rarely used in US and Canada. Under this method, the consistency between the asset and the liabilities values can be clearly observed, as the assets are discounted using closely related assumptions to the ones that are used for calculating the present values of the pension liabilities Owadally (1998). Nevertheless, the valuation of any variable income assets is based on the assumptions made for the discount rate, leading to fluctuations of gains and losses if the assumptions were not borne out.

- Book value method: it values the assets at the original purchase prices, this method is no longer widely used, as it is neither realistic nor consistent. Keeping the assets at the original prices does not show either the decreasing value of assets, because of inflation or depreciation, or the increasing value as a result of the gains obtained on assets.

- Smoothed market value as discussed by the Society of Actuaries' Committee on Retirement Systems Research (2001) [2]: this method reflects the market value with incorporating a specific algorithm for smoothing market fluctuations. It is classified into: average of cost and market values, write up (which is the most frequently used in US among the four methods in this category), deferred recognition (which is the most frequently used in this category in Canada) and average market value. Owadally (1998) suggested two reasons for smoothing asset values, they are:

    - generating a stable and smooth pattern of contribution rates;

[2] For further discussion about the smoothed value market method, see Owadally and Haberman (2004).

- generating an asset value that is more consistent with the long term assumptions used when valuing liabilities.

3.4 Dynamic Pension Funding Plan

Having studied the main concepts of pension funding and how to set up a general pension funding plan, we now consider establishing a dynamic relationship between the contribution rates and the fund levels in the scheme, i.e. how the contribution rate affects the level of fund needed in the scheme to meet its liabilities at certain time. Therefore, in this section, pension funding control systems will be explained in order to derive four dynamic pension funding models. Dynamic programming, as a mathematical tool, and control theory are used to reach the optimal contribution rate and optimal fund level in a finite time horizon.

3.4.1 Control systems

According to Hale(1973), control systems can be classified into:

- Open-Loop system: which is characterised by the input entering directly into the control elements unaffected by the output.
- Closed- loop system: in which the input is modified by the desired output before entering the controller. The main advantage of this system, which involves the introduction of feedback, is monitoring the performance of a system and using the measured performance as information for the proper control or manipulation of the system. This allows the system to respond to any disturbances that act upon it from the environment.

Therefore, we can consider the pension scheme as a system with the contributions and the fund available, which consists of the assets and the investment earnings, are the inputs to the system while the output is represented by the benefit outgo. The pension funding system can then be controlled by designing the controlling variable to produce a desired output (controlled variable) by the scheme's actuary (Voland, 1986). Here, the controlling variable can be the contribution rate and the controlled variable is the fund level.

3.4.2 Pension funding control feedback system

By definition, closed-loop control systems are more preferable than the open-loop ones because of their information feedback characteristic. In that sense, the pension fund closed-loop or feedback control system can be designed to obtain an optimal feedback control law, which represents the dynamic pension funding plan according to Sung (1997). Thus, dynamic pension funding plan is set up on the basis of information fed back about the actual output (the fund level) response, the optimal contribution rate is then derived within the context of the dynamic programming and control theory.

Therefore, applying a dynamic pension funding plan could be more effective to the funding of the scheme than using a general pension funding plan, in which the contribution rate is not determined upon the actual output response.

In addition, using dynamic pension funding plan allows for the uncertainties in the economic and demographic factors, and explains how the various variables composing the mechanism of pension funding interact with one another and evolve with time. This leads to secure optimally the promised benefits of the members, without undue financial burden being placed on the employer or employees (Sung, 1997).

Further, the dynamic pension funding plan could enable the management of the pension fund to forecast the future cash flows and adjust the contribution rates, under various scenarios based on specific plan strategies, which could help in decision making process (Chang, 1999).

In the following subsections, we consider the literature review of using the dynamic programming and control theory approach, and their applications on the dynamic pension funding models before progressing to derive our models.

3.4.3 Review of dynamic programming and control theory literature

Control theory first used in actuarial applications by Benjamin (1984) followed by Benjamin (1989) who considers the optimal control of pension funds in discrete time.

O'Brien (1987) introduces a model based on a continuous time stochastic approach, in order to minimize both the risks of the contribution rate and the solvency risk. Here, the controlled object (pension funding dynamic system) is linear assuming that: the benefit outgo is a linear growth function of time; the growth rate in membership, salary and earned rate of return on the fund are mutually independent normal random variables. The scheme is assumed to be only for active members. O'Brien uses the following performance index which is designated to evaluate the control errors and the cost of control excluding the contribution rate target level:

$$J(s, X, u) = E_{s,x} \int_s^T e^{-\rho t}(u^2 + \beta(\eta A - F)^2\, dt \qquad (3.10)$$

where:
u(t): the contribution rate at time t
A(t): the expected present value of future benefits at time t for the active members.
F(t): the fund level at time t
η : the fund ratio

Vanderbroek (1990) develops the O'Brien model and introduces a deterministic model based on continuous time where the benefit outgo, total payroll and the actuarial present value of future benefits are each exponential functions of time. Her interest is to apply the model to the national social security system in Belgium. The performance index contains a percentage of the payroll as a contribution target, as follows:

$$\underset{c,\alpha}{Min} \int_0^T e^{-\rho t}\{[C(t) - \alpha W(t)]^2 + \beta[\eta A(t) - F(t)]^2\}\, dt \qquad (3.11)$$

where:
C(t): the annual contribution rate at time t
A(t): the present value at time t of future benefits for active and retired members
F(t): the fund level at time t
α: a fixed percentage of salary to represent the contribution level
W(t): total salary rate at time t

η : the fund ratio

Later, Haberman and Sung (1994) introduce a deterministic and stochastic approach on discrete time basis, where the controlled object was linearly formulated, under the assumption that the investment returns are independent and identically distributed random variables in the stochastic case. Thus, they develop the previous models by taking into consideration both the contribution target and the fund target. They correspond to the normal cost and actuarial liability respectively in the performance index as follows:

$$J_T = \underset{C_t,\ldots\ldots,C_{T-1}}{Min} \{\sum_{t=0}^{T-1}[v^t(C_t - CT_t)^2 + v.\beta(F_t - FT_t)^2]\} \tag{3.12}$$

$$J_T = \underset{C_t,\ldots\ldots,C_{T-1}}{Min} E\{\sum_{t=0}^{T-1}[v^t(C_t - CT_t)^2 + v.\beta(F_t - FT_t)^2]\} \tag{3.13}$$

where :
C_t : contribution rate for period (t, t+1)
CT_t : contribution target at time t
F_t : fund level at time t
FT_t : fund target at time t
β : a weighting factor to reflect the relative importance of the solvency risk against the the contribution rate risk.
v : the discount rate during the period.

Among the applications of the dynamic pension funding, Cairns (1997, 2000) uses the quadratic loss function of Haberman and Sung to set up a stochastic continuous model, it optimizes the contribution rate over the range of possible asset-allocation strategies, where n risky assets and the risk free asset as well as demographic disturbances are considered:

$$L(t,c,x) = (c - c_m)^2 + 2\rho(c - c_m)(x - x_p) + (k + \rho^2)(x - x_p)^2 \tag{3.14}$$

where:
c : contribution rate
c_m : contribution target
x: the fund size
x_p : the fund target

Owadally and Haberman (2004) also consider the efficiency of methods of amortizing actuarial gains and losses in defined benefit pension plans. This is explained by the optimization of a quadratic objective criterion when the fund is invested in two assets: a random risky and a risk-free asset.

Chang (1999, 2000) follows the same approach of Haberman and Sung by setting up a model based on discrete time stochastic approach. The performance index is designed based on ratios which are used in measuring the discounted quadratic deviation over the chosen time horizon:

$$J(C_0, \ldots\ldots, C_{T-1}) = E\{\sum_{t=0}^{T-1}[v_t(1-\frac{C_t}{NC_t})^2 + v_{t+1}\beta_{t+1}(1-\frac{F_t}{AL_t})^2]\} \qquad (3.15)$$

where:
C_t: the contribution paid at time t.
NC_t: the normal cost at time t.
F_t : the fund level at time t.
AL_t : the actuarial liability at time t.
β : the risk-weighted ratio at time t.
v_t : the discount factor at time t.
η : the target funding ratio.

Chang then applies the model to the Taiwan's Public Employees Retirement System as a case study. He reaches the optimal pension funding using the ratio-induced performance measure and comparing it with the cost-induced performance measure derived earlier by Haberman & Sung.

Later, Chang et al (2002) continue his earlier work by considering investment strategies in the dynamic funding policy of defined benefit pension schemes. The model is also applied to the Taiwanese Pension Scheme. Further, Chang et al (2003) develop the ratio model by incorporating downside risks, and allowing the weighting factors in the performance criterion to belong to a broader parametric family. The rates of investment returns are assumed to follow the autoregressive process.

Thereafter, Haberman and Sung (2002) present a discrete time stochastic approach in the case of incomplete-state information. The model is based on ratio-induced performance index. This allows for a one-unit time delay due to the physical inaccessibility of some of the economic parameters or inaccuracies in the measurement procedures. Taking into consideration both the demographic and economic disturbances, the performance index used is shown below.

$$\underset{\{CR_t;t=0,1,\ldots,T-1\}}{Min} E\{\sum_{t=0}^{T-1}\theta(FR_t-fr_t)^2+(1-\theta)(CR_t-cr_t)^2\} \tag{3.16}$$

where :
FR_t : the funding ratio = F_t/AL_t
CR_t : the contribution ratio = C_t/AL_t
fr_t : fund target at time t.
cr_t : contribution target at time t.
θ : weighting risk factor.

Finally, Haberman and Sung (2005) extend their earlier work and derive a dynamic stochastic model in a discrete-time with an infinite time horizon. Both the economic and demographic random disturbances are considered. The stochastic inputs are assumed to be stationary for deriving a definite funding policy.

3.4.5 Optimization problems: discrete-time deterministic and stochastic dynamic models

Our objective is to determine an optimal feedback control law by using a performance index, which is a quantitative technique measuring the system's performance and minimizing it, and taking into consideration the two main risks of funding a defined benefit scheme, which are:

- Contribution rate risk: which refers to the instability of the contributions paid mainly by the sponsor of the scheme;
- solvency risk: this relates to the ability of the scheme to cover its liabilities towards the beneficiaries. This applies as long as the scheme operates on an on-going valuation. However, the risk of the scheme or the employer being insolvent is unpredictable, it means that the market value of the assets on a wind-up valuation does not cover the benefits payable (Haberman et al, 2003). Sung (1997) classifies the solvency risk into:

(1) The investment risk: it is simply referred to the risk associated with the expected returns from investments on different types of assets, and the separate holdings within each type of them. Thus, it could be divided into asset value risk, asset income risk and matched risk (for further detail, see Sung (1997)).

(2) The non investment risk: which deals with the liability risk of the scheme, it could include the liquidity risk where the funds may not be available at the time of the due payments.

Lee (1986) indicates that the main key characteristics of funding are: stability, security, liquidity and durability. Among these characteristics, the sponsoring employer is concerned mainly about the stability of the contribution rate, consequently, he is interested in minimising the contribution rate risk seeking for the stability. On the other hand, the trustees and the members are concerned mainly about the security of the promised benefits, which indicates their interest to minimize the solvency risk to secure their benefits. Thus, the objective of minimizing the performance index in order to obtain the optimal pension funding plan corresponds to minimizing the contribution rate risk (which corresponds to the cost of control in the control theory) and the solvency risk (which corresponds to the control errors), seeking for the achievement of stability and security of the scheme respectively.

Therefore, our models are an extension to the earlier work of Sung and Chang. We introduce four deterministic and stochastic models in discrete-time basis for a finite time span (wind-up) valuation. Both cost-induced and ratio-induced performance measures are used to derive the optimal contribution rate.

The reasons of choosing these four models are:

firstly, the deterministic models are generally used in the actuarial calculations in real practice for the purpose of simplicity. Hence, it will be helpful to derive a dynamic pension funding plan that matches with the approaches used in practice. However, stochastic models are recommended to replace the traditional deterministic approaches, as they are more realistic and correspond in a better way to the uncertainty surrounding the environment of the pension schemes.

Secondly, although the finite time span deals with wind-up situations, it will be more appropriate to use the finite time horizon in the application of the dynamic models in our research. This is because the cash flows of a particular defined benefit pension scheme will be used as time series of past data (eg: 12 years) to compare the actual cash flows with the optimal expected contribution rates and fund levels.

Thirdly, despite of similarity in composition of the cost-induced and the ratio-induced performance indexes mathematically, using both approaches enables us to understand the differences in their behaviour in practice.

Furthermore, the intersection between the contribution rate risk and the solvency risk in the performance indexes, as represented in the middle term $(C_t - CT_t)(F_{t+1} - FT_{t+1})$, is considered . This intersection term is introduced by Cairns (2000) as an extra term in the general quadratic loss function, which is proposed to provide an explicit solution for the optimal contribution and asset allocation strategies.

The explanation of adding this term to the performance index is that the interests of the employer and the members are considered simultaneously rather than separately. On other words, instead of weighing the conflicting interests between the employer, seeking for the stability of the contribution plan from one side, and the trustees and the members, seeking for the security of the pension fund from the other side, these interests could be viewed as mutual and dependent. Since discontinuance holds the employer responsible to compensate his employees and meet their rights, he should be keen to keep his company and the scheme solvent, and hence, he seeks for minimizing the solvency risk. On the other side, the members should seek the stability of the contribution rate in order to guarantee the payments of the required contributions at the specified times. This should not disrupt unduly the financial position of their employer, so that, the accrued benefits will be paid regularly without delays. Further, the stability of the contribution rate will be a main interest for the employees, if they are sponsoring the scheme as well as the employer, e.g. the defined benefit Private Pension Funds in Egypt.

In this subsection, we set up two deterministic and two stochastic performance indexes for pension funding of defined benefit scheme based on discrete time finite time horizon. Thus, the basic notations are defined, followed by the dynamic programming problems and the performance indexes of the four models.

The basic notations used in our models are:

- T : the finite time span (the control period).
- C_t : contribution paid at time t.
- F_t : Fund assets at time t.
- B_t : overall benefit outgo for the period (t, t+1).
- AL_t : actuarial liability at the end of the period (t, t+1).
- i_{t+1} : real rate of investment return earned during the period (t, t+1).
- FT_t : fund target for the period (t, t+1), which corresponds to AL_t (actuarial liability), if all actuarial assumptions will be realised exactly during the control period.
- CT_t : contribution target for the period (t, t+1), which corresponds to NC_t (normal cost) for the same condition mentioned above.
- γ : a risk measurement weighting factor to reflect the relative importance of the contribution rate risk
- α : an integrate risk weighting factor to reflect the relative importance of the contribution risk combined with the solvency risk.
- β : a risk measurement weighting factor to reflect the relative importance of the solvency risk.
- η : the fund target ratio.

Here, it is notable that the fund target refers to the standard fund (actuarial liability), and the contribution target refers to the standard contribution rate (normal cost)(Sung, 1994).

Therefore, the optimization problem is described as follows: Find the contribution rates C_t, C_{t+1}, ..., C_{T-1} For a finite time span, which minimize the quadratic performance criterion:

$$J_T = \underset{C_t,\dots,C_{T-1}}{Min} \{\sum_{t=0}^{T-1} \gamma(C_t - CT_t)^2 + \alpha(C_t - CT_t)(F_{t+1} - FT_{t+1}) + \beta(F_{t+1} - FT_{t+1})^2\} \qquad (3.17)$$

in the cost-induced deterministic case;

$$J_t = \underset{C_t,\dots,C_{T-1}}{Min} \{\sum_{t=0}^{T-1} \gamma(1 - \frac{C_t}{CT_t})^2 + \alpha(1 - \frac{C_t}{CT_t})(1 - \frac{F_{t+1}}{\eta FT_{t+1}}) + \beta(1 - \frac{F_{t+1}}{\eta FT_{t+1}})^2\} \qquad (3.18)$$

in the ratio-induced deterministic case;

$$J_T = \underset{C_t,\dots,C_{T-1}}{Min} E\{\sum_{t=0}^{T-1} \gamma(C_t - CT_t)^2 + \alpha(C_t - CT_t)(F_{t+1} - FT_{t+1}) + \beta(F_{t+1} - FT_{t+1})^2\} \qquad (3.19)$$

in the cost- induced stochastic case, and

$$J_t = \underset{C_t,\dots,C_{T-1}}{Min} E\{\sum_{t=0}^{T-1} \gamma(1 - \frac{C_t}{CT_t})^2 + \alpha(1 - \frac{C_t}{CT_t})(1 - \frac{F_{t+1}}{\eta FT_{t+1}}) + \beta(1 - \frac{F_{t+1}}{\eta FT_{t+1}})^2\} \qquad (3.20)$$

in the ratio-induced stochastic case[2].

Therefore, our approach is to use the control theory in either deterministic or stochastic environment, regarding the following recurrence relations to derive the optimal contribution rate:

$$F_{t+1} = (1 + i_{t+1})(F_t + C_t - B_t) \qquad (3.21)$$

$$AL_{t+1} = (1 + i)(AL_t + NC_t - B_t) \qquad (3.22)$$

under the assumptions that:

- An actuarial valuation is carried out to estimate them annually (at any time t for integer values, t =0, 1, 2,...)
- The contribution income and benefit outgo cash flows occur at the start of each scheme year.

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[2] The letter E in the equations refers to the expectation used for the stochastic problems.

3.4.6 Optimisation problems' solutions - deterministic cases -

(a) Cost-induced performance index

The optimal contribution rate (C_t*) will be determined sequentially according to the principle of optimality (Bellman, 1957) for a chosen time period t ε [0, T), thus, the $C^*_0, C^*_1, \ldots\ldots, C^*_{T-1}$ will depend on the observed pension fund state variables which is the response or the controlled variable of the dynamic pension system $F_0, F_1, F_2, \ldots, F_t$, and the inputs /controlling variables {CT_t, FT_t, and B_t } which are all previously given for the same time domain where t ε [0, T) , thus, our performance index in this deterministic case is:

$$J_T = \sum_{s=0}^{T-1}\{\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2\} \tag{3.23}$$

so, it could then be written as it consists of two parts:

$$J_T = \sum_{s=0}^{t-1}\{\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2\}$$
$$+$$
$$\sum_{s=t}^{T-1}\{\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2\} \tag{3.24}$$

Hence, the first part does not depend on $C_t, C_{t+1}, \ldots, C_{T-1}$, and $C_0, C_1, \ldots, C_{t-1}$ have already known at time t, we are concerned about minimizing the second part, to proceed by induction:

$$V(F_t, t) = \underset{\{C_s; s=t,t+1,\ldots,T-1\}}{Min}\{\sum_{s=t}^{T-1}\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2\} \tag{3.25}$$

From the principle of optimality, we obtain :

$$V(F_t, t) = \underset{C_t}{Min}\{\gamma(C_t - CT_t)^2 + \alpha(C_t - CT_t)(F_{t+1} - FT_{t+1}) + \beta(F_{t+1} - FT_{t+1})^2\} +$$
$$\underset{\{C_s; s=t+1,t+2,\ldots,T-1\}}{Min}\{\sum_{s=t+1}^{T-1}\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2\} \tag{3.26}$$

$$= \underset{C_t}{Min}\{(\gamma(C_t - CT_t)^2 + \alpha(C_t - CT_t)(F_{t+1} - FT_{t+1}) + \beta(F_{t+1} - FT_{t+1})^2) + V(F_{t+1}, t+1)\} \tag{3.27}$$

Hence, we can try the solution of the last equation of the form:

$V(F_t,t+1)=a_1(t)F_t^2+a_2(t)F_t+a_3(t)$ and where $a_1(T)=a_2(T)=a_3(T)=0$

For a boundary condition $V(F_T, T) = 0$, because there is no loss associated with the terminal state F_T[3].

As the pension fund process $\{F_{t+1}, t\ [0, T]\}$ is governed by a difference equation, the future states are a function of F_t only. Then, it is sufficient to choose C_t as a function of F_t, so we can have the following:

$V(F_t,t)=\underset{C_t}{Min}\{G(C_t,t)\}$

Suppose that $(1+i_{t+1}) = r_{t+1}$ and $k_{t+1} = (\beta + a(t+1))$

then,

$$G(C_t,t)=\{\gamma(C_t-CT_t)^2+\alpha((C_t-CT_t)(r_{t+1}(F_t+C_t-B_t)-FT_{t+1}))+\beta(r_{t+1}(F_t+C_t-B_t)-FT_{t+1})^2 \\ +a_1(t+1)(r_{t+1}(F_t+C_t-B_t))^2+a_2(t+1)(r_{t+1}(F_t+C_t-B_t))+a_3(t+1)\} \quad (3.28)$$

Hence, an optimal unique value of C_t^* should be derived, as $G(C_t, t)$ is a strictly convex function under the above condition, it is sufficient that the coefficient of C_t^* is positive for all $t\ \varepsilon\ [0, T)$, so that C_t^* equals:

$$C_t^* = \frac{\{(2\gamma+\alpha r_{t+1})CT_t-(\alpha r_{t+1}+2r_{t+1}^2k_{t+1})(F_t-B_t)+(\alpha+2\beta r_{t+1})FT_{t+1}-r_{t+1}a_2(t+1)\}}{2(\gamma+\alpha r_{t+1}+r_{t+1}^2k_{t+1})} \quad (3.29)$$

Since C_t^* involves both $a_1(t+1)$ and $a_2(t+1)$, by substituting C_t^* in equation (3.28), we can obtain the following recursive equations which can be used to calculate the C_t^* using the

[3] For mathematical simplicity, we could assume that there is no loss associated with the terminal state F_T, taking into consideration the different time spans that could be used in applying the models, i.e T= 30 or 40 and given that pension funds have a life cycle and are not expected to carry on their operations to infinity.

backward recursive relations:

$$a_1(t) = \frac{r_{t+1}^2(4\gamma k_{t+1} - \alpha^2)}{4(\gamma + \alpha r_{t+1} + r_{t+1}^2 k_{t+1})} \tag{3.30}$$

$$a_2(t) = \frac{(4r_{t+1}^2\gamma k_{t+1} - \alpha^2 r_{t+1}^2)(CT_t - B_t) + \alpha r_{t+1}FT_{t+1}(\alpha + 2r_{t+1}k_{t+1}) + r_{t+1}(2\gamma + \alpha r_{t+1})(a_2(t+1) - 2\beta FT_{t+1}}{2(\gamma + \alpha r_{t+1} + r_{t+1}^2 k_{t+1})} \tag{3.31}$$

(b) Ratio-induced performance index

The optimal contribution rate of ratio-induced performance index could be derived using the principle of optimality (Bellman, 1957) for a chosen time period t ε [0, T) following the same approach that has been used earlier to determine the optimal contribution rate of cost-induced performance index, thus:

$$V(F_t, t) = \underset{C_t}{Min}\{\gamma(1 - C_t/CT_t)^2 + \alpha(1 - C_t/CT_t)(1 - F_{t+1}/\eta FT_{t+1}) + \beta(1 - F_{t+1}/\eta FT_{t+1})^2 + V(F_{t+1}, t+1)\} \tag{3.32}$$

Consequently, the optimal contribution rate C_t* is:

$$C_t^* = \frac{\frac{(2\gamma+\alpha)}{CT_t} - (\frac{m_{t+1}}{CT_t} + 2r_{t+1}^2 h_{t+1})(F_t - B_t) + \frac{(\alpha r_{t+1} + 2\beta r_{t+1})}{\eta FT_{t+1}} - r_{t+1}a_2(t+1)}{2(\frac{\gamma}{(CT_t)^2} + \frac{m_{t+1}}{CT_t} + r_{t+1}^2 h_{t+1})} \tag{3.33}$$

where:

$$h_{t+1} = \frac{\beta}{(\eta FT_{t+1})^2} + a_1(t+1)$$

$$m_{t+1} = \frac{\alpha r_{t+1}}{\eta FT_{t+1}}$$

and, the recursive equations for both $a_1(t)$ and $a_2(t)$ are:

$$a_1(t) = \frac{r_{t+1}^2 h_{t+1}\frac{4\gamma}{(CT_t)^2} - (\frac{m_{t+1}}{CT_t})^2}{4(\frac{\gamma}{(CT_t)^2} + \frac{m_{t+1}}{CT_t} + r_{t+1}^2 h_{t+1})} \tag{3.34}$$

and

$$a_2(t) = \frac{(\frac{m_{t+1}}{CT_t})^2 B_t + 2r_{t+1}^2 h_{t+1}(\frac{2\gamma}{CT_t} + \frac{\alpha}{CT_t} - \frac{2\gamma B_t}{(CT_t)^2}) + \frac{r_{t+1}(\alpha^2 - 4\gamma\beta)}{\eta FT_{t+1}(CT_t)^2} - \frac{\alpha r_{t+1}^2(\alpha+2\beta)}{(\eta FT_{t+1})^2 CT_t} + r_{t+1}a_2(t+1)(\frac{2\gamma}{(CT_t)^2} + \frac{m_{t+1}}{CT_t})}{2(\frac{\gamma}{(CT_t)^2} + \frac{m_{t+1}}{CT_t} + r_{t+1}^2 h_{t+1})} \tag{3.35}$$

3.4.7 Optimisation problems' solutions - stochastic cases -

Although we have derived the optimal contribution rate in the deterministic case, where the future behaviour of the different variables assumed to be predictable and determined, the need for a stochastic approach grows in importance as it corresponds to the rapid changes in the market allowing for more accurate and realistic results as previously mentioned.

Therefore, we derive the expected optimal contribution rate for the same control problem in the discrete time incorporating only the economic disturbance by assuming that the earned real rates of investment return in each interval of time (t, t+1) are independent and identically distributed as normal random variables with mean θ and variance σ^2. Thus, i_{t+1} is the random rate of investment return for the period (t, t+1), and the observed outputs of the measurement process are equal to the fund states $\{F_0, F_1, \ldots, F_t\}$ as in the case of complete state information.

(a) Cost-induced performance index

The stochastic pension funding control problem is:

$$J_T = \underset{C_t,\ldots,C_{T-1}}{Min} E\{\sum_{t=0}^{T-1} \gamma(C_t - CT_t)^2 + \alpha(C_t - CT_t)(F_{t+1} - FT_{t+1}) + \beta(F_{t+1} - FT_{t+1})^2\}$$

Subject to:

$$F_{t+1} = (1 + i_{t+1}).(F_t + C_t - B_t)$$

where:

i_{t+1} ~ IID independent identical distributed as normal variables with mean θ and variance σ^2 and where F_0 and i_0 are given.

For minimizing J_T, the optimal control sequence $E(C_t^*)$ should be determined at time t where t ε [0,T), and where the fund state variables F_0, F_1, F_2,..., F_t have been observed and the contribution target $\{CT_t$, t ε [0,T)$\}$, the fund target $\{FT_{t+1}$, t ε [0,T)$\}$ and the benefit outgo $\{B_t$, t ε [0,T)$\}$ are known.

Therefore, J_T can be written as follows:

$$J_T = E\{\sum_{s=0}^{t-1}\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2\} +$$
$$E\{\sum_{s=t}^{T-1}\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2\} \quad (3.36)$$

The first part does not depend on C_t, C_{t+1}, ..., C_{T-1}, then to minimize J_T with respect to C_t for s ε (t, t+1, ..., T-1) is equivalent to minimizing the second part. Without loss of generality, assuming that the second part of the performance criterion has a unique minimum with respect to C_s for all F_{s+1}, s ε (t, t+1, ..., T-1) and using the property of conditional expectation (Aström, 1970), then:

$$V(F_t,t) = \underset{\{C_s;s=t,t+1,\ldots,T-1\}}{Min} E\{\sum_{s=t}^{T-1}\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2\} \quad (3.37)$$

$$= \underset{\{C_s;s=t,t+1,\ldots,T-1\}}{Min} E\{\sum_{s=t}^{T-1}\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2 | H_t\} \quad (3.38)$$

Where H_t consists of the events prior to and inclusive of time t from the measurement process$\{F_0$, F_1,...., $F_t\}$, and since$\{F_t$, t ε [0,T)$\}$is a Markov process, thus, for all t ε [0,T), the conditional probability distributions of future states $\{F_{t+1}$,...., $F_T\}$given past values $\{F_0$, F_1,...., $F_t\}$are functions of F_t only. It is then sufficient to choose C_t as a function of F_t, so that:

$$V(F_t,t) = \underset{\{C_s;s=t,t+1,\ldots,T-1\}}{Min} E\{\sum_{s=t}^{T-1}\gamma(C_s - CT_s)^2 + \alpha(C_s - CT_s)(F_{s+1} - FT_{s+1}) + \beta(F_{s+1} - FT_{s+1})^2 | F_t\} \quad (3.39)$$

Using Aström(1970) and similarly for $V(F_{t+1}, t+1)$, we obtain the following Bellman equation with the boundary condition $V(F_T,T) = 0$, because there is no expected loss associated with the terminal state F_T:

$$V(F_t,t) = \underset{C_t}{Min}\, E\{\gamma(C_t - CT_t)^2 + \alpha(C_t - CT_t)(F_{t+1} - FT_{t+1}) + \beta(F_{t+1} - FT_{t+1})^2 + V(F_{t+1}, t+1)|F_t\} \tag{3.40}$$

Referring to the funding constraint:

$F_{t+1} = (1 + i_{t+1})(F_t + C_t - B_t)$

and where i_{t+1} is IID ~ normal distribution with mean θ and variance σ^2

thus, the first two moments of i_t are:

$E[1 + i_{t+1}] = (1 + 6)$

and

$E[(1 + i_{t+1})^2] = [(1 + \theta)^2 + \sigma^2]$

Therefore, the conditional distribution of F_{t+1} given F_t is as follows:

$$E[F_{t+1}|F_t] = (1 + \theta)(F_t + C_t - B_t) \tag{3.41}$$

and

$$E[F_{t+1}^2|F_t] = [(1 + \theta)^2 + \sigma^2](F_t + C_t - B_t)^2 \tag{3.42}$$

Since C_t is a function of F_t and B_t and i_t, which are fully informed up to time t, we can try the solution of the Bellman Equation in the following form:

$V(F_t, t) = a_1(t)F_t^2 + a_2(t)F_t + a_3(t)$ with the boundary condition $a_1(T) = a_2(T) = a_3(T) = 0$

Thus,

$$V(F_t,t) = \underset{C_t}{Min}\{G(C_t,t)\}$$

$$\begin{aligned} G(C_t,t) = \{&\gamma(C_t - CT_t)^2 + \alpha((C_t - CT_{t+1})((1+\theta)(F_t + C_t - B_t) - FT_{t+1})) + \beta((d(F_t + C_t - B_t) - FT_{t+1})^2 \\ &+ a_1(t+1)(d(F_t + C_t - B_t))^2 + a_2(t+1)((1+\theta)(F_t + C_t - B_t) + a_3(t+1)\} \end{aligned} \tag{3.43}$$

where

$d = [(1 + \theta)^2 + \sigma^2]$ and $l = (1 + 6)$

In order to have a unique funding controller, it is sufficient that the coefficient of $E(C_t^*)$ in equation (3.43) is positive for all t ε [0,T), Since it is a strictly convex function under the above condition for uniqueness, the optimal pension funding controller at time t, t ε [0,T) is:

$$E(C_t^*) = \frac{(2\gamma + \alpha l)CT_t - (\alpha l + 2dk_{t+1})(F_t - B_t) + (\alpha + 2\beta l)FT_{t+1} - l(a_2(t+1)}{2(\gamma + \alpha l + dk_{t+1})} \tag{3.44}$$

The recursive equations then are:

$$a_1(t) = \frac{4\gamma dk_{t+1} - (\alpha l)^2}{4(\gamma + \alpha l + dk_{t+1})} \tag{3.45}$$

and

$$a_2(t) = \frac{1}{2(\gamma + \alpha l + dk_{t+1})}\{(4\gamma dk_{t+1} - (\alpha l)^2)(CT_t - B_t) + \alpha FT_{t+1}(\alpha l + 2dk_{t+1}) + (2\gamma l)(a_2(t+1) - 2\beta FT_{t+1}) + \alpha l^2(a_2(t+1) - 2\beta FT_{t+1})\} \tag{3.46}$$

(b) Ratio-induced performance index

Following the same approach we have used to obtain the optimal contribution rate for the cost induced performance measure, we could now derive the C_t^* for the ratio-induced performance measure in the stochastic case. Thus, we will have the following Bellman equation to be solved using the conditional distribution for F_{t+1} given F_t:

$$V(F_t, t) = \underset{C_t}{Min}\, E\{\gamma(1 - C_t/CT_t)^2 + \alpha(1 - C_t/CT_t)(1 - F_{t+1}/\eta FT_{t+1}) + \beta(1 - F_{t+1}/\eta FT_{t+1})^2 + V(F_{t+1}, t+1)|F_t\} \tag{3.47}$$

The optimal contribution rate could then be derived to be:

$$E(C_t^*) = \frac{\{\frac{(2\gamma+\alpha)}{CT_t} - (\frac{z_{t+1}}{CT_t} + 2dh_{t+1})(F_t - B_t) + \frac{\alpha l + 2\beta l}{\eta FT_{t+1}} - la_2(t+1)\}}{2(\frac{\gamma}{CT_t^2} + \frac{z_{t+1}}{CT_t} + dh_{t+1})} \tag{3.48}$$

where

$$z_{t+1} = \frac{\alpha l}{\eta FT_{t+1}}$$

Consequently, the recursive equations for both $a_1(t)$ and $a_2(t)$ will be:

$$a_1(t) = \frac{dh_{t+1}\frac{4\gamma}{(CT_t)^2} - (\frac{z_{t+1}}{CT_t})^2}{4(\frac{\gamma}{CT_t^2} + \frac{z_{t+1}}{CT_t} + dh_{t+1})} \tag{3.49}$$

and

$$a_2(t) = \frac{\{(\frac{z_{t+1}}{CT_t})^2 B_t + 2dh_{t+1}(\frac{2\gamma+\alpha}{CT_t} - \frac{2\gamma B_t}{CT_t^2}) + \frac{l(\alpha^2-4\gamma\beta}{\eta FT_{t+1}CT_t^2} - \frac{\alpha l^2(\alpha+2\beta)}{(\eta FT_{t+1})^2 CT_t} + la_2(t+1)(\frac{2\gamma}{CT_t^2} + \frac{z_{t+1}}{CT_t})\}}{2(\frac{\gamma}{CT_t^2} + \frac{z_{t+1}}{CT_t} + dh_{t+1})}$$

(3.50)

Summary

The pension funding is a major process for setting up defined benefit pension schemes. The actuarial valuations are important to be carried out by the actuaries to examine the financial position of the schemes. Thereafter, the contribution rate is needed to be determined along with the level of the fund assets to cover the promised benefits of the beneficiaries.

Due to the uncertainty about the economic and demographic assumptions - which are considered a main part of the actuarial valuation - the dynamic pension funding plans are recommended to be used to assist in the decision making process, as they based on the feedback control system which allow for the adjustment of the contribution rate according to the fund level at certain time.

In this chapter, we proposed four dynamic pension funding models in both deterministic and stochastic based on discrete-time for finite time horizon, two approaches were used: the cost-induced performance index and the ratio-induced performance index to reach the optimal contribution rate.

The properties of these models and how the optimal contribution rates and the optimal fund levels behave over time with changing the parameters used in the models worth to be explored, thus, the following chapters 4 and 5 will cover the properties of the cost-induced performance index (CIPI) and the ratio-induced performance index (RIPI) respectively.

Chapter Four
Computational experiments to reveal the underlying properties of the model and policy implications I

4.1 Introduction

The previous chapter has presented the mathematical derivation of the dynamic pension funding models in both deterministic and stochastic cases. This needs to be followed with testing of the results, by applying numerical illustrations and carrying out a sensitivity analysis, in order to interpret the different results and understand the properties of the models. In fact, the recursive nature of the mathematical results in chapter 3 make these properties difficult to identify directly, and so, the use of numerical experiments can be valuable.

Therefore, in this chapter, we will start with specifying the different values of the parameters of the numerical illustrations that are needed to carry out the sensitivity analysis. Second, the results of testing the CIPI model will be analyzed by studying the different effect of the different parameters on the optimal fund level and contribution rate.

4.2 Parameter values

To explain the patterns of the derived deterministic and stochastic models, numerical illustrations are carried out with the following parameter values:

- The control period is 15 years, i.e. $T = 15$.

as we apply our models for a finite time horizon, we have decided that $T = 15$ is a reasonable period to test the models. Increasing its value to 30 or 40 will not have a major effect on the behaviour of optimal fund level and contribution rate.

- The fund target or actuarial liability $FT_t = AL_t = 1$ for all t.
- The contribution target or normal cost $CT_t = NC_t = 0.2$ for all t.

It is important to state that the choice of the fund and the contribution targets as

(1, 0.2) is considered the default scenario of our analysis. This is only one scenario, out of five, that will be applied to test the effect of changing the fund and contribution targets on both models. The other four scenarios are: (1, 0.22), (1, 0.18), (1.2, 0.2) and (0.8,0.2), where in the first and second scenarios, we allow for changing the contribution targets up and down around 0.2 and the fund targets are equal to 1. In the last two scenarios, we allow for changing the fund targets up to 1.2 and down to 0.8 while the contribution targets remain constant equal to 0.2.

- According to the growth funding equation:

$F_{t+1}= (1+i_{t+1}) \cdot (F_t + C_t - B_t)$

The rate of return is represented by i_{t+1} and it is equal to 10% in the deterministic case. In the stochastic case, it is independent identically distributed as a lognormal variable δ with a mean θ = 10% and different values of the variance σ^2, where σ = 1%, 5%, 10%, 15% and 20% to represent a range of volatility levels.

- The value of the benefit outgo will be determined accordingly to achieve an approximate balance of the actuarial liability growth equation:

$AL_{t+1} = (1 + i_{t+1}) \cdot (AL_t + NC_t - B_t)$

where the benefit outgo B_t will be equal to 0.3 - as a round value - for all scenarios in our analysis except in section 4.5, where we allow the value of B_t to be changed in order to maintain the equilibrium of the actuarial liability growth equation. For the different scenarios, the equilibrium value of B_t – shown in Table 4.1 - is used to examine the effect of changing θ, the weighting risk factors of the contribution rate risk and the solvency risk when the cross product factor $\alpha = 0$ and $\alpha \neq 0$.

- The initial value of the fund $F_0 = 1$; this value differs when we change the fund target to 1.2 and 0.8 in the fourth and fifth scenarios. It is mainly used when we apply the growth funding equation to calculate the value of C_0^*.

- The weighting risk factors are changed as part of applying the sensitivity analysis, in order to test their effect on the models in each scenario we mentioned earlier[1], the parameters are:

γ : represents the weighting factor of the contribution rate risk and it will take the following values: 0.1, 0.3, 0.5, 0.7, 0.9.

α : represents the weighting risk factor of the middle mixed term which combines both the contribution rate risk and the solvency risk. It will take the following values: 0, 0.2, 0.4, 0.6, 0.8.

β : represents the weighting factor of the solvency risk and it will take the following values: 0.9, 0.7, 0.5, 0.3, 0.1.

For both stochastic cases, 10,000 simulations are carried out by using the Visual Basic Program (Visual Basic 6.0) and VBA (excel 2000), where δ is independent identically distributed as a lognormal variable with mean θ and different values of variance σ^2 as previously stated. Hence, the total number of cases obtained is 1250 (250 cases under each scenario). In our sensitivity analysis, we refer to some of these cases interchangeably which helps in clarifying the properties of our models.

4.3 Results of sensitivity analysis – the deterministic case-

The results of both models CIPI and RIPI, in the deterministic case, are expected to be similar to the ones we obtain in the stochastic case, when the same value of the interest rate of return - which is 10% as a mean for the distribution of δ - and the lowest level of volatility when $\sigma = 1\%$ are used. Therefore, it is preferable to comment on the results we have obtained in the stochastic case when the level of σ is low, bearing in mind that the values of the expected optimal fund level and contribution rate should be quite similar to the deterministic case. Full details of the results under the deterministic case are available but are not included in the thesis.

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[1] A combination of the three weighting risk factors γ, α and β are used, throughout this chapter and the rest of the thesis, in this form (γ, α, β) to refer to the cases we have applied and investigated in our analysis, e.g. (0.1, 0, 0.9) means that $\gamma = 0.1$, $\alpha = 0$ and $\beta = 0.9$.

4.4 Results of sensitivity analysis - Cost-induced performance index model in the stochastic case when $B_t = 0.3$ -

4.4.1 Effect of changing θ

(a) Effect of changing θ on the benefit outgo B_t

Changing the mean of the interest rate of return θ affects the balance of the actuarial liability recurrence relation equation:

$$AL_{t+1} = e^{\delta}.(AL_t + NC_t - B_t) \qquad (4.1)$$

Hence this leads to change the value of the benefit outgo B_t as shown in Table 4.1.

Table 4.1

The effect of changing θ on the benefit outgo B_t in the five different scenarios

FT_t	CT_t	θ	B_t
1	0.2	5%	0.25
1	0.2	10%	0.3
1	0.2	15%	0.34
1	0.22	5%	0.27
1	0.22	10%	0.32
1	0.22	15%	0.36
1	0.18	5%	0.23
1	0.18	10%	0.28
1	0.18	15%	0.32
1.2	0.2	5%	0.26
1.2	0.2	10%	0.31
1.2	0.2	15%	0.37
0.8	0.2	5%	0.24
0.8	0.2	10%	0.28
0.8	0.2	15%	0.31

The results in Table 4.1 show that choosing θ at a lower level equal to 5% and a higher level equal to 15% instead of 10%, with keeping the values of both the normal cost and actuarial liability as they are in the different scenarios, results in the following:

The lower level of θ equal to 5% in the first scenario - where the actuarial liability (AL = FT = 1) and the normal cost (NC = CT = 0.2) - results in changing the value of the benefit outgo to be equal to 0.25 instead of 0.3. On the other hand, choosing θ at a higher level is equal to 15% leads to increase this value to 0.34.

In (1, 0.22), if $\theta = 5\%$ we will have the value of the benefit outgo = 0.27 and if θ moves up to 15% the value will come up to 0.36.

The third scenario (1, 0.18) leads to the same results, where the values of the benefit outgo are changed to be 0.23 under 5% and 0.32 under 15%, so that, with increasing θ the benefit outgo increases, and with reducing it, B_t decreases to keep the balance of the linear equation.

Further, the changes of the actuarial liability in the two remaining scenarios have an effect on B_t. In (1.2, 0.2), the value of the benefit outgo becomes 0.26 under 5%, while applying the last scenario (0.8, 0.2) results in having a lower value of the benefit outgo equal to 0.24. In the case of θ is equal to 15%, these values become 0.37 and 0.31 respectively under both scenarios.

Finally, it is clear that changing θ has an effect on the equilibrium of the recurrence equation. When θ changes to a lower level, this results in having lower investment income which needs to be matched by having lower values of the benefit outgo. On the other hand, having a higher value of θ leads to an increase in the value of the B_t to maintain the equilibrium of the equation with different levels of normal costs and actuarial liabilities. Also, it is notable that the values of the benefit outgo in the second and the fourth scenarios are higher than the values in the third and the fifth scenarios due to the higher level of the contribution target and the fund target respectively.

(b) Effect of changing θ on $E(F_t^*)$ and $E(C_t^*)$

Here, we test the effect of changing θ on the expected optimal fund level and contribution rate when $B_t = 0.3$. It is noted that if we keep the value of $B_t = 0.3$, moving θ up to 15% and down to 5% leads to changes in $E(F_t^*)$ and $E(C_t^*)$ up and down respectively, which depend on the values of the weighting risk factors.

Considering the default scenario (1, 0.2) when $\theta = 5\%$ and 15% with the round value of $B_t = 0.3$ leads to the following results:

In the case of (0.1, 0, 0.9), the value of $E(F_t^*)$ remains close to its target without significant changes for both $\theta = 5\%$ and 15%. On the other hand, the value of $E(C_t^*)$ moves up to be around 25% when the expected rate of return is decreased to 5%, where as it moves down to be around 17% when the expected rate of return is increased to 15%. Tables 4.2 and 4.3 show these results below.

Table 4.2

The expected optimal fund level under CIPI (1, 0.2) for different levels of θ at $\sigma = 5\%$ in the case of (0.1, 0, 0.9) when $B_t = 0.3$

t	θ = 5%	θ = 10%	θ = 15%
1	100.00%	100.00%	100.00%
2	99.80%	99.81%	99.78%
3	99.76%	99.77%	99.74%
4	99.76%	99.77%	99.74%
5	99.78%	99.79%	99.76%
6	99.74%	99.76%	99.74%
7	99.76%	99.78%	99.75%
8	99.76%	99.78%	99.75%
9	99.80%	99.80%	99.77%
10	99.81%	99.79%	99.75%
11	99.75%	99.76%	99.73%
12	99.79%	99.80%	99.77%
13	99.73%	99.75%	99.72%
14	99.77%	99.78%	99.75%
15	99.75%	99.78%	99.78%

When σ is increased to 20%, the levels of $E(F_t^*)$ remain close to each other for the different values of θ but they are further away from the target (relative to the case of $\sigma = 5\%$) due to increasing the level of volatility as shown in Table 4.4. For $E(C_t^*)$, we note that it has a similar behaviour where it increases when $\theta = 5\%$ and decreases when $\theta = 15\%$. However, it moves further away from the target compared with the cases when $\sigma = 5\%$ as seen in Table 4.5.

Table 4.3

The expected optimal contribution rate under CIPI (1, 0.2) for different levels of θ at $\sigma = 5\%$ in the case of (0.1, 0, 0.9) when $B_t = 0.3$

t	θ = 5%	θ = 10%	θ = 15%
1	25.05%	20.73%	16.76%
2	25.23%	20.90%	16.96%
3	25.27%	20.94%	17.00%
4	25.27%	20.94%	17.00%
5	25.25%	20.92%	16.98%
6	25.28%	20.95%	17.00%
7	25.26%	20.93%	16.99%
8	25.27%	20.93%	16.99%
9	25.23%	20.91%	16.97%
10	25.22%	20.92%	16.99%
11	25.27%	20.95%	17.01%
12	25.24%	20.91%	16.97%
13	25.29%	20.96%	17.02%
14	25.22%	20.93%	17.01%
15	24.79%	20.86%	17.25%

Table 4.4

The expected optimal fund level under CIPI (1, 0.2) for different levels of θ at $\sigma = 20\%$ in the case of (0.1, 0, 0.9) when $B_t = 0.3$

t	θ = 5%	θ = 10%	θ = 15%
1	100.00%	100.00%	100.00%
2	96.45%	96.75%	96.97%
3	96.06%	96.40%	96.66%
4	96.00%	96.36%	96.64%
5	96.07%	96.44%	96.72%
6	95.96%	96.30%	96.57%
7	96.05%	96.39%	96.66%
8	96.07%	96.38%	96.64%
9	96.35%	96.62%	96.83%
10	96.51%	96.70%	96.87%
11	96.37%	96.49%	96.66%
12	96.22%	96.53%	96.78%
13	96.01%	96.30%	96.55%
14	96.18%	96.48%	96.72%
15	96.33%	96.62%	96.86%

Table 4.5

The expected optimal contribution rate under CIPI (1, 0.2) for different levels of θ at $\sigma = 20\%$ in the case of (0.1, 0, 0.9) when $B_t = 0.3$

t	θ = 5%	θ = 10%	θ = 15%
1	21.91%	17.98%	14.34%
2	25.17%	21.00%	17.16%
3	25.54%	21.32%	17.45%
4	25.59%	21.35%	17.47%
5	25.53%	21.28%	17.40%
6	25.62%	21.41%	17.53%
7	25.54%	21.32%	17.45%
8	25.52%	21.33%	17.47%
9	25.27%	21.11%	17.29%
10	25.12%	21.03%	17.26%
11	25.25%	21.23%	17.45%
12	25.39%	21.19%	17.34%
13	25.58%	21.41%	17.56%
14	25.41%	21.25%	17.44%
15	25.08%	21.26%	17.73%

The other four scenarios (1, 0.22), (1, 0.18), (1.2, 0.2) and (0.8, 0.2) are examined in the same case (0.1, 0, 0.9), and we find that the results of $E(F_t^*)$ and $E(C_t^*)$ are similar to the ones shown above (full details of the results are available but not shown in order to avoid repetition).

4.4.2 Effect of changing σ

The values of θ =10% and B_t = 0.3 are used in our analysis in this subsection. This is applied for all the five different scenarios in order to calculate the expected optimal fund level and contribution rate in the stochastic models for both CIPI and RIPI.

We consider the results of σ = 1%, 5%, 10%, 15% and 20% in order to allow for different levels of volatility. We anticipate that the expected values of the fund level and the contribution rate will be affected, as we increase the level of volatility of the rate of return (representing changes in the financial markets). Therefore, the effect on the expected optimal fund level and contribution rate according to the level of σ will be explored for the main five scenarios applied under CIPI below.

(a) First scenario: $FT_t = 1$ and $CT_t = 0.2$

For the optimal fund level in (1, 0.2), we find that $E(F_t^*)$ tends to decrease over the control period for the different combination of parameters. For example, in the case of (0.1, 0, 0.9), it is almost around its target and reaches 99.99% at the end of the control period when σ = 1%. When σ moves up to 5%, $E(F_T^*)$ goes down to 99.8%. As the level of volatility increases to 10%, 15% and 20%, $E(F_T^*)$ continues to move down to reach 99.1%, 98.1% and 96.6% respectively as shown in Table 4.6.

Table 4.6

The expected optimal fund level under CIPI (1, 0.2) for different levels of volatility in the case of (0.1, 0, 0.9)

t	σ =0.01	σ =0.05	σ =0.1	σ =0.15	σ=0.2
1	100.00%	100.00%	100.00%	100.00%	100.00%
2	100.00%	99.80%	99.17%	98.14%	96.74%
3	99.99%	99.76%	99.06%	97.93%	96.39%
4	99.99%	99.77%	99.06%	97.91%	96.36%
5	100.00%	99.79%	99.10%	97.97%	96.43%
6	100.00%	99.76%	99.03%	97.86%	96.29%
7	100.00%	99.77%	99.07%	97.93%	96.39%
8	100.00%	99.77%	99.06%	97.92%	96.37%
9	100.00%	99.80%	99.15%	98.08%	96.62%
10	99.99%	99.78%	99.15%	98.12%	96.70%
11	99.99%	99.75%	99.05%	97.95%	96.49%
12	100.00%	99.80%	99.13%	98.03%	96.53%
13	99.99%	99.74%	99.01%	97.84%	96.29%
14	99.99%	99.77%	99.09%	97.98%	96.48%
15	99.99%	99.78%	99.13%	98.07%	96.62%

A similar conclusion is reached when we consider the other combinations of parameters (γ, α, β), i.e. in the case of (0.3, 0, 0.7) at σ is equal to 1%, $E(F_T^*)$ reaches 99.99% and gradually goes down to 96% at σ is equal to 20%. For (0.5, 0, 0.5), $E(F_T^*)$ decreases from 99.8% to 95.3% with increasing the level of volatility from 1% to 20% where as in the case of (0.7, 0, 0.3) it is equal to 99.6% and moves down to 94.1%.

Finally, in the case of (0.9, 0, 0.1), $E(F_T^*)$ is equal to 98.7% and gradually decreases to 98.1%, 96.3%, 93.5% and 89.8% with σ = 1%, 5%, 10%, 15% and 20%. Full details are shown in Figures 4.1- 4.4[2].

Figure 4.1

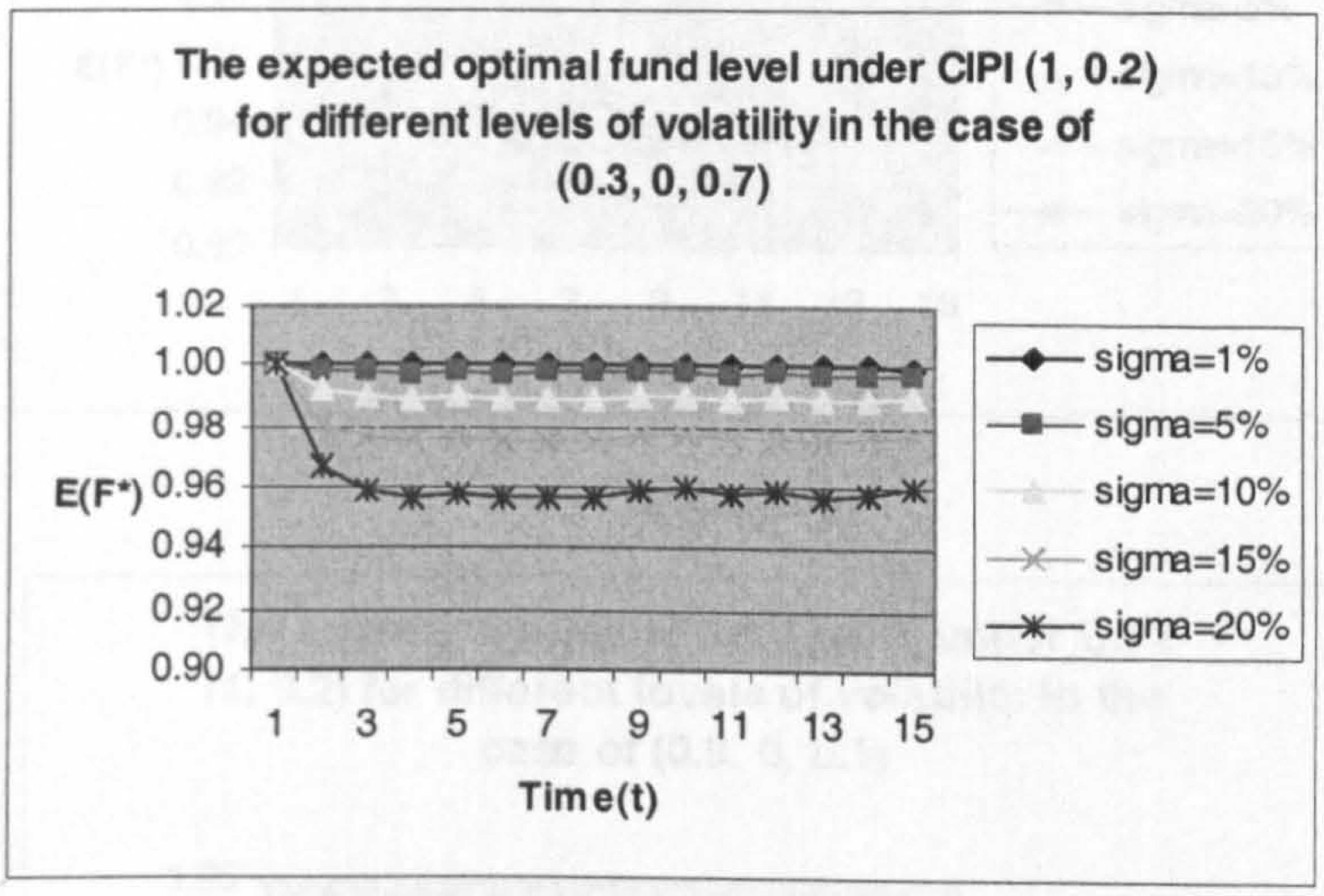
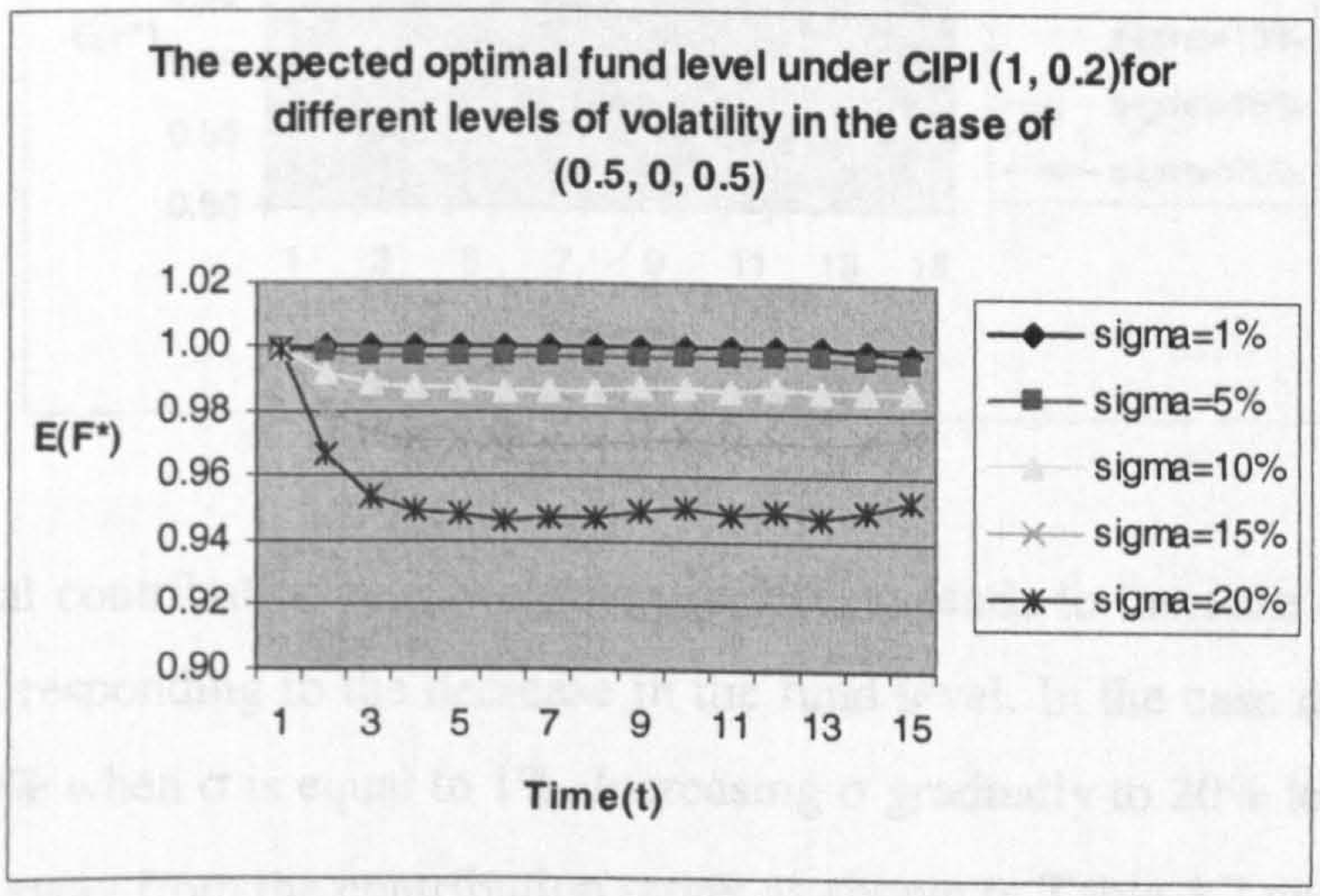


Figure 4.2



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[2] Here, it is important to mention that the scale of the y-axis (representing $E(F_t^*)$ and $E(C_t^*)$) varies from one graph to another. Also, we allow the horizontal axis t to cross at a different value from 0 to show the small differences in the five trends of the expected fund level and contribution rate.

Figure 4.3

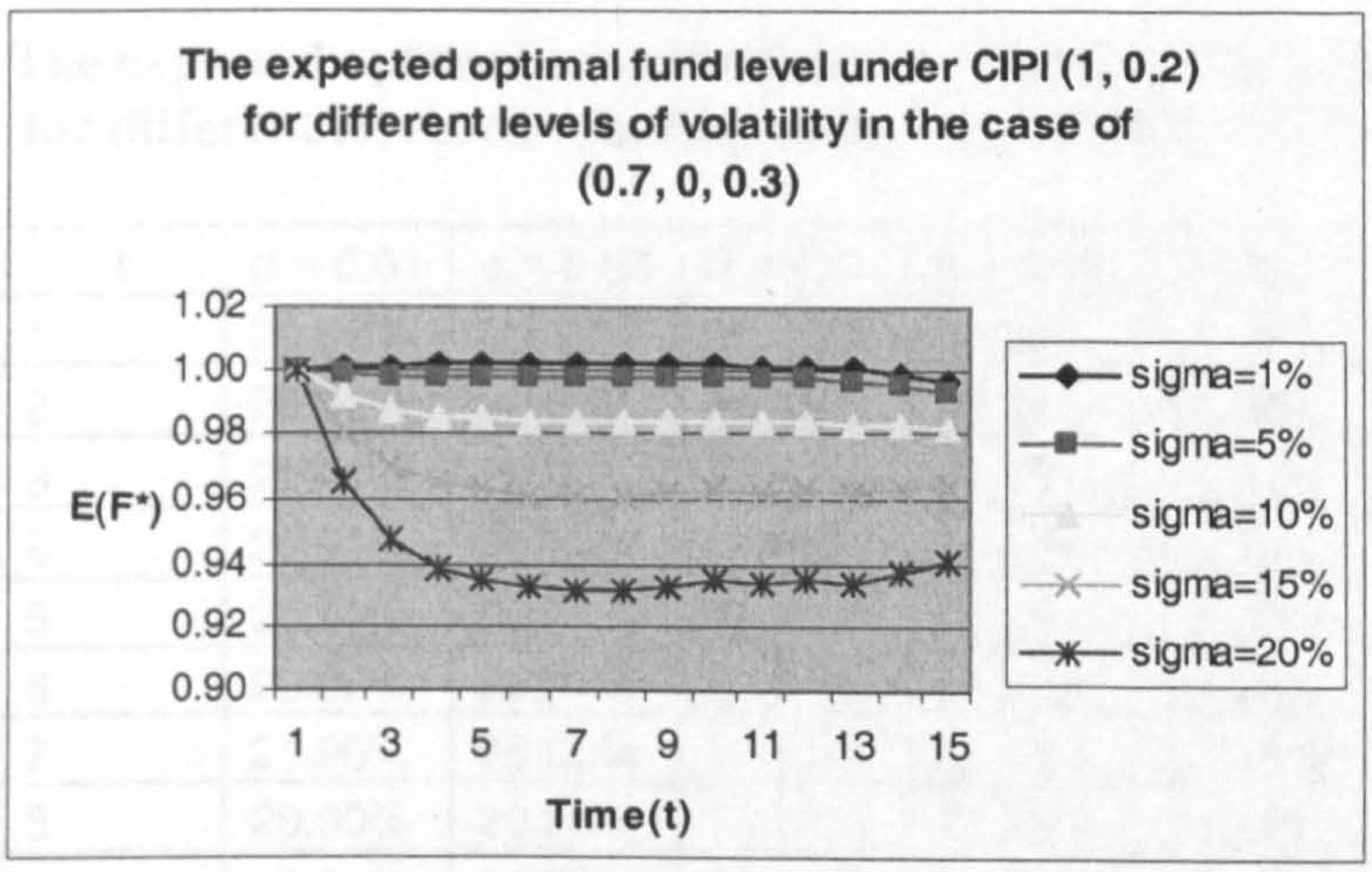
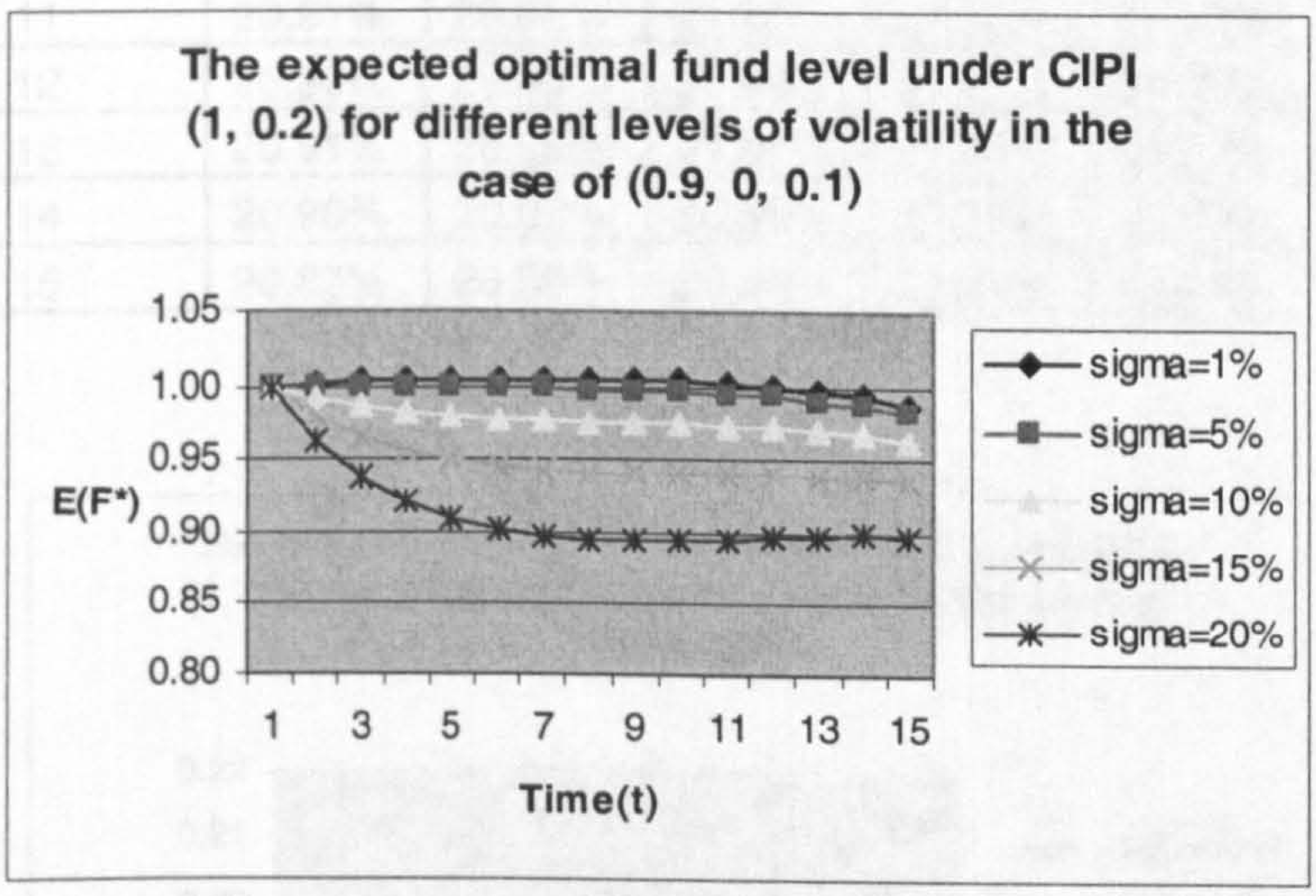


Figure 4.4



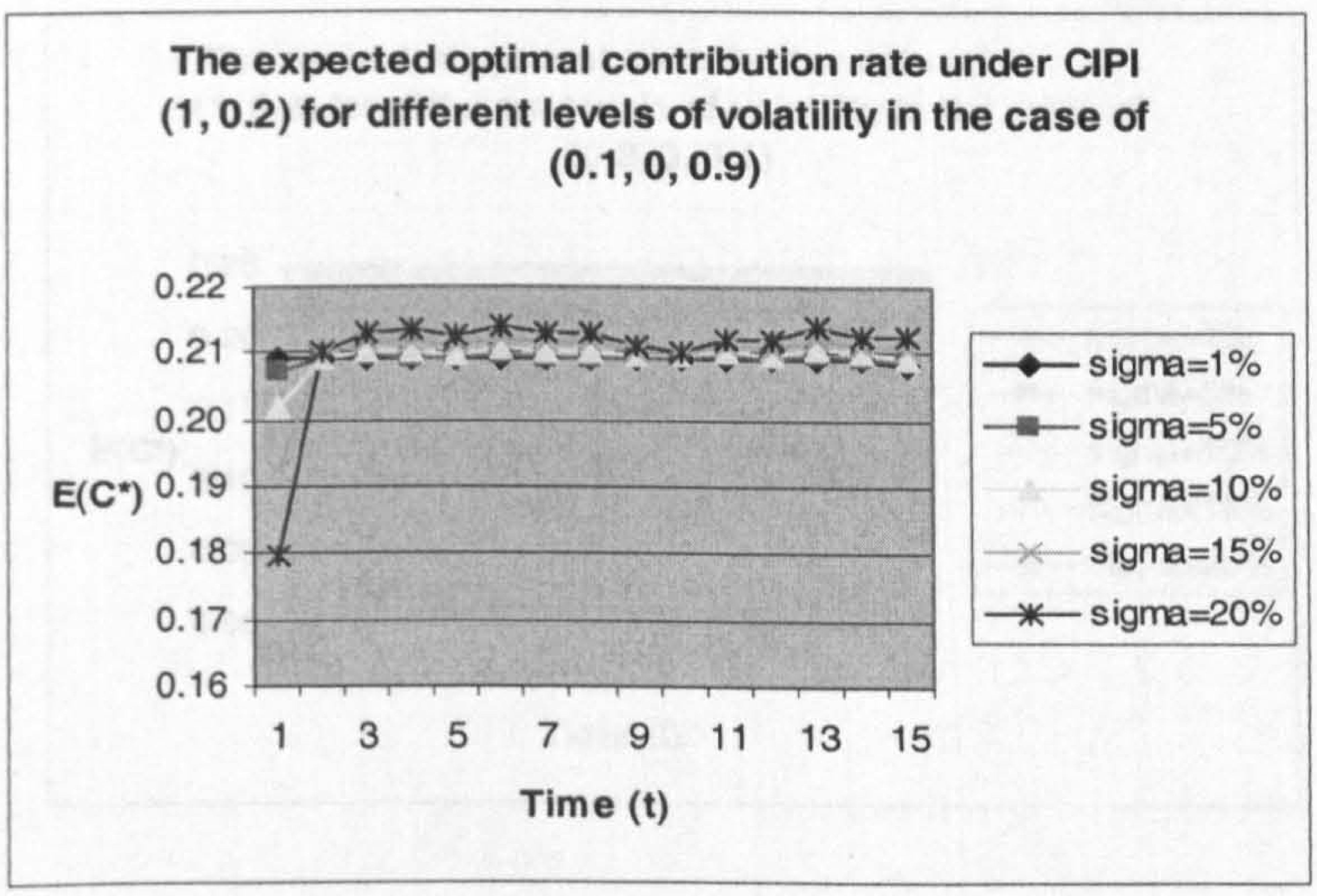
For the optimal contribution rate, we note that $E(C_t^*)$ tends to increase slightly over the control period responding to the decrease in the fund level. In the case of (0.1, 0, 0.9), it is around 20.9% when σ is equal to 1%. Increasing σ gradually to 20% leads this value to move slightly away from the contribution target as shown in Table 4.7 and Figure 4.5.

Table 4.7

The expected optimal contribution rate under CIPI (1, 0.2) for different levels of volatility in the case of (0.1, 0, 0.9)

t	σ = 0.01	σ = 0.05	σ = 0.1	σ = 0.15	σ = 0.2
1	20.91%	20.72%	20.16%	19.24%	17.98%
2	20.90%	20.90%	20.92%	20.95%	20.99%
3	20.91%	20.94%	21.02%	21.15%	21.31%
4	20.91%	20.93%	21.02%	21.16%	21.35%
5	20.90%	20.91%	20.98%	21.11%	21.28%
6	20.90%	20.94%	21.05%	21.21%	21.40%
7	20.90%	20.93%	21.01%	21.15%	21.32%
8	20.90%	20.93%	21.02%	21.16%	21.33%
9	20.90%	20.91%	20.94%	21.01%	21.10%
10	20.91%	20.92%	20.94%	20.97%	21.03%
11	20.91%	20.95%	21.03%	21.13%	21.23%
12	20.90%	20.90%	20.96%	21.06%	21.19%
13	20.91%	20.96%	21.07%	21.23%	21.41%
14	20.90%	20.92%	20.99%	21.10%	21.25%
15	20.83%	20.86%	20.94%	21.07%	21.26%

Figure 4.5



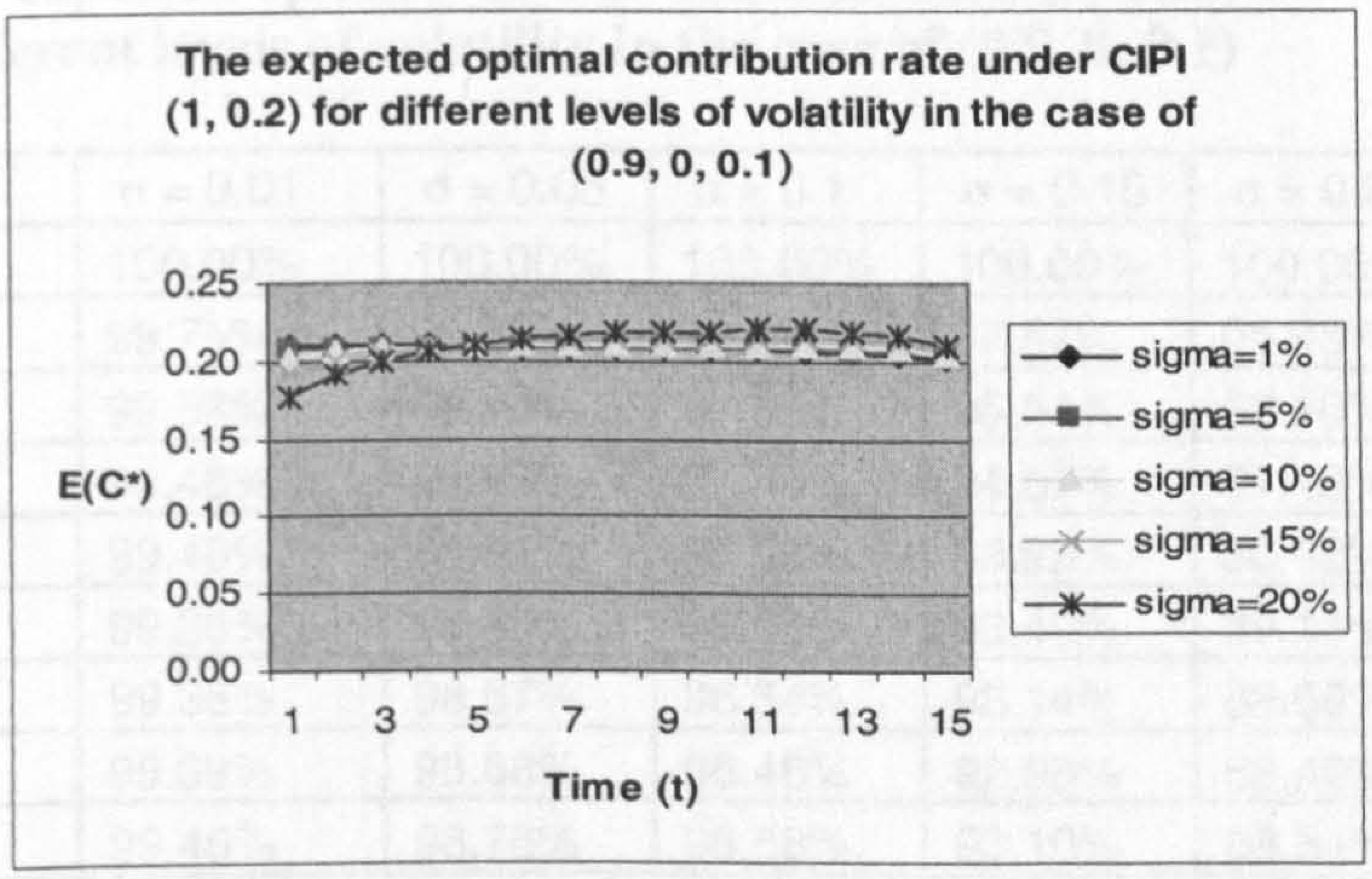
The opposite case of (0.9, 0, 0.1) also indicates the increasing trend of the optimal contribution rate; however, it is closer to the target of 20% due to our giving more importance to the contribution rate risk weighting factor γ, i.e. the $E(C_T^*)$ increases from 20.3% at σ = 1% to 21% at σ = 20%. The results are shown in Table 4.8 and Figure 4.6.

Table 4.8

The expected optimal contribution rate under CIPI (1, 0.2) for different levels of volatility in the case of (0.9, 0, 0.1)

t	σ = 0.01	σ = 0.05	σ = 0.1	σ = 0.15	σ = 0.2
1	21.08%	20.86%	20.19%	19.10%	17.62%
2	21.00%	20.87%	20.48%	19.85%	19.02%
3	20.95%	20.89%	20.70%	20.39%	20.00%
4	20.91%	20.90%	20.85%	20.77%	20.68%
5	20.88%	20.89%	20.93%	21.00%	21.10%
6	20.86%	20.90%	21.01%	21.20%	21.45%
7	20.84%	20.89%	21.05%	21.31%	21.65%
8	20.81%	20.88%	21.08%	21.41%	21.82%
9	20.79%	20.85%	21.07%	21.42%	21.87%
10	20.76%	20.83%	21.05%	21.42%	21.89%
11	20.71%	20.80%	21.06%	21.46%	21.98%
12	20.65%	20.73%	21.00%	21.42%	21.97%
13	20.56%	20.65%	20.93%	21.36%	21.92%
14	20.43%	20.51%	20.75%	21.13%	21.61%
15	20.25%	20.30%	20.45%	20.69%	21.00%

Figure 4.6



The effect of changing σ for different contribution and fund targets is explained below.

(b) Second scenario: $FT_t = 1$ and $CT_t = 0.22$

Under the second scenario where FT_t is equal to 1 and CT_t is equal to 0.22, we note that for the case of (0.1, 0, 0.9), with σ is equal to 1 %, the value of $E(F_t^*)$ is around its target during the control period. This value decreases gradually until it reaches 96.62% when σ = 20%.

In the case of (0.9, 0, 0.1), $E(F_t^*)$ keeps close to the target with the lower levels of volatility, e.g. σ = 1% and 5%. It decreases over the control period to be equal to 99.06%, 96.19%, 92.43% as we increase σ to 10%,15% and 20% respectively. Generally, $E(F_t^*)$ follows the same behaviour in the other cases. In order to illustrate the effect of increasing the value of σ on the expected optimal fund level, Table 4.9 shows the trend of the expected optimal fund level for different levels of σ in the case of (0.9, 0, 0.1).

Under the same scenario (1, 0.22) and with σ is equal to 1%, $E(C_t^*)$ is around 21% in both cases (0.1, 0, 0.9) and (0.9, 0, 0.1). But there is a slight increase (towards the target) in the latter case where more importance is given to the weighting factor of the contribution rate risk. Allowing σ to increase to 20% results in moving these percentages slightly away from the target. Table 4.10 and Figure 4.7 show the results of the expected optimal contribution rate for different levels of σ in the case of (0.9, 0, 0.1).

Table 4.9

The expected optimal fund level under CIPI (1, 0.22) for different levels of volatility in the case of (0.9, 0, 0.1)

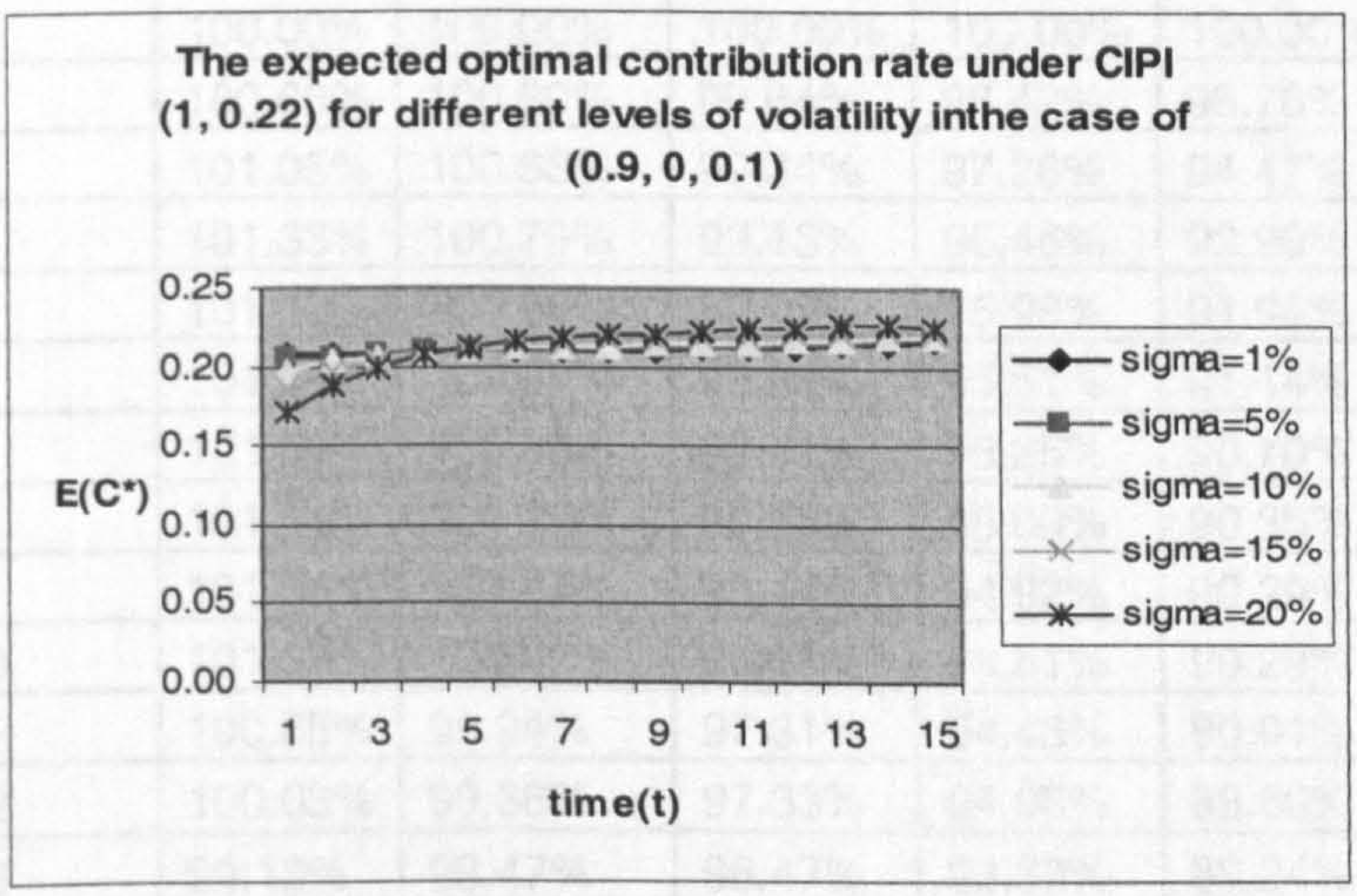
t	$\sigma = 0.01$	$\sigma = 0.05$	$\sigma = 0.1$	$\sigma = 0.15$	$\sigma = 0.2$
1	100.00%	100.00%	100.00%	100.00%	100.00%
2	99.75%	99.52%	98.77%	97.57%	95.93%
3	99.58%	99.16%	97.89%	95.84%	93.10%
4	99.46%	98.93%	97.30%	94.67%	91.18%
5	99.40%	98.81%	96.96%	93.98%	90.02%
6	99.36%	98.70%	96.66%	93.40%	89.11%
7	99.36%	98.67%	96.54%	93.14%	88.66%
8	99.39%	98.68%	96.48%	92.98%	88.40%
9	99.46%	98.76%	96.58%	93.10%	88.54%
10	99.55%	98.86%	96.74%	93.35%	88.87%
11	99.71%	99.00%	96.88%	93.52%	89.12%
12	99.95%	99.29%	97.24%	93.98%	89.70%
13	100.28%	99.62%	97.61%	94.44%	90.31%
14	100.77%	100.14%	98.23%	95.24%	91.32%
15	101.48%	100.88%	99.06%	96.19%	92.43%

Table 4.10

The expected optimal contribution rate under CIPI (1, 0.22) for different levels of volatility in case of (0.9, 0, 0.1)

t	σ = 0.01	σ = 0.05	σ = 0.1	σ = 0.15	σ = 0.2
1	20.67%	20.46%	19.80%	18.71%	17.24%
2	20.77%	20.64%	20.26%	19.63%	18.80%
3	20.84%	20.78%	20.59%	20.29%	19.90%
4	20.89%	20.87%	20.82%	20.75%	20.66%
5	20.93%	20.94%	20.97%	21.04%	21.14%
6	20.96%	20.99%	21.11%	21.29%	21.54%
7	20.99%	21.04%	21.20%	21.45%	21.79%
8	21.01%	21.08%	21.28%	21.59%	22.00%
9	21.05%	21.11%	21.32%	21.66%	22.10%
10	21.09%	21.16%	21.38%	21.73%	22.20%
11	21.14%	21.23%	21.48%	21.88%	22.38%
12	21.22%	21.30%	21.56%	21.97%	22.51%
13	21.32%	21.42%	21.69%	22.11%	22.66%
14	21.47%	21.55%	21.79%	22.16%	22.63%
15	21.69%	21.74%	21.89%	22.12%	22.43%

Figure 4.7



(c) Third scenario: $FT_t = 1$ and $CT_t = 0.18$

Keeping FT_{t+1} is equal to 1 and decreasing the contribution target CT_t to 0.18 (instead of 0.2) result in decreasing the values of $E(F_t^*)$ over the control period for different levels of σ. So that, in the case of (0.1, 0, 0.9), $E(F_T^*)$ reaches 99.99% at the end of the control period when σ = 1% and 96.6% when σ = 20%. For the other extreme case (0.9, 0, 0.1), it reaches 95.95% at the lowest level of σ and decreases gradually to 95.4%, 93.6%, 90.8% and 87.1% for σ = 5%, 10%, 15% and 20% respectively. Table 4.11 indicates these changes in the case of (0.9, 0, 0.1).

Consequently, the expected optimal contribution rate is affected by decreasing CT_t to 0.18 at different levels of σ. Taking into consideration the same cases as above: in (0.1, 0, 0.9), $E(C_T{}^*)$ reaches 20.7% when σ = to 1% and increases to 21.1% when σ = 20%. However, in the case of (0.9, 0, 0.1), $E(C_T{}^*)$ reaches 18.8%, indicating that the trend of $E(C_t{}^*)$ is getting close to the contribution target under the lowest level of volatility. Then, $E(C_T{}^*)$ increases gradually to move further away from the target and reaches 19.6% when σ = 20% as shown in Table 4.12.

Table 4.11

The expected optimal fund level under CIPI (1, 0.18) for different levels of volatility in case of (0.9, 0, 0.1)

t	σ = 0.01	σ = 0.05	σ = 0.1	σ = 0.15	σ = 0.2
1	100.00%	100.00%	100.00%	100.00%	100.00%
2	100.63%	100.39%	99.64%	98.42%	96.76%
3	101.05%	100.63%	99.34%	97.26%	94.47%
4	101.33%	100.79%	99.13%	96.46%	92.90%
5	101.50%	100.90%	99.02%	95.99%	91.95%
6	101.58%	100.91%	98.83%	95.51%	91.14%
7	101.59%	100.89%	98.72%	95.25%	90.70%
8	101.51%	100.79%	98.56%	95.00%	90.35%
9	101.35%	100.64%	98.44%	94.92%	90.29%
10	101.07%	100.37%	98.23%	94.81%	90.29%
11	100.65%	99.94%	97.81%	94.43%	90.01%
12	100.03%	99.38%	97.33%	94.08%	89.80%
13	99.13%	98.47%	96.47%	93.33%	89.24%
14	97.82%	97.20%	95.32%	92.37%	88.52%
15	95.94%	95.35%	93.57%	90.76%	87.08%

Table 4.12

The expected optimal contribution rate under CIPI (1, 0.18) For different levels of volatility in the case of (0.9, 0, 0.1)

t	σ =0.01	σ =0.05	σ =0.1	σ =0.15	σ =0.2
1	21.48%	21.26%	20.59%	19.49%	18.00%
2	21.23%	21.10%	20.71%	20.07%	19.23%
3	21.06%	21.00%	20.81%	20.50%	20.10%
4	20.94%	20.92%	20.87%	20.79%	20.69%
5	20.84%	20.85%	20.88%	20.95%	21.05%
6	20.76%	20.80%	20.92%	21.11%	21.36%
7	20.69%	20.74%	20.91%	21.17%	21.52%
8	20.62%	20.68%	20.89%	21.22%	21.64%
9	20.53%	20.60%	20.82%	21.18%	21.64%
10	20.42%	20.50%	20.73%	21.10%	21.59%
11	20.28%	20.37%	20.64%	21.05%	21.58%
12	20.08%	20.17%	20.44%	20.87%	21.43%
13	19.80%	19.89%	20.18%	20.61%	21.18%
14	19.39%	19.48%	19.72%	20.09%	20.58%
15	18.82%	18.87%	19.02%	19.26%	19.57%

(d) Fourth scenario: $FT_t = 1.2$ and $CT_t = 0.2$ and fifth scenario: $FT_t = 0.8$ and $CT_t = 0.2$

Here, we consider the other two scenarios, in which the contribution target is fixed at 20% while the fund target changes to be 120% and 80% respectively. The expected optimal fund level under both scenarios (1.2, 0.2) and (0.8, 0.2) tends to move further away from the target to different levels depending on σ. This is indicated in the following examples:

- When σ is equal to 1 % and in the case of (0.5, 0, 0.5), $E(F_T^*)$ is around its target value 120% at the end of the control period. The $E(F_T^*)$ takes lower values as we increase σ, reaching 1.15 when σ = 20%. In comparison with the fifth scenario (0.8, 0.2), $E(F_T^*)$ = 79.6% at the lowest level of σ and reduces gradually to be 75.6% with the highest level of σ. Figures 4.8 and 4.9 illustrate these results.

Figure 4.8

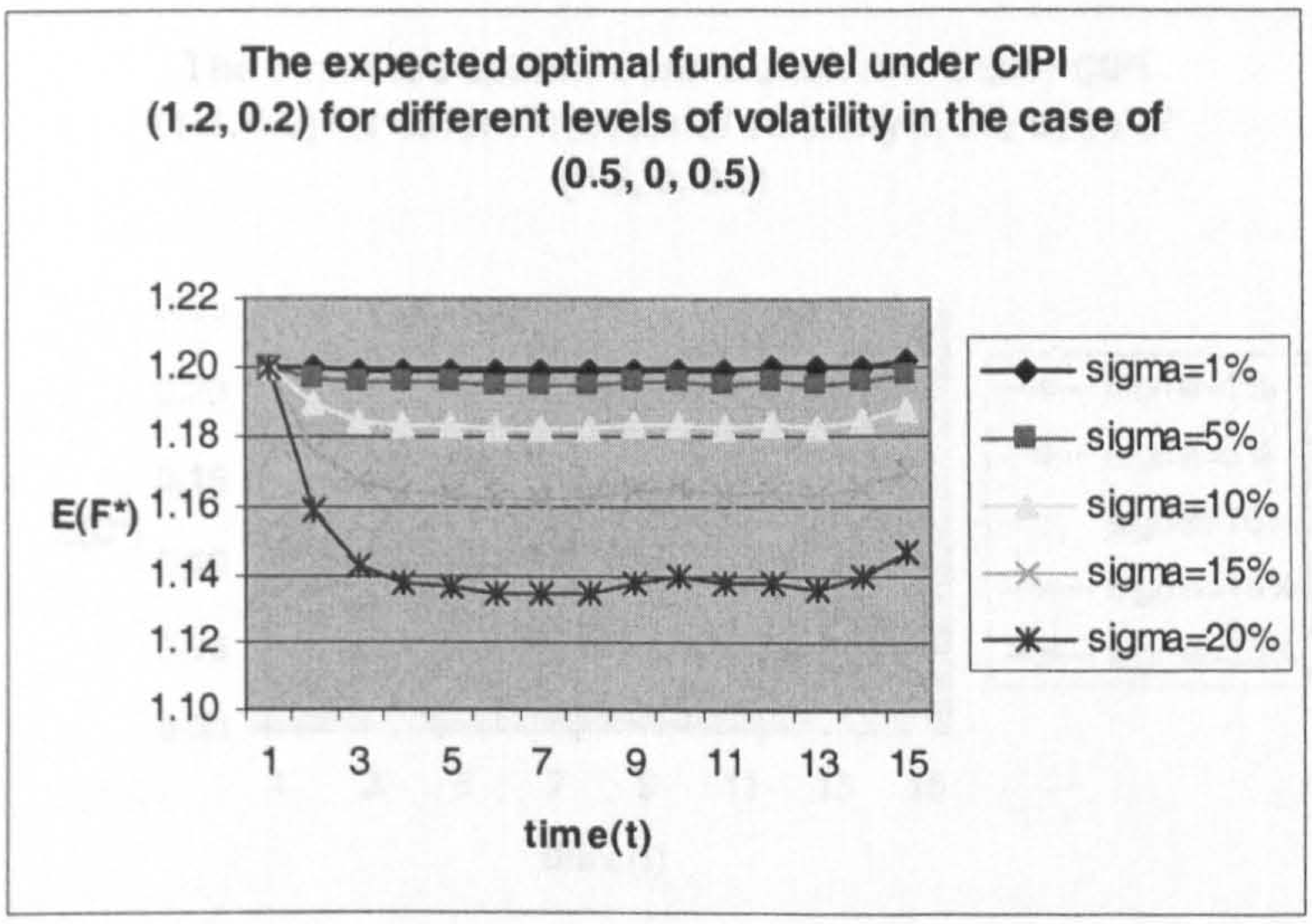
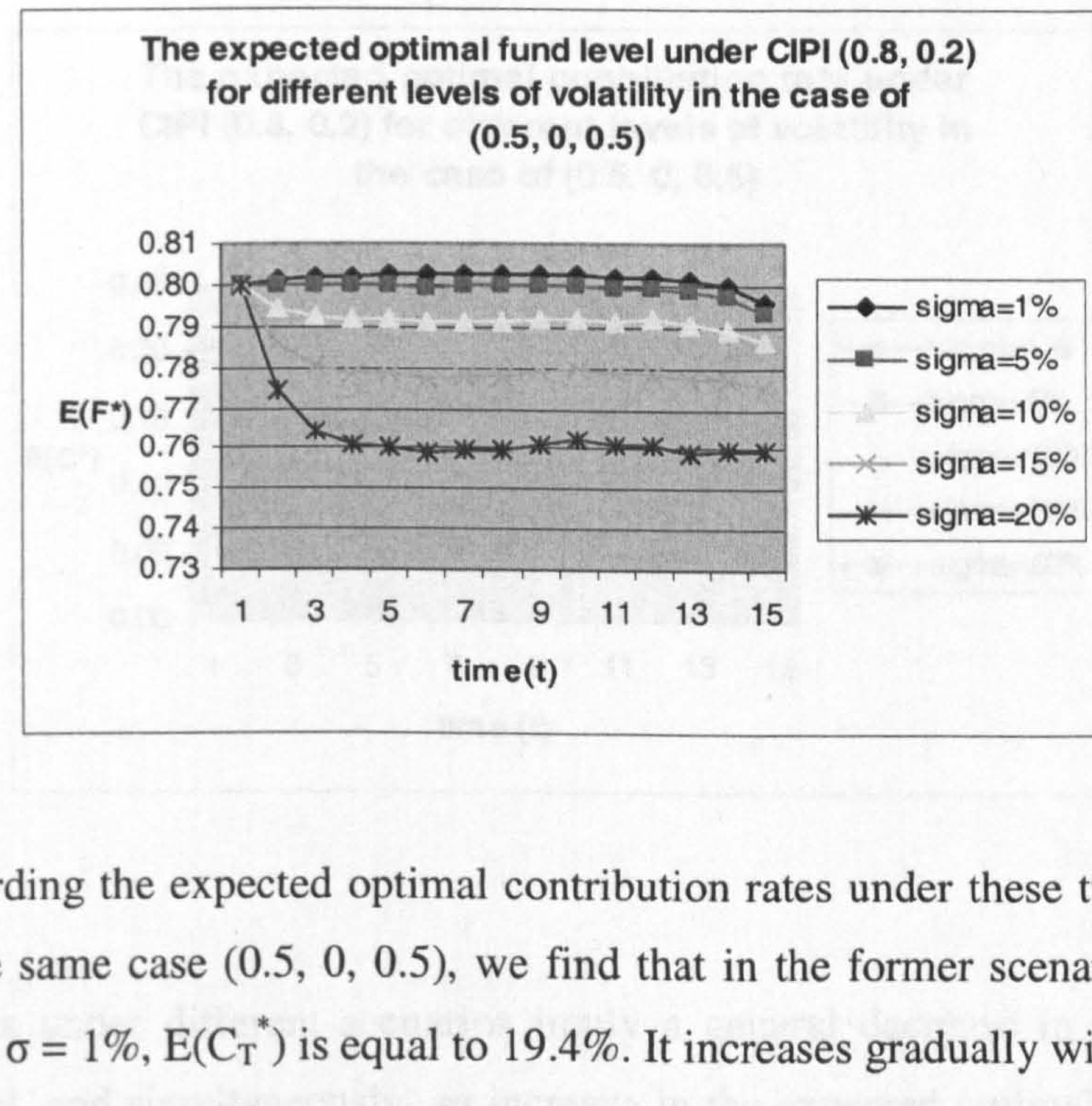


Figure 4.9



- Regarding the expected optimal contribution rates under these two scenarios in the same case (0.5, 0, 0.5), we find that in the former scenario (1.2, 0.2) when $\sigma = 1\%$, $E(C_T^*)$ is equal to 19.4%. It increases gradually with increasing the level of volatility to reach 20.5% when $\sigma = 20\%$. In the latter scenario (0.8, 0.2), it is equal to 21.7% and moves up to 22.5% at the same levels of σ, as shown in Figures 4.10 and 4.11.

Figure 4.10

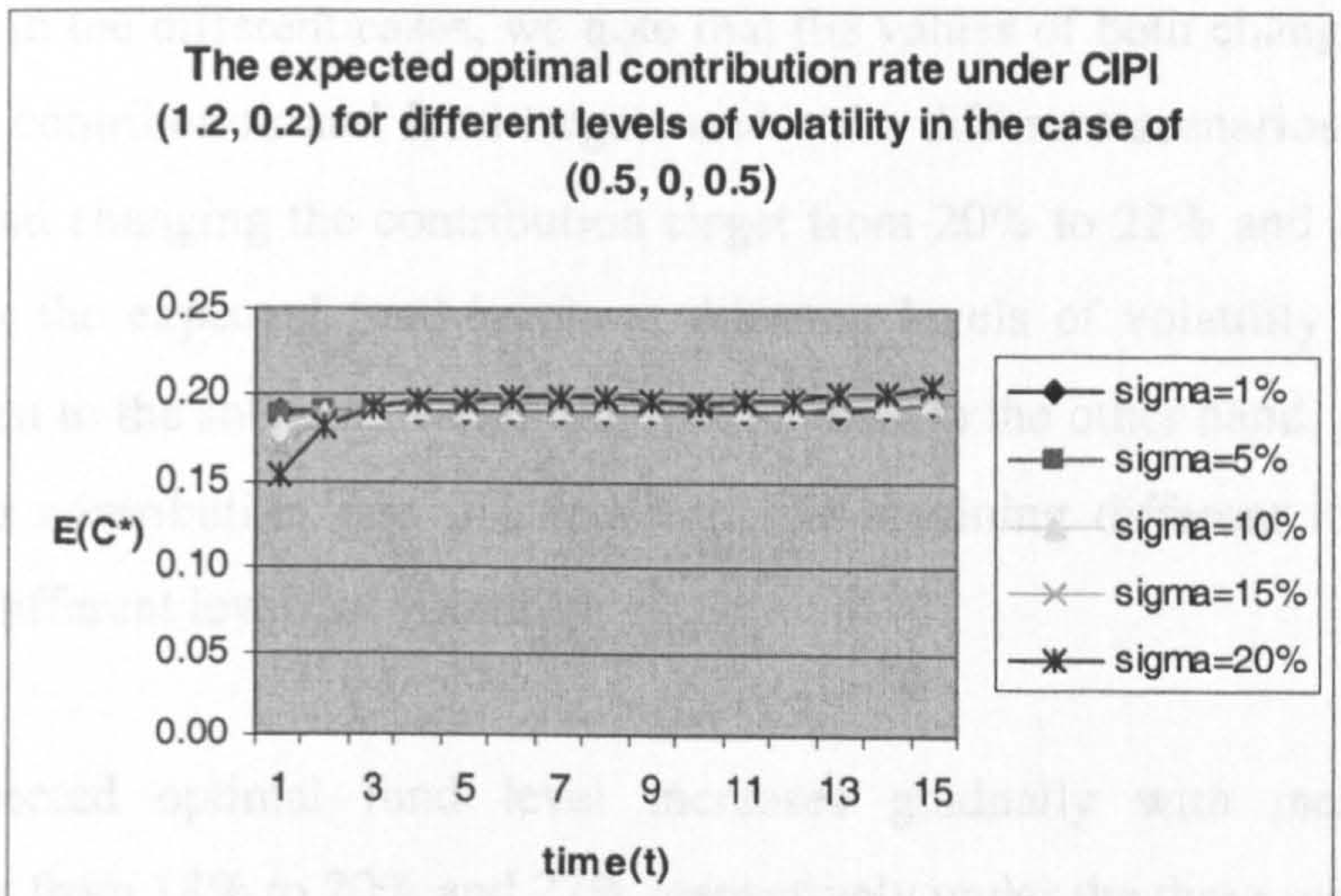
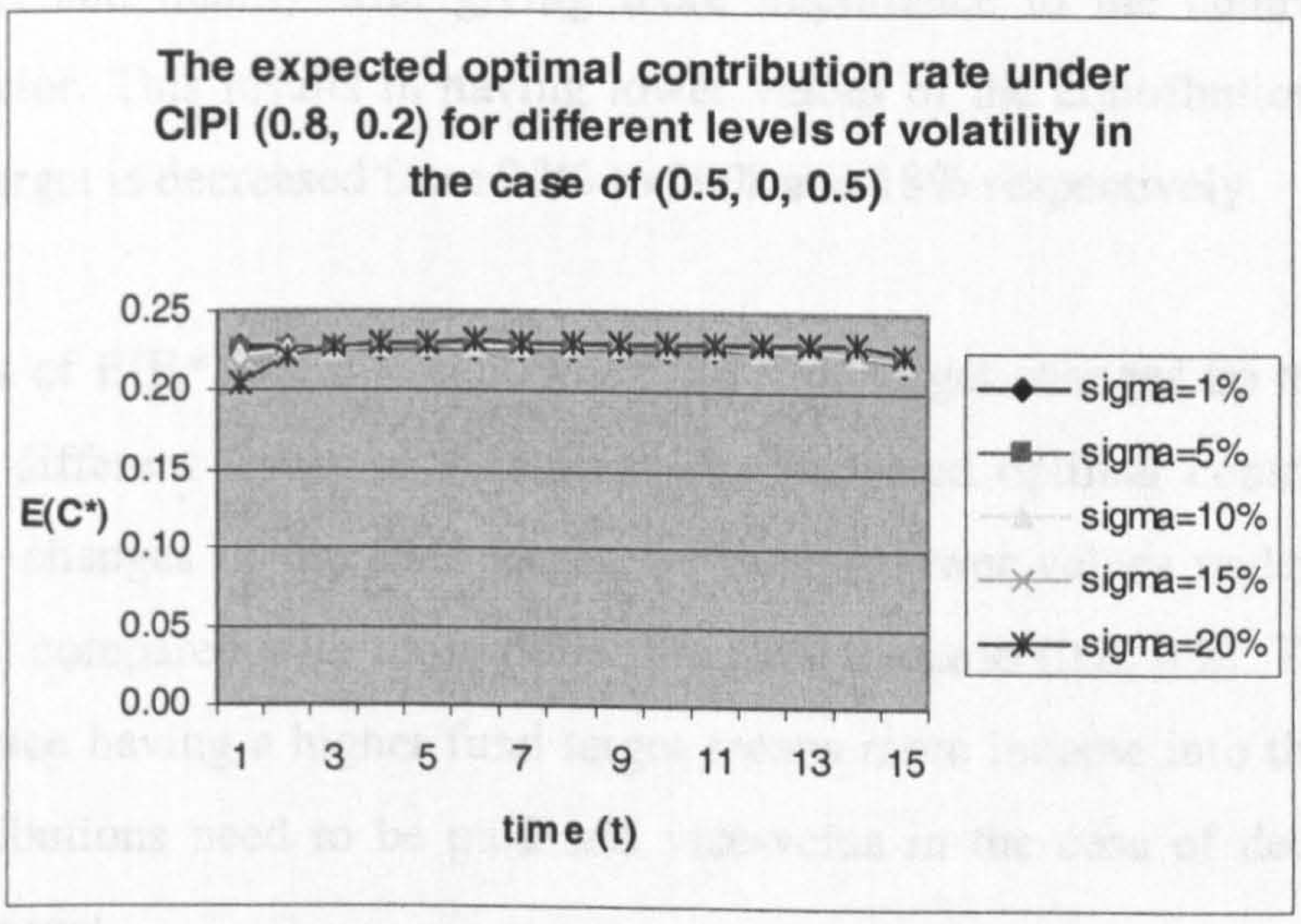


Figure 4.11



Conclusion

The above results under different scenarios imply a general decrease in the expected optimal fund level, and simultaneously, an increase in the expected optimal contribution rate. This is due to the dynamic relation between the optimal fund level and the optimal contribution rate. Changing the levels of volatility affects the expected values of both, resulting in their keeping close to their targets at the lowest level of σ, and their moving further away from their targets as the level of σ is increased up to 20%.

Although the general trends of the expected optimal fund level and contribution rate have the same patterns in the different cases, we note that the values of both change according to changes in the contribution and fund targets under the different scenarios. Fixing the fund target at 1 and changing the contribution target from 20% to 22% and 18% lead to similar results for the expected fund levels at different levels of volatility when more importance is given to the solvency weighting risk factor. On the other hand, giving more importance to the contribution rate risk leads to our obtaining different levels of the expected fund at different levels of volatility.

Further, the expected optimal fund level increases gradually with increasing the contribution target from 18% to 20% and 22% respectively under the three scenarios. Simultaneously, the expected optimal contribution rate is affected by changing the contribution target and mainly with giving more importance to the contribution rate weighting risk factor. This results in having lower values of the contribution rate when the contribution target is decreased from 22% to 20% and 18% respectively.

Clearly, the levels of $E(F_t^*)$ are different when the fund target changes up to 120% and down to 80% at different levels of volatility. The expected optimal contribution rate responds to these changes of the fund target, by having lower values under the fourth scenario (1.2, 0.2) compared with those under the fifth scenario (0.8, 0.2). These results are reasonable, since having a higher fund target means more income into the fund, and thus, lower contributions need to be paid and vice-versa in the case of decreasing the level of the fund target.

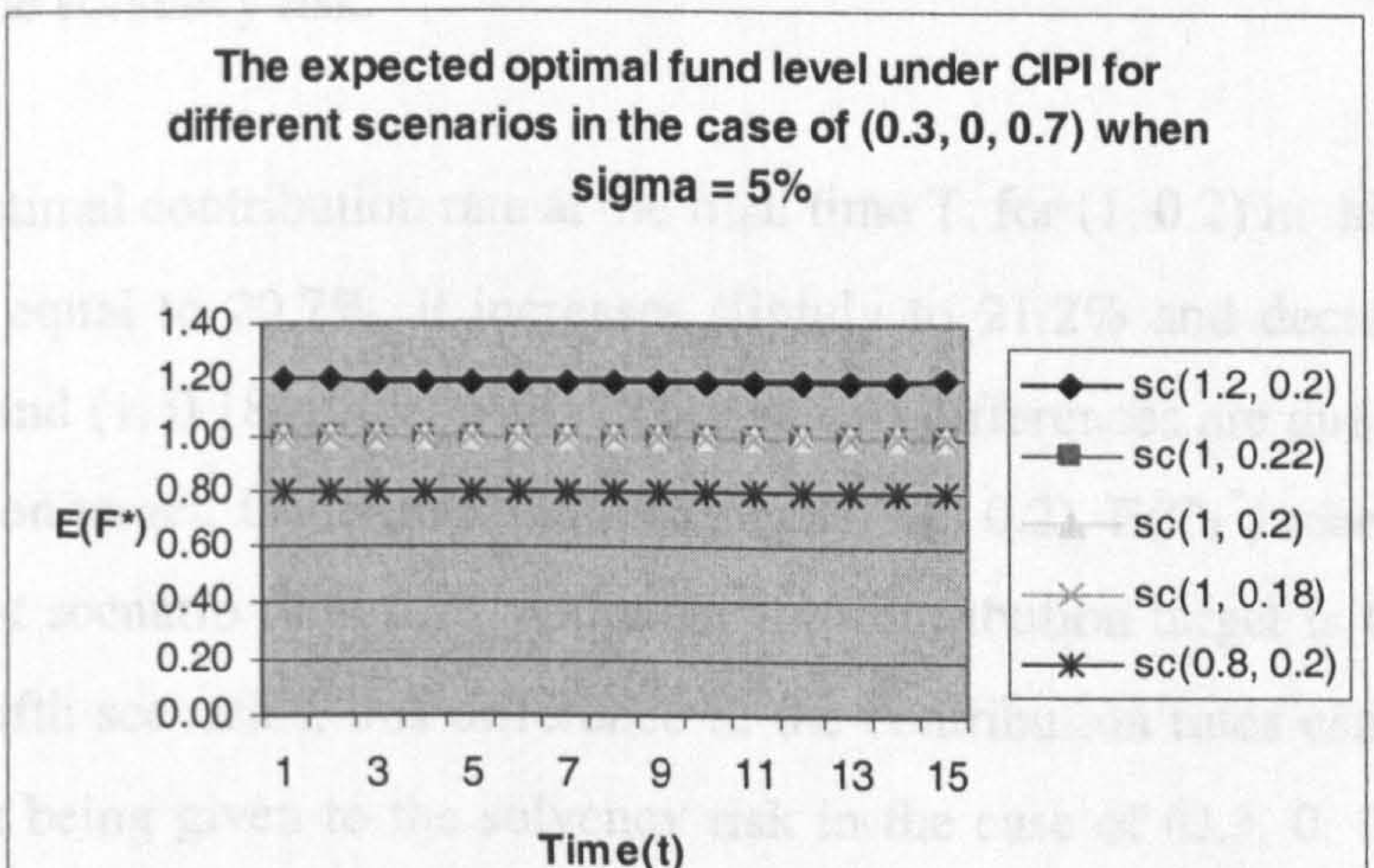
4.4.3 Effect of changing targets FT_t and CT_t

In this subsection, both the fund target FT_t and contribution target CT_t change to higher and lower values, in order to explore the effect of these changes in the models. These changes in the targets are represented in the five scenarios mentioned in section 4.2. Thus, our analysis considers the effect of these five different scenarios on $E(F_t^*)$ and $E(C_t^*)$ when level of volatility is low $\sigma = 5\%$ and when it is equal to 20%. Here, we note that we can interpret FT_t as representing the actuarial liability AL_t, and the contribution target CT_t as representing the normal cost NC_t.

(a) Level of volatility σ = 5%

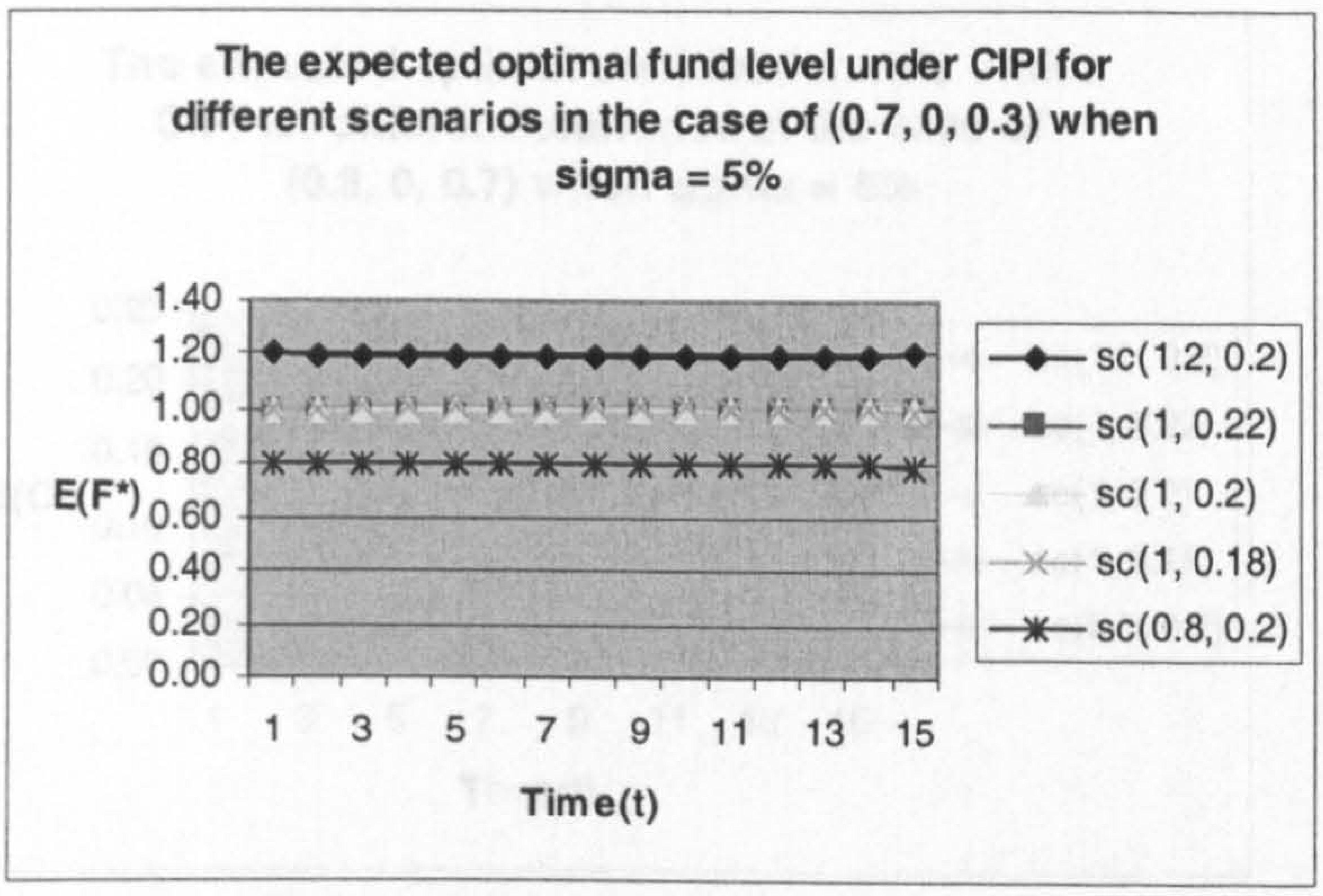
Applying the default scenario (1, 0.2), the results imply that the expected optimal fund level still tends to decrease slightly over the control period as concluded earlier. For example; in the case of (0.3, 0, 0.7), we find that $E(F_t^*)$ is around its target value 100%. The same trends are obtained under both scenarios (1, 0.22) and (1, 0.18) where the fund target is equal to 1 with σ equal to 5%. Furthermore, the same feature is applied in the same case under the fourth scenario (1.2, 0.2) and (0.8, 0.2). This can be seen in Figure 4.12

Figure 4.12



In the other extreme case of (0.7, 0, 0.3) - where there is less importance given to the solvency risk - we find that $E(F_T^*)$ reaches 99.2% under (1, 0.2) while it is equal to 100% under the second scenario (1, 0.22) and 98.4% when FT_t = 1 and CT_t = 0.18. Although the fund targets are the same in the three scenarios, these changes in the optimal fund level can be traced back to our giving more importance to the contribution rate weighting risk factor. For the fourth scenario (1.2, 0.2), $E(F_T^*)$ is approximately around its target 120%. Finally, it reduces to 78.6% under the fifth scenario (0.8, 0.2). These results are shown in Figure 4.13.

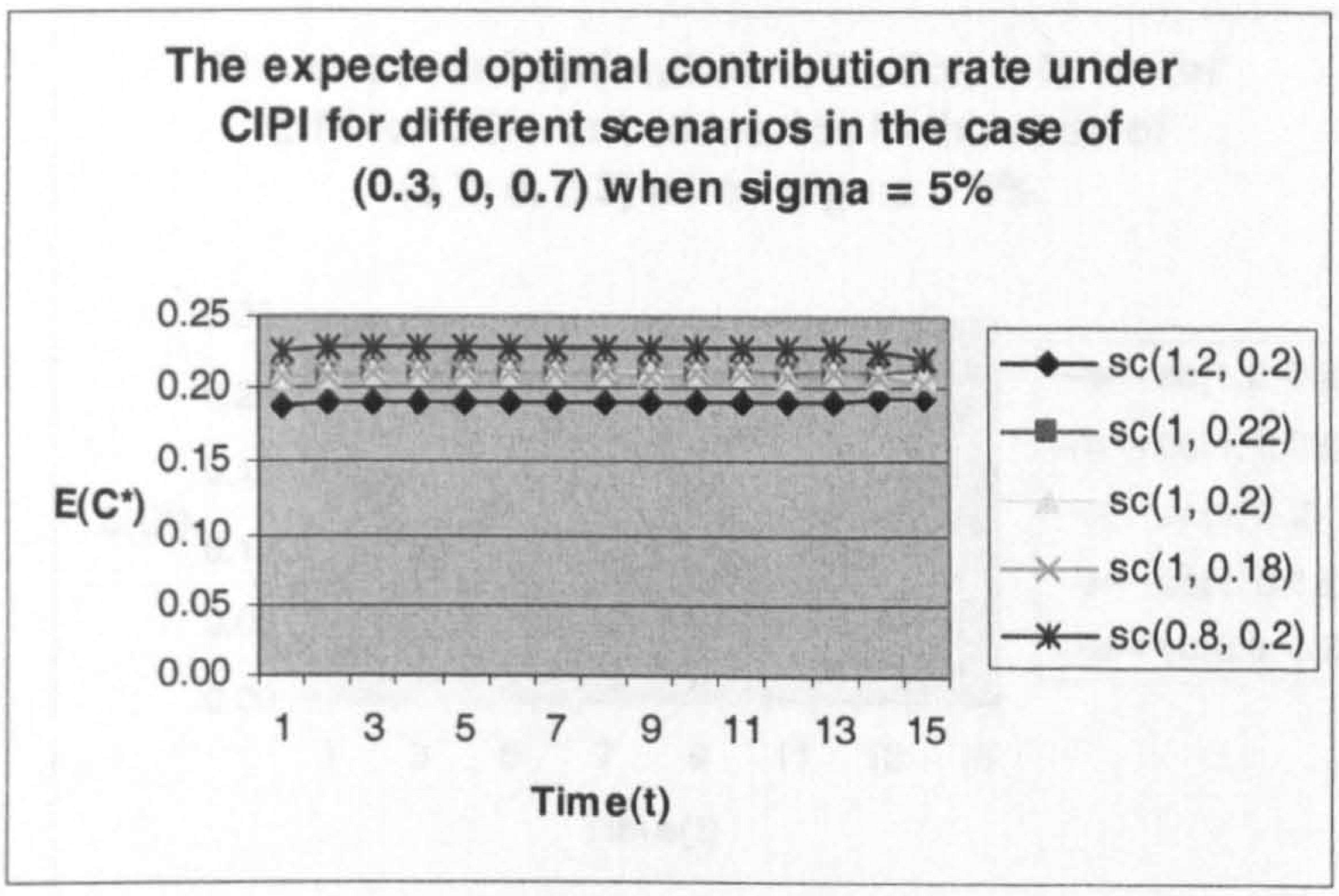
Figure 4.13



Here, it is important to mention that there is a slight decrease in the expected optimal fund levels as we move from Figure 4.12 to 4.13 arising from the reduced weighting being given to the solvency risk.

The expected optimal contribution rate at the final time T, for (1, 0.2) in the same case of (0.3, 0, 0.7), is equal to 20.7%. It increases slightly to 21.2% and decreases to 20.3% under (1, 0.22) and (1, 0.18) respectively. These small differences are due to the changes in the contribution target. Under the fourth scenario (1.2, 0.2), $E(C_T^*)$ reaches 19.4% and 22.1% in the last scenario (0.8, 0.2). Although the contribution target is the same under the fourth and fifth scenarios, this difference in the contribution rates can be referred to the extra weight being given to the solvency risk in the case of (0.3, 0, 0.7). Therefore, the value of expected optimal contribution rate decreases when we move to a higher level of the fund target and increases when FT_t is equal to 80%. Figure 4.14 shows these results.

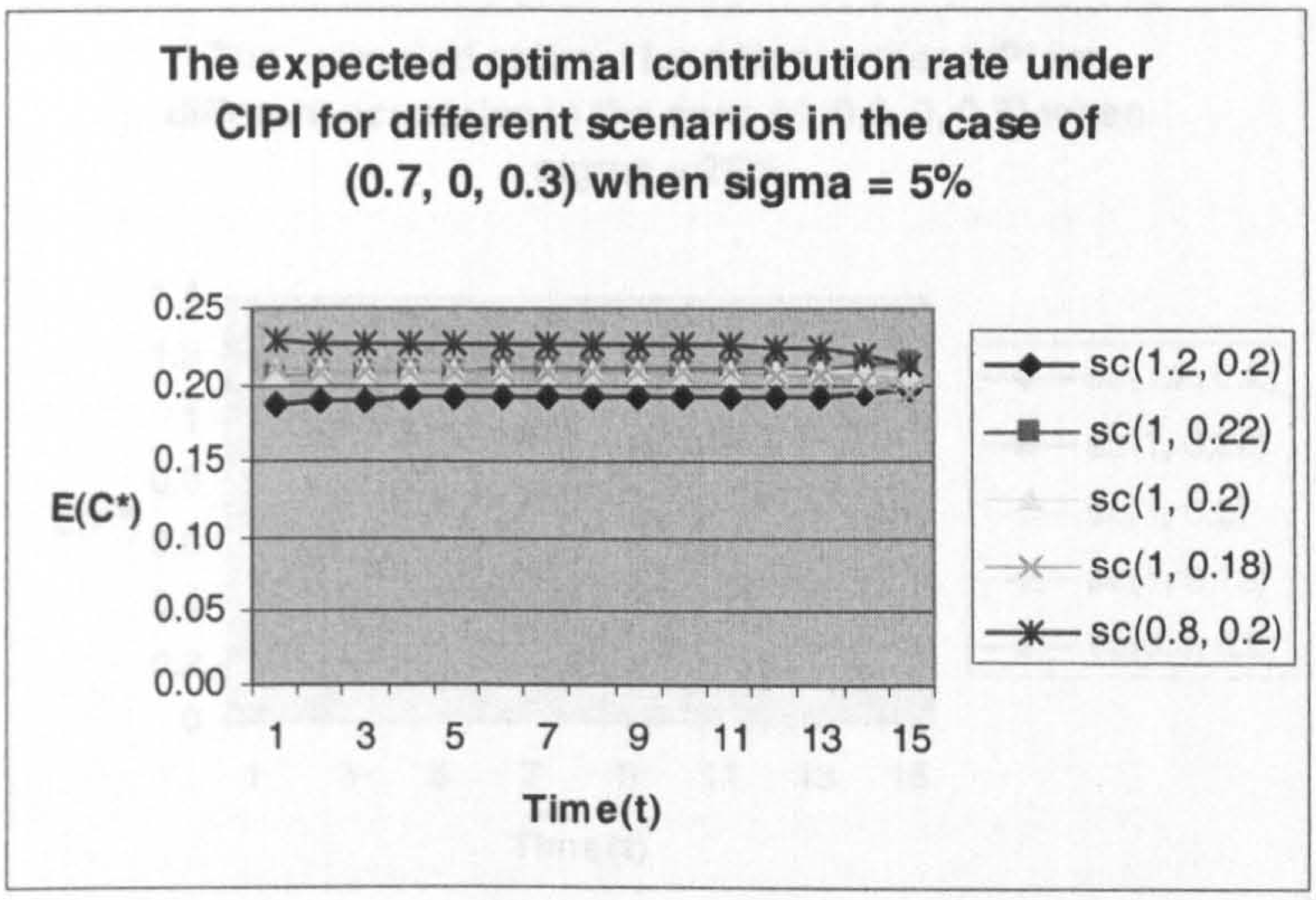
Figure 4.14



For the same five scenarios in the case of (0.7, 0, 0.3), the value of $E(C_T^*)$ is equal to 20.5%, 21.5%, 19.5%, 19.6% and finally 21.4%. For the first three values of $E(C_T^*)$, we note that the expected contribution rate increases with increasing the contribution target to 0.22 while it decreases with $CT_t = 0.18$. It is also noted that the last two values (19.6% and 21.4%) are close to each other compared with the corresponding values (19.4% and 22.1%) in the last case (0.3, 0, 0.7). This is due to our giving more importance to the contribution rate weighting risk factor alongside with the equality of the values of the contribution target in the last two scenarios. This can be observed more clearly in the case of (0.9, 0, 0.1) - where β has the highest value of 0.9 - which results in the almost equal values of $E(C_T^*)$ equal to 21% in the fourth scenario and 21.6% in the fifth scenario. Figure 4.15 shows the results of applying the case of (0.7, 0, 0.3).

Although $E(C_t^*)$ trends look similar in Figures 4.14 and 4.15, there is a slight increase observed in the contribution rates, for the case (0.7, 0, 0.3), due to increasing the weighting risk factor of the contribution rate risk.

Figure 4.15



(b) Level of volatility $\sigma = 20\%$

Increasing the level of σ to 20% will help us to understand the behaviour of the expected optimal fund level and contribution rate, when their targets are changed in a more volatile financial environment. In the following analysis, we will use the same cases mentioned when $\sigma = 5\%$. For the cases of (0.3, 0, 0.7) and (0.7, 0, 0.3), we can summarize the results of $E(F_T^*)$ under the five scenarios in Table 4.13.

Table 4.13
The value of $E(F_T^*)$ under the different five scenarios for the cases (0.3, 0, 0.7) and (0.7, 0, 0.3)

%

Cases	(1, 0.2)	(1, 0.22)	(1, 0.18)	(1.2, 0.2)	(0.8, 0.2)
(0.3, 0, 0.7)	96.0	96.1	95.9	115.3	76.6
(0.7, 0, 0.3)	94.1	94.9	93.3	113.7	74.5

From Table 4.13, it is noted that the expected fund levels are almost the same in the first three scenarios in both cases, this is due to our giving more importance to the solvency weighting risk factor and the equality of the fund target. However, under the fourth and fifth scenarios, $E(F_T^*)$ has different values because of the different values of the fund targets. In addition, the effect of changing the weighting risk factors of γ and β interchangeably can be seen on the levels of the expected funds in Figures 4.16 and 4.17, as they move further away from the target when β changes from 0.7 to 0.3.

Figure 4.16

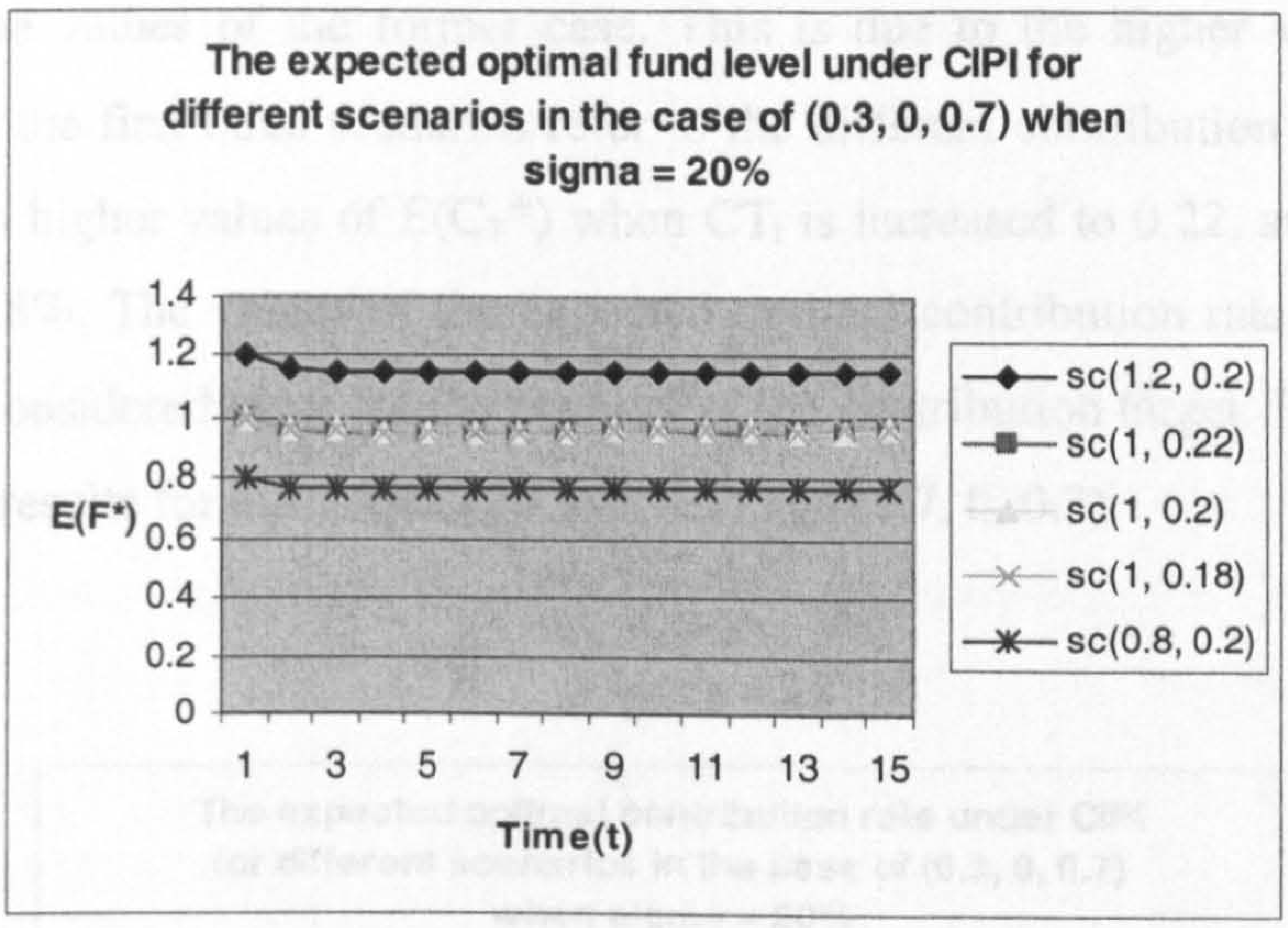
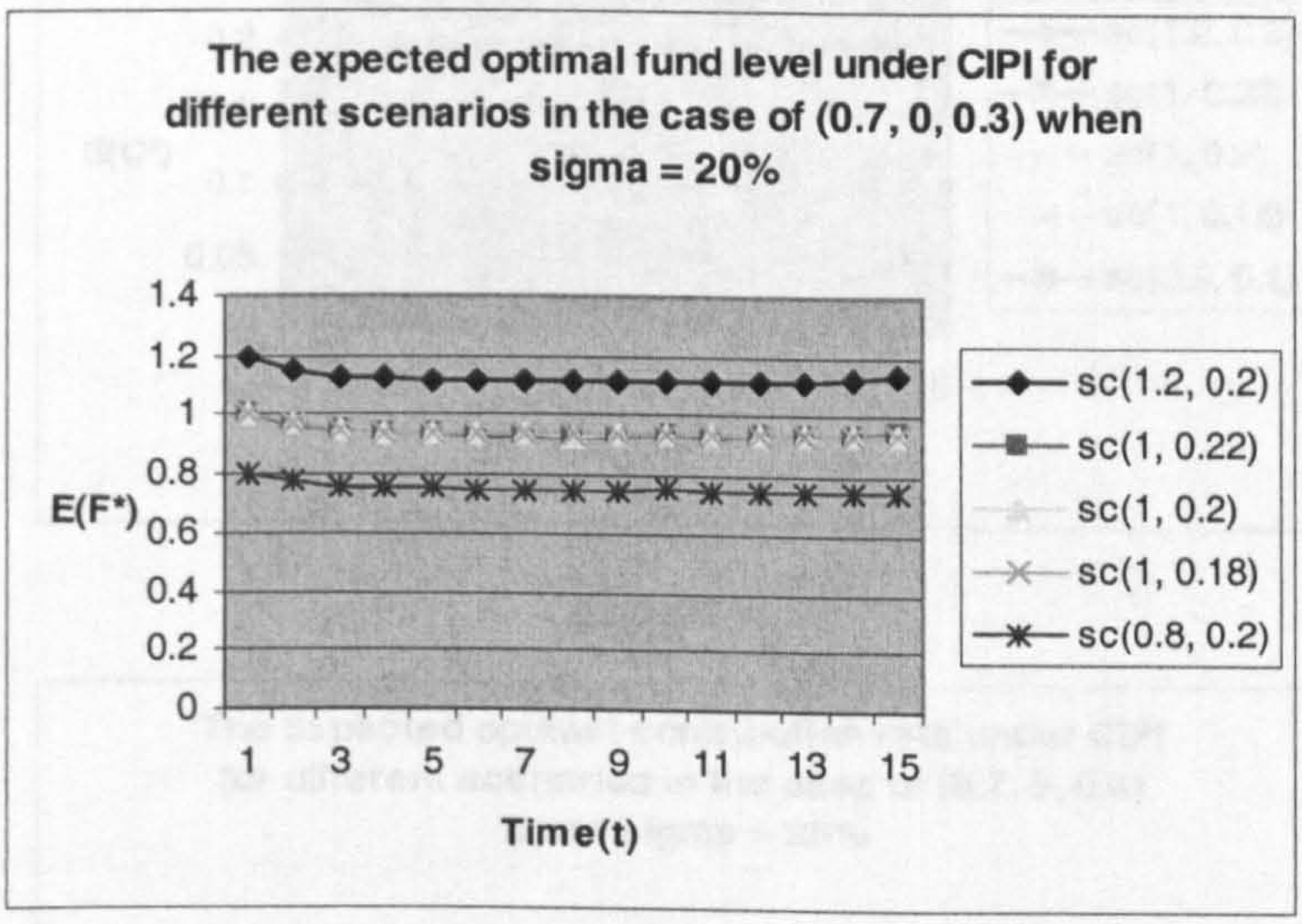


Figure 4.17



Here, we note that increasing the level of volatility results in the values of the expected optimal fund level moving away from the targets markedly compared with those when $\sigma = 5\%$.

In the case of (0.3, 0, 0.7) at the higher level of volatility, the results are: $E(C_T^*) = 21.5\%$, 21.9% and 21% under the three first scenarios, while it is equal to 20.2% and 22.7% in the fourth and the fifth scenarios respectively. On the other hand, applying the case of (0.7, 0, 0.3) leads these values to be 21.4%, 22.4%, 20.3%, 20.7% and 22.1%.

Here, we note that the values for $E(C_T{}^*)$ in the latter case are closer to the contribution targets than the values of the former case. This is due to the higher value of β. The changes under the first three scenarios refer to the different contribution targets applied, which result in higher values of $E(C_T{}^*)$ when CT_t is increased to 0.22, and lower values when CT_t is 18%. The values of the expected optimal contribution rate in the last two scenarios are considered close for the equality of the contribution target. Figures 4.18 and 4.19 show the results for the cases of (0.3, 0, 0.7) and (0.7, 0, 0.3).

Figure 4.18

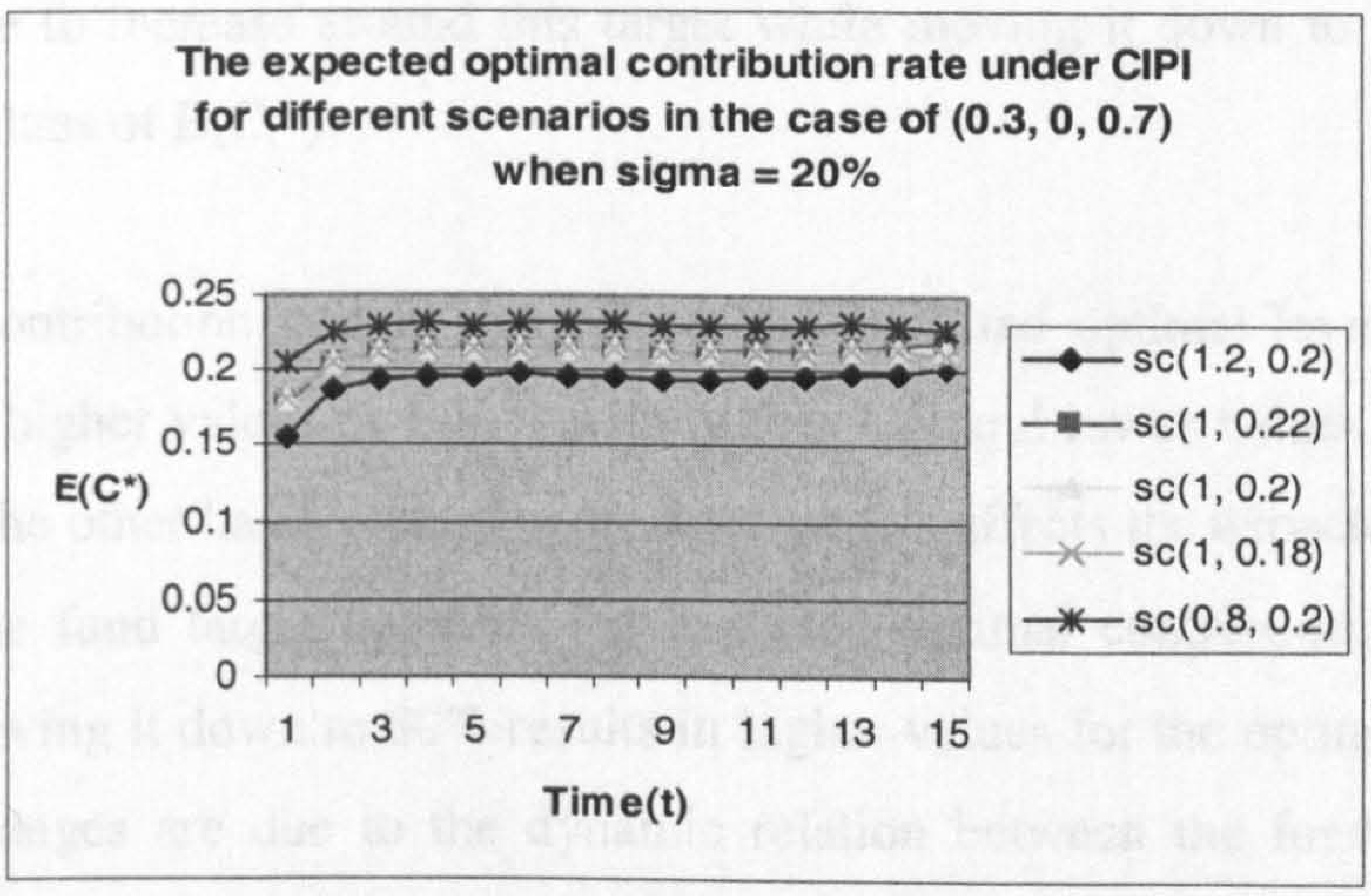
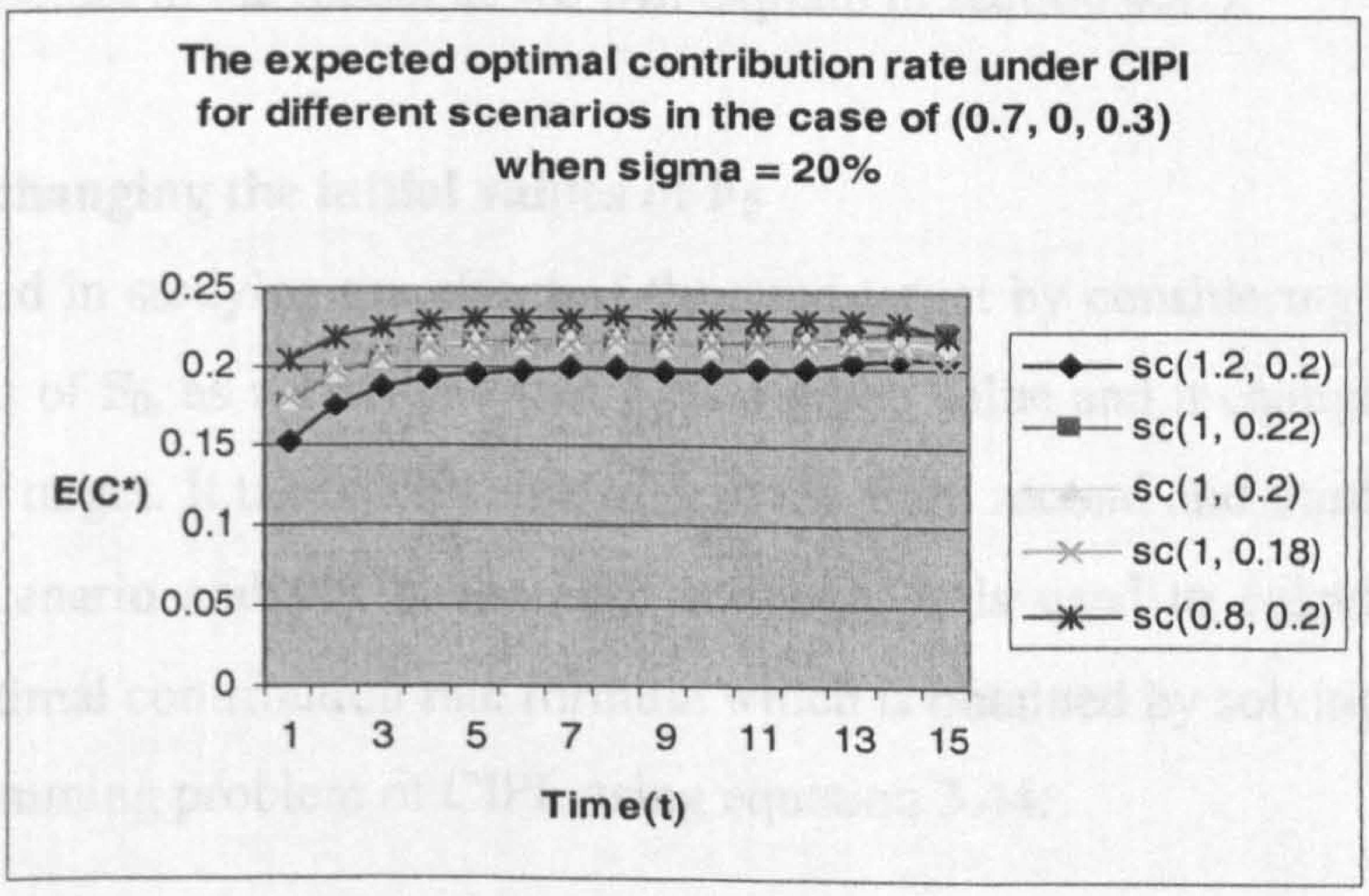


Figure 4.19



Again, we note that the expected optimal contribution rates move further away from the targets compared with their corresponding values, as the level of volatility is increased from 5% to 20%.

Conclusion

The changes of the fund target and the contribution target clearly affect the values of the expected optimal fund level and contribution rate. In other words, the expected optimal fund levels move around their targets whether the target is equal to 80%, 100% or 120%. Similarly, moving the contribution target up to 22% leads the expected optimal contribution rate to increase around this target while moving it down to 18% results in having lower values of $E(C_t^*)$.

Changing the contribution targets also affects the expected optimal levels of the fund, where we have higher values of $E(F_t^*)$ with higher CT_t and lower values of $E(F_t^*)$ with lower CT_t. On the other hand, changing the fund targets affects the expected contribution rate; moving the fund target up leads the expected optimal contribution rate to move down, while moving it down to 80% results in higher values for the optimal contribution rates. These changes are due to the dynamic relation between the fund level and the contribution rate. This dynamic relation is affected by changing the parameters of the weighting risk factors in the model as we will explain in section 4.4.5.

4.4.4 Effect of changing the initial values of F_0

Here, we proceed in studying the effect of the fund target by considering the changes in the initial values of F_0, as we assume that F_0 is a given value and it changes according to the chosen fund target. It takes the value of 1 in the first, second and third scenarios, 1.2 in the fourth scenario and 0.8 in the fifth scenario. It is used to calculate $E(C^*_0)$ by applying the optimal contribution rate formula which is obtained by solving the stochastic dynamic programming problem of CIPI, using equation 3.44:

$$E(C_t^*) = \frac{(2\gamma + \alpha l)CT_t - (\alpha l + 2dk_{t+1})(F_t - B_t) + (\alpha + 2\beta l)FT_{t+1} - l(a_2(t+1)}{2(\gamma + \alpha l + dk_{t+1})}$$

Thus, $E(C_0^*)$ changes with the values that the initial fund can take according to the different fund targets under all scenarios. We examine the case of (0.1, 0, 0.9), as an example, in order to understand the effect of changing the initial fund on the value of $E(C_0^*)$ as follows:

When $\sigma = 5\%$, $E(C_0^*)$ has the same value which is equal to 20.9% under the first three scenarios (1, 0.2), (1, 0.22) and (1, 0.18). Increasing σ to 15% leads this value to decrease to 19.2% and then to 18% when σ reaches 20%. Under the fourth and the fifth scenarios, $E(C_0^*)$ changes due to the changes in the fund targets and consequently the value of F_0 as shown in Table 4.14.

Table 4.14
The values of $E(C_0^*)$ under different scenarios for the case of (0.1, 0, 0.9) when $\sigma = 5\%, 15\%$ and 20%

%

	(1, 0.2)	(1, 0.22)	(1, 0.18)	(1.2, 0.2)	(0.8, 0.2)
$\sigma = 5\%$	20.9	20.9	20.9	18.8	22.6
$\sigma = 15\%$	19.2	19.2	19.2	17.1	21.4
$\sigma = 20\%$	18	18	18	15.6	20.4

These results indicate the fact that the contribution rates have higher values when the fund level is lower than 100%. They decrease gradually with increasing the fund level until they reach the minimum values in the range tested when $FT_t = 120\%$.

Finally, the values of $E(C_0^*)$ are used in the fund growth formula to calculate the fund at t=1 as follows:

$$E(F_1|F_0) = E(e^{\delta}) \cdot (F_0 + E(C_0|F_0) - B_0) \tag{4.2}$$

Therefore, we can conclude that the values of $E(C_0^*)$ are definitely affected by the values specified for given F_0, so that, $E(C_0^*)$ has the same value when $F_0 = 1$ in the three first scenarios, while it decreases when $F_0 = 1.2$ and moves up when $F_0 = 0.8$. Further, its values are also affected by level of volatility applied.

4.4.5 Effect of changing the weighting factors of the contribution rate risk and the solvency risk; γ and β (with $\alpha = 0$)

The contribution rate risk and the solvency risk are the two main risks involved in our models as previously mentioned in subsection 3.4.5. Thus, it is important to examine the effect of changing the two main weighting risk factors γ and β associated with these risks respectively. In this subsection, we consider the effect on both the expected optimal fund level and contribution rate over the control period, regarding the cases where the weighting risk factor of the middle term α is equal to 0, in order to understand precisely the behaviour of the dynamic models with changing these main risks.

Under each scenario there are five different cases. In each case, we allow for the weighting risk factor of the contribution rate γ to increase and the parameter of the solvency risk to decrease incrementally whereas α remains equal to 0. These cases appear in the order (γ, α, β) and they are:

The first case is: (0.1, 0, 0.9);

the second case is: (0.3, 0, 0.7);

the third case is (0.5, 0, 0.5);

the fourth case is (0.7, 0, 0.3);

the fifth case is (0.9, 0, 0.1).

(a) First scenario: $FT_t = 1$, $CT_t = 0.2$

Under our default scenario and at a low level of volatility when $\sigma = 5\%$, the expected optimal fund level decreases slightly over the control period to reach 99.8% in the case of (0.1, 0, 0.9). Considering the other four cases, $E(F_t^*)$ decreases gradually with increasing the weighting risk factor for the contribution rate. Further, $E(F_t^*)$ decreases more at a higher level of volatility together with an increased weighting risk factor for the contribution rate, e.g. for the above case(0.1, 0, 0.9), $E(F_T^*)$ reaches 96.6% when $\sigma = 20\%$.

Table 4.15 shows that (for $\sigma = 5\%$) increasing the weighting risk factor of the contribution rate leads the expected optimal fund level to decrease gradually, it reaches the lowest level at the value of 98.1% at the end of the control period when $\gamma = 0.9$.

Further, this value for $\gamma = 0.9$ declines to 89.8% when σ is increased to 20% (details not shown).

Table 4.15

The expected optimal fund level under CIPI (1, 0.2) for different combinations of γ and β when $\sigma = 5\%$

t	(0.1, 0, 0.9)	(0.3, 0, 0.7)	(0.5, 0, 0.5)	(0.7, 0, 0.3)	(0.9, 0, 0.1)
1	100.00%	100.00%	100.00%	100.00%	100.00%
2	99.80%	99.82%	99.84%	99.87%	99.96%
3	99.76%	99.75%	99.75%	99.78%	99.90%
4	99.77%	99.74%	99.73%	99.74%	99.86%
5	99.79%	99.77%	99.75%	99.75%	99.85%
6	99.76%	99.74%	99.72%	99.72%	99.81%
7	99.77%	99.75%	99.73%	99.72%	99.78%
8	99.77%	99.75%	99.73%	99.72%	99.74%
9	99.80%	99.77%	99.75%	99.73%	99.70%
10	99.78%	99.76%	99.75%	99.72%	99.62%
11	99.75%	99.73%	99.71%	99.68%	99.47%
12	99.80%	99.77%	99.74%	99.68%	99.33%
13	99.74%	99.72%	99.68%	99.58%	99.04%
14	99.77%	99.73%	99.66%	99.47%	98.67%
15	99.78%	99.70%	99.56%	99.24%	98.12%

On the other hand, the expected optimal contribution rate becomes closer to the contribution target with our giving less importance to the weighting risk factor of the solvency risk. We note that the expected optimal contribution rate is around 21% but is moving closer to the target with increasing γ as shown in Table 4.16.

When σ is increased to 20%, the expected optimal contribution rate becomes more volatile. It starts from lower levels around 18% and tends to increase slightly and move away from its target due to the decreasing trend of the optimal fund level. Nevertheless, it tends to be closer to the contribution target with increasing the value of γ at the end of the control period as can be seen in Table 4.17.

Table 4.16

The expected optimal contribution rate under CIPI (1, 0.2) for different combinations of γ and β when $\sigma = 5\%$

t	(0.1, 0, 0.9)	(0.3, 0, 0.7)	(0.5, 0, 0.5)	(0.7, 0, 0.3)	(0.9, 0, 0.1)
1	20.72%	20.74%	20.76%	20.78%	20.86%
2	20.90%	20.88%	20.86%	20.85%	20.87%
3	20.94%	20.93%	20.92%	20.90%	20.89%
4	20.93%	20.93%	20.93%	20.92%	20.90%
5	20.91%	20.92%	20.92%	20.92%	20.89%
6	20.94%	20.94%	20.94%	20.93%	20.90%
7	20.93%	20.93%	20.93%	20.93%	20.89%
8	20.93%	20.93%	20.93%	20.93%	20.88%
9	20.91%	20.91%	20.92%	20.91%	20.85%
10	20.92%	20.92%	20.92%	20.91%	20.83%
11	20.95%	20.94%	20.94%	20.92%	20.80%
12	20.90%	20.91%	20.90%	20.87%	20.73%
13	20.96%	20.94%	20.91%	20.85%	20.65%
14	20.92%	20.89%	20.84%	20.74%	20.51%
15	20.86%	20.74%	20.63%	20.50%	20.30%

Table 4.17

The expected optimal contribution rate under CIPI (1, 0.2) for different combinations of γ and β when $\sigma = 20\%$

t	(0.1, 0, 0.9)	(0.3, 0, 0.7)	(0.5, 0, 0.5)	(0.7, 0, 0.3)	(0.9, 0, 0.1)
1	17.98%	17.94%	17.89%	17.82%	17.62%
2	20.99%	20.57%	20.16%	19.71%	19.02%
3	21.31%	21.22%	21.02%	20.68%	20.00%
4	21.35%	21.37%	21.33%	21.18%	20.68%
5	21.28%	21.34%	21.38%	21.36%	21.10%
6	21.40%	21.45%	21.50%	21.54%	21.45%
7	21.32%	21.39%	21.47%	21.57%	21.65%
8	21.33%	21.39%	21.48%	21.60%	21.82%
9	21.10%	21.20%	21.32%	21.51%	21.87%
10	21.03%	21.11%	21.23%	21.44%	21.89%
11	21.23%	21.27%	21.36%	21.54%	21.98%
12	21.19%	21.25%	21.36%	21.58%	21.97%
13	21.41%	21.45%	21.56%	21.75%	21.92%
14	21.25%	21.41%	21.58%	21.73%	21.61%
15	21.26%	21.46%	21.49%	21.37%	21.00%

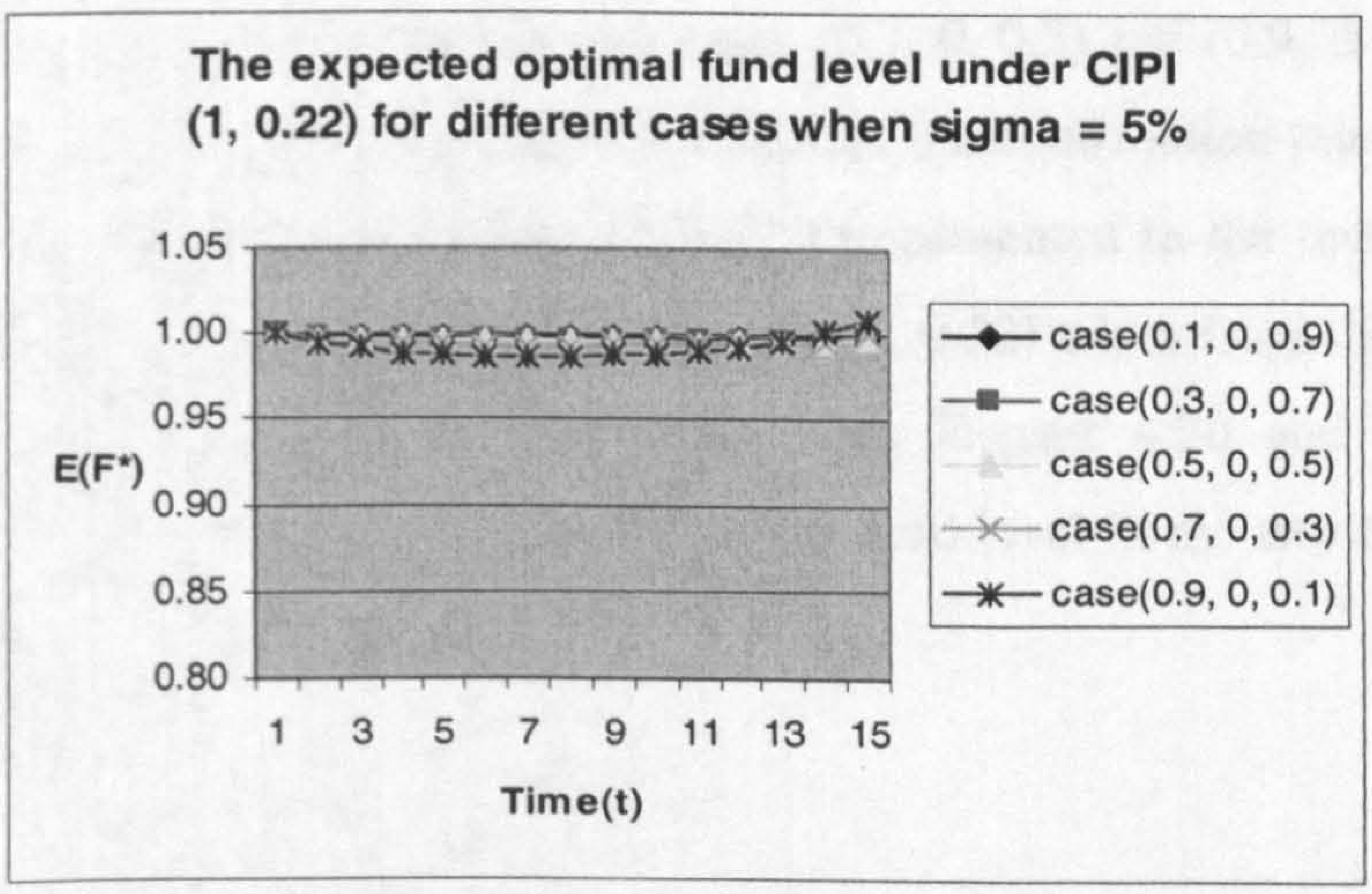
(b) Second scenario: $FT_t = 1$, $CT_t = 0.22$

Under the second scenario where the fund target is similar to the first one and the contribution target is increased to 22%, when $\sigma = 5\%$, we note that the trends of expected optimal fund level in the five cases remain close to the target with a very small tendency to decrease, even when more importance is given to the weighting risk factor of the contribution target. In fact, $E(F_t^*)$ tends to increase towards the target by the end of the control period as shown in Figure 4.20.

Over the control period, the five trends of expected optimal contribution rate are close to 20%. Then, they become closer to the target by the end of the control period particularly in the last three years as shown in Figure 4.21. This is more obvious as we increase the value of γ where the expected optimal contribution rate tends to get closer to 22%.

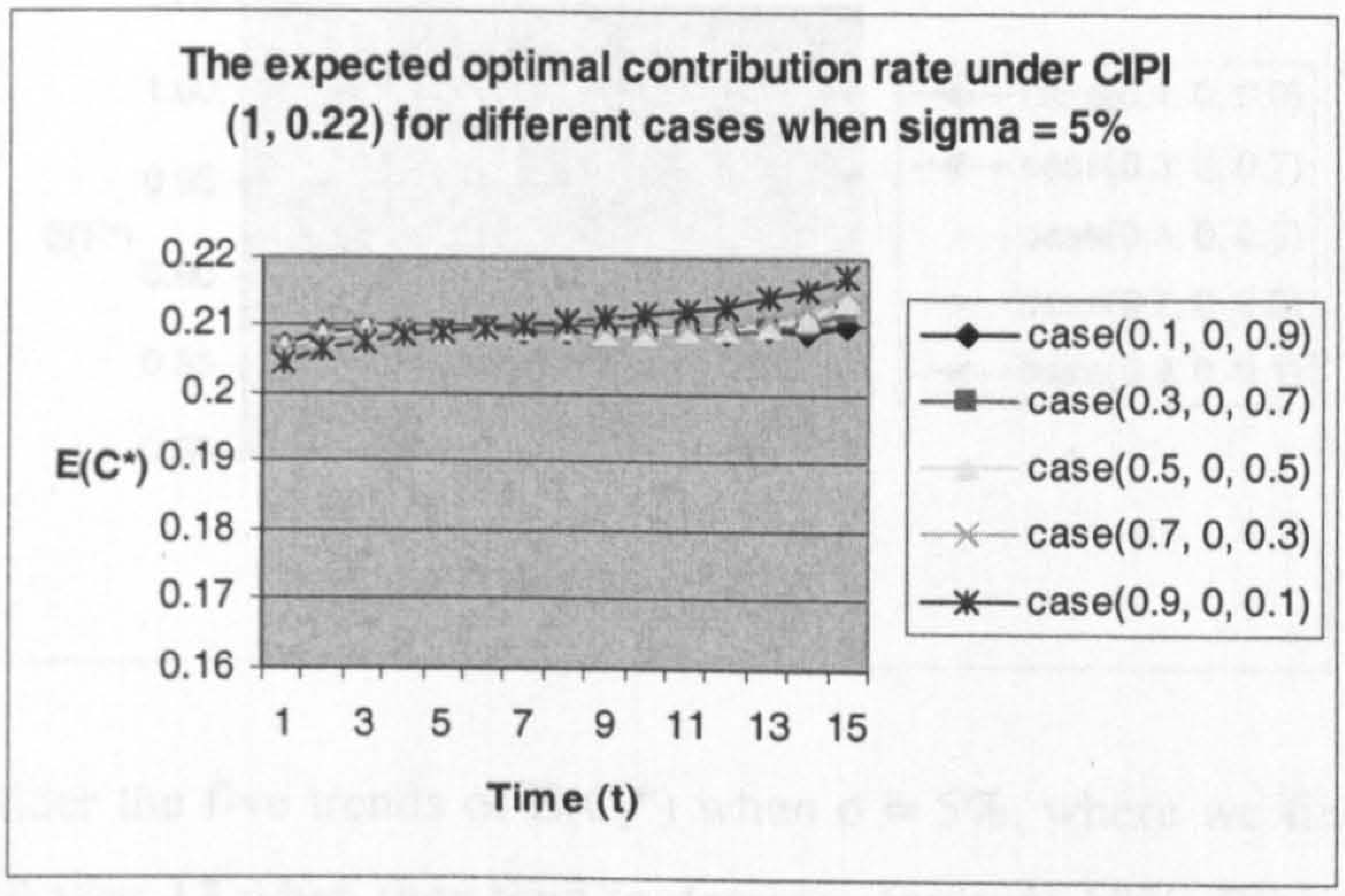
When the level of volatility is increased to 20%, we find that the trends of the expected optimal fund level decrease more over time. They move away from the target but still with higher values obtained with comparison to the ones in the first scenario. This is due to the higher level of CT_t.

Figure 4.20



When $\sigma = 20\%$, the expected optimal contribution rate tends to increase to be around 21% over time to respond to the decreasing level of the fund, and they become close to the target 22% at the end of the control period. Full results for $\sigma = 20\%$ are available but are not shown.

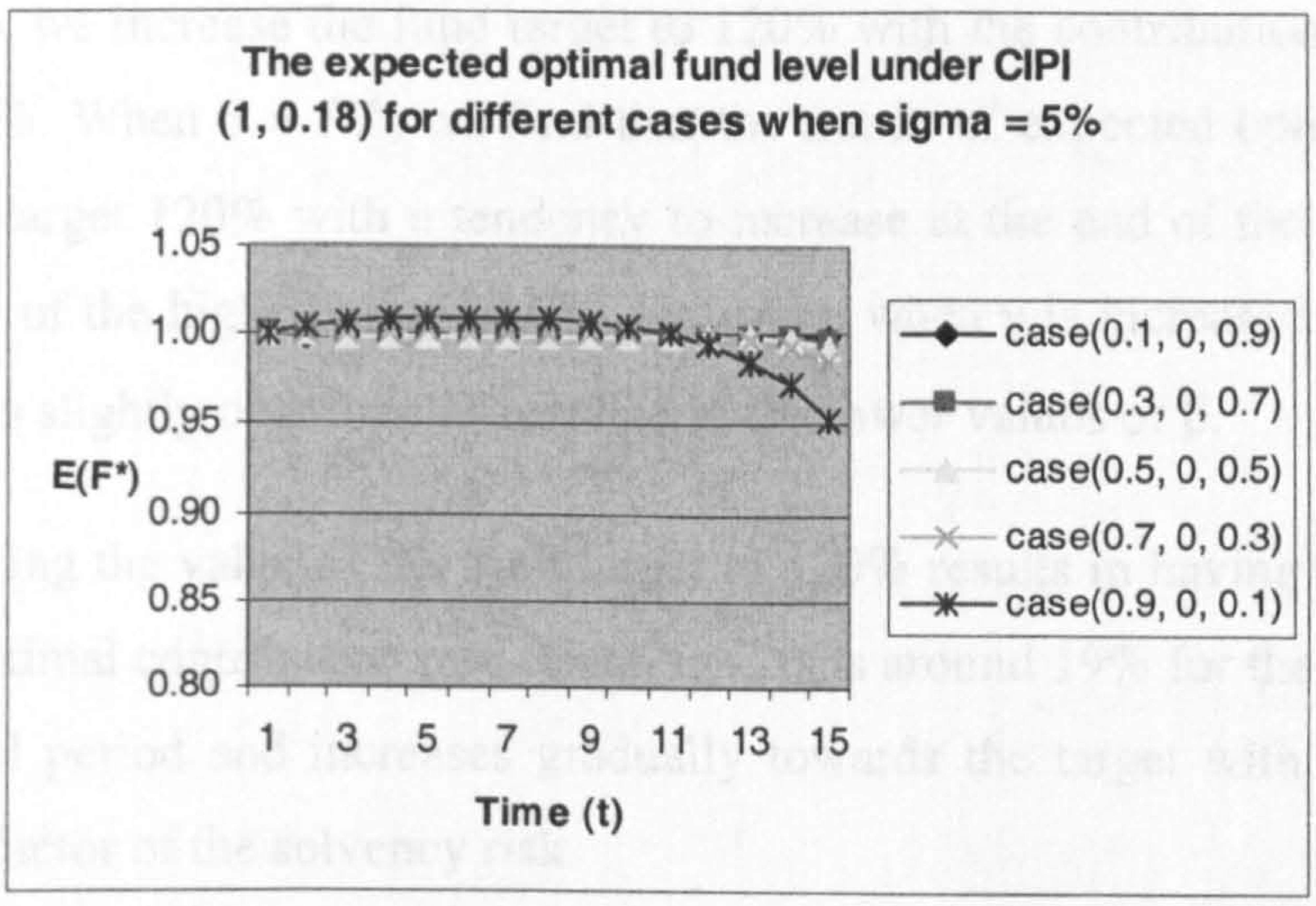
Figure 4.21



(c) Third scenario $FT_t = 1, CT_t = 0.18$

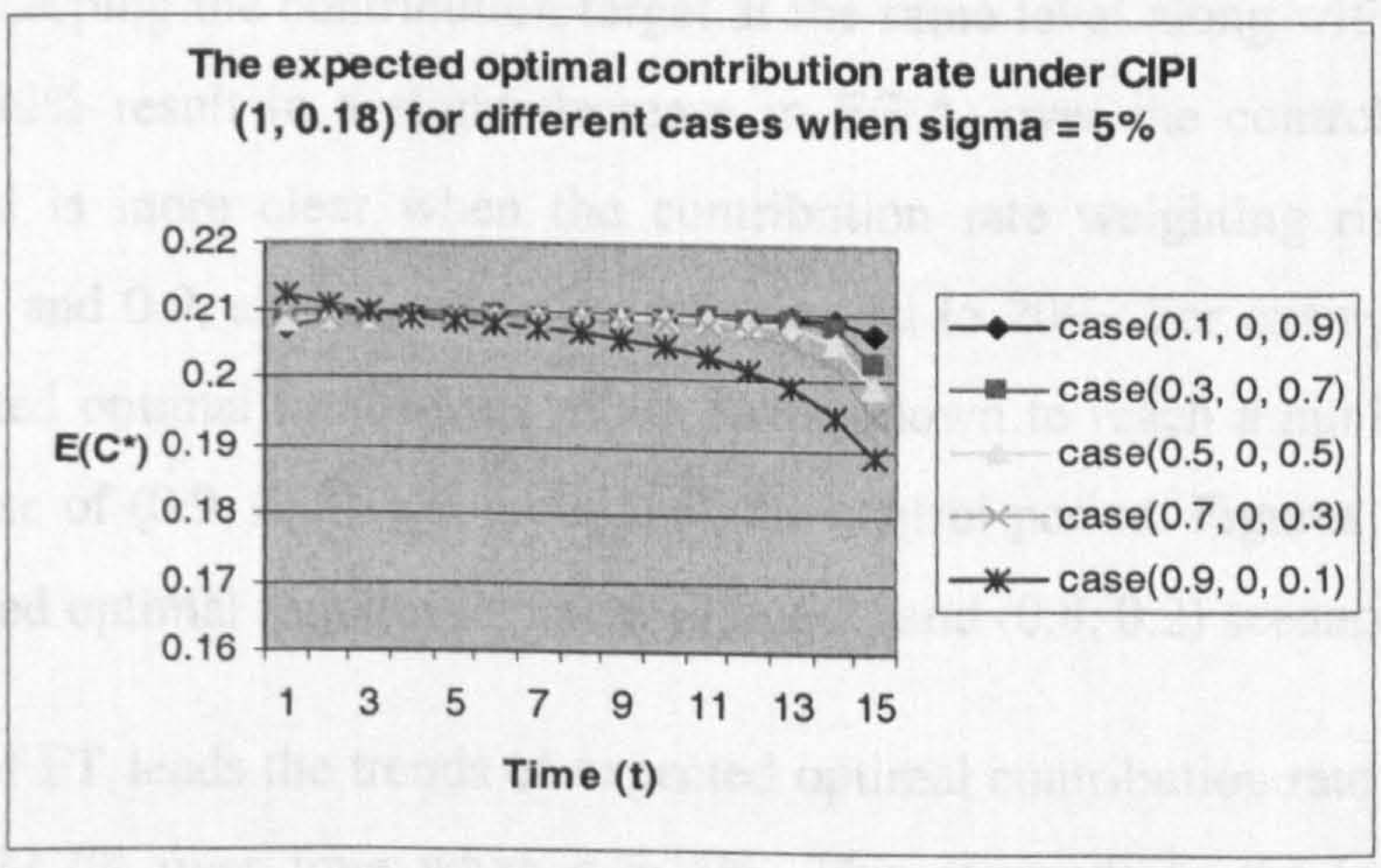
In this scenario, the fund target is still the same where CT_t is reduced to 18%. Thus, when $\sigma = 5\%$, the trends of expected optimal fund level for the five cases remain close to the target 100% over time, with a tendency to decrease by the end of the control period. This decrease is more obvious in the last two cases (0.7, 0, 0.3) and (0.9, 0, 0.1), where more importance is given to the weighting risk factor of the contribution rate as can be seen in Figure 4.22. The opposite behaviour of $E(F^*_t)$ (represented in the tendency to decrease over time) compared with the previous scenario (1, 0.22) arises from the reduction in the value of contribution target to 18% from 22%. Figures 4.20 and 4.22 indicate the difference in behaviour of the expected optimal fund level in the two scenarios (1, 0.22) and (1, 0.18).

Figure 4.22



In turn, we consider the five trends of $E(C_t^*)$ when $\sigma = 5\%$, where we find that they are around 21% until year 13 when they tend to decrease towards 18%. When γ is increased to 0.9, the expected optimal contribution rate is forced to move down to be closer to the target value of 18%. Figures 4.21 and 4.23 show the differences in the levels of $E(C_t^*)$ between the second and third scenarios.

Figure 4.23



Increasing the level of volatility to 20% leads to similar results (as pointed out in previous subsections), except that both the expected optimal fund level and the contribution rate move further away from the target compared to the lower volatility case.

- **Fourth scenario $FT_t = 1.2$ and $CT_t = 0.2$**

In this scenario, we increase the fund target to 120% with the contribution target fixed at the level of 20%. When $\sigma = 5\%$, we find that the trends of expected optimal fund level are around the target 120% with a tendency to increase at the end of the control period. This is because of the higher value of FT_t. However, when γ is increased to 0.7 and 0.9, $E(F_t^*)$ decreases slightly over time to respond to the lower values of β.

Further, increasing the value of the fund target to 120% results in having lower levels of the expected optimal contribution rate. Thus, $E(C_t^*)$ is around 19% for the different cases over the control period and increases gradually towards the target with decreasing the weighting risk factor of the solvency risk.

Increasing the level of volatility leads to lower levels of the expected optimal fund level as they move further down from the target. On the other hand, the levels of the expected contribution rate increase gradually to reach a maximum of 21%. These results assert that $E(F_t^*)$ and $E(C_t^*)$ move further away from the target when σ is increased to 20%. They are shown in Figures 4.24 and 4.26.

- **Fifth scenario $FT_t = 0.8$ and $CT_t = 0.2$**

When $\sigma = 5\%$, keeping the contribution target at the same level along with reducing the fund target to 80% result in a slight decrease in $E(F_t^*)$ over the control period. This decreasing trend is more clear when the contribution rate weighting risk factor γ is increased to 0.7 and 0.9, and also when σ is increased to 20%. For example, when $\sigma = 20\%$, the expected optimal fund levels move further down to reach a minimum level of 69.1% in the case of (0.9, 0, 0.1) at the end of the control period. Figures 4.24 and 4.25 show the expected optimal fund levels under (1.2, 0.2) and (0.8, 0.2) scenarios.

A lower value of FT_t leads the trends of expected optimal contribution rate to increase to approximately 22.7% over time when $\sigma = 5\%$. This responds to the lower levels of $E(F_t^*)$. Nevertheless, as γ increases, the levels of the expected optimal contribution rate move downwards in order to be closer to the target by the end of the control period.

As σ increases to 20%, the levels of the expected optimal contribution rate tend to increase more to be around 23%. After, they start to decrease to become closer to the contribution target at the end of the control period. Figures 4.26 and 4.27 show the differences between the trends of the expected optimal contribution rate over time in the fourth and fifth scenarios.

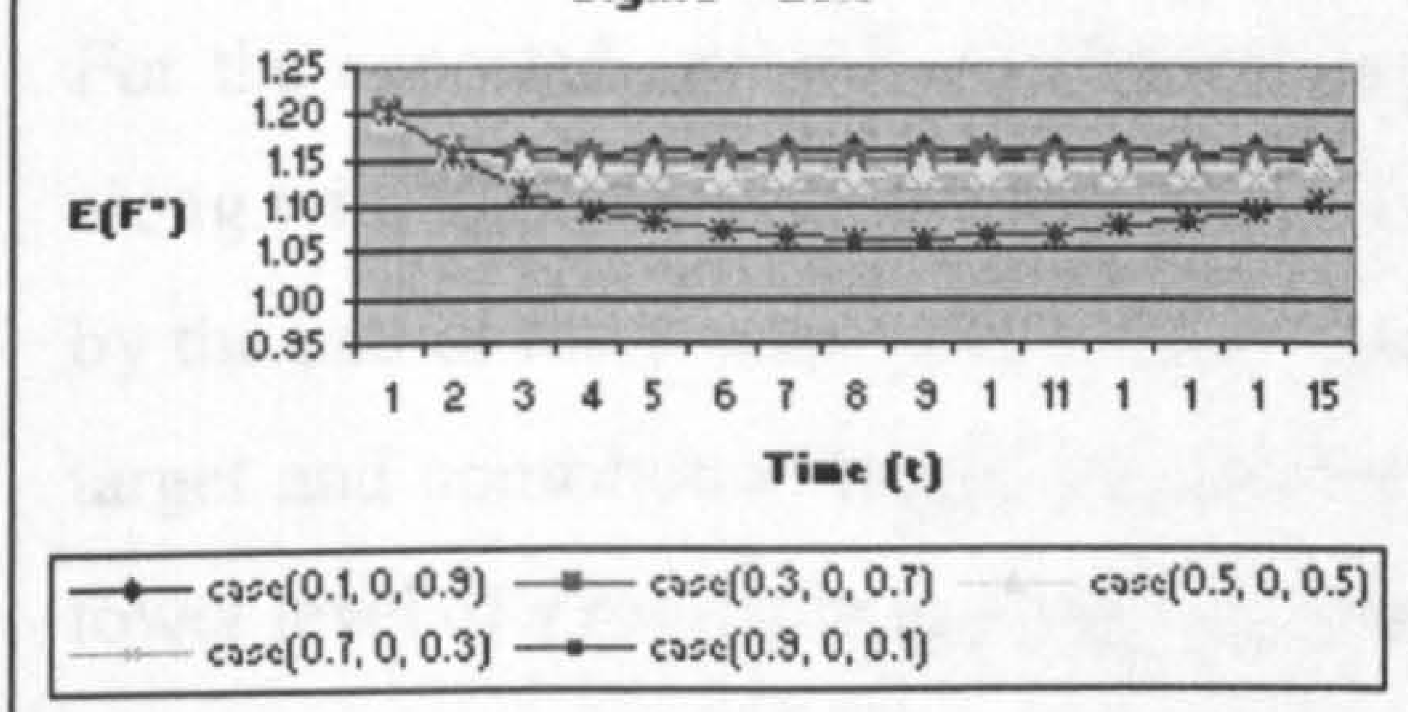


Figure 4.24
The expected optimal fund level under CIPI (1.2, 0.2) for different cases when sigma = 20%

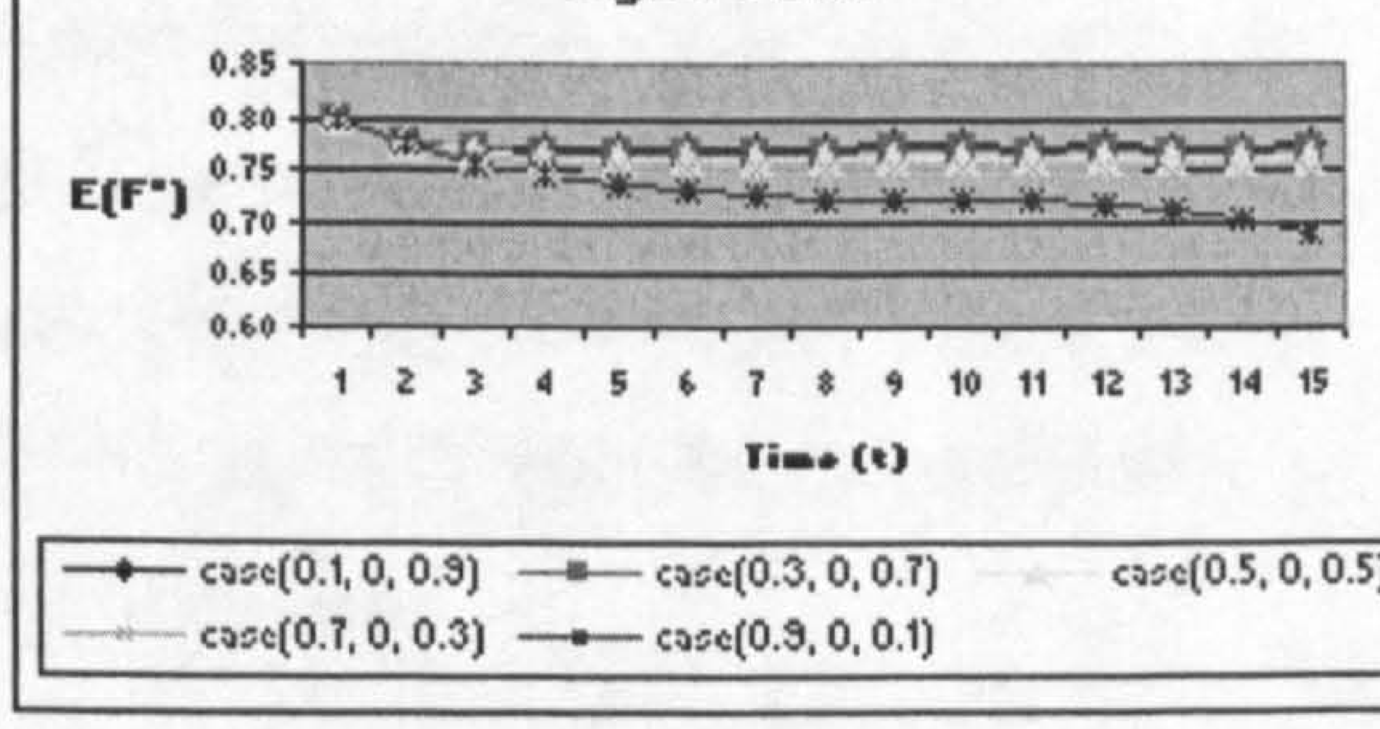


Figure 4.25
The expected optimal fund level under CIPI (0.8, 0.2) for different cases when sigma = 20%

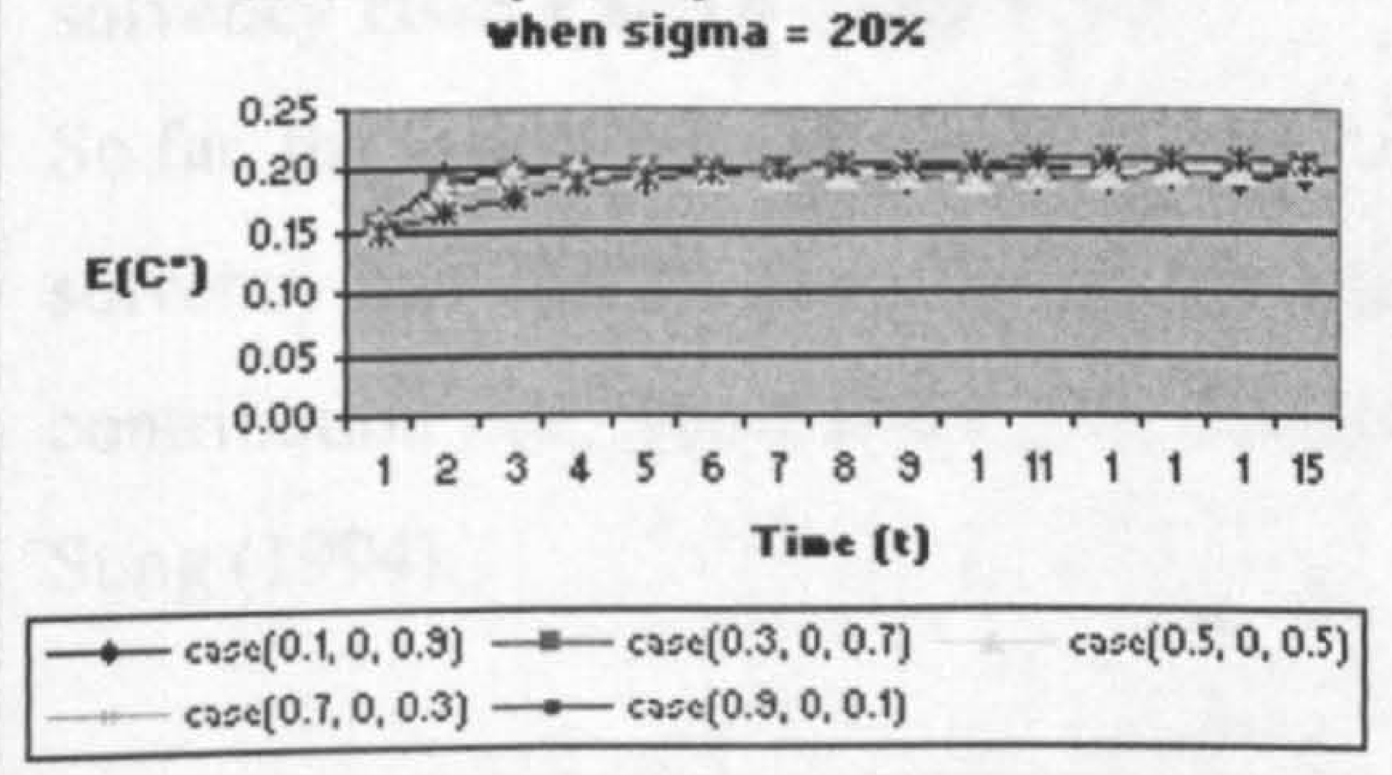


Figure 4.26
The expected optimal contribution rate under CIPI (1.2, 0.2) for different cases when sigma = 20%

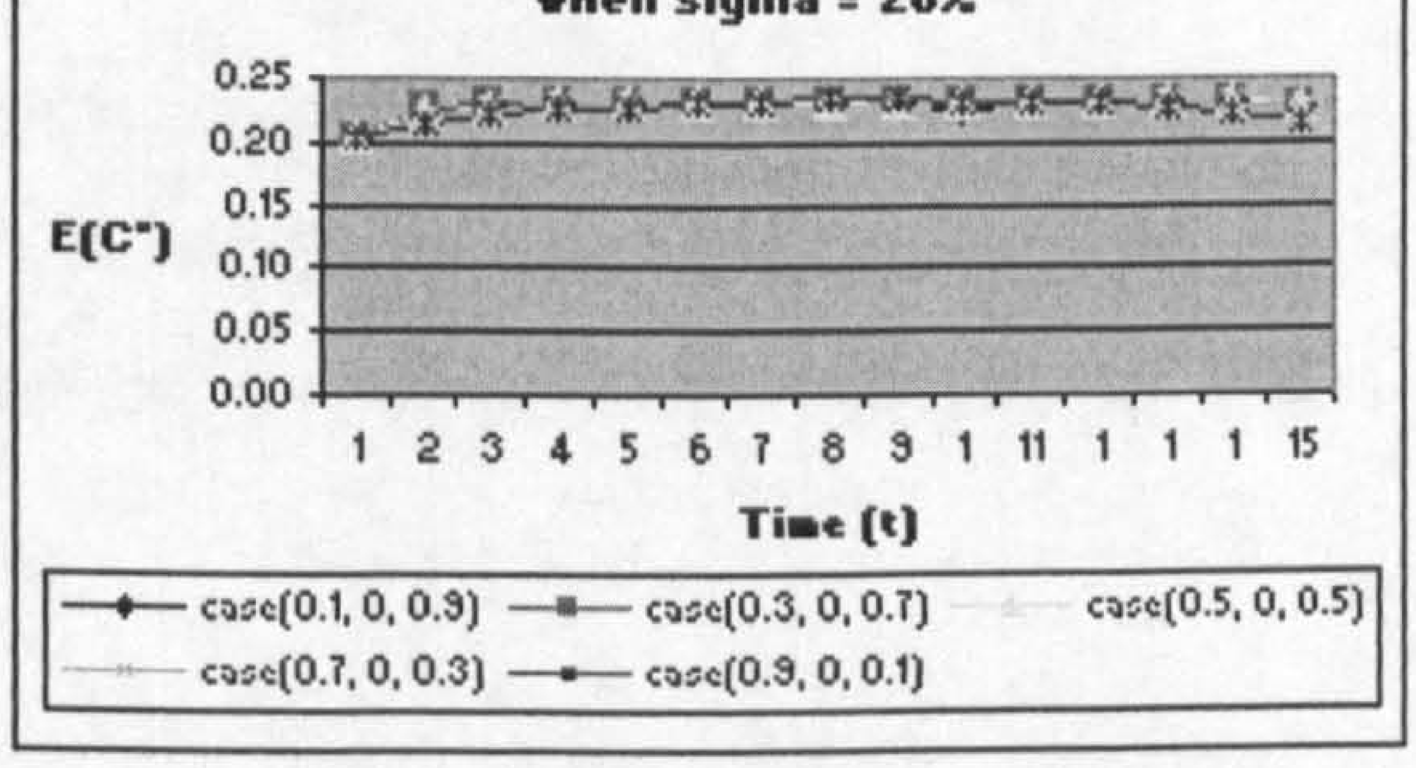


Figure 4.27
The expected optimal contribution rate under CIPI (0.8, 0.2) for different cases when sigma = 20%

Conclusion

Both parameters γ and β affect the expected optimal fund level and the contribution rate. Therefore, decreasing the weighting risk factor of the solvency risk β means giving more importance to the contribution rate risk. This results in moving $E(F^*_t)$ away from its target. While increasing the value of γ leads the expected optimal contribution rate to move towards its target. Small differences are observed among the cases with changing the parameters γ and β, so that, it was more reasonable to comment on the trends of $E(F_t^*)$ and $E(C_t^*)$ in the five cases simultaneously.

Increasing the value of either CT_t or FT_t (in the second and fourth scenarios) keeps the expected fund levels close to its target with a tendency to increase by the end of the control period. Meanwhile, decreasing both targets in the third and fifth scenarios results in having the fund levels close to its target but with a tendency to decrease by the end of the control period. Under both scenarios, a higher value of β keeps the level of the expected fund close to the target.

For the expected optimal contribution rate, a higher contribution target and fund target along with higher levels of γ lead $E(C_t^*)$ to be close to the target with an increasing trend by the end of the control period. However, a decreasing trend is observed when the fund target and contribution target are decreased to 80% and 18% respectively. Generally, a lower level of γ results in moving $E(C_t^*)$ away from the target.

4.4.6 Effect of changing the weighting factors of the contribution rate risk and the solvency risk; γ and β (with $\alpha \neq 0$)

So far, the results have implied how the different values of the parameters considering the solvency risk and the contribution rate risk affect the expected optimal fund level and the contribution rate. These results are consistent with the models derived by Haberman and Sung (1994).

Having different values of the weighting risk factor of the cross-product term deals with the effect of combining both the contribution rate risk and the solvency risk. This actually reflects the effect of the mutual interests of the employers and the employees on the expected optimal fund level and contribution rate. In this subsection, we explain this effect by assuming ascending values of the mixed term weighting risk parameter α, where it is equal to 0.2, 0.4, 0.6, 0.8. These values are applied to the different scenarios and for all cases with different levels of volatility. The following analysis considers some of these results as examples to describe the effect on both the expected fund level and the contribution rate.

(a) First scenario $FT_t = 1$, $CT_t = 0.2$

Firstly, under the default scenario, we consider a comparison between the cases - mentioned in the previous subsection 4.4.5 - when $\alpha = 0$ and the corresponding cases when $\alpha \neq 0$, in order to test the effect of the three weighting risk factors on $E(F_t^*)$ and $E(C_t^*)$. In Tables 4.18 and 4.19, we compare the expected optimal fund level and contribution rate for different cases when γ and β change with a specific non-zero value of α, i.e. $\alpha = 0.2$.

Table 4.18

The expected optimal fund level under CIPI (1, 0.2)

for different cases when $\alpha = 0.2$ and $\sigma = 5\%$

t	(0.1,0.2,0.9)	(0.3,0.2,0.7)	(0.5,0.2,0.5)	(0.7,0.2,0.3)	(0.9,0.2,0.1)
1	100.00%	100.00%	100.00%	100.00%	100.00%
2	99.74%	99.75%	99.75%	99.76%	99.75%
3	99.67%	99.65%	99.62%	99.60%	99.53%
4	99.67%	99.63%	99.58%	99.52%	99.38%
5	99.69%	99.64%	99.59%	99.50%	99.28%
6	99.66%	99.62%	99.56%	99.46%	99.16%
7	99.68%	99.63%	99.56%	99.46%	99.09%
8	99.67%	99.63%	99.56%	99.45%	99.01%
9	99.70%	99.65%	99.58%	99.46%	98.94%
10	99.69%	99.64%	99.58%	99.45%	98.85%
11	99.65%	99.61%	99.54%	99.40%	98.70%
12	99.70%	99.65%	99.57%	99.41%	98.58%
13	99.64%	99.60%	99.51%	99.31%	98.32%
14	99.67%	99.61%	99.49%	99.22%	98.03%
15	99.68%	99.58%	99.41%	99.03%	97.63%

Table 4.18 shows that changing γ and β gradually with keeping $\alpha = 0.2$ leads to similar results to the ones we get when $\alpha = 0$, i.e. decreasing the value of β results in moving the expected fund level away from its target. For example: $E(F_T^*)$ reaches the minimum level = 97.6% when $\gamma = 0.9$.

Table 4.19 emphasizes that increasing the weighting risk factor of the contribution rate (γ) leads the expected contribution rates to be closer to the target CT_t over the control period. Similar results also occur when σ is increased to 20%, but then the levels of $E(F_t^*)$ and $E(C_t^*)$ depart from their targets compared to the ones when σ = 5%, as explained in the previous sections, i.e. the minimum level of $E(F_t^*)$ = 90.9% when γ = 0.9 while $E(C_t^*)$ levels increase to approximately 22%.

Table 4.19

The expected contribution rate under CIPI (1, 0.2) for different cases when α = 0.2 and σ = 5%

t	(0.1,0.2,0.9)	(0.3,0.2,0.7)	(0.5,0.2,0.5)	(0.7,0.2,0.3)	(0.9,0.2,0.1)
1	20.66%	20.67%	20.68%	20.68%	20.67%
2	20.89%	20.86%	20.83%	20.80%	20.75%
3	20.95%	20.93%	20.91%	20.88%	20.82%
4	20.95%	20.95%	20.94%	20.92%	20.87%
5	20.93%	20.93%	20.94%	20.93%	20.89%
6	20.95%	20.96%	20.96%	20.95%	20.91%
7	20.94%	20.95%	20.95%	20.95%	20.92%
8	20.94%	20.95%	20.95%	20.96%	20.93%
9	20.92%	20.93%	20.94%	20.94%	20.92%
10	20.93%	20.94%	20.94%	20.94%	20.90%
11	20.96%	20.96%	20.96%	20.95%	20.89%
12	20.92%	20.93%	20.93%	20.91%	20.84%
13	20.97%	20.96%	20.94%	20.89%	20.80%
14	20.94%	20.92%	20.87%	20.81%	20.71%
15	20.90%	20.81%	20.73%	20.64%	20.59%

Secondly, we examine the effect of increasing the value of α on the first, third and fifth cases - which represent the two extreme cases and the middle one for the values of γ and β – in order to understand the effect of the weighting risk factor of the middle term.

In the first case, where γ is equal to 0.1 and β is equal to 0.9, we find that the expected optimal fund level decreases slightly with increasing the value of α from 0 to 0.8 when σ is equal to 5% as shown in Figure 4.28.

Figure 4.28

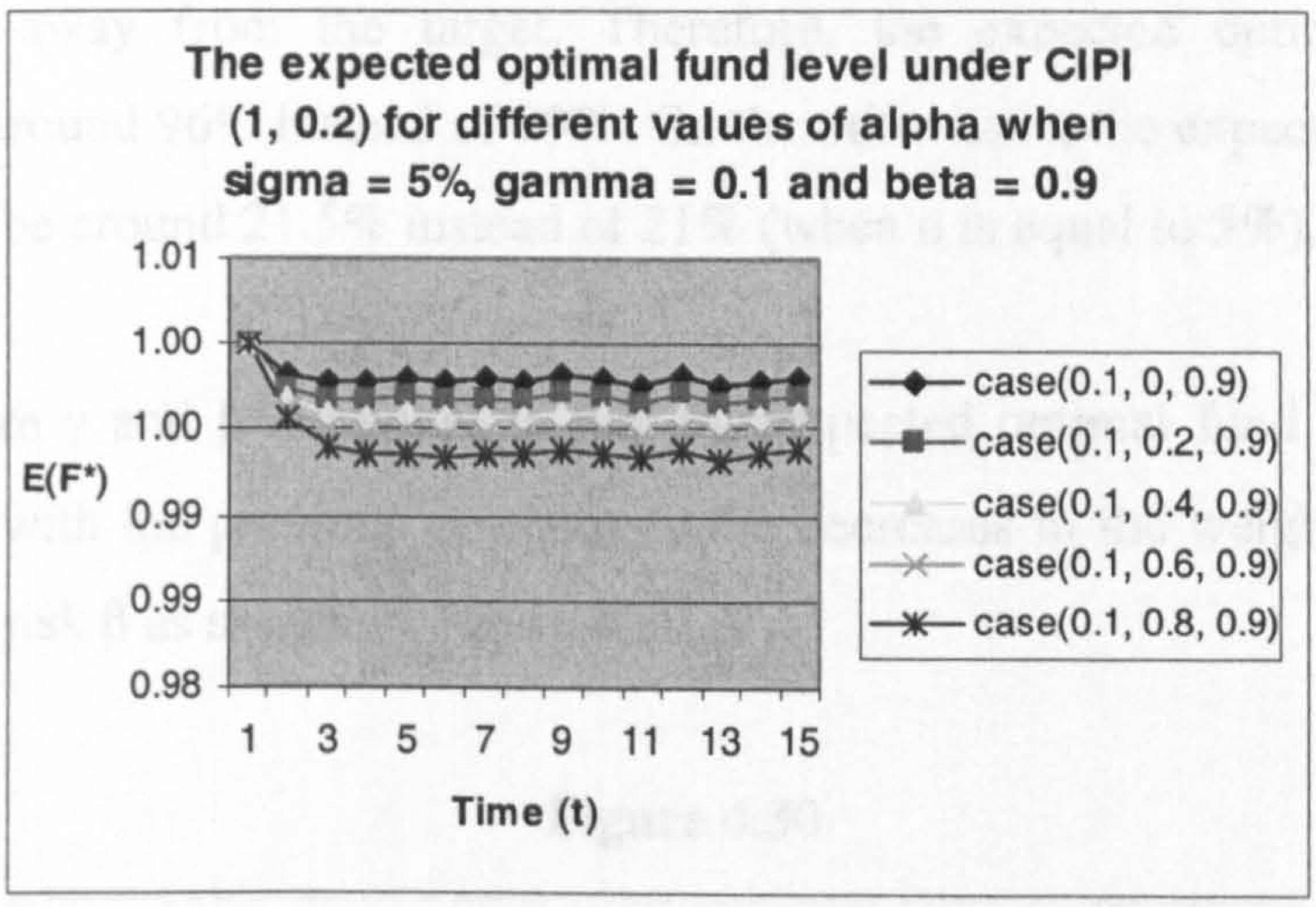
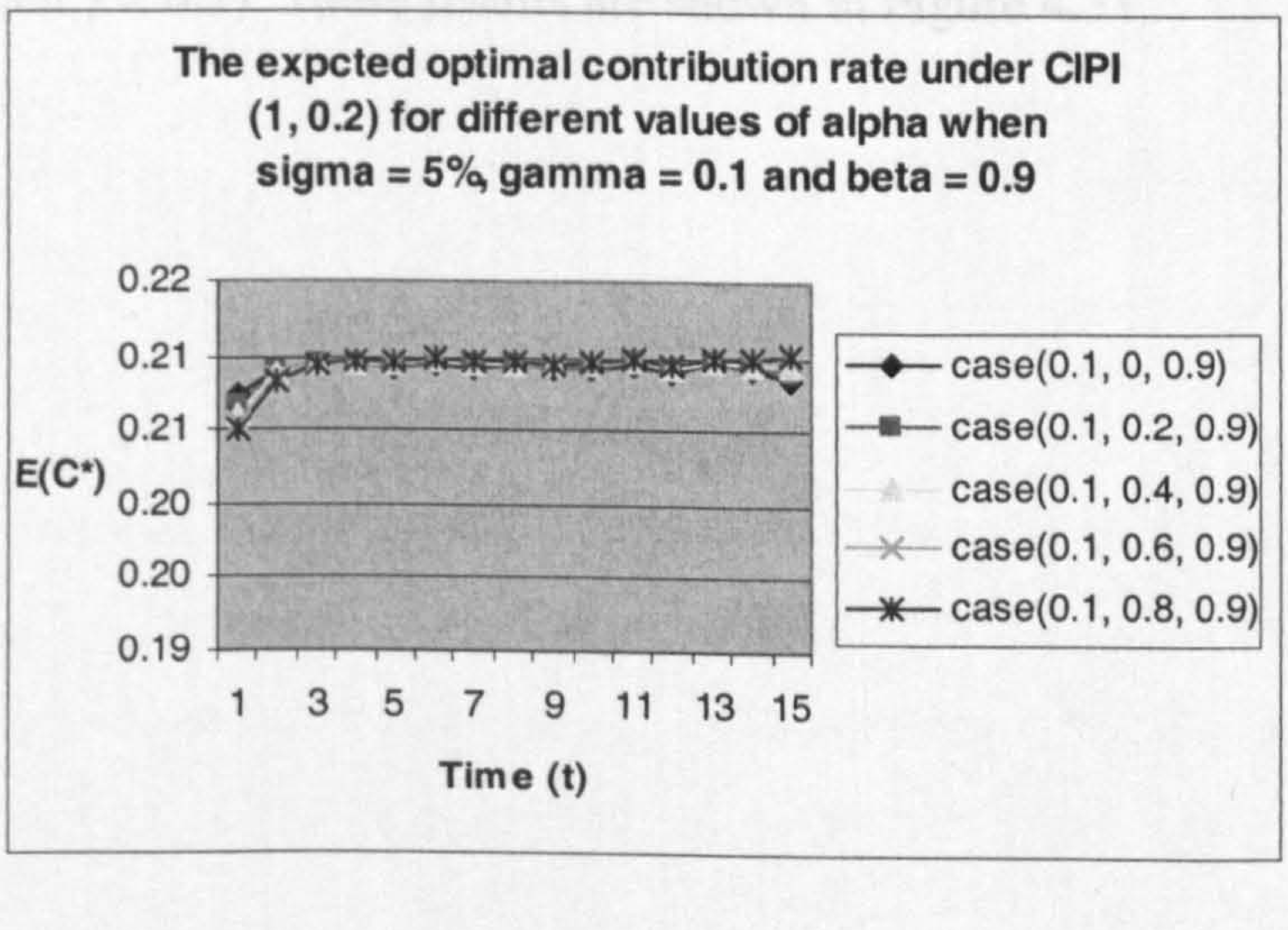


Figure 4.28 shows that increasing the weighting risk factor of the middle term results in moving the expected optimal fund level further away from its target. In fact, this refers mathematically to rewarding the deviation from the target rather than penalizing it by considering the squared deviation from the target.

For the expected optimal contribution rate and the same level of volatility, when $\beta = 0.9$, $E(C_t^*)$ increases over the control period and moves away from the target as α is increased. Here, a similar interpretation of rewarding the deviation from the contribution target through the cross-product term applies. The results are shown below in Figure 4.29.

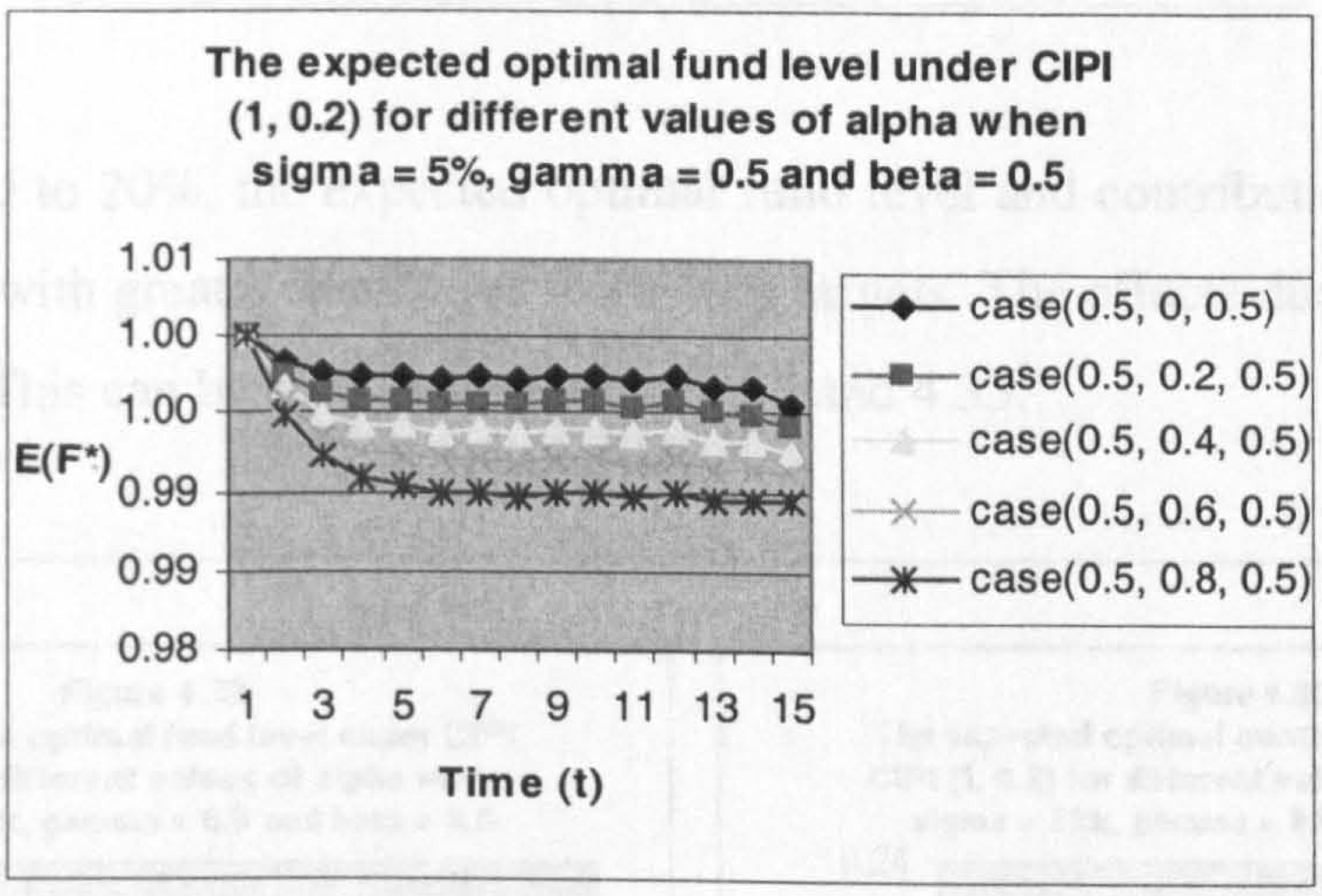
Figure 4.29



Increasing the level of volatility when σ is equal to 20% leads to the same results with moving further away from the target. Therefore, the expected optimal fund level decreases to be around 96% instead of 99%. On the other hand, the expected contribution rate increases to be around 21.5% instead of 21% (when σ is equal to 5%).

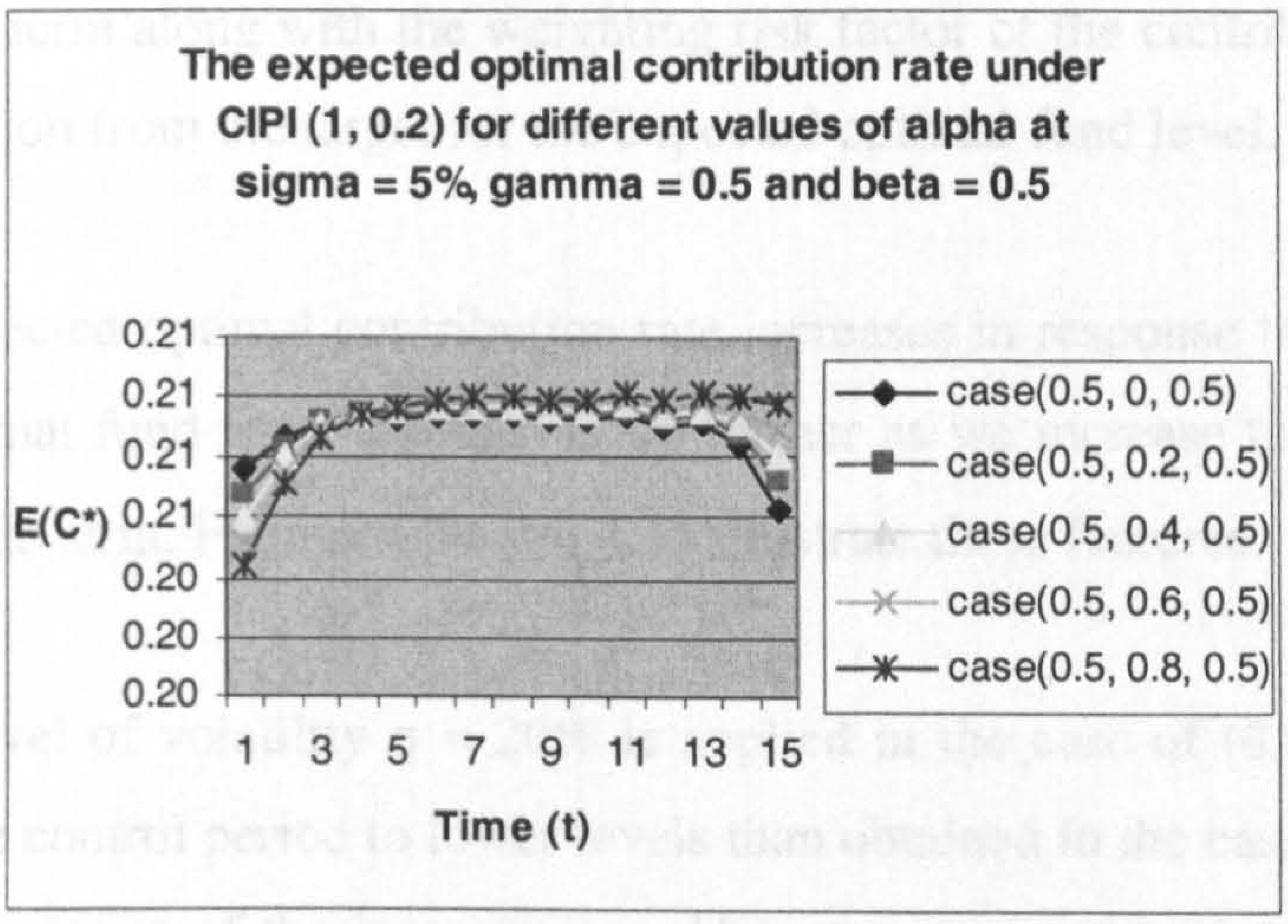
In the case where γ and β are equal to 0.5, the expected optimal fund levels decrease more compared with the previous case due to the decreases in the weighting risk factor for the solvency risk β as shown in Figure 4.30.

Figure 4.30

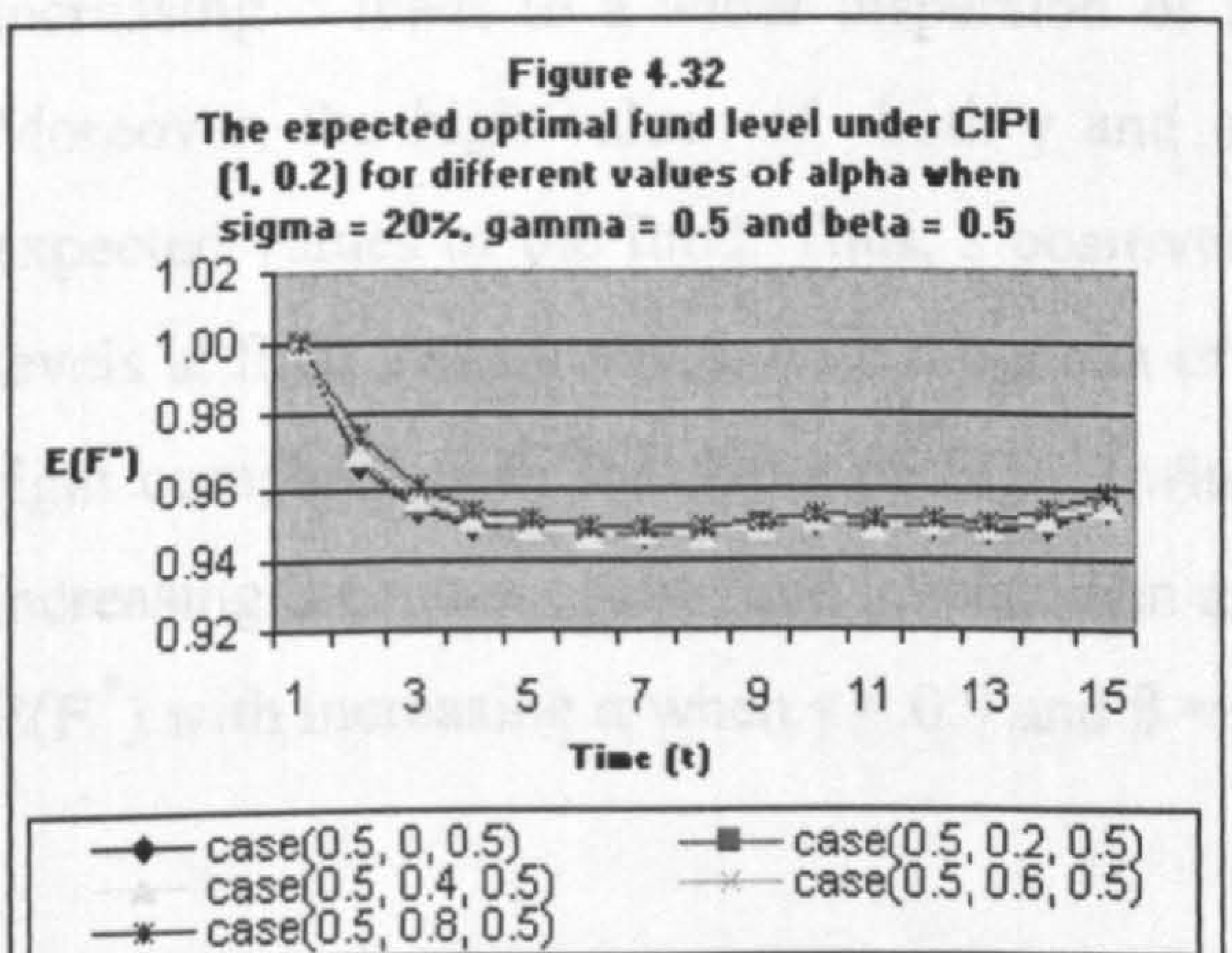


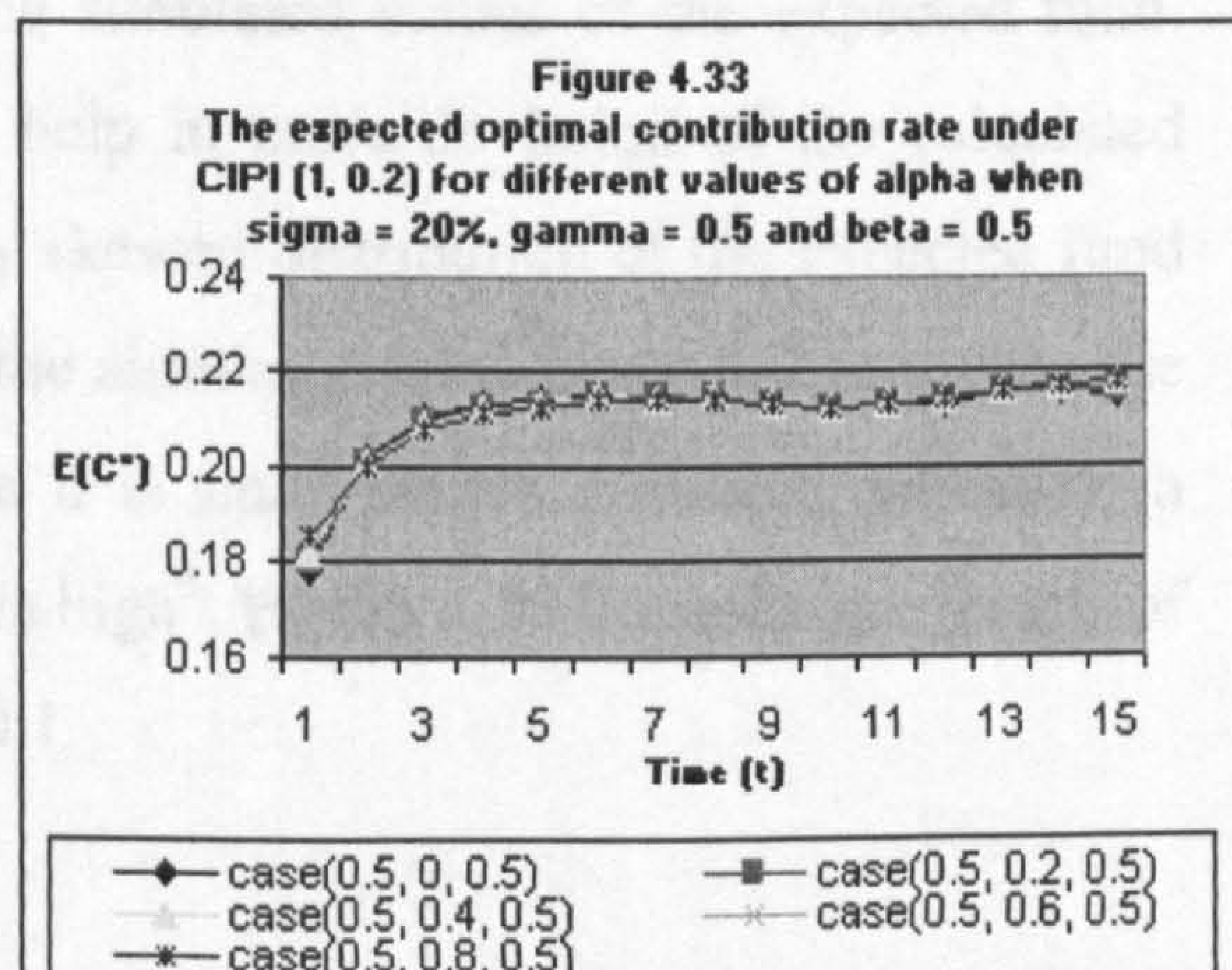
On the other hand, the expected optimal contribution rate increases slightly and moves closer to the target due to the increase in the weighting risk factor γ to 0.5(in comparison with the case when $\gamma = 0.1$). These results are shown in Figure 4.31.

Figure 4.31



As σ is increased to 20%, the expected optimal fund level and contribution rate have the same trends but with greater departures from their targets. The effects due to increasing α are quite small. This can be seen from Figures 4.32 and 4.33.





In the last case, we consider the case where $\gamma = 0.9$ and $\beta = 0.1$. When $\sigma = 5\%$, the expected optimal fund level decreases to the lowest level compared with the two previous cases as α increases to 0.8. For example, in this case (0.9, 0.8, 0.1), $E(F_T^*)$ is equal to 95.1% whereas it is equal to 99.4% and 98.95% in the cases of (0.1, 0.8, 0.9) and

(0.5, 0.8, 0.5) respectively. This is consistent with the fact that giving more importance to the middle mixed term along with the weighting risk factor of the contribution rate result in a greater deviation from the target for the expected optimal fund level.

However, the expected optimal contribution rate increases in response to the decrease of the expected optimal fund level and moves up further as we increase the weighting risk factor of the middle term. Figures 4.34 and 4.35 illustrate these features.

When a higher level of volatility $\sigma = 20\%$ is applied in the case of (0.9, 0, 0.1), $E(F_t^*)$ decreases over the control period to lower levels than obtained in the cases of (0.5, 0, 0.5) and (0.1, 0, 0.9) because of the increase in γ. However, increasing the value of α has a different effect on the results of $E(F_t^*)$ and $E(C_t^*)$. Thus, $E(F_t^*)$ starts to increase gradually when the value of α is increased.

In trying to interpret this change in the trends especially when $\alpha = 0.6$ and 0.8, we have examined the distribution of $E(F_T^*)$ at the end of the control period. We find that increasing σ leads to a wider dispersion of the simulated values of the expected fund. Moreover, the high values of both γ and α help in more deviation of the calculated expected values of the fund. Thus, a positively skewed distribution of the expected fund levels at final T is obtained, with the mean of the simulated fund levels lies further to the right compared with the mean of $E(F_T^*)$ when α is small, hence, resulting generally in increasing the mean of the fund levels when α is high[3]. Figure 4.36 illustrate the results of $E(F_t^*)$ with increasing α when $\gamma = 0.9$ and $\beta = 0.1$.

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[3]Appendix 4C shows the histograms and the summary statistics of $E(F_T^*)$ in the cases of (0.9, 0, 0.1) and (0.9, 0.8, 0.1)

Figure 4.34
The expected optimal fund level under CIPI (1, 0.2) for different values of alpha when sigma = 5%, gamma = 0.9 and beta = 0.1

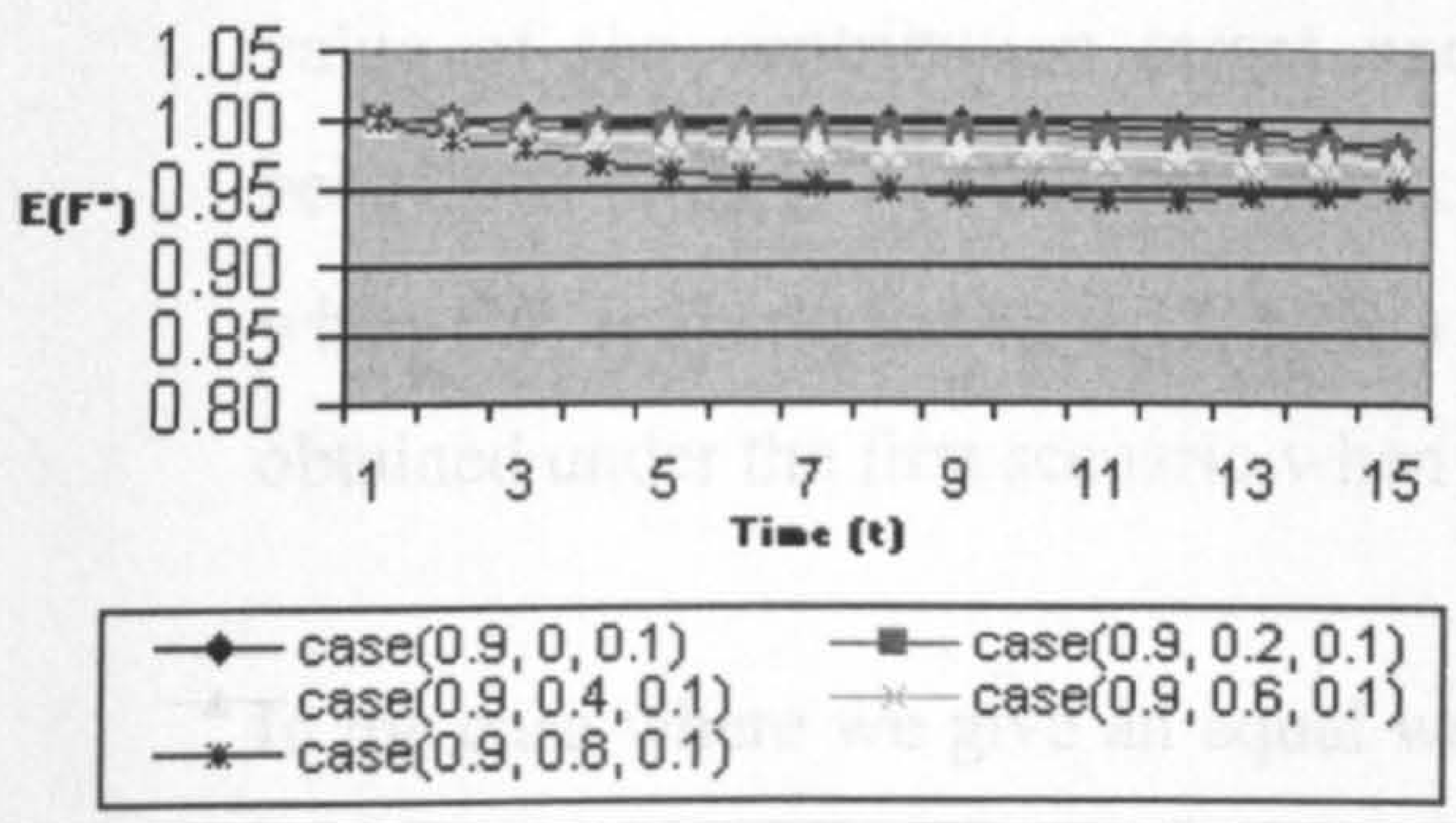


Figure 4.35
The expected optimal contribution rate under CIPI (1, 0.2) for different values of alpha when sigma = 5%, gamma = 0.9 and beta = 0.1

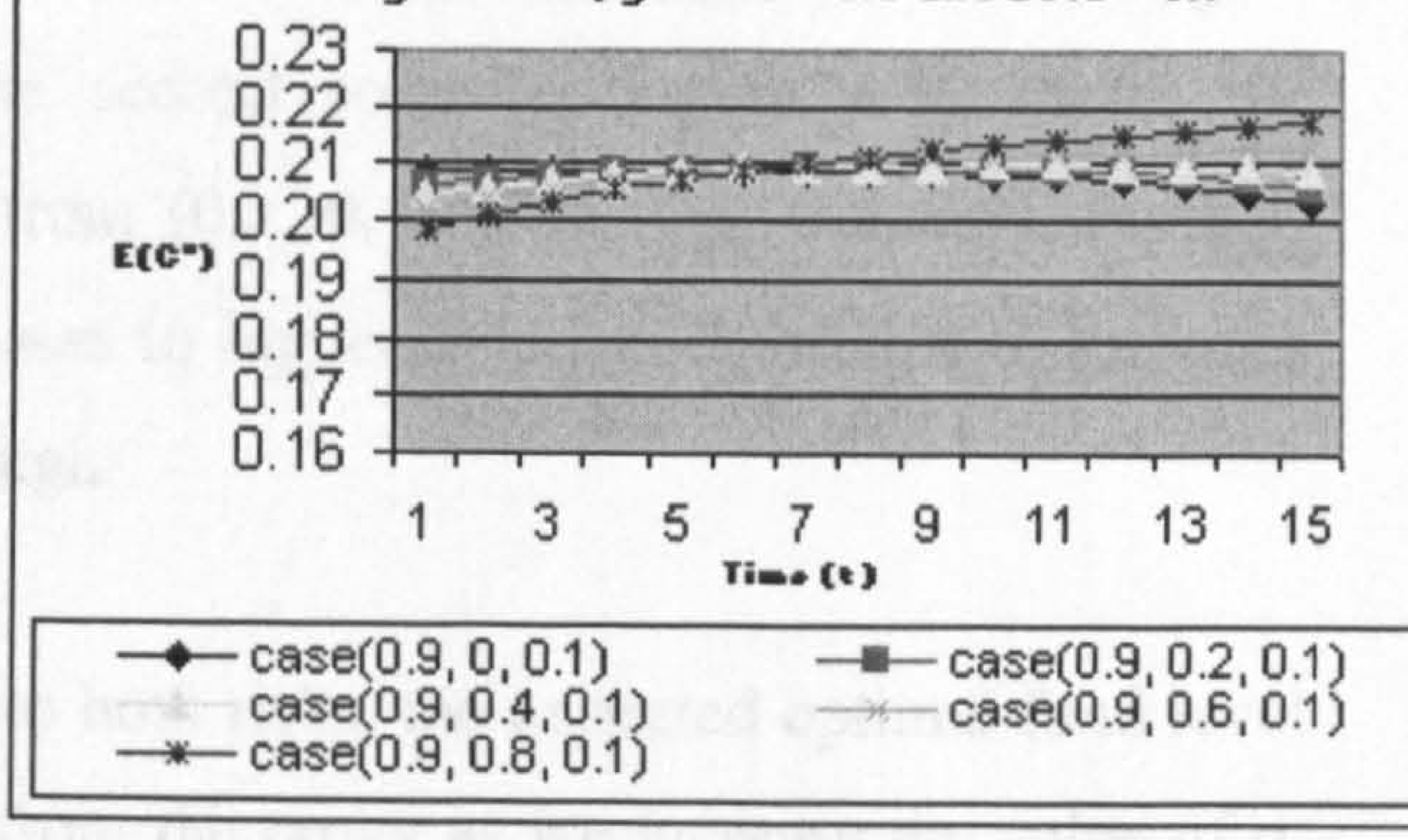


Figure 4.36
The expected optimal fund level under CIPI (1, 0.2) for different values of alpha when sigma = 20%,gamma = 0.9 and beta = 0.1

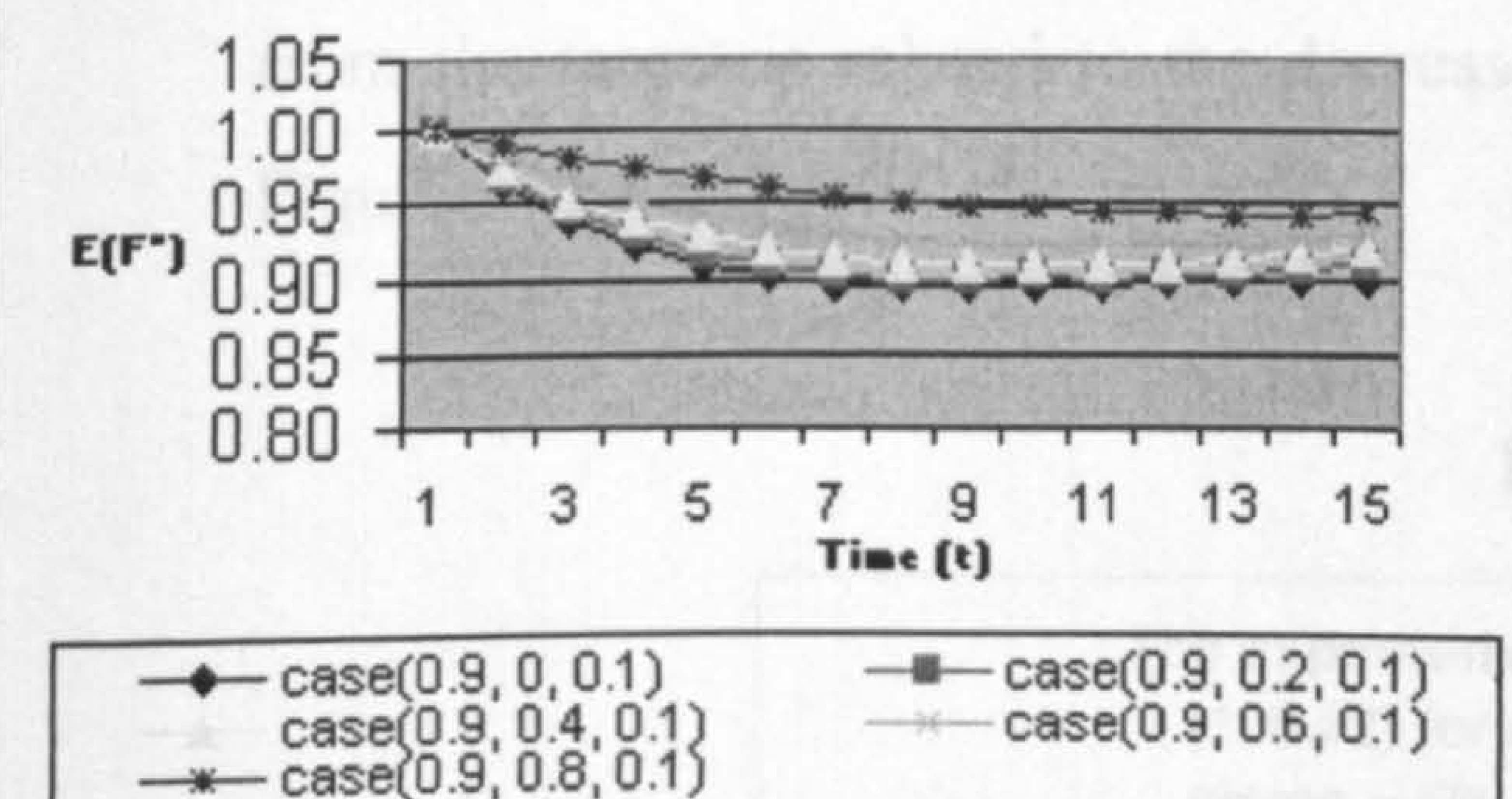
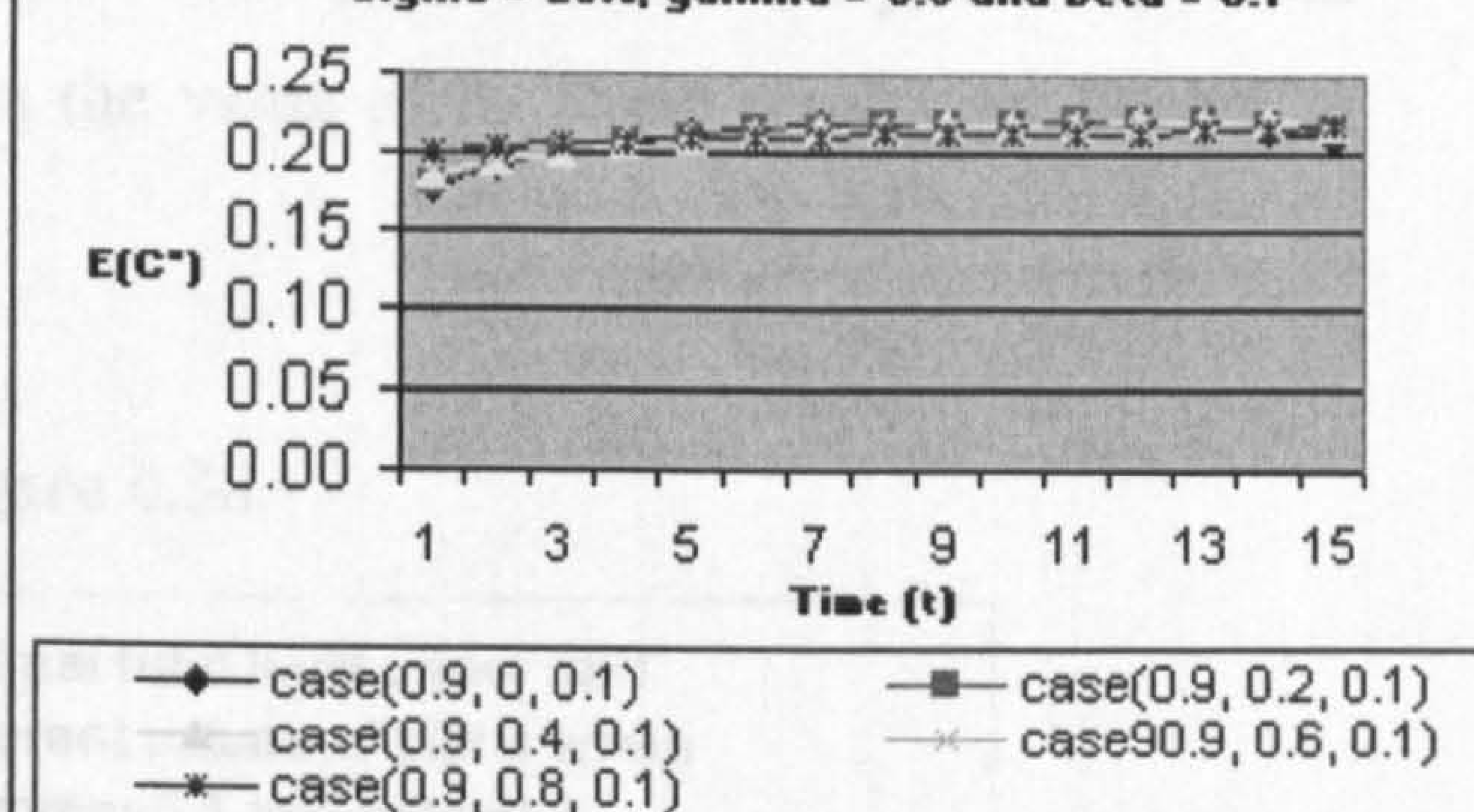


Figure 4.37
The expected optimal contribution rate under CIPI (1, 0.2) for different values of alpha when sigma = 20%, gamma = 0.9 and beta = 0.1



Consequently, this affects the trends of the optimal contribution rate which increases in response to the decrease of the fund level, however, when α is increased to 0.6 and 0.8, the expected optimal contribution rate moves closer to the contribution target. Figure 4.37 shows the results of the expected optimal contribution rate in the extreme case where γ is equal to 0.9, β is equal to 0.1 and α is increasing to 0.8.

(b) Second scenario: $FT_t = 1$ and $CT_t = 0.22$

The same three cases - that were studied under the first scenario – will be used for the analysis under the second scenario.

When σ = 5%, in the first case where γ is equal to 0.1 and β is equal to 0.9, we find that the expected optimal fund levels are around the target 100% over the control period. The differences among the cases are very small when α is increased. However, $E(F_t^*)$ tends to increase around the target when α is increased to 0.6 and 0.8. This is due to the higher value of the contribution target under the second scenario. Figure 4.38 shows the behaviour of $E(F_t^*)$ in the different cases from (0.1, 0, 0.9) to (0.1, 0.8, 0.9). Further, when CT_t is increased to 0.22, $E(F_t^*)$ increases to higher levels compared with the ones obtained under the first scenario when α is high.

In the case where we give an equal weight to both risks, the expected optimal fund level increases and indicates a greater departure from the target as we increase the value of α, compared with the cases when β = 0.9. In addition, $E(F_t^*)$ moves further up and reaches the furthest departure from the target when β is decreased to 0.1. Clearly, this deviation from the target is related to the decrease in the value of β. These results are shown in Figures 4.39 and 4.40.

Figure 4.38

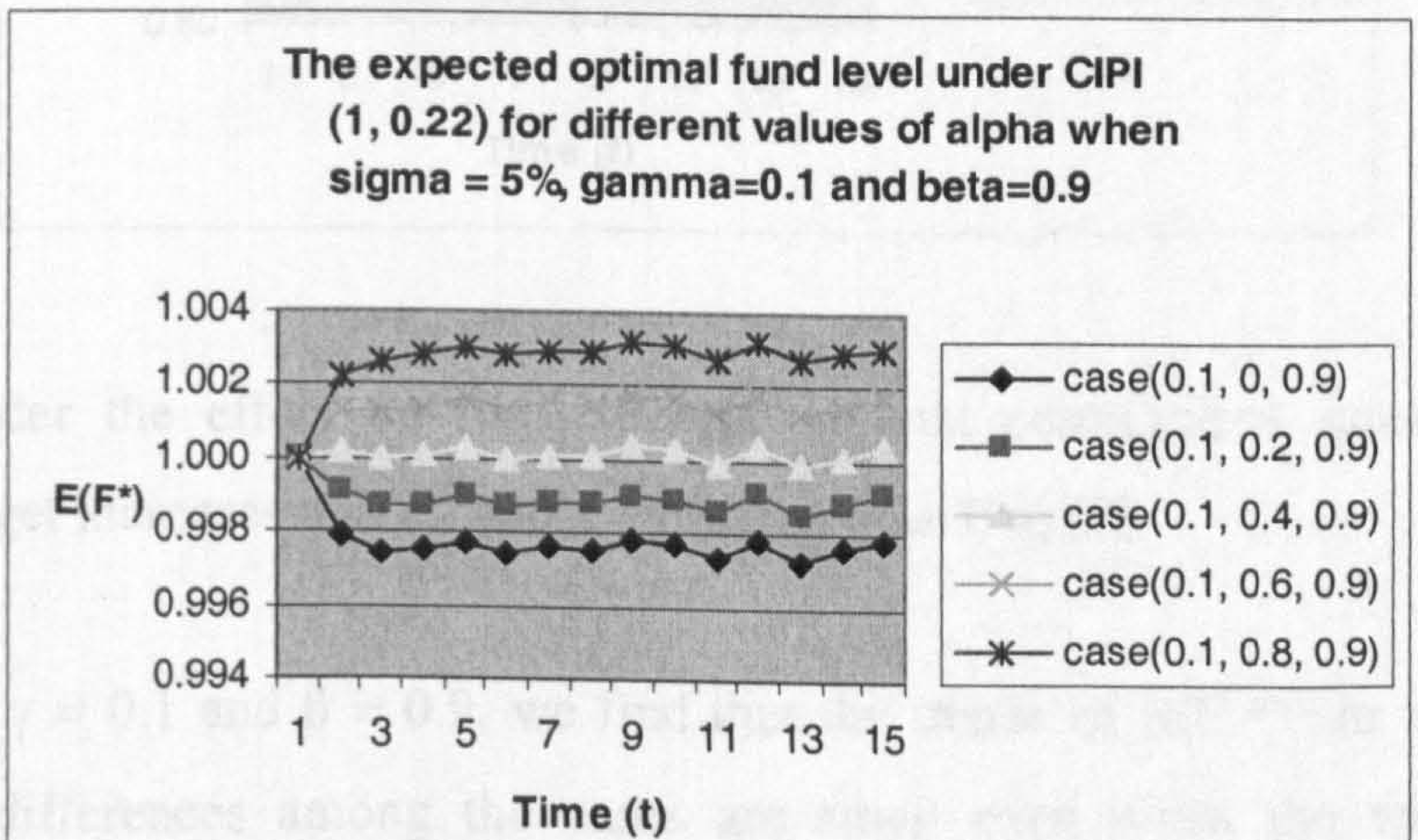


Figure 4.39

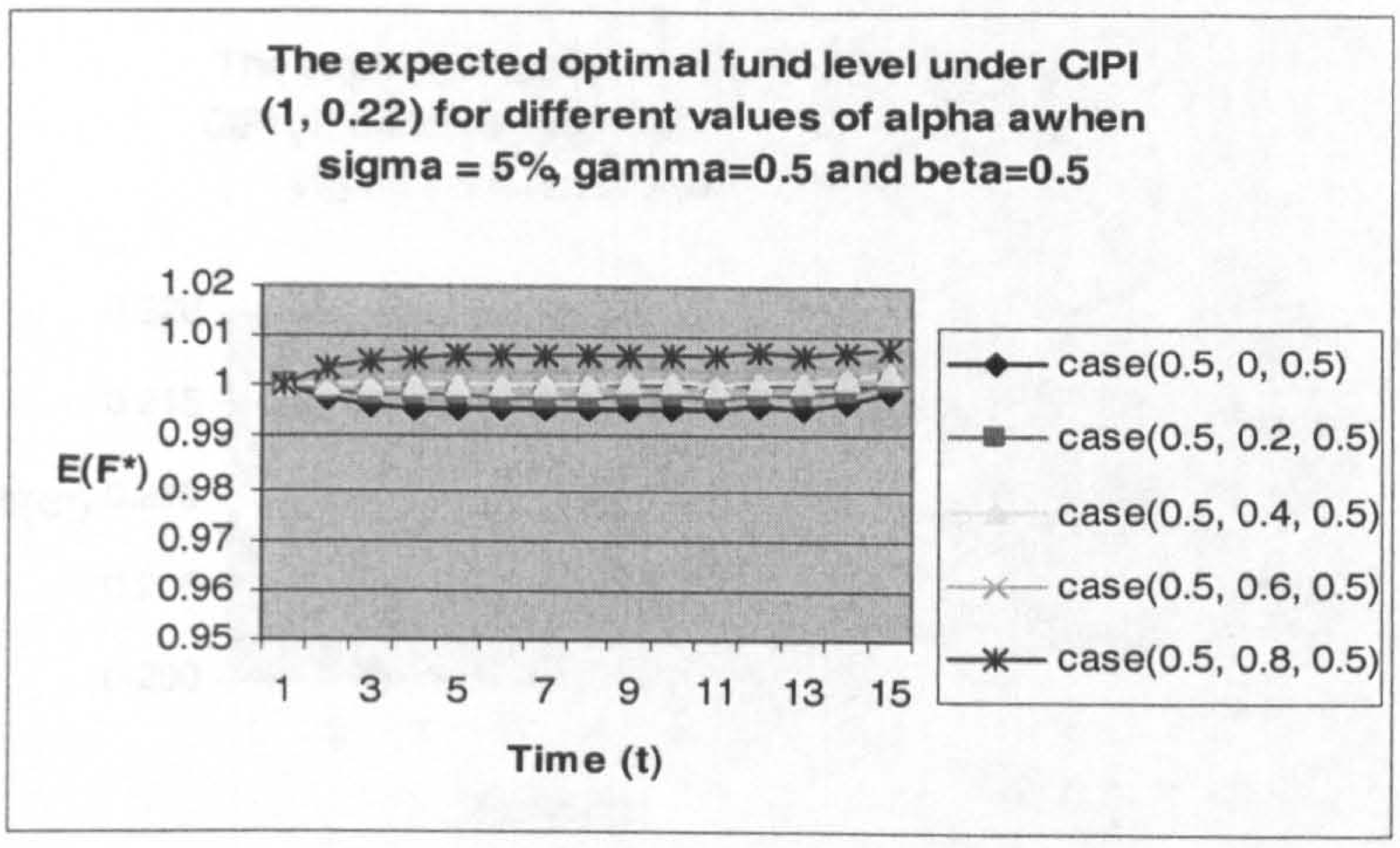
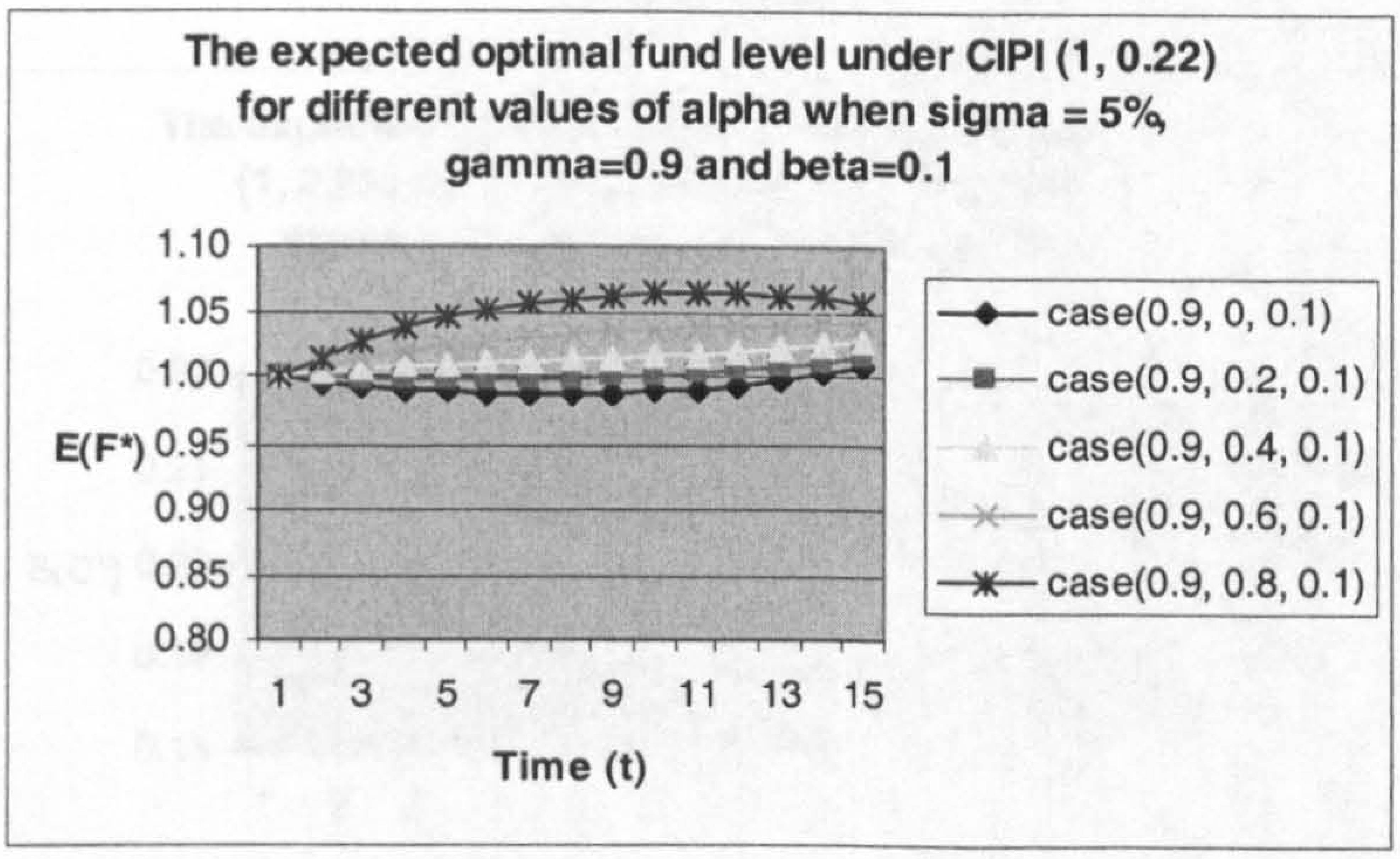


Figure 4.40



We now consider the effect on the expected optimal contribution rates, when the contribution target increases to 0.22 and α increases from 0 to 0.8.

When $\sigma = 5\%$, $\gamma = 0.1$ and $\beta = 0.9$, we find that the trends of $E(C_t^*)$ are around 21%. Generally the differences among the cases are small even when the value of α is increased to 0.8. Although $E(C_t^*)$ increases over the control period to get close to the target 22% when the value of γ is increased from 0.1 to 0.9, the trend of $E(C_t^*)$ tends to decrease over time and moves away from the target when $\alpha = 0.6$ and 0.8. This is due to the increasing trend of the expected optimal fund level. The behaviour of $E(C_t^*)$ emphasizes the fact that a higher value of α leads to a greater departure from the target as can be seen in Figures 4.41, 4.42 and 4.43.

Figure 4.41

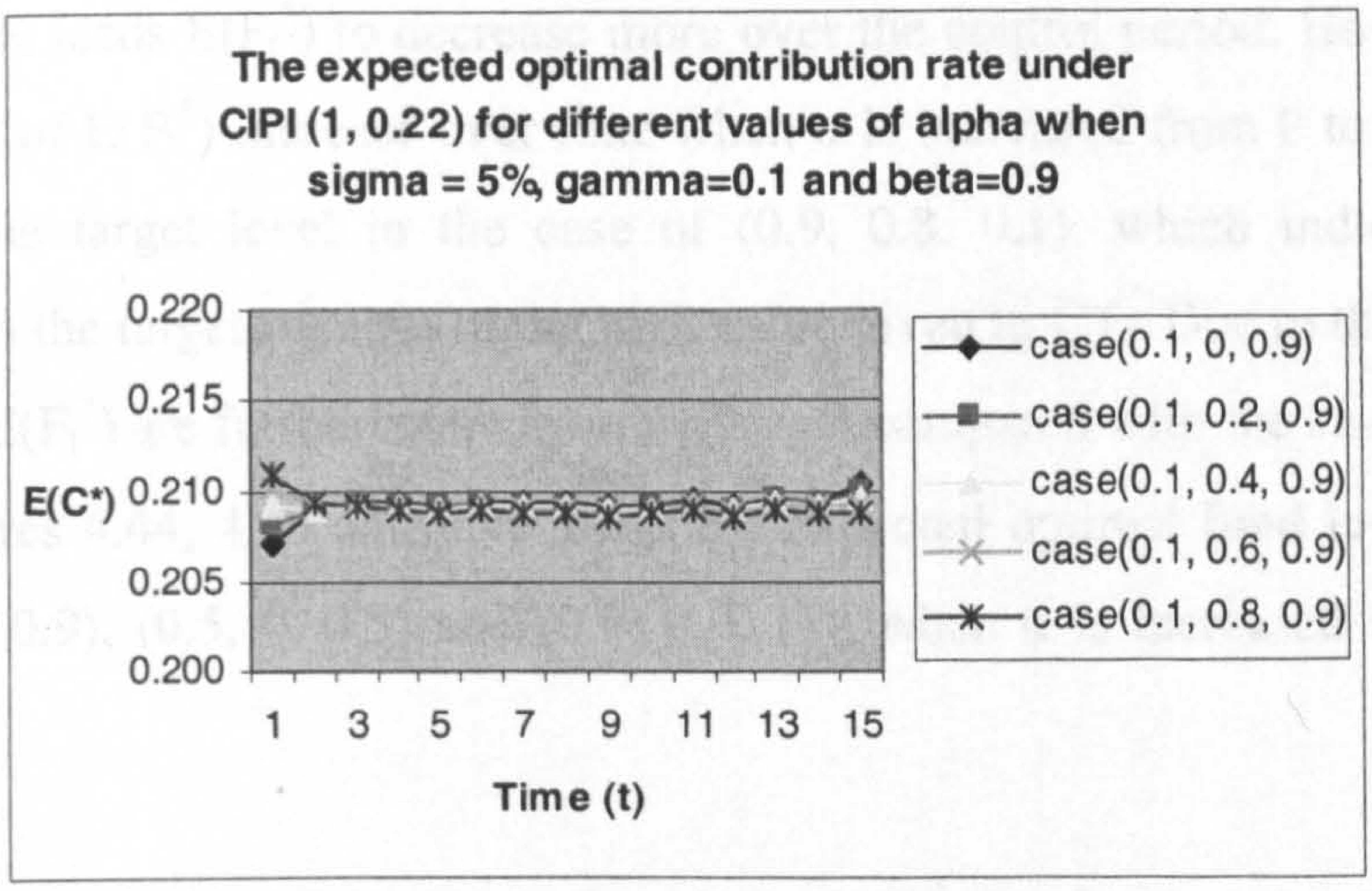


Figure 4.42

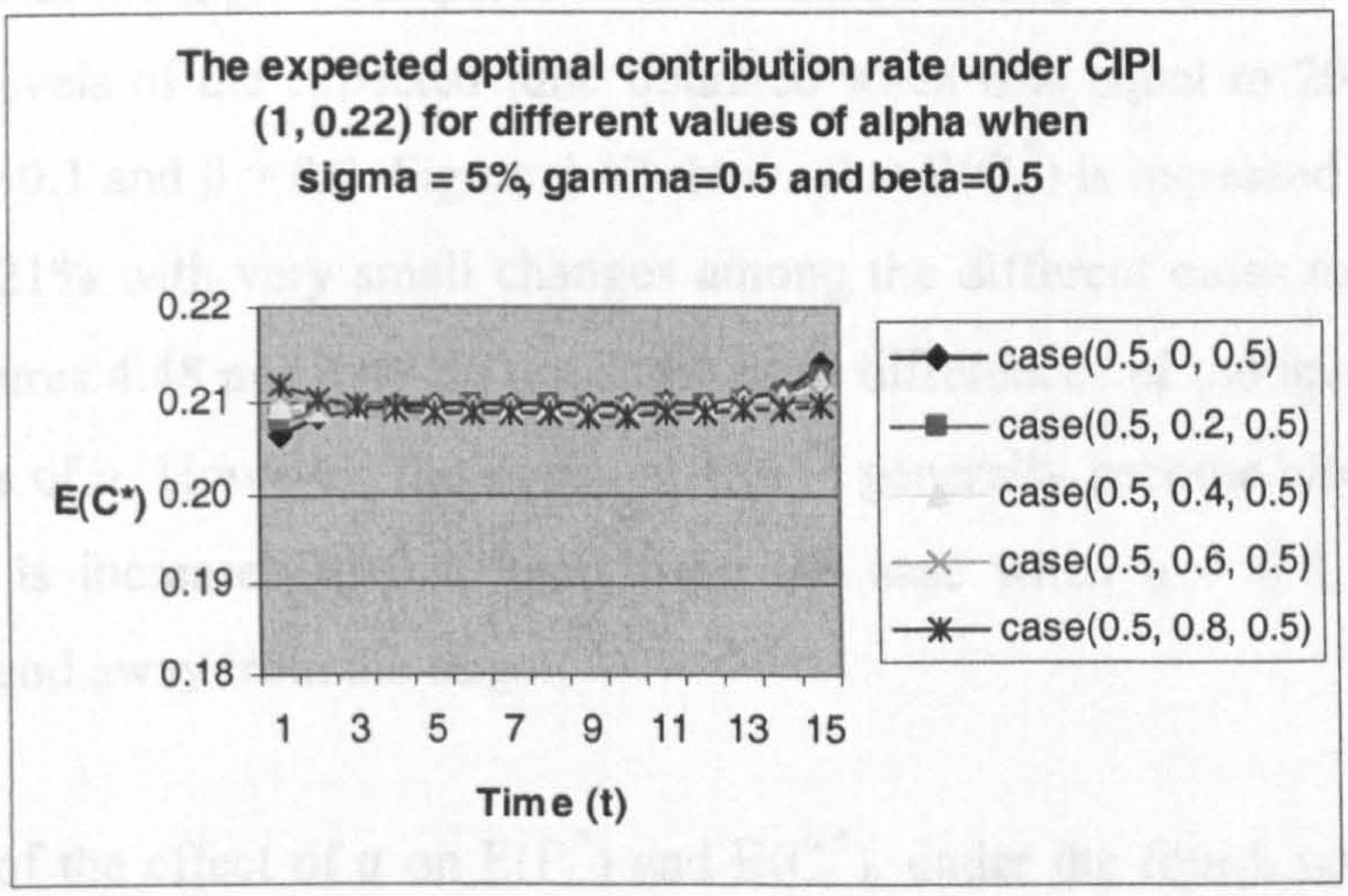
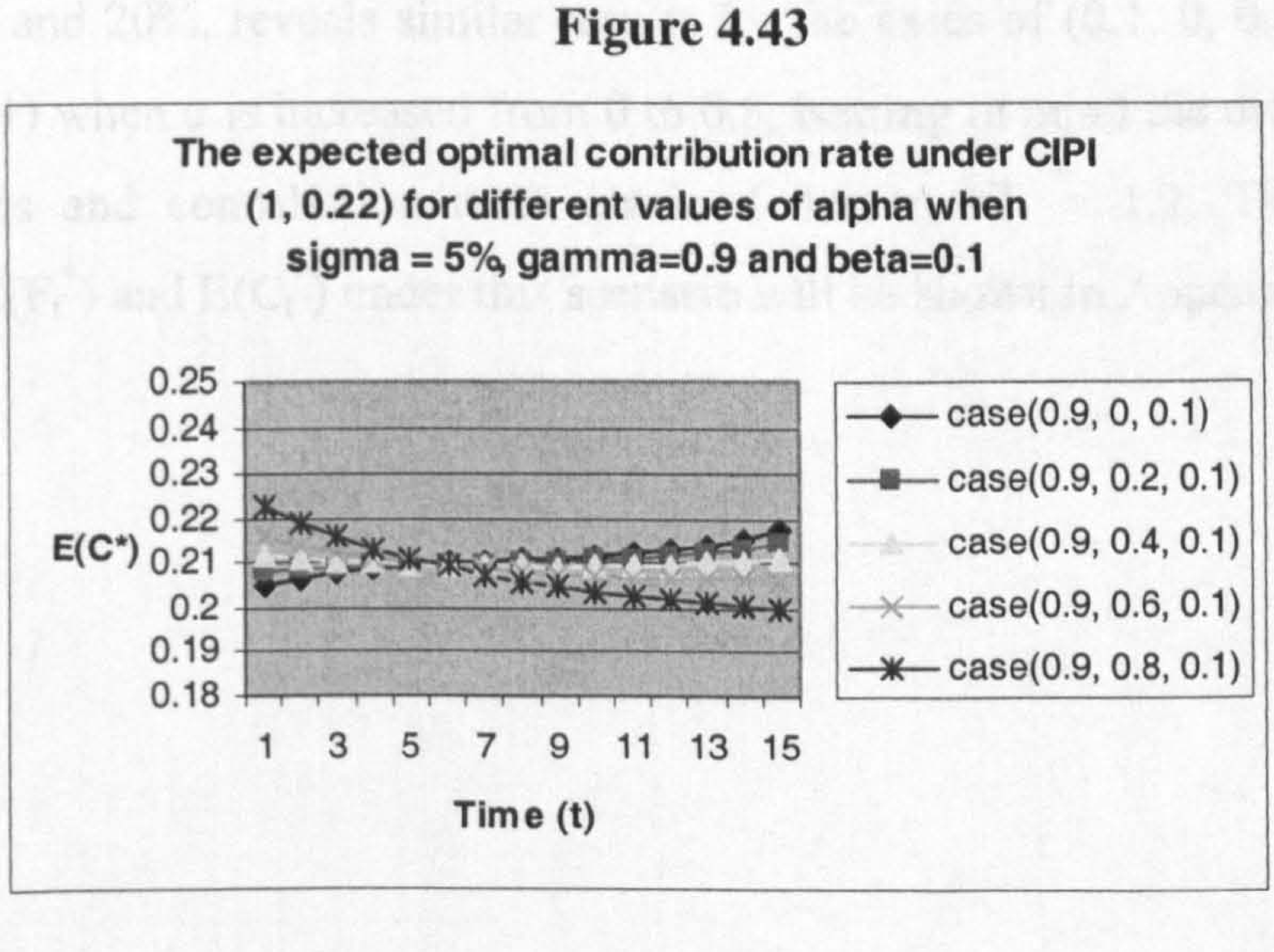


Figure 4.43



In the three cases: (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0,0.1), a higher level of volatility when $\sigma = 20$ % leads $E(F_t^*)$ to decrease more over the control period. However, we note that the levels of $E(F_t^*)$ increase over time when α is increased from 0 to 0.8. Further, it goes above the target level in the case of (0.9, 0.8, 0.1), which indicates a greater departure from the target because of the high value given to CT_t. Due to the increase of σ, the values of $E(F_t^*)$ are further away from the target compared with the cases when $\sigma = 5\%$. Figures 4.44, 4.45 and 4.46 show the expected optimal fund level in the three cases (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0, 0.1) - when α is increased from 0 to 0.8 - respectively.

For the optimal contribution rate, when $\sigma = 20\%$, the levels of expected optimal contribution rates are higher compared with the ones when σ is equal to 5%. This is due to the lower levels of the expected fund obtained when σ is equal to 20%. For the first case when $\gamma = 0.1$ and $\beta = 0.9$, Figure 4.47 shows that $E(C_t^*)$ is increased from 18%-19% to be around 21% with very small changes among the different cases as α is increased. Similarly, Figures 4.48 and 4.49 do not show large differences of the levels of $E(C_t^*)$ for different cases of α. However, the trends of $E(C_t^*)$ generally become closer to the target 22% when γ is increased to 0.9, apart from the case when $\alpha = 0.8$ which shows a downwards trend away from the target.

The analysis of the effect of α on $E(F_t^*)$ and $E(C_t^*)$, under the fourth scenario (1.2, 0.2) when $\sigma = 5\%$ and 20%, reveals similar results for the cases of (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0, 0.1) when α is increased from 0 to 0.8, bearing in mind the different levels of expected funds and contribution rates obtained due to $FT_t = 1.2$. Thus, the results obtained for $E(F_t^*)$ and $E(C_t^*)$ under this scenario will be shown in Appendix 4A.

Figure 4.44
The expected optimal fund level under CIPI (1, 0.22) for different values of alpha when sigma = 20%, gamma=0.1 and beta=0.9

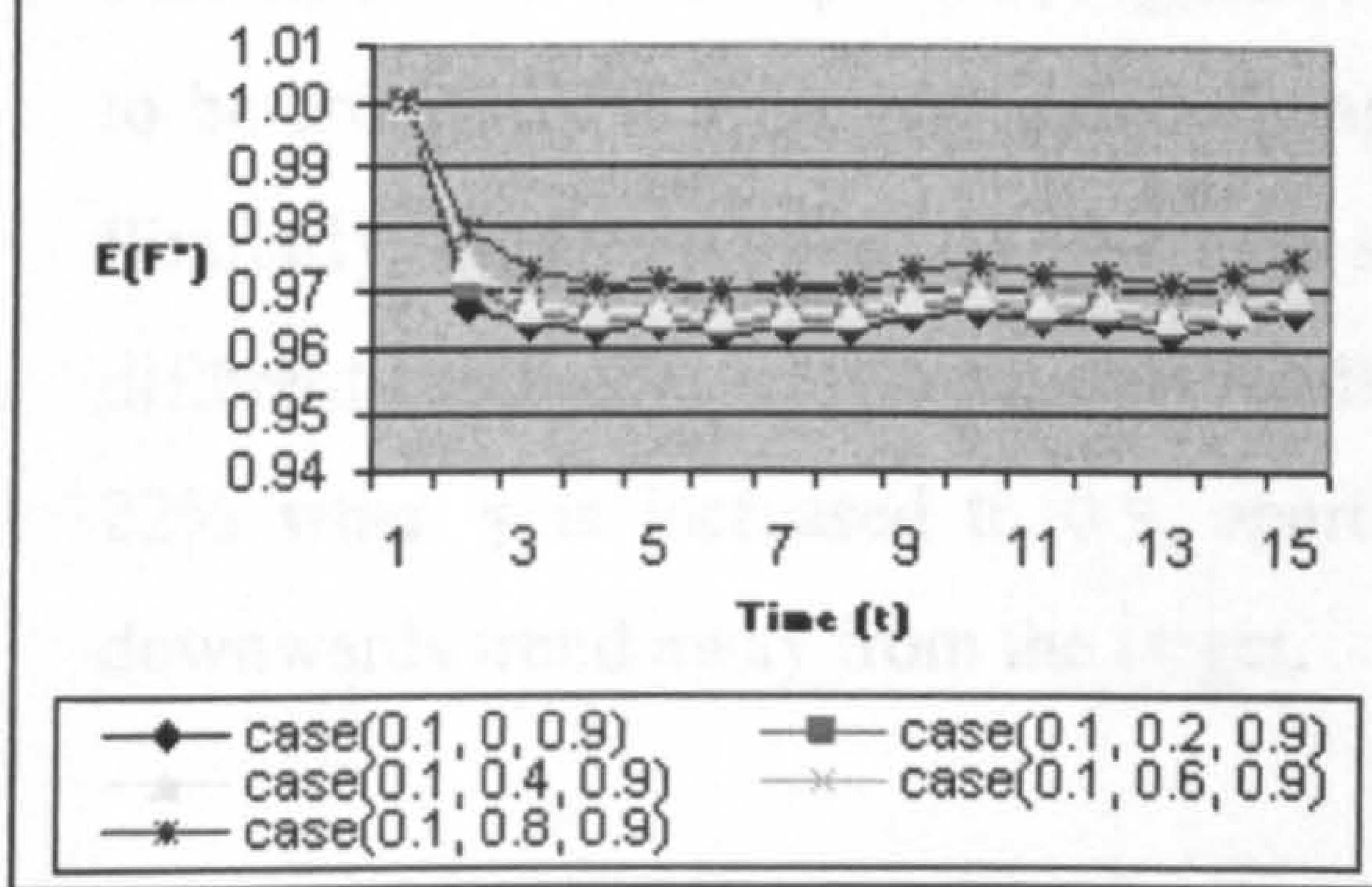


Figure 4.47
The expected optimal contribution rate under CIPI (1, 0.22) for different values of alpha when sigma = 20%, gamma=0.1 and beta=0.9

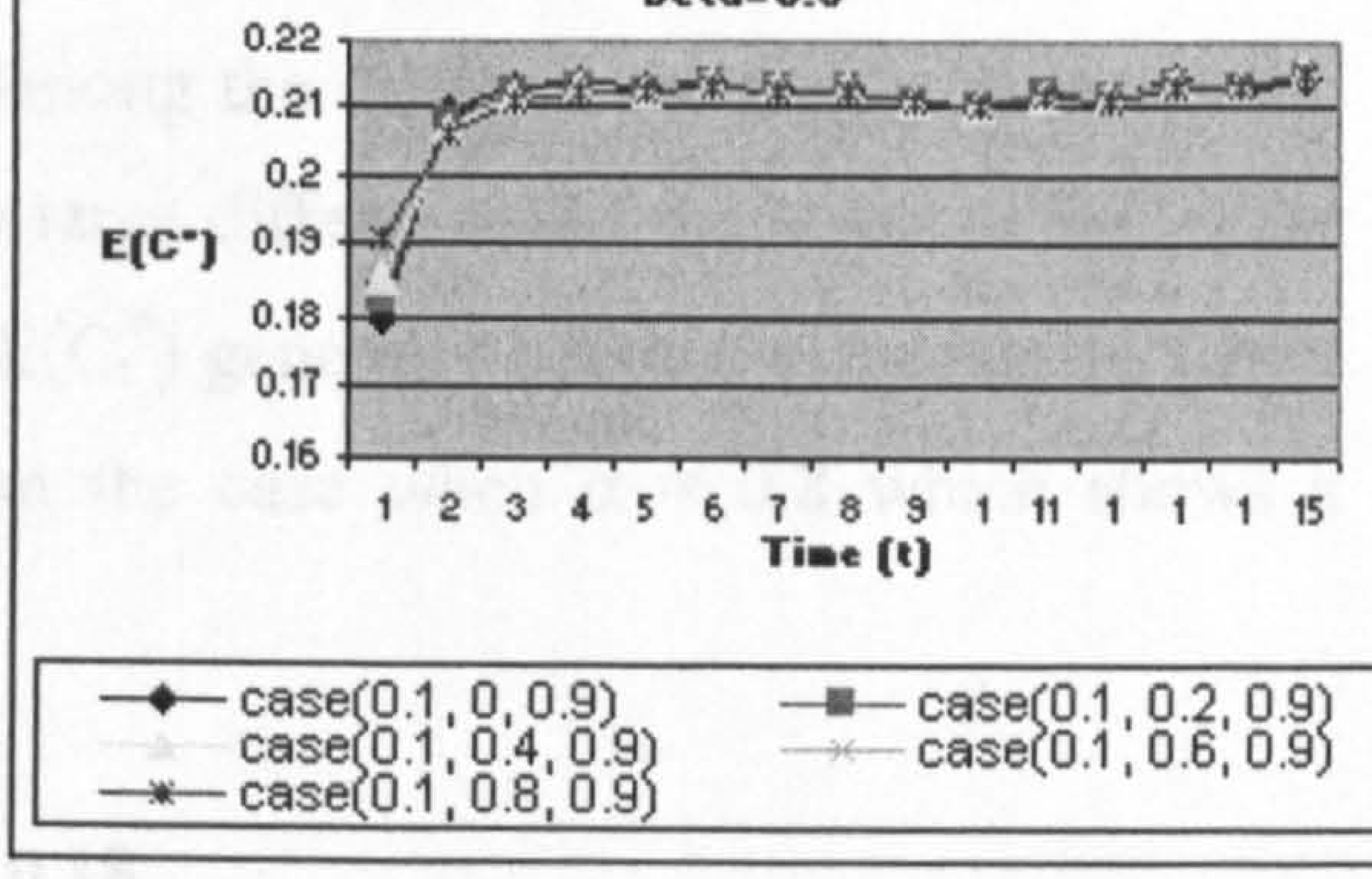


Figure4.45
The expected optimal fund level under CIPI (1, 0.22) for different values of alpha when sigma = 20%, gamma=0.5 and beta=0.5

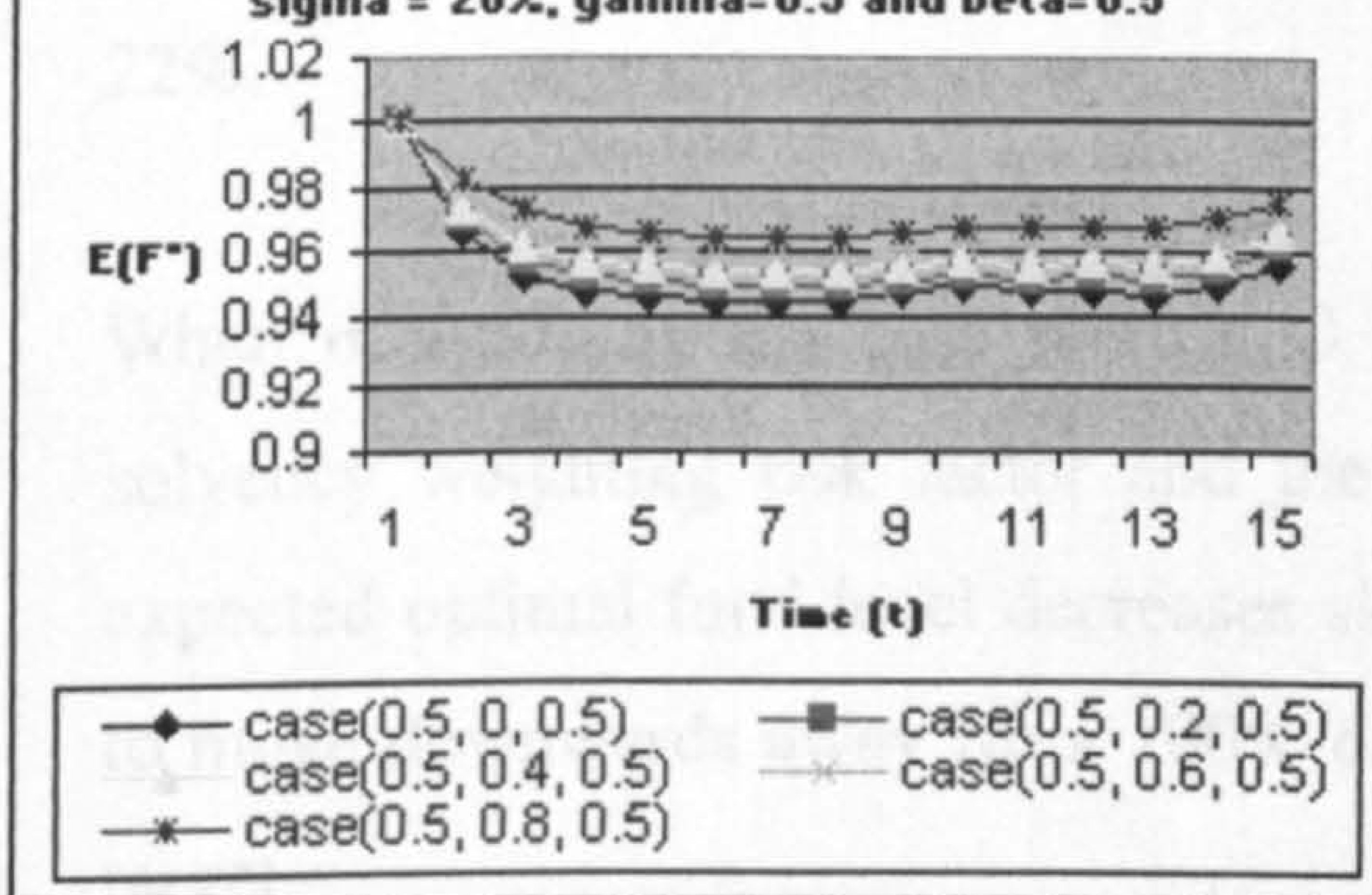


Figure 4.48
The expected optimal contribution rate under CIPI (1, 0.22) for different values of alpha when sigma = 20%, gamma=0.5 and beta=0.5

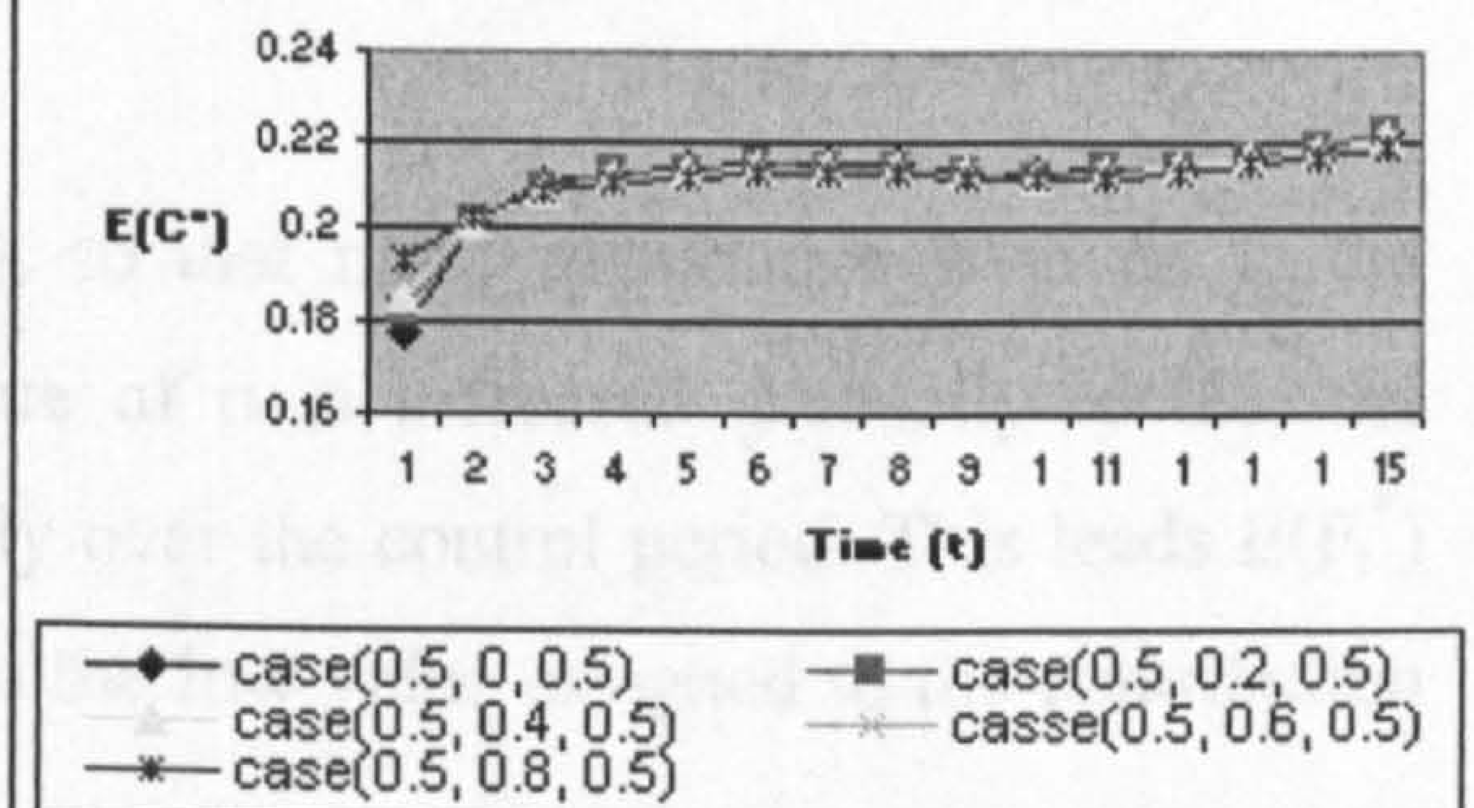


Figure 4.46
The expected optimal fund level under CIPI (1, 0.22) for different values of alpha when sigma = 20%,gamma=0.9 and beta=0.1

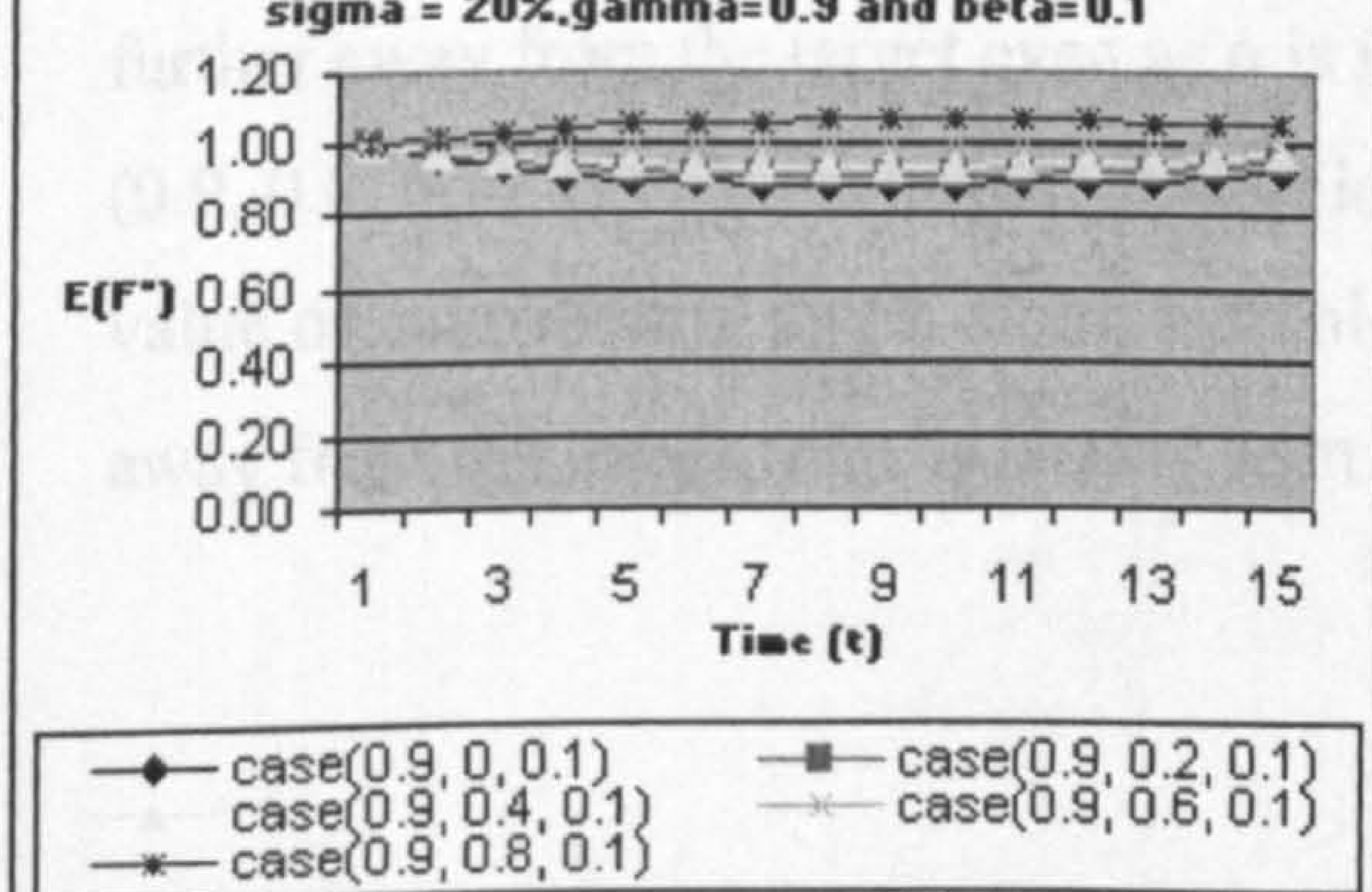
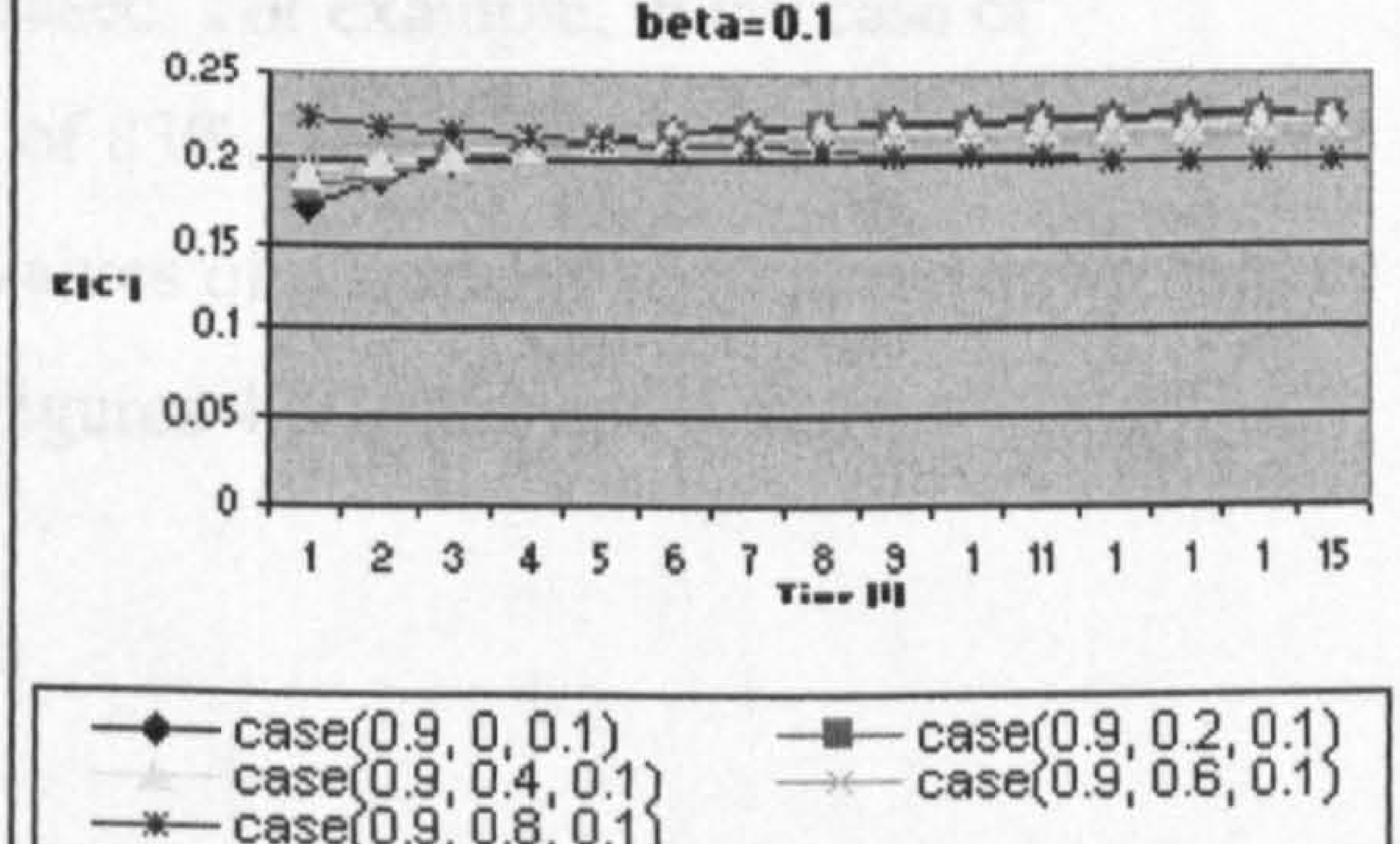


figure 4.49
The expected optimal contribution rate under CIPI (1, 0.22) for different levels of alpha when sigma = 20%,gamma=0.9 and beta=0.1



For the optimal contribution rate, when $\sigma = 20\%$, the levels of expected optimal contribution rates are higher compared with the ones when σ is equal to 5%. This is due to the lower levels of the expected fund obtained when σ is equal to 20%. For the first case when $\gamma = 0.1$ and $\beta = 0.9$, Figure 4.47 shows that $E(C_t^*)$ is increased from 18%-19% to be around 21% with very small changes among the different cases as α is increased. Similarly, Figures 4.48 and 4.49 do not show large differences of the levels of $E(C_t^*)$ for different cases of α. However, the trends of $E(C_t^*)$ generally become closer to the target 22% when γ is increased to 0.9, apart from the case when $\alpha = 0.8$ which shows a downwards trend away from the target.

(c) Third scenario $FT_t = 1$ and $CT_t = 0.18$

When the contribution target is decreased to 0.18, it leads to different results from those that we have obtained in the previous scenario where the contribution target is equal to 22%.

When $\sigma = 5\%$, in the case of (0.1, 0, 0.9), so that more importance is given to the solvency weighting risk factor and the value of α is increased gradually to 0.8, the expected optimal fund level decreases slightly over the control period. This leads $E(F_t^*)$ to move downwards away from 100% due to the low value assigned to the contribution target.

When γ is increased to 0.5 and 0.9 respectively, $E(F_t^*)$ decreases over time and moves further away from the target even as α is increased. For example, in the case of (0.9, 0.8, 0.1), $E(F_t^*)$ reaches the lowest level of 83%. Thus, we can conclude that a lower value of contribution target along with high values of α lead $E(F_t^*)$ to move down further away from the target. This is clearly seen in Figures 4.50, 4.51 and 4.52.

Figure 4.50

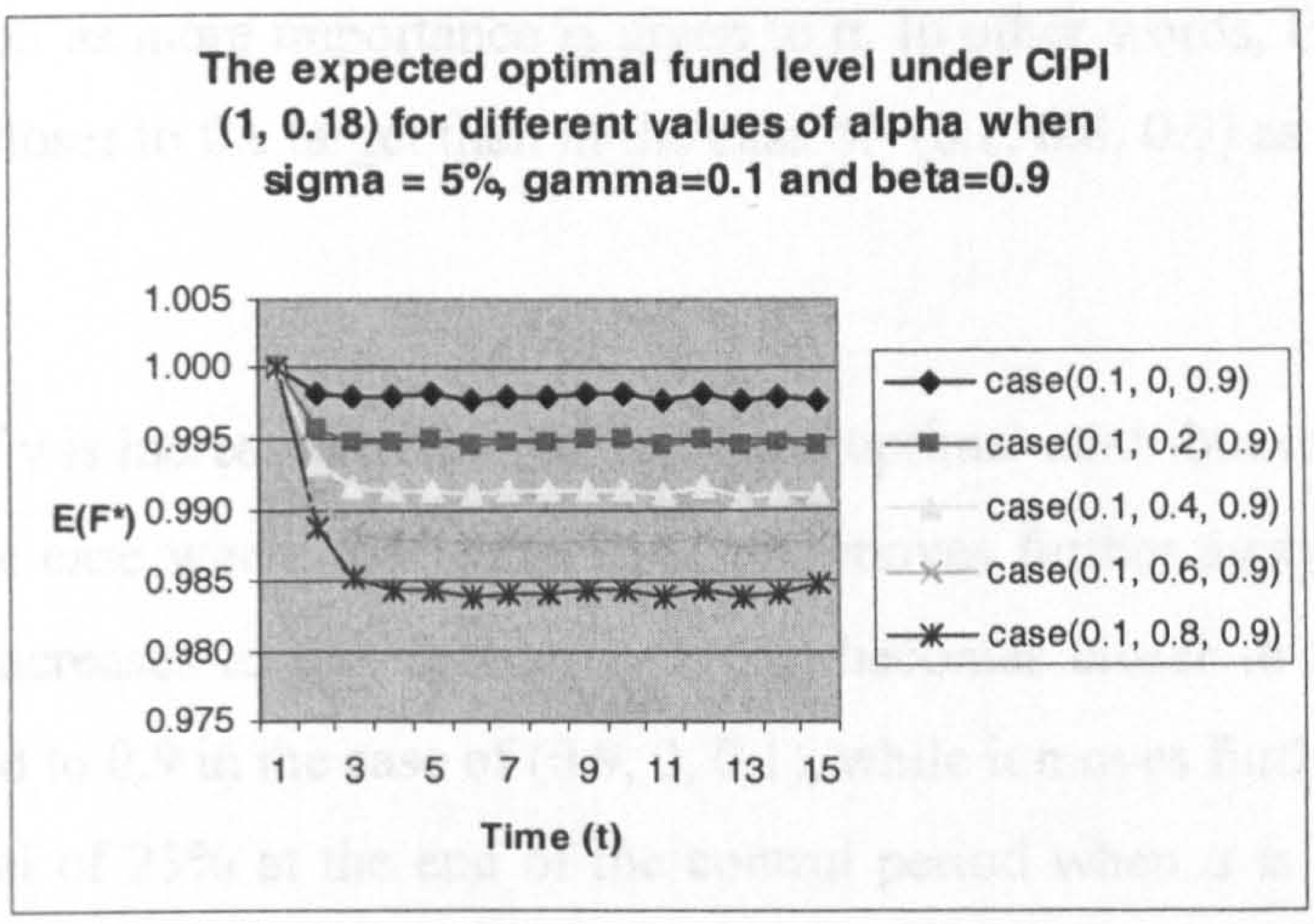


Figure 4.51

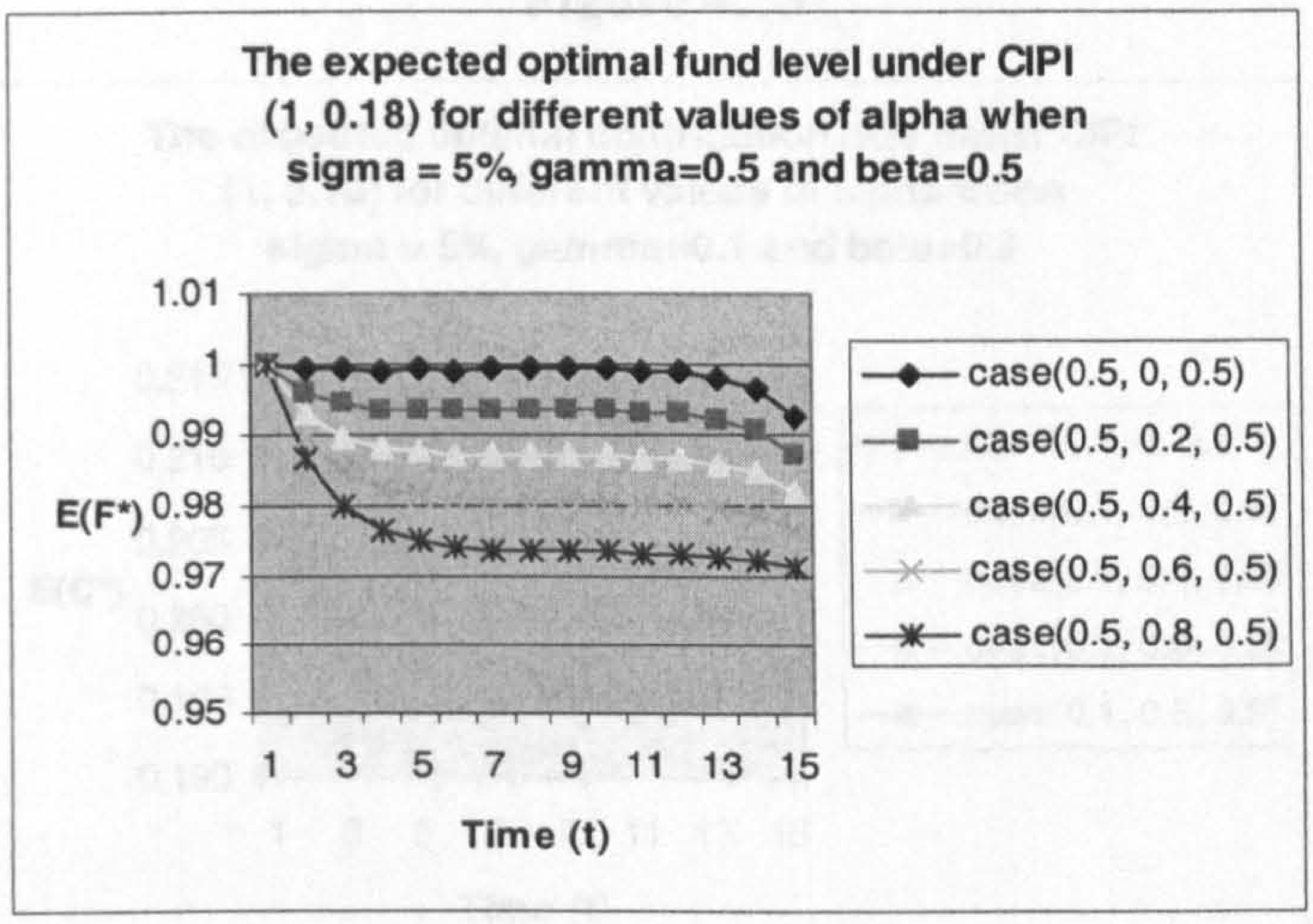
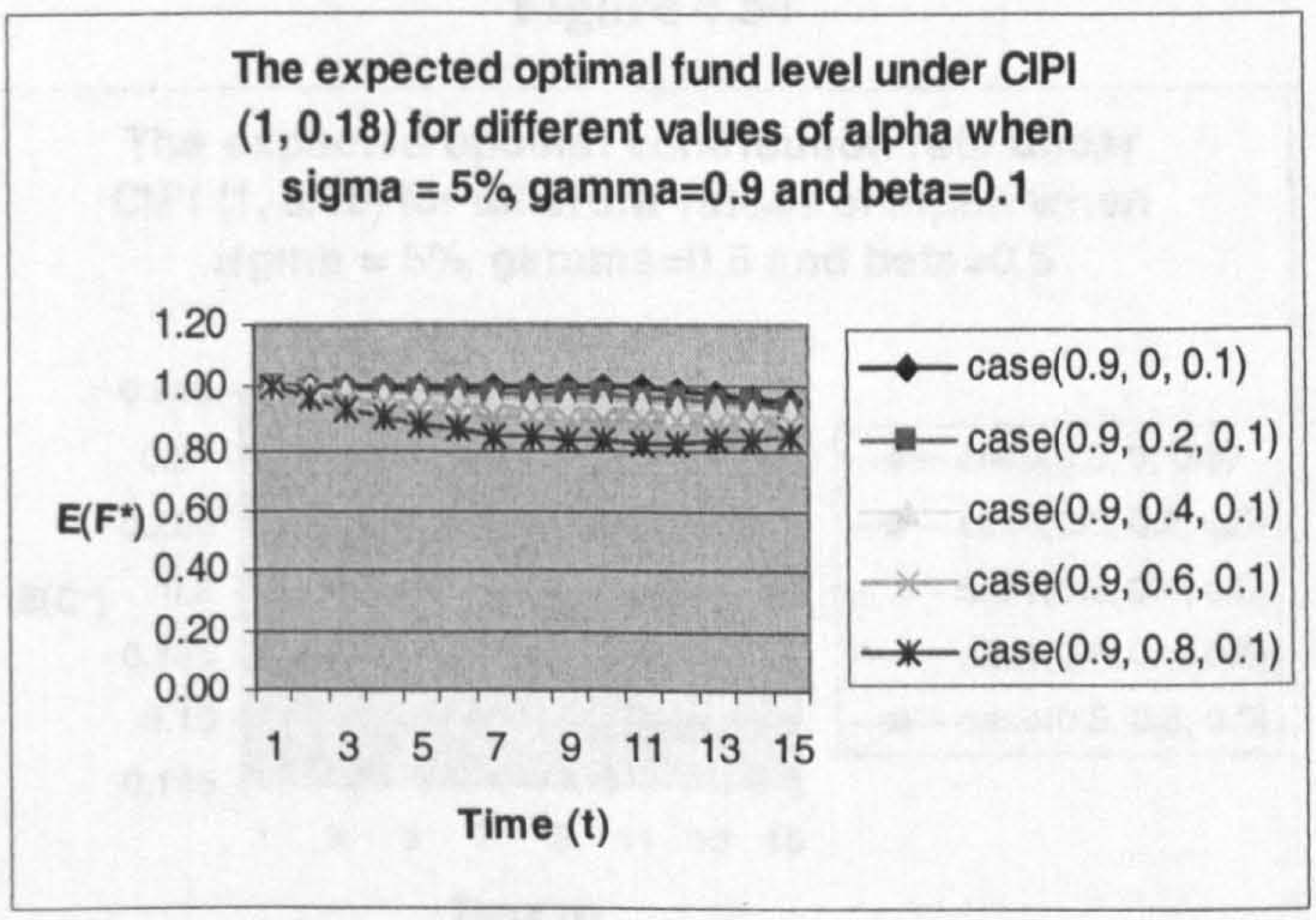


Figure 4.52



For the expected optimal contribution rate, we note that it increases over time and moves away from its target as more importance is given to α. In other words, $E(C_t^*)$ in the case of (0.1, 0, 0.9) is closer to the target than in the case of (0.1, 0.8, 0.9) as shown in Figure 4.53.

When the value of γ is increased to 0.5, the expected optimal contribution rate gets closer to the target in the case where α is equal to 0, and moves further away from the target when this value increases to 0.8. Similarly, $E(C_t^*)$ becomes closer to the target (18%) when γ is increased to 0.9 in the case of (0.9, 0, 0.1), while it moves further away to reach the maximum level of 23% at the end of the control period when α is increased to 0.8. Figures 4.54 and 4.55 show these results.

Figure 4.53

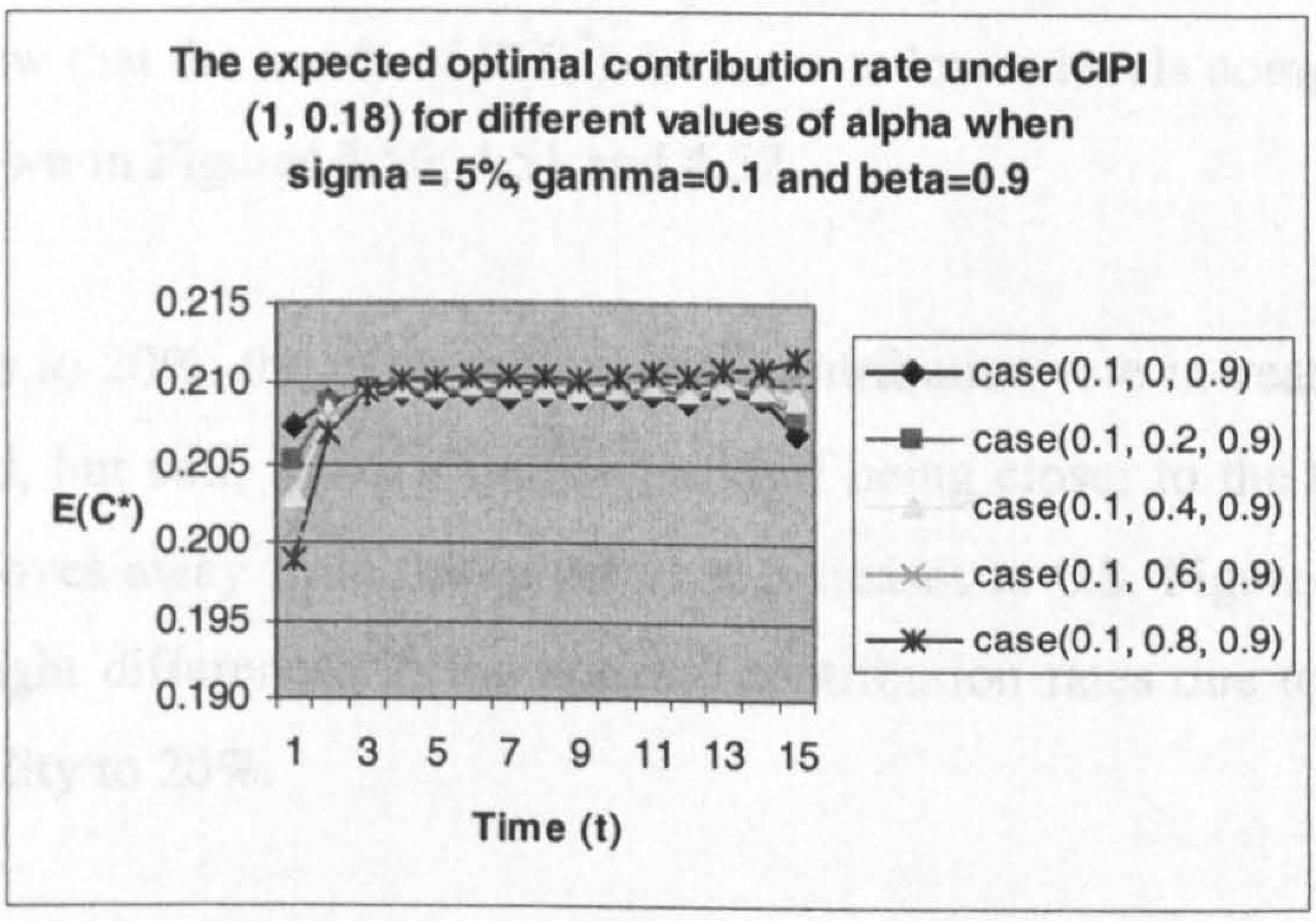


Figure 4.54

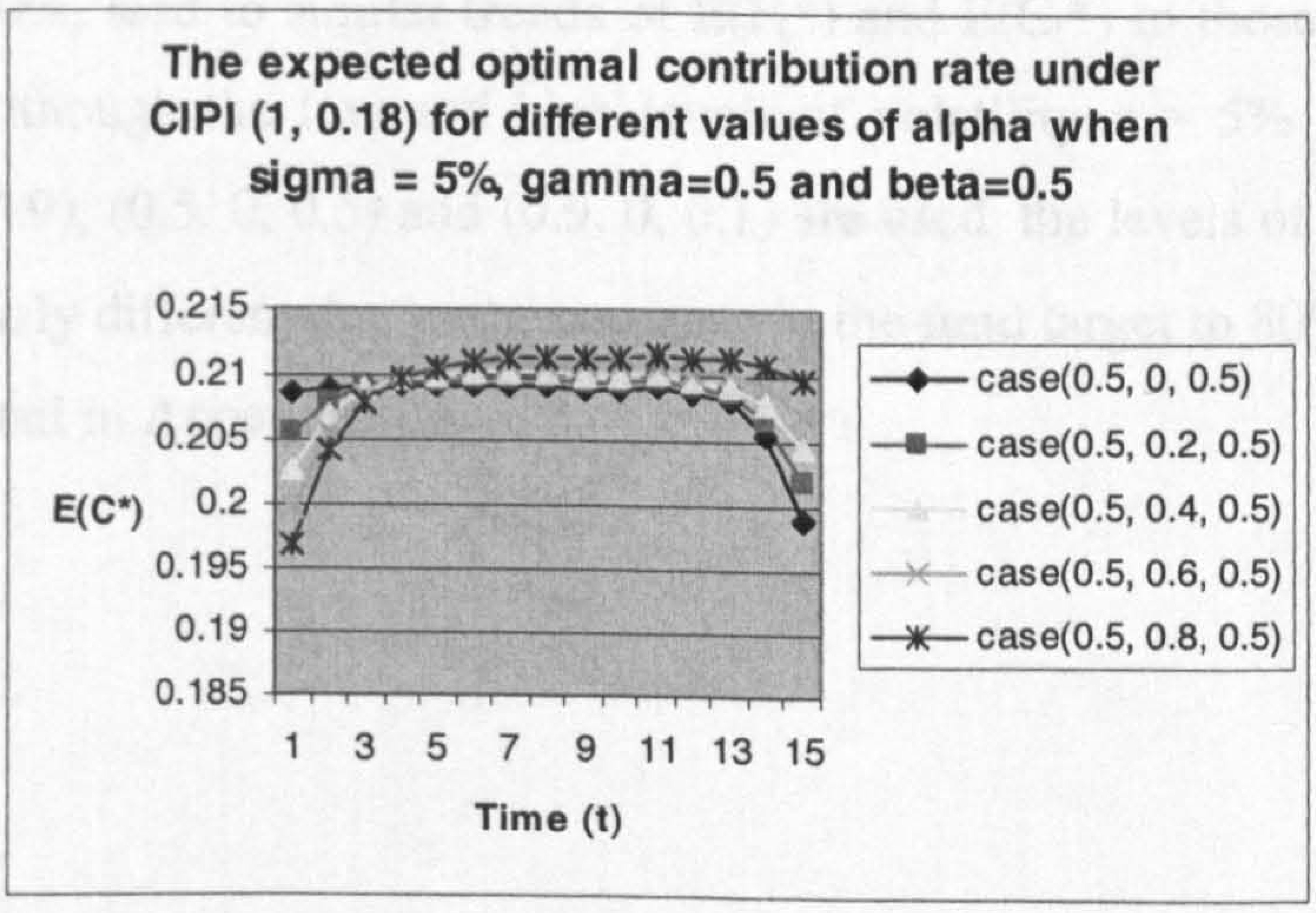
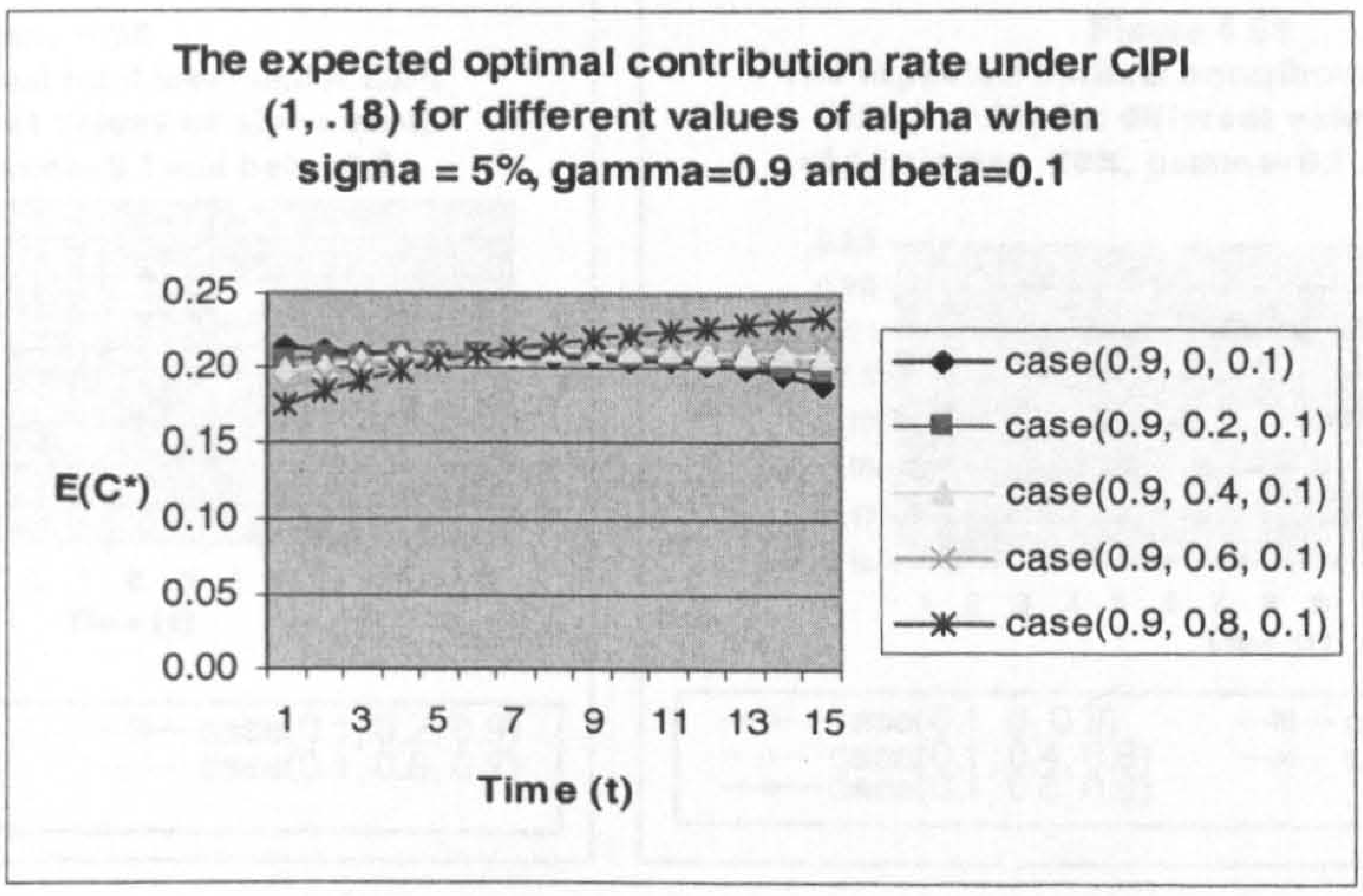


Figure 4.55



For $\sigma = 20\%$, when α is increased to 0.8 and β is decreased from 0.1 to 0.9, Figures 4.56, 4.57 and 4.58 show that the trends of $E(F_t^*)$ decrease to lower levels compared with those when $\sigma = 5\%$ shown in Figures 4.50, 4.51 and 4.52.

When σ increases to 20%, the expected optimal contribution rate increases slightly over the control period, but still, it has a similar trend of being closer to the target when α is equal to 0 and moves away from the target as α increases to 0.8. Figures 4.59, 4.60 and 4.61 show the slight differences in the optimal contribution rates due to the increase in the level of volatility to 20%.

Finally, it is important to mention that the results of applying the fifth scenario, when FT_t is decreased to 80%, lead to similar trends of $E(F_t^*)$ and $E(C_t^*)$ to those obtained in the third scenario. Although the low and high levels of volatility $\sigma = 5\%$ and 20% in the cases of (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0, 0.1) are used, the levels of both $E(F_t^*)$ and $E(C_t^*)$ are obviously different due to the decrease in the fund target to 80%. These results can be seen in detail in Appendix 4B.

Figure 4.56
The expected optimal fund level under CIPI (1, 0.18) for different values of alpha when sigma = 20%, gamma=0.1 and beta=0.9

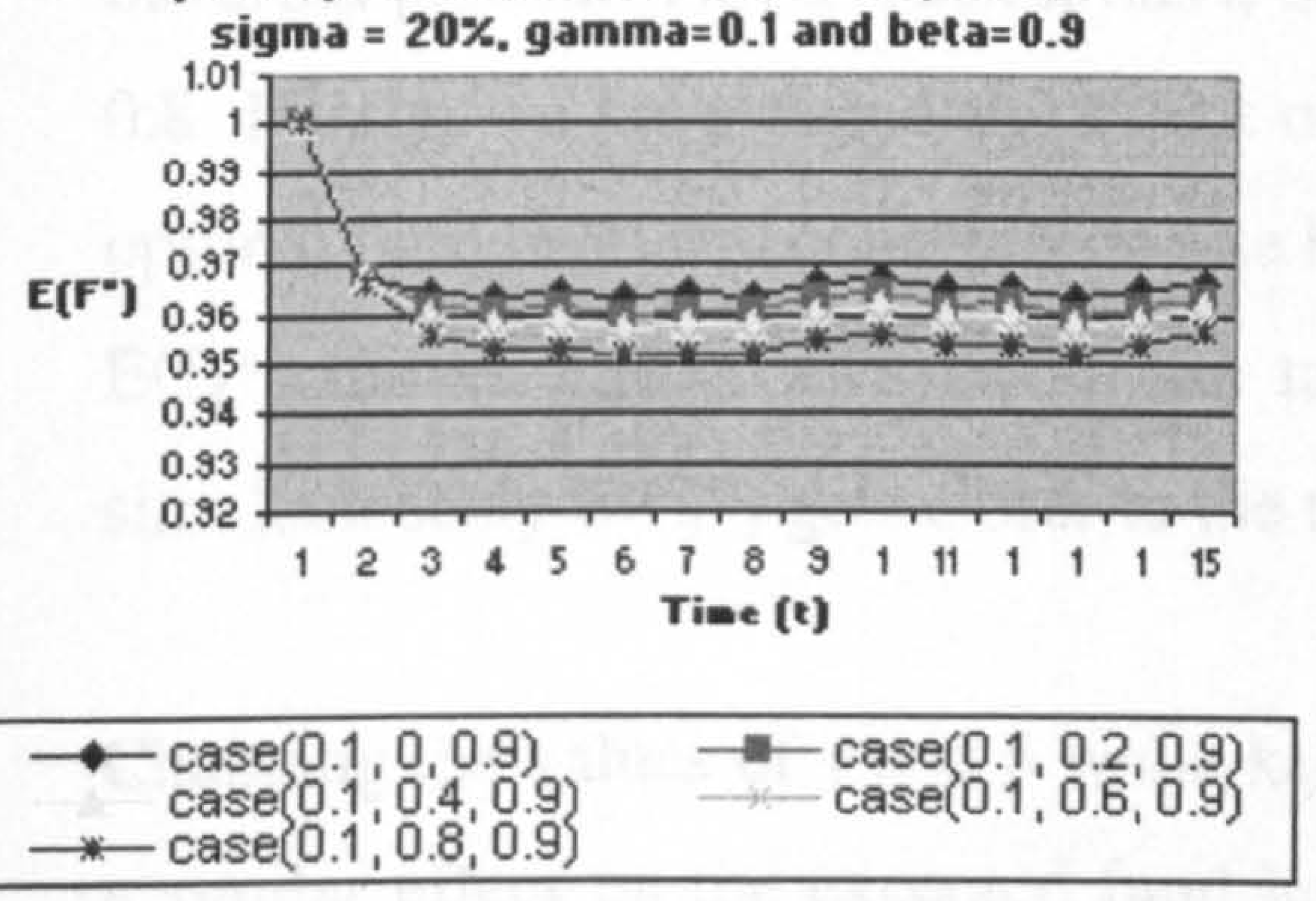


Figure 4.59
The expected optimal contribution rate under CIPI (1, 0.18) for different values of alpha when sigma = 20%, gamma=0.1 and beta=0.9

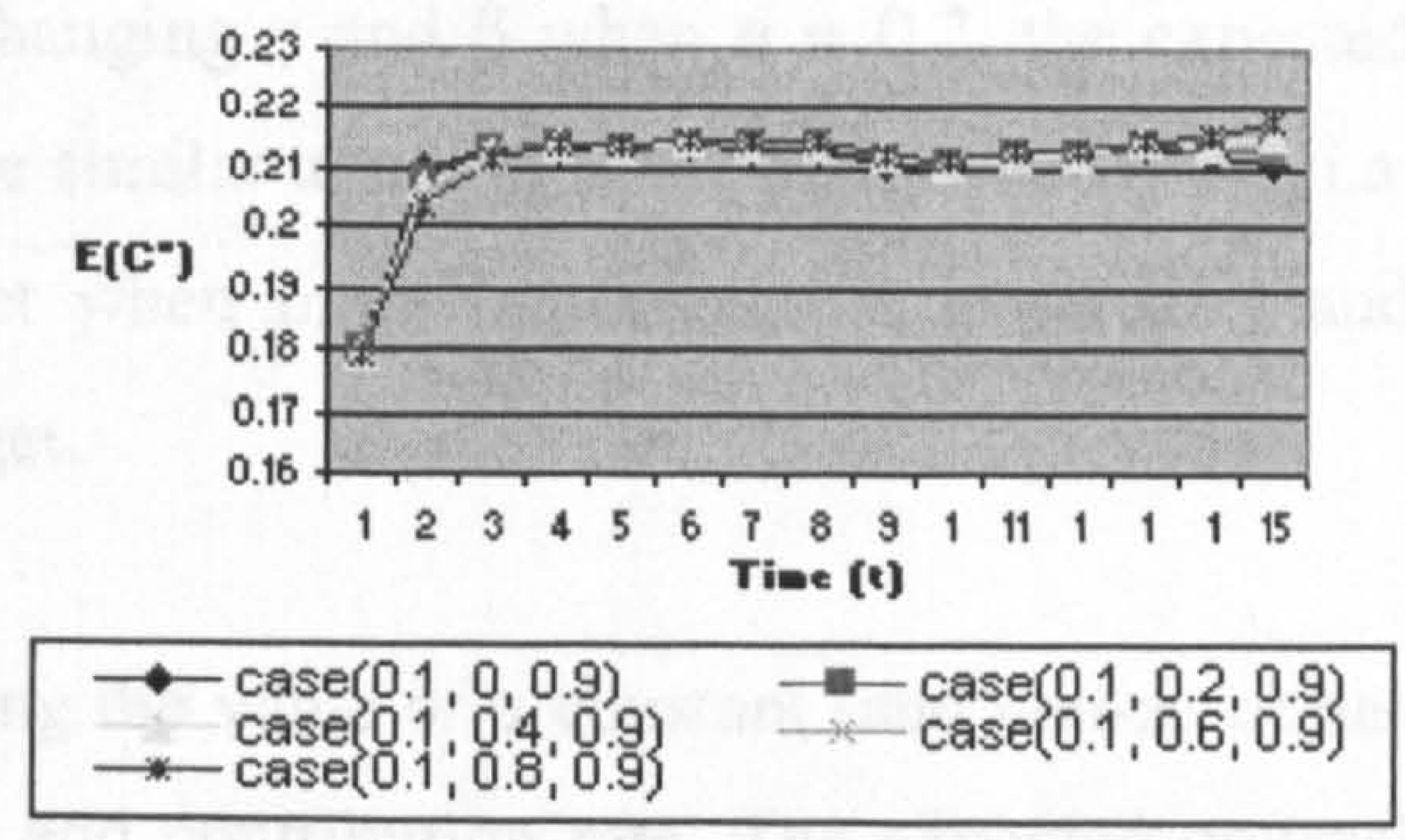


Figure 4.57
The expected optimal fund level under CIPI (1, 0.18) for different values of alpha when sigma = 20%, gamma=0.5 and beta=0.5

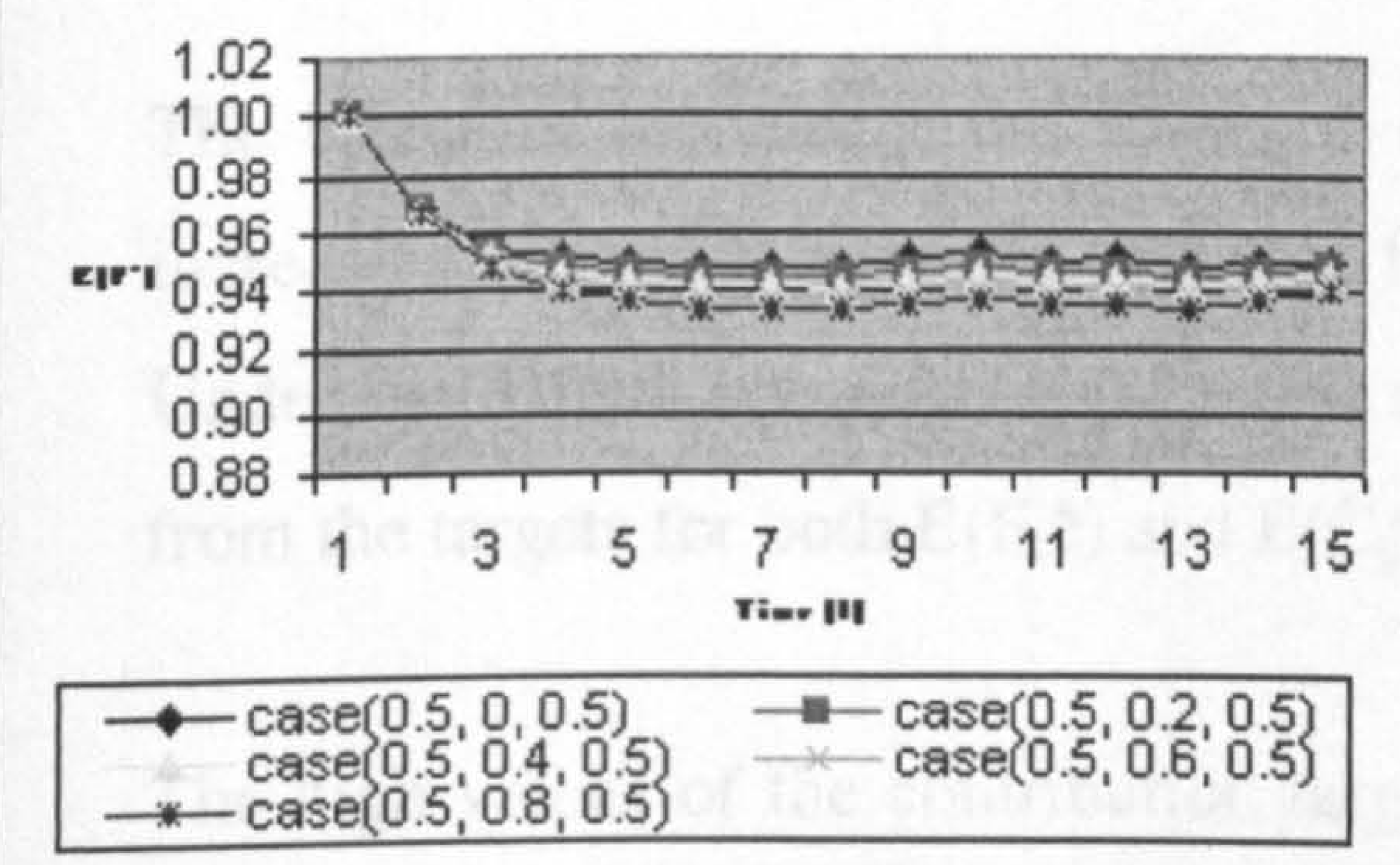


Figure 4.60
The expected optimal contribution rate under CIPI (1, 0.18) for different values of alpha when sigma = 20%, gamma=0.5 and beta=0.5

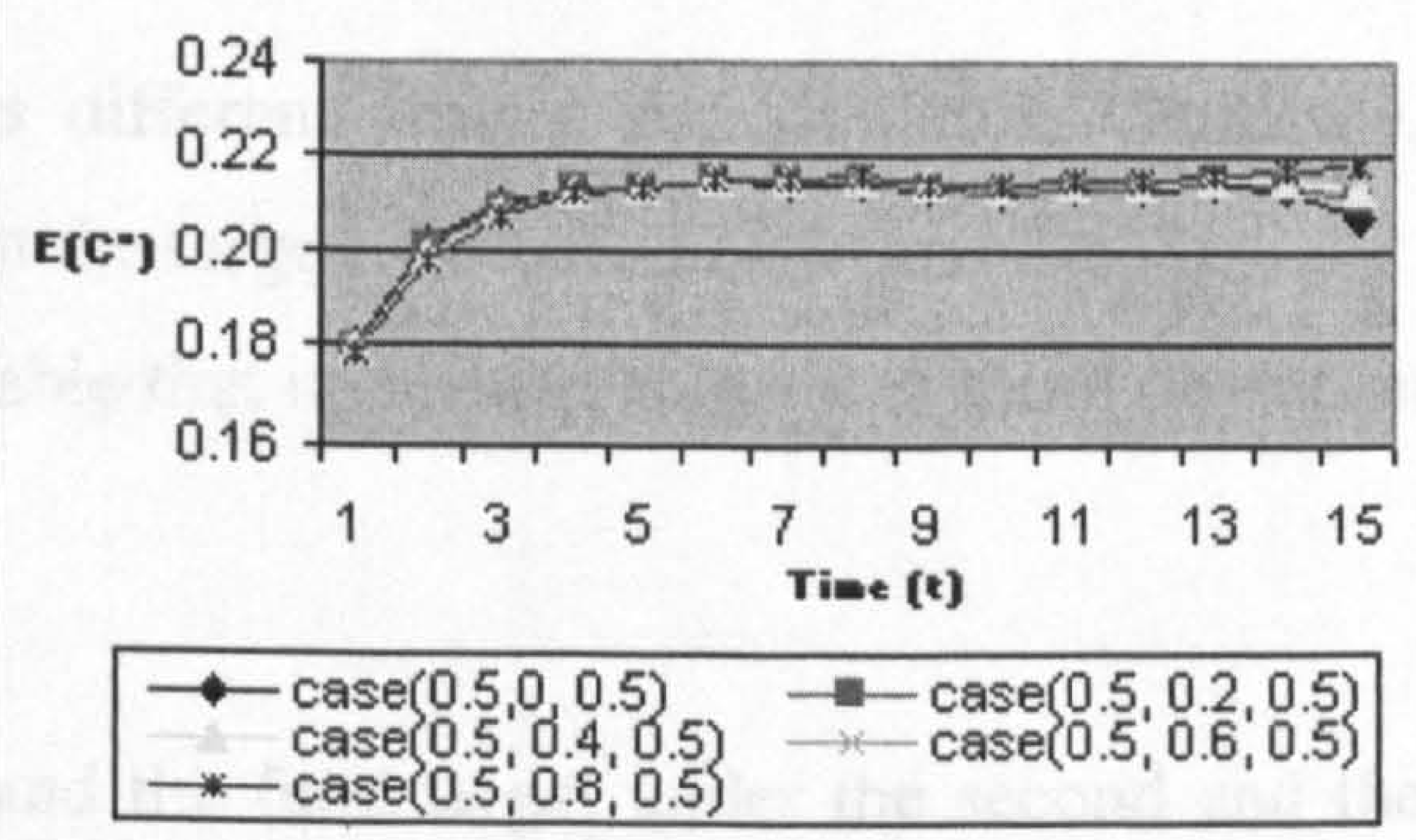


Figure 4.58
The expected optimal fund level under CIPI (1, 0.18) for different values of alpha when sigma = 20%, gamma=0.9 and beta=0.1

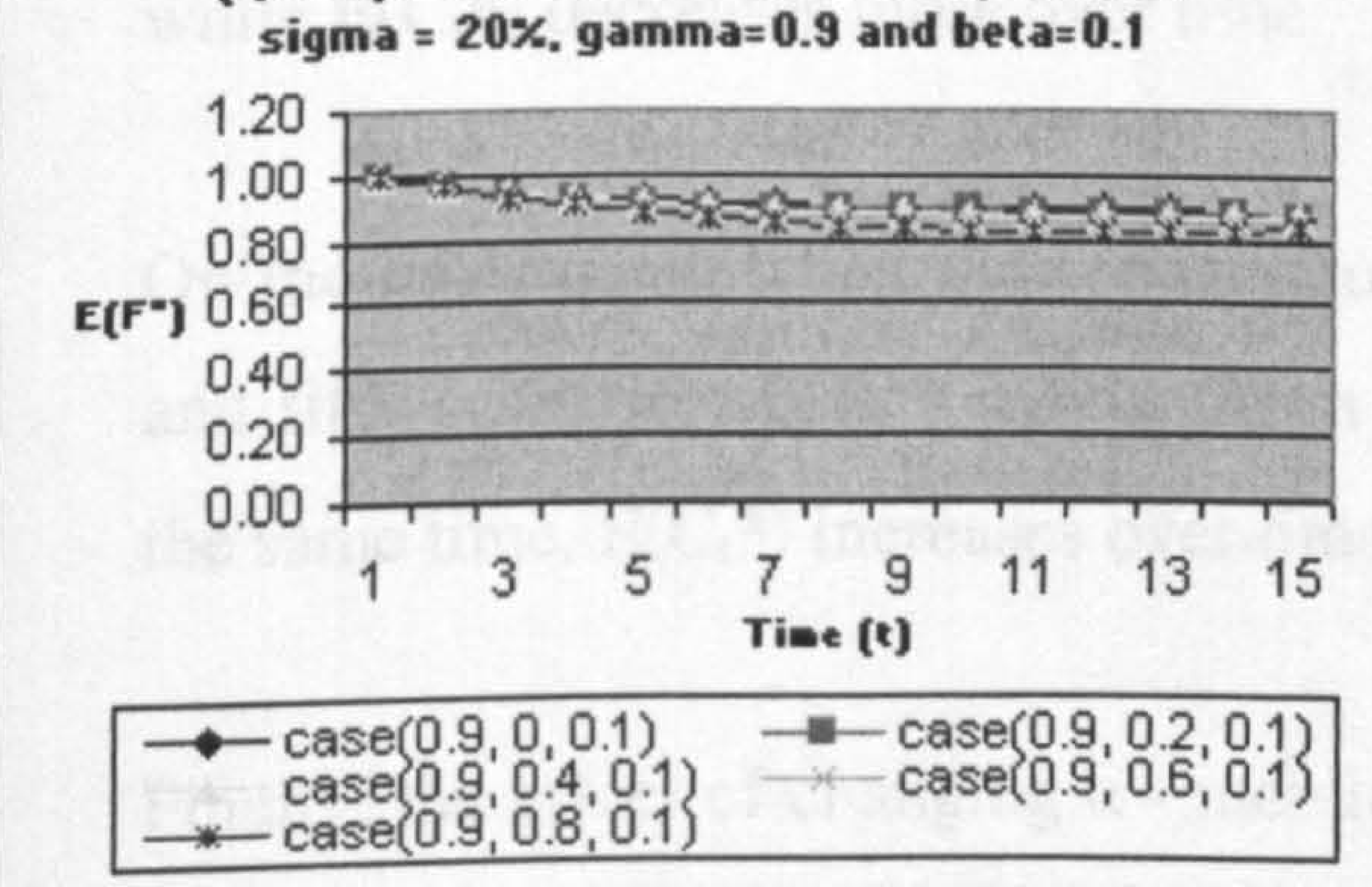
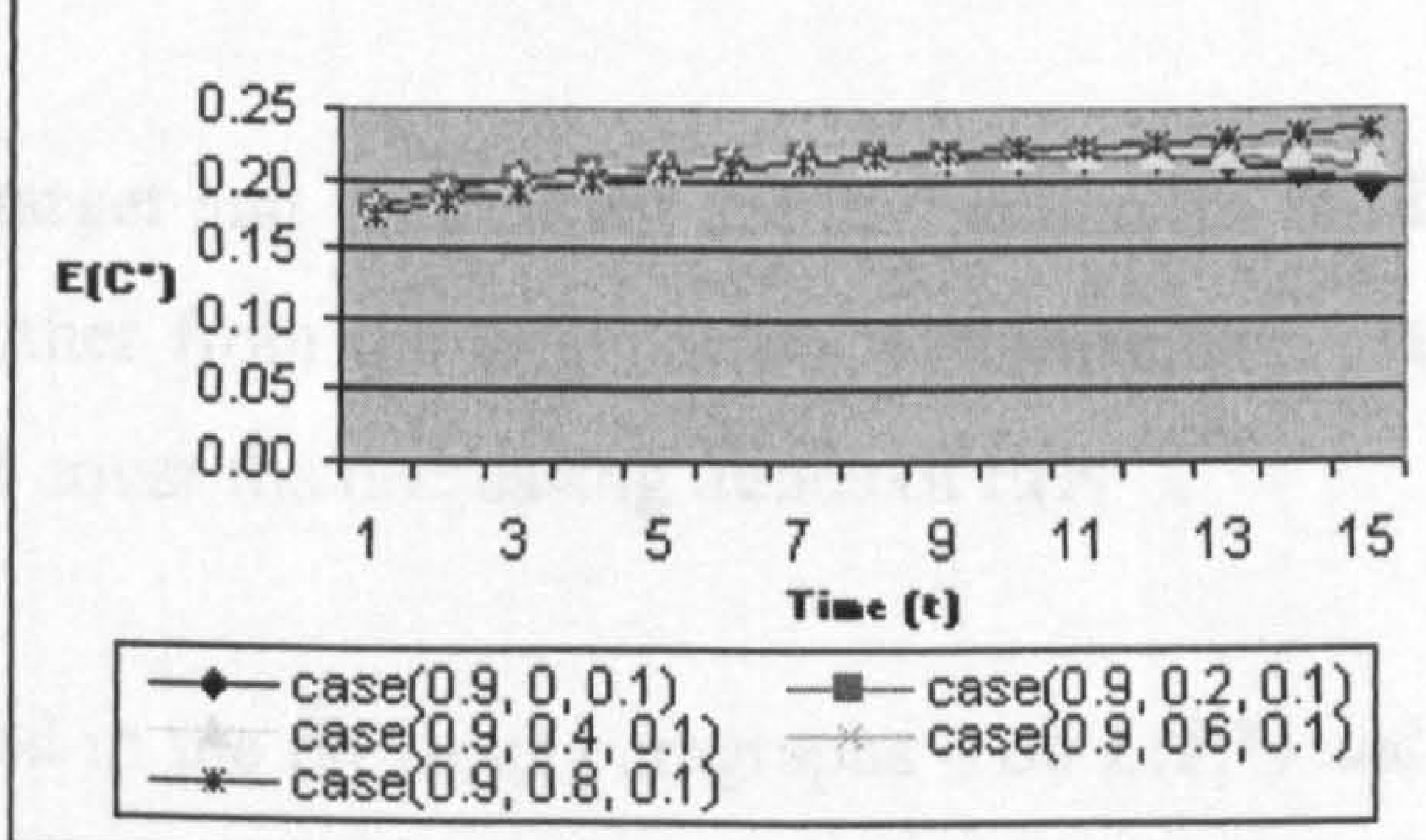


Figure 4.61
The expected optimal contribution rate under CIPI (1, 0.18) for different values of alpha when sigma = 20%, gamma=0.9 and beta=0.1



Conclusion

In this subsection, we have examined the effect of changing the weighting risk factor of the cross product term α when it has a non-zero value, i.e. when it is increased from 0 to 0.8. Firstly, we have tested the effect of changing γ and β when $\alpha = 0.2$, the expected optimal fund level and contribution rate have similar trends to those cases when $\alpha = 0$, i.e. $E(F_t^*)$ moves further away from the target when more importance is given to γ and simultaneously $E(C_t^*)$ gets closer to the target.

Changing the values of γ and β while keeping the value of α constant (and non-zero) has a similar effect on the excepted fund level and contribution rate. The expected optimal fund level is close to its target as we give more importance to β, and it moves further away from it with a high value of the weighting risk factor for the contribution rate γ.

The effect of increasing the value of α is different among the scenarios. Generally, increasing α leads to a greater deviation from the targets for both $E(F_t^*)$ and $E(C_t^*)$.
Under the default scenario (1, 0.2), it is notable that increasing α leads to more deviation from the targets for both $E(F_t^*)$ and $E(C_t^*)$.

The high values of the contribution target and the fund target under the second and the fourth scenarios have similar impact on $E(F_t^*)$ and $E(C_t^*)$. We note that the expected optimal fund level moves up further from the target when the value of α is increased, while $E(C_t^*)$ decreases more over time.

On the other hand, when the contribution target and fund target are low under the third and fifth scenarios, $E(F_t^*)$ moves down further from the target when α is increased. At the same time, $E(C_t^*)$ increases over time to cover the decreasing trends of $E(F_t^*)$.

Finally, the effect of changing α - mentioned in the previous paragraphs – on $E(F_t^*)$ and $E(C_t^*)$ is also influenced by the levels of γ and β. In other words, $E(F_t^*)$ gets close to the target when more importance is given to β and vice versa. Further, the high level of volatility leads to more deviation from the target as previously stated.

4.5 Results of sensitivity analysis - Cost-induced performance index model in the stochastic case using the equilibrium value of B_t -

4.5.1 Effect of changing θ

The effect of changing θ on the expected optimal fund level and contribution rate is tested in the first scenario (1, 0.2) – the same scenario that was considered in section 4.4.1. Here, we allow the value of θ to move down to 5% and up to 15% with changes to B_t according to the values shown in Table 4.1. This leads to the following results:

- In the case of (0.1, 0, 0.9), with $\sigma = 5\%$, the expected fund level decreases slightly when $\theta = 5\%$ compared with the values under the same case when $\theta = 10\%$. On the other hand, $E(F_t^*)$ increases with increasing θ to 15% as shown in table 4.20.

Table 4.20

The expected optimal fund level under CIPI (1, 0.2) for different levels of θ at σ = 5% in the case of (0.1, 0, 0.9)

t	θ = 5%	θ =10%	θ = 15%
1	100.00%	100.00%	100.00%
2	99.77%	99.80%	99.83%
3	99.73%	99.76%	99.79%
4	99.73%	99.77%	99.79%
5	99.75%	99.79%	99.81%
6	99.71%	99.76%	99.79%
7	99.73%	99.77%	99.80%
8	99.73%	99.77%	99.80%
9	99.77%	99.80%	99.82%
10	99.77%	99.78%	99.80%
11	99.72%	99.75%	99.78%
12	99.76%	99.80%	99.83%
13	99.70%	99.74%	99.77%
14	99.74%	99.77%	99.80%
15	99.76%	99.78%	99.80%

The expected optimal contribution rate follows a similar behaviour, i.e. it increases with increasing θ to 15%, and decreases with decreasing it to 5%, compared with the values obtained when θ is equal to 10%. These results are shown in Table 4.21.

Here, it is noted that the values of $E(F_t^*)$ and $E(C_t^*)$ when $\theta = 10\%$ in Tables 4.20 and 4.21 are similar to the ones shown in Tables 4.2 and 4.3. This is because the equilibrium value of B_t is equal to 3 when $\theta = 10\%$ under the first scenario (1, 0.2).

Table 4.21
The expected optimal contribution rate under CIPI (1, 0.2) for different levels of θ at σ = 5% in the case of (0.1, 0, 0.9)

t	θ = 5%	θ =10%	θ = 15%
1	20.02%	20.72%	20.80%
2	20.23%	20.90%	20.95%
3	20.27%	20.94%	20.99%
4	20.26%	20.93%	20.98%
5	20.24%	20.91%	20.96%
6	20.28%	20.94%	20.99%
7	20.26%	20.93%	20.97%
8	20.26%	20.93%	20.98%
9	20.22%	20.91%	20.96%
10	20.22%	20.92%	20.98%
11	20.27%	20.95%	21.00%
12	20.23%	20.90%	20.95%
13	20.29%	20.96%	21.00%
14	20.25%	20.92%	20.97%
15	20.23%	20.86%	20.91%

Increasing the level of volatility to 20% provides another example for testing the effect of changing the mean of the rate of return on $E(F_t^*)$ and $E(C_t^*)$. For the same case and under the same scenario, both the expected optimal fund level and contribution rate increase with increasing the value of θ to 15%, and decrease when θ is decreased to 5%, this is shown in Tables 4.22 and 4.23 respectively.

Although increasing σ from 5% to 20% tends to move the values of the expected optimal fund level and the contribution rate away from the targets, we note that $E(F_t^*)$ and $E(C_t^*)$ still have similar patterns with respect to t, for the different values of θ.

Thus, we can conclude that there are generally small differences among the values of the expected optimal fund level and contribution rate when θ is changed. This is due to our changing B_t to its equilibrium values of 0.25 (when $\theta = 5\%$) and 0.34 (when $\theta = 15\%$), where these values correspond to the lower and higher values of θ in order to maintain the balance in the fund growth equation.

Table 4.22
The expected optimal fund level under CIPI (1, 0.2) for different levels of θ at $\sigma = 20\%$ in the case of (0.1, 0, 0.9)

t	$\theta = 5\%$	$\theta = 10\%$	$\theta = 15\%$
1	100.00%	100.00%	100.00%
2	96.42%	96.74%	97.02%
3	96.03%	96.39%	96.71%
4	95.97%	96.36%	96.69%
5	96.04%	96.43%	96.77%
6	95.93%	96.29%	96.62%
7	96.02%	96.39%	96.71%
8	96.04%	96.37%	96.69%
9	96.32%	96.62%	96.88%
10	96.48%	96.70%	96.92%
11	96.34%	96.49%	96.71%
12	96.19%	96.53%	96.83%
13	95.98%	96.29%	96.60%
14	96.15%	96.48%	96.77%
15	96.34%	96.62%	96.87%

The results of $E(F_t^*)$ and $E(C_t^*)$ using the equilibrium value of B_t show some differences from the results obtained when $B_t = 0.3$ shown in section 4.4.1(b). Although the differences of $E(F_t^*)$ are small with increasing θ when $\sigma = 5\%$ and 20%, it is noted that the levels of $E(C_t^*)$ show a different behaviour from the one obtained when $B_t = 0.3$. Here, there is also less deviation from the targets between the levels of $E(C_t^*)$ compared with the results in section 4.4.1(b). This could be due to the direct impact of changing θ on the balance of the actuarial growth function leading to better results of $E(F_t^*)$ and $E(C_t^*)$ when using the equilibrium value of B_t.

Table 4.23
The expected optimal contribution rate under CIPI (1, 0.2) for different levels of θ at σ = 20% in the case of (0.1, 0, 0.9)

t	θ = 5%	θ =10%	θ = 15%
1	16.89%	17.98%	18.38%
2	20.17%	20.99%	21.15%
3	20.53%	21.31%	21.44%
4	20.59%	21.35%	21.46%
5	20.52%	21.28%	21.39%
6	20.62%	21.40%	21.52%
7	20.54%	21.32%	21.44%
8	20.52%	21.33%	21.46%
9	20.27%	21.10%	21.28%
10	20.12%	21.03%	21.24%
11	20.25%	21.23%	21.44%
12	20.38%	21.19%	21.33%
13	20.58%	21.41%	21.55%
14	20.44%	21.25%	21.40%
15	20.50%	21.26%	21.40%

4.5.2 Effect of changing the weighting factors of the contribution rate risk and the solvency risk; γ and β (with $\alpha = 0$)

In the same scenarios where the equilibrium value of B_t is changing as shown in Table 4.1, the effect on the expected optimal fund level and the contribution rate is examined in the following cases: (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0, 0.1). Under the first scenario (1, 0.2) when $B_t = 0.3$, the results are the same as shown in subsection 4.4.5(a). The results of the four other scenarios are explained below.

(a) Second scenario: $FT_t = 1$, $CT_t = 0.22$

In the second scenario (1, 0.22) where $B_t= 0.32$, in the three cases mentioned above when $\sigma = 5\%$, we find that $E(F_t^*)$ moves further away from the target when more importance is given to the contribution rate, i.e. the lowest level of $E(F_t^*)$ is reached in the case of (0.9, 0, 0.1) where $E(F_T^*)$ is equal to 98.12%. At the same time, $E(C_t^*)$ becomes close to the target in the case of (0.9, 0, 0.1) compared with the cases of (0.1, 0, 0.9) and (0.5, 0, 0.5). Figure 4.62 and 4.63 show the expected optimal fund level and the contribution rate when

σ = 5% for the different cases over the control period. When σ is increased to 20%, the results of $E(F_t^*)$ and $E(C_t^*)$ show more departure from the targets.

Figure 4.62

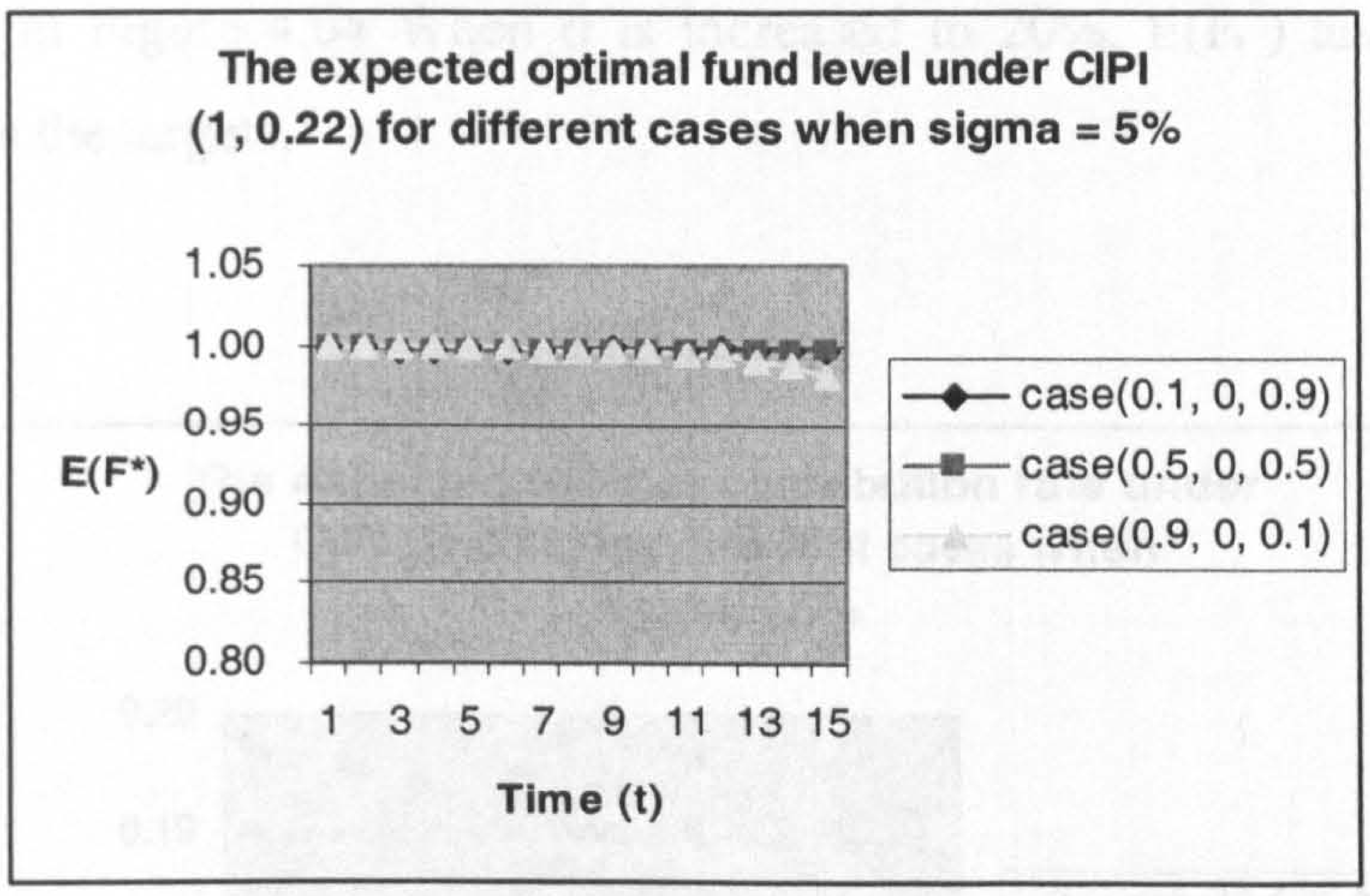
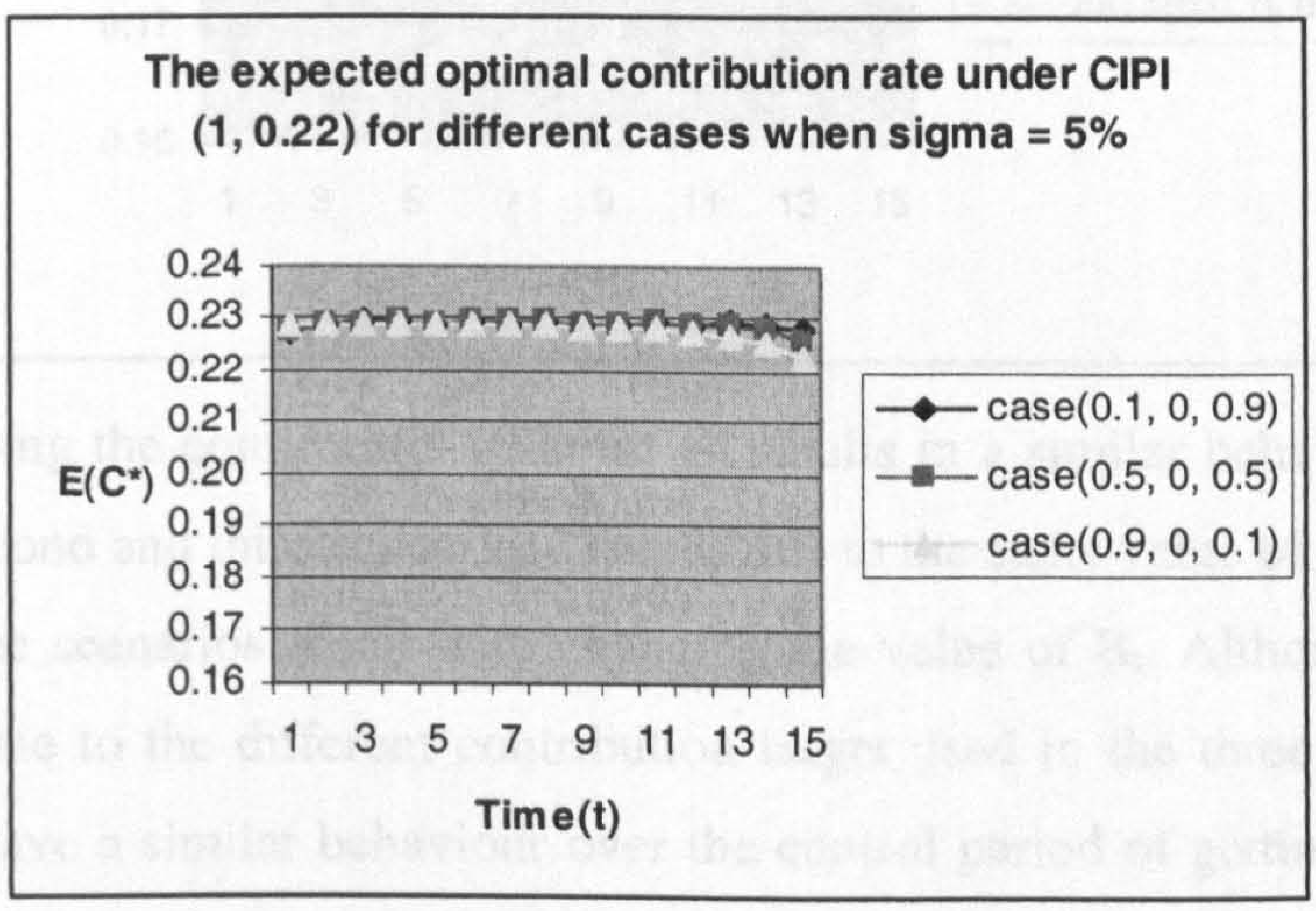


Figure 4.63

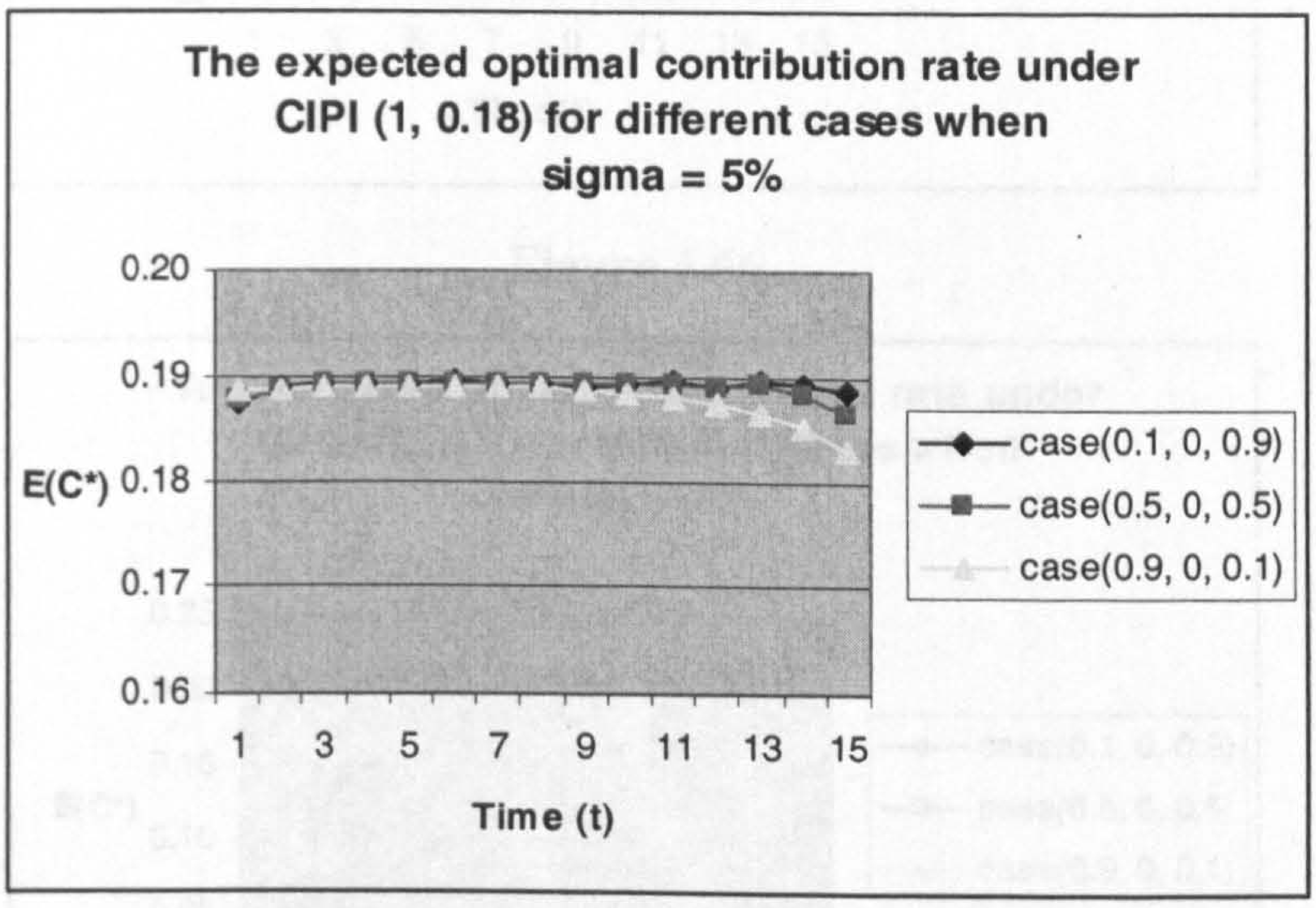


The effect of using the equilibrium value of B_t on $E(F_t^*)$ can be observed by comparing Figures 4.20 and 4.62 , where $E(F_t^*)$ follows the same behaviour with a slightly decreasing trend at the end of the control period. The decreasing trend is more obvious when more importance is given to γ. The expected optimal contribution rate becomes closer to the target when γ is increased – similar to the results of B_t = 0.3 shown in Figure 4.21- .

(b) Third scenario: $FT_t = 1, CT_t = 0.18$

Under the third scenarios (1, 0.18), $E(F_t^*)$ has the same decreasing trend obtained under the first and second scenario when γ is increased. However, $E(C_t^*)$ trends to increase around 19% with a tendency to get closer to the target of 18% at the end of the control period as shown in Figure 4.64 When σ is increased to 20%, $E(F_t^*)$ and $E(C_t^*)$ move further away from the targets.

Figure 4.64



It is noted that using the equilibrium value of B_t results in a similar behaviour of $E(F_t^*)$ under the first, second and third scenarios. This is due to the same value of the fund target used in these three scenarios along with changing the value of B_t. Although $E(C_t^*)$ has different values due to the different contribution target used in the three scenarios, the trends of $E(C_t^*)$ have a similar behaviour over the control period of getting closer to the target with the high value of γ as can be seen in Figures 4.63 and 4.64.

(c) Fourth scenario: $FT_t = 1.2, CT_t = 0.2$

In the fourth scenario (1.2, 0.2), when more importance is given to the weighting risk factor of contribution rate γ and σ is increased, $E(F_t^*)$ moves away from the target as shown in Figure 4.65. This results in moving $E(C_t^*)$ closer to the target particularly when σ is small. Figure 4.66 shows the trends of $E(C_t^*)$ for different values of γ and β when σ = 20%.

Figure 4.65

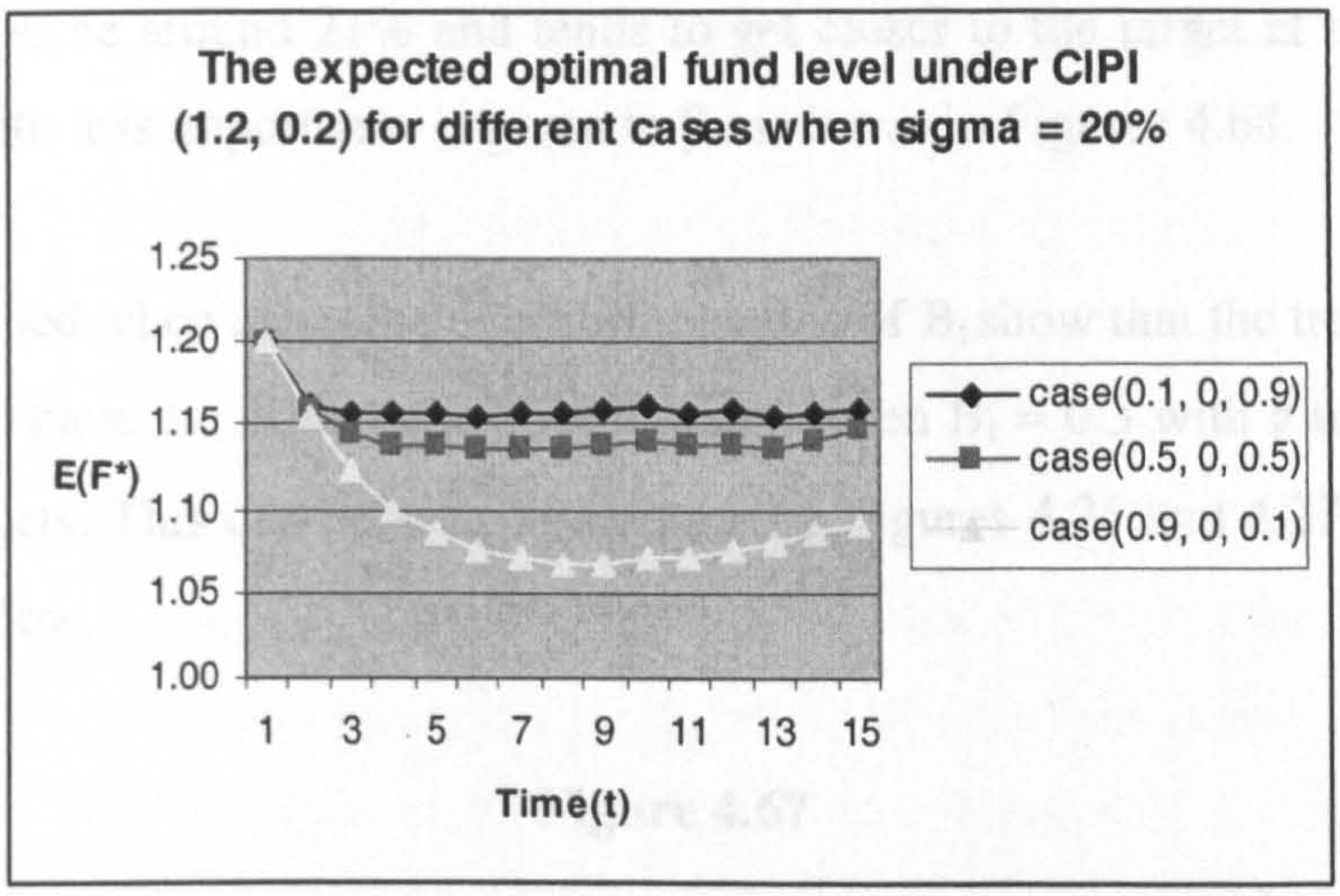
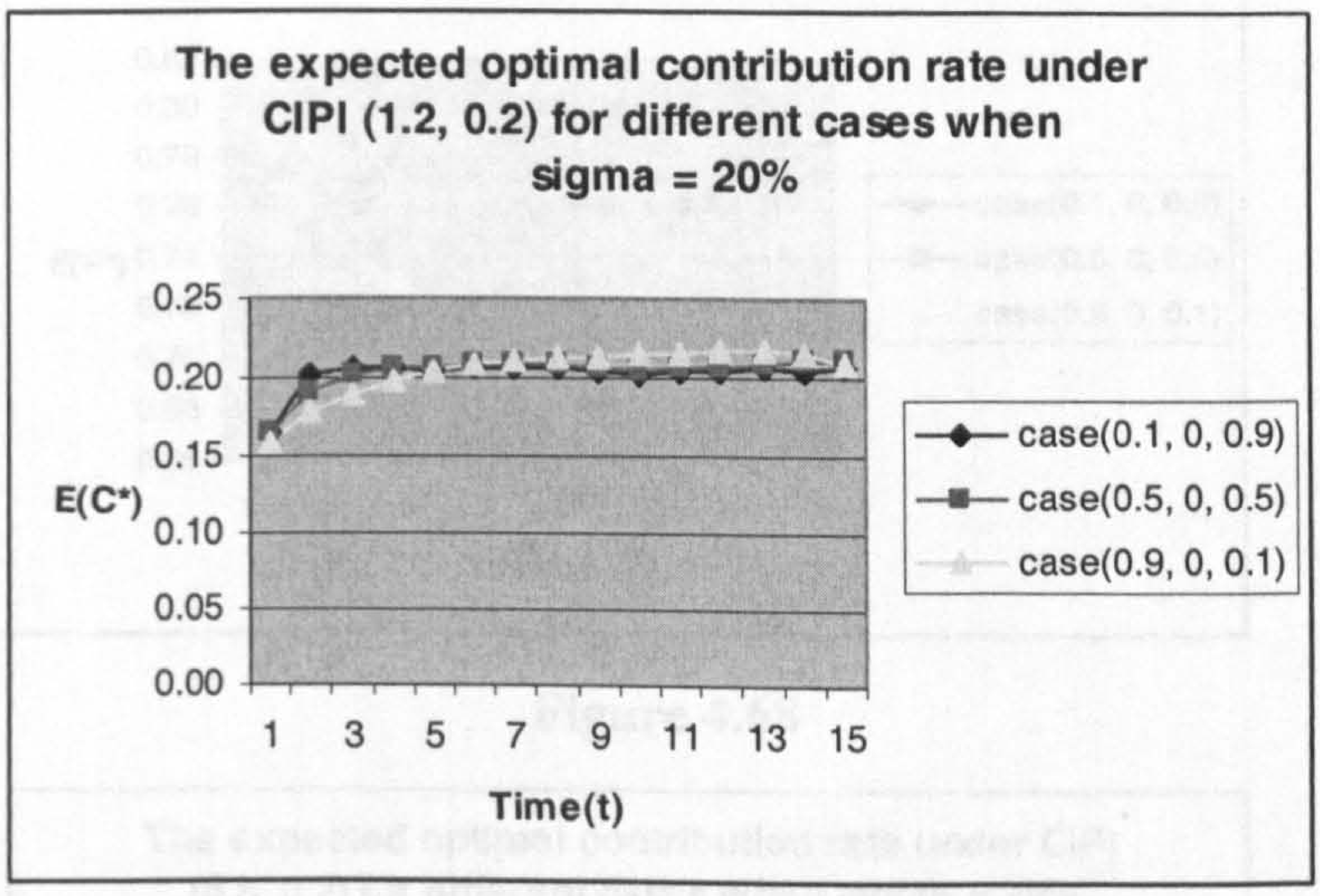


Figure 4.66



Figures 4.65 and 4.66 indicate that the behaviour of $E(F_t^*)$ and $E(C_t^*)$ over the control period is similar when it is compared with the results shown in Figures 4.24 and 4.26. It is noted that the differences in the levels of $E(F_t^*)$ and $E(C_t^*)$ for the different cases are very small when using the equilibrium value of B_t rather than the value $B_t = 0.3$.

(d) Fifth scenario: $FT_t = 0.8$, $CT_t = 0.2$

Under the fifth scenario (0.8, 0.2), $E(F_t^*)$ decreases to reach the minimum level when

$\gamma = 0.9$ and $\sigma = 20\%$, where $E(F_T^*) = 71.8\%$ as shown in Figure 4.67. On the other hand, $E(C_t^*)$ increases to be around 21% and tends to get closer to the target at the end of the control period with less importance is given to β as shown in Figures 4.68.

The results obtained when using the equilibrium value of B_t show that the trends of $E(F_t^*)$ and $E(C_t^*)$ have the same behaviour to those when $B_t = 0.3$ with a tendency to be closer to the targets. This can be seen by comparing Figures 4.25 and 4.27 with Figures 4.67 and 4.68 below.

Figure 4.67

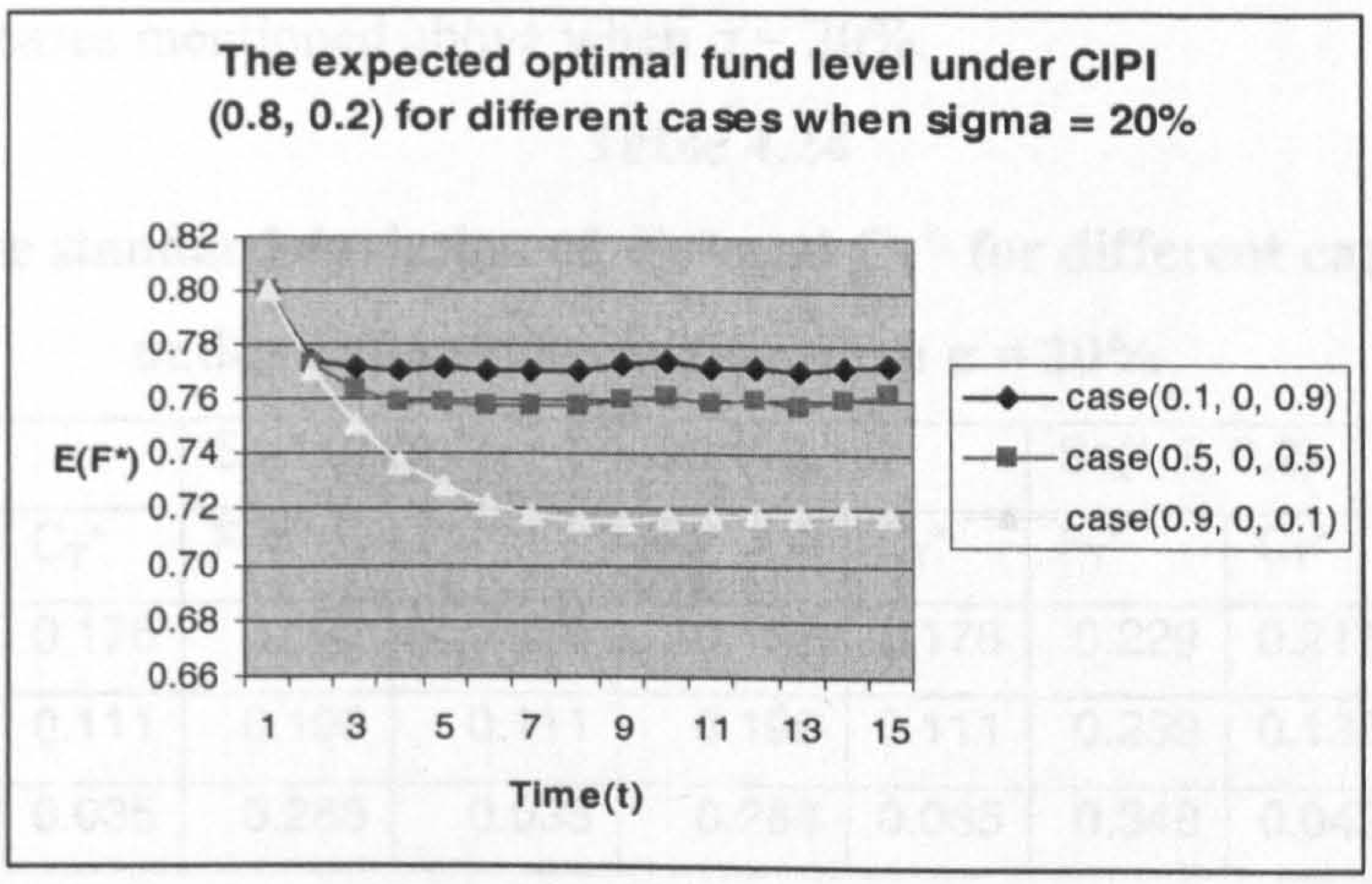
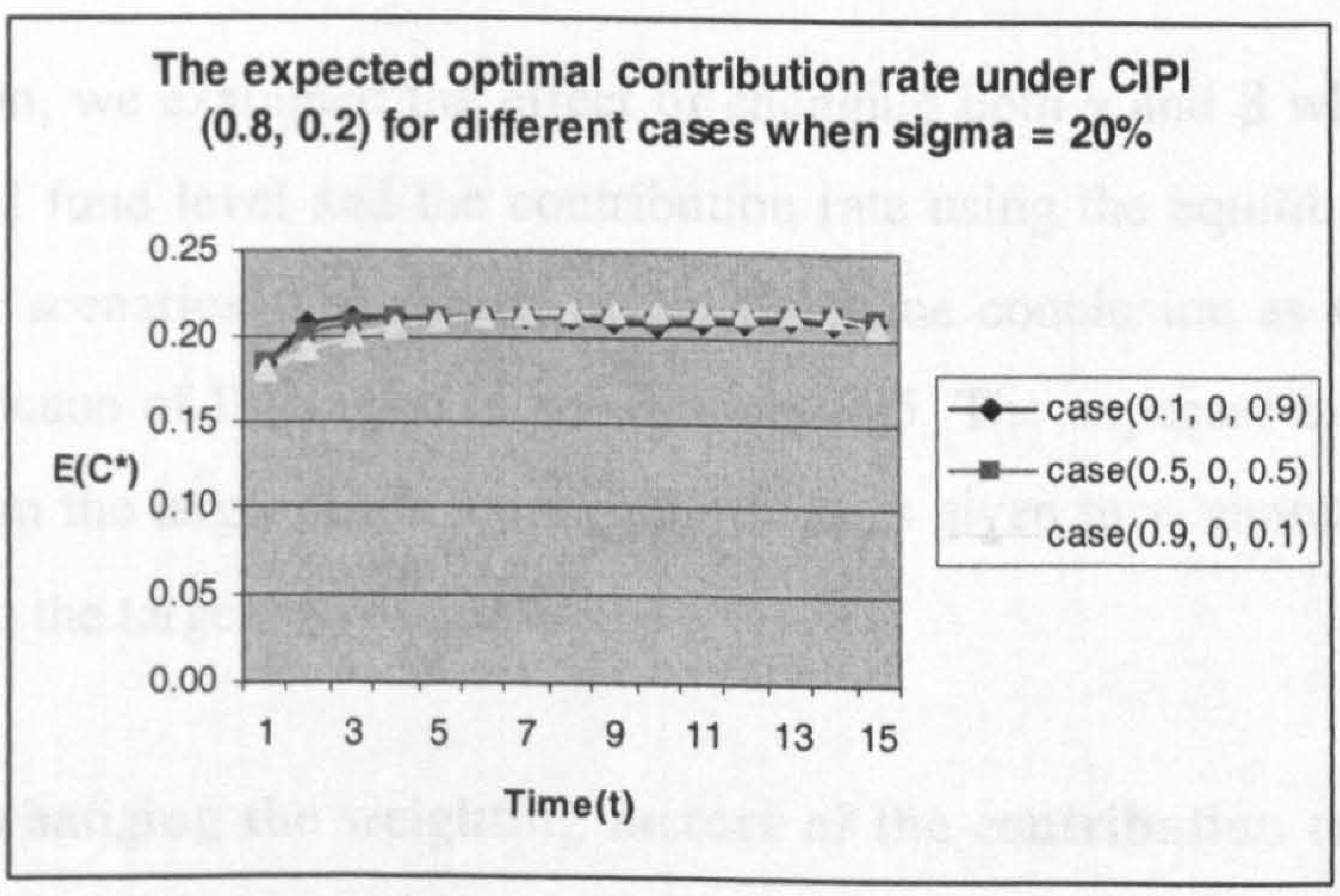


Figure 4.68



(e) The variance of F_T* and C_T*

Under the different scenarios, for the cases of (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0, 0.1) when σ = 20%, we note that the variance of F_T^* becomes higher when more importance is given to the weighting risk factor of the contribution risk γ. Simultaneously, the variance of C_T* has the smallest value in the case of (0.9, 0, 0.1).

The results for the different cases show that the variance of F_T* and C_T* are the same under the first, second and the third scenarios. The variance of both the fund level and the contribution rate is the highest under the fourth scenario and is the lowest under the fifth scenario. Table 4.24 shows the standard deviation of F_T* and C_T* under the different scenarios in the cases mentioned above when $\sigma = 20\%$.

Table 4.24

The standard deviation of F_T* and C_T* for different cases under different scenarios when $\sigma = 20\%$

	Sc(1, 0.2)		Sc(1, 0.22)		Sc(1, 0.18)		Sc(1.2, 0.2)		Sc(0.8, 0.2)	
	F_T*	C_T*	F_T*	C_T*	F_T*	C_T*	F_T*	C_T*	F_T*	C_T*
Case(0.1, 0, 0.9)	0.192	0.176	0.192	0.176	0.192	0.176	0.229	0.211	0.153	0.141
Case(0.5, 0, 0.5)	0.199	0.111	0.199	0.111	0.199	0.111	0.239	0.133	0.159	0.088
Case(0.9, 0, 0.1)	0.288	0.035	0.288	0.035	0.288	0.035	0.348	0.042	0.23	0.028

Conclusion

In this subsection, we examined the effect of changing both γ and β when $\alpha = 0$ on the expected optimal fund level and the contribution rate using the equilibrium value of B_t for the different scenarios. The results reveal the same conclusion as we have reached when the fixed value of B_t is used in subsection 4.4.5. The expected optimal fund level moves away from the target when more importance is given to γ, simultaneously, $E(C_t^*)$ becomes close to the target.

4.5.3 Effect of changing the weighting factors of the contribution rate risk and the solvency risk; γ and β (with $\alpha \neq 0$)

Here, the effect of the cross- product term risk factor α is examined when the weighting risk factors γ and β are changed with focusing on the following cases:

(0.1, 0.2, 0.9); (0.1, 0.4, 0.9); (0.1, 0.8, 0.9); (0.9, 0.2, 0.1); (0.9, 0.4, 0.1) and (0.9, 0.8, 0.1). The results of the default scenario (1, 0.2) is described in section 4.4.6 (a) since we have used the equilibrium value of B_t = 0.3. The results of the four other scenarios are explained in turn below.

(a) Second scenario: $FT_t = 1$, $CT_t = 0.22$

In the second scenario (1, 0.22), the expected optimal fund levels decrease over the control period with small changes when the value of α is increased. The decreasing trend is more obvious when more importance is given to γ although the trends of $E(F_t^*)$ are close to each other. These results are shown in Figures 4.69 and 4.70.

When σ is increased to 20%, $E(F_t^*)$ moves further away from the target with small differences between the cases when $\gamma = 0.1$ and $\beta = 0.9$ and α is increased. When more importance is given to $\gamma = 0.9$ and $\beta = 0.1$, the trends of $E(F_t^*)$ increase over the control period when α is increased to 0.8.

The distribution of F_T^* is studied in these cases in order to understand better the behaviour of $E(F_t^*)$ when the values of both α and σ are high. We find that the variance of F_T^* is high and increases to reach the maximum value when $\gamma = 0.9$ and $\alpha = 0.8$ as will be shown in subsection 4.5.3 (e).

Figure 4.69

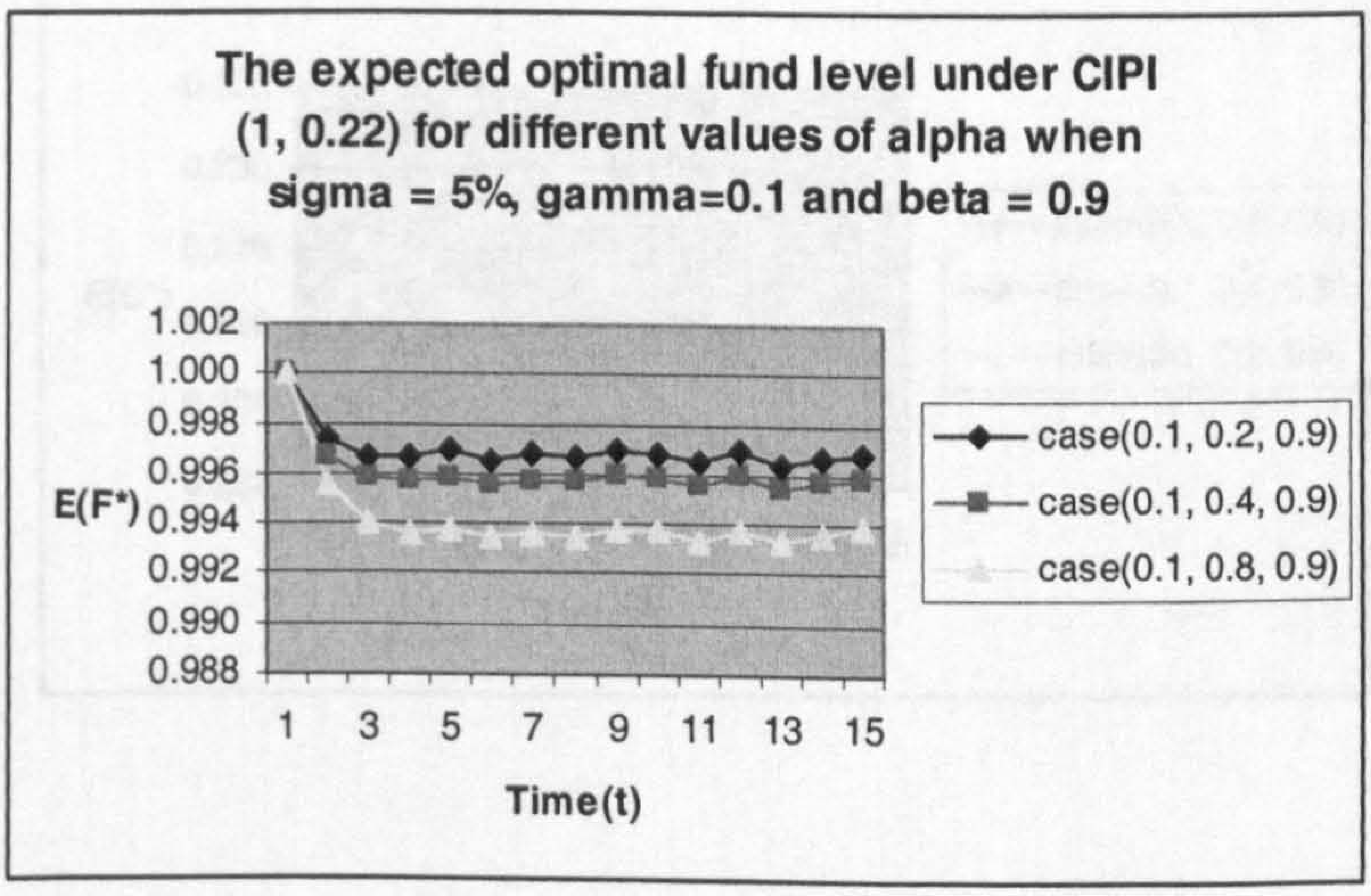
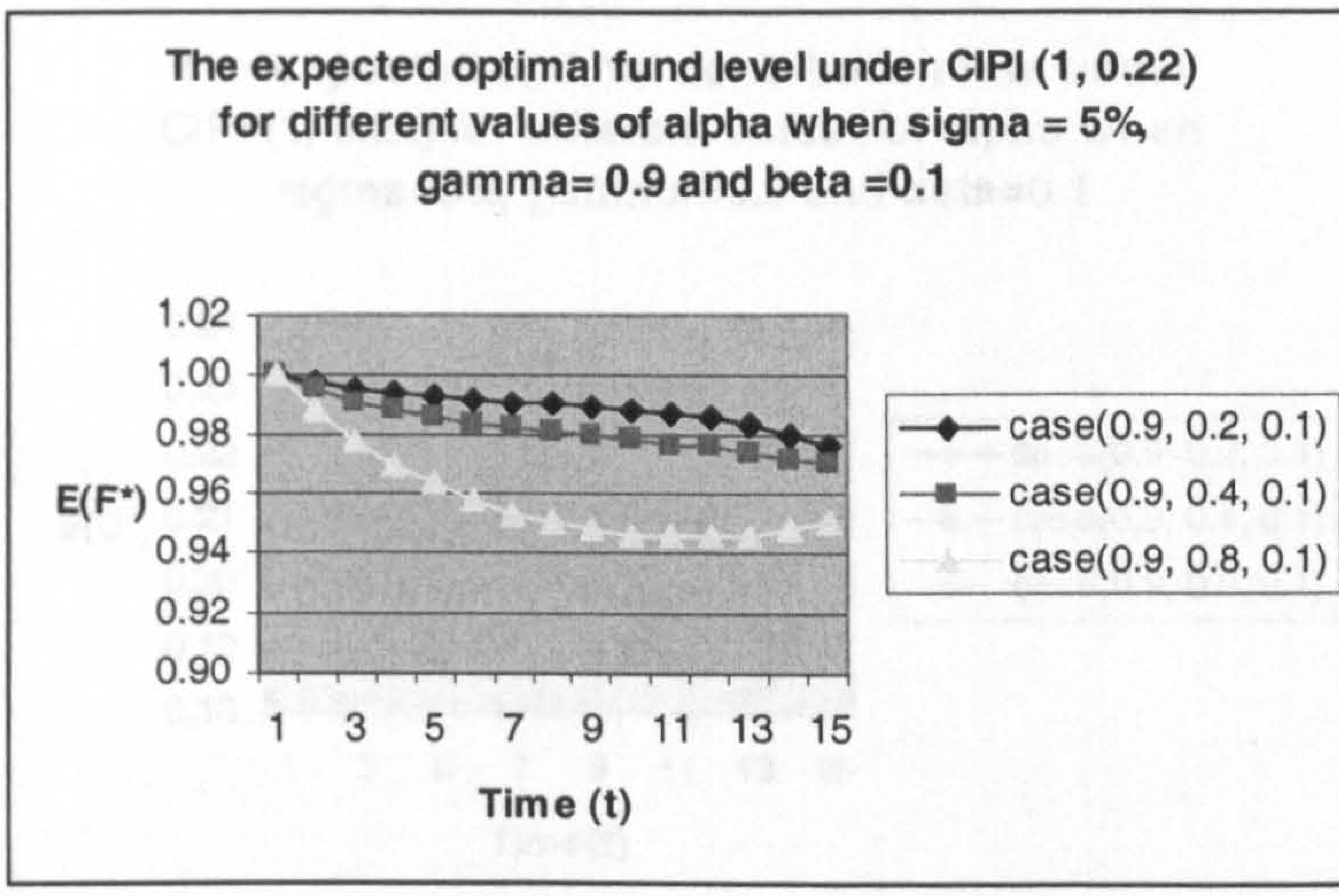


Figure 4.70



For the expected optimal contribution rate, when σ = 5% in the cases of (0.1, 0.2, 0.9); (0.1, 0.4, 0.9); (0.1, 0.8, 0.9), the trends increase with small differences between these cases, in response to the decreasing trend of $E(F_t^*)$ over the control period. It gets closer to the target when γ = 0.9 and α is small. When α is increased along with the level of volatility, it is noted that the trends of $E(C_t^*)$ are much wider in range with a tendency to move closer to the target at the end of the control period. These results are shown below.

Figure 4.71

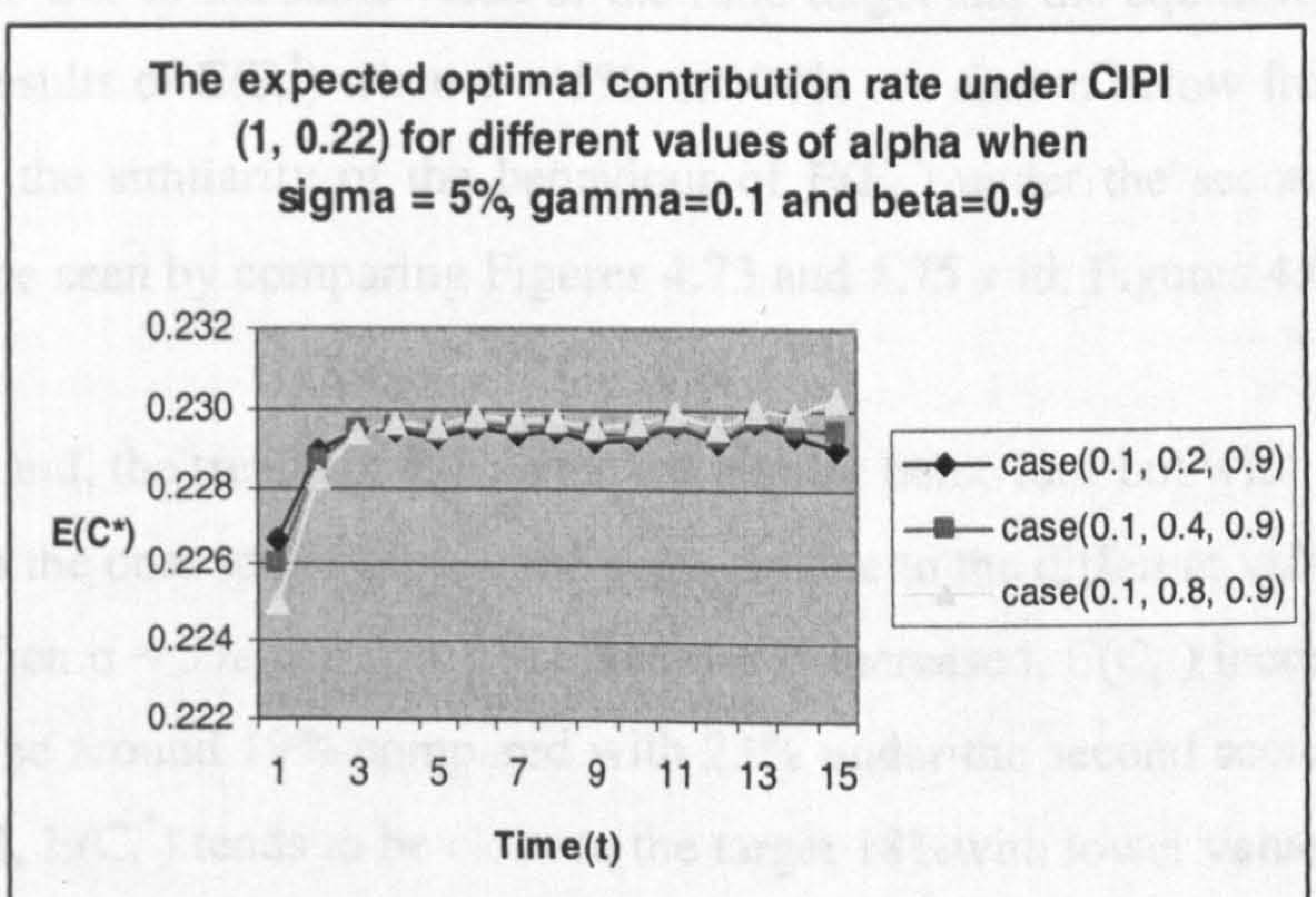
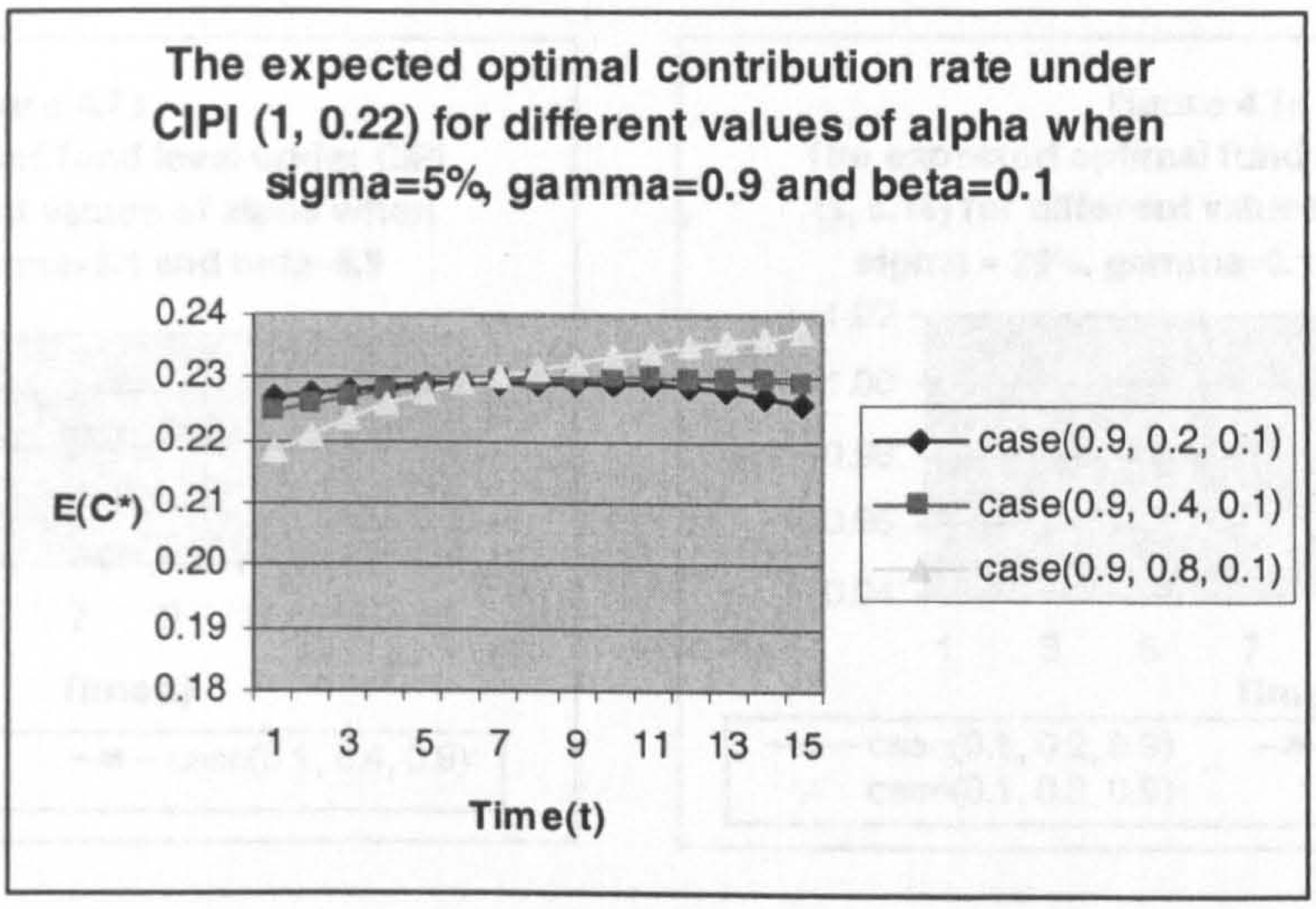


Figure 4.72



When α is increased, using the equilibrium value of $B_t = 0.32$ leads to small differences between the trends of both $E(F_t^*)$ and $E(C_t^*)$ when $\sigma = 5\%$. The trends of both $E(F_t^*)$ and $E(C_t^*)$ follow the results of the default scenario (1, 0.2), where the deviation from the targets are more obvious with a higher value of α.

(b) Third scenario: $FT_t = 1, CT_t = 0.18$

In the third scenario (1, 0.18), with different levels of volatility in the cases under study, the expected optimal fund level has a similar behaviour to the one obtained under the second scenario due to the same value of the fund target and the equilibrium value of the B_t used. The results of $E(F_t^*)$ when $\sigma = 5\%$ and 20% are shown below from Figures 4.73 to 4.76. Here, the similarity of the behaviour of $E(F_t^*)$ under the second and the third scenarios can be seen by comparing Figures 4.73 and 4.75 with Figures 4.69 and 4.70.

On the other hand, the trends of $E(C_t^*)$ have a similar behaviour but with different values compared with the ones under the second scenario due to the different value of CT_t. Thus, in the cases when $\sigma = 5\%$, $\gamma = 0.1$, $\beta = 0.9$ and α is increased, $E(C_t^*)$ increases with small differences to be around 19% compared with 23% under the second scenario. When $\gamma = 0.9$ and $\beta = 0.1$, $E(C_t^*)$ tends to be close to the target 18%with lower values of α. This can be seen in Figures 4.77 and 4.78 shown below.

Figure 4.73
The expected optimal fund level under CIPI (1, 0.18) for different values of alpha when sigma = 5%, gamma=0.1 and beta=0.9

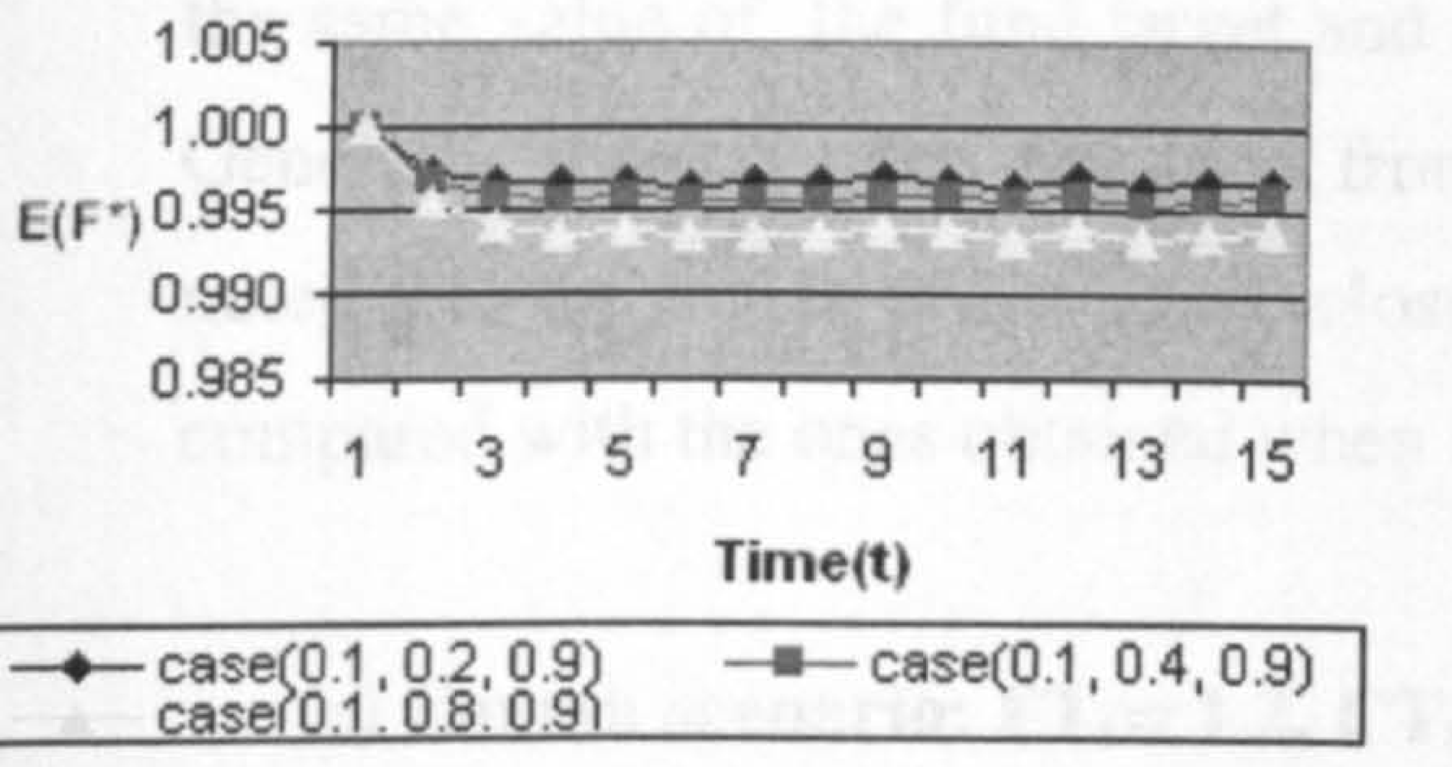


Figure 4.74
The expected optimal fund level under CIPI (1, 0.18) for different values of alpha when sigma = 20%, gamma=0.1 and beta=0.9

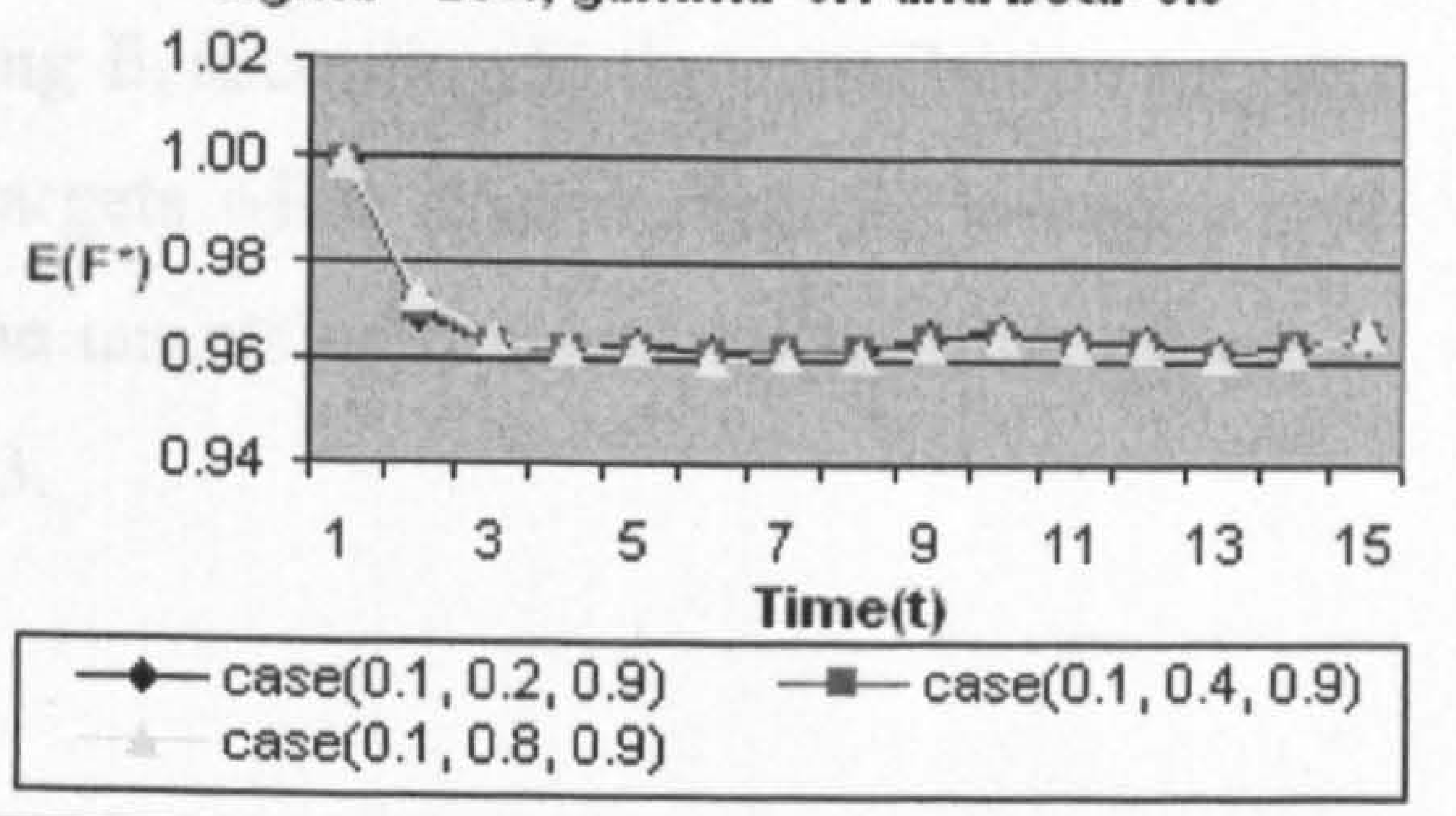


Figure 4.75
The expected optimal fund level under CIPI (1, 0.18) for different values of alpha when sigma = 5%, gamma=0.9 and beta=0.1

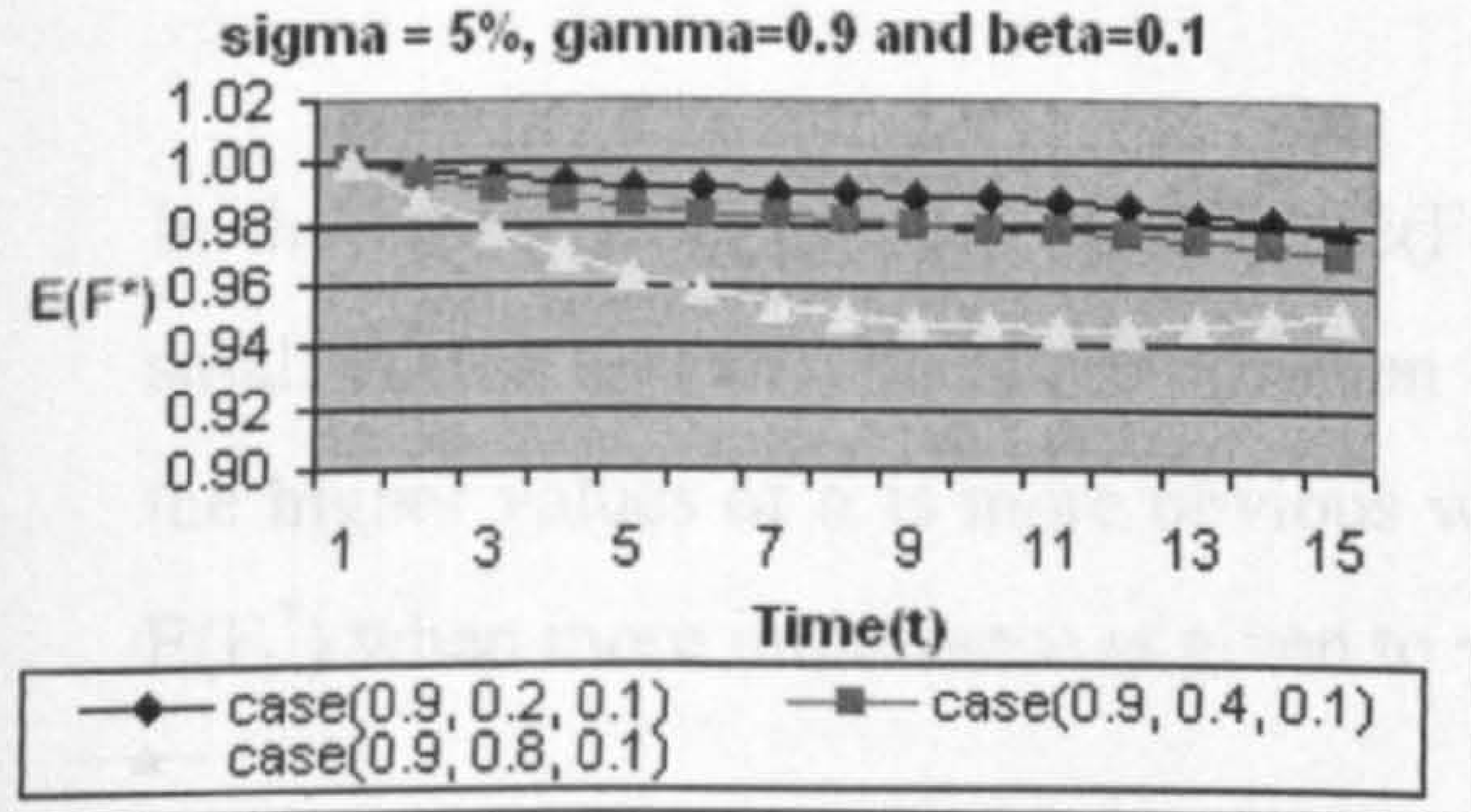


Figure 4.76
The expected optimal fund level under CIPI (1, 0.18) for different values of alpha when sigma = 20%,gamma=0.9 and beta=0.1

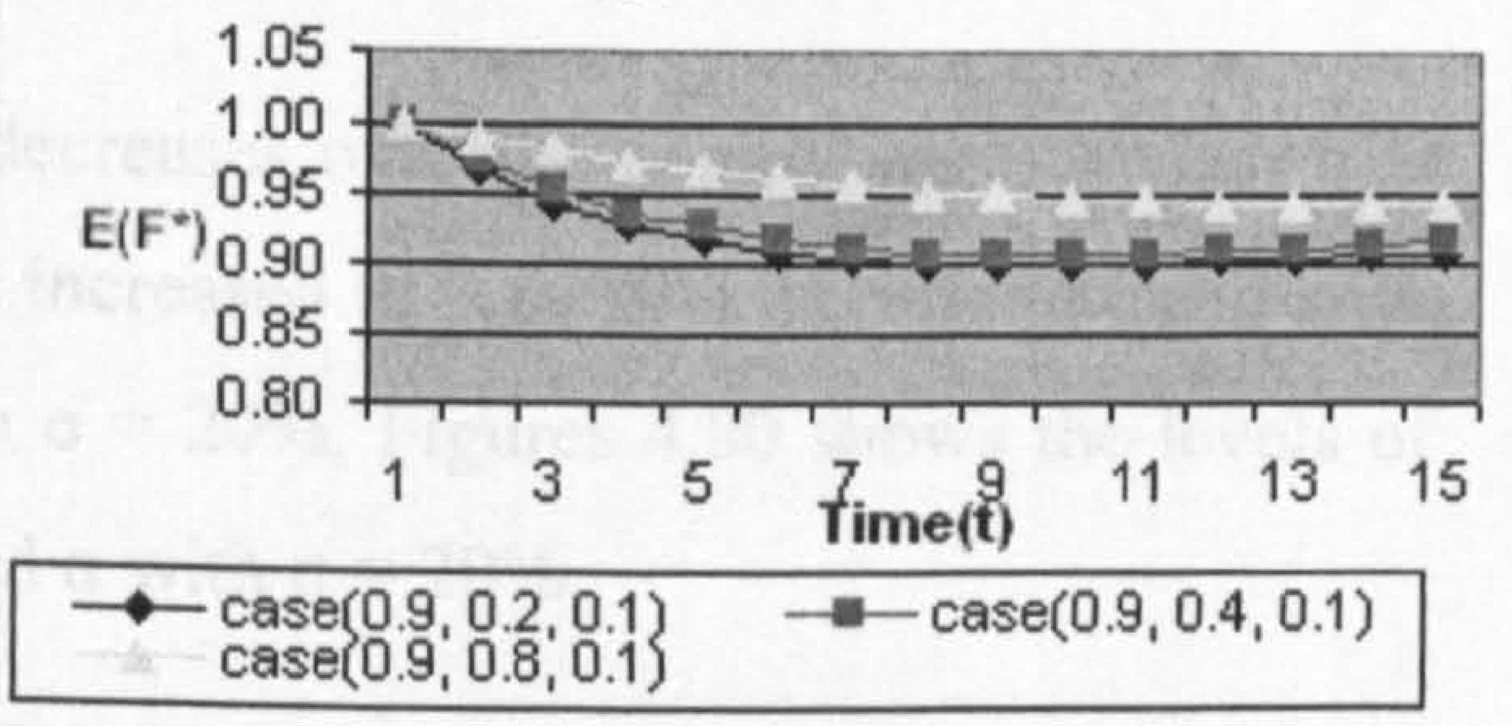


Figure 4.77
The expected optimal contribution rate under CIPI(1, 0.18) for different values of alpha when sigma = 5%, gamma=0.1 and beta=0.9

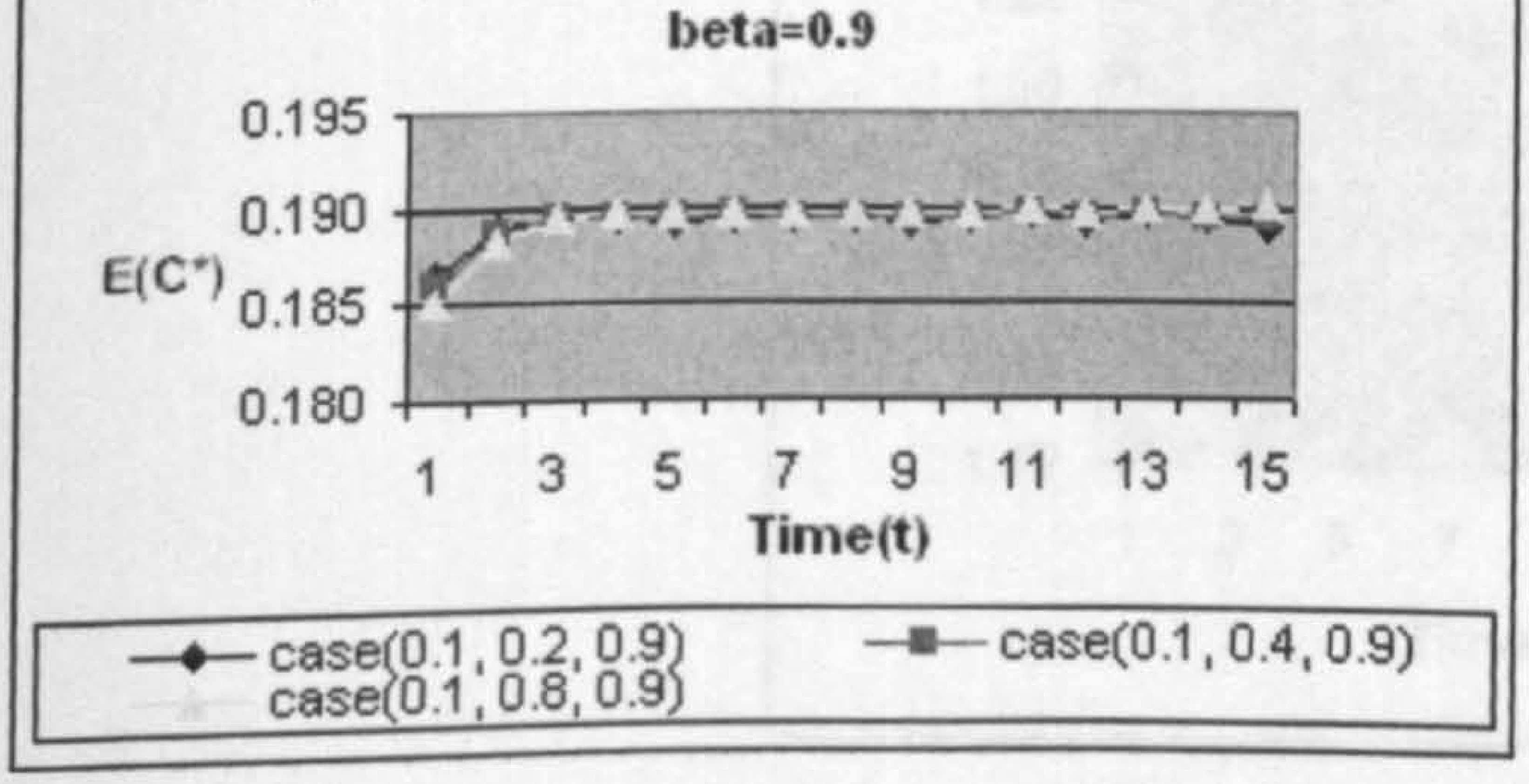
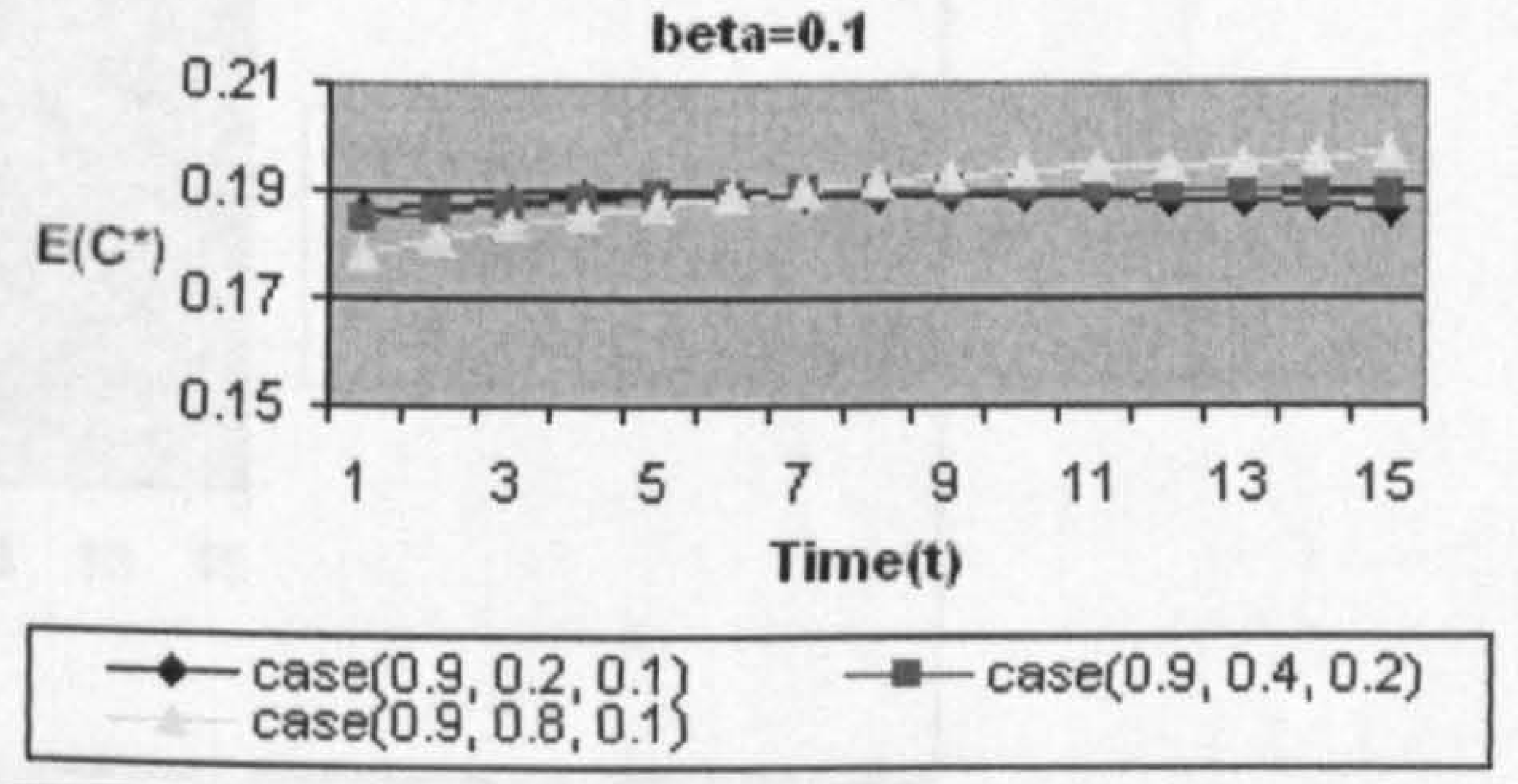


Figure 4.78
The expected optimal contribution rate under CIPI (1, 0.18) for different values of alpha when sigma = 5%, gamma=0.9 and beta=0.1



When the equilibrium value of B_t is used, the trends of both $E(F_t^*)$ and $E(C_t^*)$ have a similar behaviour to the ones when $B_t = 0.3$, except in the cases when $\sigma = 20\%$, $\gamma = 0.9$, $\beta = 0.1$ and α is increased. This is because $E(F_t^*)$, under the third scenario, follows a similar behaviour to the one obtained under the first and second scenarios, due to using the same value of the fund target and changing B_t according to the contribution targets. Generally, there is more deviation from the targets when α is increased. However, it is noted that the trends of $E(F_t^*)$ are closer to the targets with using the equilibrium value compared with the ones obtained when $B_t = 0.3$.

(c) Fourth scenario: $FT_t = 1.2$, $CT_t = 0.2$

Under the fourth scenario (1.2, 0.2), $E(F_t^*)$ decreases with small differences when $\sigma = 5\%$, $\gamma = 0.1$, $\beta = 0.9$ and α is increased. When σ is increased to 20%, $E(F_t^*)$ moves further away from the target with small differences between the cases with increasing α as shown in Figure 4.79.

In the cases when $\gamma = 0.9$, $\beta = 0.1$, $E(F_t^*)$ decreases over the control period when α is small with a tendency to move up when α is increased to 0.8. This increasing trend with the higher values of α is more obvious when $\sigma = 20\%$. Figures 4.80 shows the levels of $E(F_t^*)$ when more importance is given to γ and α with $\sigma = 20\%$.

Figure 4.79

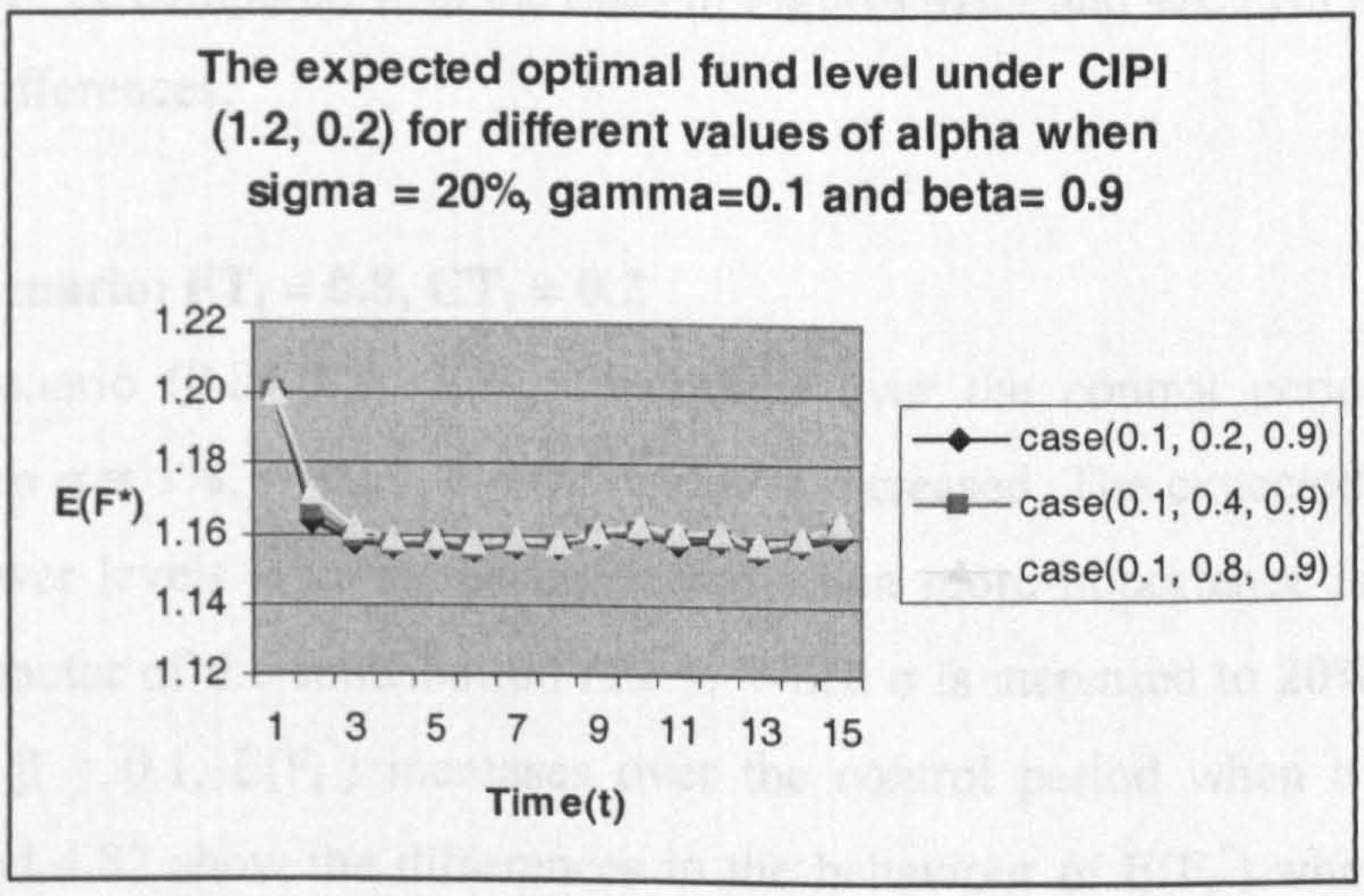
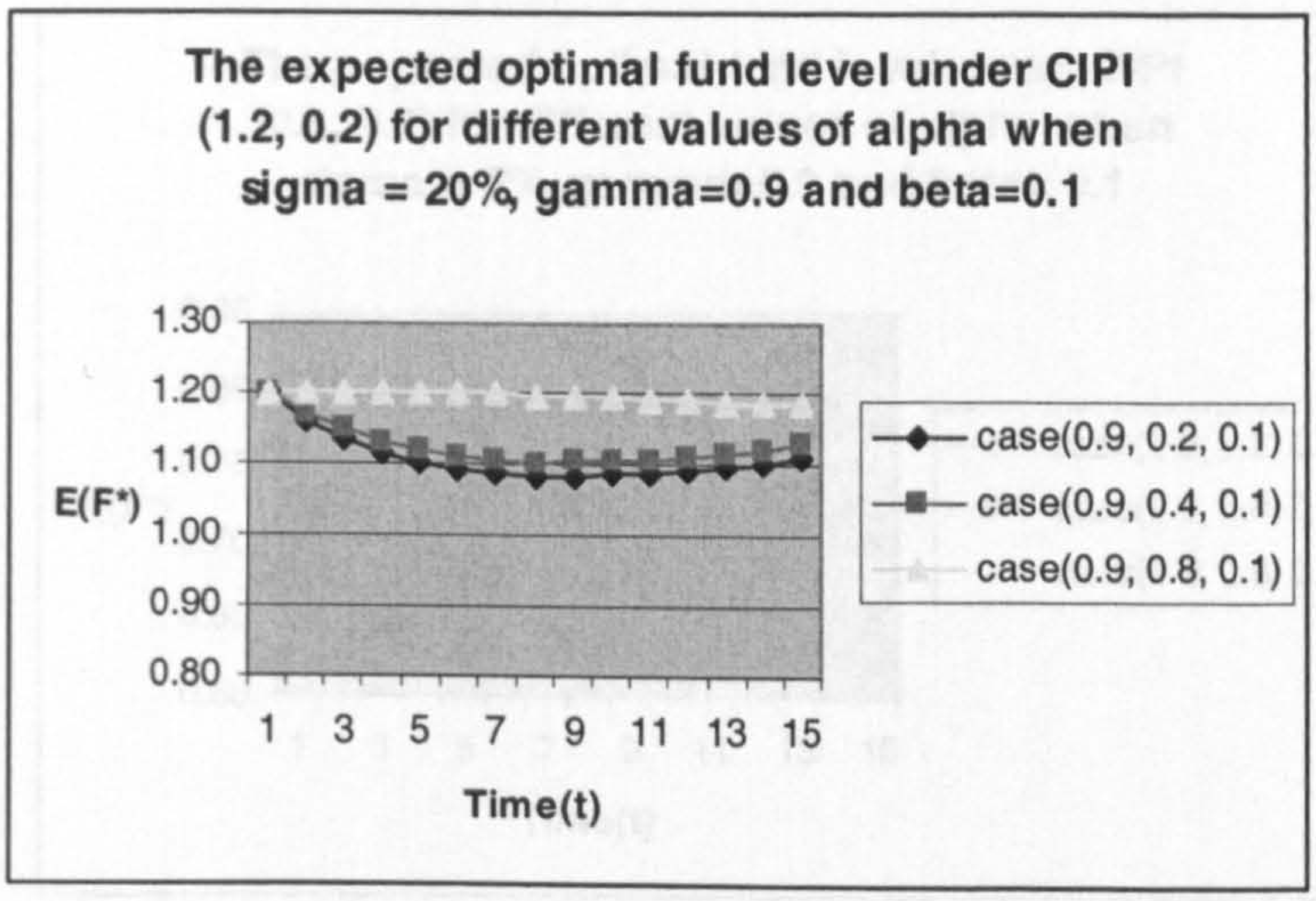


Figure 4.80



The expected optimal contribution rate is close to the target with small differences in the cases of (0.9, 0.2, 0.1), (0.9, 0.4, 0.1) and (0.9, 0.8, 0.1) when σ = 5%. The trends of $E(C_t^*)$ are wider in range when σ is increased to 20%. The values become very close to the target in the last case of (0.9, 0.8, 0.1) in response to the level of $E(F_t^*)$ due to the high value of the fund target.

When these results are compared with the ones obtained when $B_t = 0.3$, we find that $E(F_t^*)$ have similar trends where they move towards the target with increasing α. This behaviour is more obvious when both γ and σ are high. However, when B_t= 0.31, the trends have less deviation from the targets. For example, the trends of $E(F_t^*)$ in Figures 4.79 and 4.80 can be compared with the ones in Figures 4A.7 and 4A.9 (in Appendix 4A) to show these differences.

(d) Fifth scenario: $FT_t = 0.8$, $CT_t = 0.2$

In the fifth scenario (0.8, 0.2), $E(F_t^*)$ decreases over the control period with small differences when σ = 5%, γ = 0.1, β = 0.9 and α is increased. The expected optimal fund levels reach lower levels with increasing α and when more importance is given to the weighting risk factor of the contribution rate γ. When σ is increased to 20%, in the cases when γ = 0.9, β = 0.1, $E(F_t^*)$ increases over the control period when α is increased. Figures 4.81 and 4.82 show the differences in the behaviour of $E(F_t^*)$ when the level of volatility and the risk factor γ are high with increasing the value of α.

Figure 4.81

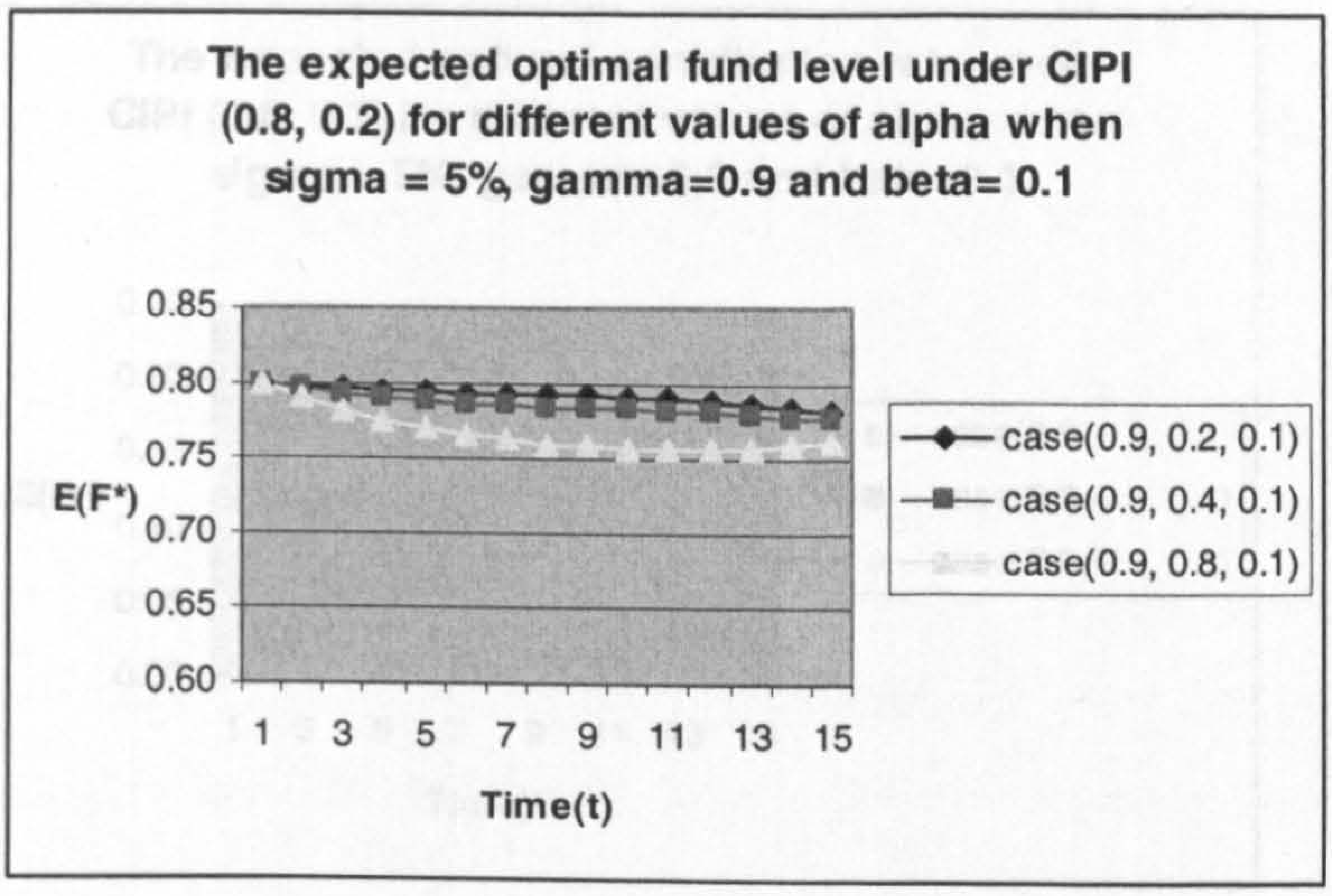
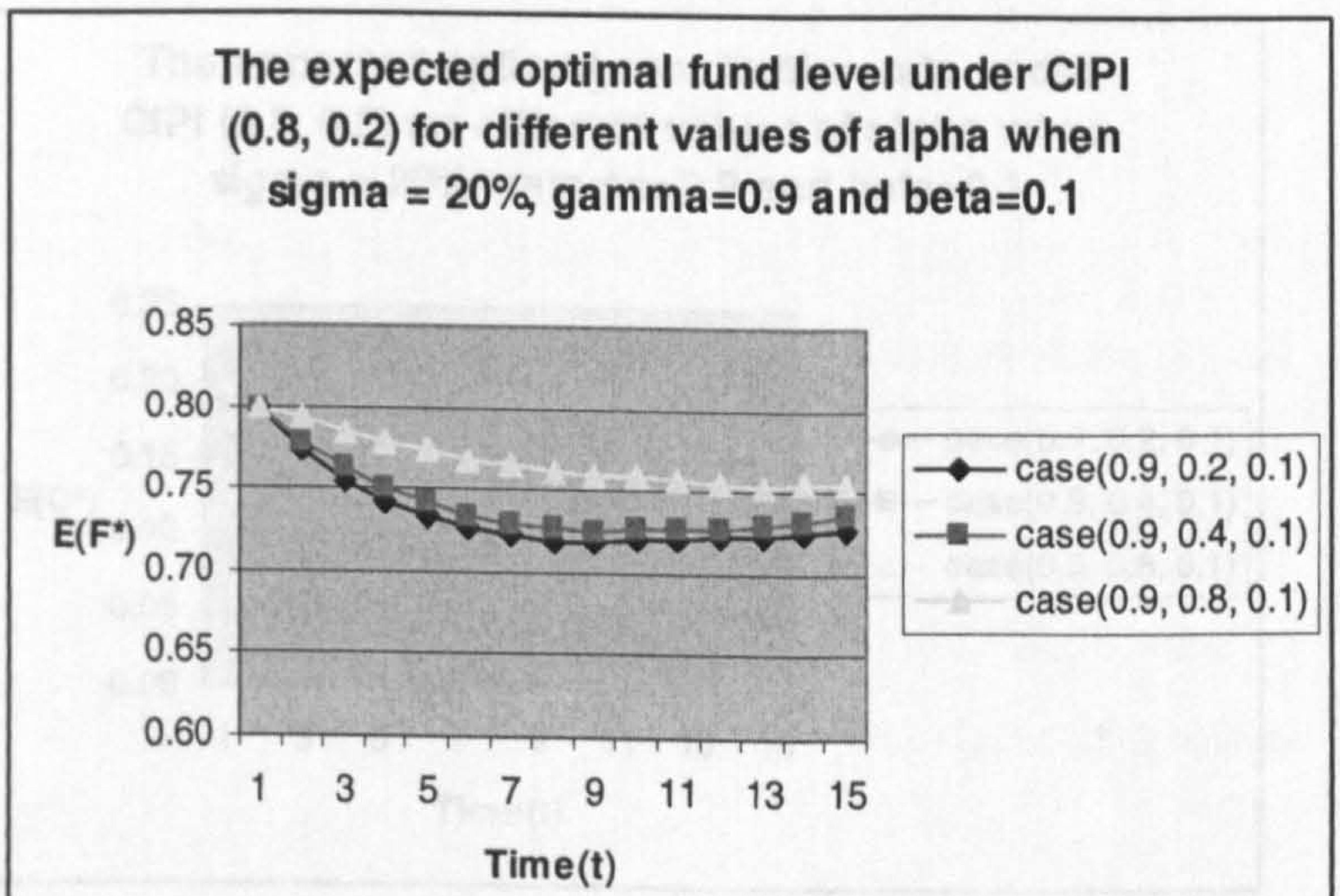


Figure 4.82



The expected optimal contribution rate increases over the control period with small differences in the cases of (0.1, 0.2, 0.9), (0.1, 0.4, 0.9) and (0.1, 0.8, 0.9) when σ = 5%. It becomes closer to the target when more importance is given to γ and with small values of α. When σ is increased to 20%, $E(C_t^*)$ deviates more from the target compared with the ones when σ = 5%, however, it becomes close to the target when γ is high and α is small. Figures 4.83 and 4.84 show the expected optimal contribution rate when γ = 0.9, β= 0.1 and α is increased with σ = 5% and 20%

Figure 4.83

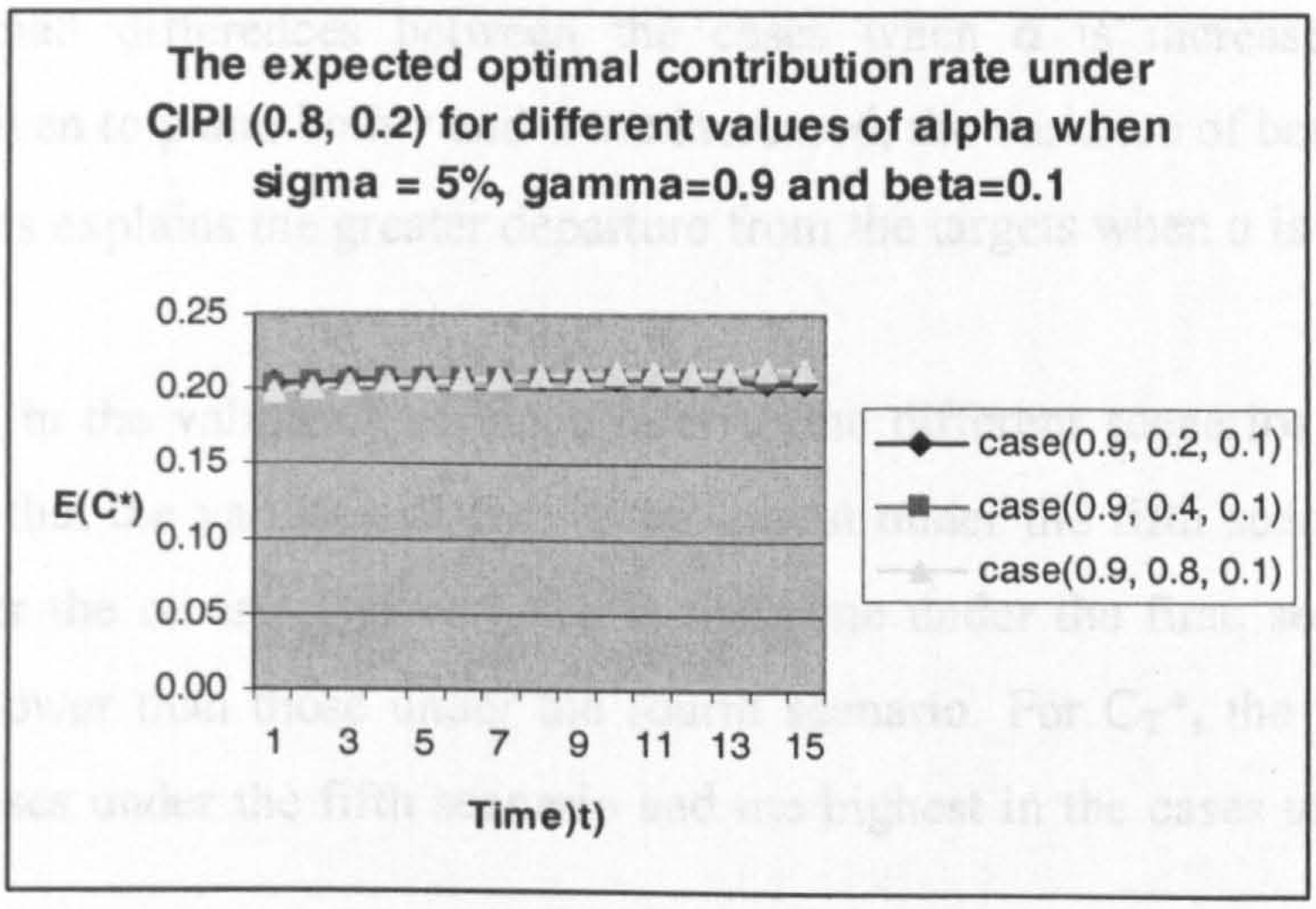
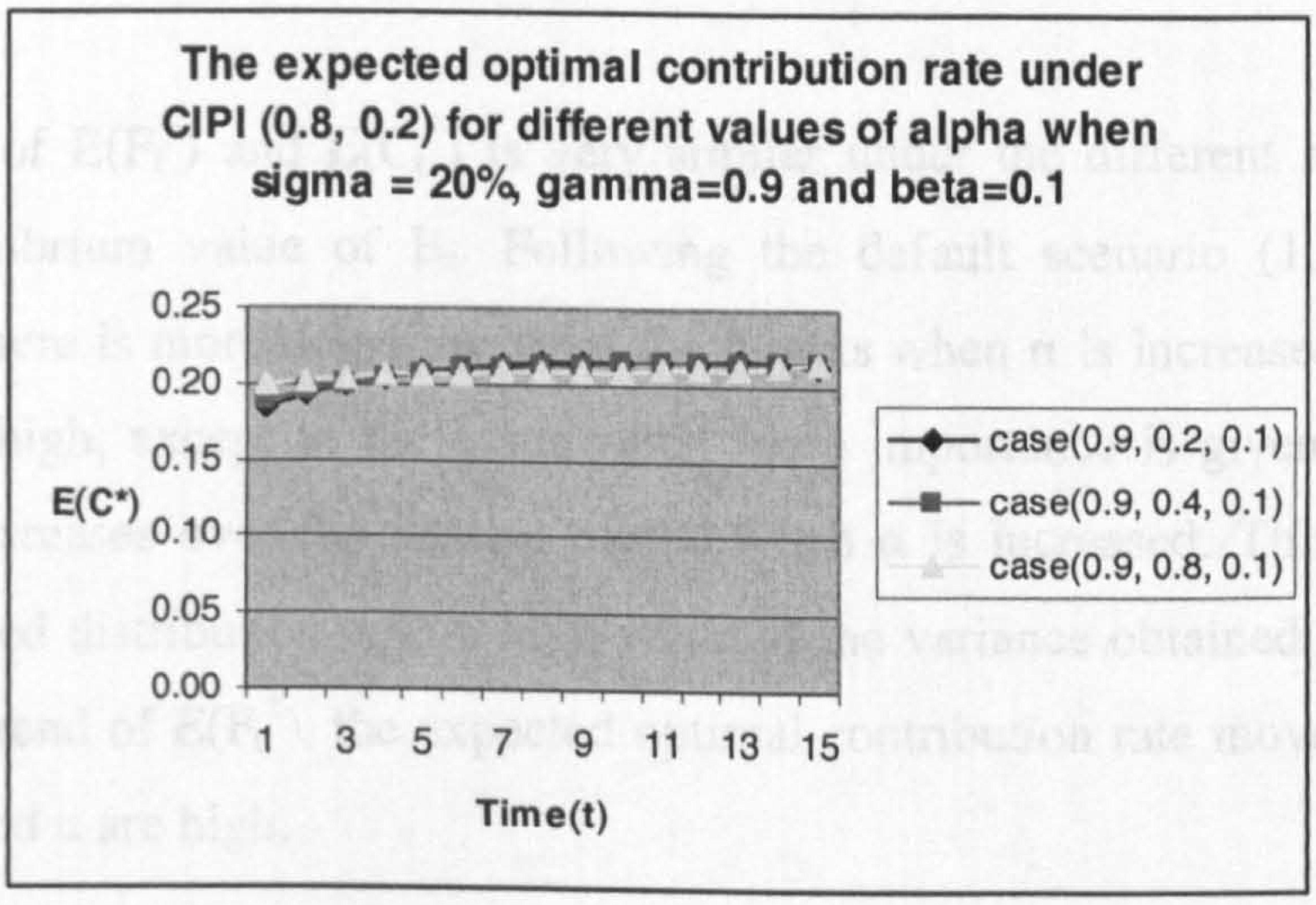


Figure 4.84



Therefore, using the equilibrium value of B_t leads to similar results to those obtained under the previous scenarios. There are small differences between the different cases. When $\sigma = 20\%$, $\gamma = 0.9$, $\beta = 0.1$ and $\alpha = 0.8$, we find that $E(F_t^*)$ moves up towards the target as can be seen from Figure 4.82. This emphasizes the fact that $E(F_t^*)$ has a similar behaviour under the five different scenarios due to using the equilibrium value of B_t.

(e) The variance of F_T* and C_T*

The variance of the optimal fund level and contribution rate for the final T is examined to understand their behaviour when $\sigma = 20\%$ and α is changed. For all the scenarios under study, we note that in the cases when $\gamma = 0.1$, $\beta = 0.9$, the variance of F_T* increases slightly when α is increased. Simultaneously, the variance of C_T* decreases slightly. This explains the small differences between the cases when α is increased. When less importance is given to β and both γ and α are increased, the variance of both F_T* and C_T* increases and this explains the greater departure from the targets when α is increased.

The differences in the values of variance refer to the different scenarios used. In other words, we find that the variance of F_T* is the lowest under the fifth scenario compared with those under the others. The variance is the same under the first, second and third scenarios, and lower than those under the fourth scenario. For C_T*, the variance is the lowest in the cases under the fifth scenario and the highest in the cases under the fourth scenario.

Conclusion

The behaviour of $E(F_t^*)$ and $E(C_t^*)$ is very similar under the different scenarios when using the equilibrium value of B_t. Following the default scenario (1, 0.2), we can conclude that there is more departure from the targets when α is increased and the level of volatility is high, except in the cases when more importance is given to γ. In these cases, $E(F_t^*)$ increases over the control period when α is increased. This is due to the positively skewed distribution with a high value of the variance obtained. In response to the increasing trend of $E(F_t^*)$, the expected optimal contribution rate moves closer to the target when γ and α are high.

Therefore, the level of $E(F_t^*)$ and $E(C_t^*)$ become the closest to the target under the fourth scenario compared with the other scenarios. This results from the highest value of the variance of F_T* and C_T* obtained under the fourth scenario.

Summary

The properties of the cost-induced performance index are explored in this chapter, in order to understand how the expected optimal fund level and contribution rate are affected by changing the parameters in the models. As a numerical illustration, a sensitivity analysis has been carried out with 10,000 simulations for the stochastic models. The effect of changing θ, σ, FT_t, CT_t and F_0 are tested first, then, followed by analysing the effect of the weighting risk factors γ, β and α. The analysis has been carried out using a round value of the benefit outgo $B_t = 0.3$ for different choices of the contribution and fund targets, different levels of volatility and different values of the weighting risk factors. We can summarise the main findings below:

- Increasing the level of volatility leads to more deviation from the target;
- giving more importance to the weighting risk factor of the solvency risk β leads $E(F_t^*)$ to become closer to the target, while $E(C_t^*)$ moves further away from the target;
- increasing α leads to more deviation from the target.

The equilibrium value of B_t is used to examine the effect of changing θ and the weighting risk factors γ, β and α. Under the different scenarios, the results of $E(F_t^*)$ and $E(C_t^*)$ follow the ones obtained under the default scenario (1, 0.2) when the value of $B_t = 0.3$ is used.

Finally, it is important to have a complete understanding of our models to apply the same procedures for the ratio-induced performance index model to order to specify the differences between both models, and that will be the issue to cover in chapter 5.

Appendix 4A

Effect of changing the weighting factors of the contribution rate risk and the solvency risk; γ and β (with $\alpha \neq 0$)

Figures of Fourth scenario (1.2, 0.2), when σ = 5% and 20%

Figure 4A.1
The expected optimal fund level under CIPI (1.2, 0.2) for different values of alpha when sigma = 5%, gamma=0.1 and beta=0.9

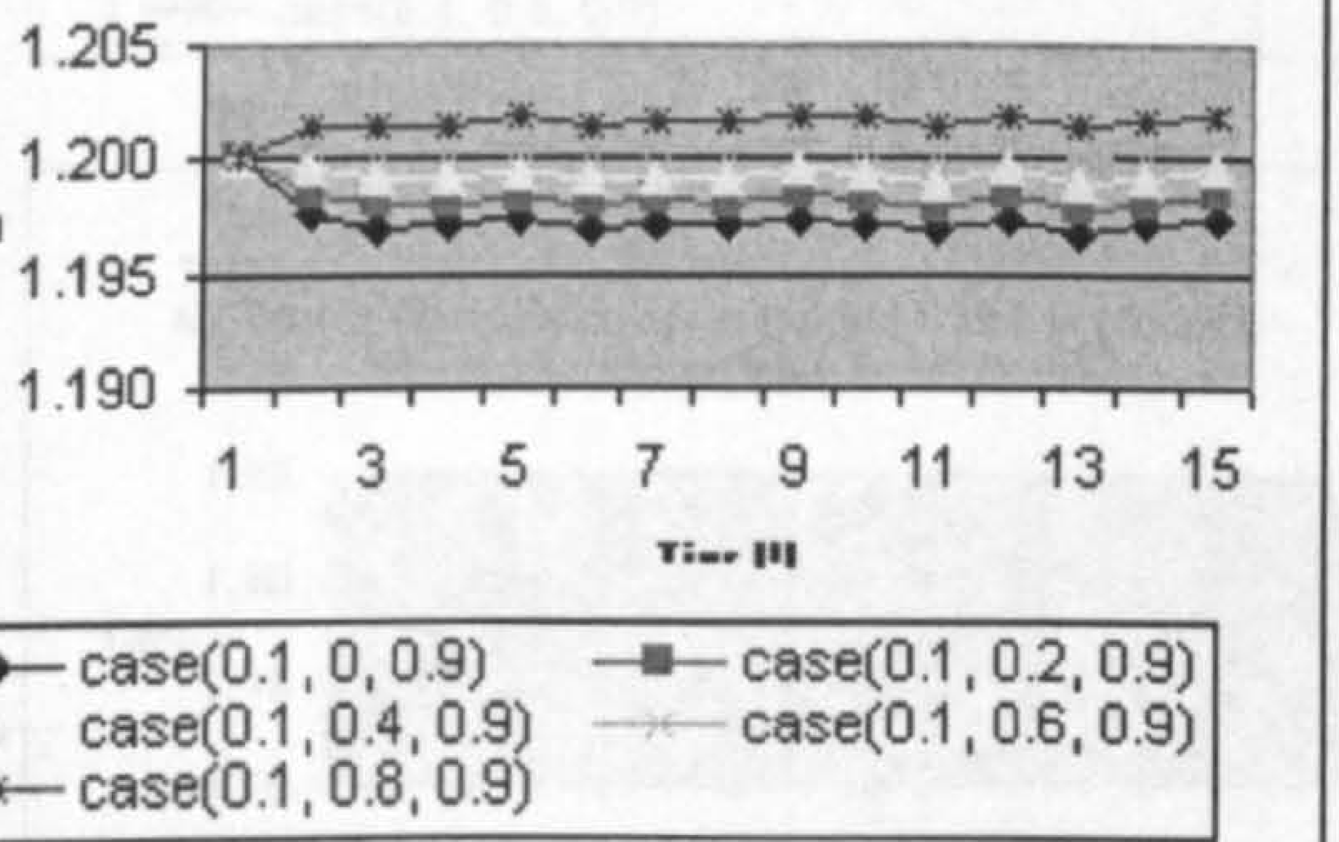


Figure 4A.2
The expected optimal fund level under CIPI (1.2, 0.2) for different levels of alpha when sigma = 5%, gamma=0.5 and beta=0.5

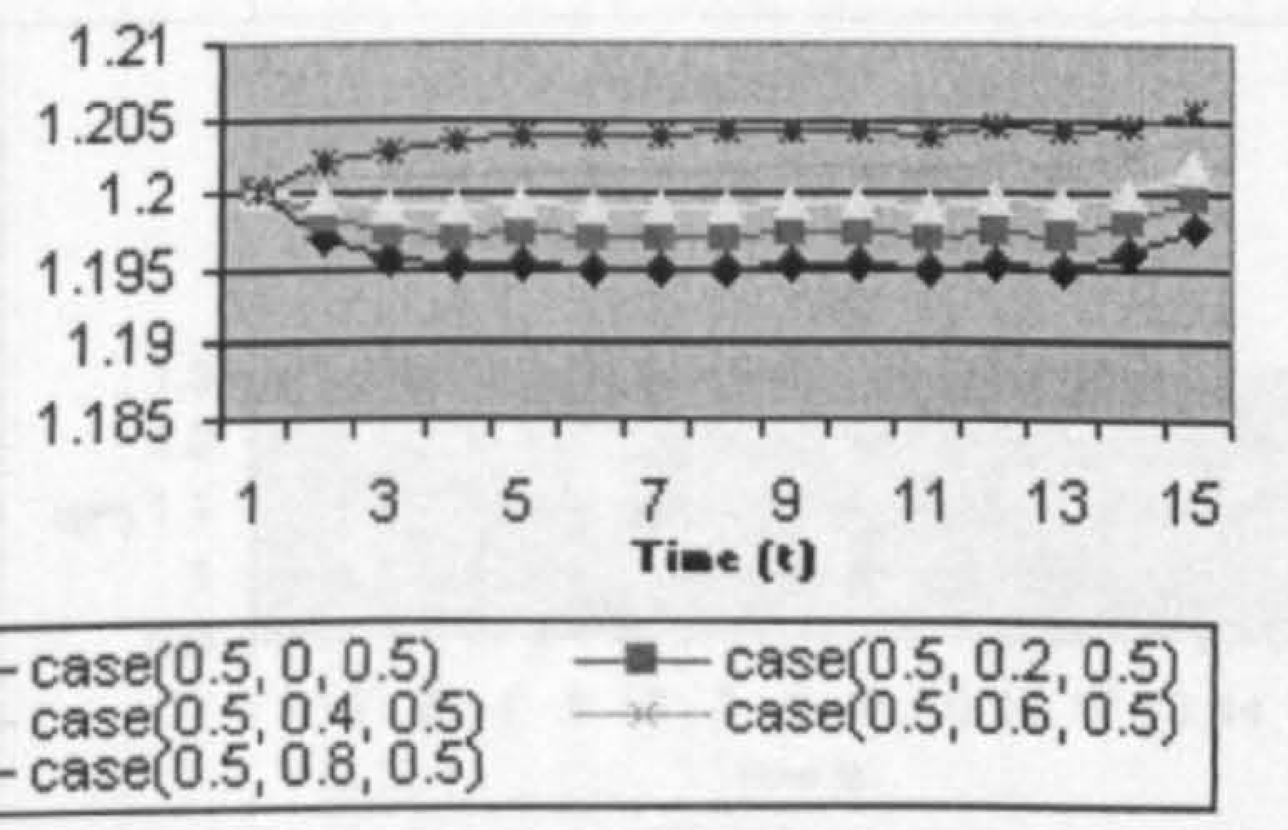


Figure 4A.3
The expected optimal fund level under CIPI (1.2, 0.2) for different values of alpha when sigma = 5%, gamma=0.9 and beta=0.1

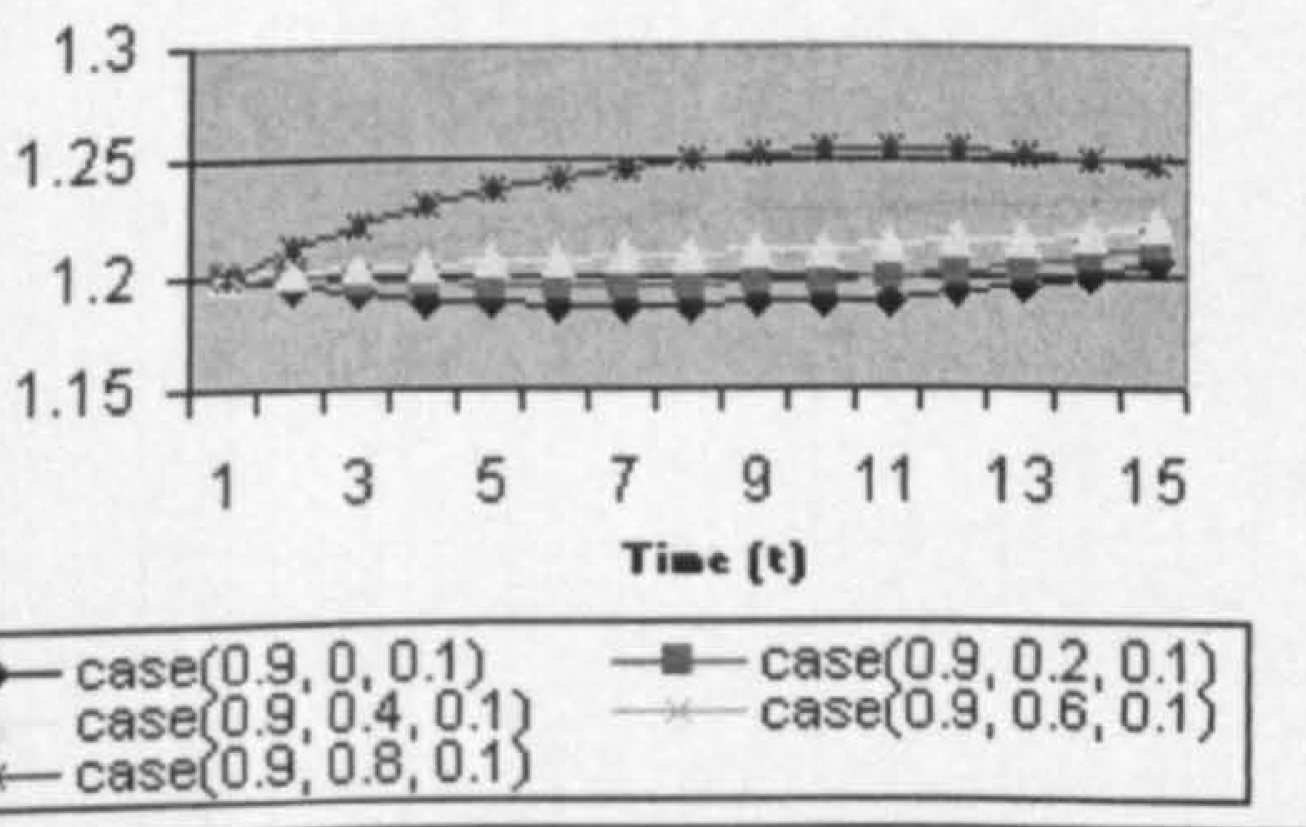


Figure 4A.4
The expected optimal contribution rate under CIPI (1.2, 0.2) for different values of alpha at sigma = 5%, gamma=0.1 and beta=0.9

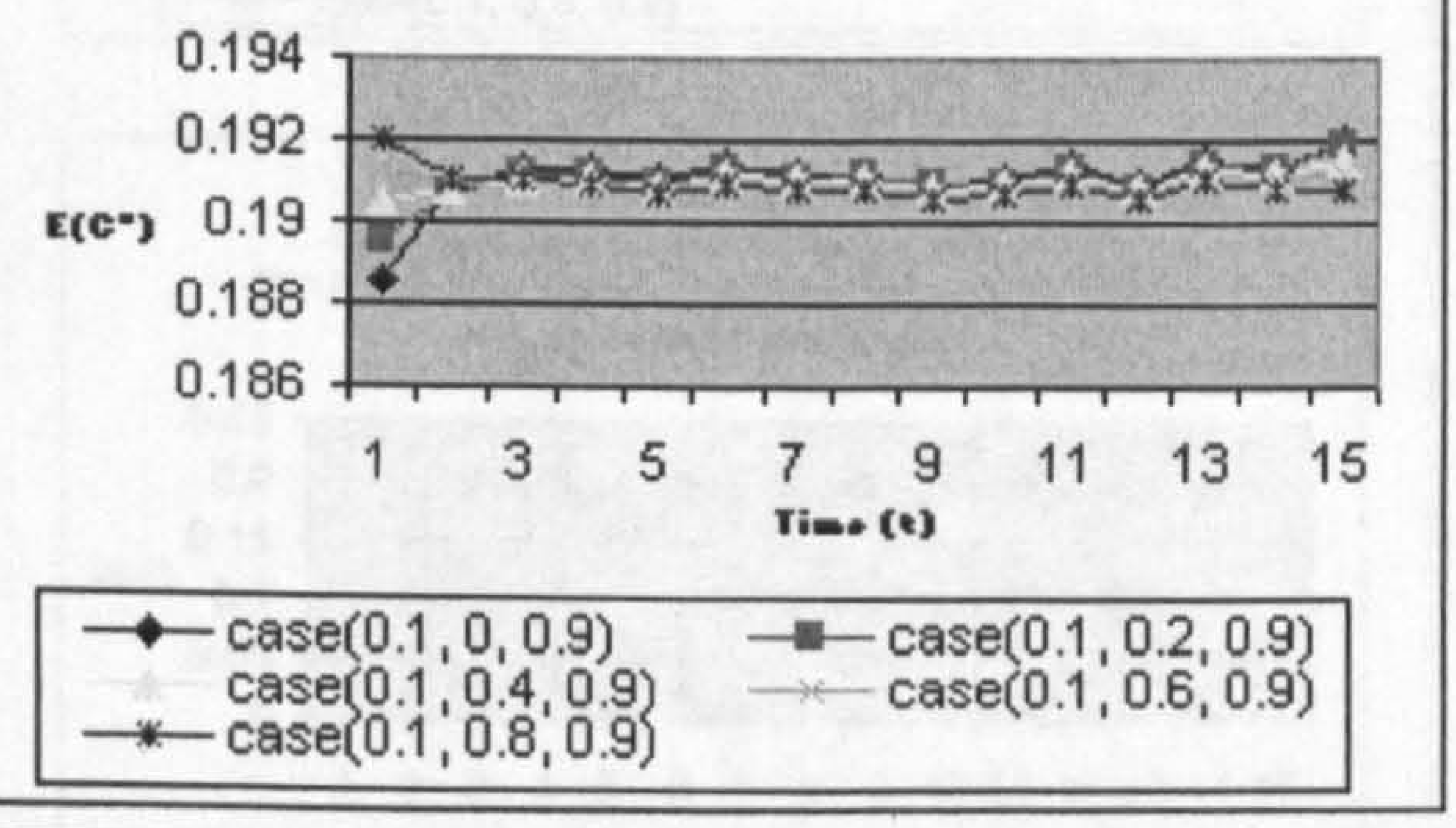


Figure 4A.5
The expected optimal contribution rate under CIPI (1.2, 0.2) for different values of alpha at sigma = 5%, gamma=0.5 and beta=0.5

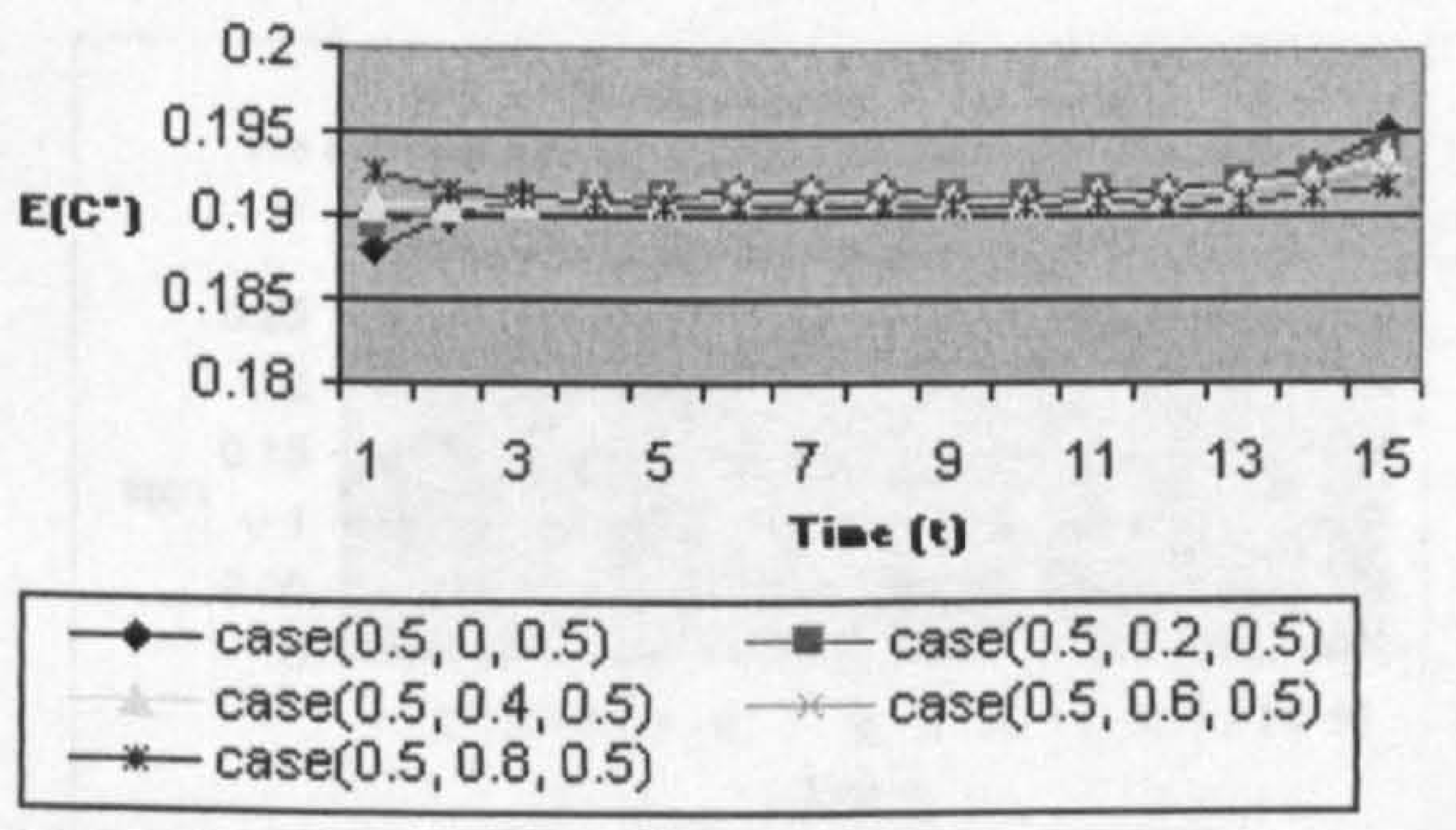


Figure 4A.6
The expected optimal contribution rate under CIPI (1.2, 0.2) for different values of alpha when sigma = 5%, gamma=0.9 and beta=0.1

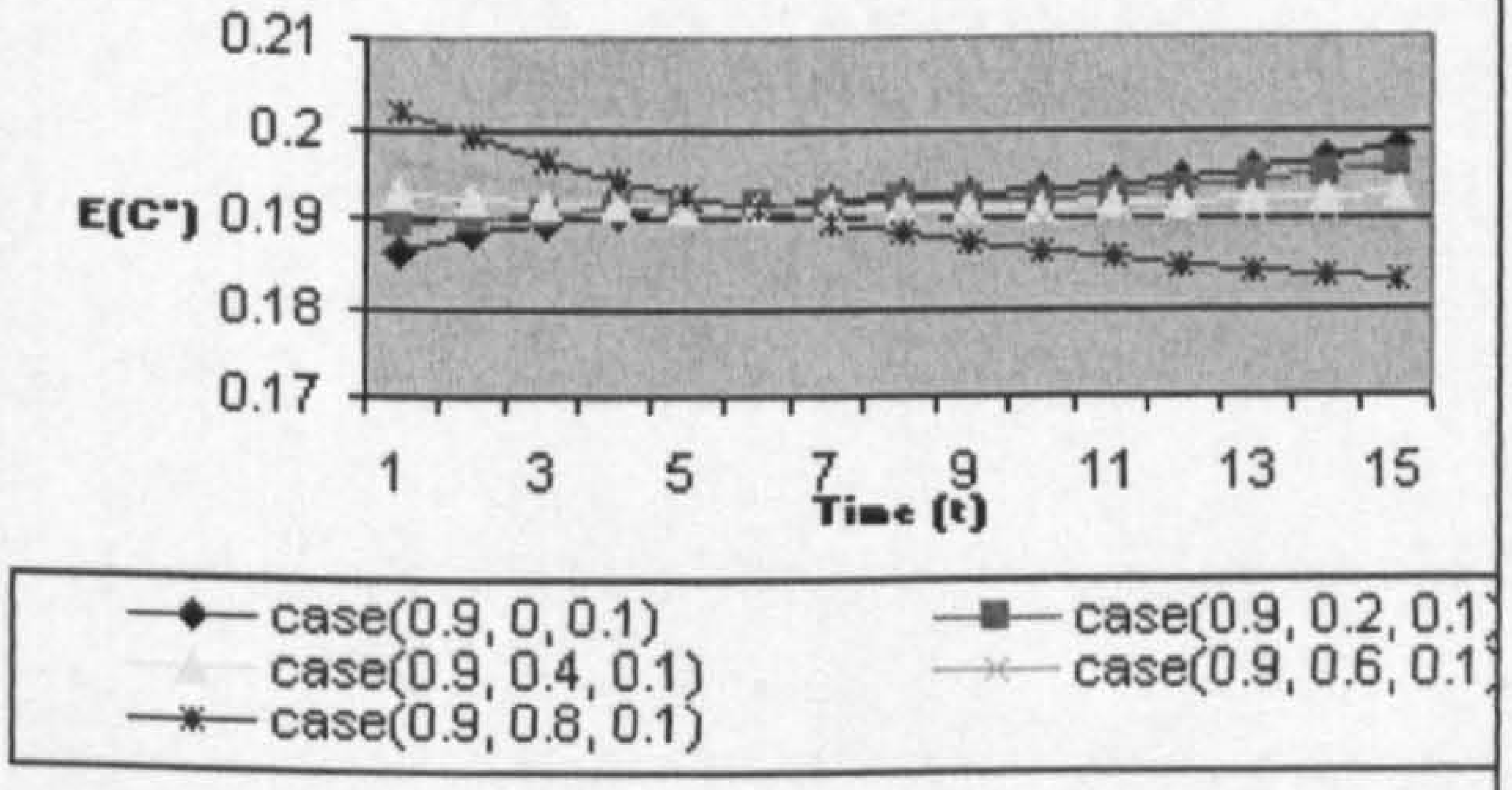


Figure 4A.7
The expected optimal fund level under CIPI (1.2, 0.2) for different values of alpha when sigma = 20%, gamma=0.1 and beta=0.9

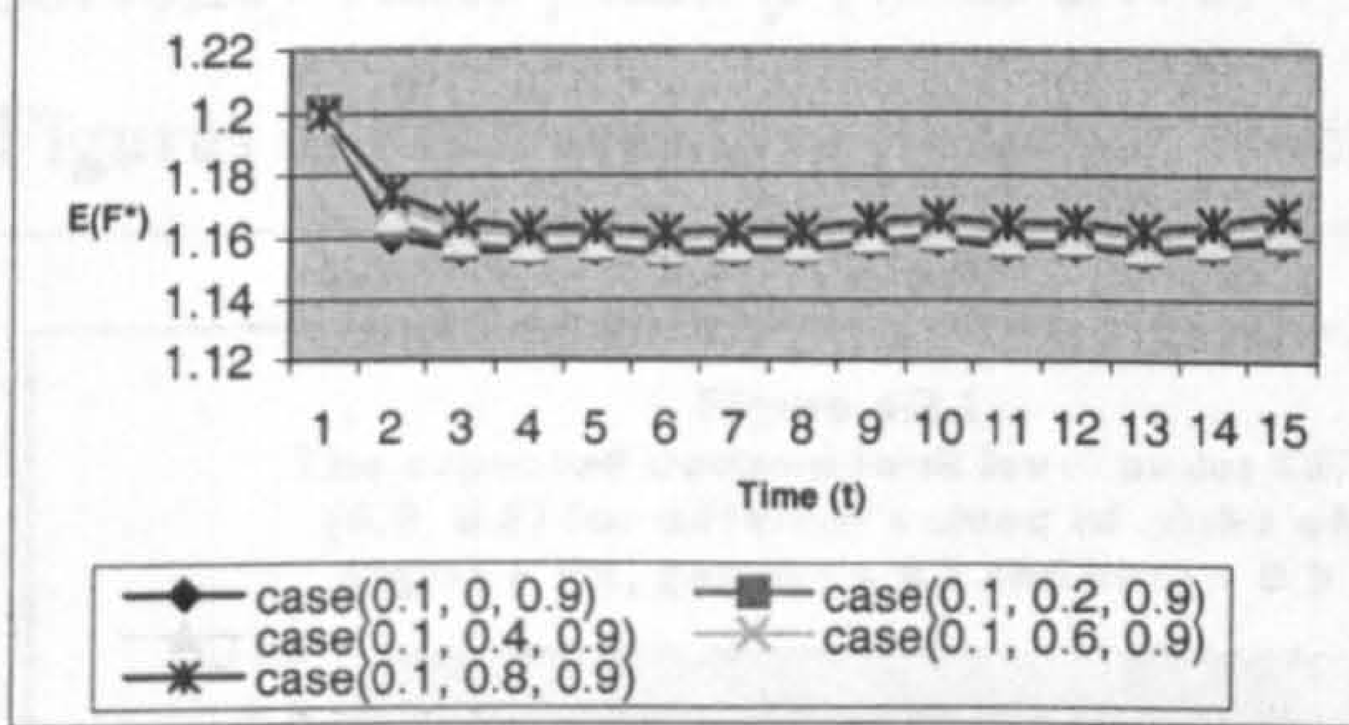


Figure 4A.10
The expected optimal contribution rate under CIPI (1.2, 0.2) for different values of alpha when sigma = 20%, gamma=0.1 and beta=0.9

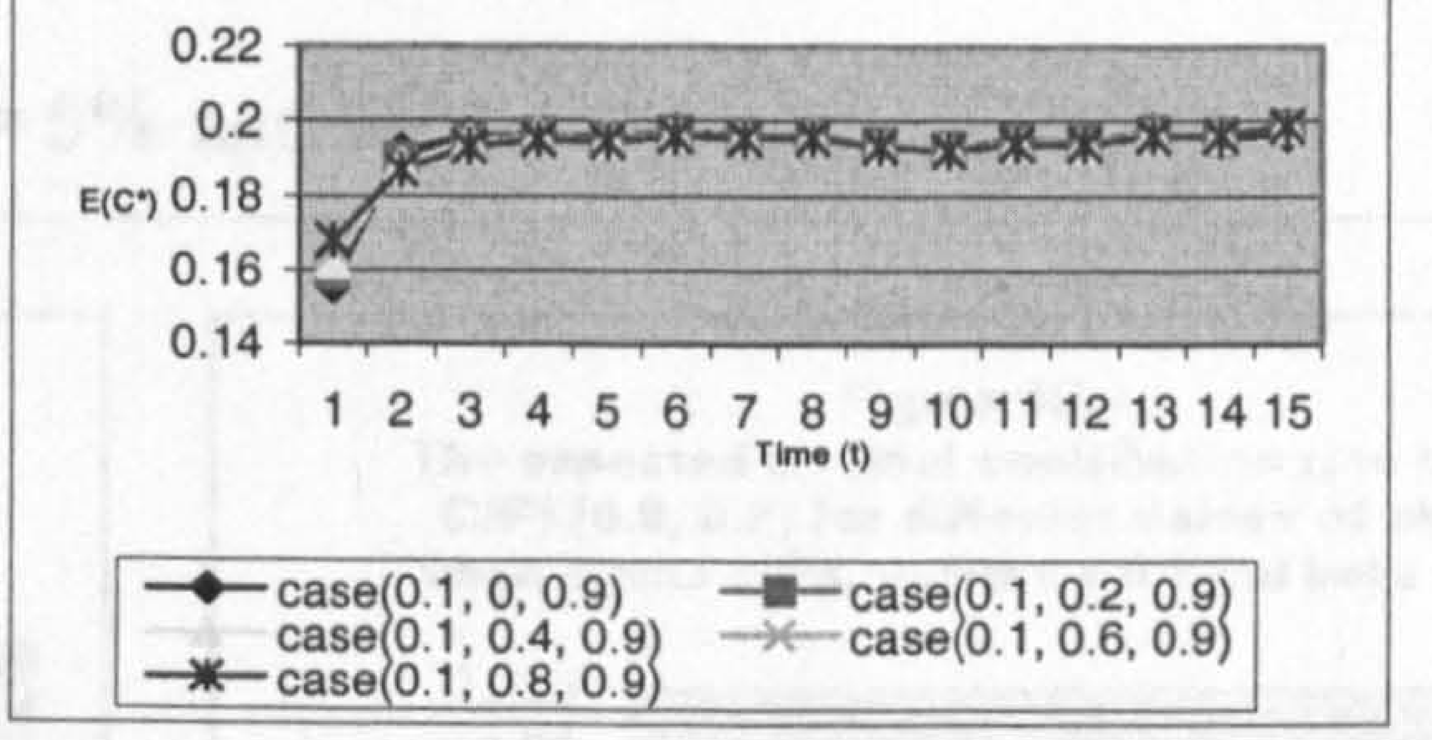


Figure 4A.8
The expected optimal contribution rate under CIPI (1.2, 0.2) for different values of alpha when sigma = 20%, gamma=0.5 and beta=0.5

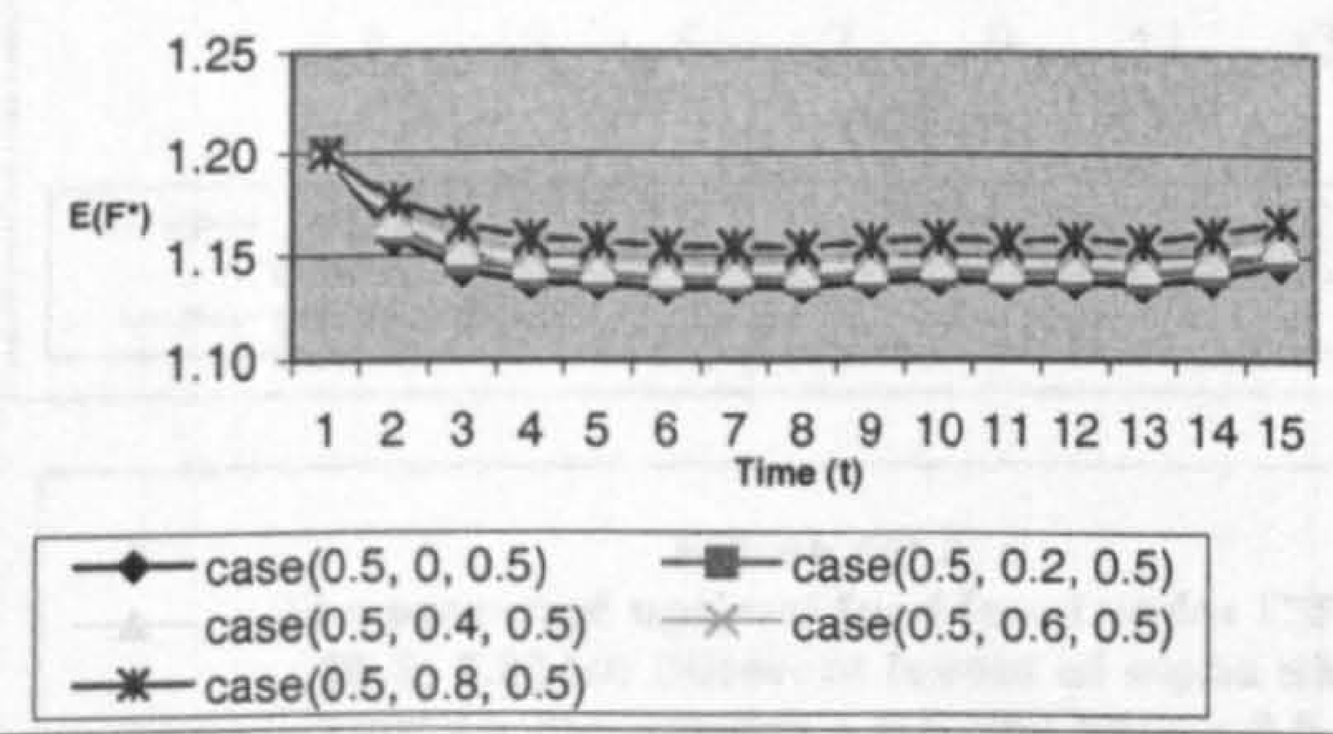


Figure 4A.11
The expected optimal contribution rate under CIPI (1.2, 0.2) for different values of alpha when sigma = 20%, gamma=0.5 and beta=0.5

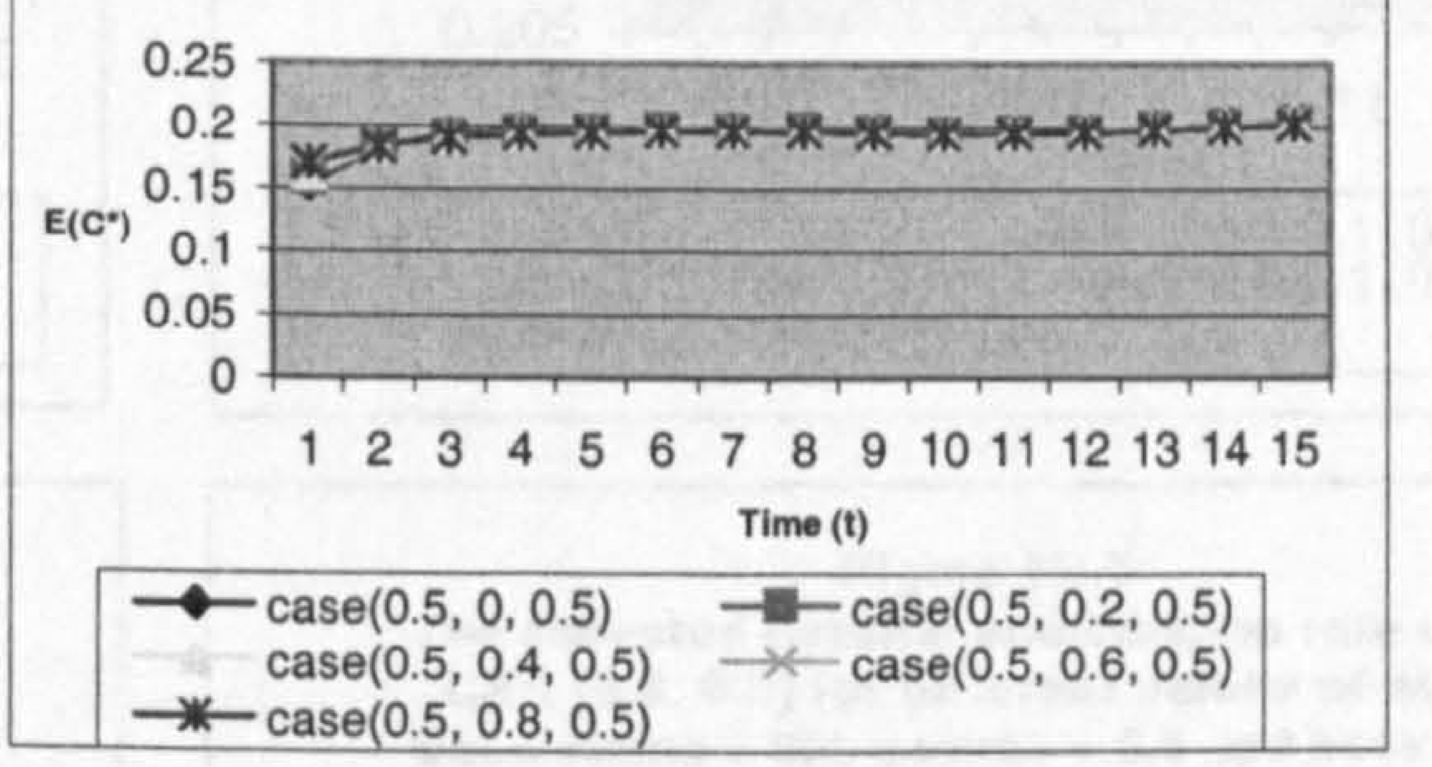


Figure 4A.9
The expected optimal fund level under CIPI (1.2, 0.2) for different levels of alpha when sigma = 20%, gamma=0.9 and beta=0.1

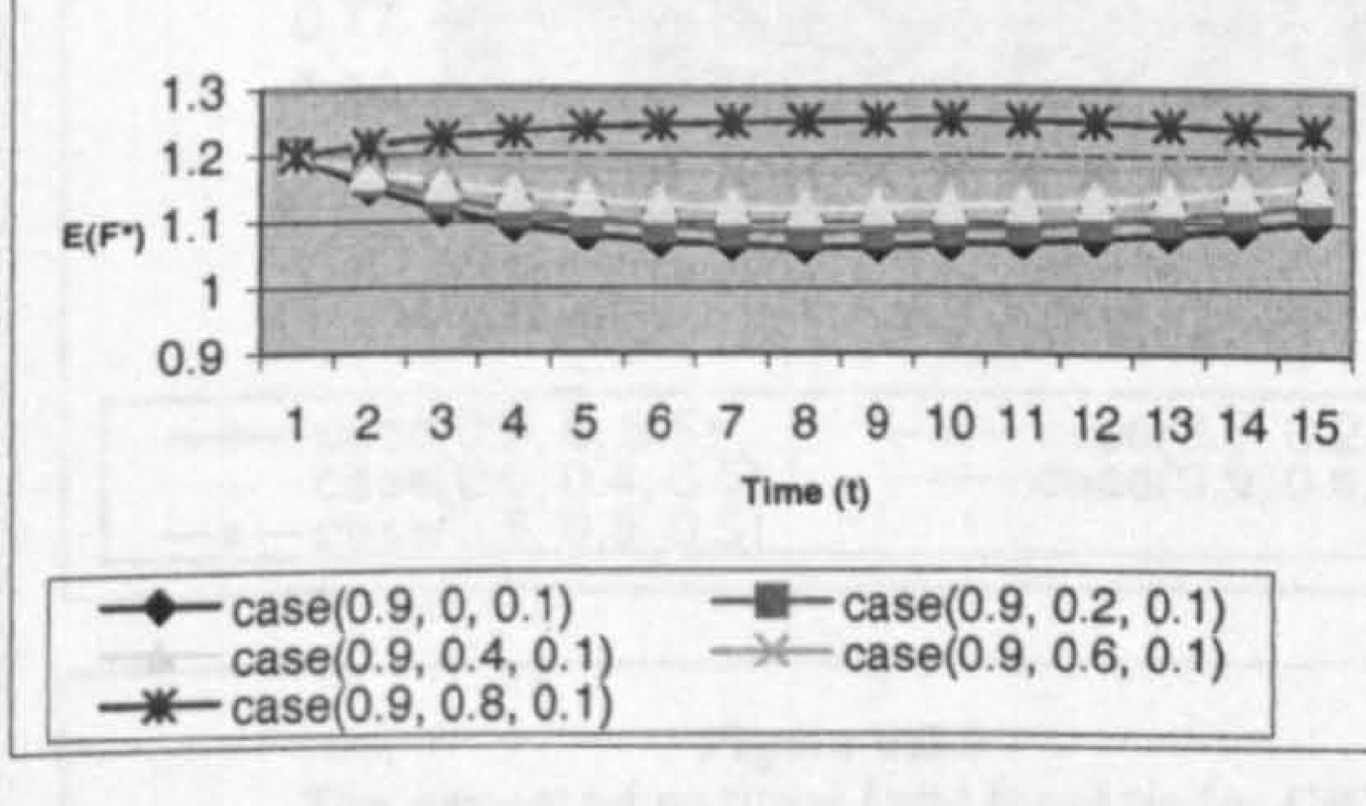
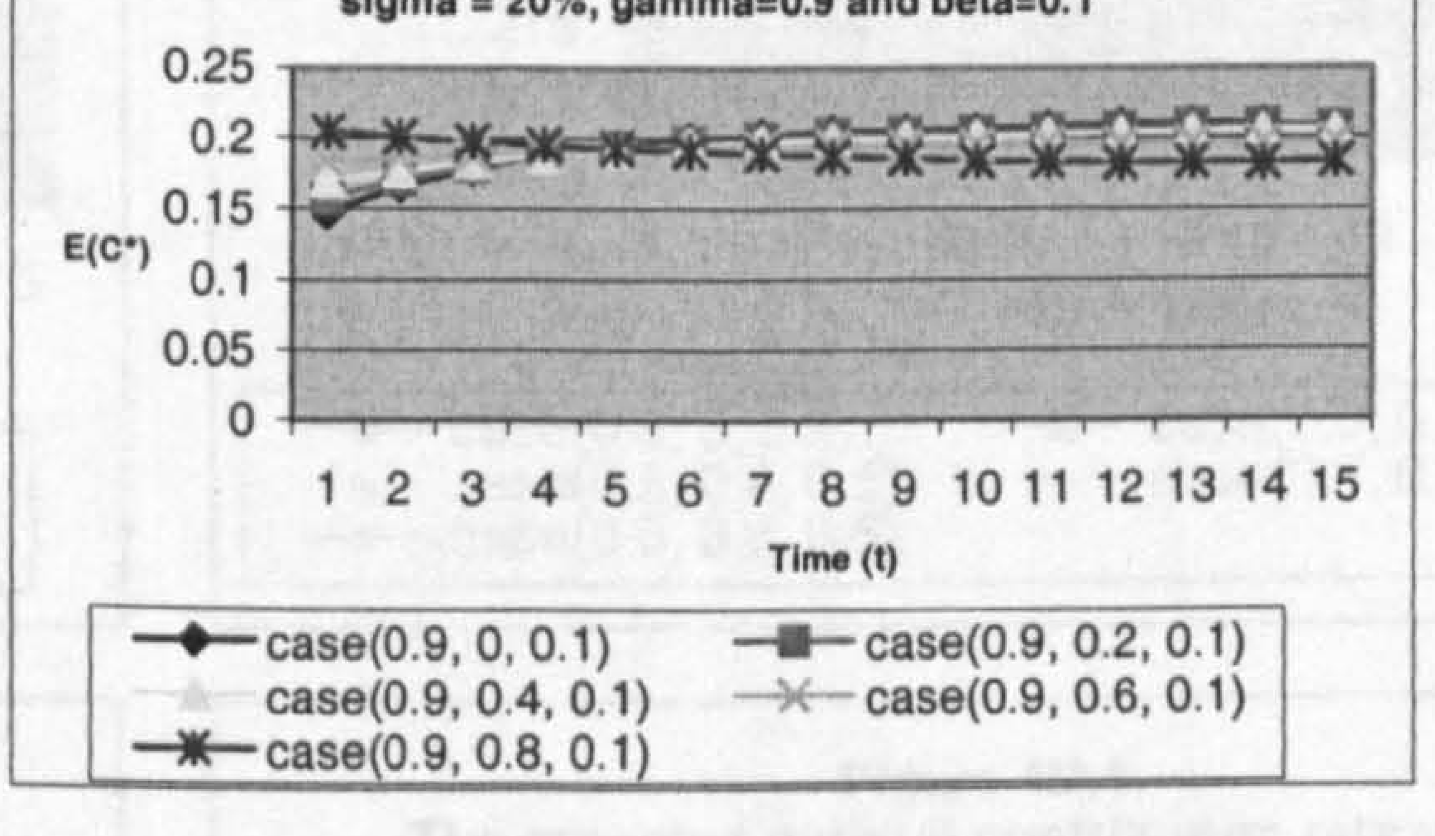


Figure 4A.12
The expected optimal contribution rate under CIPI (1.2, 0.2) for different levels of alpha when sigma = 20%, gamma=0.9 and beta=0.1



Appendix 4B

Effect of changing the weighting factors of the contribution rate risk and the solvency risk; γ and β (with $\alpha \neq 0$)

Figures of Fifth scenario (0.8, 0.2), when $\sigma = 5\%$ and 20%

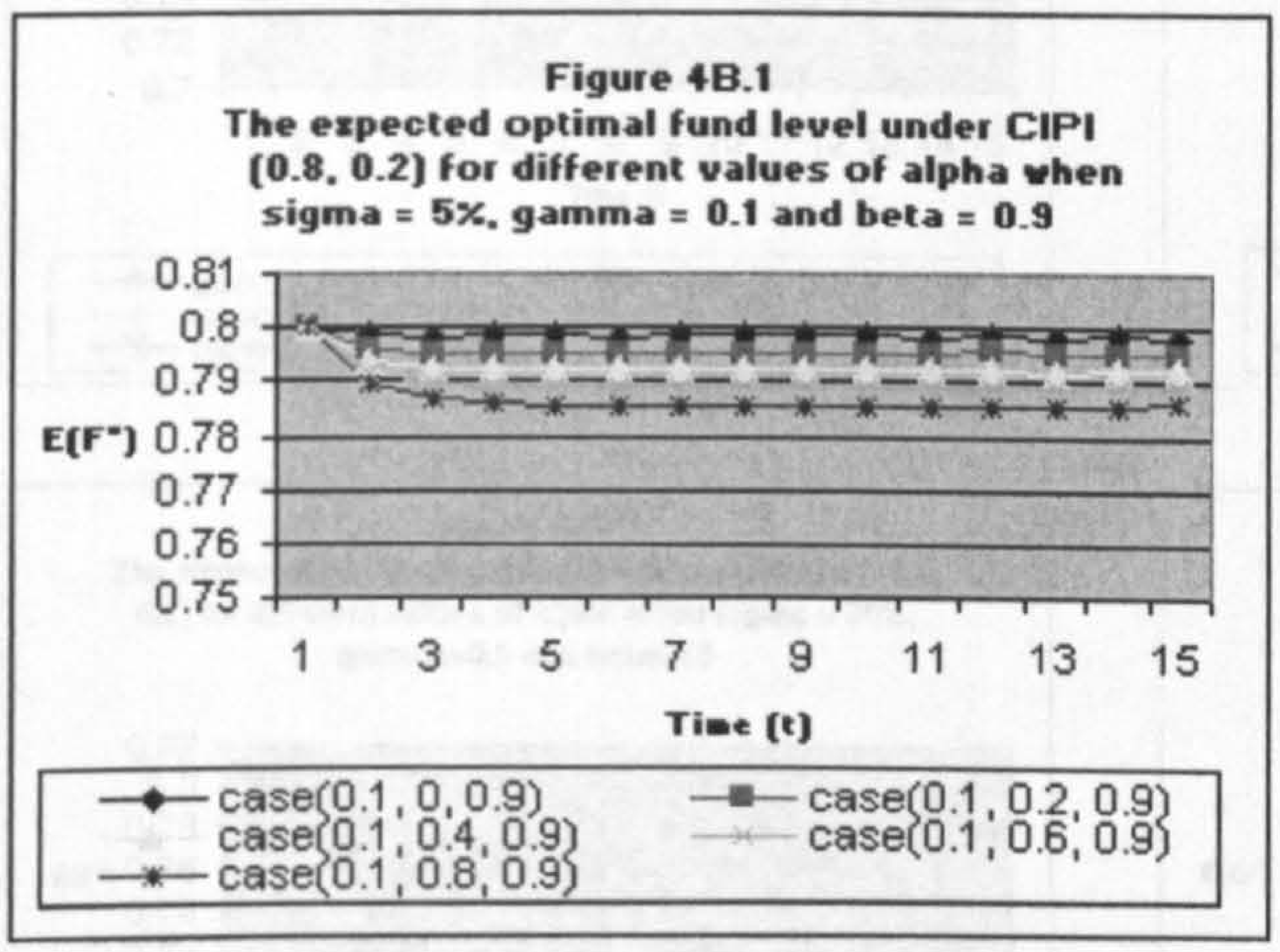


Figure 4B.1 The expected optimal fund level under CIPI (0.8, 0.2) for different values of alpha when sigma = 5%, gamma = 0.1 and beta = 0.9

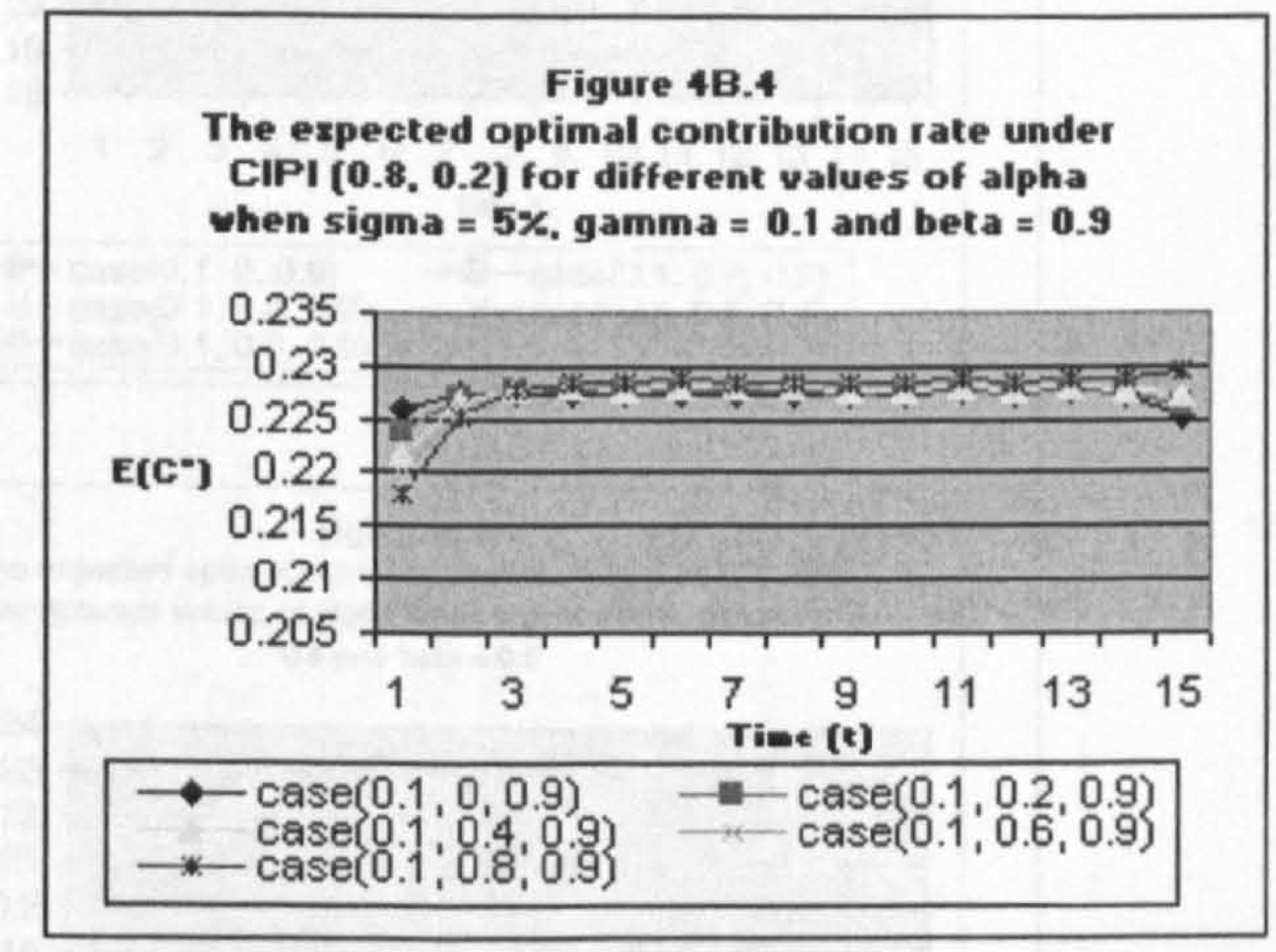


Figure 4B.4 The expected optimal contribution rate under CIPI (0.8, 0.2) for different values of alpha when sigma = 5%, gamma = 0.1 and beta = 0.9

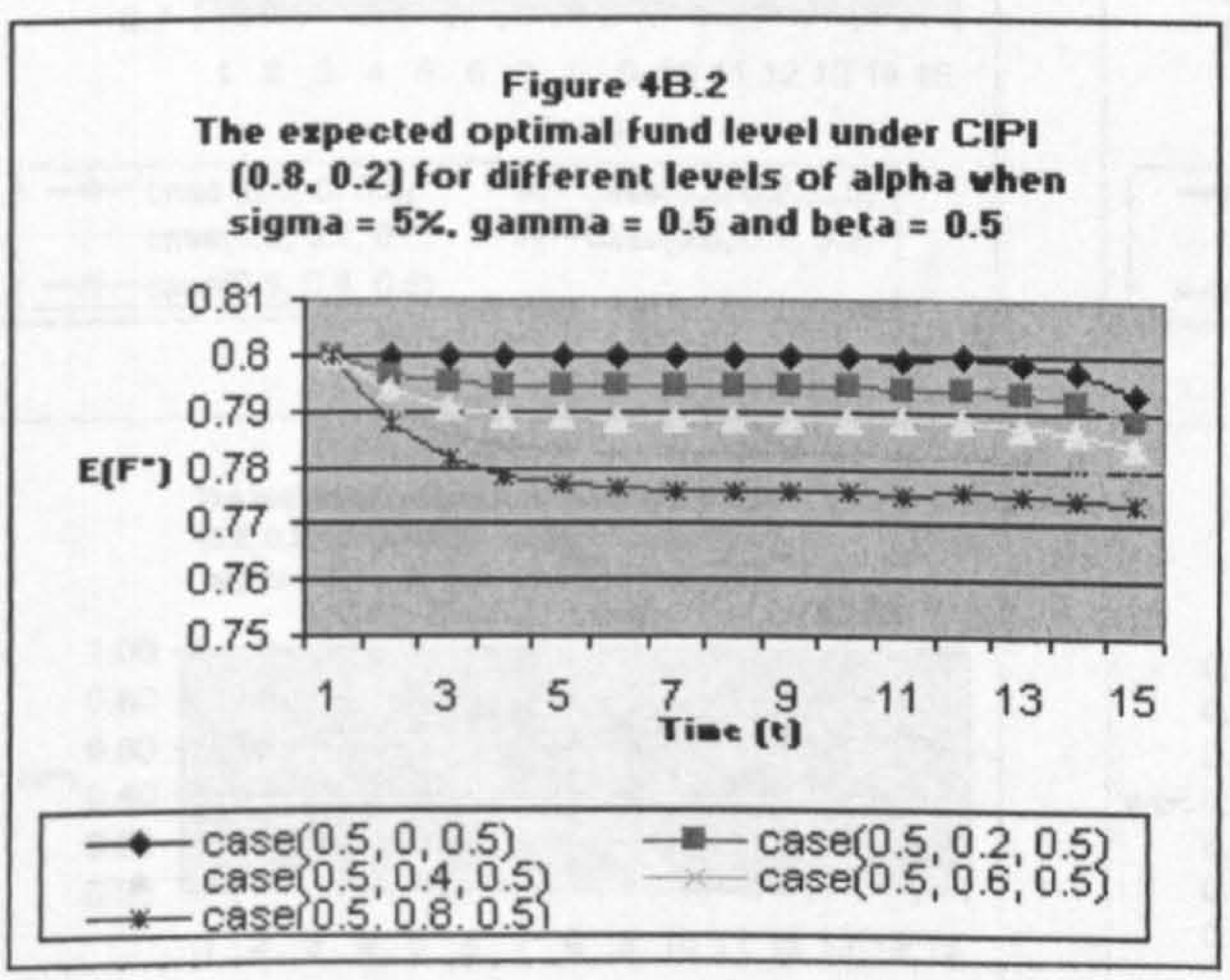


Figure 4B.2 The expected optimal fund level under CIPI (0.8, 0.2) for different levels of alpha when sigma = 5%, gamma = 0.5 and beta = 0.5

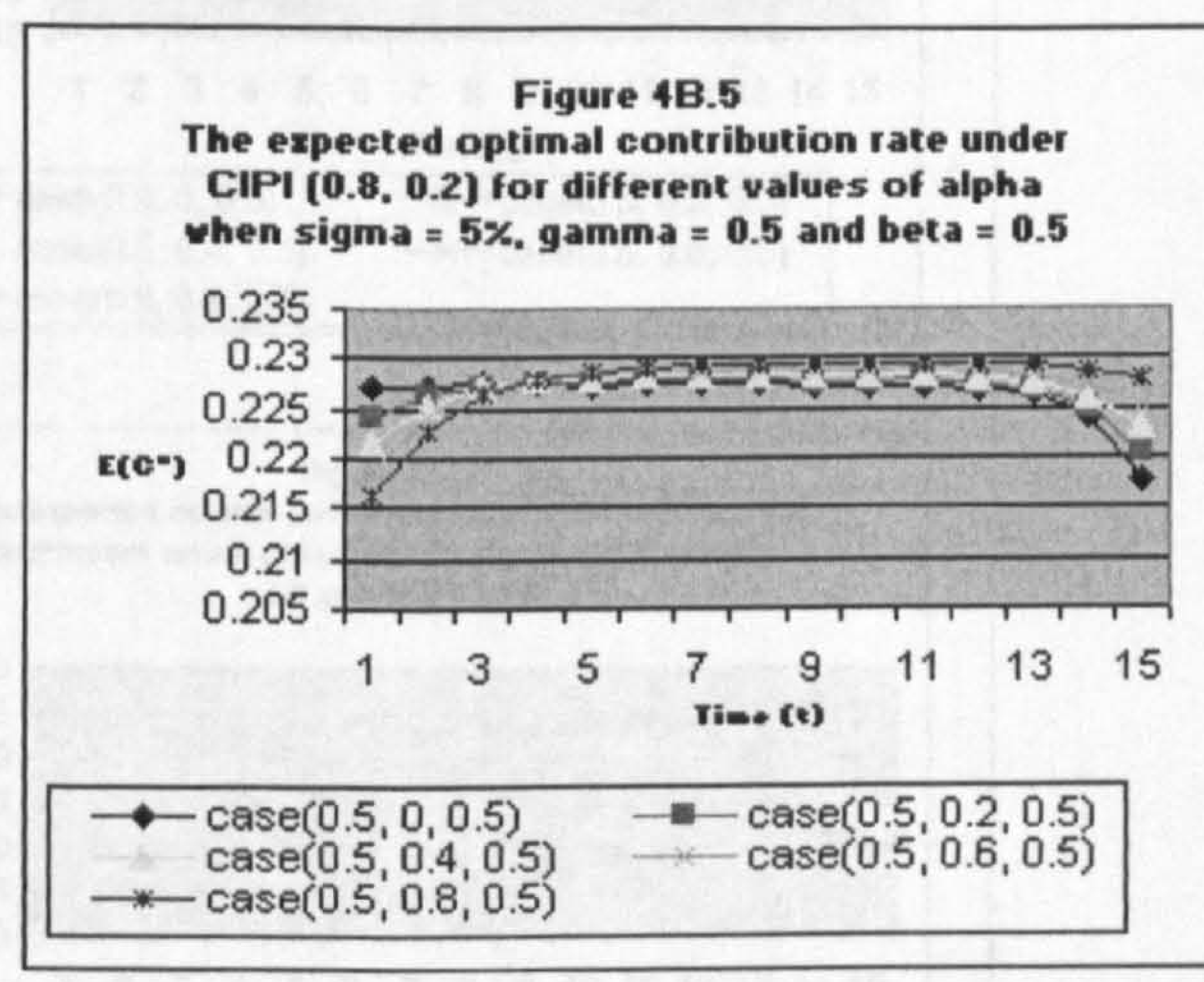


Figure 4B.5 The expected optimal contribution rate under CIPI (0.8, 0.2) for different values of alpha when sigma = 5%, gamma = 0.5 and beta = 0.5

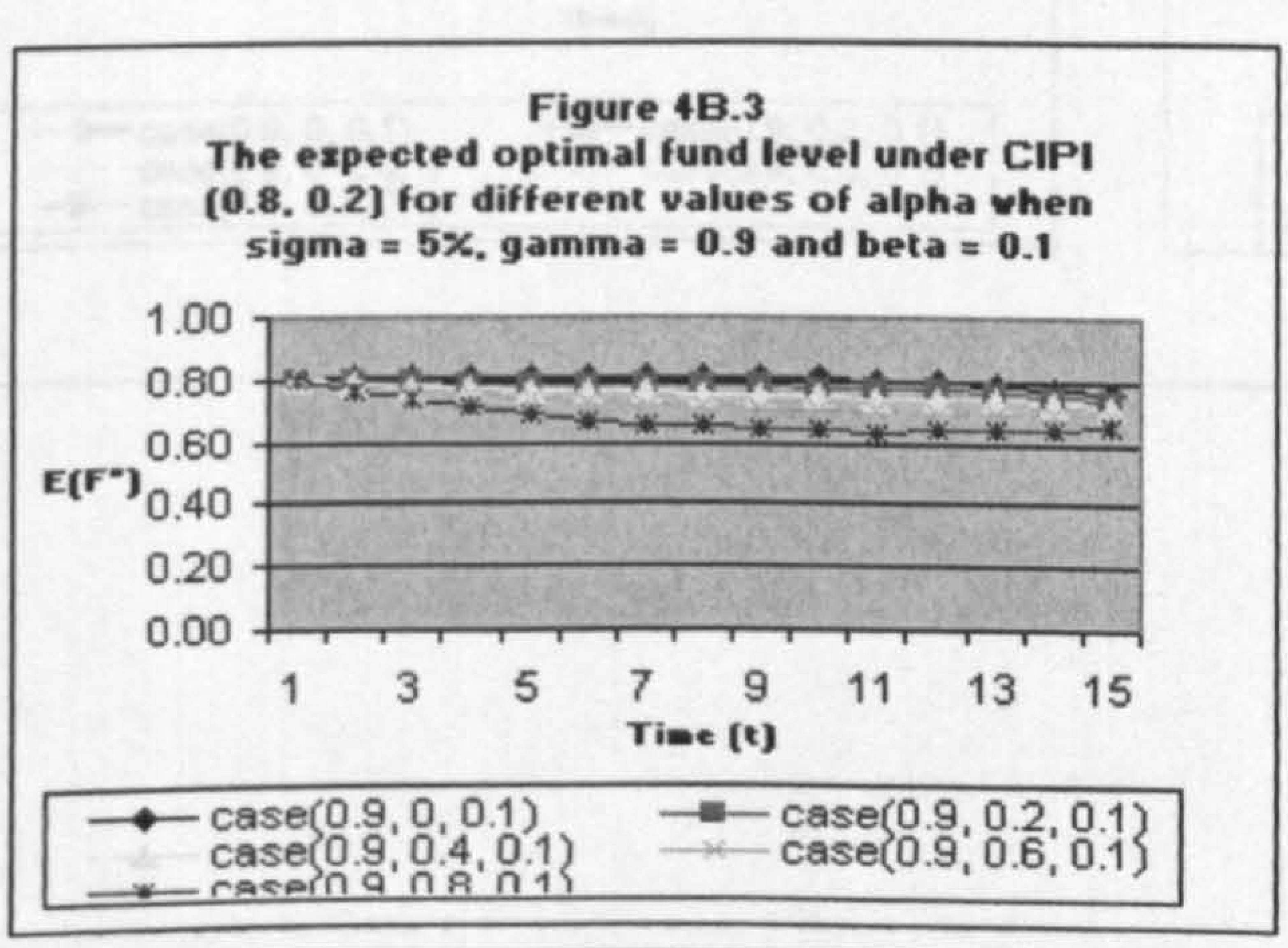


Figure 4B.3 The expected optimal fund level under CIPI (0.8, 0.2) for different values of alpha when sigma = 5%, gamma = 0.9 and beta = 0.1

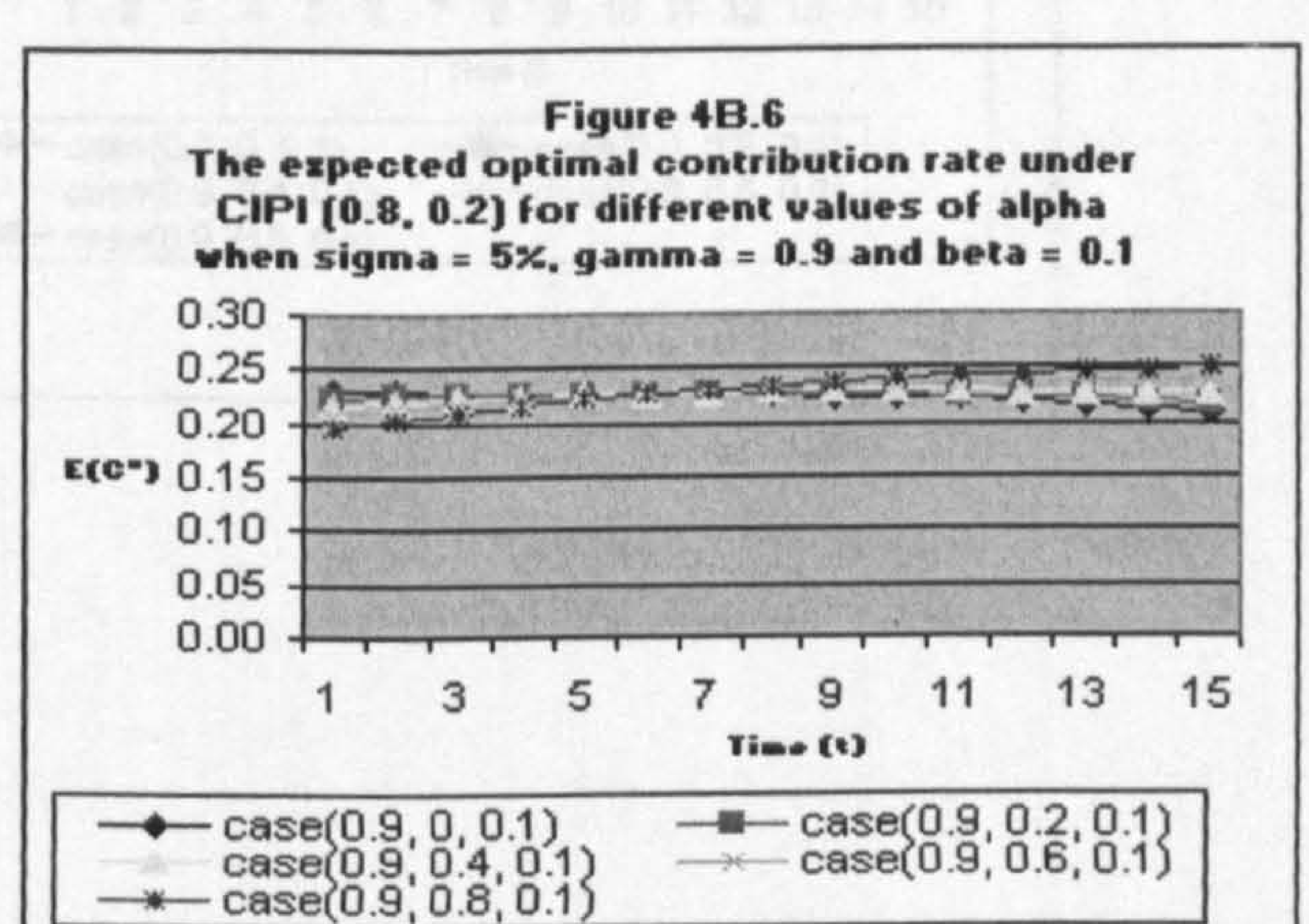


Figure 4B.6 The expected optimal contribution rate under CIPI (0.8, 0.2) for different values of alpha when sigma = 5%, gamma = 0.9 and beta = 0.1

Figure 4B.7
The expected optimal fund level under CIPI (0.8, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.1 and beta = 0.9

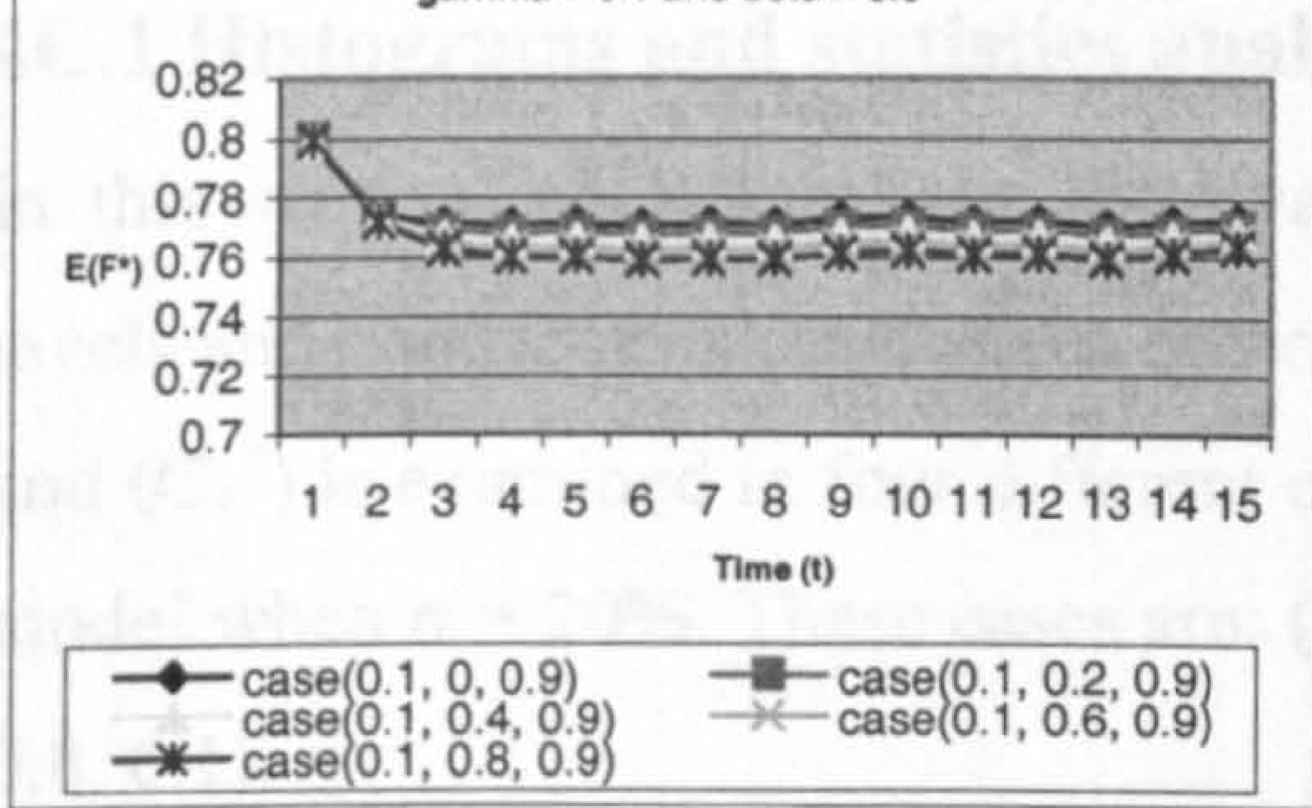


Figure 4B.10
The expected optimal contribution rate under CIPI (0.8, 0.2) for different values of alpha when sigma = 20%, gamma = 0.1 and beta = 0.9

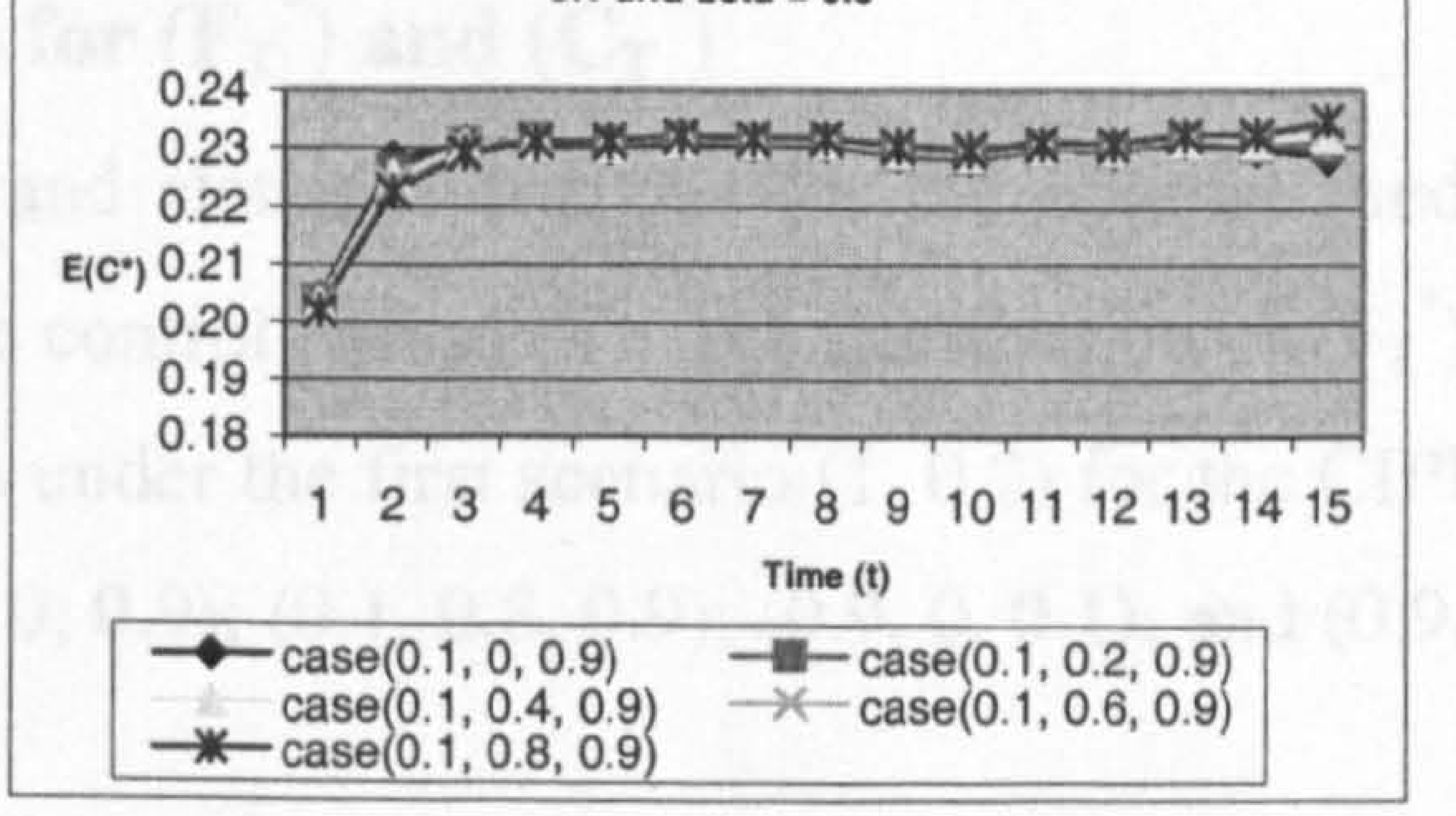


Figure 4B.8
The expected optimal contribution rate under CIPI (0.8, 0.2) for different values of alpha when sigma = 20%, gamma=0.5 and beta=0.5

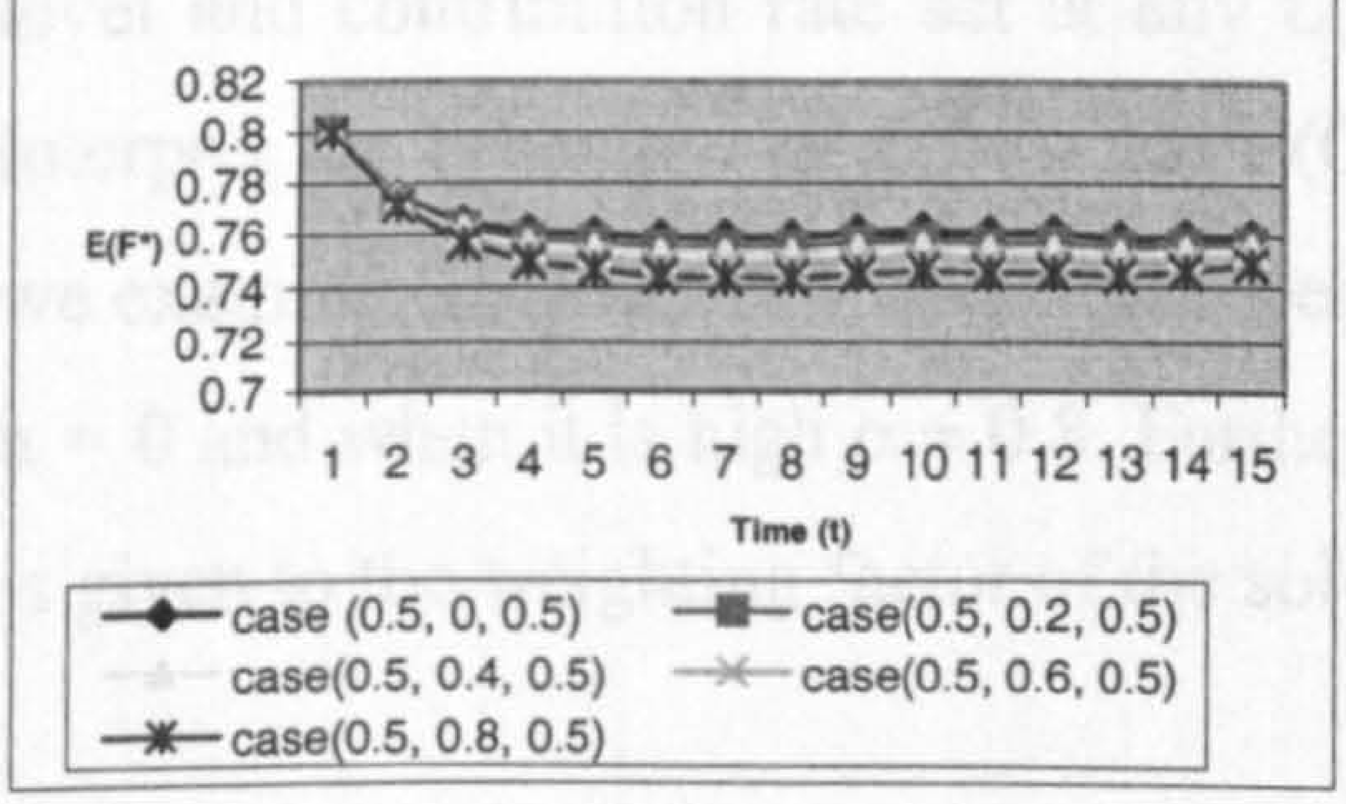


Figure 4B.11
The expected optimal contribution rate under CIPI (0.8, 0.2) for different values of alpha when sigma = 20%, gamma = 0.5 and beta = 0.5

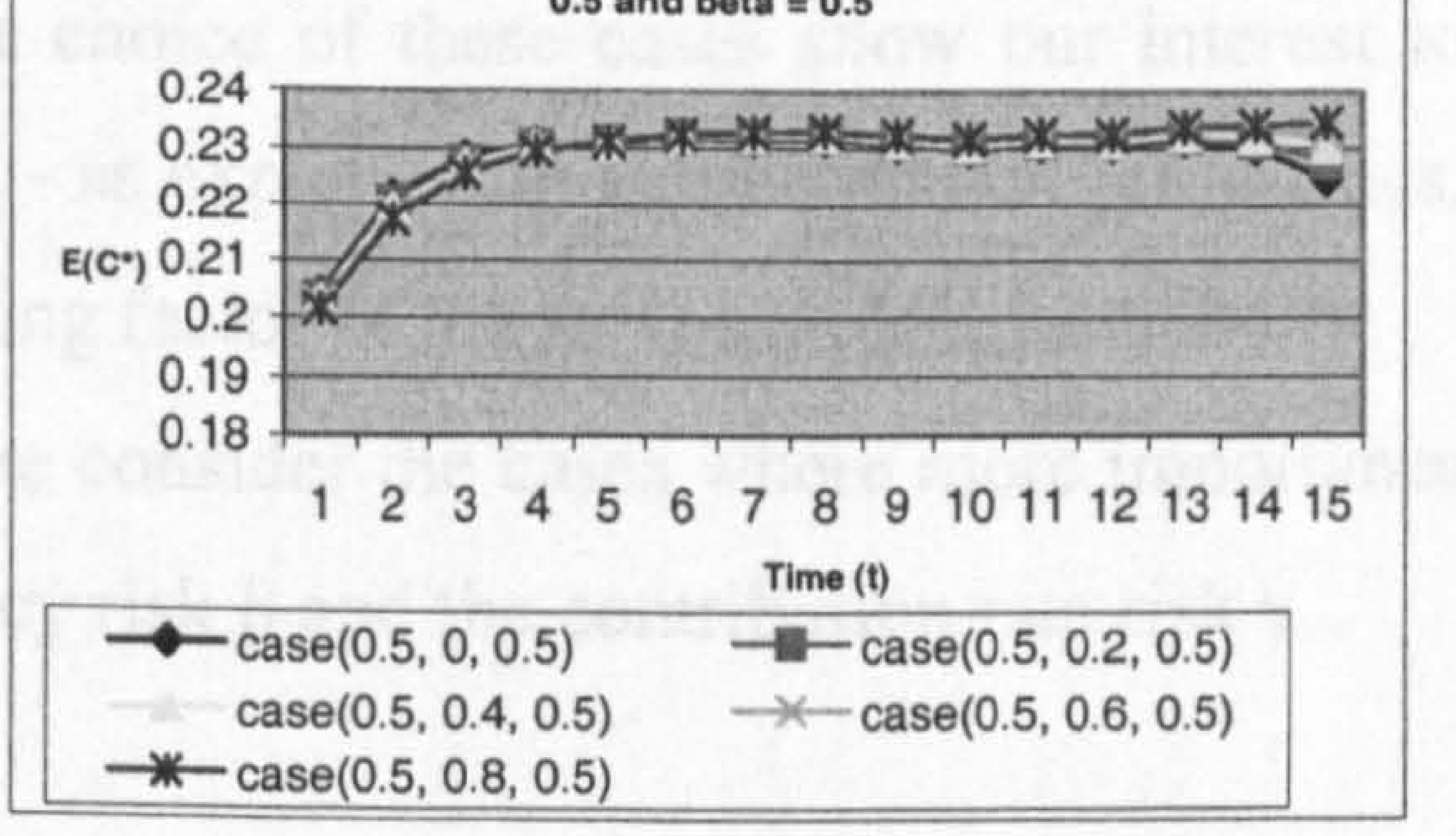


Figure 4B.9
The expected optimal fund level under CIPI (0.8, 0.2) for different values of alpha when sigma = 20%, gamma = 0.9 and beta = 0.1

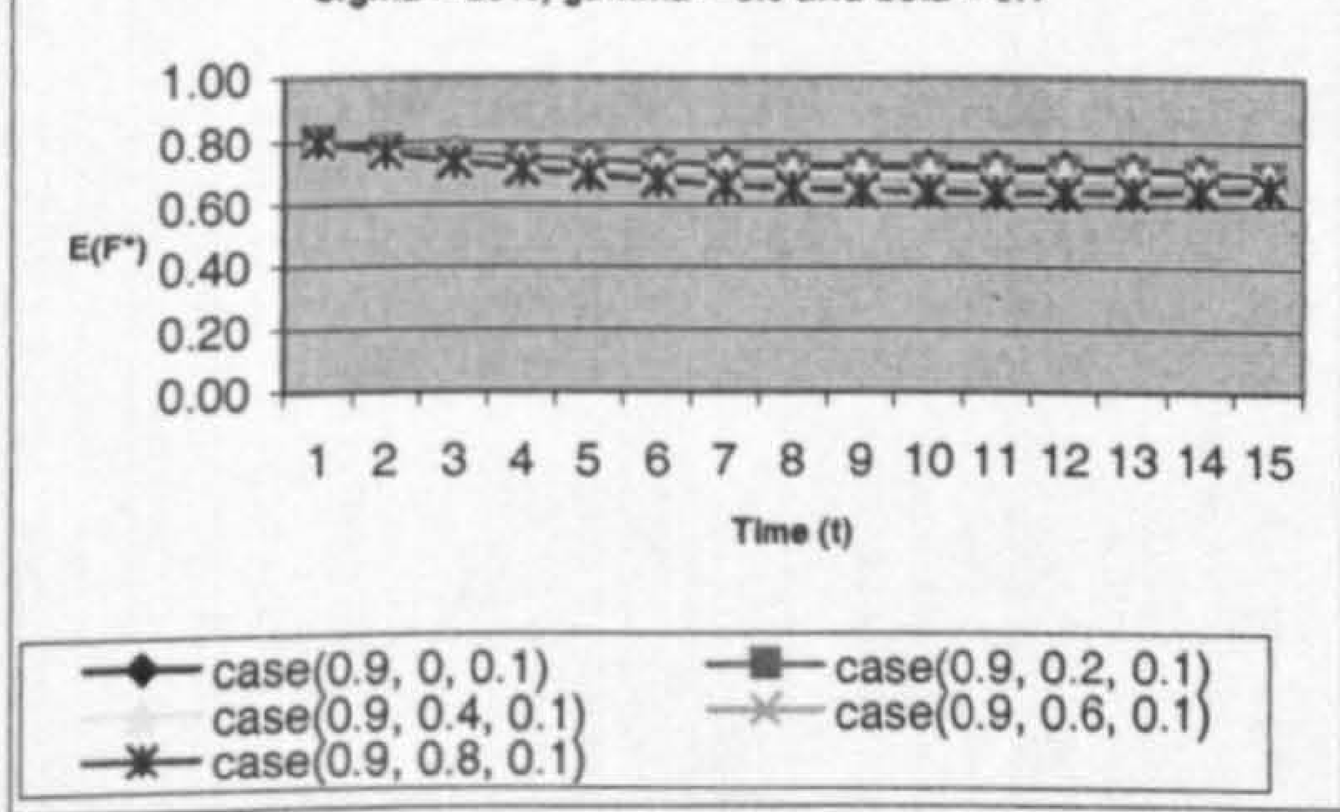
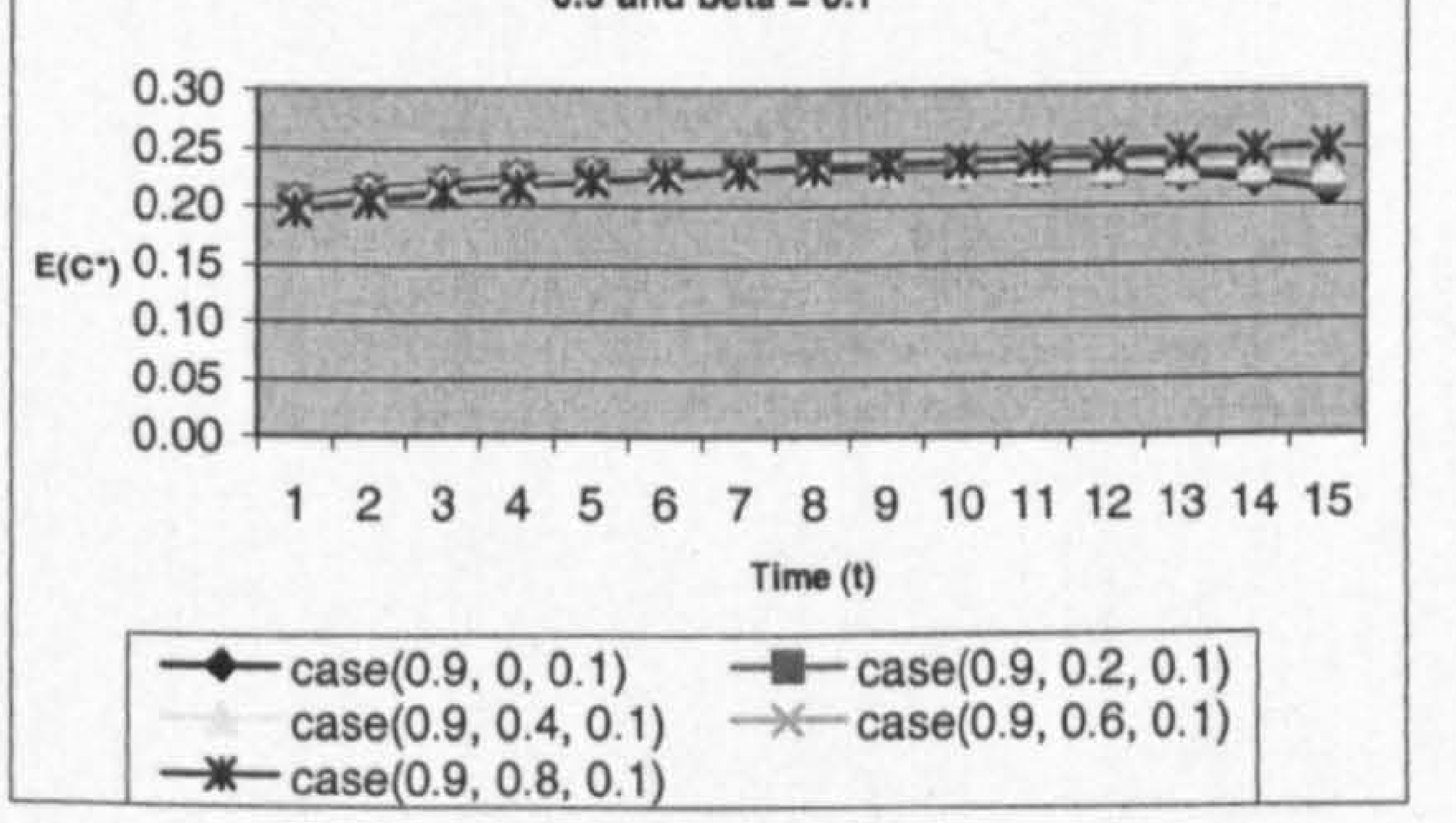


Figure 4B.12
The expected optimal contribution rate under CIPI (0.8, 0.2) for different values of alpha when sigma = 20%, gamma = 0.9 and beta = 0.1



Appendix 4C

4C.1 Histograms and statistics analysis for (F_T^*) and (C_T^*)

In this section, we present the histograms and statistics analysis for the optimal fund levels and contribution rates at the end of the control period (T). The distribution of (F_T^*) and (C_T^*) is examined in four different cases under the first scenario (1, 0.2) for the CIPI model when $\sigma = 20\%$. These cases are: (0.1, 0, 0.9); (0.1, 0.8, 0.9); (0.9, 0, 0.1); and (0.9, 0.8, 0.1).

The analysis of the behaviour of (F_T^*) and (C_T^*) is an example of how the optimal fund level and contribution rate act at any t. The choice of these cases show our interest to interpret the behaviour of $E(F_T^*)$ and $E(C_T^*)$ - as explained in subsection 4.4.6(a)-. Thus, we examine (F_T^*) and (C_T^*) when the weighting factor of the mixed middle term is low $\alpha = 0$ and when it is high $\alpha = 0.8$. Further, we consider the cases where more importance is given to the weighting factor of the solvency risk β and the contribution rate risk γ.

The results of (F_T^*) and (C_T^*) are shown, in turn, below.

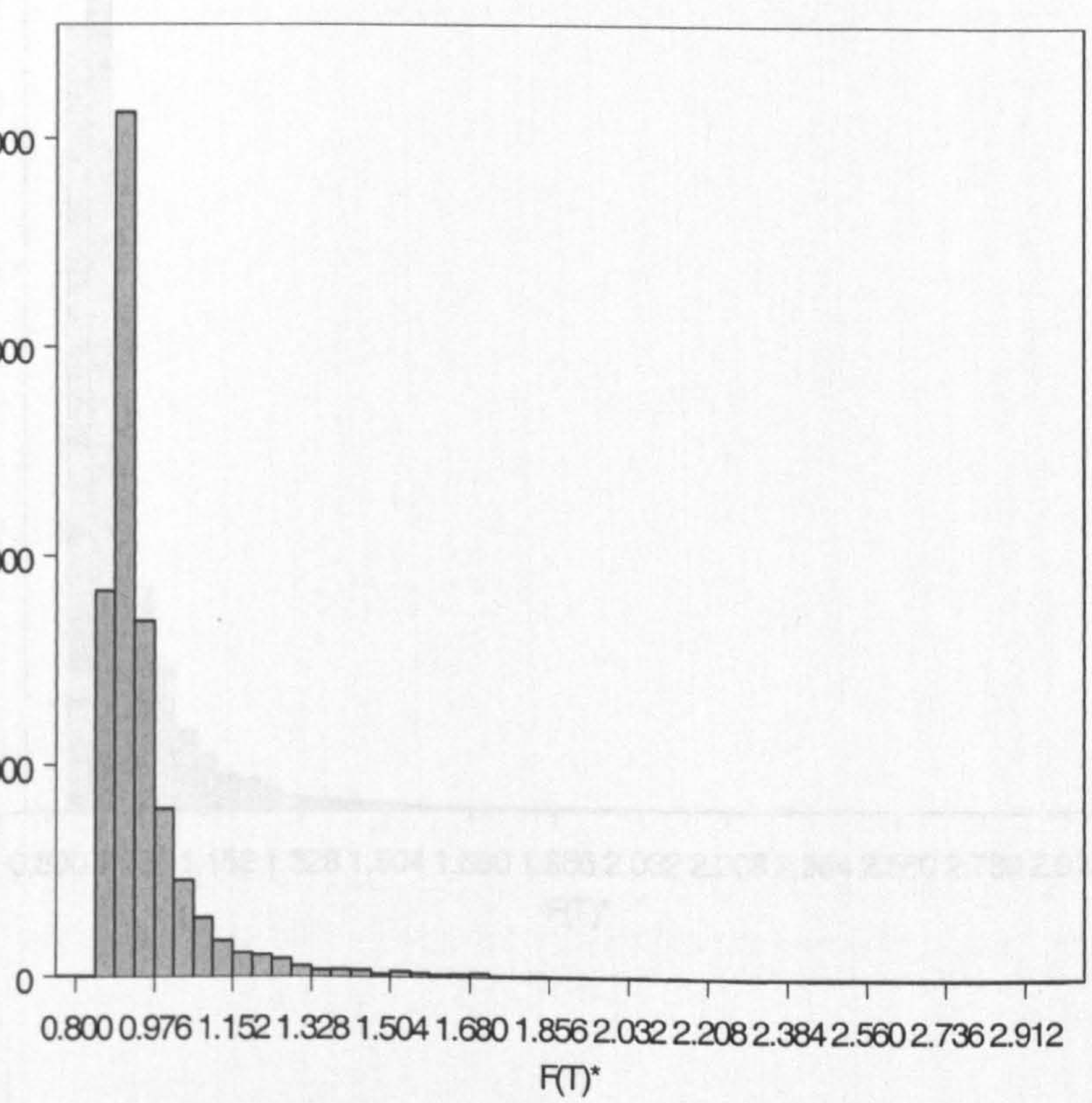
4C.1.1 The results of (F_T^*)

Case: CIPI (0.1, 0, 0.9), σ = 0.2

Summary Statistics for data in: (0.1, 0, 0.9)

```
                  F(T)*
      Min:      0.8718469
  1st Qu.:      0.8928930
     Mean:      0.9662359
   Median:      0.9175373
  3rd Qu.:      0.9705734
      Max:      8.1691670
  Total N: 10000.0000000
    NA's :      0.0000000
 Std Dev.:      0.1916204
```

Figure 4C.1
Histogram of (F_T^*) in the case of (0.1, 0, 0.9) when σ = 20%

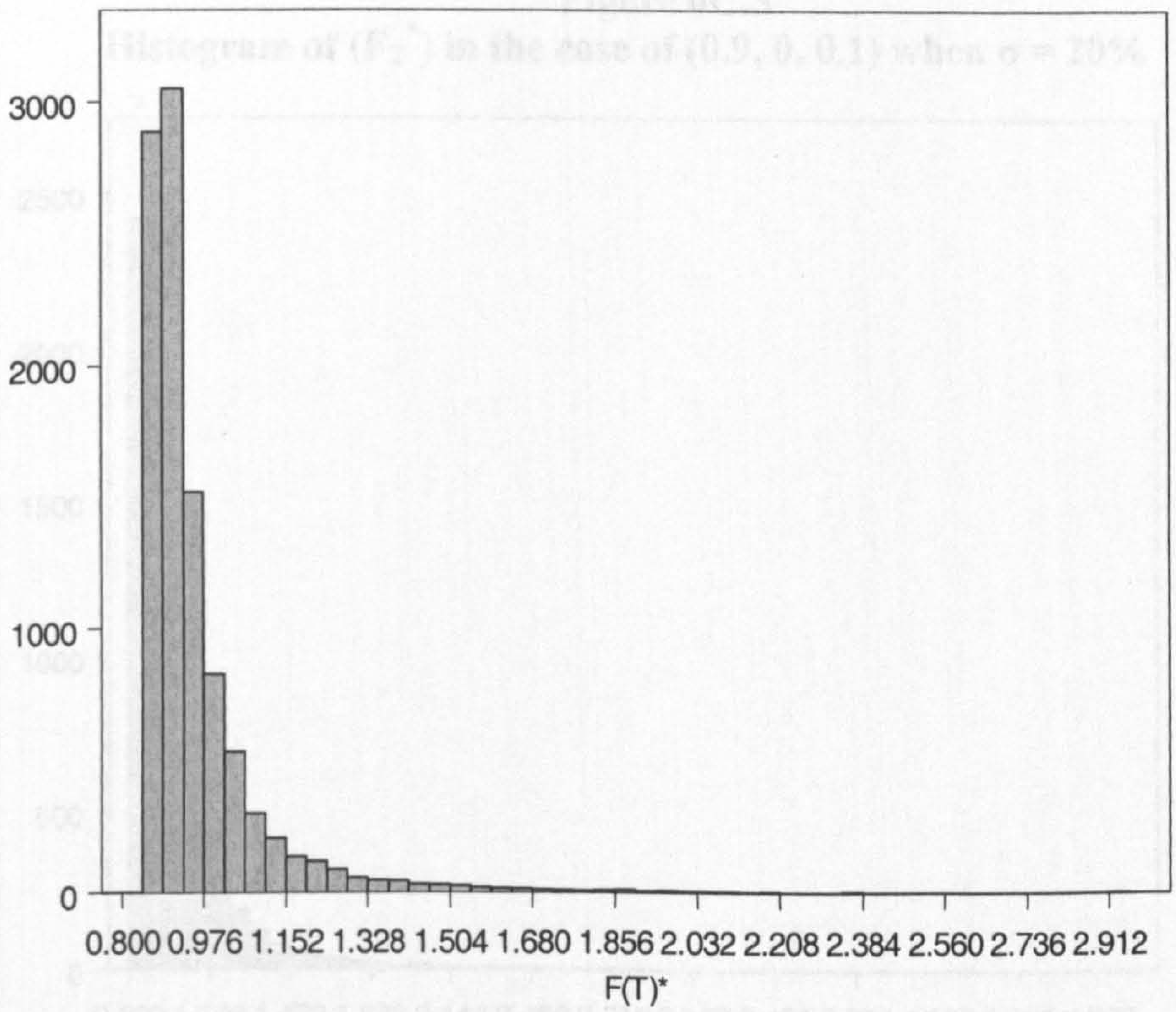


Case: CIPI (0.1, 0.8, 0.9) sigma = 0.2

Summary Statistics for data in: (0.1, 0.8, 0.9)

```
              F(T)*
     Min:     0.8495855
 1st Qu.:     0.8843292
    Mean:     0.9658564
  Median:     0.9155776
 3rd Qu.:     0.9782698
     Max:     7.9955450
 Total N: 10000.0000000
    NA's :    0.0000000
Std Dev.:     0.1969735
```

Figure 4C.2
Histogram of (F_T^*) in the case of (0.1, 0.8, 0.9) when $\sigma = 20\%$

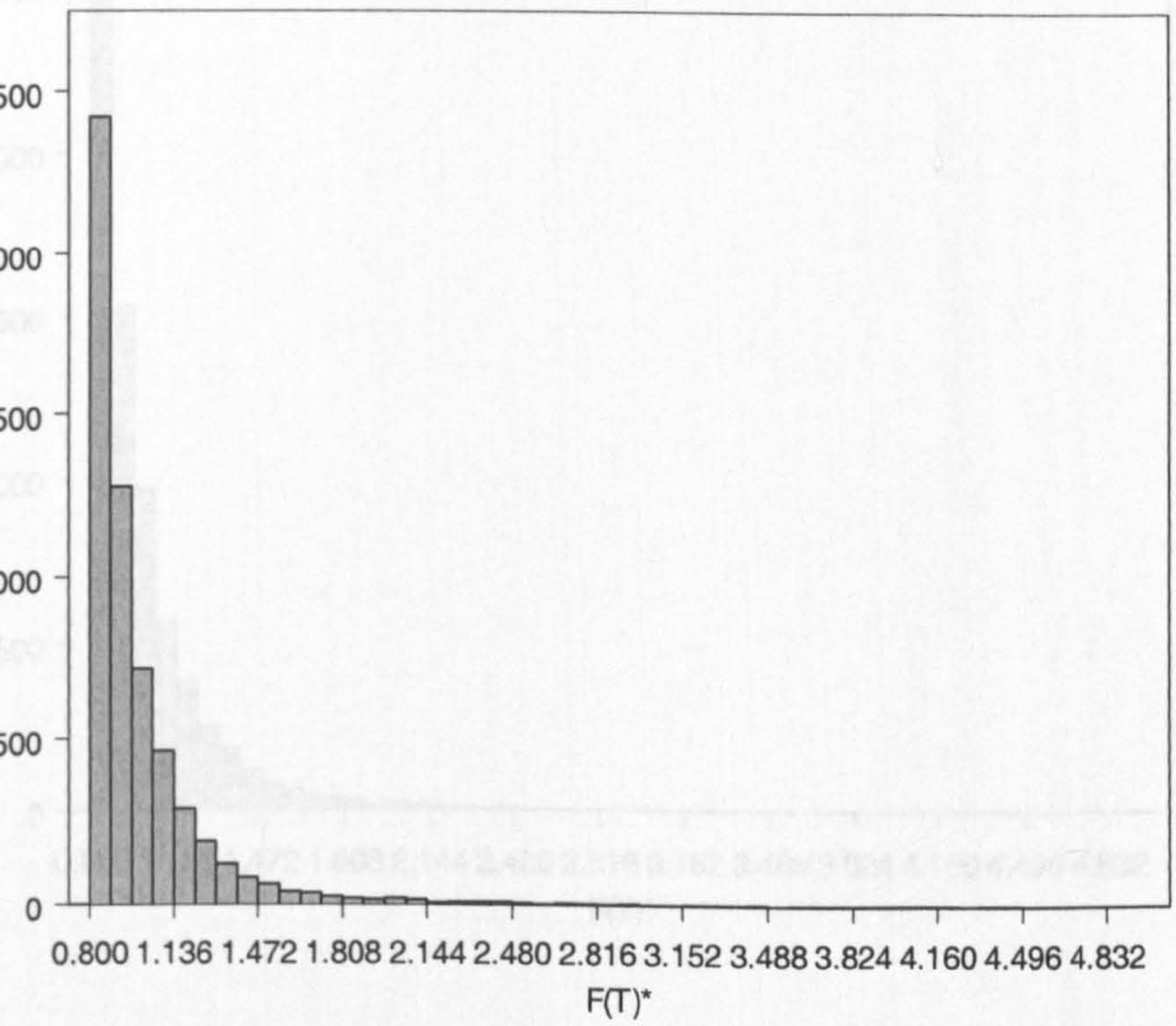


Case: CIPI (0.9, 0, 0.1) sigma = 0.2

Summary Statistics for data in: (0.9, 0, 0.1)

```
                  F(T)*
     Min:     0.6554199
 1st Qu.:     0.7585064
    Mean:     0.8975981
  Median:     0.8253162
 3rd Qu.:     0.9416538
     Max:    11.2304800
 Total N: 10000.0000000
   NA's :     0.0000000
Std Dev.:     0.2881246
```

Figure 4C.3
Histogram of (F_T^*) in the case of (0.9, 0, 0.1) when σ = 20%

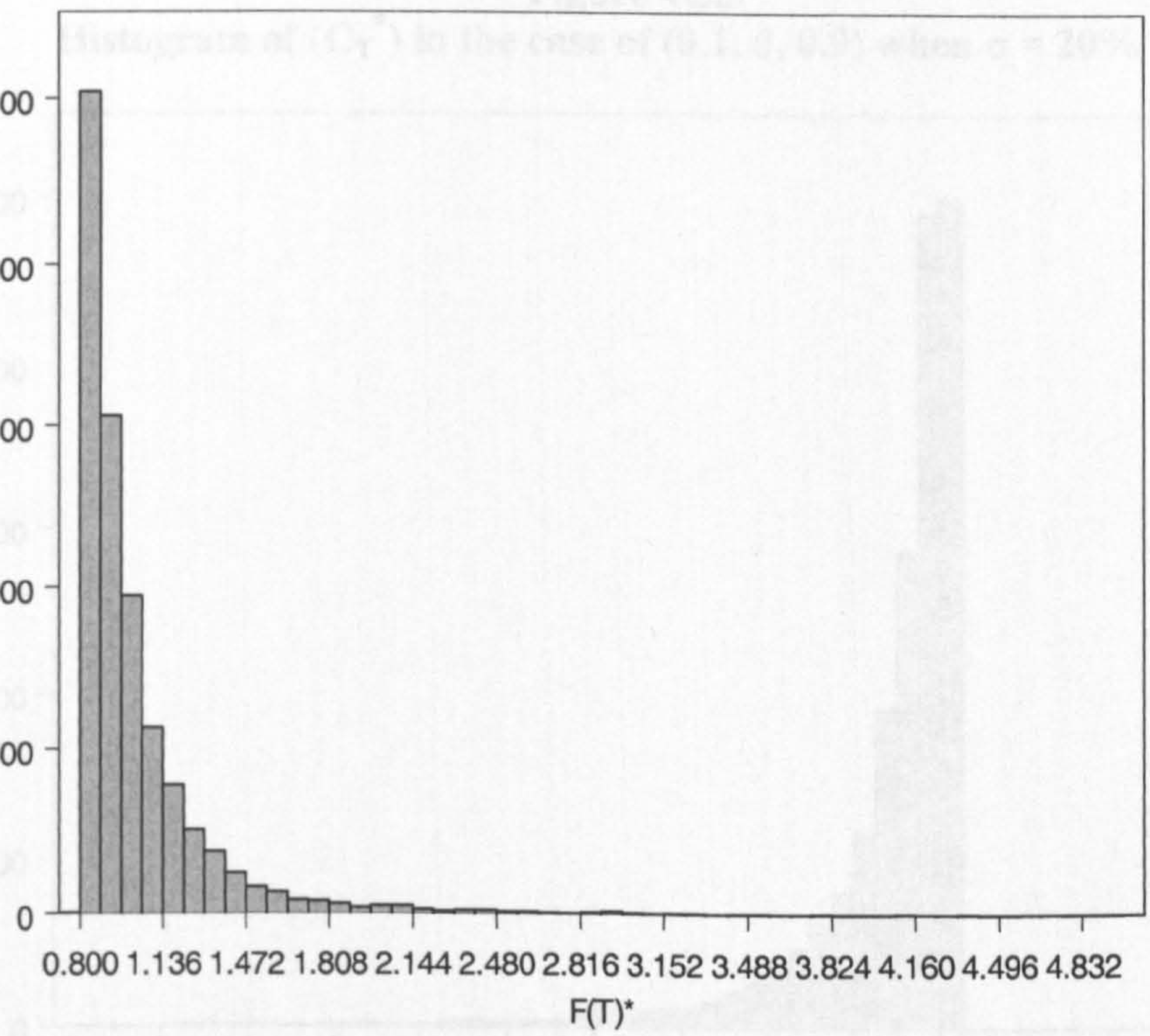


Case: CIPI (0.9, 0.8, 0.1) sigma = 0.2

Summary Statistics for data in: (0.9, 0.8, 0.1)

```
               F(T)*
      Min:     0.6544825
  1st Qu.:     0.7833344
     Mean:     0.9442336
   Median:     0.8618014
  3rd Qu.:     0.9957414
      Max:    13.1294600
 Total N: 10000.0000000
    NA's :     0.0000000
 Std Dev.:     0.3287367
```

Figure 4C.4
Histogram of (F_T^*) in the case of (0.9, 0.8, 0.1) when σ = 20%



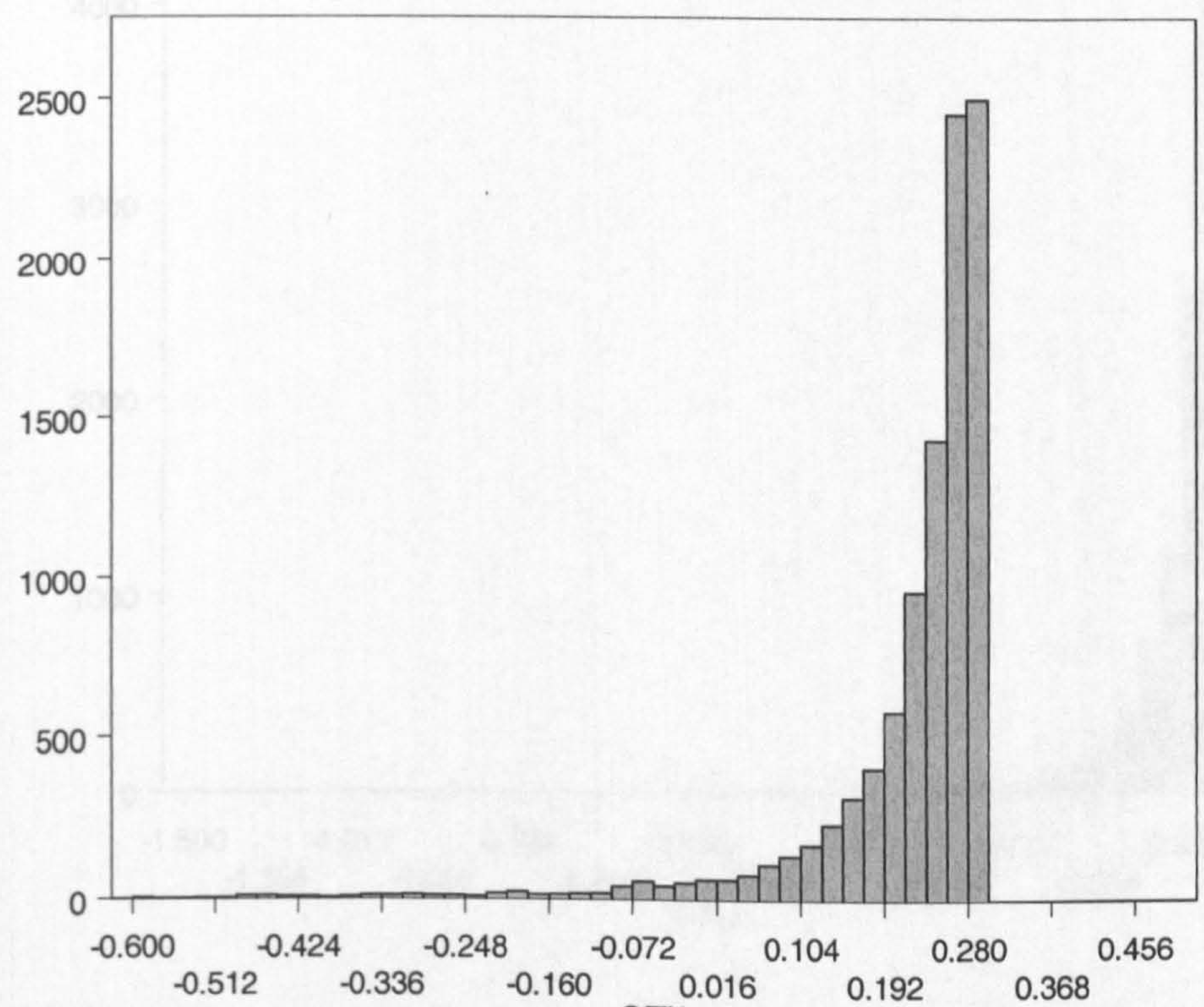
4C.1.2 The results of (C_T^*)

Case: CIPI (0.1, 0, 0.9) sigma = 0.2

Summary Statistics for data in: (0.1, 0, 0.9)

```
                  C(T)*
      Min:   -6.4022960
 1st Qu.:     0.2086571
     Mean:    0.2126404
   Median:    0.2573637
 3rd Qu.:     0.2799963
      Max:    0.2993243
  Total N: 10000.0000000
    NA's :    0.0000000
Std Dev.:     0.1759780
```

Figure 4C.5
Histogram of (C_T^*) in the case of (0.1, 0, 0.9) when σ = 20%

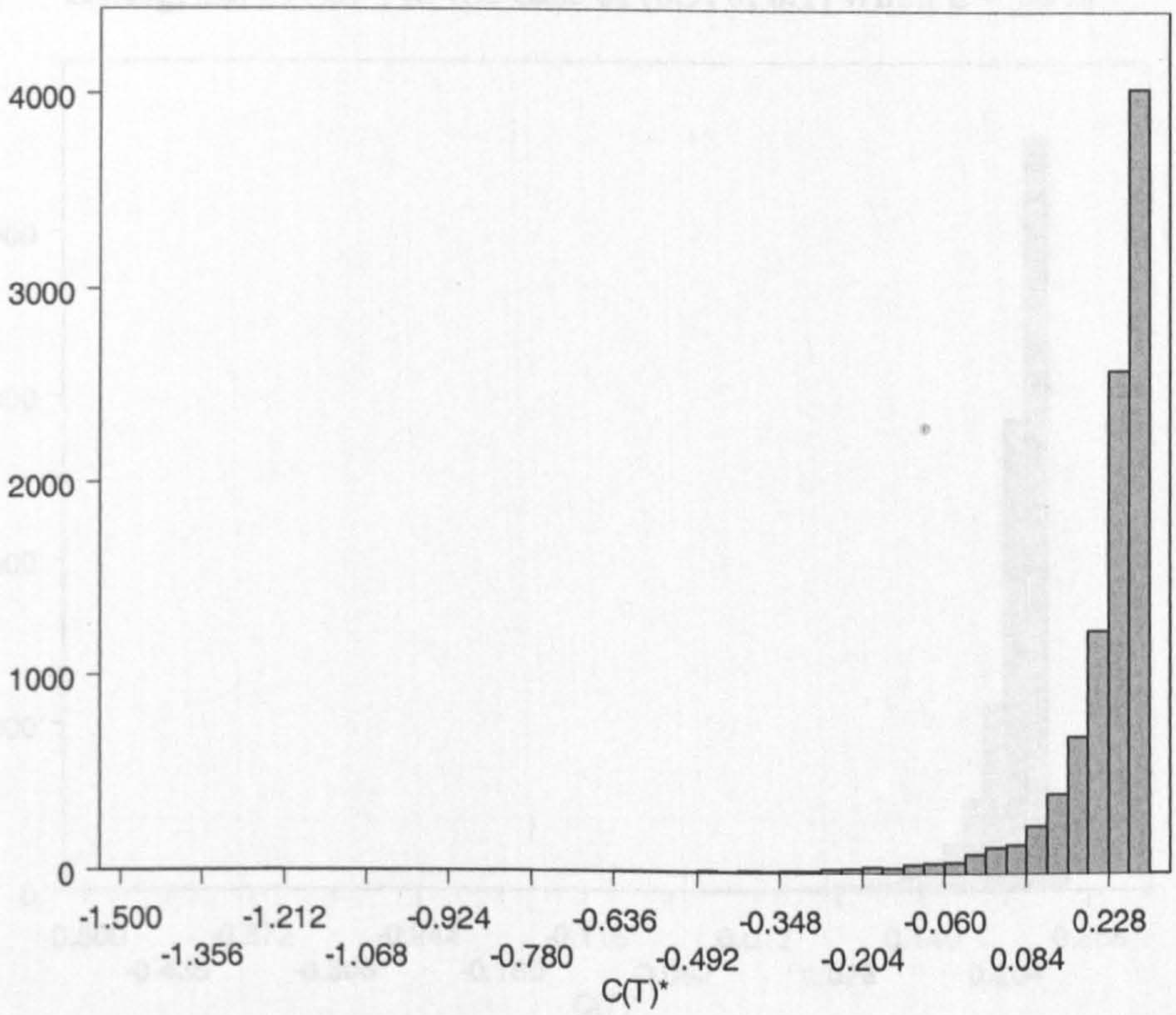


Case: CIPI (0.1, 0.8, 0.9) sigma = 0.2

Summary Statistics for data in: (0.1, 0.8, 0.9)

```
                  C(T)*
      Min:    -5.0097520
  1st Qu.:     0.2073672
     Mean:     0.2165961
   Median:     0.2539768
  3rd Qu.:     0.2772089
      Max:     0.3030398
  Total N: 10000.0000000
    NA's :     0.0000000
 Std Dev.:     0.1464434
```

Figure 4C.6
Histogram of (C_T^*) in the case of (0.1, 0.8, 0.9) when σ = 20%



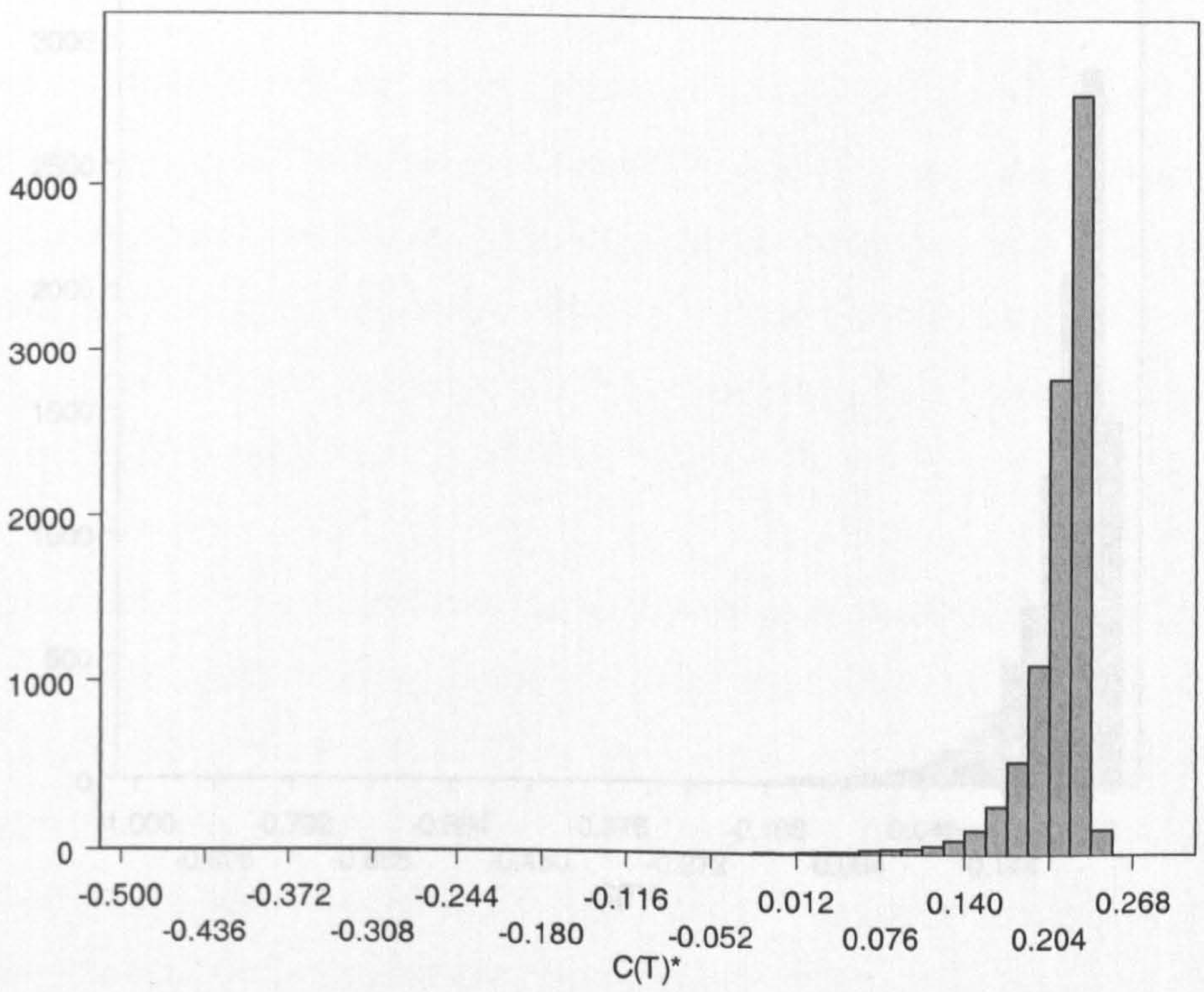
Case: CIPI (0.9, 0, 0.1) sigma = 0.2

Summary Statistics for data in: (0.9, 0, 0.1)

```
               C(T)*
      Min: -1.050058e+000
  1st Qu.:  2.046764e-001
     Mean:  2.100490e-001
   Median:  2.188639e-001
  3rd Qu.:  2.270114e-001
      Max:  2.395829e-001
  Total N:  1.000000e+004
    NA's :  0.000000e+000
 Std Dev.:  3.513714e-002
```

Figure 4C.7

Histogram of (C_T^*) in the case of (0.9, 0, 0.1) when σ = 20%

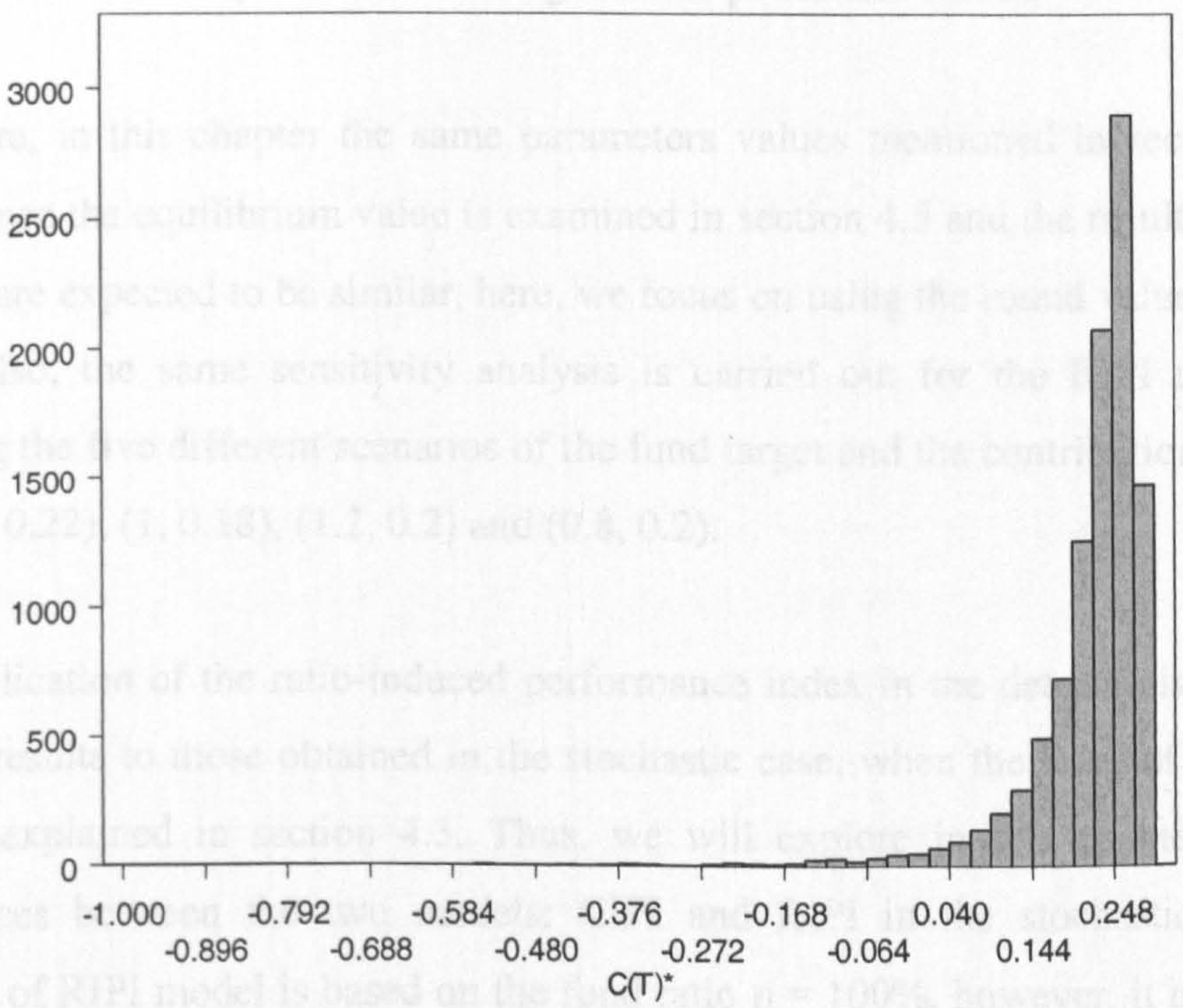


Case : CIPI (0.9, 0.8, 0.1) sigma = 0.2

Summary Statistics for data in: (0.9, 0.8, 0.1)

```
                  C(T)*
     Min: -3.396664e+000
 1st Qu.:  2.020505e-001
    Mean:  2.173270e-001
  Median:  2.417755e-001
 3rd Qu.:  2.650478e-001
     Max:  3.032638e-001
 Total N:  1.000000e+004
    NA's :  0.000000e+000
Std Dev.:  9.749935e-002
```

Figure 4C.8
Histogram of (C_T^*) in the case of (0.9, 0.8, 0.1) when $\sigma = 20\%$



Chapter Five

Computational experiments to reveal the underlying properties of the model and policy implications II

5.1 Introduction

Due to the similarity in the mathematical construction of the dynamic control problems of both models: the cost- induced performance index (CIPI) and the ratio-induced performance index (RIPI), we expect to obtain similar results for the expected optimal fund level and contribution rate with small differences between them. We highlight these differences in order to understand precisely the behaviour of the two models in response to the changes in the parameter values.

Therefore, in this chapter the same parameters values mentioned in section 4.2 are used. Since the equilibrium value is examined in section 4.5 and the results of the two models are expected to be similar, here, we focus on using the round value of $B_t = 0.3$ only. Also, the same sensitivity analysis is carried out for the RIPI model, with applying the five different scenarios of the fund target and the contribution target: (1, 0.2), (1, 0.22), (1, 0.18), (1.2, 0.2) and (0.8, 0.2).

The application of the ratio-induced performance index in the deterministic case has similar results to those obtained in the stochastic case, when the level of volatility is low as explained in section 4.3. Thus, we will explore in this chapter the exact differences between the two models: CIPI and RIPI in the stochastic case. Our analysis of RIPI model is based on the fund ratio $\eta = 100\%$, however, it is also worth examining the effect of changing the parameter η on $E(F_t^*)$ and $E(C_t^*)$ by allowing η to be equal 90% and 110% (this is discussed in section 5.2.7of this chapter).

5.2 Results of the sensitivity analysis

In the following subsections, the results of the sensitivity analysis of RIPI stochastic model will be illustrated, in order to explore the effect of changing the values of different parameters following the same approach as have introduced in the previous chapter.

5.2.1 Effect of changing θ

(a) Effect of changing θ on the benefit outgo B_t

Since the actuarial liability recurrence relation equation used for the RIPI model is the same as the one used in subsection 4.4.1(a):

$$AL_{t+1} = e^{\delta} . (AL_t + NC_t - B_t)$$

the value of the benefit outgo B_t changes as the value of θ increases from 10% to 15% or decreases to 5%. These changes are exactly similar to the ones shown in Table 4.2. Thus, B_t goes up with increasing the value of θ and moves down with decreasing the value of θ.

(b) Effect of changing θ on $E(F_t^*)$ and $E(C_t^*)$

Similarly to the previous chapter, here, we examine the effect of changing θ under the first scenario only, where the fund target is equal to 1 and the Contribution target is equal to 0.2. The value of B_t remains equal to 0.3 when θ is equal to 10%. However, B_t reduces to 0.25 as θ decreases to 5% and it increases to 0.34 when θ = 15%. Applying different values of B_t – that respond to changing θ - affects slightly the expected optimal fund and contribution rate. For example, in the case of (0.1, 0, 0.9)[1] when σ = 5%, we find that the expected fund levels remain around their target with a slight decreasing trend over the control period. On the other hand, the expected contribution rates are also around their target with an increasing trend over the control period. The effect of changing θ in the same case mentioned above is more obvious when σ is equal to 20%. Figures 5.1 and 5.2 show the differences between $E(F_t^*)$ under RIPI and CIPI as θ changes with σ equal to 20%.

[1] To recall: the form of (0.1, 0, 0.9) represents the weighting risk factors in the following order (γ,α, β).

Figure 5.1

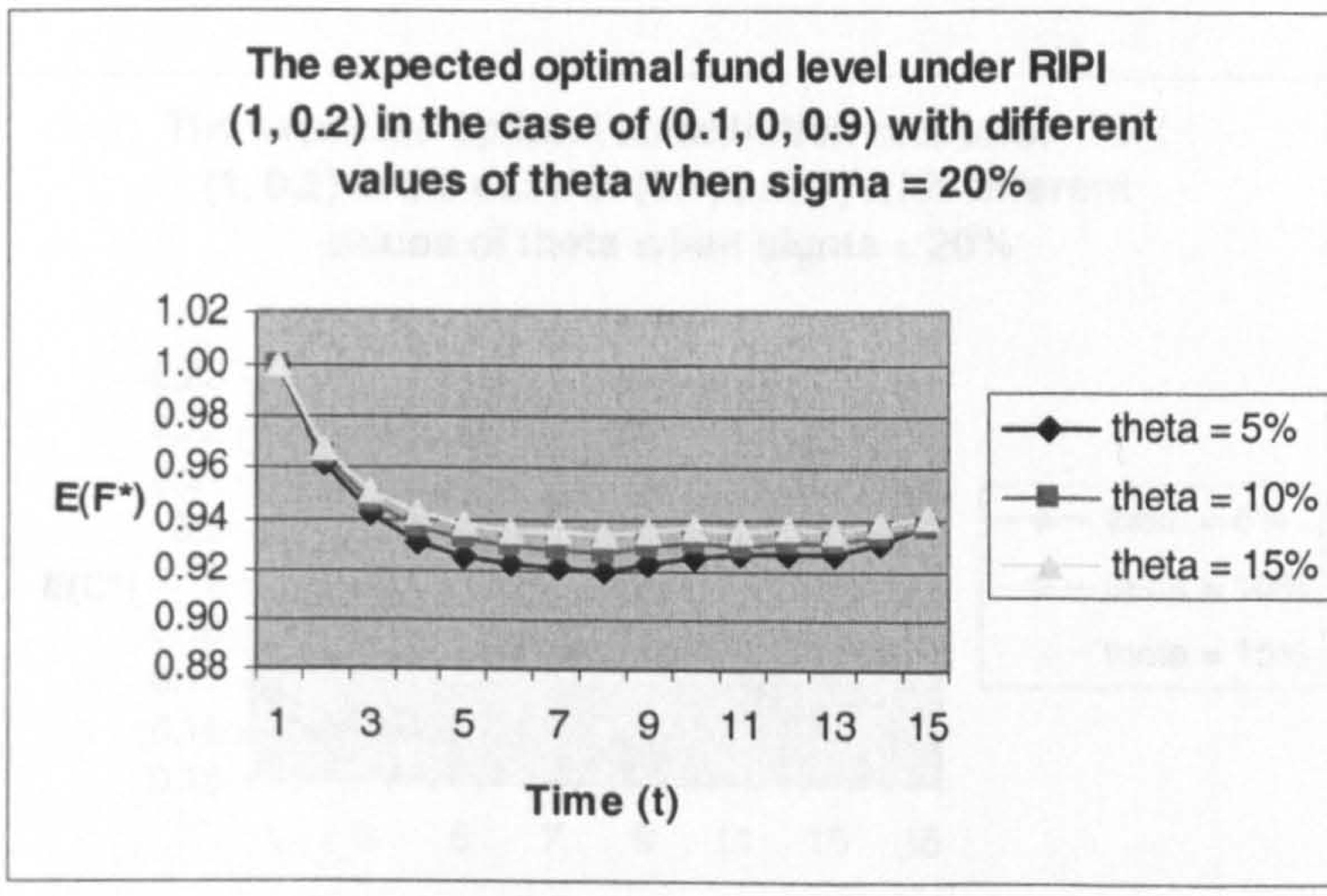
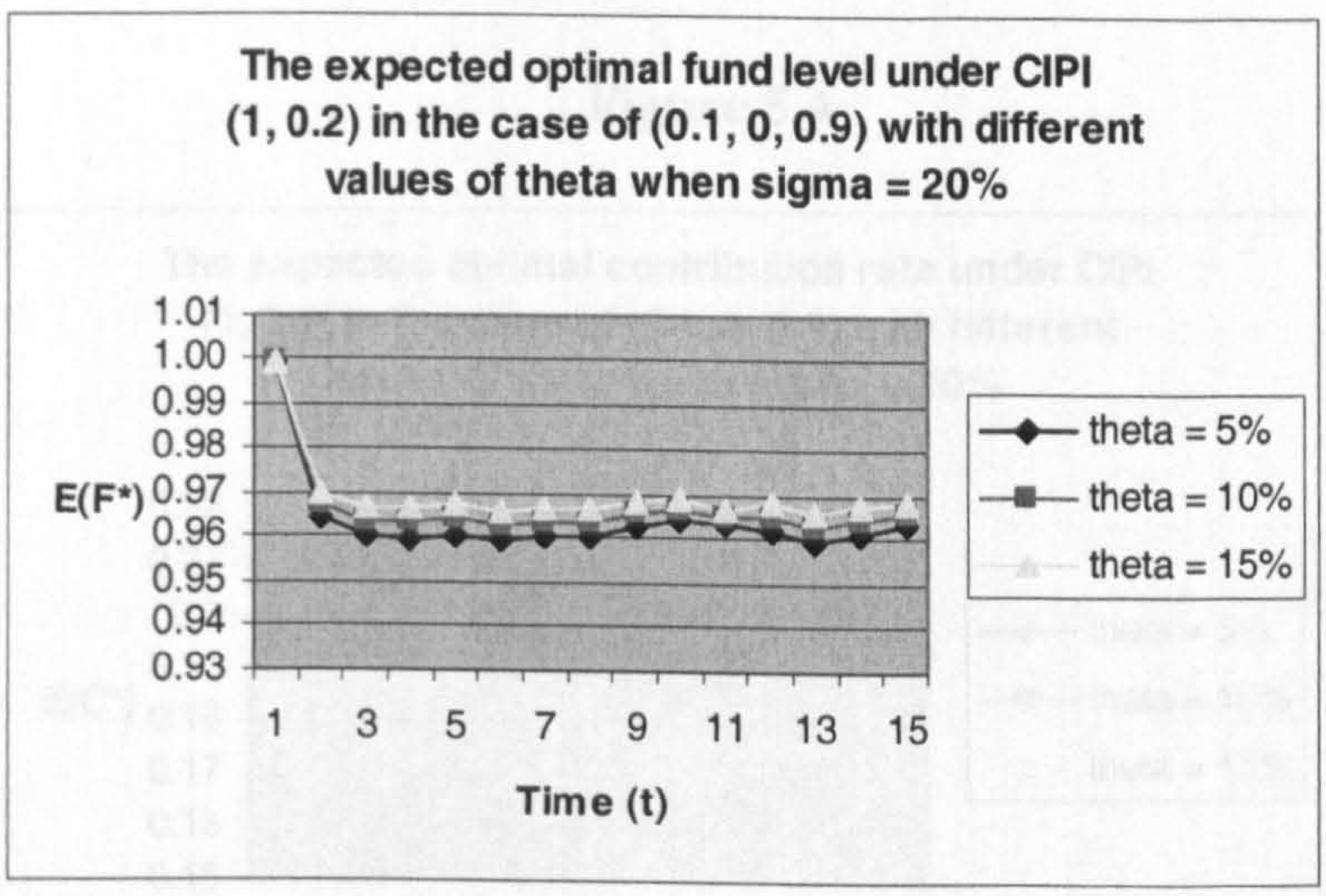


Figure 5.2



Figures 5.1 and 5.2 illustrate the similarity in the trends of expected optimal fund level between both models CIPI and RIPI. We note that they decrease over the control period with decreasing θ. Therefore, when $\theta = 5\%$, $E(F_t^*)$ reaches the lowest levels in both models. However, $E(F_t^*)$ under CIPI is closer to the target and is around 96% while it is around 92% under RIPI.

The expected optimal contribution rates in the same case (0.1, 0, 0.9) increase over the control period as θ increases from 5% to 15%. Thus, $E(C_t^*)$ increases to reach 22% under RIPI while it is around 21% under CIPI as shown in Figures 5.3 and 5.4.

Figure 5.3

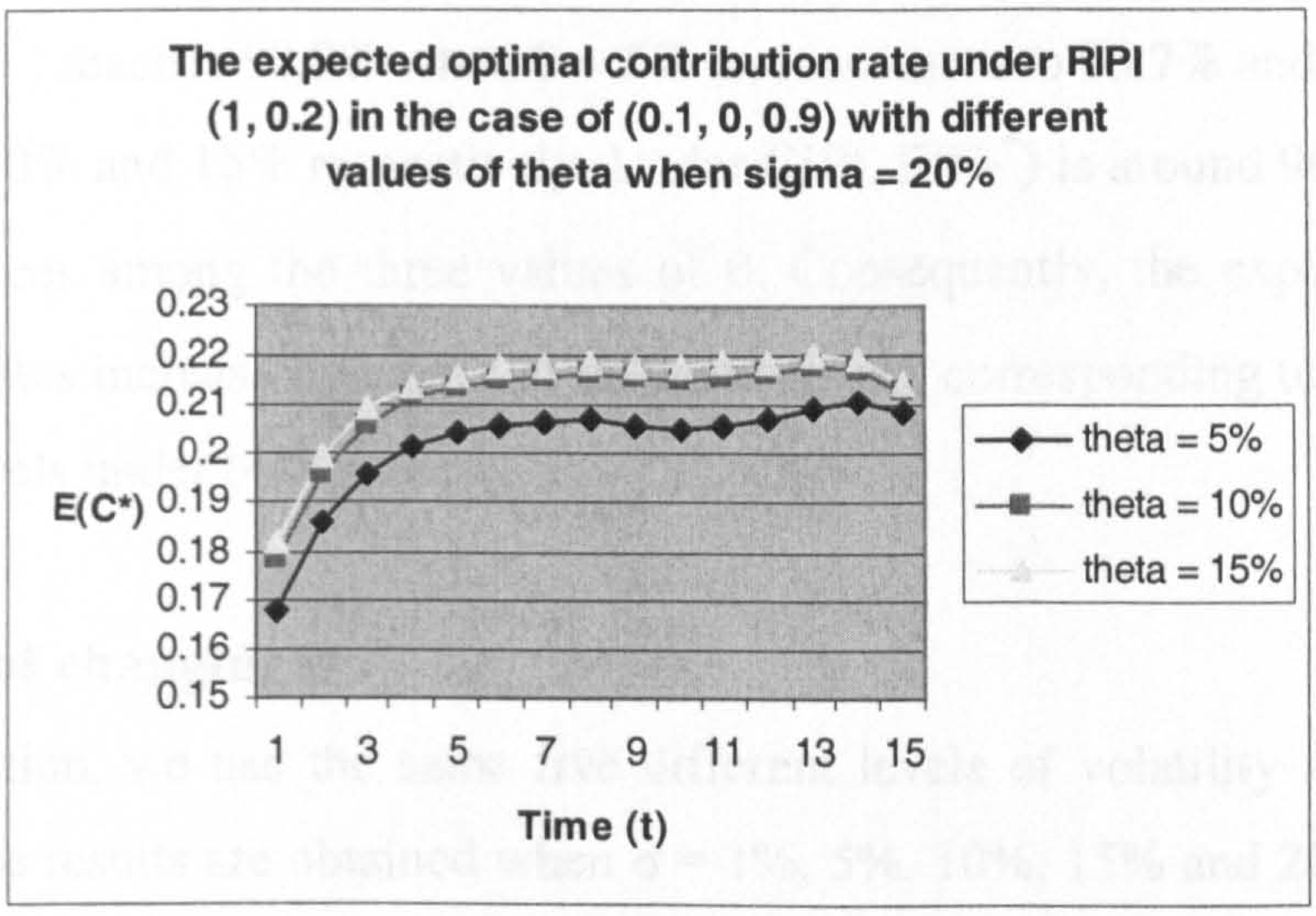
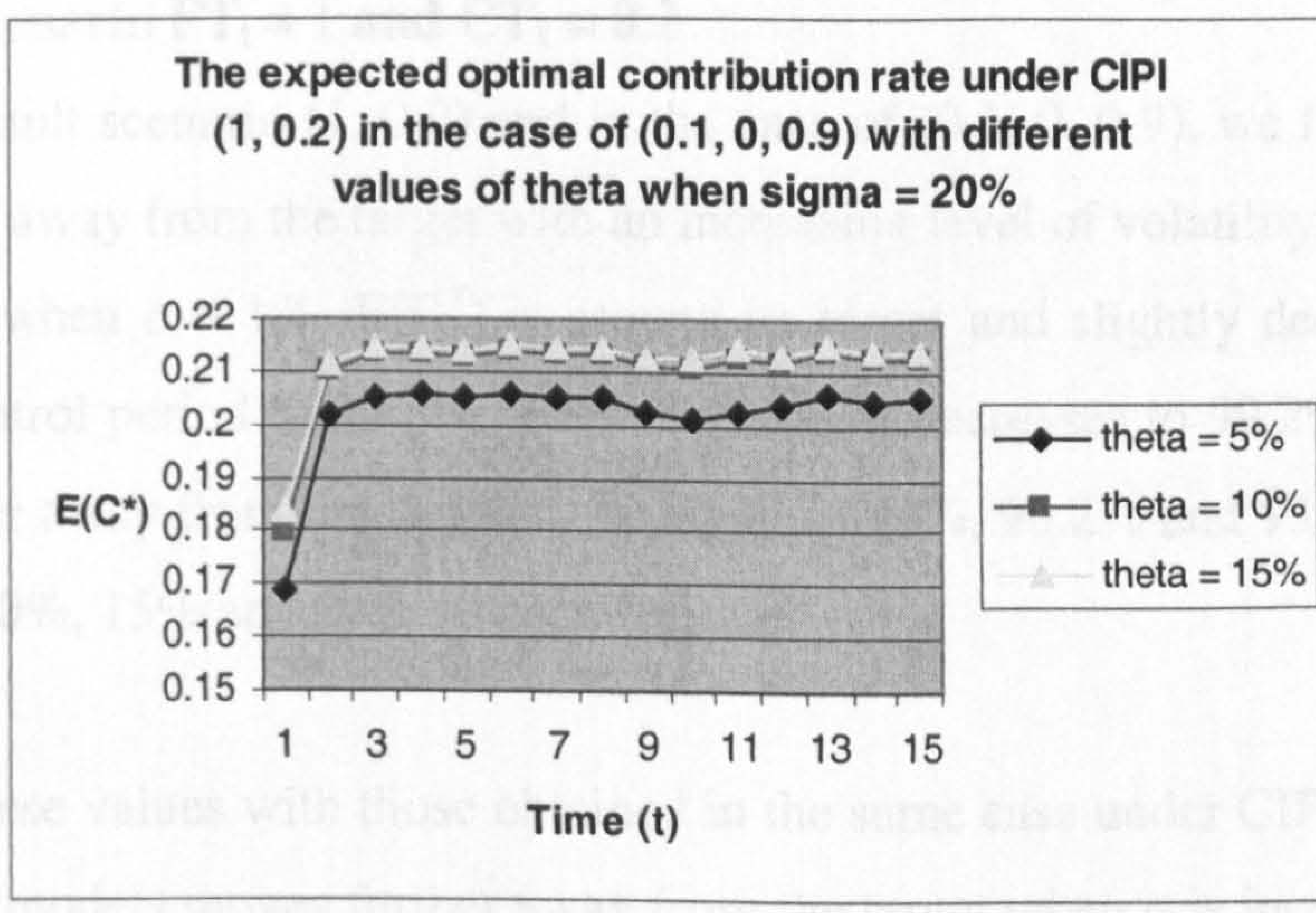


Figure 5.4



This difference between $E(C_t^*)$ under both models reflects the difference on $E(F_t^*)$ within the dynamic relationship. Thus, it is expected that the optimal contribution rate under RIPI increases more than under CIPI to cover the lower levels of the fund under RIPI.

Further, we include the results of keeping the value of B_t equal to 0.3 when the value of θ changes to 15% and 5% (instead of θ remains constant and is equal to 10%). For example, in RIPI under the first scenario (1, 0.2) and in the case of (0.1, 0, 0.9), $E(F_t^*)$ tends to increase with increasing θ and consequently $E(C_t^*)$ decreases over the control period. In comparison with CIPI, the changes in both $E(F_t^*)$ and $E(C_t^*)$ are more clear in RIPI whether σ is equal to 5% or 20%. In other words, in the latter model when $\sigma = 20\%$, $E(F_T^*)$ reaches 90.9% when $\theta = 5\%$ and increases to 93.7% and 95.6% when θ is equal to 10% and 15% respectively. Under CIPI, $E(F_T^*)$ is around 96% with very slight differences among the three values of θ. Consequently, the expected optimal contribution rates increase over time at different levels, corresponding to the decrease in the fund levels under both models.

5.2.2 Effect of changing σ

In this subsection, we use the same five different levels of volatility applied under CIPI. Thus, the results are obtained when σ = 1%, 5%, 10%, 15% and 20%, the mean of rate of return $\theta = 10\%$ and the benefit outgo $B_t = 0.3$.

(a) Default scenario $FT_t = 1$ and $CT_t = 0.2$

Under the default scenario (1, 0.2) and in the case of (0.1, 0, 0.9), we find that $E(F_t^*)$ moves further away from the target with an increasing level of volatility.
For instance, when σ = 1%, $E(F_t^*)$ is around its target and slightly decreases by the end of the control period to 99.5%. When σ = 5%, it decreases to 99.2% and it keeps moving further away from the target to be equal to 98%, 96.2% and 93.7% when σ is increased to 10%, 15% and 20% respectively.

Comparing these values with those obtained in the same case under CIPI, we find that $E(F_t^*)$ in both models moves further away from the target when σ is increased to 20%. However, $E(F_t^*)$ under RIPI has lower levels than those under CIPI, i.e. when σ = 20%, $E(F_t^*)$ reaches the lowest level 93% under the former model while it is around 96.5% under CIPI. Figures 5.5 and 5.6 illustrate the differences in the trends of $E(F_t^*)$ with different levels of volatility for both models.

Figure 5.5

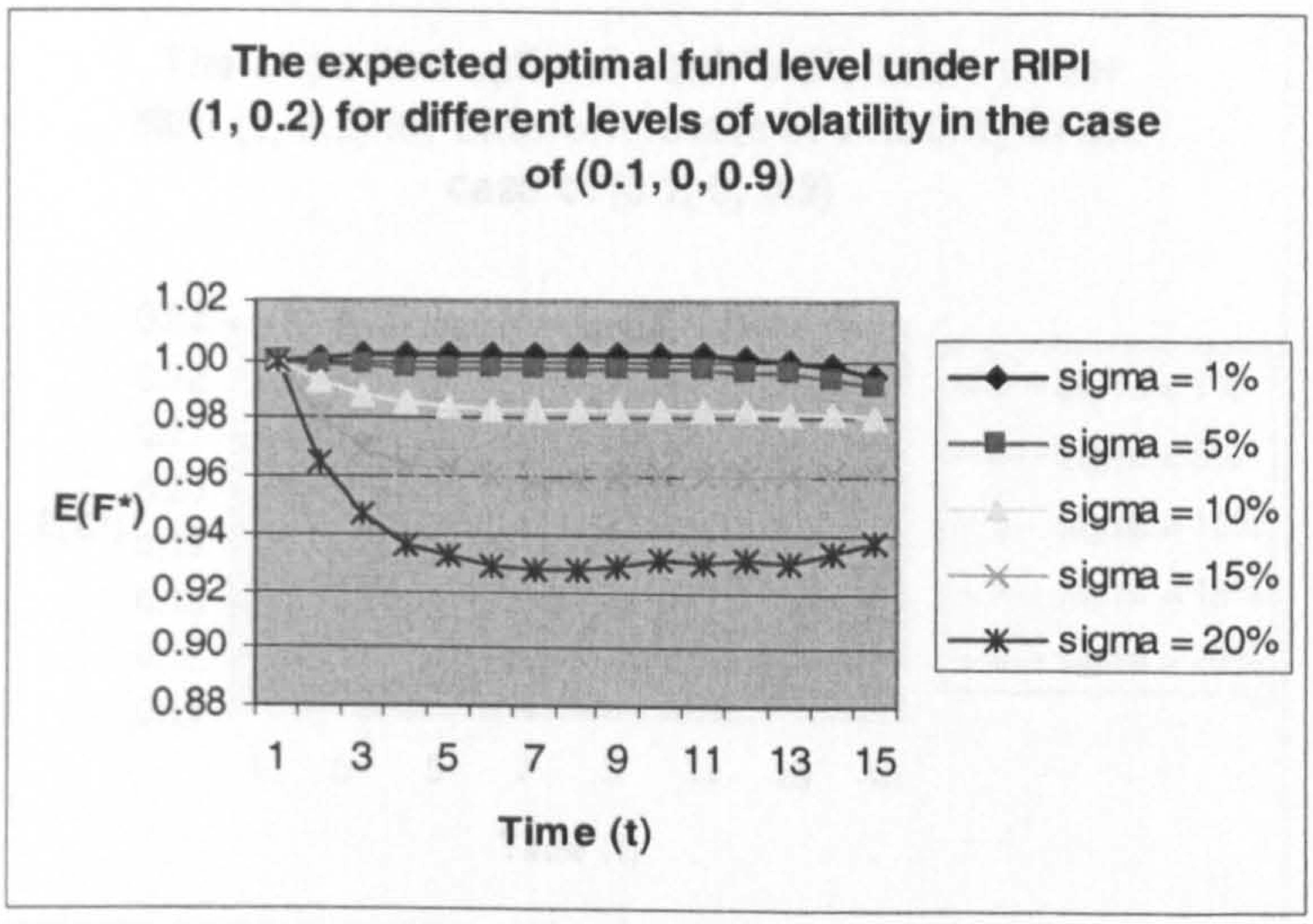
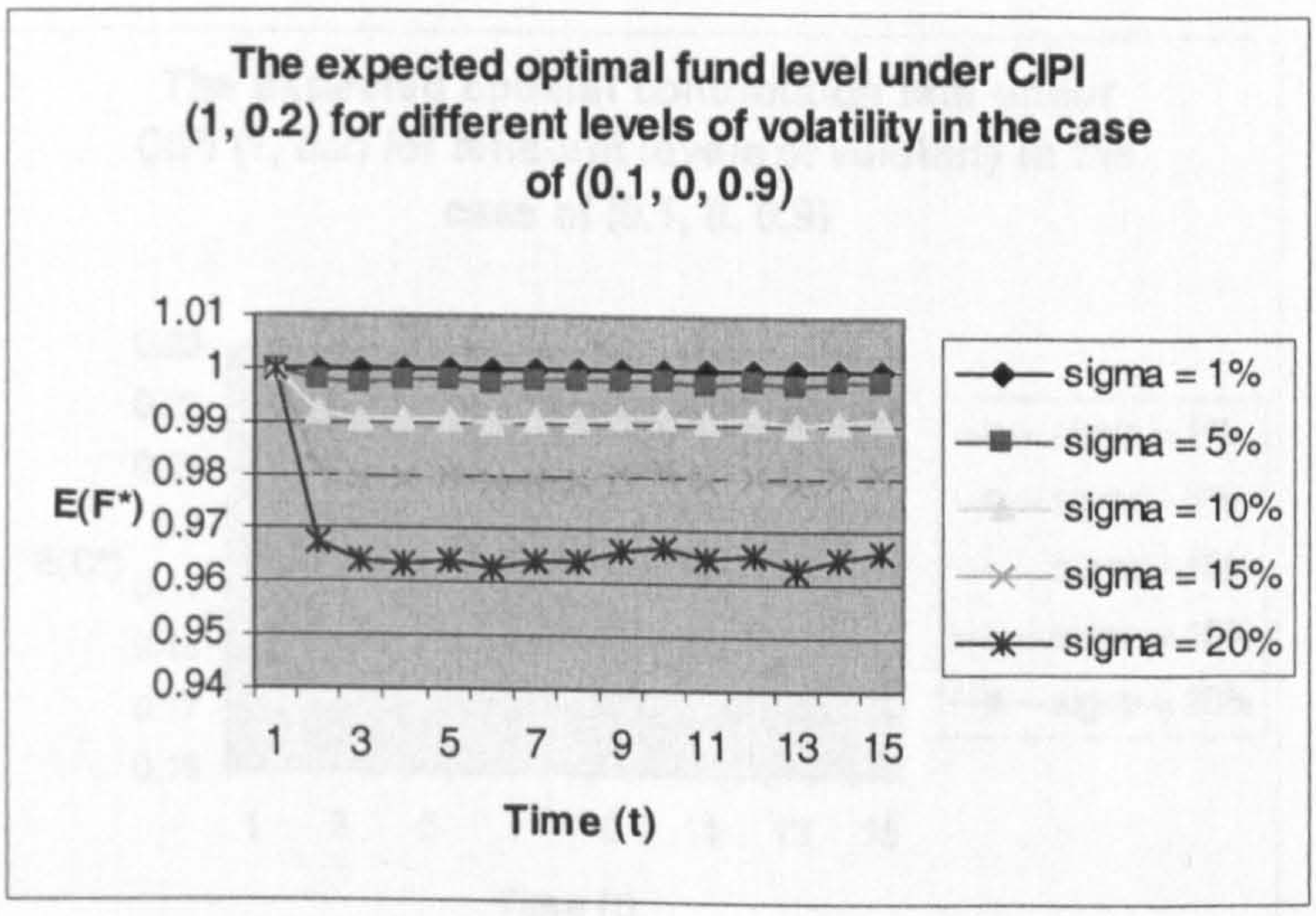


Figure 5.6



For other cases such as: (0.5, 0, 0.5) and (0.9, 0, 0.1), similar results are obtained, where $E(F_T^*)$ decreases in the former case from 97.4% when σ =1% to 83.3% when σ is increased to 20%. In the latter case where $\beta = 0.1$, it decreases more markedly from 89.5% to 65.2% at the same levels of volatility.

For the expected optimal contribution rate, we note that it also moves away from its target with increasing the level of volatility. In the first case of (0.1, 0, 0.9) when σ = 1%, $E(C_t^*)$ increases gradually over the control period to reach the value of 20.4% at the final T. When σ is increased to 20%, the value of $E(C_T^*)$ moves up to 21.3%. The difference of $E(C_t^*)$ levels between RIPI and CIPI are shown in Figures 5.7 and 5.8.

Figure 5.7

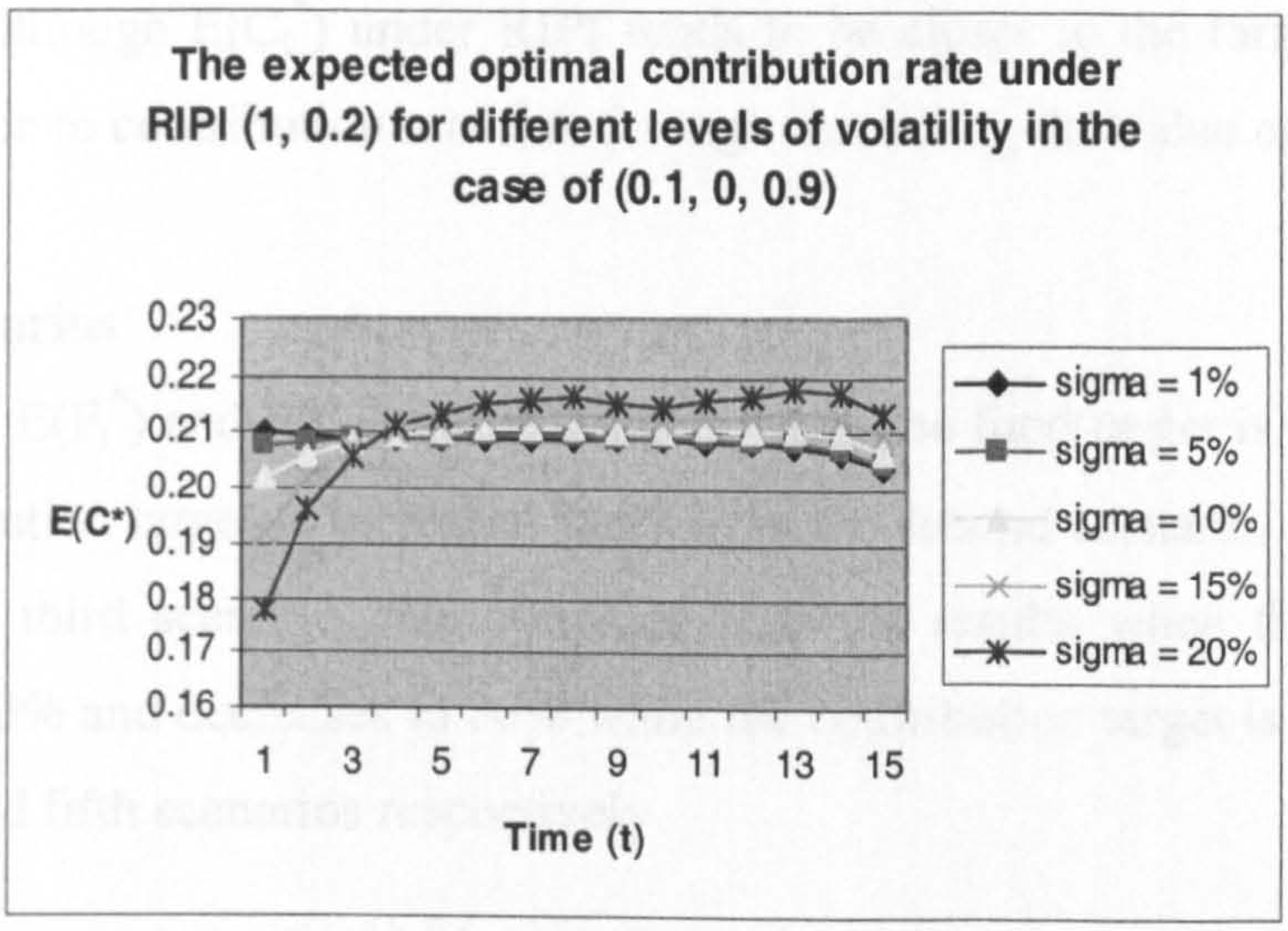
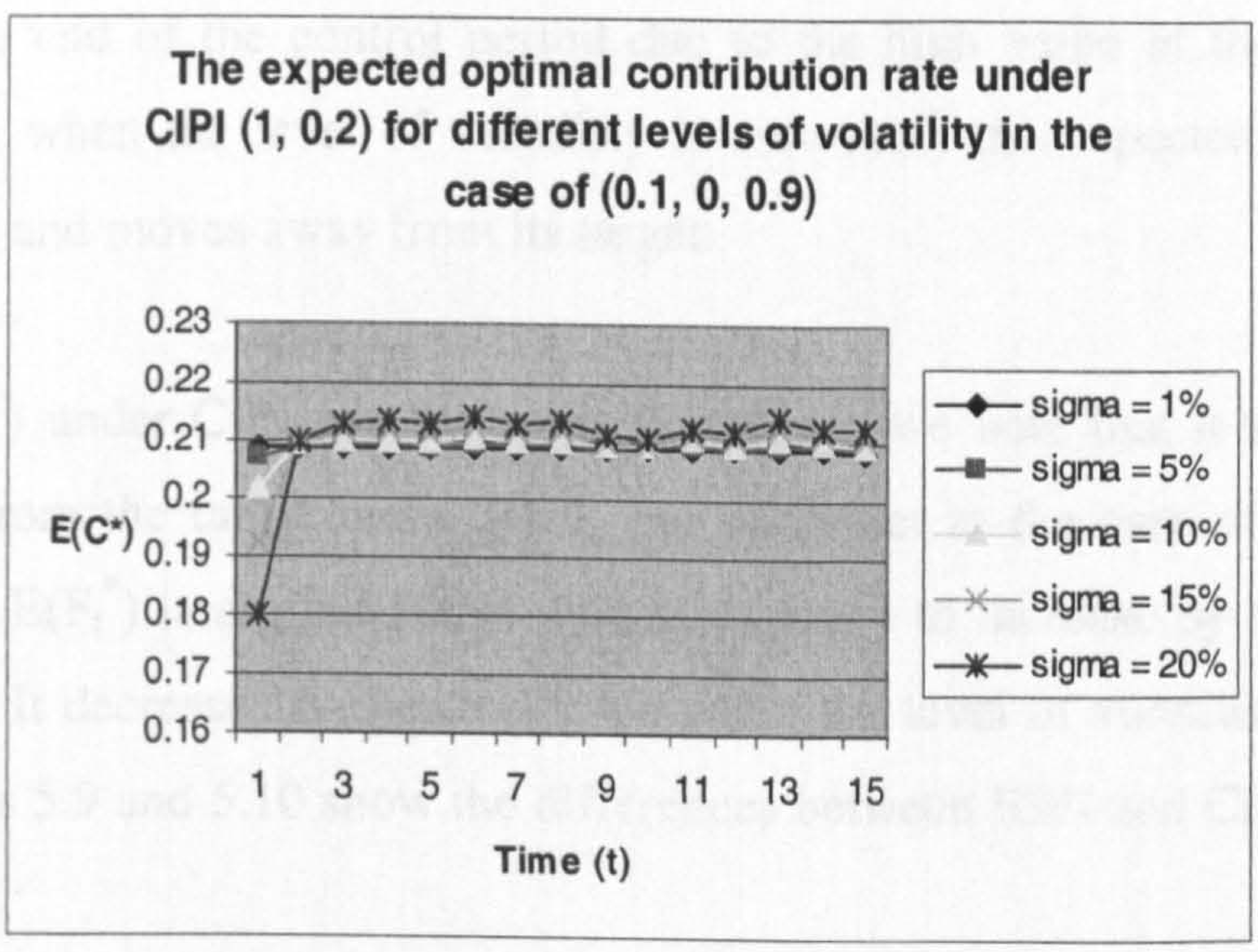


Figure 5.8



For the other two cases of (0.5, 0, 0.5) and (0.9, 0, 0.1), $E(C_t^*)$ has the same behaviour as it moves slightly away from the target with increasing σ. The expected optimal contribution rate increases gradually over the control period, and then tends to decrease to get closer to the target at the final T. Thus, in the former case, (0.5, 0, 0.5) $E(C_T^*) = 20.2\%$ and increases slightly to 20.7% when σ increases from 1% to 20%. In the latter case, although $E(C_t^*)$ becomes closer to the target due to the high value of $\gamma = 0.9$, the departure from the target is still observed when σ is increased from 1% to 20%.

Comparing these results with CIPI, we find that the differences are small between both models, although $E(C_t^*)$ under RIPI tends to be closer to the target as we give more importance to contribution rate risk through increasing the value of γ.

(b) Other scenarios

The changes in $E(F_t^*)$ and $E(C_t^*)$ are examined, when the fund target is fixed at 100% and the contribution target is increased to 22%, in the second scenario, and decreased to 18% in the third scenario. We conclude also the results when the fund target increases to 120% and decreases to 80% while the contribution target is equal to 20%, in the fourth and fifth scenarios respectively.

Under the second scenario, $E(F_t^*)$ remains around its target with a tendency to increase by the end of the control period due to the high value of the contribution target. Further, when the level of volatility is increased, the expected optimal fund level decreases and moves away from its target.

Although $E(F_t^*)$ under CIPI has the same behaviour, we note that it tends to move further away from the target under RIPI. For example: in the case of (0.9, 0, 0.1), when $\sigma = 1\%$, $E(F_t^*)$ is around 100% with a tendency to increase by the end of the control period. It decreases further to 85.4% when the level of volatility is increased to 20%. Figures 5.9 and 5.10 show the differences between RIPI and CIPI in this case (0.9, 0, 0.1).

Figure 5.9

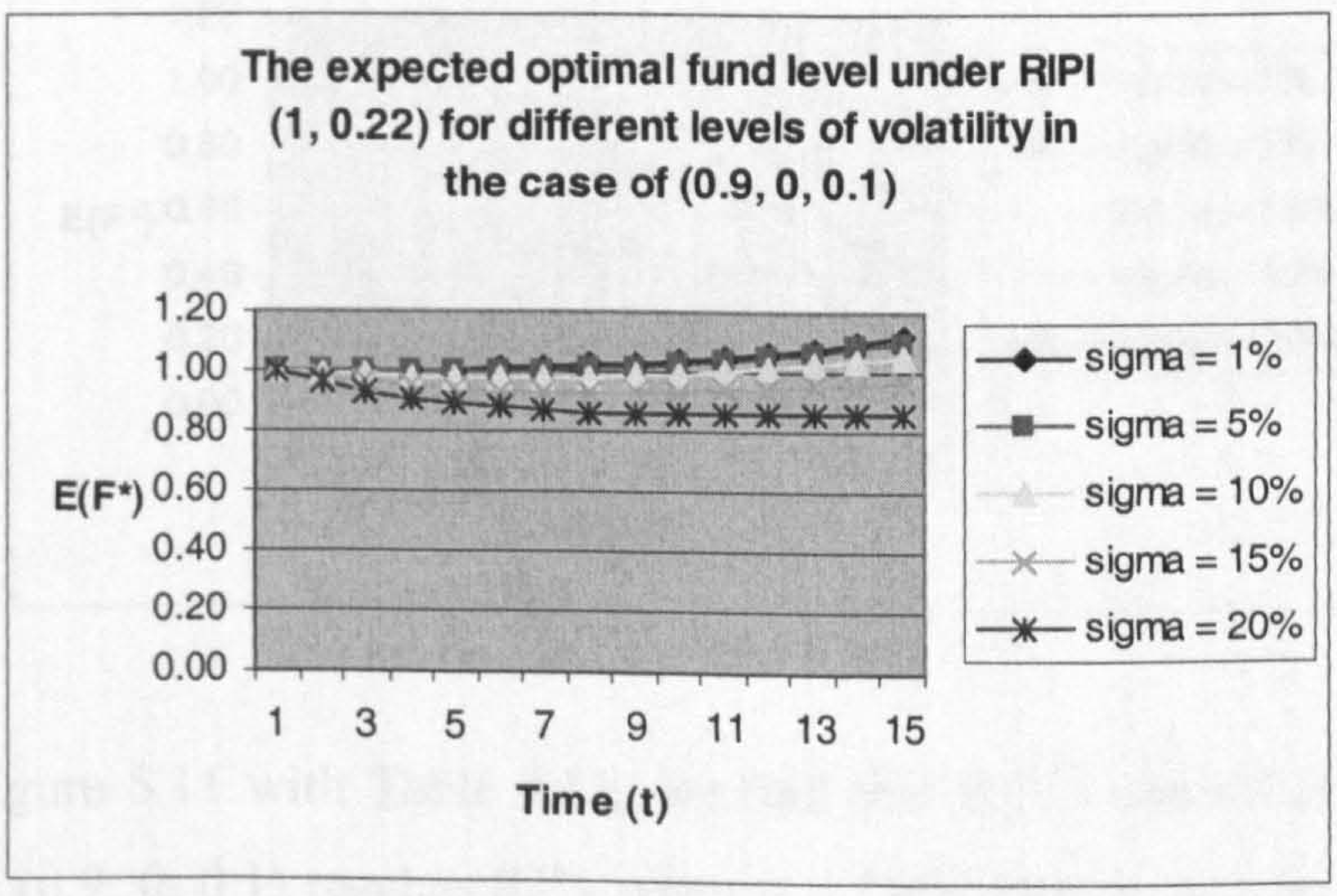
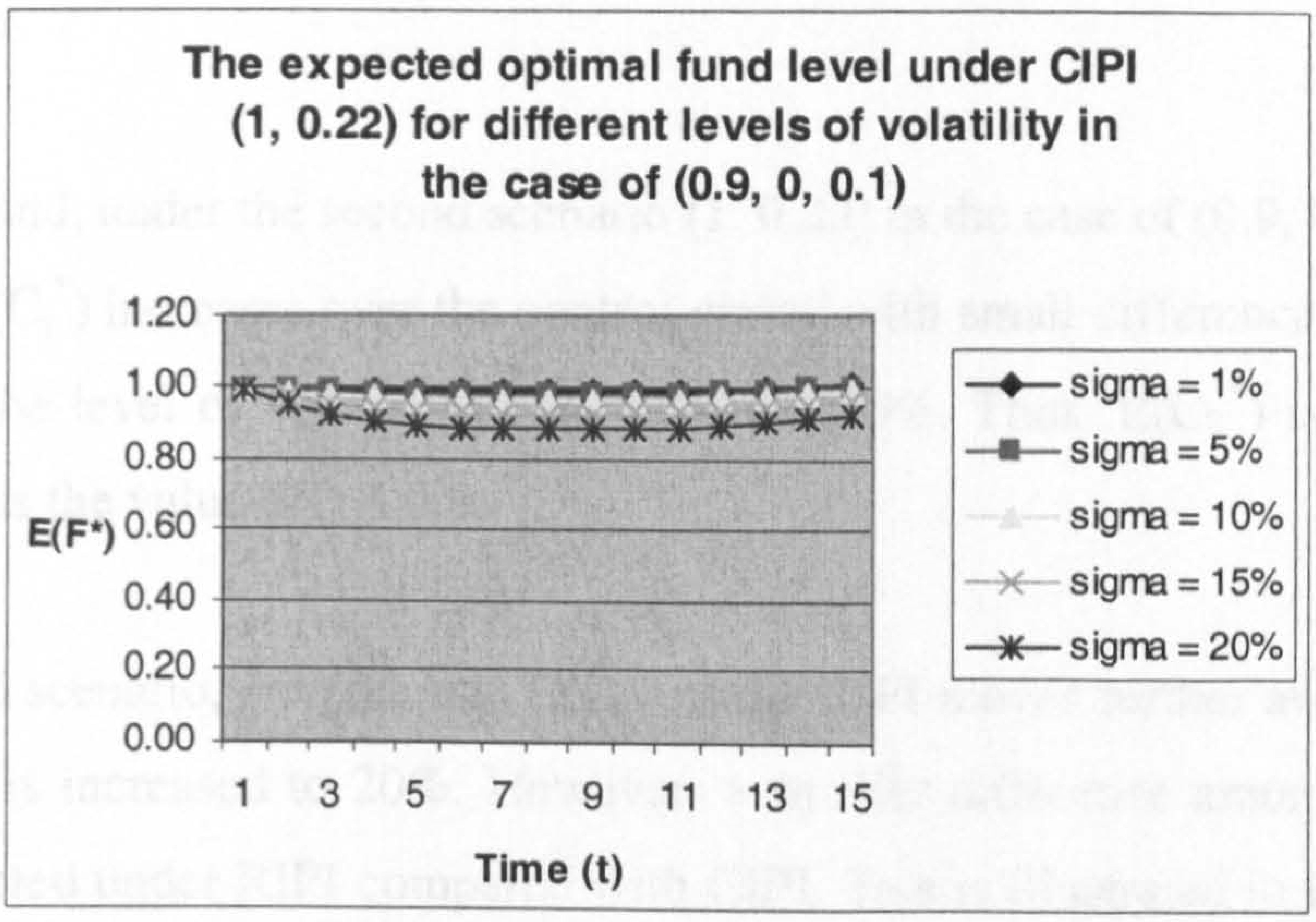
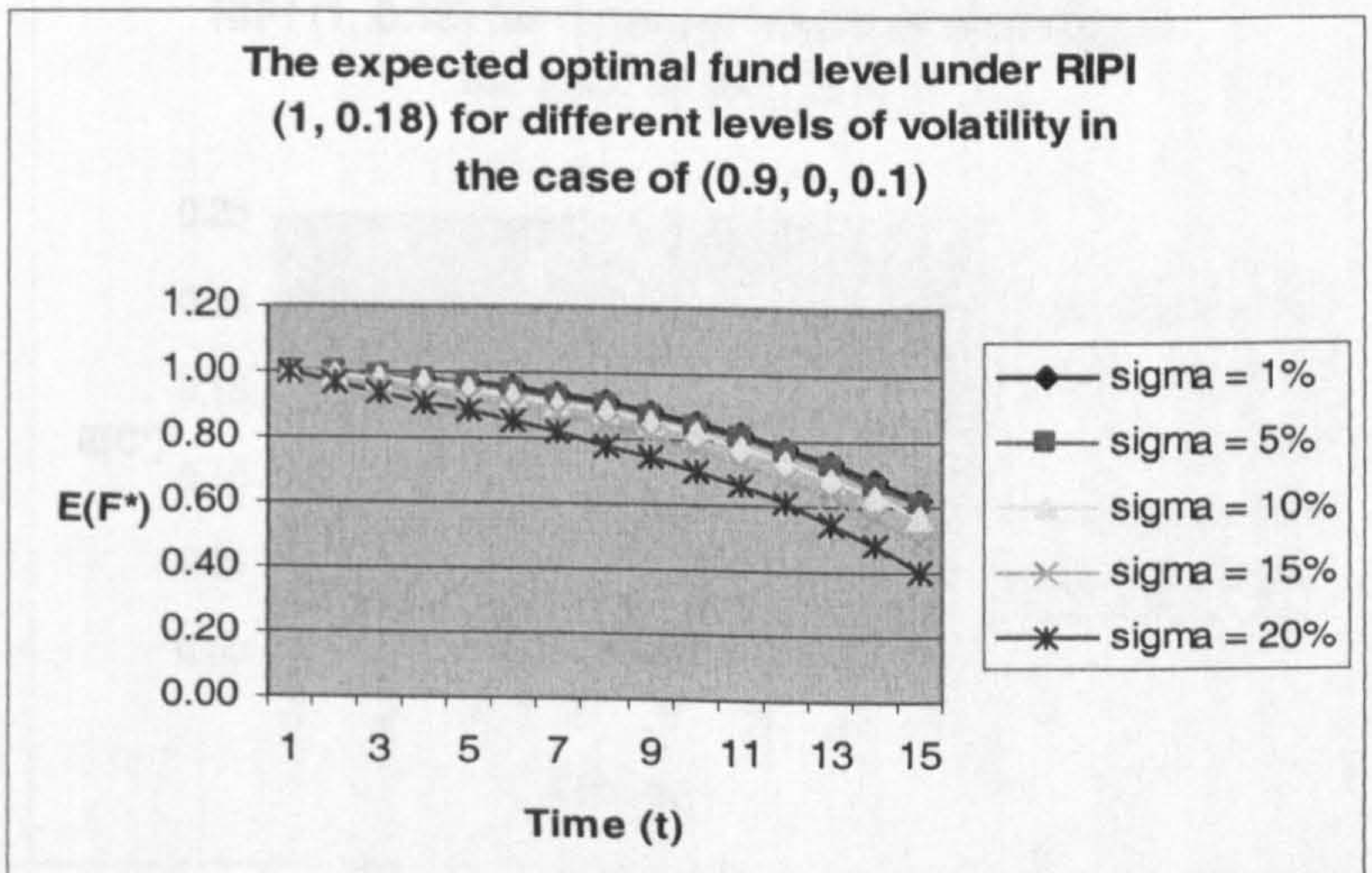


Figure 5.10



Under the third scenario (1, 0.18) in RIPI, $E(F_t^*)$ tends to decrease more markedly over the control period compared with the previous scenario. Moreover, increasing the level of volatility leads the fund level to move down further from its target. We consider the same case as above, (0.9, 0, 0.1), for the RIPI model, we find that $E(F_t^*)$ decreases dramatically to reach about 40% when σ = 20%. This is due to increasing the level of volatility along with more importance is given to the weighting factor for the contribution rate as shown in Figure 5.11

Figure 5.11



Comparing Figure 5.11 with Table 4.11, we find that $E(F_t^*)$ under CIPI (1, 0.18) in the same case (0.9, 0, 0.1) reaches 87% when σ = 20%. Hence, we can conclude that the expected optimal fund level under RIPI is more affected by the changes of the volatility levels.

On the other hand, under the second scenario (1, 0.22) in the case of (0.9, 0, 0.1), we find that $E(C_t^*)$ increases over the control period with small differences among the trends, when the level of volatility is increased to 20%. Thus, $E(C_T^*)$ is around the target of 22% as the value of $\gamma = 0.9$.

Under the third scenario, we note that $E(C_t^*)$ under RIPI moves further away from the target when σ is increased to 20%. However, a smaller difference among the trends for $E(C_t^*)$ is noted under RIPI compared with CIPI. This is illustrated in Figures 5.12 and 5.13.

Under the fourth scenario(1.2, 0.2), similar trends of $E(F_t^*)$ and $E(C_t^*)$ in RIPI to the ones under the second scenario are obtained, while the results of the fifth scenario are similar to the ones under the third scenario, with different levels of expected funds and contribution rates due to the different value of the fund target. Hence, the same conclusion about the trends of $E(F_t^*)$ and $E(C_t^*)$ in RIPI and CIPI is reached.

Figure 5.12

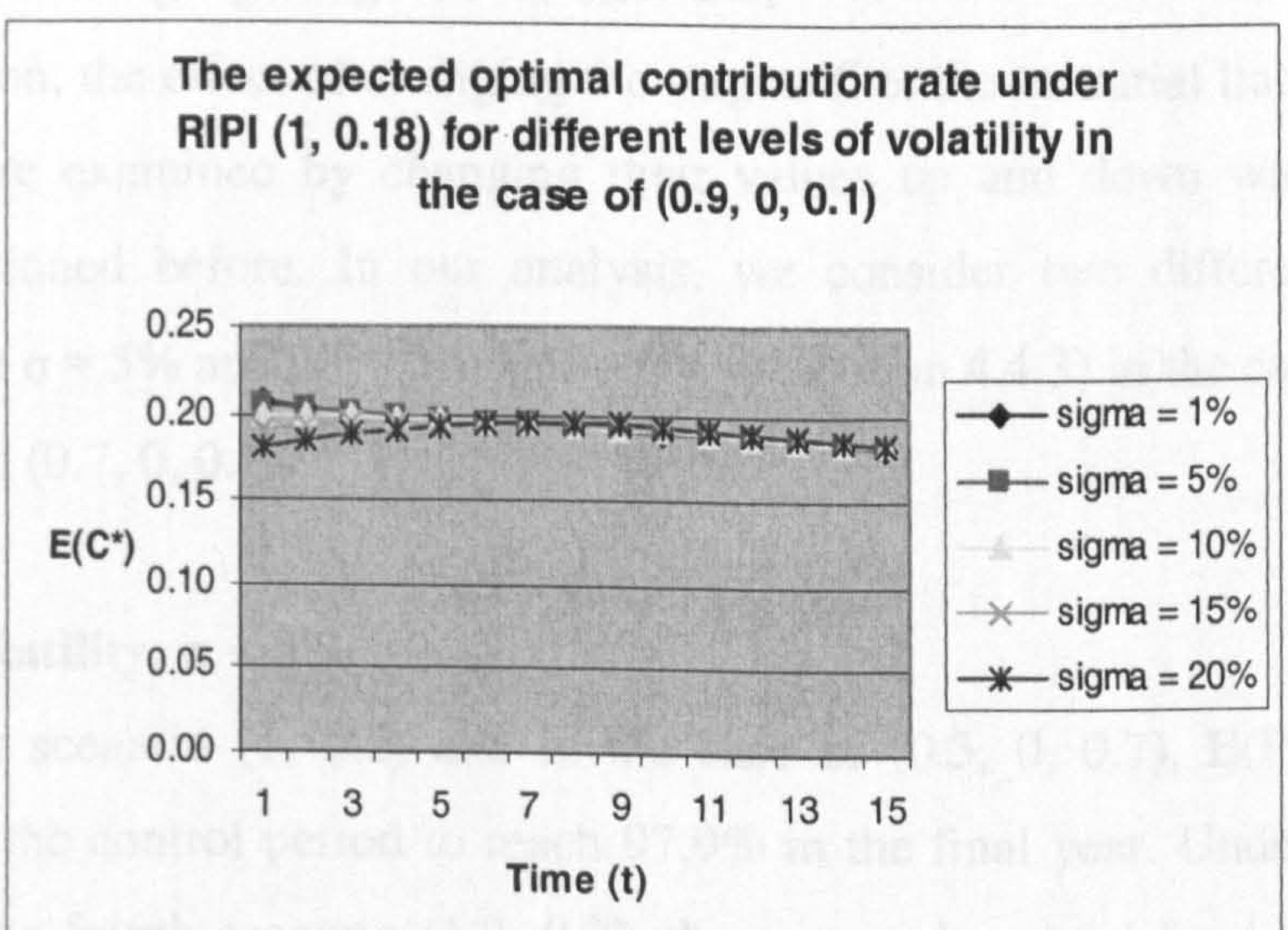
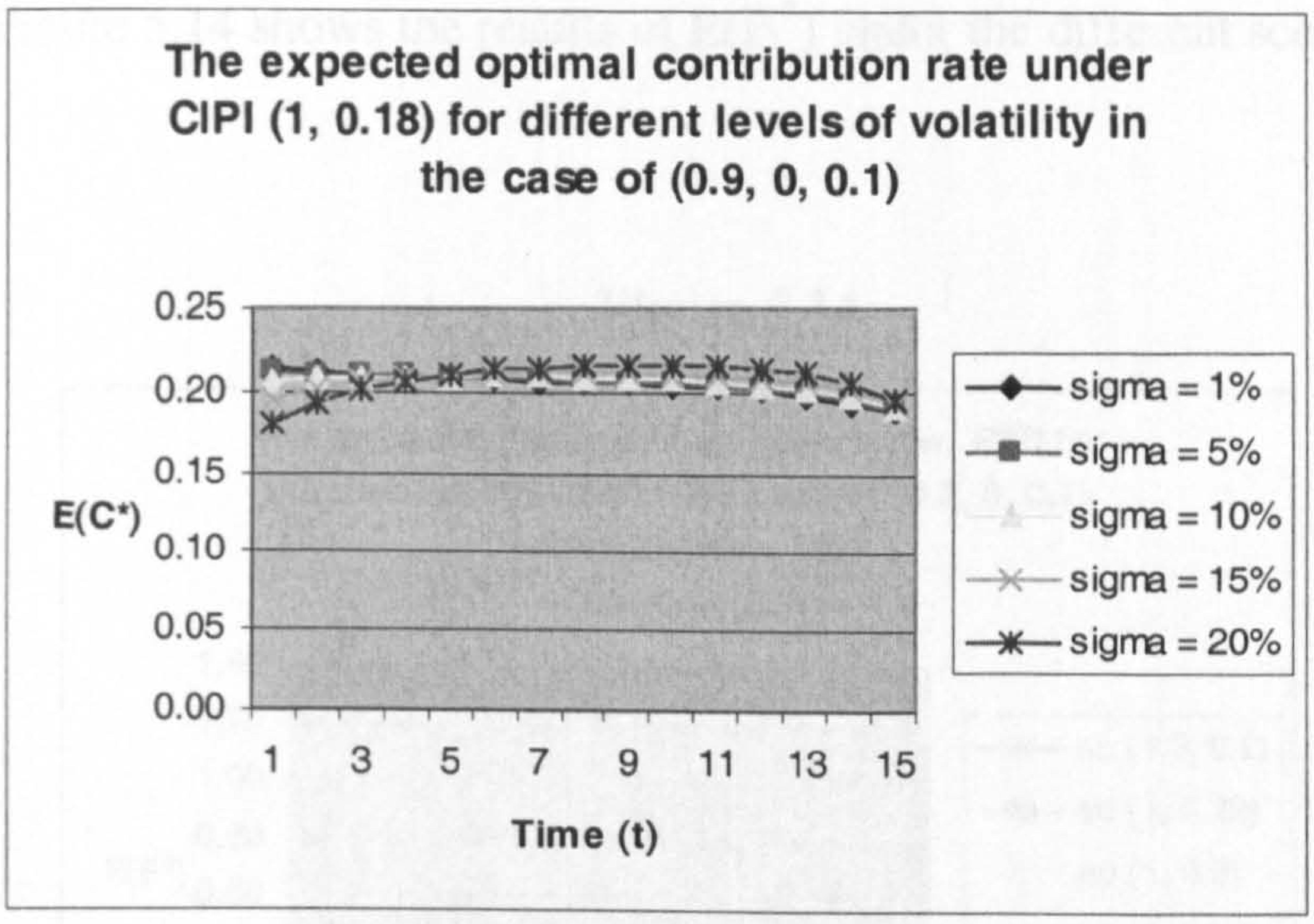


Figure 5.13



Conclusion

The results obtained under different scenarios in RIPI implies a general decrease in the expected optimal fund level over the control period, when the level of volatility increases. In response, the expected optimal contribution rate increases and moves away from the target. The comparison between RIPI and CIPI reveals that the trends of $E(F_t^*)$ and $E(C_t^*)$ in RIPI deviate more from the target than those under CIPI.

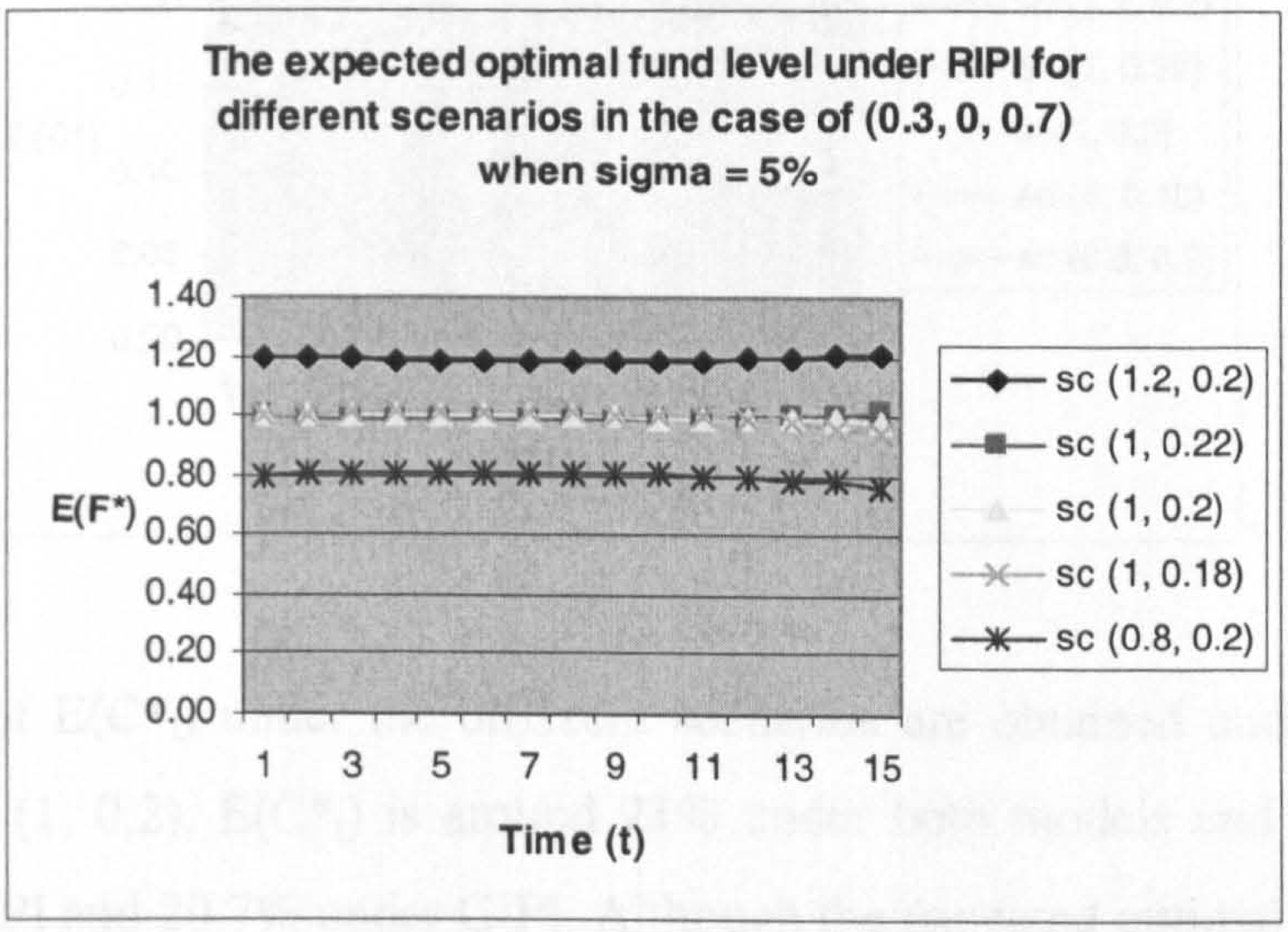
5.2.3 Effect of changing targets FT_t and CT_t

In this subsection, the effect of changing the targets (i.e. the actuarial liability and the normal cost) are examined by changing their values up and down within the five scenarios mentioned before. In our analysis, we consider two different levels of volatility, when $\sigma = 5\%$ and 20% (as applied in subsection 4.4.3) in the cases of (0.3, 0, 0.7) and (0.7, 0, 0.3).

(a) Level of volatility: $\sigma = 5\%$

Under the first scenario (1, 0.2) and in the case of (0.3, 0, 0.7), $E(F_t^*)$ decreases gradually over the control period to reach 97.9% in the final year. Under the second (1, 0.22) and the fourth scenario (1.2, 0.2), the expected optimal fund level slightly decreases with a tendency to increase in year 13 onwards. This is due to increasing the contribution and the fund targets in both scenarios respectively. On the other hand, under the third scenario (1, 0.18) and fifth scenario (0.8, 0.2), $E(F_t^*)$ remains around its targets and decreases in year 13 onwards. This is due to decreasing the values of both targets. Figure 5.14 shows the results of $E(F_t^*)$ under the different scenarios when $\sigma = 5\%$.

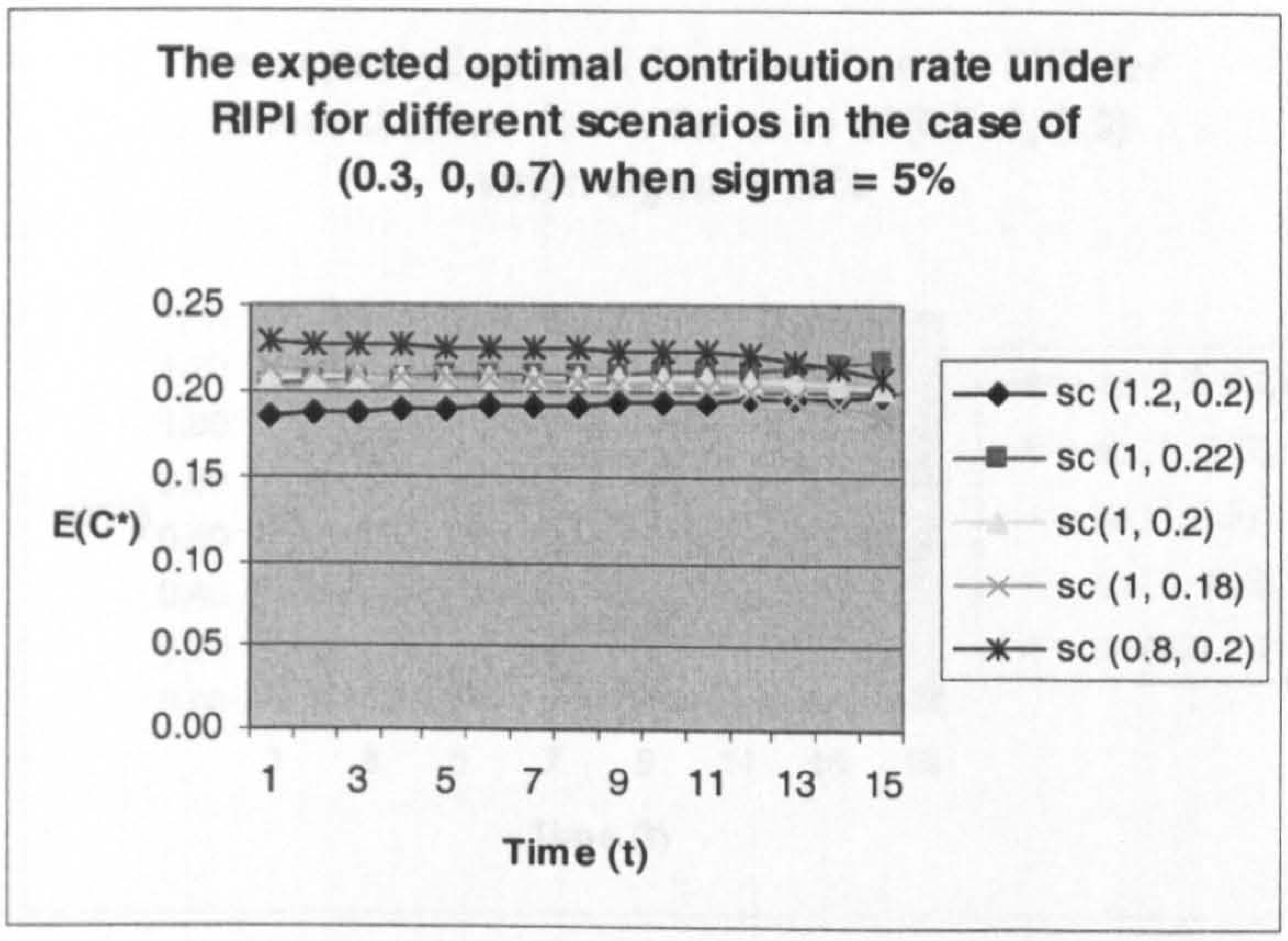
Figure 5.14



Here, it is notable that the behaviour of $E(F^*_t)$ is similar under RIPI and CIPI, where it decreases slightly around the targets as the weighting risk factor of the solvency risk is still high, i.e. $\beta = 0.7$. However, we note that the tendency for the fund level to increase or decrease by the end of the control period is more obvious under RIPI than CIPI.

For the expected optimal contribution rate under (1, 0.2), we find that it remains around the target with a slight increase responding to the slight decrease of the fund level. Under the second and fourth scenarios, there is a slight increase over the control period to get closer to the target. Nevertheless, under the third scenario (1, 0.18) and fifth scenario (0.8, 0.2), $E(C^*_t)$ starts from the higher levels 21.4% and 23% respectively and decreases over the time span to get closer to the target. This is shown in Figure 5.15.

Figure 5.15



Similar trends of $E(C^*_t)$ under the different scenarios are obtained under CIPI. For example, under (1, 0.2), $E(C^*_t)$ is around 21% under both models and decreases to 20.3% under RIPI and 20.7% under CIPI. Although the expected optimal contribution rates at the end of control period may have small differences between the two models - given that these values are simulated-based estimates of the expected contribution rates - there is, in general, a similar behaviour of $E(C^*_t)$ under both models.

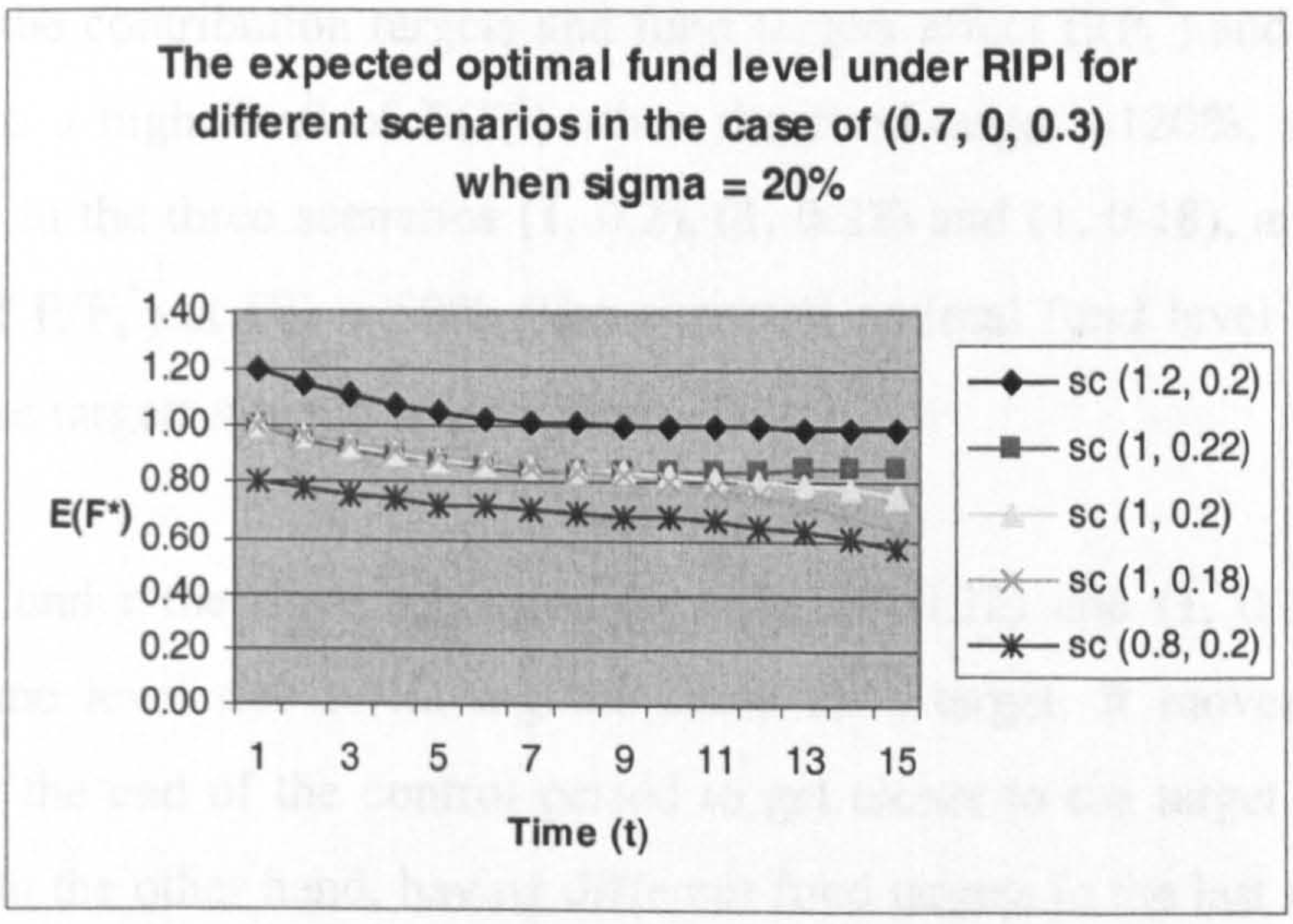
(b) Level of volatility: $\sigma = 20\%$

Here, we will use the case of (0.7, 0, 0.3) to examine the effect of increasing the level of volatility when FT_t and CT_t are changed.

According to the high level of volatility and the low value of the weighting risk factor of the solvency risk, the expected optimal fund level moves further away from the target as can be seen from Figure 5.16.

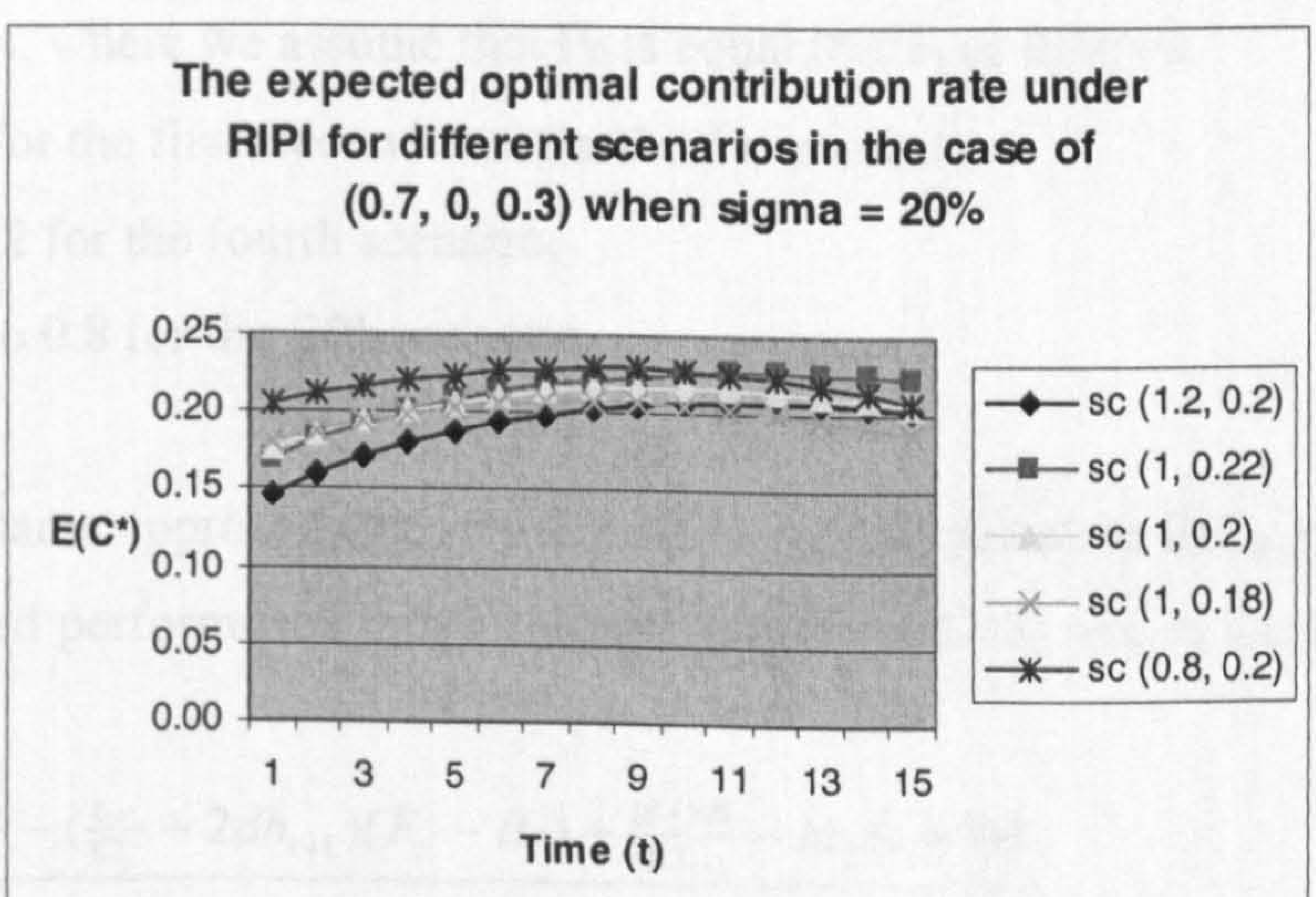
Comparing Figure 5.16 with Figure 4.17, we note that $E(F_t^*)$ in RIPI deviates more from the target. For example: under (1.2, 0.2), $E(F_t^*)$ decreases to reach 113.6% in CIPI whereas it reaches 98.9% in RIPI. Further, under (0.8, 0.2), it decreases to reach 74.5% in CIPI and 56.9% in RIPI.

Figure 5.16



On the other hand, the expected optimal contribution rate increases over the control period and moves further away from the target with a high level of σ. However, $E(C_t^*)$ tends to become closer to the target at the end of the control period, due to the high value of the contribution rate risk (γ) given in this case (0.7, 0, 0.3). Comparing the expected optimal contribution rates in Figure 5.17 and 4.18, we note that the expected contribution rates under RIPI tend to increase to approximately the same levels as CIPI. Although they decrease to get closer to the targets by the end of the time span, it is noted that $E(C_t^*)$ levels for RIPI get closer to their targets compared with those for CIPI shown in Figure 4.18.

Figure 5.17



Conclusion

The values of the contribution targets and fund targets affect $E(F_t^*)$ and $E(C_t^*)$ under RIPI leading to a high level of $E(F_t^*)$ when the fund target =120%, similar levels equal to 100% in the three scenarios (1, 0.2), (1, 0.22) and (1, 0.18), and we get the lowest level of $E(F_t^*)$ at FT_t = 80%. The expected optimal fund level move further away from these targets when σ is increased.

Consequently, under the three scenarios (1, 0.2), (1, 0.22) and (1, 0.18), $E(C_t^*)$ is around the same level due to having the same fund target. It moves upwards or downwards by the end of the control period to get closer to the target 22% or 18% respectively. On the other hand, having different fund targets in the last two scenarios (1.2, 0.2) and (0.8, 0.2) leads to lower level of the contribution rates in the former scenario and a higher level in the latter one.

Although the trends of $E(F_t^*)$ and $E(C_t^*)$ have similar behaviour in RIPI and CIPI (when the fund targets and contribution targets are changed), the differences in the levels of the expected fund and contribution rate are more obvious with a higher level of volatility.

5.2.4 Effect of changing the initial value of F_0

Here, a special case of studying the effect of changing the fund target (which has been studied in the subsection 5.2.3) to examine the effect on $E(C_0^*)$ is considered. The values of the initial fund F_0 in RIPI model are similar to those used in CIPI in subsection 4.4.4, where we assume that F_0 is equal to FT_t as follows:

F_0 is equal to1 for the first, second and the third scenarios;

F_0 is equal to 1.2 for the fourth scenario;

and it is equal to 0.8 for the fifth scenario.

Following the same approach, the initial fund is used to calculate $E(C_0^*)$ according to the ratio-induced performance index solution (equation 3.48) which has been derived in Chapter 3:

$$E(C_t^*) = \frac{\{\frac{(2\gamma+\alpha)}{CT_t} - (\frac{z_{t+1}}{CT_t} + 2dh_{t+1})(F_t - B_t) + \frac{\alpha l + 2\beta l}{\eta FT_{t+1}} - la_2(t+1)\}}{2(\frac{\gamma}{CT_t^2} + \frac{z_{t+1}}{CT_t} + dh_{t+1})}$$

The result of calculating $E(C_0^*)$ is then used to calculate $E(F_1^*)$ according to the fund growth equation as previously mentioned in section 4.4.4.

Changing the initial fund values, consequently, results in changing the calculated values of $E(C_0^*)$. For example: in the case of (0.1, 0, 0.9) and when $\sigma = 5\%$, the values of $E(C_0^*)$ are almost the same when $F_0 = 100\%$. Further, it decreases to 18.7% and increases to 22.8% when $F_0 = 120\%$ and 80% respectively. A higher level of $\sigma = 15\%$ and 20% leads these values to decrease as shown in Table 5.1.

Table 5.1
The values of $E(C_0*)$ under different scenarios for the case (0.1, 0, 0.9) when $\sigma = 5\%$, 15% and 20%

%

	1, 0.2	1, 0.22	1, 0.18	1.2, 0.2	0.8, 0.2
$\sigma = 5\%$	20.8	20.6	21	18.7	22.8
$\sigma = 15\%$	19.2	19	19.3	16.7	21.6
$\sigma = 20\%$	17.8	17.6	18	15.1	20.4

Comparing these values with those obtained under CIPI in Table 4.14, it is notable that the values of $E(C_0^*)$ are approximately the same at different levels of volatility between both models.

5.2.5 Effect of changing the weighting factors of the contribution rate risk and the solvency risk; γ and β (with $\alpha = 0$)

The effect of changing the two main parameters γ and β in RIPI model is explored in this subsection through the different scenarios. The five cases – that previously mentioned in subsection 4.4.5 - with different values of the parameters are studied, while the value of the mixed term weighting risk factor α is set to be 0.

(a) First scenario: $FT_t = 1$, $CT_t = 0.2$

Under this scenario when $\sigma = 5\%$, we allow γ to increase gradually and β to decrease simultaneously in these different cases: (0.9, 0, 0.1), (0.3, 0, 0.7), (0.5, 0, 0.5), (0.7, 0, 0.3) and (0.9, 0, 0.1). Thus, the following results are obtained and shown in Table 5.2.

Table 5.2

The expected optimal fund level under RIPI (1, 0.2) for different combinations of γ and β when $\sigma = 5\%$

t	(0.1, 0, 0.9)	(0.3, 0, 0.7)	(0.5, 0, 0.5)	(0.7, 0, 0.3)	(0.9, 0, 0.1)
1	100.00%	100.00%	100.00%	100.00%	100.00%
2	99.88%	99.97%	100.02%	100.01%	99.79%
3	99.80%	99.92%	99.99%	99.95%	99.49%
4	99.76%	99.89%	99.95%	99.86%	99.13%
5	99.76%	99.88%	99.92%	99.76%	98.73%
6	99.73%	99.83%	99.83%	99.58%	98.21%
7	99.73%	99.79%	99.74%	99.36%	97.62%
8	99.72%	99.74%	99.61%	99.08%	96.92%
9	99.74%	99.69%	99.45%	98.75%	96.11%
10	99.72%	99.59%	99.22%	98.32%	95.17%
11	99.67%	99.42%	98.89%	97.76%	94.06%
12	99.67%	99.25%	98.53%	97.13%	92.84%
13	99.55%	98.92%	97.97%	96.29%	91.37%
14	99.42%	98.50%	97.30%	95.30%	89.72%
15	99.15%	97.90%	96.43%	94.11%	87.84%

Comparing the results with those obtained in Table 4.15, we find that $E(F_t^*)$ for the RIPI model has a similar behaviour to the CIPI model. In response to our giving more importance to the weighting risk factor of the contribution target γ, the expected optimal fund level moves down further from the target over the control period. We also note that $E(F_T^*)$ reaches 87.8% when $\gamma = 0.9$ compared with 98.1% obtained in CIPI model. This indicates that $E(F_t^*)$ under RIPI tends to move further away from its target with decreasing the value of β compared with under CIPI model.

The expected optimal contribution rate has also a similar behaviour to the one under CIPI shown in Table 4.16. The levels of $E(C_t^*)$ increase to be around 21% and get close to the target with increasing the value of γ as shown in Table 5.3.

Increasing the level of volatility leads both $E(F_t^*)$ and $E(C_t^*)$ to move further away from their targets. In RIPI model, for the case (0.9, 0, 0.1) when $\sigma = 20\%$, $E(F_T^*)$ reaches 65.2% whereas it is equal to 89.8% under CIPI.

Meanwhile, $E(C_t^*)$ for the RIPI, in the same case of (0.9, 0, 0.1), starts from approximately 18% and increases gradually to 20.7% in year 8; then it remains around this value until year 13, when it starts to decrease to get closer to the target at the final T. In fact, $E(C_t^*)$ of CIPI has a similar behaviour, starting from approximately 18% and increasing gradually to be around 21.9% in year 8, it remains around this value until year 13; thereafter, it decreases to get closer to the target.

Table 5.3

The expected optimal contribution rate under RIPI (1, 0.2) for different combinations of γ and β when $\sigma = 5\%$

t	(0.1, 0, 0.9)	(0.3, 0, 0.7)	(0.5, 0, 0.5)	(0.7, 0, 0.3)	(0.9, 0, 0.1)
1	20.80%	20.87%	20.92%	20.91%	20.71%
2	20.86%	20.88%	20.90%	20.87%	20.67%
3	20.90%	20.89%	20.89%	20.85%	20.64%
4	20.92%	20.90%	20.87%	20.82%	20.61%
5	20.92%	20.89%	20.85%	20.78%	20.57%
6	20.94%	20.89%	20.84%	20.75%	20.53%
7	20.93%	20.88%	20.81%	20.72%	20.49%
8	20.93%	20.87%	20.79%	20.68%	20.45%
9	20.92%	20.84%	20.75%	20.63%	20.40%
10	20.91%	20.82%	20.70%	20.57%	20.35%
11	20.91%	20.78%	20.65%	20.51%	20.30%
12	20.87%	20.71%	20.57%	20.44%	20.25%
13	20.83%	20.63%	20.48%	20.35%	20.19%
14	20.71%	20.49%	20.36%	20.25%	20.13%
15	20.48%	20.29%	20.20%	20.13%	20.07%

(b) Second and third scenarios: $FT_t = 1$, $CT_t = 0.22$; $FT_t = 1$ and $CT_t = 0.18$

Here, we examine the results of changing the parameters γ and β under the second and third scenarios when the level of volatility is equal to 5% only. Due to the similarity between the results obtained under the second and fourth scenarios as well as the third and fifth scenarios, the results will be explained under the fourth and fifth scenarios when $\sigma = 20\%$ in the following sub-subsection (c). This is to avoid the repetition of the results interpretations in our analysis.

When $\sigma = 5\%$, $E(F_t^*)$ generally decreases slightly to be around the target in the first half of the control period, and it starts to increase afterwards due to the high value of the contribution target. However, under the third scenario, $E(F_t^*)$ remains around its target in the beginning of the control period and gradually decreases over the years of the control period.

Here, it is notable that giving less importance to β affects clearly the behaviour of $E(F_t^*)$. In other words, in the case of (0.9, 0, 0.1), we note that $E(F_t^*)$ increases from year 6 till it reaches 110% by the end of the control period under the second scenario, and it decreases from the beginning of the control period till it reaches 60.6% in the final year under the third scenario. Figures 5.18 and 5.19 show the differences between the two scenarios.

Figure 5.18

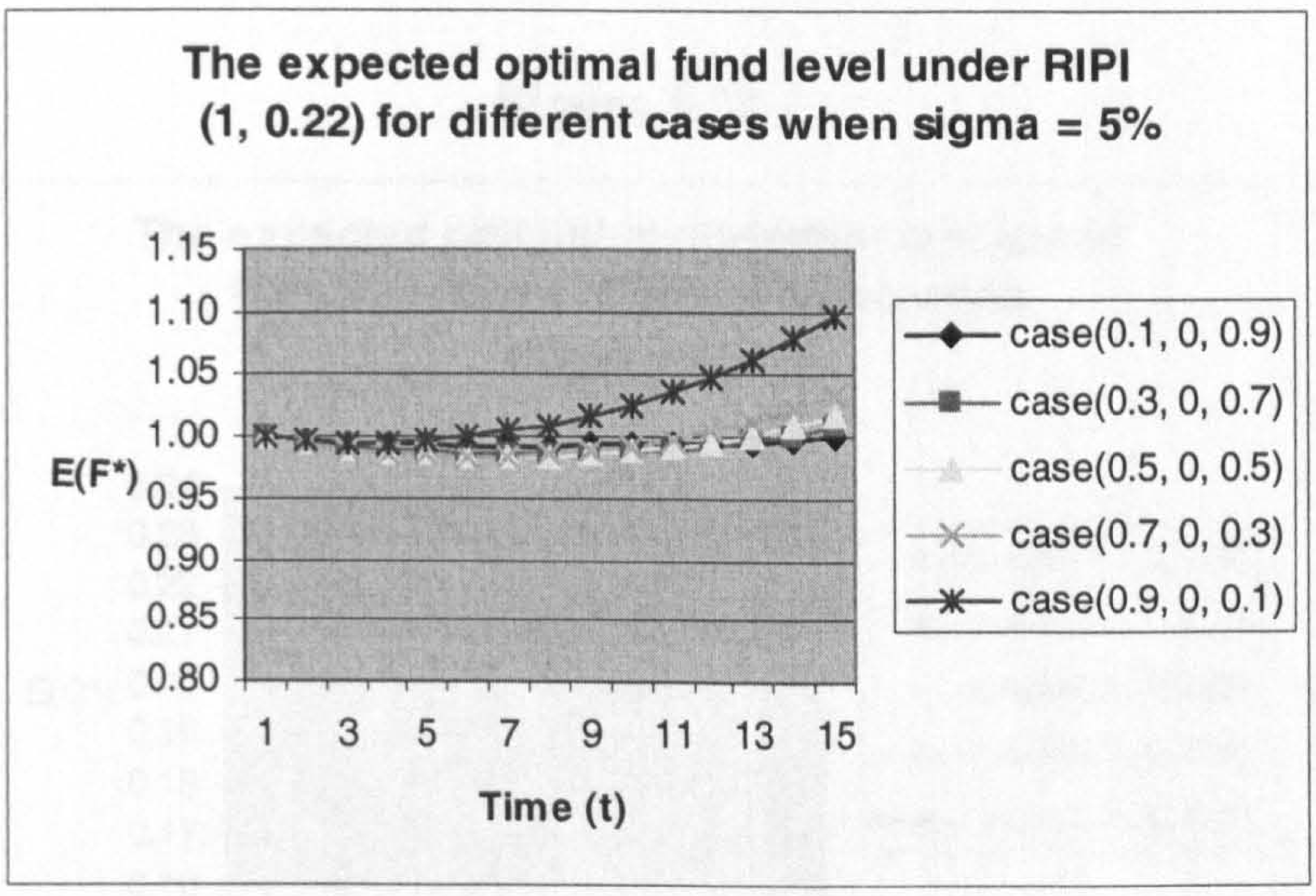
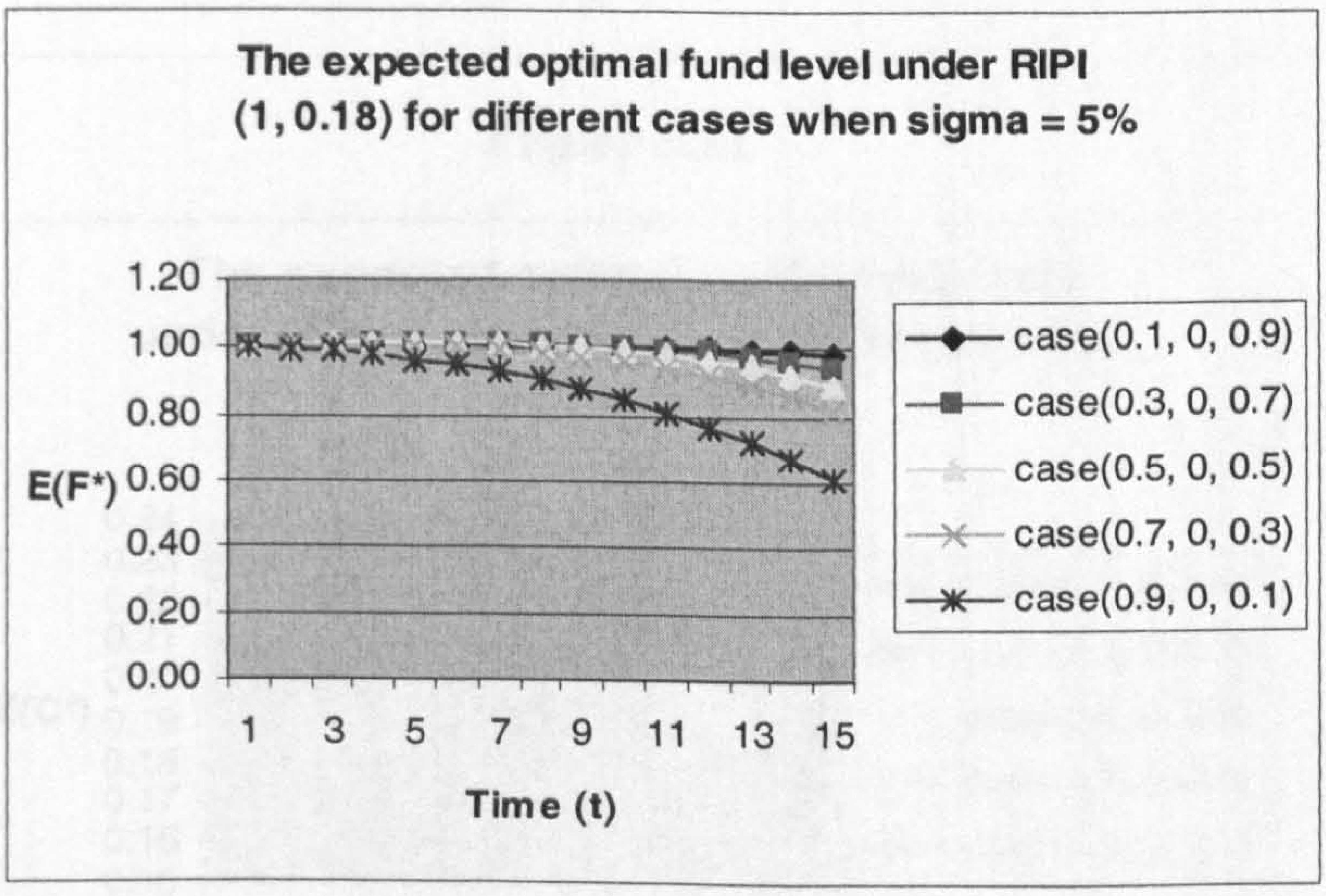


Figure 5.19



Comparing these results with Figures 4.20 and 4.21, we find that $E(F_t^*)$ in RIPI model deviates further from the targets. It can be seen in Figure 5.18 that the final level of $E(F_t^*)$ moves up to reach 110% while it reaches approximately 102% in Figure 4.20.

Furthermore, under the third scenario, the decreasing trends reach a minimum of 60.6% in the case of (0.9, 0, 0.1) in Figure 5.19 while under CIPI the minimum level of $E(F_t^*)$ is 95%, as shown in Figure 4.21.

Under (1, 0.22), for all cases, the expected optimal contribution rates start from around 20% and increase over the control period to be closer to the contribution target 22%. Under the third scenario, they start from around 21% and decrease gradually over the control period to be closer to the target 18%. From Figures 5.20 and 5.21, it is clear that giving more importance to the weighting risk factor of the contribution target leads the final level of $E(C_t^*)$ to be closer to the target.

Figure 5.20

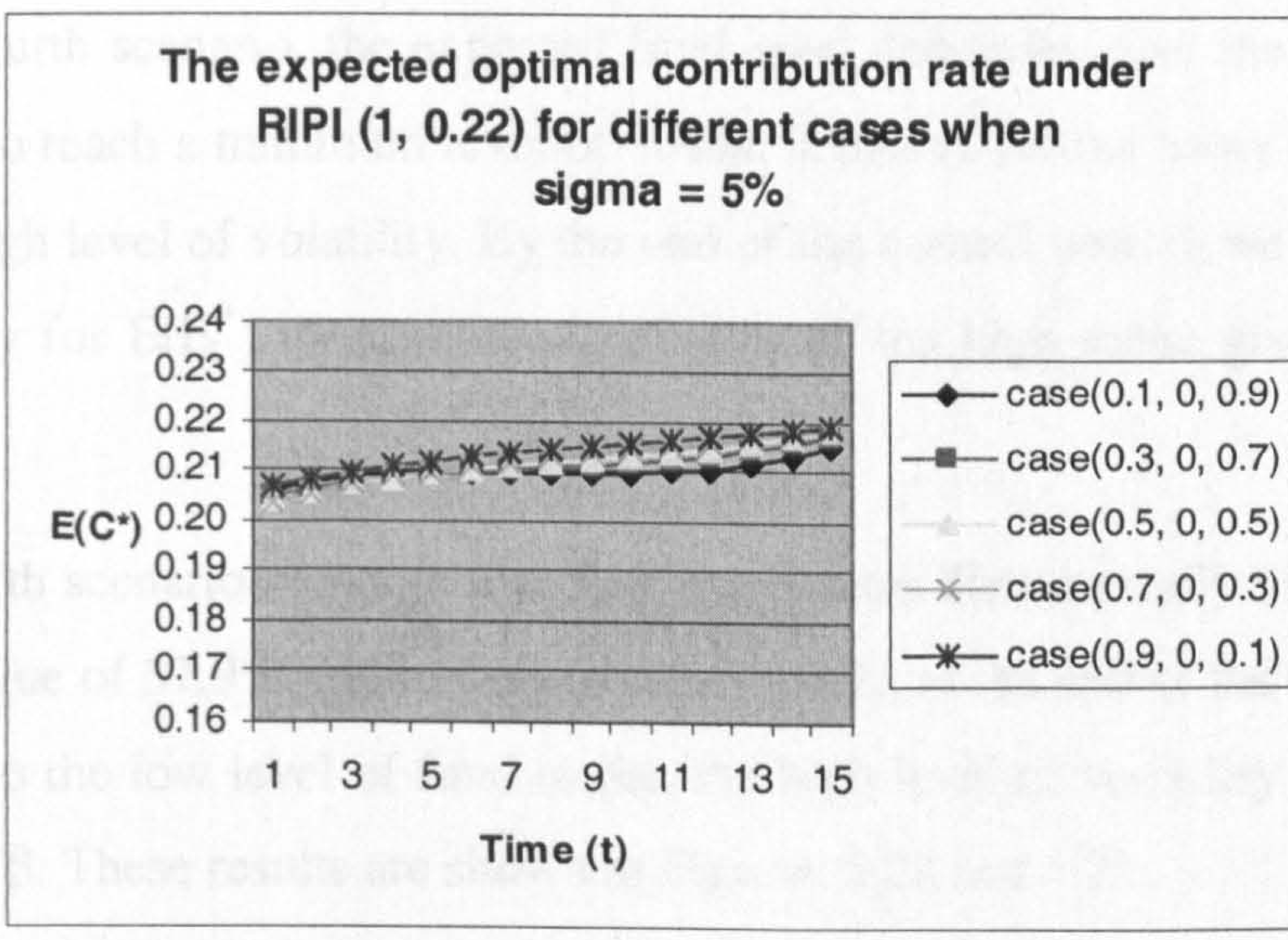
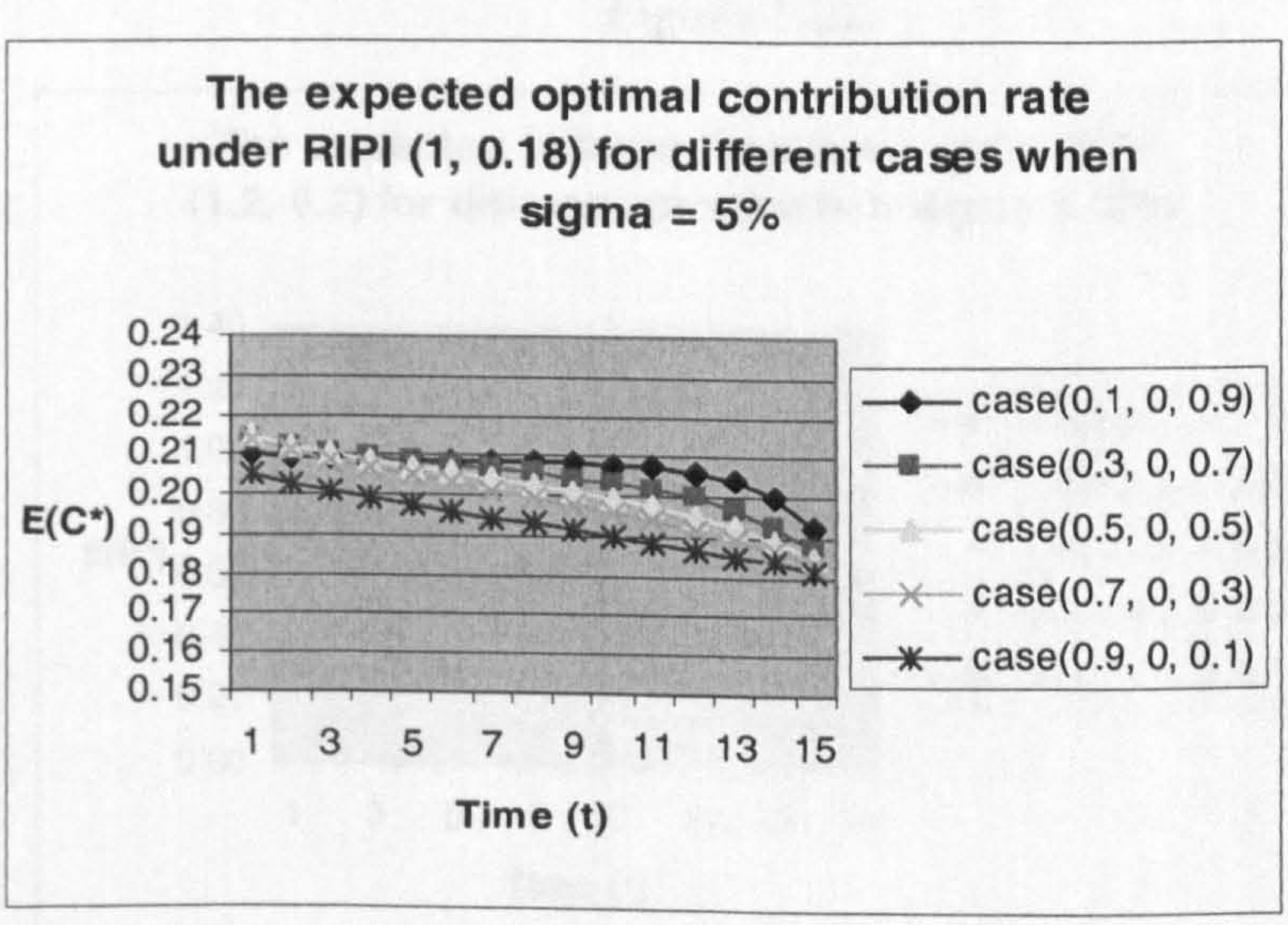


Figure 5.21



Looking at Figures 4.22 and 4.23 and comparing them with Figures 5.20 and 5.21, we find that in the CIPI model under both scenarios, the behaviour of $E(C_t^*)$ shows more stability around 21% in most of the time span compared with the trends under RIPI. However, $E(C_t^*)$ increases towards 22% under (1, 0.22) and decreases towards 18% under (1, 0.18) as expected, when approaching the end of this time span.

(c) Fourth and fifth scenarios: $FT_t = 1.2$ and $CT_t = 0.2$; $FT_t = 0.8$ and $CT_t = 0.2$

Here, we change the fund target up to 120% and down to 80% and keep CT_t at 20%. We study the effect of changing the parameters of the contribution rate risk and solvency risk in all cases when $\sigma = 20\%$ only, for the same reason explained in sub-subsection (b).

Under the fourth scenario, the expected fund level decreases over the control period from 120% to reach a minimum level of 100%. It moves further away from the target due to the high level of volatility. By the end of the control period, we note that there is a tendency for $E(F_t^*)$ to increase as a result of the high value given to the fund target.

Under the fifth scenario, we note that $E(F_t^*)$ decreases daramatically until it reaches a minimum value of 37.9%, in the case of (0.9, 0, 0.1), at the end of the control period. This is due to the low level of fund target, the high level of volatility and finally the low value of β. These results are shown in Figures 5.22 and 5.23.

Figure 5.22

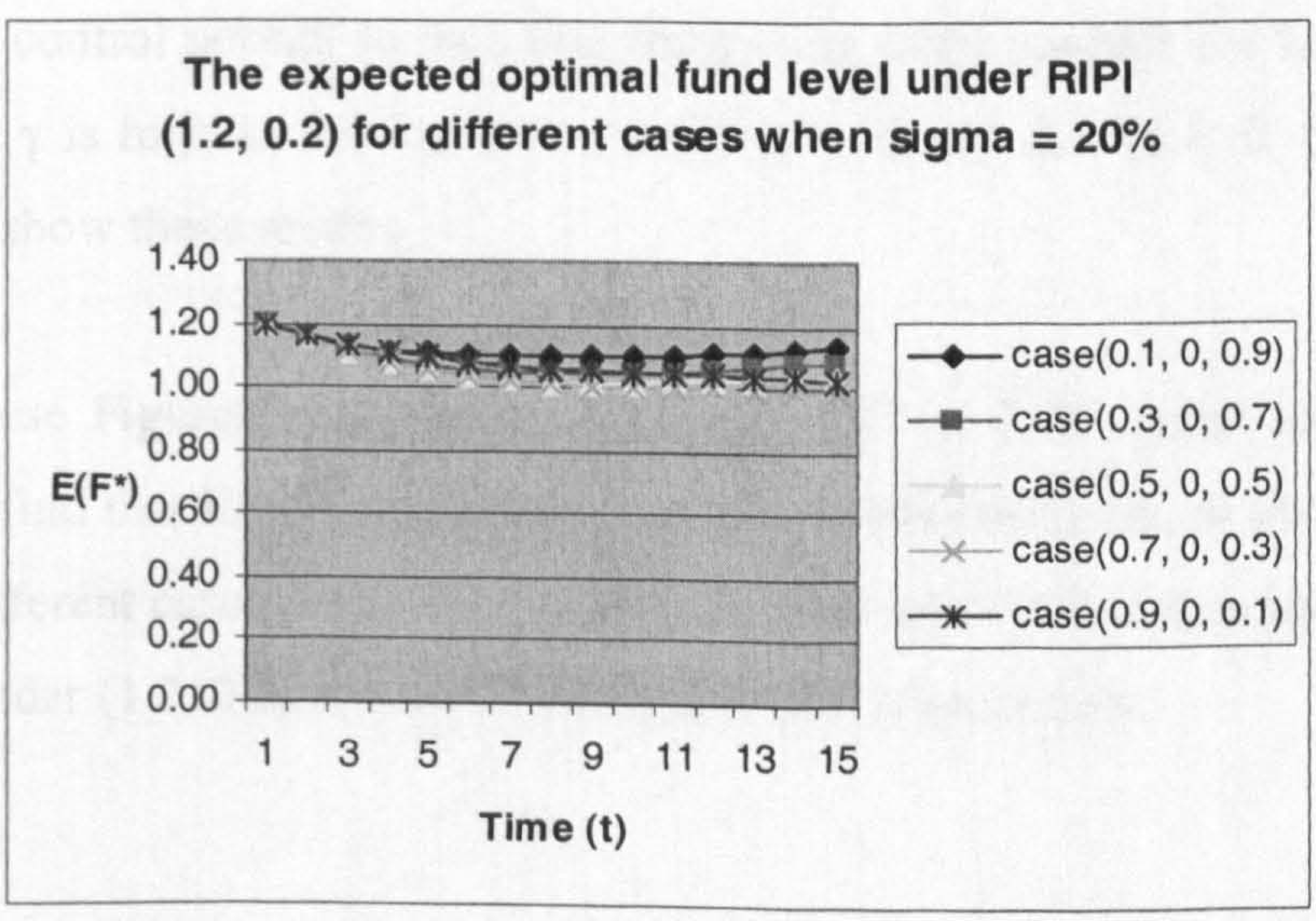
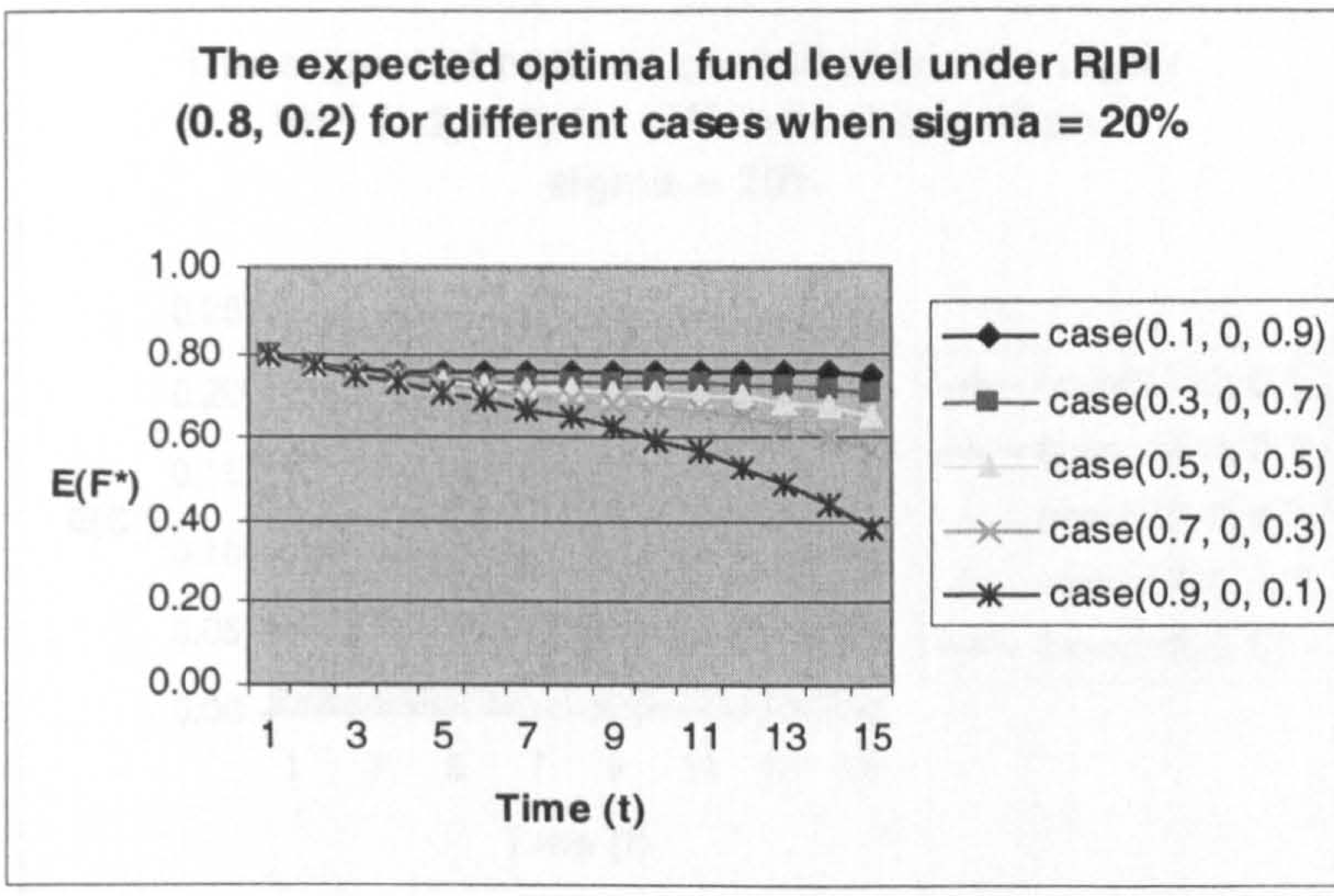


Figure 5.23



Comparing these results with those obtained under CIPI in Figures 4.24 and 4.26, it is clear that our giving more importance to the weighting risk factor of the contribution rate leads $E(F_t^*)$ to move further away from its target under RIPI. Thus, under the fourth scenario, it decreases to reach a minimum level of 106% under CIPI compared with 100% under RIPI. In the fifth scenario (0.8, 0.2) and in the case of (0.9, 0, 0.1), $E(F_t^*)$ falls to be 37.9% under RIPI compared with 69% under CIPI.

When $\sigma = 20\%$, under the fourth scenario for the same five cases, the trends of $E(C_t^*)$ start from lower levels of about 15% and increase over the control period to be around 20%. Under the fifth scenario, they start from a higher level of about 20%, then, increase until they reach a maximum of 23.2% in the first three cases; (0.1, 0, 0.9), (0.3, 0, 0.7) and (0.5, 0, 0.5). Subsequently, they decrease to be closer to the target by the end of the control period. In fact, this decreasing trend towards the target is more obvious when γ is high in the last two cases; (0.7, 0, 0.3) and (0.9, 0, 0.1). Figures 5.24 and 5.25 show these results.

Comparing these Figures with Figures 4.26 and 4.27 in CIPI model under the two scenarios, we find that $E(C_t^*)$ starts from the same levels; however, in the CIPI model for the five different cases, $E(C_t^*)$ is more stable with small differences around certain levels: 20% under (1.2, 0.2) and 23% under (0.8, 0.2) respectively.

Figure 5.24

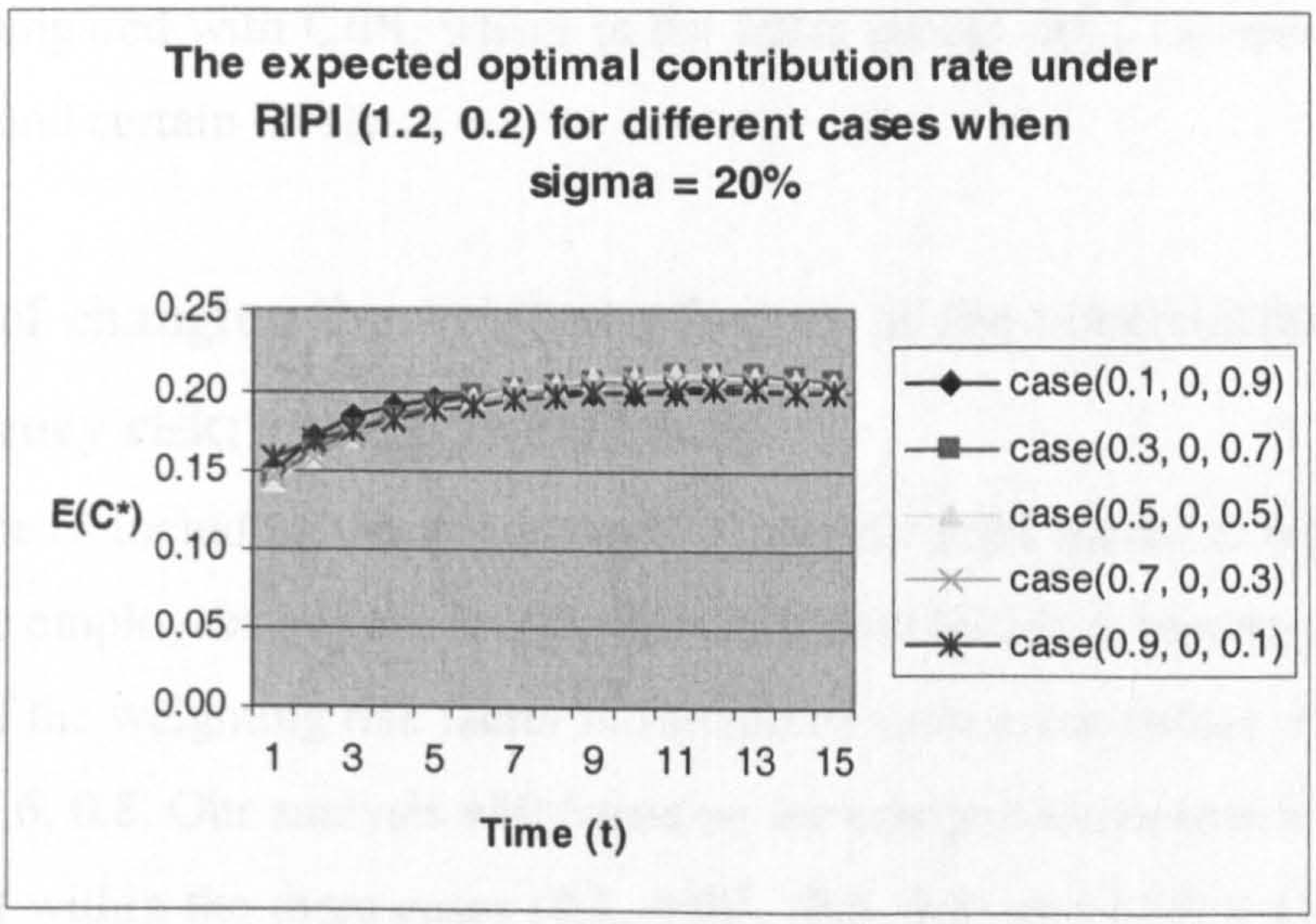
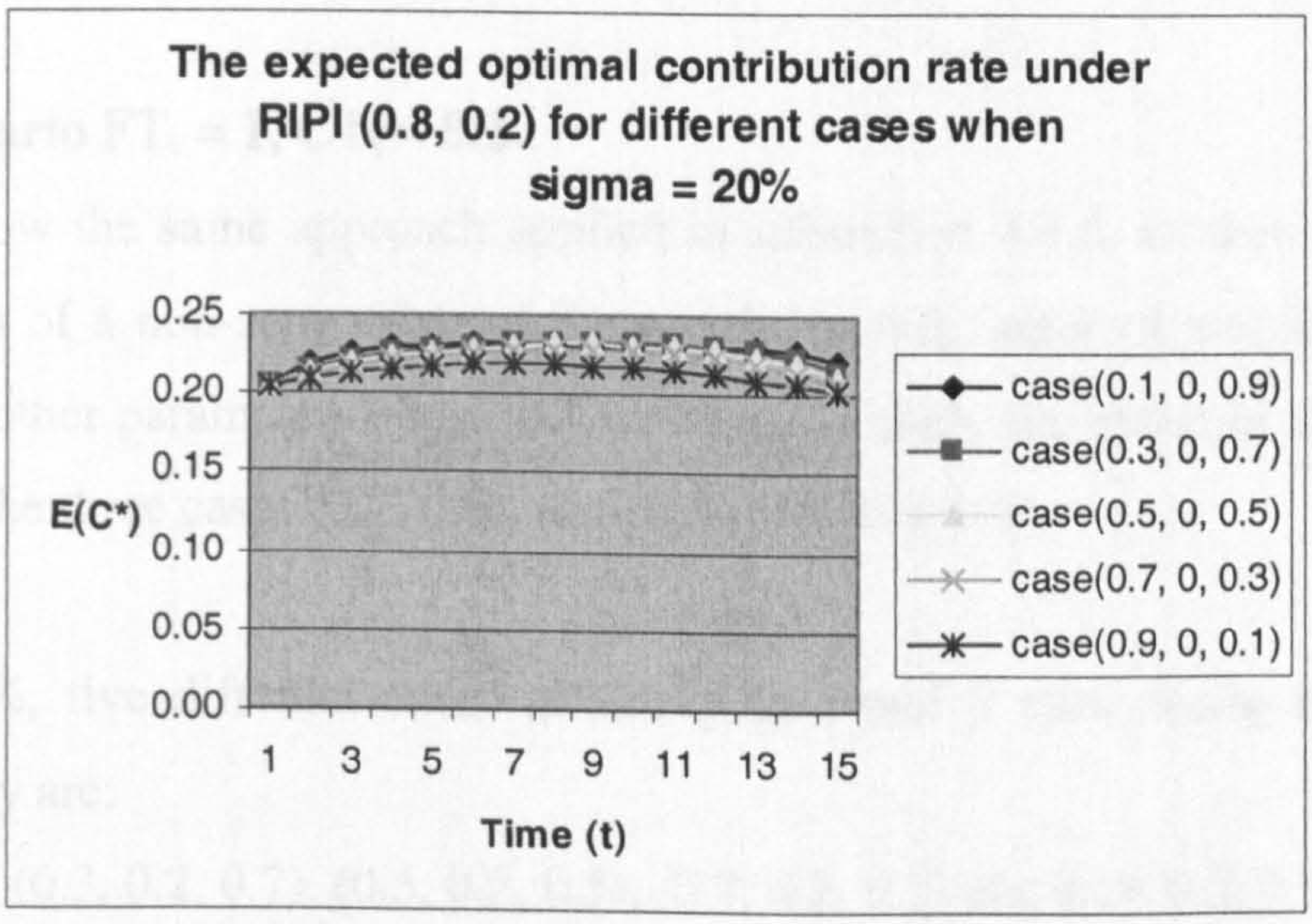


Figure 5.25



Conclusion

The effect of the weighting risk factors γ and β on $E(F_t^*)$ and $E(C_t^*)$ for the RIPI model is identified under the five different scenarios. The expected optimal fund level generally remains close to the target when less importance is given to γ. Simultaneously, $E(C_t^*)$ moves further away from the target. This deviation from the target for either $E(F_t^*)$ or $E(C_t^*)$ is more obvious with a higher level of volatility and under RIPI compared with CIPI.

It is also noted that the high value of σ results in more variation of the levels of $E(C_t^*)$ under RIPI compared with CIPI, where in the latter model $E(C_t^*)$ shows more stable behaviour around certain levels.

5.2.6 Effect of changing the weighting factors of the contribution rate risk and the solvency risk; γ and β (with α ≠ 0)

The importance of including the mixed middle term for RIPI model to test the mutual interests of the employees and the employer is explored in this subsection. We use the same values of the weighting risk factor of the mixed term given before, i.e
$\alpha = 0.2, 0.4, 0.6, 0.8$. Our analysis will focus on the comparison between both models CIPI and RIPI within the three cases (0.1, 0.9)[2], (0.5, 0.5) and (0.9, 0.1), where they represent the two extreme cases and the intermediate one.

(a) First scenario $FT_t = 1$, $CT_t = 0.2$

Here, we follow the same approach applied in subsection 4.4.6, so that, we identify first the effect of a non-zero value of the weighting risk factor of middle term with changing the other parameters γ and β. Secondly, we study the effect of changing the value of α in the three cases (0.1, 0.9), (0.5, 0.5) and (0.9, 0.1).

When σ = 5%, five different cases of changing γ and β with fixing α at 0.2 are examined, they are:
(0.1, 0.2, 0.9), (0.3, 0.2, 0.7), (0.5, 0.2, 0.5), (0.7, 0.2, 0.1) and (0.9, 0.2, 0.1),
the results of $E(F_t^*)$ and $E(C_t^*)$ are shown in Tables 5.4 and 5.6.

[2] In this part and also throughout the thesis, we refer to the weighting risk factors of solvency and contribution rate, in the cases under study, in the form of (γ, β) as the cross-product factor α is changing.

Table 5.4

The expected optimal fund level under RIPI (1, 0.2)

for different cases when $\alpha = 0.2$ and $\sigma = 5\%$

t	(0.1, 0.2, 0.9)	(0.3, 0.2, 0.7)	(0.5, 0.2, 0.5)	(0.7, 0.2, 0.3)	(0.9, 0.2, 0.1)
1	100.00%	100.00%	100.00%	100.00%	100.00%
2	99.70%	99.83%	99.89%	99.87%	99.62%
3	99.50%	99.68%	99.76%	99.70%	99.16%
4	99.39%	99.57%	99.64%	99.51%	98.66%
5	99.35%	99.50%	99.54%	99.32%	98.12%
6	99.29%	99.40%	99.39%	99.05%	97.49%
7	99.27%	99.34%	99.25%	98.78%	96.80%
8	99.26%	99.25%	99.07%	98.44%	96.02%
9	99.26%	99.18%	98.89%	98.06%	95.17%
10	99.25%	99.08%	98.64%	97.61%	94.21%
11	99.19%	98.90%	98.31%	97.04%	93.12%
12	99.19%	98.74%	97.95%	96.43%	91.96%
13	99.09%	98.44%	97.43%	95.63%	90.59%
14	98.99%	98.07%	96.83%	94.73%	89.10%
15	98.80%	97.57%	96.07%	93.67%	87.44%

Table 5.4 shows a decreasing trend of the expected optimal fund level over the control period. We also note that it decreases more when γ is increased to higher levels; for example, $E(F_t^*)$ decreases from 1 to 98.8% in the final year in the case of (0.1, 0.2, 0.9), while $E(F_T^*) = 87.4\%$ in the case of (0.9, 0.2, 0.1).

Moreover, for all cases shown above, $E(F_t^*)$ decreases slightly more compared with the results obtained in Table 5.1 when $\alpha = 0$. This is generally consistent with the results previously obtained under CIPI. However, $E(F_t^*)$ moves further away from the target under RIPI compared with the results obtained under CIPI in Table 4.18, where $E(F_T^*) = 99.7\%$ and 97.6% respectively for the two cases (0.1, 0.2, 0.9) and (0.9, 0.2, 0.1).

From Table 5.5, the expected optimal contribution rate shows a slight increase to be around 21% and there is a tendency to decrease by the end of the control period to be around 20%. Decreasing the value of β to 0.1 with $\alpha = 0.2$ leads $E(C_t^*)$ to be the closest to the target of 20% over the control period.

Comparing with Table 5.2 where $\alpha = 0$, $E(C_t^*)$ takes very similar values, except that it is slightly further away from the target by the end of the control period when $\alpha = 0.2$.

Table 5.5

The expected contribution rate under RIPI (1, 0.2)

for different cases when $\alpha = 0.2$ when $\sigma = 5\%$

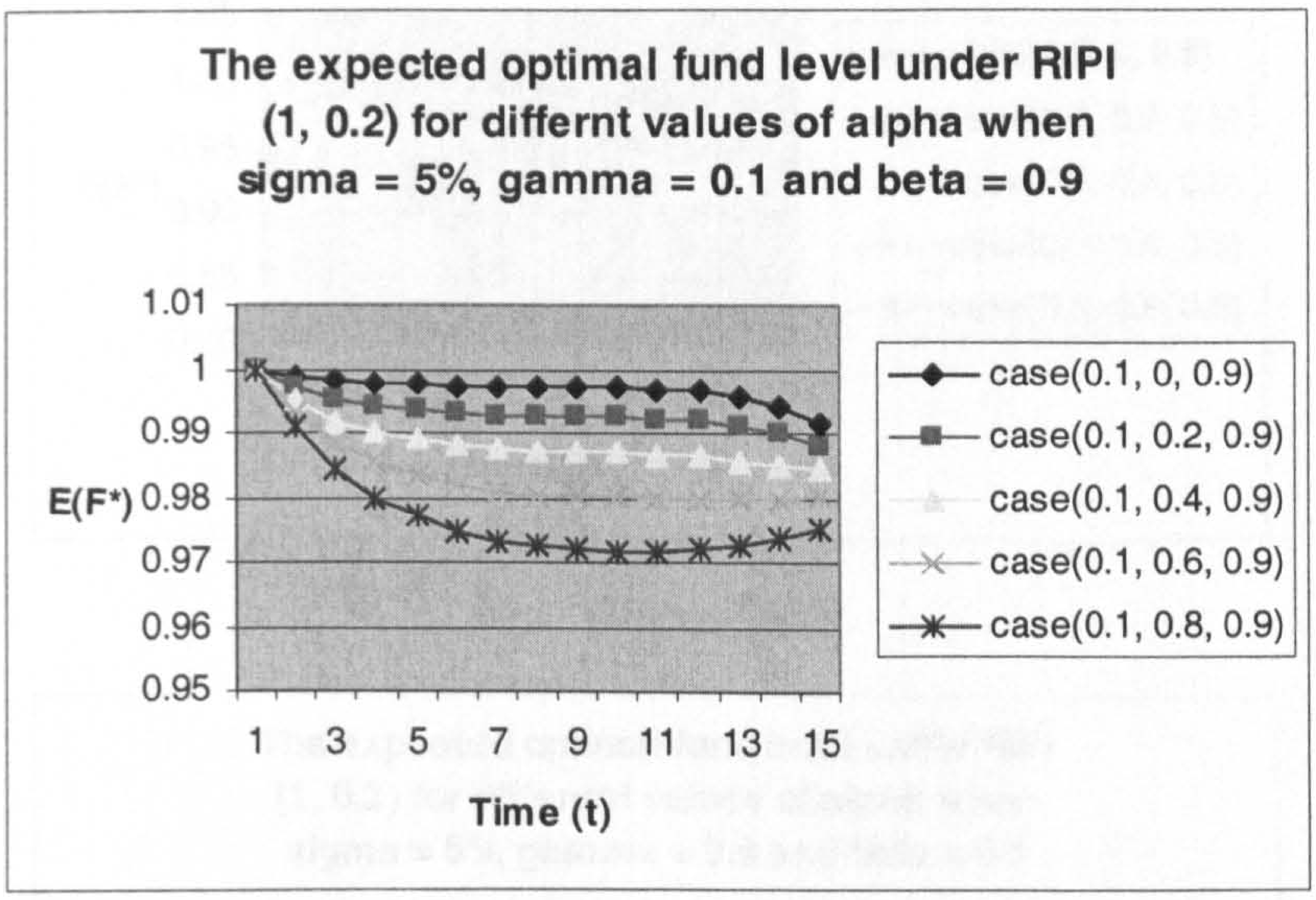
t	(0.1, 0.2, 0.9)	(0.3, 0.2, 0.7)	(0.5, 0.2, 0.5)	(0.7, 0.2, 0.3)	(0.9, 0.2, 0.1)
1	20.63%	20.75%	20.80%	20.79%	20.55%
2	20.77%	20.80%	20.82%	20.78%	20.54%
3	20.86%	20.85%	20.83%	20.77%	20.54%
4	20.92%	20.87%	20.84%	20.77%	20.53%
5	20.93%	20.88%	20.83%	20.75%	20.52%
6	20.96%	20.90%	20.83%	20.74%	20.51%
7	20.96%	20.90%	20.82%	20.72%	20.50%
8	20.97%	20.90%	20.81%	20.70%	20.48%
9	20.96%	20.88%	20.78%	20.66%	20.47%
10	20.95%	20.86%	20.75%	20.63%	20.46%
11	20.96%	20.84%	20.72%	20.59%	20.44%
12	20.92%	20.78%	20.66%	20.54%	20.42%
13	20.90%	20.72%	20.59%	20.49%	20.41%
14	20.83%	20.62%	20.50%	20.42%	20.40%
15	20.69%	20.48%	20.39%	20.35%	20.38%

Furthermore, comparing these results with the ones obtained in Table 4.19, it is more clear that $E(C_t^*)$ has an overall decreasing trend towards its target under RIPI, while it is more stable around 21% over the control period with a tendency to decrease at the end of control period under CIPI.

The second analysis in this sub-subsection considers the comparison between the two extreme cases, (0.1, 0.9) and (0.9, 0.1), and the middle case (0.5, 0.5) for different values of α. The results for each case are discussed below and also compared with the CIPI results.

When $\sigma = 5\%$, in the case of (0.1, 0.9), and α is increased to 0.8, the expected optimal fund level decreases slightly over the control period as shown in Figure 5.26. The departure from the target, when the cross-product coefficient α is increased, is explained by rewarding the deviation from the target due to the mathematical structure of the dynamic programming model. This is consistent with the results obtained under CIPI but with more deviation from the target as explained in sub-subsection 4.4.6(a).

Figure 5.26



For instance, $E(F_t^*)$ moves down to be around 97% in the case of (0.1, 0.8, 0.9), while it reaches 99.4% for the same case under CIPI in Figure 4.28.

Considering the results of applying the other cases (0.5, 0.5), (0.9, 0.1), at the same level of $\sigma = 5\%$, the levels of $E(F_t^*)$ decrease more to reach 94.3% and 81.5% in the cases of (0.5, 0.8, 0.5) and (0.9, 0.8, 0.1) respectively. This deviation of $E(F_t^*)$ from the target is due to increasing the weighting risk factors of both the contribution rate γ and the mixed middle term α. These results are shown in Figures 5.27 and 5.28. The results can be compared with Figures 4.30 and 4.34 to identify the differences between RIPI and CIPI, where under the latter model $E(F_T^*) = 98.9\%$ and 95.1% in the cases of (0.5, 0.8, 0.5) and (0.9, 0.8, 0.1) respectively.

Figure 5.27

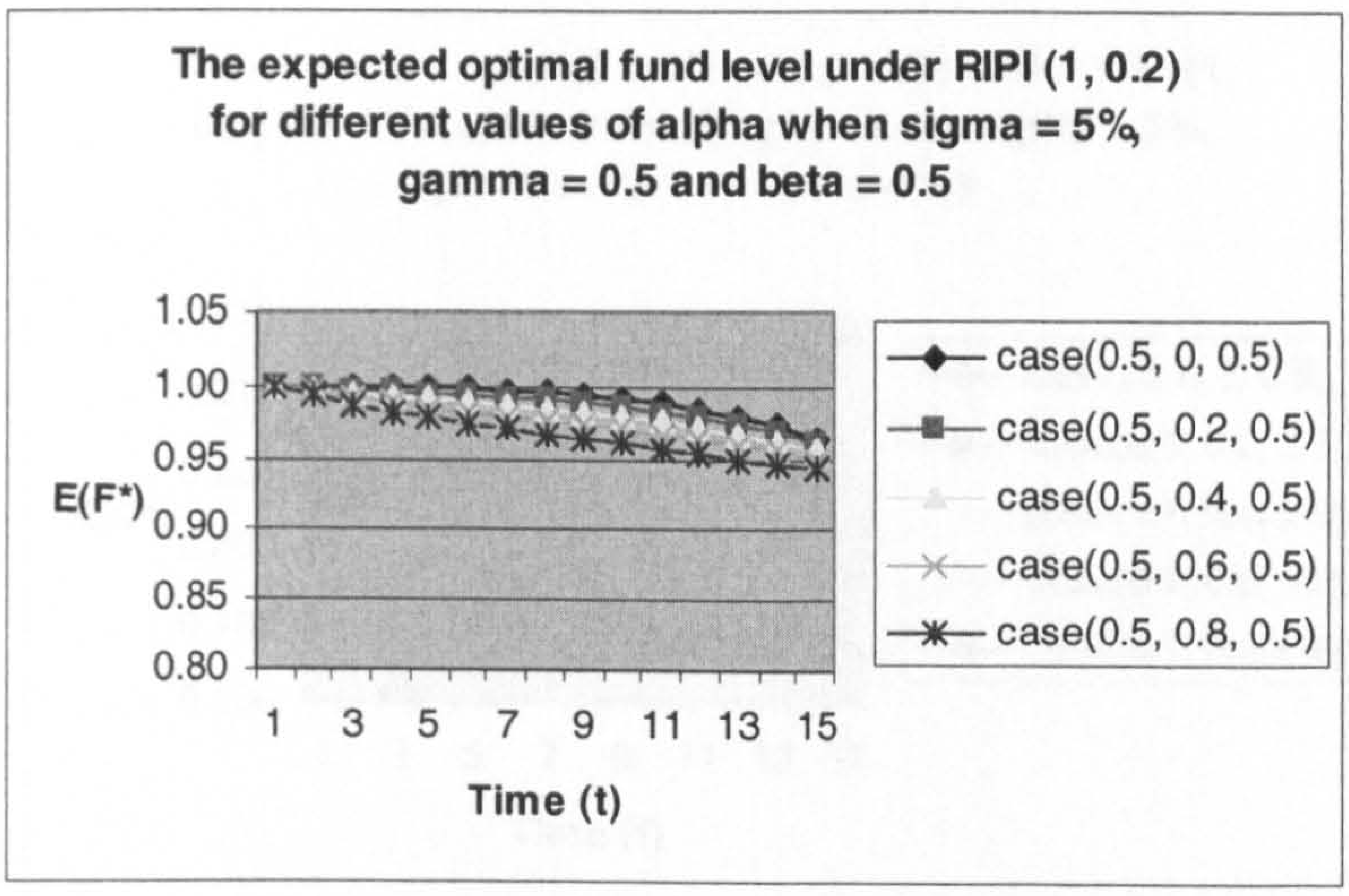
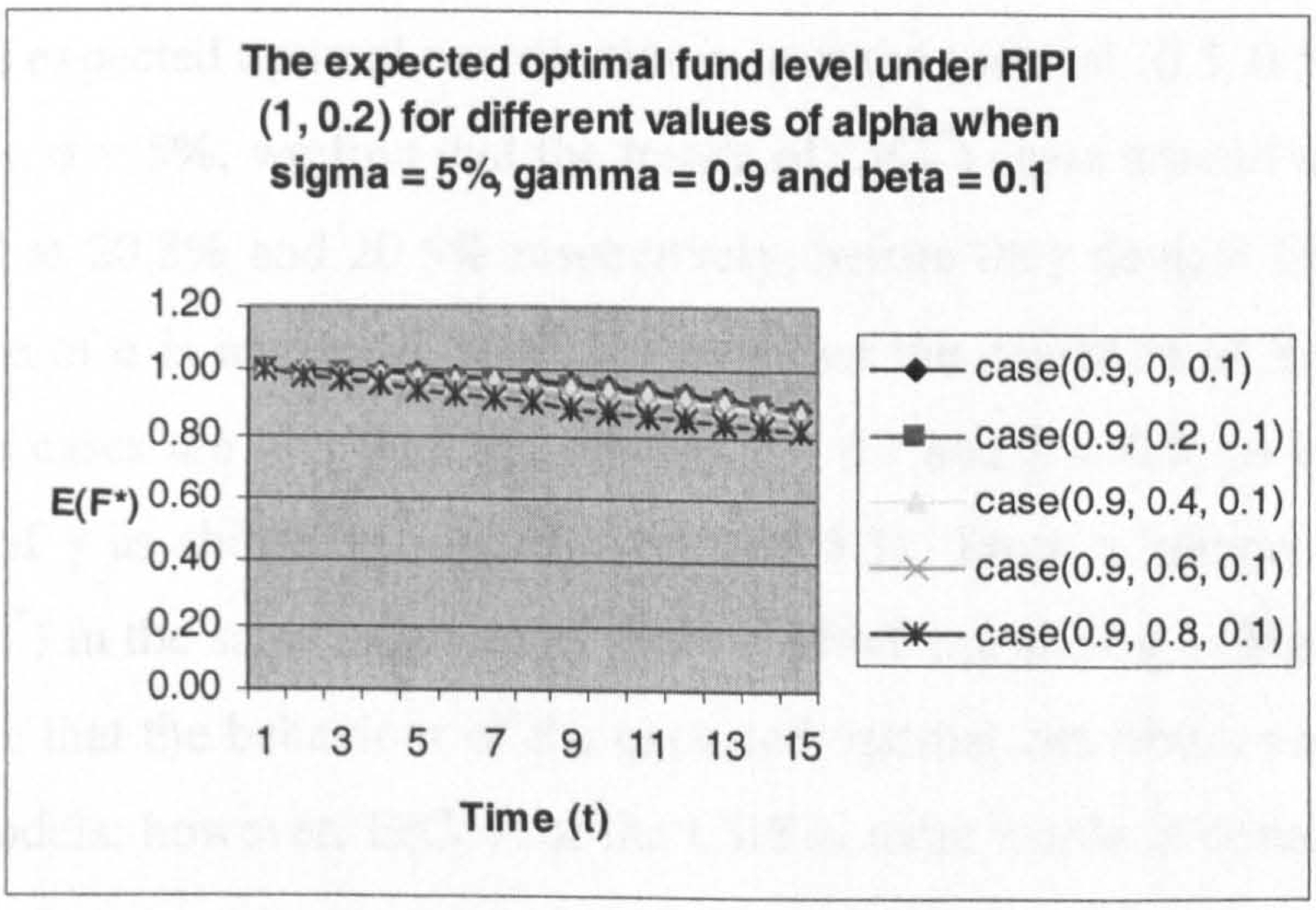


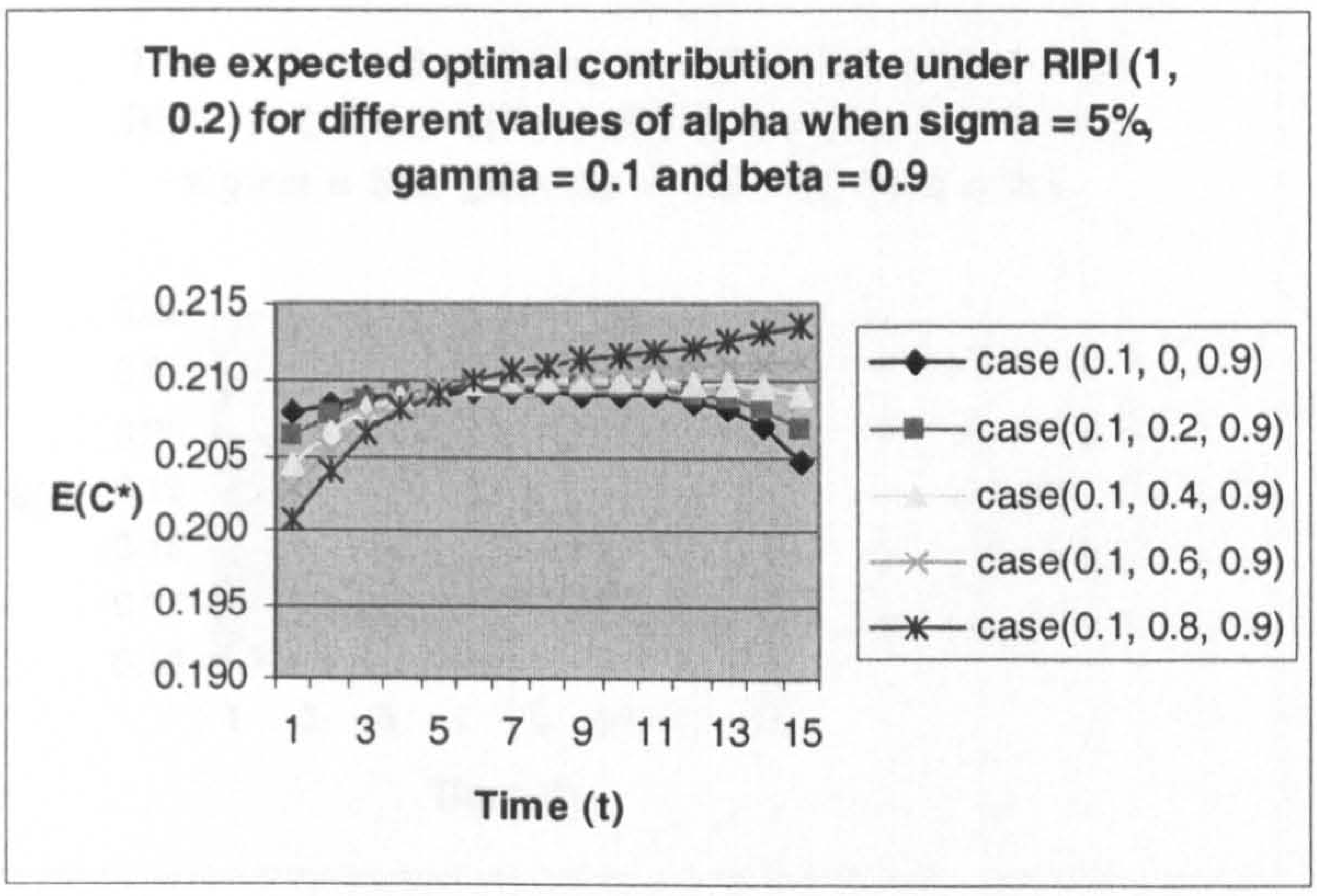
Figure 5.28



At the same level of volatility $\sigma = 5\%$, in the cases of (0.1, 0.9), the expected optimal contribution rate is almost around 21%. We also note that all trends of $E(C_t^*)$ cross at the value of 20.9% in year 5, then, they move away from the target to reach 21.4% as α is increased to 0.8. This deviation from the target reflects the mathematical structure of the dynamic programming model, in which the weighting risk factor of the middle mixed term affects both the fund level and the contribution rate, resulting in rewarding a deviation from the target rather than penalizing it.

Figure 5.29 indicates that $E(C_t^*)$ deviates more from the target with increasing α than under the CIPI model in Figure 4.29, where it remains stable around 21% until it decreases at the end of the control period.

Figure 5.29



Looking at the expected optimal contribution rate in the cases of (0.5, 0.5) and (0.9, 0.1) when σ = 5%, we find that the trends of $E(C_t^*)$ cross around the mid of the control period at 20.8% and 20.5% respectively, before they deviate from the target when the value of α is increased. Here, we note that the deviation of $E(C_t^*)$ from the target in these cases are less than the case of γ = 0.1 and β = 0.9, as a result of the higher value of γ as shown in Figures 5.30 and 5.31. From a comparison with the results of $E(C_t^*)$ in the same cases under CIPI – which are shown in Figures 4.31 and 4.34 – we note that the behaviour of the expected optimal contribution rate is similar under both models; however, $E(C_t^*)$ for the CIPI is more stable at certain levels than RIPI.

Figure 5.30

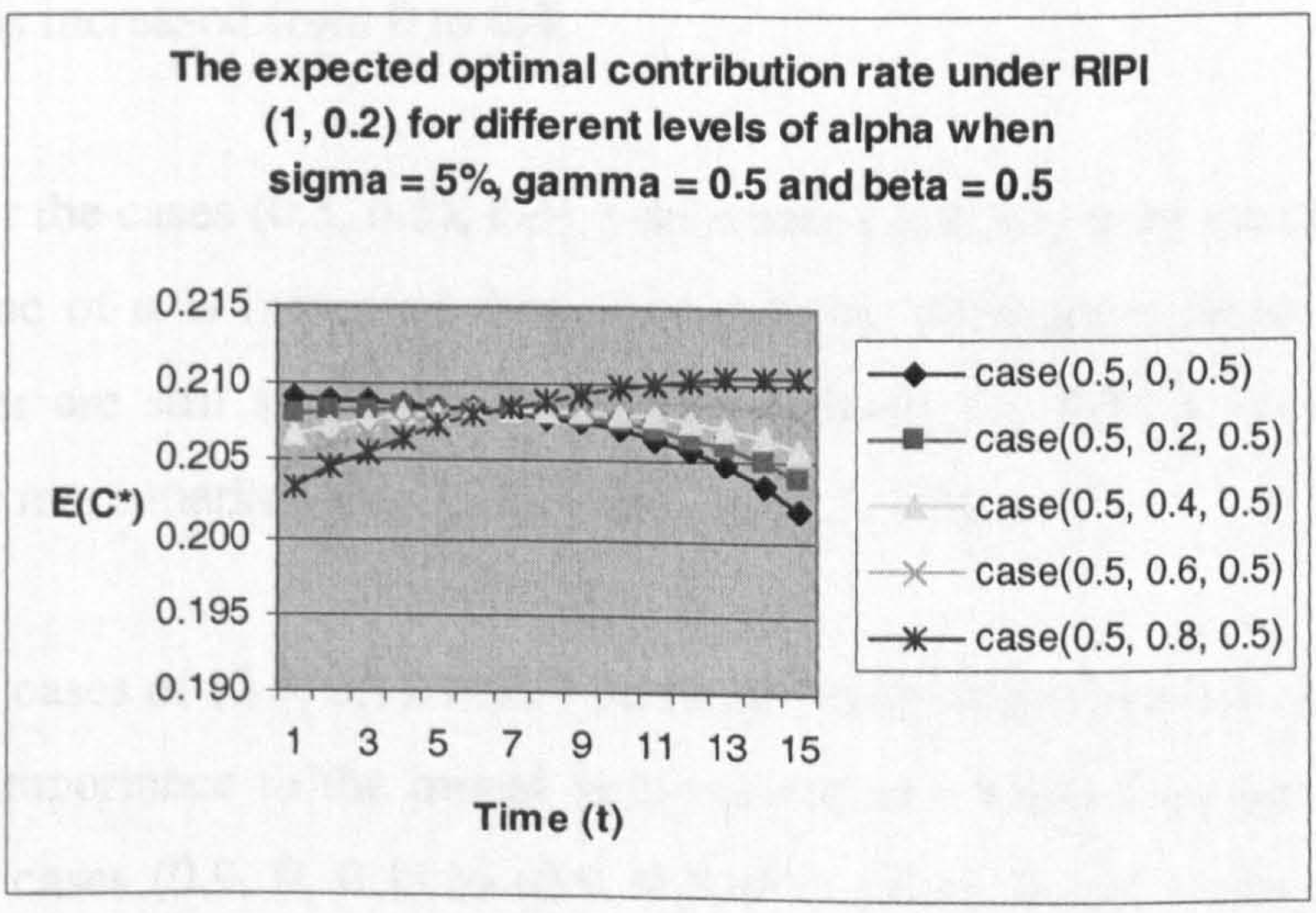
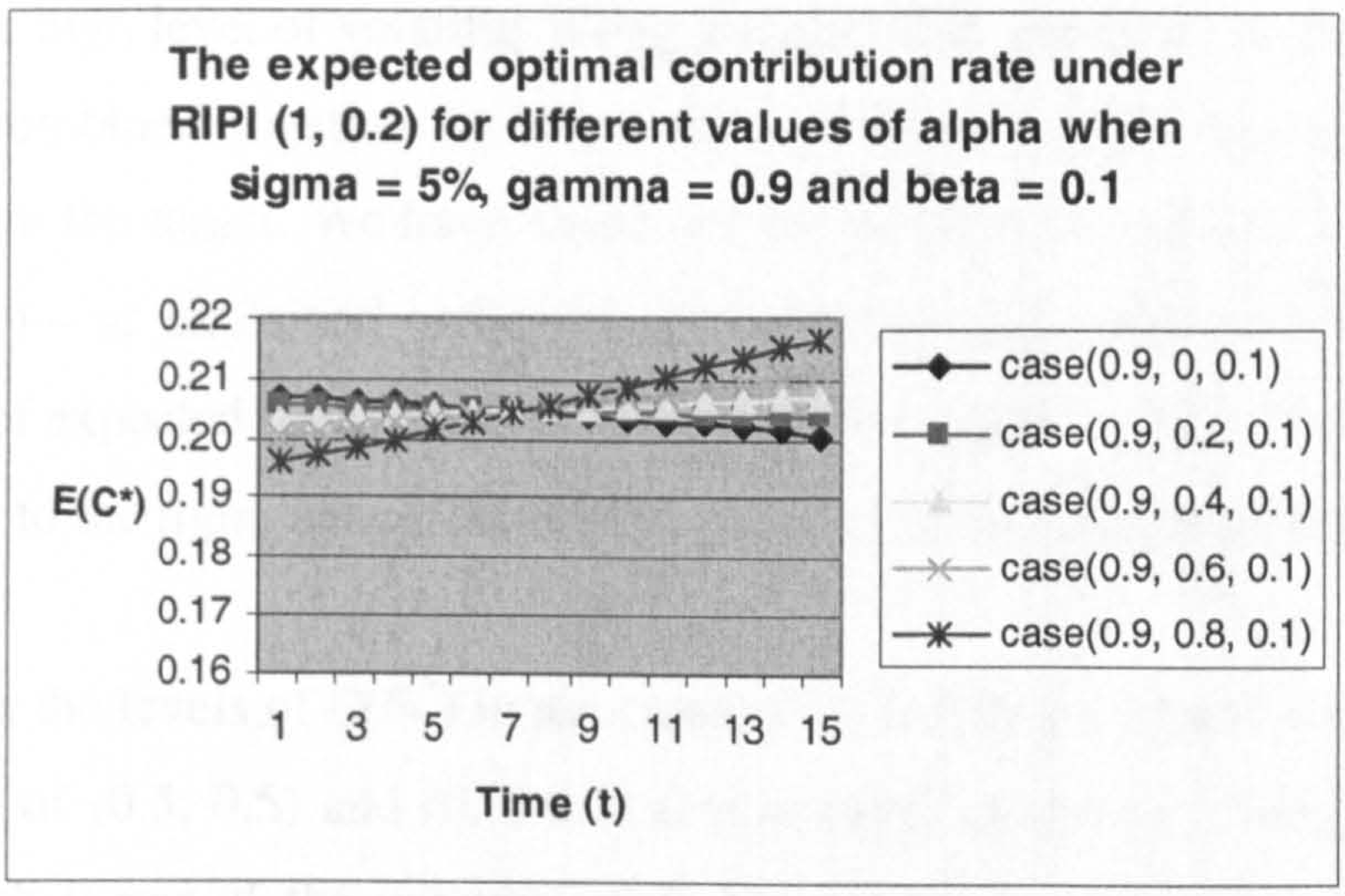


Figure 5.31



When the level of volatility is increased to 20%, the different behaviours of $E(F_t^*)$ and $E(C_t^*)$ in the three cases under study: (0.1, 0.9), (0.5, 0.5) and (0.9, 0.1) are described below.

The results of $E(F_t^*)$ show a decreasing trend to a lower level compared with the results when $\sigma = 5\%$, for each case from (0.1, 0, 0.9) to (0.1, 0.8, 0.9). This is due to the increase in the level of volatility. However, we note that $E(F_t^*)$ tends to increase with small differences with increasing α from one case to another. These differences are very small and could be referred to the estimates being simulation-based. Hence, we can note that $E(F_t^*)$ remains between 94% and 95% as α is increased.

Similar cases under CIPI show that $E(F_t^*)$ has no remarkable difference among the cases when α is increased from 0 to 0.8.

Under RIPI for the cases (0.5, 0.5), $E(F_t^*)$ decreases gradually over the control period. When the value of α is increased from 0 to 0.8, the differences observed from one case to another are still small. However, the tendency for $E(F_T^*)$ to increase with increasing α is more marked than in the cases of (0.1, 0.9).

Further, in the cases of (0.9, 0.1), $E(F_t^*)$ continues to decrease over the control period. Giving more importance to the mixed term parameter α leads the levels of $E(F_t^*)$ to increase from cases (0.9, 0, 0.1) to (0.9, 0.8, 0.1). Here, the differences among the levels are more obvious than in the other cases of (0.1, 0.9) and (0.5, 0.5). This is caused by the high level of volatility along with the high values of both γ and α, these values are combined together to affect the trends of $E(F_t^*)$ leading to a greater deviation from the target. We have examined the distribution of $E(F_T^*)$ in the case of (0.9, 0.8, 0.1) – as explained earlier in sub-subsection 4.4.6(a) – and noted that the distribution of expected fund values is more positively skewed. This leads the mean to move further to the right, hence, increasing the level of the expected fund.

It is clear that the levels of $E(F_t^*)$ in the cases of (0.1, 0.9) are closer to the target than for the cases of (0.5, 0.5) and (0.9, 0.1) due to more importance being given to the weighting risk factor of the solvency risk. Figures 5.32, 5.33 and 5.34 show these results.

Therefore, $E(F_t^*)$ indicates more volatile behaviour under RIPI compared with CIPI. The levels of $E(F_t^*)$ under CIPI remain more stable in the cases of (0.1, 0.9) and (0.5, 0.5) as α is increased; however, the behaviour of $E(F_t^*)$ shows an increasing trend in the cases of (0.9, 0.1) when α is increased as shown in Figure 4.36.

Looking at the changes of the expected optimal contribution rates when $\sigma = 20\%$, we note that $E(C_t^*)$ increases over the control period in response to the downward trend of $E(F_t^*)$. Nevertheless, as the value of α is increased from 0 to 0.8 in the cases of (0.1, 0.9), $E(C_t^*)$ takes similar values and increases to reach approximately 22%.

In the cases of (0.5, 0.5), with increasing α, $E(C_t^*)$ increases with small differences among the cases to reach approximately 22% in the cases of (0.5, 0.6, 0.5) and (0.5, 0.8, 0.5). However, we find that $E(C_t^*)$ is closer to the target in the cases where $\alpha = 0$, 0.2 and 0.4 compared with the corresponding cases of (0.1, 0, 0.9), (0.1, 0.2, 0.9) and (0.1, 0.4, 0.9) due to the greater importance being given to γ.

Finally, the cases of (0.9, 0.1) show that $E(C_t^*)$ increases to be around 21%, which is less than the cases of (0.1, 0.9) and (0.5, 0.5), again due to the greater importance being given to γ. Although the differences among the cases with increasing α are small, the level of $E(C_T^*)$ for the case of (0.9, 0, 0.1) is the closest to the target. These results are shown below in Figures 5.35, 5.36 and 5.37.

Figure 5.32
The expected optimal fund level under RIPI (1, 0.2) for different levels of alpha when sigma = 20%, gamma=0.1 and beta=0.9

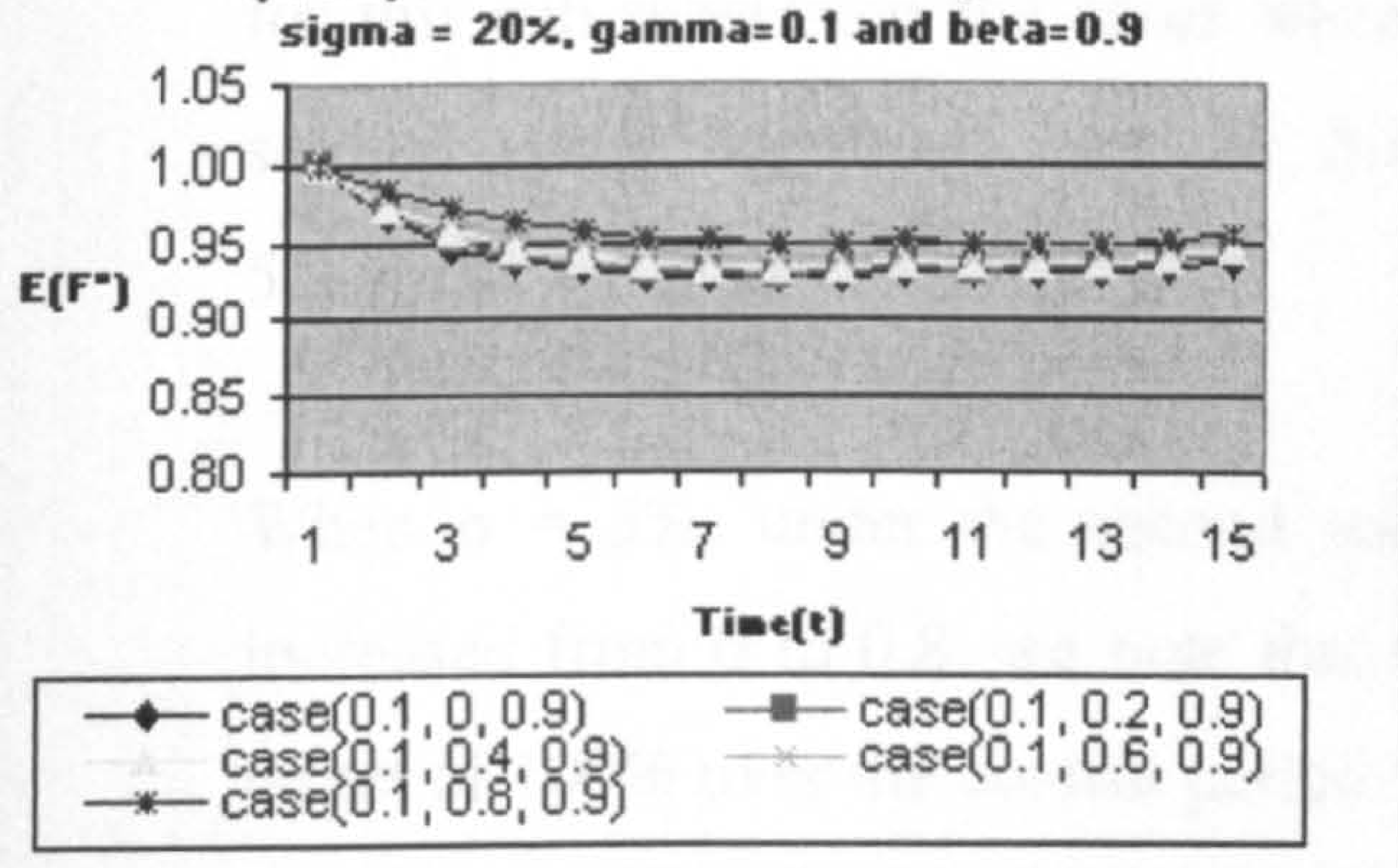


Figure 5.33
The expected optimal fund level under RIPI (1, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.5 and beta = 0.5

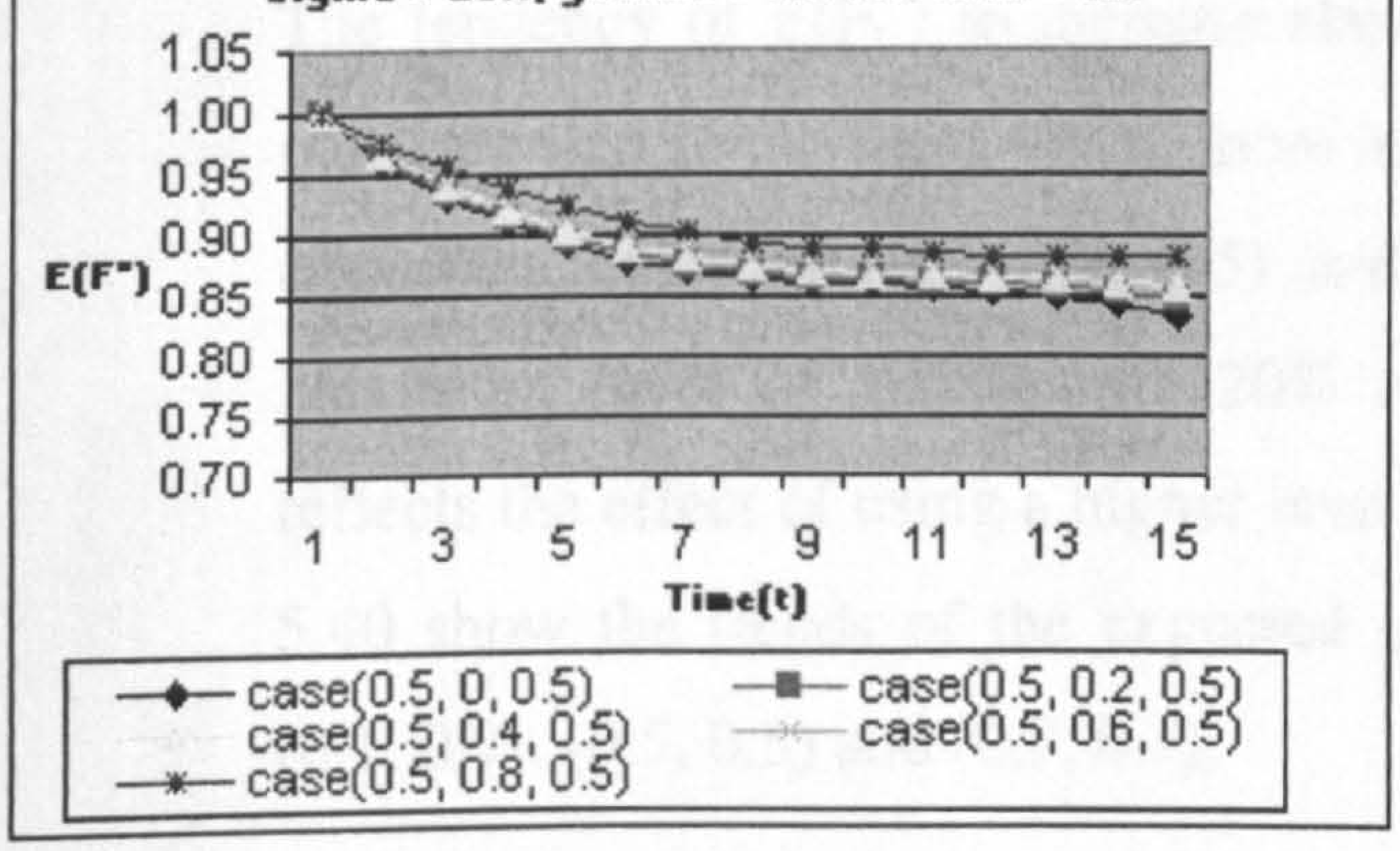


Figure 5.34
The expected optimal fund level under RIPI (1, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.9 and beta = 0.1

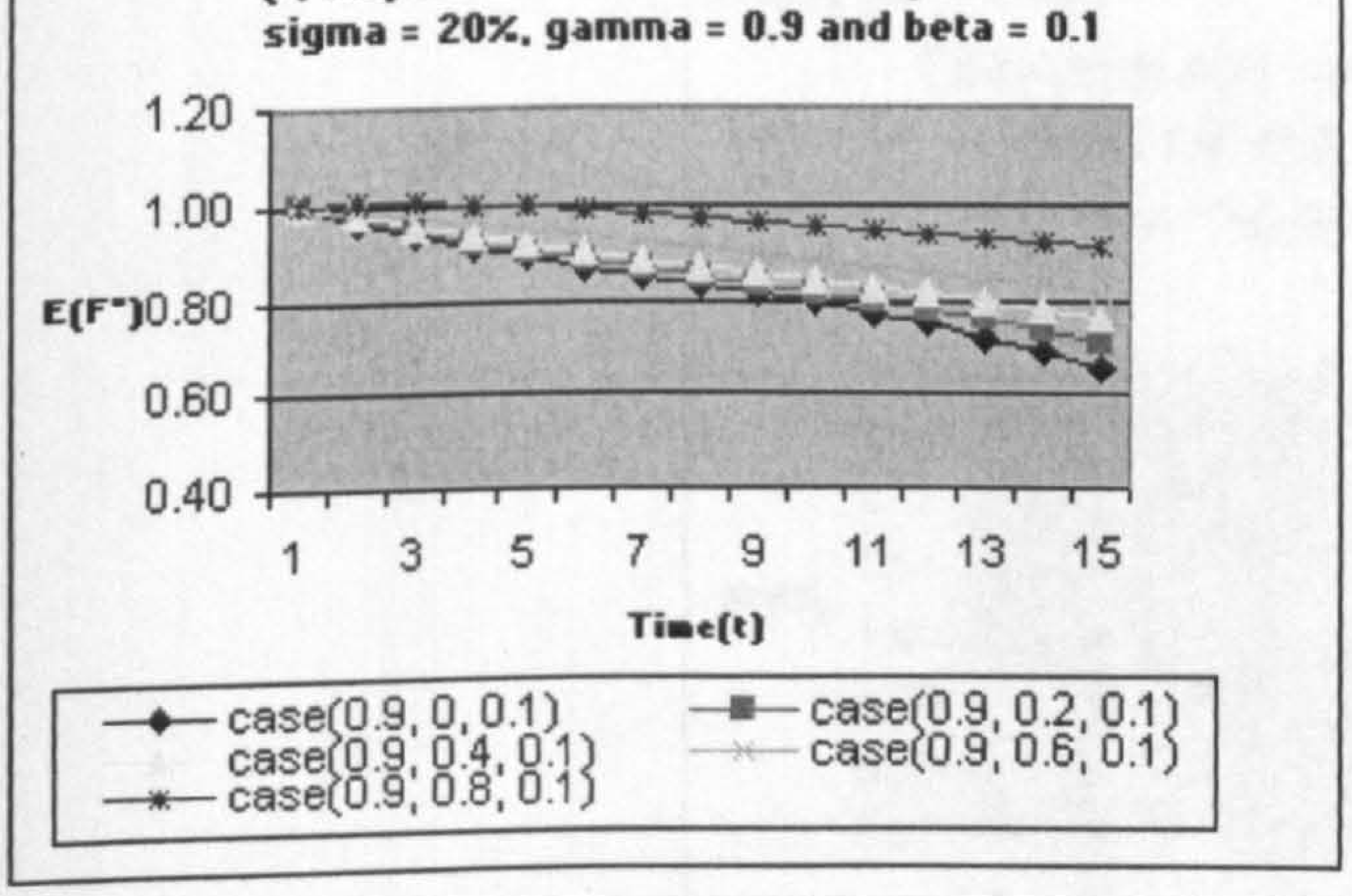


Figure 5.35
The expected optimal contribution rate under RIPI (1, 0.2) for different levels of alpha when sigma = 20%, gamma=0.1 and beta=0.9

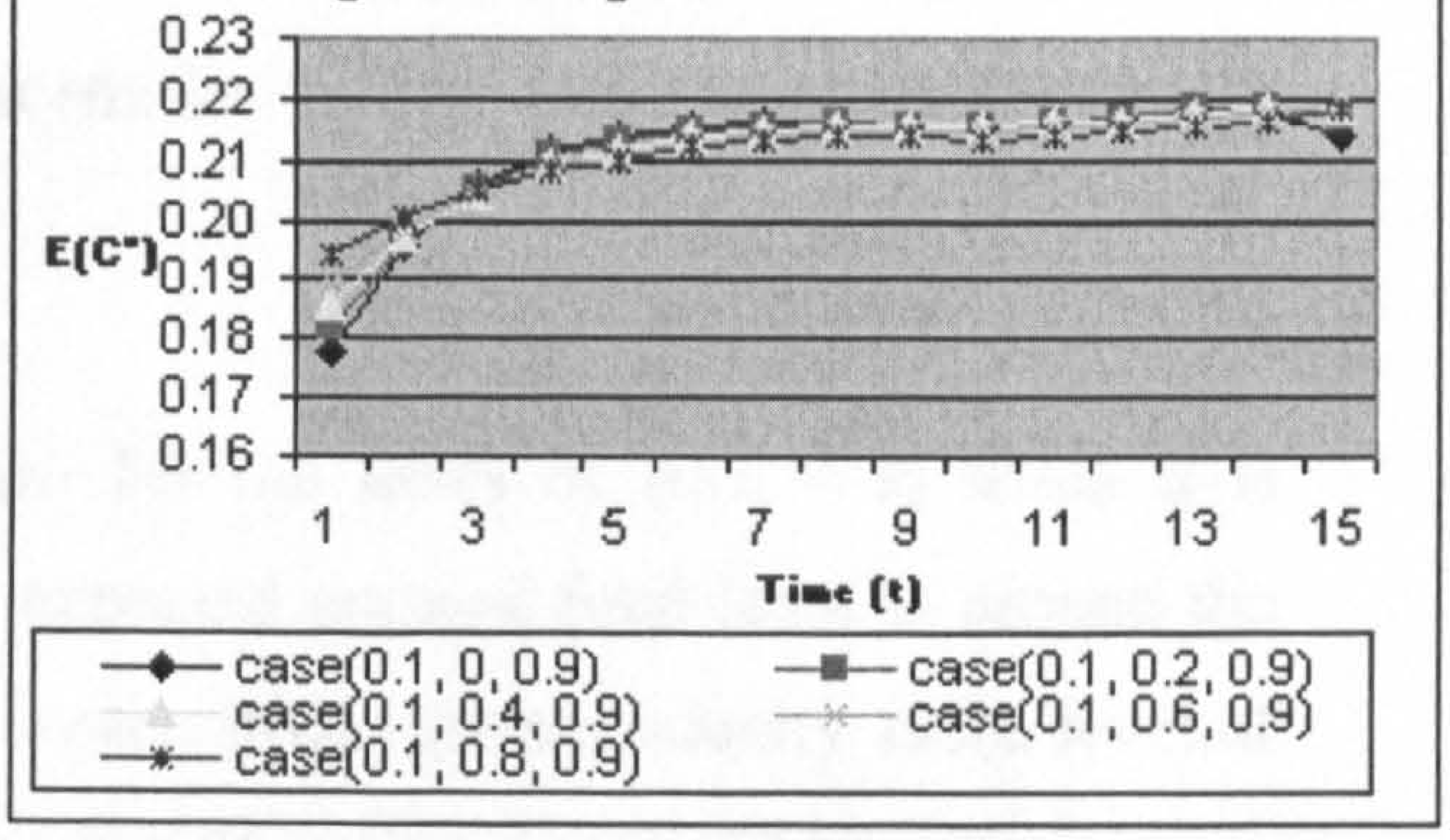


Figure 5.36
The expected optimal contribution rate under RIPI (1, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.5 and beta = 0.5

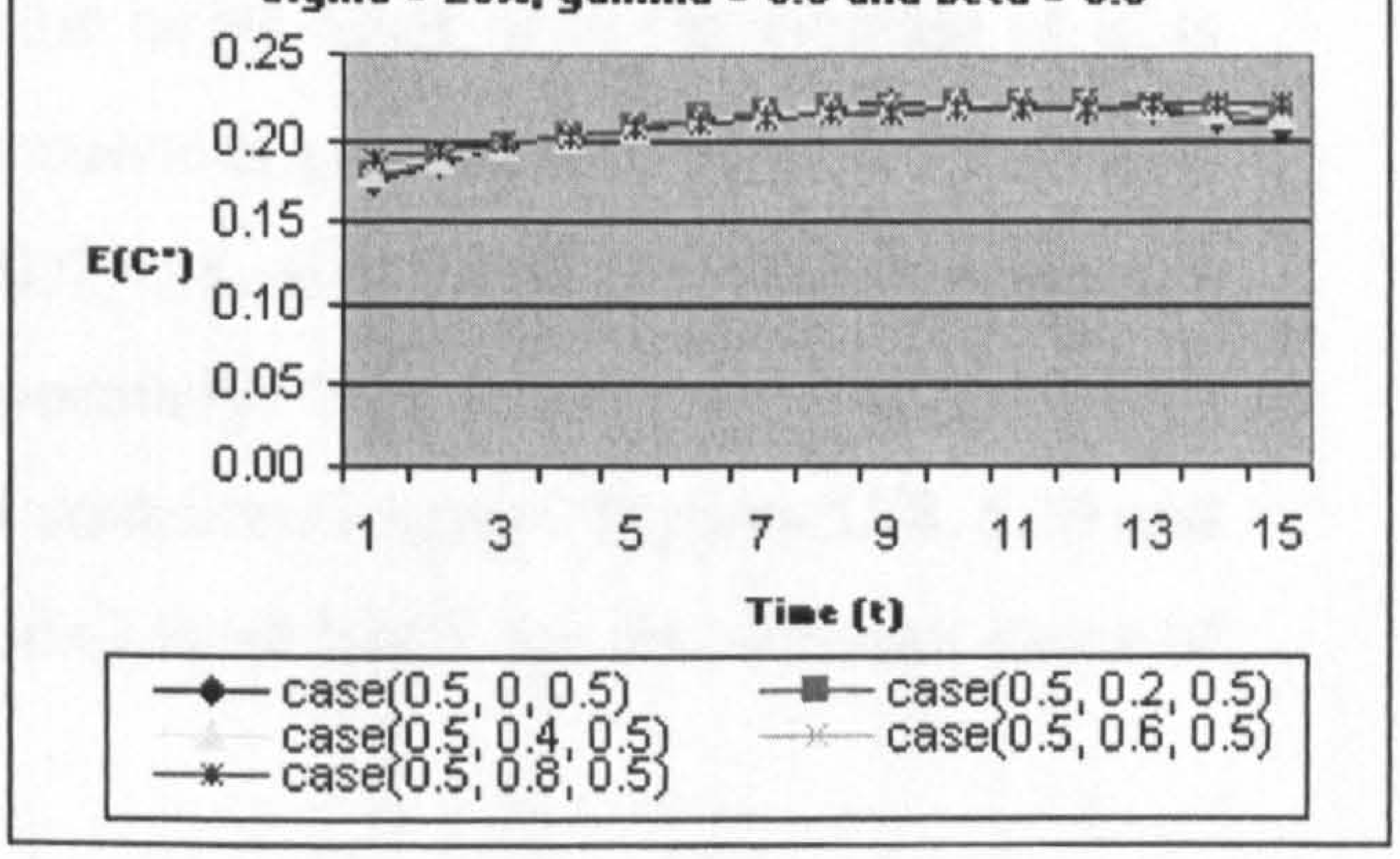
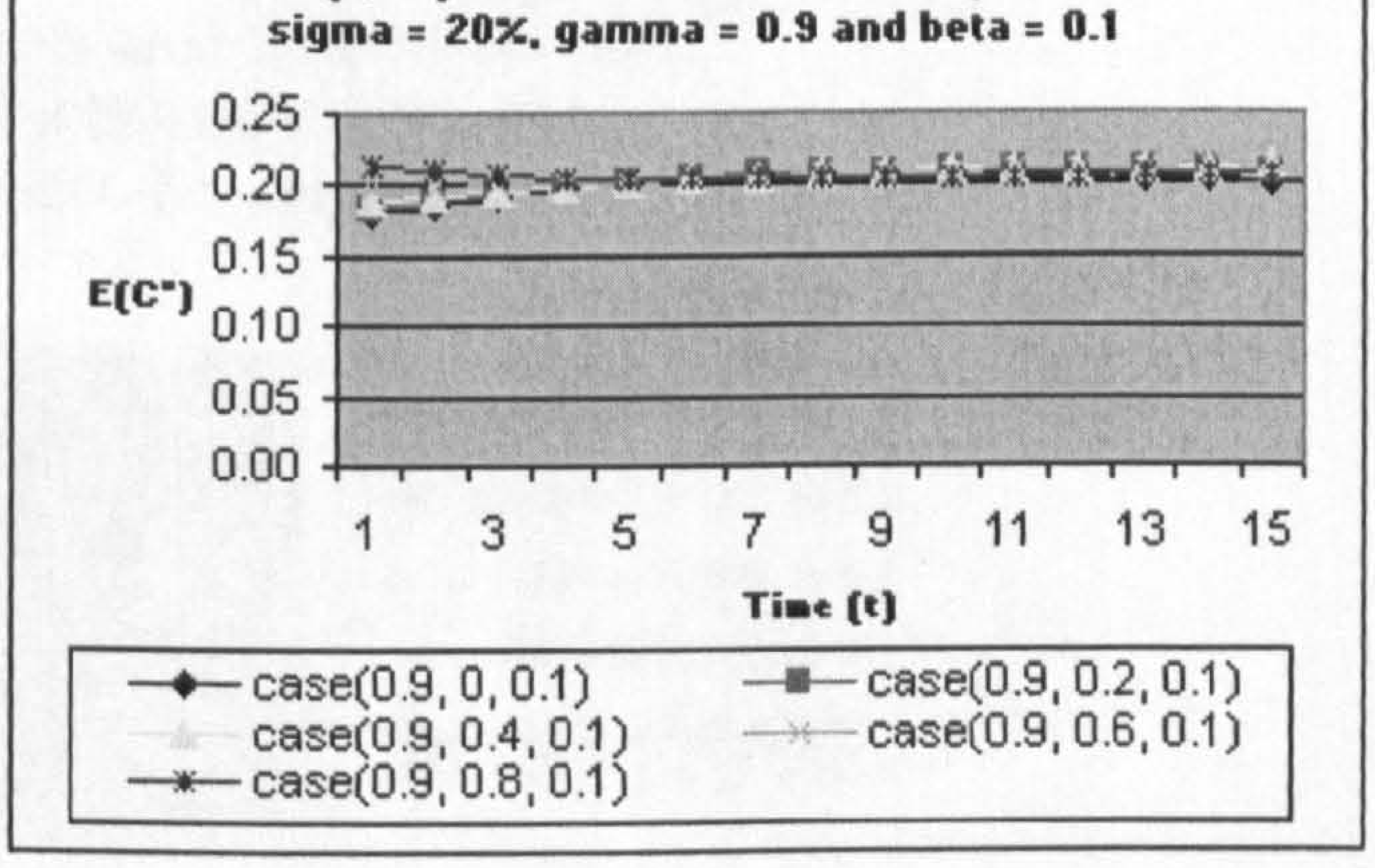


Figure 5.37
The expected optimal contribution rate under RIPI (1,0.2) for different levels of alpha when sigma = 20%, gamma = 0.9 and beta = 0.1



(b) Second and third Scenarios: $FT_t = 1$ and $CT_t = 0.22$; $FT_t = 1$ and $CT_t = 0.18$

Similarly to the approach applied in subsection 5.2.5 (b), we consider in our analysis for this sub-subsection the cases when $\sigma = 5\%$. The cases, when $\sigma = 20\%$, will be studied under the fourth and the fifth scenarios in the following sub-subsection 5.2.6(c).

When $\sigma = 5\%$, under the second scenario for the cases of (0.1, 0.9) when α is increased from 0 to 0.8, we note that the expected optimal fund level is around the target of 100% over the control period. However, it increases gradually as the level of α is increased until it reaches the maximum level of approximately 103% when $\alpha = 0.8$.

The tendency of $E(F_t^*)$ to increase above the target level, with the increase of α, is more obvious in the cases where more importance is given to both γ and α. The results under the cases of (0.5, 0.8, 0.5) and (0.9, 0.8, 0.1) show that $E(F_t^*)$ reaches a maximum level of 105% and 120% respectively. This increasing trend of $E(F_t^*)$ reflects the effect of using a higher level of contribution target. Figures 5.38, 5.39 and 5.40 show the trends of the expected optimal fund levels for the different cases of (0.1, 0.9), (0.5, 0.5) and (0.9, 0.1).

Figure 5.38

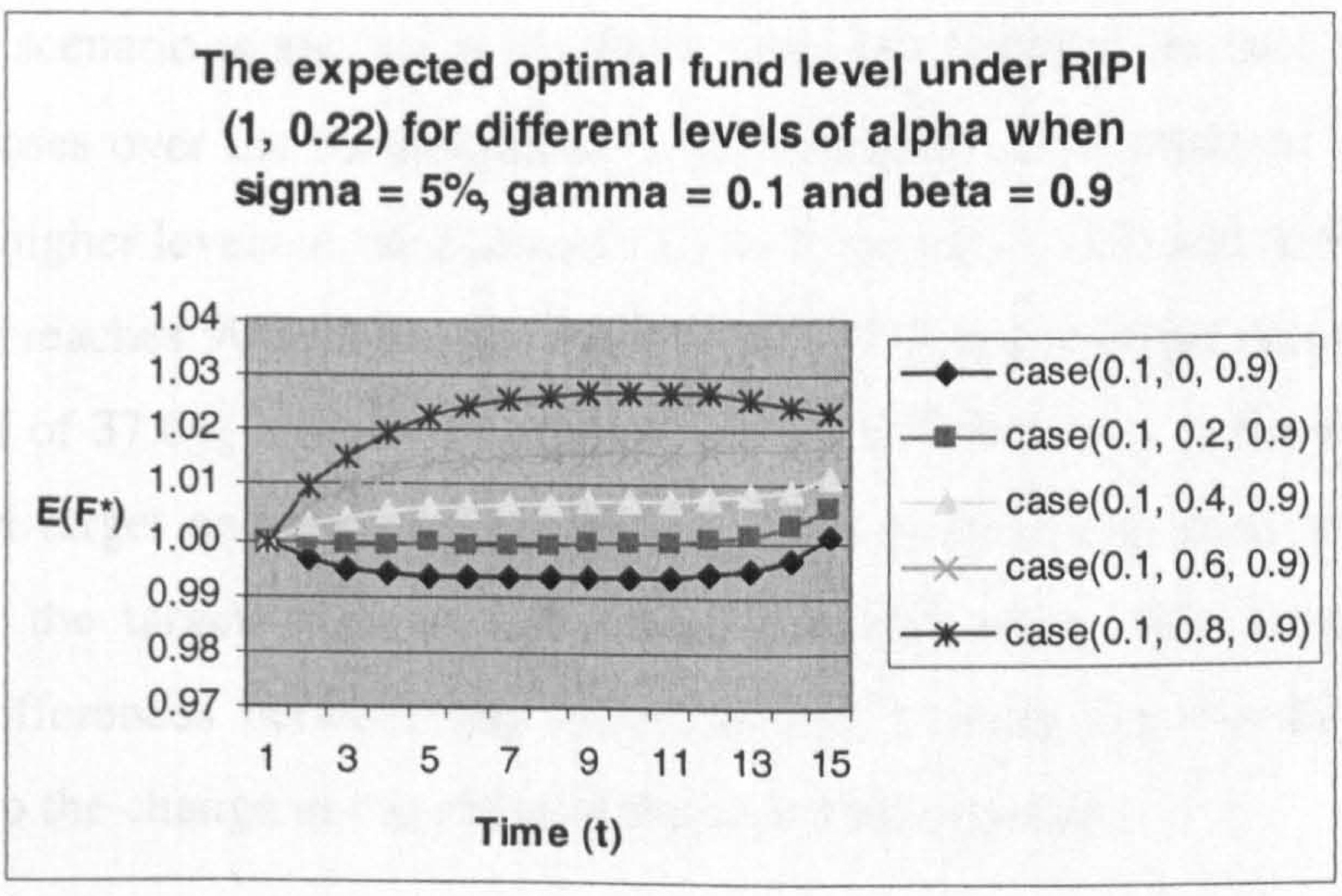


Figure 5.39

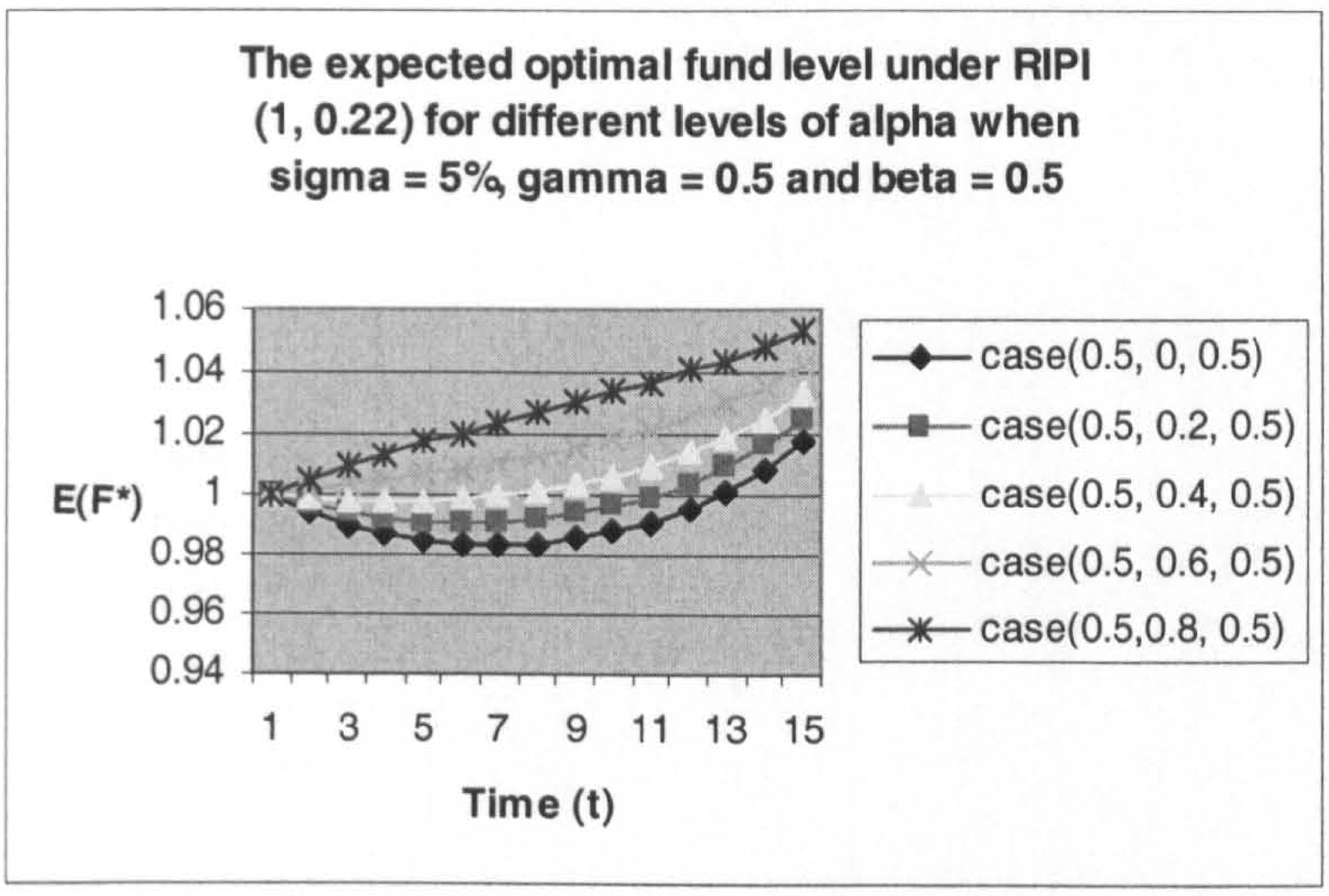
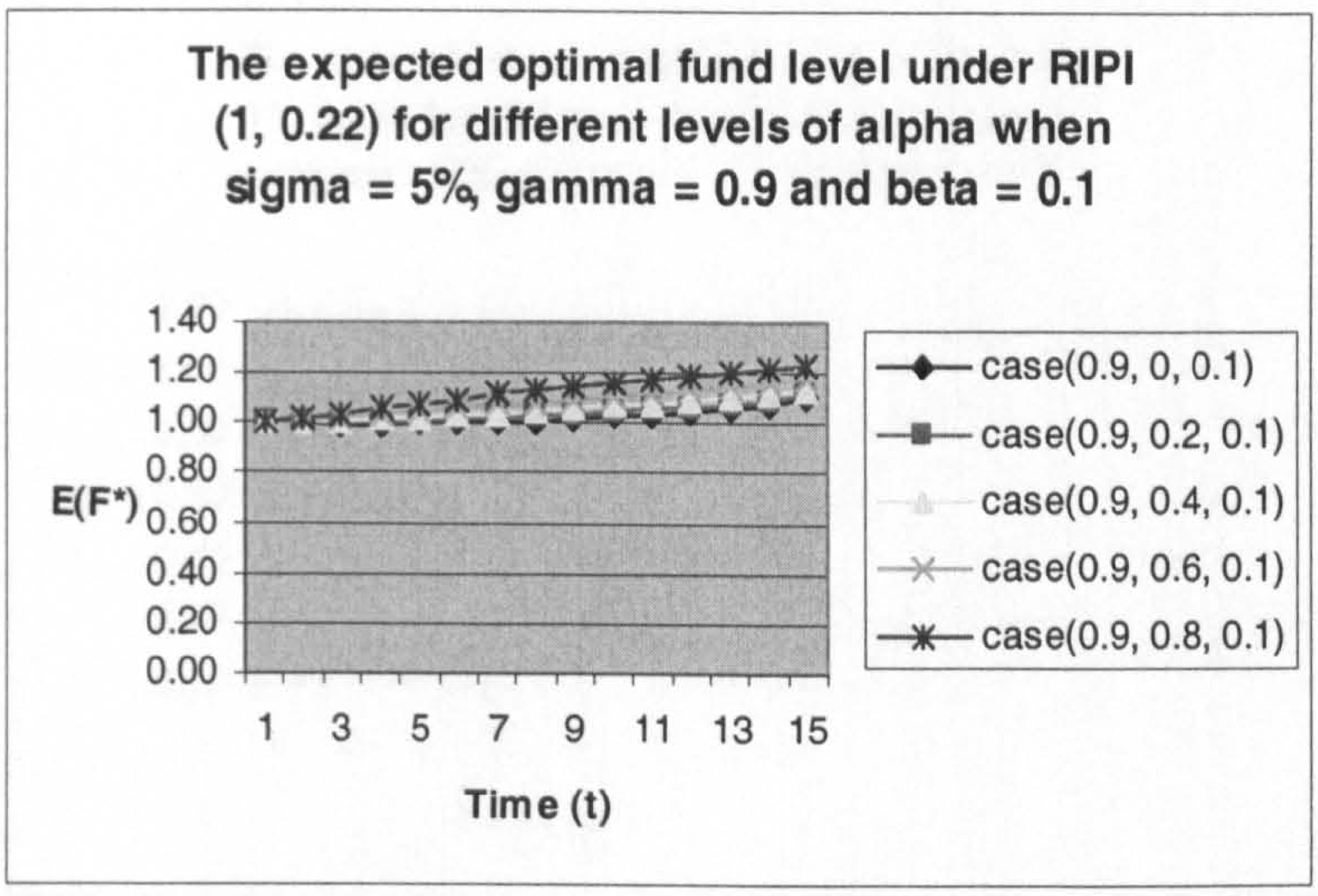


Figure 5.40



When the third scenario is applied at the same level of volatility, we note that $E(F_t^*)$ generally decreases over the control period and from one case to another. The trends of $E(F_t^*)$ are at higher levels in the cases of (0.1, 0.9) than (0.5, 0.5) and (0.9, 0.1). For example, $E(F_t^*)$ reaches 90% in the case of (0.1, 0.8, 0.9) and it drops dramatically to the lowest level of 37.6% in the case of (0.9, 0.8, 0.1). This is due to the decrease in the contribution target and the increase in the value of α, which leads to a greater departure from the target. Figures 5.41, 5.42 and 5.43 show these results which illustrate the differences between the trends of $E(F_t^*)$ under the second and third scenarios, due to the change in the value of the contribution target.

Figure 5.41

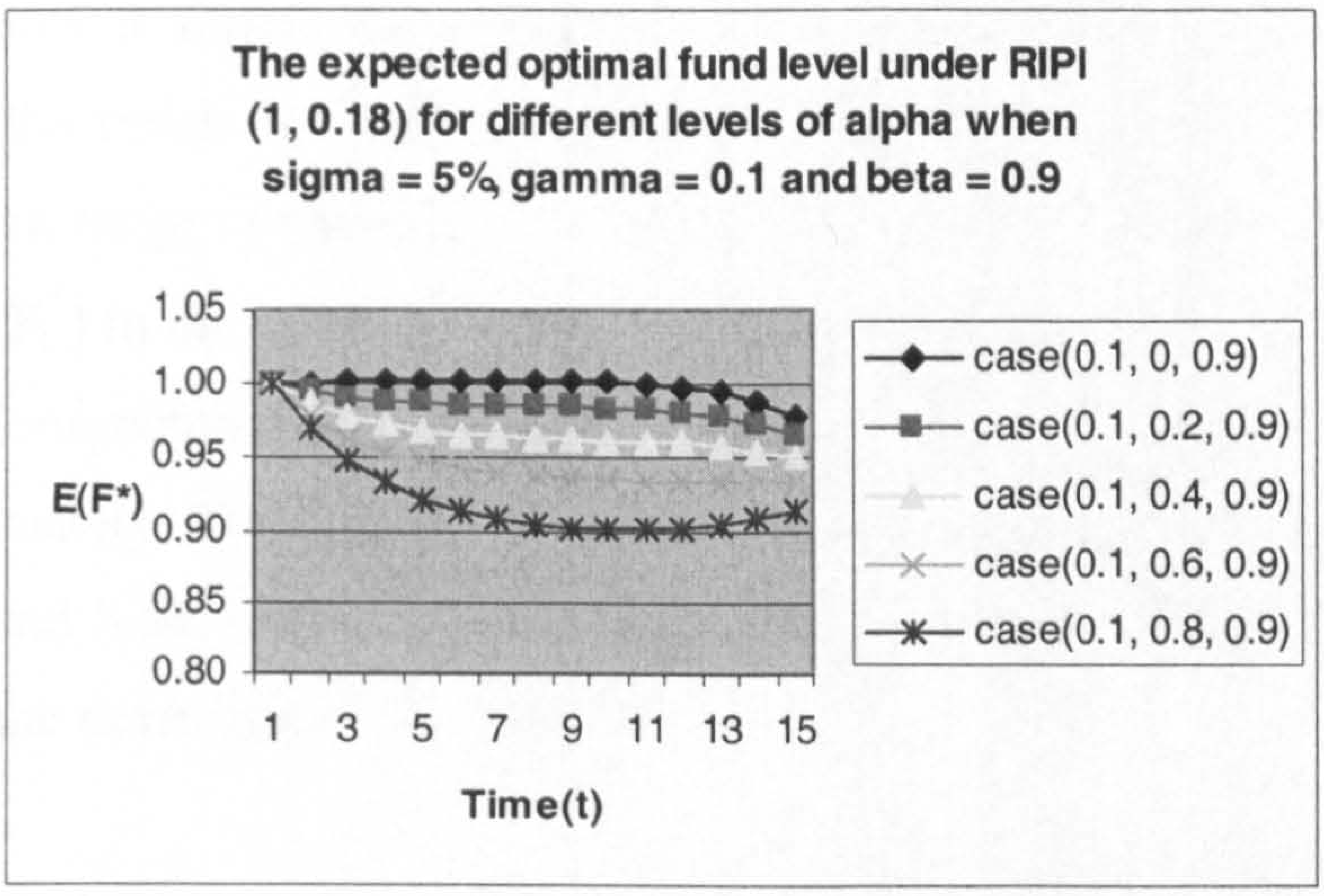


Figure 5.42

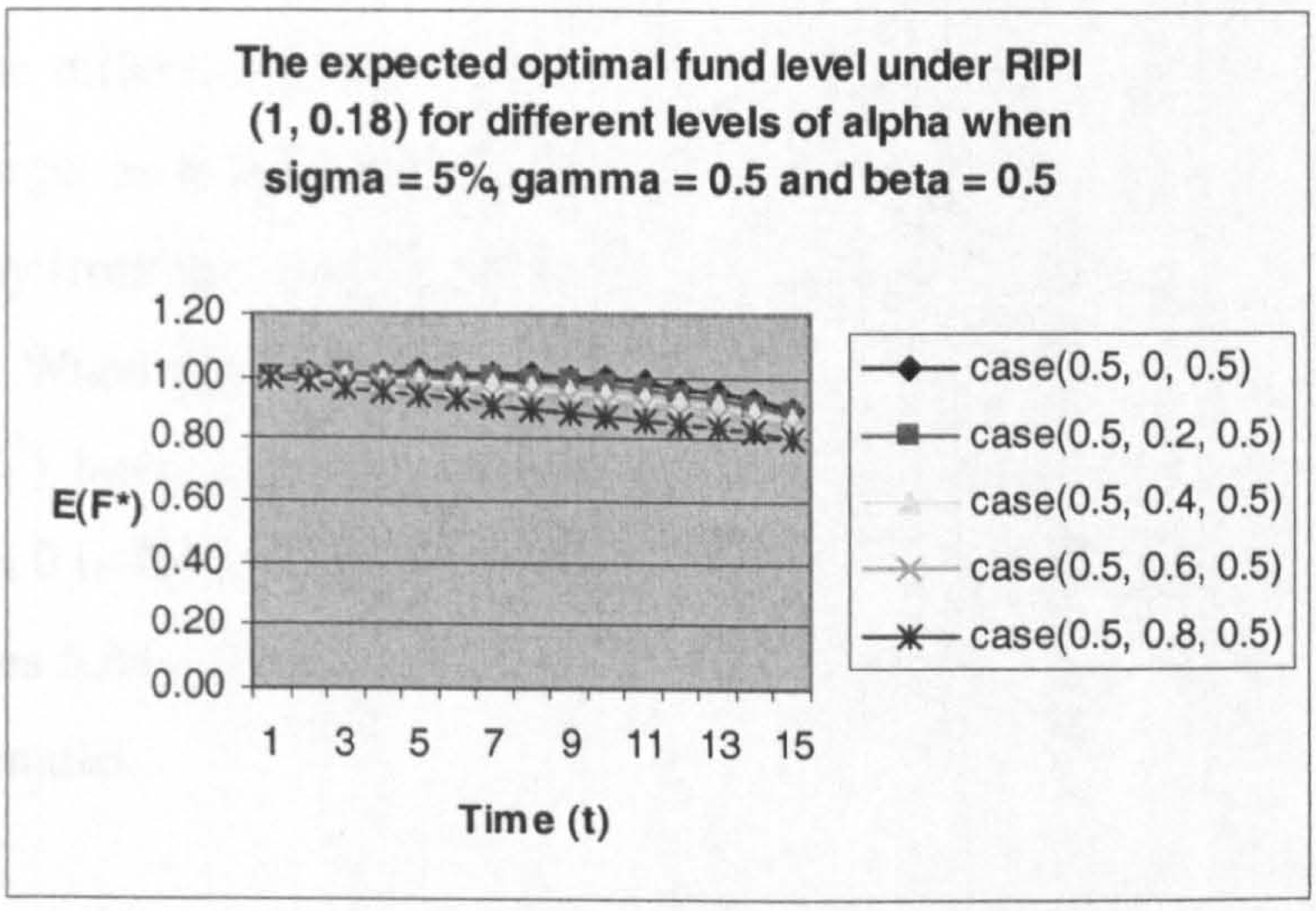
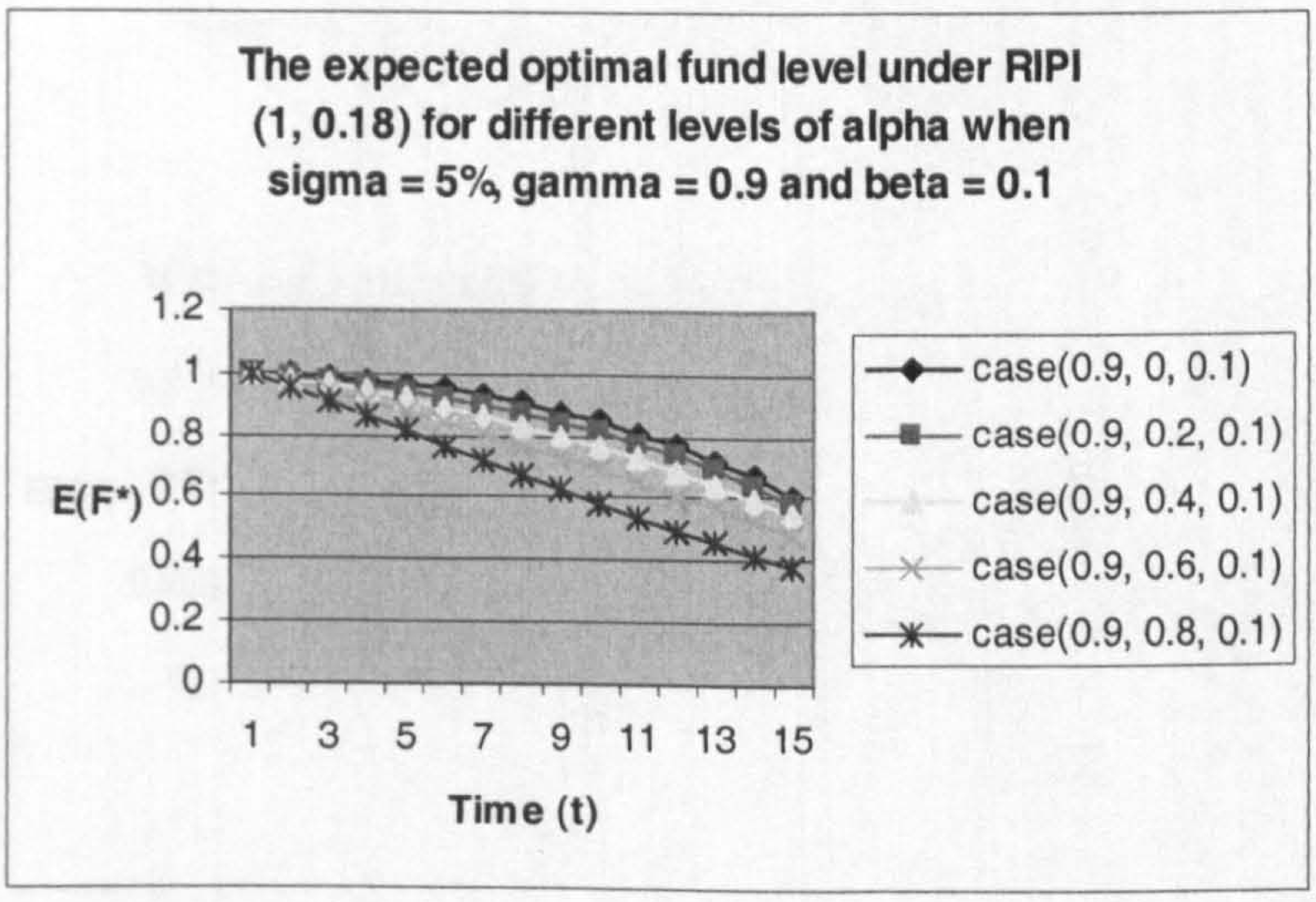


Figure 5.43



Further, we compare the trends of $E(F_t^*)$ for the CIPI and RIPI under both scenarios. The results show a similar behaviour between both models under each scenario. Nevertheless, the trends for $E(F_t^*)$ under RIPI deviate further away from the target and are wider in range compared with CIPI. For instance, under the second scenario, the level of $E(F_t^*)$ in the case of (0.1, 0, 0.9) moves down to 99.3% and 99.8% under RIPI and CIPI respectively. The level of $E(F_t^*)$ in the case of (0.1, 0.8, 0.9) moves up to be approximately 103% under RIPI and 100.3% under CIPI. This is shown in Figures 4.38 and 5.38. Under the third scenario, these values are decreased to 90% and 98.5% in the same case of (0.1, 0.8, 0.9).

On the other hand, when $\sigma = 5\%$, under the second scenario (1, 0.22) for the cases of (0.1, 0.9) and α is increased from 0 to 0.8, $E(C_t^*)$ is around 21%. However, it is notable that the different levels of $E(C_t^*)$ cross in year 5 before they tend to increase towards the target as α is increased from 0 to 0.4. Meanwhile, they tend to decrease and move away from the target in the two remaining cases (0.1, 0.6, 0.9) and (0.1, 0.8, 0.9). When β is decreased to 0.5 and 0.1 with increasing α from 0 to 0.8, the trends of $E(C_t^*)$ have a similar behaviour becoming closer to the target when α is increased from 0 to 0.4 and departing from the target with the higher values of $\alpha = 0.6$ and 0.8. Figures 5.44 – 5.46 show the behaviour of $E(C_t^*)$ for the different cases under the second scenario.

Figure 5.44

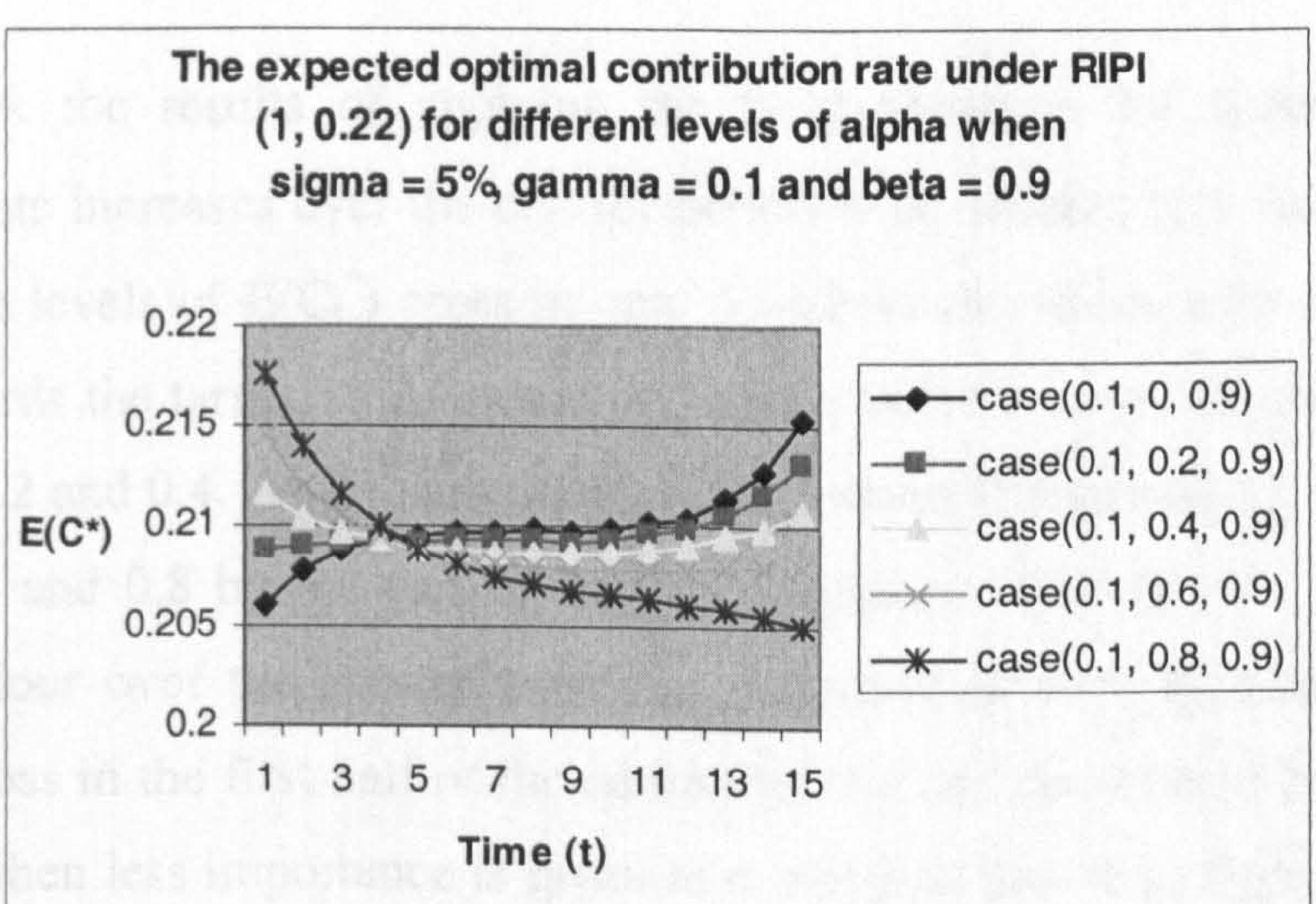


Figure 5.45

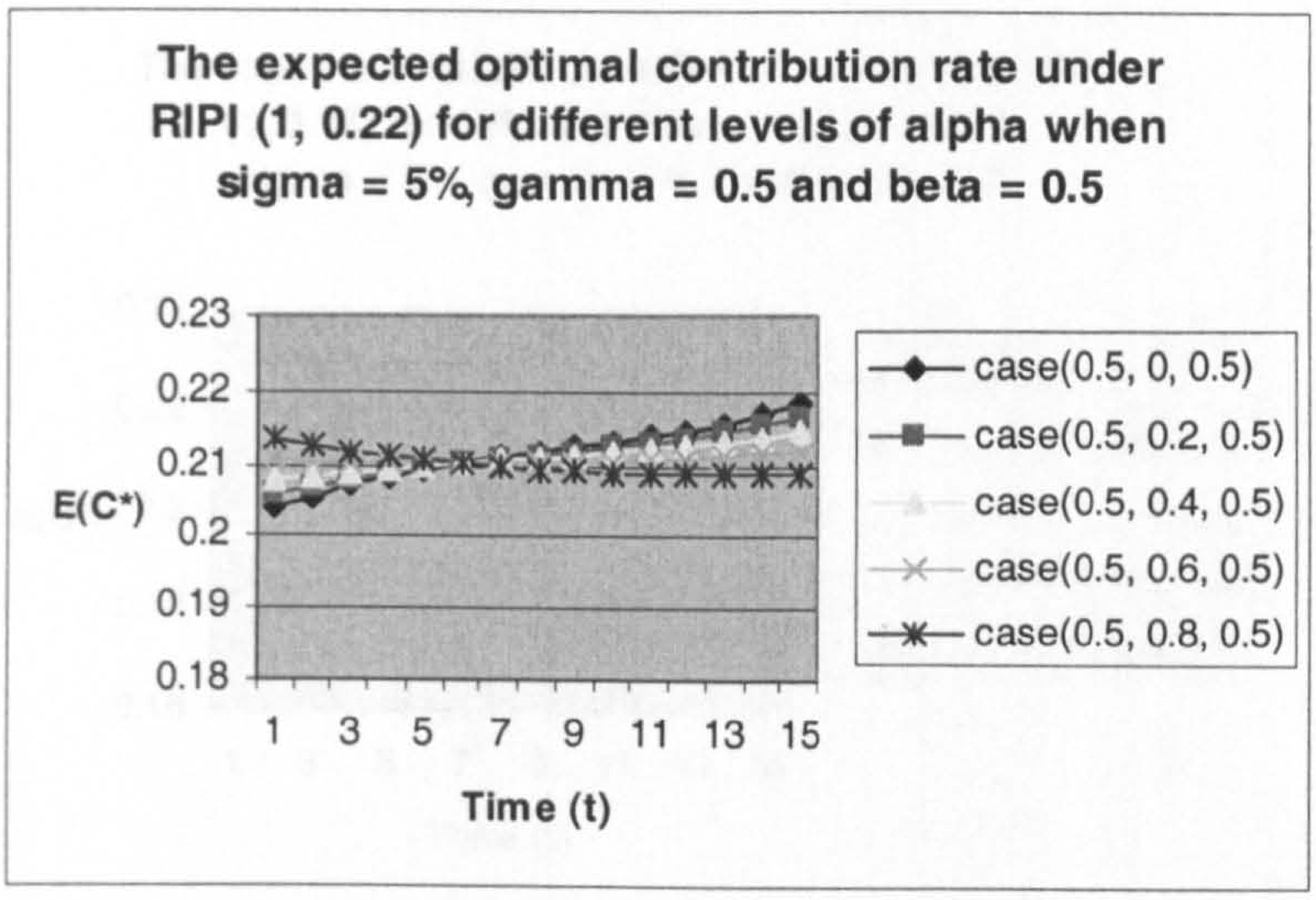
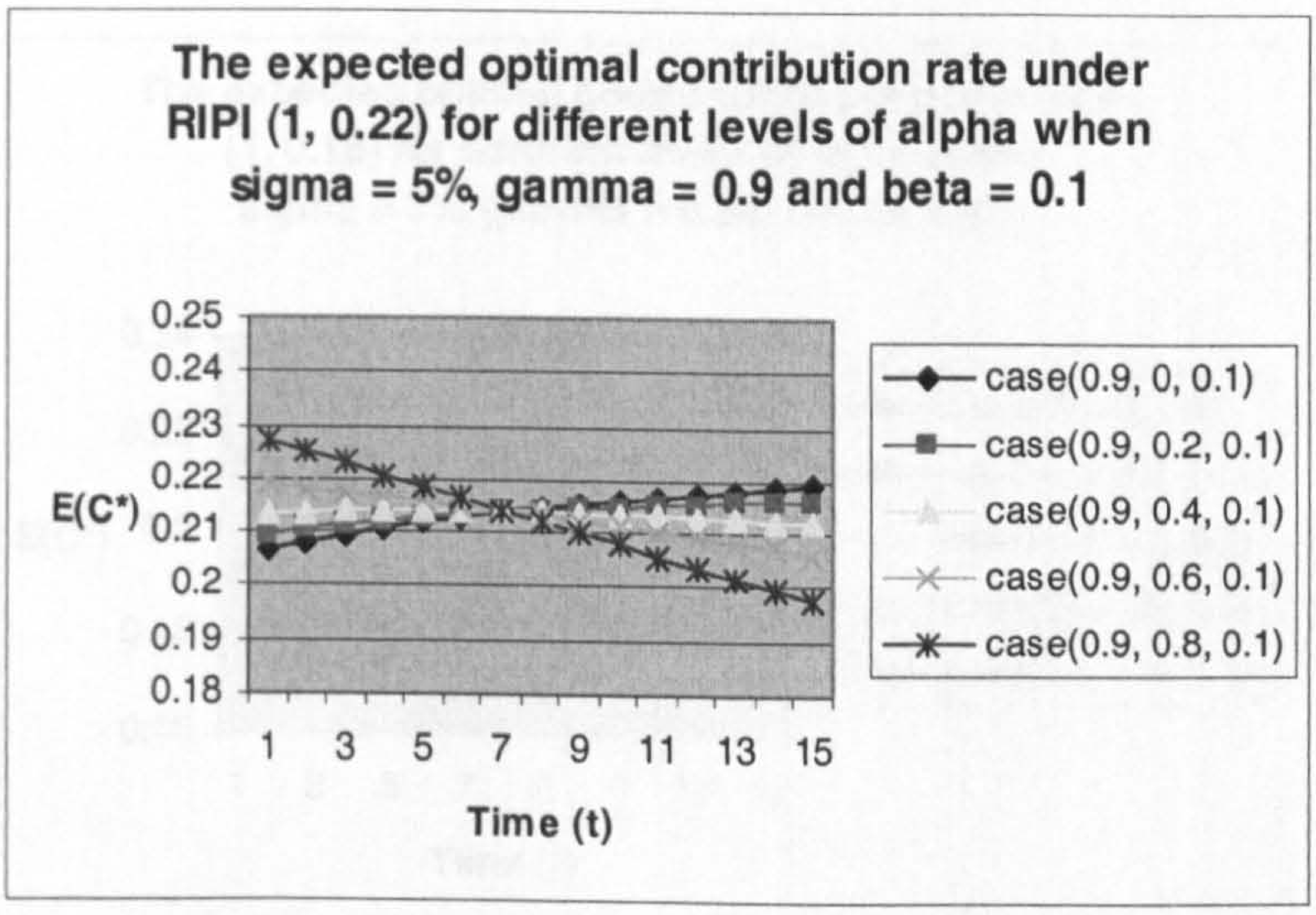


Figure 5.46



Next, we look the results of applying the third scenario, the expected optimal contribution rate increases over the control period to be around 21% for the cases of (0.1, 0.9). The levels of $E(C_t^*)$ cross in year 5 and remain stable with a tendency to decrease towards the target 18%, by the end of the control period, in the three cases when $\alpha = 0$, 0.2 and 0.4. At the same time, $E(C_t^*)$ moves further away from the target when $\alpha = 0.6$ and 0.8 by the end of the control period. The trends of $E(C_t^*)$ have similar behaviour over the control period in the cases of (0.5, 0.5) and (0.9, 0.1), where they cross in the first half of the control period and decrease to become closer to the target when less importance is given to α and β as shown in Figures 5.47, 5.48 and 5.49.

Figure 5.47

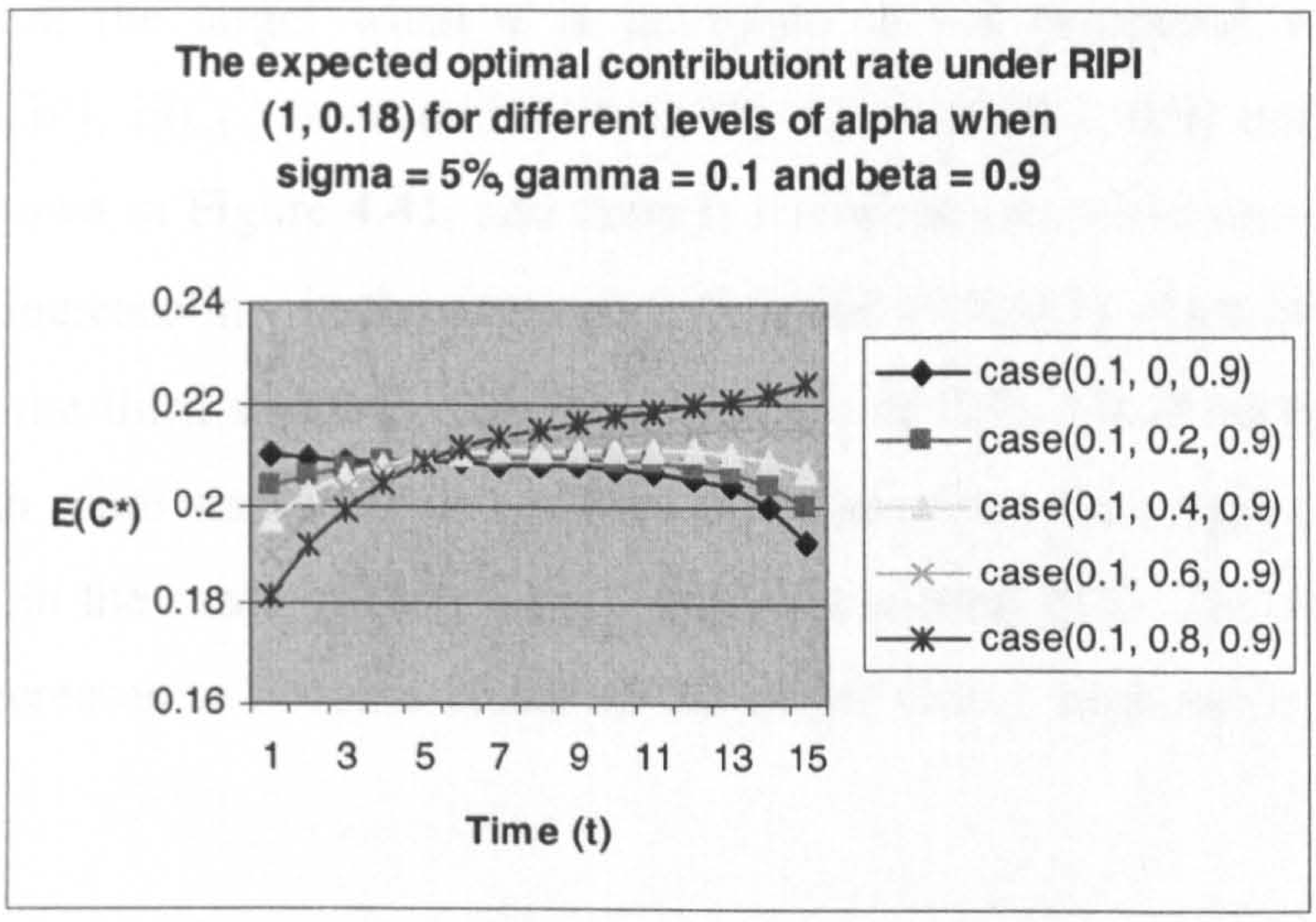


Figure 5.48

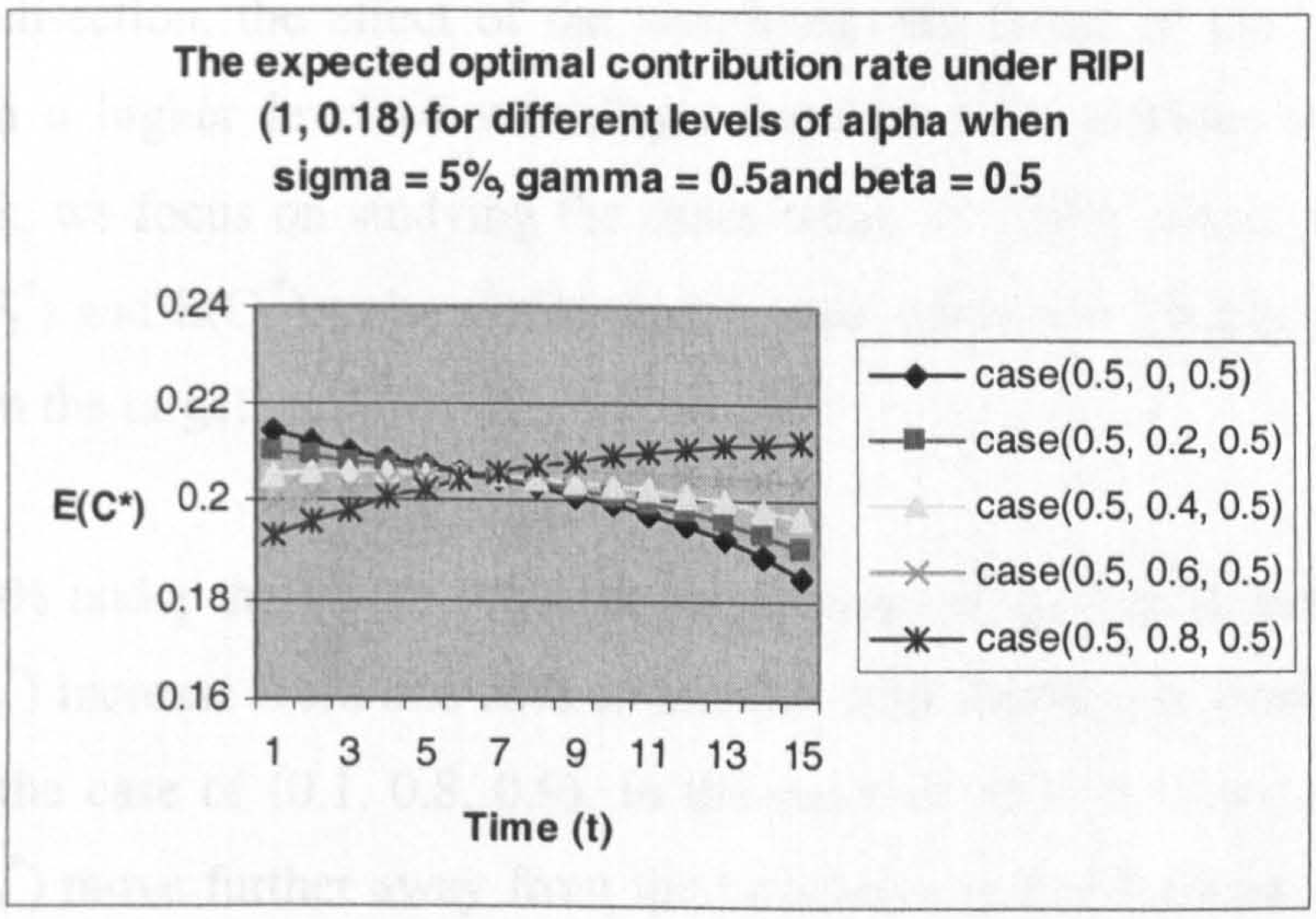
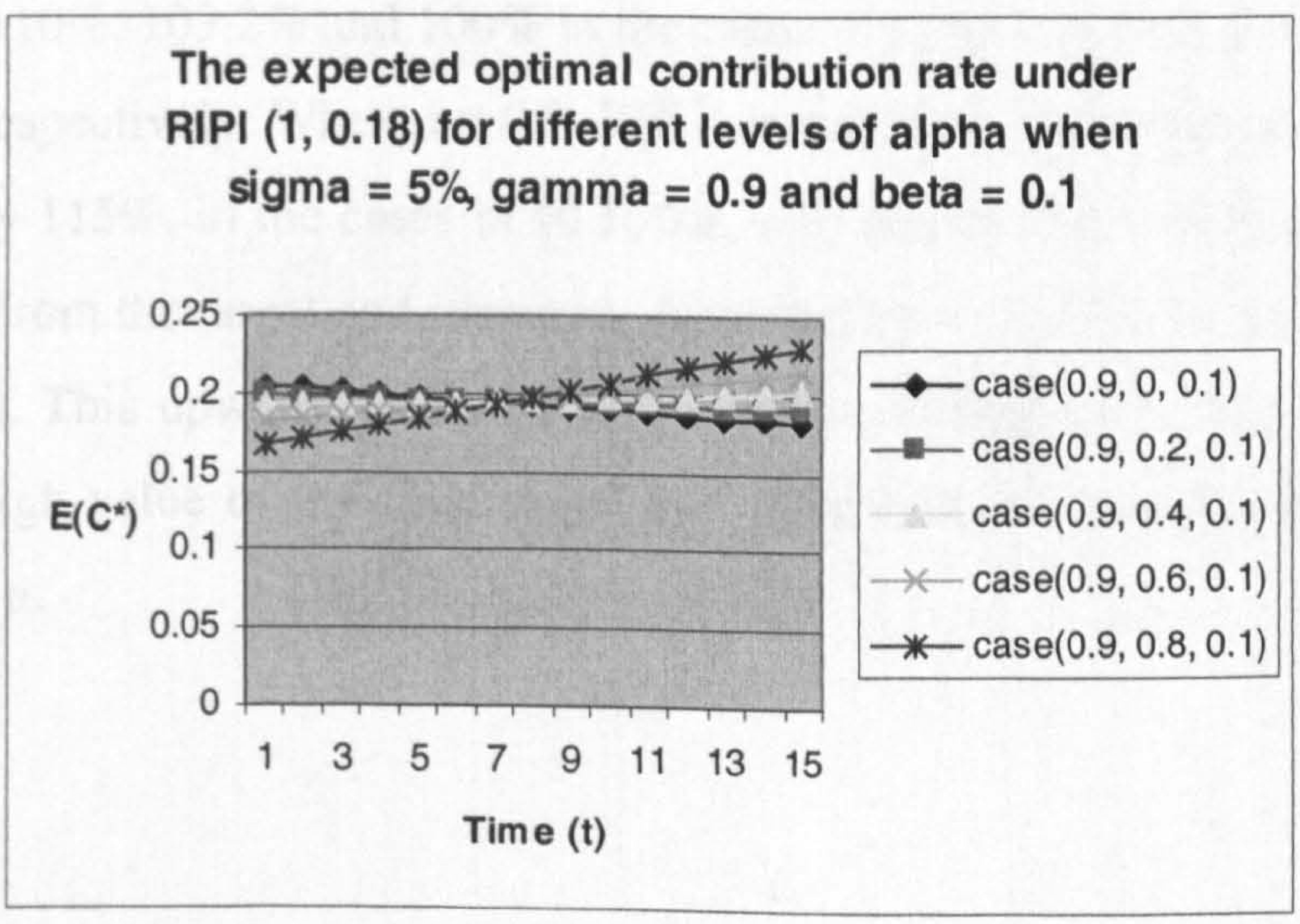


Figure 5.49



On the other hand, the trends of $E(C_t^*)$ under CIPI are more stable and show smaller deviations from the target when α is increased to 0.8 compared with RIPI. For example, in CIPI, $E(C_t^*)$ is around 21% in the cases of (0.1, 0.9) under the second scenario as shown in Figure 4.41, and there is a tendency to move closer to the target 22% with an increase in γ in the cases (0.5, 0.5) and (0.9, 0.1) when the value of α is small. Under the third scenario, similar behaviour of $E(C_t^*)$ is observed under CIPI but again with more stable trends and less deviation from the target compared with RIPI, so that, in the cases of (0.1, 0.9), $E(C_t^*)$ is around 21% - as shown in Figure 4.53 - and decreases to become closer to the target with a high value of γ and low value of α.

(c) Fourth and fifth scenarios: $FT_t = 1.2$ and $CT_t = 0.2$; $FT_t = 0.8$ and $CT_t = 0.2$

In this sub-subsection, the effect of the weighting risk factor of the mixed term is explored with a higher level of volatility compared with previous sub-subsection 5.2.6(b). Thus, we focus on studying the cases when $\sigma = 20\%$, where we expect the results of $E(F_t^*)$ and $E(C_t^*)$ to be similar to the ones when $\sigma = 5\%$ but with a greater deviation from the target.

When $\sigma = 20\%$ under the fourth scenario for all cases of (0.1, 0.9), we note that the levels of $E(F_t^*)$ increase from one case to another with increasing α until it is around the target in the case of (0.1, 0.8, 0.9). In the cases of (0.5, 0.5) and (0.9, 0.1), the trends of $E(F_t^*)$ move further away from the target due to the decrease of β. We find that the trends are wider in range when α is increased from 0 to 0.8, i.e. $E(F_t^*)$ decreases to 110%, 103.2% and 100% in the cases (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0, 0.1) respectively. When $\alpha = 0.8$, $E(F_t^*)$ moves from the target level of 120% to approximately 115%, in the cases of (0.1, 0.8, 0.9) and (0.5, 0.8, 0.5), then, it moves further away from the target and increases dramatically to 152.6% in the case of (0.9, 0.8, 0.1). This upward trend of $E(F_t^*)$ - shown in Figures 5.50, 5.51 and 5.52 - reflects the high value of the fund target and high level of volatility along with the high value of α.

Figure 5.50
The expected optimal fund level under RIPI (1.2, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.1 and beta = 0.9

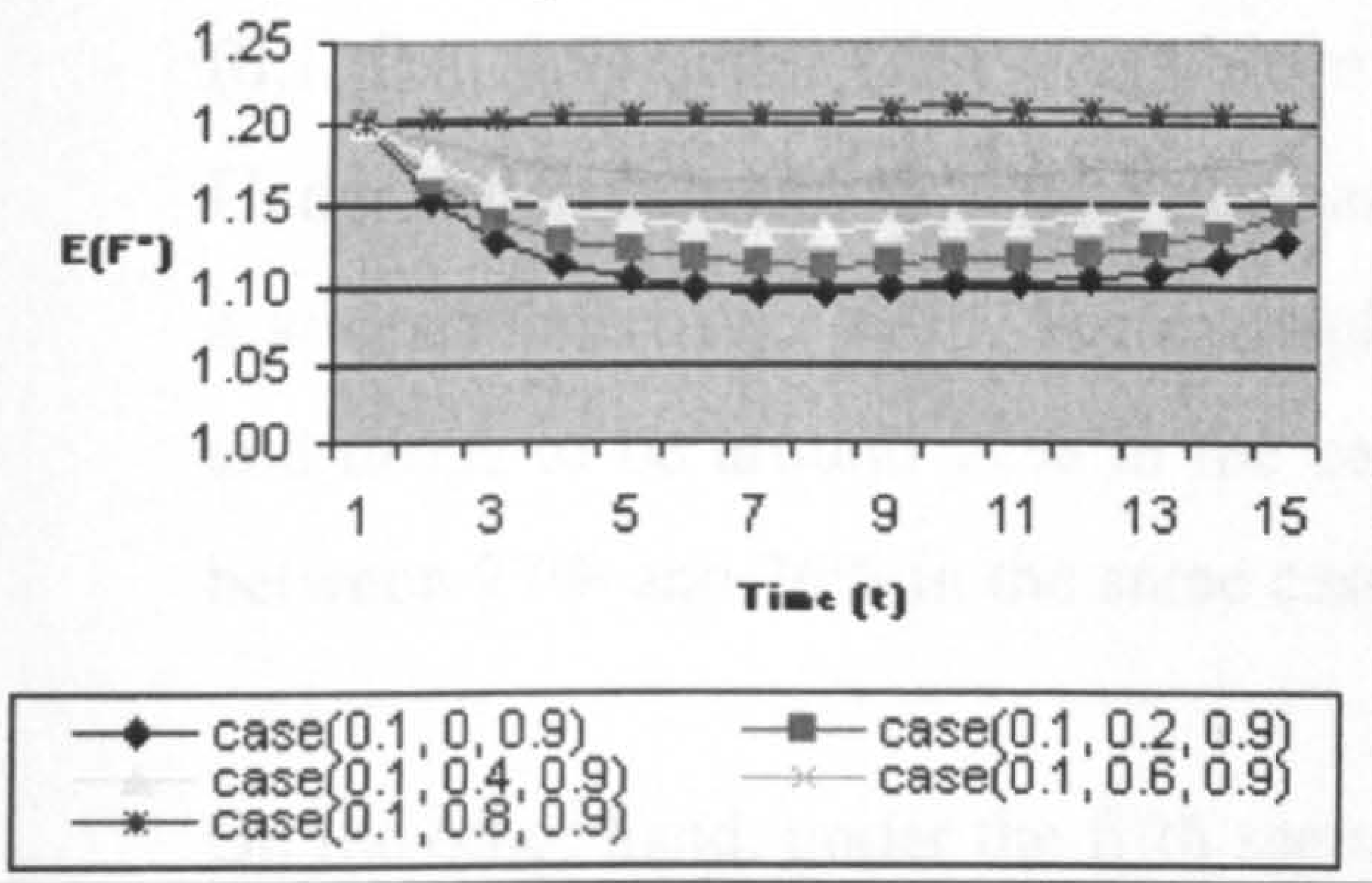


Figure 5.51
The expected optimal fund level under RIPI (1.2, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.1 and beta = 0.9

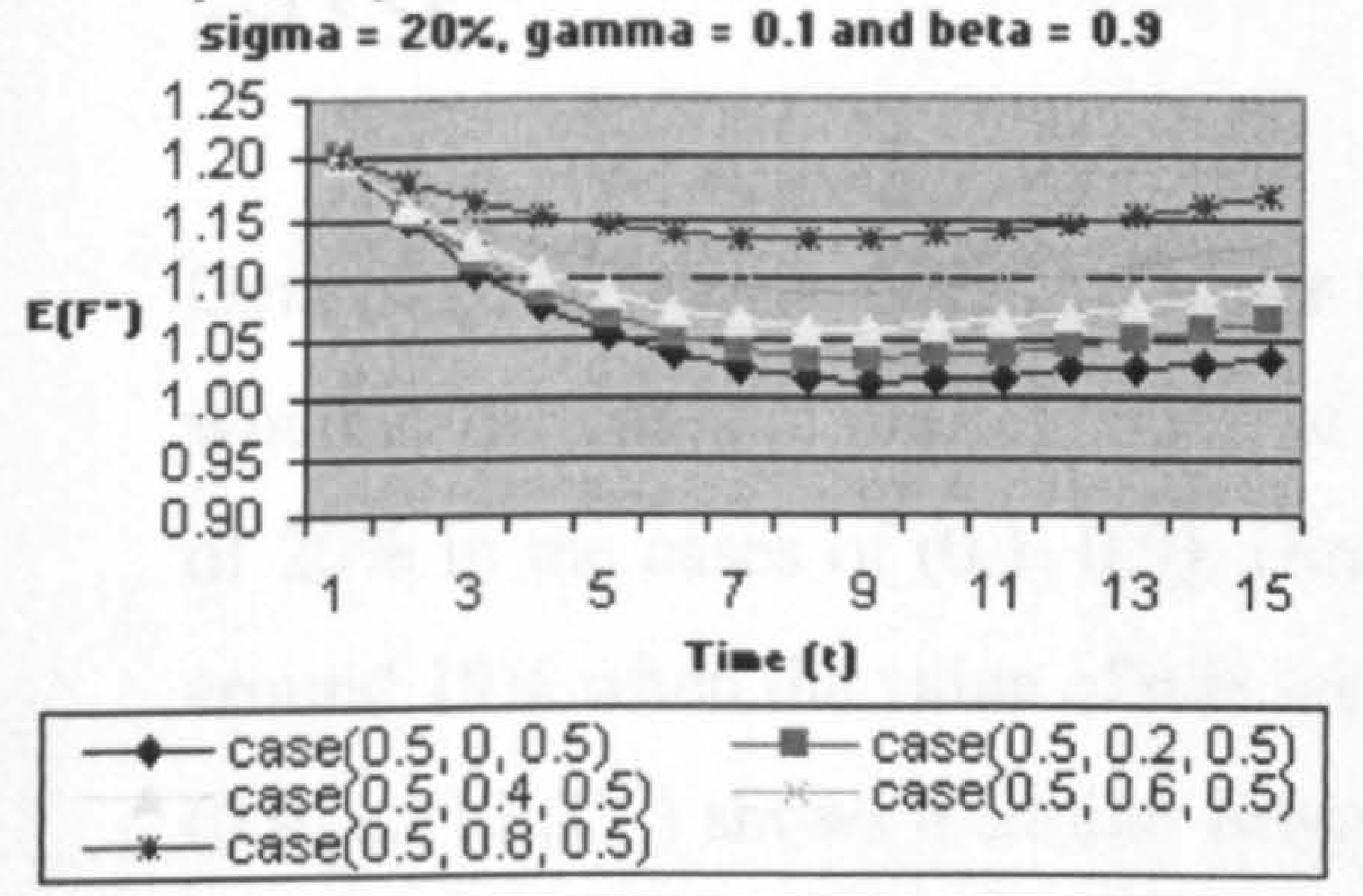


Figure 5.52
The expected optimal fund level under RIPI (1.2, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.9 and beta = 0.1

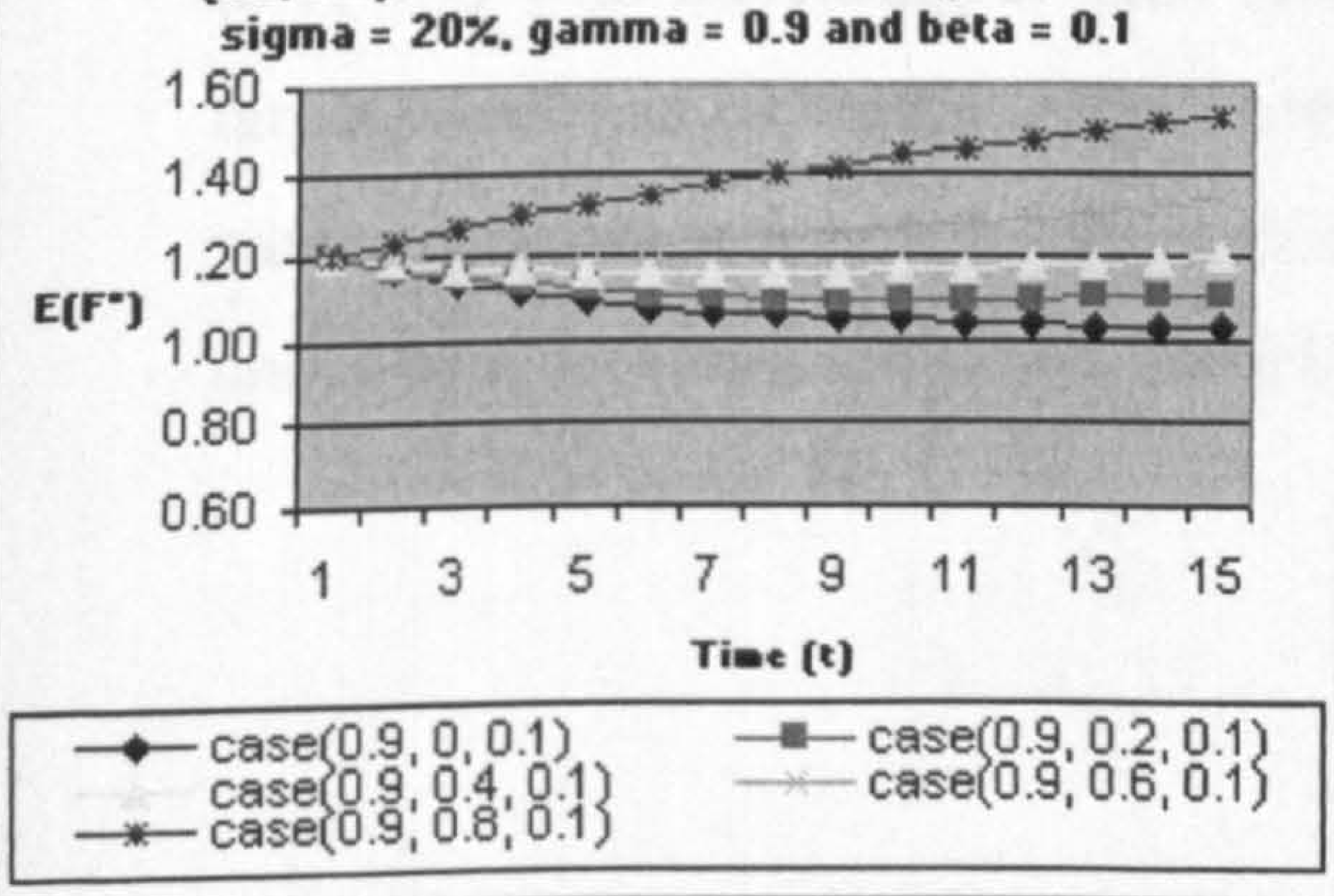


Figure 5.53
The expected optimal fund level under RIPI (0.8, 0.2) for different levels of alpha when sigma = 20%, gamma= 0.1 and beta = 0.9

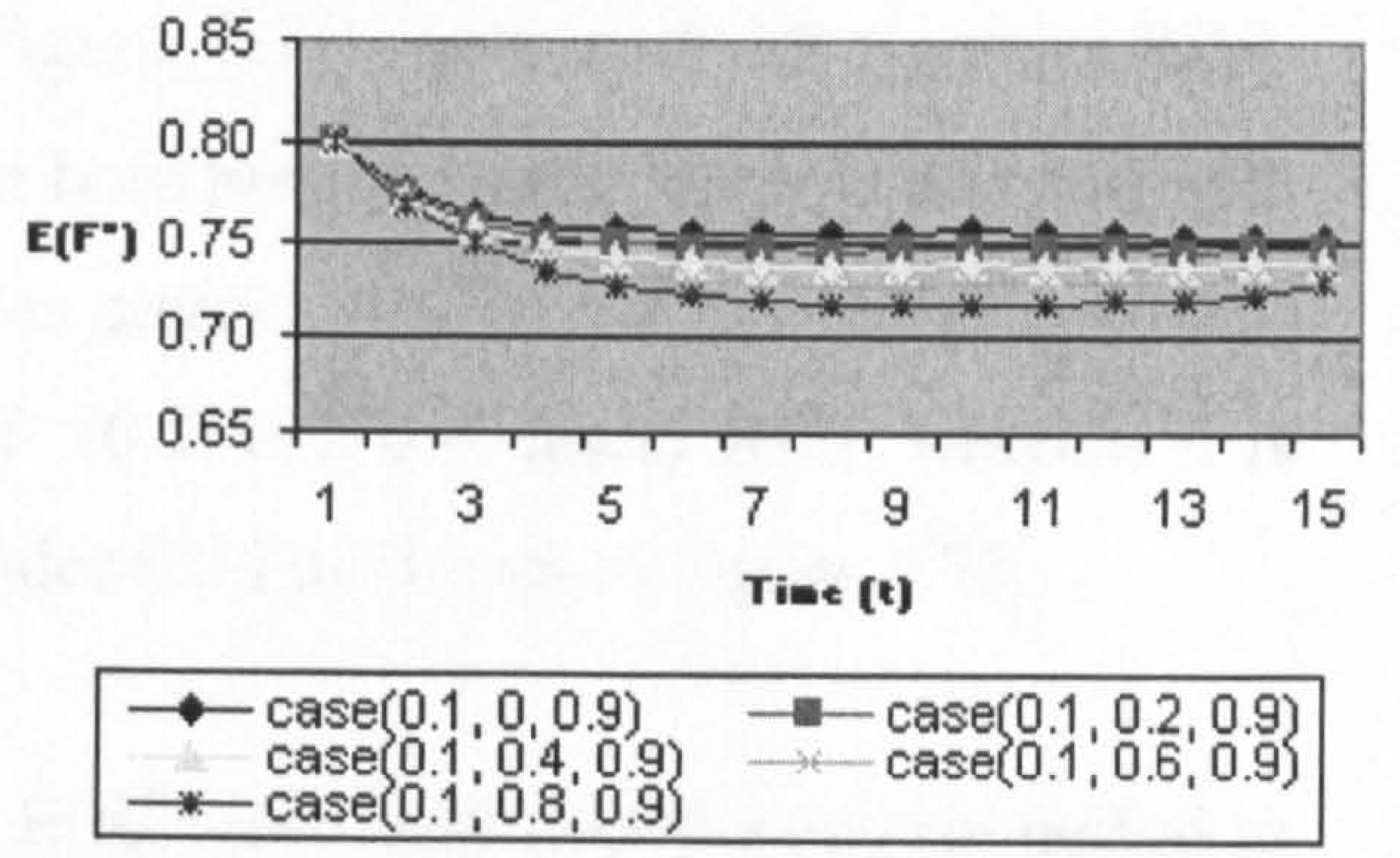


Figure 5.54
The expected optimal fund level under RIPI (0.8, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.5 and beta = 0.5

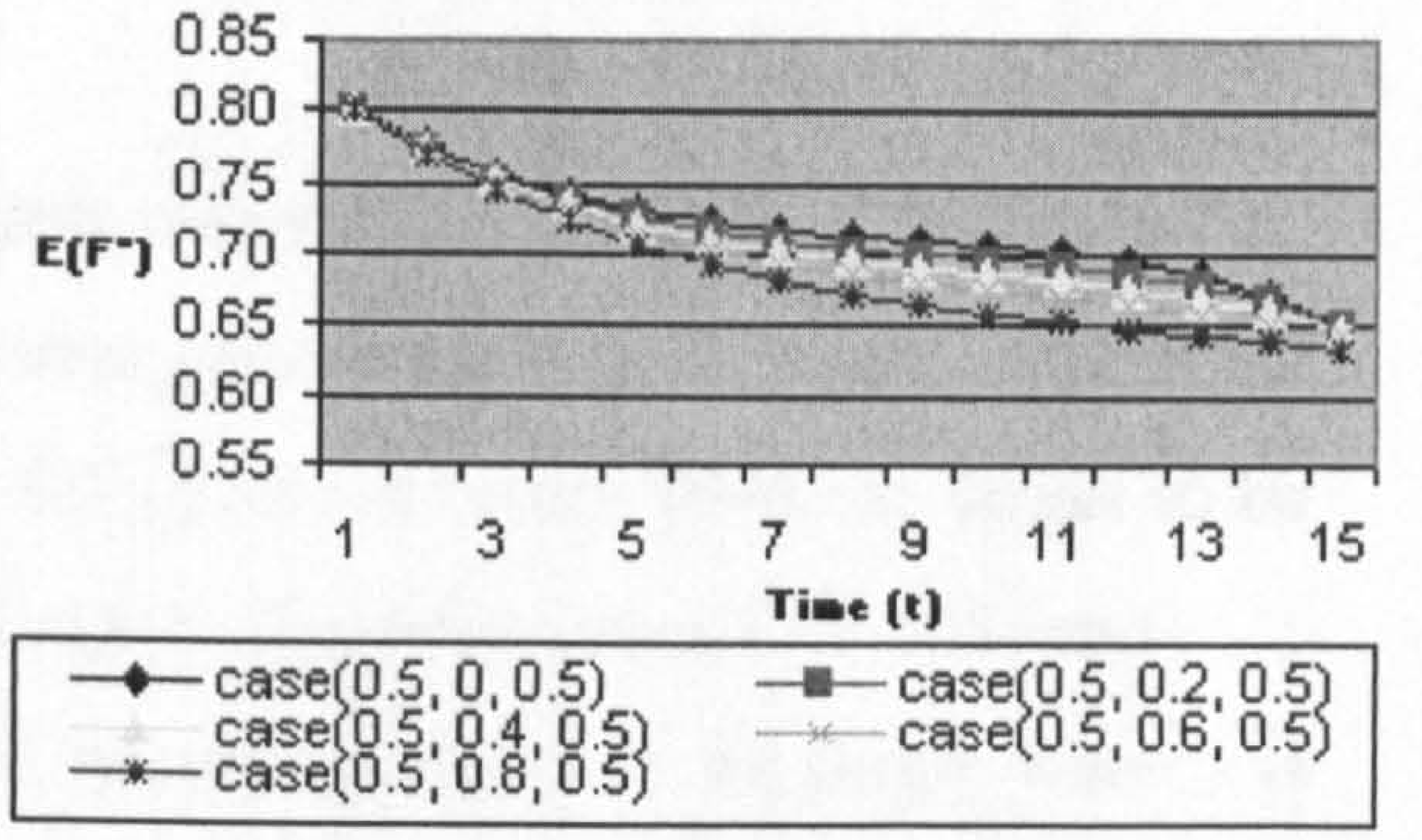
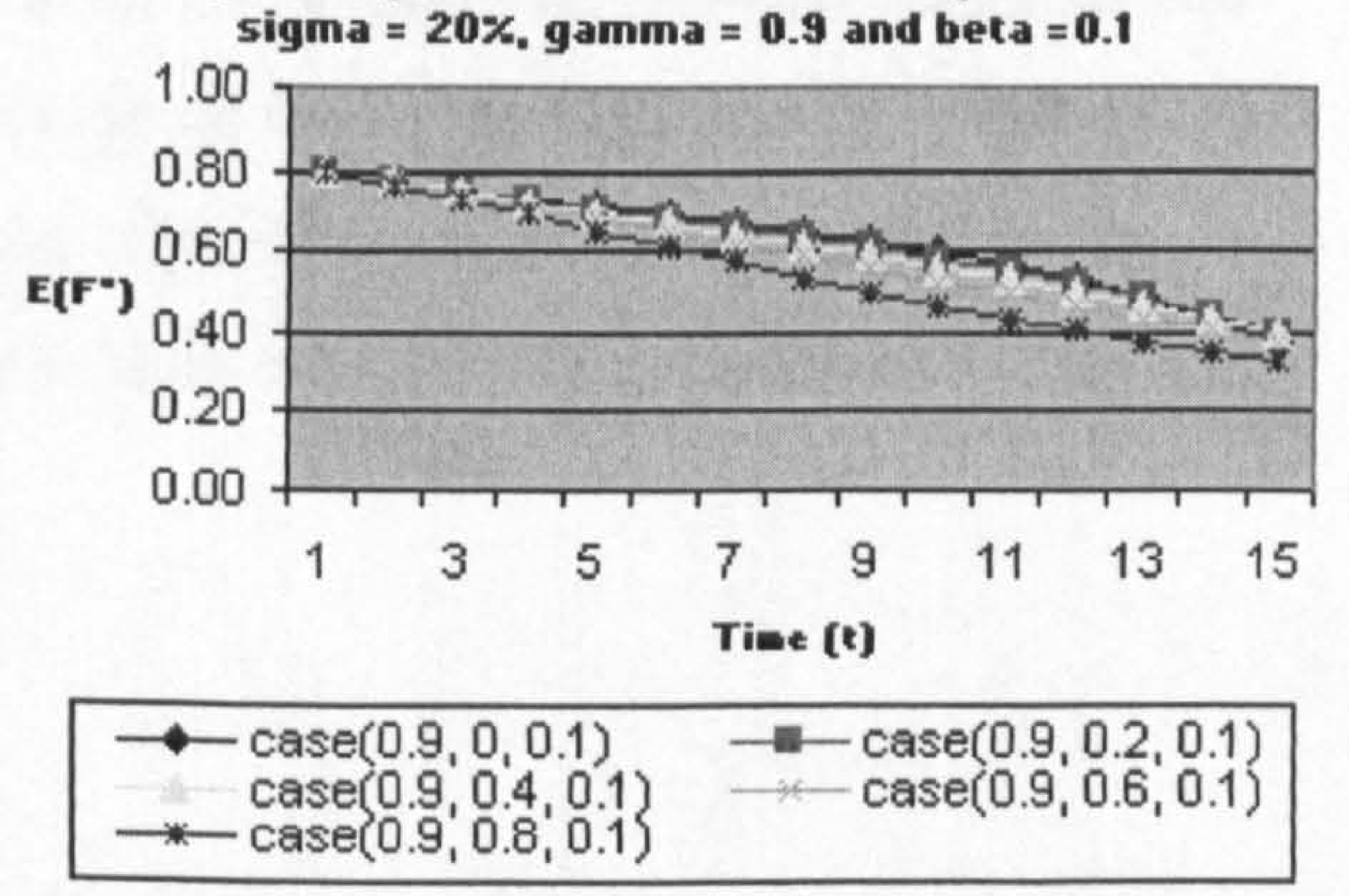


Figure 5.55
The expected optimal fund level under RIPI (0.8, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.9 and beta =0.1



Comparing these results with the ones under the CIPI model, we find that the behaviour of $E(F_t^*)$ is similar under both scenarios. However, under the fourth scenario in the CIPI model, it increases from one case to another with increasing α but in a narrower range. For example, $E(F_t^*)$ is around 116.5% in the case of (0.1, 0.8, 0.9) under CIPI – as shown in Figure 4.68 - instead of 120% under RIPI. Under the fifth scenario, $E(F_t^*)$ decreases in both models but to lower levels and with a wider range under RIPI. For example, it is around 76% in the case of (0.1, 0, 0.9) and drops to be around 72% in the case of (0.1, 0.8, 0.9) under RIPI, whereas it is between 77% and 76% in the same cases under CIPI as shown in Figure 4.75.

On the other hand, under the fifth scenario, $E(F_t^*)$ decreases over the control period as more importance is given to α and less to β . In other words, $E(F_t^*)$ reaches the minimum level of 33% in the case of (0.9, 0.8, 0.1) as shown in Figures 5.53, 5.54 and 5.55.

Generally, the trends of $E(C_t^*)$ under the fourth scenario are very close to each other, where it increases in the beginning of the control period and remain stable at the level of 20% in the cases of (0.1, 0.9). However, $E(C_t^*)$ moves away from the target to be around 19% when the value of α is increased to 0.8. For other cases (0.5, 0.5) and (0.9, 0.1) $E(C_t^*)$ shows a similar behaviour of becoming closer to the target when γ is increased to 0.5 and 0.9 with lower levels of α. On the other hand, under the fifth scenario, $E(C_t^*)$ increases to be around 23% with small differences among the cases of (0.1, 0.9) with increasing α. This level of $E(C_t^*)$ decreases to become closer to the target when γ is increased, apart from the cases of (0.5, 0.8, 0.5) and (0.9, 0.8, 0.1), where it deviates from the target due to the high level of α. The results of $E(C_t^*)$ under both scenarios are shown from Figure 5.56 to 5.61.

Figure 5.56
The expected optimal contribution rate under RIPI (1.2, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.1 and beta = 0.9

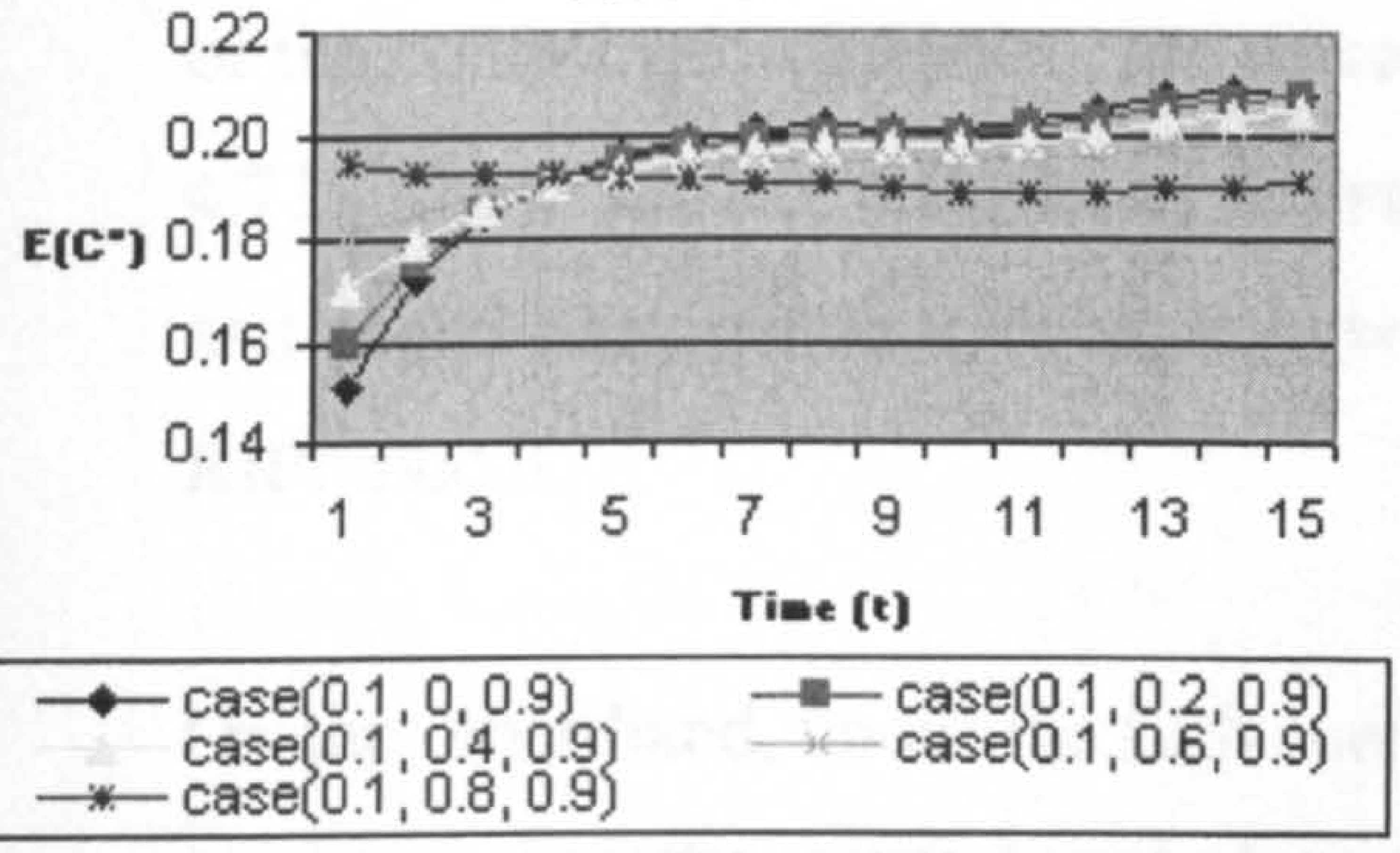


Figure 5.57
The expected optimal contribution rate under RIPI (1.2, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.5 and beta = 0.5

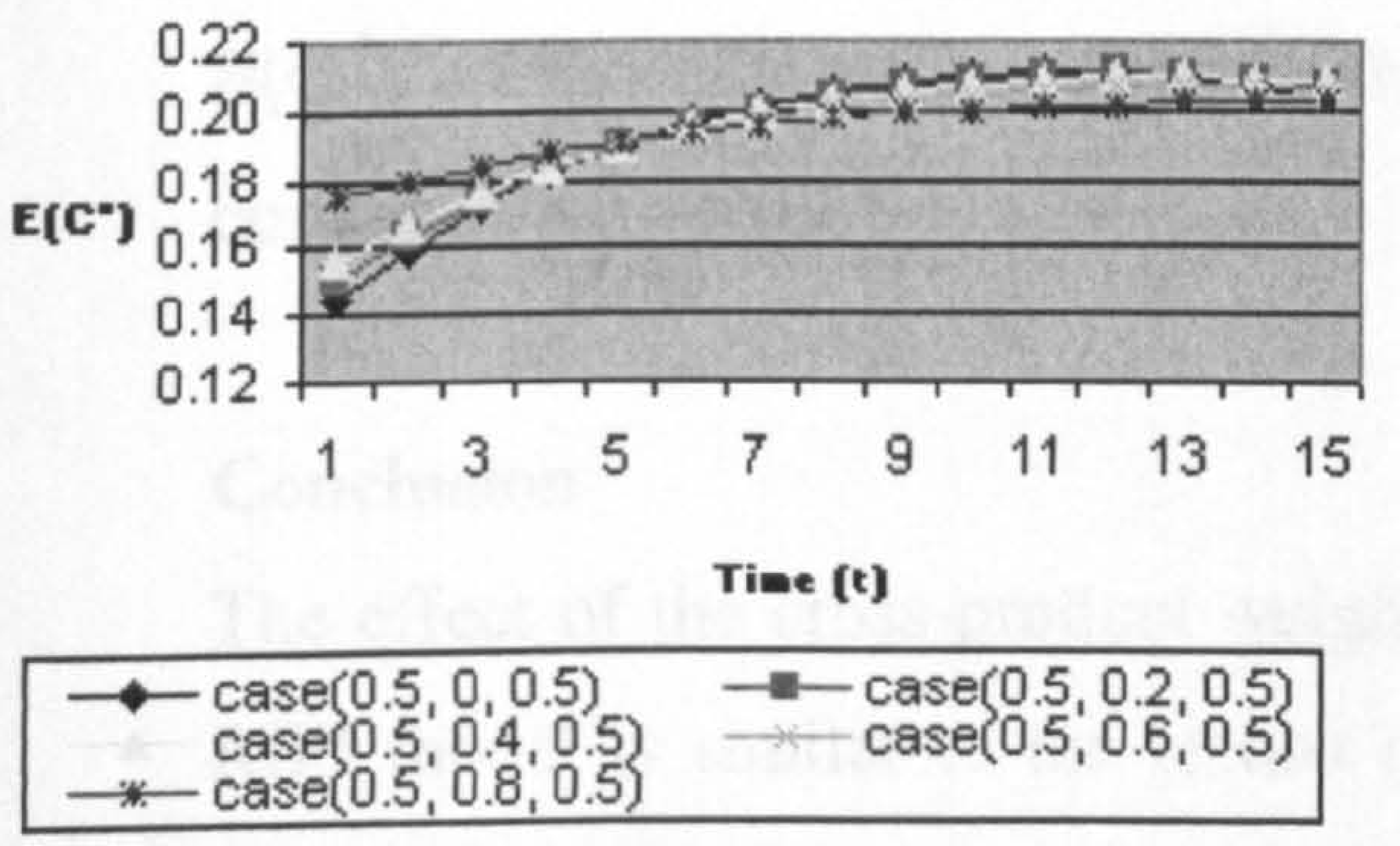


Figure 5.58
The expected optimal contribution rate under RIPI (1.2, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.9 and beta = 0.1

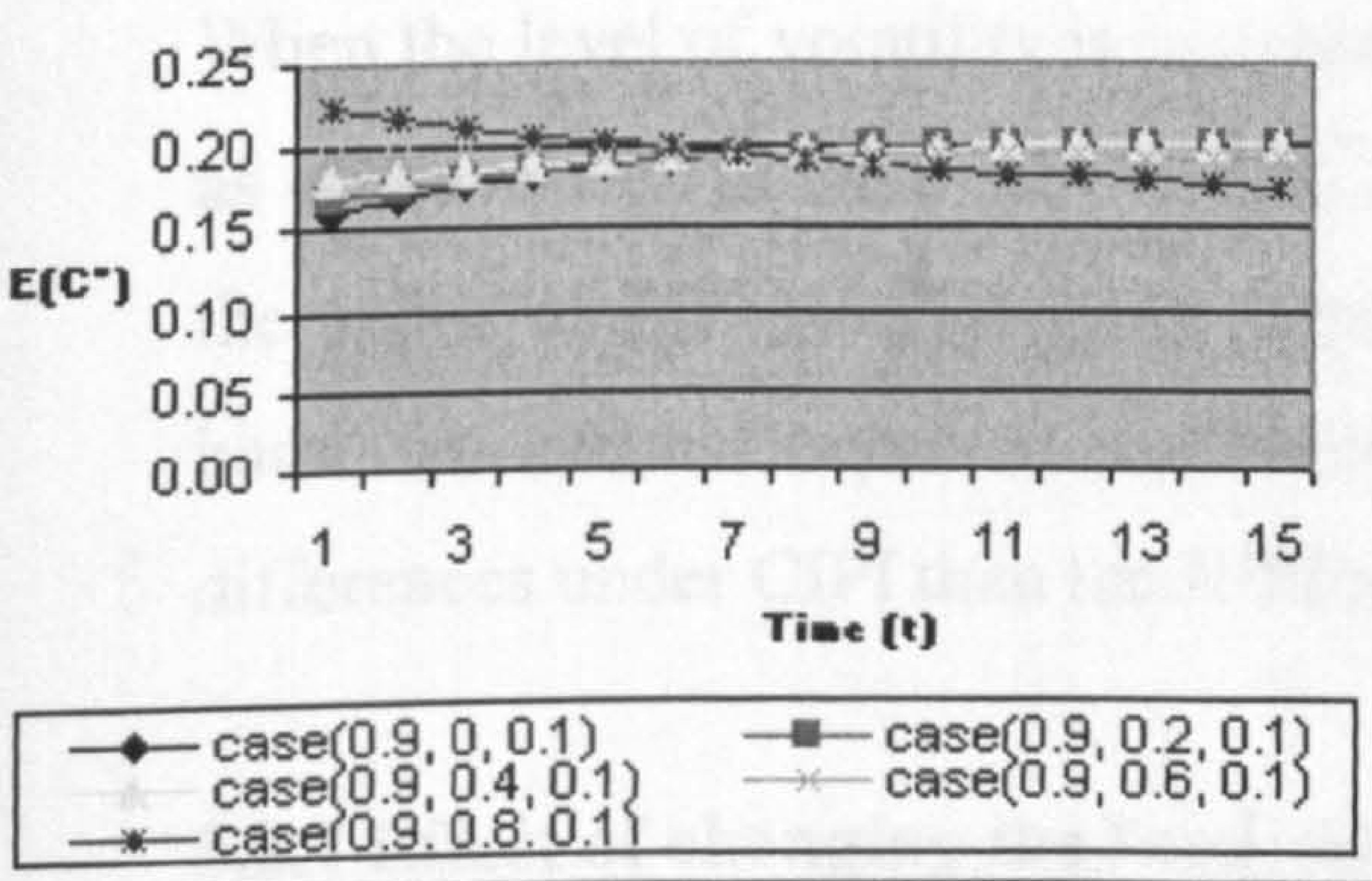


Figure 5.59
The expected optimal contribution rate under RIPI (0.8, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.1 and beta = 0.9

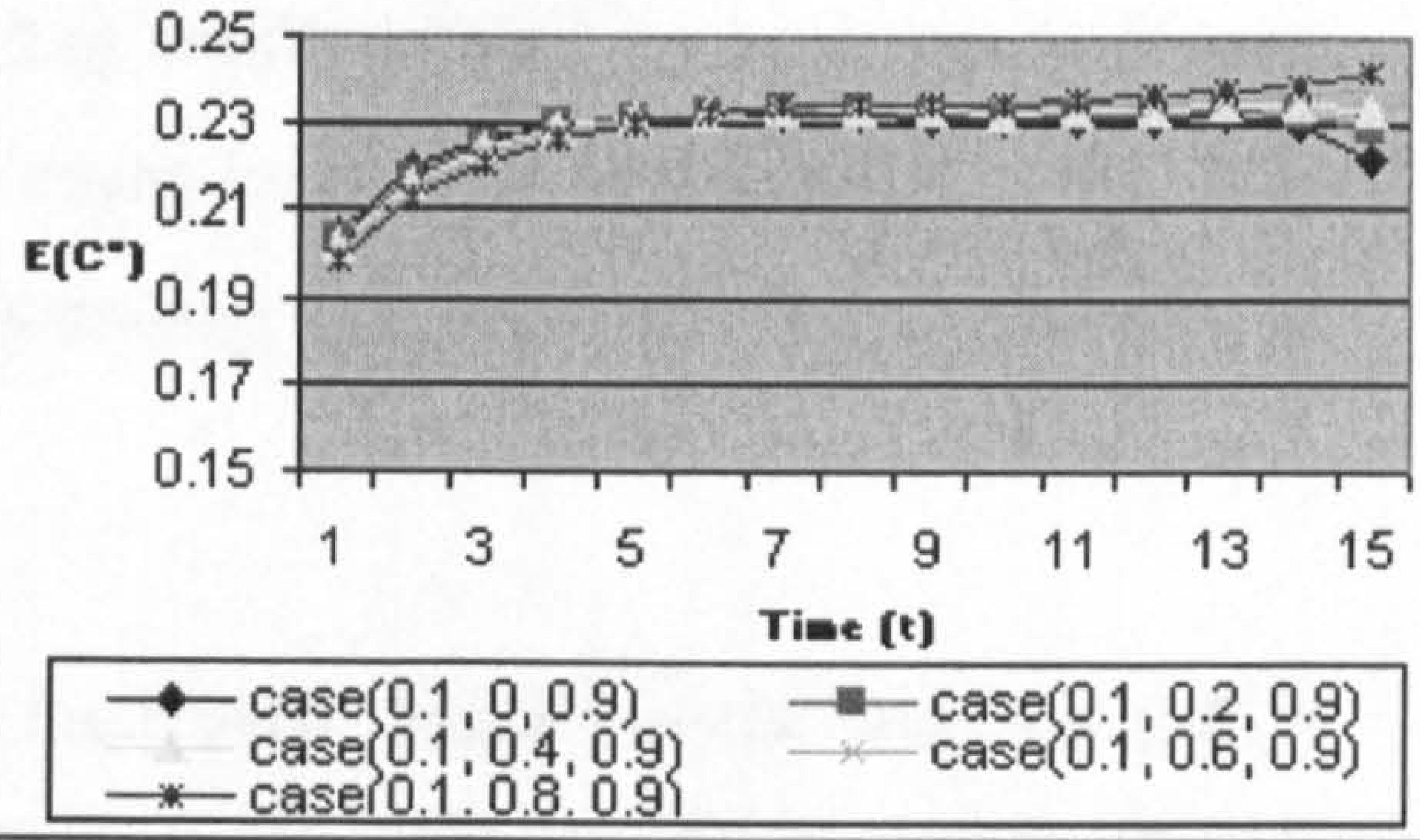


Figure 5.60
The expected optimal contribution rate under RIPI (0.8, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.5 and beta = 0.5

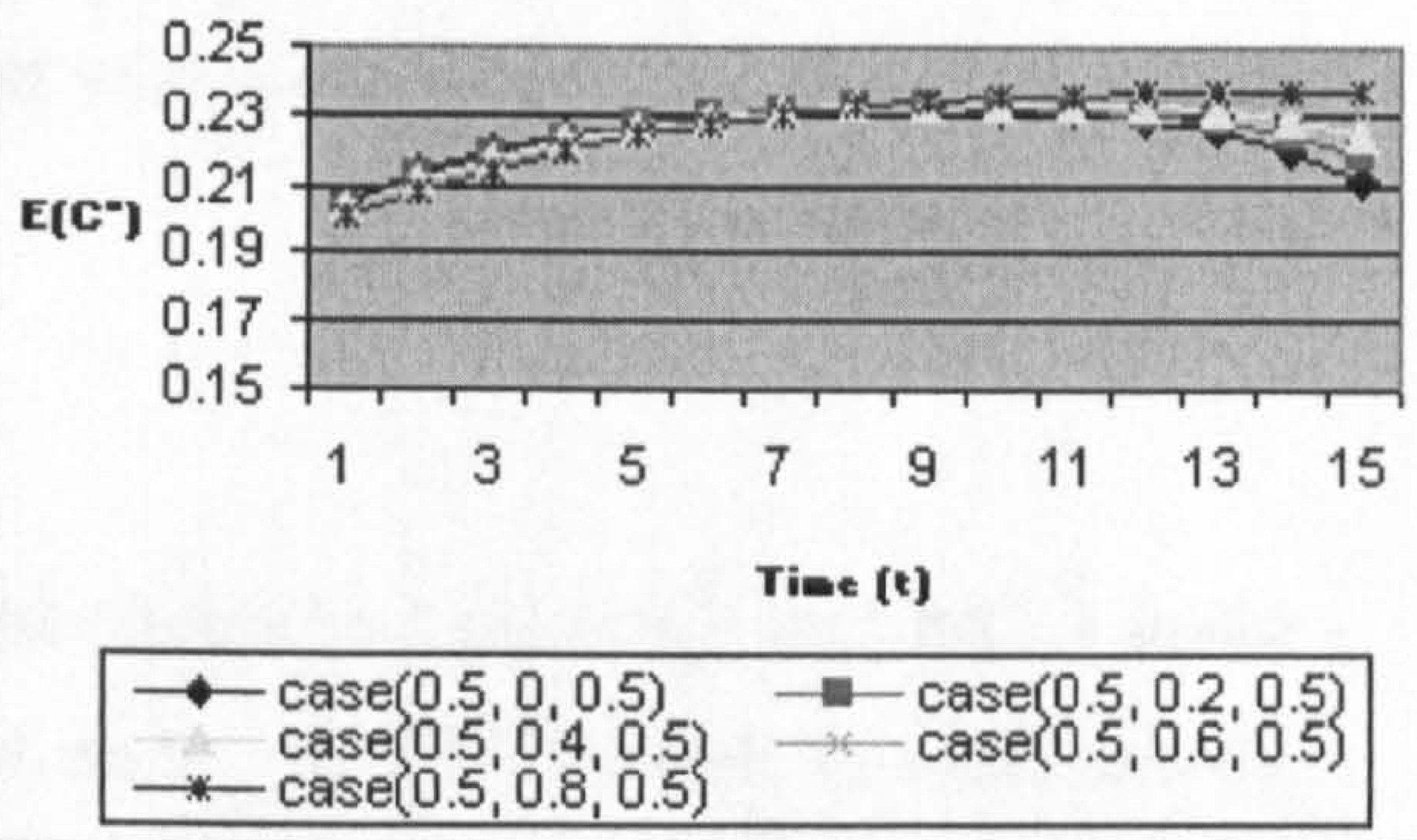
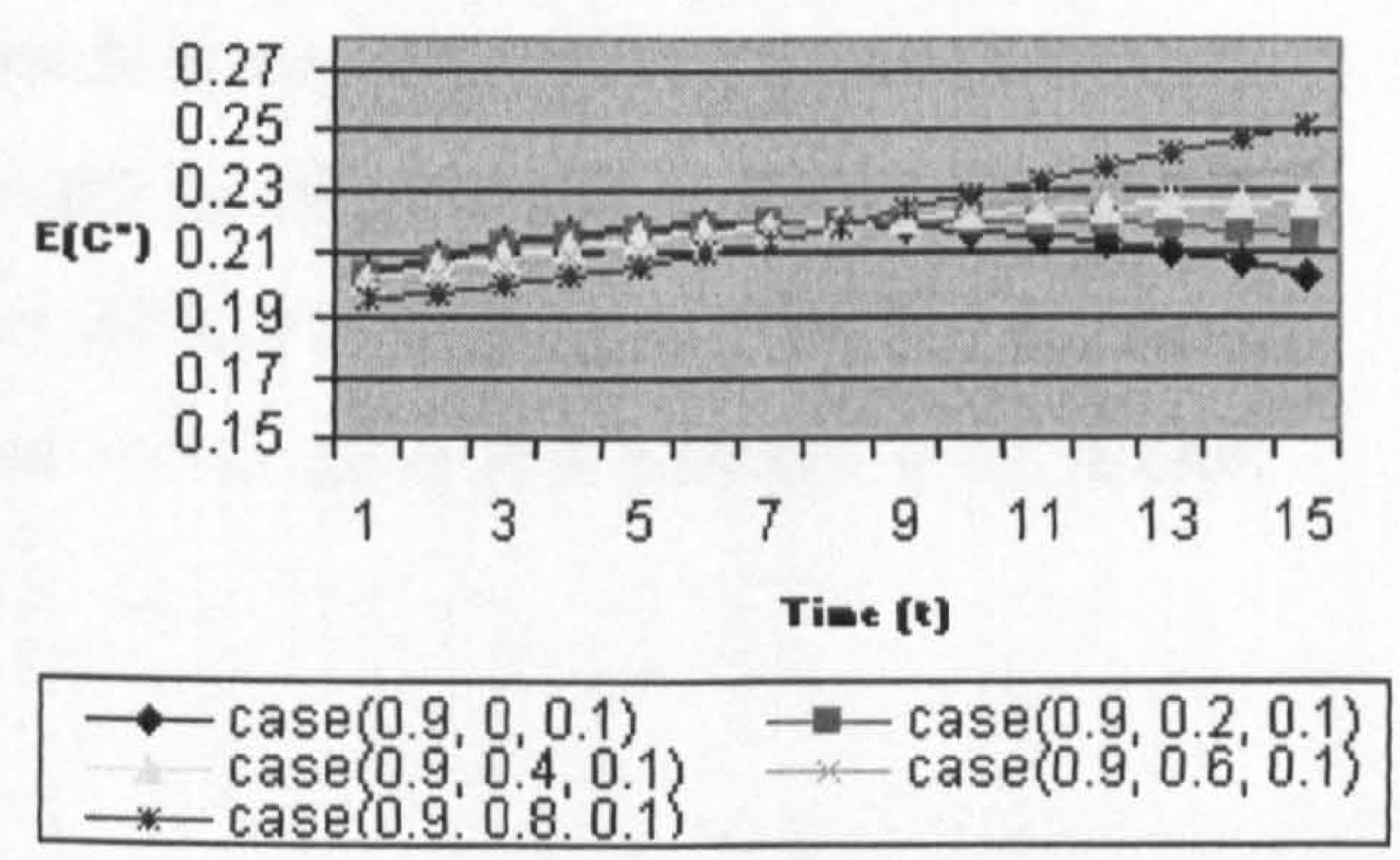


Figure 5.61
The expected optimal contribution rate under RIPI (0.8, 0.2) for different levels of alpha when sigma = 20%, gamma = 0.9 and beta = 0.1



Comparing these results with the trends of $E(C_t^*)$ under CIPI, we find that, under the fourth scenario in the cases of (0.1, 0.9) for CIPI, the trends of $E(C_t^*)$ increase over the control period around the level of 19%, and move upwards to the target by the end of the control period. Hence, the corresponding results of $E(C_t^*)$ - as shown in Figure 4.71 - show smaller differences among the cases from (0.1, 0, 0.9) to (0.1, 0.8, 0.9) and more homogeneous behaviour with increasing α compared with the results in RIPI model.

On the other hand, under the fifth scenario, the results reveal that the levels of $E(C_t^*)$ increase over the control period and to be around 23%. Similarly to the fourth scenario, we note that the levels of $E(C_t^*)$ are closer to each other with small differences when the parameter α is increased compared with the RIPI results. In the cases of (0.5, 0.5) and (0.9, 0.1), similar conclusions are reached where the trends of $E(C_t^*)$ are more stable and closer to the target with a narrower range among the cases compared with RIPI.

Conclusion

The effect of the cross-product weighting risk factor (α) on $E(F_t^*)$ and $E(C_t^*)$ under RIPI model is similar to the results reached under the CIPI model. In other words, greater deviations from the target are generally observed for the different trends of $E(F_t^*)$ and $E(C_t^*)$, when the value of α is increased, under RIPI model.

When the level of volatility is increased, it leads to marked departures from the target, as we have seen in the trends of $E(F_t^*)$ under the fourth and fifth scenarios. Moreover, the trends of $E(F_t^*)$ are wider in range under RIPI compared with CIPI. On the other hand, the optimal expected contribution rate shows more stable trends with smaller differences under CIPI than the RIPI model.

5.2.7 Effect of changing the fund ratio η

Finally, we consider studying the effect of changing the parameter η on $E(F_t^*)$ and $E(C_t^*)$ under the five different scenarios. Hence, we allow the fund ratio in RIPI model to change from 100% to 90% and 110% respectively. we examine the different cases of RIPI model – without any comparison with CIPI cases - as the parameter η exists only in RIPI model.

Here, It is important to mention that the full analysis of some parts in this subsection is available but is not shown to save space and avoid the repetition of interpretation of the results.

(a) Default scenario $FT_t = 1, CT_t = 0.2$

Under this scenario, we study the effect of changing η on the cases when $\alpha = 0$ and $\alpha \neq 0$.

Firstly, considering the cases when $\alpha = 0$ and $\sigma = 5\%$, for example, (0.1, 0, 0.9), $E(F_t^*)$ decreases to lower level around 90% over the control period when $\eta = 90\%$, compared with the level of 100% when η is increased to 100%. Similarly, it moves up slightly to 110% over the control period when $\eta = 110\%$ whereas it remains around the target when the fund ratio is equal to 100%. Table 5.6 shows the results.

Table 5.6

The expected optimal fund levels with different fund ratios in the case of (0.1, 0, 0.9) when $\sigma = 5\%$

t	η = 90%	η = 100%	η = 110%
1	100.00%	100.00%	100.00%
2	94.99%	99.88%	104.12%
3	92.45%	99.80%	106.41%
4	91.20%	99.76%	107.72%
5	90.59%	99.76%	108.47%
6	90.25%	99.73%	108.86%
7	90.10%	99.73%	109.11%
8	90.02%	99.72%	109.24%
9	89.99%	99.74%	109.34%
10	89.96%	99.72%	109.39%
11	89.89%	99.67%	109.38%
12	89.85%	99.67%	109.45%
13	89.69%	99.55%	109.43%
14	89.44%	99.42%	109.48%
15	88.94%	99.15%	109.53%

Decreasing the weighting risk factor of the solvency risk to 0.5 and 0.1 leads $E(F_t^*)$ to have lower values compared with the ones in the case of (0.1, 0, 0.9). Hence, it moves further away from the target when $\eta = 90\%$, 100% and 110%. Furthermore, when the value of σ is increased to 20%, $E(F_t^*)$ has even lower values than those obtained when $\sigma = 5\%$. For example, in the case of (0.1, 0, 0.9), $E(F_t^*)$ decreases to reach 84.3%, 93.7% when η = 90% and 100% respectively at the final T, whereas $E(F_T^*)$ = 103.2% when the fund ratio is increased to 110%.

The expected optimal contribution rate is also affected by changing the fund ratio. We examine this effect by studying the same cases used in analyzing the behaviour of $E(F_t^*)$. In the case of (0.1, 0, 0.9) and σ = 5%, we note that $E(C_t^*)$ becomes closer to the target when η is increased to 110%, compared with those when η = 90% and 100%. When more importance is given to the weighting risk factor of the contribution rate γ, $E(C_t^*)$ moves more closer to the target for the different levels of fund ratio. This can be clearly seen from Figures 5.62, 5.63, 5.64. Increasing the level of volatility to 20%, in the cases of (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0, 0.1) with changing η, leads to similar behaviours of $E(C_t^*)$, but further away from the target. Figure 5.65 show the different levels of $E(C_t^*)$ with changing η in the case of (0.9, 0, 0.1), this can be compared with Figure 5.64 to see the effect of increasing σ to 20%.

Figure 5.62

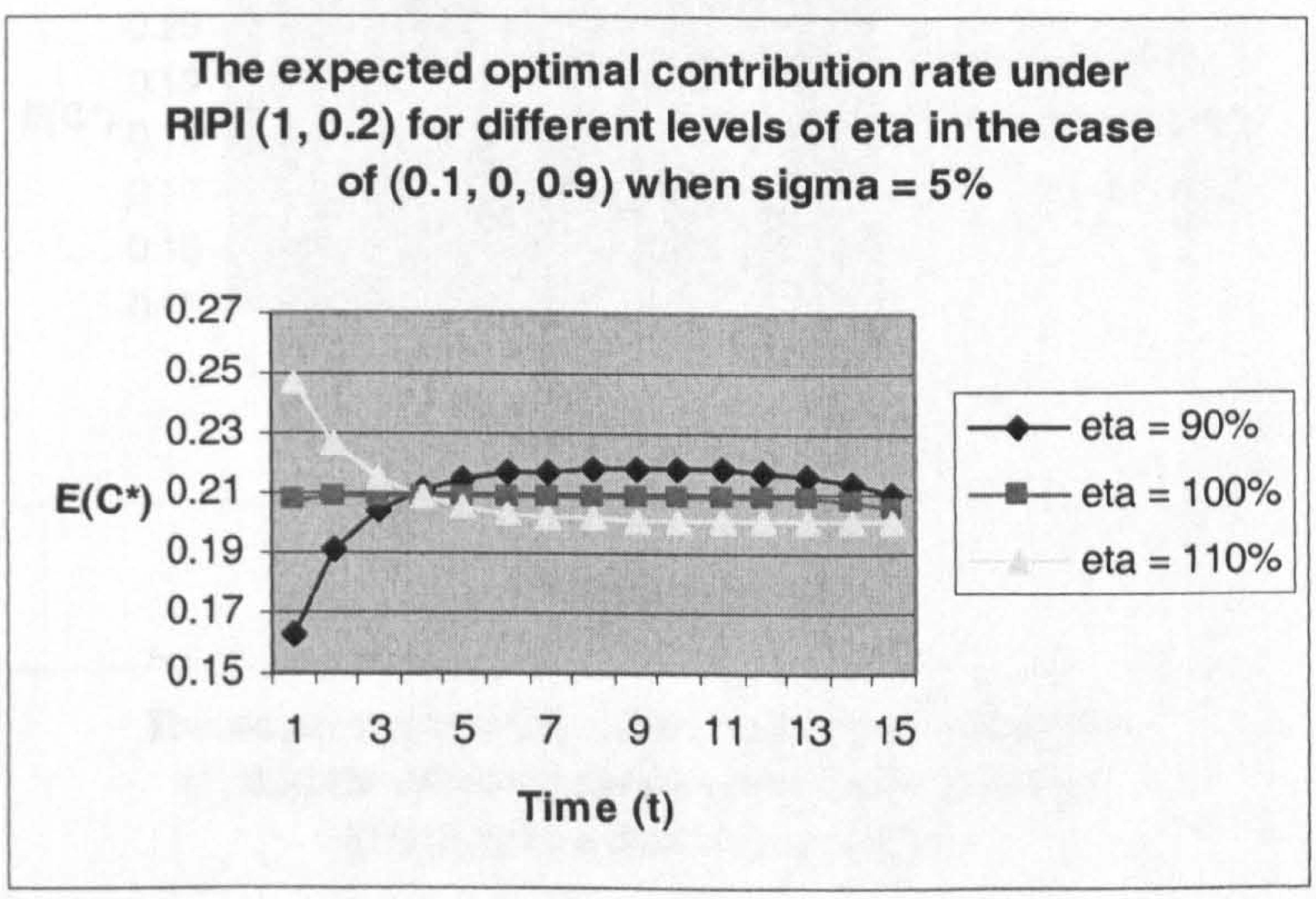


Figure 5.63

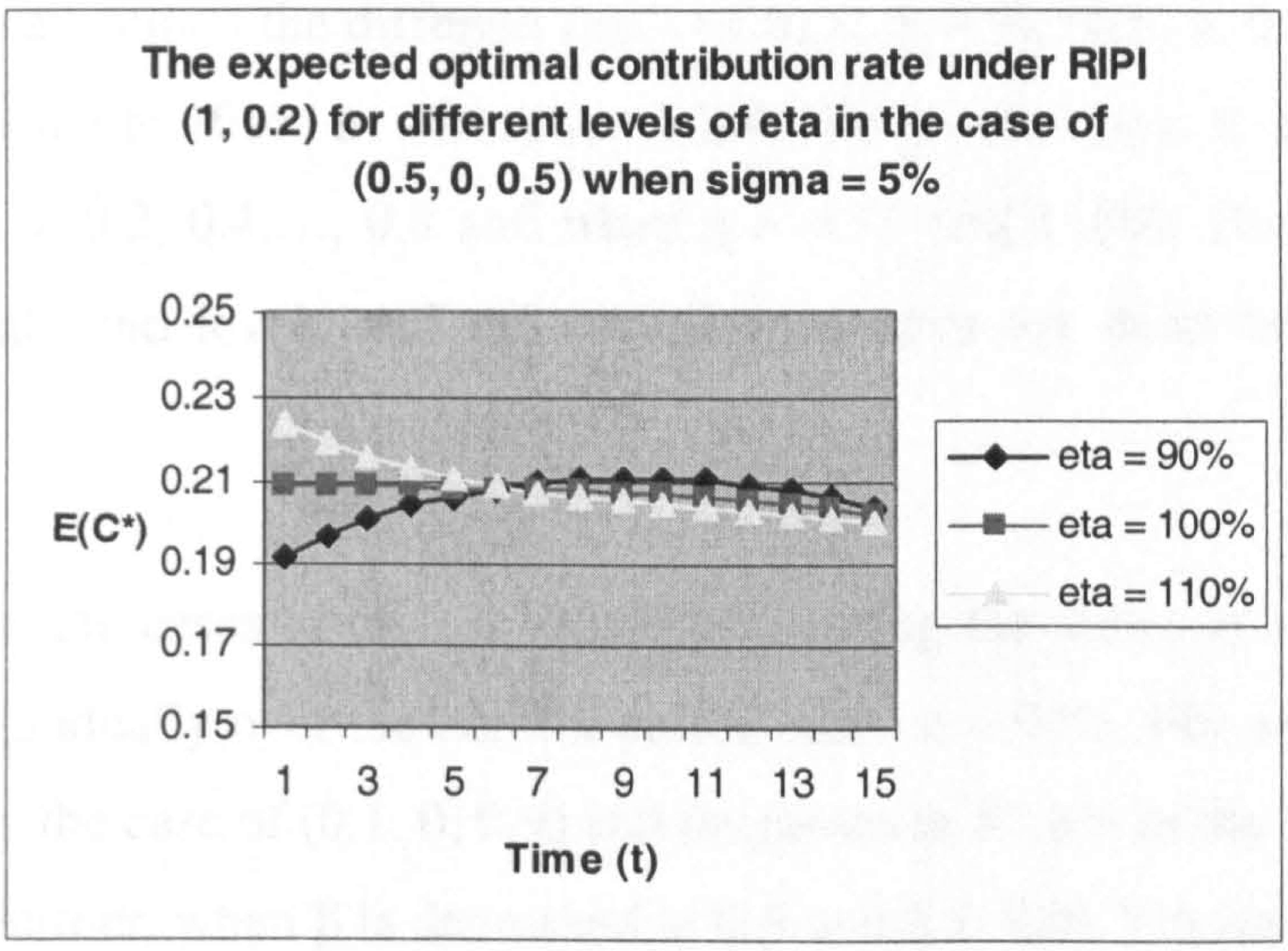


Figure 5.64

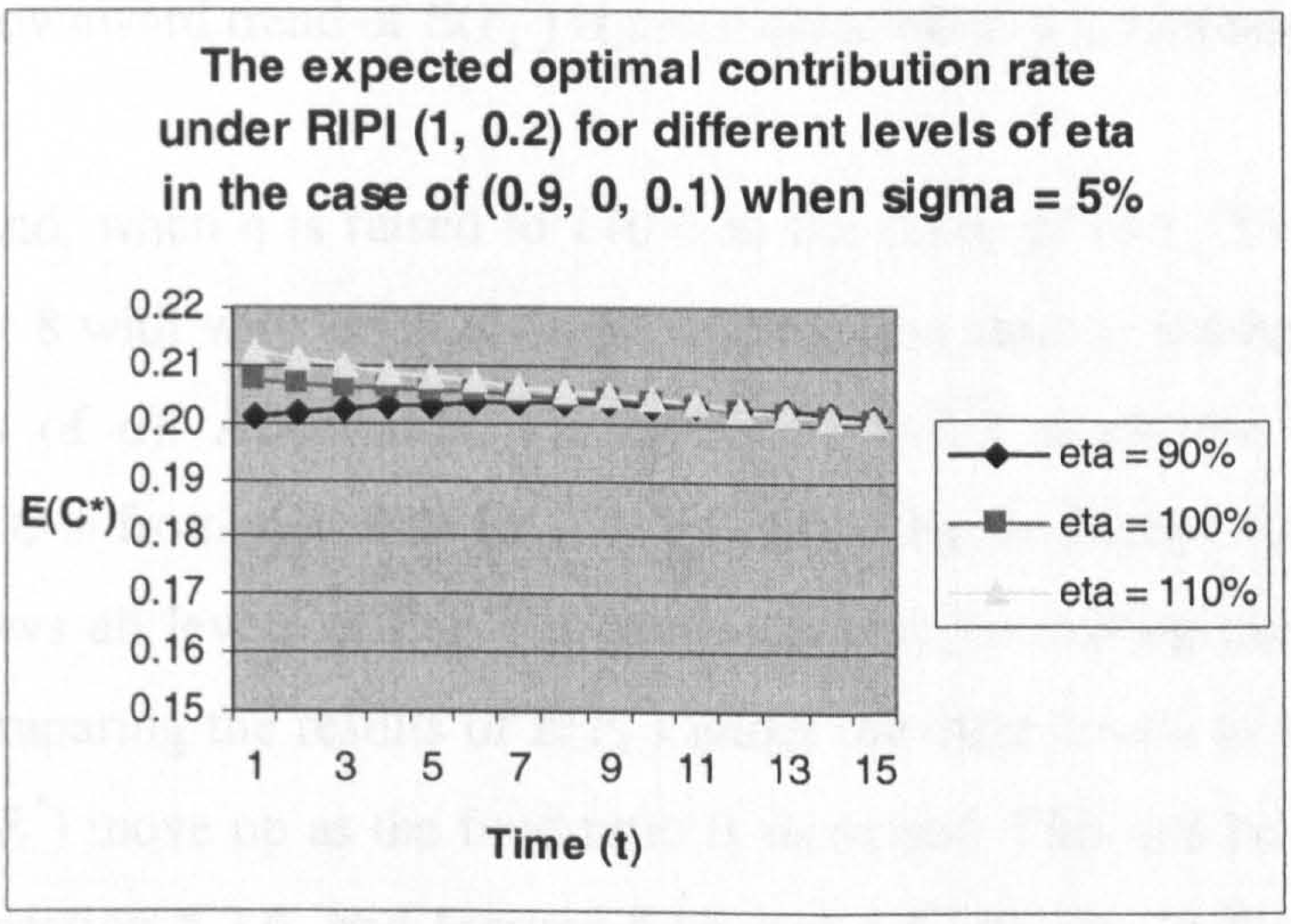
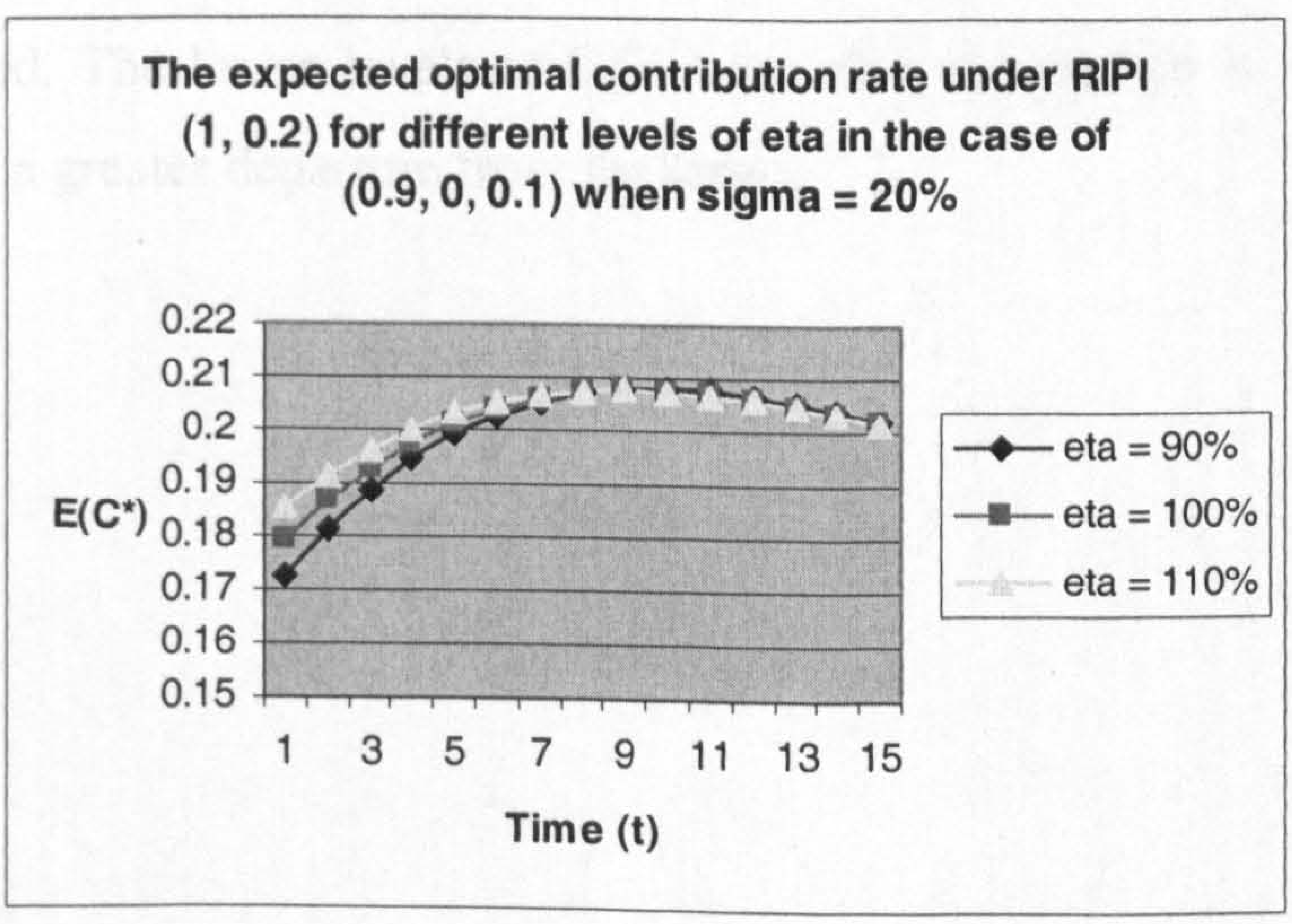


Figure 5.65



The second part of our analysis in this sub-subsection is to study the effect of η when $\alpha \neq 0$. We have examined the different cases of (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.9, 0, 0.1) when η is equal to 100% in subsection 5.2.6. We proceed now to study the same cases when α = 0, 0.2, 0.4,..., 0.8 and when η = 90% and 110%. The results of the expected optimal fund levels and the contribution rates are described in the next paragraphs.

When σ = 5% in the cases of (0.1, 0.9) with increasing the value of α to 0.8, $E(F_t^*)$ levels decrease gradually over the control period when η = 90%. For example, $E(F_T^*)$ reaches 88.9% in the case of (0.1, 0, 0.9) and decreases to 85.8% in the case of (0.1, 0.8, 0.9). Further, when β is decreased to 0.5 and 0.1, $E(F_t^*)$ moves down further as α is increased, it reaches the lowest level of approximately 70% in the case of (0.9, 0.8, 0.1). This downward trend of $E(F_t^*)$ is more clear when σ is increased to 20%.

On the other hand, when η is raised to 110% in the cases of (0.1, 0.9), $E(F_t^*)$ levels increase till year 8 with very small differences from one case to another (for the five different values of α). Afterwards, the levels of $E(F_t^*)$ reach the same level of approximately109% from year 9 to 15. This is caused by the higher value of the fund ratio which allows all levels of $E(F_t^*)$ to move up with increasing the value of α. In other words, comparing the results of $E(F_t^*)$ under the three levels of η indicates that the levels of $E(F_t^*)$ move up as the fund ratio is increased. This can be observed from Figure 5.26 in section 5.2.6, and Figures 5.66 and 5.67 below. In the cases of (0.5, 0.5) and (0.9, 0.1) when α is increased to 0.8, $E(F_t^*)$ attains lower levels as the value of β is decreased. The lower levels of $E(F_t^*)$ are also obtained as σ is increased to 20%, indicating a greater departure from the target.

Figure 5.66

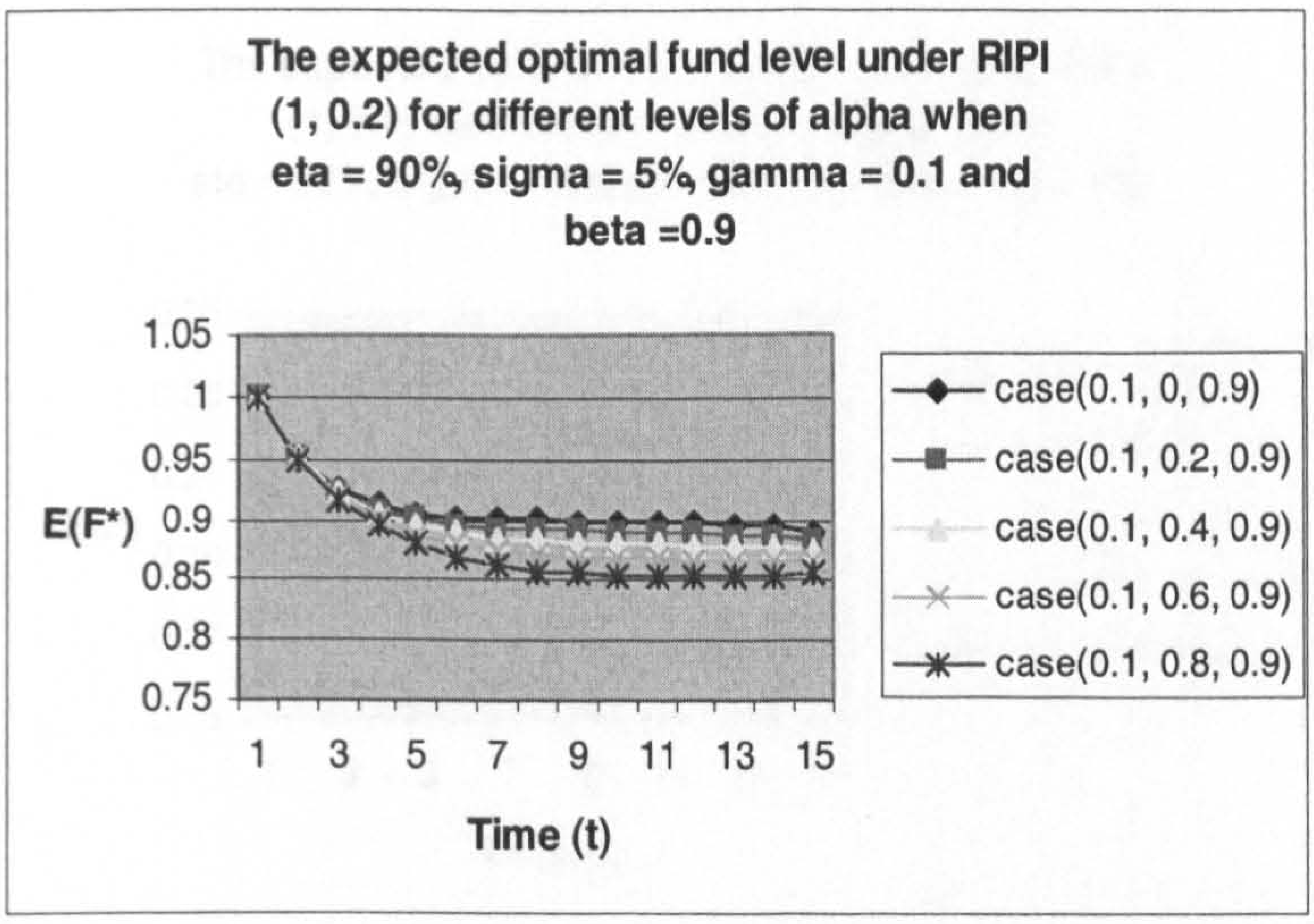
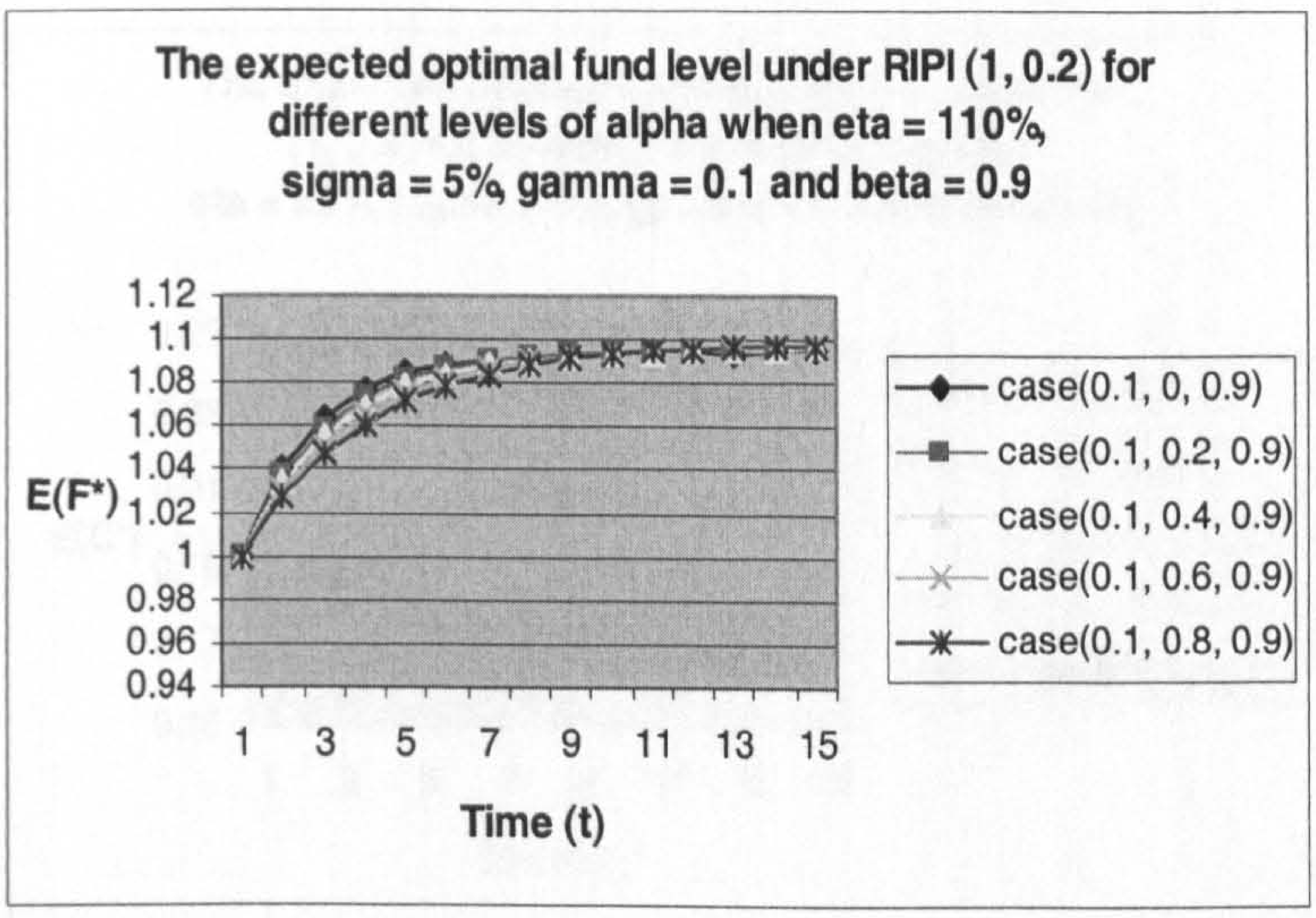


Figure 5.67



When $\sigma = 5\%$ and $\eta = 90\%$, we examine the behaviour of $E(C_t^*)$ for the cases of (0.1, 0.9) with increasing α. We find that $E(C_t^*)$ levels start from lower levels around 16% and increase over the control period. The differences are small between the cases although they move further away from the target as the value of α is increased, as shown in Figure 5.68. The cases of (0.5, 0.5) show an increasing trend of $E(C_t^*)$ over the control period, but in lower levels to get closer to the target, compared with the previous cases of (0.1, 0.9). This is due to the higher value of $\gamma = 0.5$. Further, in the cases of (0.9, 0.1) with increasing α from 0 to 0.8, $E(C_t^*)$ remains around the target of 20% when the value of α is small ($\alpha = 0, 0.2, 0.4$). However, in the cases of (0.9, 0.6, 0.1) and (0.9, 0.8, 0.1), we note that $E(C_t^*)$ tends to deviate away from the target as shown in Figure 5.69.

Figure 5.68

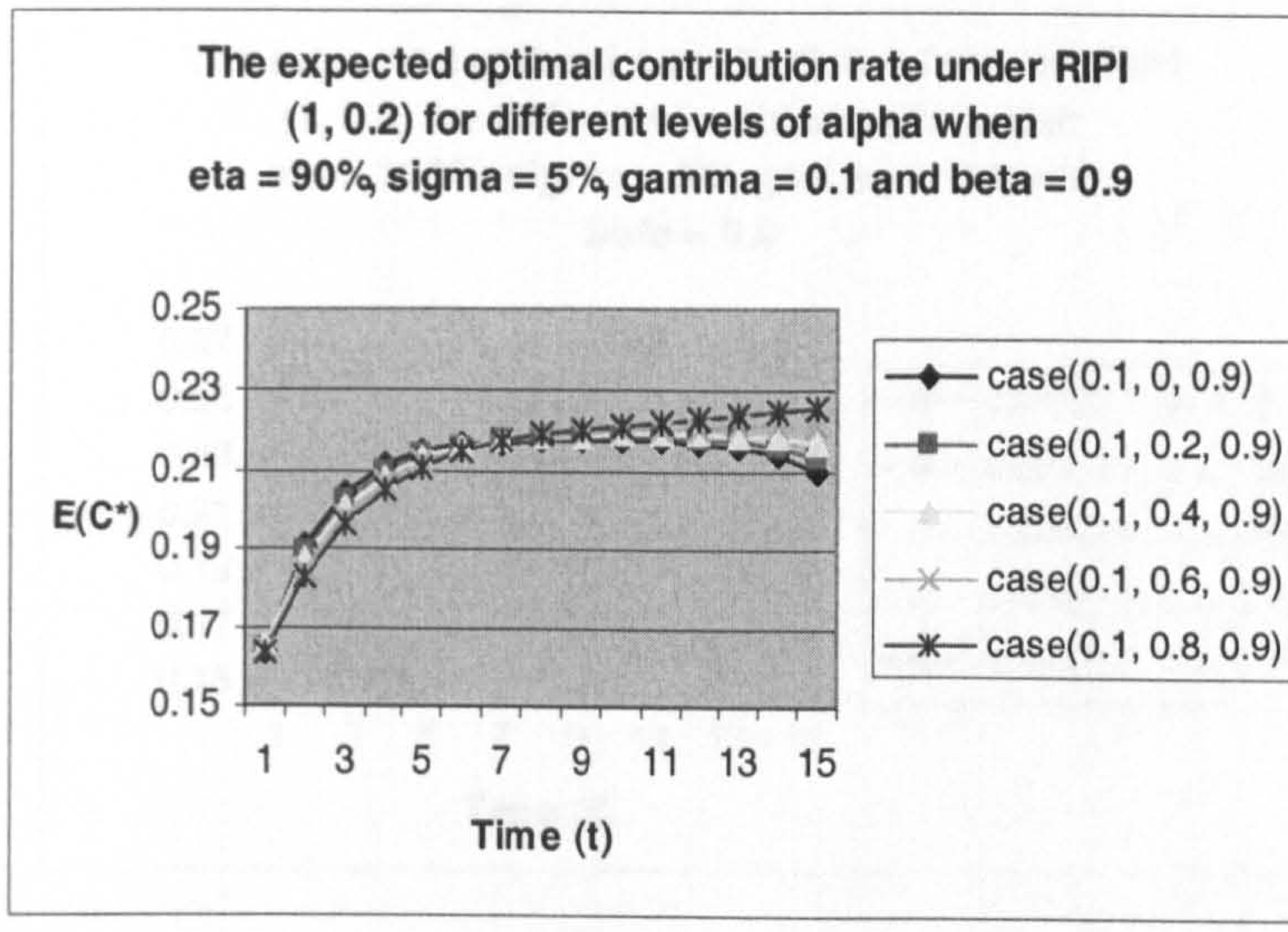
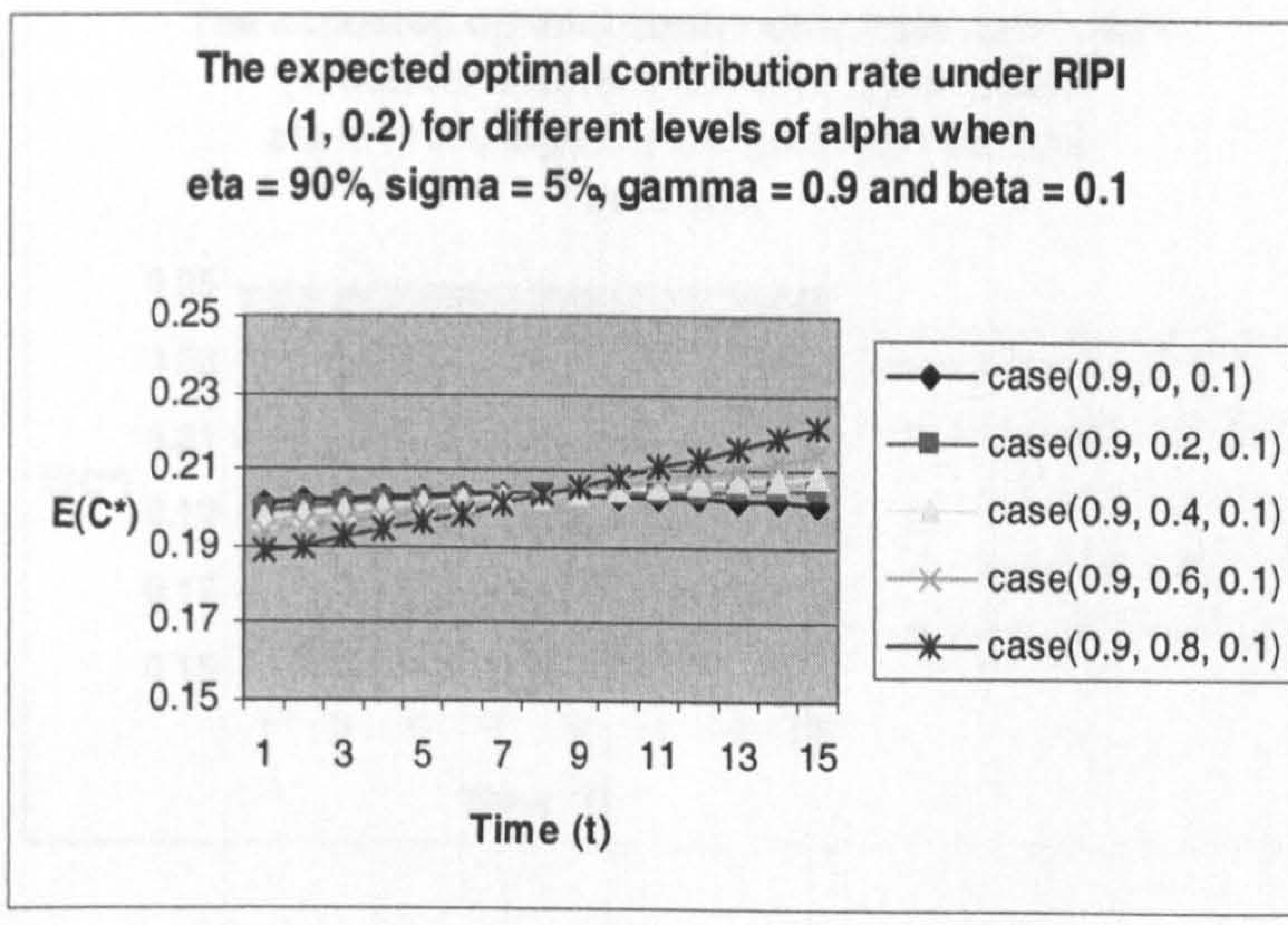


Figure 5.69



In contrast, increasing the fund ratio to 110% for the cases of (0.1, 0.9) leads $E(C_t^*)$ to decrease gradually from around 24% towards the target level of 20% as shown in Figure 5.69. When the value of γ is increased to 0.5 and 0.9, $E(C_t^*)$ moves closer to the target to be around 20% over the control period for the cases of (0.9, 0.1). Further, in the cases of (0.9, 0.6, 0.1) and (0.9, 0.8, 0.1), we note that $E(C_t^*)$ levels are closer to the target compared with those when η = 90%. This is due to the increased value of the fund ratio. Figures 5.70 and 5.71 show the difference in the levels of $E(C_t^*)$ for the cases of (0.1, 0.9) and (0.9, 0.1) when α is increased and η = 110%.

Figure 5.70

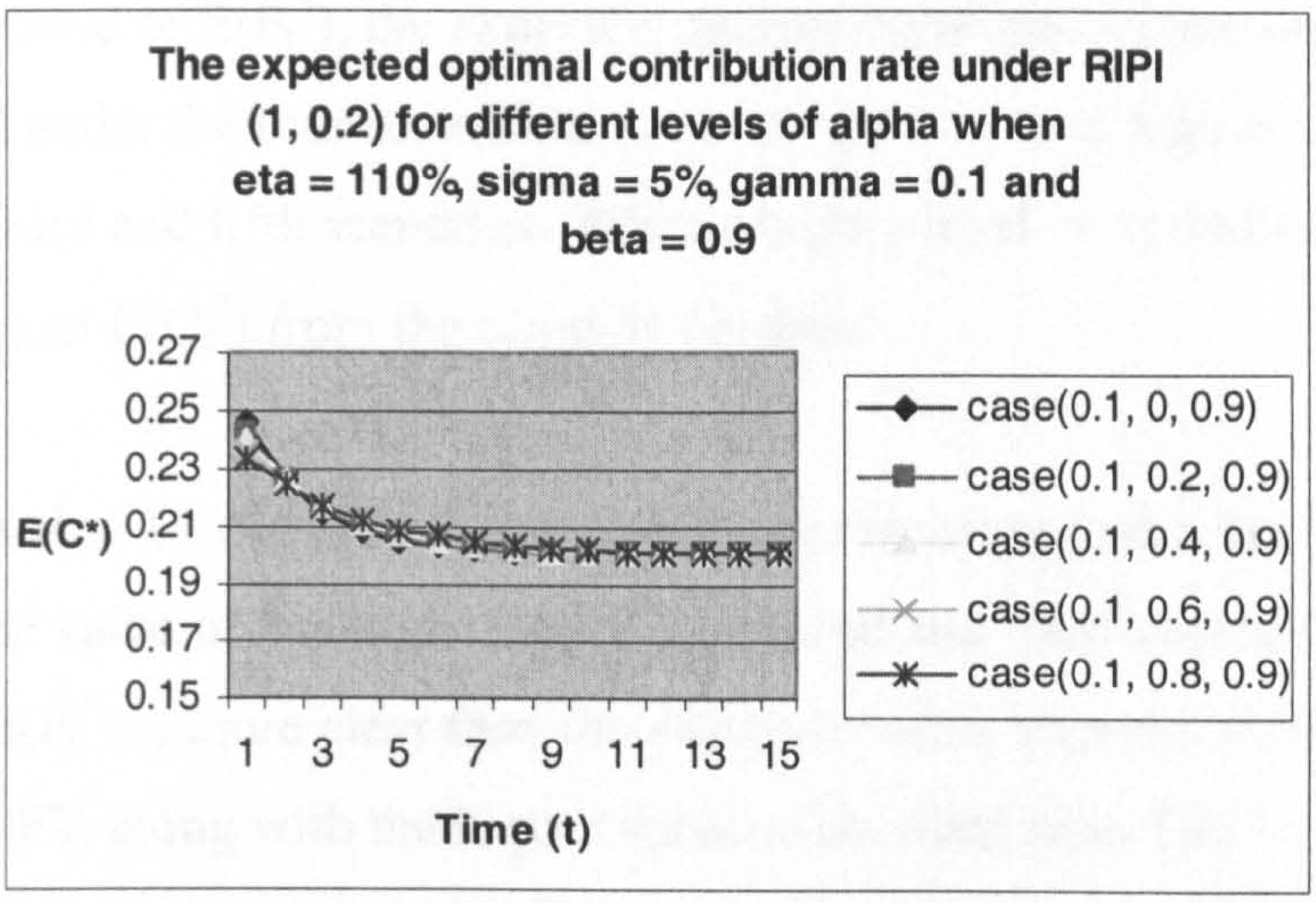
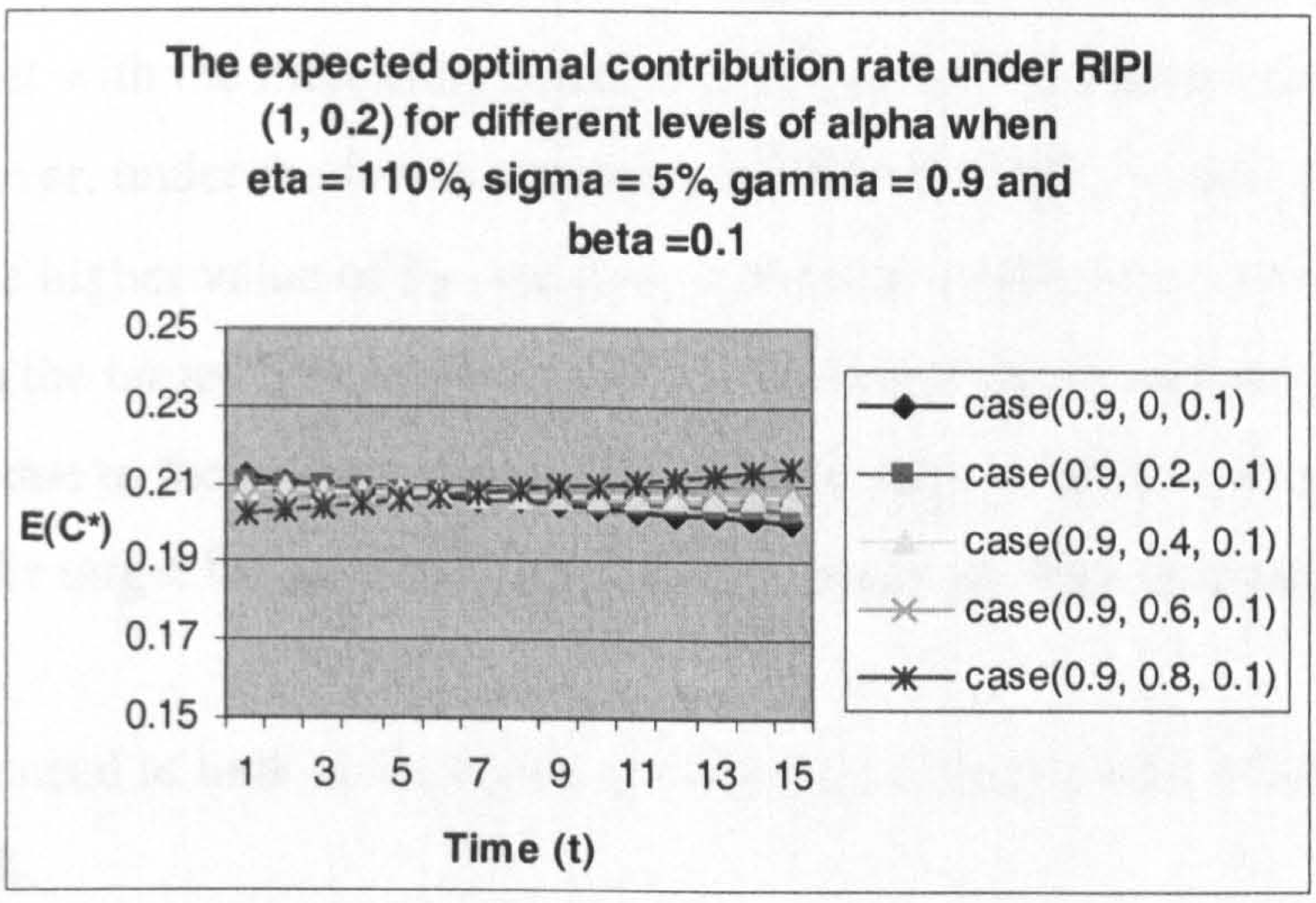


Figure 5.71



(b) Other scenarios

Here, we follow the same approach applied in section 5.2.7(a), where we test the effect on $E(F_t^*)$ and $E(C_t^*)$ when η is increased from 90% to 110%, $\alpha = 0$ and $\alpha \neq 0$ for the four scenarios. We summarize the results reached for $E(F_t^*)$ and $E(C_t^*)$ due to the similarities between the second and the fourth scenarios from one side, and the third and fifth scenarios from the other side(bearing in mind that all the results are available but are not shown for the purpose of avoiding repetition).

We start with the effect of η =90% when $\alpha = 0$, so that, under the four scenarios, we find that $E(F_t^*)$ decreases over the control period. This decreasing trend is more clear when the values of the contribution and fund targets are low, i.e. the third and fifth scenarios. Further, lower levels of $E(F_t^*)$ are observed as σ is increased. In response to the decreasing trend of $E(F_t^*)$, the expected optimal contribution rate increases over the control period under the four scenarios. Accordingly, we note higher levels of $E(C_t^*)$ under the third and fifth scenarios. When a higher level of volatility is applied, a greater departure of $E(C_t^*)$ from the target is obtained.

When η is increased to 110%, $E(F_t^*)$ increases above the target under the four scenarios when the value of β is high. Under the second and fourth scenarios, the upward trend of $E(F_t^*)$ is more clear than the other scenarios because of the higher values of CT_t and FT_t along with the higher value of the fund ratio The levels of $E(F_t^*)$ decreases over the control period as β is decreased. On the other hand, the levels of expected optimal contribution rate tends to decrease over the control period to get closer to the target with the increasing trend of $E(F_t^*)$ under the second, third and fifth scenarios . However, under the fourth scenario, we note that $E(C_t^*)$ starts from lower levels - due to the higher value of F_0 – so that, it increases over the control period to become closer to the target. The levels of $E(C_t^*)$ increase over the control period when $E(F_t^*)$ decreases due to the lower value of β. A higher value of σ indicate a greater departure from the target for both $E(F_t^*)$ and $E(C_t^*)$ under the four scenarios.

At this point, we need to look at the effect of changing the value of α when η changes to 90% and 110%.

When $\eta = 90\%$, we note that $E(F_t^*)$ decreases over the control period. The levels of $E(F_t^*)$ increases when α is increased from 0 to 0.8, under the second and fourth scenario. This is due to the higher level of the contribution and fund targets. However, $E(F_t^*)$ decreases as α increases under the third and fifth scenarios.

Increasing η to 110% leads the levels of $E(F_t^*)$ to be at higher levels as α is changed from 0 to 0.8, under the second and fourth scenarios, compared with those under $\eta = 90\%$. The levels of $E(F_t^*)$ decreases with the increased levels of α, under the third and fifth scenarios, but again with higher levels than those obtained when $\eta = 90\%$.

For different cases under the four scenarios, when $\eta = 90\%$ and 110%, smaller differences between the levels of $E(F_t^*)$ is observed when the value of α is low. A greater departure of $E(F_t^*)$ from the target is explained when a higher value of α is applied, this is also occurred as a result of increasing the level of volatility. from one case to another

Therefore, the expected optimal contribution rate responds to the decreasing trend of $E(F_t^*)$, when $\eta = 90\%$, and increases over the control period as α increases. Consequently, when η is increased to 110%, $E(C_t^*)$ increases to lower levels than those under $\eta = 90\%$ as the value of α is increased. The higher values of α show a wider range for the levels of $E(C_t^*)$ and greater deviation from the target. It is noted that the behaviour of $E(C_t^*)$ under the second and fourth scenarios is similar where as the results of $E(C_t^*)$ under the third and fifth scenarios are more consistent.

Conclusion

The effect of increasing η is realized on $E(F_t^*)$ and $E(C_t^*)$, under different scenarios, when $\alpha = 0$. Decreasing the value of η to 90% leads generally to lower levels of $E(F_t^*)$, consequently, $E(C_t^*)$ increases to be around the target. When η is increased to 110%, higher levels of $E(F_t^*)$ are obtained, hence, $E(C_t^*)$ decreases to become closer to the target.

When the value of α is increased under the different scenarios, there is always a departure from the target either for $E(F_t^*)$ or $E(C_t^*)$. Higher levels of contribution rate and fund target along with the increased level of η lead to higher levels of $E(F_t^*)$. Accordingly, $E(C_t^*)$ increases to lower levels to be close to the target.

Summary

The properties of the ratio-induced performance index (RIPI) is explained in comparison with the results of applying the CIPI model, which have been reached in chapter 5. The comparison between CIPI and RIPI reveals a major feature of the behaviour of $E(F_t^*)$ and $E(C_t^*)$ over the control period, under the different scenarios. This feature is: a greater deviation from the target under RIPI compared with the CIPI model. Other features of the trends $E(F_t^*)$ and $E(C_t^*)$ are also explored and compared with CIPI under different scenarios. Further, the effect of changing the fund ratio η - which included in RIPI model - on the expected optimal fund level and contribution rate is described when $\alpha = 0$ and $\alpha \neq 0$, for the different scenarios.

The properties of RIPI and CIPI are explained through studying the behaviour of the expected optimal fund level and contribution rate, and their sensitivity to changes in the different parameters. The practical implementation of these models reveals more about their properties, and the behaviour of $E(F_t^*)$ and $E(C_t^*)$ in the real practice.

Appendix 5A

5A.1 Histograms and statistics analysis for (F_T^*) and (C_T^*)

Similarly to Appendix 4C, the histograms and statistics analysis of (F_T^*) and (C_T^*) are considered in this section for the RIPI model. The same cases under the first scenario (1, 0.2) when $\sigma = 20\%$ are applied, and the results are shown below.

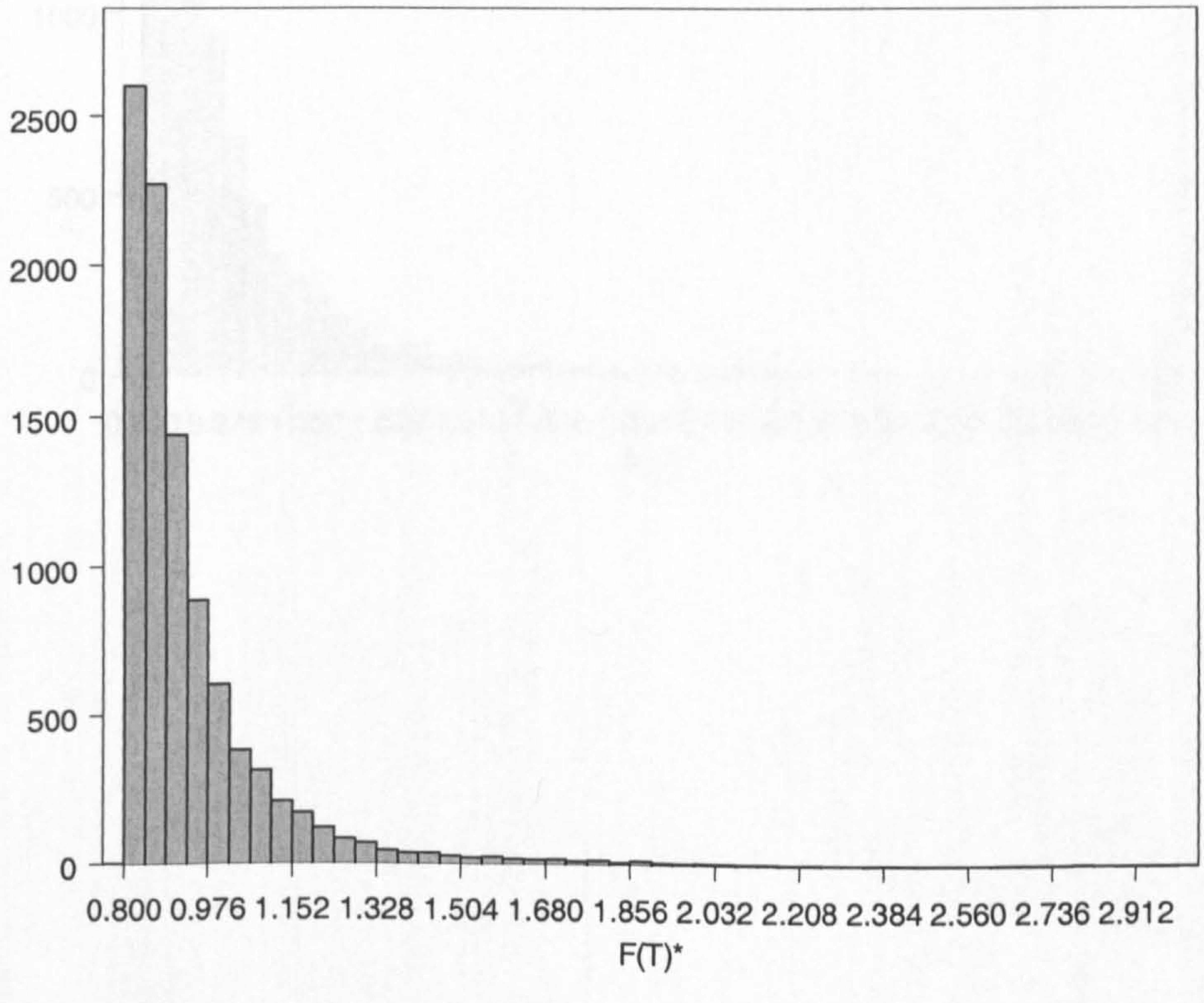
5A.1.1 The results of (F_T^*)

Case: RIPI (0.1, 0, 0.9) sigma = 0.2

Summary Statistics for data in: (0.1, 0, 0.9)

```
                 F(T)*
      Min:     0.7718250
  1st Qu.:     0.8343303
     Mean:     0.9370717
   Median:     0.8795399
  3rd Qu.:     0.9630345
      Max:     7.3868560
 Total N: 10000.0000000
    NA's :     0.0000000
 Std Dev.:     0.2199655
```

Figure 5A.1
Histogram of (F_T^*) in the case of (0.1, 0, 0.9) when σ = 20%

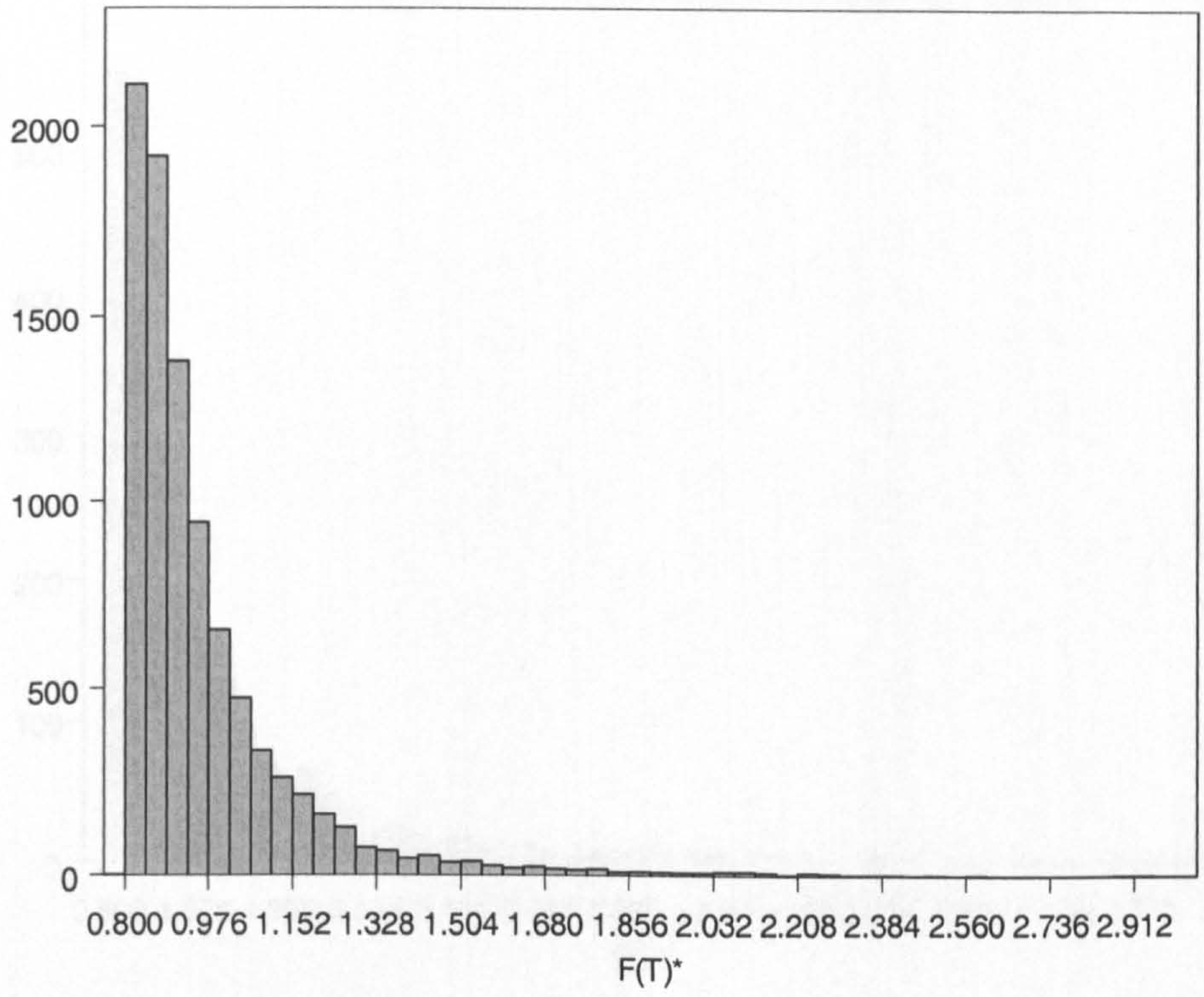


Case: RIPI (0.1, 0.8, 0.9) sigma = 0.2

```
*** Summary Statistics for data in:  (0.1, 0.8, 0.9) ***

                  F(T)*
      Min:      0.7487880
  1st Qu.:      0.8339960
     Mean:      0.9555811
   Median:      0.8900536
  3rd Qu.:      0.9913554
      Max:      8.1035050
  Total N: 10000.0000000
    NA's :      0.0000000
 Std Dev.:      0.2533422
```

Figure 5A.2
Histogram of (F_T^*) in the case of (0.1, 0.8, 0.9) when σ = 20%

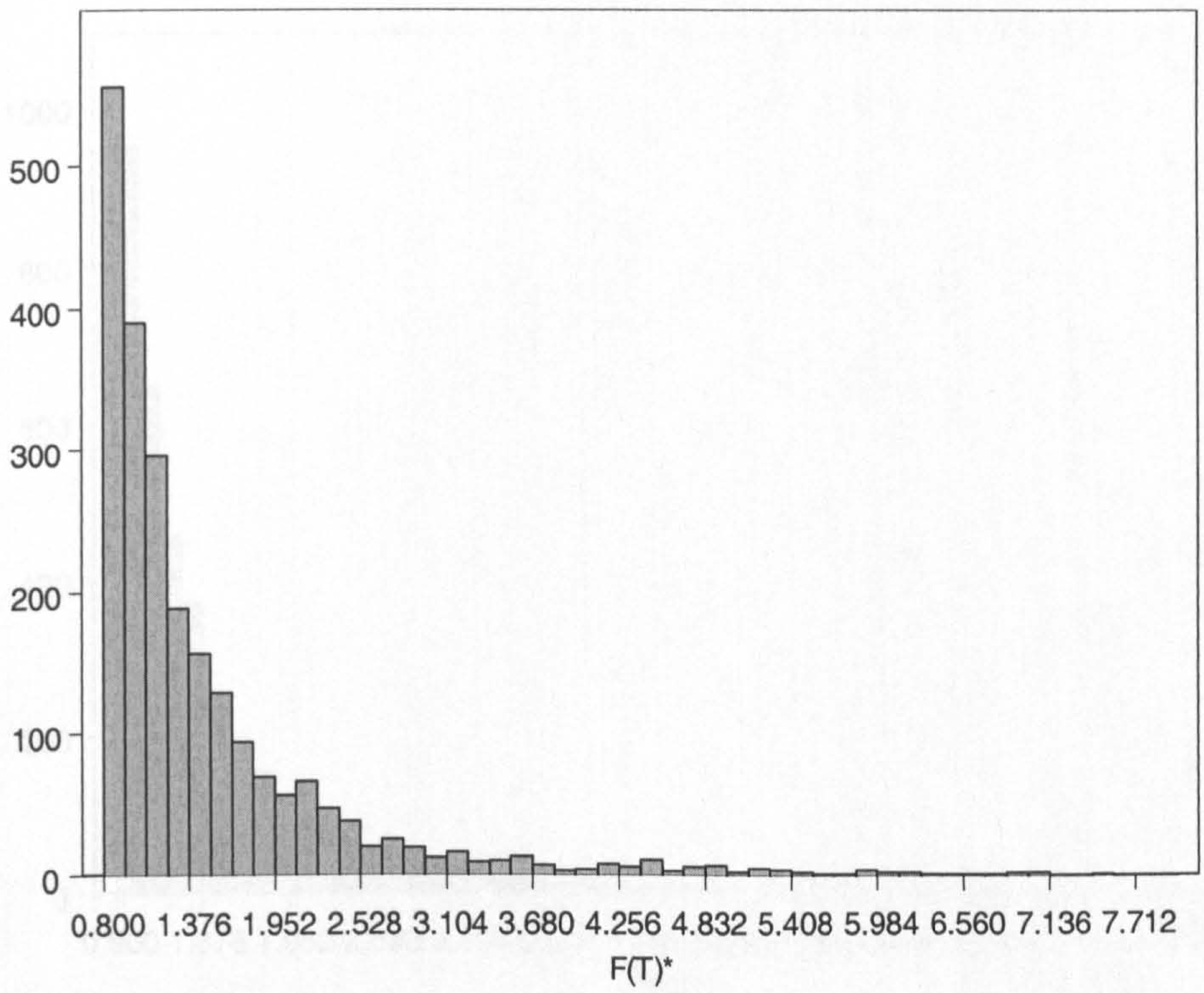


Case: RIPI (0.9, 0, 0.1) sigma = 0.2

Summary Statistics for data in: (0.9, 0, 0.1)

	F(T)*
Min:	-0.0069900
1st Qu.:	0.2655109
Mean:	0.6515287
Median:	0.4403381
3rd Qu.:	0.7602803
Max:	19.6671600
Total N:	10000.0000000
NA's :	0.0000000
Std Dev.:	0.7894561

Figure 5A.3
Histogram of (F_T^*) in the case of (0.9, 0, 0.1) when σ = 20%

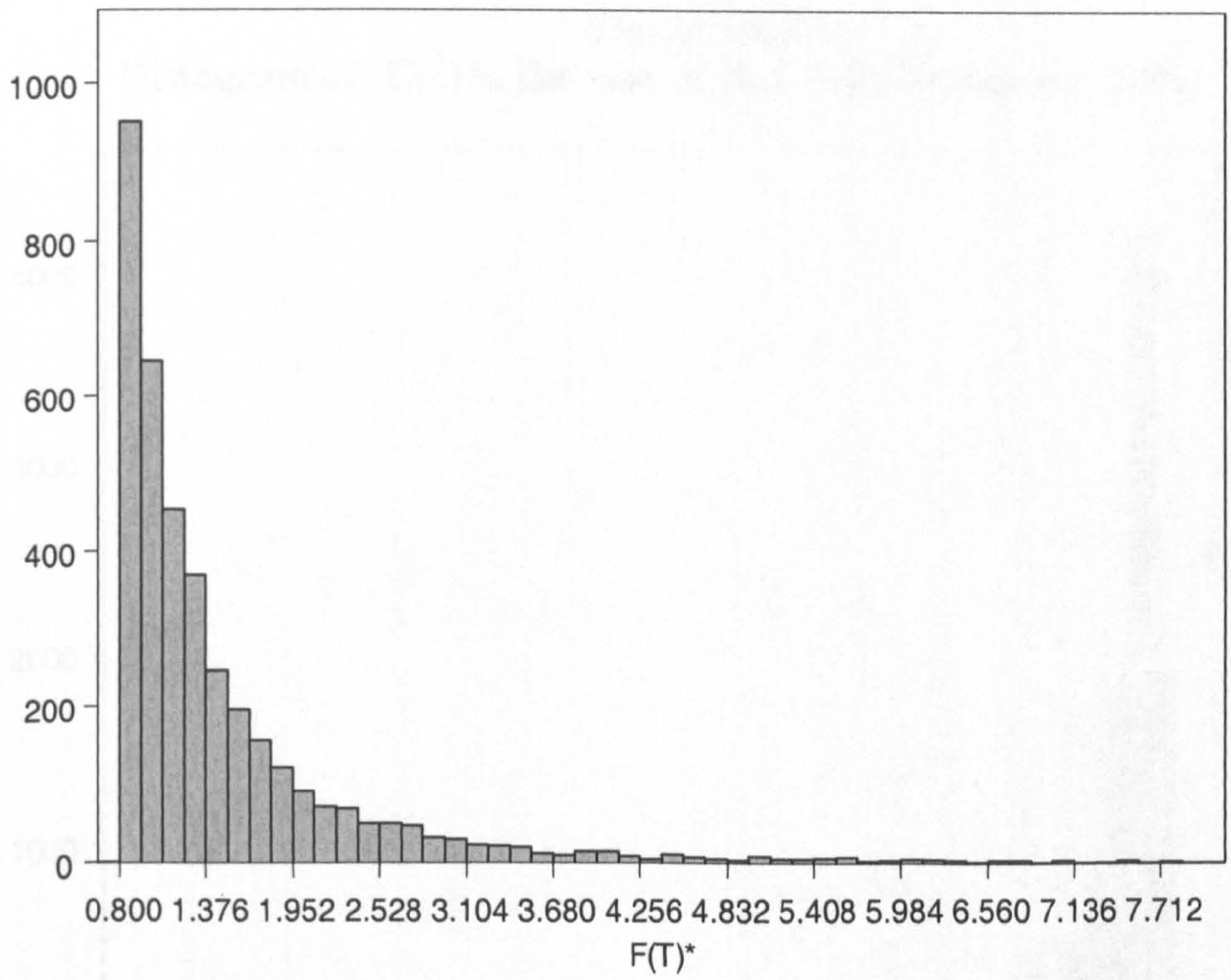


Case: RIPI (0.9, 0.8, 0.1) sigma = 0.2

Summary Statistics for data in: (0.9, 0.8, 0.1)

	F(T)*
Min:	0.1730009
1st Qu.:	0.4695316
Mean:	0.9069162
Median:	0.6662209
3rd Qu.:	1.0189867
Max:	22.8079900
Total N:	10000.0000000
NA's :	0.0000000
Std Dev.:	0.8989038

Figure 5A.4
Histogram of (F_T^*) in the case of (0.9, 0.8, 0.1) when σ = 20%



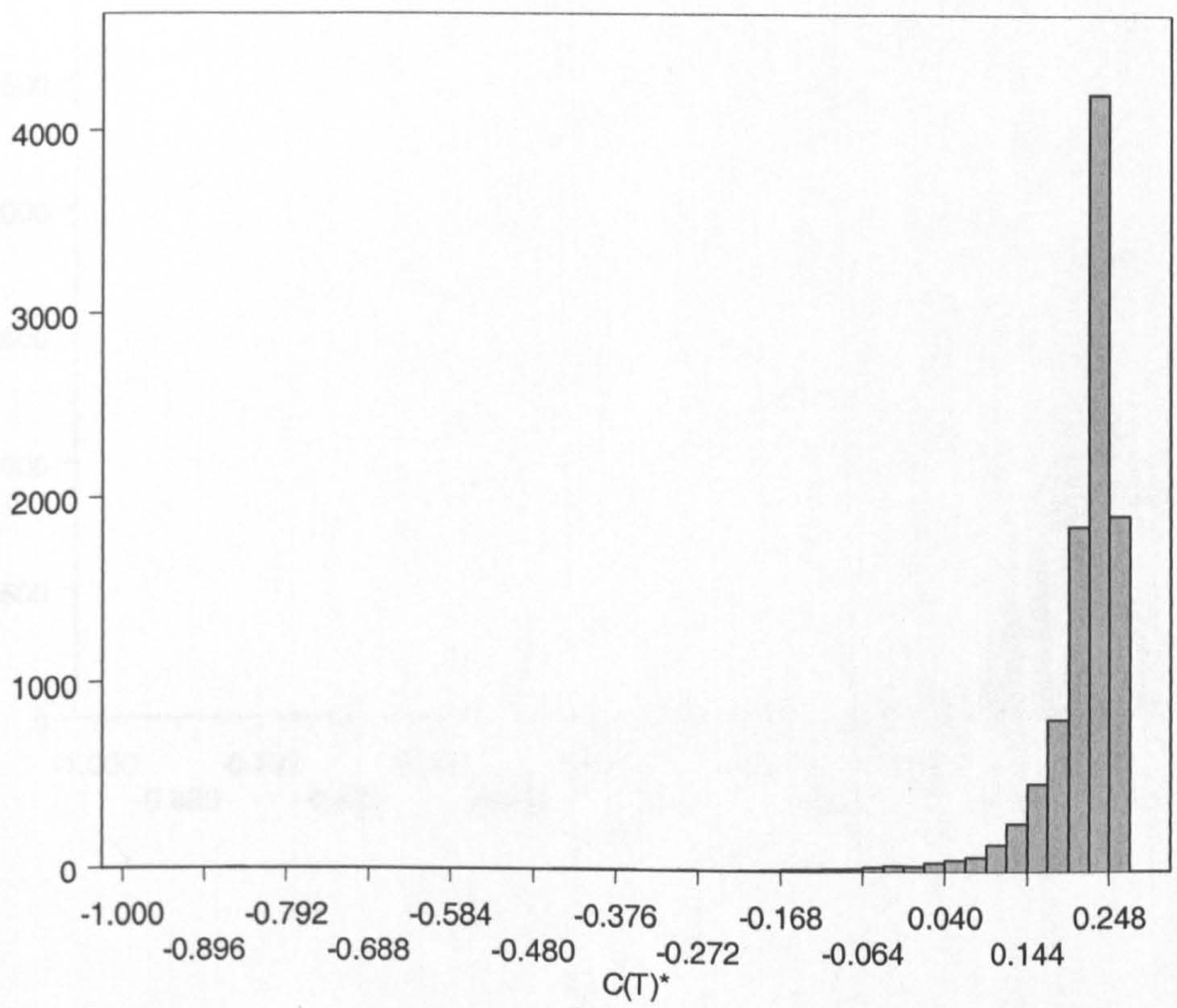
5A.1.2 The results of (C_T^*)

Case: RIPI (0.1, 0, 0.9) sigma = 0.2

Summary Statistics for data in: (0.1, 0, 0.9)

```
              C(T)*
     Min:     -1.7883340
 1st Qu.:      0.2052651
    Mean:      0.2133225
  Median:      0.2311773
 3rd Qu.:      0.2452078
     Max:      0.2646060
 Total N:  10000.0000000
   NA's :      0.0000000
Std Dev.:      0.0682653
```

Figure 5A.5
Histogram of (C_T^*) in the case of (0.1, 0, 0.9) when σ = 20%

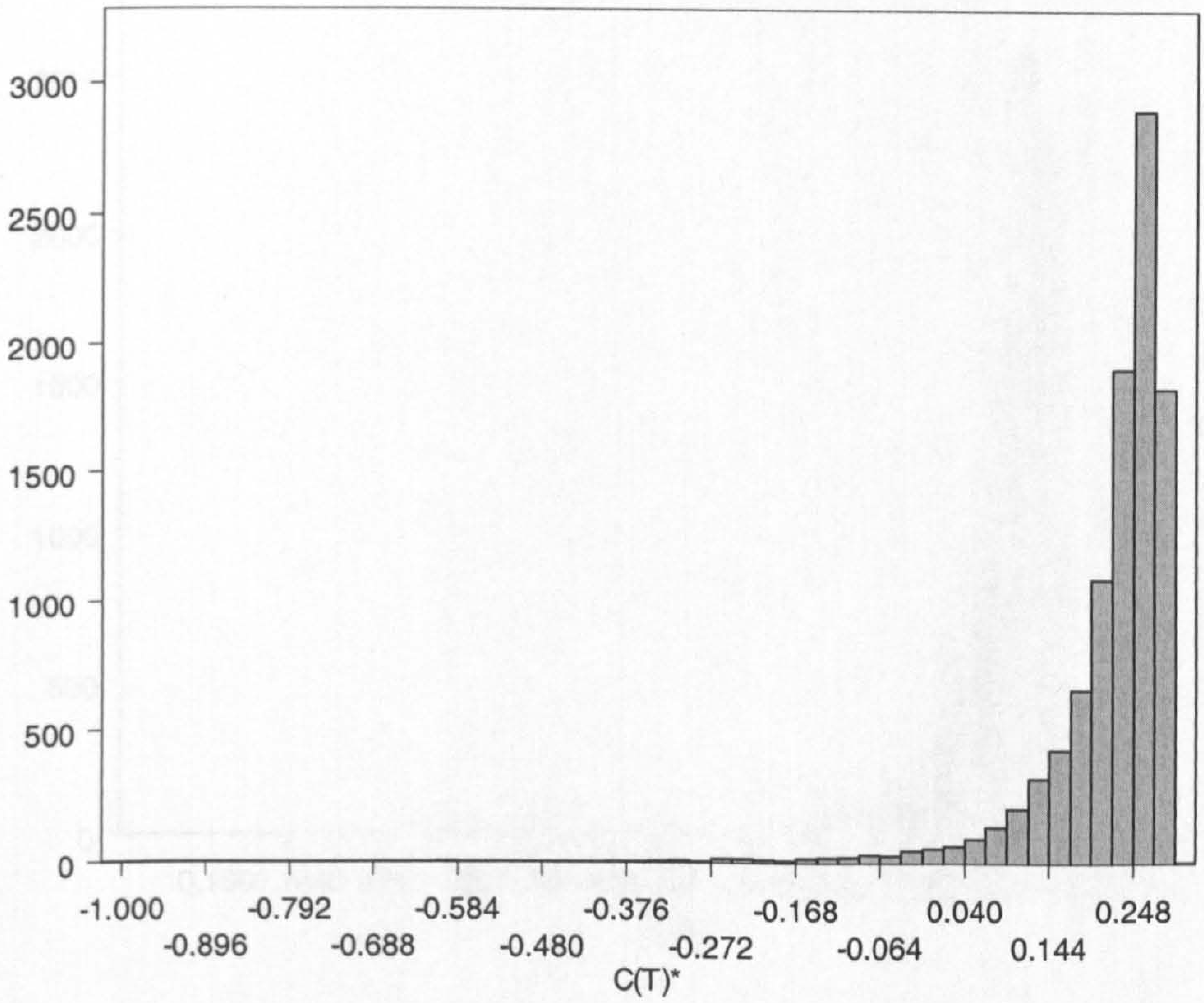


Case: RIPI (0.1, 0.8, 0.9) sigma = 0.2

```
***  Summary Statistics for data in:   (0.1, 0.8, 0.9) ***

                    C(T)*
      Min:     -2.7435080
  1st Qu.:      0.2032702
     Mean:      0.2180926
   Median:      0.2452426
  3rd Qu.:      0.2684690
      Max:      0.3037732
  Total N:  10000.0000000
    NA's :      0.0000000
 Std Dev.:      0.1049674
```

Figure 5A.6
Histogram of (C_T^*) in the case of (0.1, 0.8, 0.9) when σ = 20%

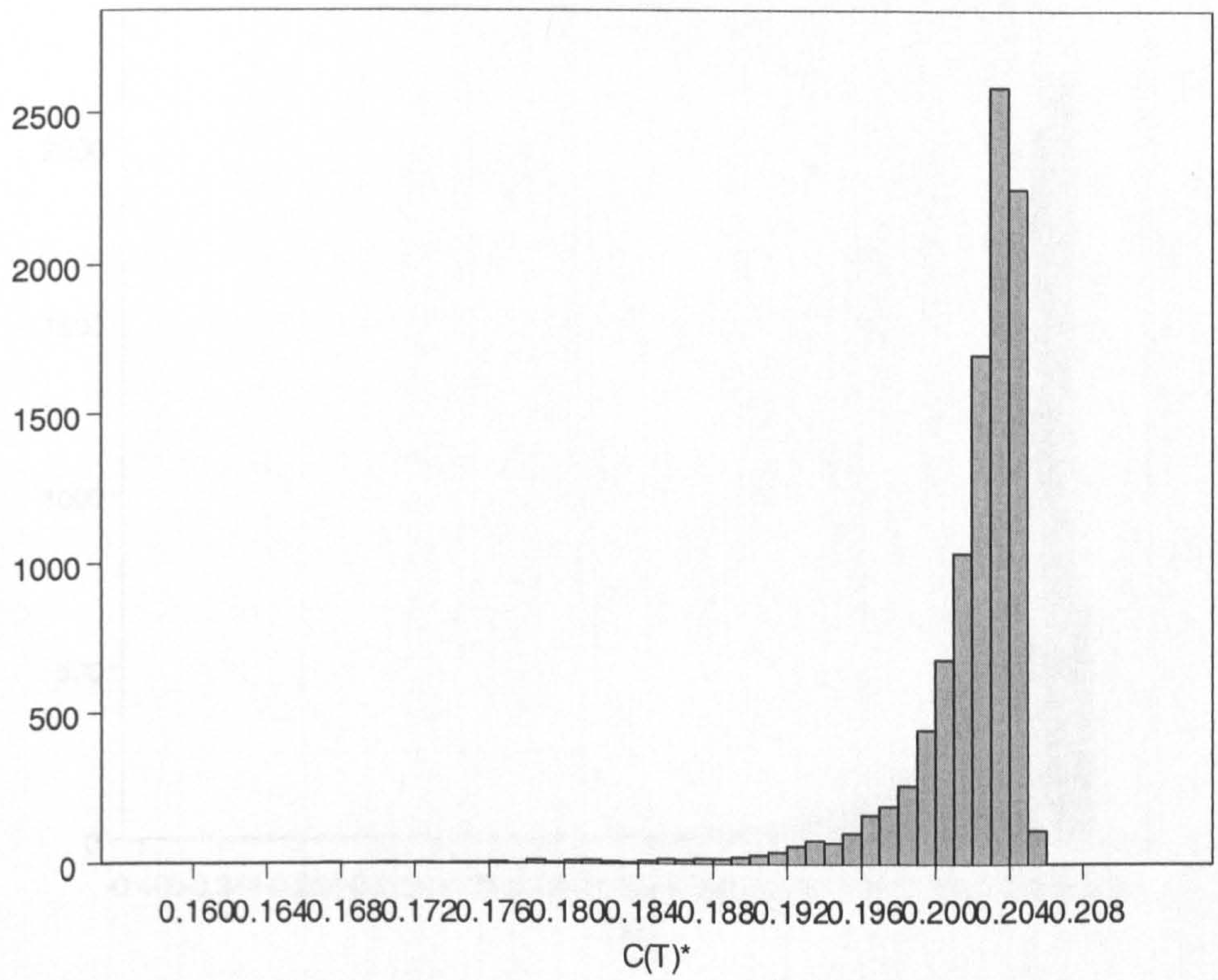


Case: RIPI (0.9, 0, 0.1) sigma = 0.2

Summary Statistics for data in: (0.9, 0, 0.1)

```
              C(T)*
      Min: 9.675600e-002
  1st Qu.: 2.012140e-001
     Mean: 2.018148e-001
   Median: 2.029816e-001
  3rd Qu.: 2.039475e-001
      Max: 2.054530e-001
  Total N: 1.000000e+004
    NA's : 0.000000e+000
 Std Dev.: 4.361636e-003
```

Figure 5A.7
Histogram of (C_T^*) in the case of (0.9, 0, 0.1) when $\sigma = 20\%$

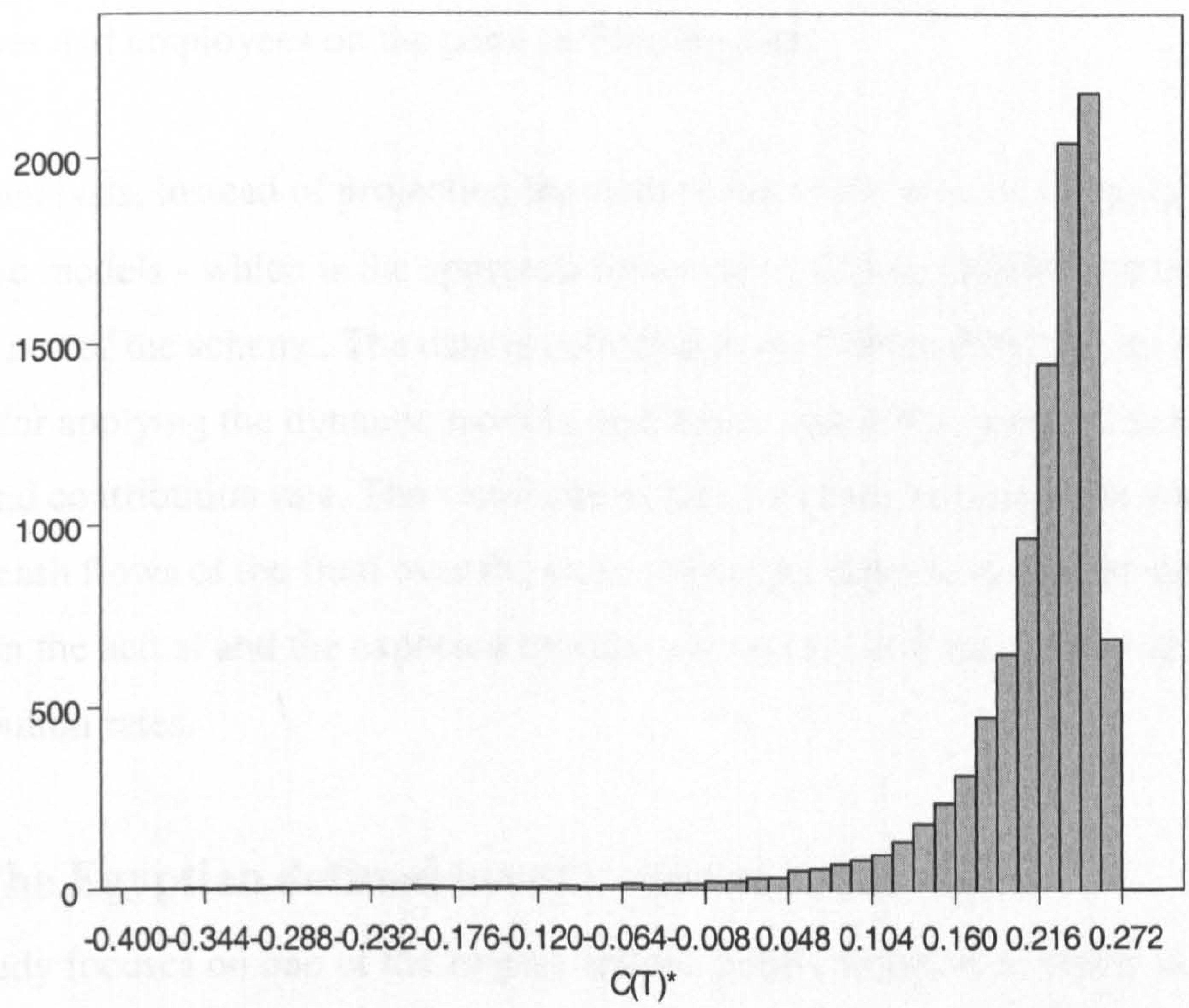


Case: RIPI (0.9, 0.8, 0.1) sigma = 0.2

Summary Statistics for data in: (0.9, 0.8, 0.1)

```
              C(T)*
      Min: -1.675526e+000
  1st Qu.:  1.990142e-001
     Mean:  2.086556e-001
   Median:  2.293631e-001
  3rd Qu.:  2.462846e-001
      Max:  2.717955e-001
  Total N:  1.000000e+004
    NA's :  0.000000e+000
 Std Dev.:  7.733409e-002
```

Figure 5A.8
Histogram of (C_T^*) in the case of (0.9, 0.8, 0.1) when σ = 20%



Chapter Six
Realistic Pension Funding Approach: A case study from Egypt and policy implications for Egyptian context

6.1 Introduction

According to Chang (1999, 2000), dynamic pension funding models help the management of a pension fund to reach the optimal contribution rates, when the disturbances to the economic factor, represented in the rate of return, are considered in the decision making process. In this chapter, we test the properties of our models by applying them to an Egyptian defined benefit pension scheme. This empirical study will mainly enable us to understand the effect of the mutual interests of the employer and employees on the pension funding plan.

In our analysis, instead of projecting the cash flows of the scheme to apply the dynamic models - which is the approach followed by Chang (1999) - we use the past experience of the scheme. The data is collected from 1990 to 2002 to provide the inputs for applying the dynamic models, and hence, reach the expected optimal fund level and contribution rate. The simulated values will then be compared with the actual cash flows of the fund over the same period, in order to detect the differences between the actual and the expected optimal values for the fund levels and the contribution rates.

6.2 The Egyptian defined benefit scheme - case study -[1]

The study focuses on one of the largest defined benefit pension schemès in Egypt. The pension fund set up in 1977 to offer benefits to its members in addition to the benefits offered by the main State Social Insurance System, which has been applied by the company since 1966. It is registered as the fourth Private Pension Fund at Egyptian Insurance Supervisory Authority (EISA) on 7/11/1977 according to the Law no.79 of 1975. At the present time, it is considered the largest scheme with total assets exceed 900 million Egyptian Pounds.

[1] The name of the Private Pension Fund will not be mentioned according to the request of the management of the fund.

The scheme has taken over the management of a total number of 34 other funds in different companies, because of its well known experience, from 1994 until now. The fund has also set up a Center of Insurance Studies in 1983 to provide advice to other pension schemes.

In the following sections, we describe the resources and the benefits of the scheme as mentioned in The Statute of the Scheme and The Actuarial Reports (1990-2002).

6.3 Resources of the scheme

The scheme covers all the employees who work in the company including the ones in the other 34 companies. Both employer and the employees contribute into the scheme. According to the statute of the fund, the pensionable salary is defined as "the basic monthly salary plus the regular bonuses and promotion bonuses".

Thus, the contributions are calculated using the pensionable salary which includes: the basic salary and the allowances that the member receives on a regular basis. These allowances include: the bonuses, high-cost of living allowance, area of work allowance (is defined according to the area of work and its destination from the capital), nature of work allowance (according to the job profiles and the type of responsibilities that should be carried out by the workers).

Therefore, the main resources of the fund are:

- Employer's contribution

  18% of allowances;

  7% of basic salaries;

  and 2.75% of basic salaries for a group life insurance policy called " 1000 Days" group insurance policies (more detail is mentioned in section 6.4).

- Employees' contribution

  10% of the allowances;

  7% of the basic salaries;

  and the premiums of insurance group policies.

- Returns on investments.

The expected present value of the contributions is calculated according to the following equations - after deducting 6% for the management expenses and excluding 2.75 % of the basic salaries for the group insurance policies -:

$$PVCA = 0.94 * 12 * 0.28 \sum_{x=20}^{60} BMS_x \frac{{}^sN_x}{{}^sD_x} \qquad \text{for allowances} \qquad (6.1)$$

$$PVCB = 0.94 * 12 * 0.14(0.12) \sum_{x=20}^{60} TMS_x \frac{{}^sN_x}{{}^sD_x} \qquad \text{for basic salaries} \qquad (6.2)$$

where:

PVCA: the present value of the contributions based on the allowances.

PVCB: the present value of the contributions based on the basic salaries.

BMS_x: is the allowances for each member at age x

TMS_x: is the basic monthly salary for each member at age x.

Note: 12 % is the percentage calculated for the 34 companies involved in the fund.

6.4 Benefits of the scheme

The scheme offers a variety of benefits to its members, this include a monthly pension and lump sum payments, as follows:

- **The social insurance (on the allowances)**

Each member is entitled to a pension at the end of the service for one of the following reasons:

- Reaching the normal retirement age (NRA = 60 years).
- Total disability or death.
- Partial permanent disability.
- End of service before NRA conditional on a minimum period of contribution equals to 240 months.

The contributions are deducted from the monthly allowances of each member. The pension is equal to 3.3% of the average pensionable salary over the last year of service, for each year of membership, and 1.7% of the same average for the period prior to the membership.

The expected present value of these benefits are calculated using the following equations:

$$PVP = 1.1 * 0.03 * 12 \sum_{x=20}^{60} (t + 60 - x) BMS_x \frac{{}^sD_{60}}{{}^sD_{x^a}} \quad (6.3)$$

60

$$PVD = 1 * 12 \sum_{x=20}^{60} BMS_x \frac{{}^sM_{x^a}}{{}^sD_x} \quad (6.4)$$

$$PVAI = 12 \sum_{x=20}^{60} BMS_x \frac{{}^sM_{x^r}}{{}^sD_x} \quad (6.5)$$

where:

PVP: the present value of the NRA pension.

PVD: the present value of death pension.

PVAI: the present value of the additional indemnity pension, where the additional indemnity: covers the total and partial disability.

(t + 60 –x): represents the number of years of service.

- **1000 Days insurance policies**

This policy is a group insurance policy which was issued by the insurance company (EL-AHLIA) and now is managed by the Fund. The benefits and the premiums of this policy are calculated separately, i.e. they are not included in the actuarial valuation reports.

- **Loyalty remuneration**

It is a lump sum remuneration equals to 3.5 months of the final salary for each year of service in the case of retirement, and 1.5 months in the cases of death or disability. In the case of early leaving, the benefits are reduced according to the age of the member and the number of years in service.

The equations are:

$$PVL = 3.5 \sum_{x=20}^{60} (t + 60 - x) TMS_x \frac{{}^sD_{60}}{{}^sD_x} \quad (6.6)$$

$$PVD_L = 1.5 \sum_{x=20}^{60} \left(tTMS_x \frac{{}^sM_x}{{}^sD_x} + TMS_x \frac{{}^sR_x}{{}^sD_x} \right) \quad (6.7)$$

where
PVL: the present value of NRA in a lump sum form.
PVD_L: the present value of death in a lump sum form.

- **The complementary pension**

It is a unified pension for all the employees and the workers and is equal to LE50 per month.

$$PVC = 12*50\sum_{x=20}^{60} m_x \frac{{}^sD_{60^a}}{{}^sD_x} \tag{6.8}$$

PVC: the present value of the complementary pension.

- **The ancillary benefits**

The present value of these benefits assumed to be LE 2 million Egyptian pounds and they are paid to the members in cases such as: the loss of home, medical surgeries among others.

- **Fidelity remuneration**

This benefit is offered by the scheme as an additional benefit as a result of achieving high surplus in the Fund. It depends on the number of years of service as follows:

10 years and less	no remuneration
from 10 to less than 20 years	one month
from 20 to less than 25	one and a half month
25 years and more	two months

$$PVF = \sum_{x=20}^{60} r_x(t+60-x)TMS_x \frac{{}^sD_{60}}{{}^sD_x} \tag{6.9}$$

where
r_x: is the rate used to calculate the benefit for each member at age x according to the number of years of service.
PVF: the present value of fidelity remuneration in the form of the lump sum.

6.5 Analysis of dynamic pension funding models applications for the Egyptian context

According to the scheme's policy, the actuarial valuation is carried out annually to calculate the normal cost and the actuarial liability of the scheme. The availability of data on annual basis for more than 10 years is another reason of choosing this scheme for our empirical study. In 2000, the number of members of the scheme were 48914, including the members of the 34 companies. In this section, we start with the actuarial assumptions used to calculate the actuarial liability and normal cost of the fund. Thereafter, we describe the data that have been used for applying the models. Finally, the results of application of both models CIPI and RIPI are analyzed.

6.5.1 Actuarial assumptions

The actuarial assumptions that have been used in the annual actuarial reports are as follows:

- The valuation rate of return = 9%;
- the life table: A49/52 (based on the experience of UK insured persons between 1949 and 1952, as collected and analyzed by the CMI Bureau);
- The salary scale: the salary scale which was used in the past was :

  $S_x = S_{20}(1+0.15(x-20)$

  then from 1997 to 1999, it was changed to be:

  $S_x = S_{20}(1.03)^{(x-20)}$

  in 2000 it has been raised to 4 % as follows:

  $S_x = S_{20}(1.04)^{(x-20)}$
- The funding method is the prospective method which depends on the difference between the expected present value of the benefits and the expected present value of the contributions.
- The entry age is the actual age of the members at the date of actuarial valuation.
- The expense rate is 6% of the contributions.
- The contributions are paid annually.

- The age at the valuation date is = valuation year − year of birth.

6.5.2 Data description

The stochastic models of CIPI and RIPI are implemented, using 10,000 simulations based on a VBA program. The following data are used to calculate the expected optimal fund levels and contribution rates over 12 years from 1990 to 2002:

- The control period T = 12;
- the Fund return parameters: $\theta = 9\%$(corresponding to the valuation rate of return), and $\sigma = 1\%, 5\%, 10\%, 15\%, 20\%$;
- the fund ratio: $\eta = 100\%$;
- risk weighting factors: γ and $\beta = 0.1, 0.3, 0.5, 0.7, 0.9$ interchangeably, and the mixed weighting factor $\alpha = 0.2, 0.4, 0.6$ and 0.8;
- initial fund: $F_0 = 271{,}980{,}000$ Egyptian Pounds.

We assume that the actuarial valuation occurs at times 0, 1,....,T-1, so that, these information are available:

- $NC_t = CT_t$: the normal cost or the contribution target at time t, where t = 0, 1,..,T;
- $AL_{t+1} = FT_{t+1}$: the actuarial liability of the fund at time t+1, where t = 0, 1,...,T;
- B_t : the total benefits paid to the beneficiaries during the year [t, t+1); where t = 0, 1,....T;

Table 6.1 shows the cash flows of the fund and the benefits outgo from 1990 to 2002 (in thousands of Egyptian Pounds), that will be used to calculate the expected optimal fund level $E(F_t^*)$ and contribution rate $E(C_t^*)$.

Table 6.1
The normal cost, actuarial liability and the benefits of the scheme from year 1990 to 2002

(in thousands LE)

Year	NC_t	AL_t	B_t
1990	145290	315000	11388
1991	179600	373600	15214
1992	181000	448000	17936
1993	142600	497900	20834
1994	181401	693480	35385
1995	203274	805872	32155
1996	243610	874833	36282
1997	290665	983412	42163
1998	311998	1108036	50611
1999	319218	1170290	53941
2000	364318	1333544	56932
2001	348220	1425492	80524
2002	360604	1484285	94248

6.5.3 Results of the sensitivity analysis for applying CIPI and RIPI

In this subsection, we study the effect of changing θ, σ, and the risk weighting factors γ, α and β on both stochastic models CIPI and RIPI. However, the effect of changing the initial fund F_0, the fund target and the contribution target are not examined due to applying one scenario and using the real data. Our analysis mainly consider the cases when $\sigma = 20\%$, as it is more realistic than the lower levels of volatility (which are more representative to the deterministic approach).

(a) Effect of changing θ

We assume that the value of θ changes up to 10% and down to 8%. The expected optimal fund levels and contribution rate are affected by changing θ up to 10% and down to 8%. For both models CIPI and RIPI, in the case of (0.1, 0, 0.9) when $\sigma = 20\%$, we find that $E(F_t^*)$ increases over the time span with a tendency to increase when the value of θ is increased. However, $E(F_t^*)$ increases to reach higher levels under RIPI compared with the ones under CIPI. Figures 6.1 and 6.2 show the similarity of the behaviour of $E(F_t^*)$ under both models.

Figure 6.1

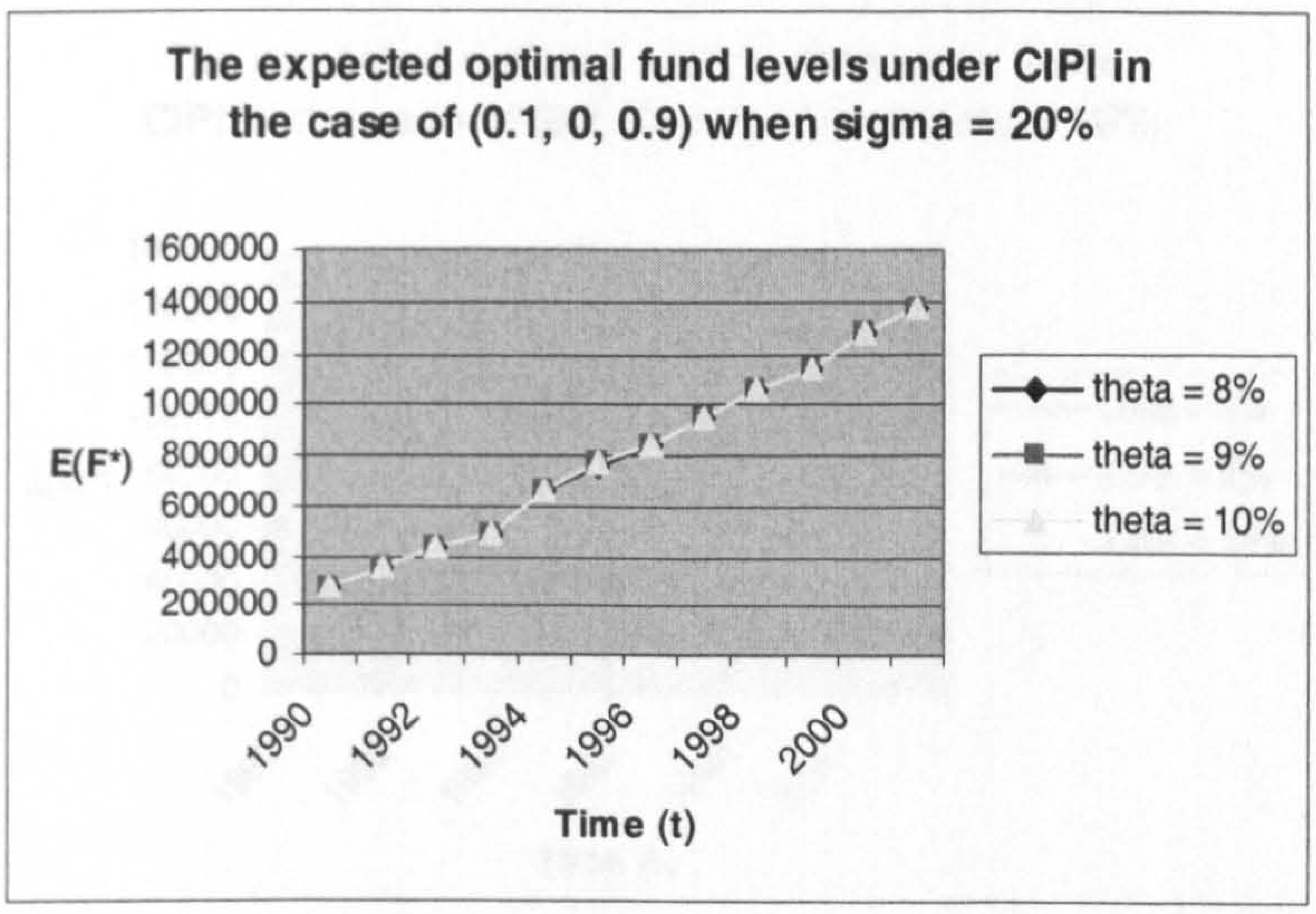
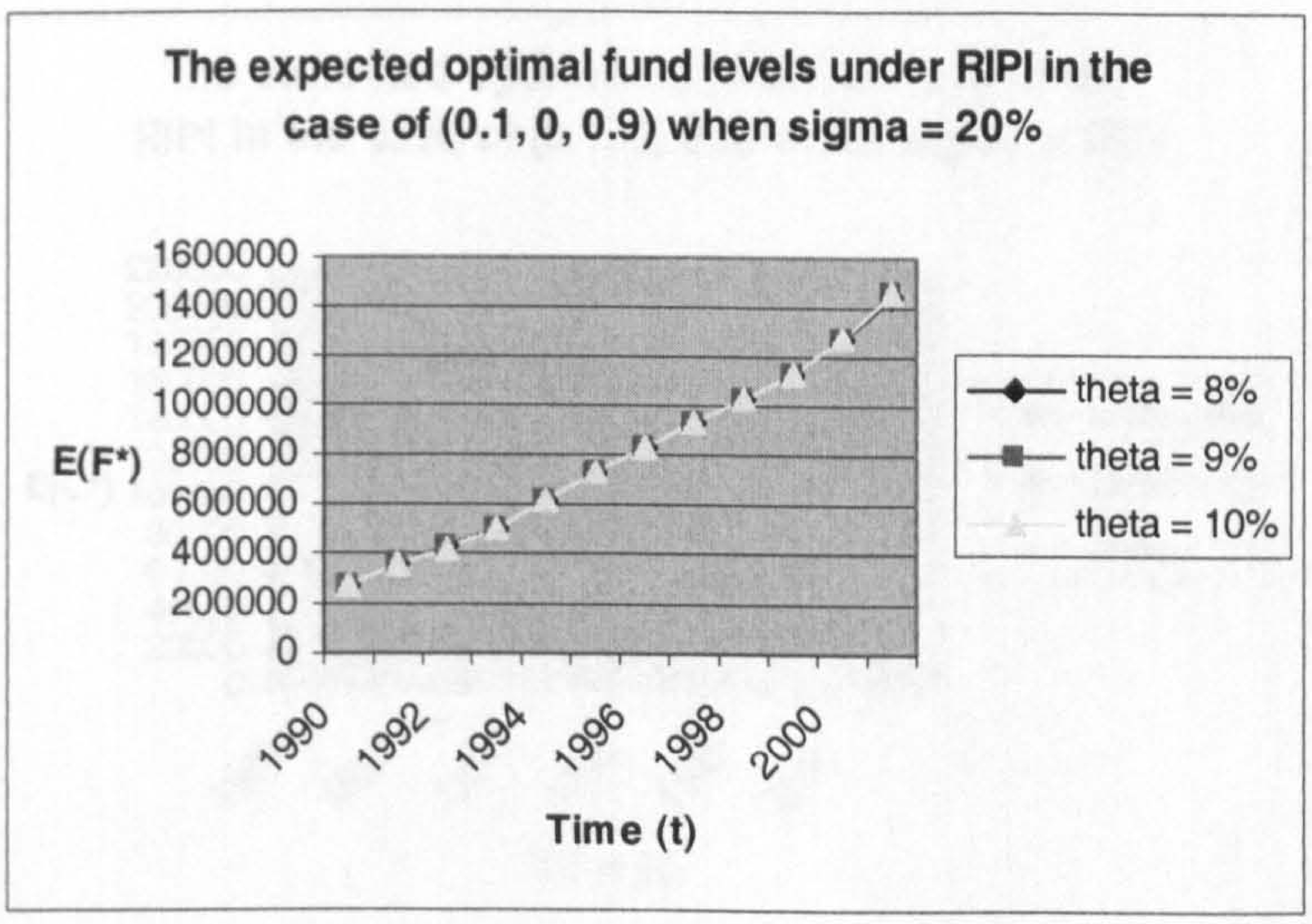


Figure 6.2



On the other hand, under CIPI, $E(C_t^*)$ tends to fluctuate over time, while under the RIPI model, it tends to decrease over the control period from year 1990 to 1999 except in year 1993 and then starts to increase in the last three years. This also indicates that $E(C_t^*)$ has a more stable and smooth behaviour under RIPI compared with the ones under CIPI. Nevertheless, the $E(C_t^*)$ levels decrease slightly over time when θ is increased under both models. Figures 6.3 and 6.4 show these results.

Figure 6.3

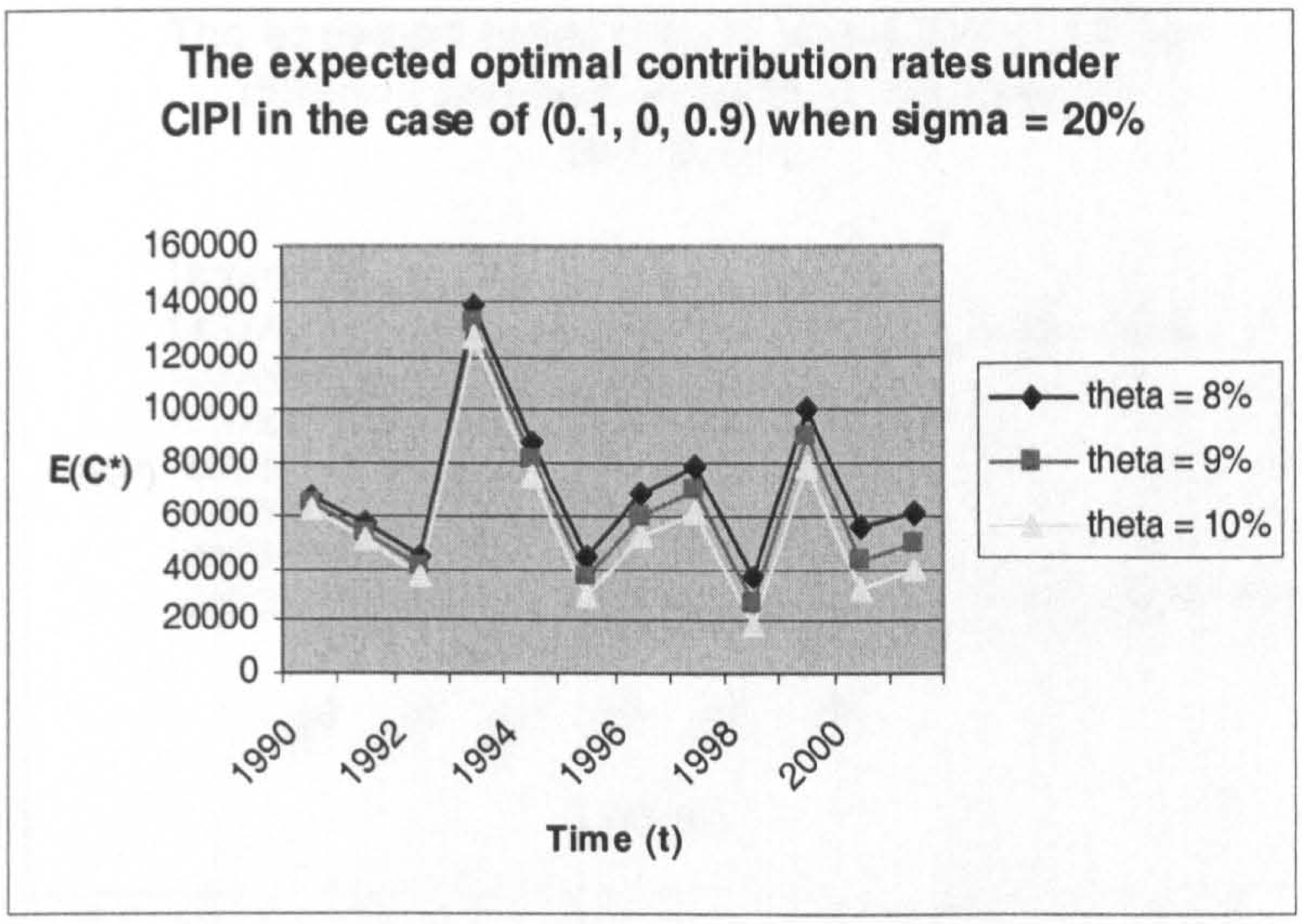
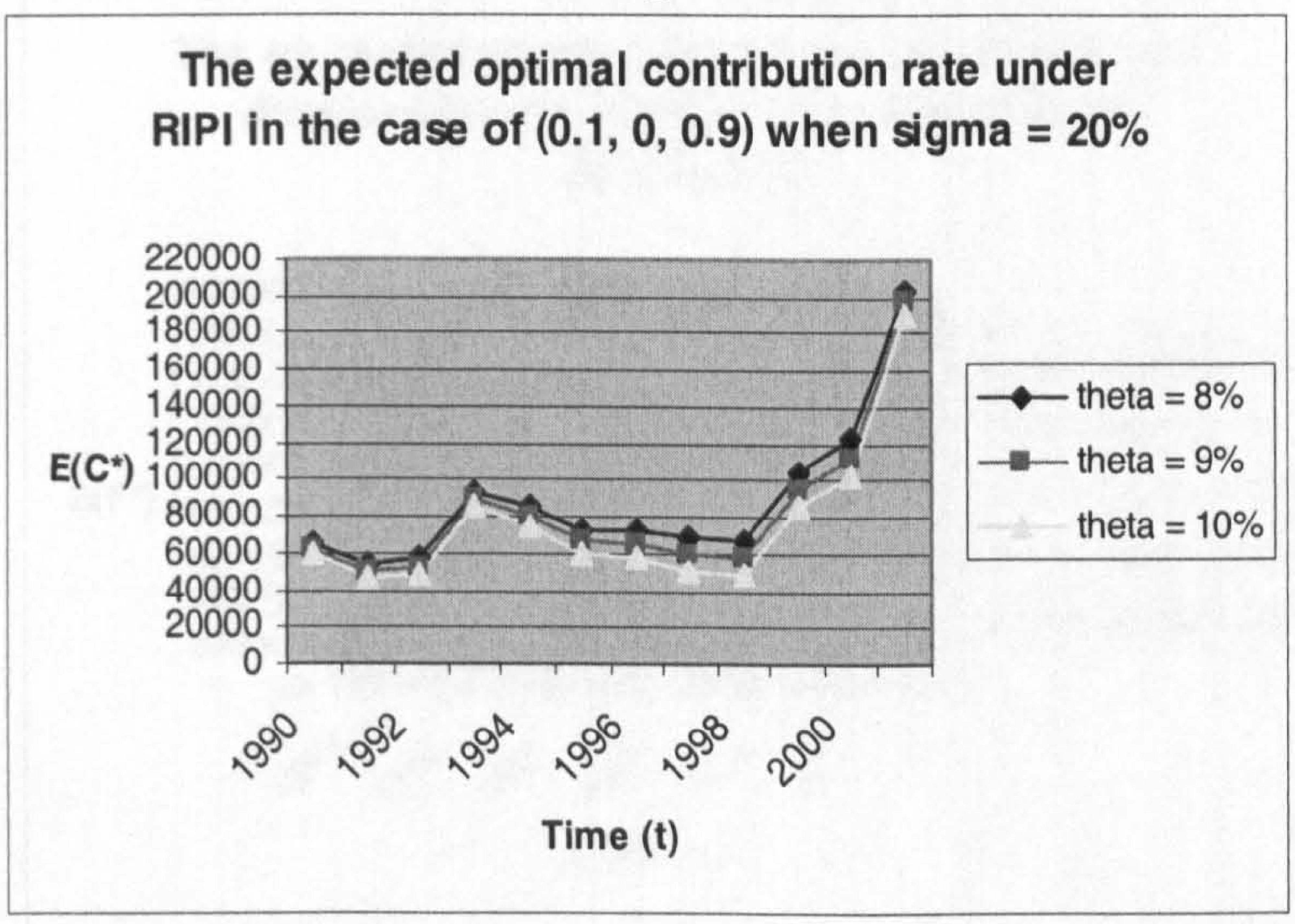


Figure 6.4



(b) Effect of changing σ

The results in chapters 4 and 5 imply that increasing the levels of volatility leads the fund levels to decrease to lower levels over time. For the scheme under study, in the case of (0.1, 0, 0.9), $E(F_t^*)$ has an upward trend over the time, but the levels of the expected fund decrease with the increased level of volatility. This applies for both models CIPI and RIPI although we find that the levels of the fund are slightly higher under RIPI as shown in Figures 6.5 and 6.6. Further, similar results are obtained when other cases such as: (0.5, 0, 0.5) and (0.9, 0, 0.1) are applied.

Figure 6.5

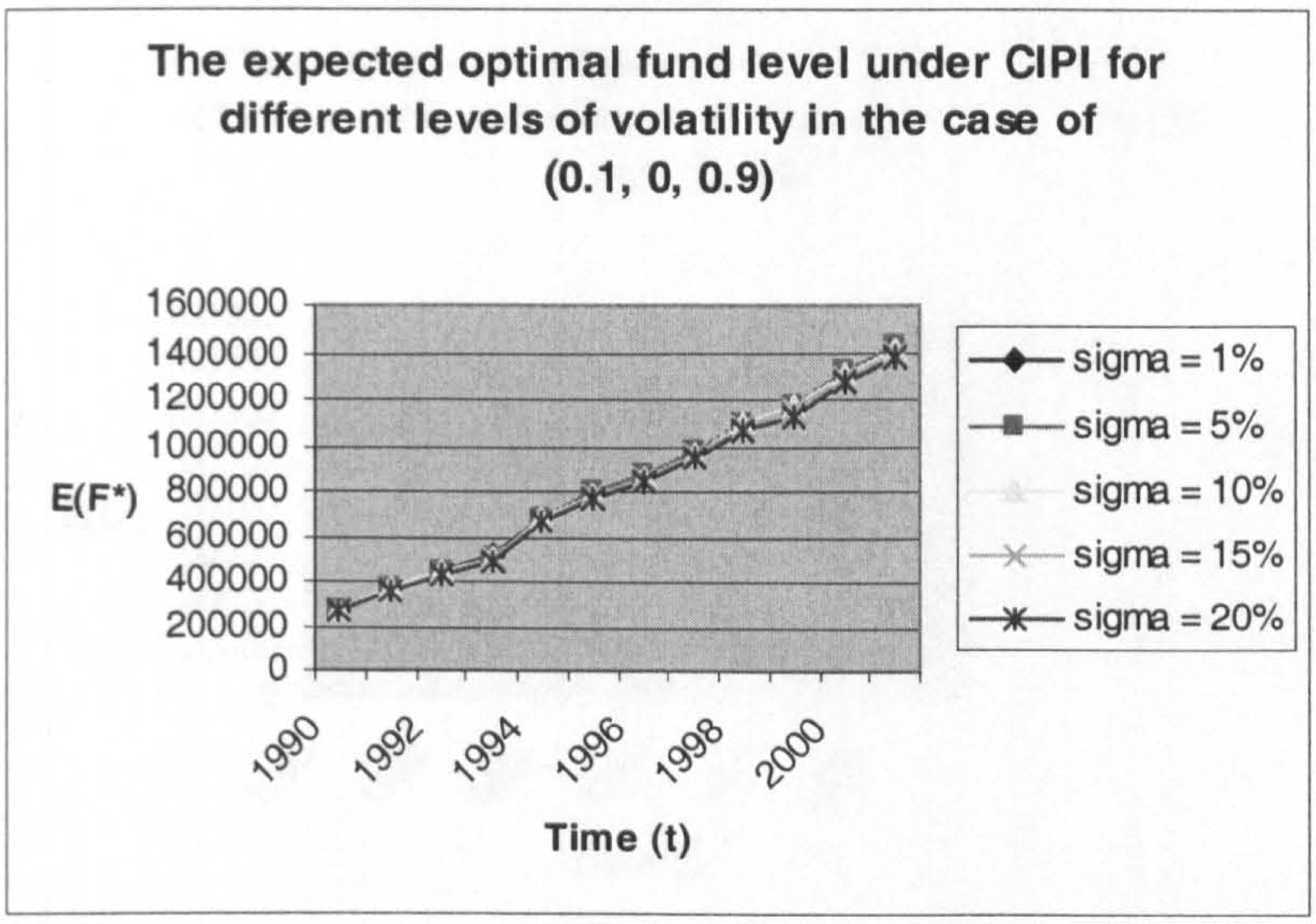
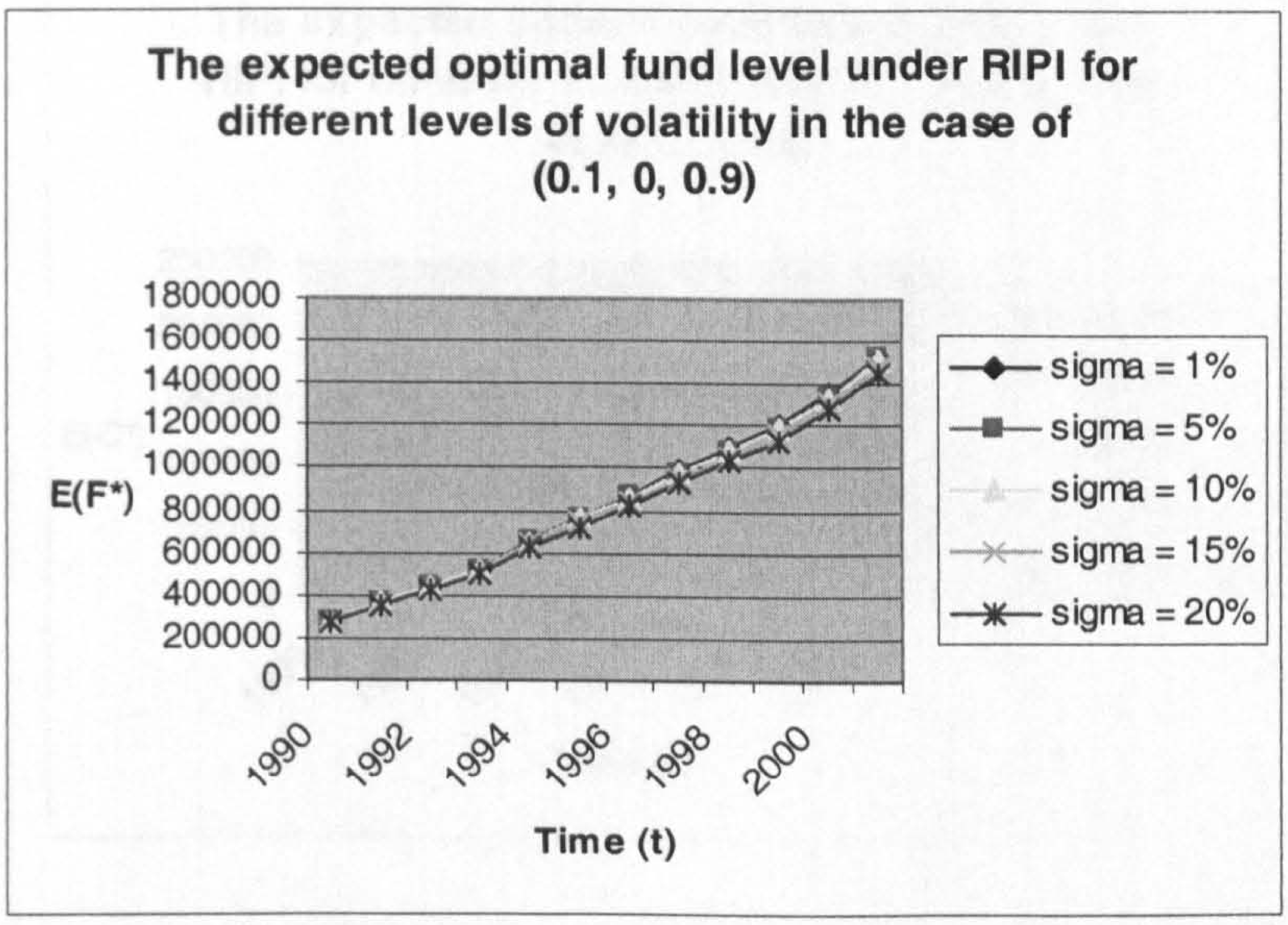


Figure 6.6



On the other hand, under CIPI, the expected optimal contribution rate fluctuates clearly over the control period, while under RIPI, $E(C_t^*)$ it is more stable with a tendency to increase in the last three years. It also decreases with increasing σ over time. Figures 6.7 and 6.8 illustrate these results.

Figure 6.7

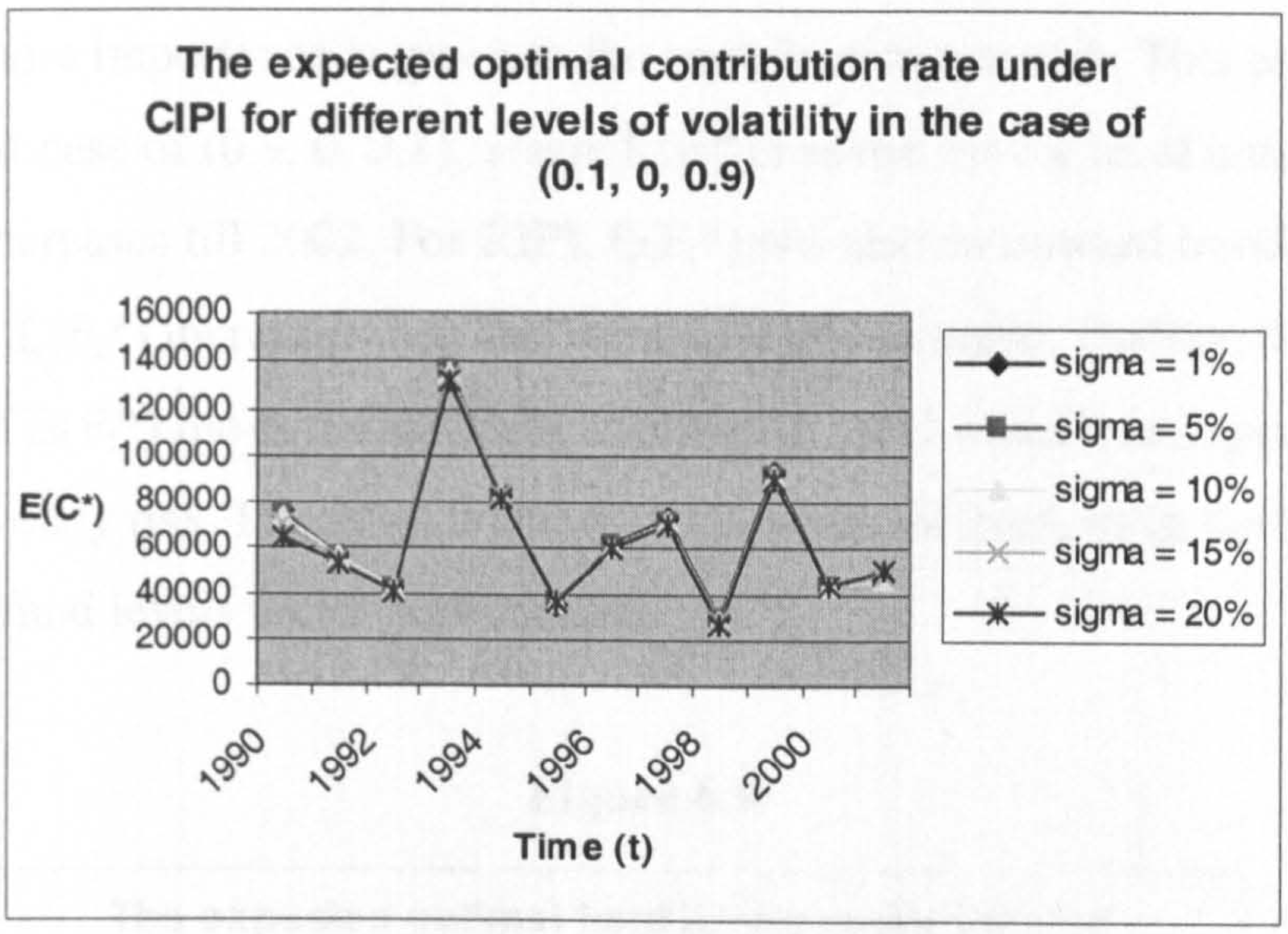
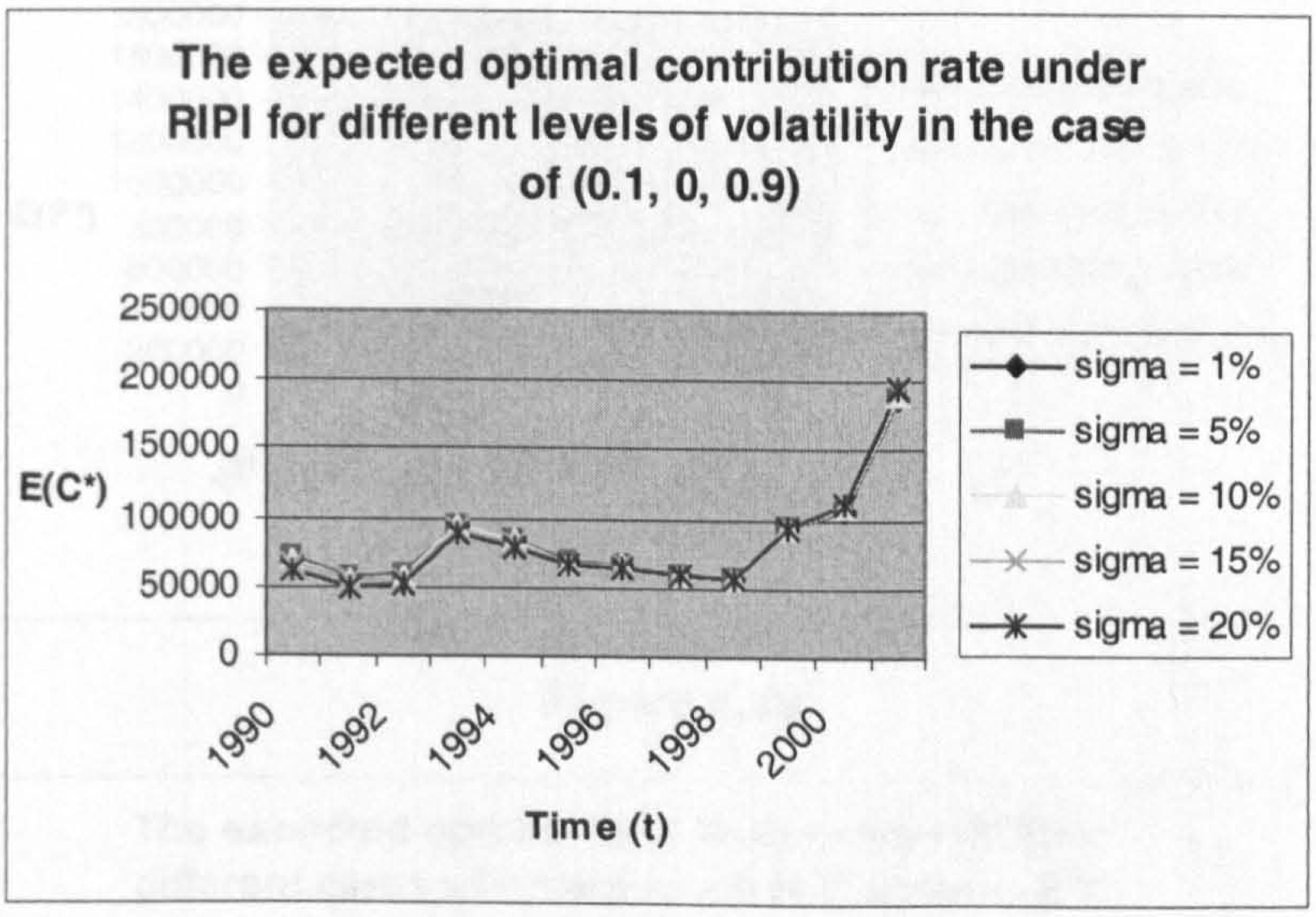


Figure 6.8



When σ is increased, small differences between the levels of expected optimal contribution rate in both models are observed. For the cases of (0.5, 0, 0.5) and (0.9, 0, 0.1), we note that $E(C_t^*)$ has a similar behaviour under both models, RIPI and CIPI, when σ is increased.

(c) Effect of changing the parameters γ and β when α = 0

The changes in $E(F_t^*)$ and $E(C_t^*)$ are analyzed regarding the different values given to the weighting factors of the contribution rate risk and the solvency risk. Here, we focus on the different cases when γ and β take the values of 0.1, 0.3, 0.5, 0.7, 0.9 interchangeably and when σ = 20%.

Under the CIPI model, although $E(F_t^*)$ has an upward trend, the levels of $E(F_t^*)$ tend to decrease as more importance is given to the contribution rate risk. This can be seen clearly in the last case of (0.9, 0, 0.1), where $E(F_t^*)$ has the lowest level until year 1999 and then increases till 2002. For RIPI, $E(F_t^*)$ has also an upward trend over time but the levels of $E(F_t^*)$ increase when the value of β is decreased. Further, we note that the levels of $E(F_t^*)$ move dramatically to a higher level when less importance is given to the solvency risk. Figures 6.9 and 6.10 show the differences in the behaviour of the expected fund levels under both models.

Figure 6.9

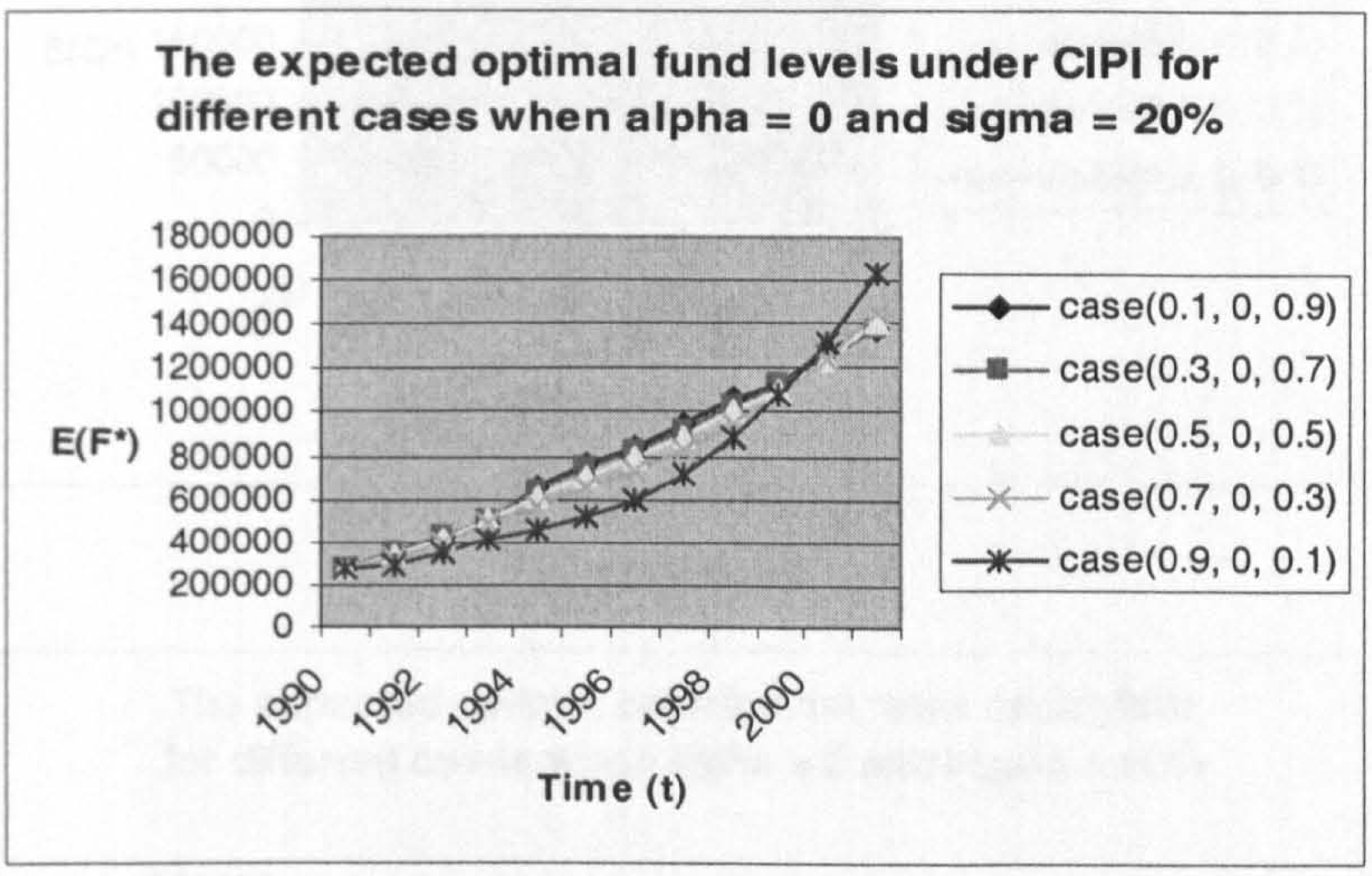
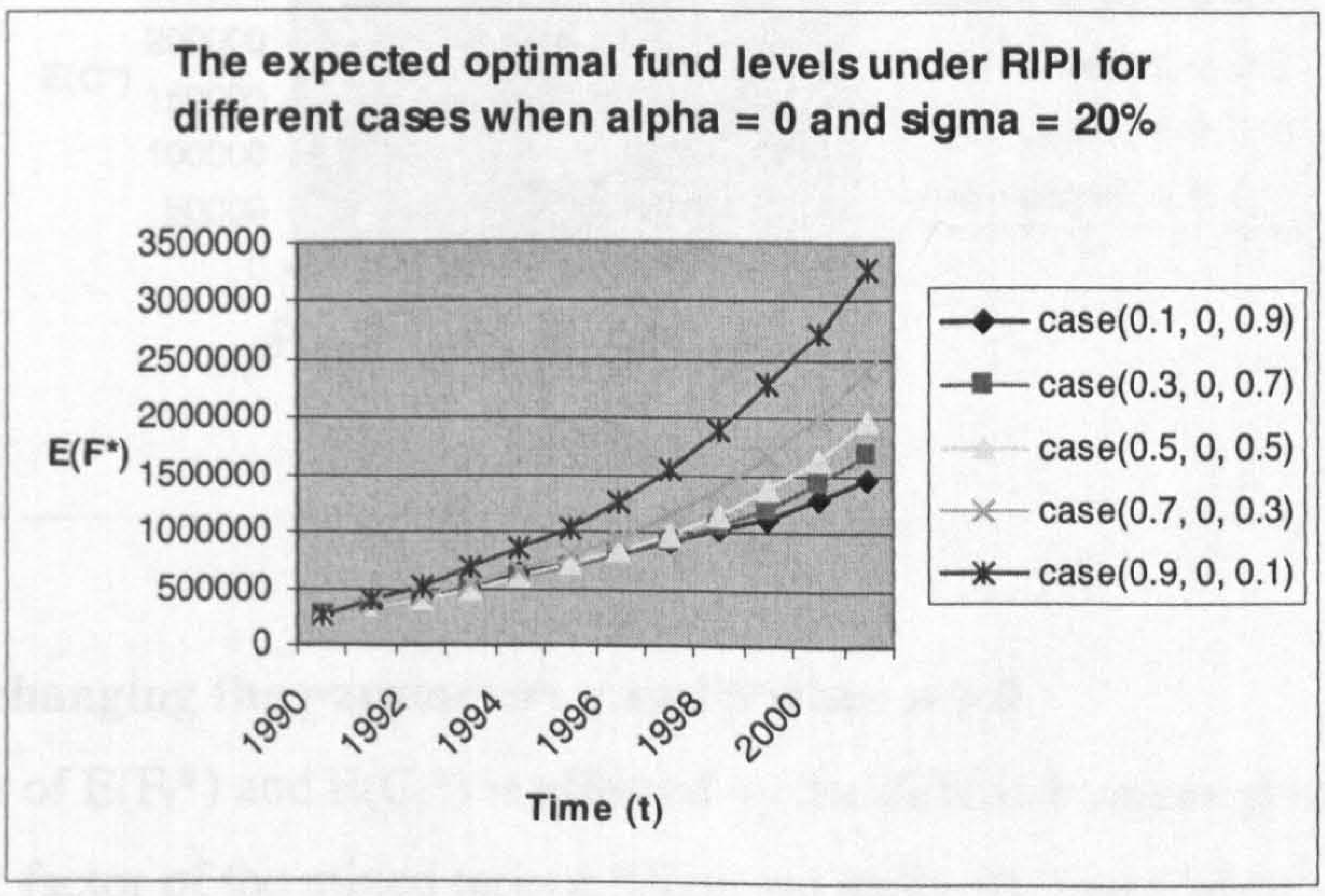


Figure 6.10



For the expected optimal contribution rate, under CIPI, we find that the levels of $E(C_t^*)$ fluctuate over time and move to higher levels when the value of γ is increased. Further, $E(C_t^*)$ has a smoother upward trend in the case of (0.9, 0, 0.1) compared with the cases when β is high. Under RIPI model, the levels of $E(C_t^*)$ increase in the second half of the control period when more importance is given to γ, i.e. $E(C_t^*)$ has the highest level in the case of (0.9, 0, 0.1). These results are shown in Figures 6.11 and 6.12.

Figure 6.11

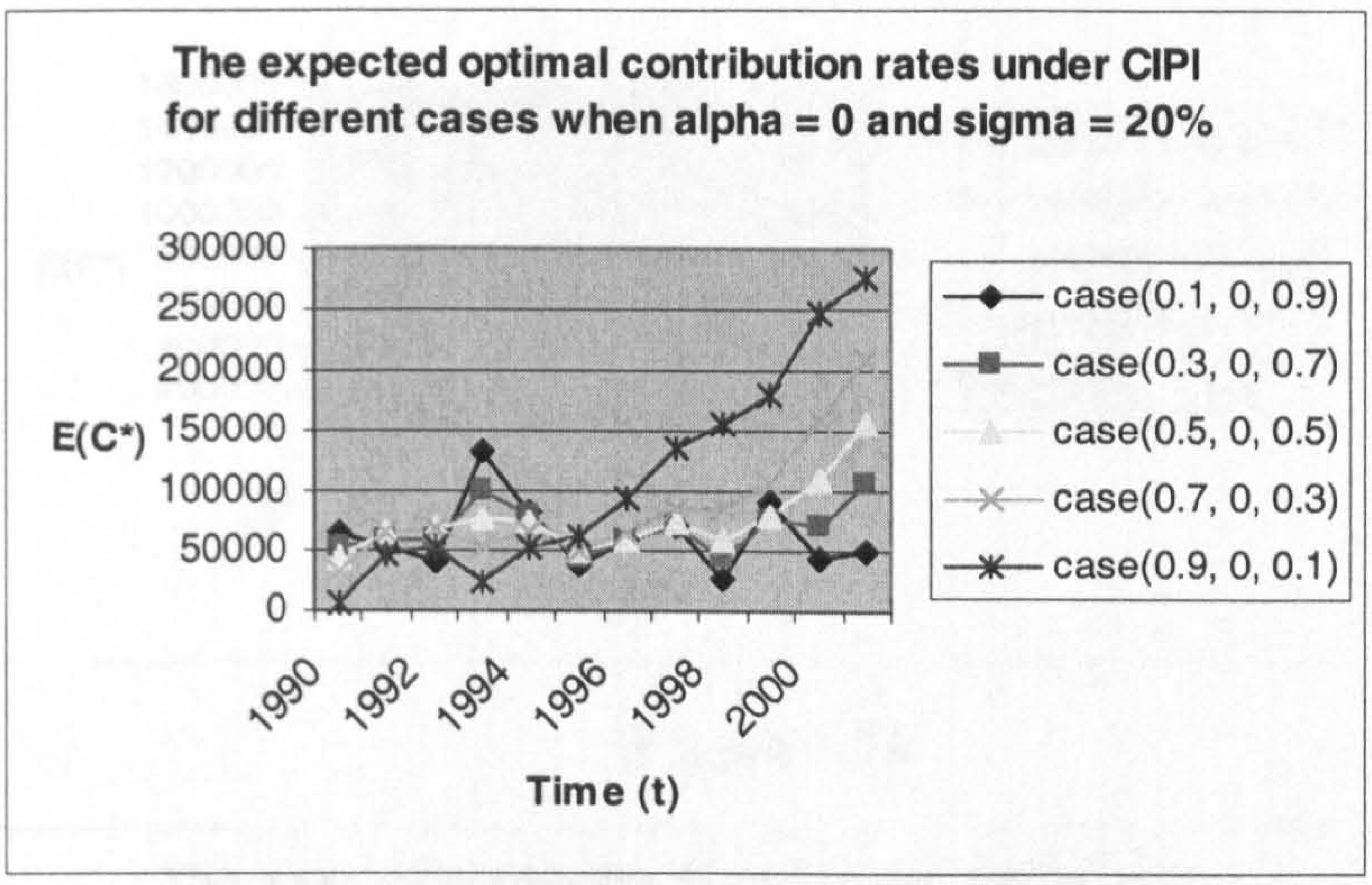
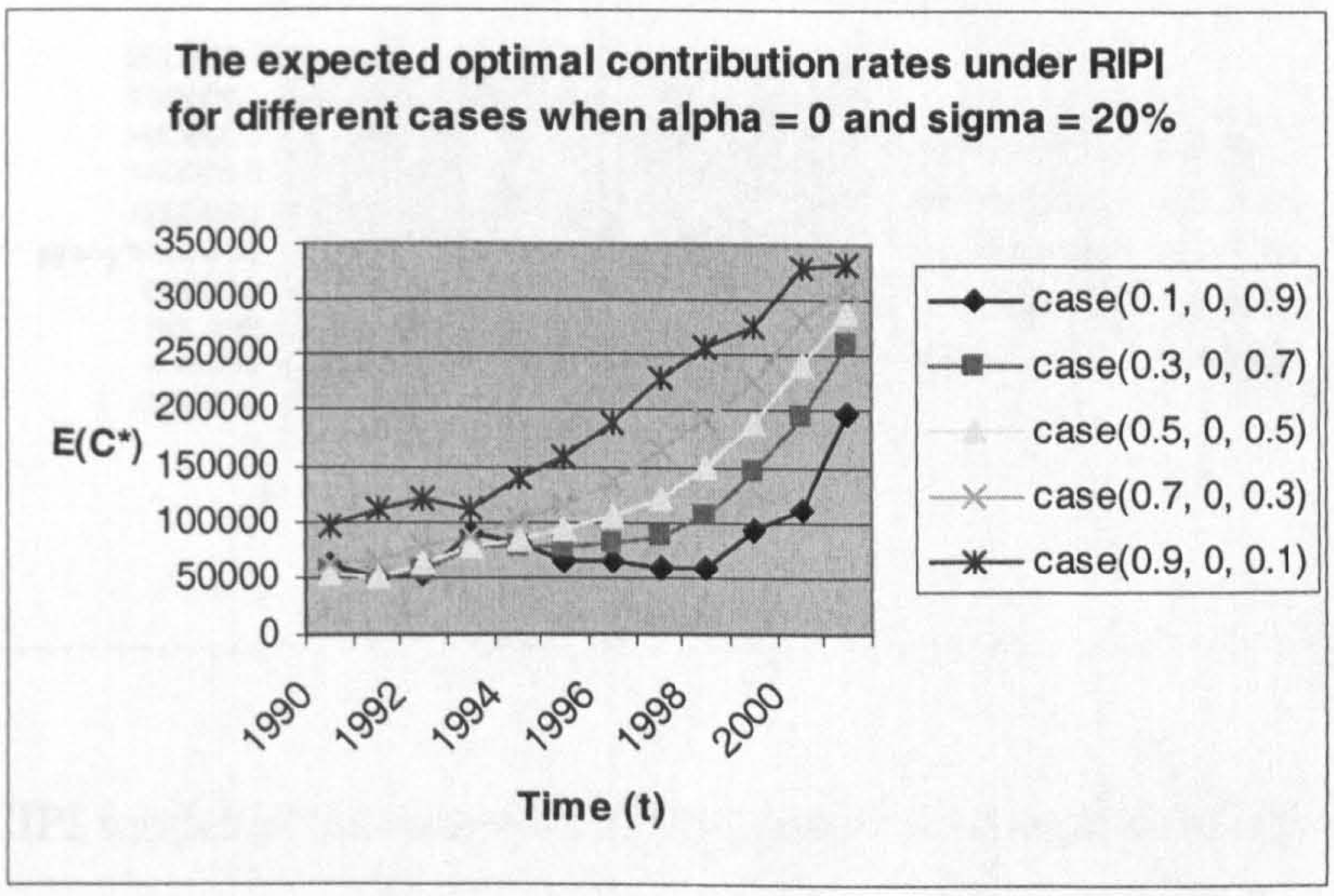


Figure 6.12



(d) Effect of changing the parameters γ and β when α ≠ 0

The behaviour of $E(F_t^*)$ and $E(C_t^*)$ is affected by the different values given to the weighting risk factor of the mixed term α. Here, we study the cases when

$\alpha = 0, 0.2, 0.4, 0.6, 0.8$ in the two extreme cases, (0.1, 0.9) and (0.9, 0.1) and the middle one (0.5, 0.5)[2].

Figure 6.13

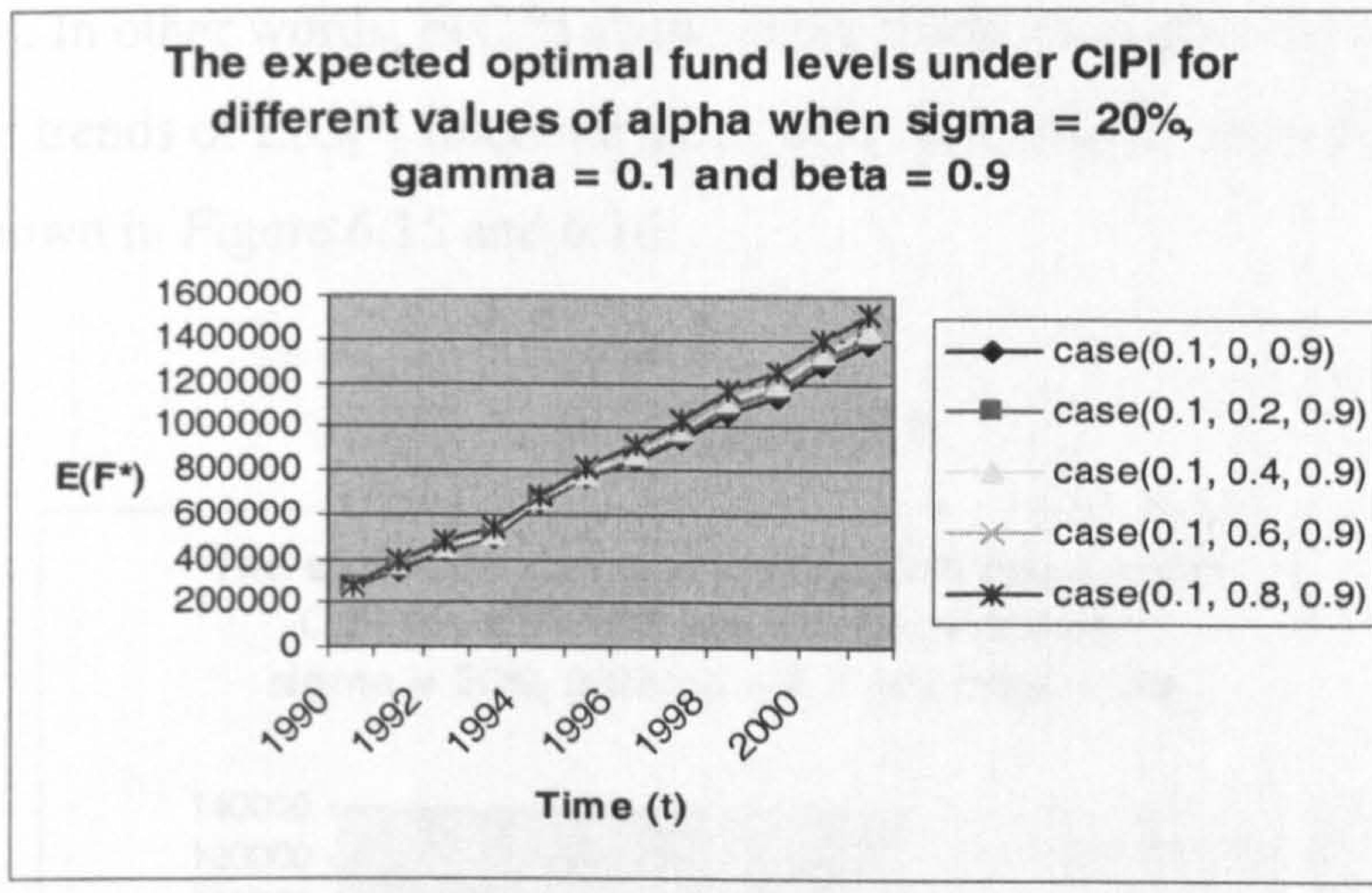
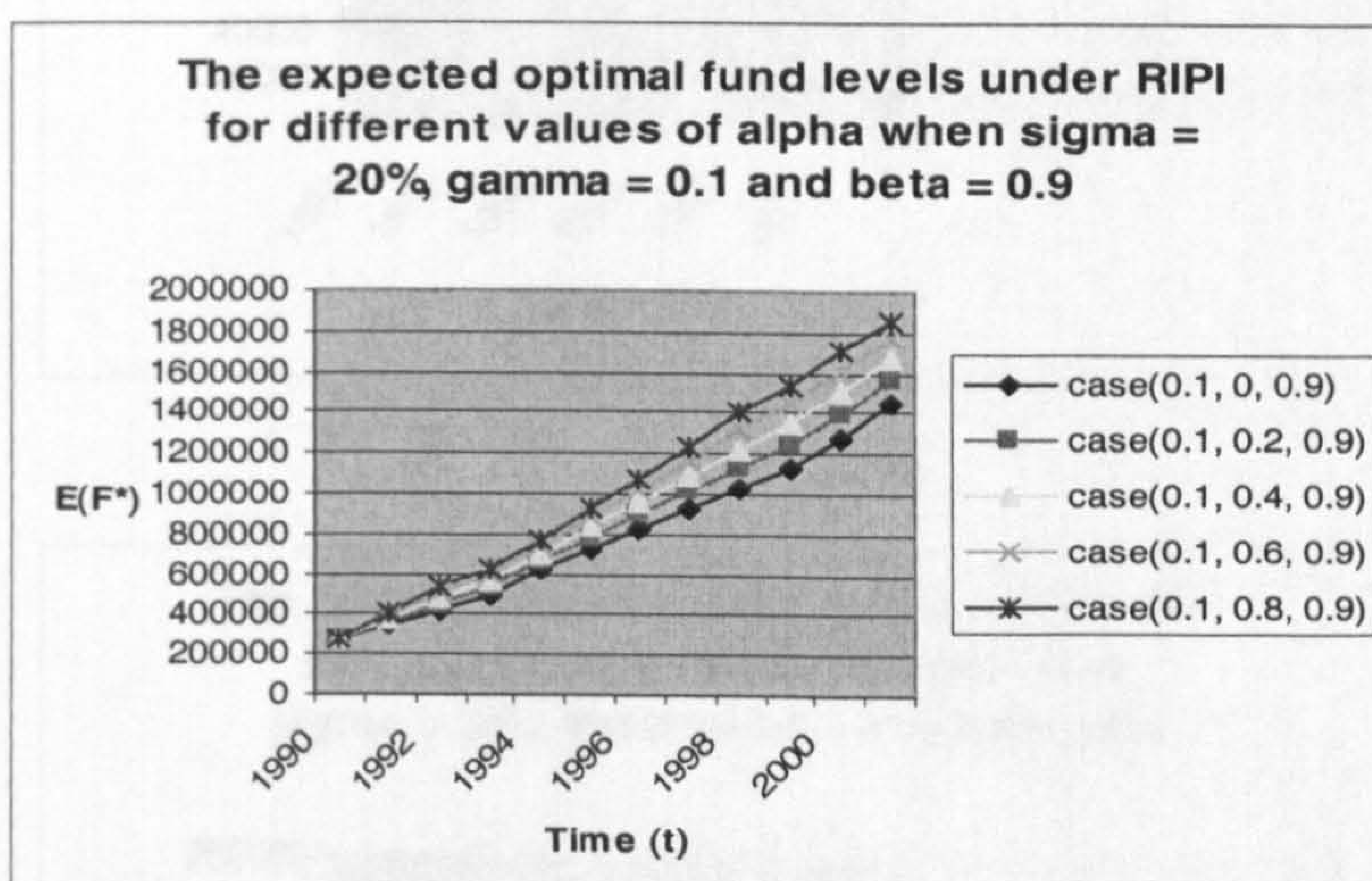


Figure 6.14



Thus, under CIPI model in the case (0.1, 0.9) when α is increased, $E(F_t^*)$ has an upward trend over the time with a tendency to increase with increasing α. Similarly, $E(F_t^*)$ has a similar behaviour under RIPI, but it reaches higher levels compared with those under CIPI as shown in Figures 6.13 and 6.14.

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[2] This to recall that the form for the cases represents the weighting risk factors is used in this order (γ, β) when the value of α is changed or (γ, α, β) with a specific value of α.

Consequently, the expected contribution rates react under both models to the changes in $E(F_t^*)$ when α is increased. Thus, under CIPI, the trends of $E(C_t^*)$ fluctuate over time. We note that there are small differences between the cases, where they increase slightly at the beginning of the control period and decrease more in the final year T as α is increased. In other words, $E(C_t^*)$ shows more fluctuation behaviour when $\alpha = 0.8$. Similarly, the trends of $E(C_t^*)$ fluctuate more with increasing α under RIPI. These results are shown in Figure 6.15 and 6.16.

Figure 6.15

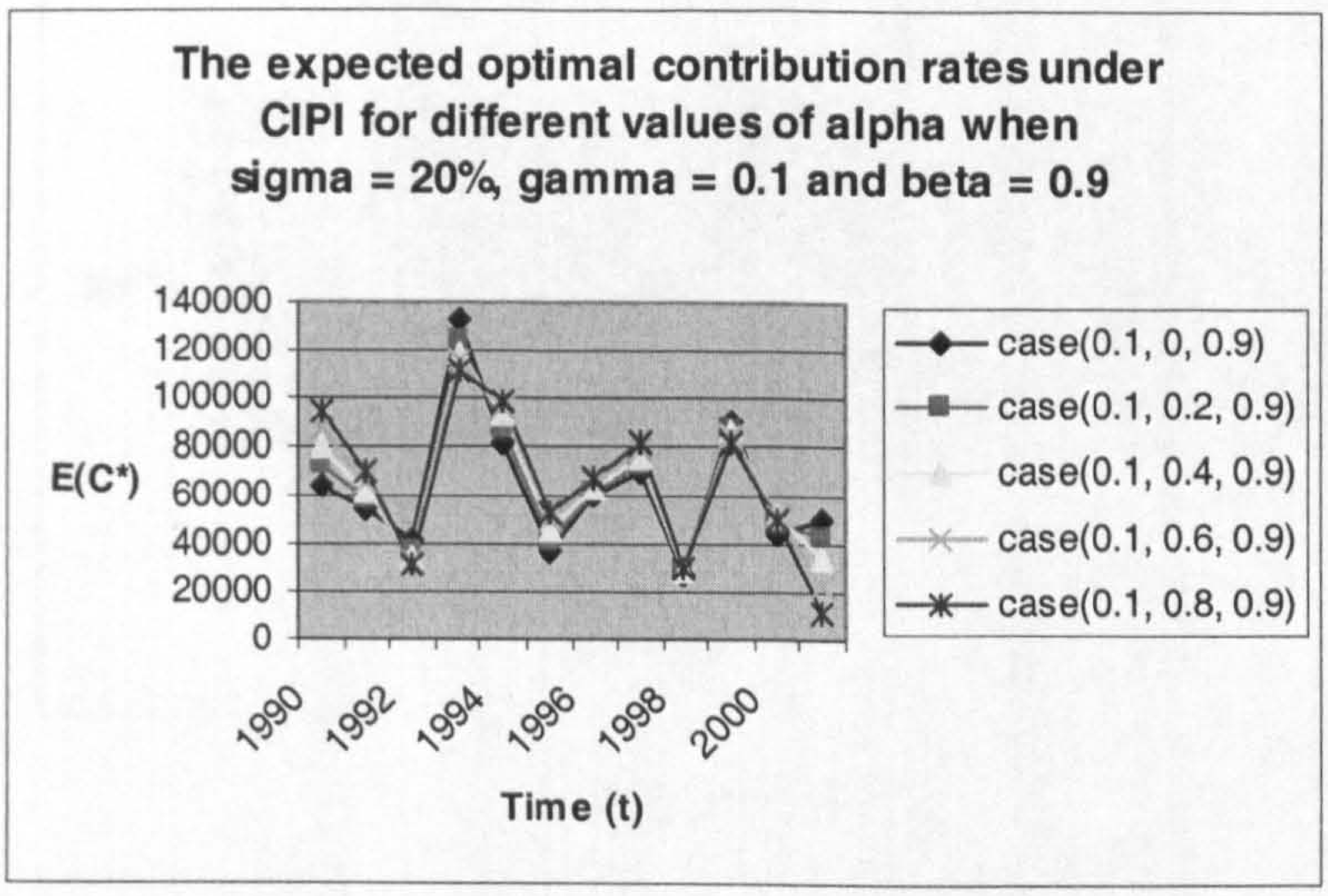
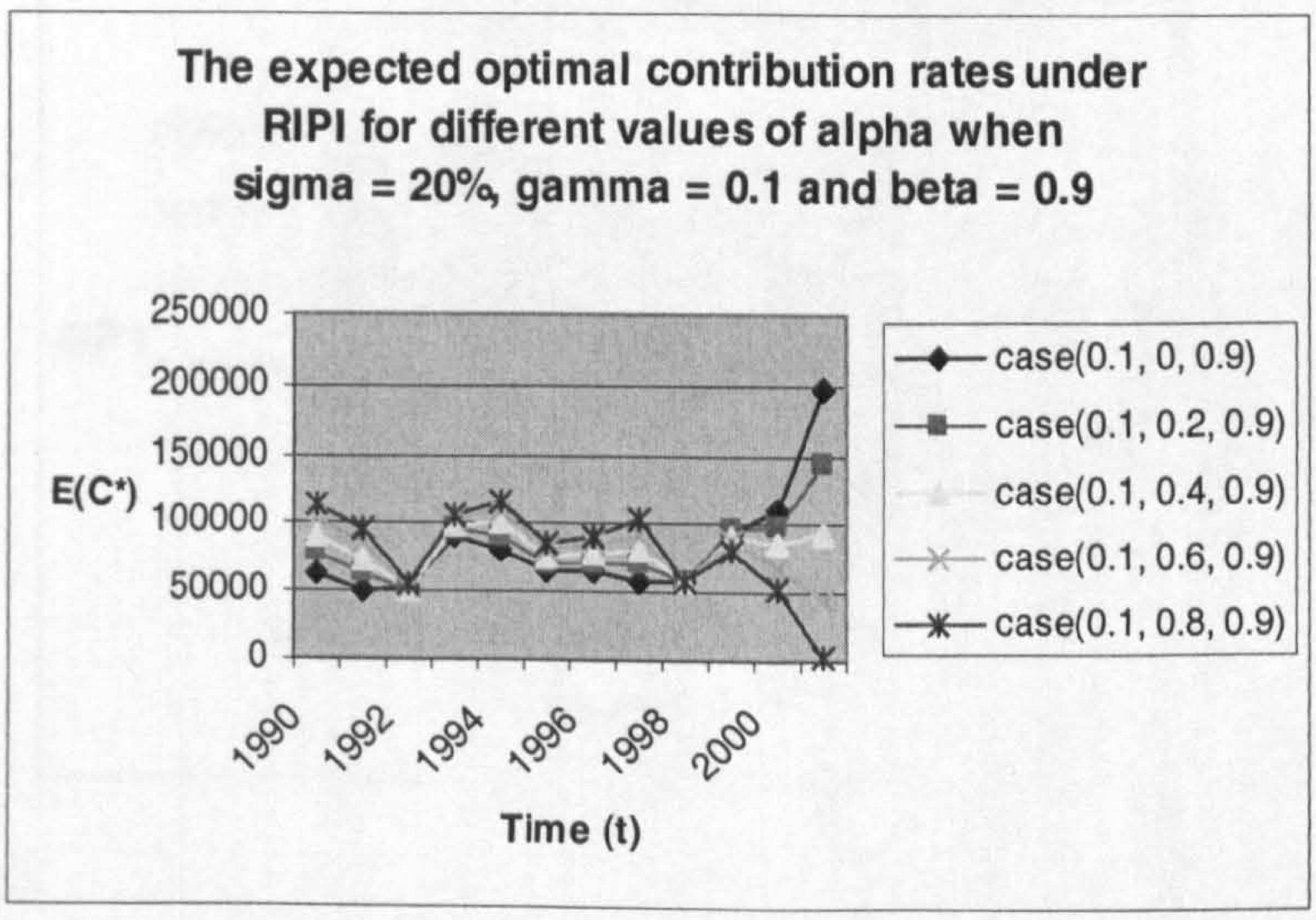


Figure 6.16



For the cases of (0.5, 0.5), increasing α leads $E(F_t^*)$ to have an upward trend under both models with higher levels compared with the ones obtained under the cases when

γ and β are equal to 0.1 and 0.9 respectively. Moreover, this increasing trend of $E(F_t^*)$ between cases reach higher levels under RIPI compared with those under CIPI. This increasing behaviour of $E(F_t^*)$ is shown in Figures 6.17 and 6.18. We can conclude that there is more deviation from the actual trend of the fund level in the scheme when α is increased.

Figure 6.17

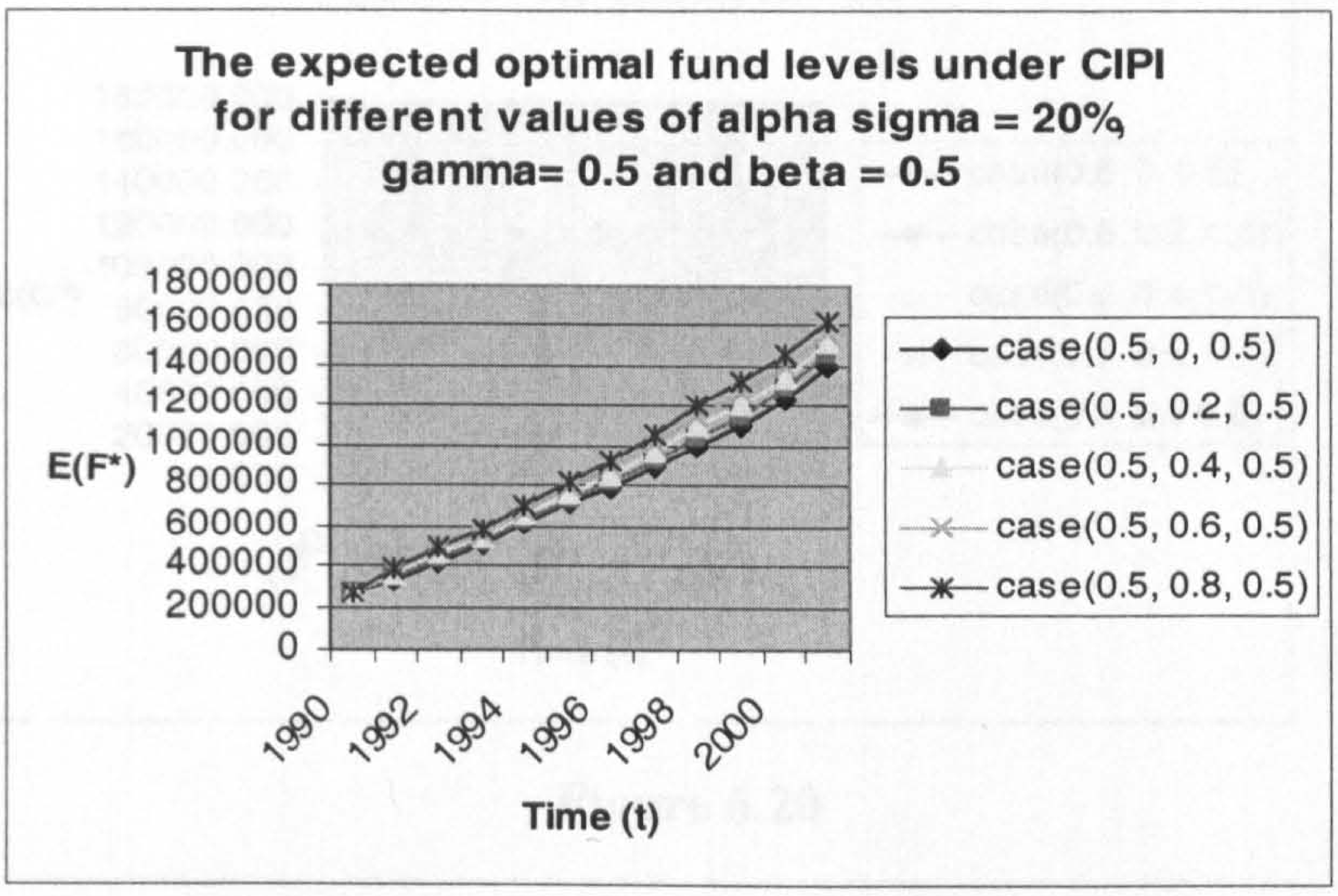
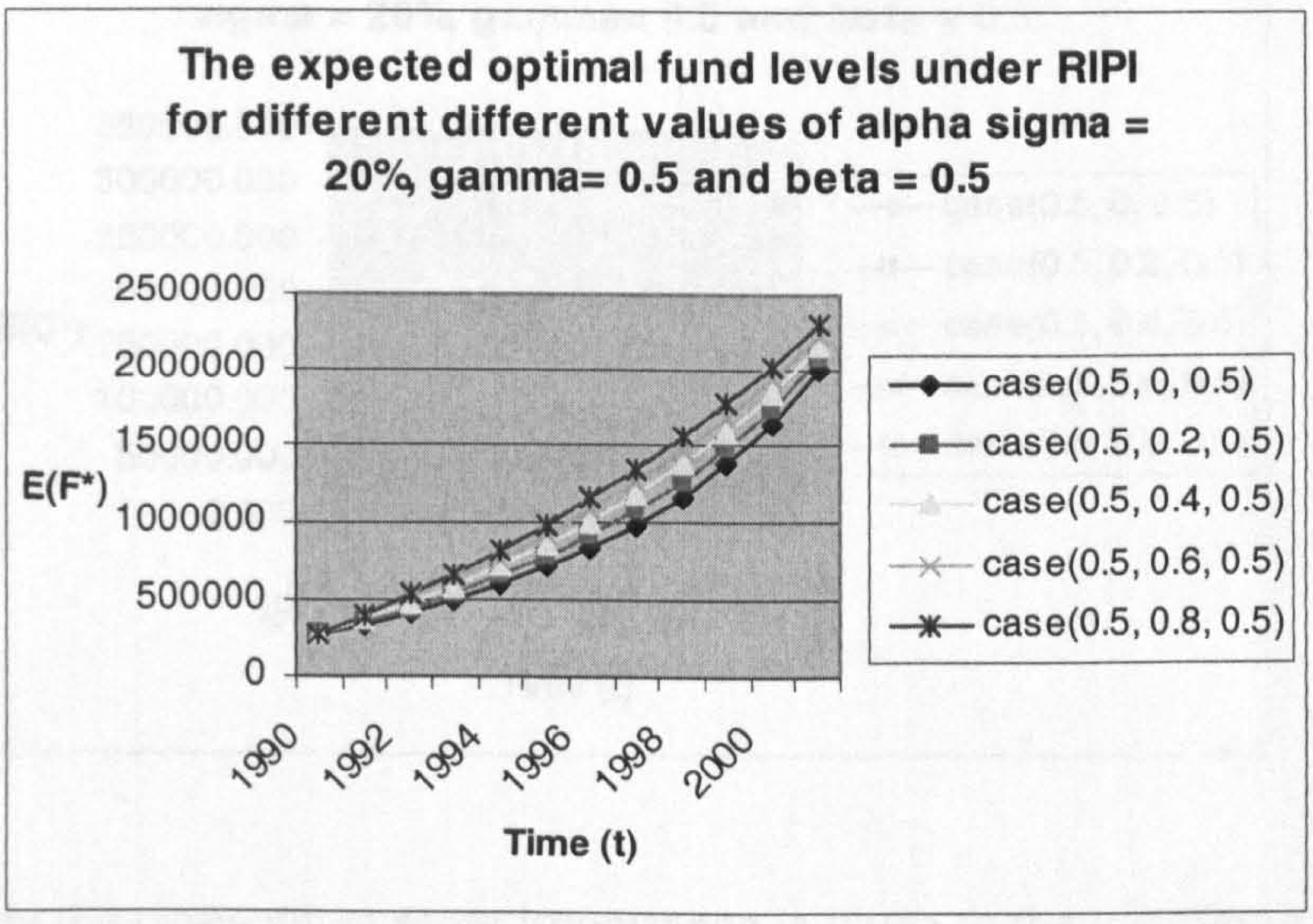


Figure 6.18



Consequently, the behaviour of $E(C_t^*)$ over time responds to the increased level of α in the cases where γ and β are equal to 0.5. Under CIPI, we note that the levels of $E(C_t^*)$ fluctuate and move further away as α is increased to 0.8.

Under RIPI, $E(C_t^*)$ has a more stable behaviour compared with CIPI. The levels of $E(C_t^*)$ decrease with less fluctuation when α is increased to 0.8. The behaviour of $E(C_t^*)$ under both models is shown in Figures 6.19 and 6.20.

Figure 6.19

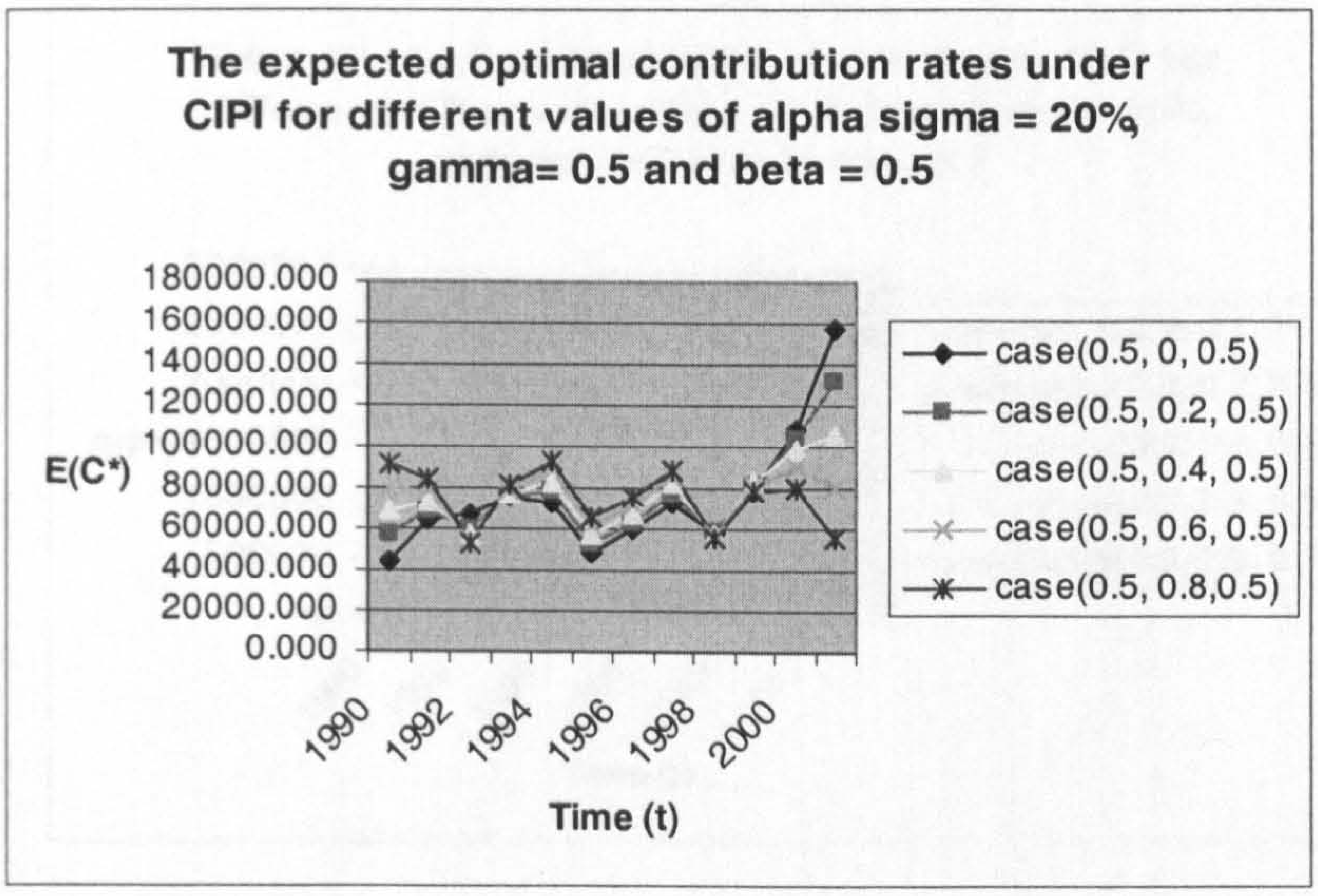
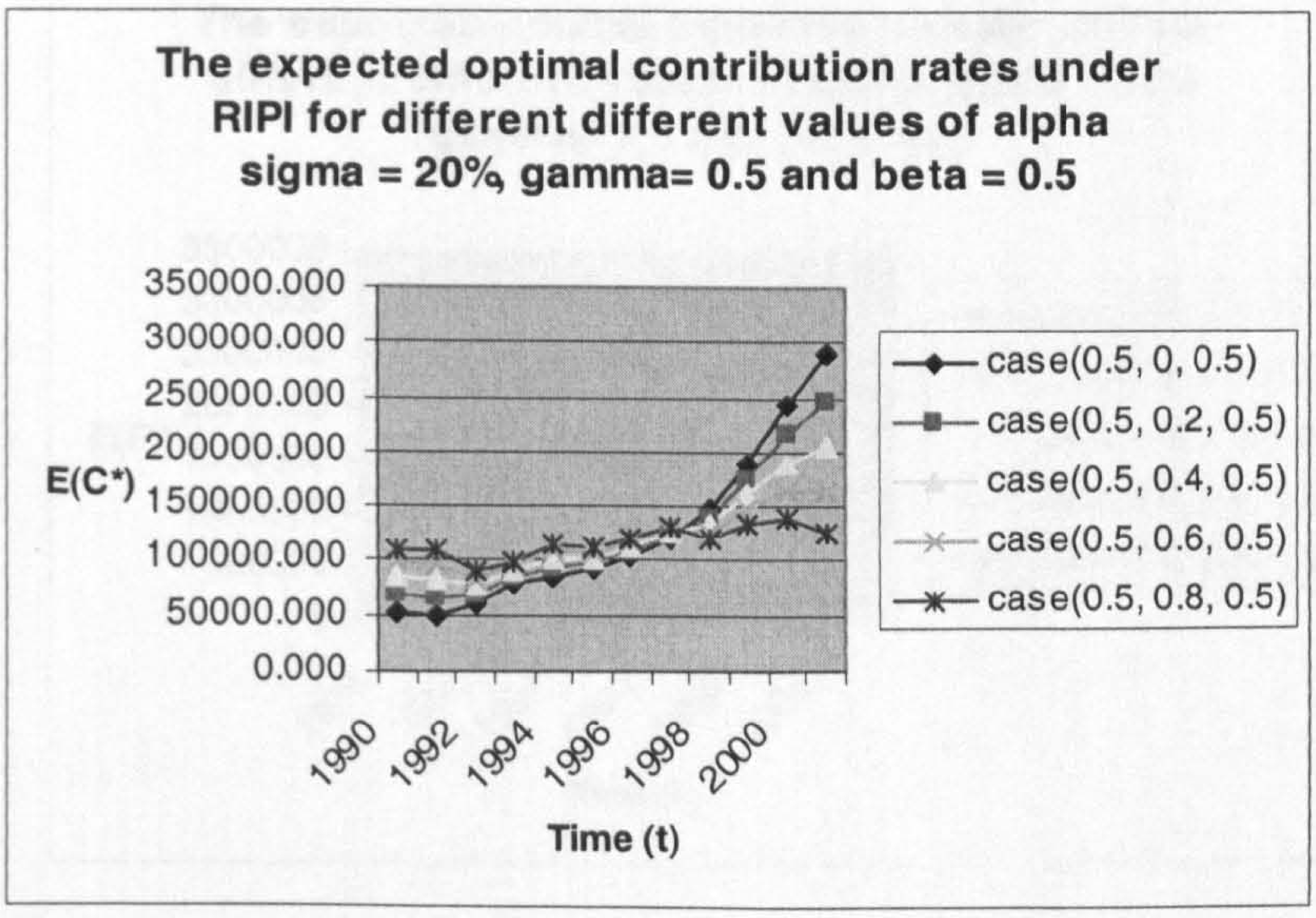


Figure 6.20



Finally, we study the cases when more importance is given to the weighting factor of the contribution rate risk along with increasing the weighting factor for the cross-product term (i.e. the cases of (0.9, 0.1)).

Thus, under both models, we find that $E(F_t^*)$ moves up to higher levels as less importance is given to β. Further, the levels of $E(F_t^*)$ increase more with the high values of α. Moreover, we note that the increasing levels of $E(F_t^*)$ are greater under RIPI compared with those under CIPI. These results are shown in Figures 6.21 and 6.22.

Figure 6.21

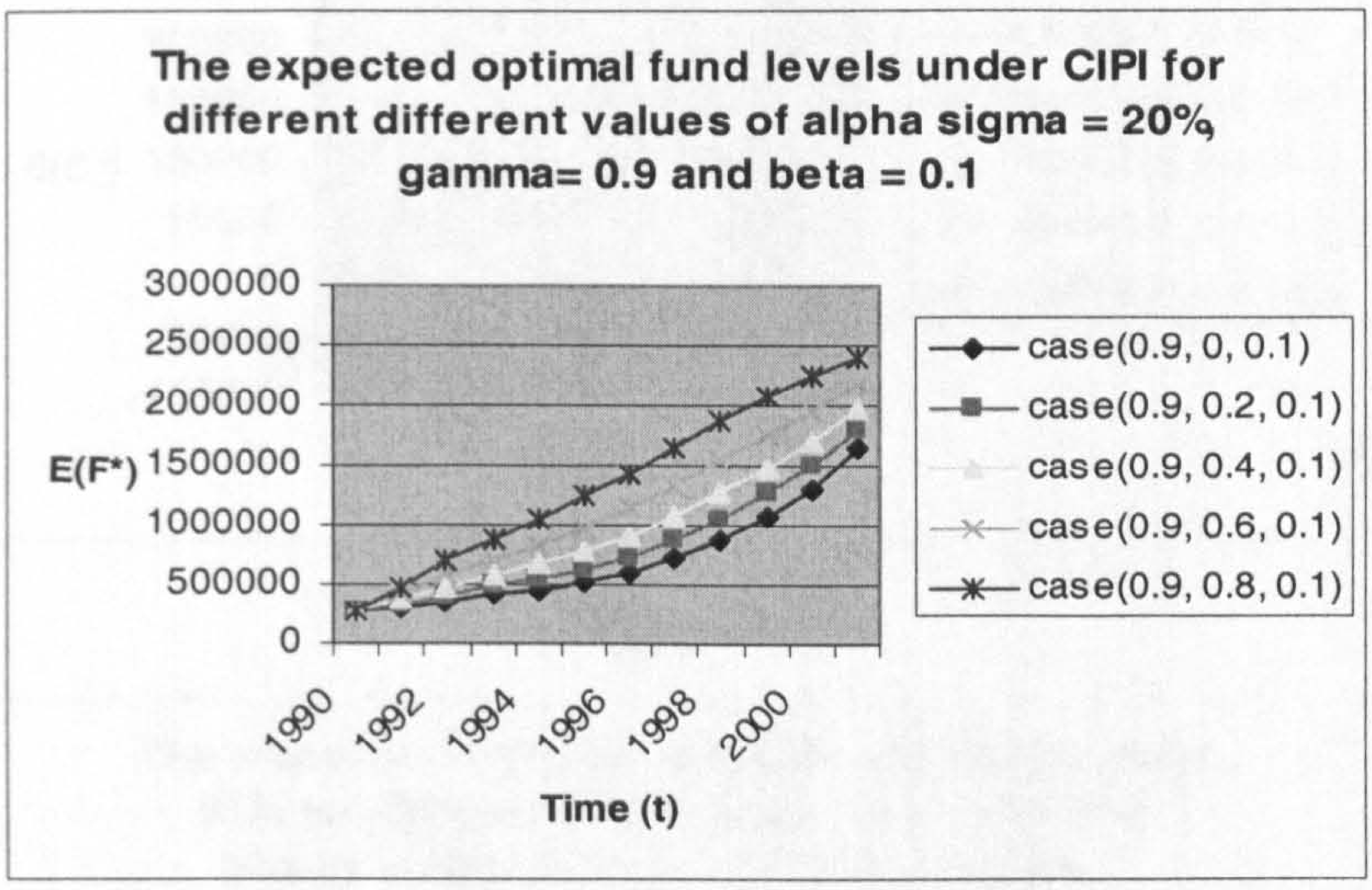
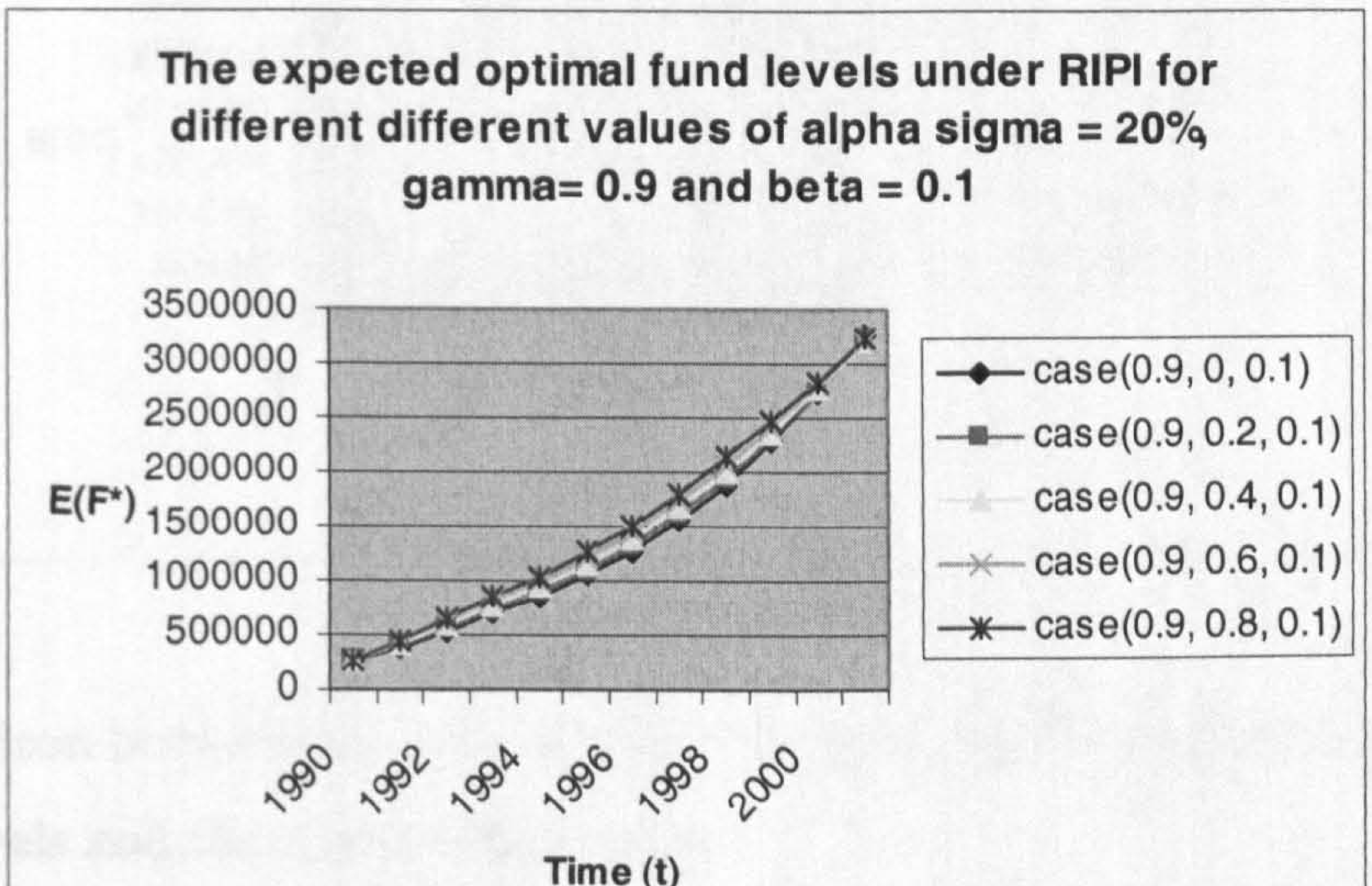


Figure 6.22



The trends of $E(C_t^*)$ fluctuate sharply when the value of α is increased to 0.8, under the CIPI model. A similar result is obtained for the behaviour of $E(C_t^*)$ under RIPI. However, the levels of $E(C_t^*)$ are more stable under RIPI compared with CIPI. These differences can be seen from Figure 6.23 and 6.24.

Figure 6.23

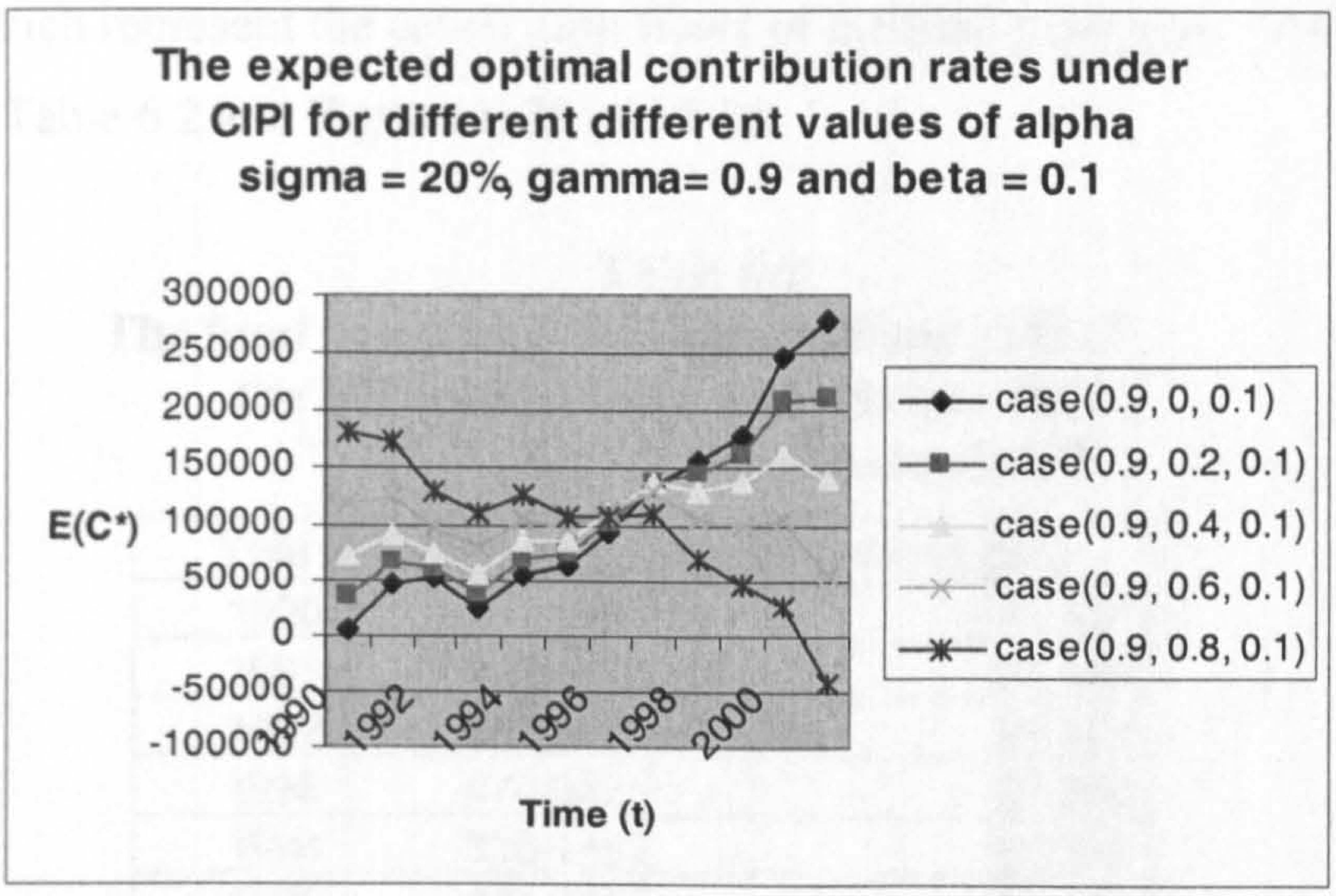
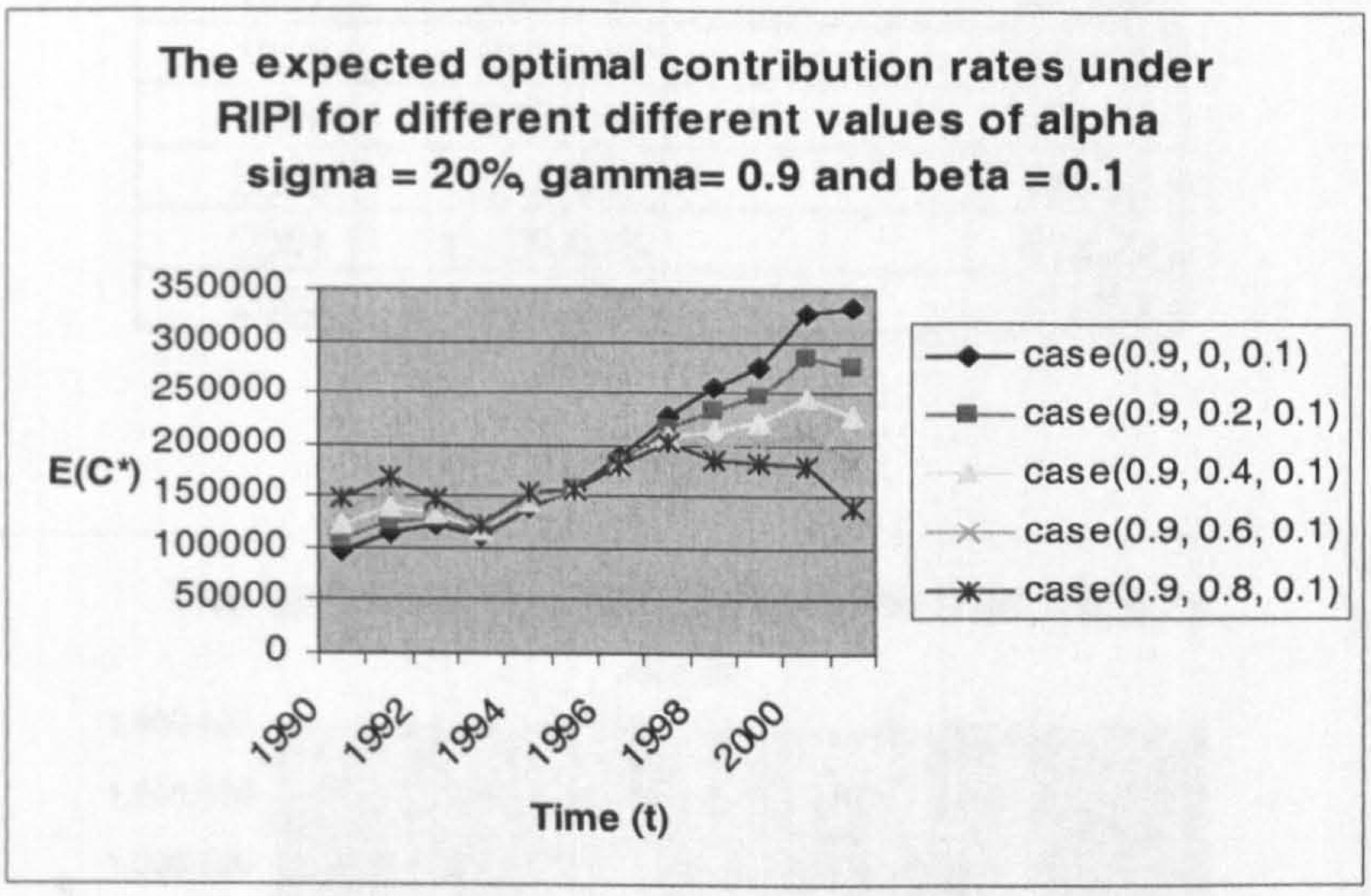


Figure 6.24



6.5.4 Comparison between the cash flows of the fund and the expected optimal fund levels and the contribution rates

The results of applying CIPI and RIPI from year 1990 to 2002 indicate different trends for $E(F_t^*)$ and $E(C_t^*)$, in response to changing the different parameters of the models. Therefore, it is interesting to finalize our analysis by comparing these trends with the actual cash flows of the fund. This comparison is useful to understand the differences between the actual cash flows and the expected ones, and to find out the best combination of the parameters which eliminate these differences.

Firstly, the fund assets and the contributions paid by both the employer and the employees which represent the actual cash flows of the fund from year 1990 to 2002 are shown in Table 6.2 and Figures 6.25 and 6.26.

Table 6.2
The fund assets and the contributions paid of the DB scheme from year 1990 to 2002
(in thousands LE)

Year	Fund assets	Contributions paid
1990	271,980	21,437
1991	323,812	22,936
1992	393,824	23,813
1993	474,692	27,374
1994	516,145	31,563
1995	570,266	28,745
1996	652,614	32,895
1997	749,233	41,288
1998	852,142	44,256
1999	946,676	47,704
2000	1,041,963	49,454
2001	1,135,018	60,523
2002	1,236,770	71,034

Figure 6.25

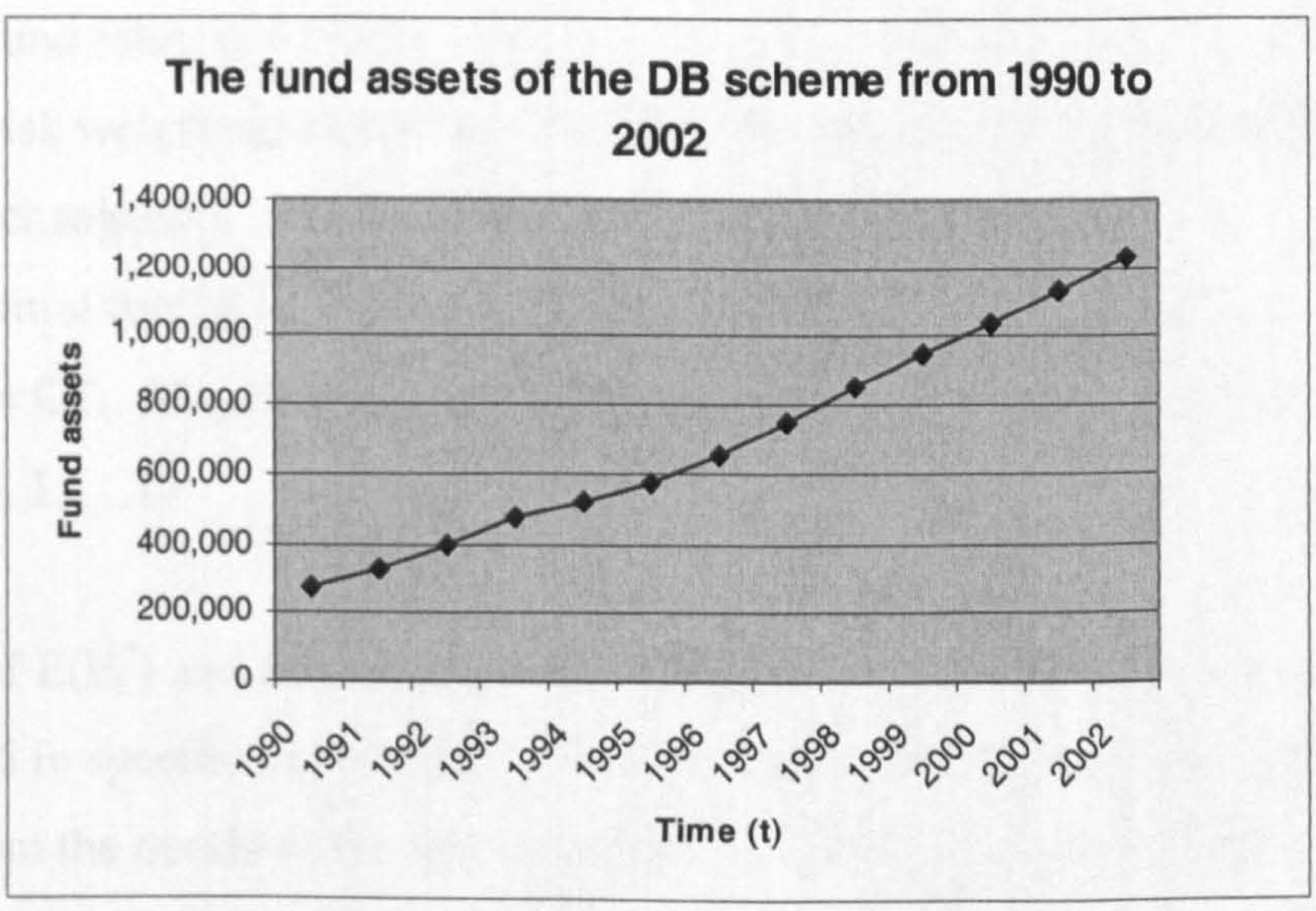
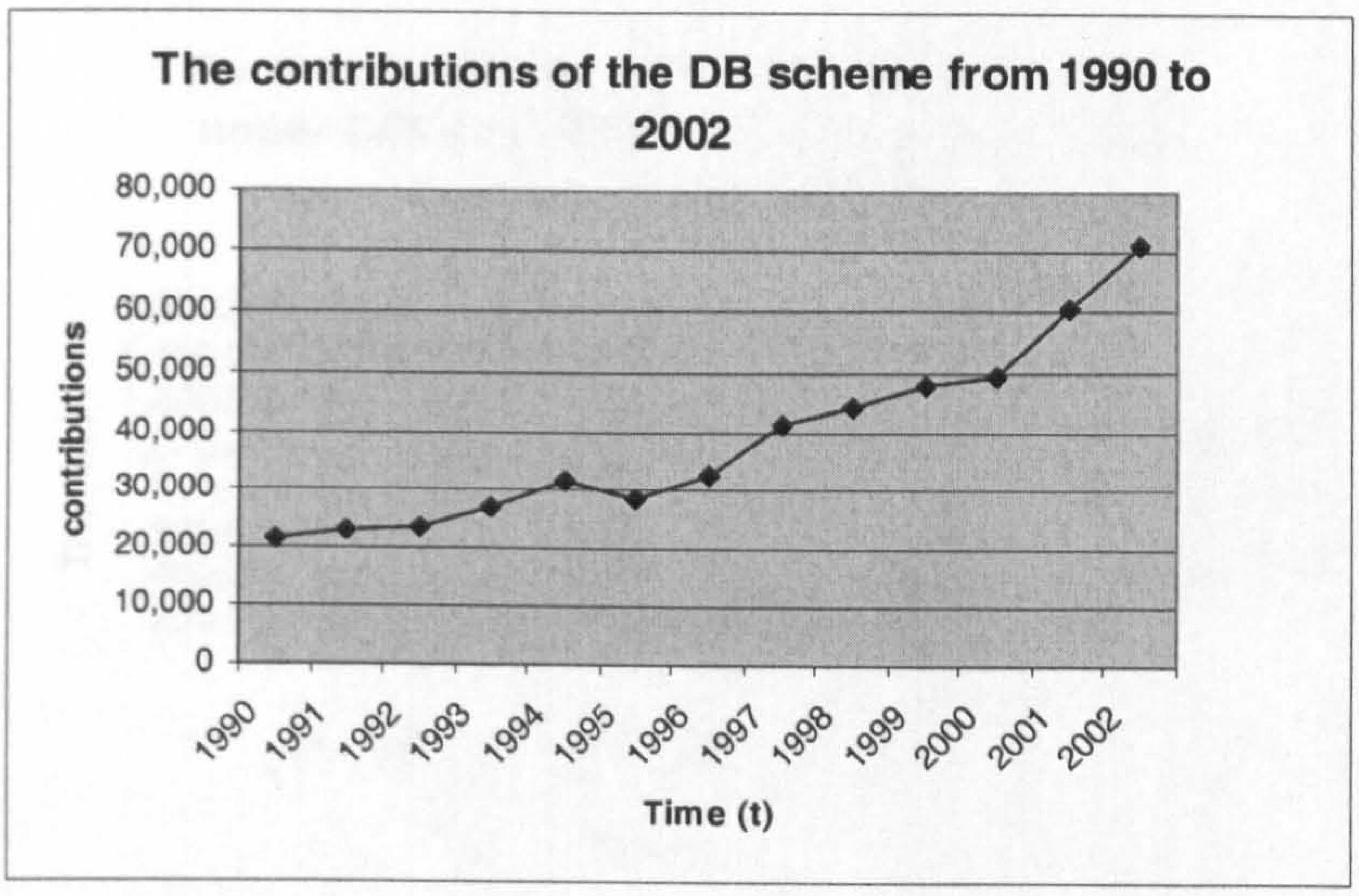


Figure 6.26



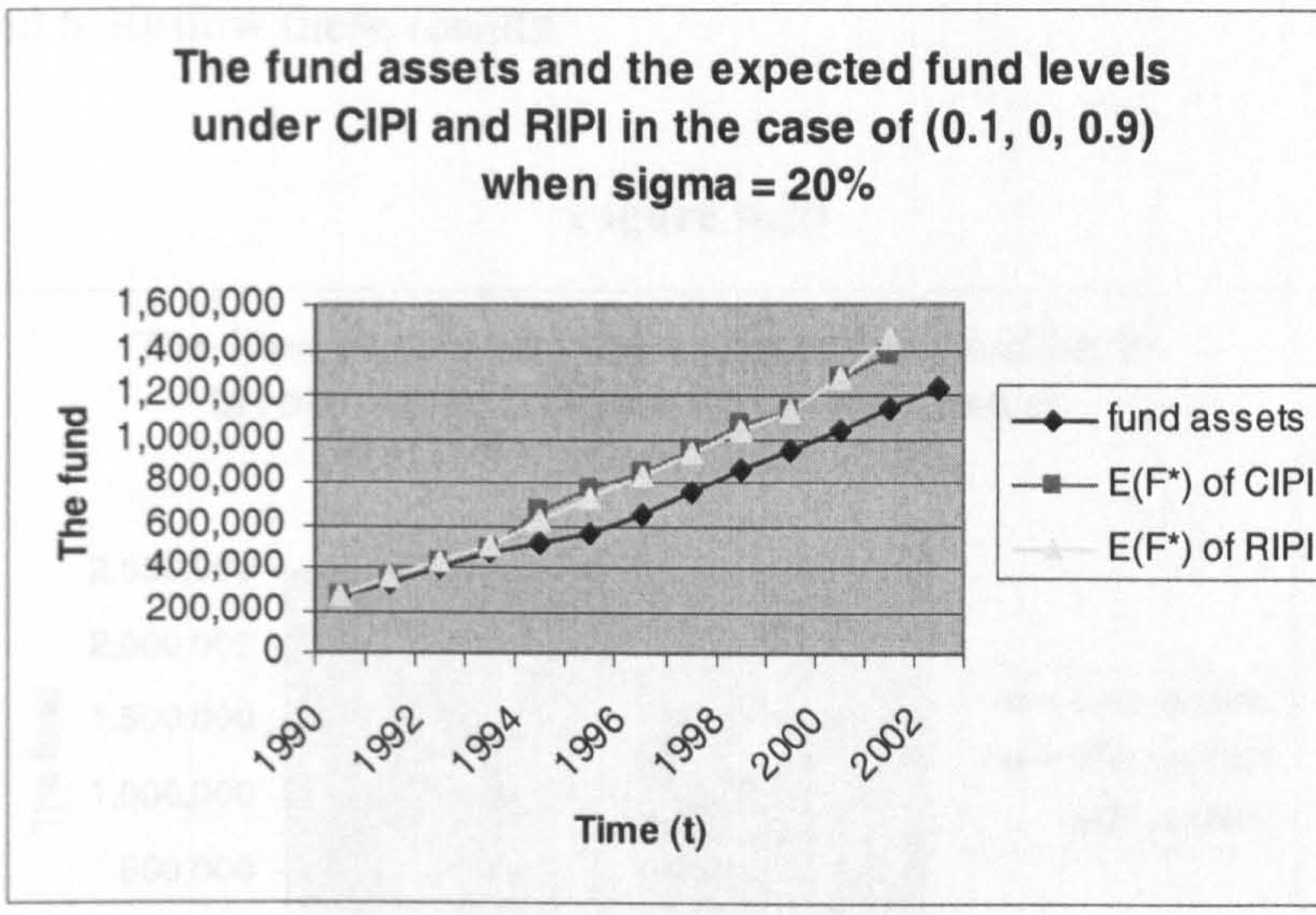
Secondly, we examine some cases to reach $E(F_t^*)$ and $E(C_t^*)$under both models CIPI and RIPI, where:

- The control period is represented from year 1990 to 2002, T = 12;
- the fund return parameters: $\theta = 9\%$, which is the valuation rate of return used in the actuarial reports of the fund, and σ is chosen to be equal to 20% to represent the volatility in the market;
- the fund ratio: $\eta = 100\%$;
- the risk weighting factors are chosen to be: γ and $\beta = 0.1, 0.5$ and 0.9 interchangeably, and the cross product risk factor $\alpha = 0, 0.4$;
- the initial fund $F_0 = F_{1990}$ = LE 271, 980,000;
- $NC_t = CT_t$, $AL_{t+1} = FT_{t+1}$ and B_t are obtained from Table 6.1 where $t = 0, 1,\ldots,T$.

The results of $E(F_t^*)$ and $E(C_t^*)$ show similar behaviour to the ones shown in Figures 6.25 and 6.26 in specific cases only. Alternatively, the other cases show more deviation from the trends of the actual cash flows - shown in Figures 6.25 and 6.26 - when different combination of the parameters are considered.

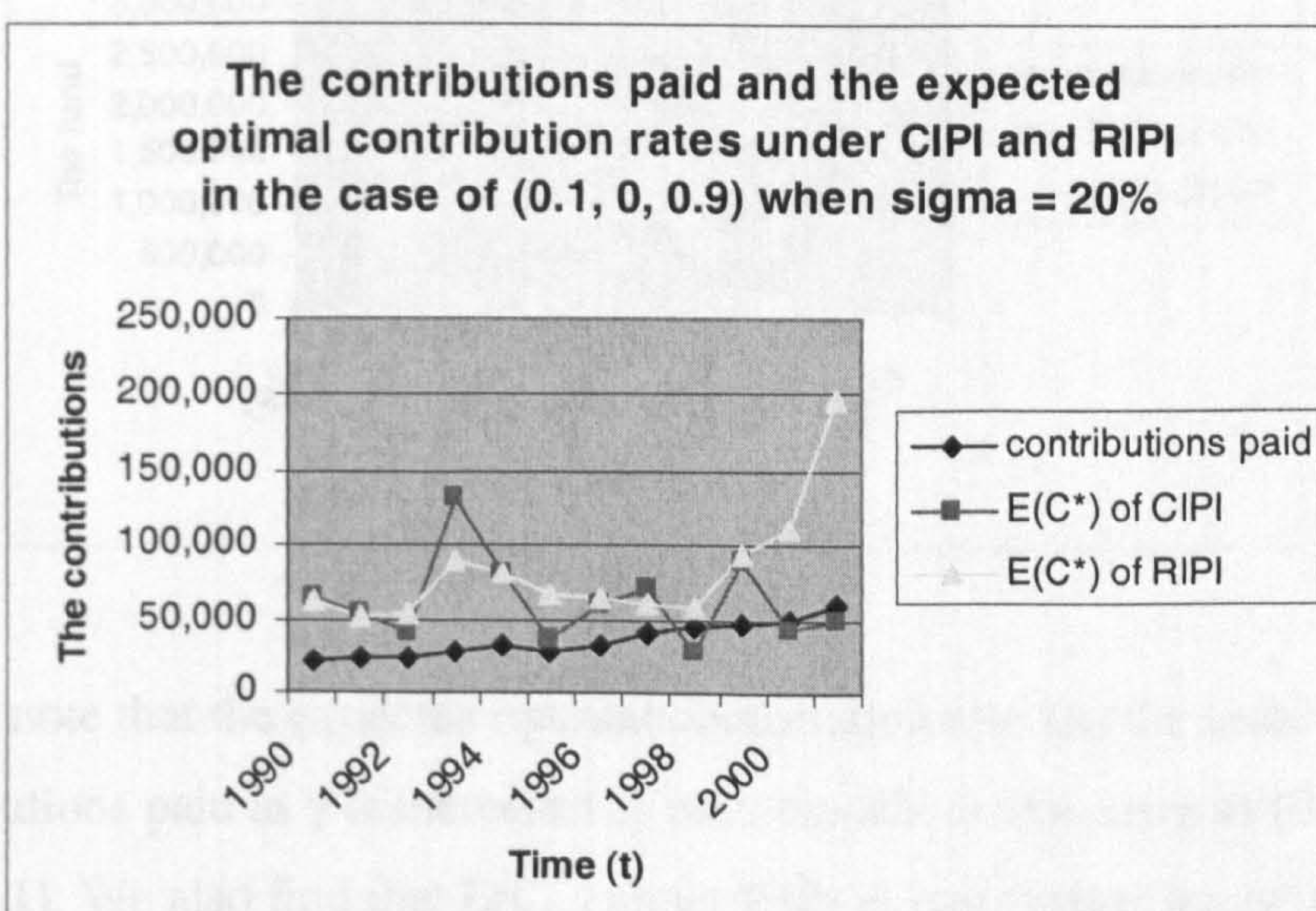
The expected optimal fund level, under CIPI and RIPI, shows that there is no major difference between both trends in the case of (0.1, 0, 0.9). This implies that when more importance is given to the weighting factor of the solvency risk, $E(F_t^*)$ has an upward trend very close to the trend of the fund assets as shown in Figure 6.27.

Figure 6.27



However, the trends of $E(C_t^*)$ in the same case of (0.1, 0, 0.9) have different behaviour under both models. Although $E(C_t^*)$ under RIPI is more similar to the contributions paid compared with the one under CIPI, both of them tend to diverge from the trend in the actual contributions paid. These different trends can be shown in Figure 6.28.

Figure 6.28



For the cases of (0.5, 0, 0.5) and (0.9, 0, 0.1), we note that $E(F_t^*)$ moves away from the fund assets trend as less importance is given to β. Furthermore, $E(F_t^*)$ under RIPI moves further away from the actual fund assets compared with the one under CIPI. Figures 6.29 and 6.30show these results.

Figure 6.29

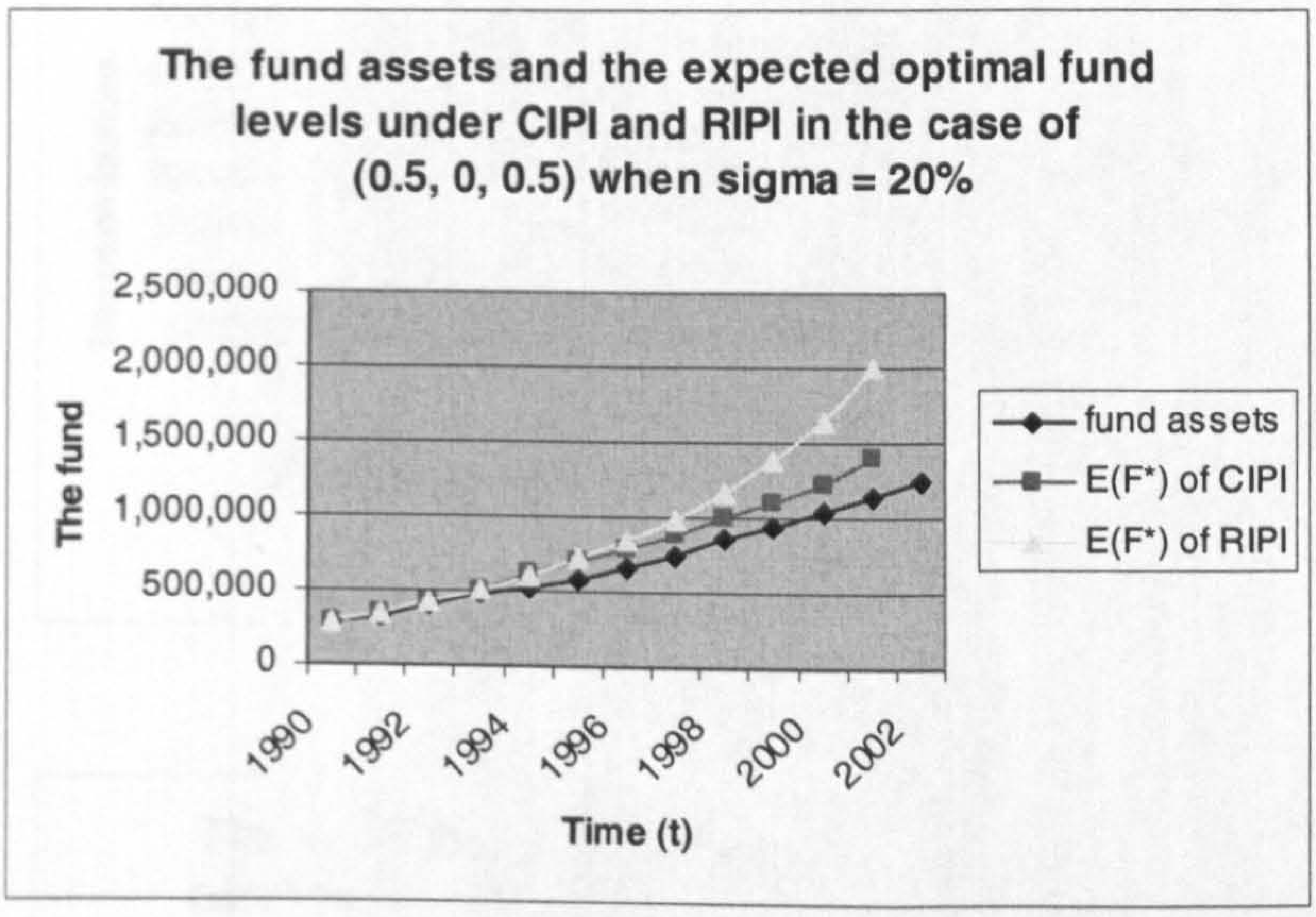
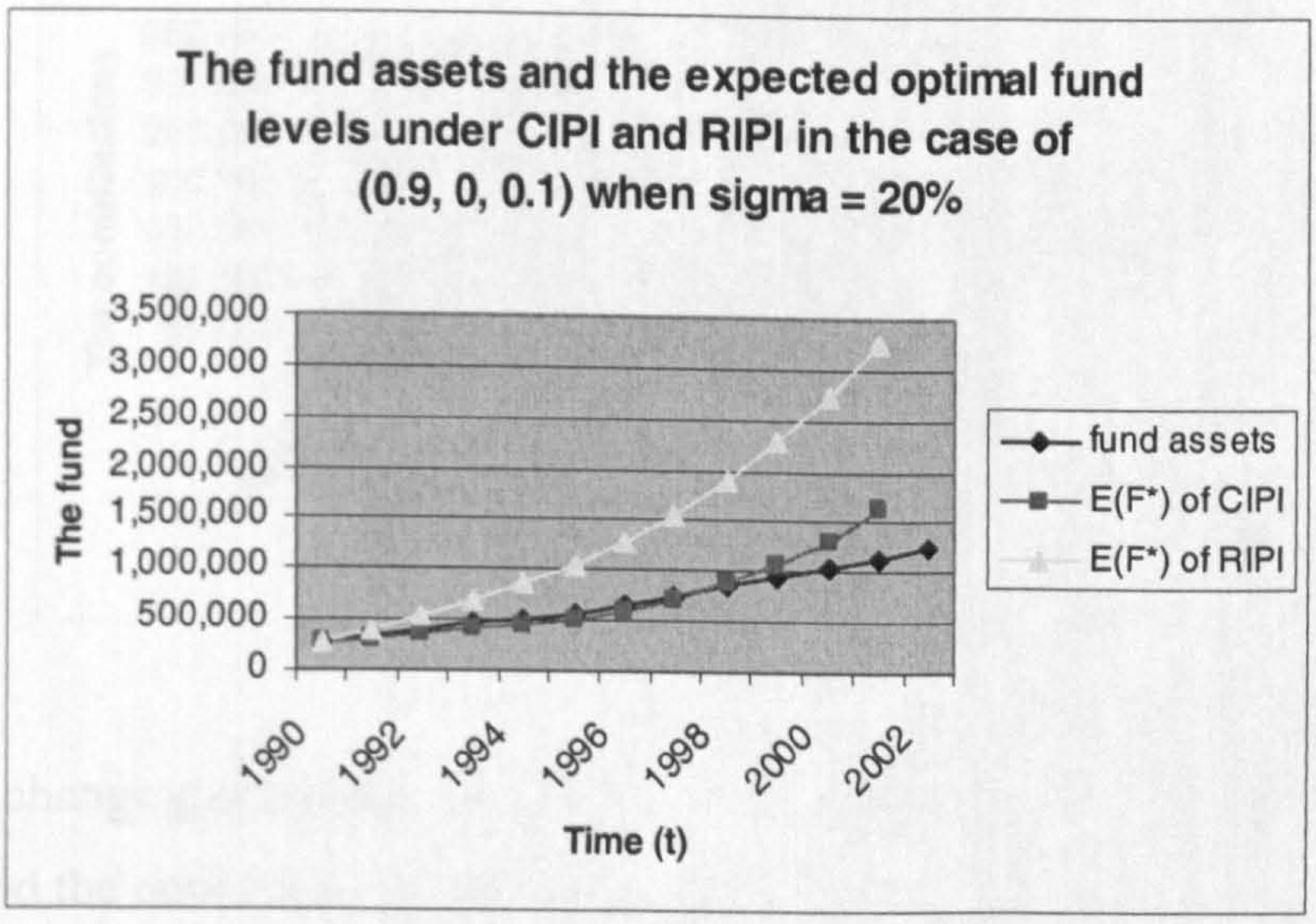


Figure 6.30



However, we note that the expected optimal contribution rate has the same behaviour of the contributions paid as γ is increased in both models, in the cases of (0.5, 0, 0.5) and (0.9, 0, 0.1). We also find that $E(C_t^*)$ under RIPI moves further away from the contributions paid compared with the one under CIPI in both cases. Figures 6.31 and 6.32 show the trends of $E(C_t^*)$ in the cases of (0.5, 0, 0.5) and (0.9, 0, 0.1) under both models.

Figure 6.31

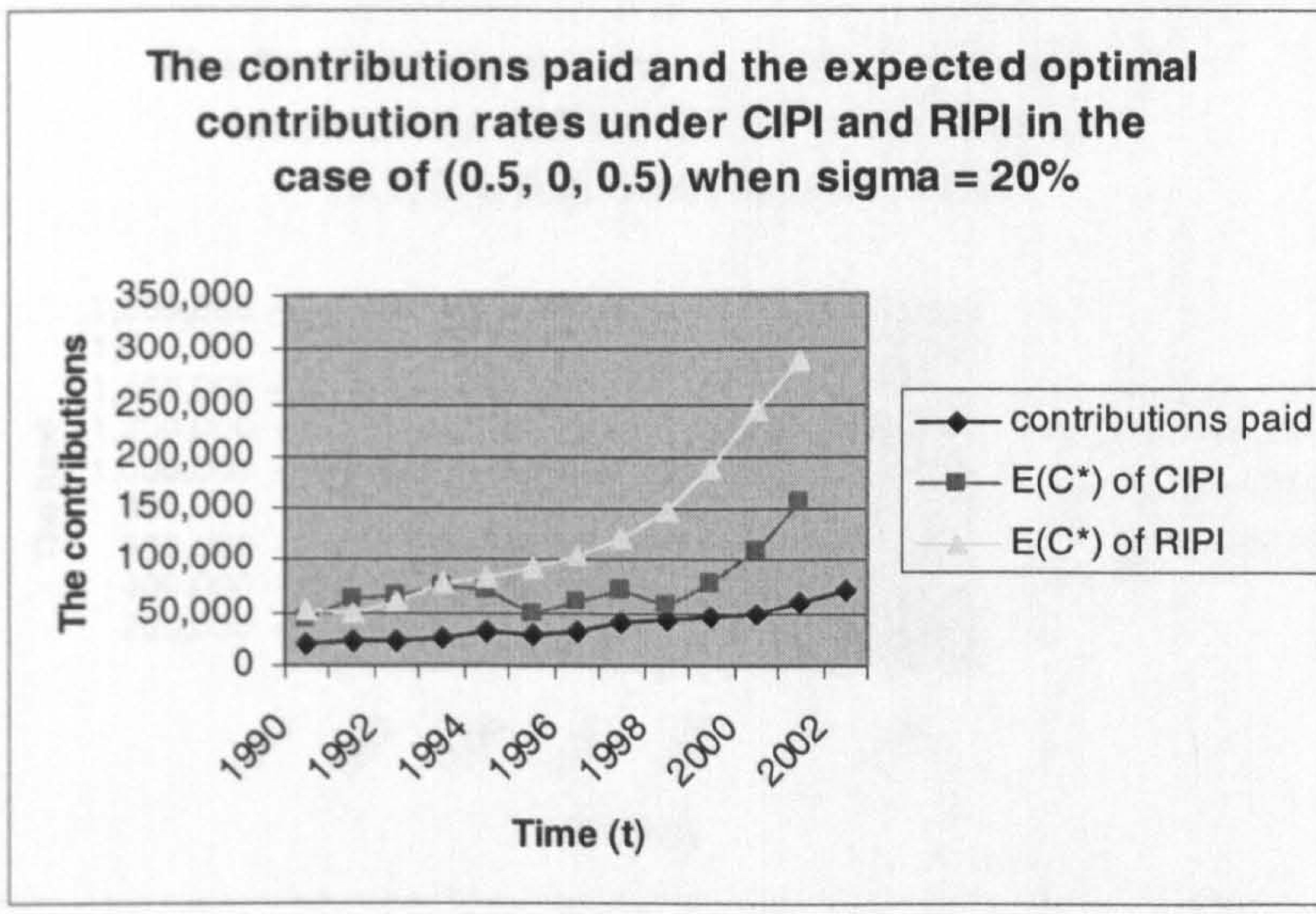
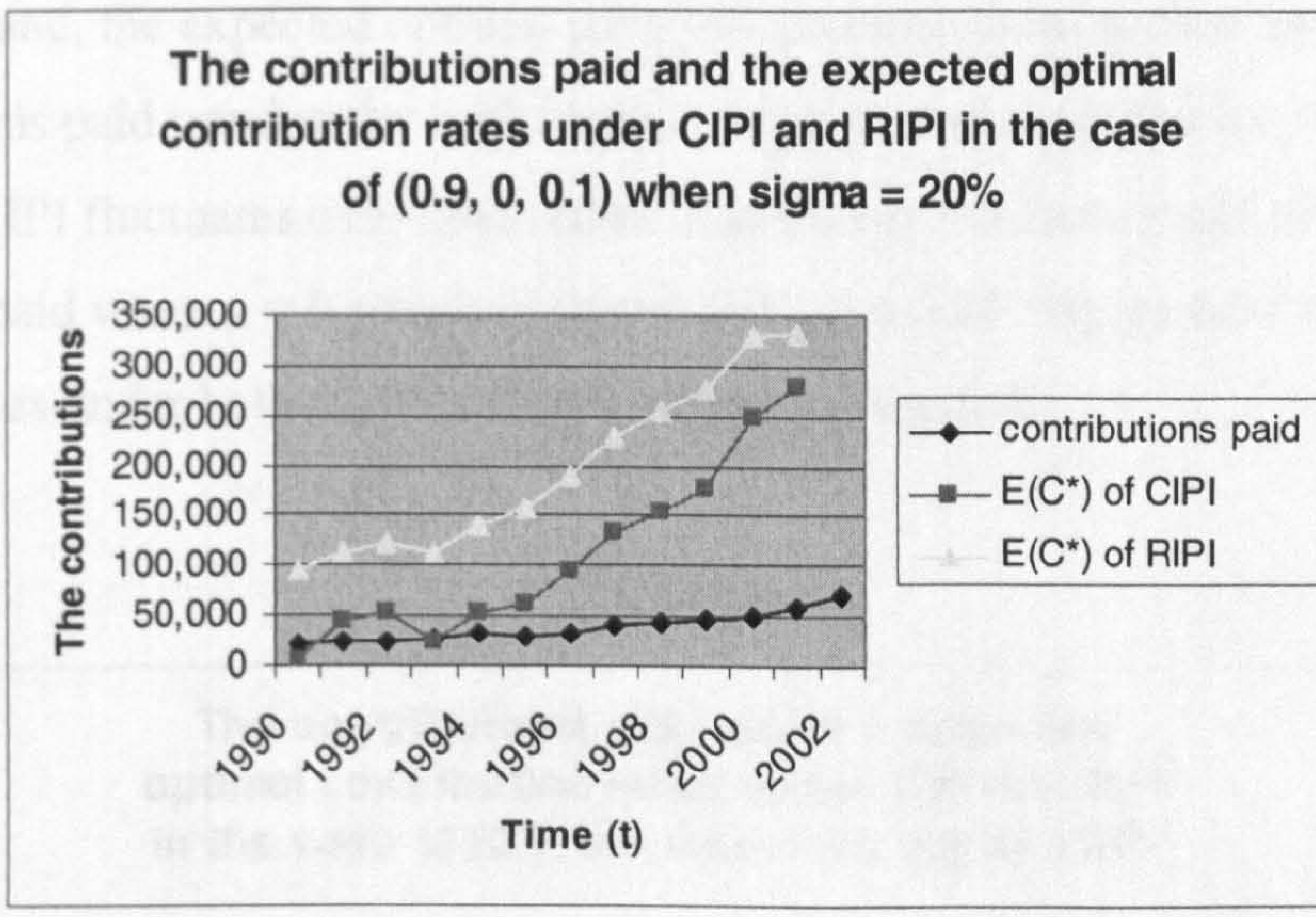
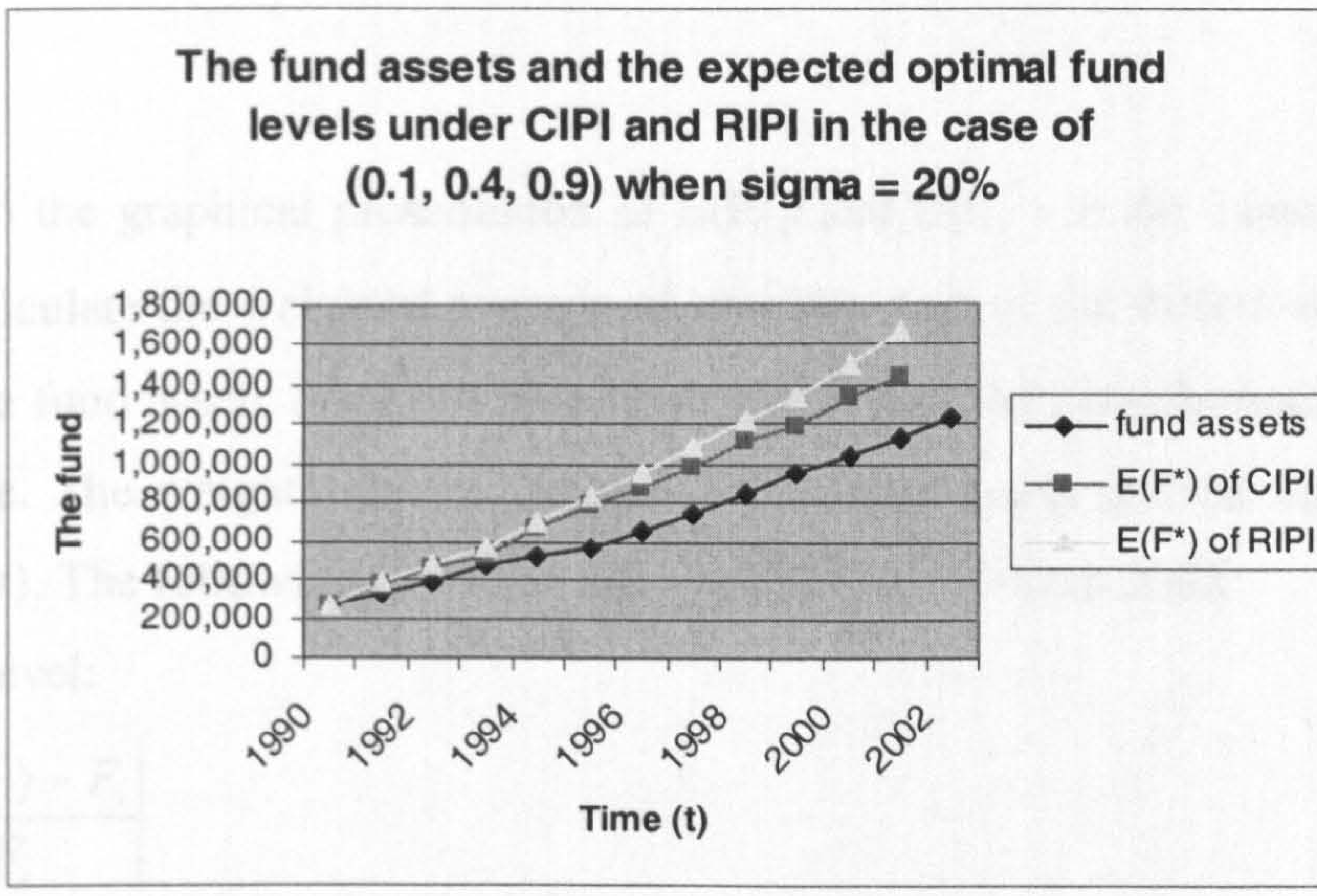


Figure 6.32



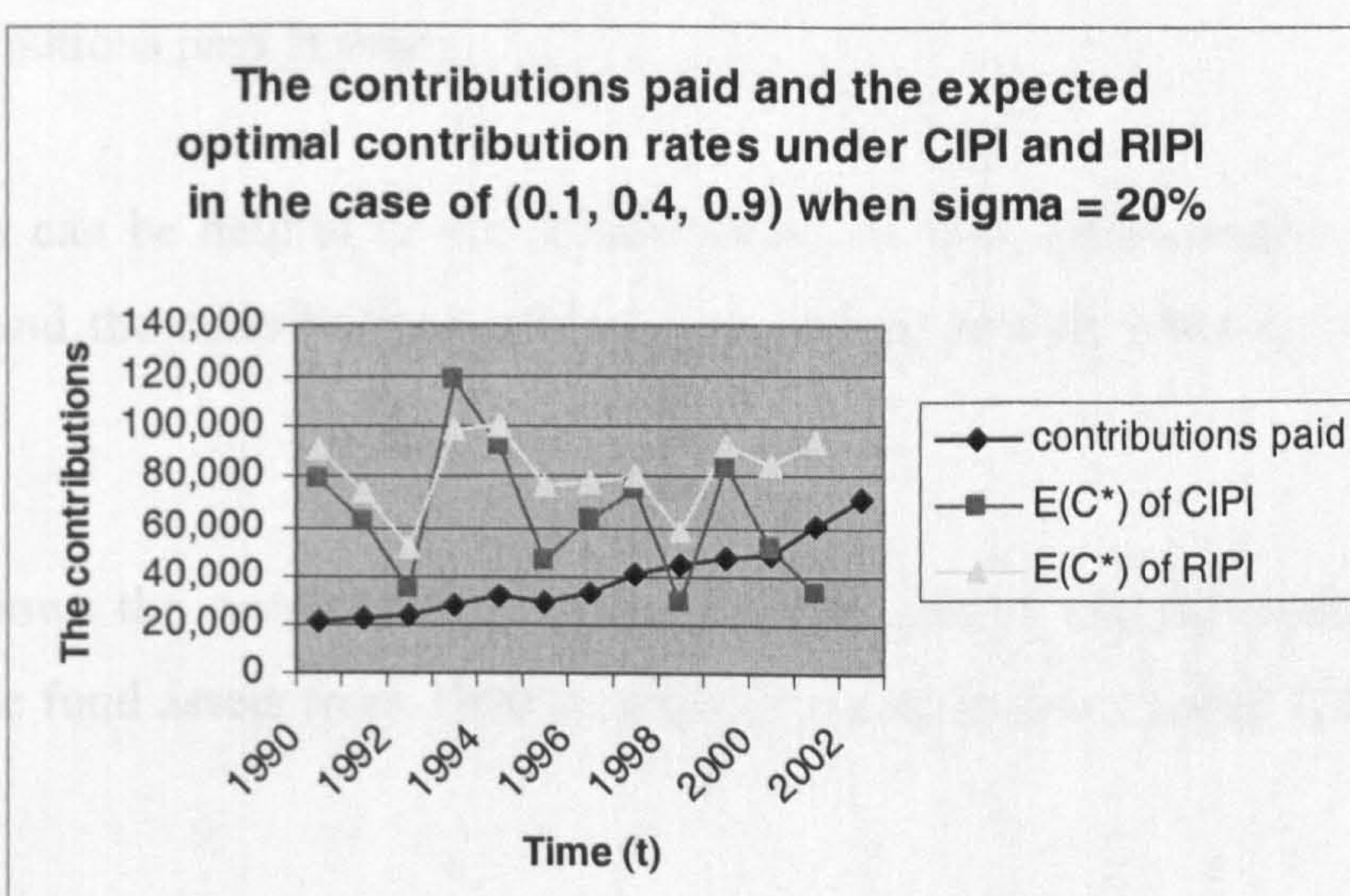
The effect of changing α leads both $E(F_t^*)$ and $E(C_t^*)$ to move further away from the fund assets and the contributions paid. For example, in the case of (0.1, 0.4, 0.9), we find that $E(F_t^*)$ deviates more from the actual fund assets and moves up under both models compared with the same case when $\alpha = 0$. Figure 6.27 can be compared with Figure 6.33 to realize the difference resulting from increasing α to 0.4.

Figure 6.33



On the other hand, the expected optimal contribution rate moves further away from the contributions paid trend under both models when α is increased to 0.4. In fact, $E(C_t^*)$ under RIPI fluctuates over time rather than having the same trend of the contributions paid when α = 0 which is shown in Figure 6.28. Figure 6.34 illustrate these differences under both models due to increasing α to 0.4.

Figure 6.34



For the cases of (0.5, 0.4, 0.5) and (0.9, 0.4, 0.1), similar results have been reached. In both cases, $E(F_t^*)$ and $E(C_t^*)$ move further away from the fund assets and the contributions paid compared with those when α = 0. Although the expected contribution rates tend to have a similar behaviour of the contributions paid when γ is increased.

In addition to the graphical presentation of $E(F_t^*)$ and $E(C_t^*)$ in the cases mentioned above, we calculate the weighted average of absolute sum of the differences between $E(F_t^*)$ and the fund assets from one side, and $E(C_t^*)$ and the contributions paid from the other side. The weights are the inverse of the fund assets and the contributions paid at time (t). The following formulae are applied from 1990 to 2002:

for the fund level:

$$\frac{1}{12}\sum_{t=1}^{12}\left|\frac{E(F_t^*)-F_t}{F_t}\right| \tag{6.10}$$

and for the contribution rate:

$$\frac{1}{12}\sum_{t=1}^{12}\left|\frac{E(C_t^*)-C_t}{C_t}\right| \tag{6.11}$$

where:

$E(F_t^*)$: the expected optimal fund level at time t;

$E(C_t^*)$: the expected optimal contribution rate at time t;

F_t: the fund assets at time t;

C_t: the contributions paid at time t.

This analysis can be helpful in our estimation to the best representative case of the fund assets and the contributions paid of this defined benefit scheme from 1990 to 2002.

Table 6.3 shows the weighted average of absolute sum of the differences between $E(F_t^*)$ and the fund assets from 1990 to 2002 for different cases under CIPI and RIPI models.

Table 6.3

The weighted average of absolute sum of the differences between $E(F_t^*)$ and the fund assets for different cases under CIPI and RIPI

Cases	CIPI model	RIPI model
(0.1, 0, 0.9)	0.1894	0.1743
(0.5, 0, 0.5)	0.1437	0.2759
(0.9, 0, 0.1)	0.0175	0.8888
(0.1, 0.4, 0.9)	0.2376	0.3405
(0.5, 0.4, 0.5)	0.2300	0.4647
(0.9, 0.4, 0.1)	0.3750	0.9958

Table 6.4 shows also the weighted average of absolute sum of the differences between $E(C_t^*)$ and the contributions paid from 1990 to 2002 for different cases under CIPI and RIPI.

Table 6.4

The weighted average of absolute sum of the differences between $E(C_t^*)$ and the contributions paid for different cases under CIPI and RIPI

Cases	CIPI model	RIPI model
(0.1, 0, 0.9)	0.9592	1.3052
(0.5, 0, 0.5)	1.1498	2.2532
(0.9, 0, 0.1)	1.6766	4.2865
(0.1, 0.4, 0.9)	1.0459	1.4945
(0.5, 0.4, 0.5)	1.2533	2.3904
(0.9, 0.4, 0.1)	2.0059	4.0426

From Table 6.3 and 6.4, we find that the average deviation of the fund level is generally less than the deviation of the contributions paid. This can be explained by the high fluctuation of the expected optimal contribution rates compared with the actual cash flows shown in the previous figures. Furthermore, the lowest weighted average in the fund assets is obtained in the case of (0.9, 0, 0.1) under CIPI, while the case of (0.9, 0.4, 0.1) under RIPI has the highest weighted average in the fund assets. On the other hand, the lowest weighted average obtained in the contributions paid is in the case of (0.1, 0, 0.9) under CIPI where as the highest is in the case of (0.9, 0.4, 0.1) under RIPI.

Therefore, there is no one case has the lowest weighted average in both the fund assets and the contributions paid according to the data analysis. Hence, the weighted average deviation of the following cases are considered relatively small for both the fund assets and the contributions paid:

- the cases of (0.1, 0, 0.9), (0.5, 0, 0.5) and (0.1, 0.4, 0.9) under CIPI model;

- the cases of (0.1, 0, 0.9) and (0.1, 0.4, 0.9) under RIPI model.

Under both models, it is noted that the weighted average deviation is small in the cases where more importance is given to the solvency risk especially when the risk factor of the cross product term $\alpha = 0$. The weighted average deviation becomes higher when γ is high and equal to 0.9.

Therefore, in this Fund, the dynamic pension funding models are close to the realistic approach, when the management of the fund seeks for the interest of the members (i.e. to keep the fund solvent) over the interest of employer to keep the contribution rate stable. It is also clear that including a fund ratio of 100% in the application of the dynamic models give more similar results to those obtained in the real practice.

Finally, it is noted that the graphical presentation of the expected and actual cash flows shows more stable trends of $E(F_t^*)$ and $E(C_t^*)$ in the different cases under RIPI model than those under CIPI model. This is more clear in the case of (0.1, 0, 0.9) when more importance is given to the solvency risk. Here, it is worth mentioning that Chang (2000) has used the projection approach to estimate the expected optimal fund level and contribution rate with one set of risk parameters. Although he has concluded that the ratio-induced performance measure is more robust than the cost-induced performance measure, our data analysis of the past experience (in this defined benefit pension scheme from 1990 to 2002) shows that the results of $E(F_t^*)$ and $E(C_t^*)$ have small weighted average deviation from the actual cash flows in some cases under both CIPI and RIPI models.

Summary

The application of the stochastic models of cost-induced performance index and ratio-induced performance index is a main objective of our research. Thus, in this chapter, we introduced an Egyptian Private Pension Fund as a case study. This scheme is a defined benefit scheme and one of the largest according to its fund assets. We have applied a different approach than the one used by Chang (1999, 2000) to test our dynamic pension funding models.

Thus, our analysis of the expected cash flows of the scheme was based on the past data achieved over the years from 1990 to 2002. The effect of changing the levels of volatility and the weighting risk parameters are examined with the effect of changing the parameter of the cross-product term.

The application of stochastic CIPI and RIPI models on the data of the Egyptian Pension Fund reveals that there are few cases in which $E(F_t^*)$ and $E(C_t^*)$ are close to the actual cash flows. These cases are obtained under both models when more importance is given to the solvency risk especially when $\alpha = 0$ and $\eta = 100\%$.

Chapter Seven
Conclusions and further work

7.1 Conclusions

The project has been undertaken to analyze dynamic pension funding models and test their application in the real practice. A full sensitivity analysis is carried out to understand the properties of the models. A defined benefit Egyptian Private Pension Fund is chosen as a case study in order to examine the behaviour of our models in reality. The main findings throughout the thesis are described below.

Chapter 1 is an introduction about the different types of pension schemes. It highlights the importance of applying dynamic pension funding plans rather than the static ones in the pension schemes. The economic changes that have taken place in the Egyptian market, and the growing role of the Private Pension Funds justify the choice of the practical implementation of our models in Egypt.

Chapter 2 summarizes the social security system in Egypt in order to provide a background of the environment where the models are applied on. The social security system in Egypt is covered through two main pillars: the State Social Insurance System and the Occupational Pension Schemes or Private Pension Funds as they are called in Egypt. Contracted-Out Schemes and personal insurance policies also offer social security coverage, but they are not as important as the former ones.

The State Social Insurance System (SSIS) is regulated by four laws in which all the working people are covered. The employees in the government, public and private sectors constitute the majority of the working people covered under the Law no 79 of 1975 (General Social Insurance System). The General Social Insurance System is considered a defined benefit pension scheme, funded by two separate funds: the Government Sector Fund and the Public and Private Sectors Fund. The pension at NRA, death and disability is offered to the members and their dependants, and it is based on the basic and variable salaries. Other benefits are also offered to cover various contingencies.

The Private Pension Funds are considered the main complementary system to the SSIS. They also cover the employees under government, public and private sectors. The Funds are governed by Law no. 54 of 1975 which requires the funds to be registered in the Egyptian Insurance Supervisory Authority (EISA). Thus, the EISA is responsible of supervising and controlling the activities of the Funds. The types of the contributions and benefits differ between schemes according to the statute of each scheme.

The investment strategies, adopted by the SSIS and Contracted-Out schemes as well as the Private Pension Funds, appear to be prudent. More than 90% of the amount invested of SSIS and COS is allocated in the National Investment Bank (NIB). Further, most of the assets of Private Pension Funds are invested in Government bonds and bank deposits. The investment in other types of assets including equities is really limited.

There is a need to reform the SSIS, as the main problem that faces the system is the distribution of the resources between the poor and the better off. The Private Pension Funds face problems of discontinuance resulting from different reasons, e.g. debts, actuarial deficit and misappropriate management. Therefore, more restrictions are needed in the law to be applied in managing the funds. Increasing the annuity and life insurance market (which is still limited) provides a good solution to the reform of the social security programs in Egypt.

The objective of Chapter 3 is to derive the dynamic pension funding models in order to reach the optimal contribution rate. Chapter 3 firstly provides the basic concept of the pension funding for the defined benefit pension schemes, the actuarial valuation and the difference between the general and dynamic pension funding plans. Secondly, it reviews the literature of using the dynamic programming and the control theory in pension funding. Finally, the dynamic pension funding models are derived.

The last part of Chapter 3 illustrates the optimisation problems in discrete finite time horizon, they are based on two approaches: the cost-induced performance index and the ratio-induced performance index. The deterministic and stochastic cases are applied to formulate two deterministic optimisation problems and two stochastic ones.

Thus, the performance indexes are set up to minimize two main risks of the pension scheme: the contribution rate risk and the solvency risk, in order to reach the optimal contribution rates.

Minimizing the contribution rate risk and the solvency risk corresponds to achieve the stability of the contribution rate and security of the promised benefits respectively. Therefore, seeking for the stability of the contribution rate, the sponsoring employer is concerned mainly about minimizing the contribution rate risk. On the other hand, the trustees and the members are concerned mainly about the security of the promised benefits, which indicates their interest to minimize the solvency risk to secure their benefits.

In our models, the mutual interest between the employer and the employees is considered. We allow for the intersection between the contribution rate risk and the solvency risk in the performance indexes. This is explained by the interest of the employer to keep his company and the scheme solvent to be able to pay the accrued rights. At the same time, the employees seek for the stability of the contribution rate, especially if they pay part of the contributions like the case of the Private Pension Fund in Egypt. This guarantees the payments of the contributions at the specified times without unduly financial burden for both the employer and the employees.

Chapters 4 and 5 consider the properties of the cost-induced performance index (CIPI) and the ratio-induced performance index (RIPI). A sensitivity analysis is carried out with assumed values for the different parameters of the models. The results of applying the two stochastic models of (CIPI) and (RIPI) are analysed. For both stochastic models, 10,000 simulations are carried out by using the Visual Basic Program (Visual Basic 6.0) and VBA (excel 2000), taking into consideration the disturbances in the economic factor only. In other words, in both stochastic models, the rate of return is assumed to be independent identically distributed as a lognormal variable with mean θ and different values of variance σ^2.

Under CIPI and RIPI, we allow the fund target and the contribution target to change in five main scenarios, they are: (1, 0.2), (1, 0.22), (1, 0.18), (1.2, 0.2) and (0.8, 0.2). The results of the expected optimal fund level and the contribution rate are analysed through the cases obtained when the weighting risk factors are changed interchangeably. Thus, the weighting factors of the contribution rate risk and the solvency risk γ and β take the values of (0.1, 0.3, 0.5, 0.7, 0.9) interchangeably, while the weighting factor of the cross-product term $\alpha = 0, 0.2, 0.4, 0.6, 0.8$.

In Chapter 4, we examine the results of applying the stochastic model of CIPI when the value of $B_t = 0.3$ (as a round value). Also, the equilibrium value of B_t is used in some cases to examine the effect of changing θ and the weighting risk factors γ, α and β. The results show the following:

- The effect of changing θ when $B_t = 0.3$ is clear for both $E(F_t^*)$ and $E(C_t^*)$, where $E(F_t^*)$ decreases when θ is low and increases when it is high, consequently, $E(C_t^*)$ increases in the former case and decreases in the latter case. However, when the equilibrium value of B_t is used, there are small differences between the trends of $E(F_t^*)$ and less deviation from the targets for $E(C_t^*)$ when θ changes up and down.

- The effect of the level of volatility is clear in all cases. Both the expected optimal fund level and contribution rate move away from the targets when the value of σ is increased.

- The expected optimal fund levels and the contribution rates are remarkably affected by changing the fund and contribution targets. The trends of $E(F_t^*)$ and $E(C_t^*)$ move around the different targets. The expected optimal fund level is increased by increasing the contribution target and vice versa. The expected optimal contribution rate increases with a lower value of the fund target, and it decreases when the fund target is high. In our analysis, the value of the initial fund F_0 is assumed to be equal to the fund targets. Thus, the value of $E(C_0^*)$ changes according to the value of the fund target.

- The expected optimal fund level becomes closer to the target, when the value of the weighting factor of the solvency risk (β) is high. In the same sense, the expected optimal contribution rate moves further away from the contribution target, when the weighting factor of the contribution rate risk is low. The same conclusion is reached when the equilibrium value of B_t is used.

- The results obtained for $E(F_t^*)$ and $E(C_t^*)$ show small differences between the cases when the value of the weighting factor of the cross-product term $\alpha = 0$ and when $\alpha \neq 0$. It is noted that $E(F_t^*)$ and $E(C_t^*)$ depart from the targets when the value of α is increased with specific values of γ and β. In other words, when more importance is given to weighting risk factor of the cross-product term, which represents the mutual interest of employers and employees, the results of $E(F_t^*)$ and $E(C_t^*)$ show a tendency to deviate away from the targets. This is also applied when we use the equilibrium value of B_t. However, the trends of $E(F_t^*)$ and $E(C_t^*)$ get close to the targets when α, σ and γ are high due to the skewed distribution with a high value of the variance obtained.

Chapter 5 considers the properties of the stochastic RIPI model by highlighting the differences between the results obtained under RIPI and CIPI when $B_t = 0.3$. Generally, the effect of changing the different parameters is similar under both models. However, the trends obtained under RIPI show more departure from the targets compared with those obtained under CIPI.

Chapter 6 explains the empirical implementation of the stochastic RIPI and CIPI models. The application of the models is based on the past data, which is collected from 1990 to 2002 from the actuarial reports of the largest Egyptian private pension fund. The effect of changing the different parameters is tested on the simulated expected cash flows. Thereafter, the actual cash flows is compared with those resulted from applying the dynamic pension funding models.

The results of the sensitivity analysis, based on the actual cash flows in our case study and when $\sigma = 20\%$, are summarized below:

- The expected optimal fund level show similar increasing trends under both CIPI and RIPI when θ is increased. However, $E(F_t^*)$ reaches higher levels under RIPI compared with those under CIPI. On the other hand, the expected optimal contribution rate decreases over time when θ is increased. Further, $E(C_t^*)$ is more smooth and less fluctuated under RIPI compared with those under CIPI.

- The effect of increasing the level of volatility show small differences between the levels of $E(F_t^*)$ and $E(C_t^*)$ under both models. However, $E(F_t^*)$ tends to decrease over time, when σ is increased, while $E(C_t^*)$ tends to increase slightly over time.

- The effect of the weighting factors of the contribution rate risk and solvency risk shows an upward trend of $E(F_t^*)$ under CIPI and RIPI. However, the levels of $E(F_t^*)$ decrease under CIPI model, when more importance is given to the weighting factor of the contribution rate, while they increase under RIPI model. The expected optimal contribution rate increases when the value of γ is high under CIPI and RIPI, although $E(C_t^*)$ levels are smoother under RIPI.

- The levels of $E(F_t^*)$ increase over time when a higher value is given to the weighting factor of the cross-product term (α). On the other hand, $E(C_t^*)$ levels show a fluctuation behaviour over time when α is increased. However, they are smoother under RIPI compared with those under CIPI.

At the end of chapter 6, the expected cash flows are compared with the actual cash flows achieved over the period from 1990 to 2002. Generally, we find that there are some cases where $E(F_t^*)$ and $E(C_t^*)$ are close to the actual cash flows under CIPI and RIPI models. According to the data of the Egyptian defined benefit private pension fund, it is noted that giving more importance to the weighting risk factor β keeps the weighted average deviation from the actual cash flows small, especially when $\alpha = 0$ and the fund ratio $\eta = 100\%$ under CIPI and RIPI models.

7.2 Further work

In this section, a few ideas are mentioned as a possible extension to the work of this thesis.

The stochastic optimization problems are formulated taking into consideration the disturbances in the economic factor only. It will be useful to modify the models by considering the disturbances in both the economic and demographic factors.

The models are set up based on a finite time span which corresponds to the wind up valuation of the scheme. Thus, the models can be developed on the basis of infinite time horizon. The empirical implementation of these models could enable us to predict the optimal contribution rates in on-going valuation of the scheme.

In the thesis, the investment strategy has not been included in the models. It could be useful to consider the asset allocation strategies and examine their effect on the expected optimal contribution rate. Thereafter, the results can be compared with the former results that obtained without considering the investment strategies of the scheme.

The fact of the growing popularity of the defined contribution schemes, and the importance of applying the dynamic pension funding plans are considered good reasons to carry out the empirical study on one of the defined contribution schemes.

Finally, applying the dynamic programming and control theory on the hybrid schemes following both approaches adopted in the defined benefit schemes and the defined contribution schemes could be an interesting area of further research.

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