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# CHAOTIC CASCADES FOR D-BRANES ON SINGULARITIES

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**Abstract** We briefly review our work on the cascading renormalization group flows for gauge theories on D-branes probing Calabi-Yau singularities. Such RG flows are sometimes chaotic and exhibit duality walls. We construct supergravity solutions dual to logarithmic flows for these theories. We make new observations about a surface of conformal theories and more complicated supergravity solutions.

## 1. INTRODUCTION

Extending the revolutionary AdS/CFT correspondence [1] beyond the original relation between  $\mathcal{N} = 4$  SYM on  $N$  D3-branes and Type IIB supergravity (sugra) in  $AdS_5 \times S^5$  with  $N$  units of RR 5-form flux on the  $S^5$  is important to understanding realistic strongly coupled field theories such as QCD.

Two standard extensions have been (1) *reducing the SUSY to  $\mathcal{N} = 1$*  by placing the D3-branes transverse to a Calabi-Yau singularity (the dual sugra background becomes  $AdS_5 \times X^5$ , where  $X^5$  is some non-spherical horizon); and (2) *breaking conformal invariance and inducing an RG flow*, by introducing fractional branes, i.e., D5-branes wrapped over collapsing 2-cycles of the singularity (in the sugra dual, 3-form fluxes are turned on). A fascinating type of RG flow is the *duality cascade*: Seiberg duality is used to switch to an alternative description whenever infinite coupling is reached. This idea was introduced in [2] for the gauge theory on D-branes probing the conifold.

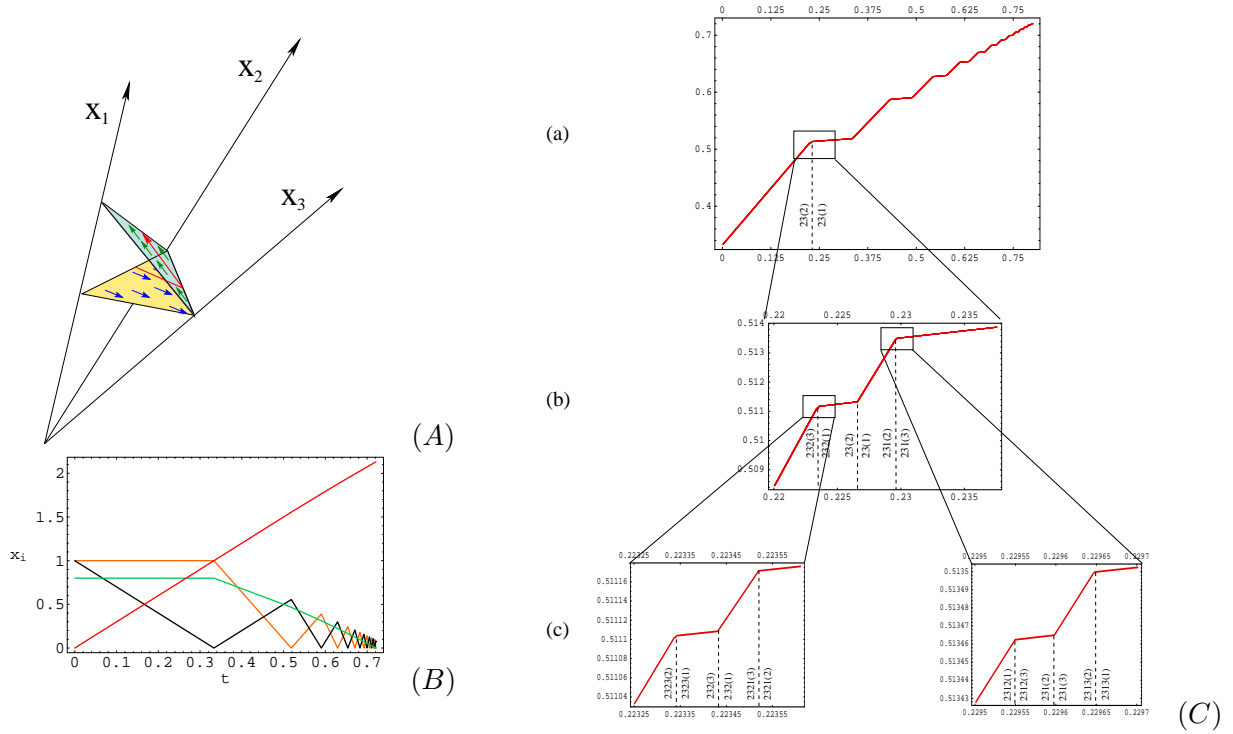


Figure 1.1

## 2. CASCADES IN COUPLING SPACE

There is an interesting way to look at cascading RG flows. In a gauge theory described by a quiver with  $k$  gauge groups, the inverse squared couplings  $x_i \equiv 1/g_i^2$  are positive and define a  $k$ -dimensional cone  $(\mathbb{R}_+)^k$ . Inside this cone, the RG flow generates a trajectory dictated by the beta functions and satisfying  $\sum_i r^i/g_i^2 = \text{constant}$ . Each step between dualizations then corresponds to a straight line in the simplex defined by the intersection between this hyperplane and the  $(\mathbb{R}_+)^k$  cone. We show such a trajectory in Figure 1.1(A). Now, each wall of the cone corresponds to one of the gauge couplings going to infinity. Therefore, whenever one of them is reached, we switch to a Seiberg dual theory at weak coupling. There will be then a different simplex associated to the dual theory. The entire cascade corresponds to a flow in the space of glued simplices. From this perspective, which resembles a billiard

bouncing in coupling space, one foresees that cascading RG flows will exhibit chaotic behavior.

### 3. DUALITY WALLS AND FRACTALS

After introducing the notion of a duality cascade, it is natural to wonder whether some supersymmetric extension of the Standard Model, such as the MSSM, can sit at the IR endpoint of a cascade. This question was posed by Matthew Strassler [3]. Generically, while trying to reconstruct such a RG flow, one encounters a UV accumulation point beyond which Seiberg duality cannot proceed. This phenomenon is dubbed a *duality wall* and has been constructed for gauge theories engineered with D-branes on singularities [4]. Figure 1.1(B) shows the behavior of couplings for a cascade with a duality wall for the theory on D-branes over a complex cone over the Zeroth-Hirzebruch surface  $F_0$ <sup>1</sup>.

Postponing the question of a possible UV completion of duality walls, we can study the dependence of its position on initial couplings. Illustrating with  $F_0$ , the result is remarkable and is presented in Figure 1.1(C). The curve is a fractal, with concave and convex cusps. Whenever we zoom in on a convex cusp, an infinite, self-similar structure of more cusps emerges.

One subtle point which was not emphasized in [5] involves the existence in coupling space of a codimension two surface of conformal theories for  $F_0$  and the other del Pezzo quiver gauge theories. If the number of gauge couplings is  $n + 2$ , then a naive counting of the linearly independent  $\beta$ -functions constrains only two combinations of gauge couplings when the theory is conformal, leaving an  $n$ -dimensional surface of conformal theories. This  $n$ -dimensional surface is parametrized on the gravity side by the dilaton and the integral of the NSNS  $B_2$  form through  $n - 1$  independent 2-cycles.

The existence of this codimension two surface may well affect the existence and behavior of the duality wall for  $F_0$ . In [4], it was assumed that a generic choice of initial couplings would lie on the conformal surface. However, if the initial conditions do not lie on the conformal surface, one expects large coupling constant corrections to the anomalous dimensions, which will in turn affect the strengths of the  $\beta$ -functions.

### 4. SUPERGRAVITY DUALS

The main support for the idea of a cascading RG flow in the original case of the conifold comes from a supergravity dual construction. This

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<sup>1</sup>We refer the reader to [4, 5] for a detailed description of the associated quiver theory.

dual reproduces the logarithmic decrease in the effective number of colors towards the IR and also matches the beta functions for the gauge couplings.

In [5], analog supergravity solutions were constructed describing logarithmic cascades for the gauge theories on D-branes probing complex cones over del Pezzo surfaces. The fact that this was possible is remarkable, since they were obtained without knowing the explicit metric. These supergravity solutions are of the general type studied by Graña and Polchinski [6].

The general form of the metric is a warped product of flat four-dimensional Minkowski space and a Calabi-Yau  $\mathbf{X}$

$$ds^2 = Z^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + Z^{1/2} ds_{\mathbf{X}}^2, \quad (1.1)$$

The solution also carries 3-form flux  $G_3 = F_3 - \frac{i}{g_s} H_3$ . In order to preserve  $\mathcal{N} = 1$  supersymmetry,  $G_3$  must be supported only on  $X$ , imaginary self-dual, a  $(2, 1)$  form and harmonic. Indeed, it is possible to construct a  $G_3$  satisfying all these conditions. It has the form

$$G_3 = \sum_{I=1}^n a^I (\eta + i \frac{dr}{r}) \wedge \phi_I \quad (1.2)$$

where the  $\phi_I$ ,  $I = 1 \dots n$ , are a basis of  $(1, 1)$  forms orthogonal to the Kähler class of the del Pezzo and  $\eta = (\frac{1}{3} d\psi + \sigma)$ . The one-form  $\sigma$  satisfies  $d\sigma = 2\omega$ , with  $\omega$  the Kähler form on  $dP_n$ , and  $0 \leq \psi < 2\pi$  is the angular coordinate on the circle bundle over  $dP_n$ .

The intersection product between the  $\phi_I$  is  $\int_{dP_n} \phi_I \wedge \phi_J = -A_{IJ}$ , where  $A_{IJ}$  is the Cartan matrix for the exceptional Lie algebra  $\mathcal{E}_n$ . There is a different type of fractional brane associated to each  $\phi_I$ , given by D5-branes wrapping the 2-cycle in the del Pezzo Poincaré dual to  $\phi_I$ .

Let us now study the number of D5-branes and D3-branes associated to these solutions

**D5-Branes:** The number of D5-branes is given by the Dirac quantization of the RR 3-form  $F_3$ :  $a^J = 6\pi\alpha' M^J$ . Hence, this family of solutions are dual to cascades in which the number of fractional branes of each type remains constant.

**D3-Branes:** Similarly, the effective number of D3-branes is computed from  $F_5 = \mathcal{F}_5 + *\mathcal{F}_5$  where  $\mathcal{F}_5 = d^4x \wedge d(Z^{-1})$  and  $Z$  is the warp factor in (1.1). The factor  $Z$  satisfies the equation

$$\nabla_{\mathbf{X}}^2 Z = -\frac{1}{6} |H_3|^2. \quad (1.3)$$

In [5],  $|F_3|^2$  was assumed to be a function only of the radius, in which case

$$Z(r) = \frac{2 \cdot 3^4}{9-n} \alpha'^2 g_s^2 \left( \frac{\ln(r/r_0)}{r^4} + \frac{1}{4r^4} \right) \sum_{i,j} M^I A_{IJ} M^J \quad (1.4)$$

and from Dirac quantization, the number of D3-branes will grow logarithmically:  $N = \frac{3}{2\pi} g_s \ln(r/r_0) \sum_{I,J} M^I A_{IJ} M^J$ . However, generically,  $Z$  may depend on other coordinates on the Calabi-Yau cone  $\mathbf{X}$ .<sup>2</sup> The function  $Z$  averaged over the other coordinates may still be logarithmic in  $r$  [7].

## 5. RECENT DEVELOPMENTS

Recently, there has been further progress in the study of quiver theories and their sugra duals. In [8], a-maximization [9] was used to compute the volume of the 5d horizon of the dual of the  $dP_1$  gauge theory, yielding an irrational value. This result corrected previous computations in the literature and was obtained by carefully taking into account the global symmetries that are actually preserved by the superpotential. In [5], the duality cascade for  $dP_1$  was analyzed using naive R-charges that did not take into account these global symmetries. A stable elliptical region in coupling space was found with a self-similar logarithmic cascade. Re-doing the analysis with the new R-charges, we find the same elliptical region albeit with a slightly different shape and center.

The 5d horizon for the complex cone over  $dP_1$  is called  $Y^{2,1}$  and is a member of an infinite family of Sasaki-Einstein geometries denoted  $Y^{p,q}$ . They have  $S^2 \times S^3$  topology. Their metrics were first found, locally, in [10] and then the global properties were analysed in [11]. In [12], their toric description was worked out. Furthermore, the gauge theory duals to the entire  $Y^{p,q}$  family have been constructed [13]. These developments change profoundly the status of the AdS/CFT, providing an infinite number of field theories with explicit sugra duals.

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