# DERIVATION OF DYNAMIC PROPERTIES OF STEEL ASYMETRIC PERFORATED ULTRA SHALLOW FLOOR BEAMS (USFB$^{TM}$) VIA FINITE ELEMENT MODAL ANALYSIS AND EXPERIMENTAL VERIFICATION

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## 1. ABSTRACT

In recent years, the incorporation of asymmetric perforated ultra shallow floor beams (USFBs) constructed from advanced UB and UC profile beams in various composite floor systems has been extensively considered in practice. To date, limited research effort has been devoted to the detailed investigation of the dynamic properties of USFBs. In this paper, modal analyses of detailed FE models of various USFBs commonly used in composite floor systems developed in ANSYS are conducted to extract their dynamical properties (i.e. natural frequencies and mode shapes). Furthermore, experimental data pertaining to the standard impact test is also considered to validate the accuracy of the aforementioned FE results. In particular, a six meter long USFB beam is subject to impulsive excitation by means of an appropriately instrumented hammer. The dynamic properties obtained by processing the recorded response signals compare well vis-a-vis the corresponding results from the FE modal analysis. Finally, effective properties of USFBs which can be readily used in the definition of beam elements of constant cross-section along their longitudinal direction are derived. This constitutes an important step to facilitate the analysis and design of USFBs against dynamic loads at the serviceability limit state using standard commercial structural analysis software.

## 2. INTRODUCTION

Requirements for maximum space utilization, efficiency during construction, and cost-effectiveness demand the use of long-spanned, shallow, light steel beams in steel and composite structures. In achieving optimum integration of structure and services within the same horizontal zone, perforated beams having large closely spaced web openings along their length is a commonly used structural element in long span light roof as well as in composite floor systems. Typically, such structural systems are sensitive to dynamic loads induced during their service life by personnel activities (e.g. walking, jumping, e.t.c.) as well as by broadband forces from turbulence in piping and ducting supported within floor slabs. In this respect, their response to such excitations needs to be considered at design.

In recent years, a new breed of composite floors have been introduced by ASDWestok Ltd. utilizing the Ultra Shallow Floor Beam (USFB). The USFB steel section is fabricated by welding two highly asymmetric cellular tees together along the webs resulting in a large bottom flange. Either precast concrete floor units or profiled steel decking rest on the bottom flange of the USFB creating a very shallow floor beam construction system, thus minimizing the overall structural depth (*Fig. 1*). As the floors are being cast, the in-situ concrete passes through the web openings, which may or may not include a tie-bar or duct. This concrete plug and tie-bar forms a unique mechanism for transferring longitudinal shear force along the beam. A special end diaphragm is used for deep decking floor applications so that the concrete fully surrounds the steel section, apart from the bottom plate. 'Arching' action is occurred through the concrete partial encasement, which is resisted by the end plate connections. The web openings provide a passage for reinforcing tie-bars and service ducts within the depth of the beam. In-situ concrete fills the web openings as the floor slabs are cast. This concrete passing through the web openings, together with the reinforcing bars and ducts, contribute to the longitudinal shear resistance when the beam is subjected to axial bending. The free web openings under the profiled steel decking are utilized for the integration of services. [1,2,3,4,5]

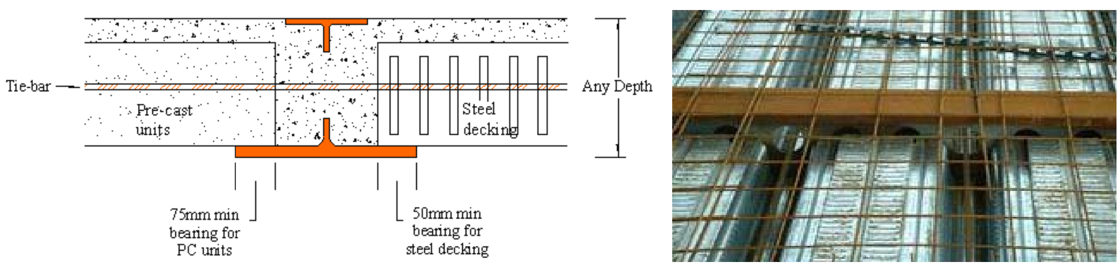


*Fig. 1: Cross-section configuration of composite floor construction using asymmetric USFBs (left panel) and deep decking floor application (right panel) (ASDWestok Ltd.)*

Herein, a first step towards the study of the vibration behavior of the aforementioned composite floor structures is undertaken by probing into the dynamic properties of commonly used bare steel USFBs. For this purpose, experimental impact hammer modal testing is conducted in the Heavy Structures Lab at City University London to characterize the first few flexural modes of vibration of a simply supported USFB steel specimen. Next, a finite element (FE) model of this specimen is developed in ANSYS and its effectiveness to capture the vibration behavior of the considered USFB is verified by comparing the experimental data with results from FE modal analysis. Then, parametric modal analyses based on the above FE model is undertaken in ANSYS to obtain the natural frequencies of several USFBs commonly used in building engineering practice. Finally, equivalent geometric properties of these USFBs corresponding to equivalent Euler-Bernoulli beams of constant cross-section are derived by relying on specific criteria of equivalency involving inertial and dynamic properties.

## 3. EXPERIMENTAL MODAL TESTING

A simply supported bare steel Ultra Shallow Floor Beam (USFB) spanning 6m is subject to impact experimental modal testing to extract the modal characteristics of the transverse flexural modes of vibration of the beam specimen. The geometry of the beam specimen and the experimental setup are shown in *Fig. 2*. Three uni-axial high sensitivity shear accelerometers, henceforth acc1, acc2, and acc3, are placed on the middle of the upper flange in the mid-span and in the quarters of the 6m (positions #2,#4 and #6 as shown in *Fig. 2*). An impact hammer (model 5803A by Dytran Instruments) equipped with an embed force sensor is used to excite the transverse flexural modes of vibration of the beam specimen by hitting it downwards along the gravitational axis at the seven positions indicated in *Fig. 2*. Five “qualified” hits are recorded at each position. An appropriately hard hammer head was used to ensure that an adequately broadband input signal is achieved to excite modes at least up to 300Hz. A USB-9234 device by National Instruments connected to a regular PC unit is used for simultaneous data acquisition of the four channels (three output/acceleration and one input/force) and storage in ASCII format using specialized software onto the hard disk of the PC. In this manner, 30s long digital recorded signals sampled at a rate of 1024Hz are obtained from all four channels for the 35 hammer hits considered in total. This duration was found to be sufficiently long for the response transient signals to die out eliminating the need to apply an exponentially decaying time window. The stored data are then processed off-line to construct a 3-by-7 matrix $\mathbf{H} = \left[ H_{ij} \right]$ containing frequency response functions (FRFs) corresponding to location $i$ due to an impact at location $j$ using a custom-made script in MATLAB. The latter follows the common analysis steps used by standard FFT analysers [6,7,8].

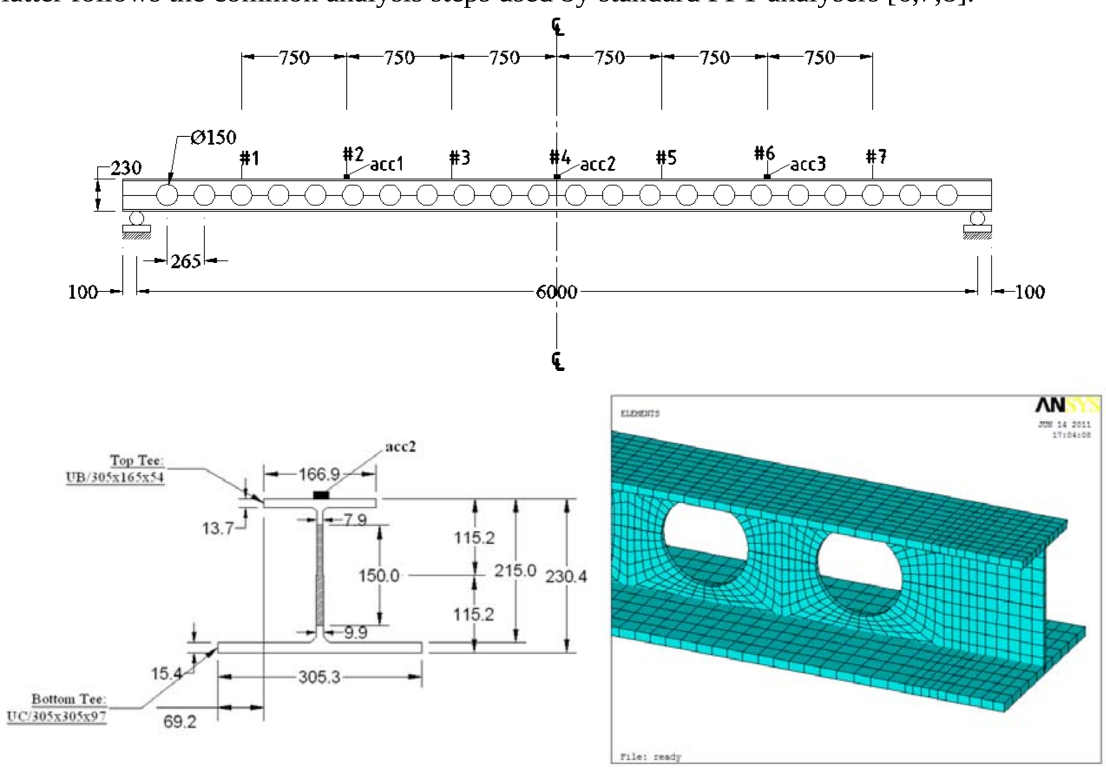


*Fig. 2: Longitudinal view, cross-sectional properties and finite element mesh of the USFB considered for experimental modal testing (units in mm)*

Specifically, the raw signals are processed by a 5th order band-pass Butterworth filter with cut-off frequencies at 5Hz and at 400Hz. The filtered signals, although deterministic and non-stationary (transient) in nature, are treated as random (i.e. as samples of certain underlying stochastic processes); a common consideration in the field of impact experimental modal testing [6]. This allows for computing FRF estimates in the domain of frequencies $\omega$ by the following widely used expressions:

$$H_1(\omega) = \frac{G_{xf}(\omega)}{G_{xx}(\omega)}, \qquad (1)$$

and

$$H_2(\omega) = \frac{G_{xx}(\omega)}{G_{xf}(\omega)}, \qquad (2)$$

where $G_{xx}$ and $G_{ff}$ are the (real) auto- power spectral density (PSD) functions of the acceleration and the input hammer force, respectively, and $G_{xf}$ is their (complex) cross-PSD.

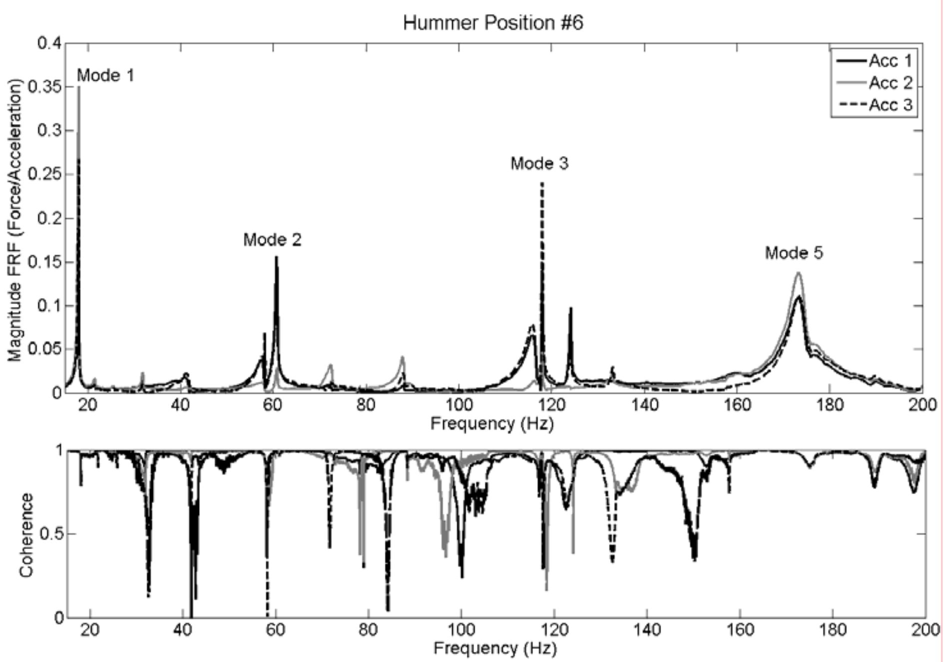


Fig. 3 : Average frequency response functions and coherence corresponding to position #6

*Fig. 3* plots the magnitude of $H_2$ estimators linearly averaged over an ensemble of five functions corresponding to the five hammer hits at position #6 obtained from the three accelerograms (the 6th column of the $\mathbf{H}$ matrix constructed from the experimental data). A plot of the coherence function defined as the ratio $H_1/H_2$ is also shown to provide an indication of the reliability of the plotted FRF curves (the closer the coherence lies to unity, the more reliable the FRF functions). The 1st, 3rd and 5th natural frequencies are clearly identified in the aforementioned FRFs. The 2nd natural frequency is picked up only by the acc1 and acc3 channels as the acc2 lies at a nodal point of the second mode (a point where the corresponding mode shape attains a zero). The 4th natural frequency cannot be determined by any of the accelerograms as they all lie at nodal points of the 4th mode shape. However, it can be retrieved by Finite Element modal analysis as discussed in the next section. Similar observations are made to the averaged sets of FRFs corresponding to the rest of the columns of the $\mathbf{H}$ matrix.

Next, the FRF data around the four natural frequencies of interest indicated in
*Fig.* 3 are isolated and a standard SDOF circle-fitting method on the complex (Argand) plane implemented in MATLAB is employed to extract estimates of natural frequencies, damping, and modal constants (coordinates of the mode shapes) corresponding to the first three and the 5$^{th}$ flexural modes of vibration. The average values obtained from the total 21 FRFs of these natural frequencies and damping are collected in *Table 1*. Furthermore, *Fig. 4* plots the first two mode shapes normalized to their peak absolute value.

| Mode | Damping (%) | | | | Natural Frequency (Hz) (in-plane flexural modes) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Experimental | | | | Experimental | | | | FE (ANSYS) | FE (ANSYS) Solid beam |
| | acc 1 | acc 2 | acc 3 | average | acc 1 | acc 2 | acc 3 | average | | |
| 1$^{st}$ | 0.98 | 0.98 | 0.98 | 0.98 | 17.954 | 17.948 | 17.948 | 17.950 | 18.184 | 18.449 |
| 2$^{nd}$ | 0.94 | 0.80 | 0.98 | 0.91 | 60.705 | 61.135 | 60.699 | 60.846 | 63.565 | 69.628 |
| 3$^{rd}$ | 0.23 | 0.24 | 0.23 | 0.23 | 118.03 | 117.93 | 118.02 | 117.99 | 120.74 | 141.87 |
| 4$^{th}$ | - | - | - | - | - | - | - | - | 167.86 | 179.37 |
| 5$^{th}$ | 1.58 | 1.58 | 1.49 | 1.55 | 173.91 | 173.9 | 173.86 | 173.89 | 184.17 | 231.94 |

*Table 1. Experimentally derived damping and natural frequencies vis-à-vis results from finite element based modal analysis.*

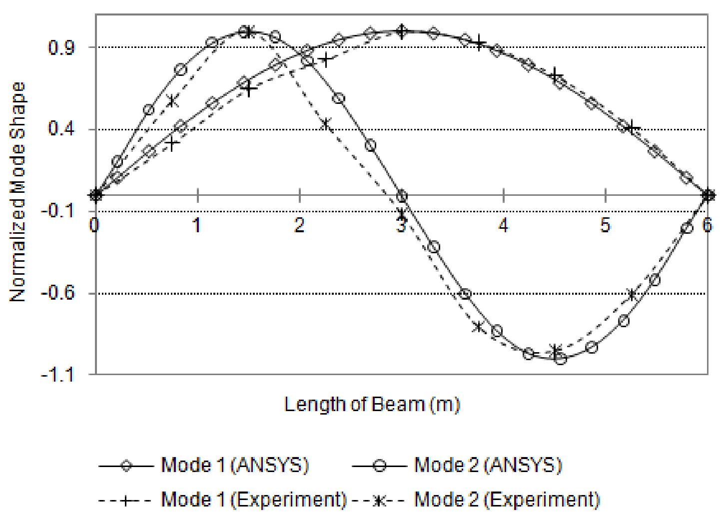


*Fig. 4 : Mode shapes normalized to their peak absolute value*

## 4. FINITE ELEMENT MODELLING AND ANALYSES

Following standard practices in modal analysis [6,9,10,11], a Finite Element (FE) model of the USFB used in the experimental modal test is developed in ANSYS. Eight-node shell elements (SHELL281) and mapped meshing is implemented to design the bare steel beams. A snapshot of the model is included in *Fig. 2*. The mass density is computed using the total measured weight of the beam specimen (404 kg) and the dimensional data of the cross sectional properties shown in *Fig. 2*. The accuracy of this FE model compares well against the experimental data (*Table 1* and *Fig. 4*). Moreover, natural frequencies for the same asymmetric beam but without web openings (solid) are also included in *Table 1*. Evidently, it is found that the existence of the web openings influence significantly the higher modes of vibration.

## 4.1 Parametric Finite Element analysis

Having established a FE model for the beam specimen which compares well with the experimental modal testing, a parametric FE modal analysis is undertaken considering four commonly used in practical applications cross-sections for USFBs of four different lengths. In total, 16 models are constructed in ANSYS. Key geometric properties of the considered models are reported in the first four columns of *Table 2*. The first five natural frequencies and typical results from modal analysis are also reported in *Table 2*.

| (FE Model) Top / Bottom Tee-Section (mm) | Volume ($m^3$) | Number of web openings | Length (m) | Natural Frequencies (Hz) (in-plane flexural modes) | | | | | $A_{eq}$ ($cm^2$) | $I_{eq}$ ($cm^4$) |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1st | 2nd | 3rd | 4th | 5th | | |
| (CAT1) UB 254x102x22/ UC 254x254x73 | 0.0231 | 15 | 4 | 36.468 | 119.68 | 214.01 | 255.57 | 306.65 | 57.750 | 3107.71 |
| | 0.0342 | 22 | 6 | 16.999 | 61.413 | 120.45 | 171.75 | 187.2 | 57.000 | 3374.05 |
| | 0.0451 | 30 | 8 | 9.7153 | 36.096 | 74.92 | 118.99 | 132.39 | 56.375 | 3444.96 |
| | 0.0561 | 37 | 10 | 6.2704 | 24.071 | 50.89 | 83.643 | 105.51 | 56.100 | 3486.41 |
| (CAT2) UB 254x102x22/ UC 254x254x73 | 0.0351 | 15 | 4 | 38.412 | 120.00 | 206.27 | 247.16 | 287.98 | 87.750 | 5238.97 |
| | 0.0518 | 22 | 6 | 18.184 | 63.565 | 120.74 | 167.86 | 184.17 | 86.333 | 5847.73 |
| | 0.0684 | 30 | 8 | 10.454 | 38.444 | 76.962 | 118.71 | 131.68 | 85.500 | 6049.45 |
| | 0.0851 | 37 | 10 | 6.7679 | 25.611 | 53.111 | 85.369 | 104.81 | 85.100 | 6161.16 |
| (CAT3) UB 254x102x22/ UC 254x254x73 | 0.0454 | 11 | 4 | 51.732 | 142.1 | 222.98 | 243.66 | 291.52 | 113.500 | 12290.77 |
| | 0.0668 | 17 | 6 | 25.518 | 80.458 | 139.74 | 168.34 | 197.18 | 111.333 | 14850.75 |
| | 0.0881 | 23 | 8 | 15.005 | 51.126 | 94.762 | 127.42 | 140.9 | 110.125 | 16052.50 |
| | 0.1100 | 28 | 10 | 9.8448 | 35.318 | 68.807 | 101.17 | 108.49 | 110.000 | 16851.22 |
| (CAT4) UB 254x102x22/ UC 254x254x73 | 0.0659 | 9 | 4 | 63.399 | 160.00 | 217.17 | 253.58 | 327.04 | 164.750 | 26795.08 |
| | 0.0966 | 14 | 6 | 32.559 | 92.416 | 150.81 | 164.37 | 218.63 | 161.000 | 34962.13 |
| | 0.1270 | 19 | 8 | 19.597 | 61.956 | 107.92 | 123.43 | 157.48 | 158.750 | 39470.96 |
| | 0.1580 | 24 | 10 | 13.005 | 43.64 | 79.941 | 99.678 | 119.67 | 158.000 | 42237.93 |

*Table 2. Natural frequencies from FE modal analysis and equivalent properties corresponding to Euler-Bernoulli beams of constant cross-section for various USFBs*

## 4.2 Equivalent Geometric Properties of USFB beams

Relying on the previously discussed FE parametric study, a further step of practical importance is taken to obtain equivalent geometric properties for the considered USFBs corresponding to a prismatic Euler-Bernoulli beam of constant cross-section along its longitudinal axis [12]. These properties can be readily used in conjunction with commercial FE codes to model the bare steel USFBs. For this purpose, an equivalent cross-sectional area $A_{eq}$ and an equivalent second moment of area $I_{eq}$ are defined based on the two following criteria: (i) The total mass of the equivalent prismatic beam is equal to the mass of the USFB assuming that both beams are of the same length, and (ii) The natural frequency of the first in-plane flexural mode shape of the USFB and its prismatic surrogate are equal.

In satisfying criterion (i), the $A_{eq}$ is determined as the ratio of the volume over the length of the USFB. The volumes of the herein considered USFBs are reported in *Table 2* along with the thus obtained $A_{eq}$. Furthermore, assuming simply supported beams, criterion (ii) is met by determining the $I_{eq}$ from the expression:

$$I_{eq} = \frac{4 f_1^2 \rho A_{eq} L^4}{E \pi^2}. \quad (3)$$

The latter equation is derived from the well known expression of continuum dynamics for the first natural frequency ($f_1$) of the free transverse vibration of a simply supported Euler-Bernoulli beam with mass density $\rho$, modulus of elasticity $E$, cross-sectional area $A_{eq}$, and length $L$. *Table 2* includes values of $I_{eq}$ for the various USFBs considered obtained using eq. (3) assuming $\rho$=7800 kg/m$^3$ and $E$=200GPa corresponding to nominal steel material properties.

It is noted in passing that other criteria of equivalency can be utilized in deriving equivalent geometric properties which will generally yield different values for $A_{eq}$ and $I_{eq}$ [12]. In this work, the aforementioned criteria have been selected as they relate to the dynamic behavior of the USFB beams under investigation in a straightforward manner.

## 5. CONCLUDING REMARKS

The vibration behavior of asymmetrical ultra shallow floor beams (USFBs) has been investigated. Experimental data obtained from impact hammer modal test on a specific simply supported USFB specimen have been considered to extract certain of the first five natural frequencies and first two mode shapes corresponding to the flexural in-plane free vibration motion using purpose-made MATLAB scripts. These compare well with results from finite element (FE) modal analysis undertaken in ANSYS on a detailed FE model developed to capture the dynamic behavior of the considered specimen. Furthermore, the first five natural frequencies of four different USFBs commonly used in practice for various lengths have been obtained via modal analyses in ANSYS. These analyses have been applied on detailed FE models produced by relying on the initial model pertaining to the experimentally tested beam. Finally, equivalent geometric properties (cross-sectional area and second moment area) for all the considered USFBs in the parametric study corresponding to prismatic Euler-Bernoulli beams with constant cross-sections have been computed. These properties are derived by enforcing equality of the total mass and the natural frequency of the first flexural mode of vibration between the USFBs and their prismatic surrogates. Clearly, these equivalent properties can be used to incorporate USFBs in FE models of structures developed in FE software commonly used by the practicing engineers for structural design against dynamic loads, such as those induced by earthquakes. Future experimental and computational work will be directed towards the characterization of the dynamic properties of composite floors incorporating USFBs similar to those shown in *Fig. 1* [13,14].

## 6. REFERENCES


| | |
|---|---|
| [1] | TSAVDARIDIS KONSTANTINOS D. **"**Structural Performance of Perforated Steel Beams with Novel Web Openings and with Partial Concrete Encasement", *PhD Thesis*, School of Engineering and Mathematical Sciences, City University London, 2010. |
| [2] | TSAVDARIDIS KONSTANTINOS D. and D'MELLO CEDRIC "Behaviour and Strength of Perforated Steel Beams with Novel Web Opening Shapes", *Journal of Constructional Steel Research*, 2010, Reference: JCSR3314, PII: S0143-974X(11)00106-4, DOI: 10.1016/j.jcsr.2011.04.004. |
| [3] | TSAVDARIDIS KONSTANTINOS D., D'MELLO CEDRIC and HUO BING Y. "Shear Capacity of Perforated Concrete-Steel Ultra Shallow Floor Beams (USFB)", *16th National Concrete Conference*, 21-23 October 2009, Cyprus, Reference no: 201101, pp.159. |
| [4] | TSAVDARIDIS KONSTANTINOS D., D'MELLO CEDRIC and HAWES MIKE "Experimental Study of Ultra Shallow Floor Beams (USFB) with Perforated Steel Sections", *Nordic Steel Construction Conference 2009 - NSCC2009*, 2-4 September 2009, Malmö, Sweden, Reference no: 128, pp. 312-319. |
| [5] | TSAVDARIDIS KONSTANTINOS D., D'MELLO CEDRIC "Experimental and Finite Element Investigation on the Vierendeel Bending of Perforated Steel Beams with Various Sizes and Novel Shapes of Web Openings", *Journal of Structural Engineering-ASCE*, 2011, In print |
| [6] | EWINS DAVID "Modal testing: Theory and Practice", *Research Studies Press Ltd,* Taunton, UK, 1995. |
| [7] | SCHWARZ BRIAN and RICHARDSON MARK "Experimental modal analysis", *CSI Reliability Week,* Orlando, USA, 1999. |
| [8] | BERCZYNSKI STEFAN and WROBLEWSKI TOMASZ "Experimental verification of natural vibration models of steel-concrete composite beams", *Journal of Vibration and Control,* Vol. 16, 2010, pp. 2057-2081. |
| [9] | JAWORSKI JUSTIN and DOWELL EARL "Free vibration of a cantilevered beam with multiple steps: Comparison of several theoretical methods with experiment", *Journal of Sound and Vibration,* Vol. 312, 2008, pp. 713-725. |
| [10] | LOTFOLLAHI-YAGHIN MOHAMMAD and AHMADI HAMID "Investigation of Dynamic Properties of Cantilever Castellated Beams in Comparison with Plain-webbed Beams using White Noise Excitation", *World Applied Sciences Journal 3*, Vol. 3, 2008, pp. 522-530. |
| [11] | REN WEI-XIN, ZHAO TONG and HARIK ISSAM E. "Experimental and Analytical Modal Analysis of Steel Arch Bridge", *Journal of Structural Engineering-ASCE*, Vol. 1031, 2004, DOI: 10.1061/~ASCE!0733-9445~2004!130:7~1022. |
| [12] | ZHENG TIANXIN and JI TIANJIAN "Equivalent representations of beams with periodically variable cross-sections", *Engineering Structures,* Vol. 33, 2011, pp. 706-719. |
| [13] | EL-DARDIRY EMAD and JI TIANJIAN "Modelling of the dynamic behavior of profiled composite floors", *Engineering Structures,* Vol. 28, pp. 567-579. |
| [14] | VENGHIAC VASILE-MIRCEA and D'MELLO CEDRIC "The influence of the thickness of the slab and concrete grade on composite floors", *Publicat de Universitatea Tehnică - Gheorghe Asachi" din Iaşi Tomul LVI (LX)*, Buletinul Institutului Politehnic Din Iasi, 2010. |