THE APPLICATION OF DYNAMIC RELAXATION

TO THE DESIGN OF

MODULAR SPACE STRUCTURES

A thesis submitted for the degree of Doctor of Philosophy by

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To my parents

CONTENTS

		Page No.
ABSTRACT		iv
ACKNOWLEDGEMENTS		v
CHAPTER 1:	INTRODUCTION	1
CHAPTER 2:	REVIEW OF TOPOLOGICAL DESIGN METHODS	9
	Introduction	10
	Review of Topological Design Methods	11
	Suitable Comparative Methods of Topological Design	20
	Linear Programming Techniques	21
	Non-Linear Programming	33
	Fully Stressed Design Methods	52
	Examples	54
	The Problems of Buckling Effects in Topological Design	59
CHAPTER 3:	DYNAMIC RELAXATION FORMFINDING	60
	Review of Dynamic Relaxation	61
	Outline of the Dynamic Relaxation Method with Finite Element idealisation	64
	The Damping Factor	67
	The Time Interval	69
	Initial Conditions	72
	Shapefinding and Formfinding with Dynamic Relaxation	73
	Michell's Theorem for Optimum Structures	74
	Formfinding for Modular Triangulated Space Structures	78
	Basis and Control of Dynamic Relaxation Formfinding for Space Structures	80
	Example Structures	83
	Comparison with other Methods	94
	Further Comments	98

Page No.

CHAPTER 4: INVESTIGATION OF THE DYNAMIC RELAXATION FORMFINDING METHOD FOR MODULAR SPACE STRUCTURES 99

A Parametric Study of Stability 100

The Effects of Topology on Stability and the Solution Convergence Rate 103

Additional Mass Components 106

The Choice of Reference Member 107

Initial Member Areas of the Ground Structure 109

More Complex Loading Conditions 109

The Inclusion of Self Weight 110

True Member Area Sizes 112

The Effects of Different Stress Constraints in Tension and Compression Members on Optimum Form 113

Conclusions 119

CHAPTER 5: THE SIZING OF MODULAR SPACE STRUCTURES OF FIXED TOPOLOGY 121

Fixed Topology Member Area Sizing 122

Concurrent Loading Vectors and Damping 124

The Effects of Initial Member Area Sizes 127

Maximum and Minimum Member Area Sizes 130

Further Comparison with Non-Linear Programming Solutions 135

Deflection Constraints 148

Further Comparisons with Non-Linear Programming Solutions 153

Conclusions 155

CHAPTER 6: FORMFINDING AND SIZING OF MODULAR SPACE STRUCTURES CONSIDERING MULTIPLE LOADING CASES 158

Introduction 159

Fully Stressed Design Solution 162

Solution by Linear Programming 164

Dynamic Relaxation Solutions assuming Fixed Trends 166

Interactive Dynamic Relaxation Formfinding 173

		Page No.
	Non-Linear Programming Solution	178
	Topological Non-Linear Design	180
	On-Off non-linearities and Dynamic Relaxation	183
	Design of structures with cable elements	184
	The use of Prestress Effects in Design	185
	Conclusions	188
CHAPTER 7:	GENERAL CONCLUSIONS AND SUMMARY	190
APPENDIX I:	APPLICATION OF THE THEOREMS OF STRUCTURAL VARIATION TO ALLOW FOR CLADDING	197
APPENDIX II:	A REVIEW OF REANALYSIS TECHNIQUES	201
APPENDIX III:	NON-LINEAR ALGORITHMS SUITABLE FOR THE DESIGN OF SPACE STRUCTURES	213
AN ANNOTATED BIBLIOGRAPHY CONCERNING THE COMPUTER DESIGN AND OPTIMISATION OF SPACE STRUCTURES		220

ABSTRACT

This thesis is concerned with the development and assessment of computer techniques for the formfinding and sizing of large modular building space structures suitable for urban development.

The contents of the Chapters are summarised as follows:

1: An introduction to conceptual and computer aided design of large building space structures.

2: A review of topological computer design methods.

3: A comparison of a dynamic relaxation method, for the formfinding of modularly constrained structures subject to a dominant design loading case, with linear programming and fully stressed design methods. This comparison shows that the dynamic relaxation method is efficient and particularly suitable for interactive use.

4: A parametric study of the effects of the iteration parameters on the stability and rate of convergence is presented. No general rules appear to be possible regarding the effects of these parameters on stability. It is noted, however, that the number of structure modifications before the solution becomes apparent is independent of the parameters. The dynamic relaxation formfinding procedure is generalised to cater for different stress constraints in tension and compression members and, for the problem considered, derives a lighter form than the fully stressed design technique. The optimum form of the D.R. solution is verified by the linear programming technique.

5: An intuitive dynamic relaxation method for the sizing of structures of fixed topology subject to multiple loading cases and stress constraints is presented. The method also caters for maximum member area sizes and deflection constraints by the use of parallel elastic and elasto-plastic analyses. Solutions derived using this method are compared with solutions derived using the non-linear program algorithm. They are shown to be of similar weight and to require similar solution times.

6: Two dynamic relaxation methods are presented for the formfinding and sizing of multiply loaded space structures. The first or parallel method is suitable for deriving and sizing forms of optimum or near optimum weight by deleting members which are small in area size and reducing in size. The second or series method is particularly suitable for interactive use and consists of testing the efficiency of each member with respect to each loading case. The final topology is then sized considering all loading cases simultaneously. These methods are both applied to a bridging ground structure subject to multiple loads and compared with solutions derived using linear, non-linear programming and topological design methods. The parallel dynamic relaxation method is then extended to cater for cable members allowing for on-off non-linearities and prestress effects. The bridging structure is subsequently redesigned using internal cable members and adjusting the prestress level to ensure that the bridge deck does not deflect vertically under the action of the primary loading case.

7: A summary of conclusions.

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CHAPTER 1: INTRODUCTION

During the last thirty years space structures have been increasingly used to construct a wide range of exciting and imaginative forms. The main distinguishing feature of such structural systems is that they are light, stiff and can sustain omni-directional forces. They have been most frequently used as roofing structures. A rapid rise in the numbers of these roofs has resulted because of economy through prefabrication and standardisation, the ability to cover large spans without internal columns and the freedom of expression allowed with this type of construction. There are now a large number of building systems available to construct these skeletal frameworks. Many of these, together with the associated architectural and engineering developments of space structures generally, have been reviewed by Makowski (9, 10, 11).

These roof structures are primarily two dimensional structures because their form is independent of the internal plan of the structure. In contrast, space structures which allow the construction of houses and offices within the skeletal grid require planning in three dimensions. An architectural study of this type of structure used for urban development was published in 1966 by Yona Friedman (49). Since then Du Chateau (111) has worked on the practical development of giant prefabricated systems for these "space towns". These structures are supported at a few points only, to allow circulation and existing building development to remain underneath and hence they are primarily bridging structures. The recent work of Gabriel (6,7) and Pearce (15) is also relevant to the architectural development of these structures.

Gabriel (6,7) showed it was possible to organise the architectural space of multi-layered space frames (with two or three way grids) by using the braces or oblique columns to divide the space into modular units. He subsequently illustrated how three-dimensional suburbs can be developed within the space frame system. Two examples of his megastructure suburbs are shown in figures 1 and 2. This form of construction results in economy and allows for adaption with changing use of the structure. Pearce looked to nature when designing the nodal connector for his space structure system (15).

Earlier work comparing and relating biological morphology to mechanical efficiency and form had been initiated by D'Arcy Thompson (19). This work was further developed by Frei Otto (17) who noted the similarity between the first man made building structures and the nests, dens and other structures built by animals. Otto pointed out that it was possible to make direct design comparisons between advanced engineering structures and the similar internal forms of living creatures and organisms. A space lattice structure with members with short buckling lengths is shown in figure 3. It was designed by Otto in 1962-3 following model studies in minimal weight form. This elegant structure has a remarkable resemblance to the unicellular biological structures studied by Otto. Otto's work led Conrad

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Roland to comment "One may venture to speak of a new era in architecture - an era that is more natural - more true to life".

Pearce's approach, although drawing parallels from nature, was based on a completely different postulate which is microscopic in comparison with Otto's. Pearce noted that nature produces snowflakes from crystals of water, no two of which have been found alike. This represents what he coins a minimum inventory maximum diversity building system. These systems are often seen in crystal and molecular structures. Pearce developed a Universal Node connector by considering the integration of crystal forms. The model Universal Node, shown in figure 4, has 26 spokes emanating from a common centre such that 6 spokes are square in cross section, 8 spokes are triangular in cross section and 12 spokes are rectangular in cross section. These different shaped spokes of the node represent axes of rotational symmetry. The nodal connector represents an inventory of alternatives of great diversity and adaptability which can be generated with simplicty and efficiency.

Pearce quoted D'Arcy Thompson who said "In short the form of an object is a diagram of forces, in this sense at least, from it we can judge or deduce the forces that are acting or have acted upon it, in this strict and particular sense it is a diagram". Pearce classified these forces as; intrinsic forces which are the governing factors which are inherent in the structural system, and extrinsic forces which are those governing influences which are classified by design criteria. A model of

an eight storey building structure designed by Pearce using the Universal Node and considering extrinsic forces is shown in figure 5. Pearce has also considered the use of this building system for constructing megastructures. For building structures the grid will be nowhere near fully connected but will still be highly redundant to ensure that the structure has an ability to redistribute loads, in the event of local failure.

In nature, forms are determined by interaction of intrinsic and extrinsic forces. Pearce was concerned that his Universal Node, an intrinsic force system was capable of effective responses "to the environmental circumstances which now prevail and will continue to prevail". Assuming that this building system exhibits this adaptability and that the practicalities of constructing the connector have been overcome; then the engineer and architect require to assess proposed design with respect to the extrinsic forces and decide which bars of the system should preferably be included and their area size. In following such a design policy the resulting form of the structure might then comply more closely with the "diagram of forces" referred to by D'Arcy Thompson.

Computer design or sythesis of this type of problem is a science in it's infancy and generally consists of an analysis with subsequent resize and reanalysis cycles. When changes in size of the members ceases or become slow the design process is terminated and assumed to have arrived at an 'optimum'. With the design of large structures each analysis can be very expensive.

Reanalysis methods which assess the effects of modification on the response of the structure in less time than a complete analysis become indispensable. These methods can also be used to assess the adaptability of the structure, the effects of support movement, the feasibility of construction methods, the evaluation of errection loads, the effects of fabrication tolerances and the "bridging action" which can be expected as a consequence of damage to any part of the structure. These reanalysis methods are discussed in more detail in Appendix II.

Methods for assessing the resize of these structures are innumerable but can be divided into a series of classes (e.g. linear programming, non-linear programming and fully stressed design techniques) which are outlined in Appendix III. It is interesting to note here that some attempts have been made to mirror the evolutionary processes of nature when synthesising structures. For example the Monte-Carlo or Random Search Method (137,144) which searches for better solutions in random directions is said to be an evolutionary strategy. However,many engineers will not accept this analogy and similarity between heuristic mathematical search strategies and evolution strategies as proof of their validity.

Methods of topological design reviewed in Chapter 2 include synthesis resize methods which consider form by deriving subsiduary structures when deleting inefficient members from a ground structure of candidate structures. These methods are particularly suitable for the design of modularly constrained space structures.

This thesis is concerned with the development and assessment of computer techniques for the formfinding of sizing of large modular building structures. With these structures the design process will ideally be a synthesis of architectural and structural design so the problem cannot be solved by simply initiating an automatic optimisation procedure. The design may become a dynamic process in which many functional or conceptual decisions may be made during the process which will repeatedly alter the design objectives and constraints. A most promising tool for this type of interactive design problem is the explicit analysis technique - Dynamic Relaxation. With this analysis method there is no overall stiffness matrix and ammendments can be made to the structure during the analysis.

In Chapter 3 a ground structure method using Dynamic Relaxation for the formfinding of modularly constrained structures subject to a dominant design load is assessed and compared with derived solutions using the Fully Stressed Design Technique and Dual Linear Programming Method. The methods are applied to a series of cantilever structures and the computation times compared. Using the dominant loading case for formfinding ensures that the form clearly expresses the main function of the structure.

In Chapter 4 the stability and rate of convergence of the Dynamic Relaxation method is investigated with respect to the iteration parameters. The method is generalised to cater for differing stress constraints in tension and compression

members.

members. The results are compared with solutions derived from the Fully Stressed Design Technique and Dual Linear Programming methods.

In Chapter 5 the form of the structure is assumed fixed having been derived using the techniques of Chapters 3 and 4 or from other architectural considerations. An intuitive method using Dynamic Relaxation for sizing structures of fixed form and subject to stress constraints is presented. The method also caters for maximum member area sizes and deflection constraints by the use of parallel elastic and elasto-plastic analyses. The method is applied to a series of problems and comparison of the solutions and computational times is made with those derived from a non-linear programming algorithm.

In Chapter 6, methods suitable for the formfinding and sizing of modularly constrained structures are applied to a ground structure to derive an efficient form for a bridging structure subject to multiple loading cases. Two alternative procedures using Dynamic Relaxation are presented and applied to the structure. The second of which is particularly suitable for interactive use. The design procedure is extended to cater for structures with cable elements. This method, which accounts for cable slackening and prestress effects, is applied to the ground structure to find an efficient form and size the members. This method is particularly suitable for the preliminary form-finding and assessment of design alternatives for composite structures composed of cable and stiff members. The method may be applied to bridging structures such as Network Arches (20) or to building structures such as the prefabricated suspended block of flats designed by Minke (13,14).

CHAPTER 2: REVIEW OF TOPOLOGICAL DESIGN METHODS

Introduction - Review of Topological Design Methods - Suitable Comparative Methods of Topological Design - Linear Programming - Non-Linear Programming - Fully Stressed Design Methods - Examples - The Problems of Buckling Effects in Topological Design.

Summary:

In this chapter computer methods for the topological design of triangulated structures are reviewed. These methods are suitable for the formfinding of optimum or near optimum topology. The methods used for comparative purposes in the following chapters are outlined in detail and applied to a series of three bar truss problems.

Introduction:

It is now generally agreed that computer analysis methods are sufficiently advanced so that nearly all structural problems can be modelled to the required degree of accuracy for practical situations. These rapid and efficient analysis techniques should pave the way for great advances in design methodology. However, current design approaches have not nearly as great a degree of sophistication and generality as the current analysis methods. During the last ten years, the increase in the number of papers published in this field has indicated the interest in and need for efficient and general methods of computer or computer aided design.

The goal of most of these published methods has been the minimisation of the structural weight or volume. Although these methods represent a considerable advance, techniques should take account of fabrication, construction and maintenance costs for practical design. Aesthetic criteria are often of prime importance too. The probability of structural failure both local and total and is consequent 'cost' must also be considered in any comprehensive design process.

For the type of structures discussed in Chapter 1 the cost of preparing the members and joints can be kept to a minimum by prefabrication. The size and cost of joints can be reduced by fitting the 'points' of the universal node only when required. In this way the cost function may possibly be close to or a linear function of the weight.

The use of linear and non-linear programming is now well established in relation to the optimisation of pin jointed frameworks. In this case non-linearity does not describe the material behaviour but the response of the structure to changes in the design variables. In fact optimisation techniques are confined to structures which are idealised with linear material behaviour. Other more intuitive optimality criteria based methods such as Fully Stressed Design Techniques are efficient and usually yield optimum or near optimum designs.

Methods which use only member areas as design variables are discussed in Appendix III. Methods which use additional design parameters to represent topology in the design process are reviewed in the following section. These methods can be used for formfinding of triangulated structures.

Review of Topological Design Methods:

For triangulated structures the topology is almost independent of the member forces and many diverse forms may be used to support given loading systems. Engineering judgement cannot always ensure that the optimum form is chosen. Methods which derive the best topology, which can be sized to give a minimum least weight design and support given loading systems while satisfying the design constraints are therefore required by the engineer.

One of the earliest approaches to optimisation of structural form was developed by Michell (194) using a theorem previously presented by Maxwell (193). Michell's theorem, for a single loading case, stated that "a frame attains the limits of economy of material possible in any frame under the same applied forces, if the space occupied by it can be subjected to an appropriate small deformation, such that the strains in all the bars of the frame are increased by equal fractions of their lengths, not less than the fractional change of length of any elements of the space". Michell structures depend on an appropriate specification of the strain field and are unfortunately usually impractical, consisting of non-standard lengths and joints. This early work was discussed and developed by others (175, 180, 195, 196) but still suffered from the same impracticalities. Parkes (197) has published a theoretical investigation in which he considered the effect of the cost of joints when deriving the optimum form of networks under simple single loading conditions.

Hemp and Chan (176, 187, 188, 189) and in parallel work Dorn, Gomory and Greenberg (182) overcame these impracticalities by considering a 'ground structure' of a grid of points which included the structural joints and loading positions. The grid was connected by many potential members. They showed that a structure subjected to a virtual displacement field which maximises the external work and complies with the strain constraints is an optimum. The virtual displacements of the points were varied using linear programming methods to make the virtual work a maximum and so a strain field was derived in which all permitted members achieved the maximum virtual strain. Other members were then removed. Some of the remaining members and nodes can also usually be removed by considering equilibrium but the reduced structure may still be indeterminate. If $\sigma_c = \sigma_T$ then the indeterminate subset/full set or determinate subset is readily sized so fully stressed to give a least weight design. Dorn, Gomory and Greenberg (182) also investigated the effect of the grid size on optimum form and weight.

Pearson (200) also used a linear technique in which he expressed the redundant member forces as the variables and varied them with a random direction search technique until a minimum was found. He noted that for a single loading case "normally only a statically determinate set of links have non-zero areas". However he was only considering a much reduced ground structure with little choice of alternative members. The important point is that if enough alternative members are available then although the optimum value will be unique the form is not necessarily unique and can become a matter for choice.

Pearson (200), Chan (176) and Dorn, Gomory and Greenberg (182) all considered the problem of multiple loading cases, in which the optimum is nearly always indeterminate. The member forces were expressed in terms of the applied loads and the redundant forces. The redundant forces were then varied until the weight function became a minimum. The problem of linear programming techniques with multiple loading cases was reviewed and developed by Hemp (188), Reinschmidt and Russell (206) and Pope (203).

The problems of fully stressing determinate and indeterminate structures were however understood in concept much earlier (177, 185, 202, 223). Schmidt (211) avoided these problems by considering the layout of statically determinate structures with multiple loading cases. However he concluded that a statically indeterminate form could sometimes give a lighter structure than a statically determinate form. Schmidt (158) later considered the problem of fully stressed indeterminate structures under multiple loading cases and stated that there will be a number of fully stressed designs and that the optimum structure will be the lightest of these.

The problems of ensuring a compatible design together with deflection constraints led to the application of non-linear programming methods to formfinding. These techniques can be divided into three main groups.

I. Methods which allow member areas to reduce to zero.

II. Methods which include nodal co-ordinates, etc., as design variables.

III. Hybrid methods which allow for topology considerations at certain points during the design.

Type I methods will, of course, include any non-linear programming technique in which the bar areas have no lower bounds. Little investigation of this type has been reported. Dobbs and Felton (181) used a 'steepest descent - constant weight algorithm' to minimise the weight of a ground structure subject to stress constraints and multiple loading. During the process some member areas reduced to zero and these were removed and not allowed to re-enter the design. There is no mathematical justification for the removal of these members and no proof that they would not subsequently help to reduce the weight of the structure. Gallagher (132) noted in his review of Fully Stressed Design Techniques that "it appears that there is a need for algorithms that incorporate the automatic removal of members as they approach zero size so as to include consideration of subsidiary structural forms".

Methods of type II include the work of Schmit et alia who included the angles between members as variables when formulating the optimisation problem of a three bar truss using the steepest descent algorithm. Schmit (212) showed that for fixed topology under multiple loads the optimum solution need not be fully stressed. Schmit and Kicher (214) considered several configurations by varying the angles and so made configuration a discrete variable. They showed that the optimum configuration can change for even slight variations in the design constraints and materials. The choice of design was made by comparison of the optimum for each configuration. Schmit and Morrow (215) introduced buckling stress constraints into the problem. No automatic method was yet available for deriving the optimum topology. Schmit and Mallett (215) considered the angles as continuous design variables. Their results indicated that the optimum objective value and displacement pattern for a problem can be achieved with different sizing of the members. The optimum form was thus shown not to be unique. The inclusion of configurational variables improved the optimum value and they showed that even with multiple loading cases the optimum may be statically determinate.

Sved and Ginos (274) investigated a three bar truss previously solved by Schmit (212) showed that a global optimum could be obtained for the triple loading case problem by removing one of the members and in effect violating the stress constraint. They suggested "if there is a single redundancy, it is necessary systematically to search all perfect structures (i.e. structures for which the stiffness matrix is not singular) that can be obtained from the original one by omitting one member". Corcoran (178, 179) reconsidered this truss problem and made it six dimensional by including three nodal co-ordinates as design variables. He showed that the problem of considering reduced topology could be avoided if the co-ordinates were considered as variables thus allowing some nodes to coalesce and the stress constraints of all members to be satisfied. Pedersen (201) also presented a technique utilising the simplex method and nodal sensitivity analysis where joint co-ordinates were included as design variables. Although the method included self weight stress and buckling constraints the solutions were statically determinate and only derived for a single loading case. Spillers (220) developed an iterative procedure based on optimality criteria for pin jointed structures under a single loading case.

Many of the design methods which use nodal co-ordinates as design variables, optimise the structure iteratively considering the two design spaces of member areas and nodal co-ordinates separately. This technique avoids the ill conditioning problems usually associated with combining area and co-ordinate variable design spaces.

Fu (186) used an iterative search technique for the design of pin jointed trusses subject to multiple loads. He showed that the response curve of an unloaded joint is unimodal. Vanderplaats and Moses (225, 226, 227) developed a separated technique for multiple loading areas by using the steepest descent method for co-ordinate variables and the stress-ratio method for stress constraints. The method was iterative and first sized the members with respect to the stress constraints and then moved the nodes to their optimum position. Lipson et alia (190, 191) used a modified 'complex' method for the design of nodal positions and a stress ratio method with displacement scaling method for member sizing. The method was developed to account for discrete member sizes throughout the process. These techniques which consider the movement of nodes to form optimum structures show significant improvement when compared with conventionally designed structures.

Methods of type III include the work of Reinschmidt and Russell (206, 207, 209) who by relaxing the compatibility requirements used an iterative linearisation procedure to remove inefficient members from an indeterminate structure with multiple loading cases. The structural form was fixed by using a dual simplex linear program. The reduced structural form was resized using the fully stressed design technique and a new estimate of the buckling stress made before the linear program was reapplied to the reduced form

Initial conservative estimates of the buckling stress must be avoided to ensure that important members are not deleted by the linear program. They showed that the method was better than the fully stressed design method alone for deriving an optimum configuration. Sheu and Schmit (217) also used a ground structure technique. They based their design on a comparison of the upper bound for the configuration derived with a feasible direction technique and lower bounds for subsets of the configuration derived with a dual simplex algorithm with compatibility relaxed. This method seems unwieldy for large problems.

Farshi and Schmit (184) developed an iterative technique using a force formulation with the simplex method to derive a solution considering equilibrium conditions only and a displacement analysis to assess the compatibility requirements that need to be imposed on the design. The compatibility requirements are thus gradually applied and a global optimum form is derived.

Spillers and Freidland (221) considered the philosophy of adding members during a design process and presented some simple examples of statically determinate structures under a single loading case.

Majid and Elliott (50, 71, 73, 74) developed a combined technique for pin jointed structures which used a non-linear programming method (gradient type) to optimise a ground structure. Theorems of structural variation were used to provide trade-off data to indicate the order in which members should be deleted from the structure. This method provides a series of local optimum designs of gradually reducing redundancy and is the only method that directly applies the criteria of Sved and Ginos.

Porter Goff (204, 205) and Palmer and Sheppard (199) applied dynamic programming tecniques to the design of the shape of pin jointed cantilever structures. This technique sequentially designs the structure in a series of decisions which gradually bridge the gap to be spanned by the structure.

Suitable Comparative Methods of Topological Design:

For the modularly constrained structures discussed in Chapter 1 topological design method which use the concept of a ground structure are most suitable. All the standard computer methods of design can be applied to this problem, i.e. linear programming, non-linear programming, and fully stressed design techniques and these methods will now be outlined in detail as used for comparative purposes in this thesis.

Linear Programming Techniques:

Linear programming techniques are methods which can be applied to optimisation problems where the objective function and the constraints are linear. These problems can be formulated in the following way:

$$\text{Maximise} \quad Z = c_1 x_1 + c_2 x_2 + \ldots \ldots + c_n x_n$$

Subject to the constraints:

$$\sum a_{ij} x_j \quad \left\{ \leqslant \text{ or } = \geqslant \right\} b_i \qquad (i = 1, \ldots \ldots m)$$

$$x_j \geqslant 0$$

Where a_{ij} , b_i and c_i are all constants and there are m constraints.

The use of the simplex method for solving linear problems of this type will be outlined but first the formulation for structural optimisation will be presented. This method of linear programming is particularly suitable for formfinding of triangulated structures because the whole design space is considered throughout the process ensuring that global optimum solutions are derived.

For a pin jointed structure with only one loading case the problem of minimising the volume of the material can be stated as:

$$\text{Minimise: } \sum_{i=1}^{m} L_i A_i$$

where:

m = the number of members

L_i = length of member i

A_i = area of member i

subject to the constraints:

$$A_i \geqslant 0 \qquad i = 1, 2, \ldots m$$

$$T_i \leqslant \sigma_T A_i \qquad i = 1, 2, \ldots m$$

$$-T_i \leqslant \sigma_c A_i \qquad i = 1, 2, \ldots m$$

where:

T_i = force in member i

σ_T = permissible tensile stress

σ_c = permissible compressive stress

subject to the equilibrium conditions

$$\sum_{i=1}^{m} K_{ij} . T_i = F_j \qquad j = 1, 2, \ldots n$$

where: K_{ij} = direction cosines of the members in the framework.

The compatibility requirements of the structure are not taken into account in this formulation.

This problem can be stated in matrix form as follows:

$$\begin{bmatrix} K_{ji} & O \\ I & -\sigma_c \\ -I & \sigma_T \\ O & [L]^T \end{bmatrix} \begin{bmatrix} T \\ A \end{bmatrix} \begin{bmatrix} =F \\ \geq 0 \\ \geq 0 \\ \min \end{bmatrix}$$

This problem is designated the primal problem. By reference to Duality principles as discussed by Dantzig (128), Hadley (136) and Wagner (171), a dual problem can be formulated in which:

(I) The jth column of coefficients in the primal is the same as the jth row of coefficients in the dual.

(II) The row of coefficients of the primal object function is the same as the column of constraints on the right hand side of the dual

(III) The column of constraints on the right-hand side of the primal is the same as the row of coefficients of the dual objective function.

(IV) The direction of the inequalities and sense of optimisation are reversed in the pair of problems.

The formal proof of these duality principles can be developed by use of Lagrange's method (138, 136).

Using these duality principles the problem can be restated as:

$$\begin{bmatrix} K & +I & -I \\ O & \left[-\sigma_c\right]^T & \left[\sigma_T\right]^T \\ \left[F\right]^T & O & O \end{bmatrix} \cdot \begin{bmatrix} V \\ LE' \\ LE'' \end{bmatrix} \begin{bmatrix} =O \\ \leqslant hL^T \\ \max \end{bmatrix}$$

where the variables are usually interpreted as:

V = a nodal virtual displacement vector

E_i' E_i'' = the compressive and tensile virtual strains in member i

The first set of equations represent a set of compatibility requirements for the structure:

$$\sum K_{ij} \cdot V_j + L_i E_i' - L_i E_i'' = 0$$

The objective function represents the external work of the virtual displacement of the nodes. It can be shown by duality principles the optimum maximum value of this function equals the optimum minimum value of the primal volume objective function.

This dual problem is to find a set of compatible joint displacements so that the bar strains are within allowable limits and the external work maximised. By way of contrast the primal problem is to find a set of bar areas and forces in equilibrium so that the bar stresses are within allowable limits and the volume minimised. The strain limits in the dual problem do not generally correspond to the stress limits in the primal problem. If (as throughout Chapter 3) $\sigma_c = \sigma_T = \sigma_a$ then the second set of equations become:

$$\sigma_a(-L_i E_i' + L_i E_i'') \leqslant hL_i$$

where:

h = the strain energy density factor (T.Δ/Volume)

The dual and primal problems should correspond then:

$$h = \frac{T\Delta}{\text{Volume}} = \frac{\sigma_a . A_i}{A_i\ L_i} \cdot \frac{\sigma_a L_i}{E_m} = \sigma_a E_a$$

E_m = Young's modulus.

and the constrains in the dual problem become:

$\sigma_a\ (L_i E_i) \leqslant \sigma_a E_a L_i$ hence: $E_i \leqslant E_a$

which is the same as the permissible stress constraint:

$$T_i / A_i \leqslant \sigma_a$$

This means that for the special case of $\sigma_c = \sigma_T$ the dual and the primal problem fully correspond and any solution whether determinate or indeterminate is both compatible and fully stressed and hence optimum.

For the case where $\sigma_T \neq \sigma_c$ then:

$$h = \sigma_T\ E_T = \sigma_T^2/E$$

and $$h = \sigma_c\ E_c = \sigma_c^2/E$$

Hence compatibility cannot be ensured for an indeterminate elastic structure and in this case the limits on the member strains do not correspond to the limits on the bar stresses in the primal problem.

The dual linear programming formulation can readily be presented for multiple loading cases. The problem with two loading cases is given below:

$$\begin{bmatrix} K_{ij} & 0 & +I & -I & +I & -I \\ 0 & K_{ij} & 0 & 0 & +I & -I \\ 0 & 0 & -\sigma_{C1} & \sigma_{T1} & -\sigma_{C2} & \sigma_{T2} \\ F_1^T & F_2^T & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ LE_1' \\ LE_1'' \\ LE_2' \\ LE_2'' \end{bmatrix} \begin{bmatrix} = 0 \\ = 0 \\ \leqslant hL \\ \max \end{bmatrix}$$

The objective function represents the sum of the work of the external forces in all loading cases. If $\sigma_c = \sigma_T$ for all loading cases then the constraints become:

$$\sum_{k=1}^{2} E_k' \leqslant \frac{\sigma_a}{E_m}$$

Which means that the sum of the absolute values of the member distortions for all loading cases cannot exceed the allowable elastic distortion.

The member distortions, E'', can be removed from the dual by substituting the equality into the inequality to give:

$$\begin{bmatrix} K_{ij} & 0 & -I & 0 \\ 0 & K_{ij} & 0 & -I \\ \sigma_{c1} K_{ij} & \sigma_{c2} K_{ij} & (\sigma_{T1} - \sigma_{c1}) & (\sigma_{T2} - \sigma_{c2}) \\ F_1^T & F_2^T & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ L E_1' \\ L E_2' \end{bmatrix} \begin{bmatrix} \lesssim 0 \\ \lesssim 0 \\ \lesssim hL \\ \max \end{bmatrix} \qquad (2.1)$$

These formulations can readily be extended to cater for any number of alternative loading cases.

Wagner (171) suggests that the computational burden of the simplex method increases as the cube of the number of constraints. The dual formulation is, therefore, preferred because it usually has far fewer constraints.

The solution of the above structural formulations is standard and can be found in books on linear programming (128, 136, 171). The method proceeds in systematic steps from an initial feasible solution to other feasible solutions in such a way that the value of the object function at each iteration is better (or at least not worse) than at the preceeding step.

The constraints can usually be expressed in the following form:

$$\sum_{i=1}^{n} a_{ij} . x_i \leqslant b_j \qquad (j = 1, \ldots m)$$

where b_j is always made positive.

These inequality constraints can be converted into equality constraints by adding m slack variables as follows:

$$\sum_{i=1}^{n} a_{ij} x_i + x_{n+j} = b_j \quad (j = 1, \ldots m)$$

A suitable feasible starting solution would be:

$$x_{n+j} = b_j \qquad (j = 1, \ldots. m)$$

and

$$x_i = 0 \qquad (i = 1, \ldots. n)$$

It is now said that the slack variables are in the basis and are called basic variables. The remaining variables are called non-basic.

The objective function can also be rearranged in a similar way:

$$\sum_{i=1}^{n} -c_i x_i + Z = 0$$

where Z is the value of the objective function which is initially zero.

It should be noted that if the objective coefficient i is negative then the value of the objective function can be improved increasing the variable x_i.

The cycle of calculation for the simplex method are as follows:

1. If there are any variables not in the basis which have negative function coefficients the one with the most negative coefficient (x_j), that is the best per unit potential gain is selected. If all non-basic variables have positive or zero coefficients an optimal solution has been obtained.

2. Calculate the ratios of the current right hand side to the coefficient of the entering variable for each constraint. (Ignore ratios with negative numbers or zeros in the denominator). Select the minimum ratio which will equal the value of x_j in the next feasible solution. The minimum occurs for the variable x_k in the present solution which is set to zero

in the next. (This variable becomes zero first if x_j is increased and if the constraints are not to be violated.) (If this minimum ratio is negative then the solution is unbounded and the objective function would be reduced by the introduction of x_j into the basis.)

The variable to be introduced into the basis is x_j and the variable to be removed is x_k. The variable x_j is removed from the constraints (or tableau) by subtracting the kth constraint multiplied by the factor (a_{ij} / a_{kj}) from the ith equation $(i = 1, \ldots m, i \neq k)$. Where a_{kj} is referred to as the pivot. The value of the objective function is revised by a similar operation with equation k. These operations eliminate the coefficients of variable x_j from the constraints with the exception of the kth equation which is divided by the pivot value. The variable x_k now has a coefficient in the objective function and the new basic variable x_j value is b_k/a_{kj}. This operation is called a change of basis. The iterative cycle now returns to stage 1 to check if any further improvement is possible.

It is possible for cycling to occur but this rarely happens with practical problems.

This formulation allows for only positive variables, however, the nodal displacements may be positive or negative.

Negative variables can be accounted for by checking at stage 1 to see if any of the objective function coefficients corresponding to displacement variables are positive. If so the objective function can possibly be improved by changing the direction of the displacement variable, by changing all the signs of the coefficients of the variable in the tableau. A record of the sign of the displacement variable can be stored.

When the optimisation is complete the coefficients of the slack variables in the objective function represent the optimal values of the dual variables. In the above formulations these are the member areas.

For the problem with multiple loading cases studied in Chapter 6, the formulation of Reinschmidt and Russell (206) as given in equation (2.1) was used. For the problems with just single loading cases studied in Chapter 3 with $\sigma_c = \sigma_T$, the problem can be simplified as given by Hemp and Chan (189) to:

$$\sum_{i=1}^{n} k_{ji} \cdot U_i + U_{n+j} = d_j \quad (j = 1, \ldots . m)$$

$$U_{n+j} + U'_{n+j} = 2d_j$$

$$U_{n+j} \, , \, U'_{n+j} \geqslant 0$$

$$\max \sum_{i=1}^{n} F_i \, U_i$$

where: $\quad d_j = hL_j$

The first three sets of equations confine the strain in each member to the bound $-d_j$ and d_j. These modifications can be catered for by the use of the standard upper bound technique given by Wagner (170).

The simplex algorithm can be modified to account for the second equation and the bounds explicitly. If the variable to be introduced into the basis has an upper bound then the procedure will be as follows:

1. Check to see if the upper bound is greater than the pivot divided by the left hand side of the constraint (which is the value x_j will take in the next tableau b_k/a_{kj}). If so then there is no need to consider the upper bound.

2. If the pivot divided by the left hand side is greater than the upper bound then the sign of the variable is made negative by changing the sign of the coefficients. The value of the left hand side of the inequality is set to its original value less the upper bound multiplied by the pivot value. In short this process consists of a set of sign changes, substitutions and relabellings, using the equation:

$$U_{n+j} = (2d_j - U'_{n+j})$$

Mathematically U is replaced with the variable U' and the necessary changes made.

Members which have no final area but have a strain of $-d_j$ or $+d_j$ are alternative optimum members. These members can be recognised in the simplex tableau by checking to see if they are at their upper bound.

Non-Linear Programming:

A non-linear programming method was used by Dobbs and Felton (181) to find the form and member sizes of a truss from a ground structure. They used a steepest descent-alternate mode algorithm and deleted any members whose cross sectional areas reduced to zero at the end of steepest descent move. The alternate move was chosen so as to move the design away from the most critically violated stress constraints while keeping the weight of the structure constant. Other methods of choosing analternate mode direction vector are discussed in Appendix III.

In this section an algorithm developed by Elliott (50) is outlined in which the alternate mode direction vector is chosen such that the design moves away from the critical constraints. This vector also considers the effects of altering each design variable on each of the constraints. This algorithm, therefore, derives a highly directed and efficient alternate mode vector. The theorems of structural variation and method of topological design as presented by Majid and Elliott (50, 71, 73, 74) are also outlined.

The problem of non-linear programming can be defined as:

$$\text{Minimise } Z = f(x)$$

Subject to the constraints:

$$g_i(x) \leqslant, \geqslant T_i \qquad (i = 1, 2, \ldots M)$$

$$x_k^u \geqslant x_k \geqslant x_k^o \qquad (k = 1, 2, \ldots n)$$

Where x_k^u, x_k^o are the upper and lower bounds on the variable x_k

The constraints can all be written in the following form:

$$G = T - g(x) \geqslant 0$$

The normalised constraints are used in this algorithm as follows:

$$G_N = D(1 - g(x)/T) \geqslant 0$$

Where D is a positive scalar constant which is set to the value 1000 throughout in the work presented in this thesis unless otherwise stated.

The design process consists of finding a series of designs on the edge of the design space called boundary solutions. These solutions must be feasible and have at least one critical constraint which has a normalised value of zero or near zero. The other normalised constraints are all positive. Boundary solutions may be derived to any degree of accuracy by averaging the values of the most recent feasible and non-feasible solution vectors. The conditions for a feasible boundary vector are of two types:

$$G_{Nj} < T1 \quad (j = 1, 2, \ldots m)$$

for at least a single j
or:

$$\mathrm{mod}(1 - x_i/y_i) < T2 \quad (i = 1, 2 \ldots n)$$

must hold true.

where:

x_i = most recent feasible value of ith design variable

y_i = most recent non feasible value of the ith design variable

T1 = 30.0 for the work in this thesis

T2 = 0.01 for the work in this thesis

Boundary solutions are not initially required to a high degree of accuracy but as the optimum solution is approached these vectors are required to a higher degree of accuracy.

The non-linear programming algorithm is divided into three main stages as follows:

I. The Search for an Initial Feasible Solutions:

This stage is carried out once only. The design variables are set to their lower bounds x_k^o and the slope of the objective function found.

For a linear objective function:

$$z(x) = \sum_{i=1}^{n} c_i x_i$$

the rate of change of the objective function is found by partial differentiation as:

$$\frac{\partial Z}{\partial x_i} = c_i$$

The design variables are then increased so that the mose costly variables with the larger objective function coefficients will increase least. The variables are increased using the equation:

$$x_i' = x_i^o + S.dx_i \qquad \ldots (2.2)$$

where:

$dx_i = +1/c_i$

S = step length = min $\left[c_i \, (x_i^u - x_i) \right]$

if $x_i^u = x_i$ then this term is ignored and

dx_i is set to zero.

To find a feasible solution which does not violate any of the constraints it may be necessary to drive one or more of the variables to their upper bounds by repeated use of equation (2.2).

II. The Alternate Mode Move:

At this stage a new direction vector is calculated so that the new design can be derived, without any change in weight, but such that the feasibility of the design is improved. This step constitutes a redistribution of the weight throughout the structure.

A slope matrix may be defined as follows:

$$S' = \begin{bmatrix} S'_{11}\ldots & \ldots S'_{1j}\ldots & \ldots S'_{1m} \\ S'_{21} & S'_{2j} & S'_{2m} \\ \vdots & \vdots & \vdots \\ S'_{i1}\ldots & \ldots S'_{ij}\ldots & \ldots S'_{im} \\ \vdots & \vdots & \vdots \\ S'_{n1}\ldots & \ldots S'_{nj}\ldots & \ldots S'_{nm} \end{bmatrix}$$

where:

$$S'_{ij} = (\partial G_{Nj} / \partial x_i).(1/(1 + G_{Nj}))$$

or

$$S'_{ij} = (G'_{Nj} - G_{Nj})/(EINF.(1 + G_{Nj}))$$

and

G_{Nj} = The value of the jth normalised constraint when the ith variable has a value x_i

G'_{Nj} = the value of the jth normalised constraint when the ith variable is increased by an increment EINF to become x_i + EINF.

EINF = 0.001 for the work in this thesis unless otherwise stated.

The elements of a direction vector dx may be obtained by adding all the elements of a whole row of the slope matrix as follows:

$$\underline{dx} = \underline{S}'\underline{i}$$

where $\underline{i}$ is a unit column vector of order m.

However, this direction vector suffers from the fact that it does not preserve a constant value of the objective function. However, it does indicate the manner in which the variables should be altered so that all the constraints are best affected and also ensures that the normalised critical constraints are increased in value.

After taking a step of the type:

$$x_i' = x_i + s dx_i \qquad (2.3)$$

the increase in the value of the objective function must equal to zero hence:

$$s \sum_{i=1}^{n} c_i dx_i = 0 = \sum_{i=1}^{n} c_i dx_i$$

The direction vector is then modified to:

$$dx_i^* = c_i dx_i \qquad (i = 1, 2 \ldots n)$$

The direction vector can then be redefined to ensure that variable bounds are not violated and to ensure constant weight:

$$dx_i = \frac{C_T}{c_i} \cdot \left[\frac{K_i}{K_S} - \frac{K_i'}{K_S'} \right] \quad (i = 1, 2 \ldots n) \quad (2.4)$$

where:

$$C_T = \sum_{i=1}^{n} c_i$$

$$K_S = \sum_{i=1}^{n} K_i$$

$$K_S' = \sum_{i=1}^{n} K_i'$$

$$K_i = a_i(x_i^u - x_i)$$

$$K_i' = b_i(x_i - x_i^o)$$

$$a_i = (dx_i^* - dx^*_{min})/(dx^*_{max} - dx^*_{min})$$

$$b_i = (dx^*_{max} - dx_i^*)/(dx^*_{max} - dx^*_{min})$$

$$a_i = 1 - b_i$$

dx^*_{max} , dx_{min} are the largest and smallest elements in the vector dx

The factor C_T/c_i preserves the constant value of the objective function by relatively scaling down the more expensive variables. It is possible for K_S' to vanish indicating that it is impossible to decrease the value of any variable without violating either a constraint or a lower bound of a variable. In this case it is impossible to move in any feasible direction without increasing the value of the objective function.

Once a new boundary solution has been found in the direction of this vector using equation (2.3) this solution is averaged with the previous boundary solution (from Stage I or III) and the solution checked for feasibility. The solution should now be as far away from constraints as practicable.

III. The Steepest Descent Move:

This move reduces the volume of the structure in the steepest descent direction such that the more costly variables are reduced at a greater rate. The direction vector used in equation (2.3) is:

$$dx_i = -c_i \quad (i = 1, 2 \ldots\ldots n) \qquad (2.5)$$

Stages II and III are employed repeatedly until an 'optimum' solution is obtained. This is checked by a tolerance test which is carried out at the end of Stage III of each cycle:

$$\left|1 - x'_{bi} / x_{bi}\right| < T3 \qquad (i = 1, 2, \ldots\ldots n)$$

and $$\left|1 - Z/Z'\right| < T4$$

where:

x_{bi}, x'_{bi} are the values of the ith design variable at the current and previous design cycle respectively

Z, Z' are the values of the objective function at the current and previous design cycle respectively.

T3, T4 = 0.001 throughout the work in this thesis unless otherwise specified.

Elliott (50) pointed out that these tests are not sufficient to ensure even a local optimum.

The first time Stage II is performed, the step length, S, is set such that at least one of the design variables is driven to a bound. For all subsequent stages the step length is selected based on the distance moved in the stage immediately previous to the current design. This distance is given by:

$$D = \sqrt{\left[\sum_{i=1}^{n} (x_i - x_{pi})^2\right]}$$

where:

x_{pi} = the commencing value of variable i in the previous stage.

For the current stage:

$$D = \sqrt{\left[\sum_{i=1}^{n} (dx_i)^2\right]}$$

Therefore:

$$S = \sqrt{\left[\sum_{=1}^{n} (x_i - x_{pi})^2 \Big/ \sum_{=1}^{n} (dx_i)^2\right]} \qquad (2.6)$$

It must also be ensured that the value of, S, calculated in this manner does not violate the limiting values of any of the design variables. This can be checked by use of the following equations in which S_i is the upper bound on S imposed by variable i.

$$\text{if } dx_i > 0 \text{ then } S_i = (x_i^u - x_i)/dx_i \qquad (2.7)$$

$$\text{if } dx_i < 0 \text{ then } S_i = (x_i^o - x_i)/dx_i \quad (i = 1,2\ldots n)$$

The minimum value of S given by equations (2.6) and (2.7) is, therefore, used as the step length.

Further computational details including flowcharts for this algorithm can be found in reference (50).

The effects of changes in the design variables and the slope matrix can be calculated using the theorems of structural variation developed by Majid and Elliott (50, 71, 72, 73, 74). In this way the structure need only be analysed once.

The effects of design changes in the member areas may be assessed in the following manner:

If the original area A, of a member is changed by dA where an increase in member size is designated positive.

The remaining Area A' is given by:

$$A' = A - dA$$

The factor is defined as:

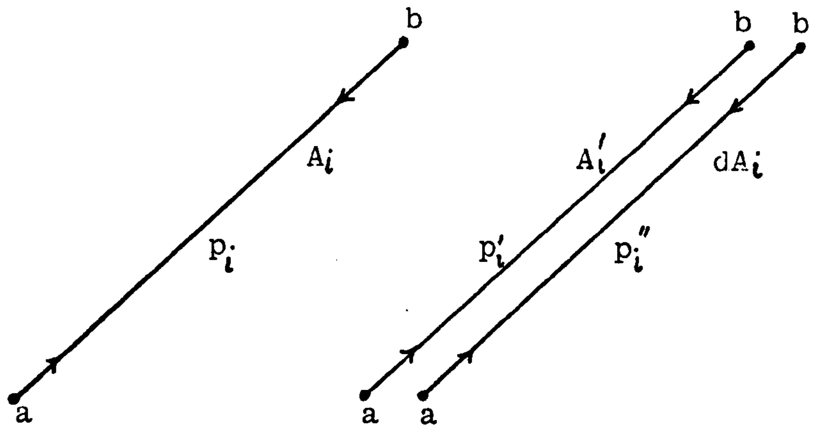
$$\alpha = -dA/A$$

and hence $A' = (1 + \alpha)A$.

For the total removal of the member:

$A - dA = 0$ then $\alpha = -1$

Consider the change in cross sectional area of member i from A_i to A_i' by an amount dA_i. If the member connects joints a and b the member can be visualised as being split into two members of area A_i' and dA_i. The corresponding member forces will be p_i' and p_i''.



For equilibrium:

$$p_i = p_i' + p_i''$$

and $p_i/A_i = p_i'/A_i = p_i''/dA_i = \sigma$

hence $p_i'' = dA_i . p_i / A_i = -\alpha p_i$

and $p_i' = A_i' . p_i / A_i = (1 + \alpha) p_i$

The member of area dA_i' can be removed without altering the member forces elsewhere provided it is replaced by two equal and opposite forces p_i'' at nodes a and b.

The case of complete removal of dA_i is to be considered without any compensation with external forces.

The resulting member forces due to an external load system of only two equal but opposite unit loads at nodes a and b are defined as:

$$f_i = \{f_{1i} , f_{2i} \ldots f_{ii} \ldots f_{ni}\}$$

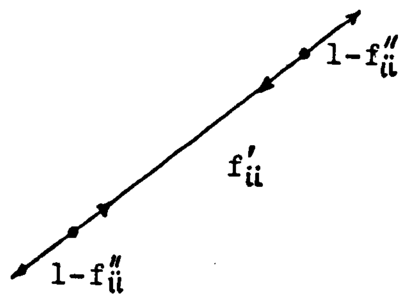
where:

f_{1i} = the force in member 1 due to unit loads acting axially to member i.

The force is in member i is split into f_{ii}' and f_i'' corresponding to A_i' and dA_i where:

$$f_{ii}'' = -\alpha f_{ii}$$

$$f_{ii}' = (1 + \alpha) f_{ii}$$



Under these unit loads the member with area dA_i and force f_{ii}'' can be removed provided it is compensated by equal and opposite forces f_{ii}''. The magnitude of the unit loads can be increased by a factor $r_{\alpha i}$. The removal of member dA_i requires compensation of $r_{\alpha i} f_{ii}''$. The net externally applied loads under actual loads if dA_i is removed must be zero. Hence:

$$r_{\alpha i} - r_{\alpha i} f_{ii}'' - p_{ii}'' = 0$$

The variation factor, $r_{\alpha i}$, for member i is, therefore,

$$r_{\alpha i} = p_{ii}'' / (1 - f_{ii}'')$$

and

$$r_{\alpha i} = -\alpha p_i / (1 + \alpha f_{ii})$$

The change in the member force under external loads = $r_{\alpha i} f_{ii}'$

Hence the final force in member i = π_i

$$\pi_i = p_i' + r_{\alpha i} f_{ii}' \qquad (2.8a)$$

$$\pi_i = p_i (1 + \alpha)/(1 + \alpha f_{ii})$$

The force in any other member is found by superposition:

$$\pi_j = p_j + r_{\alpha i} f_{ji} \qquad (2.8b)$$

If the member is completely removed:

$$\alpha = -1$$

and $$r_i = p_i/(1 - f_{ii})$$

The deflections ψ can be predicted by superposition as follows:

$$\psi_j = x_j + r_{\alpha i} \chi_{ji} \qquad (2.9)$$

where:

ψ_j = deflection of joint j after modification of member i

x_j = deflection of joint j before modification of member i

χ_{ji} = deflection of joint j due to unit force at the ends of member i.

The structure need only be analysed once under the applied loads and unit load vectors. The results of these analyses can be stored and used with equations (2.8) and (2.9) to assess the effects of design changes.

These theorems were also incorporated in a topological design process developed by Majid and Elliott (50, 71, 74). This design process can be used after each time Stage III of the non-linear algorithm is performed.

This design process is outlined below:

The stress in member j to removal of member i is given by:

$$\sigma_j = \Pi_{ji}/A_j$$

If σ_{pj} is the permissible stress then for fully stressed conditions its area may be changed by dA_j so that:

$$\sigma_{pj} = \Pi_{ji}/(A_j + dA_j)$$

Therefore:

$$dA_j = (A_j \sigma_j / \sigma_{pj}) - A_j$$

Using equation (2.8) the stress in member j is:

$$\sigma_j = (p_j + r_{\alpha i} f_{ji})/A_j$$

Therefore:

$$dA_j = (A_j (p_j + r_{\alpha i} f_{ji})/A_j \sigma_{pj}) - A_j$$

The corresponding change in the member volume is:

$$L_j \, dA_j = L_j A_j(((p_j + r\alpha_i f_{ji})/A_j \sigma_{pj}) - 1)$$

When all members are altered the total decrease in the structural volume due to the removal of i is:

$$dV_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} L_j A_j \left[\left[(p_j + r\alpha_i f_{ji})/A_j \sigma_{pj}\right] - 1\right]$$

The new volume of the structure is:

$$V_i = V_o + dV_i - L_i A_i$$

where:

$$V_o = \sum_{k=1}^{n} L_k A_k$$

The new volume of the structure can, therefore, be expressed in the following way:

$$V = \sum_{\substack{j=1 \\ j \neq i}}^{n} L_j (p_j + r\alpha_i f_{ji})/\sigma_{pj} \qquad (2.10)$$

The value of σ_{pj} will depend on the sign of the member force and will either be $-\sigma_c$ or σ_T.

A similar formula for the volume was developed by Majid and Elliott for structures subject to deflection constraints. In this thesis the problem of deflection constraints and formfinding will not be considered. The work of Majid and Elliott also considered selfweight and groups of members.

Equation (2.10) can be used to calculate the weight of the structure when each of the members has been removed. This enables a benefit order vector to be formed for the members. This benefit vector only gives the order in which members may be removed from the structure provided this results in a feasible and lighter structure. The removal of several members may alter the topology of the initial structure significantly and possibly render the forcast erroneous. The removal process is terminated as soon as this entails an increase in the overall weight of the structure.

In many cases removal of a member results in a non-feasible solution. In this case a new structure must be determined so that the weight of the structure is not increased using the equation:

$$x' = x - s.dx \qquad (2.11)$$

This can be accomplished by defining a new slope matrix

$$S^*_{ij} = (\partial G_{Nj}/\partial x_i)/(1 + (G_{Nj} - G_{N\,min}))$$

$$= (G'_{Nj} - G_{Nj})/((1 + (G_{Nj} - G_{N\,min})).EINF))$$

The elements of the direction vector, dx, are determined by summing the rows of matrix S^*_{ij}

$$dx_i = - \sum_{j=1}^{m} S^*_{ij}$$

This vector is again weighted so that the sensitivity of the most inactive constraints is damped down but the elements of the column for the most critical, negative normalised constraints, remain the same.

The vector is normalised as follows

$$dx^*_i = dx_i/e \qquad (i = 1, 2 \ldots . n)$$

where:

$$e = \sum_{i=1}^{n} dx_i/n$$

The step-length, s, is chosen so that the new volume does not exceed the original volume.

Hence:

$$s = L_i a_i / \sum_{=1}^{n} c_i.dx_i$$

As the design procedure progresses there are the following possibilities.

(a) Removal of a member results in non feasible design. In this case the member is restored and the non-linear programming algorithm proceeds with the design.

(b) The solution is local optimum with respect to the non-linear programming algorithm, however, removal of the member at the top of the benefit vector results in a non-feasible design. In this case the design is made feasible by further steps of equation (2.11). The design process again proceeds using the non-linear algorithm starting with Stage II.

Further details of this method which can be used to derive a series of local optimum design of decreasing degrees of redundancy can be found in reference (50).

In Chapter 5 the non-linear programming algorithm is used to derive comparative designs for structures of fixed topology subject to multiple loading cases with stress and deflection constraints and limits on the maximum and minimum member area sizes. In Chapter 6 the non-linear programming algorithm is applied to bridging ground structure subject to multiple loading conditions and members whose cross sectional areas are reduced to zero at the end of the steepest descent more are deleted. The non-linear programming algorithm with the topological design procedure is also applied to the bridging ground structure. In this chapter these algorithms are applied to a series of small truss problems.

Fully Stressed Design Methods:

These techniques are based on simultaneous failure mode concepts which assume that in the optimum design each member is fully stressed under at least one of the loading cases. Schmit (212) showed that for some cases this assumption is not necessarily true and the minimum weight structure is not fully stressed. This is the result of the absence of a weight objective function from the algorithm. However, for many problems the fully stressed design solution is optimum or near optimum and is always at the very least a buildable design. For a structure with only a single loading case the design will be determinate and always an optimum.

This type of design technique is said to be analysis orientated because nearly all the computational effort is spent on the analysis of new designs. The most frequently used fully stressed design technique is the stress-ratio method in which the member areas are modified after each reanalysis as follows:

$$A_{m,i+1} = A_{mi} \cdot (\sigma_m / \sigma_{mp})_{max.i}$$

where:

$A_{m,i+1}$, A_{mi} are the cross sectional areas of member m after the i + 1th and ith resize.

σ_m = the stress in member m.

σ_{mp} = the permissible stress of member m

$max.i$ = signifies the maximum ratio is taken for member m after the ith reanalysis.

Other techniques have been developed to find fully stressed design solutions. These included the work of Reinschmidt, Cornell and Brotchie (153) and Gallagher (132).

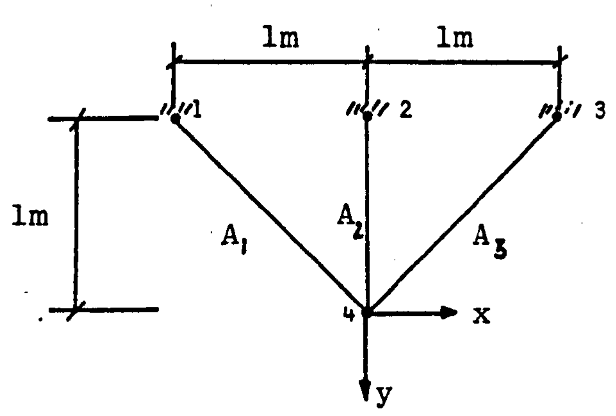
With fully stressed design techniques displacement constraints are not usually considered although approximate rules to ensure these are not violated have been formulated by Razani (151) and another technique was used by Lipson and Gwin (191).

For formfinding problems the stress ratio method can be applied to a ground structure and members areas which become zero or very small in magnitude can be deleted from the structure. The solution for a structure subject to a single loading condition is determinate and if no members are deleted it will be a global optimum. If any members are deleted or the structure is subject to more than one loading condition then there is no unique fully stressed solution and the solution derived will not necessarily be a global optimum. Barta (174) has investigated this problem and showed that for structures subject to a single loading case wide differances exist between the fully stressed designs but the optimum is always statically determinate. The formfinding of structures subject to a single loading case using the stress-ratio method is discussed in more detail in Chapters 3 and 4. In Chapter 6 the problem of formfinding of structures subject to multiple loading cases using the stress-ratio method is considered.

It should be pointed out that no formfinding work of this type appears to have been published, however, this work is a natural engineer's intuitive approach.

Examples:

Each of the above methods was applied to a series of truss problems originally studied by Schmit (212) and Razani (151). The structure studied is shown below:



A series of four different sets of loading condtions and stress constraints were used and the results for each, and published comparative solutions are tabulated below:

Problem 1:

For all members $\sigma_T = 20N/m^2$ $\sigma_c = -15N/m^2$ $E = 10^4 N/m^2$

Load Case 1: 4x = 15.0N 4y = -25.9808N

Load Case 2: 4x = -20.0N 4y = 0.0N

Member Areas m^2	F.S.D.	L.P.	N.L.P.	N.L.P.T.
1:	1.071	1.232	1.070	1.449
2:	0.544	0.307	0.544	-
3:	0.611	0.490	0.614	0.707
Volume m^3	2.922	2.742	2.925	3.049

Published Solutions.

Member Areas m^2	Schmit N.L.A.	Razani F.S.D.
1:	1.072	1.074
2:	0.544	0.546
3:	0.611	0.608
Volume m^3	2.924	2.926

F.S.D. = Fully Stressed Design Stress Ratio Program

L.P. = Dual Simplex Linear Program

N.L.P. = Non-Linear Program due to Elliott

N.L.P.T. = Non Linear Program with Topological Design Algorithm

Problem 2:

For members 1 and 3 $\sigma_T = -\sigma_c = 5N/m^2$
For member 2 $\sigma_T = -\sigma_c = 20N/m^2$ $\Big\}$ $E = 10^4 N/m^2$

Load Case 1: 4x = 28.2843N 4y = -28.2843N

Load Case 2: 4x = 0.0N 4y = -30.0N

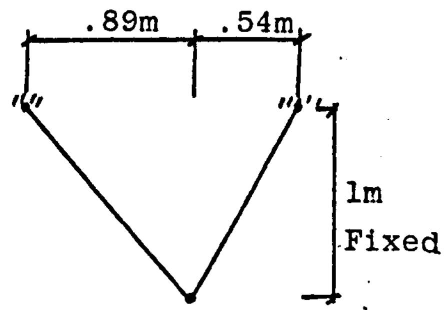
Load Case 3: 4x =-14.1421N 4y = -14.14121N

Member Areas m^2	F.S.D.	L.P.	N.L.P.	N.L.P.T.
1:	8.000	5.879	7.059	8.000
2:	0.000	.750	1.993	1.501
3:	4.242	2.121	2.827	-
Volume m^3	17.314	12.064	15.975	12.815

Published Solutions:

Member Areas m^2	Schmit- N.L.P.	Sved & Ginos by inspection*	Corcoran N.L.P. geometry variables
1:	7.099	8.00	4.241
2:	1.849	1.50	2.038
3:	2.897	Removed	
Volume m^3	15.896	12.812	7.55

Corcoran's Layout



* Also derived by Sheu and Schmit (217)

Problem 3:

For all members $\sigma_T = -\sigma_c = 10N/m^2 \quad E = 10^4 N/m^2$

Load Case 1:	4x =	14.142N	4y =	-14.142N
Load Case 2:	4x =	0.0N	4y =	-15.0N
Load Case 3:	4x =	-7.0711N	4y =	-7.071N

Member Areas, m^2	F.S.D.	L.P.	N.L.P.	N.L.P.T.
1:	1.708	1.470	1.707	2.000
2:	0.933	0.750	0.931	-
3:	0.524	0.530	0.527	1.061
Volume, m^3	4.089	3.578	4.091	4.328

Published Solutions:

Member Areas, m^2	Schmit N.L.A.	Razani F.S.D.
1:	1.707	1.706
2:	0.940	0.915
3:	0.526	0.537
Volume, m^3	4.099	4.099

Problem 4:

For all members $\sigma_T = 20N/m^2$ $\sigma_c = -15N/m^2$ $E = 10^4 N/m^2$

Load Case 1: 4x = 14.142N 4y = -14.142N

Load Case 2: 4x =-14.142N 4y = -14.142N

Member Areas, m^2	F.S.D. +	L.P.	N.L.P.	N.L.P.T.
1:	1.000	0.571	0.789	1.000
2:	0.000	0.606	0.408	-
3:	1.000	0.571	0.789	1.000
Volume m^3	2.828	2.222	2.639	3.047

Published Solutions:

Member Areas, m^2	Schmit N.L.A.	Razani F.S.D.
1	0.784	0.990
2	0.422	0.014
3	0.784	0.990
Volume, m^3	2.639	2.814

\+ Similar result found by Melosh (146) using extrapolation fully stressed design techniques.

These results generally confirm the published solutions.

The Problems of Buckling Effects in Topological Design:

Approximate formula relating the member area to its permitted load to prevent buckling have been developed for use in the design of structures of fixed topology (50, 206). Dobbs and Felton (181) showed that these types of relationships cannot be used in topological design because although inefficient members may reduce to a small cross sectional area they will not reduce to zero area. This is because the member areas must be sufficiently large to ensure buckling does not occur. Reinschmidt and Russell (206, 207) showed that the effects of buckling can be considered in an iterative linear programming formfinding procedure. This method consists of a series of linear sub-problems in which estimates of the permissible stress to prevent buckling are made. In each sub-problem members may be deleted. If, however, at any stage a too conservative estimate of the permissible compressive stress is made for any of the optimum members, they may be deleted in the linear sub problem resulting in a non-optimum form.

The above investigations revealed the difficulties of including considerations for buckling behaviour in a topological design process.

CHAPTER 3: DYNAMIC RELAXATION FORMFINDING

Review of Dynamic Relaxation - Outline of the Dynamic Relaxation Method with Finite Element idealisation - The Damping Factor - The Time Interval - Initial Conditions - Shapefinding and Formfinding with Dynamic Relaxation - Michell's Theorem for Optimum Structures - Formfinding for Modular Triangulated Space Structures - Basis and Control of Dynamic Relaxation Formfinding for Space Structures - Example Structures - Comparison with other Methods - Further Comments.

Summary:

In this chapter the method of Dynamic Relaxation (D.R.) is reviewed and outlined. The method's application to shapefinding and formfinding is discussed. The Michell criteria for optimum structures are derived and their application to modularly constrained triangulated structures discussed. An iterative procedure based on the D.R. solution method is outlined in which a ground structure is continuously modified to comply with the criteria. The D.R. procedure, Fully Stressed Design and Linear Programming Methods are applied to four problems analogous to Michell structures. Computation times are compared and D.R. is shown to be an efficient solution procedure particularly for interactive use.

Review of Dynamic Relaxation:

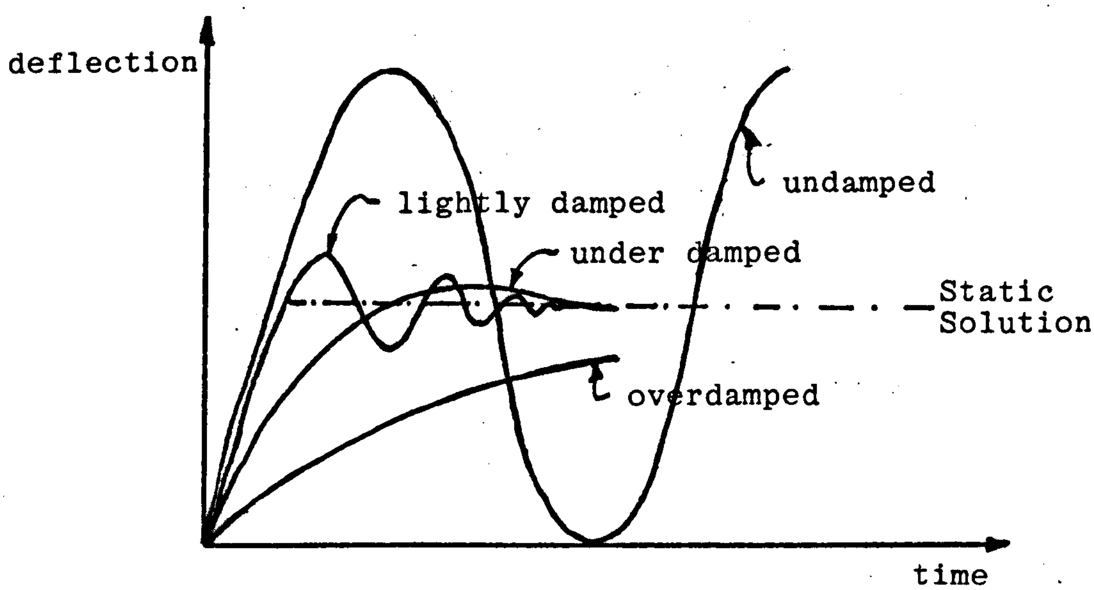
Dynamic Relaxation (D.R.) is a step by step numerical integration method for tracing the dynamic behaviour of a structure. The method directly utilises Newton's Second Law of Motion and when damping is included in the formulation the motion eventually comes to rest giving a static analysis. This explicit structural analysis technique was conceived in 1965 by A.S. Day (253), for the finite difference analysis of concrete pressure vessels. He had previously used the method for solving tidal river flow problems. Day (247-8) later illustrated its use both for finite difference and stiffness formulations. Otter et alia (253,255,256) subsequently developed the method for finite difference non-linear analysis of pressure vessels. Both Otter (254) and Wood (260) compared the method with other classical iterative methods. Rushton (257,258) developed automatic trial and correction techniques for obtaining the time interval and damping factor and optimised the iteration by using fictitious densities when analysing plates. Cassell et alia (242-5) investigated the stability of the iteration for linear and non-linear analyses and made further comparisons with other iterative methods.

The majority of early published papers were concerned with the finite difference formulation of the method. When Severn (259), in 1973, reviewed stress analysis techniques in general, he made no mention of the finite element formulation of D.R. However, Zienkiewicz and King (256) had commented, as early as 1967, on the advantages of D.R. combined with a finite element idealisation. The apparent omission was probably due to the fact that explicit methods, used with either finite difference of finite element formulation were considered inefficient due to the small time steps required for the iteration. In 1976 Cundall (246) pointed out that this was not really true if programs were written carefully. He quoted another researcher who found HEMP (a finite difference program) faster than SAP (a well established finite element program) for three dimensional problems in linear elasticity but slower in two dimensions. The advantages and scope for the finite element idealisation with explicit methods are only now being fully investigated.

In 1969 Day and Bunce (249) illustrated how D.R. could be applied to geometrically non-linear cable structures. Since 1971 Barnes (228-235) has illustrated how this method can be applied to shapefinding, static and dynamic, linear and non-linear analysis of tension structures. In 1971 Brew and Brotton (240-1) used D.R. for the static analysis of linear and non-linear frames in both successive and simultaneous forms. In comparison with the direct methods D.R. compared unfavourably for linear problems but favourably for highly non-linear problems. In addition, D.R. required less storage and the programming was simpler.

When D.R. is used with light real damping the method is equivalent to the ordinary central difference explicit integration scheme for dynamic analysis as given by Biggs (239). Belytschko et alia (237) used a direct central difference scheme using Newmarks method ($\beta = 0$, $\gamma = \frac{1}{2}$) with beam, triangular and rectangular finite elements for non-linear dynamic analysis of large problems with explosive loading. This method is equivalent to D.R. They concluded that this direct method was faster and required less core store than ficititious force methods.

In real structures the effects of damping result from various factors for example molecular friction of material and energy losses associated with friction in connections. For analysis this damping is usually assumed viscous and the damping force defined as proportional to, but in the opposite direction to the velocity. The effect of damping for an ideal one degree system is as shown on the following deflection-time trace:



The critical damping factor, i.e. the damping factor which causes the structure to approach the static position most rapidly, is used in static analysis. For dynamic analysis lighter damping is used with smaller time steps to follow the true path of the structure. The term D.R. should only be used for static analysis but the solution procedure for dynamic analysis is the same. The advantage of using D.R. for static analysis during a design process is that changes can be made during the analysis and if used in an interactive form the method allows the designer to alter the structure during the design to comply with the design constraints. Barnes has illustrated this with respect to tension structures.

Outline of the Dynamic Relaxation Method with Finite Element idealisation:

The mass of the structure is assumed to be concentrated at the nodes and the trace is calculated from when the load is initially applied.

Newton's second law of motion is applied to the nodes of the structure. A viscous damping term is included and the out of balance or residual force at node i at time t in the x direction is given by:

$$R_i^t = M_i \dot{V}_{ix}^t + C_i V_{ix}^t \qquad (3.1)$$

where:

R_{ix}^t = the residual force at node i in the x direction at time t

M_i = the mass of node i

C_i = the damping constant at node i

V_{ix}^{t} , $\dot{V}_{ix}^{t}$ = the velocity and acceleration at time t in the x direction.

Similar equations can be written for the other directions.

The finite difference form of equation (3.1) is:

$$R_{ix}^{t} = \frac{M_i}{\Delta t}\left(V_{ix}^{t+\Delta t/2} - V_{ix}^{t-\Delta t/2}\right) + \frac{C_i}{2}\left(V_{ix}^{t+\Delta t/2} + V_{ix}^{t-\Delta t/2}\right) \qquad (3.2)$$

The residuals are calculated at the ends of time intervals 0, Δt, $2\Delta t$ ...., $t-\Delta t$, t, $t+\Delta t$, and the velocities at mid points of the time intervals.

The velocity at time $t+\Delta t/2$ can be determined from the velocity at time $t-\Delta t/2$ and the residual R as follows:

$$V_{ix}^{t+\Delta t} = V_{ix}^{t-\Delta t/2}\left[\frac{Mi/\Delta t - Ci/2}{Mi/\Delta t + Ci/2}\right] + R_{ix}^{t}\left[\frac{1}{Mi/\Delta t + Ci/2}\right] \qquad (3.3)$$

The increase in the deflection of node i in the x direction during the time interval t to t + Δt is given by:

$$\delta\Delta = \Delta t \,.\, V_{ix}^{t+\Delta t/2}$$

In this way the velocity is assumed to vary linearly over a time interval and the new position of the node i is given by:

$$X_i^{t+\Delta t/2} = X_i^{o} + \Delta x_i^{t} + \Delta t . V_{ix}^{t+\Delta t/2} \qquad (3.4)$$

These calculations are performed for each node of the structure, and in each co-ordinate direction to give the complete displacement form at time t+Δt. Fixed nodes are excluded from the calculations or assigned high masses.

The current force in each bar m can be calculated from:

$$T_m^{t+\Delta t} = \frac{EA_m}{L_m} (L_m^{t+\Delta t} - L_m) + T_m^{\circ} \qquad (3.5)$$

where:

L_m = the initial length of the bar m

$L_m^{t+\Delta t}$ = the current length of the bar m

EA_m = Young's modulus multiplied by the cross sectional area of the bar m

T_m° = initial prestress in bar m

If bar m connects nodes i and k then the force in x direction at node i from bar m is given by:

$$\Delta R_{ixm}^{t+\Delta t} = \frac{T_m^{t+\Delta t}}{L_m^{t+\Delta t}} \left\{(X_k^{\circ} + \Delta X_k^{t+\Delta t}) - (X_i^{\circ} + \Delta X_i^{t+\Delta t})\right\} \qquad (3.6)$$

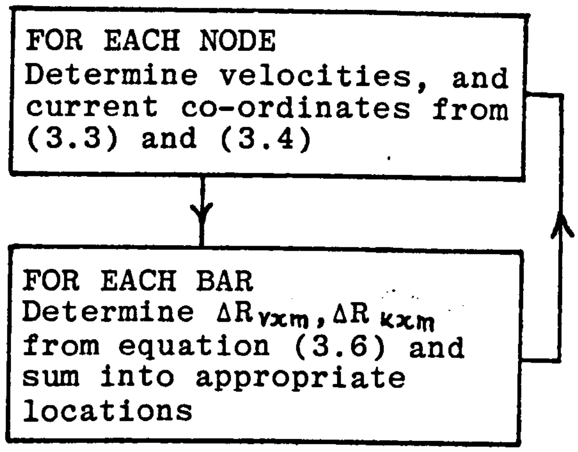
$$= -\Delta R_{kxm}^{t+\Delta t}$$

The current geometry need only be used to calculate the components of the forces if the structure is geometrically non-linear.

The contributions of each bar at each node of the structure are summed with the applied load P_{ix} to give the residual force at time t+Δt:

$$R_{ix}^{t+\Delta t} = P_{ix} + \underset{\substack{\text{for all}\\ \text{members}}}{\overset{Y}{\overset{n}{\Sigma}}} \Delta\, R_{Yxn}^{t+\Delta t} \qquad (3.7)$$

The cycle of calculations is as follows:



In this way compatibility and equilibrium conditions are considered separately until both are satisfied. Other simple elements can readily be included in the integration, for example triangular constant stress and moment elements (235).

The Damping Factor:

The motion of any structure depends on the mass, member stiffnesses and damping factor. If C_i is zero for all nodes the motion will continue indefinitely. If a damping factor is used the amplitude of deflection will decay to a static solution.

The damping constant C_i may be defined as a constant for the complete structure, or it may be defined as $C_i = M_i(C'/\Delta t)$. The damping/unit mass, $C'/\Delta t$ may be taken as constant for the whole structure.

Equation (3.3) can be rewritten as:

$$V^{t+\Delta t/2} = V^{t-\Delta t/2}\left[\frac{1 - C'/2}{1 + C'/2}\right] + R_{ix}^{t}\,\frac{\Delta t}{M_i}\left[\frac{1}{1 + C'/2}\right]$$

or as:

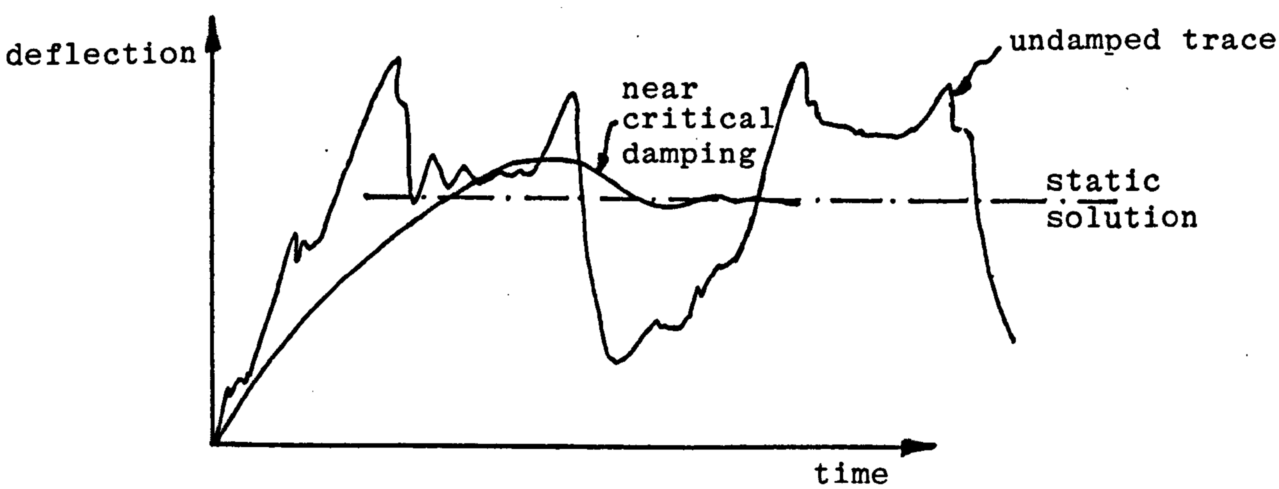
$$V_{ix}^{t+\Delta t/2} = AV_{ix}^{t-\Delta t/2} + B_i R_{ix}^{t} \qquad (3.8)$$

where:

A is a constant for the whole structure

B_i is a constant for each node.

For real structures the deflection trace will not be ideal and will normally be more like the trace shown below.



A highly fictitious damping constant is used which is just sub-critical and gives bounds to the true equilibrium state. Sometimes the structure frequency may be known otherwise the lowest frequency can be obtained from a short undamped run.

The critical damping factor for the structure can then be determined from the expressions given by Biggs (239) for a one degree system.

The critical damping factor = $C_{ci} = 2\sqrt{S_i M_i}$

The frequency = $f = \dfrac{1}{2\Pi}\sqrt{\dfrac{S_i}{M_i}}$

$$\text{Then} \quad C_{ci} = 4\Pi f M_i \qquad\qquad (3.9)$$

$$\text{and} \quad C_c' = 4\Pi f \Delta t$$

The Time Interval:

If the time interval used for the integration exceeds a certain value, instability will occur in the calculations. The following method for estimating the critical time is due to Barnes (229,235).

Consider the motion of node i relative to adjacent nodes k. If the vibrations occur only in the x direction then from equation (3.8):

$$V_{ix}^{t+\Delta t/2} = AV_{ix}^{t-\Delta t/2} + B_i R_{ix}^{t}$$

and the velocity for the next interval is:

$$V_{ix}^{t+3\Delta t/2} = AV_{ix}^{t+\Delta t/2} + B_i(R_{ix}^{t} - \sum_{\substack{\text{all links}\\ \text{at } i}} S_{xik}^{t+\Delta t} \cdot \Delta\delta_{xik}^{t+\Delta t}) \qquad (3.10)$$

where:

$S_{xik}^{t+\Delta t/2}$ is the x axis direct stiffness of node i relative to adjacent nodes k due to the structural element connecting nodes i and k.

$\Delta\delta_{xik}^{t+\Delta t} = V_{xik}^{t+\Delta t} \cdot \Delta t$ is the increment of x deflection of node i relative to adjacent nodes k during the time interval $t \rightarrow t + \Delta t$.

So if the time interval Δt is large and the stiffness/ mass ratio is large then instability may occur in the form of successive reversal and build up of amplitude and velocities and deflections may occur.

Bounds to Δt may be obtained by considering adjacent nodes I and K where the stiffness/mass ratio for one of the nodes is highest. The most critical structural configuration and state of motion will be such that all nodes k adjacent to I are different from all nodes i adjacent to K with the relative vibrations of nodes i and k exactly out of phase.

Hence from (3.10) for node I:

$$V_{Ix}^{t+3\Delta t/2} - (A+1)V_{Ix}^{t+\Delta t/2} + AV_{Ix}^{t-\Delta t/2} = -B_I \sum_{\substack{\text{for}\\ \text{all } k}} (S_{x_{Ik}}(\Delta\delta_{xI} - \Delta\delta_{xk}))^{t+\Delta t} \qquad (3.11)$$

and similarly for node K:

$$V_{Kx}^{t+3\Delta t/2} - (A+1)V_{Kx}^{t+\Delta t/2} + AV_{Kx}^{t-\Delta t/2} = -B_K \sum_{\substack{\text{for}\\ \text{all } i}} (S_{x_{Ki}}(\Delta\delta_{xK} - \Delta\delta_{xi}))^{t+\Delta t} \qquad (3.12)$$

For the most critical case assume:

$$\frac{M_i}{S_i} = \frac{M_k}{S_k}$$

$$\Delta\delta_{xi} = \Delta\delta_{xI}$$

$\Delta\delta_{x_k} = \Delta\delta_{x_K}$ for all k and i.

subtracting equation (3.12) from (3.11) gives:

$$V_{x_{IK}}^{t+\Delta t/2} - (A+1)V_{x_{IK}}^{t+\Delta t/2} + AV_{x_{IK}}^{t-\Delta t/2} = -B\ (S_{x_I}\,2\Delta\delta_{x_{IK}})^{t+\Delta t/2}$$

where:

$V_{x_{IK}}$ = velocity of I relative to K

S_{x_I} = the direct stiffness of node I relative to all adjacent nodes (where direction x is assumed to be the maximum principal stiffness).

The limiting case for stability is when $V_{x_{IK}}$ during one time increment produces relative deflection changes $\Delta\delta x$ such that V_x in the next time increment is equal and opposite to the previous value, i.e.

$$V_{x_{IK}}^{t-\Delta t/2} = -V_{x_{IK}}^{t+\Delta t/2} = +V_{x_{IK}}^{t+3\Delta t/2}$$

Hence:

$$-2(A+1)V_{x_{IK}}^{t+\Delta t/2} = -B\ S_{x_I}\,2\Delta\delta_{x_{IK}}^{t+\Delta t}$$

Then:

$$\frac{(A+1)}{B_I} = S_{xI}\,\Delta t$$

Therefore: $$\Delta t_{critical} = \sqrt{\frac{2M_I}{S_{xI}}} \qquad (3.13)$$

Alternatively assuming when subtracting equation (3.12) from (3.11) that:

$$\frac{S_{xK}}{M_K} \leqslant \frac{S_{xI}}{M_I}$$

i.e. node K is effectively fixed then:

$$\Delta t_{critical} = \sqrt{\frac{4M_I}{S_{xI}}}$$

In practice these limits provide bounds within which the true critical time lies. The lowest value must be used if no other guide is available and when calculating Δt the highest ratio of S/M at any node in any direction must be considered.

Initial Conditions:

To satisfy conditions $V^{o}_{ix} = 0$ and $R^{o}_{ix} = P^{o}_{ix}$:

$$V_{ix}^{\Delta t/2} = \frac{B_i P^{o}_{ix}}{(1+A)}$$

Shapefinding and Formfinding with Dynamic Relaxation:

The expression for calculating the residuals will depend on the problem being considered. One of the main advantages of the method is that because of its explicit nature, with separated conditions of equilibrium and compatibility, the effects of buckling, creep, plasticity and on-off non linearities can be accounted for when calculating residuals. For formfinding it is more convenient to express the equation for the residuals as:

$$R_{ix}^{t} = P_{ix} + \sum_{\substack{\text{all links } m \\ \text{at node } i}} \left\{ T_m^{o} + K_m \left\{ L_m^{t} - L_m^{o} \right\} \pm Q_m \right\} \frac{DX}{L_m}^{t} \qquad (3.14)$$

where:

T_m^{o} = Specified pretension (which may be held constant by setting K_m equal to zero)

K_m = Specified Elastic Stiffness of link m (EA/L^{o})

L_m^{o} = Slack length of member

Q_m = Traction force applied along link

DX = Current x co-ordinate difference.

The majority of published papers on this topic relate to formfinding of funicular structures. In these cases the topology is often fixed and the problem is one of shapefinding for a system which is a structural mechanism and the shape depends solely on the equilibrium of internal forces and dead loads. The factor, Q_m, is useful for the derivation of free form plan momentless boundary contours for the support of tension systems such as networks or membranes; with boundary tractions applied by supporting shear walls (234). The other factors T_m^o, K_m and L_m are used for the derivation of geodesic networks (with $K_m = 0$) and uniform mesh networks with ($T_m^o = 0$, K_m and L_m held constant)(230,233).

For triangulated structures the choice of form is not necessarily dictated by mechanical equilibrium of applied loads and member forces. However, for economy the designer will require the best geometry which can be sized to give a least weight design and support given loading systems. In formfinding this can be achieved by adjustment and control of the K_m factor. One of the earliest approaches to optimisation was developed by Michell (194). His criteria are used in a modified form to guide the changes in the K_m factor.

Michell's Theorem of Optimum Structures:

Michell's work is based on a theorem given in 1854 by Maxwell (193) which states:

> "In any system of points in equilibrium in a plane under the action of repulsions and attractions, the sum of the products of each attraction multiplied by the distance of the points between which it acts, is equal to the sum of the products of the repulsions multiplied each by the distance of the points between which it acts."

This theorem can be shown to be true for all frames in equilibrium under the action of the same loads and reacticns.

Then:

$$\Sigma \ell_p f_p - \Sigma \ell_q f_q = C$$

where:

f_p = absolute tension in tie bar of length ℓ

f_q = absolute thrust in strut bar of length ℓ

C can be shown by virtual work principles to be a constant and independent of the form of the frame.

This theorem is important because in general the strength of a member is proportional to its cross-sectional area, and so if the strength of each strut is proportional to the tress it has to bear, its weight will be proportional to the product of the stress multiplied by the length of strut. The sum of these products gives an estimate of the quantity of material required to support the external loads.

In 1904 Michell (194) developed Maxwell's work further by showing that special classes of truss can be defined which support a given loading system with the required material being at a lower limit.

If the greatest allowable tensile stress is P and the greatest compressive stress Q the least volume of material in a fully stressed frame is:

$$V = \Sigma \ell_p \frac{f_p}{P} + \Sigma \ell_q \frac{f_q}{Q}$$

For the framework in which V is least:

2PQV + (P - Q) is also least.

Thus $$2PQ\left[\Sigma\, L_p\frac{F_p}{P} + \Sigma L_q\frac{F_q}{Q}\right] + (P-Q)\left[\Sigma L_p F_p - \Sigma L_q F_q\right]$$

$$= (P+Q)\left[\Sigma L_p F_p + \Sigma L_q F_q\right] \text{ is least}$$

or $\Sigma L|F|$ is least.

If the frames are subject to an arbitrary deformation such that no element of space suffers an extension or contraction numerically greater than $\varepsilon\delta\ell$ where $\pm\varepsilon$ are the limits of strain; the virtual work done by the applied loads is equal to the sum of increases of energy stored in the bars viz:

$$\Sigma e\ell f = dW$$

where $-\varepsilon \leqslant e \leqslant \varepsilon$ and f may be of different sign to e.

Hence:

$$\delta W = \Sigma eLf \ngtr \Sigma|e|.L|F| \ngtr \varepsilon\Sigma L.|F|$$

For frame A:

$$\Sigma L_A.|F_A| \nless \frac{\delta W}{\varepsilon}$$

If a particular frame, M can be found such that for any member $e = \pm\varepsilon$ and the signs of F and e correspond then:

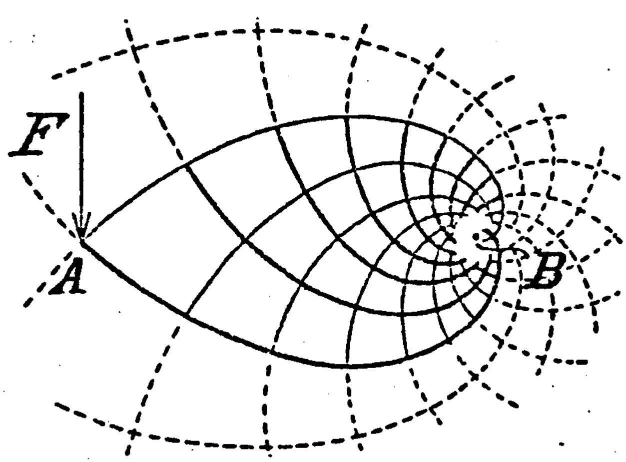
$$\Sigma L_M.|F_M| = \frac{\delta W}{\varepsilon}$$

Hence $\Sigma L_M|F_M|$ is a minimum and consequently the volume of frame M is also a minimum.

Michell stated his theorem in the following way:

> "A frame therefore attains the limits of economy of material possible in any frame-structure under the same applied forces, if the space occupied by it can be subjected to an appropriate small deformation, such that the strains in all the bars of the frame are increased by equal fractions of their lengths, not less than the fractional change of length of any element of the space."

If the space subjected to the deformation extends to infinity then the structure is an absolute optimum otherwise it is a minimum relative to those within the same boundary. For a compatible deformation field such that $+\varepsilon$ and $-\varepsilon$ strains coincide with tension and compression members, the members must coincide with mutually orthogonal principal trajectories of a virtual strain field compatible in sign with member forces. One example given by Michell is a cantilever formed from two equiangular spirals.



Formfinding for Modular Triangulated Space Structures:

The Michell form represents an aesthetic ideal and standard by which to measure other candidate structures. However, its member areas, lengths and joints are non-standard making these structures impractical to build.

Methods which utilise the concept of a modular ground structure have been discussed in Chapter 2. These methods automatically avoid the problem of non-standard connections and member lengths by specifying that the structure must comply with a particular building system. Nearly all these methods are automatic with little or no man-machine interaction. Configurations of unbalanced and unaesthetic appearance can easily be derived if particularly stringent constraints are specified in some regions of the structure. For this reason it may in many practical cases be beneficial to restate the design form-finding of space structures as: Determine the most economical structural form, subject to the dominant design loading which expresses the function of the structure, using a building system consisting of a particular type of space-node connector allowing for the junction of many potential membes and sets of discrete member sizes which comply in length with the space filling capabilities allowed by the nodes.

Hemp (187) gave the criteria for a least weight solution of a problem of this type assuming a continuous range of area sizes:

> "The necessary and sufficient conditions that a pinjointed framework, selected from a given framework, should have a minimum volume of material are that it should be capable of carrying its given forces with stresses σ_T in its tension members and $-\sigma_c$ in its compression members and should allow a virtual displacement of its nodes, which produces a strain of $\sigma\varepsilon/\sigma_T$ in its tension members and a strain of $-\sigma\varepsilon/\sigma_c$ in its compression members and strains not outside this range in members of the given framework, which are not present in the optimum design." (where $\sigma = (\sigma_c + \sigma_T)/2$ and ε is a virtual strain parameter).

This is analogous to the Michell criteria but cast in a form applicable to geometrically modular structures. It was originally developed for use with linear programming.

A formfinding method for modular structures was proposed and applied by Barnes (235). The method was based on Dynamic Relaxation and utilised the original Michell criteria and the concept of a changing ground structure. In the present chapter this method is applied to a series of problems and compared in efficiency with solutions obtained from fully stressed design and linear programming. This work was subsequently incorporated in reference (236). In the following chapter the method is more fully investigated with respect to the choice of iteration parameters to achieve optimum convergence and stability. The effects of more complex loading and differing stress constraints in tension and compression are also considered.

Basis and Control of Dynamic Relaxation Formfinding for Space Structures:

For a single dominant design loading, selection of a least-weight structural form from a large set of possibilites may be achieved without minimizing a weight function if the analysis procedure leads to a direct physical solution which complies with the Michell theorem. Since deflections and rates of deflection of nodes are controlled by dynamic equations of motion the possibility of ill conditioning maybe precluded by suitable control of the fictitious nodal mass components which govern the acceleration. Additionally the lack of an overall stiffness matrix with the D.R. explicit solution method makes the technique ideally suitable for man-machine interaction.

If a multiply connected ground structure in which member areas may change continuously (as loads are taken up and transmitted to the supports) is to be modified to comply with the theorems, then the areas must be modified by a function which tends to make them proportional to the current ratio of their own strain magnitude, $|e_c|$, to that in a reference member, $|\varepsilon_c|$, which is assumed to participate in the optimum solution. If $|e_c|$ exceeds $|\varepsilon_c|$ the member area, A_c, is increased and conversely if $|e_c| < |\varepsilon_c|$ the area is reduced.

The full ground structure topology must be retained throughout the analysis to ensure that the design complies with the Michell theorem. However, a converged solution implies $|e_c| = |\varepsilon_c|$ for all members and this is impossible if

E is the same for all members since nodal displacements and hence member strains, are determined by any reduced subset of bars which give a determinate structure. This problem is avoided by retaining the complete topology until after sufficient modifications have occurred such that the areas of members not required are insignificant or small and reducing and required members are significant or large and increasing. Thus if the method yields monotonic convergence then the members of the first type can be excluded and the solution gives an optimum structural form. The formfinding analysis provides a virtual strain field which gives the optimum layout. The constant, E, is a fictitious elastic modulus to ensure that deflections are not excessive. If the solution is of determinate form the member areas can readily be sized otherwise methods of Appendix III must be utilised. In some cases fictitious and realareas may coincide so that the design and analysis coincide as shown in Chapter 4.

To enable a solution by Dynamic Relaxation three stabilizing conditions should be observed:

(a) The analysis should be well damped to ensure monotonic convergence and to account for changes in the lowest natural frequency of the structure due to the area modifications.

(b) The changes in stiffness should lag behind the resulting rates of strain. This can be ensured by modifying the areas at each stage according to the relation:

$$A_{c+1} = \frac{A_c}{2}\left[1 + \frac{|e_c|}{|\varepsilon_c|}\right] \qquad (3.15)$$

For members which participate in the optimum solution the cross sectional areas thus become converged when $|e_c| \rightarrow |\varepsilon_c| =$ constant. For bars which remain unstrained the areas reduce exponential from stage to stage whilst other strained members not in the optimum reduce more slowly.

(c) The ficititious mass component at each node in each co-ordinate direction is adjusted at each modification stage so that it is proportional to the direct stiffness component. Each node has the same relative response and the same critical time interval in each co-ordinate direction. The time interval used is to be 90% of the critical time interval, and the required mass component given by re-arranging equation (3.13) is:

$$(M_{ix}) = \left[\frac{\Delta t}{0.9}\right]^2 \left[\frac{S_{ix}}{2}\right] \qquad (3.16)$$

If the co-ordinate directions, x,y,z, coincide with the principal direct stiffness directions and the system geometry does not change significantly during the analysis the mass components for each co-ordinate direction could be set according to equation (3.16) with a higher proportion of the critical time interval.

In practice, however, the principal directions will differ from those of a convenient co-ordinate system and the principal stiffnesses will change throughout the analysis as areas are modified. The masses must therefore be adjusted to give a lower proportion of the critical time interval (0.9 in equation 3.16) and in an extreme case this may need to be reduced to 0.5. An alternative which ensures stability is to use in place of the diagonal stiffnesses at a node S_{ixx}, S_{iyy}, S_{izz}, (in 3.16) the row sums of the direct stiffness sub-matrix (S_{ixx} + S_{ixy} + S_{ixz}), (S_{iyy} + S_{iyx} + S_{iyz}), (S_{izz} + S_{izx} + S_{izy}). For the well triangulated type of structural examples analysed below, however, it has been found that the masses given by equation 3.16 are sufficient.

Example Structures:

The application of the formfinding process is illustrated in the following simple examples which are plane trusses analogous to the Michell cantilever. Each is subject to a single load cantilevered from two hinge supports. They have also been designed by the dual linear programming technique (176,186-188) and the fully stressed design stress ratio method (132). The first three examples were designed by the D.R. procedure in reference (235) using a full ground structure. Here symmetry has been taken into account and only the upper half of the structure considered. The final example was first considered by Hemp and Chan (186-8) in relation to linear programming.

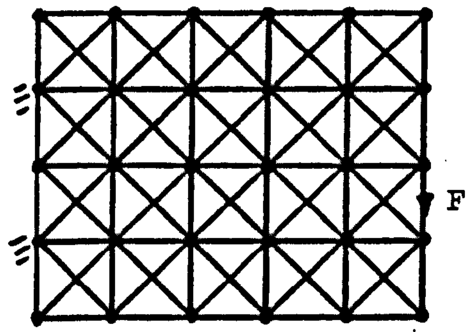
The time interval used in the D.R. procedure for all the designs was 0.1 seconds and the EA values of all members were initially set to 1000 N with the grid set at 1 metre centres and the cantilever load to 1 N.

Five Bay Sparse Ground Structure:

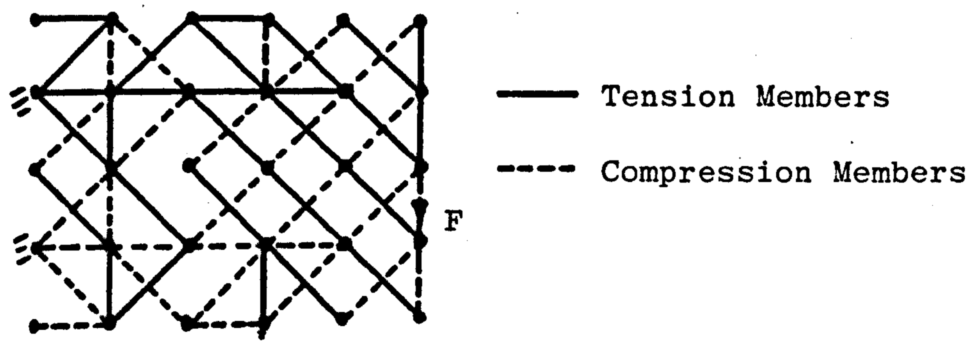
An initial undamped run without modifying member areas gave a fundamental frequency of 0.1 which yields an estimate of the critical damping per unit mass of $C_c'/\Delta t = 4\Pi f = 1.26$.

The formfinding process was carried out using a damping factor/unit mass of 10.0 and areas were modified according to equation (3.15) after every 10 iterations. The tensile strain in the member marked 'c' was used as the reference strain, ε_c, the area of the member being held constant throughout. After sixteen stages of modification (equivalent to 0.300 seconds of CDC execution time) the pattern of the most efficient members had become clear. For the D.R. design, the EA values of members indicated in the figure by a full line were all stable or increasing and lay in the range 700 N - 3000 N. The EA values of members

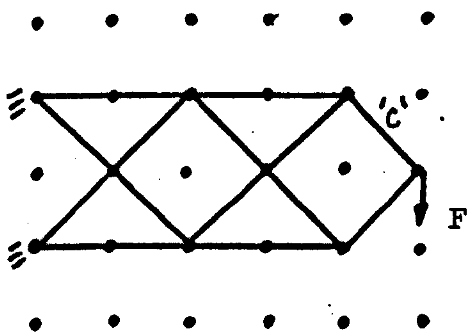
FIVE BAY SPARSE GROUND STRUCTURE



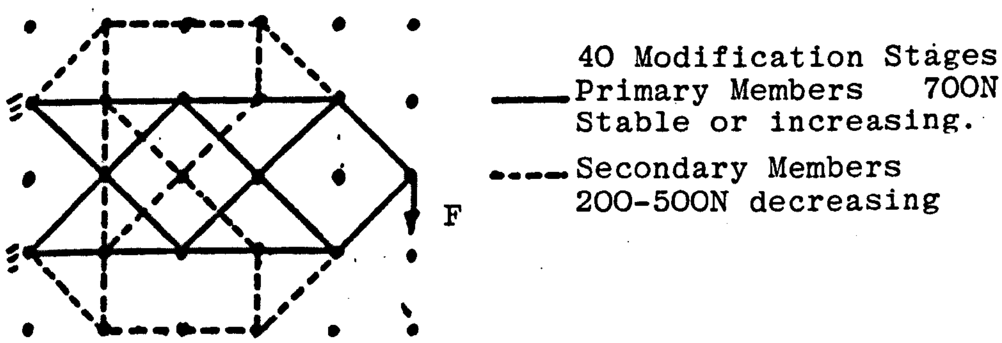
Ground Structure



Dual Linear Programming Solution



Linear Programming Statically Determinate
Optimum Solution



Dynamic Relaxation Design

shown dashed were in the range 200 - 500 N and decreasing. And all other members had values less than 200 N, half of which were less than 10 N. After ninety stages the full members all had EA values greater than 980 N and the remainder were less than 10 N (with 70% less than 0.01 N). At a later stage, after 194 modification stages, the analysis became unstable due to the fact that at certain nodes which did not participate in the optimum structure the mass components as given by equation (3.16) were no longer adequate. Instability at this stage, however, is not in practice a problem since the solution becomes apparent at a much earlier stage.

With the same number of time intervals, 10, between each modification stage the analyses were convergent with damping factors as low as 1.0 per unit mass. But analyses run with a damping factor less than 1.0 per unit mass diverged at an early stage.

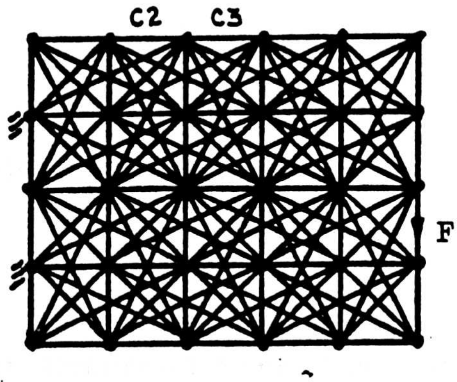
Finally the analysis was run with a damping factor per unit mass of 6.0 and with 4 time intervals between each modification stage. The solution was evident by the 40th stage and the execution time taken was .295 seconds. If either the damping factor or the number of iterations between modification stages was reduced the design became numerically unstable at an early stage. Comparison with other computer runs indicated that if the number of intervals between modification stages was increased the damping factor could be reduced without loss of stability. However, if the damping factor was reduced to its optimum value then the solution CDC execution time was always of the order .295 seconds.

Five Bay Extended Ground Structure:

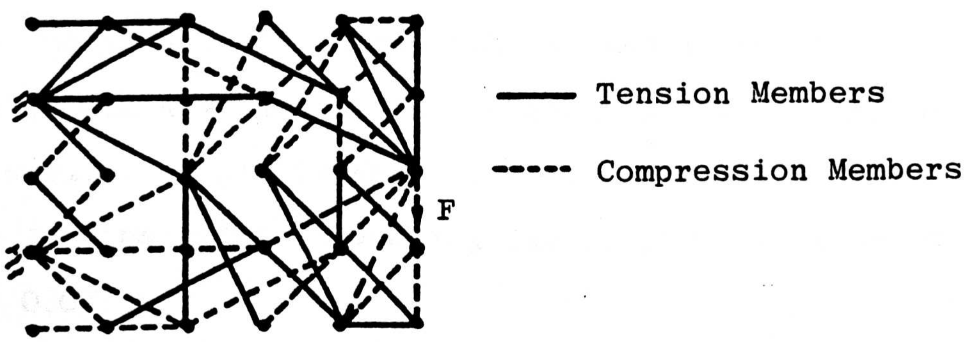
The more extensively coupled ground structure shown below was analysed using the same initial values and time interval. A preliminary undamped analysis gave an estimate for the critical damping per unit mass of 1.5 (f = .12).

Using a damping factor of 6.0 and modifying member areas after every 4 time intervals the primary members shown in the figure were evident after only 40 modification stages (equivalent to 0.504 seconds of CDC execution time). The form of the structure by full, dashed and dotted lines which correspond respectively to EA values 600 N (increasing or stable), 170 N (reducing), 50 N (reducing). Of the remaining 109 members 87 were less than 3 N. After 110 modification stages only the primary members had significant EA values the rest being 75 N. The analysis did not become unstable until after 2010 modification stages. It was also noted that members C2 and C3 could be used as reference members for this structure although these members are outside the optimum. The analysis was stable for at least 157 modification stages with a damping factor of 80.0 and 10 time intervals between

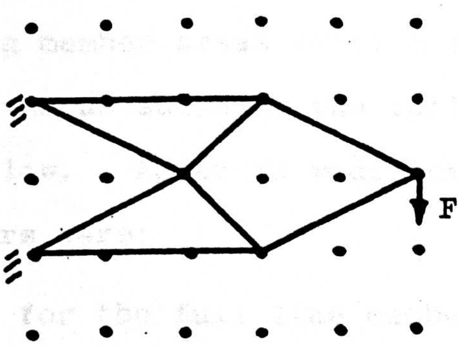
FIVE BAY EXTENDED GROUND STRUCTURE



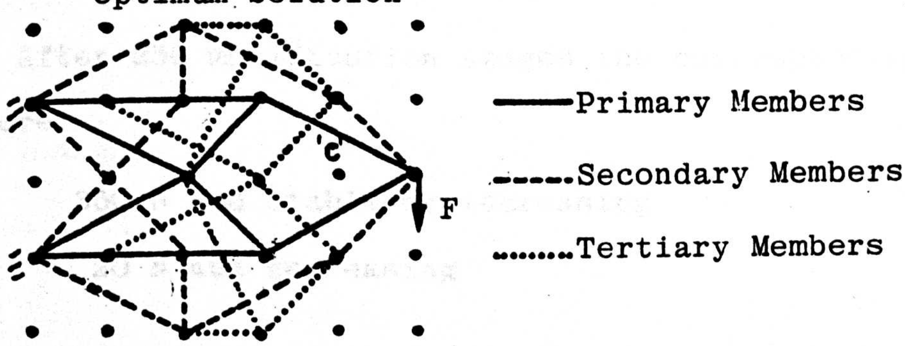
Ground Structure



Dual Linear Programming Solution



Linear Programming Statically Determinate
Optimum Solution



Dynamic Relaxation Design

each modification stage. It is interesting to note that C2 is within the Michell field but C3 outside it.

This structure shows a 4.5% reduction in volume compared with the structure generated from the reduced ground structure in the last section.

Seven Bay Extended Ground Structure:

The ground structure with two additional bays shown below was analysed using the same initial values and time interval. A preliminary undamped analysis gave an estimate for the critical damping factor per unit mass of 1.0 (f = 0.077).

Using a damping factor per unit mass of 6.0 and modifying member areas every 5 time intervals the optimum derived was as shown by the full line members in the figure shown below. After 70 modification stages the EA values of members were:

a) for the full line members: 270 N stable or increasing

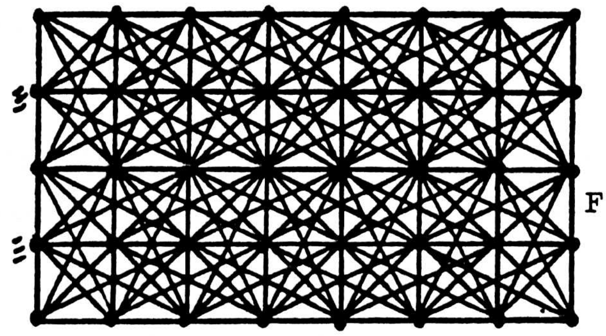
b) for all other members: 270 N and decreasing.

After 250 modification stages the corresponding EA values were:

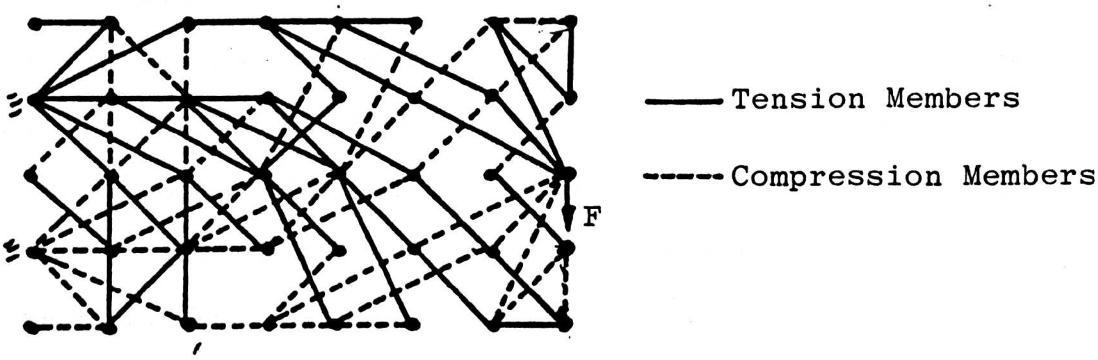
a) 350 N and stable or increasing

b) 20 N and decreasing

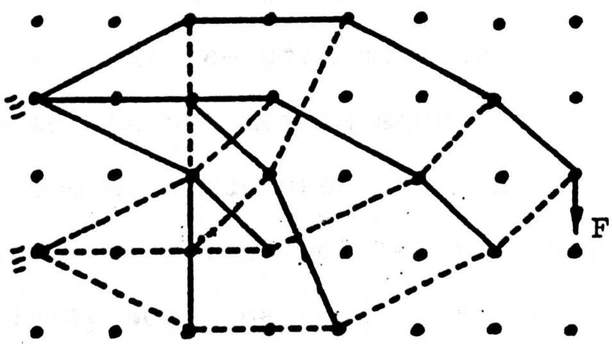
SEVEN BAY EXTENDED GROUND STRUCTURE



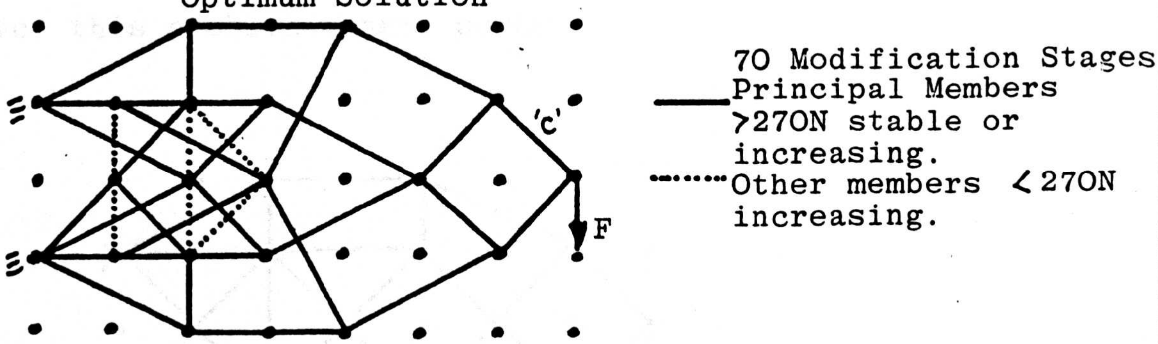
Ground Structure



Dual Linear Programming Solution



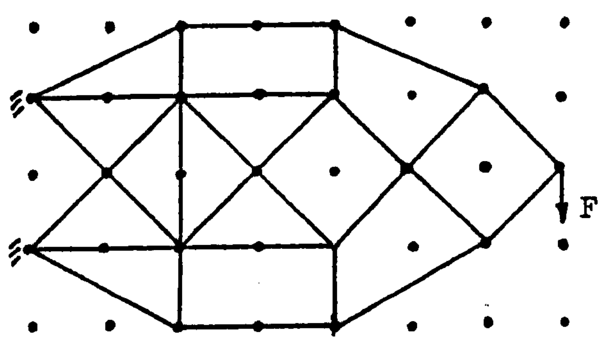
Linear Programming Statically Determinate
Optimum Solution



Dynamic Relaxation Design

The only exceptions to this classification were the dotted line members which, whilst not complying with categories (a), had EA values which were slowly increasing. The reason for this is that there are several statically determinate or indeterminate structures of equal optimum weight. These structures are sub-sets of the Linear Programming Dual Strain Field. Convergence of the analysis was comparatively slow because in addition to the optima of equal weight, numerous other structures have weights close to that of the optimum and consequently their member areas decreased slowly. The time for 70 modification stages at which the optimum could be obtained, was equivalent to 1.068 seconds CDC execution time.

If one imposes the restriction that members may only cross at the grid node points, a structure which is close to the optimum is shown below. This is a simpler and more elegant structure. If automatic checks for this condition were incorporated in the D.R. procedure the method's simplicity would be lost. For the main application of formfinding of three dimensional modular space structures however this problem cannot occur.



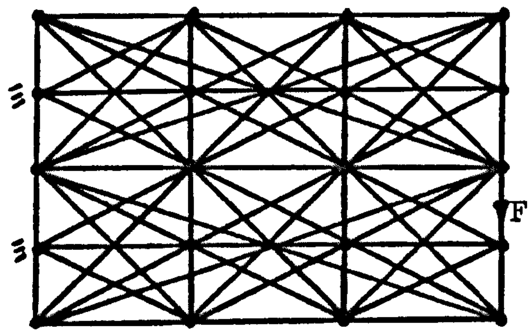
Three Bay Fully Connected Ground Structure:

The fully connected 3 bay ground structure shown below has a 2 metre spacing horizontally and a 1 metre spacing vertically. It was analysed using the same initial EA values and time step.

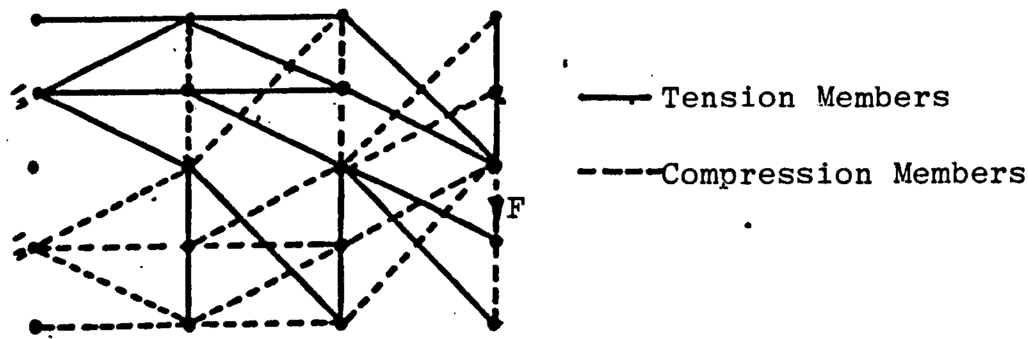
A preliminary undamped analysis gave an estimate a the critical damping factor per unit mass of 1.74 ($f = 0.138$). The structure was analysed using a damping factor of 6.0 and modifying the areas after every third time interval. The form shown below became apparent after 50 modification stages which was equivalent to 0.295 seconds of CDC execution time. The members within the optimum form all had EA values greater than 350 N and the deleted members had values less than 120 N. The optimum form for this structure is statically indeterminate indicating there are a series of statically determinate optimum forms. The form is therefore a matter of choice and some of the alternative structures are shown below. After 675 modification stages the statically determinate form given by the dual linear program becomes apparent. The reason for this is uncertain.

The volume of these modularly constrained optimum structures is 29.0 WL/σ compared with 26.04 WL/σ for the equivalent Michell structure (where L = modular spacing = 1.0 metre).

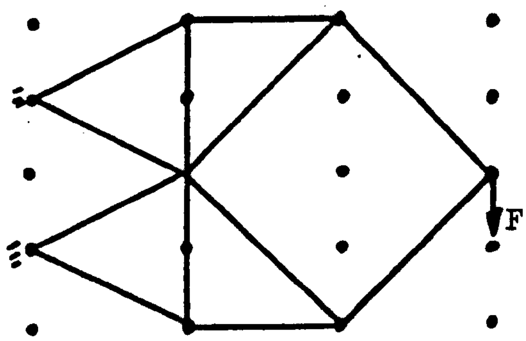
THREE BAY FULLY CONNECTED GROUND STRUCTURE



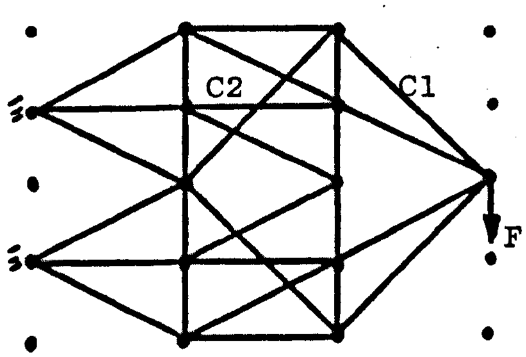
Ground Structure



Dual Linear Programming Solution

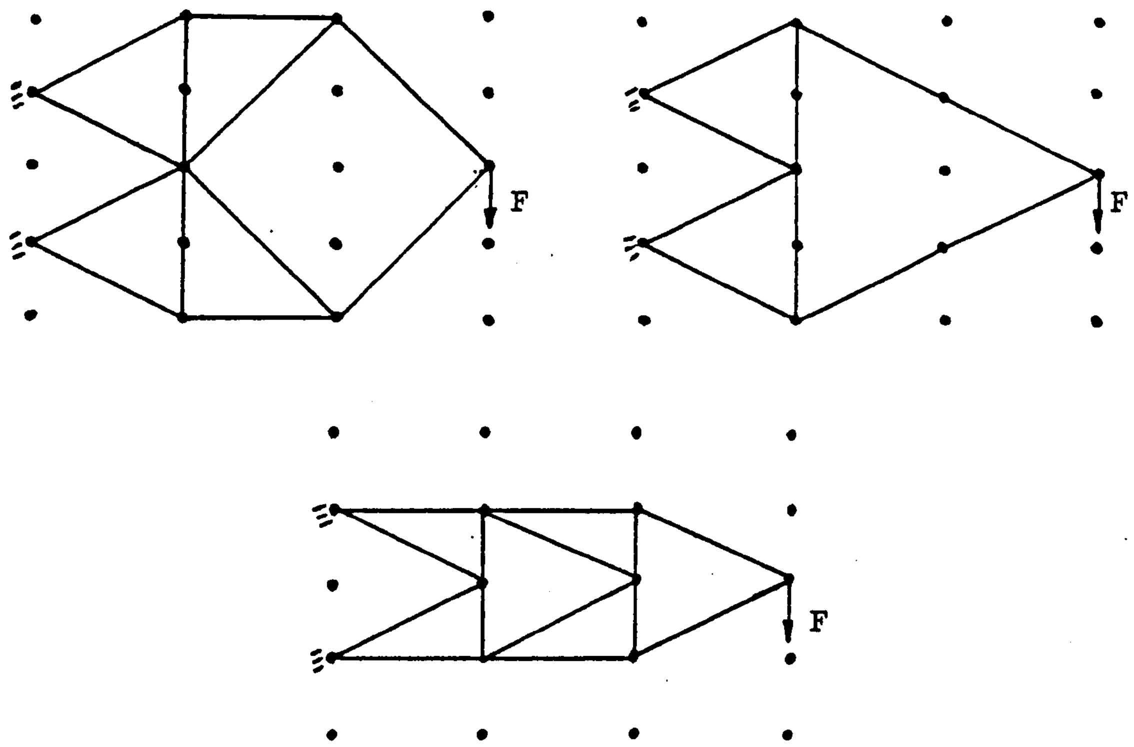


Linear Programming Statically Determinate
Optimum Solution



50 Modification Stages
Members $>$ 350N stable
or increasing.
All other members not
shown $<$ 120N

Dynamic Relaxation Design



Some alternative statically determinate optimum structures.

Comparison with other methods:

The forms derived from the dual linear programming technique (L.P.) and the Fully Stressed Design Method (F.S.D.) using the stress ratio method have been shown in the previous sections.

The three bay structure was designed using the F.S.D. method and the modified Newton or effective load method for the reanalyses. The out of balance forces were initially large due to large changes in the stiffness and the calculations diverged after a few modifications. The out of balance force permitted on resizing was varied and the following table produced to assess the cause of divergence. (Load = 100 N, EA start = 1000 N, Permissible Stress = 100 N/m^2 E=1000 N).

Max. Permissible Out of Balance force after reanalysis before resizing		5N	0.5N	0.1N	0.05N	0.01N	0.005N
No. of cycles before resize no: (initial maximum out of balance force shown in brackets)	2:	19	36	52(138N)	59	75(138N)	82
	3:	4	4	4(6.4N)	4	4(.55N)	4
	4:	5	4	3(3.4N)	3	3(.27N)	3
	5:	12	4	4(3.9N)	4	4(.31N)	4
	6:	D	6	7(14.9N)	6	6(1.25N)	6
	7:		22	10(8.8N)	10	9(2.2N)	9
	8:		D	17(13.8N)	22	16(2.7N)	15
	9:			45(28.4N)	D	54(3.3N)	52
	10:			D(35.5N)		D(4.7N)	D

D = Diverges

The optimum permitted out of balance force for this problem is 0.1 N. Below this value there is no increase in stability and insignificant increase in accuracy in the area sizes. It appears that large initial out of balance forces due to radical changes in the actual stiffness of the structure are the cause of the instability. To avoid this problem a new stiffness matrix was formed when the out of balance forces were larger than 10 N. The resulting modification sequence was as follows:

Before Resizing No:	Max. Out of Balance Force:	
2	138 N	Stiffness matrix reformed
3	55.1 N	" " "
4	28.1 N	" " "
5	18.4 N	" " "
6	12.2 N	" " "
7	9.35 N	Reanalysis by effective load technique 3 Iterations
8	.30 N	Reanalysis by effective load technique 2 Iterations
9	2.42 N	Reanalysis by effective load technique 3 Iterations
10	4.87 N	Reanalysis by effective load technique 4 Iterations

Following the above modification the calculations remained stable. General rules for the stability for this reanalysis technique which depend on the changes in or non-linearity of the stiffness matrix have not been developed. In their absence the Newton-Raphson procedure was used which completely avoided the need to calculate out of balance forces. With this type of formfinding problem some nodes may eventually have no stiffness which will result in the stiffness matrix becoming singular. This was prevented by fixing all degrees of freedom where the leading diagonal of the stiffness matrix was less than 10^{-120}.

For the examples considered, with magnitudes of tensile and compressive stress limits identical, and provided all possible members are included throughout the design, the

F.S.D. gives a global optimum because real strains in the fully stressed links are $\pm\varepsilon$ with $|e| < |\varepsilon|$ in all other possible links thus satisfying the Michell criteria.

In all cases the identical D.R. and F.S.D. solutions are subsets of the strain field derived by the L.P. technique. If there are no alternative optima the D.R. and F.S.D. solutions are identical to the L.P. strain field with unloaded members removed. If there are alternative optima (as with the 3 and 7 bay problems) the D.R. and F.S.D. solutions are indeterminate since there are several alternative statically determinate structures of equal elast weight. For these problems the L.P. technique used extracts one of the possible determinate forms. The CDC execution times for all the problems, with symmetry accounted for in each analysis are listed below. Times quoted for the D.R. and F.S.D. methods are those required for the solution to become apparent. The general trend indicates that the D.R. method becomes more efficient for larger structures.

	CDC 7600 Executions Times (seconds)		
	L.P.	F.S.D.	D.R.
3 Bay Fully Connected Structure	0.146	0.499	0.295
5 Bay Sparse Structure	0.345	0.796	0.292
5 Bay Extended Structure	0.460	1.165	0.504
7 Bay Extended Structure	1.581	10.268	1.068

Further Comments:

The D.R. design procedure provides an efficient and simple method for deriving an optimum form for a dominant loading condition. The method indicates solution trends before the final solution is arrived at. Using these trends the engineer does not only observe the developments of the final solution but can alter the problem if unforeseen or undesirable trends are noticed during the process. The explicit separated form of D.R. allows the structure to be changed during the process, making it ideally suitable for interactive use. Members and nodes can be added or deleted, fixity conditions altered and nodal co-ordinates adjusted if required. With this interactive facility the number of alternative solutions could be large and aesthetic criteria would become more significant during the design process. Such man/machine interaction is more than a simple matter of choosing alternative members from a statically indeterminate optimum form. It enables the conceptual design to be more fully integrated with the initial structural design than is the case with more automatic optimization methods such as Linear Programming.

While working on the above problems it was noticed that numerical instability might be more effectively controlled, and in the following chapter these problems are more fully investigated.

CHAPTER 4: INVESTIGATION OF THE DYNAMIC RELAXATION FORMFINDING METHOD FOR MODULAR SPACE STRUCTURES

A Parametric Study of Stability - The Effects of Topology on Stability and the Solution Convergence Rate - Additional Mass Components - The Choice of Reference Member - Initial Member Areas of the Ground Structure - More Complex Loading Conditions - The Inclusion of Self Weight - True Member Area Sizes - The effects of different stress constraints in Tension and Compression Members on Optimum Form - Conclusions.

Summary:

In this chapter a full investigation of the effects of the iteration parameters on the stability and convergence of the Dynamic Relaxation Method is reported. The problems of the choice of reference member, initial member areas, complex loading cases, the inclusion of self weight and the use of time area sizes are studied. The Michell theorem, adapted to cater for differing stress contraints in tension and compression members, is discussed. And the effects of these more general constraints on the optimum form are compared with the results from a dual linear program and a Fully Stressed Design program.

A Parametric Study of Stability:

The three bay fully connected ground structure used in Chapter 3 was chosen to carry out a parametric study of the stability of the D.R. procedure to assess the effects of:

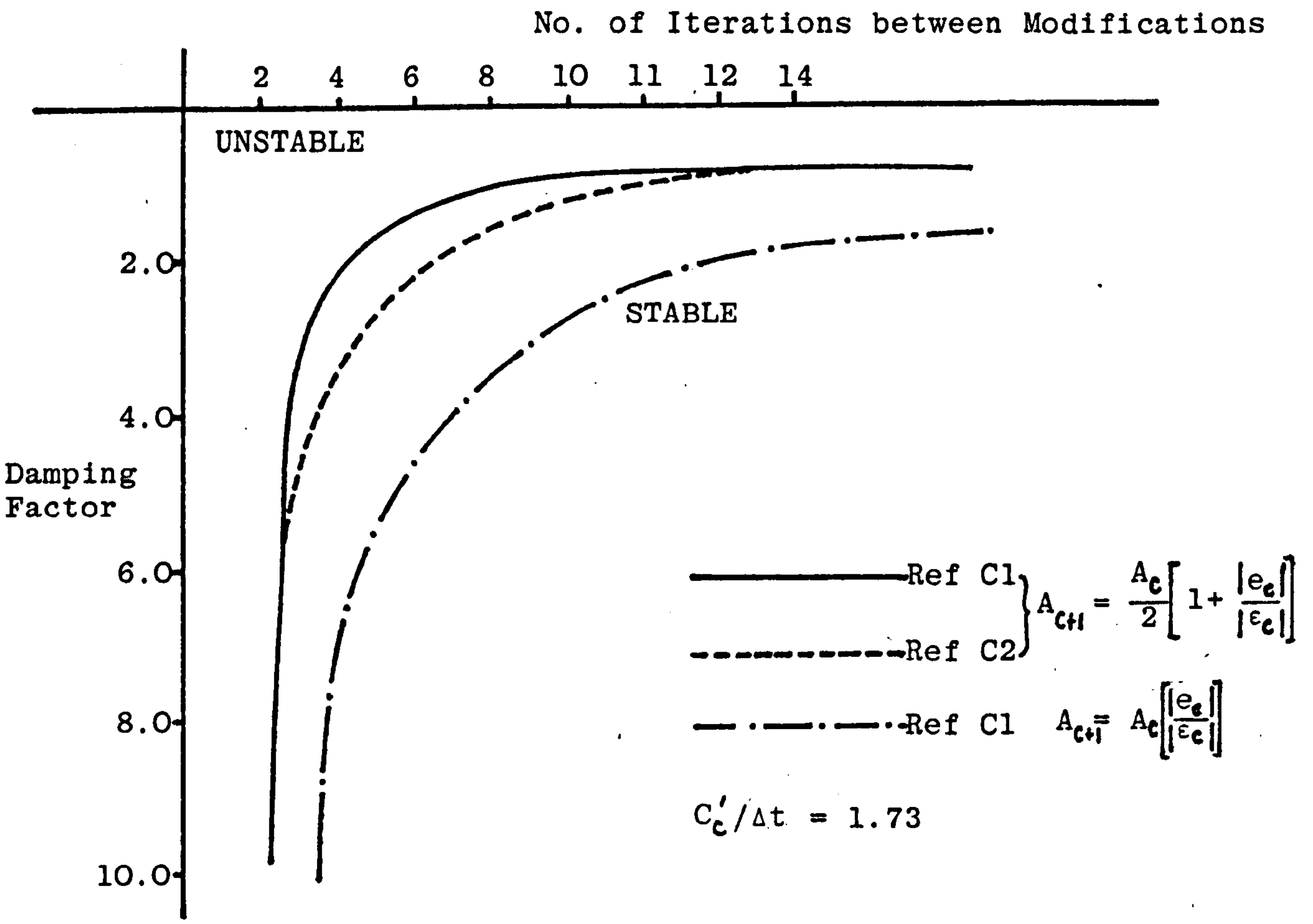
a) the damping factor

b) the number of intervals between area modifications

c) the function used to modify the member areas

d) the member chosen as a reference member.

All four of these factors were varied and approximate stability curved were plotted for the problem:



The mass components used for all these analyses were as given by equation (3.16). When the masses were all doubled; the damping factor or number of time intervals between modifications could be further reduced without loss of stability. This, together with the above graph, indicates that if the modifications are initially too rapid the iteration becomes unstable. Although results are difficult to quantify, it can be concluded that stiffness modifications should lag behind the changes in strain and that the damping factor should be increased for reduced numbers of time intervals between modification stages. This aspect is important for efficiency since it was found that the solution becomes apparent after a particular number of modification stages. This number was found to be independent of the number of time intervals between modification stages and the damping factor (for the range of values considered). When the function used to modify the areas was of the form $A_{C+1} = A_C . |e/\varepsilon|$ the number of modification stages for the solution to become apparent was cut by 50%. However, as shown on the graph the range for which the problem was stable was much reduced.

The D.R. procedure was also used to derive a form for which the solution was a structural mechanism. In this case the problem had to be over damped to achieve convergence and avoid instability.

In an effort to cut the number of time intervals between modification stages towards the end of the solution procedure the number of time intervals was varied linearly and exponentially as the calculations proceeded. In both cases the increase in computational efficiency was insignificant and avoiding instability became a greater problem.

For different damping factors it was noticed that the precise order of member sizes was not the same but members could always be assigned into three distinct groups:

a) Bars which are within the optimum and play an important role in the structure with large areas which are stable or increasing slowly.

b) Bars which have medium sized areas and are reducing in size.

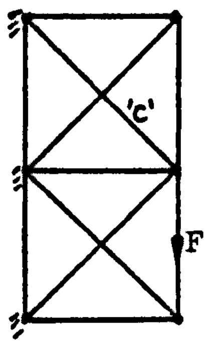
c) Bars which have insignificant areas and are reducing in size.

The slight variations in order which occurred are probably due to changed in the critical damping factor resulting in the structure being only lightly damped. For example, the critical damping for the 3 bay ground structure is 1.74. Whereas the critical damping factor when the statically indeterminate form is just apparent is 3.93 and for the determinate form it is 5.71. The solution can therefore be arrived at even though the system becomes increasingly under-damped.

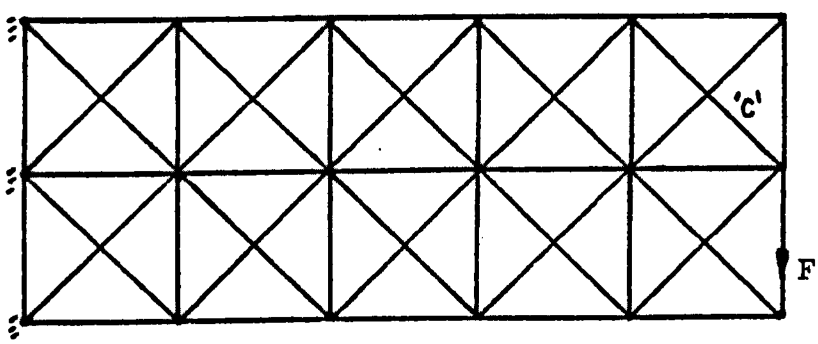
Further attempts were made to control the stability and reduce the solution time by defining the damping per unit mass as inversely proportional to the residual force. In this way it was hoped that out of balance forces due to stiffness changes would be quickly accounted for in the analysis. The larger forces appear to lead to rapid stress changes resulting in spurious stiffness modifications. Instability was thus more difficult to control with this form of D.R.

The Effects of Topology on Stability and Solution Convergence:

A series of cantilevers increasing in size from one bay to five bays were designed to investigate the effects of topology path length on stability and convergence. As with the previously studied cantilever structures only the upper half of these structures was considered.



One Bay Cantilever



Five Bay Cantilever

The initial EA values were 1000 N and the cantilever nodal spacing 1.0 m. The time interval used was 0.1 seconds and the critical damping factors were as follows:

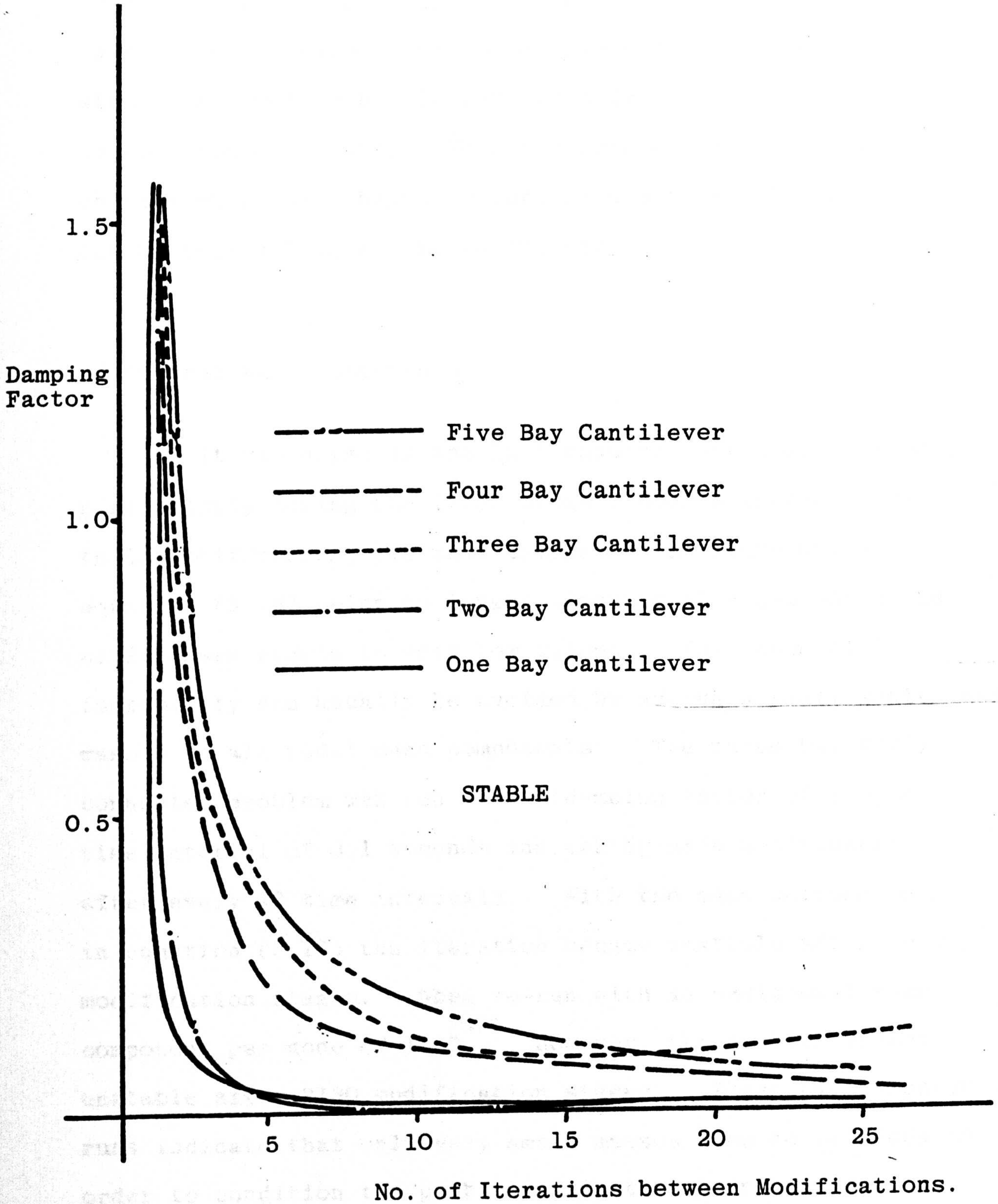
No. of Cantilever Bays	Critical Damping Factor per unit mass
1	10.47
2	6.98
3	3.49
4	2.17
5	1.5

The approximate stability curves for each problem are shown on the following graph. The smaller structures have a greater range of stable parameters. The overall effects of topology on stability, however, do not appear significant.

The number of modification stages before the solution of a particular problem becomes apparent has already been shown to be constant. The number of mocification stages for each cantilever solution to become apparent are tabulated below:

No. of Cantilever Bays	No. of Modification Stages before solution is apparent
1	14
2	35
3	38
4	44
5	76

STABILITY CURVES FOR ONE TO FIVE
BAY CANTILEVERS



The rate at which convergence occurs is shown to be related to topology, generally being slower and taking more modifications for a longer cantilever. The criteria for rate of convergence cannot be easily related to the ground structure topology but is more readily related to the form of solution structure. This was apparent in the last chapter when the 7 bay structure with several alternative optima took a longer time to converge.

Additional Mass Components:

It was noted in the last chapter that some problems, particularly during the later stages, become unstable due to low stiffnesses, the mass components assigned using equation (3.16) being no longer adequate at nodes where the stiffnesses reduce to very low values. This type of instability can usually be avoided by adding a small additional mass s to all nodal mass components. The three bay fully connected problem was run with a damping factor of 2.0, a time interval of 0.1 seconds and making area modifications after every 10 time intervals. With the mass defined as in equation (3.16) the iteration became unstable after 1960 modification stages. When re-run with an additional mass component per node of 10.0^{-276} however, the problem became unstable after 2130 modification stages. These two computer runs indicate that only very small masses need to be added in order to condition the problem against such instability.

The Choice of Reference Member:

Barnes (235) suggested that any member could be used as the reference member. However if the reference member is not within the optimum form then the areas of optimum members tend to become very large or infinite. (The converse is if the reference member is in the optimum form then the areas of members not in the optimum become infinitesimal.) The problem is that the former case will become unstable although the optimum form usually becomes apparent before this happens. In general, however, the optimum form can be more easily seen if the reference member is within the optimum form. For this reason it is preferable, particularly with large structures, to choose a reference member, within the optimum form, that is attached to a heavily loaded node which attains a significant strain in the early stages of the D.R. analysis. This fact is also illustrated by the parametric study graph in the first section of this chapter. It is possible to ensure that the reference member is always within the optimum form by changing the reference member to that member with the highest strain each time the member areas are modified.

The three bay fully connected ground structure was redesigned using this technique modifying member areas every 10 time intervals. The initial reference member chosen was 5(Link 6-7) and the reference member on final convergence was 21 (Link 3-10). Subsequently two other computerruns, with damping factors of 40.0 and 10.0 were made with member 21 as the initial reference member. In all three runs the reference members were usually different at a particular iteration but always converged with member 21 as the reference member. Changes in the reference member for each design are tabulated below:

Initial Reference Member:	5	21	21
Damping Factor:	10.0	10.0	40.00
Iteration at which the reference member is changed: New reference member	10:49	10:45	10:45
	20:45	20:43	20:19
	30:43	30:41	30:34
	40:21	40:21	40:41
	50:15	60:16	50:15
	80:13	80:13	60:43
	290:14	310:14	70:21
	1060:21	830:21	340:15
			700:21

No further changes

Key to Nodal positions:

1 • 2 • 3 • 4 •
5 • 6 • 7 • 8 •
9 • 10 • 11 • 12 • $\downarrow$ F

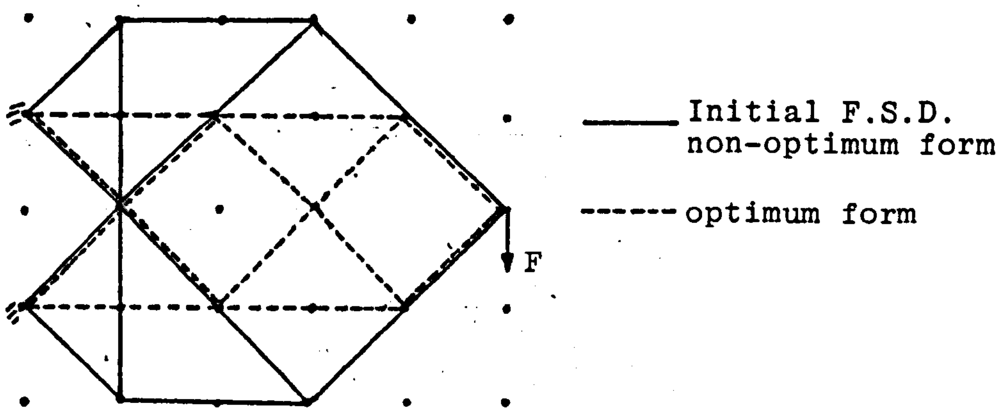
Link No: Nodal Connections:

5:6/7	21:3/10
13:7/11	34:1/6
14:2/6	41:6/9
15:6/10	43:7/10
19:1/10	45:8/11
	49:9/8

A further computer run was made printing out values of strain e, areas A and strain ratios e/ε . Although member areas converge and eventually achieve constant values the strain ratios always vary. Members within the optimum have strain values nearer 1.0 but the overall range for all members is 0.02 to 1.9.

Initial Member Areas of the Ground Structure:

A fully stressed but non-optimum form solution was presented within the ground structure for the sparse five bay structure. This form is illustrated below:



All the other members of the ground structure were included with initially very small areas and it was found that the final optimum form emerged as designed in chapter 3. This indicates, as concluded in chapter 3, that an optimum structure can be ensured provided that all the members of the ground structure are retained throughout the process.

More Complex Loading Conditions:

In the previous examples only simple loading conditions have been considered. In the following the effect of simultaneous application of loads at different positions is considered.

Two undamped trial runs were performed for the three bay fully connected structure first with a load applied at node 6 alone and then at node 12. The critical damping factors for the two cases were found to be respectively 12.57 and 1.75 per unit mass. With node 6 loaded only and modifying member areas after every 10 time intervals the analysis was stable using damping factors of 2.0 and above. With both nodes 6 and 12 loaded simultaneously the analysis was stable for damping factors of 3.0 and above. The critical damping factor for the latter converged solution, with node 12 loaded only was 3.69.

These runs suggest that with more complex loading cases the excitation of higher modes of vibration is not a problem and it appears that to ensure stability it is sufficient to critically damp the fundamental mode.

The Inclusion of Self Weight:

Self weight was included in the analysis by forming a new self weight loading vector each time the areas were modified and adding this to the applied load vector at each iteration. To examine the efficiency of this technique the five bay sparse structure and the seven bay structure were run modifying the areas after every 4 and 5 time intervals respectively and using a damping factor of 6.0. The initial EA values used were 2.48×10^{6} kN, the Youngs modulus was

was 207.0 kN/mm^2. The ground structure spacings were 1000 mm, and the applied cantilever load for each structure was 100 kN. The masses given by equation (3.16) were multiplied by 1.5 and additional mass components of 0.5 were added to increase stability. The critical damping factors were estimated from undamped runs to be 0.64 for the five bay and 0.92 for the seven bay structure.

The CDC execution times and number of modification stages for the solutions to become apparent with and without the self weight considerations are tabulated below:

Specific Weight $= 7.98 \times 10^{-8}$ KN/mm^2	No. of Modification Stages:	Time Seconds:
5 Bay Sparse Ground Structure:	40	0.295
5 Bay Sparse Ground Structure with Self Weight:	32	0.276
7 Bay Ground Structure:	70	1.068
7 Bay Ground Structure with Self Weight:	60	0.964

These comparisons indicate that the solution times do not increase when self weight is considered despite the extra computational operations required for the calculation of the self weight vector. It should be noted that the effective weight derived in the above manner is related to initial EA values assigned to the members. If these are not realistic, or do not remain realistic, the forms derived may be incorrect.

True Member Area Sizes:

If true member sizes are used the self weight vector becomes the true self weight vector. Using the method of chapter 3 the resulting member areas are approximately in their correct proportions and, to arrive at EA values consistant with safety for the dominant load case, only the reference member need be resized when member areas are modified. If reference member resizing takes place from the first modification stage this is liable to lead to excessive cuts in the reference member area, and hence all member areas at the next modification stage, eventually resulting in instability. It was found that this problem of instability was less likely to occur when the initial EA values of all members were set rather low and the reference member resized only after its stress level became inconsistent with safety.

An alternative procedure consists of modifying all member areas by the function:

$$A_{c+1} = \frac{A_c}{2.0}\left[1.0 + \left|\frac{e_m}{\varepsilon_c}\right|\right]\left[\frac{Arnew}{Arold}\right]$$

where:

e_m = current strain of member m

ε_c = current strain of the reference member

Arold = area of the reference member before modification

Arnew = area of the reference member after resizing to be consistent with stress constraint.

In this case, because modifications are made continuously it is necessary to have a higher damping factor to ensure stability.

The rate of convergence of both these procedures (unlike the procedure of Chapter 3) does not depend only on the relative EA values but, in addition, on arriving at a constant value for the reference area. For solution efficiency the iteration parameters must be adjusted so that members converge to their final force level as soon as possible. This is normally achieved by using damping as near the critical value as possible. But this is not always possible if the number of modifications between area changes are to be kept small. Problems can also be run changing the reference member as discussed in a previous section.

The Effects of different stress constraints in Tension and Compression members on Optimum Form:

The criteria for optimum form have already been discussed in Chapter 3. The strain bounds quoted by Michell (194) were given as $+\varepsilon$ and $-\varepsilon$ for tension and compression members respectively but Hemp (187) quoted more general strain constraints which he derived when considering linear programming techniques (see section entitled "Linear Programming Techniques" in Chapter 2 and Chapter 3). These were $+\sigma\varepsilon/\sigma_T$ for tension members and $-\sigma\varepsilon/\sigma_c$ for compression members.

where:

σ_T = permissible tensile stress

σ_c = permissible compressive stress

σ = $(\sigma_c + \sigma_T)/2$

ε = a virtual strain parameter.

The significance of these bounds in relation to the Michell proof are illustrated below:

Suppose the candidate frames for optimum form are subject to an arbitrary deformation such that no element of space suffers an extension greater than $\varepsilon_T \delta L$ or contraction greater than $-\varepsilon_c \delta L$ where ε_T and $-\varepsilon_c$ are the limits of strain.

The virtual work done by the applied load is equal to the sum of the increases of the applied energy stored in the bars viz:

$$\sum eLf = dW$$

If a particular frame, M, can be found such that for any member e equals ε_T or $-\varepsilon_c$ and the signs F and e correspond, then:

$$-\varepsilon_c \Sigma L_c F_c + \varepsilon_T \Sigma L_T F_T = \delta W$$

The partial derivatives of the virtual work with respect to the strain bounds are as follows:

$$\frac{\partial \delta W}{\partial \varepsilon_c} = -\Sigma L_c F_c$$

$$\frac{\partial \delta W}{\partial \varepsilon_T} = \Sigma L_T F_T$$

Which for a fully stressed design gives:

$$\frac{\partial \delta W}{\partial \varepsilon_T} = A\Sigma L_T \sigma_T$$

$$\frac{\partial \delta W}{\partial \varepsilon_c} = -A\Sigma L_c \sigma_c$$

This indicates that the strain constraints vary inversely with respect to the stress constraints. If the strain constraints are to be dimensionless and virtual then they can be expressed in terms of a stress parameter σ and a virtual strain constant ε:

$$\varepsilon_T = \sigma\varepsilon/\sigma_T \qquad (4.2)$$

and $$\varepsilon_c = -\sigma\varepsilon/\sigma_c$$

A convenient quantity for σ, as suggested by Hemp (187), is the mean of the stress limits, i.e.: $(\sigma_c + \sigma_T)/2$. In comparison with (4.2) the real strains in a fully stressed design are for tension members σ_T/E and for compression members $-\sigma_c/E$.

If the form derived using the strain criteria of equations (4.2) is determinate then, for a single loading condition, it is always possible to find a set of real member areas which ensure that the structure is fully stressed. For indeterminate structures, however, this can only be so when the virtual strains can be proportional to the real strains (assuming Young's modulus is the same for all members).

In this case the stress limits σ_T and σ_C must be equal, giving also that $\varepsilon_T = \varepsilon_C$.

The above discussion indicates that a design derived using the modified virtual strain criterial of (4.2) may differ from a form derived using the Fully Stressed Design technique.

The Dynamic Relaxation formfinding procedure can be generalised to account for the modified virtual strain criteria as follows:

Firstly the strain parameter ε can be calculated from the current strain value in the reference member as follows:

If the reference member is in tension:

$$\varepsilon = \frac{\varepsilon_{ref}\ \sigma_T}{\sigma}$$

but if the reference member is in compression:

$$\varepsilon = \frac{\varepsilon_{ref}\ \sigma_c}{\sigma}$$

The function for modifying the member areas to comply with the strain criteria is as follows:

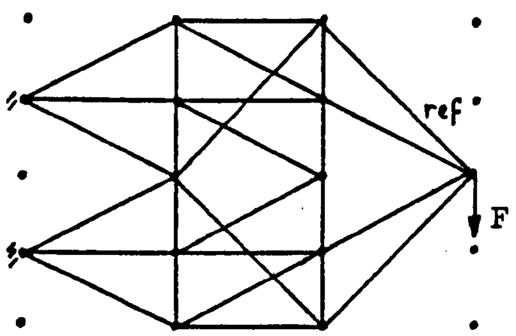
For a tension member:

$$A_{C+1} = \frac{A_C}{2}\left[1 + \left|\frac{e}{\sigma\varepsilon/\sigma_C}\right|\right]$$

And for a compression member:

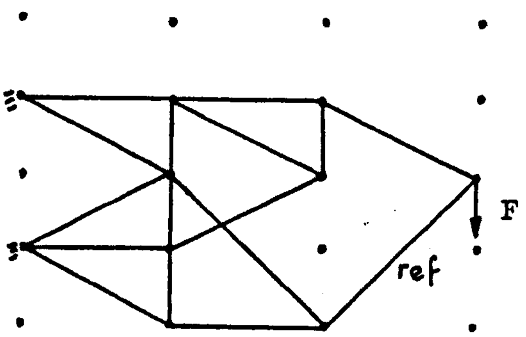
$$A_{c+1} = \frac{A_c}{2}\left[1 + \left| \frac{e}{\sigma\varepsilon/\sigma_c} \right| \right]$$

To investigate the use of differing stress constraints in tension and compression the 3 Bay fully connected ground structure was chosen for more detailed consideration without using symmetry. Exactly the same conditions were used as in Chapter 3 but the problem was now run with $\sigma_c = 0.01\ \text{N/m}^2$ and $\sigma_T = 0.1\ \text{N/m}^2$. The form previously derived with $\sigma_T = \sigma_c = 0.1\ \text{N/m}^2$ is shown here for comparison:



$\sigma_T = \sigma_c = 0.1\text{N/m}^2$

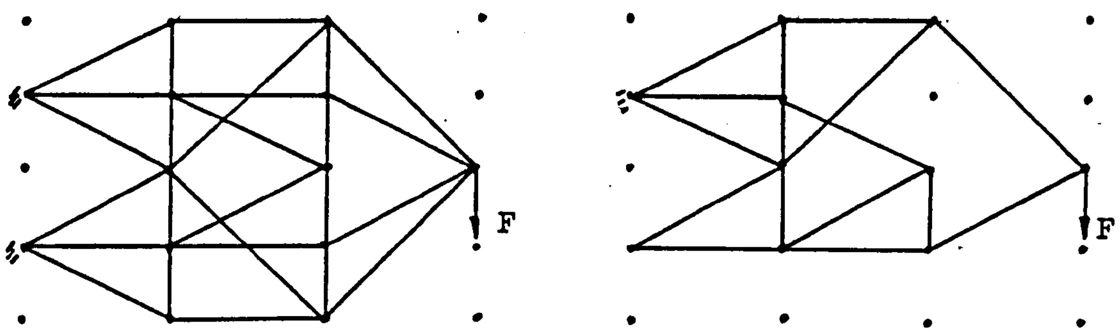
Volume = 290 m^3



$\sigma_c = 0.01\text{N/m}^2 \quad \sigma_T = 0.1\ \text{N/m}^2$

Volume = 1550 m^3

The problem was also designed using the fully stressed design stress ratio technique, which corresponds to a strain field of σ_T/E and $-\sigma_C/E$, and gave the result shown in the figure below:



$\sigma_T = \sigma_C = 0.1\ N/m^2$
Volume = 290 m^3

$\sigma_C = 0.01N/m^2 \quad \sigma_T = 0.1N/m^2$
Volume = 1640 m^3

As was expected the forms are inverse because the criteria are inverse. A dual linear program using the formulation of Reinschmidt and Russell (206) gave the same form as the Dynamic Relaxation procedure with a volume of 1550 m^3. This form was subsequently resized by Fully Stressed Design to ensure compatibility and safety under the applied loading, and, as expected with a determinate form the volume remained at 1550 m^3. This shows a saving of 5.8% compared with the form directly derived using fully stressed design.

Conclusions:

The following can be concluded about the method outlined in Chapter 3:

a) The number of modification stages before the optimum form becomes apparent independent of the damping factor and the number of time intervals between modification stages.

b) The choice of reference member, damping factor, number of intervals between modifications, area modification equation, and conditioning masses all affect the stability of the process.

c) If the stress constraints in tension and compression are not equal then it is possible for a least weight form structure to be derived using the modified strain criteria with D.R. procedure. Subsequent resizing using the Fully Stressed Design technique can be used to ensure safety.

In the last two chapters a method for determining the optimum form for a structure subject to a dominant design load has been investigated and further developed. Most structures are subject to many design criteria. For example; multiple loading cases, different stress constraints in tension and compression, maximum and minimum member area sizes and deflection constraints. In the following Chapter a method is presented in which the form derived using the dominant loading case is sized to efficiently comply with all of these additional design constraints.

CHAPTER 5: THE SIZING OF MODULAR SPACE STRUCTURES OF FIXED TOPOLOGY

Fixed Topology Member Area Sizing - Concurrent Loading Vectors and Damping - The Effects of Initial Member Area Sizes - Maximum and Minimum Member Area Sizes - Further Comparison with Non-Linear Programming Solutions - Deflection Constraints - Further Comparisons with Non-Linear Programming Solutions - Conclusions.

Summary:

In this chapter the Dynamic Relaxation method is developed with the use of concurrent vectors, to cater for multiple load cases simultaneously. The effects of kinetic damping, viscous damping and initial member areas are studied in relation to a design procedure based on the fully stressed criteria. This procedure is developed to cater for both maximum and minimum area sizes and deflection constraints. Example structures are designed and compared with non-linear programming solutions.

Fixed Topology Member Area Sizing:

If a structure of fixed topology is to be sized subject to multiple loading cases and stress constraints only, one of the most attractive computer design methods is the Fully Stressed Design technique. This method has immediate appeal to engineers because it is an iterative procedure of size, reanalysis and resize type which is akin to the trial and correct procedures of manual design. The method assumes that an optimum design is one in which all the member stress constraints for at least one of the loading cases bound the design. For multiply loaded structures this is not always the case but the method does lead to safe designs which are often close in weight to optimum value. The simplicity of the method together with a normally high speed of convergence ensures that this method is usually prefered to non-linear programming techniques. If deflection constraints are to be considered then the method developed by Razani (151) can be used to adjust the final member area sizes so that the deflections are within the constraints. The method essentially consists of calculating the sensitivity of the constrained nodes with respect to each member area. Areas are then reallocated by inspection or by using an optimal search method. Allocation is difficult because the sensitivities are not constant and if the process is to be accurate second derivatives must be calculated. This method therefore approaches a safe design from the unsafe side using an approximate optimisation procedure. For most problems with stress and deflection constraints it is usually considered better to approach an optimum design from the safe side using a non-linear programming algorithm. Unfortunately most non-linear programming algorithms do not always converge to global optimum solutions, particularly if the problem is large or the design space fragmented or excessively constrained. Siddall (27) suggests starting the solution from different feasible points or using several algorithms on the problem and then adopting the least weight design. Unfortunately these algorithms use large amounts of core store and often take considerable time to converge. This makes several solutions impractical for large one off designs. It appears that non-linear programming techniques are best confined to component optimisation where the number of variables is small and the cost of the design can be spread over many similar components.

The methods of designing fixed topology structures are discussed in more detail in Appendix III. One of the most popular types of non-linear algorithm is the steepest descent -alternate mode algorithm. A particular form of this algorithm which caters for maximum and minimum area sizes and deflection constraints was developed by Elliott (50). The problem of design is further complicated if a complete range of area sizes are not available. It is usually assumed that if nearest reasonable sizes are used to replace the optimum sizes then this will not invalidate the design procedure. Elliott suggested that this assumption was valid for most practical purposes.

In contrast to the methods discussed above a method is presented in the following sections which is based on the criteria of Fully Stressed Design utilising the Dynamic Relaxation analysis procedure and continuously modifying member areas. An approximate method catering for maximum and minimum member sizes and deflection constraints is also presented. Elliott's algorithm has been programmed to enable comparisons to be made with solutions derived from the method proposed in this Chapter. It is not suggested that the solutions from the non-linear program are always optimum but they do represent typical solutions such as would be expected from a non-linear algorithm.

Concurrent Loading Vectors and Damping:

For the type of design problem considered here, the derived stiffness of the members depend on the level of load in all loading cases and concurrent vectors for each case must therefore be considered simultaneously. Each of these load cases will require additional vectors for the current co-ordinates, nodal residuals and velocities. The advantages of using an explicit analysis become more apparent when maximum and minimum member area sizes and deflection constraints are considered.

Before any modifications are made it is important that the members are significantly stressed so that their areas are not all cut to very small values. In an attempt to derive a damping method which would ensure that the members quickly achieved significant stress values and that convergence was steady, two types of damping were considered. The use of a trial run to provide an estimated critical viscous damping factor and a subsequent damped analysis is likely to be slow in most cases. The method of Kinetic Damping employed by Cundall (246) was therefore investigated. In this method the analysis is not viscously damped but every time the structure reaches a kinetic energy peak the velocities are set to zero. In this way all modes are eventually damped out. This method was dismissed for this problem because considerations of maximum and minimum area sizes and deflection constraints involve plasticity effects. In these cases it was thought that the solution path could become critical if convergence was to be ensured. The kinetic energy method can often result in an unsteady and non monotonic convergence which is characterised by significant jumps in the design parameters. An alternative procedure using kinetic damping and load increments could, however, have been devised to ensure that the correct solution path is followed.

It was therefore decided to use a combined kinetic and viscous damping scheme in which the analysis was initially undamped and each loading case was run only until it reached a kinetic energy peak. When all loading cases reached a peak the nodal velocities were set to zero and the iteration restarted using a viscous damping factor based on the number of iterations required to reach the first energy peak. The damping factor per unit mass was then set equal to:

$$\frac{C_p'}{\Delta t} = 4\Pi f = \frac{\Pi}{Np.\Delta t} \qquad (5.1)$$

where:

Np = the number of iterations to the first kinetic energy peak for any loading case (assumed ¼ of of the fundamental period).

In this way the damping factor is always such that the analyses are heavily damped, unlikely to diverge, and convergence is steady. This method differs from that used by Rushton (257,258) for the analysis of plates for a single load case in that his damping factor was based on the first true kinetic energy peak and his analysis was then completely restarted using this damping factor.

The member areas are resized according to the following equation:

$$A_{c+1} = A_c \, (\sigma_m/\sigma_{pm}) \qquad (5.2)$$

where:

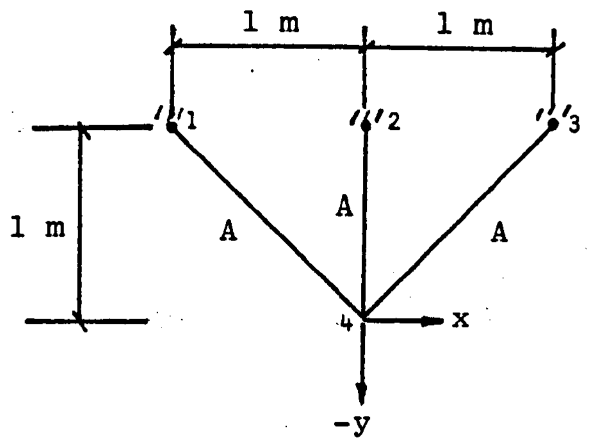
σ_m = the current stress in the member m

σ_{pm} = the tensile or compressive permissible stress for the member m.

For multiple loads the maximum value of the ratio (σ_m/σ_{pm}) under all loading cases is used to ensure safety.

The Effects of Initial Member Area Sizes:

The effects of varying the initial member sizes and factoring the damping factor, Cp', on the rate of convergence were assessed by studying the following problem. This problem was first studied by Schmit (212) using a steepest descent alternate mode algorithm and subsequently by Razani (151) using the fully stressed design method.



Load case 1:
4x = 15.0N 4y = -25.9808N
Load Case 2:
4x = -20.0N 4y = 0.0N

The analysis was run using damping factors factored 8, 4, 2, 1 and 0.5 times and a time step of 0.1 seconds. The damping factor derived using equation (5.1) was 15.7. The member areas were resized every 10 iterations after the energy peak. The permissible stresses in tension and compression were 20.0 N/m^2 and -15.0 N/m^2 respectively and the Young's

Modulus for all members was 100,000 N/m^2. The masses used in this and the following analyses were assigned as in equation (3.16). The converged solution had a volume of 2.922 m^3 with forces and areas as tabulated below:

Areas/Forces:	Member Area, m^2	Forces Load Case 1,N	Forces Load Case 2,N
Member 1:	1.072	21.29	-16.06
Member 2:	0.544	10.87	2.72
Member 3:	0.611	.08	12.22

Solution time = 0.096 CDC seconds

This is exactly the same result as Razani obtained using the F.S.D. method.

The problem was also run using different elastic moduli and initial member areas. A comparison of the effects on convergence rate is given in the following table. The effects of initial area sizes are given by quoting the member force at the iteration after the first modification stage.

Although these forces vary significantly the convergence of areas to three decimal places always occurs between 8 and 18 modification stages. The minimum being when the damping factor is $C\acute{p}$ and the initial member areas are set equal to 0.1. Generally, however, it appears that

Table showing Member Forces after first iteration after first resize
and the Number of area resizes before areas converge to 3 figures (NC)

Initial Member Areas: Elastic Modulus:	Damping Factor Cp′ * 8.0		Damping Factor Cp′ * 4.0		Damping Factor Cp′ * 2.0		Damping Factor Cp′ * 1.0		Damping Factor Cp′ * 0.5	
A = 10.0	NC = 14		NC = 9		NC = 9		NC = 9		NC = 10	
E = 1000.	6.88	-5.28	9.5	-7.2	13.9	-10.6	19.3	-14.5	24.2	-18.3
	4.75	.08	6.6	.14	9.7	.28	13.6	.5	17.1	.74
	-.75	4.02	-1.0	5.5	-1.4	8.1	-1.9	11.2	-2.3	14.14
A = 1.0	NC = 14		NC = 9		NC = 9		NC = 9		NC = 9	
E = 10000.	26.27	-20.52	35.1	-27.2	18.7	-14.6	19.3	-15.1	19.8	-15.6
	17.88	.51	29.4	.13	13.0	.9	13.5	1.2	14.0	1.7
	-3.10	15.83	-5.7	27.3	-2.0	11.8	-2.0	12.5	-1.9	13.2
A = 0.1	NC = 14		NC = 10		NC = 8		NC = 8		NC = 9	
E = 100000.	220.2	-172.9	123.8	-98.3	66.8	-54.7	20.1	-19.6	-24.1	13.26
	149.3	4.9	84.2	6.0	46.1	8.4	14.9	12.0	-14.4	16.11
	-26.5	134.0	-14.6	79.2	-7.3	48.9	-1.2	25.9	4.6	5.02
A = 0.01	NC = 18		NC = 11		NC = 9		NC = 9		NC = 13	
E = 1000000.	2159.8	-1696.6	1162.8	-925.9	547.7	-455.9	27.4	-64.1	-463.2	301.8
	1463.5	48.7	790.4	59.9	377.2	83.9	29.0	119.9	-293.3	161.3
	-260.9	1316.0	-137.9	749.3	-60.6	419.8	6.4	159.6	70.4	-76.7

the initial areas should be set to a large value to ensure a fast rate of convergence. Gnerally efficiency is better if the areas are initially set to their maximum value. It was therefore decided to set all initial areas to their maximum value and to use C_p' as the damping factor in all subsequent analyses.

It is noticeable that for the lower elastic modulus values the areas did not converge to exact values due to the effects of geometric non-linearity which are automatically accounted for in the D.R. analysis.

Maximum and Minimum Member Area Sizes:

The work reported in the literature using maximum and minimum member sizes is confined to non-linear programming and the possibility of introducing these constraints into fully stressed design techniques does not appear to have been investigated.

In this section an intuitive method based on the D.R. procedure is developed. The method was developed after detailed consideration of the problem investigated in the last section. The problem was first run without area constraints using the F.S.D. technique and the Elliott Non-Linear Programming Algorithm.

The solutions for each method were as follows:

	F.S.D. Solution			N.L.P. Solution		
Areas/Forces:	Areas m^2	Load Case 1.N	Load Case 2.N	Areas m^2	Load Case 1.N	Load Case 2.N
Member 1:	1.071	21.29	-16.06	1.070	21.28	-16.05
Member 2:	0.544	10.87	2.72	.544	10.89	2.69
Member 3:	0.611	0.075	12.22	.614	.0671	12.33
	Volume = 2.922 m^3			Volume = 2.925 m^3 D = 1000. EINF = 1.0 Solution Time = 0.259 CDC seconds		

These analyses agree with the work of Razani, Schmit and the D.R. solution and confirms that the F.S.D. solution for this problem is an optimum.

It can easily be seen that if a constraint on the maximum area is to be set below 1.071 m^2 the most likely member to be overstressed is member 1, where the compression force in load case 2 will be most critical. It was first hoped that the relative deflections of the nodes of member 1 could be reduced to reduce this compression force. This would in effect correspond to pretensioning member 1 by shortening it. However, for multiply loaded structures, pretension effects would obviously be best arranged differently for each loading case. The best distribution of prestress can only be determined by an optimal search method.

It was decided to make the analysis elasto-plastic by setting the maximum force in any member which is at its maximum size equal to the maximum area multiplied by the permissible stress. This can be accomplished efficiently in the explicit D.R. method by setting the maximum force sustained by any member to $\sigma_T . A_{max}$ in tension or $\sigma_c \cdot A_{max}$ in compression when calculating the residuals. The results for this design using a maximum area of $.95m^2$ are tabulated with the results of a subsequent elastic analysis for the structure in the table below:

	D.R. Elasto-Plastic Solution			Elastic Analysis	
Areas/Forces	Area m^2	Load Case 1.N.	Load Case 2.N.	Load Case 1.N.	Load Case 2.N.
Member 1:	0.950	19.000(P)	-14.25(P)	19.70	-15.32
Member 2:	0.705	14.108	.15	13.11	1.661
Member 3:	0.702	-2.208	14.034	-1.509	12.92
	Volume = 3.041 m^3 Solution Time = .081 CDC seconds				

The subsequent elastic analysis shows that the areas of member 2 and 3 have not been increased enough to ensure that member 1 is not overstressed.

It was noted that if the plastic force carried by the members of maximum size had been reduced then even more load would have been transferred to the elastic members producing an increase in their member areas on resizing. The extent to which the plastic force should be reduced to avoid overstress in the elastic analysis can be determined by running an elastic analysis simultaneously with the elasto-plastic analysis. The plastic force in members of maximum area was therefore defined as below:

$$\text{Plastic Force} = \text{Area}_{max.} \cdot \text{Permissible Stress} . AC_{mic} \qquad (5.3)$$

where:

AC_{mic} = a factor for member m, under the action of load case i at modification stage C.

This factor has an initial value of 1.0, must always remain less than or equal to 1.0 and is always reset equal to 1.0 if the area reduces to a value less than the maximum permitted area.

The value of the factor is periodically amended throughout the procedure to ensure that the plastic members are not overstressed in the elastic case. The AC factor can be updated in the following way:

$$AC_{mic+1} = AC_{mic} \cdot \text{Abs} \left\{ \frac{\text{Permissible Stress of member m}}{\text{Elastic Stress of member m under load case i}} \right\} \qquad (5.4)$$

Areas are modified using equation (5.2) considering member stresses under the action of all loading cases in both the elastic and elasto-plastic analyses. At convergence, all the areas and AC factors must be constant.

The above technique ensures that any members which are overstressed and are of maximum area are relieved of their overstress by increasing other members. In some cases where alternative load paths are not available, or strain compatibility requirements might prohibit solution, convergence may not be possible. But for the sizing of three dimensional highly redundant space frames this should not generally be a problem.

The technique was applied to the above problem, modifying member areas and the AC factors after every 10 time intervals. The results for this are tabulated below:

	Modified D.R. Elastic Plastic Design			Elastic Analysis	
Areas/Forces:	Areas m^2	Load Case 1,N	Load Case 2,N	Load Case 1,N	Load Case 2,N
Member 1:	0.950	19.00(P)	-14.25(P)	19.00	-14.25
Member 2:	0.8013	16.02	-6.11	14.10	.159
Member 3:	0.9232	-3.56	18.46	-2.21	14.032

Volume = 3.451 m^3

Solution Time = .354 CDC seconds

A comparative solution derived using the non-linear programming algorithm is as given below:

N.L.P. Solution			
Areas/Forces:	Areas ,m^2	Load Case 1,N	Load Case 2,N
Member 1:	.950	19.00	-14.24
Member 2:	.8014	14.11	.136
Member 3:	.927	-2.21	14.05

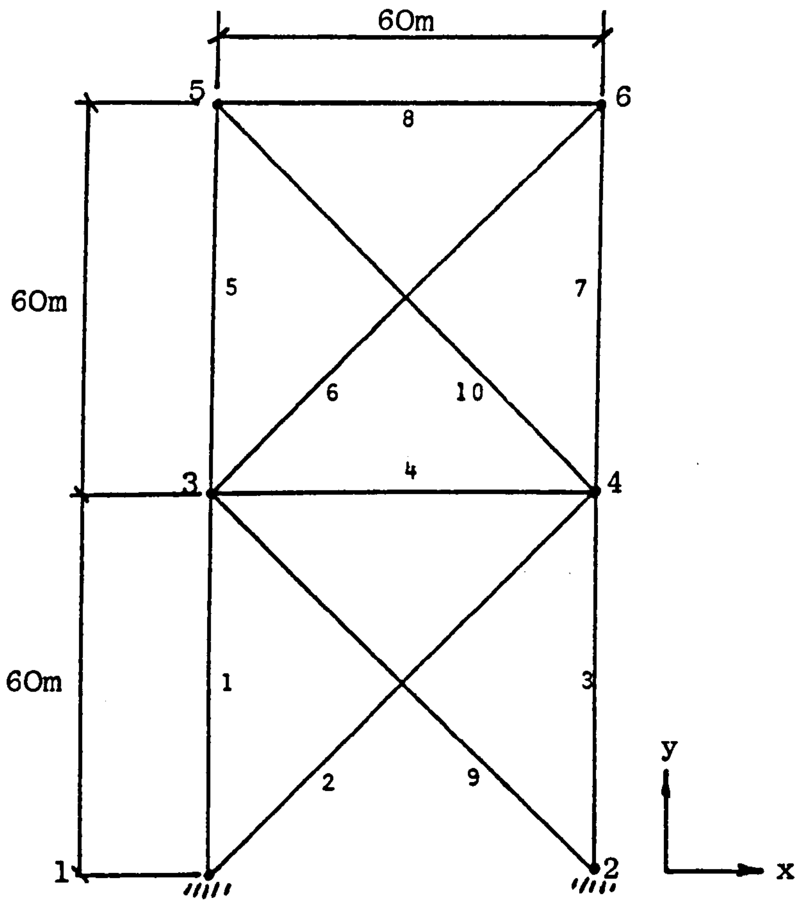
Volume = 3.455 m^3

Solution Time 1 .970 CDC seconds
(D = 1000. EINF = 1.0)

It is interesting to note that despite the large change in volume due to the constrain on maximum member size the results for the two methods are the same. The D.R. method, however, is at least twice as fast as the N.L.P. algorithm.

Further Comparison with Non-Linear Programming Solutions:

The structure considered for these comparisons is shown below. The permissible stresses used were σ_T = 21.6 N/m^2 and σ_c = -10.0 N/m^2 and the Young's modulus for all members was 10000 N/m^2.



The structure was subjected to three different sets of loading conditions and various constraints on the maximum and minimum member areas were applied. It was noted that if maximum and minimum size contraints were active and a solution was possible the D.R. iteration oscillated about a mean point. However, if a solution was not possible without at least slightly overstressing one of the members at a maximum area size then convergence to the solution was rapid. This oscillating instability resulted because the AC factors were modified at the same time as the member areas. This means that the plasticity effects were readjusted before the effects of the last adjustment had been taken into account by area modifications. If these area modifications had been made they would have resulted in changes in the stress levels in both the elastic and elasto-plastic analyses. It was therefore decided to adjust the areas after every 10 time intervals and the AC factors after every 50 time intervals. To ensure that the plasticity effects were not too rapid the AC factors were updated using the following expression:

$$AC_{mic+1} = \frac{AC_{mic}}{2.0}\left[1 + \text{Abs}\left\{\frac{\text{Permissible Stress of member m}}{\text{Elastic Stress of member m under load case i}}\right\}\right] \tag{5.5}$$

Masses were assigned using the equation (3.16) factored by 2.0. An added mass component of .005 was also used.

The structures were designed using both the D.R. procedure and the non-linear programming algorithm.

The first set of loading conditions was as follows:

Load Case 1:

3x = 5.0 N 3y = 0.0 N

6x = -10.0 N 6y = 0.0 N

Load Case 2:

4x = 5.0 N 4y = 0.0 N

5x = -6.0 N 5y = 4.0 N

6y = 0.0 N 6y = -12.0 N.

The D.R. solutions with maximum member area sizes of 3.0 m^2 and 1.3 m^2 are tabulated below:

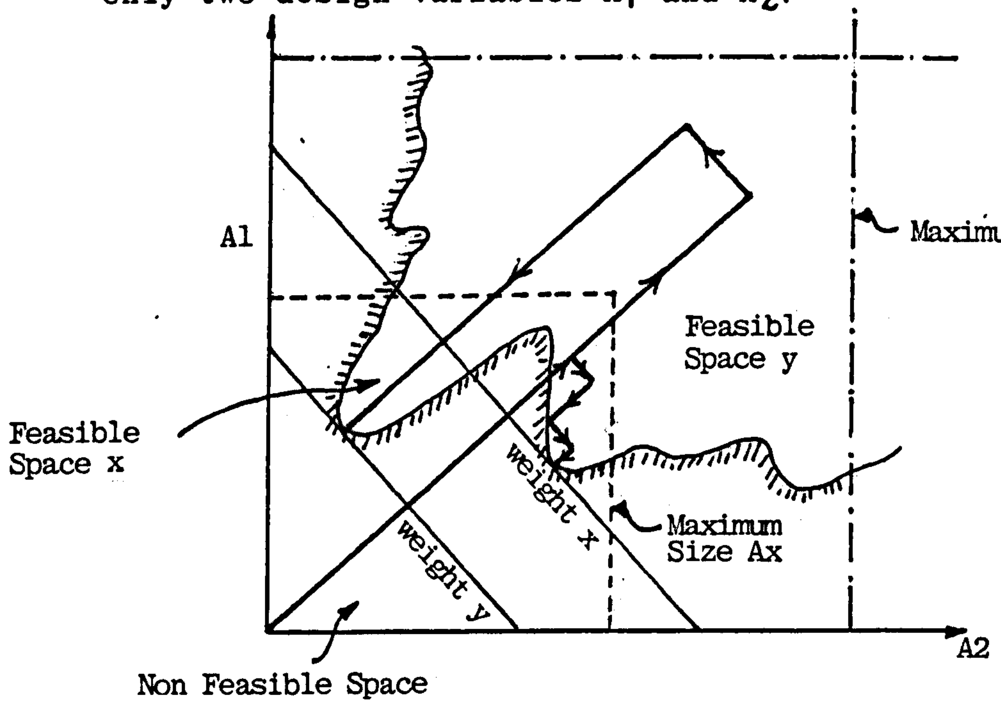
	D. R. Solution			D. R. Solution		
Max/Min Areas	3.0 m^2 0.0 m^2			1.3 m^2 0.0 m^2		
Areas/ Forces	Areas, m^2	Load Case 1,N	Load Case 2,N	Areas, m^2	Load Case 1,N	Load Case 2,N
Member 1:	1.161	-11.61	-10.33	1.155	-11.50	-10.32
2:	.484	-4.84	-1.00	0.505	-5.04	-1.03
3:	.623	13.46	-5.27	0.632	13.62	-5.25
4:	.245	2.86	5.29	0.234	2.73	5.06
5:	.443	-0.530	-4.43	0.469	-.805	-4.69
6:	1.336	-13.36	-7.91	1.300	-13.00	-7.55
7:	.641	9.50	-6.41	0.667	9.25	-6.66
8:	.258	-0.530	5.57	0.246	-.79	5.32
9:	.104	2.24	0.43	0.096	2.07	0.405
10:	0.035	0.75	0.61	0.056	1.13	0.970
	Volume = 368.51 m^3 Solution Time = 1.171 CDC seconds (Changes every 10 iterations)			Volume = 370.01 m^3 Solution Time = 12.070 CDC seconds		

The first D.R. solution is the same as a solution resulting from a F.S.D. stress ratio program. However, the second run with the maximum areas constrained at 1.3 m^2 shows that member 6 reaches its stress limit and other members are increased to relieve the overstress.

The results of two comparative runs using the N.L.P. algorithm are tabulated below:

	N.L.P. Solution			N.L.P. Solution		
Max/Min Sizes	3.0 m^2 0.0 m^2			1.3 m^2 0.0 m^2		
Areas/ Forces	Areas, m^2	Load Case 1,N	Load Case 2,N	Areas, m^2	Load Case 1,N	Load Case 2,N
Member 1:	1.157	-11.57	-10.31	1.030	-10.30	-9.96
2:	.487	-4.85	-.98	.666	-6.65	-1.48
3:	.622	13.43	-5.31	.699	14.70	-4.96
4:	.214	2.201	4.601	.583	1.32	4.30
5:	.510	-1.23	-5.09	.665	13.39	-5.74
6:	1.240	-12.40	-6.94	.935	-9.35	-6.02
7:	.710	8.77	-7.09	.927	6.611	-7.74
8:	.261	-1.23	4.907	.653	-3.39	4.26
9:	.106	2.217	.432	.025	.42	-0.06
10:	.098	1.74	1.546	.502	4.79	2.46
	Volume = 372.23 m^3 (D = 1000. EINF = 1.0) Solution Time = 11.775 CDC seconds			Volume = 453.97 m^3 (D = 1000. EINF = 1.0) Solution Time = 9.018 CDC seconds		

The first of these two runs derives a solution of similar volume to the D.R.solutions although the member areas were all less than 1.3 m^2 . The solution derived using a maximum area constraint of 1.3 m^2 was not similar although the areas were all less than 1.3 m^2 and were generally nearer the average size. With this constraint the algorithm arrived at a non-optimum solution from which the algorithm failed to move away. The changes in volume and area sizes became small and the algorithm was terminated. This is probably due to the area of the search being cut by the maximum area constraint. A two dimensional representation of this type of problem is illustrated below considering only two design variables A_1 and A_2.



When the D.R. solution with the 1.3 m^2 area constraint is compared with the first non-linear programming solution it is shown to have a slightly lower volume and a comparable solution time.

The second set of loading conditions was as follows:

Load Case 1:

$5x = 0.0$ N $5y = -10.0$ N

$6x = 0.0$ N $6y = -10.0$ N

Load Case 2:

3x = 10.0 N   3y = 0.0 N

5x = 10.0 N   5y = 0.0 N

D.R. solutions were run using a maximum area of 3.0 m^2 firstly with a minimum of 0.0 m^2 and secondly with a minimum of 0.5 m^2. The results are tabulated below:

	D. R. Solution			D. R. Solution		
Max/Min Sizes	3.0 m^2 0.0 m^2			3.0 m^2 0.5 m^2		
Areas/ Forces	Areas, m^2	Load Case 1,N	Load Case 2,N	Areas, m^2	Load Case 1,N	Load Case 2,N
Member 1:	1.0	-10.0	20.0	1.121	-8.84	24.21
2:	0.655	0.0	14.142	0.500	-1.66	8.29
3:	2.0	-10.0	-20.0	1.587	-8.82	-15.87
4:	0.0	0.0	0.0	0.500	2.57	0.81
5:	1.0	-10.0	10.0	0.861	-8.61	6.71
6:	0.0	0.0	0.0	0.500	-1.98	4.71
7:	1.0	-10.0	0.0	0.860	-8.60	-3.33
8:	0.0	0.0	0.0	0.500	1.39	-3.33
9:	1.414	0.0	-14.142	2.000	-1.65	19.99
10:	1.414	0.0	-14.142	0.942	-1.96	9.42
	Volume = 595.56 m^3 Solution Time = 0.585 CDC Seconds			Volume = 660.21 m^3 Solution Time = 0.580 CDC Seconds		

The first solution was the same as a F.S.D. solution and is a mechanism. In the second solution the process retains all members and fixes the structure topology by always setting members which may become less than the minimum area equal to 0.5 m^2.

A further series of D.R. solutions was derived with a minimum area of 0.5 m^2 and maximum areas of 1.8, 1.9 and 2.0 m^2. The results are tabulated below:

	D.R. Solution			D.R. Solution			D.R. Solution		
Max/Min Sizes	1.8 m^2 0.5 m^2			1.9 m^2 0.5 m^2			2.0 m^2 0.5 m^2		
Areas/ Forces	Areas m^2	Load Case 1,N	Load Case 2,N	Areas m^2	Load Case 1,N	Load Case 2,N	Areas m^2	Load Case 1,N	Load Case 2,N
Member 1:	1.056	-8.71	22.80	1.088	-8.84	23.51	1.121	-8.84	24.21
2:	0.659	-1.85	10.28	0.559	-1.66	9.28	0.500	-1.66	8.29
3:	1.800	-8.68	-17.28	1.855	-8.81	-16.57	1.587	-8.82	-15.87
4:	0.500	2.68	-0.37	0.500	2.59	0.29	0.500	2.57	0.81
5:	0.862	-8.62	6.95	0.859	-8.59	6.89	0.861	-8.61	6.71
6:	0.500	-1.96	4.38	0.500	-2.006	4.46	0.500	-1.98	4.71
7:	0.861	-8.61	-3.10	0.858	-8.58	-3.15	0.860	-8.60	-3.33
8:	0.500	1.38	-3.09	0.500	1.41	-3.15	0.500	1.39	-3.33
9:	1.800	-1.84	-18.01	1.900	-1.65	-19.00	1.999	1.65	-19.99
10:	0.976	-1.9	-9.76	1.005	-1.99	-9.68	0.942	-1.96	9.42
	Volume=668.62 m^3 Solution Time= 0.756 CDC Seconds			Volume=676.07 m^3 Solution Time= 2.027 CDC Seconds			Volume=660.21 m^3 Solution Time= 1.524 CDC Seconds		

A solution was not possible for the first set of constraints because the further required reduction in the AC factor for member 9, load case 2, did not result in any further load being transferred to any of the elastic members. A comparative non-linear program was run which did not yield a solution because a feasible starting point could not be found. This program took 0.107 CDC seconds. In contrast the D.R. solution took 0.756 CDC seconds but did provide a solution in which the only overstressed member was member 9 (0.01%). This design, which for most practical purposes is acceptable, indicated why a feasible design was not possible.

A D.R. solution with a maximum area size of 1.9 m^2 was possible. In this case member 9 was at its maximum size. The D.R. solution with the maximum member area size set at 2.0 m^2 had no members at the maximum size and therefore no plasticity effects were considered. A comparative N.L.P. solution was run with maximum and minimum member area sizes set at 2.0 m^2 and 0.5 m^2. The results are tabulated on the following page.

This solution is not a global optimum. It indicates, however, that a solution can be found in which 1.9 m^2 and 2.0 m^2 maximum area constraints are not active. The weight of the structure is respectively 12.4% and 15.1% greater than the D.R. solutions. In addition the N.L.P. solution time was considerably greater than the D.R. solution times.

N.L. Program Solution			
Max/Min Areas	$2.0 m^2$	$0.5 m^2$	
Areas/ Forces	Areas m^2	Load Case 1,N	Load Case 2,N
Member 1:	1.096	-8.40	21.9
2:	.815	-2.26	11.38
3:	1.804	-8.40	-18.04
4:	.797	3.26	-16.95
5:	1.004	-8.34	6.35
6:	.771	-2.35	5.16
7:	1.002	-8.34	-3.65
8:	.844	1.66	-3.65
9:	1.691	-2.26	-16.91
10:	1.046	-2.35	-8.98
Volume = 759.61 m^3 D = 1000.0 EINF = 1.0 Solution Time = 9.062 CDC seconds			

The third set of loading of conditions was as follows:

Load Case 1:

4x = -5.0 N 4y = 0.0 N

5x = 0.0 N 5y =-4.0 N

6x =-10.0 N 6y =-4.0 N

Load Case 2:

3x =	-5.0 N	3y =	0.0 N
4x =	5.0 N	4y =	0.0 N
5x =	10.0 N	5y =	0.0 N
6x =	10.0 N	6y =	-10.0 N

A comparison of F.S.D., D.R. and N.L.P. solutions for non active maximum area size constraints are as shown in the table below:

	F.S.D. Solution			D.R. Solution			N.L. Program		
Max/Min Sizes	-	$0.0m^2$		$10.0m^2$	$0.0m^2$		$10.0m^2$	$0.0m^2$	
Areas Forces/	Areas m^2	Forces Case 1	Load Case 2	Areas m^2	Load Case 1	Load Case 2	Areas m^2	Load Case 1	Load Case 2
Member 1:	1.471	-14.702	20.121	1.400	-14.000	20.049	2.189	-20.827	27.685
2:	2.021	-20.220	28.113	2.126	-21.262	28.415	1.156	-11.558	17.416
3:	4.988	20.298	-49.879	5.001	21.056	-50.094	4.232	14.172	-42.315
4:	0.0	0.0	0.0	0.054	1.162	.766	.212	.610	1.839
5:	1.329	-13.298	14.879	1.288	-12.882	15.931	.687	-6.562	9.154
6:	.335	-.993	7.243	.271	-1.580	5.860	1.139	-10.519	15.338
7:	1.512	-3.298	-15.121	1.409	-2.861	-14.093	2.178	3.438	-20.846
8:	.929	-9.298	4.879	.890	-8.898	5.923	.300	-2.562	-.846
9:	.046	.993	-.172	0.000	0.000	0.000	1.55	9.655	-10.868
10:	2.104	13.149	-21.042	2.246	12.571	-22.463	1.496	-3.623	-12.964
	Volume = 996.26 m^3			Volume = 997.07 m^3 Solution Time = 2.514 CDC Seconds			Volume = 1007.5 m^3 D=1000 EINF=1.0 Solution Time = 3.255 CDC Seconds		

Member 9 has an area of 0.046 m^2 in the F.S.D. solution and zero area in the D.R. solution. Although these solutions are therefore of different form their volumes are almost identical. The N.L.P. solution is once again not a global optimum solution and requires a longer solution time than the D.R. method.

Solutions were also derived with maximum and minimum member area constraints of 4.2 m^2 and 0.5 m^2 . These are tabulated below:

	N.L. Program			D.R. Solution		
Max/Min Sizes	4.2 m^2 0.5 m^2			4.2 m^2 0.5 m^2		
Areas/ Forces	Areas m^2	Load Case 1.N	Load Case 2.N	Areas m^2	Load Case 1.N	Load Case 2.N
Member 1:	2.545	-23.82	33.54	2.0195	-20.15	27.79
2:	.732	-7.32	9.13	1.245	-12.51	17.37
3:	3.646	11.18	-3.65	4.200	14.85	-42.23
4:	1.505	-2.56	7.87	.500	.67	2.68
5:	1.697	-6.74	9.33	.715	-7.17	10.01
6:	1.922	-10.27	15.09	.969	-9.67	14.24
7:	2.434	3.26	-20.67	2.015	2.86	-19.99
8:	1.223	-2.74	-.67	.500	-3.15	-.03
9:	2.315	13.89	-19.15	1.159	8.67	-10.94
10:	1.882	3.87	-13.19	1.406	4.47	-14.10
	Volume = 1364.31 m^3 D=1000. EINF= 1.0 Solution Time = 4.687 CDC Seconds			Volume = 1001.6 m^3 Solution Time = 3.035 CDC Seconds		

A D.R. solution is not possible because the A.C. factors are still increasing but the members are of constant area indicating that the excessive force in member 3 cannot be transferred to other members. A N.L.P. solution in which the maximum area constraint is not active is possible.

The maximum member area constraint was relaxed to 4.3 m^2 and a D.R. solution derived in which the maximum member area constraint was active for member 3. The maximum area constraint was further relaxed to 4.5 m^2 and a N.L.P. solution derived in which the maximum member area was 4.080 m^2. This solution has a lower volume than the N.L.P solution run using a maximum member area constraint of 4.2 m . This is because the constraint of 4.5 m^2 allowed a greater area of search. However, the D.R. solution is lighter than the N.L.P. solutions and the solution time is not significantly longer.

The results for these solutions are tabulated on the following page.

	N.L.P. Solution			D.R. Solution		
Max/Min Areas	$4.5\ m^2$	$0.5\ m^2$		$4.3\ m^2$	$0.5\ m^2$	
Areas/ Forces	Areas m^2	Load Case 1,N	Load Case 2,N	Areas m^2	Load Case 1,N	Load Case 2,N
Member 1:	2.127	-21.21	29.20	1.968	-19.68	27.14
2:	1.105	-11.01	15.27	1.323	-13.23	18.36
3:	4.080	13.79	-40.80	4.300	15.40	-43.00
4:	0.504	.165	3.25	0.500	0.98	2.308
5:	.882	-6.63	9.05	0.738	-7.38	10.37
6:	1.257	-10.43	15.49	0.939	-9.39	13.76
7:	2.100	3.37	-20.96	1.972	2.66	-19.68
8:	0.500	-2.63	-.95	0.500	-3.37	.339
9:	1.491	10.20	-13.02	1.032	8.00	-10.01
10:	1.289	3.72	-12.79	1.460	4.78	-14.60
	Volume = 1047.98 m^3 Solution Time = 6.814 CDC Seconds			Volume = 1002.06 m^3 Solution Time = 7.275 CDC Seconds		

Deflection Constraints:

In addition to stress constraints and restrictions on maximum and minimum area sizes many structures are designed so that deflections at certain or all nodes are constrained.

The method for designing structures accounting for deflection constraints presented in this section was developed by further consideration of the three bar truss problem. It was noted that if the deflection of a node was excessive this could be reducted by increasing the nodal stiffness in the appropriate direction. The extent of the increase in stiffness can be determined by applying a fictitious force in the elasto-plastic analysis which, on resizing, will result in an appropriate increase in stiffness. The magnitude of the fictitious force can be estimated from the current values of nodal elastic stiffness and deflection in the elastic analysis.

The fictitious forces for node i, load case l, in the elasto-plastic analysis are defined as below:

$$FF_{ixlc+1} = FF_{ixlc} - \left(\frac{S_{ix}}{2.0}\left(DC_{ixl} - \Delta x_{il}\right)\right)_c$$

where:

S_{ix} = the stiffness of node i in the x direction. (This has been calculated already to condition the fictitious masses)

DC_{ixl} = the constrained value of node i in the x direction for load case l. (The sign of the deflection is set the same as Δx_{il})

Δx_{il} = the deflection of node i, in the x direction, under load case l, in the elastic analysis.

FF_{ixlc+1} = ficititious force at modification stage c + 1, for node i, in the x direction, under load case l.

The factor of 2.0 ensures that changes are not too rapid and that instability is avoided. All fictitious forces are initially set to zero and each time they are modified, checks are made to ensure that:

a) if the deflection is greater than 0.0 and the fictitious force is negative, the fictitious force is set equal to 0.0.

or b) if the deflection is less than 0.0 and the fictitious force is positive, the ficititious force is set equal to 0.0.

These checks ensure that if the deflection is less than the constrained value the fictitious forces are either set to zero or ultimately reduced to zero.

When the three bar truss was sized without any maximum or minimum area constraints the horizontal deflections were for load case 1, +0.00020 m and for load case 2, -0.00035 m. The results for a D.R. solution and a Non Linear Programming solution with the horizontal deflection constrainted at 0.0003 m are tabulated on the following page. With the D.R. method fictitious forces were reset every 10 iterations when the AC factors were reset and the member areas resized.

Although these results are different the weights and solution times are similar.

	D.R. Solution			N.L.P. Solution		
Max/Min Sizes	2.0 m^2 0.0 m^2			2.0 m^2 0.0 m^2		
Areas/ Forces	Areas m^2	Load Case 1,N	Load Case 2,N	Areas m^2	Load Case 1,N	Load Case 2,N
Member 1:	1.210	22.350	-15.561	1.131	21.79	-15.24
2:	0.469	9.372	2.009	0.508	10.16	1.55
3:	0.742	1.139	12.724	0.789	0.578	13.05
Deflection node 4 x:		0.00017	-0.0030		0.00019	-0.00030
	Volume = 3.230 m^3 Solution Time = .460 CDC seconds			Volume = 3.224 m^3 Solution Time = .319 CDC seconds (D=1000.0 EINF=1.0)		

The structure was redesigned constraining the deflections of node 4 in both directions to 0.0002 m. The non-linear algorithm failed to converge rapidly with a maximum area constraint of 2.0 m^2 so this was relaxed to 3.0 m^2. The results are as tabulated below:

	D.R. Solution			N.L.P. Solution		
Max/Min Size	2.0 m^2 0.0 m^2			3.0 m^2 0.0 m^2		
Areas/ Forces	Areas m^2	Load Case 1,N	Load Case 2,N	Areas m^2	Load Case 1,N	Load Case 2,N
Member 1:	1.691	26.465	-14.499	1.565	25.754	-14.401
2:	0.178	3.551	0.507	0.228	4.559	.365
3:	1.206	5.253	13.785	1.285	4.541	13.884
Deflection Node 4 x:		0.00011	-0.00020		0.00013	-.0002
Deflection Node 4 y:		-0.00020	-0.00003		-.0002	-.00002
	Volume = 4.276 m^3 Solution Time = .636 CDC seconds			Volume = 4.259 m^3 Solution Time = 0.406 CDC seconds (D=1000.0 EINF=1.0)		

These solutions indicate that if deflection constraints are made more severe, such that the members have low stresses, then the solution by D.R. for small problems takes longer. However, small problems do not necessarily give good indications of a method's relative efficiency for larger problems. Although the solutions are different there is not a significant difference in their volume.

The structure was again redesigned with the same deflection constraints but using a maximum member area of 1.5 m^2. The results are as follows:

	D.R. Solution			N.L.P. Solution		
Max/Min Size	1.5 m^2	0.0 m^2		1.5 m^2	0.0 m^2	
Areas/ Forces	Areas m^2	Load Case 1,N	Load Case 2,N	Areas m^2	Load Case 1,N	Load Case 2,N
Member 1:	1.500	25.352	-14.308	1.431	24.86	-14.179
2:	0.256	5.125	0.237	0.291	5.818	0.0519
3:	1.336	4.140	13.976	1.398	3.651	14.105
Deflection Node 4x:		0.00014	-0.0002		0.00015	0.0002
Deflection Node 4y		-0.0002	-0.00001		-0.0002	0.000002
	Volume = 4.267 m^3 Solution Time = 0.766 CDC Seconds			Volume = 4.291 m^3 Solution Time = 0.402 CDC Seconds (D=1000.0 EINF=1.0)		

These runs indicate that even if some bars are plastic in the elastic plastic analysis then the method yields a reasonable solution. The problem was again redesigned using a maximum member area of $1.4 m^2$. The non-linear program failed to find a feasible solution to enable it to start its search. The D.R. solution showed that if both members 1 and 3 were not to be overstressed in the second loading case with the deflection constraints satisfied then the member areas would have to be greater than $1.4 m^2$.

Further Comparison with Non-Linear Programming Solutions:

The two bay tower structure with the second set of loading conditions was reconsidered with deflection contraints in addition to maximum and minimum member area size restrictions.

Using a maximum area of $3.0 m^2$ and a minimum area of $0.5 m^2$ the unconstrained deflections of nodes 5 and 6 obtained from a D.R. solution were:

Load Case 1:

5x = -0.055 m 5y = -.107 m

6x = -0.039 m 6y = -.093 m

Load Case 2:

5x = .616 m 5y = 0.175 m

6x = .575 m 6y = -0.085 m

Comparative deflection constrained designs were made using the D.R. procedure and the non-linear programming algorithm. A constraint of 0.5 m on x and y deflections was applied at nodes 5 and 6. The maximum and minimum member areas were set at 3.0 m^2 and 0.5 m^2. Areas were modified after every 10 time intervals and fictitious forces and AC factors reset after every 50 time intervals. The results for the solutions are tabulated below:

	D.R. Solution			N.L.P. Solution		
Max/Min Sizes	3.0 m^2	0.5 m^2		3.0 m^2	0.5 m^2	
Areas/ Forces	Areas m^2	Load Case,1,N	Load Case,2,N	Areas m^2	Load Case,1,N	Load Case,1,N
Member 1:	1.489	-9.18	24.80	1.468	-8.775	22.373
2:	0.500	-1.18	7.43	0.904	-1.732	10.786
3:	2.114	-9.15	-15.25	1.791	-8.775	-17.627
4:	0.500	2.41	1.80	0.500	2.568	-.421
5:	0.844	-8.44	7.086	0.926	-8.657	7.206
6:	0.500	-2.227	4.166	0.519	-1.899	3.951
7:	0.843	-8.426	-2.946	0.970	-8.657	-2.794
8:	0.500	1.571	-2.944	0.500	1.343	-2.794
9:	2.498	-1.176	-20.86	1.813	-1.732	-17.498
10:	0.142	-2.214	-9.96	1.216	-1.899	-10.191
Deflection 5x		-0.0547	0.5000		-0.0374	0.5000
5y		-0.0970	0.1493		-0.0920	0.1381
6x		-0.0358	0.4643		-0.0213	0.4665
6y		-0.0860	-0.0652		-0.0629	-0.0763
	Volume = 794.42 m^3 Solution Time = 5.227 CDC Seconds			Volume = 747.04 m^3 Solution Time = 3.012 CDC Seconds (D=1000.0 EINF = 1.0)		

With this problem plasticity effects need not be considered because the maximum area is less than the maximum area constraint. The structure can be designed by modifying member areas and ficititious forces after every 10 time intervals and the solution time is then 2.773 CDC seconds. This solution time is less than the N.L.P. solution time, however, the D.R. solution is slightly heavier.

The structure was redesigned with the maximum member area reduced to $2.4m^2$. This constraint ensured that plasticity effects were considered. The results for the comparative solutions are tabulated on the following page:

The maximum area used in the N.L.P. solution was less than $2.0\ m^2$, and, although the solution was different to the D.R. solution, the volumes were within .63%. The D.R. solution time was less than the N.L.P. solution time.

Conclusions:

The volumes of solutions derived using the D.R. method were often less than the N.L.P. solution volume. This is because the N.L.P. algorithm frequently derives local optimum solutions and fails to find the globally optimum solutions. The D.R. solution times for problems with area size and stress constraints were faster than the N.L.P. times. But with the addition of deflection constraints the N.L.P. solution was generally faster. In all cases, however, solution times for the two methods were of similar order.

	D.R. Solution			N.L.P. Solution		
Max/Min Sizes.	2.4 m^2	0.5 m^2		2.4 m^2	0.5 m^2	
Areas/ Forces	Areas m^2	Load Case 1,N	Load Case 2,N	Areas m^2	Load Case 1,N	Load Case 1,N
Member 1:	1.461	-9.19	24.55	1.293	-8.511	21.552
2:	0.500	-1.17	7.59	0.996	-2.106	11.947
3:	2.205	-9.15	-15.31	1.849	-8.511	-18.448
4:	0.500	2.40	1.69	0.679	2.979	-1.775
5:	0.843	-8.43	7.09	1.096	-8.510	6.673
6:	0.500	-2.24	4.09	0.758	-2.107	4.705
7:	0.842	-8.42	-2.88	1.071	-8.510	-3.327
8:	0.500	1.58	-2.91	0.728	1.490	-3.327
9:	2.400	-1.16	-20.60	1.640	-2.106	-16.337
10:	1.447	-2.22	-9.99	1.136	-2.107	-9.437
Deflections 5x		-0.0575	0.5001		-0.0340	0.5000
5y		-0.0978	0.1503		-0.0861	0.1366
6x		-0.0385	0.4649		-0.0217	0.4726
6y		-0.0849	-0.0631		-0.0753	-0.0784
	Volume = 792.31 m^3 Solution Time = 6.396 CDC Seconds			Volume = 787.310 m^3 Solution Time = 7,152 CDC Seconds (D=1000.0 EINF=1.0)		

In the problems considered, different but constant permissible stresses were considered in tension and compression. Approximate expressions relating compressive permissible stresses and member areas have been developed by Reinschmidt and Russell (206) and Elliott (50). These relationships could readily be incorporated into the D.R. method to cater for the effects of buckling. It is also possible to incorporate deflection constraints which are different for positive and negative co-ordinate directions. If required, self weight can also be accounted for during the process.

In this chapter the fixed topology was assumed to have been derived either by using the D.R. method of Chapter 2 with the dominant loading case or from other aesthetic or architectural considerations. However, if either a dominant loading case or a loading case which expresses the function of the structure is not available, then more than one loading case might be considered when the form is derived. In the following chapter the effects of sizing the structure while making changes in form with respect to all of the loading cases is considered.

CHAPTER 6: FORMFINDING AND SIZING OF MODULAR SPACE STRUCTURES CONSIDERING MULTIPLE LOADING CASES

Introduction - Fully Stressed Design Solution - Solution by Linear Programming - Dynamic Relaxation Solutions assuming Fixed Trends - Interactive Dynamic Relaxation Formfinding - Non-Linear Programming Solution - Topological Non-Linear Design - On-off non-linearities and Dynamic Relaxation - Design of structures with cable elements - The use of Prestress Effects in Design - Conclusions.

Summary:

In this chapter efficient bridging forms are extracted from a ground structure subject to multiple loads by using published fully stressed design, linear programming and non-linear programming techniques. In contrast to these automated methods Dynamic Relaxation is used to derive a series of designs by continuously deleting members which have small member areas and are reducing in size when modified to comply with the fully stressed criteria. The method is subsequently modified to cater for cable members and prestress effects. In addition, an interactive scheme is demonstrated where each loading case is applied in series and a few test modifications made which comply with fully stressed design trends for the single loading case. In this way the significance of each member for each loading case can be measured intuitively by

considering the member area sizes, changes and trends. A final topology can be fixed by the engineer and topology resized considering all load cases simultaneously.

Introduction:

Computer methods suitable for the topological design of space structures were reviewed in Chapter 2. These methods were used to minimise the weight of the structure by using member areas, topology and nodal co-ordinates as design variables. This thesis is concerned with the design of modular systems where nodal co-ordinates must comply with a grid system. For structures of this type topology is considered by taking a ground structure composed of a fully connected grid or a structure combining several candidate structures. During the design procedure inefficient members are removed and the members are sized to ensure that the design constraints are not violated. The most suitable methods for this type of design are:

a) A Fully Stressed Design stress-ratio procedure which allows inefficient members to reduce in area until they are zero and can be deleted.

b) A technique by which the form is derived by using a Linear Programming algorithm and the member areas are sized by a subsequent fully stressed design.

c) A non-linear programming algorithm of the steepest descent - alternate mode type in which members which reduce to zero at the end of steepest descent moves are deleted from the structure.

d) A non-linear programming topological design method using trade-off calculations to decide the order in which inefficient members should be deleted.

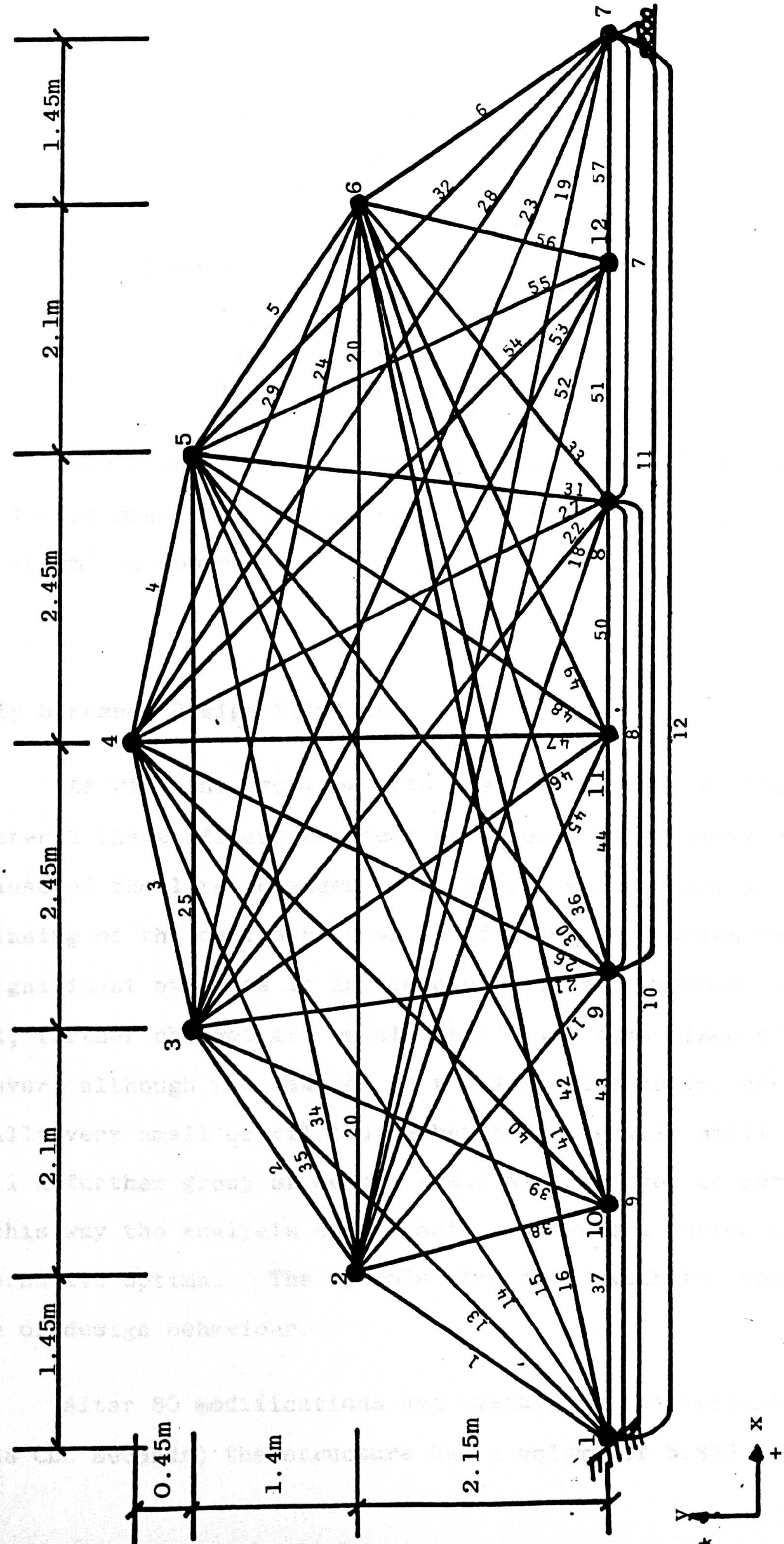
In Chapter 2 these methods were applied to a series of small problems. In this chapter they are applied to a large ground structure being used to derive an efficient bridging structure. The ground structure layout is shown in the following figure. This ground structure is based on a structure designed by Majid and Elliott (50,71), but with only two simple loading cases and without nodes 10, 11 and 12. The additional members and nodes have been incorporated to allow more loading cases and make the problem more complex and representative of a real structure. The four loading cases for the structure are as follows:

Load Case 1:

8y	=	-50 kN
9y	=	-50 kN
10y	=	-50 kN
11y	=	-50 kN
12y	=	-50 kN

Load Case 2:

10y	=	-100 kN
9y	=	-100 kN
11y	=	-100 kN


1.45m
2.1m
2.45m
2.45m
2.1m
1.45m
0.45m
1.4m
2.15m
x
y

Load Case 3:

$$9y = -100 \text{ kN}$$
$$11y = -100 \text{ kN}$$
$$8y = -100 \text{ kN}$$

Load Case 4:

$$11y = -100 \text{ kN}$$
$$8y = -100 \text{ kN}$$
$$12y = -100 \text{ kN}$$

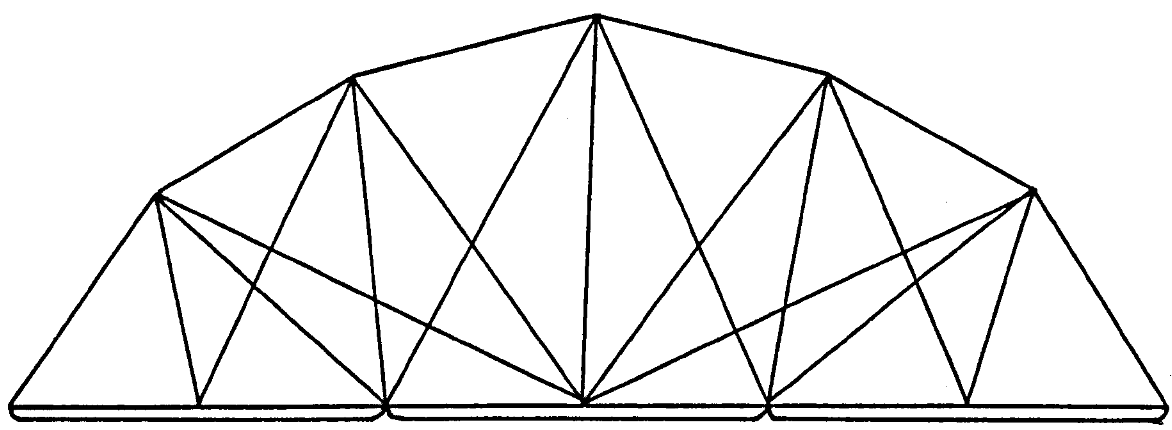
The Young's modulus for all members was 207 kN/mm^2 and the permissible stresses were 0.16 kN/mm^2 in tension and 0.1 kN/mm^2 in compression.

Fully Stressed Design Solution:

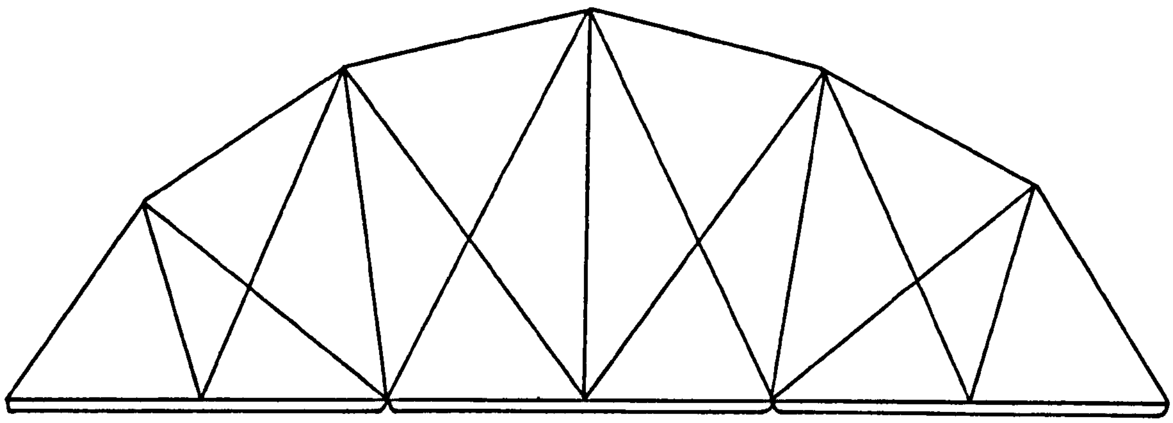
As with the problems with one loading case studied in Chapter 3 the stiffness matrices were reset after resizing because of the large changes in stiffness which occur at the beginning of the design process. After members which have insignificant stresses in any loading case had reduced to zero area, further changes are usually small and take place slowly. However, although the changes in the structure volume are usually very small or nil the member areas usually still change until a further group of member areas have reduced to zero. In this way the analysis can be said to derive a series of alternative optima. The example structure exhibited this type of design behaviour.

After 80 modifications and reanalyses (equivalent to 2.638 CDC seconds) the structure had a volume of $5.823 \times 10^7 \text{ mm}^3$.

The members with the maximum area were 1 or 6 with an area of 2412 mm^2. The form is shown below where all excluded members had an area less than 0.411 mm^2, and all included members had areas greater than 10 mm^2.



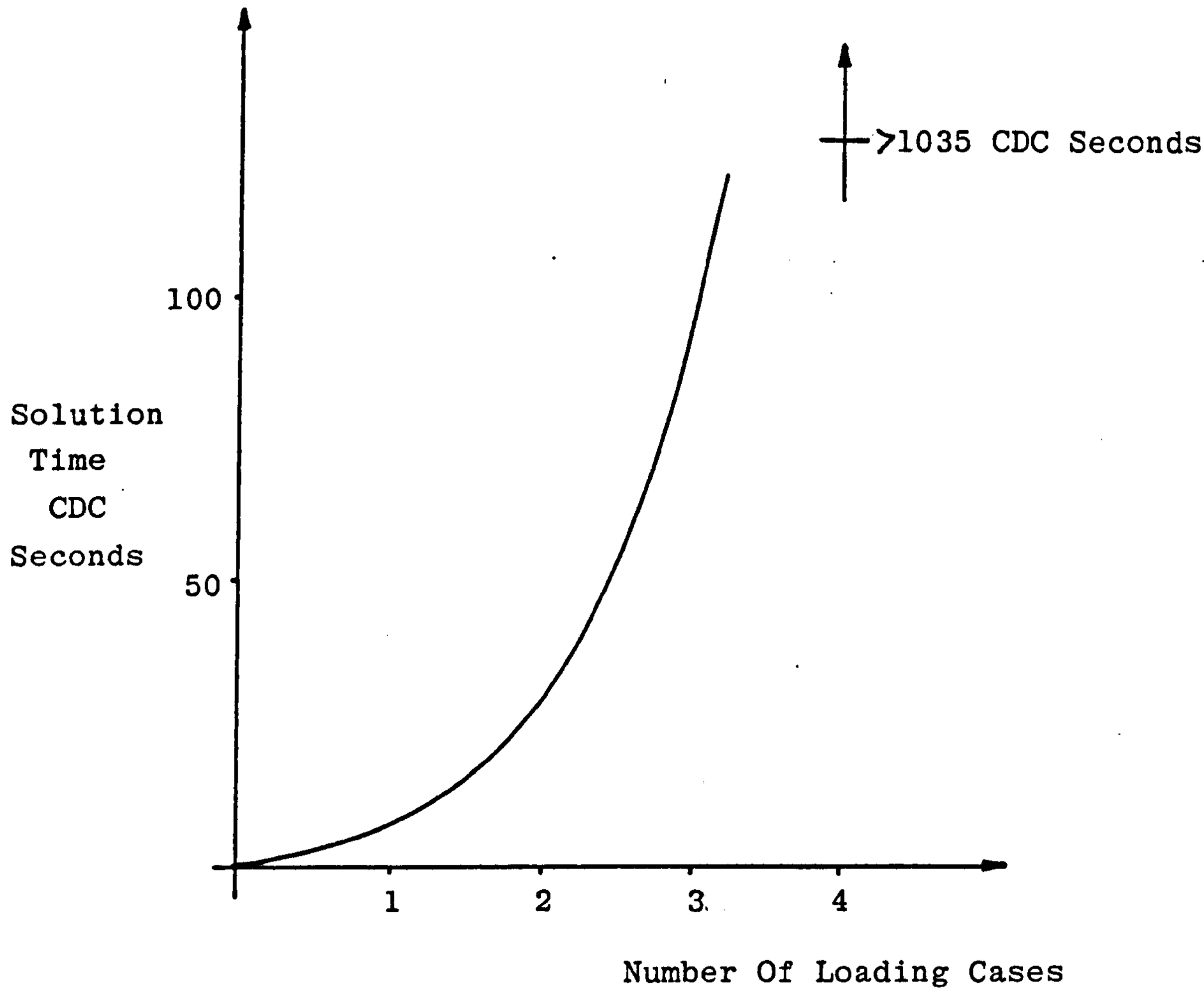
After 1120 modifications and reanalyses (equivalent to 34.834 CDC seconds) the form of the structure reduced to that shown on the following page. All excluded members had areas less than 0.00015 mm^2 and all included members had areas greater than 90 mm^2. The members with maximum areas were again 1 and 6 with areas of 2412 mm^2, and the volume of the structure reduced only fractionally to $5.821 \times 10^7 mm^3$.



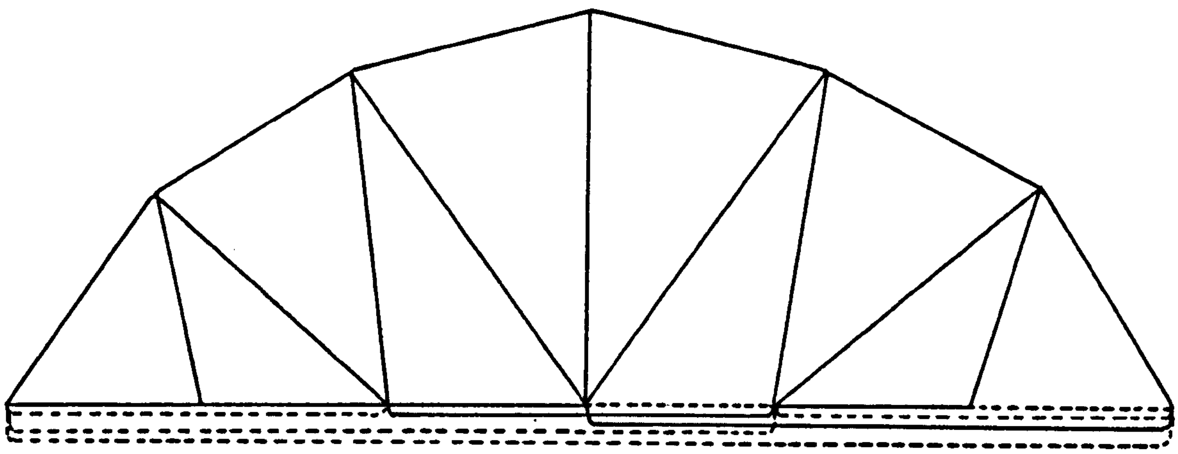
Solution by Linear Programming:

A linear program based on the formulation of Reinschmit and Russell (206) was written. As indicated in the review of Chapter 2, the linear programming method is more likely to derive a globally optimum form. However, the structure has to be resized to ensure safety because the solution does not ensure compatibility.

The solution of the bridge problem with four loading cases required consideration of 375 constraints with 312 variables and solution was not complete after 1036 CDC seconds. This indicates that the method is not practical for problems of this size with large numbers of loading cases. To assess the effects of the number of loading cases on the solution time a series of programs was run with fewer loading cases. The solution times were practically independent of the load cases used but increased rapidly as the number of loading cases increased.



The form derived using the first loading case only is shown in the diagram on the following page. The volume was 3.221×10^{7} mm^{3}. After this form had been resized using the F.S.D. technique with all four loading cases the volume was 5.932×10 mm^{7}.



------ Alternative optimum members with zero area.

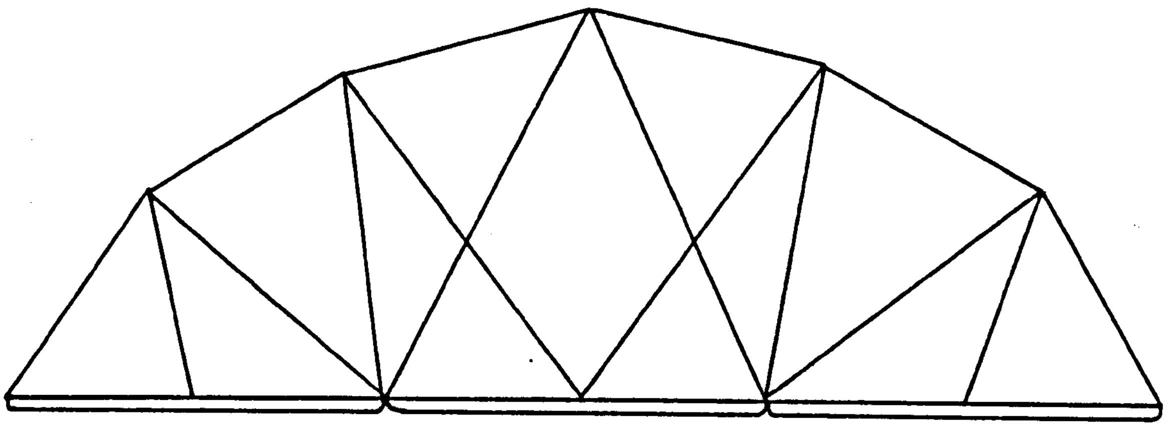
Dynamic Relaxation Solutions assuming Fixed Trends:

The Dynamic Relaxation method using the damping procedure developed in Chapter 5 was applied to the bridging ground structure. Member areas were continuously modified according to equation (5.2). Maximum and minimum area sizes were not set and member areas if necessary were allowed to reduce to zero. Masses were assigned using equation (3.16) and then doubled and an additional mass component of 0.5 was also used. The initial size of all member areas was 966 mm^2. A series of programs was run in which areas were modified after every 100, 50, 15, 10, 5, 4, 3, 2 and 1 time intervals. Modifying areas after every 3, 2 or 1 time intervals the analysis oscillated and would not converge. Analyses run modifying areas after every 10, 5 and 4 modification stages did converge, but the volume converged to 5.821×10^7 mm^3 only after 172, 239 and 143 modification stages respectively (at which stages uneccessary alternative members had been completely removed). The early iteration paths of these analyses were marked by the oscillation of member areas and the volume of the structure. Making modifications after every 100, 50 and 15 time intervals the analysis did not oscillate and the path to convergence was stable. Changing areas after every 15 time intervals the volume converged to $5.821 \times 10^7 mm^3$ after 160 modifications (24.196 CDC seconds). However, for practical purposes, convergence occurred after 84 modifications (8.537 CDC seconds) when the form finally derived using the fully stressed design technique became apparent. The volume at this stage was $5.822 \times 10^7 mm^3$. All members excluded from this form had areas less than 5 mm and all members included had areas greater than 90 mm .

With these analyses it was noticed that trends in the changes of areas indicated which members should be deleted. And if each member trend is assumed to be generally the same throughout the design of the structure then a good indication of which members can be eventually deleted is given at an early stage. It was therefore decided to remove members which were reducing in area and were less than a certain percentage of the maximum area size. These members were deleted and the topology reordered.

The percentage was fixed at 0.05% and the areas modified after every 100 time intervals. The solution derived was of the following form (Topology A).

Topology A

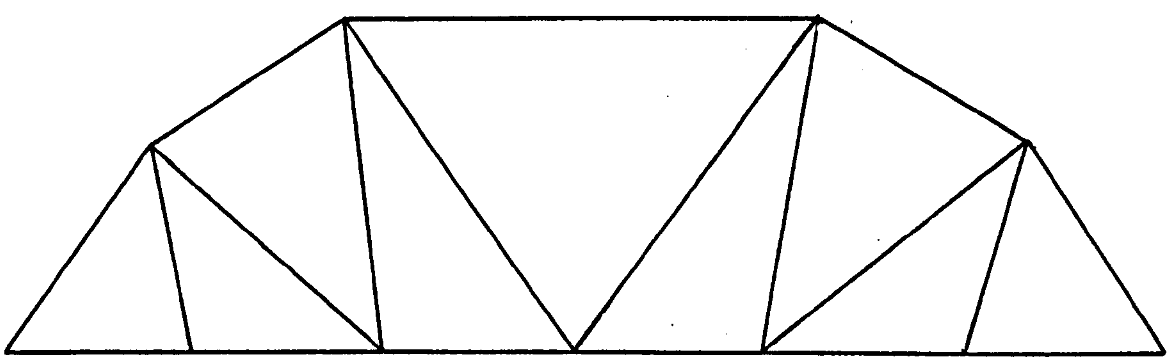


The structure had a volume of 5.816×10^7 mm^3, which is lighter than the fully stressed design or previous D.R. solution, and the solution was converged to 5 significant figures after only 24 modifications (14.287 CDC seconds). (This analysis simulated a full reanalysis procedure at each stage because after each modification a fully converged solution was obtained in less than 80 time intervals).

The number of intervals between modification stages was cut to 5 and the problem rerun. Unfortunately, due to low member areas, all the bridge deck members which were in tension were removed. For future runs members 37, 43, 44, 50, 51 and 57 were therefore not deleted. The solution then converged to the same form (Topology A)(except that member 8 was deleted) and had converged to five figure accuracy after 52 modification stages (1.687 CDC seconds). With the number of time intervals before each modification fixed at 15, the solution converged to the same form after 32 modification stages (2.947 CDC seconds).

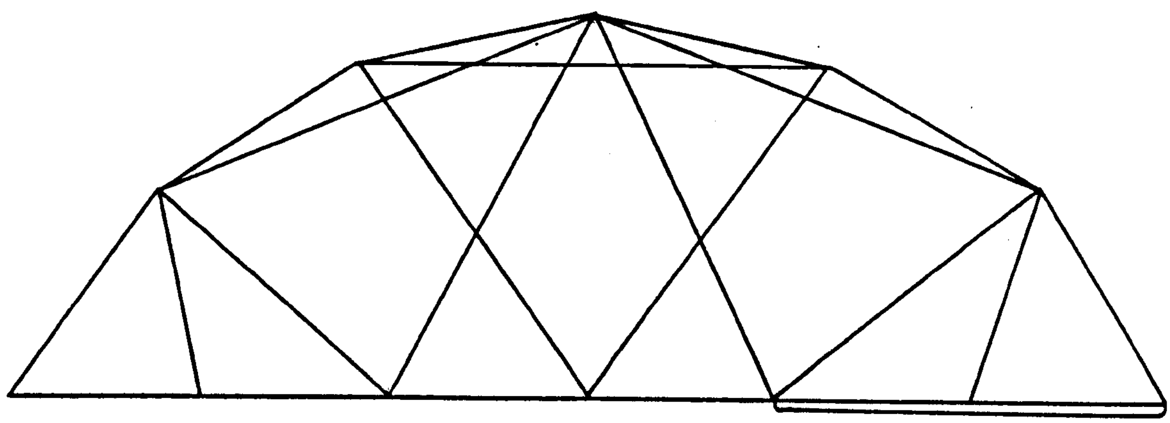
The program was rerun with the deletion percentage fixed at .1% and modifying areas after every 5 modification stages. The following form (Topology B) was derived and the solution converged to five significant figures after 55 modification stages (1.496 CDC seconds). The volume of the solution was $6.019 \times 10^7 mm^3$.

Topology B



A series of programs were also run with the percentage fixed at 0.2%, 0.15% and 0.1%, with member areas modified in each case after every 15 time intervals. The first two runs failed to produce a solution because too many members were deleted and the solutions became mechanisms. The third run derived the following form (Topology C) and the solution converged to five significant figure accuracy after 46 modification stages (3.511 CDC seconds). The volume of the structure was 6.014×10^7 mm^3.

Topology C



The variation of the deletion percentage leads to the derivation of structures of different form but similar weight.

An alternative damping procedure called 'kinetic damping' was developed by Cundall (246). With this type of damping there is no viscous damping factor but the velocity vector is set to zero every time the structure reaches a kinetic energy peak. In this way all the modes of vibration are eventually damped out. An approximation to the position of the energy peak can be made by resetting the current co-ordinates to their position in the middle of the time interval in which the structure is found to reach the peak. This should be done each time the velocities are reset to zero to ensure that the analysis converges properly. This technique was suggested by Wakefield (270) when developing the use of kinetic damping for formfinding of cable networks. Another scheme for ensuring convergence was developed by Papadrakakis (269).

For problems with multiple loading cases it was found more efficient to reset the velocities and co-ordinates when all loading cases had reached their kinetic energy peak. The analyses of the loading cases which had reached their kinetic energy peak were held fixed until all other loading cases had reached a peak.

The bridging structure was analysed using masses assigned according to equation (3.16) and modifying member areas after every 5, 10 and 20 energy peaks. All these analyses quickly diverged. The analyses were therefore rerun with the masses doubled and an additional mass factor of 0.005. The first analysis failed to converge. The second and third analyses converged to the form of the fully stressed design, but the five significant figure accuracy required a solution time in excess of 60 CDC seconds. This showed that the method was less efficient than the viscous damping solution process.

The efficiency of the method deleting members which were reducing in area and were less than a certain percentage of the maximum area was assessed by a series of runs, the results of which are tabulated on the following page.

Run No.:	1	2	3	4	5	6
No. of Kinetic Energy peaks before modifications:	5	10	20	20	20	20
Percentage area deletion:	0.1%	0.1%	0.1%	0.05%	0.015%	0.2%
Topology derived:	C	C	C	A	C Without member 25	
Volume x $10^7 mm^3$:	Volume Oscillat-ing	6.036	6.024	5.813	6.138	Fails
No. of modifications before solution to 5 significant figures:	-	19	15	24	13	
CDC solution time: Secs. 4 figs.	11.117	8.327	10.173	22.481	6.546	

These comparisons indicate that if the percentage used in deletion is 0.05% the process will derive Topology A regardless of types of damping, numeber of intervals or peaks between modifications. Likewise using 0.15% or .1% the topology derived will be C or possibly B.

The viscous damping method appears to be the most efficient way of deriving different forms of approximately the same weight. The solution times and core store requirements are low which means that it would be possible to derive several forms using the method and then choose the most attractive or lightest form. Alternatively the solution can be derived interactively using the method as a guide and allowing the engineer freedom to alter the structure during the process. Members and nodes could be added or deleted, fixity conditions altered and nodal co-ordinates adjusted if required. Simple tests performed on altering nodal co-ordinates suggest it is beneficial to adjust current co-ordinates by a similar amount to avoid any instability effects. An alternative formfinding procedure which is particularly suitable for interactive use is outlined in the following section.

Interactive Dynamic Relaxation Formfinding:

Methods for interactive design are often of an intuitive nature requiring a heuristic approach. Their mathematical basis can be nebulous and can defy traditional analytical techniques. In this case successful demonstration of a technique is the only way to establish a new method.

It was shown in Chapter 4 that if a structure was subject to a single loading case and the areas iteratively adjusted to comply with optimum strain criteria then the members can always be divided into three distinct groups:

a) Bars which are within the optimum and play an important role in the structure with large areas which are stable or increasing slowly.

b) Bars which have medium sized areas and are reducing in size.

c) Bars which have insignificant areas and are reducing in size.

It is more convenient to consider stress constraints in which the true member areas are considered (although in Chapter 4 it was shown that strain criteria may sometimes lead to the derivation of lighter layouts). However, the members can still be divided into three groups.

If a structure is subject to multiple loading the importance of a member with respect to one of the loading cases can be assessed by applying the criteria and seeing with which group the member is classified. If the loading cases are successively applied in order of their decreasing importance and members of type c deleted when their relevance to the loading cases has been determined, a form can be derived which can subsequently be sized by considering all load cases simultaneously. The use of stress criteria ensures that the relative stress levels of the loading cases is accounted for. Members which have no significance for the primary loading can be deleted immediately if the designer considers them of little relevance to other loading cases. However, members of type b should be retained until their relevance to other loading cases has been tested by applying the criteria. Members which are repeatedly classified in group b for each loading can be deleted at the designer's discretion. This process is particularly suitable for interactive use where a a continuously updated picture which could be modified by the engineer with the use of a light pen. A storage tube could be used to provide tabulated information about each member.

For the bridging structure, loads will be applied in the order 1, 2, 4 and 3. (Loading cases 2 and 4 are mirror images of each other so only load case 2 need be applied). The initial EA values for the structure are .2 x 10^6kN and the masses are assigned as in the last section. The areas are modified after every 15 time intervals according to equation (5.2). Each loading case is applied in turn and run using the damping procedure of Chapter 5 for 10 modification stages. The progress of this design is outlined in the following table. The current member EA values and trends (either increasing (I) or decreasing (D)) have been recorded after the application of each loading case.

Decisions by the engineer to delete members are also noted. After applying load cases 2 and 3 it was decided to delete all duplicate members which idealised the bridge deck (retaining only members 37, 43, 44, 50, 51 and 57). The initial EA values used when applying loading case 4 were the maximum areas derived using either loading case 2 or 3. After applying all four loading cases a final decision was made on whether to delete any other members by considering their EA values, increase or decrease in value, and trend after applying each of the loading cases. Members which tend to have low EA values and generally decreasing trends can be deleted. Throughout this process the engineer is in control and ensures that enough members are retained so that the structure does not become a mechanism. The final topology is as shown on the following page.

Member	Nodal Connections	EA values after applying Load Case 1	Trend after LCI	EA values after applying Load Case 2	Trend after LC2	Load Case 3 Symmetric value of LC2		Trend after LC3	EA values after applying Load Case 4	Trend after LC4	Decision to Retain or Delete
1	1-2	$.3051\times10^{6}$	I	$.4759\times10^{6}$	D	member 6	$.4759\times10^{6}$	I	$.3908\times10^{6}$	I	Ret
2	2-3	$.2147\times10^{6}$	I	$.3666\times10^{6}$	D	member 5	$.3666\times10^{6}$	I	$.4007\times10^{6}$	I	Ret
3	3-4	$.1594\times10^{6}$	I	$.3715\times10^{6}$	I	member 4	$.3715\times10^{6}$	I	$.3723\times10^{6}$	I	Ret
4	4-5	$.1585\times10^{6}$	I	$.3307\times10^{6}$	I	member 3	$.3715\times10^{6}$	I	$.3462\times10^{6}$	I	Ret
5	5-6	$.2126\times10^{6}$	I	$.1751\times10^{6}$	I	member 2	$.3666\times10^{6}$	I	$.3429\times10^{6}$	I	Ret
6	6-7	$.3045\times10^{6}$	I	$.1757\times10^{6}$	I	member 1	$.4759\times10^{6}$	I	$.3169\times10^{6}$	I	Ret
7	7-8	$.2926\times10^{5}$	D	$.1728\times10^{5}$	I	member 9	$.4017\times10^{5}$	I	Dlt		
8	9-8	$.2741\times10^{5}$	D	$.2298\times10^{5}$	D				Dlt		
9	1-9	$.3117\times10^{5}$	D	$.4017\times10^{5}$	I	member 7	$.1728\times10^{5}$	I	Dlt		
10	1-8	$.3183\times10^{5}$	D	$.3183\times10^{5}$	I	member 11	$.2832\times10^{5}$	I	Dlt		
11	7-9	$.2982\times10^{5}$	D	$.2832\times10^{5}$	I	member 10	$.7046\times10^{5}$	I	Dlt		
12	1-7	$.3093\times10^{5}$	D	$.657\ \times10^{5}$	I				Dlt		
13	1-3	$.7716\times10^{4}$	D	$.4738\times10^{-1}$	D	member 32	$.1592\times10^{6}$	I	$.4281\times10^{5}$	D	Dlt
14	1-4	$.8286\times10^{1}$	D	Dlt							
15	1-5	$.1423\times10^{-4}$	D	Dlt							
16	1-6	$.2904\times10^{-2}$	D	Dlt							
17	2-9	$.1343\times10^{5}$	I	$.7424\times10^{4}$	I	member 33	$.4690\times10^{5}$	I	$.1141\times10^{4}$	D	Ret
18	2-8	$.1576\times10^{2}$	D	Dlt							
19	2-7	$.2972\times10^{-1}$	D	Dlt							
20	2-6	$.5786\times10^{0}$	D	Dlt							
21	3-9	$.5459\times10^{5}$	I	$.6788\times10^{5}$	D	member 31	$.6544\times10^{3}$	D	$.5374\times10^{5}$	I	Ret
22	3-8	$.2354\times10^{2}$	D	Dlt							
23	3-7	$.1061\times10^{-6}$	D	Dlt							
24	3-6	$.2102\times10^{4}$	D	$.9300\times10^{1}$	D	member 34	Dlt				
25	3-5	$.4395\times10^{5}$	D	$.6740\times10^{4}$	D				$.2394\times10^{4}$	D	Dlt
26	4-9	$.1941\times10^{4}$	D	$.7981\times10^{5}$	I	member 27	$.5281\times10^{5}$	I	$.1247\times10^{4}$	D	Ret
27	4-8	$.1352\times10^{4}$	D	$.5281\times10^{5}$	D	member 26	$.7981\times10^{5}$	I	$.1141\times10^{5}$	D	Ret
28	4-7	$.3254\times10^{2}$	D	Dlt							
29	4-6	$.3969\times10^{2}$	D	$.1317\times10^{2}$	D	member 35	$.5708\times10^{2}$	D	$.6264\times10^{0}$	D	Dlt
30	5-9	$.4467\times10^{2}$	D	Dlt							

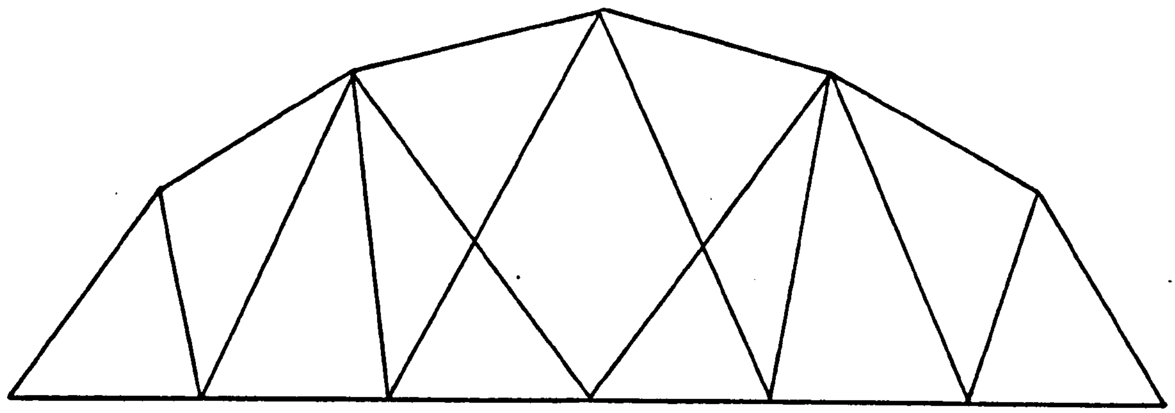
I = Increasing Trend   Ret = Member Retained

D = Decreasing Trend   Dlt = Member Deleted

Member	Nodal Connections	EA Values after applying Load Case 1	Trend after LC1	EA Values after applying Load Case 2	Trend after LC2	Load Case 3 Symmetric value of LC2	Trend after LC3	EA values after applying Load Case 4	Trend after LC4	Decision to Retain or Delete
31	5-8	$.5335x10^{5}$	D	$.6544x10^{3}$	D	member 21 $.6788x10^{5}$	D	$.7030x10^{5}$	I	Ret
32	5-7	$.9987x10^{4}$	D	$.1592x10^{6}$	I	member 13 $.4738x10^{-1}$	D	$.1532x10^{4}$	D	Dlt
33	6-8	$.1582x10^{5}$	D	$.4690x10^{5}$	I	member 17 $.4690x10^{5}$	I	$.7162x10^{5}$	I	Ret
34	2-5	$.1910x10^{4}$	D	$.1817x10^{1}$	D	member 24 Dlt				
35	2-4	$.3785x10^{5}$	D	$.5708x10^{2}$	D	member 29 $.1317x10^{2}$	D	$.1172x10^{0}$	D	Dlt
36	9-6	$.1853x10^{2}$	D	Dlt						
37	10-1	$.2089x10^{5}$	D	$.8836x10^{5}$	I	member 57 $.2508x10^{5}$	D	$.3974x10^{3}$	I	Ret
38	10-2	$.6694x10^{5}$	D	$.1306x10^{6}$	D	member 56 $.1917x10^{5}$	I	$.4587x10^{2}$	I	Ret
39	10-3	$.1612x10^{2}$	D	Dlt						
40	10-4	$.1832x10^{-1}$	D	Dlt						
41	10-5	$.2149x10^{-1}$	D	Dlt						
42	10-6	$.3472x10^{1}$	D	Dlt						
43	10-9	$.3733x10^{5}$	D	$.1179x10^{6}$	I	member 51 $.2681x10^{5}$	D	$.4188x10^{3}$	I	Ret
44	11-9	$.2736x10^{5}$	D	$.7145x10^{5}$	I	member 50 $.3931x10^{4}$	D	$.9975x10^{5}$	I	Ret
45	11-2	$.2438x10^{3}$	D	Dlt						
46	11-3	$.6850x10^{4}$	I	$.2544x10^{3}$	I	member 48 $.1144x10^{6}$	I	$.5933x10^{5}$	D	Ret
47	11-4	$.5114x10^{5}$	I	$.3559x10^{5}$	D			$.6767x10^{5}$	D	Dlt
48	11-5	$.9251x10^{4}$	D	$.1144x10^{6}$	I	member 46 $.2644x10^{6}$	I	$.1438x10^{5}$	D	Ret
49	11-6	$.2869x10^{3}$	D	Dlt						
50	11-8	$.2591x10^{5}$	D	$.3931x10^{4}$	D	member 44 $.7143x10^{5}$	I	$.1377x10^{6}$	I	Ret
51	12-8	$.3772x10^{5}$	D	$.2681x10^{5}$	D	member 43 $.1179x10^{6}$	I	$.9336x10^{5}$	I	Ret
52	12-2	$.3559x10^{5}$	D	Dlt						
53	12-3	$.1017x10^{-1}$	D	Dlt						
54	12-4	$.6716x10^{-2}$	D	Dlt						
55	12-5	$.8088x10^{1}$	D	Dlt						
56	12-6	$.6656x10^{5}$	I	$.1917x10^{5}$	D	member 38 $.1306x10^{6}$	D	$.6929x10^{0}$	D	Ret
57	12-7	$.2133x10^{5}$	D	$.2508x10^{5}$	I	member 37 $.8836x10^{6}$	D	$.1011x10^{6}$	I	Ret

I = Increasing Trend　　Ret = Member Retained

D = Decreasing Trend　　Dlt = Member Deleted

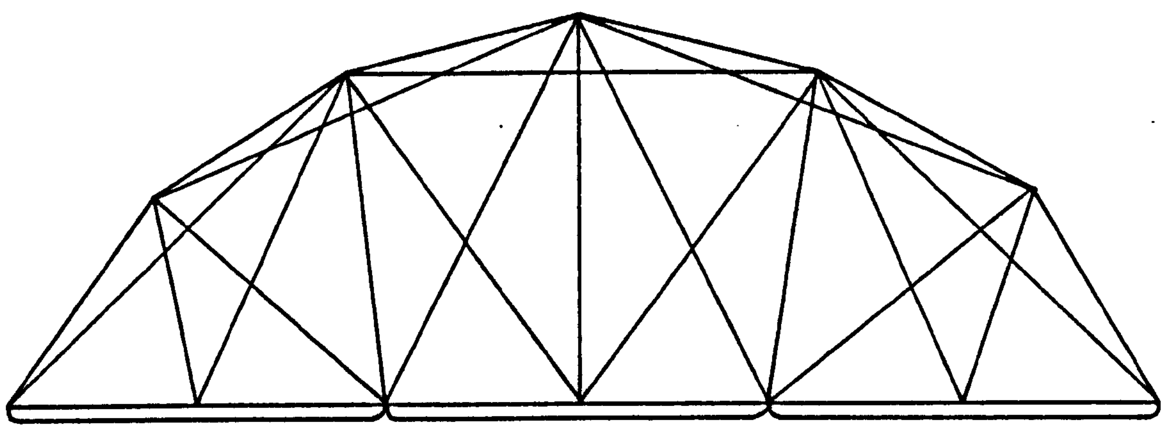


The volume of the structure when subsequently sized using all four loading cases simultaneously was 5.809×10^{7} mm^{3}. The total solution time for formfinding was 1.749 CDC seconds, and using the final sizing solution time was 2.538 CDC seconds.

Non-Linear Programming Solution:

The non-linear program using Elliott's algorithm was modified so that any member which reduced to zero area after a steepest descent move was deleted from the structure. The maximum member area permitted was 4000 mm^{2}.

The form of the final solution was:



The volume of the solution was 7.385×10^{7} mm^{3} and the solution time was 394.740 CDC seconds. The most critically constrained members were members 1 and 6 under load case 1. These members had the greatest areas of 1933 mm^{2}. In contrast, these members had an area of 2412 mm^{2} when sized using the fully stressed design technique.

Using a maximum member area size of 3000 mm^{2} the final topology of the structure was the same but the volume was 8.119×10^{7} mm^{3} and the area of members 1 and 6 were 1888 mm^{2}. The solution time was 528.290 CDC seconds.

The above work simulated the method of non-linear programming used by Dobbs and Felton (181).

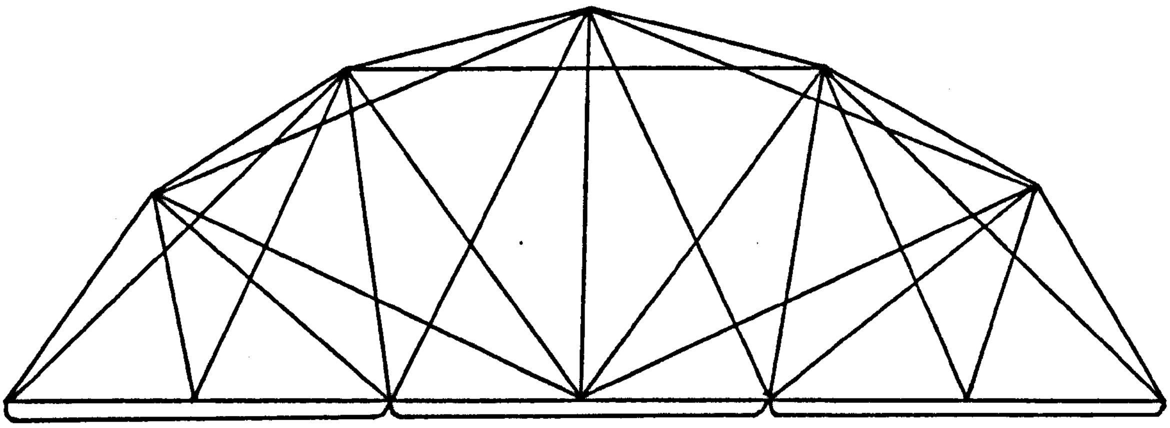
Topological Non-Linear Design:

A non-linear program with topological design algorithm as devised by Majid and Elliott (50,71,74) and outlined in Chapter 2 was written and applied to the bridging structure. The maximum permitted area used was set at 3,000 mm^2. The program was initially run performing six cycles of the alternate mode-steepest descent algorithm before using the topological algorithm. The algorithm was then used after every design cycle. This procedure ensures that the structure is close to a local optimum before any members are removed. The topological algorithm produces a preference list of the members to be removed. Members at the top of this list are only removed if this does not require an increase in the structural volume or the structure is a local non-linear optimum. The degree of convergence required for a structure to be defined as a local optimum was reduced by setting the convergence test factors T3 and T4 to 0.005. The use of the topological design algorithm was found to use large amounts of computing time and it was decided to use it only after every 3 cycles of the non-linear program or when the structure reached a local non-linear optimum.

Symmetry was not directly taken into account while deriving the structures. However, members of similar benefit (within $\pm$ .08%) were removed at the same time.

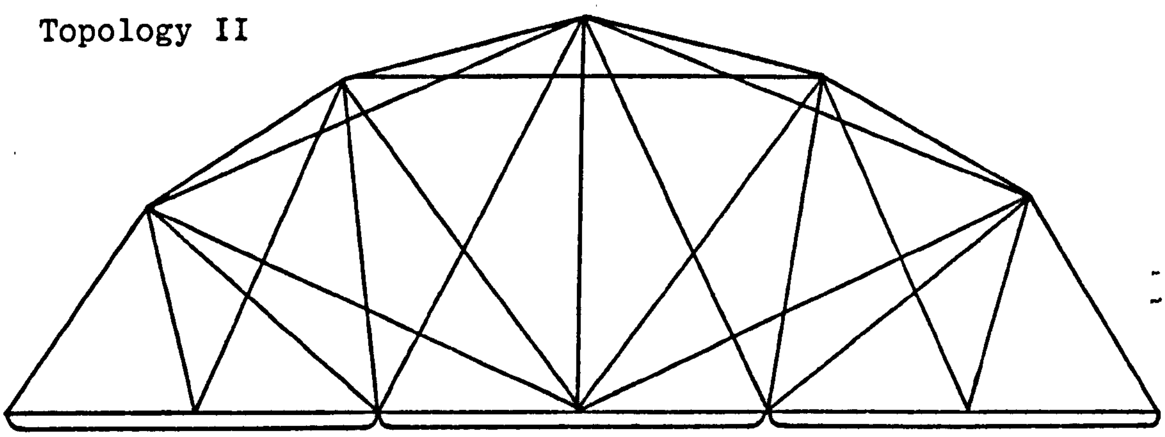
The series of structures derived using the method are shown below.

Topology I



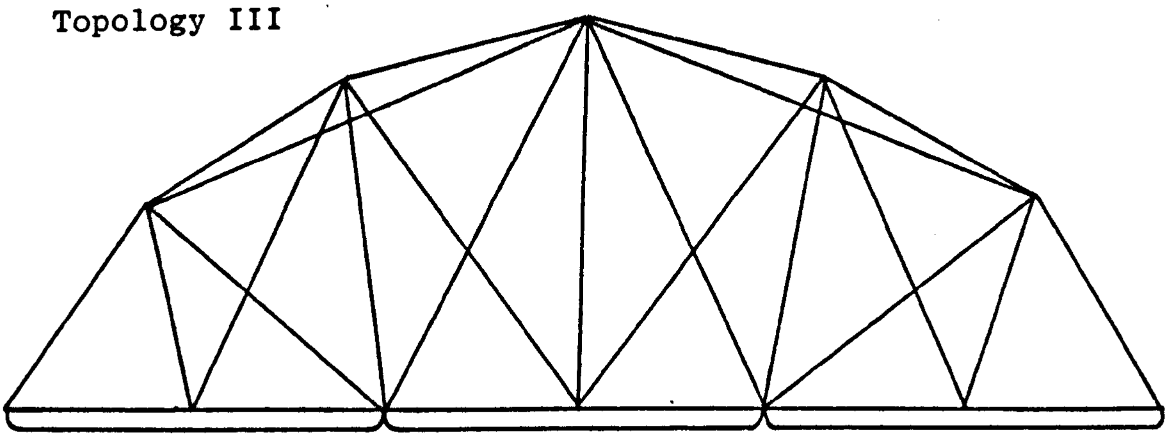
Volume = $9.405 \times 10^7 \text{ mm}^3$

Topology II



Volume = $10.776 \times 10^7 \text{ mm}^3$

Topology III



Volume = $7.969 \times 10^7 \text{ mm}^3$

Topology I was derived using just the non-linear programming method. The volume, $9.405 \times 10^7 mm^3$, was higher than that derived in the last section because the problem was not so fully converged. Topology II was derived using the topological algorithm to remove two members. This resulted in a higher volume immediately after the removal of these members. After the use of the non-linear algorithm the volume was reduced to $10.776 \times 10^7 mm^3$. Removal of two other members using the topological algorithm to give Topology III resulted in no further increase in volume. The volume was subsequently reduced by use of the non-linear algorithm to $7.969 \times 10^7 mm^3$. At this stage a benefit vector indicated that members 35(Link 2-4) and 29 (Link 4-6) should be removed next. No further computer time was available to obtain member areas. The program at this stage had already used 1200 CDC seconds.

It is interesting to note that the area of members 1 and 6 were 1998 mm^2 for Topology I but gradually increased throughout the design to 2428 mm^2 for Topology III. This is similar to the area 2412 mm^2 which these members had when sized using the fully stressed design technique.

These trends indicate that the program would eventually derive an optimum of similar form to those derived using the Fully Stressed Design Technique. The time used to derive this form with the Non-Linear technique would, however, be prohibative.

The results of this section confirm the view that the algorithm is not suitable for large problems. Majid and Elliott (50, 71, 74) did not consider very large problems. Their bridging structure had 36 members which were divided into 13 groups. The problem had therefore only 13 design variables which were considered under the action of two simple loading cases. The problem considered in this chapter is more representative of large practical structures.

On-Off Non-Linearities and Dynamic Relaxation:

Dynamic Relaxation is an explicit method which is ideally suitable for catering with on/off non-linear effects and particularly slackening of cable members.

To analyse a structure with cable elements only two modifications are required to account for the effects:

a) A one dimensional array NS is established which is used to keep a record of the members which are cables.

b) To account for the fact that cable members cannot sustain compression when the residuals are calculated compression member forces are modified as follows:

$$Tm' = Tm \cdot NS(m)$$

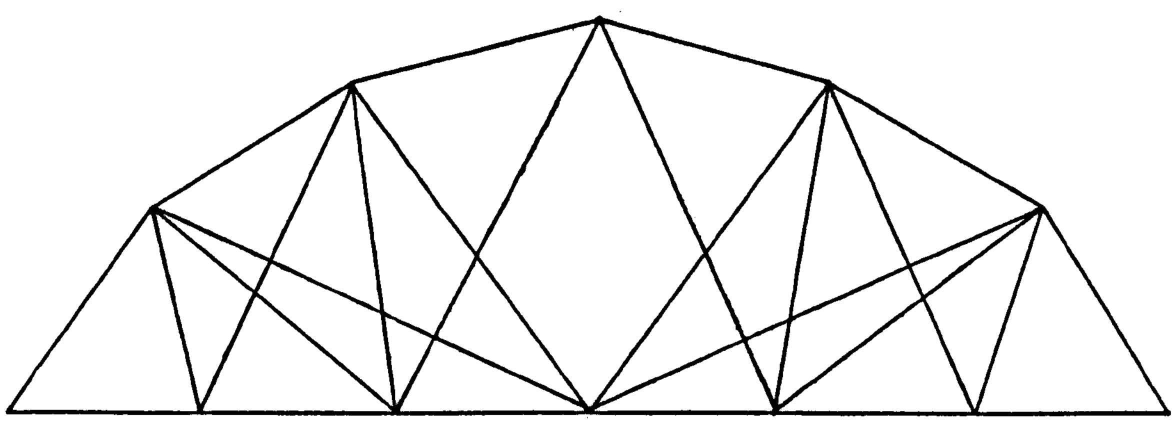
where: $NS(m) = 1.0$ for bar members

$NS(m) = 0.0$ for cable members.

Cable members can be accounted for in the design process by one further modification of type b when the member force is calculated to assess the amount of resizing. This ensures that any modification to the cable member areas only considers tension forces and accounts for cable members slackening.

Design of Structures with Cable Elements:

The structure was redesigned with all internal members replaced with cables. The only stiff members were members 37, 43, 44, 50, 51, 57 and members 1 to 12. Member properties were the same as for the rigid structure. The member areas were modified after every 15 time intervals. The masses derived using equation (3.16) were doubled and an additional mass of 0.005 was used. After 84 modification stages the form was as follows:



The maximum area was 2410 mm^2 and the total volume of the structure was $5.974 \times 10^7 mm^3$. All members with areas less than 2.5 mm^2 were excluded and the smallest member included had an area of 127 mm^2 (Link 12-15). The solution time at this stage was 10.035 CDC seconds, and no further changes in form were apparent after an additional 135 modification stages.

Although the form of this structure is different from the rigidly based D.R. design the volumes of the solutions are similar.

Another analysis was performed assuming fixed trends and deleting members which were reducing and had areas less than 0.05% of the maximum member area. The final form of this solution was the same but member 3-10 was deleted. The volume of the final structure was $5.925 \times 10^7 mm^3$. The solution conveyed to five significant figures after only 25 modifications (2.421 CDC seconds).

The Use of Prestress Effects in Design:

The use of prestress in computer aided design and optimisation appears not to have been considered by previous workers with the exception of Hofmeister and Felton (271) who, however, did not consider the extra weight of cables required to provide an optimised prestress distribution. In this section the Dynamic Relaxation design method is expanded to account for prestress effects and in particular, to ensure that the prestress is adjusted so that the bridge deck under a primary loading condition remains horizontal.

For design with concurrent loading vectors, including the above deflection constrained case, it is convenient to refer prestress modifications to initial, or slack, member lengths as follows:

$$T_m^{t+\Delta t} = \frac{EA_m}{L_{ms}}(L_{mc} - L_{ms}) \qquad (6.2)$$

where: $T_m^{t+\Delta t}$ = current force in member m

L_{mc} = current length of member m

L_{ms} = initial or slack length of member m

EA_m = elastic modulus multiplied by member area

During the formfinding, prestress levels may be controlled by the designer. Alternatively they may be adjusted to ensure that the final structure conforms with deflection criteria. For example the bridging structure can be redesigned to ensure that under load case 1 the vertical deflections of all nodes along the bridge deck are zero. This was done by continuously recalculating the initial lengths (Ls) of the cable members assuming that the initial co-ordinates are moved in the y direction as much as they are deflected down under the first loading case. The initial lengths of the stiff members are kept constant.

Before applying this process some initial runs were made to find how frequently the member lengths could be adjusted without causing instability. Member areas were kept constant but cable lengths were adjusted so the bridge deck did not deflect vertically under the first loading case. Member lengths were modified after every 10, 8, 6, 4, 2 and 1 time intervals. The last two calculations became unstable and it was noted that the process was slightly more efficient when the lengths were modified after every 6 time intervals.

Three programs were then run using the complete process modifying lengths after every 6 time intervals and areas after every 2, 5 and 10 time intervals. To avoid quasi-stability effects the mass components derived using equation (3.16) were multiplied by 4.0. An additional mass component of 0.005 was also used. The first two analyses failed to converge but the third converged to the same form derived for the cable bridge without prestress. The solution converged to five figure accuracy after 1245 iterations, which was equivalent to 10.351 CDC seconds. The volume of the solution was 5.980×10^7 mm^3, indicating that the prestress had little effect on the volume of the structure. After convergence no further modifications were made and the loads were removed from each of the loading vectors. Each of these concurrent solutions converged to a slightly different equilibrium position. This is because the structure is a mechanism when the structure is unloaded and the cables are not in tension. The final position for each concurrent vector is therefore dependent on the path that the structure takes to equilibrium.

Conclusions:

Both the Dynamic Relaxation methods presented in this chapter provide alternative forms which are of similar or lower weight than those derived by the comparative methods. The first D.R. or parallel method is suitable for automatic use but is best performed with the engineer keeping a watching eye on the process to ensure that the form derived is suitable. The second D.R. or series method is especially suitable for interactive design. The solution times for these methods are competitive enabling several runs to be made using different parameters to generate slightly different forms of similar weight. These solutions can then be studied and the lightest suitable form adopted.

Solution times for linear and non-linear programming techniques are prohibitive and the non-linear program derived a solution which was not a global optimum. The failure of the non-linear algorithm to arrive at non-optimum solutions was also noted in Chapter 5.

The results of this chapter confirm the view that optimisation methods either non-linear or linear are not suitable for large problems with multiple loading cases. Intuitive methods based on fully stressed criteria, of which the Dynamic Relaxation based methods are examples, appear to be more practical design methods for these large problems.

CHAPTER 7: GENERAL CONCLUSIONS AND SUMMARY

This thesis has been concerned with the assessment and development of computer aided design methods for the design of large modularly constrained building structures.

In Chapter 2, computer methods suitable for the topological design of space structures were reviewed. Methods which remove inefficient members from a ground structure were shown to be well established and these techniques are particularly suitable for modularly constrained structures.

In Chapter 3 a ground structure method of formfinding using Dynamic Relaxation was assessed and compared with solutions derived using the Fully Stressed Design Technique and Dual Linear Programming Method. A series of cantilever structures were derived and the Dynamic Relaxation Method was shown to be computationally efficient. The method is particularly suitable for modularly constrained structures subject to a dominant design loading case, is simple to program and requires less core store because an overall stiffness matrix need not be stored. For the formfinding of the three dimensional building space structures discussed in Chapter 1 which are primarily bridging structures, the method is ideal because a dominant loading case will be available. Use of this technique ensures that the formfinding process will derive a form which clearly expresses the main function of the structure. Because Dynamic Relaxation is an explicit method it allows modifications to be made to the structure during the analysis. This method of formfinding is therefore particularly suitable for interactive design and allows the engineer complete freedom during the conceptual design.

In Chapter 4 a full investigation of the effects of the iteration parameters on the stability and convergence of the Dynamic Relaxation Formfinding method was reported. General rules governing the effects of these parameters on stability cannot easily be formulated. However, it was noted that the number of structure modifications before the solution became apparent was independent of the iteration parameters. The formfinding method was generalised to cater for differing stress constraints in tension and compression members. The effects of these constraints on optimum form were compared with the forms derived using the Fully Stressed Design Technique and the Dual Linear Programming Method. With this modified D.R. method the form derived was the same as that derived using the Dual Linear Programming Method. The Fully Stressed Design Technique, however, derived a form of heavier weight for the problem considered.

In Chapter 5 an intuitive Dynamic Relaxation method was presented to size structures of fixed topology subject to multiple loading cases. The method caters for constraints on maximum and minimum member area sizes, together with stress and deflection constraints, by the use of parallel elastic and elasto-plastic analyses. This method was applied to a series of problems and compared with solutions derived using a non-linear programming method. These solutions were of comparable weight and required similar solution times, but the non-linear programming method frequently failed to converge to a global optimum and further program runs had to be made to check several solutions.

In Chapter 6 two Dynamic Relaxation formfinding and sizing methods were applied to a bridging ground structure subject to multiple loading cases. Both methods were based on the Fully Stressed Design Criteria.

The first or parallel method was suitable for deriving and sizing forms of optimum or near optimum weight by deleting members which were small in area size and reducing in size. Variation of the size at which members are deleted resulted in a series of near optimum designs. The method is computationally efficient and could be used to derive any designs which might then be assessed on the basis of other criteria such as aesthetic or functional requirements.

The second or series method is particularly suitable for interactive use and consists of testing the efficiency of each member with respect to each loading case by making a few test modifications which comply with the fully stressed criteria. In this way the loading cases are first applied in series to fix the form. The final topology is then fixed and sized by considering all loading cases simultaneously. The method appears computationally efficient and allows the engineer freedom to fix the form using the tests as a guide.

These Dynamic Relaxation Methods should be particularly attractive to engineers because they are simple, intuitive and allow the engineer to gain a feel for the response of the structural form to the loading cases.

Comparative solutions derived using Linear and Non-Linear Programming Methods require excessive amounts of computational time and core store. By comparison the Dynamic Relaxation techniques are efficient and do not require the storage of an overall stiffness matrix. The Non-Linear program often converged to a solution which was not a global optimum. The Topological Non-Linear design method used excessive amounts of computing time but appears to be better at deriving a suitable form.

The Fully Stressed Design technique derived a suitable form in a solution time which was faster than the Linear and Non-Linear Programming Techniques but slower than both Dynamic Relaxation techniques. In addition, solution with an overall stiffness matrix does not allow the same degree of interaction or flexibility as the Dynamic Relaxation based methods.

The parallel Dynamic Relaxation method was extended to cater for on-off slackening of cable members and applied to the bridging ground structure. The method was further extended to cater for prestress effects and the bridging structure was designed to ensure that the deck did not deflect vertically under the primary loading case. Unfortunately no methods appear available for comparison with this method.

The method of using a fully or well connected structure to represent a series of alternative candidate structures often results in a highly redundant ground structure. In 1936 Hardy Cross (22) discussed the problems of designing highly indeterminate structures which he designated "hybrid type". He commented "Probably the chief identifying characteristic of the type is that it responds sluggishly or erratically to traditional methods of structural design. Successive cycles of design and analysis may indicate a trend, but produce only slowly a definite and satisfactory conclusion. If there are discontinuities in this design procedure the traditional process might be quite misleading. Traditional processes are not very helpful in this field, although they still have their place". Highly connected ground structures required for topological considerations have been shown in this thesis to exhibit the same sluggish response to computer design methods which usually mirror the traditional design methods. The Dynamic Relaxation Methods used in Chapter 6 seek to remove inefficient members to ensure that design behaviour speeds up or becomes "normal". Hardy Cross implies that this might reduce the validity of the solution but for the design of large structures a rigorous design is not always possible. The most efficient and sometimes the only practical way to tackle the problem with the present computing power is to reduce its size as soon as possible. For the large modular building structures discussed in Chapter 1 Hardy Cross's approach appears outmoded. In addition, adhering to Hardy Cross's precept would result in members which are only subject to low compatibility stresses being retained. These members should ideally be removed. Majid and Elliott's Non-Linear Topological Design method removes these and other inefficient members in a ground structure. Although this method may be thought rigorous and exact it can be exposed to the same type of criticism. For there is no assurance that any member which is removed may not, if it were to be subsequently reintroduced into the design, result in a substantial reduction in the structural weight. For this method to be rigorous the feasibility of reintroducting deleted members should therefore be considered at each redesign stage. This would require large amounts of computing time and core store for even small problems.

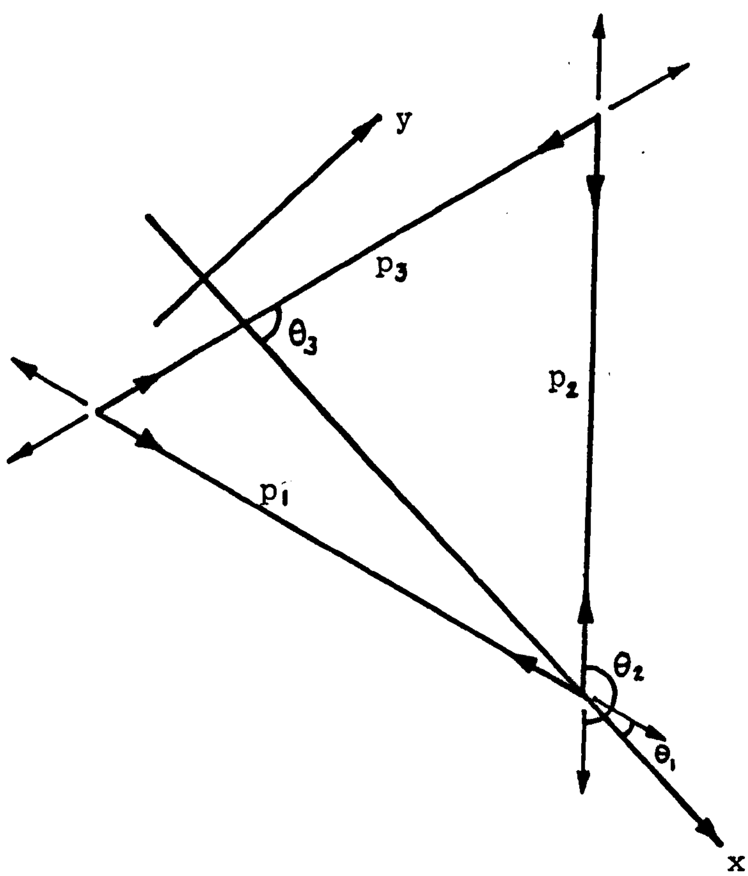
A study of the literature shows that most of the research effort in computer aided structural design has been channelled into producing non-linear optimisation algorithms. The over riding goal being the development of the "perfect" algorithm which can automatically derive global optimum solutions. Unfortunately most authors only apply their algorithms to simple small truss problems with few design variables. Few methods appear to have been applied to large problems and it can only be assumed that in most cases they are too expensive or fail to converge to global optimum solutions for such problems. In 1974, Schmit and Farshi (103) commented "A widely held current view point is that while mathematical programming methods are at present well suited for detailed component optimization they are not practical when dealing with large structural systems".

This thesis reinforces this viewpoint and indicates that more intuitive design methods controlled by explicit design criteria appear to be the most efficient and practical approach to formfinding and sizing of large modular space structures.

APPENDIX I: APPLICATION OF THE THEOREMS OF STRUCTURAL VARIATION TO ALLOW FOR CLADDING

The theorems of structural variation developed by Majid and Elliott (50, 71, 72, 73, 74) can also be extended to cater for cladding elements by using a 3 by 3 natural triangular stiffness matrix as first developed by Argyris (31).

If the stiffness matrix is defined by considering a triangular element in which the state of stress is defined by the edge extensions of the triangle (instead of the three nodes). This reduces the stiffness matrix to a 3 x 3 size as compared with the usual 6 x 6. A typical element is shown below:



The displacement of the edges are defined as:

$$\begin{bmatrix} d^e \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

The corresponding element forces or tensions along each edge are:

$$\begin{bmatrix} p^e \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

The strains are assumed constraint along each edge and throughout the panel and may be expressed in terms of $\begin{bmatrix} d^e \end{bmatrix}$. The edge strains E_1, E_2, E_3 may be expressed in terms of the element strains E_x, E_y, ∂_{xy} as follows:

$$E_i = E_x \cos^2\theta_i + E_y \sin^2\theta_i + \partial_{xy} \cos\theta_i \sin\theta_i$$

$$(i = 1,\ 2,\ 3)$$

The terms for each edge strain can similarly be developed and rearranged to give:

$$\begin{bmatrix} E_x \\ -E_y \\ \partial_{xy} \end{bmatrix} = D \begin{bmatrix} (b_2 c_3 - b_3 c_2) & -(b_1 c_3 - c_1 b_3) & (b_1 c_2 - b_2 c_1) \\ (a_2 c_3 - a_3 c_2) & -(a_1 c_3 - a_3 c_1) & (a_1 c_2 - a_2 c_1) \\ (a_2 b_3 - a_3 b_2) & -(a_1 b_3 - a_3 b_1) & (a_1 b_2 - a_2 b_1) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{I.1}$$

where:

$$D = a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1) + a_3(b_1 c_2 - b_2 c_1)$$

and $a_i = \cos^2\theta_i$, $b_i = \sin^2\theta_i$ and $c_i = \sin\theta_i \cos\theta_i$

The side strains can be determined in terms of the side lengths L_1, L_2 and L_3 and extension as follows:

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 1/L_1 & 0 & 0 \\ 0 & 1/L_2 & 0 \\ 0 & 0 & 1/L_3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \qquad (I.2)$$

The element stresses can be related to the element strains using the standard plane stress relations:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = (E/(1 - V^2)) \begin{bmatrix} 1 & V & 0 \\ V & 1 & 0 \\ 0 & 0 & (1-V)/2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ \gamma_{xy} \end{bmatrix} \qquad (I.3)$$

where:

E = Young's Modulus

V = Poisson's Ratio

These relationships represented in the standard finite element formulation (119) are as follows:

$$\left[E_x\right] = \left[B\right]\left[E_i\right] \qquad (I.1b)$$

and $$\left[E_i\right] = \left[L\right]\left[e_i\right] \qquad (I.2b)$$

and $$\left[G\right] = \left[B\right]\left[L\right]$$

and $$\left[\sigma\right] = \left[D\right]\left[E_x\right] \qquad (I.3b)$$

The stiffness matrix is given by the following standard relationship (119):

$$\left[p^e\right] = \int \left[G\right]^T \left[D\right]\left[G\right]\left[d^e\right] .dvol \qquad (I.4)$$

where:

d vol = thickness x element area.

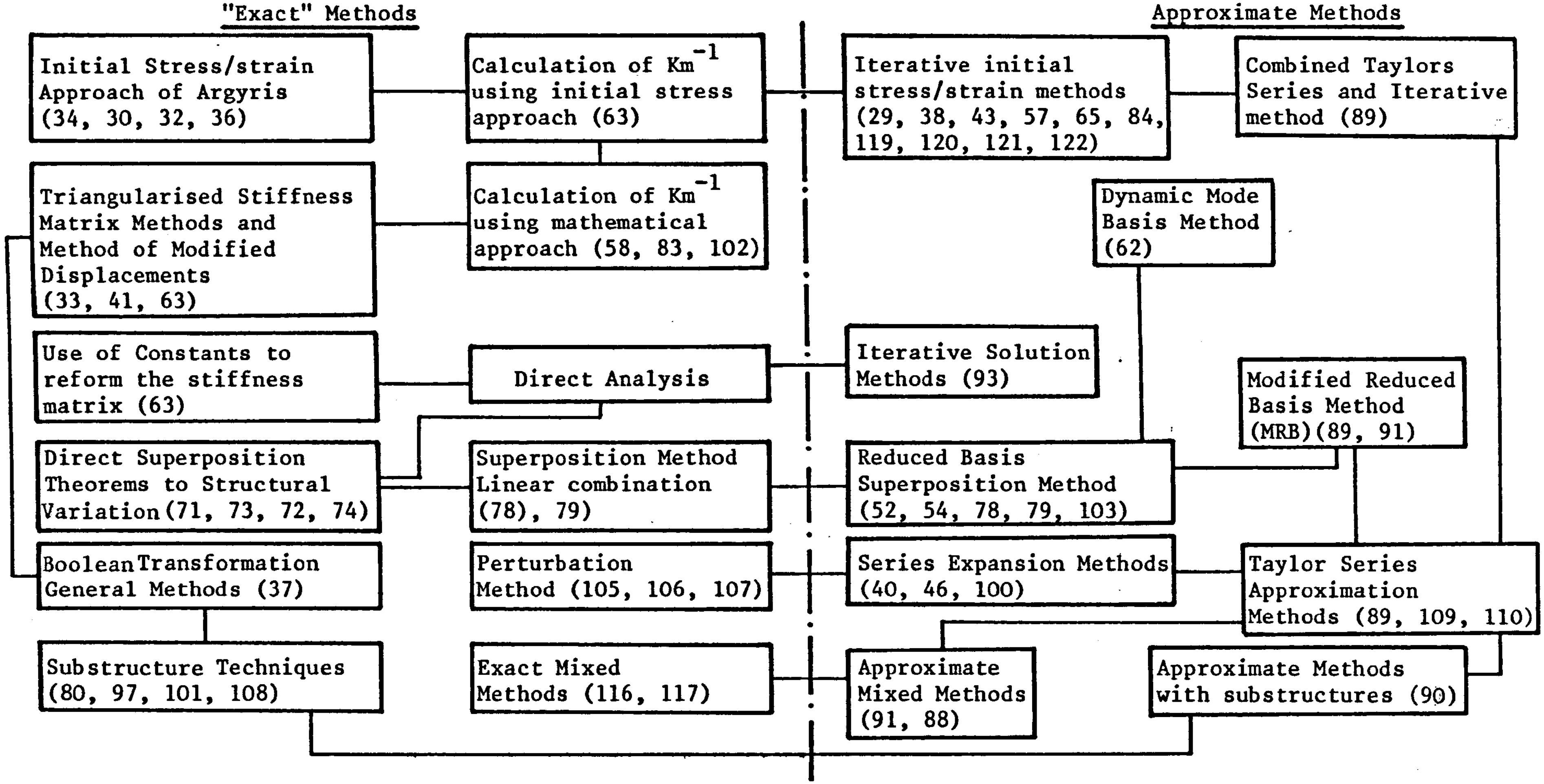
For a structure which is analysed using an idealisation of pin jointed bars and triangular constant stress elements an exact re-analysis is possible using the theorems of structural variation and the results of an initial analysis. The original structure is analysed under the actual loads and pairs of equal but opposite unit forces applied at pairs of nodes in the direction of each edge of all elements. The deflections of this conventional analysis can be used to calculate the element side forces using equation (I.4). These three forces now describe the element stress and can be interpreted as the forces in three bars connecting the nodes. In this way the forces for each element in each loading case can be determined and tabulated as flexibility or influence coefficients. The theorems of structural variation as given by equations (2.8) and (2.9) can thus be used to reanalyse the structure and predict the response after any changes have been made.

APPENDIX II: A REVIEW OF REANALYSIS TECHNIQUES

A reanalysis technique is a method of solving a structure given the solution of a slightly different structure to the same loads, without completely resolving the structure. Arora (39) gave a most succinct definition of the problem when he stated that it was "to find the response of a structure after modifications using the original response of the structure such that the computational time of reanalysis is less than the complete analysis time". The use of these methods is diverse. However, considerable effort has been made to develop suitable techniques for non-linear optimisation methods where rapid reanalyses are required to give design parameter sensitivities and to assess the effects of design changes. Several attempts have been made to find the most efficient technique (51, 65, 78, 106) and the main types of algorithm are shown in the table on the following page.

The methods have been classified either as "exact" or approximate. "Exact" methods are those in which no further approximations are made after those which are required for the initial finite element analysis. The finite element method is well established in both Force (70, 98, 99) and Displacement (68, 97, 119) forms, but the latter is usually preferred because it does not require selection of redundancies. This selection can now be performed

Table of Techniques for Reanalysis



automatically (70, 99). Both methods result in the formation of large linear equation systems. The available methods for solving these equations were studied by Fox (53) and the computational aspects reviewed in two excellent papers by Meyer (80, 81) and in a paper by Melosh and Bamford (77). Iterative solution methods generally tend to become inefficient if there are more than a few loading cases. The choice of the original solution method is particularly important if an appropriate reanalysis technique is to be used.

Ab initio or direct reanalysis by solution of the modified stiffness matrix is usually expensive and inefficient except for simple structures however the possibility of this approach must not be excluded.

For a structure where the modifications consist of factoring element stiffness matrices by constants, these matrices can be stored to reform the overall stiffness matrix with a minimum of effort. However this approach (63) will not be practical for large structures with higher order elements where computer storage requirements will be considerable. Another disadvantage of this method is that the stiffness matrix must be completely resolved.

The first matrix reanalysis method, published in 1956, by Argyris and Kelsey (34), was based on the initial strain concept. The germ of the method was developed earlier in non-matrix form by Best (42), Cicala (48) and Michielson and Dijk (82) when they applied fictitious forces equal to, but in an opposite direction to those resulting from the removal of the material such that the stresses and deflections are exactly the same as for the original structure. Argyris and Kelsey (30) used a matrix force method and applied initial strains to account for cut outs. Argyris (34) later generalised the method to cater for modifications and Poppleton (95, 96) used the method to redesign structures with undesirable stress distributions. Argyris (32) subsequently developed a parallel displacement reanalysis method which used initial stresses to derive expressions for the modified forces and displacements in terms of the reduced stiffness matrix. Unfortunately, both of these methods, which have been discussed and republished (35, 39, 55, 56, 75, 92, 94), are not suitable for optimisation problems because the inverse of large matrices have to be calculated if the modifications are extensive. In addition, all subsequent modifications must be referred to the original analysis and the area of modifications must be known before the initial analysis so that the problem can be efficiently ordered. Kalvie and Powell (63) reformulated the method relating the inverse of the modified stiffness matrix to the modifications and the original inverse stiffness matrix. They concluded that the method was inefficient compared with ab initio analysis for the typical problem they considered.

Similar formulae to those developed using the initial strain concepts have been developed for a mathematical basis without physical reasoning. An identity relating the inverse of a modified matrix to the modifications and the inverse of the original matrix was published by Householder (58). It was attributed to Woodbury but based on the Sherman-Morrison Formula. The identity was subsequently used by Sack, Carpenter and Hatch (102) for the modification of the inverse of the stiffness matrix accounting the non-zero rows in ΔK one at a time (where ΔK = the change in the modified stiffness matrix). MacNeal (69) presented a similar method based on the compensation theorem for electrical networks. His work was developed by Kosko (67) and used by Schmit (212) when minimising the weight of a three bar truss.

Kalvie and Powell (63) developed a method based on the approach of Sack, Carpenter and Hatch that did not require explicit formation of the inverse stiffness matrix. For this method, in which the modified displacements are calculated directly, it is only assumed that the stiffness matrix is in triangularised form. However the method does not compare favourably with ab initio analyses except in the case where successive changes in the stiffness matrix differ by a constant. Kirsch and Rubinstein (64) have also explored the use of the Sherman-Morrison formula for structural modifications.

Argyris et alia (33) developed another technique based on Householder's identity which used a triangularised stiffness matrix formulation. This method accounts for changes in all non-zero columns of ΔK simultaneously and takes less than half the number of operations required using the Kalvie and Powell technique. The method has similarities with the substructure techniques used by Rosen and Rubinstein (101), and the partitioning methods of Stewart and Baty (108). Further work relating to the modification of triangularised factors of modified stiffness matrices can be found in the paper by Bennett (41).

Mohraz and Wright (83) developed an algorithm using Householders identity to derive the inverse stiffness matrix after joints had been deleted and members modified. Argyris and Roy (37) presented a comprehensive treatment of general structural modifications which accounts for addition of degrees of freedom and segments of the structure and modification of members. Their method utilises modification of the triangularised stiffness matrix together with modification of substructures with Boolean Transformations to partition unmodified degrees of freedom. The method is completely general and is particularly suitable for fast computers with large core facilities with matrix handling schemes. Other references to substructure techniques can be found in references (80, 97, 101, 108).

Sobiezczanski's parallel element or perturbation method (105, 106, 107) is similar to the initial strain concept but subsequent reanalyses are not referred to the initial analysis. All modifications are interpreted as the introduction of parallel elements to the modified members. The method can be formulated with both stiffness and flexibility methods and new elements may be introduced between nodes not previously connected.

Other exact methods include the superposition methods of Majid and Elliott (71, 73, 74) and Melosh and Luik (78, 79). Majid and Elliott's method utilises theorems of structural variation, with a set of displacement and force vectors for unit loads applied at the ends of each member, to predict the displacements and forces of modified pin jointed structures. The method is particularly suitable for the optimisation of pin jointed structures because the sensitivities of the design parameters and the design changes can be quickly assessed. The method can also be used to provide trade off data in the topological design of pin jointed structures. The method was subsequently extended by Majid (72) to cater for rigidly jointed frames and has been further extended in Appendix I to cater for constant stress triangular elements. Melosh and Luik used the initial response and a series of member self straining vectors as a basis to assess the response of the modified structure. The complementary energy of the modified structure was minimised to calculate the linear combination coefficients and ensure compatibility. The method can be made approximate by reducing the basis or the number of self straining vectors to less than the degree of redundancy of the structure (52). Majid and Elliott's method, however, is always exact and the method explicitly evaluates the member coefficients with respect to their unit vectors and modifies the members sequentially.

Approximate methods can be divided into three main classes; iterative, series expansion and reduced basis techniques.

With iterative techniques the degree of approximation can often be readily measured and further iteration can be undertaken if the accuracy is not high enough. Iterative techniques include the modified Newton-Raphson method in which the out of balance force due to modifications is calculated and applied as an effective load to the original stiffness relations. The process requires iteration to ensure that the load is transferred to the other members and is therefore often called the 'stress transfer' or 'residual force' method. The use of the original stiffness matrix means that the matrix need only be inverted or reduced once. The method is particularly suitable for non-linear analysis where modifications and member stiffnesses depend on the level of loading in each element. The method has now been applied to a large range of problems (29, 38, 43, 57, 84, 119, 120, 121, 122) and several attempts have been made to accelerate this type of iteration (45, 60, 61, 85). The use of the method is confined to small changes because with large changes convergence may be slow or the analysis may diverge as illustrated in Chapter 3. Kirsch and Rubinstein (65) proposed a method to improve convergence by expressing ΔK as the sum of two matrices.

Phansalkar (93) considered splitting the stiffness matrix in different ways and concluded that different iterative solution methods could be generated in this way. Simple, Jacobi and Gauss-Seidel iterative methods were generated and their convergence investigated. The use of Block Gauss-Seidel (where groups of unknowns were adjusted) greatly improved convergence and was shown to be an effective tool for initial and reanalysis problems.

Other approximate techniques include the use of Binomial or Taylor Series Expansions of the modified equilibrium equations. Kirsch and Rubinstein (46) investigated a binomial expansion technique which was shown to be a truncated form of the modified Newton-Raphson technique. The method was shown by Arora and Rim (40) to be only suitable for small changes in stiffness.

Romstad et alia (100) also investigated the use of power series expansions for static, dynamic and stability reanalysis and derived similar expressions. A truncated Taylor series expansion method was used by Storasli and Sobieszczanski (109, 110) to reanalyse structures with large modifications. The method requires calculation of the first order sensitivities and give reasonable accuracy ($\pm$ 15% maximum), in only a fraction of the time for a complete reanalysis. Noor and Lowder (89) investigated the use of Taylor series expansions for the reanalysis of truss problems and developed its use to give first approximations in an iterative process. Later they developed its use with a mixed method of analysis (91) and with substructures (90).

Reduced Basis Methods were investigated by Melosh and Luik (78, 79) and Fenves and Ertas (52). These techniques are based on the fact that the number of variables required for solution are usually far smaller than the degrees of freedom of the system and that often the behaviour of large numbers of the degrees of freedom are dictated by the topology of the design rather than by the complexity of its behaviour. The work of Melosh and Luik has already been discussed and the contribution of Fenves and Eratas noted. It is important to note however that this method in its approximate form does not necessarily indicate when a mechanism has been formed. Fox and Muira (54) and Schmit and Farshi (103) have also used this technique for the optimisation of space trusses.

Noor and Lowder (89) developed a modified reduced basis method using sensitivity vectors and compared the results with a Taylor series expansion method.

Kavanagh (62) presented an approximate reanalysis technique suitable for a small capacity computer using the normal mode method (59) as a basis. The method assumes that normal modes of the modified structure can be approximated to that of the initial structure using a reduced eigenvector basis. Modifications were introduced into the normal mode equations as a structural non linearity and the equations solved by dynamic relaxation. The method gave errors of up to 15% for the example studied.

As indicated by this review the range of methods is apparently very wide but nearly all technieques are based on superposition principles. In general exact techniques tend to be slow and are only efficient for small modifications. Approximate techniques are efficient but modified solutions can exhibit errors up to 16%.

Comparisons between methods are limited (40, 51, 63, 64, 89, 106) and inconclusive and it appears that there is no overall 'best' technique in either class. The adopted method for any problem should ideally only use additional information that can be readily generated or is already defined for the problem. This indicates that the choice of algorithm is often problem related.

APPENDIX III: NON-LINEAR ALGORITHMS SUITABLE FOR THE DESIGN OF SPACE STRUCTURES.

In Chapter 2 the use of three methods suitable for the topological design of highly connected ground structures were outlined in detail. These methods and many of the other reviewed in Chapter 2 were initially developed for use with problems of fixed topology. There is now a wide range of optimisation methods which may be applied to the design of structures of fixed topology. The following comments and the annotated bibliography (Section E) give an indication of the range of algorithms described in the literature. The reader is referred to the reviews by Gallagher and Zienklewicz (132) and Gellatly and Berke (134, 135) for further details and comparisons of many of these methods.

In the field of linear programming the dual simplex algorithm, as outlined in Chapter 2, is predominantly used. However, many of the techniques applied to non-linear problems can also be used for linear problems (142, 200). Linear methods can also be applied iteratively to non-linear problems. Examples of these techniques include piecewise linearisation and the cutting plane method (70, 71, 139, 145, 147, 148). In the cutting plane method the constraints are linearised and the problem solved, at which point the non-linear constraints are tested and the most seriously violated constraint is linearised and the problem resolved. In this way the problem is iteratively solved and non-linear constraints are gradually added to the problem, until the solution is of satisfactory accuracy. A move limit method was also developed which involves a complete linearisation of the non-linear problem before each linear sub-problem is solved. This method makes use of limits on the permitted variation of the design variables in a typical linearised design field.

Mathematical programming methods which explicitly consider the non-linear response of the structure to changes in the design variables throughout the design process should ideally require less computational time. With non-linear problems most of these methods are liable to lead to local optimum solutions and it is difficult to ensure that any solution is a global optimum. For small problems repeated use of an algorithm from a different starting position gives some confidence to a solution that is repeatedly derived.

The most important concept used to explain these methods is that of the design space, described by axes representing the design variables. The number of design variables, n, is usually large and therefore, the design space defies illustration. This n dimensional space is called a hyperspace.

The simplest non-linear algorithm is that of Hooke and Jeeves (27) in which a direct search is conducted in each variable direction. For a search and move it must be confirmed that an improvement in the objective function will occur and that no constraints will be violated. As the optimum is approached the step length must be reduced. The effects of constraints can be incorporated in the objective function by the use of a penalty function which ensures that the weight of the structure is increased if any of the constraints are violated. Monte Carlo or random search techniques have been used with varying success by some researchers (137, 144). Both these methods have the advantage that derivatives of the constraints need not be calculated.

Other techniques have been devised in which the weight of the structure is reduced in a highly directed way. Once an initial feasible design has been found it is usual to reduce the weight of the structure in the most efficient way by using the modification $A_{i+1} = A_i + s.D_i$ in which the direction vector, D, is the slope of the objective function. When the weight of the structure cannot be reduced any further, without violating the constraints, another direction vector must be chosen. There are many methods available for selecting this vector. Three of the most important types of algorithm are:

(a) Constant Weight Methods:

With these techniques the direction vector is such that after the change in design variables the weight of the structure remains constant. There are an infinite number of directions which can be taken in this hyperplane. Schmit (212) appears to have been the first to use this type of algorithm. Elliott's non-linear programming algorithm outlined in Chapter 2 used this side step type procedure and adopted a direction which moved the design away from the most critically violated constraints and considered the effects of the variation of the design parameters on all the constraints.

(b) Rosen's Gradient Projection Method:

This technique (155) was first developed by Brown and Ang (124) for structural optimisation of elastic rigid frames. This method adopts a vector which moves along the gradient of the design space boundary (constraint) and further reduces the weight of the structure. If the design space is convex this results in a non-feasible point which must be modified and made feasible.

(c) Zoutendijk's method of feasible directions:

This method due to Zoutendijk was developed for structural design by Vanderplaats and Moses (169) and Kowalik (141). The aim of this algorithm is to derive a useable (reduces the weight of the structure) and feasible (will not result in the violation of the constraints) vector. The chosen direction vector is a linear combination of the slope of the objective function and the slope of the active normalised constraints.

Other directed algorithms for unconstrained minimisation have been developed with the use of penalty functions for structural optimisation (132).

All of the above algorithms are suitable for problems where a continuous range of member sizes is available. For most problems only a discrete range of sizes have to be rounded up to the nearest available size. There are however a limited number of specialist algorithms for problems where only discrete member sizes are available (125, 152, 168, 190, 191).

The objective function of the above algorithms is usually taken to be the weight of the structure and the function is therefore normally linear. However if the cost of the structure is taken to be the objective function then the objective function often becomes non-linear. Geometric programming has been developed for structural optimization where the objective function is a polynomial (164, 165). This method converts the primal problem into the dual problem which is usually easier to solve.

The method is efficient when there are fewer terms in the problem than there are variables.

Another specialist optimisation technique is Dynamic Programming which can be used to solve a multi-stage process in a systematic way. At each stage decisions are made, which influence the design procedure for the next. Structural problems solved with this technique include pin jointed cantilevers and multi-storey frames (198, 199, 204, 205). With this method it is not easy to formulate problems in a general way and for large problems storage requirements may become excessive.

For some problems it is possible to find, in closed form, the conditions that the optimum design has to fulfil. These conditions, called optimality criteria, appear as relations between stresses and displacements and the design parameters. These criteria have been used by Gellatly and Berke (132, 134, 135) to derive optimum structures. The techniques of fully stressed design (123, 129, 131, 132, 140, 146, 149, 151, 153, 158) represent optimality criteria which are not necessarily correct except in the case of statically determinate structures subject to a single loading case.

For large structures the above algorithms (perhaps with the exception of fully stressed design techniques) may become unwieldy. The use of approximating techniques for design and analysis must then be considered (see Appendix II and 103, 78, 79). Although many of the above algorithms are automated their use must be carefully assessed. Perhaps the greatest future for these methods in Civil Engineering is for the design of small components and for interactive computer design where these methods can be used as a guide by the engineer/designer.

AN ANNOTATED BIBLIOGRAPHY CONCERNING THE COMPUTER DESIGN AND OPTIMISATION OF SPACE STRUCTURES

A. SPACE STRUCTURES, ARCHITECTURAL AND NATURAL FORM

B. CONCEPTS OF DESIGN

C. ANALYSIS AND REANALYSIS

D. FULLY STRESSED DESIGN, LINEAR AND NON LINEAR OPTIMISATION TECHNIQUES

E. TOPOLOGICAL DESIGN OF SPACE STRUCTURES

F. DYNAMIC RELAXATION

G. INTERACTIVE DESIGN

H. ADDITIONAL REFERENCES

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40. Arora, J.S., Rim, K., "An Algorithm for Fail Safe Structural Optimization and a Review of Reanalysis Techniques", Tech. Report, n.11, Dept. of Mechanics and Hydraulics, College of Eng., The Univ. of Iowa, U.S.A., March, 1974.

(Review of reanalysis techniques with special reference to their use for fail-safe optimisation)

41. Bennett, J.M., "Triangular Factors of Modified Marrices", Numerische Mathematik, V.7, 217-221, 1965.

(Further work relating to householders' identity, but working with Gaussian triangularised matrices)

42. Best, G.C., "The Stress Area Method for Designing Beams", J. of the Aeronautical Sciences, V.13, n.3, 151-155, March, 1946.

(Development of non-matrix re-analysis technique based on the use of fictitious forces)

43. Best, G.C., "A Method of Structural Weight Minimization suitable for High Speed Digital Computers", AIAA Journal, V.1, 478-9, Feb. 1963.

(An iterative reanalysis technique using the inverse of the original stiffness matrix and an equivalent load formulation suitable for small changes in the stiffness matrix)

44. Bhatia, K.G., "Rapid Iterative Reanalysis for Automated Design", NASA Technical Note, NASA TN D-7357, NASA, Washington D.C., October, 1973.

(Dynamic Reanalysis in which the generalised stiffness and inertia matrices are expressed as functions of the structural design parameters. These matrices are expanded with a Taylors series about the initial design. The method is approximate because it uses static condensation, modal reduction and linear Taylor series expansions. An extension of the method to static analysis is outlined but not tested on any numerical example)

45. Boyle, E.F., Jennings, A., "Accelerating the Convergence of Elastic-Plastic Stress Analysis", Int. J. of Num. Methods in Eng., V.7, 232-235, 1973.

(Application of modified Aitken acceleration method to modified Newton Raphson non-linear analysis (see reference (61))

47. Cambell, J.S., Roy, J.R., Cook, R.D., discussion of "Efficient Solution of Load Deflection Equations", by Melosh, R.J., Bamford, R.M., J. of Struct. Div. ASCE, V.95, n.ST12, 2993-2997, Dec., 1969.

48. Cicala, P., "Effects of Cut-Outs in Semi-monoque Structures", J. of Aeronautical Sciences, V.15, n.3, 171-179, March 1948.

(Application of 'fictitious force' non-matrix method of reanalysis to semi-monocoque structures)

49. Davies, R.M.(ed), "Space Structures", A study of methods and developments in three-dimensional construction resulting from The Int. Conf. on Space Structures, Univ. of Surrey, Sept. 1966., Blackwell Scientific Publications, Oxford.

(Space Structures - state of the art conference)

50. Elliott, D.W.C., "Structural Optimisation", A thesis submitted for the degree of Doctor of Philosophy to the Univ. of Aston, Birmingham, 1971.

(Thesis concerning non-linear optimisation including Chapters on topological design of pinjointed structures using theorems of structural variation for reanalysis)

51. Ertas, R., Fenves, S.J., "Automatic Analyser for Iterative Design", Civ. Eng. Studies, Structural Research Series n. 352, Univ. of Illinois, Illinois, Sept., 1969.

(Development of modified stiffness, modified flexibility and modified Gauss methods of reanalysis. Similar to the work of McNeal, Kosko and Argyris. The area of modifications must be defined prior to the original analysis when using the Gauss method)

52. Fenves, S.J., Ertas, R., Discussion of "Multiple Configuration Analysis of Structures", by Melosh,R.J., Link, R., J. of Struct. Div. ASCE, V.95, 1586-1589, July, 1969.

(Discusses the required number of self straining vectors required in the reduced basis technique of reference (78,79). Suggests that only r vectors are required for an exact analysis where r is the degree of redundancy of the structure.)

53. Fox, L., "An Introduction to Numerical Linear Algebra", Clarendon Press, Oxford, 1964.

(Classical text which includes all basic methods for solution of linear equations)

54. Fox, R.L., Muira, H., "An Approximate Analysis Technique for Design Calculations", AIAA Journal, V.9., n.1, Jan., 1971.

(Reduced basic method of reanalysis using a series of "basic" designs to represent the behaviour of the modified structure as a linear combination of the resulting independent displacement vectors)

55. Grzedzielski, A.L.M., "Note of some Applications of the Matrix Force Method of Structural Analysis", J. of Royal Aeronautical Soc., V. 64, 354-357, Aug., 1960.

(Criticism of the initial concept for modifications. See references (35,56))

56. Grzedzielski, A.L.M., Argyris, J.H., Kelsey, S., Discussion on "The Initial Strain Concept", J. of Royal Aeronautical Soc., V.65, 127-138, Feb., 1961.

57. Gupta, A.K., Moliraz, B., Schnobrich, W.C., "Elasto-Plastic Analysis of Three Dimensional Structures using the Isoparametric Element", Nuclear Eng. and Design, V.22, 305-317, 1972.

(Application of modified Newton-Raphson method (constant stiffness) to non-linear analysis using higher order elements, with incremental loading techniques)

58. Householder, A.S., "The Theory of Matrices in Numerical Analysis", Blaisdell Pub. Co., New York, 1964.

(Includes an identity for the inverse of a modified matrix (attributed to Woodbury but based on Sherman and Morrison formula))

59. Hurty, W.C., Rubenstein, M.F., "Dynamics of Structures", Prentice-Hall Inc., Englewood-Cliffs, New Jersey, 1964.

(Basic text on structural dynamics)

60. Irons, B.M., Tuck, R.C., "A version of the Aitken Accelerator for Computer Iteration", Int. J. Num. Meth. Eng., V.1, 275-277, 1969.

(Application of acceleration techniques to power iteration of eigen values, modified Newton-Raphson non-linear analysis and relaxation type iteration (see reference (120))

61. Jennings, A., "Accelerating the Convergence of Matrix Iterative Processes", J.Inst. Maths. Applics., V.8, 99-110, 1971.

(Application of modified Aitken δ^2 method for accelerating iterative matrix processes (see reference 45))

62. Kavanagh, K.T., "An Approximate Algorithm for the Reanalysis of structures by the Finite Element Method", Computers and Structures, V.2, 713-722, 1972.

(Approximate re-analysis method suitable for a small capacity computer using the normal mode method as a basis. Modifications are introduced into the normal mode equations as a structural nonlinearity. Solution of these non-linear equations is found by using Dynamic Relaxation. The example structure exhibited errors of up to 15%)

63. Kavlie, D., Powell, G.H., "Efficient Reanalysis of Modified Structures", J. Struct. Div. ASCE, V.97, n.ST1, 377-392, Jan. 1971.

(Discusses work of Argyris (32), Sack, Carpenter & Hatch(102) and Melosh and Luik(79). A reanalysis method is presented using the technique of Sack, Carpenter and Hatch which does not require the explicit formation of the inverse stiffness matrix. An iterative method is also outlined)

64. Kursch, U., Rubinstein, M.F., "Modification of Structural Analysis by the Solution of a Reduced Set of Equations", UCLA Paper ENG-0570, School of Eng. and Applied Sc., Univ. of Cal., Los Angeles, Dec., 1970.

(Comparison of methods based on Sherman-Morrison formula and an explicit reduced equation technique using the identity $K\delta = (K + \Delta K)(\delta + \Delta\delta)$)

65. Kursch, U., Rubinstein, M.F., "Structural Reanalysis by Iteration", Computers and Structures, V.2, 497-510, 1972.

(Iterative reanalysis method using a power series expansion of the effective load technique. Convergence was improved by expressing the matrix of stiffness changes into a linear combination of two matrices)

66. Klein, S., Discussion of "Efficient Solution of Load-Deflection Equations", Melosh, R.J., Bamford, R.M., J. of Struct. Div. ASCE, V.96, n.ST5, 983-985, May, 1970.

67. Kosko, E., "Effect of Local Modifications in Redundant Structures", J. of Aeronautical Sciences, V.21, n.3, 206-207, March, 1954.

(Development of reference (69) for the calculation of the modified inverse stiffness matrix for redundant structures)

68. Livesley, R.K., "Matrix Methods of Structural Analysis", Pergamon Press, Oxford, 1969.

(Basic text on matrix methods of analysis including force and displacement methods)

69. MacNeal, R.H., "Application of the Compensation Theorem to the Modification of Redundant Structures", J. of Aeronautical Sciences, V.20, n.10, 726-727, Oct., 1963.

(Method for calculating the modified inverse stiffness matrix)

70. Majid, K.I., "Non-Linear Structures - Matrix Methods of Analysis and Design by Computers", Butterworths, London, 1972.

(Non-Linear analysis including; Force and Displacement analysis methods, Stability Functions, Elastic-Plastic analysis and Linear Optimisation methods)

71. Majid, K.I., "Optimum Design of Structures" Newnes-Butterworths, London, 1974.

(Basic text on optimum design of structures (mainly linear techniques) which includes an outline of a topological design method which uses theorems of structural variation for the reanalysis of pin jointed structures)

72. Majid, K.I., "Generalised theorems of structural variation for rigidly jointed frames", Int. Conf. on the Behaviour of Slender Structures, London, Sept. 1977.

(Extension of theorems of structural variation to cater for rigidly jointed structures)

73. Majid, K.I., Elliott, D.W.C., "Forces and Deflexions in Changing Structures", The Structural Engineer, V.51, n.3, March, 1973.

(Outline of theorems of structural variation for pin jointed structures)

74. Majid, K.I., Elliott, D.W.C., "Toplogical Design of Pin Jointed Structures by non-linear programming", Proc. I.C.E., V.55, Pt.2, 129-149, Paper n.7539, March, 1973.

(Outline of topological design method using theorems of structural variation)

75. Meek, J.K., "Matrix and Structural Analysis", McGraw Hill Book Co., New York, 1971.

(Basic text on matrix methods of analysis includes sections on Band Solvers, Cholesky Decomposition and the inverse of modified stiffness matrices using the initial strain approach of Argyris)

76. Melosh, R.J., "Structural Analysis, Fraility Evaluation and Redesign", Technical Report No. TR-70-15, Vol.1, Air Force Fligh Dynamics Laboratory, Wright Patterson Air Force Base, Ohio, July, 1970.

(Investigation of Fully Stressed Design Techniques. Uses an exact reanalysis procedure (in preference to iterative techniques) which is a hybrid method involving alternately using an influence method and partially reforming, modifying and redecomposing the stiffness matrix)

77. Melosh, R.J., Bamford, R.M. "Efficient Solution of Load-Deflection Equations", J. Struct. Div. ASCE, V.95, n.ST4, 661-676, April 1969 and discussion reply V.97, n.ST2, 713-717, Feb., 1971.

(Development of integrated procedure for solving load-deflection equations involving three passes, using wavefront processing and a modified Gauss algorithm. This paper together with the discussion (47, 66, 113, 114) represented the State of the art for solution of linear equation systems)

78. Melosh, R.J., Luik, R., "Approx. Multiple Configuration Analysis and Allocation for Least Weight Structural Desing", Wright Patterson, AFFDL-TR-67-59, Ohio, April, 1967.

(Two approximate reduced basis methods are presented. The first is based on a complementary energy approach and the second on a potential energy approach. Minimisation is performed by a sequential search process in which design changes minimise the merit function with respect to each parameter iteratively)

79. Melosh, R.J., Luik, R., "Multiple Configuration Analysis of Structures", J. of Struct. Div. ASCE, V.94, n.ST11, 2581-2596, Nov. 1968.

(Reduced Basis Reanalysis techniques of reference (78))

80. Meyer, C., "Solution of Linear Equations - State of the Art", J. of Struct. Div. ASCE, V.99, n.ST7, 1507-1526, July, 1973.

(Reviews application of methods of Gauss (with and without symmetry) and Decompostion (Gauss, Cholesky, Crout, Doolittle, etc) together with Bond, Frontal, Substructuring and Iterative methods)

81. Meyer, C., "Special Problems Related to Linear Equation Solvers", J. of Struct. Div. ASCE, V.101, n.ST4, 869-890, April 1975.

(Updated review including sections on Reanalysis and Error Techniques)

82. Michielson, H.F., Dijk, A., "Structural Modifications in Redundant Structures", J. of Aeronautical Sciences, V.20, n.4, 286-288, April, 1953.

(Use of ficititious force method for non-matrix reanalysis of redundant structures)

83. Mohraz, B, Wright, R.N., "Solving Toplogically Modified Structures", Computers and Structures, V.3, 341-353, 1973.

(Use of Householders identity to calculate the modified inverse stiffness matrix after joints have been deleted and members modified)

84. Nam, C.J., Salmon, C.G., "Finite Element Analysis of Concrete Beams", J. Struct. Div., ASCE, V.100, n.ST12, 2419-2432, Dec., 1974.

(Application of Newton-Raphson and modified Newton-Raphson Techniques to the non-linear analysis of reinforced concrete beams. Concludes that the modified N-R procedure provides neither an efficient or correct solution)

85. Nayak, G.C., Zienkiewicz, O.C., "Note on the Alpha-Constant Stiffness Method for the Analysis of Non-Linear Problems", Int. J. Num. Meth. in Eng., V.4, 579-582, 1972

(Use of α constant (which is dependent on the current tangent stiffness (uninverted)) to accelerate the Newton Raphson iteration for non-linear problems)

86. Nayak, G.C., Zienkiewicz, O.C., "Elasto-Plastic Stress Analaysis - A Generalisation for various Constitutive Relations including strain softening", Int. J. Num. Meth. Eng., V.5, 113-135, 1972.

(Comparison of Modified Newton-Raphson and Newton Raphson techniques for the analysis of non-linear problems)

87. Noor, A.K., "Non-Linear Analysis of Space Trusses", J. Struct. Div. ASCE., V.100, n,ST3, 533-546, March, 1974.

(Application of Mixed Methods of Analysis to geometric and material non-linear problems. Uses modified versions of the effective load technique with load increments and the Newton-Raphson procedure to solve the non-linear equations)

88. Noor, A.K., "Multiple Configuration Analysis via Mixed Method", J.Struct.Div. ASCE, V.100, n.ST9, 1991-1997, Sept., 1974.

(Mixed method of analysis and reanalysis applied to space truss problems. Suitable for the computer design of structures in which configuration (coordinate) design variables are included)

89. Noor, A.K., Lowder, H.E., "Approximate Techniques of Structural Reanalysis", Computers and Structures, V.4, 801-812, 1974.

(Application of (1) Reduced Basis Methods, (2) Taylor's Series Expansion, (3) Iterative techniques with Taylor series, expansions to the reanalysis problem. Results from space truss structures are presented which show that major problems of reanalysis can be overcome by Iterative-Taylor's series method. A Modified Reduced Basis Method was presented in which the reduced basis was a normalised set of vectors consisting of the original analysed design and the first-order sensitivity analysis vectors. These vectors form a good basis for the modified solution of displacements)

90. Noor, A.K., Lowder, H.E., "Approximate Reanalysis Techniques with substructuring", J. Struct. Div., ASCE, V.101, n.ST8, 1687-1698, Aug., 1975.

(Presents a reduced basis technique with substructuring and asseses the merits of Taylor series expansion and reduced basis-substructuring reanalysis techniques for large complex structures)

91. Noor, A.K., Lowder, H.E. "Structural Reanalysis via a Mixed Method", Computers and Structures, V.5, 9-12, 1975.

(Application of the Taylor's series expansion Mixed Method and comparison with the Modified Reduced Basis Method using the displacement formulation)

92. Pestel, E.C., Leckie, F.A., "Matrix Methods in Elasto-Mechanics", McGraw Hill, New York, 1963.

(Basic Text on matrix analysis of structures includes a section on the reanalysis of modified structures using the initial strain method of Argyris)

93. Phansalkar, S.R., "Matrix Iterative Methods for Structural Reanalysis", Computers and Structures,V.4, 779-800, 1974.

(Detailed study of iterative techniques showing that simple, Jacobi and Gauss-Seidel iterations are the consequence of splitting the coefficient matrix in different ways. Block Guass-Seidel iteration techniques are shown to be effective for reanalysis and initial analysis)

94. Pipes, L.A., "Matrix Methods of Engineering", Prentice Hall, Inc., Englewood Cliffs, N.J., 1963.

(Basic Text on matrix methods of analysis includes a section on the analysis of modified structures using initial strain or stress techniques of Argyris)

95. Poppleton, E.D., "Note on the Design of Redundant Structures", UTIA Tech. Note, n.36, Inst. of Aerophysics, Univ. of Toronot, July, 1960.

(Redesign of structures having undesirable stress distributions using the method of Argyris and Kelsey (34))

96. Poppleton, E.D., "The Redesign of Redundant Structures Having Undesirable Stress Distributions", J. of Aerospace Sciences, V.28, n.5, 347-8, April, 1961.

(Precis of reference (95))

97. Przemieniecki, J.S. "Theory of Matrix Structural Analysis", McGraw Hill Book Co., New York, 1968.

(Basic text on matrix structural analysis includes a section on substructures)

98. Robinson, J., "Structural Matrix Analysis for the Engineer", John Wiley and Sons, New York, 1966.

(Elementary text on finite element methods)

99. Robinson, J., "Integrated Theory of Finite Element Methods", John Wiley and Sons, New York, 1973.

(Advanced text on finite element methods)

100. Romstad, K.M., Hutchinson, J.R., Runge, K.H., "Design Parameter Variation and Structural Response", Int. J. Num. Meth. Eng., V.5, 337-349, 1973.

(Power series reanalysis technique for static, dynamic and stability problems)

101. Rosen, R., Rubinstein, M.F., "Substructure Analysis by Matrix Decompostion", J. Struct. Div. ASCE, V.96, n.ST3, 663-670, March, 1970.

(Substructure analysis techniques using Cholesky decomposition)

102. Sack, R.L., Carpenter, W.C., Hatch, G.L., "Modification of Elements in the Displacement Method", AIAA Journal, V.5, n.9, 1708-1710, 1967.

(Describes the use of Householder's identity for calculating the inverse of modified stiffness matrices)

103. Schmit, L.A., Farshi, B., "Some Approximation Concepts for Structural Synthesis", AIAA Journal, V.12, n.5, 692-699, 1974.

(Use of the Reduced Basis method for reanalysis and Taylor series to approximate the dependence of the structure response on the design variables)

104. Schwarz, H.R., "Numerical Analysis of Symmetric Matrices", Prentice Hall, Englewood Cliffs, N.J.

(Fundamental concepts and theories of matrix techniques. Includes methods for calculating the inverse of symmetric matrices including Cholesky techniques)

105. Sobiezczanski, J., "Structural Modification by Perturbation Method", J.Struct. Div. ASCE, V.94, n.ST12, 2799-2816, Dec., 1968.

(Method of reanalysis similar to initial strain concept but subsequent analyses are referred to the modified solution and the changes are idealised as parallel elements)

106. Sobiezczanski, J., "Evaluation of Algorithms for Structural Modifications", Proc. ASCE Conf. on Appl. of Finite Element Methods in Civ. Eng., Nashville, Tenn., Nov., 1969.

(Assesses efficiency of parallel element method compared with the initial strain method of Argyris (34))

107. Sobiezczanski, J., "Matrix Algorithm for Structural Modification Based upon the Parallel Element Concept", AIAA, J. V.7, n.11, 2132-2139, 1969.

(Same paper as reference (106)).

108. Stewart, K.L., Baty, J., "Dissection of Structures", J. Struct. Div. ASCE, V.93, n.ST5, 217-232, Oct. 1967.

(Partitioning techniques for substructure analysis)

109. Storaasli, O.O., Sobieszczanski, J., "Design Oriented Structural Analysis", AIAA/ASME/SAE 14th Structures, Structural Dynamics and Materials Conf. Williamsburg, Virginia, March, 1973.

(The use of first order terms of Taylor's expansion is proposed for the reanalysis of large structures. An aircraft fuselage structure is used to demonstrate the process. Modifications ranging from -100% (element removal) to +2000% for an individual element and -50% to +50% for multi element removal resulted in errors of less than about 16%)

110. Storaasli, O.O., Sobieszczanski, J., "On the Accuracy of the Taylor Approximation of Structure Resizing", AIAA Journal, V.12, n.2, 231-233, Feb., 1974.

(Precis of reference (110)).

111. Supple, W.S., (ed.), "Proc. 2nd Int. Conf. on Space Structures", Dept. of Civ. Eng., Univ. of Surrey, Guildford, England, Sept., 1975.

(Space Structures - state of the art conference)

112. Tarpy, T.S., Rowan, W.H., Discussion of "Substructure Analysis by Matrix Decompostion" by Rosen, R. and Rubinstein, M.F., J.Struct. Div. ASCE., n.ST10, 2249-2251, Oct., 1970

113. Tezcan, S.S., Kostro, G., Felippa, C.A., Tocher,J.L., Discussion of "Efficient Solution of Load Deflection Equations", Melosh, R.J., Bamford, R.M., J. of Struct. Div. ASCE., V.96, n.ST2, 421-426, Feb., 1970.

114. Venkayya, V.B., Irons, B.M., Discussion of "Efficient Solution of Load Deflection Equations", by Melosh, R.J., Bamford, R.M., J. Struct. Div. ASCE, V.96, n.ST1, 148-153, Jan., 1970.

115. Wiberg, N-E., Discussion of "Substructure Analysis by Matrix Decomposition", by Rosen R. and Rubinstein, M.F., J. Struct. Div. ASCE, V.97, n.ST2, 724-726, Feb., 1971.

116. Wiberg, N-E., Discussion of "Efficient Reanalysis of Modified Structures" by Kavlie, D., Powell, G.H., J. Struct. Div. ASCE, V.97, n.ST2, 2612-2619, Oct., 1971.

(Discussion includes the presentation of a mixed method of reanalysis)

117. Wiberg, N-E., "System Analysis in Structural Mechanics by use of Force and Displacement Variables" Thesis presented to Chalmers Univ. of Technology, Gothenburg, Sweden, 1970.

118. Zeilke, G., "Numerical Computation of Modified Inverse Matrices and Linear Systems of Equations" Verlag Friedrich Viehweg und Sohn, GmbH, Braunschweig, Germany, 1970.

(See reference (81))

119. Zienkiewicz, O.C., "The Finite Element Method in Engineering Science", McGraw Hill, London, 1971.

(Basic text on Finite Element Displacement method. Includes a section on Non-Linear Analysis using Newton-Raphson techniques, modified Newton-Raphson etc.)

120. Zienkiewicz, O.C., Irons, B.M., "Matrix Iteration and Acceleration Processes in Finite Element Problems of Structural Mechanics", Chapt.9, of "Numerical Methods for Non Linear Algebraic Equations", Ed. Rabinowitz, P., Gordon and Beach Science Pub., London, 1970.

(Outlines the acceleration of modified Newton-Raphson iteration using the Aitken δ^2 method)

121. Zienkiewicz, O.C., Valliappan, S., King, I.P., "Stress Analysis of Rock as a 'No Tension' Material", Geotechnique, V.18, 56-66, 1968.

(Modified Newton-Raphson iteration using constant stiffness. Stresses in cracked elements relieved using load transfer process)

122. Zienkiewicz, O.C., Valliappan, S., King, I.P., "Elasto-Plastic Solutions of Engineering Problems 'Initial Stress' Finite Element Approach", Int. J. Num. Meth. Eng., V.1, 75-100, 1969.

(Application of modified Newton-Raphson iteration using constant stiffness to elasto-plastic problems)

D. FULLY STRESSED DESIGN, LINEAR AND NON-LINEAR OPTIMISATION TECHNIQUES.

(Also see section E and references 76, and 271)

123. Bartholomew, P., Morris, A.J., "A Unified Approach to Fully-Stressed Design", Engineering Optimization, V. 2, 3.- 15, 1976.

(Application of fully stressed methods of design. Considers the problems of slack constraints when minimum member sizes are used. Comparisions between the convergence rates and stability of various fully stressed methods are also presented.)

124. Brown, D.M., Ang, A.H.S., "Structural Optimization by Non-Linear Programming", J. Struct. Div. ASCE., V. 92, n. ST6, 319 - 340, Dec. 1966.

(Application of Rosen's gradient projection method (155) to non-linear structural optimization of elastic rigid frames. Suggest starting the algorithm at a few widely different but feasible points to ensure that a global optimum is derived. This technique is necessary because the design space is not necessarily convex.)

125. Cella, A., Logcher, R.D., "Automated Optimum Design from Discrete components", J. Struct. Div. ASCE., V. 97, n. ST1, 175-189, Jan. 1971.

(Non-Linear algorithm for the design of structures from discrete components using an integer programming search technique. Displacement constraints are not considered.)

126. Chao, H., discussion of "Optimum Structural Design using Linear Programming", Moses, F., J. Struct. Div. ASCE., V. 91, n. ST3, 292 - 296, June, 1965.

127. Cooper, L., Steinberg, D., "Introduction to Methods of Optimization", W.B. Saunders Co,, London, 1970.

(Basis Introduction to Optimization Methods includes sections on; Linear Programming, Search Techniques (Unconstrained Problems), Non-Linear Programming and Integer Programming.)

128. Dantzig, G.B., "Linear Programming and Extensions", Princeton Univ. Press, Princeton, New Jersey, U.S.A., 1963.

(Basic texton Linear Programming includes a Chapter on the proof of the simplex algorithm and the duality theorems.)

129. Dayaratnam, P., Patnaik, S., "Feasibility of Full Stress Design", AIAA Journal, V. 7, n. 4, 773-774, April, 1969.

(Shows that for a structural framework with N members and M degrees of freedom with only one load condition, only M member stresses can arbitrarily be selected as fully stressed the remaining (N-M) member stresses are automatically fixed. (N-M) is the degree of redundancy. If all N members are to be fully stressed then the minimum number of loading cases required is N/M, taking M independent members to be fully stressed in each loading case.)

130. Dorn, W.S., Greenberg, H.J., "Linear Programming and Plastic Limit Analysis of Structures", Q. Appl. Maths., V. 15, n.2, 155-167, 1957.

(Outlines method of linear programming for structural analysis. Suggests methods to keep the problem as small as possible. Includes a discussion of problems with bounded variables.)

131. Dwyer, W., Rosenbaum, J., Shulman, M., Pardo, H., "Fully-stressed Design of Airframe Redundant Structures", AFFDL-TR-68-150, 155-180, Proc. 2nd. Conf. on Matrix Methods in Structural Mechanics, Dayton, Ohio, Oct. 15-17, 1968.

(Two methods of fully stressing structures, one based on the 'average stress' in each element and another based on the 'nodal stresses'. The first method is shown to result in large discontinuities in material distribution, the second requires more programming.)

132. Gallagher, R.H., Zienkiewicz, O.C., "Optimum Structural Design-Theory and Applications", John Wiley and Sons, London, 1973.

(Review and State of the art papers on most of the aspects of optimization including: Basic Concepts, Fully Stressed Design, Optimality Criterion based algorithms, Mathematical Programming Methods, Dynamic Programming, Discrete Variable Optimization - Computer Aided Design, etc.)

134. Gellatly, R.A., Berke, L., "Optimal Structural Design", Technical Report AFFDL-TR-70-165, AD 725745, USAF, April, 1971.

(Application of an optimality criteria method to indeterminate structures under multiple loading with stress and displacement constraints.)

135. Gellatly, R.A., Berke, L., "A Review of Some Aspects of Structural Optimization", World Congress on Finite Element Methods in Structural Engineering, Bournemouth, Dorset, England, Oct. 1975.

(Over view of three optimization methods: Optimality criteria, mathematical programming and sieve search.)

136. Hadley, G., "Linear Programming", Addison-Wesley Pub., Co., Reading, Massachusetts, 1962.

(Basic text on linear programming.)

137. Hartmann, D., "Optimization of Sheet Structures illustrated with Reference to Long Cylindrical Shells", (translation British Library) Internationales Symposium Weitgespannte Flachentragwerke, SFB 64, Work Document 4.8, University of Stuttgart, 1976.

(Application of Monte Carlo, random direction search, method to the optimization of cylindrical shell structures.)

138. Joos, G., "Theoretical Physics", Blackie and Son Ltd., London, 1965, 113-117.

(Outline of Lagrange's method applied to systems of linear equations.)

139. Karihaloo, B.L., Pathare, P.R., Ramesh, C.K., "The optimum design of space structures by linear programming using the stiffness matrix method of analysis", Chapter 25, "Space Structures" ed. Davies, R.M., Int. Conf., Univ. of Surrey, Sept. 1966, Blackwell Scientific Publications, Oxford.

(Use of first order Taylor's series expansions to express constraints as linear functions. The optimization is then performed iteratively using the simplex method with modifications for upper and lower bounds. This is a further application of the cutting-plane method.)

140. Kicher, T.P., "Optimum Design-Minimum Weight verses Fully Stressed", J. Struct. Div. ASCE,, V. 92, n. ST6, 265-279, Dec. 1966.

(Investigation of fully stressed or simultaneous failure mode design compared with non-linear optimisation. Concludes that the minimum weight design of a statically determinate stress limited truss will be constrained by at least as many constraints as there are members. (That is the stress in each member must be at its bound in at least one load condtion.) The minimum weight design of structures subject to multiple loading cases are either (a) statically indeterminate or (b) have buckling modes dependent on the loading condtions will not, in general, be fully stressed.)

141. Kowalik, J., "Feasible Direction Methods", AGARD Conf. Proc. n. 36 on Symposium on Structural Optimization, AGARD-CP-36-70, Istanbul, 1969.

(Survey of feasible direction methods for non-linear programming with an outline of the methods of Zoutendijk (which leads to a linear sub-problem to calculate a useable feasible direction which reduces the objective function), Rosen (gradient projection which uses the gradient of the active constraint functions) and Gellatly (using a linear combination of the gradients of the objective function and the active constraint functions.)

142. La Pay, W.S., Goble, G.G., "Optimum Design of Trusses for Ultimate Loads", J. Struct. Div. ASCE., V.97, n. ST1, 157-174, Jan. 1974.

(Limit analysis based design of transmission towers accounting for buckling and multiple loading. Buckling is accounted for by repeated use of the simplex algorithm using improved estimates of the buckling stress from the previous simplex solution. Comparison with an unconstrained minimisation penalty function algorithm using the redundant forces as design variables. Application of methods to the design of a ten member truss is presented. Concludes that linear methods are not suitable for large problems with a large number of loading cases.)

143. Livesley, R.K., "The Automatic Design of Structural Frames", Quart. Journ. Mech. and Applied Math., V. 9, Pt. 3, 257-278, 1956.

(Application of a linear method to the design of Structural frames. Defines the sum of the maximum plastic moment sustained by any member in a group multiplied the group lengths to be the weight or objective function to be minimised. This presentation considered a force formulation without compatibility requirements.)

144. Lootsma, F.A., "Numerical Methods for Non-Linear Optimization", Conf. Sponsored by The Science Research Council, Univ. of Dundee, Scotland, Academic Press, London, 1972.

(Conference outlines the state of the art of the most important Non-Linear Optimization Methods. Includes papers on Unconstrained Methods, Conjugate Gradients, Random Search Methods, Gradient Projection Methods, etc.)

145. Majid, K.I., Anderson, D., "Optimum Design of Hyperstatic Structures", Int. J. for Num. Methods in Eng., V. 4, 561-578, 1972.

(Piecewise linearisation techniques for the optimisation of frames using the matrix force method to formulate the constraints.)

146. Melosh, R.J., "Convergence in Fully-Stressed Designing", AGARD Conf. Proc. n.36 on Symposium on Structural Optimisation, AGARD-CP-36-70, Istanbul, 1969.

(An investigation of optimality, convergence and rate for convergence of the fully stressed design technique. Modifications of the process are described to treat discrete sizes, multiple degree of freedom elements, group sizes and convergence characteristics.)

147. Moses, F., "Optimum Structural Design using Linear Programming", J. Struct. Div. ASCE., V. 90, n. ST6, 89-104, Dec. 1964.

(Use of first order Taylor's series approximations to define constraints near the current best solution. This defines a linear programming problem which is referred to as the cutting plane method. The method is used iteratively to solve the optimization problem. A three bar truss and a one storey rigid frame are presented as examples.)

148. Moses, F., Onoda, S., "Minimum Weight Design of Structures with Application to Elastic Grillages", Int. J. for Num. Methods in Eng., V. 1, 311-331, 1969.

(Outlines the stress-ratio, cutting plane and useable feasible directions methods. A comparison for the efficiency of these methods is given for elastic grillage designs which are shown to have non-convex stress constraints and numerous relative optima.)

149. Patnaik, S., Dayaratnam, P., "Behaviour and Design of Pin Connected Structures", Int. J. for Num. Methods in Eng., V. 2, 579-595, 1970.

(Investigates the use of a force method to achieve fully stressed designs. Concludes that necessary condition to achieve a preassigned stress in an indeterminate truss by changing member areas is $L \gg M/(M - R)$. Where L is the number of load conditions, M are the number of members and R the number of redundants in the truss. Investigates the relations between fully stressed and minimum weight structures.)

150. Peterson, E.L., "An Introduction to Mathematical Programming", from "Optimization and Design", ed. Avriel, M., Rijckaert, M.J., Wilde, D.J., Prentice Hall Inc., Englewood Cliffs, New Jersey, 1973.

(Summary of Mathematical programming concepts.)

151. Razani, R., "Behavior of Fully Stressed Design of Structures and Its Relationship to Minimum-Weight Design", AIAA Journal, V. 3, n. 12, 2262-2268, Dec. 1965.

(Use of Kuhn-Tucker optimality condtion of non-linear programming to find the necessary condition for the equivalence of fully stressed and minimum weight designs. Method for verification of the optimality of a fully stressed design is developed and a method for determining the optimum is presented.)

152. Reinschmidt, K.F., "Discrete Structural Optimisation" J. Struct. Div. ASCE., V. 97, n. ST1, 133-156, Jan. 1971.

(Discrete optimisation using integer linear programming for plastic design and a first order term Taylor series formulation with integer linear programming for elastic design.)

153. Reinschmidt, K.F., Cornell, C.A., Brotchie, J.F., "Iterative Design and Structural Optimisation", J. Struct. Div., ASCE, V. 92, n. ST6, 281-318, Dec., 1966.

(Comparison of fully stressed design techniques including (a) the stress-ratio technique (b) a technique using partial derivatives (with Taylor series expansion to assess the effects of changes in the member stress due to changes in the member area) and (c) a technique which considers the effects of the other member forces varying due to changes in a member. Each technique often derives differant but fully stressed designs. A technique using the sensitivities and linear programming, to ensure minimum weight, was also presented.)

154. Reinschmidt, K.F., Norabnoompipat, T., "Structural Optimisation by Equilibrium Linear Programming", J. Struct. Div. ASCE., V. 101, n. ST4, 921-938, April, 1975.

(Comparison of the equilibrium linear programming techniques (which iteratively ensure compatibility) applied to elastic planar grillages with the results of Moses and Onoda (148). The iterative linear programming techniques in which compatibility is initially relaxed appear more likely to derive globally optimum solutions.)

155. Rosen, J.B., "The Gradient Projection Method for Non-Linear Programming. Part II. Non-Linear Constraints", J. Soc. Indust. Appl. Math. V.9, N.4, Dec. 1961.

(Algorithm of the type $x_{i+1} = x_i + S.dx$. In which the gradient of the objective function is used until a boundary design is achieved. The vector is then modified to move along the gradient of the design space boundary while further reducing the weight. If the boundary is concave this results in an infeasible point which must be modified so feasible. The cycle must then return to the steepest descent mode.)

156. Rubinstein, M.F., Karagozian, J., "Building Design using Linear Programming", J. Struct. Div. ASCE., V.92, n.ST6, 223-245, Dec, 1966.

(A Linear Programming Method is presented with facilities to control deflections in the rigid-plastic minimum weight design of tall building frames by assuming that 60% of the sway deflection was due to beam flexibility. This method is suitable for preliminary design. The member area sizes were assumed to vary linearly with the storey height.)

157. Russell, A.D., Reinschmidt, K.F., discussion of "Optimum Design of Trusses for Ultimate Loads", Lapay, W.S., Goble, G.G., J. Struct. Div. ASCE, V. 97, n. ST9, 2437-2442, Sept. 1971.

158. Schmidt, L.C., "Fully Stressed Design of Elastic Redundant Trusses under Alternative Load Systems", Australian J. of Applied Science, V. 9, 337-348, Melbourne, 1958.

(Notes that there will be a number of fully stressed designs and states that the 'optimum' structure will be the lightest of these. Suggests that any one fully stressed design will give a good, efficient structure.)

159. Schmidt, L.A., "Structural Engineering Applications of Mathematical Programming Techniques", AGARD Conf. Proc. n. 36 on Symposium of Structural Optimization, AGARD-CP-36-70, Istanbul, 1969.

(Review of Mathematical programming techniques.)

160. Schmidt, L.A., Fox, R.L., "An Integrated Approach to Structural Synthesis and Analysis", AIAA Journal, V. 3, n. 6, 1104-1112, June, 1965.

(Steepest descent type algorithm with an unconstrained minimisation formulation which reduces the weight and uses a penalty function to account for constraints.)

161. Shen, C.Y., Prager, W., "Recent Developments in Optimal Structural Design", Applied Mechanics Reviews, V. 21, n. 10, 985-992, October, 1968.

(Review of optimization methods. (only includes papers published after 1962))

162. Spillers, W.R., Al-Banna, S., "Optimization Using Iterative Design Techniques", Computers and Structures V. 3, 1263-1271, 1973.

(Application of iterative design techniques using Newton's method and the Kuhn-Tucker Conditions. The scheme is generalised for sandwich plate and frame design where Hooke's Law is relaxed.)

163. Spunt, L., "Optimum Structural Design", Prentice Hall, Inc., Englewood Cliffs, New Jersey.

(Chapter 1 includes an outline of design methodology and some standard optimisation techniques.)

164. Templeman, A.B., "The use of Geometric Programming Methods for Structural Optimization", AGARD Lecture Series n. 70, on Structural Optimization, AGARD-LS-70, Istanbul, 1969.

(Application of geometric programming to component design and trusses.)

165. Templeman, A.B., "Structural Design for Minimum Cost using the method of geometric programming", Proc., ICE, Paper n. 7295, 459-472, 1970.

(Introduction to geometric programming which is a technique for finding the minimum of a polynomial function subject to several polynomial constraints. This method transforms the primal problem into a dual problem with only linear constraints. This technique becomes inefficient if the number of terms in problem is large compared with the number of variables.)

166. Toakley, A.R., "The Design of Optimum Triangulated Frameworks", Research Report n. 3, Dept. of Civ. Eng., Univ. of Manchester, March, 1967.

(Design of statically determinate pin jointed space frames subject to deflection controls using piecewise linearisation. The deflection constraints are linearised and the objective function becomes non-linear but convex. However, for statically indeterminate structures the procedures presented do not necessarily derive global optimum solutions.)

167. Toakley, A.R., "Optimum Design of Triangulated Frameworks", Int. J. Mech. Sci., V. 10, 115-127, 1968.

(Precis of above report.)

168. Toakley, A.R., "Optimum Design using Available Sections", J. Struct. Div. ASCE., V. 94, n. ST5, 1219-1241, May, 1968.

(A method for the optimisation of statically determinate frameworks with discrete member sizes using 'zero-one variables' and linear programming is presented in this paper. Two methods for rigidly jointed frame works are presented (a random search method and Gomory's algorithm.))

169. Vanderplaats, G.N., Moses, F., "Structural Optimisation by Methods of Feasible Directions", Computers and Structures, V. 3, 739-755, 1973.

(Application of Zoutendijk's method of feasible directions to structural optimisation of redundant trusses. The algorithm uses the gradient of the objective function and active constraint functions to calculate a useable feasible direction vector which also reduces structural weight.)

170. Wagner, H.M., "The Dual Simplex Algorithm for Bounded Variables", Naval Research Logistics Quarterly, V. 5, 257-261, 1958.

(Application of the dual simplex algorithm with bounded variables.)

171. Wagner, H.M., "Principles of Operations Research with Applications to Managerial Decisions", Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

(Basic text on linear programming methods which outlines the simplex technique. Excellent section on duality principles.)

172. Wolfe, P., "Recent Developments in Non Linear Programming", A Report prepared for USAF Project Rand, R-401-PR, May, 1962.

(Review of the early developments in Non-Linear optimisation techniques mainly iterative linear approximations.)

173. Young, J.W., Christiansen, H.N., "Synthesis of a Space Truss based on Dynamic Criteria", J. Struct. Div. ASCE., V. 92, n. ST6, 425-442, Dec. 1966.

(Synthesis technique based on fully stressed design technique and the resonant frequency requirements. The structure is designed for two alternative inertial loadings produced by resonances. The member areas, mode shapes at resonance, inertial loads and design stresses are unknowns which must all be iterated upon.)

E. TOPOLOGICAL DESIGN OF SPACE STRUCTURES

(Also see references: 50, 70, 71, 72, 73, 74, 78, 88)

174. Barta, J., "On the Minimum Weight of Certain Redundant Structures", Acta. Tech. Acad. Sci. Mungar., V. 18, 67-76, 1957.

(Generalises the theorem of Sved (223) to account for the variation of compressive stress due to buckling effects and shows that although wide differences exist between within a class of structure for alternative fully stressed designs, the optimum is always statically determinate.)

175. Chan, A.S.L., "The Design of Michell Optimum Structures", College of Aeronautics, Cranfield, Report n. 142, Dec. 1960.

(Application of Michell's Theorem (194) to design of structures using the analogy of the theory of plane plastic flow. Expressions for calculating sizes and volumes are developed along with a graphical construction method for structural layout.)

176. Chan, H.S.Y., "Optimum Structural Design and Linear Programming", College of Aeronautics, Cranfield, Report n. 175, Sept, 1964.

(Application of linear programming to pin jointed structures subject to single loading cases. Multiple loading systems also considered by varying the force in the redundants until structure is an optimum.)

177. Cilley, F.H., "The Exact Design of Statically Indeterminate Frameworks. An Exposition of its Possibility, but Futility", ASCE, V. 43, (with discussion), 351-443, June, 1900.

(Nineteenth century views on the superiority of statically determinate structures compared with indeterminate structures.)

178. Corcoran, P.J., "Configurational Optimization of Structures", Int. J. Mech. Sci., V. 12, 459-462, 1970.

(Study of three bar truss previously studied by Schmit (212) and Sved and Ginos (224) using both member areas and nodal coordinates as design variables. Shows that optimum geometry (position) and topology can be derived by variation of nodal coordinates.)

179. Corcoran, P.J., "The Design of Minimum Weight Structures of Optimum Configuration" A thesis submitted for the degree of Doctor of Philosophy of The City University, 1970.

(Computational details of reference (178) plus additional examples. Outlines the use of Rosen's gradient projection method (155) and sequential unconstrained minimisation technique to configurational optimisation of pin jointed structures subject to multiple loading cases.)

180. Cox, H.L., "The Design of Structures of Least Weight", Pergamon Press, London, 1965.

(Includes a chapter on Layout which is based on Michell's work.)

181. Dobbs, M.W., Felton, L.P., "Optimization of Truss Geometry", J. Struct. Div. ASCE, V.95, n. ST10, 2105-2118, Oct, 1969.

(Application of non-linear programming to the optimisation of pin jointed ground structures subject to multiple loading cases. A steepest descent-alternate mode algorithm was used and member areas which were zero at the end of a steepest descent stage were deleted. The effects of local buckling on compressive stresses were only considered when the topology was fixed. The effects of varying the nodal positions of the ground structure, on topology and optimum volume were also investigated in a parametric study.)

182. Dorn, W.S., Gomory, R.E., Greenberg, H.J., "Automatic Design of Optimal Structures", Journal de Mecanique, V. 3, n 1, Mars, 1964.

(Application of dual linear programming techniques to pin jointed ground structures. The variation of the effects of the possibility of accounting for multiple loading cases is discussed but no examples are presented.)

183. Estrada-Villegas, J.E., "Optimum Design of planar trusses using linear programming", A thesis submitted to the Massachusetts Institute of Technology, at Cambridge, Mass, in partial fulfillment of the requirements for the degree of Master of Science, 1965.

(Describes the repeated application of Linear programming techniques to the optimisation of plane trusses using member areas and nodal co-ordinates as design variables. The effects of buckling are also studied.)

184. Farshi, B., Schmit, L.A., "Minimum Weight Design of Stress Limited Trusses", J. Struct. Div. ASCE, V. 100, n. ST 1, 97-107, Jan, 1974.

(Optimisation of indeterminate trusses under multiple loading conditions with member size and stress constraints using a force formulation with the simplex method is uded to derive a solution using the displacement method. Unnecessary members are removed from the structure to enable global optimum structures to be derived.)

185. Francis, A.J., "Direct Design of Elastic Statically Indeterminate Triangulated Frameworks for single Systems of Loads", Australian J. Appl. Sci. V. 4, 175-85, 1953.

(Hand design method for indeterminate trusses under a single loading case. Physical compatibility approach (c.f. reference (202)). Notes that it is not usually possible to fully stress an indeterminate structure unless prestress effects are considered.)

186. Fu, Kuan-Chen, "An Application of Search Techniques in Truss Configuration Optimization", Computers and Structures, V. 3, 315-328, 1973.

(Use of an iterative search technique for the design of pin jointed trusses subject to multiple loads with member areas and nodal position design variables. The response surface of a joint is shown to be unimodal and comparison of relevant structures shows similarity with Michell structures.)

187. Hemp, W.S., "Studies in the theory of Michell Structures", Proc. Int. Congr. Appl. Mech., Munich, 1964.

(Includes a section on the derivation of approximate Michell structures using the dual linear programming technique to maximise the virtual work and hence minimise the volume using a ground structure. Unnecessary joints and members were deleted from a ground structure subject to a single load case.)

188. Hemp, W.S., "Optimum Structures", Clarendon Press, Oxford, 1973.

(Fundamental text on optimum structures. Includes a section on linear programming techniques and a chapter on layout and Michell structures.)

189. Hamp, W.S., Chan, H.S.Y., "Optimum Design of Pin Jointed Frameworks", Ministry of Technology, Aeronautical Research Council, R. & M. n. 3632, H.M.S.O., London, 1970.

(Application of dual linear programming techniques to the design of pin jointed structures subject to a single trading case. Includes a program written in Algol.)

190. Lipson, S.L., Agrawal, K.M., "Weight Optimisation of Plane Trusses", J. Struct. Div. ASCE., V. 100, n. ST5, 865-879, May, 1974.

(Minimisation of plane trusses considering member areas and nodal co-ordinates as design variables under multiple loading cases. Members and nodes were deleted if required and the degrees of freedom modified if two or more joints coalesce. A modified 'complex' method was used to optimise the structure. This method assumes that the design space is convex. Discrete member sizes and displacement constraints can be accounted for throughout this design process. This method does not ensure a global optimum. design problems are compared with the results of reference (227).)

191. Lipson, S.L., Gwin, L.B., "Discrete Sizing of Trusses for optimal Geometry", J. Struct. Div. ASCE., V. 103, n. ST5, 1031-1046, May, 1977.

(Further work relating to reference (190). Considers separation of the design spaces using the 'complex' method for geometry optimisation and the stress ratio method and displacement scaling for member sizing. The method accounting for discrete member sizes with stress and displacement constraints is outlined in detail.)

192. Mayerjak, R.J., "On the Weight and Design of a Redundant Truss", Aeron. Res. Lab., Office of Aerospace Res., U.S.A.F., ARL 62-338, AD 278 293, April, 1962.

(Discusses the characteristics of elastic fully stresses and plastic optimum design in relation to a three bar truss subject to multiple loading cases. Points out that the removal of members in the fully stressed case is a consequence of the compatibility requirements.)

193. Maxwell, J.C., "On Reciprocal Figures, Frames and Diagrams of Forces", Scientific Papers, Cambridge Univ. Press, V. 2, 175-177, 1890.

(Theorem relating sum of products of each attraction and length of each member to the sum of products of each repulsion and length of each member of pin jointed structure in equilibrium under the action of external loads.)

194. Michell, A.G.M., "The Limits of Economy of Materials in Frame Structures", Phil. Mag., S.6, V. 8, n. 47, 589-597 1904.

(Development of Maxwell's theorem. Presents theorem of minimum structures for the layout of pin jointed structures subject to a single loading condition.)

195. Owen, J.B.B., "The Analysis and Design of Light Structures' Edward Arnold, London, 1965.

(Includes a chapter on the design of structures using Michell's theorem.)

196. Parkes, E.W., "Braced Frameworks", Pergamon Press, London, 2nd Edn., 1974.

(Introduction to the theory of frameworks includes a section on the minimum weight layout of trusses.)

197. Parkes, E.W., "Joints in Optimum Frameworks", Int. J. Solids Structures, V. 11, 1017-1022, 1975.

(Theoretical investigation considering the effects of the cost of joints when deriving the optimum form of networks under simple loading conditions.)

198. Palmer, A.C., "Optimum Structure Design of Dynamic Programming", J. Struct. Div. ASCE, V. 94. n. ST8, Aug, 1968.

(Application of dynamic programming to continuous beams and frames. The method of dynamic programming is shown to be only efficient when the interaction between different parts of the structure can be expressed with a few variables to ensure that the minimisation of each stage is efficient.)

199. Palmer, A.C., Sheppard, D.J., "Optimizing the shape of pin jointed Structures", Proc. ICE, V. 47, 363-376, 1970.

(Application of techniques of Dynamic programming to the shape design of pin jointed cantilever structures. The method shows up to a 20% saving on conventional shaped structures. The method is also extended to cater for asymmetry, alternate loading conditions and discrete member dizes. A solution of this type is shown to have a 9% saving.)

200. Pearson, C.E., "Structural Design by High-Speed Computing Machines", Conf. on Electronic Computation, Structural Division, ASCE., Kansas City, Missouri, November, 1958.

(Optimisation of pin jointed structures considering equilibrium conditions only and using the force in the redundant members as the design variables. The algorithm uses a random direction vector to derive optimum solutions. Multiple loading cases were considered assuming that each member must sustain the maximum stress in at least one of the loading cases. Pearson showed that if only one loading case was considered then only a statically determinate set of members have non-zero areas and if two loading cases are considered then r members are at a maximum stress for both loads. (Where r is the degree of redundancy for the structure.))

201. Pedersen, P., "On the Minimum Mass Layout of Trusses", AGARD Conf. Proc. n. 36, Symposium on Structural Optimisation, AGARD-CP-36-70, 1970.

(Application of linear programming and sensitivity analysis to the configurational optimisation of pin jointed structures subject to a single loading case. Calculates the sensitivity of the nodal co-ordinate movements with respect to the mass of the structure and uses an iterative simplex procedure to account of stability and self weight.)

202. Pippard, A.J.S., "On a method for the direct design of framed structures having redundant bracing", Aero, Res. Committee, R. & M. n. 793, May, 1922.

(Describes a process using the flexibility method for the hand design of statically indeterminate pin jointed structures under the action of one loading case.)

203. Pope, G.G., "The Application of Linear Programming Techniques in the Design of Optimum Structures", Symposium on Structural Optimization, Conf. Proc. n. 36, AGARD-CP-36-70,

(Reviews linear programming techniques suitable for the repeated application to non-linear problems and the work of Hemp on the strictly linear problem of optimising elastic or perfectly plastic pin jointed frameworks. Includes a short discussion of the problem of multiple loads.)

204. Porter Goff, R.F.D., "Decision Theory and Shape of Structures", J. Roy. Aero. Soc., Vol. 70, 448-452, March, 1964.

(Describes the application of the sequential decision making technique called Dynamic Programming to the design of the shape of structures. Examples include layout of two classes of cantiliver, the constant depth beam and the statically determinate truss. Comparisons with Michell structures are given.

205. Porter Goff, R.F.D., "Dynamic Programming and the Shape of a Bridge Truss", Int. Conf. Computer Aided Design, Univ. of Southampton, April, 1969.

(Extends work of reference (204) to include the live loads when optimising the layout of a bridge truss. The effects of instability and yield design criteria are also considered.)

206. Reinschmidt, K.F., Russell, A.D., "Linear Methods in Structural Optimization", Research Report R 70-41, Dept. of Civ. Eng., Cambridge, Mass, July, 1970.

(Reviews linear programming techniques in structural optimisation. Presents an iterative method to cater for buckling compressive stresses which encourages inefficient members to be removed.)

207. Reinschmidt, K.F., Russell, A.D., "Applications of Linear Programming in Structural Layout and Optimization", Computers and Structures, V. 4, 855-869, 1974.

(Describes an iterative technique using the simplex linear programming method to derive the form of the structure and the fully stressed design technique to assess the buckling stress and ensure compatibility.)

208. Richards, D.M., Chan, H.S.Y., "Developments in the theory of Michell optimum structures", AGARD Report n. 543, April, 1966.

(Development of Michell's theorem for use in designing practical structures composites, variable thickness plates and shells etc. Includes a section on the use of the linear programming method for designing approximate Michell structures by maximising the Virtual Work of the applied loads while constraining the member strains.)

209. Russell, A.D., Reinschmidt, K.F., Discussion on "Optimum Design of Trusses for Ultimate Loads", Lapay, W.S., Goble, G.G., J. Struct. Div. ASCE., V. 97, n. ST9, 2437-2442, 1971.

(Outlines the iterative method described in detail in references (206, 207))

210. Scheffey, C.F., "Optimization of Structures by Variation of Critical Parameters", Proc. 2nd Nat. Conf. on Electronic Computation, Pittsburgh, ASCE, 135-144, Sept. 1960.

(Qualitative study of design parameter hierarchy on the effects of efficiency in optimisation methods. Suggests using a method of rate of slope changes along each parameter plane for the adjustment of geometrical design factors.)

211. Schmidt, L.C., "Minimum Weight Layouts of Elastic, Statically Determinate, Triangulated Frames under Alternative Load Systems", J. Mech. Phys. Solids., V. 10, 139-149, 1962.

(Extension of Michell theorem to the layout of statically determinate structures with multiple loading cases. Concludes that a statically indeterminate from could sometimes give a lighter structure than a statically determinate form.)

212. Schmit, L.A., "Structural Design by Systematic Synthesis", Proc. 2nd. Nat. Conf. on Electronic Computation, Pittsburgh, ASCE, 105-132, Sept, 1960.

(Application of steepest descent-alternate mode, non-linear programming algorithm to the optimisation of a three member pin jointed truss subject to multiple loading cases. Shows that a minimum weight structure is not necessarily one in which each member is fully stressed in at least one loading case.)

213. Schmit, L.A., "Structural Synthesis 1959-1969: A Decade of Progress", from "Recent Advances in Matrix Methods of Structural Analysis and Design", Univ. of Alabama Press, 565-634, August, 1969.

(Reviews advances in structural synethesis 1959-1969. Includes a discussion of the work based on plastic analysis by Heyman, Foulkes, Livesley, etc. and the topological design of pin jointed structures using plastic analysis by Dorn, Gomory and Greenberg (182))

214. Schmit, L.A., Kicher, T.P., "Synthesis of Material and Configuration Selection", J. Struct. Div. ASCE, V. 88, n. ST3, 79-102, June, 1962.

(Considers the effects of the angles between members and the material used in the design of the three bar truss studies in reference (212). A series of candidate optimum designs using different angles and materials were compared.)

215. Schmit, L.A., Morrow, W.M., "Structural Synthesis with Buckling Constraints", J. Struct. Div. ASCE., V. 89, n. ST2, 107-102, April, 1963.

(Similar study to reference (214) but considers the effects of buckling when fixing permissible compressive stresses.)

216. Schmit, L.A., Mallett, R.H., "Structural-Synthesis and Design Parameter Hierarchy," J. Struct. Div. ASCE., V. 89, n. ST4, 269-299, Aug. 1963.

(Treats angles between members of a three member truss as continuous variables. Approximates material selection by assuming an interpolated materials concept which contends that there exists a continuous spectrum of materials between existing materials.)

217. Sheu, C.Y., Schmit, L.A., "Minimum Weight Design of Elastic Redundant Trusses under Multiple Static Loading Conditions", AIAA Journal, V. 10, n.2, 155-162, Feb. 1972.

(Presents a method for deriving the optimum configuration from a primary truss using a subset of configurations. The basic approach is (a) find using the simplex linear programming technique lower bounds to the minimum for those subtrusses which have an equal chance to become a most promising candidate among those still under consideration; (b) using a non-linear programming algorithm a reduced upper bound for the global minimum if found for the most promising candidate still surviving; then return to (a) and continue. In this way the linear programming solutions guide the search for the global optimum.)

219. Soosaar, K., Cornell, A.C., "Optimization of Topology and Geometry of Structural Frames", ASCE., Joint Specialty Conf. on Optimization and Non Linear Problems, Chicago, Illinois, April, 1968.

(In a private communication, Professor Cornell indicated that only abstracts of this paper were prepared. Although there is a thesis by Soosaar in the M.I.T. Library.)

220. Spillers, W.R., "Iterative Design for Optimal Geometry", J. Struct. Div. ASCE., V. 101, n. ST7, 1435-1442, July, 1975.

(Considers variable node locations in an iterative design process based on optimality criteria for the pin jointed structures under a single loading case.)

221. Spillers, W.R., Friedland, L., "On Adaptive Structural Design", J. Struct. Div. ASCE., V. 98, n. ST10, 2155-2163, Oct. 1971.

(Considers philosophy of adding members and nodes in geometrical and topological optimisation. Illustrates the process with some simple examples for statically determinate structures under a single loading case.)

223. Sved, G., "The Minimum Weight of Certain Redundant Structures", Australian J. Appl. Sci., V. 5, 1 - 9, 1954.

(Shows that the weight of a pin jointed structure constructed from n bars with r redundancies under a single loading case is a minimum when the forces in the redundants are adjusted so that the forces in r members is zero.)

224. Sved, G., Ginos, Z., "Structural Optimization under Multiple Loading", Int. J. Mech. Sci., V. 10, 803-805, 1968.

(Considers removal of a member from a three bar truss subject to three loading cases and previously optimised by Schmit (212). Shows that a statically determinate optimum may exist even when multiple loads are considered. Suggests that searches of all perfect structures obtained by omitting members are necessary to ensure that the global optimum is reached.)

225. Vanderplaats, G.N., "Automated Design of Elastic Trusses for Optimum Geometry", thesis presented to Case Western Reserve Univ. at Cleveland, Ohio, in June 1971, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

226. Vanderplaats, G.N., "Design of Structures for Optimal Geometry", National Aeronautics and Space Administration, TMX-62 462, Aug. 1975.

(Similar to reference (227) except a constrained minimisation technique based on Zoutendijk's method of feasible directions is used to optimise the problem in the member area design space. The method is also sufficiently generalised to include finite element problems.)

227. Vanderplaats, G.N., Moses, F., "Automated Design of Trusses for Optimum Geometry", J. Struct. Div. ASCE, V. 98, n. ST3, 671-690, March, 1972.

(Optimisation of trusses, using member areas and nodal coordinates as design variables, with multiple loading cases. It was not possible to automatically delete nodes or members with this method. The optimisation is performed iteratively in each design space (i.e. areas and coordinates) using a fully stressed design technique for the area sizing and a steepest descent vector to direct the adjustment of nodal co-ordinates. The force method of analysis was used throughout the process. The ill-conditioning problems usually associated with combining area and co-ordinate variables in the process are seldom encountered.)

F. DYNAMIC RELAXATION

(Also see references: 62, 269, 270)

228. Barnes, M.R., "Pretensioned Cable Networks", CONRAD, V.3, n.1, 17-22, April, 1971

(Comparison of D.R. analysis and formfinding with the results of model studies)

229. Barnes, M.R., "Dynamic Relaxation Analysis of Tension Networks", Int.Conf. on Tension Structures, London, April, 1974.

(Application of D.R. to pretensioned calbe structures utilising: triangular constant stress cladding elements, an approximate method of estimating the critical time interval and fictitious masses for static analyses. Formfinding of pretension geometry of a geodesic structure together with model test comparisons for static and dynamic analyses)

230. Barnes, M.R., "Applications of Dynamic Relaxation to the Topological Design and Analysis of Cable, Membrane and Pneumatic Structures", 2nd Int.Conf. on Space Structures, Guildford, Sept., 1975.

(Application of D.R. to formfinding pretension geometries of cable and pneumatic structures and to the static analysis of tension structures dealing with on-off non-linearities)

231. Barnes, M.R., "Explicit Dynamic Analysis and Model Correlation of Tension Structures", Int. Symp. on Wide Span Surface Structures, Stuttgart, April, 1976.

(Inclusion of visco-elastic elements in D.R. formfinding, static and dynamic analyses of tension cable and membrane structures)

232. Barnes, M.R., "An Investigation of Vibration Decay in a Model Pneumatic Dome", Int. Symp. on Wide Span Surface Structures, Stuttgart, April, 1976.

(Comparison of visco-elastic dynamic behaviour of a model pneumatic dome subject to a simple symmetric loading with D.R. analysis utilising a simple Kelvin model)

233. Barnes, M.R., "Interactive Graphical Design of Tension Surface Structures", Int. Symp. on Wide Span Surface Structures, Stuttgart, April, 1976.

(Application of D.R. to formfinding of uniform or variable stress membranes and geodesic cable networks with funicular tension or compression boundaries inclues use of traction forces)

234. Barnes, M.R., "Formfinding of Minimum Surface Membranes", World Congress on Structures for Space Enclosures, Montreal, July, 1976.

(Application of D.R. to formfinding considering problems of neutral or quasi-instability)

235. Barnes, M.R., "Form-Finding and Analysis of Tension Space Structures by Dynamic Relaxation", A thesis submitted on the basis of published papers for the degree of Doctor of Philosophy, Dept. of Civil Eng., The City University, London, Oct., 1977.

(Includes refs. 228-35, together with additional results not included in papers plus state-of-the-art reviews of techniques for formfinding and analysis of tension structures)

236. Barnes, M.R., Topping, B.H.V., Wakefield, D.S., "Aspects of Form-Finding by Dynamic Relaxation", Int. Conf. on the Behaviour of Slender Structures, London, Sept., 1977.

(Application of D.R. to formfinding of (a) a funicular lattice shell structure using 'kinetic' damping and (b) triangulated modular space structures subject to a dominant load condition using viscous damping comparisons with linear programming and fully stressed design solutions)

237. Belytshko, T., Chiapetta, R.L., Bartel, H.D., "Efficient Large Scale Non-Linear Transient Analysis by Finite Elements", Int. J. for Num. Meth. in Eng., V.10, 579-596, 1976.

('Direct' explicit integration non-linear analysis of soil-structure interaction problems using linear displacement triangular and rectangular elements, including sliding debonding interfaces and artificial damping. Comparison of the computational efficiency of explicit and implicit methods using stiffness matrices)

238. Bergan, P.G., Soreide, T., "A Comparative Study of Different Numerical Solution Techniques as Applied to a Non-Linear Structural Problem", Comp. Meth. in Appl. Mech. and Eng., V.2, 185-201, 1973.

(Classification of non-linear methods into minimisation, iterative and incremental techniques. An example truss spring problem with three possible equilibrium positions is studied. Concludes that strongly non-linear problems require incremental solution methods (which closely follow the equilibrium path) combined with minimisation or iterative techniques)

239. Biggs, J.M., "Introduction to Structural Dynamics", McGraw-Hill, 1964.

(Basic text on structural dynamics with examples illustrating numerical integration methods)

240. Brew, J.S., "The Application of Dynamic Relaxation to the Solution of Non-Linear Structural Plane Frames", A thesis presented for the Degree of Master of Science, Dept. of Civ. Eng., U.M.I.S.T., Manchester, Oct., 1968.

(As (241) but applies D.R. in both simultaneous and successive forms. Considers Successive Over Relaxation to be a special case of the latter with similar convergence rate)

241. Brew, J.S., Brotton, D.M., "Non-Linear Structural Analysis by Dynamic Relaxation", Int. J. for Num. Meth. in Eng., V.3, 463-483, 1971.

(D.R. finite element solution of framed structures accounting for large deflections, instability, 'bowing', and plasticity using unassembled stiffness matrices. Block operations used for linear sway frames show the Direct Stiffness Method to be more efficient. The unassembled D.R. method was shown to be more efficient for highly non-linear problems than iterated direct solutions. Theoretical analysis for optimum convergence and iteration parameters given for linear problems on the basis of an eigen value analysis for error vectors)

242. Cassell, A.C., "Shells of revolution under arbitrary loading and the use of fictitious densities in dynamic relaxation", Proc. I.C.E., V.45, 65-78, Paper n.7210, 1970.

(D.R. finite difference analysis of elastic shells using fictitious densities to give $\Delta t = 1$ second everywhere. The critical damping estimated from a trial run. Compares D.R. with Frankel's method to derive optimum values of Δt and damping factor in terms of smallest and largest eigen values of the structure)

243. Cassell, A.C., Hobbs, R.E., "Dynamic Relaxation", Proc. Symp. on High Speed Computing of Elastic Structures, Univ. of Liege, 787-808, 1970.

(Review of D.R. and comparison with Frankel iteration and other iterative methods. Fictitious density proportional to the row sum of the modulus of the stiffness matrix. Summarises use of D.R. for finite difference and finite element formulation including non-linear effects)

244. Cassell, A.C., Hobbs, R.E., "Numerical Stability of Dynamic Relaxation Analysis of Non-Linear Structures", Tech. Note, 1407-1410

(Extends use of optimal parameters, i.e. densities and damping factor to non-linear problems. The optimum densities can be predicted from the row sums of the stiffness matrix. For computational efficiency this is divided into constant and variable parts)

245. Cassell, A.C., Kinsey, P.J., Sefton, D.J., "Cylindrical shell analysis by dynamic relaxation", Proc. I.C.E., V.39, 75-84, Paper n. 7033, 1968.

(Finite difference analysis of cylindrical shells using an interlacing grid and full shell equations. Damping factor estimated from a trial run. Stability criteria for Δt is investigated with respect to the velocity of the pressure wave)

246. Cundall, P., "Explicit Finite Difference Methods in Geomechanics", Proc. E. F. Conf. Num. Meth. in Geomechanics, Blacksburg, Va., June, 1976.

(Comparison of D.R. and explicit central difference analyses of nonlinear continua problems (soil-structure interaction) and discontinua (studies of rock blast surfaces idealised as a series of rigid blocks). Utilised a system of 'kinetic damping' in which the kinetic energy of a system is followed to a maximum at which point the velocities are set to zero and the system restarted. The process is continued until all modes of vibration are eliminated and the kinetic energy on resetting is small)

247. Day, A.S., "An Introduction to Dynamic Relaxation", The Engineer, Jan., 218-221, 1965.

(Outlines the method of D.R. for structural analysis illustrating the method by a stiffness line bending element idealisation of a portal frame and a finite difference idealisation of a plate bending problem)

248. Day, A.S., "Analysis of plates by dynamic relaxation with special reference to boundary conditions", Symp. on the Use of Electronic Digital Computers in Struct. Eng., Dept. of Civ. Eng., Working Session n.4, Paper n.10, Univ. of Newcastle, July, 1966.

(D.R. finite difference analysis of linear elastic plates)

249. Day, A.S., Bunce, J., "The Analysis of Hanging Roofs", Arup Journal, Sept. 1969, 30-31.

(Application of D.R. to simple cable structures using pinjointed stiffness element idealisation, prestress effects not included)

250. Holland, J.A., "Dynamic Relaxation applied to local effects", Proc. Conf. on Prestressed Concrete Pressure Vessels, ICE, London, 587-595, March, 1967.

(Linear and Non-Linear three dimensional stress analysis of prestressed concrete end blocks including cracking effects using an approximate finite difference formulation)

251. Iwegue, I.E., Brotton, D.M., "A Numerical Integration method for computing the flutter speeds of suspension bridges in erection conditions", Proc. ICE., V.63, Pt2, Paper n. 8042, 785-802, Dec., 1977.

(Calculation of flutter speeds of suspension bridges using D.R. to determine equilibrium geometry of the erection conditions. The response of the structure to an initial displacement and increasing wind speeds was investigated using Newmarks generalised acceleration method (with linear variation of acceleration within a time step). The flutter speed, defined as the speed at which the damping was zero, was determined from a graph of windspeed against damping factor and checked by further analysis)

252. Newmark, N.M., "A Method of Computation for Structural Dynamics", J. Eng. Mech. Div. ASCE., V.85, n.EM3, 67-94, July, 1959.

(General numerical integration techniques for linear and non-linear dynamic analysis. Velocity and displacement parameters depend on: the velocity and displacement at the beginning and acceleration at the end and the beginning of the time interval. Two forms depending on the parameters are the conditionally stable linear acceleration method and the unconditionally stable constant average acceleration method)

253. Otter, J.R.H., "Computations for Prestressed Concrete Reactor Pressure Vessels using Dynamic Relaxation", Nuclear Structural Eng., V.1, 61-75, 1965.

(Analysis of axi-symmetric thick walled pressure vessel using D.R. in finite difference and incremental form. Appendix discusses the origin of the method with respect to tidal flow equations)

254. Otter, J.R.H., "Dynamic Relaxation compared with other Iterative Finite Difference Methods", Nuclear Eng. and Design, V.3, 183-185, 1966.

(Compares D.R. with classical explicit methods of iterative finite difference analysis. Outlines some of the advantages of D.R.)

255. Otter, J.R.H., Cassell, A.C., Hobbs, R.E., "Dynamic Relaxation", Proc. ICE., V.35, 633-656, Paper n.6986, Dec., 1966.

(D.R. finite difference analysis using cartesian and polar co-ordinates illustrated with elastic arch dam analyses. Comparison with Frankel iteration showed D.R. to be an equivalent iteration. Comparison with Jacobi, Gauss Seidel and Successive Over Relaxation methods applied to Laplaces Equation in two dimensions. Using the damped wave equation S.O.R. was shown to converge more rapidly than D.R. However D.R. has the advantage of separated equations and simplified boundary conditions)

256. Otter, J.R.H., Cassell, A.C., Hobbs, R.E., Discussion of "Dynamic Relaxation", Proc. ICE., V.36, 723-750, 1967.

(King, I.P., Zienkiewicz, O.C., - possible use of finite element idealisation, Hussey, M.J.L., Wood, W.L., - further comparison with other iterative methods, R.W. Postlethwaite et alia, Welch, A.K., - use of ficititious densities to optimise the iteration, Cubbitt, N.J., - practical tests for convergence, Rushton, K.R., - underdamping to arrive at bounds of solution)

257. Rushton, K.R., "Large Deflection Analysis of Variable - Thickness Plates", Int. J. Mech., Sci., V.10, 723-735, 1968.

(Finite difference D.R. analysis of rectangular plate problems. Automatic derivation of Δt by restarting calculations and setting $\Delta t = \Delta t * 0.8$ on divergence. Damping factor derived from a trial undamped run tracing kinetic energy until reaches true peak. Restarting the calculations using the estimated damping and tracing the damped motion for the equivalent of two cycles (when the problem was considered solved). The residuals were then calculated to give estimate of the error)

258. Rushton, K.R., "Dynamic Relaxation Solutions of Elastic-Plate Problems", J. of Strain Analysis, V.4, n.1, 23-32, 1968.

(Non-Linear finite difference D.R. analysis of plates using fictitious masses in each direction to accelerate the calculations)

259. Severn, R.T., "Numerical Methods of Calculation of stress and strain", Phil. Trans. R. Soc. Lond. A, V.274, 339-350, 1973.

(Review paper giving a comparison of the methods of stress analysis for linear, non-linear, large-displacement, elasto-plastic problems, i.e. reviews finite element methods and finite difference D.R. procedures for static problems. Unfortunately the paper does not explore D.R. with finite element idealisation)

260. Wood, W.L., "Comparison of Dynamic Relaxation with three Iterative Methods", The Engineer, Nov., 1967.

(Unified comparison of D.R. with Successive Over Relaxation, Jacobi and Gauss-Seidel iterative methods)

261. Zienkiewicz, O.C., "A New Look at the Newmark, Houbolt and other time Stepping Formulas. A Weighted Residual Approach", Earthquake Eng. and Structural Dynamics, V.5, 413-418, 1977.

(Classification of Newmarks integration method and other iterative methods according to shape functions and γ and β parameters. Higher order formulas using more time stations are considered)

G. INTERACTIVE DESIGN

262. Butlin, G.A., "Man-Machine Interative Structural Analysis as a Preliminary-Design Aid", AGARD Conf. Proc. n. 36, on Symposium on Structural Optimization, AGARD-CP-36-70.

(Describes early development of an interactive unit using a refresh cathode ray tube and light pen. Existing finite element software is integrated with a system for generating and manipulating data)

263. Cope, R.J., Bungey, J.H., "An Examination of interactive computer usage in structural design, with special reference to bridges", Proc. ICE., V.61, Part 2, 525-538, Sept., 1976.

(Describes an investigation of computer aided design of r.c. prestressed bridges using engineer initiation of a series of program modules as required (stored data for suitable optimum bridge sections used as a starting point))

264. Cope, R.J., Bungey, J.H., Dickin, E.A., Winterbottom, S.K., "Some Lessons from Interactive Computer Usage", 3rd Int. Conf. on Computers in Eng. and Building Design, CAD 78, Metropole Centre, Brighton, Sussex, March 1978.

(Discusses the philosophy of the last paper above in more detail)

265. Douty, R., Shore, S., "Technique for Interactive Graphics in Design", J. Struct. Div. ASCE., V.97, n.ST1, 273-288, Jan., 1970.

(Discusses use of interaction to help aid convergence and allow primary decisions to be made during a computer design or optimisation process)

266. Rabe, G., Trossello, V.A., "Man-Orientated Program System for Engineering Design", J. Struct. Div. ASCE, V.97, n.ST1, 223-235, Jan., 1971.

(Describes an interactive modular design program, allowing the user to develop further modules. Interactive use is a major feature of the program)

267. Sambura, A.S., "Computer graphic techniques in Structural Optimization", Proc. 2nd. Int. Conf. on Computers in Eng. and Building Design, CAD 76, Imperial College, London, March, 1976.

(Suggests looking for local optima which are often of similar weight and using an interactive process to decide on which design to adopt from aesthetic criteria. Methods for deriving local optima are summarised)

268. Wilson, J.L., "Criteria for interatice computer aided design", Proc. 2nd. Int. Conf. on Computers in Eng. and Building Design, CAD 76, Imperial College, London, March 1976.

(Discusses basic concepts of interactive computer aided design graphics)

H. ADDITIONAL REFERENCES

269. Papadrakakis, E., "Gradient and Relaxation Non-Linear Techniques for the Analysis of Cable Supported Structures", a thesis submitted for the degree of Doctor of Philosophy, The Dept. of Civ. Eng., The City University, London, July, 1978.

270. Wakefield, D.S., "Dynamic Relaxation Analysis of Spatial Networks supported by Compression Arches", a thesis submitted for the degree of Doctor of Philosophy, The Dept. of Civ. Eng., The City University, London, Autumn, 1978.

271. Hofmeister, L.D., Felton, L.P., "Prestressing in Structural Synthesis", AIAA Journal, V.8, n.2, 363-4, Feb., 1970.

(Magnitude of prestress included as design variables in a constrained minimisation problem. The prestress effects were assumed not to contribute to the weight of the problem. Reconsiders a simple 3 bar truss example of Schmit (212))

Published Work

Sections of Chapters 2, 3 and 4 were incorporated by the author in the paper entitled:

"Aspects of Form-Finding by Dynamic Relaxation"
Barnes, M.R., Topping, B.H.V. and Wakefield, D.S.
Int. Conf. on the Behaviour of Slender Structures,
London, September, 1977.