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A Standard Model from the $E_8 \times E_8$ Heterotic Superstring

Volker Braun[1,2], Yang-Hui He[1], Burt A. Ovrut[1], and Tony Pantev[2]

[1] Department of Physics, [2] Department of Mathematics

University of Pennsylvania

Philadelphia, PA 19104–6395, USA

Abstract

In a previous paper, we introduced a heterotic standard model and discussed its basic properties. This vacuum has the spectrum of the MSSM with one additional pair of Higgs-Higgs conjugate fields and a small number of uncharged moduli. In this paper, the requisite vector bundles are formulated; specifically, stable, holomorphic bundles with structure group $SU(N)$ on smooth Calabi-Yau threefolds with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group. A method for computing bundle cohomology is presented and used to evaluate the cohomology groups of the standard model bundles. It is shown how to determine the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on these groups. Finally, using an explicit method of "doublet-triplet splitting", the low-energy particle spectrum is computed.

*vbraun, yanghe, ovrut@physics.upenn.edu; tpantev@math.upenn.edu

1 Introduction:

In [1], we presented a standard model within the context of the $E_8 \times E_8$ heterotic superstring. These vacua are $N = 1$ supersymmetric and have the following properties.

- The observable sector has gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, *three* families of quarks and leptons, each with a *right-handed neutrino*, and *two* Higgs-Higgs conjugate pairs. There is *no exotic matter*. In addition, there are 6 geometric moduli and a *small number* of vector bundle moduli. That is, the observable sector has exactly the spectrum of the MSSM with one additional Higgs-Higgs conjugate pair.

Within our context, the visible sector vector bundle is unique. All other bundles lead to an observable sector spectrum that is not realistic, having large numbers of exotic multiplets and Higgs-Higgs conjugate pairs.

- The structure of the hidden sector depends on whether one considers the weakly or strongly coupled regime. In the strongly coupled context, we find a minimal hidden sector with gauge group $E_7 \times U(6)$, and *no matter fields*. For weak coupling, one finds a minimal hidden sector with gauge group $Spin(12)$ and *two* matter multiplets, each in the $\mathbf{12}$ of $Spin(12)$. In both cases, there is a *small number* of vector bundle moduli.

There is flexibility in choosing the hidden sector vector bundles since one can always perform small instanton transitions, see [2–4]. However, those leading to the minimal spectra just presented are, essentially, unique.

In [1] we presented the basic structure of the heterotic standard model, but only briefly outlined the requisite technical results. The properties of the smooth compactification manifold, Calabi-Yau threefolds with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group, as well as the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on the associated Wilson lines were discussed in detail in [5]. However, the construction of the standard model vacua requires three other ingredients; first, stable, holomorphic vector bundles with $SU(N)$ structure groups over this threefold, second, the cohomologies associated with these bundles and third, the explicit representations of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on these cohomology groups. The low energy spectrum is then identified with the subspace invariant under the product of these representations with the action on the Wilson lines. In this paper, we will discuss these three ingredients in more detail. This will establish the technical basis for our results in [1] and provide the context for assessing their uniqueness.

The standard model vector bundles are *not* constructed from spectral covers [6–14]. Rather, they are produced using a generalization of the method of "bundle extensions" introduced in [15–21]. The techniques for explicitly computing bundle cohomologies, and for finding the representations of a finite group on the cohomology groups, were presented [22, 23]. Standard model vacua require a significant extension of the methods discussed in [24, 25]. Finally, we emphasize that our computation of the spectrum as the invariant subspace under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on the cohomology groups and Wilson lines represents an explicit method of "doublet-triplet splitting" [26, 27]. It is this technique which allows us to project out all exotic matter and to arrive at the minimal MSSM spectrum with one additional pair of Higgs-Higgs conjugate fields.

In this paper, we present our computations and discuss the extensive search that led to the heterotic standard model. However, the full technical details will be left to future publications [28, 29]. For example, the computation of vector bundle moduli is more involved than for other fields and will be presented elsewhere. In this paper, we simply point out that the $\mathbb{Z}_3 \times \mathbb{Z}_3$ projection greatly reduces the number of such moduli.

2 Requisite Data:

We begin with the $E_8 \times E_8$ heterotic string compactified on a smooth Calabi-Yau threefold X. This manifold admits stable, holomorphic vector bundles V in the observable E_8 sector and V' in the E_8' hidden sector.[1] It follows that the four-dimensional effective theory will exhibit $N = 1$ supersymmetry.

2.1 The Observable Sector Spectrum

Consider the minimal supersymmetric standard model, the MSSM. It is well-established that neutrinos have a non-vanishing mass [30]. Since the MSSM has no exotic multiplets, $N = 1$ supersymmetry will suppress any purely left-handed Majorana neutrino mass to be too small by several orders of magnitude [31, 32]. It follows that the MSSM must be extended by adding a right-handed neutrino to each family of quarks/leptons.

[1] We will distinguish the E_8 gauge group of the hidden sector by denoting it with a prime.

We would like to find a vacuum of the $E_8 \times E_8$ heterotic string whose observable sector is as close to this extended MSSM as possible. To do this, it is useful to recall that each generation of quarks/leptons with a right-handed neutrino fits exactly into the **16** spin representation of $Spin(10)$. It is compelling, therefore, to try to spontaneously break the E_8 gauge group of the observable sector to $Spin(10)$ as already suggested in [26]. This can be accomplished if we choose V to have structure group $SU(4)$. Then

$$E_8 \longrightarrow Spin(10), \tag{1}$$

as desired. With respect to the maximal subgroup $SU(4) \times Spin(10)$, the adjoint **248** of E_8 decompose as

$$\mathbf{248} \quad \rightarrow \quad \big(\mathbf{1}, \mathbf{45}\big) \oplus \big(\mathbf{15}, \mathbf{1}\big) \oplus \big(\mathbf{4}, \mathbf{16}\big) \oplus \big(\overline{\mathbf{4}}, \overline{\mathbf{16}}\big) \oplus \big(\mathbf{6}, \mathbf{10}\big). \tag{2}$$

The $\big(\mathbf{1}, \mathbf{45}\big)$ contain the gauginos of $Spin(10)$, the $\big(\mathbf{15}, \mathbf{1}\big)$ correspond to vector bundle moduli and the remaining representations are the matter fields.

If X is not simply connected, one can introduce, additionally, a Wilson line W to further reduce the gauge group. It was shown in [5] that to break $Spin(10)$ to a group containing the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, the simplest possibility is to require that X have first fundamental group

$$\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3. \tag{3}$$

Calabi-Yau threefolds with this property were explicitly constructed in [5]. On such manifolds, one can choose Wilson lines with the property that their holonomy group is $hol(W) = \mathbb{Z}_3 \times \mathbb{Z}_3$. It was shown in [5] that W will then spontaneously break

$$Spin(10) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}, \tag{4}$$

where, in addition to the standard model gauge group, there is a gauged $U(1)_{B-L}$ symmetry. With respect to this low energy gauge group, the $Spin(10)$ matter fields decompose as

$$\begin{aligned} \mathbf{16} \ &\rightarrow \big(\mathbf{3}, \mathbf{2}, 1, 1\big) \oplus \big(\overline{\mathbf{3}}, \mathbf{1}, -4, -1\big) \oplus \big(\overline{\mathbf{3}}, \mathbf{1}, 2, -1\big) \oplus \big(\mathbf{1}, \mathbf{2}, -3, -3\big) \oplus \\ &\qquad \oplus \big(\mathbf{1}, \mathbf{1}, 6, 3\big) \oplus \big(\mathbf{1}, \mathbf{1}, 0, 3\big), \\ \mathbf{10} \ &\rightarrow \big(\mathbf{3}, \mathbf{1}, -2, -2\big) \oplus \big(\overline{\mathbf{3}}, \mathbf{1}, 2, 2\big) \oplus \big(\mathbf{1}, \mathbf{2}, 3, 0\big) \oplus \big(\mathbf{1}, \overline{\mathbf{2}}, -3, 0\big) \end{aligned} \tag{5}$$

where we have displayed the quantum numbers $3Y$ and $3(B-L)$ for convenience. The $\overline{\mathbf{16}}$ decomposition is obtained by conjugation. We see from eq. (5) that the standard model fermions, including a right-handed neutrino, arise from the decomposition of the $\mathbf{16}$, as expected. Similarly, Higgs doublets occur in the $\mathbf{10}$ of $Spin(10)$.

Note, however, that there may be extra, exotic matter multiplets in the spectrum. These include all fields arising from the decomposition of a $\overline{\mathbf{16}}$. Additionally, any of the color triplets in the decomposition of a $\mathbf{10}$ are unobserved. Therefore, if one is to be successful in finding a heterotic standard model, these exotic matter multiplets must be projected out.

2.2 The Calabi-Yau Threefold X

The above discussion implies that one must construct Calabi-Yau threefolds X with fundamental group $\mathbb{Z}_3 \times \mathbb{Z}_3$. This was carried out in detail in [5]. Here, we simply outline those properties of the construction that are required for the analysis in this paper.

The requisite Calabi-Yau threefolds, X, are constructed as follows. We begin by considering a simply connected Calabi-Yau threefold, $\widetilde{X}$, which is an elliptic fibration over a rational elliptic surface, $d\mathbb{P}_9$. It was shown in [5] that there are special $d\mathbb{P}_9$ surfaces which admit a $\mathbb{Z}_3 \times \mathbb{Z}_3$ action. Furthermore, in a six-dimensional region of moduli space, $\widetilde{X}$ admits an induced $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action which is fixed point free. The quotient $X = \widetilde{X}/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ is a smooth Calabi-Yau threefold that is torus-fibered over a singular $d\mathbb{P}_9$ and has non-trivial fundamental group $\mathbb{Z}_3 \times \mathbb{Z}_3$, as desired.

Specifically, $\widetilde{X}$ is a fiber product

$$\widetilde{X} = B_1 \times_{\mathbb{P}^1} B_2 \tag{6}$$

of two $d\mathbb{P}_9$ special surfaces B_1 and B_2. Thus, $\widetilde{X}$ is elliptically fibered over both surfaces with the projections

$$\pi_1 : \widetilde{X} \to B_1, \quad \pi_2 : \widetilde{X} \to B_2. \tag{7}$$

The surfaces B_1 and B_2 are themselves elliptically fibered over $\mathbb{P}^1$ with maps

$$\beta_1 : B_1 \to \mathbb{P}^1, \quad \beta_2 : B_2 \to \mathbb{P}^1. \tag{8}$$

The invariant homology ring of each special $d\mathbb{P}_9$ surface is generated by two $\mathbb{Z}_3 \times \mathbb{Z}_3$

invariant curve classes f and t with intersections

$$f^2 = 0, \quad ft = 3t^2. \tag{9}$$

Using projections (7), these can be lifted to divisor classes

$$\tau_1 = \pi_1^{-1}(t_1), \quad \tau_2 = \pi_2^{-1}(t_2), \quad \phi = \pi_1^{-1}(f_1) = \pi_2^{-1}(f_2) \tag{10}$$

on $\widetilde{X}$ satisfying the intersection relations

$$\phi^2 = \tau_1^3 = \tau_2^3 = 0, \quad \phi\tau_1 = 3\tau_1^2, \quad \phi\tau_2 = 3\tau_2^2. \tag{11}$$

These three classes generate the invariant homology ring of $\widetilde{X}$. For example,

$$span_{\mathbb{C}}\{\phi, \tau_1, \tau_2\} = H_4(\widetilde{X}, \mathbb{C})^{\mathbb{Z}_3 \times \mathbb{Z}_3} \simeq H^{1,1}(X). \tag{12}$$

It follows that $h^{1,1}(X) = 3$. Similarly, one can show that $h^{1,2}(X) = 3$. Hence, X has six geometric moduli; three Kahler moduli and three complex structure moduli. Finally, the Chern classes of $\widetilde{X}$ can be shown to be

$$c_1(T\widetilde{X}) = c_3(T\widetilde{X}) = 0, \quad c_2(T\widetilde{X}) = 12(\tau_1^2 + \tau_2^2). \tag{13}$$

2.3 The Holomorphic $SU(4)$ Bundle V

Next, we produce the requisite observable sector bundles V on X. This is accomplished by constructing stable, holomorphic vector bundles $\widetilde{V}$ with structure group $SU(4)$ over $\widetilde{X}$ that are equivariant under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$. Then $V = \widetilde{V}/(\mathbb{Z}_3 \times \mathbb{Z}_3)$.

The vector bundles $\widetilde{V}$ are constructed using a generalization of the method of "bundle extensions" introduced in [15–21]. Specifically, $\widetilde{V}$ is the extension

$$0 \longrightarrow V_1 \longrightarrow \widetilde{V} \longrightarrow V_2 \longrightarrow 0 \tag{14}$$

of two rank two bundles V_1 and V_2 on $\widetilde{X}$. These bundles are of the form

$$V_i = \mathcal{L}_i \otimes \pi_2^* W_i, \quad i = 1, 2 \tag{15}$$

for some line bundles $\mathcal{L}_i$ on $\widetilde{X}$ and rank 2 bundles W_i on B_2. The rank two bundles W_i are themselves extensions

$$0 \longrightarrow \mathcal{O}_{B_2}(a_i f_2) \longrightarrow W_i \longrightarrow \mathcal{O}_{B_2}(b_i f_2) \otimes I_{k_i} \longrightarrow 0, \tag{16}$$

where a_i, b_i are integers and I_{k_i} is the ideal sheaf of some k_i-tuple of points on B_2. That is, (16) gives us a prescription to build rank two bundles on B_2, (15) to produce two rank two bundles on $\widetilde{X}$ and, finally, we use (14) to construct $\widetilde{V}$.

One must specify not only the bundles $\widetilde{V}$, but their transformations under $\mathbb{Z}_3 \times \mathbb{Z}_3$ as well. To do this, first notice that for the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on the space of extensions to be well-defined, the line bundles $\mathcal{O}_{B_2}(a_i f_2)$, $\mathcal{O}_{B_2}(b_i f_2)$ and $\mathcal{L}_i$ must be equivariant under the finite group action. In this case, the space of extensions will carry a representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$. An equivariant rank four vector bundle will be any $\widetilde{V}$ that does not transform under this action. A $\widetilde{V}$ with this property will inherit an explicit equivariant structure from the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on its constituent line bundles. Having found such a $\widetilde{V}$, one can construct $V = \widetilde{V}/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ on X, as required.

To proceed, therefore, one must consider the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on line bundles and show how to construct line bundles that are equivariant. Two natural one-dimensional representations of $\mathbb{Z}_3 \times \mathbb{Z}_3$ are defined by

$$\chi_1(g_1) = \omega, \quad \chi_1(g_2) = 1; \qquad \chi_2(g_1) = 1, \quad \chi_2(g_2) = \omega, \tag{17}$$

where $g_{1,2}$ are the generators of the two $\mathbb{Z}_3$ factors, $\chi_{1,2}$ are two group characters of $\mathbb{Z}_3 \times \mathbb{Z}_3$ and $\omega = e^{\frac{2\pi i}{3}}$ is a third root of unity. All other one-dimensional representations are products of (17) and, in any case, do not appear in our construction. Note that none of these representations is faithful.

Let us consider an explicit example of a $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on a line bundle. Recall that $\mathcal{O}_{\widetilde{X}} \simeq \widetilde{X} \times \mathbb{C} \ni (p, v)$ is the trivial line bundle on $\widetilde{X}$. The simplest action of a group element $g \in \mathbb{Z}_3 \times \mathbb{Z}_3$ on $\mathcal{O}_{\widetilde{X}}$ is by translation of p to $g(p)$, with no action on v. However, for any representation χ we can define a twisted action on $\mathcal{O}_{\widetilde{X}}$ by

$$(p, v) \mapsto (g(p), \chi(g)v). \tag{18}$$

In this paper, we denote $\mathcal{O}_{\widetilde{X}}$ carrying this twisted action by $\chi\mathcal{O}_{\widetilde{X}}$. It is straightforward to show that $\chi\mathcal{O}_{\widetilde{X}}$ is equivariant under $\mathbb{Z}_3 \times \mathbb{Z}_3$, as desired. We may similarly define line bundles $\chi\mathcal{L}$ on $\widetilde{X}$, $\chi\mathcal{O}_{B_i}(nf)$ on B_i and $\chi\mathcal{O}_{\mathbb{P}^1}(n)$ on $\mathbb{P}^1$.

Finally, having constructed equivariant holomorphic bundles $\widetilde{V}$ with structure group $SU(4)$ over $\widetilde{X}$, one must ensure that they are stable. For an arbitrary holomorphic vector bundle $\mathcal{F}$, a complete proof of stability is extremely complicated. However, one can show that a bundle $\mathcal{F}$ with vanishing first Chern class is stable only if

$$H^0(\widetilde{X}, \mathcal{F}) = H^0(\widetilde{X}, \mathcal{F}^*) = 0, \quad H^0(\widetilde{X}, \mathcal{F} \otimes \mathcal{F}^*) = 1. \tag{19}$$

We will use these criteria as highly non-trivial checks on the stability of $\widetilde{V}$, as well as on the hidden sector bundle $\widetilde{V}'$.

2.4 Computing the Particle Spectrum

A method for computing the low-energy particle spectrum after compactification on X with a holomorphic vector bundle V and possible Wilson line W was presented in [22,23,33–39] and will be used in this paper. The spectrum is identified with the zero modes of the Dirac operator on X "twisted" by the bundle $V \oplus W$. The zero modes are the invariant elements of certain bundle cohomology groups. In this method, one does not actually make use of X and V, all computations being performed on the covering space $\widetilde{X}$ with bundle $\widetilde{V}$.

To be specific, let us consider the observable sector discussed above. In this case, $\widetilde{V}$ has structure group $SU(4)$ which breaks E_8 to $Spin(10)$. Furthermore, $\widetilde{X}$ admits a free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action and $\widetilde{V}$ is equivariant under this action. Let R be a representation of $Spin(10)$ and denote the associated $\widetilde{V}$ bundle by $U_R(\widetilde{V})$. One first constructs $H^1(\widetilde{X}, U_R(\widetilde{V}))$ for all non-trivial bundles $U_R(\widetilde{V})$. When $U_R(\widetilde{V})$ is trivial, one considers $H^0(\widetilde{X}, \mathcal{O}_{\widetilde{X}})$ which is always one-dimensional and carries the trivial representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$. The next step is to find the representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on $H^1(\widetilde{X}, U_R(\widetilde{V}))$. Choosing $\widetilde{V}$ to be equivariant guarantees that these actions exit. Finally, tensor each such representation with the action of the Wilson line on R. The zero mode spectrum is then the invariant subspace under this joint group action. In summary, the particle spectrum is

$$\begin{aligned}\ker(\not{D}_{\tilde{V}}) = & \Big(H^0(\widetilde{X}, \mathcal{O}_{\widetilde{X}}) \otimes \mathbf{45}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \oplus \Big(H^1\big(\widetilde{X}, \mathrm{ad}(\tilde{V})\big) \otimes \mathbf{1}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \oplus \\ & \oplus \Big(H^1(\widetilde{X}, \tilde{V}) \otimes \mathbf{16}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \oplus \Big(H^1(\widetilde{X}, \tilde{V}^*) \otimes \overline{\mathbf{16}}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \oplus \Big(H^1(\widetilde{X}, \wedge^2\tilde{V}) \otimes \mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3},\end{aligned} \tag{20}$$

where the superscript indicates the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant subspace.

Although we have illustrated our method for the observable sector, it is completely general, applying to the hidden sector as well. It follows that the computation of cohomology groups, and the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on these groups, is a major ingredient of our construction.

2.5 Physical Constraints

Obtaining realistic particle physics in the observable sector requires the the following additional constraints on $\widetilde{V}$.

1. **Three Generations:** To ensure that there are three generations of quarks and leptons in the low-energy spectrum, one must require that

$$h^1(X,V) - h^1(X,V^*) = 3. \tag{21}$$

Using Serre duality, and assuming $\widetilde{V}$ satisfies eq. (19), the Atiyah-Singer index theorem implies

$$-h^1(X,V) + h^1(X,V^*) = \int_X \mathrm{ch}(V)\mathrm{td}(TX) = \frac{1}{2}\int_X c_3(V) = -3. \tag{22}$$

Therefore, one must demand $c_3(V) = -6$ or, equivalently, that

$$c_3\big(\widetilde{V}\big) = -6 \times |\mathbb{Z}_3 \times \mathbb{Z}_3| = -54. \tag{23}$$

2. **No Exotic Matter in the Observable Sector:** The previous constraint ensures that there are precisely three chiral generations descending from the **16** representations. However, there remains, in general, a large number of additional low energy multiplets which descend from vector-like $\overline{\mathbf{16}}-\mathbf{16}$ pairs. These "exotic multiplets" are unobserved. Therefore, we place a very strong restriction on $\widetilde{V}$ and demand that there be no exotic multiplets in its low-energy spectrum. Referring to (20), we see that the simplest way to ensure this is to require

$$h^1(\widetilde{X},\widetilde{V}^*) = 0. \tag{24}$$

To our knowledge, this has never been accomplished in any other phenomenological string vacua. These typically have exotic multiplets in vector-like pairs which, it is hoped, acquire heavy masses. In our work, we constrain our spectrum to be as close to the MSSM as possible.

3. **Small Number of Higgs Doublets:** The number of **10** zero modes is given by $h^1(\widetilde{X},\wedge^2\widetilde{V})$. Since the Higgs fields arise from the decomposition of the **10**, we must not set the associated cohomology to zero. Rather, we restrict $\widetilde{V}$ so that

$$h^1(\widetilde{X},\wedge^2\widetilde{V}) \quad \text{is minimal,} \tag{25}$$

but non-vanishing.

4. Doublet-Triplet Splitting: Inspecting (5), we see that the decomposition of the $\mathbf{10}$ representation contains, in addition to Higgs fields, unwanted "exotic" color triplet multiplets. We require, therefore, that $(H^1(\widetilde{X}, \wedge^2\widetilde{V})\otimes\mathbf{10})^{\mathbb{Z}_3\times\mathbb{Z}_3}$ contain only the Higgs-doublets $\big(\mathbf{1}, \mathbf{2}, 3, 0\big) \oplus \big(\mathbf{1}, \mathbf{2}, -3, 0\big)$, thus projecting out the color triplets at low energy. This provides a natural solution to the doublet-triplet splitting problem. Note that this mechanism is not confined to doublets/triplets. It applies to the components of any multiplet, greatly reducing the spectrum after taking the $\mathbb{Z}_3 \times \mathbb{Z}_3$ quotient.

The vector bundle $\widetilde{V}'$ of the hidden sector must also obey the following constraint.

5. Anomaly Cancellation: For the theory to be consistent, one must require the cancellation of all anomalies. Through the Green-Schwarz mechanism, this requirement relates the observable and hidden sector bundles, imposing the constraint on the second Chern classes that

$$[\mathcal{W}_5] = c_2\big(T\widetilde{X}\big) - c_2\big(\widetilde{V}\big) - c_2\big(\widetilde{V}'\big) \tag{26}$$

be an effective class. In the strongly coupled heterotic string, $[\mathcal{W}_5]$ is the class of the holomorphic curve around which a bulk space five-brane is wrapped. In the weakly coupled case, $[\mathcal{W}_5]$ must vanish. In either case, $c_2(T\widetilde{X})$ and $c_2(\widetilde{V})$ are fixed by previous considerations. Therefore, (26) becomes a constraint on the second Chern class of $\widetilde{V}'$.

3 The Solution:

In this section, explicit bundles $\widetilde{V}$ and $\widetilde{V}'$ satisfying the requisite data outlined above are constructed. We begin by considering the observable sector bundle.

3.1 The Observable Sector Bundle $\widetilde{V}$

After an extensive search, we found a unique solution for $\widetilde{V}$ that is compatible with all of our constraints. It is constructed as follows. First consider the rank two bundles W_i for $i = 1, 2$ on B_2. Take W_1 to be

$$W_1 = \mathcal{O}_{B_2} \oplus \mathcal{O}_{B_2}. \tag{27}$$

Note that this is the trivial extension of (16) with $a_1 = b_1 = k_1 = 0$. Now let W_2 be an equivariant bundle in the extension space of

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow W_2 \longrightarrow \chi_2\mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0, \tag{28}$$

where for the ideal sheaf I_9 of 9 points we take a generic $\mathbb{Z}_3 \times \mathbb{Z}_3$ orbit. Second, choose the two line bundles $\mathcal{L}_i$ for $i = 1, 2$ on $\widetilde{X}$ to be

$$\mathcal{L}_1 = \chi_2\mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \tag{29}$$

and

$$\mathcal{L}_2 = \mathcal{O}_{\widetilde{X}}(\tau_1 - \tau_2) \tag{30}$$

respectively. Then, the two rank 2 bundles $V_{1,2}$ defined in eq. (15) are given by

$$\begin{aligned} V_1 &= \chi_2\mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \oplus \chi_2\mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \\ V_2 &= \mathcal{O}_{\widetilde{X}}(\tau_1 - \tau_2) \otimes \pi_2^* W_2. \end{aligned} \tag{31}$$

Note that V_1 is of a special form, having no ideal sheaves and being itself the trivial extension, namely, a direct sum of two line bundles. The observable sector bundle $\widetilde{V}$ is then defined as an equivariant element of the space of extensions (14). We now show that $\widetilde{V}$, so-defined, satisfies all of the requisite constraints.

Let us begin with the three generation condition. Computing the Chern classes of $\widetilde{V}$, we find that

$$c_1(\widetilde{V}) = 0, \quad c_2(\widetilde{V}) = -2\tau_1^2 + 7\tau_2^2 + 4\tau_1\tau_2, \quad c_3(\widetilde{V}) = -54. \tag{32}$$

Note that $c_3(\widetilde{V}) = -54$, as required by the three generation condition (23).

To count the number of exotic multiplets in the observable sector, it follows from (20) that one must compute $h^1(\widetilde{X}, \widetilde{V}^*)$. We find it more convenient to calculate $h^2(\widetilde{X}, \widetilde{V})$ and then use Serre duality to find $h^1(\widetilde{X}, \widetilde{V}^*)$. Furthermore, to discuss the stability of $\widetilde{V}$ as well as the number of **16** representations, we see from (19) and (20) that we need to know $h^i(\widetilde{X}, \widetilde{V})$ for $i = 0, 1, 3$ as well. To do this, recall that $\widetilde{V}$ is in the short exact bundle sequence (14). This induces a long exact sequence involving the desired cohomology groups $H^i(\widetilde{X}, \widetilde{V})$ for $i = 0, 1, 2, 3$. These groups can be calculated if we can compute the adjacent terms in the long exact sequence, namely $H^i(\widetilde{X}, \mathcal{F})$ where $\mathcal{F} = V_1$ and V_2. This can indeed be accomplished using Leray spectral sequences. Exploiting the fact that $\widetilde{X}$ is "doubly" elliptic, with π_i in (7) projecting $\widetilde{X}$ to B_i and β_i in (8) mapping B_i to $\mathbb{P}^1$, the spectral sequence for any sheaf $\mathcal{F}$ simplifies to

$$H^0(\widetilde{X},\mathcal{F}) = H^0(\mathbb{P}^1, \beta_{i*}\pi_{i*}\mathcal{F}) \tag{33}$$

and

$$\begin{tikzcd}
& 0 \arrow[d] & & & & \\
& H^0(\mathbb{P}^1, R^1\beta_{i*}\pi_{i*}\mathcal{F}) \arrow[d] & & H^0(\mathbb{P}^1, \beta_{i*}R^1\pi_{i*}\mathcal{F}) \arrow[d, equal] & & \\
0 \arrow[r] & H^1(B_i, \pi_{i*}\mathcal{F}) \arrow[r] \arrow[d] & \boxed{H^1(\widetilde{X},\mathcal{F})} \arrow[r] & H^0(B_i, R^1\pi_{i*}\mathcal{F}) \arrow[r] & H^2(B_i, \pi_{i*}\mathcal{F}) \arrow[r] & \cdots \\
& H^1(\mathbb{P}^1, \beta_{i*}\pi_{i*}\mathcal{F}) \arrow[d] & & & & \\
& 0 & & & &
\end{tikzcd} \tag{34}$$

where we have boxed the term we wish to compute in (34). By $R^1\pi_*$ and $R^1\beta_*$ we mean the first higher images of the push-down maps π_* and β_* respectively. To calculate the cohomology spaces H^i for $i = 2, 3$ one can simply use Serre duality which, on a Calabi-Yau threefold, $\widetilde{X}$ states that

$$H^i(\widetilde{X},\mathcal{F}) \simeq H^{3-i}(\widetilde{X},\mathcal{F}^*)^*, \quad i = 0,1,2,3. \tag{35}$$

Equations (33) and (34) reduce the computation of $H^i(\widetilde{X},\mathcal{F})$ for $i = 0,1,2,3$ to the evaluation of certain cohomology spaces on $\mathbb{P}^1$. In the present case, $\mathcal{F} = V_1, V_2$. Using (31) and the expressions for the push-downs given by

$$\begin{gathered}
\mathcal{O}_{B_i}(nf) = \beta_i^*\mathcal{O}_{\mathbb{P}^1}(n)\,,\ n \in \mathbb{Z} \\
\beta_{i*}\mathcal{O}_{B_i}(2t) = 6\mathcal{O}_{\mathbb{P}^1} \qquad \beta_{i*}\mathcal{O}_{B_i}(-2t) = 0 \qquad R^1\beta_{i*}\mathcal{O}_{B_i}(2t) = 0 \\
\beta_{i*}\mathcal{O}_{B_i}(t) = 3\mathcal{O}_{\mathbb{P}^1} \qquad \beta_{i*}\mathcal{O}_{B_i}(-t) = 0 \qquad R^1\beta_{i*}\mathcal{O}_{B_i}(t) = 0 \\
R^1\beta_{1*}\mathcal{O}_{B_1}(-t) = 3\chi_1\mathcal{O}_{\mathbb{P}^1}(-1) \qquad R^1\beta_{1*}\mathcal{O}_{B_1}(-2t) = 6\chi_1\mathcal{O}_{\mathbb{P}^1}(-1) \\
R^1\beta_{2*}\mathcal{O}_{B_2}(-t) = 3\mathcal{O}_{\mathbb{P}^1}(-1) \qquad R^1\beta_{2*}\mathcal{O}_{B_2}(-2t) = 6\mathcal{O}_{\mathbb{P}^1}(-1)\,,
\end{gathered} \tag{36}$$

the cohomology spaces on $\mathbb{P}^1$ can easily be computed.

Putting everything together, we find that

$$
\begin{aligned}
h^0(\widetilde{X},\widetilde{V}) &= h^3(\widetilde{X},\widetilde{V}^*) = 0 \\
h^1(\widetilde{X},\widetilde{V}) &= h^2(\widetilde{X},\widetilde{V}^*) = 27 \\
h^2(\widetilde{X},\widetilde{V}) &= h^1(\widetilde{X},\widetilde{V}^*) = 0 \\
h^3(\widetilde{X},\widetilde{V}) &= h^0(\widetilde{X},\widetilde{V}^*) = 0.
\end{aligned}
\tag{37}
$$

Note that these results are consistent with equation (24) for the absence of exotic multiplets arising from vector-like $\overline{\mathbf{16}} - \mathbf{16}$ pairs. They also satisfy the necessary conditions, given in (19), for $\widetilde{V}$ to be a stable bundle. Finally, cohomology (37) is consistent with the Atiyah-Singer index theorem for $\widetilde{V}$ on $\widetilde{X}$ and the three generation condition (23).

Next, consider the $\mathbf{10}$ representations of $Spin(10)$ which, from (5), give rise to Higgs doublets. It follows from (25) that one must compute $h^1(\widetilde{X},\wedge^2\widetilde{V})$ and show it to be minimal, but non-vanishing. To do this, note that $\wedge^2\widetilde{V}$ lies in the intertwined sequences

$$
\begin{tikzcd}
 & & & 0 \arrow[d] & \\
 & & & \wedge^2 V_2 \arrow[d] & \\
0 \arrow[r] & \wedge^2 V_1 \arrow[r] & \wedge^2\widetilde{V} \arrow[r] & Q \arrow[r] \arrow[d] & 0\,, \\
 & & & V_1 \otimes V_2 \arrow[d] & \\
 & & & 0 &
\end{tikzcd}
\tag{38}
$$

where Q is the quotient of the map $\wedge^2 V_2 \to \wedge^2\widetilde{V}$. Since $V_{1,2}$ are rank 2, $\wedge^2 V_{1,2}$ are line bundles and, using (31), are given by

$$
\wedge^2 V_1 = \chi_2^2 \mathcal{O}_{\widetilde{X}}(-2\tau_1 + 2\tau_2), \quad \wedge^2 V_2 = \mathcal{O}_{\widetilde{X}}(2\tau_1 - 2\tau_2). \tag{39}
$$

The bundle sequences (38) give rise to two long exact cohomology sequences. To compute $H^i(\widetilde{X},\wedge^2\widetilde{V})$ for $i = 0,1,2,3$, one must compute the adjacent terms in these sequences, namely, $H^i(\widetilde{X},\mathcal{F})$ for $\mathcal{F} = \wedge^2 V_1$, $\wedge^2 V_2$ and $V_1 \otimes V_2$. This can be accomplished using the Leray spectral sequences given in (33) and (34). We find, happily, that the entire cohomology of both $\wedge^2 V_{1,2}$ vanish. It follows that

$$
H^i(\widetilde{X},\wedge^2\widetilde{V}) \simeq H^i(\widetilde{X}, V_1 \otimes V_2), \quad i = 0,1,2,3. \tag{40}
$$

Finally, setting $\mathcal{F} = V_1 \otimes V_2$ in (33) and (34), we find

$$
h^0(\widetilde{X},\wedge^2\widetilde{V}) = h^3(\widetilde{X},\wedge^2\widetilde{V}) = 0, \quad h^1(\widetilde{X},\wedge^2\widetilde{V}) = h^2(\widetilde{X},\wedge^2\widetilde{V}) = 14. \tag{41}
$$

Although not immediately apparent, an exhaustive search reveals that

$$h^1(\widetilde{X}, \wedge^2 \widetilde{V}) = 14 \tag{42}$$

is the minimal number of **10** representations within our context.

As discussed previously, knowledge of the bundle cohomology groups corresponding to the **16** and **10** representations is not sufficient to determine the low energy spectrum. One must also evaluate the explicit action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on these spaces. First consider the cohomology space $H^1(\widetilde{X}, \widetilde{V})$ associated with the **16** representation. In this case, one can determine the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action using a simple argument. Note from (37) that

$$h^1(\widetilde{X}, \widetilde{V}) = 27. \tag{43}$$

Furthermore, (37) specifies that $h^1(\widetilde{X}, \widetilde{V}^*)$ vanishes and, hence, $h^1(X, V^*) = 0$. Then (22) becomes

$$h^1(X, V) = 3. \tag{44}$$

Comparing (43) to (44), it follows that the invariant subspace of the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $H^1(\widetilde{X}, \widetilde{V})$ must be three-dimensional. That is,

$$h^1(\widetilde{X}, \widetilde{V})^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 3. \tag{45}$$

Now, $\widetilde{V}$ is equivariant under the explicit action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ discussed earlier. However, as far as cohomology is concerned, one can consider nine equivariant actions specified by the characters ${\chi_1}^p {\chi_2}^q$ for $p, q = 0, 1, 2$, on $\widetilde{V}$. Since the bundle is the same, $H^1(\widetilde{X}, \widetilde{V})$, (43) and (44) remain unchanged. However, the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on $H^1(\widetilde{X}, \widetilde{V})$ will be altered for each choice of ${\chi_1}^p {\chi_2}^q$. Specifically, the original representation will be multiplied by the character. Since (43) and (44) remain unchanged, we conclude that

$$h^1(\widetilde{X}, {\chi_1}^p {\chi_2}^q \widetilde{V})^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 3 \tag{46}$$

for each choice of $p, q = 0, 1, 2$. The only way this can be true is if the original $\mathbb{Z}_3 \times \mathbb{Z}_3$ action is

$$H^1(\widetilde{X}, \widetilde{V}) = \mathrm{Reg}(\mathbb{Z}_3 \times \mathbb{Z}_3)^{\oplus 3}, \tag{47}$$

where the regular representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ is given by

$$\mathrm{Reg}(\mathbb{Z}_3 \times \mathbb{Z}_3) = 1 \oplus \chi_1 \oplus \chi_2 \oplus {\chi_1}^2 \oplus \chi_1 \chi_2 \oplus {\chi_2}^2 \oplus {\chi_1}^2 \chi_2 \oplus \chi_1 {\chi_2}^2 \oplus {\chi_1}^2 {\chi_2}^2. \tag{48}$$

Note that (48) contains all of the irreducible representations of $\mathbb{Z}_3 \times \mathbb{Z}_3$.

Now consider the the cohomology space $H^1(\widetilde{X}, \wedge^2\widetilde{V})$ associated with the **10** representation. We know from (42) that $h^1(\widetilde{X}, \wedge^2\widetilde{V}) = 14$. One can find the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on this space as follows. Recall from (40) that

$$H^1(\widetilde{X}, \wedge^2\widetilde{V}) \simeq H^1(\widetilde{X}, V_1 \otimes V_2). \tag{49}$$

It follows from (31) that

$$V_1 \otimes V_2 = (\pi_2^*(\chi_2 W_2))^{\oplus 2}, \tag{50}$$

where W_2 is defined by (28). Note that the χ_2 action on the line bundles in (31) modifies the equivariant structure of W_2, which we indicate by $\chi_2 W_2$. Then

$$H^1(\widetilde{X}, V_1 \otimes V_2) \simeq H^1(\widetilde{X}, \pi_2^*(\chi_2 W_2))^{\oplus 2}. \tag{51}$$

For ease of notation, we will, henceforth, denote $\chi_2 W_2$ simply as W_2. To proceed, one must calculate $H^1(\widetilde{X}, \pi_2^* W_2)$. This can be accomplished using (34) with $i = 2$ and $\mathcal{F} = \pi_2^* W_2$, as well as the push-down formulas

$$\begin{aligned} \beta_{2*} W_2 &= \chi_2 \mathcal{O}_{\mathbb{P}^1}(-2) \oplus {\chi_2}^2 \mathcal{O}_{\mathbb{P}^1}(-1), \\ R^1\beta_{2*} W_2 &= {\chi_2}^2 \mathcal{O}_{\mathbb{P}^1}(1) \oplus \chi_2 \mathcal{O}_{\mathbb{P}^1} \oplus \bigoplus_{i=1}^{3} \mathcal{O}_{\beta_2(p_k)}, \end{aligned} \tag{52}$$

where p_k are points in B_2 associated with the ideal sheaf I_9 in the definition of W_2. Using (52), we find that the terms adjacent to $H^1(\widetilde{X}, \pi_2^* W_2)$ in (34) are

$$H^0(\mathbb{P}^1, \beta_{2*} R^1\pi_{2*}(\pi_2^* W_2)) = 0 \tag{53}$$

and

$$\begin{aligned} H^1(\mathbb{P}^1, \beta_{2*} W_2) &= \chi_1^2 \chi_2 \\ H^0(\mathbb{P}^1, R^1\beta_{2*} W_2) &= (\chi_2^2 \oplus \chi_1 \chi_2^2) \oplus \chi_2 \oplus (1 \oplus \chi_1 \oplus \chi_1^2). \end{aligned} \tag{54}$$

Expression (53) cuts off the horizontal sequence in (34), yielding

$$H^1(\widetilde{X}, \pi_2^* W_2) \simeq H^1(B_2, W_2). \tag{55}$$

On the other hand, (54) inserted into the vertical sequence of (34) implies, using (55), that

$$H^1(\widetilde{X}, \pi_2^* W_2) = 1 \oplus \chi_1 \oplus \chi_2 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1 \chi_2^2 \oplus \chi_1^2 \chi_2. \tag{56}$$

Putting (49), (51) and (56) together, we find that the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $H^1(\widetilde{X}, \wedge^2 \widetilde{V})$ is

$$H^1(\widetilde{X}, \wedge^2 \widetilde{V}) = 2 \oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2. \tag{57}$$

Having determined $\widetilde{V}$, the cohomology groups $H^i(\widetilde{X}, U_R(\widetilde{V}))$ and the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on these spaces, it remains to compute the low energy spectrum of the observable sector. To do this, one must give the representation of $hol(W) = \mathbb{Z}_3 \times \mathbb{Z}_3$ on each multiplet R. We can choose the Wilson line W to have the following actions.

First consider $R = \mathbf{16}$. Then

$$\mathbf{16} = \left(\chi_1^2\chi_2(\mathbf{3}, \mathbf{2}) \oplus \chi_1^2\chi_2^2(\overline{\mathbf{3}}, \mathbf{1}) \oplus \chi_1^2(\mathbf{1}, \mathbf{1})\right) \oplus \left(\chi_2^2(\overline{\mathbf{3}}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2})\right) \oplus (\mathbf{1}, \mathbf{1}). \tag{58}$$

The terms are grouped according to the $\mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1}$ decomposition of $\mathbf{16}$ under $SU(5)$. For simplicity, we have only given the $SU(3)_C \times SU(2)_L$ quantum numbers in (58). Tensoring this with the action (47), (48) of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on $H^1(\widetilde{X}, \widetilde{V})$, we find that the invariant subspace is spanned by three families of quarks/leptons, each family transforming as

$$\left(\mathbf{3}, \mathbf{2}, 1, 1\right), \quad \left(\overline{\mathbf{3}}, \mathbf{1}, -4, -1\right), \quad \left(\overline{\mathbf{3}}, \mathbf{1}, 2, -1\right) \tag{59}$$

and

$$\left(\mathbf{1}, \mathbf{2}, -3, -3\right), \quad \left(\mathbf{1}, \mathbf{1}, 6, 3\right), \quad \left(\mathbf{1}, \mathbf{1}, 0, 3\right) \tag{60}$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. We have displayed the quantum numbers $3Y$ and $3(B-L)$ for convenience. Note from eq. (60) that each family contains a right-handed neutrino, as desired.

Now consider $R = \mathbf{10}$. We find that

$$\mathbf{10} = \left(\chi_1^2(\mathbf{1}, \mathbf{2}) \oplus \chi_1^2\chi_2^2(\mathbf{3}, \mathbf{1})\right) \oplus \left(\chi_1(\mathbf{1}, \overline{\mathbf{2}}) \oplus \chi_1\chi_2(\overline{\mathbf{3}}, \mathbf{1})\right). \tag{61}$$

where we have grouped the terms in the $\mathbf{5} \oplus \overline{\mathbf{5}}$ decomposition of $\mathbf{10}$ under $SU(5)$. Tensoring this with the the action (57) of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on $H^1(\widetilde{X}, \wedge^2 \widetilde{V})$, one finds that the invariant subspace consists of *two* copies of the vector-like pair

$$(\mathbf{1}, \mathbf{2}, 3, 0), \quad \left(\mathbf{1}, \overline{\mathbf{2}}, -3, 0\right). \tag{62}$$

That is, there are two Higgs-Higgs conjugate pairs occurring as zero modes in the observable sector. Note that the unobserved color triplet multiplets have been projected out, as desired. This is an explicit mechanism for "doublet-triplet" splitting.

We conclude that the zero mode spectrum of the observable sector 1) has gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, 2) contains *three families* of quarks and leptons each with a *right-handed neutrino*, 3) has *two* Higgs-Higgs conjugate pairs and 4) *contains no exotic fields* of of any kind. Additionally, there are 5) a *small number* of uncharged vector bundle moduli. These arise from the invariant subspace of $H^1\big(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*\big)$ under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ and will be computed elsewhere.

3.2 The Hidden Sector Bundle $\widetilde{V}'$

The vacuum also contains a stable, holomorphic vector bundle, V', on X whose structure group is in the E_8' of the hidden sector. Additionally, there can be a Wilson line W' on X whose $\mathbb{Z}_3 \times \mathbb{Z}_3$ holonomy group is contained in E_8'. However, to allow for spontaneously breaking of the $N = 1$ supersymmetry via gaugino condensation in the hidden sector, it is expedient to reduce E_8' as little as possible. With this in mind, we will choose W' to be trivial.

As for V, we construct V' by building stable, holomorphic vector bundles $\widetilde{V}'$ over $\widetilde{X}$ which are equivariant under $\mathbb{Z}_3 \times \mathbb{Z}_3$ using the method of "bundle extensions". V' is then obtained as the quotient of $\widetilde{V}'$ by $\mathbb{Z}_3 \times \mathbb{Z}_3$. This bundle must satisfy the anomaly cancellation condition (26). The simplest possibility is that $\widetilde{V}'$ is the trivial bundle. However, in this case, we find that $[\mathcal{W}_5]$ is not effective. Instead, we find the following minimal solutions, depending on whether one works in the strongly or the weakly coupled regime of the heterotic string.

3.2.1 Strong Coupling: Bulk Five-branes

The minimal vector bundle $\widetilde{V}'$ that is consistent with anomaly constraint (26) is found to have structure group $SU(2)$. For this bundle,

$$[\mathcal{W}_5] \neq 0 \tag{63}$$

and, hence, this hidden sector is compatible only with the strongly coupled heterotic string. $\widetilde{V}'$ spontaneously breaks the hidden sector E_8' gauge symmetry to

$$E_8' \longrightarrow E_7. \tag{64}$$

With respect to $SU(2) \times E_7$, the adjoint representation of E_8' decomposes as

$$\mathbf{248}' = \big(\mathbf{1}, \mathbf{133}\big) \oplus \big(\mathbf{3}, \mathbf{1}\big) \oplus \big(\mathbf{2}, \mathbf{56}\big)\,. \tag{65}$$

The $\big(\mathbf{1}, \mathbf{133}\big)$ contain the gauginos of E_7, the $\big(\mathbf{3}, \mathbf{1}\big)$ correspond to vector bundle moduli and $\big(\mathbf{2}, \mathbf{56}\big)$ represent charged exotic matter fields. In addition to demanding that $\widetilde{V}'$ satisfy the stability conditions (19), we require that there be no exotic matter in the hidden sector. This is most easily accomplished by imposing the constraint that

$$h^1(\widetilde{X}, \widetilde{V}') = 0\,. \tag{66}$$

The requisite $SU(2)$ bundle $\widetilde{V}'$ is any element of the space of extensions

$$0 \longrightarrow \mathcal{O}_{\widetilde{X}}(2\tau_1 + \tau_2 - \phi) \longrightarrow \widetilde{V}' \longrightarrow \mathcal{O}_{\widetilde{X}}(-2\tau_1 - \tau_2 + \phi) \longrightarrow 0. \tag{67}$$

One can easily show that the entire cohomology ring vanishes. That is

$$h^i(\widetilde{X}, \widetilde{V}') = 0, \qquad i = 0, 1, 2, 3. \tag{68}$$

Note that this result is consistent with the necessary conditions (19) that $\widetilde{V}'$ be stable. Furthermore, it follows that (66) is satisfied and, hence, there is no exotic matter in the hidden sector.

The five-brane wrapped on a holomorphic curve associated with $[\mathcal{W}_5]$ contributes non-Abelian gauge fields, but no matter fields, to the hidden sector. Following the results in [40,41], we find that the five-brane gauge group is $U(6)$. Moving in the moduli space of the holomorphic curve, this group can be maximally broken to $U(1)^6$.

We conclude that, within the context of the strongly coupled heterotic string, our observable sector is consistent with a hidden sector 1) with gauge group $E_7 \times U(6)$ and 2) *no exotic matter*. In addition, 3) there is a *small number* of vector bundle moduli arising from the invariant subspace of $H^1(\widetilde{X}, \widetilde{V}' \otimes \widetilde{V}'^*)$ under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$, as well as some five-brane moduli. These will be computed elsewhere.

3.2.2 Weak Coupling: No Five-branes

We now exhibit a hidden sector, consistent with our observable sector, that has no five-branes; that is, for which

$$[\mathcal{W}_5] = 0. \tag{69}$$

This hidden sector is compatible with both the weakly and strongly coupled heterotic string. We are unable to satisfy (69) for any bundle with an $SU(2)$ structure group. From the results in [42,43], we expect that the appropriate structure group may be the product of two non-Abelian groups, the simplest choice being $SU(2) \times SU(2)$. This bundle, which is the sum of two $SU(2)$ factors, $\widetilde{V}' = \widetilde{V}_1' \oplus \widetilde{V}_2'$, spontaneously breaks

$$E_8' \longrightarrow Spin(12). \tag{70}$$

With respect to $SU(2) \times SU(2) \times Spin(12)$, the adjoint representation of E_8' decomposes as

$$\mathbf{248}' = \big(\mathbf{1},\mathbf{1},\mathbf{66}\big) \oplus \big(\mathbf{3},\mathbf{1},\mathbf{1}\big) \oplus \big(\mathbf{1},\mathbf{3},\mathbf{1}\big) \oplus \big(\mathbf{2},\mathbf{1},\mathbf{32}\big) \oplus \big(\mathbf{1},\mathbf{2},\mathbf{32}\big) \oplus \big(\mathbf{2},\mathbf{2},\mathbf{12}\big)\,. \tag{71}$$

Representations $\big(\mathbf{1},\mathbf{1},\mathbf{66}\big)$ and $\big(\mathbf{3},\mathbf{1},\mathbf{1}\big) \oplus \big(\mathbf{1},\mathbf{3},\mathbf{1}\big)$ contain the $Spin(12)$ gauginos and vector bundle moduli. Exotic matter in the hidden sector can arise from $\big(\mathbf{2},\mathbf{1},\mathbf{32}\big)$, $\big(\mathbf{1},\mathbf{2},\mathbf{32}\big)$, and $\big(\mathbf{2},\mathbf{2},\mathbf{12}\big)$, corresponding to the cohomology spaces $H^1(\widetilde{X},\widetilde{V}_1')$, $H^1(\widetilde{X},\widetilde{V}_2')$, and $H^1(\widetilde{X},\widetilde{V}_1' \otimes \widetilde{V}_2')$ respectively. Unlike the case in the strong coupling regime, subject to (69) and the stability conditions eq. (19) applied to $\widetilde{V}_{1,2}'$, we are unable to find a hidden sector bundle for which all exotic matter is absent.

However, relaxing the constraints so that a small amount of hidden exotic matter may exist, one finds the following minimal solution. It turns out that $\widetilde{V}_1'$ is the bundle $\widetilde{V}'$ introduced in eq. (67) and $\widetilde{V}_2'$ is the pullback of an extension on B_1. Specifically, $\widetilde{V}_2' = \pi_1^* \mathcal{S}_B$, where

$$0 \longrightarrow \mathcal{O}_{B_1}(-2f_1) \longrightarrow \mathcal{S}_B \longrightarrow \mathcal{O}_{B_1}(2f_1) \otimes I_6 \longrightarrow 0. \tag{72}$$

Here, I_6 is the ideal sheaf of 6 points on B_1 which are a single orbit of $g_2 \in \mathbb{Z}_3 \times \mathbb{Z}_3$ with multiplicity 2.

Recall from eq. (68) that $h^i(\widetilde{X},\widetilde{V}_1')$ for $i = 0,1,2,3$ vanish. Therefore, $\widetilde{V}_1'$ satisfies the stability conditions (19) and there is no matter in the $\big(\mathbf{2},\mathbf{1},\mathbf{32}\big)$ representation. For $\widetilde{V}_2'$, we find that

$$h^0(\widetilde{X},\widetilde{V}_2') = h^3(\widetilde{X},\widetilde{V}_2') = 0, \quad h^1(\widetilde{X},\widetilde{V}_2') = h^2(\widetilde{X},\widetilde{V}_2') = 4. \tag{73}$$

Furthermore, for $\widetilde{V}_1' \otimes \widetilde{V}_2'$ one can show

$$h^0\big(\widetilde{X},\widetilde{V}_1' \otimes \widetilde{V}_2'\big) = h^3\big(\widetilde{X},\widetilde{V}_1' \otimes \widetilde{V}_2'\big) = 0 \tag{74}$$

and

$$h^1\big(\widetilde{X},\widetilde{V}_1' \otimes \widetilde{V}_2'\big) = h^2\big(\widetilde{X},\widetilde{V}_1' \otimes \widetilde{V}_2'\big) = 18. \tag{75}$$

If follows from (73) that $\widetilde{V}_2'$ also satisfies the stability constraints (19). However, $h^1(\widetilde{X}, \widetilde{V}_2')$ and $h^1\big(\widetilde{X}, \widetilde{V}_1' \otimes \widetilde{V}_2'\big)$ do not vanish and may give rise to hidden sector exotic matter in the representations $\big(\mathbf{1}, \mathbf{2}, \mathbf{32}\big)$ and $\big(\mathbf{2}, \mathbf{2}, \mathbf{12}\big)$ respectively.

To analyze this, it is necessary to explicitly compute the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on these cohomology spaces. This can be accomplished using methods similar to those discussed previously. Here, we simply state the results. The action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on $H^1(\widetilde{X}, \widetilde{V}_1')$ and $H^1\big(\widetilde{X}, \widetilde{V}_1' \otimes \widetilde{V}_2'\big)$ is found to be

$$H^1(\widetilde{X}, \widetilde{V}_1') = 2\chi_1 \oplus 2\chi_1^2 \tag{76}$$

and

$$H^1\big(\widetilde{X}, \widetilde{V}_1' \otimes \widetilde{V}_2'\big) = \mathrm{Reg}(\mathbb{Z}_3 \times \mathbb{Z}_3)^{\oplus 2} \tag{77}$$

respectively. It follows from (76) that $H^1\big(\widetilde{X}, \widetilde{V}_2'\big)$ has no invariant subspace. Since there is no Wilson line in the hidden sector, all $\big(\mathbf{1}, \mathbf{2}, \mathbf{32}\big)$ exotic matter fields are projected out of the low energy spectrum. Unfortunately, this is not the case for $H^1\big(\widetilde{X}, \widetilde{V}_1' \otimes \widetilde{V}_2'\big)$. Action (77) implies that there remain two exotic $\mathbf{12}$ multiplets of $Spin(12)$ after projection.

We conclude that, for vacua with no five-branes, our observable sector is consistent with a hidden sector 1) with gauge group $Spin(12)$ and 2) *two* $\mathbf{12}$ multiplets. We emphasize that these are not charged under the observable sector gauge group. There are also vector bundle moduli arising from the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant subspace of $H^1(\widetilde{X}, \widetilde{V}' \otimes \widetilde{V}'^*)$, which will be computed elsewhere. These vacua can occur in the context of both the weakly and strongly coupled heterotic string.

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