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DERIVATION OF EUROCODE 8 SPECTRUM-COMPATIBLE TIME-HISTORIES FROM RECORDED SEISMIC ACCELEROGRAMS VIA HARMONIC WAVELETS

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Abstract. A computationally efficient harmonic wavelet-based iterative procedure is proposed to modify suites of recorded accelerograms to be used in the aseismic design of critical structures regulated by the European code provisions (EC8). Special attention is focused on assessing the potential of appropriately defined orthogonal harmonic wavelet basis functions to derive design spectrum compatible time-histories which preserve the non-stationary characteristics of the original recorded signals. This is a quite desirable attribute in the practice of the aseismic design of yielding structures. In this regard, seven recorded accelerograms recommended for the design of base-isolated structures are modified via the proposed procedure and base-line adjusted to meet the pertinent EC8 compatibility criteria. The instantaneous energy (IE) and the mean instantaneous frequency (MIF) of the modified EC8 compatible time-histories extracted from appropriate wavelet-based signal time-frequency analyses are compared vis-à-vis the IE and MIF of the corresponding original accelerograms. Examining these numerical results, it is established that the herein proposed procedure is a useful tool for processing recorded accelerograms in cases where accounting for the time-varying energy content and frequency composition of strong ground motions associated with historic seismic events is deemed essential in aseismic design.

1 INTRODUCTION

Contemporary code provisions allow for the exclusive consideration of response spectrum-based kind of analyses incorporating appropriate modal combination rules for the aseismic design of ordinary structures. To facilitate this practice, the shaking severity is represented by means of smooth elastic and inelastic response (design) spectra. However, in designing critical and certain non-conventional/special structured facilities it is commonly deemed essential to perform additional dynamic time-history analyses. In the context of such analyses, aseismic code provisions require representing the input seismic action by suites of acceleration time-histories (accelerograms) whose average response spectrum is in a close agreement (compatible) with a prescribed elastic design spectrum.

Subject to availability, the option of judicially chosen “real” field recorded accelerograms pertaining to historic seismic events is preferred over “artificial” numerically simulated signals to be used as input for time-history analyses[1]. This is due to the fact that real accelerograms capture reliably the time-varying attributes of the actual strong ground motion in terms of intensity and frequency content. Nevertheless, recorded accelerograms do not usually satisfy the pertinent compatibility criteria posed by code regulations. In this regard, several researchers have adopted the use of various wavelet families in conjunction with the continuous wavelet transform (CWT) to decompose and to modify iteratively real accelerograms such that their response spectra matches a given response/design spectrum[2,3]. In accomplishing this task, limited attention has been given to what extent the evolutionary characteristics of the original records are properly preserved in the modified signals[4,5]. This is an important consideration for the structural aseismic design, since it is known that these time-varying attributes influence the inelastic response of yielding structural systems[6,7].

In a recent paper[8], the authors have proposed the use of a harmonic wavelet-based iterative procedure to modify artificial accelerograms generated via a stochastic approach so that they meet the pertinent compatibility criteria of the current European aseismic code provisions (EC8)[9]. Herein, this wavelet-based procedure is adopted to treat field recorded accelerograms in conjunction with the continuous wavelet transform (CWT) to decompose and to modify iteratively real accelerograms such that their response spectra matches a given response/design spectrum[2,3]. In accomplishing this task, limited attention has been given to what extent the evolutionary characteristics of the original records are properly preserved in the modified signals[4,5]. This is an important consideration for the structural aseismic design, since it is known that these time-varying attributes influence the inelastic response of yielding structural systems[6,7].

In a recent paper[8], the authors have proposed the use of a harmonic wavelet-based iterative procedure to modify artificial accelerograms generated via a stochastic approach so that they meet the pertinent compatibility criteria of the current European aseismic code provisions (EC8)[9]. Herein, this wavelet-based procedure is adopted to treat field recorded accelerograms to comply with the EC8 compatibility criteria in a deterministic context. Specifically, a suite of recorded accelerograms is modified via the adopted procedure and base-line adjusted to be used for the design of a base-isolated structure according to the EC8. Special attention is given to assessing the potential of orthogonal harmonic wavelets to derive EC8 compatible time-histories which retain the time-varying attributes of the original signals. To this aim, joint time-frequency representations of the above original recorded and the modified EC8 compatible signals are compared. These representations are obtained by using non-orthogonal Hanning filtered harmonic wavelets[10,11]. Furthermore, the instantaneous energy[12] and the mean instantaneous frequency[13,14] associated with the aforementioned wavelet-based time-frequency signal representations are also considered to monitor the time-variation of the average energy and frequency content of
the original vis-à-vis the modified signals.

2 THEORETICAL BACKGROUND

2.1 The Harmonic wavelet transform as a joint time-frequency signal representation tool

Consider a real-valued continuous-time signal/function \( s(t) \) of finite energy \( E \). Further, consider the continuous-time Fourier transform (FT) defined by the equation

\[
\hat{S}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt.
\] (1)

This transform decomposes the signal \( s(t) \) by projecting it onto a basis of sinusoidal functions with varying frequencies \( \omega \). In mapping \( s(t) \) in the domain of frequencies, the FT preserves the signal energy and the following relation holds

\[
E = \int_{-\infty}^{\infty} s(t)^2 dt = 2\pi \int_{-\infty}^{\infty} |\hat{S}(\omega)|^2 d\omega.
\] (2)

In this regard, a plot of the squared magnitude of the FT versus frequency shows how the energy of the signal is distributed on the frequency axis. However, this distribution furnishes no temporal information since sinusoids are non-decaying waveforms in the time domain. In this respect, the squared magnitude of the FT cannot trace the evolutionary characteristics of the energy of signals whose intensity and frequency content vary in time. An example of such non-stationary signals is the acceleration time-history of the strong ground motion pertaining to an earthquake event. Obviously, alternative analyzing functions featuring finite effective support in both the time and the frequency domain must be employed to obtain a reliable distribution of the energy of non-stationary signals on the time-frequency plane.

To this end, the harmonic wavelet transform (HWT) has been proved to yield useful time-frequency representations of non-stationary signals encountered in the field of earthquake engineering and structural dynamics\(^1\[11,14\]. Specifically, the HWT incorporates a basis of complex-valued analyzing functions termed harmonic wavelets attaining a band-limited Fourier spectrum and observing a decaying oscillatory waveform in time. A “general” harmonic wavelet \( \psi_{(m,n),k} \) of \((m,n)\) scale centered at the \( k/(n-m) \) position in time is defined as a function possessing a constant spectrum within the frequency band \([m2\pi, n2\pi)\), where \( m \) and \( n \) are real positive numbers\(^1\[16\]. It is expressed in the frequency domain by the equation

\[
\hat{\psi}_{(m,n),k}(\omega) = \begin{cases} 
\frac{1}{(n-m)2\pi} \exp\left(-i\omega k\right), & m2\pi \leq \omega < n2\pi \\
0, & \text{otherwise}
\end{cases}
\] (3)

Consider a collection of such functions spanning adjacent non-overlapping bands of arbitrary widths at different \((m,n)\) scales along the whole frequency axis. At each scale \((m,n)\), the considered collection encompasses wavelets centered at all possible \( k/(n-m) \) time instants, where \( k \) is an integer. Newland\(^1\[16\] has shown that such a collection constitutes a complete and orthogonal basis for finite energy signals. By utilizing this basis, the HWT given by the formula\(^1\[16\]

\[
W^{\psi}_{(m,n),k} = (n-m)^2 \int_{-\infty}^{\infty} s(t) \psi_{(m,n),k}^{*}(t) dt,
\] (4)

where the bar over a symbol denotes complex conjugation, yields a signal representation via the wavelet coefficients \( W^{\psi}_{(m,n),k} \) which preserves the signal energy. This can be mathematically expressed by the equation

\[
E = \int_{-\infty}^{\infty} s(t)^2 dt = 2\sum_{n,m} \sum_{k} |W_{(m,n),k}|^2,
\] (5)

where the summation over \( m,n \) accounts for all \((m,n)\) scales considered and the summation over \( k \) accounts for all wavelets located at \( k/(n-m) \); \( k \in \mathbb{Z} \) time instants of each \((m,n)\) scale.

In the following section, an iterative procedure employing the general harmonic wavelets to produce design spectrum compatible seismic accelerograms is reviewed. The assessment of the merits of this scheme in treating field recorded accelerograms necessitates the consideration of enhanced time-frequency signal representations. In this junction, note that the discontinuity of the box-shaped spectrum of the harmonic wavelets defined by Eq. (3) yields waveforms with a relatively slow rate of decay in time and thus with poor time-localization.
capabilities. In this respect, harmonic wavelets incorporating a smooth Hanning window function in the frequency domain have been considered to improve the time-localization attributes of the HWT\(^{[10,11,14]}\). In particular, a Hanning filtered harmonic wavelet \(ψ_{f,m,n,k}(ω)\) is defined in the frequency domain by the equation\(^{[10]}\)

\[
ψ_{f,m,n,k}(ω) = \begin{cases} 
\frac{1}{(n-m)2\pi} \left( 1 - \cos \frac{ω-m2π}{n-m} \right) \exp \left( -i\omega k \right), & m2π ≤ ω < n2π \\
0, & \text{otherwise}
\end{cases}
\]

The wavelet coefficients \(W_f(m,n,k)\) obtained by substituting in Eq. (4) the Hanning filtered harmonic wavelets do not preserve the energy of the signal as in the case of the general harmonic wavelets. However, the associated filtered harmonic wavelet spectrogram defined as\(^{[11,14]}\)

\[
SP\left( t = \frac{k}{m-n}, ω = \frac{m+n}{2} \right) = \frac{2}{(n-m)} |W_f(m,n,k)|^2,
\]

provides a representation of the signal energy on the time-frequency plane of enhanced time and frequency resolution. The “instantaneous” signal energy (IE) as captured by the filtered harmonic wavelet coefficients normalized by the total signal energy is then given by the equation\(^{[12]}\)

\[
IE(t) = \frac{\int SP(t,ω) dω}{E}.
\]

Furthermore, by treating the wavelet spectrogram as a time-evolving energy distribution the normalized mean “instantaneous” frequency (MIF) of a broadband signal can be defined at each time instant by the expression\(^{[13]}\)

\[
MIF(t) = \frac{\int ωSP(t,ω) dω}{\int SP(t,ω) dω}.
\]

The latter quantity captures the average value of the signal energy normalized by the total energy of the signal released at each time instant as captured by the wavelet spectrogram. The concept of the MIF has been recently used for structural damage detection purposes\(^{[13,14]}\).

2.2 Harmonic wavelet-based iterative response spectrum matching procedure

It can be deduced from the preceding exposition that the HWT given by Eq. (4) can decompose an accelerogram \(s(t)\) into band-limited sub-signals \(s_{m,n}(t)\) expressed by\(^{[11]}\)

\[
s_{m,n}(t) = 2 \text{Re} \left\{ \sum_k W_{m,n,k} ψ_{G,m,n,k}(t) \right\}.
\]

In light of Eq. (5), it can be readily shown that the original accelerogram \(s(t)\) can be reconstructed by summing up the above sub-signals. That is,\(^{[11,16]}\)

\[
s(t) = \sum_{m,n} s_{m,n}(t).
\]

Note that the energy of each \(s_{m,n}(t)\) component of \(s(t)\) is concentrated within the \([m2π, n2π]\) interval in the frequency domain. Consequently, the response of a lightly damped single-degree-of-freedom oscillator of natural period \(T_n\) lying within the interval \((1/n_j, 1/m_j]\) excited by the accelerogram \(s(t)\) would be mainly influenced by the \(s_{m,n}(t)\) component of \(s(t)\). Thus, an iterative modification procedure can be devised to scale all sub-signals at the \(v\)-th iteration according to the equation\(^{[2,8,17]}\)

\[
s_{m,n}(t) = \frac{1}{\int_{T_n}^{1/m} D^{(v)}(T) dT} \int_{T_n}^{1/m} S_{f}(T) dT)
\]

to improve the agreement of the response spectrum of an accelerogram \(s(t)\) with any given response/design spectrum \(S_d\). In Eq. (12), \(D^{(v)}(T)\) is the response spectrum of the \(s^{(v)}(t)\) obtained by Eq. (11) at each iteration.
practice, band-limited discrete-time finite-duration recorded accelerograms sampled at a constant frequency/time-step are to be processed. To this aim, the fast Fourier transform based algorithm presented in Newland[10] can be employed for the efficient computation of the convolution integral of Eq. (4) in the frequency domain. Furthermore, a sufficient number of scales/bins” must be considered to cover all frequencies of interest. A related detailed discussion along with numerical evidence on the impact of the width of these bins on the efficiency of the above described iterative procedure can be found in Giaralis and Spanos[8].

Finally, it is noted that the modified spectrum compatible signals need to be base-line adjusted to suppress any spurious low-frequency content that may have been introduced during the aforementioned iterative scaling procedure[8,17]. For this purpose, a-causal zero-phase high pass filtering using standard infinite impulse response (IIR) filters is considered[18]. This method of base-line correction does not alter significantly the response spectral ordinates of the accelerograms[19] and, thus, it does not influence the quality of the spectrum matching achieved by the proposed procedure[8,17].

3 NUMERICAL APPLICATION TO THE EUROPEAN CODE PROVISIONS (EC8)

3.1 Derivation of EC8 compatible field recorded accelerograms

In this section the applicability and the usefulness of the previously described harmonic wavelet-based matching procedure to satisfy the compatibility criteria of the current European aseismic code (EC8)[9] are assessed by considering a specific design scenario. In particular, it is set to design a base-isolated building for a design ground acceleration \( \alpha_g \) equal to 0.36g (g=9.81m/sec\(^2\)), and for soil conditions “B” as classified by the EC8[9]. Suppose that it has been deemed essential to include a nonlinear dynamic time-history analysis in the design process. Then, according to the EC8, a suite of at least three accelerograms (i.e. \( \alpha_j(t) \), \( j=1,\ldots,N \) with \( N\geq3 \)) need to be considered to represent the input seismic action. These time-histories must observe the following two rules/compatibility criteria. First, the average of the absolute peak value of each time-history must be greater than the design ground acceleration \( \alpha_g \) times a soil-dependent amplification factor which is equal to 1.2 for soil conditions B. Second, the average response spectrum for 5% ratio of critical damping of the suite of the accelerograms considered must be greater than 90% of the EC8 elastic design (target) spectrum for the same level of damping within a specific range of periods. This range/interval is defined as \([0.2T_1, 2T_1]\), where \( T_1 \) is the fundamental period of the structure in the direction where the accelerograms will be applied. For the purpose of this study, focus is given to one principal horizontal axis of the structure to be designed assuming that \( T_1=2s \) along the direction of this axis. In this case, the preceding criteria can be mathematically expressed by

\[
\frac{1}{N} \sum_{j=1}^{N} \left( \frac{\max_{t} \alpha_j(t)}{1.2 \alpha_g} \right) > 1 \quad \text{and} \quad \min_{T} \left\{ \frac{1}{N} \sum_{j=1}^{N} D_j(T) \right\} > 0.90, \quad 0.4 \leq T \leq 4.0.
\]  

(13)

Figure 1. Field recorded accelerograms considered as input to the harmonic wavelet based matching procedure

To this end, it is chosen to perform time-history analyses for seven field recorded accelerograms pertaining to historical seismic events along the considered direction of the structure. This choice is justified by the fact that EC8 allows using the average of the pertinent peak response quantities as the design value when at least seven time-history analyses are performed. The seven accelerograms shown in Figure 1 are selected out of a data-bank of strong ground motion records specifically proposed for the design of base isolated structures[20]. The individual and the average response spectra of these signals in terms of displacement and pseudo-acceleration are...
plotted in Figure 2. In the latter figure, the target EC8 design spectrum is also superimposed.

Figure 2. Displacement and pseudo-acceleration response spectra of the original accelerograms of Figure 1

The accelerograms considered must be modified prior to their use for time-history analyses since they do not satisfy the EC8 compatibility criteria. Specifically, the ratios appearing in Eq. (13) equal to 0.41 and 0.71, respectively. To satisfy the second more stringent criterion, the considered signals are treated using a harmonic wavelet basis featuring scales of non-uniform bandwidths across the frequency axis. The objective is to take advantage of the versatility of the harmonic wavelets to obtain a more detailed “discretization” of the frequency domain in the range [1.57, 15.71] (rad/s). The latter range is mapped to the interval [0.4, 4] (s) on the axis of natural periods. In particular, scales of 0.0785 rad/s width are defined inside the above range of interest, and scales 16 times wider are considered outside this range. In this fashion, more weight is assigned to obtaining enhanced agreement of the response spectral ordinates of the accelerograms with the target spectrum for the oscillators whose natural frequencies lie closer to the fundamental period $T_1$ as has been proposed by Giaralis and Spanos [8]. Clearly, this renders the iterative matching procedure governed by Eq. (12) quite efficient in meeting the compatibility criterion of Eq. Error! Reference source not found. Indeed, within two iterations the resulting modified seismic signals satisfy the compatibility criteria of EC8: the ratios in Eq. (13) are computed as 1.10 (>1) and 0.91 (>0.90), respectively. These signals have also been base-line corrected via high-pass zero-phase acausal filtering using a Butterworth filter of order 4 and of cut-off frequency 0.1Hz. The latter base-line correction step ensures that the velocity and the displacement traces obtained by integration of the modified accelerograms are physically meaningful [8]. The individual and the average displacement and pseudo-acceleration response spectra of the seven EC8 compatible accelerograms are shown in Figure 3 along with the target spectrum.

Figure 3. Displacement and pseudo-acceleration response spectra of the modified EC8 compatible accelerograms
Besides the efficiency of the proposed matching procedure, what is perhaps more important in the case of considering real recorded accelerograms is whether this procedure alters the time-varying characteristics of the original signals severely. This point is discussed in the next section using numerical results pertaining to the above suites of original and modified EC8 compatible accelerograms.

3.2 Time-varying frequency and energy content of the EC8 compatible accelerograms

This section probes joint time-frequency representations of the original vis-à-vis the previously derived modified signals to ascertain that the herein proposed modification procedure preserves the main time-varying features of real accelerograms. These representations have been obtained by means of the HWT using Hanning filtered harmonic wavelets (Eqs. (6) and (7)). A uniform grid in the frequency domain has been considered defining intervals of width equal to 1.25rad/sec. This discretization has been found to yield a satisfactory resolution on the time-frequency plane for all the frequency bands of interest. For brevity, results pertaining to two representative records, namely the Century and the Petrolia accelerograms, are included in this paper.

In Figures 4 and 5 contour plots of wavelet spectrograms pertaining to the original and the modified Century and Petrolia signals, respectively, are juxtaposed; the time histories of the accelerograms are shown as well to expedite the physical interpretation. The original Century signal constitutes a typical example of a recorded accelerogram characterized by a single burst of energy. The higher frequencies are more dominant during the fast “growth phase” of the signal and then fade away as the intensity of the strong ground motion decays slowly with time. The Petrolia record features a more complex frequency content containing three distinct bursts of energy, the first being the most dominant one. Notably, the above time-varying patterns of the frequency content of the original signals are, evidently, maintained in the modified records as well. Although the overall intensity level of the modified Century record is considerably larger than the original one, the energy distribution along both the frequency and the time axes remains practically identical. In the case of the Petrolia record, the overall energy of the modified signal is decreased compared to the original record and shifted towards higher frequencies. However, this shift occurs uniformly in time and the evolutionary energy pattern of the original signal is not altered. The three energy bursts are maintained and their relative significance remains the same. To further support the above observations, plots of the normalized instantaneous energy calculated by Eq. (8) for the original and the modified Century (Figure 6(a)) and Petrolia (Figure 6(b)) records are compared. Evidently, the difference of the average time-varying energy content of the original and the EC8 compatible signals is practically negligible.

Finally, the MIF of the original and the modified signals obtained by Eq. (9) are plotted to monitor the evolution of the average frequency content for the Century (Figure 6(c)) and for the Petrolia (Figure 6(d)) cases. Clearly, the MIF of the Century accelerograms are in a close agreement. Moreover, the shift of the average frequency content towards higher frequencies is readily deduced in the case of the modified Petrolia record. However, the number and the time location of the troughs and valleys of the original and the modified Petrolia MIF time-histories coincide. Evidently, no significant change in the time-varying patterns of the average frequency content has been introduced to the modified EC8 compatible signal.

Figure 4. Joint time-frequency representation of the original and the modified Century record via Hanning filtered harmonic wavelets
4 CONCLUDING REMARKS

A harmonic wavelet-based iterative procedure has been proposed for modifying suites of field recorded accelerograms to derive time-histories compatible with the European aseismic code provisions (EC8). The derived suites of accelerograms can be used as input for dynamic time-history analysis of critical structures whose aseismic design is regulated by the EC8 code. Specifically, it has been pointed out that the pertinent EC8 compatibility criteria require the average response spectrum of such a suite of signals to be in a prescribed level of agreement with the EC8 design spectrum at only a specific interval on the axis of natural periods. This interval depends on the fundamental natural period of the structure to be designed. In this regard, the versatility of the orthogonal harmonic wavelets characterized by a band-limited box-shaped spectrum has been utilized to “surgically” modify the energy and frequency content of the original recorded signals to meet the EC8 compatibility criteria.

In this regard, a specific real-life design scenario has been considered involving the application of the proposed modification procedure to seven recorded accelerograms recommended for the design of base-isolated structures. In particular, a suite of seven EC8 compatible accelerograms has been derived within only two iterations to be used for the design of a base-isolated building of a certain fundamental natural period. In deriving these accelerograms a case-dependent orthogonal harmonic wavelet basis encompassing functions spanning non-uniform non-overlapping bandwidths have been appropriately constructed to discretize the frequency axis so that a finer grid is assigned within the interval that spectral matching is required according to the EC8. Furthermore, the evolutionary patterns of the instantaneous energy and the mean instantaneous frequency pertaining to the original recorded and the thus derived EC8 compatible accelerograms have been found to be in a reasonable
agreement. These quantities have been obtained from appropriate wavelet-based time-frequency signal analyses employing non-orthogonal Hanning filtered harmonic basis functions. In light of these numerical results, it can be argued that the herein adopted harmonic wavelet based procedure constitute a quite effective tool for processing real recorded accelerograms to be used in aseismic design. This is particularly true in cases where accounting for the evolutionary features of the energy and frequency composition of strong ground motions associated with specific earthquake events is deemed essential in the design process of special civil infrastructure.

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